GAMMA-RAY STUDIES RELATING TO THE $^{16}_N(p, y)^{15}_O$
REACTION AT STELLAR ENERGIES

by

Christopher H Osman

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TO

MY PARENTS

AND

LIESL
PREFACE

This thesis describes a number of experiments which were carried out within the Department of Nuclear Physics, at the Australian National University.

The $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction measurements, which were suggested by Dr. D.F. Hebbard, were carried out in collaboration with Dr. F.C.P. Huang, with whom the work was approximately equally shared. At a later stage of the experiment, Dr. T.R. Gophel was actively involved in the coincidence measurements, and their analysis. The remainder of the work on this reaction, concerning the theoretical determination of cross-sections, and the extrapolation to stellar energies was carried out by the author alone. Dr. D.J. Baugh and Mr. D.H. Rosalky assisted with the collection of data for the $^{19}\text{F}(p,\gamma)^{16}\text{O}$ measurements. The analysis was done by the author, as was the investigation into the characteristics of the Ge(Li) detector.

No part of this thesis has been submitted towards a degree at any other university.

C. H. OSHAN
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INTRODUCTION

The $^{14}_{\text{He}} N(p,\gamma)^{15}_{\text{O}}$ reaction is a member of the C.N.O cycle of nuclear reactions, involved in the production of energy in stars. It has the lowest cross-section, at stellar energies, of these reactions, and hence has a controlling influence on the rate of energy production.

Due to the exponential decrease in the nuclear potential-barrier penetrability as the bombarding energy is decreased, it is not possible to determine reaction cross-sections at stellar energies directly. The lowest energies at which measurements of the $^{14}_{\text{He}} N(p,\gamma)$ cross-section have been made, are in the range 100-135 keV (La57). A beam current of 25 ma. was used in this case. In the absence of any lower energy resonances, however, it is expected that a reasonable extrapolation can be made. The cross-section factor

$$S(E) = E\sigma(E)\exp(2\pi n),$$

where $E$ is the channel energy,

$\sigma(E)$ is the cross-section,

$n$ is the usual coulomb field parameter,

is expected, in these circumstances, to vary reasonably slowly with energy.

The most detailed experimental determination of the $^{14}_{\text{He}} N(p,\gamma)^{15}_{\text{O}}$ reaction cross-section at stellar energies has been carried out by Hebbard and Bailey (He 63). They observed that the cross-section factor between the 278 keV and 1061 keV resonances could not be accounted for by the tails of these resonances alone and postulated that a direct-capture mechanism was responsible. A measurement
of the angular distribution of the radiation leading to the 6.789 MeV. state in $^{15}O$, supported this point of view. Using the theoretical formalism of Christy and Duck (Ch 61), Hebbard and Bailey performed calculations of the cross-section factors for direct-capture transitions to all the bound states in $^{15}O$. Cross-section measurements between 400 and 1000 keV. $E_p$, combined with these calculations, and the calculated tails of the 278 and 1061 keV. resonances, enabled Hebbard and Bailey to extrapolate the cross-section factor, for each transition, to stellar energies.

However, there were two main sources of uncertainty in their results, which could not be resolved at that time. First, the direct-capture calculations could not be satisfactorily normalised, owing to a lack of information on the reduced widths of the bound states in $^{15}O$. Second, apart from the transition to the 6.789 MeV. state, for which measurements had established a direct-capture mechanism, it was not known whether contributions from distant broad levels could also account for the measured cross-section factors, between 400 and 1000 keV. proton energy. Duncan and Perry (Du 51) had shown that there was a broad resonance at $\sim 2.35$ MeV., but had measured only the total width, and the sum of all radiative widths. Furthermore, elastic scattering measurements indicated a much smaller total width than had been found by Duncan and Perry.

In view of these uncertainties, Hebbard and Bailey were unable to decide definitely whether it was the tail of the distant broad resonance or direct capture that was responsible for the bulk of the cross-section
factor between 400 and 1000 keV. for some transitions. The extrapolation to stellar energies was achieved by fitting their experimental data with the known contributions from the tails of the 278 and 1061 keV. resonances, plus the calculated direct capture cross-section factors, or a constant cross-section factor (to simulate the tail of the 2.35 MeV. resonance). The magnitudes of the direct capture or distant-resonance contributions were determined by the fit. Hebbard and Bailey found that satisfactory fits to the data could be obtained in either case, but the extrapolated cross-section factors at stellar energies differed substantially.

The present work was undertaken to resolve the uncertainties arising from the work of Hebbard and Bailey. Measurements of the cross-section factors between 2-3 MeV. were performed in order to establish the contributions of the 2.35 MeV. resonance to the cross-section factors below 1000 keV. In addition, a comprehensive set of direct-capture calculations was carried out. This included calculations for several transitions which had not been considered by Hebbard and Bailey. Furthermore, certain features of the calculations of Hebbard and Bailey appeared unsatisfactory, so that it was decided to consider all possible transitions. These new calculations, the results of which differed significantly from those of Hebbard and Bailey in some cases, considered together with the new information on the broad 2.35 MeV. level, necessitated a complete re-assessment of the analysis carried out by Hebbard and Bailey.

As a consequence of the experimental work on the $^{14}_N(p,\gamma)$ reaction, a study of the radiation produced
by the $^{10}\text{F}(p,\alpha\gamma)^{16}\text{O}$ reaction was subsequently undertaken. The strong 6-7 MeV. radiation from this reaction was found to be a troublesome contaminant in $^{14}\text{N}(p,\gamma)^{15}\text{O}$ spectra. Exhaustive examination of the published literature revealed little detailed information concerning the radiation from $^{16}\text{F}(p,\alpha\gamma)^{16}\text{O}$, though there exists considerable information on the yield of some particle groups and the total yield of gamma radiation. The use of a Ge(Li) detector enabled a study of individual transitions to be made. This had not previously been possible, owing to the relatively poor resolution of NaI(Tl) detectors.

Following these measurements, the Ge(Li) detector was damaged, resulting in severe loss of resolution. It was therefore decided to study the distribution of the pulse rise times for this detector, in order to determine whether the resolution could be improved by selecting only pulses with short rise times. This possibility seemed quite likely, in view of work by Alexander et al. (Al 64) and others, which showed that defective pulses, associated with incomplete charge collection, were commonly associated with abnormally long rise times.
Chapter 1

THE ANALYSIS OF GAMMA-RAY SPECTRA
1.1. Introduction:

The analysis of gamma-ray spectra requires a totally different approach to that required for particle spectra. This is made necessary by the complexity of the interaction of radiation with matter, and the consequent structure of detector line shapes.

Depending on the size and type of the detector, the number of transitions in the spectrum, and the energies of the radiation, it is in some cases possible to measure the areas under the photopeak or escape peaks of each line. Knowledge of the appropriate peak to total ratios then enables the determination of the total detected intensity of each transition. For NaI(Tl) detectors, this approach is generally possible only at low energies (less than several MeV.), where the photopeak to total ratio is large, and the energy resolution is good. For Ge(Li) detectors, the requirements are much less stringent, since at higher energies, the double-escape peak becomes prominent, and is well separated from the other peaks in the line shape. However, even if at least one peak in a line shape is prominent (so that its area may be accurately obtained) the above method becomes less useful when even a few lines are present in the spectrum, and unusable if the lines strongly overlap. In most cases, the method is not possible for the analysis of NaI(Tl) spectra, but is normally suitable for Ge(Li) spectra.

Even if the above requirements are satisfied, the method is still intrinsically unsatisfactory, since only part of the line shape is being utilised, and information is lost. This is particularly so with Ge(Li) detectors, since peak to total ratios are rather small. Furthermore, radiative transitions are usually weak, so that one can often not afford to throw away a large number of the counts.
The alternative approach is to employ line-shape analysis. This requires the measurement and/or calculation of the line shape for each transition in the spectrum. The intensity of each line is then obtained either by spectrum stripping (starting with the highest energy line), or by fitting the data with the line shapes, using the linear least-squares method. This approach gives reliable results in most circumstances, and fails only when lines become too close in energy compared with the detector resolution. In this case, however, no method is suitable (excluding the possibility of using coincidence methods).

The method of line-shape analysis was employed in the work described in this thesis. The procedure consisted of three main steps:

1. A set of standard line shapes was generated, covering the energy region of interest. Each line shape was then converted to a functional representation, which enabled the spectrum gain and zero to be changed as desired, without any distortion in shape. Such a set of line shapes was generated for each detector used.

2. The line shape for each transition, in the spectrum under analysis, was obtained by interpolating between the standard line shapes. The computer program, which performed this calculation, also changed the gain and zero of the line shape to that of the spectrum.

3. The intensity of each transition was then obtained by finding the best fit to the data, using the interpolated line shapes. The criterion of goodness-of-fit was the minimisation of the sum of the squares of the differences between the data and fit at each channel in the spectrum.
1.2: The determination of detector line shapes for mono-energetic radiation

1.2.1 Measurement of a set of standard line shapes:

Information about the reactions used to provide standard line shapes in the present work is summarised in table 1.1. Particular care was taken in measuring spectra with the Ge(Li) detector so that the lines were not appreciably broadened by high counting rates, or target thickness. The relatively poor resolution of NaI(Tl) detectors make these considerations less important. The generation of pure line shapes from these spectra involved the removal of background radiation, and contributions from transitions other than the one of interest. The resultant spectra were subsequently smoothed. The precise details varied from line to line, depending on what subsidiary radiation was present, and the proportions relative to the main transition, but the general procedure was as follows.

Time-dependent background radiation was measured, and subtracted from each line shape spectrum, according to the relative live times. An initial approximation to each standard line shape was then made by estimating the height of the Compton/bremsstrahlung tail (underlying the small quantities of other radiation) and extending this to the energy zero of the spectrum. This approximate set of curves was then fitted with functions (as described in the next section), and used to obtain, by interpolation, the line shapes of the subsidiary radiation which was present in the raw line-shape spectra. It was then possible to calculate a second approximation to the set of standard line shapes, by fitting each spectrum, over the appropriate limited region, with line shapes for the subsidiary radiation and the Compton/bremsstrahlung tail of the principal radiation. The set of second approximation line shapes
<table>
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<th>Reaction</th>
<th>$E_p$ (KeV.)</th>
<th>$\Gamma_{lab}$ (KeV.)</th>
<th>Principal $\gamma$-ray (MeV.)</th>
<th>$%$</th>
<th>Other radiation</th>
<th>Special target requirements</th>
<th>Special beam requirements</th>
<th>Detector</th>
<th>Broadening</th>
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<tr>
<td>$^{12}\text{C}(p,\gamma)$</td>
<td>459</td>
<td>32</td>
<td>2.367</td>
<td>100</td>
<td>—</td>
<td>thin</td>
<td>precisely known</td>
<td>both</td>
<td>target/beam</td>
</tr>
<tr>
<td>$^{13}\text{C}(p,\gamma)$</td>
<td>554</td>
<td>30</td>
<td>8.060</td>
<td>82</td>
<td>2 - 6 MeV.</td>
<td>thin</td>
<td>precisely known</td>
<td>both</td>
<td>target/beam</td>
</tr>
<tr>
<td>$^{13}\text{C}(p,\gamma)$</td>
<td>3,110</td>
<td>30</td>
<td>10.43</td>
<td>90</td>
<td>4.0, 6.44</td>
<td>thin</td>
<td>precisely known</td>
<td>NaI</td>
<td>target/beam</td>
</tr>
<tr>
<td>$^{15}\text{N}(p,\alpha\gamma)$</td>
<td>3,000</td>
<td>45</td>
<td>4.433</td>
<td>~100</td>
<td>weak $^{14}\text{N}(p,\gamma)$</td>
<td>—</td>
<td>—</td>
<td>NaI</td>
<td>strongly doppler broadened</td>
</tr>
<tr>
<td>$^{18}\text{F}(p,\alpha\gamma)$</td>
<td>596</td>
<td>30</td>
<td>6.129</td>
<td>~100</td>
<td>very weak $^{16}\text{O}$</td>
<td>front layer of gold to stop loss of $^{16}\text{O}$ ions</td>
<td>—</td>
<td>both</td>
<td>no broadening due to long lifetime</td>
</tr>
<tr>
<td>$^{11}\text{B}(p,\gamma\gamma)$</td>
<td>680</td>
<td>300</td>
<td>4.433</td>
<td>~50</td>
<td>12 MeV.</td>
<td>—</td>
<td>—</td>
<td>Ge(Li)</td>
<td>—</td>
</tr>
<tr>
<td>$^{27}\text{Al}(p,\gamma\gamma)$</td>
<td>992</td>
<td>80</td>
<td>$\begin{cases} 1.772 \ 10.76 \end{cases}$</td>
<td>~100</td>
<td>—</td>
<td>thin</td>
<td>precisely known</td>
<td>Ge(Li)</td>
<td>target/beam</td>
</tr>
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were considered to be sufficiently accurate to use in the analysis of the data.

Standard curves, generated by the above method, for the 20 cc. Ge(Li) detector are shown in figure 1.1, and those for a 5" x 4" NaI(Tl) detector are shown in figure 1.2. The detectors were uncollimated in both cases, and the source of radiation lay on the detector axis. It is essential, when measuring line shapes, that the radiation-source/detector geometry corresponds as closely as possible to that used in measuring the spectra to be analysed.

It is not possible, however, to reproduce the desired experimental line shapes exactly, since one generally does not know the angular distributions of the radiation being analysed. Thus the angular distribution of radiation across the face of the detector in the line-shape measurements is in general different from that in the experimental spectra. In any case, it would be difficult (though not impossible) to utilise such information if it were available. However, provided that the solid angle subtended by the detector is not too large, the errors involved are not expected to be important. In general, the target/detector distance is of a reasonable magnitude, in order to lessen the consequent smoothing of the angular distribution (when this is being measured), or to lessen the contribution of sum peaks (from the simultaneous detection of cascade radiation). The question of errors in line shapes is discussed further in section 1.3.3.

1.2.2 Functional representation of line shapes:

In the use of measured line shapes for the analysis of gamma-ray spectra, there are three main advantages in employing functional representations of the line shapes:
Figure 1.1. Line shapes for the 20 cc. Ge(Li) detector.
Figure 1.2: Line shapes for a 5" x 4" NaI(Tl) detector.
(1) The parameterisation, in terms of functions, eliminates the inherent statistical fluctuations present in the measured line-shape spectra. Consequently, there is a reduction in the errors obtained when analysing data.

(2) The gain and zero of line-shape spectra may be varied by any amount (in order to reproduce the gain and zero of the spectrum to be analysed), without incurring any distortion.

(3) When a computer of moderate core size is employed in the calculations, the consequent saving in memory space may be very valuable (as it was in the present case). Thus a set of 5 line shapes, each measured in ~4000 channels, could be stored in ~400 words (as functions), compared with ~20,000 words as spectra.

The parameterisation of each line-shape spectrum was effected by the following method.

The spectrum was split up into a set of intervals, such that the variation of the line shape within each interval was simple. That is to say, it was arranged that the shape of the curve (in each interval) was such that it appeared possible (by inspection) to produce an adequate fit with a small number of terms of the series expansion:

\[ f_i(x) = A_0^i + A_1^i (x - x_0^i) + \sum_{k=1}^{n^i} R_k^i \sin\left(\frac{k\pi(x-x_0^i)}{(x_1^i - x_0^i)}\right), \quad (1.1) \]

where the index \( i \) denumerates the intervals.

\( x_0^i \) and \( x_1^i \) are the lower and upper bounds of fit (channel numbers) of the \( i \)-th interval,

\( x \) is a general channel number,

\( A_0^i, A_1^i, R_k^i \) are coefficients to be determined by the fit.
The coefficients, $n^i$, were determined by inspection. If the resulting fit within an interval was found to be inadequate, a larger value for $n^i$ was specified, and the calculation repeated. In general, values between 1 and 4 were used.

In order to minimise discontinuities at the interval boundaries, lower and upper bounds of use, $X^i_0$ and $X^i_1$, in addition to the bounds of fit, were defined by

$$
x^i_0 \leq x^i_1, \\
x^i_1 \leq x^i_1, \\
x^i_1 = x^{i+1}_0.
$$

By thus allowing sufficient overlap of intervals of fit, a smooth transition was obtained between intervals.

Having fitted the series (1.1) to each interval of the line-shape spectrum, the parameterisation was complete. The line shape was then completely specified by the set of numbers

$$X^i_0, X^i_1 \text{ (for the last interval only, since } X^i_{1-1} = X^i_0),$$

$$A^i_0, A^i_1, B^i_k \ (1 \leq k \leq n^i).$$

In order to alter the gain and zero of the spectrum ($G_1$ KeV./channel, $Z_1$ channels), as represented by these functions, to some other gain and zero ($G_2$ KeV./channel, $Z_2$ channels), it was merely necessary to multiply the above parameters by simple functions of $G_1$, $G_2$, $Z_1$, and $Z_2$. Evaluation of the resultant functions at each channel then produced the line-shape spectrum with the required gain and zero. The new values of the parameters were given by the expressions
The radiation present in the spectra under analysis were normally of different energies to those of the measured line shapes. It was thus necessary to interpolate between the standard line shapes. Figure 1.3 gives an indication of the way in which this was achieved. The line shapes in this diagram are approximately those which were measured for a 5" x 4" NaI(Tl) detector, as used in the measurements described in chapter 2.

The calculation was carried out from the zero channel, in steps of 1 channel, to some maximum just beyond the full-energy peak in the interpolated line shape.

The dashed lines in figure 1.3 join a selection of the sets of points (in the standard curves), the intensities at which were fitted by a polynomial, to determine the appropriate intensity for the interpolated curve. From the full energy peak to a point 1.5 MeV. lower in the line shape, the expression

\[ x_\lambda = x_\gamma + (E_\lambda - E_\gamma) \times (x_\gamma - Z)/G \] (1.4)
Figure 1.3: Scheme for interpolation of line shapes. Dotted lines join a sample of the sets of points on standard curves, for the interpolation of each point on the calculated line shape. Circles represent the channel positions, and crosses the appropriate intensities at these points. The standard curves are all normalised to the same total intensity, and are for a 5" x 4" NaI(Tl) detector.
was used to calculate the set of channel numbers, $x_{\lambda}$, of the standard curves, which were associated with the channel number, $x_{\gamma}$, in the interpolated curve.

$E_{\lambda}$ and $E_{\gamma}$ are respectively the energies of the radiation associated with the standard and interpolated line shapes,

$G$ and $Z$ are the gain and zero, respectively, and must be the same for all line shapes.

The use of this expression arises from the fact that the shape, of that part of the line shape for which it is used, is dominated essentially by three equally spaced gaussian peaks, whose spacing is independent of the line shape energy. Thus the sets of points, $x_{\lambda}$, lie on straight lines, which are parallel to the lines joining full energy and escape peaks.

For channels appropriate to an energy more than 1.5 MeV. below the full-energy peak, the expression

$$x_{\lambda} = x_{\gamma}(E_{\lambda} - E_{\gamma}) \times (x_{\gamma} - Z_{\gamma})/(E_{\gamma} - 1500)$$

was used instead of (1.4), where $E_{\lambda}$ and $E_{\gamma}$ are in KeV. The sets of $x$ still lie on straight lines, but the slope varies continuously with $x_{\gamma}$. When

$$(E_{\gamma} - 1500)/(x_{\gamma} - Z) = G,$$

(the channel, $x_{\gamma}$, at which the transition is made between equations (1.4) and (1.5)), the slope of the line is parallel to that of the line joining the full-energy peaks on each line shape. When $x_{\gamma} = Z_{\gamma}$, the $x_{\lambda}$ are all zero, as required.

Having defined the sets of channel numbers, $x_{\lambda}$, for a given $x_{\gamma}$, the next step was to use the intensities, $f_{\lambda}(x_{\lambda})$, at each of these points, to derive the intensity of the interpolated line, $f(x_{\gamma})$. The method used was that due to Lagrange, wherein an $n$'th degree polynomial is set up, which takes the values $f_{\lambda}(x_{\lambda})$
at the \( n + 1 \) points, \( x_\lambda \). The value of the polynomial, \( F(x) \), at the point \( x = x_\gamma \) was taken to be the required interpolated intensity for that channel. The polynomial, of lowest order, which passes through the points \( \{x_\lambda, f_\lambda(x_\lambda)\} \) is given by (1a52):

\[
f(x_\gamma) = F(x_\gamma) = \frac{x_\gamma - x_1}{x_0 - x_1} x \frac{x - x_2}{x_0 - x_2} \cdots \frac{x - x_n}{x_0 - x_n} f_0(x_0) \\
+ \frac{x_\gamma - x_0}{x_1 - x_0} x \frac{x - x_2}{x_1 - x_2} \cdots \frac{x - x_n}{x_1 - x_n} f_1(x_1) \\
+ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (1.6)
\]

By the above means, the value of the line-shape function was determined at each channel. Only three standard curves were used in interpolating each line shape, the three selected being the closest in energy. It was found that the use of five curves tended to produce anomalous bumps in some cases. This was probably due partly to irregularities in the individual standard curves. On the other hand, as there is no a priori reason for expecting the sets of \( f_\lambda(x_\lambda) \) to be reproduced accurately by polynomials, it is to be expected that irregularities will occur when the standard curves span a wide energy range. In the present work, the radiation of interest lay almost entirely between 5 and 10 MeV. The line shapes are changing rather slowly in this region, so that it was felt that the interpolation procedure used was satisfactory. It was found that extrapolation (at the high energy end of the set of standard curves) was possible only over a very limited range (\( \sim 200 \) KeV.). Beyond this range, extrapolated curves had very irregular shapes.
1.2.4 Calculations of line shapes:

A detailed knowledge of the dynamic and kinematic features of the processes whereby photons lose energy in a detector, together with the geometry of the active region of the detector, in principle enable the theoretical prediction of line shapes via Monte-Carlo techniques. The method consists of tracing the histories of a large number of photon interactions. Thus the probability distribution of energy deposited in the detector is generated, which is essentially the line shape, apart from broadening effects. The type and geometrical details of each interaction are selected on a random basis. The dynamical details then follow from the theoretical differential cross sections for each type of interaction. Several such calculations of line shapes and efficiencies have been performed (Ca66, Wa66).

There are, however, a number of important difficulties:

(1) The line shapes for a Ge(Li) detector can be considerably altered as a result of incomplete charge collection (as discussed in chapter 6). It has been found that pulses with abnormally long rise times, or rise-time components, are often associated with incomplete charge collection. There does not appear to be any way in which such effects could adequately be included in the calculation.

(2) Owing to the enormous complexity of the primary interaction processes, as well as secondary events such as bremsstrahlung production and re-absorption of secondary radiation, it is generally necessary to make substantial approximations in the theoretical cross-section expressions. The result is that considerable uncertainties exist in the resultant line shapes.
The statistical effects which determine the energy resolution cannot be calculated \textit{a priori}, so that measurement is necessary to determine this aspect of the line shapes.

It does not appear that sufficient information on the calculation of line shapes is available, to determine whether it is actually possible to calculate line shapes as accurately as they can be measured (aside from resolution considerations). Until detailed comparisons have been made, it would be unwise to rely solely on theoretically determined line shapes.

1.3. Factors affecting line shapes:

1.3.1 Lifetime and Doppler shift:

It has thus far been assumed that radiation emitted in a nuclear transition is very nearly mono-energetic. This is often not the case, however, depending on the type of reaction involved in the emission of radiation, and the lifetime of the initial state. Radiation may be emitted (a) from a compound nucleus state, or another state after one or more radiative transitions, or (b) from a state formed by the emission of an intermediate particle. These two situations may both result in a considerable dispersion in energy of the detected radiation.

In the first case, if the lifetime of the initial state in the transition is sufficiently short, so that decay occurs before the emitting nucleus has come to rest, a \textit{discrete} shift in energy is observed. This is due principally to the motion imparted to the nucleus in the initial collision, with a lesser contribution from the recoil of the nucleus away from the photon. The shift is a function of the angle of emission with respect to the incident particle beam, and is maximum at $0^\circ$, and minimum at $90^\circ$. 


If the lifetime is sufficiently long so that the nucleus has come to rest before emission, the only shift observed is that due to the recoil of the nucleus on emission. In the intermediate lifetime region, where the decay time is comparable with the slowing-down time of the nucleus within the target, a distribution of photon energies results, with a consequent smearing of the observed line shape. If the initial state is particle unbound, the lifetime will be of the order of $10^{-18}$ sec. or less (corresponding to a width of ~1 KeV. or more). The full Doppler shift will then be present. Broadening may occur only in a transition from a bound state.

In the second case, where an intermediate particle transition is involved, the effects are quite different. The variation in kinetic energy, and direction of motion of the excited nucleus as a consequence of the particle emission, result in a considerable dispersion in energy of the radiation observed at a given angle. The resultant line-shape broadening may thus be rather large. For example, it has been calculated that 6-7 MeV. radiation from the $^{18}\text{F}(p,\alpha\gamma)^{16}\text{O}$ reaction is broadened by ~140 KeV. at a proton energy of 4 MeV. (see chapter 5). The maximum broadening is observed, in this case, if the lifetime of the photon-emitting state is short compared with the nuclear slowing-down time (in contrast to the first case). If the lifetime of the state is sufficiently long so that the nucleus is at rest at the time of emission, only the small shift due to recoil is observed. In the intermediate-lifetime region, the effects considered in the first case will also be present.

1.3.2 Simultaneous detection of cascade radiation:

The possibility of simultaneous detection of two or more members of a set of cascade transitions, results in the presence of convoluted line shapes in gamma-ray spectra. It is generally only
necessary to consider pair-wise coincidence, since the intensity of successive orders is less by a factor corresponding to the detection efficiency (for uncorrelated radiation).

The convoluted line shape in this case, \( G_{12}(x) \), has the form

\[
G_{12}(x) = \int_0^x F_1(x_1)F_2(x_2)\,dx_1,
\]

where \( x_1, x_2, X \) are channel numbers,

\( F_1 \) and \( F_2 \) are the line shape functions of the cascade members (and are normalised to unity),

\[
x_2 = X - x_1.
\]

It is assumed that the successive transitions are uncorrelated in angle. The full-energy peak corresponds to the sum of the energies of the two transitions. There are, in general, five peaks in \( G_{12}(X) \), corresponding to the loss of 0, 1, 2, 3 or 4 annihilation quanta.

