A STUDY OF THE DISINTEGRATION OF EXCITED STATES

IN $^8$Be AND $^{16}$O

by

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Doctor of Philosophy

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PREFACE

This thesis describes a series of experiments which were performed using the 12 MeV Tandem Van de Graaff accelerator in the Department of Nuclear Physics at the Australian National University.

All the experiments described here were carried out under the supervision of Dr. P. B. Treacy, and in collaboration with others. The $^6\text{Li}(d,\alpha)\alpha$ experiment described in Chapter I was undertaken jointly with Dr. Treacy and Dr. D. J. Sullivan, the work being shared approximately equally among us.

The $^8\text{B}$ $\beta$-decay experiment discussed in Chapter II is part of a program being undertaken at the Australian National University to look at details of $^8\text{B}$ via scattering and reactions. The experiment was designed and analysed by myself with Dr. P. B. Treacy and Mr. S. N. Tucker assisting in the data collection. The theoretical basis of the analysis and the principal computer program used in the analysis were developed by Dr. F. C. Barker, of the Department of Theoretical Physics.

The experimental work of the $^{12}\text{C}(\alpha,\alpha')$ study, which is described in Chapter III, was shared among Dr. Treacy, Dr. Sullivan and myself. The analysis was carried out by me.

Some of the work in this thesis has appeared in the following publications:
"The $^6\text{Li}(d,\alpha)^4\text{He}$ Reaction and States of $^8\text{Be}$ between 22 and 32 MeV".


Nuclear Physics, A98 (1967) 473.

"$^{12}\text{C} - \alpha$ Elastic Scattering and States of $^{16}\text{O}$ between 9.16 and 12.11 MeV".


Nuclear Physics (in press).

No part of this thesis has been submitted for a degree at any other university.

Gregory J. Clark.
ACKNOWLEDGEMENTS

It is a pleasure to thank Dr. P. B. Treacy for his valued assistance and supervision during the course of this work. Thanks are also due to Dr. D. J. Sullivan and Mr. S. N. Tucker for their enthusiastic co-operation.

I am grateful to Professor E. W. Titterton for his interest and the opportunity of using the facilities of the Department of Nuclear Physics.

I am particularly indebted to Dr. F. C. Barker of the Department of Theoretical Physics for many illuminating discussions and suggestions, particularly in regard to the experiment described in Chapter II.

I am indebted to the Workshop Staff and the Technical Staff for assistance in various ways.

The award of a Commonwealth Post-Graduate Scholarship is gratefully acknowledged.

Finally, I wish to express my appreciation to Miss Margot Mackie for her patience and care in typing this thesis.
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CHAPTER I

THE \(^6\text{Li}(d,\alpha)\text{He}\) REACTION AND STATES OF \(^8\text{Be}\) BETWEEN 22 AND 32 MEV

1.1 Introduction

The \(\alpha\)-particle emitting states of \(^8\text{Be}\) above 22 MeV have been investigated via \(\alpha-\alpha\) elastic scattering (Da 65), as well as by the \(^6\text{Li}(d,\alpha)\alpha\) (Fr 65, Ma 65: , Je 62) and the \(^7\text{Li}(p,\alpha)\alpha\) (Ma 64a) reactions. The phase shifts deduced from an analysis of the elastic scattering data exhibit broad anomalies, indicating (Da 65) the existence of broad \(6^+\) and \(8^+\) states near 30 MeV and 60 MeV respectively. A detailed analysis of the \(^6\text{Li}(d,\alpha)\alpha\) process (Fr 65) gave evidence for states at 22.54 MeV (\(2^+\)), 24.02 MeV (\(0^+\)) and 25.23 MeV (\(2^+\)), but no further obvious states up to 30 MeV. In this analysis, Legendre polynomial coefficients of order higher than four were neglected. The polynomial coefficients which correspond to the \(^8\text{Be}\) levels as suggested by Freeman et al. (Fr. 65) gave acceptable fits to the experimental data up to an excitation energy of 25 MeV. Above this energy the fits were not satisfactory, especially for order four, and it is not certain that higher order effects were negligible here.

If the interpretation of the broad \(I = 6\) \(\alpha-\alpha\) scattering anomaly in terms of a \(6^+\) state in \(^8\text{Be}\) is meaningful, then provided the appropriate particle widths are finite, it should be possible to observe the effects of this state with a reaction. To test this hypothesis, detailed angular distributions of the \(\alpha\)-particles from the reaction \(^6\text{Li}(p,\alpha)\text{He}\) were studied
in the region above 25 MeV $^8$Be excitation (beam energies 3 to 12 MeV).

The data are reported in section 1.3, an analysis in terms of Legendre polynomial expansions is discussed in section 1.4, and in section 1.5 an interpretation in terms of states in the intermediate $^8$Be nucleus is given.

1.2 Experimental Procedure

1.2.1 Target Chamber

A deuteron beam was provided by the A.N.U. Tandem Van de Graaff accelerator. After magnetic analysis, the beam entered a scattering chamber through two 1.5 mm collimators placed 12.5 cm apart and a 2.0 mm anti-scatter baffle mounted 12.5 cm beyond the final collimator. A beam of approximately 0.2 μA was obtained on target.

The target chamber used in this experiment (Fig. 1.2 and Fig. 1.3) was a 51 cm diameter chamber designed by Ohlsen and Young (Oh 64). The chamber has provision for mounting detectors in the plane of the beam on each of the two independently rotatable lids. Each detector was collimated by a 1 mm x 5 mm vertical slit which was attached to the counter block immediately in front of the detector. A 3 mm x 8 mm slit, placed 5.5 cm in front of the defining slit, acted as an anti-scatter baffle. In the reaction plane the angle subtended by the counter at the target with this collimation arrangement was 0.5°. With the use of a vernier scale,
FIGURE 1.1 Energy level diagram of $^8$Be. As well as the levels given in (La 66), the diagram includes additional levels discussed in this thesis.
FIGURE 1.2  Vertical cross section of the 51 cm scattering chamber.

The numbered parts are:

(1) Beam collimating system
(2) Target holder
(3) Faraday cup
(4) Suppression magnet
(5) Pointer giving angle position of target
(6) Gears by which upper and lower lids are rotated
(7) Vernier scales from which the angles of the counter are read.
FIGURE 1.3  Horizontal cross section of the 51 cm scattering chamber.

The numbered parts are:

(1) Rotating lid
(2) Vernier scale from which counter angle is read
(3) Beam collimating system
(4) Target
(5) Faraday cup housing
(6) Solid state counter mounting block
(7) Counter collimating slits
(8) Solid state counter.
angle settings could be set to an accuracy of ± 0.2°. Up to four targets may be mounted on a frame located centrally in the upper lid. Initial optical alignment of the chamber and collimating system established coincidence of a point on the beam axis, the centres of rotation of the counters and the target holder to within 0.005 cm.

The beam, after passing through the target, was collected in an insulated Faraday cup, employing both electrostatic and magnetic electron suppression and located immediately behind the scattering chamber. The charge was measured with a current integrator, known to be accurate to about 0.5%.

Detailed checks of the alignment, accuracy of the angular scales and beam current integration are discussed in section 3.3.3.

1.2.2 Foil Targets

Thin (20 - 60 µg/cm²) self-supporting carbon foils were prepared by evaporating carbon, using an arc struck, in vacuum, between two spectroscopically pure carbon rods, on to glass slides coated with a thin layer of detergent (De 60). These films were then carefully floated off the slides in a bath of warm distilled water and mounted on aluminium frames with 1.6 cm diameter holes. The natural carbon foil targets used in the experiment described in Chapter III were prepared in this manner. Targets prepared by this method contained ¹⁶O as a contaminant in sufficient
concentration to introduce significant uncertainties in the $^{12}\text{C}(\alpha,\alpha)$ scattering yield at angles $< 35^\circ$, because of the difficulties in resolving the $^{16}\text{O}$ and $^{12}\text{C}$ groups. Two commercial detergents (Teepol and R.B.S. 25) were tried as substrates for the evaporation. All measurements were made with foils prepared with R.B.S. 25 detergent, as these showed substantially reduced $^{16}\text{O}$ concentration and floated off the backing more readily.

For the experiment described in this chapter, the $^6\text{Li}$ targets were prepared by evaporating the lithium isotope (in the form of lithium oxide) onto carbon backings of surface density $30 - 60 \, \mu\text{g/cm}^2$. The lithium oxide was prepared by heating on a platinum wire $\text{Li}_2\text{CO}_3$, which had been enriched to 99.6% $^6\text{Li}$. Deposits of $\text{Li}_2\text{O}$ up to $60 \, \mu\text{g/cm}^2$ could be obtained, after which the carbon foils commenced to break.

For the experiment described in Chapter II, the $^6\text{Li}$ targets were prepared by evaporating the lithium oxide onto high grade nickel foils* of thickness $565 \, \mu\text{g/cm}^2$ and $1130 \, \mu\text{g/cm}^2$. The thickness of the $\text{Li}_2\text{O}$ deposit as determined by a weighing procedure, was $20 - 70 \, \mu\text{g/cm}^2$. The nickel foils were supported on aluminium frames with an epoxy resin. To prevent formation, by deliquescence, of the less stable lithium hydroxide, the targets were kept in a dry atmosphere.

* The Chromium Corporation of America, Waterbury, Connecticut, U.S.A.
1.2.3 **Detectors and Electronics**

Pulses from solid state detectors were amplified by "Ortec" charge sensitive preamplifier-linear amplifier systems and then fed directly to R.I.D.L. 400 channel pulse height analysers. A schematic representation of one such system is given in Fig. 1.4. Because of the high count rate, the amplifiers were operated in the double-delay line pulse-shaping mode.

A typical spectrum of the detected reaction products (Fig. 1.5) shows two strong groups of elastically scattered deuterons not completely resolved above a background and, in addition, the alpha group from the (d,α) reaction. This group was clearly resolved for angles down to 20°, below which pile-up effects from the deuteron group commenced to engulf it. However, observations down to 9° were achieved by using reduced beam intensity and an Ortec time pick-off and inspector unit (Fig. 1.4), which rejects pulse pairs, above an adjustable bias level, whose time separation is less than 2.2 µs.

The pulse rejection system operates as follows. A small pulse transformer between the detector and the preamplifier permits fast detection of the arrival of a particle. A fast amplifier followed by a tunnel diode discriminator produces a suitable logic signal and permits rejection of noise signals. The logic signal is then used to generate another logic pulse corresponding to separation in time of any two by less than 2.2 µs (but
FIGURE 1.4 A block diagram of the electronics used with one solid state detector in the $^6\text{Li}(d,\alpha)\alpha$ experiment.
FIGURE 1.5  A spectrum at a laboratory angle of 90° obtained from a thin $^6\text{Li}$ target bombarded with 10.75 MeV deuterons.
greater than 100 nanoseconds). This latter pulse can then be used to prevent analysis of the distorted linear signal. Since the probability of pile-up occurring between an alpha pulse and a deuteron pulse is comparatively low, the number of alpha pulses lost is insignificant. There is an output available from the time pick-off units, corresponding to every input above the level of the tunnel diode discriminator setting; this was used to obtain gated spectra which facilitated setting the level above noise. In some cases the beam was reduced to an extremely low intensity and the performance of the units was checked.

The "inhibit" pulse provided by the analysers while 'busy', was used to gate the output of a free-running oscillator. Comparison of the scaled gated counts with ungated counts from the same oscillator provided a measure of the analyser dead time. For measurements made at forward angles, the 'percentage dead time' was reduced by biasing out the elastically-scattered deuterons.

1.3 Experimental Results

1.3.1 Measurement of the Excitation Function

An excitation function was measured at 90° in the laboratory system over the range 3.0 to 12.0 MeV in 0.2 MeV steps. To remove any systematic errors arising from target deterioration, several runs were taken over the whole energy region with positive and negative energy increments. The
average values are plotted in Fig. 1.6.

The yield at each energy was obtained by summing the number of counts in the α-particle peak, correcting for dead time and normalizing to the beam charge. When present, spectra backgrounds were accounted for by subtracting an inferred linear background from the total peak count.

It was found that, independently of counting statistics, the data could only be measured to an accuracy of 3%. This 'residual' error, which may be attributed mainly to beam-spot density variations across an irregular target, was estimated by examining the variation in yield at a particular energy, obtained from several runs for the same integrated current. The distribution of yields included an extra three per cent beyond the standard deviation due to counting statistics. The errors plotted in Fig. 1.6 take account of counting statistics and the residual error.

In this experiment no attempt was made to measure absolute yields.

1.3.2 Measurement of Angular Distributions

Angular distributions were measured at nine energies in the energy range covered by the excitation function, viz: at 3.0, 3.93, 5.0, 6.0, 7.0, 8.0, 9.5, 10.75 and 12.0 MeV. For each angular distribution, a rotating counter and a monitor counter, fixed at a laboratory angle of 90° were used. The measurements were made in 2.5° increments for laboratory angles less than 20° and in 5° increments for angles between 20° and 75°. Use of the
FIGURE 1.6 Excitation function for the $^6\text{Li}(d,\alpha)\alpha$ reaction at a laboratory angle of $90^\circ$. The points include probable errors where these are significant (see Section 1.3.1). The solid curve was used to normalize the Legendre polynomial coefficients derived from the angular distributions to a common energy scale (see Section 1.4).
time pick-off system described in section 1.2.3 enabled laboratory angles to be reached down to 9° without pile-up effects engulfing the α-particle group. The yield at each angle was normalized to the 90° laboratory yield. This effectively cancelled 'residual' errors introduced by target thickness changes and beam shifts. The 'residual' errors incurred from the chamber geometry uncertainties were less than 0.5 per cent (a quantitative discussion is given in Chapter III). The rotating counter (R) and monitor counter (M) yields were corrected for dead times and spectra background when present.

The ratio of the two yields \( \frac{Y_R(\psi)}{Y_M(\psi)} \) in the laboratory system was converted to the centre-of-mass system by the non-relativistic kinematic relation:

\[
\frac{I_{\exp}(E_1, \theta)}{G(E_1, \theta)} = \frac{Y_R(E_1, \psi)}{Y_M(E_1, \psi)} = K(E_1) \frac{d\sigma}{d\Omega}(E_1, \theta),
\]

where \( G \) is the normal solid angle correction factor used when converting from laboratory to centre-of-mass angles. \( I(E_1, \theta) \) is then related to the centre-of-mass differential cross-section, \( \frac{d\sigma}{d\Omega}(E_1, \theta) \), by a factor, \( K(E_1) \), that depends only on energy \( E_1 \). The various \( I(\theta, E_1) \) are plotted as a function of the centre-of-mass angle, \( \theta \), in Fig. 1.7. The errors are approximately the size of the points in Fig. 1.7 and they contain only significant contributions from counting statistics. To minimize the effect of any systematic errors in setting the counter angles, several runs were taken over the whole
Angular distributions for the $^6$Li(d,α)α reaction between 3.0 and 12.0 MeV deuteron beam energies. The full curves correspond to Legendre polynomial fits as discussed in Section 1.4. In the case of the 12.0 and 10.75 MeV data, curves of $L_{\text{max}} = 8$ (dashed) are plotted for direct comparison to the accepted fits corresponding to $L_{\text{max}} = 12$ (full curves).
range with both positive and negative angle increments.

Because the particles detected belonged to a two-alpha-particle final state, the angular distributions must be symmetrical about 90° when plotted in centre-of-mass co-ordinates. This was checked at some points, but it was found most accurate in practice to detect particles in the forward hemisphere, as indicated in Fig. 1.7.

1.4 Reduction of Data

The excitation function exhibits a broad resonance near 4.0 MeV which is identified with the \( J^\pi = 2^+ \) state at 25.23 MeV excitation in \(^8\text{Be}\) (Fr 65). Unless otherwise stated, the energies quoted are laboratory energies. There is some indication of a resonance in the vicinity of 6 - 7 MeV, but no further resonances are evident. The effect of the \( 6^+ \) state seen with \( \alpha-\alpha \) scattering is not observed directly in the excitation function because of the extremely broad nature of the state. For evidence of broad levels it is more appropriate to examine the structure of the angular distributions.

The angular distributions data,

\[
I_{\text{exp}}(E_1, \theta) = K(E_1) \frac{d\sigma}{d\Omega}(E_1, \theta),
\]

were fitted with an even-order Legendre polynomial expansion:
\[ I_{\text{th}}(B_L, \theta) = \sum_{L = 0}^{L_{\text{max}}} B_L(E_1) P_L(\cos \theta) , \]  

(1.2)

where \( \theta \) is the centre-of-mass angle of observation. Using a least-squares fitting criterion, the linearity of the above expression enabled a unique set of \( B_L \) coefficients to be obtained for each angular distribution, once a value for \( L_{\text{max}} \) had been chosen. The full curves of Fig. 1.7 represent fits obtained from the linear least squares procedure of Rose (Ro 53). The derived polynomial coefficients, \( B_L \), were normalized to a common energy scale using the \( 90^\circ \) laboratory excitation function of Fig. 1.6 to obtain the \( K(E_1) \) within an arbitrary constant. The normalized polynomial coefficients, \( A_L \), are listed in Table 1.1 and plotted as a function of energy in Fig. 1.8. The errors assigned to the \( A_L \) coefficients are the direct result of the experimental uncertainties in the data points (statistical errors), which in this case contain only significant contributions from counting statistics.

The value of \( L_{\text{max}} \) used in each expansion was obtained by examining the variation of \( X^2(E_1, B_L) \) with \( L_{\text{max}} \). The quantity \( X^2(E_1, B_L) \) is defined in terms of the data points \( I_{\exp}(E_1, \theta) \), their respective errors \( \epsilon \) and the function \( I_{\text{th}} \) discussed above by:

\[ X^2(E_1, B_L) = \frac{1}{n - m} \sum_{j = 1}^{n} \left[ \frac{I_{\exp}(E_1, \theta) - I_{\text{th}}(B_L, \theta)}{\epsilon} \right]^2 , \]  

(1.3)
FIGURE 1.8  The $A_L$ coefficients derived from the Legendre polynomial expansions of the angular distributions in Figure 1.7. Probable errors are plotted where significant. The solid curves serve only to connect the points, in accordance with the discussion of Section 1.5.
Polynomial Coefficients

$A_0 x^2$

$A_1$

$A_8$

$A_{10}$

$A_{12}$

Deuteron Energy (MeV)
TABLE 1.1

<table>
<thead>
<tr>
<th>E MeV</th>
<th>( A_0 )</th>
<th>( A_2 )</th>
<th>( A_4 )</th>
<th>( A_6 )</th>
<th>( A_8 )</th>
<th>( A_{10} )</th>
<th>( A_{12} )</th>
<th>( A_{14} )</th>
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<tr>
<td>3.0</td>
<td>8.951 ± 0.070</td>
<td>4.640 ± 0.215</td>
<td>0.658 ± 0.298</td>
<td>0.279 ± 0.342</td>
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<tr>
<td>3.93</td>
<td>12.676 ± 0.115</td>
<td>-2.463 ± 0.218</td>
<td>0.154 ± 0.321</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>5.0</td>
<td>6.032 ± 0.076</td>
<td>-3.159 ± 0.127</td>
<td>-1.469 ± 0.184</td>
<td>0.399 ± 0.234</td>
<td>-0.190 ± 0.247</td>
<td></td>
<td></td>
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<tr>
<td>6.0</td>
<td>3.979 ± 0.022</td>
<td>-2.182 ± 0.035</td>
<td>-1.602 ± 0.052</td>
<td>0.388 ± 0.061</td>
<td>-0.052 ± 0.061</td>
<td></td>
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<tr>
<td>7.0</td>
<td>3.090 ± 0.057</td>
<td>-1.968 ± 0.100</td>
<td>-0.426 ± 0.140</td>
<td>0.376 ± 0.140</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>2.195 ± 0.013</td>
<td>-1.581 ± 0.024</td>
<td>0.091 ± 0.032</td>
<td>0.392 ± 0.040</td>
<td>-0.048 ± 0.048</td>
<td></td>
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<tr>
<td>9.5</td>
<td>1.689 ± 0.007</td>
<td>-0.829 ± 0.015</td>
<td>-0.212 ± 0.020</td>
<td>0.243 ± 0.026</td>
<td>0.081 ± 0.026</td>
<td>-0.113 ± 0.027</td>
<td>-0.020 ± 0.031</td>
<td></td>
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<tr>
<td>10.75</td>
<td>1.404 ± 0.009</td>
<td>-0.455 ± 0.023</td>
<td>-0.141 ± 0.032</td>
<td>0.234 ± 0.040</td>
<td>0.166 ± 0.043</td>
<td>-0.027 ± 0.043</td>
<td>0.128 ± 0.052</td>
<td>0.005 ± 0.061</td>
</tr>
<tr>
<td>12.0</td>
<td>1.159 ± 0.009</td>
<td>-0.233 ± 0.017</td>
<td>-0.184 ± 0.029</td>
<td>-0.008 ± 0.031</td>
<td>0.337 ± 0.033</td>
<td>-0.200 ± 0.039</td>
<td>0.097 ± 0.046</td>
<td>-0.021 ± 0.051</td>
</tr>
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</table>

The \( A_L \) coefficients derived from the Legendre polynomial expansions of \( ^6\text{Li}(d,\alpha)^\prime \) angular distributions described in section 1.4. In some cases higher order coefficients than that required by the analysis, have been included to illustrate the consistency of the analysis.
where \( n \) is the number of data points and \( m \) is the number of polynomials used in the expansion \( m = \frac{1}{2} L_{\text{max}} + 1 \). Plots of \( X^2 \) as a function of \( L_{\text{max}} \) for the two most complicated cases at 10.75 and 12.0 MeV are shown in Fig. 1.9. The use of \( X^2 \) in this situation has been placed on a formal basis by the analysis of Rose (Ro 53). Rose estimates the total error in the \( A_L \) coefficients due to statistical errors in data, as well as errors introduced by the use of an inappropriate functional form in fitting the data. The square of the ratio of this total error to the statistical error is simply the quantity \( X^2 \). The criterion for the choice of \( L_{\text{max}} \) follows from the expectation that \( X^2 \) be approximately unity provided the correct function has been used to describe the data.

It is clear from Fig. 1.9 that the 12.0 MeV angular distribution data requires an \( L_{\text{max}} \) of at least 10 and possibly 12. A choice of \( L_{\text{max}} = 12 \) was made, since, as discussed in section 1.5 below, any \(^8\)Be level requiring \( P_{10} \) will also require, in principle, \( P_{12} \). For the 10.75 MeV angular distribution a similar argument again led to the choice of \( L_{\text{max}} = 12 \). In terms of this procedure, polynomial contributions of higher order than \( L_{\text{max}} \) are identically zero. In this manner a unique value of \( L_{\text{max}} \) was chosen for each angular distribution. Corresponding to the increasing structure in the curves of Fig. 1.7 with rising energy, it was found that \( L_{\text{max}} \) increased monotonically with energy. The actual values chosen for \( L_{\text{max}} \) vary from 4 (with beam energy 3 - 4 MeV) to 12 (with beam energy 8 - 12 MeV). A comparison of the quality
Plots of $X^2$ as a function of $L_{\max}$, the upper limit for the Legendre polynomial expansions used to fit the angular distributions. Only the cases corresponding to 12.0 and 10.75 MeV beam energies are given.
of fits with \( L_{\text{max}} = 8 \) and \( L_{\text{max}} = 12 \) can be made with reference to Fig. 1.7, where fits with \( L_{\text{max}} = 8 \) (dashed curves) have been included for the 12.0 and 10.75 MeV cases.

