Fusion Barrier Distributions and Fission Fragment Anisotropies in Heavy-Ion Fusion Reactions

by

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Preface

This thesis describes the measurement of the fusion excitation functions for four reactions, $^{16}\text{O},^{17}\text{O} + ^{144}\text{Sm}$ and $^{16}\text{O},^{28}\text{Si} + ^{208}\text{Pb}$. All four experiments were carried out using heavy-ion beams from the 14UD Pelletron accelerator operated by the Department of Nuclear Physics at the Australian National University. All measurements documented in this work were made with the Fission Reactions group, with assistance from the academic and technical support staff.

This project was suggested by Drs J.R. Leigh and D.J. Hinde. The evaporation residue measurements were carried out using the existing velocity filter/MWPC arrangement and data analysis techniques. The fission fragment angular distributions were measured using the fission fragment spectrometer which was designed and built by Dr D.J. Hinde and the Department of Nuclear Physics. The front window and energy cathodes of the MWPCs were designed and manufactured by Dr D.J. Hinde and the author. A data analysis program was developed by the author to analyse and present the fission fragment data.

Assistance with the interpretation of the results presented here was received from Drs J.R. Leigh, D.J. Hinde and M. Dasgupta.

An exact coupled-channels calculation for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction was carried out by Drs N. Rowley and A.T. Kruppa.

The following publications, directly related to the work described in this thesis, have been or will be published:


No part of this thesis has been submitted for a degree at any other university.

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Abstract

Fusion excitation functions were measured for the reactions $^{16}\text{O},^{17}\text{O} + ^{144}\text{Sm}$ and $^{16}\text{O},^{28}\text{Si} + ^{208}\text{Pb}$ with high precision in small energy steps spanning the barrier region. The distribution of fusion barriers for each system was obtained from these detailed cross-section measurements. The barrier distributions for the $^{16}\text{O},^{17}\text{O} + ^{144}\text{Sm}$ reactions revealed a double-peaked structure that was interpreted in terms of weak coupling to collective excitations of $^{144}\text{Sm}$. The effects of single-neutron stripping channels were evident in the comparison of the barrier distributions for the $^{17}\text{O}$ and $^{16}\text{O}$ induced reactions.

The experimental barrier distributions were compared with coupled-channels models of the fusion process. Both simplified and exact coupled-channels calculations were performed for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction. The eigenchannel representation gave an excellent qualitative description of the data, despite the approximations used in this approach. However, small quantitative differences were found in comparison with the exact coupled-channels calculations. Experimentally, the effects of projectile excitation on fusion are not apparent, in contrast with the clear signatures in all calculations which included this channel.

The measured barrier distribution for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction could not be interpreted using the simple coupling scheme evident in the $^{16}\text{O},^{17}\text{O} + ^{144}\text{Sm}$ reactions. A better representation of the experimental barrier distribution was obtained with a coupling scheme that included the multiple-phonon excitations in $^{208}\text{Pb}$. Further evidence for such complex excitations associated with the $^{208}\text{Pb}$ was found in the barrier distribution for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction, although the interpretation of this reaction was complicated by the significant influence of the structure of the $^{28}\text{Si}$ projectile.

The fusion measurements for the $^{16}\text{O},^{28}\text{Si} + ^{208}\text{Pb}$ reactions also yielded the fission fragment angular distributions and their anisotropies. Fission fragment anisotropies were calculated using the fission transition state model, making use of the angular momentum distributions more accurately determined from the new cross-section data, and a more realistic calculation of the saddle-point temper-
ature. Comparison of these calculations with the new anisotropies for the $^{16}$O + $^{208}$Pb reaction showed no evidence for the anomalously large fission fragment anisotropies, which were previously seen at energies below the fusion barrier. Thus, the standard models of fusion and fission are able to describe the $^{16}$O + $^{208}$Pb data.

The equivalent calculations for the $^{28}$Si + $^{208}$Pb reaction underpredicted the anisotropies at essentially all energies measured. This failure was attributed to the presence of quasi-fission. In light of these and other recent measurements of fission fragment anisotropies, the applicability of the fission fragment angular distribution technique for obtaining information on fusion angular momentum distributions is discussed and compared to an alternative method of precise fusion cross-section measurements. It is shown that fusion angular momentum distributions could be extracted using the latter method even when there is a significant contribution from quasi-fission.
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Chapter 1

INTRODUCTION

In a macroscopic description of the fusion process, the effective nucleus-nucleus potential depends upon the gross properties of the colliding nuclei. The simplest form of the effective potential is a function of the inter-nuclear separation and the orbital angular momentum \( l \hbar \) of the projectile. It is assumed that the reaction partners in the collision are spherical and no other nuclear degrees of freedom play a role in the reaction.

The 'hot' compound nucleus formed following the fusion reaction, thermally equilibrates by sharing the excitation energy statistically amongst the constituent nucleons. The subsequent decay of the compound nucleus is independent of its formation. The total energy, parity, and the linear and angular momentum are all conserved in the reaction. The compound nucleus can decay by particle evaporation, gamma-ray emission, or by fission. The latter process, which may be preceded by particle emission, dominates in heavy systems where the fission barrier is comparable to the neutron binding energy. In general, the competition between these decay modes depends upon factors such as the excitation energy, the angular momentum and the mass of the compound system.

For reactions that involve nuclei with a large charge product, fission may proceed without the complete shape equilibration of the compound nucleus. This process is called quasi-fission and can be thought of as an intermediate process between complete fusion and deep inelastic collisions. The study of these fusion and fission reaction processes should ultimately lead to a more complete description of the dynamical processes involved in the formation, evolution and decay of the compound system.
In heavy-ion fusion reactions, the fusion process has been described in terms of quantum mechanical tunnelling through a one-dimensional barrier \[1\]. Here, the single barrier, represented by \( B_0 \), is the fusion barrier calculated for the interaction of two spherical nuclei without consideration of any dynamical effects. This approach, referred to as the one-dimensional barrier penetration model (BPM), describes well the fusion of light systems \[2,3\], but for heavier reactions, measured fusion cross-sections at energies below the single barrier exceed predictions based on this model \[4,5\]. The observed enhancement in the fusion probability can be explained by taking into account the internal structure of interacting nuclei.

A simple example of the effects of the nuclear structure on fusion is static nuclear deformation. If one of the reaction partners is classically deformed, then the fusion barrier will depend on the relative orientation of the deformed target nucleus and the direction of the incoming projectile. This is illustrated in Fig. 1.1 in a simple geometric interpretation of the fusion reaction. When the projectile is incident on the target in a direction along its symmetry axis, then the fusion barrier is lower in energy relative to the single or 'spherical' barrier. This configuration is shown in Fig. 1.1 (A) where \( B_1 \) represents the fusion barrier for this 'lowest energy' configuration. When the projectile is incident on the target in a direction perpendicular to the symmetry axis, the barrier is increased relative to the single barrier. This is shown by configuration (B) in Fig. 1.1, where \( B_2 \) is now at an energy greater than the single barrier. Fusion occurs for all orientations intermediate between these two limiting cases, and thus there is a continuous distribution of barriers from the lowest to the highest barrier, instead of the single barrier \( B_0 \). This continuous distribution is represented by the broken line in Fig. 1.1. It is then the passage over the lower barriers in this distribution that gives rise to the enhancement in fusion observed at energies below the single barrier.

The enhancement of fusion cross-sections is a general phenomenon and not just associated with static deformation of the reaction partners. The coupled-channels description of the fusion process, in principle, considers coupling of the relative motion of the two nuclei to all other degrees of freedom. In practice, theoretical descriptions can only, and often need only, include a few of the most important degrees of freedom. These can be divided into two classes: the collective nuclear-shape degrees of freedom, and the single particle processes, such as the single or multiple transfer of nucleons between the reaction partners \[6\]. The former classification encompasses either the static deformation of the target and/or projectile \[7\], or the collective surface vibrations of the nucleus \[8,9\].
FIG. 1.1: The distribution of barriers arising from the fusion of an inert projectile with a classically deformed target. The lowest barrier, $B_1$, corresponds to fusion between the projectile and target aligned along the direction of the nuclear symmetry axis (A). The highest barrier $B_2$ is from the other limiting orientation, where the projectile is incident in a direction perpendicular to the symmetry axis (B). The broken line is the classical distribution of barriers. The barrier for the spherical target is represented by $B_0$.

When the above degrees of freedom are taken into account, the single barrier is split into a distribution of barriers. The concept of a distribution of fusion barriers [10,1,2,8,11-14] replaces the one-dimensional barrier penetration model and is a more realistic description of heavy-ion fusion.

Various simple forms of the barrier distribution have been assumed in order to represent the actual distribution of barriers. These included rectangular [2], Gaussian [11,12] and a 'modulated' flat distribution [13,14] [see the continuous curves in Fig 1.2(a)]. Even though these proposed barrier distributions were distinct from each other, the experimental cross-sections were not determined to a high enough precision to test the applicability of these representations [14]. Recently, there has been renewed interest in the study of heavy-ion fusion, and this is partly due to the application of a new technique for determining the distribution of fusion barriers directly from experimental data [15]. The distribution of barriers is directly proportional to the second derivative with respect to energy of the quantity $E\sigma$, where $\sigma$ is the fusion cross-section at energy $E$. Here the quantity $d^2 (E\sigma)/dE^2$ is referred to as the distribution of barriers. This novel technique does not rely on the assumption of a specific form of the distribution. However, at the time of Rowley, Satchler and Stelson's publication [15] this technique could...
FIG. 1.2: (a) An example of three different forms of the barrier distribution obtained from different models; the symmetric rectangular and Gaussian distributions (broken lines) and the asymmetric distribution discussed in the text (solid line). The data points are the experimental barrier distributions evaluated from the cross-sections of Ref. [7]. (Taken from Ref. [16]). (b) The experimental barrier distribution for the $^{16}$O + $^{154}$Sm reaction (solid squares) [16]. The solid line is a calculation performed including the quadrupole deformation of the target.

not be successfully applied to the existing experimental data because it lacked the necessary precision. The shapes of the barrier distributions determined from the existing data were not well defined and could be equally well described by a multitude of diversely shaped functions [15,16]. An example of this is illustrated in Fig. 1.2(a) where the barrier distributions shown by the solid squares and open circles were obtained from the cross-sections of Ref. [7]. All three barrier distributions are equally consistent with the experimental data. Fusion excitation functions needed to be measured to unprecedented precision and with sufficiently small energy steps in order to obtain meaningful barrier distributions.
The distribution of fusion barriers was successfully determined in an experiment in 1991 from precise measurements of the fusion excitation function for the $^{16}O + ^{154}Sm$ reaction [16]. The asymmetric shape of the barrier distribution, shown by the square data points in Fig. 1.2(b), confirmed that the fusion enhancement was largely due to the statically deformed $^{154}Sm$ nuclei. This was the first time a well defined barrier distribution was obtained directly from experimental data. The measured barrier distribution is well represented by the classical barrier distribution, as shown by the broken line in Fig. 1.1, smoothed over $\approx 2$ MeV because of quantal barrier penetration effects.

The advantage of measuring the distribution of fusion barriers is illustrated in Fig. 1.3 where the fusion excitation functions from model calculations are displayed along with their barrier distributions. The theoretical calculations are for a fictitious system with three different coupling schemes: (a) a negative Q-value reaction, (b) a positive quadrupole deformation, and (c) a positive Q-value reaction. The broken line in Fig. 1.3 represents the excitation function for the no coupling case (the one-dimensional BPM), and is the same in all panels. The degree of enhancement at the lower energies is approximately the same for each case, although the energy at which the no coupling calculation merges with the coupled-channels calculation is different. When the fusion cross-sections are measured to a precision of $\pm 10\%$, and at intervals of several MeV, it is very difficult to distinguish between these three markedly different coupling schemes. However, when the fusion cross-sections are measured with a precision of around 1%, with an energy interval of $\lesssim 1$ MeV, the barrier distributions are well defined and can be determined directly from the fusion cross-sections.

The advantage of this representation is illustrated in the lower three panels of Fig. 1.3, where the barrier distributions are plotted for the three coupling schemes. The barrier distributions for each coupling scheme are different from the spherical barrier distributions, the Gaussian shape shown by the broken lines in Fig. 1.3(d)–(f). Also, each barrier distribution is distinctly different from each other. The height of the main peak in the distributions depends on the strength of the coupling present. The strength of the coupling interaction relative to the Q-value of the reaction channel can be used to categorise the nature of the fusion reaction. The term strong coupling is used when the strength of the coupling interaction is large compared to the Q-value of the reaction [8]. The barrier distribution in Fig. 1.3(e) is an example of a strong coupling case, where the distribution of barriers arises from coupling between the successive members of a rotational band. Note that the height of the barrier distribution in Fig. 1.3(e)
is the smallest of the three distributions. An example of this type of barrier
distribution is the $^{16}\text{O} + ^{154}\text{Sm}$ reaction [see Fig. 1.2(b)], where the coupling is
between successive states of the ground-state rotational band in $^{154}\text{Sm}$.

The term weak coupling describes a reaction in which the coupling strength is
small compared the Q-value. The barrier distributions in Fig. 1.3(d) and (f) are
examples of a weak coupling scheme. The shape of the barrier distribution given
in Fig. 1.3(d) is due to the coupling effects of inelastic channels. In Fig. 1.3(f),
the enhancement is due to the presence of a negative Q-value transfer channel.
These theoretical coupling schemes demonstrate that the distribution of barriers
has a shape or 'fingerprint' which is characteristic of the important channels
affecting fusion. These barrier distributions thus provide a further constraint
on theoretical models. A complete understanding of the fusion process requires
detailed knowledge of all the barriers in the interaction, and so any model which
successfully reproduces the fusion cross-sections must also reproduce the relevant
shape of the distribution of fusion barriers.

Since the first observation of enhanced fusion cross-sections at energies below
$B_0$, the phenomenon has been found in many fusion reactions. The calcula­
tions which considered coupling to the extra degrees of freedom mentioned above
have been generally successful is explaining the enhancement observed. Having
reached this point, it is of interest to turn the problem around and ask what can
be learnt about the role of the nuclear structure of the participating species in
fusion. The availability of many different nuclear beams and targets enables the
experimenter to select from a wide choice of various nuclear properties. There
now exists the possibility that new aspects of nuclear structure effects in fusion
can be systematically uncovered through the determination of the distribution of
fusion barriers.

An example of this approach is demonstrated in the experiments involving
the reaction of $^{16}\text{O}$ on the deformed nuclei of $^{154}\text{Sm}$ and $^{186}\text{W}$. Not only was
the distribution of barriers sensitive to the quadrupole deformation, but also
the effects of hexadecapole deformation were seen [16,18,19]. Simple geometric
calculations were able to explain the general shape of the barrier distribution after
inclusion of these higher order deformation parameters. It was suggested [19] that
the relatively weak effects of inelastic couplings could also be present for these
systems. The inclusion of additional coupling to weaker vibrational channels
was able to improve the agreement between the data and a calculation using
reasonable values of the deformation parameters for the $^{16}\text{O} + ^{186}\text{W}$ reaction.
The equivalent calculation did not significantly affect the agreement for the $^{16}\text{O}$
FIG. 1.3: Fusion excitation functions calculated for a fictitious reaction. The curves in the top panels are the fusion cross-sections for three different coupling schemes (a) $Q < 0$ (b) $\beta_2 > 0$ and (c) $Q > 0$. The broken line is the calculation with no coupling. The lower three panels (d)–(f) are the same couplings schemes as above, but the fusion cross-sections are represented in terms of their distribution of fusion barriers. Taken from Ref. [17].

However, in the presence of the strong coupling between the states of the ground-state rotational band in the $^{154}$Sm and $^{186}$W nuclei, the effects of these weaker couplings are difficult to isolate.

To observe the effects of weak coupling, it is necessary to select a system where such strong coupling effects are absent. The first excited state in the neutron-magic $^{144}$Sm nucleus is at 1.660 MeV. Hence, this reaction should be a good candidate for the investigation of the influence of the more weakly coupled channels on the fusion process. Altering the system by adding a neutron to the $^{16}$O projectile should provide further insight into the sensitivity of fusion to this minor change in the nuclear structure. The effects on the fusion process were examined by determining the distribution of barriers for the $^{17}$O + $^{144}$Sm reaction [19].
reaction, using the $^{16}$O + $^{144}$Sm reaction as a comparison.

Fusion barrier distributions provide a sensitive tool for clearly determining the role of nuclear structure effects on fusion. The information obtained from the details present in the barrier distributions has wider implications for the study of angular momentum distributions for fusion. Many investigations [20] have studied the fusion angular momentum distributions in terms of the mean angular momentum, $\langle l \rangle$. Comparison of experimental values of $\langle l \rangle$ with theoretical calculations have been used as another test of various models of fusion. As described below, one such technique for determining the mean-square angular momentum for fusion, $\langle l^2 \rangle$, uses measurements of the fission fragment angular distributions. The assumption in this technique is that the standard model of the fission process can be used as a reliable probe of the fusion angular momentum distributions. In this work, precise measurements of the fusion excitation functions are used to test the fission fragment angular distribution technique for determining the $\langle l^2 \rangle$ for fusion.

Techniques that have been developed to measure details of the fusion angular momentum distributions include the isomer ratio method [21], $\gamma$-ray multiplicity measurements [22,23], populations of the ground-state rotational bands [24], angular distributions of evaporated $\alpha$-particles [25], partial cross-sections of evaporation residues [26] and fission fragment angular distributions [27]. The last method assumes that a measurement of the anisotropy $A$ of an angular distribution is proportional to

$$A \approx 1 + \frac{\langle J^2 \rangle}{(T J_{\text{eff}}^2)},$$  \hspace{1cm} (1.1)

where $J\hbar$ is the total angular momentum of the nucleus, $T$ is the temperature and $J_{\text{eff}}$ is the effective moment of inertia, both measured at the saddle point of the fissioning system [25]. The fission fragment angular distribution method is the only one available for heavier systems, where fission is the dominant decay mode and the other techniques are not applicable. However, the results from this method have not been in agreement with standard fusion models, particularly at energies below $B_0$, where the measured fission fragment anisotropies are significantly larger than expected [20]. This disagreement is in contrast to the findings from analyses using other techniques for determining $\langle l \rangle$. In these analyses, there has been general agreement between theoretical models and the data for all but the most mass symmetric systems [28–31].

Possible reasons for the disagreement could include problems with the fusion models or with some of the assumptions in the calculation of fission fragment
anisotropies. These could include such fundamental assumptions as the use of the reduced mass of the compound system as the effective mass [27], or in the simplicity of the fission model itself [32]. The problem was compounded by measurements of fission fragment anisotropies which did not discriminate against transfer-induced fission. The fission fragment anisotropies for the reactions $^{16}$O + $^{234}$Th [33] and $^{16}$O + $^{238}$U [34,33] will, in general, be different from the anisotropies obtained from an analysis which considers fusion-fission only. More recent measurements [35], have excluded transfer-induced fission from their analyses, but the larger than expected anisotropies persist. Another possible explanation for the large anisotropies is the presence of quasi-fission [36,27]. In a quasi-fission reaction [37], the dinuclear system evolves along the mass asymmetry degree of freedom at deformations more compact than the entrance-channel mass asymmetry, but never as compact as the unconditional saddle-point configuration. The experimental implication of this is a larger than normal anisotropy.

The failure of the fission fragment angular distribution technique is most acute for the $^{16}$O + $^{208}$Pb reaction. For this reaction, which involves a doubly magic projectile and target, standard fusion models are expected to be applicable. Since the product of the projectile and target charges is small compared to the thorium and uranium systems with the same or heavier projectiles, quasi-fission might be expected to be an insignificant reaction process at energies near the single barrier. Supporting evidence for this claim comes from the experiment of Back et al. [34], where there was no evidence for any quasi-fission for energies above $B_0$. Despite these favourable factors, at energies below $B_0$, the measured fission fragment anisotropies exceeded those calculated by a significant amount [27]. Since the anisotropy in the standard transition state model depends upon the temperature of the system at its saddle point, a measurement was made of the pre-scission neutron multiplicity [38]. From this, the average excitation energy removed by evaporation of neutrons before the system crossed the saddle point was determined, where it was assumed that all the emission was pre-saddle emission. A colder fissioning system results in a larger calculated anisotropy [see Eq. (1.1)], nevertheless, an anomaly still remained at energies below $B_0$. This is shown in Fig. 1.4, where the calculation underpredicts the measured anisotropy. If it is assumed that the fusion models are correct, and the general success of these models does nothing to suggest otherwise, then the anomaly for the $^{16}$O + $^{208}$Pb reaction casts doubt on the technique of using fission fragment angular distributions to determine the angular momentum distributions for fusion. To test this technique, detailed measurement and analysis of the $^{16}$O + $^{208}$Pb system is warranted.
FIG. 1.4: The experimental fission fragment anisotropies (closed data points) as a function of $E_{\text{c.m.}}$. The solid stars are the results from the transition state model calculation of Rossner et al. [38]. This calculation underpredicts the measured anisotropies.

If the anisotropy for fusion-fission can be reliably calculated for a system where there exists no quasi-fission, then the results of the fission model can be more accurately compared with a system that exhibits truly anomalous anisotropies. A recent measurement [39] of the fission fragment anisotropies for the $^{16}\text{O} + ^{238}\text{U}$ reaction (which excluded transfer fission), found that the anisotropy rose rapidly as the beam energy decreased through the barrier region. This effect was found to be correlated with the relative orientation of the projectile and the direction of the nuclear symmetry axis in the deformed $^{238}\text{U}$ nuclei. Collisions between the projectile and the tips of the deformed nuclei in the target produce a ‘stretched’ dinuclear system, which undergoes quasi-fission, whilst collisions with the sides produce a more compact dinucleus, which equilibrates thermally inside its fission barrier. This compound nucleus will then decay statistically, dominantly by fission, resulting in fusion-fission. Thus, fission fragment anisotropies are a very sensitive observable for the presence of quasi-fission.

In an earlier measurement [34] of the fission fragment anisotropies for the $^{28}\text{Si}$
+ 208Pb reaction, there was evidence for a significant presence of quasi-fission at energies above the single barrier. To examine the behaviour of the anisotropies at energies below $B_0$, further detailed measurements are required. The fission fragment anisotropies can then be compared with the improved statistical model calculations, in order to test the applicability of this technique for determining information on the angular momentum distributions for fusion in heavier systems.

In this thesis, the fusion excitation functions for the $^{16}$O, $^{17}$O + $^{144}$Sm and $^{16}$O, $^{28}$Si + $^{208}$Pb reactions have been measured to high precision. This involved measuring the evaporation residue and fission cross-sections, as relevant to each system. The precision of the measurements enabled the distribution of barriers to be obtained. The shape of each barrier distribution is indicative of the couplings effects in fusion. The measured barrier distributions are compared with the distributions from theoretical models in the coupled-channels representation. These models, and those describing the fission process, are discussed in Chapter 2.

The experimental methods used in this work are described in Chapter 3. For the $^{16}$O + $^{208}$Pb reaction, both the ER and fission cross-sections were measured. The ER cross-sections were measured in order to reconcile the significant differences in previous experiments for this system [40–42]. The results of the ER and fission measurements are presented in Chapter 4. The results are discussed in Chapter 5. The barrier distributions for the $^{16,17}$O + $^{144}$Sm reactions show the effects of specific inelastic and transfer channels on fusion. In the $^{16}$O, $^{28}$Si + $^{208}$Pb reactions, calculations involving the expected inelastic channels, as observed in the $^{16,17}$O + $^{144}$Sm reactions, are a poor representation of the data, suggesting that other more complex channels are present.

Fission fragment anisotropies were obtained for the $^{16}$O, $^{28}$Si + $^{208}$Pb reactions. It is shown that there is no anomaly in the fission fragment anisotropies for the $^{16}$O + $^{208}$Pb reaction. The accurate determination of the ER cross-section is crucial for the comparison of the data and fission model. A summary of this work in given in Chapter 6.
Chapter 2

THEORY

In this Chapter, the theoretical background necessary for a description of the fusion process is presented. The coupled-channels model of fusion is detailed and the method for obtaining the distribution of fusion barriers from the fusion cross-sections is discussed. In the second part of this Chapter, the standard model of fission fragment angular distributions, the transition state model, is given.

2.1 Heavy-ion fusion reactions

In heavy-ion collisions, for a given projectile and target, the type of reaction can be classified largely according to the impact parameter of the collision. This is related to the amount of orbital angular momentum $l\hbar$ brought in by the projectile. Classically, the orbital angular momentum is equal to the product of the impact parameter and the linear momentum of the projectile. For collisions with large impact parameters, and correspondingly large $l$-values, the distance of closest approach is so large that only the Coulomb field acts and the dominant reaction process is Rutherford scattering. Due to the finite extent of the interacting nuclei, the Coulomb field may cause strong excitations of the low-lying states in both the projectile and target. This type of reaction is called Coulomb excitation.

For smaller impact parameters, in which the surfaces of the two nuclei graze one another, a moderate amount of energy is lost from the relative motion and there is only a minor rearrangement of the constituent nucleons of the colliding species. These reactions can be termed quasielastic or direct reactions. The angular momentum where the two nuclei just touch one another is called the
grazing angular momentum \( l_g \hbar \), which can be defined as the value of \( l \) at which the energy of relative motion equals the maximum of the interaction potential (see Fig. 2.1). For these collisions, both the Coulomb and nuclear fields influence the collision process. Inelastic and transfer reactions all take place for \( l \)-values in the region of \( l_g \).

If the two colliding nuclei have sufficient energy to overcome the interaction barrier that exists between them, then there is a large overlap in the densities of the two nuclei. A large amount of energy and angular momentum is lost from the relative motion to the internal degrees of freedom of the system, and the two nuclei fuse to form a composite system. The reaction cross-section at energy \( E \) is the incoherent sum of the inelastic, transfer and fusion cross-sections, respectively:

\[
\sigma_{\text{react}}(E) = \sigma_{\text{inel}}(E) + \sigma_{\text{tran}}(E) + \sigma_{\text{fus}}(E).
\]  

(2.1)

The measurement of \( \sigma_{\text{fus}}(E) \), and the effects of the other reaction channels on the fusion process, is the central concern of this thesis.

2.1.1 The fusion cross-section

In the partial wave expansion, the fusion cross-section can be written as

\[
\sigma_{\text{fus}}(E) = \sum_{l=0}^{\infty} \sigma_l(E),
\]  

(2.2)

where \( \sigma_l(E) \) is the fusion cross-section for the \( l \)-th partial wave. Hereafter, the subscript 'fus' is dropped for convenience. The partial cross-section is given by

\[
\sigma_l(E) = (\pi \lambda^2)(2l + 1)T_l(E),
\]  

(2.3)

where \( \lambda \) is the reduced de Broglie wavelength and \( T_l(E) \) is the transmission coefficient, which is defined as the probability that the \( l \)-th partial wave will fuse at energy \( E \).

The effective interaction potential \( V_l(r) \) between the two interacting nuclei at a separation \( r \) (in one dimension) can be written as the sum of the Coulomb potential \( V_{\text{Coul}}(r) \) and the nuclear potential \( V_n(r) \) and the centrifugal term \( V_{\text{cent}}(r) \):

\[
V_l(r) = V_{\text{Coul}}(r) + V_n(r) + V_{\text{cent}}(r),
\]  

(2.4)
where

\[ V_{\text{Coul}}(r) = \begin{cases} 
Z_1 Z_2 e^2 (3R_c^2 - r^2) / 8\pi\varepsilon_0 R_c^3 & \text{for } r < R_c, \\
Z_1 Z_2 e^2 / 4\pi\varepsilon_0 r & \text{for } r \geq R_c,
\end{cases} \tag{2.5} \]

and

\[ V_{\text{cen}}(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2}. \tag{2.6} \]

In Eq. (2.5), \( Z_1 \) and \( Z_2 \) are the charges of the projectile and target, respectively. The radius, \( R_c = 1.2(A_1^{1/3} + A_2^{1/3}) \), is the characteristic radius of the charge distribution (in fm), where \( A_1 \) and \( A_2 \) are the masses of the projectile and target, respectively. In Eq. (2.5), \( e^2 / 4\pi\varepsilon_0 = 1.44 \text{ MeV fm} \) and \( \mu \) is the inertia parameter, taken to be the reduced mass of the system. In this work, the nuclear potential is taken to be of the Woods-Saxon form [43], given by

\[ V_n(r) = \frac{-V_0}{1 + \exp[(r - R_0)/a]^3}, \tag{2.7} \]

where \( V_0 \) is the potential depth, \( R_0 = r_N(A_1^{1/3} + A_2^{1/3}) \) and \( a \) is the surface diffuseness parameter. Since the region of interest for the interaction is at distances around the tail of this potential, the depth of the Woods-Saxon potential is not critical and the main dependence is in the surface diffuseness of the tail. A schematic plot of \( V_n(r) \) versus \( r \) is shown in Fig. 2.1 for various partial waves. For \( l = 0 \) there exists a maximum in the interaction potential which occurs at the radius \( R_B \) where the Coulomb and nuclear potentials exactly balance each other. This is called the fusion barrier or the single barrier and is represented by the symbol \( B_0 \). Then, if it is assumed that the barrier height does not change with \( l \), the barrier height for \( l > 0 \) is given by

\[ V_l(r) = B_0 + \hbar^2 l(l+1)/2\mu r^2. \tag{2.8} \]

### 2.1.2 The one-dimensional barrier penetration model

The effects of quantum tunnelling through the fusion barrier can be approximately taken into account as follows. Consider the case of two spherical nuclei with no dynamical distortions between them. Since \( V_l(r) \) in the region of the maxima in Fig. 2.1 approximate the shape of a parabola, the Coulomb and nuclear terms in the interaction potential can be replaced by an inverted harmonic oscillator.
FIG. 2.1: The interaction potential $V_l(r)$ plotted for various partial waves. At $l = 0$ there exists a maximum in the interaction potential (the fusion barrier) where the repulsive Coulomb potential exactly balances the attractive nuclear potential. This occurs at the radius $R_B$. Also shown is the potential for the grazing $l$-wave. As $l$ increases the barrier shifts to a smaller radius $R_E$. From Ref. [44].

The potential to give

$$V_l(r) = V_l(R_l) - \frac{1}{2} \mu \omega_l^2 (r - R_l)^2,$$

where $R_l$ is the barrier radius for the $l$-th partial wave. The barrier curvature $\hbar \omega_l$ is related to $V_l(r)$ by

$$\hbar \omega_l = \left( \frac{\hbar^2}{\mu} \frac{d^2 V_l(r)}{dr^2} \right)^{1/2} \bigg|_{r=R_l}.$$ (2.10)

In this approximation the transmission coefficients can then be written as [45]

$$T_l(E) = \{1 + \exp[(2\pi/\hbar \omega_l)(V_l(R_l) - E)]\}^{-1}. \quad (2.11)$$

If it is also assumed that the barrier curvature and radius are independent of $l$, so that $\hbar \omega_l = \hbar \omega_0$ and $R_l = R_B$ [1], then substituting Eq. (2.11) into Eqs. (2.3) and (2.2), and replacing the sum over $l$ with an integral, the fusion cross-section becomes

$$\sigma_w(E) = \frac{\hbar \omega_0 R_B^2}{2E} \ln\{1 + \exp[(2\pi/\hbar \omega_0)(E - B_0)]\}. \quad (2.12)$$

The cross-section $\sigma_w(E)$ in Eq. (2.12) is often referred to as the Wong cross-section [1]. For energies well above the single barrier, $E \gg B_0$, Eq. (2.12) reduces
to the classical result
\[ \sigma(E) \approx \pi R_B^2 (1 - B_0/E). \] (2.13)

For energies below the single barrier, \( E < B_0 \), Eq. (2.12) can be written
\[ \sigma(E) \approx \frac{\hbar \omega_0 R_B^2}{2E} \exp \left[ \frac{2\pi}{\hbar \omega_0} (E - B_0) \right], \] (2.14)

thus \( \sigma(E) \) is an exponential function of \( (E - B_0) \) below the single barrier.

The cross-section given by Eq. (2.12) has been used extensively in comparisons with measured fusion cross-sections at energies near \( B_0 \). For reactions involving light ions, this expression successfully reproduces the experimental data but for reactions involving heavier ions it significantly underestimates the experimental cross-sections, particularly at energies below \( B_0 \) [3]. The failure of the one-dimensional barrier penetration model is due to several factors.

The above derivation of the Wong cross-section does not take into account the fact that the barrier radius and barrier curvature depend on \( l \). As \( l \) increases the position of the barrier shifts to smaller values of \( r \), as shown in Fig. 2.1. Also, it was shown [44], from analysis of experimental data based on the proximity potential, that neglect of the \( l \)-dependence in the energy region \( E > B_0 \) leads to values for the surface diffuseness parameter \( a \) which are larger than expected. The shift in the barrier can be taken into account by considering the tail of the Woods-Saxon nuclear potential to have an exponential form [46,47]. The nuclear potential is then given by [44]
\[ V_n(r) = -V_0 \exp[-(r - R_B)/a]. \] (2.15)

The constant \( V_0 \) is determined using the fact that at the \( l = 0 \) barrier \( V_n'(R_B) = -V_0'C_{\text{Coul}}(R_B) \), where the prime denotes the derivative with respect to \( r \). This condition gives \( V_0 = -1.44aZ_1Z_2/R^2(B) \). For \( l > 0 \), or equivalently for \( E > B_0 \), the barrier position for the grazing \( l \)-wave occurs at a new radius \( R_E \) where \( R_E < R_B \), see Fig. 2.1. It can be shown [44] that at the position of the new barrier
\[ \frac{d}{dr} \left[ r^2(E - V_0(r)) \right] \bigg|_{r=R_E} = 0. \] (2.16)

Using Eqs. (2.15) and (2.5) it can be shown that the new radius can be written as
\[ R_E = R_B - a \ln \{1 + 2(E - B_0)/B_0\}, \] (2.17)

for shifts \( (R_B - R_E) \) that are not too large.
In the derivation of the Wong cross-section, it was assumed that the discrete variable \( l \) could be replaced with a continuous variable of \( l \). For reactions involving lighter nuclei, like \(^{12}\text{C} + ^{12}\text{C}\), this approximation may not be reasonable and the quantised nature of \( l \) gives rise to fusion oscillations in \( \sigma(E) \) [48]. However, for the heavy reactions in this work the above approximation is reasonable.

For the reactions studied here, the third and most important factor neglected in deriving the cross-section given by Eq. (2.12), is the effect of coupling to additional degrees of freedom. Such couplings are required to explain the experimental observation of significantly larger than predicted cross-sections at energies below the single barrier [4, 5]. The effect of coupling to other degrees of freedom is to replace the single fusion barrier of the one-dimensional BPM with a distribution of barriers. This concept is central to the study of low-energy heavy-ion fusion.

### 2.1.3 Coupled-channels formalism

The problem of barrier penetration in the presence of coupling to the other degrees of freedom was investigated by Dasso, Landowne and Winther [8] using a coupled-channels framework. In this section, the coupled-channels formalism, as applied to fusion, is detailed. Consider two incoming nuclei with masses \( A_1 \) and \( A_2 \), with the internal wave functions \( \psi_1 \) and \( \psi_2 \). If \( \hat{H}_1 \) and \( \hat{H}_2 \) represent the internal Hamiltonians (in the rest frame of each particle) then the Schrödinger equations for each system are

\[
\hat{H}_1 \psi_1 = \varepsilon_1 \psi_1, \quad \hat{H}_2 \psi_2 = \varepsilon_2 \psi_2,
\]

where \( \varepsilon_1 \) and \( \varepsilon_2 \) are the eigenvalues of the non-interacting system, which correspond to the internal energy states of the nuclei. If \( r \) is the distance between mass centres, then the interaction between the two nuclei can be described by the coupling potential \( V_{\text{cpl}}(r) \). In general, \( V_{\text{cpl}}(r) \) describes the excitation of one nucleus or both nuclei, or rearrangement of their constituent nucleons. The total Hamiltonian of the system \( \hat{H} \) is then the sum of the internal Hamiltonians, \( \hat{H}_1 \) and \( \hat{H}_2 \), the kinetic energy of the relative motion \( T(r) \), the nuclear and Coulomb potentials, and the coupling potential \( V_{\text{cpl}}(r) \):

\[
\hat{H} = \hat{H}_1 + \hat{H}_2 + T(r) + V_n(r) + V_{\text{Coul}}(r) + V_{\text{cpl}}(r).
\]

The kinetic energy of relative motion is given by

\[
T(r) = \frac{\hbar^2}{2\mu} \left[ \nabla^2 - \frac{I(I + 1)}{r^2} \right],
\]
where \( \mu = \frac{A_1 A_2}{A_1 + A_2} \) is the reduced mass of the system. If the spins of the interacting nuclei are not too large, then \( I \) is identified with the total angular momentum of the system. This approximation is called the \textit{isocentrifugal approximation} \cite{49-51}. For heavy-ions, where \( \mu \) is large and the fusion barrier occurs at a large radius, the difference in the centrifugal barriers for the various channels is small \cite{49}. So for angular momenta that are not too large, the isocentrifugal approximation is good for these heavy systems.

To simplify Eq. (2.19), let \( \hat{H}_0(\xi) \) be the Hamiltonian for both nuclei whose internal structure is represented by the variable \( \xi \). The internal system then satisfies the equation \( \hat{H}_0|n\rangle = \varepsilon_n|n\rangle \), where \( \varepsilon_n = \varepsilon_1 + \varepsilon_2 \). Also, consider only one spatial dimension, \( x \). The total wavefunction of the interacting system is then expanded to give

\[
\Psi(x) = \sum_n \chi_n(x)|n\rangle,
\]

(2.21)

where \( \chi_n(x) \) are the relative motion wavefunctions. The time-independent Schrödinger equation for the set of coupled equations is then

\[
[-\frac{\hbar^2}{2\mu} \frac{d^2}{dx^2} + V_i(x) - E]\chi_n(x) = -\sum_m \langle n|\hat{H}_0 + V_{\text{cpl}}(x;\xi)|m\rangle \chi_m(x),
\]

(2.22)

where \( \left( \frac{\hbar^2 k_n^2}{2\mu} \right) = E - \varepsilon_n \) and \( k_n \) is the wavenumber of the relative motion.

