THE EVOLUTION OF A TOKAMAK DISCHARGE

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PREFACE

Inevitably in a close-knit team such as the Plasma Physics Group, in which the present research was conducted, the achievements reflect not merely the author's work but also what he has learned from others. I should like to acknowledge, therefore, from the outset, the encouragement, assistance and stimulation provided by my colleagues.

Dr. J. D. Strachan first suggested the importance of studying the early stages of the discharge and made numerous helpful suggestions. My supervisor, Dr. A. H. Morton, has encouraged and guided me with a light hand throughout and I thank him especially for his constant willingness to interrupt his own train of thought to listen to one of mine. Discussions with Dr. M. G. Bell and Dr. R. L. Dewar have also been of great benefit.

Nevertheless, the research reported herein represents almost entirely my own work except, obviously, when I have attempted to summarise other results, to which appropriate reference has been made. The chief exception is the investigation of perturbation mode structure by external probes (section 5.3) which was performed by Dr. Morton.

The technical assistance particularly of R. M. McLeod, who made the probes, was invaluable to me and I thank also Mrs. H. Hawes who typed the manuscript. I am grateful to Professor S. Kaneff for the opportunity to work in the Department of Engineering Physics. Acknowledgement is made for the financial support of the Australian Government in the form of a scholarship under the Commonwealth Scholarship and Fellowship Plan and also for the supplementation by the Australian National University.
Finally I should like to express my dependence upon the one who first devised plasmas. It is to this end that the inscriptions to the chapters are intended.

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I. H. Hutchinson.
ABSTRACT

The evolution of the LT-3 Tokamak discharge is investigated, particularly in its early development and during the disruptive instability.

When the toroidal electric field is initially applied, under certain circumstances, a delay is observed before the main current rise and bulk ionisation onset. During this 'prebreakdown' phase it is shown that transverse drifts of runaway electrons dominate the discharge behaviour until a certain critical electron line density is exceeded.

During the period of ionisation of the filling gas, the electron energy balance between ohmic heating input and inelastic collisional losses leads to the development of a hollow 'skin like' current profile. This effect depends only upon the presence of a centrally peaked electron density and so is expected to occur widely in Tokamaks.

Subsequent to the completion of ionisation the hollow current profile relaxes rapidly to a more uniform distribution. Simultaneously, helical perturbations of the plasma column and rapid runaway electron diffusion are observed. These are interpreted as showing the relaxation to be due to the development of an MHD (probably tearing mode) instability.

The expansion of the current channel at the disruptive instability is observed using an internal magnetic probe. The results confirm the triggering to be MHD in origin, probably through the interaction of different helical instabilities. However, an extreme resistivity anomaly exists in the discharge during the expansion which suggests the importance of microinstabilities. Preliminary investigations reveal high frequency plasma fluctuations which point to the presence of ion cyclotron waves.
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1.1 GENERAL

A major motivation for plasma physics research continues to be the prospect of producing energy in a controlled manner from the thermonuclear fusion of light elements in a plasma which is sufficiently well contained at sufficiently high temperature.

Broadly speaking we may divide fusion prospects into two major confinement schemes: (1) Magnetic Confinement. (2) Inertial Confinement. The latter is something of a late starter and it is only since the declassification of laser fusion work in the United States\(^1\) that the concept has appeared attractive. Very little experimental investigation of laser compression has been performed and it is therefore difficult to assess the prospects of the approach as a whole.

Magnetic confinement, however, has been the subject of detailed experimental and theoretical study in a wide variety of configurations. In what may be classified as 'open' geometries (ones in which the magnetic surfaces on which the plasma is confined intersect the chamber walls at some point), for example the 'mirror' machine\(^2\), the 'z-pinch'\(^3\) or the 'g-pinch'\(^4\), the confinement is limited, in general, by plasma loss along the magnetic field. For this reason 'closed' geometries with magnetic surfaces which are closed within the plasma present an attractive prospect. The 'toroidal z-
pinch', of which Zeta\textsuperscript{5} is the most famous example, the Stellarator\textsuperscript{6}, the 'toroidal $\theta$-pinch' (Scyllac\textsuperscript{7}) and the Tokamak\textsuperscript{8} are examples of closed systems. In any closed system it may be shown that the magnetic surfaces are topologically equivalent to a torus.

Since the confirmation of temperatures of the order of keV in Tokamak T-3\textsuperscript{9}, the Tokamak has appeared an attractive approach to containment and heating. In this device the toroidal magnetic field is considerably larger than the poloidal field. The confinement scheme may be regarded in one of two equivalent ways. Either from the MHD viewpoint as a toroidal $z$-pinch with a very strong stabilising toroidal field or, from the particle orbit viewpoint, as a toroidal trap formed by the toroidal field with the transverse particle drifts due to the curvature of the magnetic field (inevitable in toroidal geometry) being compensated by a rotational transform arising from the toroidal current flowing in the plasma itself. This distinguishes it from the Stellarator where the rotational transform is produced by external windings. Heating is also provided in a Tokamak by the toroidal plasma current.

The toroidal electric field required to maintain the plasma current is induced by a change in the magnetic flux linking the torus; effectively the plasma becomes a one-turn secondary of a transformer. Herein lies a major limitation because the total change in magnetic flux, often referred to as the 'Volt-seconds', is limited to a finite value. This means that unless the effective resistance of the plasma column falls to zero a Tokamak cannot run in a steady state; it must have a repeated cycle of operation whose length is limited by the Volt-seconds available. It has been suggested\textsuperscript{10} that the effective plasma resistance may fall to zero because of an effect known as
the 'bootstrap current' predicted by 'Neoclassical Transport Theory'; however, this remains unverified experimentally.

An issue which may prove to be an even more stringent limitation upon the length of a reactor discharge is the problem of impurities. The interaction of energetic plasma particles with the walls or limiter causes the sputtering of heavy impurity atoms. These can then enter the plasma and reach high degrees of ionization. The presence of such high-\(Z\) ions is generally deleterious to plasma heating and energy confinement, principally because of the increased line and recombination radiation for which they are responsible.

The heart of the problem is that Neoclassical theory predicts a preferentially inward diffusion of impurities leading to a build-up of high-\(Z\) materials near the plasma centre. Experimental results suggest\(^{11}\), though not unequivocally, that such build-up does occur. The removal of impurities once they penetrate the plasma appears to be an insurmountable problem, in which case the discharge duration is limited by the time taken to reach unacceptable levels of plasma impurities.

The fact that Tokamaks almost certainly have to run in a cyclic mode immediately enhances the importance of studying the way in which the discharge develops in time. Not only must we understand the behaviour of the plasma in a 'quasi-steady' state but we must also examine the stages through which the plasma evolves in approaching that state, and, of course, how it 'devolves' from that state back to the original state as cold gas.

The early stages of the discharge are particularly important because the conditions of the formation of the discharge can have a persistent in-
fluence upon the whole character of the discharge throughout its duration. One obvious possibility is that an interaction of the plasma with the walls or limiter early on can significantly alter the impurity content of the plasma. Again, the population of runaway electrons or the level of microturbulence may significantly alter the character of the discharge at later times. It is in this context that the present thesis seeks to investigate certain features of the evolution of the plasma discharge in the Canberra Tokamak LT-3 in an attempt to develop a broader understanding of the general principles underlying Tokamak discharge evolution.

1.2 TOKAMAK EVOLUTION

The normal sequence of events in the initiation of a tokamak discharge may be summarised as follows:

(a) The toroidal magnetic field $B_\phi$ is applied by energising the windings. $B_\phi$ is normally slowly varying with respect to most other processes.

(b) Some form of preionisation is usually applied to provide an initial small degree of ionisation. This is very often in the form of a radio frequency (R. F.) discharge.

(c) The toroidal electric field ($E_\phi$) is applied leading to:

(d) Ionisation of the filling gas, and

(e) Rise of the toroidal plasma current ($I$).

Until fairly recently very little attention has been paid to understanding the processes occurring during the early stages, ionisation and initial current rise of Tokamaks. Interest has been centred upon the hottest plasma produced at around peak toroidal current. It is becoming recognised, however,
for the reasons outlined in the preceding section that an understanding of these stages is essential for the effective operation of the Tokamak. There is also considerably greater confidence in the results concerning the hot quasi-steady-state stages so increasing attention is now being paid to evolution and the early stages.

The processes taking place upon the initial application of the toroidal electric field are far from fully understood. The free electrons present are accelerated and cause further ionisation by impact with neutrals but beyond this the processes governing the rate of ionisation and current growth in the specifically toroidal situation have yet to be clarified. Chapter 3 is concerned with investigating this situation and a theoretical model is developed which provides quite good agreement with the experimental observations. This model can predict optimum fields for gas breakdown as well as the minimum degree of preionisation required for immediate breakdown.