The \(^6\text{Li}(d,\alpha)\alpha\) reaction data has been fitted (El 63) using a direct reaction model at deuteron energies below 3 MeV. To check on the reaction model in this present work the magnitude of the ratio of the \( A_L \) coefficients to their associated errors was plotted as a function of \( L \). (The 12 MeV case is shown in Fig. 1.10). At each energy, the curves fluctuate rapidly in magnitude and tend to zero quickly with increasing \( L \) indicating that the reaction proceeds through states of well-defined angular momentum, i.e., the reaction mode is predominantly a compound nucleus one. Conventional direct-reaction behaviour results in angular distribution polynomial expansions whose \( A_L \) coefficients for higher \( L \) values decrease slowly as a function of \( L \) (Ho 66).

### 1.5 Interpretation in Terms of \(^{8}\text{Be}\) States

It is assumed, following the discussion in section 1.4 that the \(^6\text{Li}(d,\alpha)\alpha\) process is a two stage reaction proceeding through intermediate states of definite spin parity \( J^\pi \). On the basis of this assumption the \( R \)-matrix theory of nuclear reactions (La 58) was used to interpret the \( A_L \) coefficients in terms of levels in the compound nucleus \(^{8}\text{Be}\). The formulae
FIGURE 1.10  Plot of $|A_L|/\text{Error}$ as a function of $L$ for the Legendre polynomial expansion used to fit the 12.0 MeV angular distribution.
and notation of this theory which are relevant in the following discussion, are given in Appendix I.

1.5.1 Evidence for a $J^\pi = 6^+$ State

At low energies, the analysis described in section 1.4 agrees with that of Freeman and Mani (Fr 65) in that only $A_L$ coefficients up to order 4 are significant and the energy dependence of these coefficients is similar in the two analyses. However, $A_4$ shows a pronounced anomaly near 6 MeV, which may explain the deviation between the data and fit of Freeman and Mani. There is a broad maximum in $A_6$ near 8 MeV falling to zero near 12 MeV, while the magnitudes of higher order coefficients increase up to the highest energy investigated.

Because of the satisfactory fit of the low energy data, the three levels $(2^+, 0^+, 2^+)$ postulated by Freeman and Mani seem acceptable. It is then natural to interpret the tail of $A_0$ as that of the broad $0^+$ state at 24.02 MeV excitation in $^8$Be. However, the presence at higher energies of $A_L$ coefficients above order 4, indicates the existence of at least one other level. The following is an identification of this level.

The $^8$Be states participating in the $^6$Li(d, $\alpha$)$\alpha$ reaction are restricted to even spins and parities because of the decay into the symmetrical final state of two $\alpha$-particles. The incoming channel spin ($S_d$) can have three possible values (0, 1, 2) while the outgoing spin ($S_\alpha$) is zero. The selection rules
listed in equation (I.8) of Appendix I restrict both channel orbital angular
momenta to even values.

The participation of $^8\text{Be}$ states above 5 MeV deuteron energy may be
investigated with the aid of Table 1.2 which was compiled using the relation
(I.8) of Appendix I. Table 1.2 lists the possible $A_L$ coefficients associated
with a level of a particular spin and parity and also lists terms arising from
interference between levels.

The presence of terms up to $A_{12}$, but not beyond, assures the exis-
tence of a $J^\pi = 6^+$ state. Since there is no evidence for the state in $A_0$ it
must be a very broad level. This is in agreement with the $\alpha-\alpha$ scattering
data. The peak in $A_6$ is qualitatively as expected from $0^+, 6^+$ interference.
The fact that $A_6$ appears to change sign near 31 MeV excitation in $^8\text{Be}$ may
be used to obtain some estimate for the position of the $6^+$ level.

Referring to equation (I.28) of Appendix I, the energy dependence
of $A_6$ due to interference between two broad levels of different spins 0 and
6 is determined essentially by the factor:

$$\cos(\xi_{d0} + \xi_{a0} - \xi_{d6} - \xi_{a6} + \beta_0 - \beta_6), \quad (1.4)$$

where $d$ and $\alpha$ refer to channels and

$$\xi_{c\lambda} = \omega_c - \vartheta_c.$$
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Angular momentum quantum numbers associated with transitions in the \( ^6\)Li(d,\( d\)) reaction proceeding through levels of the compound nucleus \(^8\)Be. The notation used is:

- \( J \): spin of compound nucleus level
- \( t \): channel orbital angular momentum
- \( S \): channel spin
- \( d \): \(^6\)Li + d channel
- \( \sigma \): \( \sigma + \sigma \) channel
- \( L \): Legendre polynomial order
- \( \lambda \mu \): denote levels.

(Note: \( S = 0 \))
(The Coulomb-hard sphere and resonance phase shifts \( \omega_c, \phi_c \) and \( \beta_\lambda \) are defined in Appendix I). Using the Freeman and Mani parameters for the \( 0^+ \) state (Fr 65), the associated resonance phase factor \( \beta_0 \) can be calculated for a particular channel radius. It is then possible to confirm that the above phase factor should be small at 31 MeV excitation in \( ^8\text{Be} \) if a channel radius of 5.5 fm is assumed for a \( 6^+ \) state with properties (Table 1.3) as determined from a recent re-analysis (Ba 67a) of the \( \alpha-\alpha \) scattering data of Darriulat et al. (Da 65). That is, provided a channel radius of 5.5 fm is assumed, the \( 6^+ \) level seen with the reaction \( ^6\text{Li}(d,\alpha)\alpha \) may be identified with the \( 6^+ \) level seen in \( \alpha-\alpha \) scattering. Unfortunately it is not possible to obtain from the data presented here an independent estimate of the \( 6^+ \) level parameters. This is because of the sensitivity of the \( \xi \)'s (Coulomb minus hard sphere phases) in the above expression to changes in the channel radius.

1.5.2 Evidence for a \( J^\pi = 4^+ \) State

The variation of the coefficients \( A_2 \) and \( A_4 \) in the region between 6 and 8 MeV deuteron energies, indicates a level of width about 1 MeV situated at 27.5 MeV in \( ^8\text{Be} \). As higher-order terms (except possibly \( A_6 \), in which a sharp threshold is observed) do not display the anomaly, it is natural to assume that it is of character either \( 2^+ \) or \( 4^+ \), interfering with the broad \( 0^+ \) level at 24.0 MeV, and the \( 6^+ \) level near 30 MeV. It is convenient to denote the spin of this 27.5 MeV level by \( J \).
With reference to Table 1.2 it is seen that, in principle, both the 
\((J, 0^+)\) and \((J, 6^+)\) interference can contribute to \(A_2\) and \(A_4\). However, the high \(l\)-values required to populate the \(6^+\) state lead to small penetration 
factors \((P_c)\) in the corresponding interference terms (equation (I.28), Appendix I). This is in direct contrast to the population of the \(0^+\) state. On these 
grounds the \((J, 6^+)\) interference is disregarded as a major contribution to \(A_4\).

Initially, consider the level \(J\) to be of character \(2^+\). The differential 
cross-section in the region of 7 MeV deuteron energy may then be expressed 
in terms of \(0^+\) and \(2^+\) intermediate levels. The \(2^+\) contribution to \(A_0^{(2^+)}\), \(A_4^{(2^+)}\), 
will add incoherently to the Breit-Wigner tail of the \(0^+\) state. From Fig. 1.8 
an upper limit of 0.2 may be placed on the positive Breit-Wigner "bump" in 
\(A_0\) at 7.0 MeV deuteron energy. The \(A_4^{(2^+)}\) anomaly (which must also be 
incoherent in this case) is at least 1.7. Thus the ratio \(A_0^{(2^+)} / A_4^{(2^+)}\); deter-
mined from the present experiment is approximately 0.12.

From equation (I.10) of Appendix I, and using Table 1.2, the 
differential cross-section may be written:

\[
\frac{d\sigma}{d\Omega} \frac{d\alpha}{d\alpha} = \text{const} \frac{1}{k^2} \sum_{L=0,2,4} A_L(E) P_L(\cos \theta). \tag{1.5}
\]

Assuming the level to be of \(2^+\) nature, then \(A_0^{(2^+)}\) and \(A_4^{(2^+)}\) may be expressed 
in terms of the collision matrices \(U^{2}_{S',S},S\). Such a procedure indicates that
$A_0^{(2^+)}$ would be at least 0.39 times $A_4^{(2^+)}$. Thus the possibility of the level at 7.0 MeV being $2^+$ is ruled out.

The assignment $J = 4^+$ to this level cannot be ruled out. In this case $(0^+, 4^+)$ interference terms can contribute to $A_4$ and the magnitude of the observed anomaly in $A_4$ is consistent with this effect. The existence of a $4^+$ level at 7 MeV deuteron energy implies that all other terms up to $A_8$ should contribute to some extent. The absence of $A_8$ is consistent with the large angular momentum barriers for $g$-wave deuteron and $\alpha$-particles. All the other terms exhibit some anomaly near 7 MeV. If the possibility of $(4^+, 6^+)$ interference is now admitted, then the negative excursion in $A_2$ and the sharp threshold observed in $A_6$ are reasonable. Thus, there is direct qualitative evidence that the 27.5 MeV level in $^8\text{Be}$ is of $4^+$ character.

### 1.6 $^8\text{Be}$ Levels Above 22 MeV Excitation

Table 1.3 is a summary of the data on the known $\alpha$-emitting states in $^8\text{Be}$ above 22 MeV excitation, including information from the experiment described in this chapter.

In this table $\Gamma$ is the total width of the level as defined in equation (I.26) of Appendix I, and $E_{\text{res}}$ is the resonance energy in $^8\text{Be}$ defined in terms of the shift factor $\Delta \lambda$ and the energy eigenvalue $E_\lambda$ by (equation (I.20), Appendix I),

$$E_\lambda + \Delta \lambda (E_{\text{res}}) = E_{\text{res}}.$$  \hspace{1cm} (1.10)
TABLE 1.3

$^8$Be Levels Above 22 MeV Excitation in $^8$Be

<table>
<thead>
<tr>
<th>Beam Energy $E_d$ (MeV)</th>
<th>Excitation Energy $E_{res}$ (MeV)</th>
<th>$J, \pi$</th>
<th>$\Gamma$(C.M.) (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.35(a)</td>
<td>22.54</td>
<td>$2^+$</td>
<td>0.80</td>
</tr>
<tr>
<td>2.31(a)</td>
<td>24.02</td>
<td>$0^+$</td>
<td>1.36</td>
</tr>
<tr>
<td>3.93(a)</td>
<td>25.23</td>
<td>$2^+$</td>
<td>0.27</td>
</tr>
<tr>
<td>7.0(b)</td>
<td>27.5</td>
<td>$4^+$</td>
<td>1.0</td>
</tr>
<tr>
<td>4.95(b,c)</td>
<td>26.0</td>
<td>$6^+$</td>
<td>14.4</td>
</tr>
<tr>
<td>27.6(c)</td>
<td>43.0</td>
<td>$8^+$</td>
<td>23.0</td>
</tr>
</tbody>
</table>

The parameters of the levels denoted (a) are from an analysis (Fr 65) using a channel radius of 4.5 fm. Those denoted (b) have parameters derived from the present experiment, using a channel radius of 5.5 fm. The $6^+$ and $8^+$ levels (denoted (c)) have the properties required for a three level fit (Ba 67) to the $\alpha-\alpha$ scattering data (Da 65) with the boundary condition parameter $B$ set to zero and a channel radius of 5.5 fm (see section 1.5.1). It should be noted that the optimum fit to the $\alpha-\alpha$ scattering data was obtained with a channel radius of approximately 6.5 fm rather than 5.5 fm. However, a channel radius of 5.5 fm is justified for a 1-level fit to the $\alpha-\alpha$ scattering
data over a limited energy region (Ba 67). A radius of 5.5 fm was used here, as with this radius the parameters obtained for the $6^+$ state from the $^6\text{Li(d,}\alpha)^4\text{He}$ reaction data and the $\alpha-\alpha$ scattering data are compatible.

The broad $6^+$ and $8^+$ states, together with the well known $0^+$, $2^+$, $4^+$ states below 16 MeV have large alpha particle reduced widths compared to the Wigner limit. These states form a sequence whose positions are approximately proportional to $\frac{\hbar^2}{2\mu} J(J + 1)$. This suggests (Da 65) a rotational model description of $^8\text{Be}$. In the shell model scheme, however, these states cannot all belong to the same configuration (Ba 66). Calculations for the 1-p shell (Ba 66) yield states which are in good agreement with the $0^+$, $2^+$, $4^+$ sequence below 16 MeV, but mixing of higher configuration shell model states is required to account for the $6^+$ and $8^+$ states.

A simple model has been proposed by Temmer (Te 64) for the $^8\text{Be}$ states near 22 MeV excitation in $^8\text{Be}$. It involves coupling an s-wave deuteron to a $^6\text{Li}$ core. Considering that $^6\text{Li}$ may be in the ground or first few excited states Temmer predicted one $0^+$, one $4^+$ and three $2^+$ levels in the region of 20 to 25 MeV. Some of these may well be identified with those states observed.

F. C. Barker's intermediate-coupling shell-model calculations for the 1p shell (i.e., the lowest configuration) (Ba 66) predict levels near 22 MeV excitation in $^8\text{Be}$ which may be identified with the experimental ones. In particular $T = 0$, $2^+$ levels are predicted at 23.42 and 24.50 MeV and a
$T = 0$, $4^+$ level is predicted at 25.84 MeV. The predicted $0^+$ level nearest to that suggested at 24.0 MeV by the $^6\text{Li}(d,\alpha)^4\text{He}$ data is at 28.1 MeV. These identifications can only be tentative however, because of the uncertainty in associating levels at such high excitations with predicted shell-model states of the lowest configuration.

A recent re-analysis (Ke 67) of the g-wave phase shift for $\alpha-\alpha$ scattering suggests a $4^+$ state at 26.7 MeV. Since the width of this level is approximately 25 MeV, it follows that the states at 26.7 MeV and 27.5 MeV are distinct. The present analysis is not sufficiently detailed to identify the broad $4^+$ state.
CHAPTER II

THE β-DECAY OF $^8$B AND THE $2^+$ STATES OF $^8$Be

2.1 Introduction

The nucleus $^8$Be has been studied in considerable detail (La 66) and its level structure below the first $T = 1$ state at about 17 MeV appears to possess the simplicity expected of it on either the alpha-particle model or the lowest shell model configuration, namely a $0^+$ ground state, a broad $2^+$ state at 2.9 MeV and a very broad $4^+$ state at 11.4 MeV. This structure is shown in Fig. 1.1, together with other $^8$Be levels discussed in this thesis. However, other light even-even nuclei from $^{10}$Be to $^{16}$O have additional $0^+$ and $2^+$ excited states at excitation energies of order 6 MeV and 10 MeV respectively, with little correlation to the mass number $A$ or to $Q_\alpha$, the $Q$-value for $\alpha$-particle decay. Candidates for such levels are shown in Table 2.1, together with the energies of the corresponding $T = 1$ states in their odd-odd isobars. These states probably do not belong to the lowest shell model configuration. This is obviously the case for the $^{16}$O state which appears to contain an appreciable 4p-4h component (Br 66). Also 2p-2h configurations have been suggested for the $^{14}$N state (Un 58) and for the $^{10}$Be and $^{10}$B states (Tr 61). In Table 2.1 two $2^+$ levels are listed for some nuclei with $A = 10$ or 14. One of these levels is expected to belong to the lowest configuration; for $A = 8$, 12 and 16 no such level of the
TABLE 2.1

Excitation energies of $0^+$ and $2^+$ excited states of light even nuclei

<table>
<thead>
<tr>
<th></th>
<th>$E_x$ (MeV)</th>
<th>$Q_\alpha$ (MeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$0^+$</td>
<td>$2^+$</td>
</tr>
<tr>
<td>Be$^8$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Be$^{10}$</td>
<td>6.18</td>
<td>5.96</td>
</tr>
<tr>
<td>B$^{10}$ (T = 1)</td>
<td>5.82</td>
<td>7.16</td>
</tr>
<tr>
<td>C$^{10}$</td>
<td>5.6</td>
<td>7.2</td>
</tr>
<tr>
<td>C$^{12}$</td>
<td>7.66</td>
<td>$\sim$10.0</td>
</tr>
<tr>
<td>C$^{14}$</td>
<td>6.59</td>
<td>7.01</td>
</tr>
<tr>
<td>N$^{14}$ (T = 1)</td>
<td>6.31</td>
<td>6.86</td>
</tr>
<tr>
<td>O$^{14}$</td>
<td>5.91</td>
<td></td>
</tr>
<tr>
<td>O$^{16}$</td>
<td>6.06</td>
<td>6.92</td>
</tr>
</tbody>
</table>
lowest configuration is expected (Ku 56). The states probably also have large 
$\alpha$-particle reduced widths. Of the $2^+$ levels, the $\alpha$-particle reduced width 
for $^{10}$B is known to be large (La 66), and an appreciable reduced width has 
been obtained for the $^{16}$O state from the $^{6}\text{Li}(^{12}\text{C},d)^{16}\text{O}^*$ reaction (Lo 67).

Because of the large energy available for $\alpha$-particle decay, one 
would expect such states in $^8\text{Be}$ to be extremely broad. Although this would 
make their identification difficult, the systematics in neighbouring nuclei 
indicate the probable existence of such states in $^8\text{Be}$.

The work described in this chapter is part of a program being under-
taken at A.N.U, to look at details of $^8\text{Be}$ via scattering and reactions of the 
type $D(d,c)^8\text{Be}(\alpha)\alpha$. All states of $^8\text{Be}$ are unbound against break-up into two 
$\alpha$-particles, the ground state itself being unbound by 95 keV. It is therefore 
of considerable interest to correlate the level structure of $^8\text{Be}$, as seen in 
nuclear reactions with the $\alpha-\alpha$ elastic scattering phase shifts. In a recent 
paper (Ba 68) on the $0^+$ states of $^8\text{Be}$, the $s$-wave $\alpha-\alpha$ scattering phase shift 
and data from the $^9\text{Be}(p,d)^8\text{Be}$ reaction were analysed using three-level 
R-matrix formulae. A consistent fit required a channel radius, $a_\alpha$, of 
order 7 fm, implying a second $0^+$ level in $^8\text{Be}$ at about 6 MeV excitation. 
Such a level, although not previously identified, is consistent with the 
systematics of neighbouring nuclei.

A three-level analysis of the $d$-wave $\alpha-\alpha$ scattering phase shift,
made by Barker (Ba 68a), indicates a second $2^+$ level at approximately 10 MeV excitation in $^8$Be. Restrictions may be placed on the acceptable range of channel radii $a_2$ and $^8$Be level parameters by obtaining simultaneous fits to reaction data. Available reaction data are the $^9$Be(p,d)$^8$Be data for $^8$Be excitation energies, $E_x$, up to 4 MeV, the data from $^{10}$B(d,α)$^8$Be, $^7$Li(d,n)$^8$Be, $^9$Be(p,d)$^8$Be and $^9$Be(3He,α)$^8$Be for $E_x$ from 15 to 17 MeV, the $^8$Li(β)$^8$Be data for $E_x \leq 14$ MeV and the $^8$B(β)$^8$Be data for $E_x \leq 17$ MeV.

The β-decays of $^8$Li and $^8$B are suitable means for studying the $2^+$ states of $^8$Be for several reasons:

(a) The allowed decays populate only the $2^+$ states. Consequently there is no distortion of the 2.9 MeV state by the "ghost" of the ground state. The reaction $^9$Be(p,d)$^8$Be shows evidence for the ghost (Ba 62).

(b) The strong interaction effects would probably be minimal.

(c) There is no competing mode in which the α-particle is emitted before the β-particle.

(d) The effective "penetration factor" for β-emission is well known.

Many studies have been made of $^8$Li and $^8$B β-decay as seen through the distribution of α-particles from the subsequent break-up of $^8$Be (La 66). The most accurate experimental results for the $^8$Li decay appear to be those of Alburger et al. (Al 63), who measured the α-spectrum following $^8$Li β-decay for energies corresponding to $^8$Be excitation energies $E_x \leq 7.4$ MeV.
Matt et al (Ma 64) measured the spectrum of high energy $\alpha$-particles following $^8$B $\beta$-decay for energies corresponding to $E_x = 14.9 - 16.9$ MeV. Previously Farmer and Class (Fa 60) measured this spectrum for $E_x \leq 16$ MeV, although the results are not very useful because of uncertainties in corrections made to the data.

Analysis using the one-level approximation did not adequately fit the observed spectrum of $\alpha$-particles following the $^8$Li and $^8$B $\beta$-decays for $E_x \geq 3.5$ MeV (Al 63). It has been suggested that the contribution from the 16.6 MeV level can account for the discrepancy (Al 63, Ma 64). No such detailed fit has been published, apart from the early one by Griffy and Biedenharn (Gr 60), who used a form of the yield curve that is probably incorrect (Al 63). Barker (Ba 68a) has performed a multi-level fit to the $^8$Li($\beta^-\ ^8$Be data (Al 63) using his form of the cross section for a three-stage process (Ba 67). This work indicates that levels in addition to the 16.6 MeV and 2.9 MeV ones must be included, if the discrepancy is to be explained. However, the extracted level parameters are incompatible with those extracted from the d-wave $\alpha-\alpha$ scattering analysis, unless an energy shift of order $+100$ keV is applied to the experimental $\alpha$-particle spectrum subsequent to the $^8$Be break-up. This apparent inconsistency could arise from data errors or from limitations in the analysis.

To test if such an energy shift is reasonable and to obtain further
information on the $^8\text{Be}$ states as seen through final state interactions, the $^8\text{B}$ $\beta$-decay was studied. This work, described here, consisted of a measurement of the $\alpha$-particle spectrum following the $^6\text{Li}(^3\text{He},n)^8\text{B}(\beta^+)^8\text{Be}(\alpha)^4\text{He}$ reaction for energies corresponding to the $^8\text{Be}$ excitation energies $E_x \leq 14.1 \text{ MeV}$. The data was analysed using Barker's multi-level density of states function and the results correlated to those obtained from analysis of the $\alpha-\alpha$ scattering data, the $^9\text{Be}(p,d)$ reaction data and the $^8\text{Li}$ $\beta$-decay data.

### 2.2 Experimental Procedure

The $\alpha$-particle spectrum following the $^8\text{B}$ $\beta$-decay to states in $^8\text{Be}$ was measured. The $^8\text{B}$ was formed using the reaction $^6\text{Li}(^3\text{He},n)^8\text{B}$. The half-life of $^8\text{B}$ is 0.77 sec. (La 66).

A $^3\text{He}$ beam from the A.N.U. Tandem Van de Graaff generator was passed through a $90^\circ$ analysing magnet and into a 16 cm target chamber (Fig. 2.1), via two 0.8 mm collimating slits placed 30 cm apart. The beam had an analysed energy of 3.5 MeV and approximate intensity of 0.05 $\mu\text{A}$ on target. A $\text{Li}_2\text{O}$ target of $\sim 20 \mu\text{g/cm}^2$ surface density on a nickel backing of surface density 1130 $\mu\text{g/cm}^2$ was used. The $\text{Li}_2\text{O}$ was enriched to 99.6% $^6\text{Li}$. The preparation of these targets is discussed in section 1.2.1. The target was placed in the chamber with the $\text{Li}_2\text{O}$ side facing the beam and the
FIGURE 2.1  Vertical cross section of the 16 cm scattering chamber.