In writing Eq. (2.22), the nuclear and Coulomb potentials have been combined with the centrifugal term and written as the interaction potential \( V_i(x) \). In the above coupled-channels equations, it is assumed that the reduced mass and the interaction potential are the same for all channels \cite{9} and there is no explicit reference to angular momentum couplings or rearrangement collisions \cite{8}.

The sum in Eq. (2.22) is over an infinite set of equations describing the complete reaction. In practical solutions of the coupled-channels equations, the infinite number of equations is truncated to the relatively few channels that are expected to be important. To determine the cross-section for fusion an assumption has to be made as to what defines fusion. It is usually assumed that fusion occurs when flux penetrates some distance into the nuclear interior. This can be modelled in two ways. Either a localised imaginary potential is introduced to model the loss of flux from the direct channels to the fusion channel, or an ingoing-wave boundary condition (IWBC) is used. The latter, applied to the coupled differential equations given by Eq. (2.22), gives the following boundary
conditions

\[ \chi_n(x) \rightarrow \begin{cases} 
\delta_{n0} \exp(-ik_n x) + r_n \exp(ik_n x) & x \rightarrow +\infty, \\
t_n \exp(-ik_n x) & x \rightarrow -\infty, 
\end{cases} \quad (2.23) \]

where \( r_n \) and \( t_n \) are the reflection and transmission coefficients, respectively. The label '0' denotes the plane wave in the entrance channel incident on the barrier \( V_i(x) \) from right to left, and it is assumed that there are reflected waves to the right and transmitted waves to the left. The IWBC is imposed in the simplified coupled-channels model of Ref. [8]. In this model, the coupled equations are decoupled using certain approximations, as described below.

Decoupling the equations

First consider the case where the coupling potential factorises into two parts, the intrinsic-motion, described by \( G(\xi) \), and the relative-motion, described by \( F(x) \) [9]. The coupling interaction is then

\[ (n|V_{\text{cpl}}(x, \xi)|m) = F(x)(n|G(\xi)|m) = F(x)G_{nm}, \quad (2.24) \]

so that the coupling form factor \( F(x) \) is independent of the intrinsic states being coupled. The coupled equations in Eq. (2.22) are then decoupled if \( F(x) \) is treated as being spatially constant in the region of the barrier, giving

\[ (n|\hat{H}_0 + F(x)G(\xi)|m) \approx \varepsilon_n \delta_{nm} + F_0 G_{nm} \equiv M_{nm}, \quad (2.25) \]

where \( F_0 \) is considered as the representative value of the coupling strength and \( M_{nm} \) is the coupling matrix. This approximation is often referred to as the constant coupling approximation. The decoupled equations are now solved by introducing the unitary transformation

\[ Y_m(x) = \sum_n U_{mn}(x)\chi_n(x), \quad (2.26) \]

where \( U(x) \) diagonalises the matrix \( M_{nm} \),

\[ \sum_{ik} U_{ni} M_{ik} U_{km}^{-1} = \lambda_m \delta_{nm}, \quad (2.27) \]
to give the set of eigenvalues $\lambda_m(x)$. Using Eqs. (2.25) and (2.26) the set of coupled equations in Eq. (2.22) can be decoupled to give

$$\left[-\left(\frac{\hbar^2}{2\mu}\right)\frac{d^2}{dx^2} + V_l(x) + \lambda_m(x) - E\right]Y_m(x) = 0. \quad (2.28)$$

The total transmission function for fusion is then the sum of the transmission coefficients for each eigenbarrier weighted by the factors $|U_{m0}(x)|^2$:

$$T_l(E) = \sum_m |U_{m0}(x)|^2 T_l(E, V_l(x) + \lambda_m(x)), \quad (2.29)$$

where the transmission coefficients are now functions of $E$ and the quantity $V_l(x) + \lambda_m(x)$. Note that in the constant coupling approximation, the unitary matrix and the eigenvalues do not depend on $x$, so that $U_{m0}(x) = U_{m0}$ and $\lambda_m(x) = \lambda_m$. This result demonstrates that the effect of coupling is to replace the single (uncoupled) barrier with a distribution of barriers $V_l(x) + \lambda_m(x)$ \[9\]. Thus, the distribution of barriers arises naturally from the coupled-channels picture. For situations where at least one of the eigenvalues is negative, there will be enhancement in the transmission at energies below the uncoupled barrier.

If the excitation energies of the intrinsic states are small compared to the coupling strength $F(x)$, then they can be neglected altogether. In this case it is not necessary to use the constant coupling approximation, and the coupled equations can be solved using

$$\langle n|\hat{H}_0 + F(x)G(\xi)|m\rangle \approx F(x)\langle n|G(\xi)|m\rangle. \quad (2.30)$$

This approximation is called the sudden approximation and usually holds for deformed nuclei, like $^{154}$Sm. The solution of the coupled-channels equations is exact when $\epsilon_n = 0$ \[52\]. As mentioned in Chapter 1, the term strong coupling is used to describe situations in which $F(x) > |Q|$. An example is the $^{16}$O + $^{154}$Sm reaction, where the coupling strength arises from coupling between the elastic channel and successive members of the ground-state rotational band of $^{154}$Sm \[49\]. The term weak coupling \[53\] is defined as $F(x) < |Q|$.

### 2.1.4 The simplified coupled-channels code CCMOD

The codes CCFUS \[54\] and CCDEF \[55\], which are based on the above eigenchannel formalism, have been used extensively to model the effects of inelastic and transfer couplings on fusion. In the original versions of these codes, the matrix equation
[Eq. (2.27)] was solved two channels at a time\(^1\) to obtain the eigenvalues \(\lambda_m(x)\) and weights \(w_m(x) = |U_m(x)|^2\) of each eigenbarrier. The weights were evaluated at the position of the single barrier, \(x = R_B\). The eigenvalues were evaluated either at \(x = R_B\) or using a second order correction to the eigenradius, which applied for radii no larger than 1 fm from \(R_B\). In the version of the code used in this work, CCMOD, the matrix diagonalisation is performed exactly and the eigenvalues are calculated taking into account the shift in the radius for each eigenbarrier [56]. Before discussing the methods used in CCMOD, the approach of the codes CCFUS and CCDEF is reviewed.

When the coupling effects are relatively strong, use of the constant coupling approximation can lead to significant overpredictions of the fusion cross-sections at energies below the single barrier [50]. This occurs because the constant coupling approximation does not take into account the shift in the barrier positions. To rectify this problem, Dasso and Landowne [54] extended the model of Refs. [8,9] to include variations in the radius of the eigenbarriers. The eigenpotentials \(V_i(x) + \lambda_m(x)\) in the vicinity of the unperturbed barrier \(V(R_B) = B_0\) are expanded to give

\[
V_i(x) + \lambda_m(x) = V(R_B) + \lambda_m(R_B) + \lambda'_m(R_B)(x - R_B) + \frac{1}{2}[V''(R_B) + \lambda''_m(R_B)](x - R_B)^2, \tag{2.31}
\]

where the primes denote the derivatives with respect to \(r\). The radii of the eigenbarriers, \(R_m\), are then given by

\[
R_m = R_B - \frac{\lambda'_m(R_B)}{[V''(R_B) + \lambda''_m(R_B)]}. \tag{2.32}
\]

This procedure works well if the shifts \((R_m - R_B)\) are not too large [54]. When Eq. (2.32) is included in the model, it is often said that finite range effects have been taken into account. This procedure is used in the codes CCFUS and CCDEF.

The code CCMOD [56] does not follow this method, rather it solves the matrix equations as follows. When the excitation energies of the intrinsic states are finite, they can be included in the couple-channels equations using the constant coupling approximation, described earlier. If the coupling strength is also assumed to vary with radius, then the coupled-channels equations cannot be solved exactly. However, the approach of CCMOD includes both finite excitation energies

\(^1\)A consequence of solving the matrix problem two channels at a time, is that for the three channel problem, the cross coupling (two-phonon) term is also included along with the two one-phonon terms.
and coupling strengths that vary with radius. The coupled equations are thus solved at the expense of the condition of unitarity. This approach works well provided the excitation energies of the states are not too large. For large values of the excitation energy, this approximation is poor [57].

In the modified version [56] of the codes CCFUS and CCDEF, the coupling matrix is diagonalised at each value of the inter-nuclear separation \( x \). The barrier radius is obtained by finding the value of \( x \) for which \( V_i(x) + \lambda_a(x) \) is a maximum, instead of using the correction in Eq. (2.32). Then the eigenbarriers \( V_i(R_{\alpha}) + \lambda_a(R_{\alpha}) \) are evaluated at the new eigenradii \( R_{\alpha} \). In the code CCMOD, the weights are still evaluated at \( R_B \), as done in CCFUS and CCDEF. The transmission coefficients in Eq. (2.29) are then calculated using the inverted harmonic oscillator approximation, Eq. (2.11), and the cross-sections are calculated using Eqs.(2.3) and (2.2).

The code CCMOD also includes the correction for the shift in the radius for higher values of \( l \), as given by Eq. (2.17) above.

**Inputs to the code CCMOD**

The potential parameters for CCMOD are specified by the user. The code requires the depth \( V_0 \) of the nuclear potential, a potential radius \( r_V \) and the diffuseness \( a \). These parameters are usually determined from fits to experimental data in the energy region well above the single barrier, where the effects due to coupling are expected to be small. In the code CCMOD, static deformation of nuclei is treated classically by considering the different orientations of the interacting nuclei. The quadrupole and hexadecapole deformation parameters \( \beta_2 \) and \( \beta_4 \) of the deformed nuclei are specified as input. The Coulomb and nuclear potential terms are calculated as described in Ref. [55]. For inelastic excitations, the coupling form factor \( F(r) \) is given by [53,54]

\[
F(r) = \frac{\beta_\lambda}{\sqrt{4\pi}} \left[ -R \frac{dV_n(r)}{dr} + \frac{3Z_1Z_2e^2}{(2\lambda + 1)r^{\lambda+1}} \right],
\]

where \( \lambda \) is the multipolarity of the transition, \( \beta_\lambda \) is the deformation parameter of the mode and \( R \) is the radius of the nucleus which is excited.

Additional couplings can be included in CCMOD by directly entering values for the coupling strength and the Q-value of the channel. Such channels are assumed to represent one-particle transfer reactions whose form factors are given by [58,53]

\[
F_{\text{tran}}(r) = \frac{\kappa}{\sqrt{4\pi}} \exp \left[ -\frac{(r - R_B)}{1.2 \text{ fm}} \right],
\]
where $\mathcal{K}$ is the transfer coupling strength. The (net) $Q$-value for the transfer reaction is given by $Q = Q_t - Q_{\text{opt}}$, where $Q_t$ is the difference in the binding energies of the initial and final products and $Q_{\text{opt}}$ is the optimum $Q$-value, which takes into account the change in the Coulomb energy as a result of the transfer, and is given by [59]

$$Q_{\text{opt}} = \left(\frac{Z'_1 Z'_2}{Z_1 Z_2} - 1\right) V_B,$$

(2.35)

where $Z'_1$ and $Z'_2$ are the charges of the nuclei after the transfer of the nucleons and $V_B = 1.44 Z_1 Z_2 / R_B$. For neutron transfers, $Z'_1 Z'_2 = Z_1 Z_2$ and $Q_{\text{opt}} = 0$.

The channels that couple to the elastic channel are entered into CCMOD in a coupling matrix which has the form

$$M = \begin{pmatrix}
0 & F_1(r) & F_2(r) & \ldots & F_n(r) \\
F_1(r) & -Q_1 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
F_n(r) & 0 & 0 & \ldots & -Q_n
\end{pmatrix},$$

(2.36)

where the coupling form factors $F_i(r)$ are calculated as described above. For inelastic excitations, $Q_i$ is the excitation energy of the state and for transfer reactions, $Q_i = Q_{\text{opt}}$.

Two-channel coupling

Consider a simple coupling scheme where the elastic channel is coupled to an inelastic channel which represents an excited state in a nucleus. In the constant coupling approximation, the coupling matrix

$$M = \begin{pmatrix}
0 & F_0 \\
F_0 & -Q
\end{pmatrix},$$

(2.37)

is diagonalised to obtain the eigenvalues

$$\lambda_{\pm} = \frac{1}{2}(-Q \pm \sqrt{Q^2 + 4F_0^2}).$$

(2.38)

The corresponding weights are given by [9]

$$U^2_{\pm} = \frac{4F_0^2}{4F_0^2 + (-Q \pm \sqrt{Q^2 + 4F_0^2})^2}.$$  

(2.39)
FIG. 2.2: The transmission functions for a simple coupling scheme in the eigenchannel representation. The classical sharp-cutoff transmission is represented by the dotted line. The transmission through the single (uncoupled) barrier is represented by the dashed line. The quantum mechanical tunnelling distributes the probability about the single barrier $B_0$. When coupling is included in the eigenchannel model, the transmission is enhanced at energies below $B_0$ and suppressed at energies above it (solid line). Adapted from Ref. [8].

Figure 2.2 illustrates schematically how the transmission coefficients change for coupling in this simple 2-channel case. The dashed line in Fig. 2.2 is the transmission through the uncoupled barrier and the solid line demonstrates how the transmission increases at energies below $B_0$ when coupling is included. This is interpreted as transmission through the barrier at $[V_i(x) + \lambda_-]$. Correspondingly, the barrier at energies above $B_0$ is denoted by $[V_i(x) + \lambda_+]$.

2.1.5 The distribution of fusion barriers

It was shown in the previous Section that coupling the elastic channel to an inelastic channel splits the single barrier into two barriers. More generally, coupling to several channels will split the single barrier into a distribution of barriers. For a continuous distribution of fusion barriers denoted by $D(B)$, $\sigma(E)$ is assumed [13] to be given by

$$\sigma(E) = \int_0^\infty \sigma(E, B) D(B) dB,$$

(2.40)

where $\sigma(E, B)$ is the cross-section, summed over all $l$ for the barrier $B$, and

$$\int D(B) dB = 1.$$  

(2.41)
In the eigenchannel representation described above, the cross-section for fusion is given by the weighted sum of each of the eigenchannel contributions:

$$\sigma(E) = \sum_\alpha w_\alpha \sigma_\alpha(E), \quad (2.42)$$

where $w_\alpha$ is the weight for the eigenchannel $\alpha$ [15]. If it is assumed that fusion occurs in each eigenchannel whenever the incoming flux penetrates the corresponding eigenbarrier $B_\alpha$, then Eq. (2.42) can be identified with Eq. (2.40). Now the distribution of barriers corresponds to a discrete spectrum of barriers given by

$$D(B) = \sum_\alpha w_\alpha \delta(B - B_\alpha). \quad (2.43)$$

An example of a discrete barrier distribution is shown schematically in Fig. 2.3(a) where the thick solid lines represent the positions of the eigenbarriers and the length of the line their weights.

In the approach by Stelson et al. [13, 14], continuous barrier distributions $D_c(B)$ were used to fit the experimental cross-sections using Eq. (2.40) and the classical expression for the fusion cross-section

$$\sigma(E, B) = \sigma_c(E, B) = \pi R^2 (1 - B/E) \quad \text{for } E > B,$$
$$= 0 \quad \text{for } E \leq B, \quad (2.44)$$

where it is assumed the $R$ is independent of $B$. The assumed form of the continuous barrier distribution was obtained largely by trial and error. The technique of Rowley et al. [15] demonstrated that the distribution of barriers could be extracted from experimental data without introducing a particular parametrisation of the distribution. Following Ref. [15], the second derivative with respect to $E$ of Eq. (2.44) gives the expression

$$\frac{1}{\pi R^2} \frac{d^2 (E \sigma_c)}{dE^2} = \delta(E - B) \quad (2.45)$$

for a barrier distribution with only one barrier. Repeating this procedure for Eq. (2.40) gives

$$\frac{d^2 (E \sigma)}{dE^2} = \frac{d^2}{dE^2} \left\{ \int_0^\infty E \sigma(E, B) D(B) dB \right\} \quad (2.46)$$
FIG. 2.3: (a) Here \( D(B) \) is a discrete distribution of barriers. The thick vertical lines represent the position and weights of the eigenbarriers. For simplicity, the barriers in this diagram have the same separation in energy and the same weight. (b) The thin broken lines are the quantities \( E\sigma \), calculated from the classical cross-sections for the eigenbarriers \( B_1, B_2 \) and \( B_3 \). The thicker broken line is the sum of each \( E\sigma \). (c) The first derivative of \( E\sigma \) is a series of step functions (thick broken line). (d) The second derivative of \( E\sigma \) returns the original discrete distributions of barriers. When the effects of quantum mechanical tunnelling are included, \( E\sigma \) and its derivatives are a smooth function of \( E \) (thick shaded curves).
which is equal to
\[ \frac{d^2(E\sigma)}{dE^2} = \frac{d^2}{dE^2} \left\{ \int_0^E \pi R^2 (E - B) D(B) dB \right\} \] (2.47)
upon substitution of Eq. (2.44). Then using Eq. (2.45), the expression in Eq. (2.47) becomes
\[ \frac{1}{\pi R^2} \frac{d^2(E\sigma)}{dE^2} = D(E). \] (2.48)
So if Eqs. (2.40) and (2.44) are valid, then the distribution of barriers \( D(E) \) can be extracted directly from \( \sigma(E) \).

The above technique is applied to the example shown in Fig. 2.3 for three eigenbarriers with equal weights. The quantity \( E\sigma \) is plotted as a function of \( E \) in Fig. 2.3(b) for the three barriers using the classical expression Eq. (2.44). The first derivative of \( E\sigma \) produces the step functions shown in Fig. 2.3(c). This function is related to the transmission coefficients for fusion. The second derivative of \( E\sigma \) produces the original three barriers, as shown in Fig. 2.3(d).

When the classical form of the cross-section, Eq. (2.44), is replaced by the expression that includes quantum mechanical tunnelling, Eq. (2.12), then the equivalent expression for Eq. (2.45) is
\[ \frac{1}{\pi R^2} \frac{d^2(E\sigma)}{dE^2} = G(x) = \frac{2\pi e^x}{\hbar \omega (1 + e^x)^2}, \] (2.49)
where \( x = (2\pi/\hbar \omega)(E - B) \). The function \( G(x) \) can be thought of as a generalisation of the delta function \( \delta(E - B) \), smeared by the effects of quantum mechanical tunnelling. In the limit of \( \hbar \omega \to 0 \), the function \( G(x) \) becomes a delta function and Eq. (2.49) gives the discrete relation, Eq. (2.45). Substitution of Eq. (2.49) into Eq. (2.40) produces the analogous expression to Eq. (2.48):
\[ \frac{1}{\pi R^2} \frac{d^2(E\sigma)}{dE^2} = \tilde{D}(E) \equiv \int_0^\infty D(B)G(x) dB. \] (2.50)
Thus, the quantity on the LHS of Eq. (2.50) is related to \( \tilde{D}(B) \), which is in turn related to the 'true' barrier distribution \( D(B) \) smoothed by the quantum tunnelling. The effects of this smoothing applied to the example in Fig. 2.3 are shown by the thick shaded lines. The quantity \( E\sigma \) is now a smooth function of \( E \), as shown in Fig. 2.3(b). The second derivative now gives the Gaussian-like barriers in Fig. 2.3(d).

The effects of the smoothing function \( G(x) \) were investigated by Rowley et
al. [15]. Since $G(x)$ is derived using the Wong approximation to the barrier penetration, this approximation was investigated by comparing it to the results of a more exact calculation. In Fig. 2.4 the solid line represents the quantity $\pi R^2 G(x)$ as a function of $E$, obtained from the LHS of Eq. (2.49), for an exact calculation of the fusion cross-section with one channel only. The barrier parameters for this calculation were $B = 58.2$ MeV, $R = 10.51$ fm and $\hbar \omega = 4.2$ MeV. The broken curve is the same quantity calculated using the RHS of Eq. (2.49) with the above barrier parameters. The full-width half maximum (FWHM) of the function is $0.56\hbar \omega$. The good agreement between the two calculations means that the smoothing of $D(B)$ associated with the function $G(x)$ is realistic.

![Graph](image)

**FIG. 2.4:** The smoothing function $G(x)$ evaluated using the RHS of Eq. (2.49) with the barrier parameters given in the text (solid line). The broken line is $G(x)$ evaluated from the LHS of Eq. (2.49) where the cross-sections were determined from an exact model calculation of the barrier penetration. From Rowley et al. [15].

The FWHM of the smoothing function has an important bearing on the structure present in the barrier distribution. Consider the case of a discrete barrier distribution with two eigenbarriers. If $\Delta B_\alpha$ is defined as the difference in energy between these two barriers, then there are two limiting cases.

1. $\Delta B_\alpha \leq 0.56\hbar \omega$, implying that the barrier distribution is dense. Here the barrier distribution is not too different from the spherical barrier distribution. This is shown in Fig. 2.5 where the solid line is the spherical barrier
distribution and the broken line is the barrier distribution where $\Delta B_\alpha \approx 1$ MeV.

2. $\Delta B_\alpha > 0.56\hbar\omega$, implying the barrier distribution is not dense. In this case the two barriers will be resolved and the structure will be evident in $d^2(E\sigma)/dE^2$. The dotted line in Fig. 2.5 demonstrates this case, where the spread in the barriers is $\approx 4$ MeV.

**FIG. 2.5:** A comparison between the barrier distributions obtained for a single barrier (solid line), a dense spectrum of barriers (broken line) and a spectrum that is not dense (dotted line). The calculations were performed with CCMD. The barrier distribution shown by the dotted line is asymmetric because the curvature for each barrier is evaluated for a different value of the potential, see Eq. (2.10).

For all reactions considered here, $\hbar\omega \approx 4$ MeV, and so the single barrier width is around 2 MeV. If the experimental barrier distribution has a width much larger than the width of a single barrier, then the quantity $d^2(E\sigma)/dE^2$ obtained from the measured fusion cross-sections should reveal the coupled-channels effects on fusion. To obtain a meaningful barrier distribution, the cross-sections must be measured over the whole energy range of the distribution and in energy steps much smaller than typically done in earlier experiments. As shown below, the cross-sections must also be measured with high precision.
Obtaining the barrier distribution from experimental data

It was shown in the previous Section that the quantity \( d^2(E\sigma)/dE^2 \) is directly related to the distribution of barriers smoothed by barrier penetration [15]. The smoothing of the barrier distribution was taken into account using Eq. (2.12). An example of a barrier distribution which consists of a set of eight barriers is shown in Fig. 2.6. The broken line in Fig. 2.6 represents the quantity \( d^2(E\sigma)/dE^2 \) with the smoothing effects of Eq. (2.12). However, Eq. (2.12) does not take into account the change in the barrier radius with \( l \), and each barrier is assumed to have the same radius and curvature \( \hbar \omega \). In Fig. 2.6, the solid line is the result of a more realistic calculation [17] that takes these factors into account. The difference between these two calculations is not large. This demonstrates that the quantity \( d^2(E\sigma)/dE^2 \), the solid line in Fig. 2.6, is a good representation of the smoothed barrier distribution shown by the broken line in Fig. 2.6. Since the calculations performed include the \( l \)-dependence of the eigenradii, it is convenient to refer to \( d^2(E\sigma)/dE^2 \) as the barrier distribution.

![Image of Fig. 2.6](image)

**FIG. 2.6:** A fictitious example of a barrier distribution for a system with many eigenbarriers. The broken line is the 'true' barrier distribution, as represented by the vertical solid lines, smoothed by the effects of quantum tunelling. The solid line is the quantity \( d^2(E\sigma)/dE^2 \) calculated from a more realistic calculation that includes the \( l \)-dependence of \( R_\alpha \). The barrier distribution becomes negative for large \( E \) because the barrier radius shifts to smaller values with the increase in \( l \). From Ref. [17].
To calculate the barrier distribution from the measured fusion cross-sections, the second derivative of $E\sigma$ is approximated using a multi-point derivative formula [60]. For the cross-sections $\sigma_i$ measured at energies $E_i$ (in the c.m.) with a discrete constant separation in energy $\Delta E$, the 3-point derivative formula is given by

$$\frac{d^2(E\sigma)}{dE^2}\bigg|_{E=E_i} = \frac{(E\sigma)_{i-1} - 2(E\sigma)_i + (E\sigma)_{i+1}}{(\Delta E)^2}.$$  \hspace{1cm} (2.51)

The statistical uncertainty $\delta_c$ for Eq. (2.51) is approximately given by [16]

$$\delta_c \simeq \left( \frac{E}{\Delta E^2} \right) [(\delta\sigma_{i-1})^2 + 4(\delta\sigma_i)^2 + (\delta\sigma_{i+1})^2]^{1/2},$$  \hspace{1cm} (2.52)

where $\delta\sigma_i$ are the uncertainties in the experimental cross-sections. Equation (2.52) shows that very good statistics are required in the higher energy range of the measurement. This is because the statistical uncertainty, in millibarns, is directly proportional to the cross-section which increases with $E$. So to maintain a constant value for the statistical uncertainty $\delta_c$, over the whole energy range of the barrier distribution, an increase in the precision of the measurement is required as $E$ increases. Usually, the cross-sections for the high energy region are measured with a fixed percentage uncertainty of $\pm 1\%$, which means $\delta_c$ will be an order of magnitude larger when the cross-section is $10^3$ mb compared to $10^2$ mb. Fortunately, the barrier distribution is well defined at energies below $B_0$, even with relatively large percentage uncertainties.

Ideally, smaller energy steps $\Delta E$ give a better estimate of $d^2(E\sigma)/dE^2$ and this quantity becomes exact in the limit of $\Delta E \to 0$. However, as can be seen from Eq. (2.52), $\delta_c$ is inversely proportional to $\Delta E^2$ and, for a given level of precision, as $\Delta E$ is decreased the barrier distribution becomes more and more ill defined. The opposite is also true. If $\Delta E$ is increased then $\delta_c$ is reduced and the quantity $d^2(E\sigma)/dE^2$ is better defined, but this leads to an increased damping of the features present in the barrier distribution and information about the structure present on an energy scale smaller than $\Delta E$ will be lost. The choice of the size of $\Delta E$ is thus a compromise between these two considerations. A step length of 2 MeV in the laboratory frame is adopted for most of the reactions presented here. This does not result in significant additional smoothing of the barrier distribution since it is already smoothed by $0.56\hbar \omega \approx 2$ MeV because of the effects of quantum tunnelling. All barrier distributions calculated from theory were determined in a manner identical to their corresponding experimental barrier distributions.
2.2 Fission fragment angular distributions

In this Section, the standard model of fission fragment angular distributions is described. The model assumes that the direction of the fission fragments is determined at the system's saddle point [61]. If \( Jh \) is defined as the total angular momentum of the system, then the component angular momentum \( Kh \) is the projection of angular momentum quantum number \( J \) on the nuclear symmetry axis. These angular momenta are shown schematically in Fig. 2.7 for an elongated system. The angular distribution of the fission fragments depends upon both \( J \) and the value of \( K \) at the saddle point. A quantitative description of the fission fragment angular distributions relies on the assumption that the fragments are emitted along the nuclear symmetry axis and that the value of \( K \), determined by the system at the saddle point, is not altered as the system proceeds from saddle to scission and then separation [25].

When the excitation energy of the system is large enough, the fission transition state can be described by statistical methods and there is a simple relationship [62] between the nuclear temperature \( T \), the shape of the elongated system and the distribution of \( K \), all determined at the saddle point. If it is further assumed that complete fusion has taken place, then the angular distribution of the fission fragments is given by

\[
W_{MK}^J(\theta) = \frac{2J + 1}{2} |D_{MK}^J(\phi, \theta, \psi)|^2,
\]

where \( \theta \) is the angle between the nuclear symmetry axis and the space-fixed axis. Here, the space-fixed axis is taken to be the beam axis as shown in Fig. 2.6. In Eq. (2.53), \( M \) is the projection of \( J \) on the beam axis and \( D_{MK}^J(\phi, \theta, \psi) \) are the symmetric-top wavefunctions [25,34]. For the fusion of systems in which the target and projectile have zero spins, then \( M = 0 \) and Eq. (2.53) reduces to

\[
W_{0K}^J(\theta) = \frac{2J + 1}{2} |d_{0K}^J(\theta)|^2.
\]

The dependence of the angular distributions on the other two Euler angles \( \phi \) and \( \psi \) disappears when taking the absolute value of the \( D \) functions. This means that the angular distribution for fission fragments depends only on the angle \( \theta \) and is isotropic in the azimuthal angle \( \phi \). Note also, that the assumption \( M = 0 \) is not strictly true when particle emission occurs before fission. However, in the reactions considered here, this effect is small.

The statistical description of the fission transition state is given below, fol-
FIG. 2.7: A schematic representation of the angular momentum vectors used in describing the fission fragment angular distributions. The total angular momentum quantum number is given by $J$. The projection of $J$ on the nuclear symmetry axis defines $K$. The angle $\theta$ is defined as the angle between the nuclear symmetry axis and the space-fixed axis which is taken to be the beam axis. In this figure $M = 0$. The quantity $R$ is the collective rotational angular momentum quantum number, defined in a direction perpendicular to the nuclear symmetry axis. Taken from Ref. [25].

Following the approach of Ref. [25]. The level density of states with total angular momentum $Jh$ and projection $Kh$ on the nuclear symmetry axis is given by

$$\rho(J, K) \propto \exp\left[\frac{(E - E^J_K)}{T}\right],$$  \hspace{1cm} (2.55)

where $E$ is the total energy and $E^J_K$ is the rotational energy of the system at its saddle point and $T$ is assumed to be constant for small changes in the excitation energy around $E$. The rotational energy is given by

$$E^J_K = \frac{\hbar^2}{2J_\perp}(J^2 - K^2) + \frac{\hbar^2}{2J_\parallel}K^2,$$  \hspace{1cm} (2.56)

where $J_\perp$ and $J_\parallel$ are the moments of inertia perpendicular and parallel to the symmetry axis, respectively. Substitution of Eq. (2.56) into Eq. (2.55) gives

$$\rho(J, K) \propto \exp\left\{ \frac{E}{T} - \frac{\hbar^2}{2J_\perp T}J^2 - \frac{\hbar^2}{2T}K^2 \left[ \frac{1}{J_\parallel} - \frac{1}{J_\perp} \right] \right\}.$$  \hspace{1cm} (2.57)
For a fixed value of $E$, $T$ and $J$, the level density dependence on $K$ becomes

$$\rho(K) \propto \exp \left\{ -\frac{\hbar^2}{2T} K^2 \left[ \frac{1}{J_{||}} - \frac{1}{J_{\perp}} \right] \right\},$$

(2.58)

or equivalently [62]

$$\rho(K) \propto \exp[-K^2/(2K_0^2)], \quad \text{for } K \leq J,$$

(2.59)

$$0, \quad \text{for } K > J,$$

where

$$K_0^2 = \frac{J_{\text{eff}}}{\hbar^2} T, \quad \text{and} \quad J_{\text{eff}}^{-1} = J_{||}^{-1} - J_{\perp}^{-1},$$

(2.60)

where $J_{\text{eff}}$ is the effective moment of inertia. The distribution of $K$ values is a Gaussian with a variance of $K_0^2$.

The fission fragment angular distributions following fusion are then obtained by summing over all values of $J$ and $K$ to give [25]

$$W(\theta) \propto \sum_{J=0}^{\infty} (2J + 1)T_J \frac{\sum_{K=-J}^{J} (2J + 1)|d_{JK}^{J}(\theta)|^2 \exp[-K^2/(2K_0^2)]}{\sum_{K=-J}^{J} \exp[-K^2/(2K_0^2)]},$$

(2.61)

where $T_J$ are the transmission coefficients for the fusion of the $J$th partial wave. Usually $J$ can be identified with the orbital angular momentum quantum number $l$ of the projectile, although when pre-scission emission occurs, the value of $J$ is equal to $l$ modified by the angular momentum carried away by the evaporated particles. Because of level density effects, on average $J$ is less than $l$ after particle emission. The quantity $d_{JK}^{J}(\theta)$ in Eq. (2.61) is given by [34]

$$d_{JK}^{J}(\theta) = J! \sqrt{(J - K)!(J + K)!} \sum_{x=K}^{J} (-1)^x \frac{(\sin \frac{\theta}{2})^{2x-K}(\cos \frac{\theta}{2})^{2J+K-2x}}{(J - x)!(J + K - x)!(x - K)!}.$$

(2.62)

The curves shown in Fig. 2.8 are examples of the fission fragment angular distributions evaluated using Eqs. (2.61) and (2.62) for different values of the ratio $J/K_0$. These calculations were performed using the expressions given in Appendix A of Ref. [34]. The $d$-functions were calculated for $\theta = 0^\circ, 5^\circ, 10^\circ, \ldots, 90^\circ$ in the range of $J = 0-150$ and $K = 0-J$ and the results stored in an array. This array was referenced by a subroutine which returned $W(\theta)$ for given values of $K_0$ and $J$. A small correction to $K_0$ was made before $W(\theta)$ was determined to take into account any changes in the trajectory of the fragments due to Coulomb reorientation effects [34]. For the range of angular momenta covered in this work,
this correction is around 1% or less.

When $K = 0$, the fragments will be emitted in the plane perpendicular to $J$ which is in turn perpendicular to the beam axis. Here, the fission fragment angular distribution is the classical case, proportional to $1/(\sin \theta)$. This angular distribution is shown by the solid line in Fig. 2.8. For a given $J$, larger values of $K_0$ result in angular distributions that are less forward peaked, as shown by the dotted and broken curves in Fig. 2.8.

![Diagram showing fission fragment angular distributions](image)

**FIG. 2.8:** A plot of the fission fragment angular distributions for ratios of $J/K_0$. When the ratio is infinity, then angular distribution behaves as $1/(\sin \theta)$.

### 2.2.1 The fission fragment anisotropy

The fission fragment anisotropy is defined as the ratio of the fission fragment yield at $\theta = 180^\circ$ (or $\theta = 0^\circ$) to that at $\theta = 90^\circ$. A useful approximate expression for the anisotropies is

$$A = \frac{W(180^\circ)}{W(90^\circ)} \approx 1 + \frac{(J^2)\hbar^2}{4TJ_{\text{eff}}} = 1 + \frac{(J^2)}{K_0^2},$$

(2.63)
where \( \langle J^2 \rangle \) is the mean-square angular momentum of the system. If two of the variables in Eq. (2.63) are known, then the third quantity can be calculated from measurements of the fission fragment anisotropies. This approach has been used by many authors to extract information about the mean-square angular momentum for fusion. (See Ref. [20], Refs. [63] and [64] and references therein). The quantity \( K_q^2 \) can be determined in two ways. Either the value of \( K_q^2 \) is obtained from model calculations of \( J_{\text{eff}} \) and \( T \), or experimentally from a calibration reaction [20]. Frequently, data from \( \alpha \)-induced reactions are used. These data are for reactions that make the compound system of interest at energies roughly twice the barrier energy of the \( \alpha \)-induced reaction, so that excitation energies and angular momenta are comparable.

Where there are no suitable calibration reactions, models must be used to determine \( K_q^2 \). This is the case for the \(^{16}\text{O} + ^{208}\text{Pb} \) reaction. The effective moment of inertia at the saddle point can be estimated from macroscopic liquid-drop models [65] and the saddle-point temperature can be determined from statistical model calculations [66]. The relevant aspects of these models are discussed briefly below.

### 2.2.2 The statistical model of compound nucleus decay

As mentioned above, when the excitation energy of the compound system is large, then the high density of states means that the decay of the system can be described by statistical methods. It is assumed that the compound system is in equilibrium after its formation and that the decay is independent of the formation. The decay is assumed to proceed either through fission or the emission of light particles, neutron emission being dominant for the reactions considered here. The amount of \( \gamma \)-emission in competition with particle emission is very small, so this is not considered in the calculations in this thesis. The statistical models used here calculate only the early stages of decay. Once the excitation energy of the system is smaller than the particle binding energy and below the fission threshold, the modelling of the decay ceases. The various parameters used in the statistical model calculations are described below.

Consider the formation of a hot compound system with excitation energy \( E^* \) and angular momentum \( J\hbar \). If the compound system fissions before any neutrons are emitted, then it is referred to as first chance fission. Fission can also occur after the emission of one neutron or two neutrons, and so on. The average number of neutrons emitted before fission is called the pre-scission neutron multiplicity. The emission of a succession of neutrons leads to the formation of evaporation
residues.

In the liquid drop model [67, 68], the stability of a system with respect to small changes in the Coulomb and surface energies is measured in terms of the fissility parameter $x$. Consider axially symmetric distortions of the compound system about its spherical shape. If $\Delta E_c$ and $\Delta E_s$ represent small changes in the Coulomb and surface energies, respectively, then the nucleus is stable for [25]

$$x = \frac{|\Delta E_c|}{\Delta E_s} < 1. \quad (2.64)$$

In the liquid drop model of Refs. [67, 68], the fissility is proportional to $Z_{CN}^2/A_{CN}$ where $Z_{CN}$ and $A_{CN}$ are the charge and mass of the compound system, respectively. For the two fissile systems studied here, the $^{16}\text{O} + ^{208}\text{Pb}$ has $x = 0.76$ and for $^{28}\text{Si} + ^{208}\text{Pb}$ $x = 0.82$. The compound systems formed in heavy-ion collisions are also rotating rapidly. Thus, the stability of compound system against fission also depends on its rotational energy [69]. The rotational energy is given by

$$E_{rot} = \frac{J(J + 1)h^2}{2J}, \quad (2.65)$$

where $J$ is the moment of inertia.

