ISOBARIC ANALOGUE RESONANCE STUDIES IN SOME RARE-EARTH NUCLEI

by

Nolan Howard Merrill

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PREFACE

This thesis relates to work which was carried out in the Department of Nuclear Physics at the Australian National University under the supervision of Dr. S. Whineray.

The theme of the work is a study of isobaric analogue resonances in heavy nuclei of the rare-earth region. After an introductory chapter describing the physical concepts associated with the analogue resonance phenomenon, the thesis is divided into two parts: part A which describes cross-section measurements of analogue resonances in elastic and inelastic proton scattering and part B which covers beam optics calculations which were performed in preparation for the use of the ANU polarised ion source for further analogue resonance studies. Unfortunately, experimental difficulties with the source have prevented any polarisation measurements through analogue resonances being carried out.

The subject of the work was suggested by Dr. Whineray with whom the experiments in part A were carried out. Additional assistance during the data collection was provided by Drs. D.C. Weisser and W.M. Zuk. All the data reduction and analysis was performed by the author with advice from Dr. Whineray. Most of the computer programmes used in the analysis were written by the author except for Dr. H.J. Hay's FITTEM search routine, Dr. T.R. Ophel's INCRED and the single-particle widths programme which was provided in Algol by Dr. F. Folkmann of the Niels Bohr Institute. The necessary back-up techniques such as the target preparation and detector redrift process were adapted and carried out by the author. The beam optics calculations in part B were performed by the author with supervision by Dr. Whineray.

The work described in part B has appeared in the publication:
"Acceptance Calculations for the Australian National University
Model EN (HVEC) Tandem Accelerator", N.H. Merrill and S. Whineray,
Nucl. Instr. and Meth. 91 (1971) 613.

The redrift technique used for the rejuvenation of Si(Li)
particle detectors has been described in:

"Redrift Treatment of Radiation-Damaged Si(Li) Detectors",
N.H. Merrill, D. Thomas and S. Whineray, ANU Report ANU-P/541
(1971).

No part of the work has been submitted for a degree at any
other university.

N.H. Merrill

N.H. Merrill

Canberra,

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ABSTRACT

Isobaric analogue resonances have been observed in the elastic and inelastic proton scattering from rare-earth nuclei of ^{160}Gd , ^{162}Dy , ^{164}Dy , ^{166}Er , ^{168}Er , ^{170}Er and ^{174}Yb . From the elastic scattering cross-section excitation functions, it has been possible to assign orbital angular momenta to 21 resonances, thus providing a useful check on the Nilsson assignments in existence for the parent states. In the case of $^{162}\text{Dy}(p,p_0)$, the observed analogue states have shown up inadequacies in the Nilsson assignments in ^{163}Dy based on $^{162}\text{Dy}(d,p)$ measurements. In all seven cases, the Coulomb displacement energy for the parent nucleus has been found to exhibit the systematic lowering, relative to that of a spherical nucleus, which arises from the deformation of the nuclear charge distribution. From the partial proton widths fitted to the resonance data, the $C_{j\ell}^2$ values for the $\frac{3}{2} \frac{1}{2}^-$ (510), $\frac{1}{2} \frac{1}{2}^-$ (521) and $\frac{7}{2} \frac{5}{2}^-$ (512) states have been obtained in several nuclei. The $C_{j\ell}^2$ values derived for the $\frac{3}{2} \frac{1}{2}^-$ (510) state in the erbium and dysprosium nuclei are about 25% of the Nilsson model estimate in accordance with the strong particle-vibration interaction which was evident in experimental (d,p) cross-sections to this state.

In preparation for polarisation measurements through isobaric analogue resonances, the phase-space acceptance of the Australian National University EN tandem accelerator has been calculated to facilitate the matching of the recently acquired polarised ion source onto the machine.

CHAPTER 1

INTRODUCTION

Within a year of Chadwick's discovery of the neutron [Ch32], Heisenberg introduced the concept of isospin (or isobaric spin) to specify whether a nucleon was a proton or a neutron [He32]. The original idea arose from the similarity between the two particles; although they have different charge and magnetic moment, their masses differ by only $\sim 0.14\%$ and both have an intrinsic spin of $\frac{1}{2}$. Heisenberg suggested that the proton and neutron could be regarded as different states of the same entity, the nucleon, and he went on to describe this concept with a mathematical formalism analogous to that developed by Pauli for the description of the spin properties of spin- $\frac{1}{2}$ particles [Fe65b]. At that time the isospin formalism was simply a labelling device for distinguishing between neutrons and protons.

The full usefulness of the concept of isospin was realised only after the analysis of the binding energies of ${}^3\text{H}$ and ${}^3\text{He}$, together with the results of p-p and n-p scattering experiments, led Breit et al. [Br36] to hypothesise that nuclear forces are charge independent, that is, the n-n, p-p and n-p interactions are equal after electromagnetic effects, the most important of which is the Coulomb interaction, are removed. Cassen and Condon [Ca36] extended the isospin formalism to many-nucleon systems and Wigner [Wi37] subsequently pointed out the usefulness of isospin in the description of isobaric multiplets. Wigner thereby established isospin as a useful quantum number for the description of nuclear states. Shortly afterwards, Oppenheimer and Serber [Op38], in a discussion of the small decay width of a resonant state excited in ${}^{12}\text{C}$ by ${}^{11}\text{B} + \text{p}$, concluded that the state was of a

different isospin from the lower-lying levels. This was the very first recognition of what was later to be called an isobaric analogue resonance.

Isospin to some extent then "lost its way" until the 1950s when accelerator developments, together with improved techniques for determining spins and parities, enabled comparisons to be made of energy levels in isobars; many $T = \frac{1}{2}$ and $T = 1$ multiplets were identified in light nuclei ($A \leq 25$). At this time it was widely assumed that similar multiplets would not exist in heavy nuclei because the viability of isospin as a good quantum number was thought to depend upon the Coulomb energy being small compared with the nuclear energy. However, the discovery in the early 1960s of isobaric analogue states, both as final [An61] and resonant states [Fo64] in proton-induced reactions on heavy nuclei, produced a radical change of thinking and a resurgence of interest. Isobaric analogue states have now been observed and used as a spectroscopic tool in all regions of the mass table. Such is the present importance of isospin in nuclear structure physics that two international conferences have been held on the subject within the last six years [Fo66, An69].

The approach adopted in the work described here has been one of locating and using analogue resonances as a means of obtaining spectroscopic information rather than an attempt to further the understanding of the analogue resonance phenomenon itself. It will therefore be useful, before considering the work in detail, to give a brief description of the more relevant concepts involved.

1.1 ISOSPIN FORMALISM

The nucleon is defined to have an isospin vector $\underline{t} = \frac{1}{2}$ lying parallel or anti-parallel to the z axis of an abstract three-dimensional

charge space. The binary nature of the nucleon is then associated with the two projections of its isospin vector along the z axis and, by the most commonly accepted convention in low-energy nuclear physics, the state with $t_z = +\frac{1}{2}$ is the neutron while that with $t_z = -\frac{1}{2}$ is the proton. In complete analogy with the Pauli formalism for spin- $\frac{1}{2}$ particles, it is then possible to construct an isospin operator for the nucleon with the component matrix operators,

$$\hat{t}_x = \begin{pmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & 0 \end{pmatrix}, \quad \hat{t}_y = \begin{pmatrix} 0 & -\frac{1}{2}i \\ \frac{1}{2}i & 0 \end{pmatrix} \quad \text{and} \quad \hat{t}_z = \begin{pmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{pmatrix}. \quad 1.1$$

Then, the spinor $\begin{pmatrix} 1 \\ 0 \end{pmatrix}_n$ represents the neutron and $\begin{pmatrix} 0 \\ 1 \end{pmatrix}_p$ the proton. Ladder operators can also be constructed which either raise or lower the t_z value of a nucleon. These are

$$\hat{t}^{\pm} = \hat{t}_x \pm i\hat{t}_y, \quad 1.2$$

so that

$$\begin{aligned} \hat{t}^+ \begin{pmatrix} 1 \\ 0 \end{pmatrix}_n &= 0, & \hat{t}^+ \begin{pmatrix} 0 \\ 1 \end{pmatrix}_p &= \begin{pmatrix} 1 \\ 0 \end{pmatrix}_n, \\ \hat{t}^- \begin{pmatrix} 0 \\ 1 \end{pmatrix}_p &= 0 \quad \text{and} \quad \hat{t}^- \begin{pmatrix} 1 \\ 0 \end{pmatrix}_n &= \begin{pmatrix} 0 \\ 1 \end{pmatrix}_p. \end{aligned} \quad 1.3$$

For a system consisting of N neutrons and Z protons, the total isospin vector and its z component are constructed by the usual angular momentum coupling laws,

$$\underline{T} = \sum_{i=1}^{N+Z} \underline{t}(i) \quad 1.4$$

and

$$T_z = \sum_{i=1}^{N+Z} t_z(i) = \frac{N - Z}{2}. \quad 1.5$$

The simultaneous eigenvalues of $\hat{T}^2$ and $\hat{T}_z$ are $T(T+1)$ and $-T \leq T_z \leq T$.

Thus, for a particular value of T , there are $2T+1$ possible values of T_z .

corresponding to the $2T+1$ member nuclei of an isobaric multiplet.

These two additional quantum numbers then serve to label isospin states which can be written as $|T, T_z\rangle$. The ladder operators for a system of nucleons are

$$\hat{T}^{\pm} = \hat{T}_x \pm i\hat{T}_y \quad 1.6$$

and these connect members of an isobaric multiplet which differ in T_z by ± 1 , since

$$\hat{T}^{\pm} |T, T_z\rangle = \sqrt{(T \mp T_z)(T \pm T_z + 1)} |T, T_z \pm 1\rangle. \quad 1.7$$

1.2 ISOBARIC ANALOGUE STATES

1.2.1 The Purity of Isospin

The validity of T and T_z as quantum numbers depends upon the commutation relations between the total nuclear Hamiltonian and the isospin operators $\hat{T}^2$ and $\hat{T}_z$ [Me61]. T_z is clearly a good quantum number since charge is conserved in nuclear interactions. However, $\hat{T}^2$ will only commute with the Hamiltonian if the forces between nucleons are charge independent. This independence is known to be violated by the Coulomb (and magnetic) forces which can mix, into a state of isospin T , components of states with different T values. Fortunately, as electromagnetic forces are relatively weak compared with hadronic forces, these admixtures are relatively small and the description of a state in terms of T remains useful. The Coulomb effects do, however, manifest themselves in an energy splitting between adjacent members of a T -multiplet. Similarly, the difference in mass between the neutron and the proton also produces a charge dependent term in the Hamiltonian which will not commute with $\hat{T}^2$. This is also small and produces only a small perturbation in the energies of the members of a T -multiplet.

There are in addition some very weak charge-dependent effects in the strong hadronic forces which arise mainly from the mass difference between neutral and charged mesons.

1.2.2 Isobaric Multiplets

As mentioned above, the charge-dependent effects, while not strong enough to completely obliterate the regularities between multiplet members expected from the charge-independent part of the Hamiltonian, do give rise to an energy splitting of the levels of a multiplet in neighbouring isobaric nuclei. The energy splitting Δ_{T_z, T_z-1} between two members of a multiplet is given by

$$\Delta_{T_z, T_z-1} = \Delta E_c - \Delta m_{pn} , \quad 1.8$$

where ΔE_c is the Coulomb displacement energy between the two nuclei concerned and Δm_{pn} is the proton-neutron mass difference (in energy units). When this splitting is allowed for in the energy-level diagrams of light nuclei, the remarkable isobaric multiplets become very obvious [Bu63].

1.2.3 Isobaric Analogue States in Heavy Nuclei

In heavier nuclei the Coulomb potential becomes very strong with values of the order of 20 MeV for the heaviest nuclei. Thus, for a long time it was expected that the mixing of states of different T values would be such as to render T useless as a quantum number in heavy nuclei and, consequently, isobaric analogue states would not be observed. However, this assumption was shown to be incorrect following the discovery by Anderson and Wong [An61] of well-defined isobaric analogue states in medium-mass nuclei.

Lane and Soper [La61] promptly noted that the excitation of analogue states via the (p,n) reaction did not of itself imply that either the target or analogue had pure isospin, but rather it demonstrated the "dynamic" validity of isospin. In other words, the relationship between neighbouring levels of a multiplet is determined by the relative isospin impurity between the levels rather than the absolute impurity. Whereas all Coulomb forces contribute to the total isospin impurity, the difference between neighbouring members of a multiplet arises from the Coulomb interaction of a single proton. Thus, most of the Coulomb interaction is common to both systems and produces the same mixing effects in each without disturbing the relationship between them.

The really surprising result came when Lane and Soper [La62b] showed that the absolute isospin impurity of the low-lying states of heavy nuclei near to the stability line was in fact quite small. They pointed out that the excess neutrons have pure isospin $T = \frac{N - Z}{2}$ which serves to dilute the isospin impurity of the core to give a reasonably pure total isospin. If the nuclear wave function is written as

$$\psi = |T, T\rangle + \gamma |T+1, T\rangle, \quad 1.9$$

then current estimates of the isospin impurity of nuclear ground states throughout the mass table show that γ^2 remains < 0.003 [Bo69a]; moreover, any decrease in the neutron excess results in increased isospin mixing in the ground state. The effect of the excess neutrons can be regarded as creating a large energy gap over which a proton must be raised when changing it to a neutron to introduce the (T+1) admixtures.

1.2.4 The Lane Equations

The excitation of analogue states in the (p,n) reaction [An61] was described by Lane [La62a] as being "quasi-elastic" in that the final state was the isobaric analogue of the target ground state. Lane suggested that such a process could be explained by writing the nucleon-nucleus optical potential in the form

$$V = V_0 + V_1 (\underline{t} \cdot \underline{T}) , \quad 1.10$$

where $\underline{t}$ and $\underline{T}$ are respectively the isospins of the nucleon and nucleus and V_0 contains the usual central and spin-orbit terms which are independent of isospin. The isospin-dependent term in equation 1.10 is, in fact, a more fundamental form of the symmetry term (proportional to $\frac{N - Z}{A}$) which has been used to account for the difference between the neutron and proton well depths in the nuclear optical potential [Sa58a]. Moreover, when summed over all particles this leads to the asymmetry term in Weizsäcker's semi-empirical mass formula [We35].

The usefulness of the above potential for describing the (p,n) reaction can be seen by rewriting the isospin-dependent part in terms of isospin operators:

$$V_1 (\underline{t} \cdot \underline{T}) = V_1 [\hat{t}_z \hat{T}_z + \frac{1}{2}(\hat{t}^+ \hat{T}^- + \hat{t}^- \hat{T}^+)] . \quad 1.11$$

The $V_1 \hat{t}_z \hat{T}_z$ term is just the symmetry term mentioned above while the $\hat{t}^+ \hat{T}^-$ term gives rise to the quasi-elastic (p,n) reaction where $\hat{t}^+$ changes a proton into a neutron and $\hat{T}^-$ converts the target nucleus into its isobaric analogue. Similarly, the $\hat{t}^- \hat{T}^+$ term produces the inverse quasi-elastic (n,p) reaction.

Including the Coulomb potential V_c , the total nucleon-nucleus potential may be written as

$$V_o + V_1 (\underline{t}, \underline{T}) + (\frac{1}{2} - t_z) V_c \quad 1.12$$

so that the Schrödinger equation for an incident proton of energy E is

$$\{\mathcal{J} + V_o + V_1 (\underline{t}, \underline{T}) + (\frac{1}{2} - t_z) V_c\} \psi = \{E - (\frac{1}{2} - t_z) \Delta E_c\} \psi, \quad 1.13$$

where $\mathcal{J}$ is the kinetic energy term and ΔE_c is the Q value for the quasi-elastic (p,n) process.

The two isospin channels for such a system consist of the elastic channel with the proton plus target and the analogue channel which is a neutron of energy $(E_p - \Delta E_c)$ and the analogue of the target. The solution of equation 1.13 will therefore be of the form

$$\psi = \chi_p(r) \alpha_p \phi_{T_o, T_o} + \chi_n(r) \alpha_n \phi_{T_o, T_o-1}, \quad 1.14$$

where $\chi_p(r)$ and $\chi_n(r)$ are the space-spin wave functions and α_p , α_n and ϕ_{T,T_z} are respectively the isospin states of the proton, neutron and target nuclei. Substitution of 1.14 into the Schrödinger equation, 1.13, leads to two coupled differential equations:

$$\left(\mathcal{J} + V_o - E + V_c - \frac{T_o}{2} V_1 \right) \chi_p(r) = - \left(\frac{T_o}{2} \right)^{\frac{1}{2}} V_1 \chi_n(r) \quad 1.15$$

$$\left(\mathcal{J} + V_o - E + \Delta E_c + \frac{T_o-1}{2} V_1 \right) \chi_n(r) = - \left(\frac{T_o}{2} \right)^{\frac{1}{2}} V_1 \chi_p(r). \quad 1.16$$

These are commonly known as the Lane equations and are the basis of the Lane model description of isobaric analogue states.

1.3 ISOBARIC ANALOGUE RESONANCES

In 1963 Fox et al. [Fo64] made the second important experimental breakthrough when they discovered isobaric analogue states as compound nucleus resonances in the (p,p) and (p,n) reactions. Rather than considering the nuclei actually investigated by Fox et al., we

shall take as an example the proton bombardment of ^{160}Gd which was studied in this work. The compound nucleus formed in this case is $^{161}_{65}\text{Tb}$ which has $T_z = \frac{31}{2}$. The $T = \frac{31}{2}$ states at low excitation energies in ^{161}Tb are widely spaced but as the excitation energy increases the density of states becomes very large. However, superimposed on the high density $T = \frac{31}{2}$ states are widely spaced $T = \frac{33}{2}$ states which are the $T_z = \frac{31}{2}$ analogues of the low-lying $T = \frac{33}{2}$ states in the $T_z = \frac{33}{2}$ nucleus $^{161}_{64}\text{Gd}$. Owing to the large Coulomb displacement energy (15.86 MeV), the analogue states in ^{161}Tb are above the threshold for proton emission and they can thus be seen as resonances in elastic proton scattering from ^{160}Gd . The high-lying $T = \frac{33}{2}$ states in ^{161}Tb are said to be the "analogue states" corresponding to the low-lying $T = \frac{33}{2}$ "parent states" in ^{161}Gd . For obvious reasons, the $T = T_z + 1$ states in ^{161}Tb are referred to as $T_>$ states while the background states with $T = T_z$ are the $T_<$ states.

For the general case of elastic proton scattering on the target nucleus (Z,N) , the $T_>$ states seen in the compound nucleus $(Z+1,N)$ are the analogues of the low-lying bound states in the parent nucleus $(Z,N+1)$ which are populated by the (d,p) reaction on the same target. The analogue and parent states, as well as having the same T value, will also have the same spin and parity. Indeed, if charge independence was completely valid the analogue and parent state would show an exact correspondence between their space-spin wave functions. Thus, any spectroscopic information which can be acquired for the analogue state will also relate to the parent state. This particular aspect has been the basis of much of the interest shown in isobaric analogue resonances since 1963.

The question arises of why the analogue state is observable as a narrow resonance, or alternatively why the $T_>$ state is not

completely dissolved in the $T_{<}$ states by the Coulomb interaction. The explanation can be found in the complexity of the $T_{<}$ states compared to the relatively simple structure of the analogue state. Adopting the approach of first-order perturbation theory, the mixing strength is proportional to

$$\sum_{T_{<}} \frac{\langle \psi_{T_{>}} | V_c | \psi_{T_{<}} \rangle}{|E_{T_{>}} - E_{T_{<}}|} .$$

Since the simple states which would overlap strongly with $\psi_{T_{>}}$ are energetically distant the mixing strength is small.

1.3.1 Energy Relationship between Parent and Analogue States

If the isospin of the target is T_o , then the parent and analogue states are adjacent members, $|T_o + \frac{1}{2}, T_o + \frac{1}{2}\rangle$ and $|T_o + \frac{1}{2}, T_o - \frac{1}{2}\rangle$ respectively, of a $T = T_o + \frac{1}{2}$ multiplet. They are therefore split in energy, according to equation 1.8, by $\Delta E_c - \Delta m_{pn}$. The energy relationship between the two states is shown diagrammatically in figure 1.1. It can be seen that the proton bombarding energy E_p , in the centre-of-mass system, for exciting the analogue of the parent ground state is given by

$$E_p = \Delta E_c - S_n , \quad 1.17$$

where S_n is the neutron separation energy of the parent nucleus. Thus, the Coulomb displacement energy ΔE_c can be obtained from a measurement of the resonance energy and a knowledge of S_n .

1.3.2 Composition of Analogue Resonances

If the proton + target and neutron + target systems are denoted by $|pC\rangle$ and $|nC\rangle$ respectively and assuming that the target system C has pure isospin T_o , then $|pC\rangle$ and $|nC\rangle$ can be expanded in

Figure 1.1: The energy relationship between states in the parent and analogue systems.

terms of the isobaric spin states which are formed by coupling a proton and a neutron to the target [Ro65]:

$$|pC\rangle = \frac{1}{(2T_o+1)^{\frac{1}{2}}} [|T_o+\frac{1}{2}, T_o-\frac{1}{2}\rangle + (2T_o)^{\frac{1}{2}} |T_o-\frac{1}{2}, T_o-\frac{1}{2}\rangle] \quad 1.18$$

$$|nC\rangle = |T_o+\frac{1}{2}, T_o+\frac{1}{2}\rangle . \quad 1.19$$

It can be seen that, while the $|nC\rangle$ system has pure isobaric spin $T_o+\frac{1}{2}$, the $|pC\rangle$ system has two components of isobaric spin $T_> = T_o+\frac{1}{2}$ and $T_< = T_o-\frac{1}{2}$. Constructing states of definite isobaric spin, one obtains

$$|T_>\rangle = \frac{1}{(2T_o+1)^{\frac{1}{2}}} [|pC\rangle + (2T_o)^{\frac{1}{2}} |nA\rangle] \quad 1.20$$

and

$$|T_<\rangle = \frac{1}{(2T_o+1)^{\frac{1}{2}}} [(2T_o)^{\frac{1}{2}} |pC\rangle - |nA\rangle] , \quad 1.21$$

where $|nA\rangle$ is a neutron coupled to the analogue of the target $|T_o, T_o-1\rangle$. The $|T_>\rangle$ are the isobaric analogues of the $|nC\rangle$ states while the $|T_<\rangle$ are often known as anti-analogue states and have the same isospin as the ground state of the nucleus (p+C). The anti-analogue states lie about 10 MeV below the analogue states in heavy nuclei and consequently are either bound or inaccessible to proton resonance formation because of the Coulomb barrier. However the anti-analogue states can be reached in proton transfer reactions such as $(^3\text{He}, d)$ [Vo67].

The decomposition of the analogue states in equation 1.20 is represented symbolically in figure 1.2 for a case where the parent state is a pure single-particle state. While the analogue state does have a single-particle component $|pC\rangle$ of amplitude $\frac{1}{(2T_o+1)^{\frac{1}{2}}}$, it is dominated by the more complex $|nA\rangle$ components which are of two particle (proton and neutron)-one hole (neutron hole) character. It is these

PARENT:

Figure 1.2: A symbolic representation of the composition of the analogue of a pure single-particle parent state.

latter components which cause resonant effects to appear also in inelastic proton scattering [Jo64] leading to excited states of the target which have the same neutron-neutron hole components. These resonant effects in the inelastic proton channels are particularly important since they spectroscopically relate the parent states to excited states of the target rather than to the ground state as in the (p, p_0) and (d, p) cases.

