THE COMPOUND NUCLEUS PROCESS IN REACTION MECHANISM
AND NUCLEAR STRUCTURE STUDIES

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ABSTRACT AND ACKNOWLEDGEMENTS

The compound nucleus process, as described in the theory of Wolfenstein, Hauser and Feshbach and Satchler, has been used from two different points of view. In part I of this thesis the theory has been used in combination with the optical-model and the distorted-wave Born approximation for reaction mechanism studies, the main objective being to allow optical-model parameters for different nuclei to be extracted and to compare predictions of the above mentioned models to those of the statistical fluctuation model of Ericson and Brink and Stephen. If the fluctuation model and the other mentioned models give consistent results for elastic and inelastic proton scattering experiments, then the fluctuation theory will be suitable for estimating the amount of a reaction which proceeds by a direct process in reactions for which the direct reaction theory is not well known.

The mean level widths at high excitation energies in the compound nuclei $^{24}$Mg and $^{32}$S have been investigated as a function of energy and angle and compared to theoretical predictions given by Ericson.
In calculating the mean level width and the fraction of direct reaction Fourier methods are used as well as the method given by Ericson and Brink and Stephen.

Evidence for intermediate structure is found in two of the reactions considered. Theoretical analyses show that it is possible to understand the observed selective occurrence of the gross structure. The presence of intermediate structure complicates the analyses and the interpretation of the results from the different models. However, it is shown that the intermediate structure is treated differently by each model, and by redefining appropriate parameters of the models consistency between the models results.

In part II of the thesis the theory of the compound nucleus process is employed to investigate spins of levels in the nuclei $^{62}$Cu and $^{64}$Cu. Gamma-ray angular distributions following (p,n) reactions are compared to theoretical predictions for different spin sequences. The spin of the 426keV level in $^{62}$Cu was measured to be $3^+$. Unique spin assignments were obtained for the 159 keV ($2^+$), 278 keV ($2^+$) and 609 keV ($2^+$) levels in $^{64}$Cu, and the most probable spins for the other levels were found to be $344$ keV ($1^+$), $362$ keV ($3^+$), $663$ keV ($1^+$), $739$ keV
(2\(^+\) or 3\(^+\)), 746 keV (3\(^+\)), 878 keV (1\(^+\) or 2\(^+\)), 895 keV (3\(^+\)) and 927 keV (1\(^+\)). Multipole mixing ratios of the γ-rays observed were also obtained.

Some collaboration was involved in all experiments to be considered. All laboratory work was equally shared between my supervisor, Dr P.J. Dallimore, and myself during the first year. When Dr W.F. Davidson joined the group after this period all remaining running time was shared equally between the three of us. I was responsible for all the remaining work in connection with the analyses of proton elastic and inelastic scattering from \(^{23}\)Na and did approximately half of the work involved in the analyses of the other experiments. The more complicated computer programs necessary for the analyses were written by Dr Dallimore (the combined optical-model and Hauser-Feshbach program used in chapter 1; the program for calculating the mean level width \(\Gamma_j\) described in section 3.5.2.) and myself (the Fourier analysis program used in chapter 3; the program for calculating the nuclear damping coefficient described in section 3.3). All other computer programs necessary for the analyses were written by Dr Dallimore, myself or were already available in the department with the exception of the DWBA program.
It is a pleasure to thank first and foremost my supervisor Dr P.J. Dallimore for help with the analyses, daily discussions and friendship over the past three years. I also want to thank Dr W.F. Davidson for his encouragement and help since joining the group. Drs B.A. Robson, T.R. Ophel and P.B. Treacy, Professor D.C. Peaslee, other staff members and fellow students are thanked for numerous helpful discussions. I am deeply indebted to Professor K.J. Le Couteur for his great interest and friendly help on several occasions. Thanks are also due to Dr G.M. Crawley at Michigan State University for carrying out the DWBA calculations, to Dr T. Ericson at CERN for helpful comments on parts of this work, to Miss N. Chin for typing the rough draft and Miss P. Fryer for typing the final manuscript. I record my gratitude to the Australian National University for the award of the Research Scholarship. Finally I wish to thank Professor Sir Ernest Titterton for his detailed constructive criticism on several occasions, for inviting me to work in his laboratory and for firm support and encouragement during my stay here.

Part of the work described in this thesis has been reported in the following publications:

1) Level widths in the compound nucleus $^{32}$S from the reaction $^{31}$P(p,p$_0$), J. Hellström and P.J. Dallimore, Nucl. Phys. A125 (1969) 684-694


4) The reaction $^{23}$Na(p,p) in the energy range 8.0 to 12.0 MeV. (I) Analysis of average elastic cross sections, J. Hellström, P.J. Dallimore and W.F. Davidson, Nucl. Phys. A132 (1969) 581

5) Evidence for intermediate structure in the compound nucleus $^{24}$Mg, K.J. Le Couteur, P.J. Dallimore, W.F. Davidson and J. Hellström, Contribution to the International Conference on Properties of Nuclear States, Montreal, Canada 1969


7) The reaction $^{23}$Na(p,p) in the energy range 8.0 to 12.0 MeV. (II) Evidence for and interpretation of intermediate structure, J. Hellström, P.J. Dallimore and W.F. Davidson, accepted for publication in Nucl. Phys.
8) Gamma-ray angular distributions following the $^{64}\text{Ni}(p,n\gamma)^{64}\text{Cu}$ reaction, W.F. Davidson, P.J. Dallimore and J. Hellström, accepted for publication in Nucl. Phys.

9) The reaction $^{23}\text{Na}(p,p)$ in the energy range 8.0 to 12.0 MeV. (III) Analysis of average inelastic cross sections, fluctuations and model comparisons, J. Hellström, P.J. Dallimore and W.F. Davidson, accepted for publication in Nucl. Phys.

10) Analysis of the reaction $^{31}\text{P}(p,p_0)$ in the energy range 8-10 MeV, P.J. Dallimore and J. Hellström, accepted for publication in Nucl. Phys.

No part of this thesis has been submitted for a degree at any other University.

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INTRODUCTION

Man had sought to understand his environment for thousands of years with limited success until the EXPERIMENT was introduced in the 16th century and physics was born. Mere observation of phenomena was replaced by manipulation of the environment which ever since has been the basis of physics. To learn how bodies fell the generation of Galilei let them fall and studied their motion, and experiences from such observations formed the basis for the laws of physics.

Nowadays, to describe phenomena in atomic, nuclear or elementary particle physics, physicists often introduce concepts and analogies from previous generations and from everyday life to make the equations more comprehensible. In this way the theoretical physicist provides the experimentalist with a model, which might exaggerate certain characteristics, give birth to new concepts, or form a basis for dialogue. In nuclear physics, concepts such as nuclear radius, potential wells and surface absorption have all been introduced this way. The models may describe what happens but do not explain why, and must not be taken to be necessarily true. This would presuppose
that the statements of physics are of analytic origin. No doubt, the laws of physics are summarized in a compact mathematical language of symbols, but their contents stem from an entirely different basis. Mathematical theorems are of analytic structure, i.e. of the form 'all husbands are married' and do not contribute anything new that was not there from the beginning. The laws of physics on the other hand tell us something about nature and add to our knowledge. They are synthetic statements, i.e. of the form 'Jan has black hair', and therefore have the undesirable property of not necessarily being true. Therefore, it is excellent if the models can describe certain characteristics but if not, it does not imply, at least from the experimental point of view, catastrophe or the need for undue concern. Plato tried at all costs to preserve the phenomena, but this must never be done with nuclear models; rather it is of primary importance to check all possible aspects of models and point out any defects.

No-one knows what really happens in an atomic collision, but it is hoped to find a description of this drama in terms of concepts that are understood. In a sense it is therefore necessary to enter a theatre where the stage is being held by actors who may be called V, W and R. They will perform according to the script where
their roles and functions are stated. Depending on how the roles are being interpreted the result is a failure or a success. Either way, in order to judge, the play has to be written and who denies Brecht, Strindberg or Shakespeare their right of interpreting reality!

Models are judged by their ability to describe experimental data; but once general applicability has been established, a model may be used to predict more detailed properties of quantities under consideration, or for comparing similar aspects of other models. Thus different models might directly or indirectly be tested, resulting in a broader understanding of the problems involved. In the following sections average effects of nuclear reactions will be studied, different models and their predictions compared, and individual properties of nuclei investigated. In all cases statistical models will at least partly be used to predict, determine, and compare properties of nuclei.

The need for statistical models was clearly illustrated by Ericson (Er 66a) who considered the two neighbouring nuclei $^{237}$Np and $^{239}$U. He pointed out that there is one level of spin $1/2^+$ in the first 0.5 MeV excitation energy of $^{237}$Np and that one sees, by slow neutron capture, 27 levels of spin $1/2^+$ at 4.8 MeV in an
energy interval of 0.5 keV in \(^{239}\text{U}\). Thus there are about 27,000 times more levels per unit excitation energy at around 5 MeV and the detailed properties of the 27,032\(^{\text{nd}}\) level are of no particular interest. Rather the average properties of these levels are investigated. The situation is rather similar for some light nuclei when they are bombarded with protons of approximately 10 MeV bombarding energy, since in the region of excitation attained, the levels are overlapping i.e. the level widths are greater than the level spacings.

The first part of this thesis is devoted to investigation of average properties of the nuclei \(^{23}\text{Na},\) \(^{27}\text{Al},\) \(^{31}\text{P}\) and \(^{62}\text{Ni}\) and the search for any systematics as a function of mass, energy or angle which may occur. These properties include the fraction of the total differential cross section that is of direct reaction nature (from now on called the fraction of direct reaction), the mean level width in the compound nucleus and the optical-model parameters. The theoretical predictions are calculated from different models and the results compared. The experimental data to be considered are as follows:

1. elastic and inelastic scattering of protons from \(^{23}\text{Na}\) over the energy range 8.0 to 12.0 MeV, measured in 20 keV steps at 23 different angles;
(2) elastic scattering of protons from $^{27}$Al over the energy range 7.8 to 8.2 MeV, measured in 25 keV steps at 27 different angles;

(3) elastic scattering of protons from $^{31}$P over the energy range 8.0 to 10.0 MeV, measured in 10 keV steps at 14 angles and in 50 keV steps for 13 angles;

(4) elastic scattering of protons from $^{62}$Ni over the energy range 8.00 to 8.05 MeV, measured in 10 keV steps and at 27 different angles.

In chapter 1 the analysis of the above mentioned proton elastic scattering experiments is presented, together with a short section on the analysis of inelastic scattering data. The theories used are the optical-model, Hauser-Feshbach and distorted wave Born approximation (DWBA). The main problem is to separate the compound nucleus contribution, as given by the Hauser-Feshbach theory from the direct reaction contribution, given by either the optical-model or the DWBA theories. A reduction factor is introduced in the Hauser-Feshbach calculation to compensate for the fact that in this theory no direct reaction mechanism in the reaction channels is considered. The reduction factor will be given appropriate attention.
in this work, and will serve as a link between chapters 1 and 2. It will be shown that in some cases the reduction factor indicates that the sharp division between direct and compound nucleus processes is not sufficiently accurate and that intermediate states, observed as intermediate structure, must be considered. This is discussed in chapter 2.

Chapter 3 contains the fluctuation analysis of the measured cross sections using models developed by Ericson and Brink and Stephen. The main results are summarized in chapter 4, and include a comparison of the fraction of direct reaction as predicted by both the fluctuation analysis and the combined optical-model and Hauser-Feshbach theory or the combined DWBA and Hauser-Feshbach theory. Chapter 4 concludes part I of this thesis.

Part II contains chapter 5 in which the results from γ-ray angular distributions following the reactions $^{62}\text{Ni}(p,n\gamma)$ and $^{64}\text{Ni}(p,n\gamma)$ are presented. These angular distributions allowed the spins of some levels in the nuclei $^{62}\text{Cu}$ and $^{64}\text{Cu}$ to be determined.
PART I
CHAPTER 1

ANALYSES OF AVERAGE ANGULAR DISTRIBUTIONS FROM PROTON ELASTIC AND INELASTIC SCATTERING

1.1 INTRODUCTION

For medium-weight and heavy nuclei, the average experimental cross sections for proton elastic scattering have been well described by generalized optical-model parameters (Pe 63, Ro 65). Lighter nuclei, however, have not been studied to such an extent that it is possible to generalize the optical-model parameters in such a compact form. At tandem Van de Graaff energies, there is generally a significant compound nucleus contribution to elastic scattering measurements (Ga 66, Ha 68 and Hu 69) on nuclei of mass $A \leq 30$; this must be considered when searching on optical-model parameters to obtain fits to the experimental data. Thus, for reactions which proceed through regions of many overlapping levels in the compound nucleus, the experimental data should be analyzed by using a combination of the Hauser-Feshbach and optical-model theories in order to extract meaningful optical-model parameters. The applicability of this method, and some of the errors involved, depend on several factors; some of
these are the energy of the incident particle, the mass of the target nucleus, the energy range over which the angular distribution is averaged, and the value of the nuclear damping coefficient for the statistical part of the reaction. However, before discussing these factors in more detail a brief account of the optical-model and the Hauser-Feshbach theories will be given.

1.2 THEORY

In a measurement of the average elastic scattering cross section, the observable quantity is the average direct reaction cross section plus the average compound elastic cross section, i.e.

\[ \langle \sigma_{EL} \rangle = \langle \sigma_{DR} \rangle + \langle \sigma_{CE} \rangle. \]

Although the two processes are quite different and at least in theory could be separated on a time scale (We 61), this is not yet experimentally possible. Instead the separation has to be done with the aid of different models.

The direct reaction contribution to the cross section can be calculated using the optical-model. According to this model, the particles are scattered by a field represented by an average potential \( U \) of complex form,
which averages over resonances and gives rise to a scattered wave of amplitude $S$. The $S$ is related to the measured averaged elastic cross section $\langle \sigma_{EL} \rangle$ by (Bl 52, Ho 63)

$$\langle \sigma_{EL} \rangle = \frac{\pi}{k^2} \left\{ \left| 1 - S \right|^2 \right\} = \frac{\pi}{k^2} \left\{ \left| 1 - \langle S \rangle \right|^2 - \left| \langle S \rangle \right|^2 + \langle |S|^2 \rangle \right\}$$

where the term $\frac{\pi}{k^2} \left\{ 1 - \langle S \rangle \right\}^2$ is defined as the direct reaction cross section and is referred to as the shape elastic cross section, and $k$ is the wave number of the channel considered. The nuclear field is assumed to be represented by the short-range potential $U$ and the Schrödinger equation can be solved in the usual manner (Ho 63) to give $\langle S \rangle$.

Likewise the compound nucleus cross section is estimated using a model. The theory developed by Wolfenstein (Wo 51) and Hauser and Feshbach (Ha 52) from now on referred to as the Hauser-Feshbach theory, enables the average differential compound nucleus cross section to be calculated using the formula (Ri 65)

$$\langle \frac{d\sigma}{d\omega}(\theta) \rangle = \frac{\chi_\alpha^2}{4(2I_a+1)(2I_a+1)} \sum_{CC'} A_{CC'}(\theta) \frac{T_C}{\Sigma} \frac{T_{C'}}{T_{C''}}$$

(1.1)

where $\chi_\alpha$ is the wave length (divided by $2\pi$) for the entrance channel,
the spin of the incoming particle, 

the spin of the target nucleus, 

quantum numbers of the entrance channel, 

quantum numbers of the exit channel, 

transmission coefficients, 

means summation over all possible exit channels, and 

is a geometrical factor which will be further discussed in chapter 3.

The transmission coefficients, for a given channel \( \alpha \) and orbital angular momentum \( \ell \), were calculated from the expression

\[
T_{\alpha \ell} = 1 - |\langle S_{\alpha \ell} \rangle|^2
\]

1.3 THE WIDTH FLUCTUATION CORRECTION FACTOR

In order to arrive at formula 1.1, the compound nucleus process must be assumed to be independent of its formation and decay. This enables the term containing the transmission coefficients in the Hauser-Feshbach expression to be written

\[
\frac{\langle T_C \frac{T_{C'}}{C''} \rangle}{\langle T_C \rangle \langle T_{C'} \rangle} = \frac{T_C}{\Sigma} \frac{T_{C'}}{C''} = \frac{T_C}{\Sigma} \frac{T_{C'}}{C''}
\] (1.2)
There is, however, some evidence that expression 1.2 is not sufficiently accurate because of fluctuations in the widths of the compound nucleus (La 57, Mo 61, Mo 64, Tu 65, Ga 66). By assuming a linear relationship between the level width $\Gamma_\alpha$ and the transmission coefficient $T_\alpha$, i.e.

$$T_\alpha = \frac{2\pi \langle \Gamma_\alpha \rangle}{\langle D_\alpha \rangle}$$

the inaccuracy in equation 1.1 due to fluctuating level widths can be removed by multiplying by the correction factor

$$\tilde{W}_{\alpha\beta} = \frac{\langle \Gamma_\alpha \Gamma_\beta \rangle}{\langle \Gamma \rangle} \frac{\langle \Gamma_\alpha \langle \Gamma_\beta \rangle \rangle}{\langle \Gamma \rangle} .$$

Hodgson (Ho 67a) has carried out the evaluation of the factor $\tilde{W}_{\alpha\beta}$ under the assumption of a Porter-Thomas distribution for the level widths $\Gamma_\alpha$, and the same width fluctuating correction factor was used in all subsequent calculations of the compound nucleus cross sections. Under the above assumptions the width fluctuation correction factor is given by

$$\tilde{W}_{\alpha\beta} = (1+2\delta_{\alpha\beta}) \left( \sum_\gamma \langle \Sigma_\gamma \rangle \right) \int_0^\infty (1+2x\Gamma_\alpha)^{-1} (1+2x\Gamma_\beta)^{-1} \prod_\gamma (1+2x\Gamma_\gamma)^{-1/2} \, dx$$

where $\gamma$ is the summation index for all open channels, $x = \Gamma / \langle \Gamma \rangle$ and $\delta_{\alpha\beta}$ is the usual Kronecker symbol.
1.4 ESTIMATION OF ALL POSSIBLE EXIT CHANNELS

It is necessary to estimate all possible exit channels in the Hauser-Feshbach calculation as can be seen from equation 1.1. Ericson (Er 59) pointed out that if \( N(E) \) is the number of excited states with energy less than \( E \), then in many cases a plot of \( \log N(E) \) versus \( E \) gives a straight line, i.e.

\[
\frac{d}{dE} \log N(E) = \frac{1}{T}
\]

According to this formula, the number of excited states varies exponentially and describes what in many cases is experimentally observed. On the other hand, at higher excitation energies, the Fermi gas model can be used and predicts that \( \log N(E) \) is proportional to \( E^{1/2} \). Thus the constant temperature model predicts that \( \log N(E) \) is linear in \( E \) whereas the Fermi gas model specifies the relationship as proportional to \( E^{1/2} \).

Gilbert and Cameron (Gi 65a) combined the two models in a semi-empirical level density formula in which the constant temperature model is used for low excitations and the Fermi gas model for high excitations. The parameters were calculated in such a way that the two models were continuous at the cross over energy. Both
formulae include corrections for shell and pairing effects. The level density formula was used at high excitation energies where all possible exit channels were not known.

1.5 THE REDUCTION FACTOR

The imaginary part of the optical-model potential accounts for the fraction of the incident flux removed from the shape elastic channel. Explicitly, the absorption cross section

\[ \sigma_a = \frac{\pi}{k^2} \sum_l (2l+1)(1 - |\langle S_{\alpha l}' \rangle|^2) \]

and the cross section for formation of the compound nucleus

\[ \sigma_c = \frac{\pi}{k^2} \sum_l (2l+1) T_{\alpha l} \]

will differ if part of the absorbed flux goes into the direct reaction processes. Here, the dependence of the orbital angular momentum has been introduced whereas in section 1.2 this was included in S. Weisskopf (We 61) has given an interpretation of the different processes and this is shown in figure 1.5.1. Since the optical-model potentials are used in the Hauser-Feshbach calculation without considering that direct reactions also take place in the reaction channels, the calculated compound
Figure 1.5.1

Different reaction modes according to Weisskopf.
The top part of the figure shows how the different processes can be imagined to take place, whereas the bottom part of the figure shows where, on a time scale, the different processes take place. Observe the difference in the use of the word direct reaction according to Weisskopf and throughout this work, where all processes different from the compound nucleus process are labelled direct reactions.
Direct Inelastic Scattering

Compound Nucleus Formation (1 step)

Direct Inelastic Scattering (Collective)

Compound Nucleus Formation (2 steps)

Direct Reaction

Shape Direct Elastic Scattering Inelastic Scattering Compound Nucleus Scattering

V-IW

Same particle leaving

No Collision I Coll 2 Coll 3 Coll n th Coll

Other particles leaving

Shape Direct Elastic Reaction Compound Nucleus Reaction
nucleus contributions to the cross sections are generally too high. If some of the flux contributes to direct reaction processes, it is necessary to compensate the calculated Hauser-Feshbach cross sections. Hodgson and Wilmore (Ho 67b) introduced the concept of the reduction factor by which the calculated compound nucleus cross sections are corrected. If the total reaction cross section is \( \sigma_R \), it might be expected that the reduction factor would be \( 1 - \frac{\sigma_{DR}}{\sigma_R} \) where \( \sigma_{DR} \) is the direct part of the total cross section. However, besides the inability of calculating \( \sigma_{DR} \), it has been found experimentally that this factor may vary for different reaction channels (Ho 67b). It is therefore impossible to estimate theoretically the value which should be used. Instead, the reduction factor must be included as a variable in the fitting procedure.

1.6 THE APPLICABILITY OF THE THEORIES AND ERRORS DUE TO A COMPOUND NUCLEUS CONTRIBUTION

In using the combined optical-model and Hauser-Feshbach theory it must be remembered that these apply only to the average differential cross sections. It is impossible to combine the theories for isolated angular distributions because of the interference effects between
the two competing mechanisms. However, for average angular distributions, the interference effects cancel out and the addition of the two independent differential cross sections is possible (Br 63).

Careful consideration must be given to the choice of target nucleus and to the incident particle energy when carrying out measurements to be analysed by the optical-model, the Hauser-Feshbach theory and the theory of fluctuations and when predictions of these theories are to be compared, as the compound nucleus energy must correspond to a region of many overlapping levels. However, this criterion is generally fulfilled when the compound nucleus energy is approximately 3 MeV above neutron threshold (Er 63).

Because the compound nucleus mechanism in the region of overlapping levels is of statistical nature, it is necessary to measure the differential cross sections over a sufficiently large energy range to obtain a good estimate of the average cross section before using the combined optical-model and Hauser-Feshbach theory. Also, because of the necessary high excitation energy in the compound nucleus it is possible to apply the statistical theory of fluctuations (to be discussed in chapter 3) to estimate the errors involved.
Dallimore (Da 69) investigated the errors under the above mentioned assumptions. It should be pointed out that the errors do not include any contribution due to the experimental measurement of the mean cross section but are only due to the presence of a compound nucleus contribution. The approximate expression for the finite range of data (FRD) error is

\[ \langle \sigma \rangle = \bar{\sigma} \{1 \pm \sqrt{\frac{a}{N}(1 - Y_D^2)}\}; \]

where \( \langle \sigma \rangle \) = measured mean differential cross section in the energy range \( \Delta E \),

\( \bar{\sigma} \) = theoretical mean differential cross section in the range \( \Delta E \),

\[ a = \frac{2}{n^2} \tan^{-1}n - \frac{1}{n} \ln(1 + n^2), \]

\[ n = \Delta E / \Gamma, \]

\( \Gamma \) = the mean level width,

\( N \) = the nuclear damping coefficient (see chapter 3), and

\[ Y_D = \frac{\langle \sigma_{\text{direct}} \rangle}{\langle \sigma \rangle} \]

The main results are shown in figures 1.6.1 and 1.6.2. It can be seen that the statistical error is largely dependent on the spin of the target nucleus and to a lesser
Dependence of the percentage error, $100 \left[ \frac{a}{N(1-Y_D^2)} \right]^{1/2}$, on the target thickness $t$ ($\mu g/cm^2$) and mass number $A$ for 10 MeV protons, a compound nucleus excitation energy of 20 MeV, and a 50% direct reaction contribution.