For deformations along some minimum energy path, there is a point on the potential energy surface that is a maximum. This is defined as the saddle or transition point. The angular momentum dependent fission barrier $B_f(J)$ is defined as the difference in the energies of the rotating system at its saddle point and at its equilibrium deformation. As the angular momentum of the system increases, the rotational energy of the equilibrium deformation increases more rapidly than that of the deformed saddle point, and eventually for large enough $J$, $B_f(J)$ goes to zero and the system is unstable against fission. These rotational effects were incorporated in the rotating liquid drop model (RLDM) of Ref. [69]. Many statistical model analyses [70] of experimental fission and ER cross-sections, for nuclei with $A_{CN} \approx 200$, used the values of the RLDM fission barriers [69] scaled by $B'_f(J) = k_f B_f(J)$, where $k_f$ is a $J$-independent fission barrier scaling factor. Values as low as $k_f = 0.5$ were required to fit the experimental data. A reasonable description of the reduction was obtained when the finite range of the nuclear force was taken into account [65]. This effect was incorporated into Sierk's [65] finite range rotating liquid drop model (FRRRLDM or more commonly RFRM).

In the statistical model, the competition between the fission and particle evaporation is determined by the level density of the fissioning system at the saddle
point and the level density of the nuclear states fed by the neutron decay. The ratio of the fission to neutron decay widths can be written as [71]

$$\Gamma_f/\Gamma_n \propto \exp[2\sqrt{a_f(E^* - k_fB_f)} - 2\sqrt{a_n(E^* - B_n)}],$$  \hspace{1cm} (2.66)

where $B_n$ is the particle (neutron) binding energy, $a_f$ is the level density parameter at the saddle point of the fissioning system and $a_n$ is the level density parameter of the residual nucleus, at its equilibrium deformation. The actual forms for the particle and fission decay width are more complicated than given by Eq. (2.66), but it is instructive to consider the factors that this ratio depends upon. If the ratio $(\Gamma_f/\Gamma_n)^{1st}$ represents the probability of first chance fission evaluated at $E^*$, then the probability for second chance fission at the excitation energy $E^* - \Delta E$ is given by $(\Gamma_f/\Gamma_n)^{2nd}$, where $\Delta E$ is the average energy loss from the system following neutron evaporation. Then, with the assumption that $E^* \gg E^* + \Delta E$, the ratio of the first chance to second chance fission can be written as [71]

$$\left(\frac{(\Gamma_f/\Gamma_n)^{1st}}{(\Gamma_f/\Gamma_n)^{2nd}}\right) \approx \exp \left[ \Delta E \left(\frac{a_n}{E^*}\right)^{1/2} \{(a_f/a_n)^{1/2} - 1\} \right],$$  \hspace{1cm} (2.67)

where it is assumed that the fission barrier is the same for the two fissioning nuclei. The ratio in Eq. (2.67) demonstrates that the pre-scission neutron multiplicity is independent of $k_f$, weakly dependent on $a_n$, but very sensitive to changes in $a_f/a_n$. Even when the fission barriers are assumed to vary with mass, the pre-scission neutron multiplicity is still insensitive to changes in $k_f$. This result is important to the statistical model fits of the ER cross-sections for the $^{16}$O + $^{208}$Pb reaction, since the saddle-point temperature depends critically on the value of $a_f$. This point is discussed further in Chapter 5.

In the statistical model, the level density $\rho(E^*)$ at the excitation energy $E^*$ is determined by counting the number of different ways the nucleons can be arranged in the various single-particle states. For temperatures $T$ small compared to the Fermi energy, the dominant term in the level density can be written as [72]

$$\rho(E^*) \propto \exp[2\sqrt{aE^*}],$$  \hspace{1cm} (2.68)

where $a$ is the level density parameter. The temperature of the compound nucleus is given by

$$E^* = aT^2.$$  \hspace{1cm} (2.69)

As a guide to the choice of the nuclear level density parameter, the parametriza-
tion of Töke and Swiatecki [73] was followed.

The decay of the compound systems is modelled using statistical model codes. There are two different approaches taken by the codes in common use today [66]. In first approach, a grid in $Z_{CN}$ and $A_{CN}$ is constructed, and for each nucleus the distribution of events is calculated using two parameters, the excitation energy and the angular momentum. The calculation continues down the decay chain for each new daughter nucleus until further decay is energetically forbidden. The code ALERT1 [74], which is based on the code MBII [75], is of this type. It is used in Chapter 4 to model the ER cross-sections for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction. In the second approach, the decay of the compound system is treated using Monte Carlo techniques. The precision of the results from these calculations depends on the number of initial events in the cascade. The codes PACE [76] and JOANNE [77,78] are of this type. The ability of PACE to determine ER angular distributions is exploited in the calculations for the $^{16,17}\text{O} + ^{144}\text{Sm}$ reactions (see Chapter 3). All of the above statistical model codes, except JOANNE, use the RLDM fission barriers.

The Monte Carlo code JOANNE is used in this work to calculate the distribution of events with a given $T$ and $J$, in order to evaluate the fission fragment angular distributions. The code JOANNE uses the RFRM fission barriers as discussed above. The optical model transmission coefficients in JOANNE are derived, using the method of PACE2, from the ‘universal’ optical models parameters of Refs. [79, 80]. The Q-value for this calculation was, unless otherwise stated, evaluated relative to the liquid-drop ground state using

$$Q_{LD}^{\text{LD}} = E_{1}^{\text{exp}} + E_{2}^{\text{exp}} - M_{CN}^{\text{LD}} - 3(1/A^{1/2}),$$

(2.70) where $E_{1}^{\text{exp}}$ and $E_{2}^{\text{exp}}$ are the experimental ground state masses for the projectile and target, and $M_{CN}^{\text{LD}}$ is the liquid-drop mass evaluated using the Lysekil parameters [68]. The last term in Eq. (2.70) is a pairing energy term [81]. The theoretical calculations of the fission fragment anisotropies are explained in more detail in Chapter 5.
Chapter 3

EXPERIMENTAL METHODS

To obtain a well-defined distribution of fusion barriers, the fusion excitation functions must be measured to high precision. The overall aim of the experimental methods, was to measure the fusion cross-sections over a wide range of energies to a precision of ±1% or better. The experiments were performed with oxygen and silicon beams provided by the 14UD Pelletron accelerator at the Australian National University (ANU). For precise measurements of $\sigma_{\text{ fus}}(E)$, an accurate knowledge of the beam energy was required. The magnetic field of the analysing magnet was measured with a nuclear magnetic resonance (NMR) probe. The energy $E$ of the analysed beam is given by the relativistic expression

$$E = C H^2 \left( \frac{Z_{\text{eff}}^2}{m} \right) \left( 1 + \frac{E}{2mc^2} \right)^{-1},$$

where $C$ is the magnet constant, $H$ is the magnetic field strength, $Z_{\text{eff}}$ is the effective charge of the particle and $m$ is its mass [82]. The 'constant' $C$ has been calibrated [82] more than 20 times over a period of 15 years. Two calibrations of the NMR probe were performed during the course of these measurements, and no change was required in the magnet constant [83]. The standard deviation of the series of calibrations was 0.04%, thus a beam energy of 100.0 MeV has an absolute uncertainty of ±40 keV. The relative uncertainty in defining the beam energy intervals is better than 5 keV. All beams were pulsed using the ANU beam pulsing system which consists of a room temperature pre-tandem buncher [84], which compresses the DC beam into pulses of 1 ns width with separation 106.3 ns, and a post-tandem beam chopper, which removes the background of the unbunched
Table 3.1: The range of beam energies for the projectile/target combinations used in these experiments and the areal densities of the targets. The areal densities of the PbS targets were measured using the offset in the excitation functions and the relative yields in the two targets for a reaction at the same beam energy and intensity.

<table>
<thead>
<tr>
<th>Projectile</th>
<th>Target</th>
<th>Areal density (μg cm⁻²)</th>
<th>Beam energies (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>¹⁶O, ¹⁷O</td>
<td>¹⁴⁴Sm</td>
<td>40</td>
<td>61.0 to 100.0</td>
</tr>
<tr>
<td>¹⁶O</td>
<td>²⁰⁸Pb</td>
<td>350, 23 (fission)</td>
<td>76.0 to 92.0</td>
</tr>
<tr>
<td></td>
<td></td>
<td>350 (ERs)</td>
<td>76.0 to 96.0</td>
</tr>
<tr>
<td>²⁸Si</td>
<td>²⁰⁸Pb</td>
<td>23</td>
<td>135.0 to 176.0</td>
</tr>
</tbody>
</table>

ions between pulses. A pre-tandem 'slow' chopper was used to remove pulses in order to increase their separation by a chosen value (in multiplies of 106.3 ns). The slow chopper is operated in phase with the buncher with variable pulse widths and pulse separations in the 0.1 μs to 500 ms range.

The energy range of the projectiles for each of the four reactions are listed in Table 3.1. The targets were prepared by evaporation of the target material onto carbon backing foils. The samarium targets, isotopically enriched to 96.5 % in ¹⁴⁴Sm, had areal densities of ≈40 μg cm⁻² deposited on a ¹²C backing of ≈10 μg cm⁻². For the lead reactions, targets of PbS, 99.0 % enriched in ²⁰⁸Pb were used, with areal densities of 350 μg cm⁻² and 23 μg cm⁻², deposited on the carbon backings. The PbS targets were orientated with the carbon backings facing downstream relative to the beam so as to minimise the projectile energy loss. The Sm targets were orientated with the carbon backings facing upstream so as to minimise the effects of multiple scattering of the forward going ERs.

Three different experimental setups were used in this work. A compact velocity filter and multi-wire proportional counter arrangement was used to detect the evaporation residues from the ¹⁶,¹⁷O + ¹⁴⁴Sm reactions. The evaporation residue excitation function for the ¹⁶O + ²⁰⁸Pb reaction was determined by detecting the α-activity from the decay of the ERs and their daughters using an annular silicon detector. The fission fragment angular distributions, and hence the fission cross-sections, were measured with the fission fragment spectrometer. In this Chapter, each of these setups is described and the method for obtaining the relevant cross-section is detailed.
3.1 Evaporation residue detection

Evaporation residues formed in heavy-ion fusion reactions can be detected either by direct or indirect techniques. With the indirect detection techniques, ERs can either be measured in-beam using γ-ray spectroscopy, or out-of-beam by the detection of the α-particles, γ-rays or X-rays from the decay of the evaporation residues and their daughters. The α-decay technique was used in this work to measure the ERs for the $^{16}$O + $^{208}$Pb reaction. For direct detection of ERs, the type of technique used depends on the scattering angle at which the residues are measured. At the larger scattering angles, evaporation residues can be identified directly using simple energy and time-of-flight techniques. At angles close to the beam axis, electrostatic and/or magnetic separators are required to separate the ERs from the intense elastically scattered beam. These two methods are described in Section 3.1.1.

3.1.1 Direct detection of evaporation residues

The ERs at scattering angles greater than 10° were detected in a Si surface-barrier (SSB) detector. At angles forward of 10°, a compact velocity filter [85] was used to transmit the ERs whilst suppressing the intense elastic scattering yield for angles close to the beam axis.

The principle of operation of the compact velocity filter is based on the Wien filter [86]. With the addition of a magnetic field, the large angular range of dispersion associated with electrostatic deflection alone can be significantly reduced. The ions are transmitted through a region of electric field and magnetic field, which have directions perpendicular to each other. An ion with a particular velocity moving through the filter will not be deflected if the magnetic and electric forces acting on the ion balance one another. Thus, by selecting a particular ratio of the electric to magnetic field strengths, only particles of a certain velocity will be transmitted through the filter. In practice, the magnetic field strength is fixed and the electric field is varied in order to minimise the deflection of the ERs. The elastically scattered particles suffer more deflection and their intensity can be significantly suppressed by preventing most of them entering the particle detector.

The velocity filter used in this work consists of two electrostatic plates and two permanent Co/Sm magnets, housed in a nickel plated soft iron box, 200 mm in length, which acts as a yoke for the magnet. The magnetic field strength in the gap between the permanent magnets was 1.4 kG and the electric field was adjusted
to obtain the desired particle deflection with a ±20 kV high voltage supply. The filter was mounted on a moveable robotic arm inside the 2 m scattering chamber, which was kept at a vacuum of ~10^-6 Torr. The position of the arm can be set remotely over an angular range of -15° ≤ θ ≤ +30° with respect to the beam axis. At the entrance to the velocity filter, a circular Ta collimator, 1.5 mm in diameter and 250 mm from the target, defined the solid angle of the ER detector. Particles were detected in a position sensitive multiwire proportional counter (MWPC), positioned behind the filter, as shown in Fig. 3.1.

![Diagram of the velocity filter and MWPC](image)

FIG. 3.1: A schematic view of the velocity filter and MWPC used to detect evaporation residues [85]. The whole apparatus is mounted on a moveable arm for measurement of ER angular distributions. The monitors are fixed at ±30°.

The evaporation residues were separated from the elastically scattered particles during their flight through the filter. Most of the elastically scattered particles were prevented from entering the MWPC by a Ta ‘finger’ which was brought to the required distance across the entrance aperture of the detector. The efficiency for transmission of the ERs through the velocity filter and subsequent detection in the MWPC is expected to be 98%. The missing 2% is due to the area of the wires which shadow the ΔE cathode in the MWPC. The efficiency was measured to be 103 ± 4% [85], a value close to the expected efficiency.

The large angle SSB detector was positioned at a nominal angle of 20° from the axis of the velocity filter. The large angle detector was mounted on the same rotating arm as the velocity filter so that the angle between them was fixed. The large angle SSB detector had a solid angle around ten times that defined by the entrance aperture of the velocity filter. The elastic scattering rate was
significantly less at these angles, allowing residues to be identified by their total energy and time-of-flight, without suppression of the scattered beam particles.

Two SSB detectors were fixed at \( \pm 30^\circ \) to the beam axis to monitor Rutherford scattering. At the end of each run, a fourth SSB detector was moved immediately behind the velocity filter’s entrance aperture. In this position, Rutherford scattering, measured by the calibration detector, was used to determine the solid angle of the velocity filter’s entrance aperture relative to the monitor detectors, and also, the true angle of the filter with respect to the beam axis. Henceforth, reference to the solid angle of the velocity filter means the solid angle as defined by the entrance aperture to the velocity filter.

**Principles of the operation of MWPCs**

The evaporation residues are detected in a multi-wire proportional counter and identified by their energy loss and their time-of-flight. The principles involved here are the same as those applied in the detection of fission fragments. The large-area MWPCs used in the fission experiments are described in Section 3.2.

The basic design of a MWPC [87] consists of an anode and a cathode plane separated by a distance of typically 3 mm. The region between the electrodes is filled with a gas. The electrodes are operated at a potential difference such that the number of ions collected is directly proportional to the energy loss in the gas. This is called the proportional region [88]. In this region, the potential difference is large enough to cause secondary ionisation from collisions between the accelerated electrons and the molecules in the gas. These additional electrons are accelerated and cause further ionisation producing a cascade effect. This effect is responsible for the multiplication or gain of the output signal. The choice of gas is governed by several factors. These include the operating voltage, the desired gain of the output signal of the detector, the ‘degree’ of proportionality and the ability to withstand high particle rates. Noble gases such as Ar are often used, however they are limited to moderate gains because of discharge triggered by photons from the decay of the excited Ar atoms. To prevent this a quenching gas such as methane, isobutane or propane is mixed with the Ar to absorb the radiated photons and thus increase the gain possible. For heavily ionising particles, 100% pure isobutane has very good quenching properties and, for a given pressure, has a higher gain and stopping power than the other gases [89]. For these reasons, isobutane was chosen for use in these ER experiments. The isobutane was flowed through the MWPC to prevent contamination by electronegative gas molecules, outgassed from the detector body, which suppress dramatically the efficiency of
the charge collection.

The cathode is usually made from a thin foil coated with a thin layer of metal such as Au or Al. Particles which pass through the gas lose energy according to the Bethe-Bloch equation for stopping powers [88]. The positive signal induced in the cathode can be amplified and used as an energy loss signal, $\Delta E$. As well as providing a $\Delta E$ signal, the cathode has excellent timing properties with a signal rise-time of a few ns.

The MWPC can be made position sensitive by replacing the anode plane by a grid of thin metal wires. Now the signal from the passage of an ion through the ionisation gas will be collected primarily near one of the wires. The spacing between the wires is typically 1 mm and they are connected to each other via a series of delay chips to form a delay line. The wire from which the signal originated can be determined by measuring the difference in the time for the propagation of the signal to opposite ends of the delay line. A signal is also generated on neighbouring wires, however this is generally smaller in amplitude. Thus, the position of the particle can be determined to a resolution equal to the separation of the wires in the anode.

**ER identification with the MWPC**

The MWPC used in this work consisted of two proportional cells with an ionisation region in between. The first cell, as shown in Fig. 3.2, consisted of a vertical plane of Au coated W wires, 20 $\mu$m thick, and a cathode made from polypropylene coated with a thin layer of Au. The wires in the anode were connected via delay chips (with a nominal delay of 5 ns per wire), which were used to determine the position signal of the particles as described in the Section above. Situated a distance of 3.2 mm behind the wire anode, the cathode provided the TOF signal and the first of the two energy loss signals, $\Delta E_1$. The energy resolution of the $\Delta E_1$ signal is poor because of the small effective ionisation path ($\approx 0.3$ mm) of the first cell. The cathode was common to the second cell of the MWPC, which consisted of a mesh Frisch grid connected to ground and a second Au coated anode. Electrons drift from the common cathode towards the second anode. The region between the common cathode and the Frisch grid acts as a large ionisation region. Here the potential difference is such that the electron-ion recombination forces are overcome and the charge collected is independent of the applied voltage (the ionisation region). Electrons then pass through the transparent grid and into the second proportional region. The purpose of the Frisch grid is to screen the second anode from the ionisation region between the grid and the common
FIG. 3.2: A cross-sectional view of the MWPC counter. When a particle enters the counter, it ionises both regions on either side of the common cathode, and then stops in the metal plate anode. Electrons from this ionisation drift towards the wire anode (position signal), and the metal plate anode ($\Delta E_2$ signal).

cathode. The second anode energy signal, $\Delta E_2$, has a better energy resolution than the $\Delta E_1$ signal because now the effective ionisation path is much larger, $\approx 12$ mm. The typical operating voltages applied to each electrode are given in Table 3.2.

The evaporation residues transmitted through the velocity filter were identified in the MWPC which generated four signals: two signals from the $\Delta E$ of the two separate electrodes, the horizontal position of the particles and their TOF relative to the pulsed beam. The signal generation is discussed below.

Electronics for ER detection in the MWPC

Each of the four signals from the MWPC was amplified by charge sensitive preamplifiers, which were located outside the 2 m scattering chamber. The electronics consisted of standard modular NIM components, where the 'fast' units processed
Table 3.2: The operating voltages applied to the electrodes of the MWPC:

<table>
<thead>
<tr>
<th>Electrode</th>
<th>Applied voltage</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wire anode</td>
<td>+250 V</td>
</tr>
<tr>
<td>Common cathode</td>
<td>-280 V</td>
</tr>
<tr>
<td>Second anode</td>
<td>+390 V</td>
</tr>
<tr>
<td>with 3 Torr of isobutane</td>
<td></td>
</tr>
</tbody>
</table>

the position and TOF signals, and 'slow' units handled the $\Delta E$ and logic signals. The position signal from the wire anode of the MWPC was generated using a Time to Amplitude Converter (TAC) which was started and stopped by the signals from either end of the delay line. The time-of-flight (TOF) of the particle was measured using another TAC. The fast signal from the cathode was used to start the TOF TAC and the RF signal of the pulsed beam provided the stop pulse. A plot of $\Delta E$ versus TOF can be used to distinguish ERs (and fission fragments) from target recoils and slit scattered particles that accompany the reaction. Full details of the electronics layout are given in Ref. [90]. The two $\Delta E$ signals, the position and TOF of the evaporation residues, the elastic events in the monitor detectors and the pulser signal, were all recorded event by event via Analogue to Digital Converters using the ANU data acquisition system.

In Fig. 3.3, the energy loss from the second anode versus time-of-flight is plotted for (a) the $^{16}\text{O} + ^{144}\text{Sm}$ reaction at an energy of $E_{\text{beam}} = 68.0$ MeV. The residues are well separated in both energy and time from the slit scattered beam particles. Also shown in the plot is the pulser signal which was used to determine the computer and electronic deadtime during each energy measurement. The pulser signal was generated from a Pulse Generator triggered by a scaled signal from a monitor detector. The pulser signal was then pre-scaled to give a rate similar to the ER collection rate. This ensured that the deadtime pertaining to all beam related events was taken into account.

In Fig. 3.3(b), $\Delta E_2$ versus TOF is plotted for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction. The identification of the ERs in this plot is not as clear as for the samarium reactions, for a variety of reasons. Firstly, since the residue cross-section is only a small fraction of the total fusion cross-section (the fission cross-section dominates as the energy increases), the yield is smaller than for the samarium reactions for similar beam currents and target thicknesses. Secondly, the possible contamination of the $\Delta E_2$-TOF spectrum from C or O impurities in the target made the identification
FIG. 3.3: Plot of the energy loss $\Delta E_2$ versus the time-of-flight TOF for (a) the $^{16}\text{O} + ^{144}\text{Sm}$ reaction at $E_{\text{beam}} = 68.0$ MeV. The TOF runs from right to left. The scales on the axes are in channels. The ERs are tightly grouped in energy and time, clearly separated from the elastic tail. (b) $\Delta E_2$ versus TOF for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction at $E_{\text{beam}} = 82.0$ MeV. The ERs are less clearly identified in this case because of the contamination from light ion impurities in the target, as shown by the small boxed group located just under the ERs. The $\Delta E_2$ gain was larger for this measurement compared to the measurement in (a). Although it appears otherwise, the pulser signal was precisely defined, with 99% of the pulser events within a couple of pixels of each other.

of ERs more difficult. As shown in Fig. 3.3(b), the tightly grouped particles at a $\Delta E_2$-TOF just under the ERs, may be associated with $^{16}\text{O}$ reactions on light nuclei present in the PbS target. The final problem was due to difficulty in measuring the full angular distributions for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction. The fission competition present in the $^{16}\text{O} + ^{208}\text{Pb}$ reaction caused problems in theoretical calculations of the differential to total cross-section ratio. For these reasons, the ERs for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction were not determined using the above technique. This is discussed in more detail in Section 3.1.2.

Obtaining the differential cross-sections

Before detailing the specific equations used to obtain the ER cross-sections, the general principle involved in measuring relative cross-sections is described. The number of particles produced in a nuclear reaction per unit time is given by

$$Y(\theta, E) = [I][N] \frac{d\sigma(\theta, E)}{d\Omega} d\Omega, \quad (3.1)$$
where $I$ is the number of beam particles per unit time, $N$ is the number of target nuclei (in units of cm$^{-2}$), $d\sigma/d\Omega$ is the differential cross-section (mb sr$^{-1}$) and $d\Omega$ is the solid angle in steradians (sr). If the yield for a particular reaction of interest is measured relative to the yield of Rutherford scattering in a monitor detector, then the factors in the brackets in Eq. (3.1) cancel when the ratio of the two yields is taken. The differential cross-section is then given by

$$\frac{d\sigma(\theta, E)}{d\Omega} = \frac{Y(\theta, E)}{Y_M} \frac{d\sigma_R}{d\Omega},$$

(3.2)

where $d\sigma_R/d\Omega$ is the Rutherford differential cross-section, as measured in the monitor detector, and $d\Omega_M$ is its solid angle. If two monitor detectors are used, located either side of the beam axis, any changes in the beam spot position during the course of the cross-section measurements can be accounted for. Even though the yield in both monitor detectors is affected, for small angle shifts corresponding to small movements in the beam spot, the sum of the yields remains essentially unaffected. Equation (3.2) describes the basic form for determining cross-sections measured relative to Rutherford scattering and is used throughout this work in converting both ER and fission yields.

The evaporation residue cross-sections for the $^{16,17}$O + $^{144}$Sm reactions were determined at each beam energy $E$ by measuring the yield of ERs relative to the Rutherford scattering yields in the two monitor detectors at an angle of $\theta_M = \pm 30^\circ$ to the beam axis. The differential cross-section for ERs in the laboratory reference frame, is given by

$$\frac{d\sigma(S, E)}{d\Omega} = \frac{T(E)}{Y_{ML} + Y_{MR}},$$

(3.3)

where $Y_{VP}$ is the yield of ERs as measured by the velocity filter, $T(E)$ is the normalisation factor, and $Y_{ML}$ and $Y_{MR}$ are the Rutherford scattering yields in the left and right monitors, respectively. The ER yield was obtained by integrating the number of counts in the $\Delta E_2$-TOF spectrum, and the yield in the monitor detectors was easily identified since the elastic events were well resolved from the reaction products from light-ion contaminants in the target. Both the monitor and ER yields were corrected for any deadtime. The energy dependent normalisation factor $T(E)$ is given by

$$T(E) = \frac{T \times (E_{cal})^2}{E^2},$$

(3.4)
where \( E_{\text{cal}} \) is the energy at which the calibration reaction was performed. The normalisation constant \( T \) in Eq. (3.4) is defined as

\[
T := \frac{1}{d\Omega_{\text{VF}}} \left( \frac{d\sigma_R(\theta_{M_L}, E_{\text{cal}})}{d\Omega} d\Omega_{M_L} + \frac{d\sigma_R(\theta_{M_R}, E_{\text{cal}})}{d\Omega} d\Omega_{M_R} \right),
\]

where \( d\Omega_{\text{VF}} \) and \( d\Omega_M \) are the solid angles of the velocity filter and monitors, respectively, and the other terms in Eq. (3.5) are the Rutherford scattering cross-sections in the monitor detectors.

The normalisation factor \( T \) was determined in a calibration measurement with the calibration SSB detector in position behind the velocity filter's entrance aperture. The yield \( Y_{\theta_{\text{cal}}} \) of Rutherford scattering in the calibration detector at angle \( \theta_{\text{cal}} \), was measured relative to the yields in the monitor detectors at \( \theta_M = \pm 30^\circ \). The yield for the normalisation reaction is then given by

\[
\frac{d\sigma_R(\theta_{\text{cal}}, E_{\text{cal}})}{d\Omega} = T \cdot \frac{Y_{\theta_{\text{cal}}}}{Y_{M_L} + Y_{M_R}}.
\]

The factor \( T \) can then be calculated from the ratio of the measured yields, provided the angle \( \theta_{\text{cal}} \) is well known. The actual angle of the velocity filter was determined by measuring the ratio of the Rutherford cross-sections, relative to the monitors, at two angles with a known angular separation. The calibrations were performed at the nominal angles \( \pm 8^\circ, \pm 10^\circ \) and \( \pm 12^\circ \). Calibration at several angles ensured consistency between these measurements, and also meant that a clean separation between Rutherford scattering from the target and the scattering from the carbon backing was achieved. The angle of the velocity filter was determined to an accuracy of \( < \pm 0.05^\circ \) [90].

A similar procedure was carried out to determine the calibration factor for the large angle SSB ER detector and the actual value of its angular separation from the velocity filter, nominally 20°. Again, by using the ratio of Rutherford scattering in this detector to that measured in the monitor detectors.

**Differential cross-sections and ER angular distributions**

The differential cross-sections, as determined by Eq. (3.3), were measured for the \(^{16,17}\text{O} + ^{144}\text{Sm} \) reactions at nominal angles of \( \theta = \pm 2^\circ \) to the beam axis for the range of beam energies given in Table 3.1 at intervals of 0.5 MeV. For beam energies above 80.0 MeV, the energy intervals for the differential cross-sections were 5.0 MeV. Full ER angular distributions were measured at intervals of typically 5.0 MeV. This involved detecting ERs by moving the velocity filter.
to angles in the range \(-5^\circ \leq \theta \leq +10^\circ\). The angular distributions for the \(^{16}\text{O} + ^{144}\text{Sm}\) reaction at the energies 68.0, 70.0, 75.0, 80.0, 85.0 and 100.0 MeV are shown in Fig. 3.4. The triangles in Fig. 3.4 represent the differential cross-sections from the large angle SSB detector.

Qualitatively, the shape of the angular distribution can be understood by consideration of the recoil angle of the residues, after some combination of neutrons, protons and \(\alpha\)-particles have been evaporated. For angles close to the beam axis, the main contribution to the differential cross-section is from ERs recoiling after neutron or proton emission. At the larger angles, it is the \(\alpha\)-emission that is responsible for the shape of the angular distributions [91,92] because of the larger momentum transfer associated with the \(\alpha\)-particles. This contribution increases with energy, in agreement with the expected probability for \(\alpha\)-emission. Each ER angular distribution was fitted using the sum of two Gaussian functions, as shown by the solid lines in Fig. 3.4. An example of the component Gaussians is shown by the dotted lines in Fig. 3.4. The total cross-section for each full angular distribution, \(\sigma_{\text{FAD}}(E)\), was taken to be the area under the Gaussian curves multiplied by \(2\pi \sin \theta\). The uncertainty in this method was determined by fitting two Gaussian functions to theoretical angular distributions with a known cross-section. The theoretical distributions were produced using calculations from the statistical model code PACE2 [76]. The fits to the theoretical distribution reproduced the known cross-sections to within 1% [90], with a scatter substantially less than 1%.

**Interpolation of the differential cross-sections**

To convert the differential cross-sections \(d\sigma(\theta, E)/d\Omega\), measured at \(\pm 2^\circ\) to the beam axis, into total cross-sections, the ratio

\[
R(E) = \frac{d\sigma(\theta, E)/d\Omega}{\sigma_{\text{FAD}}(E)}
\]

was plotted at each energy where the full angular distribution was determined. Although the total fusion cross-section in Eq. (3.7) can be determined by any technique, the differential cross-sections depend on the target used and its orientation with respect to the beam axis. This is because the forward going ERs can be multiply-scattered in the target material and this alters their angular distribution. These effects were minimised by the use of thin targets. Over the course of these measurements, target breakage required different targets (from the same batch) to be used. Because of the dependence of the differential cross-sections
FIG. 3.4: The evaporation residue angular distributions for the $^{16}$O + $^{144}$Sm reaction at the energies indicated. The circles are the velocity filter measurements for positive angles (solid) and negative angles (open) and the triangles are the results from the large angle SSB detector. The solid lines are the two-Gaussian fits to the angular distribution. For the 68.0 MeV angular distribution, each component Gaussian is shown by the dotted lines. The total cross-section is the area under these two Gaussian curves multiplied by $2\pi \sin \theta$. 
Table 3.3: The beam energies at which the full angular distributions were performed during each of the two separate experimental runs.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>run one</th>
<th>run two</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{O} + ^{144}\text{Sm}$</td>
<td>70.0, 80.0 MeV</td>
<td>68.0, 75.0, 85.0, 100.0 MeV</td>
</tr>
<tr>
<td>$^{17}\text{O} + ^{144}\text{Sm}$</td>
<td>-</td>
<td>70.0, 80.0, 90.0, 100.0 MeV</td>
</tr>
</tbody>
</table>

on the target used, care was taken to ensure the ratio in Eq. (3.7) was evaluated without mixing values for the differential cross-sections measured from different targets. The measurements for the $^{16,17}\text{O} + ^{144}\text{Sm}$ reactions were each performed with two separate experimental 'runs'. The energies at which the full angular distributions were measured are given in Table 3.3.

An example of $R(E)$ is shown in Fig. 3.5 for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction from the run one experiment (solid circles) and the run two experiment (open circles). As a guide to how this ratio should fall as a function of energy, statistical model calculations were performed, again with the Monte Carlo code PACE2. For the $^{16,17}\text{O} + ^{144}\text{Sm}$ reactions, the PACE2 calculation showed an approximately linear energy dependence. The least-squares fit to the PACE2 calculation is shown by the dotted line in Fig. 3.5. This smooth variation in $R(E)$ was also seen in the $^{16}\text{O} + ^{154}\text{Sm}$ reaction [90]. The behaviour of the PACE2 calculation and the data are similar, although there is some scatter in the run two data. Also, the PACE2 calculation underestimates the magnitude of $R(E)$. This is most likely due to the underestimation of the $\alpha$-emission component in the statistical model calculation. A change in the magnitude of $R(E)$ will shift the cross-sections in a uniform way. However, this systematic shift in the cross-sections will only scale the distribution of barriers, with no effect on their overall shape. If the data define the magnitude of $R(E)$, and the experimental slope is reasonably well reproduced by the statistical model calculation, then the interpolation procedure will only cause a small systematic uncertainty in the extracted cross-sections. A linear least-squares fit to $R(E)$ is given by the solid line in Fig. 3.5. By interpolation of this fit, the total cross-section was determined at the energies where only the (average) differential cross-sections were measured.

The interpolation procedure is not likely to introduce any significant random uncertainties, at least no larger than the ±1 % values from the counting statistics. The possible systematic uncertainties are from the conversion of the differential cross-section to the total cross-section and from the efficiency of the detection.
3.1.2 Evaporation residue detection via their $\alpha$-decays

An alternative method for the detection of evaporation residues, for favourable cases, is through measurement of their $\alpha$-decay. There were two reasons why a method alternative to the velocity filter technique was used for the $^{16}$O + $^{208}$Pb reaction. Firstly, there was a problem in identifying the ERs in the energy loss versus time-of-flight spectrum. (See the earlier discussion in Section 3.1.1 and Fig. 3.3). Secondly, the competition between the evaporation decay mode and the dominant fission decay mode conspired to produce structure in the ER excitation function (see Fig. 4.10 in Chapter 4). This meant that it was difficult to accurately determine the behaviour of $R(E)$ [Eq. (3.7)]. In the lighter $^{16,17}$O + $^{144}$Sm systems, statistical model calculations of $R(E)$ were useful because the fission competition is negligible. Since the velocity filter measurements rely on knowledge of the behaviour of this function, the interpolation of the differential cross-sections to total cross-sections for the $^{16}$O + $^{208}$Pb reaction would have been unreliable. One way around this problem would involve measuring full angular distributions at considerably more energies. However, the would have been a very
time consuming task.

Hence the evaporation residue excitation function for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction was determined by detecting the $\alpha$-decay of the ERs and their daughters. Each evaporation channel has a characteristic decay-chain, thus enabling the cross-section per evaporation channel to be determined. In the Subsections that follow, the $\alpha$-decay setup is described, and the method for obtaining the ER cross-section is detailed.

The experimental setup

The recoiling evaporation residues were stopped in an Al catcher foil of $\approx 800 \, \mu g \, cm^{-2}$, positioned immediately behind the $350 \, \mu g \, cm^{-2}$ PbS target. The range $R$ of recoiling $^{232}\text{Th}$ ions (which are heavier than the system of interest, $^{224}\text{Th}$) in Al is $420 \, \mu g \, cm^{-2} \lesssim R \lesssim 520 \, \mu g \, cm^{-2}$ for the minimum and maximum energies of this experiment [93]. The longitudinal straggling (the standard deviation of the final-depth distribution), which is a measure of the distribution about the average range, is $\approx 152 \, \mu g \, cm^{-2}$ for $50 \, MeV \, ^{238}\text{U}$ ions in Al [94]. It is expected to be substantially less for the $\sim 10 \, MeV \, ^{224}\text{Th}$ ions in this experiment. These ranges are such that all the ERs will be stopped in the Al catcher foil. Their subsequent $\alpha$-decays were detected using an annular Si surface barrier detector which viewed the target and catcher foil directly. A schematic view of this detector system is shown in Fig. 3.6. A Ta shield, with an aperture 3 mm in diameter, was mounted upstream of the target. A Pb absorber directly behind the Ta aperture shielded the angular SSB detector from X-rays produced upstream. A small permanent magnet was placed near the Al catcher foil in an attempt to reduce some of the large electron background.

The detector was placed at a mean angle of $\theta_D = 164^\circ$ to the beam direction in the laboratory frame. In this position, the solid angle of the annular counter was 2.7% of $4\pi$ sr. Two monitor detectors were positioned in an ‘up-down’ configuration at an angle of 22° to the beam axis. The beam was pulsed with a 3.12 $\mu s$ beam on period and 21.3 $\mu s$ beam off, giving a total cycle time of 24.4 $\mu s$. To enable the annular counter to recover from the irradiation period, the data collection began after a delay of 1.22 $\mu s$. Then, the energy and arrival time of the $\alpha$-decays were collected for 19.5 $\mu s$ during the beam off period (see Fig. 3.8). The large background flux of electrons and scattered particles during the beam on period, and the long fall time of the preamplifier ($\approx 200 \, \mu s$), caused a shift in the baseline of the main amplifier. The baseline shift became worse with larger beam currents. Consequently, beam currents for these measurements were kept
FIG. 3.6: The cross-sectional view of the experimental setup for α-decay measurement.
to a tolerable minimum. Under these circumstances, the $\alpha$-energy resolution was $\approx 180$ keV for the annular counter.

**ER cross-sections from the $\alpha$-decay technique**

The differential cross-section for ERs measured by the annular counter at an energy $E$ is given by

$$\frac{d\sigma}{d\Omega}(\theta_D, E) = S(E) \frac{Y_\alpha}{(Y_{M_+} + Y_{M_-})} T, \quad (3.8)$$

where $S(E)$ is the normalisation factor, which was determined in the calibration reaction as described below. In Eq. (3.8), $Y_\alpha$ is the yield of $\alpha$-decays in the annular counter, and $Y_{M_+}$ and $Y_{M_-}$ are the Rutherford yields in the monitors. The factor $T$ is the ratio of the cycle time to the data collection time; for these measurements $T = 1.25$.

