THE $^{50,52,54}$Cr($\alpha$,p)$^{53,55,57}$Mn REACTIONS

AT $E_\alpha = 18$ AND 26 MeV

by

Konrad Anton Aniol

A thesis submitted to the
Australian National University
for the degree of Doctor of Philosophy

Canberra,
February, 1977
To Kathye
ACKNOWLEDGEMENTS

Contemporary nuclear physics research is by its very complexity necessarily a co-operative undertaking of many people. It is not possible to list all the individuals for whose help I am indebted. However, I would like to thank: my supervisor Dr. Jan Nurzynski, for his support and cheerful countenance during many overnight shifts; Dr. Dennis Gebbie my acting supervisor during Dr. Nurzynski's sabbatical, especially for his careful reading of this thesis as well as for his invaluable assistance in collecting the data; Dr. Charles Hollas for his assistance in collecting the data, especially during two quite fruitful Christmas runs; Prof. John Newton, whose willingness to discuss with me topics of interest other than my immediate research is greatly appreciated; Dr. Brian Robson for his helpful comments on and reading of Chapter 2 and Appendix 1.

Thanks are also due to Dr. David Weisser and the technical staff for the reliably good condition of both the EN tandem and 14 UD Pelletron accelerators. Of my fellow students I would like to thank in particular A.D. Frawley and E.C. Pollacco for their companionship and for the interesting discussions we had which were useful to me in interpreting some of my data. I also wish to thank my wife Katherine, whose support can not be overestimated. The care with which N. Chin has typed this thesis is appreciated.

All of the experiments described in this thesis were proposed by me. However, (p,α) measurements had been proposed earlier by various members of the Direct Reaction group. The 18 MeV data were obtained jointly by Drs. Nurzynski, Gebbie, Hollas, Barbopoulos and me. Dr. Nurzynski assisted in the preliminary measurements at 26 MeV and Drs. Barbopoulos and Bartle also assisted during a few runs at 26 MeV. However, the bulk of the measurements at 26 MeV were undertaken by Drs. Hollas, Gebbie and me. The (p-γ) coincidence measurements were carried
out by Dr. Gebbie and me. All of the data reduction and analysis was
performed by me.

This research was possible due to the generous support of the
Australian National University through an A.N.U. Scholarship.

Some of this work has already been reported at the AINSE

Koarad A. Black
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACKNOWLEDGEMENTS</td>
<td>ii</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>vii</td>
</tr>
<tr>
<td>CHAPTER 1. INTRODUCTION</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 2. THE DWBA THEORY OF THE A(α,p)B REACTION</td>
<td>13</td>
</tr>
<tr>
<td>2A. INTRODUCTION</td>
<td>13</td>
</tr>
<tr>
<td>2B. EFFECTS OF ANTISYMMETRIZATION</td>
<td>13</td>
</tr>
<tr>
<td>2C. APPROXIMATIONS OF THE POTENTIAL V IN THE STRIPPING AMPLITUDE</td>
<td>23</td>
</tr>
<tr>
<td>2D. SPECIFIC EXAMPLES OF THE (α,p) FORM FACTOR</td>
<td>25</td>
</tr>
<tr>
<td>2D1. Shell Model States</td>
<td>25</td>
</tr>
<tr>
<td>2D2. Microscopic Form Factors</td>
<td>31</td>
</tr>
<tr>
<td>2D3. Triton-Cluster Form Factor</td>
<td>33</td>
</tr>
<tr>
<td>2E. L-SPACE LOCALICATION AND RADIAL CUT-OFFS</td>
<td>37</td>
</tr>
<tr>
<td>2F. COMPARISON OF RELATIVE CROSS SECTIONS IN THE (α,p) REACTION</td>
<td>41</td>
</tr>
<tr>
<td>2F1. Relative Cross Sections in the (α,p) Reaction</td>
<td>41</td>
</tr>
<tr>
<td>2F2. Comparison of the (α,p) Data with Single Proton Transfer Data</td>
<td>42</td>
</tr>
<tr>
<td>2G. SELECTION RULES FOR THE DIRECT (α,p) REACTION</td>
<td>46</td>
</tr>
<tr>
<td>CHAPTER 3. THE $^{50,52,54}$Cr(α,p)$^{53,55,57}$Mn REACTIONS AT 18 AND 26 MeV AND THE $^{55}$Mn(p,p'γ) P-γ COINCIDENCE EXPERIMENT AT $E_p = 7.975$ MeV</td>
<td>49</td>
</tr>
<tr>
<td>3A. INTRODUCTION</td>
<td>49</td>
</tr>
<tr>
<td>3B. $^{50,52,54}$Cr(α,p)$^{53,55,57}$Mn AT $E_α = 18$ MeV</td>
<td>50</td>
</tr>
<tr>
<td>3C. $^{50,52,54}$Cr(α,p)$^{53,55,57}$Mn AT $E_α = 26$ MeV</td>
<td>68</td>
</tr>
<tr>
<td>(1)</td>
<td>Proton Spectra</td>
</tr>
<tr>
<td>(2)</td>
<td>J-Dependence of the (α, p) Reaction</td>
</tr>
<tr>
<td>(3)</td>
<td>Population of Hole States in 53, 55Mn</td>
</tr>
<tr>
<td>(D)</td>
<td>DWBA ANALYSIS</td>
</tr>
<tr>
<td>(E)</td>
<td>DISCUSSION 53Mn</td>
</tr>
<tr>
<td>(F)</td>
<td>DISCUSSION 55Mn</td>
</tr>
<tr>
<td>(G)</td>
<td>55Mn(p,p'γ) p-γ COINCIDENCE EXPERIMENT</td>
</tr>
<tr>
<td>(H)</td>
<td>DISCUSSION 57Mn</td>
</tr>
<tr>
<td>(I)</td>
<td>SUMMARY AND CONCLUSIONS</td>
</tr>
</tbody>
</table>

CHAPTER 4. THEORETICAL INTERPRETATIONS OF THE NUCLEAR STRUCTURE OF 53, 55Mn

(A) THEORETICAL STUDIES OF 53Mn

(1) Shell Model Interpretations | 150 |
(2) Unified Rotational Model of 53Mn | 158 |

(B) THEORETICAL STUDIES OF 55Mn

(1) Shell Model Interpretations | 159 |
(2) Unified Rotational Model of 55Mn | 163 |

(C) CONCLUSIONS | 164 |

CHAPTER 5. SUMMARY AND CONCLUSIONS | 167 |

APPENDIX 1. RECURSION RELATIONS FOR THE TRANSFORMATION COEFFICIENTS FROM SPHERICAL TO CARTESIAN COORDINATES FOR THE HARMONIC OSCILLATOR

(A) INTRODUCTION | 172 |
(B) RECURSION RELATIONS | 173 |
(C) CLOSED FORM FOR Azzz, oqsq | 176 |

APPENDIX 2. EXPERIMENTAL DETAILS

(A) A PRODUCTION TECHNIQUE FOR SELF-SUPPORTING Cr FOILS | 178 |
(B) TARGET ANGLE EFFECTS ON RESOLUTION | 181 |
(C) NORMALIZATION AND ABSOLUTE CROSS SECTIONS | 184 |
ABSTRACT

The $^{50,52,54}$Cr($\alpha$,p)$^{53,55,57}$Mn reactions were studied at 18 and 26 MeV bombarding energies. From the 26 MeV data angular distributions for 95 levels were obtained. Nearly all these angular distributions can be described by the distorted wave Born approximation theory assuming a quasi-triton transfer process. In contrast to this, very few of the angular distributions measured at 18 MeV could be described by the DWBA theory.

The spectroscopic knowledge of $^{57}$Mn has been greatly enhanced because for the first time excitation energies and tentative spin-parity assignments for levels above 2.2 MeV are proposed. The $Q_{\alpha p}$-value for the $^{54}$Cr($\alpha$,p)$^{57}$Mn has been measured and is $-4.302 \pm 0.008$ MeV. A total of 42 new states in $^{57}$Mn have been observed. In $^{55}$Mn two new levels at 6.069 and 6.164 MeV were located. Using the J-dependence of the ($\alpha$,p) reaction spin assignments were made for levels in $^{53,55}$Mn for which previous single particle transfer data had established the orbital angular momentum transfers. In $^{53}$Mn 8 proton unbound states were populated, the angular distributions for which showed no marked differences from those for the bound states.

The present (\alpha,p) data disagree with the hypothesized passive role of the transferred neutrons in determining the cross section. Large differences were found for the relative excitation strengths of levels in the (\alpha,p) reaction compared to the relative strengths in the ($^3$He,d) reaction. In particular, several states in $^{53,55}$Mn were excited in the (\alpha,p) reaction which were not populated in the ($^3$He,d) reaction. A notable example of this is a spin 1/2$^+$ level at about 1.292 MeV in $^{55}$Mn which is excited in multi-nucleon transfer but not in single proton transfer.

Several interesting aspects of the (\alpha,p) reaction mechanism
have also been observed in the 26 MeV data. An \( L = 3 \) J-dependence has been found by comparing states in \(^{53,55}\text{Mn}\) which were excited by \( L_p = 3 \) transfer in single proton transfer reactions. The well established \( L = 1 \) J-dependence has also been observed in the present work. The mechanism apparently is stripping rather than knock-out because of the absence of suspected hole states in \(^{53,55}\text{Mn}\) from the experimental spectra. The \((\alpha,p)\) reaction must occur near the nuclear surface as opposed to the interior because of the marked similarity of the angular distributions for states in all three isotopes.

The data have been discussed in relation to the theoretical studies of \(^{53,55}\text{Mn}\). The work of Benson [1975] seems best able to explain the present results for \(^{53}\text{Mn}\) although the deficiencies of their analysis, which they themselves pointed out, are apparent in the \((\alpha,p)\) data.
CHAPTER 1
INTRODUCTION

The study of nuclear structure is heavily dependent upon nuclear reactions. It is only by observing the nucleus in its various states that any substantial understanding, beyond that determined from its ground state properties, can be obtained. In nature the nucleus is almost invariably in its ground state. This is because the ambient temperature, even in the most extreme conditions, is usually far smaller than the energy of the first excited state. Even in the stellar medium, only a small fraction of the total mass is involved at any moment in thermonuclear reactions. Any gamma emission from nuclear states produced in the stellar interior suffers sufficient collisions before it reaches the surface that the electromagnetic spectrum contains negligible information about the excited nuclear states from which the \( \gamma \)-rays originated. Consequently, excited nuclear states must be artificially produced in the laboratory.

The many-body nature of the nucleus has several important consequences upon the types of nuclear reactions that can occur. One of the most general properties of the few "many"-body systems is the spacing between adjacent energy levels. For particle stable states, i.e., bound states, the wave function is \( \psi(\xi_1, \ldots, \xi_A, t) = \phi(\xi_1, \ldots, \xi_A) e^{-iEt/\hbar} \). The probability, \( |\psi|^2 = \phi(\xi_1, \ldots, \xi_A)|^2 \), of finding the particles with coordinates \( \xi_1, \ldots, \xi_A \) is independent of time. For
unbound states the probability does not possess such a simple time
dependence. The unbound states have much larger widths than the bound
states and whereas the bound states can usually decay only by \( \gamma \)-emission,
the particle unstable states have many more channels of decay open to
them. Hence, if we introduce an energy \( E \) into the system beyond the
binding energy of a nucleon, we can not say that the nucleus is in a
particular state because we can generally expect that several overlap­
ning states are present. This feature of overlapping states introduces
a radically different time dependence for the nuclear system. This can
be seen from the following argument of Weisskopf [1950] and Friedmand
and Weisskopf [1955]. In the region of strongly overlapping levels the
system can be described by

\[
\psi(\tau_1, \ldots, \tau_A, t) = \sum_n a_n \phi_n(\tau_1, \ldots, \tau_A) e^{-iE_n t/\hbar}.
\]

Suppose that the levels are separated by an average spacing \( \Delta \varepsilon \) so that
\( E_n = E_0 + n\Delta \varepsilon \). Then

\[
\psi = e^{-iE_0 t/\hbar} \sum_n a_n \phi_n e^{-i\Delta \varepsilon t/\hbar}
\]

and

\[
|\psi|^2 = \left| \sum_n a_n \phi_n e^{-i\Delta \varepsilon t/\hbar} \right|^2.
\]

From (1.3) we see that the system has a period of

\[
T = \frac{2\pi\hbar}{\Delta \varepsilon}.
\]

The period \( T \) in (1.4) actually corresponds to the time required for the
A particles to regain the same coordinates they occupied at time \( t - T \).
Since the level spacing becomes smaller as the energy of the system is
increased, the repetition rate in (1.4) becomes longer. The motion of the system becomes more complicated because (1.1) will contain more terms.

In order for the system to actually pass through a cycle of such complicated motion it must not have decayed before reaching $T$. If the system has been able to go through several periods $T$, then the subsequent emission of particles is liable to leave the residual nucleus in a complicated state since the decaying nucleus started in a complicated configuration. At higher bombarding energies one can expect that the system will decay in a much shorter relative time.

If the residual state has a complicated structure such that it requires the compound system to pass through several equally complicated configurations, then the production rate of the residual state will decline as the bombarding energy is increased because of the greater number of accessible channels open for decay with which it must compete. On the other hand, a state with a simple configuration does not require that the compound system proceeds through the cycle $T$, and hence its production rate will not decline as dramatically as that for the complicated state.

The reactions producing these two types of states are actually the two extreme kinds of nuclear reaction. The statistical nuclear reactions require the long-lived compound system. The direct reactions responsible for the production of "simple" states are characterized by short times, i.e., much smaller than $T$. Between these two extremes lies a complicated body of reactions that are not in fact very well understood due to the complexities of the processes.

The preferential reaction mode depends then both on the
bombarding energy and the mass of the compound system since \( \Delta \epsilon \) is a function of mass. The compound systems of interest for the \( ^{50,52,54}\text{Cr}(\alpha,p) \) reactions are \( ^{54,56,58}\text{Fe} \). The level spacing for \( ^{56}\text{Fe} \) can be estimated by using the results of Huizenga [1969] and Katsanos [1970] on the level densities of \( ^{56}\text{Fe} \). These authors studied the level density of \( ^{56}\text{Fe} \) to about 23 MeV excitation. If their results are extrapolated to the 25.6 and 32.6 MeV excitation appropriate for 18 and 26 MeV \( \alpha \)-particles, then the level densities yield

\[
2 \times 10^{-3} \text{ keV} \leq \Delta \epsilon \leq 3 \times 10^{-3} \text{ keV} \quad \text{for } 3 \leq J \leq 10 \text{ at } E_x = 25.6 \text{ MeV}
\]

and

\[
\Delta \epsilon \approx 10^{-4} \text{ keV} \quad \text{for } 3 \leq J \leq 10 \text{ at } E_x = 32.6 \text{ MeV}.
\]

The corresponding times from (1.4) are

\[
T_{18} \approx 2 \times 10^{-15} \text{ sec and } T_{26} \approx 4 \times 10^{-14} \text{ sec.}
\]

From Huizenga [1969] we can also estimate the ratio of the width \( \Gamma \) of the levels to the level spacing \( D \) for levels of spin \( J^* = 0^* \). These ratios are

\[
1,200 \leq \Gamma / D_{0^*,\pi} \leq 2,500 \quad \text{for } E_x = 25.6 \text{ MeV}
\]

and

\[
100,000 \leq \Gamma / D_{0^*,\pi} \leq 250,000 \quad \text{for } E_x = 32.6 \text{ MeV}.
\]

At both bombarding energies the ratios of \( \Gamma / D_{0^*,\pi} \) are high. However, from these ratios we can not easily decide which reaction mode will take precedence. This question will be pursued further in Chapter 3 where the experimental results for the 18 and 26 MeV \((\alpha,p)\) reactions are presented.

The interests in studying the \( ^{50,52,54}\text{Cr}(\alpha,p)^{53,55,57}\text{Mn} \) reactions are twofold, namely, the spectroscopy of the \( ^{53,55,57}\text{Mn} \)
isotopes and the \((\alpha, p)\) reaction mechanism itself. The single particle structure of \(^{53,55}\text{Mn}\) has been well studied using the \((^3\text{He}, d)\) reaction by O'Brien [1967], Rapaport [1969] and Gunn [1976]. The \((^7\text{Li}, ^6\text{He})\) reaction was also performed by Gunn [1976] to exploit the J-dependence of that reaction to measure spins in \(^{53}\text{Mn}\). Deuteron pick-up reactions leading to \(^{55}\text{Mn}\) have also been performed by Peterson [1971] with some highly surprising results relating to an unexpected \(1/2^-\) state degenerate with the known \(11/2^-\) state at 1.292 MeV. This \(1/2^-\) state was not detected in single proton transfer reactions. The greater degrees of freedom allowed by the \((\alpha, p)\) reaction can perhaps elucidate the nature of this state. A detailed discussion of this state and the experimental results obtained in \((\alpha, p)\) and \(^{55}\text{Mn}(p, p'\gamma)^{55}\text{Mn}\) can be found in Chapter 3.

Two high resolution reactions designed to measure the excitation energies of \(^{55}\text{Mn}\) and \(^{53}\text{Mn}\) have also been done; for \(^{55}\text{Mn}\) the \((p, p')\) reaction of Katsanos [1967], and for \(^{53}\text{Mn}\) the \(^{56}\text{Fe}(p, \alpha)^{53}\text{Mn}\) reaction of Tarara [1976]. Other spectroscopic studies of \(^{53,55}\text{Mn}\) will be mentioned where appropriate in Chapter 3 where the individual levels of \(^{53,55,57}\text{Mn}\) are discussed.

The level structure of \(^{53}\text{Mn}\) has received a great deal of theoretical attention. Some of the theoretical work on this nucleus is considered in Chapter 4. However, it is worthwhile mentioning that Benson [1975] concluded that neutron excitations were the dominant portions of the \([(1f_{7/2}^2)_{j}^{l}]\) configurations. The single proton transfer studies mentioned above are sensitive to the excited proton configurations. The \((\alpha, p)\) reaction, since it transfers three particles, can excite such configurations. There is also evidence that two particle excitations out of the \(1f_{7/2}\) shell should appear at around 2.5 MeV excitation in \(^{53}\text{Mn}\). Such configurations are ideally suited for
investigation by the \((\alpha,p)\) reaction because the neutrons can be transferred to any unoccupied orbit. The insight that the \((\alpha,p)\) reaction has contributed to such states plus the other evidence is discussed in Chapter 4. The use of the \((\alpha,p)\) reaction can then be viewed as a test of the assumed inert nature of the \(^{40}\text{Ca}\) core used in many of the theoretical interpretations of \(^{53}\text{Mn}\) detailed in Chapter 4. In particular, since single proton transfer studies are available, deviations in the relative strengths of states populated in the \((^3\text{He},d)\) as compared to \((\alpha,p)\) reactions will point to an active role for the transferred neutrons. The comparison of the present \((\alpha,p)\) data and the previous \((^3\text{He},d)\) studies is made in Chapter 3, and the interpretation of this comparison can be found in Chapter 4.

In contrast to \(^{53}\text{Mn}\), \(^{55}\text{Mn}\) has not received as extensive a theoretical treatment. The treatment by Peterson [1971] which included \((3p-4h)\) and \((2p-3h)\) configurations is the most extensive shell model analysis reported, although they were primarily interested in explaining the doublet at 1.292 MeV which they discovered. The proton transfer reaction \((^3\text{He},d)\) can only test a portion of the wave function. The influence of the extra core neutrons beyond the \(N=28\) core is liable to add \(I_n > 0\) components into the wave functions, even for the low lying states. For example, based on the shell model, the states \(11/2^-\), \(9/2^-\), \(3/2^-\), \(5/2^-\) and \(7/2^-\) in both \(^{53,55}\text{Mn}\) depicted in Fig. 1.1 could be composed of the following configurations:
\[ \psi_{J}^{(53\text{Mn})} = a \left\{ \pi \, f_{7/2}^{-3} \, v_{p} = 1 \right\} 7/2^{-} \, \delta_{J,7/2} + b \left\{ \pi \, f_{7/2}^{-3} \, v_{p} > 1 \right\} J \] (1.5)

and

\[ \psi_{J}^{(55\text{Mn})} = \left\{ a_{0} \left\{ \pi \, f_{7/2}^{-3} \, v_{p} = 1 \right\} 7/2^{-} \, \delta_{J,7/2} + a_{1} \left\{ \pi \, f_{7/2}^{-3} \, v_{p} > 1 \right\} J \right\} \]

\[ \times (v_{j}^{2})_{0} + b \left\{ \left( \pi \, f_{7/2}^{-3} \, v_{p} = 1 \right) 7/2^{-} \, (v_{j_{1},j_{2}})_{I_{P}} \right\} J \]

\[ + c \left\{ \left( \pi \, f_{7/2}^{-3} \, v_{p} > 1 \right) I_{P} \, (v_{j_{1},j_{2}})_{I_{P}} \right\} J \] (1.6)

If the ground states of \(^{50,52}\text{Cr}\) were purely \(v_{p} = 0, \, v_{n} = 0\), then only the \(7/2^{-}\) state in \(^{53}\text{Mn}\) could be excited since the added proton can only change the seniority by 1. Thus from (1.5) both the \(^{3}\text{He},d\) and \((\alpha,p)\) reactions should give similar results. For the states in \(^{55}\text{Mn}\) however the \(v_{n} = 2\) components in (1.6) are available. The \(^{3}\text{He},d\) reaction would still not excite any state described by (1.6) except for the \(7/2^{-}\) state via the configuration of the \(a_{0}\) term. However, the \((\alpha,p)\) reaction allows one to investigate the \(a_{0}\) and \(b\) terms in (1.6) and hence all the
states of spin $J$ can be produced. There is in fact no other method of investigating the neutron configurations of the excited states of $^{55}\text{Mn}$ (other than the states in the neutron core) than by multi-nucleon transfer. The ($\alpha$,p) reaction is the simplest tool available to study the distribution of the neutrons beyond the $N = 28$ core in $^{55}\text{Mn}$. The data on the population of the levels in Fig. 1.1 are in Chapter 3, and the discussion in Chapter 4.

Besides the accessibility of the more complex configurations, the $J$-dependence of the ($\alpha$,p) reaction makes it an attractive choice for assigning spins. The $L_p$ transfers measured in the ($^3\text{He},d$) reactions allow unambiguous $J''$ assignments to be made in conjunction with the ($\alpha$,p) data. Both $^{53,55}\text{Mn}$ have been investigated by ($^3\text{He},d$) with the preponderance of measured $L_p$ transfers being $L_p = 1$ or $L_p = 3$. The $J$-dependence for $L_p = 1$ has already been established in several studies of the ($\alpha$,p) or (p,\alpha) reactions. The availability of $L_p = 3$ transitions in $^{53,55}\text{Mn}$ allow for any $L_p = 3$ J-dependence to be tested experimentally. The DWBA calculations using a triton cluster transfer indicate that a $J$-dependence for $0 \leq 40^\circ$ exists for $L_p = 3$. The angular distributions for the $L_p = 3$ transfers in $^{53,55}\text{Mn}$ are shown in Chapter 3.

The possibility of investigating the level structure of $^{57}\text{Mn}$ via the $^{54}\text{Cr}(\alpha,p)^{57}\text{Mn}$ reaction was a great attraction. At the beginning of the $^{54}\text{Cr}(\alpha,p)$ experiment no information was available on the excited state of $^{57}\text{Mn}$. During the course of the present investigation a report on the levels of $^{57}\text{Mn}$ below 2.2 MeV was published [Mateja, 1976]. These authors studied the $\gamma$-decay of $^{57}\text{Mn}^*$ following $^{54}\text{Cr}(\alpha,p)$. Their spin assignments are based upon p-$\gamma$ angular correlations with protons detected near 180$^\circ$. They reported no proton angular distributions. The only single particle transfer reaction capable of producing $^{57}\text{Mn}$ is
$^{58}\text{Fe}(d,^3\text{He})^{57}\text{Mn}$. The measurement of the $^{53,55}\text{Mn}$ spectra in conjunction with those for $^{57}\text{Mn}$ by the $(\alpha,p)$ reaction enables an experimental comparison of the various $J^\pi$ transfers. It is not, a priori, obvious that the DWBA theory can fit the angular distributions, especially at the lower energy, i.e., 18 MeV. Previous $(\alpha,p)$ and $(p,\alpha)$ studies in other nuclei were generally well described by DWBA for higher $\alpha$-particle energies. The 26 MeV data are likely to be amenable to a DWBA treatment. The experimental and theoretical angular distribution for the three isotopes are presented in Chapter 3. It is also of interest to determine whether or not the low-lying level structure of $^{57}\text{Mn}$ resembles that of $^{53,55}\text{Mn}$ depicted in Fig. 1.1.

Another useful property of the $(\alpha,p)$ reaction is its selectivity with regard to isospin. As will be described in Chapter 2, the $(\alpha,p)$ reaction can only excite states with $T = T_3 = (N-Z)/2$ in the residual nucleus. Here $T_3$ is the isospin projection of the residual nucleus. This feature enables the $T_3 = T_3$ and $T_3 = T_3 + 1$ states populated in $(^3\text{He},d)$ to be sorted from one another. For low-lying states this is usually not a problem since they are generally $T_3$ states. However, when trying to locate the total $T_3$ strength the higher excited states present an ambiguity since $(^3\text{He},d)$ can excite both $T_3$ and $T_3$ states.

As well as nuclear structure aspects, the use of the $(\alpha,p)$ reactions on $^{50,52}\text{Cr}$ allows for further insight into the reaction mechanism. For example, does the reaction occur by stripping three nucleons from the incident $\alpha$-particle or does it proceed through knock-out whereby a proton from the target is ejected and the incident $\alpha$-particle is captured? This question can be partially answered using the $(\alpha,p)$ reaction by noting the absence or presence of suspected hole states in $^{53,55}\text{Mn}$. The $(^3\text{He},d)$ reactions have located states via $L_p = 0$
and $L_p = 2$ transitions in both $^{53,55}\text{Mn}$ and their interpretation has been that of holes in the s-d shell.

Another question is that of the role of the transferred neutrons in the reaction cross section. A comparison of the $^{64}\text{Zn}(d,^3\text{He})^{63}\text{Cu}$ and $^{66}\text{Zn}(p,\alpha)^{65}\text{Cu}$ reactions reported by Schiffer [1967] suggested that the transferred neutrons play a passive role in determining the cross section. That is, the relative intensity of levels produced in proton and triton pick-up are quite similar. Bucurescu [1972] compared the $^{64}\text{Ni}(^3\text{He},d)^{65}\text{Cu}$ and $^{62}\text{Ni}(\alpha,p)^{65}\text{Cu}$ cross sections and also concluded that the neutrons play a minor role in determining the relative cross sections. On the other hand, a comparison of $^{206}\text{Pb}(t,\alpha)^{205}\text{Tl}$ and $^{208}\text{Pb}(p,\alpha)^{205}\text{Tl}$ by Glashausser [1974] indicated that the neutrons were important in determining the relative cross sections. Smits [1976] arrived at the same conclusion by studying $^{118}\text{Sn}(p,\alpha)^{115}\text{In}$ and comparing the cross sections to $^{116}\text{Sn}(d,^3\text{He})^{115}\text{In}$. The relative strengths differed greatly. In the study of $^{12}\text{C}(\alpha,p)^{15}\text{N}$, Falk [1975] found it necessary to include two-neutron configurations involving $I_n > 0$ to account for the relative cross sections. The availability of ($^3\text{He},d$) data for $^{53,55}\text{Mn}$ enables a comparison to be made for these nuclei. The question of the transferred neutrons' role in the reaction obviously has a great deal to do with the structure of the states. A comparison of the $(\alpha,p)$ and ($^3\text{He},d$) data for $^{53,55}\text{Mn}$ is made in Chapter 3.

Yet another question regarding the $(\alpha,p)$ reaction is the bearing various regions of configuration space have upon the differential cross section. The similarity or lack thereof in the shapes of the angular distributions is an indicator of the importance of the nuclear interior as will be discussed in Chapter 2. One might also
consider the success or failure of the triton cluster transfer model in the DWBA theory as a measure of the importance of the different regions of configuration space. Falk [1975] has shown that a microscopic treatment of the three particle transfer produces in general a quite different form factor in the nuclear interior than one generated by the simpler triton-cluster transfer. It is only beyond the nuclear surface that the microscopic and point-triton form factors agree in shape. Experimentally one can expect quite different form factors for $^{53,55}\text{Mn}$ because of the closed neutron shell in $^{53}\text{Mn}$ compared to the f-p orbitals available to the neutrons in $^{55}\text{Mn}$. A comparison of the low-lying levels of $^{53,55,57}\text{Mn}$ will therefore be of considerable value in answering this question.

In summary, the employment of the ($\alpha,p$) reaction to study $^{53,55,57}\text{Mn}$ provides the opportunity for a greatly increased understanding of these nuclei. The single particle aspects of $^{53,55}\text{Mn}$ having been extensively studied, the next logical step in a theoretical understanding is to investigate the multi-particle excitations. For $^{57}\text{Mn}$ the ($\alpha,p$) reaction will provide information on the level structure of this nucleus which has only recently been studied by Mateja [1976].

Various aspects of the reaction mechanism can also be investigated by using the previously obtained information on $^{53,55}\text{Mn}$.

The remainder of this thesis consists of a theoretical development of the ($\alpha,p$) reaction with a description of the triton cluster transfer model and its approximations in Chapter 2. The experimental results of the $^{50,52,54}\text{Cr}(\alpha,p)^{53,55,57}\text{Mn}$ reactions at 18 and 26 MeV and the $^{55}\text{Mn}(p,p'\gamma)$ p-\gamma coincidence experiment are presented in Chapter 3. A review of the theoretical work done on $^{53,55}\text{Mn}$ and a
comparison with experiment is presented in Chapter 4. The conclusions reached in these experiments are presented in Chapter 5. Appendix 2 discusses the mechanics of how the (α,p) experiments were performed and Appendix 1 demonstrates that recursion relations can be simply obtained for the coefficients that transform the harmonic oscillator states from spherical to cartesian coordinates. These coefficients are useful for a particular approach to the microscopic treatment of the three particle transfer as explained in Chapter 2.
CHAPTER 2

THE DWBA THEORY OF THE A(α,p)B REACTION

INTRODUCTION

The distorted wave Born approximation of the A(α,p)B reaction is discussed in this chapter. The importance of using antisymmetrized wave functions and their relationships to the various reaction processes are discussed in section (A). Sections (B) and (C) consider the approximations of the potential V and the form factors. The lack of L-space localization for most of the states excited in the (α,p) reaction and its consequences upon the reliability of the calculated cross sections is discussed in section (D). A comparison of the single proton and triton transfer relative cross sections is made in section (E). The selection rules for the (α,p) reaction are presented in section (F).

(A) EFFECTS OF ANTISYMMETRIZATION

In Chapter 1 nuclear reactions were broadly classified into two types which were categorized by a characteristic reaction time. Direct reactions occur in short times, i.e., of the order of the transit time of the incident projectile across the target. Direct reactions can be further classified into various processes. Consider the A(α,p)B reaction illustrated in Figs. 2.1.
The outgoing proton can either arise from the target A as illustrated in Fig. 2.1(a) or from the incident α-particle as illustrated in Fig. 2.1(b). The process illustrated in Fig. 2.1(b) is referred to as the stripping process. The processes in Fig. 2.1(a) are known as exchange processes. A complete DWBA treatment of the (α,p) reaction requires the calculation of all the transition amplitudes associated with the processes in Figs. 2.1. Generally only the stripping process of Fig. 2.1(b) is calculated, but it is not clear that
this is a good approximation for the (α,p) reaction because the high
stability of the α-particle is conducive to α-particle clustering on the
nuclear surface, which favours the capture of the α-particle as depicted
in Fig. 2.1(a).

Austern [1970] has demonstrated that all the processes in
Figs. 2.1 are automatically included in the DWBA theory if a totally
antisymmetrized entrance channel wave function is employed. This will
be demonstrated for the A(α,p)B reaction.

Direct nuclear reaction theories have been developed in
several articles; see Austern [1970] or Hodgson [1971] for example.
However, it will be useful to calculate the transition amplitude
\[ T_{αp}(Ω, k_p) \]
explicitly including the effects of antisymmetrization, since
it is not readily available in the literature. The general expressions
given by Austern [1970] are the starting point.

The transition amplitude \( T_{αp}(Ω, k_p) \) is related to the
differential cross section by

\[ \sigma_{αp}(Ω, k_p) = \frac{M_α M_p}{(\pi h^2)^2} \left( \frac{k_p}{k_α} \right) \left( \frac{Z_A}{Z_B} \right)^{\frac{1}{2}} \left( \frac{N_A}{N_B} \right)^{\frac{1}{2}} | T_{αp}(Ω, k_p) |^2. \]  

The quantities \( M_α \) and \( M_p \) are the reduced masses in the entrance and exit
channels; \( k_α \) and \( k_p \) are the magnitudes of the wave numbers in the α-
particle and proton channels; \( Z \) and \( N \) are the proton and neutron
numbers of the target and residual nucleus. The factorials in (2.1)
arise from the method of antisymmetrizing the incoming and outgoing
channel wave functions assuming that protons and neutrons are distinct
particles. The antisymmetrization is accomplished by permutations of
the particle labels. If the labelling is chosen so that protons \( 0_p \) and
\( 1_p \) and neutrons \( 0_n \) and \( 1_n \) are in the incident α-particle and protons
and neutrons \( Z_p, n_1, \ldots, n_n \) are in the target nucleus \( A \), then the DWBA transition amplitude is

\[
\mathcal{T}_{ap} = \sum_{P, N, Z} (-1)^{P + N + Z} \left( \chi_p^{(-)}(0_p, \mathcal{B}) \phi(0_p) \psi_B(1_p \ldots Z_p; 0_1 n \ldots n_n) |V| \right.
\]

\[
\times P_{Z} P_{N} \psi_\alpha(0_1 p_1; 0_1 n_1) \psi_A(2_p \ldots Z_p; 2_n \ldots n_n) \chi_\alpha^{(+)}(0_1 p_1; 0_1 n_1; \mathcal{A}) \bigg),
\]

where

\( \chi_p^{(-)}(0_p, \mathcal{B}) \) is the proton distorted wave with proton \( 0_p \) outgoing and \( \mathcal{B} \) is the location of \( B \),

\( \phi(0_p) \) is the spin wave function of outgoing proton \( 0_p \),

\( \psi_B \) is the antisymmetrized residual state wave function,

\( \psi_\alpha \) is the antisymmetrized \( \alpha \)-particle internal wave function,

\( \psi_A \) is the antisymmetrized target ground state,

\( \chi_\alpha^{(+)} \) is the incoming \( \alpha \)-particle distorted wave and \( \mathcal{A} \) is the location of target \( A \),

\( P_{Z} \) permutes proton labels \textit{between} the \( \alpha \)-particle and the target,

\( P_{N} \) permutes neutron labels \textit{between} the \( \alpha \)-particle and the target

\[
V = \sum_{i=1_p}^{Z_p, N_n} (v_{0_i p}^i + v_{0_j p}^j) - U_B(0_p),
\]

in which

\( v_{0_p k} \) is the interaction between the outgoing proton \( 0_p \) and nucleon \( k \) in the residual nucleus \( B \) and \( U_B(0_p) \) is the optical model potential in the proton channel.

The permutation operators \( P_{Z} \) and \( P_{N} \) produce a totally antisymmetrized wave function in the entrance channel. The effects of these operators will be dealt with individually.
First consider the neutron permutations. For \( N_A \) neutrons in the target \( A \) and 2 neutrons in the \( \alpha \)-particle we obtain a total of \( (N_A + 2)!/(2!N_A! \) permutations of the neutron labels. A typical permutation is \( 0_n \leftrightarrow 2_n \) which adds a term to \( T_{\alpha p} \) equal to

\[
- \left( \phi(p) \chi_p^{(-)}(0_p, \alpha_p)\psi_B(1_p \ldots Z_p; 0 \ldots n \ldots N_p) \right| V \right| \psi_A(0_p, 1_p, 2_p; 1_n) \chi_A^{(+)}(0_{1_n}, 2_{1_n}) \psi_A(2_p \ldots Z_p; 0 \ldots n \ldots N_p) | V | \psi_A(0_p, 1_p, 2_p; 1_n) \chi_A^{(+)}(0_{1_n}, 2_{1_n}; \alpha_A) \right)
\]

But if we interchange \( 0_n \leftrightarrow 2_n \) in \( \psi_B \) on the left hand side of expression (2.4) we obtain

\[
\left( \phi(p) \chi_p^{(-)}(0_p, \alpha_p)\psi_B(1_p \ldots Z_p; 0 \ldots n \ldots N_p) \right| V \right| \psi_A(0_p, 1_p, 2_p; 1_n) \chi_A^{(+)}(0_{1_n}, 2_{1_n}) \psi_A(2_p \ldots Z_p; 0 \ldots n \ldots N_p) | V | \psi_A(0_p, 1_p, 2_p; 1_n) \chi_A^{(+)}(0_{1_n}, 2_{1_n}; \alpha_A) \right)
\]

Note that the sign changes from (2.4) to (2.5) because \( \psi_B \) is already antisymmetric. It will be noted that (2.5) has the same form as the zero order permutation of (2.2) except that what was called neutron \( 0_n \) in (2.2) is called neutron \( 2_n \) in (2.5). Obviously this can not have any effect on the physical process. Hence, all the neutron permutations yield the same result as the zero order permutation which means that the effect of \( P_N \) is simply to multiply the zero order permutation by the total number of neutron permutations. The transition amplitude then is

\[
T_{\alpha p} = \frac{(N_A + 2)(N_A + 1)}{2} \sum_{P_Z} (-1)^{P_Z} \left( \phi(p) \chi_p^{(-)}(0_p, \alpha_p)\psi_B(1_p \ldots Z_p; 0 \ldots n \ldots N_p) \right| V \right| \psi_A(0_p, 1_p, 2_p; 1_n) \chi_A^{(+)}(0_{1_n}, 2_{1_n}; \alpha_A) \right)
\]

For the case of the proton permutations there are as before \( (Z_A + 1)(Z_A + 2)/2 \) total number of permutations but these do not all yield the same result as the zero order permutation. There are three
types of proton permutations to consider:

\[ 0_p \leftrightarrow 2_p \] \hspace{1cm} (2.7a)

\[ 1_p \leftrightarrow 2_p \] \hspace{1cm} (2.7b)

\[ (0_p \leftrightarrow 2_p)(1_p \leftrightarrow 3_p) \] \hspace{1cm} (2.7c)

The number of permutations of type (2.7a) is \( Z_A \) because \( 0_p \) can be interchanged with any proton in target A. The same applies to permutation (2.7b). The null permutation contributes one term. If \( n_C \) is the number of permutations of type (2.7c) then

\[ 1 + Z_A + Z_A + n_C = \frac{(Z_A + 2)(Z_A + 1)}{2} \] \hspace{1cm} (2.8)

or

\[ n_C = \frac{Z_A(Z_A - 1)}{2} \] \hspace{1cm} (2.9)

By the same arguments as used in the neutron permutations the permutations of (2.7b) are all equivalent to the zero order permutation and hence there are another \( Z_A \) null permutation terms to (2.6).