Figure 1.6.1
% ERROR = 100 \sqrt{\frac{a}{N(1 - y_D^2)}}

y_D = 0.5

E_p = 10.0 MeV

E_{c.n.} = 20.0 MeV
Figure 1.6.2

Dependence of the percentage error on the fraction of direct reaction $Y_B$ and on the mass and spin $I_a$ of the target nucleus for 10 MeV protons and 200 $\mu g/cm^2$ targets.
\[ f = 200 \, \mu g/cm^2 \]

<table>
<thead>
<tr>
<th>N</th>
<th>Target</th>
<th>Spin</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>1/2</td>
<td></td>
</tr>
<tr>
<td>32</td>
<td>3/4</td>
<td></td>
</tr>
<tr>
<td>72</td>
<td>5/6</td>
<td></td>
</tr>
</tbody>
</table>

\[ y_D \]

\[ \% \text{ ERROR} \]
extent on the fraction of direct reaction and the target thickness. For example, elastic scattering measurements on $^{27}\text{Al}(5/2^+)$ should have FRD errors which are a factor of six smaller than measurements on nearby spin zero targets (e.g. $^{26}\text{Mg}$, $^{28}\text{Si}$). Furthermore, Dallimore suggests that for medium mass nuclei ($\Gamma \approx 3$ keV) the averaging should be done by measuring several individual angular distributions separated in energy by at least $\pi \Gamma$ to obtain a good average representation of the cross section, whereas for light nuclei ($\Gamma \approx 100$ keV) any such averaging will be over energy ranges where the compound nucleus and the direct reaction mechanisms might exhibit significant variations. In the latter case it is therefore more appropriate to measure excitation functions and average them by a smoothing procedure.

1.7 EXPERIMENTAL TECHNIQUES

The ANU 12 MeV tandem Van de Graaff accelerator provided a proton beam which was magnetically analysed by a 90° magnet, and entered a 51 cm diameter scattering chamber through a narrow collimation system optically aligned with the chamber centre. The scattering chamber and the equipment inside the chamber were gradually improved during the first of the proton scattering
experiments $^{31}\text{P}(p,p_0)$. The final assembly of the scattering chamber and associated equipment is shown in figure 1.7.1. The beam entered the scattering chamber through the collimation system from the left and the scattered protons passed through anti-scattering slits and were detected in surface barrier detectors for which the solid angles were defined by rectangular apertures mounted in the counter blocks. The angular resolution was approximately 0.5° and Rutherford scattering has shown the overall accuracy to be less than 0.2°. The charge was collected in a Faraday cup using magnetic and electric suppression and integrated in an ELCOR A3093 current integrator. An array of eight 1000 μ ORTEC silicon surface barrier detectors was used to detect the scattered protons.

To obtain the best resolution, the detectors were generally overbiased to improve the charge collection efficiency, and cooled with alcohol to dry ice temperature to reduce the intrinsic noise. The cooling system can be seen in figure 1.7.1. The alcohol circulated through the copper tube which was clamped to the brass blocks (of which two are shown in the figure) in which the detectors were mounted. To avoid thermal contact with the scattering chamber the brass blocks were insulated from the holders by Lucite.
Figure 1.7.1

The final assembly of the scattering chamber and some of its associated equipment used in the proton scattering experiments.
The resolution of the counters was further improved by placing magnets in front of the counter blocks (see figure 1.7.1) to deflect the large number of electrons produced at the target; these electrons would otherwise have produced low energy pulses in the detectors and the consequent pile-up would have worsened the resolution.

1.7.1 ELECTRONICS

The pulses were amplified in charge sensitive preamplifiers and main amplifiers and the combination of biased amplifiers and R-C main amplifiers enabled the desired parts of the spectra to be expanded over the full range of an analogue to digital converter (ADC) before being fed into an IBM 1800 computer. A schematic diagram of the electronics is shown in figure 1.7.2.

Each amplified pulse from a particular counter was associated with two pulses which were fed to two CA13 ADC's; one was a linear pulse (ADC1), of height proportional to the energy of the detected particle, and the other was a logic pulse (ADC2), whose height was dependent on the particular counter in which it was detected. The two ADC's worked in conjunction so that only pulses within the time coincidence resolution of the pair of the ADC's
(200ns) were accepted. If a pulse was sensed in ADC1 it started conversion. Providing ADC2 also sensed a pulse, the pulse was converted to a digital number, and as a result there were two addresses to store in the memory of the computer. The memory was divided into 8 blocks of 512 channels and the address from ADC2 indicated in which of the spectrum block the address from ADC1 was to be stored. The address from ADC1 indicated where in the spectrum it was to be stored.

It is clear that if ADC1 and ADC2 are converting pulses to digital numbers, other pulses arriving during this interval will be lost. To estimate the fraction of lost pulses, a source of clock pulses (100/s) was used to feed two scalers. One of the scalers was gated to ignore clock pulses occurring during the dead time of the ADC's. The other scaler counted all clock pulses. The dead time was reduced by the use of biased amplifiers, which excluded pulses that were of no interest.

1.8 DATA REDUCTION

Following the arguments put forward in section 1.6, excitation functions were measured for all reactions except for the reaction $^{62}\text{Ni}(p,p_0)$, for which angular
distributions were measured. In each set of angles at least one angle was repeated to check on consistency and reproducibility. The peaks in the spectra were added either by hand or using a computer program which included background subtraction. After having corrected for dead time the excitation functions were normalized relative to each other by measurements over a short energy range (≈ 1/10 of the full energy range) for all angles at which complete excitation functions had been measured. These cross section measurements were repeated at least twice with different counters for all angles, again to check on consistency. Absolute differential cross sections were determined by using the formula

\[ \frac{d\sigma}{d\Omega} = \frac{nAe}{td\Omega qN} , \]

where \( n \) is the number of counts in the peak considered, \( A \) the atomic weight of the element of interest in the target, \( e \) the charge of the electron, \( t \) the target thickness, \( d\Omega \) the solid angle, \( q \) the charge incident on the target, and \( N \) Avogadro's number.
The value of \( t \, d\Omega \) for each detector was calculated by measuring angular distributions and excitation functions at approximately 2 MeV for the elastically scattered protons and then comparing the results with the Rutherford scattering formula. A typical example of the result of such a measurement is shown in figure 1.8.1 where the values \( t \, d\Omega \) are plotted versus angle.

Since the optical-model and the compound nucleus theories describe average cross sections, the fluctuations were averaged out by using a linear least-squares program to fit smooth curves of the form \( AE^2 + BE + C \), where \( E \) is the energy and \( A, B \) and \( C \) are constants, to the excitation functions measured over large energy ranges. Thus average angular distributions at any energy could be read off from the smooth fits. For the reactions measured over a short energy range \((^{27}\text{Al}, ^{62}\text{Ni})\) the average angular distributions were determined from the experimental data points by summing all data points and dividing these by the number of data points measured.

1.9 THE FITTING PROCEDURE

The fitting procedure for the angular distributions consisted of searching for the minimum value of \( \chi^2 \) defined by
\[ \chi^2 = \frac{1}{M} \sum \left( \frac{\sigma_{\text{exp}} - \sigma_{\text{th}}}{\Delta \sigma_{\text{exp}}} \right)^2 , \]

where \( M \) = the number of data points in the angular distribution

\( \sigma_{\text{exp}} \) = the experimentally measured value at a given angle

\( \sigma_{\text{th}} \) = the calculated value at this angle

\( \Delta \sigma_{\text{exp}} \) = the error in \( \sigma_{\text{exp}} \).

Each value \( \sigma_{\text{th}} \) was the sum of an optical-model calculation and a Hauser-Feshbach calculation using the transmission coefficients from the optical-model calculation.

The optical-model program used in the analysis was the code of Perey (Pe 63) which uses a potential of the form

\[ V(r) = V_c(r) + V_s f(r,r_{os},a_s) + 4a_i W_D \frac{d}{dr} f(r,r_{oi},a_i) \]

\[ + \sigma \cdot \mathbf{L} \left( \frac{\hbar}{m_r c} \right)^2 V_{so} \frac{d}{dr} f(r,r_{os},a_s) \]

where \( V_c(r) \) is the Coulomb potential and \( V_s, W_D \) and \( V_{so} \) are the depths of the real, imaginary and spin-orbit parts of the potential respectively. The function \( f(r,r_{o},a) \) is the Saxon-Woods form factor.
Figure 1.8.1

Values of $td\Omega$ derived from comparison with the Rutherford scattering formula.
\[ f(r, r_0, a) = \left[ 1 + \exp \left[ \frac{(r - r_0 A^{1/3})}{a} \right] \right]^{-1} \]

and all notations are the same as those given in reference (Pe 63). As was mentioned earlier, the compound nucleus calculation used transmission coefficients from the optical-model calculation. However, the compound nucleus program did not allow for the inclusion of \( j \)-dependent transmission coefficients and instead generalised transmission coefficients were used. These are defined by (Sh 63)

\[
T_{\ell} = \left\{ (\ell + 1)T_{\ell}^{(+)} + \ell T_{\ell}^{(-)} \right\} / (2\ell + 1)
\]

where the superscript \((\pm)\) refers to the vector composition of spin and orbital angular momenta to a resultant \( j = \ell \pm 1/2 \).

It is possible that some systematic error may occur when measuring absolute cross sections for the angular distributions, even though the relative errors between the data points are well established. Therefore a normalization parameter, by which all the experimental cross sections were multiplied, was initially introduced as a variable parameter in the search for the minimum value of \( \chi^2 \).
It was also necessary to include the reduction factor $R$ as a variable in the fitting procedure as was pointed out in section 1.5. However, it should be emphasized that considerable care must be taken when varying the reduction factor, especially for angular distributions for which there is a small compound nucleus component. This is because any systematic error in the determination of the absolute cross section will affect the derived value of the reduction factor. If the error in the absolute differential cross section ($\sigma_T = R\sigma_{HF} + \sigma_{OM}$) is defined as $\Delta\sigma_T$, then the error $\Delta R$ in the reduction factor $R$ for an elastic scattering process is given by

$$\frac{\Delta R}{R} = \frac{\Delta\sigma_T}{\sigma_T} (1 - \frac{\sigma_{OM}}{\sigma_T})^{-1}$$

where $\sigma_{OM}$ is the optical-model cross section and $\sigma_{HF}$ is the Hauser-Feshbach cross section. A 5% error in $\sigma_T$ produces a 5.6% error in the reduction factor for a 10% direct (optical-model) reaction contribution, whereas for a 90% direct reaction contribution the error in the reduction factor becomes 50%. This point will further be illustrated in the analysis of the average angular distribution for the reaction $^{62}\text{Ni}(p,p_0)$. 
The parameters included in the search, apart from the normalization and the reduction factor were the real and imaginary parts of the optical-model potential. All the other parameters were fixed at the values used by Perey to enable the results to be compared with previously predicted mass and energy dependent formulae for \( V \) and \( W \) although, as has been pointed out by Perey and Perey (Pe 68), there are indications of a variation of the shape of the potential as a function of energy, so that the geometrical parameters used might not be the most suitable. However, it was not feasible to carry out searches over all parameters because of the prohibitive time taken in performing the Hauser-Feshbach calculation.

1.10 ANALYSES OF AVERAGE ANGULAR DISTRIBUTIONS FROM PROTON ELASTIC SCATTERING ON \( ^{62}\text{Ni} \), \( ^{31}\text{P} \), \( ^{27}\text{Al} \), AND \( ^{23}\text{Na} \)

1.10.1 INTRODUCTION

It is generally accepted that optical-model calculations will be more satisfactory for heavier mass nuclei than for lighter nuclei because of the improvement in the nucleon-nucleus interaction approximation. However, even if it is assumed that this approximation is equally valid for all nuclei, it may still be possible to explain the relatively poor agreement for light mass
nuclei by the neglect of the compound nucleus contribution. The fraction of direct reaction will generally increase with increasing mass number because of the increase in the number of possible exit channels which removes the compound nucleus contribution. It may therefore be possible to obtain good fits from optical-model calculations on heavy and medium-weight nuclei even though the compound nucleus contribution is neglected. Moreover, the mean level width in the compound nucleus increases with decreasing mass number. For example, in two of the reactions to be considered, the mean level widths for \(^{31}\text{P}(p,p'_{o})\) and \(^{62}\text{Ni}(p,p'_{o})\) are approximately 30 keV and 4 keV respectively. This means that the excitation functions will exhibit fluctuations with these widths, and the amplitude of these fluctuations depends on the fraction of direct reaction present, and on the spins of the particles involved. If, for example, the combined energy resolution due to beam spread and target thickness is 30 keV, then for an isolated angular distribution the energy averaging is over approximately one fluctuation for \(^{31}\text{P}(p,p'_{o})\) and approximately seven fluctuations for \(^{62}\text{Ni}(p,p'_{o})\). The interference terms in the latter angular distribution will therefore be considerably reduced compared with the former, and the
angular distribution for the reaction $^6_{20}$Ni(p,p) will be a better representation of the average angular distribution. However, if all these facts are taken into consideration, it may be possible to obtain equally good fits for light nuclei as for medium and heavy nuclei.

It was mentioned earlier that one requirement of this type of experiment was that the excitation energy in the compound nucleus should be approximately 3 MeV above neutron threshold. The most suitable target nuclei fulfilling this requirement for proton bombarding energies of 10 MeV, mass between 15 and 35, and excitation energy of approximately 20 MeV in the compound nucleus are $^{15}_N$, $^{19}_F$, $^{23}_Na$, $^{26}_Mg$, $^{27}_Al$, and $^{31}_P$.

Elastic scattering measurements are considered to be the most sensitive for testing the applicability of the width fluctuation correction factor, since the cross section should be enhanced by approximately a factor of two or three over the standard calculation (Sa 63).

1.10.2 THE REACTION $^6_{20}$Ni(p,p) AT 8 MeV

A $^6_{20}$Ni target of approximately 20 $\mu$g/cm$^2$ was prepared by vacuum evaporation of isotopically enriched (>99%) $^6_{20}$Ni onto a 10 $\mu$g/cm$^2$ carbon backing. The
target was placed at $45^\circ$ to the beam direction (as were all other targets to be discussed) at the centre of the scattering chamber.

Six angular distributions were measured for incident proton energies from 8.00 to 8.05 MeV in 10 keV intervals. The target thickness of 20 $\mu g/cm^2$ gave an energy loss of less than 1 keV so that the overall beam resolution should have been approximately that normally found on a tandem Van de Graaff; that is, less than 5 keV. Normally elastic scattering measurements in this mass region should be taken with thick targets to obtain averaged cross sections; however, one of the reasons for doing this experiment was to investigate the effect when the energy resolution and the mean level width in the compound nucleus are comparable.

The steps of 10 keV used in measuring the angular distributions correspond to approximately two to three times the mean level width in the compound nucleus $^{63}\text{Cu}$. Thus it is expected that there should be very little correlation in the angular distributions due to the statistical nature of the compound nucleus contribution to the cross sections (Gi 65b). In figure 1.10.2.1 three excitation functions taken in 5 keV steps from 8.000 to 8.355 MeV for angles of $100^\circ$, $120^\circ$ and $140^\circ$ (1ab) are
Figure 1.10.2.1

Three excitation functions for the reaction $^{62}\text{Ni}(p,p_0)$ measured in 5 keV steps between 8.000 and 8.355 MeV.
$^{62}\text{Ni}(p, p')$ EXCITATION FUNCTIONS

Counts (Arbitrary Units)

8.1 8.2 8.3

PROTON ENERGY IN MEV

140° (lab)

120° (lab)

100° (lab)
shown. It is evident from these that the above conclusions are valid.

In figure 1.10.2.2a, three of the angular distributions have been superimposed and it is seen that considerable variations in the cross sections do occur for angles greater than approximately 60°. Thus it was necessary to average all the angular distributions before trying to fit the data with the optical-model and Hauser-Feshbach theories. The optical-model parameters used were those of Perey (Pe 63) for protons and neutrons and of McFadden and Satchler (Mc 66) for alphas. A total of 155 open channels, consisting of 89 proton channels, 14 alpha channels and 52 neutron channels were used in the Hauser-Feshbach calculations.

The errors on the data points were assumed to arise from

1. background subtraction and peak summation errors,
2. relative errors between angles due to the normalization of the different counters and solid angles used, and
3. finite range of data.

The last error, due to the statistical nature of the compound nucleus part of the cross section, used the assumption that each angular distribution was independent.
Figure 1.10.2.2

(a) Three individual angular distributions measured at 8.00, 8.02 and 8.04 MeV for the reaction $^{62}\text{Ni}(p,p_0)$

(b) The average angular distribution and the best fit obtained using the combined shape elastic (SE) and the compound elastic calculations (CE).
Figure 1.10.2.3 shows the results for the best fits as a function of the normalization and the minimum value of $\chi^2$ occurs for a normalisation of 0.80. One of the reasons for introducing the normalization factor was an error found in the calibration of the current integrator after completion of the $^{62}$Ni(p,p$_o$) and $^{31}$P(p,p$_o$) measurements. It will be seen how favourably the value 0.80 compares with the value of the normalization factor for the reaction $^{31}$P(p,p$_o$) in the next section, and that indeed a value of 1.0 is found for the reaction $^{23}$Na(p,p$_o$) when the error had been corrected. The proton optical-model potentials obtained were $V = 55.2$ MeV and $W = 10.8$ MeV together with a reduction factor of 0.45 for the Hauser-Feshbach calculation (including the width fluctuation correction factor). The fit is shown in figure 1.10.2.2b and it would appear that the pure shape elastic cross section gives a better fit, especially for the very backward angles. However, this is not so because the combined theory fits better for all angles between $50^\circ$ and $125^\circ$ and because of the different weighting of the points due to the FRD errors. If there is no compound elastic cross section, these contributions to the errors become zero. As a test, a pure optical-model search was done on the average angular distribution yielding
Four parameter search on the average angular distribution for proton elastic scattering from $^{62}$Ni. For each value of the normalization, the parameters $W$, $V$ and $R$ were varied to minimize $\chi^2$. 
$^{62}\text{Ni}(p,p\alpha)$

- $\chi^2$
- $W$ (MeV)
- $V$ (MeV)
- $R$

vs. Normalisation
V = 55.2 MeV, W = 10.1 MeV and $\chi^2 = 2.2$ as compared with $\chi^2 = 1.0$ obtained using the combined calculations.

It is interesting to note that, for example, at 90° (lab) where the calculated average compound nucleus contribution is only 2%, there is a maximum variation of 40% between the cross sections for the individual angular distributions. This illustrates the point that, because of interference effects between the compound and direct processes, it is impossible to do either a pure optical-model, or a combined optical-model plus Hauser-Feshbach analysis on individual angular distributions, if the energy resolution is comparable to the mean level width in the compound nucleus. This type of analysis should only be done for averaged cross sections obtained from several angular distributions.

Table 1.10.2.1 summarises the results and includes the fits to the individual angular distributions assuming that there is only shape elastic present. Although this is incorrect, it does illustrate the errors which can occur if only isolated angular distributions are fitted. That is, the apparent fluctuations in the potentials are due to the interference effects between the direct and compound nucleus mechanisms.
**TABLE 1.10.2.1**

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Type of Analysis</th>
<th>V (MeV)</th>
<th>W (MeV)</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.00</td>
<td>OM</td>
<td>52.9</td>
<td>8.3</td>
<td>8.7</td>
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<td>OM</td>
<td>55.4</td>
<td>8.9</td>
<td>7.8</td>
</tr>
<tr>
<td>8.03</td>
<td>OM</td>
<td>56.6</td>
<td>9.3</td>
<td>8.1</td>
</tr>
<tr>
<td>8.04</td>
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<td>$\langle 8.025 \rangle$</td>
<td>OM</td>
<td>55.2</td>
<td>10.1</td>
<td>2.2</td>
</tr>
<tr>
<td>$\langle 8.025 \rangle$</td>
<td>OM + HF</td>
<td>55.2</td>
<td>10.8</td>
<td>1.0</td>
</tr>
</tbody>
</table>

It may be said that generally much thicker targets are used for these types of analyses, so that the averaging is effectively done over many compound nucleus states. However, it has been shown (Gi 65b) that the important parameter is not the energy range $\Delta E$ but the ratio $\Delta E/\Gamma$, where $\Gamma$ is the mean level width in the compound nucleus. Thus the present results may be compared to scattering from lighter nuclei, where $\Gamma$ is much larger, and using very thick targets.

The reduction factor obtained of 0.45 illustrates the importance of the width fluctuation correction factor; if it were not included then a value greater than unity
would be obtained, since for these measurements the basic Hauser-Feshbach cross section is approximately one-third of the modified value. However, for angular distributions where there is only a very small compound nucleus component the estimation of the reduction factor can contain very large errors. This is shown in figure 1.10.2.4 where the best fit values for $\chi^2$, $V$ and $W$ are compared as functions of the reduction factor. It is apparent that in this experiment the reduction factor has very little meaning.

1.10.3 THE REACTION $^{31}$P(p,$p_0$) AT 8 MeV

Although data were obtained over the range 8-10 MeV proton bombarding energy for this reaction, the normalization for the absolute cross section determination was made between 8.00 and 8.20 MeV. This was done primarily to investigate the applicability of the arguments put forward in the introduction to this section, i.e. to compare the two reactions $^{31}$P(p,$p_0$) and $^{62}$Ni(p,$p_0$) when averaged over a short energy range. Therefore, the short energy range of the reaction $^{31}$P(p,$p_0$) will be presented as such before considering the full range 8-10 MeV.
The dependence of $\chi^2$ on the reduction factor with a normalization of 0.80 for the elastic scattering of protons from $^{62}$Ni. For each value of the reduction factor the parameters $W$ and $V$ were varied to minimize $\chi^2$. 
\[ ^{62}\text{Ni}(p, p^0) \]

\[ \chi^2, W(\text{MeV}), V(\text{MeV}) \]

REDUCTION FACTOR

0.2  0.4  0.6  0.8
The $^{31}$P target of approximately 50 $\mu$g/cm$^2$ was prepared by vacuum evaporation of natural phosphorous onto a $\approx$10 $\mu$g/cm$^2$ carbon backing (Ho 64).

Excitation functions were measured in 10 keV steps from 8.00 to 8.20 MeV incident proton energy, and for angles from 30$^\circ$ (lab) to 160$^\circ$ (lab) in 5$^\circ$ intervals. The cross sections from these excitation functions were averaged to obtain the angular distribution corresponding to a mean energy of 8.10 MeV used in the analysis.

Typical examples of the fluctuations are shown in the excitation functions of figure 1.10.3.1. The crosses indicate the mean values used in the average angular distribution, and the error bars contain only the estimated finite range of data (FRD) errors. The possible exit channels were obtained from Endt and Van der Leun (En 67). Altogether 48 exit channels were used, consisting of 35 proton channels, 11 alpha channels and 2 neutron channels. Where the spins were unknown they were again assigned on the basis of the Gilbert and Cameron level density formula.

The alpha particle parameters used were from Weiss and Davies (We 62) and the neutron parameters were from Perey and Buck (Pe 62). It was found that varying these parameters did not significantly effect the proton elastic and inelastic cross sections, although varying
Three excitation functions for $^{31}\text{P}(p,p_0)$ measured in 10 keV steps between 8.00 and 8.20 MeV. The crosses at 8.10 MeV indicate the mean cross sections from the full energy range 8.00-8.20 MeV and the error bars are due only to the FRD effects. The absolute values include the normalization factor.
$^{31}\text{P}(p,p\alpha)$

**30° (lab)**

**110° (lab)**

**160° (lab)**

**PROTON ENERGY (MeV)**
the alpha potentials did effect the alpha channel cross sections.

The sources of errors considered were the same as in the previous section.

Figure 1.10.3.2 shows the best fits obtained for various values of the absolute normalization. The best fit corresponded to a normalization of 0.81. It was expected, from the error in the current integrator, that a normalization of less than unity would be needed and the value obtained here is in excellent agreement with that obtained in the $^{62}$Ni(p,p$_o$) analysis.

Having estimated the normalization parameter, it was necessary to check the accuracy of the fitting procedure on the variations in the reduction factor (R), V and W. Figure 1.10.3.3 shows the values of $\chi^2$, V and W for the best fits as a function of the reduction factor. It can be seen that the method is quite sensitive to the value of the reduction factor and enables an accurate estimate of its value to be determined.

Generally, fits with a value $\chi^2$ less than one cannot be meaningfully distinguished. However, when the FRD errors are a major contribution, the errors for nearby angles are correlated and the experimental points will no longer have a statistical distribution about the
Four parameter search on the average angular distribution for proton elastic scattering from $^{31}\text{P}$. For each value of the normalization the parameters $W$, $V$ and $R$ were varied to minimize $\chi^2$. 
The dependence of $\chi^2$ on the reduction factor with a normalization of 0.81 for the elastic scattering of protons from $^{31}\text{P}$. For each value of the reduction factor the parameters $W$ and $V$ were varied to minimize $\chi^2$. 