The total ER cross-section for isotropic emission of the $\alpha$-particles from the catcher foil is,

$$\sigma_{ER}(E) = \left( \frac{4\pi}{M_\alpha} \right) \frac{d\sigma}{d\Omega}(\theta_D, E),$$

where $M_\alpha$ is the $\alpha$-multiplicity for each $\alpha$-decay chain. Note that $M_\alpha$ is not the number of $\alpha$-particles evaporated from the compound nucleus. Substitution of Eq. (3.8) into the above expression gives,

$$\sigma_{ER}(E) = \left( \frac{4\pi}{M_\alpha} \right) S(E) \frac{Y_\alpha}{(Y_{M_+} + Y_{M_-})} T \text{ mb.} \quad (3.9)$$

So by measuring the ratio of the yield of $\alpha$-decays corresponding to a given evaporation channel to the sum of the elastic yields in the monitor detectors, the partial evaporation residue cross-section was determined. The signature for a certain evaporation channel was via its characteristic $\alpha$-decay chain.

**Calibration of the annular SSB detector**

The monitor detectors were used to determine in situ the relative solid angle of the annular counter using Rutherford scattering for the reaction $^{16}$O + $^{208}$Pb at a calibration energy $E_{\text{cal}} = 60.0$ MeV, an energy well below the average fusion barrier. The energy dependent normalisation factor can be written as

$$S(E) = S \left( \frac{E_{\text{cal}}}{E} \right)^2, \quad (3.10)$$

58
where $S$ is defined as

$$S := \left[ \frac{d\sigma_R(\theta, E)}{d\Omega} \right] \frac{d\Omega_M}{d\Omega_D}, \quad (3.11)$$

where $d\Omega_M$ and $d\Omega_D$ are the solid angles of the monitors and annular SSB detector, respectively. The term in brackets in Eq. (3.11) is the Rutherford cross-section at the angle of the monitors, $\theta_M$. The normalisation factor $S$ in Eq. (3.11) was determined experimentally by measuring the yield of Rutherford scattering in the annular counter relative to the yields in the monitor detectors. That is, $S$ was determined from the equation

$$\frac{d\sigma_R(\theta_D, E_{\text{cal}})}{d\Omega} = S \frac{Y_{\text{cal}}}{(Y_{M_+} + Y_{M_-})}, \quad (3.12)$$

where $Y_{\text{cal}}$ is the Rutherford yield in the annular counter. The LHS of Eq. (3.12) is the Rutherford cross-section in the annular counter, where $\theta_D$ was determined geometrically. The precision to which this angle was measured is not too critical since the Rutherford cross-section at $E_{\text{cal}}$ varies by only 1% from $163^\circ$ to $165^\circ$.

**Identification of the $\alpha$-decay chains**

The $\alpha$-decay chains were identified by their energies $E_\alpha$ and their half-lives $t_{1/2}$. The $\alpha$-transitions in this region of the thorium and actinium decay series have been determined previously in a series of measurements [95–97]. A summary of these activities is given in Table 3.4. As can be seen in Table 3.4, the $t_{1/2}$ of the first member in the decay chain in all channels is 1.05 $\mu$s or longer. This allows detection of all subsequent daughter decays, since the data collection commenced only a period of 1.22 $\mu$s after irradiation ceased.

Since the $\alpha$-decay energies are well known, it was possible to define sets of $\alpha$-lines with fixed relative intensities corresponding to given evaporation channels. The beam-energy independent relative intensities, were calculated from the known branching ratios, the half-lives and the experimental counting period. The decay multiplicity $M_\alpha$ in Table 3.4 is the number of $\alpha$-particles per ER in the energy interval $7.8 \leq E_\alpha \leq 10$ MeV. There is an upper energy threshold because $\alpha$-decays that pile-up in the annular counter were excluded from the analysis. The probability for pile-up for a given decay chain was calculated by consideration of the geometry of the annular counter. This value is given in the last column of Table 3.4. The correction for pile-up was small because of the small solid angle of the annular counter. Also included in the ‘deadtime’ of the detector was the loss in yield due to the difference in the half-lives between ‘short’ and ‘long’ decays.
This is explained below.

At each bombarding energy, the contributions of the possible evaporation channels to the α-spectrum, were obtained by fitting the α-yields using the peak fitting routine. For this fitting procedure, the energies of the lines and their relative intensities were given as initial parameters. The α-spectrum was fitted only for energies above 7.8 MeV because of the presence of an unidentified long-lived contaminant activity ($t_{1/2} > 0.5$ s) at $E_\alpha \approx 7.4$ MeV. An example of the α-spectrum observed at a beam energy of 84.0 MeV is given in Fig. 3.7. At this beam energy, the major contribution to the total α-spectrum fit (solid line) is due to the 3n-channel (broken line in Fig. 3.7). This analysis determined the cross-section corresponding to each evaporation channel.

![Graph](image_url)

**FIG. 3.7:** The measured α-spectrum obtained at an energy $E_{\text{lab}} = 84.0$ MeV. The solid line is the peak fit to the data, fitted for α-energies above the threshold energy of 7.8 MeV. The lower panel shows the breakdown of this fit into its constituent evaporation channels. The 3n-channel dominates at this energy. The energy resolution of the annular counter was $\approx 180$ keV because of the large flux of electrons and background present during the irradiation period.
Table 3.4: The α-decay chains for the xn-, axn- and pxn-evaporation channels. The α-spectrum was fitted for energies $7.8 \leq E_\alpha \leq 10$ MeV. Only decays with half-lives longer than $0.1 \mu$s were observed. The last column includes both the corrections for electronic pile-up and the loss in yield due to the different $t_{1/2}$ of each decay chain.

<table>
<thead>
<tr>
<th>Nuclide (channel)</th>
<th>$E_\alpha$ (MeV)</th>
<th>$t_{1/2}$</th>
<th>Branch. (%)</th>
<th>$M_\alpha$</th>
<th>D.T.</th>
</tr>
</thead>
<tbody>
<tr>
<td>222Th family (2n)</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>222Th</td>
<td>7.982*</td>
<td>2.8 ms</td>
<td>100</td>
<td></td>
<td></td>
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<tr>
<td>218Ra</td>
<td>8.390*</td>
<td>14 µs</td>
<td>100</td>
<td></td>
<td></td>
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<tr>
<td>214Rn</td>
<td>9.037*</td>
<td>270 ns</td>
<td>100</td>
<td></td>
<td></td>
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<td>221Th family (3n)</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>221Th</td>
<td>8.146*</td>
<td>1.68 ms</td>
<td>56</td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>8.472*</td>
<td></td>
<td>39</td>
<td></td>
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<td>7.733</td>
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<td>6</td>
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<td></td>
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<tr>
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<td>100</td>
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<td></td>
</tr>
<tr>
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<td>8.087*</td>
<td>25 ms</td>
<td>99</td>
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<tr>
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<td>7.552</td>
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<td></td>
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<td>9.6 µs</td>
<td>100</td>
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<td>182 ns</td>
<td>100</td>
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<td>212Rn</td>
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<td>24 min</td>
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<td>1.05 µs</td>
<td>100</td>
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<tr>
<td></td>
<td>8.700*</td>
<td>1.6 ms</td>
<td>96</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8.171*</td>
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<td>1.4</td>
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<tr>
<td></td>
<td>7.883*</td>
<td></td>
<td>2.8</td>
<td></td>
<td></td>
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<tr>
<td>219Ra family (αn)</td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>219Ra</td>
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<td>10 ms</td>
<td>35</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7.680</td>
<td></td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>215Rn</td>
<td>8.674*</td>
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<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>218Ra family (α2n)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>217Ra family (α3n)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>221Ac family (p2n)</td>
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<tr>
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<td>100</td>
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<td></td>
</tr>
<tr>
<td>217Fr</td>
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<td>22 µs</td>
<td>100</td>
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</tr>
<tr>
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<td>9.08*</td>
<td>110 ns</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>220Ac family (p3n)</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>220Ac</td>
<td>7.70*</td>
<td>26.1 ms</td>
<td>100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>216Fr</td>
<td>9.005*</td>
<td>700 ns</td>
<td>100</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* α-lines included in the fitting procedure.
† average.
‡ part of the decay chain listed above.
Correction of the yield for different activities

Because of the 1.22 \mu s delay before data acquisition began, corrections were made to the yields for decay chains whose half-lives of the first daughter in the chain were shorter than \( \sim 1 \) ms. This situation is shown graphically in Fig. 3.8, where curve (1) is the equilibrium yield after \( n \) discontinuous cycles of irradiation time \( t_{on} \) and decay time \( t_{off}. \) If, after \( n \) cycles, \( N_0 \) is the number of radioactive nuclei at the end of the beam off period, then curve (2) in Fig. 3.8 can be written as

\[
N_0 = N(t_{on}) \exp(-\lambda t_{off}),
\]

where \( N(t_{on}) \) is the number of radioactive nuclei at the end of the irradiation time \( t_{on}, \) and \( \lambda = \ln 2/t_{1/2} \) is the decay constant. The subsequent accumulation curve for the next cycle, curve (3), is

\[
N(t_{on}) = \frac{F}{\lambda} [1 - \exp(-\lambda t_{on})] + N_0 \exp(-\lambda t_{on}),
\]

which is the sum of terms involving the production of the radioactive nuclei and the decay of the residual nuclei during the irradiation time. In Eq. (3.14), \( F \) is the rate of production of the activity per unit time and depends upon the particular
Table 3.5: The calculated yields for various evaporation channels. The calculations were performed with the half-lives in units of μs.

<table>
<thead>
<tr>
<th>Channel</th>
<th>$t_{1/2}$ (μs)</th>
<th>$N_0$</th>
<th>$N(t_{on})$</th>
<th>$\Delta N$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3n</td>
<td>1680</td>
<td>308.3$F$</td>
<td>311.0$F$</td>
<td>2.487$F$</td>
</tr>
<tr>
<td>4n</td>
<td>9.6</td>
<td>0.7235$F$</td>
<td>3.374$F$</td>
<td>2.331$F$</td>
</tr>
<tr>
<td>5n</td>
<td>1.05</td>
<td>1.03 × 10^{-6}$F$</td>
<td>1.322$F$</td>
<td>0.5097$F$</td>
</tr>
<tr>
<td>$\alpha$3n</td>
<td>1.60</td>
<td>1.68 × 10^{-4}$F$</td>
<td>1.7109$F$</td>
<td>1.0083$F$</td>
</tr>
</tbody>
</table>

irradiation conditions, such as the beam flux and target thickness. Here, $F$ is the same for both the short and long activities.

When the time for a number of cycles is long compared to the half-life of the first $\alpha$-decay in the chain, which is the case for all decays considered here, the rate of production of new activity must be equal to the rate of decay, and the equilibrium value for the number of radioactive nuclei remaining can be calculated by solving Eqs. (3.13) and (3.14) simultaneously for $N_0$, giving

$$N_0 = \frac{F[1 - \exp(-\lambda t_{on})]\exp(-\lambda t_{off})}{\lambda[1 - \exp(-\lambda(t_{on} + t_{off}))]}.$$  \hspace{1cm} (3.15)

The correction required for the yield from different $\alpha$-activities depends upon the counting period over which the $\alpha$-decays were collected. The counting period for this experiment is shown graphically in the inset of Fig. 3.8. Here, $t_1$ and $t_2$ are defined as the time that data collection began and finished relative to the time the irradiation ceased, respectively. To compare the yields of the $\alpha$-decays from the differing activities, the ‘difference’ between the yields during $t_1$ and $t_2$ is given by

$$\Delta N = N(t_{on})[\exp(-\lambda t_1) - \exp(-\lambda t_2)],$$

where $t_1 = 1.22$ μs and $t_2 = 20.7$ μs. The difference in yields for the 3n-, 4n-, 5n- and $\alpha$3n-channels are summarised in Table 3.5. As an example, the ratio of $\Delta N$s in the last column of Table 3.5, require that the yield for the 4n-channel be scaled by 1.067 relative to the yield for the 3n-channel. For decays with $t_{1/2} > 1$ ms, the correction to the yield was negligible.
ER cross-sections from the average decay multiplicities

The complex nature of the $\alpha$-spectrum, and the poor energy resolution, made it difficult to precisely identify the contribution of the weak evaporation channels. The fitting procedure generated unrealistic fluctuations in the cross-sections for the $\alpha xn$- and $pxn$-channels. See Fig. 4.7 in Chapter 4. In order to reduce the dependence of the ER cross-sections on the fitting procedure, the mean decay chain $\alpha$-multiplicities $M_\alpha(E)$, were evaluated at each energy using

$$M_\alpha(E) = \sum_i f_i(E) M^i_\alpha,$$

where $f_i(E)$ is the fraction of the ER cross-section in channel $i$ and $M^i_\alpha$ are the decay $\alpha$- multiplicities in channel $i$. The extracted mean decay $\alpha$- multiplicities are shown in Fig. 3.9. This function cannot change rapidly with energy since the excitation functions of each channel vary slowly and smoothly over an energy range of $\approx 10$ MeV (see Fig. 4.7 in Chapter 4). As shown in Fig. 3.9, $M_\alpha(E)$ falls by only 30% over an energy range of 15 MeV. Smoothing of $M_\alpha(E)$ in order to minimise scatter in this function is then reasonable. A second ER excitation function was then generated using the total yield between energies in the range of 7.8 to 10 MeV of the $\alpha$-spectrum divided by the smoothed mean decay $\alpha$- multiplicities (the solid line in Fig. 3.9). The analysis technique is now independent of fluctuations resulting from the fitting procedure at particular energies. The ER cross-sections obtained using this analysis procedure were adopted for this work.

### 3.2 Fission fragment detection

Fission fragments were detected using either one or two large-area multiwire proportional counters that make up part of the fission fragment spectrometer (see frontispiece). These detectors were positioned in a square formation around the target (see Fig. 3.10) inside a vacuum chamber that operated at pressures of around $2 \times 10^{-6}$ Torr. The detectors were arranged so that they were in opposite hemispheres. The large active area of each detector enabled very efficient data collection. Thus data with high statistical precision were obtained for the fission cross-section measurements in a relatively short period of time. The detectors could either be operated in singles or coincidence mode. Both modes of operation were used here. The Section below describes some specific details relating to the
FIG. 3.9: The mean decay chain $\alpha$-multiplicities as a function of lab. energy. The solid line represents the smoothed multiplicity, used in the second analysis procedure.

design and operation of these detectors. Then, a description is given on the identification of fission fragments, followed by an account of the procedure for the extraction of the total fission cross-sections.

3.2.1 The large-area position sensitive MWPCs

The large-area MWPCs work on the same principle as the MWPC used for ER detection, as described in Section 3.1.1. The MWPCs of the fission fragment spectrometer have an additional wire plane, in order to obtain position information in two dimensions. Each large-area MWPC consists of a plane of vertical (X-plane) and horizontal (Y-plane) wires each located 3 mm either side of a central cathode, as shown in Fig. 3.10. The Au coated W wires are separated from each other by a distance of 1 mm, there being 284 in the X-plane and 357 in the Y-plane. Each wire is connected to a delay chip to form the delay line for the position signal of each plane. The delay between each wire is nominally 1 ns. The intrinsic position resolution obtained with these detectors was 1 mm. The cathode was made from Au coated polyethylene terephthalate (PET) 0.9 $\mu$m thick. This cathode was divided into four equal segments by the shadow cast from a cross-shaped mask placed in front of the cathode during its thin film coating with $\approx 20 - 50$ $\mu$g cm$^{-2}$ of Au. The purpose of the subdivided cathode was to reduce the capacitance of each segment. The cathode was coated with Au on both sides. The operating voltage applied to the cathode of each detector was typically $-500$ V.
FIG. 3.10: A plan view of the fission fragment spectrometer, showing the layout of each of the two MWPCs in relation to the target. Each detector is mounted at 45° to the beam axis and 180 mm from the target. The 'middle' of the detector has coordinates (0,0,180) as explained in the text. The X-position is asymmetric about the middle of the detector, and has the values $-130 \leq X \leq +154$ mm. The Y-position is symmetric about (0, 0, 180), and has the range $-178.5 \leq Y \leq +178.5$ mm. The lower part of the figure depicts an exploded view of the position grids and the central cathode of one MWPC (not to scale).
Each MWPC sits in its detector housing which is mounted vertically with the cathode a distance of 180 mm, and at an angle of 45°, to the beam direction (see Fig. 3.10). Propane was used as the ionisation gas because of its availability, and the pressure inside the detectors were maintained at 4 Torr by a gas-flow system. The front window of each detector was also made from the 0.9 μm PET, with a coating of ≈50 μgcm⁻² Cu over the inside surface. The purpose of this is twofold. Firstly, it dissipates any undesirable charge build-up on the inside of the detector window and secondly, it reduces the loss of propane gas from the detector to the vacuum chamber. It does this by inhibiting the molecular exchange between the PET foil and the propane gas [98]. The major design challenge in the manufacture of these metalised foils, was obtaining an adequate metal coating without perforating the thin foil in the evaporation process. The best method proved to be evaporation using a cylindrical evaporation boat, with a small hole in the top of the cylinder, mounted at an angle around 45° to the horizontal. In this configuration, the Cu atoms were able to escape, but the Cu droplets ejected from the molten metal had a very small probability of passing out through the hole and piercing the thin PET foil.

Representation of the position data

The large continuous angular coverage of each MWPC means a choice must be made on how to best represent the position information. The first step was to convert the position signals from each plane into position coordinates on the active area of the detector. This was done by assuming a linear transformation between the position signals and the distance across the detector measured in mm. The gain and offset of the linear calibration were calculated using the detector widths. Corrections were made to the linear relationship in bins of Δθ_{lab} = 5°, with an elastic scattering calibration measurement. This calibration measurement showed that the differential nonlinearity [88] was < 5%. The elastic scattering calibration is discussed in more detail in Section 3.2.3.

To convert the position information into the scattering angles, a right-hand coordinate system was defined, as shown in Fig. 3.11(a), where the origin is at the target position. If (X,Y,Z) defines a point on the active area of Detector 1, then the cathode of that detector is defined by the plane (X,Y) and Z = 180 mm, the perpendicular distance from the target to the cathode. Here, -130 ≤ X ≤ +154 mm and the active area for the Y-position has the coordinates -178.5 ≤ Y ≤ +178.5 mm. The centre of the cathode for Detector 1 has the coordinates (0,0,180) mm.
Table 3.6: The angular limits of the ‘axe’-shaped position spectrum for Detector 1. The angles ($\theta_{lab}$, $\phi_{lab}$) were obtained using Eqs. (3.17) and (3.18).

<table>
<thead>
<tr>
<th>Location</th>
<th>X (mm)</th>
<th>Y (mm)</th>
<th>$\theta_{lab}$ (°)</th>
<th>$\phi_{lab}$ (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>backward, top</td>
<td>-130</td>
<td>-178.5</td>
<td>140.3</td>
<td>168.8</td>
</tr>
<tr>
<td>backward, bottom</td>
<td>-130</td>
<td>+178.5</td>
<td>140.3</td>
<td>11.2</td>
</tr>
<tr>
<td>forward, top</td>
<td>+154</td>
<td>-178.5</td>
<td>93.6</td>
<td>127.1</td>
</tr>
<tr>
<td>forward, bottom</td>
<td>+154</td>
<td>+178.5</td>
<td>93.6</td>
<td>52.9</td>
</tr>
<tr>
<td>backward, in-plane</td>
<td>-130</td>
<td>0</td>
<td>170.8</td>
<td>90</td>
</tr>
<tr>
<td>forward, in-plane</td>
<td>+154</td>
<td>0</td>
<td>94.5</td>
<td>90</td>
</tr>
</tbody>
</table>

Each event ($X,Y,180$) on the active area of the detector was then transformed to laboratory coordinate system ($\theta_{lab}$, $\phi_{lab}$). The scattering angle $\theta_{lab}$ is defined as the angle between the vector describing the beam axis $B = (1,0,-1)$ and the vector locating the position of the event, $A = (x,y,1)$, as shown in Fig. 3.11(a). Here, $(x,y,z)$ are defined as the coordinates $(X,Y,Z)$ divided by 180 mm. The azimuthal angle $\phi_{lab}$ is defined in the plane perpendicular to the beam axis, as shown in Fig. 3.11(b), and at 45° to the detector plane. Specifically, it is the angle between the y-axis and the projection of the vector $A = (x,y,1)$ onto the plane perpendicular to the beam axis. The angles ($\theta_{lab}$, $\phi_{lab}$) are given by

$$\cos \theta_{lab} = \frac{x - 1}{\sqrt{2(x^2 + y^2 + 1)}}$$

$$\cos \phi_{lab} = \frac{y}{\sqrt{0.5(x + 1)^2 + y^2}},$$

where the $(x,y,1)$ are the coordinates of the active area, in units of 180 mm.

The transformations described in Eqs. (3.17) and (3.18) applied to the position data collected in Detector 1, produced the $\phi$ versus $\theta$ plot shown in Fig. 3.12. The active (coloured) region represents 68% of the black region, which has a solid angle of $\pi$ sr. The four corners of the detector plane correspond to the angular limits given in Table 3.6. Note how the angular range of $\phi_{lab}$ increases for the more backward scattering angles. In the following analysis, cuts in both $\theta_{lab}$ and $\phi_{lab}$ were made to ensure that there was no contribution from ‘edge’ effects at the limits of the detection area.
Plan view: Detector 1 \((x=X/180, y=Y/180, z=1)\)

Detector 1

\(-130 < X < +154\)
\(-178.5 < Y < +178.5\)

Detector 2

Cathode

(b)

FIG. 3.11: (a) A plan view of the RH coordinate system \((x, y, z)\) used to convert the position of an event on the active area into the scattering angles. In this definition, the cathode of Detector 1 is defined by the plane \((x, y)\) and \(z = 1\), where the coordinates \((x, y, z)\) are defined as \((X, Y, Z)\) divided by 180 mm. (b) A schematic illustration of the scattering angles \(\theta_{\text{lab}}\) and \(\phi_{\text{lab}}\).
FIG. 3.12: A plot of the laboratory azimuthal angle as a function of the laboratory scattering angle for a detector in the backward hemisphere. The solid angle of the detector is $0.68\pi$ sr (coloured region).
Signal generation and electronics

For one detector, there are a total of four parameters of interest: the X and Y positions of the event and its $\Delta E$ and TOF relative to the pulsed beam. All signals were initially amplified by voltage sensitive preamplifiers which were mounted on the back of the detector housing inside the vacuum chamber. The position signals were generated from a TAC using two fast logic pulses from both ends of the position delay line (see Section 3.2.1). The electronics circuit is shown schematically Fig. 3.13. The $\Delta E$ and timing signals for each segment of the cathode are summed in a linear fan-in unit. The slow $\Delta E$ signal is then amplified using a spectroscopy amplifier. The fast timing signal from the cathode is then used to start the TOF TAC whose stop signal comes from the beam RF. In addition to the above four parameters, two monitor detector signals and a pulser signal are also collected by the data acquisition system.

3.2.2 Fission fragment identification

The fission fragments were identified by their energy loss $\Delta E$ in the cathode and by their time-of-flight TOF with respect to the pulsed beam. Typical plots of $\Delta E$ versus TOF for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction at $E_{\text{beam}} = 86.0 \text{ MeV}$ are shown in Fig. 3.14(a)-(c), for three different cuts in $\theta_{\text{lab}}$. The $\Delta E$ has been corrected for the geometric variation of the raw energy pulse-height ($\Delta E'$) with the incident angle $\psi$ of the particle in the cathode:

$$\Delta E = \Delta E' \cos \psi,$$

where $\tan \psi = (x^2 + y^2)^{1/2}$. It was initially puzzling to observe the apparent increase in the elastic scattering rate at the angle $\theta_{\text{lab}} = 162.5^\circ$ compared with the elastic yield at $\theta_{\text{lab}} = 132.5^\circ$. This is because the $\Delta E'$ signal for the scattered particles is larger at the more forward and backward angles compared to that from particles that enter the detector at the normal angle. Since the lower level discriminator on the raw $\Delta E$ signal was fixed near the top of the elastic events, more events at $\theta_{\text{lab}} = 162.5^\circ$ cross the threshold than at $\theta_{\text{lab}} = 132.5^\circ$, even though the elastic rate falls with increasing $\theta_{\text{lab}}$.

Examples of the $\Delta E$-TOF spectra for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction, at a beam energy $E_{\text{beam}} = 168.1 \text{ MeV}$, well above the average barrier, are shown in Fig. 3.14(d)-(f). At this beam energy, the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction has a very small elastic scattering yield at angles backward of $\approx 130^\circ$. Note the $\Delta E$ tail on the fission events.
FIG. 3.13: A schematic representation of the electronics for one detector operated in singles mode. The quantities $X_L$ and $X_R$ ($Y_T$ and $Y_B$) denote the left and right (top and bottom) position signals from either end of the X-position (Y-position) delay line. The time signals from each quadrant of the cathode are denoted by $T_1, \ldots, T_4$. 

<table>
<thead>
<tr>
<th>Symbol</th>
<th>NIM Unit</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>PRE</td>
<td>Fast preamplifier</td>
<td>HMI IV71A</td>
</tr>
<tr>
<td>T-AMP</td>
<td>Fast Amplifier</td>
<td>PS Model 778</td>
</tr>
<tr>
<td>CFD</td>
<td>Constant Fraction Discriminator</td>
<td>EG+G-ESN CF 8000</td>
</tr>
<tr>
<td>FAN-IN</td>
<td>Linear Fan-in/Fan out</td>
<td>PS Model 740</td>
</tr>
<tr>
<td>HPF</td>
<td>High pass filter</td>
<td>—</td>
</tr>
<tr>
<td>AMP</td>
<td>Linear Amplifier</td>
<td>Tennelec TC 203BLR</td>
</tr>
<tr>
<td>Logic</td>
<td>2 Fold Logic</td>
<td>Canberra Coinc. 1446</td>
</tr>
<tr>
<td>TSCA</td>
<td>Timing Single Channel Analyser</td>
<td>Canberra 1437</td>
</tr>
<tr>
<td>L+GS</td>
<td>Linear Gate and Stretcher</td>
<td>Canberra 1454</td>
</tr>
<tr>
<td>TAC</td>
<td>Time to amplitude converter</td>
<td>Ortec 467</td>
</tr>
<tr>
<td>ADC</td>
<td>Analogue to digital converter</td>
<td>Silena 7423 UHS or Canberra 8077</td>
</tr>
</tbody>
</table>
FIG. 3.14: The energy-loss signal $\Delta E$ versus the TOF signal for the $^{16}$O + $^{208}$Pb (a)-(c) at $E_{\text{beam}} = 86.0$ MeV and (d)-(f) $^{28}$Si + $^{208}$Pb reactions at $E_{\text{beam}} = 168.1$ MeV. The $\Delta E$ has been corrected for the angle of incidence of the particle in the cathode. The plots are for 5° bins at the average angle indicated at the top of each panel.
at the backward angles. At the higher energies, the fission fragments were well separated from the elastically scattered projectiles. However, at the very lowest beam energies and most backward angles, the $\Delta E$-TOF information alone could not be used to identify all the fission events. The $\Delta E$ tail of the slowest fission fragments was not separated from the slit-scattered particles which contaminated the spectra. In this case, a second MWPC was used and fission fragments were detected in coincidence with the first detector. The coincidence requirement then allowed the reliable identification of fission fragments at the most backward angles.

The $\Delta E$-TOF identification technique, used in singles or coincidence modes, allows accurate determination of the yield of fission fragments. A pulser was used to correct the yield for the electronic and data acquisition deadtime as described earlier for the ER measurement. The pulser was located in a channel position clear of the $\Delta E$-TOF of the particles of interest, as shown in Fig. 3.14(a). Section 3.2.3 below describes how the fission fragment yields were converted to angular distributions and total fission cross-sections.

### 3.2.3 Determination of fission fragment angular distributions

The large solid angle of each detector enables the collection of a large number of counts. This allows the data to be divided into relatively small bins of width $\Delta \theta_{\text{lab}}$. Here, for a constant cut in $\Delta \phi_{\text{lab}}$, a bin width of $\Delta \theta_{\text{lab}} = 5^\circ$ was chosen. The differential cross-section was then calculated for each bin by measuring the yield of fission fragments concurrent with the Rutherford scattering yield in the monitors.

The ratio of the yield in the fission fragment detector for the laboratory energy $E$ at angle $\theta_{\text{lab}}$, $Y_{\text{FF}}(\theta_{\text{lab}}, E)$, to the sum of the elastic yield in the monitor detectors, $Y_{\text{M}}(\theta_{\text{M}}, E)$, is given by

$$
\frac{Y_{\text{FF}}(\theta_{\text{lab}}, E)}{Y_{\text{M}}(\theta_{\text{M}}, E)} = \frac{d\Omega_{\text{FF}}}{d\Omega_{\text{M}}} \frac{d\sigma(\theta_{\text{lab}}, E)/d\Omega}{d\sigma(\theta_{\text{M}}, E)/d\Omega}.
$$

In Eq. (3.19), $\theta_{\text{M}}$ is the laboratory angle of the monitor detectors with respect to the beam axis and $d\Omega_{\text{FF}}$ and $d\Omega_{\text{M}}$ are the solid angles of the fission fragment detector and monitor detectors, respectively. The solid angle of the fission fragment detector is given by $d\Omega_{\text{FF}} = \sin \theta_{\text{lab}} d\theta_{\text{lab}} d\phi_{\text{lab}}$. The laboratory energy $E$, is defined as the beam energy after correcting for the energy loss in the target.
Since the bins have a width of 5°, an angle quoted as $\theta_{\text{lab}} = 162.5°$ refers to the angular range $\theta_{\text{lab}} = 160°-165°$.

To calculate the differential cross-section for fission fragments, a normalisation reaction was carried out for each experimental run. The detector was calibrated using a Rutherford scattering reaction which enabled the normalisation factor per bin, $A(\theta_{\text{lab}})$, to be determined. At the energy of the calibration reaction, $E_{\text{cal}}$, Eq. (3.19) can be written

$$\frac{d\Omega_{\text{M}}}{d\Omega_{\text{FF}}} = \frac{Y_{\text{M}}(\theta_{\text{M}}, E_{\text{cal}}) \frac{d\sigma(\theta_{\text{lab}}, E_{\text{cal}})}{d\Omega}}{Y_{\text{cal}}(\theta_{\text{lab}}, E_{\text{cal}}) \frac{d\sigma(\theta_{\text{M}}, E_{\text{cal}})}{d\Omega}}, \tag{3.20}$$

where $Y_{\text{cal}}(\theta_{\text{lab}}, E_{\text{cal}})$ is the elastic yield in the fission fragment detector. Then, by writing $d\Omega_{\text{FF}} = (2\pi \sin \theta_{\text{lab}} d\theta_{\text{lab}} \Delta \phi_{\text{lab}})/2\pi$, Eq. (3.20) becomes

$$\frac{d\Omega_{\text{M}}}{2\pi d\theta_{\text{lab}}} = \frac{\sin \theta_{\text{lab}} \Delta \phi_{\text{lab}}}{2\pi} \frac{Y_{\text{M}}(\theta_{\text{M}}, E_{\text{cal}}) \frac{d\sigma(\theta_{\text{lab}}, E_{\text{cal}})}{d\Omega}}{Y_{\text{cal}}(\theta_{\text{lab}}, E_{\text{cal}}) \frac{d\sigma(\theta_{\text{M}}, E_{\text{cal}})}{d\Omega}}. \tag{3.21}$$

The RHS of Eq. (3.21) is defined as the normalisation constant per bin, which is given by

$$A(\theta_{\text{lab}}) = \left( \frac{\sin \theta_{\text{lab}} \Delta \phi}{2\pi} \right) \frac{Y_{\text{M}}(\theta_{\text{M}}, E_{\text{cal}}) \frac{d\sigma(\theta_{\text{lab}}, E_{\text{cal}})}{d\Omega}}{Y_{\text{cal}}(\theta_{\text{lab}}, E_{\text{cal}}) \frac{d\sigma(\theta_{\text{M}}, E_{\text{cal}})}{d\Omega}}. \tag{3.22}$$

The differential cross-section for fission fragments at an energy $E$, is then given by

$$\frac{d\sigma(\theta_{\text{lab}}, E)}{d\Omega} = \frac{2\pi A(\theta_{\text{lab}}) Y_{\text{FF}}(\theta_{\text{lab}}, E)}{\Delta \phi \sin \theta_{\text{lab}} Y_{\text{M}}(\theta_{\text{M}}, E)} \left[ \frac{d\sigma(\theta_{\text{M}}, E)}{d\Omega} \right], \tag{3.23}$$

where the term in brackets is the Rutherford cross-section at the monitor angle $\theta_{\text{M}}$ and $\Delta \phi$ is some constant cut in $\phi$. For the $^{16}\text{O} + ^{208}\text{Pb}$ and $^{28}\text{Si} + ^{208}\text{Pb}$ reactions $\Delta \phi = 57°$ and $70°$, respectively. The exact angle $\theta_{\text{M}}$ of the monitor detectors is not critical since the dependence of $d\sigma(\theta_{\text{lab}}, E)/d\Omega$ on $\theta_{\text{M}}$ in Eq. (3.23) cancels out with the term $d\sigma(\theta_{\text{M}}, E_{\text{cal}})/d\Omega$ in Eq. (3.22). Note that the normalisation constants per bin, Eq. (3.22), are independent of the energy and the $Z_1Z_2$ product of the calibration reaction. Since the calibration run was made for some particular 'up-down' monitor ratio, $M_+/M_-\,$, which defines the position of the beam spot relative to the beam axis, subsequent yields in the monitor detectors were renormalised to this original ratio using the relation

$$Y_{\text{M}}(\theta_{\text{M}}, E) = f(M_+/M_-)Y_{\text{M}}(\theta_{\text{M}}, E_{\text{cal}}). \tag{3.24}$$
The function \( f(M_+/M_-) \) was determined at the end of run, by steering the beam off axis, and measuring the ratio of the Rutherford scattering yield in the fission fragment detector to the average yield in the monitor detectors. An example of this function is shown in Fig. 3.15. This correction was typically 0.1%.

**FIG. 3.15:** The ratio of the Rutherford yield as measured in the MWPC to the average yield in the monitor detectors as a function of the ratio of the monitor yields (open circles). The solid curve guides the eye. The function was produced during a calibration run by moving the beam spot off axis, both in the left and right directions, with the switching magnet. The monitor ratio of the actual calibration run is shown by the large closed circle. The points at the bottom of the figure indicate how the ratio of the monitors varied during the actual fission cross-section measurements. This function allowed the fission cross-sections to be corrected for any movement of the beam spot on the target relative to the position it had during the calibration run.

**Obtaining the total fission cross-sections**

The differential cross-sections for each experiment were calculated from Eq. (3.23) using the normalisation factors obtained during the calibration runs. The Sub-section below describes the details of the bin normalisation factors. In this work, there were two separate measurements for the \(^{16}\text{O} + ^{208}\text{Pb}\) system, each with their own calibration runs, with three individual passes through the excitation function. For the \(^{28}\text{Si} + ^{208}\text{Pb}\) reaction, there were three separate measurements, but owing to a normalisation problem during one experiment, the fission cross-sections were obtained for only two of these measurements. This problem did not affect the fission fragment anisotropies for the third measurement. These runs are labelled in Table 3.7.
Table 3.7: The $\chi^2$ for each reaction and each experimental run.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>Experiment</th>
<th>$\chi^2 \pm$ s.d.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O + $^{208}$Pb</td>
<td>run one (thick)</td>
<td>2.1 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>run one (thin)</td>
<td>1.4 ± 0.5</td>
</tr>
<tr>
<td></td>
<td>run two (thin)</td>
<td>1.2 ± 0.4</td>
</tr>
<tr>
<td></td>
<td>total:</td>
<td>1.6 ± 0.5</td>
</tr>
<tr>
<td>$^{28}$Si + $^{208}$Pb</td>
<td>run one</td>
<td>1.3 ± 0.5</td>
</tr>
<tr>
<td></td>
<td>run two</td>
<td>1.1 ± 0.6</td>
</tr>
<tr>
<td></td>
<td>total:</td>
<td>1.2 ± 0.6</td>
</tr>
</tbody>
</table>

The differential cross-section in the lab. reference frame was converted to the centre-of-mass system assuming symmetric fission with the total kinetic energies from the Viola systematics [99]. An example of the differential cross-sections for the $^{16}$O + $^{208}$Pb system are shown in Fig. 3.16 as a function of the centre-of-mass scattering angle $\theta_{c.m.}$. The total fission cross-section was obtained by integrating the differential cross-section multiplied by $2\pi \sin \theta_{c.m.}$ over $\theta_{c.m.}$ between 90° and 180°. To do this, it was necessary to extrapolate to regions beyond the detector's angular coverage. Two techniques were used to do this. In the first, the angular distributions were plotted as $d\sigma/d\theta_{c.m.}$ and fitted with a cubic spline function assuming that $d\sigma/d\theta_{c.m.}$ goes to zero at $\theta_{c.m.} = 180°$ and remains almost constant for $\theta_{c.m.}$ near 90°.

The second method involved fitting the differential cross-section using the transition state model procedure of Back et al. [34], assuming that the angular distributions in the extrapolated region can be so described. Both methods were always in agreement within a fraction of a percent. The broken lines in Fig. 3.16 are the fits to the angular distributions using the model based procedure, specifically Eqs. (2.61) and (2.62) in Chapter 2. For the $^{16}$O + $^{208}$Pb reaction, the $\chi^2$ per degree of freedom, $\chi^2_{d.o.f.}$, averaged over both runs was 1.6 ± 0.5. Table 3.7 lists the breakdown for each particular experiment for both the $^{16}$O + $^{208}$Pb and $^{28}$Si + $^{208}$Pb reactions. It is interesting to note how the $\chi^2$ has improved with each subsequent experiment. This is possibly due to an increase in experience and refinement of the operation of the fission fragment spectrometer.