However, the permutations of (2.7a) are not equivalent to the null permutation because \( 0_p \) and \( 2_p \) on the left hand side of (2.6) appear in two separate wave functions. The above arguments about both the permutations of (2.7a) and (2.7b) lead to the conclusion that

\[ (1_p \leftrightarrow 3_p)(0_p \leftrightarrow 2_p) = Z_A(0_p \leftrightarrow 2_p) \] \hspace{1cm} (2.10)

for the (2.7c) permutations. Hence, the permutations of the proton labels are

\[ \sum (-1)^P_{Z} P_{Z} = 1 + Z_A - \frac{Z_A(Z_A - 1)}{2} P_0 2_p \]

\[ = \left( 1 + Z_A - \frac{Z_A(Z_A + 1)}{2} P_0 2_p \right), \] \hspace{1cm} (2.11)
where $p_0^2$ interchanges protons $0$ and $2$. If (2.11) is substituted into (2.6) two terms of the transition amplitude arise which can be associated with the two types of processes in Figs. 2.1. The so-called direct, or stripping, term corresponding to Fig. 2.1(b) is

$$T_{\text{dir}}^{\text{direct}} = \frac{(1 + Z_a^1)(N_a^1 + 2)(N_A^2 + 1)}{2} \left[ x_p(-)(0, p, p, p, p, p, n, n, n) \phi(0, p, p, p, p, p, n, n, n) \right] \times |V| \psi_A(0, p, p, p, p, n, n, n, n) \phi_p(0, p, p, p, p, p, n, n, n) \chi_p^+(0, p, p, p, p, p, n, n, n, n, n)$$

$$+ \frac{Z_A(Z_A^1 + 1)(N_A^1 + 2)(N_A^2 + 1)}{4} \left[ x_p(-)(0, p, p, p, p, p, n, n, n) \phi(0, p, p, p, p, p, n, n, n) \right] \times |V| \psi_A(0, p, p, p, p, n, n, n, n) \phi_p(0, p, p, p, p, p, n, n, n) \chi_p^+(0, p, p, p, p, p, n, n, n, n, n)$$

(2.12)

from which we see that three nucleons from the incident $\alpha$-particle are transferred to the target nucleus $A$ and the remaining proton continues on. The exchange terms corresponding to Fig. 2.1(a) are

$$T_{\text{ex}}^{\text{exchange}} = \frac{Z_A(Z_A^1 + 1)(N_A^1 + 2)(N_A^2 + 1)}{4} \left[ x_p(-)(0, p, p, p, p, p, n, n, n) \phi(0, p, p, p, p, p, n, n, n) \right] \times |V| \psi_A(0, p, p, p, p, n, n, n, n) \phi_p(0, p, p, p, p, p, n, n, n) \chi_p^+(0, p, p, p, p, p, n, n, n, n, n)$$

(2.13)

from which we see that proton $0^p$ initially in the target $A$ is ejected and the four nucleons in the incident $\alpha$-particle are captured.

Equation (2.13) contains several exchange processes which depend upon various parts of the potential $V$. From (2.3) we deduce that the knock-out process is mediated by the interaction between the incident $\alpha$-particle and proton $0^p$ in the target, i.e.,

$$V_{\text{KO}} = V_{0^p 1^p} + V_{0^p 2^p} + V_{0^p 0^p} + V_{0^p 1^p} \cdot \quad (2.14)$$

A qualitative measure of the relative importance of the exchange and direct terms can be obtained by noting that the exchange processes are more complicated than the direct process [Austern, 1970] and by
considering the asymptotic character of the form factors. Consider the
knock-out amplitude in zero range, i.e.,

$$V_{KO} = V_0 \delta(r_\alpha - r_0)$$  \hspace{1cm} (2.15)$$

and write the target and residual states as parentage expansions of a
proton plus core C and a four particle state plus core C as illustrated
in Fig. 2.1(a):

$$\psi_A(p, \ldots Z, p; 0, 1, \ldots N) = \sum_{k, m} A_{km} \left[ \psi^k(p) \psi^m_c(p, \ldots Z, p; 0, 1, \ldots N) \right] J_A$$  \hspace{1cm} (2.16)$$

and

$$\psi_B(p, \ldots Z, 0, 1, \ldots N) = \sum_{n, m} B_{nm} \left[ \psi^n(p, \ldots Z, p; 0, 1, \ldots N) \right] J_B$$  \hspace{1cm} (2.17)$$

where the brackets \( [ \) indicate angular momentum coupling. If (2.15),
(2.16) and (2.17) are substituted into (2.13) then

$$T_{\alpha p}^{KO} \propto V_0 \sum_{k, n, m} A_{km} B^*_{nm} \left[ \chi^{(-)}(r_\alpha, r_B) \phi(0) \psi^n(p, \ldots Z, p; 0, 1, \ldots N) \right] \psi^k(r_\alpha)$$

$$\times \psi_\alpha(2; p, 1, \ldots N) \chi^{(+)}(2; p, 1, \ldots N) J_A$$  \hspace{1cm} (2.18)$$

For the stripping term in the zero range approximation

$$V_{\text{strip}} = V_0 \delta(r_t - r_0)$$  \hspace{1cm} (2.19)$$

where \( r_t \) is the triton coordinate. If the residual state is written as
an expansion of three particle states plus states of target A as in Fig.
2.1(b) then

$$\psi_B(p, \ldots Z, 0, 1, \ldots N) = \sum_{p, r} S_{pr} \psi^p(1; 0, 1, \ldots N) \psi^r_A(p, \ldots Z, p; 0, 1, \ldots N)$$  \hspace{1cm} (2.20)$$

Substitution of (2.19) and (2.20) into (2.12) yields
The asymptotic behaviour of the form factors for the two processes are

\[ \psi_n^* (1, 2 : 0, 1) \psi^k (r_\alpha) \sim e^{-k_\alpha r_\alpha} e^{-k_p r_\alpha} \]  

and

\[ \psi_p^* (1 : 0, 1) \sim e^{-k_t r_t} , \]  

where

\[ k_\chi = \sqrt{2M_\chi S_\chi / \hbar^2} . \]  

The quantities \( M_\chi \) and \( S_\chi \) are the reduced masses for particle \( \chi \) and the separation energy of \( \chi \) from the appropriate nucleus as depicted in Figs. 2.1. For the case of the \( ^5\text{Cr}(\alpha,p)^{57}\text{Mn} \) reaction say, the numerical values for the form factors \( F \) for the ground state transition are

\[ F_{KO} \sim e^{-2r_\alpha} \]  

and

\[ F_{strip} \sim e^{-1.5 r_t} . \]
Consequently in the important region outside the nuclear surface the knock-out form factor falls off more rapidly than the stripping form factor. On the other hand, it will be noted that the coefficient of the integral for the exchange processes in (2.13) is larger by a factor of $Z_A/2$ than the coefficient in (2.12) for the stripping process. If (2.23a) is increased by a factor of $12 (Z_A = 24$ for Cr) then the asymptotic values in (2.23) are equal at $r = 5.0 \text{ fm}$ or $12 F_{KO}/F_{strip} > 1$ for $r < 5.0 \text{ fm}$. The half density radius for $^{57}$Mn using $R = 1.2 A^{1/3}$ is $4.62 \text{ fm}$. Since $Z_A \approx 1/2 A$ and $R = 1.2 A^{1/3}$ it will be seen that the factors in front of the knock-out integral in (2.13) become progressively larger as the target mass increases and the knock-out form factor extends further and further beyond the half density radius relative to the stripping form factor.

These considerations can only be taken as a guide. Most calculations assume that the exchange terms are small and retain only the direct term [Austern, 1970].

Other than by explicit calculations of the knock-out amplitude it is possible to gauge the relative importance of the various processes experimentally. The knock-out process is capable of producing hole states. Small cross sections to known hole states are taken to mean that the dominant mechanism is stripping.

Previous work in the $1f_{7/2}$ shell on the Ca($\alpha,p$)Sc reactions by Ginaven [1970] has shown that the hole states in the scandium isotopes are weakly excited if at all. In $^{53,55}$Mn, O'Brien [1967] and Rapaport [1969] have located $L_p = 0$ and 2 states using the ($^3$He,d) reactions. These states are believed to be hole states. The excitation of the hole states in the ($\alpha,p$) reaction will be discussed in Chapter 3.
Only the stripping term is used in the present work and the remainder of this chapter deals with the stripping amplitude.

(B) APPROXIMATIONS OF THE POTENTIAL V IN THE STRIPPING AMPLITUDE

Before (2.12) is calculated, further approximations are generally made to the potential $V$. For brevity's sake, the explicit particle labelling in (2.12) will be reduced to $p = \xi_p$, $n = \xi_t$ and $2 \ldots Z \ldots Z p; 2 \ldots Z n = \xi_A$. The form factor of (2.12) is then

$$F(R_{pB}, R_{\alpha A}) = \langle \phi(\xi_p) \psi_B(\xi_t, \xi_A) | V | \psi_\alpha(\xi_p, \xi_t) \psi_A(\xi_A) \rangle. \quad (2.24)$$

The integration in (2.24) is over the internal quantum numbers and $R_{pB}, R_{\alpha A}, \Gamma_{pt}$ are the centre of mass separations of the subscripted nuclei as illustrated in Fig. 2.2.

FIG. 2.2

From (2.3) the potential $V$ is

$$V = V(\xi_p, \xi_t) + V(\xi_p, \xi_A) - U(R_{pB}) \quad (2.25)$$

where

$$V(\xi_p, \xi_A) = \sum_{i=2}^{Z_p} \sum_{j=2}^{N_n} (v_{0_i} v_{0_j}) \quad (2.26)$$
for example.

Substituting (2.25) into (2.24) yields

\[
F(R_{PB}, R_{A}) = \langle \phi(\xi_p) \psi_B(\xi_t, \xi_A) | V(\xi_p, \xi_t) | \psi(\xi_p, \xi_t) \psi_A(\xi_A) \rangle \\
+ \langle \phi(\xi_p) \psi_B(\xi_t, \xi_A) | V(\xi_p, \xi_A) - U(R_{PB}) | \psi(\xi_p, \xi_t) \psi_A(\xi_A) \rangle.
\]  

(2.27)

Both terms of (2.27) describe the disintegration of the \( \alpha \)-particle and the capture of the triton-cluster by the target \( A \). However, the potentials which mediate this process are quite different in the first and second terms. In the first term the interaction \( V(\xi_p, \xi_t) \) between the outgoing proton and the three particles in the transferred triton causes the \( \alpha \)-particle to break up. In the second term the potential \( V(\xi_p, \xi_A) \) is the interaction between the outgoing proton and the target nucleus \( A \). The predominant effect of this interaction is elastic scattering of protons off \( A \). The potential \( U(R_{PB}) \) is chosen to reproduce the elastic scattering of protons off \( B \) and hence

\( V(\xi_p, \xi_A) - U(R_{PB}) \) should largely cancel [Austern, 1970].

Smith [1969] has considered the effect of all three terms of \( V \) for the \( (d,p) \) reaction. The inclusion of the term \( V_{PA} - V_{PB} \) has appreciable effect only for light nuclei, for example \(^{12}\text{C}\). For \(^{28}\text{Si}\) Smith found that the difference between using \( V_{pn} \) and \( V_{pn} + V_{PA} - U_{PB} \) became negligible except for an imaginary term \( iW = V_{PA} - U_{PB} \) which arises because \( U_{PB} \) is complex whereas \( V_{PA} \) is the summed interaction of the proton with the nucleons in \( A \).

The \( (\alpha,p) \) reaction form factor is then approximated by the first term of (2.27). In order to avoid doing a finite range calculation the zero range form (see (2.19)) of the potential \( V(\xi_p, \xi_t) \) is often used [Sherr, 1962; Falk, 1973, 1975]. The stripping amplitude
is then given by

\[ T_{\text{strip}}^{\alpha p} = \frac{(Z_A + 1)(N_A + 2)(N_A + 1)}{2} \int dR_B dR_A \chi_p (R_A) \chi_p (R_B) \]

\[ \times \langle \phi (E_p, E_A) \psi_A (E_p, E_A) | V (E_p, E_A) | \psi_A (E_p, E_A) \rangle \psi_A (E_A) \chi_A (R_A) \). \]

(2.28)

(C) SPECIFIC EXAMPLES OF THE (\(\alpha, p\)) FORM FACTOR

(1) Shell Model States

If \(\psi_A\) and \(\psi_B\) are pure shell model states the integration over
the \(\xi_A\) variables is straightforward. Suppose in target A that \(Z_C\) and \(N_C\)
protons and neutrons are coupled to spins \(J^P_C\) and \(J^N_C\) respectively and
that \(z_a\) and \(n_a\) protons and neutrons in shells \(j_p\) and \(j_n\) couple to spins
\(J^P_p\) and \(J^N_n\) respectively. For the reaction \(A(\alpha, p)B\) we will assume that
the transferred proton goes into shell \(j_p\) and the neutrons go into shell
\(j_n\).

In order to bring \(\psi_A(Z_C, N_C; z_a, n_a)\) and \(\psi_B(Z_C, N_C; z_a + 1, n_a + 2)\)
into forms amenable to integration over the core variables \(Z_C\) and \(N_C\) we
can use the permutation operations \(P^A_Z\) and \(P^A_N\) of section (A). Then

\[ \psi_A^J = C_A \sum_{P^A_Z, P^A_N} (-1)^{P^A_Z + P^A_N} P^A_Z P^A_N \psi_C (Z_C, N_C) \psi_{j_p j_n} (z_a, n_a) \] \(J_A\) \]

(2.29)

and

\[ \psi_B^J = C_B \sum_{P^A_Z, P^A_N} (-1)^{P^A_Z + P^A_N} P^A_Z P^A_N \psi_C (Z_C, N_C) \psi_{j_p j_n} (z_a + 1, n_a + 2) \] \(J_B\) \]

(2.30)

where

\[ C_A = \sqrt{Z_C! Z_A! / Z_A!} \sqrt{N_C! N_A! / N_A!} \] \(2.31\)
and a similar expression holds for $C_B$. The core wave function $\Psi_C$ and the wave functions $\Psi_{jp}^{jn}$ are already individually antisymmetric. As before, the permutations are between the labels of $\Psi_C$ and $\Psi_{jp}^{jn}$.

To evaluate the integral over $\xi_A$,

$$\int d\xi_A \frac{J^*_B}{\Psi_B(\xi_t, \xi_A)} \frac{J}{\Psi_A(\xi_A)}$$

we note that for every permutation in (2.29) there is one permutation of (2.30) with the same arrangement of particle labels. For these permutations the factors $(-1)^{P_Z+P_N}$ yield a value of plus one. However, for the permutations of $\Psi_B^B$ in (2.30) which interchange a transferred particle with a core particle there is no labelling of $\Psi_A$ which will yield a non-zero overlap integral [Macfarlane, 1960]. Therefore there are as many terms in (2.32) as there are permutations of the particle labels in (2.29), and all the terms have the same value as that due to the initial labelling. Hence

$$\int d\xi_A \frac{J^*_B}{\Psi_B(\xi_t, \xi_A)} \frac{J}{\Psi_A(\xi_A)} = C_{C_B}^C \frac{Z_a!}{Z_a!} \frac{N_a!}{Z_a!} \frac{N_a!}{Z_a!}$$

$$\times \int d\xi_A \left[ \psi_C(Z_C^N) \psi_{jp}^{jn}(z_a+1, n_a+2) \right]^*$$

$$\times \left[ \psi_C(Z_C^N) \psi_{jp}^{jn}(z_a, n_a) \right]_{J_B}.$$ (2.33)

If we let

$$C = \frac{C_{C_B}^C \frac{Z_a!}{Z_a!} \frac{N_a!}{Z_a!} \frac{N_a!}{Z_a!}}{Z_a!} = \sqrt{\frac{(z_a+1)(n_a+2)(n_a+1)}{(z_a+1)(n_a+2)(n_a+1)}},$$ (2.34)

then for the case where $J^B = J^n_C = 0$ (2.33) reduces to:

$$C \int dz_a dz_a \left[ \psi_{jp}^{jn}(z_a+1, n_a+2) \right]^* \left[ \psi_{jp}^{jn}(z_a, n_a) \right]_{J_B}.$$ (2.35)
where the \( z_{a+1} \) protons and \( n_{a+2} \) neutrons in shells \( j_p \) and \( j_n \) in B couple to spin \( J^P_B \) and \( J^n_B \) respectively.

Since only the shells \( j_p \) and \( j_n \) now appear in (2.35) the integration over the \( z_{a} \) and \( n_{a} \) coordinates can be accomplished by using one and two particle coefficients of fractional parentage (fpc) [de-Shalit, 1963]. The integration is easily performed using the notation and techniques of Macfarlane [1960]. Briefly, in this notation angular momentum coupling is indicated by triangles; for example \( J^P_a + J^n_a = J_A \) is denoted by

\[
|J^P_a; J^n_a; M, M' : J, M_A \rangle = \begin{array}{ccc}
\hline
& J^P_a & J^n_a \\
\hline
J_A & & \\
\hline
\end{array} \tag{2.36}
\]

An antisymmetric wave function of \( z_{a} \) particles in the shell \( j_p \) coupled to spin \( J^P_a \) is denoted by a curved line

\[
\psi^a_{j_p}(z_a) = \begin{array}{ccc}
\hline
& z_a & \\
j_p & & \\
\hline
j^P_a & & \\
\end{array} \tag{2.37}
\]

Combining (2.36) and (2.37) the integral (2.35) is

\[
\begin{array}{ccc}
\hline
& j^P_a & j^n_a \\
\hline
J_B & & \\
\hline
j^P_B & & \\
\hline
J_A & & \\
\hline
\end{array} \tag{2.38}
\]
where the integration is over the $z_a$ and $n_a$ coordinates. If the fpc's are now used the left hand side of (2.38) becomes

\[
\sum_{\nu_p, I_p, J_p, I_n, J_n} \left\{ \begin{array}{c} z^{a+1}_p \\ j_p \end{array} \right| j_p (\nu_p I_p) j_p J_p B \left| \begin{array}{c} z^n \nu_p J_p B \\ j_n \end{array} \right\}, (2.39)
\]

where the $\nu$'s are seniority numbers. In order to integrate (2.38) using (2.39) the angular momenta $I_p$ and $I_n$ must be coupled. This can be done in two steps using the recoupling diagram of Macfarlane [1960],

\[
d = \sum_a \sigma_a \left( \begin{array}{c} b \\ a \end{array} \right), (2.40)
\]

where $U(abcdef)$ is the normalized Racah coefficient. Using (2.40) twice the wave function in (2.39) becomes
If (2.41) and (2.39) are now used in (2.38) the integral (2.32) can finally be written as

$$C \left( j_p^a \left( v_p^a j_p^a j_p^B \right) \right) z_{a+1}^a \sum_{I_1, I_2} \left( j_n^a \left( v_n^a j_n^a j_n^B \right) \right) \sum_{I_1, I_2} \left( J_{a+1}^a \left( v_{a+1}^a j_{a+1}^a \right) \right) \sum_{I_1, I_2} \left( J_{a+2}^a \left( v_{a+2}^a j_{a+2}^a \right) \right)$$

where $v_p^a$ and $v_n^a$ are the seniority numbers for the $z_a^p$ and $n_a^n$ protons and neutrons in shells $j_p^a$ and $j_n^a$.

If the target has zero spin then from (2.42) $I_n^I = I_1$ and $j_p^I + I_n^I = J_B^I$. In this case the form factor from (2.24) can be written
\[
F(R_{PB}, R_{QA}) = C \cdot fpc(1) \sum fpc(2) \cdot U \cdot U \int dS_0 d\xi'_1 d\xi'_0 d\xi'_1 n \quad \text{(2.43)}
\]

where \( C \) is given in (2.34), \( fpc(1) \), \( fpc(2) \) and \( U \) are the 1-particle, 2-particle and \( U \) coefficient of (2.42), \( S_0 \) is the spin of the outgoing proton and the transferred proton and neutrons \( l_p, 0 \) and \( l_n \) have been labelled explicitly in the single particle wave functions \( \psi_j^p \) and \( \psi_j^n \).

As a particular example, if \( J^p_a = J^n_a = I'_n = 0 \) and \( \tilde{\nu}_p = 0, \tilde{\nu}_n = 1 \) in (2.42) then the explicit forms of the \( fpc \) given by Glendenning [1965] and Schwartz [1954] yield

\[
\frac{d\sigma_{\alpha \beta}}{d\Omega} = \frac{M_a M_p}{(2\pi \hbar)^2} \left( \frac{k_p}{k_\alpha} \right) (n_a + 2)(2j + 1 - z_a)(2j + 1 - n_a) \frac{(2j + 1)(2j + 1)}{21_{a}^{n} - 1 + 1} \sum M_B \left| I_{\alpha \beta} \right|^2, (2.44)
\]

where

\[
I_{\alpha \beta} = \int dR_{PB} dR_{QA} \chi'_p(-)^*(R_{PB}) \bar{F}(R_{PB}, R_{QA}) \chi_{\alpha}^+(R_{QA}), (2.45)
\]

where \( \bar{F} \) in (2.45) is the integral in (2.43) exclusive of the \( C, fpc \) and \( U \) coefficients.

The differential cross section in (2.44) is that for the case when the two neutrons couple to total spin equal to zero in a target where the protons and neutrons have also coupled to spin zero. The seniority of the residual state is \( \tilde{\nu}_p = 1 \) and \( \tilde{\nu}_n = 0 \). The coefficient

\[
S = \frac{(n_a + 2)(2j + 1 - z_a)(2j_n + 1 - n_a)}{(2j + 1)(2j_n + 1)}
\]

is sometimes referred to as a spectroscopic factor. It is valid only within the context of the derivation given above since it is a
consequence of the assumption of shell model states. Accordingly, it is not logical to calculate the form factor using a triton cluster (see section C.3 of this chapter) and then multiply the resulting cross section by (2.46) which has been done in some cases, for example Bucurescu [1972]. The computational techniques which were required to maintain the antisymmetrical nature of the wave functions in deriving (2.46) are not applicable to the triton cluster since the triton is certainly distinguishable from the nucleons in the residual nucleus.

(2) Microscopic Form Factors

The most detailed calculation of the (α,p) form factor has been carried out in zero range by Falk [1973, 1975]. In this calculation single particle wave functions produced in a Woods-Saxon well are expanded in harmonic oscillator functions. The product of single particle wave functions in (2.43) involves a sum of products of harmonic oscillator functions of the form

\[
\left\{ \psi_{n_1} z_{1 m_1} (r_{1 p}) \psi_{n_2} z_{2 m_2} (r_{0 n}) \psi_{n_3} z_{3 m_3} (r_{1 n}) \right\}_{L_n L}, \tag{2.47}
\]

where \( r_{1 p} \) is the proton coordinate and \( r_{0 n} \) are the neutron coordinates. The relative and centre of mass (c.m.) motion of the neutron pair can be easily found using the standard Talmi-Moshinsky brackets. However, the proton and di-neutron relative and c.m. motion must now be found and the standard brackets are inapplicable because the proton and di-neutron are mass 1 and mass 2 particles respectively. In order to obtain the relative and c.m. motion of the proton and di-neutron generalized Talmi-Moshinsky brackets must be employed. A computer code had been written by Sotona [1972] to obtain these generalized brackets.
Alternatively, Smirnov's [1962] technique of transferring to Cartesian coordinates to obtain the relative and c.m. motion can be used. To transfer from spherical to Cartesian coordinates coefficients \( A_{pq \sigma}^{n \ell m} \) [Smirnov, 1962] given by

\[
\psi_{n \ell m}(x,y,z;\nu) = \sum_{pq \sigma} A_{pq \sigma}^{n \ell m} \psi_{pq \sigma}(x,y,z;\nu) \tag{2.48}
\]

are used. These coefficients have been obtained in closed form by Falk [1973]. However, it is demonstrated in Appendix 1 that the \( A_{pq \sigma}^{n \ell m} \) coefficients are connected by simple recursion relations so that starting with \( A_{pq \sigma}^{n \ell m} \) all the other coefficients for a given set \((n,\ell)\) can be computed. These recursion relations are original as far as the author is aware. The recursion relations from Appendix 1 are

\[
A_{n \ell m-1}^{n \ell m,\alpha \beta \gamma} = \frac{1}{\sqrt{\ell(\ell+1) - m(m-1)}} \left\{ \sqrt{\gamma(\gamma+1)} A_{n \ell m,\alpha-1,\beta,\gamma+1}^{n \ell m,\alpha,\beta,\gamma-1} - \sqrt{\gamma(\gamma+1)} A_{n \ell m,\alpha+1,\beta,\gamma-1}^{n \ell m,\alpha,\beta,\gamma+1} \right\} \tag{A1.15}
\]

and

\[
m A_{n \ell m}^{n \ell m,\alpha \beta \gamma} = i \left\{ \sqrt{\beta(\beta+1)} A_{n \ell m,\alpha+1,\beta-1,\gamma}^{n \ell m,\alpha,\beta+1,\gamma} - \sqrt{\alpha(\alpha+1)} A_{n \ell m,\alpha-1,\beta+1,\gamma}^{n \ell m,\alpha,\beta-1,\gamma} \right\}. \tag{A1.17}
\]

Various other properties of the \( A_{n \ell m}^{n \ell m,\alpha \beta \gamma} \) coefficients are also discussed in Appendix 1.

A semi-microscopic form factor has been suggested by Suck [1971] who proposed using a product of proton and two neutron form factors. This procedure was used by Vergnes [1974] who reported no improvement in the shapes of the angular distribution over those obtained by a triton cluster form factor.
Triton Cluster Form Factor

The most common form factor used to date is the triton cluster form factor. This form factor assumes that a quasi-triton is transferred. The form factor is essentially the bound state wave function of a spin 1/2, mass three particle bound in a Woods-Saxon well with a separation energy equal to that of a triton from the residual state.

The notion of a cluster transfer originates easily if one considers the average density distribution of the residual nucleus and the proximity of the three transferred particles to one another due to their origin in the α-particle. Using a Woods-Saxon density distribution for $^{55}$Mn say, the particle density is

$$\rho(r) = 0.1032 [1 + \exp\left((r - 4.754)/0.65\right)]^{-1}$$

in which the usual parameters $r_0 = 1.25$ fm and $a_0 = 0.65$ fm are used. The density is normalized to give 55 particles when integrated over all space. The average number of particles $\tilde{n}(r)$ in a volume $dV$ is roughly

$$\tilde{n}(r) \approx \rho(r) \, dV .$$

If $dV$ is chosen as the volume to which the transferred nucleons are confined in the α-particle or the triton then $dV \approx 20$ fm$^3$ based on the r.m.s. radius of the triton ($r = 1.68$ fm) given by Glendenning [1965]. Hence

$$\tilde{n}(r) \approx 2[1 + \exp\left((r - 4.754)/0.65\right)]^{-1} .$$

Consequently, at the half density radius we can expect to find on average one nucleon occupying the same volume as that of the three transferred nucleons. Thus the transferred particles constitute a density anomaly of a factor of three compared with the average density.
The number of nodes \( N_t \) in the quasi-triton bound state wave function is determined by the harmonic oscillator rule

\[
N_t = n_1 + n_2 + n_3 + \frac{1}{2}(l_1 + l_2 + l_3 - L_L),
\]

(2.52)

where \( n \) is the number of nodes excluding the origin. The microscopic calculations of Falk [1975] show that (2.52) correctly gives the number of nodes of the form factor with the difference that the exact shapes of the cluster and microscopic form factors differ in the nuclear interior and surface regions. The difference in form factors in the nuclear interior produces essentially only a difference in magnitude of the cross section and has little influence on the shape of the angular distribution. This effect was found by Falk [1975] and in the present study. Specifically, once the correct parameters for the bound state well were found the shape of the angular distribution did not alter substantially. The susceptibility of the cross section to changes in the form factor in the nuclear interior is illustrated in Figs. 2.3 for the particular case of the \( J^e_T = 7/2^- \) transfer to the 1.883 MeV level in \(^{55}\text{Mn}\). The form factors for \( N_t = 3 \) and \( N_t = 4 \) using a well geometry \( r_0 = 1.45 \) and \( a_0 = 0.35 \) fm are depicted in Fig. 2.3(a). The two form factors are roughly out of phase in the interior and are identical except for a constant factor beyond 7 fm. The angular distributions calculated with these two form factors are depicted in Fig. 2.3(b). The shapes of the angular distributions could not be distinguished by the level of experimental accuracy.

Since the differential cross section consists of a weighted sum of Legendre polynomials the similarity of the two calculated angular distributions suggests that the weighting factors of the Legendre polynomials for \( N_t = 4 \) are increased over those for the \( N_t = 3 \) case by a
FIGS. 2.3: Form factors and angular distributions for $^{52}\text{Cr} (\alpha, p) ^{55}\text{Mn}$
\( J^* = 7/2^+ \), \( Q = -4.454 \) MeV, \( E_{\alpha} = 26 \) MeV.

TRITON WELL
\( r_0 = 1.45 \)
\( a_0 = 0.35 \)

\[ \left( \frac{A_u}{A_3} \right)^2 = 1.65 \]

\[ F_3 (r) = A_3 e^{-kr} \]

\[ F_u (r) = A_u e^{-kr} \]

\[ \frac{\sigma_u}{\sigma_3} \approx 2.2 \]
constant factor which is the same for all the polynomials in the cross section. The weighting factors in turn depend upon the radial integrals and the only region for which the form factors in Fig. 2.3(a) have the same shape is for the $r \geq 6$ fm. Hence, the exterior region must make the dominant contribution to the radial integrals.

If we let $F_{N_t}$ represent the form factor for $N_t$ nodes then

$$\frac{\sigma_h}{\sigma_3} = \frac{|\langle \chi_p^{(-)} | F_{N} | \chi_{\alpha}^{(+)} \rangle|^2}{|\langle \chi_p^{(-)} | F_3 | \chi_{\alpha}^{(+)} \rangle|^2}.$$  \hspace{1cm} (2.53)

For $r \geq 7$ fm, $F_{N} \approx A_N e^{-kr}$, and therefore

$$\frac{\sigma_h}{\sigma_3} = \frac{|A_h|^2}{|A_3|^2} = 1.65$$ \hspace{1cm} (2.54)

as illustrated in Fig. 2.3(a). The actual difference in magnitude is about 2.2 from the DWBA calculation. However, we see that the external region can account for 75% of the relative increase of cross section going from $N_t = 3$ to $N_t = 4$.

In agreement with previous analyses using the triton cluster form factor the present study indicates that a smaller diffuseness parameter $a_0$ is needed for the bound "triton" than that deduced from triton elastic scattering. This feature is illustrated in the next chapter where the triton well parameters used in the various calculations are presented.
The stability of the shape of the angular distribution against changes of the form factor in the interior may at first seem surprising because it is well known that in cases of angular momentum mismatch the use of a radial cut-off can produce drastic changes in the cross section [e.g., Hooper, 1966; Stock, 1967]. The effect of a radial cut-off can be seen in Fig. 2.4(a) for the 2.562 MeV state in $^{55}\text{Mn}$. On the left hand side is the variation of the $J_T^\pi = 1/2^+$ angular distribution. The cut-off radii for the $1/2^+$ calculation are nodes of the form factor. The right hand side of Fig. 2.4(a) shows the effect of the radial cut-off on a large $J_T^\pi$ transfer, $J_T^\pi = 17/2^-$. The cross sections for $R_C = 0$ and 1 fm are identical and it is apparent that there is far less variation in the case of large $J_T^\pi$ as compared to small $J_T^\pi$ transfer. The partial wave scattering amplitudes for the 2.562 MeV state are plotted in Fig. 2.4(b) from which an angular momentum mismatch $L_C = 8$ or 9 is deduced.

The explanation [e.g., Stock, 1967] for the difference in sensitivity to the radial cut-off between $J_T^\pi = 1/2^+$ and $17/2^-$ can be seen in Figs. 2.5 which show the radial matrix elements for the two cases. For the $J_T^\pi = 1/2^+$ calculation the low $L_T$ partial waves contribute to the cross section. These partial waves have reflection coefficients close to zero as seen in Fig. 2.4(b) and consequently they are approximated by incoming exponentials [Austern, 1961]. Due to the attractive nuclear potential the wave number increases and the phase of the exponential changes rapidly with $R$ producing an integrand in the radial integral which likewise varies rapidly with $R$. Due to the rapid phase change of the integrand the interior contribution to the radial matrix element is small although highly dependent upon the location of $R_C$. Beyond the nuclear radius the wave number decreases so the
FIGS. 2.4: (a) Influence of radial cut-off $R_C$ on $\sigma(\theta)$; (b) $^{52}\text{Cr}(\alpha,p)^{55}\text{Mn}$, 26 MeV, $|A_L| = |(\eta_L - 1)/2i|$.
$J_T^\pi = 17/2^-$

$L_T = 9$

$Q = -5131 \text{ keV}$

$\hbar \omega p$

$16 15 14 13 12 11$

$\hbar L_T$

$Q = -5131 \text{ keV}$

[Cr(α,p)$^{55}\text{Mn}$

$E_\alpha = 26 \text{ MeV}$

FIGS. 2.5: $\bullet = \left\vert \sqrt{(2L_p+1)(2L_\alpha+1)} \langle \chi^p_j \chi^T_L \chi^L_\alpha \rangle \right\vert$. Weighted radial matrix elements for $j_p = L_p + \frac{1}{2}$. 
integrand suffers less rapid phase changes.

For the $J_T^* = 17/2^-$ calculation an inspection of Fig. 2.5(a) reveals that the dominant partial waves in the radial matrix elements are $L_\alpha \approx 13$ and $L_p \approx 3$. These values of $L_\alpha$ and $L_p$ correspond to partial waves for which the reflection coefficient $\eta_L$ (see Fig. 2.4(b)) falls to about 0.5 and which hence contain ingoing and outgoing parts that do not average to zero in the integration [Austern, 1961]. Moreover, the centrifugal barrier keeps these waves away from the nuclear interior and hence the form factor near the surface is the most critical part; this is intuitively expected for the $(\alpha, p)$ reaction.

From the foregoing it appears that the best policy with regards to a radial cut-off is not to use it at all, that is set $R_C = 0$. Firstly, there is no intelligent way of choosing $R_C$. Secondly, the interior contribution cancels out largely as seen in the example of Figs. 2.3. Letting $R_C = 0$ then simulates the physical process that we expect intuitively for the $(\alpha, p)$ reaction, namely that it is a surface reaction, not because the calculated $\alpha$-particle partial waves have zero amplitude in the interior, but rather that the rapid fluctuations of the low $L_\alpha$ waves cause a net decrease in the interior's contribution to the cross section.

Another source of ambiguity due to the angular momentum mismatch is the poor determination of the partial waves used in the reaction calculation. In the present study most of the orbital angular momentum transfers $L_t$ were for $L_t \leq 4$. This represents a substantial mismatch of angular momentum since the matching angular momentum $L_C$ is about 8 or 9.

However, the elastic scattering is not strongly influenced by
the details of the low L partial waves [Stock, 1967]. The partial waves that are well determined by the optical model analysis are those for which the reflection coefficient is approximately 0.5. In the present experiments these correspond to $L_{\alpha} = 13$ or 14 as seen in Fig. 2.4(b).

This difficulty in the normalization of the ($\alpha$,p) reaction theory is in addition to that caused by the ambiguities introduced by various optical model sets which predict the $\alpha$-particle elastic scattering equally well.

A more complete discussion of L-space localization can be found in Austern [1961, 1970], Hooper [1966] and Stock [1967].

(E) COMPARISON OF RELATIVE CROSS SECTIONS IN THE ($\alpha$,p) REACTION AND IN SINGLE PROTON TRANSFER REACTIONS

(1) Relative Cross Sections in the ($\alpha$,p) Reaction

With the above ambiguities in mind a comparison of the relative strength of states excited in the ($\alpha$,p) reaction is more likely to be accurate than the determination of absolute cross sections. The hope is that the DWBA can at least account for the relative kinematics if not the absolute kinematics.