The parts are as designated in the diagram.
normal to the target at $50^\circ$ to the beam direction. The kinematics of the reaction $^6\text{Li}(^3\text{He},n)^8\text{B}$ indicate that at energies just above the neutron threshold at 2.966 MeV all the $^8\text{B}$ nuclei are thrown forward in a cone and stopped in the Ni backing. The half-angle of the cone increases with energy above threshold. For the conditions described here the half-angle of the cone was $10.9^\circ$ and the $^8\text{B}$ nuclei were stopped close to the centre of the nickel backing. The $\alpha$-particles following the radio-active decay of $^8\text{B}$ were detected with a surface barrier detector placed 4 cm from the target centre and at an angle of $90^\circ$ to the target. The solid angle subtended by the detector at the target was defined by a collimator of diameter 0.475 cm placed immediately in front of the counter. An anti-scatter collimator of diameter 0.635 cm was placed 1.9 cm in front of the collimator. Pulses from the surface barrier detector were amplified by a charge sensitive pre-amplifier-linear amplifier system operated in the double-delay line pulse-shaping mode. The amplified pulses were fed directly to an I.B.M. 1800 computer operated as a 512 channel pulse height analyser.

The $^3\text{He}$ beam was interrupted by a shutter that rotated at 85 r.p.m., 10 ft. upstream from the target. An analyser gate, synchronized with the beam shutter enabled the target to be irradiated for approximately the lifetime of $^8\text{B}$ and then the alpha-particle spectrum following the radio-active decay of $^8\text{B}$ to be accumulated for a similar period. As the analyser was
gated by a pulse from the shutter to reject pulses during the "beam-on" period, reaction products other than those from or following a radio-active decay were not detected. During the irradiation the counter was protected by a pneumatically operated shutter that was synchronized with the beam shutter.

During the beam-on cycle, protons which populated the ground state of $^8$Be in the reaction $^6$Li($^3$He, p) were analysed with a $180^\circ$ double focussing spectrometer placed at $15^\circ$ to the beam direction and detected in a solid state counter placed in the focal plane of the spectrometer. The proton yield was used to monitor the beam. Initially an attempt was made to monitor the beam by detecting neutrons from the ($^3$He, n) reaction using a boron loaded plastic scintillator. However, this method was not satisfactory because of the high neutron background.

The experimental arrangement is illustrated in Fig. 2.2, together with a diagramatic description of the synchronized operation of the beam shutter, the analyser gate and the pneumatic shutter.

In this experiment it is important to know the absolute energy of the $\alpha$-particles. The energy scale of the 512 channel analyser was calibrated using the 8.776 MeV and 6.043 MeV $\alpha$-particle groups from a ThC source. (The 6.043 MeV group is actually a doublet of energies 6.082 and 6.043 MeV and branching ratios of 0.272 and 0.698 respectively). The half-life of the source is 12.1 hours. For the calibration measurements, the target
FIGURE 2.2 (a) Diagramatic description of the experimental arrangement.
A pulse from the rotating shutter was used to gate the pulse height analyser and to operate a pneumatic shutter which protected the counter during the "beam-on" period.

(b) Diagram showing the sequential operation in time of the beam shutter, the analyser gate and the pneumatic shutter.
was replaced by a small brass disc on which the source had been deposited.

The $\alpha$-particle spectrum from the ThC source was then recorded using identical experimental conditions to that used for recording the $\alpha$-particle spectrum following the $^8$Be break-up. The differential linearity of the analyser energy scale was checked using a mercury-relay pulse generator. The variations in the linearity were less than 2%.

The following procedure was used in collecting an alpha-particle spectrum:

(a) The energy scale of the analyser was calibrated with a ThC source.

(b) The linearity of the energy scale was checked.

(c) The alpha-particle spectrum was recorded.

(d) At frequent intervals the pulser was used to check for any energy shift in the analyser scale.

(e) After the spectrum had been recorded, the energy scale was recalibrated and checked for linearity.

A raw $\alpha$-particle spectrum, which was recorded in this manner, is shown in Fig. 2.3, where the energy scale is 50.3 channels/MeV.

A general check on the system was made by rotating the target through 180° and increasing the beam energy to allow for beam energy loss in the Ni backing. An absence of counts indicated that, as required, the
FIGURE 2.3 The α-particle spectrum following the $^8$B β-decay to states in $^8$Be. There have been no corrections applied to the data. The energy scale is 50.3 channels/MeV.
α-particles detected were from the $^8$B nuclei that had decayed after stopping in the Ni backing. It also indicated that <1% of the $^8$B nuclei was stopped in the $^6$Li$_2$O target.

2.3 Treatment of Experimental Spectrum

2.3.1 Consideration of Other Reactions

Any distortion of the α-particle spectrum by "prompt" particles emitted in subsidiary nuclear reactions, e.g., $^6$Li($^3$He,α)$^5$Li was avoided by gating the pulse height analyser to receive pulses only during the "beam off" period. Only particles involved in or following a radio-active decay could be detected. Other than the α-particles of interest, the only particles with any significant possibility of detection were the β-particles from the radio-active decay of $^{18}$F and $^{14}$O. These may be formed in the reactions $^{16}$O($^3$He,p)$^{18}$F and $^{12}$C($^3$He,n)$^{14}$O. The lifetimes of $^{18}$F and $^{14}$O are 111 minutes and 71 seconds respectively (La 66). The maximum β-particle energy for $^{18}$F decay is 0.64 MeV and as $^{14}$O decays primarily (99.4%) to the 2.31 MeV state in $^{14}$N, its maximum β-particle energy is 1.84 MeV. In the raw α-particle spectrum, Fig. 2.3, there is a sharp peak at channel ~10. This corresponds to ~0.20 MeV on the α-particle energy scale. The intensity of the group was strongly dependent on the target being used. As the group had a half-life of approximately 100 minutes, it was assumed to be the $^{18}$F(β$^+$)
activity. There was no evidence for the presence of $^{14}\text{O}(\beta^+)\!$ activity. The distortion of the spectrum resulting from the $^{18}\text{F}(\beta^+)\!$ activity (and also the $^{14}\text{O}(\beta^+)\!$, if present) was avoided by analysing the spectrum only above the alpha-particle energy of 1.1 MeV.

2.3.2 Correction for $\alpha$-particle Energy Loss and Straggling

The kinematics of the reaction $^6\text{Li}(^3\text{He},n)^8\text{B}$ indicate that at 3.5 MeV, all the $^8\text{B}$ nuclei are thrown forward in a cone of half-angle 10.9°. Thus the $^8\text{B}$ nuclei come to rest, neglecting straggling in a truncated cone whose base is approximately the beam spot area. The kinematic spread in the $^8\text{B}$ energies is from 1.479 MeV to 0.672 MeV, corresponding to the $^8\text{B}$ ions being emitted at 0° or 180° respectively in the C.M. system. The corresponding $^8\text{B}$ ranges in nickel are 43.8 $\mu$in. and 27.5 $\mu$in. respectively. A discussion of the $^8\text{B}$ ion ranges in nickel is given in section 2.3.5.
As illustrated in Fig. 2.4, suppose the $^8\text{B}$ ion comes to rest and subsequently decays at a point $P$ in the nickel. $P$ is defined by the co-ordinates $(I, \theta, \phi)$, where $I$ is the range of the $^8\text{B}$ ions in the nickel, $\theta$ is the polar angle and $\phi$ is the azimuthal angle. Suppose the thickness of the nickel backing is $t$, the perpendicular distance from $P$ to the front of the nickel is $d$, and the perpendicular distance from $P$ to the emergent face of the target is $h$. The energy loss of $\alpha$-particles emitted through the nickel was initially calculated for $^8\text{B}$ ions that decay at the mean rest position of the $^8\text{B}$ ions in the nickel. To calculate this position, a knowledge of the density distribution $\eta(h)$ of the $^8\text{B}$ ions in the nickel backing as a function of $h$ was required:

$$
\eta(h) = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \int_{0}^{\theta_{\text{max}}} N(\theta) \sin \theta \, d\theta \, d\phi \, \delta(t - d - h),
$$

(2.1)

where $N(\theta)$ is the angular distribution of $^8\text{B}$ nuclei in the laboratory system. The values of $h$ corresponding to the maximum and minimum $^8\text{B}$ ranges $h_{o1}$ and $h_{o2}$ were 19.8 $\mu$m and 34.8 $\mu$m, respectively.

As the beam energy used was only 534 keV above threshold, it is probable that in the $^6\text{Li}(^3\text{He},n)^8\text{B}$ reaction predominantly $s$-wave neutrons are emitted. The ratio of the penetration factors for $s$-wave and $p$-wave neutrons at this energy is approximately 3.5. Van de Merwe et al. (Va 67)
have measured the neutron angular distribution for the reaction $^6\text{Li}(^3\text{He},n)^8\text{B}$ at beam energies of 4.8 - 5.8 MeV. These angular distributions which peak at forward angles, show smooth systematic variations with energy, becoming more isotropic in shape as the energy is decreased. The results could not be reliably extrapolated to a beam energy of 3.5 MeV, and because of this, in the absence of direct information, it was assumed in the determination of $\eta(h)$, that the angular distribution was isotropic,

i.e.,

$$N(\theta) = \text{const.} \quad (2.2)$$

Initially the density distribution $\eta(h)$ was calculated for particles in the reaction plane only. Ignoring $^8\text{B}$ range straggling, the mean distance through which the alpha-particles must pass, $\bar{h}$, was estimated to be 27.8 $\mu$m. The following analysis of the $\alpha$-particle spectrum was made using this estimation of $\eta(h)$. To check the accuracy of this estimate, the calculation was subsequently repeated by Barker (Ba 68a) for $^8\text{B}$ particles in and out of the reaction plane, and the mean distance $\bar{h}$ was estimated to be 27.3 $\mu$m. The difference between the two calculations of $h$ corresponds to a difference of 3.2 keV in the energy lost by a 1.5 MeV $\alpha$-particle as it emerges through the nickel. As $N(\theta)$ is not known precisely, there is no reason why the second estimate of $\bar{h}$ should be used in preference to the first. When $N(\theta)$ has been measured accurately at 3.5 MeV, $\bar{h}$ could be determined exactly
from equation (2.1) and the α-particle spectrum reanalysed. Unfortunately it was not feasible to do this in the present experiment.

Range straggling of the \(^8\)B ions as they come to rest distorts the calculated density distribution \(\eta(h)\). A discussion of range straggling is given in section 2.3.6, where it is estimated that for 0.672 MeV and 1.479 MeV \(^8\)B ions, the straggling is \(\sim 19.5\%\) and \(\sim 12.5\%\) respectively. Although there may be a slight asymmetry in the shape of the range straggling curve (\(\Delta y \approx 48\)), it was assumed for the analysis described here that the straggling was Gaussian in shape. The straggling was 'folded' into the density distribution of \(^8\)B ions in nickel \(\eta(h)\) to give a more valid description of the density distribution:

\[
\rho(h) = \int_{h_{o1}}^{h_{o2}} \eta(h_o) \left( \frac{-(h-h_o)^2}{2\sigma^2} \right) dh_o,
\]

(2.3)

where \(h_{o1} = 19.8\ \mu\text{in.}, \ h_{o2} = 34.8\ \mu\text{in.}\) and \(\sigma\) is the standard deviation of the Gaussian describing the straggling. As the range straggling could not be determined accurately (see section 2.2.6), \(\sigma\) was effectively allowed to vary in the final analysis. The resulting density distribution \(\rho(h)\) is shown as a function of \(h\) in Fig. 2.5, where the position of the mean \(^8\)B ion, \(\bar{h}\), was 27.6 \(\mu\text{in.}\).

Once the position of the mean \(^8\)B particle in the nickel backing had
been established, the experimental alpha-particle spectrum was corrected for the energy loss experienced by the α-particles as they are emitted from this point and pass through the nickel, and the 'dead layer' on the surface of the solid state counter. The surface density of the 'dead layer' on the counter and the nickel backing were both determined experimentally, as discussed in section 2.3.4. The stopping power tables of Williamson and Boujot (Wi 66) were used for these calculations.

The resultant α-particle spectrum, denoted B, is the 'true' alpha-particle spectrum, denoted A, smeared out, i.e., broadened by an 'effective' energy straggling in the alpha-particles. The spatial distribution of $^8\text{B}$ nuclei in the nickel foil shown in Fig. 2.5, produces a variation in the energy loss of the α-particles of a particular energy which are emitted through a nickel backing. This distribution in energy, together with the actual α-particle straggling in the nickel was regarded as an 'effective' straggling in the alpha-particle energy. It was assumed that the straggling was Gaussian. At each alpha-particle energy the standard deviation of the Gaussian was calculated by estimating the difference in energy loss experienced by an α-particle of that particular energy when it was emitted following the decay of a $^8\text{B}$ nucleus, at points in the nickel corresponding to half maximum points on the $^8\text{B}$ density distribution curve $\rho(h)$. The 'real' energy straggling of the alpha-particles in the nickel foil was estimated from the data of Comfort
The density distribution, $\rho(h)$, of $^8$B ions at rest in the nickel. The perpendicular distance to the emergent face of the nickel is denoted $h$. The mean value of $h$, $\bar{h}$, is 27.6 $\mu$m. The values $h_{01}$ and $h_{02}$ correspond to the maximum and minimum ranges of $^8$B ions in nickel respectively. Range-straggling of the $^8$B ions in the nickel has been folded into the distribution. The calculation is for particles in the reaction plane only.
et al. (Co 66). These authors experimentally determined the energy straggling of \( \alpha \)-particles in nickel for the energy range 1.0 to 8.8 MeV. The 'effective' straggling was calculated by summing these two types of straggling.

The alpha-particle spectrum 'B' was corrected for the 'effective' straggling by using a perturbation procedure. The procedure is based on an assumption that the relative change in the alpha-particle yield at a particular energy ordinate resulting from the introduction of straggling, is proportional to the magnitude of the straggling for small changes in the latter. Suppose the true spectrum A is broadened by straggling, to give spectrum B. If spectrum B is then further broadened by the same straggling to give spectrum C, then the assumption is that at each energy ordinate \( E \)

\[
\frac{N_A(E)}{N_B(E)} = \frac{N_B(E)}{N_C(E)},
\]

i.e.,

\[
N_A(E) = \frac{(N_B(E))^2}{N_C(E)},
\]

where \( N(E) \) is the alpha-particle yield at energy ordinate \( E \).

Initially spectrum C was obtained by smearing the spectrum B. This was done by replacing the yield curve at each energy ordinate of B by a Gaussian energy distribution, whose standard deviation, \( \sigma \), was estimated
from the 'effective' straggling at that energy and then integrating over the energy range to obtain the yield of spectrum C at a particular ordinate.

\[ E_0 = 14.2 \text{ MeV} \]

i.e.,

\[ N_C(E) = \int_{E_0=0.5 \text{ MeV}}^{E_0} N_B(E_0) \left( \frac{(E - E_0)^2}{2\sigma^2} \right) dE_0. \]  

(2.7)

As \( N_B(E) \) was known from the experimental spectrum and \( N_C(E) \) was calculated using equation (2.7), the 'true' alpha-particle spectrum following the \( \beta \)-decay of \(^8\text{B}\) to \(^8\text{Be}\) was calculated using equation (2.6). The perturbation implied on the spectra by introducing the straggling was only of order 4% at an energy ordinate of \( \sim 3.0 \text{ MeV} \).

The validity of the assumption was tested by 'adding' straggling to the spectrum A and comparing the result to the experimental spectrum B. The difference between the two spectra at an energy ordinate of 3.0 MeV was less than 0.3% of the ordinate value justifying the use of the perturbation procedure. As previously discussed, in the final analysis, the 'effective' straggling was allowed to vary as a free parameter to obtain optimum fits to the data.

The final spectrum is shown in Fig. 2.6. The energy scale shown is the alpha-particle channel energy, i.e., the excitation energy in \(^8\text{Be}\) defined relative to two free\(\alpha\)-particles, and not to the ground state of \(^8\text{Be}\).
FIGURE 2.6  The final corrected alpha-particle spectrum. The energy scale shown is the alpha-particle channel energy, i.e., the excitation energy in $^{8}$Be defined relative to two free $\alpha$-particles. The $^{8}$Li(β$^{-}$)$^{8}$Be data of Alburger et al., (Al 63), has also been plotted for comparison to the present data.

The solid line represents a multi-level fit to the data using a channel radius of 6.0 fm. This is the optimum fit to the data.
ALPHA PARTICLE SPECTRUM FROM

\(^{6}\text{Li}(^{3}\text{He},n)^{9}\text{Be}(\beta^{+})^{7}\text{Be}(\alpha)^{4}\text{He}\)

'2.9' MeV Level Parameters

\(E_r = 2.95\) MeV
\(\gamma = 0.54\) MeV

* Present Experiment
- Li-6 \(\beta\)-Decay Data (Al63)
--- Theoretical Fit to *

NUMBER OF COUNTS

- 30,000
- 20,000
- 10,000
- 0

CHANNEL ENERGY (MeV)

- 14.0
- 12.0
- 10.0
- 8.0
- 6.0
- 4.0
- 2.0
2.3.3 Consideration of Neutrino-Electron Recoil

It has been assumed in section 2.3.1 that the alpha-particle channel energy is twice the alpha-particle energy. Because of the electron-neutrino recoil, this is not strictly true, as the leptons take away momentum. A distribution in α-particle energies follows the β-transitions to a fixed excitation in $^{8}\text{Be}$. Thus the α-particle spectrum does not directly image the β-transition probability, a knowledge of which was required.

For a $\beta^+$ transition to a level in $^{8}\text{Be}$ of excitation 2.9 MeV, the effect will cause a variation of ± 13 keV in the energy of the alpha-particles decaying from this level. Although this is a reasonably large effect, it was effectively accounted for in this work by allowing the straggling to vary as a free parameter in the final analysis (section 2.3.2).

2.3.4 Measurement of Thickness of Counter Dead Layer

An independent measurement was made to determine the thickness of the dead layer (window) on the surface of the solid state counter. The dead layer is defined as the thickness of any insensitive layer through which the incident particle must pass in order to reach the sensitive volume. The counter was exposed to an alpha-particle beam from a ThC source. The beam was defined by two 0.8 mm collimating slits placed 3.5 cm apart. Initially a pulse height spectrum was recorded with the normal to the counter surface along the beam direction. The counter was tilted through 60° and the
effect on the pulse height spectrum noted. The shifts in the 8.776 MeV and
6.043 MeV α-particle groups was 6.9 keV and 8.5 keV respectively. These
energy losses correspond to a gold layer of thickness $41 \pm 2 \mu g/cm^2$
(calculated using Williamson and Boujot's (Wi 66) stopping power tables).
This result is consistent with the value quoted by the manufacturer* for the
nominal thickness of gold on the surface of the counter.

2.3.5 Measurement of Thickness of Nickel Backing

Using the same geometry arrangement as described in section 2.3.4, the
thickness of the nickel target backing was determined. Initially a spectrum
was recorded without the nickel foil. The foil was then placed in the beam,
its normal lying along the beam direction, and the effect on the pulse height
spectrum noted. The shifts in the 8.776 MeV and 6.082 MeV α-particle
groups were 348 keV and 444 keV corresponding to a nickel backing thickness
of $1095 \pm 39 \mu g/cm$. This value is consistent with the nominal thickness of
$1130 \mu g/cm^2$ quoted by the manufacturer**.

2.3.6 Calculation of the Range and Range-Straggling of $^8$B Ions in Nickel

In this section, estimates of the range-energy relation and the range
straggling-energy relation are made for $^8$B ions in nickel.

There are effectively no experimental data available for the range

---

* Ortec, Oak Ridge, Tennessee, U.S.A.
** Chromium Corporation of America, Waterbury, Connecticut, U.S.A.
or stopping power of B ions in nickel. Warburton et al. (Wa 66, Wa 63) obtained stopping power curves for B ions in Ni by interpolation of existing stopping-power and charge state data for other ions in nickel. They used the stopping-power and charge state data cited in the review article of Northcliffe (No 63) as well as the stopping-power versus velocity data of Porat and Ramavataram (Po 61) in these interpolation procedures. The results obtained are shown in Fig. 2.7. The stopping-power is plotted versus ion velocity in units of c/137 (v_o = c/137), so that the curves are valid for all isotopes of B. The error bars attached to these points show estimates of the combined uncertainties of the experimental data and the interpolation procedure. The dashed curve in Fig. 2.7 is a fit to the data using the function form for the stopping-power dE/dx:

$$\frac{dE}{dx} = \frac{K_n}{v/v_0} + K_e (v/v_o) - K_3 (v/v_o)^3.$$  \hspace{1cm} (2.8)

The curve corresponds to $K_n = 0.05$, $K_e = 0.96$ and $K_3 = 0.016$. All constants are in units of (keV cm$^2$/μg. The leading term in equation (2.11) is the nuclear stopping power term which is due to ion-atom collisions and gives rise to the upturn of dE/dx at low values of v/v_o.

The range, R, of the $^8$B ions is given by:

$$R = \int_0^E (-\frac{dE}{dx})^{-1} \, dE ,$$ \hspace{1cm} (2.9)
FIGURE 2.7 Stopping power curves for B ions in Ni, showing the differential energy loss as a function of ion velocity (in units of $v_o = c/137$). The data were obtained by Warburton et al., (Wa 66) using an interpolation procedure. The solid curve is a fit of

$$\frac{dE}{dx} = K_2/(v/v_o) + K_e(v/v_o) - K_3(v/v_o)^3$$

to the region $1.0 \leq v/v_o \leq 6.0$ ($K_e = 0.96$, $K_n = 0.05$, $K_3 = 0.016$). All constants are in units of $(\text{keV cm}^2)/\mu\text{g}$. 
where $E_0$ is the incident energy. From equations (2.8) and (2.9) Warburton obtained an analytic approximation for the range $R$. To obtain a more accurate estimate of $R$, the data of Fig. 2.7 was replotted as $(dE/dx)^{-1}$ versus $E$ and the curve was numerically integrated. The estimated distribution of range of the $^8$B ions in the Ni backing was 43.8 μin. to 27.5 μin., corresponding to the kinematic spread in $^8$B energies from 0.672 MeV to 1.479 MeV respectively. Using the errors assigned by Warburton et al. to their data, these ranges were estimated to be accurate to approximately ± 5%. This result agrees with an estimation made by Northcliffe (No 63) who, by using a similar procedure, obtained a value of 40 μin. for the range of a 1.479 MeV $^8$B ion in nickel. However, the curves containing the results of Northcliffe can only be read to an accuracy of ± 20%.

There are practically no experimental data on the range straggling of low energy ions in matter. In particular there are no data directly related to $^8$B ions in nickel. Powers and Whaling (Po 62) have measured the range straggling of N, Ne, Ar, Kr and Xe ions in Be, B, C and Al for incident energies of 50 to 500 keV. In the present case an estimate of the straggling is required at slightly higher energies. Comfort et al. (Co 66) have measured the energy straggling of alpha-particles in Ni, Al, Ag and Au at energies from 1 to 8.78 MeV and compared the results to theoretical predictions of Bohr, Bethe and Livingston and Titeica. Although the theoretical fits were
satisfactory at high energies, such large discrepancies were found at low energies, that it was thought inadvisable to use a theoretical estimate of the straggling in the present case.