The quality of these fits gave confidence that the angular range covered by the MWPC detectors defined the angular distributions, and hence anisotropies, to the desired precision. From these fits to the angular distributions, the total fission
FIG. 3.16: The fission fragment angular distributions for $^{16}\text{O} + ^{208}\text{Pb}$ plotted as a function of the c.m. scattering angle. The broken lines are the fits to the angular distributions which enabled the anisotropies and the total cross-section to be obtained.
cross-sections were obtained. The results of this analysis are given in Chapter 4.

The bin normalisation

The detector calibrations were carried out using either $^{28}$Si or $^{34}$S projectiles elastically scattered from target of $\approx 200 \mu$g.cm$^{-2}$ of $^{197}$Au. An example of a normalisation run for the $^{16}$O + $^{208}$Pb system is shown in Fig. 3.18, obtained using Eq. (3.22). The normalisation factors are plotted in the laboratory reference frame. The assumption of a linear calibration of the position information was true to around $\pm 3\%$ for this normalisation. The structure in $A(\theta_{lab})$ was attributed to non-linearities in either the delay line response of the detector, or the external electronics which further process the position signals. These non-linearities were accounted for by using the measured $A(\theta_{lab})$ function, assuming that the response of the detector did not change during the measurements of the fission fragment angular distributions. Any difference between these non-linear corrections as determined during the calibration run and those applicable during the actual fission measurements, will cause small distortions in the fission fragment angular distributions. This effect can be seen from the small scatter in the differential cross-section in Fig. 3.16. The consequences of this effect are minimal. The total cross-section remained essentially unchanged when the scatter was removed by hand and the cross-section re-fitted. The anisotropy is also reasonably insensitive to variations in $A(\theta_{lab})$ on an individual bin basis.

The success of the above technique for correcting for non-linearities in the detector response, was checked by calculating the Rutherford cross-section using a 'mixture' of calibration runs. In other words, using the $A(\theta_{lab})$ given in Fig. 3.17, the differential cross-section for elastic scattering was obtained for the run one calibration run, made during a separate experiment, with a different energy and beam species and also for a different monitor angle, $\theta_M$. The ratio of the differential cross-section to the Rutherford cross-section for $^{34}$S + $^{197}$Au at $E_{lab} = 90.0$ MeV, is shown in Fig. 3.18. To calculate this ratio precisely, the angle of the monitor detectors must be known because this angle was not the same for each calibration run. The differential cross-sections per bin scatter about the average value, as shown by the solid line in Fig. 3.18. There is also a systematic increase of $\approx 3\%$ from the expected Rutherford cross-section, the broken line in Fig. 3.18. This systematic difference arises because the monitors were moved to a different position for the second calibration run, a difference that is consistent with the uncertainty in determination of the monitor angle. These effects are not too large, demonstrating that the normalisation per bin was successful in
Fig. 3.17: The bin normalisation factors $A(\theta_{\text{lab}})$ for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction. The width of each bin is 5° in the lab reference frame. The broken line is the average normalisation factor, and the histograms show the deviation of the detector response from the assumption of a linear calibration.

correcting for the detector response; and that between runs, the normalisation does not shift dramatically.

3.3 General measurement practice

The following practices were adhered to during the measurements of the ER and fission cross-sections in order to minimise the size of the random and systematic uncertainties.

1. The beam energy was always set by monotonically increasing the magnetic field in the analysing magnet, after the magnet had been 'recycled'. The latter involved smoothly increasing the magnet current from 0 to the supply maximum of 200 A and then reducing it back to zero; this was done three times and then the field was set to the required value approaching from below. Since the magnet 'constant' may depend weakly on the magnetic field, due to saturation and fringing effects [82], this procedure minimised any effects of differential hysteresis. Whenever a second excitation function was measured, or a datum point was repeated, the magnet was recycled.
The reproducibility of the beam energy has been checked by re-measuring data points in the ER excitation function for $^{16}\text{O} + ^{154}\text{Sm}$ after a magnet recycling, and the cross-sections agreed with statistical uncertainties of less than 1% [16].

2. For the fission measurements and the ERs measured in the velocity filter, the beams were 'tuned-up' by focussing them through a Ta collimator which was moved into the target position between each new energy setting. This focussing ensured a maximum amount of beam current at the target position. In the velocity filter measurement, the number of slit scattered particles entering the MWPC was minimised by observing the count rate in the MWPC at an angle of 5°. In this case, the beam was tuned up when the rate in the MWPC was at a minimum and the beam current was at a maximum.

3. To set the angle of the velocity filter, a consistent procedure was followed where the detector arm was always rotated into its final position from the one direction. This procedure minimised the uncertainty in setting the angle of the detector by minimising the effects of backlash in the detector arm mechanism. Again, the uncertainty in this method was not significant. The ER differential cross-section at ±2° for $^{16}\text{O} + ^{154}\text{Sm}$ at 90 MeV has
been measured nine times, with a standard deviation of 0.3% [17].

4. The counting rates in the MWPCs used in the fission measurements were kept to a level where the deadtime of the data acquisition system was around 1%.

The above routine, established over a period of several years, gives confidence in the results obtained and the estimation of the size of their random and systematic uncertainties.
Chapter 4

RESULTS AND ANALYSIS

The results of the fusion cross-section and fission fragment angular distribution measurements are presented in this Chapter. The total fusion cross-sections were taken to be equal to the ER cross-sections for the $^{16,17}O + ^{144}Sm$ systems since the fission component is negligible. The $^{16}O + ^{208}Pb$ system had significant contributions from both the fission and ER decay modes, whereas for the $^{28}Si + ^{208}Pb$ system, fission dominates at all energies measured. The fusion excitation functions for each system were then fitted with a one-dimensional barrier penetration model for energies greater than 6% above their single barriers, which corresponds to a cross-section of $\gtrsim 200$ mb. In this high energy region, the effects of coupling on the fusion are minimal, and so the parameters defining the 'uncoupled' nuclear potential for fusion can be obtained. The fusion cross-sections are then used to obtain the distribution of barriers for each system and the results are compared to the one-dimensional barrier penetration calculation.

4.1 The $^{16}O + ^{144}Sm$ reaction

The fusion cross-sections $\sigma(E)$ for the $^{16}O + ^{144}Sm$ reaction are shown in Fig. 4.1 as a function of the centre-of-mass energy $E_{c.m.}$ of the system. The uncertainties in the cross-sections in Fig. 4.1 were less than 1 mb for energies up to $E_{c.m.} = 63$ MeV and were taken to be $\pm 1\%$ for the higher energies. The values for $\sigma(E)$ are also tabulated in the Appendix. The uncertainty on the tabulated cross-sections consists of contributions from the random uncertainties only. The systematic uncertainties are small, and since they have a minimal effect on the
FIG. 4.1: The experimental fusion excitation function for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction as a function of the centre-of-mass energy. The uncertainties in the fusion cross-sections are ±1% for energies above $E_{\text{c.m.}} = 63$ MeV, and less than 1 mb for energies below this. The dotted line is the single barrier calculation with $B_0 = 61.10$ MeV.

Since the target contained small quantities of all the other stable isotopes of samarium, a correction had to be made to the measured cross-sections. This was carried out as follows. Let $\sigma^{\text{Sm}}(E)$ represent the cross-section for the reaction $^{16}\text{O} + ^A\text{Sm}$, where A is the mass of the target. The isotopic breakdown of the target [100] is $^{144}\text{Sm} (96.47\%), ^{147}\text{Sm} (1.08\%), ^{148}\text{Sm} (0.56\%), ^{149}\text{Sm} (0.54\%), ^{150}\text{Sm} (0.84\%)$.
(0.24%), $^{152}\text{Sm}$ (0.65%) and $^{154}\text{Sm}$ (0.46%), with the percentage composition shown in the parentheses. Let $\sigma(E)$ represent the cross-section for the reaction of interest, $^{16}\text{O} + ^{144}\text{Sm}$. The effects of the contaminants on $\sigma(E)$ were then estimated from model fits to the experimental cross-sections for the reactions on all samarium isotopes. The cross-sections for the samarium isotopes were taken from Refs. [16,101,17] and the model assumed that the contaminant samarium nuclei were statically deformed, with the deformation parameters changing smoothly and systematically with mass.

From these model fits to all the samarium data, it was recognised that the excitation function for the $^{16}\text{O} + ^{148}\text{Sm}$ provided a good representation of the cross-sections for all samarium isotopes, for all but the lowest three data points. Thus, corrections to $\sigma^{\text{Sm}}(E)$ were made assuming that the cross-sections for the $^{16}\text{O} + ^{148}\text{Sm}$ reaction could be used to represent the average cross-sections for the other contaminants. That is,

$$\sigma^{148\text{Sm}}(E) = 1.08%\sigma^{147\text{Sm}}(E) + \ldots + 0.46%\sigma^{144\text{Sm}}(E).$$ (4.1)

Then, the cross-section for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction is given by

$$\sigma(E) = (1/0.965)[\sigma^{\text{Sm}}(E) - 0.035\sigma^{148\text{Sm}}(E)],$$ (4.2)

where $\sigma^{148\text{Sm}}(E)$ is the cross-section for the $^{16}\text{O} + ^{148}\text{Sm}$ reaction from the precisely measured experimental results of Ref. [17].

The uncertainties quoted on the cross-sections include the uncertainties from the isotopic correction. The uncertainties on the cross-sections at the lowest three beam energies were increased to account for the doubt in the target contaminant correction. The insensitivity of the barrier distributions to this correction can be seen in Fig. 4.3(a). This comparison shows the two barrier distributions calculated before and after the corrections for target contamination were made. The effect on the shape of the barrier distributions is minimal. The effects of the contaminant correction at high energies is very small $\lesssim 1%$.

The fusion excitation function was fitted for energies $E_{\text{c.m.}}/B_0 \geq 1.06$ with a one dimensional barrier penetration model (see Chapter 2). The depth $V(r)$ and diffuseness $a$ of the Woods–Saxon form of the nuclear potential were varied in this fitting procedure. The best fit parameters are listed in Table 4.1 along with the corresponding s-wave parameters for the single barrier. The $\chi^2$-per degree of freedom for this fit was 0.88 for energies $E_{\text{c.m.}}/B_0 \geq 1.06$. The data were also fitted for a range of $B_0$ and $a$ values which increased the total $\chi^2$ by 1,
Table 4.1: Parameters for the nuclear potential obtained from the fits to the high energy region for the $^{16,17}$O + $^{144}$Sm reactions. The depth is given in column 2 for the radius $r_V$ in fm. Columns 4 to 6 are the parameters for the single barrier, its position and its curvature, $\hbar \omega_0$.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>$V(r_V)$ (MeV)</th>
<th>$a$ (fm)</th>
<th>$B_0$ (MeV)</th>
<th>$R_B$ (fm)</th>
<th>$\hbar \omega_0$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O + $^{144}$Sm</td>
<td>-1195 (0.8 fm)</td>
<td>0.84</td>
<td>61.10</td>
<td>10.8</td>
<td>4.23</td>
</tr>
<tr>
<td>range</td>
<td>±0.02</td>
<td>±0.03</td>
<td>±0.01</td>
<td>±0.04</td>
<td></td>
</tr>
<tr>
<td>$^{17}$O + $^{144}$Sm</td>
<td>-772.5 (0.8 fm)</td>
<td>0.92</td>
<td>60.68</td>
<td>10.8</td>
<td>3.89</td>
</tr>
<tr>
<td>range</td>
<td>±0.02</td>
<td>±0.04</td>
<td>±0.02</td>
<td>±0.06</td>
<td></td>
</tr>
</tbody>
</table>

corresponding to one standard deviation. The change in the fit parameters as a result of such variation is also given in Table 4.1. For example, using a diffuseness of $a = 0.86$ results in a decrease in the single barrier to $B_0 = 61.07$. The change in the fitted barrier radius and curvature is also shown in Table 4.1. The range gives some indication of the sensitivity of these fit parameters, demonstrating that very precise data in the high energy region provides a tight constraint on the potential parameters for the single barrier.

The dotted line in Fig. 4.1 is the excitation function calculated using the single barrier obtained from this fit. As expected, the calculation underpredicts the measured cross-sections for energies below the single barrier, indicating the presence of additional degrees of freedom. At this point in the analysis, the usual approach was to redo the theoretical calculations, with the inclusion of some combination of additional couplings, and to continue until the appropriate degree of enhancement was obtained. The agreement between the calculation and the measured excitation function can also, in principle, be improved over the full energy range by altering the parameters of the single barrier. Such an approach, used in combination with the less well defined potential parameters, can lead to incorrect conclusions about the reasons for the fusion enhancement. However, if the data and calculation are represented in terms of their distribution of barriers, the reasons for the enhanced fusion cross-sections become more apparent. Knowledge of the distribution of barriers is a more transparent approach to understanding the important channels affecting the fusion process.

In figure 4.2 the quantity $d^2(E\sigma)/dE^2$ is plotted as a function of the centre-of-mass energy, obtained from the cross-sections presented in Fig. 4.1, using the point difference formula, Eq. (2.51) in Chapter 2 with $AE_{c.m.} = 1.80$ MeV. As
explained in Chapter 2, the quantity $d^2(E_\sigma)/dE^2$, is directly related to the barrier distribution. The barrier distribution consists of two peaks, the smaller one at higher energies is almost completely resolved from the larger peak at the lower energies. Also shown in Fig. 4.2 is the barrier distribution for the single barrier calculation. A comparison of the measured and calculated barrier distributions suggests that the single barrier is split into two separate barriers, the height and separation of each barrier reflecting the strength of the coupling. The previous analysis of Ref. [102], fitted less precise data with a single barrier in order to obtain the parameters of a spherical potential. In a subsequent analysis of the same system [101], the cross-sections for the $^{16}$O + $^{144}$Sm reaction were re-fitted with a distribution of barriers due to a small prolate deformation and a smaller compensating value for $\hbar \omega_0$. Both these approaches are inconsistent with the results presented here, and demonstrate the benefit of precise data in the energy region above $B_0$ which place a strict limit on the potential parameters.

The structure observed in the barrier distribution is characterised by a distinct peak at energies above the single barrier. As shown in Chapters 1 and 2, to produce a barrier distribution with this feature, coupling to a negative Q-value.

FIG. 4.2: The experimental fusion barrier distribution for the $^{16}$O + $^{144}$Sm reaction (open squares). The dotted line is the distribution for the single barrier calculation with the s-wave parameters given in Table 4.1. The uncertainties in the barrier distribution are calculated with the uncertainty in the cross-sections assumed to be ±1% for energies above $E_{\text{c.m.}} = 63$ MeV.
FIG. 4.3: The experimental barrier distributions before and after correction for the isotopic contamination of the samarium target. Panel (a) is the $^{16}$O + $^{144}$Sm reaction and (b) the $^{17}$O + $^{144}$Sm reaction. The correction has a minimal effect on the shape of the barrier distributions.

channel (or channels) is required. Coupling to the low-lying vibrational states in $^{144}$Sm will split the single barrier into separate barriers with distributed weights. The effect can be modelled using the coupled-channels approach. Each inelastic channel can be included state by state, and the results compared with the measured barrier distribution. This comparison with coupled-channels models is carried out in Chapter 5.

The experimental uncertainties

The uncertainties can be divided into two main categories: (i) the systematic uncertainties and (ii) the random uncertainties. The random uncertainties include those uncertainties that are statistical in origin and those that are classed as non-statistical. These two main categories are discussed below, starting with the random uncertainties.

In the earlier measurements [16,19] of the barrier distributions for the $^{16}$O + $^{154}$Sm and $^{16}$O + $^{186}$W systems, scatter was evident in the second derivative for energies above the single barrier. The scatter was greater than that expected on the basis of the statistical uncertainty alone. For this reason the uncertainties on the second derivative were calculated assuming a random uncertainty of ±1%, even though the statistical uncertainties were significantly smaller [16]. However,
in the present work, the quality of the data for the $^{16,17}$O + $^{144}$Sm reactions suggest that the assumption of a random uncertainty of ±1% is not warranted (see Fig. 4.2). Of course this conclusion is based on the assumption that the true behaviour of the data should be smooth and Gaussian-like in shape, an assumption that derives from experience gained from theoretical calculations.

As discussed above, the magnitude of the random uncertainties on the fusion cross-sections are important for defining the distribution of barriers. The systematic uncertainties are less important since they only introduce a small shift in the normalisation of the distribution and do not affect the definition of its overall shape. In fact, the addition of factors with the form $E\sigma = a(E\sigma) + b$ do not affect the shape of the barrier distribution since their second derivative with respect to energy is zero. Hence, providing the systematic uncertainties are small, it is possible to evaluate the uncertainty on the second derivative using fusion cross-sections with their random uncertainties only.

This argument can be examined by comparing the distributions of barriers at energies in the region of the second peak for the $^{16}$O + $^{144}$Sm and $^{17}$O + $^{144}$Sm reactions, Fig. 4.6(a). The error bars on the $^{16}$O + $^{144}$Sm barrier distribution are calculated assuming a somewhat arbitrary uncertainty of ±1% for energies above 63 MeV. For the $^{17}$O + $^{144}$Sm reaction, measured in an independent experiment, the error bars on the barrier distribution are calculated using the random uncertainties only. There is very good agreement between these two distributions in this region. If it is assumed that the couplings in this region have a similar effect on the barrier distribution, then this comparison demonstrates that the smaller error bars, associated with the random uncertainties on $\sigma_{\text{IA}}(E)$, are a reasonable estimate of the uncertainty on the barrier distribution.

In the comparisons between the data and the calculations that follow, the barrier distributions for the $^{16,17}$O + $^{144}$Sm reactions are calculated with the random uncertainties only. There are two sources of random uncertainty in the $^{16,17}$O + $^{144}$Sm reactions. The first is the random uncertainty from the counting statistics, and the second comes from the uncertainty in the target contamination correction. The uncertainties from the latter source were calculated using the statistical uncertainties from the measured cross-section for the $^{16}$O + $^{148}$Sm reaction. The uncertainties for the lowest three points in the excitation function were increased, since, as discussed earlier, the effect of the contamination correction was the largest here. The second contribution decreases rapidly with increasing energy because the correction for the target contamination is already as small as 1% at an energy of $E_{\text{lab}} \approx 63$ MeV.
4.2 The $^{170}O + ^{144}Sm$ reaction

The measured fusion cross-sections for the $^{170}O + ^{144}Sm$ reaction are shown in Fig. 4.4, along with the cross-sections for the $^{160}O + ^{144}Sm$ measurement. In order to display these differing mass systems on the one figure, the cross-sections are plotted as a function of the difference between the centre-of-mass energy and the single barrier $(E_{\text{c.m.}} - B_0)$ for each system. The cross-sections for the $^{170}O + ^{144}Sm$ reaction have also been corrected for the presence of the heavier samarium isotopes. The correction was made in a manner similar to the $^{160}O + ^{144}Sm$ reaction, but with additional complications. Since the $^{170}O + ^{A}Sm$ reaction has not been measured, again it was assumed that the $^{170}O + ^{148}Sm$ reaction was a good representation of the effects of the target contaminants. This excitation function was then estimated by performing a calculation with the code CCMOD. Since the $^{170}O + ^{148}Sm$ reaction has a positive Q-value for the neutron stripping reaction, the theoretical calculation included this channel, with a coupling strength taken from the fits to the barrier distribution, as discussed later. A CCMOD calculation was also performed for the $^{160}O + ^{148}Sm$ reaction by fitting the data from the measured cross-sections [17]. The potential parameters for both these reactions were the same. The generated excitation function $\sigma_G$ for the $^{170}O + ^{148}Sm$ is given by

$$\sigma_G(170 + 148Sm) = \sigma_{148Sm}(E) \frac{\sigma_{th}(170 + 148Sm)}{\sigma_{th}(160 + 148Sm)},$$

(4.3)

where $\sigma_{148Sm}(E)$ is the measured excitation function for the $^{160}O + ^{148}Sm$ reaction [17], and $\sigma_{th}$ are the theoretical calculations for the $^{17,160}O + ^{148}Sm$ reactions. The ratio of the theoretical calculations was taken to reduce any systematic bias in the CCMOD calculations. The cross-section, corrected for the isotopic contamination, is then given by

$$\sigma(E) = (1/0.965)[\sigma_{Sm}(E) - 0.035\sigma_G(170 + 148Sm)],$$

(4.4)

where $\sigma_{Sm}(E)$ is the measured excitation function for the $^{170}O + ^{144}Sm$ reaction and $\sigma_G(170 + 148Sm)$ is the excitation function generated from Eq. (4.3). As for the $^{160}O + ^{144}Sm$ case, the above corrections had little effect on the shape of the barrier distributions, as shown in Fig. 4.3(b).

The comparison between the $^{160}O + ^{144}Sm$ and $^{170}O + ^{144}Sm$ fusion excitation functions in Fig. 4.4 reveals a significant difference in the low energy cross-sections. This difference cannot be explained by the difference in the potential for the $^{170}O + ^{144}Sm$ reaction, since the shift in the single-barrier parameters, due
predominantly to the slight change in the radius of the $^{17}\text{O}$ projectile, is small. The potential parameters from the fit to the high energy data were obtained as described in Section 4.1. Their values are given in Table 4.1, along with the parameters for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction for comparison. Since the charge for both $\text{O} + \text{Sm}$ reactions is the same, $B_0$ for the $^{17}\text{O} + ^{144}\text{Sm}$ reaction should scale with the nuclear radius factor $r_0(A_1^{1/3} + A_2^{1/3})$. The single barrier expected for the $^{17}\text{O}$ induced reaction, evaluated by scaling the single barrier for the $^{16}\text{O}$ induced reaction, is $B_0 = 60.70$ MeV. This agrees well with the fitted value of $B_0 = 60.68$ MeV given in Table 4.1, and so most of the difference in the single
barriers can be attributed to the change in the nuclear radii between systems. There is also a difference in the diffuseness parameters in Table 4.1 for the two reactions. This difference cannot be explained by allowing the $\chi^2$ of the fit to increase by one. It is clear that the difference between the diffusenesses for the $^{16}$O + $^{144}$Sm and $^{17}$O + $^{144}$Sm reactions is significant. A larger $a$ for the $^{17}$O + $^{144}$Sm reaction could be explained in terms of $^{16}$O core plus an additional neutron giving a more diffuse surface for the $^{17}$O projectile. The difference in the diffuseness parameters is also reflected in the curvatures of the two systems. A larger $a$ means that the potential changes less rapidly with $r$, see Eq. (2.10) in Chapter 2, and so the curvature $\hbar \omega$ decreases for larger values of $a$. Thus, $\hbar \omega$ for the $^{16}$O + $^{144}$Sm reaction is larger than that for the $^{17}$O + $^{144}$Sm reaction.

The dot-dashed line in Fig. 4.4 is the excitation function for the s-wave barrier for the $^{17}$O + $^{144}$Sm reaction. For $E_{c.m.} \gg B_0$, the two single barrier calculations converge, as expected. For $E_{c.m.} < B_0$, the calculated cross-sections for the $^{17}$O + $^{144}$Sm reaction fall more rapidly than the $^{16}$O + $^{144}$Sm reaction. This is because of the smaller value of $\hbar \omega_0$, which means a 'flatter' parabolic barrier and hence reduced penetrability.

The distribution of barriers for the $^{17}$O + $^{144}$Sm reaction is shown in Fig. 4.5, evaluated with an energy step of $\Delta E_{c.m.} = 1.79$ MeV. The two barrier distributions are overlayed in Fig. 4.6(a), again with the $B_0$ dependence removed. The double-peaked feature is also observed in the barrier distribution for the $^{17}$O + $^{144}$Sm reaction. The similarity between these two distributions in the region of 65 MeV is remarkable considering that they were derived from two different measurements. This agreement, in the energy region above the single barrier between two independently measured reactions, is compelling evidence that the double-peaked barrier distribution is due to the inelastic excitations in the target nucleus.

There is, however, a small but significant difference in the barrier distribution for the $^{17}$O induced reaction at energies below the single barrier. The size of the main peak for the $^{17}$O + $^{144}$Sm reaction is smaller than that for the $^{16}$O + $^{144}$Sm reaction. The strength missing from this peak is present at lower energies, reflected in the less rapidly decreasing $d^2(E\sigma)/dE^2$ for the $^{17}$O + $^{144}$Sm reaction. In contrast, $d^2(E\sigma)/dE^2$ for the $^{16}$O + $^{144}$Sm reaction is smaller at lower energies. To emphasise the difference, the barrier distribution for the $^{16}$O induced reaction has been subtracted from the barrier distribution for the $^{17}$O induced reaction. This difference in the barrier distributions is shown in Fig. 4.6(b), and clearly illustrates the shift in barrier weight from the main peak, $(E_{c.m.} - B_0) \approx -2$ MeV.
FIG. 4.5: The experimental fusion barrier distribution for the $^{170} + ^{144}\text{Sm}$ reaction (solid circles). The dot-dashed line is the calculation using the single-barrier parameters given in Table 4.1. Note that the maximum of the second derivative for this calculation is larger than that for the $^{160} + ^{144}\text{Sm}$ reaction (see Fig. 4.2) because of the difference in the curvatures. The error bars are the random uncertainties only.

To the lower energies, $(E_{c.m.} - B_0) \approx -5$ MeV. Since in these two reactions, the only possible coupling mechanisms are weak couplings, barrier weight at energies below the main barrier can only arise from coupling to a positive Q-value transfer reaction [8]. Coupling to negative Q-value channels will push the main barrier to lower energies relative to the single barrier, but it cannot redistribute weight from the main barrier to lower energies.

To summarise the results thus far, the distribution of fusion barriers has been measured for the $^{16,170} + ^{144}\text{Sm}$ reactions. Both distributions show a double-peaked structure consistent with a negative Q-value reaction process, such as the excitation of states in $^{144}\text{Sm}$. The barrier distribution for the $^{170} + ^{144}\text{Sm}$ reaction differs from the $^{160} + ^{144}\text{Sm}$ barrier distribution since weight from main barrier has been redistributed to energies below the main barrier. This is evidence for the effects of a positive Q-value reaction on fusion. It is worth stressing here, that without resort to further calculations, evidence for the presence and relative importance of certain couplings can be inferred directly from the comparison of these fusion barrier distributions. In Chapter 5, with the aid of a variety of fusion models, it is shown that theoretical calculations support the above assignments to inelastic and transfer couplings.
FIG. 4.6: (a) A comparison between the barrier distributions for the $^{16}$O + $^{144}$Sm (open squares) and the $^{17}$O + $^{144}$Sm (closed circles) reactions. The error bars for the $^{16}$O + $^{144}$Sm reaction are for the somewhat arbitrary uncertainties of ±1% above $E_{\text{c.m.}} = 63.0$ MeV. The error bars on the $^{17}$O + $^{144}$Sm reaction are from the random uncertainties only. They are shown with horizontal cross-bars to distinguish them from the $^{16}$O + $^{144}$Sm reaction. (b) The difference between the $^{17}$O + $^{144}$Sm and $^{16}$O + $^{144}$Sm barrier distributions. This difference plot reveals the difference in the shape of the two barrier distributions at energies below $B_0$. 
4.3 The $^{16}\text{O} + ^{208}\text{Pb}$ reaction

Both the ER and fission cross-sections for the $^{16}\text{O} + ^{208}\text{Pb}$ system have been measured on several occasions. Whilst the fission cross-section measurements [103,34,41,27] have been in reasonable agreement with each other, the ER cross-sections have proved difficult to determine accurately. The measurements of Hartel [40] gave up to four times the peak cross-section of the data of Vulgaris et al. [41]. The latter experiment used a recoil mass selector to filter the ERs from the beam scattered particles, whereas Hartel determined $\sigma_{ER}(E)$ from the $\alpha$-decay of the residues. Recently, Brinkmann et al. [42] have re-measured the ER excitation function using electrostatic deflection and Si surface-barrier detectors, obtaining cross-sections that are larger still, with their maximum value around four times the maximum of Hartel [40]. It is frustrating that such measurements vary so widely. Because of the nature of the $^{16}\text{O} + ^{208}\text{Pb}$ reaction, with both nuclei having closed shells, and with competition between both the ER and fission modes, this system is important in the study of many aspects of the fusion and fission processes [104,20,105]. The magnitude of the ER cross-section is very important in defining the temperature of the fissioning system at its saddle point [91]. In order to more accurately define the fusion excitation function for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction, both the ER and fission cross-sections have been re-measured in detail. The results of these measurements and comparisons with the previous data are discussed below.

The evaporation residue cross-section

The results of the $\alpha$-decay technique are presented in terms of the cross-sections for each individual evaporation channel and total ER cross-sections. The cross-sections for each evaporation channel, obtained from the fits to the $\alpha$-spectrum, are shown in Fig. 4.7 in terms of their fraction of the total cross-section at each energy $E_{lab}$. The cross-sections for the $xn$-channels, Fig. 4.7(a), are typically an order of magnitude greater than those for the $\alpha xn$- and $pxn$-channels, Fig. 4.7(b). These cross-sections are compared to a statistical model calculation in Fig. 4.8, where the ratio of each $xn$-channel to the sum of all $xn$-channels is plotted. The statistical model code ALERT1 [75], modified to treat the fusion angular momentum distributions in a more realistic way, was used for this comparison. The parameters of the ALERT1 calculation were $k_f = 1.14$, where $k_f$ scaled the RLDM fission barriers (see Chapter 2), $a_f/a_n = 1.00$ and $a_n = A/9$ MeV$^{-1}$. These parameters where obtained by fitting total ER cross-section at one energy,
FIG. 4.7: (a) The ER cross-sections as a fraction of the total ER cross-section, for the xn-evaporation channels in the $^{16}$O + $^{208}$Pb reaction obtained by fitting the $\alpha$-particle energy spectra. (b) Same as in (a) for the $\alpha$xn- and pxn-channels. The lines guide the eye.

$E_{\text{lab}} = 87.8$ MeV. These parameters were kept constant when calculating the cross-sections at the other energies. For ratios greater than 10%, the ALERT1 calculation reproduces the behaviour of the xn-channels well, particularly the cross-over energies between 2n- and 3n-, and 3n- and 4n-evaporation channels. However, the ALERT1 calculation underpredicts the cross-sections for the $\alpha$xn- and $\alpha$3n-channels, by up to an order of magnitude. This is possibly due to incorrect values for the optical model transmission coefficients for $\alpha$-emission in ALERT1.

The total ER cross-sections were obtained by summing each evaporation channel in Fig. 4.7. The result is shown by the triangles in Fig. 4.9(a). As mentioned in Chapter 3, the fitting procedure generated unrealistic fluctuations in the cross-sections for the weaker $\alpha$xn- and pxn-channels, see Fig. 4.7. These fluctuations were reduced by fitting the total number of events in each $\alpha$-spectrum divided by the smoothed $\alpha$-multiplicity, as described in Section 3.1.2. The results from this second analysis technique are shown in Fig. 4.9(a) by the open circles. The consistency between the two techniques for obtaining $\sigma_{\text{ER}}(E)$ is il-
FIG. 4.8: The ratio of the ER cross-sections for each $x_n$-channel to the sum of all $x_n$-channels. The symbols are as defined in Fig. 4.7(a). The lines are these ratios calculated from the statistical model code ALERT1; broken line (2n), solid line (3n), dotted line (4n) and dot-dashed (5n) channels. For ratios greater than 10%, the agreement between the experiment and statistical model is very good. Note also that the ALERT1 calculations reproduce reasonably well the cross-over energies of the $x_n$-evaporation channels.

illustrated in Fig. 4.9(b), where the ratio of the two excitation functions is plotted. The smoothed cross-sections are typically 2% larger than those obtained from the fitting procedure, perhaps reflecting the effects of weak evaporation channels omitted in the fitting process, or imperfect matching of the peak shapes. The smoothed excitation function, denoted by the open circles in Fig. 4.9(a), was the one adopted in this work.

The ER excitation function in Fig. 4.9(a) is distinguished by a dip in $\sigma_{ER}(E)$ at an energy around $E_{lab} = 88$ MeV. At energies above 88 MeV, the cross-section rises again to peak at 94 MeV. This structure arises from the relative strengths of the 3n- and 4n-channels, and the increasingly significant contribution of the $\alpha 3n$ channel at the higher energies. Similar structure in the ER excitation function is also predicted in the statistical model calculations of ALERT1. The solid line in Fig. 4.10(a) is the total ER cross-section from ALERT1. Although the ALERT1 calculations and the data disagree in magnitude, qualitatively the model calculations produce the dip and rise in the excitation function. The peak at 94 MeV is not as pronounced in the ALERT1 calculation because it underpredicts the cross-sections for the $\alpha x_n$-channels. To check these features in $\sigma_{ER}(E)$, additional calculations were performed with the Monte Carlo evaporation code PACE2 [76]. The same parameters used in the ALERT1 calculation were also used for PACE2. To demonstrate that the 3n- and 4n-channels are largely responsible for the structure, the results for the PACE2 calculation are given as the sum of
FIG. 4.9: (a) The ER excitation functions from the two analyses. The sum of each evaporation channel as determined from the peak fitting analysis of the $\alpha$-spectrum (triangles), and the smoothed excitation function obtained using the total $\alpha$-yield and the $\alpha$-decay multiplicities (open circles). The latter was adopted for this work. Note the linear scale. (b) The ratio of the adopted to the peak fitted excitation functions. This demonstrates that the fitting procedure accounts for all the evaporation channels down to the $\approx 2\%$ level (broken line).

the 3n- and 4n-channels only, as shown by the dotted line in Fig. 4.10(a). Again, the dip is qualitatively reproduced. Further calculations were performed with ALERT1, for energies greater than those measured here. Structure in $\sigma_{\text{ER}}(E)$ was also observed, so it would be interesting to extend these detailed measurements of $\sigma_{\text{ER}}(E)$ to higher bombarding energies.

In Fig. 4.10(b), the results from the $\alpha$-decay technique are compared with the cross-sections obtained from previous measurements for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction [41,40,42]. The cross-sections from this work and the data of Brinkmann et al. [42] are in reasonable agreement, confirming that the previous measurements of Vulgaris et al. [41] and Hartel [40] were too low. The reasons are not clear, but could be associated with the uncertainty in the efficiency of the recoil mass selector for the experiment of Vulgaris et al.. Or, in the case of the measurement of Hartel, a stopper foil that was too thin.
As mentioned in Section 3.1.1, an initial attempt at measuring $\sigma_{ER}(E)$ was made with the compact velocity filter arrangement. The results from this measurement were found to be around 15% below the cross-sections from the $\alpha$-decay technique. This is most likely because of the difficulty in resolving the ERs from slit scattered particles and other contaminants in the $\Delta E$-TOF spectra. The velocity filter method also relies on knowledge of the behaviour of $R(E)$, Eq. (3.7) in Chapter 3. The structure observed in $\sigma_{ER}(E)$ meant the theoretical calculations of $R(E)$ varied strongly with energy, making it difficult to extract reliable interpolated cross-sections. This is why the $\alpha$-decay technique was used, since with this method the cross-sections are obtained at each energy without reliance on the ratio $R(E)$. 
The fission cross-section

The fission cross-sections for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction were obtained by integrating fitted angular distributions, multiplied by $2\pi \sin \theta$, over all angles from $90^\circ$ to $180^\circ$. An example of the fission fragment angular distributions was shown in Fig. 3.16, Chapter 3. The fission cross-sections are shown in Fig. 4.11, where each open symbol represents one of the three passes through the fission excitation function. Also shown are the previous measurements of $\sigma_{\text{fis}}(E)$ for this system [103,34,41,27]. There is good agreement between this work (open circles) and the previous measurements (solid points), except for the data of Murakami et al. [27], where an energy shift of $-0.7$ MeV was required to obtain agreement with the other data sets.

**FIG. 4.11:** The fission cross-sections from this work (open symbols) as a function of $E_{\text{lab}}$. Also shown are the data of Vulgaris et al. [41] (solid squares), Videbaek et al. [103] (solid triangles), Back et al. [34] (inverted solid triangle), and Murakami et al. [27] (solid circles). The data of Murakami et al. has been offset by $-0.7$ MeV.
Table 4.2: The nuclear potential parameters from the fits to the high energy region for the $^{16}$O, $^{28}$Si + $^{208}$Pb reactions. Also shown are the values for the single barrier, its position and its curvature, for the s-wave.

<table>
<thead>
<tr>
<th>Reaction</th>
<th>V(r) (MeV)</th>
<th>a (fm)</th>
<th>$B_0$ (MeV)</th>
<th>$R_B$ (fm)</th>
<th>$\hbar \omega_0$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}$O + $^{208}$Pb</td>
<td>$-697.2$ (0.8 fm)</td>
<td>1.05</td>
<td>74.62</td>
<td>11.5</td>
<td>3.91</td>
</tr>
<tr>
<td>$^{28}$Si + $^{208}$Pb</td>
<td>$-544.2$ (0.8 fm)</td>
<td>1.23</td>
<td>128.1</td>
<td>11.5</td>
<td>3.55</td>
</tr>
</tbody>
</table>

The total fusion cross-section for the $^{16}$O + $^{208}$Pb reaction was obtained by summing $\sigma_{\text{fit}}(E)$ shown in Fig. 4.11, and $\sigma_{\text{ER}}(E)$ from the $\alpha$-decay technique (Fig. 4.10). At energies where the ER cross-section was not available, an interpolated value was used. This procedure gave a negligible additional uncertainty in the fusion cross-sections, since the ER cross-sections are well defined, and represent a small fraction, decreasing with increasing energy, of the fusion cross-section. The fusion excitation function is shown in Fig. 4.12(a). The cross-sections for energies $E_{\text{c.m.}}/B_0 > 1.06$ were fitted with a single barrier and the potential parameters from this fit are given in Table 4.2. The broken line in Fig. 4.12(a) is the excitation function for the single barrier calculation.