In a Tokamak, provided a certain initial degree of preionisation is present, the toroidal electric field will normally cause bulk ionisation to occur over a time dependent upon $E_\phi$ and the initial gas filling pressure. During this time and subsequent to it the toroidal plasma current rises and the resistance of the plasma column falls, as total ionisation is approached and the electron temperature, $T_e$, begins to rise. An important question concerning the current rise is how the current arrives at its final distribution over the torus minor cross-section. In any highly conducting medium we expect to observe a 'skin effect' for sufficiently rapid changes in current. For a material of constant electrical conductivity such as copper wire the penetration time may be calculated by solving the simple diffusion equation for the magnetic
field (or equivalently the current density). For a plasma, however, the conductivity is proportional to $T_e^{3/2}$ and this effect greatly complicates the issue. In fact the temperature dependence will tend in general to enhance any skin effect that may be present. This occurs because any concentration of current flow will cause preferential ohmic heating of the plasma in that area leading to temperature rise, conductivity rise and therefore current density rise, providing positive feed-back.

The presence of a broad, or even hollow, current density profile in a Tokamak is generally deleterious to energy confinement, peak temperature and overall stability as well as being a cause of energy wastage. There have been several computational simulations of Tokamak discharges in which the (coupled) equations governing heat flux and magnetic field diffusion are solved in order to predict the development of the current profile. In nearly all cases a pronounced and persistent skin effect is predicted, in sharp contrast with the experimentally observed results of rapid current diffusion and central peaking of the electron temperature. The explanation of this discrepancy must lie in the enhancement of diffusion above the Neoclassical values presumably because of instabilities.

The evolution of the LT-3 discharge in this period subsequent to the commencement of bulk ionisation is investigated in Chapters 4 and 5. Energy balance in the ionisation stage leads to the formation of a hollow current profile. The observed rapid relaxation of this profile is attributed to the development of resistive MHD helical instabilities which cause break-up of the magnetic surfaces leading to rapid current diffusion.

Under what may be considered normal, stable Tokamak conditions
the evolution of the discharge subsequent to the current-rise stage is largely quiescent. It consists in a steady rise of electron and ion temperature until a state of equilibrium is attained. The current profile usually retains a roughly constant shape, though some slight 'peaking' in the centre may occur.

In contrast, under certain conditions the Tokamak suffers what is known as the disruptive instability. This is characterised by the appearance of negative-going spikes on the externally measured toroidal voltage, expansion of the current channel minor radius and decrease of the major radius. There is a considerable deterioration in the discharge confinement properties at the disruption, whose occurrence is contingent upon high particle densities and low values of the safety factor, \( q \). It therefore constitutes a major limitation to the increase of particle density and ohmic heating input.

The disruptive instability is essentially a problem of discharge evolution. Firstly because there are gross changes in the plasma through the instability itself and secondly because the occurrence of the instability usually follows a period of contraction of the current distribution. The causes of the disruption itself or of the current contraction leading to it are still a matter of debate even though their effects have been known since the early years of Tokamak research.

In Chapter 6 we focus upon the instability itself and the rapid processes taking place through it. An extremely high anomalous resistivity is observed at the disruptive instability which, together with observations of ion heating points to a conclusion that microinstabilities play an important role in the instability development.
CHAPTER 2.

THE TOKAMAK LT-3

Isaiah 46:6

2.1 ELECTRICAL AND MECHANICAL CONFIGURATION

LT-3 is a modification of the machine LT-1 whose construction was initiated in the Department of Engineering Physics in 1964. Full details of the design and operation of LT-1 have been published\(^{19-20}\). We recapitulate them only insofar as is necessary for the understanding of the operation of LT-3 required here.

A schematic diagram of the machine is shown in Fig 2.1. The inconel vacuum vessel, circular in minor cross-section, is divided into quadrants, electrically insulated from one another by alumina spacers, vacuum sealed by atmospheric pressure on 'Viton-A' O-rings. Surrounding the vacuum vessel is a $\frac{3}{4}$" thick copper shell, which is also divided into quadrants, each of which is further divided by a horizontal slit to allow the penetration of the main toroidal magnetic field. This field is generated by current in a toroidal coil wound over the copper shell. Its winding is somewhat uneven but the consequent inhomogeneities in the magnetic field are compensated by currents in the copper shell. Over the toroidal winding lies a primary winding which is linked to the plasma as secondary by an iron core. The core may be biassed prior to the discharge by an appropriate winding in order to allow a greater flux swing (i.e. more Volt-seconds). No mechanical limiter is
Fig 2.1. The Tokamak LT-3
normally provided although molybdenum rods may be inserted through ports above and below the plasma. However perturbations in the magnetic field are found to create a 'magnetic aperture' somewhat smaller than the vacuum vessel (c.f. Table 1).

Fig 2.2 shows the electrical circuits for the operation of LT-3. The circuits powering the primary and toroidal coils are entirely separated. Capacitor bank 2 provides a typical clamped current through the toroidal coil. Bank 1 is likewise clamped and powers the primary winding in series with an inductive choke. Several auxiliary parts of the circuit are included. The spark-gap prevents damage to the circuit caused by extremely high voltages which can occur across the primary if the plasma suddenly extinguishes. Its role was previously fulfilled by the bypass 1 which was switched once bulk ionisation was completed but normally this bypass is now only used for altering the standard wave-form. Bypass 2 enables the discharge to be chopped after the period of interest. An auxiliary 'dummy' capacitor may be included in parallel with the choke in order to simulate the operation of the circuit of LT-1. Its presence leads to high initial voltages and rapid initial current rise facilitating the investigations of Chapter 3.

Preionisation of the gas is provided by a \(< 300\ \mu\text{sec}\) pulse of 2 MHz radio frequency (R. F.) power capacitively coupled at \(< 2\ \text{kV}\) to opposite inconel quadrants. The pulse is applied after the toroidal magnetic field has begun to rise but ends before the application of the toroidal electric field. At low pressures the R. F. breakdown can become erratic and recently an additional electron source in the form of a glowing filament just outside the main vessel in an access port has been introduced. This has greatly improved the
Fig 2.2. LT-3 Electrical Circuits
consistency of the R. F. breakdown.

Table 1 summarises the parameters of the machine and plasma as a whole. Fig 2.3 gives the typical sequence of the operation of the machine. Normally the shape and magnitude of the plasma current is determined by the external circuit while the toroidal electric field adjusts itself according to plasma conditions.

The systems of coordinates used hereafter are illustrated in Fig 2.4. Both cylindrical \((R, \phi, z)\) about the torus vertical major axis and toroidal \((r, \phi, \theta)\) coordinates are referred to, as convenient. The minor axis is usually taken as the vacuum vessel axis unless the shift of the plasma is explicitly referred to.

2.2 DIAGNOSTICS

The diagnosis of Tokamak plasmas constitutes a major problem to the experimenter. The combination of high temperature, high magnetic field and low plasma density leads to a situation where the introduction of probes can lead to unacceptable perturbation of the plasma or else probe damage. Likewise, most of the radiation from the plasma is in the vacuum ultraviolet or x-ray regions of the spectrum where instrumentation is difficult, whilst scattering experiments require high power densities for detectability.

The difficulties are magnified on LT-3 because access to the plasma is provided only via ports 6 mm in diameter and 10 cm long. However the electron temperature is generally rather lower than in most Tokamaks and this allows probes to be used in certain circumstances. We summarise here the various diagnostics available on LT-3. Further details of their interpretation and implementation are given at other points in the thesis where necessary.
TABLE 1

The Parameters of LT-3

<table>
<thead>
<tr>
<th>Major Radius</th>
<th>R</th>
<th>40 cm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vessel Minor Radius</td>
<td>w</td>
<td>10 cm</td>
</tr>
<tr>
<td>Copper Shell Radius</td>
<td>b</td>
<td>11.5 cm</td>
</tr>
<tr>
<td>Aperture Radius</td>
<td></td>
<td>~ 9 cm</td>
</tr>
<tr>
<td>Hydrogen Filling Pressure</td>
<td>$p_0$</td>
<td>~ 0.2 - 1.0 mtorr</td>
</tr>
<tr>
<td>Toroidal Magnetic Field</td>
<td>$B_\phi$</td>
<td>~ 1 T</td>
</tr>
<tr>
<td>Perpendicular Magnetic Field</td>
<td>$B_\perp$</td>
<td>~ 6.10^{-4} T</td>
</tr>
<tr>
<td>Plasma Toroidal Current</td>
<td>I</td>
<td>~ 25 kA</td>
</tr>
<tr>
<td>Discharge Duration</td>
<td></td>
<td>~ 8 msec</td>
</tr>
<tr>
<td>Electron Temperature</td>
<td>$T_e$</td>
<td>~ 100 eV</td>
</tr>
<tr>
<td>Ion Temperature</td>
<td>$T_i$</td>
<td>~ 60 eV</td>
</tr>
<tr>
<td>Electron Density</td>
<td>$n_e$</td>
<td>~ 3.10^{19} m^{-3}</td>
</tr>
<tr>
<td>Energy Confinement Time</td>
<td>$\tau_e$</td>
<td>~ 200 μsec</td>
</tr>
</tbody>
</table>
Fig 2.3. The typical operating sequence, indicating the times at which various switches are closed.