A precise knowledge of the range-straggling of the $^8$B ions was not required as the 'effective' straggling $S$ was allowed to vary in the final analysis. An estimate of the range-straggling was made by interpolating between the Powers and Whaling data and results from a theoretical calculation made for high energy particles by Steinheimer (St 60) using a relativistic form of the Bethe and Livingston formulae. Define the percentage range straggling $\epsilon$ as

$$\epsilon = 100 \left( \frac{\sigma}{R} \right),$$

where $R$ is the range and $\sigma = \left[ \langle (\Delta R)^2 \rangle \right]^{\frac{1}{2}}$ is the root mean square deviation of the range. Except at very high energies for particles of a given velocity $v$ and in a given material, $\epsilon$ is proportional to $M^{\frac{1}{2}}$, where $M$ is the mass of the ion, i.e., for two particles, i, j (St 60),

$$\left( M_i \right)^{\frac{1}{2}} \epsilon_i(v) = \left( M_j \right)^{\frac{1}{2}} \epsilon_j(v).$$

In Fig. 2.8 the data of Powers and Whaling and Steinheimer are represented by plotting $\left( M_i \right)^{\frac{1}{2}} \epsilon_i(v)/n$ versus $v$. The number of electrons/
The range–straggling data of Powers and Whaling (Po 62) and Sternheimer (St 60) are represented by plotting
\[
\left( \frac{\sigma_i}{R_i} \right) \sqrt{\frac{M_i}{n}} . 6.0 \text{ versus ion velocity.}
\]
The range, range-straggling and mass (in a.m.u.) of the ion, \( i \), are represented by \( R_i, \sigma_i \) and \( M_i \) respectively. The number of electrons/atom in the stopping material is denoted by \( n \).
The inclusion of the \( n^{-1} \) term allows for straggling variations with stopping material. The errors on the Powers and Whaling data are those quoted by those authors. The smooth curve represents a visual fit to the data. These interpolated results indicate that the percentage straggling of the 0.672 MeV and 1.479 MeV \(^8\)B ions in Ni are 19.5% and 12.5% respectively.
atom in the stopping material is denoted by \( n \). The inclusion of the \( n^{-1} \) term allows for straggling variations with stopping material. These interpolated results indicate that the percentage straggling of the 0.672 MeV and 1.479 MeV \( ^8 \)B ions in Ni is 19.5% and 12.5% respectively. The nature of the calculations made it difficult to assign an error to these estimates.

2.4 Analysis

2.4.1 R-Matrix Parameters from \( \alpha-\alpha \) Scattering

Barker (Ba 68a) has analysed the d-wave \( \alpha-\alpha \) scattering phase shift for channel energies \( E \leq 17 \) MeV using three-level one-channel R-matrix formulae. For the present case, definitions of the phase shift in terms of the R-matrix and of the R-matrix in terms of the level parameters \( \gamma_\lambda^2 \) and \( E_\lambda \) are given in equation (I.15) and (I.17) of Appendix I. \( E_\lambda \) and \( \gamma_\lambda^2 \) represent the energy eigenvalue and reduced width of the level \( \lambda \). The rapid changes in \( \delta_2^{\text{exp}} \) due to the narrow \( 2^+ \) levels at 16.6 and 16.9 MeV excitation in \(^8\)Be were neglected in the analysis. The parameter values which gave best fits to \( \delta_2^{\text{exp}} \) for various channel radii \( a_2 \) are given in Table 2.2. These values were obtained by taking the boundary condition parameter \( B_2 \) (discussed in Appendix I) equal to zero and minimizing \( X_2^2 \) by varying the level parameters \( E_\lambda \) and \( \gamma_\lambda^2 \) \( (\lambda = 1, 2, 3) \), except for the case \( a_2 = 5.5 \) fm when \( E_3 \) was kept fixed. The quantity \( X_2^2 \) is defined as:
TABLE 2.2

Parameter values for best fits to $\delta_{2}^{\text{exp}}$ in the three-level approximation

for various channel radii and $B_{2} = 0$

<table>
<thead>
<tr>
<th>$a_{2}$</th>
<th>$E_{1}$ (MeV)</th>
<th>$\gamma_{1}^{2}$ (MeV)</th>
<th>$E_{2}$ (MeV)</th>
<th>$\gamma_{2}^{2}$ (MeV)</th>
<th>$E_{3}$ (MeV)</th>
<th>$\gamma_{3}^{2}$ (MeV)</th>
<th>$X_{2}^{2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>2.630</td>
<td>0.592</td>
<td>13.46</td>
<td>1.212</td>
<td>47.0</td>
<td>3.19</td>
<td>0.45</td>
</tr>
<tr>
<td>6.0</td>
<td>2.675</td>
<td>0.467</td>
<td>10.75</td>
<td>0.941</td>
<td>196.8</td>
<td>18.42</td>
<td>0.49</td>
</tr>
<tr>
<td>6.5</td>
<td>2.650</td>
<td>0.390</td>
<td>8.87</td>
<td>0.809</td>
<td>45.2</td>
<td>3.83</td>
<td>0.48</td>
</tr>
<tr>
<td>7.0</td>
<td>2.599</td>
<td>0.339</td>
<td>7.43</td>
<td>0.686</td>
<td>27.5</td>
<td>1.98</td>
<td>0.44</td>
</tr>
<tr>
<td>7.5</td>
<td>2.526</td>
<td>0.306</td>
<td>6.34</td>
<td>0.560</td>
<td>20.2</td>
<td>1.20</td>
<td>0.39</td>
</tr>
<tr>
<td>8.0</td>
<td>2.436</td>
<td>0.286</td>
<td>5.56</td>
<td>0.455</td>
<td>16.3</td>
<td>0.83</td>
<td>0.53</td>
</tr>
</tbody>
</table>
where $\delta_2^{\text{calc}}(E_i)$ is the phase shift calculated using the R-matrix formulae at energy $E_i$ ($i = 1 \ldots m$) and $\epsilon_2(E_i)$ is the error in the measured phase shift, $\delta_2^{\text{exp}}(E_i)$. Identical fits can be obtained for any other value of $B_2$ (Ba 68). From Table 2.2 it is seen that acceptable fits to $\delta_2^{\text{exp}}$ can be obtained for a wide range of channel radii.

For each channel radius, variations of the parameter values about those given in Table 2.2 can still lead to acceptable fits, defined as fits with $X_2^2 \lesssim 1$, to $\delta_2^{\text{exp}}$. Parameter values for fits with $X_2^2 \lesssim 1$ are given in Table 2.3 for the $a_2 = 6.0$ fm case. Barker obtained this result by taking a set of fixed $E_i$ values and varying the remaining level parameters.

2.4.2 R-Matrix Parameters from $^8\text{B(}\beta^+\text{)}^8\text{Be}$

Restrictions on the acceptable values of the channel radius and level parameters of $^8\text{Be}$ may be obtained by requiring them to give simultaneous fits to the $\alpha-\alpha$ scattering data and to other data obtained from a reaction which proceeds through an intermediate stage involving $2^+$ states of $^8\text{Be}$. With this in mind, an analysis of the $\alpha$-particle spectrum following the $\beta^+$ decay of $^8\text{B}$ to $2^+$ states in $^8\text{Be}$ was made. The measurement of the spectrum and corrections applied to it are given in section 2.2 and section 2.3.
### TABLE 2.3

Parameter values for fits to $\delta_2^{exp}$ in the three-level approximation for $a_2 = 6.0$ fm, $B_2 = 0$ and various fixed values of $E_1$.

As defined in the text, the $X^2$ value signifies the quality of the fit. The quality of the fit to the $^8B$ $\beta$-decay data, $Y_1^2$,

with this set of parameters, is also given for various energy shifts to the data and various values of $S$,

the 'effective' straggling of the $\alpha$-particles. $S$ is defined in the text. The optimum results were obtained with $S = 2.1$.

<table>
<thead>
<tr>
<th>$a$ (fm)</th>
<th>$E_1$ (MeV)</th>
<th>$\gamma_1$ (MeV)</th>
<th>$E_2$ (MeV)</th>
<th>$\gamma_2$ (MeV)</th>
<th>$E_3$ (MeV)</th>
<th>$\gamma_3$ (MeV)</th>
<th>$X^2$</th>
<th>$S = 1$</th>
<th>$E_{shift} = 0$</th>
<th>$S = 2.1$</th>
<th>$S = 3$</th>
<th>$E_{shift} = 0.0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.0</td>
<td>2.50</td>
<td>0.570</td>
<td>10.706</td>
<td>1.039</td>
<td>51.47</td>
<td>4.42</td>
<td>2.28</td>
<td>11.55</td>
<td>27.71</td>
<td>13.25</td>
<td>8.34</td>
<td>2.12</td>
</tr>
<tr>
<td></td>
<td>2.25</td>
<td>0.554</td>
<td>10.719</td>
<td>1.022</td>
<td>59.38</td>
<td>5.21</td>
<td>1.78</td>
<td>6.31</td>
<td>15.40</td>
<td>4.23</td>
<td>1.84</td>
<td>3.97</td>
</tr>
<tr>
<td></td>
<td>2.55</td>
<td>0.538</td>
<td>10.729</td>
<td>1.004</td>
<td>67.30</td>
<td>6.00</td>
<td>1.38</td>
<td>6.98</td>
<td>5.89</td>
<td>1.28</td>
<td>3.48</td>
<td>12.64</td>
</tr>
<tr>
<td></td>
<td>2.575</td>
<td>0.524</td>
<td>10.741</td>
<td>0.976</td>
<td>106.46</td>
<td>10.15</td>
<td>1.04</td>
<td>14.65</td>
<td>5.17</td>
<td>5.95</td>
<td>13.72</td>
<td>30.47</td>
</tr>
<tr>
<td></td>
<td>2.60</td>
<td>0.507</td>
<td>10.741</td>
<td>0.981</td>
<td>80.97</td>
<td>7.22</td>
<td>0.79</td>
<td>26.31</td>
<td>14.51</td>
<td>15.11</td>
<td>37.94</td>
<td>50.00</td>
</tr>
<tr>
<td></td>
<td>2.625</td>
<td>0.492</td>
<td>10.747</td>
<td>0.962</td>
<td>107.16</td>
<td>9.80</td>
<td>0.51</td>
<td>47.27</td>
<td>23.36</td>
<td>38.63</td>
<td>63.76</td>
<td>81.74</td>
</tr>
<tr>
<td></td>
<td>2.65</td>
<td>0.480</td>
<td>10.754</td>
<td>0.948</td>
<td>163.99</td>
<td>15.37</td>
<td>0.49</td>
<td>74.47</td>
<td>45.14</td>
<td>59.39</td>
<td>103.97</td>
<td>120.64</td>
</tr>
<tr>
<td></td>
<td>2.675</td>
<td>0.467</td>
<td>10.759</td>
<td>0.941</td>
<td>196.82</td>
<td>18.43</td>
<td>0.56</td>
<td>108.41</td>
<td>72.14</td>
<td>92.36</td>
<td>159.32</td>
<td>166.64</td>
</tr>
<tr>
<td></td>
<td>2.70</td>
<td>0.453</td>
<td>10.764</td>
<td>0.935</td>
<td>218.79</td>
<td>20.34</td>
<td>0.72</td>
<td>149.76</td>
<td>108.76</td>
<td>133.25</td>
<td>206.28</td>
<td>220.58</td>
</tr>
</tbody>
</table>

---
respectively. This spectrum for $E \leq 14.2$ MeV is shown in Fig. 2.4, where the energy scale $E = 2E_\alpha$, $E_\alpha$ being the $\alpha$-particle energy. If the electron-neutrino recoil, discussed in section 2.2.6, is neglected, then $E$ represents channel energy. The spectrum of high energy $\alpha$-particles following $^8_B \beta$-decay to the region in $^8$Be of 14.9 - 16.9 MeV excitation (from data of Matt et al. (Ma 64)), was also used in the analysis. Errors were assigned to the present data and to the data of Matt et al. on the basis of statistical errors.

This analysis of the $\alpha$-particle spectrum is based on R-matrix theory using Barker's form of the cross section for a three-stage process (Ba 67). The computer program used in the analysis was written by Barker for his similar treatment of the $^8$Li($\beta^-$) data (Ba 68a). An outline of the analysis is now given. The form of the $\alpha$-spectrum calculated includes interfering contributions from the levels $\lambda = 1, 5$ with the Gamow-Teller and the Fermi contributions (denoted by G and F) adding incoherently. Levels denoted $\lambda = 4, 5$ are the 16.6 and 16.9 MeV levels respectively. As a function of channel energy $E$, the $\alpha$-spectrum may be written:

$$N(E) = C_f \beta^2 P_2 \left[ \frac{5}{\Sigma} \frac{g_{\lambda G} \gamma_\lambda}{E_\lambda - E} \right]^2 + \left[ \frac{5}{\Sigma} \frac{g_{\lambda F} \gamma_\lambda}{E_\lambda - E} \right]^2$$

$$1 - (S_2 - B_2 + iP_2) \left[ \frac{5}{\Sigma} \frac{\gamma_\lambda}{E_\lambda - E} \right]^2,$$

(2.13)

where
$E_\lambda$ and $\gamma_\lambda^2$ represent the energy eigenvalue and reduced width of the level $\lambda$.

$\beta_\lambda$ is the integrated Fermi function as defined by Bachall (Ba 66a),

g_\lambda is the amplitude of the feeding factor to the level $\lambda$,

$P_2$, $S_2$ and $B_2$ are the $\alpha-\alpha$ penetration factor, shift factor and boundary condition respectively, (these are defined in Appendix I)

and $C$ is a normalization parameter.

It was assumed that $B_2 = 0$ and since level 3 is not expected to be strongly populated, it was assumed $g_{3G} = 0$. It was also assumed that levels 1, 2 are pure $T = 0$, so that $g_{1F}^\lambda = 0$ ($\lambda = 1, 2$).

The levels 4 and 5 may be described as mixtures of basic $T = 0$ and $T = 1$ states labelled 0 and 1' respectively, so that the wave functions of these states may be written as (Br 66, Ba 66):

$$\Psi_4 = \alpha\Psi_0 + \beta\Psi_1$$

$$\Psi_5 = \beta\Psi_0 - \alpha\Psi_1$$

with $\alpha^2 + \beta^2 = 1$. Then one may write (Ba 68a):

$$g_{4G} = \alpha g_{0G} + \beta g_{1'G} , \ g_{4F} = \beta g_{1'F} , \ \gamma_4 = \alpha\gamma_0$$

$$g_{5G} = \beta g_{0G} - \alpha g_{1'G} , \ g_{5F} = -\alpha g_{1'F} , \ \gamma_5 = \beta\gamma_0$$

(2.14)
For $^8B(\beta^+)^8Be$ decay with energies $E \leq 14.2$ MeV,

$|E_4 - E| \gg |E_4 - E_5|$, so one may write $E_4 - E = E_5 - E = E_0 - E$,
i.e., levels $\lambda = 4, 5$ are considered as a single level $\lambda = 0$. Equation (2.13) may then be written, using equation (2.14):

$$N(E) = f_{\beta} P^2 \frac{\left[ \sum_{\lambda = 0} \frac{A_{\lambda}}{E_{\lambda} - E} \right]^2}{1 - (S_2 + iP_2) \sum_{\lambda = 0} \frac{\gamma_{\lambda}^2}{E_{\lambda} - E}}$$

where the amplitudes $A_{\lambda}$ are defined as:

$$A_{\lambda} = C_{\lambda} G_{\lambda} \gamma_{\lambda} \quad (\lambda = 0, 1, 2) .$$

Normally $\beta$-decay transition probabilities are specified by $ft$ values. However, for transitions to interfering levels, which is the case here, one cannot strictly speak of $ft$ values. However, the following definition of an $ft$ value was used. The branching ratio to the level $\lambda$ was defined as the ratio of the fictitious cross section when only the level $\lambda$ is being populated to the cross section when all levels $\lambda$ are being populated, i.e.,

$$b_{\lambda} = \frac{\int_{0}^{\infty} N(E)^{\lambda} dE}{\int_{0}^{\infty} N(E) dE} .$$
where

\[ N(E) = f_\beta P_2 \left[ \left( \frac{A_\lambda}{E_\lambda - E} \right)^2 \right] \left( \frac{1}{1 - (S_2 + iP_2)^3} \sum_{\lambda = 0}^{\gamma_\lambda} \frac{\gamma_\lambda}{E_\lambda - E} \right)^2 \]  

(2.17)

Note that \( \sum_{\lambda = 0}^{2} b_\lambda \neq 1 \), so that the \( b_\lambda \) are "un-normalized" branching ratios.

If \( t_\frac{1}{2} \) is the half-life for the \( \beta \)-decay (\( t_\frac{1}{2}^{\exp} = 0.774 \) sec) then the partial half-life for the level \( \lambda \) is defined as:

\[ t_\frac{1}{2}^\lambda = \frac{1}{b_\lambda} t_\frac{1}{2} \]  

(2.18)

The \( f_\lambda \) value for the level \( \lambda \) is defined as:

\[ f_\lambda = \frac{\int_0^\infty N(E) \lambda^\lambda dE}{\int_0^\infty \frac{N(E)}{f_\beta} dE} \]  

(2.19)

Then the \( f^\lambda \) value for the level \( \lambda \) may then be derived as:

\[ (f^\lambda) = f_\lambda t_\frac{1}{2} = \frac{t_\frac{1}{2}}{\frac{1}{f_\lambda} \int_0^\infty N(E) dE} \]  

(2.20)

\[ \int_0^\infty dE \left( P_2 - \frac{(A_\lambda / E_\lambda - E)^2}{1 - (S_2 + iP_2)^3} \sum_{\lambda = 0}^{\gamma_\lambda} \frac{\gamma_\lambda}{E_\lambda - E} \right)^2 \]
A shell model calculation enables \((ft)_4\) and \((ft)_5\) to be calculated from \((ft)_0\).

Using a linear least squares fitting routine, the parameters \(A_{\lambda} (\lambda = 0, 1, 2)\) were varied to obtain best fits to the \(^8\)B \(\beta\)-decay data and subsequently to obtain \((ft)_\lambda\) values for \(\lambda = 0, 1, 2\). Initially \(A_0\) was chosen to fit the high energy \(^8\)B \(\beta\)-decay data of Matt et al. (Ma 64). In this case the contribution to the cross section from the levels \(\lambda = 1, 2\) was negligible. The optimum value of the amplitude \(A_0\) was estimated by minimizing the quantity \(Y_2^2\) defined as:

\[
Y_2^2 = \frac{1}{m} \sum_{i=1}^{m} \left( \frac{N^{\text{exp}}(E_i) - N^{\text{calc}}(E_i)}{\eta(E_i)} \right)^2,
\]

where \(m\) is the number of data points and \(N^{\text{calc}}(E_i)\), \(N^{\text{exp}}(E_i)\) and \(\eta(E_i)\) are the calculated yield and the measured yield with its associated error at the channel energy \(E_i\). The 16.9 and 16.6 MeV level parameters used in fitting the data of Matt et al. were those estimated by Browne and Callender (Br 66) from their \(^{10}\)B(\(d,\alpha\))\(^{8}\)Be data.

Results of fitting the data of Matt and Pfander are listed in Table 2.4, where for various channel radii \(a_2\) the minimum values \(Y_2^2\) are given, together with the corresponding values of \(\log (ft)_0\) and the derived values of \(\log (ft)_4\) and \(\log (ft)_5\).
TABLE 2.4

Best fits to the $^8\text{Be}(\beta^+)^8\text{Be}$ data of Matt et al. (Ma 64) for various channel radii together with log ft values for transitions to the 16.6 and 16.9 MeV levels

<table>
<thead>
<tr>
<th>$a_2$ (fm)</th>
<th>$Y_2^2$</th>
<th>log (ft)$_o$</th>
<th>log (ft)$_4$</th>
<th>log (ft)$_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>2.67</td>
<td>3.32</td>
<td>3.40</td>
<td>3.40</td>
</tr>
<tr>
<td>6.0</td>
<td>2.59</td>
<td>3.33</td>
<td>3.41</td>
<td>3.40</td>
</tr>
<tr>
<td>6.5</td>
<td>2.78</td>
<td>3.31</td>
<td>3.40</td>
<td>3.39</td>
</tr>
<tr>
<td>7.0</td>
<td>2.94</td>
<td>3.34</td>
<td>3.41</td>
<td>3.41</td>
</tr>
</tbody>
</table>

The log (ft)$_o$ values given in Table 2.4 are very close to a calculated shell model value of 3.33 (Ba 66).

The low energy $^8\text{B}$ $\beta$-decay data measured in this experiment were fitted by minimizing a quantity $Y_1^2$ defined in an identical manner to $Y_2^2$ (equation (2.21)) with $N(E)$, given by equation (2.15). The analysis was performed for channel radii of 6.0, 6.5 and 7.5 fm using, for each radius, a range of $E_\lambda$ and $\gamma^2_\lambda$ ($\lambda = 1, 2$) values determined from the three-level analysis of the $\alpha-\alpha$ scattering data. These data are given in Table 2.3 for the case $a_2 = 6.0$ fm. In the fitting procedure $A_o$ was kept fixed at the value corresponding to the log (ft)$_o$ value determined from the
analysis of the high energy data (Table 2.4). \( A_1 \) and \( A_2 \) were then varied to minimize \( Y_1^2 \).

As discussed in section 2.3 an additional parameter \( S \) was introduced into the analysis to account for uncertainties in the calculation of the range-straggling of the \( ^8B \) ions in nickel and the effect on the spectrum of the electron-neutrino recoil when the \( ^8B \beta \) decays. It was assumed that \( S = 1 \) for the calculated 'effective' straggling. The above analysis, which was initially performed with \( S = 1 \) was repeated for several values of \( S \) between 1 and 3. The optimum fits to the data were found with \( S = 2.1 \), i.e., the estimates of the 'effective straggling' made in section 2.3 were too small. The quality of the fits \( Y_1^2 \) for the \( 6.0 \) fm case and \( S = 1, 2.1 \) and 3 are shown in Table 2.3.

Barker (Ba 68a) found that the \( Y^2 \) values for his fits to the \( ^8Li \beta \)-decay data of Alburger et al. (Al 63) could be appreciably reduced (a factor of 3) by applying an energy shift of \( \sim 100 \) keV to the experimental data. To see if the same effect applied in the present case, the analysis described above was repeated with \( S = 2.1 \) and with energy shifts of \(+0.05, -0.025, -0.05 \) and \(-0.1 \) MeV. The results for the \( a_2 = 6.0 \) fm case are given in Table 2.3.

In order to obtain an idea of the best overall fit to the \( \delta_{2}^{\text{exp}} \) and \( ^8B \beta \)-decay data, the quantity \( Z \) was introduced where,
\[ Z = X + 0.5Y_1 + 0.2Y_2. \]

\( Z \) was defined in this manner to give approximately equal weights to the three sets of data. For the \( S = 2.1 \) case the smallest values of \( Z \) are given in Table 2.5, for various channel radii and energy shifts. It is seen from Table 2.5 that no energy shifts are required to obtain optimum fits to the data. If an acceptable fit to the data is defined rather arbitrarily as a fit with \( Z \leq 3.0 \), then from Table 2.5 and Table 2.6 it is seen that acceptable fits were obtained with a range of channel radii from 5.5 to 6.5 fm.

**TABLE 2.5**

Smallest values of \( Z = X + Y_1 + Y_2 \) for simultaneous fits to the \( \delta_2^{\text{exp}} \) data, low energy \( \beta \)-decay data and high energy \( \beta \)-decay data for various channel radii and energy shifts to the \( \beta \) data

<table>
<thead>
<tr>
<th>( a_2 ) (MeV)</th>
<th>( E_{\text{shift}} ) (MeV)</th>
<th>+0.05</th>
<th>0.0</th>
<th>-0.25</th>
<th>-0.05</th>
<th>-0.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td></td>
<td>4.16</td>
<td>2.71</td>
<td>3.11</td>
<td>3.38</td>
<td>4.78</td>
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<tr>
<td>6.0</td>
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<td>3.73</td>
<td>2.54</td>
<td>3.02</td>
<td>3.28</td>
<td>5.98</td>
</tr>
<tr>
<td>6.5</td>
<td></td>
<td>4.07</td>
<td>2.84</td>
<td>3.24</td>
<td>3.59</td>
<td>6.29</td>
</tr>
<tr>
<td>7.0</td>
<td></td>
<td>3.98</td>
<td>3.10</td>
<td>3.17</td>
<td>3.23</td>
<td>5.23</td>
</tr>
</tbody>
</table>
Level parameters and fit values corresponding to the best fits for channel radii 5.5, 6.0, 6.5 and 7.0 fm are given in Table 2.6. The overall best fit to the data, obtained with a channel radius of 6.0 fm, is shown as a solid line in Figure 2.4. The $E_1 = 2.55$ MeV value for this fit would correspond to a resonance energy of 2.95 MeV if only this level was being fed.