Figure 1.10.3.3
theoretical values. Accordingly, it is difficult to attach any confidence limit to the absolute value of $\chi^2$, although it might be expected that for good fits values of $\chi^2 \ll 1.0$ would be significant.

Figure 1.10.3.4 shows the final results with $V = 53.1$, $W = 8.3$ and a reduction factor of 0.90. These values have been obtained using the width fluctuation correction factor in the compound nucleus calculations. If this is not included then the calculated compound nucleus cross sections are approximately half their value from the standard calculation. Therefore, it may be assumed that a fitting procedure would give a reduction factor of approximately 1.8 in the standard case. However, this is unrealistic as a reduction factor of greater than 1.0 would be impossible for a normal process. Thus, it is necessary to include the width fluctuation correction in the Hauser-Feshbach calculations. The value of 0.90 for the reduction factor indicates that only a small direct reaction component is present in the reaction channels. A previous analysis of the $^{31}$P(p,$\alpha$) reaction between 8.5 and 12.3 MeV for the ground and first excited state groups found no evidence for a direct reaction contribution (Da 68). Also, because of the small cross sections observed in the proton inelastic
The best fit obtained for the average angular distribution for the reaction $^{31}\text{P}(p,p_\theta)$ at a mean incident proton energy of 8.10 MeV. The calculated shape elastic (SE), compound elastic (CE) and the total differential elastic (SE + CE) cross sections are as indicated.
AVERAGED ANGULAR DISTRIBUTION

$^3\text{P}(p,p')$

$E_p(\text{mean}) = 8.10 \text{ MeV}$

$V = 53.1 \text{ MeV}$

$W = 8.3 \text{ MeV}$

$R = 0.90$
channels, it is considered that there is very little direct reaction present. The value of 0.90 is thus a realistic estimate for the reduction factor. As has been mentioned previously, if the measurement of the absolute cross section is assumed accurate, and if the compound and total differential cross sections have similar angular distributions, then any error in the absolute cross section will appear as a compensating error in the reduction factor. For cases where these angular distributions are very different, as in the present example, this will be a small effect. However, it may be significant in the analysis of inelastic scattering data.

1.10.4 THE REACTION $^{31}$P(p,p$_0$) IN THE ENERGY RANGE 8.00-10.00 MeV

The reaction $^{31}$P(p,p$_0$) was previously discussed in the range 8.0 to 8.2 MeV. The work was extended by measuring excitation functions over the range 8.0 to 10.0 MeV at angles between 30° and 160° (lab). Fourteen excitation functions were measured in 10 keV steps over this range and thirteen excitation functions were measured in 50 keV steps. It was found that the 50 keV steps were sufficient to derive the average cross sections.
Excitation functions were measured in 10 keV steps over the full energy range for laboratory angles of 30°, 40°, 50°, 60°, 65°, 70°, 75°, 80°, 85°, 90°, 95°, 100°, 105°, 110°, 115°, 120°, 125°, 130°, 135°, 140°, 145° and 150° and in 50 keV steps for angles of 35°, 45°, 55°, 65°, 75°, 85°, 95°, 100°, 115°, 125°, 130°, 135°, 145° and 155°. Some of the excitation functions are shown in figure 1.10.4.1, together with the fitted curves. It is apparent that the quadratic expression satisfactorily estimates the energy dependence of the mean cross section, although at 110° (lab) the smooth curve increases in value between 9.5 and 10.0 MeV. This is probably due to the form of the expression used for fitting the excitation functions. However, because of the general behaviour of the fits obtained and, because of the fact that a more complicated function might overestimate the effects due to statistical fluctuations, the quadratic expression was considered satisfactory for the analysis.

The errors in the average cross sections consisted of the errors associated with the determination of the absolute cross sections in the previous section and with the normalization of the present data to these measurements, plus a finite range of data (FRD) error due to the statistical component of the reaction. The total FRD error was obtained by combining the standard deviation
Excitation functions for the reaction $^{31}\text{P}(p,p_0)$ measured in 10 keV steps from 8.0 to 10.0 MeV. The smooth curves are least squares fits of the form $AE^2 + BE + C$. 
of all the points about the smooth fit with the calculated FRD error in the average cross section over the full energy range. Thus the errors in the points obtained from the 50 keV excitation functions are larger than in those obtained from the 10 keV excitation functions.

From the smooth fits to the excitation functions, average angular distributions were calculated every 0.25 MeV from 8.00 to 10.00 MeV and analysed using the combined optical-model and Hauser-Feshbach theory. The results are shown in figure 1.10.4.2 as a function of the incident proton energy. The solid lines have been drawn to guide the eye.

The reduction factor exhibits the expected decrease with increasing incident particle energy from 8.0 MeV to approximately 9.25 MeV due to the increased amount of direct reaction taking place in the reaction channels. However, from 9.25 to 10.00 MeV the reduction factor increases slightly. This is unrealistic and is probably due to the overestimation of the average cross sections at the minima in the angular distributions, resulting from the expression chosen for fitting the excitation functions. Similarly, the slight increase in W with increasing energy and the decrease in the goodness of
Figure 1.10.4.2

Dependence of the reduction factor, the real and imaginary optical-model potential and $\chi^2$ on the incident proton energy.
fit are probably due to the overestimation of the cross sections. These effects are only small but serve to illustrate the importance of obtaining true representations of the average cross sections.

The real proton optical potential \( V \) shows a very pronounced energy dependence, decreasing from 53.6 MeV at 8.0 MeV to 48.6 MeV at 10.0 MeV. The variation is considerably greater than the \(-0.55 \varepsilon\) dependence predicted by Perey, and illustrates the importance of obtaining detailed results on several nuclei at around these energies before deriving any conclusions on the systematics of the parameters involved.

The final fits, calculated every 0.25 MeV, are shown in figure 1.10.4.3 together with the values of \( V, W \) and \( R \) obtained. It can be seen that at all energies the compound elastic cross section must be considered when analysing the experimental data.

1.10.5 THE REACTION \( ^{27}\text{Al}(p,p') \) AT 8 MeV

Excitation functions for the reaction \( ^{27}\text{Al}(p,p') \) were measured between 7.8 and 8.2 MeV in 0.25 keV steps and at 27 different angles. The experimental method, data reduction and the theoretical analysis were the same as
The best fits to the angular distributions measured every 0.25 MeV. The dotted lines are the compound elastic calculations, the dashed lines are the shape elastic calculations, and the solid lines are the resulting total differential cross section calculations.
$^3$P(p, p')

**Absolute Cross Section**

- $E_p = 8.00$ MeV
  - $V = 53.6$
  - $W = 7.9$
  - $R = 0.75$

- $E_p = 8.25$ MeV
  - $V = 52.7$
  - $W = 7.9$
  - $R = 0.65$

- $E_p = 8.50$ MeV
  - $V = 52.0$
  - $W = 8.0$
  - $R = 0.50$

- $E_p = 8.75$ MeV
  - $V = 51.3$
  - $W = 8.0$
  - $R = 0.40$

- $E_p = 9.00$ MeV
  - $V = 50.6$
  - $W = 8.2$
  - $R = 0.30$

- $E_p = 9.25$ MeV
  - $V = 50.1$
  - $W = 8.6$
  - $R = 0.25$

- $E_p = 9.50$ MeV
  - $V = 49.5$
  - $W = 9.0$
  - $R = 0.25$

- $E_p = 9.75$ MeV
  - $V = 49.0$
  - $W = 9.3$
  - $R = 0.30$

- $E_p = 10.00$ MeV
  - $V = 48.6$
  - $W = 9.6$
  - $R = 0.40$

Theta (degrees)
discussed in the previous sections. The best fit to the average angular distribution is shown in figure 1.10.5.1 where the free parameters also are indicated. The alpha-particle and neutron potentials were those of So (So 66) and Perey and Buck (Pe 62) respectively.

The high value of the reduction factor obtained can be seen as a consequence of the reported evidence for intermediate structure in the compound nucleus $^{28}\text{Si}$ at the obtained energy of excitation (Si 65). This will be discussed further in the following sections.

1.10.6 THE REACTION $^{23}\text{Na}(p,p')$ IN THE ENERGY RANGE 8.0-12.0 MeV

The targets were made of metallic sodium evaporated onto $\approx 10 \ \mu\text{g/cm}^2$ carbon backings and were transported to the scattering chamber via a vacuum lock. Excitation functions were measured in 20 keV steps between 8.0 and 12.0 MeV, and at 23 angles between 35° and 155° (lab).

It was extremely important to check on consistency and reproducibility of the data obtained since the melting point of Na is approximately 97°C. However, the beam was always kept of the order of 0.03 μA and no deterioration of the target was detected.
Figure 1.10.5.1

The best fit to the average angular distribution for the reaction $^{27}\text{Al}(p,p')$ at 8 MeV.
\( ^{27}\text{Al}(p,p') \) \( E_p=8\text{MeV} \)

\( V=52 \quad W=13 \quad R=0.05 \quad \chi^2=0.64 \)
The excitation functions were normalized relative to each other by measurements between 9.8 and 10.2 MeV in 20 keV steps for all angles at which complete excitation functions had been measured previously. The cross section measurements were repeated at least twice with different counters for all angles and the values agreed within 2.5%.

The value of $\Delta Q$ for each detector was calculated by measuring angular distributions at 1.8 MeV and by measuring excitation functions at four different angles between 1.8 and 2.0 MeV for the elastically scattered protons, and comparing the results with the Rutherford scattering formula. An approximate estimate for the target thickness is obtained by assuming $\Delta Q = 0.4$ mster yielding $t = 300 \pm 8 \mu g/cm^2$.

The fluctuations were averaged out by using a linear least squares program to fit smooth curves of the form $AE^2 + BE + C$, where $E$ is the energy and $A$, $B$ and $C$ are constants, to the measured excitation functions. Three of the excitation functions and their smooth fits are shown in figure 1.10.6.1.

The most forward angle at which measurements were possible was $35^0$. At more forward angles, it was impossible to separate the oxygen and carbon elastic
Smooth curves of the form $AE^2 + BE + C$ fitted to three excitation functions for the reaction $^{23}\text{Na}(p,p\alpha)$. 

Figure 1.10.6.1
scattering from the sodium elastic scattering. For example, at \(30^0\), the separation between the oxygen and sodium peaks at 8.0 MeV proton energy is only 30 keV. The angular distributions at 8, 9, 10, 11 and 12 MeV are shown in figure 1.10.6.2.

The errors considered were due to statistics, background subtractions, target thickness measurements, relative and absolute cross section measurements, FRD errors and fitting of smooth curves. All but the last contribution were straight-forward estimations. Here the smooth fits were considered to truly represent the average over the whole region, although one expects the representation to be better in the middle of the energy range, and in a standard way the errors were calculated by considering all points and their deviations from the average.

As can be seen from figure 1.10.6.3, the errors at \(65^0\) were generally larger than at other angles, since the counter used at this angle was not of the same quality as the others and there were difficulties with background subtractions and peak separations. However, the cross sections between 9.8 and 10.2 MeV are considered accurate. A steadily increasing error, reaching a maximum value of
Angular distributions at 1 MeV intervals between 8.0 and 12.0 MeV. These were obtained from the smooth fits to all excitation functions, three of which are shown in figure 1.10.6.1.
The fits obtained to the experimental average angular distributions in 0.5 MeV steps from 8.0 to 12.0 MeV.
$^{23}\text{Na}(p,p_\alpha)$

<table>
<thead>
<tr>
<th>Energy</th>
<th>Absolute Cross-Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_p = 8.0$ MeV</td>
<td>$V = 46.8$</td>
</tr>
<tr>
<td>$W = 10.0$</td>
<td>$R = 0.60$</td>
</tr>
<tr>
<td>$E_p = 8.5$ MeV</td>
<td>$V = 47.0$</td>
</tr>
<tr>
<td>$W = 10.5$</td>
<td>$R = 0.70$</td>
</tr>
<tr>
<td>$E_p = 9.0$ MeV</td>
<td>$V = 47.3$</td>
</tr>
<tr>
<td>$W = 11.0$</td>
<td>$R = 0.80$</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>Angle</th>
</tr>
</thead>
<tbody>
<tr>
<td>120°</td>
</tr>
<tr>
<td>160°</td>
</tr>
</tbody>
</table>

Laboratory Angle
Typical errors for the other angles are of the order of 6% to 7%.

From the smooth fits to the excitation functions, average angular distributions were derived at 0.5 MeV intervals between 8.0 to 12.0 MeV, and analysed using the method described previously.

As the absolute cross sections were determined in the region of 10 MeV, the first search on V, W, R and N was done at this energy and then repeated at two neighbouring energies of 9.5 and 10.5 MeV. The results are shown in figure 1.10.6.4 and it is apparent that the lowest value of $\chi^2$ occurs for $N = 1.0$. The values of $\chi^2$ were calculated in steps of 0.3 MeV for V and W, and steps of 0.05 for the reduction factor and the normalization parameter, and these are indicated as error bars in figure 1.10.6.4. The value of 1.0 obtained for N established that the absolute cross sections were determined correctly. Thus N was kept constant and equal to 1.0 in the analysis of all the other angular distributions.

Figures 1.10.6.3 and 1.10.6.5 summarize the best fits obtained for all angular distributions, together with the values of V, W, R and $\chi^2$. Good fits were
Dependence of the minimum values of $\chi^2$ as a function of the normalization $N$ with the corresponding values obtained for $V$, $W$ and $R$. 
Variation of the parameters $V, W, R$ and $\chi^2$ with proton bombarding energy.
obtained at all energies and with systematic variations in the parameters. Table 1.10.6.1 lists the values of $V$ and $W$ obtained from these angular distributions together with the values of $R$ and $\chi^2$. The neutron optical-model parameters were those obtained by Perey and Buck (Pe 62) and the alpha particle parameters those of Bourke (Bo 68).

Table 1.10.6.1

<table>
<thead>
<tr>
<th>$E_p$ (lab)</th>
<th>$V$</th>
<th>$W$</th>
<th>$V$</th>
<th>$W$</th>
<th>$R$</th>
<th>$\chi^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.0</td>
<td>51.6</td>
<td>13.5±2</td>
<td>46.8</td>
<td>10.0</td>
<td>0.60</td>
<td>1.14</td>
</tr>
<tr>
<td>8.5</td>
<td>51.3</td>
<td>13.5±2</td>
<td>47.0</td>
<td>10.5</td>
<td>0.70</td>
<td>0.71</td>
</tr>
<tr>
<td>9.0</td>
<td>51.0</td>
<td>13.5±2</td>
<td>47.3</td>
<td>11.0</td>
<td>0.80</td>
<td>0.54</td>
</tr>
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<td>9.5</td>
<td>50.7</td>
<td>13.5±2</td>
<td>47.3</td>
<td>11.3</td>
<td>0.90</td>
<td>0.45</td>
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<tr>
<td>10.0</td>
<td>50.5</td>
<td>13.5±2</td>
<td>47.5</td>
<td>11.5</td>
<td>1.00</td>
<td>0.46</td>
</tr>
<tr>
<td>10.5</td>
<td>50.2</td>
<td>13.5±2</td>
<td>47.5</td>
<td>11.5</td>
<td>1.05</td>
<td>0.65</td>
</tr>
<tr>
<td>11.0</td>
<td>49.9</td>
<td>13.5±2</td>
<td>47.5</td>
<td>11.3</td>
<td>1.10</td>
<td>0.87</td>
</tr>
<tr>
<td>11.5</td>
<td>49.7</td>
<td>13.5±2</td>
<td>47.3</td>
<td>10.8</td>
<td>1.00</td>
<td>1.03</td>
</tr>
<tr>
<td>12.0</td>
<td>49.4</td>
<td>13.5±2</td>
<td>46.8</td>
<td>10.0</td>
<td>0.90</td>
<td>2.46</td>
</tr>
</tbody>
</table>

$E_p$, $V$ and $W$ are in MeV

From the optical-model and Hauser-Feshbach calculations, the average direct and compound nucleus components of the cross sections as a function of energy
and angle were extracted. Some of them are shown in figure 1.10.6.6. It is evident that, for all angles, the average compound nucleus component decreases with increase in energy. This is to be expected as, with increasing energy, there is an increase in the number of open channels by which the compound nucleus can decay, so that the compound nucleus cross section in any one channel should decrease, provided that both the incident and outgoing particles are sufficiently above the Coulomb barrier of the target and residual nuclei respectively. The direct reaction component, however, does not show any such trend with energy and angle. Thus, at forward angles it decreases with increasing energy, while at backward angles it increases. Also, the fraction of direct reaction, defined by the average direct reaction cross section divided by the average differential cross section, does not show any systematic behaviour. This is shown in figure 1.10.6.7 and it is evident that at these energies and in this mass region the fraction of direct reaction as defined previously need not increase with increasing energy. For example, at 50° the fraction of direct reaction decreases with increasing energy. This is because the diffraction
The direct reaction component (open circles) and the compound nucleus component (filled circles) as given by the two theories employed at eight different lab angles for the reaction $^{23}\text{Na}(p,p_0)$. 
Figure 1.10.6.7

Variation of the fraction of direct reaction with lab angles at 8, 10 and 12 MeV proton bombarding energy obtained from the theoretical calculations.
pattern for the direct component changes with energy whereas the shape of the compound nucleus distribution remains fairly uniform.

It is interesting to note the surprisingly large values obtained for the reduction factor, and the unexpected variations of the reduction factor and W with energy. However, if the compound nucleus process is correctly described by the Hauser-Feshbach theory, then a third reaction mechanism besides the compound nucleus and the direct reaction process could have the observed effect on R and W. Such a mechanism could be that observed as intermediate structure, which the Hauser-Feshbach theory would attempt to account for by increasing the reduction factor. Indeed, evidence for intermediate structure has been seen in this mass region, and in particular in the compound nucleus $^{24}_{\text{Mg}}$. Therefore, the high values of the reduction factor found here might be regarded as further evidence for the presence of intermediate structure in the compound nucleus $^{24}_{\text{Mg}}$ at the considered excitation energies. This point will be further discussed in chapter 2.
1.11 ANALYSES OF AVERAGE ANGULAR DISTRIBUTIONS FROM PROTON INELASTIC SCATTERING REACTIONS ON $^{23}\text{Na}$

1.11.1 INTRODUCTION

From the spectra of the proton induced reactions on $^{23}\text{Na}$ it was possible to obtain excitation functions for the $p_2$ and $p_3$ channels. Due to problems arising from separating carbon and oxygen impurity peaks from the $p_1$ group it was impossible to extract angular distributions for the $p_1$ channel. For the reaction $^{23}\text{Na}(p,p_2)$ the compound nucleus contribution was calculated using the Hauser-Feshbach theory and the direct inelastic differential cross sections were calculated using the distorted wave Born approximation (DWBA) (Le 57). Although the DWBA theory must be considered as one of the most popular tools for estimating direct interactions and consequently has been well documented in the literature, it is considered worth while mentioning how the direct interaction is defined in the DWBA theory, thus elucidating the differences in definition of the direct process as derived from an optical-model, DWBA or fluctuation analysis (chapter 3).
1.11.2 THE DWBA THEORY

The distorted wave theory assumes that the elastic scattering process is the dominant one, but there is a small perturbation which gives rise to the nonelastic processes. Couplings between other channels than the elastic channel and the inelastic channel under consideration are neglected. The inelastic differential cross section for exciting the target from an initial state $|i\rangle$ to a final state $|f\rangle$ is given by Mott and Massey (Mo 49) as

$$\frac{d\sigma}{d\Omega} = \left(\frac{m}{2\hbar^2}\right)^2 \frac{k_f}{k_i} \sum_{\text{Av}} |T_{fi}|^2$$

where $k_f$ and $k_i$ are the final and initial wave numbers respectively for the inelastically scattered particle of reduced mass $m$, and $\sum_{\text{Av}}$ means that the sum is taken over the unobserved quantum numbers in the exit channel and the average is taken over the unobserved quantum numbers in the incident channel. $T_{fi}$ is the transition amplitude used in the DWBA theory for a one-step direct reaction (Sa 64)

$$T_{fi} = \int \chi_f^*(-\mathbf{r}_f) \langle \psi_f | V | \psi_i \rangle \chi_i^{(+)}(\mathbf{r}_i) \, d\mathbf{r}_i \, d\mathbf{r}_f$$
The $\chi_i$ and $\chi_f$ are distorted waves describing the elastic scattering before and after the inelastic event which excites the target from the state $\psi_i$ to the state $\psi_f$ under influence of the perturbing interaction $V$. It remains to specify the nuclear matrix elements $\langle \psi_f | V | \psi_i \rangle$.

In doing so two different approaches can be made. Either one pictures the projectile to interact with only a single target nucleon (the microscopic description), or the projectile may interact with the nucleus as a whole (the collective model) (Sa 66). The microscopic approach to inelastic scattering assumes that the interaction is a sum, $V$, of two-body potentials and the matrix elements $\langle f | V | i \rangle$ connect shell model wave functions $| f \rangle$ and $| i \rangle$.

In some respects the nucleus $^{23}_{\text{Na}}$ shows collective properties and the nuclear matrix elements can be derived in the collective model from a deformed nonspherical potential well. The potential is supposed to be a function of the distance between the projectile and the nuclear surface and is expanded in a Taylor series. The first term in this expansion is identified with the spherical optical-model potential and the second term is taken to be the potential responsible for
the inelastic scattering. Terms of higher order in the expansion are neglected. The first-order term yields a matrix element (Sa 66)

$$\langle \psi_f | V | \psi_i \rangle \sim \beta \frac{\partial U}{\partial r}$$

where \( \beta \) is the deformation parameter and \( U \) the optical-model potential.

For the actual calculation of the DWBA cross section, the collective model and the computer code JULIE were used. Three calculations were performed for incoming particle energies of 8, 10 and 12 MeV. Strictly one should use the optical-model at the correct energy of the incoming and outgoing particle, i.e. at 10 MeV incident energy the outgoing particle has an energy of approximately 8 MeV (the Q-value for the second excited state in \(^{23}\)Na is approximately \(-2\) MeV). This was done for 10 and 12 MeV but for 8 MeV the same optical-model potentials were used for the incoming and outgoing channels. Both the real and imaginary parts of the optical-model potential were deformed, and the optical-model parameters were those found from the elastic scattering data. The transmission coefficients used in the Hauser-Feshbach calculations were also obtained from the elastic scattering data.
1.11.3 THE REACTION $^{23}\text{Na}(p,p_2)$

The predicted results from the combined DWBA and Hauser-Feshbach theory together with the experimental data are shown in figure 1.11.3.1. The angular distributions were obtained from smooth fits of the form $A\theta^2 + B\theta + C$ to the excitation functions, and the errors were assigned in a similar way as in the elastic scattering analysis.

Besides the reduction factor $R$, the deformation parameter $\beta$ in the DWBA calculation may be regarded as a free parameter when fitting experimental data of this kind to the combined DWBA and Hauser-Feshbach theory. Therefore, for the 8.0 MeV angular distribution the reduction factor was fixed at the value obtained from the elastic scattering analysis, leaving $\beta$ as the only free parameter. The fit obtained is the one shown in figure 1.11.3.1 and agrees well with the experimental data at all angles. The value of 0.6 used for the reduction factor gave a corresponding value for $\beta$ of 0.4, which compares favourably with similar analyses in this mass region (Cr 68). The parameter $\beta$ was thereafter kept fixed at 0.4 for all other analyses of the angular distributions. The only parameter varied in fitting the angular distributions at 10.0 and 12.0 MeV was the reduction factor. For both
Average angular distributions obtained from the smooth fits to the excitation functions and the theoretical predictions using the combined DWBA and Hauser-Feshbach theory for the reaction $^{23}\text{Na}(p,p_2)$. 
these energies the resulting theoretical fits to the experimental angular distributions underestimated the cross section at the backward angles. However, this is probably due to the effect of the intermediate structure (chapter 2). It is apparent that the fitting of smooth curves to the excitation functions to obtain the average angular distributions, and the effect of direct reaction, especially at the forward angles, reduced the effect of the intermediate structure. The values of the calculated reduction factors are also shown in figure 1.11.3.1.