1.3.3 Mixing with Background States

The mixing of the analogue $T_>$ states with the closely spaced $T_<$ background states was demonstrated on the very first occasion that analogue resonances were observed by Fox et al. [Fo64] by the appearance of the resonances in the (p, n) reaction. Although the neutron decay of an analogue resonance to low-lying states in the final nucleus may be energetically possible, it violates the conservation of isospin unless the $T_>$ analogue state has $T_<$ impurities.

More-direct evidence for mixing of the $T_>$ analogue state with the $T_<$ background compound states has been provided by the observation of the fine structure of analogue resonances. The ability to observe this structure depends upon the density of the background states and the beam energy resolution. The group at Duke [Ke66] were successful in observing the fine structure for favourable cases involving light nuclei with low level-density at the excitation energy of the analogue state. They used a proton beam of energy resolution < 250 eV to show that single analogue resonances observed with normal resolutions of ~ 5 keV are in fact many much narrower resonances which appear to have the same J^π as that assigned to the gross structure.

The isospin mixing of the analogue state is often described by introducing a partial spreading width $\Gamma^\downarrow$ for the decay of the

resonance into the background states of lower isospin. The total width Γ of the resonance is then given by

$$\Gamma = \sum_i \Gamma_i + \Gamma^\downarrow, \quad 1.22$$

where Γ_i are the partial widths for the open direct channels such as elastic and inelastic proton scattering. An estimate of $\Gamma^\downarrow$ can thus be obtained from equation 1.22 if Γ and the main Γ_i are measured. A compilation of the few available spreading widths by Mekjian [Me70] suggests that $\Gamma^\downarrow$ is a regular function of mass number for heavy nuclei and from this Bondorf [Bo71] has proposed an empirical relationship of the form

$$\Gamma^\downarrow \approx 3.8 (T_o + \frac{1}{2}) \text{ keV}, \quad 1.23$$

where $(T_o + \frac{1}{2})$ is the analogue state isospin. Thus, for nuclei with $A \sim 170$ the predicted $\Gamma^\downarrow$ is about 70 keV which would amount to about 50% of the total width.

Because of the mixing of the analogue and background compound states the possibility exists of seeing compound nucleus contributions to the proton cross-sections. However, for heavy nuclei many neutron channels are open at the analogue-state energy and the branching ratio for proton decay of the compound nucleus is therefore small.

The Robson theory of isobaric analogue resonances [Ro65] based on a Lane isospin-dependent optical potential demonstrates clearly that the Coulomb forces are responsible for the isobaric-spin mixing. By introducing basis states of definite isospin into the Lane equations, equations 1.15 and 1.16, Robson showed that the coupling between the $T_>$ and $T_<$ states arises from a term $(\Delta E_c - V_c(r))$. The radial dependence of the Coulomb displacement energy ΔE_c and the Coulomb potential $V_c(r)$, as predicted for a uniform charge distribution,

Figure 1.3: Schematic representation of the radial variation of the Coulomb potential $V_c(r)$ and the Coulomb displacement energy ΔE_c for a proton moving in the field of a spherical uniform charge distribution. The approximate cancellation of the terms in the factor $(\Delta E_c - V_c(r))$ in the region $r < a_c$ illustrates the weakness of the isospin mixing in the internal region compared with that in the external region [Ro65].

is shown in figure 1.3; it can be seen that $V_c(r) \approx \Delta E_c$ inside the nuclear charge radius a_c . Therefore, the coupling between the $T_>$ and $T_<$ states in this "internal" region, $r < a_c$, will be weak compared with the coupling in the "external" region, $r > a_c$. This dominance of external mixing over internal mixing reflects the difference in isospin purity between bound and continuum states. For bound states, with the system almost completely confined to the internal region, external mixing is not expected to be important whereas the unconfined continuum states will undergo appreciable external mixing.

1.3.4 Analogue States and Doorway States

The isobaric analogue states seen in proton scattering being mainly of 2p-1h character, as shown in figure 1.2, are doorway states for the elastic and inelastic proton channels. Consequently isobaric analogue resonances constitute some of the best examples of intermediate structure yet observed.

The total width of an intermediate structure resonance can be expressed as

$$\Gamma = \Gamma^\uparrow + \Gamma^\downarrow, \quad 1.24$$

where $\Gamma^\uparrow$ is the escape width for decay of the doorway state into the various direct channels and $\Gamma^\downarrow$ is the spreading width resulting from the coupling of the doorway state to the more complex hallway states of 3p-2h character. The conditions required for a doorway state to produce an observable intermediate structure resonance are:

(a) the separation between doorway states should be larger than the sum $\Gamma^\uparrow + \Gamma^\downarrow$, and

(b) the escape width for decay into the channel of observation should be at least comparable with the spreading width $\Gamma^\downarrow$.

In the case of analogue resonances both (a) and (b) are satisfied for the special reason that the doorway and hallway states are of different isospin.

1.4 ANALYSIS OF RESONANCES IN ELASTIC SCATTERING

The natural division of space, created by the "internal" and "external" mixing regions, suggests a description of isobaric analogue resonances by the R-matrix theory of nuclear reactions [La58], in which configuration-space is split into an inner and an outer region separated at some matching radius a_m . In this approach all unknown details about the interaction are confined to the inner region $r \leq a_m$ whereas known quantities such as Coulomb or optical model wave functions are applicable in the outer region $r \geq a_m$. Such an R-matrix approach has been applied by Robson [Ro65], and later by Thompson et al. [Th68], to the analogue resonances observed in elastic proton scattering from spin-zero nuclei. Thus, the differential cross-section, at energy E and centre-of-mass angle θ , is given in terms of the non-spin-flip and spin-flip amplitudes as [Th68]

$$\frac{d\sigma}{d\Omega}(E, \theta) = |a'(E, \theta)|^2 + |b'(E, \theta)|^2, \quad 1.25$$

where

$$a'(E, \theta) = - \frac{\eta}{2k \sin^2 \frac{\theta}{2}} \exp \left[-i\eta \ln \left(\sin^2 \frac{\theta}{2} \right) \right] + \frac{1}{2ik} \sum_{\ell j} (j + \frac{1}{2}) [U_{\ell j} - \exp(2i\omega_{\ell})] P_{\ell}(\cos \theta) \quad 1.26$$

and

$$b'(E, \theta) = \frac{1}{2ik} \sum_{\ell j} (-1)^{j-\ell-\frac{1}{2}} [U_{\ell j} - \exp(2i\omega_{\ell})] P_{\ell}^1(\cos \theta). \quad 1.27$$

The first term in $a'(E, \theta)$ represents pure Rutherford scattering with η being the Coulomb parameter and k the wave number of the incident

proton. $P_\ell(\cos\theta)$ and $P_\ell^1(\cos\theta)$ are respectively ordinary and associated Legendre polynomials while j and ℓ are the total spin and orbital angular momentum of the resonance. The Coulomb phase-shift is denoted by ω_ℓ and $U_{\ell j}$ is the collision matrix [La58] which, for an isolated Breit-Wigner resonance with a non-resonant background of optical and Coulomb potential scattering, has the form

$$U_{\ell j} = \exp(2i\omega_\ell) \left[\exp(2i\xi_{\ell j}) + \exp(2i\phi_{\ell j}) \frac{i\Gamma_p}{E_r - E - \frac{1}{2}i\Gamma} \right]. \quad 1.28$$

$\xi_{\ell j}$ and $\phi_{\ell j}$ are respectively the optical and background resonance phase-shifts, and Γ_p and Γ are the partial and total widths of a resonance at energy E_r . As the spin-flip contribution to the non-resonant scattering is ignored the phase-shifts $\xi_{\ell j}$ and $\phi_{\ell j}$ will depend only on ℓ .

In a development given by Zaidi [Za68], the potential scattering is written in terms of an amplitude ρ and phase γ with

$$\begin{aligned} \rho e^{i\gamma} = & - \frac{\eta}{2k \sin^2 \frac{\theta}{2}} \exp \left[-i\eta \ln \left(\sin^2 \frac{\theta}{2} \right) \right] \\ & + \frac{1}{2ik} \sum_{\ell j} (j+\frac{1}{2}) [\exp(2i\omega_\ell) \exp(2i\xi_\ell) - \exp(2i\omega_\ell)] P_\ell(\cos\theta). \end{aligned} \quad 1.29$$

Multiplying both a' and b' by $e^{-i\gamma}$ gives

$$a(E, \theta) = \rho + \frac{1}{2k} \sum_{\ell j} (j+\frac{1}{2}) e^{i\alpha_\ell} \frac{\Gamma_p}{E_r - E - \frac{1}{2}i\Gamma} P_\ell(\cos\theta) \quad 1.30$$

and

$$b(E, \theta) = \frac{1}{2k} \sum_{\ell j} (-1)^{j-\ell-\frac{1}{2}} e^{i\alpha_\ell} \frac{\Gamma_p}{E_r - E - \frac{1}{2}i\Gamma} P_\ell^1(\cos\theta), \quad 1.31$$

where

$$\alpha_\ell = 2\omega_\ell + 2\phi_\ell - \gamma.$$

The amplitudes a and b , together with equation 1.25, are frequently

used for the analysis of excitation functions (for example, in the BRIGIT code [Mo67a]), with each observed resonance being represented by a Breit-Wigner term inside the summation signs of equations 1.30 and 1.31. It should be noted that such a summation will be incorrect if resonances of the same spin and parity overlap. In these circumstances a multi-level resonance formula is required to describe the cross-section [Th68].

1.4.1 Determination of Orbital Angular Momentum and Spin of Analogue Resonances

The resonant amplitudes in equations 1.30 and 1.31 are generally small compared to the non-resonant amplitude ρ and therefore, when the cross-section is evaluated, the dominant term responsible for the resonance structure will be the product of ρ and the resonant part of a . The magnitude of the structure is therefore approximately proportional to $(j+\frac{1}{2}) \Gamma_p P_\ell(\cos\theta)$ and at the angles where P_ℓ vanishes the resonant structure in the cross-section, produced by a partial wave of order ℓ , will be very weak. This provides a useful method of determining the ℓ value associated with a particular resonance.

Although the cross-section is j -dependent through the $(j+\frac{1}{2})$ term, Γ_p is generally unknown and it is therefore impossible to distinguish between $j = \ell - \frac{1}{2}$ and $j = \ell + \frac{1}{2}$ by fitting cross-section data. However, j can be uniquely determined by measuring the polarisation of the scattered protons [Ad66]. In terms of the amplitudes, $a(E, \theta)$ and $b(E, \theta)$ in equations 1.30 and 1.31, the polarisation is given by

$$P(E, \theta) = 2 \frac{\text{Im}[a(E, \theta) b^*(E, \theta)]}{|a(E, \theta)|^2 + |b(E, \theta)|^2} . \quad 1.33$$

As the $b(E, \theta)$ amplitude is proportional to $(-1)^{j-\ell-\frac{1}{2}}$, the anomaly in the polarisation at the resonance energy will be of opposite sign for

$j = \ell \pm \frac{1}{2}$ thus providing a means of making definite spin assignments for the resonance.

1.4.2 Coulomb Displacement Energies

As mentioned earlier in section 1.3.1, if the neutron binding energy in the parent state is known, then the Coulomb displacement energy can be calculated from the energy of an analogue resonance. This has proved to be a much simpler and more accurate method of determining Coulomb displacement energies than the direct (p,n) reaction which requires higher beam energies and a neutron time-of-flight system. Extensive investigations of Coulomb displacement energies have been carried out by the analogue resonance method with observations being made in both the proton [Lo66] and neutron exit channels [Ha66].

1.4.3 Spectroscopic Factors

The observed partial proton width Γ_p , obtained by fitting the expression 1.25 to the experimental excitation function near to an isolated resonance, can be written as

$$\Gamma_p = S_p \Gamma_p^{S.P.}, \quad 1.34$$

where S_p is a spectroscopic factor and $\Gamma_p^{S.P.}$ is the theoretical partial proton width of a resonance, at the same energy and in the same partial wave, which is the analogue of a single-particle neutron state in the parent nucleus. This spectroscopic factor S_p should be identical to the neutron spectroscopic factor, relating the parent state and the target ground state, which is obtained from a distorted wave (DWBA) analysis of (d,p) measurements. Thus, while the elastic scattering measurements do not provide any information which cannot in principle be obtained from the (d,p) measurements, the uncertainties in extracting

spectroscopic factors by the two methods are quite unrelated and a comparison of results provides a mutual check on the analogue resonance and DWBA theories. The determination of spectroscopic factors from analogue resonance measurements does, however, have the advantage that only the elastic cross-section need be known whereas the DWBA analysis requires knowledge of the (d,p) cross-section as well as the elastic deuteron and proton cross-sections, which determine the input parameters for the DWBA calculations. On the other hand, the intrinsic resolution with which states can be resolved in an analogue resonance experiment is much worse than in the corresponding (d,p) experiment.

1.5 ANALOGUE RESONANCE STUDIES IN RARE-EARTH NUCLEI

The majority of analogue resonance experiments have so far been carried out on nuclei with neutron numbers near to the magic numbers of 50, 82 and 126. In these regions of neutron shell closures the parent states are almost pure single-particle states with large separations so that the analogue resonances will be correspondingly large and isolated. However in the regions of stable deformations, such as the higher Z rare-earth nuclei, the experimental situation is not so favourable because of the high density of states in the odd-mass parent nuclei. Fortunately only two, or at most three, members of each rotational band are strong enough to be seen as resonances and by making observations at angles where the Legendre polynomials vanish it is possible to eliminate the resonances of the corresponding partial waves. Thus, both Whineray *et al.* [Wh70] and Foissel *et al.* [Fo72] have succeeded in obtaining useful results from analogue resonance measurements with targets of the ytterbium isotopes. The information which can be obtained from elastic scattering excitation functions is particularly valuable for nuclei of this mass region for the reasons which will be given in the following subsections.

1.5.1 Spectroscopy of Parent States

Many of the spin and parity assignments for the odd-mass parent nuclei in the rare-earth region have come from high resolution (d,p) measurements obtained using broad range magnetic spectrographs. Both the Tallahassee and Copenhagen groups have been very prolific in this area with the latter carrying out extensive energy level studies by both the (d,p) and (d,t) reactions of the odd-mass nuclei of gadolinium [Tj67], dysprosium [Gr70b], erbium [Tj69] and ytterbium [Bu66]. In most cases (d,p) angular distributions have not been measured for the very good reason that while the DWBA theory does reproduce the data it does not do so for a unique ℓ value [Ve63]. Consequently, the assignments have been based on the Nilsson model predictions for the intensity and spacing of the different members of a rotational band (the "finger-print" technique). While this has undoubtedly been very successful, complications can arise from inter-band Coriolis coupling and target excitation effects leading to two-step stripping. Thus the possibility of determining ℓ values and confirming the assignments for some of the parent states makes the analogue resonance experiments very worthwhile.

1.5.2 Deformation Effects on Coulomb Displacement Energies

One effect of the deformation of a nucleus is that, relative to a spherical nucleus of the same mass number, the mean square charge radius becomes larger with a resultant decrease in the Coulomb displacement energy. Macfarlane [Ma66] pointed out that the Coulomb displacement energy for an ellipsoidal charge distribution would be given by

$$\Delta E_c(\delta) = \Delta E_c(0) \left(1 - \frac{4}{45} \delta^2 \right), \quad 1.35$$

where δ is the quadrupole deformation parameter and $\Delta E_c(0)$ is the

Coulomb displacement energy for a spherical charge distribution. This formula predicts a lowering of ΔE_c by about 135 keV for a deformation parameter of 0.3 which is the typical maximum value for rare-earth nuclei. Thus, with a typical error of ± 20 keV on the values of ΔE_c obtained from analogue resonance experiments, it has been possible to observe this effect in several nuclei of the rare-earth region (for example [Ca68]). There are several experimentally viable regions between the $N = 82$ and $N = 126$ closed shells where there is still a scarcity of Coulomb displacement energy data and the nuclei studied in this work ($A = 161$ to 171) were chosen to fill one of these gaps.

PART A

CROSS-SECTION MEASUREMENTS THROUGH ISOBARIC ANALOGUE RESONANCES

CHAPTER 2

EXPERIMENTAL TECHNIQUES AND INITIAL TREATMENT OF DATA

Isobaric analogue resonances have been observed in the elastic and inelastic scattering of protons from targets of ^{160}Gd , ^{162}Dy , ^{164}Dy , ^{166}Er , ^{168}Er , ^{170}Er and ^{174}Yb . For each target, the expected proton bombarding energy for the excitation of the analogue of the ground state of the parent nucleus was calculated from equation 1.16 using tabulated neutron binding energies [Wa71] and Coulomb displacement energies obtained from the empirical formula [Wh70]

$$\Delta E_c = -0.479 + 1.398 Z/A^{1/3}, \quad 2.1$$

where Z and A are respectively the atomic number and the mass number of the parent nucleus. As the nuclei studied are all permanently deformed ($\delta \sim 0.3$) and equation 2.1 relates to a spherical charge distribution, it was necessary to lower ΔE_c by about 100 keV to allow for the deformation. For each target, excitation functions were measured for proton energies ranging from at least 600 keV below the predicted energy of the ground-state analogue to that corresponding to an excitation of about 1.5 MeV in the parent nucleus. The deformed nature of the target nuclei used means that the first excited state of the target, the 2^+ member of the ground-state rotational band, occurs typically at an excitation energy of about 75 keV and is readily excited by Coulomb excitation. Thus, in order to resolve the first inelastic group from the elastic peak in the proton spectrum, it was necessary to achieve resolutions better than 30 keV; this need for high resolution was an important consideration in the experimental approach.

2.1 EQUIPMENT

2.1.1 Proton Beam

Proton beams of up $0.6 \mu\text{A}$ on target and ranging in energy from 9.5 MeV to 12.5 MeV were obtained from the ANU EN (HVEC) tandem accelerator. Energy selection of the beam was made by setting the frequency of an NMR system on a 90° inflection magnet. The magnet calibration, based on (p,n) threshold measurements on ^{11}B and ^{27}Al , was previously estimated to be accurate to ± 5 keV with relatively small hysteresis effects [Mo67b]. The spread in energy of the proton beam from the ANU tandem has been found to be $\lesssim 3$ keV from observations of very narrow resonances and of the slope in (p,n) cross-sections just above threshold [Op72]. A simple energy-proportional extrapolation would then predict a spread of $\lesssim 5$ keV for beams of the energy used in these experiments.

After analysis, the beam was magnetically deflected through a further 25° and focused by a magnetic quadrupole doublet into a 51 cm diameter scattering chamber. Beam current monitoring and integration were carried out using a 5 cm diameter Faraday cup, situated about 90 cm downstream from the target, in conjunction with either an Elcor A309A integrator or an Ortec 439 current digitizer. Permanent magnets at the entrance to the cup served both to suppress the collection of electrons knocked out of the target and to prevent electrons leaving the cup.

2.1.2 Scattering Chamber

A general view of the interior of the scattering chamber, set up for an experiment, is shown in figure 2.1. The proton beam entered the chamber from the right through defining collimators which were

Figure 2.1: Photograph of the interior of the scattering chamber set up for an experiment.

The beam enters the chamber from the right through the defining collimators marked T.

Five detector blocks are being used and are each clamped to the copper cooling loop.

In the centre of the chamber are the electrostatic deflector plates used to suppress secondary electrons produced by atomic collisions of the beam in the target.

optically aligned to the chamber. The whole collimator assembly was mounted in a brass barrel and consisted of four 0.5 mm thick tantalum discs 10.48 cm apart, with circular apertures of diameters 1.5 mm, 2.3 mm, 1.5 mm and 3.2 mm, the first of these being 52 cm from the target. The first and third apertures defined the beam while the remaining two served to reduce the scattered particles which reached the target. The beam spot at the target position was found to be < 2 mm in diameter from both the burn-mark on a paper target exposed to the beam for a few seconds and the beam marks on targets irradiated for 2-3 days during the measurement of an excitation function.

Semiconductor detectors were held in special mounting blocks [Op70a] located in a 32 cm diameter annular groove in the rotatable base of the chamber. The blocks also held the defining slits in front of each detector. The slits were constructed from 0.5 mm thick tantalum with rectangular apertures ranging from 9 mm (vertical) $\times$ 5 mm (horizontal) to 1 mm $\times$ 1 mm. For ease of polishing, which is important in order to minimise slit-edge scattering [Re69], the set of rectangular tantalum slits was partially replaced during the course of the experiments by circular apertures, up to 7 mm in diameter, in 0.5 mm thick brass sheet. The angle subtended at the target by a defining slit of width 1 mm was 0.31° so that, assuming a beam spot diameter of 2 mm, the maximum angular spread for the apertures used was $\pm 1.4^\circ$. In all cases the kinematic spread in energies of the detected particles due to the finite slit width was < 3 keV. The accuracy of the angular scale of the chamber has been found to be of the order of 0.2° from the observation of Rutherford scattering [Op70a].

It is well known that to obtain high resolution with solid-state particle detectors it is necessary to cool the detectors in order to reduce the thermal noise and allow the application of high biases

[An67]. Thus, the detectors were cooled to about -20°C by clamping the mounting blocks onto a loop of 0.63 cm diameter copper pipe through which refrigerated alcohol was circulated from the high-energy cold trap of the tandem vacuum system. This cooling system proved to be unreliable and was later replaced by a freon expansion refrigerator, with similar cooling capabilities, circulating through the same loop.

Another important factor in improving resolution is the suppression of the low-energy electrons produced by atomic collisions of the beam in the target [An67]. For a 12 MeV proton beam these secondary electrons have energies up to about 25 keV and are easily swept away by electric or magnetic fields. An electrostatic system was designed and consisted of two 0.63 cm thick annular aluminium plates, of inner and outer diameters 10.2 cm and 17.8 cm respectively, separated by 1.0 cm thick teflon spacers. The plates were mounted symmetrically above and below the reaction plane and between the target and detectors. The lower plate was grounded while voltages up to -4 kV were applied to the upper plate via a high-voltage feed-through in the wall of the chamber. For a field of 4 kV/cm between the plates, the calculated deflections, after a single passage through the gap, of a 25 keV electron and a 10 MeV proton are 0.3 and 8×10^{-4} radians respectively. The effectiveness of the system is demonstrated in figure 2.2 by the spectra obtained for the elastic and first inelastic proton groups of ^{174}Yb both with and without voltage applied to the upper plate.

2.1.3 Detectors

The scattered protons were detected using solid-state detectors of both the lithium-drifted silicon (Si(Li)) and the silicon surface-barrier types with depletion depths in the range 1.5 - 2.0 mm. By employing the previously mentioned techniques, resolutions, quoted

Figure 2.2: Typical spectra for the elastic and first inelastic proton groups scattered from ^{174}Yb , obtained both with and without voltage between the electron suppression plates.

as the FWHM of 11 MeV protons scattered off a heavy target, of 25 keV and 20 keV respectively were obtained for the two types of detectors. The number of detectors mounted in the chamber was normally five: a monitor at 40° and four other detectors, for measuring the excitation functions, at 160.0° , 140.5° , 125.0° and 89.6° in the laboratory system. The last three angles were chosen as, in the centre-of-mass system, they correspond to zeros of Legendre polynomials.

A severe problem encountered with the Si(Li) detectors was their susceptibility to radiation damage with average lifetimes of about 14 days general use at EN tandem energies; this figure is consistent with the figure of $10^8 - 10^9$ particles/cm² quoted by Dearnaley and Northrop [De66] as the "significant-damage dose" for lithium-drifted detectors. Rejuvenation of the detectors was carried out by means of a redrift technique, similar to that described by Jaskola [Ja66], in which the damaged detectors were maintained under reverse bias conditions at a temperature of about 60°C for several days. Details of the redrift operations are given in Appendix 1. While this technique was invaluable when working with a limited number of Si(Li) detectors, the radiation-damage problem has been somewhat alleviated recently by the availability of surface-barrier detectors, manufactured at the ANU, which are inherently able to withstand much larger radiation doses.