Fig 2.4. The Coordinate Systems.
2.21 Electrical

The total toroidal plasma current is measured by a Rogowski coil encircling the torus between the toroidal and primary windings. The Rogowski coil output, proportional to the time derivative of the current linking it, is actively integrated using an operational amplifier to give the current waveform.

A single loop linking the core, adjacent to one set of primary windings measures the volts per turn \( V_\phi \) around the torus. Combining this with the plasma current we obtain the plasma resistance which provides an estimate of the mean plasma electron temperature.

A loop encircling the liner measures the poloidal e.m.f. \( V_\theta \) or equivalently the rate of change of toroidal magnetic flux. This is the basis of the measurement of the so called diamagnetic effect \(^{21}\), giving the value of the poloidal beta \( \beta_1 \) of the plasma. The implementation of this measurement and some results are presented in appendix 1.

2.22 Probes

The use of magnetic probes is made possible on LT-3 because of the comparatively low electron temperatures. Even so, during the main part of the discharge introduction of the probes to closer than \( \sim 3.5 \) cm from the axis perturbs the plasma so much as to suppress the disruptive instability. During the cooler early stages of the discharge the perturbation is less severe and probes can be inserted right on to axis.

Three-dimensional probes, measuring the magnetic field components in three directions simultaneously have been used on LT-1. However, as the extra information gained was not considered to justify the increased interpret-
ation difficulties, in the present work only the poloidal component was measured. Fig 2.5 shows the construction of the two types of probe used. Both have the same design of probe coil using enamelled copper wire on a boron nitride former. The first consists of a single coil, while the second has an array of 18 individual coils spaced along its length. Both are housed in silica tubes of ~6 mm outer diameter. The coil outputs are integrated to give response proportional to the magnetic field.

Because the toroidal magnetic field is about ten times as large as the poloidal field the coils must be carefully aligned to avoid any pick-up of the toroidal field. This is achieved by adjusting the probe alignment for zero signal when the toroidal field is applied without a plasma. Owing to slight misalignments of the coils of the multi-coil probe it is impossible to align all the coils simultaneously. However, by taking a shot with the toroidal magnetic field only, we can obtain the toroidal field pick-up which may be subtracted to obtain the true poloidal field.

Floating double Langmuir probes may be used on LT-3. Their construction is shown schematically in Fig 2.6. The current-voltage characteristic may be obtained either statically, varying the applied voltage from shot to shot, or else by a rather novel swept-probe technique described in appendix 2. In addition, the probe may be used to observe electrostatic fluctuations in the plasma to provide information on instabilities.

Information on runaway electrons is obtained by a runaway probe. This technique, developed on LT-1\textsuperscript{22}, consists of introducing a wire probe, sometimes with a target on the end, into the plasma. Runaway electrons on surfaces that intersect the probe collide with the probe, emitting thick target
Fig 2.5. The Two Types of Magnetic Probe.
Fig 2.6. The Langmuir Probe Construction.
bremsstrahlung x-rays. These are detected through a port opposite that through which the probe is introduced by a NaI (TI) crystal and photomultiplier. In this way information upon the spatial distribution and diffusion of runaways may be obtained. Their energies may be determined by pulse-height analysis or absorber filtering.

The structure of perturbations of the plasma may be determined using arrays of magnetic probes situated outside the plasma at various values of $\theta$ and $\phi$. These are similar to the single-coil magnetic probes previously described.

2.23 Spectroscopic

Line radiation from the hydrogen filling gas and impurities such as oxygen and carbon is observed with monochromators. This can provide information upon such questions as electron temperature and density, neutral particle density and impurity influx.

The radial profile of the emission of a line may be obtained by abelling the emission observed through seven ports distributed across the discharge.

A technique for obtaining the line profile has been developed using a high-resolution McPherson Model 2051 monochromator with specialised exit optics known as the 'polychromator' to observe simultaneously seven adjacent spectral bands of width $\sim 0.25 \, \text{Å}$. For lines such as those of oxygen impurities for which the spectral shape is primarily caused by doppler broadening an ion temperature may immediately be deduced.

The emission of energetic neutral particles has been observed by a stripping cell and ion energy analyser, thus providing information on the high energy tail of the ion velocity distribution.
CHAPTER 3.

PARTICLE DRIFTS IN THE PREBREAKDOWN PHASE

Ecclesiastes 3:2

3.1 INTRODUCTION

In several investigations of the ionisation stages of toroidal discharges, a delay has been observed between the application of the toroidal electric field, $E$, and the rise in current as ionisation occurs\textsuperscript{25-28}. We find that LT-3 exhibits a similar delay which is the concern of this chapter. This "prebreakdown" phase is not characterised by an exponential growth in current. There appears to be a definite transition between the low-current prebreakdown phase, where runaway electrons dominate the discharge, and the rapidly increasing current of the "breakdown" phase during which complete ionisation occurs.

Measurement of gas current and runaway electron energy indicates that the development of the discharge is strongly affected by the drift of the runaways across the magnetic field during the prebreakdown stage. The drift is not compensated by the rotational transform or the image currents in the conducting casing whilst the current is low. When the current reaches a certain critical value the transition to breakdown occurs. This is interpreted as the onset of particle containment by the poloidal field.

We obtain theoretical predictions of the critical current above which compensation of the drifts should occur, and the expected maximum energy
which runaway electrons attain before being lost, during the phase below critical current. In addition the effect of a superimposed perpendicular magnetic field is considered. These predictions are compared with the experimental results and explain a number of features of the discharge, confirming the dominance of the drift of runaway electrons in the prebreakdown phase.

3.2 GENERAL

The results of this chapter refer to the operation of LT-3 with the dummy capacitor included in the primary circuit (c.f. Fig 2.2). This has two major effects. Firstly it means that prior to any appreciable current flow in the primary the toroidal electric field, \( E \), is rather higher \( \sim 500 \text{ Vm}^{-1} \) than is usual in a Tokamak. Once breakdown occurs and the plasma current rises the electric field falls to more typical values. Secondly the plasma current rises much more rapidly, once breakdown is established, than is the case when the dummy capacitor is excluded. Both these effects enhance the abruptness of the transition between the prebreakdown and the breakdown phases, thus facilitating the investigations.

Molybdenum rods are used as limiters above and below the plasma at positions \( z = \pm 7 \text{ cm} \).

A 250 \( \mu \text{sec} \) pulse of r.f. provides an initial degree of ionisation and about 50 \( \mu \text{sec} \) later the primary winding is energised, inducing the toroidal electric field. The subsequent delay of the major current rise occurs at initial hydrogen pressures, \( p_0 \), below about 0.7 mTorr. At higher pressures the current rise becomes practically immediate. A lower limit to \( p_0 \) is imposed by the failure of the r.f. pre-ionisation below about 0.25 mTorr. The
The majority of the measurements are made at 0.42 mTorr. The duration, $t_B$, of the delay is extremely sensitive to variations of $p_0$ and becomes effectively infinite for $p_0 \leq 0.35$ mTorr, when the current disappears in about $10\mu$sec. Typically, through, the delay, $t_B$, is between 20 and 100 $\mu$sec. During this prebreakdown period B and E are substantially constant chosen in the ranges $0.15 - 0.4$ tesla and $200 - 700 \text{ V} \cdot \text{m}^{-1}$ respectively. The gas current, $I$, is about $50 - 400$ A.

A runaway electron probe, consisting of a 1 mm diameter tungsten wire, reveals the presence of considerable numbers of runaways. These have comparatively low energies. The thick-target bremsstrahlung X-rays emitted on their collision with the probe are indicative of runaway energies around 10 keV.

Fig 3.1 shows a typical discharge. The end of the r.f. pre-ionisation pulse is visible on the E trace which subsequently disappears off-scale when the primary is fired. For about $60\mu$sec following, during the prebreakdown phase, the X-ray signal and current remain low until the transition to breakdown occurs when they rise steeply and the electric field falls.

The prebreakdown phase of another discharge is shown in greater detail in Fig 3.2. The current shows noticeable oscillations which the X-ray signal follows (the lag of the X-rays behind the current is due to the decay time of the photomultiplier circuit.) At the time of breakdown, when the X-ray signal increases rapidly, the X-ray energy also shows a sharp increase corresponding to electron energies $\geq 50$ keV. At this time the current is between 500 and 1000 A.