**TABLE 2.6**

Level parameters for best fits to $\delta_2^{\text{exp}}$ and the $\beta$-decay data together with the corresponding log ft values

<table>
<thead>
<tr>
<th>$a_2$ (fm)</th>
<th>$E_1$ (MeV)</th>
<th>$\gamma_1$ (MeV)</th>
<th>$E_2$ (MeV)</th>
<th>$\gamma_2$ (MeV)</th>
<th>$Y_1$</th>
<th>log (ft)$_1$</th>
<th>log (ft)$_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>2.55</td>
<td>0.656</td>
<td>13.52</td>
<td>1.22</td>
<td>1.73</td>
<td>5.78</td>
<td>5.08</td>
</tr>
<tr>
<td>6.0</td>
<td>2.55</td>
<td>0.538</td>
<td>10.72</td>
<td>1.00</td>
<td>1.28</td>
<td>5.80</td>
<td>4.91</td>
</tr>
<tr>
<td>6.5</td>
<td>2.55</td>
<td>0.429</td>
<td>8.77</td>
<td>0.83</td>
<td>1.96</td>
<td>5.85</td>
<td>5.26</td>
</tr>
<tr>
<td>7.0</td>
<td>2.525</td>
<td>0.36</td>
<td>7.35</td>
<td>0.67</td>
<td>3.10</td>
<td>5.90</td>
<td>5.35</td>
</tr>
</tbody>
</table>

The log ft values for the levels 1 and 2 are 5.80 and 4.91 respectively. For the 2.9 MeV level this value is larger than the value of 5.64 obtained by Bahcall (Ba 66a) on the assumption that all the $^8$B decays go
to a sharp level at 2.9 MeV and the value of 5.45 obtained from a shell model
calculation by Barker (Ba 66).

2.4.3 Comparison of the $^8_\text{B} (\beta^+) ^8_\text{Be}$ Data with Other Data

The $^8_\text{Li} \beta$-decay data of Alburger et al. (Al 63) peaks at about 50 keV
lower than the data measured in the present experiment; of this 7 keV may be
attributed to the different Fermi functions for $^8_\text{B}$ and $^8_\text{Li}$ decay. The Alburger
data which indicates that the width of the '2.9' MeV peak is somewhat narrower
than that indicated by the present data, is plotted for comparison to the $^8_\text{B}$
$\beta$-decay data in Figure 2.4. In an earlier experiment Farmer and Class (Fa 60)
found the $^8_\text{B}$ $\beta$-decay peak $\sim$ 30 keV higher than that measured here.

Barker (Ba 68a) has analysed the Alburger data in a manner similar
to that described in this chapter for the $^8_\text{B}$ $\beta$-decay data. The level parameters
which gave the best fits were incompatible with those that gave acceptable fits
to the $\alpha-\alpha$ scattering data, unless an energy shift of $+100$ keV was applied to
the data. With a 100 keV energy shift, the best fits were obtained with a
channel radius of 6.5 fm and an $E_1$ value of 2.62 MeV. These parameters
are very close to those that gave the best fit to the $\delta_2^{\exp}$ data for $a_2 = 6.5$ fm.
Barker (Ba 68a) has also performed a multi-level R-matrix fit to the $^9_\text{Be}(p,d)$
data of Hay et al. (Ha 67). The best overall fit to these data and to the $\alpha-\alpha$
scattering data is obtained for $a_2 = 7.0$ fm and $E_1 = 2.58$ MeV. Acceptable
fits can be obtained for the $^8_\text{Li} \beta$-decay and the $^9_\text{Be}(p,d) ^8_\text{Be}$ data with a
channel radius of 6.0 fm. Although the $E_1$ values are somewhat higher, $(E_1 = 2.70$ for $^8$Li $\beta$-decay and $E_1 = 2.63$ for $^9$Be(p,d)) the parameters still give good fits to the $\alpha-\alpha$ scattering data.

2.5 Discussion

Analyses of both scattering and reaction data were necessary to limit the acceptable values of channel radii and level parameters which describe the $2^+$ states in $^8$Be below 17 MeV excitation. The set of level parameters which gave the best fit to the $\alpha-\alpha$ scattering data also gave near-optimum fits to the $^9$Be(p,d)$^8$Be data and to the $^8$Li($\beta^-$)$^8$Be data. The fit to the $^8$B($\beta^+$)$^8$Be data with this set of parameters was an acceptable, although not an optimum one. This uncertainty in defining the level parameters was to be expected, because of experimental limitations inherent in the data.

It was difficult to specify a particular set of data as giving the most reliable estimates of the level parameters. The $^8$Li($\beta^+$)$^8$Be data had the disadvantage that an energy shift of 100 keV to the data was required before it would be analysed. The reliability of parameter values extracted from the $^9$Be(p,d)$^8$Be data was uncertain because of an unknown effect on the $\alpha$-particle spectrum, due to interaction between the three particles in the final state. With the scattering data, one had the advantage that it was only
a two-body problem, although in analysing this data, the uncertainty in defining the hard sphere phase shift was a complication. Because of this, the fits to the \( \alpha-\alpha \) scattering data were far less critical to changes in level parameters than the fits to the \( ^8B(\beta^+)\ )^8\) Be data, indicating that parameters extracted from the latter set of data may be more reliable. Possible errors in the \( ^8B(\beta^+)\ )^8\) Be data measured in the experiment arise from the uncertainty in the calculation of the \( ^8\) B ions' mean rest position in the nickel. The uncertainty in the mean position of the \( ^8\) B ion causes an uncertainty in the energy loss of the \( \alpha \)-particles coming from that position. A measurement of the angular distribution of neutrons emitted in the reaction \( ^6Li(\ ^3He,n)\ )^8\) B with a beam energy of 3.5 MeV, would enable this energy loss to be calculated exactly. The validity of the assumptions made in the present analysis could then be checked and if necessary, the \( ^8B(\beta^+)\ )^8\) Be data could be reanalysed. Initially the 'effective' straggling could not be estimated to a high degree of accuracy. It was therefore allowed to vary freely in the analysis and as it did not deviate wildly from its estimated value, there seems some justification for treating it as an independent parameter.

Thus, with the three-level approximation of R-matrix theory, it appears that all the experimental information on the \( ^2+ \) states in \( ^8\) Be below 17 MeV obtained from \( \alpha-\alpha \) scattering, \( ^8B(\beta^+)\ )^8\) Be, \( ^8Li(\beta^-)\ )^8\) Be and \( ^9\) Be(p,d) can be fitted with the one set of parameters which are reasonably well
defined. As well as justifying the use of a many-level R-matrix formalism, this fact indicates that the inclusion of the \( \sim 10 \text{ MeV} \, 2^+ \) level in the analysis is not just a means of adding extra parameters in the analysis to improve the fit to the data, but justifies the identification of the \( 2^+ \) level as a 'true' R-matrix level. As indicated in Table 2.1, the existence of such a level is expected from the systematics of other light even nuclei.
CHAPTER III

$^{12}$C - $\alpha$ ELASTIC SCATTERING AND STATES OF $^{16}$O

3.1 Introduction

In recent years many calculations (Br 63, Ei 65, Br 66, Bo 66, Se 66, Ph 67) have been performed in an attempt to determine the structure of $^{16}$O. Experimental determination of the positions and characteristics of $^{16}$O states is therefore of particular interest. The level structure of $^{16}$O below the $^{15}$N + p threshold at 12.126 MeV has previously been studied through a number of reactions. These investigations are summarized in Table 3.1. The excitation energies of $^{16}$O levels and the corresponding spin-parities are listed in the first and second columns respectively. Reactions through which levels have been seen are represented by asterisks (*) in the following columns. The various reactions are designated by letters as indicated in the footnote. References are also given in the footnote. Certain levels cannot be seen with some reactions; for example, the $^{12}$C($\alpha,\alpha$) reaction may only excite states of natural parity, as both the target and bombarding nuclei have zero spin. Also, $T = 1$ states cannot be excited by this process. The known energy levels of $^{16}$O in the energy region up to the first $T = 1$ state at 13.26 MeV are shown schematically in Fig. 3.1 (which is primarily taken from the latest compilation of Lauritsen et al. (La 62)).
TABLE 3.1

Reactions used to excite $^{16}$O levels. The letters at the top of columns designate reactions as indicated in the footnote. Reactions through which levels have been seen are represented by asterisks (*) in the body of the table. References are given in the footnote.

<table>
<thead>
<tr>
<th>Excitation in $^{16}$O (MeV)</th>
<th>$J^\pi$</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
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</thead>
<tbody>
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<td>6.052</td>
<td>0$^+$</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<td>*</td>
<td>*</td>
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<td></td>
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<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td></td>
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<td></td>
</tr>
<tr>
<td>6.917</td>
<td>2$^+$</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
<td>*</td>
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<td>*</td>
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<td>2$^+$</td>
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</tbody>
</table>

A. $^{12}$C($\alpha,\alpha$) $^{12}$C (Hi 53, Bi 54, Jo 62, La 66)
B. $^{12}$C($\alpha,\gamma$) $^{16}$O* (La 64)
C. $^6$Li($^3$C,d) $^{16}$O* (Lo 67)
D. $^3$N($^4$He,p) $^{16}$O* (Br 64)
E. $^{16}$O($p,p'$) $^{16}$O* (Cr 67, Ho 55)
F. $^{16}$O($\alpha,\alpha$) $^{16}$O* (Co 58)
G. $^{19}$F($p,\alpha\gamma$) $^{16}$O* (Yo 57)
H. $^{15}$N(d,n) $^{16}$O, $^{15}$N(d,\,n\,$\gamma$) $^{16}$O (We 57)
I. $^8$O(n,n'$\gamma$) $^{16}$O (Da 56)
J. $^{16}$N(\,$\beta$,\,$\gamma$) $^{16}$O (Al 59)
K. $^{12}$C($^7$Li,d) $^{16}$O (Be 67)
L. $^{12}$C($^7$Li,t) $^{16}$O (Be 67)
FIGURE 3.1 Energy level diagram of $^{16}$O showing levels up to the first $T = 1$ state at 13.26 MeV. The diagram is primarily taken from the latest compilation of Lauritsen (La 62).
Despite these extensive investigations there still exist uncertainties in the level structure of $^{16}$O.

The elastic scattering of alpha particles by $^{12}$C below the $^{15}$N + p threshold has been experimentally studied and the data interpreted with a compound nucleus analysis by Hill (Hi 53) (0.4 to 4.0 MeV beam energy), Jones et al. (Jo 62) (2.5 to 4.8 MeV) and Bittner and Moffat (Bi 54) (4.0 to 7.6 MeV). The data indicate the presence of narrow natural-parity levels in $^{16}$O at excitation energies 9.85 MeV ($2^+$), 10.36 MeV ($4^+$) and 11.52 MeV ($2^+$). In addition, Larson and Tombrello (La 66) have assigned spins and parities to two very narrow levels in $^{16}$O at 11.09 MeV ($4^+$) and 12.05 MeV ($0^+$) excitation. The data also indicate the existence of three broad levels in $^{16}$O at excitation energies of 9.59 MeV ($1^-$), 11.26 MeV ($0^+$) and 11.63 MeV ($3^-$).

The $s$-, $p$- and $f$-wave phase shifts extracted by Jones et al. (Jo 62) are not consistent with those calculated from the level parameters of Bittner and Moffat. For instance, in the energy region under consideration, the $0^+$ level parameters of Bittner and Moffat predict a positive $s$-wave phase shift, while the analysis of Jones et al. indicates a negative phase shift. In fact, the results of Bittner and Moffat are the only evidence for the levels at 11.26 MeV and 11.63 MeV. Anomalies related to the narrow levels below the $^{15}$N + p threshold were removed by the work of Larson and Tombrello (La 66a).
The reaction \(^{12}\text{C}(\alpha, \gamma)^{16}\text{O}\) is believed to play an important role (Bu 57) in the production of \(^{16}\text{O}\) in stellar interiors. Radiative capture into the 7.12 MeV (1\(^-\)) level and possibly the 9.59 MeV (1\(^-\)) level, is of significance at stellar temperatures (Fo 64). An accurate estimate of the alpha-particle reduced widths of both these levels is required to estimate reaction rates.

The 9.59 MeV (1\(^-\)) level has been identified previously using the reactions \(^{12}\text{C}(\alpha, \gamma)\) (La 64), \(^{12}\text{C}(\alpha, \alpha)\) (Jo 62), \(^{14}\text{N}(\text{He}, p)^{16}\text{O}\) (Br 63), \(^{12}\text{C}(\text{Li}, t)^{16}\text{O}\) (Be 67) and \(^{6}\text{Li}(^{12}\text{C}, d)^{16}\text{O}\), but the assigned widths differ by up to 100 keV. Prior to the present experiment, no direct measurement of the width of the 7.12 MeV level had been made.

The inconsistencies discussed above suggested a further study of the broad \(^{16}\text{O}\) levels. The work described here consisted of an investigation of the region below the threshold and in particular, measurements of angular distributions at energies away from well known narrow resonances. The experimental procedure and results are described in sections 3.2 and 3.3 respectively. A single channel phase shift analysis was performed on these results (section 3.4) and subsequently level parameters were extracted from the phases using both single and multi-level analysis (section 3.5). In section 3.6, the level parameters, including those of the 7.12 MeV level, are discussed and compared with those deduced by other authors.
3.2 Experimental Procedure

The results of Ferguson and McCallum (Fe 61, McC 65) indicate that the $^{12}$C($\alpha$,\,$\alpha$) differential cross section in the beam energy region 7.2 to 11 MeV is strongly energy and angle dependent. Unless otherwise stated all energies and angles quoted in this chapter are beam energies and laboratory angles. The authors (McC 66) attributed difficulties in obtaining satisfactory theoretical fits to the data to experimental inaccuracies in absolute normalization of the data as well as to systematic errors such as buckling of the foil target. The latter introduced errors of order 1° in the measured scattering angles. To overcome such experimental difficulties, it was decided in the present work to use CO$_2$ gas targets for measurements at higher energies (above 4.745 MeV). At the same time, measurements of the absolute cross section were feasible.

The use of the CO$_2$ gas target enabled the $^{16}$O($\alpha$,\,$\alpha$) data to be collected simultaneously with the $^{12}$C($\alpha$,\,$\alpha$) data. Initially it was intended to assign absolute values to the $^{12}$C($\alpha$,\,$\alpha$) data by normalizing the $^{16}$O($\alpha$,\,$\alpha$) data to the absolute measurements of McDermott et al. (McD 60). However, inconsistencies between the two sets of $^{16}$O($\alpha$,\,$\alpha$) data appeared (see section 3.3.4). Because of this, it was necessary to make a direct determination of the absolute cross section from measured yields, gas pressures and temperatures.
At low beam energies and backward angles, it became difficult to resolve the elastic alpha-particle groups. These difficulties were avoided by using a foil target when measurements were made below 4.745 MeV. In the course of these measurements it became evident that, with appropriate precautions, data of the required accuracy could be obtained with foils. To exploit the advantages of using a foil target, viz: superior energy resolution, ease and speed of data collection, all subsequent measurements of both angular distributions and excitation functions were made with a foil target.

3.2.1 Scattering Chamber and Detectors

An alpha-particle beam of approximately 0.1 μA from the A.N.U. Tandem Van de Graaff accelerator was passed through a 90° analysing magnet and into a 51 cm scattering chamber (described in section 1.2.1) via two 1.5 mm collimating slits, placed 12.5 cm apart, and a 2.5 mm anti-scatter slit, placed 12.5 cm in front of the final collimator. Several types of target arrangement (section 3.2.2) were used.

The counter mounting blocks (Item 6, Fig. 1.3) were reversed from the position shown in Fig. 1.3, to increase the target-counter distance and thus improve the angular resolution. When foil targets were used the solid angle subtended by a detector was defined by a 1 mm x 5 mm rectangular slit (Item 7, Fig. 1.3) attached to the counter block immediately in front of the counter. A second slit (2 mm x 5 mm) placed 5 cm in front of the collimator
acted as an anti-scatter slit. For gas target measurements two 1 mm x 5 mm slits placed 5 cm apart defined the target volume "seen" by the detector. Initial optical alignment of the chamber and collimator established coincidence of a point on the beam axis, the centres of rotation of the counters and of the target holder to within 0.005 cm.

Pulses from the surface barrier detector were amplified by a charge sensitive preamplifier-linear amplifier system operated in the double-delay-line pulse-shaping mode. The amplified pulses were fed directly to a 400 channel pulse height analyser. Three such systems were used for all measurements. Analyser dead times were determined by the method described in section 1.2.3 and were usually less than 1%.

3.2.2 Targets

3.2.2.1 Foil Targets

Thin self-supporting carbon foils of approximate surface density 20 µg/cm² (prepared in the manner outlined in section 1.2.2) were used with the arrangement described in section 1.2.1. Reaction products were collimated within 0.5° in the reaction plane in each of the three detection systems. A typical energy spectrum of the scattered particles, shown in Fig. 3.2(a), displays the well-resolved alpha peak from ¹²C(α,α). Difficulty was experienced in resolving the alpha-particle groups, elastically scattered from ¹²C and ¹⁶O at angles forward of 35°.

The beam was integrated in the normal manner (section 1.2.1) with
FIGURE 3.2 Typical energy spectra of $\alpha$-particles elastically scattered from: (a) a thin $^{12}$C foil target, and (b) a $\text{CO}_2$ gas target.
FOIL TARGET
$E_{\text{beam}} = 5.135$ MeV
$\psi_{\text{lab}} = 65^\circ$

GAS TARGET
$E_{\text{beam}} = 5.163$ MeV
$\psi_{\text{lab}} = 65^\circ$
an insulated Faraday cup employing both magnetic and electrostatic suppression.

3.2.2.2 Gas Targets

Two types of gas target were used: one for the initial angular distribution measurements above 4.745 MeV and one for measurements of absolute cross sections.

For angular distribution measurements, the beam entered the scattering chamber through a nickel foil of surface density 565 µg/cm². The chamber was filled to a pressure of 15 mm Hg with CO₂ gas of purity 99.0%. The surface-barrier counters were operated in the gas, thus avoiding the disadvantages associated with detecting low-energy alpha-particles passed through a foil. Reaction products were collimated within 0.5° in the reaction plane. A typical energy spectrum of the detected particles is shown in Fig. 3.2(b). Use of a gas target in this situation limited the maximum forward angle to 60° because of the inability to separate the elastic groups from \(^{12}\text{C}\) and \(^{16}\text{O}\) beyond this point.

To measure absolute cross sections a gas cell (Yo 65) was mounted inside the 51 cm scattering chamber (Fig 3.3 and Fig. 3.4). The cell was filled through two copper tubes which passed through the base of the chamber and were connected to an external gas handling system. The gas
FIGURE 3.3  Vertical cross section of the scattering chamber and gas cell.

The numbered parts are:

(1) Collimator tube
(2) Gas cell
(3) Nickel windows
(4) Suppressor magnet
(5) Faraday cup
(6) O-ring seal
(7) Teflon bearing rings
(8) Drive sprocket
FIGURE 3.4  Horizontal cross section of the scattering chamber and gas cell. The numbered parts are:

(1) Collimator tube  
(2) Gas target  
(3) Mylar foil  
(4) Nickel windows  
(5) Detector slit assembly  
(6) Solid state counter  
(7) Faraday cup.
pressure was monitored with a precision aneroid manometer* which had been calibrated using an oil (dibutyl phthalate) manometer. A thermometer was inserted with the copper tubing to make thermal contact with the base of the gas chamber. The cell was filled with CO$_2$ to a pressure of 22 mm Hg. A mass spectrometer analysis** of the gas showed impurities of N$_2$, CO (0.8%) and O$_2$ (0.2%) totalling 1%.

The beam entered and emerged from the gas cell through nickel foils of surface density 565 μg/cm$^2$. The beam was integrated in the normal manner with an insulated Faraday cup. The scattered particles emerged from the cell through a Mylar foil of thickness 635 μcm and were detected externally to the cell. Angular distribution data could not be collected using this gas cell because of the large energy losses experienced by the low energy reaction products on passing through the Mylar.

3.3 Experimental Results

3.3.1 Excitation Functions

Excitation functions were measured in the energy range 2.8 MeV to

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** The mass spectrometer analysis was performed by Dr. F.J. Bergerson of the C.S.I.R.O., Canberra.
6.6 MeV at the angles 70°, 90° and 110°. The data were taken in energy steps of 400 keV as well as at the intermediate energies where angular distribution and absolute cross section measurements were made. Each datum point was corrected for analyser dead time and normalized to the beam charge. The final set of data are shown in Fig. 3.5. These points have been normalized to an absolute cross section and corrected for the $^{13}$C impurity (discussed in section 3.3.6). In the present work the excitation functions were only used to normalize the angular distributions.

3.3.2 Angular Distributions

Angular distributions were measured at seven energies: 4.745, 5.165, 5.670, 6.000, 6.250, 6.510, and 6.600 MeV using a gas target. With foil targets, angular distributions were measured at eight energies: 2.800, 3.100, 3.400, 4.000, 5.135, 6.250, 6.510 and 6.600 MeV. These data were collected in 5° increments between 60° and 165°. In addition, the foil target measurements at 6.250 and 6.510 MeV were extended forward to 35°. The angular ranges $35° \leq \theta_{lab} \leq 95°$ and $85° \leq \theta_{lab} \leq 160°$ were covered using reflection and transmission geometry respectively when foil targets were used. Each datum point was corrected for analyser dead time and normalized to the yield at 165°, measured with a monitor counter. The final data are shown in Fig. 3.6. These points have been normalized to an absolute cross section and corrected for the $^{13}$C impurity (discussed in section 3.3.6).
FIGURE 3.5  Excitation functions of α-particles elastically scattered from
$^{12}$C using a foil target. Also shown are the relevant excitation
function data of Jones et al. (Jo 62) and Bittner and Moffat (Bi 54).
The triangles represent absolute cross sections measured
in the present experiment.
Angular distributions of $\alpha$-particles elastically scattered from $^{12}$C using both solid (thin foil) and gas targets. The full curves represent fits obtained from the analysis discussed in the text. The corresponding phase shifts are given in Figure 3.7. The hollow circles represent cross sections measured using a foil target when the remainder of the angular distribution had been measured using a gas target.
3.3.3 **Accuracy of Data**

As was mentioned in section 3.2, earlier experimental results (McC 65), (McC 66) suffered from the sensitivity of data to experimental conditions. In the present experiment, therefore, a systematic attempt was made to establish and monitor these effects.

3.3.3.1 **Determination of Beam Energies**

The thickness of the nickel foil windows used in gas target measurements was determined by using a "thin" carbon foil target and comparing the position of a known (Bi 52) narrow 2+ resonance at 5.82 MeV with or without the nickel foil. The chamber was then filled with CO₂ and, by using the gas target, the thickness of gas between the foil and target was similarly measured. The surface density of the nickel foil and gas were then estimated from stopping-power data (Wi 66, Wh 58). The measured value of the surface density of the nickel foil was consistent with that quoted by the manufacturers*. Once the surface densities were determined, the beam energies at the target centre were estimated from the stopping power data. This procedure introduced an effective uncertainty of 15 keV in the beam energy at 4.745 MeV.

Studies of the Tandem beam analysing system (Mo 67) have shown that for constant accelerator conditions, reproducibility of better than 3 keV may be obtained. However, larger changes (5 - 10 keV), attributed to small variations

* Chromium Corporation of America, Waterbury, Connecticut, U.S.A.
in the beam trajectory through the analyser, have been observed when the accelerator conditions have been altered. A check on the energy consistency was made by comparing excitation function and angular distribution data at common angles and energies. No significant variations were recorded. The energy scales are thus internally consistent with a possible error of ±10 keV in the absolute energy.