If \( \varepsilon_1 \) and \( \varepsilon_2 \) are the total detection efficiencies for the two transitions, then the probability of simultaneous detection (i.e. the intensity of \( G_{12}(X) \)) is \( \varepsilon_1 \varepsilon_2 \). Thus the probability of detection of the first transition (singly) is \( \varepsilon_1 - \varepsilon_1 \varepsilon_2 = \varepsilon_1 (1 - \varepsilon_2) \). Similarly, the probability of detection of the second transition (singly) is \( \varepsilon_2 (1 - \varepsilon_1) \). In the analysis of the \(^{14}N(p,\gamma)^{15}O\) reaction data (chapter 2), single line-shape components for cascade transitions were formed, by combining the single and convoluted line shapes in the correct proportions. Such a component, \( C(x) \), had the general form

\[
C(x) = \varepsilon_1 (1 - \varepsilon_2)F_1(x) + \varepsilon_2 (1 - \varepsilon_1)F_2(x) + \varepsilon_1 \varepsilon_2 G_{12}(x). \tag{1.8}
\]

In that work, only the intensity of the second cascade was of interest. Dividing \( C(x) \) in (1.8) by \( \varepsilon_2 \) gave a line-shape component
corresponding to unit intensity of the second transition, as required.

As previously mentioned, the above considerations strictly apply only to uncorrelated transitions. In general, this assumption is not fulfilled, so that the correct convoluted line shape will be somewhat different in form from that given by equation (1.7). In order to minimise the effect of this uncertainty on the overall analysis of a spectrum, it is therefore necessary that the proportion of $G_{12}(x)$ in equation (1.8) be small. In that case, small errors in $G_{12}(x)$ will have a negligible effect. In the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ measurements (see chapter 2), $\varepsilon_1$ and $\varepsilon_2$ were less than 0.1, so that this requirement was satisfied.

1.3.3 Summary of sources of errors in line shapes:

The line-shape components used in fitting a gamma-ray spectrum inevitably differ to some extent from those actually present in the spectrum. These differences vary in importance, depending on the relative intensities and energies of transitions. Their effect on the fitted intensities is somewhat unpredictable, and they tend not to be adequately covered by the errors indicated by the least-squares fitting calculation (see next section). The main sources of line-shape errors are as follows:

1. Initial spectra for producing standard line shapes may be broadened because of target thickness, Doppler effects and various electronic defects, (for example, excessive noise in the pre-amplifier - especially when measuring Ge(Li) line shapes, and excessive count rate).

2. The line shapes in the spectrum under analysis may be broadened, relative to the standard line shapes, for the same reasons as mentioned in (1). Since many overlapping lines are often present in this case, the presence of broadening is generally
not so readily noticed. As a particular example, the 6.92 and 7.117 MeV. radiation from $^{19}F(p,\gamma)^{16}O$ is strongly Doppler broadened, and its presence in experimental spectra (see chapter 2) is therefore more disadvantageous than if it were not broadened. Most of the above effects cause shifts in the line shapes, in addition to the broadening.

(3) The removal of background and other unwanted radiation from standard line-shape spectra, will introduce error mainly in the relative heights of the Compton/bremsstrahlung tail, and the photopeak and escape peaks. Hence the desirability of choosing reactions which produce as little radiation as possible, aside from the desired transition. This requirement is not easily satisfied for the energy range of interest in the present work. ($\sim$ 4-10 MeV.).

(4) The interpolation procedure is a very likely source of error. The detailed shape of interpolated line shapes was found to vary significantly, depending on which of the standard curves were used in the calculation, and the number used. This is a consequence, primarily, of the necessity of using some arbitrary analytic curve (in the present case, a polynomial in the channel number) in fitting sets of points on the standard curves, to deduce the appropriate intensity in the interpolated curve (see section 1.2.3). Hence the desirability of having many closely-spaced standard curves. The generation of the standard curves also incurs errors from this source, due to the necessity of interpolating line shapes for the unwanted subsidiary radiation.

(5) The angular distribution of the radiation (often unknown), both in line-shape spectra and experimental spectra may cause distortions. It is thus desirable, from this point of view, to use as large a target-to-detector distance as possible, consistent with adequacy of count rate.
(6) The calculation of convoluted line shapes (discussed in the previous section) normally involves a number of approximations. Firstly, one must generally assume an isotropic correlation of the successive transitions. Secondly it may be necessary to ignore some pairs of transitions, (from the point of view of simultaneous detection) if the decay scheme of the levels being studied is too complex.

1.4. The least-squares analysis of spectra:

1.4.1 Detection of efficiency:

In order to establish absolute cross-sections, the detection efficiency of the 5" × 4" NaI(Tl) detectors, which were used in much of the present work, was calculated using the known gamma-ray attenuation coefficients for NaI (Gr57).

The fraction of isotropically emitted radiation $\varepsilon(E)$, of energy $E$, which interacts with a detector is given by:

$$
\varepsilon(E) = \frac{1}{4\pi} \int_\Omega \left(1 - e^{-\tau(E)x(\theta,\varphi)}\right) d\Omega, \quad (1.9)
$$

where $\tau(E)$ is the linear attenuation coefficient (for all incoherent processes) of the material of the detector, $x(\theta,\varphi)$ is the distance which a photon, emitted in a direction $(\theta,\varphi)$, may travel within the detector, $\Omega$ is the solid angle subtended by the detector.

The geometry for a right cylindrical detector is shown in figure (1.4).
Figure 1.4: Detector geometry, for the calculation of efficiency.
In this case, it may be shown that

\[ x(\theta, \varphi) = h \sec \theta \quad 0 \leq \theta \leq \alpha \]

\[ = r \cosec \theta - d \sec \theta \quad \alpha \leq \theta \leq \beta \tag{1.10} \]

where \( \alpha = \tan^{-1} \left( \frac{r}{d + h} \right) \)

\( \beta = \tan^{-1} \left( \frac{r}{d} \right) \tag{1.11} \)

Inserting (1.11) and (1.10) into equation (1.9), and integrating over \( \varphi \), one obtains:

\[ \varepsilon(E) = \frac{1}{2} \left\{ \int_{0}^{\alpha} (1 - e^{-\tau(E)h\sec \theta}) \sin \theta \, d\theta ight. \\
\left. + \int_{\alpha}^{\beta} (1 - e^{-\tau(E)(r\cosec \theta - d\sec \theta)}) \sin \theta \, d\theta \right\} \tag{1.12} \]

Calculation of the detection efficiency for the NaI(Tl) detector, using equation (1.12) was straightforward, since the geometry is well known. This was not so in the case of the Ge(Li) detector, however, which was of the coaxially-drifted type. However, in this case (see chapter 5), only the energy variation of the detection efficiency was required. Consequently, the detector was approximated by a right cylinder of the same volume as the detector, and equation (1.12) used. It was found that the relative efficiencies thus obtained were in close agreement with the values obtained using a more realistic geometry (Hu68).
1.4.2 Derivation of the fundamental equations:

The calculation of the intensities of a set of transitions in a spectrum, using linear least-squares fitting with line shapes, is quite straightforward. Put

\[
A = \begin{pmatrix}
    f_1(x_1) & f_2(x_1) & \ldots & f_m(x_1) \\
    f_1(x_2) & f_2(x_2) & \ldots & f_m(x_2) \\
    \vdots & \vdots & \ddots & \vdots \\
    f_1(x_n) & f_2(x_n) & \ldots & f_m(x_n)
\end{pmatrix},
\]

(1.13)

where the \( f_i(x) \) are a set of \( m \) interpolated line shapes, defined over a range of \( n \) channels. It is required to find a set of intensities, \( a_i \), represented by the column matrix, \( \underline{a} \), such that the column matrix, \( \underline{W} \), given by

\[
\underline{W} = A \underline{a},
\]

(1.14)

is as close as possible to the matrix, \( \underline{V} \), of measured channel populations, in the \( \gamma \)-ray spectrum under analysis. The usual criterion of least-squares requires that

\[
\Delta = \sum_{i=1}^{n} \omega_i (W_i - V_i)^2
\]

(1.15)

is a minimum, where \( \omega_i \) are weights.

(1.15) is satisfied by

\[
\frac{\partial \Delta}{\partial a_j} \bigg|_{j=1,m} = 0.
\]

(1.16)
From (1.14) and (1.15)

\[ \Delta = \sum_{i=1}^{n} \omega_i \left( \sum_{j=1}^{m} A_{ij} a_j - V_i \right)^2. \]  

(1.17)

Then

\[ \frac{\partial \Delta}{\partial a_{il}} = \sum_{i=1}^{n} \omega_i \frac{\partial}{\partial a_{il}} \left\{ \left( \sum_{j=1}^{m} a_{ij} A_{ij} \right) \left( \sum_{k=1}^{m} a_{ik} A_{ik} \right) - 2V_i \sum_{j=1}^{m} a_{ij} A_{ij} \right\} \]

\[ = \sum_{i=1}^{n} \omega_i (A_{il} (\bar{W}_i - V_i)) \]

\[ = (\bar{A}^T \omega \bar{A} - \bar{A}^T \omega \bar{V}) = 0, \]

(1.18)

where \( \omega \) is a diagonal matrix of weights,

\( \bar{A}^T \) is the transpose of \( \bar{A} \).

Then

\[ \bar{a} = (\bar{A}^T \omega \bar{A})^{-1} \bar{A} \omega \bar{V}. \]

(1.19)

For a given line-shape matrix, \( \bar{A} \), equation (1.19) gives the best estimate of the intensity of each radiation component in the spectrum, according to the criterion (1.15).

The solution (1.19) is only unique for a given set of weights, \( \omega_i \). The most common choice is

\[ \omega_i = S_i^{-1} \]

where \( S_i \) is the variance associated with the \( i \)'th channel.

Following the usual practice, these weights were used in the present work. The variances, \( S_i \), were taken to be the channel populations. Weighting according to the inverse of the variance has certain
advantages (Fe 65), from the point of view of error analysis. It can then be shown that the matrix \((A^T \omega A)^{-1}\) in equation (1.19) is the matrix of the variances and co-variances of the intensities, \(a_i\).

1.4.3 Estimation of errors

Birge (Bi 32) has noted that it is necessary to evaluate not only the errors on the extracted intensities, but also the reliability of these errors. An attempt has been made, in the present work, to take account of both statistical and non-statistical errors.

Following Hebbard (He 67a): the factor

\[
(n-m)^{-1} \sum_{i=1}^{n} \omega_i (v_i - V_i)^2
\]

was included in the expression for the errors on the intensities, \(a_i\), derived from the least squares fit. This factor was intended to take some cognizance of other than statistical errors, for example errors in the line shapes used in fitting.

Combining this factor with the elements of the matrix \((A^T \omega A)^{-1}\) in equation (1.19) (as previously mentioned), the variances, \(\sigma_k^2\), on the intensities, \(a_k\), were taken to be

\[
\sigma_k^2 = (A^T \omega A)^{-1} \times (n-m)^{-1} \sum_{i=1}^{n} \omega_i (v_i - V_i)^2
\]

It is felt, however, that (1.21) does not always accurately reflect the true errors in the intensities. The problem is most acute in the analysis of NaI(Tl) spectra, where the resolution is relatively poor. Where even a
A moderate number of overlapping lines is present in the spectrum, a combination of errors in the line shapes (as discussed in section 1.3.3) can result in a better fit than if the exact line shapes were present. This is particularly so if unknown contaminants are present in a spectrum, and extra components are included in the fit, on the basis of reasonable (but not certain) assumptions as to the sources of the extra radiation. The point of view has therefore been taken, in the present work, that the errors extracted from the least-squares analysis of spectra do not completely cover the actual errors.
Chapter 2

THE LEVEL STRUCTURE OF $^{150}$, AND THE $^{14}_N(p,\gamma)^{150}$ REACTION BETWEEN 1.7 AND 3.1 MeV.
2.1. Introduction:

The aim in this chapter is, firstly, to review the available information relating to levels in $^{150}$, with reference to their effect on the gamma-ray yield from $^{14}_N(p,\gamma)^{150}$ at stellar energies. A resonance level may contribute in two ways:

1. directly through the $^{14}_N(p,\gamma)^{150}$ reaction.
2. indirectly, through direct capture of protons into bound states of $^{150}$.

The theory of the direct-capture process is discussed in detail in chapter three. It is only necessary to note here that elastic scattering from both bound and unbound levels in $^{150}$ modifies the initial-state wave function, and hence the direct-capture cross-section. The bound levels are of additional importance, since these constitute the final states in direct-capture transitions. Their structure is ultimately the most important factor in determining the transition cross sections. In discussing the structure of the bound states, the jj-coupling representation has been used. This is not, of course, intended to imply that pure jj coupling occurs. It is just a convenient representation for discussing the phenomena of stripping and direct capture.

2.2. Properties of states in $^{150}$:

2.2.1 Bound states - general characteristics

On the basis of the shell model, the ground state and 6.18 MeV state (Wa65) should consist predominantly of $1p_{\frac{1}{2}}$ and $1p_{\frac{3}{2}}$ holes respectively, in an $^{16}_O$ ground-state core (assumed to be mainly of the closed $1p$-shell configuration). Their spins and parities
would then be \( \frac{1}{2}^- \) and \( \frac{3}{2}^- \) (they are the only -ve parity bound states). This prescription is supported by the study of the reactions \( ^{16}\text{O}(^{3}\text{He},\alpha)^{15}\text{O} \) (Wa65a) and \( ^{16}\text{O}(p,d)^{15}\text{O} \) (Ma66b, Ba67). It was found that particle spectra were dominated by transitions leading to these states, and that the yields of the appropriate particle groups were close to the expected ratio of 2:4. Angular distributions showed the expected \( l = 1 \) stripping/pick-up patterns for the reactions \( ^{14}\text{N}(^{3}\text{He},d)^{15}\text{O} \) and \( ^{16}\text{O}(^{3}\text{He},\alpha)^{15}\text{O} \) (Bo68), and \( ^{14}\text{N}(d,n)^{15}\text{O} \) (Mu67). Povh and Hebbard (Po59), verified that the spin of the 6.18 MeV. level is \( \frac{3}{2} \).

In order that stripping and direct-capture reactions may take place leading to the 6.18 MeV. final state, it is necessary that configurations other than \( 1p_2^{-1} \) be present in this state (on the assumption that the \( ^{14}\text{N} \) ground state configuration is of the form \( 1p_2^{-2} \)). Alternatively, the \( ^{14}\text{N} \) ground-state wave function may be more complex. A number of calculations have been performed, the results of Kumar (Ku68) being the most recent. The relative intensities of the configurations \( 1p_2^{-2}, 1p_2^{-1} 1p_2^{-1} \) and \( 1p_2^{-2} \) in the \( ^{14}\text{N} \) ground-state wave function were found to be 66%, 14% and 20%, when transformed from the LS-coupling representation to the equivalent jj-coupling representation. Similarly, Visscher and Ferrell (Vi57) give 85.5%, 13% and 1.5% respectively.

The positions of the 5.2 MeV. doublet levels have been determined, with high precision, using Ge(Li) detectors (Al66a, Ch67, Al68), and are taken to be 5.191 ± 0.005 MeV. and 5.241 ± 0.0005 MeV. The \( ^{16}\text{O}(^{3}\text{He},\alpha)^{15}\text{O} \) reaction has been studied by Hinds and Middleton (Hi59) and Bohne et al. (Bo68). An \( l = 0 \) pick-up pattern for the angular distribution of the alpha group leading to the 5.191 MeV. level was observed and therefore \( J^\pi = \frac{1}{2}^+ \). The fact that the pick-up reaction has a high cross section suggests that the wave function of this state has a
large $1s_2^{-1}$ component. The angular distribution of the $^{14}_N(^3He,d)^{15}O$ reaction leading to the 5.191 MeV. level (Bo68) has none of the characteristics of an $\ell = 0$ stripping pattern. This suggests that the reduced width of this level for the channel comprising $^{14}_N + p$ with $\ell = 0$ is not large. However Robson (Ro66a) has found that $^{14}_N(d,p)^{15}N$ data, at lower energies, for the transition to the assumed mirror level at 5.31 MeV., can be accounted for by D.W.B.A. stripping theory. It appears that the wave function may contain at least a small amount of a configuration with an $\ell = 0$ proton coupled to a $^{14}_N$ ground-state core. This suggests that the direct capture transition is possible, though likely to be weak. As discussed in chapter 3, the cross section for a direct-capture transition is proportional to the reduced width of the final state, in the appropriate channel.

The spin and parity of the 5.241 MeV. state are generally assumed to be $\frac{3}{2}^+$, in accord with the Halbert and French (Ha57) model, in which the structure is taken to be $1p_{1/2}2d$. This assignment is supported by the angular correlation measurements of Gor letzky et al. (Go66, Go67). $^{14}_N(^3He,d)^{15}O$ data (Bo68) and $^{14}_N(d,n)^{15}O$ data (Ev53) are consistent with a stripping mechanism with $\ell = 2$, as are the data for the $^{14}_N(d,p)^{15}N$ reaction to the assumed mirror level at 5.27 MeV. (Ro66b). The spin of the latter state has been independently determined to be $\frac{5}{2}^+$ (He60, Wa65b).

The fourth and fifth excited states have been found to have excitation energies $6.789 \pm 0.007$ MeV. and $6.857 \pm 0.003$ MeV. (Wa65). The 6.789 MeV. state has been assigned $J^\pi = \frac{3}{2}^+$ (Po59) on the basis of an angular correlation measurement. The angular distribution measurements of Bohne et al. (Bo68) are interesting because they show an $\ell = 0$ stripping pattern for the $^{14}_N(^3He,d)^{15}O$ reaction, but an $\ell = 2$ pick-up pattern for $^{16}O(^3He,\alpha)^{15}O$. This also fixes $J^\pi = \frac{3}{2}^+$, and points to the
necessity of an additional configuration other than the closed p-shell, in the $^{16}O$ ground-state wave function. The $l=0$ stripping pattern suggests the presence of a configuration comprising an $l=0$ proton coupled to the $^{14}N$ ground-state core. The $^{14}N + p$ channels appear to be almost entirely $l=0$.

Hebbard and Bailey measured the angular distribution of radiation leading to the 6.789 MeV. state from $^{14}N(p,γ)^{15}O$, and found it to have a $\sin^2θ$ dependence within experimental error. This angular distribution is predicted for a p to s direct capture transition. An upper limit of 6% has been assigned (Wa65) to transitions from the 6.789 MeV. state to lower-energy states other than the ground state.

Warburton et al. (Wa65) found that the 6.857 MeV. state decays to the 5.241 MeV. state with no direct evidence for transitions to any other states. They have established that the 6.857 $→$ 5.241 transition is E1, requiring that the spin be $\frac{3}{2}$ or $\frac{5}{2}$ (taking into account previous work). The data of Bohne et al. (Bo68) show $l=2$ angular distribution patterns for both the $^{14}N(^3He,d)^{15}O$ and $^{16}O(^3He,α)^{15}O$ reactions, which requires the parity to be positive. Angular correlation measurements (Ga66, Gi67) favour an assignment $\frac{5}{2}^-$, as does the measurement for the corresponding state in $^{15}N$, and in the following, $J^\pi = \frac{5}{2}^+$ has been assumed.

The energy of the sixth excited state was measured by Warburton et al. (Wa65) to be 7.284 ± 0.007 MeV., and later more accurately (Al66) as 7.276 ± 0.0006 MeV. An angular correlation measurement was performed by Hensley (He67) using the $^{16}O(^3He,α)^{15}O$ reaction, in order to determine the spin of the state. An unambiguous assignment of $\frac{7}{2}^-$ was obtained, supporting its identification as the analogue of the $\frac{7}{2}^-$ state in $^{15}N$ at 7.563 MeV. Measurements of the $^{14}N(^3He,d)^{15}O$ (Bo68) and $^{14}N(d,n)^{15}O$ (Mu67) angular distributions of groups populating this state, indicate $l=2$ for the captured proton, hence
supporting even parity for the state. The well defined stripping patterns indicate that direct radiative capture of protons in $^{14}\text{N}$ is possible, with the proton being captured into an $l = 2$ orbit. The 7.276 MeV state has been found to decay entirely to the 5.241 MeV state, within experimental errors (He67, Wa65).

### 2.2.2 Bound states - reduced widths.

The reduced width for a particular channel (as defined by Lane and Thomas (La58)) may be split up into two factors (Fr60): (1) a spectroscopic factor which is a measure of how much the state resembles the channel configuration, and (2) a single-particle reduced width, which is related to the probability of finding the constituent particles separated when the configuration occurs.

In principle, analysis of stripping data can provide the desired information, but in general only spectroscopic factors can be obtained with any reliability. The plane-wave stripping theory of Butler contains the total reduced width as a factor (Ma60), but it is well known that the theory greatly over-estimates cross sections, resulting in too-small reduced widths. On the other hand, the D.W.B.A. theory contains only the spectroscopic factor as a parameter, and has nothing to say about single-particle reduced widths, since the wave functions over all space are involved (Gl63).

In recent years, stripping reactions leading to bound states in $^{15}\text{O}$ and $^{15}\text{N}$ have been comprehensively studied and analysed in terms of the D.W.B.A. theory. Bohne et al. (Bo68) have deduced spectroscopic factors for the $^{14}\text{N}(^3\text{He},d)^{15}\text{O}$ reaction leading to all bound states, and their results have been used
in the present work. For the purpose of normalisation, the spectroscopic factors were multiplied by 1.45 so as to make the spectroscopic factor for the ground-state transition the same as that calculated by Cohen and Kurath (Co67). It is found that the results are very close to the theoretical values for $^{15}_N$ states of positive parity, calculated by Halbert and French (Ha57).

Robson (Ro68) has derived very similar values from the D.W.B.A. analysis of $^{15}_N(d,p)^{15}_N$ angular distributions, in which Hauser-Feshbach (Ha52a) calculations have been performed to derive energy-averaged compound-nucleus angular distributions, which have been subtracted from energy-averaged data before analysis. Thus it was felt that the spectroscopic factors of Bohne et al. could be used with reasonable confidence.

The situation with regard to single-particle reduced widths is not so well defined. What has been done is to use the calculations for a square-well potential given by Macfarlane and French (Ma60). These figures are similar to values suggested by Barker (Ba63b).

The data on bound states in $^{15}_O$ is collected in table 2.1. As the stripping analysis does not differentiate between different channel spins, it is necessary to make an arbitrary decision (depending on the circumstances) as to the distribution of the reduced width between the two channels with spins $s = \frac{1}{2}$ and $s = \frac{3}{2}$. This matter is discussed further in chapter 4, where the results of the direct-capture calculations are considered.
Table 2.1: Properties of the $^{15}$O bound states.

<table>
<thead>
<tr>
<th>Excitation Energy (MeV.)</th>
<th>$J^\pi$</th>
<th>$l$ channel spin</th>
<th>Binding Energy (MeV.)</th>
<th>Spectroscopic Factor $S_{ls}$</th>
<th>Dimensionless single-particle reduced width $\sigma^2_{ol}$</th>
<th>Dimensionless Reduced width $(\text{MeV.})$ $\gamma_{ls}^2 = S_{ls} \cdot \sigma^2_{ol}$</th>
<th>Reduced width $\left(\frac{A^2}{A_{\text{MeV.}}^2} \right) \sigma^2_{ls}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>$\frac{1}{2}^-$</td>
<td>1</td>
<td>$\frac{1}{2}, \frac{3}{2}$</td>
<td>7.293</td>
<td>1.45</td>
<td>0.45</td>
<td>0.65</td>
</tr>
<tr>
<td>5.191</td>
<td>$\frac{1}{2}^+$</td>
<td>0</td>
<td>$\frac{1}{2}$</td>
<td>2.102</td>
<td>0.01</td>
<td>0.5</td>
<td>0.005</td>
</tr>
<tr>
<td>$\pm 0.005$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5.241</td>
<td>$\frac{1}{2}^+$</td>
<td>2</td>
<td>$\frac{1}{2}, \frac{3}{2}$</td>
<td>1.952</td>
<td>0.06</td>
<td>0.65</td>
<td>0.04</td>
</tr>
<tr>
<td>$\pm 0.0005$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.180</td>
<td>$\frac{1}{2}^-$</td>
<td>1</td>
<td>$\frac{1}{2}, \frac{3}{2}$</td>
<td>1.133</td>
<td>0.07</td>
<td>0.5</td>
<td>0.04</td>
</tr>
<tr>
<td>$\pm 0.004$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.789</td>
<td>$\frac{1}{2}^+$</td>
<td>0</td>
<td>$\frac{3}{2}$</td>
<td>0.504</td>
<td>0.56</td>
<td>0.45</td>
<td>0.25</td>
</tr>
<tr>
<td>$\pm 0.007$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6.857</td>
<td>$\frac{1}{2}^+$</td>
<td>2</td>
<td>$\frac{1}{2}, \frac{3}{2}$</td>
<td>0.436</td>
<td>0.52</td>
<td>0.6</td>
<td>0.3</td>
</tr>
<tr>
<td>$\pm 0.003$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7.276</td>
<td>$\frac{1}{2}^+$</td>
<td>2</td>
<td>$\frac{3}{2}$</td>
<td>0.015</td>
<td>0.42</td>
<td>0.6</td>
<td>0.25</td>
</tr>
<tr>
<td>$\pm 0.0006$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2.2.3 Unbound states:

The lowest-lying unbound state, which is of great importance for the present study, is excited via the $^{14}\text{N} + p$ channel at a proton energy of 278 KeV. (Pi57). Taking the $^{14}\text{N} + p$ threshold to be 7.293 MeV. (Ma65a), the excitation energy is 7.552 MeV. From the study of the reactions $^{14}\text{N}(p,p)^{14}\text{N}$ and $^{14}\text{N}(p,\gamma)^{15}\text{O}$ (Pi57, Ov56, Ta60), it is known to have $J^\pi = \frac{1}{2}^+$ and to be formed by $\ell = 0$ protons. Its total width (Pi57) is $1.7 \pm 0.5$ KeV. Significant formation of the 7.552 MeV. state by d-waves is unlikely, since a partial width of as little as 1% of the full width would require a reduced width equal to the Wigner limit. Thus it can reasonably be assumed that the state is formed only via the channel with $\ell = 0$ and channel spin $s = \frac{1}{2}$. The gamma-ray branching ratios are well known (He63, Ta60, Wa65). Povh and Hebbard (Po59) have shown that the transitions through the doublets at 5.2 and 6.8 MeV. involve the 5.191 MeV. and 6.789 MeV. states.

The 8.283 MeV. level, which is excited by 1.061 MeV. protons (Ev66a) has been studied through the reactions $^{14}\text{N}(p,p)^{14}\text{N}$ (Ta56, Ha57a, 0158, Ba59) and $^{14}\text{N}(p,\gamma)^{15}\text{O}$ (Du51, Go57, He59, He63, Ev66a). The analysis by Hagedorn et al. (Ha57a) indicated $J^\pi = \frac{3}{2}^+$, with total width $\Gamma = 3$ KeV., and formation through s-waves. $J = \frac{3}{2}$ has been confirmed by an angular distribution measurement of the ground-state transition, by Gorodetzky et al. (Go57). The fact that the angular distributions is non-isotropic means that the state may be formed by d-waves as well as s-waves. In fact three channels are possible: $(\ell = 2, s = \frac{1}{2})$, $(\ell = 0, s = \frac{3}{2})$, $(\ell = 2, s = \frac{3}{2})$. If it is assumed that only channel spin $s = \frac{3}{2}$ is involved, then it may be shown that the intensity of d-wave formation is only 0.25% of that for s-wave formation. It may also be shown that no combination of the
channels \((l = 2, s = \frac{1}{2})\) and \((l = 0, s = \frac{3}{2})\) can account for the measured angular distribution, so that the assumption that formation occurs only via the channel \((l = 0, s = \frac{3}{2})\) is a good one.