1.11.4 THE REACTION $^{23}$Na(p, p$_3$)

The angular distributions for the p$_3$ channel exhibit similar shapes to those predicted by the Hauser-Feshbach theory and it was found unnecessary to include any direct reaction component in the fitting procedure. Although the angular distributions for this channel are not very extensive, the results are shown in figure 1.11.4.1 together with the theoretical fits obtained using the parameters from the elastic scattering analysis and only varying the reduction factor. The angular distributions were obtained in the same way as the p$_0$ and p$_2$ angular
Figure 1.11.4.1

Average angular distributions obtained from the smooth fits to the excitation functions and the theoretical predictions using the Hauser-Feshbach theory for the reaction $^{23}\text{Na}(p,p_3)$. 
$^{23}\text{Na}(p,p_3)$

$E_p = 9 \text{ MeV}, \quad R = 0.65$

$E_p = 10 \text{ MeV}, \quad R = 0.74$

$E_p = 11 \text{ MeV}, \quad R = 0.77$
distributions and a typical excitation function and its smooth linear least squares fit, using a parabola, are shown in figure 1.11.4.2.

Significant discrepancies between the theory and the experimental points due to the effect of intermediate structure are not observed for any of the angular distributions. However, the reduction factor does increase with energy. This is not to be expected and is most certainly due to the effect of intermediate structure.
Smooth curve of the form $AE^2 + BE + C$ fitted to the excitation function measured at $150^\circ$ (lab) for the reaction $^{23}\text{Na}(p,p_3)$. 
CHAPTER 2

INTERMEDIATE STRUCTURE IN EXCITATION FUNCTIONS FROM THE REACTION $^{23}\text{Na}(p,p')$

2.1 INTRODUCTION

Broad resonances found in excitation functions, additional to fine structure fluctuations, are generally referred to as intermediate structure resonances and can be interpreted theoretically either in terms of 2p-1h states as in the theory developed by Feshbach (Fe 58a, Fe 62), Block and Feshbach (Bl 63) and Kerman et al. (Ke 63), or by statistical models of highly excited nuclear states as in the theory developed by Moldauer (Mo 67).

In the theory of Feshbach, the complex states of the compound nucleus are pictured as being formed by a chain of two-body collisions. The first step towards the formation of a complex compound nucleus state is the formation of a two-particle-one-hole (2p-1h) state. Once the state has been formed the two-body interaction can act again, and the 2p-1h configuration proceed further towards the formation of a more complex compound nucleus configuration via a 3p-2h state, or the 2p-1h state may decay by emission of a free particle or $\gamma$-ray. In either
case the 2p-lh state is involved and such states are therefore referred to as doorway states. Thus the concept of doorway states refers to the interpretation of the experimentally observed gross structure - the intermediate structure.

Since the lifetime of a doorway state is much shorter than the ordinary complex compound nucleus states, decay by emission of a free particle or a γ-ray should be apparent as gross structure in measured excitation functions. This is then the most immediate indication of the presence of doorway states.

Moldauer (Mo 67), on the other hand, has shown that gross structure of the kind discussed, also can be accounted for by statistical models of highly excited nuclear states. For highly absorbed channels, in the region of overlapping levels, he has shown that resonances occur with widths much larger than the widths of the surrounding resonances. The large widths occur preferentially in conjunction with large strengths of the absolute resonance amplitudes, and the net effect will be observed as intermediate structure resonances. What is involved is not the accidental 'lumping' of resonance contributions to the reaction amplitude which
has been found to arise from conventional statistical assumptions; instead, the gross structure as interpreted by either model discussed here should be associated with a definite spin in the compound nucleus. However, it is often difficult to extract the spins of these states because the effects in most cases are rather small and interference from the statistical fluctuations makes it very difficult to extract cross sections and angular distributions representing only the intermediate structure.

2.2 EVIDENCE FOR INTERMEDIATE STRUCTURE IN THE COMPOUND NUCLEUS $^{24}\text{Mg}$

In exploring the giant dipole resonance in $^{24}\text{Mg}$ through the reaction $^{23}\text{Na}(p,\gamma)$, Bearse et al. (Be 68) found evidence for intermediate structure. This was supported further when the reaction $^{23}\text{Na}(p,p_\gamma)$ was analysed in the previous chapter, using the optical-model and Hauser-Feshbach theory. It was found that the reduction factor in the Hauser-Feshbach calculation showed a peculiar variation with energy. Starting off with a value of 0.6 at 8 MeV proton bombarding energy, the reduction factor gradually increased and reached a maximum value of 1.10 at 11 MeV. The absorption parameter
W in the optical-model calculation also increased and reached a maximum value between 10 and 11 MeV indicating strong absorption at this energy. Because of these facts and the general trend of the cross sections at backward angles, which show an increase with energy not to be expected for a reaction where the compound nucleus process plays a significant part, it may be concluded that intermediate structure could produce such effects on W and the reduction factor. Further evidence for the presence of intermediate structure is forthcoming from the analysis of the inelastic proton scattering from $^{23}$Na.

Figures 2.2.1 and 2.2.2 show excitation functions for the reaction $^{23}$Na(p,p$_2$) measured in 20 keV steps between 8.0 and 12.0 MeV and a small section of a p$_1$ excitation function, together with p$_3$ excitation functions, the latter averaged over a 200 keV interval to illustrate the gross structure effects. Unfortunately, 150° was the only backward angle over the energy range for which it was possible to measure the p$_1$ excitation function because of oxygen and carbon impurity peaks. The p$_2$ excitation functions exhibit gross structure over two regions, from 8.0 to 10.0 MeV and from 10.0 to 11.0 MeV, similar to that found by Bearse et al. A theoretical calculation of the correlation angle (chapter 3) gives a
Figure 2.2.1

Excitation functions measured in 20 keV steps from 8.0 to 12.0 MeV for the reaction $^{23}_{\text{Na}}(p,p_2)$ and a small section of the $p_1$ excitation function at $150^\circ$ lab.
Smooth excitation functions obtained by using a 200 keV averaging interval for the excitation functions measured in 20 keV steps from 8.3 to 12.0 MeV for the reaction $^{23}\text{Na}(p,p_3)$. 
value of approximately $25^\circ$. Therefore there should be little statistical correlation between the excitation functions at the angles shown in figure 2.2.1. Because of the magnitude of the fluctuation and the widths of the compound nucleus states ($\approx 100$ keV (chapter 3)), apparent shifts in the position of the gross structure may be observed due to interference effects when comparing excitation functions for which the fluctuations are uncorrelated. This may be seen by comparing the $55^\circ$ excitation function with the two at backward angles, where the maximum of the gross structure between 8.0 and 10.0 MeV is shifted to a slightly higher energy. The $p_3$ excitation functions show gross structure effects at all angles at approximately 9.0, 10.0 and 11.5 MeV.

2.3 INVESTIGATION OF APPARENT SHIFTS OF GROSS STRUCTURE OBSERVED IN THE MEASURED EXCITATION FUNCTIONS

In order to check whether the shifts of the gross structure observed were of a statistical nature, artificial excitation functions were created using a scattering amplitude of the form

$$f = -i \sum_{\lambda} a_\lambda \frac{\Gamma_\lambda}{E - E_\lambda + i \frac{\lambda}{2\pi}}$$
Artificial excitation functions including gross structure resonance terms. The first four excitation functions correspond to $N = 1$. The fifth excitation function is formed by the coherent addition of the first four excitation functions ($N = 4$) and illustrates the large shifts of the gross structure.
Artificial excitation functions including gross structure resonances

10, 20, 30
to give the differential cross section (Br 63)

\[ \sigma = \left| \sum_{\lambda} \frac{a_{\lambda}}{E - E_{\lambda}^\prime + i \frac{\Gamma_{\lambda}}{2}} \right|^2 \]

The following assumptions were used: \( \Gamma/D = 20 \), \( \Gamma_{\lambda} \) = constant, and \( a_{\lambda} \) were taken as real random numbers with a Gaussian distribution of mean zero and a standard deviation of unity. At intervals of \( 20 \Gamma \), a gross structure resonance term with \( \Gamma_g = 7 \Gamma \) was inserted and the \( a_{\lambda} \) for this term were chosen such that the average cross section for the statistical fluctuation component given by (St 63)

\[ \bar{\sigma} = \frac{4\pi |a_{\lambda}|^2}{\Gamma_D} \]

was equal to the cross section for the isolated gross structure at the resonance energy. All parameters are thus chosen to correspond to those experimentally observed in the reaction \( ^{23}\text{Na}(p, p_2) \). The result is shown in figure 2.3.1 where the top four excitation functions are calculations for \( N=1 \) (Chapter 3) and it can be seen that, in some cases, the gross structure is shifted 3-4 \( \Gamma \) and, in other cases, there is no evidence for the gross structure. The bottom excitation function in figure 2.3.1.
is formed by the coherent addition of the top four excitation functions and therefore corresponds to the N=4 case since the statistical cross sections for the first four figures are independent. Thus, it is assumed in the bottom figure that the gross structure term interferes coherently in all these cases. If this is not so, the shifts may be even larger than those observed in the last figure. It is clear that shifts of the magnitude observed in the excitation functions from the reaction $^{23}\text{Na}(p,p_2)$ can be accounted for by interference between the statistical fluctuations and the gross structure.

2.4 INTERPRETATION OF THE SELECTIVE OCCURRENCE OF INTERMEDIATE STRUCTURE IN EXCITATION FUNCTIONS FROM PROTON INDUCED REACTIONS ON $^{23}\text{Na}$

As can be seen from figures 2.2.1 and 2.2.2, there is strong correlation between the gross structure over all angles for each channel considered, but there is very little correlation between the $p_2$ and $p_3$ channels. This is surprising since it is generally assumed that there should be correlation between intermediate structure in different channels as well as over different angles. However, it will be shown that, in certain reactions,
correlation between different channels need not be present, although in all cases it is necessary to have correlation over all angles of each channel.

Figure 2.4.1a shows that the transmission coefficients for $p_0$, $p_3$ and $p_2$ at 10 MeV bombarding energy for the reaction $^{23}\text{Na}(p,p)$ are very large, indicating strong absorption in these channels. The presence of such channels is sufficient to predict intermediate structure resonances by statistical models according to Moldauer. Accordingly, it seems reasonable to try to understand the relative intensity of decay of the intermediate states on the basis of a statistical model as well. A Hauser-Feshbach calculation was performed to calculate the partial differential cross sections $\frac{d\sigma}{d\Omega}$ for the decay from a compound nucleus state of spin $J$ of both positive and negative parity to a series of final states of different spins. The result is shown in figure 2.4.1b where the probability defined as $100 \frac{d\sigma_J}{d\Omega} / \frac{d\sigma}{d\Omega}$ is shown as a function of $J$ for even parity. The odd parity case is almost identical. From the figure, it can be seen clearly how the relative decay from compound nucleus states of varying spins to the $1/2^+(p_3)$ state is quite different compared to that for the $7/2^+(p_2)$ state. On the other hand, the relative decay to the $3/2^+$ ground state
Figure 2.4.1

(a) Transmission coefficients as a function of the orbital angular momentum at 10 MeV for the \( p_0, p_3 \) and \( p_2 \) exit channels.

(b) Dependence of the percentage probability for decay of the compound nucleus as a function of the spin \( J \) of the compound nucleus.
overlaps both of the previous decay modes. Thus, if it is assumed that the intermediate structure states have a similar J dependence for the partial differential cross sections, and if there are several intermediate structure resonance states of different spins present in the energy range over which the excitation functions have been measured, states of a certain spin may have a higher cross section than states of different spins for decay to particular final levels. Therefore, it is reasonable to conclude that the excitation functions for different exit channels will show intermediate structure effects which vary for each channel, depending on the spin of the final level.

2.5 OTHER PROPERTIES OF THE INTERMEDIATE STRUCTURE

Because of the selective nature of the reaction mechanism occurring here and because of the observed separation of the gross structure (≈1.5 MeV), it is probable that the states in the compound nucleus responsible for the intermediate structure observed in a particular exit channel are not overlapping. Therefore, angular distributions obtained from the gross structure should correspond to states of definite spins. The
angular distribution for the gross structure occurring at approximately 10.5 MeV in the \( p_2 \) channel was obtained by calculating the average cross sections in the energy range 9.5-10.0, 10.0-11.0 and 11.0-11.5 MeV and assuming that the average cross section for the ranges 9.5-10.0 and 11.0-11.5 MeV represented the average cross section for the range 10.0-11.0 MeV without intermediate structure present.

The angular distribution defined by

\[
\langle \frac{d\sigma}{d\Omega} \rangle_{10-11 \text{ MeV}} = \frac{1}{2} \left( \langle \frac{d\sigma}{d\Omega} \rangle_{9.5-10 \text{ MeV}} + \langle \frac{d\sigma}{d\Omega} \rangle_{11-11.5 \text{ MeV}} \right)
\]

is shown in figure 2.5.1a and is approximately symmetric about 90°. This is consistent with a compound nucleus process, but as may be seen from figure 2.5.1b the calculated Hauser-Feshbach cross section is approximately isotropic. The angular distribution could be fitted with Legendre polynomials with \( \nu_{\text{max}} = 4 \) as is shown in figure 2.5.1a. Channel spin considerations lead to spin limits of \( 2 \leq J \leq 6 \). The average angular distribution \( \langle \frac{d\sigma}{d\Omega} \rangle_{10-11 \text{ MeV}} \) (including data from 30° lab) exhibits a similar shape as that shown in figure 2.5.1a and it can be seen from this figure that it is impossible to predict angular distributions of this form by combining the DWBA
and Hauser-Feshbach calculations. However, when the averaging is done over a large energy range, as in section 1.11, the gross structure effects will effectively be eliminated and reasonably good fits can be obtained. It should also be noted that angular distributions similar to the one shown in figure 2.5.1a have been obtained for DWBA calculations when exchange terms have been included (Am 67).
Figure 2.5.1

(a) Angular distribution for the intermediate structure obtained between 10.0 and 11.0 MeV for the $^{23}\text{Na}(p,p_2)$ reaction

(b) Predicted DWBA and Hauser-Feshbach cross sections for the reaction $^{23}\text{Na}(p,p_2)$ at 10.5 MeV.
Ang. dist. for gross structure in $^{23}\text{Na}(p,p')$

$\nu_{\text{max}} = 2$ (even and odd)

$\nu_{\text{max}} = 4$

Total theoretical cross section

$df/d\theta$ arb. units

$df/d\theta$, mb/stér

$\Theta_{\text{CM}}$
CHAPTER 3

FLUCTUATION ANALYSES AND MODEL COMPARISONS

3.1 INTRODUCTION

In chapters 1 and 2 the analysis was concerned with properties of the average cross sections. However, for reactions which proceed partly through a compound nucleus mechanism and for which the excitation energy in the compound nucleus corresponds to the region of overlapping levels, the excitation functions exhibit rapid fluctuations with energy. The fluctuations were first predicted by Ericson (Er 60a, Er 60b) and a model describing them developed by Ericson (Er 63) and Brink and Stephen (Br 63).

In this model of statistical fluctuations, the differential cross section \( \sigma(E) \) is defined by

\[
\sigma(E) \sim |S(E)|^2
\]

The scattering amplitude \( S(E) \) consists of two parts; one which varies rapidly with energy \( S_R \) and one which varies slowly with energy \( S_S \). Thus

\[
S = S_R + S_S \tag{3.1.1}
\]
It is now useful to assume that the time dependent wave equation representing a decaying quantum mechanical system can be written as

\[ \psi(t) = \begin{cases} \lambda e^{-iE_0 t/H} e^{-t/2T} & ; \ t \geq 0 \\ 0 & ; \ t < 0 \end{cases} \]

where \( E_0 \) is the total energy of the stationary state \( \lambda \exp(-iE_0 t/H) \). The wave function squared represents the probability for the number of particles in the state at the time \( t \).

\[ |\psi(t)|^2 = \lambda^2 e^{-t/T} \]

Instead of studying the wave function \( \psi \) as a function of \( t \), the energy dependence may be obtained by a Fourier transformation of \( \psi(t) \) to the energy plane

\[ \psi(E) = \int_{-\infty}^{\infty} \psi(t) e^{iEt/H} dt = -\frac{i\lambda A}{E-E_0+i\pi/2T} \]

The probability for the state to have an energy between \( E \) and \( E + dE \) is then

\[ |\psi(E)|^2 \sim \frac{1}{(E-E_0)^2 + \pi^2/4} \]
where $\Gamma = \hbar$. This is the energy distribution to be expected when a quantum mechanical system with mean energy $E_0$ and mean life time $\tau$ decays.

The scattering amplitude $S_R$ in equation 3.1.1 can be expressed as (St 63)

$$S_R(E) = -i \sum_{\lambda} \frac{\gamma^{i}_{\lambda} \gamma^{f}_{\lambda}}{E - E_{\lambda} + i\Gamma_{\lambda}/2} = -i \sum_{\lambda} \frac{A_{\lambda}}{E - E_{\lambda} + i\Gamma_{\lambda}/2}$$

where $S_R(E)$ is the scattering amplitude from an initial state $|i\rangle$ to a final state $|f\rangle$ via the intermediate states $|\lambda\rangle$, and $\gamma^{i}_{\lambda}$ and $\gamma^{f}_{\lambda}$ are the amplitudes for formation and decay of the compound nucleus states $|\lambda\rangle$ respectively.

The statistical model assumes that $S_R(E)$ may be divided into two terms; $S_C(E)$ which fluctuates rapidly with energy and with mean value zero and $S_I(E)$ which is constant with energy. $S_I(E)$ is combined with $S_S(E)$ of equation 3.1.1 to give the amplitude $S_D(E)$. Thus the scattering amplitude is

$$S = S_D + S_C$$

where $S_D$ is identified with the direct reaction mechanism and $S_C$ with the compound nucleus mechanism. The scattering amplitude $S_C$ is given by
\[ S_c(E) = -i \sum_{\lambda} \frac{A_{\lambda}^i}{E - E_{\lambda} + i\Gamma_{\lambda}/2} \] (3.1.2)

In the region of many overlapping levels, i.e. when \( \Gamma \gg D \) (D is the level spacing), the number of compound nucleus states \( |\lambda\rangle \) within the coherence energy \( \Gamma \) which contribute significantly to the cross section will be large. The central limit theorem of probability (e.g. (Fe 58b)) states that if the \( A_{\lambda}^i \) have random phases then the sum over the \( \lambda \)'s will have, to a very good approximation, a Gaussian distribution with mean value zero. Thus the assumption of random phases of the \( A_{\lambda}^i \) is equivalent to the assumption of the mean value of \( S_c \) being zero. Equation 3.1.2 has been used extensively to generate artificial excitation functions by which many of the predictions of the statistical model of fluctuations have been tested (e.g. Da 66, Wo 66, Ho 69a). The method was used in chapter 2 and will be used in the following sections.

### 3.2 PREDICTIONS OF THE STATISTICAL FLUCTUATION MODEL

The differential cross section to a specific final state can be expressed as the sum of \( N \) independent partial cross sections \( (\sigma_\mu) \) (Bo 64, Bo 65)
Thus the fluctuations observed in the cross section \( \sigma(E) \) will decrease with increasing \( N \). The number \( N \) is dependent on the angle of observation of the cross section \((Br 64)\) and on the projection on the z-axis of the spins of the particles in the reaction \((Er 66b)\). Therefore, in a proton elastic or inelastic reaction, the fluctuations will decrease with increase of the target spin and the spin of the residual nucleus. The value \( N \) is often referred to as the nuclear or fluctuation damping coefficient.

Ericson, and Brink and Stephen used this definition of \( N \) and the statistical model assumptions to derive the approximate expression for the autocorrelation function

\[
C(E, \varepsilon) = \frac{\langle \sigma(E+\varepsilon) \sigma(E) \rangle}{\langle \sigma(E+\varepsilon) \rangle \langle \sigma(E) \rangle} - 1 = \frac{1}{N} \frac{1-Y_D^2}{1+\varepsilon^2} \quad (3.2.1)
\]

where \( \langle \rangle \) stands for energy averaged values and \( Y_D \) is defined as \( \langle \sigma \rangle_{\text{direct}} / \langle \sigma \rangle_{\text{total}} \).

According to the above formula, the autocorrelation function will have a Lorentzian form as a function of \( \varepsilon \) with the mean level width \( \Gamma \) equal to the half width at
half height. The quantity $\frac{1}{N}(1-Y_D^2)$ is determined when $\epsilon = 0$, i.e.

$$C(\epsilon=0) = \frac{1}{N}(1-Y_D^2) \quad (3.2.2)$$

To extract the fraction of direct reaction, $Y_D$, from excitation functions it is first necessary to estimate $N$.

### 3.3 Calculation of the Nuclear Damping Coefficient N

The nuclear damping coefficient $N$ may be calculated using the formalism given by Brink et al. (Br 64).

The differential cross sections are expressed as

$$\langle \frac{d\sigma}{d\eta}(\theta) \rangle = \frac{\chi^2}{4(2I_a+1)(2i_a+1)} \sum \frac{A}{CC'; CC'}\left(\theta\right) \frac{T_C T_{C'}}{\sum_{C''} T_C''}$$

$$= \frac{\chi^2}{4(2I_a+1)(2i_a+1)} \langle \omega(\theta) \rangle$$

The correlation function can be written as

$$C(\theta, \theta') = \frac{\langle \omega(\theta) \omega(\theta') \rangle}{\langle \omega(\theta) \rangle \langle \omega(\theta') \rangle} - 1$$
which Brink et al. have shown yields the expression

$$C(\theta, \theta') = \frac{\sum_{\alpha\beta\gamma} |A_{\alpha\beta\gamma}(\theta)|^2 K_{\alpha\gamma}}{\sum_{\alpha\beta\gamma} |A_{\alpha\beta\gamma}(\theta)|^2 K_{\alpha\gamma} \sum_{\alpha'\beta'\gamma'} |A_{\alpha'\beta'\gamma'}(\theta')|^2 K_{\alpha'\gamma'}}$$

The factor $K_{\alpha\gamma}$ includes all the terms containing the transmission coefficients. In the calculation of $C(\theta, \theta')$ the sums over all the exit channels were replaced by (E69)

$$\sum_{C''} T_{C''} = \frac{2\pi}{D_J} \Gamma_J$$

where $\Gamma_J$ is the level width and $D_J$ the level spacing of compound nucleus states of spin $J$. Richter et al. (Ri 65, Ri 66) have given these as

$$\Gamma_J = \Gamma_0 \exp\left(-J(J+1)/2\sigma_c^2\right)$$

$$D_J = D_0 / \left\{(2J+1) \exp\left(-J(J+1)/2\sigma_c^2\right)\right\}$$

Here

$$\frac{1}{\sigma^2} = \frac{1}{\sigma_{\text{res}}^2 + mR^2 T} - \frac{1}{\sigma_c^2}$$
where $\sigma^2_{\text{res}}$ and $\sigma^2_{\text{c}}$ are the spin cut-off parameters of the residual nucleus and of the compound nucleus respectively, $m$ is the mass of the neutron, $R$ is the radius of the residual nucleus and $T$ is its nuclear temperature.

Assuming that $2mR^2T \ll 2\sigma^2_{\text{res}}$, $K_{\alpha\gamma}$ can be written as

$$K_{\alpha\gamma} = \frac{T_{\ell_a} T_{\ell_b}}{(2J+1) \exp(-J(J+1)/2\sigma^2_{\text{res}})}$$

and the different summations in the expression for $C(\theta,\theta')$ can be carried out, e.g.

$$\sum_{\alpha\beta\gamma} |A_{\alpha\beta\gamma}(\theta)|^2 K_{\alpha\gamma}$$

$$= \sum S_a S_b n_a n_b \ell_a J \ell_b$$

$$\times \left( S_b \ell_b m_b | J n_a \right) Y_{\ell_b} m_b (\theta,\phi) \frac{D_0}{2\pi T_0} T_{\ell_a} T_{\ell_b}$$

$$\times \exp(J(J+1)/2\sigma^2_{\text{res}})/(2J+1)|^2$$

where

$S_a$ is the spin of the incident channel

$S_a = |i_a^a - I_a|$, \ldots, $(i_a^a + I_a)$

$i_a$ the spin of the incoming particle and $I_a$ the spin of the target nucleus;
\( S_b \) is the spin of the exit channel
\[
S_b = |i_b - I_b|, \ldots, (i_b + I_b)
\]
i_b the spin of the outgoing particle and \( I_b \) the spin of the residual nucleus;

\( n_a \) and \( n_b \) the z-components of \( S_a \) and \( S_b \) respectively;
\[
n_a = -S_a, \ldots, S_a; \quad n_b = -S_b, \ldots, S_b
\]
\( \ell_a \) is the angular momentum of the incoming particle
\[
\ell_a = 0, 1, \ldots, \ell_{\text{max}};
\]
\( J \) is the spin of the compound nucleus
\[
J = |S_a - \ell_a|, \ldots, (S_a + \ell_a);
\]
\( \ell_b \) is the angular momentum of the outgoing particle
\[
\ell_b = |S_b - J|, \ldots, (S_b + J);
\]
\( m_b \) is the z-component of \( \ell_b \).