Energy straggling in the nickel foil and CO₂ gas was calculated using the theory of Symon (Sy 48). Although this theory is primarily for calculating straggling of high energy particles in matter, Kerr (Ke 68a) has shown it applicable in the present conditions. The calculations agree with the experimental values measured by Comfort et al. (Co 66) at 6.25 MeV. The energy resolution of the data is limited by straggling in the nickel foil (23 keV) and the CO₂ gas (20 keV) and the finite thickness of gas viewed by the detectors (6 keV).

These uncertainties led to a net energy spread of 36 keV. The values quoted are for measurements made at 4.745 MeV, the lowest energy used with a gas target.

3.3.3.2 Determination of Angle

Kerr et al. (Ke 68) have checked the absolute calibration of the counter angles in the 51 cm chamber, by comparison of the observed scattering from gold with calculated Rutherford scattering. The results indicated that the angular scale was accurate to ±0.2°. At each energy used for angular distribution measurements, the symmetry of the collimator, target and detector
arrangements was checked by observation of the scattering yields at equal angles \((\Psi_{lab} = 65^\circ)\) to the left and right of the centre line. The results indicated an uncertainty of \(\pm 0.15^\circ\) in the angle at \(65^\circ\). The location of the foil targets relative to the centre of rotation of the counters and the effect of foil buckling were checked by taking a measurement at a certain angle, then rotating the target through \(180^\circ\) and repeating the measurement. The results indicated a foil movement of less than 0.4 mm.

The net uncertainty in the calibration of the counter angles when gas targets were used was \(\pm 0.25^\circ\) and for foil targets, \(\pm 0.3^\circ\).

### 3.3.3.3 Beam Integration

Initial calibration and routine checks on the integrator stability, made using a constant current source and electronic timer, showed the instrument to be accurate to better than 1%. A test was performed to ensure that the suppression potential of \(-300\) volts applied to the Faraday cup was adequate. In the test the \(^{12}\text{C}(\alpha,\alpha)\) scattering yield was measured under constant conditions for a number of suppressor voltages between \(-900\) and \(+300\) volts. As the voltage was increased from \(-900\) volts, the yield remained constant, within statistics, until the suppressor voltage was between \(-100\) and 0 volts. The effect became greater as the voltage became more positive.

### 3.3.3.4 Target Thickness

For removal of systematic errors arising from carbon build-up on the foil target, the excitation functions were measured over the whole energy
range several times with both positive and negative energy increments. The final excitation functions were obtained by averaging the results at each energy. Normally the build-up was negligible, although at times the angular distribution monitor yield indicated carbon deposits of up to 5% over a 10-hour run. The reason for the differing amounts of build-up was not obvious, but it was believed to be related to the quality of the chamber vacuum.

3.3.4 Absolute Cross Section

As previously mentioned, it was initially intended to assign absolute values to the $^{12}\text{C}(\alpha,\alpha)$ cross sections by normalizing the $^{16}\text{O}(\alpha,\alpha)$ data, collected simultaneously, to the absolute measurements of McDermott et al. (McD 60). However, the two sets of $^{16}\text{O}(\alpha,\alpha)$ data are not consistent, as is shown in Table 3.2, where the McDermott data are compared to the data from the present experiment (normalized to the McDermott data at 5.670 MeV beam energy).

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>Cross Section (mb/sr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>McDermott</td>
</tr>
<tr>
<td>6.510</td>
<td>-</td>
</tr>
<tr>
<td>6.250</td>
<td>17.</td>
</tr>
<tr>
<td>5.670</td>
<td>11.</td>
</tr>
<tr>
<td>5.135</td>
<td>4.</td>
</tr>
<tr>
<td>4.745</td>
<td>18.</td>
</tr>
</tbody>
</table>
These inconsistencies may be attributed entirely to systematic errors of up to 90 keV in the energies quoted by McDermott. The errors are a result of saturation in the $90^\circ$ analysing magnet that was used (McD 63). The recent work of Hunt et al. (Hu 67) also indicates this systematic error. Because of the inconsistencies in the data and difficulties in obtaining accurate numerical values of McDermott's data, a direct determination of the $^{12}\text{C}(\alpha,\alpha)$ absolute cross section was attempted as described below.

The absolute measurements were made at angles of $70^\circ$, $90^\circ$ and $125^\circ$ using "corrected" beam energies (i.e., beam energy at the target) of $6.510 \pm 0.028$ and $5.670 \pm 0.030$ MeV. The gas cell used for the measurements is described in section 3.2.2.2.

The absolute cross section $\sigma(\Psi)$ at lab angle $\Psi$ was obtained from the actual yield $Y$, the number of bombarding particles $n$, and the number of target nuclei per unit volume $N$, by the relation (Si 59):

$$\sigma(\Psi) = \frac{Y \sin \Psi}{n.N.G}.$$  \hspace{1cm} (3.1)

The factor $G$ depends on the geometry of the detecting system, the derivatives of the cross section with respect to angle and the beam dimensions:

$$G = G_{oo}(1 + \Delta_o + \frac{\sigma'}{\sigma} \Delta_1 + \frac{\sigma''}{\sigma} \Delta_2 + \ldots).$$ \hspace{1cm} (3.2)
where
\[ G_{oo} = \frac{4b_1 b_2 \ell}{R_0 h}, \]  
(3.3)

\[ \Delta_0 = \frac{1}{3} \frac{\cos^2 \psi}{\sin^2 \psi} \frac{b_2^2}{R_2^2} - \frac{1}{2h^2} (b_1^2 + b_2^2) - \frac{1}{8} \frac{\ell^2}{R_0^2}, \]  
(3.4)

and the terms \( b_1, b_2 \) etc., are defined in Table 3.3. The higher order terms involving \( \Delta_1, \Delta_2 \) etc., were neglected here.

**TABLE 3.3**

Nominal Slit Geometry

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Dimension (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2b_1</td>
<td>Front slit width</td>
<td>1</td>
</tr>
<tr>
<td>2b_2</td>
<td>Rear slit width</td>
<td>1</td>
</tr>
<tr>
<td>( \ell )</td>
<td>Rear slit height</td>
<td>5</td>
</tr>
<tr>
<td>R_0</td>
<td>Distance from centre of target to rear slit</td>
<td>107.5</td>
</tr>
<tr>
<td>h</td>
<td>Distance between slits</td>
<td>50.5</td>
</tr>
</tbody>
</table>

In equation (3.1) the number of bombarding particles \( n \) was determined from the charge collected for each datum point. The quantity \( N \) was determined from the measured pressure and temperature of the \( \text{CO}_2 \) gas,
using the ideal gas law. As the gas cell had a leak rate of approximately 0.5 mm per hour, the pressure and temperature were recorded at the beginning and end of each run. The yield, Y, was corrected for dead time, in the usual way.

The cross sections calculated using equation (3.1) are in the laboratory system. The results were converted to the c.m. system. Each datum point was corrected, using the $^{13}$C($\alpha$,\(\alpha\)) results of Kerr et al. (Ke 68), for the 1% of $^{13}$C present in natural carbon. All the measurements were repeated three times. The averaged estimates of the $^{12}$C($\alpha$,\(\alpha\)) absolute cross sections and the associated errors are given in Table 3.4. The error in the measurements was estimated (see section 3.3.5) to be 3.2%.

<table>
<thead>
<tr>
<th>Energy (MeV)</th>
<th>$\psi = 70^0$</th>
<th>$\psi = 90^0$</th>
<th>$\psi = 125^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.510 ± 0.027</td>
<td>56.1 ± 1.8</td>
<td>23.4 ± 0.8</td>
<td>29.0 ± 1.0</td>
</tr>
<tr>
<td>5.670 ± 0.029</td>
<td>25.9 ± 0.9</td>
<td>63.9 ± 2.1</td>
<td>-</td>
</tr>
</tbody>
</table>
3.3.5 **Accuracy of Absolute Measurements**

3.3.5.1 **Energy Determination**

The experimental method, previously described, of determining nickel foil and gas surface densities was not suitable to be used with the gas cell, as foil targets could not be placed in the cell. Instead the surface density of the nickel foils quoted by the manufacturers and the gas surface density determined directly from pressure and temperature measurements were assumed in the calculations. The 'target' energies were determined in the manner described for angular distribution measurements. This procedure introduced an effective uncertainty of 10 keV in the beam energy at 6.510 MeV.

The energy resolution of the measurements (see section 3.3.3.1) was limited by straggling in the nickel foil (20 keV) and in the CO\textsubscript{2} gas (14 keV), instability in the 90° magnet (10 keV) and the finite thickness of gas viewed by the detectors (6 keV).

These uncertainties, estimated for the 6.510 MeV measurement, gave a net energy spread of 28 keV.

3.3.5.2 **Determination of n**

As discussed in section 3.3.3, the integrator accuracy was better than 1%. An additional error was introduced with the use of the gas cell because of the inability to collect all the charge multiply scattered from the entrance and exit foils. At 6.5 MeV beam energy the effect resulted in a
0.05% beam loss at the Faraday cup (Yo 65).

3.3.5.3 Geometry of the Detection System

The dimensions of the slit system (nominal values given in Table 3.1) were measured using a calibrated microscope and were estimated to be accurate to 0.6%. The approximations made introduced an uncertainty in the value of G of ~ 0.05% (section 3.3.4). At the angles used for absolute cross section measurements the error due to uncertainties in the chamber geometry is less than 0.5% (section 3.3.3).

3.3.5.4 Determination of N

The error in the measurement of the CO\textsubscript{2} gas pressure is estimated to be 0.8%. This is a compounded error based on a 0.3% uncertainty in reading the gauge and a 0.5% uncertainty in the gauge calibration. The error resulting from the correction for impurities in the gas was estimated to be 1%, which assumes that the actual correction made in the worst case was in itself accurate to only 100%. The measured temperature of the gas target was typically about 1°C above the ambient room temperature. The uncertainty in the temperature resulted from local heating along the beam path through the gas. Young (Yo 65) has had some success in using the heat equation to estimate this effect. His results indicate that an error of less than 0.5% is introduced by ignoring the effect in the present case.

The estimated errors in the measured absolute cross section are
3.3.6 Normalization of Data

To assign absolute values to the angular distribution data, the following procedure was used:

(a) The excitation function at 90° was normalized to the measured value of the absolute cross section at 6.510 MeV. (This was done prior to the absolute measurement being corrected for 13C impurity).

(b) The angular distributions were normalized to the 90° excitation function.

(c) The remaining two excitation functions at 70° and 90° were normalized to the 5.135 MeV angular distribution.

(d) Using the 13C(α,α) results of Kerr et al. (Ke 67a), each datum point was corrected for the 1% of 13C present in natural carbon.

(e) Each angular distribution datum point was assigned an error compounded from the statistical error, the error in the measurement of the absolute cross section, geometrical errors, etc.

Cross checking the angular distribution data, the excitation function data, and the absolute cross section data showed the results to be 'internally consistent' to within 5%.

The final excitation function and angular distribution data are shown in Fig. 3.5 and Fig. 3.6 respectively. The 6.600 MeV angular distribution is not illustrated but is very similar to that at 6.510 MeV. When a particular angular distribution was measured with both foil and gas targets (e.g., 6.250
**TABLE 3.5**

Estimated Errors in the Measured Absolute Cross Section

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Source of Error</th>
<th>Error %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid angle</td>
<td>Uncertainty in G</td>
<td>0.05</td>
</tr>
<tr>
<td></td>
<td>Slit width measurements</td>
<td>0.6</td>
</tr>
<tr>
<td></td>
<td>Chamber geometry</td>
<td>0.5</td>
</tr>
<tr>
<td>Beam intensity</td>
<td>Integrator absolute error</td>
<td>1.0</td>
</tr>
<tr>
<td></td>
<td>Neglect of multiple scattering</td>
<td>0.05</td>
</tr>
<tr>
<td>Target thickness</td>
<td>Error in pressure measurement</td>
<td>0.8</td>
</tr>
<tr>
<td></td>
<td>Error in temperature measurement</td>
<td>2.0</td>
</tr>
<tr>
<td></td>
<td>Effect of local heating along beam path</td>
<td>0.5</td>
</tr>
<tr>
<td>Target purity</td>
<td>$^{13}$C contamination</td>
<td>1.0</td>
</tr>
<tr>
<td>Yield</td>
<td>Statistical error</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>Resultant Error</td>
<td>3.2</td>
</tr>
</tbody>
</table>
MeV), the variation between the two sets of results was within 3%. With such measurements, the gas target data were plotted in preference to the foil target data. In Fig. 3.6 the forward angle data, collected with foil targets at 6.250 and 6.510 MeV, are shown as open circles.

When possible (i.e., at common angles and energies) the data of Jones et al. (Jo 62) and Bittner and Moffat (Bi 54) are plotted in Fig. 3.5, for comparison to the present data. These comparisons, together with comparisons made by extrapolating the data at other energies and angles, indicate satisfactory agreement at backward angles. However, at forward angles, particularly at low energies, systematic differences with the Jones results, corresponding to an error in the scattering angle of up to 4° appears. Bittner and Moffat reported no forward angle data to compare with the present work. The experimental estimates of the absolute cross sections made in this work are also plotted in Fig. 3.5 for comparison to the normalized cross section.

3.4 Phase Shift Analysis

3.4.1 Analytical Expression for Cross Section

The differential scattering cross section $\sigma(\theta)$, in the centre of mass system is given by equation (I.9) of Appendix I as:
\[ \sigma(\theta) = \frac{1}{2k^2} \left[ -\frac{n}{2} \csc^2 \left( \frac{\theta}{2} \right) \exp \left[ i\eta \log \csc^2 \left( \frac{\theta}{2} \right) \right] + \frac{i}{2} \sum_l (2l + 1) \right] \]  

(3.5)

\[ (e^{2i\omega} - U_l) P_l (\cos \theta) \right] ^2, \]

with the symbols as defined in Appendix I. The diagonal elements of the collision matrix may be written:

\[ U_l = e^{2i\delta_l}, \]

(3.6)

where the nuclear phase shift \( \delta_l \) is in general complex, the imaginary part arising from absorption of particles out of the elastic channel into non-elastic channels. For the special case when elastic scattering is the only open channel \( \delta_l \) is real. In the general case, writing

\[ \delta_l = \Delta_l + i\epsilon_l, \]

(3.7)

with \( \Delta_l \) and \( \epsilon_l \) both real and

\[ A_l = e^{-2\epsilon_l}, \]

(3.8)

the expression for \( U \) becomes:

\[ U_l = e^{2i\omega} A_l e^{2i\Delta_l}, \]

(3.9)
where $A_L$ appears as a damping parameter with values $0 \leq A_L \leq 1$.

As discussed in section 3.4.4, the form of the collision matrix given by equation (3.6), with $\delta_L$ real, was used in the phase shift analysis, described below.

It should be noted that fitting the experimental data with the general cross section expression involved no specific assumptions regarding the detailed form of the collision matrix (i.e., no assumption as to the nature of the scattering process or adoption of any particular reaction theory). The phase shift extraction is thus to be regarded as a simple parameterization of the data in terms of the real phase shifts $\delta_L$. The interpretation of the resulting parameters is treated as a separate problem.

3.4.2 Non-Linear Least Squares Fitting Routine

As the phase shifts appear non-linearly in the cross section expression (equation (3.5)), a conventional linear 'least squares' method cannot be applied. In the present case, numerical techniques must be used. This section briefly describes a general non-linear 'least squares' fitting routine developed by Hay and Barker (Ha 67) which is suitable for use on an I.B.M. 360/50 computer.

The numerical routine finds a minimum of the multi-dimensional surface $X^2(\delta_L)$ in the parameter space $\delta_L$ by assuming a quadratic approximation to the surface in the neighbourhood of the minimum.

$$X^2(\delta_L) = \frac{1}{n - m} \sum_k \left( \frac{\text{observed value (k)} - \text{calculated value (k)}}{\epsilon(k)} \right)$$, \hspace{1cm} (3.10)
where \( m \) represents the number of free parameters and \( n \) the number of data points. The quantity \( \epsilon(k) \) is the error associated with the observed value \( (k) \). 

\( X^2(\delta_f) \) is thus the mean square ratio of the residuals to the experimental errors. A satisfactory fit yields a value of \( X^2(\delta_f) \approx 1 \), provided realistic errors have been assigned.

The function \( X^2 \) is minimized by making parameter changes in two ways:

1. By a descent cycle in which parameters are changed one at a time, in rotation, until the value of \( X^2 \) does not change rapidly.

2. By a global cycle, in which all the parameters are changed simultaneously following an estimation of the \( X^2 \) surface contours by changing the parameters singly and in pairs.

In both cases, the parameter changes are made by fitting a parabolic surface to points on the \( X^2 \) surface, within a region specified by a scale factor, and then proceeding to the minimum in the parabolic surface. As the scale factor is chosen to correspond to a statistically significant increase in the value of \( X^2 \), the absolute minimum of the \( X^2 \) surface is not necessarily found, but a region near the minimum is reached such that the difference between it and the actual minimum is of no statistical significance. On exit from the routine, the 'step sizes' for adjustment of the parameters to the values which change \( X^2 \) from the minimum value to the minimum value times the scale factor, provide an estimate of the accuracy with which the parameters are determined.
The above procedure obviously requires selection of a starting set of parameters in the region of the minimum; in the case of complicated $X^2$ surfaces, it is necessary to try many starting positions in order to ascertain the "correct" or lowest $X^2$.

3.4.3 Phase Shift Extraction from Angular Distribution Data

No attempt was made to extract phase shifts from the excitation function data.

Phase shifts were extracted from each of the angular distributions using the non-linear least squares fitting routine described above. The phases of Jones et al. (Jo 62) and phases calculated from the level parameters of Bittner and Moffat (Bi 54) were used as two independent sets of initial starting values for fitting the angular distributions. The requirement that phase shifts be continuous in energy was used to determine a further set of starting phases.

The fits to the data required significant amounts of $\delta_0$, $\delta_1$, $\delta_2$, $\delta_3$, $\delta_4$, and $\delta_5$ only. The continuous lines shown in Fig. 3.6, represent the best fits to the experimental data points. The corresponding phase shifts are shown in Fig. 3.7 and listed in Table 3.6. Also shown in Fig. 3.7 are the phase shifts calculated from the level parameters of Bittner and Moffat (Bi 54) and the phases quoted by Jones et al. (Jo 62). The positions of the narrow $2^+$ and $4^+$ levels have been indicated in Fig. 3.7 by $180^0$ displacements of the
<table>
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<tr>
<th>Beam Energy</th>
<th>$\delta_0$</th>
<th>Error</th>
<th>$\delta_1$</th>
<th>Error</th>
<th>$\delta_2$</th>
<th>Error</th>
<th>$\delta_3$</th>
<th>Error</th>
<th>$\delta_4$</th>
<th>Error</th>
<th>$\delta_5$</th>
<th>Error</th>
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<td>2.4</td>
<td>351.7</td>
<td>1.1</td>
<td>126.4</td>
<td>1.0</td>
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<td>2.5</td>
<td>0.9</td>
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<tr>
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<td>1.9</td>
<td>356.1</td>
<td>0.7</td>
<td>134.8</td>
<td>1.1</td>
<td>178.9</td>
<td>1.5</td>
<td>3.2</td>
<td>0.7</td>
</tr>
<tr>
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<td>121.5</td>
<td>2.1</td>
<td>357.5</td>
<td>0.9</td>
<td>133.1</td>
<td>0.9</td>
<td>179.1</td>
<td>1.0</td>
<td>2.8</td>
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</table>
Experimental phase shifts derived from fitting the angular distributions of α-particles elastically scattered from $^{12}$C. The solid dots correspond to fits to the angular distributions measured in the present experiment. Phase shifts calculated from the level parameters of Bittner and Moffat (Bi 54) and the phases quoted by Jones et al. (Jo 62) are also shown. The curves representing the hard sphere phase shift $- \varrho \ell = - \tan^{-1} (F_\ell / G_\ell)$ were calculated using a channel radius of 5.3 fm. Both single and multi-level fits to the phase shifts are shown. These fits are discussed in the text and the corresponding level parameters are given in Table 3.8 and Table 3.11.
corresponding phase shifts.

An examination of the topology of the $X^2$ surface was made to ensure that the routine was not 'trapped' in a secondary minima. This was done by using an optimized grid search program in which one parameter was stepped over the range of possible values and held constant after each change, while all other parameters were varied freely to minimize $X^2$. This is illustrated in Fig. 3.8 where for the 5.135 MeV angular distribution, minimized $X^2$ values are plotted as a function of the stepped phase. The complexity of the $X^2$ surface is indicated by the several minima in the $\delta_o$ curve. Although the surface complexity increases with energy, the criterion that the phases must be continuous in energy enabled a consistent final set of values for the phase shifts to be determined.

The error assigned to a particular phase was chosen somewhat arbitrarily to correspond to the variation in the value of the phase that is necessary to produce a 50% change in the value of $X^2$ for the fit, the other parameters being kept fixed at their true values. The deterioration that a 50% change in $X^2$ produces in the fits is visible. The curves obtained by this procedure for the 5.135 MeV angular distribution are shown in Fig. 3.9.

The sensitivity of the phases to uncertainties in the absolute cross section was investigated by examining the variations in the phases and $X^2$ values when the angular distributions were refitted with the absolute cross
FIGURE 3.8 Curve of minimum $X^2$ for fixed values of the phase shift parameters $\delta_0$, $\delta_1$, $\delta_2$ and $\delta_3$. The contours correspond to the case of the 5.135 MeV angular distributions.
FIGURE 3.9 Curve of $X^2$ for variations in a particular parameter while the other parameters were kept at their optimum values.

The curves correspond to the case of the 5.135 MeV angular distributions. The error assigned to a particular phase was the variation in phase required to produce a 50% change in $X^2$. 
sections varied by up to ± 20% of the nominal value. No critical effects other than the expected dependence on $\delta_0$ were observed. Although these variations produced superior fits for some angular distributions, this did not infer that the absolute cross section was incorrect, as the sign and magnitude of the variations in the absolute cross section which produced the best fits, varied from energy to energy.

The use of a gas target for the measurements made at higher energies precluded detection at angles forward of 60°. It can be seen (Fig. 3.6) that the fits to the angular distributions with beam energies higher than 5.5 MeV imply the existence of some structure at forward angles. This structure was checked with a foil target at 6.250 MeV and 6.510 MeV. As shown in Fig. 3.6, satisfactory agreement between the measured and predicted cross section was obtained. In Fig. 3.6 the open circle points at forward angles represent data collected with foil targets.

3.4.4 Justification of Single-Channel Assumption

As mentioned in section 3.4.1, the assumption was made that in the energy region considered in this analysis, the elastic scattering channel is the only open channel. In a strict sense above the threshold of the inelastic channel, $^{12}$C* + $\alpha'$, at 5.907 MeV a complex phase shift analysis should be performed using the form of the collision matrix given by equation (3.9). However, the penetration factor for the inelastically scattered $\alpha'$ group is
much smaller (a factor of $10^5$ for s-wave particles at a beam energy of 6.5 MeV) than that of the elastically scattered group. Also data on the $^{12}$C($\alpha,\alpha'\gamma$) process (Mi 64) shows a very small cross section for the ($\alpha,\alpha'$) reaction at the energies under consideration. These facts indicated that a real phase shift analysis of the $^{12}$C($\alpha,\alpha$) process below the $^{15}$N + p threshold in $^{16}$O is acceptable. This assumption was substantiated by performing a complex phase shift analysis of the 6.6 MeV data. The damping parameter $A_\ell$ of equation (3.9) was always constant at $A_\ell = 1$ within the experimental errors. The error assigned to the parameter $A_\ell$ was determined in a similar manner to that for the real phases $\delta_\ell$.