The radiative widths have been measured by Evans et al. (Ev66a), using the absolute normalisation of Duncan and Perry (Du51), and the doublet members taking part in the decay of the level have been identified as the 5.241 and 6.857 MeV. states.

The 8.749 level is excited by 1.55 MeV. protons on \(^{14}\text{N}\), and has a total width of \(35 \pm 5\) KeV. (0158). Proton elastic scattering measurements (Ta56, Ha57a) indicate that the spin and parity are \(\frac{1}{2}^+\), and that the entrance channel has \(l = 0\) and hence \(s = \frac{1}{2}\). There is no indication in the literature of any formation via the channel \((l = 2, s = \frac{3}{2})\). Decay of the state is found to take place via the states at 5.191 and 6.18 MeV. (Ev66a).

The properties of the states at 8.915 and 8.972 MeV. (Ev66a) are not definitely fixed. Cohen-Ganouna et al. (Co63) assigned the 8.915 MeV. level a spin and parity of \(\frac{3}{2}^+\) with \(\frac{1}{2}^+\) a possibility. Hagedorn et al. (Ha57a), however, found their \(^{14}\text{N}(p,p)^{14}\text{N}\) data to be most consistent with \(J^\pi = \frac{3}{2}^+\), with formation through the \((l = 2, s = \frac{1}{2})\) channel. The angular distribution measurement by Evans et al. (Ev66a), of the ground-state transition, is incompatible with \(J^\pi = \frac{3}{2}^+\) and \(\frac{3}{2}^-\). It is compatible with \(J^\pi = \frac{1}{2}^+\), with formation through channel spin \(\frac{3}{2}\) and an \(l = 2\) to \(l = 0\) intensity ratio of 35\%. Cohen-Ganouna et al. found the total width to be 1.6 KeV., though this may be an underestimation, since their intrinsic resolution was 4 KeV. However the width of 11 KeV. found by Duncan and Perry (Du51), from the \(^{14}\text{N}(p,\gamma)^{15}\text{O}\) reaction, is curiously large compared with values obtained from \(^{14}\text{N}(p,p)^{14}\text{N}\) measurements.
Cohen-Ganouna et al. (Co63) derived a spin and parity of $\frac{5}{2}^-$ for the 8.972 MeV. state, whilst Hagedorn et al. (Ha57a) found both $\frac{3}{2}^-$ and $\frac{1}{2}^-$ equally probable. Evans (Ev66a) has pointed out that an assignment of $J^\pi = \frac{5}{2}^-$ would be improbable on the basis of the strength of the transition to the 5.191 MeV. state.

The lack of agreement with respect to the 8.915 and 8.972 MeV. levels may possibly be due to the effects of interference with the broad level at 9.490 MeV. It may also be due to the presence of another level in the vicinity. Young and Steerman (Yo67, St68) have pointed out that the branching ratios of the 9.16 MeV. level in $^{15}\text{N}$ vary with the mode of excitation, and they give convincing evidence that in fact the level is an unresolved doublet. This being the case, another level must exist in $^{15}\text{O}$, and would be expected to lie near 8.8 MeV. on the basis of the relative excitation energies of other pairs of mirror levels in $^{15}\text{O}$ and $^{15}\text{N}$ (Wa65).

The final group of levels below 10 MeV. excitation energy occur within the energy range examined in the present measurements, the aim of which was to determine the properties of the broad 2.35 MeV. resonance level. During the course of this work, detailed measurements of the elastic scattering (La67, La67a) and radiative capture (Ev66) of protons on $^{14}\text{N}$ was reported in the literature. This work was complementary to the work described in the next section.

The elastic scattering measurements of Lambert and Durand (La67, La67a) resulted in the assignment $J^\pi = \frac{3}{2}^-$ for the 2.35 MeV. resonance level, with a total width of $300 \pm 30$ KeV. The level was found to be populated via three channels ($\ell = 0$, $s = \frac{3}{2}$), ($\ell = 2$, $s = \frac{3}{2}$), ($\ell = 2$, $s = \frac{1}{2}$). There is, however,
an error in the published reduced widths for each channel (Du68),
which was caused by an error in the calculation of the coulomb
wave functions. The published full width and partial widths
are correct, but the reduced widths are incorrect. On calculating
the values of the partial widths for each channel in the region
20-1000 KeV., it was found that the s-wave channel (as might
be expected) is dominant, even though its reduced width is only
\( \sim \frac{1}{6} \)th of that of the channels with \( l = 2 \). The calculated
ratios of the three partial widths to the full width are shown in
table 2.2.

Table 2.2: Ratios of partial widths to the full width for the
9.49 MeV. level.

<table>
<thead>
<tr>
<th>Energy (MeV.)</th>
<th>( l = 0 )</th>
<th>( l = 2 )</th>
<th>( l = 2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( s = \frac{1}{2} )</td>
<td>( s = \frac{1}{2} )</td>
<td>( s = \frac{1}{2} )</td>
</tr>
<tr>
<td>( \gamma^2 )</td>
<td>109 KeV.</td>
<td>104 KeV.</td>
<td>570 KeV.</td>
</tr>
<tr>
<td>1000</td>
<td>0.85</td>
<td>0.02</td>
<td>0.13</td>
</tr>
<tr>
<td>500</td>
<td>0.945</td>
<td>0.008</td>
<td>0.047</td>
</tr>
<tr>
<td>200</td>
<td>0.977</td>
<td>0.003</td>
<td>0.02</td>
</tr>
<tr>
<td>20</td>
<td>0.986</td>
<td>0.002</td>
<td>0.012</td>
</tr>
</tbody>
</table>

Owing to the presence of fluorine and other contaminants,
Evans (Ev66) was unable to determine the decay modes of the level,
but did note that it appeared to decay largely to the ground state.

The other three levels in this region are quite narrow,
and occur at 9.487, 9.613 and 9.670 MeV. They have been
studied in detail by Lambert and Durand (La67, La67a) and Evans (Ev66).
The published reduced widths for these levels are also in error (Du68), but the full widths are correct. Data on the unbound levels in $^{150}O$, up to 10 MeV. excitation energy, are shown in Table 2.3. Gamma-ray branching ratios of both bound and unbound levels are shown in figure 2.1.

Table 2.3: The unbound states of $^{150}O$.

<table>
<thead>
<tr>
<th>Proton Energy (MeV.)</th>
<th>Excitation Energy (MeV.)</th>
<th>$J^\pi$</th>
<th>Total width, (KeV.)</th>
<th>channels $(l,s)$</th>
<th>Reduced width, $\bar{\Gamma}$ (KeV.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.278 ± 0.0004</td>
<td>7.552 ± 0.001</td>
<td>$1^+$</td>
<td>1.7 ± 0.5</td>
<td>$(0,\frac{1}{2})$</td>
<td>1260</td>
</tr>
<tr>
<td>1.061 ± 0.002</td>
<td>8.283 ± 0.003</td>
<td>$2^+$</td>
<td>3 ± 1</td>
<td>$(0,\frac{3}{2})$</td>
<td>7.3</td>
</tr>
<tr>
<td>1.550 ± 0.005</td>
<td>8.749 ± 0.006</td>
<td>$\frac{1}{2}^+$</td>
<td>35 ± 5</td>
<td>$(0,\frac{1}{2})$</td>
<td>38</td>
</tr>
<tr>
<td>1.739 ± 0.002</td>
<td>8.915 ± 0.003</td>
<td>$2^+ (5^+)$</td>
<td>(1.6)</td>
<td>$(2,\frac{3}{2})$</td>
<td>(2,\frac{3}{2})</td>
</tr>
<tr>
<td>1.800 ± 0.004</td>
<td>8.972 ± 0.005</td>
<td>$\frac{1}{2}^- (\frac{3}{2}^-)$</td>
<td>7 ± 3</td>
<td>$(1,\frac{3}{2})$</td>
<td>7.8</td>
</tr>
<tr>
<td>2.342 ± 0.004</td>
<td>9.487 ± 0.005</td>
<td>$\frac{5}{2}^-$</td>
<td>7 ± 2</td>
<td>$(1,\frac{3}{2})$</td>
<td>7.8</td>
</tr>
<tr>
<td>2.347 ± 0.04</td>
<td>9.49 ± 0.04</td>
<td>$\frac{3}{2}^-$</td>
<td>300 ± 30</td>
<td>$(0,\frac{3}{2})$</td>
<td>(0,\frac{3}{2})</td>
</tr>
<tr>
<td>2.479 ± 0.006</td>
<td>9.613 ± 0.007</td>
<td>$\frac{3}{2}^-$</td>
<td>7 ± 2</td>
<td>$(1,\frac{3}{2})$</td>
<td>7.1</td>
</tr>
<tr>
<td>2.547 ± 0.006</td>
<td>9.670 $(\frac{7}{2},\frac{9}{2})^-$</td>
<td>2 ± 1</td>
<td>$(3,\frac{3}{2})$</td>
<td>(3,\frac{3}{2})</td>
<td></td>
</tr>
<tr>
<td>2.607 ± 0.006</td>
<td>9.760 $(\frac{7}{2},\frac{9}{2})^-$</td>
<td>2 ± 1</td>
<td>$(3,\frac{3}{2})$</td>
<td>(3,\frac{3}{2})</td>
<td></td>
</tr>
</tbody>
</table>
Figure 2.1: Energy levels and decay schemes of $^{15}$O.
2.3. Experimental Considerations:

2.3.1 General details.

The A.N.U.'s tandem Van de Graaff accelerator provided a beam of protons in the energy range 1.7 - 3.1 MeV. Gamma-ray spectra were collected at 20 KeV. intervals using a 5" x 4" uncollimated NaI(Tl) detector at 55° to the beam direction, and an R.I.D.L. pulse-height analyser in the 200 channel mode. Three sets of runs were taken with different targets. The first set of runs was discarded owing to rather severe fluorine contamination and inadequate detector resolution. The fluorine contamination (resulting in strong 6-7 MeV. radiation from the $^{19}$F(p,γ)$^{16}$O reaction) was also evident in the other runs, but to a lesser degree.

A 20cc Ge(Li) detector was used at several energies to obtain more detailed information on the radiation present in the spectra; because of its inherently low efficiency and the low reaction cross sections, runs of ~8 hours were necessary to obtain reasonable statistics.

In order to eliminate radiation from the $^{19}$F(p,γ)$^{16}$O reaction, and the inevitable strong 4.433 MeV. radiation from the $^{15}$N(p,γ)$^{12}$C reaction, due to the 0.4% isotopic abundance in natural nitrogen, a number of coincidence spectra were taken. Two 5" x 4" NaI detectors were placed on either side of the target chamber, and as close to the target as possible. Timing single-channel analysers (T.S.C.A's) enabled a lower-level cut off to be applied, and provided timing pulses for the fast coincidence unit. The block circuit diagram is shown in figure 2.2.

By setting the lower-level discriminators just above the 0.511MeV. annihilation line, only radiation from cascade transitions of two or more members were recorded, eliminating the radiation
Figure 2.2: Experimental arrangement for the coincidence measurements.
from the $^{19}\text{F}(p,\alpha\gamma)^{16}\text{O}$ and $^{15}\text{N}(p,\alpha\gamma)^{12}\text{C}$ reactions. However, a component due to these reactions was present in spectra, owing to the escape of Compton radiation and/or fast electrons from one detector to the other. This component was of a structureless nature, and hence did not confuse the picture.

**2.3.2 Targets.**

The choice of targets was decided by three main considerations:

1. The need to avoid fluorine contamination.
2. The necessity for targets to be stable under beam bombardment, since the low cross section for the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction necessitated moderate beam currents ($\sim 0.5-1.0 \mu$amp.). This requirement is not easy to satisfy, as most nitrogen compounds which are suitable from a nuclear physics point-of-view are unstable - sometimes explosively.
3. Isotopically enriched targets would have been desirable, since the strong $^{15}\text{N}(p,\alpha\gamma)^{12}\text{C}$ 4.433 MeV. radiation tends to obscure the ground-state radiation from the 5.2 MeV. doublet in $^{15}\text{O}$, as well as totally swamping the low-energy members of cascade transitions.

There is no source of nitrogen with an $^{15}\text{N}$ concentration less than the naturally occurring 0.4%, so that some workers have attempted to make isotopically enriched targets using magnetic separation of nitrogen isotopes from an ion source. Evans et al. (Ev66a) used 400 KeV. ($^{14}\text{N}$)$^+_2$ ions from a Van de Graaff generator ion source to bombard a tantalum foil. However, it was found that the targets lacked uniformity, making them unsuitable for angular-distribution or excitation-function measurements.

Targets made by heating tantalum strips in an atmosphere of nitrogen or ammonia have been used successfully (Go58, Po59, He59, He63), and this type of target was used in most of the present work.
Evans et al. (Ev66, Ev66a) have made targets by sputtering tantalum, in a nitrogen atmosphere, onto a tantalum backing. The disadvantage of tantalum targets is that fluorine tends to be a rather common contaminant, and the heated targets sometimes suffer from a tail of nitrogen deep in the target. Recent work in this laboratory (He63, Ba63a) indicated that the nitrided-tantalum targets, which had been used in $^{14}\text{N}(p,\gamma)^{15}\text{O}$ studies, were quite satisfactory, both as regards fluorine contamination and the nitrogen distribution. The availability of such targets, their stability under beam bombardment, and their other good characteristics, made them a logical choice. It was later discovered, however, that these targets were quite unsatisfactory, since they possessed a very long tail in their nitrogen distribution. This is discussed in more detail in chapter 4.

Platinum has been used as a backing material in an attempt to circumvent the problem of fluorine contamination in tantalum backed targets (Ev66a). It was found, however, that radiation from the 350 KeV. state, populated by coulomb excitation, caused severe dead time problems, for proton energies above ~1.8 MeV.

For some of the coincidence measurements in the present work, a target made by evaporating a thin layer of adenine (C$_5$H$_5$N$_5$) onto a water-cooled gold-on-copper backing was used. Beam currents had to be kept low, as the target tended to decompose.

Gas targets have been used (Du51, Du51a) with some success, though they are less convenient to use than solid targets. Nevertheless, for quantitative measurements, a gas target is probably the best choice, as it is now known that the nitrided tantalum targets are unsuitable.
2.3.3 Measurement of TaN target thickness.

Nitrogen-diffused tantalum targets were used for all the singles NaI(Tl) measurements, from which it was desired to obtain absolute cross sections. The distribution of nitrogen within the target was determined, by measuring an excitation function of the 4.433 MeV. radiation from the 429 KeV. resonance (\( \Gamma \sim 0.9 \) KeV.) in \(^{15}\text{N}(p,\gamma)^{12}\text{C}\). The excitation function was extended up to 900 KeV., so that the distribution of nitrogen in the tail could be determined. To do this, it was necessary to subtract out the contribution to the yield from off-resonance reactions. The excitation function for the \(^{15}\text{N}(p,\gamma)^{12}\text{C}\) normalisation is only roughly known.

The yield curve for the target used in obtaining the 2nd and 3rd sets of singles NaI spectra is shown in figure 2.3. In order to determine the number of nitrogen atoms in the target, the yield curve was divided into 5 KeV. intervals, corresponding to the yield of resonance radiation from successive layers within the target, plus non-resonant radiation from all the layers nearer the front of the target.

For each layer, the yield of resonance radiation, \( Y \), was given by

\[
Y = \frac{2\pi^2 \kappa^2 \omega \Gamma \Gamma \alpha}{\Gamma} \frac{1}{E \cdot W \cdot Q} \text{ (15N)}
\]

where \( \frac{2\pi^2 \kappa^2 \omega \Gamma \Gamma \alpha}{\Gamma} \) is the resonance integrated cross section, for which a value of 0.26 KeV. barn was used. Heppard and Bailey (He63) have discussed the corrections involved in obtaining this figure from the data of Schardt et al. (Sc52).
Figure 2.3: Excitation function of the 4.433 MeV. $\gamma$-ray from the $^{15}\text{N}(p,\alpha\gamma)^{12}\text{C}$ reaction, in the nitried tantalum target used in the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ measurements.
\( \varepsilon(^{15}\text{N}) \) is the stopping cross section per \(^{15}\text{N} \) atom in the layer, which is to be determined,

\( E \) is the detector efficiency,

\( W \) is the correction factor to allow for the known \((\text{Kr53})\) angular distribution of radiation from the \(429 \text{ KeV. resonance} \),

\( q \) is the number of protons incident on the target.

The yield, \( Y \), was obtained by reading off the average value from the target yield curve, and applying a correction for the non-resonant yield from all layers in the target nearer the front than the one being considered. Having thus obtained the value of \( \varepsilon(^{15}\text{N}) \) for the layer, the number of nitrogen and tantalum atoms, per unit area within the layer, could be determined, using the known values of the stopping cross sections for nitrogen and tantalum \((\text{Wh58, De62})\), and the known isotopic abundance of \(^{15}\text{N} \) in natural nitrogen \((\text{He63})\). This procedure was carried out for all layers in the target. Unfortunately, the nitrogen distribution curve was not uniquely determined, because of the uncertainty in the absolute normalisation of the non-resonant \(^{15}\text{N}(p,\gamma)^{12}\text{C} \) reaction cross-section curve. The main effect was to make uncertain the extent of the tail.

In the work of Hebbard and Bailey \((\text{He63})\), this problem had been solved as a by-product of the analysis of their \(^{14}\text{N}(p,\gamma)^{15}\text{O} \) spectra at energies between \(400 \text{ and } 1000 \text{ KeV} \). This is discussed in detail in chapter 4, but the method consisted essentially of making a correction for excitation of the \(278 \text{ KeV. resonance} \) in the tail of the target. It was assumed that the deviation of the excitation function, of one of the transitions, from the theoretical excitation function, was due to excitation of the resonance, enabling the shape of the tail to be determined. As the target used in the present measurements was made at the same time and by the
same methods as those used by Hebbard and Bailey, and as the shape of the nitrogen distribution near the front surface of the target was very similar in both cases, it was reasonable to assume that the shape of the nitrogen distribution deep in the target was also very similar in both cases. As the analysis of Hebbard and Bailey had shown that the tail was rather short, and contributed only a small percentage of the total nitrogen concentration, it was apparent that the present uncertainty in the shape of the tail was not important. The target analysis was thus carried out with an absolute normalisation on the cross section of the non-resonant $^{15}N(p,\alpha y)^{12}C$ excitation function, such as to derive a nitrogen distribution similar to that obtained by Hebbard and Bailey.

It is now known that the assumptions used by Hebbard and Bailey in deriving their target nitrogen distribution were unjustified, and that the tail extends deep into the target. Fortunately, this has the effect only of increasing the uncertainty of the results, both for the present measurement and also those of Hebbard and Bailey, and does not make the measurements worthless. This is discussed further in chapter 4.

2.4. Analysis of data:

2.4.1 Singles NaI spectra:

The general principles used in the line-shape analysis have been discussed in chapter 1, but there are a number of specific details to be mentioned.

Data were fitted from just above the full-energy peak of the $^{15}N(p,\alpha y)^{12}C$ 4.433 MeV. gamma ray, to somewhat beyond the full-energy peak of the ground-state radiation from $^{14}N(p,\gamma)^{15}O$. It was not found possible to produce a satisfactory fit over the 4.433 MeV. line, because it was strongly Doppler broadened,
and the broadening increased with beam energy. This made it
difficult to fit adequately over the 5.2 MeV. line, so the best
solution appeared to be to include the 4.433 MeV. line in the
calculation, but to include only the high energy tail in the fit.

The line-shape components used in the fits were those
appropriate to the following transitions:

(1) The ground-state transitions from $^{14}\text{N}(p,\gamma)^{15}\text{O}$,
through the 5.2 MeV. doublet (unseparated), and the 6.18 and
6.789 MeV. states. Convoluted line shapes, corresponding to the
simultaneous detection of both cascade members, were also generated
for these transitions (as described in chapter 1). These were
combined, in the relevant proportions, with the appropriate
single-detection line shapes, to form a single component for each
cascade.

(2) The full energy transition from $^{14}\text{N}(p,\gamma)^{15}\text{O}$ to the
ground state. It was also found necessary to include full-energy
components appropriate to the excitation of lower-energy resonances
in the tail of the target.

(3) In the first set of runs, it was found necessary to
include the full energy transition from $^{13}\text{C}(p,\gamma)^{14}\text{N}$. This was
not found to be necessary in the second and third sets of runs.

(4) The ground state transitions from the 6.129 ,
6.92 and 7.117 MeV. states in $^{16}\text{O}$, excited via the $^{18}\text{F}(p,\alpha\gamma)^{16}\text{O}$
reaction.

Because of the 6.129 MeV. radiation arising from fluorine
contamination, it was not possible to obtain meaningful results
for the 6.18 MeV. transition. The two lines were too close
in energy to be separated by the detector, so that only an
upper limit on the intensity of radiation from the 6.18 MeV.
transition could be obtained. The presence of 6.92 and 7.117 MeV.
radiation was a rather serious matter, because of their strength, the proximity to the 6.789 MeV. line, and because they were strongly Doppler broadened. The errors associated with the ground-state transition from the 6.789 MeV. state were thus appreciably increased. When the analysis of the $^{19}$F(p,γ)$^{16}$O reaction data (described in chapter 5) had been completed, the ratios of the fitted intensities of 7.117/6.129 and 7.117/6.92 MeV. radiation were calculated. These ratios were then compared with the equivalent ratios derived from the analysis of the $^{14}$N(p,γ)$^{15}$O data. The general trend of the 7.112/6.92 ratios were sufficiently similar, especially near a large peak at ~2.05 MeV., to indicate that the contaminant radiation came from $^{19}$F(p,γ)$^{16}$O, as assumed. However, in some regions, the ratios were very different. This was probably mainly due to distribution of fluorine within the body of the target, rather than just at the front surface. For this reason, it was not considered worthwhile to attempt to re-analyse the $^{14}$N(p,γ)$^{15}$O data using a single component for the $^{19}$F(p,γ)$^{16}$O radiation, in the form of the sum of 7.117, 6.92 and 6.129 MeV. line shapes weighted according to the measured proportions.

Figure 2.4 shows the data and fits for the spectra at 1.80, 2.48 and 2.90 MeV.

2.4.2 Ge(Li) spectra:

Spectra obtained at proton energies of 2.51 and 2.60 MeV. are shown in figure 2.5. In these and in other spectra, careful examination of the energy calibration showed that the lines in the 5-7 MeV. range were the 5.241 MeV. member of the doublet, the 6.129 MeV. line from $^{19}$F(p,γ)$^{16}$O rather than the 6.18 MeV. line, and the 6.789 MeV. member of the doublet. The latter result is in agreement with the finding by Warburton et al. (Wa65)
Figure 2.4: Gamma-ray spectra from the $^{14}\text{N}(\text{p},\gamma)^{15}\text{O}$ reaction, using a NaI(Tl) detector, at 1.80, 2.48 and 2.90 MeV. The circles are the data points, and the solid lines are the least-squares fits.
Figure 2.5: Gamma-ray spectra from the \( ^{14}\text{N}(p,\gamma)^{15}\text{O} \) reaction, using a Ge(Li) detector, at 2.51 and 2.60 MeV.
that the 6.857 MeV. level decays exclusively to the 5.241 MeV. level. There was evidence for 6.18 MeV. radiation in all spectra, but the intensity was substantially lower than that of the 6.129 MeV. line.

Calculation of the areas under double-escape peaks enabled a comparison to be made with the results from the analysis of the singles NaI data. This was true also of the full-energy region of these spectra, where separate double-escape peaks could be seen from the 9.487 and 9.613 levels excited in the tail of the nitrogen distribution within the target.

**2.4.3 Coincidence spectra:**

Spectra were taken with both the nitrided tantalum and adenine targets at proton energies of 2.36 and 2.48 MeV., exciting the levels at 9.487 and 9.613 MeV., and also at 2.40 and 2.60 MeV. to examine the underlying contributions. Because of the presence of a component in the spectra arising from coincidences between radiation scattered from one crystal to the other, line-shape analysis of the data at $E_p = 2.36$ and 2.48 MeV. was performed using the spectrum at 2.40 MeV. as a component.

The Compton-scattered coincidence counting rate was rather high when using adenine targets, due to the strong 6-7 MeV. radiation from $^{19}F(p,\gamma)^{16}O$, as well as the 4.433 MeV. radiation from $^{15}N(p,\gamma)^{12}C$. The latter was the main contributing reaction when using nitrided tantalum targets. The fluorine contamination in the adenine targets was much worse than in the nitrided tantalum targets. This is now thought to be due to the use of a fluorine contaminated sand blaster, in cleaning the gold layer before evaporation.

The coincidence data were also analysed without the 2.40 MeV.
spectrum as a component, allowing the relative intensities of higher energy radiation to be determined. Figure 2.6 shows a number of the spectra obtained. The spectrum in figure 2.6(d) is actually a little below resonance, which accounts for much of the difference from the spectrum in figure 2.6(b).

2.4.4 Correction for the nitrogen-distribution tail in the nitrided-tantalum target.

The excitation functions for the ground-state transitions from the full-energy and 5.241 MeV. states were distorted, at proton energies above the narrow resonances at 2.36 and 2.48 MeV. and the broad resonance at 2.35 MeV., due to the excitation of these levels within the tail of the target nitrogen distribution. A reasonable estimate of the correct behaviour of the excitation functions could be obtained from the NaI and Ge(Li) singles spectra, and similar results were obtained in each case.

Line-shape components corresponding to the full-energy transitions from the 2.36 and 2.48 MeV. resonance levels were included in the least-squares analysis of the singles NaI data where appropriate, and it was felt that their combined fitted intensities were a good approximation to the yield of radiation from these resonances and the broad resonance, for proton energies above ~2.6 MeV. The intensity of the line-shape component of energy appropriate to the beam energy, was then taken to be that of the radiation from the region of high nitrogen concentration at the surface of the target. Between ~2.36 MeV. and 2.6 MeV., this technique was not very satisfactory, since the detector energy resolution was not sufficiently good to enable the separation of lines too close in energy. Below 2.36 MeV., contributions from some of the lower-energy resonances were significant, and line shape
Figure 2.6: Coincidence spectra. The solid lines are least squares fits to the data. The spectrum in figure (d) is for an energy just below resonance.
components appropriate to these transitions were included in the fits. Radiation from these resonances could thus also be separated from that due to reactions at the front surface of the target.

In the Ge(Li) spectra, the double-escape peaks of the 2.36 and 2.48 MeV. resonances could be clearly discerned at bombarding energies above these levels, enabling an estimate to be made of their contribution to the total yield of full-energy radiation. The results were in agreement with those derived from the NaI data, and also qualitatively with the results of Evans (Ev66).