Since a variety of triton well geometries and form factor restrictions (i.e., number of nodes) can predict the correct shape of the angular distributions it is important to determine whether the relative cross sections as a function of the Q-value calculated from the various triton well geometries are consistent with each other. For example, if a triton geometry called set A predicts $[\sigma(Q_1)/\sigma(Q_2)]_A$ to be wildly different than $[\sigma(Q_1)/\sigma(Q_2)]_B$ predicted by set B then even the determination of relative cross section is dubious.
This possibility is considered in Figs. 2.6 using the triton cluster form factor calculated with a variety of well geometries and form factor restrictions. Two $J_T^\pi$ transfers of $3/2^-$ and $7/2^-$ are considered. From Fig. 2.6(a) it is seen that the absolute cross sections compared between different wells vary by a factor of $10$. In Fig. 2.6(b) the $\sigma(Q)$ are normalized at a value $Q_0$ and for both $J_T^\pi = 7/2^-$ and $3/2^-$ the relative variations differ by about $11\%$. The DWBA calculations are probably better than $11\%$ because only a few well geometries predict the correct shape of the angular distribution. The DWBA calculations can account for the relative kinematics therefore to at least $11\%$ despite the multiplicity of suitable triton well geometries and form factor restrictions.

(2) Comparison of $\alpha,p$ Data with Single Proton Transfer Data

It was pointed out in the Introduction that certain $(\alpha,p)$ or $(p,\alpha)$ studies obtained relative cross sections which were nearly identical to those for the corresponding single proton transfers exciting the same states. This feature has led to the so-called spectator model of the $(\alpha,p)$ reaction [Schiffer, 1967] in which the transferred neutrons play a passive role in determining the cross section. The conditions under which the $(\alpha,p)$ and single proton transfer reactions will excite states with the same relative strength will be considered with $^{55}\text{Mn}$ as the residual nucleus as an example.

In the Macfarlane [1960] notation discussed in section C.1 of this chapter a state $e$ of $^{55}\text{Mn}$ is written as
FIGS. 2.6: (a) \( \sigma(Q) = \sum \sigma(\theta_i) \), \( 20.5^\circ \leq \theta_i \leq 41^\circ \), arbitrary units; relative \( \sigma(Q) \) normalized to 1 at \( Q = -4.45 \) (\( J^\pi = 7/2^- \)) or \( Q = -4.1 \) (\( J^\pi = 3/2^- \)).
Here $a$, $b$, $c$, $d$ and $e$ are sets of quantum numbers needed to describe the states. For example, $e = (J_e, T_e, X_e)$, where $J_e$ and $T_e$ are the total spin and isospin and $X_e$ are any other labels needed to distinguish state $e$ from other states with the same $J$ and $T$. In (2.55a) we view the state of 55 nucleons as an expansion of 54 nucleons whose quantum numbers couple to $b$ plus a proton with quantum numbers $a$. The 54 nucleons in turn are considered as an assemblage of 52 nucleons coupled to $c$ plus two neutrons coupled to $d$ in (2.55b). Whichever expansion we use depends upon the experiment.

For the single proton transfer the only configuration of (2.55a) that is excited is that for which $b = b_0$, where $b_0$ is the ground state of $^{54}$Cr. In that case the cross section is proportional to $|\alpha_{a_0 b_0}|^2$ where the quantum numbers of the proton are limited to one set $a_0$. In principle since $a = (n,j,t,t_z)$, where $n$ is the number of nodes of the single particle state, $j$ is the angular momentum, $t = 1/2$ and $t_z = -1/2$ are the isospin and isospin projection of the proton, a sum over $n$ is possible. But since the states of $n$ and $n \pm 1$ are far removed
from each other in energy only one value of n predominates.

For the \((\alpha,p)\) reaction the only configurations in (2.55b) that can be excited are those with \(c = c_0\), where \(c_0\) is the ground state of \(^{52}\text{Cr}\). However, this places no restriction on the values which the quantum numbers of the two neutrons can assume. In particular the total angular momentum of the neutron pair is not restricted to \(L_{2n} = 0\). The total intrinsic spin of the neutrons \(S_{2n}\) must be zero however, since they must overlap with the motion in the \(\alpha\)-particle.

The form factor derived from (2.55b) does not factor into a spectroscopic amplitude and radial function as in the case of (2.55a), and hence the transition amplitude is a coherent sum of terms which correspond to the various configurations in (2.55b) with \(c = c_0\). Since different configurations are accessible by the transfer of the neutrons it would be surprising if the neutrons played a passive role.

To decide when the spectator model is applicable we must determine the nature of the excited states of the 54 nucleons coupled to \(b\) in (2.55a). If \(b \neq b_0\) in (2.55a) then the state \(e\) of \(^{55}\text{Mn}\) has as a parent an excited state of \(^{54}\text{Cr}\). From (2.55b) this parent is accessible via the \((\alpha,p)\) reaction if \(b = c_0 + d\), where the intrinsic spin of the neutrons \(S_{2n} = 0\). Then the \((\alpha,p)\) transition amplitude is a coherent sum of terms which in general can not produce the same relative cross sections as the single proton transfer. However, if all the excited parent states \(b\) were of such a nature that \(b \neq c_0 + d\) for any allowed \(d\) then these parent states can not be excited in the \((\alpha,p)\) reaction. In this case the only configuration in (2.55b) which can contribute is that based upon the \(^{54}\text{Cr}\) ground state with \(b = b_0\). Those are the terms
where $a$ has been restricted to $a_0$ by the same arguments employed previously. In this case the cross section will be proportional to $|\alpha_{a_0 b_0}^e|^2$ again.

If the state $e$ is compared to another state $e'$ with the same restriction then the relative cross sections in $(\alpha,p)$ and single proton transfer will be the same only if the proton occupies the same state $a_0$ in both cases; otherwise the $\alpha$-particle overlap with the $p$ and $2n$ motion in (2.55c) will be different. In practice this means that relative cross sections can only be the same for states with the same spin.

In summary, we can expect the spectator model to be valid only when comparing states of the same spin and which have a parentage based on the $^{54}$Cr ground state (i.e., $b = b_0$) plus any excited states of $^{54}$Cr which can not be constructed from the $^{52}$Cr ground state plus a pair of neutrons coupled to spin $S_{2n} = 0$ (i.e., $b \neq 00 + d$).

(F) SELECTION RULES FOR THE DIRECT $(\alpha,p)$ REACTION

Angular Momentum:

Since $J_f = J_1 + J_T$, the final state angular momentum is uniquely fixed by $J_T$ if $J_1 = 0$. Since $S_{\alpha} = S_{0} + S_{1} + S_{T} = S_{0} + S_{T} = 0$, $p$
the transferred spin $S_T = \frac{1}{2}$.

**Seniority:**

Only one proton is transferred, hence $\Delta \nu_p = 1$.

Two neutrons are transferred, hence $\Delta \nu_n = 0, 2$.

**Parity:**

Since the three transferred nucleons spend most of their time in the $\alpha$-particle in a spatially symmetric state their relative angular momentum $L_T = 0$. If $L_T$ is the total transferred angular momentum and $\Lambda_T$ is the centre of mass angular momentum then $L_T = L_1^p + L_0^p + L_1^n = \Lambda_T + \Lambda_T^L$, so $\Delta \nu = (-1)^L = (-1)^{\Lambda_T}$. This condition also fixes $L_T$ since $J_T = J_f = L_T + S_T$ for $J_1 = 0$.

**Isospin:**

Since $T_\alpha = 0 = \ell_0^p + \ell_1^p + \ell_0^n + \ell_1^n = \ell_0^p + \ell_T$ the transferred isospin $T_T = \frac{1}{2}$. This is an important selection rule because it requires that only states with $T = T_z$, where $T_z$ is the isospin projection of the residual nucleus, can be excited. This follows because

$$T_f = T_i \pm \frac{1}{2} \quad (2.56)$$

but since $T_i = (N - Z)/2$ and $T_f \geq [(N + 2) - (Z + 1)]/2$ we have

$$T_f \geq (N - Z)/2 + 1/2 = T_i + 1/2 \quad (2.57)$$

The two conditions (2.56) and (2.57) limit $T_f$ to $T_f = T_i + 1/2 = T_z$. A comparison with the single proton transfer data will indicate which states are likely $T_z$ states because these states cannot be excited in the $(\alpha, p)$ reaction. Conversely, states that are excited in the $(\alpha, p)$
reaction are necessarily T< states since T_f = T_Z. In practice this prohibits excitation of the isobaric analog states in (Z+1, N+2) nuclei based on (Z, N+3) parents because the IAS are T> states.
CHAPTER 3

THE $^{50,52,54}$Cr(α,p)$^{53,55,57}$Mn REACTIONS
AT 18 AND 26 MeV AND THE $^{55}$Mn(p,p'γ)p-γ
COINCIDENCE EXPERIMENT AT $E_p = 7.975$ MeV

(A) INTRODUCTION

The experiments were performed on the ANU EN tandem accelerator at 18 MeV and on the ANU 14 UD pelletron at 26 MeV. Typical beam currents were 400 nA of He$^{++}$ on target at both energies. The targets varied in thickness from approximately 40 to 200 μg/cm$^2$ with the thinner gold-backed chromium targets used at 18 MeV and the thicker self-supporting targets used at 26 MeV. The accumulated charge varied between 6,000 and 40,000 μC depending upon angle and energy. The experimental resolution at 18 MeV was typically around 32 keV, whereas the resolution at 26 MeV varied between 45 and 60 keV depending upon the availability of detectors at the time of the experimental run. Further experimental details can be found in Appendix 2.

The first five states of $^{53}$Mn from the $^{50}$Cr(α,p)$^{53}$Mn reaction at 18 MeV were also measured in the ANU 24" double focussing spectrometer for a range of lab angles between 5° and 90°. This enabled the strong $J^M = 7/2^-$ transition to the $^{53}$Mn ground state to be measured to quite forward angles. The angular distributions measured for these five states using the spectrometer and the scattering chamber were in agreement with each other both in shapes and relative magnitudes. The
points in the angular distribution were normalized at the peak. The combined set of angular distributions for the $^{53}$Mn ground state at 18 MeV can be found in Fig. 3.1.

The spectrometer was also employed to search for a possible doublet in $^{55}$Mn at 1.292 MeV. The search for this supposed doublet was extended to a $^{55}$Mn (p,p'γ)p-γ coincidence measurement. The p-γ experiment and the speculated doublet at 1.292 MeV in $^{55}$Mn will be discussed in section (G) of this chapter.

(B) $^{50,52,54}$Cr(α,p)$^{53,55,57}$Mn AT $E_α = 18$ MeV

Spectra for these three reactions at 18 MeV can be seen in Figs. 3.11 and 3.12. It is expected for this bombarding energy and mass region that compound nuclear reactions are not negligible. Indeed, the selection rules described in Chapter 2 for the direct (α,p) stripping reaction would inhibit the excitation of states with large proton seniority three components such as the 5/2−, 9/2− and 11/2− states in $^{53}$Mn at 0.376, 1.618 and 1.438 MeV respectively. A discussion of the theoretical interpretation of these three states is in Chapter 4. Nevertheless, these states were populated with enough strength to obtain angular distributions. For the first isotope measured, $^{53}$Mn, preliminary analyses of the angular distributions of groups going to the ground state and the states at 0.376, 1.287, 1.438 and 1.618 MeV were attempted using an incoherent sum of a statistical compound nuclear reaction plus the DWBA angular distributions. The compound nuclear angular distribution was calculated using the Hauser-Feshbach theory with the code HAUSER [DAL] while the DWBA calculations were performed with the code DWUCK [Kunz, 1969]. Linear least squares fitting either did not
improve the agreement with the experimental data or produced non-physical (e.g., negative) coefficients for the presumed reaction components. This procedure was then abandoned and all the remaining angular distributions, for states which seemed to be amenable to this approach, were calculated using only the DWBA.

Excitation functions for the $^{52,54}\text{Cr}(\alpha,\alpha')$ reaction at about 18 MeV bombarding energy were briefly investigated during one run to ascertain the nature or indeed the existence of fluctuations in the elastic scattering. If the excitation functions were a smoothly varying function of bombarding energy, then the elastic scattering, at least, should be described by an incoherent sum of the Hauser-Feshbach and DWBA cross sections [see for example Hellström, 1970]. The pertinent quantity in determining the type of fluctuations measured is $\Gamma(U)/\Delta E$, where $\Gamma(U)$ is the average level width at the excitation energy $U$ in the compound nucleus and $\Delta E$ is the energy spread due to the beam spread and the target thickness. Statistical fluctuations in the excitation function are expected to occur if $\Delta E \leq \Gamma$ [Ericson, 1960]. From Chapter 1, the average level width $\Gamma$ is probably between 5 and 10 keV in the compound nucleus $^{56}\text{Fe}$ at $U = 25.6$ MeV. The targets used for the excitation function measurements were approximately 7 keV thick for the $^{52}\text{Cr}$ target and 27 keV thick for the $^{54}\text{Cr}$ target, as estimated from Rutherford scattering. These target thicknesses straddled the range of target thicknesses used for the 18 MeV $(\alpha,p)$ measurements. The energy spread in the beam from the tandem was approximately 13 keV at the time of these measurements. This rather large spread was later traced to periodic variations in the current supply for the 90° analysing magnet [Bartle, 1975] which were shorter than the time required to measure a point in the excitation function. The total energy spread $\Delta E$ was
therefore between 15 and 33 keV for both the excitation function and
(a,p) measurements. Since it was expected that $\Gamma \leq 10$ keV, step sizes of
25 to 50 keV were used in the excitation function except for a few
points measured with 10 keV steps in $^{54}$Cr. The excitation functions for
45°, 90°, 120° and 150° are displayed in Figs. 3.0(a) and 3.0(b). The
observed structure has a width of between 50 and 200 keV. Although the
excitation functions were not measured over a very large energy range
there appears to be a correlation in the $^{52}$Cr case (Fig. 3.0(a)) between
the 90°, 120° and 150° fluctuations. In fact, 90° and 150° are better
correlated with one another than the 120° excitation function. For the
$^{54}$Cr target, excitation functions at 120° and 150° also seem to be
correlated. The correlations between the various excitation functions
at the different angles is suggestive of intermediate structure
[Hodgson, 1971, Chapter 18]. If the fluctuations observed were purely
statistical they should be uncorrelated for $\Delta \theta >> (kR)^{-1}$ [Brink, 1964].
Using $R = 1.2 A^{1/3}$, $(kR)^{-1} \approx 7^\circ$ under the conditions of these measure-
ments. The observed fluctuations appear to be correlated over $\Delta \theta = 30^\circ$
and 60° so therefore some intermediate process, as well as the
statistical compound nuclear and direct (or shape elastic) reaction,
will probably be needed to describe the elastic scattering. If inter-
mediate processes are occurring in the elastic channel then one might
expect that these processes should be included in the analysis of the
reaction channels, in this case (a,p). This possibility affords an
explanation of the failure of the Hauser-Feshbach plus DWBA analysis
referred to earlier for the case of $^{53}$Mn. The excitation functions
presented in Figs. 3.0(a) and 3.0(b), although suggestive, are not
conclusive evidence of intermediate structure. The energy range studied
should be extended to determine whether or not the correlations are

* D.M. Brink, R.O. Stephen, N.W. Tanner, Nucl.Phys. 54 (1964) 577
\( ^{52}\text{Cr}(\alpha,\alpha_0) \text{ EXCITATION FUNCTION} \) (a)

FIG. 3.0(a): See text for description.
$^{54}\text{Cr}(\alpha,\alpha')$ EXCITATION FUNCTION

FIG. 3.0(b): See text for description.
accidental; for example, the existence of intermediate structure in the excitation functions of $^2^3\text{Na}(p,p')$ was found using $8 \text{ MeV} \leq E_p \leq 12 \text{ MeV}$ by Hellström [1970].

The angular distributions for most of the groups in Figs. 3.11 and 3.12 could not be described by DWBA. Examples of these states are the low-lying $5/2^-$ states shown in Fig. 3.6 and the $3/2^-$ state at 1.286 MeV in $^{53}\text{Mn}$ shown in Fig. 3.8.

States which could be described by DWBA are displayed in Figs. 3.1 to 3.10. The $L_p$ values specified in the figures come from the $(^3\text{He},d)$ studies of O'Brien [1967], Gunn [1976] and Rapaport [1969]. In fact, states with large spectroscopic factors in $(^3\text{He},d)$ are strongly excited in $(\alpha,p)$ and even at 18 MeV their angular distributions in $(\alpha,p)$ indicate that a substantial portion of their production, at least for $0 < 90^\circ$, is via the direct reaction mechanism. The notable examples of this are the strong transitions to the $7/2^-$ and $3/2^-$ states in $^{53}\text{Mn}$ at $E_x = 0$ and 2.406 MeV and at $E_x = 0.123$ and 2.255 MeV in $^{55}\text{Mn}$ (see Fig. 3.12). The lowest $7/2^-$ states in $^{53,55}\text{Mn}$ are compared with the first excited state of $^{57}\text{Mn}$ in Fig. 3.1. The similarities in the experimental angular distributions for these states suggest that the first excited state of $^{57}\text{Mn}$ is $7/2^-$. The DWBA calculations fit reasonably well out to at least $90^\circ$. Calculations using $J_T^\pi = 5/2^-$ are also included in Fig. 3.1 for states in $^{53,55}\text{Mn}$ and show that the $7/2^-$ transition is favoured for these states. It is observed that the angular distribution of the second excited state in $^{57}\text{Mn}$ (peak number 3 in Fig. 3.11) resembles the three known $J_T^\pi = 3/2^-$ angular distributions in $^{53,55}\text{Mn}$.

The 18 MeV data reveal that it is not a necessary condition for a state to have a large spectroscopic factor in single proton transfer in order to be strongly excited with a direct angular
$^{50,52,54}$Cr(a,p)$^{53,55,57}$Mn
$E_a = 18$ MeV

$^{53}$Mn $J^\pi = 7/2^-$, $EX = 0.00$, $L_p = 3$

$^{57}$Mn $Q^* = 4.386$
TRITON WELL PARAMETERS
$R_0 = 1.45$, $A_0 = 0.30$

$^{55}$Mn $EX = 2.197$

$^{55}$Mn $J^\pi = 7/2^-$, $EX = 0.126$, $L_p = 3$

$\sigma(\theta)$

100 
10 
\(\mu b/sr\)

20 40 60 80 100 120 140 160
$\theta_{cm}$

FIG 3.1

CURVES ARE DWBA TRITON TRANSFERS FOR $J^\pi_r = 7/2^-$ (SOLID)
$J^\pi_r = 5/2^-$ (DASHED)

$L_p$ VALUES ARE FROM $^3$He,d REACTIONS
FIG. 3.2
CURVES ARE DWBA TRITON TRANSFERS FOR $J^p = 3/2^-$. $L_p$ VALUES ARE FROM $({}_3\text{He},d)$ REACTIONS
52.54 \text{Cr(\alpha,p)} 55.57 \text{Mn}

E_\alpha = 18 \text{ MeV}

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig3-3}
\caption{Curves are DWBA Triton Transfers for $J^+_T = 3/2^{-}$ (Solid) and $J^+_T = 1/2^{-}$ (Dashed). $L_p$ value from $^3\text{He},d$ reaction.}
\end{figure}
59

\[ E_q = 18 \text{ MeV} \]

**TRITON WELL PARAMETERS**

- \( R_0 = 1.35 \)
- \( A_0 = 0.23 \)

---

**FIG. 3.4**

**CURVES ARE DWBA TRITON TRANSFERS**
TRITON WELL PARAMETERS

$R_0 = 150$, $A_p = 0.30$

CURVES ARE DWBA TRITON TRANSFERS

FIG 3.5
50, 52, 54 Cr(α, p) $^{53, 55, 57}$ Mn

$E_α = 18$ MeV

$^{55}$ Mn $\text{EX} = 0.0$

$J^\pi = 5/2^-$

$^{53}$ Mn $\text{EX} = 0.376$, $J^\pi = 5/2^-$

$^{57}$ Mn $Q = -4.302$

(Ex = 0.0, $J^\pi = 5/2^-$)

$\sigma(\theta) \, \text{σb/sr}$

$\theta_{cm}$

FIG. 36
55Mn, EX = 3104, $L_p = 1 + 3$
$J^{\pi} = 7/2^-$

$55Mn$, EX = 1883
$L_p = 3$
$J^{\pi} = 5/2^-$

$57Mn$, $Q = -6.910$
$J^{\pi} = 7/2^-$
$J^{\pi} = 5/2^-$

TRITON WELL PARAMETERS
$R_0 = 1.45$
$A_0 = 0.30$

FIG 3.7
CURVES ARE DWBA TRITON TRANSFERS
$^{50,52}$Cr$(\alpha,p)^{53,55}$Mn

$E_\alpha = 18$ MeV

FIG. 3-8

L_p VALUES FROM ($^3$He,d) REACTION
52, 54 Cr(α, p) 55, 57 Mn

$E = 18$ MeV

57 Mn $Q = -6540$

$J^P = 9/2^+$

$J^P = 11/2^-$

57 Mn $Q = -7760$

55 Mn

$EX = 3.126$

$L_p = 3$

$J^P = 5/2^-$

$J^P = 11/2^-$

55 Mn

$EX = 3.607$

$L_p = 3$

$J^P = 5/2^-$

$J^P = 11/2^-$

FIG. 3.9

POTENTIAL HIGH SPIN STATES IN 57 Mn $J \geq 9/2$
$^{50,52}$Cr($\alpha$, $p$)$^{53,55}$Mn

$E'_{\alpha} = 18$ MeV

**Figure 3.10**

Curves are DWBA triton transfers.
FIG. 3.11

PROTON SPECTRA FROM THE $^{54}$Cr($\alpha$,p) REACTION AT $E_\alpha = 18$ MeV. EXPERIMENTAL RESOLUTION IS ~32 keV. THE EXCITATION ENERGIES AND SPIN ASSIGNMENTS COME FROM THE ANALYSIS OF 20 PROTON SPECTRA MEASURED BETWEEN 10° TO 127° LAB. THE STATE WITH Q$_{exp}$~6302 MeV IS LABELLED PEAK NO 1. THE PROTON SPECTRA FROM THE $^{52}$Cr CONTAMINANT HAVE ALREADY BEEN SUBTRACTED FROM THE SPECTRA IN THE FIGURE.

54Cr($\alpha$,p)57Mn 18 MeV 52°50°G Lab

54Cr($\alpha$,p)57Mn 18 MeV 450°G Lab
PROTON SPECTRA FROM THE $^{50,52}$Cr($\alpha$,p) REACTIONS. THE NUMBERS ABOVE THE PEAKS ARE EXCITATION ENERGIES OF STATES IN $^{53,55}$Mn. THE L=0,2 TRANSITIONS AT 2 426,2 994 MeV IN $^{55}$Mn SEEN IN THE ($^3$He,d) REACTION ARE ABSENT IN THE ($\alpha$,p) SPECTRA. HOWEVER, THE 2 197 MeV STATE, WHICH IS WEAKLY POPULATED IN ($^3$He,d) IS EXCITED WITH REASONABLE STRENGTH IN THE ($\alpha$,p) REACTION AND EXHIBITS A $j^\pi=7/2^-$ ANGULAR DISTRIBUTION.

THE L=0 TRANSITION REPORTED IN ($^3$He,d) TO THE 2 720 MeV STATE IN $^{53}$Mn IS ABSENT IN THE ($\alpha$,p) REACTION. THE L=1 TRANSITION AT 2 678 MeV IS MASKED BY THE PEAK LABELLED 2 692 WHICH APPEARS TO BE A DOUBLET WHOSE MEMBERS ARE ABOUT 14 keV APART.

THE EXPERIMENTAL RESOLUTION IS $\approx$ 32 keV.

FIG. 3.12

50CR($\alpha$,p)53MN 18 MEV 41-50E6 LAB

52CR($\alpha$,p)55MN 18 MEV 450E6 LAB
distribution in \((α,p)\). This point is strikingly made by the 2.197 MeV level in \(^{55}\text{Mn}\). Rapaport [1969] report this state in \((^3\text{He},d)\) to have a maximum cross section nearly fifty times smaller than that to the 0.123 MeV \(7/2^-\) state. It is seen in Fig. 3.12 that this state has a cross section in \((α,p)\) comparable to that of the 0.123 MeV state, and moreover, the angular distribution (Fig. 3.1) in \((α,p)\) is that of a \(7/2^-\) state.

Whether or not there are more states which exhibit such a spectacular difference in relative cross section between \((^3\text{He},d)\) and \((α,p)\) is difficult to ascertain due to the uncertainty of the \((α,p)\) reaction mechanism at this energy and also to the high density of states. Nevertheless, the number of states excited at 18 MeV is still smaller than that expected if all known states [Nucl. Data Sheets, 1970] were excited. In \(^{53}\text{Mn}\) for example, there are twelve groups excited up to 3.1 MeV whereas there are nineteen states according to the adopted level scheme.

\(\text{(C)} \quad ^{50,52,54}\text{Cr}(α,p)^{53,55,57}\text{Mn} \text{ AT } E_α = 26 \text{ MeV}\)

1) Proton Spectra

Examples of proton spectra measured at 26 MeV are displayed in Figs. 3.13 to 3.16. The labelling of the peaks in these figures corresponds to the labelling in Tables 3.1, 3.2 and 3.3 and to the angular distributions in Figs. 3.18 to 3.31. In order to facilitate comparison, angular distributions have been grouped according to similarities of shape rather than by isotope.

Among the salient features of the spectra is the reduction in the number of states excited at 26 MeV as compared to 18 MeV. In \(^{53}\text{Mn}\)
FIG. 3.13

Proton spectra from the $^{52,54}$Cr($\alpha$,p)$^{55,57}$Mn reactions at 26 MeV bombarding energy. The experimental resolution is ~50 keV FWHM. The labelling is the same as in Fig. 3.11. The arrow points to the expected location of the $^{57}$Mn ground state group based on the mass tables. Note the near absence of the ground state groups in both cases.
Peaks labelled as in Table 3.1.*

FIG. 3.14: Peaks labelled as in Table 3.1.
FIG. 3.15. Peaks labelled as in Table 3.2.

$^{52}$CR (A, P) $^{55}$MN

26MEV 36° 5DEG
FIG. 3.16: Peaks labelled as in Fig. 3.11.
for example, at 18 MeV there are twenty-seven proton groups (Fig. 3.12) which can be assigned to this nucleus up to 4.079 MeV excitation. At 26 MeV there are ten groups which can be identified with $^{53}$Mn states in the same energy span. The influence of the seniority selection rule can now be seen by the near absence of the first four excited states of $^{53}$Mn at 26 MeV as compared with 18 MeV. The reduction in the compound nuclear contribution to the cross section has allowed states with predominantly direct reaction production modes to emerge above the background. Thus, at 26 MeV an interesting group of states (numbers 23 to 27) in $^{55}$Mn is seen at about 5.5 MeV excitation, whereas at 18 MeV the analysis of data stopped at about 4.7 MeV due to the high density of states. In contrast to the situation at 18 MeV nearly all the states excited at 26 MeV have angular distributions which can be described by DWBA.

(2) J-Dependence of the ($\alpha$,p) Reaction

Based on general arguments Henley [1966] have concluded that the ($\alpha$,p) and (p,$\alpha$) angular distributions should show a dependence upon the transferred J as well as on the L transfer. They define two quantities $\sigma^R(\Omega)$ and $\delta\sigma^R_J(\Omega)$ which are related to the differential cross section $\sigma_J(\Omega)$ by

$$\sigma_J(\Omega) = (2J+1)[\sigma^R(\Omega) + \delta\sigma^R_J(\Omega)] , \quad (3.1)$$

where $\sigma^R(\Omega)$ is the cross section due to the central terms of the optical model potential and $\delta\sigma^R_J(\Omega)$ arises from the spin-orbit L*S term. The quantities $\delta\sigma^R_J(\Omega)$ are simply related to one another by

$$\delta\sigma^R_{L+1/2} = -\left(\frac{L}{L+1}\right) \delta\sigma^R_{L-1/2} \quad (3.2)$$

From (3.2) it can be seen that the contributions due to the spin-orbit
term are out of phase and that the deviation of the cross section from that due solely to the central term of the optical potential is largest for the $L-\frac{1}{2}$ transfer. For example, $\delta_{3/2}^R = (-1/2)\delta_{1/2}^R$ for $L=1$. In particular, Henly [1966] have concluded that near $\theta = 0^\circ$ (3.2) predicts that the $L-\frac{1}{2}$ transfer is enhanced relative to the $L+\frac{1}{2}$ transfer.

This general property of the differential cross section introduced by the spin-orbit potential can be observed, of course, only if the quantities $\delta_{0}^R$ are large enough. Experimentally a prominent $L=1$ $J$-dependence has been observed for a wide range of targets and bombarding energies; for example in $^{58,60}\text{Ni}(\alpha,p)^{61,63}\text{Cu}$ at $E_\alpha = 18$ MeV [Lee, 1965], in $^{66,68}\text{Zn}(p,\alpha)^{63,65}\text{Cu}$ at $E_p = 17.5$ MeV and $^{66}\text{Zn}(p,\alpha)^{63}\text{Cu}$, $^{92}\text{Zr}(p,\alpha)^{89}\text{Y}$ at 28 MeV [Nolen, 1966], in $^{107}\text{Ag}(p,\alpha)^{104}\text{Pd}$ at 12 MeV [Dittmer, 1969], in $^{108}\text{Pd}(p,\alpha)^{105}\text{Rh}$ at $E_p = 15$ MeV [Dittmer, 1970] and in several other studies as well. An $L=2$ $J$-dependence was measured by Glenn [1971] at $E_\alpha = 35.5$ MeV in $^{26}\text{Mg}(\alpha,p)^{29}\text{Al}$ and $^{28}\text{Si}(\alpha,p)^{31}\text{P}$. They observed the predicted enhancement of the $3/2^+$ transition relative to the $5/2^+$ transitions for $\sigma_J(\theta)$ measured as far forward as $2^\circ$. However, August [1969] did not observe an $L=2$ $J$-dependence for $^{28}\text{Si}(\alpha,p)^{31}\text{P}$ at $E_\alpha = 28.8$ MeV and $25^\circ \leq \theta \leq 140^\circ$. Nevertheless, August [1971] did observe a $J$-dependence for $L=2$ in the $^{32}\text{S}(\alpha,p)^{35}\text{Cl}$ and $^{24}\text{Mg}(\alpha,p)^{27}\text{Al}$ reactions at $E_\alpha = 28.8$ MeV. The differential cross sections for $40^\circ \leq \theta \leq 130^\circ$ were out of phase for $J=3/2^+$ and $5/2^+$ transitions. An $L=3$ $J$-dependence has been measured in the $^{40,42}\text{Ca}(\alpha,p)^{43,45}\text{Sc}$ reactions at 31 MeV [Ginaven, 1970].

The $L=1$ $J$-dependence found in the $^{50,52}\text{Cr}(\alpha,p)^{53,55}\text{Mn}$ reactions is in agreement with the previously mentioned studies. The $J$-dependence for $L=1$ is characterized by a relatively smooth decline of cross section with angle for $J^\tau = 3/2^-$ as compared with the pronounced
oscillations found for $J_T^* = 1/2^-$. An example of this $J$-dependence can be found in Fig. 3.20.

A test of $L=3$ $J$-dependence is possible for the $^{50,52}$Cr$(\alpha,p)^{53,55}$Mn reactions because of the $L=3$ assignments made from single proton transfer reactions. In $^{53}$Mn O'Brien [1967] and Gunn [1976] have made $L_p=3$ assignments to the states at $E_x = 0.0, 3.110, 3.670$ and $4.067$ MeV. In $^{55}$Mn Rapaport [1969] have made $L_p=3$ assignments using ($^3$He,d) for the states at $E_x = 0.128, 1.883, 3.136, 3.608$ and $5.498$ MeV. The proton angular distributions for these states using the $(\alpha,p)$ reaction can be categorized into two groups. One group consists of the ground and $4.067$ MeV states in $^{53}$Mn and the $0.128, 1.883$ and $5.498$ MeV states in $^{55}$Mn. This group is characterized by a peak in the cross section at about $35^\circ$ (see Fig. 3.18). The second group consists of the $3.110$ and $3.670$ MeV states in $^{53}$Mn and the $3.136$ MeV state in $^{55}$Mn. The angular distributions for these states show less structure than do those for the first group. In particular this group does not exhibit a maximum in the cross section at $\theta = 35^\circ$. Based on the DWBA calculations, the second group is preferentially $5/2^-$ and the first group consists of $7/2^-$ transfers. These spin assignments are in accord with previous studies, except for the assignment of $7/2^-$ at $4.067$ MeV in $^{53}$Mn. A discussion of these states is in Tables 3.1 and 3.2 and in sections (E) and (F) of this chapter. An $L=3$ $J$-dependence exists for transitions to these states which the triton cluster transfer model is able to predict.

(3) Population of Hole States in $^{53,55}$Mn

As was discussed in Chapter 2, the presence or absence of hole states excited in $(\alpha,p)$ is an indicator of the reaction mechanism. The
L_p = 0 transitions reported in (³He,d) in ⁵³,⁵⁵Mn by O'Brien [1967] and Rapaport [1969] are not excited with a substantial cross section, if at all, in the (α,p) reaction. The L_p = 2 transition to the 2.980 MeV state in ⁵⁵Mn (peak #10) is somewhat obscured by a neighbouring group. The angular distribution of this state in Fig. 3.24 is not fitted by a 3/2⁺ or 5/2⁺ transition using DWBA. It is possible that peak #10 in ⁵⁵Mn is an unresolved group, or that the 2.980 MeV level is being excited in some way other than by the direct stripping mode. It is the only obvious candidate for a knock-out reaction since it is believed to be a hole state [Rapaport, 1969]. The L_p = 0 states are also believed to be hole states. The knock-out reaction is believed to produce the same shaped angular distribution as stripping [Sherr, 1962; Kelley, 1964; Hodgson, 1971, Chapter 16]; the difference in the two modes being only one of magnitude of the cross section. If this is indeed true then group #10 in ⁵⁵Mn probably does not originate from knock-out or stripping processes. Nevertheless, due to the interference of peak #11 which is nearly three times more intense than #10, this conclusion must be qualified. A knock-out process is still possible for the 2.980 MeV state in ⁵⁵Mn but does not seem likely due to the absence of the other hole states (i.e., L_p = 0) in ⁵³,⁵⁵Mn in the (α,p) spectra.

(D) DWBA ANALYSIS

The computer code DWUCK [1969] was used to analyse the (α,p) data. A quasi-triton cluster transfer, as described in Chapter 2, was employed to generate the form factor. The proton optical model parameters were determined from the Becchetti-Greenlees formulae. These formulae and the alpha particle optical parameters can be found in the
Perey [1974] compilation. The alpha particle parameters were obtained by Lemos [1972] from the scattering of 21 MeV α-particles on $^{52}\text{Cr}$ and 25 MeV α-particles on $^{50}\text{Cr}$. The optical model parameters and triton well geometries used for the DWBA calculations in Figs. 3.1 to 3.10 (18 MeV data) are presented in Table 3.0. Volume absorption was used for the α-particle optical parameters and the depth of the triton well was chosen so that the triton was bound by an energy $S_B = S_t - E_x$, where $S_t$ is the separation energy of the triton from the residual nucleus and $E_x$ is the excitation energy of the residual nucleus.

Table 3.0
18 MeV Alpha Optical Parameters and Triton Well Geometries

<table>
<thead>
<tr>
<th></th>
<th>$V$ (MeV)</th>
<th>$r_0$ (fm)</th>
<th>$a_0$ (fm)</th>
<th>$W$ (MeV)</th>
<th>$r_w$ (fm)</th>
<th>$a_w$ (fm)</th>
<th>$r_c$ (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>alpha</td>
<td>-189.3</td>
<td>1.36</td>
<td>0.57</td>
<td>-24.9</td>
<td>1.36</td>
<td>0.57</td>
<td>1.40</td>
</tr>
<tr>
<td>triton ($^{53}\text{Mn}$)</td>
<td>1.38</td>
<td>0.23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>triton ($^{55}\text{Mn}$)</td>
<td>1.40</td>
<td>0.35</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>triton ($^{57}\text{Mn}$)</td>
<td>1.40</td>
<td>0.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

These values for the triton well parameters were found to give generally good fits and were the ones used at 18 MeV except where indicated otherwise in the figures.

At 26 MeV fourteen of the optical model parameter sets listed in the Perey [1974] compilation in the Cr region were investigated vis-à-vis the elastic scattering. The Lemos parameter set αx which gave the best fit to the elastic scattering at 26 MeV is presented in Fig. 3.17 along with the experimental and theoretical angular distributions. The errors in the elastic scattering contain a 4% normalization error.
due to the use of several detectors simultaneously. The absolute cross section scale is believed to be correct to within 7%. The errors in the reaction cross sections contain both the statistical error and an estimate of the error introduced by the spectrum fitting procedure. A detailed discussion of the error analysis and absolute cross section determinations can be found in Appendix A2, section (C). The triton well geometries had to be changed at 26 MeV from the values used at 18 MeV. The values of the triton well parameters used in the calculations in Fig. 3.18 to 3.30 are listed in Tables 3.1, 3.2 and 3.3.

The calculated angular distributions have been normalized by eye to the experimental distributions. The experimental distributions to some of the known $L_p = 1$ states in $^{55}$Mn are well described by the calculations except for a fall in cross section at about 23° c.m. An example of this is the state at 3.041 (peak #11 in Fig. 3.25). These states also differ from other known $L_p = 1$ transitions; for example, the 1.528 MeV level depicted in Fig. 3.20, which continues to show a rising cross section forward of 25°. The experimental angular distributions to known $L_p = 1$ states in $^{53}$Mn also continue to rise forward of 25°.