The summations were restricted by parity conservation by imposing the condition \( (\pi_a + \ell_a + \pi_b + \ell_b) \) must be even where \( \pi_a \) and \( \pi_b \) are parities of the entrance and exit channels respectively.

It is clear that the nuclear damping coefficient can be calculated for cross correlations \((\Theta \neq \Theta')\) as well as autocorrelations \((\Theta = \Theta')\) using the same formalism.

In the last three sections a brief account of the statistical model of fluctuations has been given together with its predictions. It has been shown how the mean
level width $\Gamma$ and the fraction of direct reaction $Y_D$ can be determined from excitation functions using the autocorrelation function (equation 3.2.1). It is also possible to extract these two quantities from excitation functions by other means; how this can be done will be discussed in the sections to follow.

3.4 THE ANALYSIS OF EXCITATION FUNCTIONS USING DIFFERENT TRANSFORMS

3.4.1 THE FOURIER TRANSFORM

From the middle of the 18th century and onwards Bernoulli, D'Alembert, Langrange, Euler and Fourier set out to find a representation of an arbitrary function as a trigonometric series. As a result of their efforts, it is possible to use what is now known as the Fourier transform. Omitting details of mathematical rigour, the purpose of the Fourier transform is to decompose a given function $f$ into purely harmonic oscillations, i.e. to find a function $g$ such that

$$f(X) = \int_{-\infty}^{\infty} g(\varepsilon) e^{2\pi i X \varepsilon} d\varepsilon,$$
where \( \langle X, \varepsilon \rangle = X_1 \varepsilon_1 + X_2 \varepsilon_2 + \ldots + X_n \varepsilon_n \). It may be shown (e.g. Ho 64) that all sufficiently regular functions that decrease sufficiently rapidly at infinity can be written with the density

\[
g(\varepsilon) = \int_{-\infty}^{\infty} f(X) e^{-2\pi i \langle X, \varepsilon \rangle} dX
\]

\( g \) is called the Fourier transform of \( f \) and vice versa.

For example, if

\[
f(X) = e^{-aX^2} \text{ then } (X \in \mathbb{R})
\]

\[
g(\varepsilon) = \int_{-\infty}^{\infty} e^{-aX^2} e^{-2\pi i \varepsilon X} dX = \frac{-\pi^{2\varepsilon^2}}{a} \int_{-\infty}^{\infty} e^{-a(X+i\varepsilon)}^2 dX
\]

\[
= \frac{-\pi^{2\varepsilon^2}}{a} \int_{-\infty}^{\infty} e^{-aX^2} dX = \frac{\sqrt{\pi}}{a} e^{-\pi^2 \varepsilon^2 / a}
\]

In particular for \( a = \pi \), \( g = f \).

### 3.4.2 THE POWER SPECTRA

If a rapidly varying excitation function \( \sigma(E) \) is expanded as a Fourier series

\[
\sigma(E) = \sigma_0 + \sum_k a_k \cos(2\pi k \frac{E}{\mathcal{E}}) + \sum_k b_k \sin(2\pi k \frac{E}{\mathcal{E}})
\]
it has been shown (Bö 65) that the expectation value \( \overline{S_k} \) of the power spectra, i.e. \( \overline{S_k} = a_k^2 + b_k^2 \) yields

\[
\overline{S_k} \sim \frac{1}{N} (1 - Y_D^2) \Gamma e^{-2\pi k \Gamma / I}
\]

where \( I \) is the energy interval. Thus by plotting \( \log \overline{S_k} \) as a function of \( k \) it is possible to determine both the mean level width \( \Gamma \) and the fraction of direct reaction \( Y_D \).

So far two different methods have been discussed for determining \( \Gamma \) and \( Y_D \) from fluctuating excitation functions. It is also possible to write the normalized autocorrelation function in other ways. One is to use the convolution integral and Laplace transforms and another to multiply the Fourier transform by its complex conjugate and take the inverse of the result.

### 3.4.3. THE LAPLACE TRANSFORM

Here, as was the case in the previous section, mathematical rigour is omitted by saying that if \( f \) is a 'reasonable' function, one can define

\[
\tilde{f}(s) = \int_0^\infty e^{-st} f(t) \, dt
\]
where \( f(s) \) is the Laplace transform of \( f(t) \). An identical method of expressing this is

\[
\tilde{f}(s) = L[f(t)]
\]

where \( s \) is a complex number. For example, if \( f(t) = 1 \) then

\[
L(1) = \int_{0}^{\infty} e^{-st} \, dt = \frac{1}{s}
\]

Writing (Ga 42)

\[
(f*g)(t) = \int_{0}^{t} f(\tau)g(t-\tau) \, d\tau = \int_{0}^{t} g(\tau)f(t-\tau) \, d\tau = (g*f)(t)
\]

it is seen that \( f*g = g*f \) is a commutative operation. Using Laplace transforms it can be shown (appendix 1) that the autocorrelation function can be written

\[
C(\tau) \sim L^{-1} L(f*g) = L^{-1} (\tilde{f}(s) \tilde{g}(s)).
\]

The rather complex operation of the convolution integral is, when Laplace transformed, simply represented by multiplication of two Laplace transforms.

3.4.4. ANOTHER FOURIER REPRESENTATION OF THE AUTOCORRELATION FUNCTION

By denoting \( F \) as the Fourier transform of a function, \( F^* \) as its complex conjugate and \( F^{-1} \) as the inverse, it is
possible to write the autocorrelation function in a very simple operator form \((Le 60)\)

\[
\langle \sigma(E+\varepsilon) \sigma(E) \rangle = F^{-1}_{\varepsilon} \left( F^*_k F_k \left( \sigma(E) \right) \right)
\]  
\((3.4.4.1)\)

This fact is well known and has been applied in, for example, communication problems.

Knowing the theoretical form of the autocorrelation function as a function of \(\varepsilon\) \((equation \ 3.2.1)\), the mean level width \(\Gamma\) can be determined from expression \(3.4.4.1\). Thus, when properly normalized, it is possible to write the autocorrelation function as

\[
F^{-1}(F^*F)
\]

Moreover, if this expression is normalized in the following way then

\[
\frac{F^{-1}(F^*F)}{\langle \sigma(E+\varepsilon) \rangle \langle \sigma(E) \rangle} - 1 = \frac{1-Y_D^2}{N} \quad \frac{\Gamma^2}{\Gamma^2+\varepsilon^2}
\]

Again, it is possible to extract the mean level width \(\Gamma\) and the fraction of direct contribution \(Y_D\) from these transforms.
3.4.5 TESTING THE METHODS

The mean level width and the amount of direct reaction were extracted from artificial excitation functions using the formalism given in the previous sections. A computer program was written to evaluate the finite Fourier transform

\[ F(k) = \sum_{j=0}^{n-1} \sigma(E_j) \exp\left(2\pi i \frac{k}{n} j\right) \]

and a series of eight artificial excitation functions were employed to determine \( \Gamma \) and \( Y_D \). The mean of the determined values of \( \Gamma \) and \( Y_D \) agreed to within 10-15% for all the different methods. Generally \( Y_D \) was associated with smaller errors than \( \Gamma \) and the spread in \( Y_D \) and \( \Gamma \) were smaller for the method discussed in section 3.4.2 than for the other methods. It should also be pointed out that the methods described in sections 3.2 and 3.4.4 gave almost identical results, while the method described in section 3.4.2 often deviated from the others. However, any trends, such as increase or decrease of \( Y_D \) or \( \Gamma \), generally occurred simultaneously for all of the methods.
The methods for calculating $\Gamma$ and $Y_D$ as discussed in sections 3.2, 3.4.2 and 3.4.4 will be referred to as methods (i), (ii) and (iii) respectively.

3.5 EXPERIMENTAL INVESTIGATION OF MEAN LEVEL WIDTHS

3.5.1 THE MEAN LEVEL WIDTH IN THE COMPOUND NUCLEUS $^{32}\text{S}$

The mean level width in the compound nucleus $^{32}\text{S}$ has been obtained from an analysis of the excitation functions measured for the reaction $^{31}\text{P}(p,p_0)$ for incident proton energies between 8.0 and 10.0 MeV. This corresponds to excitation energies in the compound nucleus $^{32}\text{S}$ of between approximately 16.5 and 18.5 MeV.

The expression (equation 3.2.1) for the autocorrelation function has been derived using the assumptions of constant mean differential cross sections $\langle \sigma_D \rangle$ and $\langle \sigma_C \rangle$ which are generally not valid. It might therefore be expected that the experimental cross sections ($\sigma$) should be divided by the estimated mean compound nucleus component to obtain corrected excitation functions which have a varying direct component, but which have a constant mean compound nucleus component. If it is assumed that the average compound nucleus contributions calculated in chapter 1 ($R \times \sigma_{\text{HF}}$) show the same energy
dependence as that predicted for $\langle \sigma_0 \rangle$ by the fluctuation theory, then these may be used to correct the experimental excitation functions. It is then possible to apply the theory of Hall (Ha 64) to the corrected excitation functions to obtain the required autocorrelation function $C(\varepsilon)$ from

$$C(\varepsilon) = C'(\varepsilon) - K(\varepsilon) \quad (3.5.1.1)$$

where $C'(\varepsilon)$ is the autocorrelation for the corrected excitation function and $K(\varepsilon)$ is the autocorrelation for the smooth curve which describes the energy dependence of the excitation function. In the following this will be referred to as Method 1.

An alternative method (Method 2) is to remove the energy dependence of the experimental excitation functions by dividing through by the smooth fits. Although excitation functions will generally result in which the mean compound and direct cross sections vary with energy, the long range correlation effects present in Method 1 will be removed.

A comparison of the two methods is shown in figure 3.5.1.1 where autocorrelation functions are calculated for angles of $30^\circ, 70^\circ, 110^\circ$ and $150^\circ$ (lab). In all cases, the autocorrelation functions obtained by dividing
Figure 3.5.1.1

Autocorrelation functions calculated by Methods 1 and 2 for angles 30°, 70°, 110° and 150° (lab). The dashed line is for Method 1, the solid line for Method 2 (in both cases method (i) was used to calculate the autocorrelation function) and the dotted line (for the 70° (lab)) shows the result using Method 2 for removing the energy dependence and method (iii) for calculating the autocorrelation function.
AUTOCORRELATION ANALYSIS

\[ C(E,E+E) \]

\[ 3^1p(p,p)\ 3^1p \]

\[ \theta(\text{lab}) = 30^\circ \]

\[ \theta(\text{lab}) = 70^\circ \]

\[ \theta(\text{lab}) = 110^\circ \]

\[ \theta(\text{lab}) = 150^\circ \]

\[ \varepsilon(\text{keV}) \]

\[ \varepsilon(\text{keV}) \]

METHOD 1

METHOD 2

\[ C(E,E+E) \]
through by the smooth fits (Method 2) exhibit typical patterns - decreasing from a maximum at $\epsilon = 0$ with a Lorentzian shape of width $\Gamma$ and oscillating about the base line for large $\epsilon$. However, the autocorrelation functions calculated from the excitation functions, formed by dividing through by the estimated Hauser-Feshbach cross sections and using equation 3.5.1.1 to obtain $C(\epsilon)$ (Method 1), exhibit very peculiar behaviour at $30^\circ$ and $70^\circ$. For $110^\circ$ and $140^\circ$, the results are similar to the autocorrelation functions obtained using Method 2 although the oscillations about the base line for large $\epsilon$ are larger. It is therefore concluded that Method 1, although obeying the requirements of a constant mean compound nucleus component, is completely unsatisfactory for estimating either the mean width $\Gamma$ or the fraction of direct reaction $Y_D$.

Thus, for all fluctuation analyses, the energy dependence of the mean cross section should be removed by dividing through by the smooth fits to the excitation functions and the standard theory used to calculate the required quantities. The resulting excitation functions, after removal of the energy dependence, have constant mean cross sections of approximately unity.
For the estimation of $r$ using Method (i), no further corrections were made but when using Methods (ii) and (iii) the d.c. component was removed to give corrected excitation functions with average values of zero. At this point it could be argued that manipulating the excitation functions in this way might result in an incorrect determination of the mean level width. To check this, one artificial excitation function, from which the d.c. component had been removed, was multiplied by an energy-dependent factor which steadily increased the amplitude of the fluctuations over the range. The average amplitude of the fluctuations at the end of the range was about twice that at the beginning. It should be noted that, experimentally, this corresponds to an excitation function for a reaction in which energy-dependent compound and direct processes are occurring.

The results of the Fourier analysis (ii) of this excitation function before multiplication gave a mean level width of 50 keV and, after multiplication, it gave 49 keV. The error in the second case was about 10% greater than in the first case due to large low frequency components.

The Fourier analysis Method (ii) is illustrated in figure 3.5.1.2 where the line fitted to all the
frequency components for the corrected $140^\circ$ (lab) excitation function is shown. The calculated widths using Methods (i) and (ii) are shown in table 3.5.1 for all angles at which excitation functions were measured. The results using Method (iii) are almost identical to those obtained using Method (i). From table 3.5.1 it is seen that the mean level width shows roughly the same trend with angle when calculated by the two methods although the variations are not as large for the Fourier analysis (ii) results.

3.5.2 STATISTICAL CALCULATION OF THE MEAN LEVEL WIDTH

The mean level width $\Gamma$ of the compound nucleus may be calculated theoretically by combining the partial width $\Gamma_J$ with the partial cross sections $\sigma_J(\theta)$ by (Fe 65)

$$
\Gamma(\theta) = \frac{\sum_J \sigma_J(\theta)}{\sum J \sigma_J(\theta)/\Gamma_J},
$$

(3.5.2.1)

where $J$ is the spin of the compound nucleus.

The partial cross sections were derived by the standard Hauser-Feshbach method including the width fluctuation correction factor. The $\Gamma_J$ may be calculated
The straight-line fit to all the frequencies for the $140^\circ$ (lab) excitation function obtained from the reaction $^{31}\text{P}(p,p_0)$. The fit gives a value for the mean level width of 29 keV.
Table 3.5.1

Comparison of the mean level widths in the compound nucleus $^{32}$S obtained by the Fourier analysis method (ii) and by the autocorrelation function method (i)

<table>
<thead>
<tr>
<th>Angle (lab)</th>
<th>Mean level width (keV)</th>
<th>Method (ii)</th>
<th>Method (i)</th>
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</table>
by two methods. The first method (Method I) uses the expression given by Ericson (Er 63)

$$
\Gamma_J = \frac{D_J(E_{\text{CN}})}{2\pi} \sum_{\ell=0}^{\infty} \sum_{S=|J-\ell|}^{S+i\gamma} \sum_{j=|S-i\gamma|}^{E_{\text{CN}}-Q} \int \rho_R(E_R,j)T_{\ell,j}^\gamma(E)dE_R,
$$

where $E_{\text{CN}}$ is the energy of the compound nucleus, $D_J(E_{\text{CN}})$ the spacing of levels at this energy in the compound nucleus, $i_Y$ the spin of the outgoing particle, $Q$ the binding energy of this particle to the compound nucleus, $\rho_R(E_R,j)$ the level density in the residual nucleus at an excitation energy of $E_R$ for levels of spin $j$, the transmission coefficients $T_{\ell,j}^\gamma(E)$ correspond to particle $Y$ going to these levels, $S$ the exit channel spin and $\ell$ the orbital angular momentum in the exit channel.

This method has been used by Leachman et al. (Le) for the analysis of the mean level width in the compound nucleus $^{32}\text{S}$ obtained from the excitation functions for the reaction $^{31}\text{P}(p,\alpha)^{28}\text{Si}$ and $^{16}\text{O}(^{16}\text{O},\alpha)^{28}\text{Si}$. Therefore, to enable a comparison with their results, the same expression for the level density has been adopted with the parameters of Gilbert and Cameron. Also, identical energy-dependent spin cut-off parameters have been used.
The second method (Method II) of calculating $\Gamma_j$ uses the expression

$$\Gamma_j = \Gamma_0 \exp(-J(J+1)/2\sigma^2),$$

where

$$S \equiv \frac{1}{\sigma^2} = \frac{1}{\sigma_r^2 + mR^2T} - \frac{1}{\sigma_c^2}.$$

Here $\Gamma_0$ is the width of the levels of lowest $J$-value in the compound nucleus, $\sigma_r^2$ the spin cut-off parameter of the residual nucleus, $\sigma_c^2$ the spin cut-off parameter of the compound nucleus, $m$ the mass of the neutron, $R$ the radius of the residual nucleus and $T$ its nuclear temperature.

This method does not allow the absolute value of $\Gamma_j$ to be calculated unless the value of $\Gamma_0$ obtained from Method I is used. However, it does enable the relative angular dependence of $\Gamma(\theta)$ to be obtained, together with its variation on the values of the spin cut-off parameters. Figure 3.5.2.1 shows the values of $\Gamma(\theta)/\Gamma_0$ calculated for various values of $1/\sigma^2$ and also the results obtained when Method I is used. By comparison of these results, it is seen that good agreement is obtained when $1/\sigma^2 \approx 0.033$. 
Comparison of Methods I and II for calculating the ratio $\Gamma(0)/\Gamma_0$, where $\Gamma_0$ is the partial width for $J = 0$. 

**Figure 3.5.2.1**
The value of $\sigma_c^2 = 4.82$, which is the value used in Method I, gives $\sigma_r^2 + mR^2T = 4.16$. A value of 3.50 for $\sigma_r^2$ is then obtained by assuming $R = 1.25 \text{ A}^{1/3} \text{ fm}$ and $T = 1.78 \text{ MeV}$ (Gi 65a). This value is in agreement with the energy-dependent value of Method I, which varies from 1.52 at 0.4 MeV to 4.52 at 9.0 MeV.

To obtain a minimum in the mean level width near $90^\circ$ as is evident from the experimental data, it is necessary to have $\sigma_r^2 + mR^2T > \sigma_c^2$. Generally this is impossible because the compound nucleus is excited approximately 10 MeV above the residual nucleus, therefore $\sigma_c^2 > \sigma_r^2$. The term $mR^2T$ is a small correction and does not affect this result. However, Le Couteur (Le 69) has pointed out that for a ground state transition, $mR^2T$ is replaced by $m_0R^2E$, where $m_0$ is the mass of the outgoing particle and $E$ its energy, therefore $m_0R^2E$ is no longer a small correction, and $\sigma_r^2 + mR^2E$ can be greater than $\sigma_c^2$.

For the ground state transition, $\sigma_r^2 \approx 1$ and $m_0R^2E \approx 3.4$, therefore a negative value of $S$ would require $\sigma_c^2 < 4.4$. This is possible though less than the above estimate. As may be seen from figure 3.5.2.1, a negative value of $S$ predicts a minimum at $90^\circ$, although it still
does not predict the large angular dependence observed experimentally.

In the calculation of the $\sigma_J$ used in equation 3.5.2.1, it is necessary to know all the possible exit channels; for incident proton energies up to 10 MeV these are all known (En 67), although in some cases, the spins and parities of the final states had to be assigned on the basis of the Gilbert-Cameron level density formula. A check was made on these assignments by removing several of the levels of the final nucleus $^{31}$P and replacing them with levels generated from the level density formula. The resulting variation in $\sigma_J$ was insignificant when compared with the variation obtained by using different optical-model parameters to generate the transmission coefficients.

The optical-model parameters chosen were either those of Perey (Pe 63), Rosen (Ro 65) or those from section 1.10.4 for protons, of Weiss (We 62) for alpha particles and of Rosen (Ro 65) for neutrons. The effect of the different choices of the proton optical-model parameters may be seen in figures 3.5.2.2-5. Figure 3.5.2.2 shows the respective $\Gamma_J$ for incident proton energies of 8, 9 and 10 MeV; figure 3.5.2.3 shows the $\sigma_J(\theta)$ for $\theta = 90^\circ$
Comparison of the theoretical calculations of $\Gamma_J$ by method I for different proton optical-model parameters and for different excitation energies of the compound nucleus $^{32}\text{S}$.\vspace{1em}

**Figure 3.5.2.2**
The dependence of $\sigma_J(\theta)$ on $J$ when calculated by the Hauser-Feshbach theory and including the width fluctuation correction factor. The solid line represents the Perey proton optical-model parameters and the dashed line the Rosen parameters.
and 30° (lab); figure 3.5.2.4 shows the resultant mean level width as a function of angle together with the experimental values obtained using Method (ii); and figure 3.5.2.5 shows the energy dependence when averaged over all the angles for which the excitation functions had been measured. To obtain the energy dependence from the experimental data, the excitation functions were subdivided into two 1 MeV ranges and the widths calculated. These were then assumed to be the widths at the mid-points of the energy ranges. A similar assumption is used for the data over the full range.

From a comparison of all these results, it is evident that the statistical method of calculating the mean level width does not predict the experimentally observed angular dependence of $\Gamma$. Although the angular dependence may be statistical only, the relative symmetry about 90° implies that it may be due to some compound nucleus process.

3.5.3 THE MEAN LEVEL WIDTH IN THE COMPOUND NUCLEUS $^{24}\text{Mg}$

The excitation functions for the reactions $^{23}\text{Na}(p,p_0,2,3)$ have been analysed in a similar manner as the $^{31}\text{P}(p,p_0)$ excitation functions to obtain the mean
The experimental mean level widths $\Gamma(\varrho)$ obtained from the Fourier analysis (method (ii)) compared with the theoretical values calculated by method I.
Figure 3.5.2.5

The dependence of $\langle \Gamma(\theta) \rangle$ on the excitation energy of the compound nucleus $^{32}\text{S}$. 
level widths in the compound nucleus $^{24}_{\text{Mg}}$. The incident proton energies of between 8.0 and 12.0 MeV correspond to excitation energies in the compound nucleus of between approximately 19.4 and 23.2 MeV. The average results for the mean level widths calculated from the excitation functions over the energy ranges 8.0-10.0 MeV, 8.0-12.0 MeV and 10.0-12.0 MeV, are shown in figures 3.5.3.1, 3.5.3.2 and 3.5.3.3. The results are the average values obtained using the three methods of analysis and the error bars are due to the deviations from the mean value of $\Gamma$ when calculated by the three methods. The large variations of these errors indicate the uncertainties involved when calculating parameters from the experimentally measured excitation functions.

The results for the average values and the standard deviations obtained are summarized in table 3.5.3.1. The results from the three methods are generally in good agreement although, as has been pointed out previously, for any one excitation function, large variations can occur.

The average mean level width obtained from the $p_0$ channel exhibits the expected increase with excitation energy, as may be seen from figure 3.5.3.4, although it was not possible to theoretically predict
The mean level width $\Gamma$ in the compound nucleus $^{24}_{\text{Mg}}$ obtained from the excitation functions for the reaction $^{23}_{\text{Na}}(p,p_0)$ as a function of angle.

Figure 3.5.3.1
The mean level width $\Gamma$ in the compound nucleus $^{24}_{\text{Mg}}$ obtained from the excitation functions for the reaction $^{23}_{\text{Na}}(p,p_2)$ as a function of angle.
Figure 3.5.3.3

The mean level width $\Gamma$ in the compound nucleus $^{24}\text{Mg}$ obtained from the excitation functions for the reaction $^{23}\text{Na}(p,p_3)$ as a function of angle.
$^{23}\text{Na}(p,p_3)$
Figure 3.5.3.4

The dependence of $\langle \Gamma(\Theta) \rangle$ on the proton bombarding energy.
shown that the calculation of $\Gamma$ is very sensitive to this parameter ($V_0$ 68).

For the $p_2$ and $p_3$ exit channels, the experimentally determined mean level widths show peculiar behaviour with energy. In the $p_2$ case the value obtained from the range 8.0-12.0 MeV is less than that obtained in both the ranges 8.0-10.0 MeV and 10.0-12.0 MeV, while in the $p_3$ case the reverse occurs. However, for these two channels the effects of intermediate structure in the excitation functions are more pronounced than in the $p_0$ channel and will therefore interfere with the statistical calculations of the mean level widths to a greater extent.