Correction of the excitation function of 5.241 MeV. radiation was achieved by using the correction for the full-energy radiation, and taking into account the known branching ratios of the 2.36 and 2.48 MeV. levels.

2.5. Results:

2.5.1 General considerations:

Excitation functions for the full-energy, 5.241, 6.129/6.180 and 6.789 MeV. radiation are shown in figures 2.7 - 2.9. Although the data are not as good as they might have been, they are sufficient for the purpose for which the experiment was initially planned.

The extracted intensities of ground-state cascade radiation derived from analysis of the coincidence data were corrected for detection efficiency and the loss of counts due to the settings of the lower-level discriminators in the single-channel analysers. The results were then normalised to the singles NaI data for the 5.241 MeV. gamma-ray yield at
Figure 2.7: The excitation functions of the transitions to the ground state and 5.241 MeV. level. The curves have been corrected for the tail in the target nitrogen distribution.
Figure 2.8: The excitation function of the transition to the 6.18 Mev. level.
Figure 2.9: The excitation function of the transition to the 6.79 MeV level.
\( E_p = 2.36 \text{ MeV} \). The Ge(Li) data were normalised to the singles NaI data, at an energy of 2.36 MeV., in the full-energy gamma-ray excitation function.

### 2.5.2 Properties of the broad level:

The radiative partial widths, \( \Gamma_y \), for decay of the broad level (with total width \( \Gamma \)) to each final state were obtained by comparing the peak cross section obtained experimentally with that given by a Breit-Wigner resonance curve. At the resonance energy, the angle-integrated cross section is given by:

\[
\sigma = \frac{2}{3} \pi \lambda^2 \cdot \frac{4\pi s}{\Gamma} \sum \frac{l^3 \Gamma_l \Gamma_y}{l^2},
\]

where the factor \( \frac{2}{3} \) is the statistical factor for a target spin 1, projectile spin \( \frac{1}{2} \), compound state spin \( \frac{3}{2} \), and the summation is over all channels populating the level. \( l \) and \( s \) are respectively the orbital angular momentum (0 or 2) and channel spin (\( \frac{1}{2} \) or \( \frac{3}{2} \)) of these channels.

For the full-energy transition, the cross-section at an energy of \( \sim 2.4 \text{ MeV} \) (between the two sharp resonances) should be a reasonable estimate of the peak cross section due to the broad level, and this was taken to be \( 40 \pm 10 \mu\text{barn} \).

Only an upper limit could be set for the transitions through the 5.241, 6.18 and 6.789 MeV. states because of inaccuracies in the data, the unknown intensity of 6.129 MeV. radiation from fluorine, and the unknown contribution from direct processes. These have been estimated as 5, 5 and 10 barn respectively.

Hebbard and Bailey (He63) noted that the results of Duncan and Perry (Du51) for the thick target yield of radiation from the 278 KeV. resonance in \( ^{14}\text{N}(p,\gamma)^{15}\text{O} \), appear to be too high.
(by a factor of $\sim 1.5$), when compared with the work of other authors. The same also applies to their values for the total cross-section factors in the energy range 700 - 1000 KeV. Assuming that a renormalisation should be applied to all their data, the present result for the peak cross-section of the broad resonance agrees well with their value, assuming that the partial widths for other than the ground-state transition are very small. Indeed, there is little doubt that the upper limits proposed are a generous over-estimate. The gamma-ray partial widths obtained, using equation 2.2, were $14 \pm 3$ eV. (full energy), $< 4$ eV. (6.789 MeV.), $< 2$ eV. (6.18 MeV), $< 2$ eV (5.2 MeV.).

Using the Breit-Wigner expression with full energy dependence included, the contribution to the cross-section factor below 1000 KeV. was found to be negligible ($< 0.008$ KeV. barn at 1000 KeV. proton energy), for the transitions through the 6.789, 6.18 and 5.2 MeV. states. The contribution to the yield of full-energy radiation is significant, however ($\sim \frac{1}{4} - \frac{1}{3}$ of the yield found by Hebbard and Bailey (He63)). In calculating the resonance cross-section factors, it was assumed that the transitions were all dipole. In this case, using the long-wavelength approximation, a gamma-ray partial width, $\Gamma_\gamma(E)$, at any energy, $E$, may be expressed as

$$\Gamma_\gamma(E) = \left(\frac{E - E_f}{E_R - E_f}\right)^3 \Gamma_\gamma(E_R)$$  \hspace{1cm} (2.3)

where $\Gamma_\gamma(E_R)$ is the width at the resonance energy, $E_R$, and $E_f$ is the energy of the final state (see chapter 3). This assumption is in doubt only for some of the transitions for which the calculated yield is already negligible. Since the energy term is raised to the power $2L + 1$, in the general case, the non-applicability of this assumption for some transitions would merely have the effect of reducing the yield still further.
2.5.3 Nitrogen distribution in the nitrided tantalum target

The analysis of the singles NaI spectra showed that the yield of full-energy radiation, from the front surface of the target, fell off rapidly above the 2.48 MeV. resonance (see figure 2.7). The corresponding yield of all high-energy radiation may then be regarded as a fairly good indication of the shape of the nitrogen-distribution tail within the target, since the excitation function is effectively the thick-target yield of the three resonances near 2.4 MeV. It is evident that the nitrogen concentration falls off rather slowly within the target, in marked contrast with the experience of Hebbard and Bailey (He63). This matter is discussed more fully in chapter 4, where evidence is presented to show that the target analysis conducted by Hebbard and Bailey is in error.

2.6. Conclusions:

The results obtained from this study are of great importance in the re-assessment of the results of Hebbard and Bailey, concerning the extrapolation of the $^{14}$N(p,γ)$^{15}$O cross-section factor to stellar energies. Indeed, the conclusions previously reached (He63) as to the relative importance of resonant and non-resonant processes, for each transition, have had to be substantially revised.

Intimately connected with this re-assessment, is the question of the usefulness of nitrided tantalum targets. The conclusion reached from the present study, is that they are much less satisfactory than previously thought. It is worth noting that the reason why the presence of a long nitrogen tail would not have been readily deduced from the previous work of Hebbard and Bailey, is that the
full energy transition from the 278 KeV. resonance is very weak. In contrast to that situation, the 2.36 and 2.48 MeV. resonances have very strong ground-state transitions, and the excitation of these resonances was thus clearly evident in the present work.
Chapter 3

THEORY OF DIRECT RADIATIVE CAPTURE
3.1. Introduction:

The calculation of electromagnetic-transition matrix elements may in principle be carried out using perturbation theory. The coupling between the motion (and intrinsic spins) of nuclear particles and the electromagnetic field is weak, so that only the first non-zero term in the perturbation expansion need be considered. However, even though the interaction potential may readily be derived, the form of the nuclear wave functions is not generally known, so that in practice such calculations may be of dubious accuracy.

An exception to this rule is the case where the bulk of the transition matrix element arises from interactions outside the range of the strong nuclear forces. In this situation the form of the initial and final-state functions is well known, allowing an accurate calculation to be performed. The possibility of significant extra-nuclear processes was first suggested by Thomas (Th51), in connection with the radiative capture of thermal neutrons by Li. The reaction mechanism is generally known as direct radiative capture. The incident particle is captured directly into a bound state (but see section 3.6) of the composite nuclear system, without the formation of an intermediate compound nuclear state, and with the simultaneous emission of radiation.

Calculations (Ch61, To61, To63, Ba63, He63) have shown that under certain conditions (low incident-projectile energy, weak binding of the final state), the process takes place predominantly outside the nuclear radius.

In the following sections, expressions are derived for direct capture with the emission of electric multipole radiation. The assumption is made in these expressions that the coupling between
nuclear motion and the electromagnetic field is much stronger outside the nuclear radius than within. Thus the wave functions used are appropriate to a system where only the coulomb field is important. Conversely, when the calculations are performed, this assumption is found to be valid in most cases. The source of radiation is no longer just the nuclear volume, but extends to a considerable distance beyond, necessitating a close examination of the usual assumptions made when calculating electromagnetic matrix elements.

3.2. The general expression for the cross section:

The probability per unit time of emission of electric \(2^L\)-pole radiation (with magnetic quantum number \(M\)), from a nuclear system with an initial wave function, \(\Phi_i\), and a final wave function, \(\Phi_f\), is given by Blatt and Weisskopf (Bl52):

\[
T_{LM}^E = \frac{8\pi(L + 1)}{L[2L + 1]} \frac{E^{2L+1}}{\hbar c} \frac{1}{\hbar} |Q_{LM}|^2,
\]

where \(Q_{LM}\) is the electric multipole moment, which for a system of \(Z\) protons and \(A-Z\) neutrons has the form

\[
Q_{LM} = e \sum_{k=1}^{Z} \int_{\tau} r_k^L \gamma_{LM}^*(\theta_k, \phi_k) \Phi_f^* \Phi_i \, d\tau
\]

\[
-\frac{iE}{\hbar c(L + 1)} \times \frac{e\hbar}{2mc} \sum_{k=1}^{A} \mu_k \int_{\tau} r_k^L \gamma_{LM}^*(\theta_k, \phi_k) \nabla \cdot (\Phi_f^* r_k^L \times \hat{\sigma}_k \Phi_i) \, d\tau,
\]

where the summation is over protons in the first integral, and over all nucleons in the second, and the integration is over all space.
\( Y_{LM} \) is a spherical harmonic,

\( \mu_k \) and \( \sigma_k \) are respectively the magnetic moment (in Bohr magnetons) and Pauli spin operator for the \( k' \)th nucleon,

\( m \) is the nucleon mass,

\( E_\gamma \) is the photon energy.

Equation (3.2) is not exact, and relies on several simplifying approximations, the main one being that the wavelength of the emitted radiation is large compared with the source of radiation \( (k >> r_{\text{max}} \), or \( K_{r_{\gamma}} \) max \( << 1 \), where \( r_{\text{max}} \) is the extent of the source and \( K_{r_{\gamma}} \) is the wave number of the radiation). This is the long wavelength approximation. In the present context, this requirement needs to be investigated more closely, since the source of radiation may be considerably larger than the extent of the nucleus itself.

The second term in equation (3.2) is smaller (by inspection) than the first term by a factor \( \sim K_{r_{\gamma}} \) \( (K_{r_{\gamma}} = E_{\gamma} / \gamma c) \). Thus it seems reasonable to neglect this term altogether in the long wavelength approximation, and this has been done here.

If \( K_{r_{\gamma}} \) \( << 1 \), the spherical bessel function, \( j_{L_{\gamma}}(K_{r_{\gamma}}) \), (which arises in the expression for the vector potential of the electromagnetic field) may be replaced by the expression \( (K_{r_{\gamma}})^{L_{\gamma}} / (2L+1)^{1/2} \). This in turn allows the integrands in (3.2) to be obtained by the use of certain identities involving vector products (see B152).

Finally, if \( K_{r_{\gamma}} \) becomes greater than 1, the multipole expansion does not converge, and a different method of calculation must be used. For a 1 MeV. photon and an extension of 20 fm. (typical values in the present case), \( K_{r_{\gamma}} = 0.1 \).
Other assumptions used in the above expressions are that relativistic effects need not be taken into account, and that meson-exchange currents are not of importance. The latter assumption has been investigated by Sachs (Sa51, Sa53), who found that such currents do not appreciably affect electric multipole transitions, though they may be of importance for magnetic radiation.

Transition matrix elements have been calculated for both E1 and E2 radiation, and as expected, the E1 matrix elements dominate in all cases. Since the current density which gives rise to magnetic radiation is smaller than the charge density (which gives rise to electric radiation) by a factor $v/c$, where $v$ is the speed of the radiating charged particle, it was expected that M1 radiation would be weaker than E1 radiation by a factor of order $(v/c)^2 \sim 0.05$ (Wi60). For this reason, there seemed to be no need to extend the calculations to the more complex problem of magnetic multipole transitions.

Expressions (3.1) and (3.2) may be specialised to give the cross section for the direct-capture process. The $A$ nucleons are considered to be concentrated in a projectile of charge $Z_1 e$ and mass $M_1$, and a target of charge $Z_2 e$ and mass $M_2$, and the transition probability (3.1) is divided by the flux of projectiles interacting with the target, namely the velocity of relative motion, $v$.

\[
\sigma_{LM}^{E} = \frac{1}{v} \frac{T_{LM}^{E}}{\rho} = \frac{8\pi(L + 1)}{L[(2L + 1)!!]^2} \frac{E}{(\hbar c)^2} \frac{1}{nv} |Q_{LM}|^2 ,
\]

where

\[
Q_{LM} = e[Z_1 (\frac{L}{M_1})^{L} + Z_2 (\frac{L}{M_2})^{L}] \int_\tau r L_{IM} (\theta, \phi) \phi_i \phi^*_j d\tau ,
\]
\( \mu \) is the reduced mass,

\((r, \theta, \phi)\) are the relative co-ordinates of the centres of mass of target and projectile. It is assumed that the summations over the intermediate and final states, and the averaging over the initial magnetic substates is included in the expression for \( Q_{LM} \).

3.3. The form of the wave functions:

3.3.1 Derivation of the initial-state wave function:

The initial-state wave function, \( \phi_i \), is asymptotically a superposition of an incoming plane wave plus outgoing spherically scattered waves. The phases of the outgoing waves are shifted relative to that of the incoming wave by the coulomb and nuclear forces. The radial dependence of the wave function is given by a suitable combination of the regular and irregular coulomb wave functions, since the nuclear interaction is assumed to be zero for \( r \) greater than the nuclear radius. The amplitude of the wave function is strongly dependent on the details of the specifically nuclear interaction, since at a resonance in the elastic scattering, the nuclear phase shift normally changes from near 0 to \( \pi \) over an energy region comparable with the width of the resonance. In fact the situation is more complex, since the phase shift must be determined for each value of \( \ell, S, J \), and the energy dependence of the phase shift depends on the partial width of each channel in relation to the total width of the level.

The following derivation is essentially along the lines used in the review article by Iane and Thomas (Ia58). A channel is formally defined by the set of quantum numbers

\[ c \equiv \{\alpha, s, \ell, J, m_J\} \],
where \( \alpha \) defines the pair of interacting particles and their internal modes of motion, and \( s, l, J, m_J \) are the channel spin, orbital angular momentum, total spin and Z-component thereof. In the following, \( m_J \) has been dropped from the list on the understanding that summation over \( m_J \) will be carried out at the appropriate point in the development. This is satisfactory because nothing ultimately depends on \( m_J \). Furthermore, since the parameters associated with \( \alpha \) are assumed fixed, this label will generally not be referred to. Thus as a general rule, a channel \( c \) means the set of quantum numbers \( s, l, J \).

Lane and Thomas ([La58]) define a wave function in the channel \( c, \psi_c \), by

\[
\psi_c = \sum_{m,J}^r \chi_{s,J} \Phi_{l,J} Y_{l,J} \chi_{s,J} \tag{3.5}
\]

where the summation is over values of \( m_J \) such that \( m_l + m_s = m_J \).

\[
\chi_{s,J} = \sum_{m,I,T,I} (I_T, I_P, m_T, m_P | s_J) \Phi_{l,T} \Phi_{l,P} \tag{3.6}
\]

is the channel-spin wave function,

\[
\Phi_{l,T} \quad \text{and} \quad \Phi_{l,P}
\]

are spin wave functions for the target and projectile nuclei respectively,

\[
m_T + m_P = m_s ,
\]

\( Y_{l,J} \) is a Clebsch-Gordon coefficient,

The wave function of the internal structure of the target and projectile nuclei has been omitted, as it is not changed by the interaction.
Two linearly independent solutions to the radial wave equations, corresponding to ingoing and outgoing waves are given by

\[ I_c = (G_c - iF_c)e^{i\omega_c}, \]
\[ O_c = (G_c + iF_c)e^{-i\omega_c} \]

where \( F_c \) and \( G_c \) are respectively the regular and irregular coulomb wave functions, and

\[ \omega_c = \sigma_l - \sigma_0 = \sum_{n=1}^{l} \tan^{-1}(\eta/n), \]

where \( \eta \) is the usual coulomb-field parameter.

Ingoing and outgoing spherical waves of unit flux, \( i_c \) and \( o_c \), are then defined by

\[ i_c = \psi_c I_c v_c^{-\frac{1}{2}}, \]
\[ o_c = \psi_c O_c v_c^{-\frac{1}{2}} \]

The total wave function may then be written as

\[ \Phi_1 = \sum_c (A_c i_c - B_c o_c), \]

where \( B_c \) and \( A_c \) are related by elements of the scattering matrix, \( U_{cc'} \):

\[ B_c = \sum_{c'} U_{cc'} A_{c'}, \]
where $c$ and $c'$ must have the same $J$ and $m_J$, since these are conserved in any reaction, and

$$A_c = i(k_c)^{-1}(\pi v_c)^{\frac{1}{2}}(\ell_{\text{som}} | Jm)(2\ell + 1)^{\frac{1}{2}}, \quad (3.11)$$

where $k_c$ is the channel wave number.

The total wave equation may now be written, using equations (3.5)-(3.11). However, the expression is rather involved, and it is desirable to make the assumption that the scattering matrix is diagonal, and hence that only one channel exists for the formation of each resonance level in the compound system. In this case, one obtains the result

$$\phi_i = \sum_c A_c (i_c - U_{cc} c_c)$$

$$= \sum_c i \frac{[\pi(2\ell+1)]^{\frac{1}{2}}}{k_c r} \{(\ell_{\text{som}} | Jm)$$

$$\times \sum_{m',m''} (\ell_{m',m''} | Jm)i^{\ell} Y_\ell \chi_{m'} \chi_{m''}$$

$$\times \{[G_c(\eta,\rho) - iF_c(\eta,\rho)]e^{i\omega} - U_{cc}[G_c(\eta,\rho)$$

$$+ iF_c(\eta,\rho)]e^{-i\omega}\}, \quad (3.12)$$

where $\rho = k_c r$.

As it is assumed that only pure elastic scattering can take place ($c = c'$), the scattering-matrix elements may be represented as phase factors:

$$U_{cc} = e^{i(\omega - \Phi_c)} e^{2i\beta} \quad (3.13)$$
where $\varphi_c$ is the hard-sphere phase evaluated at the nuclear channel radius, and $\beta_c$ is the nuclear phase. All the phases are real. Equations (3.12) and (3.13) then give the required expression for $\Phi_i$:

$$
\Phi_i^{sm} = \sum_{l,j} \frac{[4\pi(2l+1)]^{\frac{1}{2}}}{kr} (lsom | Jm) \\
\times \sum_{m',m''} (lsm'_{l}m'_{s} | Jm) i^l Y_{lm} m'_{s} (\chi_{sm'}) \\
i(\omega + \delta_c) \\
\times \{e^{-i\delta_c} (F_c(\eta,\rho)\cos\delta_c + G_c(\eta,\rho)\sin\delta_c)\},
$$

(3.14)

where $\delta_c = \beta_c - \varphi_c$.

The quantities $l, s, m, J, c$ all refer to the initial state of the direct-capture transition. The summation over $s$ and $m'$ is included at a later stage.

3.3.2 Phase shifts:

The nuclear phase shifts, $\beta_c$, which enter into equation (3.14), may be represented in terms of the energy eigenvalues and reduced widths of levels in the compound system. If only one level (labelled by $\lambda$), and one channel contribute to the scattering, then (La58)

$$
e^{2i\beta_c} = 1 + \frac{i\Gamma_\lambda}{(E_\lambda + \Delta_\lambda - E) - \frac{1}{2}i\Gamma_\lambda},
$$

where $\Gamma_\lambda$ is the total level width, and $\Delta_\lambda$ is the usual shift factor.
Then
\[
\tan \beta_c = \frac{\frac{1}{2} \Gamma_{\lambda}}{E_{\lambda} + \Delta_{\lambda} - E} \tag{3.15}
\]

Mitchell (Mi62) has shown that for many levels (one channel), the total resonant phase shift is given by the expression

\[
\tan \beta_c = \frac{P_c}{\left( \sum_{\lambda} \frac{\gamma_{\lambda c}}{E_{\lambda} - E} \right) - S_c} \tag{3.16}
\]

where \( S_c \) and \( P_c \) are the usual real and imaginary parts of the logarithmic derivative of the outgoing wave in channel \( c \).

If more than one channel (different \( \ell \) and \( s \)) can contribute to the scattering from a resonance level, the situation becomes much more complex. Off-diagonal elements of the scattering matrix are then present, equation (3.14) no longer holds as a result of this, and the scattering-matrix elements can evidently no longer be represented in terms of real phase shifts. The diagonal elements may be represented by complex phase shifts, however, which simply means that the modulus of the matrix element is no longer unity.

For one level, a diagonal matrix element may be written (La58) as:

\[
U_{cc} = e^{2i(\omega - \varphi_c)} \left( 1 + \frac{i \Gamma_{\lambda c}}{(E_{\lambda} + \Delta_{\lambda} - E) - \frac{1}{2}i \Gamma_{\lambda}} \right) \tag{3.17}
\]

where \( \Gamma_{\lambda c} \) (\( \not= \Gamma_{\lambda} \)) is the partial width for channel \( c \).

Putting \( \beta_c \equiv \xi_c + i \zeta_c \) (\( \xi_c, \zeta_c \) real), one obtains

\[
e^{2i\beta_c} = e^{2i\xi_c} e^{-2i\zeta_c} = 1 + \frac{i \Gamma_{\lambda c}}{(E_{\lambda} + \Delta_{\lambda} - E) - \frac{1}{2}i \Gamma_{\lambda}}.
\]
By equating real and imaginary parts, it may then be shown that

$$\tan 2\xi_c = \frac{(E \lambda + \Delta \lambda - E)\Gamma_{\lambda c}}{(E \lambda + \Delta \lambda - E)^2 + \frac{1}{2} \Gamma_{\lambda} (\frac{1}{2} \Gamma_{\lambda} - \Gamma_{\lambda c})^2}, \quad (3.18)$$

and

$$-2\xi_c = \frac{[(E \lambda + \Delta \lambda - E)^2 + \frac{1}{2} \Gamma_{\lambda} (\frac{1}{2} \Gamma_{\lambda} - \Gamma_{\lambda c})^2 + (E \lambda + \Delta \lambda - E)\Gamma_{\lambda c}]^{\frac{1}{2}}}{(E \lambda + \Delta \lambda - E)^2 + (\frac{1}{2} \Gamma_{\lambda})^2} \quad (3.19)$$

The behaviour of $\xi_c$ and $-2\xi_c$ is rather interesting, and is shown in figure (3.1), in which $a = \Gamma_{\lambda c}/\Gamma_{\lambda}$, and $R = [E - (E \lambda + \Delta \lambda)]/\Gamma_{\lambda}$. For values of $a > 0.5$, the real phase shift, $\xi_c$, behaves in the normal way, changing through $\pi/2$ at the resonance energy ($R = 0$). For $a < 0.5$, however, the behaviour is quite different, and the amplitude of the phase-shift anomaly decreases with decreasing $a$, becoming zero for $a = 0$. The factor $e^{-2\xi_c}$ acts as an attenuation factor.

It can therefore be seen that the contribution to the total initial wave function, from pure elastic scattering ($c = c'$) in a particular channel from a resonance level, approaches zero as the partial width approaches zero. To evaluate the complete effect of several channels, one would have to go back to equation (3.10) and proceed in all generality, but the above analysis indicates (as expected) that, provided the reduced width of a particular channel is small, its effect on the total wave function may be neglected.

In the experimental situation in hand ($^{14}\text{N} + p$), two channel spins ($\frac{1}{2}$ and $\frac{3}{2}$), and in general two $l$-values, are possible for the formation of each level in $^{15}\text{O}$, allowing up to 4 energy-elastic channels to contribute. Photon channels will have a negligible effect on the phase shifts, since the radiative widths are much less than the proton widths. No other
Figure 3.1: Behaviour of the real and imaginary parts of the complex phase-shift, which represents a diagonal element of the scattering matrix when the partial width for the channel is less than the full width.
channels are open up to \( \sim 2.48 \text{ MeV} \) proton energy. Although the details vary from level to level, it was found, in all cases, that the partial width for one particular channel was much larger than the others (especially at lower energies). It was thus possible to treat each level as if it were populated via one channel only.

Thus the total phase shift for each channel was described adequately by equation (3.16) (the summation being over those levels populated through the channel in question), with equation (3.14) giving the total wave function.

### 3.3.3 Final-state wave function:

The radial wave function in the external region, for a bound state, is a purely outgoing wave, and the only acceptable solution to the wave equation is the function which vanishes at infinity. This is proportional to the Whittaker function, \( W_{\eta',\ell}(Kr) \), defined by

\[
W_{\eta',\ell}(Kr) = \frac{(2Kr)^{-\eta'}e^{-Kr}}{\Gamma(\ell + \eta' + 1)} \int_{0}^{\infty} t^{\ell+\eta'}e^{-\eta'(1 + \frac{t}{2Kr})}dt ,
\]

where

\[
\eta' = \frac{Z_1Z_2e^{2\mu}}{\hbar^2K},
\]

\[
K = \frac{[2\mu E_B]^{1/2}}{\hbar}
\]

\( E_B \) is the binding energy of the state, and

\( \Gamma(\quad) \) is the gamma function.
Lane and Thomas (La58) have considered the problem of the correct normalisation of this wave function in terms of the dimensionless reduced width

$$
\theta_c^2 = \left( \frac{M^2\rho^2}{\pi^2} \right) \gamma_c^2,
$$

where $\gamma_c^2$ is the reduced width, and $\rho$ is the nuclear radius.

On setting

$$
u(r) = C \left[ \frac{1}{\eta'_{\ell}}(K) \right],
$$

where $r^{-1}u(r)$ is the correct radial wave function in the external region, and $C$ is the correct normalising factor, one obtains (from p. 280 in La58)

$$
C^2 = \frac{\theta_c^2}{\frac{1}{2} a \eta'_{\ell}(K) + \theta_c^2 \int_{a}^{\infty} \frac{x^2}{\eta'_{\ell}(K)} x \, dx} \tag{3.21}
$$

In practice, it is found that the integral term in (3.21) is somewhat less than the first term. On ignoring this term, one obtains the same normalisation as used by Tombrello et al. (To63), and Christy and Duck (ch61).

The total wave function for the final state (for a given $\ell, S, J, m_J$) may then be written as

$$
\phi_f^{\ell, S, J, M_J} = \left[ \frac{\theta_c^2}{\frac{1}{2} a \eta'_{\ell}(K) + \theta_c^2 \int_{a}^{\infty} \frac{x^2}{\eta'_{\ell}(K)} x \, dx} \right]^{\frac{1}{2}} \times \frac{W_{\eta'_{\ell}}(K)}{r}
$$

$$
\times \sum_{m_{\ell}, m''_{S}} \left( \ell S_{\ell S} m_{\ell S} | m_{J} \right) Y_{\ell m_{\ell}} \left( \theta, \phi \right) \chi_{m''_{S}} \tag{3.22}
$$
The quantum numbers \( \ell, s, J, m_J, c \) all refer to the final state, and hence are (in general) different from the analogous quantities in equation (3.14).