In an R-matrix analysis, as described in section 3.5, closed channels can have an effect on the energy dependence of the cross section through the level shift factor $\Delta_\lambda \ell$ which is defined in equation (I.21) of Appendix I. This dependence occurs independently of whether the phase shift $\delta_\ell$ is real or imaginary. The apparent energy dependence is removed if the shift factor for the closed channel varies linearly with energy (Ba 68).

3.4.5 Discussion of Phase Shifts

Except for the anomaly near 6.5 MeV, the s-wave phase shift decreases monotonically with energy. At low energies it is in agreement with the work of Jones et al. (Jo 62). It is approximately half the hard sphere value $\varphi_0$ (defined in Appendix I, equation (I.15)), calculated with a channel
radius of 5.3 fm. This is shown as a dotted curve in Fig. 3.7. The anomaly of δ_o observed in the energy region 6.250 to 6.600 MeV was reproduced using a foil target. There is a known (La 66) narrow δ_o level at 12.05 MeV excitation in \(^{16}\text{O}(E_\alpha = 6.518 \text{ MeV})\), however the detailed behaviour of δ_o in this region could not be attributed entirely to the influence of this level.

The p-wave phase shift exhibits a resonance behaviour in the vicinity of \(E_\alpha = 3.23 \text{ MeV}\) similar to that observed by Hill (Hi 53) and Jones et al. (Jo 62). At higher energies, there is a variation in the p-wave phase shift from the hard sphere value, \(\varnothing_1\), indicating the influence of broad levels in the energy region. The f-wave phase shows a broad resonance behaviour covering most of the energy region under investigation in the experiment.

The primary aim of this work is to parameterize the broad levels in \(^{16}\text{O}\) that are influential in this region. Consequently no study of the phase shift variation across the narrow 2^+ and 4^+ levels was made. The positions of narrow 2^+ and 4^+ levels have been indicated in Fig. 3.7 by 180° displacements of the corresponding phase. The d-, g- and h-wave background phase shifts vary slightly, although not radically from the hard-sphere phases calculated with a channel radius of 5.3 fm.
3.5 Level Parameters

The level parameters assigned to broad $0^+$, $1^-$ and $3^-$ levels contributing to the $^{12}\text{C}(\alpha,\alpha)$ process in the region investigated were obtained from R-matrix fits to the phase shifts. Although, for reasons outlined in section 3.4.4, only the one-channel case of elastic scattering was considered, analyses were made with the one-, two- and three-level approximations.

The analytic form of the phase shift in terms of level parameters (equation (1.15) of Appendix I) is:

$$\delta_{\ell} = \beta_{\ell} - \varphi_{\ell} + \omega_{\ell}, \quad (3.11)$$

where the resonant phase shift $\beta_{\ell} = \tan^{-1} \frac{RP}{1 - RS}$ and

$$R = \sum_{\lambda} \frac{\gamma_{\lambda\ell}^2}{(E_{\lambda\ell} - E)}.$$

The energy eigenvalue and the reduced width of the level $\lambda$ are denoted by $E_{\lambda\ell}$ and $\gamma_{\lambda\ell}^2$ respectively. The hard sphere phase shift $\varphi_{\ell}$, the Coulomb phase shift $\omega_{\ell}$, the penetration factor $P$ and the shift factor $S$ are defined in Appendix I. The boundary condition parameter $B$ which is discussed in Appendix I, was defined to be zero for the present analysis, i.e., $B = 0$. 


In the special case of only one level contributing to the phase shift, equation (3.11) may be written:

\[ \delta_l = \tan^{-1} \frac{\gamma^2 P}{E - \gamma^2 S - E} - \phi_l + \omega_l \]  

(3.12)

A background phase shift, resultant from the effects of distant levels may be accommodated in R-matrix theory by including an extra level in the analysis (Ba 68). The multi-level fits thus allowed the influence of distant 1^- and 3^- levels on the p- and f-wave phase shifts respectively to be included.

The phases were fitted with the above expression, using the non-linear least squares fitting program described in section 3.4.2. The program minimizes the quantity \( X^2 \) which is defined in equation (3.10). In the case of complicated \( X^2 \) surfaces, the fitting procedure requires selection of a starting set of parameters in the region of the "correct" solution. The level parameters of the Rice group (Jo 62) and Bittner and Moffat (Bi 54) were used as starting parameters. These level parameters are listed in Table 3.7. The various starting values gave consistent final values.

The resonance energy \( E_{rl} \) where the resonant phase shift \( \beta_l \) passes through 90° was determined from the energy eigenvalue \( E_{\lambda l} \), using the expression:

\[ E_{rl} = E_{\lambda l} + \Delta_{\lambda l} (E_{rl}) \]
where the level shift $\Delta \lambda_{\ell}$ is defined in equation (I.21) of Appendix I. The level width $\Gamma_{\lambda_{\ell}}$ was obtained from a knowledge of the reduced width and the penetration factor;

$$\Gamma_{\lambda_{\ell}} = 2\gamma_{\lambda_{\ell}}^2 P_{\ell}.$$  \hspace{1cm} (3.14)

The error assigned to a parameter was chosen to correspond to the variations in the value of the parameter that was necessary to produce a 50% change in the value of $X^2$ for the fit, the other parameters being kept fixed at their 'true' values. All the parameters are defined in the centre-of-mass system.

3.5.1 Level Parameterization of the $\delta_0$ Data

Using the one-level one-channel approximation, the minimum $X^2_{\delta_0}$ values and the 'best fit' values of the level parameters were obtained for a range of channel radii. The results of this survey are given in Fig. 3.10, where $X^2_{\delta_0}$, $E_{\delta_0}$ and $\gamma^2_{\delta_0}$ are plotted as a function of channel radius $a_{\delta_0}$. The optimum channel radius is in the region 5.0 to 5.5 fm. The $0^+$ level parameters calculated using a channel radius of 5.3 fm are given in Table 3.8 and the corresponding calculated phase shifts are shown as solid lines in Fig. 3.8. The single level fits to the $\delta_0$ phase shift are of extremely high quality; $X^2_{\delta_0} \simeq 0.8$. Values of $X^2_{\delta_0} < 1$ indicate an overestimation of the magnitude of the errors.

A two-level analysis of the data was attempted but the level
FIGURE 3.10 Plots of minimum $X^2$ values and the corresponding energy eigenvalues $E_\lambda$, and reduced width, $\gamma_\lambda^2$, as a function of the channel radius a. These data were obtained from one-level fits to the s-, p- and f-wave phase shifts.
### Table 3.7

<table>
<thead>
<tr>
<th></th>
<th>$E_r$ (LAB) (MeV)</th>
<th>$\Gamma_\lambda$ (LAB) (MeV)</th>
<th>$J^\pi$</th>
<th>$\gamma^2$ (C.M.) (MeV)</th>
<th>Excitation in $^{16}$O (MeV)</th>
<th>Dimensionless Reduced Width*</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jones et al. (1)</td>
<td>3.205</td>
<td>0.792</td>
<td>1$^-$</td>
<td>0.608</td>
<td>9.552</td>
<td>0.85</td>
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<td>3.556</td>
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<td>0.001</td>
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<td>0.0015</td>
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<td>0.033</td>
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<td>5.47</td>
<td>3.300</td>
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<td>0.55</td>
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<td>0.67</td>
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<td>12.43</td>
<td>0.04</td>
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<td>Larson and Tombrello (La 66)</td>
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<td>11.094</td>
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<td>0.0001</td>
<td>12.050</td>
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(1) Channel radius $a_o = 5.43$ fm.

(2) Channel radius $a_o = 5.4$ fm.

* Ratio $\gamma^2/\gamma_o^2$ where $\gamma_o^2$ is the single $\alpha$-particle reduced width $\frac{3\hbar^2}{2Ma^2}$
parameters extracted were not physical in that negative reduced widths were obtained.

The single level analysis described above was repeated with the energy range over which the $\delta_0$ phases were fitted, extended to 8.0 MeV by the inclusion of phases extracted from the $^{12}_C(\alpha,\alpha')$ excitation function data of Kerr et al. (Ke 68b). This data had a quoted accuracy of $\pm 10^0$. Such an analysis is not strictly valid, as the single channel assumption is not justified in this region. However, an excellent fit to the data ($X^2 \sim 0.4$) was once again obtained. The level parameters extracted are given in Table 3.8.

As the narrow $0^+$ level at 6.518 MeV has been studied in detail (La 66), no attempt was made to extract its level parameters.

3.5.2 Level Parameterization of the $\delta_1$ Data

Plots of optimum $X^2_1$, $E_{11}$, and $\gamma^2_{11}$ values as a function of channel radius $a_1$, are given in Fig. 3.10. These results were obtained from a single level fit to the p-wave phase shifts. The level parameters of the $1^-$ level, assuming a channel radius of 5.3 fm, in the region of 3.23 MeV, are given in Table 3.8 and the corresponding calculated phase shifts are shown as a solid line in Fig. 3.7. From Fig. 3.7, it can be seen that although the fit ($X^2 = 10.6$) is acceptable near the resonance energy, it is unsatisfactory away from the resonance indicating that distant $1^-$ levels are influencing the p-wave phase shift in this energy region.
TABLE 3.8

Level parameters from single-level fits to phase shifts derived from the present experiment using a channel radius of 5.3 fm.

<table>
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<tr>
<th>$E_\alpha^{(LAB)}$ (MeV)</th>
<th>Excitation in $^{16}$O (MeV)</th>
<th>$J^\pi$</th>
<th>$\Gamma$(C.M.) (MeV)</th>
<th>$2\gamma$(C.M.) (MeV)</th>
<th>Dimensionless Reduced Width$^+$</th>
<th>$X^2$</th>
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<tr>
<td>3.23 ± 0.08</td>
<td>9.58</td>
<td>1$^-$</td>
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<td>0.52 ± 0.06</td>
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<td>0.83</td>
<td>0.52 ± 0.04</td>
<td>0.69</td>
<td>7.3</td>
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<tr>
<td>9.1 ± 0.25 (10.25)*</td>
<td>14.0</td>
<td>0$^+$</td>
<td>4.8 (5.4)*</td>
<td>0.63 ± 0.08</td>
<td>0.85</td>
<td>0.8</td>
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</table>

* The bracketed figures represent the $0^+$ level parameters extracted from a single level fit to the $\delta_0$ phase over the energy region 2.8 to 8.0 MeV as discussed in the text.

$^+$ Ratio $\gamma^2 / \gamma_0^2$ where $\gamma_0^2$ is the single $\alpha$-particle reduced width $3/2 \hbar^2 / M\alpha^2$. 
To include the effects of higher $1^-$ levels, a multi-level fit to the p-wave phase was performed. Initially a three level analysis was made in which the levels at 3.23 MeV and 7.04 MeV ($\Gamma_{lab} = 0.13$ MeV) (Ke 68a) were included with a background level. Subsequently it was shown that the parameters describing the background could be adjusted to include the influence of the 7.04 MeV level. The quality of the analysis made with the two level approximation is illustrated in Table 3.9, where some parameter values are given for the case $a_1 = 5.3$ fm. The table gives the results of various fits in which a fixed value of $E_{21}$ was assumed for each fit, while $\gamma_{11}^2$, $E_{11}$ and $\gamma_{21}^2$ were varied. The inclusion of a background level in the analysis is justified by the fact that the $X^2$ value obtained with the two-level approximation is a factor of 2.5 smaller than that obtained with the one-level. It is seen that acceptable fits (say $X_1^2 \leq 4.0$) can be obtained for a wide range of $E_{21}$ values. It is noted that $E_{11}$ and $\gamma_{11}^2$ remain fairly constant as $E_{21}$ is varied, although there is a dramatic change in the value of $\gamma_{11}^2$ compared to the value obtained with a single-level fit. These results are for a channel radius of 5.3 fm. The dependence of the 'minimum' $X^2$ values on the channel radius $a_1$ is shown in Fig. 3.11.

The next step in improving the fit to the p-wave phase shift data was to extend the two-level fit to a three-level fit, by including the effect of the $1^-$ level at 7.116 MeV excitation in $^{16}$O. This level is suspected of
TABLE 3.9

Level parameters obtained from fits to $\delta_1^{\exp}$ in the region $2.8 \leq E \leq 6.6$ MeV using the two-level approximation with $a_1 = 5.3$ fm

<table>
<thead>
<tr>
<th>$E_{11}$</th>
<th>$\gamma_{11}^2$</th>
<th>$E_{21}$</th>
<th>$\gamma_{21}^2$</th>
<th>$X_1^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.29</td>
<td>0.53</td>
<td>5.0</td>
<td>3.79</td>
<td>13.3</td>
</tr>
<tr>
<td>1.24</td>
<td>1.00</td>
<td>10.0</td>
<td>1.21</td>
<td>3.9</td>
</tr>
<tr>
<td>1.30</td>
<td>0.91</td>
<td>20.0</td>
<td>2.83</td>
<td>2.8</td>
</tr>
<tr>
<td>1.34</td>
<td>0.90</td>
<td>30.0</td>
<td>4.95</td>
<td>3.4</td>
</tr>
<tr>
<td>1.44</td>
<td>0.85</td>
<td>40.0</td>
<td>7.17</td>
<td>3.8</td>
</tr>
<tr>
<td>1.45</td>
<td>0.82</td>
<td>50.0</td>
<td>8.19</td>
<td>3.5</td>
</tr>
<tr>
<td>1.43</td>
<td>0.81</td>
<td>60.0</td>
<td>8.91</td>
<td>3.7</td>
</tr>
<tr>
<td>1.41</td>
<td>0.79</td>
<td>70.0</td>
<td>9.74</td>
<td>3.9</td>
</tr>
<tr>
<td>1.44</td>
<td>0.76</td>
<td>80.0</td>
<td>10.18</td>
<td>4.1</td>
</tr>
</tbody>
</table>
FIGURE 3.11  Plots of minimum $X^2$ values as a function of channel radius $a$. These data were obtained from two-level fits to the $p$- and $f$-wave phase shifts.
having a large alpha-particle reduced width (Fo 64) and consequently, although
energy bound by 46 keV, its influence on the $^{12}$C + $\alpha$ reaction may be substantial.
As the resonance energy, $E_{r1}$, of the level is well known (Br 64), the eigen-
value energy, $E_{31}$, may be calculated using equations (I.20) and (I.21) of
Appendix I, provided the reduced width of the level $\gamma_{31}^2$ and the shift factor
$S^-$ are known. The shift factor, $S^-$, which is non-zero for the case of nega-
tive energy channels and charged particles, is defined in terms of Whittaker
functions in equation (I.16) of Appendix I. As no tabulations of Whittaker
functions for the appropriate $\eta$ and $\rho$ values are available, $S^-$ was calculated
using a W.K.B. approximation, as outlined by Lane and Thomas (La 58a).

As $E_{r1}$ is known, it was kept fixed during the fitting procedure while
the alpha-particle reduced width $\gamma_{31}^2$ and consequently the energy eigenvalue
$E_{31}$ were varied. Lobenstein et al. (Lo 67) extracted from their $^6$Li($^{12}$C, d)$^{16}$O
data a value of $\theta^2_{\alpha} = 0.1^{+} 0104$ for the dimensionless reduced width of the
7.116 MeV level. The corresponding value of $\gamma_{31}^2$, together with the values
of $E_{11}$, $\gamma_{11}^2$, $E_{21}$ and $\gamma_{21}^2$ given in Table 3.8, were used as starting values
in the fitting program. Initially $\gamma_{31}^2$ was optimized while the other parameters
were kept fixed at their initial values. Subsequently, the "final" level para-
meters were obtained by allowing $E_{11}$, $\gamma_{11}^2$, $E_{21}$, $\gamma_{21}^2$ and $\gamma_{31}^2$ to vary
simultaneously. The quality of the fits for the case of $a_1 = 5.3$ fm is illus-
trated in Table 3.10. The factor of 3 improvement in $X^2$ that occurs when
TABLE 3.10

Level parameters obtained from fits to $\delta_1^{\text{exp}}$

in the region $2.8 \leq E \leq 6.6$ MeV,

using the three-level approximation with $a_1 = 5.3$ fm.

As discussed in the text, the energy eigenvalue $E_{31}$

is not a free parameter.

<table>
<thead>
<tr>
<th>E_{11}</th>
<th>$\gamma_{11}^2$</th>
<th>E_{21}</th>
<th>$\gamma_{21}^2$</th>
<th>$E_{31}^2$</th>
<th>$\gamma_{31}^2$</th>
<th>$X_1^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.12</td>
<td>1.02</td>
<td>18.35</td>
<td>3.39</td>
<td>-0.99</td>
<td>0.26</td>
<td>1.5</td>
</tr>
<tr>
<td>1.17</td>
<td>0.99</td>
<td>28.90</td>
<td>5.42</td>
<td>-0.81</td>
<td>0.21</td>
<td>1.1</td>
</tr>
<tr>
<td>1.23</td>
<td>0.94</td>
<td>40.34</td>
<td>7.15</td>
<td>-1.61</td>
<td>0.43</td>
<td>0.9</td>
</tr>
<tr>
<td>1.23</td>
<td>0.93</td>
<td>49.24</td>
<td>9.05</td>
<td>-1.95</td>
<td>0.53</td>
<td>0.9*</td>
</tr>
<tr>
<td>1.21</td>
<td>0.95</td>
<td>58.01</td>
<td>11.87</td>
<td>-1.99</td>
<td>0.54</td>
<td>1.0</td>
</tr>
<tr>
<td>1.05</td>
<td>1.13</td>
<td>79.41</td>
<td>23.38</td>
<td>-2.23</td>
<td>0.60</td>
<td>1.0</td>
</tr>
<tr>
<td>0.95</td>
<td>1.25</td>
<td>99.73</td>
<td>33.87</td>
<td>-3.02</td>
<td>0.83</td>
<td>1.6</td>
</tr>
</tbody>
</table>
the effect of the 7.116 MeV level is included, justifies the three-level analysis. Acceptable fits ($X^2 \leq 1$) were obtained over a wide range (40 to 80 MeV) of $E_{21}$ values with $\gamma_{31}^2$ varying by about $\pm 10\%$ of its mean value in the range. Similarly $E_{11}$ and $\gamma_{11}^2$ vary by $\pm 15\%$ from their mean value in the range.

To determine the sensitivity of the fits to variations in the quantity $\gamma_{31}^2$, $X^2$ was plotted as a function of $\gamma_{31}^2$, the other parameters being kept fixed at the set of values marked with an asterisk (*) in Table 3.10. As shown in Fig. 3.12 variations of $-26\%$ or $+53\%$ are necessary to produce a $50\%$ increase in $X_{1}^2$. The reduced width $\gamma_{31}^2$ also lacks sensitivity to changes in the channel radius. This is illustrated in Fig. 3.13 where $\gamma_{31}^2$ is plotted as a function of $a_1$. The curve was obtained by minimizing $X^2$ for various channel radii; the set of values marked (*) in Table 3.10 being used as starting values.

Summarizing, it may be said that the inclusion of the bound level is necessary to obtain an optimum fit to the p-wave phase shift data, but unfortunately, the reduced width of the level cannot be determined to a high degree of precision.

Table 3.12 contains the optimum level parameters ($a_3 = 5.3 \text{ fm}$) assigned to the $1^-$ levels contributing to the $^{12}\text{C}(\alpha,\alpha)$ process in the region investigated. The parameters describing the broad background level are not included. The corresponding calculated phase shifts which give the
FIGURE 3.12 Plot of $X_1^2$ for variations in the parameter $\gamma_{31}^2$, the other parameters being kept fixed at the set of values marked with an asterisk (*) in Table 3.10.

FIGURE 3.13 Plot of $\gamma_{31}^2$ as a function of channel radius $a_1$. 
best fit to the $\delta_1$ data, are shown in Fig. 3.7.

3.5.3 Level Parameterization of $\delta_3$ Data

Plots of optimum $X^2_3$, $E_{13}$ and $\gamma^2_{13}$ values as a function of channel radius $a_3$ are given in Fig. 3.10. These results were obtained from a single level fit to the f-wave phase shifts in the energy region from 2.80 to 6.60 MeV. The level parameters ($a_3 = 5.3$ fm) which describe the broad $3^-$ level in the region of 5.71 MeV are given in Table 3.8 and the corresponding phase shifts are shown as a solid line in Fig. 3.7 ($X^2 = 7.3$).

Although a reasonable fit was obtained with the single-level approximation, the influence of higher $3^-$ levels was included in the analysis by performing a two-level fit to the data. Checks were made to verify the fact that the effects of the known $3^-$ levels at 7.960 and 8.130 MeV were included in the analysis by the introduction of the second level. The quality of the two-level analysis is illustrated in Table 3.11 where some parameter values are given for the case $a_3 = 5.3$ fm. This table gives the results for a series of fits to the data in which a fixed value of $E_{23}$ was assumed for each fit and the remaining parameters varied. Acceptable results (say $X^2_3 < 2.5$) were obtained for a wide range of $E_{23}$ values (30 to 100 MeV). Although the improvement in $X^2$ by a factor of 3.6 justifies the two-level analysis, it is notable that the level parameters obtained with the two different approximations do not differ dramatically. The corresponding minimum $X^2_3$ values
Level parameters obtained from fits to $\delta_3^{\exp}$ in the region $2.8 \leq E \leq 6.6$ MeV, using the two-level approximation with $a_1 = 5.3$ fm.

TABLE 3.11

<table>
<thead>
<tr>
<th>$E_{11}$</th>
<th>$\gamma_{11}$</th>
<th>$E_{21}$</th>
<th>$\gamma_{21}$</th>
<th>$X_3^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.83</td>
<td>0.53</td>
<td>10.0</td>
<td>1.69</td>
<td>20.7</td>
</tr>
<tr>
<td>3.36</td>
<td>0.89</td>
<td>20.0</td>
<td>4.93</td>
<td>8.9</td>
</tr>
<tr>
<td>3.49</td>
<td>0.55</td>
<td>30.0</td>
<td>2.48</td>
<td>2.1</td>
</tr>
<tr>
<td>3.52</td>
<td>0.54</td>
<td>40.0</td>
<td>3.70</td>
<td>2.3</td>
</tr>
<tr>
<td>3.44</td>
<td>0.58</td>
<td>50.0</td>
<td>6.38</td>
<td>1.9</td>
</tr>
<tr>
<td>3.51</td>
<td>0.57</td>
<td>60.0</td>
<td>8.58</td>
<td>2.4</td>
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<td>3.18</td>
<td>0.62</td>
<td>70.0</td>
<td>10.90</td>
<td>2.4</td>
</tr>
<tr>
<td>3.22</td>
<td>0.60</td>
<td>80.0</td>
<td>12.61</td>
<td>2.5</td>
</tr>
<tr>
<td>3.32</td>
<td>0.59</td>
<td>100.0</td>
<td>10.42</td>
<td>2.9</td>
</tr>
</tbody>
</table>
TABLE 3.12

The optimum level parameters from multi-level fits to the phase shifts derived from the present experiment using a channel radius of 6.3 fm.