In general, the experimental angular distributions are similar in shape. The most structured angular distribution is for the $J_T = 1/2$ transition. For many states it is difficult to make spin assignments due to the similarities in the angular distributions or to poor statistics. For transitions with previously determined $L_p$ values it is usually easy to distinguish between the two possible $J_T$ values. If $L$ is to be determined from the $(\alpha, p)$ angular distributions there is no clear cut choice for $J_T$. An example of this ambiguity is presented in Fig. 3.31 for peak numbers 32 and 33 in $^{53}$Mn. Calculated angular distributions for $J_T = 5/2^-$, $5/2^+$ and $7/2^+$ all fit the experimental
Table 3.1
$^{50}$Cr(α,p)$^{53}$Mn, 26 MeV, Triton Well Parameters

<table>
<thead>
<tr>
<th>Peak #</th>
<th>Fig. #(6)</th>
<th>$-Q \pm Q_{\text{rms}}$ (1) keV</th>
<th>$E_x$ (3)(10) keV</th>
<th>$J^\pi_p$ or $J^\pi$ (3)(10)</th>
<th>Triton Well Parameters(15)</th>
<th>$\sigma(\theta_0) \pm \Delta\sigma$(8) µb/sr</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A(7)</td>
<td>3.18</td>
<td>432 ± 11</td>
<td>0</td>
<td>$f_{7/2}$</td>
<td>7/2$^-$, 3, 1.45, 0.35</td>
<td>36.7, 2</td>
</tr>
<tr>
<td>1B(7)</td>
<td>3.32(b)</td>
<td>1732</td>
<td>1288</td>
<td>1</td>
<td>3/2$^-$, 4, 1.48, 0.10</td>
<td>10.9, 1.3</td>
</tr>
<tr>
<td>2</td>
<td>3.19</td>
<td>2845 ± 7</td>
<td>2413</td>
<td>$P_{3/2}$</td>
<td>3/2$^-$ #1, 4, 1.48, 0.10</td>
<td>119, 4</td>
</tr>
<tr>
<td>3</td>
<td>3.18</td>
<td>3010 ± 8</td>
<td>2578</td>
<td>7/2$^-$</td>
<td>3, 1.45, 0.35</td>
<td>7.0, 1.8</td>
</tr>
<tr>
<td>4</td>
<td>3.20</td>
<td>3116 ± 5</td>
<td>2684</td>
<td>$P_{1/2}$</td>
<td>1/2$^-$ #1, 4, 1.45, 0.25</td>
<td>50.1, 3.6</td>
</tr>
<tr>
<td>5(9)</td>
<td>3.29</td>
<td>3542 ± 4</td>
<td>3110(4), 3061,3104</td>
<td>1,3</td>
<td>5/2$^-$, 3, 1.45, 0.35</td>
<td>13.7, 2.2</td>
</tr>
<tr>
<td>6</td>
<td>3.21</td>
<td>3892 ± 15</td>
<td>3460</td>
<td>$P_{1/2}$</td>
<td>1/2$^-$ #1, 4, 1.45, 0.30</td>
<td>40.5, 3.3</td>
</tr>
<tr>
<td>7</td>
<td>3.22</td>
<td>3993 ± 7</td>
<td>3561</td>
<td>11/2$^-$</td>
<td>3, 1.45, 0.35</td>
<td>44.4, 3.5</td>
</tr>
<tr>
<td>8</td>
<td>3.23</td>
<td>4102 ± 10</td>
<td>3670</td>
<td>$f_{5/2}$</td>
<td>5/2$^-$, 3, 1.50, 0.35</td>
<td>37.2, 3.3</td>
</tr>
<tr>
<td>9</td>
<td>3.21</td>
<td>4325 ± 10</td>
<td>3893</td>
<td>1</td>
<td>7/2$^-$, 3, 1.50, 0.35</td>
<td>43.2, 3.5</td>
</tr>
<tr>
<td>10</td>
<td>3.29</td>
<td>4499 ± 9</td>
<td>4067(5), 4070</td>
<td>$f_{5/2}$</td>
<td>1/2$^-$ #1, 4, 1.45, 0.30</td>
<td>65.5, 5.2</td>
</tr>
<tr>
<td>11</td>
<td>3.23</td>
<td>4737 ± 11</td>
<td>4305(4), 4278,4304</td>
<td>3</td>
<td>7/2$^-$, 3, 1.50, 0.35</td>
<td>14.1, 1.8</td>
</tr>
<tr>
<td>Peak #</td>
<td>Fig. #(6)</td>
<td>$\bar{Q} \pm Q_{rms}$ keV</td>
<td>$E_x$ keV</td>
<td>$E'_x$ keV</td>
<td>$L_p$ or $J^\pi_\rho$ (3)(10)</td>
<td>Triton Well Parameters</td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
<td>------------------</td>
<td>---------</td>
<td>---------</td>
<td>-----------------</td>
<td>----------------------</td>
</tr>
<tr>
<td>12</td>
<td>3.24</td>
<td>4994</td>
<td>9</td>
<td>4562</td>
<td>4569</td>
<td>1</td>
</tr>
<tr>
<td>13</td>
<td>3.24</td>
<td>5058</td>
<td>13</td>
<td>4626</td>
<td></td>
<td>$3/2^-$ 4 1.48 0.10</td>
</tr>
<tr>
<td>14</td>
<td>3.24</td>
<td>5219</td>
<td>11</td>
<td>4787</td>
<td>4788 (14) (0),(1)</td>
<td>$5/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>15</td>
<td>3.24</td>
<td>5272</td>
<td>10</td>
<td>4840</td>
<td></td>
<td>$7/2^-$ 3 1.50 0.35</td>
</tr>
<tr>
<td>16</td>
<td>3.19</td>
<td>5382</td>
<td>6</td>
<td>4950(4)</td>
<td>4963,4936</td>
<td>$1,3$</td>
</tr>
<tr>
<td>17</td>
<td>3.24</td>
<td>5464</td>
<td>11</td>
<td>5032</td>
<td></td>
<td>$5/2^-$ 3 1.50 0.35</td>
</tr>
<tr>
<td>18</td>
<td>NONE</td>
<td>5644</td>
<td>(2)</td>
<td>5112</td>
<td>5085</td>
<td>1</td>
</tr>
<tr>
<td>19</td>
<td>3.24</td>
<td>5687</td>
<td>12</td>
<td>5155</td>
<td></td>
<td>$5/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>20</td>
<td>NONE</td>
<td>5672</td>
<td>(2)</td>
<td>5240</td>
<td></td>
<td>$9/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>21</td>
<td>3.24</td>
<td>5788</td>
<td>7</td>
<td>5356</td>
<td></td>
<td>$13/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>22</td>
<td>3.18</td>
<td>5878</td>
<td>9</td>
<td>5446</td>
<td></td>
<td>$7/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>23</td>
<td>3.21</td>
<td>5924</td>
<td>14</td>
<td>5492</td>
<td>5485</td>
<td>1</td>
</tr>
<tr>
<td>24</td>
<td>3.22</td>
<td>6246</td>
<td>5</td>
<td>5814</td>
<td>5800</td>
<td>$11/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>25</td>
<td>3.30</td>
<td>6313</td>
<td>7</td>
<td>5881</td>
<td>5886</td>
<td>$5/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>26</td>
<td>3.27</td>
<td>6438</td>
<td>4</td>
<td>6006</td>
<td>6005</td>
<td>$3/2^-$ 4 1.48 0.10</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5/2^-$ 3 1.45 0.35</td>
</tr>
<tr>
<td>Peak #</td>
<td>Fig. #</td>
<td>( Q(T) ) ( \pm Q_{\text{rms}} ) (1)</td>
<td>( E_x ) keV</td>
<td>( J_p ) or ( J_p ) (3)(10)</td>
<td>( N )</td>
<td>( N_0 )</td>
</tr>
<tr>
<td>-------</td>
<td>-------</td>
<td>-------------------------------</td>
<td>-----------</td>
<td>----------------</td>
<td>-----</td>
<td>------</td>
</tr>
<tr>
<td>27</td>
<td>3.27</td>
<td>6551 10</td>
<td>6119</td>
<td>6.15</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>28</td>
<td>3.27</td>
<td>6609 5</td>
<td>6177</td>
<td>6.24</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>29</td>
<td>3.27</td>
<td>6688 7</td>
<td>6256</td>
<td>6.32</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>30</td>
<td>3.27</td>
<td>6774 4</td>
<td>6442</td>
<td>6.41</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>31</td>
<td>3.27</td>
<td>6850 8</td>
<td>6418</td>
<td>6.54</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>32</td>
<td>3.27</td>
<td>6971 4</td>
<td>6542</td>
<td>6.41</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>33</td>
<td>3.27</td>
<td>7033</td>
<td>6601</td>
<td>6.87</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>34</td>
<td>3.27</td>
<td>7176 10</td>
<td>6744</td>
<td>6.97</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>35</td>
<td>3.28</td>
<td>7305 11</td>
<td>6873</td>
<td>7.10</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>36</td>
<td>3.28</td>
<td>7422 11</td>
<td>6990</td>
<td>7.54</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>37</td>
<td>3.28</td>
<td>7515 11</td>
<td>7083</td>
<td>7.91</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>38</td>
<td>3.28</td>
<td>8006 14</td>
<td>7574</td>
<td>8.05</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>39</td>
<td>3.28</td>
<td>8331 19</td>
<td>7899</td>
<td>8.05</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>40</td>
<td>3.28</td>
<td>8448 13</td>
<td>8016</td>
<td>8.05</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>(11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(12)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(13)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Table 3.1 (cont'd)

NOTES:

1. Average $\bar{Q}$ and $Q_{rms}$ deviation over 10 to 14 angles at 26 MeV.
2. Peaks seen at too few angles to obtain an angular distribution.
3. Measured by O'Brien et al. [1967].
4. Suspected unresolved doublet in $^{50}$Cr($\alpha$,p)$^{53}$Mn 26 MeV based on ($^3$He,d) data.
5. O'Brien et al. report an unresolved doublet in ($^3$He,d).
6. Proton angular distributions for these states are to be found in the specified figure.
7. Due to instrumental effects peaks #1A and #1B have a separate energy calibration from the rest of the spectrum in Fig. 3.14.
8. $\theta_0$ is the most forward angle plotted in Figs. 3.18 to 3.32. For most states this corresponds to $\sim 20.8^\circ$ c.m.
9. Tarara [1976] suggest that the energies of these levels determined by O'Brien [1967] are in error. Tarara [1976] claim that the L = 3 level should be at $\sim 3100$ keV instead of at 3061 keV as given by O'Brien et al. This re-assignment is in closer accord with the ($\alpha$,p) data presented here.
10. For $E_x > 6$ MeV these values come from Gunn [1976] and $J^\pi$ assignments from Gunn [1976] from ($^7$Li,$^4$He).
11. Gunn [1976] believe this to be IAS of $^{53}$Cr g.s.
12. Gunn [1976] believe this to be the IAS of $^{53}$Cr 1st excited state.
13. Gunn [1976] believe this to be the IAS of $^{53}$Cr 2nd excited state but the IAS cannot be excited in ($\alpha$,p).
14. Čujec [1969] claim that $L_p = 1$ is also possible.
15. Parameters used for angular distributions in Figs. 3.18 to 3.30.
Table 3.2
$^{52}\text{Cr}(\alpha,p)^{55}\text{Mn}$, 26 MeV, Triton Well Parameters

<table>
<thead>
<tr>
<th>Peak #</th>
<th>Fig. #</th>
<th>$-Q_{\text{TMS}}$ (keV)</th>
<th>$E_x$ (keV)</th>
<th>$E_x$ (keV)</th>
<th>$L_p$ (3)</th>
<th>Triton Well Parameters (4)</th>
<th>$\sigma(\theta_0) \pm \Delta\sigma$ (6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.18</td>
<td>2697 7</td>
<td>128</td>
<td>127</td>
<td>3</td>
<td>$7/2^-$ 3 1.50 0.35</td>
<td>18.9 1.6</td>
</tr>
<tr>
<td>2</td>
<td>3.22</td>
<td>3552 6</td>
<td>983</td>
<td></td>
<td></td>
<td>$9/2^-$ 3 1.45 0.35</td>
<td>5.3 0.8</td>
</tr>
<tr>
<td>3</td>
<td>3.22</td>
<td>3860 5</td>
<td>1291</td>
<td></td>
<td></td>
<td>$3/2^-$ 4 1.45 0.35</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.20</td>
<td>4097 2</td>
<td>1528</td>
<td>1527</td>
<td>1</td>
<td>$11/2^-$ 3 1.45 0.35</td>
<td>22.3 1.5</td>
</tr>
<tr>
<td>5</td>
<td>3.18</td>
<td>4452 4</td>
<td>1883</td>
<td>1881</td>
<td>3</td>
<td>$1/2^-$ 4 1.45 0.35</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3.18</td>
<td>4766 7</td>
<td>2197</td>
<td>2198</td>
<td></td>
<td>$3/2^-$ 4 1.45 0.35</td>
<td>46.0 2.2</td>
</tr>
<tr>
<td>7</td>
<td>3.19</td>
<td>4827 5</td>
<td>2258</td>
<td>2250</td>
<td>1</td>
<td>$7/2^-$ 3 1.45 0.35</td>
<td>6.5 0.9</td>
</tr>
<tr>
<td>8</td>
<td>3.19</td>
<td>5131 6</td>
<td>2562</td>
<td>2560</td>
<td>1</td>
<td>$3/2^-$ 4 1.45 0.35</td>
<td>18.6 1.5</td>
</tr>
<tr>
<td>9</td>
<td>3.29</td>
<td>5310</td>
<td>2741</td>
<td>2742</td>
<td></td>
<td>$3/2^-$ 4 1.45 0.35</td>
<td>36.6 2.0</td>
</tr>
<tr>
<td>10</td>
<td>3.24</td>
<td>5549 14</td>
<td>2980</td>
<td>2984</td>
<td>2</td>
<td>$7/2^-$ 3 1.45 0.35</td>
<td>7.6 1.1</td>
</tr>
</tbody>
</table>

83
<table>
<thead>
<tr>
<th>Peak #</th>
<th>Fig. #</th>
<th>$\bar{Q} \pm Q_{\text{rms}}$ keV</th>
<th>$E_x$ keV</th>
<th>$E_x$ keV</th>
<th>$L_p$</th>
<th>Triton Well Parameters</th>
<th>$\sigma(\theta_0) \pm \Delta \sigma$(6) µb/sr</th>
</tr>
</thead>
<tbody>
<tr>
<td>11</td>
<td>3.25</td>
<td>5610</td>
<td>3041</td>
<td>3028</td>
<td>1</td>
<td>$3/2^-$</td>
<td>30.0  1.9</td>
</tr>
<tr>
<td>12</td>
<td>3.23</td>
<td>5705</td>
<td>3136</td>
<td>3147</td>
<td>3</td>
<td>$5/2^-$</td>
<td>56.0  2.5</td>
</tr>
<tr>
<td>13</td>
<td>3.26</td>
<td>5832</td>
<td>3263</td>
<td>3260</td>
<td>3</td>
<td>$3/2^-$</td>
<td>12.1  1.3</td>
</tr>
<tr>
<td>14</td>
<td>3.26</td>
<td>5923</td>
<td>3354</td>
<td>3354</td>
<td>1</td>
<td>$1/2^-$</td>
<td>22.9  1.8</td>
</tr>
<tr>
<td>15</td>
<td>3.20</td>
<td>6000</td>
<td>3431</td>
<td>3429</td>
<td>1</td>
<td>$7/2^-$ #1</td>
<td>28.2  1.9</td>
</tr>
<tr>
<td>16</td>
<td>3.30</td>
<td>6177</td>
<td>3608</td>
<td>3608</td>
<td>3</td>
<td>$7/2^-$ #2</td>
<td>4.15  0.20</td>
</tr>
<tr>
<td>17</td>
<td>3.21</td>
<td>6416</td>
<td>3847</td>
<td>3847</td>
<td>3</td>
<td>$9/2^+$ #1</td>
<td>17.2  1.6</td>
</tr>
<tr>
<td>18</td>
<td>3.21</td>
<td>6558</td>
<td>3989</td>
<td>3998</td>
<td>1</td>
<td>$9/2^+$ #2</td>
<td>17.2  1.6</td>
</tr>
<tr>
<td>19</td>
<td>3.25</td>
<td>6655</td>
<td>4086</td>
<td>4086</td>
<td>1</td>
<td>$5/2^-$</td>
<td>20.6  2.4</td>
</tr>
<tr>
<td>20</td>
<td>3.25</td>
<td>6835</td>
<td>(2) 4266</td>
<td>4266</td>
<td>1</td>
<td>$3/2^-$ #1</td>
<td>43.7  1.9</td>
</tr>
<tr>
<td>21</td>
<td>3.26</td>
<td>7209</td>
<td>4640</td>
<td>4638</td>
<td>1</td>
<td>$3/2^-$ #2</td>
<td>15.9  1.9</td>
</tr>
</tbody>
</table>

Table 3.2 (cont'd)
<table>
<thead>
<tr>
<th>Peak #</th>
<th>Fig. #</th>
<th>( E_x ) (keV)</th>
<th>( J^\pi )</th>
<th>( Q_{rms} ) (keV)</th>
<th>( I_p )</th>
<th>Triton Well Parameters 1</th>
<th>( \sigma(0_0^+; r_0) ) (ub/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>22</td>
<td></td>
<td>7305</td>
<td>3/2^-</td>
<td>2.25</td>
<td>1</td>
<td>4.145 0.35</td>
<td>34.7 2.8</td>
</tr>
<tr>
<td>23</td>
<td></td>
<td>7829</td>
<td>2/1^-</td>
<td>3.50</td>
<td>8</td>
<td>3.145 0.01</td>
<td>39.7 3.0</td>
</tr>
<tr>
<td>24</td>
<td></td>
<td>7334</td>
<td>7/2^-</td>
<td>3.19</td>
<td>6</td>
<td>3.145 0.10</td>
<td>87.4 3.7</td>
</tr>
<tr>
<td>25</td>
<td></td>
<td>8067</td>
<td>5/2^-</td>
<td>3.18</td>
<td>7</td>
<td>3.145 0.10</td>
<td>172.7 4.9</td>
</tr>
<tr>
<td>26</td>
<td></td>
<td>8638</td>
<td>7/2^-</td>
<td>3.50</td>
<td>20</td>
<td>3.145 0.35</td>
<td>39.9 5.5</td>
</tr>
<tr>
<td>27</td>
<td></td>
<td>8733</td>
<td>11/2^-</td>
<td>3.28</td>
<td>22</td>
<td>3.145 0.35</td>
<td>132.3 5.5</td>
</tr>
</tbody>
</table>

NOTES:
1. R.m.s. deviation over 15 to 20 angles.
2. Suspected unresolved doublet in \( ^5\text{Cr}(d,p)^{12}\text{Cr} \).
3. Parameters used for angular distributions are those found in Figs. 3.18 to 3.30.
4. Parameters based on \( ^3\text{He},d \) data.
5. Proton angular distribution for these peaks are to be found in the specified figure.
<table>
<thead>
<tr>
<th>Peak #</th>
<th>Fig. #</th>
<th>$-\bar{Q}^* \pm Q_{rms}^{(1)}$ keV</th>
<th>$E_x^{(2)}$ keV</th>
<th>$E_x^{(2)}$ keV</th>
<th>Triton Well Parameters$^{(7)}$</th>
<th>$\sigma(\theta^0) \pm \Delta \sigma^{(6)}$</th>
<th>( \mu b/sr )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>3.18</td>
<td>4386 6</td>
<td>84</td>
<td>84</td>
<td>7/2$^-$</td>
<td>3  1.50 0.35</td>
<td>40.0 1.8</td>
</tr>
<tr>
<td>3</td>
<td>3.19</td>
<td>5154 6</td>
<td>852</td>
<td>851</td>
<td>3/2$^-$ #1 4</td>
<td>1.45 0.20</td>
<td>63.5 2.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3/2$^-$ #2 4</td>
<td>1.45 0.10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>3.22</td>
<td>5366 6</td>
<td>1064</td>
<td>1074</td>
<td>9/2$^-$</td>
<td>3  1.45 0.35</td>
<td>9.5 1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/2$^-$</td>
<td>4  1.45 0.35</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>3.22</td>
<td>5531 7</td>
<td>1229</td>
<td>1227</td>
<td>9/2$^-$</td>
<td>3  1.45 0.35</td>
<td>22.2 1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>11/2$^-$</td>
<td>3  1.45 0.35</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>3.30</td>
<td>5680 6</td>
<td>1378</td>
<td>1376</td>
<td>NONE</td>
<td></td>
<td>2.4 0.6</td>
</tr>
<tr>
<td>12</td>
<td>3.23</td>
<td>6227 7</td>
<td>1925</td>
<td></td>
<td>5/2$^-$ #1 3</td>
<td>1.45 0.35</td>
<td>12.6 1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5/2$^-$ #2 3</td>
<td>1.40 0.30</td>
<td></td>
</tr>
<tr>
<td>14</td>
<td>3.20</td>
<td>6490 14</td>
<td>2188</td>
<td>2188</td>
<td>1/2$^-$</td>
<td>4  1.45 0.35</td>
<td>35.8 1.8</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1/2$^+$</td>
<td>5  1.45 0.35</td>
<td></td>
</tr>
<tr>
<td>15</td>
<td>3.26</td>
<td>6540 10</td>
<td>2238</td>
<td>2234</td>
<td>3/2$^-$ #1 4</td>
<td>1.48 0.15</td>
<td>28.1 1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>3/2$^-$ #2 4</td>
<td>1.45 0.20</td>
<td></td>
</tr>
<tr>
<td>18</td>
<td>3.29</td>
<td>6827 10</td>
<td>2525</td>
<td></td>
<td>9/2$^+$</td>
<td>3  1.45 0.10</td>
<td>16.1 1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5/2$^-$ #1 3</td>
<td>1.45 0.35</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>5/2$^-$ #2 3</td>
<td>1.40 0.30</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.3

$^{54}$Cr($\alpha$,p)$^{57}$Mn, 26 MeV, Triton Well Parameters
### Table 3.3 (cont'd)

<table>
<thead>
<tr>
<th>Peak #</th>
<th>Fig. #(5)</th>
<th>$-Q \pm Q_{rms}$ keV</th>
<th>$E_x$ keV</th>
<th>$E_x^{(2)}$ keV</th>
<th>Triton Well Parameters</th>
<th>$\sigma(\theta_0) \pm \Delta \sigma$(6) μb/sr</th>
</tr>
</thead>
<tbody>
<tr>
<td>19 + 20</td>
<td>3.29</td>
<td>6920</td>
<td>(3)</td>
<td>2618</td>
<td>$9/2^+$</td>
<td>3  1.45  0.10   17.9  1.4</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5/2^-$ #1</td>
<td>3  1.45  0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5/2^-$ #2</td>
<td>3  1.40  0.30</td>
</tr>
<tr>
<td>23</td>
<td>3.19</td>
<td>7150</td>
<td>14</td>
<td>2848</td>
<td>$3/2^-$ #1</td>
<td>4  1.48  0.15   38.3  1.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$3/2^-$ #2</td>
<td>4  1.45  0.10</td>
</tr>
<tr>
<td>25</td>
<td>3.19</td>
<td>7425</td>
<td>(3)</td>
<td>3123</td>
<td>$3/2^-$</td>
<td>4  1.48  0.15   47.0  2.2</td>
</tr>
<tr>
<td>27 + 28</td>
<td>3.30</td>
<td>7534</td>
<td>(3)</td>
<td>3232</td>
<td>NONE</td>
<td></td>
</tr>
<tr>
<td>30 + 31</td>
<td>3.26</td>
<td>7770</td>
<td>(3)</td>
<td>3468</td>
<td>$9/2^+$</td>
<td>3  1.45  0.10   74.9  2.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$3/2^-$</td>
<td>4  1.45  0.20</td>
</tr>
<tr>
<td>32</td>
<td>3.23</td>
<td>7844</td>
<td>10</td>
<td>3542</td>
<td>$3/2^-$</td>
<td>4  1.45  0.10   29.0  1.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5/2^-$</td>
<td>3  1.45  0.10</td>
</tr>
<tr>
<td>34 + 35</td>
<td>3.26</td>
<td>8000</td>
<td>(3)</td>
<td>3698</td>
<td>$3/2^-$</td>
<td>4  1.48  0.15   35.6  1.6</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$9/2^-$</td>
<td>3  1.45  0.35</td>
</tr>
<tr>
<td>38</td>
<td>3.29</td>
<td>8140</td>
<td>10</td>
<td>3838</td>
<td>$9/2^+$</td>
<td>3  1.45  0.10   19.9  1.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5/2^-$ #1</td>
<td>3  1.45  0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5/2^-$ #2</td>
<td>3  1.40  0.30</td>
</tr>
<tr>
<td>41</td>
<td>3.29</td>
<td>8249</td>
<td>11</td>
<td>3947</td>
<td>$3/2^-$</td>
<td>4  1.48  0.15   15.7  1.1</td>
</tr>
<tr>
<td>42</td>
<td>3.29</td>
<td>8331</td>
<td>4</td>
<td>4029</td>
<td>$3/2^-$</td>
<td>4  1.48  0.15   14.7  1.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$5/2^-$</td>
<td>3  1.50  0.35</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$9/2^-$</td>
<td>3  1.45  0.35</td>
</tr>
</tbody>
</table>
Table 3.3 (cont'd)

<table>
<thead>
<tr>
<th>Peak #</th>
<th>Fig. #(5)</th>
<th>-Q ± Q_{rms} (1) keV</th>
<th>E_x (2) keV</th>
<th>Triton Well Parameters</th>
<th>(\sigma(\theta_0) ± \Delta\sigma(6)) ub/sr</th>
</tr>
</thead>
<tbody>
<tr>
<td>43</td>
<td>3.25</td>
<td>8470 ± 10</td>
<td>4168</td>
<td>(3/2^-) 4 1.48 0.15</td>
<td>26.0 1.4</td>
</tr>
<tr>
<td>44</td>
<td>3.25</td>
<td>8513 ± 10</td>
<td>4211</td>
<td>(9/2^-) 3 1.45 0.35</td>
<td>20.8 1.3</td>
</tr>
<tr>
<td>46</td>
<td>3.25</td>
<td>8665 ± 11</td>
<td>4363</td>
<td>(11/2^-) 3 1.45 0.35</td>
<td>37.9 1.7</td>
</tr>
<tr>
<td>47</td>
<td>3.26</td>
<td>8774 ± 9</td>
<td>4474</td>
<td>(3/2^-) 4 1.48 0.15</td>
<td>38.5 1.8</td>
</tr>
<tr>
<td>48(4)</td>
<td>3.26</td>
<td>8825 ± 23</td>
<td>4523</td>
<td>(9/2^-) 3 1.45 0.35</td>
<td>58.3 2.1</td>
</tr>
<tr>
<td>49</td>
<td>3.23</td>
<td>8928 ± 15</td>
<td>4626</td>
<td>(5/2^-) 3 1.45 0.35</td>
<td>43.5 1.9</td>
</tr>
<tr>
<td>50</td>
<td>3.23</td>
<td>9050 ± 15</td>
<td>4748</td>
<td>(5/2^-) 3 1.50 0.35</td>
<td>19.9 2.2</td>
</tr>
<tr>
<td>51</td>
<td>3.30</td>
<td>9143 ± 19</td>
<td>4841</td>
<td>(9/2^+) 3 1.45 0.10</td>
<td>20.5 2.1</td>
</tr>
</tbody>
</table>

NOTES:  
(1) The Q values come from the analysis of the 18 MeV data due to the better resolution at 18 MeV. See Fig. 3.11.  
(2) From Mateja et al. [1976].  
(3) States at this energy were partially resolved at 18 MeV.  
(4) Q values for peaks #48 - #51 determined by 26 MeV data.  
(5) Proton angular distribution for these peaks are to be found in the specified figure.  
(6) \(\theta_0 = 23.4^\circ\) c.m.  
(7) Parameters used for angular distributions in Figs. 3.18 to 3.30.
FIG. 3.17: Elastic scattering at 26 MeV. Optical model calculations were done using set ax from Lemos [1972].
FIG. 3.18

DWBA calculations peak #A in $^{1}{}^{50}_{Mn}$, A(B)$^{J+}$ See Tables 3.1, 3.2, 3.3
$^{50,52,54}_{Cr(\alpha,p)^{53,55,57}}_{Mn}$, $E_\alpha = 26$ MeV
FIG. 3.19: DWBA calculations peak #A in $^5$Mn, A(B)J$^\pi$. See Tables 3.1, 3.2, 3.3. $^{50,52,54}$Cr(α,p)$^{53,55,57}$Mn, $E_\alpha = 26$ MeV.
FIG. 3.20: DWBA calculations peak #A in \(^{88}\text{Mn}\), \(A(B)J\). See Tables 3.1, 3.2, 3.3. \(^{50,52,54}\text{Cr(α,p)}^{53,55,57}\text{Mn}\), \(E_\alpha = 26\) MeV.
FIG. 3.21: DWBA calculations peak #A in $^{55}$Mn, A(B)J*. See Tables 3.1, 3.2. $^{50,52}$Cr($\alpha$,p)$^{53,55}$Mn, $E_\alpha = 26$ MeV.
FIG. 3.21: DWBA calculations peak #A in $^{55}$Mn, $A(J^\pi)$. See Tables 3.1, 3.2. $^{50,52}$Cr($\alpha$,p)$^{53,55}$Mn, $E_\alpha = 26$ MeV.
FIG. 3.22: DWBA calculations peak #A in $^{55}$Mn, A(B)J$^*$. See Tables 3.1, 3.2, 3.3. $^{50,52,54}$Cr($\alpha,p$)$^{53,55,57}$Mn, 26 MeV.
FIG. 3.23: DWBA calculations peak #A in $^{55}$Mn, A(B)J". See Tables 3.1, 3.2, 3.3. $^{53,55,57}$Cr(α,p)$^{53,55,57}$Mn, 26 MeV.
FIG. 3.24: DWBA calculations peak #A in $^5$Mn, A(B)J. See Tables 3.1, 3.2. $^{50,52}$Cr(α,p)$^{53,55}$Mn, E$_\alpha$ = 26 MeV.
FIG. 3.25: DWBA calculations peak #A in $^{52}$Mn, $A(B)J^\pi$. See Tables 3.2, 3.3. $^{52,54}$Cr(α,p)$^{55,57}$Mn, $E_\alpha = 26$ MeV.
FIG. 3.26: DWBA calculations peak #A in $^1$Mn, A(B)J$. See Tables 3.2, 3.3. $^{52,54}$Cr(α,p)$^{55,57}$Mn, E$^\alpha = 26$ MeV.
FIG. 3.27: DWBA calculations peak #A in $^{53}$Mn, A(B)J$^\pi$. See Table 3.1. $^{50}$Cr($\alpha$,p)$^{53}$Mn, $E_\alpha = 26$ MeV.
FIG. 3.28: DWBA calculations peak #A in $^58_{\text{Mn}}$, A(B)J$^*$. See Tables 3.1, 3.2. $^{50,52}_{\text{Cr}}(\alpha,p)^{53,55}_{\text{Mn}}$, $E_\alpha = 26$ MeV.
FIG. 3.29: DWBA calculations peak #A in $^{57}$Mn, A(B)J°. See tables 3.1, 3.2, 3.3. $^{50,52,54}$Cr(α,p)$^{53,55,57}$Mn, $E_α = 26$ MeV.
FIG. 3.30: DWBA calculations peak #A in $^B_{\text{Mn}}$, A(B)J$^\pi$. See Tables 3.1, 3.2, 3.3. $^{50,52,54}\text{Cr}(\alpha,p)$$^{53,55,57}\text{Mn}$, $E_\alpha = 26$ MeV.
FIG. 3.31: Ambiguities in $J^\pi$ assignments from DWBA calculations.
$^{50}$Cr$(\alpha,p)^{53}$Mn, 26 MeV.
(a) Calculated Angular Distributions for Various $J^\pi$ for peak # 32 in $^{53}$Mn

Triton Well

$r_0 = 1.45$ fm
$a_0 = 0.35$ fm

FIG. 3.32: $^{50}$Cr(α,p)$^{53}$Mn, $E_\alpha = 26$ MeV.
distributions equally well. The calculated distributions presented in Figs. 3.18 to 3.31 are then not necessarily the only ones which can fit the data. However, the possible set is limited as can be seen in Fig. 3.32 for various calculations for peak #32 in $^{53}\text{Mn}$. In the cases where only one angular distribution is presented, this $J^\pi_T$ transfer produced the best fit to the data and is thus the preferred value for the spin assignment. Usually several $J^\pi_T$ values were tried for any given state. In cases where no best choice is obvious more than one calculation is plotted. In some cases it is possible to decide between the various calculations on the basis of experimental similarity. For example, peak #20 in $^{55}\text{Mn}$ (Fig. 3.25) has an angular distribution similar to peak #11 in $^{55}\text{Mn}$ which is known to be $I_p^\pi = 1$. The calculated angular distributions for $J^\pi_T = 3/2^-$ fit both states equally well, although the experimental cross section falls short of the calculation in both cases at 23°. An assignment of $J^\pi_T = 3/2^-$ is therefore the most likely one for peak #20 in $^{55}\text{Mn}$.

Calculations were performed for a few transitions that included a spin-orbit potential in the triton well. The use of the spin-orbit potential did not alter the predicted shape of the angular distributions but it did affect the magnitude. All the calculations presented in Tables 3.1 to 3.3 were performed without a spin-orbit term for the triton form factor.

(E) DISCUSSION $^{53}\text{Mn}$

The nuclear structure of $^{53}\text{Mn}$ has been extensively studied in the past by single proton transfer reactions. The ($^3\text{He},d$) reaction has been studied by Armstrong [1965], O'Brien [1967], Ćujec [1969] and Gunn
[1976] and analysed using the DWBA theory. The \((\alpha,t)\) reaction was also studied by Armstrong [1967]. The \(J\)-dependence of the \((^7\text{Li},^6\text{He})\) reaction (34 MeV bombarding energy) was also exploited by Gunn [1976] for \(J\) assignments. Other investigations of \(^{53}\text{Mn}\) have been performed by West [1971] using the \(^{53}\text{Cr}(p,\gamma\gamma)^{53}\text{Mn}\) reaction for \(2.6 \leq E_p \leq 5.9\) MeV. Neutron and \(\gamma\)-ray angular distributions were measured by these authors for states in \(^{53}\text{Mn}\) below 3.25 MeV. Chung [1973] also measured the \((p,\gamma\gamma)\) reaction to obtain \(\gamma\)-ray angular distributions and yield curves for \(3.0 \leq E_p \leq 5\) MeV. The \(^{50}\text{Cr}(\alpha,\gamma)^{53}\text{Mn}\) reaction was studied by Sawa [1973] at 14.2, 13.2 and 12.2 MeV to obtain \(\gamma\)-ray angular distributions and \(\gamma\)-ray yields. This study enabled Sawa [1973] to confirm or assign new spins, especially high spin states at 1.441 \((11/2^-)\), 1.621 \((9/2^-)\), 2.564 \((13/2^-)\), 2.693 \((15/2^-)\), 2.697 \((11/2^-)\), 3.426 \((13/2^-)\) and 3.440 \((17/2^-)\) MeV in \(^{53}\text{Mn}\). States above 2.0 MeV up to 5.6 MeV were investigated by Shulte [1975] using the \(^{52}\text{Cr}(p,\gamma)^{53}\text{Mn}\) reaction. Angular distributions were measured at four resonances with \(0.9 \leq E_p \leq 1.03\) MeV. Many other studies of \(^{53}\text{Mn}\) have also been performed and an extensive list of investigations can be found in the article by Shulte [1975].

The availability of proton transfer data enables definite spin assignments to be made by using the \(J\)-dependence of the \((\alpha,p)\) reaction. It is also possible to compare the relative transition strengths between \((\alpha,p)\) and the proton transfer studies. It will be recalled from Chapter 2 that the ratio of transition strengths will be different in the two reactions if the residual states have parentages of the \(^{50}\text{Cr}\) excited states based upon the \(^{52}\text{Cr}\) ground state plus two neutrons, and will be the same if the excited states of \(^{50}\text{Cr}\), which form the parentage of the residual states in \(^{53}\text{Mn}\), cannot be described as the \(^{50}\text{Cr}\) ground state plus two neutrons coupled to an intrinsic spin equal to zero.
A comparison of the spectroscopic factors $S$ obtained by O'Brien [1967] and Gunn [1976] and the reduced strengths in $(\alpha,p)$ is made for the 26 MeV data in Fig. 3.33. The reduced strength for the $(\alpha,p)$ data is defined as

$$\frac{\sum_\theta \sigma_{\alpha p}^{\text{exp}}(\theta)}{\sum_\theta \sigma_{\alpha p}^{\text{DWBA}}(\theta)},$$

where $\sigma_{\alpha p}^{\text{DWBA}}(\theta)$ has been calculated using a triton transfer form factor with a triton well geometry of $r_0 = 1.45 \text{ fm}$ and $a_0 = 0.35 \text{ fm}$ for all the states shown in Fig. 3.33. The numbers above the bars are the values $2J^*$ used in calculating $\sigma_{\alpha p}^{\text{DWBA}}(\theta)$. The $(\alpha,p)$ data are depicted by the solid bars and the spectroscopic factors by the dashed bars; the two sets of data have been normalized at the ground state. Only those states for which forward angle data were collected have been included in Fig. 3.33.