3.4. Detailed expression for the cross section:

Using equations (3.3), (3.4), (3.14) and (3.22), the total cross section may now be derived on performing the necessary integration and summation. As mentioned earlier, it is assumed that the channel spin remains constant during the interaction, so that \( S_i = S_f = S \).

For a given \( S \) and \( m \) in the initial state, and a given \( \ell, s, J, m_J \) in the final state, equation (3.4) becomes

\[
Q^0_{LM} = e[Z_1(\frac{\mu}{M_1})^L + Z_2(\frac{\mu}{M_2})^L] \int r^L Y^*_{LM}(\theta, \phi) \\
\times \left\{ \left[ \frac{\theta^2_{\ell_f S J_f}}{\frac{1}{2} a W_s^2 (Kr)} + \theta^2_{\ell_f S J_f} \int_0^\infty W_s^2 (Kr) dr \right] \frac{1}{r} \right\} W_{\eta', \ell_f} (K r) \\
\times \sum_{m_s} \left\{ (\ell_1 S m_s m'' | J m_{f_s}) Y_{\ell_1 m_1} (\theta, \phi) \chi_{m_s} \right\} \\
\times \sum_{m_{s'}} \left[ 6\pi(2\ell_1 + 1) \right]^{\frac{1}{2}} (\ell_1 S m_{s'} | J_{1 s'}) \sum_{m_{s'}} (\ell_1 S m' m''_s | J_{1 m'_{s'}}) \\
\times l_{i_1} l_{i_1}^* c_i \chi_{m_{s'}} e^{i(\omega_{c_s} + \delta_{c_i})} \\
\times [F_{\ell_1}(\eta, \rho) \cos \delta_{c_i} + G_{\ell_1}(\eta, \rho) \sin \delta_{c_i}] \right\} d\tau \quad (3.23)
\]
where $\delta_{c_1} = \beta - \varphi_{c_1}$ is the total phase shift in the incident channel. The superscript on $Q_{LM}$ denotes that the various summations have yet to be performed.

As only the total cross section is required, one may now integrate over $\theta$ and $\varphi$ and sum over $M$ in order to obtain $\sum_M |Q_{LM}|^2$ as desired. At the same time, an average over the initial substates, and a summation over the final substates are performed. It is also necessary to make explicit recognition of the fact that the phase shifts in the initial-state wave function are dependent on $S$, $l_1$ and $J_1$. One then obtains (C61) for the direct capture cross section:

$$\sigma_L = \frac{8\pi(L + 1)}{L[(2L + 1)]^2} \frac{E c_{2L+1}}{(\hbar c)} \frac{1}{h \nu} e^{2} \left[ \frac{Z_{1}}{M_{1}} \frac{L}{L + \frac{1}{2}} + \frac{Z_{2}}{M_{2}} \left( \frac{L}{L + \frac{1}{2}} \right) \right]^2$$

$$\times \frac{(2L + 1)(2J_{f} + 1)(2l_{f} + 1)}{(2I_{T} + 1)(2I_{P} + 1)}$$

$$\times \sum_{s} \left[ \frac{\sigma_{sll_{f}J_{f}}^{2}}{\frac{1}{2} \omega_{l_{f}J_{f}}(K) + \sigma_{sll_{f}J_{f}}^{2} \int_{a}^{\infty} \omega_{l_{f}J_{f}}(K) dr} \right]^{1/2}$$

$$\times (l_{f}l_{100} | l_{10})^{2} \sum_{J_1} (2J_{1} + 1) \left\{ \frac{L_{f} l_{f} l_{1}}{s J_{1} J_{f}} |R_{sll_{f}J_{f}}^{L}|^2 \right\}$$

where
\[
\mathcal{R}_{s l_1 J_i}^L = \int_{a}^{\infty} \frac{r}{k} W_0 (K_r) (F_{l_i} (\eta, \rho) \cos \delta_{s l_1 J_i} + G_{l_i} (\eta, \rho) \sin \delta_{s l_1 J_i}) \, dr,
\]

(3.25)

\{ \} is a 6-j coefficient.

The integrals \(\mathcal{R}_{s l_1 J_i}^L\) must be evaluated separately for each value of \(s, l_i, J_i\), corresponding to the different phase shifts in each channel.

The expression (3.24) is the cross section for capture of a particle with orbital angular momentum, \(l_i\), into a bound state with orbital angular momentum \(l_f\) and total spin \(J_f\), with the emission of electric 2\(^{-}\)pole radiation.

3.5. Contribution from the nuclear interior:

The above calculations are only reliable in the external region where the wave functions are well known. However, a rough estimate of the contribution from the internal region can be made, which serves to indicate how reliable the overall calculations are (apart from the uncertainty associated with the reduced widths of the final states).

Specifically, (3.25) was altered by replacing \(\mathcal{R}_L\) by \(\mathcal{R}^L + i^L\), where \(i^L\) is given by

\[
i^L = \int_{0}^{a} \frac{r}{k} \psi_i \psi_f \, dr,
\]

(3.27)

where \(r^{-1} \psi_i\) and \(r^{-1} \psi_f\) are the initial and final-state radial wave functions for the internal region. The simplest approach (and the one which was used here), is to approximate the nuclear
potential by a square well. In this case, the regular solution to the wave equation is the spherical bessel function, 

\[ A_{j}^{L}(k'r) \] (where \( A \) is a normalising coefficient), and the value and first derivative of the wave functions in the external and internal regions are required to be equal.

The discrete ambiguity in the value of \( k' \) was resolved (for the final bound state), by requiring that the wave function, \( \Psi_{f} \), have the correct number of nodes, corresponding to one less than the principal quantum number of the orbital angular momentum shell to which the extra particle in the bound state is added. It was assumed that the bound states may be described by configurations with the captured particle in a 2s, 1p, 1d or 1f orbit around the \( 1^n \text{N} \) ground-state core. For the wave function \( \Psi_{i} \), it was assumed that the same criteria applied (by a continuity argument), though the principal quantum number has no meaning for continuum states.

The reliability of the overall calculation was assessed by calculating

\[ |R^{L}|^2 / |R^{L} + I^{L}|^2 . \]

When this quantity was near 1, the contribution from the interior region was small, and the calculation was expected to be reliable.

3.6 Direct capture to unbound final states:

As has already been stated, direct capture may occur to a state which is particle unbound (just as may a radiative transition from a compound nucleus state), but because the particle widths for an unbound state are usually much larger than the radiative
widths, subsequent gamma radiation of a significant strength would not be observed. Such a reaction would thus be a type of inelastic scattering (in the present case, \( ^{14}\text{N}(p,\gamma)^{15}\text{N} \)).

Although this reaction has a negligible direct effect on the production of \(^{15}\text{O}\) nuclei for stellar reactions, inelastic scattering via direct capture to an unbound state nevertheless could have an indirect effect. This is because the initial wave function for direct capture is altered via the phase shifts introduced into the outgoing waves associated with each incoming partial wave. It is thus important to see what order of magnitude might be expected for the cross section for this reaction.

In gamma-ray spectra from the proton bombardment of carbon targets, Woodbury et al. (Wo 54) noted the presence of a weak low-energy component whose energy was a linear function of the beam energy. It was deduced that the radiation arose from a direct capture reaction in which the final state was the 2.369 MeV. state in \(^{13}\text{N}\), which is unbound for proton emission. Spectra were collected at 0° and 90° to the beam axis, the results being consistent with zero yield at 0°, which is inconsistent with the formation of a compound-nucleus state. It was thus thought probable that the angular distribution was of the form \( \omega(\theta) \sim \sin^2 \theta \), which is appropriate to a p to s direct capture transition.

The differential cross section at 90° was found to be \( \sim 0.2 \mu \text{b/steradian} \) at a proton energy around 2 MeV., giving a total cross section of \( \sim 1.6 \mu \text{b} \) (on the basis of a \( \sin^2 \) angular distribution). This is the order of magnitude expected for a direct-capture transition to a bound state, so that the effect on the elastic-scattering phases, and hence indirectly on the direct capture to bound states may reasonably be expected to be negligible.
Examination of the expressions relevant to the calculation of direct-capture cross sections, in the case of unbound final states, shows that the present model is inadequate for the purpose. The final-state wave function is no longer the exponentially-decaying Whittaker function (which is responsible for the well-defined region of integration in the calculation of the matrix element). Instead, the wave function is of a similar nature to the initial-state wave function, and consists of a plane wave plus ingoing spherically scattered waves (see, for example, To61a). Thus the range of integration is no longer restricted so as to ensure the validity of the long wavelength approximation. This being the case, none of the expressions used in this work are valid. Indeed, the multipole expansion no longer converges, indicating that a totally different approach is necessary.
Chapter 4

THE DIRECT-CAPTURE CALCULATIONS, AND THE EXTRAPOLATION TO STELLAR ENERGIES OF THE $^{14}\text{N}(p,\gamma)^{15}\text{O}$ REACTION CROSS-SECTION FACTORS.
4.1. Introduction:

The aim in this chapter is to achieve a synthesis of three sets of information relating to the extrapolation of the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ reaction to stellar energies. These are:

a) The data measured in the present work, which pertains to the gamma-ray branching ratios of the broad 9.49 MeV. level in $^{15}\text{O}$.

b) The results of the present calculations of cross-section factors for the direct capture of protons by $^{14}\text{N}$.

c) The cross-section measurements of Hebbard and Bailey (He63, Ba63), in the proton energy range 400 - 1000 KeV.

These data have been completely re-assessed in this chapter, since certain information is now available which casts doubt on some aspects of both the measurements and analysis. In particular, the question of the extent of the tail in the nitrogen distribution of the nitrided tantalum targets is of the greatest relevance to these considerations.

All energies referred to in this chapter are in the laboratory system, except for excitation energies. The calculations of direct capture and resonance cross sections have been performed in the centre-of-mass system, however, and the results have simply been plotted as a function of laboratory energy for comparison with experimental results. The cross-sections have been converted to cross-section factors, as defined in the introduction.

A figure of 20 KeV. has been assumed as representative of the range of energies of importance in stellar reactions (Sc58). Whenever an energy of 20 KeV. is quoted, it is intended to have this meaning.
4.2. The direct-capture calculations:

4.2.1 Angular momentum considerations:

The relevant equations determining the allowed values of the various angular momenta are:

\[
S = I_T + I_p, \quad (4.1)
\]

\[
J_1 = S + \ell_1, \quad (4.2)
\]

\[
J_1 = L + J_f, \quad (4.3)
\]

\[
J_f = S + \ell_f, \quad (4.4)
\]

where the various symbols are as defined in chapter 3, and the initial and final channel spins are assumed equal. The angular momenta are all vectors and are added as such.

Possible values of \( S \) are \( \frac{1}{2} \) and \( \frac{3}{2} \) in the present case, and \( \ell_1 \) has been allowed to take the values 0, 1, 2 and 3. \( L \) is equal to 1 or 2.

The equations above determine that for \( L = 1 \), \( |\ell_1 - \ell_f| = 1 \), and for \( L = 2 \), \( |\ell_1 - \ell_f| = 0 \) or 2.

4.2.2 Reduced widths:

The reduced widths for the bound states were taken from table 2.1. Analysis of stripping data does not distinguish between channel spins in the final state, so that there exists an ambiguity for the present calculations. It was found, however, that the calculated cross-section factors were generally not sensitive to the assumptions made regarding the distribution of the reduced width between the two channel spins. The two exceptions to this rule are discussed in a later section.
As discussed in chapter 3, the initial-state wave function for the direct-capture process is modified by the phase shifts in each channel, which are determined by the elastic scattering from both bound and unbound levels. Of the unbound levels, the only one which is definitely known to be populated via more than one channel is the 9.49 MeV. level. However the partial width in the channel with \( l = 0 \) and \( s = \frac{3}{2} \) is > 85% of the total width, for energies less than 1000 KeV. (see table 2.2), and only this channel was considered. The other unbound states were either found to have a negligible effect on the initial-state wave function, or were known to be populated essentially through only one channel.

4.2.3 Results:

Table 4.1 gives a list of the parameters considered in the present calculations. Only a small number of these calculations yielded a significant cross-section factor. In particular, transitions with \( l_1 = 3 \) were negligible, and in general quadrupole transitions were weak, though not necessarily negligible.

The individual transitions are discussed in greater detail in a later section, but several broad features may be noted here. Insofar as the reliability of the calculations is affected by the contribution to the radial integrals from the interior region, the most satisfactory calculations were those for high \( l_1 \), low \( l_f \), high \( L \), and low binding energy of the final state. These factors influence the position of peak of the radial integrand—the further out this is, the less the contribution from the interior region. From this point of view, the ground-state calculations were the least reliable.

The other important factor is the relative magnitudes of the contributions of the regular and irregular coulomb wave functions
### Table 4.1: Allowed direct-capture transitions

<table>
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<th>Final State</th>
<th>$j_f$</th>
<th>$l$</th>
<th>$s$</th>
<th>$l_1$</th>
<th>$l_f$</th>
<th>States contributing to phase shifts in $\Phi_1$</th>
<th>$j_1$</th>
<th>Possible values of $j_1$</th>
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<td>1</td>
<td>$\frac{1}{2}$</td>
<td>0</td>
<td>1</td>
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<td>8.749</td>
<td>$\frac{1}{2}$</td>
</tr>
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<td>$\frac{1}{2}$</td>
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<td>1</td>
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<td></td>
<td></td>
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<td>$\frac{1}{2}$</td>
<td>0</td>
<td>1</td>
<td>8.283, 9.490</td>
<td>6.789</td>
<td></td>
</tr>
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<td>$\frac{1}{2}$</td>
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<td>1</td>
<td></td>
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<td></td>
</tr>
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<td>2</td>
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<td>S</td>
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<td>$l_f$</td>
<td>States contributing to phase shifts in $\phi_i$</td>
<td>$j_1$</td>
<td>Possible values of $j_1$</td>
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<td>1</td>
<td>7.552</td>
<td>-</td>
<td>$\frac{1}{2}$</td>
</tr>
<tr>
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<td>$^-_{\pi}$</td>
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<td>2</td>
<td>1</td>
<td>8.749</td>
<td>-</td>
<td>$\frac{1}{2}$</td>
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<tr>
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<td>$^-_{\pi}$</td>
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<td>$\frac{1}{2}$</td>
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<td>8.283</td>
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<tr>
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<td>$\frac{1}{2}$</td>
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<td>1</td>
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<td>5.241</td>
<td>-</td>
</tr>
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<td>2</td>
<td>$\frac{1}{2}$</td>
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<td>$^2$, $^3$</td>
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<td>$^-_{\pi}$</td>
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<td>$^2$, $^3$</td>
</tr>
<tr>
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<td>$^+_{\pi}$</td>
<td>1</td>
<td>$\frac{1}{2}$</td>
<td>1</td>
<td>0</td>
<td>-</td>
<td>-</td>
<td>$\frac{1}{2}$, $^3$</td>
</tr>
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<td>( J_f^\pi )</td>
<td>( L )</td>
<td>( S )</td>
<td>( \ell_1 )</td>
<td>( \ell_f )</td>
<td>States contributing to phase shifts in ( \Phi_i )</td>
<td>( J_1 )</td>
<td>Possible values of ( J_1 )</td>
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<td>-------------</td>
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<td>1 * *</td>
<td>1</td>
<td>2</td>
<td>6.180</td>
<td>*</td>
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</tr>
<tr>
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<td>1 * *</td>
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</tr>
<tr>
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<td>6.857 *</td>
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<tr>
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<td>2</td>
<td>7.552</td>
<td>8.749</td>
<td>( \frac{1}{2} )</td>
<td>( \frac{1}{2} )</td>
<td></td>
</tr>
<tr>
<td>6.857 *</td>
<td>2 * *</td>
<td>2 * *</td>
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<tr>
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<td>2 * *</td>
<td>0 * *</td>
<td>2 * *</td>
<td>8.283</td>
<td>9.490</td>
<td>6.789</td>
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</tr>
<tr>
<td>6.857 *</td>
<td>2 * *</td>
<td>2 * *</td>
<td>2 * *</td>
<td>6.857</td>
<td>5.241</td>
<td>7.276</td>
<td>( \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} )</td>
<td></td>
</tr>
<tr>
<td>7.276 *</td>
<td>1 * *</td>
<td>1 * *</td>
<td>1</td>
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<td>*</td>
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</tr>
<tr>
<td>7.276 *</td>
<td>1 * *</td>
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<td>2</td>
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<td></td>
<td></td>
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<tr>
<td>7.276 *</td>
<td>2 * *</td>
<td>0 * *</td>
<td>2 * *</td>
<td>8.283</td>
<td>9.490</td>
<td>6.789</td>
<td>*</td>
<td></td>
</tr>
<tr>
<td>7.276 *</td>
<td>2 * *</td>
<td>2 * *</td>
<td>2 * *</td>
<td>6.857</td>
<td>5.241</td>
<td>7.276</td>
<td>( \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2} )</td>
<td></td>
</tr>
</tbody>
</table>
to the initial-state wave function at the nuclear radius. This is determined by the elastic-scattering phase shifts in each channel. If only the hard-sphere phase is present, the total wave function in that channel is zero at the nuclear radius. Since the irregular function falls off rapidly outside the nuclear radius, this guarantees that the peak of the radial integrand will be outside the nucleus. Where the elastic scattering from the 7.552 MeV. level (and at some energies, the 8.283 MeV. level) contributed to the phase shift, the peak was found to occur inside the nuclear radius, and these calculations were rejected.

Figure 4.1 shows the form of the radial integrand for various angular momenta and final states. It is interesting to note that the source of radiation may extend to a very great distance from the nuclear radius.

4.3. The effect of resonances on the gamma-ray yield, in the proton energy range 20 - 1000 KeV.

The discovery of a new level in $^{150}$ (Wa65, Al66, He67) at an energy of $7.276 \pm 0.0006$ MeV. raised the possibility of a significant contribution to the $^{14}_N(p,\gamma)^{150}$ reaction at stellar energies. This level is only 17 KeV. below the proton binding energy.

Alburger et al. (Al66) have measured the mean lifetime, and obtained the result $(1.25 \pm 0.3) \times 10^{-12}$ sec. This implies a total gamma-ray partial width of $(5.3 \pm 1.4) \times 10^{-4}$ eV. The spin is known to be $7/2$, with formation via the channel with $\ell = 2$ and $s = \frac{3}{2}$.

Using the normal Breit-Wigner resonance formula, and assuming a proton reduced width equal to the Wigner limit (2.9 MeV.), the cross-section factor at 20 KeV. was found to be $\sim 1.4 \times 10^{-3}$ KeV. barn.
Figure 4.1: A selection of radial integrands for direct-capture matrix elements, for various final states and values of angular momenta.
The effect of this level on stellar reactions is thus negligible.

The 278 KeV. and 1061 KeV. resonances have a very strong effect over most of the energy region from 20 - 1000 KeV. The more important of these is the 278 KeV. resonance, because of the necessity of taking account of its effect in extrapolating to low energies. It was also found to be excited deep in the target, even at the highest energies considered in this work.

The parameters for these two resonances have been taken from the work of Hebbard and Bailey (He63), and are listed in table 4.2.

Table 4.2: Partial widths of the 278 and 1061 KeV. resonances

<table>
<thead>
<tr>
<th>Resonance Energy</th>
<th>Radiative widths ( \Gamma_p ) (eV.) for transitions to states at</th>
</tr>
</thead>
<tbody>
<tr>
<td>(MeV. ) (KeV.)</td>
<td>0.0 MeV. 5.191 MeV. 5.241 MeV. 6.180 MeV. 6.789 MeV.</td>
</tr>
<tr>
<td>278 1.7</td>
<td>1.53x10^{-3} 6.84x10^{-3} —</td>
</tr>
<tr>
<td>1061 3.0</td>
<td>0.32 —</td>
</tr>
</tbody>
</table>

The broad 2.35 MeV. resonance, which was discussed in chapter 2, was found to have a gamma-ray width of 14 ± 3 eV. for the ground-state transition. The upper limits assigned to the widths for transitions to the other bound states were found to be too small to allow a significant yield from this source. No other levels can have a significant effect on the gamma-ray yield in the energy region under consideration (He63).
4.4. The experimental results and analysis of Hebbard and Bailey:

For ease of reference, the cross-section factors obtained for the transitions to the 6.789 MeV. and 6.18 MeV. levels, the doublet at 5.2 MeV. and the ground state, by Bailey (Ba63) are listed in Table 4.3.

Table 4.3: Experimental data of Hebbard and Bailey (He63).

<table>
<thead>
<tr>
<th>Incident energy (KeV.)</th>
<th>mean energy within target (KeV.)</th>
<th>ground state</th>
<th>5.2 MeV. levels</th>
<th>6.18 MeV. level</th>
<th>6.789 MeV. level</th>
</tr>
</thead>
<tbody>
<tr>
<td>400</td>
<td>370</td>
<td>0.664</td>
<td>2.064</td>
<td>6.75</td>
<td>3.365</td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 0.054</td>
<td>± 0.096</td>
<td>± 0.090</td>
<td>± 0.072</td>
</tr>
<tr>
<td>500</td>
<td>475</td>
<td>0.175</td>
<td>0.461</td>
<td>1.525</td>
<td>1.296</td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 0.072</td>
<td>± 0.123</td>
<td>± 0.141</td>
<td>± 0.115</td>
</tr>
<tr>
<td>600</td>
<td>575</td>
<td>0.228</td>
<td>0.310</td>
<td>0.870</td>
<td>1.100</td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 0.033</td>
<td>± 0.037</td>
<td>± 0.033</td>
<td>± 0.033</td>
</tr>
<tr>
<td>700</td>
<td>680</td>
<td>0.390</td>
<td>0.240</td>
<td>0.694</td>
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</tr>
<tr>
<td></td>
<td></td>
<td>± 0.027</td>
<td>± 0.037</td>
<td>± 0.037</td>
<td>± 0.032</td>
</tr>
<tr>
<td>800</td>
<td>780</td>
<td>0.315</td>
<td>0.157</td>
<td>0.336</td>
<td>0.959</td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 0.014</td>
<td>± 0.026</td>
<td>± 0.031</td>
<td>± 0.026</td>
</tr>
<tr>
<td>900</td>
<td>880</td>
<td>0.316</td>
<td>0.149</td>
<td>0.340</td>
<td>0.928</td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 0.016</td>
<td>± 0.025</td>
<td>± 0.024</td>
<td>± 0.028</td>
</tr>
<tr>
<td>1000</td>
<td>980</td>
<td>0.451</td>
<td>0.210</td>
<td>0.257</td>
<td>0.817</td>
</tr>
<tr>
<td></td>
<td></td>
<td>± 0.012</td>
<td>± 0.018</td>
<td>± 0.016</td>
<td>± 0.018</td>
</tr>
</tbody>
</table>
The errors quoted by Bailey are those derived from the linear least-squares analysis of the spectra.

In order to allow for the presence of radiation from the 278 KeV. resonance excited deep in the target, Hebbard and Bailey used the following procedure. The cross-section factor for direct capture to the 6.789 MeV. level (with $l_1 = 1$, $l_2 = 0$, $L = 1$, $S = \frac{3}{2}$) was calculated, and normalised to the experimental data above 500 KeV. The contribution from the tail of the 278 KeV. resonance level, excited in the front nitrogen-dense layer of the target, was included in the fit, using the parameters listed in table 4.2.

They assumed that the excess yield (see figure 5 in reference He63) at bombarding energies of 400 and 500 KeV. was due to the 278 KeV. resonance being excited in the tail of the nitrogen distribution in the target. They were thus able to calculate the shape of this tail (see figure 4 in reference He63). Using the differences between the measured cross-section factors and those predicted by the fitted curve, and the known gamma-ray partial widths of the 278 KeV. resonance, they then calculated the corrections for the other transitions (see figures 6-8 in reference He63). They argued that because their corrected 6.789 MeV. data was well fitted by the theoretical curve, and the excitation functions for the other transitions were smoothly varying after the corrections had been applied, there was negligible excitation of the 278 KeV. resonance above 500 KeV. This assumption implies that the nitrogen-distribution tail of the target extended only a short distance into the target.

On the other hand, other data of Hebbard and Bailey tend to refute this latter assumption. In a previous paper (Ba63a), Hebbard and Bailey measured the angular distribution, at a proton energy of 800 KeV., of the direct-capture radiation leading to
the 6.789 MeV. level. The target used in this measurement was not the same one that was used in the excitation function measurements, but was of the same type and constructed in a similar fashion. The spectrum of low energy radiation in coincidence with all radiation above \( \sim 5 \) MeV. is shown in figure 5 in this reference (and in figure 4.9 in reference Ba63). In addition to the 1.23 MeV. line from the direct-capture transition in question, a gamma ray of energy \( \sim 1.4 \) MeV. is also apparent, which was identified as being due to the transition from the 278 KeV. resonance (excited deep in the target) to the 6.18 MeV. level. Using the 1.23 MeV. and 1.4 MeV. photopeak areas (obtained from these diagrams), it has been calculated that \( \sim 10\% \) of the yield of 6.789 MeV. radiation at 800 KeV. would be due to excitation of the resonance. Since it is now known that a long nitrogen-distribution tail was also present in the target used to perform the measurements described in chapter 2, it is likely that the presence of a long tail is an inevitable feature of this type of target.

Nevertheless, there is no direct evidence to show that the particular target used by Hebbard and Bailey in their excitation-function measurements did in fact possess a similar tail. In the present re-assessment of their data, therefore, the point of view has been taken that the assumptions made by Hebbard and Bailey cannot be completely ruled out (i.e. no excitation of the 278 KeV. resonance at 800 KeV.). In the light of the above discussion, however, it is very likely that \( \sim 10\% \) of the yield of 6.789 MeV. radiation at 800 KeV. arises from excitation of the resonance, though the data do not rule out an even greater contribution. It has been assumed that the correct figure lies in the range 0 - 20\%.

The most unsatisfactory feature of the analysis of Hebbard and Bailey is the assumption that the 278 KeV. resonance is not excited within the target, at energies greater than 500 KeV.
The basis for this assumption is quite insubstantial and appears to be solely that the corrected data lie on smooth curves. However, this criterion is just as well satisfied on the assumption of considerable excitation of the resonance at all energies up to 1000 KeV. As well as the rather arbitrary nature of this assumption, the corrections at the lowest energy data points are, in some cases, very large. The magnitude of the corrections thus depends sensitively on the detailed assumptions made regarding the excitation of the 278 KeV. resonance, and the partial widths for gamma-decay thereof. Moreover, it is just these low energy data points which are of greatest importance, since they give evidence for interference between the 278 KeV. resonance and non-resonant amplitudes.