3.5.4 APPLICATION OF FILTERS TO MAGNIFY OR REDUCE GROSS STRUCTURE FOUND IN EXCITATION FUNCTIONS

It should be emphasized that the Fourier analysis method might be a valuable tool for the interpretation of gross structure. Thus, if large gross structure effects are present in an excitation function then, after transposing the cross section from the energy plane to the frequency plane, one can, if the high frequencies tend to blur the underlying structure, filter out these frequencies before transforming again to the energy plane. A filter of the form
\[
\frac{1}{1 + j\omega t},
\]
where \( j = \sqrt{-1}, \) \( \omega \) the frequency and \( t \) a constant may then be used. On the other hand, if one wants to decrease the gross structure effects, then another kind of filter should be used to suppress the frequencies of the structure. A possible filter is then

\[
\frac{1}{1 + t/j\omega}.
\]

Both filters were used in the analysis of the excitation functions from the reaction \(^{23}\text{Na}(p, p_2)\). However, because of the shifts of the excitation functions and the relatively small gross structure effects, it was found more appropriate to average the excitation functions over several energy ranges as in section 2.5 when extracting an angular distribution for the gross structure. Also, when removing the effects of the gross structure in the excitation functions the results are very sensitive to the value of \( t \) used.

This would probably not be so for larger gross structure effects (e.g., giant resonance effects). It must therefore be concluded that although the filtering
methods are possible for magnifying or reducing the gross structure effects, these must be considerably more pronounced than those observed in the excitation functions from the reaction $^{23}\text{Na}(p,p_2)$ before filtering methods can be used.

3.6 SEPARATION OF COMPOUND AND DIRECT PROCESSES USING THE STATISTICAL MODEL OF FLUCTUATIONS AND COMPARISONS WITH THE OPTICAL-MODEL AND HAUSER-FESHBACH THEORY FOR THE REACTION $^{31}\text{P}(p,p_0)$

As in the analysis of the mean level widths, the fraction of direct reaction $Y_D$ may be calculated by the three methods described in section 3.4. If bias effects due to the finite range of data of the excitation functions involved are considered then equation 3.2.2 of Method (i) may be expressed as (Da 66)

$$Y_D = \sqrt{\frac{1-a-aC(\varepsilon=0)-NC(\varepsilon=0)}{1-a-aC(\varepsilon=0)}}$$

where $a = \frac{2}{n} \tan^{-1} n - \frac{1}{n^2} \ln(1-n^2)$

and $n = \text{energy range/mean level width (}\Gamma\text{)}$. 
Besides calculating \( Y_D \) for the full energy range the excitation functions were divided into two regions from 8.0 to 9.0 MeV and from 9.0 to 10.0 MeV and a similar analysis carried out. The results are shown in figure 3.6.1.

The comparison of the results from the fluctuation analysis and from the average angular distribution analysis from chapter 1 for the three energy ranges show reasonably good agreement and generally the different models agree.

3.7 SEPARATION OF COMPOUND AND DIRECT PROCESSES USING THE STATISTICAL MODEL OF FLUCTUATIONS AND COMPARISONS WITH OTHER MODELS FOR PROTON ELASTIC AND INELASTIC SCATTERING ON \( {}^{23}\text{Na} \)

The amount of direct reaction \( (Y_D) \) contributing to the differential cross sections was calculated using the three methods discussed earlier and the results have been compared with those found from the analysis of the average angular distributions (sections 1.10.5 and 1.11). The deviations between the results from the different models are very pronounced. However, this is not surprising as intermediate structure has been shown to be present in all proton exit channels considered, and the reduction factor
The fractions of direct reaction \( (Y_D) \) calculated from the optical-model plus Hauser-Feshbach analysis (solid lines) and from the fluctuation analysis (points). The estimated standard deviation errors due to the finite ranges of data are shown by the dashed lines.
in the Hauser-Feshbach theory is expected to be most sensitive to such phenomena. For example, in the analysis of the proton elastic scattering data using the combined optical-model and Hauser-Feshbach theory, the compound nucleus mechanism as defined by the analysis has been shown to predominate at the minima in the angular distributions. However, because of the symmetry of the intermediate structure around $90^\circ$ and because of the general trend of the cross sections at backward angles, it is obvious that the Hauser-Feshbach calculation (in the sense of allowing the reduction factor as a free parameter in the fitting procedure) will treat intermediate structure as a compound nucleus process.

The fluctuation theory, on the other hand, separates the scattering amplitude into a slowly varying part (the direct part) and a rapidly fluctuating part connected with the compound nucleus process. It might therefore be expected that the fluctuation theory will treat the intermediate structure, at least partly, as a direct process.

From the above arguments it is appropriate to redefine the reduction factor for the optical-model plus Hauser-Feshbach theory and the $\text{DWBA} + \text{Hauser-Feshbach}$ theory such that $R = R_{IS} + R_{CN}$, where $R$ is the
reduction factor obtained from, for example, the optical-model and Hauser-Feshbach theory, $R_{IS}$ is the reduction factor due to intermediate structure, and $R_{CN}$ is to be identified with the ordinary reduction factor for the compound nucleus process. The average differential cross section can now be written as

$$\langle \frac{d\sigma}{d\Omega} \rangle = \langle \frac{d\sigma}{d\Omega} \rangle_{\text{direct}} + R \langle \frac{d\sigma}{d\Omega} \rangle_{\text{HF}}$$

$$= \langle \frac{d\sigma}{d\Omega} \rangle_{\text{direct}} + (R_{IS} + R_{CN}) \langle \frac{d\sigma}{d\Omega} \rangle_{\text{HF}}$$

The predictions of $Y_D$ given by the optical-model plus Hauser-Feshbach theory and the fluctuation theory were found to agree generally within errors in the reaction $^{31}\text{P}(p,p\alpha)$. This fact, together with the above discussion, then makes it possible to derive at least a lower limit of the reduction factor $R_{CN}$ by redefining the fraction of direct reaction $Y_D$ such that

$$Y_D = \frac{\langle \frac{d\sigma}{d\Omega} \rangle_{\text{direct}}}{\langle \frac{d\sigma}{d\Omega} \rangle_{\text{total}}} = 1 - \frac{\langle \frac{d\sigma}{d\Omega} \rangle_{\text{HF}}}{\langle \frac{d\sigma}{d\Omega} \rangle_{\text{total}}} R_{CN}$$

and varying $R_{CN}$ until agreement is found for the different models. The results are shown in figures 3.7.1, 3.7.2 and 3.7.3 where the circles with error bars are from the
fluctuation analyses and the curves are the results from the average cross section analyses, using various reduction factors. The error bars are a combination of the errors due to FRD effects and the use of three methods in analyzing the excitation functions. The largest fraction of the errors arise from FRD effects, especially for the angles for which the fraction of direct reaction in the total differential cross section is small. In a similar manner to the calculation of the mean level widths, the values of $Y_D$ have been calculated for the energy ranges 8.0-10.0 MeV, 8.0-12.0 MeV and 10.0-12.0 MeV.

The comparison of the results for the $p_0$ exit channel (figure 3.7.1) indicate that the values of $Y_D$ obtained by the fluctuation analysis are generally much larger than those obtained from the optical-model and the Hauser-Feshbach analysis ($R_{CN} = R$), especially for the backward angles. However, by reducing the reduction factor ($R_{CN}$) and assuming that the intermediate structure condition is treated as a direct reaction component in the fluctuation analyses, reasonable agreement is obtained between the two models for values of $R_{CN}$ in the range 0.2 to 0.4. These values are consistent with what is to be expected, because of the large direct reaction components in the $p_1$ and $p_2$ channels.
The predictions of the redefined $Y_D$ given by the optical-model and the Hauser-Feshbach theory (curves) and the theory of Ericson and Brink and Stephen (circles) for the $p_0$ channel.
The predictions of the redefined $Y_D$ given by the DWBA and the Hauser-Feshbach theory (curves) and the theory of Ericson and Brink and Stephen (circles) for the $p_2$ channel.
The values of $Y_D$ obtained for the $p_2$ exit channel also indicate that the results should be greater than those predicted by the DWBA + Hauser-Feshbach analysis. The corrected reduction factor $R_{CN}$ is in the range 0.2-0.4 although the results are not as conclusive as for the $p_0$ channel.

Figure 3.7.3 compares the results for the $p_3$ channel and indicates a reduction factor of $\leq 0.2$, and values of $Y_D$ of approximately 0.8. In section 1.11 it was shown that the angular distributions could be fitted without assuming any direct reaction contribution and therefore the value of $Y_D$ obtained from the fluctuation analysis gives a measure of the contribution to the average cross section from the intermediate structure. It is probable that the large estimate of $\approx 80\%$ of the average cross section being due to intermediate structure includes a small percentage due to a direct reaction component.

3.8 CROSS CORRELATION ANALYSIS

3.8.1 INTRODUCTION

The general expression for the cross correlation function is
Figure 3.7.3

The predictions of the redefined $Y_D$ given by the Hauser-Feshbach theory (curves) and the theory of Ericson and Brink and Stephen (circles) for the $p_3$ channel.
$^{23}\text{Na}(p,p_{3})$

- $8-10 (9) \text{ MeV}$

- $8-12 (10) \text{ MeV}$

- $10-12 (11) \text{ MeV}$

$\theta_{CM}$
The theoretical value for $C(\theta, \theta')$, for the particular case of only a compound nucleus contribution to the reaction mechanism, may be calculated by the method described in section 3.3. This value will now be defined as $1/M$. It was found impossible to predict theoretically the expression when a direct reaction component is present, although by comparison with the theoretical value for the autocorrelation function when a direct reaction mechanism is present, i.e.

$$C(\theta, \theta) = \frac{1}{N}(1 - Y_D^2)$$

and by the use of boundary condition considerations it was assumed that the general cross correlation expression could be written as

$$C(\theta, \theta') = \frac{1}{M} \sqrt{(1 - Y_D^2(\theta))(1 - Y_D^2(\theta'))}$$

(3.8.2)

For $\theta = \theta'$ or for $Y_D = 0$ the expression reduces to the normal autocorrelation function.
3.8.2 THE REACTIONS $^{23}\text{Na}(p,p_0)$ AND $^{23}\text{Na}(p,p_2)$

Cross correlation analyses are generally assumed to be sensitive tests for the presence of intermediate structure, although any direct reaction contribution complicates the interpretation of the results. Equations 3.8.2 and 3.8.1 were used to calculate the 'theoretical' and experimental values of $C(\theta,\theta')$. The values of $Y_D$ used were the average values obtained from the three methods of analyzing the excitation functions. As in other analyses of the excitation functions, the energy dependence of the mean cross sections was first removed by dividing through by the smooth fits to the excitation functions.

The result for $\theta = 155^\circ$ (lab) is shown in figure 3.8.2.1. The error bars are due to finite range of data effects (Gi 65c, Da 66). The agreement between the experimental data points and the theoretical curve is good and does not show any evidence for intermediate structure. However, as has been pointed out previously, the removal of the energy dependence of the mean cross sections also reduces the effects due to intermediate structure. Furthermore, the apparent energy shifts observed in the intermediate structure will reduce the cross correlations due to the gross structure.
Cross correlation analysis for $\theta = 155^\circ$ (lab) for the reaction $^{23}\text{Na}(p,p_0,2)$ over the energy range 8.0-12.0 MeV.
$^{23}\text{Na}(p,p\alpha)$

$\theta = 155^\circ$

$C(\theta,\theta')$

$^{23}\text{Na}(p,p_2)$

$\theta = 155^\circ$

$\Theta_{lab}$
3.8.3 THE REACTION $^{31}\text{P}(p,p_0)$

The experimental cross correlations were calculated after removal of the energy dependence of the mean cross section and compared with the theoretical predictions including FRD bias effects as described in the previous section. The values of $Y_D$ were from the average cross section analysis (chapter 1). The results are shown in figure 3.8.3.1 for values of $\theta$ of $90^\circ$ and $160^\circ$ (lab) and for the energy ranges 8.0-9.0 MeV, 9.0-10.0 MeV and the combined range 8.0-10.0 MeV. The error bars shown are due to FRD effects and are therefore the errors in the theoretical predictions. As in the previous section the errors for the particular cases of $\theta = \theta'$ (i.e. the autocorrelation) and for angles for which the excitation functions are uncorrelated (i.e. $|\theta - \theta'| \gg kR^{-1}$) are shown and have been calculated using the formulae of (Da 66) including the effect of the direct reaction component. The errors are largest for large $N$ and small $Y_D$ and vary systematically with angle because both $N$ and $Y_D$ are angle dependent.

At $\theta = 160^\circ$, the experimental results are in good agreement with the theory and are generally within the estimated FRD errors. The typical decrease with
Cross correlation analysis for \( \theta = 90^\circ \) and \( 160^\circ \) (lab) and for energy ranges of 8.0-9.0 MeV, 9.0-10.0 MeV and 8.0-10.0 MeV.
CROSS CORRELATION ANALYSIS

$^{3}\text{P}(p,p')$
increasing $|\theta - \theta'|$ is exhibited and the correlation angle is approximately $20^\circ$.

For $\theta = 90^\circ$, the theoretical curve displays unusual behaviour in that a higher cross correlation is predicted for $\theta' = 100^\circ$ than for the autocorrelation. However, this is to be expected because of the angular dependence of $Y_D$ and the fact that it is much smaller at $100^\circ (\approx 0.65)$ than at $90^\circ (\approx 0.86)$. The experimental points are in good agreement with the theoretical predictions except for angles of $105^\circ$, $110^\circ$ and $120^\circ$ (lab). However, since these angles correspond to the region for which the fraction of direct reaction is a minimum, it may be that the values of $Y_D$ used in calculating the cross correlations are too small; this would result in the theoretical curve being greater than the experimental points. Indeed, as may be seen from figure 3.6.1, the values of $Y_D$ obtained from the optical-model plus Hauser-Feshbach analysis for these angles are generally much smaller than those obtained from the autocorrelation analysis. Therefore, the theoretical calculations may be too large for these angles.

The overall agreement between the experimental points and the predicted values indicates that expression
3.8.2 is a good approximation for calculating the expected cross correlations when a direct reaction component is present.
CHAPTER 4

SUMMARY AND CONCLUSIONS

It must again be emphasized that the optical-model and Hauser-Feshbach theories apply only to average differential cross sections. All of the reactions analyzed have shown that it would be meaningless to carry out the analyses for isolated angular distributions because of the interference effects between the competing mechanisms. The energy averaging of the differential cross sections must be done over a sufficiently large energy range to remove the interference effects. Thus the mean level width in the compound nucleus and the target thickness must be considered when analysing experiments of this type.

For heavy nuclei, it may be possible to effect the averaging with thick targets although this must be investigated very carefully. In light nuclei it is impossible to obtain a sufficient energy averaging with thick targets and thus it is necessary to do the averaging by either measuring separate angular distributions or by measuring excitation functions at
all angles and then obtaining the average cross sections by fitting smooth curves to the experimental excitation functions.

The choice of the smooth function is most important. It should not be so detailed as to reproduce any large fluctuations in the cross sections, but it must be able to accurately represent the energy dependence of the mean cross section. A simple quadratic expression of the form $AE^2 + BE + C$ was found to be satisfactory for predicting the mean energy dependence of the differential cross sections in both the $^{23}\text{Na}(p,p)$ and $^{31}\text{P}(p,p_0)$ experiments. One exception was noted. This was an abnormal energy dependence of the reduction factor between 9.5 and 10.0 MeV for the reaction $^{31}\text{P}(p,p_0)$. Since no evidence for intermediate structure was found in this reaction or in the reaction $^{31}\text{P}(p,\alpha)$ (Da 68) the behaviour was attributed to the form of the function used for estimating the mean cross sections at the angles corresponding to the minima in the angular distributions. However, any more detailed function would have only complicated the analysis and may even have resulted in over fitting the excitation functions. For all other excitation functions, as can be seen from all figures of
the excitation functions and their smooth fits, there is no doubt that the analyses were done with angular distributions representing true average values.

Figures 1.10.3.4, 1.10.4.3, 1.10.5.3 and 1.10.6.3 indicate that the optical-model and the Hauser-Feshbach theories describe the experimental data very well, and it is evident that the effect of the compound nucleus contribution must be considered, even for reactions consisting of more than 90% direct reaction. For the reactions $^{31}\text{P}(p,p_0)$, $^{27}\text{Al}(p,p_0)$ and $^{23}\text{Na}(p,p_{0,2,3})$ the compound nucleus contribution must be included directly as a Hauser-Feshbach calculation. However, for the reaction $^{62}\text{Ni}(p,p_0)$ the analysis of a sufficient number of angular distributions enabled good representations to be made with only an optical-model calculation. Thus, depending on the relative magnitude of the fluctuations, the compound nucleus contribution to a particular reaction must be considered either explicitly by a Hauser-Feshbach calculation (as in the $^{31}\text{P}(p,p_0)$, $^{27}\text{Al}(p,p_0)$ and $^{23}\text{Na}(p,p_0)$ reactions) or implicitly by removing the interference effects but without any Hauser-Feshbach calculation (as in the $^{62}\text{Ni}(p,p_0)$ analysis). The method of analysis and the energy range over which the excitation functions must be averaged should be carefully investigated in each specific case.
Examples of the large fluctuations possible in proton elastic and inelastic scattering are shown in the excitation functions for the reaction $^{24}\text{Mg}(p,p)$ measured for incident proton energies between 11.0 and 13.0 MeV (figure 4.1). It would be meaningless to measure individual angular distributions at, for example, 13.0 MeV and analyze them using the optical-model, DWBA and the Hauser-Feshbach theories. If the excitation functions were measured at higher energies, for example with incident proton energies of around 15 MeV and a 1 mg/cm$^2$ target, the energy averaging due to the target would be approximately 20 keV compared with a mean level width of greater than 50 keV. Thus if fluctuations are present, and if the combined energy resolution due to the beam spread and the target thickness is less than a few $\Gamma$, the analysis of the angular distributions must make allowance for the compound nucleus contribution. The questions now arise:

(1) What is the behaviour of the excitation functions at higher energies and when can the compound nucleus contribution be neglected?

(2) Are the fluctuations still present and if so what is their magnitude?
Figure 4.1

Excitation functions for proton elastic and inelastic scattering on $^{24}\text{Mg}$ at $90^\circ$ (lab) and for proton bombarding energies between 11 and 13 MeV.
It is obvious that elastic and inelastic proton scattering excitation functions should be measured on a series of nuclei to at least 20 MeV incident proton energy to answer the above questions.

An example of the large fluctuations observed in a (p,α) reaction for incident proton energies from 10.9 to 19.7 MeV has been given by Put et al. (Pu 68). They measured excitation functions for the reaction $^{27}\text{Al}(p,\alpha)^{24}\text{Mg}$ and found large fluctuations over the complete energy range. Although (p,α) reactions do not have large direct reaction components it is obvious that there will still be significant fluctuations observed in proton elastic and inelastic scattering measurements up to approximately 20 MeV for nuclei of around mass 30. This is particularly so for reactions which have only a few independent partial cross sections contributing (see sections 3.2 and 3.3). Thus proton elastic scattering on spin zero targets, where $N$ at 90° is approximately 2, should exhibit large fluctuations.

Even larger fluctuations should be observed for alpha particle elastic scattering from spin zero targets; for such reactions $N$ is approximately one at 90°. Also, for alpha scattering, the calculated Hauser-Feshbach
angular distributions exhibit considerably more structure than proton scattering because of the higher orbital angular momentum of the alpha particle compared to a proton of similar energy. For comparison, a Hauser-Feshbach calculation was done for the reaction \( ^{20}\text{Ne}(\alpha,\alpha_0) \) for the same excitation energy in the compound nucleus \( ^{24}\text{Mg} \) as the reaction \( ^{23}\text{Na}(p,p_0) \) at 10 MeV bombarding energy. The result is shown in figure 4.2. It is clear that the same criteria as for the analysis of proton scattering apply to the analysis of alpha scattering but in alpha scattering measurements the diffraction patterns due to the compound nucleus process must not be confused with those due to the direct reaction process. In figure 4.2 the absolute values do not include the reduction due to the effect of direct reaction processes in the reaction channels; i.e. the calculation corresponds to a reduction factor of unity. The relative importance of the diffraction patterns due to the two processes will obviously depend on the accuracy with which the reduction factor can be determined.

From the proton scattering measurements, the reduction factor was found to be meaningful whenever there was a significant compound nucleus contribution at
Figure 4.2

Hauser-Feshbach calculation for the reaction $^{20}\text{Ne}(\alpha,\alpha_0)$ at 14.4 MeV alpha particle bombarding energy.
$^{20}\text{Ne}(\alpha,\alpha_0)$

$E_\alpha = 14.4 \text{ MeV}$
some angles. However, when the compound nucleus contribution is small, as for the reaction $^{62}\text{Ni}(p,p_0)$, it is impossible to obtain an accurate estimate of the reduction factor. The analyses of the two reactions $^{23}\text{Na}(p,p_0)$ and $^{27}\text{Al}(p,p_0)$ gave reduction factors which were greater than unity and this was interpreted as being due to the presence of intermediate structure. The fact that evidence for intermediate structure resulted from these analyses indicates that the combined optical-model and Hauser-Feshbach theory is a very sensitive and reliable model in this mass region and at the energies considered.

Evidence for intermediate structure in this mass region has been reported previously by several authors (Si 65, El. 66, Le 66, Be 68). From the analysis of the reactions $^{23}\text{Na}(p,p_0,2,3)$ it was shown that the separation of the three reaction mechanisms, the direct, the compound nucleus and the intermediate structure process, depends on the models used. Furthermore it was shown that the apparent shifts of the gross structure seen in some excitation functions could be accounted for by considering the interference effects between the intermediate structure and the statistical fluctuations. When four
artificial excitation functions were added, each consisting of statistically uncorrelated terms and identical gross structure resonance terms, apparent shifts of the gross structure of the order of magnitude observed in the experimental excitation functions resulted. The combined excitation function corresponded to the case of $N = 4$. The approximate values of $N$ at $90^\circ$ for experimental excitation functions may be calculated from the expression (Er 66b)

$$N = \frac{1}{2}(2i_a + 1)(2I_a + 1)(2I_b + 1)(2i_b + 1)$$

where $i_a$, $I_a$, $I_b$ and $i_b$ are the spins of the incoming particle, target nucleus, residual nucleus and outgoing particle respectively. The resulting values of $N$ are 64 for the reaction $^{23}\text{Na}(p,p_2)$ and 16 for the $p_3$ channel. Thus the $p_2$ and $p_3$ channels correspond to the addition of 64 and 16 statistically independent partial cross sections respectively. If the intermediate structure interferes with all the partial cross sections then the shifts of the position of the resonances of the gross structure should be smaller for the $N = 64$ than for the $N = 16$ case. However, this is not observed experimentally and the shifts in the $p_3$ channel are considerably less
than in the $p_2$ channel. The disagreement could be accounted for by assuming a selective interference between the gross structure and the statistical fluctuations, and thereby limit the number of such interfering partial cross sections.

Bearse et al. (Be 68) claim that, for the excitation energies achieved in the compound nucleus $^{24}\text{Mg}$, the intermediate states overlap and therefore several states of different spins are accessible in the incident proton energy range of 8.0-12.0 MeV. By assuming that the intermediate states could be treated as the compound nucleus states in the Hauser-Feshbach theory, it was possible to show that the excitation functions for the $p_2$ channel will show gross structure due to higher spin resonance states than the $p_3$ channel. Similarly, the $p_0$ channel should show structure due to all states so that it would be difficult to distinguish the effects of individual intermediate structure resonances from the overall gross structure (figure 2.4.1b). The general trend of the excitation functions for the $p_0$ channel at backward angles can be seen as a direct consequence of figure 2.4.1b and the above discussion. Furthermore, the similarity between the excitation
functions for the \((\gamma_2 + \gamma_3)\) group in the reaction \(^{23}\text{Na}(p,\gamma)\) measured by Bearse et al. (Be 68) and the \(p_2\) excitation functions shown in figure 2.2.1 are accounted for since both lead to final states of high spins.

Apart from the possible statistical nature of intermediate structure, the exact origin of the gross structure seen in excitation functions is not yet fully understood. However, the importance of such structure has been discussed by Feshbach (Fe 67): "... will excited states which are simple because they in some sense maintain the symmetry of the ground state act as doorway states? Or turning this thought around, the intermediate resonance may represent an important way to determine the symmetry of nuclear systems. In the various shells it may be the case that a certain set of nuclear configurations is more stable because the strong nuclear forces have weak matrix elements with configurations outside the set. The ground state of the nucleus might be made up of these configurations and the doorway states of a similar set of related configurations. We can expect that these principal configurations will be representations of some symmetry group."
Before the mechanisms for the occurrence of intermediate structure are fully understood it is evident that many more studies, both theoretical and experimental, must be carried out, and perhaps some resistance will have to be overcome: 'These experiments must be among the dullest experiments you can conceive of doing ....' Gove (Mo 65). - This might or might not be so but if the speculations of Feshbach are proved to be correct, then the analysis and interpretation of such experiments will definitely be some of the most exciting one can conceive of doing. Meanwhile what could be more appropriate than to repeat what Socrates said about Heraclitus' book: 'What I have understood is excellent, what I have not understood is probably excellent as well, but I believe we do need a diver from Delos.' (Ge 63).