The ionisation growth in a toroidal discharge is governed by the
Fig 3.1. The initiation of the discharge $E$, $17 \text{ V m}^{-1}$ per division; 
$I$, $2.5 \text{ kA}$ per division; X-ray signal, arbitrary scale;
Time: $20 \mu\text{sec}$ per division.

Fig 3.2. The prebreakdown phase X-ray signal (inverted); $E$, 
$330 \text{ V m}^{-1}$ per division; $I$, $250 \text{ A}$ per division; Time
$10 \mu\text{sec}$ per division.
characteristic time, $\tau_1$, which is the reciprocal of the number of ionising collisions per electron per second. It has been determined theoretically and experimentally that for the parameters here, $\tau_1$ has a value $2 \mu$sec. This is very much shorter than the duration, $t_B$, of the prebreakdown phase, so appreciable loss mechanisms must be operating.

The rate of runaway of electrons in a slightly ionised gas has been estimated theoretically. In our regime we expect all the electrons to run away almost immediately even at the lowest values of our electric field. We should expect all the current in the initial stages, therefore, to be carried by runaways. This is confirmed by the similarity of the X-ray and current traces.

3.3 THEORY

3.3.1 Drift Compensation by the Rotational Transform

In a magnetic field with radius of curvature $R$, charged particles experience a drift across the magnetic field with velocity:

$$v = \frac{u^2}{R \Omega}$$

where $u$ is the longitudinal velocity along the magnetic field and $\Omega = eB/m$ is the gyro frequency. For runaway electrons this rapidly becomes the dominant drift owing to the second power of the constantly increasing velocity $u$. Under normal conditions in a tokamak the loss of particles due to such a drift is compensated by the rotational transform caused by the toroidal current. We may describe this in terms of the safety parameter $q(r) = rB/RB_\theta (r)$ where $r$ is the minor radius, $R$ the major radius and $B_\theta$ the poloidal magnetic field. In the prebreakdown phase the rotational transform is very small, corresponding to
\[ q(a) \sim 100 \text{ where } a \text{ is the radius of the plasma. In such circumstances particle loss may still occur. Assuming approximate cylindrical symmetry about the minor axis we have} \]

\[
B_\theta (a) = \frac{\mu I_0}{2\pi a} \tag{3.2}
\]

The condition for the drift \( v \) of an individual particle to be able to be compensated by \( B_\theta \) is that the transverse velocity due to the poloidal magnetic field should be greater than the drift velocity, i.e.

\[
v < u \frac{B_\theta}{B} \tag{3.3}
\]

On substituting from Eqs (3.1) and (3.2) this becomes

\[
\frac{2\pi ma}{\mu_0 eR} u < I \tag{3.4}
\]

3.32 Equilibrium

We have so far considered the behaviour of the electrons only under the influence of their own magnetic field and derived a condition for the current channel to hold together by the mutual attraction of current elements. However, it is important to consider also whether the currents induced in the metal casing are able to balance the drift of the electrons as a body. That is, do we have overall equilibrium?

The condition for equilibrium to be obtainable is that the magnetic field produced by an imaginary image current outside the casing should balance the drift velocity in the plasma. The distance between the centres of image current and plasma current is at least \( 2a \) and so the transverse magnetic field is half that given in Eq 3.2. The current required is therefore at least twice that given by Eq 3.4.33.
We may obtain a condition more rigorously from the expression given by Shafranov for the displacement, \( \Delta \), of the plasma inside a conducting shell together with the condition \( \Delta < w-a \) (where \( w \) is the vacuum wall radius) that the plasma should not intersect the walls. For a purely runaway discharge the longitudinal momentum is the dominant term and the condition reduces to

\[
(3.5) \quad \frac{2\pi m(a+w)}{u_0 eR} u < I
\]

If this is satisfied then so is Eq (3.4). We shall therefore use Eq (3.5) in comparison with experiment.

If we substitute typical parameters for our regime \( R = 0.4 \text{ m} \), \( a = 0.07 \text{ m} \), \( w = 0.1 \text{ m} \) and \( I \sim 100 \text{ amperes} \) we find that Eq (3.5) indicates that runaway electron drifts are compensated only for energies less than \( \sim 100 \text{ eV} \). Our runaways have energies \( \sim 10 \text{ keV} \), however. We therefore conclude that in the prebreakdown phase there is neither overall equilibrium due to the conducting casing nor self-containment of the electrons due to the rotational transform.

Equation (3.5) likewise expresses the condition for the transition to breakdown, since when the current rises to satisfy it we begin to contain the runaways. This, in turn, further increases the current and so containment and bulk ionisation are rapidly "switched on".

3.33 Electron Drift Lifetimes

In view of the absence of containment during prebreakdown, we expect the electrons, as they accelerate along the magnetic field, to drift across it until they are lost to the limiter.

For a freely accelerating runaway electron
(3.6) \[ u = \frac{eE}{m} \cdot t \]

so

(3.7) \[ v = \frac{1}{R \Omega} \cdot \left( \frac{eEt}{m} \right)^2 \]

and the distance drifted across the magnetic field is

(3.8) \[ d = \int vdt = \frac{1}{R \Omega} \left( \frac{eE}{m} \right)^2 \frac{t^3}{3} \]

This may be rewritten, using Eq (3.6) in terms of the electron energy \( W \) as

(3.9) \[ W = \frac{m}{2} \left( \frac{3R \frac{e^2}{m^2} d}{(EB)^{2/3}} \right)^{2/3} \]

which expresses the energy an electron has reached when it has drifted a distance \( d \). A typical distance drifted before being lost to the limiter is \( d \sim a \), so we have

(3.10) \[ W \approx 540(EB)^{2/3} \text{ eV} \]

as the maximum expected runaway energy. This may also be expressed as a lifetime, \( \tau_p \), i.e. the time before an electron is lost. This is

(3.11) \[ \tau_p = \left( \frac{3RdBm}{eE^2} \right)^{1/3} \approx 7.8 \times 10^{-5} \left( \frac{B}{E^2} \right)^{1/3} \]

This is typically about 1 \( \mu \)sec which is less than the ionisation time \( \tau_i \sim 2 \mu \)sec.

3.34 The Influence of Perpendicular Magnetic Fields

If we impose an externally generated vertical magnetic field, \( B_\perp \), perpendicular to the minor axis in an attempt to compensate the toroidal drift, then the total transverse velocity becomes

(3.12) \[ v = \frac{u^2}{R} - \frac{B_\perp}{B} \cdot u \]
The first term is the curvature drift and the second is the transverse motion due to the vertical field, applied in such a direction as to compensate the first term. Evidently if we wish \( v \) to be zero then we require \( B_\perp \) to be proportional to \( u \). During the prebreakdown phase, however, a whole spectrum of velocities exists as electrons are created, accelerate and are then lost.

Nor do we have the necessary windings inside the conducting casing to produce the rapidly varying vertical field required to compensate a monoenergetic beam.

We consider, therefore, the effect of a static field. For a runaway starting at time \( t = 0 \), Eq (3.12) becomes (substituting from Eq (3.6))

\[
(3.13) \quad v(t) = \frac{eE}{mB} t \left( \frac{E}{R} t - B_\perp \right)
\]

and the perpendicular distance drifted is

\[
(3.14) \quad d(t) = \frac{eE}{mB} t^2 \left( \frac{E}{3R} t - \frac{B_\perp}{2} \right)
\]

This is stationary when \( v = 0 \), i.e. \( t = RB_\perp/E \); then

\[
(3.15) \quad d = \frac{d}{m} = \frac{eR^2}{mEB} \cdot \frac{B_\perp^3}{6}
\]

which may be written

\[
(3.16) \quad B_\perp = \left( \frac{6d}{mEB} \frac{mEB}{eR^2} \right)^{1/3}
\]

If \( d/m \) is greater than the transverse dimensions of the plasma, most electrons will drift out under the influence of the perpendicular magnetic field, while if it is less than the transverse dimensions they will tend to drift out in the opposite direction, as the toroidal drift overcomes the compensation.

This may be expressed as a condition on \( B_\perp \) using Eq (3.16). With \( d/m \sim a \) our case gives a critical value:
To obtain the corrected particle containment time, $\tau_p^\prime$, for a given $B_\perp (< B_{lc})$, we put $d(t) = a$; then Eq (3.14) may be rearranged by comparison with Eqs (3.11) and (3.16) to give

$$1 = \frac{t^3_p}{\tau^3_p} - \frac{B_\perp}{B_{lc}} \cdot \frac{3}{4} \cdot \frac{t^2_p}{\tau^2_p}$$

whose solution is the required containment time:

$$t = \frac{\tau^\prime_p}{\tau_p} = \alpha \frac{\tau_p}{\tau_p} \text{ (say)}$$

For $0 < B_\perp \leq B_{lc}$ we find

$$1 < \alpha < 2.1$$

Here $\alpha$ is the proportional increase in containment time due to the perpendicular field and depends only upon the ratio $B_\perp / B_{lc}$; not on other machine parameters. The corresponding electron energy is

$$W' = \alpha W$$

For $B_\perp > B_{lc}$ we expect that there should be a rapid deterioration once more in breakdown characteristics because of the overcompensation.