2.1.4 Targets

Thin foil targets, with thicknesses between 50 and 150 $\mu\text{g}/\text{cm}^2$, of ^{162}Dy , ^{164}Dy , ^{166}Er , ^{168}Er , ^{170}Er and ^{174}Yb were prepared by the vacuum evaporation of small amounts of the enriched metal onto 20-40 $\mu\text{g}/\text{cm}^2$ thick carbon films. The metal of each separated isotope was prepared in quantities of 10-20 mg by a preliminary reduction of

the oxide[†] using lanthanum or thorium powder as a reducing agent. The preparation technique closely followed that of Westgaard and Bjørnholm [We66] and is described in Appendix 2. In the case of gadolinium, the yield of metal from the reduction process was so low that the ^{160}Gd targets were prepared by evaporating the enriched oxide $^{160}\text{Gd}_2\text{O}_3$ from a tantalum crucible directly onto carbon films. The isotopic composition of the oxides used is given in table 2.1.

Table 2.1
Isotopic composition of target materials

Target	Isotopic composition of oxide (%)							
^{160}Gd	^{156}Gd	0.8	^{157}Gd	1.2	^{158}Gd	2.6	^{160}Gd	94.8
^{162}Dy	^{161}Dy	5.1	^{162}Dy	91.0	^{163}Dy	2.8	^{164}Dy	0.9
^{164}Dy	^{161}Dy	1.0	^{162}Dy	3.2	^{163}Dy	12.4	^{164}Dy	83.2
^{166}Er	^{166}Er	94.9	^{167}Er	3.5	^{168}Er	1.2	^{170}Er	0.3
^{168}Er	^{166}Er	1.4	^{167}Er	2.4	^{168}Er	95.5	^{170}Er	0.6
^{170}Er	^{166}Er	0.9	^{167}Er	0.7	^{168}Er	1.5	^{170}Er	96.9
^{174}Yb	^{172}Yb	1.0	^{173}Yb	2.2	^{174}Yb	95.8	^{176}Yb	0.6

The mean thicknesses of the targets used for the experiments were obtained to an accuracy of about 4% from the Rutherford scattering of 4.5 MeV protons at 90° measured at the end of each excitation function. The general expression for the Rutherford scattering yield Y from a nucleus of mass number A is

$$Y = \frac{N T N_o}{A} \left(\frac{d\sigma}{d\Omega} \right)_R \Delta\Omega, \quad 2.2$$

[†] The oxides of the enriched isotopes were obtained from the Oak Ridge National Laboratory, Tennessee, U.S.A.

where N is the number of incident particles, T is the thickness of the target (gm/cm^2) and N_0 is Avogadro's number. $\left(\frac{d\sigma}{d\Omega}\right)_R$ is the differential cross-section (cm^2/sr) for Rutherford scattering at the angle concerned and $\Delta\Omega$ is the solid angle (sr) subtended by the detector at the target. The major contributions to the error in the thickness determined by the above formula come from the uncertainties in the beam-charge integration ($\sim 2\%$) and in the solid angle subtended by the detector defining slit. This latter contribution was minimised by determining all thicknesses from the yield observed through one particular slit whose area had been measured to $\lesssim 2\%$ using a travelling microscope.

During the excitation functions, the targets were oriented in a reflection geometry with the metal facing the beam and the normal to the plane of the foil at 45° to the beam direction. The effective thickness of the metal deposit in this orientation is given in table 2.2, together with the energy loss experienced by an 11 MeV proton beam in passing through the full effective thickness, calculated using

Table 2.2
Effective thicknesses of targets

Target	Effective thickness ($\mu\text{g}/\text{cm}^2$)	Energy loss for 11 MeV protons (keV)
^{160}Gd	128 ± 5	3.0 ± 0.3
^{162}Dy	147 ± 6	2.7 ± 0.3
^{164}Dy	53 ± 3	1.0 ± 0.1
^{166}Er	77 ± 4	1.4 ± 0.2
^{168}Er	150 ± 6	2.7 ± 0.3
^{170}Er	130 ± 5	2.3 ± 0.3
^{174}Yb	110 ± 4	2.0 ± 0.2

tabulated proton stopping cross-sections [Wi66]. The energy loss in the oxide target of ^{160}Gd includes a 0.7 keV contribution from the oxygen. While the thicknesses were not used directly in the calculation of the absolute differential cross-sections they were required for a slight correction to the resonance energies arising from the beam energy loss in the targets.

2.1.5 Electronics and Data Storage

A block diagram of the electronics and pulse analysis system employed is shown in figure 2.3. The output pulses from each detector were amplified in Ortec 109A charge-sensitive preamplifiers before being fed into Ortec 410 linear amplifiers with RC pulse-shaping time-constants of 0.5 μs . The unipolar outputs of the 410 amplifiers were then fed into Ortec 408 biased amplifiers which served both to reject the noise and the unwanted low-energy part of the spectrum and to increase the energy dispersion for pulse-height analysis. The output of the biased amplifier for each of the detectors was fed into both a linear summing amplifier and an analogue routing system which produced a logic pulse whose height was related to the detector from which the pulse originated. As the length of the output pulse from the biased amplifiers was a function of the height of the input pulse above the bias level, an Ortec 411 pulse stretcher (not shown in figure 2.3) was usually employed after the summing amplifier to produce a uniform pulse length. This is known to reduce the bandwidth used in the analogue-to-digital converter (ADC) with a subsequent improvement in linearity. The linear and logic pulses were analysed separately in two Intertechnique CA13 ADCs which were interfaced to an IBM 1800 computer. If a logic and a linear pulse were converted in the ADCs within a period of about 500 ns then the event was accepted by the interface for further analysis

Figure 2.3: Block diagram of the electronics and pulse-routing system, shown here for a four-detector array, as used for the analogue resonance measurements [Ro71].

and routed into the part of the 1800 core appropriate to the size of the logic pulse. With the usual arrangement of five detectors, data was stored in five separate arrays of 512 channels. A dead-time correction for the ADCs was determined by feeding pulses from a standard clock into two scalers, one of which was gated open only when the ADCs were live.

On completion of the data collection at a particular energy, the spectra stored in the computer were transferred to magnetic disc storage and the core cleared in preparation for the next energy. The data were retained on disc at the local 1800 computer for preliminary analysis before being transferred via a high-speed link to magnetic tape storage at the central IBM 360/50 computer.

2.2 EXPERIMENTAL PROCEDURE

2.2.1 Excitation Functions

Although the 90° analysing magnet was reported to be free from hysteresis effects [Mo67b], a standard procedure [Sn67] for its use was adopted: before setting the magnet current for the lowest energy of an excitation function (about 3.5 A for 10 MeV protons), it was cycled three times between 0 and 10 A with a waiting period of about one minute between each half-cycle. During the course of the excitation function the magnet current was never decreased.

The beam energy was initially incremented in steps of 50 keV, when several hundred keV below the predicted ground-state resonance energy, but this was reduced to 10 keV once the resonance region was reached. At each energy spectra were collected for 300-500 μC of integrated beam charge so that the counts in the rare-earth elastic peak exceeded 20,000 in all detectors. This minimum number of counts

was often increased to 30,000 once the energy location of a resonance had been established. The number of counts in the elastic peak in the monitor detector was usually 10^5 and often 2×10^5 when remeasuring.

Samples of spectra recorded in the computer on two different occasions are shown in figures 2.4 and 2.5. Figure 2.4 was collected at 141° from a ^{168}Er target using a Si(Li) detector and shows up impurities of ^{13}C , ^{14}N , ^{16}O , ^{23}Na , ^{28}Si , ^{31}P , ^{32}S and ^{39}K . The proton spectrum in figure 2.5 is from a ^{170}Er target and was collected by a surface-barrier detector at 125° . The beam energy is quite close to a resonance and the second inelastic group, going to the 4^+ member of the ground-state rotational band in the target nucleus, is visible. The impurities in the target were very similar to those found in the ^{168}Er target. A comparison of the dip between the elastic and first inelastic peaks in figures 2.4 and 2.5 illustrates the slightly better performance of the surface-barrier detectors.

2.2.2 Rutherford Scattering

In order to obtain a cross-section calibration, measurements were made of the Rutherford scattering of 4.5 MeV protons from the rare-earth target at the end of every excitation function. Because of the high scattering cross-section at this energy (about six times that at 11 MeV), the measurement was made with less than 50 nA of beam and usually for a total beam charge of about 100 μC to obtain at least 10^5 counts in the elastic peak for each detector. As already mentioned, this run also served to measure the target thickness.

Figure 2.4: Pulse-height spectrum from a Si(Li) detector for the scattering of 11.6 MeV protons at $\theta_{\text{lab}} = 141^\circ$ from a ^{168}Er target. Proton groups arising from impurities in the target are labelled.

Figure 2.5: Pulse-height spectrum from a surface-barrier detector for the scattering of 11.71 MeV protons at $\theta_{\text{lab}} = 125^\circ$ from a ^{170}Er target. The proton bombarding energy is close to that of a resonance and both the 2^+ and 4^+ inelastic groups are visible.

2.3 INITIAL TREATMENT OF DATA

2.3.1 Unfolding of Spectra

As previously mentioned, the separation of the elastic proton group from the first inelastic group was typically ~ 75 keV and even with a peak FWHM of 25 keV the low-energy tail of the elastic (0^+) peak extended right under the inelastic (2^+) peak. A computer programme (SKEWT) was therefore written to unfold the peaks using the fit function which was found to reproduce the spectra best. This is a Gaussian with an exponential tail of the form [Li66]

$$f(x) = \frac{A}{(2\pi)^{\frac{1}{2}}\sigma} \exp[-(x-\bar{x})^2/2\sigma^2] + AC \exp[D(x-\bar{x})] \cdot \{1 - \exp[-(x-\bar{x})^2/2G^2\sigma^2]\} , \quad 2.3$$

where A is the area of the Gaussian centred at $x = \bar{x}$, C is the tail height at $x = \bar{x}$ expressed as a fraction of A (~ 0.1), D is the slope of the exponential (~ 0.2), G is a parameter (~ 1) which rounds off the tail where it joins the Gaussian and σ is a width ($\sim \text{FWHM}/2.4$). In this application of the function, all lengths such as x , $\bar{x}$ and σ are given in channels and the area is in counts. The first term in equation 2.3 is the simple Gaussian while the second is the exponential tail which is, of course, only added to the low-energy side of the Gaussian ($x < \bar{x}$). Both the elastic and inelastic groups were fitted with the same width and tail parameters but differed in position and intensity. As the elastic peak from the rare-earth nucleus was the highest-energy peak in the spectrum it was not necessary to allow for a background from higher-energy groups.

While it is possible to use such a fit function in a linear least squares method when the parameters C, D and G are fixed, this was found to be inadequate to fit the spectra and a non-linear approach

with the ability to vary all parameters was adopted. The computer programme, which employed a general-purpose search routine, FITTEM [Ha68], read spectra off magnetic tape in a sequential fashion and used the fitted parameters from one spectrum as the starting values for the next. As the beam energy was increasing throughout the excitation function, the position of the elastic peak moved consistently in the direction of increasing channel number and was thus easily tracked. A starting value for the peak position was obtained by calculating the position of the maximum of the parabola which passed through the three largest channel counts in the peak; the value of the maximum was also used as a normalisation factor to obtain starting values for the peak area. After a number of trial fits in which all parameters were varied, it was usually found possible to fix C, G and the separation between the 0^+ and 2^+ peaks at average values and then vary only A, $\bar{x}$, σ and D. The time taken by the IBM 360/50 computer for these limited fits, over 24 channels with two component peaks, was about 7 seconds per spectrum.

The above fitting procedure was used to unfold the 0^+ and 2^+ peaks at 90° , 125° , 141° and 160° in all cases except for that of the 90° data collected for $^{164}\text{Dy}(p,p)$. A particularly poor detector made unfolding impossible in this case and a simple summation was made over both peaks.

2.3.2 Monitor Counts

The analysis of the monitor spectra collected at 40° was much simpler than for the spectra at other angles because of the negligible cross-section for Coulomb excitation of the 2^+ state, with $\sigma^{2^+}/\sigma^{0^+}$ being ~ 0.001 at this angle. As the rare-earth elastic peak was well separated from the main impurities, the counts in the peak, which were used to normalise the counts at the other angles, were obtained by a

simple integration over a fixed number of channels, above and below the peak position, using the spectrum analysis program INCRED [Op70b]. As for the spectra collected at other angles, no background correction arising from higher-lying peaks was necessary.

2.3.3 Rutherford Scattering Spectra

Since no 2^+ inelastic group was observed at 4.5 MeV beam energy, the elastic peak areas were all obtained by a simple summation of the counts in the peak as carried out for the monitor detector.

2.3.4 Inelastic Peaks

Although the elastic peaks were of primary interest in this work, the yields to the first two inelastic groups were also extracted in some cases. The yield to the 2^+ state of the ground-state rotational band in the target was obtained along with the elastic yield from the peak-unfolding procedure described in section 2.3.1. The third member of the rotational band, a 4^+ state, lies at an excitation energy of about 250 keV so that, although the inelastic group to this state was reasonably well separated from the other peaks in the spectra, there was usually a large underlying background from the elastic peak. Consequently, a simple peak integration routine [Op70b] was applied with a background correction based on the counts in the channels immediately on either side of the group. As the 4^+ yield away from any resonances was scarcely visible above the background, the limits for the summation were set at fixed numbers of channels below the elastic peak; these were determined from spectra collected near to a resonance where the 4^+ yield was enhanced (see figure 2.5). If the summation was performed from, for example, channels NL to NH inclusive, then the counts were

summed over $\frac{NH - NL + 1}{2}$ channels on either side of the main summation region to obtain an average background correction.

2.3.5 Calculation of Absolute Cross-Sections

The number of particles $N(E, \theta)$ scattered at angle θ from a target of thickness t , which is bombarded by a beam of total charge Q and energy E , can be written in a general form as

$$N(E, \theta) = A(\theta) t Q \sigma(E, \theta), \quad 2.4$$

where $\sigma(E, \theta)$ is the differential cross-section for the scattering process and $A(\theta)$ is a constant for the detector at angle θ which includes the solid angle, Avogadro's number and the density of the target. For the case of proton scattering off a heavy target, the scattering at 4.5 MeV can be described by the Rutherford cross-section $\sigma_R(4.5, \theta)$ and the detector constant $A(\theta)$ can then be expressed as

$$A(\theta) = \frac{N(4.5, \theta)}{t' Q' \sigma_R(4.5, \theta)}, \quad 2.5$$

where the primes indicate the thickness and charge during the 4.5 MeV run. At an arbitrary energy E in the bombarding-energy region where isobaric analogue resonances are excited, the cross-section at 40° is almost, but not quite, completely due to Rutherford scattering and we can write

$$\sigma(E, 40) = f(E, 40) \frac{4.5^2}{E^2} \sigma_R(4.5, 40), \quad 2.6$$

where $f(E, 40)$, the ratio of the scattering cross-section to the Rutherford cross-section, is typically about 0.9. The counts collected in the detectors at the angles θ and 40° with a proton energy E are respectively

$$N(E, \theta) = A(\theta) t'' Q'' \sigma(E, \theta) \quad 2.7$$

and

$$N(E, 40) = A(40) t'' Q'' \sigma(E, 40) , \quad 2.8$$

where the double primes indicate the thickness and charge during the run at energy E. The final expression for the cross-section at energy E and angle θ can be obtained from equations 2.5 to 2.8 in the form

$$\sigma(E, \theta) = 20.25 \frac{N(E, \theta) N(4.5, 40) f(E, 40) \sigma_R(4.5, \theta)}{N(E, 40) N(4.5, \theta) E^2} . \quad 2.9$$

Several important points about equation 2.9 are worth noting:

(a) If all data pass through the same ADC then dead-time corrections will cancel out;

(b) There is no implicit assumption of constant target thickness throughout the excitation function;

(c) The cross-section $\sigma(E, \theta)$ does not depend directly on the Rutherford cross-section at 40° , where $\frac{d\sigma_R}{d\theta}$ is large, but only indirectly through the term $f(E, 40)$. As the latter varies relatively slowly with angle, any error in setting the monitor angle will not greatly affect the calculated absolute cross-sections.

The first two of these points are regarded as good reasons for using a monitor detector rather than relying on beam charge for normalisation of data.

The factors $f(E, 40)$ were calculated using JIB3FP, a slightly modified version of the optical model programme JIB3 written by Perey [Pe63]. The well parameters used for the calculations were obtained from the Perey prescriptions [Pe63] based on 9-22 MeV proton scattering data.

2.4 EXCITATION FUNCTIONS

2.4.1 Elastic Scattering

The experimental excitation functions for $^{160}\text{Gd}(p,p_0)$, $^{162}\text{Dy}(p,p_0)$, $^{164}\text{Dy}(p,p_0)$, $^{166}\text{Er}(p,p_0)$, $^{168}\text{Er}(p,p_0)$, $^{170}\text{Er}(p,p_0)$ and $^{174}\text{Yb}(p,p_0)$ are presented in order of increasing target mass in figures 2.6 to 2.12. The differential cross-sections are calculated in the centre-of-mass system using equation 2.9 while the proton energies are plotted in the laboratory system. The statistical errors in the cross-section values are indicated, for three or four points in each excitation function, as fine vertical lines which are typically about three times as long as the diameter of the plotting symbols. These correspond to $\lesssim 1\%$ relative error in the points and are based on counting statistics only. However, additional systematic errors will be present from slight variations in the beam spot on the target arising from changes in the focusing and steering of the beam. The number of counts recorded in the monitor detector will be most influenced by these effects and periods of beam instability should affect the calculated cross-section at all angles. The overall error in the values of the absolute cross-section is estimated to be less than 10%, based on the reproducibility of cross-sections measured on different occasions.

The vertical lines below each set of excitation functions are drawn so that their height is proportional to the measured differential cross-section at 60° for the (d,p) reaction leading to the states in the parent nucleus [Bu66, Tj67, Tj69, Gr70b]. Moreover, the lines are at the predicted energies of the analogue states which were calculated from equations 1.16 and 2.1. The same Coulomb displacement energy was assumed for all states and a lowering by about 100 keV was included to allow for the effect of the deformation. The numbers above some of the larger lines indicate the expected orbital angular momentum transfers

Figure 2.6: Elastic scattering excitation functions for $^{160}\text{Gd}(p, p_0)$ from 9.8 to 11.5 MeV measured at $\theta_{\text{lab}} = 89.6^\circ, 125.0^\circ, 140.5^\circ$ and 159.9° .

Figure 2.7: Elastic scattering excitation functions for $^{162}\text{Dy}(p,p_0)$ from 9.8 to 11.8 MeV measured at $\theta_{\text{lab}} = 89.6^\circ$, 125.0° , 140.5° and 159.9° .

Figure 2.8: Elastic scattering excitation functions for $^{164}\text{Dy}(p,p_0)$ from 10.0 to 12.4 MeV measured at $\theta_{\text{lab}} = 125^\circ$, 141° and 160° . The data shown for $\theta_{\text{lab}} = 90^\circ$ is the summed cross-section of the elastic and first inelastic proton groups.

Figure 2.9: Elastic scattering excitation functions for $^{166}\text{Er}(p, p_0)$ from 10.0 to 12.4 MeV measured at $\theta_{\text{lab}} = 90^\circ, 125^\circ, 141^\circ$ and 160° .

Figure 2.10: Elastic scattering excitation functions for $^{168}\text{Er}(p, p_0)$ from 10.0 to 12.5 MeV measured at $\theta_{\text{lab}} = 90^\circ, 125^\circ, 141^\circ$ and 160° .

Figure 2.11: Elastic scattering excitation functions for $^{170}\text{Er}(p,p_0)$ from 10.0 to 12.1 MeV measured at $\theta_{\text{lab}} = 89.6^\circ$, 125.0° , 140.5° and 159.9° .

Figure 2.12: Elastic scattering excitation functions for $^{174}\text{Yb}(p, p_0)$ from 11.0 to 12.3 MeV measured at $\theta_{\text{lab}} = 89.6^\circ$, 125.0° , 140.5° and 159.9° .

in the (d,p) reaction according to the Nilsson assignment for the parent states. Each set of elastic scattering excitation functions will be treated in detail in the next chapter but a comment can be made that, with the exception of the $^{162}\text{Dy}(p,p_0)$ data, there is general agreement between the positions of the observed anomalies in the scattering cross-section and the predicted energies of the analogues of the states which are strongly excited in the (d,p) reaction. The 90° excitation functions are generally featureless at energies corresponding to the low-lying parent states, indicating the absence of any strongly-excited positive-parity states; this is characteristic of the odd-mass nuclei of this region where the majority of low-lying states originate from the $N = 5$ oscillator shell. However, anomalies do appear in the 90° excitation functions at higher energies; for example, a clear effect is present at $E_p \simeq 12.15$ MeV in the $^{168}\text{Er}(p,p_0)$ data (figure 2.10) which suggests a positive parity for the state at 1.488 MeV excitation in ^{169}Er whose analogue is predicted to occur close to this energy. Similar effects were observed in the ytterbium isotopes by both Whineray et al. [Wh70] and Foissel et al. [Fo72].

2.4.2 Inelastic Scattering

The differential cross-section for inelastic scattering to the first excited state of ^{168}Er is shown in figure 2.13 at the laboratory angles of 125° , 141° and 160° . The predicted resonance positions, corresponding to the parent states excited in the (d,p) reaction, are indicated in the same manner as for the elastic scattering excitation functions by vertical lines at the bottom of the diagram. The data, which are typical of the results obtained for all the targets used, show a large non-resonant background arising from Coulomb excitation. The solid lines indicate the Coulomb excitation

Figure 2.13: Inelastic scattering excitation function for $^{168}\text{Er}(p,p_1)^{168}\text{Er} [2^+, 80 \text{ keV}]$ from 10.0 to 12.5 MeV measured at $\theta_{\text{lab}} = 125^\circ, 141^\circ$ and 160° . The straight line shown for each angle represents the calculated cross-section for Coulomb excitation to the 2^+ state.

cross-section calculated for an E2 excitation using the semi-classical method of Alder et al. [A156]. The agreement between the experimental and calculated values below the resonance region is quite reasonable at 125° with the experimental values being slightly low at 141° and 160° . However, at higher energies the experimental values exhibit the interference effects between the direct and resonant scattering which have also been observed in the inelastic scattering from both the ytterbium [Fo72] and osmium [Wi67] isotopes.

Figure 2.14 shows the excitation functions for the scattering from ^{170}Er at $\theta_{\text{lab}} = 159.9^\circ$ for the elastic and first two inelastic groups. The most noticeable feature of the 4^+ data is the small yield below the resonance region as opposed to the appreciable non-resonant cross-sections for both the elastic and 2^+ inelastic groups. The $\ell = 3$ resonance which appears quite strongly in the elastic channel at 11.01 MeV is only weakly excited in the 2^+ and 4^+ channels. The first really large effects in the inelastic channels occur at about 11.7 MeV where a very pronounced interference dip in the 2^+ curve is accompanied by an appreciable rise in the 4^+ yield. Unfortunately, the density of states at this energy is such that the resonances in the 4^+ channel are overlapping and it was not possible to measure their total widths. It may be possible to see individual resonances corresponding to lower-lying states in the parent but much better counting statistics would be required.

Figure 2.14: Elastic and inelastic scattering excitation functions for $^{170}\text{Er}(p,p')$, leading to the 0^+ (g.s.), 2^+ (79 keV) and 4^+ (261 keV) states of the target, from 10.0 to 12.1 MeV measured at $\theta_{\text{lab}} = 159.9^\circ$.