The use of the triton-cluster form factor hopefully can remove at least some of the dependence of $\sigma_{\alpha p}(\theta)$ on the mass, $Q$ and $J$ values of the residual state. As was pointed out in Fig. 2.3, two form factors which differ substantially in the nuclear interior can still produce angular distributions with the same shape. The magnitude of the cross section is therefore more sensitive to the microscopic details of the transfer process than is the shape. Consequently, Fig. 3.33 shows $^{53}\text{Mn}$ according to the $(\alpha,p)$ reaction once the reaction kinematics have been removed. As such, the reduced strength is determined by both the nuclear structure of the residual state and the overlap of the relative motion of the three transferred particles in the alpha particle and in the final state. To draw any detailed conclusions about the nuclear structure from Fig. 3.33 requires that some plausible connection be made
FIG. 3.33: Comparison of reduced strengths and spectroscopic factors from the \((\alpha,p)\) and \((^3\text{He},d)\) Reactions.
between the microscopic form factor and the point-triton form factor. In the first place, the triton-cluster form factor, which is an approximation for the microscopic form factor calculated assuming the transfer of three separate nucleons, must be a good representation of the microscopic form factor near the nuclear surface. With this in mind it does not make sense to assume the single particle orbitals in the residual nucleus to be harmonic oscillator eigenfunctions in order to calculate the form factor. The form factor from such an approach falls off too rapidly beyond the surface [Smits, 1976] and it differs in precisely that region of configuration space from the triton cluster transfer form factor (using a Woods-Saxon well) which is most important for the (α,p) reaction, namely the surface region. Since no adequate connection between the point triton and microscopic form factors has yet been presented, it seems necessary to use the microscopic form factor, starting with realistic single particle states [Falk, 1973, 1975], if a detailed comparison of the (α,p) data and theoretical nuclear structure calculations is to be made.

Nevertheless, Fig. 3.33 does indicate that the neutrons are playing an active role in determining the cross section. The relative cross sections in (α,p) and (3He,d) differ widely. In fact, even if only states with the same J are compared so that the contributions from the relative motion overlaps are likely to cancel in the ratios of the reduced strengths, only the 1/2− states at 2.684 and 3.460 MeV have the same relative strengths in (α,p) and (3He,d). Moreover, states are excited in the (α,p) reaction which are absent from the (3He,d) spectrum; for example the 7/2− state at 2.578 MeV.

Not all the known states of 53Mn have been excited in the (α,p) reaction. A recent high resolution 56Fe(p,α)53Mn experiment by
Tarara [1976] at $E_p = 14$ to 16 MeV indicates that there are at least 81 states in $^{53}\text{Mn}$ up to 5.092 MeV excitation. The same excitation energy span in the $(\alpha,p)$ reaction at 26 MeV reveals only 19 resolvable groups in the present work and it does not seem probable from the spectra or angular distributions that any group has a sizeable contribution from more than one state. In the first place, the density of states reported by Tarara [1976] is large enough that the 50 keV resolution obtained in the $(\alpha,p)$ experiments would have produced very few isolated groups above 2.4 MeV if the $(\alpha,p)$ reaction at 26 MeV were not selective. Secondly, all the angular distributions for the groups excited in $(\alpha,p)$ at 26 MeV could be placed into relatively few categories based on shape. These shapes could in turn be reproduced by DWBA calculations with very little variation in the triton well parameters. The great bulk of the angular distributions, even up to 8 MeV excitation, were fitted by $L = 1, 3$ or 4 as one would expect from a naive model of stripping into the $p, f$ and $g$ subshells. As was noted previously there is some ambiguity in the $J^\pi_T$ assignments from DWBA, but in no case was an experimental angular distribution observed which resembled that calculated for $J^\pi_T > 13/2$. In particular, the high spin states reported by Sawa [1973] were not observed in the $(\alpha,p)$ experiment at 26 MeV. This is interesting because the $11/2^-$ and $15/2^-$ states at 2.697 and 2.693 MeV would have been unresolved from the $1/2^-$ state at 2.684 MeV in the present work. The angular distribution of the corresponding proton group (peak #4 in Fig. 3.20) is well reproduced by a $J^\pi_T = 1/2^-$ transfer although there does appear to be a filling in of the minimum at 70°. Hence, the $(\alpha,p)$ reaction at 26 MeV to $^{53}\text{Mn}$ is highly selective. The enhancement of the large $J$-transfer cross sections due to the angular momentum mismatch (see Fig. 3.32) apparently can not compensate for the loss of cross
section due to the state's structure.

A level scheme with proposed spin assignments based on the 
(α,p) reaction at 18 and 26 MeV is shown in Fig. 3.40. A summary of the 
states populated in the present work is given below and in Table 3.1:

Peak #1A, $E_x = 0.0$ MeV: $L_p = 3$ from ($^3$He,d). $J'' = 7/2^-$ from the J-
dependence of ($^7$Li,$^6$He) [Gunn, 1976]. The (α,p) data at both 18 
and 26 MeV favour $J'' = 7/2^-$ based on the calculated J-dependence.

Peak #1B, $E_x = 1.288$ MeV: $L_p = 1$ from ($^3$He,d). One of the six states 
possible with ($f^3_{7/2}$) configurations $3/2^- < J < 15/2^-$ [Lips, 
1970; Benson, 1975]. Due to the proximity of the $2p_{3/2}$ single 
particle state this state presumably has a large enough $p_{3/2}$ 
component to be detected in ($^3$He,d) and (α,p). At 18 Mev this 
state (Fig. 3.8) has an angular distribution different from that of 
the 2.413 MeV level which is a 3/2$^-$ state. At 26 MeV the angular 
distribution of this state is in closer accord with that of the 
2.413 MeV state, although there still appears to be some divergence 
for $\theta > 60^\circ$. The J-dependence in ($^7$Li,$^6$He) [Gunn, 1976] and (α,p) 
makes a definite $J'' = 3/2^-$ assignment possible.

Peak #2, $E_x = 2.413$ MeV: $L_p = 1$ from ($^3$He,d). This state has been 
assigned a spin $J'' = 3/2^-$ by Schulte [1975], Wiest [1971] and Chung 
[1973]. The J-dependence of the ($^7$Li,$^6$He) [Gunn, 1976] and the 
(α,p) reactions at both 18 and 26 MeV makes a definite $J'' = 3/2^-$ 
possible.

Peak #3, $E_x = 2.578$ MeV: Not populated in ($^3$He,d). Wiest [1971] and 
Chung [1973] assign a spin $J'' = 7/2^-$. The (α,p) data at 26 MeV 
suggests that this state has a $J'' = 7/2^-$ because of the similarity
of its angular distribution with the $7/2^-$ ground state. The calculated $L = 3$ J-dependence also favours the $J^\pi = 7/2^-$ assignment.

Peak #4, $E_x = 2.684$ MeV: $L_p = 1$ from $(^3\text{He},d)$. The J-dependence of the $(\alpha,t)$ [Armstrong, 1965], the $(^7\text{Li},^6\text{He})$ [Gunn, 1976] and the $(\alpha,p)$ reaction at 26 MeV makes a definite $J^\pi = 1/2^-$ assignment possible.

Peak #5, $E_x = 3.110$ MeV: $L_p = 1+3$ from $(^3\text{He},d)$. Čujec [1969] and O'Brien [1967] report two states at this excitation energy; an $L_p = 1$ at 3.113 MeV and an $L_p = 3$ at 3.065 MeV. However, Tarara [1976] repeated the $(^3\text{He},d)$ experiment using the same reaction conditions of O'Brien [1967] and failed to observe a state at 3.065 MeV. Tarara [1976] concluded that O'Brien [1967] and Čujec [1969] misidentified the levels near 3.100 MeV by three of their standard deviations. The 26 MeV $(\alpha,p)$ data favour the interpretation of Tarara by placing the $L = 3$ level at 3.110 MeV. This group can be fitted with solely a $J^\pi_T = 5/2^-$ transfer. Use of both $3/2^-$ and $5/2^-$ in a least squares fit produced a negative coefficient for the $3/2^-$ component. The absence of the characteristic peak at 35° for a $J^\pi_T = 7/2^-$ transition plus the predicted J-dependence for $L = 3$ makes a definite $J^\pi = 5/2^-$ possible.

Peak #6, $E_x = 3.460$ MeV: $L_p = 1$ from $(^3\text{He},d)$. From the J-dependence of the $(^7\text{Li},^6\text{He})$ [Gunn, 1976] and the $(\alpha,p)$ reactions a definite spin assignment of $J^\pi = 1/2^-$ is possible. The existence of a neighbouring state as a high energy shoulder on peak #6 (Fig. 3.14) probably is responsible for filling in the minimum at 70°.

Peak #7, $E_x = 3.561$ MeV: A tentative $J^\pi = 11/2^-$ assignment can be made for this group because of the good DWBA fit.
Peak #8, $E_x = 3.670$ MeV: $L_p = 3$ from $(^3\text{He},d)$. The J-dependence of the $(^7\text{Li},^6\text{He})$ [Gunn, 1976] and $(\alpha,p)$ reactions makes a definite $J^* = 5/2^-$ assignment possible.

Peak #9, $E_x = 3.893$ MeV: $L_p = 1$ from $(^3\text{He},d)$. The J-dependence of the $(\alpha,p)$ reaction enables a definite $J^* = 1/2^-$ assignment to be made.

Peak #10, $E_x = 4.067$ MeV: $L_p = 1+3$ from $(^3\text{He},d)$. The $L = 3$ state dominates in the $(\alpha,p)$ reaction at 26 MeV. This group has been assigned as $J^* = 3/2^- + 5/2^-$ by Gunn [1976] from the $(^7\text{Li},^6\text{He})$ reaction, although these authors do not present a theoretical fit to the angular distribution for this particular state. The presence of a maximum in the cross section at $35^\circ$ in the $(\alpha,p)$ data favours a $J^* = 7/2^-$ for the $L = 3$ component. The inclusion of $J^* = 1/2^-$ or $3/2^-$ does not improve the fit and fills in the minimum observed at $30^\circ$. A tentative value of $J^* = 7/2^-$ based on the $(\alpha,p)$ data is made.

Peak #11, $E_x = 4.305$ MeV: $L_p = 3$ from $(^3\text{He},d)$. Due to instrumental effects forward angle data for peaks #11 to 15 were not obtained. No $J^*$ assignment can be made for this state.

Peak #12, $E_x = 4.562$ MeV: $L_p = 1$ from $(^3\text{He},d)$. The lack of a minimum at $70^\circ$ favours $J^* = 3/2^-$; however this assignment is tentative.

Peak #13, $E_x = 4.626$ MeV: Not excited in $(^3\text{He},d)$.

Peak #14, $E_x = 4.787$ MeV: O'Brien and Čujec tentatively assign $L_p = 0$ and 1 to this state respectively. The $(\alpha,p)$ data also allow for $J^*_T = 5/2^-$ or $9/2^-$. The angular distribution for this state from the $(\alpha,p)$ data favours the $L_p = 1$ assignment because of the absence
of prominent oscillations associated with $J_T = 1/2$ transfers.

Peak #15, $E_x = 4.840$ MeV: Not populated in $(^3\text{He},d)$. $J_T^m = 7/2^-$ is favoured but no spin assignment is made.

Peak #16, $E_x = 4.950$ MeV: O'Brien [1967] and Čujec [1969] reported an $L_p = 3$ transition to a state at 4.936 MeV and an $L_p = 1$ to a state at approximately 4.963 MeV using $(^3\text{He},d)$. The group in $(\alpha,p)$ presumably consists of both states. However the similarity of the angular distribution of this peak to that for the known $3/2^-$ state at 2.413 MeV allows a tentative $J_T^m = 3/2^-$ assignment to be made.

Peak #17, $E_x = 5.032$ MeV: Not reported in $(^3\text{He},d)$. Both $J_T^m = 9/2^-$ and $5/2^-$ produce reasonable fits except that $5/2^-$ does slightly better for $\theta < 45^\circ$.

Peak #18, $E_x = 5.112$ MeV: Possibly the $L_p = 1$ seen by O'Brien [1967] at $E_x = 5.085$ MeV or the $L_p = 1$ seen by Čujec [1969] at 5.105 MeV. This state is too weakly excited in $(\alpha,p)$ to obtain an angular distribution.

Peak #19, $E_x = 5.155$ MeV: Not reported in $(^3\text{He},d)$. Both $5/2^-$ and $9/2^-$ transitions can fit this group but the $9/2^-$ is slightly favoured due to the better fit at $45^\circ$ and $50^\circ$.

Peak #20, $E_x = 5.240$ MeV: Group seen at too few angles to obtain an angular distribution.

Peak #21, $E_x = 5.356$ MeV: Not reported in $(^3\text{He},d)$. The broad flat angular distribution favours a high spin $J \geq 9/2$ based on the DWBA calculations.
Peak #22, $E_x = 5.446$ MeV: Not reported in $(^3\text{He},d)$. The angular distribution for this state resembles the ground state. Schulte [1975] have limited the spin to $3/2 \leq J \leq 7/2$. A tentative $J^\pi = 7/2^-$ assignment is possible from the 26 MeV data.

Peak #23, $E_x = 5.492$ MeV: $L_p = 1$ from $(^3\text{He},d)$. Schulte [1975] claims $J \leq 5/2$. The $J$-dependence of the $(\alpha,p)$ reactions enables a definite $J^\pi = 1/2^-$ assignment to be made.

Peak #24, $E_x = 5.814$ MeV: Seen in $(^3\text{He},d)$ but no $L_p$ assignment made. A tentative $J^\pi = 11/2^-$ assignment can be made due to the good DWBA fit.

Peak #25, $E_x = 5.881$ MeV: Seen in $(^3\text{He},d)$ but no $L_p$ value obtained. Both $J_T^\pi = 5/2^-$ and $11/2^-$ can fit the angular distribution but these are both tentative.

Peak #26, $E_x = 6.006$ MeV: Seen in $(^3\text{He},d)$ but no $L_p$ value obtained. This state has an angular distribution resembling that of the other states near this excitation energy as evidenced by Fig. 3.27. The small cross section prohibits any definite spin assignment but $J_T^\pi = 5/2^-$ produces a better fit than $J_T^\pi = 3/2^-$. 

Peaks #27 - #40, $E_x = 6.119$ to 8.016 MeV: These groups are beyond the range of the high resolution $(^3\text{He},d)$ studies of O’Brien [1967]. However, most of these groups can be identified with levels deduced by Gunn [1976] who state an accuracy of $\pm 15$ keV for their levels. A comparison of the Gunn [1976] data with the 26 MeV $(\alpha,p)$ data is made in Table 3.1. The angular distributions in $(\alpha,p)$ for peaks #27 to 33 are well described by $J_T^\pi = 5/2^-$. The level at 6.542 MeV is reported as $L_p = 4$ in $(^3\text{He},d)$ by Gunn [1976]. However, from the
angular distribution and fits that they present it is obvious that \( L_p = 3 \) better describes the data. They have favoured the \( L_p = 4 \) assignment on the argument that if the state were populated by \( L_p = 3 \) the total expected \( f \) strength would be exceeded. The \((\alpha,p)\) data are presented in Fig. 3.27 and it is apparent that a \( J^* = 5/2^- \) transfer certainly provides a good description of the angular distribution for the 6.542 MeV level. A \( J^*_T = 9/2^+ \) transfer, however also provides an adequate description of the angular distribution.

Unfortunately the \((\alpha,p)\) data can not clarify the \( J^* \) or \( L \) ambiguity here because of the similarity between the calculated angular distributions for the \((\alpha,p)\) reaction.

The sensitivity of the calculated angular distributions to the triton well parameters is exemplified by the \( 9/2^+ \) transitions when comparing the calculations made using diffuseness parameters of \( a_0 = 0.10 \) and 0.35 fm respectively for peaks #21 and 32 in Figs. 3.24 and 3.27. The radius parameter is the same in both cases, \( r_0 = 1.45 \) fm. The smaller diffuseness parameter causes the calculated cross section to fall relative to that for the larger parameters. Several other \( J^*_T \) transfers fit the angular distributions for the 6.542 MeV level as well, as is illustrated in Fig. 3.31. Thus, despite the reasonably good statistics for this state the similarities in the calculated angular distributions preclude any definite spin assignments. The possible set of \( J^*_T \) is not unlimited as witnessed by Fig. 3.32(a) where various \( J^*_T \) calculations are presented. The spin of this state, as well as that of all the peaks from 27 to 40 is probably limited to \( 5/2 \leq J \leq 11/2 \).

Above the 6.542 MeV level the states are proton unbound because the proton separation energy is 6.561 MeV. Due to instrumental limitations it was not possible to obtain angular distributions for
states above 8 MeV excitation energy. The angular distributions for these unbound states do not exhibit any obvious departures from the shapes observed for the bound states. The DWBA calculations for the unbound states were performed with the same triton transfer model as for the bound states. Since $^{53}\text{Mn}$ is still stable against triton decay at this energy there were no problems connected with unbound wave functions in the radial integrals.

Most of the peaks #27 to #40 were also seen by Gunn [1976]. Due to the $(\alpha,p)$ isospin selection rule these states must be $T=3/2$. Gunn [1976] assign as isobaric analog states of the ground, first and second excited states of $^{53}\text{Cr}$ levels at 6.97, 7.54 and 8.03 MeV. Schulte [1975] also assign a level at 7.546 MeV as the isobaric analog state of the first excited state of $^{53}\text{Cr}$. If these levels reported by Gunn [1976] and Schulte [1975] are the same as peaks #36, #38 and #40 at 6.990, 7.574 and 8.016 MeV seen in $(\alpha,p)$, then the assignment of these levels as isobaric analog states of $^{53}\text{Cr}$ can not be correct because these would be $T=5/2$ states.

As can be seen from Table 3.1 the agreement between Gunn [1976] and the 26 MeV data is good with regard to the number of states and their energies. There were only two states seen in $(\alpha,p)$ which were not reported by Gunn [1976] and of the seventeen states between 6 and 8 MeV reported by Gunn [1976], thirteen have been observed at 26 MeV in the $(\alpha,p)$ reaction.
A recent compilation for $A = 55$ by Kocher [1976] contains references for those authors not explicitly mentioned in this text.

The 26 MeV ($\alpha,p$) data are summarized in Table 3.2, Fig. 3.34 and Fig. 3.40. The quantities referred to in Fig. 3.34 are the same as those in Fig. 3.33 and the values of $S$ have been taken from the ($^3$He,d) data of Rapaport [1969].

In contrast to the $^{50}\text{Cr}(\alpha,p)^{53}\text{Mn}$ reaction, the reduction in the number of states excited going from 18 to 26 MeV is not as great in the $^{52}\text{Cr}(\alpha,p)^{55}\text{Mn}$ reaction. A comparison of Figs. 3.12 and 3.15 reveals that up to 4.727 MeV in $^{55}\text{Mn}$ twenty-nine groups were excited at 18 MeV and 22 groups at 26 MeV. Above 4.727 MeV however, the density of states is very high at 18 MeV compared to the density at 26 MeV. In fact, at 26 MeV the spectrum is dominated by only a few groups above 4.727 MeV which consist of peaks #23 to 27. The larger density of states in $^{55}\text{Mn}$ as compared to $^{53}\text{Mn}$ is not surprising considering that $^{53}\text{Mn}$ has a closed $N=28$ neutron core. The prominent gap in the $^{53}\text{Mn}$ 26 MeV spectra between the ground state and 2.413 MeV level is broken only by the weakly excited $3/2^-$ level at 1.288 MeV. In $^{55}\text{Mn}$ there are five states excited between the strongly excited $7/2^-$ and $3/2^-$ states at 0.128 and 2.258 MeV. In both nuclei however, the low lying $5/2^-$ states at 0.376 MeV in $^{53}\text{Mn}$ and the ground state in $^{55}\text{Mn}$ are absent from the 26 MeV spectra although they are present at 18 MeV. The density of states excited in the $^{52}\text{Cr}(\alpha,p)^{55}\text{Mn}$ reaction is still considerably smaller than that expected from the adopted level scheme of Kocher [1976]. At 18 MeV 19 states up to 3.429 MeV were excited and 15 states were excited in the same excitation energy range at 26 MeV. The adopted level scheme lists 47 states up to 3.429 MeV. Thus the ($\alpha,p$) reaction is highly selective.
for $^{55}$Mn as well as for $^{53}$Mn.

The states seen in the $^{52}$Cr($\alpha$,p)$^{55}$Mn reaction are discussed below.

Peak #1, $E_x = 0.128$ MeV: $L = 3$ in ($^3$He,d) and (d,n). The $L = 3$ $J$-dependence favours the adopted [Kocher, 1976] spin assignment of $7/2^-$. 

Peak #2, $E_x = 0.983$ MeV: Not seen in proton transfer. This state exhibits a marked relative decline in strength going from 18 to 26 MeV. The large statistical errors do not allow a spin assignment to be made. As can be seen in Fig. 3.22 both $J_T^* = 3/2^-$ and $9/2^-$ can fit the data and $J_T^* = 11/2^-$ is not excluded either. The adopted [Kocher, 1976] level scheme makes a tentative $J^* = (9/2)^-$ assignment.

Peak #3, $E_x = 1.291$ MeV: $L_p = 1? \ in \ (d,n); \ not \ seen \ in \ (^3$He,d). This group is ambiguous. Spin assignments made in the past range from $1/2^-$ to $11/2^-$. Peterson [1971] found it necessary to include an $L = 0$ component in the angular distribution to this group from the $^{57}$Fe(p,$^3$He)$^{55}$Mn reaction and concluded that they were exciting a $J^* = 1/2^-$ state unresolved from a state with $J^* = 11/2^-$. Using 5 to 7 MeV $\alpha$-particles, Kulkarni [1974] found thick target $\gamma$-ray yields for a 1293 keV $\gamma$-ray which could be fitted by first order Coulomb excitation theory for a $\lambda = 2$ transition. They believed that they were exciting a state at 1.293 MeV with $1/2^- \leq J^* \leq 9/2^-$. The observation of $\gamma$-ray transitions to the $7/2^-$ and $(9/2)^-$ states at 0.128 and 0.983 MeV further limited their spin assignments to $5/2^- \leq J^* \leq 9/2^-$. They also found that the yields for the 307 and 1167 keV $\gamma$-rays which originate from the 1.293 MeV level could be
accounted for by E2 Coulomb excitation theory. Unfortunately some degree of confusion exists about the interpretation of these Coulomb excitation data due to the publication of a second paper on the same reaction [Kulkarni, 1976]. The Coulomb excitation measurements of $^{55}$Mn were extended to 8 MeV by Kulkarni [1976] who observed the same $\gamma$-ray spectrum as Kulkarni [1974] except for the 307 and 1167 keV $\gamma$-rays which could possibly be identified with the 304 and 1164 keV $\gamma$-rays observed by Kulkarni [1976]. However, Kulkarni [1976] found that the yield for the 1164 keV $\gamma$-ray rose too steeply to be accounted for by a single E2 excitation and thus this $\gamma$-ray can not be identified with the 1167 keV $\gamma$-ray of Kulkarni [1974]. Kulkarni [1976] concluded that there were two levels separated by 3 keV; a $J^\pi = 11/2^-$ state at 1.290 MeV and a $1/2^-$ state at 1.293 MeV. The assignment of spin $1/2^-$ for the state at 1.293 MeV was made on the basis of both the $\gamma$-ray yields and a limited $\gamma$-ray angular distribution measured at $\theta_\gamma = 0^\circ$ and $90^\circ$.

According to Kulkarni [1976] the 1.293 MeV level de-excites 100% to the ground state. A 1293 keV $\gamma$-ray has not been seen in the several ($\alpha$,p$\gamma$) experiments [Kocher, 1976] or in singles or coincidence p-$\gamma$ spectra measured via the $^{55}$Mn(p,p$'\gamma$) reaction described in section (G) of this chapter.

The ($\alpha$,p) data at 26 MeV shows a pronounced minimum (Fig. 3.22) and maximum at the same angles observed for other $J^\pi_T = 1/2^-$ transitions. A least squares combination of $J^\pi_T = 1/2^-$ and $11/2^-$ produces a good fit as shown in Fig. 3.22. The intensities of the two components at $\theta_{cm} = 23^\circ$ are 12 $\mu$b/sr for $J^\pi = 1/2^-$ and 10 $\mu$b/sr for $J^\pi = 11/2^-$. It is interesting to note the relative intensities of the
0.983 and 1.291 MeV proton groups between the 18 and 26 MeV data in Figs. 3.12 and 3.15. At 26 MeV the 0.983 MeV level is weakly excited relative to the 1.291 MeV level, whereas at 18 MeV it is excited with comparable strength. The 9/2\(^-\) and 11/2\(^-\) states at this excitation energy are theoretically expected to have large (f\(^{\pm}\))\(^{1}\)\(_{7/2}\) J components [McGrory, 1967]. These (f\(^{\pm}\))\(^{1}\)\(_{7/2}\) J components should not be excited if the \(^{52}\)Cr ground state has purely proton seniority zero components. Hence, the marked decline in strength of the (9/2\(^-)\) level at 0.983 MeV from 18 to 26 MeV is understandable since the compound nuclear cross section has apparently declined substantially compared with the direct reaction cross section as discussed earlier. The decline in strength of the \(J^\pi = (11/2^-)\) 1.291 MeV level is much smaller and hence suggests that another state is also being excited in the (α,p) reaction.

Peak #4, \(E_x = 1.528\) MeV: \(L_p = 1\) from \((^3\text{He},d)\). The J-dependence of the (α,p) reaction corroborates the adopted spin assignment of \(J^\pi = 3/2^-\). This state and the 2.258 MeV 3/2\(^-\) level are excited with the same relative intensity in (α,p) and \((^3\text{He},d)\).

Peak #5, \(E_x = 1.883\) MeV: \(L_p = 3\) from \((^3\text{He},d)\). The similarity of this angular distribution with that of the other 7/2\(^-\) states (Fig. 3.18) suggests that \(J^\pi = 7/2^-\).

Peak #6, \(E_x = 2.197\) MeV: Not populated strongly in \((^3\text{He},d)\). This state has quite a distinctive \(J^\pi_T = 7/2^-\) angular distribution both at 18 and 26 MeV. The assignment agrees with the proposed 7/2\(^-\)\(^(-)\) assignment from Kocher [1976].

Peak #7, \(E_x = 2.258\) MeV: \(L_p = 1\) from \((^3\text{He},d)\) and (d,n). The J-dependence at both 18 and 26 MeV fix the spin at \(J^\pi = 3/2^-\).
Peak #8, \( E_x = 2.562 \) MeV: \( L_p = 1 \) in \(^{3}\text{He},d\) and \((d,n)\). From the J-dependence \( J_T^* = 3/2^- \) both at 18 and 26 MeV.

Peak #9, \( E_x = 2.741 \) MeV. No \( L_p \) value reported in \(^{3}\text{He},d\). The adopted level scheme [Kocher, 1976] places two states at 2.727 and 2.753 MeV with spins of 7/2 and (5/2, 7/2)\(^-\) respectively. An assignment of \( J_T^* = 3/2^- \) however, produces a reasonable fit except that the experimental and calculated curves diverge for \( \theta_{\text{cm}} < 29^\circ \) (see Fig. 3.29). However other known \( L = 1 \) transitions such as the 3.041 MeV level (Fig. 3.25) also diverge for \( \theta_{\text{cm}} < 29^\circ \). A tentative \( J_T^* = 3/2^- \) assignment is made for this state.

Peak #10, \( E_x = 2.980 \) MeV: \( L_p = 2 \) in \(^{3}\text{He},d\). This state represents a potential knock-out candidate. The inability of the DWBA to reproduce the shape of the angular distribution suggests that the production mechanism is not direct, as was discussed in section (C.3) of Chapter 3.

Peak #11, \( E_x = 3.041 \) MeV: \( L_p = 1 \) in \(^{3}\text{He},d\) and \((d,n)\). The (\( \alpha,p \)) J-dependence enables a firm \( J_T^* = 3/2^- \) to be made.

Peak #12, \( E_x = 3.136 \) MeV: \( L_p = 3 \) in \(^{3}\text{He},d\). The experimental angular distribution is different from that for known \( J_T^* = 7/2^- \) states implying an \( L = 3 \) J-dependence. The DWBA gives a best fit for \( J_T^* = 5/2^- \).

Peak #13, \( E_x = 3.263 \) MeV: No \( L_p \) value from \(^{3}\text{He},d\). The similarity of the angular distribution for this group to that of the 3.041 MeV level suggests \( J_T^* = 3/2^- \).

Peak #14, \( E_x = 3.354 \) MeV: Not reported in \(^{3}\text{He},d\). A tentative
$J_T^* = 3/2^-$ is made based on the comparison of the $(\alpha,p)$ data from this state to that of the 3.041 MeV level.

Peak #15, $E_x = 3.431$ MeV: $L_p = 1$ in $(^3\text{He},d)$ and $(d,n)$. The J-dependence fixes the spin of this state as $1/2^-$.  

Peak #16, $E_x = 3.608$ MeV: $L_p = 3$ in $(^3\text{He},d)$. The angular distribution of this state in $(\alpha,p)$ is unusual both at 18 and 26 MeV. Neither $J_T^* = 5/2^-$ or $7/2^-$ calculations produce good fits (see Figs. 3.9 and 3.23). The 18 MeV data favour $5/2^-$ but the calculated fit is not good beyond 60°. Rapaport [1969] suggests that $J_T^* = 5/2^-$ because an assignment of $J_T^* = 7/2^-$ would exceed the sum rule strength expected for the $T = 5/2$, $f_{7/2}$ transitions. The expected total summed strength is 4.0 and the summed strength for the $7/2^-$ levels at 0.128 and 1.883 MeV is 3.5. The strength of the 3.608 MeV level $[(2J+1)C^2S]$ is 1.5 in $(^3\text{He},d)$.  

Peak #17, $E_x = 3.847$ MeV: Not reported in $(^3\text{He},d)$. This state, along with two groups in $^{57}\text{Mn}$ (see Fig. 3.30) is one of the few states observed at 26 MeV with good enough statistics that could not be fitted using DWBA with a triton-cluster transfer model. This state is populated by an $L = 2$ transfer in the $^{55}\text{Mn}(p,p')^{55}\text{Mn}$ reaction [Kocher, 1976] suggesting $J_T^* = (1/2, 3/2, 5/2, 7/2, 9/2)^-$ since the $^{55}\text{Mn}$ ground state has $J_T^* = 5/2^-$.  

Peak #18, $E_x = 3.989$ MeV: $L_p = 1$ in $(^3\text{He},d)$. The J-dependence enables a definite $J_T^* = 3/2^-$ assignment to be made.  

Peak #19, $E_x = 4.086$ MeV: $L_p = 1$ for a state at 4.100 MeV from $(d,n)$. This and the similarity of the angular distribution to that of the 3.041 MeV level enables a tentative $J_T^* = 3/2^-$ assignment to be made.
Peak #20, $E_x = 4.266$ MeV: No $L_p$ reported. The compilation [Kocher, 1976] reports positive parity states at about this energy. The $3/2^+$ DWBA angular distribution is not too different from that for $J_T^* = 3/2^-$ but there are no $3/2^+$ states against which to make an experimental comparison. The angular distribution, however, resembles that of the known $L_p = 1$ state at 3.041 MeV. Spin 3/2 is most probable.

Peak #21, $E_x = 4.640$ MeV: $L_p = 1$ from $(^3\text{He},d)$ and (d,n). The DWBA calculations do not fit very well, but the lack of the $J = 1/2$ oscillations favours $J^* = 3/2^-$. 

Peak #22, $E_x = 4.736$ MeV: $L_p = 1$ in $(^3\text{He},d)$. This group is partially resolved in $(\alpha,p)$. The J-dependence establishes the spin of this state as $J^* = 3/2^-$. 

Peak #23, $E_x = 5.260$ MeV: No $L_p$ reported. The experimental angular distribution of this state resembles that of the known $L_p = 1$ level at 3.041 MeV. However, it is possible to find a triton well geometry which also produces a comparable fit for $J_T^* = 7/2^-$ as shown in Fig. 3.30. Except for the point at $\theta = 45^\circ$ it also resembles the angular distribution for the level at 5.365 MeV (Fig. 3.19, peak #24) which is an $L_p = 1$ from the $(^3\text{He},d)$ and (d,n) data. Based on the experimental similarities a tentative $J = 3/2$ assignment is made.

Peak #24, $E_x = 5.365$ MeV: $L_p = 1$ in $(^3\text{He},d)$. This group does not have an angular distribution that resembles that for other known $L_p = 1$ states. In particular the angular distribution for this level falls off too slowly between 28° and 45° compared to the other $L_p = 1$ states. It is possible to fit the angular distribution for
$28^\circ \leq \theta \leq 60^\circ$ if a smaller diffuseness parameter $a_0$ (see Table 3.2) is used. The $J^*_T = 3/2^-$ calculation continues to rise compared with the data forward of $28^\circ$; however several known $L_p = 1$ states exhibit this feature, for example peaks #11 and 18. A tentative $J^*_T = 3/2^-$ assignment is made based on the lack of the characteristic $J^*_T = 1/2^-$ oscillations in the angular distribution.

Peak #25, $E_x = 5.498$ MeV: $L_p = 3$ in $(^3\text{He},d)$. This group dominates the spectrum at all angles measured at 26 MeV. Its strength has increased by a factor of 28 over that to the $7/2^-$ state at 0.128 MeV when compared to the same ratio from $(^3\text{He},d)$. As with the 5.365 MeV state, the triton well diffuseness parameter has to be decreased to fit the angular distribution. However, the angular distribution is qualitatively similar to the other known $J^*_T = 7/2^-$ states in that there is a falling off of cross section forward of $30^\circ$. The calculated $J^*_T = 5/2^-$ transitions exhibit a steadily increasing cross section between $30^\circ$ and $20^\circ$ even for the smaller value of $a_0$. A tentative $J^*_T = 7/2^-$ assignment can be made based on the observed $L = 3$ J-dependence.

Peak #26, $E_x = 6.069$ MeV. No $L_p$ reported. This state is a new state beyond the excitation energy range covered in the compilation [Kocher, 1976]. The experimental angular distribution is similar to some of the known $L_p = 1$ levels, for example the 3.041 MeV level. It is also similar to the $L_p = 3$ transition at 3.608 MeV (see Fig. 3.23) except that the peak at $55^\circ$ in the angular distribution for the 3.608 MeV level is not present in the case of the 6.069 MeV level. A tentative $J = 3/2$ assignment is made for the 6.069 MeV level based on the experimental similarities between the angular
FIG. 3.34: Comparison of reduced strengths and spectroscopic factors from the (α,p) and (³He,d) reactions.
distribution of this state and the known \( L = 1 \) levels at 3.041 and 3.989 MeV (see Figs. 3.25 and 3.21, peaks #11 and 18).

Peak #27, \( E_x = 6.164 \) MeV: No \( L \) reported. Another new state. The angular distribution does not fall off quickly enough for \( 30^\circ \leq \theta \leq 40^\circ \) to identify this level as \( J^{\pi} = 5/2^- \) although the peak at 30° is reminiscent of the \( L = 1 \) transitions at 3.041 and 3.989 MeV. A \( J^{\pi}_T = 7/2^- \) calculation falls off too slowly for \( \theta < 30^\circ \). The best fit is obtained with \( J^{\pi}_T = 11/2^+ \), however this must be regarded as tentative since reasonable fits with other combinations of triton well parameters and \( J^{\pi}_T \) values could probably also be found.

(G) \(^{55}\text{Mn}(p,p'\gamma)\) p-\( \gamma \) COINCIDENCE EXPERIMENT

A great deal of confusion surrounds the level structure of \(^{55}\text{Mn}\) at about 1.29 MeV excitation. In a high resolution \((p,p')\) study using 7.975 MeV protons Katsanos [1967] observed a doublet at this energy although they made no comment about it. Peterson [1971] reported the existence of a 1/2\(^-\) state nearly degenerate with the known 11/2\(^-\) state at 1.29 MeV. This was based on their analysis of the \(^{57}\text{Fe}(p,^3\text{He})^{55}\text{Mn}\) and \(^{57}\text{Fe}(d,\alpha)^{55}\text{Mn}\) reactions at \( E_p = 27 \) MeV and \( E_d = 16.5 \) MeV. Hichwa [1973] saw no evidence that they were populating a 1/2\(^-\) level at 1.29 MeV in their \((\alpha,\gamma)\) measurements at \( E_\alpha = 10.5 \) and 11.1 MeV. Their spin assignment agreed with the previously assigned 11/2\(^-\) value. They did not observe a 1292 keV \( \gamma \)-ray transition which is the only likely mode of decay open to the conjectured 1/2\(^-\) state as can be seen in Fig. 3.37(b). The observation of a 1293 keV \( \gamma \)-ray by Kulkarni [1974, 1976] was discussed in the previous section. The branching ratios for the 1.29 MeV state \( \gamma \)-decays by Kulkarni [1974, 1976] are in serious
disagreement with those of Hichwa [1973]. However, it is not clear that the 1293 keV γ-ray observed by Kulkarni [1974, 1976] originates from a level at 1.29 MeV because they did not report particle-gamma measurements or observe any γ-rays in coincidence with the 1293 keV γ-ray.

The \( ^{52}\text{Cr}(\alpha,p)^{55}\text{Mn} \) data at 26 MeV described previously required a mixture of spin 1/2 plus something else to reproduce the angular distribution.

To establish the existence of a 1.29 MeV γ-ray originating from a level at 1.29 MeV it was considered necessary to do an experiment in which the conjectured 1/2 state was populated with sufficient strength to enable a particle-gamma coincidence measurement to be made. Since the (p,p') experiment of Katsanos [1973] showed a doublet at the right excitation energy, the \( ^{55}\text{Mn}(p,p') \) experiment was repeated at the same bombarding energy as Katsanos. A preliminary measurement was performed at \( E_p = 7.975 \text{ MeV} \) using the 24" double focussing spectrometer. The best resolution attained was 12 keV and no doublet was evident at the 1293 keV level. Measurements were made at 30°, 40°, 50° and 140° lab.