3.4 LT-3 RESULTS

3.41 Initial Electron Density

Since we may assume that on the application of the electric field all the electrons immediately run away, we may obtain a measure of the degree of pre-ionisation as follows. If $N$ is the number of electrons per unit length of
the column

\[(3.22) \quad I = \text{Neu}\]

For times such that \(t < t_p < t_I\), \(N\) is approximately constant and the electron stream is approximately monochromatic, with \(u\) given by Eq (3.6). Therefore,

\[\frac{dI}{dt} = \frac{d}{dt} (\text{Neu}) = N \frac{e^2}{m} E\]

This shows that we expect initially \(dI/dt \propto E\) and their ratio gives a measure of the initial line density \(N\). Fig 3.3 shows a typical shot and it can be seen that the expected proportionality holds quite well for the first 0.5 \(\mu\)sec or so.

A plot against filling pressure of the calculated line density and the electron density, \(n\), corresponding to a uniform column of radius 7 cm is shown in Fig 3.4; there are considerable random variations. We indicate the pressure, \(p_c\), at which the delay to breakdown disappeared. This corresponds, as indicated, to \(N \approx 3.5 \times 10^{13} \text{m}^{-1}\) or \(n \approx 2.3 \times 10^{15} \text{m}^{-3}\).

We may rewrite the condition, Eq (3.5), in terms of the line density thus:

\[(3.24) \quad N = \frac{I}{e u} > \frac{2\pi m(a^+w)}{\mu_0 e^2 R}\]

which for our values is

\[(3.25) \quad N > 7.5 \times 10^{13} \text{m}^{-1}\]

This expresses the condition for particle drifts to be compensated and so should be the condition for the absence of the delay to breakdown. There is a short time before the electrons begin to be lost, however, (roughly \(\tau_p\)) after the electric field is switched on. During this period the initial line density
Fig 3.3 The initial electron density measurements $\frac{dl}{dt}$ (inverted), $10^7 \text{ A. s}^{-1} \text{ per division}; E, 165 \text{ V. m}^{-1} \text{ per division}; \text{ Time 0.5 } \mu\text{sec per division.}$
increases, owing to ionisation, by a factor \( \exp \left( \frac{\tau_p}{\tau_i} \right) \). Our measured initial line density is increased by this factor before particles begin to be lost.

The theoretical condition on the initial line density which corresponds to Eq (3.24) is therefore

\[
N > \frac{2 \pi m(a+w)}{\mu_0 e^2 R} \exp \left( -\frac{\tau_p}{\tau_i} \right) = 4.5 \times 10^{13} \text{ m}^{-1}
\]

which should be compared with the measured value.

Fig 3.4. The initial electron density produced by r.f. pre-ionisation versus filling pressure. The pressure \( p_c \) (at which the delay vanishes), and the corresponding density are shown. (\( B = 0.17 \) tesla, \( B_\perp = 3.7 \) G).

3.42 Electron Energies

Runaway electron energies have been measured as function of \( E \) and \( B \) by pulse-height analysis during the prebreakdown phase. Resolution of individual pulses was achieved by placing a variable (< 4 mm) thickness of aluminium absorber in front of the detector, attenuating the signal. This has the effect of absorbing mostly the low-energy X-rays; the pulses recorded therefore give an estimate of the maximum X-ray energy, and correspondingly the maximum electron energy.
The results are shown in Fig 3.5. Each point represents the mean of a number of pulses (about 15 on average) and the error bars indicate one standard deviation of the mean. We also show for comparison the theoretically predicted maximum electron energy given by Eq (3.21). No significant variation of energy was observed with variations of filling pressure, nor with variations of radial position of the target wire. This latter may be understood as due to the fact that X-rays may be observed from runaways which collide with the wire at any point of its length (unlike the situation in Chapter 5 where a target is fixed to the end of the wire). Since the direction of curvature drift is in our case upwards, towards the wire, the maximum-energy X-rays arise from that part of the wire which is at the largest radius.

![Graphs showing measured maximum X-ray energies compared with theoretically predicted energies](image)

Fig 3.5. Measured maximum X-ray energies (points) compared with the theoretically predicted maximum runaway electron energy,

(a) versus electric field ($B = 0.23$ tesla)

(b) versus magnetic field ($E = 450$ volt m$^{-1}$)

$p_0 = 0.42$ m Torr; $B_\| = 3.7$ G.
3.43 Critical Current and Delay Time

The gas current, \( I_B \), at the time of breakdown, defined quite accurately as the time of sharp rise in runaway energy, is plotted as a function of \( B \) in Fig 3.6(a) and \( E \) in Fig 3.6(b). If we substitute the predicted maximum velocity corresponding to the energy in Eq (3.21) into the condition, Eq (3.5), for breakdown we obtain

\[
I > \alpha \frac{4\pi a}{\mu_0} \left( \frac{3 m d}{e R^2} \right)^{1/3} (EB)^{1/3}
\]

as the theoretically predicted critical current condition. This is included in Fig 3.6 for comparison with the measured values.

Fig 3.6. Observed current at the time of breakdown and the theoretically predicted critical current,

(a) versus electric field \( (B = 0.23 \text{ tesla}) \)
(b) versus magnetic field \( (E = 450 \text{ V} \cdot \text{m}^{-1}) \)

\( p_0 = 0.42 \text{ mTorr}; \quad B_\perp = 3.7 \text{ G} \).

Figure 3.7 shows \( t_D \), the time from the beginning of the discharge
to the moment of breakdown. No increase of $t_B$ with $B$ such as occurred in other investigations was observed. The increase of $t_B$ with $E$ is to be expected since $\tau_p \propto E^{-2/3}$. It should be emphasized, however, that $t_B \gg \tau_1$.

![Graph showing relationship between $t, \mu s$ and $E, V/m$](image)

**Fig 3.7. Delay to breakdown, $t_B$ (same conditions as Fig 3.6)**

### 3.44 Effect of Vertical Magnetic Fields

The effect of a static vertical magnetic field upon the delay time $t_B$ is shown in Fig 3.8. The improvement obtained is greater than considerations of confinement times might lead one to expect. The issues are complicated by the fact that the vertical field is observed to affect the degree of ionisation produced by the r.f. Whilst the delay was effectively zero, electron energies obviously could not be measured and when $t_B$ begins to increase again as over-compensation occurs there are very few X-rays observed, corresponding to the loss of the majority of electrons in the opposite direction to the wire. Applying
an adverse field increases $t_B$ and a decrease in runaway energy was observed.

Fig 3.8. The effect of a constant vertical magnetic field upon the delay to breakdown, $t_B$. $E = 430 \text{ V m}^{-1}$; $B = 0.2 \text{ tesla}; p_0 = 0.55 \text{ mTorr}$. (Error bars show the standard deviation of the mean.)

3.5 DISCUSSION

Our theoretical model supposes that, during the prebreakdown phase, electrons drift freely under the influence only of the curvature drift and therefore their maximum energy is determined by the time this drift takes to carry them to the walls. The condition for the transition to breakdown to occur is that the current should be great enough to compensate these drifts. We have ignored questions of electron density profile and current profile and freely used "typical" dimensions without detailed consideration.

Nevertheless, the theoretical predictions concerning the initial line density, the runaway electron energies during prebreakdown, and the critical current for breakdown are well confirmed by experimental measurements. Investigation of the effect of perpendicular magnetic fields also appears to confirm the model.

The mechanisms enabling the rise to the critical current and determining the delay of such a rise remain obscure. This is especially true in view of
the fact that the particle lifetime is less than the ionisation time and that the delay is much greater than either of them. In previous studies\textsuperscript{1-4} the delay has been of the right order to be produced by exponential increase in ionisation. This mechanism alone cannot explain our results.

It seems that we require some mechanism whereby electrons can be re-introduced into the discharge; secondary emission from the walls and limiter may provide this. We might also expect electric fields to provide other particle drifts. These could arise from the overall displacement of electrons in the discharge or from a build-up of excess charge on the plasma giving a radial electric field. These fields are greatly affected (through the effective permittivity of the plasma) by the plasma density and so may provide a feedback mechanism producing the oscillations in current and X-rays previously noted. It may be that the rise of the breakdown current above the theoretical in Fig 3.6 (b) is due to the increasing importance of other drifts.

The possibility of runaways being braked by plasma instabilities\textsuperscript{35} cannot be excluded, though at these very low degrees of ionisation it seems likely that plasma turbulence is ineffective in producing such braking\textsuperscript{36}. Once the critical current is exceeded, the discharge enters a period of exponentially increasing ionisation and more typical tokamak behaviour ensues.