It is relevant at this point to remark on certain features of the direct-capture calculations made by Hebbard and Bailey. The formula they used was equation (25) from a paper by Christy and Duck (Ch61). The derivation of this formula relies on the nuclear phase shifts, \( \delta_{J_i} \), being independent of \( J_i \). One may then remove the summation over \( J_i \) (see equation (8) in reference Ch61), by writing

\[
\sum_{J_i} (2J_i + 1) \left\{ \sum_{J_f} \delta_{J_i J_f} \right\} = \frac{1}{2\ell_f + 1} . \tag{4.5}
\]

This summation cannot be performed in the more general case where the elastic scattering from bound or unbound levels contributes significantly. The \( \delta_{J_i} \) then depend on \( J_i \), and hence the radial integral must be calculated separately for each value of \( J_i \) (see chapter 3).

Furthermore, Christy and Duck absorb a factor
into the dimensionless reduced width, \( \sigma^2 \) (equation (25) in reference Ch61). In the present case, where the reduced widths have been obtained from stripping data, there appears to be no justification for this step. The result is that the present direct-capture calculations differ in magnitude from those of Hebbard and Bailey by the factor (4.6). The latter calculations (for transitions through the 6.789 and 6.18 MeV. levels) appeared to agree rather well with the experimental data, but the present conclusion is that this finding is incorrect.

4.5. Details of the individual transitions:

4.5.1 The transition to the 6.789 MeV. level:

The only significant direct-capture transition to the 6.789 MeV. level was found to be the E\(^1\) transition with \( l_1 = 1, l_f = 0, S = \frac{3}{2} \). Calculation of the quadrupole transition with \( l_1 = 2 \) showed it to be weaker by a factor of \( \sim 1000 \). The result of Bailey and Hebbard (Be63a), that the angular distribution of the direct-capture radiation is very nearly \( \sin^2 \theta \), indicates that transitions with \( l_f = 2 \) need not be considered. Only channel spin \( S = \frac{3}{2} \) is possible, so that there is no ambiguity as to the relative magnitudes of the reduced widths for different channel spins.

The 278 KeV. resonance level has a relatively strong transition to the 6.789 MeV. level. There is no interference with the direct-capture amplitude, however, since the resonance level is populated via the channel with \( l = 0 \) and \( S = \frac{1}{2} \), whereas the
channel with \( l = 1 \) and \( S = \frac{3}{2} \) is involved in the direct capture. The transition to the 6.789 MeV. level is not noticeably excited at the 1061 KeV. resonance.

Figure 4.2 shows the data of Hebbard and Bailey, together with the results of the present calculations of the direct capture, and normalised direct capture plus resonance excitation functions. The calculated direct capture is lower than the data (near 800 KeV.) by a factor of \( \sim 1.6 - 1.8 \). Some fraction of this discrepancy (between 0 and 20%) is due to excitation of the 278 KeV. resonance deep in the target. Most of the rest is probably due to error in the reduced width (which determines the normalisation of the final-state wave function). As the single-particle reduced widths (see chapter 2) have been calculated on the basis of a very simple model of the nucleus (square-well potential), this result is not surprising.

The calculated direct-capture cross-section factor has been normalised by two factors. Using a value of 1.58, the calculated curve (Curve 'A' in figure 4.2) is \( \sim 10\% \) lower than the data at 800 KeV., and is thus thought to be a realistic representation of the actual cross-section factor energy distribution. The difference between each datum point and the calculated curve then represents the contribution from the 278 KeV. resonance at that energy. On this basis, using the known resonance gamma-ray widths, the data for the other transitions have also been corrected.

A curve is also plotted with a normalising factor of 1.75 (Curve 'B' in figure 4.2), which is nearly equivalent to assuming that the nitrogen distribution within the target is as suggested by Hebbard and Bailey. In this case also, corrections were calculated for the transitions to the other bound states.
Figure 4.2: Cross-section factors for the transition through the 6.789 MeV state, as a function of the mean proton energy in the nitrogen-rich layer at the front of the target. The corrected data points are displaced in energy for the sake of clarity.
When extrapolated to 20 KeV., these curves indicate a cross-section factor of ~1.2 and 1.3 KeV. barn respectively. If the assumption is made that as much as 20% of the cross-section factor at 800 KeV. is due to excitation of the 278 KeV. resonance, the extrapolated value becomes 1.1 KeV. barn. The cross-section factor at 20 KeV. has been taken to be 1.2 ± 0.1 KeV. barn.

4.5.2 The transition to the 6.18 MeV. level:

Of the transitions listed in table 4.1 in which the final state is the 6.18 MeV. level, only the first four, associated with the emission of E1 radiation, were found to be significant. As both channel spins are possible in this case, it was necessary to decide what proportion of the reduced width for the 6.18 MeV. level was associated with each channel spin.

Figure 4.3 shows the results obtained assuming a value of $\sigma^2 = 0.04$ for both transitions with $S = \frac{1}{2}$ and $S = \frac{3}{2}$. The d-p transition with $S = \frac{3}{2}$ is somewhat weaker than the s-p transition, and the sum of these two transitions is plotted in the diagram. The s-p transition is affected by the elastic-scattering phase shift due to the 8.283 MeV. level, and the calculation in the 900 - 1100 KeV. region is of rather poor accuracy because of this.

The s-p transition with $S = \frac{1}{2}$ is very strongly influenced by the 7.552 MeV. level. It was found that the reliability index (chapter 3, section 3.5), namely the ratio of the squares of the external radial integral to the integral over all space, was very different from 1 at most energies. Consequently, the results were very unreliable, and their use cannot be justified.
Figure 4.3: Theoretical direct-capture cross-section factors for the transition to the 6.18 MeV. state. Curves for both channel spins are plotted for $\theta^2 = 0.04$. 

In the present, it is assumed that the reduced width of the 6.18 MeV. level, the channel spins $\frac{\gamma}{2}$, is zero.

Data of related and Bailey are plotted to our curve. The new data, except that of the 2.0 MeV. energy, agrees very well with the results by another point. The decay due to the same energy is not unreasonable, when it is remembered that the theory is not the fitting procedure tends to produce correlated energy. The 6.18 and 6.70 MeV. states overlap by a large energy, and a positive error in the number of either times should be large. Another point is that the full energy and the determination of energy by the case respectively, indicating an error of $\theta^2 = 0.04$. 

\[ S(E) \quad \text{(Kev. barn)} \]

\[ 6.180 \text{ Mev.} \]

\[ S(E) \]

DIRECT CAPTURE

\[ S = \frac{1}{2} \text{(R.H. SCALE)} \]

\[ S = \frac{1}{2} \text{(L.H. SCALE)} \]

\[ S = \frac{3}{2} \text{(L.H. SCALE)} \]
It is evident that this transition is rather weak, since a very large cross-section factor is predicted near the resonance. For the present, it is assumed that the reduced width of the 6.18 MeV. level, for channel spin \( \frac{1}{2} \), is zero.

The data of Hebbard and Bailey are plotted in figure 4.4. The same data, corrected for the 278 KeV. resonance, are also plotted. The correction procedure used was as described in the previous section. The error bars were obtained by combining the errors on the data points (from table 4.3) with the errors on the 6.789 MeV. transition data points (multiplied by \( \frac{\Gamma_{6.18}}{\Gamma_{6.789}} \), the ratio of the gamma-ray partial widths for decay of the 278 KeV. resonance via the 6.18 and 6.789 MeV. levels).

There is a further source of error arising from the correction for the 278 KeV. resonance. Any error in \( \frac{\Gamma_{6.18}}{\Gamma_{6.789}} \) has the effect of shifting all the corrected data points up or down, by a fixed fraction of the correction applied. In the case of the 400 KeV. datum point, where the correction is \( \sim 500\% \) of the final value, an error of 10% in \( \frac{\Gamma_{6.18}}{\Gamma_{6.789}} \) would result in a shift in the corrected value of 50%. Such an error in \( \frac{\Gamma_{6.18}}{\Gamma_{6.789}} \) is not unreasonable, when it is remembered that the least-squares fitting procedure tends to produce correlated errors in the intensities of lines nearby in energy. The 6.18 and 6.789 MeV. lines overlap to a large degree, and a positive error in the intensity of one line would tend to be coupled with a negative error in the other line, so that the total number of counts in the region of the full energy and escape peaks remains approximately the same. One should therefore simply add the fractional errors in the partial widths, rather than treat them as probable errors. Hebbard and Bailey (He63) obtained errors of \( \sim 3\% \) and 1% for, \( \Gamma_{6.789} \) and \( \Gamma_{6.18} \) respectively, indicating an error of \( \sim 4\% \) in \( \frac{\Gamma_{6.18}}{\Gamma_{6.789}} \).
Figure 4.4: Cross-section factors for the transition through the 6.18 MeV state, as a function of the mean proton energy in the nitrogen-rich layer at the front of the target. The corrected data points are displaced in energy for the sake of clarity.
Since the inevitable imperfections in the interpolated line shapes used in fitting a gamma-ray spectrum tend not to be reflected in the extracted errors (as discussed in chapter 1), it is likely that the actual error is somewhat larger.

A source of error of a different nature is the possibility of the presence, in the $^{14}\text{N}(p,\gamma)^{15}\text{O}$ spectra, of contaminant 6.129 MeV. radiation from $^{18}\text{F}(p,\alpha\gamma)^{16}\text{O}$ or $^{15}\text{N}(p,\gamma\gamma)^{16}\text{O}$. Such radiation would be indistinguishable from the 6.18 MeV. radiation. Hebbard and Bailey found it necessary to include line shape components appropriate to radiation from these sources, in the fits to their data at 500, 700 and 800 KeV., and assumed that the $^{15}\text{N}(p,\gamma\gamma)^{16}\text{O}$ reaction was responsible. No comment was made on the reason for making this assumption, though the possibility of the presence of radiation from $^{18}\text{F}(p,\alpha\gamma)^{16}\text{O}$ was mentioned (He63).

It is known (Aj59) that the low-lying resonances in $^{18}\text{F}(p,\alpha\gamma)^{16}\text{O}$ tend to have much larger partial widths for decay via the 6.129 MeV. level than via the 6.92 or 7.12 MeV. levels, so that even a small intensity of the higher energy lines would tend to be associated with quite a large intensity of 6.129 MeV. radiation. It is known that the target used for the measurements described in chapter 2 suffered from fluorine contamination. Since this target was made from the same materials as those used by Hebbard and Bailey, it is likely that the source of contaminant 6.129 MeV. radiation is the $^{18}\text{F}(p,\alpha\gamma)^{16}\text{O}$ reaction. The data in figure 4.4 at 700 and 800 KeV. differ by a factor of ~2 in cross-section factor, indicating the existence of a substantial contribution of 6.129 MeV. radiation.

The theoretical curves plotted in figure 4.4 are the contributions from the 278 and 1061 KeV. resonances, as well as an estimated upper limit on the direct-capture cross-section factor,
assuming only channel spin \( \frac{1}{2} \) to be present. Robson (Ro68) has estimated an upper limit of 0.15 for the spectroscopic factor of the 6.33 MeV. level in \( ^{15}\text{N} \) (which is the analogue of the 6.18 MeV. level in \( ^{15}\text{O} \)). Assuming the same upper limit for the 6.18 MeV. level, and allowing a factor of 2 in the single-particle reduced width (see table 2.1), an upper limit of 0.16 is obtained for the dimensionless reduced width. It is this reduced width which has been used to calculate the curve plotted in figure 4.4.

In summary, the data of Hebbard and Bailey for the 6.18 MeV. transition are subject to considerable uncertainty, arising from

a) the unknown contribution of the 278 KeV. resonance level, excited deep in the target, to the yield of 6.789 MeV. radiation, and hence, via the correction factor, to the yield of 6.18 MeV. radiation;

b) the sensitivity of the correction factor (for the 278 KeV. resonance) on the value of \( \frac{\Gamma_{6.18}}{\Gamma_{6.789}} \), principally at the 400 and 500 KeV. data points;

c) the unknown contribution from contaminant 6.129 MeV. radiation, arising from the \( ^{19}\text{F}(p,\alpha\gamma) \) or \( ^{15}\text{N}(p,\gamma\gamma)^{16}\text{O} \) reactions.

The analysis of Hebbard and Bailey can be regarded as giving only an upper limit on the cross-section factor at 20 KeV., since

a) the minimum possible correction for the 278 KeV. resonance was applied, the only data points affected being those at 400 and 500 KeV.;

b) the possible presence of 6.129 MeV. contaminant radiation indicates that some or all of the data points should be reduced according to the intensity of this radiation. The fitted theoretical curve should thus be lowered appropriately.
At the other extreme, one could assume that as much as 20% of the yield of 6.789 MeV. radiation at 800 KeV. is due to excitation of the 278 KeV. resonance. One could also assume that the value of $\Gamma_{6.18}/\Gamma_{6.789}$ is 5-10% higher than assumed in the present calculations (and those of Hebbard and Bailey). Taking into account, also, the presence of 6.129 MeV. radiation, it is felt that the theoretical curve in figure 4.4 gives a not unreasonable lower limit on the cross-section factor at 20 KeV.

It could be assumed that the discrepancy between the shape of the theoretical curve (only $S = \frac{3}{2}$ direct capture considered), and the trend of the data, is partly due to interference between the 278 KeV. resonance amplitude and the rapidly-varying $S = \frac{1}{2}$ direct-capture amplitude (see figure 4.3). This was assumed by Hebbard and Bailey in their fit to the data. In view of the large uncertainty in the corrected data, however, there is no basis for deciding what proportion of the $S = \frac{1}{2}$ direct-capture amplitude is actually present. This uncertainty has little effect on the estimation of the lower limit for the extrapolated cross-section factor, however.

In connection with this estimate, examination of figures 4.9 and 4.11 in reference Ba63 shows that the yield of direct-capture radiation to the 6.18 MeV. level at 800 KeV. (the line labelled 1.89 MeV.) is very small. It is considerably smaller, in fact than the resonance radiation from the 278 KeV. resonance (labelled 1.43 MeV.). It is extremely small compared with the yield of direct capture radiation to the 6.789 MeV. level (1.23 MeV. radiation), whereas the data of Hebbard and Bailey (see figures 4.2 and 4.4 give a ratio of 3:1.

It is relevant, at this point, to note that from the measurements of the gamma-ray partial widths of the broad 9.49 MeV. level (see chapter 2), a contribution of not more than 0.006 KeV. barn to the cross-section factor at 1000 KeV. is possible from this source.
One of the reasons for examining the radiation from the 9.49 MeV. level was to test whether the yield was sufficient to be able to account for the measured cross-section factors for the 6.18 MeV. transition (see figure 6 in reference He63). It is now possible to rule out this alternative completely.

In view of all the above considerations, the cross-section factor for the transition through the 6.18 MeV. level at 20 KeV. is known only with very poor accuracy. The value of $0.6 \pm 0.4$ KeV. barn has been assigned.

4.5.3 The transition to the 5.2 MeV. levels:

5.2 MeV. radiation may arise from transitions direct to the 5.191 or 5.241 MeV. levels, or indirectly to the 5.241 MeV. level via the levels at 6.857 and 7.276 MeV. Figure 4.5 shows the results of the direct-capture calculations for the significant transitions. The spread in values shown is due to the unknown distribution of the reduced widths for the 6.857 and 5.241 MeV. levels between the channels with $S = \frac{1}{2}$ and $S = \frac{3}{2}$. The uncertainty is quite small, however.

The relative proportions of the transitions through the various states are undoubtedly somewhat different from these calculated, since one must expect reasonable errors in the assumed reduced widths. However, it is clear that even if one generously allows the reduced width of the 5.191 MeV. level to be 10 times the value assumed in the calculation (see table 2.1), the yield of 5.2 MeV. direct-capture radiation would still be only $\sim 50\%$ due to the 5.191 MeV. level. This is contrary to the assertion by Hebbard and Bailey (He63). This disagreement stems mainly from the absorption of the factor (4.6) into the dimensionless reduced
Figure 4.5: Theoretical direct-capture cross-section factors for transitions through the 5.191, 5.241, 6.857 and 7.276 MeV states. The spread in values for each curve is due to the unknown distribution of the reduced widths between the two channel spins.
widths $\theta^2$ (as discussed at the end of section 4.4) by Hebbard and Bailey, in their calculations.

The data of Hebbard and Bailey for the transitions through the 5.2 MeV. levels is shown in figure 4.6. The data have been corrected in the same manner as for the 6.18 MeV. transition. These data suffer from the same types of uncertainty as are associated with the 6.18 MeV. data, except that the problem of contaminant radiation of similar energy is not present.

In their analysis of these data, Hebbard and Bailey considered the possibility that the broad 2.35 MeV. resonance, interfering with the 278 KeV. resonance, could account for the measured cross-section factors. The present measurements of the gamma-ray branching ratios of the broad level have enabled this possibility to be rejected.

The theoretical curves in figure 4.6 are for the 278 and 1061 KeV. resonances singly, and combined with the calculated direct capture using the 'total' curve in figure 4.5. The general trend of the data is not well described by the theoretical curve, though the shape of the latter curve could be altered somewhat by varying the relative proportions of the transitions through the 7.276, 6.857, 5.241 and 5.191 MeV. levels (see figure 4.5). On the other hand, the agreement could be improved on the assumption of a 5-10% increase in $\Gamma_{5.191}/\Gamma_{6.789}$ (which determines the size of the corrections for the 278 KeV. resonance). The measurements of Hebbard and Bailey (He63) indicate errors of $\sim 3\%$ and $4\%$ for $\Gamma_{6.789}$ and $\Gamma_{5.191}$ respectively. It does not seem probable that interference between the direct-capture and 278 KeV. resonance can be invoked to explain the trend of the data (as assumed by Hebbard and Bailey (He63)), since the present calculations indicate that the transition through the 5.191 MeV. level is less important than those directly or indirectly through the 5.241 MeV. level.
Figure 4.6: Cross-section factors for the transitions giving rise to 5.2 MeV. radiation, as a function of the mean proton energy in the nitrogen-rich layer at the front of the target. The corrected data points are displaced in energy for the sake of clarity.
Nevertheless, the transitions through the 5.2 MeV. levels are considerably weaker than that through the 6.789 MeV. level, and the possible range of the cross-section factor at 20 KeV. is appreciably less than the uncertainty for the 6.18 MeV. transition.

The cross-section factor at 20 KeV. is estimated to be $0.1 \pm 0.05$ KeV. barn. The occurrence of constructive interference above the 278 KeV. resonance would result in an even smaller value.

4.5.4 The transition to the ground state:

The broad 2.35 MeV. resonance level is important for this transition, and the amplitude for the channel with $\ell = 0$ and $S = \frac{3}{2}$ interferes with the 1061 KeV. resonance amplitude, assuming the spin is $\frac{3}{2}$. The broad level does not interfere with the 278 KeV. resonance level, since the spin of the latter is $\frac{1}{2}$.

The theoretical resonance curves are plotted in figure 4.7, for both constructive and destructive interference below the 1061 KeV. resonance (the former option appears to be favoured by the data). The corrections to the data of Hebbard and Bailey to allow for the 278 KeV. resonance are small, with the exception of the lowest-energy data point.

In agreement with the experience of Hebbard and Bailey, the direct-capture calculations for the transitions to the ground state proved very unsatisfactory. The peak of the radial integrand in general occurred near the nuclear radius, and the contribution from the nuclear interior was then considerable. If the full reduced width for the ground state were assumed to be for the channel with $S = \frac{1}{2}$, the $S - p$ transition gave extremely large cross-section factors ($> 4000$ KeV. barn at 200 KeV.) because of the resonating phase shift in the channel with $\ell = 0$, $S = \frac{1}{2}$, $J_1 = \frac{1}{2}$ (due to the 278 KeV. resonance level).
Figure 4.7: Cross-section factors for the full-energy transition, as a function of the mean proton energy in the nitrogen-rich layer at the front of the target. The corrected data points are displaced in energy for the sake of clarity.
The calculations assuming $S = \frac{3}{2}$ were somewhat more reasonable, with the exception of the $S-p$ transition, in which the 1061 KeV. resonance level affects the initial wave function. However it is not felt that one can justify the use of the calculations in determining the cross-section factor at 20 KeV. The analysis of Hebbard and Bailey of the ground-state transition, in terms of resonance contributions only, is now known to be incorrect on two counts. The broad level interferes with the 1061 KeV. resonance, and not that at 278 KeV. The resonance yield is also insufficient to account for the measured data. On the other hand, it cannot be assumed that the contribution from direct capture is negligible simply because it is not possible to perform an accurate calculation.

Because the gamma-ray partial width for the ground-state decay of the 278 KeV. resonance is so small, the data of Hebbard and Bailey are quite insensitive to the assumptions made regarding the nitrogen-distribution tail in the target. If one assumes that the actual direct-capture cross-section factor is constant with energy, then an approximate fit to the data in figure 4.7, including the resonance contributions, gives a value of $0.25 \pm 0.1$ KeV. barn for the cross-section factor at 20 KeV. In fact the situation is more complicated, since the direct-capture amplitudes for $S = \frac{1}{2} \& \frac{3}{2}$ may interfere with the 278, 1061 and 2350 KeV. resonances, and are not necessarily constant. To allow for these possibilities, the error on the extrapolated cross-section factor has had to be increased. A value of $0.25 \pm 0.2$ KeV. barn has been assumed. The large errors on the data points does not permit the resolution of the uncertainty.
4.6. Conclusions and discussion:

The detailed analysis in the preceding sections lead to a number of conclusions.

Firstly, the experimental results and analysis of Hebbard and Bailey are associated with a large uncertainty, associated with the excitation of the 278 KeV. resonance deep in the target. This then leads to the second major conclusion, namely that targets made by heating tantalum in nitrogen or ammonia are of questionable value where quantitative results are required. This might not be the case, however, if examination of low energy radiation from the bombardment of such a target failed to reveal the presence of radiation due to transitions from resonance levels at lower energies.

As far as the measurements described in chapter 2 are concerned, it is clear that an unfortunate choice of target was made. At the time, the full extent of the nitrogen-distribution problem for nitrided tantalum targets was not realised, though of course the problem was known to exist. Nevertheless, the measurements were quite adequate for the purpose for which they were originally undertaken. As discussed in section 4.5, it is now possible definitely to reject the hypothesis that the broad 2.35 MeV. resonance contributes significantly to the transitions through the 6.789, 6.18 or 5.2 MeV. levels, in the energy range 20-1000 KeV. Furthermore, the 2.35 MeV. resonance has been shown to be only partially responsible for the ground-state radiation, as measured by Hebbard and Bailey. The use of a more satisfactory target would have enabled a better estimate to be made of the partial widths for decay of the 2.35 MeV. resonance through the 6.789, 6.18 and 5.2 MeV. levels. The error on the partial width for the ground-state decay could also have been reduced.
The present estimate for the \( ^{14}\text{N}(p,\gamma)^{15}\text{O} \) cross-section factor at 20 KeV. is \( 2.15 \pm 0.75 \text{ KeV. barn} \). This is appreciably lower, though consistent with, the original value obtained by Hebbard and Bailey (He63), and is generally lower than other estimates.

Lamb and Hester (La57) measured the \( ^{14}\text{N}(p,\gamma)^{15}\text{O} \) reaction cross section in the energy range 100-135 KeV. Cross-section factors were calculated for these data, and extrapolated to stellar energies by fitting a straight line. The cross-section factor was found to be \( 2.8 \text{ KeV. barn} \). The targets used in these measurements were water-cooled sheets of titanium, which had been heated in \( \text{NH}_3 \) to produce a nitrided surface layer.

Calculation of cross sections from the measured yield curve required the knowledge of the stopping cross section per nitrogen atom within the target. It was assumed that the surface layer was entirely composed of the nitride TiN. However, it is well known that titanium, like tantalum, may combine with nitrogen in various proportions, depending on the temperature and gas pressure employed. Whilst the nitride TiN appears to be the most common, there are indications that TiN\(_2\) may be formed too (Ho62). Other proportions or Ti and N are also found to occur. Using the data of Whaling (Wh58, De62), it is found that the stopping cross section per nitrogen atom is \( 47 \text{ eV. cm}^2. \) for TiN, and \( 32 \text{ eV. cm}^2. \) for TiN\(_2\). If the nitride layer were indeed closer to TiN\(_2\) than TiN in composition, the result of Lamb and Hester could be reduced to as low a value as \( 1.8 \text{ KeV. barn} \).

No indication is given in the paper by Lamb and Hester as to how they checked the thickness of the nitride layer, and what tests (if any) were conducted to confirm the assumed composition. Their result must consequently be held to be in doubt. It is of interest to note that in the analogous case of nitrided tantalum targets, although the composition TaN is normally assumed,
the nitride $\text{TaN}_2$ is also found (Os63), as well as other proportions of nitrogen and tantalum. Targets made in this laboratory have been found to have a surface composition corresponding to $\text{TaN}_{1.6}$.

The results of Duncan (Du51, Du51a) are considerably higher than those of Hebbard and Bailey, or Pixley (Pi57) (see for example, figure 9 in reference He63). It has also been noted that the result for the thick-target yield of radiation from the 278 KeV resonance level is more than 50% higher than the other published values (He63).

The measurements were carried out with three types of targets:

1. Thin gas targets, the yield from which was converted directly into cross sections,
2. thick $\text{Be}_3\text{N}_2$ targets, the yield from which was normalised to the yield from the thin gas targets, and
3. thick gas targets, the yield from which was normalised to the yield from the $\text{Be}_3\text{N}_2$ targets. The yield calibration of all data was related to a single target. It is therefore expected that any calibration errors in the thick target yield of radiation from the 278 KeV resonance were also present in the cross-section data for the energy region of present interest.

The value obtained by Duncan for the yield of radiation from the 278 KeV resonance disagrees with the results of three other independent workers (He63), which are in mutual agreement. It therefore seems reasonable to apply a correction factor to all the data of Duncan. His cross-section factor data in the proton-energy range 700-1000 KeV then agree quite well with those of Hebbard and Bailey.

The width of 1200 KeV., which was obtained by Duncan for the broad 2.35 MeV. resonance level, is now known to be in error. Much of the low-energy yield is due to direct capture, and the broad level has a width of $\sim 300$ KeV.
The work of Pixley (Pi57) included the measurement of the $^1H(p,\gamma)^{15}O$ reaction cross section in the proton energy range 400-650 KeV. The beryllium nitride targets which were used in these measurements were made by heating beryllium metal in $NH_3$. According to Pixley, these targets did not suffer from deeply penetrated nitrogen (in contrast to nitrided tantalum targets).