CHAPTER 3

ANALYSIS OF EXCITATION FUNCTIONS AND DISCUSSION OF RESULTS

The excitation functions for elastic proton scattering were presented as figures 2.6 to 2.12 and discussed briefly in section 2.4. It was noted that, in general, the positions of the anomalies present in the cross-sections agree reasonably well with the predicted energies for the analogues of the low-lying parent states which are strongly populated in the (d,p) reaction on the same target. Moreover, the behaviour of the anomalies at the four backward angles does suggest an angular dependence which is characteristic of that predicted by equation 1.25 for an analogue resonance. For example, in figure 2.6, the anomaly found in $^{160}\text{Gd}(p,p_0)$ at a proton energy of 11.18 MeV coincides with the predicted resonance energy for the analogue of an $\ell = 3$ parent state. In addition, there is little or no effect at $\theta_{\text{lab}} = 89.6^\circ$ and 140.5° with a mirror reflection of the anomaly above and below 140.5° . This is typical of the $P_3(\cos\theta)$ dependence which would be expected for the analogue resonance corresponding to an $\ell = 3$ parent state.

Such a qualitative comparison suggests quite good agreement between the (p,p₀) data and the assignments for the parent states in all cases except $^{162}\text{Dy}(p,p_0)$. The excitation functions for this case are shown in figure 2.7 and a pronounced dip is visible in the 140.5° curve at a proton energy of about 10.9 MeV. This suggests that the state seen in the (d,p) reaction, for which the analogue is predicted at 10.88 MeV, is in fact two unresolved states, one of which is definitely not of $\ell = 3$. However, to make a proper quantitative comparison between the (p,p₀) data and the parent state assignments it is

necessary to attempt to fit the experimental resonance shapes with equation 1.25 and extract a set of resonance parameters which reproduce the data at all angles.

3.1 RESONANCE FITTING PROCEDURE

The analysis of the elastic scattering excitation functions was carried out using two separate fitting routines. The first of these, PREFIT, performed a fit to the data at a single angle. This was used for fitting the 141° excitation functions which are inherently less complicated than those at other angles because of the very small effect of $\ell = 3$ resonances in the neighbourhood of $\theta_{\text{CM}} = 140.7^\circ$ where $P_3(\cos\theta)$ vanishes. This thinning out of the resonance structure near to 141° proved to be very useful in fitting the data, since the observed resonances were overlapping in nearly all cases. In the absence of the $\ell = 3$ resonances, there were typically about three strong $\ell = 1$ resonances remaining in the first MeV of the excitation function above the position of the ground-state analogue.

The second fitting routine, DOUBFIT, performed a theoretical fit to the data at two angles simultaneously and this was used for the 125° and 160° excitation functions. Because of the additional presence of the $\ell = 3$ resonances, in some cases as many as eight resonances had to be included in the fitting in order to reproduce the first MeV of the excitation function above the position of the ground-state analogue.

The philosophy in both fitting procedures was to attempt to use the spin-parity assignments which already existed for the parent states. If the data could be reproduced reasonably well at all angles, with agreement between the resonance parameters obtained from both PREFIT and DOUBFIT, then the orbital angular momenta of the stronger resonances could be established, thus providing a partial confirmation

of the Nilsson assignment for the parent state. If a satisfactory fit could not be obtained then judicious changes of spin and parity were made for the resonances concerned. Fits were only attempted for the part of the excitation function corresponding to about the first MeV of excitation in the parent nucleus; at higher excitations the density of states becomes prohibitively large for resonance fitting.

3.1.1 Theoretical Fit Function

The theoretical expression used for the elastic scattering cross-section in the resonance region was that given earlier as equations 1.25, 1.30 and 1.31. It is a superposition of individual resonance amplitudes and a non-resonant background amplitude arising from potential scattering. Figure 3.1 shows a set of typical $\ell = 1$ and $\ell = 3$ resonance shapes calculated by equation 1.25 for the three angles at which data were fitted. An energy-independent, non-resonant background is assumed and the interference shapes are drawn from $E_r - 2\Gamma$ to $E_r + 2\Gamma$. The small effect discernible at 140.8° for the $\ell = 3$ resonance arises from the b amplitude in equation 1.25 which has an angular dependence determined by $P_3^1(\cos\theta)$.

As the experiments were all performed in the neighbourhood of the proton Coulomb barrier of the target nuclei, the non-resonant background amplitude arose mainly from Rutherford scattering and consequently decreased monotonically with increasing proton energy. The form of the background amplitude at each angle was therefore taken to be

$$\rho(E) = \sum_{n=0}^3 \frac{a_n}{E^n}, \quad 3.1$$

where a_n are the constants and E is the energy of the incident proton in the centre-of-mass system. This type of empirical background

Figure 3.1: Sample resonance shapes calculated using equation 1.25 for the $l = 1$ and $l = 3$ partial proton waves at $\theta_{CM} = 125.3^\circ$, 140.8° and 160.0° . The non-resonant background was taken to be energy independent with total phases typical of those obtained from fitting the experimental excitation functions.

description has been used by several authors (for example, [Za68, Ab69, Bu70, Gr70a]) and has been found to be equivalent to the slightly more rigorous optical-model description, which is often employed, with well parameters obtained from off-resonance angular distributions.

Both fitting routines varied the energy E_r , total width Γ , partial width Γ_p and total phase α_ℓ for each resonance, together with the background parameters a_n , in order to minimise the quantity

$$\chi^2 = \frac{1}{N_f} \sum \frac{\left[\frac{d\sigma}{d\Omega}_{\text{ex}} - \frac{d\sigma}{d\Omega}_{\text{th}} \right]^2}{\left[\Delta \left(\frac{d\sigma}{d\Omega}_{\text{ex}} \right) \right]^2} . \quad 3.2$$

The summation extends over all data points in the fitting region at a single angle for PREFIT and at two angles for DOUBFIT. $\frac{d\sigma}{d\Omega}_{\text{ex}}$ and $\Delta \left(\frac{d\sigma}{d\Omega}_{\text{ex}} \right)$ are respectively the experimental differential cross-section and its estimated error, based on counting statistics, while $\frac{d\sigma}{d\Omega}_{\text{th}}$ is the theoretical differential cross-section calculated by means of equation 1.25. The number of degrees of freedom is represented by N_f , which is the number of experimental points less the number of free parameters. The minimisation of χ^2 for both PREFIT and DOUBFIT was carried out by the general search routine FITTEM [Ha68].

The coding of the fit function in PREFIT was checked by calculating the theoretical cross-section through a $1/2^+$ resonance in $^{92}\text{Mo}(p, p_0)$ [Mo66] and a $3/2^-$ resonance in $^{142}\text{Nd}(p, p_0)$ [Gr70a] using the resonance parameters fitted by the authors. In both cases, the calculated resonance shape matched the data as well as the original fit.

3.1.2 Simplifying Assumptions and Restraints

In some cases up to eight resonances were included in the fit and, with four free parameters per resonance, the computer time required

to vary all parameters was prohibitive. The number of free parameters was therefore reduced by the following assumptions, both of which have been shown to be reasonable in previous analogue resonance studies by other authors:

(a) The spacing of the analogue states was taken to be the same as the spacing of the parent states; this has been found to be true, within the experimental errors, in the study of analogue resonances in many nuclei including the ytterbium isotopes [Ca68]. Consequently, only one resonance energy (that of the ground-state analogue) was varied and all other resonance positions were fixed with respect to it. In all cases the excitation energies used for the parent states were those obtained from high resolution (d,p) experiments performed with a magnetic spectrograph.

(b) At a particular angle, the total phase α_ℓ in equations 1.30 and 1.31 was assumed to be the same for all resonances of the same ℓ value. This assumption is in accord with the work of Grosse et al. [Gr70a] whose analysis of analogue resonances in the elastic proton scattering from ^{142}Nd demonstrates that α_ℓ is essentially a function of ℓ . This assumption also implies a negligible energy-dependence of the phase over the energy region of the fit. A check on the validity of this energy-independence was made by fitting the $^{166}\text{Er}(p,p_0)$ and $^{168}\text{Er}(p,p_0)$ resonance data with an energy-dependent phase of the form

$$\alpha_\ell(E) = p + q(E - 11) , \quad 3.3$$

where p and q are variable parameters and E is the proton bombarding energy in the centre-of-mass system. In both cases, the fitted value of p was very close to the value found for the energy-independent phase and q was (0.1 ± 0.8) . Thus, while this does not rule out the possibility of the phase being energy-dependent, it does show that the

present data are not sensitive to any energy-dependence and the assumption employed here was justified.

Some temporary additional restraints were placed on the widths of the resonances corresponding to weaker parent states. The total width was taken to be equal to the total width of the nearest strong resonance of the same ℓ value. In addition, the partial width was fixed, relative to that of the strong resonance, in the ratio of the differential cross-sections for the (d,p) reaction leading to the corresponding parent states. Once the fit was converging, the partial width of the weaker resonance was then allowed to vary independently; however, the total width usually remained tied to that of the stronger resonance.

In fitting the 125° and 160° data, the parameters of the $\ell = 1$ resonances were initially fixed at the values obtained from the fit to the 141° data. However, this restraint was removed and the parameters varied freely in the final stages of the fitting.

Starting values for the background parameters at each angle were obtained from a least squares fit to the excitation function with a heavy weighting being placed on the data points below the ground-state analogue. In no case were terms of higher order than $1/E^3$ required.

The main criterion for "goodness of fit" was, of course, the minimisation of χ^2 , given by equation 3.2, but a careful check had to be kept on the parameters to ensure that they remained sensible. If a parameter tried to change in an unphysical manner, then it was temporarily fixed at a reasonable value until a different point on the χ^2 surface was reached. Furthermore, the phase parameter α_ℓ was expected to vary smoothly with angle [Wi70] and checks were made to ensure that

this was so in the fitted values. Thus, the search by FITTEM was carried out in a very controlled manner in as many as twenty or thirty separate stages with a visual inspection of the parameters and fits at each interruption.

3.1.3 Uncertainties in Fitted Parameters

Although the search code FITTEM does not give a true standard error for the fitted parameters, it can be made to calculate the change in a parameter required to increase χ^2 by a given factor above its minimum value. The magnitude of this change obviously depends on the required increase in χ^2 but it should give a reasonable estimate of the relative uncertainties among the different parameters. True standard errors on the resonance parameters are, however, obtainable from a fit to the data by the non-linear least squares method [Fe65a]. In this approach, a set of linear normal equations are obtained by approximating the non-linear fit function by a truncated Taylor series in the fitted parameters. The standard errors σ_i are then calculated, as for a linear least squares procedure, from the diagonal elements of the inverse of the normal matrix G by the equation

$$\sigma_i^2 = \chi^2 G^{-1}_{ii}, \quad 3.4$$

where χ^2 is as defined in equation 3.2 and G^{-1} is known as the variance-covariance matrix. A resonance fitting programme which employed this method was used to calculate the standard errors on a set of fitted resonance parameters [Wh72] for comparison with the uncertainties estimated by FITTEM. In table 3.1, the standard errors are shown together with the uncertainties estimated by FITTEM for a 50% increase in χ^2 above its minimum. It can be seen that the 50% criterion employed by FITTEM slightly underestimates the standard error on E_r but

overestimates for Γ_p and Γ . From these results, it can be concluded that the 50% criterion gives reasonable error estimates and all errors on resonance parameters quoted in this work have been calculated in this way.

Parameter	Value	FITTEM uncertainty	Standard error
E_r (MeV)	10.635	0.016	0.020
Γ_p (keV)	7.2	2.6	1.9
Γ (keV)	112	57	44
α_ℓ (rad)	-0.94	0.26	0.26

Table 3.1. Comparison of the uncertainties in resonance parameters calculated by FITTEM, for a 50% increase in χ^2 above its minimum, with the standard errors obtained by a non-linear least squares method [Wh72].

The off-diagonal elements of the variance-covariance matrix of the least squares method are also useful as they describe the correlations between the parameters being varied. A correlation matrix can be calculated with the elements $\rho_{ii'}$, given by

$$\rho_{ii'} = \chi^2 G^{-1}_{ii'} / (\sigma_i \sigma_{i'}) . \quad 3.5$$

The element $\rho_{ii'}$, gives the correlation between the parameters designated by the subscripts i and i' ; comparison with equation 3.4 shows that if $i = i'$ then the matrix element is unity. For the resonance used above in the standard error calculation, the correlation matrix is

$$\begin{array}{c}
 E_r \quad \Gamma_p \quad \Gamma \quad \alpha_\ell \\
 \begin{array}{c} E_r \\ \Gamma_p \\ \Gamma \\ \alpha_\ell \end{array} \left[\begin{array}{cccc} 1 & 0.2 & 0.03 & 0.7 \\ 0.2 & 1 & 0.7 & 0.3 \\ 0.03 & 0.7 & 1 & 0.2 \\ 0.7 & 0.3 & 0.2 & 1 \end{array} \right] .
 \end{array}$$

The parameters corresponding to each row and column are indicated on the left and top sides of the matrix. It can be seen that there exist relatively strong correlations between Γ_p and Γ and between E_r and α_ℓ . Because these correlations are so strong, it would be very valuable to have inelastic scattering or (p,n) data through the same resonances in order to obtain independent estimates of E_r and Γ . Unfortunately, this was not possible here because of high level-densities and interference effects in the excitation functions for the 2^+ inelastic channel and the overlapping complexity of the 4^+ channel data. Although the neutron decay of the resonances was not investigated, it is anticipated that the large neutron background and high level-density would make the extraction of resonance parameters from (p,n) data equally difficult. A check can be made, however, on whether the phases α_ℓ are in approximate agreement with those obtained in cases where E_r could be obtained independently from inelastic data.

3.2 RESULTS OF FITTING

The results of the fits to the resonances are given in this section, in order of increasing target mass, in figures 3.2 to 3.8 and tables 3.2 to 3.8. Taking $^{160}\text{Gd}(p,p_0)$ as an example, the 141° fit is presented in both graphical and tabular form in figure 3.2(a) and table 3.2(a) while the results obtained at 125° and 160° using DOUBFIT are shown in figure 3.2(b) and table 3.2(b). The data points shown in the

figures are those included in the fitting procedure to obtain the fits represented by the solid lines. The position of each resonance included in the fit is indicated by an arrow labelled with a capital letter. The label is then used to identify the resonance in column 1 of the corresponding table. Columns 2 and 3 give the existing Nilsson assignment and the excitation energy of the parent state associated with each resonance; these are taken in most cases from the references [Bu66, Tj67, Tj69, Gr70b] for the (d,p) work on the same target. Columns 4, 5 and 6 contain the fitted resonance energy, partial proton

Target	θ_{CM} (deg)	a_0	a_1	a_2	a_3
^{160}Gd	125.3	3.07	-73.3	1106.9	0.0
	140.7	-5.07	81.0	283.9	0.0
	160.0	2.97	-101.3	1268.1	-441.4
^{162}Dy	125.3	34.58	-1077.9	11644.6	-35704.4
	140.7	9.05	-218.1	1917.2	0.0
	160.0	5.64	-150.3	1532.7	-114.7
^{164}Dy	125.3	0.61	8.5	269.7	2951.5
	141.2	-11.57	228.2	-482.1	0.0
	160.1	2.74	-82.9	1115.9	0.0
^{166}Er	125.3	-3.23	63.5	475.3	0.0
	141.2	-16.20	324.9	-936.3	0.0
	160.1	-3.01	39.1	515.9	0.0
^{168}Er	125.3	-4.59	93.9	255.6	0.0
	141.2	-7.69	155.6	-142.3	0.0
	160.1	-5.79	105.4	132.1	0.0
^{170}Er	125.3	101.58	-3036.1	30159.7	-91262.5
	140.7	4.50	-123.3	1460.0	0.0
	160.0	-9.56	171.8	-153.9	0.0
^{174}Yb	125.3	-41.84	885.5	-3874.5	0.0
	140.7	7.60	-206.0	2033.8	0.0
	160.0	-8.91	169.7	-160.3	0.0

Table 3.9. Coefficients used to describe the non-resonant scattering.

width and total width while column 7 shows the assumed spin and parity. An entry in the last column, headed ℓ_{pp} , indicates that an ℓ value assignment was possible from the (p, p_0) data. In the subsidiary table the fitted values of the total phase α_ℓ are given for each angle and ℓ value. Between the main and subsidiary tables, the fitted energy (in centre-of-mass co-ordinates) of the ground-state analogue, with respect to which all resonance energies were fixed, and the values of χ^2 per degree of freedom for the best fit are also shown.

The parameters used to describe the non-resonant background scattering are summarised in table 3.9.

3.2.1 $^{160}\text{Gd}(p, p_0)$

The experimental data and theoretical fits for $^{160}\text{Gd}(p, p_0)$ are shown in figures 3.2(a) and 3.2(b) and the tabulated resonance parameters in tables 3.2(a) and 3.2(b). An initial comment can be made that the assignments already in existence for the parent states were all quite adequate as a basis for fitting the resonance data.

The 140.7° excitation function is dominated by the two $\ell = 1$ resonances, C and D, which are the analogues of the $\frac{3}{2} \frac{3}{2}$ - (521) and $\frac{1}{2} \frac{1}{2}$ - (521) states both of which are strongly populated in the (d, p) reaction. An additional dip in the 140.7° data at about 11.06 MeV (in centre-of-mass co-ordinates) was attributed to the state at 0.834 MeV excitation in ^{161}Gd ; this state was observed in the (d, p) reaction [Tj67] but no assignment was made for it. As there is no effect in the 90° data (figure 2.6) at this energy, an $\ell = 1$ resonance, G, was tried and, although the parameter uncertainties are large, an $\ell = 1$ assignment for the 0.834 MeV state seems quite reasonable. A similar state was observed in the $^{162}\text{Dy}(p, p_0)$ data at 0.817 MeV in the isotone ^{163}Dy and this was also found to have $\ell = 1$ characteristics.

Figure 3.2(a): Theoretical fit to the elastic scattering excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{160}\text{Gd}(p,p_0)$. The energies of the resonances included in the fit are indicated by arrows and the fitted parameters are given in table 3.2(a).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
C	$\frac{3}{2} \frac{3}{2}^-$ - (521)	0.313	10.539	5.1 ± 1.7	186 ± 75	$3/2^-$	1
D	$\frac{1}{2} \frac{1}{2}^-$ - (521)	0.356	10.582	5.5 ± 2.4	110 ± 64	$1/2^-$	1
G	-	0.834	11.060	2.0 ± 2.0	84 ± 107	$1/2^-$	1

$$E_r^{\text{g.s.}} = (10.226 \pm 0.020) \text{ MeV} \quad \chi^2 = 2.3$$

θ_{CM}	ℓ	Phase (rad)
140.7°	1	-0.88 ± 0.18

Table 3.2(a): Resonance parameters used to calculate the theoretical excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{160}\text{Gd}(p,p_0)$ as drawn in figure 3.2(a). An entry in the column headed ℓ_{pp} indicates that an ℓ value assignment for the resonance is possible from the elastic scattering data.

Figure 3.2(b): Theoretical fits to the elastic scattering excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.0° for $^{160}\text{Gd}(p, p_0)$. The energies of the resonances included in the fits are indicated by arrows and the fitted parameters are given in table 3.2(b).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
A	$\frac{5}{2} \frac{5}{2}^-$ (523)	0.0	10.231	0.4	77	$5/2^-$	-
B	$\frac{7}{2} \frac{5}{2}^-$ (523)	0.073	10.304	0.5 ± 0.9	77 ± 70	$7/2^-$	-
C	$\frac{3}{2} \frac{3}{2}^-$ (521)	0.313	10.544	5.6 ± 2.4	219 ± 115	$3/2^-$	1
D	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.356	10.587	5.6 ± 3.3	123 ± 93	$1/2^-$	1
E	$\frac{5}{2} \frac{1}{2}^-$ (521)	0.438	10.669	2.8 ± 1.5	121 ± 84	$5/2^-$	3
F	$\frac{7}{2} \frac{1}{2}^-$ (521)	0.529	10.760	0.2 ± 0.4	121	$7/2^-$	-
G	-	0.834	11.065	2.0 ± 2.4	82 ± 115	$1/2^-$	1
H	$\frac{7}{2} \frac{5}{2}^-$ (512)	0.889	11.120	3.0 ± 1.1	123 ± 55	$7/2^-$	3

$$E_r^{g.s.} = (10.231 \pm 0.018) \text{ MeV} \quad \chi^2 = 4.0$$

θ_{CM}	ℓ	Phase (rad)
125.3°	1	-1.26 ± 0.56
	3	2.54 ± 0.70
160.0°	1	-0.28 ± 0.26
	3	3.22 ± 0.26

Table 3.2(b). Resonance parameters used to calculate the theoretical excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.0° for $^{160}\text{Gd}(p, p_0)$ as drawn in figure 3.2(b). An entry in the column headed ℓ_{pp} indicates that an ℓ value assignment for the resonance is possible from the elastic scattering data.

In the 125.3° and 160.0° excitation functions, the two $\ell = 1$ resonances, C and D, produced marked effects which are reproduced by parameters which agree, within the uncertainties, with those obtained at 140.7° . Strong characteristic $\ell = 3$ interference effects are quite consistent with resonances E and H being the analogues of the $\frac{5}{2} \frac{1}{2}^-$ (521) and $\frac{7}{2} \frac{5}{2}^-$ (512) states in ^{161}Gd . The weak resonances, A, B and F, all correspond to states with $\ell = 3$ assignments in the parent and a slight improvement in χ^2 was made by their inclusion in the fit. However, this cannot be regarded as a clear confirmation of the assignments as, for example, in the case of the parents of resonances E and H.

3.2.2 $^{162}\text{Dy}(p, p_0)$

The elastic scattering data collected from ^{162}Dy has undoubtedly been the most interesting of this work because of the fact that the resonance structure has shown marked differences from that expected from the (d,p) measurements of the parent ^{163}Dy [Gr70b]. The data and the best fits obtained are shown in figures 3.3(a) and 3.3(b) with the corresponding parameters in tables 3.3(a) and 3.3(b). It must be emphasised that these fits are not regarded as completely satisfactory although they do allow some conclusions to be drawn from the data.

The most obvious evidence of a discrepancy between the elastic scattering data and the assignments made after the most recent (d,p) measurements [Gr70b] is provided by the dip in the 140.7° excitation function at 10.836 MeV. At the corresponding excitation in ^{163}Dy , no states with $\ell = 1$ assignments were observed in the (d,p) reaction although the strongest state populated, with a $\frac{7}{2} \frac{5}{2}^-$ (512) assignment, is quite close at 0.801 MeV excitation. A state at 0.821 MeV excitation in ^{163}Dy was, however, observed in (n, γ) studies on ^{162}Dy [Sc67] and its J^π was assigned to be $3/2^-$. The fitting of the 140.7° data

Figure 3.3(a): Theoretical fit to the elastic scattering excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{162}\text{Dy}(p, p_0)$. The resonance parameters used for the fit are given in table 3.3(a).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
A	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.350	10.369	1.6 ± 1.7	63 ± 93	$1/2^-$	-
B	-	0.437	10.456	2.2 ± 1.1	95 ± 60	$3/2^-$	-
F	-	0.817	10.836	3.2 ± 1.1	125 ± 62	$3/2^-$	1
H	$\frac{3}{2} \frac{1}{2}^-$ (510)	1.199	11.218	3.6 ± 0.9	101 ± 36	$3/2^-$	1

$$E_r^{\text{g.s.}} = (10.019 \pm 0.013) \text{ MeV} \quad \chi^2 = 2.2$$

θ_{CM}	ℓ	Phase (rad)
140.7°	1	-0.78 ± 0.18

Table 3.3(a): Resonance parameters used to calculate the theoretical excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{162}\text{Dy}(p, p_0)$ as drawn in figure 3.3(a).