Since the overall behaviour of a tokamak plasma may be affected by the conditions of its initiation, it may be that prebreakdown conditions have a rather extensive significance\textsuperscript{37-38}. It appears undesirable to have a long delay, $t_B$, since during this period the induced voltage is high, and extensive saturation of the iron core may occur, shortening the discharge. The present results indicate that this may be avoided by pre-ionisation to densities above...
the critical or by reduction of the initial electric field. For larger machines
the critical initial electron density, $n$, corresponding to Eq (3.26), is
smaller, being inversely proportional to the square of the torus linear dimen-
sions.

The presence of a prebreakdown phase may bear directly upon such
factors as the initial runaway population and the level of plasma turbulence
and may therefore be a factor contributing to the observed dependence of
discharge character on pre-ionisation$^{37,38}$. 
CHAPTER 4.

THE IONISATION STAGE

Ecclesiastes 3:3

4.1 INTRODUCTION

Once particle confinement is established, the exponential growth in ionisation begins and the electron density rises from the low values of the pre-ionisation and prebreakdown phases until practically all the neutral gas is ionised. It is this ionisation stage which is the concern of the present chapter.

The behaviour of the discharge during this period is dominated by complex atomic processes such as collisional excitation, ionisation and dissociation. Nevertheless we find that many of the broad features of the discharge development may be understood in terms of a simple theoretical model of electron energy balance.

There are some discrepancies in the quantitative details between theory and experiment, due possibly to runaway electron current or thermal conductivity whose complete modelling would require a far more elaborate theory. Nevertheless the theory models very closely, in the main, the experimentally observed evolution. In particular it is found that when the electron density is peaked on axis, as is the case in LT-3, the current density evolves from an initial centrally peaked distribution to a hollow current profile.

The theory appears to have a rather wide applicability in Tokamak-
type discharges, depending, as it does, only upon the dominance of inelastic collisions in the electron energy loss.

4.2 EXPERIMENTAL RESULTS

4.21 Gross Properties

In Fig 4.1 we show the time development of the gross properties of the discharge during the first millisecond of its evolution.

The toroidal magnetic field, $B_\phi$, and the plasma current, $I$, are controlled essentially by the external circuit. For the conditions shown, which are the main concern of this and the next chapter, the dummy capacitor (c.f. Fig 2.2) is not included in the primary circuit. The plasma current is thereby constrained to rise almost linearly during this period. The toroidal electric field, $E_\phi$, is much lower than in the previous chapter, its precise value being determined by the total plasma resistance.

The poloidal beta, $\beta_p$, which is a measure of the total plasma kinetic energy, is deduced from the diamagnetic measurement. The accuracy of this measurement is severely restricted at early times when the current is small, as illustrated by the bars representing an estimate of the total uncertainty due mainly to systematic errors.

Monochromator measurements at 6563 Å and 3947.5 Å show radiation from Hydrogen ($H_\alpha$) and neutral Oxygen atoms in the discharge. The emitted $H_\alpha$ intensity is approximately proportional to the ionisation rate for these conditions. The main period of $H_\alpha$ emission may therefore be identified with the bulk ionisation of the filling gas. We shall restrict our attention here mainly to this period, i.e., the first 500 µsec. The next chapter will discuss more fully the subsequent period. The ionisation rate for Oxygen is
Fig. 4.1. The Bulk Properties of the Plasma
very similar to that of Hydrogen and so radiation from any Oxygen present at the discharge initiation should exhibit a time profile similar to that of $H_\alpha$.

The low level during the first 300 µsec corresponds to this. The rapid increase in O I signal from 400 - 500 µsec indicates an impurity influx of gas presumably sputtered from the chamber walls.

The mean conductivity temperature, $<T_o>$ is deduced from magnetic probe measurements, described in section 4.22, assuming $Z_{\text{effective}} = 1$, while $\tau_E$, the gross energy confinement time, is obtained from the ohmic heating input and the total kinetic energy deduced from $\beta_I$. From the total energy and the mean temperature, $<T_o>$, may be deduced a mean electron density because

$$<n_e> = \frac{\beta_I \mu_0 I^2}{(1+\alpha) 8\pi a^2 e}$$

and therefore

$$<n_e> = \frac{8\pi a^2 <n_e (T_e + T_i) e\mu_0 I^2}{<T_o>^{2} 8\pi a^2 e}$$

where $\alpha$ is the ratio of mean ion temperature to electron temperature. The final trace of Fig 4.1 shows $<n_e>$ defined by equation 4.2 with $\alpha = 1/3$. It should be treated with some caution, however, since equation 4.2 does not accurately include the effects of the plasma profile and $\alpha$ is uncertain during this period.

4.22 Magnetic Probe Measurements

During the comparatively cool early stages it is possible to employ magnetic probes extended right to the centre of the discharge. Thermal damage to the probe may be avoided by terminating the discharge once the period
of interest is past. Both single- and multi-coil probes have been used, introduced vertically into the plasma from above.

The integrated signals from the coils (proportional to $B_R$) in either case are recorded photographically and subsequently digitised into a (PDP 15) computer for automatic processing. The processing program recovers from disk the value of $B_R$ at all relevant radii at a given time for the appropriate discharge conditions and fits a least squares polynomial to these points. The polynomial is constrained to be antisymmetric about the zero crossing, as $B_R$ would be expected to be if cylindrically symmetric. In general a polynomial of 8 or 9 terms was found to give the most satisfactory fit to the data. Data from the single coil probe is shown in Fig 4.2 for a period of 500 $\mu$sec from the application of the toroidal electric field. Profiles are stepped to give a 3-dimensional visual representation of the time-evolution. In the interests of clarity the actual measurements are plotted only for a small number of profiles. The scatter of these points gives an indication of the overall random errors involved which are typically ~ 5% of the maximum value.

The least squares curve can easily be differentiated to give the current density from the relation

$$j(r) = \frac{1}{\mu_0 r} \frac{\partial}{\partial r} (r B_\theta)$$

($B_\theta = B_R$ assuming cylindrical symmetry). The resulting current profile is shown in Fig 4.3. The current density is peaked on axis for the first 100 $\mu$sec and then a hollow profile rapidly develops which persists until after the $H_\alpha$ radiation indicates ionisation to be complete.

The effect of probe perturbations upon the plasma and the overall accuracy of the analysis of the results is discussed in appendix 3 where it is
Fig 4.2 EVOLUTION OF POLOIDAL MAGNETIC FIELD
Fig 4.3. EVOLUTION OF CURRENT DENSITY
argued that the single-coil probe results approximate quite closely the profile in the absence of the probe. An estimate of the accuracy of the results is included in Fig 4.3.

The radial profile of the toroidal electric field may be deduced from the time derivative of $B_R$. It is obtained from Faraday's law in the form:

\[
E_\phi(z_2) = E_\phi(z_1) - \int_{z_1}^{z_2} \dot{B}_R \, dz
\]

assuming toroidal symmetry. Given $E_\phi$ outside the discharge, measured by the $V_\phi$ loop, we can then determine $E_\phi$ at any radius inside the plasma. The time derivative of the magnetic field is obtained by fitting a polynomial to experimental measurements of $B_R$ at times $t \pm \Delta t/2$, where $t$ is the time of interest and $\Delta t$ is a time interval chosen short relative to the time over which field changes. The two least squares curves are then subtracted to obtain the derivative. This scheme has the advantage over the alternative of differentiation at each point separately that it automatically smooths high frequency components which may vary from shot to shot. Rounding errors are also averaged out over the whole profile.

The results are shown in Fig 4.4. It is only during the first $\sim 100 \mu$ sec that any appreciable variation of $E_\phi$ across the discharge occurs. The inductive effect due to the rising current causes the electric field to peak towards the outside of the discharge. Thereafter this effect is greatly reduced owing to the development of a broad current profile.

From $j_\phi$ and $E_\phi$ we may immediately obtain the conductivity, $\sigma$, of the plasma. The Spitzer conductivity,\(^{12}\),

\[12\]
Fig 4.4. The Radial Profile of the Electric Field.
\[ \sigma = 1.6 \times 10^3 \frac{T^{3/2}}{\sigma} \Omega^{-1} \text{ m}^{-1} \]

(Where \( T \) is in eV and we have taken \( Z_{\text{effective}} = 1 \) and the coulomb logarithm, \( \ln \Lambda = 12 \)) defines a conductivity temperature which we denote by \( T_\sigma \). Insofar as the assumption \( Z_{\text{effective}} = 1 \) holds we may identify \( T_\sigma \) with the electron temperature. During these early stages, when the temperature is low, this is probably a good approximation when ionisation is complete provided a high level of turbulence does not exist within the plasma. However when the degree of ionisation is small (\( \lesssim 0.5 \)) collisions with neutral atoms and molecules make a substantial contribution to the resistivity and so \( T_\sigma \) underestimates the true temperature. This is further discussed in section 4.3. Fig 4.5 shows the development of the temperature profile. Only subsequent to about 200 \( \mu \) sec may the conductivity temperature be taken to reflect at all realistically the electron temperature.