The \( ^{55}\text{Mn}(p,p'\gamma) \) p-γ coincidence measurements were made at the same bombarding energy. Singles γ-ray spectra were collected with the target in and out of the beam. The spectra were calibrated using a \( ^{152}\text{Eu} \) source located in the same position as the target. The experimental details can be found in Fig. 3.35. The γ-ray detector was located at 90° with respect to the beam to minimize Doppler shift and the total acceptance angle of the Ge(Li) detector was ±32°. No 1293 keV γ-ray was observed in the singles spectrum. A portion of the γ-ray spectrum is shown in Fig. 3.36. According to Kulkarni [1974] the branch to the ground state of the 1293 keV state is five times more likely than
the branch to the first excited state which produces an 1165 keV γ-ray. The relative efficiency for detecting a 1293 as compared to a 1165 keV γ-ray was 0.9 for this experimental configuration. The observed 1280 keV γ-ray has about 1/5 the intensity of the 1165 keV γ-ray in Fig. 3.36. If a 1293 keV γ-ray were present in this spectrum its intensity at \( \theta_\gamma = 90^\circ \pm 32^\circ \) is less than 1/5 that of the 1165 keV transition. A list of γ-rays seen in singles, along with their relative intensities is presented in Table 3.4. Gamma rays which were present in the singles spectrum when the target was removed from the beam have not been included in the table.

According to the manufacturer's specifications the manganese target had a minimum assay of 99% with maximum limits of iron 0.002%, lead 0.001%, nickel 0.002% and zinc 0.05%.

The p-γ coincidence data were recorded event by event on magnetic tape and the measurement took approximately 45 hours of running time with 2 to 8 na on a target of 150 μg/cm\(^2\) \(^{55}\)Mn. Fig. 3.37 shows the proton projection and the γ-rays in coincidence with the 1292 keV proton group. The TAC time window was set to 200 ns and the true to chance ratio was 20/1 with a 12 ns FWHM for the TAC peak. From Fig. 3.37(b) it is apparent that no γ-ray was detected at 1292 keV. In fact, the (α,γ) decay scheme of Hichwa [1973] for the 1293 keV level was confirmed. Coincident γ-rays for the other groups in Fig. 3.37(a) were also measured and there was no transition seen from known low spin states to the 1293 keV level.

Because of the broad peaks in the proton projection spectrum the composition of the particle groups is uncertain. For levels above

Table 3.4

Gamma rays observed in singles measurement at $\theta_\gamma = 90^\circ \pm 32^\circ$ in the $^{55}\text{Mn}(p, p'\gamma)$ reaction at $E_p = 7.975$ MeV. Thin target $\sim 150$ $\mu$g/cm$^2$ on C foil backing. Known impurities or non-prompt $\gamma$-rays excluded.

<table>
<thead>
<tr>
<th>$E_\gamma^+$ (keV)</th>
<th>$\pm \Delta E_\gamma$ (keV)</th>
<th>Relative Intensity**</th>
<th>$\pm$ $\Delta RI$</th>
</tr>
</thead>
<tbody>
<tr>
<td>129.7</td>
<td>*</td>
<td>301</td>
<td>13</td>
</tr>
<tr>
<td>158.2</td>
<td>.2</td>
<td>99</td>
<td>11</td>
</tr>
<tr>
<td>238.0</td>
<td>.2</td>
<td>147</td>
<td>22</td>
</tr>
<tr>
<td>273.3</td>
<td>.2</td>
<td>688</td>
<td>40</td>
</tr>
<tr>
<td>307.4</td>
<td>.2</td>
<td>109</td>
<td>15</td>
</tr>
<tr>
<td>385.2</td>
<td>.2</td>
<td>534</td>
<td>44</td>
</tr>
<tr>
<td>411.3</td>
<td>.2</td>
<td>224</td>
<td>36</td>
</tr>
<tr>
<td>416.7</td>
<td>.5</td>
<td>8.7</td>
<td>2.5</td>
</tr>
<tr>
<td>442.0</td>
<td>.2</td>
<td>221</td>
<td>33</td>
</tr>
<tr>
<td>477.0</td>
<td>.2</td>
<td>3094</td>
<td>214</td>
</tr>
<tr>
<td>482.9</td>
<td>.7</td>
<td>10</td>
<td>3</td>
</tr>
<tr>
<td>532.0</td>
<td>.2</td>
<td>209</td>
<td>29</td>
</tr>
<tr>
<td>743.8</td>
<td>.6</td>
<td>65</td>
<td>21</td>
</tr>
<tr>
<td>765.1</td>
<td>.2</td>
<td>76</td>
<td>16</td>
</tr>
<tr>
<td>803.2</td>
<td>.2</td>
<td>2172</td>
<td>132</td>
</tr>
<tr>
<td>810.6</td>
<td>.2</td>
<td>352</td>
<td>38</td>
</tr>
<tr>
<td>826.7</td>
<td>.2</td>
<td>400</td>
<td>44</td>
</tr>
<tr>
<td>846.0</td>
<td>.2</td>
<td>1014</td>
<td>79</td>
</tr>
<tr>
<td>857.7</td>
<td>.2</td>
<td>1000</td>
<td></td>
</tr>
<tr>
<td>895.1</td>
<td>.2</td>
<td>252</td>
<td>55</td>
</tr>
<tr>
<td>910.5</td>
<td>.8</td>
<td>68</td>
<td>28</td>
</tr>
<tr>
<td>930.8</td>
<td>.2</td>
<td>7964</td>
<td>491</td>
</tr>
<tr>
<td>962.9</td>
<td>.8</td>
<td>103</td>
<td>30</td>
</tr>
</tbody>
</table>
Table 3.4 (cont’d)

<table>
<thead>
<tr>
<th>$E_\gamma$</th>
<th>$\pm \Delta E_\gamma$ (keV)</th>
<th>Relative Intensity**</th>
<th>$\pm$ ΔRI</th>
</tr>
</thead>
<tbody>
<tr>
<td>983.7</td>
<td>.5</td>
<td>250</td>
<td>35</td>
</tr>
<tr>
<td>1165.1</td>
<td>.3</td>
<td>333</td>
<td>49</td>
</tr>
<tr>
<td>1212.2</td>
<td>.3</td>
<td>567</td>
<td>75</td>
</tr>
<tr>
<td>1221.9</td>
<td>.4</td>
<td>1622</td>
<td>162</td>
</tr>
<tr>
<td>1236.9</td>
<td>.5</td>
<td>154</td>
<td>31</td>
</tr>
<tr>
<td>1280.4</td>
<td>.5</td>
<td>68</td>
<td>20</td>
</tr>
<tr>
<td>1315.8</td>
<td>.3</td>
<td>5542</td>
<td>512</td>
</tr>
<tr>
<td>1326.5</td>
<td>.3</td>
<td>412</td>
<td>64</td>
</tr>
<tr>
<td>1369.1</td>
<td>.2</td>
<td>1067</td>
<td>116</td>
</tr>
<tr>
<td>1378.6</td>
<td>1.3</td>
<td>105</td>
<td>48</td>
</tr>
<tr>
<td>1407.7</td>
<td>.2</td>
<td>2067</td>
<td>215</td>
</tr>
<tr>
<td>1419.2</td>
<td>.7</td>
<td>169</td>
<td>51</td>
</tr>
<tr>
<td>1433.3</td>
<td>.3</td>
<td>958</td>
<td>127</td>
</tr>
<tr>
<td>1459.8</td>
<td>.4</td>
<td>375</td>
<td>62</td>
</tr>
<tr>
<td>1505.2</td>
<td>.8</td>
<td>191</td>
<td>52</td>
</tr>
<tr>
<td>1527.7</td>
<td>.4</td>
<td>678</td>
<td>103</td>
</tr>
<tr>
<td>1554.7</td>
<td>.8</td>
<td>280</td>
<td>72</td>
</tr>
<tr>
<td>1572.0</td>
<td>.8</td>
<td>356</td>
<td>84</td>
</tr>
<tr>
<td>1620.7</td>
<td>.5</td>
<td>315</td>
<td>68</td>
</tr>
<tr>
<td>1638.9</td>
<td>.4</td>
<td>531</td>
<td>94</td>
</tr>
<tr>
<td>1663.4</td>
<td>.5</td>
<td>287</td>
<td>63</td>
</tr>
<tr>
<td>1882.4</td>
<td>.5</td>
<td>379</td>
<td>93</td>
</tr>
</tbody>
</table>

* Spectrum non-linear for low channel numbers. This is probably the 126 keV transition from the 126 keV level to the ground state.

** Relative intensities normalized to the 857.7 keV $\gamma$-ray.

† Measured $125 \text{ keV} \leq E_\gamma \leq 1900 \text{ keV}$. 
Beam: 7.975 MeV protons, 2 to 8 nA on target

Target: $^{55}$Mn strip target, $\sim 150 \mu g/cm^2$, $\sim .2$ wide, C foil backing

Detectors: $\gamma$ - Ge(Li), 60 cm$^3$, 4.9$^\phi \times 3.3$ dimension
$\theta_\gamma = 90^\circ \pm 32^\circ$

$\theta_p = -80^\circ \pm 5.4^\circ$

p - Si surface barrier, ANU #277, 1.9$^\phi$
- magnetic electron suppression
- cooled by Cu strap connection to cold finger

N.B.: All dimensions in cm, $\gamma$-rays traverse 1 cm perspex window.
FIG. 3.36: $^{55}\text{Mn}(p, p'\gamma)\gamma$ singles spectrum, $\theta_\gamma = 90^\circ \pm 32^\circ$, $E_p = 7.975$ MeV.
Full TAC particle projection (excitation energies are approximate)

Full TAC (200 ns) — Gamma-rays in coincidence with 1292 keV particle group. 984→0 keV transition is based on a window set on 984 keV particle group. Arrow points to expected location of a 1292 keV γ-ray.

FIG. 3.37: (a) Particle projection before subtraction of chance coincidence. 1431 counts full scale.
(b) Gamma-rays seen in 1292 keV particle group window. Subtraction of chance spectrum does not alter the conclusion of inset decay scheme. 115 counts full scale.
FIG. 3.38: $^{55}$Mn($p,p'\gamma$) $p-\gamma$ coincidence scheme.
2200 keV the origins of γ-ray transitions were based on the known levels in $^{55}$Mn and the energy difference to other known levels in order to reproduce the observed γ-ray energies in the proton group windows.

The p-γ coincidence scheme obtained from this experiment is depicted in Fig. 3.38. Firmly assigned transitions are indicated by solid lines and dashed lines correspond to coincident γ-ray transitions of doubtful statistical significance.

The p-γ coincidence scheme obtained from the present work is in substantial agreement with that determined by Hichwa [1973] and Hichwa, Lawson and Chagnon [1973] for levels below approximately 2800 keV excitation obtained using the (α,py) p-γ coincidence technique.

(H) DISCUSSION $^{57}$Mn

At the commencement of the (α,p) measurements no information was available about the excited states of $^{57}$Mn. The $Q_{\alpha p}$ value given by Gove [1972] was -4.170(0.050) MeV.

The energy calibration for the $^{53,55}$Mn isotopes was comparatively easy because both these isotopes have well known levels up to 5.5 MeV. Since this was not the case for $^{57}$Mn, it was decided to run the $^{54}$Cr(α,p)$^{57}$Mn experiment consecutively with the $^{55}$Mn measurements at both 18 and 26 MeV. This procedure enabled the $^{55}$Mn energy calibrations to be used for the $^{57}$Mn spectra. Thin $^{52,54}$Cr targets of about 50 µg/cm$^2$ on gold backings were prepared under identical conditions for use at 18 MeV. Differences in energy loss due to different target thicknesses for the $^{52}$Cr or $^{54}$Cr targets were minimized in this way. A check of the relative target thicknesses for the $^{52,54}$Cr targets using
elastic scattering showed that the pair of targets did not differ from one another for the (α,p) reaction measurements by more than about 6 keV. The difference was generally smaller than this and hence no correction to the 55Mn energy calibration was used to determine Q-values in the 57Mn spectra.

At 26 MeV self-supporting 52,54Cr targets were obtained from two different sources and target thickness corrections were made to the energy calibration. Differences in target thickness were again estimated by elastic scattering yields. The largest difference ΔE was 11 keV, and usually much smaller. The 57Mn spectra were therefore shifted by an amount ΔE compared to the 55Mn spectra.

The Q-values for 18 and 26 MeV were in agreement for states produced in 57Mn as can be seen in Table 3.5. The r.m.s. deviations for the Q-values of 57Mn states were of the same size as the r.m.s. deviations for the Q-values of known levels in 55Mn. The ground state Q-value for the 54Cr(α,p)57Mn reaction using the 18 MeV data was determined to be

\[ Q_{\alpha p_0} = -4.302 \pm 0.008 \text{ MeV} . \]

This value is in agreement with a recently reported measurement [Mateja, 1976] of the same reaction using a 100 cm broad range spectrograph. Their value is

\[ Q_{\alpha p_0} = -4.308 \pm 0.008 \text{ MeV} . \]

The discrepancy between \( Q_{\alpha p_0} \) determined from (α,p) and that given by Gove [1972] can be removed if the data of Ward [1969] on the β-decay of 57Mn is re-interpreted by requiring the 83% branching mode in the β-decay to populate the 136 keV level in 57Fe instead of the 14 keV level. When this is done the \( Q_{\alpha p_0} \)-value predicted on the basis of the
more accurate measurement by Vasiley [1964] of the $\beta^-$-end point energy is $-4.299 \pm 0.050$ MeV.

The collection of consecutive spectra for $^{55,57}$Mn under identical conditions facilitated the subtraction of the $^{55}$Mn contaminant peaks in the $^{57}$Mn spectra. The $^{54}$Cr targets were 91% $^{54}$Cr and 7% $^{52}$Cr. All of the $^{57}$Mn spectra shown already have the $^{52}$Cr contaminant spectra subtracted.

During the course of this investigation measurements [Mateja, 1976] were reported on the low lying levels of $^{57}$Mn using the ($\alpha$,p) reaction at 15, 21 and 24 MeV. This work reported levels in $^{57}$Mn up to 2.234 MeV. The spin assignments made on the basis of ($\alpha$,p$\gamma$) measurements are in agreement with the spin assignments made based on the proton angular distributions shown in Figs. 3.1 to 3.31. Of particular interest in the results of Mateja [1976] is a pair of states at about 1.06 MeV separated by 15 keV. Mateja [1976] assigned spins of (1/2$^-$) and (9/2$^-$) to these states. This feature should be compared with the search for a (1/2$^-$, 11/2$^-$) doublet in $^{55}$Mn at 1293 keV.

The reduced strength spectrum for $^{57}$Mn at 26 MeV in the ($\alpha$,p) reaction is presented in Fig. 3.39. The quantities in the figure are the same as in Figs. 3.33 and 3.34. The triton well used for the calculations in Fig. 3.39 is $r_0 = 1.45$ fm and $a_0 = 0.35$ fm. The spectrum has been normalized to place the 7/2$^-$ state at 100 on the scale.

A discussion of the $^{57}$Mn states seen in the ($\alpha$,p) reaction follows below and a summary is given in Table 3.3. The peak labelling in Table 3.3 corresponds to the labelling in the 18 MeV spectra of Fig. 3.11. Peaks have been referred to as doublets in the 26 MeV data on the basis of comparison with the 18 MeV data where better resolution prevailed.
Peak #1, $E_x = 0.0$ MeV: This state is not excited with substantial strength at 18 or 26 MeV. The $\beta^-$-decay studies favour a spin of $5/2^-$. The close similarity of the relative intensities and position of the lowest lying states of $^{53,55,57}$Mn suggests that this is the $(f_{7/2}^3)_{5/2}$ state.

Peak #2, $E_x = 0.084$ MeV: Both at 18 MeV (Fig. 3.1) and 26 MeV (Fig. 3.18). This state dominates the spectrum. The angular distribution at both energies resembles that of the other two low lying $7/2^-$ states in $^{53,55}$Mn and is well fitted by a $J^{\pi}_T = 7/2^-$ triton transfer. Mateja [1976] also make an assignment of $7/2^-$. 

Peak #3, $E_x = 0.852$ MeV: Both at 18 MeV (Fig. 3.2) and 26 MeV this state has an angular distribution resembling that of the strong $J^{\pi}_T = 3/2^-$ transitions below 2.4 MeV in $^{53,55}$Mn. Mateja [1976] also make a $3/2^-$ assignment.

Peak #4, $E_x = 1.064$ MeV: At 18 MeV this group has an angular distribution fitted by $J^{\pi}_T = 3/2^-$ (Fig. 3.5). At 26 MeV (Fig. 3.22) the angular distribution of this group resembles that of the 1.292 MeV state in $^{55}$Mn. This feature is of considerable interest because Mateja [1976] have found a doublet separated by 15 keV at this excitation energy in $^{57}$Mn to which they tentatively assign spins of $1/2^-$ and $9/2^-$. Fig. 3.22 displays the angular distribution of the group and the DWBA least squares fit of $9/2^-$ plus $1/2^-$. 

Peak #5, $E_x = 1.229$ MeV: At 18 MeV this group is well fitted by a $3/2^-$ distribution (Fig. 3.4). At 26 MeV the angular distribution is better described by an $11/2^-$ transfer. Because of the greater
confidence that the direct production mode contributes more at 26 MeV than at 18 MeV, the 26 MeV assignment is more reliable. Mateja [1976] also make an 11/2 assignment for this state.

Peak #6, $E_x = 1.378$ MeV: At 18 MeV this group is fitted by a 3/2$^-$ transfer (Fig. 3.5). However, at 26 MeV the angular distribution is radically altered (Fig. 3.30) and was not fitted. Mateja [1976] made no $J^\pi$ assignment. It is one of three groups found in $^{55,57}$Mn with similar angular distributions. It resembles the 3.847 MeV state in $^{55}$Mn.

Peak #12, $E_x = 1.925$ MeV: Not reported by Mateja [1976]. At 26 MeV the angular distribution of this state is best fitted by $J_T^\pi = 5/2^-$ and is similar to that for the 5/2$^-$ state at 3.136 MeV in $^{55}$Mn (Fig. 3.23).

Peak #14, $E_x = 2.188$ MeV: At both 18 and 26 MeV this state shows the characteristic oscillation of a low $J_T$ transfer (Fig. 3.5 and Fig. 3.20). On the basis of the $(\alpha,p)$ angular distributions the spin of this state is 1/2. Since the 1/2$^+$ states in $^{53,55}$Mn were not excited in $(\alpha,p)$, the parity of this level is likely to be negative. This level was reported by Mateja [1976] but no spin assignment was made.

Peak #15, $E_x = 2.238$ MeV: The angular distribution of this group at 18 MeV seems to require a large $J_T^\pi$ transfer (> 9/2). At 26 MeV the rapidly falling angular distribution is similar to that for the 2.413 MeV level in $^{53}$Mn. This state probably has $J^\pi = 3/2^-$.  

Peak #18, $E_x = 2.525$ MeV: No spin assignment was possible but $J_T^\pi = 5/2^-$.
using the triton well \((r_0, a_0) = (1.45, 0.35)\) produces the best fit.

Peak #19 + 20, \(E_x = 2.618\) MeV: Doublet at 18 MeV. The peak at 47° in the experimental angular distribution was not fitted by any \(J_T^x\) used but \(J_T^x = 5/2^-\) gives the best fit for the rest of the data.

Peak #23, \(E_x = 2.848\) MeV: At 18 MeV (Fig. 3.3) and 26 MeV (Fig. 3.19) the angular distribution is best described by \(J_T^x = 3/2^-\) transfer.

Peak #25, \(E_x = 3.123\) MeV: The 26 MeV data favour \(J_T^x = 3/2^-\).

Peak #27 + 28, \(E_x = 3.232\) MeV: No DWBA fit for this group was possible. The angular distribution closely resembles that of the 1.378 MeV level in \(^{57}\text{Mn}\) and the 3.847 MeV level in \(^{58}\text{Mn}\).

Peak #30 + 31, \(E_x = 3.468\) MeV: At 18 MeV a large \(J_T^x\) transfer \((J \geq 9/2)\) is favoured (Fig. 3.9). At 26 MeV a large \(J_T^x\) transfer or \(J_T^x = 3/2^-\) fit equally well. No spin assignment is possible.

Peak #32, \(E_x = 3.542\) MeV: The \(J_T^x = 5/2^-\) calculation shown used a triton well unlike most of the other \(5/2^-\) calculations. The \(J_T^x = 3/2^-\) calculation used a well that is not very different from the other \(3/2^-\) calculations. The \(J_T^x = 3/2^-\) is slightly favoured.

Peak #34 + 35, \(E_x = 3.698\) MeV: No spin assignment was possible but \(J_T^x = 9/2^-\) gives the best fit.

Peak #38, \(E_x = 3.838\) MeV: The \(J_T^x = 5/2^-\) transfer with \((r_0, a_0) = (1.40, 0.30)\) produces the best fit.

Peak #41, \(E_x = 3.948\) MeV: A \(J_T^x = 3/2^-\) transfer gives a good fit to the data.
Peak #42, $E_x = 4.029$ MeV: Although no spin assignment was possible due to the multiplicity of theoretically possible fits, $J^* T = 3/2^-$ is slightly favoured.

Peak #43, $E_x = 4.168$ MeV: No spin assignment was possible but $J^* T = 9/2^-$ provides the best fit for $\theta < 45^\circ$.

Peak #44, $E_x = 4.211$ MeV: No spin assignment was possible. Both $J^* T = 3/2^-$ and $11/2^-$ can fit the data but these are probably not unique.

Peak #46, $E_x = 4.363$ MeV: A $J^* T = 3/2^-$ transfer fits the data well and a tentative $J^* = 3/2^-$ assignment is made.

Peak #47, $E_x = 4.474$ MeV: Except for the two points at $67^\circ$ and $72^\circ$ the angular distribution is reproduced by a $J^* T = 3/2^-$ transfer.

Peak #48, $E_x = 4.523$ MeV: A $J^* T = 3/2^-$ transfer provides a good fit for $\theta$ between $23^\circ$ and $67^\circ$. A tentative $J^* = 3/2^-$ assignment is made. Peaks #48 to 51 were only seen at 26 MeV.

Peak #49, $E_x = 4.626$ MeV: The $J^* T = 5/2^-$ transfer is favoured over that for $J^* T = 3/2^-$ and a tentative $J^* = 5/2^-$ assignment is made.

Peak #50, $E_x = 4.748$ MeV: Both $J^* T = 3/2^-$ and $5/2^-$ fit the data but the $J^* T = 3/2^-$ provides the best fit.

Peak #51, $E_x = 4.841$ MeV: From the DWBA calculations the broad slow decline of this angular distribution favours a large $J^* T$ transfer ($J \geq 9/2$).

A comparison of Fig. 3.11 which summarizes the 18 MeV $^{54}$Cr($\alpha,p$) data with the spectrum of Mateja [1976] reveals a discrepancy
in the number of states excited between 1.378 and 2.188 MeV. In particular, the present work indicates the existence of six excited states between these levels, whereas Mateja found none. An examination of the spectrum they present for $\theta = 15.5^\circ$, $E_\alpha = 24$ MeV reveals an unspecified peak which probably corresponds to the 1.925 MeV level seen in the present $(\alpha, p)$ experiments at 18 and 26 MeV. The peaks labelled in Fig. 3.11 are kinematically consistent with protons coming from a mass 58 nucleus. However, with the given r.m.s. deviations for $Q$ it would still be possible to misidentify protons from mass 53 or mass 55 targets as products from the $^{54}$Cr$(\alpha, p)$ reaction. The $^{52}$Cr contaminant spectrum was subtracted before $Q$ values were determined, even though protons originating from mass 55 could have been kinematically distinguished. The isotopic analysis of the $^{54}$Cr target material provided by the supplier* in atomic per cent is:

<table>
<thead>
<tr>
<th>ISOTOPE</th>
<th>ATOMIC per cent</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>0.25</td>
</tr>
<tr>
<td>52</td>
<td>6.96</td>
</tr>
<tr>
<td>53</td>
<td>2.18</td>
</tr>
<tr>
<td>54</td>
<td>90.60</td>
</tr>
</tbody>
</table>

The only likely remaining contaminant is $^{53}$Cr. But there are two reasons why $^{56}$Mn is not a likely source of these groups. Firstly, there would be many states in $^{56}$Mn below this group (peaks #6-11) if in fact they arise from $^{56}$Mn but the other states of $^{56}$Mn are missing. Secondly, the cross section is likely to be comparable for both $^{53,54}$Cr$(\alpha, p)^{56,57}$Mn but from the isotopic abundances this group is then far too strongly excited. For example, in the $45^\circ$ spectrum (Fig. 3.11)

* Oak Ridge.
Table 3.5

\(^{54}\text{Cr}(\alpha,p)^{57}\text{Mn}\) Q-values for \(E_\alpha = 18, 26\) MeV

<table>
<thead>
<tr>
<th>18 MeV</th>
<th>26 MeV</th>
<th>Excitation Energy keV</th>
<th>18 MeV</th>
<th>26 MeV</th>
<th>Excitation Energy keV</th>
</tr>
</thead>
<tbody>
<tr>
<td>(-Q \pm \Delta Q)</td>
<td>(-Q \pm \Delta Q)</td>
<td>keV</td>
<td>(-Q \pm \Delta Q)</td>
<td>(-Q \pm \Delta Q)</td>
<td>keV</td>
</tr>
<tr>
<td>4302</td>
<td>8</td>
<td>0</td>
<td>7556</td>
<td>13</td>
<td>7558</td>
</tr>
<tr>
<td>4386</td>
<td>6</td>
<td>4395</td>
<td>7</td>
<td>84</td>
<td>7673</td>
</tr>
<tr>
<td>5154</td>
<td>6</td>
<td>5162</td>
<td>7</td>
<td>852</td>
<td>7760</td>
</tr>
<tr>
<td>5366</td>
<td>6</td>
<td>5373</td>
<td>8</td>
<td>1064</td>
<td>7784</td>
</tr>
<tr>
<td>5531</td>
<td>7</td>
<td>5540</td>
<td>9</td>
<td>1229</td>
<td>7844</td>
</tr>
<tr>
<td>5680</td>
<td>6</td>
<td>5680</td>
<td>7</td>
<td>1378</td>
<td>7910</td>
</tr>
<tr>
<td>5789</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5835</td>
<td>10</td>
<td>5824</td>
<td>19</td>
<td>1533</td>
<td>8017</td>
</tr>
<tr>
<td>5932</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6036</td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6132</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6227</td>
<td>7</td>
<td>6233</td>
<td>11</td>
<td>1925</td>
<td>8172</td>
</tr>
<tr>
<td>6318</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6490</td>
<td>14</td>
<td>6499</td>
<td>8</td>
<td>2188</td>
<td>8249</td>
</tr>
<tr>
<td>6540</td>
<td>10</td>
<td>6544</td>
<td>11</td>
<td>2238</td>
<td>8303</td>
</tr>
<tr>
<td>6645</td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6729</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6827</td>
<td>10</td>
<td>6831</td>
<td>10</td>
<td>2525</td>
<td>8513</td>
</tr>
<tr>
<td>6910</td>
<td>9</td>
<td>6926</td>
<td>8 (^{(2)})</td>
<td>2608</td>
<td>8594</td>
</tr>
<tr>
<td>6933</td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7002</td>
<td>11</td>
<td>7022</td>
<td>25 (^{(2)})</td>
<td>2700</td>
<td>8774</td>
</tr>
<tr>
<td>7043</td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7074</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7150</td>
<td>14</td>
<td>7143</td>
<td>7</td>
<td>2848</td>
<td></td>
</tr>
<tr>
<td>7230</td>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7389</td>
<td>12</td>
<td>7381</td>
<td>15</td>
<td>3087</td>
<td></td>
</tr>
<tr>
<td>7425</td>
<td>10</td>
<td>7429</td>
<td>15</td>
<td>3123</td>
<td></td>
</tr>
<tr>
<td>7466</td>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7514</td>
<td>10</td>
<td>7511</td>
<td>9</td>
<td>3212</td>
<td></td>
</tr>
</tbody>
</table>

\(^{(1)}\) \(\Delta Q\) is the r.m.s. deviation for Q over the angles for which the peak was seen at each energy.

\(^{(2)}\) Doublet at 26 MeV from comparison to the 18 MeV data.
5l*Cr (a,p) 57Mn reaction at 26 MeV. The numbers above the bars are the 2J' values used in calculating oDWBA(e).

FIG. 3.39: Reduced strength R = Σ_o o^DWBA_o(θ)/Σ_o o^DWBA(0) vs. excitation energy in 67Mn from the 54Cr(a,p)57Mn reaction at 26 MeV. The numbers above the bars are the 2J' values used in calculating o^DWBA(θ).
the largest cross section is 36.5 \( \mu b/sr \) for peak #2. If a \(^{56}\)Mn level were misidentified the expected cross section would be about 0.9 \( \mu b/sr \), but for peak #10 the measured cross section is 3.3 \( \mu b/sr \), larger by approximately a factor of four than would be expected for a misidentified \(^{56}\)Mn level.

A comparison of the Q-values for \(^{57}\)Mn at 18 and 26 MeV can be found in Table 3.5. The 18 MeV Q-values have been used to calculate the excitation energies.

(I) SUMMARY AND CONCLUSIONS

The strong forward peaking, the marked reduction in the number of states excited at 26 MeV as compared to 18 MeV and the generally good fits afforded by the DWBA to the angular distributions indicate that the states observed at 26 MeV are predominantly excited by the direct reaction mode for the angular ranges studied.

In agreement with previous studies of triton transfer reactions in this mass region, the mechanism seems to be stripping rather than knock-out. This is based on the absence in the spectra of suspected hole states.

The \( L = 1 \) J-dependence was exploited and enabled many spin assignments to be made in \(^{53,55}\)Mn. In particular four \( J^\pi = 1/2^- \) states were located in \(^{53}\)Mn. An \( L = 3 \) J-dependence was also found from a comparison of known \( L_p = 3 \) transitions.

The relative strengths of levels populated in \((\alpha,p)\) can vary radically from the relative strengths populated in \((^3\text{He},d)\). The most noteworthy example of this feature is the 2.197 MeV \( 7/2^- \) level in \(^{55}\)Mn.
Many new states were observed. In $^{57}\text{Mn}$, 42 new states were observed; in $^{55}\text{Mn}$ two new states above 5.5 MeV were observed and in $^{53}\text{Mn}$ 14 levels above 6 MeV were located, most of which were proton unbound. There is no apparent change in either the magnitude or shape of the differential cross section for the proton unbound states as compared to the bound states.

Further evidence was provided for the existence of a close doublet at 1.292 MeV in $^{55}\text{Mn}$ from the oscillatory character of the 26 MeV ($\alpha$,p) angular distribution. An attempt to observe a 1.292 MeV $\gamma$-ray in singles from the $^{55}\text{Mn}(p,p'\gamma)$ reaction at $E_p = 7.975$ MeV proved to be unsuccessful. The p-$\gamma$ coincidence experiment also did not reveal a 1.292 MeV $\gamma$-ray to be in coincidence with the 1.292 MeV proton group. The p-$\gamma$ coincidence scheme obtained from (p,p$'\gamma$) was in substantial agreement with that deduced by Hichwa [1973] using the ($\alpha$,p$\gamma$) reaction.

A summary of the 26 MeV data is given in Fig. 3.40 in which the energy levels and proposed $J^\pi$ values for the three Mn isotopes studied are depicted. Tentative $J^\pi$ values are in brackets.

The seniority selection rule $\Delta \nu_p = 1$ weighs heavily against the excitation of the seniority three states below 2 MeV in $^{53}\text{Mn}$. Presumably the 3/2$^-$ state at 1.288 MeV in $^{53}\text{Mn}$ is excited through a small single particle seniority one component. This is plausible due to the proximity of the 2p$^{5/2}$ single particle state. The influence of the closed neutron shell in $^{53}\text{Mn}$ seems to reduce the number of states excited in $^{53}\text{Mn}$ at 26 MeV. There are 10 states excited in $^{53}\text{Mn}$ up to 4 MeV as compared with 18 states in $^{55,57}\text{Mn}$. In both $^{53,55}\text{Mn}$ the strongest $J^\pi = 3/2^-$ transition is in the proximity of a 7/2$^-$ state. However in $^{53}\text{Mn}$ the 7/2$^-$ state is weakly excited compared with the
<table>
<thead>
<tr>
<th>Energy (keV)</th>
<th>Spin Assignments</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>7-</td>
</tr>
<tr>
<td>1000</td>
<td>3-</td>
</tr>
<tr>
<td>2000</td>
<td>3-</td>
</tr>
<tr>
<td>3000</td>
<td>5-</td>
</tr>
<tr>
<td>4000</td>
<td>7-</td>
</tr>
<tr>
<td>5000</td>
<td>9</td>
</tr>
<tr>
<td>6000</td>
<td>(7-)</td>
</tr>
<tr>
<td>7000</td>
<td>(9-)</td>
</tr>
<tr>
<td>8000</td>
<td>(11-)</td>
</tr>
<tr>
<td>9000</td>
<td>(11-)</td>
</tr>
</tbody>
</table>

**NOTE:**

1. Only states for which angular distributions at 26 MeV were obtained are included here.
2. Where 2 or more assignments are made these represent the best fit but do not exhaust the possibilities.
3. $x = (5, 7^-, 9^+)$.

FIG. 3.40: Spin assignments based on the ($\alpha$,p) data.
lowest $7/2^-$ state ($E_x = 0.0$), whereas in $^{55}\text{Mn}$ this state is excited with comparable strength. Indeed, below 2 MeV the $7/2^-$ strength seems to be concentrated in one state in $^{53}\text{Mn}$ and also in $^{57}\text{Mn}$. In $^{55}\text{Mn}$ however, there are three $7/2^-$ states below 2 MeV. The centre of gravity of the $3/2^-$ strength shifts down in energy going from $^{53}\text{Mn}$ to $^{57}\text{Mn}$. The first $3/2^-$ state acquires strength at the expense of the second $3/2^-$ state. In $^{53}\text{Mn}$ the 1.288 MeV state is very weakly excited compared to the 2.413 MeV state. In $^{55}\text{Mn}$ the 1.528 MeV state is not as strongly excited as the 2.258 $3/2^-$ level, but is still a strongly excited state. In $^{57}\text{Mn}$ the first $3/2^-$ state is one of the most strongly excited states in the spectrum. The distribution of the $3/2^-$ and $7/2^-$ strengths observed in the $(\alpha, p)$ reaction for excitation energies below about 2.4 MeV can be seen in Figs. 3.33, 3.34 and 3.39.

Most of the angular distributions could be accounted for by $L = 1, 3$ or 4 transfers. The assignment of spin on the basis of the $(\alpha, p)$ reaction is not unambiguous however, unless the $L$ value is already known. Spin $1/2$ is easy to recognize though due to the pronounced oscillations in the differential cross section.

Since the $(\alpha, p)$ reaction can only excite $T_f = T_s$ states, the states at 6.990, 7.574 and 8.016 MeV in $^{53}\text{Mn}$ can not be isobaric analog states if they are, in fact, the same states seen in $(^3\text{He}, d)$ and $(^7\text{Li}, ^6\text{He})$ by Gunn [1976].
CHAPTER 4
THEORETICAL INTERPRETATIONS OF THE NUCLEAR STRUCTURE OF $^{53,55}$MN

(A) THEORETICAL STUDIES OF $^{53}$Mn

(1) Shell Model Interpretations

The theoretical interpretation of the $^{53}$Mn nuclear structure has proceeded along two lines; a description in terms of the shell model or in terms of the unified model. Early shell model treatments by McCullen [1964] considered only $1f^{n}_{7/2}$ proton configurations. This model was able to predict low-lying levels for $E_x \leq 2$ MeV. The basic feature of this simplified treatment displayed in the ($\alpha$,p) reaction is the near absence of the proton seniority ($\nu_p$) 3 states in $^{53}$Mn below 2.4 MeV for the 26 MeV data.

This highly restricted basis is inadequate [Lips, 1970] to reproduce other features of the experimental information; notably, M1 gamma transitions are forbidden between $j^n$ configurations, whereas experimentally they are found to occur. The shell model calculation was extended to include $1f^{n}_{7/2}2p_{3/2}$ and $1f^{n}_{7/2}1f^{5/2}_{5/2}$ proton configurations by Lips [1970]. They found that the ground state was more than 98% $1f^{5}_{7/2}$ and that the lowest excited state for each J was more than 80% $1f^{5}_{7/2}$ except for the first 9/2$^{-}$ state which was 40% $1f^{5}_{7/2}$ and 50% $1f^{4}_{7/2} (\nu = 2, J = 6)2p_{3/2}$. As can be seen from their calculation the $1f^{5}_{7/2}$ configuration dominates the low-lying spectrum and hence the low-lying
states are weakly excited, if at all, except for $J^m = 7/2^-$. Although the inclusion of $1f_{7/2}^4 2p_{3/2}^0$ configurations in the basis enabled reasonable spectroscopic factors for ($^3$He,d) to be calculated, the basis was still too restricted to fit M1 transition rates. Proton excitations into the $1f_{5/2}$ shell were needed to account for the M1 transition rates.