Figure 3.3(b): Theoretical fits to the elastic scattering excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.0° for $^{162}\text{Dy}(p, p_0)$. The resonance parameters used for the fits are given in table 3.3(b).

	Parent		Analogue					
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}	
A	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.350	10.373	2.9 ± 2.4	71 ± 64	$1/2^-$	-	
B	-	0.437	10.460	2.0 ± 1.1	87 ± 60	$3/2^-$	-	
C	$\frac{7}{2} \frac{1}{2}^-$ (521)	0.517	10.540	1.0 ± 0.9	116 ± 131	$7/2^-$	3	
D	$\frac{7}{2} \frac{3}{2}^-$ (521)	0.556	10.579	0.7 ± 0.9	116	$7/2^-$	-	
E	$\frac{7}{2} \frac{5}{2}^-$ (512)	0.801	10.824	2.5 ± 0.9	115 ± 53	$7/2^-$	3	
F	-	0.817	10.840	3.2 ± 1.3	138 ± 78	$3/2^-$	1	
G	-	0.949	10.972	0.7 ± 0.8	115	$7/2^-$	-	
H	$\frac{3}{2} \frac{1}{2}^-$ (510)	1.199	11.222	3.7 ± 1.1	111 ± 46	$3/2^-$	1	

$$E_r^{g.s.} = (10.023 \pm 0.013) \text{ MeV} \quad \chi^2 = 2.4$$

θ_{CM}	ℓ	Phase (rad)
125.3°	1	-1.08 ± 0.36
	3	2.58 ± 0.40
160.0°	1	-0.28 ± 0.22
	3	3.32 ± 0.26

Table 3.3(b). Resonance parameters used to calculate the theoretical excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.0° for $^{162}\text{Dy}(p, p_0)$ as drawn in figure 3.3(b).

showed that the unexpected dip could indeed be reproduced by the inclusion of an $\ell = 1$ resonance, F, at an energy which corresponds to (0.817 ± 0.029) MeV in ^{163}Dy . This is in good agreement with the (n,γ) data. At the other two angles it was found necessary to include both an $\ell = 3$ and an $\ell = 1$ resonance, E and F respectively, to reproduce this region of the excitation function. Thus, it seems likely that the strong state observed at 0.801 MeV excitation in $^{162}\text{Dy}(d,p)$ is in fact an unresolved doublet with $\ell = 3$ and $\ell = 1$ components. It is striking that the analogue resonance experiment, with its inherently worse resolution, was able to separate two states which were not resolved in the (d,p) measurements. The $\ell = 1$ member of the doublet may well have the same configuration as the unassigned $\ell = 1$ state at 0.834 MeV excitation in ^{161}Gd which was seen in the $^{160}\text{Gd}(p,p_0)$ data.

The second conflict with the existing assignments for the parent states in ^{163}Dy arises from the anomaly in the cross-section near to 10.4 MeV. The depression at 140.7° is very broad and attempts to fit it solely with the analogue of the $\frac{1}{2} \frac{1}{2}^-$ (521) state at 0.350 MeV in ^{163}Dy proved unsuccessful. It appears that the effect arises from more than one resonance. The only other state in this energy region which is strongly excited in the (d,p) reaction is at 0.425 MeV and carries a $\frac{5}{2} \frac{1}{2}^-$ (521) assignment. However, deviations from the Nilsson model predictions for the intensity of the different members of the $\frac{1}{2}^-$ (521) band prompted Grottdal et al. [Gr70b] to suggest that part of the strength of the 0.425 MeV state may arise from the $\frac{3}{2} \frac{3}{2}^-$ (521) state. Thus, two $\ell = 1$ resonances, A and B, corresponding to the 0.350 MeV and 0.425 MeV states respectively, were included in the fitting of the 140.7° data and a great improvement was seen. The energy of the unpredicted $\ell = 1$ resonance, B, was varied and found to correspond to an excitation of (0.437 ± 0.031) MeV in ^{163}Dy . However, when the same

two resonances, A and B, were included in the fits at 125.3° and 160.0° the data were not satisfactorily reproduced and the fitted parameters for resonance A are not consistent with those obtained at 140.7° . The inclusion at 125.3° and 160.0° of an $\ell = 3$ resonance corresponding to the $\frac{5}{2} \frac{1}{2}^-$ (521) state, previously assigned at 0.425 MeV in ^{163}Dy , was found not to be required as the fitted partial width was invariably less than 0.1 keV. A small anomaly in this energy region of the 89.6° data in figure 2.7 could indicate a positive parity resonance but this was not tried in the fitting.

The resonance, H, corresponding to the $\frac{3}{2} \frac{1}{2}^-$ (510) state in ^{163}Dy was well fitted at all angles and its $\ell = 1$ character is certainly confirmed.

The $\ell = 3$ effect at 10.540 MeV is adequately reproduced by the analogue, C, of the $\frac{7}{2} \frac{1}{2}^-$ (521) state with a small contribution from the $\ell = 3$ resonance, D, corresponding to the $\frac{7}{2} \frac{3}{2}^-$ (521) state. There is a suggestion of a small $\ell = 3$ effect in the 160.0° data at 10.97 MeV and an attempt was made to reproduce this by including a tentative $7/2^-$ resonance corresponding to an unassigned state at 0.949 MeV in ^{163}Dy . However, the resonant effect is small and no definite conclusions can be made about it.

3.2.3 $^{164}\text{Dy}(p, p_0)$

The experimental and fitted excitation functions for the elastic scattering from ^{164}Dy are shown in figures 3.4(a) and 3.4(b) with the respective resonance parameters in tables 3.4(a) and 3.4(b). In this case, the data were reproduced theoretically without any modifications or additions to the assignments which already existed for the parent states in ^{165}Dy [Gr70b]. The main features of the excitation functions are the strong resonances, A and E, whose $\ell = 1$ character

Figure 3.4(a): Theoretical fit to the elastic scattering excitation function at $\theta_{\text{CM}} = 141.2^\circ$ for $^{164}\text{Dy}(p,p_0)$. The resonance parameters used for the fit are given in table 3.4(a).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_P (keV)	Γ (keV)	J^π	ℓ_{PP}
A	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.109	10.651	9.7 ± 2.8	131 ± 51	$1/2^-$	1
E	$\frac{3}{2} \frac{1}{2}^-$ (510)	0.606	11.148	3.6 ± 1.4	105 ± 57	$3/2^-$	1

$$E_r^{\text{g.s.}} = (10.542 \pm 0.016) \text{ MeV} \quad \chi^2 = 1.8$$

θ_{CM}	ℓ	Phase (rad)
141.2°	1	-0.76 ± 0.26

Table 3.4(a): Resonance parameters used to calculate the theoretical excitation function at $\theta_{\text{CM}} = 141.2^\circ$ for $^{164}\text{Dy}(p,p_0)$ as drawn in figure 3.4(a).

Figure 3.4(b): Theoretical fits to the elastic scattering excitation functions at $\theta_{\text{CM}} = 125.3^\circ$ and 160.1° for $^{164}\text{Dy}(p,p_0)$. The resonance parameters used for the fits are given in table 3.4(b).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
A	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.109	10.645	8.4 ± 2.4	134 ± 46	$1/2^-$	1
B	$\frac{5}{2} \frac{1}{2}^-$ (521)	0.182	10.718	0.9 ± 0.9	83 ± 95	$5/2^-$	-
C	$\frac{7}{2} \frac{5}{2}^-$ (512)	0.262	10.798	1.8 ± 0.6	90 ± 42	$7/2^-$	3
D	$\frac{7}{2} \frac{1}{2}^-$ (521)	0.298	10.834	0.9 ± 0.6	73 ± 57	$7/2^-$	3
E	$\frac{3}{2} \frac{1}{2}^-$ (510)	0.606	11.142	3.0 ± 1.1	101 ± 55	$3/2^-$	1

$$E_r^{g.s.} = (10.536 \pm 0.011) \text{ MeV} \quad \chi^2 = 1.5$$

θ_{CM}	ℓ	Phase (rad)
125.3°	1	-1.30 ± 0.62
	3	2.88 ± 0.52
160.1°	1	-0.58 ± 0.26
	3	3.26 ± 0.22

Table 3.4(b). Resonance parameters used to calculate the theoretical excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.1° for $^{164}\text{Dy}(p, p_0)$ as drawn in figure 3.4(b).

agrees with the assignments of $\frac{1}{2} \frac{1}{2}^-$ (521) and $\frac{3}{2} \frac{1}{2}^-$ (510) for the corresponding parent states. At 125.3° and 160.1° , the inclusion in the fitting of the $\ell = 3$ resonances, B, C and D, was found to reproduce adequately the measured cross-section between the two $\ell = 1$ effects. As the fit function used here assumes isolated resonances, it appears that any interference between C and D, which both have spin and parity of $7/2^-$ and are only 36 keV apart, does not have a large effect on the cross-section.

3.2.4 $^{166}\text{Er}(p, p_0)$

The experimental and fitted excitation functions for $^{166}\text{Er}(p, p_0)$ and the corresponding parameters are shown in figures 3.5(a) and 3.5(b) and tables 3.5(a) and 3.5(b) respectively. The parent nucleus in this case is an isotone of ^{165}Dy and the resonance effects in the scattering from ^{164}Dy and ^{166}Er are quite similar. The 141.2° curve is again dominated by the analogues, A and F, of the $\frac{1}{2} \frac{1}{2}^-$ (521) and $\frac{3}{2} \frac{1}{2}^-$ (510) states although a weak resonance, E, arising from the $\frac{3}{2} \frac{3}{2}^-$ (521) state in ^{167}Er , was also included in the fitting. The same state probably exists in ^{165}Dy but, as it is predominantly a hole-state and ^{165}Dy is not accessible via the (d,t) reaction, it has not been identified there. At 125.3° and 160.1° , there is again a close similarity with the $^{164}\text{Dy}(p, p_0)$ data with the $\ell = 3$ resonances, B, C and D, arising respectively from the same $\frac{5}{2} \frac{1}{2}^-$ (521), $\frac{7}{2} \frac{1}{2}^-$ (521) and $\frac{7}{2} \frac{5}{2}^-$ (512) states observed in ^{165}Dy .

3.2.5 $^{168}\text{Er}(p, p_0)$

The data and fits for $^{168}\text{Er}(p, p_0)$ are shown in figures 3.6(a) and 3.6(b) with the resonance parameters in tables 3.6(a) and 3.6(b). The two strong $\ell = 1$ effects, A and D, arising from the analogues of

Figure 3.5(a): Theoretical fit to the elastic scattering excitation function at $\theta_{\text{CM}} = 141.2^\circ$ for $^{166}\text{Er}(p,p_0)$. The resonance parameters used for the fit are given in table 3.5(a).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
A	$\frac{1}{2} \frac{1}{2}^-$ - (521)	0.208	10.465	9.2 ± 3.3	120 ± 51	$1/2^-$	1
E	$\frac{3}{2} \frac{3}{2}^-$ - (521)	0.750	11.007	1.4 ± 1.3	73	$3/2^-$	-
F	$\frac{3}{2} \frac{1}{2}^-$ - (510)	0.802	11.059	3.1 ± 1.1	73 ± 37	$3/2^-$	1

$$E_r^{\text{g.s.}} = (10.257 \pm 0.018) \text{ MeV} \quad \chi^2 = 2.2$$

θ_{CM}	ℓ	Phase (rad)
141.2°	1	-0.92 ± 0.26

Table 3.5(a): Resonance parameters used to calculate the theoretical excitation function at $\theta_{\text{CM}} = 141.2^\circ$ for $^{166}\text{Er}(p,p_0)$ as drawn in figure 3.5(a).

Figure 3.5(b): Theoretical fits to the elastic scattering excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.1° for $^{166}\text{Er}(p, p_0)$. The resonance parameters used for the fits are given in table 3.5(b).

Figure 3.6(b): Theoretical fits to the elastic scattering excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.1° for $^{168}\text{Er}(p,p_0)$. The resonance parameters used for the fits are given in table 3.6(b).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
A	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.0	10.637	6.9 ± 2.4	120 ± 55	$1/2^-$	1
B	$\frac{7}{2} \frac{5}{2}^-$ (512)	0.176	10.813	2.6 ± 0.8	134 ± 60	$7/2^-$	3
C	$\frac{7}{2} \frac{1}{2}^-$ (521)	0.225	10.862	0.9 ± 0.8	134	$7/2^-$	-
D	$\frac{3}{2} \frac{1}{2}^-$ (510)	0.599	11.236	2.6 ± 0.8	80 ± 35	$3/2^-$	1
E	$\frac{5}{2} \frac{1}{2}^-$ (510)	0.654	11.291	1.1 ± 1.3	134	$5/2^-$	-
F	$\frac{3}{2} \frac{3}{2}^-$ (521)	0.714	11.351	0.4 ± 0.9	80	$3/2^-$	-

$$E_r^{g.s.} = (10.637 \pm 0.013) \text{ MeV} \quad \chi^2 = 1.1$$

θ_{CM}	ℓ	Phase (rad)
125.3°	1	-1.52 ± 0.62
	3	2.96 ± 0.52
160.1°	1	-0.44 ± 0.26
	3	3.68 ± 0.26

Table 3.6(b). Resonance parameters used to calculate the theoretical excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.1° for $^{168}\text{Er}(p, p_0)$ as drawn in figure 3.6(b).

the $\frac{1}{2} \frac{1}{2}^-$ (521) and $\frac{3}{2} \frac{1}{2}^-$ (510) states in the parent are again present. With two more neutrons in ^{169}Er than in ^{167}Er , the two states in this case are at lower excitations with the $\frac{1}{2} \frac{1}{2}^-$ (521) state being the ground state of ^{169}Er . The analogue, F, of the $\frac{3}{2} \frac{3}{2}^-$ (521) hole-state was also included but found to be very weak. Similarly, the large $\ell = 3$ anomaly at 125.3° and 160.1° was again satisfactorily fitted by including the analogues, B and C, of the $\frac{7}{2} \frac{5}{2}^-$ (512) and $\frac{7}{2} \frac{1}{2}^-$ (521) states. The lower excitation energy of the $\frac{1}{2}^-$ (510) band also brings the analogue, E, of the $5/2^-$ member down into the fitting region but no strong $\ell = 3$ effects are visible because of the close proximity of the large $\ell = 1$ resonance arising from the $3/2^-$ member of the same band.

3.2.6 $^{170}\text{Er}(p, p_0)$

The data and fits for $^{170}\text{Er}(p, p_0)$ are shown in figures 3.7(a) and 3.7(b) and the resonance parameters in tables 3.7(a) and 3.7(b). The major anomaly in the 140.7° data is that produced by resonance C which is the analogue of the $\frac{3}{2} \frac{1}{2}^-$ (510) state in ^{171}Er . Smaller effects in the data are attributed to resonances B and F, corresponding to the $\frac{1}{2} \frac{1}{2}^-$ (521) and $\frac{3}{2} \frac{3}{2}^-$ (512) states in the parent, but the uncertainties in the parameters are large and there are some unexplained oscillations in the data close to 11.7 MeV. Resonance C has a similar strong effect on the cross-section at 125.3° and 160.0° . The strong $\ell = 3$ interference pattern at about 10.88 MeV was well reproduced by resonance A corresponding to the $\frac{7}{2} \frac{5}{2}^-$ (512) state in ^{171}Er . Three additional $\ell = 3$ resonances, D, E and G, corresponding to the $\frac{5}{2} \frac{1}{2}^-$ (510), $\frac{7}{2} \frac{1}{2}^-$ (510) and $\frac{5}{2} \frac{3}{2}^-$ (512) states, were also included in the fit but their characteristic $\ell = 3$ interference shape is not so apparent to the eye.

Figure 3.7(a): Theoretical fit to the elastic scattering excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{170}\text{Er}(p,p_0)$. The resonance parameters used for the fit are given in table 3.7(a).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
B	$\frac{1}{2} \frac{1}{2}^-$ - (521)	0.195	11.063	2.5 ± 2.8	146 ± 225	$1/2^-$	-
C	$\frac{3}{2} \frac{1}{2}^-$ - (510)	0.745	11.613	5.6 ± 1.3	148 ± 46	$3/2^-$	1
F	$\frac{3}{2} \frac{3}{2}^-$ - (512)	0.906	11.774	1.5 ± 1.1	122 ± 173	$3/2^-$	-

$$E_r^{\text{g.s.}} = (10.868 \pm 0.024) \text{ MeV} \quad \chi^2 = 3.1$$

θ_{CM}	ℓ	Phase (rad)
140.7°	1	-0.90 ± 0.22

Table 3.7(a): Resonance parameters used to calculate the theoretical excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{170}\text{Er}(p,p_0)$ as drawn in figure 3.7(a).

Figure 3.7(b): Theoretical fits to the elastic scattering excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.0° for $^{170}\text{Er}(p, p_0)$. The resonance parameters used for the fits are given in table 3.7(b).

	Parent		Analogue					
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}	
A	$\frac{7}{2} \frac{5}{2}^-$ (512)	0.076	10.956	1.7 ± 0.8	126 ± 86	$7/2^-$	3	
B	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.195	11.075	2.8 ± 3.1	149 ± 191	$1/2^-$	-	
C	$\frac{3}{2} \frac{1}{2}^-$ (510)	0.745	11.625	6.5 ± 1.1	144 ± 33	$3/2^-$	1	
D	$\frac{5}{2} \frac{1}{2}^-$ (510)	0.795	11.675	1.4 ± 0.8	105 ± 89	$5/2^-$	-	
E	$\frac{7}{2} \frac{1}{2}^-$ (510)	0.880	11.760	0.6 ± 0.7	105	$7/2^-$	-	
F	$\frac{3}{2} \frac{3}{2}^-$ (512)	0.906	11.786	2.1 ± 1.1	138 ± 100	$3/2^-$	-	
G	$\frac{5}{2} \frac{3}{2}^-$ (512)	0.972	11.852	1.7 ± 1.3	113 ± 107	$5/2^-$	-	

$$E_r^{g.s.} = (10.880 \pm 0.016) \text{ MeV} \quad \chi^2 = 1.3$$

θ_{CM}	ℓ	Phase (rad)
125.3°	1	-1.28 ± 0.26
	3	2.74 ± 0.44
160.0°	1	-0.36 ± 0.16
	3	3.44 ± 0.30

Table 3.7(b). Resonance parameters used to calculate the theoretical excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.0° for $^{170}\text{Er}(p,p_0)$ as drawn in figure 3.7(b).

3.2.7 $^{174}\text{Yb}(p, p_0)$

The elastic scattering of protons from ^{174}Yb was measured to enable a comparison to be made of the resonance parameters determined here with those obtained by other experimenters; both Foissel et al. [Fo72] and Whineray et al. [Wh70] have studied the analogue states in $^{174}\text{Yb}(p, p_0)$ and extracted resonance parameters.

The data and fitted excitation functions for $^{174}\text{Yb}(p, p_0)$ in the region of the first large $\ell = 1$ resonance are shown in figures 3.8(a) and 3.8(b). Tables 3.8(a) and 3.8(b) give the resonance parameters. The large $\ell = 1$ resonance, A, which dominates the excitation function at all angles is the analogue of the $\frac{3}{2} \frac{1}{2}^-$ (510) state in ^{175}Yb ; the same state was also found to have a large effect on the $^{170}\text{Er}(p, p_0)$ excitation function. A weak $\ell = 1$ resonance, C, arising from the $\frac{3}{2} \frac{3}{2}^-$ (512) state was also included in the 140.7° fit and the experimental cross-section was reproduced quite satisfactorily. For the two angle fit at 125.3° and 160.0° the analogue, B, of the $\frac{5}{2} \frac{1}{2}^-$ (510) state was included but no characteristic $\ell = 3$ effects are discernible because of the large $\ell = 1$ resonance only 48 keV less in excitation.

The parameters obtained from the data at 140.7° for the analogue of the $\frac{3}{2} \frac{1}{2}^-$ (510) state in ^{175}Yb are presented in table 3.10 for comparison with the parameters determined for the same resonance by Foissel et al. [Fo72] and Whineray et al. [Wh70]. It can be seen that the resonance energies are in reasonable agreement which is a useful check on the accuracy of the energy calibration of the ANU beam analysing magnet upon which all stated energies are based. Similarly, there is agreement, within the stated errors, between the different values for the total width of the resonance. However, a large discrepancy exists in the partial proton widths; the value for Γ_p/Γ obtained here

Figure 3.8(a): Theoretical fit to the elastic scattering excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{174}\text{Yb}(p,p_0)$. The resonance parameters used for the fit are given in table 3.8(a).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
A	$\frac{3}{2} \frac{1}{2}^-$ - (510)	0.552	11.636	7.7 ± 0.8	136 ± 20	$3/2^-$	1
C	$\frac{3}{2} \frac{3}{2}^-$ - (512)	0.809	11.893	0.8 ± 0.8	100 ± 158	$3/2^-$	-

$$E_r^{\text{g.s.}} = (11.084 \pm 0.011) \text{ MeV} \quad \chi^2 = 1.6$$

θ_{CM}	ℓ	Phase (rad)
140.7°	1	-0.88 ± 0.12

Table 3.8(a): Resonance parameters used to calculate the theoretical excitation function at $\theta_{\text{CM}} = 140.7^\circ$ for $^{174}\text{Yb}(p,p_0)$ as drawn in figure 3.8(a).

Figure 3.8(b): Theoretical fits to the elastic scattering excitation functions at $\theta_{\text{CM}} = 125.3^\circ$ and 160.0° for $^{174}\text{Yb}(p,p_0)$. The resonance parameters used for the fits are given in table 3.8(b).

	Parent		Analogue				
Res.	Previous assignment	E_x (MeV)	E_r (CM) (MeV)	Γ_p (keV)	Γ (keV)	J^π	ℓ_{pp}
A	$\frac{3}{2} \frac{1}{2}^-$ (510)	0.552	11.637	7.8 ± 0.6	133 ± 15	$3/2^-$	1
B	$\frac{5}{2} \frac{1}{2}^-$ (510)	0.600	11.685	3.3 ± 0.8	188 ± 55	$5/2^-$	-

$$E_r^{g.s.} = (11.085 \pm 0.011) \text{ MeV} \quad \chi^2 = 1.1$$

θ_{CM}	ℓ	Phase (rad)
125.3°	1	-1.56 ± 0.22
	3	3.04 ± 0.52
160.0°	1	-0.48 ± 0.08
	3	3.44 ± 0.22

Table 3.8(b). Resonance parameters used to calculate the theoretical excitation functions at $\theta_{CM} = 125.3^\circ$ and 160.0° for $^{174}\text{Yb}(p,p_0)$ as drawn in figure 3.8(b).

Author	E_r (MeV)	Γ_p (keV)	Γ (keV)	Γ_p/Γ
Present work (140.7°)	11.636 ± 0.011	7.7 ± 0.8	136 ± 20	0.057
Whineray [Wh70]	11.632 ± 0.020	8.4 ± 1.7	150 ± 30	0.056
Foissel [Fo72]	11.611 ± 0.020	14 ± 4	150 ± 30	0.093
PREFIT - Foissel	11.611	10.3 ± 2.6	170 ± 60	0.061

Table 3.10. Comparison of resonance parameters for the analogue of the $\frac{3}{2} \frac{1}{2}-$ (510) state in ^{175}Yb .

agrees well with that of Whineray but Foissel's value is about 65% larger. This disagreement was noted by Foissel et al. [Fo72] who suggested that it was experimental in origin. This idea was checked by performing a fit to Foissel's data using PREFIT and the parameters obtained in this case are shown in the bottom line of table 3.10. The agreement with the present values and with those of Whineray is then quite reasonable so that the discrepancy appears to come from the different fitting programmes used to analyse the data.