The distribution of fusion barriers was obtained from the fusion excitation function by evaluating the point difference formula, Eq. (2.51) in Chapter 2, using an energy step of 1.86 MeV in the c.m. frame. The barrier distribution is shown in Fig. 4.12(b), for each of the three passes through the excitation function. The uncertainties in the fusion cross-sections were ±1%, or, at the lowest beam energies, as given by the counting statistics, whichever was larger. The point denoted by the diamond in Fig. 4.12(b) at ≈71 MeV was evaluated using one cross-section from the measurement of Murakami et al. [27], shifted by −0.7 MeV, and two data points from this work. Although the points in the barrier distribution scatter, particularly at the higher energies, the overall shape of the distribution is well defined. The broken line is the barrier distribution from the single barrier calculation which is a very poor representation of the experimental data. The measured barrier distribution demonstrates that there is considerable weight at energies above $B_0$ and that there is also evidence for some barrier weight at the lower energies. The experimental barrier distribution is compared to model calculations in Chapter 5.
FIG. 4.12: (a) The experimental fusion excitation function obtained from summing the ER and fission excitation functions. Each symbol represents one of the three different passes through the excitation function. The broken line is the single barrier calculation with the parameters given in Table 4.2. (b) The distribution of fusion barriers for the $^{16}$O + $^{208}$Pb reaction. The symbols are defined in panel (a). The energy step for calculating the second derivative is 1.86 MeV. The diamond was obtained using the cross-section from the data of Murakami et al. [27], shifted by −0.7 MeV. The single barrier calculation does not fit the data; the maximum of the second derivative is at 1200 mb/MeV for an energy of 74.6 MeV.
4.3.1 Fission fragment anisotropies

Fission fragment angular distributions for the $^{16}$O + $^{208}$Pb data, obtained using Eq. (3.23) in Chapter 3, are shown in Fig. 4.13(a). The error bars in the data points are the statistical uncertainties. The data in Fig. 4.13(a) were taken from the second of the two experimental runs. The angular distributions are very well defined. In Fig. 4.13(b), the anisotropies corresponding to the angular distributions are shown. The anisotropies show a linear dependence with energy, although the two data points at $E_{\text{beam}} = 86.5$ and 79.5 MeV are high. However, these two points are consistent within one standard deviation.

In Fig. 4.13(c), the angular distributions from run two are overlayed with the angular distributions obtained in a separate experiment, labelled run one. The run one data provided the first excitation function measured after the fission fragment spectrometer became operational. The corresponding anisotropies are shown in Fig. 4.13(d). The run one data show more irregularities in the energy dependence of the differential cross-sections at all angles. These irregularities are particularly evident for the higher beam energies, see Fig. 4.13(c). The angular distributions have been re-analysed; there was no loss in the fission yield in the $\Delta E$-TOF analysis. Thus, the fluctuations may be related to rate or noise problems during the collection, or may simply reflect less familiarity with the fission fragment detectors for this inaugural run. The systematic differences in the angular distributions are responsible for a small 'stagger' in the fission cross-sections between the run one and run two data (see Fig. 4.11). This difference is small, around 3%, but because of it, experimental runs are not 'mixed' for the evaluation of the barrier distribution.

The irregularities in the energy dependence of the angular distributions, and hence the anisotropies, arise from a shift in the value of $A(\theta_{\text{lab}})$ obtained during the calibration run and the bin normalisation applicable during the fission measurements. However, the effect of the change in bin normalisations is not much larger than would be expected simply on the basis of the statistical error bars shown, and an additional ±3% uncertainty would encompass this scatter.

The fission fragment anisotropies from this work are compared with the previous work of Back et al. [34], Vulgaris et al. [41], and Murakami et al. [27] in Fig. 4.14. The overall agreement is reasonable. However, the previously measured anisotropies at $E_{\text{c.m.}} = 71$ and 72 MeV, which are at energies below the single barrier, are $\approx 10\%$ larger than the present measurements. This may be already responsible for some of the suggested anomaly [27]. This is discussed further in Chapter 5.
FIG. 4.13: (a) The fission fragment angular distributions for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction plotted as a function of the centre-of-mass scattering angle. (b) The fission fragment anisotropies obtained from the fits to the angular distributions in (a). The error bars are from the statistical uncertainties. The results for a different experimental run are shown in panels (c) and (d). There are systematic differences between the two runs at the level of $\approx 3\%$. 

EC.m. (MeV)
FIG. 4.14: The experimental fission fragment anisotropies for the $^{16}$O + $^{208}$Pb reaction as a function of $E_{c.m.}$. Each open symbol represents one of the three passes through the excitation function. The solid points are the results of previous measurements. The data of Murakami et al. [27] have been offset by -0.7 MeV. An anisotropy from Murakami at 73.8 MeV is not shown since it falls exactly on the point represented by the triangle at this energy.

4.4 The $^{28}$Si + $^{208}$Pb reaction

The fission excitation function for the $^{28}$Si + $^{208}$Pb reaction was obtained in the manner of the $^{16}$O + $^{208}$Pb reaction. The ER cross-sections are negligible for this reaction, so the fusion cross-sections are equated with the fission cross-sections. The results are shown in Fig. 4.15(a) by the circular and square data points, which represent two independent passes through the excitation function. A third pass was made but because of problems with the normalisation of the data, the total fission cross-sections could not be extracted. This problem did not affect the anisotropies, see Section 4.4.1 below. In Fig. 4.15(a) the triangles are from the previous measurement of Back et al. [34]. There is a significant difference in the cross-sections of Ref. [34] and those from this work. This may be due to uncertainty in the beam energy calibration for the measurements of Ref. [34]. The broken line in Fig. 4.15(a) is the excitation function from the calculation assuming a single barrier with the s-wave parameters in Table 4.2.
The distribution of barriers, calculated with a step length of $\Delta E_{\text{c.m.}} = 3.53$ MeV, is shown Fig. 4.15(b). The uncertainties on the fusion cross-sections were ±1%. The step length is larger than that used in the $^{16}\text{O} + ^{208}\text{Pb}$ reaction and this can be justified as follows. The spread in the barrier heights, $\Delta B_\alpha$, is defined as difference between the maximum and minimum eigenbarriers in the barrier distribution [49]. This is a measure of the overall width of the barrier distribution. The coupled-channels picture gives rise to a set of eigenbarriers with heights $B_\alpha = B_0 + \lambda F(r)$, and so the spread in eigenbarriers $\Delta B_\alpha$ is proportional to the coupling strength $F(r)$, Eq. (2.33) in Chapter 2. Considering the nuclear term in $F(r)$, then

$$\Delta B_\alpha \propto \beta R \frac{dV_n(r)}{dr}. \quad (4.5)$$

Since at the barrier $dV_n(r)/dr = 1.44Z_1 Z_2 / R^2$, the spread in the eigenbarriers, and hence the width of the barrier distribution, increases with $Z_1 Z_2$ [15,49]. Thus, the effect of the $^{28}\text{Si}$ in the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction is to ‘magnify’ the features due to the $^{208}\text{Pb}$ in the $^{16}\text{O} + ^{208}\text{Pb}$ reaction.

To determine the magnitude of this effect is somewhat more difficult. However, an estimate can be made by assuming that the $^{28}\text{Si}$ projectile is spherical and that there are only two channels to couple to. The spread in eigenbarriers is then given by

$$\Delta B_{21} = (\lambda_2 - \lambda_1) F(r), \quad (4.6)$$

where $\lambda_\alpha$ are the eigenvalues of the two-channel problem. In a calculation with 3-channel in $^{208}\text{Pb}$, $\Delta B_{21}$ for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction is 1.5 times $\Delta B_{21}$ for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction. This implies that since the $^{16}\text{O} + ^{208}\text{Pb}$ barrier distribution was determined with a step length of 2.0 MeV (lab), to produce a barrier distribution with an equivalent amount of smoothing, a step length 3.0 MeV (lab) could be used for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction. It is likely that the structure of the $^{28}\text{Si}$ projectile adds additional smoothing. Thus a step length of 4.0 MeV in the laboratory frame was used. This procedure should not reduce the ability to resolve features present in the experimental barrier distribution. All corresponding theoretical calculations for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction were also evaluated with the step length of 4.0 MeV (lab).

Again, the single barrier calculation shown in Fig. 4.15 fails to reproduce the experimental data. This barrier distribution differs quite markedly from the $^{16}\text{O} + ^{208}\text{Pb}$ distribution, indicating that the $^{28}\text{Si}$ projectile is playing a significant role in the fusion reaction. The extent of the projectile’s role in the fusion process, and model comparisons with the $^{16}\text{O} + ^{208}\text{Pb}$ data are made in Chapter 5.
FIG. 4.15: (a) The experimental fusion excitation function for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction over the full energy range. The squares and circles represent two separate measurements of the excitation function. The triangles are the data of Back et al. [34]. The broken line is the single barrier calculation with the parameters given in Table 4.2. (b) The distribution of fusion barriers for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction, displayed over a narrower energy range than in (a). The energy step here is 3.53 MeV. The single barrier calculation peaks at 880 mb/MeV for an energy of 128.1 MeV.
4.4.1 Fission fragment anisotropies

Fission fragment angular distributions for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction are shown in Fig. 4.16. In this example, the angular distributions are for energies that range from $E_{\text{beam}} = 142.0$ to 178.0 MeV in steps 4.0 MeV. The broken lines in Fig. 4.16 are the fits to the angular distributions using the procedure described in Section 3.2.3. In reactions where quasi-fission is significant, this fitting procedure may not be appropriate for modelling the asymmetric angular distributions observed [37]. Inspection of the fits to the angular distributions for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction showed that the procedure used here gave a good description of the data, and this allowed the anisotropy to be well defined.

The fission fragment anisotropies for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction are shown in Fig. 4.17. The anisotropies determined from the 'singles' measurement in the back counter alone are shown by the open symbols. The open squares correspond to singles data where the fission cross-sections were obtained, and the open triangles represent the measurement which had no absolute normalisation for the cross-sections. The solid circles are the anisotropies obtained from the coincidence experiment. Although the statistics are worse for the coincidence measurement, these anisotropies should be more reliable than the singles data at the lower beam energies because of the better fission fragment identification. As show in Fig. 4.17, the singles data (open symbols) have anisotropies smaller than the coincidence data (solid circles). Hence, below $E_{\text{c.m.}} \approx 126$ MeV only the coincidence data should be used, whilst above this energy all points should be reliable.

Also shown in Fig. 4.17 are three data points from Back et al. [34]. The agreement between these two data sets is satisfactory. The anisotropies for both the $^{28}\text{Si} + ^{208}\text{Pb}$ and $^{16}\text{O} + ^{208}\text{Pb}$ reactions fall monotonically as a function of the beam energy. However, there are quantitative differences in anisotropies for both reactions. The $^{28}\text{Si} + ^{208}\text{Pb}$ data are more anisotropic at $E_{\text{c.m.}}/B_0 = 1.10$ than the $^{16}\text{O} + ^{208}\text{Pb}$ reaction. The anisotropies for the $^{28}\text{Si}$ induced reaction rise more rapidly with beam energy than the $^{16}\text{O}$ induced reaction. The fission fragment anisotropies for both reactions are compared to transition state model calculations in the next Chapter.
FIG. 4.16: The fission fragment angular distributions for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction plotted as a function of the centre-of-mass scattering angle.

FIG. 4.17: The experimental fission fragment anisotropies for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction. The open symbols are from the singles measurements and the solid circles represent the anisotropies for the coincidence measurement. The solid squares are the results of a previous measurement from Back et al. [34].
Chapter 5

DISCUSSION

As shown in Chapter 4, the barrier distributions for each reaction are different and their shapes clearly reveal the influence on fusion of channels with both positive and negative Q-values. In this Chapter, theoretical models which incorporate various coupling schemes are compared with the measured barrier distributions in order to gain further insight into which channels are dominant.

The study of the fusion process using barrier distributions also has implications for the parallel study of fusion angular momentum distributions. It has been proposed by several authors [11, 106–108] that a fusion model which successfully reproduces the energy dependence of the fusion cross-sections should also correctly predict the angular momentum distributions. Recently [109], the precise measurements of the fusion cross-sections for the $^{16}$O + $^{154}$Sm reaction have been used to obtain the angular momentum distributions through a 'mapping' procedure. The mean angular momentum ($\langle l \rangle$) obtained using this technique was found to be consistent with the values of $\langle l \rangle$ measured using the ground-state rotational band populations for the 4n-evaporation channel [24]. This mapping procedure is equivalent to fitting the measured $\sigma(E)$ with a fusion model and then calculating the angular momentum distribution with this model. Thus, precise measurements of the fusion excitation function allow a determination of the distribution of fusion barriers in a reaction, and if both the cross-sections and barrier distributions can be interpreted with a fusion model, then the method can be used to obtain angular momentum distributions.

In this Chapter, the precisely measured $\sigma(E)$ for the $^{16}$O + $^{208}$Pb reaction are used to obtain the angular momentum distributions for fusion. These results
are then used to test the fission fragment angular distribution technique as a method for determining the mean-square angular momentum for fusion. It will be shown that there is agreement between the fusion and fission models for the $^{16}O + ^{208}Pb$ reaction, but, because of doubts in some parameters in the fission model, and in the experimental effort in obtaining them, the fission fragment angular distribution technique is a more involved method of obtaining information on angular momentum distributions compared to measuring fusion excitation functions. Furthermore, using the $^{28}Si + ^{208}Pb$ results, it will be shown that the applicability of the fission fragment angular distribution technique is restricted to reactions that are free from contamination of quasi-fission. It will be argued that in reactions where quasi-fission is significant, it is still valid to obtain the barrier distribution, and hence the angular momentum distribution associated with the formation of the composite system.

Each of the four reactions is discussed in turn below.

5.1 The $^{16,17}O + ^{144}Sm$ reactions

As discussed in Chapter 2, the simplified coupled-channels model of Dasso, Landowne and Winther [8,9] has been widely used in calculations of fusion excitation functions. The codes CCFUS [54] and CCDEF [55] are based on this eigenchannel representation, where the equations of relative motion are decoupled by treating the coupling form factor as constant in the region of the single barrier. The relative simplicity and negligible time for the computations (compared to exact coupled-channels calculations) have made this code a standard and popular tool for exploring the contributions to enhanced fusion cross-sections. In the calculations that follow, most of the comparisons with the experimental barrier distributions are made using the code CCMOD [56], a modified version of CCDEF. As explained in Chapter 2, CCMOD includes the excitation energies in the coupling matrix in a similar manner to CCDEF, but the matrix is diagonalised at each value of the inter-nuclear separation $r$, not at the single barrier radius.

For selective comparisons, the more exact coupled-channels code of Kruppa and Rowley [110] was used. This approach is based on the framework of direct reaction theory [111,112], where reaction channels which are not included explicitly in the model are described by an imaginary term in the potential. A comparison is also made with the exact coupled reaction channels (CRC) code FRESCO [113]. The code FRESCO can calculate the transfer channels explicitly in addition to the elastic, inelastic and fusion channels. The addition of coupling to the transfer
channels adds considerably to the complexity of the coupled-channels problem, and dramatically increases the immensity of the computation. Consequently, calculations were only performed for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction, and transfer was not included in the FRESCO calculations.

5.1.1 The $^{16}\text{O} + ^{144}\text{Sm}$ reaction

The experimental distribution of fusion barriers for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction is characterised by two Gaussian-like peaks almost resolved from each other. This feature, which is also present in the $^{17}\text{O} + ^{144}\text{Sm}$ reaction, strongly suggests that the double-peaked barrier distribution is due to coupling to negative Q-value channels associated with inelastic states in $^{144}\text{Sm}$. The effect of coupling to the inelastic channels was modelled with the code CCMOD. The potential parameters used for these calculations were those determined from the single-barrier fits to the high energy data, as given in Table 4.1 of Chapter 4. The spectroscopic parameters for the inelastic states of the $^{144}\text{Sm}$ nucleus were obtained from the tabulated values in Refs. [114,115]. No optimisation of the parameters was performed, and since the potential parameters have been determined by the single barrier fit to the high energy data, the following calculations were in this sense parameter free. The spin and parity of each state, its excitation energy, and the ground-state transition strengths are listed in Table 5.1. The second last column in Table 5.1 gives the deformation parameters which are evaluated using

\[
\beta_\lambda = \frac{4\pi}{3Z R^2} \left[ \frac{B(E\lambda)}{e^2} \right]^{1/2},
\]

where $R = 1.06A^{1/3}$ fm. The choice of 1.06 fm for the nuclear radius parameter is discussed later. In all the following calculations, the Coulomb and nuclear deformation parameters are taken to be equal, as in Ref. [54].

The $3^-$ inelastic channel in $^{144}\text{Sm}$ is expected to be the most important channel since it has the largest $B(E\lambda)$ $\uparrow$. The two-channel CCMOD calculation, with coupling between the elastic channel and the $3^-$ inelastic channel in $^{144}\text{Sm}$, splits the single barrier into two separate eigenbarriers. The weights $w_\alpha$ and barrier heights $B_\alpha$ are given in Table 5.2, where $R_\alpha$ are the radii of the shifted barriers. The barrier distribution from this CCMOD calculation is shown in Fig. 5.1(a) by the broken line, with the spherical calculation for comparison. The theoretical calculation with coupling to the single $3^-$ phonon gives a reasonably good reproduction of the shape of the measured barrier distribution.
FIG. 5.1: (a) The distribution of barriers from the theoretical CCMOD calculation with coupling to the $3^-$ state in $^{144}$Sm only (broken line). The dotted line is the calculation for the single barrier, that is, no coupling. (b) The cross-sections for two-channel calculation with the $3^-$ state (broken line) is compared with the single barrier excitation function (dotted line). Coupling to the $3^-$ state in $^{144}$Sm provides a reasonably good representation of the barrier distribution and the fusion cross-sections. The theoretical barrier distributions were derived from the calculated excitation functions with the same energy step as the experimental distributions.
Table 5.1: The spin and parity, excitation energy, and reduced transition strengths of the states considered in the coupled-channels calculations. Except where indicated, the deformation parameters were calculated with the nuclear radius parameter of \( r_0 = 1.06 \, \text{fm} \). The states in \( ^{16}\text{O} \) are also given here. The effects of coupling to the projectile channel is discussed in a later Section.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>( J^\pi )</th>
<th>( E^* ) (MeV)</th>
<th>( B(E\lambda) \uparrow )</th>
<th>( \beta_\lambda )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>( ^{144}\text{Sm} )</td>
<td>2(^+)</td>
<td>1.660</td>
<td>0.266 e(^2)b(^2)</td>
<td>0.113</td>
<td>Ref. [114]</td>
</tr>
<tr>
<td></td>
<td>3(^-)</td>
<td>1.810</td>
<td>0.27 e(^2)b(^3)</td>
<td>0.205</td>
<td>Ref. [115]</td>
</tr>
<tr>
<td>( ^{16}\text{O} )</td>
<td>2(^+)</td>
<td>6.917</td>
<td>0.0040 e(^2)b(^2)</td>
<td>0.362(^\dagger)</td>
<td>Ref. [114]</td>
</tr>
<tr>
<td></td>
<td>3(^-)</td>
<td>6.130</td>
<td>0.0015 e(^2)b(^3)</td>
<td>0.733(^\dagger)</td>
<td>Ref. [115]</td>
</tr>
</tbody>
</table>

\( \dagger \) calculated with \( r_0 = 1.2 \, \text{fm} \).

Table 5.2: The barrier heights and their weights for the two- and three-channel coupling schemes calculated using CCMOD. The coupling here refers to the inelastic channels in the target only. The eigenradii are given in the last column.

<table>
<thead>
<tr>
<th>State</th>
<th>( \alpha )</th>
<th>( B_\alpha ) (MeV)</th>
<th>( w_\alpha )</th>
<th>( R_\alpha ) (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3(^-) only</td>
<td>1</td>
<td>64.46</td>
<td>0.27</td>
<td>10.3</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>60.28</td>
<td>0.73</td>
<td>11.0</td>
</tr>
<tr>
<td>3(^-,) 2(^+)</td>
<td>1</td>
<td>64.78</td>
<td>0.287</td>
<td>10.2</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>62.78</td>
<td>0.0007</td>
<td>10.7</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>60.20</td>
<td>0.711</td>
<td>11.1</td>
</tr>
<tr>
<td>2(^+) only</td>
<td>1</td>
<td>63.12</td>
<td>0.13</td>
<td>10.6</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>60.85</td>
<td>0.87</td>
<td>10.9</td>
</tr>
</tbody>
</table>

The coupling scheme that best reproduces the experimental distribution of barriers should also reproduce the magnitude of the fusion cross-sections. The fusion cross-sections from the above CCMOD calculations are shown in Fig. 5.1(b), and are in good agreement with the experimental data. The calculated fusion cross-sections were always checked against the experimental ones, although in the comparisons that follow only the barrier distributions are illustrated.

The inclusion of the 2\(^+\) state as well as the 3\(^-\) state in the CCMOD calculation does not alter the theoretical barrier distribution dramatically. The three-channel calculation produces three eigenbarriers, as shown in Table 5.2, but the weight of
the third barrier is very small. The reason why the weight of the third barrier is so small can be understood in terms of the solution to the eigenchannel problem which involves only coupling between the elastic and inelastic channels. There is no coupling between inelastic channels. The coupling matrix (see Section 2.1.4 in Chapter 2) for the three channel case is given by

\[
M = \begin{pmatrix}
0 & \beta_1 & \beta_2 \\
\beta_1 & -Q_1 & 0 \\
\beta_2 & 0 & -Q_2
\end{pmatrix},
\]

where \( \beta_i \) represents the coupling strength in each channel. If the energies of the inelastic states are degenerate, \( Q_1 = Q_2 = Q \), then it can be shown that the solution of the coupled equations consists of three barriers. However, the energy of one of these eigenbarriers is \( Q \) and its weight is always zero. In the equivalent 4-channel problem, the solution returns 4 barriers but two of them have energies \( Q \) and zero weights, and so on. In the 3-channel case, as the 'degree of degeneracy' changes, weight is progressively shared with the third barrier. Since the difference in the energies of the \( 3^- \) and \( 2^+ \) states in \(^{144}\text{Sm} \) is only 150 keV, the weight of the third barrier is very small.

The barrier distribution for the three-channel coupling is shown in Fig. 5.2 by the solid line, in comparison with the previous calculation which included the \( 3^- \) state only, the broken line. The additional coupling pushes the main barrier and the second barrier apart by a small amount, and redistributes weight from the main barrier to the barrier at higher energies. In doing so, the dip in the experimental barrier distribution is, albeit marginally, better reproduced. For the \(^{16}\text{O} \to ^{144}\text{Sm} \) reaction, these calculations show that the \( 3^- \) state is the dominant coupling. This is because the coupling strength due to the \( 3^- \) state is much larger since its deformation parameter is 1.8 times that for the \( 2^+ \) state.

The relative weights of the two barriers, and their separation in energy, are directly related to the coupling strength of the inelastic couplings. As given by Eq. (2.33) in Chapter 2, the coupling strength \( F(r) \) is proportional to both the deformation parameter and the nuclear potential:

\[
F(r) \propto \beta \lambda R \frac{dV_n(r)}{dr}.
\]

Thus, the above calculations depend on \( \beta_\lambda \) and the choice of the nuclear potential. These two factors are discussed in detail below.

If \( B(E\lambda) \) in Eq. (5.1) is taken from the adopted values, as is the case in this
FIG. 5.2: The distribution of barriers from the theoretical CCMOD calculations. The solid line is the barrier distribution with coupling to the $3^{-}$ and $2^{+}$ channels. It is not very different from the barrier distribution with the $3^{-}$ channel only (broken line). The dot-dashed line is the barrier distribution with the $2^{+}$ channel only.

analysis, then $\beta_{A}$ depends only on the choice of the nuclear radius parameter. The radius parameter of $r_{0} = 1.06$ fm used in this work was initially chosen for consistency, since it was used in previous calculations for the reactions $^{16}$O on $^{154}$Sm and $^{186}$W [18,19]. This differs from the value of $r_{0} = 1.2$ fm which should be used in the sharp-cutoff model for converting $B(E\lambda)$ values to deformation parameters [116]. A larger nuclear radius parameter reduces $\beta_{A}$, see Eq. (5.1), and this reduces the separation of the two barriers in the theoretical calculation, altering their weights to give a poorer quality reproduction of the experimental data. Although a value of $r_{0} = 1.06$ fm gives better agreement with the experimental barrier distribution, as shown below, this result is not necessarily suggestive of any underlying physics because there is some doubt in the real nuclear potential.

The nuclear potential used in this analysis was obtained by fitting a single barrier to the high energy fusion cross-sections. Since the fit was done without consideration of the direct reaction channels, the potential obtained depends on the fusion model, and may not necessarily represent the true interaction potential. An example of this can be seen by comparing the values of the potential parameters obtained from analyses of different reaction channels. The diffuseness obtained from the fits to the fusion channel [18,19,57] is often significantly
larger than that obtained from an optical model analysis [104,117,118]. For example, compare $a = 0.84$ fm from the analysis of the $^{16}\text{O} + ^{144}\text{Sm}$ reaction in this work, with $a = 0.3$ fm from the analysis of elastic scattering data by Kittl et al. [117]. A possible explanation for this difference in diffuseness parameters is due to Leigh [17], and is described below.

Dependence of the diffuseness parameter on the model analysis

The difference in the value for the diffuseness of the real potential derived from fusion analyses and those values extracted from elastic scattering data could arise from the differing treatment of the partial waves for each reaction channel. In an optical model analysis, the flux which is reflected from the real potential represents elastic scattering, whilst the imaginary potential represents the absorption of all other flux not included explicitly in the model. When the analysis is concerned with obtaining a good fit to the fusion cross-sections, high $l$-waves, such as those associated with transfer reactions, are prevented from contributing to fusion by reflecting them from the real potential barrier. Use of a large diffuseness gives barriers which increase more rapidly with $l$ and the effect of a larger barrier means that the rejected $l$-waves contribute to the elastic channel. In contrast, the scattering analyses must prevent these same $l$-waves from contributing to the elastic channel. This is achieved by allowing them to pass over a smaller barrier and to be absorbed, contributing to the fusion channel. Thus, it is not clear what is the appropriate nuclear potential to use, since there is a dependence on the type of data that is being analysed. A different value of the diffuseness will alter $dV_{n}(r)/dr$ and could change the coupling strength, $F(r)$. This problem could be pursued by carrying out a global analysis of all reaction channels simultaneously. The potential extracted then should represent the true interaction potential. Such an analysis is beyond the scope of this work.

The inclusion of other channels

Although the simplified coupled-channels calculations provide a good reproduction of the experimental barrier distribution, there are still inadequacies in the above model calculations. The calculation does not reproduce correctly the width of the main peak of the barrier distribution, see Fig. 5.2. Correspondingly, there is too much weight in the theoretical calculation at energies around the main peak and perhaps not enough in the second peak. This could indicate that there are some channels still missing from the coupling scheme. For the $^{16}\text{O} + ^{144}\text{Sm}$
system, channels that could effect the fusion process include: (a) transfer channels (b) projectile excitation and (c) other inelastic channels. The only single-particle transfer reaction with a \(|Q| < 5\, \text{MeV}\) is the proton stripping channel, with \(Q = -1.7\, \text{MeV}\). As shown later, the effect of a negative \(Q\)-value transfer channel in the presence of an inelastic channel is small. Projectile excitation does not have a significant effect in this reaction either; see a later discussion.

The next lowest lying inelastic states in \(^{144}\text{Sm}\) are the \(4^+\) at \(2.19\, \text{MeV}\) and \(2^+\) at \(2.45\, \text{MeV}\). A CCM0D calculation including these channels, and the proton stripping channel, is shown in Fig. 5.3 by the broken line. The coupling strength for the transfer channel was \(\mathcal{K} = 2.8\, \text{MeV}\), a value determined from the fits to the \(^{170}\, +\, ^{144}\text{Sm}\) barrier distribution, as discussed later. The deformation parameters for the two extra states were obtained from the \((\alpha, \alpha')\) scattering measurement of Ref. [119]. The combined effect of these extra channels does alter the shape of the barrier distribution, improving the agreement in the region of the second barrier. Inclusion of higher lying states will have an even smaller effect because of their larger excitation energies and the weakness of their transition strengths.

It can be concluded from the above calculations that the effects from the other channels are small, although not insignificant when combined. However, compared to the dominant \(3^-\) channel, the effects of the extra channels are not too large. Also, an overriding consideration is the effects of the approximations made in the code CCM0D. The data may not support the need for the extra channels if their effects are similar in size to those due to the approximations. This is examined below using the results of exact couple-channels calculations.

Correct treatment of the excitation energies of the excited states

An approximation in the above CCM0D calculations is the treatment of the excitation energies of the inelastic states. In the eigenchannel representation, on which the code CCFUS is based, the excitation energies of the inelastic states are treated in an approximate manner in order to facilitate decoupling of the coupled-channels equations. As discussed in Chapter 2, the excitation energy is included explicitly in the coupling matrix and the form factors are assumed to vary in the region of the barrier. The coupled equations are then solved using the unitary matrix that was used to solve the equations for the zero excitation energy case. As the excitation energy increases, the approximation will gradually worsen. This approximate treatment of the excitation energies in CCM0D is expected to be valid for excitation energies \(< 2\, \text{MeV}\) [120].

To examine the approximate treatment of the excitation energies, the results
FIG. 5.3: The distribution of barriers calculated with the $3^-$ and $2^+$ states only (solid line) and also with 2 extra inelastic channels and the proton transfer channel (broken line). The 5-channel calculation improves the agreement in the region of the second barrier, but the area of the main barrier is still not correct.

of the CCM0D calculation are compared to the barrier distribution calculated using more exact codes. The main difference between CCM0D and the more exact codes is in the approach to solving the coupled-channels equations. The exact codes solve the coupled-channels equations by means of numerical integration. The reaction cross-section is given by

$$
\sigma_{\text{react}}(E) = \sigma_{\text{inel}}(E) + \sigma_{\text{abs}}(E),
$$

(5.3)

where $\sigma_{\text{abs}}(E)$ is the absorption cross-section, which is all the flux that is not included in the inelastic and elastic reactions. What the absorption term describes depends upon whether or not the code includes the transfer channels explicitly. To model the absorption of the other reaction channels not included in the coupled-channels space that accounts for the elastic and inelastic reactions, an imaginary term in the potential may be introduced. In the code of Kruppa and Rowley [110], transfer reactions are not included explicitly and so $\sigma_{\text{abs}}(E) = \sigma_{\text{fus}}(E) + \sigma_{\text{tran}}(E)$. Also, their code calculates the fusion cross-section using an ingoing-wave boundary condition (IWBC). At a radius that lies well inside the fusion barrier radius,
it is assumed that there are only ingoing waves, see Eq. (2.23) in Chapter 2. In this case, an explicit imaginary term in the optical potential is redundant. The code FRESCO differs in its approach in that it can include the transfer reactions explicitly [104], and so in Eq. (5.3) $\sigma_{\text{abs}}(E) = \sigma_{\text{fus}}(E)$. The depth of the imaginary term in the potential is adjusted to simulate the IWBC. The approach of FRESCO differs from some other methods which describe transfer reactions using a short-range or surface imaginary term [117] or two imaginary terms [121,122], an 'inner' term and a more peripheral term.

The complete coupled-channels codes solve the coupled equations treating the excitation energies of the inelastic channels exactly. Both codes include the effects of coupling to all orders in the inelastic form factor $F(r)$. In contrast, CCMOD truncates this expansion to include only the first term, $dV_{\text{n}}/dr$. Both CCMOD and the code of Kruppa and Rowley solve the coupled equations in the isocentrifugal approximation, and in FRESCO there is a choice of either using this approximation or solving the equations without it. Although the simple eigenchannel picture is lost when using the more exact calculations, it is shown below that it is still instructive to interpret the distribution of fusion barriers in this picture.

The barrier distribution calculated using the code of Kruppa and Rowley is shown by the solid line in Fig. 5.4. Also shown is the barrier distribution for $^{16}$O + $^{144}$Sm as determined by the approximate code CCMOD (dashed line). Both calculations included the $2^+$ and $3^-$ inelastic states in $^{144}$Sm, given in Table 5.1. The potential parameters for both these calculations were equal to each other but slightly different to those optimum parameters given in Table 4.1. This is not important since the difference in $B_0$ and $R_B$ is small and it is the comparison between the two codes that is of interest here. The exact code of Kruppa and Rowley also reproduces the main features of the experimental barrier distribution. The main peak of the barrier distribution is larger than the CCMOD calculation and the second peak is pushed to slightly larger energies, with better reproduction of the 'dip' between the two barriers. Qualitatively the barrier distribution is similar to the CCMOD calculation, but they do differ in detail.

In the eigenchannel approach, the energies of the excited states play a major role in determining the relative weights of the barriers. A two-channel calculation, with the excitation energy of the state set to zero, yields two barriers with equal weights. This implies that the inelastic state is excited as easily as the ground state. When the excitation energy is included, the weight of the second barrier decreases, and the separation in energy between the barriers increases. The separation and weight of the two barriers from the CCMOD calculation in Fig. 5.4
FIG. 5.4: The theoretical distribution of barriers for the $^{16}$O + $^{144}$Sm reaction calculated using two different approaches. The dashed line represents the barrier distribution using CCMOD, which solves the coupled equations in the eigenchannel representation. The solid line is the barrier distribution from the calculation of Kruppa and Rowley [110], where the excitation energies of the inelastic states were treated exactly. The dotted line is from the exact coupled reaction channels code FRESCO.

is in reasonable agreement with the exact calculations. This suggests that the approximate treatment of the excitation energies in CCMOD is reasonable for the energies of the first 3− and 2+ states in $^{144}$Sm. However, the situation is expected to be different as the excitation energy increases above several MeV.

The experimental barrier distribution for the $^{16}$O + $^{144}$Sm reaction was also compared with a calculation using the code FRESCO [113]. The real and imaginary potential terms of the optical potential are summarised in Table 5.1.1. The depth and diffuseness of the real potential are again different from those parameters given in Table 5.1 but the barrier height, radius and curvature given in Table 5.1.1 are very similar to the best fit values. This is possible since different sets of potential parameters $(U, r_V, a)$ can approximate the same barrier parameters even though the shape of the real potential is different. The code FRESCO solves the coupled differential equations by step-by-step numerical integration, calculating the scattering wavefunctions over the interval $\Delta R$ up to some maxi-
Table 5.3: The real (U) and imaginary (W) potential parameters used in the CRC code FRESCO. The form for the imaginary potential was a Woods-Saxon squared. The real potential had the usual Woods-Saxon form. In these calculations the deformation parameters for the Coulomb, real and imaginary potentials were equal.

<table>
<thead>
<tr>
<th>U (MeV)</th>
<th>r_V (fm)</th>
<th>a (fm)</th>
<th>W_i (MeV)</th>
<th>r_i (fm)</th>
<th>a_i (fm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-105.1</td>
<td>1.10</td>
<td>0.75</td>
<td>-10.0</td>
<td>1.00</td>
<td>0.40</td>
</tr>
</tbody>
</table>

\[ B_0 = 61.23 \quad R_B = 10.8 \quad \hbar \omega = 4.27 \]

\[ \beta_3 = 0.205 \quad \beta_2 = 0.113^\dagger \]

\( ^\dagger \beta_{nuc}^W = \beta_{nuc}^U = \beta_{Coul} \)

The results in Fig. 5.4 reveal small differences in the shape of the barrier distributions between the calculations from CCMOD and the exact codes. Given that the coupling schemes and deformation parameters used are identical in all three calculations, this comparison should test the validity of the approximations made in the eigenchannel approach of CCMOD. The validity of these approximations are examined in Fig. 5.5, where the solid line represents the barrier distribution from a FRESCO calculation without the isocentrifugal approximation and with coupling to all orders in the inelastic form factor. The equivalent calculation, but with the isocentrifugal approximation, is barely distinguishable from the solid line and is not shown in Fig. 5.5. The barrier distribution represented by the dotted line in Fig. 5.5 is a FRESCO calculation with the first term \( dV_n/dr \) only in \( F(r) \), and the isocentrifugal approximation. The calculation suggests that this approximation is not too severe for the relatively small deformations of the inelastic channels considered here. However, these approximations still do not account for the differences in the CCMOD calculation (dashed line in Fig. 5.5) and
the FRESCO calculations.

The remaining differences could be due to the approximate treatment of the excitation energies by CCMOD, although the energies of the inelastic states are not too large. There is, however, one other approximation that CCMOD makes. When evaluating the weights of the eigenbarriers, CCMOD does so at a particular radius [52] taken to be the radius of the single barrier $R_B$. This is done to ensure that the sum of the weights of all eigenbarriers is unity. Since in the eigenchannel solution to the coupled equations, the weights are functions of $r$ (see Chapter 2), there is nothing unique about evaluating them at the radius $R_B$. It was shown in Ref. [57], that in the vicinity of the lowest eigenbarriers, the weights were changing rapidly, so small changes in $r$ can have a significant effect on the weight of each eigenbarrier. This ambiguity in calculating the weight of each eigenbarrier is inherent in the eigenchannel representation, whenever the constant coupling approximation is not being used. This factor, along with the approximate treatment of the excitation energies, is the most likely explanation for the differences between the approximate code and exact coupled-channels calculations in Fig. 5.5. Although very detailed comparisons of experimental data should be made using models that do not use such approximations, the code CCMOD is invaluable for making fast qualitative interpretations of the channels influencing fusion.