The profile of the safety factor, \( q \), is obtained directly from the value of \( B_\theta \). Fig 4.6 shows its variation across the plasma during the period of interest.

4.23 Langmuir Probes

A swept double Langmuir probe technique may be used during the ionisation stages. Its implementation is described in appendix 2. Normal symmetric double probe characteristics are obtained. The spatial profile is obtained by varying the probe position from shot to shot.

The interpretation of Langmuir probe characteristics in a magnetic field is notoriously difficult. In principle it is possible to determine the electron temperature and the ion density from the ion saturation current and the slope of the characteristic at the origin. In practice, however, even for very simple
Fig 4.5. EVOLUTION OF ELECTRON TEMPERATURE
Fig 4.6. The Safety Factor Profile
geometries the analysis is subject to large uncertainties.

A double probe when biassed to saturation draws essentially the ion saturation current and provided the magnetic field is weak enough that the ion larmor radius is much larger than the dimensions of the probe we may expect the usual theories of ion collection to apply. In particular we shall use the Bohm relationship

\[ i_s = \frac{1}{2} A n_1 e \left( \frac{T_e e}{m_i} \right)^{1/2} \]

where \( i_s \) is the ion saturation current, \( A \) the probe area, \( n_1 \) the ion density, \( e \) the electronic charge, \( m_i \) the ion mass and \( T_e \) the electron temperature in electron-Volts. The slope at the origin may also be expected to be given correctly by

\[ \frac{d i}{d V} \bigg|_o = \frac{i_s}{2 T_e} \]

where \( i \) is the probe current and \( V \) the probe voltage. Concerning equation (4.6) is should be noted (a) that the ion larmor radius in the present conditions is \( \sim 1 \text{ mm} \) which is only about twice the probe radius and (b) that the equation applies to cylindrical probes whereas ours are effectively plane. For these reasons we can place little confidence in the absolute value of the electron density deduced. We expect the relative values to be considerably more accurate, however, since the factors mentioned above are essentially geometric and therefore systematic. The electron temperature deduced from equation (4.7) probably is not so seriously affected by these factors. The introduction of the probe into the plasma may perturb both the density and the temperature although the more serious problem is probably with the latter since the main effect of probe perturbation is to cool the plasma locally.
The characteristics obtained, allow determination of $T_e$ through equation (4.7) with an uncertainty of up to 50% (ignoring perturbation effects). Fig 4.7 shows an example of the temperature profile deduced at 300 μsec together with the conductivity profiles deduced from data obtained with the single- and multi-coil magnetic probes. It may be seen that the Langmuir probe results agree reasonably well with the multi-coil probe results but fall below those obtained with the single-coil probe. This is as expected since the multi-coil results reflect the temperature in the presence of the probe, as do the Langmuir probe results. At later times as the temperature rises the Langmuir probe results fall below even the multi-probe deduced temperature. This is probably due to local cooling at the Langmuir probe tip. The temperature estimate provided by the Langmuir probe therefore confirms that provided by magnetic probes within experimental error, although it is considered generally less reliable. However, it is always used in equation (4.6) in deducing the electron density since the equation refers to the local temperature. Errors in $T_e$ are, in any case, less important in this equation since they enter only within the square root.

The electron density obtained is shown in Fig 4.8. The absolute calibration is obtained by comparison with the results of the diamagnetic measurement as follows. Assuming the density to be given correctly by equation (4.6) apart from a constant multiplicative factor to be determined, we employ this density together with the electron temperature determined by the magnetic probe measurements to obtain the average

$$
\left\langle n_e T_e \right\rangle = \int_0^a 2\pi r n_e(r) T_e(r) \, dr / \pi a^2 .
$$

(4.8)
Fig 4.7. Comparison of the temperatures deduced from magnetic and electric probes at time 300 μsec.
Fig 4.8. The electron density profile determined by the Langmuir probe, corrected by comparison with $\beta_1$. 
Hence we have the electron poloidal beta:

\[
\beta_e = \frac{8\pi^2 a^2 \langle n_e T_e \rangle e}{\mu_0 I^2}
\]

This is then scaled to agree with the total poloidal beta, \( \beta_I \), determined from the diamagnetic effect and the scaling factor to be applied to \( n_e \) thus determined.

The result of this scaling is shown in Fig 4.9 where we compare \( \beta_e \) and \( \beta_I \). During the first 400 \( \mu \) sec neutral gas dominates the discharge and we may expect the ion temperature to be very low. During this period, therefore, the contribution of the ions to the poloidal beta should be small and \( \beta_I \approx \beta_e \).

The figure shows the good agreement in the relative time variation of \( \beta_e \) and \( \beta_I \) during this period. This affords us greater confidence in our assumption that equation (4.6) is correct apart from a multiplicative factor and allows us to perform the scaling consistently. Subsequently \( \beta_e \) falls below \( \beta_I \). This may be interpreted as due to the rise in the ion temperature once the neutral gas density falls, as ionisation is completed. This effect may be written

\[
\frac{\langle T_i \rangle}{\langle T_e \rangle} \approx \frac{\beta_I}{\beta_e} - 1
\]

The data of Fig 4.9 would then imply an ion temperature equal to about 6 \( T_e \), at time 700 \( \mu \)sec, or \( \langle T_i \rangle \approx 8 \) eV.

The scaling factor required to produce agreement in the poloidal betas is nearly 6. In other words equation (4.6) leads to a density estimate which is nearly six times too small. This sort of error is not unreasonable considering the uncertainty of the theory. For example, Balfour \(^{42}\), using probes of a similar design, obtained a density corresponding to only one third of the filling pressure of the A.E.I. Levitron in magnetic fields smaller than those...
Fig 4.9. The poloidal beta from diamagnetism ($\beta_1$) and from the (corrected) Langmuir probe results ($\beta_e$).
here. The corrected density we obtain is of the same order of magnitude as
the filling pressure (0.5 mtorr H\textsubscript{2}) once ionisation is complete. This adds to
our confidence concerning the scaling process.

4.24 H\textsubscript{\alpha} Radiation

Diagnostic ports in LT-3 allow spectroscopic observations to be
made along seven chords perpendicular to the plasma axis. These are situated
at $z = 0, \pm 2.5, \pm 5.0, \pm 7.5$ cm. Observations of H\textsubscript{\alpha} radiation have been
made through these ports in order to determine the radial variation of the radia-
tion across the discharge.

The optical system employed, consisting of a single collimating lens
between the port and the monochromator entrance slit, is designed to ensure
that the effective aperture (namely the lens aperture) is external to the port.
This is in order to ensure consistency from port to port. A mean of three
separate shots is taken through each port. Shot to shot reproducibility is very
good ($\lesssim 10\%$).

The results are 'abelised' as follows. First the observations are
'folded' about the centre, i.e. the mean of measurements above and below the
axis is taken. This is possible since there appears to be little vertical shift
of the plasma and it provides a check on alignment. Observations above and
below the axis agree to within $\lesssim 15\%$. This results in four independent
measurements. These are then transformed by a simple matrix program to
obtain an estimate of the radial variation.

The transformation scheme assumes the radial profile to depend
upon the value at four points: $r = 0, 2.5, 5.0, 7.5$ cm and to vary linearly
between these, being zero at $r = 10$. There is then a one to one relationship
between the vector $\mathbf{f}$ consisting of the values at $r = 0, 2.5, 5.0, 7.5$ cm and the vector $\mathbf{F}$ consisting of the observations along the chord at $z = 0, 2.5, 5.0, 7.5$ cm, viz:

\[
\mathbf{F} = A \mathbf{f}
\]

or

\[
\mathbf{f} = A^{-1} \mathbf{F}
\]

where $A$ and $A^{-1}$ are four by four matrices. $A^{-1}$ is found to be:

\[
A^{-1} = \begin{bmatrix}
1 & -.935 & .0948 & -.0823 \\
0 & .467 & -.397 & .0422 \\
0 & 0 & .350 & -.293 \\
0 & 0 & 0 & .292
\end{bmatrix}
\]

The radial profiles ($\mathbf{f}$) obtained by this transformation are shown in Fig 4.10. Prior to the formation of the hollow current profile, the $H_\alpha$ radiation peaks on axis but subsequently it develops a second peak in the $r = 5$ cm region.

The accuracy of these profiles is fairly limited. The transformation scheme tends to enhance the angularity of the distribution but this may be corrected visually i.e. round off the corners. More important is the sensitivity of the distribution particularly near the centre to small errors in the observations. This is illustrated by Fig 4.11 where we show the effect of increasing the 2.5 cm observation by 15% (time 300 $\mu$sec). This removes the central peak almost entirely. Our estimated uncertainty in the observations ($\sim 10\%$) leads to an uncertainty at the profile centre of about 50%. Nevertheless the broad features of the emission are felt to be correct.