An extension of this model to include $2p_{1/2}$ proton excitations was made by Saayman [1976]. Instead of using the two particle matrix elements as adjustable parameters to fit the energy levels they used a surface delta interaction

$$V_{ij}^{(SDI)} = -4\pi V_0 \delta(\rho_{ij}) \delta(r_i - R) \delta(r_j - R)$$

in which the particles $i$ and $j$ are assumed to interact by a delta function interaction only when they are both at the nuclear surface $R$. By varying $V_0$ alone they could account for 40 energy levels of $^{50}$Ti, $^{51}$V, $^{52}$Cr, $^{53}$Mn, $^{54}$Fe and $^{55}$Co with an r.m.s. deviation of 450 keV. They were also able to reproduce the E2 gamma transition rates. Their conclusion about the nature of the low-lying states is quite similar to that of Lips [1970]. Of particular interest is their decomposition of the first and second excited states of $^{53}$Mn. The $5/2^-$ state is 4% $1f_{7/2}^4 1f_{5/2}^0$ compared to the $3/2^-$ state which is 11% $1f_{7/2}^4 2p_{3/2}^0$. That is, the probability of exciting the $3/2^-$ state through a single particle component is nearly three times larger than for exciting the $5/2^-$ state through a single particle component. As can be seen in the proton spectrum of Fig. 3.14 the $3/2^-$ level at 1.288 MeV is considerably weaker than many other states in the spectrum. Thus, it is no surprise that the $5/2^-$ state at 0.38 MeV is not seen in the $(\alpha,p)$ reaction.

All the above calculations assumed only single proton excitation into the f-p shell. The neutrons have been confined to the
N = 28 core. Osnes [1971] has objected to such an approach because the states constructed only on proton excitations do not have good isospin. The main feature of the use of states with definite isospin is that the M1 transition rates can alter dramatically due to the inclusion of the excited neutron configurations needed to produce good isospin. The use of wave functions with good isospin does not substantially alter the single particle transfer spectroscopic factors or the E2 transition rates for states with dominant 1f_{7/2}^{5} configurations. Osnes' [1971] basis was limited to the so-called "antianalog" states. These are states constructed from 1f_{7/2}^{5} proton configurations by application of the operator \( P_{T<} \) such that

\[ \psi(T_{<}T_{3}) = P_{T<}[1f_{7/2}^{5}] \]  

(4.2)

where

\[ P_{T<} = \sqrt{2j+3 - \frac{3}{2}} \left( 1 - \frac{T_{-}T_{+}}{2j+3 - \frac{3}{2}} \right), \]  

(4.3)

where \( \tilde{z} \) is the number of protons in the 1f_{7/2} shell and \( T_{\pm} \) are the isospin projection raising and lowering operators. For \( ^{53}\text{Mn} \) \( \tilde{z} = 5 \) and the antianalog state can be depicted as

\[ \psi(T = 3/2) = \sqrt{\frac{4}{5}} \begin{array}{c} \times \end{array} \begin{array}{c} \text{****} \end{array} - \sqrt{\frac{1}{5}} \begin{array}{c} \text{*****} \end{array} \begin{array}{c} 0 \end{array}. \]  

(4.4)

The factors in front of the configurations in (4.4) are the Clebsch-Gordan isospin coupling coefficients.

According to Benson [1975] the antianalog states account for only a small fraction of the \( f_{7/2}^{5} T = 3/2 \) states. They expanded the calculation considerably to include other \( T = 3/2 \) neutron hole states.
As can be seen from (4.4) the antianalog states consist of 80% proton excitation and 20% neutron excitation. However, Benson [1975] using $1f_{7/2}^3$ and $1f_{7/2}^4j$ configurations for $j = 2p_{3/2}, 2p_{1/2}$ and $1f_{5/2}$ showed that the lowest one particle-four hole (1p-4h) states are predominantly neutron excitations. In accord with previous calculations the first five states are mainly $f_{7/2}^3$ configurations.

Besides explaining the absence of three out of this group of five states in the $(\alpha,p)$ reaction, their calculation predicts a $3/2^-$ state at about 2.5 MeV with a large $2p_{3/2}$ proton component which can be identified with the $3/2^-$ state at 2.413 MeV. The only other state in the calculated spectrum below 3.5 MeV with a large single particle component is a $5/2^-$ state at 3.1 MeV. In fact, at 3.110 MeV in $^{53}\text{Mn}$ the 26 MeV $(\alpha,p)$ data reveal a $5/2^-$ state, but Benson [1975] prefer to associate the calculated level with the observed $5/2^-$ state at 3.670 MeV because of the similarity of the proton spectroscopic factor to that measured in $(^3\text{He},d)$. Their lowest $1/2^-$ state is calculated to be at 2.56 MeV and is associated with the $1/2^-$ level at 2.684 MeV seen in the $(\alpha,p)$ reaction. The proton spectroscopic factors agree. Their second calculated $1/2^-$ state lies at 3.4 MeV and the $(\alpha,p)$ data at 26 MeV reveal a $1/2^-$ state at 3.460 MeV. The proton spectroscopic factors also agree.

The experimental spectrum has several levels below 3.5 MeV which are not predicted by Benson's [1975] calculation. The $7/2^-$ state at 2.578 MeV is of particular interest since it was not excited in single proton transfer. Their explanation of this feature is that two particle excitations should begin to appear at this excitation energy. In $^{54}\text{Fe}$ there is a $0^+$ state at 2.56 MeV which is believed to be a $(2p-4h)$ state. On this basis the observation of the $7/2^-$ state at 2.578
MeV in the (α,p) reaction is quite understandable since the transferred neutrons can go into any unoccupied orbit. It is of interest to note here that the 7/2⁻ state at 2.197 MeV in ⁵⁵Mn probably also has a component of two particle excitation from the 1f⁷/₂ shell. Although the relative excitation strengths of these two 7/2⁻ states in ⁵³,⁵⁵Mn differ greatly when compared to the first 7/2⁻ states in the respective spectra, their proximity to the strongly excited 3/2⁻ states and their absence in the (³He,d) reaction is suggestive of this explanation.

The theoretical proton spectroscopic factors mentioned above were calculated assuming that the ⁵²Cr ground state is pure 1f⁷/₂⁻. For the sake of illustration let us assume that the ⁵⁶Cr ground state is pure 1f⁹/₂⁻. The types of configuration which can be excited by the (α,p) reaction are depicted in Figs. 4.1. The dashed lines in these
configurations delineate the target ground state; in this case $^{50}\text{Cr}$ with $J_0 = 0$ and $T_0 = 1$. What might be expected, a priori, is that the target can contain several non-zero seniority components. All that is required is that $T_0 = 1$ and $J_0 = 0$. In fact, the wave function for the $^{50}\text{Cr}$ ground state given by McCullen [1964] is

$$
\Psi^{(50}\text{Cr})_{\text{g.s.}} = 0.8224(j_0^+j_0^-)_0 + 0.5420(j_2^+j_2^-)_0 + 0.0563(j_2^-j_2^+)_0 + 0.0861(j_4^-j_4^-)_0 - 0.1383(j_4^+j_4^+)_0 - 0.0127(j_6^-j_6^-)_0,
$$

(4.5)

where $j = 1f_{7/2}$. The seniority zero component in (4.5) represents 67% of the wave function. Hence, 33% of this wave function is available for production of the first $5/2^-$, $3/2^-$, $9/2^-$ and $11/2^-$ states in $^{53}\text{Mn}$ which according to Benson [1975] are over 75% of the $f_{7/2}^3$. The fact that the transferred neutrons must couple to a total spin of $L_n = 2$, 4 or 6 to populate these five states is not in fact a hindrance as can be estimated if the single particle wave functions are assumed to be harmonic oscillator wave functions. To find the relative and centre of mass motion of the transferred neutrons the Talmi-Moshinsky brackets are used as described in Chapter 2 section (C.2). The transformation to relative and centre of mass motion is given by

$$
|n_1L_1,n_2L_2,L_nM_n\rangle = \sum_{n\lambda,N\Lambda,L_nM_n} \langle n\lambda,N\Lambda,L_n|n_1L_1,n_2L_2,L_n\rangle, \quad (4.6)
$$

where

$n_1L_1$ and $n_2L_2$ are the number of nodes and orbital angular momenta of the single particle states,

$L_n = L_1 + L_2$ and

$n\lambda$ and $N\Lambda$ are the number of nodes and orbital angular momenta of the relative and centre of mass motion.
Since the neutrons have $\lambda = 0$ in the $\alpha$-particle only those components in (4.6) for which $\lambda = 0$ are populated in the reaction. The amplitude of these components from the tables of Brody [1967] are tabulated in Table 4.1, in which the convention that $n$ is one less than the number of nodes used above is employed, i.e., $1f_{7/2} = 0f_{7/2}$. For neutrons going into the $0f_{7/2}$ shell with total orbital angular momentum $L_n$ we abbreviate $|03,03,L_n\rangle = |L_n\rangle$.

Table 4.1

<table>
<thead>
<tr>
<th>$L_n$</th>
<th>$n\lambda = \lambda_n$</th>
<th>$L_{n1}$</th>
<th>$L_{n2}$</th>
<th>$L_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0$</td>
<td>$00,30,0_0\rangle$</td>
<td>0.2236</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$2$</td>
<td>$10,20,0_2\rangle$</td>
<td>-0.5916</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$20,00,0_2\rangle$</td>
<td>0.5196</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$30,00,0_2\rangle$</td>
<td>-0.2236</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$4$</td>
<td>$00,14,4_4\rangle$</td>
<td>0.1936</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$10,04,4_4\rangle$</td>
<td>-0.3708</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$6$</td>
<td>$00,06,6_6\rangle$</td>
<td>0.5590</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(1) Taken from Brody [1967].

Of course, a truly quantitative determination needs to be made, probably along the lines suggested by Falk [1975]. But even the simple estimate of the relative importance of the $L_n$ components in Table 4.1 shows that $L_n = 0$ is not favoured over other couplings for the transferred neutrons and other couplings require $\lambda_n = 2$ or $4$ in the $^{50}$Cr ground state. From the 26 MeV data it is quite apparent that the four states mentioned above as 76% $f_{7/2}^3$ are populated with far less strength than the ground state. Added to the fact that the McCullen [1964] $^{50}$Cr
ground state wave function contains 33% of \( \nu_p = 2 \) and 4 components must be included the feature of the kinematically favoured large J-transfers for the 9/2\(^-\) and 11/2\(^-\) states. Since the 7/2\(^-\) ground state of \( ^{51}\text{Mn} \) can be excited through a \( \nu_p = 0 \) component in the \( ^{50}\text{Cr} \) ground state and since the 5/2\(^-\), 9/2\(^-\) and 11/2\(^-\) states require \( \nu_p = 3 \) components (barring unlikely single particle state configurations), the \( \nu_p = 2 \) and 4 components in the \( ^{50}\text{Cr} \) ground state must be smaller than indicated by the McCullen [1964] wave function. Exactly how much smaller requires an exact calculation of the transfer process as well as detailed wave functions for the states in question.

All the configurations in Fig. 4.1 can be excited. Using the same criteria given above for considering the relative importance of the different \( (1f_{7/2}^\pi L_n) \) components, the tables of Brody [1967] show that the transformation brackets do not favour the \( (j^2) L_n \) configurations over the \( (j_1 j_2) L_n \) configurations in terms of \( \lambda = 0 \) content for the transferred neutrons. The Benson [1975] calculation allows configurations such as Figs. 4.1(a), (b) and (c) to be tested. As noted earlier they recognized the inadequacy of the \( 1f_{7/2}^\pi j \) configurations and proposed that configurations 4.1(d) and (e) are needed. This still does not exhaust the components that can be tested as indicated by configurations 4.1(f) and (g). If the \( ^{52}\text{Cr} \) ground state were pure \( 1f_{7/2}^\pi \), then only configurations 4.1(a) and (b) could be excited in \( (^3\text{He},d) \). Hence it would be surprising indeed if the \( (^3\text{He},d) \) and \( (\alpha,p) \) reactions populated the same final states with the same relative strength. As indicated by Fig. 3.33 the variation between the \( (^3\text{He},d) \) and \( (\alpha,p) \) reactions is large. The comparison between the p and \( (p+2n) \)-transfers shows that models which consider only proton excitations out of the \( 1f_{7/2} \) shell are inadequate. The Benson [1975] calculation is the only one which has a
chance of explaining the \((\alpha,p)\) data since it accounts for at least part of the neutron "core" excitations.

The 7/2\(^{-}\) state at 2.578 MeV, as noted previously is an example of a probable neutron excitation. From the analysis of the \(^{53}\text{Cr}(p,d)^{52}\text{Cr}\) and \(^{54}\text{Cr}(p,t)^{52}\text{Cr}\) reactions Chapman [1968] concluded that the 0\(^{+}\) state at 2.650 MeV in \(^{52}\text{Cr}\) is predominantly a \((\pi 1f_{7/2}^{8})(\nu 1f_{7/2}^{-2}2p_{3/2}^{2})\) neutron excitation. This 7/2\(^{-}\) state in \(^{53}\text{Mn}\) is then probably represented by configuration (d) in Fig. 4.1. This configuration explains rather nicely why this state is not seen in the \(^{3}\text{He},d\) reaction but is seen in the \((\alpha,p)\) reaction; that is, it has a parentage based upon an excited state of \(^{52}\text{Cr}\) (see the discussion in Chapter 2, section (E.2)).

A further piece of evidence indicating that \(^{48}\text{Ca}\) is not a good core is that from the \(^{52}\text{Cr}(p,d)^{51}\text{Cr}\) reaction exciting the doublet at 0.77 MeV in \(^{51}\text{Cr}\) with an \(I_n = 1\) transfer [Whitten, 1967; Chapman, 1968].

The inclusion of all two particle excitations from the \(1f_{7/2}\) shell requires a basis of 1400 [Benson, 1975]. Even this however, can not explain effects in the \((\alpha,p)\) data attributable to the three particle excitations such as in Figs. 4.1(f) and (g).

(2) Unified Rotational Model of \(^{53}\text{Mn}\)

Scholz [1966, 1967] have calculated the properties of \(^{53}\text{Mn}\) using the Coriolis coupling model. A description of the Coriolis coupling model can be found in Scholz [1966]. The calculated level sequence [Malik, 1966] inverts the observed positions of the first 11/2\(^{-}\) and second 7/2\(^{-}\) states although the rest of the levels up to 2.5 MeV are in reasonable agreement with experiment. No other properties were
calculated in the first paper by Malik [1966]. In their second paper [Scholz, 1967] the ground state magnetic dipole moment was calculated to be 4.623 n.m. as compared with the experimental value of 5.050 n.m.

(B) THEORETICAL STUDIES OF $^{55}$Mn

(1) Shell Model Interpretations

Early shell model calculations by Vervier [1966] and McGrory [1967] confined the protons to the $1f_{7/2}$ shell. The assumption of a closed $^{40}$Ca core was again employed. Vervier [1966] restricted his basis to $(\pi l^f_{7/2} v^2p^2_{3/2})$ configurations. The level ordering for the first $5/2^-$ and $7/2^-$ states as well as for the first $9/2^-$ and $3/2^-$ levels is reversed. A more serious failure of this restricted basis is its inability to place the $7/2^-$ state at 2.197 MeV observed in the experimental spectrum in proximity to the strongly excited $3/2^-$ level. It will be recalled that the experimental spectrum has three $7/2^-$ states below 2.2 MeV, whereas Vervier's calculation places the third $7/2^-$ state at about 2.6 MeV. Consequently, the theoretical level at 2.2 MeV was identified with the experimental $7/2^-$ state at 1.884 MeV. Despite the level inversions, the calculated E2 and M1 transition rates favoured the correct spin assignments for the 1.527 MeV $3/2^-$ state and for the 1.884 MeV $7/2^-$ state. Tentative spins for the 2.197 and 2.564 MeV levels were $5/2^-$ and $7/2^-$; however these were based on the assumption of 100% $\gamma$-decay to the ground state because there was no available experimental evidence. As can be seen in Fig. 3.38 the 2.197 MeV level decays to the 0.985 MeV level.

McGrory [1967] extended the basis to allow the two extra-core neutrons access to the $2p_{3/2}$, $2p_{1/2}$ and $1f_{5/2}$ orbits. Unfortunately
McGrory compared only the first six experimental and theoretical states for which the correct ordering of spins was predicted. No γ-ray transitions were calculated, but spectroscopic factors for $^{55}\text{Mn}(p,d)^{54}\text{Mn}$ were calculated and compared to experiment. For states below 1 MeV in $^{54}\text{Mn}$ the theoretical and experimental spectroscopic factors agree. They did not expect to see any $\lambda_n = 1$ levels above 1 MeV in $^{54}\text{Mn}$ and also did not expect $\lambda_n = 3$ transitions to the experimentally observed levels at 1.12 and 1.46 MeV. They concluded that either their model Hamiltonian does not mix enough $\text{vlf}_{5/2}$ component into their low-lying states or that neutron core excitations must be included.

The results of their $^{57}\text{Fe}(p,^3\text{He})^{55}\text{Mn}$ and $^{57}\text{Fe}(d,\alpha)^{55}\text{Mn}$ data prompted Peterson [1971] to repeat the calculations of McGrory and to extend their basis. The detection of a $1/2^-$ state nearly degenerate with the $11/2^-$ state at 1.291 MeV in $^{55}\text{Mn}$ is in contradiction to both Vervier's and McGrory's spectra. Use of the Kuo-Brown [1968] matrix elements in a $(2p - 3h)$ basis produces a spectrum which is in good agreement with experiment for the first eight states, excluding the $1/2^-$ level at 1.291 MeV. Peterson showed that the use of $(3p - 4h)$ configurations was able to produce a low-lying $1/2^-$ state near the $11/2^-$ level. Proton excitations were found to be the most important of the $(3p - 4h)$ configurations. They allowed proton excitations into the $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$ orbitals. Although they present no wave function their calculations give components of 42% $(2p - 3h)$ and 58% $(3p - 4h)$ for this level. The $(3p - 4h)$ components allow for some excitation into the $2p_{1/2}$ orbital. The spectroscopic factor is so small for single proton stripping that the state would not likely have been seen by Rapaport [1969] in the $(^3\text{He},d)$ reaction.

Both the $(2p - 3h)$ and $(3p - 4h)$ configurations depicted in Figs.
4.2 can however be excited by the \((\alpha,p)\) reaction. Hence the \(1/2^-\) state could be excited through a component such as in Fig. 4.2(a), for example \([([\pi f_{7/2}^3]7/2^-)(\nu f_{5/2}^2)_{4^+}]_{1/2^-}\). The \((3p-4h)\) components of Fig. 4.2(b) exhibit greater variety since many combinations of three particles in the \(f-p\) shell can produce spin \(1/2^-\).

![Diagram of Fig. 4.2](image)

The \((2p-3h)\) plus \((3p-4h)\) calculations fail to predict the close proximity of the strong \(3/2^-\) state at 2.258 MeV and the \(7/2^-\) state at 2.197 MeV. They also predict that much of the \(2p_{1/2}\) strength is at 2.46 MeV. An examination of Fig. 3.40 reveals that the \(L=1\) transfers at this energy are in fact \(J^*=3/2^-\). In spite of this, Peterson [1971] claim that the calculated spectroscopic factors are in good agreement with the \((^3\text{He},d)\) data. Wave functions for these states were not published so further comparisons with the \((\alpha,p)\) data can not be made.

Some qualitative features of the structure of various states can be gleaned from a comparison of the \((^3\text{He},d)\) and \((\alpha,p)\) data. For example, the lowest \(5/2^-\) state in \(^{53,55}\text{Mn}\) is not excited in single proton stripping and hence one can conclude that the single particle \(\pi f_{5/2}\) component is small. Moreover, the \(5/2^-\) states are not excited in the \((\alpha,p)\) reaction either despite the greater number of configurations.
accessible to this reaction. This means that configurations such as

\[ \{ (f^\pi_{7/2})_0 \nu(f^n_{7/2})_0 (j_1 j_2)_I \} \]

are the only configurations accessible if the target ground state has solely \( \nu_{p,n} = 0 \) components, must be small. For these low-lying \( 5/2^- \) states the dominant configurations must contain \( \{ (f^\pi_{7/2})_I \} \) components with \( I_p > 0 \) in order to invoke the \( \Delta \nu = 1 \) seniority selection rule discussed in Chapter 2 section (F). Therefore, the lowest type of excitation in \( ^{53,55,57}\text{Mn} \) seems to consist of the breaking of proton pairs, i.e., creating \( \nu_p = 3 \) states.

This argument can also be extended to the \( 11/2^- \) and \( 9/2^- \) states at 1.441 and 1.621 MeV in \( ^{53}\text{Mn} \). The \((\alpha,p)\) spectra exhibit a considerable gap of 2.413 MeV between the \( 7/2^- \) ground state and the strongly excited \( 3/2^- \) state. Only the \( 3/2^- \) level at 1.288 MeV is weakly excited, and this presumably occurs through a small single particle component. Therefore, the \( 11/2^- \) and \( 9/2^- \) states at \( 1.441 \) and \( 1.621 \) MeV must be dominated by \( \{ (f^\pi_{7/2})_I \} \) components with \( I_p > 0 \).

This large gap does not exist in the \( ^{55,57}\text{Mn} \) spectra presumably because the extra-core neutrons can couple to a variety of spins without disturbing the neutron core of \( N = 28 \). The \( 9/2^- \) and \( 11/2^- \) states in \( ^{55}\text{Mn} \) along with the several other states in \( ^{57}\text{Mn} \) are excited at excitation energies less than 2 MeV and this can occur through

\[ \{ (f^\pi_{7/2})_0 \nu(j_1 j_2)_I \} \]

components. Thus, the extra degrees of freedom introduced by the extra-core neutrons in \( ^{55,57}\text{Mn} \) allow the \( \nu_p = 1 \) and \( \nu_n = 2 \) components to be excited in the low-lying states.

The additional \( \nu_n = 2 \) components in \( ^{55}\text{Mn} \) may also explain the difference between the \((\alpha,p)\) relative reduced strengths and the relative spectroscopic factors from \((^3\text{He,d)}\) for the first \( 7/2^- \) states in \( ^{53,55}\text{Mn} \). From \((^3\text{He,d)}\), \( S_{\pi d}(53)/S_{\pi d}(55) = 1.36 \) and from the \((\alpha,p)\) data...
$R_{\alpha p}(53)/R_{\alpha p}(55) = 0.93$. Thus the relative strength of the 0.128 MeV level in $^{55}$Mn has increased. This can be attributed to either the additional $\nu_n = 2$ components or to a better overlap of the di-neutron relative motion with the di-neutron motion in the $\alpha$-particle for neutrons going into the $2p_{3/2}$, $2p_{1/2}$ of $1f_{5/2}$ orbitals as compared to the $1f_{7/2}$ orbital.

Despite the large variation in relative strengths, it is worthwhile noting the great similarity in angular distribution shapes between the three isotopes. This feature strongly supports the conjectured importance of the surface region because it is only beyond the nuclear surface that the form factors have any degree of similarity. The dominant effect of configuration mixing seems to be limited to influencing the magnitude of the cross sections.

(2) Unified Rotational Model of $^{55}$Mn

A treatment of the $^{55}$Mn structure using the Nilsson model plus Coriolis coupling has been presented by Comfort [1971]. These authors maintain that the ground state spin of $5/2^-$ (possible with prolate deformation) plus the large ground state quadrupole moment, $Q \approx +0.46$, raises the possibility of a static nuclear deformation. The only free parameter used in these calculations was the deformation parameter $\beta$, which essentially changes the single particle wave functions through its connection with the harmonic oscillator frequencies along the three nuclear axes [Scholz, 1966]. The level sequence calculated by Comfort shows a poorer correspondence with the experimental sequence than does the $(2p - 3h)$ calculations of Peterson. The Coriolis coupling calculation does not predict a low-lying $1/2^-$ state in the vicinity of the $11/2^-$ state at 1.292 MeV; however Comfort state that the exact
location of the $1/2^-$ state they calculate at 2.09 MeV depends strongly on the spin-orbit and well-flattening coefficients C and D appearing in the single particle Hamiltonian given by

$$H = -\frac{\hbar^2}{2\mu} \Delta + \frac{1}{2} \xi \left( \omega_0^2 x^2 + \omega_0^2 y^2 + \omega_0^2 z^2 \right) + C \xi^I s + D \xi^I z .$$  (4.7)

The other levels (except for a $3/2^-$ state calculated to be at 2.17 MeV) are fairly insensitive to C and D.

The ground state magnetic dipole and electric quadrupole moments are well reproduced by the calculations of Comfort [1971]; however these quantities were also reproduced by Vervier [1966] using $(\pi_{1f_{7/2}^3})(\nu_{2p_{3/2}^2})$ shell model configurations. Although the Coriolis coupling model is able to calculate the electromagnetic transition rates and some of the proton stripping spectroscopic factors for states below 2.25 MeV reasonably well, it underestimates by a factor of 100 the spectroscopic factor for the $^{54}\text{Cr}(^3\text{He},d)^{55}\text{Mn}$ reaction leading to the $J^* = 3/2^-$ level at 1.528 MeV.

Comfort [1971] suggest that the use of an extended shell model basis could simulate the deformed shell model plus Coriolis coupling calculation by increasing the configuration mixing. Indeed it seems necessary to use the shell model to discuss the $(\alpha,p)$ data because of the necessity to consider the degrees of freedom of at least three particles, as opposed to the rotation-particle coupling model which accounts only for the motion of a single particle about a deformed core.

(C) CONCLUSIONS

Excitation of the neutron core, $N=28$, seems to be important for the description of $^{53}\text{Mn}$ above 2.5 MeV. The $7/2^-$ state at 2.578 MeV
probably contains an excited-core component consisting of two neutrons
coupled to $I_n = 0$ elevated to the $f$-$p$ shell. In this regard the
calculations of Benson [1975] probably quantitatively best describe the
$(\alpha, p)$ data. In $^{53}\text{Mn}$ only the $3/2^-$ state at 1.288 MeV is excited in the
gap between the ground state and the strongly excited $3/2^-$ level at
2.413 MeV. The situation is different in $^{55,57}\text{Mn}$ as seen in Fig. 3.40.
The sparsity of levels excited in the $(\alpha, p)$ reaction between 0 and 2.4
MeV in $^{53}\text{Mn}$ can be understood by assuming that the $5/2^-, 11/2^-$ and $9/2^-$
states at 0.378, 1.441 and 1.621 MeV arise mainly from the $\pi f_7^{-3}/2$
configuration. Indeed, all the shell model calculations indicate that
this is so. However, there is an added condition that the $^{50}\text{Cr}$ ground
state wave function calculated by McCullen [1964] to consist of 33%
$v_p = 2$ or 4 components should actually be nearly pure $v_p = 0$.

In $^{55,57}\text{Mn}$ the extra degrees of freedom enjoyed by the
neutrons beyond the $1f_{7/2}$ shell enables many more states to be excited
through the $(\pi(1f_{7/2}^2)_{0,j}v_{j_1}j_2)_{I_n}$ components. The calculations by
McGrory [1967] of the $^{55}\text{Mn}$ structure are incapable of explaining the
particle transfer data of Rapaport [1968], Peterson [1971] and the 26
MeV $(\alpha, p)$ data. These reactions indicate the importance of single
proton excitation into the $f$-$p$ shell whereas McGrory's calculation
limits the protons to the $1f_{7/2}$ shell. The deuteron pick-up reactions
This state could only be explained by inclusion of $(3p - 4h)$
configurations in the basis. Thus future shell model calculations of
$^{55,57}\text{Mn}$ will need to account for proton excitation out of the $1f_{7/2}$
shell.

The unified rotational model has also been applied to $^{53,55}\text{Mn}$,
but not with any greater success than the more elaborate shell model
calculations. Moreover, this model is not readily tested by multi-nucleon transfer since it accounts for only one, or at most two [Comfort, 1971], particles outside of a deformed core.
CHAPTER 5
SUMMARY AND CONCLUSIONS

The $^{50,52,54}$Cr($\alpha,p$)$^{53,55,57}$Mn reactions were studied at 18 and 26 MeV bombarding energy. Analysis of the proton angular distributions revealed that compound nuclear reactions were not negligible at 18 MeV, although a combination of direct plus statistical compound reactions did not improve the fit to the angular distributions.

In contrast, at 26 MeV nearly all the proton groups detected had angular distributions amenable to a triton cluster transfer using the DWBA theory. The angular distributions of forty-one proton groups were obtained in $^{53}$Mn, twenty-seven groups in $^{55}$Mn and twenty-seven groups in $^{57}$Mn. Nearly all the calculated $J^T$ transfers were for $L=1$ or $L=3$. Using the well established $L=1$ $J$-dependence of the ($\alpha,p$) reaction, many spin assignments could be made for states whose $L_p$ value had already been determined from previous ($^3$He,d) studies in $^{53,55}$Mn. In addition, an $L=3$ $J$-dependence was also found for ($\alpha,p$) in this energy-mass region.

Excitation energies for states beyond 6 MeV in $^{53}$Mn agree well with recent values of Gunn [1976]. Excited states up to 8 MeV were observed, including eight proton unbound states. The angular distributions for the proton unbound states showed no departure from those observed for groups leading to bound states. If the states above 6 MeV are in fact the same as observed by Gunn [1976], then no isobaric
analog states \((T = 5/2)\) were excited in these \((^3\text{He},d)\) measurements because the \((\alpha,p)\) reaction can excite only the \(T = 3/2\) states in \(^{53}\text{Mn}\). There is a large gap in the proton spectrum from \(^{53}\text{Mn}\) in the 26 MeV data between the \(7/2^-\) ground state and the \(3/2^-\) state at 2.413 MeV. Only the \(3/2^-\) state at 1.288 MeV is weakly excited in this gap. The known \(5/2^-\), \(9/2^-\) and \(11/2^-\) states below 2.413 MeV are not excited. This can be interpreted to mean that these states have a nearly pure \(F_{7/2}^3\) configuration, i.e., \(\nu_p = 3\), and that the \(^{50}\text{Cr}\) ground state is nearly pure \(\nu_p = 0\). The \((\alpha,p)\) seniority selection rule, \(\Delta \nu_p = 1\), thus prohibits the production of these states. Two particle excitation out of the neutron "core" is probably needed to explain the \((\alpha,p)\) data above 2.5 MeV in \(^{53}\text{Mn}\).

In \(^{55}\text{Mn}\) two new states above 6 MeV were excited for which proton angular distributions were obtained. A strongly excited \(L = 3\) group at 5.498 MeV excitation dominates the \((\alpha,p)\) spectra. This group can be associated with a state measured in \((^3\text{He},d)\). However, in \((\alpha,p)\) it is excited with a factor of 28 increase in relative strength when compared to the \(7/2^-\) state at 0.128 MeV. The \(7/2^-\) strength is spread out among more states in \(^{55}\text{Mn}\) than in \(^{53}\text{Mn}\), although not all the \(7/2^-\) strength in \(^{55}\text{Mn}\) can be associated with the single particle \(7/2^-\) state. The addition of two neutrons increases the number of configurations that the \((\alpha,p)\) reaction can excite as opposed to the configurations accessible in single proton transfer. These extra degrees of freedom also allow for an explanation of the larger number of states seen in \(^{55,57}\text{Mn}\) below 2.4 MeV as compared with \(^{53}\text{Mn}\). The \(9/2^-\) and \(11/2^-\) states at 0.983 and 1.292 MeV in \(^{55}\text{Mn}\) are presumably excited through \([(\pi\nu_{3/2}^3)^{7/2}(\nu_j j_2)\nu_n]_J\) components where \(j_1, j_2 = 2p_{3/2}\) or \(1f_{5/2}\) and \(J_n = 2\) or 4. The importance of \((3p-4h)\) configurations in the low-lying
The \( \alpha \) reaction at 26 MeV also corroborates the assignment of a \( \frac{1}{2}^- \) state nearly degenerate with the \( \frac{11}{2}^- \) state at 1.292 MeV as suggested by Peterson [1971]. A \( ^{55}\text{Mn}(\alpha,\gamma)\) \( \gamma \)-\( \gamma \) coincidence measurement failed to detect a 1.292 MeV \( \gamma \)-ray in singles or in coincidence with protons exciting the assumed 1.292 MeV doublet.

In \( ^{57}\text{Mn} \), \( Q_{\alpha\gamma} \)-values for fifty-seven states were determined using both the 18 and 26 MeV data. The ground state \( Q_{\alpha\gamma} \)-value was determined to be \(-4.302 \pm 0.008 \) MeV, in agreement with a recently reported measurement by Mateja [1976]. The spin assignments made by Mateja [1976] using the \( ^{54}\text{Cr}(\alpha,\gamma)\) \( ^{57}\text{Mn} \) reaction are in agreement with those made on the basis of the proton angular distributions in \( (\alpha,p) \). The states measured in \( (\alpha,p) \) for excitation energies greater than 2.2 MeV have not been reported previously. Comparisons of the experimental and theoretical angular distributions between \( ^{53,55,57}\text{Mn} \) allow several \( J^\pi \) values to be proposed for states in \( ^{57}\text{Mn} \). The spectrum of \( ^{57}\text{Mn} \) is similar to that of \( ^{55}\text{Mn} \) in terms of numbers of states excited below 2.4 MeV. There is also a suspected \( \frac{1}{2}^- \) state low in the spectrum for which both the \( (\alpha,p) \) data and the data of Mateja lend support. However, only one \( \frac{7}{2}^- \) state was found in \( ^{57}\text{Mn} \) as opposed to four in \( ^{55}\text{Mn} \). The 26 MeV \( (\alpha,p) \) data contain 7 possible \( \frac{3}{2}^- \) states. Angular distributions that could be analyzed by DWBA were obtained for states up to 4.8 MeV although excited states in \( ^{57}\text{Mn} \) were measured up to 5.167 MeV.

The similarity of the angular distributions for all three isotopes indicates that the \( (\alpha,p) \) reaction is dominated by the region near the nuclear surface. The great variety of contributing configurations evidenced by the dramatic variation of the relative intensity of the states excited in \( (\alpha,p) \) compared to \( (^3\text{He},d) \) requires
that the nuclear interior be disadvantaged as compared to the surface. Although the form factors will probably exhibit great dissimilarity in the interior, their behaviour near the surface will be much more uniform and hence produce similarly shaped angular distributions. The influence of the configuration mixing seems to consist mainly in changing the magnitude of the cross section. The $J_T = 1/2$ transfers are readily recognized. The $J_T^e = 7/2^-$ transfers show greater similarity than the $J_T^e = 3/2^-$ transfers. The $3/2^-$ transfers in $^{53,57}$Mn are closer in shape than the $3/2^-$ transfers in $^{55}$Mn, albeit the differences are not great.

Somewhat surprising because of the results of Schiffer [1967] comparing the $^{64}\text{Zn}(d,^3\text{He})^{63}\text{Cu}$ and $^{62}\text{Zn}(p,\alpha)^{63}\text{Cu}$ reactions, is the large variation in relative intensity of states excited in ($^3\text{He},d$) compared to ($\alpha,p$). This variation occurs in both $^{53,55}$Mn. In the proton and triton pick-up to $^{63}$Cu the relative intensities of states excited shows that the neutrons are transferred in a way that has very little effect on the reaction [Schiffer, 1967]. This feature is not present in the ($\alpha,p$) reactions to the Mn isotopes because states are observed to alter their relative intensities by factors of 50 between ($^3\text{He},d$) and ($\alpha,p$). Of course, $a$ priori, one would have expected the added degrees of freedom introduced by the transferred neutrons to produce substantial effects. This assumption is borne out in the $^{50,52}\text{Cr}(\alpha,p)^{53,55}$Mn reactions.

A rather curious feature of the ($\alpha,p$) data is the lack of the suspected $J^e = 1/2^+$ or $L=2$ hole states in $^{53,55}$Mn in the experimental proton spectra. This feature is curious because these states are populated in ($^3\text{He},d$). One could explain their absence in ($\alpha,p$) by assuming that the dominant direct reaction mode is stripping instead of knock-out. However, how does one explain their production in ($^3\text{He},d$)? Invoking the isospin selection rule will be of no avail since these
states are probably \( T < \) states and not prohibited in \((\alpha, p)\) on this ground.

The simple model of a triton cluster transfer accounts very well for the shapes of the angular distributions, probably because the form factor in the nuclear interior is not of much importance. A quantitative understanding of the \((\alpha, p)\) reaction probably will proceed along a microscopic treatment of the transfer process as suggested by Falk [1975]. At present, such an involved calculation is only justifiable in the case of \(^{53}\text{Mn}\) using the wave functions of Benson [1975]. However, even this calculation probably will not be able to account for some of the \((\alpha, p)\) data because of its neglect of two particle excitations out of the \(1f_{7/2}\) shell. Adequate theoretical calculations of the structures of \(^{55,57}\text{Mn}\) still remain to be done in order to fully utilize the information available in the three nucleon transfer reaction.
APPENDIX 1

RECURSION RELATIONS FOR THE TRANSFORMATION COEFFICIENTS
FROM SPHERICAL TO CARTESIAN COORDINATES
FOR THE HARMONIC OSCILLATOR

(A) INTRODUCTION

Smirnov [1962] has discussed the coefficients \( A_{pq^s}^{nlm} \) which effect the transformation between spherical and Cartesian coordinate systems for the harmonic oscillator. These are defined by

\[
\psi_{nlm}(\nu r) = \sum_{pq^s} A_{pq^s}^{nlm} \psi_{pq^s}(x,y,z;\nu) .
\]

He has shown that the inverse transformation is

\[
\psi_{pq^s}(x,y,z;\nu) = \sum_{nlm} A_{pq^s}^{nlm*} \psi_{n^l m}(\nu r) ,
\]

where

\[
A_{pq^s}^{nlm*} = (-1)^m A_{pq^s}^{nlm} .
\]

Expressions for the harmonic oscillator functions can be found in Bertsch [1972] for example. In Cartesian coordinates

\[
\phi_p(x) = \sqrt{\frac{\nu}{\sqrt{\pi}}} \frac{\nu^2}{2^n n!} H_p^{\frac{1}{2}}(x) \exp(-\nu x^2/2) ,
\]

where

\[
H_p^{\frac{1}{2}}(x) = \sum_{i=0}^{[p/2]} \frac{2^{p-i} i^{p-i}}{2^i (2i-1)!! (\nu^{1/2} x)^{p-2i}} .
\]

In spherical coordinates
\[ \psi_{n \ell m}(\nu r) = \sqrt{\frac{3/2}{\pi^{\ell/2}}} \frac{2^{\ell-n+2}(2n+2\ell+1)!}{n!((2\ell+1)!!)^2} \frac{\nu^{\ell/2}}{\sqrt{r}} Y_{\ell}^{m}(\Omega) L^{\ell+\frac{1}{2}}_{n}(\nu r^2) \]
\[ \times \exp(-\nu r^2/2) , \quad (A1.6) \]

where
\[ L^{\ell+\frac{1}{2}}_{n}(\nu r^2) = \sum_{k=0}^{n} (-1)^{k} 2^{k} \binom{n}{k} \frac{(2\ell+1)!!}{(2\ell+2k+1)!!} (\nu r^2)^{k} . \quad (A1.7) \]

The quantities \( \nu \) and \( (\nu/2) \) are \( \nu = m\omega/\hbar \) and \( [\nu/2] = a \) where \( p = 2a \) or \( 2a+1 \).