Coupling to the projectile

Implicit in the discussion thus far, is the assumption that the projectile is inert in terms of its effect on the fusion process. In this Section, the effect of projectile excitation on the distribution of barriers is examined. The barrier distribution shown by the solid line in Fig. 5.6(a) is a calculation using the code CCMOD which includes the first 3− excited state in $^{16}$O. The energy of the 3− state in $^{16}$O is at 6.130 MeV with a deformation parameter $\beta_3 = 0.733$. The 2+ and 3− inelastic states in $^{144}$Sm were also included in the coupling scheme. It is clear from Fig. 5.6(a) that projectile excitation destroys the agreement between the measured and calculated barrier distributions. The extra coupling strength from the inelastic state in $^{16}$O pushes the second barrier to energies much higher than observed, depleting the region around 65 MeV. This also forces the main barrier to lower energies producing a barrier distribution inconsistent with the measured one.

The above result conflicts with the approach taken by some analyses [123,124], which included excitation of the $^{16}$O projectile and of other projectiles close in
FIG. 5.5: The barrier distributions evaluated using various approximations in the coupled-channels calculations. The solid line is an exact FRESCO calculation with no isocentrifugal approximation and coupling to all orders. The same calculation, with the isocentrifugal approximation, differs by less than 1% in the cross-sections (not shown). The dotted line is a FRESCO calculation with the first term $dV_n/dr$ in $F(r)$ only and the isocentrifugal approximation. The dashed line is the CCMOD calculation of Fig. 5.4.

mass. Often, the inelastic channels in the projectile have been automatically included in order to increase the theoretical cross-sections to provide better agreement with the data at energies below the single barrier. Figure 5.6 shows that incorrect conclusions can be drawn from this approach when the data and theory are compared in terms of the conventional logarithmic plot of $\sigma(E)$. The excitation function shown by the dashed line in Fig. 5.6(b) is the above CCMOD calculation, which includes projectile excitation, but with an arbitrary increase in $B_0$ of 0.5 MeV to $B_0 = 61.60$ MeV. Note that this is the sort of increase possible if the fusion cross-sections above $B_0$ are ill defined. This calculation appears to reproduce the data as well as the calculation with the best fit barrier parameters [as depicted by the dashed line in Fig. 5.1(b)] because of the compensating shift in $B_0$. But of course the shape of the barrier distribution is not correct [broken line in Fig. 5.6(a)]. It is almost the same shape as the initial calculation with projectile excitation and the best fit parameters, shifted by $+0.5$ MeV.

This demonstrates not only the importance of accurately defining the single
barrier parameters, but also shows the benefits of identifying the distribution of barriers in the reaction. It is conceivable that a statistical analysis of two different theoretically based calculations could, for $\chi^2 \gg 1$, produce identical $\chi^2$-fits to the fusion cross-sections, but have barrier distributions that are qualitatively different. In this case, it is the calculation that best reproduces the shape of the barrier distribution that is more likely to have included the correct coupling scheme.

The CCMOD calculation in Fig. 5.6 implies that the coupling strength for the inelastic states in the projectile is far too strong. A possible explanation for this is due to Esbensen et al. [125]. They suggest that the effects of coupling in $^{16}$O are suppressed for two reasons. The first is due to the dynamical effects associated with the large excitation energies of the states in $^{16}$O, and the second relates to 'proximity' effects between the surfaces of the two potentials [126]. Esbensen et al. [125] found that the collective states in the lighter ion, in an asymmetric collision, were strongly suppressed.

The dynamical argument is based on the classical zero-point motion model for collective surface vibrations [127]. Although this model has been superseded by the coupled-channels approach, it is instructive to consider its central idea. The surface vibrations correspond to oscillations in the nuclear radius, with a characteristic energy $\hbar \omega$, where $\omega$ is the frequency of the mode of oscillation. In order for a given collective state to have a significant effect on the relative motion of the two interacting nuclei, one requires $\hbar \omega \ll \hbar \omega_{\text{collision}}$, where $\omega_{\text{collision}}$ is the characteristic frequency for the collision. If this condition holds, then the dynamical effects of the vibration can be neglected ($\omega = 0$) and the states are treated as frozen or static [120]. This is sometimes called the *frozen shapes approximation* and is implicitly assumed in the geometric model of fusion for statically deformed nuclei. If, however, the excitation energy of the collective mode is large, $\hbar \omega \sim 10$ MeV [125], then the frozen shapes approximation is no longer valid. In this case, the radius changes during the time of the collision and each individual collision samples all radii giving rise to an effective *average* barrier. The effective coupling strength for states with large excitation energies is then reduced considerably. The energy of the state at which this reduction becomes significant is not obvious; Esbensen et al. suggest a few MeV [125].

A second calculation was performed with the code FRESCO to examine its treatment of projectile excitation. The dashed line in Fig. 5.7 shows the result of the FRESCO calculation when the $3^-$ channel in $^{16}$O is included in the coupling scheme, in addition to the first two excited states in $^{144}$Sm. Again, there is
FIG. 5.6: (a) The solid line is the barrier distribution from a CCMOD calculation with the $3^-$ state in $^{16}$O included in addition to the two low-lying states in $^{144}$Sm. The measured barrier distribution demonstrates that the coupling strength for projectile excitation is dramatically overestimated in the simplified coupled-channels approach. The dashed line is the equivalent calculation but with the single barrier shifted by +0.5 MeV. The dotted line is a calculation that also includes projectile excitation but with the deformation parameter for the $\beta_2$ in $^{16}$O reduced by one fifth. Panel (b) displays the same calculations in terms of their cross-sections. This figure illustrates how it is possible to draw incorrect conclusions about the significance of a particular channel coupling when the data and calculation are compared in the usual manner.
a striking disagreement with the experimental barrier distribution and a large difference from the calculation without projectile excitation (solid line in Fig. 5.7). A similar result was also obtained when projectile excitation was included in a calculation using the code of Kruppa and Rowley (not shown). These calculations suggest that either there is still something missing in the exact coupled-channels treatment of the $^{16}$O inelastic channels or that the effect of projectile excitation is overestimated. An arbitrary reduction in the deformation parameter for the $^{16}$O nucleus produced the barrier distribution shown by the dotted line in Fig. 5.7. A second calculation was also performed with CCMOD. Here projectile excitation was included in the coupling scheme but with a deformation parameter one fifth its original value (dotted line in Fig. 5.6). The effect of the reduced coupling strength in both these calculations is to put the theoretical barrier distribution back in better agreement with the experimental one, though the shape of the second barrier is still mismatched.

Clearly there is an unresolved problem associated with the coupling to the inelastic channel in $^{16}$O. What is certain is that including this channel in the CCMOD calculations is not appropriate. In all subsequent coupled-channels calculations the $^{16}$O projectile is treated as inert. Also, the inelastic channels in the
\[ ^{17}\text{O} \text{ projectile are not included in the following calculations since the collective properties for this nucleus are expected to be similar to those of } ^{16}\text{O}. \]

Although there maybe some uncertainties regarding the inputs to the coupled-channels codes, it has been shown that the general features of the measured barrier distribution are very well reproduced by such models. The double-peaked barrier distribution was observed in two different approaches to solving the coupled-channels problem, using three distinctly different codes. The remaining discrepancies between the measured barrier distributions and the calculations pose an interesting question. To what level of detail can the current theoretical coupled-channels models describe the experimental barrier distributions?

5.1.2 The \( ^{17}\text{O} + ^{144}\text{Sm} \) reaction

As shown in Chapter 4, the shape of the measured barrier distribution for the \( ^{17}\text{O} \) induced reaction is qualitatively different to that for the \( ^{16}\text{O} + ^{144}\text{Sm} \) reaction. At energies below the main barrier, there exists a tail on the barrier distribution not observed in the \( ^{16}\text{O} + ^{144}\text{Sm} \) reaction. At energies around 5 MeV above the single barrier, the two barrier distributions are equivalent, within the statistical uncertainties. The first step in theoretical description of the barrier distribution for \( ^{17}\text{O} + ^{144}\text{Sm} \) is to carry out the equivalent calculation done in Section 5.1.1. The barrier distribution from the CCM0D calculation with the \( 3^- \) and \( 2^+ \) states in \( ^{144}\text{Sm} \) is shown by the dashed line in Fig. 5.8. The calculation was performed with the fusion potential parameters for the \( ^{17}\text{O} + ^{144}\text{Sm} \) reaction given in Table 4.2 and the same coupling strengths for the inelastic channels as for the \( ^{16}\text{O} + ^{144}\text{Sm} \) calculation. Again, the double-peaked feature in the barrier distribution is well reproduced. However, the calculation does not put enough weight in the energy region below the main barrier. Note how the size difference between the main barrier in this calculation and that for the equivalent calculation for the \( ^{16}\text{O} + ^{144}\text{Sm} \) reaction (see Fig. 5.2 in this Chapter) survives after the addition of the inelastic couplings. Thus, the shape of the experimental barrier distribution for the \( ^{17}\text{O} + ^{144}\text{Sm} \) reaction cannot be explained by considering inelastic channels only.

It is also worth emphasising that there is no change in the above conclusion when the wrong single-barrier parameters are used in the CCM0D calculation. This can be seen by comparing the barrier distributions shown in in Fig. 5.8. The barrier distribution represented by the dashed line in Fig. 5.8 is the calculation performed with the best-fit potential parameters. The dotted line in Fig. 5.8 is the barrier distribution obtained with a calculation using a diffuseness \( a = 0.84 \)
fm and a corresponding barrier $B_0 = 60.8 \text{ MeV}$, obtained from re-fitting the high energy $^{17}O + ^{144}\text{Sm}$ data. (The $\chi^2_\nu$ for this less the ideal fit was almost twice the best fit values given in Table 4.1). This variation in the potential parameters does not alter the shape of the double-peaked barrier distribution dramatically. More importantly, there is only a very small change in the shape of the barrier distribution for energies below the main barrier. The solid line in Fig. 5.8 is a second calculation with an even more severe change in the potential parameters. This calculation was performed with $a = 0.62$ and $B_0 = 61.1 \text{ MeV}$, giving a $\chi^2_\nu$ fit to the high energy data an order of magnitude worse than the best-fit parameters.

The peak of the main barrier is now smaller, but the whole barrier distribution is shifted to higher energies because of the increase in the single barrier. If the diffuseness was larger than the best-fit value then the barrier distribution would shift to lower energies, but now the peak of the main barrier would be larger because of the increase in $a$. Neither of these approaches will produce a 'tail' on the low energy side of the main barrier. In otherwords, the disagreement at energies below the main barrier between the calculation including the inelastic channels and the $^{17}O + ^{144}\text{Sm}$ barrier distribution cannot be due to incorrect single-barrier parameters.

Barrier weight can be shifted from the main barrier to lower energies by the inclusion of a positive $Q$-value channel in the coupling scheme. The $Q$-value for the single-neutron transfer to the ground-state in $^{145}\text{Sm}$, $^{144}\text{Sm}(^{17}O,^{16}O)^{145}\text{Sm}$, has $Q = +2.6 \text{ MeV}$. Note that there are no positive single particle transfer reactions for the $^{16}O + ^{144}\text{Sm}$ reaction. To examine the effects of transfer on the barrier distribution, the simplified coupled-channels code CCMOD was used. In this code, additional couplings are specified by directly including the transfer coupling strength, defined as $K/\sqrt{4\pi}$ [54] at the position of the single barrier $R_B$. The spatial variation of this strength is assumed to be exponential [58, 53] with the form factor given by

$$F_{\text{tran}}(r) = \frac{K}{\sqrt{4\pi}} \exp \left[ -\frac{(r - R_B)}{1.2 \text{ fm}} \right]. \tag{5.4}$$

In this analysis, $K$ was adjusted in order to provide the best representation of the data. The effect of the single-neutron stripping reaction is to introduce a third barrier, with a weight of $\approx 12\%$, at an energy below the main barrier. Most of the weight gained by the lowest barrier comes from the main barrier, leaving the weight of the second barrier essentially unchanged. The barrier distribution from this calculation is shown by the solid line in Fig. 5.9 for a value of $K = 2.8 \text{ MeV}$. 130
FIG. 5.8: The dashed line is the barrier distribution from a CCMOD calculation with the two inelastic states in $^{144}$Sm. The main peak of the theoretical distribution overpredicts the data by a larger amount than the $^{16}O + ^{144}Sm$ case. This is because of the difference in the penetrability between the two systems. The dotted line is a CCMOD calculation with the inelastic channels, but for different potential parameters. The solid line shows the effect on the barrier distribution for a dramatic shift in the potential parameters. This demonstrates that incorrect values for the nuclear potential cannot be responsible for the low energy tail observed in the experimental barrier distribution.

The theoretical calculation now reproduces, at least qualitatively, the tail present in the $^{17}O + ^{144}Sm$ reaction and offers a much better estimate of the main peak in the barrier distribution.

Transfer to excited states in $^{145}$Sm

The above calculation, where it was assumed that transfer proceeded to the ground-state in $^{145}$Sm, is a simplification. It is expected that the transfer will occur to several single-particles states in $^{145}$Sm. An example of this is the reaction $^{144}$Sm($^{13}$C,$^{12}$C)$^{145}$Sm [128], where transfer to four states, 2f$_{7/2}$, 3p$_{3/2}$, 3p$_{1/2}$ and 2f$_{5/2}$, was observed. The calculation in the previous Section was repeated, this time including four transfer channels with Q-values at +2.6, 1.2, 1.0 and 0.8 MeV. The transfer strength was adjusted to provide the best representation of the barrier distribution, as shown by the dotted line in Fig. 5.9. The strength was kept the same for each transfer channel and $\mathcal{K}$ was found to have the value
FIG. 5.9: The barrier distribution as shown by the dashed line is the same as the calculation in Fig. 5.8. The solid line is the barrier distribution which also includes the $Q = +2.6$ MeV transfer channel. This barrier distribution gives a better reproduction of the shape of the experimental distribution. The dotted curve is the barrier distribution calculated with transfer to the ground-state and three excited states in $^{145}$Sm.

1.4 MeV. Although each individual coupling strength is reduced, the total coupling strength (the sum of each coupling strength added in quadrature) remains unchanged. The estimate of the total coupling strength is consistent with the average value of $\approx 3$ MeV from analysis of a number of single-particle transfer reactions, though there are considerable fluctuations in these values from case to case [58].

Relative significance of inelastic and transfer channels

The ability to see the effects of the neutron stripping reaction in the barrier distribution is due to the fact that the Q-value for the reaction is positive. The weight of the barrier below the main peak is about a factor of 3 smaller than that above the main peak, which is largely due to coupling to inelastic channels. This result supports the analyses given in Refs. [53,54,108], where it was suggested that the transfer coupling strength should be weaker (by up to one fifth the value of the inelastic coupling strength [53,20]). This implies that the effects of an equally strong transfer channel, but with a negative Q-value, will not alter significantly the distribution of barriers in the region above the main barrier. A
calculation using CCMOD confirmed this conclusion, the barrier distribution being essentially unchanged when a fictitious negative Q-value channel was added to the inelastic coupling scheme, with a coupling strength of 2.8 MeV. Even in a reaction which has relatively weak inelastic channels, such as the $^{17}$O + $^{144}$Sm reaction, the effects of negative Q-value transfer will be difficult to observe in fusion.

The more complete coupled-channels models, in principle, determine the cross-sections for each reaction channel. In the above calculations, the focus has been on the fusion cross-sections and the model's ability to reproduce the measured barrier distribution for fusion qualitatively. A simultaneous description of each reaction channel would provide a more complete picture of the effects of the inelastic and transfer channels in the reaction. Transfer cross-sections for the $^{17}$O + $^{144}$Sm reaction would be particularly valuable in light of the weakness, relative to the inelastic processes, of the effect of the transfer channels on fusion.

In summary, the effects of inelastic channels on the barrier distribution were observed in both the $^{16}$O + $^{144}$Sm and $^{17}$O + $^{144}$Sm reactions. The effects of the neutron stripping channels were also identified in the latter reaction. A simplified coupled-channels calculation was able to reproduce the shapes of these barrier distributions qualitatively, providing strong support for the identification of these individual inelastic and transfer channels. The effects of the transfer channel are considerably weaker than the inelastic channel. The coupled-channels calculations suggest that the effects of projectile excitation on fusion are small or absent. A comparison of the simplified coupled-channels code was made with more exact codes. The approximations made in the eigenchannel representation applied in the code CCMOD were thus tested and found to be good.

The next Section of this Chapter is devoted to the interpretation of the $^{16}$O + $^{208}$Pb reaction. Based on the conclusions above, the coupling scheme for the $^{16}$O + $^{208}$Pb reaction could be expected to be similar to that for the $^{16}$O + $^{144}$Sm reaction. The $^{16}$O + $^{208}$Pb fusion reaction involves both neutron and proton closed shells in the target and projectile. Does the coupling scheme for this reaction follow from the insights gained into the behaviour of the $^{16,17}$O + $^{144}$Sm reactions?

### 5.2 The $^{16}$O + $^{208}$Pb reaction

The barrier distribution for the $^{16}$O + $^{208}$Pb reaction is displayed in Fig. 5.10(a), with the $^{16}$O + $^{144}$Sm reaction in panel (b) for comparison. The energy axes
Table 5.4: The ground-state transition strengths and the deformation parameters for $^{208}$Pb. The deformation parameters were calculated with a nuclear radius parameter of $r_0 = 1.06$ fm.

<table>
<thead>
<tr>
<th>Nucleus</th>
<th>J$^\pi$</th>
<th>$E^*$ (MeV)</th>
<th>$B(E\lambda) \uparrow$</th>
<th>$\beta_\lambda$</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{208}$Pb</td>
<td>$2^+$</td>
<td>4.085</td>
<td>0.29 $e^2b^2$</td>
<td>0.066</td>
<td>Ref. [114]</td>
</tr>
<tr>
<td></td>
<td>$3^-$</td>
<td>2.615</td>
<td>0.611 $e^2b^3$</td>
<td>0.150</td>
<td>Ref. [115]</td>
</tr>
</tbody>
</table>

have been renormalised by dividing by the $B_0$ for each system. Now consider the inelastic channels in the Pb nucleus. The solid line in Fig. 5.10(a) is the barrier distribution from a CCMOD calculation with coupling to the $3^-$ and $2^+$ states in $^{208}$Pb. The spectroscopic information for these states is summarised in Table 5.4. As in the $^{16}$O + $^{144}$Sm reaction, the largest effect on the barrier distribution comes from the $3^-$ state. This calculation, for a simple inelastic coupling scheme, does not reproduce the experimental barrier distribution very well. There are three major differences between the two barrier distributions. For the $^{16}$O + $^{208}$Pb reaction (i) the main peak from the CCMOD calculation is too large; (ii) the weight of the second peak in the experimental barrier distribution is much more significant and occurs at energies closer to the main peak, and (iii) there appears to be evidence for significant barrier weight at energies below the main barrier.

The above result suggests that there exists stronger coupling than the model calculation indicates. In order to improve the agreement between theory and experiment, more coupling must be included. Are there any 'strong' particle transfer reactions that may influence the fusion? There are two possibilities, a neutron pickup reaction with $Q = -3.2$ MeV and a proton stripping channel with $Q = -0.6$ MeV (the net $Q$-value corrected for the change in Coulomb energies, see Chapter 2). The broken line in Fig. 5.10(a) is the barrier distribution with these two transfer channels included as well as the inelastic channels. The coupling strength for both transfer channels in the calculation was taken to be the same value as for the $^{17}$O + $^{144}$Sm reaction. The positive $Q$-value transfer channel reduces the size of the main peak in the barrier distribution putting a barrier at energies below the main barrier. However, the calculation still fails to reproduce the low-energy edge of the experimental barrier distribution. As noted in the previous Section, the effects of the negative $Q$-value transfer, in the presence of inelastic channels, are small. The calculation does not predict enough barrier weight at energies $\approx 2-5\%$ above $B_0$. The agreement with the experimental bar-
FIG. 5.10: (a) The experimental barrier distribution for the $^{16}$O + $^{208}$Pb reaction is compared to a CCMOD calculation using the coupling scheme that was successful in describing the $^{16}$O + $^{144}$Sm reaction, see the solid line in (b). The barrier distribution for the $^{16}$O + $^{208}$Pb reaction is not well reproduced by this coupling scheme. Inclusion of two transfer channels improves the agreement marginally (broken line) but the shape of the barrier distribution is still not correct. (b) The two-channel inelastic coupling scheme for the $^{16}$O + $^{144}$Sm reaction.
rier distribution is only marginally improved with the addition of the transfer channels. Considering that the approximations made in the code CCMOD are reasonably reliable, it seems unlikely that they can be entirely responsible for the disagreement.

What is required is a mechanism whereby coupling strength is concentrated at energies close to the single barrier. As shown in the next Section, any additional increase in the coupling strength of the inelastic or transfer channels will push the existing barriers further apart in energy, an effect opposite to what is required. As a guide to the height and weight of the barriers needed to describe $^{16}$O + $^{208}$Pb reaction, the experimental fusion cross-sections were fitted using an analytical approach. Here, the fusion cross-sections are written as

$$\sigma(E) = \sum_{\alpha} w_\alpha \sigma(E, B_\alpha),$$

(5.5)

where $\sigma(E, B_\alpha)$ is the cross-section for each eigenbarrier with height $B_\alpha$, and $w_\alpha$ is the weight for each eigenbarrier in eigenchannel $\alpha$. The radius and curvature of the single barrier, as determined from the fit to the high energy data, were used as inputs to the Wong form of the fusion cross-section, as given by Eq. (2.12) in Chapter 2. Then, a $\chi^2$-fit to the experimental cross-sections over the entire energy range was performed to determine the optimum number of barriers and their weights. The fit was performed with the eigenbarrier energies as free parameters and the $\chi^2$ was minimised with respect to $w_\alpha$. The best fit was found for three barriers, giving rise to the distribution of barriers shown by the continuous solid line in Fig. 5.11. The barriers and their weights are also indicated in Fig. 5.11. The theoretical fit to the fusion cross-sections suggests that the $^{16}$O + $^{208}$Pb reaction requires at least three barriers, two of which are reasonably close in energy to each other. This is in contrast to the $^{16}$O + $^{144}$Sm reaction which was successfully described using this analytical approach with two barriers only.

**Multiphonon coupling**

One way of producing a barrier distribution that has barriers in close proximity is by using a multiple-phonon coupling scheme. The effects of multiphonon excitations on the fusion barrier distributions for the $^{16}$O + $^{92}$Zr have been studied recently by Kruppa et al. [51]. For a vibrational nucleus like $^{92}$Zr, with a low lying quadrupole phonon at 0.934 MeV and an octupole phonon at 2.339 MeV, it is expected that a multiplet of states should exist, corresponding to the double-phonon excitations at an energy approximately twice that of the single-phonon
FIG. 5.11: The results of an analytical eigenbarrier fit to the experimental fusion cross-sections for the $^{16}O + ^{208}Pb$ reaction (solid line). The best fit was obtained for 3 eigenbarriers at $(B_\alpha, w_\alpha) = (72.1, 0.29), (74.4, 0.49)$ and $(78.3, 0.26)$, as shown by the thick vertical lines. This is in contrast to the $^{16}O + ^{144}Sm$ reaction where the best fit was obtained with two barriers only.

excitations. The barrier distribution from the single-phonon coupling scheme for the $^{16}O + ^{92}Zr$ reaction is shown by the solid line in Fig. 5.12. As in both the $^{16}O + ^{144}Sm$ and $^{16}O + ^{208}Pb$ cases, this coupling scheme produces only two barriers of appreciable weight. Here $(B_\alpha, w_\alpha) = (41.2, 0.76)$ and $(45.3, 0.20)$, where these two barriers account for 96\% of the total barrier weight. The effect on the barrier distribution of including the $2^+ \otimes 3^-$ coupling term in $^{92}Zr$ is shown by the broken line in Fig. 5.12. The multiphonon coupling brings two of the barriers closer together in energy, producing the shoulder at 43 MeV in Fig. 5.12. There is also another much weaker barrier at an energy around 45 MeV. The reason for this effect can be explained by examining the behaviour of the eigenvalues for the single and multiple-phonon couplings [57].

First consider the 2-channel coupling case. The matrix is given by

$$M_2 = \begin{pmatrix} 0 & \beta \\ \beta & 1 \end{pmatrix}, \quad (5.6)$$

where $Q = 1$ MeV and $\beta$ is coupling strength. Here the Q-value is set to unity for simplicity. The solution for this case was given in Chapter 2; the eigenvalues
are
\[ \lambda_{\pm} = \frac{1}{2} (1 \pm \sqrt{1 + 4\beta^2}). \] (5.7)

The eigenvalues are plotted in Fig. 5.13 as a function of \( \beta^2 \) (solid lines). From this function it is clear why larger values for the coupling strength force the eigenvalues, and hence the eigenbarriers, further apart in energy. This is the reason why simply increasing the coupling strength in a system such as \(^{16}\text{O} + ^{208}\text{Pb}\) will never produce the desired barrier distribution. The double-phonon coupling matrix, for phonons of the same type, is given by

\[
M_3 = \begin{pmatrix}
0 & \beta & 0 \\
\beta & 1 & \beta\sqrt{2} \\
0 & \beta\sqrt{2} & 2
\end{pmatrix}.
\] (5.8)

Note the difference in the position of the off-diagonal terms compared to Eq. (5.2). This gives a third order polynomial whose three eigenvalues are represented by the broken lines in Fig. 5.13. The positive eigenvalue solution 'repels' the middle eigenvalue solution more strongly than the negative one. This results in two eigenbarriers close together in energy and a third eigenbarrier at higher energies [57]. This qualitatively explains the distribution of barriers shown in Fig. 5.12. The addition of another type of double-phonon state will complicate the above situation, but the explanation remains unchanged.

There is some theoretical support for the existence of multiphonon states in \(^{208}\text{Pb}\) [129], although as yet there has been no definitive identification of these states in spectroscopic studies [130], despite a recent claim [131]. However, in a recent measurement [132] of the fusion cross-sections for \(^{58}\text{Ni} + ^{60}\text{Ni}\) reaction, the structure present in the barrier distribution was characteristic of a strong phonon coupling scheme. The shape of the experimental barrier distribution was reproduced only when the double-phonon excitations in the target and projectile were taken into account.

In a reaction like \(^{16}\text{O} + ^{208}\text{Pb}\), where the coupling strength of the octupole phonon is larger than the quadrupole phonon, the \(3^- \otimes 3^-\) multiple-phonon state may be an important term in the overall coupling scheme. The effects of the multiphonon states were examined using CCMPH, a version of CCMOD, modified to include a multiple phonon coupling matrix. Before examining the effects of multiphonon coupling on the \(^{16}\text{O} + ^{208}\text{Pb}\) reaction, the influence of this type of coupling scheme is examined for the \(^{16}\text{O} + ^{144}\text{Sm}\) reaction. In Fig. 5.14, the solid line is a CCMPH calculation with the \(3^- \otimes 3^-\), \(2^+ \otimes 2^+\) and \(2^+ \otimes 3^-\) multiple-
FIG. 5.12: A comparison between the barrier distribution calculated with a single-phonon coupling scheme (solid line) and that from a multiple-phonon case (broken line). The multiple-phonon coupling decreases the separation in energy of the two lowest eigenbarriers.

FIG. 5.13: The eigenvalues plotted as a function of $\beta^2$ for the single-phonon (solid lines) and double-phonon (broken lines) coupling schemes. From Ref. [57].
FIG. 5.14: The barrier distribution for a multiphonon calculation for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction. The previous good agreement between the calculation and the measured barrier distribution is lost, indicating that multiphonon excitations are very weak or absent in $^{144}\text{Sm}$.

The barrier distribution shown by the solid line in Fig. 5.15 includes, in addition to the single-phonon excitations in $^{208}\text{Pb}$, both the double-phonon states, $3^{-} \otimes 3^{-}$ and $2^{+} \otimes 2^{+}$, and the cross-coupling state $2^{+} \otimes 3^{-}$. This calculation now produces a second barrier much closer in energy to the first, however the weight in this shoulder is overpredicted. Inclusion of the two transfer channels produces the barrier distribution shown by the broken line in Fig. 5.15. The size of the main barrier is well matched by this calculation, but it is still short of a complete description of the shape of the barrier distribution. A better representation of the barrier distribution was obtained with an arbitrary reduction in the coupling strength of the double-octupole phonon state. Further investigation of the multiple-phonon channels is needed.

It is worth noting that the shape of the barrier distribution for this reaction is also similar to the measured barrier distribution for the $^{16}\text{O} + ^{92}\text{Zr}$ system [133]. The experimental distribution for the $^{16}\text{O} + ^{92}\text{Zr}$ reaction can also be fitted...
FIG. 5.15: The experimental barrier distribution for the $^{16}$O + $^{208}$Pb reaction is compared with a CCMPH calculation (solid line) which includes a full multiphonon coupling scheme (see text). The 3 dominant eigenbarriers have weights and heights (73.8, 0.69), (76.7, 0.26) and (77.6, 0.03). The broken line is the same calculation but with the two transfer channels included in addition to the multiphonon channels. These calculations are still short of a complete description of the coupling scheme for the $^{16}$O + $^{208}$Pb reaction.

Although the above calculations suggest there is evidence for a multiple phonon coupling scheme in the $^{16}$O + $^{208}$Pb reaction, the high excitation energies of the two-phonon states means that the approximations made in the CCMPH code may be unreliable. It was shown in Ref. [51] that calculations which treated the excitation energy exactly, redistributed the barrier weights and their positions, and such an effect needs to be examined for the $^{16}$O + $^{208}$Pb case. It is possible that such a redistribution may improve the description of the barrier distribution. Nevertheless, the results presented here confirm that the single-phonon coupling schemes are not sufficient to reproduce the barrier distribution for the $^{16}$O + $^{208}$Pb reaction. The simple inelastic coupling scheme, that was successful in the
description of the $^{16}$O + $^{144}$Sm reaction, could not explain the stronger coupling present in the $^{16}$O + $^{208}$Pb reaction. This suggests that there are more complex channels associated with the $^{208}$Pb target.

5.2.1 Fission fragment anisotropies

The measured fusion excitation function for the $^{16}$O + $^{208}$Pb reaction, and the barrier distribution discussed in the previous Section, are used here to obtain accurate estimates of the angular momentum distributions for fusion. These angular momentum distributions are then used as inputs to the transition state model (TSM) calculations for determining the theoretical fission fragment anisotropies. The TSM is tested against the experimental anisotropies obtained from the fission fragment angular distributions for the $^{16}$O + $^{208}$Pb reaction. Again, the value of accurate fusion cross-sections are evident in the TSM calculations. Not only are they necessary for defining the angular momentum distributions, but also for obtaining an accurate estimate of the saddle-point temperature of the fissioning nuclei.

As detailed in Chapter 2, the TSM requires knowledge of the total angular momentum $J_h$ of the fissioning nucleus, the temperature $T$ of the fissioning nuclear system at its transition state, taken to be the saddle point, and the effective moment of inertia at the saddle point $J_{\text{eff}}$. In this Section, a description is given of how each of these quantities was determined. Then, comparisons between the experimental data and the theory are made in terms of the reduced anisotropy $(A - 1)$, which is approximately proportional to $(J^2)/(TJ_{\text{eff}})$, see Eq. (2.63) in Chapter 2.

Effective moment of inertia at the saddle point

To determine the spin-dependent moments of inertia perpendicular and parallel to the symmetry axis, a simple parameterization [134] of the rotating finite range model [65] (RFRM) was used. This provides a fast way of approximating a full RFRM calculation, and is adequate for the heavy systems studied here. The value of the effective moment of inertia for zero angular momentum at the saddle point was $J_{\text{eff}} = 4600$ u fm$^2$. This value is $\approx 3\%$ larger than the RFRM value. Since the reduced anisotropy is approximately proportional to $1/J_{\text{eff}}$, use of the RFRM value would scale the reduced anisotropy in an approximately linear way, resulting in an equivalent increase in $(A - 1)$. The true value of the effective moment of inertia is one of the remaining uncertainties in these TSM calculations.
Temperature at the saddle point

The temperature of the fissioning nucleus at its saddle point is reduced by neutron emission that occurs before the system passes over its own saddle. Since in practice measurements can only be made of the pre-scission neutron multiplicity, some assumption about its division into pre- and post-saddle contributions must be made. In the analysis of Rossner et al. [38] for the reaction $^{16}\text{O} + ^{208}\text{Pb}$, it was shown that for energies up to $E_{\text{c.m.}} \approx 92$ MeV, most of the measured pre-scission neutrons should be emitted before the saddle point is reached, and that the lower the energy, the more reliable this assumption. It was demonstrated in Ref. [38] that the calculated anisotropies were in better agreement with the data at low energies when the loss of excitation energy due to neutron emission was accounted for, but deviated from experimental values at higher energies ($E_{\text{c.m.}} \gtrsim 92$ MeV).

A recent calculation [135] has shown that for these higher energies, an appreciable number of neutrons may be emitted during the saddle-to-scission transition time, thus putting the theory back in better agreement with the data in the high energy region. However, an anomaly still persists at the lower energies, as shown in Fig 1.4 in Chapter 1.

An approximation of the analysis of Rossner et al. [38] was that the appropriate temperature at the saddle point was obtained for the mean value of the excitation energy of the fissioning systems. This approach results in a smaller anisotropy compared to a more realistic calculation which evaluates the anisotropy for a distribution of excitation energies. The approximation becomes worse as the spread in the range of excitation energies increases. In this work, the compound nucleus decay was modelled with the Monte Carlo evaporation code JOANNE [78], which gave a distribution of excitation energies, allowing the temperature at the saddle point to be determined on an event-by-event basis. With this approach, all correlations of the excitation energy, $J_{\text{eff}}(J)$ and $J$ are retained.

The Monte Carlo evaporation code models the decay of the compound nuclei in order to determine the distribution of events into the competing fission and ER modes. The pre-scission neutron multiplicity data of Ref. [38] and the evaporation residue cross-sections from this work were fitted with the code JOANNE. This was done by varying the level density parameter at the saddle point $a_f$, and the Sierk fission barrier scaling factor $k_f$. The level density parameter at the equilibrium deformation $a_n$, was taken to be $a_n = A/8.8$ MeV$^{-1}$. The temperature at the saddle point for a nucleus with an excitation energy $E^*$ was obtained from

$$T_f^2 = \frac{[E^* - k_f B_f(J) - E_{\text{rot}}(J) - E_n(J)]}{a_f}, \quad (5.9)$$

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where \( B_f(J) \) is the Sierk fission barrier [65], \( E_{rot}(J) \) is the rotational energy of the equilibrium configuration from the RFRM [65], and \( E_n(J) \) is the reduction in excitation energy due to the evaporation of neutrons. The excitation energies were calculated using liquid-drop masses for the compound nuclei, with \( Q^{LD} = -49.6 \) MeV [see Eq. (2.70) in Chapter 2]. Fits to the pre-scission neutron multiplicities and the ER excitation function were obtained for average values of \( a_f/a_n = 0.90 \) and \( k_f = 0.88 \). Because of the significant contribution of fission from very low excitation energies, which is not well modelled in this code since shell effects are not included, the parameters cited above can only be considered as fit parameters which result in the correct energy loss before fission, and the correct survival probability.

The effect on the distribution of excitation energies, and hence the saddle-point temperature, due to variation of these statistical model parameters was also examined. For example, the pre-scission neutron multiplicities and the ER cross-section were fitted with \( a_f/a_n = 1.00 \) and \( k_f = 0.70 \), after hindering the fission process with the introduction of a constant Kramers scaling factor [136]. Because of the uncertainty in the excitation energy due to shell effects, other calculations were performed with \( Q^{GS} = -46.5 \) MeV, the Q-value determined using the measured ground-state mass of \( ^{224}\text{Th} \), a shift of 3.1 MeV. The anisotropies from these calculations never varied by more than 3%, provided that the pre-scission neutron multiplicities and the ER cross-sections were fitted. This indicates a reasonable insensitivity of the anisotropy to different starting assumptions, as long as the relevant experimental quantities are reproduced.

In the comparison of the mean-square angular momentum obtained from the anisotropies and those values from various fusion models presented in Ref. [27], there was an ambiguity in the evaporation residue cross-sections. The measurements of \( \sigma_{ER}(E) \) by Hartel [40] were up to four times larger than those of Vulgaris et al. [41]. Similarly, the analysis of Ref. [38] depended upon knowledge of the evaporation residue survival probabilities, for two reasons. The values chosen for the ER cross-sections have an influence on the angular momentum distributions for fusion, and also on the saddle point temperature of the fissioning nuclei. As shown below, the latter has the most significant effect on the anisotropy.

**Angular momentum distributions of the fissioning nuclei**

The compound nucleus angular momentum distributions were determined from the precise fusion excitation function, as shown in Fig. 4.12, assuming that a model which fits the fusion cross-sections will correctly predict the angular mo-
momentum distributions [11, 106–108, 57]. If the model calculation was an ideal reproduction of the fusion excitation function, then this model should give a complete description of the fusion reaction. However, in the analysis of the cross-sections in this work, the model fits are not ideal, and, as discussed earlier, it may be possible that two different models will produce similarly large $\chi^2$-fits to the data. In this case, the barrier distribution can be used to distinguish between the different models, without relying solely on the $\chi^2$-fits of the analysis. It is for this reason that the barrier distribution is very useful in determining the angular momentum distributions for fusion.