The analysis of the average angular distributions for the proton inelastic scattering data on $^{23}$Na, calculated from the smooth fits to the excitation functions, did not appear to be affected significantly by the intermediate structure. Good fits were obtained for the $p_2$ channel using a deformation parameter of 0.4 in the DWBA calculation and reduction factors in the Hauser-Feshbach calculation of 0.6, 0.6 and 0.4 at 8.0, 10.0 and 12.0 MeV respectively. The angular
distributions for the $p_3$ channel were fitted with only a compound nucleus contribution and gave values for the reduction factor of 0.65, 0.74 and 0.77 at 9.0, 10.0 and 11.0 MeV respectively. It was concluded that the effect of the intermediate structure had been compensated for by both the smooth fitting of the excitation functions, and by the large values of the reduction factors used in the Hauser-Feshbach calculations.

The fluctuation analysis was done on excitation functions from proton scattering data on $^{31}$P and $^{23}$Na, of which the former showed no evidence for intermediate structure while gross structure effects were present in the latter. Because $^{31}$P, in this respect, must be regarded as simpler than $^{23}$Na, and because of the energy dependence of the average compound nucleus and direct reaction components it would be thought possible and necessary to take account of both the reaction modes explicitly. An attempt was made using the results of the average angular distributions to estimate the compound nucleus energy dependence and, using the theory of Hall, to calculate the correct autocorrelation functions. The method was unsatisfactory because of the effects of long range correlations produced in the
corrected excitation functions. However, the removal of the combined energy dependence by dividing through by the smooth fits to the excitation functions gave satisfactory results for all autocorrelation functions.

The assumption made in chapter 3 that the fluctuation analysis treats intermediate structure as a direct reaction process is also dependent on the method of removal of the mean energy dependence of the excitation functions. For example, the $130^\circ$ excitation function in figure 2.2.1 appears to consist of a relatively uniform background onto which two large gross structure resonances have been added. The smooth fit to the excitation function has a maximum at about 10 MeV and therefore the corrected excitation function, formed by dividing through by the smooth fit, will exhibit reduced gross structure effects. Autocorrelation functions for the experimental and corrected excitation functions are shown in figure 4.3. For large $\varepsilon$ they are very similar, indicating that there is no large range correlation due to the intermediate structure, which appears when there is a very pronounced energy dependence of the mean cross section ($^{\text{Al 65}}$). However, for the $70^\circ$ excitation function the background does appear to have some energy dependence,
Figure 4.3

Autocorrelation functions for $\theta = 130^\circ$ and $70^\circ$ (lab) for the reaction $^{23}\text{Na}(p,p_2)$. The solid lines are for the corrected excitation functions where the energy dependences have been removed and the dashed lines are for the uncorrected excitation functions.
with a maximum at 8.0 MeV. The autocorrelation functions for this excitation function are also shown in figure 4.3. The one from the experimental excitation function is generally greater than zero for large ε indicating some long range correlations, whereas that from the corrected excitation function oscillates about zero. Thus for the 70° data it is absolutely necessary to remove the energy dependence of the mean cross section before doing the fluctuation analysis; it would, therefore, be inconsistent not to apply the same conditions on all the excitation functions.

For both excitation functions, the mean level widths obtained from the autocorrelation functions are dependent on whether the mean energy dependence is removed or not. For the 70° data the mean level widths are 270 keV and 130 keV when calculated from the uncorrected and corrected excitation functions, respectively. For the 130° data the corresponding values are 200 keV and 130 keV for the reaction $^{23}$Na(p,p$_2$). The values obtained from the corrected excitation functions are in good agreement with other analyses around this mass region (Er 66b).

The mean level widths in the compound nuclei $^{24}$Mg and $^{32}$S showed a peculiar trend of symmetry around 90°, indicating that the trend could be of a compound nucleus
origin. This trend could not be predicted theoretically.
Although the approximate variation of $\Gamma$ with energy could be predicted in absolute terms for the reaction $^{31}\text{P}(p,p_0)$, only the relative energy dependence was given by the statistical model for the reaction $^{23}\text{Na}(p,p_0)$. By using three different methods in analyzing the excitation functions, it was possible to get an indication of the errors involved when experimentally measured excitation functions are analyzed. When the fraction of direct reaction was calculated with the three different methods the deviations between results from each calculation were negligible, whereas the deviations of the mean level width were substantial.

The cross correlation analysis gave good agreement with the predicted values obtained using the method of Brink, Stephen and Tanner, after inclusion of the direct reaction component. However, as in the case of autocorrelation analysis and fraction of direct reaction calculations for the reaction $^{31}\text{P}(p,p_0)$, the agreement was not as good at angles for which $\gamma_D$ was small. Previous comparisons of the different models have not been very extensive, and Moldauer (Mo 64) has shown that the estimations of the compound and direct components can differ in the two theories. However, when the fraction
of direct reaction is large the two theories should give similar results.

The cross correlation analysis showed no evidence for intermediate structure, even for the reaction $^{23}\text{Na}(p,p')$ and this could be understood by considering the apparent shifts of the gross structure and the removal of the energy dependence of the mean cross section before doing the analysis. Also, because the fluctuation analysis treats the intermediate structure as a predominantly direct process, it would not be expected to greatly affect the cross correlations.

Large discrepancies were evident when comparing the results for the amount of direct reaction obtained by optical-model plus Hauser-Feshbach analysis, with those obtained from the fluctuation analysis using excitation functions from proton scattering on $^{23}\text{Na}$. The discrepancies were interpreted as being due to the effects of gross structure observed in the excitation functions. Thus the optical-model plus Hauser-Feshbach analysis treated the gross structure as a compound nucleus process whereas the fluctuation analysis treated it as a direct process.
The validity of separating the gross structure in this manner depends on the width of the gross structure and on the relative angular distributions of the average compound, direct and intermediate structure cross sections. For example, if the width of the intermediate structure is large compared with the width of the fluctuations then the fluctuation analysis will treat intermediate structure as a predominantly direct reaction component. Alternatively, if the contribution to the average cross section from the intermediate structure is large where the direct reaction component is small, then the combined optical-model plus Hauser-Feshbach analysis will treat intermediate structure as a predominantly compound nucleus process and therefore will overestimate the reduction factor.

To compensate for treating the intermediate structure as a compound nucleus process in the analysis of the average angular distributions, the reduction factor was redefined as consisting of two components - one associated with the compound nucleus cross section ($R_{CN}$), and one associated with the intermediate structure cross section ($R_{IS}$). Thus it was possible, by comparing the values of $Y_D$ obtained from the average angular distributions and from the excitation functions, to
obtain estimates of both $R_{CN}$ and $R_{IS}$. Consistent reduction factors $R_{CN}$, having values within the approximate limits of 0.2 and 0.4 were found in all channels and for all energy regions under consideration. These small values for the reduction factor ($R_{CN}$) are more realistic than the values of $R$ obtained from the angular distributions of the average cross sections because of the large direct reaction component in the $p_1$ and $p_2$ channels. From the comparison of the values of $R_{CN}$ and $R_{IS}$ it is seen that the contribution to the average cross sections from the intermediate structure is approximately the same, or slightly larger, than that from the statistical compound nucleus process.
PART II
5.1 INTRODUCTION

The compound nucleus process has been discussed in the previous chapters from the point of view of reaction mechanism studies, and the Hauser-Feshbach formalism was used to estimate the compound nucleus cross section. The same theory has been incorporated in a general formalism to describe reactions such as $(p,n\gamma)$ where the nucleon emission occurs by a compound nucleus mechanism, and excited states in the final nucleus decay by $\gamma$-ray emission. By measuring angular distributions for the $\gamma$-rays, it is possible to determine spins of these states.

Few experimental studies have been made of $\gamma$-ray angular distributions following $(p,n)$ reactions, especially in the region near threshold. However, the recent experimental investigation of Birstein et al. (Bi 68), in which Ge (Li) detectors were used to measure the $\gamma$-ray angular distributions following $(p,n)$ and $(\alpha,n)$ reactions, has paved the way for a whole range of further studies,
As a result of the success achieved by Birstein et al. studies of the reactions \(^{62}\text{Ni}(p,n\gamma)\) \(^{62}\text{Cu}\) and \(^{64}\text{Ni}(p,n\gamma)\) \(^{64}\text{Cu}\) were undertaken. However, before presenting these in some detail, the theoretical formalism describing such reactions will briefly be outlined.

5.2 THEORY

The underlying theory has been discussed by Sheldon and Van Patter (Sh 66) and is based on the Hauser-Feshbach-Satchler statistical model (Ha 52, Sa 54, Sa 56, Sa 58). To fix notations consider the following diagram:

Here s, l and j are the spin, orbital and total angular momenta respectively of the particles involved in the reaction. The various compound nucleus states of spin
The differential cross section for the compound nucleus process in a (p,n) reaction involving the sequence $J_0(j_1 = \ell_1 \pm \frac{1}{2}) J_1(j_2 = \ell_2 \pm \frac{1}{2}) J_2$ is given by

$$\frac{d\sigma}{d\Omega_1} = \frac{\pi^2}{4} \sum_{j_1 j_2 j_3} g \eta \left( j_1 j_1 J_0 J_1 \right) \eta \left( j_2 j_2 J_2 J_1 \right) \tau P_\nu (\cos \theta_1)$$

This expression is equivalent to the Hauser-Feshbach expression given earlier (Sa 56, Wi 66, Sh 69). Here $g$ is a statistical spin factor

$$g = \frac{J_1^2}{(\mathcal{J} J_0)^2} = \frac{(2J_1+1)}{(2s+1)(2J_0+1)},$$

$\tau$ is the Hauser-Feshbach penetrability term

$$\tau = T_{\ell_1} (E_1) T_{\ell_2} (E_2) / \sum_{\ell J E} T_{\ell} (E),$$

and $P_\nu (\cos \theta_1)$ are Legendre polynomials as a function of the scattering angle referred to the incident beam direction. The momentum dependent terms $\eta$ are derived by multiplying $\gamma$-ray transition parameters $F_\nu$ by
particle parameters $b_\nu$ (Bi 53, De 57). In the case of a $\gamma$-ray transition of pure multipolarity $L$ linking nuclear levels of spin $J_i$ and $J_f$, the parameter $F_\nu$ is given by (Fe 55)

$$F_\nu(LLJ_fJ_i) = (-)^{J_f-J_i-1} \frac{\hat{J}_L (\hat{L})^2}{J_i} \times (LL1-1|\nu0)W(LLJ_iJ_i;\nu J_f)$$

where the bracket is a Clebsch-Gordan coefficient and $W$ a Racah coefficient. For mixed multipolarity

$$\delta \equiv \langle J_f | L' | J_i \rangle / \langle J_f | L | J_i \rangle$$

the linking parameter, $A_\nu$, is a sum of generalized $F_\nu$'s, where the $F_\nu$'s are defined as (Bi 60)

$$F_\nu(LL'J_fJ_i) = (-)^{J_f-J_i-1} \frac{\hat{J}_L (\hat{L})^2}{J_i} \times (LL'1-1|\nu0)W(LL'J_iJ_i;\nu J_f)$$

$$A_\nu(LL'J_fJ_i) = (1 + \delta^2)^{-1}$$

$$\times [F_\nu(LLJ_fJ_i) + 2\delta F_\nu(LL'J_fJ_i) + \delta^2 F_\nu(L'L'J_fJ_i)]$$

An unobserved $\gamma$-ray transition of mixed multipolarity is described by
Finally for a transition involving a particle rather than a $\gamma$-ray, the appropriate angular momentum term is

$$\eta_v(jj'f_i^J) \equiv b_v(jj'; N) F_v(LL'f_i^J)$$

$$= (-)^{j_i - J_i - \frac{1}{2}} \hat{J}_i \hat{J}_i' (jj'|-|jj') W(jj'f_i^J, vj_i)$$

The factor $b_v(jj'; N)$ for particles $N$ of total angular momentum $j, j'$ have been defined by Biedenharn and Rose (Bi 53).

Sheldon and Van Patter give the differential cross section for the $\gamma$-ray angular distribution for a $(p, n\gamma)$ reaction as shown in figure 5.2.1 and where the neutrons are unobserved as

$$\frac{d\sigma}{d\Omega} = \frac{1}{4} \chi^2 \sum g \eta_v(j_1j_1^0j_1) U_v(j_2j_2^0j_1j_2)$$

$$\times A_v(LL'J_2J_1) \tau P_v(\cos \theta_2)$$

This can be condensed to

$$\frac{d\sigma}{d\Omega} = \frac{1}{8} \chi^2 \sum N'C'W'M(\delta) \tau P_v(\cos \theta_2)$$
The summation is over $j_1$, $j_2$ and $\nu$ ($\nu$ restricted to $0 \leq \nu \leq 2j_1$, $2j_1$, $2j_2$, $2L'$) and

$$\begin{align*}
N' &\equiv (-)^{J_1^0+J_3^0-J_2^0+1} \frac{1}{2} (\hat{j}_1)^4 (\hat{j}_1^2) (\hat{j}_2^2)/(\hat{j}_0^2) \\
C' &\equiv \left( j_1 j_1 \frac{1}{2} \frac{1}{2} \mid \nu 0 \right) \\
W' &\equiv W(J_1^1 J_1^1 j_1 j_1; \nu j_0^0) W(J_1^1 J_1^1 J_2^0 J_2^0; \nu j_2^0) \\
M(\delta) &\equiv (1 + \delta^2)^{-1} \left[ M(LL) + 2 \delta M(LL') + \delta^2 M(L'L') \right] \\
M(LL') &\equiv \hat{L}_1 L_1 \left( LL' 1 \mid J^0 0 \right) W(J_2^1 J_2^1 LL'; \nu J_3^0).
\end{align*}$$

The Legendre polynomials are functions of the direction, $\Theta_2$, of the emitted $\gamma$-rays referred to the incident beam direction.

As can be seen from the above expression, the angular distribution of the $\gamma$-ray emission is given as a function of Legendre polynomials weighted by a product of energy-dependent and momentum-dependent terms. Thus, by measuring angular distributions and fitting them to Legendre polynomials, comparisons between experimental data and theoretical predictions involving different spin sequences and different mixing ratios can be made. Where the theoretical prediction for a particular spin sequence coincides with the experimental data a definite
spin assignment may result. Sheldon and Van Patter embodied all the fundamental expressions needed in a general computer code MANDY which was used to evaluate distributions for different spin sequences.

The MANDY code calculated not only absolute cross sections but also normalized angular distributions such that $W(90^\circ) = 1$. There are several advantages in not having to compare the angular distributions to theoretical predictions in absolute cross section units. Firstly, as was mentioned in section 1.10.2, absolute cross section predictions are sensitive to the width fluctuation correction factor for a compound nucleus process. The MANDY code did not include this correction factor. However, the relative shape of the angular distribution with or without the width fluctuation correction factor remains essentially the same (Ma 69). Secondly, it was also shown in the previous sections that a systematic error may occur when measuring absolute cross sections but the relative errors between data points may be well established.
5.3 THE REACTION $^{62}_{\text{Ni}}(p,n\gamma)^{62}_{\text{Cu}}$

5.3.1 INTRODUCTION

The low-lying levels of the odd-odd nucleus $^{62}_{\text{Cu}}$ have been investigated most thoroughly through the $\beta^+$/EC decay of 9.3 h $^{62}_{\text{Zn}}$ by Roulston et al. (Ro 67a), Antman et al. (An 67), Hoffman and Sarantites (Ho 69b), and also through the $^{63}_{\text{Cu}}(d,t)^{62}_{\text{Cu}}$ and $^{61}_{\text{Ni}}(^{3}\text{He},d)^{62}_{\text{Cu}}$ reactions by Hjorth and Allen (Hj 67) and Morrison and Schiffer (Mo 66) respectively. The most recent study, that of Hoffman and Sarantites, of the gamma rays following the $^{62}_{\text{Zn}}$ decay leads not only to a consistent decay scheme in which five excited states in $^{62}_{\text{Cu}}$ up to 637.20 keV are populated but also to unique spin-parity assignments for all these states. In the course of the study of the $^{62}_{\text{Ni}}(p,n\gamma)^{62}_{\text{Cu}}$ reaction, it was found that an extra level at 426.1 keV, not seen in the radioactive decay study, was strongly populated. This level decayed by emission of a 385.2 keV $\gamma$-ray to the first excited state in $^{62}_{\text{Cu}}$ at 40.8 keV. Evidence for the new state has also come from the other two reactions mentioned above where the level energy has been given as $410 \pm 25$ keV in the $(d,t)$ work, and $426 \pm 5$ keV in the $(^{3}\text{He},d)$ study.
The decay scheme of the levels in $^{62}$Cu according to Hoffman and Sarantites is summarized in figure 5.3.1.1 where for the purpose of completeness the additional level at 426.1 keV has been included.

5.3.2 EXPERIMENTAL METHOD

The experimental method used and the theoretical analysis of the results resemble closely the work of Birstein et al. A beam of protons from the ANU tandem Van de Graaff accelerator was directed onto an approximately 0.9 mg/cm$^2$ isotopically enriched $^{62}$Ni target prepared by vacuum evaporation of the material onto a backing of 0.005" tantalum foil. The target was placed in an accurately centred thin walled stainless steel scattering chamber of radius 8.0 cm and was located at 45° to the incident beam. The effective target thickness of 0.9 mg/cm$^2$ corresponds to a proton energy loss of approximately 60 keV, which was considered to give a sufficient statistical averaging over the compound nucleus states.

The Q-value for the $^{62}$Ni(p,n) reaction is -4.721 MeV. By using suitably selected proton bombarding energies, different levels in the final nucleus $^{62}$Cu
Figure 5.3.1.1

Decay scheme of $^{62}$Cu according to Hoffman and Sarantites. The additional level at 426.1 keV and its de-excitation gamma-ray observed in the present work are also included. The spin assignment of $3^+$ results from the present work.
can be populated by low energy (50-100 keV) neutrons. This has several advantages. Firstly, a given level in the final nucleus is fed by the neutrons and not by γ-ray transitions from higher states. Secondly, for outgoing neutrons of energy less than approximately 500 keV conditions for a compound nucleus process are ideal (Sh 66, Bi 68). Finally, the gamma-ray angular distribution following a compound nucleus (p,n) reaction has maximum anisotropy for the outgoing neutrons since they will be mainly s-wave; thus the de-alignment of the compound state is kept to a minimum.

To detect the γ-rays, two Ge(Li) detectors with active volumes of 40 cm³ and 30 cm³ were used. One counter was placed at 90° to the beam (at a distance of ≈10.0 cm from the target) to act as a monitor, while the other counter (at a distance of ≈9.0 cm from the target) could be rotated to any angular position between 0° and 100° with respect to the beam axis. The pulses were amplified by Tennelec preamplifiers and Ortec amplifiers and the γ-ray spectra, taken at 10° intervals between 0° and 100° in random order, were recorded with a Nuclear Data ND 2200 analyser. The angular distributions of the γ-rays, obtained by integrating
the number of counts in the full energy peak for each spectrum and then normalizing to the monitor, were corrected for self-absorption of the $\gamma$-rays in the tantalum backing and for slight instrumental anisotropies.

The angular distributions were fitted with even order Legendre polynomials

$$W(\theta) = 1 + A_2 P_2(\cos \theta) + A_4 P_4(\cos \theta)$$

using the method of linear least squares. The coefficients $A_2$ and $A_4$ were corrected for the small attenuation caused by the finite size of the Ge(Li) counter (Ro 53, Hu 68). The estimated errors arose from integration of the full energy peaks.

5.3.3 RESULTS

Although angular distributions for several different $\gamma$-rays were obtained, the main emphasis has been placed on the determination of the spin of the level at 426.1 keV, since the spins of the other levels are already known. The additional angular distributions were measured mainly to check that the method employed
predicted spins consistent with those previously
determined in $^{62}\text{Cu}$. Four $\gamma$-ray angular distributions
for different transitions in $^{62}\text{Cu}$ are shown in figure
5.3.3.1.

A precise determination of the energy of the 385.2
keV $\gamma$-ray was made by simultaneously measuring the
gamma spectrum with $\gamma$-rays from the sources $^{51}\text{Cr}$ (320.08
keV) and $^{198}\text{Au}$ (411.795 keV). The final value was
385.23 $\pm$ 0.15 keV. The resulting coefficients in the
Legendre polynomial expansion for the 385.2 keV $\gamma$-ray
were

$$A_2 = -0.53 \pm 0.03 \quad A_4 = -0.03 \pm 0.04$$

The above values of $A_2$ and $A_4$ were compared with
theoretical predictions of $A_2$ and $A_4$ for different spin
sequences. To do so, the penetrability term $\tau$ had to
be specified. This was composed of transmission
coefficients $T(E)$ derived from optical-model
calculations using potentials of Perey (Pe 63) for
protons, Wilmore and Hodgson (Wi 64) for neutrons and
Bock (Bo 67) for alpha particles. The theoretical
predictions together with the experimental data are
depicted in figure 5.3.3.2. Theoretically, a particular
Figure 5.3.3.1

Four γ-ray angular distributions for different transitions in $^{62}\text{Cu}$.
$^{62}\text{Ni}(p,\gamma)^{62}\text{Cu}$

- $E_p = 5.40\text{MeV}$
- $E_\gamma = 548.3\text{keV}$

$W(\theta)\text{ arb.units}$

- $E_p = 5.34\text{MeV}$
- $E_\gamma = 385.2\text{keV}$

- $E_p = 5.15\text{MeV}$
- $E_\gamma = 243.4\text{keV}$
- $E_\gamma = 247.0\text{keV}$

$\theta = 0^\circ, 30^\circ, 60^\circ, 90^\circ$
A comparison between the theory and the experimental values of $A_2$ and $A_4$ for different spin sequences for the 385 keV $\gamma$-ray in $^{62}$Cu.
spin sequence results in an ellipse in \( A_2 \) versus \( A_4 \) space, where the ellipse is the locus of points of different \( \delta \), the multipole mixing ratio \( (\delta^2 = E_2/M_1) \). In the case of an intermediate spin of 1, the triangle condition precludes the coefficient \( A_4 \) having a non-zero value and hence the ellipse collapses to a straight line.

The sequence \( 3^+ \rightarrow 2^+ \) is the only one to satisfy the experimental value; the value of the mixing ratio was obtained as \( \delta = 0.12 \pm 0.02 \). Here the sign of the mixing ratio follows the formalism of Rose and Brink (Ro 67b). The proximity of the most negative point on the \( A_2 \) axis of the \( 2^+ \rightarrow 2^+ \) sequence was investigated carefully. The only variable parameters are the transmission coefficients. After repeating the calculation with other transmission coefficients derived from a whole series of different optical-model potentials, it was impossible to make the coefficients \( A_2 \) more negative than \(-0.452\) (the value in figure 5.3.3.2 is \(-0.447\)). In other words, the theoretical ellipses are very insensitive to the choice of transmission coefficients. It is thus reasonable to state that the experimental evidence uniquely determines the spin assignment of the 426.1 keV state in \(^{62}\text{Cu}\) as
3\(^{+}\); the positive parity of the level results from the fact that the transferred neutron in the \(^{63}\text{Cu}(d,t)^{62}\text{Cu}\) reaction has \(\ell_n = 1\) (Hj 67).

Besides the 385.2 keV transition, the remaining three gamma-ray angular distributions shown in figure 5.3.3.1 were analyzed in the same way. The results yielded spin assignments in accordance with those proposed by Hoffman and Sarantites. The final results are summarized in table 5.3.3.1. As can be seen from table 5.3.3.1 and figure 5.3.3.1 the angular distributions for the two \(2^{+}\) levels at 243.43 keV and 287.86 keV are quite different. This is further illustrated in figure 5.3.3.3 where the two peaks are shown at 90° and 0°.