Smirnov [1962] suggests making a direct comparison of the right and left hand sides of (A1.1) to obtain the \( A_{pq}^n \) coefficients.

Falk [1973] has obtained a closed form expression for the \( A_{pq}^n \) coefficients. It is the purpose of this appendix to demonstrate that recursion relations can be obtained in a straightforward manner for these coefficients and that in fact only the \( A_{pq}^{n\ell} \) coefficients are needed to generate the remaining coefficients. These recursion relations are original as far as the author is aware.

(B) RECURSION RELATIONS

Let \( |n \ell m\rangle \) represent the oscillator state in spherical notation, \( |px\rangle, |qy\rangle \) and \( |sz\rangle \) represent one-dimensional oscillators along the \( x, y \) and \( z \) axes with \( p, q \) and \( s \) number of excitations. The coefficients \( A_{pq}^{n\ell m} \) are so constructed that the combination of one-dimensional oscillators in (A1.1) has the same rotational properties as the three-dimensional oscillator.

The recursion relations are obtained by using the angular momentum raising (or lowering) operator,
\[ L_{+} = L_{x} + iL_{y} . \quad (A1.8) \]
The $L_+$ operator changes the magnetic quantum number by one unit (see Messiah [1970] for example),

$$L_+ |n\ell m\rangle = \sqrt{\ell(\ell+1) - m(m+1)} |n\ell m+1\rangle . \quad (A1.9)$$

The operator $L_+$ can be expressed as a combination of position and momentum operators,

$$L_+ = yP_z - zP_y + i(zP_x - xP_z) . \quad (A1.10)$$

In Dirac's treatment of the harmonic oscillator the position and momentum operators are related to annihilation and creation operators by

$$X = \sqrt{\frac{\hbar}{2m\omega}} (a^+_x + a_x) \quad (A1.11a)$$

and

$$P_x = i \sqrt{\frac{m\omega\hbar}{2}} (a^+_x - a_x) . \quad (A1.11b)$$

Consequently the raising operator is

$$L_+ = (a^+_x a^+_z - a_x a^+_z) - i(a^+_y a^+_z - a_y a^+_z) . \quad (A1.12)$$

Application of the operators $a$ and $a^+$ to the one-dimensional oscillator functions have the following result;

$$a^+ |n\rangle = \sqrt{n+1} |n+1\rangle \quad (A1.13a)$$

$$a |n\rangle = \sqrt{n} |n-1\rangle . \quad (A1.13b)$$

The recursion relations follow immediately if (A1.12) is applied to both sides of (A1.1).

$$L_+ |n\ell m\rangle = \sqrt{\ell(\ell+1) - m(m+1)} |n\ell m+1\rangle$$

$$= \sqrt{\ell(\ell+1) - m(m+1)} \sum_{\alpha\beta\gamma} A_{\alpha\beta\gamma}^{n\ell m+1} |\alpha x\rangle |\beta y\rangle |\gamma z\rangle$$

$$= \sum_{pq} A_{pq}^{n\ell m} \{ \sqrt{p(s+1)} |p-1, x\rangle |s+1, z\rangle |q y\rangle - \sqrt{s(p+1)} |p+1, x\rangle |s-1, z\rangle |q y\rangle$$

$$+ i(\sqrt{s(q+1)} |q-1, y\rangle |s+1, z\rangle |p x\rangle - \sqrt{q(s+1)} |p x\rangle |q+1, y\rangle |s-1, z\rangle) \}. \}$$
If $|\alpha\rangle|\beta\rangle|\gamma\rangle$ is integrated out we obtain
\[
A_{\alpha\beta\gamma}^{n+1} = \frac{1}{\sqrt{\ell(\ell+1) - m(m+1)}} \left\{ \sqrt{\gamma(\gamma+1)} A_{\alpha+1,\beta,\gamma-1}^{n,\gamma+1} - \sqrt{\alpha(\alpha+1)} A_{\alpha-1,\beta,\gamma+1}^{n,\gamma+1} + i\sqrt{\gamma(\gamma+1) - \alpha(\alpha+1)} A_{\alpha,\beta+1,\gamma-1}^{n,\gamma+1} - i\sqrt{\beta(\beta+1) - \gamma(\gamma+1)} A_{\alpha,\beta-1,\gamma+1}^{n,\gamma+1} \right\}. \tag{A1.14}
\]

If we use the lowering operator $L_-$ instead of $L_+$ we obtain
\[
A_{\alpha\beta\gamma}^{n-1} = \frac{1}{\sqrt{\ell(\ell+1) - m(m-1)}} \left\{ \sqrt{\alpha(\alpha+1)} A_{\alpha-1,\beta,\gamma+1}^{n,\gamma+1} - \sqrt{\gamma(\gamma+1)} A_{\alpha+1,\beta,\gamma+1}^{n,\gamma+1} + i\sqrt{\beta(\beta+1) - \gamma(\gamma+1)} A_{\alpha,\beta+1,\gamma-1}^{n,\gamma+1} - i\sqrt{\alpha(\alpha+1) - \beta(\beta+1)} A_{\alpha,\beta-1,\gamma+1}^{n,\gamma+1} \right\}. \tag{A1.15}
\]

By use of the $L_z$ operator,
\[
L_z = i(a^+_x a_x - a^+_y a_y), \tag{A1.16}
\]
on (A1.1) we can determine the relationships between the coefficients for fixed $m$. This gives
\[
ma^{n,\gamma+1}_{\alpha\beta\gamma} = i\left( \sqrt{\beta(\beta+1)} A_{\alpha+1,\beta-1,\gamma}^{n,\gamma+1} - \sqrt{\alpha(\alpha+1)} A_{\alpha-1,\beta+1,\gamma}^{n,\gamma+1} \right). \tag{A1.17}
\]

Hence, from (A1.15) we can determine all the coefficients if the first set $A_{\alpha\beta\gamma}^{n,\gamma+1}$ are known. But from (A1.17) we see that the $A_{\alpha\beta\gamma}^{n,\gamma+1}$ for fixed $m$ are not independent of each other. In fact, only the $A_{\alpha\beta\gamma}^{n,\gamma+1}$ are needed to determine the remaining $A_{\alpha\beta\gamma}^{n,\gamma+1}$. The relations (A1.15) and (A1.17) can be applied in the following manner; given $A_{\alpha\beta\gamma}^{n,\gamma+1}$, the $A_{\alpha\beta\gamma}^{n,\gamma}$ can be obtained from (A1.17)
\[
2A_{\alpha\beta\gamma}^{n,\gamma+1} = i\sqrt{\beta} A_{\alpha+1,\beta-1,\gamma}^{n,\gamma+1} \tag{A1.18a}
\]

With $A_{\alpha\beta\gamma}^{n,\gamma+1}$ determined proceed to calculate $A_{\alpha\beta\gamma}^{n,\gamma+1}$ using (A1.17),
\[
\sqrt{2\beta} A_{2,\beta-1,\gamma}^{n,\gamma+1} = \sqrt{\beta+1} A_{2,\beta+1,\gamma}^{n,\gamma+1} - i2A_{1\beta\gamma}^{n,\gamma} \tag{A1.18b}
\]

This process is carried through until the $\alpha$ index equals $2n+\ell$. In fact,
from energy conservation
\[ \alpha + \beta + \gamma = 2n + l. \]  \hfill (A1.19)

Equation (A1.15) can now be used to generate the remaining coefficients.

(C) CLOSED FORM FOR \( A_{\text{pq}s}^{n_lZ} \)

The harmonic polynomial appears in (A1.6) and is
\[ \chi^m_z = r^z \chi^m_z. \]  \hfill (A1.20)

In order to integrate \( \psi_{\text{pq}s} \) out of (A1.1) to obtain \( A_{\text{pq}s}^{n_lZ} \) we need an explicit Cartesian form for the harmonic polynomial (A1.20). Using the relations found in Edmonds [1960] it is easy to demonstrate that
\[ \chi^z_z(x) = (-1)^z \sqrt{\frac{(2l+1)!!}{4\pi(2l)!!}} (x + iy)^z. \]  \hfill (A1.21)

Using (A1.21) we are able to evaluate \( A_{\text{pq}s}^{n_lZ} \) in a straightforward manner by integrating out \( |px| |qy| |sz> \) in (A1.1). The explicit form for \( A_{\text{pq}s}^{n_lZ} \) is
\[
A_{\text{pq}s}^{n_lZ} = C \sum_{a=0,b=0,k=0,\alpha=0}^{[q/2],[s/2],n,l} \frac{q!}{2a!} \frac{s!}{2b!} \frac{n!}{k!} \frac{l!}{\alpha!} (-1)^{a+b+k} \\
\times \frac{(2a-1)!!(2b-1)!!}{(2l+2k+1)!!} (-1)^{a/2} \frac{k!}{\beta!} \frac{k-\beta}{\gamma} \sum_{\beta=0}^{k} \left( \sum_{\gamma=0}^{k-\beta} \frac{(-1)^{\gamma+2a+2k+2\gamma+2l+\alpha-1}!!}{(s-2b+2\gamma-1)!!} \right), \hfill (A1.22)
\]

where
\[ C = (-1)^{l/2} \sqrt{\frac{(2l+1)!!(2n+2l+1)!!}{2^n(2l)!!n!s!q!}}. \]  \hfill (A1.23)

The evaluation of the overlap integrals in deriving (A1.22)
places the following restrictions on the indices;

\[ s \text{ is even} , \]  
\[ \alpha \text{ is even} , \]  
\[ \text{and } 2n+\ell = q+s . \]

(A1.24a)  
(A1.24b)  
(A1.24c)

These restrictions on the indices have the consequence

that \( A_{\ell s}^{n} \) is purely imaginary if \( \ell \) is odd,  
and that \( A_{\ell s}^{n} \) is purely real if \( \ell \) is even.

(A1.25a)  
(A1.25b)  
(A1.25c)

The following general properties of the \( A_{\ell s}^{n} \) coefficients follow from (A1.25), (A1.17) and (A1.15);

\[ A_{\ell s}^{n} = 0 \text{ if } \ell - m + s \text{ is odd} , \]  
(A1.26a)

\[ A_{\ell s}^{n} \text{ is purely real if } p + m \text{ is even} , \]  
(A1.26b)

\[ A_{\ell s}^{n} \text{ is purely imaginary if } p + m \text{ is odd} , \]  
(A1.26c)

\[ A_{1s}^{0} = 0 , \]  
(A1.26d)

\[ A_{1s}^{0} = 0 , \]  
(A1.26e)

\[ \text{and } n_{\ell s}^{m} = 0 . \]  
(A1.26f)

Relations (A1.26) have been tested against the table of coefficients presented by Smirnov [1962] and are in agreement. The recursion relations (A1.15) and (A1.17) and the explicit form (A1.22) for \( A_{\ell s}^{n} \) have also been tested and found to agree with the coefficients explicitly calculated by Smirnov [1962].
APPENDIX 2
EXPERIMENTAL DETAILS

(A) A PRODUCTION TECHNIQUE FOR SELF-SUPPORTING Cr FOILS

The author is indebted to E.C. Pollacco for demonstrating a method of producing self-supporting Cr foils of thickness greater than about 50 \( \mu g/cm^2 \) and for supplying the \(^{52}\)Cr targets used at 26 MeV. The self-supporting \(^{50,54}\)Cr targets were prepared by the author using the technique described below which has been found to be very reliable.

Equipment list:

- (1) chloroethene
- (2) alcohol
- (3) \( H_2O \)
- (4) trichloroacetic acid
- (5) \( NH_3 \) solution
- (6) smooth copper foil about 12 mg/cm\(^2\)
- (7) scalpel
- (8) optical pyrometer
- (9) apparatus for heating Cu foil

Successful results were obtained at temperatures below the Cr melting point. In fact, most of the target was deposited via sublimation of the Cr from a tantalum crucible of 1 cm diameter at 6 cm from the supporting Cu foil. The Cu foil should be pre-heated before the Cr is sublimed.

Procedure

(1) Select a wrinkle-free Cu foil and clean in chloroethene bath.
(2) Alternately wash the foil in alcohol, H₂O, dilute HNO₃, H₂O and alcohol. Leave foil in alcohol until it is ready to be placed in the evaporator.

(3) Spread a charge of Cr₂O₃ as evenly as possible over the surface of the tantalum boat. For the experiments described in this thesis 10 mg of oxide were normally used. This produced Cr layers of 50 to 75 µg/cm².

(4) The Cu foil should be suspended tautly about 6 cm from the boat.

(5) Preheat Cu foil to 240 °C.

(6) Slowly heat crucible to reduce Cr₂O₃ to Cr. The material will change colour from green to black.

(7) The crucible should now be heated accordingly.

<table>
<thead>
<tr>
<th>Time (min)</th>
<th>Temperature</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>1260 °C</td>
</tr>
<tr>
<td>5</td>
<td>1350 °C</td>
</tr>
<tr>
<td>20</td>
<td>1420 °C</td>
</tr>
<tr>
<td>30</td>
<td>1480 °C</td>
</tr>
</tbody>
</table>

These temperatures are those measured with an optical pyrometer with a tungsten element. For true temperatures the emissivity of the W must be considered. The Cu foil will probably sag at some stage during the heating.

(8) Allow the Cu foil to cool before cutting it with the scalpel into the desired sizes. The foil may be conveniently cut on blotting paper using glass slides as guiding edges. The Cr will not be damaged by uniform pressure of the glass slide.

(9) Turn up one corner of the small Cu plus Cr segments to facilitate
handling before dropping the small foils (with Cu face in contact with etching solution) into the etching solution. Self-supporting Cr foils that can be suspended across 9 mm diameter target frames can be made.

(10) Gently drop the small foils into the Cu etching solution so that the foils float on the surface. The following etching solution has been used:

- 30 g trichloroacetic acid
- 800 ml H₂O
- 60 ml NH₃ solution

The NH₃ will have to be added with time. This solution should be kept in a fume cupboard.

(11) When the Cu has dissolved, the Cr foil can be picked up on a glass slide. If a bubble of air was trapped under the segment the portion of Cu in contact with the bubble will not have been dissolved. By gentle manipulation with the glass slide the bubble can generally be slipped out from beneath the foil and the foil returned to the etching solution to remove the Cu spot.

(12) When the Cr foil is clear of Cu, the foil can be transferred to a large beaker of H₂O to allow the etching solution to be washed off. If a single layer target is being prepared the foil can be removed on a target frame. If thicker targets are desired the foil can be picked up on another Cr plus Cu segment. This double thickness foil can then be returned to the etching solution. The layering process can be continued up to the desired thickness. On rare occasions the layers will peel off from one another. Starting with 50 μg/cm² thick Cr, 200 μg/cm² thick targets were prepared by
layering the Cr three times.

(B) TARGET ANGLE EFFECTS ON RESOLUTION

Of the several factors that determine the experimental energy resolution there is one which the experimenter has under ready control. Cohen [1959] has discussed the fact that the target angle with respect to the beam can be adjusted to eliminate the energy spread due to the finite thickness of the target. This is to be distinguished from straggling which can not be eliminated. The pertinent quantities are illustrated in Fig. A2.1. Here

\[ \Delta E(y) = Q_a \frac{y}{\cos \alpha} + Q_b \frac{t - y}{\cos \gamma}. \]  

FIG. A2.1

The energy loss of particle b due to the reaction occurring at point y is

Because the reaction can occur at any depth there will be, for
arbitrary angle $\alpha$, a spread in the energy of $b$. However, it is easy to
demonstrate that $\Delta E$ can be made independent of $y$ and thus eliminate this
source of energy broadening. Writing (A2B.1) in the form

$$\Delta E(y) = \left[ \frac{Q_a}{\cos \alpha} - \frac{Q_b}{\cos (\theta - \alpha)} \right] y + \frac{Q_b}{\cos (\theta - \alpha)} t ,$$  \hspace{1cm} (A2B.2)

all particles will lose the same amount of energy, independent of the
reaction location, if the bracketed term in (A2B.2) is set equal to zero.
The target angle that eliminates this contribution to the broadening is

$$\alpha = \tan^{-1} \left( \frac{Q_b/Q_a - \cos \theta}{\sin \theta} \right) .$$  \hspace{1cm} (A2B.3)

Because the beam has a finite width the extra angular spread
due to the finite beam spot will cause kinematic broadening beyond that
due to the defining slit in front of the detector. The angular spread
$\Delta \phi$ is defined in (A2B.5) in Fig. A2.2 where

- $S$ is the slit size,
- $\alpha$ is the target angle,
- $R_d$ is the target to detector distance,
- $\Delta \phi$ is the angular spread of a finite beam,
- $b$ is the beam width,
- $\theta$ is the detector angle,

and

$$R = \sqrt{R_d^2 + S^2/4}$$

$$\beta_{2,1} = \theta (+, -) S/2R .$$

The angles $\theta_2$ and $\theta_1$ which are the extreme backward and forward
acceptance angles due to the finite beam spot size are given in (A2B.4).

The target thickness spread and the kinematics both contribute
to the resolution linearly, not in quadrature. This is to be expected
\[ \theta_{2,1} = \tan^{-1} \left( \frac{X}{Y} \right) = \tan^{-1} \left( \frac{R \sin \beta_{2,1}(+,-) b/2}{R \cos \beta_{2,1}(-,+)(b/2) \tan \alpha} \right) \] (A2B.4)

\[ \Delta \phi = \theta_2 - \theta_1 . \] (A2B.5)

**ANGULAR SPREAD FOR FINITE BEAM**

**FIG·A2·2**
because neither of the effects are statistical. The contribution of both these effects to the resolution can be expressed by

$$\Delta E(\theta,\alpha) = \Delta \phi \frac{dE}{d\theta} + t \left| \frac{Q_a}{\cos \alpha} - \frac{Q_b}{\cos(\theta - \alpha)} \right|. \quad (A2B.6)$$

Note that the absolute value of the second quantity in (A2B.6) is used.

For the (\(\alpha,p\)) reaction measurements the angles \(\alpha\) were selected for each particular run so that the spread described by (A2B.6) was minimized for all the detector telescopes used in the scattering chamber. Since targets are seldom free from wrinkles it should be understood that (A2B.6) can only be regarded as an intelligent estimate of the best target angle.

(C) NORMALIZATION AND ABSOLUTE CROSS SECTIONS

(1) Technique

The differential cross sections for the \(^{50,52,54}\text{Cr}(\alpha,p)^{53,55,57}\text{Mn}\) reactions at 18 and 26 MeV were determined by measuring them relative to a known cross section. This is a standard method and requires the use of a monitor detector to observe the known reaction simultaneously during the measurement. For the measurements described in this thesis the monitor was set at 45° and counted both the elastic events and the inelastic scattering events to the first 2\(^{+}\) state of the target. Briefly, the determination of the differential cross section requires two simultaneous measurements of \(N_p\) and \(N_\alpha\) where

$$N_p(\theta) = k \sigma_{p\alpha}(\theta) \, d\Omega_T \quad (A2C.1)$$

$$N_\alpha(45^\circ) = k \sigma_{\alpha\alpha}(45^\circ) \, d\Omega_M. \quad (A2C.2)$$

Here
k is a constant depending upon target thickness and the number of particles/cm² that passed through the target,

\( N_p(\theta) \) is the number of protons in a particular group,

\( N_\alpha(45°) \) is the number of \( \alpha \)-particles detected in the monitor during the measurement,

\( d\Omega_T \) is the solid angle of the proton detector defining slit and

\( d\Omega_M \) is the solid angle of the monitor defining slit.

The differential cross section \( \sigma_{\alpha p}(\theta) \) is then given by

\[
\frac{\sigma_{\alpha p}(\theta)}{\sigma_{\alpha\alpha}(45°)} = \left( \frac{d\Omega_M}{d\Omega_T} \right) \left( \frac{N_p(\theta)}{N_\alpha(45°)} \right).
\] (A2C.3)

The normalization \( d\Omega_M/d\Omega_T \) can be determined by simultaneously measuring the elastic scattering at the same angle in both the detector of interest and the monitor.

For the 26 MeV data the elastic scattering was measured simultaneously in each detector with the reaction. Hence it was possible for each detector to monitor itself via the elastic or inelastic scattering. This method has definite advantages over the use of an independent monitor but care must be taken at certain angles (see the discussion in the next section on errors).

The absolute cross sections for the elastic scattering at 18 MeV were determined from a comparison of the relative yield at angles between 20° and 30° in 2.5° steps to the yield at the same angles measured at 9 MeV. The scattering at 9 MeV is Rutherford scattering for angles less than about 35° LAB. This was determined by plotting the number of counts divided by the Rutherford cross section as a function of angle. The absolute cross sections for the 45° elastic scattering along with the r.m.s. deviations are
for $^{50}\text{Cr}$, $\sigma_{\alpha\alpha}(45^\circ) = 123 \pm 6 \text{ mb/sr}$,

for $^{52}\text{Cr}$, $\sigma_{\alpha\alpha}(45^\circ) = 125 \pm 6 \text{ mb/sr}$,

and for $^{54}\text{Cr}$, $\sigma_{\alpha\alpha}(45^\circ) = 120 \pm 10 \text{ mb/sr}$.

Because the 14 UD can not be operated readily with 3 MV on terminal, the elastic scattering cross sections at 26 MeV had to be measured relative to the 18 MeV cross sections. For $^{50}\text{Cr}$ and $^{52}\text{Cr}$ a set of optical model parameters also exist for the elastic scattering of 25 MeV $\alpha$-particles. These two sets of parameters were obtained by Lemos [1972] and can be found in the Perey and Perey [1974] compilation. The consistency of the elastic scattering cross sections can be seen in Tables A2C.1 and A2C.2. The first two columns are the 26 MeV lab cross sections determined from a yield comparison at 25 MeV. The 25 MeV cross sections were determined by calculations using two parameter sets, $x$ and $y$, or 8 and 9, for $^{50}\text{Cr}$ or $^{52}\text{Cr}$ respectively. The last column gives the 26 MeV cross section as determined from a yield comparison with the previously measured 18 MeV cross sections. From both the 18 and 25 MeV measurements the 26 MeV cross sections were deduced from the relative yields using the charge integrator.

The 25 MeV comparison was made as a check on the consistency of the results. The actual determination of the 26 MeV elastic scattering cross sections was based solely on the yield comparison with the 18 MeV scattering which had been measured absolutely. The yields at 12 angles between 20° and 47.5° in 2.5° steps were measured at 26 and 18 MeV. This gave an absolute cross section measurement at 12 separate points. Since the relative angular distribution can be measured, these twelve points implied a certain value for the 45° cross section. The 45° elastic scattering lab cross sections at 26 MeV along with their r.m.s. deviation for the 12 angles are:
Table A2C.1
The 26 MeV $^{50}$Cr($\alpha$,\(\alpha_0\)) Lab Differential Cross Sections (mb/sr) from a Comparison of Relative Yields at 26, 25 and 18 MeV

<table>
<thead>
<tr>
<th>$\theta_{\text{LAB}}$</th>
<th>25 MeV Comparison</th>
<th>18 MeV Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{\text{calc}}$ Set X</td>
<td>$\sigma_{\text{calc}}$ Set Y</td>
</tr>
<tr>
<td>20</td>
<td>2535</td>
<td>2561</td>
</tr>
<tr>
<td>25</td>
<td>712</td>
<td>727</td>
</tr>
<tr>
<td>30</td>
<td>187</td>
<td>197</td>
</tr>
<tr>
<td>35</td>
<td>113</td>
<td>118</td>
</tr>
</tbody>
</table>

Table A2C.2
The 26 MeV $^{52}$Cr($\alpha$,\(\alpha_0\)) Lab Differential Cross Sections (mb/sr) from a Comparison of Relative Yields at 26, 25 and 18 MeV

<table>
<thead>
<tr>
<th>$\theta_{\text{LAB}}$</th>
<th>25 MeV Comparison</th>
<th>18 MeV Comparison</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_{\text{calc}}$ Set 8</td>
<td>$\sigma_{\text{calc}}$ Set 9</td>
</tr>
<tr>
<td>20</td>
<td>2558</td>
<td>2571</td>
</tr>
<tr>
<td>25</td>
<td>787</td>
<td>775</td>
</tr>
<tr>
<td>30</td>
<td>191</td>
<td>191</td>
</tr>
<tr>
<td>35</td>
<td>123</td>
<td>126</td>
</tr>
</tbody>
</table>

for $^{50}$Cr, $\sigma_{\alpha\alpha_0}(45^\circ) = 23.9 \pm 0.9$ mb/sr,
for $^{52}$Cr, $\sigma_{\alpha\alpha_0}(45^\circ) = 24.6 \pm 1.0$ mb/sr,
and for $^{54}$Cr, $\sigma_{\alpha\alpha_0}(45^\circ) = 24.9 \pm 1.1$ mb sr.

(2) Errors
The use of a monitor for normalization purposes over several independent runs requires that the monitor position be accurately
reproduced each time. At 26 MeV the Cr(α,α₀) reaction exhibits a maximum at 45° in the lab and here the cross section varies slowly around the peak. The monitor was kept at 45° for all runs, and thus any uncertainty in the monitor position would be compensated for by the slowly varying cross section. The accuracy of the absolute angle scale on the scattering chamber has been measured at best as 0.1° [Ophel, ANU-P/502]; however typically an accuracy of 0.2° has been observed. Such errors are negligible for the reaction cross sections because these change approximately by only a factor of 10 from 20° to 90°. However the elastic scattering cross sections vary extremely rapidly, by about a factor of 10⁴ in the same angular range.

The errors stated for the elastic scattering contain only the relative normalization error of 4% plus the statistical error. An estimate of the largest error expected for a point can be obtained by multiplying the gradient of the angular distribution by Δθ ≈ 0.2°. The minimum in the 26 MeV elastic cross section measured at 70° lab for all three even chromium isotopes has a much larger possible error than quoted in Fig. 3.17. From 67.5° to 70° the ⁵²Cr(α,α₀) cross section decreases by a factor of 28 so that a 0.2° uncertainty at 70° can produce an error of approximately 230% in the measurement. The cross section measurements at 70° lab have been included in fact simply to illustrate the extreme nature of the minimum. The measurement of a minimum will be, in any case, an overestimate due to the finite acceptance angle of the detector slits and beam spot. For the elastic scattering measurements the combination of slit size and beam spot produced an acceptance angle of about 0.6° at most. For typical reaction cross section measurements the telescopes had acceptance angles of 1.2° and 1.8°.
The absolute cross section scale is believed to be accurate to about 7%. This was determined by the consistency of the 18 MeV elastic angular distribution. In fact, all the errors quoted for the reaction cross sections and the energy calibrations were determined on the basis of reproducibility of the results. This criterion alone does not eliminate the possibility of systematic errors. However, the use of the Lemos [1972] α-particle optical model parameters to calculate the 25 MeV (α,α₀) cross sections is quite reasonable for θ ≤ 40° because these parameter sets were obtained by fitting 25 MeV data. Hence, there is an independent check, i.e. the calculated cross sections due to Lemos [1972], on the 26 MeV and 18 MeV data from the present work. As can be seen in Tables AC2.1 and AC2.2 the agreement between the 25 and 18 MeV yield comparisons is good and therefore the systematic errors in the present work are believed to be small. For the energy calibration the procedure used, described in Chapter 3, has also reduced the possibility of systematic error.

The quoted errors are root mean square deviations. The possible sources of error in placing a point in the angular distributions are:

1. the statistical error,
2. the fitting program including background determination,
3. impurities or tailing of the monitor peak which have caused fluctuations of up to 2% in the monitor peak area, and
4. normalization between telescopes.

It is not practical to take all these factors into account individually for each point in the reaction angular distribution because there are approximately 1600 points for the 18 MeV data and 1200 points
for the 26 MeV data. The fitting program used, AUTOFIT [Spink, 1965], produces a chi-squared value for a group of fitted peaks. However, chi-squared is not always a good measure of the error in the intensity of the component of an unresolved group [Comfort, 1971, or Ophel, ANU-P/1587(3)]. The only error readily available is the statistical error which can be expected to be an underestimate of the true standard deviation. To obtain the true standard deviation $\sigma_t$, one would need to measure a single point many times. This is clearly impractical.

However, one can envisage a "typical" peak in the spectra, and for this typical peak the distribution of measured points around the true point is probably gaussian. If there were $N_0$ points measured and the difference from the true cross section, call it $x$, is measured in units of the true standard deviation $\sigma_t$, then the number of points a distance $x/\sigma$ away from the true point is expected to be

$$N(x/\sigma) = N_0 \sqrt{2/\pi} \int_{x/\sigma}^{\infty} dy \, e^{-y^2/2} . \quad (A2C.4)$$

The procedure used to estimate the true $\sigma_t$ for the points in the angular distribution is as follows. Imagine all the peaks in all the spectra to be the "typical" peak. The typical peak has therefore been measured 1600 times in the case of the 18 MeV data. We can estimate the distance $x$ from the true cross section by drawing smooth curves through the points. As a trial $\sigma_t$ let us use the statistical error, $\sigma_s$. The ratio $x/\sigma_s$ can thus be obtained for the hypothetical typical peak 1600 times. If the distribution of the ratios is plotted we should expect to find the number of points for which $x/\sigma$ exceeds a given value given by (A2C.4). Since $\sigma_s$ is in fact an underestimate, the distribution of ratios $x/\sigma_s$ will not follow (A2C.4). We can increase $\sigma_s$ by some constant $k$, say
until we find a value of k such that the distribution of \( x/\sigma_t \) most closely approximates the expected statistical distribution of (A2C.4).

The question as to what is a typical peak is the point of contention. Most of the peaks analyzed were members of multiplets at 18 MeV and hence any peak is really typical. The error for isolated peaks will probably be overestimated and for weakly excited groups in multiplets the error will be underestimated. Since we are not trying to determine 1600 individual errors this simplification is reasonable. Thus what has been done is to determine the reliability of the data as a whole with all the errors external to the statistical one approximated by the constant k. The individual peak errors enter only through the statistical error. This type of analysis has been carried through on the 18 MeV data. The results are:

\[
\begin{align*}
\text{k = 2} & \quad \text{for the } ^{54}\text{Cr}(\alpha,p)^{57}\text{Mn spectra ,} \\
\text{k = 1.9} & \quad \text{for the } ^{52}\text{Cr}(\alpha,p)^{55}\text{Mn spectra ,} \\
\text{and k = 2.3} & \quad \text{for the } ^{50}\text{Cr}(\alpha,p)^{53}\text{Mn spectra .}
\end{align*}
\]

These values of k are consistent with the notion of the typical peak because the spectra from the three Mn isotopes are similar in regards to density of states and statistics. The value k = 2 has been adopted. Hence, all values of error quoted in this thesis with regards to reaction cross section are determined by increasing the statistical error by a factor of 2.

This same factor of 2 was employed in the 26 MeV data although the error analysis was not repeated. Fewer states were excited at 26 MeV, but the energy resolution at 26 MeV was not as good as at 18 MeV, i.e., 50 keV compared with 32 keV. Hence, this estimate of the
standard deviation was considered reasonable for 26 MeV as well.

(3) Spectrum Reconstruction after a Base Line Shift

Drifting of gain or base line in the course of a long run can obliterate the spectra collected. However in the case of a base line shift it is a simple matter to reconstruct the spectra, provided that the fraction of the shift is known. This can be demonstrated by considering the unshifted spectrum $F_0(x)$ where the symbols represent counts (F) and position (x). If the base line has shifted by an amount c and $a$ is the intensity of the shifted spectrum, then the measured spectrum $F_m(x)$ is

$$ F_m(x) = F_0(x) + aF_0(x - c) , \quad (A2D.1) $$

or

$$ F_0(x) = F_m(x) - aF_0(x - c) . \quad (A2D.2) $$

From (A2D.2) we conclude that

$$ F_0(x - c) = F_m(x - c) - aF_0(x - 2c) . \quad (A2D.3) $$

If (A2D.3) is substituted into (A2D.2)

$$ F_0(x) = F_m(x) - aF_m(x - c) + a^2F_0(x - 2c) . \quad (A2D.4) $$

This iteration process can be continued as long as necessary to make the coefficient of the unknown contribution $F_0(x - nc)$ as small as desired. The general expression then becomes

$$ F_0(x) = \sum_{k=1}^{n-1} (-a)^k F_m(x - kc) + (-a)^nF_0(x - nc) . \quad (A2D.5) $$

When $a^n$ is small enough, $F_0(x)$ can be reconstructed from the measured spectrum. This is easy to accomplish with a computer. The method breaks down if $a = 1$. 
A baseline shift occurred during one of the runs and the spectra were reconstructed using (A2D.5). The reconstructed spectra were found to be in agreement with the subsequently remeasured spectra.

(D) DETECTORS AND ELECTRONICS

The basic experimental configuration for the $(\alpha,p)$ measurements consisted of two or three detector telescope packs in the 50 cm diameter scattering chambers. A collimated beam of 18 or 26 MeV alpha particles impinged on chromium targets whose thicknesses varied between 50 and 200 $\mu$g/cm$^2$. The thicker targets, i.e. 120 to 200 $\mu$g/cm$^2$ were used at 26 MeV. The collimation system consisted of four consecutive circular tantalum apertures of diameters 1.5 mm, 2.3 mm, 1.5 mm and 2.3 mm. The first aperture (1.5 mm) was 47.5 cm from the target.

Reaction products were detected in Si surface barrier detector telescope packs consisting of 2 or 3 detectors depending upon the beam energy and $Q_{\alpha p}$ values. The defining slits in front of the first detector (i.e., $\Delta E$) were 1 mm, 1.5 mm or 2 mm wide. These defining slits produced acceptance angles between 1.2° and 1.8° when folded in, as described in appendix A2.B, with the finite beam width of 1.5 mm.

The Si surface barrier detectors were manufactured in the laboratory. For the reaction measurements they were cooled using the freon cooling system available in the scattering chamber. A description of the scattering chamber and the detector mounting blocks can be found in Ophel [ANU-P/502], Borsaru [1974] and Barbopoulos [1976].

Explicit particle identification was not required because the range of all particles produced in the reaction, other than for protons,
fell within the $\Delta E$ detector. Coincident events in the $\Delta E$ and $E$ detectors were used to define the protons. When three detector telescopes were employed for the $^{50}$Cr($\alpha$,p)$^{53}$Mn reaction at 26 MeV, coincident events in $\Delta E$ and the second detector produced a gating pulse which allowed through any events occurring in the third detector. Hence, the only protons that were not detected were those whose range was entirely within the $\Delta E$ detector. At 18 MeV the $\Delta E$ detector was typically 500 $\mu$m thick and at 26 MeV the $\Delta E$ detector was anywhere between 1000 and 1500 $\mu$m thick, depending upon the availability of detectors at the time of the run.

A diagram of the basic electronic configuration can be found in Fig. A2.3. The left hand wing was used to collect the elastic scattering simultaneously with the reaction. For three detector operation another wing on the right hand side is added for which gating pulses are obtained from coincident events in the first two detectors.
1 — Si surface barrier detectors
2 — Preamp ORTEC 125
3 — Linear amp TENNELEC TC203BLR
4 — Timing single channel analyzer CANBERRA 1437
5 — Slow coincidence CANBERRA 1446
6 — Logic shaper and delay CANBERRA 1455A To ADC
7 — Linear gate and stretcher CANBERRA 1454
8 — Sum amp CANBERRA 1465A
9 — Biased amp ORTEC 408A

SCHEMATIC OF BASIC ELECTRONICS

FIG·A2·3
REFERENCES


C.M. Bartle, Research Meeting, Dept. of Nuclear Physics, A.N.U., August 1975.


J.R. Comfort, A.N.L. Physics division informal report PHY-1970B and
Additions and Changes to the Manual of Data-Analysis Programs for


1963.


DWUCK, Computer Code by P.D. Kunz (1969), University of Colorado, Spin-
Orbit Version.


F.L. Friedmand, V.F. Weisskopf, *Niels Bohr and the Development of


65.


HAUSER(DAL), Computer Code by P.J. Dallimore, ANU-P/512.

J.O.V. Hellström, *The Compound Nucleus Process in Reaction Mechanism and
University (unpublished).


(1973) 364.


Z.P. Sawa, Phys. Scr. 7 (1973) 5.