The cross section at 500 KeV. (figure 11 in reference Pi57) is equivalent to a cross-section factor of $\sim 1.35$ KeV. barn. In order to extrapolate the cross section to stellar energies, Pixley wrote the cross section, $\sigma(E)$, in the form

$$\sigma(E) = \frac{\sqrt{E} P_0}{E} \left[ \frac{k_1}{E-278+i0.85} + k_2 e^{i \delta_2} \right]^2 + k_3^2$$

where $P_0$ is $(F_0^2 + G_0^2)^{-1}$. Thus $\sqrt{E} P_0$ has the energy dependence of the normal penetration factor for $\ell = 0$, as defined by Lane and Thomas (La58). $k_1$, $k_2$, $\delta_2$, $k_3$ are parameters (to be obtained from fitting the data) associated with the 278 KeV. resonance level, a constant phase background for the channel with $\ell = 0$ and $S = \frac{1}{2}$, and a constant phase background for the channel with $\ell = 0$ and $S = \frac{3}{2}$.

Pixley noted that the resonance and interference terms appeared to be negligible above $\sim 450$ KeV., and thus wrote the cross section in the form

$$\sigma(E) = \frac{\sqrt{E} P_0}{E} (k_2^2 + k_3^2)$$

$k_2^2 + k_3^2$ was expected to be the same in the energy range 0-130 KeV. and near 500 KeV., providing there are no undetected low-energy resonances.
Calculation shows that \( \sqrt{E \rho_0} \) (as expected) has almost exactly the same energy dependence as \( e^{-2\pi \eta} \), between 20 and 500 KeV., hence the cross-section factor is proportional to \( (k_2^2 + k_3^2) \). If this term is constant, then so is the cross-section factor, and hence the extrapolated value is also \( \sim 1.35 \) KeV. barn.

Pixley, however, stated that the cross-section factor at stellar energies is 2.8 KeV. barn if \( (k_2^2 + k_3^2) \) is constant. His result is clearly incorrect if his calculations are based on the data mentioned above. In fact, the cross-section factors at 20 KeV. and \( \sim 500 \) KeV. will differ according to the relative proportion of direct capture to the various bound states, and the shape of the excitation function for each of these transitions. From the shape of the 6.789 MeV. curve (figure 4.2), it seems likely that the total cross-section factor at 20 KeV. should be \( \sim 0.3 \) KeV. barn higher than that at \( \sim 500 \) KeV., indicating that Pixley's data are consistent with a value of \( \sim 1.7 \) KeV. barn at stellar energies.

Caughlan and Fowler (Ca62) used the above-mentioned results of Lamb and Hester, and Pixley, to calculate the cross-section factor at 20 KeV. on the basis of a straight line fit. This procedure is not justified, in view of the more or less unknown effect of the 278 KeV. resonance on the cross-section factor curve (for example, the calculations of Hebbard and Bailey predict a drop in the cross-section factor between 130 KeV. and 20 KeV.). Moreover, as discussed above, the results of Lamb and Hester are in doubt.

In conclusion, the cross-section factor for the \(^{14}\text{N}(p,\gamma)^{15}\text{O}\) reaction at stellar energies is not well-known. The present reassessment of the measurements of Hebbard and Bailey gives a value of \( 2.15 \pm 0.75 \) KeV. barn, which is lower than, but consistent with, the previous result of Hebbard and Bailey \( (2.75 \pm 0.5 \) KeV. barn),
those of Lamb and Hester (La57), and Duncan (Du51, Du51a). The result is also consistent with the work of Pixley (Pi57).

In order to determine the cross-section factor more accurately, the measurements of Hebbard and Bailey need to be repeated using a different type of target. A gas target appears to be the most suitable. The bulk of the measurements could be taken with a single detector, but a number of coincidence measurements should also be performed in order to test for the presence of 6-7 MeV. radiation from $^18\text{F}(p,\gamma)$. A coincidence measurement with a Ge(Li) detector, at a single energy, would enable the determination of whether the 5.2 MeV. radiation is due to direct transitions to the 5.191 or 5.241 MeV. levels, or transitions through the 5.241 MeV. level via the 6.857 or 7.276 MeV. levels.
Chapter 5

RADIATIVE TRANSITIONS IN $^{16}\text{O}$, FOLLOWING THE $^{13}\text{F}(p,\alpha)^{16}\text{O}^*$ REACTION
5.1 Introduction:

The $^{19}$F($p,\alpha$)$^{16}$O$^*$ reaction has previously been studied in some detail, both by detecting alpha particles and the subsequent gamma radiation. Up to a proton energy of $\sim$1.75 MeV., the excitation functions are dominated by a series of strong narrow resonances, whose positions, widths and branching ratios have been measured (He 49, Bo 48, Mo 49, Ar 50, Hu 52, Sa 52, Se 52, Hu 53, Fa 53, Ki 55, Hu 55, Ra 55, Is 56, Bu 56, Li 59, As 61).

At higher bombarding energies, the excitation functions for alpha-particle transitions to the ground state and low-lying excited states of $^{16}$O (Cl 57, Ra 58, Is 58, Wa 62, Er 64), and for the sum of all gamma-ray transitions (and pairs from the 6.06 MeV. level) (Ra 58, Hu 55, Ph 51, Sa 52, Wi 52) show broad fluctuations of width $\sim$200 KeV. Little work has been done however, on the yield of individual gamma-ray transitions, except at a few isolated energies (As 61). A brief report by Trail et al. (Tr59) indicated that work was in progress on these measurements, but no further information has appeared in the literature.

The peak cross sections for the $^{19}$F($p,\alpha$)$^{16}$O$^*$ reaction are high, and the occurrence of small traces of fluorine in common target-backing materials is widespread. It is thus common to observe an appreciable yield of 6-7 MeV. radiation from this source, in gamma-ray spectra from proton-induced reactions. An examination of the literature, prompted by the work discussed in chapter 2, showed an almost complete absence of detailed gamma-ray cross-section data, for this reaction, at proton energies above $\sim$1.9 MeV. The present work was undertaken to obtain this information. Moreover, the lifetimes of the 6.92 and 7.117 MeV. levels are sufficiently short for considerable Doppler broadening to be present. At a proton energy of $^4$MeV., the
broadening was found to be \( \sim 140 \) KeV. for both these lines.

In view the inadequate energy resolution of NaI(Tl) detectors, and the presence of considerable Doppler broadening (whose effect on the line shape is unknown), it was felt that use of a Ge(Li) detector was essential.

5.2. Properties of levels in \( {}^{16}O \):

Energy conservation allows the excitation of levels in \( {}^{16}O \) up to 12.07 MeV. above the ground state, for the proton-energy range considered in this work (1.9 - 4.16 MeV.). The presently available data on spins, parities, branching ratios and lifetimes of those states which have been found to be populated via the \( {}^{19}F(p,\alpha)\,{}^{16}O \) reaction, are summarised in table 5.1. (See for example, Wi56, Be57, Sw57, Ko58, Go61, Go62, Go62a, Go63, Me63, Ga64, Ch67, Go67b, Gi67, Pi67, Wi68.) A number of levels which occur within this energy range, but are populated only by other reactions are not included in this summary.

Only the 6.129, 6.92, 7.117 and 8.88 MeV. levels were observably populated in the present energy range. Only the ground-state transitions and the 8.88 \( \rightarrow \) 6.129 transition were significant. Examination of table 5.1 shows that the lifetimes of these levels are such that radiation from the 6.92, 7.117 and 8.88 MeV. levels should be strongly Doppler broadened, whilst that from the 6.129 MeV. should not. This was indeed found to be so.
### Table 5.1

Decay Schemes and Properties of $^{160}$ states populated via $^{18}$F($p,\alpha\gamma$)$^{160}$

<table>
<thead>
<tr>
<th>Emitting State (MeV)</th>
<th>Final State (MeV)</th>
<th>% Branch</th>
<th>Lifetime (sec.)</th>
<th>$E_\gamma$ (MeV.)</th>
<th>Decay mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.06</td>
<td>0.0</td>
<td>~ 100</td>
<td>-</td>
<td>-</td>
<td>$\pi$</td>
</tr>
<tr>
<td>6.06</td>
<td>0.0</td>
<td>0.25 ± 0.11</td>
<td>-</td>
<td>-</td>
<td>$\gamma\gamma$</td>
</tr>
<tr>
<td>6.129</td>
<td>0.0</td>
<td>100</td>
<td>$(1.2 \pm 0.6) \times 10^{-11}$</td>
<td>6.129</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>6.92</td>
<td>0.0</td>
<td>~ 100</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>6.92</td>
<td>6.06</td>
<td>$(7.1 \pm 2.5) \times 10^{-3}$</td>
<td>$(1.2 \pm 0.3) \times 10^{-14}$</td>
<td>0.86</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>6.92</td>
<td>6.129</td>
<td>$(2.5 \pm 1.5) \times 10^{-2}$</td>
<td>-</td>
<td>0.79</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>7.117</td>
<td>0.0</td>
<td>~ 100</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>7.117</td>
<td>6.06</td>
<td>$\leq 3.5 \times 10^{-3}$</td>
<td>$(1.0 \pm 0.3) \times 10^{-14}$</td>
<td>1.06</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>7.117</td>
<td>6.129</td>
<td>$(8 \pm 2) \times 10^{-2}$</td>
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<td>0.99</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>8.88</td>
<td>0.0</td>
<td>$7 \pm 3$</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>8.88</td>
<td>6.06</td>
<td>$\approx 1.4 \times 10^{-3}$</td>
<td>-</td>
<td>2.82</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>8.88</td>
<td>6.129</td>
<td>$77 \pm 5$</td>
<td>$(1.92 \pm 0.29) \times 10^{-13}$</td>
<td>2.75</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>8.88</td>
<td>6.92</td>
<td>&lt; 5</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>8.88</td>
<td>7.117</td>
<td>16 ± 5</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>9.85</td>
<td>0.0</td>
<td>77</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>9.85</td>
<td>6.06</td>
<td>23</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>10.36</td>
<td>6.129</td>
<td>9</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>10.36</td>
<td>6.92</td>
<td>91</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>10.98</td>
<td>7.12</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>11.52</td>
<td>0.0</td>
<td>95</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
<tr>
<td>11.52</td>
<td>6.06</td>
<td>5</td>
<td>-</td>
<td>-</td>
<td>$\gamma$</td>
</tr>
</tbody>
</table>
5.3. Experimental Considerations:

5.3.1 Collection of Spectra

A 20 cc. lithium-drifted germanium detector, and a 5" x 4" NaI(Tl) scintillation detector were used in these measurements. The detectors were placed at an angle of 55° to the beam axis (on either side) with the Ge(Li) detector at a distance of 4 cm. from the target, and the NaI(Tl) detector at a distance of ~20 cm.

Singles spectra were recorded using conventional electronics. The NaI(Tl) spectra were collected in 512 channels, and the Ge(Li) spectra in 2048 channels.

A proton beam was supplied by the tandem electrostatic accelerator at the A.N.U. The beam energy was known to within 5 KeV. with a total spread in energy of ~5 KeV. Energy analysis was performed by means of a 90° magnet with N.M.R. field measurement and stabilisation. Beam currents in the range 50-150 nA. were employed, higher currents being precluded on the grounds of pulse pile up and excessive dead time in the pulse-height analyser.

5.3.2 Targets

Fluorine targets were made by evaporating a layer of barium fluoride (BaF₂) onto a piece of tantalum. While measuring Ge(Li) detector line shapes, it was noticed that the sharp peaks in a spectrum of 6.129 MeV. radiation from the 596 KeV. resonance in ¹⁸F(p,αγ)¹⁶O were distorted. The distortion took the form of broadening on the low or high energy side of each peak, depending on whether the detector was facing the back or the front of the target respectively.
The broadening was reduced by evaporating a thin layer of gold onto the front surface of the target. It was hence deduced that the effect was caused by excited oxygen nuclei escaping from the front surface of the target before emission of radiation. The 6.129 MeV. line is not normally Doppler broadened due to the long lifetime (~10⁻¹¹ sec.) of the initial state compared with the slowing-down time of the nuclei in the target material. However, those excited nuclei which escaped from the surface of the target were only partly slowed down before decaying, thus producing a Doppler-broadened contribution to the line shape. The gold layer was made just thick enough to eliminate most of the Doppler-broadened component.

The thickness of gold and barium fluoride layers were determined by measuring an excitation function of the yield of radiation from the 0.9 KeV. wide, 483 KeV. resonance in the ¹⁸F(p,αγ)¹⁶O reaction, both for the target of interest and another target with no gold layer. Using the stopping cross-section curves of Demirlioglu and Whaling (De62), the target was found to contain $(9.8 \pm 0.6) \times 10^{17}$ atoms of fluorine per cm². This corresponds to an energy thickness of 9.8 KeV. at a proton energy of 2 MeV. The gold layer was 3.1 KeV. thick at the same energy.

5.3.2 Experimental Method:

Spectra were collected for proton energies between 1.90 and 4.16 MeV. in 20 KeV. steps. In order to obtain the best energy resolution for the Ge(Li) spectra, double R.C. pulse shaping was used, with differentiation and integration time constants set at 1 μsec. Experience showed that loss of energy resolution due to pulse pile-up was not important providing the dead time was less than ~5%.
It was planned to derive relative intensities only from the Ge(Li) spectra since, as previously mentioned, the uncertain extent of the active area of this detector and the complicated shape thereof made accurate calculation of efficiency difficult. The NaI(Tl) spectra were then to be used to normalise these relative intensities to a total cross-section, without attempting to extract relative intensities from these data. For this reason, no attempt was made to optimise energy resolution for the NaI(Tl) spectra, and double delay-line clipping was used to minimise pulse pile-up. Analyser dead time was normally \( \sim 5-8\% \), and the beam current was kept as constant as possible to minimise the errors in dead-time correction arising out of simply using the clock-time to live-time ratio.

Each spectrum took \( \sim 1 \) hour to collect, giving a maximum channel count of \( \sim 8-10 \text{ K/channel} \) in both Ge(Li) and NaI(Tl) spectra. The beam current and collected charge were varied from run to run to maintain the dead time at the levels mentioned above.

5.4. Analysis of spectra:

5.4.1 General considerations

A typical spectrum obtained with the Ge(Li) detector is shown in figure (5.1). The most obvious feature of the spectrum is the strong Doppler broadening of the 6.92 and 7.117 MeV. lines. In view of the inherent advantages of line-shape fitting, and the obvious difficulty of accurately obtaining the areas under any of the peaks of the 6.92 and 7.117 MeV. lines, the fitting method was used to obtain the intensities of the 6.129, 6.92 and 7.117 MeV. transitions to the ground state.
Figure 5.1: High-energy portion of the gamma-ray spectrum from $^{16}\text{F}(p,\gamma)^{16}\text{O}$ at 2.90 MeV, using the Ge(Li) detector. The solid line is the least-squares fit to the data.
After this analysis had been completed, it was decided to obtain the excitation function for populating the 8.88 MeV. level also. In view of its secondary importance, and the fairly substantial outlay of computer time necessary to re-analyse the spectra, a different method of analysis was used in this case.

The intensity of the main branch (8.88 → 6.129) was obtained by summation of the photo peak, which was prominent above ~3 MeV. bombarding energy. At lower energies, the small cross section for populating the 8.88 MeV. level, coupled with moderate Doppler broadening, caused the line to be obscured. Below ~2.8 MeV., only an upper limit on the cross section could be obtained. The photo peak area of the 8.88 → 6.129 MeV. line was converted into the desired cross section by using the known peak to total ratio, and the 8.88 MeV. level branching ratios.

5.4.2 Ge(Li) spectra:

The major problem in the analysis was to devise a method for simulating the strong line-shape broadening present. In principle, if the alpha-gamma angular-correlation functions were known for the transitions through the 6.92 and 7.117 MeV. levels, it would be possible to deduce the shape of the broadened line. As such information was not available, a method was devised whereby the correct broadening was simulated within the fitting calculation. It is worth noting that the angular-correlation changes with energy, so that an enormous effort would be involved, not just at one energy, but at a reasonable number of energies spanning the range of the data.

It is evidently possible to construct a broadened line shape by folding a suitable function into an unbroadened line shape, obtained by interpolation in the normal way. Such a broadening function
could consist of a series of suitable component functions, the coefficients of which would be varied in order to give the correct shape for the total broadening function, and hence the line shape.

The properties of the Legendre polynomials seemed to be suitable for the purpose. A broadened line shape, \( G(x) \), was constructed by using the formula

\[
G(x) = \sum_{x' = -n}^{+n} f(x')F(x - x'),
\]

(5.1)

where the broadening function, \( f(x') \), is given by

\[
f(x') = \sum_{m=0}^{N} a_m P_m(g(x')).
\]

(5.2)

\( x, x' \) are spectrum channel numbers, \( F(x) \) is the unbroadened line shape, and \( g(x') \) is some function of channel number which allows the argument of the polynomials, \( P_m \), to span a suitable range of values (for example, 0-1). The range of channels from \(-n\) to \(+n\) represents the width of a broadened line, as deduced by examining (say) the double-escape peak of the appropriate line in the spectrum being analysed.

Least-squares fits to the data in hand were attempted, using equations (5.1) and (5.2) to construct broadened line shapes. Various combinations of the coefficients \( a_m \) were tried in order to optimise the fit, but it soon became apparent that the above method would be very time consuming, and would in any case not yield a very good fit. Consequently a different approach was tried.

The method is as follows. For each Doppler-broadened line, a set of normalised partial broadening functions, \( f_k(x) \) were constructed according to the formula
within the range of channels \(-n \leq x \leq n\), where \(k\) was within the range \(0 \leq k \leq 8\). The constant, 0.5, was added to remove problems associated with negative values of the polynomials. The normalisation ensured that no change in normalisation of the line shape occurred after folding in the \(f_k(x)\). This operation resulted in the formation of a set of partial broadened line shapes (one set for each line in the spectrum), \(G_k(x)\), given by

\[
G_k(x) = \sum_{x'=n}^{+n} f_k(x') F(x-x') .
\]  

(5.4)

To compensate for a small amount of broadening (due mainly to pulse pile-up) in the 6.129 MeV. line, the line shape was broadened by an asymmetric gaussian-broadening function, \(f^G(x)\), of the form

\[
f^G(x) = \frac{\exp\left(-k_1x^2/2\sigma_1^2\right) + \exp\left(-k_2x^2/2\sigma_2^2\right)}{\int_{-5\sigma_1}^{+5\sigma_2} \left[\exp\left(-k_1x^2/2\sigma_1^2\right) + \exp\left(-k_2x^2/2\sigma_2^2\right)\right]dx} .
\]  

(5.5)

defined in the range \(-5\sigma_1 \leq x \leq 5\sigma_2\), where \(k_1 = 1\) for \(x < 0\), \(k_1 = 0\) for \(x \geq 0\);

\[
k_2 = 0\) for \(x < 0\), \(k_2 = 1\) for \(x \geq 0\),
\]

\(\sigma_1\) and \(\sigma_2\) are the variances of each half of the function (determined by inspection).
The result of folding this function into the unbroadened line shape, $F(x)$, was to derive a gaussian-broadened line shape $G^g(x)$ (as in equation 5.4).

Each gamma ray spectrum was analysed by the linear least-squares method, using the sets of $G_k(x)$ for the 6.92 and 7.117 MeV. lines, and the $G^g(x)$ for the 6.129 MeV. line. A constant component was also included to take account of the Compton/bremsstrahlung tail of the 8.88 MeV. ground-state transition, and any other high-energy radiation.

The intensities, $a_k$, of each component broadened line shape, $G_k(x)$, derived from the fit, were then combined to give a normalised total line shape

$$G^d(x) = \frac{\sum_k a_k G_k(x)}{\sum_k \sum_{x'} a_k G_k(x')}$$  \hspace{1cm} (5.6)

where the summation over $k$ involves just those $G_k(x)$ belonging to the particular line in the spectrum under analysis, and the range of $x'$ is just the extent of the line shape $G_k(x')$. The functions, $G^d(x)$, are then the best estimates of the detector line shapes for Doppler-broadened radiation, according to the least-squares criterion. The sum of the $a_k$'s is the required intensity for each transition.

As the $G_k(x)$ in equation (5.6) were found to be rather similar in shape, it was expected that there would be strong correlations between their intensities. This would imply significant positive values of the off-diagonal elements of the variance-covariance matrix, $C$, derived in the fit. The variance, $\sigma_s^2$, of the sum of the intensities $a_k$ for each transition, should be given by
In practice, however, there were found to be alternate large negative as well as positive values of the elements $C_{\ell\ell'}$, resulting in unrealistic values of $\sigma_s^2$. To overcome this difficulty, as well as to check that the sets of similar components in the fit had not caused precision errors in the inversion of the normal matrix, a second fit was performed using the $G_d(x)$ instead of the $G_k(x)$ as components. The derived transition intensities were the same as for the first fit. As each transition was now represented by a single component, rather than a set of components, the variances were now given simply by the diagonal elements of the variance-covariance matrix (as discussed in chapter 1).

The set of intensities, $a_k$, in equation (5.6) do not appear to have any physical meaning, other than being just one set of parameters simulating the broadening of the line shape. It appears that no information about the alpha-gamma angular correlation can be obtained from them, and in practice they were found to form a sequence of numbers of alternate sign, and monotonically decreasing magnitude.

All Ge(Li) spectra were analysed in the same way. The value of 'n' in equation (5.3) was kept the same for the 6.92 and 7.117 MeV. lines, and increased gradually from 14 at 1.9 MeV. to 17 of 4.16 MeV. This parameter could be readily determined by inspection of the double-escape peaks, and a variation of $\pm 1$ channel was found to have a rather small effect on the extracted intensities of each transition. The variances in the gaussian-broadening function ($\sigma_1$, $\sigma_2$ in equation (5.5))
for the 6.129 MeV. line, were adjusted as required to improve the fit in the region of the peaks of this line shape. Once again, the fitted intensities were not very sensitive to changes in these parameters.

Time-dependent background radiation was measured periodically, but was found to be completely insignificant over the region of fit (~2.8-7.3 MeV.) of the spectra.

The full line drawn through the points in figure 5.1 is the fit obtained by the above method. The corresponding line shapes constructed in the fitting calculation (i.e. the \( G^d(x) \) and \( G^G(x) \)) are shown in figure 5.2.

In order to obtain the total detected yield of 2.75 MeV. radiation (from the 8.88 \( \rightarrow \) 6.129 MeV. transition), the 2.75 MeV. line shape was interpolated, and the photo peak to total ratio was calculated. The measured photo-peak area of this line in the data then gave the desired result.

### 5.4.3 NaI(Tl) spectra:

Analysis of these data was considerably simpler than that of the Ge(Li) spectra, since only the total intensity was of interest. Beam-independent background was subtracted from each spectrum, and the average intensity of radiation higher than 8.88 MeV. was estimated and also subtracted. The flat portion of the Compton/bremsstrahlung distribution from the 6-9 MeV. radiation was then extrapolated to the energy zero of the spectrum, and the total detected intensity, \( I_d \), determined.

Let \( I_{2.75}, I_{6.129} \ldots \) be the relative detected intensities of the 2.75, 6.129 MeV. \ldots radiation derived from the corresponding Ge(Li) spectrum. If \( E_{2.75} \ldots \) are the corresponding relative detection efficiencies (of the Ge(Li) detector), then
Figure 5.2: Total line shapes for the 6.13, 6.92 and 7.12 MeV. radiation, as deduced by the least-squares fitting calculation. The relevant gamma-ray spectrum is shown in figure 5.1.
the relative intensities of the ground-state transitions of the 6.129 ... 8.88 MeV. levels are

\[ \frac{I_{6.129}}{E_{6.129}} , \frac{I_{6.92}}{E_{6.92}} , \frac{I_{7.117}}{E_{7.117}} , \frac{I_{2.75}}{E_{2.75}} \times R_1 , \]

where \( R_1 \) is the ratio of the probability of decay of the 8.88 MeV. level to the ground state, to the probability of decay via the 6.129 MeV. level.

If \( e_{6.129} \ldots e_{8.88} \) are the total detection efficiencies of the NaI(Tl) detector for 6.129 ... 8.88 MeV. radiation, then the cross sections for the sum of all ground-state decays of the 6.129 ... 8.88 MeV. levels, \( \sigma_s \), and for individual transitions, \( \sigma_{6.129} \ldots \), are given by

\[ \sigma_s = \frac{I_d \times S_1 \times C_d}{S_2 \times C \times N_p \times N_T} \text{ cm}^2. \]

\[ \sigma_{6.129} = \frac{I_{6.129}}{E_{6.129}} \times \frac{\sigma_s}{S_1} \text{ cm}^2. \]

\[ \ldots \]

\[ \sigma_{8.88} = \frac{I_{2.75}}{E_{2.75}} \times R_1 \times \frac{\sigma_s}{S_1} \text{ cm}^2. \]

where \( C_d \) is the dead-time correction factor for the NaI(Tl) spectrum,

\( C \) is the charge collected in \( \mu \)-coulomb,

\( N_p \) is the number of protons/\( \mu \)-coulomb,

\( N_T \) is the number of fluorine atoms/cm\(^2\). in the target,

\[ S_1 = \frac{I_{6.129}}{E_{6.129}} + \ldots + \frac{I_{2.75}}{E_{2.75}} \times R_1 , \]
Multipli cati on of $\sigma_{8.88}$ by the appropriate branching ratio, $R_2$, gave the cross section for population of the 8.88 MeV. level. The values of $R_1$ and $R_2$ were taken from the data of Gill et al. (Gi67) as follows:

$$R_1 = \frac{8.88 \rightarrow \text{G.S}}{8.88 \rightarrow 6.129} = 0.091 \pm 0.045$$

$$R_1 \times R_2 = \frac{8.88 \rightarrow \text{all states}}{8.88 \rightarrow 6.129} = 1.24 \pm 0.06$$

The photopeak-total ratio of the 2.75 MeV. Ge(Li) line was found to be 0.053.

5.4.4 Errors:

The statistical errors associated with the linear least-squares analysis of the Ge(Li) data have been estimated in accordance with the discussion in chapter 1, and were generally $\sim 1\%$.

An investigation into the effects of varying the range of the broadening functions ('n' in equations (5.4)) and the maximum order of these functions (the range of values of 'k' in equation (5.4)) was carried out. The same applies to the variances $\sigma_1$ and $\sigma_2$ in the gaussian-broadening function (5.5). It appears that errors of $\sim 3-4\%$ in the extracted intensities might be possible from incorrect choice of these parameters. Larger errors are unlikely since the fits became noticeably worse when these parameters were varied such as to produce still greater
deviations in intensities. As discussed in chapter 1, it is difficult to estimate errors in fitted intensities reliably when there are errors in the component line shapes.

The calculation of the statistical errors in the intensities of the 2.75 MeV. line involved the estimation of the variances of the photopeak counts and the underlying background (Compton/bremsstrahlung distributions from higher energy lines). At proton energies below ~2.8 MeV., only an upper limit on the intensity could be placed, as discussed earlier in this chapter.

Random errors in the total intensity of ground-state radiation, derived from the NaI(Tl) spectra, were due to several causes. The large total number of counts (~10^6) in each spectrum resulted in a negligible statistical error. Subtraction of background radiation and the subsequent averaging and extrapolation of the Compton/bremsstrahlung distribution resulted in a possible error of ~0.5%.