3.2.8 Summary of Orbital Angular Momentum Assignments

As noted in tables 3.2 to 3.7, the present measurements have confirmed the orbital angular momenta of 21 levels in the parent nuclei from ^{161}Gd to ^{171}Er . The criterion for making a definite ℓ value assignment was that the analogue of the level in question should have some visible effect on the cross-section and be satisfactorily fitted at all three angles. Thus, the total number of states for which an ℓ value assignment has been made is very much less than the number of states included in the fitting.

The states for which the orbital angular momentum has been confirmed are summarised in table 3.11. It can be seen that the

Previous assignment	ℓ_{pp}	Parent nucleus
$\frac{3}{2} \frac{1}{2}^-$ (510)	1	^{163}Dy , ^{165}Dy , ^{167}Er , ^{169}Er , ^{171}Er
$\frac{1}{2} \frac{1}{2}^-$ (521)	1	^{161}Gd , ^{165}Dy , ^{167}Er , ^{169}Er
$\frac{3}{2} \frac{3}{2}^-$ (521)	1	^{161}Gd
$\frac{7}{2} \frac{5}{2}^-$ (512)	3	^{161}Gd , ^{163}Dy , ^{165}Dy , ^{167}Er , ^{169}Er , ^{171}Er
$\frac{7}{2} \frac{1}{2}^-$ (521)	3	^{163}Dy , ^{165}Dy
$\frac{5}{2} \frac{1}{2}^-$ (521)	3	^{161}Gd
No assignment	1	^{161}Gd , ^{163}Dy

Table 3.11. States for which the orbital angular momentum has been confirmed by the analogue resonance measurements.

$\frac{3}{2} \frac{1}{2}^-$ (510), $\frac{1}{2} \frac{1}{2}^-$ (521) and $\frac{7}{2} \frac{5}{2}^-$ (512) states are all well-established in several nuclei. Among the other states for which ℓ values have been assigned is the state at 0.834 MeV excitation in ^{161}Gd for which no previous assignment existed. The $\ell = 1$ assignment for this state makes it highly likely that it is of the same configuration as the 0.817 MeV state in the isotone ^{163}Dy . No $\ell = 1$ state in this excitation region was identified in $^{162}\text{Dy}(d,p)$ but a $3/2^-$ state has been reported in neutron capture studies.

Considerable difficulty was encountered in fitting the lowest energy anomaly in the elastic scattering data from ^{162}Dy . However, it can be stated that no evidence was found to confirm the $\frac{5}{2} \frac{1}{2}^-$ (521) assignment for the 0.425 MeV state in ^{163}Dy . A close re-investigation of the assignments for all low-lying states in ^{163}Dy thus seems highly desirable.

3.2.9 Resonance Phases

As mentioned in section 3.1.3, the correlation which exists between E_r and α_ℓ in the parameter space makes it desirable that the values obtained for α_ℓ be compared with the resonance phases obtained for other nuclei where an independent estimate of the resonance energies could be made from the inelastic scattering data. This has been done in figure 3.9 where the total phase-shifts from this work are plotted, as a function of angle, together with the phase-shifts obtained from the fitting of analogue resonances observed in ^{143}Pm by $^{142}\text{Nd}(p, p_0)$ [Gr70a] and in ^{139}La by $^{138}\text{Ba}(p, p_0)$ [Wi70].

The analogue resonances in ^{143}Pm were seen in both the elastic and inelastic proton scattering from ^{142}Nd with the resonance energies in the p_0 and p_1 channels agreeing to within 5 keV. Thus, the phases quoted in that case can be regarded as reasonably reliable and free from any errors which might arise from the correlation between E_r and α_ℓ . The elastic scattering measurements on ^{138}Ba were not accompanied by inelastic scattering observations but are included in figure 3.9 as they are from the same mass region and they constitute one of the few cases where phases have been quoted together with the other fitted parameters. The phases in ^{139}La have been modified, by the addition of $+\pi$ to the quoted values, to allow for a difference of sign in the resonant term of the expression corresponding to equation 1.26 used for the scattering amplitude. In spite of a difference in target mass of 20-30 mass units, the phases obtained here for the rare-earth nuclei are seen to agree, within the errors quoted in tables 3.2 to 3.8, with the other data. The phases for the $\ell = 1$ and $\ell = 3$ partial waves are seen to lie in two separate bands. If more data were available, it might be possible to establish an empirical criterion for the

Figure 3.9: The total phase-shifts α_ℓ , obtained from fitting the elastic scattering excitation functions, plotted as a function of θ_{lab} . The phase-shifts obtained from fits to analogue resonances in ^{143}Pr [Gr70a] and ^{139}La [Wi70] are also shown.

acceptability of resonance phases based on the values lying within certain numerical limits.

3.3 COULOMB DISPLACEMENT ENERGIES

3.3.1 Present Results

As mentioned in section 3.1.2, only the energy of the ground-state analogue was varied in the fitting of the data and all other resonance energies were fixed with respect to it, according to the excitation energy of the parent state. Consequently, the Coulomb displacement energies, calculated from the results reported here, are averaged over the various resonances included in the analysis. Thus, any possible state-dependence of ΔE_c is ignored. However, no information is really being lost here because the magnitude of the state-dependence for low-lying parent states is estimated to be no greater than 10 keV [Ca68, Se72]. The complex nature of analogue states in heavy nuclei, involving the superposition of $(2T_0 + 1)$ proton wave functions coupled to the ground and excited states of the target, leads to an averaging out of any differences in charge radius between individual states so that ΔE_c is not expected to vary appreciably from one parent state to another.

The Coulomb displacement energies between the parent-analogue nuclei studied here were calculated using equation 1.17 and are presented in column 4 of table 3.12. Also shown in the table are the energies of the ground-state analogues, obtained from the fits to the ^{141}O data, and the neutron separation energies of the parent nuclei as derived from the 1971 atomic mass evaluation [Wa71]. The quoted resonance energies have been corrected for finite target thickness by subtracting half the energy lost by an 11 MeV proton in passing through the full target thickness, calculated as described in section 2.1.4;

Parent nucleus	$E_r^{g.s.}$ (MeV)	S_n (MeV)	ΔE_c (MeV)
^{161}Gd	10.224 ± 0.021	5.636 ± 0.010	15.860 ± 0.023
^{163}Dy	10.018 ± 0.015	6.272 ± 0.002	16.290 ± 0.015
^{165}Dy	10.542 ± 0.017	5.715 ± 0.002	16.257 ± 0.017
^{167}Er	10.256 ± 0.019	6.436 ± 0.001	16.692 ± 0.019
^{169}Er	10.634 ± 0.017	6.003 ± 0.001	16.637 ± 0.017
^{171}Er	10.867 ± 0.025	5.681 ± 0.005	16.548 ± 0.025
^{175}Yb	11.083 ± 0.019	5.820 ± 0.002	16.903 ± 0.019

Table 3.12. Coulomb displacement energies

in all these cases the correction amounted to less than 2 keV. The errors quoted for the resonance energies were obtained by summing in quadrature the uncertainties arising from the fitting of the resonances, the energy spread of the beam from the tandem, the reproducibility of the beam analysing magnet settings and the effective energy-thickness of the target.

It is interesting to present the Coulomb displacement energies determined here together with those already measured in analogue resonance studies on other nuclei in this mass region. A simple graphical representation of the gross features of the data is shown in figure 3.10 where $\Delta E_c \times A^{\frac{1}{3}}$ is plotted as a function of the mass number A of the parent nucleus. The data shown here, for elements between barium and lead, are taken mainly from the compilation of Coulomb displacement energies by Nolen and Schiffer [No69]; the new data from this work are identified by circles around the points. The fragmentation of the data into families of points corresponding to the different isotopes of each element is quite apparent. The lines drawn

Figure 3.10: Coulomb displacement energies $\times A^{1/3}$ plotted as a function of the mass number A of the parent nucleus for A = 131 to A = 209. The new data from this work are indicated by circles around the points.

between the members of each family have no significance other than to link the different isotopes of the element. The factor $A^{\frac{1}{3}}$ included in the ordinate of figure 3.10 is seen to effectively remove the isotope shift in ΔE_c which arises, within a family of isotopes, from an increase in the charge radius as the neutron number increases.

3.3.2 Deformation Effects

The effect of deformation on the Coulomb displacement energy, mentioned in section 1.5.2, is best illustrated by plotting the difference between the observed Coulomb displacement energy and that predicted for a spherical nucleus with the same proton and neutron numbers [Wh70, Fo72]. The predicted spherical value can be obtained from an empirical formula such as [Wh70]

$$\Delta E_c = -0.479 + 1.398 Z/A^{\frac{1}{3}}, \quad 3.6$$

where Z and A are respectively the proton number and the mass number of the parent nucleus. This formula was obtained from a least squares fit to the available Coulomb displacement energy data in the region of the closed neutron shells at $N = 82$ and $N = 126$. It is felt that this particular formula is more suitable for use in the mass region of interest than that of Long *et al.* [Lo66] which was based on data for lighter nuclei with mass numbers between 69 and 145. Thus, the difference between the experimental Coulomb displacement energies and the values predicted for a spherical nucleus by equation 3.6 are plotted against the mass numbers of the parent nuclei in figure 3.11. Some Coulomb displacement energies, not included in figure 3.11, have been measured by the observation of analogue states in the direct (p,n) reaction but the experimental errors on the values (typically 150 keV) are such that the data are unsuitable for demonstrating deformation effects.

Figure 3.11: Plot of the difference $\Delta E_c - (1.398 Z/A^{1/3} - 0.479)$ as a function of the mass number A of the parent nucleus from $A = 139$ to $A = 209$. The new data from this work are indicated by circles.

Referring to figure 3.11, the lowering of the Coulomb displacement energy is very evident with a sharp fall away from the spherical values in the mass region between 140 and 150, where the onset of deformation occurs, and a rise back to the spherical values in the reverse transition region between mass numbers 180 and 190. The decrease in the Coulomb displacement energy, predicted by equation 1.35 [Ma66] for a quadrupole deformation parameter of 0.3, is indicated by the dashed line. This agrees fairly well with the general trend of the data. The data points added from this work, again identified by small circles, are in the region of permanent deformation and follow the general lowering of ΔE_c away from the spherical values.

There are, however, several open problems relating to the Coulomb displacement energy data shown in figure 3.11. The most obvious of these is the rise of ΔE_c above the estimated spherical values for masses around 200; such an effect cannot be explained by the classical expression in equation 1.35. A slight rise in ΔE_c between mass numbers 165 and 171 could have similar origins to those of the effect around mass 200. Experimentally, there are still mass regions where there is a need for more data; for example, in the mass ranges 150-160 and 180-190. It would also be instructive to measure some ΔE_c values in the region of permanent deformation above the $N = 126$ closed shell; the lowering of ΔE_c below the spherical value for ^{238}U appears to be about three times that predicted by equation 1.35 [Wh70].

The Coulomb displacement energies obtained from isobaric analogue resonance studies and the conclusions to be drawn from them obviously depend very critically on the energy calibration of the beam analysing magnet used for the experiment. This dependence becomes even more acute when the data for different nuclei are collected at different

laboratories. It is, therefore, very desirable to have some overlap in the investigations carried out by different groups and, if possible, a check on the calibration of the analysing magnet. The calibration accuracy of the analysing magnet used for this work was recently verified by the excellent agreement between measurements on a 6^+ state in ^{16}O , which is seen in $^{12}\text{C}(\alpha, \alpha)$, made both here and in other laboratories; the resonance energy measured at the ANU was (10.198 ± 0.010) MeV [Op71] while, for example, the value (10.199 ± 0.005) MeV was obtained in Kalamazoo [Ra71].

3.3.3 Recent ΔE_c Data

A paper by Seitz [Se72], which appeared in the literature very recently, contains a compilation of the experimental Coulomb displacement energies for non-magic nuclei between ^{108}Ag and ^{207}Pb . Included in the compilation are the displacement energies obtained by analogue resonance measurements for two of the parent nuclei, ^{167}Er and ^{169}Er , which have been studied in this work. The measurements, made at Austin, Texas, agree well with the present results as shown in table 3.13 where the energies of the ground-state analogues are compared. In addition, a note is made by Seitz that the energy of the $\frac{3}{2} \frac{1}{2}^-$ (510) resonance in $^{174}\text{Yb}(p, p_0)$ was found to be in excellent agreement with

Parent	$E_r^{g.s.}$ (MeV)	
	Present work	Austin [Se72]
^{167}Er	10.256 ± 0.019	10.247 ± 0.020
^{169}Er	10.634 ± 0.017	10.617 ± 0.020

Table 3.13. Comparison of the energies of the ground-state analogues of ^{167}Er and ^{169}Er as measured here and at Austin [Se72].

the value obtained by Whineray et al. [Wh70]. A measurement made here of the same resonance also agreed very well with Whineray's result (see table 3.10). However, there is some discord in that the Austin values of ΔE_c for the erbium nuclei were based on the energy of the analogue of the $\frac{3}{2} \frac{1}{2}^-$ (521) state in the parent nuclei; this state was not observed in any of the nuclei studied here. The $\frac{3}{2} \frac{1}{2}^-$ (521) state is only weakly excited by the (d,p) reaction compared with the $\frac{1}{2} \frac{1}{2}^-$ (521) state and it is expected that the population of the analogues will be similar.

Seitz stresses the point made in section 3.3.2 with regard to the energy calibrations of the analysing magnets used in different laboratories. To overcome erring energy calibrations, he used a correction factor obtained by remeasuring one of the resonances using the Austin tandem as a standard. From the set of corrected displacement energies, Seitz calculated the quadrupole deformation parameters and compared them with values obtained from other methods such as the observation of μ -mesonic X-rays and Coulomb excitation. The results were found to be quite consistent. However, the deformation parameters calculated for ^{167}Er , ^{169}Er , ^{169}Yb and ^{171}Yb were about 20% lower than the parameters for neighbouring nuclei. This is the same effect as the slight rise in the ΔE_c values between the mass numbers 165 and 171 which was noted in section 3.3.2.

3.4 SPECTROSCOPIC FACTORS

It was mentioned earlier in section 1.4.3 that the partial proton width fitted to analogue resonance data can be used to calculate a spectroscopic factor S_p which relates the parent state to the ground state of the target. It is convenient, for deformed nuclei, in order to facilitate a direct comparison with the Nilsson model, to express the spectroscopic factors as the Nilsson C_{jl}^2 coefficients [Sa58b]:

$$C_{j\ell}^2 = \frac{(j+\frac{1}{2}) S_p}{U^2}, \quad 3.7$$

where j is the spin of the parent state and the pairing factor U^2 takes into account the partial filling of the neutron orbital in the target nucleus. The $C_{j\ell}$ are the normalised coefficients of the expansion of the Nilsson wave function, for a nucleon in a deformed well, onto a set of spherical basis states of definite j .

3.4.1 Calculation of Theoretical Single-Particle Widths

The theoretical single-particle widths $\Gamma_p^{S \cdot P}$ required for the calculation of S_p were obtained here by a numerical solution of the coupled Lane equations, equations 1.15 and 1.16, for the proton and neutron channels. The computer programme used for the solution was originally written by Bondorf et al. [Bo66], modified by Folkmann [Fo71] and translated into Fortran IV at the ANU. A description of the potentials used in the Lane equations, together with the method of solution, is given in Appendix 3.

The values of $\Gamma_p^{S \cdot P}$ calculated for the $\ell = 1$ and $\ell = 3$ partial proton waves are shown in figure 3.12 as a function of the resonance energy in the centre-of-mass system. The spin-orbit potential produces a slight j dependence in the calculated widths with $\Gamma_p^{S \cdot P}(\ell, j+\frac{1}{2})$ being greater than $\Gamma_p^{S \cdot P}(\ell, j-\frac{1}{2})$. The splitting increases with increasing ℓ and for $\ell = 3$ the difference here was about 7%. The average curves in figure 3.12 were obtained by fitting a cubic expression to the widths calculated at four energies for both $j = \ell+\frac{1}{2}$ and $j = \ell-\frac{1}{2}$.

Figure 3.12: Proton single-particle widths $\Gamma_P^{S.P.}$ as a function of the resonance energy E_{CM} in the centre-of-mass system for $^{174}\text{Yb} + p$. The curve for each l value gives the mean of the widths calculated for $j = l \pm \frac{1}{2}$.

3.4.2 Calculation of $C_{j\ell}^2$

The experimental proton widths and the calculated single-particle widths were substituted into equations 1.34 and 3.7 to obtain the $C_{j\ell}^2$ for the $\frac{3}{2} \frac{1}{2}^-$ (510), $\frac{1}{2} \frac{1}{2}^-$ (521) and $\frac{7}{2} \frac{5}{2}^-$ (512) states. These states were chosen because their ℓ values were confirmed in a number of the nuclei studied and they had the smallest errors (about 30-40%) on the fitted proton widths. The experimental proton widths used here were those fitted to the 141° data for the $\ell = 1$ resonances and those fitted to the 125° and 160° data for the $\ell = 3$ resonances. A simple scaling factor was applied to the fitted proton widths to allow for the presence in the target of other isotopes of the target element (as listed in table 2.1). The U^2 factor in equation 3.7 was taken to be unity in all cases. This approximation should be correct to within about 20% except where the excitation energy of the state in the parent nucleus is below about 200 keV; the worst case is expected to be the $\frac{1}{2} \frac{1}{2}^-$ (521) state in ^{169}Er where U^2 is ~ 0.62 [Is63]. The results of the calculations are shown in tables 3.14(a) and 3.14(b). The first three columns give the parent nucleus, the Nilsson assignment and the excitation energy of the state while the experimental proton width (corrected for the isotopic composition of the target) and the calculated single-particle width are given in columns 4 and 5. The values of $C_{j\ell}^2$ calculated from the experimental and single-particle widths are shown in column 6 under the heading (p,p_0) . Column 7 contains, where available, the absolute value of $C_{j\ell}^2$ obtained from the (d,p) cross-section to the state [Gr70b, Tj69] while column 8 lists the values of $C_{j\ell}^2$ predicted by the Nilsson model for a deformation parameter $\delta = 0.3$ [Ve63].

It can be seen from tables 3.14(a) and (b) that the $C_{j\ell}^2$ values determined from both the (p,p_0) and (d,p) data are appreciably

Parent nucleus	Nilsson assignment	E_x (MeV)	Γ_p (keV)	$\Gamma^{S.p.}_p$ (keV)	$C_{j\ell}^2$		$\frac{C_{j\ell}^2(p,p_0)}{C_{j\ell}^2(d,p)}$
					(p,p ₀)	(d,p)	
^{163}Dy	$\frac{3}{2} \frac{1}{2}^-$ (510)	1.199	4.0 ± 1.0	79.4	0.10 ± 0.03	0.18	0.56 ± 0.14
^{165}Dy	"	0.606	4.3 ± 1.7	77.7	0.11 ± 0.04	0.14	0.78 ± 0.31
^{167}Er	"	0.802	3.3 ± 1.2	75.5	0.087 ± 0.031	0.10	0.87 ± 0.30
^{169}Er	"	0.599	2.5 ± 0.8	79.7	0.063 ± 0.021	0.10	0.63 ± 0.21
^{171}Er	"	0.745	5.8 ± 1.3	88.6	0.13 ± 0.03	0.17	0.76 ± 0.18
^{175}Yb	"	0.552	8.0 ± 0.8	89.1	0.18 ± 0.02	-	-
^{165}Dy	$\frac{1}{2} \frac{1}{2}^-$ (521)	0.109	11.7 ± 3.4	65.5	0.18 ± 0.05	0.19	0.95 ± 0.27
^{167}Er	"	0.208	9.7 ± 3.5	60.9	0.16 ± 0.06	0.14	1.14 ± 0.41
^{169}Er	"	0.0	7.5 ± 2.7	65.1	0.12 ± 0.04	0.14	0.86 ± 0.31

Table 3.14(a). Comparison of $C_{j\ell}^2$ values from (p,p₀), (d,p) and the Nilsson model for the $\frac{3}{2} \frac{1}{2}^-$ (510) and $\frac{1}{2} \frac{1}{2}^-$ (521) states.

Parent nucleus	Nilsson assignment	E_x (MeV)	Γ_p (keV)	$\Gamma_p^{s.p.}$ (keV)	$C_{j\ell}^2$			$\frac{C_{j\ell}^2(p,p_o)}{C_{j\ell}^2(d,p)}$
					(p,p _o)	(d,p)	Nilsson	
^{161}Gd	$\frac{7}{2} \frac{5}{2}^- (512)$	0.889	3.2 ± 1.2	29.0	0.44 ± 0.16	-	0.79	-
^{163}Dy	"	0.801	2.7 ± 1.0	24.5	0.44 ± 0.16	0.54	"	0.81 ± 0.30
^{165}Dy	"	0.262	1.9 ± 0.7	24.1	0.32 ± 0.10	0.38	"	0.84 ± 0.30
^{167}Er	"	0.430	2.8 ± 1.2	22.6	0.49 ± 0.20	0.54	"	0.91 ± 0.37
^{169}Er	"	0.176	2.7 ± 0.8	24.4	0.44 ± 0.14	0.57	"	0.77 ± 0.24
^{171}Er	"	0.076	1.8 ± 0.8	26.5	0.27 ± 0.13	0.39	"	0.69 ± 0.33

Table 3.14(b). Comparison of $C_{j\ell}^2$ values from (p,p_o), (d,p) and the Nilsson model for the $\frac{7}{2} \frac{5}{2}^- (512)$ state.

less than the Nilsson model predictions. This has been interpreted in the (d,p) case as arising from a coupling of the single-particle motion to gamma-vibrational modes causing a distribution of the single-particle strength over several states. The $C_{j\ell}^2$ determined from the (p,p₀) data, although carrying errors of 30-40% from the uncertainty in the experimental proton widths, certainly seem to reflect the particle-vibration interactions which show up in the $C_{j\ell}^2$ from the (d,p) data. The $C_{j\ell}^2$ (p,p₀) are on average about 25% lower than the $C_{j\ell}^2$ (d,p). Some of this discrepancy will come from the assumption that $U^2 = 1$ but this contribution is not expected to be as high as 25% for all the nuclei. A likely source of error is in the calculation of the single-particle widths by the numerical solution of the Lane equations; Bondorf and Bund [Bo69b], using the same method, found that the single-particle widths calculated for $^{208}\text{Pb}(p,p_0)$, where the parent states should be purely single particle in nature, were twice the size of the experimentally determined partial widths.

However, the consistency of the ratio $C_{j\ell}^2(p,p_0)/C_{j\ell}^2(d,p)$ does indicate that there is a correspondence between the parent and analogue states even in the presence of strong collective effects such as gamma vibrations.

PART B

PREPARATORY WORK FOR POLARISATION MEASUREMENTS

CHAPTER 4

PHASE-SPACE ACCEPTANCE CALCULATIONS FOR A TANDEM ACCELERATOR

This chapter describes calculations which have been made to match an atomic beam polarised ion source, recently acquired from the Auckland Nuclear Accessory Company (ANAC), to the Australian National University EN (HVEC) tandem accelerator.

The matching of an ion source to an accelerator means, in practical terms, the determination of the location of and requirements for the ion optical elements to be used between the ion source and accelerator so that the beam transmitted through the accelerator is maximised.