<table>
<thead>
<tr>
<th>E (LAB) α</th>
<th>(MeV)</th>
<th>Excitation in (^{16})O</th>
<th>(J^\pi)</th>
<th>(\Gamma) (C.M.) (MeV)</th>
<th>(\gamma^2) (C.M.) (MeV)</th>
<th>Dimensionless Reduced Width *</th>
<th>(X^2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.24 ± 0.04</td>
<td>7.116</td>
<td>1^{-}</td>
<td></td>
<td>0.91</td>
<td>0.53 ± 0.28</td>
<td>0.71</td>
<td>0.9</td>
</tr>
<tr>
<td>5.80 ± 0.06</td>
<td>9.59</td>
<td>1^{-}</td>
<td></td>
<td>0.97</td>
<td>0.93 ± 0.05</td>
<td>1.2</td>
<td>0.9</td>
</tr>
<tr>
<td>9.1 ± 0.25</td>
<td>11.54</td>
<td>3^{-}</td>
<td></td>
<td>4.8</td>
<td>0.58 ± 0.04</td>
<td>0.78</td>
<td>1.9</td>
</tr>
</tbody>
</table>

* Ratio \(\gamma^2/\gamma_o^2\) where \(\gamma_o^2\) is the single \(\alpha\)-particle reduced width \(\frac{3 + h^2}{2M_o^2}\)

---

---
for other channel radii are shown in Fig. 3.11.

A three-level analysis of the data was attempted but the parameter values extracted were not physical in that negative reduced widths were obtained.

Table 3.12 contains the optimum level parameters \(a_3 = 5.3\) fm) assigned to the \(3^-\) level contributing to the \(^{12}\text{C}(\alpha,\alpha)\) process in the region investigated. The parameters describing the broad background level are not listed. The corresponding calculated phase shifts which give the best fit to the \(\delta_3\) data, are shown in Fig. 3.7.

Table 3.12 contains a list of optimum level parameters assigned from the present experimental data to the broad \(0^+, 1^-\) and \(3^-\) levels contributing to the \(^{12}\text{C}(\alpha,\alpha)\) process in the region from 9.16 to 12.11 MeV excitation in \(^{16}\text{O}\). The corresponding results of other authors are given in Table 3.7. As mentioned in section 3.4.4, these level parameters may have an inherent energy dependence unless the level shift \(\Delta_f\), for closed channels varies linearly with energy. However in the present case the non-linear component of \(\Delta_f\) is so small that its effect on the parameters is within the experimental error assigned to them.

3.6 Discussion

The behaviour of the s-wave phase shift may be interpreted as a broad level \(\Gamma(C.M.) = 4.8\) MeV) in the vicinity of 14 MeV excitation in \(^{16}\text{O}\)
(E$_{\alpha}$ = 9.1 MeV), but because of the broad nature of this level, these parameters are critically dependent on the channel radius chosen (e.g., for a channel radius of 5.5 fm $\Gamma$(C.M.) = 5.0 MeV and $E_{\alpha}$ = 8.9 MeV). There is no evidence for the 0$^+$ level predicted by Bittner and Moffat at 11.25 MeV excitation in $^{16}$O ($E_{\alpha}$ = 5.47 MeV).

Since experimental evidence (Ca 64) has established that the excited states of $^{16}$O have properties appropriate to members of rotational bands in deformed nuclei, extensive calculations (e.g., Ei 65, Bo 66, Br 66, Se 66) have been made in an attempt to explain theoretically these bands. These calculations are based on extensions of the shell model and in particular on the model of Green and Brown (Br 66), which proposes that the bands are mixtures of deformed two-particle, two-hole (2p-2h) and four-particle, four-hole (4p-4h) states with the spherical shell model ground state. The presence of the 0$^+$ state at 11.26 MeV excitation is critical for the existence of one of the rotational bands. The calculations described above predict its existence and usually predict its position to a high degree of accuracy. Some authors (Ph 67) have used this level for normalization purposes. The non-existence of the 0$^+$ level at 11.26 MeV, as indicated in the present work, thus detracts from the acceptability of these calculations, which otherwise enjoy a reasonable amount of success. These difficulties may possibly be removed by identifying the calculated 0$^+$ state with the narrow 0$^+$ state that has been
observed at 12.05 MeV excitation (La 66). Since the non-existence of the broad 0\(^+\) level has been reported Eisenberg (Ei 68) has indicated that the 0\(^+\) level predicted in this region from his shell model calculation, is probably a very narrow one and therefore should perhaps be identified with the 12.05 MeV state.

Qualitatively the level parameters of the 1\(^-\) level at 9.59 MeV excitation are in agreement with previous results (Jo 62), but this work indicates a resonance energy \(E_{\text{ex}} = 0.59\) MeV, \(\gamma = 0.940\) MeV) 40 keV higher than that assigned by Jones. This brings the scattering data into agreement with the reaction data (La 64, Lo 67), although the \(^{15}\text{N}(^{3}\text{He},p)\) data (Br 63) indicates that the level is still 30 keV higher.

The importance in astrophysical calculations of a quantitative knowledge of the reduced width of the 7.12 MeV level in \(^{16}\text{O}\) has often been discussed (Fo 64, St 66). Lobenstein (Lo 67) has estimated the width of this level from the \(^6\text{Li}({^{12}\text{C},d})^{16}\text{O}\) reaction data and assigned an extracted dimensionless reduced width\(\dagger\) of \(\theta_{\alpha} = 0.1 \pm 0.04\) to the level. This result is based on a Hauser-Feshbach calculation. Calculations (St 66) based on the model of \(^{16}\text{O}\) proposed by Brown and Green (Br 66) predict a dimensionless reduced width, defined as the ratio \(\gamma^2/\gamma_o^2\), where \(\gamma_o^2\) is the single alpha-particle value \(3/2\hbar^2/\text{Ma}^2\).

\(\dagger\) Defined as the ratio \(\gamma^2/\gamma_o^2\), where \(\gamma_o^2\) is the single alpha-particle value \(3/2\hbar^2/\text{Ma}^2\).
\[ \theta_\alpha = 0.08 \pm 0.04. \] Fowler (Fo 64) has estimated a reduced width \( \theta_\alpha \approx 0.78 \) for the state by using a cluster model of \( ^{16}\text{O} \). Prior to the present experiment, no direct measurement of the width had been made. The value assigned to the dimensionless reduced width from the present experiment is:

\[ \theta_\alpha = 0.71^{+0.37}_{-0.18}. \]

This value is considerably larger than that assigned by Lobenstein (Lo 67) and Stephenson (St 66). However, as this result was obtained from the experimental data by a more direct method than the estimates of previous workers, it is considered more reliable. The result is in agreement with that of Fowler.

This analysis confirms the prediction (Bi 54) of a \( 3^- \) level at 11.54 MeV excitation in \( ^{16}\text{O} \). The level parameters obtained from the fits to the f-wave phase shifts are qualitatively in agreement with the previous work. However, the present analysis places the level \( (E_{\text{ex}} = 11.54 \text{ MeV}, \gamma^2 = 0.58 \text{ MeV}) \) 80 keV lower than the previous measurement and assigns a width 70 keV wider.

The variation of the d-, g- and h-wave background phase shifts from the hard sphere value may be interpreted as an influence of background states.
APPENDIX I

It is assumed in Chapters I and III that the $^6$Li($d,\alpha$)$^4$He and $^{12}$C($\alpha,\alpha$)$^{12}$C reactions are two-stage processes proceeding through intermediate states of definite spin parity in $^8$Be and $^{16}$O respectively. On the basis of this assumption the R-matrix theory of nuclei reactions (La 58) was used to interpret the experimental results in terms of levels in the compound nucleus $^8$Be and $^{16}$O.

This appendix presents a list of formulae used in the analysis of these experiments and defines the terms used in these formulae. All the following expressions are either implicitly or explicitly derived by Lane and Thomas (La 58) or Blatt and Biedenharn (Bl 52).

I.1 Analytic Form of the Differential Cross Section

Suppose the wave function of a pair of particles, $\alpha$, which interact is characterized by the set of quantum numbers $\alpha s l \nu m$, where $s$ is the channel spin (and component $\nu$) and $l$ is the relative angular momentum (and component $m$).

The channel $\alpha s l \nu m$ (denoted by the symbol $c$) may alternatively be written $\alpha s l J M$, where $J$ is the total spin quantum number ($J = s + l$) with component $M$. The intermediate states of definite spin-parity $J^\pi$ are denoted by $\lambda$ and $\mu$.

Initially the basic channel characteristics will be defined:
\( E_c = E'\alpha', \) the energy of relative motion of the particles of the pair \( \alpha; \)

\[
M_c = M'\alpha = \frac{M'\alpha_1 M'\alpha_2}{M'\alpha_1 + M'\alpha_2}, \text{ the reduced mass;}
\]

\[
k_c = k'\alpha = \left| \frac{2M'\alpha |E'\alpha|}{\hbar^2} \right|^{1/2}, \text{ the wave number;}
\]

\[
\nu_c = \nu'\alpha = \frac{\hbar k}{M'\alpha}, \text{ the relative velocity;}
\]

\[
\eta_c = \eta'\alpha = \frac{Z_1'\alpha Z_2'\alpha e^2}{\hbar \nu'\alpha}, \text{ the Coulomb field parameter;}
\]

\( r_c = r'\alpha, \) the interaction radius;

\[
\rho_c = \rho'\alpha = \frac{k_c r_c}{\alpha};
\]

\[
\sigma_c = \sigma'\alpha = \arg \Gamma(1 + \ell_c + i\eta_c); \]

\[
\omega_c = \omega'\alpha = \sigma_c - \sigma'\alpha = \sum_{m=1}^{l} \tan^{-1} \left( \frac{\eta_c}{m} \right). \quad (I.1)
\]

For unpolarized incident and target particles the differential cross section for the process \( \alpha \rightarrow \alpha' \) is:

\[
\frac{d\sigma}{d\Omega} = \left[ (2l_1 + 1)(2l_2 + 1) \right]^{-1} \sum_{ss', \nu \nu'} |A_{\alpha'\alpha', \nu \nu}(\Omega')|^2 d\Omega', \quad (I.2)
\]
where $A_{\alpha's',\nu'}, \alpha s_{\nu'}^{(\theta)}}$ are the complex scattering or reaction amplitudes of the outgoing waves in the channel $\alpha's',\nu'$ which are associated with a unit-flux incident plane wave in the channel $\alpha s_{\nu'}$. They are:

$$A_{\alpha's',\nu'}, \alpha s_{\nu'}^{(\theta)}} = \frac{\pi^{\frac{1}{2}}}{k} \left[ - C_{\alpha'(\alpha')} \delta_{\alpha's',\nu'}, \alpha s_{\nu'} + i \sum_{JM'\ell'm'} (2\ell + 1)^{\frac{1}{2}} \right. $$

$$ \cdot (s_{\ell'\nu'} | JM)(s's'\nu'm' | JM) T_{\alpha's',\nu'}^{J}, \alpha s_{\ell'm'}^{(\theta)}} \right],$$

where

$$T_{\alpha's',\nu'}^{J}, \alpha s_{\ell'm'}^{(\theta)}} = e^{2i\omega_{\alpha's'}} \delta_{\alpha's',\nu'}, \alpha s_{\ell'm'}^{(\theta)}} - U_{\alpha's',\nu'}^{J}, \alpha s_{\ell'm'}^{(\theta)}} \right],$$

and

$$C_{\alpha'(\alpha')} = (4\pi)^{\frac{1}{2}} \eta_{\alpha'} \csc^{2} \left( \frac{\theta_{\alpha}}{2} \right) \exp \left\{ - 2i\eta_{\alpha'} \log \sin \left( \frac{\theta_{\alpha}}{2} \right) \right\}. \quad (I.5)$$

$U_{\alpha's',\nu'}^{J}, \alpha s_{\ell'm'}^{(\theta)}}$ is the collision matrix, discussed in section (I.2), which contains the nuclear properties of the problem.

The squaring operation in equation (I.2) introduces two sets of summing integers $\{J M' \ell' m'\}$ and $\{J M \ell m\}$, to give:

$$\left(2s+1\right) \frac{k^{2}}{\Pi} \frac{d\sigma}{d\theta} \bigg|_{\alpha's',\nu'}^{\alpha' \nu'} = \left(2s+1\right) \left| C_{\alpha'(\alpha')} \right|^{2} \delta_{\alpha's',\nu'} \cdot \alpha s$$

$$+ \frac{1}{\Pi} \sum_{J \ell} B_{\ell}(\alpha's', \nu) P_{\ell}(\cos \theta_{\alpha'}) + (4\pi)^{-\frac{1}{2}} \sum_{(2J+1)2. Re} \left[ i T_{\alpha's',\nu'}^{\ell'}, \alpha s_{\ell}^{(\theta)}} C_{\alpha'(\alpha') \ell}^{J}(\cos \theta_{\alpha'}) \right]. \quad (I.6)$$
where

\[ B_L (\alpha's', \alpha s) = \frac{1}{\Xi} (-1)^{s-s'} \sum_{J J' \lambda' \lambda \mu' \mu} \overline{Z} (l J, \lambda \mu J', \lambda' \mu, s L) \]

(I.7)

\[ \cdot \overline{Z} (l', \lambda', J', \lambda' \mu, s' L) (T^\lambda \alpha's' l', \alpha s L) (T^\lambda \alpha's' l', \alpha s L)^* \]

The summations extend over all terms corresponding to the transitions satisfying the conservation of angular momentum and parity. In particular the sum over the integer L includes all terms that satisfy the three conditions:

1. \( \max (|l^\lambda - l^\mu|, |l'^\lambda - l'^\mu|, |J^\lambda - J^\mu|) \leq L \)

2. \( \min (|l^\lambda + l^\mu|, |l'^\lambda + l'^\mu|, |J^\lambda + J^\mu|) \geq L \)

3. \((l^\lambda + l^\mu - L)\) and \((l'^\lambda + l'^\mu - L)\) are both even.

The \( \overline{Z} \) coefficients in equation (I.9) are purely geometrical factors and contain no specific nuclear properties. They differ from the \( \overline{Z} \)-coefficient defined by Blatt and Biedenharn (Bl 52) by a phase factor. This is due to a difference in the time reversal conventions.

\[ \overline{Z} (l, J l, J, s L) = i^{l^\lambda - l^\mu - L} Z (l, J l, J, s L) \]

In equation (I.6) the first term may be identified as pure Coulomb
scattering, the second as a nuclear scattering and reaction term and the last as an interference term from the first two.

**SPECIAL CASES**

**Case 1: Elastic Scattering with s = 0**

For spin zero incident particles scattered from a spin zero target, conservation of angular momentum requires that \( l = l' = J, s = s' = 0 \) and \( \alpha = \alpha' \). Then equation (1.6) becomes:

\[
\sigma_{el}^0(\theta) = \frac{1}{k^2} \left( - \frac{\eta_\alpha}{2} \csc^2 \left( \frac{\theta}{2} \right) \exp \left[ i \eta_\alpha \log \csc^2 \left( \frac{\theta}{2} \right) \right] \right) 
\]

\[(I.9)\]

\[
+ \frac{i}{2} \sum_{l} (2l + 1)(e^{2i\omega_l} - U_l) P_l(\cos \theta) \right)^2.
\]

The analytic form of \( U \) is given in section I.2.

**Case 2: Reaction Cross Section**

For reactions, \( \alpha \neq \alpha' \), the Coulomb scattering and interference terms of (I.6) vanish. Then:

\[
\sigma(\theta)_{\text{reaction}} = \frac{1}{(2s + 1)k^2} \sum \beta L(\alpha's', \alpha s)P_L(\cos \theta_{\alpha'}), \quad (I.10)
\]
I. 2 Collision Matrix and Level Parameters - Introduction of R-Matrix

The expression (I.9) and (I.10) for the differential elastic and reaction cross sections each involve the collision matrix $U$ which contains the details of the physical problem. To give $U$ an explicit form the assumption is made that there exists for any pair of nuclei in channel $c$ a finite radius $a_c$, beyond which neither nucleus experiences any polarizing potential from the other. Since $U_c$ has the properties of unitary and symmetry, it may be expressed in terms of a phase shift $\delta_c$,

$$U_c = e^{2i\delta_c}.$$

If elastic scattering is the only open channel $\delta_\ell$ is real. If more than one channel is open, then $\delta_\ell$ is complex.

A matrix in channel space, $R_{c'c}$, is defined by equating the logarithmic derivative of the internal wave function to zero at the surface $r = a_c$,

$$R_{c'c} = \sum_\lambda \gamma_{c'} \gamma_c / (E_{c',c} - E),$$

(I.12)
where $E_{\lambda c}$ is the energy eigenvalue and

$$\gamma_\lambda = \left( \frac{\hbar^2}{2Ma^2} \right)^{\frac{1}{2}} u_\lambda^{(a)}$$  \hspace{1cm} (I. 13)

is the reduced width of the level $\lambda$. The term $u_\lambda^{(a)}$ denotes the radial part of the internal wave function. The quantity

$$\gamma_0^2 = \frac{3\hbar^2}{2Ma^2}$$  \hspace{1cm} (I. 14)

is the "Wigner limit" of the reduced width. It is, on the average, the upper limit that may be placed on the numerical value of the reduced width. The dimensionless reduced width is normally defined as:

$$\theta_\lambda^2 = \gamma_\lambda^2 \left( \frac{Ma^2}{\hbar^2} \cdot \frac{2}{2} \right),$$

although at times the ratio of the reduced width to the "Wigner limit" is called the dimensionless reduced width.

For the single channel case the channel $c$ can be denoted by $\ell$, the angular momentum quantum number. The phase $\delta_\ell$ is then given by:

$$\delta_\ell = \beta_\ell - \phi_\ell + \omega_\ell,$$  \hspace{1cm} (I. 15)
where

\[ \beta_\ell = \tan^{-1} \left( \frac{R_\ell P_\ell}{1 - R_\ell S_\ell} \right), \]

the resonant phase shift,

\[ \varphi_\ell = -\tan^{-1} \left( \frac{F_\ell}{G_\ell} \right), \]

the hard sphere phase shift, and

\[ \omega_\ell = \sum_{m=1}^{\ell} \tan^{-1} \frac{\eta_m}{m}, \]

the Coulomb phase shift.

The terms \( P_\ell \) and \( S_\ell \) are known as the penetration factor and the shift factor respectively. They are defined in terms of the regular and irregular Coulomb functions, \( F_\ell \) and \( G_\ell \), and the Whittaker function \( W \) by:

\[ S^+_c = \left[ \rho_c \left( \frac{F F' + G G'}{F + G} \right) \right]_{r_c = a_c} \]

\[ S^-_c = \left[ \rho_c W' / W \right]_{r_c = a_c} \]

\[ P^+_c = \left[ \rho_c / \left( \frac{F^2 + G^2}{F} \right) \right]_{r_c = a_c} \]

\[ (I.16) \]
The signs + and - refer to positive and negative energy channels respectively.

In general $R$ is a sum over levels. However, if the energy is sufficiently close to one level, $E_\lambda$ say, then,

$$R_{\ell} \sim \frac{2}{\gamma_{\lambda\ell} / (E_\lambda - E)}, \quad (I. 17)$$

and one obtains the one-level approximation to the phase shift

$$\beta_{\ell} = \tan^{-1}\left(\frac{\frac{1}{2} \Gamma_{\lambda\ell}}{E_{\ell} + \Delta_{\lambda\ell} - E}\right), \quad (I. 18)$$

where the level width

$$\Gamma_{\lambda\ell} = 2\gamma_{\lambda\ell}^2 p_{\ell} \quad (I. 19)$$

determines how fast the phase changes when $E$ passes through the resonance energy,

$$E_{r\ell} = E_{\lambda\ell} + \Delta_{\lambda\ell}, \quad (I. 20)$$

and the level shift

$$\Delta_{\lambda\ell} = -\gamma_{\lambda\ell}^2 S_{\ell} \quad (I. 21)$$
is the amount by which the resonant energy is shifted from the eigenvalue position $E_{\lambda'\mu'}$.

The conventional variation of a resonant phase shift, $\beta_{\lambda'\mu'}$, as a function of energy is thus to increase through 180° across the resonance with a value of 90° at the resonance energy.

The definitions (I.21), (I.20) and (I.19) also apply for the many-level case.

The logarithmic derivative at the surface may, in general, be equated to any constant $B$. The previous formulae, in which $B$ was taken as being equal to zero, may be adapted to this condition by replacing $S$ by $S^0$ and $L$ by $L^0$, where

$$L^0 = L - B$$
$$S^0 = S - B$$  \hspace{1cm} (I.22)

For the more general case of particles with spin and more than one channel open $U$ is given an explicit form in terms of matrices $\Omega$ and $W$,

$$U = \Omega W \Omega$$  \hspace{1cm} (I.23)

where

$$W_{cc'} = \delta_{cc'} + 2i P_c^\frac{1}{2} (\sum_{\lambda \mu} \gamma_{\lambda \mu} A_{\lambda \mu}) P_{c'}^\frac{1}{2}$$  \hspace{1cm} (I.24)

and
\[ \Omega = \exp i(\omega_c + \phi_c) . \]

The level matrix \( A_{\lambda \mu} \) is given by:

\[ (A^{-1})_{\lambda \mu} = (E_\lambda - E)\delta_{\lambda \mu} + \Delta_{\lambda \mu} - \left(\frac{i}{2}\right) \Sigma_c \Gamma^\frac{1}{2}_c \Gamma^\frac{1}{2}_{\lambda c} \mu_c . \]  \hspace{1cm} (I. 25)

For elastic scattering in the one level approximation, the collision matrix has the form:

\[ U_{cc} = \exp 2i(\omega_c + \phi_c) \left[ 1 + \frac{2i\Gamma^\frac{1}{2}_c \Gamma^\frac{1}{2}_{\lambda c'}}{(E_\lambda + \Delta - E)^2 + \frac{1}{4} \Gamma^2_{\lambda c'}} \right] . \]  \hspace{1cm} (I. 26)

where

\[ \Gamma_\lambda = \Sigma_c \Gamma_{\lambda c} , \]

and

\[ \Delta_\lambda = -\Sigma_c (S_c - B_c) \gamma^2_{\lambda c} . \]

The one-level approximation for reactions \((c \neq c')\) is:

\[ U_{cc'} = \exp i(\omega_c + \phi_c + \omega_{c'} + \phi_{c'}) \left[ \frac{2i\Gamma^\frac{1}{2}_c \Gamma^\frac{1}{2}_{\lambda c'}}{E_\lambda - E - i\Gamma_{\lambda c} / 2} \right] . \]  \hspace{1cm} (I. 27)

The energy dependence of the general many-level many-channel term in equation (I. 7) is contained in the factor
\[ \frac{J_{\lambda}}{R_{\lambda}(U_{cc}^\prime)}(U_{dd}^\prime)^* = \frac{4 \cos(\xi_c + \xi_{c'} - \xi_d + \xi_{d'}) - \beta_{\mu} + \beta_{\lambda}}{\lambda}(\Gamma_{\lambda c})^{\frac{1}{2}}(\Gamma_{\lambda c'})^{\frac{1}{2}}(\Gamma_{\lambda d})^{\frac{1}{2}}(\Gamma_{\lambda d'})^{\frac{1}{2}} \left[ (E_\lambda + \Delta_\lambda - E)^2 + \frac{1}{4} \Gamma_\lambda^2 \right]^{\frac{1}{2}} \left[ (E_\mu + \Delta_\mu - E)^2 + \frac{1}{4} \Gamma_\mu^2 \right]^{\frac{1}{2}} \]  

(I. 28)

where

\[ \xi_c = \omega_c - \phi_c. \]  

(I. 29)

In the case of pure terms involving no interference effects, this becomes:

\[ \left| \frac{J_{\lambda}}{U_{cc}^\prime} \right|^2 = \frac{4 \Gamma_{\lambda c}}{(E_\lambda + \Delta_\lambda - E)^2 + \frac{1}{4} \Gamma_\lambda^2} \left( 4 \Gamma_{\lambda c'} \gamma_{\lambda c} \gamma_{\lambda c'} \right) \frac{4 \Gamma_{\lambda c}}{(E_\lambda + \Delta_\lambda - E)^2 + \frac{1}{4} \Gamma_\lambda^2}. \]  

(I. 30)

This is the Breit-Wigner form.
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