5.4 THE REACTION \(^{64}\text{Ni}(p,n\gamma)^{64}\text{Cu}\)

5.4.1 INTRODUCTION

The nucleus \(^{64}\text{Cu}\) has been investigated previously. Buccino et al. (Bu 68) studied the \(^{64}\text{Ni}(p,n\gamma)^{64}\text{Cu}\) reaction in the range \(E_p = 2.5-3.3\) MeV, but restricted their study to elucidating the level and \(\gamma\)-ray decay structure of \(^{64}\text{Cu}\); no spins were deduced. Hjorth and
Table 5.3.3.1

Summary of the $^{62}\text{Ni}(p,n\gamma)$ Experiment

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>$E_p$ (MeV)</th>
<th>$A_2$</th>
<th>$A_4$</th>
<th>Spin</th>
<th>$\delta$</th>
<th>Multipolarity</th>
</tr>
</thead>
<tbody>
<tr>
<td>243.43</td>
<td>5.15</td>
<td>-0.303±0.045</td>
<td>-0.060±0.050</td>
<td>$2^+\rightarrow 1^+$</td>
<td>-0.03±0.03</td>
<td>M1+0.1%E2</td>
</tr>
<tr>
<td>246.7</td>
<td>5.15</td>
<td>0.001±0.044</td>
<td>-0.016±0.054</td>
<td>$2^+\rightarrow 2^+$</td>
<td>0.33±0.04</td>
<td>90%M1+10%E2</td>
</tr>
<tr>
<td>385.2</td>
<td>5.34</td>
<td>-0.53±0.03</td>
<td>-0.03±0.04</td>
<td>$3^+\rightarrow 2^+$</td>
<td>0.12±0.02</td>
<td>99%M1+1%E2</td>
</tr>
<tr>
<td>548.3</td>
<td>5.45</td>
<td>0.014±0.033</td>
<td>0.006±0.039</td>
<td>$(1^+\rightarrow 1^+)$</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>
Figure 5.3.3.3

The 243.4 and 247.0 keV doublet in $^{62}\text{Cu}$ at 90° and 0°.
243.4–247.0 keV DOUBLET IN $^{62}_{\text{Cu}}$

AT 90°

AT 0°

3.6 keV

CHANNEL NUMBER (ARBITRARY)
Allen (Hj 67) studied the $^{63}\text{Cu}(d,p)^{64}\text{Cu}$ and $^{65}\text{Cu}(d,t)$ $^{64}\text{Cu}$ reactions; however, they were unable to give definite spin assignments to many of the levels in the final nucleus $^{64}\text{Cu}$. The most thorough investigation of $^{64}\text{Cu}$ derives from the thermal neutron capture work of Shera and Bolotin (Sh 68) whose final decay scheme plus their tentative spin assignments are shown in figure 5.4.1.1. This scheme provided the basis for the $\gamma$-ray angular distribution measurements described here.

5.4.2 EXPERIMENTAL METHOD

The targets were prepared by vacuum evaporation of (98.16%) isotopically enriched $^{64}\text{Ni}$. Initially, in the study of the level scheme, a self-supporting foil $\approx 150 \ \mu g/cm^2$ was used. However, in the measurement of the angular distributions, a target $\approx 0.8 \ mg/cm^2$ was prepared by successive evaporation onto a tantalum backing sufficiently thick (0.05 mm) to stop the incident protons.

The experimental method, data deduction and theoretical analysis were the same as those discussed previously.
Decay scheme of $^{64}_{\text{Cu}}$ put forward by Shera and Bolotin with two modifications as discussed in the text. The level of excitation reached at six proton bombarding energies is shown at the left, and spin assignments arising from the present work are shown on the right of the figure. The $\gamma$-rays for which angular distributions were extracted are indicated by thicker lines.
5.4.3 RESULTS

The final values of the coefficients $A_2$ and $A_4$ in the Legendre polynomial expansions resulting from the present work are summarized in table 5.4.3.1 and a sample of eight different angular distributions is shown in figure 5.4.3.1.

The Hauser-Feshbach penetrability terms needed for the theoretical analysis consisted of transmission coefficients derived from optical-model parameters. The proton and neutron parameters were taken from the work of Perey (Pe 63), and Wilmore and Hodgson (Wi 64), respectively. It was found that, at the exit neutron energies used in the present reaction, the theoretical predictions were insensitive to the choice of transmission coefficients.

Shera and Bolotin (Sh 68), on the basis of the $\gamma - \gamma$ coincidence evidence, deduced a decay scheme as shown in figure 5.4.1.1. The present study, in which each level in $^{64}$Cu was populated in the (p,n) reaction successively, confirms the decay scheme, with the exception of two details. In the first instance, the 896 keV transition was not observed and, therefore, is denoted by a dashed line in figure 5.4.1.1. In the
Table 5.4.3.1

Resulting Coefficients $A_2$ and $A_4$ for the reaction $^{64}\text{Ni}(p,n\gamma)$

<table>
<thead>
<tr>
<th>Bombarding Energy (MeV)</th>
<th>$E_\gamma$ (keV)</th>
<th>Transition</th>
<th>$A_2$</th>
<th>$A_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.75</td>
<td>159</td>
<td>159 $\rightarrow$ 0</td>
<td>$-0.34\pm0.03$</td>
<td>$-0.02\pm0.04$</td>
</tr>
<tr>
<td>2.94</td>
<td>278</td>
<td>278 $\rightarrow$ 0</td>
<td>$-0.34\pm0.03$</td>
<td>$-0.03\pm0.03$</td>
</tr>
<tr>
<td></td>
<td>185</td>
<td>344 $\rightarrow$ 159</td>
<td>0.01$\pm0.05$</td>
<td>$-0.06\pm0.07$</td>
</tr>
<tr>
<td></td>
<td>344</td>
<td>344 $\rightarrow$ 0</td>
<td>$-0.01\pm0.02$</td>
<td>0.02$\pm0.03$</td>
</tr>
<tr>
<td></td>
<td>203</td>
<td>362 $\rightarrow$ 159</td>
<td>$-0.40\pm0.06$</td>
<td>$-0.10\pm0.09$</td>
</tr>
<tr>
<td>3.16</td>
<td>449</td>
<td>609 $\rightarrow$ 159</td>
<td>0.40$\pm0.08$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>609</td>
<td>609 $\rightarrow$ 0</td>
<td>$-0.43\pm0.05$</td>
<td>0.02$\pm0.05$</td>
</tr>
<tr>
<td>3.24</td>
<td>385</td>
<td>663 $\rightarrow$ 278</td>
<td>0.00$\pm0.04$</td>
<td>0.07$\pm0.04$</td>
</tr>
<tr>
<td></td>
<td>663</td>
<td>663 $\rightarrow$ 0</td>
<td>0.03$\pm0.04$</td>
<td>0.00$\pm0.05$</td>
</tr>
<tr>
<td>3.36</td>
<td>580</td>
<td>739 $\rightarrow$ 159</td>
<td>0.31$\pm0.04$</td>
<td>0.04$\pm0.06$</td>
</tr>
<tr>
<td></td>
<td>461</td>
<td>739 $\rightarrow$ 278</td>
<td>$-0.06\pm0.10$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>377</td>
<td>739 $\rightarrow$ 362</td>
<td>$-0.07\pm0.11$</td>
<td>$-0.05\pm0.15$</td>
</tr>
<tr>
<td></td>
<td>468</td>
<td>746 $\rightarrow$ 278</td>
<td>$-0.40\pm0.06$</td>
<td>0.06$\pm0.07$</td>
</tr>
<tr>
<td>3.50</td>
<td>878</td>
<td>878 $\rightarrow$ 0</td>
<td>$-0.01\pm0.05$</td>
<td>0.00$\pm0.06$</td>
</tr>
<tr>
<td></td>
<td>534</td>
<td>878 $\rightarrow$ 344</td>
<td>0.06$\pm0.03$</td>
<td>$-0.02\pm0.04$</td>
</tr>
<tr>
<td></td>
<td>617</td>
<td>895 $\rightarrow$ 278</td>
<td>$-0.40\pm0.09$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>768</td>
<td>927 $\rightarrow$ 159</td>
<td>0.12$\pm0.14$</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>649</td>
<td>927 $\rightarrow$ 278</td>
<td>0.01$\pm0.06$</td>
<td>0.04$\pm0.07$</td>
</tr>
</tbody>
</table>
Figure 5.4.3.1

Eight different γ-ray angular distributions resulting from the reaction $^{64}\text{Ni}(p,n\gamma)$.
$^{64}_{\text{Ni}}(p,n)^{64}_{\text{Cu}}$

- $E_p = 3.36\text{ MeV}$
  - $E_{\gamma} = 468\text{ keV}$

- $E_p = 3.18\text{ MeV}$
  - $E_{\gamma} = 609\text{ keV}$

- $E_p = 2.94\text{ MeV}$
  - $E_{\gamma} = 344\text{ keV}$

- $E_p = 2.75\text{ MeV}$
  - $E_{\gamma} = 159\text{ keV}$

- $E_p = 3.36\text{ MeV}$
  - $E_{\gamma} = 580\text{ keV}$

- $E_p = 3.24\text{ MeV}$
  - $E_{\gamma} = 663\text{ keV}$

- $E_p = 2.94\text{ MeV}$
  - $E_{\gamma} = 203\text{ keV}$

- $E_p = 2.94\text{ MeV}$
  - $E_{\gamma} = 278\text{ keV}$
second, the 534 keV $\gamma$-ray, for which there was no coincidence evidence in the (n,$\gamma$) work of Shera and Bolotin, is due to the 878 → 344 keV cascade rather than the 895 → 362 keV cascade. By setting up a Ge(Li)-Ge(Li) coincidence system, and gating on the 534 keV $\gamma$-ray it was found that the 534 keV $\gamma$-ray was coincident with the 344 keV $\gamma$-ray.

Since no emphasis was given in the present study to precise energy determination, the values of Shera and Bolotin were used.

A schematic diagram of the electronics used in the coincidence study is shown in figure 5.4.3.2. The final result of the coincidence investigation is shown in figure 5.4.3.3. and a spectrum taken at 3.50 MeV proton bombarding energy is shown in figure 5.4.3.4.

The different levels of excitation in $^{64}$Cu, at the six proton energies used in the experiment, are indicated in figure 5.4.1.1. The close spacing of several levels prevented the (p,n) reaction populating individual levels in the group, as the exit neutrons needed at least 50 keV energy to ensure adequate energy averaging.

The possible spin sequences and multiple mixing ratios derived from the present work are given in
Figure 5.4.3.2

Schematic diagram of the electronics used in the coincidence study.

Nuclear Diodes Inc.
Detector (2)

ORTEC 410
Linear (2) Amplifier

ORTEC 420
Timing Single-(2) Channel analyser

ORTEC 414
Fast coincidence

ORTEC 426
Linear Gate

P.H.A.
Nuclear Data ND 2200

Nuclear Diodes Inc.
Detector (1)

Linear (1a) Amplifier

Linear (1b) Amplifier

Timing Single-(1) Channel analyser

Delay Amplifier
The coincidence spectrum obtained when gating on the 534 keV γ-ray in $^{64}$Cu.
The γ-ray spectrum obtained from the reaction $^{64}\text{Ni}(p,n\gamma)$ at 3.50 MeV proton bombarding energy. Also shown are the tantalum x-rays, three γ-rays labeled Ta from Coulomb excitation of tantalum and γ-rays from neutron inelastic scattering on Ge (Ch65,Ro69). The γ-rays labeled Al correspond to γ-rays from the first and second excited states in $^{27}\text{Al}(843$ and 1013 keV).
64\text{Ni}(p,\gamma)64\text{Cu}

E_p = 3.50 \text{ MeV}

Ta backing
In some cases, especially for the levels above 700 keV, unique spin assignments were not possible. However, most likely assignments could be deduced by considering the magnitudes of the multipole mixing ratios. Several studies in this mass region, where the final nuclei are odd or odd-odd, including the present reaction, the $^{62}$Ni(p,n)$^{62}$Cu reaction described previously, and the work of Birstein et al. (Bi 68), have shown that where spin sequences were previously known, or where unique assignments were possible, the accompanying mixing ratio was such that $|\delta| \leq 0.4$ in all cases. It should be emphasized that, although multipole mixing ratios higher than this cannot be excluded in absolute terms, this evidence suggests that where there is ambiguity as to the correct spin sequence deduced from the present work, the spin sequence involving a low value of the mixing ratio ($|\delta| \leq 0.4$) is the more likely. Accordingly, the tentative nature of the assignments arising from the use of this criterion are indicated on figure 5.4.1.1 by placing the spin values in parentheses.

5.4.3.1 The 159 and 278 keV levels

Both these levels decay by a single $\gamma$-ray to the ground state, for which a spin-parity assignment of $1^+$
Table 5.4.3.2

Possible spin sequences and corresponding mixing ratios for γ-rays from $^{64}$Cu.

<table>
<thead>
<tr>
<th>$E_\gamma$ (keV)</th>
<th>Spin sequence</th>
<th>Mixing ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>159</td>
<td>$2^+ \rightarrow 1^+$</td>
<td>$-0.01\pm0.02$</td>
</tr>
<tr>
<td>278</td>
<td>$2^+ \rightarrow 1^+$</td>
<td>$0.025\pm0.025$</td>
</tr>
<tr>
<td>185</td>
<td>$1^+ \rightarrow 2^+$</td>
<td>-</td>
</tr>
<tr>
<td>344</td>
<td>$1^+ \rightarrow 1^+$</td>
<td>$(\delta &gt; 0)$</td>
</tr>
<tr>
<td>203</td>
<td>$3^+ \rightarrow 2^+$</td>
<td>$0.040\pm0.035$</td>
</tr>
<tr>
<td></td>
<td>$2^+ \rightarrow 2^+$</td>
<td>$0.85 \leq \delta \leq 3.5$</td>
</tr>
<tr>
<td>609</td>
<td>$2^+ \rightarrow 1^+$</td>
<td>$0.03\pm0.03$</td>
</tr>
<tr>
<td>449</td>
<td>$2^+ \rightarrow 2^+$</td>
<td>$-0.05\pm0.10$</td>
</tr>
<tr>
<td>663</td>
<td>$1^+ \rightarrow 1^+$</td>
<td>-</td>
</tr>
<tr>
<td>385</td>
<td>$1^+ \rightarrow 2^+$</td>
<td>-</td>
</tr>
<tr>
<td>580</td>
<td>$3^+ \rightarrow 2^+$</td>
<td>$-0.34\pm0.06$</td>
</tr>
<tr>
<td></td>
<td>$2^+ \rightarrow 2^+$</td>
<td>$0.05\pm0.04$</td>
</tr>
<tr>
<td>377</td>
<td>$3^+ \rightarrow 3^+$</td>
<td>$\approx 0.40$</td>
</tr>
<tr>
<td></td>
<td>$2^+ \rightarrow 3^+$</td>
<td>$\approx 0.05$</td>
</tr>
<tr>
<td>461</td>
<td>$3^+ \rightarrow 2^+$</td>
<td>$-0.15\pm0.07$</td>
</tr>
<tr>
<td></td>
<td>$2^+ \rightarrow 2^+$</td>
<td>$0.4\pm0.1$</td>
</tr>
<tr>
<td>468</td>
<td>$3^+ \rightarrow 2^+$</td>
<td>$-0.040\pm0.035$</td>
</tr>
<tr>
<td></td>
<td>$2^+ \rightarrow 2^+$</td>
<td>$0.85 \leq \delta \leq 3.3$</td>
</tr>
<tr>
<td>$E_\gamma$ (keV)</td>
<td>Spin sequence</td>
<td>Mixing ratio</td>
</tr>
<tr>
<td>-----------------</td>
<td>---------------</td>
<td>--------------</td>
</tr>
<tr>
<td>534</td>
<td>$1^+ + 1^+$</td>
<td>$\delta &lt; 0$</td>
</tr>
<tr>
<td></td>
<td>$2^+ + 1^+$</td>
<td>$-0.25 \pm 0.03$</td>
</tr>
<tr>
<td>878</td>
<td>$1^+ + 1^+$</td>
<td>$-0.20 \pm 0.05$</td>
</tr>
<tr>
<td></td>
<td>$2^+ \rightarrow 1^+$</td>
<td></td>
</tr>
<tr>
<td>617</td>
<td>$3^+ + 2^+$</td>
<td>$0.0 \pm 0.1$</td>
</tr>
<tr>
<td></td>
<td>$2^+ + 2^+$</td>
<td>$\approx 1.1$</td>
</tr>
<tr>
<td>768</td>
<td>$1^+ + 2^+$</td>
<td>$-0.22 \pm 0.04$</td>
</tr>
<tr>
<td></td>
<td>$2^+ + 2^+$</td>
<td></td>
</tr>
<tr>
<td>649</td>
<td>$2^+ + 1^+$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$1^+ + 1^+$</td>
<td></td>
</tr>
</tbody>
</table>
has been deduced from magnetic moment measurements (Le 54, Do 66). The present work uniquely assigns a spin of $2^+$ for both levels. In figure 5.4.3.1.1, the different theoretical spin sequences are compared with the experimental datum for the 159 keV γ-ray. The spin of the 159 keV level had not been measured previously, but a study of the circular polarisation of the 278 keV γ-ray after capture of polarised neutrons had indicated that the 278 keV level was $2^+$ (Ko 65). The anisotropy of the 159 keV distribution, which indicates that little or no attenuation has taken place, is compatible with the lifetime measurement $\tau < 0.3$ nsec of du Toit and Bollinger (To 61).

5.4.3.2 The 344 and 362 keV levels

The sequences $2^+ + 1^+$ and $2^+ + 2^+$ for the 344 keV and 185 keV γ-rays respectively fit the data albeit with very specific values of $\delta$. The sequences $1^+ \rightarrow 1^+$ and $1^+ \rightarrow 2^+$ for the 344 keV and 185 keV γ-rays also fit the data, each with a much larger range of possible values of $\delta$, and as a result the $1^+$ assignment for the 344 keV level seems the more probable.
Comparison of the experimental $A_2$ and $A_4$ coefficients for the 159 keV $\gamma$-ray in $^{64}$Cu with theoretical predictions of different spin sequences.
Comparison of the experimental $A_2$ and $A_4$ coefficients for the 344 and 185 keV $\gamma$-rays in $^{64}$Cu and theoretical predictions of different spin sequences.
$^{64}_{\text{Ni}}(\alpha \gamma)^{64}_{\text{Cu}}$

$E_x = 2.94 \text{ MeV}$
$E_\gamma = 344 \text{ keV}$

$E_\gamma = 185 \text{ keV}$
The 362 keV level de-excites by a 203 keV γ-ray to the 159 keV level, and was tentatively assigned as 3⁺ in ref. (Sh 68). From table 5.4.3.2, it is seen that both 2⁺ and 3⁺ are admissible. However, from the dominance of pure and almost pure M1 transitions in the decay scheme, the 3⁺ assignment appears the more likely. Angular distributions to investigate the spin of the 574 keV level were not possible to obtain because of the low intensity of the 212 keV γ-ray.

5.4.3.3 The 609 and 663 keV levels

The present results provide a unique assignment of 2⁺ for the 609 keV level. The angular distributions of the 385 and 663 keV γ-rays were isotropic within close limits and for reasons similar to those given for the 344 keV level, an assignment of 1⁺ is suggested.

5.4.3.4 The 739 and 746 keV levels

Results for the 377, 461 and 580 keV γ-rays are consistent with either 2⁺ or 3⁺ for the 739 keV state. The fact that this level decays to levels at 344 (1⁺), 278 (2⁺) and 362 (3⁺) suggests that, if M1 multipolarities dominate, the spin is 2⁺.
The 468 keV angular distribution is compared with the theoretical predictions in figure 5.4.3.4.1 and is consistent with a $3^+$ assignment although, from the proximity of the $2^+ \rightarrow 2^+$ sequence, $2^+$ cannot be ruled out. However, the $3^+$ is probably correct since the $2^+$ assignment would be associated with a large ($\delta \approx 1.1$) mixing ratio.

5.4.3.5 The 878, 895 and 27 keV levels

The 534 keV γ-ray gave an anisotropic distribution ($A_2 = 0.06^{±0.03}$) which precludes a $0^+$ value for the 878 keV level. Although $1^+$ and $2^+$ values are permissible, the isotropy of the 878 keV γ-ray suggests a $1^+$ assignment. The 617 keV data are compatible with $2^+$ ($\delta \approx 1.1$) and $3^+$ ($\delta = 0.0^{±0.1}$) for the 895 keV level. Again the $3^+$ assignment is suggested. The fact that the 895 keV transition to the ground state was not observed tends to substantiate this spin.

Finally, spins of both $1^+$ and $2^+$ for the 927 keV level fit the experimental data; the case is similar to the 344 keV level and leads to a tentative assignment of $1^+$. 
Comparison of the experimental $A_2$ and $A_4$ coefficients with theoretical predictions for the 468 keV $\gamma$-ray in $^{64}$Cu.
5.5 SUMMARY AND CONCLUSIONS

Spins in the nuclei $^{62}$Cu and $^{64}$Cu have been investigated by measuring $\gamma$-ray angular distributions following $(p,n)$ reactions and comparing the $\gamma$-ray distributions with the statistical model of Hauser and Feshbach, and Satchler. The model assumes that the $(p,n)$ reaction proceeds as a compound nucleus process and consequently it is again important to ensure that sufficient energy averaging has occurred before an analysis is carried out. Careful investigations of both reactions established that the thickness of the targets used provided sufficient averaging over the compound nucleus states.

The method of determining spins using the above mentioned model has not been extensively documented in the literature and thus it was felt necessary to ensure that consistent results were achieved when levels of known spins were investigated. The model is concluded to be reliable and moreover has the desirable property of enabling spins to be determined in a relatively simple fashion. In all cases it was found that the final results were quite insensitive to the optical model parameters.
A whole range of further studies on medium weight and heavy nuclei using the above model is thus possible. Other reactions could be used and, in particular, it may be fruitful to use \((\alpha,n)\) reactions to reach nuclei removed from the stability line.

The spin of the 426.1 keV level in \(^{62}\text{Cu}\), is consistent with the absence of any detectable \(\beta\)-feed from \(^{62}\text{Zn}\) to the level since a spin change of \(\Delta I = 3\) would be required. Again it is not fed by gamma decay from the two higher \(I^+\) levels, possibly because M1 multipolarities dominate throughout the decay structure of \(^{62}\text{Cu}\).

The level scheme of \(^{62}\text{Cu}\) below 640 keV excitation investigated in this work is in complete accord with that proposed by Hoffman and Sarantites (Ho69) but differs considerably from that furnished by Nuclear Data Sheets (Ve67). In particular, the existence of a level at 435 keV, which decays by emission of a 394.12 keV \(\gamma\)-ray, conflicts with the recent \(\beta\)-decay study (Ho69) in which a \(\gamma\)-ray of the above energy was identified as the transition \(637.20 \rightarrow 243.43\) keV.

The spins deduced for the reaction \(^{64}\text{Ni}(p,n\gamma)\(^{64}\text{Cu}\) compare well with the tentative assignments of Shera and Bolotin (Sh68). Apart from the unique assignments
for the 159, 278 and 609 keV levels, the isotropy of
the γ-decay from both the 344 and 663 keV levels points
to a 1+ assignment for both levels. The assignments
for the remaining states are much more tentative. The
609 keV level, originally suggested as 3+ by Kopecky
et al. (Ko 65), was given a unique assignment of 2+ in
the present study.
APPENDIX 1

It is required to show that

\[ L\{f * g\} = \tilde{f}(s) \tilde{g}(s) \]

where

\[ f * g = \int_{0}^{t} f(\tau) g(t - \tau) \, d\tau = \int_{0}^{t} g(\tau) f(t - \tau) \, d\tau = g * f \]

Now,

\[ L\{f * g\} = \int_{0}^{\infty} \int_{0}^{t} e^{-st} f(\tau) g(t - \tau) \, d\tau \, dt \]

\[ = \iint_{\Omega} e^{-st} f(\tau) g(t - \tau) \, d\tau \, dt \]

where \( \Omega \) is the area \( \{ 0 < \tau < t \} \) in the \( (\tau, t) \) plane.
Let $u = \tau$ and $v = t-\tau$, i.e. $\tau = u$ and $t = u + v$. The area over which the integration has to be performed in the $(u,v)$-plane is the first quadrant, which can be seen from

$$\Omega = \left\{ \begin{array}{l} 0 < \tau < \infty \\ \tau < t < \infty \end{array} \right\} \text{ or } \left\{ \begin{array}{l} 0 < \tau < \infty \\ 0 < t-\tau < \infty \end{array} \right\}$$

The functional determinant is

$$\frac{\partial (\tau, t)}{\partial (u, v)} = \begin{vmatrix} 1 & 0 \\ 1 & 1 \end{vmatrix} = 1$$

Thus

$$L[f*g] = \int_0^\infty \int_0^\infty e^{-s(u+v)} f(u) g(v) \, du \, dv$$

$$= \int_0^\infty e^{-su} f(u) \, du \int_0^\infty e^{-sv} g(v) \, dv$$

$$= \tilde{f}(s) \tilde{g}(s)$$

Q.E.D.
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