4.1 PHASE-SPACE REPRESENTATION OF CHARGED PARTICLE BEAMS

4.1.1 The Liouville Conservation Theorem

Any member of a collection of charged particles with no inter-particle forces is completely specified by its three cartesian co-ordinates x, y, z together with the corresponding momentum components p_x, p_y, p_z . It is thus possible to represent such a member by a single point in a six-dimensional phase space with co-ordinates x, y, z, p_x, p_y, p_z . As the particle moves in real space so the single point will traverse the phase space in a related manner. On extending this to the whole assembly of particles the single point becomes a six-dimensional hypervolume which contains the points corresponding to every member of the assembly. An important property of the hypervolume is given by the theorem due to Liouville which states that, provided the forces acting on a group of particles are derived from a Hamiltonian function, then the corresponding points inside the hypervolume behave as an

incompressible fluid so that, although the shape of the boundaries may change with time, the density of points and therefore the total volume will remain constant. In the case of a beam of charged particles, the above theorem is generally valid for conservative forces such as those arising from external electric and magnetic fields but the non-conservative interactions with radiation or targets do not possess Hamiltonians and under these circumstances the point density in phase space will change.

The transport of a charged particle beam is essentially concerned with the manipulation of the shape of the hypervolume representing the beam in such a way as to minimise beam current losses. In many beam transport devices the x , y and z motions can be considered independently and the complete motion thus treated in the three phase planes (x, p_x) , (y, p_y) and (z, p_z) . Liouville's theorem can then be simplified to the statement that the areas of the regions containing the representative points in each plane remain constant. In most cases only the transverse motion of the beam need be considered and thus only two of the planes are important. In situations where the axial momentum of a beam moving in the z direction is constant the transverse momentum components p_x and p_y can be replaced by the angular divergences $\frac{dx}{dz}$ and $\frac{dy}{dz}$ respectively. This is conceptually simpler as both axes in a phase plane then represent directly observable quantities.

4.1.2 Emittance and Acceptance

The area of the region in the phase plane describing the beam is known as the emittance of the beam in the corresponding plane in real space and, as this value remains constant with time, it provides a very useful measure of the overall beam width and divergence. The units of emittance are usually cm.milliradians or cm.degrees but in

connection with accelerators where the axial momentum of the beam may be changed it is better to use units which include the beam energy such as $\text{cm.rad.eV}^{\frac{1}{2}}$.

Complementary to the emittance of an ion beam is the acceptance of a particular beam-handling device. This is the phase-plane area containing all the points whose position and divergence at the entrance to the device are such that the particles they represent will be transmitted. An important consequence of Liouville's theorem is that, if the emittance of a beam is greater than the acceptance of a device to which it is presented, then there will be an inevitable loss of beam current no matter how the shape of the emittance is changed. On the other hand, if the acceptance exceeds the emittance, then complete transmission can be achieved provided the shapes are "matched". This is illustrated in figure 4.1, after Banford [Ba66].

4.2 GENERAL REMARKS

As all emittance measurements for the ion source were carried out by the manufacturer, the information necessary for the matching was the phase-space acceptance of the accelerator at some point on the beam path in the field-free region near to the low-energy end of the machine. This information has been obtained by first establishing a realistic description of the electrostatic field in the accelerating tubes and then calculating charged particle trajectories in this field.

4.2.1 Accelerating Tubes

Generally the requirements of voltage insulation have been the dominant factor in the design of accelerating tubes for electrostatic accelerators with the optical properties being of secondary importance. The introduction of the inclined-field electrode structure

Figure 4.1: Matched and mismatched beam emittances (dashed rectangles) for a given device acceptance (circles). The shading represents transmitted beam. In (a) the emittance is too large and complete transmission is impossible. In (b), (c) and (d) the emittances are all equal and less than the acceptance but the matching is only correct, and complete transmission possible, in case (d) [Ba66].

[Va62, Pu65] to overcome the "total voltage effect" was certainly an example of this, with the complexity of the field experienced by the ion beam being increased appreciably from that present in the conventional axial-field tube. Figure 4.2 shows schematically the electrode structure through a typical length of inclined-field tube as presently installed in the ANU tandem. The beam "stiffening" section at the entrance to the tube is composed of conventional axial-field electrodes with either circular or slotted apertures. Following these are lengths of electrodes having the inclined-field geometry with the transverse field direction changing from one set to the next. As the ion is accelerated the lengths of the inclined-field sections become progressively larger to take account of the increased rigidity of the beam. In between sections with opposite electrode inclination are two wedge-shaped gaps with the field direction changing by about 24° . With various drift spaces and axial-field electrodes interposed the inclined-field sections extend up to the terminal. Thus, for the field description the major part of the accelerating tube can be decomposed into axial-field sections, inclined-field sections, wedge-field sections and drift spaces.

The part of the accelerating tubes which has the largest effect on the focusing properties of the accelerator is the region close to the entrance to the first low-energy tube where the low energy of the ion beam causes it to be very strongly focused. Because of the finite extent of the fringe field here, the effect is analogous to that of a thick convex lens in light optics. It is therefore most important to have an accurate description of the field in this region.

Cramer [Cr68] and Gyarmati [Gy69] have published calculations of particle trajectories in inclined-field structures in which they used matrix methods to "transfer" the beam through each accelerating

Figure 4.2: Schematic vertical section through a typical length of inclined-field accelerating tube. The various co-ordinate axes ($X1 - Y1$, etc.) used in the calculations are shown.

gap. Similar first order methods were also used in unpublished work by Galejs [Ga70] at HVEC and by Larson [La69] at Brookhaven. However, at the time this work was commenced, apart from unpublished work at HVEC, there was no record of any attempt being made to calculate the acceptance properties of accelerating tubes. Van der Heide [Va71] has recently reported similar acceptance calculations for the Utrecht State University EN tandem. However, little attention has been paid to obtaining an accurate field description in the entrance region.

All the calculated results presented here are for particle trajectories through standard HVEC inclined-field tubes (Mark II). The assumed resistor configuration in the low-energy column was that recommended by the manufacturer where the first 14 resistors from the tube entrance are nominally 200 M Ω and the remainder are 430 M Ω . In the high-energy column the resistor values were all assumed to be 970 M Ω . The above resistances were based on an average of about 20 values for each type measured by the manufacturer. Any shorted accelerating gaps recommended by HVEC were regarded as such in the calculations.

No account was taken of the variation of the resistance value with applied voltage although a check on ten resistors revealed a reduction of 15-20% in resistance in going from 500 V to 15 kV. This reduction will obviously not affect the high-energy column but will cause a slight change with terminal voltage in the voltage distribution along the low-energy tubes; however the effect is negligible compared with an approximately 25% tolerance in the nominal resistance.

4.3 ELECTROSTATIC FIELD DESCRIPTION

4.3.1 Fringe-Field Region

The importance of the field description in this region has already been stressed; a useful simplification is that the electrode

structure in this region has cylindrical symmetry. This enabled a two-dimensional field description to be computed by a relaxation type solution of Laplace's equation [We67] with the electrode potentials as boundary conditions. The relaxation method, which is based on a finite difference approximation to Laplace's equation, was also used by Rose et al. [Ro64] to obtain field descriptions for their calculations of exact trajectories through axial-field tubes.

The region covered by the relaxation calculation, shown in figure 4.3 as extending from the seventh electrode of the tube out to a point where the field is effectively zero, was divided into a mesh of spacing 0.1 cm. The potentials at the mesh points coinciding with electrodes were fixed throughout the calculations at the values found on the electrodes for a terminal voltage of 3 MV. Initial potential values for the remaining mesh points were determined from an analogue field-plotting experiment[†] in which a full scale diagram of the electrode structure in the entrance region was drawn with silver-based paint on Teledeltos conducting paper. While this method does not give a perfect analogue of a cylindrical potential distribution, the rate of convergence of the relaxation calculation was much improved over a first attempt in which the floating mesh points were initialised to zero. Successively better approximations for the potentials at each mesh point were calculated using the standard five-point difference equations for a system with cylindrical symmetry [We67]. For the sample mesh in figure 4.4, where ϕ_i are the potentials at points P_i , the difference equations are

$$\phi_o = \frac{1}{4} \left[\phi_1 + \phi_3 + \phi_2 \left(1 + \frac{h}{2r_o} \right) + \phi_4 \left(1 - \frac{h}{2r_o} \right) \right]$$

[†] The field plotter used was a Servomex FP92, available together with paper from R.J.T. Payne Pty. Ltd., Richmond, Victoria. The paint was DAG915 Silver in MIBK.

Figure 4.3: Entrance region of the first low-energy accelerating tube showing the boundaries of the coarse and fine mesh calculations. Electrodes 1 and 2 are directly grounded and electrode 3 is grounded through the column-current meter. Electrode 4 is the first active electrode.

for $r_0 \neq 0$, and

$$\phi_0 = \frac{1}{6} [\phi_1 + 4\phi_2 + \phi_3]$$

on the axis of symmetry.

The method of cycling through the grid adopted here was that of successive displacement in which the potential values are changed immediately after an improved value is calculated; the computation time for this method is greatly affected by the order in which the points are calculated [We67] and the order used here was right to left and top to bottom of figure 4.3. This iterative procedure with the coarse mesh size of 0.1 cm was allowed to carry on until the maximum change in potential between iterations, at any one mesh point, was less than 0.5%. The calculation was then repeated over a limited region near the axis of the tube with a mesh spacing of 0.05 cm, using as additional boundary conditions the results of the coarse-mesh calculation along the boundary of this new region. The fine-mesh calculation quickly converged until the maximum change in the potential between iterations was about 0.02%. The final potential matrix containing about 21,600 elements was stored on magnetic disc for later use in the trajectory calculations.

Similar fringe fields with both convex and concave lens action occur at several places along the tubes where the potential gradient changes. However, these other lenses are weak compared with the one at the entrance fringe-field because of the higher beam energy. These additional focusing effects were approximated by the aperture lens formula of Davisson and Calbick [Da32] which is valid when the ratio of aperture potential to aperture radius is large compared to the fields on either side of the aperture [Zw45]; the change in direction $\Delta\theta$, experienced by an ion at a radius r , while passing through a

circular aperture of potential ϕ between regions of uniform electric field E_1 and E_2 , is thus given by

$$\Delta\theta = r \cdot \frac{E_1 - E_2}{4\phi} . \quad 4.1$$

4.3.2 Axial-Field Sections

For axial-field elements well away from the tube entrance (electrode no. 6 et seq., figure 4.3), Rose et al. [Ro64] have shown that the potential can be adequately described by a ramp function, provided that the electrode thickness is much less than the diameter of the apertures. Thus, for the region between two electrodes with potentials ϕ_n and ϕ_{n+1} and spacing L , the potential at a distance x from the first electrode is given by

$$\phi(x) = \phi_n + (\phi_{n+1} - \phi_n)(x/L) , \quad 4.2$$

where one uses a rectangular co-ordinate system, such as $X1-Y1$ in figure 4.2, with the abscissa along the tube axis.

4.3.3 Inclined-Field Sections

By a simple rotation of the co-ordinate system so that the abscissa remains parallel to the field direction ($X3-Y3$ or $X5-Y5$ in figure 4.2), equation 4.2 above can also be used to describe the potential distribution in the inclined-field gaps.

4.3.4 Wedge-Field Sections

In the wedge-field elements, between sections of the tube where the electrodes have opposite inclinations, it is convenient to choose co-ordinate systems in a vertical plane, such as $X2-Y2$ or $X4-Y4$ in figure 4.2, which have origins at the point where the two electrodes

of opposite inclination would intersect. Thus, assuming that the equipotentials lie radially with respect to the origin of the co-ordinate system, the potential distributions in the vertical plane through the elements are given by

$$\phi(x_2, y_2) = \phi_o + \frac{\Delta\phi}{\alpha} \left(\frac{\pi}{2} + \tan^{-1} \left(\frac{y_2}{x_2} \right) \right) \quad 4.3$$

and

$$\phi(x_4, y_4) = \phi_o + \frac{\Delta\phi}{\alpha} \left(\frac{\pi}{2} - \tan^{-1} \left(\frac{y_4}{x_4} \right) \right), \quad 4.4$$

where ϕ_o is the potential of the centre electrode, α is the angle in radians between the inclined electrodes and the vertical and $\Delta\phi$ is the potential difference between neighbouring electrodes. The methods for obtaining these expressions are essentially the same; equation 4.3 is derived in Appendix 4, section A4.1.

4.3.5 Drift Spaces

These field-free regions occur at both ends of the accelerator, inside the terminal and at the low-energy and high-energy mid-sections. As mentioned in 4.3.1, the focusing effect at the ends of all drift spaces except at the low-energy tube entrance was described by the aperture lens formula of Davisson and Calbick.

4.4 TRAJECTORY CALCULATIONS

The vertical plane through the axis of an inclined-field accelerating tube is a plane of symmetry for which the field component perpendicular to the plane is zero. This means that an ion which enters the accelerator moving in this plane will remain so throughout acceleration. Thus, by a two-dimensional treatment in this plane, it has been possible to consider the vertical motion of an ion divorced from its horizontal transverse motion. Unfortunately, there is no

equivalent horizontal plane of symmetry and the horizontal transverse motion can only be considered properly by treating the problem in three dimensions. However, as the inclined-field elements have no transverse field components in a horizontal plane, the horizontal trajectory can be expected to be very similar to that in a horizontal (or vertical) plane through a conventional axial-field tube with the same voltage gradients. Calculations have therefore been made for hypothetical axial-field tubes with the same axial potential gradients as the Mark II inclined-field tubes.

Any particle which is to be transmitted through the accelerator must pass through the stripper assembly which consists of the stripper canal together with radiation shielding at either end; for the ANU tandem this has an overall length of 85 cm and internal diameter of 0.63 cm. Thus the geometrical acceptance of the stripper, which is determined by its dimensions, provides an upper limit for the acceptance of the whole accelerator. This geometrical acceptance is represented at the mid-point of the stripper by a diamond-shaped area of phase space as shown in figure 4.5. Using the position and slope parameters of the trajectories which define the boundary of the diamond, trajectories were calculated backwards from the stripper to the low-energy baseplate. However, not all trajectories on the stripper acceptance can be propagated through the low-energy accelerating tubes without striking an electrode. By what was initially a trial and error procedure, the trajectories that pass unhindered were calculated and the phase-space acceptance at the entrance to the accelerator determined.

Figure 4.5: Geometrical acceptance of the stripper canal at its mid-point. Only the vertical-plane trajectories defined by the shaded area will pass unhindered through the accelerating tubes for a terminal voltage of 3 MV and an injection energy of 60 keV.

4.4.1 The Vertical Plane in Inclined-Field Tubes

If the vertical plane of symmetry is taken to be the x-y plane of a rectangular co-ordinate system, then the motion of a non-relativistic charged particle within the plane is governed by the exact electrostatic ray equation [Zw45],

$$\frac{d^2y}{dx^2} = \left(1 + \left(\frac{dy}{dx}\right)^2\right) \left(\frac{\partial\phi}{\partial y} - \frac{dy}{dx} \frac{\partial\phi}{\partial x}\right) / 2\phi, \quad 4.5$$

where ϕ is the electrostatic potential of the particle measured on a scale such that the particle velocity is zero when ϕ is zero. Introducing the slope z of the trajectory, the above second-order differential equation can be written in the form of two simultaneous first-order equations

$$\frac{dy}{dx} = z \quad 4.6$$

and

$$\frac{dz}{dx} = \left(1 + z^2\right) \left(\frac{\partial\phi}{\partial y} - z \frac{\partial\phi}{\partial x}\right) / 2\phi. \quad 4.7$$

Equations 4.6 and 4.7 are in a suitable form for being solved numerically by a Runge-Kutta technique [Hi56] and a fourth-order method was programmed to achieve this. The step length chosen for the integration procedure was 0.01 cm except in the wedge-field elements where a step length of 0.002 cm was used; further reduction in the step length did not alter the results significantly (see table 4.1). For comparison, the ray equations 4.6 and 4.7 were also solved by means of HPCG, an IBM-supplied Hamming modified predictor-corrector routine [Ib68]; the agreement between the two methods, as shown in table 4.1, was within 0.01 cm and 0.1 mrad respectively for the final displacement and slope of a trajectory calculated through the low-energy tubes. Also shown in table 4.1 are the computation times for the two methods. Ideally, the predictor-corrector method, which makes two evaluations of the right-

hand side of equations 4.6 and 4.7 per step, should be faster than the Runge-Kutta method which requires four; however, the generality of HPCG severely affects its speed. Consequently, the Runge-Kutta routine written by the author was used throughout the calculations.

Table 4.1

Comparison of Runge-Kutta and Hamming
modified predictor-corrector solutions

Method	Step size (cm)	Final displacement (cm)	Final slope (mrad)	CPU time (min)
Runge-Kutta	0.01	0.9337	2.117	1.90
Runge-Kutta	0.002	0.9151	2.022	4.47
HPCG	0.01	0.9226	2.093	3.82

A very useful feature of the Runge-Kutta method has been its ability to integrate in both positive and negative x directions by simply changing the sign of the step length. This has thus enabled trajectories to be calculated in the direction of both increasing and decreasing energy. When a charged particle trajectory, calculated from the ion source to the terminal, has been reversed and calculated back to the ion source, the initial trajectory has been retraced to better than 1 part in 10^3 .

The potentials and potential gradients required for the solution of equations 4.6 and 4.7 were determined, for all parts of the accelerating tube other than the entrance fringe-field region, from the previously described analytical expressions. To every calculated potential it was necessary to add the energy of the injected particle to satisfy the condition that ϕ was zero at zero particle velocity.

In the entrance region of the accelerating tube the required potentials and potential gradients were obtained by linear interpolation between the points of the potential matrix obtained from the relaxation calculation. The method, which is basically the same as that used by Kneff in an electron ballistics programme [Kn69], is described in Appendix 4, section A4.2. A quadratic interpolation between the points of the matrix was also tried but this did not significantly alter the trajectories. The fringe-field description for terminal voltages other than that assumed in the relaxation calculation was obtained by a simple scaling of the potential matrix.

4.4.2 Axial-Field Tubes

The calculations were exactly as described above for the inclined-field tubes except that all accelerating gaps were considered to be of the axial-field type. Owing to the cylindrical symmetry of the entrance fringe-field the same potential matrix was used as above.

4.4.3 Computer Programmes

The calculations have been carried out using programmes written in Fortran IV for an IBM 360/50 computer. The core storage required was about 135 K bytes, most of which was occupied by the potential matrix used to describe the entrance fringe-field. As 208 K bytes of memory space was available in the local computer, no attempt was made to reduce the core requirements but it is expected that simple overlaying procedures could have reduced the above figure to about 100 K bytes. The IBM-supplied routines used for comparison purposes are all members of the IBM Scientific Subroutines Package [Ib68].

The sequence of operations in a typical run of the programme to calculate a trajectory back from the stripper through the low-energy accelerating tubes is summarised by the flowchart in figure 4.6.

4.5 RESULTS

In the following results the injection energy of the beam into the accelerating tubes has been taken to be 60 keV in all cases except where otherwise stated. The terminal voltages considered have been in the range 1 to 6 MV.

4.5.1 Axial-Ray Calculations in Inclined-Field Tubes

Since previously published trajectory calculations in inclined-field tubes have not included a proper treatment of the entrance fringe-field region, a check for agreement with other calculations can only be made for the special case of a particle which enters the accelerator moving along the tube axis. The motion of such a particle is, of course, unaffected by the fringe field. To investigate this case, trajectories were calculated in the direction of increasing energy as far as the centre of the stripper canal. Figure 4.7 shows the trajectory in the vertical plane of an inclined-field tube for a particle entering on-axis with a terminal voltage of 3 MV. It can be seen that the inclined fields produce excursions of up to 0.4 cm from the axis and the particle enters the terminal region 0.1 cm off-axis. A slightly modified version of Cramer's programme [Cr68] produced similar oscillations in the trajectory with the final steering effect at the terminal being about 0.03 cm greater than calculated here. However, an adaptation of Cramer's programme which used beam transport equations derived by Larson [La69] reproduced the steering effect at the terminal to within 0.01 cm.

Figure 4.6: Flowchart of the computer programme used to calculate particle trajectories back from the stripper canal towards the ion source.

Figure 4.7: Calculated trajectories in a vertical plane for an ion entering the accelerator on-axis.

(a) Conventional resistor configuration, (b) all resistors of same value. The terminal voltage is 3 MV and the injection energy is 60 keV in both cases.

A comparison has also been possible with a calculated axial-ray trajectory through Mark II type accelerating tubes provided by Mrs. A. Galejs of HVEC. Her example was for a terminal potential of 6 MV with all column resistors having the same value and again the deflection of the particle at the terminal was reproduced by the ANU programme to better than 0.01 cm.

This overall steering effect of the inclined-field tubes arises from incomplete cancellation of the upward and downward steering produced by the inclined fields. One possible way of rectifying this would be to change the resistor configuration so that the energy of the beam is higher when it experiences its first excursion above the axis. For example, if the half-value resistors in the entrance section were replaced by full-value resistors the steering effect would be reduced by a factor of five. The dashed curve in figure 4.7 illustrates the improvement. However, the advantage to be gained is slight since steering effects in the beam are to some extent compensated for by pre-acceleration beam steering with electrostatic or magnetic deflectors. In addition, the increased strength of the fringe-field lens may lead to matching difficulties.

4.5.2 Calculation of the Vertical-Plane Acceptance for Inclined-Field Tubes

As mentioned in 4.4, the overall acceptance of the accelerator has an upper limit which is the geometrical acceptance of the stripper assembly; this is represented by the diamond-shaped contour in figure 4.5. In fact, for a terminal voltage of 3 MV, only trajectories defined by the shaded area in figure 4.5 are acceptable to both the low and high-energy accelerating tubes; rays outside this area strike the electrodes near the entrance to the first low-energy tube. When

selected trajectories on the boundary of the shaded area are propagated back towards the ion source, a beam waist is formed, as shown in figure 4.8, (60.0 ± 0.2) cm from the plane of the inner part of the first active electrode, which is no. 4 in figure 4.3. The uncertainty in the stated waist position is not a standard error in the usual sense but reflects the difference in the position of the waist above and below the axis; this arises because of the asymmetry of the acceptable rays with respect to the axis, caused by the steering effect of the inclined-field tubes. The phase-space diagram in figure 4.9(a) represents the beam at the waist and summarises the vertical-plane acceptance of the accelerator. The energy-invariant acceptance figure for this diagram is $5.3 \text{ cm.rad.eV}^{\frac{1}{2}}$.

In projecting the phase-space acceptance, defined by the shaded area in figure 4.5, backwards through the accelerator it was found that the straight line boundaries were maintained until the entrance fringe-field was reached when the distortion present in figure 4.9(a) was produced. This indicates the presence of aberrations in the entrance lens, particularly as the distortion is most enhanced for the large slope trajectories which pass closest to the electrodes in the fringe field.

As the strength of the entrance lens is proportional to the voltage on the first active electrode, the waist formed by the acceptable trajectories moves towards the accelerating tube entrance for increasing terminal voltage and consequently more divergent trajectories can be accepted. Thus, for a terminal voltage of 6 MV, the waist in the vertical plane is located (33.2 ± 0.3) cm from the first active electrode. The phase-space acceptance at the waist for this case is shown in figure 4.9(b) and the energy-invariant area is $7.4 \text{ cm.rad.eV}^{\frac{1}{2}}$.

Figure 4.8: Waist formation near the low-energy baseplate by acceptable trajectories defined by the boundary of the shaded area in figure 4.5. Distances along the axis are measured from the first active electrode.

Figure 4.9: Phase-space diagrams for the vertical-plane acceptance at the beam waist for terminal voltages of (a) 3 MV and (b) 6 MV.