A fit to the experimental fusion cross-sections was performed using the 'analytical eigenbarrier' approach described in Section 5.2. The angular momentum distributions were then calculated using the Wong expression, see Eq. (2.12) in Chapter 2. An ambiguity in this approach, is in the choice of the radii of the eigenbarriers. For this calculation of the angular momentum distributions, all barrier radii were set to the single barrier radius. The effect on the angular momentum distributions of using a radius that depends on each eigenbarrier, instead of the single barrier radius, was investigated. A new radius was calculated using the following approximation, which takes into account the variation in the radius of each barrier due to coupling effects [109]. The tail of the potential is assumed to have the exponential form given by Eq. (2.15) in Chapter 2. Writing the barrier height as the sum of $V_n(R(B))$ and $V_{\text{Coul}}(R(B))$, then

$$B = \frac{1.44Z_1Z_2}{R(B)} \left(1 - \frac{a}{R(B)}\right),$$

(5.10)

where the radius $R(B)$ is now a function of the barrier $B$. Eq. (5.10) is a quadratic equation in $R(B)$ which has the solution

$$R(B) = \frac{R_c}{2} [1 + (1 - 4a/R_c)^{1/2}],$$

(5.11)

where $R_c = 1.44Z_1Z_2/B$ is the distance of closest approach for a particle with energy $E = B$ in a pure Coulomb field [109]. Each eigenradius was then corrected using the approximate expression in Eq. (5.11). For the $^{16}\text{O} + ^{208}\text{Pb}$ reaction the difference in the smallest and the largest radii is $\approx 1$ fm. The difference between the $\langle l \rangle$ calculated using a fixed radius, and those calculated using the radius correction, differed by $\approx 4\%$ at the lowest energy, the energy where this effect is the largest. This corresponding difference in the anisotropies between the two approaches was found to be small, and not significant, considering the
uncertainties in the effective moment of inertia in the transition state model.

The angular momentum distributions of the fissioning nuclei are slightly different to those of the compound nuclei because of the pre-saddle particle emission. This effect was taken into account in the JOANNE calculation which corrected the above angular momentum distributions. It was assumed that all the compound nuclei are formed in complete fusion reactions.

Comparison with the experimental anisotropies

The evaporation code JOANNE was used to calculate the distribution of saddle-point temperatures and angular momenta of the fissioning systems, after the evaporation of the pre-scission neutrons. Fission events were put into bins of $J$ and $T$, with width 1 and 0.1 MeV, respectively. Angular distributions were then calculated for each value of $J$ and $T$ using Eqs. (2.61) and (2.62) in Chapter 2, with the effective moments of inertia obtained as described in the previous Subsection. The total angular distribution was obtained by taking the sum of the angular distributions for all values of $J$ and $T$, weighted by the number of fission events in each bin. The anisotropy was evaluated at the energies shown by the stars in Fig. 5.16(a), the points of which correspond to the experimental pre-scission neutron multiplicity data. The uncertainties shown for the calculated anisotropies arise mainly from the uncertainties in the measured pre-scission neutron multiplicities; there is also a contribution from the statistical nature of the Monte Carlo calculations. These calculations are in good agreement with the experimental anisotropies at all energies. The TSM calculations do not extend below $E_{c.m.} = 71.5$ MeV because the pre-scission neutron multiplicity was not determined below this energy. This result resolves the previous disagreement [27,38] of the calculated and measured anisotropies, for energies below the single barrier. Discrepancies are now at a level which is less than, or of the order of the uncertainty in the inputs to the transition state model calculations.

Reasons for the present agreement

The good agreement between the data and these TSM calculations of the fission fragment anisotropies, in comparison with the previous disagreement [38, 20], can be attributed to three factors. Firstly, the new experimental anisotropies of this work are lower than those previously measured at energies below the single fusion barrier, see Section 4.3.1. Secondly, the new ER cross-sections are much larger. Finally, there is an effect due to the use of the distribution of saddle-
FIG. 5.16: (a) Comparison between the measured (open circles) and calculated (stars) fission fragment anisotropies for the $^{16}$O + $^{208}$Pb reaction. The experiment and the calculations are plotted in terms of the reduced anisotropy $(A - 1)$. The solid line is a straight line fit to the TSM calculations. (b) The ratio of $(A - 1)$ as represented by the solid line in panel (a) to $(A - 1)$ as calculated using different input parameters. The broken line, curve (2), is the ratio obtained when the anisotropies were calculated using the low values of $\sigma_{ER}(E)$ from Ref. [41]. The effect on curve (2) of using the mean value of the excitation energy, instead of the distribution of excitation energies, is shown by the dotted line, curve (3) [see text].
point temperatures in the TSM calculations. These last two factors are discussed below.

The larger values of $\sigma_{\text{ER}}(E)$ of this work have two effects on the theoretical anisotropies. Because of the resulting larger fusion cross-sections, there is a small increase in the values of $\langle J^2 \rangle$ which increase the calculated anisotropies, although only slightly. But more importantly, the new larger ER cross-sections can only be reproduced in the JOANNE calculations by increasing $k_f$ by a significant amount, whilst $a_f/a_n$ is changed by a small amount in order to preserve the agreement with the experimental pre-scission neutron multiplicities. These parameters change in this way because the pre-scission neutron multiplicity is very sensitive to $a_f/a_n$ but only weakly dependent on $k_f$ [71]. See Eq. (2.67) in Chapter 2. The effect of the increase in the fission barrier height is to reduce the saddle-point temperature, which has a significant effect on the calculated anisotropies.

In an attempt to demonstrate the size of these effects, the plot in Fig. 5.16(b) shows the ratio of the reduced anisotropy for the best calculation to the reduced anisotropy of the other calculations. A ratio of unity means there is no anomaly, curve (1) in Fig. 5.16(b). A ratio greater than unity means the calculated anisotropies underpredict the experimental anisotropies. In the following, it is worth emphasising that there is a complex interdependence between each of the parameters in these calculations. For example, it is difficult to isolate the effects of the incorrect compound nucleus angular momentum distribution on the calculated anisotropies, since this also means fitting the lower values of $\sigma_{\text{ER}}(E)$ (which are responsible for the incorrect angular momentum distribution) and this alters the saddle-point temperature dramatically. Since these calculations were performed with a Monte Carlo code, the curves in Fig. 5.16(b) have a statistical uncertainty of $\approx 2\%$. The broken line in Fig. 5.16(b) represents the results of a calculation that was performed in a manner identical to the best calculation, except that the statistical model parameters of JOANNE have been adjusted to fit the smaller values [41] of $\sigma_{\text{ER}}(E)$. This calculation also includes the small effects of the different $\langle J^2 \rangle$ values. Above the single barrier, the reduced anisotropies from the smaller $\sigma_{\text{ER}}(E)$ are too low by $\approx 20\%$. The ratio is reasonably constant for the energies above the single barrier, reflecting the fact that the energy dependence above $B_0$ of the evaporation residue excitation functions of Ref. [41] and this work [see Fig. 4.10(b)] are similar. On the basis of Eq. (2.63) in Chapter 2, this ratio can be interpreted as the ratio of $T$ extracted by fitting the larger ER cross-section, to the $T$ from the smaller values of $\sigma_{\text{ER}}(E)$. Indeed, this was supported by the observation that the ratios of the mean saddle-point temperatures
from these two calculations had a very similar magnitude and energy dependence to the ratios of the reduced anisotropies, curve (2) in Fig. 5.16(b).

Calculating the anisotropy with the assumption of a distribution of excitation energies, accounts for some of the increase in the calculations of the theoretical anisotropies. The dotted line in Fig. 5.16(b) is similar to the TSM calculation of Ref. [38], where the temperature was evaluated assuming a mean value of \( E^* \) at the saddle point, and the low values of \( \sigma_{ER}(E) \) from Ref. [41]. This has, approximately, an additional 10% effect on the reduced anisotropies.

In summary, fission fragment anisotropies for the reaction \( ^{16}\text{O} + ^{208}\text{Pb} \) have been compared with theoretical values obtained from improved transition state model calculations. The data and the calculations are in agreement, even at energies below the single fusion barrier. It has been shown that these comprehensive experimental anisotropies and the more accurate inputs to the transition state model calculations contribute incrementally to this agreement. In particular, reproducing the correct evaporation residue survival probability has a significant effect on the calculated fission fragment anisotropies. The resolution of the anomaly in the fission fragment anisotropies for the \( ^{16}\text{O} + ^{208}\text{Pb} \) reaction implies that this technique can be used for determining the mean-square angular momentum for fusion. To obtain this information, the effective moment of inertia and the saddle-point temperature must be known accurately, and for systems like \( ^{16}\text{O} + ^{208}\text{Pb} \), which has no suitable \( \alpha \)-calibration reaction, one must rely on model calculations to determine these quantities. However, because of the remaining uncertainties in the statistical model calculations, namely in the parameters \( J_{\text{eff}} \) and \( T \), the information on angular momentum distributions obtained using fission fragment angular distributions is less reliable than precise fusion cross-section measurements. Furthermore, it is demonstrated in the next Section, using the \( ^{28}\text{Si} + ^{208}\text{Pb} \) reaction, that the utility of this technique is limited because of the prevalence of quasi-fission.

### 5.3 The \( ^{28}\text{Si} + ^{208}\text{Pb} \) reaction

In the discussion of the \( ^{16}\text{O} + ^{208}\text{Pb} \) barrier distribution, there was some evidence that the \( ^{208}\text{Pb} \) may sustain a double-phonon excitation. Further evidence of the role of multiphonon coupling in fusion can be obtained by studying a different reaction, using a heavier projectile incident on the same target. The heavier projectile could be expected to excite the double-phonon mode in \( ^{208}\text{Pb} \) more strongly. In the \( ^{28}\text{Si} + ^{208}\text{Pb} \) reaction, the projectile should play a larger role.
than the $^{16}$O did in the $^{16}$O + $^{144}$Sm and $^{16}$O + $^{208}$Pb reactions. The approaches taken in previous analyses of fusion reactions involving the $^{28}$Si nucleus have varied. Some authors [56, 137, 31] have treated it as statically deformed, while Ref. [137] also considered the effects of coupling only to the first 2$^+$ and 4$^+$ excited states in $^{28}$Si. It will be shown below that $^{28}$Si should not be treated like a true rotational nucleus, nor can it be described in terms of a pure vibrator.

**Coupled-channels analysis of the $^{28}$Si + $^{208}$Pb reaction**

In the first step in the analysis of the $^{28}$Si + $^{208}$Pb reaction, $^{28}$Si is considered to be inert. The barrier distributions were again calculated using the codes CCMOD and CCMPH, with the single barrier parameters given in Table 4.2 in Chapter 4. The solid line in Fig. 5.17(a) represents the barrier distribution produced by coupling to the 3$^- \otimes 3^-$, 2$^+ \otimes 2^+$ and 3$^- \otimes 2^+$ multiphonon states in $^{208}$Pb. As shown in the previous Section, this coupling scheme produces essentially three barriers. For the $^{28}$Si + $^{208}$Pb reaction, two of them are very close together giving the large peak around 130 MeV seen in Fig. 5.17(a). For comparison, the calculation with just the single-phonon states is shown by the broken line in Fig. 5.17(a). It is clear from these calculations that neither description is an adequate representation of the experimental barrier distribution.

As expected, the above result suggests that the structure of the $^{28}$Si is also influencing the fusion process. The next step is to consider the excited states in $^{28}$Si. The two barrier distributions in Fig. 5.17(b) are calculations with the first excited 2$^-$ state in $^{28}$Si, at 1.779 MeV with $\beta_2 = 0.522$, evaluated using a radius parameter of 1.06 fm, for the single-phonon (broken line) and the double-phonon (solid line) coupling schemes in $^{208}$Pb. Again, both calculations fail to reproduce the main features of the experimental barrier distribution. The inclusion of the projectile state increases the coupling present, forcing the two barriers further apart in energy giving a barrier distribution in disagreement with the experimental one. Consequently, $^{28}$Si does not behave like vibrational nucleus in fusion.

The barrier distributions shown in Fig. 5.17(c) result from treating the $^{28}$Si as statically deformed. The deformation parameter for $^{28}$Si was taken to be $\beta_2 = -0.552$. The barrier distributions in Fig. 5.17(c) are for the single-phonon (broken line) and double-phonon (solid line) coupling schemes in $^{208}$Pb. The latter barrier distribution now reproduces some of the basic features present in the data, although there is clearly still some disagreement. The high energy of the excited states, and the poor rotational nature of $^{28}$Si, may mean coupling to fewer states is required to explain the data. Such calculations are currently not
FIG. 5.17: The experimental barrier distribution for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction is compared with calculations assuming a single-phonon (broken line) and multiple-phonon (solid line) coupling scheme in $^{208}\text{Pb}$. In these calculations the $^{28}\text{Si}$ is treated (a) as inert, (b) as a quadrupole vibrator and (c) with static oblate deformation.
possible with codes such as CCMOD. Further investigation is warranted. There is also some degree of scatter in the data points in the high energy region of the barrier distribution and it would be advantageous to repeat the measurement here. It is possible to conclude, however, that, as in the $^{16}\text{O} + ^{208}\text{Pb}$ reaction, single-phonon coupling schemes are inadequate for reproducing features of the measured barrier distributions. It has been shown that multiphonon excitations can provide a better description of the barrier distributions for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction. The situation is made more complex because of the significant influence of the structure of the $^{28}\text{Si}$ projectile.

5.3.1 Fission fragment anisotropies

In this Section, the sensitivity of the fission fragment anisotropies to the presence of quasi-fission is discussed. The $^{28}\text{Si} + ^{208}\text{Pb}$ reaction is then used to compare the relative merits of two techniques for determining information on fusion angular momentum distributions.

Comparison of the experimental anisotropies with the TSM

The experimental fission fragment anisotropies for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction are compared to the theoretical TSM calculations in Fig. 5.18. The inputs were generated in a similar manner to the $^{16}\text{O} + ^{208}\text{Pb}$ calculations. The effective moment of inertia was obtained as described in Section 5.2.1, with $J_{\text{eff}} = 7600 \text{ u fm}^2$ for the zeroth partial wave.

When calculating the temperature at the saddle point for the heavier $^{28}\text{Si} + ^{208}\text{Pb}$ reaction, the following factors must be taken into account. The pre-scission neutron multiplicity for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction is larger than that for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction because, for the same ratio $E_{\text{c.m.}}/B_0$, the former system has more excitation energy than the latter system. At $B_0$, the $^{16}\text{O} + ^{208}\text{Pb}$ reaction has an excitation energy $E^* = 25 \text{ MeV}$ compared to the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction, with $E^* = 39 \text{ MeV}$. However, for the $^{28}\text{Si} + ^{208}\text{Pb}$ system, an appreciable fraction of these pre-scission neutrons is emitted during the saddle-to-scission time. This is due to the larger excitation energy, and also to the larger fissility for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction. The larger fissility means the saddle point shape is more compact. If the shape of the scission configuration does not change, then a larger fissility implies a larger difference between the saddle and scission shapes, and thus a longer saddle to scission time [138]. The system will also gain more excitation energy as it proceeds from saddle to scission due to the larger fission Q-value. If
the scission point was more compact for larger fissilities, then this argument may not necessary be true. However, if this was the case, then the total kinetic energy for fission would increase more strongly than $Z_{\text{CN}}^{2}/A_{\text{CN}}^{1/3}$. Since the experimental evidence from the Viola systematics shows that this is not the case [99], therefore the above argument is valid.

The statistical model time scale for the decay of the $^{236}$Cm compound system is likely to be less than the dynamical fission time scale, completely opposite to the case for the $^{16}$O + $^{208}$Pb reaction. In the former case, to obtain an accurate estimate of the saddle-point temperature, the statistical model calculations must fit the observed pre-scission neutron multiplicities less the saddle-to-scission contribution, the latter of which is unfortunately not measurable. In the following TSM calculations, two extreme assumptions regarding the neutron emission are made. In the first, it is assumed that the pre-saddle delay time is zero and the statistical model is used to estimate the saddle-point temperature. The calculations were performed with $a_{f}/a_{n} = 1 = k_{f}$ [81], since there are no data available for the ER cross-sections. In the second, it is assumed that all the fission delay time is pre-saddle, and so all the neutrons are emitted before the system crosses the saddle point.

The angular momentum distributions for the $^{28}$Si + $^{208}$Pb reaction were obtained by fitting the measured fusion cross-sections using the eigenbarrier method described in Section 5.2. The statistical model code JOANNE was used to fit the only available measurement of the pre-scission neutron multiplicity [139]. The measured pre-scission neutron multiplicity at $E_{\text{lab}} = 159$ MeV was $1.8 \pm 0.4$. For this fit $a_{n} = 8.8$ MeV$^{-1}$. The temperature distributions from the JOANNE code were then used to calculate the fission fragment anisotropies over the range of energies measured in this work. These TSM calculations are compared with the experimental data in Fig. 5.18, where the dotted line represents the anisotropy assuming the pre-saddle delay time to be zero, and the broken line, when all the neutron emission is taken to be pre-saddle. Since the actual breakdown of the number of pre-scission neutrons is unknown, the true calculation lies somewhere between these two extremes.

The anisotropies from the TSM calculations underpredict the experiment at essentially all energies. It is likely that the uncertainty in the effective moment of inertia could be at most ~10%, so this cannot account for the anomaly. There is also no reason to suspect an incorrect estimate of the saddle-point temperature, since this was calculated successfully for the $^{16}$O + $^{208}$Pb reaction. As suggested in Refs. [34,36], the most likely explanation for this disagreement is the presence of
FIG. 5.18: The experimental fission fragment anisotropies are compared with the transition state model calculations corresponding to the two extreme treatments of the excitation energy lost to pre-saddle emission. The dotted line assumes that the pre-saddle delay is zero, which results in a larger saddle-point temperature and hence a smaller anisotropy. The broken line assumes all the delay time is pre-saddle leading to a smaller temperature and larger anisotropy.

quasi-fission. In the earlier measurement of Ref. [34], quasi-fission was claimed to be a significant fraction of the total fusion-fission yield at energies above $E_{c.m.} = 140$ MeV. If the anisotropy for quasi-fission were known then the relative fusion-fission and quasi-fission yields could be obtained from these anisotropies.

Implications from the prevalence of quasi-fission

The fact that the anisotropies for the $^{28}$Si + $^{208}$Pb reaction are anomalous at all energies measured here implies that they cannot be used to extract the mean-square angular momentum for complete fusion. This will also be the case in all other systems where quasi-fission is significant. According to Ref. [36], reactions on heavy targets ($A > 230$), involving projectiles around $^{24}$Mg and heavier, roughly mark the onset of quasi-fission reactions. Recently, a measurement [39] of the fission fragment anisotropies for the $^{16}$O + $^{238}$U reaction found that quasi-
fission is significant, even for a projectile as light as the $^{16}$O nucleus. The anisotropies were measured for energies around the single barrier region using the folding angle technique to select those fission events associated with full momentum transfer only. These anisotropies were anomalously large compared with the TSM calculations by around 20% for energies $E_{\text{c.m.}}/B_0 \sim 1.10$. As the bombarding energy decreased through the barrier region, the anisotropy rose sharply then ultimately seemed to saturate at energies around 10% below $B_0$. This was suggested to be evidence for a relationship between the anisotropy and the height of the fusion barrier encountered in a collision [39]. The distribution of barrier heights is due to the static deformation of the target nucleus. The shape of the barrier distribution for the $^{16}$O + $^{238}$U reaction was described using a simple geometric calculation, which included the prolate deformation of the $^{238}$U nucleus. The observed feature in the anisotropies can thus be explained. Collisions between the projectile and the tips of the deformed nuclei in the target produce an elongated dinuclear system, which undergoes quasi-fission, whilst collisions with the sides produce a more compact dinucleus, which equilibrates thermally inside its fission barrier, leading to a true compound fusion-fission reaction.

In systems similar to the $^{16}$O + $^{238}$U reaction, anomalous anisotropies have also been observed at energies below $B_0$. The reactions $^{16}$O + $^{232}$Th [34,140,27,63], $^{19}$F + $^{232}$Th [63,141] and $^{12}$C + $^{236}$U [140,27] all exhibit some anomaly. These targets have similar deformations to the $^{238}$U nucleus, so the feature in the fission fragment anisotropies could also be expected for these systems. This would also imply the presence of quasi-fission. Thus, the fission fragment technique for extracting $\langle l^2 \rangle$ could not be applied to these reactions. However, this is not the case if fusion cross-sections are precisely determined. Following the arguments of Hinde [139], it is shown below that even for systems where quasi-fission is significant, information on the angular momentum distributions can be obtained using measured fusion cross-sections and barrier distributions.

The fusion and quasi-fission processes

In order to fuse, two nuclei must overcome the interaction potential that exists between them. The interaction potential, or fusion barrier, is the sum of the Coulomb and nuclear potentials, and a centrifugal term. In terms of the coupled-channels formalism discussed in Chapter 2, the interaction potential couples the relative motion to the internal degrees of freedom. For collisions that involve relatively small charge products ($Z_1Z_2 < 1600$, although the distinction is not likely to be sharp), the density of overlap between the two nuclei is not great and
the coupling to the internal degrees of freedom occurs via relatively few channels. The distribution of barriers arising from such couplings is centred about the uncoupled barrier. The potential which applies in the above situation has been called the *sudden potential* [142] or *fusion potential* [139]. It is ‘sudden’ in the sense that the relative motion between the two nuclei is fast, and that their densities remain frozen during the collision.

For reactions with large charge products \(Z_1Z_2 > 1600\), there is an increasingly large density overlap and the coupling will involve many more channels. The coupled-channels picture becomes no longer tractable and a macroscopic description [143,142] is required. For these heavy systems, as the two nuclei approach each other in the initial stages of the collision, the energy of the relative motion is lost to internal frictional forces. Extra kinetic energy is required, the ‘extra-push’ energy [143], to overcome the frictional energy losses and ensure the system proceeds to fusion. The effect of the extra-push energy is to shift the fusion barrier to higher energies.

The fusion cross-sections measured in this work are defined in terms of the end products of the reaction, the fission fragments or evaporation residues. However, these products are not determined solely by the fusion potential. Fusion is not necessarily identical to compound nucleus formation. During the evolution of the fused system, the two nuclei share thermal energy, mass and charge between each other, and during this process, the system evolves to form a necked and elongated dinucleus. The relevant interaction potential for this situation is the called the *adiabatic or fission potential*, since all degrees of freedom, except those describing the shape, have had time to equilibrate [142]. Further evolution of the dinucleus can be described, for example, in terms of a sequence of shapes consisting of two spheroids, with a separation \(D\), connected by a neck [144]. There are three major shape configurations. The *contact* configuration (at the fusion barrier), where the nuclei are considered as two spheres. This configuration can be quantified in terms of the mass asymmetry of the entrance channel, defined as \(\alpha = \frac{(A_1 - A_2)}{(A_1 + A_2)}\). The *conditional* saddle point configuration, defined as the maximum in the potential energy as \(D\) is varied and all other shape parameters are adjusted to minimise the potential energy, except for a particular (frozen) mass asymmetry. The true (unconditional) saddle point configuration is the corresponding maximum in the potential energy when \(\alpha\) is unconstrained. For heavy nuclei, this occurs at \(\alpha = 0\); that is, for a symmetric configuration.

For lighter reactions like \(^{16}\text{O} + ^{144}\text{Sm}\), the contact configuration is more compact (smaller \(D\)) than the unconditional saddle point. So if the system has enough
FIG. 5.19: A schematic illustration of the potential energy as a function of the distance between mass centres of two colliding nuclei. The solid curve represents the fusion potential. The broken curve represents the fission potential. In panel (a) the contact configuration is more compact than the (unconditional) saddle point configuration, and so if fusion occurs, then so does compound nucleus formation. In panel (b) the saddle point of the fission potential is too compact for the trajectory shown and the system ‘fissions’ (quasi-fissions) without passing through the compound nucleus configuration.

energy to reach the contact point, then it will fuse inside its unconditional saddle point and form a compound nucleus. This is shown schematically in Fig. 5.19(a). The compound nucleus decays by evaporation, leaving behind an evaporation residue which is indisputable evidence of fusion. For heavier systems like $^{16}$O + $^{208}$Pb, fission becomes the dominant decay mode. Here, the fission products are still associated with the decay of a fully equilibrated compound system, as attested by the anisotropies. For the $^{16}$O + $^{144}$Sm and $^{16}$O + $^{208}$Pb collisions, the definition of fusion in terms of the fusion potential and the definition in terms of the fission potential are equivalent.

However, in a collision that involves nuclei that are heavier still, like the $^{28}$Si + $^{208}$Pb reaction, quasi-fission becomes significant [37]. In terms of the above description, quasi-fission occurs when the system fuses (i.e. is trapped inside the fusion potential), but the unconditional saddle point for the fission potential is too compact, and the dinucleus reseparates into two fragments, as shown schematically in Fig. 5.19(b). In this case the system does not form a true compound system. Here the above two definitions of fusion are at odds, since in
terms of the fusion potential quasi-fission constitutes fusion, whilst in the fission potential it does not [139].

This conflict can be resolved by ensuring the appropriate data are compared with the appropriate model. If the model considers only reactions concerned with the fusion potential, then quasi-fission is part of the fusion yield. If the model describes the evolution of the system over the fission potential, quasi-fission is not part of fusion. Thus, this definition shows why it is valid to discuss 'fusion' excitation functions and 'fusion' barrier distributions, even when they include a significant proportion of quasi-fission, as in the $^{28}\text{Si} + ^{208}\text{Pb}$ case. Consequently, the fusion cross-sections and barrier distributions can be used to obtain information about the angular momentum distributions for fusion, even in such cases where quasi-fission is significant.

Summary of fission fragment anisotropy measurements

Fission fragment angular distributions have been measured for the $^{16}\text{O} + ^{208}\text{Pb}$ and $^{28}\text{Si} + ^{208}\text{Pb}$ reactions at energies around the fusion barrier. The experimental anisotropies were compared to transition state model calculations. The importance of precise determination of the fusion cross-sections was underscored in the TSM analyses. The measured fusion cross-sections and the barrier distributions enabled an accurate estimate of the angular momentum distributions for fusion, a quantity essential for the description of the decay of compound nuclei. Often, the diffuseness of the angular momentum distribution is a quantity that is fitted as a parameter in statistical model analyses. As demonstrated here, there is now one less uncertain parameter in these types of statistical model analyses. The ER cross-section has also been shown to be critical in determining the saddle-point temperature of the fissioning system. For the $^{16}\text{O} + ^{208}\text{Pb}$ reaction, agreement between the measured and calculated anisotropies was obtained using the standard models of fusion and fission.

The same models were used to compare the anisotropies for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction. The disagreement observed between the data and TSM calculations is evidence for quasi-fission in the $^{28}\text{Si} + ^{208}\text{Pb}$ system [34]. This implies that the fission fragment angular distribution technique cannot be used to extract information on the angular momentum distributions for fusion. It was demonstrated that it is still valid to do so using precise measurements of the fusion excitation functions, even in a reaction which includes quasi-fission. Therefore, this technique is a better approach for extracting information on fusion angular momentum distributions.
Chapter 6

CONCLUSION

Fusion excitation functions have been measured for the reactions $^{16,17}$O $+$ $^{144}$Sm and $^{16,28}$Si $+$ $^{208}$Pb, for energies from around 10% below the fusion barrier to 20% above the barrier in energy steps of typically 0.5 MeV. The range of $Z_1 Z_2$ of the above reactions meant both the evaporation residue and fission cross-sections had to be measured. All cross-sections were determined relative to the Rutherford scattering cross-sections measured in two monitor detectors located either side of the beam axis. The ER cross-sections for the $^{16,17}$O $+$ $^{144}$Sm reactions were determined using a velocity filter to separate the forward-going ERs from the elastically scattered beam. The ERs were identified by their energy loss and time-of-flight in a MWPC located behind the velocity filter. The evaporation residue cross-sections for the $^{16}$O $+$ $^{208}$Pb reaction were measured by detecting the α-decay of the residues and their daughter nuclei.

The fission cross-sections were measured using one or two large-area MWPCs. The fission fragments were identified using their energy loss and time-of-flight with respect to the pulsed beam. Each detector covered a large angular range in both θ and φ, and this allowed the fission fragment angular distributions to be determined efficiently with high precision.

The distribution of fusion barriers [15] for each system was determined from the precise fusion cross-section data. A similar feature was observed in the barrier distributions for both the $^{16,17}$O $+$ $^{144}$Sm reactions. In both reactions, the barrier distribution was split into two almost separate components, a main peak and a second peak at a higher energy. This double-peaked feature was attributed to the effects of the excitations of $^{144}$Sm. The barrier distribution for the $^{17}$O $+$ $^{144}$Sm
reaction was clearly different at energies below the main barrier compared to the $^{16}\text{O} + ^{144}\text{Sm}$ reaction. The shape of the main peak of the barrier distribution for the $^{170}\text{O}$-induced reaction was smaller in size and extended to lower energies than the $^{16}\text{O}$-induced reaction. Such a shape could not be reproduced by altering the nuclear potential parameters in a coupled-channels analysis. This feature can only be due to the effects of a positive Q-value reaction. The only candidate is the single-neutron stripping reaction.

Simplified coupled-channels calculations were performed to model the effects of these inelastic and transfer channels on fusion. A calculation including the first two excited states in $^{144}\text{Sm}$ was found to be in very good agreement with the data. This calculation demonstrated that the inelastic state with the largest coupling strength is largely responsible for the double-peaked feature in the barrier distribution. A coupled-channels calculation, including the single-neutron transfer channel, gave a good representation of the shape of the barrier distribution for the $^{170} + ^{144}\text{Sm}$ reaction, allowing an estimate of the transfer coupling strength.

These calculations also showed that the effects on the barrier distribution of individual transfer channels was minor relative to the inelastic channels. Whilst it was possible to observe the effects of a positive Q-value transfer, a transfer channel which has a negative Q-value is very difficult to identify, even in the presence of only weakly coupled inelastic channels, such as those in $^{144}\text{Sm}$. Given the weak signature of the transfer channel in the fusion barrier distribution, a measurement of the $^{144}\text{Sm}(^{170},^{16}\text{O})^{145}\text{Sm}$ transfer cross-section would be valuable.

Exact coupled-channels calculations were performed for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction. The isocentrifugal approximation, often used in the coupled-channels calculations, was shown to be good for the $^{16}\text{O} + ^{144}\text{Sm}$ reaction. The exact calculations were also compared with calculations from the simplified code, which solved the coupled-channels problem in the eigenchannel representation. Qualitatively, there was found to be good agreement between the barrier distributions from the two different coupled-channels codes. The quantitative differences between the simplified and exact coupled-channels calculations were suggested to be due to the simplified model’s method of evaluating the weights of the eigenbarriers. Some of the difference is also likely to be due to the approximate treatment of the excitation energies, although this approximation appears to be reasonable for excited states with energies $\lesssim 2\text{ MeV}$.

Projectile excitation appears to play an insignificant role in the $^{16}\text{O} + ^{144}\text{Sm}$ fusion reaction. Both barrier distributions, from the simplified and exact coupled-channels calculations, were found to be in disagreement with the experimental
data when coupling to the projectile state was included in the coupling scheme. This indicates that there is still something lacking in the understanding of the behaviour of the lighter partner in heavy-ion fusion reactions. To remedy this situation, further measurements of the barrier distribution are required for a range of projectiles, such as $^{12}$C, $^{24}$Mg and $^{28}$Si, incident on a 'simple' target such as $^{144}$Sm. Any deviation in the measured barrier distribution from the shape expected with the simple inelastic coupling scheme in $^{144}$Sm may reflect the influence of the structure of the projectile.

The distribution of fusion barriers for the $^{16}$O $+$ $^{208}$Pb reaction could not be successfully interpreted using a simple inelastic coupling scheme, in contrast to the $^{16}$O $+$ $^{144}$Sm reaction. A better representation of the experimental barrier distribution was achieved when multiphonon states in $^{208}$Pb were included in a simplified coupled-channels calculation. Evidence for the presence of multiphonon channels also came from the shape of the barrier distribution for the $^{28}$Si $+$ $^{208}$Pb reaction. Although further more exact calculations are needed to confirm the presence of the multiphonon channels, the results presented here indicate that fusion barrier distributions can reveal interesting details about more complex properties of the interacting nuclei.

There is no longer an anomaly in the fission fragment anisotropies for the $^{16}$O $+$ $^{208}$Pb reaction. Using the angular momentum distributions obtained from the precise fusion cross-section data, and a more accurate estimate of the saddle-point temperature, the transition state model was able to reproduce the experimental anisotropies to within the level of uncertainty in the input parameters. It was shown that the ER cross-section is critical for determining the correct saddle-point temperature of the fissioning system. The ER cross-sections for the $^{16}$O $+$ $^{208}$Pb reaction have been difficult to determine accurately [40, 41]. The consistency of the results of this work and those of Ref. [42], confirm that these cross-sections are now known to a satisfactory accuracy. Structure in the ER excitation function for the $^{16}$O $+$ $^{208}$Pb reaction was observed. This was suggested to be evidence for the variation in cross-section due to the xn-evaporation channels in competition with fission. An extension of this excitation function to higher energies would be interesting. The ER cross-sections for the $^{16}$O $+$ $^{208}$Pb reaction provided an additional constraint on the statistical model description of its decay.

The success of the transition state model in reproducing the fission fragment anisotropies for the $^{16}$O $+$ $^{208}$Pb reaction, establishes this as a method for determining the mean-square angular momentum distributions for fusion. However, this method can only be used to obtain angular momentum distributions to the
level of uncertainty in the input parameters of the model, namely the saddle-point temperature and the effective moment of inertia. For reactions inaccessible by α-calibration measurements, statistical models are required to estimate these quantities. This estimation in turn requires knowledge of the amount of excitation energy removed by particle emission before fission and, as shown here, an accurate estimate of the ER cross-section for systems where this cross-section is significant. The fission fragment angular distribution technique must also discriminate against contamination from transfer-induced fission reactions. These factors require considerable experimental effort, and perhaps reduce the attractiveness of this method.

The application of the fission fragment angular distribution technique was found to be more limited than previously thought. The $^{28}$Si + $^{208}$Pb reaction is not amenable to this technique because of the effects of quasi-fission [34]. In reactions such as $^{16}$O + $^{238}$U, the deformation of the heavier partner has a dramatic influence on the fission fragment anisotropies [39] and hence on the competition between the fusion-fission and quasi-fission processes. The presence of quasi-fission means that information about the angular momentum distributions for fusion cannot be extracted from reactions where this feature in the anisotropies is observed. It was shown, however, that information about the fusion angular momentum distributions could still be extracted using precise fusion cross-section data, even when quasi-fission was significant. Thus, this method can be applied to study the angular momentum distributions for heavier systems.

The distribution of fusion barriers is fundamental to the description of the fusion process. Its shape reflects the relevant degrees of freedom of the colliding system and their couplings to the relative motion. The fact that this distribution can be determined directly from the experimental data is significant. This enables detailed information about nuclear structure effects on fusion to be determined. By representing both the experimental data and theoretical calculations in this way, the effects of the couplings are much more clearly seen. The barrier distributions, obtained from the precise fusion cross-sections, were shown to have applications in other aspects of fusion and fission reactions. Thus, fusion barrier distributions are an important tool in the development of a more complete description of the dynamics of the fusion and fission reaction processes.
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[100] ORNL sample no. 167090.


Appendix A

TABULATED DATA

A.1 The fusion cross-sections

The fusion cross-sections for all four reactions are listed in the following tables, for convenience. The fusion cross-sections for the $^{16,17}$O + $^{144}$Sm reactions are given in Table A.1. The beam energies have been corrected for the energy loss in the target. The fusion cross-section is assumed to be equal to the evaporation residue cross-section. The fusion cross-sections have been corrected for the target contamines, as discussed in Chapter 4. The uncertainties quoted are the random uncertainties only. The random uncertainties consist of contributions from the statistical uncertainties and the correction for the target contamination. Where two measurements were made at the one energy, the average cross-section is given.

The fusion cross-sections for the $^{16}$O + $^{208}$Pb reaction are the sum of the ER cross-sections and the fission cross-sections. The ER cross-sections, as determined from the $\alpha$-decay technique, are given in Table A.2. The beam energy has been corrected for the energy loss in the PbS target. The quoted uncertainties were calculated from the counting statistics in the $\alpha$-spectrum and the monitor detectors. The fission cross-sections for the $^{16}$O + $^{208}$Pb reaction are given in Table A.3. The cross-sections are listed for each of the three passes through the excitation function. The total fusion cross-sections, taken as the sum of the ER and fission cross-sections, are given in Table A.4. At energies where the ER cross-section was not available, an interpolated ER value was used. The uncertainties on the fusion cross-sections are taken to be $\pm 1\%$, or, at the lowest beam energies as given by the counting statistics, whichever was larger.

The fission cross-sections for the $^{28}$Si + $^{208}$Pb reaction are given in Table A.5. For this reaction, the fusion cross-section was equated with the fission cross-sections. The uncertainties are $\pm 1\%$. 

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Table A.1: The fusion cross-sections for the $^{16,17}$O + $^{144}$Sm reactions. The cross-sections have not been renormalised for the 2% correction due to the efficiency of the MWPC. The uncertainties quoted are the random uncertainties only. The cross-sections and their uncertainties are in millibarns.

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<th>$\delta\sigma_{\text{fus}}$</th>
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A.2 The fission fragment anisotropies

The fission fragment anisotropies, obtained from the measured angular distributions for the $^{16}\text{O} + ^{208}\text{Pb}$ and $^{28}\text{Si} + ^{208}\text{Pb}$ reactions, are given in Tables A.6 and A.7, respectively. The error bars on the anisotropies are from the statistical uncertainties in the angular distributions.
Table A.3: The fission cross-sections for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction for each of the three separate passes through the excitation function.

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174
Table A.4: Total fusion cross-sections for the $^{16}\text{O} + ^{208}\text{Pb}$ reaction. The uncertainties are assumed to be ±1%, for all but the lowest energies. Although the statistical uncertainties were much more precise than this, the uncertainties were assumed to be ±1% because of the scatter between experimental runs.

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Table A.6: Fission fragment anisotropies for the $^{16}$O + $^{208}$Pb reaction. The uncertainties are due to the counting statistics.

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Table A.7: Fission fragment anisotropies for the $^{28}\text{Si} + ^{208}\text{Pb}$ reaction. The uncertainties are due to the counting statistics.

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