A more important source of error was the dead-time correction. With a little effort, it can be shown that the pulse-height analyser clock-time/live-time ratio gives the correct dead-time correction only if the beam current is constant (assuming a constant yield per beam particle from the reaction). In fact, for arbitrary fluctuations in the beam current, the pulse generator supplying live-time and dead-time scalers should have a repetition rate which is a linear function of the beam current.

In the absence of such equipment, it was endeavoured to maintain the beam current as constant as possible within any run. If one assumes a maximum possible dead-time correction-factor error of 30% (probably a generous estimate), the errors involved in the total intensity of all ground-state radiation amount to ~2.5% (the pulse-height analyser recording spectra from the NaI(Tl) detector had a maximum dead time of ~8%).
Two sources of a constant error for all cross sections are target thickness and detector efficiency. An error of 6% has been assigned to the number of target atoms on the basis of the spread of points on the stopping cross-section curves used in the calculation. Systematic errors (nearly the same for the 6-8 MeV. radiation) in the Ge(Li) detector efficiency from similar causes are not important, but such errors are important in calculating the efficiency of the NaI(Tl) detector. For a 5% error in stopping cross-section, the calculated efficiency error is \( \sim 2.5\% \). Thus the total systematic error applicable to all cross-sections (and not affecting relative cross-sections) is \( \sim 9\% \).

In summary, the three main types of error affecting the cross-section data are:

a) Systematic errors, uniformly affecting all cross-section measurements, due to target thickness and detector efficiency errors: \( \sim 9\% \).

b) Partly systematic errors, affecting the sum of the intensities of all ground-state transitions, but not affecting the relative intensities of the transitions at a given energy. These are partly statistical and partly dead-time correction errors, and amount to \( \sim 3\% \).

c) Statistical errors arising out of the linear least-squares analysis, which have little effect on the total intensity. These are in some doubt for the 6.129, 6.92 and 7.117 MeV. transitions, owing to the presence of errors in the calculated Doppler-broadened line shapes. From the previous discussion, errors of \( \sim 3\% \) appear to be possible as a result of this.

To these main sources of error should be added two more. Firstly, the total integrated cross sections are calculated on the assumption that yield of radiation at 55° (a zero of the
2nd-order Legendre polynomial) is directly related, via the detector efficiency, to the total cross section. The possible errors arising out of this approximation cannot be estimated without knowing the angular distribution of radiation arising from each transition. However, if the cross-section measurements are to be used mainly in conjunction with the analysis of spectra from other reactions, in which the 6-7 MeV. radiation from \(^{18}\text{F}(p,\gamma)^{18}\text{O}\) reaction is a contaminant, this uncertainty is not important. In this case, the other data would also be collected at 55° in general (as for example, in the work described in chapter 2).

Secondly, the cross sections for transitions via the 8.88 MeV. level have a uniform uncertainty, due to errors in the branching ratios which have been used in the analysis.

5.5. Results:

The final excitation functions are shown in figures 5.3-5.7. Figure 5.7 gives the total cross section for all decays direct to the ground state, and figure 5.6 gives the total cross section for the formation of the 8.88 MeV. level (not just the ground-state transition).

The errors in figure (5.6) are merely the statistical errors in obtaining the photopeak area of the 2.75 MeV. line from the Ge(Li) spectra. The data from which these curves were plotted are listed in table 5.2. The errors are again just those derived from the analysis of the Ge(Li) spectra. Depending on the use to which these data are to be put, further errors should be added to those quoted, in accordance with the discussion in section 5.4.4.
Figure 5.3: 6.129 MeV. excitation function for the $^{19}\text{F}(p,\alpha\gamma)^{16}\text{O}$ reaction.
Figure 5.4: 6.92 MeV. excitation function for the $^{19}_F(p,\alpha^0)^{16}_O$ reaction.
Figure 5.5: 7.117 MeV excitation function for the $^{19}\text{F}(p,\alpha)^{16}\text{O}$ reaction.
Figure 5.6: Excitation function for formation of the 8.88 MeV level in $^{16}O$, in the $^{19}F(p,\alpha\gamma)^{16}O$ reaction.
Figure 5.7: Excitation function for the sum of the ground-state transitions from the 6.729, 6.92, 7.117 and 8.88 MeV. levels in $^{16}O$, from the $^{19}_p(\alpha \gamma)^{16}O$ reaction.
Table 5.2
Total Cross sections for $^{19}$F(p,αγ)$^{16}$O (milli-barn)

<table>
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<tr>
<th>Proton Energy (MeV.)</th>
<th>6.129 MeV. $\rightarrow$ G.S.</th>
<th>6.92 MeV. $\rightarrow$ G.S.</th>
<th>7.117 MeV. $\rightarrow$ G.S.</th>
<th>8.88 MeV. $\rightarrow$ all states</th>
<th>Total</th>
</tr>
</thead>
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<tr>
<td>1.90</td>
<td>16.4 ± 0.2</td>
<td>7.6 ± 0.3</td>
<td>72.9 ± 0.3</td>
<td>≤ 0.7</td>
<td>96.9</td>
</tr>
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<td>1.92</td>
<td>18.7 ± 0.2</td>
<td>11.9 ± 0.3</td>
<td>85.7 ± 0.3</td>
<td>116</td>
<td></td>
</tr>
<tr>
<td>1.94</td>
<td>23.1 ± 0.2</td>
<td>25.3 ± 0.5</td>
<td>111.0 ± 0.5</td>
<td>159</td>
<td></td>
</tr>
<tr>
<td>1.96</td>
<td>21.0 ± 0.2</td>
<td>46.8 ± 0.5</td>
<td>121.5 ± 0.6</td>
<td>189</td>
<td></td>
</tr>
<tr>
<td>1.98</td>
<td>16.7 ± 0.2</td>
<td>32.2 ± 0.6</td>
<td>133.0 ± 0.6</td>
<td>182</td>
<td></td>
</tr>
<tr>
<td>2.00</td>
<td>18.0 ± 0.2</td>
<td>29.5 ± 0.6</td>
<td>150.0 ± 0.6</td>
<td>198</td>
<td></td>
</tr>
<tr>
<td>2.02</td>
<td>21.3 ± 0.2</td>
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<td>236</td>
<td></td>
</tr>
<tr>
<td>2.04</td>
<td>26.3 ± 0.2</td>
<td>25.2 ± 0.8</td>
<td>202.0 ± 0.8</td>
<td>253</td>
<td></td>
</tr>
<tr>
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<td>26.7 ± 0.7</td>
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<td>248</td>
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<tr>
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<td>27.3 ± 0.6</td>
<td>142.0 ± 0.6</td>
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</tr>
<tr>
<td>2.10</td>
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<td>130.0 ± 0.6</td>
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<tr>
<td>2.12</td>
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<td>30.7 ± 0.6</td>
<td>122.2 ± 0.6</td>
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<td></td>
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Table 5.2 (continued)

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Table 5.2 (continued)

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Chapter 6

IMPROVEMENTS IN Ge(Li) DETECTOR LINE SHAPES

BY PULSE RISE-TIME SELECTION
6.1 Introduction:

The good energy resolution of lithium-drifted germanium detectors and the consequent advantage for γ-ray spectroscopy are offset to a certain extent by two factors:

a) The small volume of available devices results in a low probability of a second interaction of compton scattered or bremsstrahlung radiation, with a consequent lessening of peak to total ratios (since part of the incident γ-ray energy is lost to the detector). Thus a low intensity line may be obscured by the compton/bremsstrahlung tail of a higher energy line.

b) Owing to the relatively modest diffusion rates of charge carriers produced by an interaction with a photon (~4 × 10⁷ cm./sec. in pure Germanium at 77⁰K, for a field of 1000 V/cm.) (Go66a), pulse rise times are dependent on the position of interaction within the detector (due to the collection of both electrons and holes, and the variation in their transit distances) and the distribution of the electric field, which will be lowered near the junctions with undepleted material. Problems can then arise in large crystals (> 20 cc.) both with respect to timing and energy resolution (in the latter case, the use of RC clipping will result in a spread of output pulse heights as a result of different rise times for a given input pulse height).

In order to improve peak-total ratios, a number of workers have developed anti-coincidence shielded detector assemblies (Mi67, Al67, Au67, Ka66) in which a small Ge(Li) detector is surrounded by NaI or organic scintillators. Any pulses produced by the central crystal which are in coincidence with pulses from the surrounding scintillators (from Compton scattering, annihilation radiation or bremsstrahlung) are rejected. This method is most useful at low energies where the photoelectric effect
is strong. At higher energies where the cross section for pair production is larger than for photoelectric absorption, the use of a three-crystal pair spectrometer is indicated, and some assemblies have been designed to function in either of these modes of operation (A167). Gruhn et al. (Gr67) have manufactured a special detector with two N-type electrodes, by which means the same type of operation may be achieved.

Alexander et al. (A164) have shown that a large proportion of the pulses underlying full energy and escape peaks in gamma-ray spectra are defective, in the sense that they are associated with incomplete charge collection within the detector, and have a long rise-time component. By employing pulse-shape discrimination so as to eliminate such pulses, a substantial improvement in peak-total ratios was obtained. Tamm et al. (Ta67a), using a different form of pulse-shape discrimination, have obtained very similar results. Strauss et al. (St67) have used a somewhat different criterion for rejection of pulses, namely that the percentage of the total pulse height reached in a certain time interval (usually 40 nsec.) be less than some value (~40%). This should more properly be termed rise-time rather than pulse-shape discrimination.

These results are of importance with respect to timing with semiconductor counters. Zero-cross timing with large volume detectors is unsatisfactory since the time jitter is approximately equal to half the rise time, and even an ideal detector has an irreducible spread in pulse rise times which may be of the order of 30-40 nsec. Consequently leading-edge timing must be used, and a number of workers (Ew66, Ma66a, Go67a) have investigated the timing distributions for both planar and co-axially drifted detectors. Rejection of pulses with long rise times results in improved timing, as expected.

Most of the work that has so far been performed in this field has been carried out using planar detectors, and the characteristics of large
volume co-axially drifted detectors have not been so well defined in this respect. In addition, the 20 cc. detector of this type which is in use in this laboratory, was recently damaged when vacuum was lost whilst the full bias voltage was applied. Certain phenomena relating to the performance of the detector since that accident provided added stimulus to further investigations.

6.2 Electrical characteristics of Lithium-drifted detectors:

Figure 6.1(a) shows schematically (not to scale) the internal configuration of a co-axially drifted detector, with the approximate dimensions of the device which was used in the present measurements. As is common in these detectors, the cross section is of trapezoidal form.

The method of construction has been discussed in the literature (Go66a, Ma65), but several points are of interest here. Starting with low resistivity (~10 Ω cm.) p-type germanium (designated as p⁺), a region of acceptor-compensated material is produced by diffusing lithium from a highly doped outer region (on all but one side of the germanium ingot), which is designated as N⁺. The diffusion is carried out at moderate temperatures with electrodes and their polarities as shown (figure 6.1(a)). The most important point here is that providing the material is free from trapping sites such as oxygen atoms, crystal imperfections etc., a thick depletion layer (> 1 cm.) may be obtained where, as various authors such as Goulding (Go66a) have shown, almost exact compensation of the bulk acceptors obtains. Under these circumstances, very moderate fields will result in total depletion of this layer.

At the junction with the p⁺ core, where the concentration of lithium ions drops to zero, a narrow region of high negative
Figure 6.1: Schematic diagram of the construction of the Ge(Li) detector, together with the approximate forms of the charge and electric field variation with bias applied.
charge density is generated so as to produce zero field in the rest of the P⁺ material (a non-zero field in this region would result in a movement of holes away from the junction until the field was again zero).

Similarly, a region of high positive charge density is created at the function of the compensated and N⁺ regions, where the concentration of lithium ions rises above the concentration of acceptor sites in the original P⁺ material.

The approximate form of the charge and electric field distributions are shown schematically in figure 6.1 (b) and (c) for a section A-A as marked. The exact form of the electric field variation within the compensated region would be difficult to calculate, due to the trapezoidal cross section of the detector and incomplete knowledge of the shape of the undepleted core, but as a rough estimate, it will decrease with the inverse of the distance from the centre of the detector. The behaviour of the field in the regions near the top of the detector will be even more uncertain. However, despite the variation, the field is everywhere high.

Within the regions of high charge density, the field drops rapidly to zero in a somewhat unpredictable fashion, the fall-off being a function of the shape of the undepleted core, the doping level in the raw detector material, and the variation in lithium concentration (falling to zero at the inner junction and rising to same unknown value at the outer junction). It is expected that the depth of these regions will be small (≪ 1 mm.), and it is desirable that this should be so, since the field is low and hence charge diffusion rates are low.
6.3. Charge collection and pulse shapes:

If the fast electron(s) created by a photon interaction are stopped completely within the fully compensated region of the depletion layer, where the applied field is large, the free electrons and holes which are created in the process are swept rapidly toward the appropriate electrodes. At liquid nitrogen temperatures, the electron and hole mobilities are very similar (~30,000 cm²/V·sec.) (Go66a), and for a field of 1000V/cm. (roughly the average field in the present circumstances), the charge carriers have a velocity of ~0.3 mm./nsec. Due to the form of the field variation, the electron velocities decrease with time, and vice-versa for the holes (the field would be ~3 times greater at the inner junction than at the outer junction).

Pulses with the fastest rise times are those in which the charge carriers reach their respective electrodes at the same time, those with the slowest rise times being due to interactions near either junction. The range of electrons in germanium is appreciable (1.2 mm. for 1 MeV. electrons, 6 mm. at 5 MeV. and 12 mm. at 10 MeV.) (Be64), so that not only the point of interaction, but also the direction of the primary electrons affects the shape and rise time. If multiple photon interactions occur (e.g. multiple Compton scattering), the pulse shape will be the sum of contributions from electrons and holes generated at a number of points within the detector where photons have successively interacted.

The fast primary electrons can lose part or all of their energy in the undepleted N⁺ or P⁺ regions. Since there is no field to separate them, the electrons and holes eventually recombine without producing any current in the external circuit. Finally, any electron-hole pairs created in the high charge density regions will initially have low drift velocities, thereby increasing charge-collection times.
Thus it can be seen that the pulse shape and rise time depends on many factors, and a large spread is possible. For the purposes of good timing characteristics, the spread in rise times ought to be reduced as much as possible. Malm (Ma66a) has improved the timing behaviour of large volume co-axial detectors by cutting off the closed end, thereby removing a region of the detector where field variations in the depleted region are especially pronounced. For single spectra, however, it is only those slow pulses which are associated with incomplete charge collection that need to be rejected.


Zero-cross timing using double delay-line clipped pulses is ideally equivalent to leading-edge timing with the discriminator set at 50% of the pulse height (except that the time is also independent of the pulse amplitude over a wide range). For pulses of constant shape, a trigger pulse is generated at a time $\Delta + \tau_R/2$ after the leading edge of the pulse at the detector (where $\Delta$ is a time delay due mainly to the amplifier delay-line, and $\tau_R$ is the pulse rise time). With leading-edge timing, a trigger pulse is generated at a time $\sim x \times \tau_R/h$ after the leading edge of the pulse (where $x$ is the discriminator setting, and $h$ is the pulse height). Thus the time delay between trigger pulses from leading-edge and cross-over timing is $\approx \Delta + \tau_R/2$ (for $x/h$ small).

This simple result is of course only approximately true, since the pulses have a time delay associated with the leading edge timing. In the present work, the latter was accomplished by means of a time pick-off unit, the pulse height produced by which being a function of the derivative of the incoming linear pulse, so that the time jitter as a function of pulse height and rise time could not be accurately gauged.
Figure 6.2. shows the block diagram of the electronics used in the present measurements. 'Start' and 'stop' pulses for the time-to-amplitude converter (T.A.C) were provided by the time pick-off unit and a timing single-channel analyser (T.S.C.A) respectively. The T.S.C.A., in addition to delivering a trigger pulse at the zero-cross time of the linear pulse, also set an energy window on the linear pulse-height spectrum, by means of lower-level and window discriminators. Another T.S.C.A. provided the means for setting a window on the rise-time spectrum produced by the T.A.C., so that the linear spectrum could be gated by any desired rise-time interval.

ORTEC modular electronics was used throughout, and proved adequate for the investigation, although the pre-amplifier rise time (~ 70 nsec. for 20 pf. detector capacitance) and main amplifier rise-time (~ 100 nsec.) were rather large compared with the expected detector pulse rise times (~ 50 nsec.).

The capabilities of the circuitry were checked with a precision pulser, with variable rise time from ~2 nsec.-250 nsec. (checked with a fast oscilloscope), and this proved to be a convenient method of calibrating the detector rise-time distributions. Figure 6.3. shows the results of an investigation, using the precision pulser, of the relationship between channel number in the rise-time spectrum, and the pulse height and rise time at the input to the pre-amplifier (with the detector connected and 1200 V. bias applied).

The variation in channel number as a function of pulse-height reflects mainly the timing uncertainty in the leading-edge trigger, and it was not possible to obtain accurate pulse rise-time distributions over a wide energy range with the present method. However the broad details were not obscured, and in particular any pulses with abnormally long rise times were clearly evident.
Figure 6.2: Circuit arrangement for the rise-time distribution measurements.
Figure 6.3: Calibration curves for rise-time versus rise-time distribution channel number, as a function of pulse-height.
6.5. Results:

Figure 6.4 (a) shows a ThC" spectrum with the positions of 3 energy windows, within which pulse rise-time distributions were measured. These distributions are shown in figure 6.4 (b), including one for all pulses above \( \sim 1 \) MeV. The most interesting feature of these curves is the very large spread in rise times compared with those expected on the basis of the discussion in section 6.3. The rise-time calibration applies to pulses of \( \sim 2.9 \) MeV, and is thus only approximate for energy windows set lower down in the spectrum. Nevertheless it can be seen that the sharp peak at the low rise-time end of the distribution corresponds roughly to the expected rise times for pulses caused by photon interactions within the fully compensated region of the detector (\( \sim 50 \) nsec.). The majority of the pulses, however, fall within a broad distribution centred at \( \sim 170 \) nsec., (and extending beyond 250 nsec.), but the ratio of 'fast' (first group) to 'slow' (2nd group) pulses increases at higher pulse heights, and is largest for the pulses contributing to the full-energy peak alone.

On the assumption that pulses with very long rise times are due to electron-hole pair creation in regions where the electric field is low - i.e. near undepleted regions, many of these pulses should be associated with incomplete charge collection. The rejection of these pulses should then greatly improve the peak-total ratio. Figure 6.5 (a) shows a rise-time distribution, with three windows which were used to gate a linear spectrum (ThC" source). The gated linear spectra are shown in figure 6.5 (b), as well as one ungated spectrum.

Two effects are apparent. One is that the resolution is marginally improved for the short rise time gated spectrum, and considerably worsened for the long rise time gated spectra. This is presumably mainly due to the effect of an RC shaping network on pulses with different rise times - longer pulses give a smaller output.
Figure 6.4: Distributions of rise times, (b), observed at different parts of a ThC" spectrum, (a). The region of the spectrum relevant to each distribution is indicated in (a). The straight lines in (b) are calibration lines (centre scale).
Figure 6.5: ThC'' spectra, (b), gated by various portions of the rise-time distribution (a). The straight line in (a) is a calibration line (right-hand scale).
than shorter pulses for the same amplitude at the input to the network. This can also be seen by examining the photopeak positions. The peak is shifted down in energy in those spectra gated by the slow portion of the rise-time distribution.

The other effect noticed in the gated spectra is a moderately small increase in the photopeak to total ratio (counts above ~1.5 MeV.), for the spectrum gated by the fast rise-time group of pulses, and a decrease for the spectrum gated by the long rise-time section of the distribution. It is evident that not all the pulses with abnormally long rise times are associated with incomplete charge collection. An interesting point is that in the spectrum gated by the slowest pulses, though the photopeak to total ratio has only been reduced by ~25%, the escape peaks have almost vanished.

Using the \(^{19}\text{F}(p,\alpha\gamma)^{16}\text{O}\) reaction at the 596 KeV. proton-energy resonance to generate nearly pure 6.13 MeV. radiation, further spectra and rise-time distributions were measured - primarily to see the effect of an increase in electron path length within the detector. The rise-time distributions are shown in figure 6.6, and the gated \(\gamma\)-ray spectra in figure 6.7. The proportion of slow pulses in the rise-time distributions in all parts of the spectrum is considerably greater than for the 2.62 MeV. ThC\(^\prime\) radiation, particularly at the lower energies. The variation in the shape of the gated spectra as a function of the region of the rise-time distribution selected is similar to the case of 2.62 MeV. radiation. Both photopeak to total and 2nd escape peak to total ratios are strongly reduced for slow pulses, but only the photopeak to total ratio is substantially improved (by ~50%) by selecting only the fast pulses. Unfortunately, the rise-time distributions were not calibrated, but it is expected that the spread in times are similar to that obtained with 2.62 MeV. radiation.

Finally, the effect of varying the detector bias voltage on the distribution of rise times was studied for 2.62 MeV. radiation (ThC\(^\prime\) source). Data were taken at 1200 V., 600 V. and 300 V.
Figure 6.6: Distributions of rise times, (b), observed at parts of a 6.13 MeV γ-ray spectrum, (a). The region of the spectrum relevant to each distribution is indicated.
Figure 6.7: 6.13 MeV $\gamma$-ray spectrum, (b), gated by various portions of the rise-time distribution, (a).
Assuming that electron and hole mobilities are independent of the internal field, the rise time of pulses formed within the fully compensated region of the detector should be proportional to the applied voltage. In figure 6.8 it can be seen that the 'fast' peak moves across the distribution in the expected fashion as the bias is decreased, but the broad distribution of slower pulses which is peaked in the vicinity of \(~170\) nsec. appears to be practically independent of the applied voltage.

6.6. Discussion:

In order to understand the present results and their relation to those of other workers, it is as well to briefly summarise the methods which have thus far been used in the examination of pulses produced by Ge(Li) detectors, and the relevant experimental conditions.

Alexander et al. (A164) used a 5 mm. thick x 19 mm. diam. (sensitive volume 1.4 cc.) planar detector with radiation (\(~5-11\) MeV.) entering from the side (i.e. in a direction parallel to the long dimension of the detector). The timing circuit which they employed recognised the presence of a slow pulse component, and such pulses were rejected. The method would not be suitable for an examination of the distribution of rise times in pulses generated by a detector. Tamm et al. (Ta67a) achieved essentially the same result with a different technique. Measurements were carried out with two planar diodes, one of which had a 2 mm. thick depletion layer (sensitive volume 0.56 cc.), and the other a 5 mm. thick depletion layer (sensitive volume 2 cc.). 7-8.5 MeV radiation was examined (as well as that from a \(^{60}\)Co source). The method is energy-independent over a very wide energy range.
Figure 6.8: Three rise-time distributions for a ThC\textsuperscript{2+} spectrum, with the detector bias voltage set at 300, 600 and 1200 V. The straight lines are calibration lines, which are slightly different for each distribution.
Strauss et al. (St67, St67a) used an entirely different approach, in which each pulse contributed to a timing distribution at a point given by the percentage of the final pulse height reached in a certain fixed time (generally 40 nsec.). This method discriminates pulse rise time and is not sensitive to the presence of a slow component. Investigations were carried out using two planar detectors (one of which was 10.6 mm. in thickness with a sensitive volume of 5.9 cc., the other with thickness 11.2 mm. and sensitive volume of 3.5 cc.), and a co-axial detector of volume 20 cc.

The method of Strauss et al. (St67) is similar to the method used in the present work, in which each pulse contributes to a timing distribution at a point given by the time taken to reach a fixed fraction of the final height (i.e. 50%).

It would appear, from comparison of these four methods, that the elimination of pulses with a slow component gives a greater improvement in the shape of spectra than merely the elimination of slow pulses as such, since Alexander et al. (Al64) and Tamm et al. (Ta67a) see a reduction in the intensity of the background distribution by a factor of between 2 and 3. On the other hand, the detectors used by these workers were very small and only high energy radiation was observed, so that the range of the fast electrons in germanium would have been of the order of the dimensions of the detector. In addition, the thickness of dead-layers at the electrodes of detectors depends on the method of fabrication and varies from detector to detector. To compare the different techniques, then, it would be necessary to carry out tests with each method on the same, or at least similar detectors.

In the work of Strauss et al. (St67, St67a) as in the present work, the reduction in the background tends to be less, and is most
pronounced in the lower channels of spectra, becoming less at higher channels. Strauss et al. achieved a 50% reduction in the bremsstrahlung tail of a 3-crystal pair spectrum, but this is to be expected, since the improvement is a function of the intensity of bremsstrahlung and defective pulses in this case, as distinct from Compton, bremsstrahlung and defective pulses in a singles spectrum.

The most important finding of the present investigation is the very strong broad distribution of slow (~170 nsec.) rise-time pulses, most of which appear to be normal so far as their heights are concerned. It is unfortunate that rise-time characteristics of the detector before the accident are not known, so that no comparison can be made with these results, but it seems reasonable to assume that some fairly drastic redistribution of lithium ions within the detector has taken place. It is not at all clear how this accounts for the fact that the rise times of these slow pulses are nearly independent of the applied bias voltage.

The main effect of selecting a narrow band of rise times (whether fast or slow) was to improve the energy resolution (this is particularly evident in figure 6.7 (b)), and this is to be expected on the basis of the wide distribution of pulse rise times. In addition, the photopeak to total ratios were significantly improved by selecting only the fast pulses, and worsened by selecting only slower pulses (the effect becomes more pronounced at higher energies). The effect on the 2nd-escape peak to total ratios in selecting only fast pulses is not so obvious, but the ratio was reduced for the slower pulses.

Much more work needs to be done, particularly to compare results for the detector currently in use with those of a good detector of similar dimensions. Study of the effects of warming up the detector for short periods (and the application of a small bias voltage) would shed further light on the causes of the slow pulses observed in this work. A further development would be...
the use of timing circuitry in which the rise time between any two selected fractions of the final pulse-height (and independent thereof) could be examined.

One method by which this could in principle be achieved would be to use cross-over timing in association with double delay-line clipped pulses, where the second delay line in each of two main amplifiers is incorporated into an amplifying network of gain respectively > 1 and < 1. If the ratio of positive excursion to negative excursion of the two doubly-clipped pulses thus produced for each detector pulse are respectively \( R_1 \) (\( < 1 \)) and \( R_2 \) (\( > 1 \)), then the delay between the leading edge of each clipped pulse and its zero-cross point is respectively \( \frac{R_1 \tau}{1+R_1} \) and \( \frac{R_2 \tau}{1+R_2} \), where \( \tau \) is the pulse rise time. Thus a delay (which is in principle energy independent) of \( \frac{R_2}{1+R_2} - \frac{R_1}{1+R_1} \) between the two zero-cross points obtains. By varying \( R_1 \) and \( R_2 \), the distribution of rise times between any two fractions of detector pulse heights could be measured over a wide energy range.
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