The relationship between waist position and waist thickness was found to be independent of both terminal voltage and injection energy as shown in figure 4.10. It is, therefore, convenient to present the position of the waist and the beam width at the waist in figure 4.11 as a function of the ratio, Q , of the beam energy at the terminal to the energy of the beam leaving the ion source.

4.5.3 Calculation of the Acceptance for Axial-Field Tubes

In the case of axial-field tubes, the situation again arises where not all trajectories which are acceptable to the stripper assembly will pass unhindered through the accelerating tubes. In fact, the usable part of the geometrical acceptance of the stripper is virtually the same for axial-field tubes as for inclined-field tubes. For a terminal voltage of 3 MV, the calculated position of the waist formed by the axial-field tubes is 60.5 cm in front of the first active electrode and the area of the acceptance contour shown in figure 4.12 is $5.3 \text{ cm} \cdot \text{rad} \cdot \text{eV}^{\frac{1}{2}}$.

4.6 DISCUSSION

4.6.1 Errors

No simple expressions are known for the precise truncation errors which arise in the Runge-Kutta method; however, an estimate of the error can be obtained by calculating a particular ordinate $y(x_o+2h)$ from $y(x_o)$ in the usual manner with two steps of length h and, in addition, with one step of double length $2h$ [Hi56]. For a fourth-order Runge-Kutta method, the estimated local truncation error is given by

$$\delta = \frac{1}{15} |y_h(x_o+2h) - y_{2h}(x_o+2h)| . \quad 4.8$$

The IBM-supplied fourth-order Runge-Kutta routine, RKGS [Ib68], makes

Figure 4.10: Relationship between waist position and waist thickness for varying terminal voltage and injection energy.

Figure 4.11: Variation of (a) waist position, measured from the first active electrode, and (b) waist thickness with both the terminal voltage for a fixed injection energy of 60 keV and Q , the ratio of the beam energy at the terminal to the beam energy at the tube entrance.

Figure 4.12: Phase-space diagram for the acceptance at the waist formed by an axial-field tube with a terminal voltage of 3 MV.

use of equation 4.8 to optimise the step size so that the truncation error at each step is kept below some chosen maximum. Using RKGS the estimated local error for the step lengths used in the calculation has been found to be less than 1 part in 10^7 . With a total of about 50,000 steps in the calculation of a trajectory from the terminal to the ion source, this would mean a maximum possible accumulated truncation error of 5×10^{-3} cm in the calculated displacement of a ray outside the accelerator. However, because of the oscillatory nature of the trajectories it is to be expected that there will be some cancellation of local errors so that the actual error in the final position may be much less.

The computer programme used single precision variables with 7 digits being carried; an estimate of the rounding errors was obtained by comparison of single precision trajectories with those calculated using double precision variables of 16 digits. For a particle entering the accelerator on-axis, the difference in the final displacement from the axis at the centre of the stripper was 6×10^{-4} cm which is negligible compared with the overall beam size.

4.6.2 The Acceptance Diagrams

On comparing the acceptance diagrams, figure 4.9(a) and figure 4.12, for inclined-field and axial-field tubes respectively, the most noticeable feature is the asymmetry about the slope axis present in the inclined-field case with the centroid of the figure being about 0.1 cm above the axis. As stated before, this arises from the steering effect produced by the inclined-field sections. In all other aspects the acceptance diagrams for the inclined-field and axial-field tubes are identical and, moreover, the beam waists are virtually in the same position in front of the tube. This allows the conclusion to be drawn

that the inclined fields have, in fact, very little effect on the focusing and acceptance properties of the tubes.

An interesting point about the acceptance results is that the clipping action of the electrodes occurs after the region of strong focusing in the entrance fringe-field and so the area of the acceptance figure is fairly insensitive to the fringe-field description. However, the position of the waist, which is an important parameter for matching applications, is certainly affected by the way in which the fringe field is treated. This is illustrated in figure 4.13 by the acceptance diagrams, at a point 60 cm from the first active electrode and for a terminal voltage of 3 MV, obtained with the fringe field being described by (a) the potential matrix and (b) the Davisson-Calbick formula.

The recent work of Van der Heide [Va71] has produced, by matrix methods, similar acceptance information to that calculated here although there are discrepancies in the predicted waist positions and thicknesses. In general, Van der Heide's waist positions are 25-30% further away from the tube entrance than those calculated here; this may be explained by his representation of the accelerating tubes by a thick bipotential lens and his approximation of the acceptance at the stripper by an ellipse. Van der Heide also obtains an acceptance diagram at the centre of the stripper which is similar to that shown here in figure 4.5 apart from a notable reflection in the slope axis; this latter point may be connected with the apparent discrepancy in the co-ordinates of rays A and B as shown in his figure 1 and figure 2.

A final practical point emerging from the present results is that, for terminal voltages above 3 MV, the waist position in front of the tube entrance is well inside the accelerator tank. Consequently, the acceptable beam diameter outside the tank can become impracticably large; for example, the projection of the acceptance diagram for 6 MV

Figure 4.13: Comparison of the vertical-plane acceptance diagram at a point 60 cm from the first active electrode calculated, for a terminal voltage of 3 MV, using (a) the potential matrix and (b) the Davisson-Calbick formula to describe the entrance fringe field.

to the outside of the low-energy baseplate gives a beam diameter of 4.4 cm. If the source emittance is comparable to the accelerator acceptance, it is clearly important that matching elements not located near to the baseplate should have substantial internal dimensions.

4.7 APPLICATION OF THE RESULTS

The emittance of the 50 keV H^- beam from the ANU polarised ion source was measured in both vertical and horizontal planes by ANAC to be as shown in figure 4.14. The final positioning of the source with respect to the tandem was a compromise between the matching requirements and the physical constraints imposed by the tandem building. Thus, the location of the source is such that the source beam waist is about 380 cm away from the first active electrode of the accelerating tubes with the beam moving perpendicular to the tandem axis. Figure 4.15 shows the accelerator and source waists together with the positions and internal dimensions of the ion-optical elements installed for the matching. The focusing properties of the 90° inflection magnet required to bend the beam onto the tandem axis imposes additional restraints on the matching. The magnet, which deflects the beam with a radius of curvature of 18 cm, is double-focusing for image and object positions at the points IP and OP on figure 4.15 [Wh71]. Thus, the acceptance waists of the accelerator are refocused nearer to the source waist and an approximate idea of the acceptable envelopes for 3 MV and 6 MV terminal voltages is given by the cones drawn at OP. The elements between OP and the source consist of two einzel lenses of 10 cm internal diameter and electrode geometry similar to example 3 in the work by Liebmann [Li49], together with two sets of electrostatic vertical-steering plates with an inter-electrode gap of 7.6 cm. Any horizontal steering can be achieved by means of the 90° magnet and

Figure 4.14: Emittance of the 50 keV H^- beam from the ANU polarised ion source in (a) the vertical plane and (b) the horizontal plane, as measured by ANAC.

Figure 4.15: Vertical-plane schematic diagram of the layout of the ion-optical elements installed between the ANU polarised ion source and the tandem accelerator. The positions of the source and accelerator waists are also shown. The numbers, for example 6/50, labelling the accelerator waists indicate the terminal voltage (in MV) and injection energy (in keV) respectively.

elements inside the ion source. The monitoring of the beam current is carried out at the points indicated by PLATE and CUP; these are, respectively, a flat plate perpendicular to the beam inside a 35 cm length of insulated 10 cm diameter beam pipe and a cup of diameter 2.1 cm and length 12.7 cm.

Operation of the polarised ion source has not yet reached a stage where reliable numbers can be given for the transmission through the accelerator but one set of beam currents recently obtained at various monitoring points is shown in table 4.2. The terminal voltage

Table 4.2

Polarised beam current at various monitoring points
when being accelerated by the EN tandem

Monitoring point	Beam current (nA)	Electrostatic electron suppression
Plate before 90° inflection magnet	34	no
Cup after 90° inflection magnet	25	yes
Cup at H.E. end of tandem	15	no
Quartz immediately before 90° analysing magnet	6.5	no
Stop immediately after 90° analysing magnet	5	no

and injection energy in this case were 3.37 MV and 50 keV. The transmission through the accelerator appears quite reasonable at about 60% but a significant amount of beam is being lost in the 90° inflection magnet between the polarised ion source and the tandem. This loss is thought to be due to the aperture effect of the magnet vacuum box and instability in the energy of the beam leaving the source. Steps to overcome these problems are presently being considered.

APPENDIX 1

REDRIFT TREATMENT OF RADIATION-DAMAGED Si(Li) DETECTORS

Work by Ammerlaan [Am68] has shown that irradiation of lithium-drifted p-i-n junctions using both high-energy electrons and γ -rays produces negative space-charge in the formerly compensated intrinsic region. The space-charge arises mainly from the loss of Li^+ ions by precipitation, often in conjunction with other impurities such as oxygen or boron, onto irradiation defects. For example, if V represents a vacancy,

and

These complexes formed by precipitation can act as charge-trapping centres to produce the long rise-time pulses which are a characteristic of radiation-damaged detectors. However, the main consequence of the loss of compensation is that the intrinsic region becomes weakly p-type, resulting in a p-n junction being formed at the back of the detector. Satisfactory operation of the detectors will then be impossible unless sufficient bias is applied to deplete the whole of the formerly intrinsic region. As the damage worsens, the bias required soon becomes impractically large.

Jaskola [Ja66] has described a redrift technique for restoring the compensation in the intrinsic region of irradiated detectors which involves maintaining the detectors under reverse bias conditions at a temperature of about 60 °C for several days. This method was successfully used on several occasions to restore Kevex Si(Li) detectors after radiation damage rendered them unusable.

Figure A1.1: Over-all view of equipment for redrifting Si(Li) detectors. The redrift oven with its lid removed (slightly left of centre) is shown together with the power supply and control unit (lower left), thermocouple potentiometer (right) and Ortec 210 bias supply.

The redrift oven, shown together with various ancillary equipment in figure A1.1, can operate with up to five detectors mounted in recesses in a 1.27 cm thick mild steel hot-plate which stands on three thermally insulating teflon pillars inside an evacuated chamber. Two power transistors, mounted with their collectors grounded and in tight thermal contact with the plate, provide the heating for the redrift process. A thermistor is also attached to the plate and, through a feedback circuit, variations in its resistance are used to control the base bias current fed to the power transistors. In this way, the collector power dissipation and, therefore, the plate temperature can be controlled to maintain a steady temperature. The plate temperature was monitored throughout the redrift using a copper-constantan thermocouple and a millivolt potentiometer. Another convenient indication of the temperature was provided by the leakage current which could be read separately for each detector from an Ortec 210 bias supply. Resolution tests on the detectors were carried out in situ in the redrift chamber by means of an α -particle source mounted on a movable plunger which could be positioned under each of the detectors in turn without breaking vacuum. A composite α -source[†], with ^{239}Pu , ^{241}Am and ^{244}Cm activities, was found to be very suitable where detectors of widely differing resolution were being tested.

Alpha spectra obtained from a Si(Li) detector before and after a 48 hour redrift period are shown in figure A1.2. The improvement is typical of that obtained for 4 out of the 8 detectors treated and the resolution, for a nominal bias of 400 V, is about 30 keV for 5.8 MeV α -particles which is close to that of a new detector. In the other cases, 40 keV resolution was obtained and this only after

[†] Code no. AMR33, obtained from The Radiochemical Centre, Amersham, Bucks., U.K.

Figure A1.2: Typical α -particle spectra obtained from a radiation-damaged Si(Li) detector (a) before and (b) after redrift treatment. The three main α -groups are labelled with their parent nuclei and have energies 5.15, 5.48 and 5.80 MeV. The doublet from ^{244}Cm is separated by 40 keV.

increasing the bias to 550 V. It may be relevant to note that the detectors which failed to recover completely were all members of a second batch of detectors purchased approximately six months after those which were successfully redrifted.

The useful lifetime of the redrifted detectors was about 10 days with 10-12 MeV protons before radiation-damage effects reappeared. However, a second redrift of 24 hours was found sufficient to restore the detectors to their original state once more; some detectors were successfully redrifted three times.

APPENDIX 2

PREPARATION OF METALLIC RARE-EARTH TARGETS

The preparation of the targets was usually carried out in two separate stages, the first of which was a reduction of the oxide to the metal and the second was the redistillation of the metal produced in the first stage onto carbon films.

A2.1 REDUCTION OF OXIDE TO METAL

The most effective reducing agent for the oxides of the rare-earth elements is lanthanum which produces metallic ytterbium, for example, by the reaction

However, in the case of dysprosium and erbium, for instance, the temperature necessary for the evaporation of the metal is high enough for the metal to become contaminated with lanthanum and thorium metal powder is then preferable as the reducing agent.

As the targets required are usually those of separated isotopes, the apparatus used for the reduction is designed for working with small quantities of oxide. A mixture of about 25 mg of oxide and about 28 mg of freshly prepared lanthanum filings (or ~ 35 mg of thorium powder) is sealed into a tantalum crucible, 0.317 cm O.D. and 1.778 cm high, with a small conical tantalum stopper having a 0.076 cm diameter central hole. The loaded crucible is then mounted in the electron bombardment apparatus as shown in figure A2.1 and heated to bring about the reduction. The volatile metal emerges through the hole in the stopper of the crucible and condenses as a small nipple on a highly

Figure A2.1: Sketch of the electron bombardment apparatus, with all radiant-heat shields removed, used for the preparation of the metallic rare-earth targets.

polished 0.5 mm thick tungsten plate suspended about 3 mm above the crucible. The metal deposit is easily peeled from the tungsten plate using a scalpel blade and yields of ≥ 15 mg of metal from 25 mg of oxide are usually achieved.

A2.2 REDISTILLATION OF METAL

The metal obtained from the reduction process is cut into small pieces and loaded into an unstoppered crucible of the same size as that used for the reduction. The electron bombardment apparatus is then used to redistill the metal onto 20-40 $\mu\text{g}/\text{cm}^2$ thick carbon films positioned about 6.3 cm above the top of the crucible. The carbon films, deposited by a carbon-arc process onto detergent-coated glass slides, are then cut into pieces of 1.3 cm $\times$ 1.9 cm and floated off the glass, using deionised water, before mounting onto standard aluminium target frames. Because of the collimating effect of the narrow crucible, the distribution of the metal on the carbon slides is not uniform and targets of thicknesses ranging from 250 $\mu\text{g}/\text{cm}^2$ to 50 $\mu\text{g}/\text{cm}^2$ might be expected from a 15 mg load of metal. The thickness variation across the width of a single target frame was found, however, by measuring the energy loss of a collimated beam of α -particles transmitted through the foil, to be at worst 10%.

APPENDIX 3

CALCULATION OF SINGLE-PARTICLE PROTON WIDTHS

The single-particle proton widths, required for the derivation of spectroscopic factors from the data, were obtained here by a direct numerical solution of the coupled Lane equations. This was achieved using a Fortran version of the programme of Bondorf et al. [Bo66] with modifications by Folkmann [Fo71].

A3.1 THE POTENTIALS IN THE LANE EQUATIONS

Referring to the Lane equations, expressed in the form of equations 1.15 and 1.16, the isospin-independent part V_o of the potential is, in general, complex, with real and imaginary parts of the form:

$$\text{Re } V_o \equiv U_o f(r) + \left(\frac{\hbar}{m_\pi c} \right)^2 U_{so} \frac{1}{r} \frac{df(r)}{dr} (\underline{\sigma} \cdot \underline{l}) \quad \text{A3.1}$$

and

$$\text{Im } V_o \equiv W \frac{T_o}{2T_o + 1} g(r) \quad , \quad \text{A3.2}$$

where U_o , W and U_{so} are respectively the depths of the real central, imaginary central and spin-orbit potentials. T_o is the isospin of the target nucleus and $\frac{\hbar}{m_\pi c}$ is the pion wavelength while $\underline{\sigma}$ and $\underline{l}$ are respectively the Pauli spin vector and the orbital angular momentum of the incident proton.

Similarly, the isospin-dependent symmetry potential V_1 is also complex and of the form:

$$\text{Re } V_1 \equiv U_1 f(r) \quad \text{A3.3}$$

and

$$\text{Im } V_1 \equiv -W \frac{2}{2T_o + 1} g(r) \quad . \quad \text{A3.4}$$

The form factors, $f(r)$ and $g(r)$, are of the Woods-Saxon and Woods-Saxon derivative types:

$$f(r) = 1/[1 + \exp[(r - R)/a_d]] \quad A3.5$$

and

$$g(r) = -4a_d \frac{df(r)}{dr}, \quad A3.6$$

where a_d is the diffuseness parameter and the nuclear radius

$$R = r_0 A^{\frac{1}{3}} \quad A3.7$$

for a target of mass number A .

The Coulomb potential V_c is written

$$\frac{Ze^2}{2R} \left(3 - \frac{r^2}{R^2} \right) \quad \text{for } r < R \quad A3.8$$

and

$$\frac{Ze^2}{r} \quad \text{for } r \geq R, \quad A3.9$$

where Z is the atomic number of the target nucleus.

It can be seen, by comparing the above with equations 1.15 and 1.16, that the proton in the proton plus target channel experiences a complex potential as in the usual optical-model scattering description. The neutron in the neutron plus target-analogue channel is, however, bound in a real potential which is the same as that used to calculate the binding energy of the parent analogue system. When the neutron energy ($E_p - \Delta E_c$) is near the energy of a bound state, $\chi_n(r)$ becomes large and thus affects $\chi_p(r)$, through the coupling, to produce an analogue resonance.

The optical parameters and other constants used here for a ^{174}Yb target nucleus are listed in table A3.1. The parameters are based on the set of proton parameters which have been used on a number of occasions in DWBA calculations on the ytterbium isotopes [Ve63, Bu66].

U_o (MeV)	W (MeV)	U_{so} (MeV)	U_1 (MeV)	r_o (fm)	a_d (fm)	ΔE_c (MeV)
55	15	7.5	104/A	1.25	0.65	16.903

Table A3.1. Constants used in the calculations

A3.2 SOLUTION OF THE LANE EQUATIONS

The numerical solution of the Lane equations involves a Runge-Kutta integration of the neutron and proton wave functions out to a matching radius where the optical potential is negligible. At the matching radius, the proton and neutron wave functions obtained from the integration must be continuous with the Coulomb and Hankel wave functions respectively [Bo69b]. By solving the matching equations, the scattering amplitude η for each value of ℓ and j of the nucleon is obtained.

The evaluation of η as a function of the proton bombarding energy is used to locate the resonances which are the analogues of the single-particle states in the bound neutron well. The required proton widths of these resonances are then obtained by parametrising the scattering amplitude in the neighbourhood of a resonance with an expression of the form [Bo69b]

$$\eta(E) = \eta_o + \eta_1 (E - E_o) - \frac{i\Gamma_p \exp(i\phi_p)}{E - E_o + \frac{1}{2}i\Gamma}, \quad A3.10$$

where η_o and η_1 describe the background amplitude as a linear function of energy and E_o , Γ_p , Γ and ϕ_p are the energy, partial proton width, total width and phase of the resonance. Once a resonance has been established, its energy may be varied, for comparison with an experimentally observed resonance, by adjusting the real central well depth.

APPENDIX 4

FIELD DESCRIPTIONS IN ACCELERATING TUBES

A4.1 POTENTIAL DISTRIBUTION IN A VERTICAL PLANE
THROUGH WEDGE-FIELD ACCELERATING GAPS

The case considered here is that for which the field inclination changes from down to up as shown by the vertical-plane section in figure A4.1 where the thick lines represent the electrodes. Assuming the equipotentials between the electrodes lie radially with respect to the point O where the electrodes would intersect and are a linear function of the polar angle θ_2 , the potential distribution between the electrodes is described by

$$\phi = k\theta_2 + c, \quad \text{A4.1}$$

where k and c are constants. Substituting the electrode potentials into equation A4.1 gives

$$\phi_o - \Delta\phi = k\left(-\frac{\pi}{2} - \alpha\right) + c \quad \text{A4.2}$$

and

$$\phi_o + \Delta\phi = k\left(-\frac{\pi}{2} + \alpha\right) + c. \quad \text{A4.3}$$

The elimination of k and c gives

$$\phi(\theta_2) = \phi_o + \frac{\Delta\phi}{\alpha}\left(\frac{\pi}{2} + \theta_2\right) \quad \text{A4.4}$$

or

$$\phi(x_2, y_2) = \phi_o + \frac{\Delta\phi}{\alpha}\left(\frac{\pi}{2} + \tan^{-1}\left(\frac{y_2}{x_2}\right)\right). \quad \text{A4.5}$$

Figure A4.1: Vertical-plane section of a wedge-field accelerating gap showing the electrode potentials and the co-ordinate axes used for the field description.

A4.2 CALCULATION OF POTENTIALS AND POTENTIAL GRADIENTS IN THE ENTRANCE FRINGE-FIELD

The potential at an arbitrary point in the fringe field is calculated by assuming a linear fit between the mesh points of the potential matrix obtained from the relaxation calculation. Thus, if the mesh spacing in the special case shown in figure A4.2 is h and the potentials are ϕ_A and ϕ_B at the points A and B respectively, then the potential ϕ at the point P on the line joining A and B is given by

$$\phi = \left(\frac{\phi_B - \phi_A}{h} \right) \Delta x + \phi_A . \quad \text{A4.6}$$

The potential gradients at any point in the fringe field are calculated by first determining the potentials at two nearby positions chosen to be equidistant from the point and the mean gradient between these two positions is then calculated. In order to avoid possible errors with the computer if the gradient was calculated between points very close together, the potentials used to obtain the gradient are chosen to be greater than $\frac{h}{2}$ from the point in question. Thus, to obtain $\frac{\partial \phi}{\partial y}$ at the point P in figure A4.3, one of two calculations is made depending on whether the distance Δy , from the point to the nearest row of the matrix with smaller y co-ordinate, is greater or less than $\frac{h}{2}$. Similarly, there are two possible calculations for $\frac{\partial \phi}{\partial x}$.

The above is clarified by the various expressions for the potentials required to obtain $\frac{\partial \phi}{\partial y}$ at P in figure A4.3. These are set out below in the order in which they would be calculated. As Δy is greater than $\frac{h}{2}$, $\frac{\partial \phi}{\partial y}$ is calculated from the potentials ϕ_{K3} and ϕ_{K4} at points each a distance Δy from P. However, it is first necessary to calculate ϕ_{K1} and ϕ_{K2} to obtain ϕ_{K3} . Thus, using equation A4.6,

$$\phi_{K1} = (\phi(JX + 1, KY + 2) - \phi(JX, KY + 2)) \frac{\Delta x}{h} + \phi(JX, KY + 2) \quad \text{A4.7}$$

and

$$\phi K2 = (\phi(JX + 1, KY + 1) - \phi(JX, KY + 1)) \frac{\Delta x}{h} + \phi(JX, KY + 1). \quad A4.8$$

Using $\phi K1$ and $\phi K2$, we obtain

$$\phi K3 = (\phi K1 - \phi K2) \frac{(2\Delta y - h)}{h} + \phi K2. \quad A4.9$$

Now,

$$\phi K4 = (\phi(JX + 1, KY) - \phi(JX, KY)) \frac{\Delta x}{h} + \phi(JX, KY), \quad A4.10$$

so that the final expression for the y gradient at the point P is

$$\left(\frac{\partial \phi}{\partial y} \right)_P = \frac{\phi K3 - \phi K4}{2\Delta y}. \quad A4.11$$

If point P was such that Δy was less than $\frac{h}{2}$, then $\frac{\partial \phi}{\partial y}$ would be calculated from the potential $\phi K2$, which is at a point $(h - \Delta y)$ above P, and the potential at a point $(h - \Delta y)$ below P.

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