This sensitivity is inherent in any abelisation process and not peculiar
Fig 4.11. The effect of errors upon the $H_\alpha$ profile.
Fig 4.10. Evolution of the radial profile of $H_{\alpha}$ emission.
to our scheme alone. It arises partly from the paucity of our observational
access (only four independent chords) which is also a severe limitation on the
spatial resolution possible ( ~2.5 cm).

4.3 THEORETICAL

4.3.1 Resistivity in a Partially Ionised Plasma

The transport processes in a plasma discharge, such as current
conduction, particle diffusion and thermal conduction are governed primarily
by the collisions between the particles present. In a fully ionised plasma
electrons collide with ions and with other electrons, both of which processes
must be taken into consideration. Owing to the property of 'quasi-neutrality'
possessed by most plasmas, which expresses the electrostatically enforced
equality of electron and ion charge densities to a high degree of approximation,
the number densities of ions and electrons are held in constant ratio; for a
singly charged ion plasma such as hydrogen \( n_i = n_e \). For this reason the
effective total collision frequency is directly proportional to the electron
density.

In a partially ionised plasma, however, where neutral atoms and
molecules are present in appreciable numbers, collisions with these neutrals
must also be considered. Moreover there is no direct constraint upon the
proportion of neutrals in the discharge; therefore the total collision frequency
is no longer proportional to the electron density. Indeed in the limit of small
degrees of ionisation, when collisions with neutrals dominate, the collision
frequency of an electron is independent of the electron density.

In particular we may consider the electrical resistivity of the plasma
in the two extreme cases. The resistivity is:

\[ \rho = \frac{m_e \nu}{n_e} \]  

(4.14)

where \( \nu \) here is the momentum transfer collision frequency. For the fully ionised case \( \nu = n_e T_e^{-3/2} \) and we recover the usual Spitzer\(^{12}\) form, which for our regime may be expressed:

\[ \rho = 6.3 \times 10^{-4} T_e^{-3/2} \frac{\Omega - m}{n_e} \]  

(4.15)

where \( T_e \) is in eV. On the other hand when neutral collisions dominate we require the momentum transfer cross-section in neutral gas; this has been determined from mobility measurements\(^{43}\). A good fit to the data is\(^{44}\)

\[ \nu_N = 1.5 \times 10^{-13} n_N \]  

(4.16)

where \( n_N \) is the density of \( \text{H}_2 \) molecules. To use equation (4.16) for our values of \( E/p \) represents a slight extrapolation of the experimental data but this should not cause serious errors. Substitution in equation (4.14) gives

\[ \rho = 5.2 \times 10^{-6} \frac{n_N}{n_e} \]  

(4.17)

At intermediate degrees of ionisation the collision frequencies and hence the resistivities are additive and so the total resistivity is

\[ \rho = \rho_p + \rho_N = 6.3 \times 10^{-4} T_e^{-3/2} + 5.2 \times 10^{-6} \frac{n_N}{n_e} \]  

(4.18)

and the conductivity is

\[ \sigma = \frac{1}{\rho} = \frac{1.6 \times 10^3 T_e^{3/2}}{1 + 8.3 \times 10^{-3} T_e^{3/2} \frac{n_N}{n_e}} \]  

(4.19)
4.32 Electron Temperature

During the ionisation period the temperature of the electrons is maintained at a fairly low value of the order of 5 - 10 eV. The primary reason for this is the rapid variation with electron energy of the collisional ionisation and excitation cross-sections. Any rise of electron temperature leads to a rise in ionisation rate. The rate of energy loss from the electrons is approximately proportional to the ionisation rate; therefore the electron temperature adjusts itself until the rate of energy loss balances the ohmic heating energy input.

We write the total energy loss rate through inelastic collisions with neutrals in the form:

\[(4.20) \quad W S_i n_e n_N \quad \text{ev m}^{-3} \text{sec}^{-1}\]

where \(S_i\) is the maxwellian rate coefficient for ionisation at temperature \(T_e\) and \(W\) is the mean total energy loss per ionising collision. Terms contributing to \(W\) are:

1. Ionisation energy of \(H_2 \sim 15\) eV
2. Relative energy lost in non-ionising collisions, \(\sim 10\) eV
3. Energy required to heat the newly created free electron to thermal energies, \(\sim 5 - 10\) eV.

Evidently, then, \(W\) is of the order of 30 eV per ionisation.

We note also that for \(T_e \lesssim 20\) eV the rate coefficient \(S_i\) may be approximated by:

\[(4.21) \quad S_i = 5.6 \times 10^{-18} T_e^3 \quad \text{m}^{-3} \text{sec}^{-1}\]

In order to obtain the predicted electron temperature we ignore other energy losses and equate the ohmic heating power input derived from the
conductivity, equation (4.19), with the collisional energy loss rate equation (4.20), thus

\[
(4.22) \quad \frac{1.6 \times 10^3 T_e^{3/2}}{1 + 8.3 \times 10^{-3} T_e^{3/2} \frac{n_N}{n_e}} E^2 = eW S_i n_e n_N
\]

or

\[
(4.23) \quad S_i = \frac{1.6 \times 10^3 T_e^{3/2}}{eW n_N} \left( 1 + 8.3 \times 10^{-3} T_e^{3/2} \frac{n_N}{n_e} \right) \left( \frac{E}{n_N} \right)^2;
\]

\( n_e/n_N \) measures the degree of ionisation. Equation (4.23) may be solved to obtain \( T_e \) as a function of \( n_e/n_N \) and \( E/n_N \). To do this we take \( W = 30 \) and use the approximate expression of equation (4.21) for \( S_i \). Then we have

\[
(4.24) \quad T_e = \left( \frac{T_e^{3/2}}{n_e/n_N} \left( 5.9 \times 10^{37} \left( \frac{E}{n_N} \right)^2 \right) \left( 1 + 8.3 \times 10^{-3} T_e^{3/2} \frac{n_N}{n_e} \right) \right)^{1/3}
\]

The parameter \( E/n_N \) may be expressed in the more convenient form \( E/p \) where \( p \) is the filling pressure to which the neutral density corresponds viz :

\[
(4.25) \quad E/p = 3.3 \times 10^{19} E/n_N
\]

with \( p \) in mtorr.

Equation (4.24) is solved iteratively to obtain the electron temperature \( T_e \) as a function of \( n_e/n_N \) for various values of \( E/p \). The results are shown in Fig 4.12. We have also plotted the corresponding predicted conductivity temperature, \( T_\sigma \), which would be deduced by the use of equation (4.5). Naturally \( T_\sigma \), which essentially measures the conductivity \( (\sigma \propto T_\sigma^{3/2}) \), is not expected to be equal to \( T_e \) for small \( n_e/n_N \) when neutral collisions dominate the resistivity.
Fig. 4.12. The electron temperature and corresponding conductivity temperature as a function of ionisation degree for different values of $E/p$. 
Only when \( n_e / n_N \geq 1 \) can we identify the conductivity temperature with the actual electron temperature.

The accuracy of the predictions is limited by the assumption that such processes as electron heat conduction and thermal equilibration with ions are negligible. These factors only become important when \( n_e / n_N \) is large. For the range plotted their effects should be small although we shall see in section 4.4 that conduction may be important at later times. The approximations made for \( W \) and \( S_1 \) are another important possible source of error. For temperatures as high as \( T_e \sim 20 \text{ eV} \) we are almost certainly underestimating \( W \) since the contribution (3), heating to thermal energies, is now 20 eV. However our approximation for \( S_1 \) is an overestimate at these temperatures so the errors will cancel to some extent. The error limits may be estimated from Fig 4.12 by noting that in equation (4.23) an error in \( S_1 \) \( W \) is precisely equivalent to a proportional error in \( (E/p)^{-2} \). Our total uncertainty of about 50% in \( S_1 \) \( W \) is therefore equivalent to an error of \( \sim 25\% \) in \( E/p \). Thus, for example, for the curve of Fig 4.12 for \( E/p = 20 \) the lower error limit is approximately the curve for \( E/p = 15 \). The uncertainty in \( T_e \) and \( T_o \), therefore, is \( \sim 20\% \).

An independent check on the accuracy of the data of Fig 4.12 and hence equation (4.24) may be obtained for small \( n_e / n_N \) by comparison with theoretical and experimental determinations of the electron temperature in hydrogen gas subject to an electric field. This comparison is illustrated in Fig 4.13 where the present calculations are shown together with the upper and lower limits of \( T_e \) from a more rigorous theoretical calculation \(^{46}\) of the distribution function and, at the low \( E/p \) end, the available experimental results \(^{43}\). The agreement appears to be satisfactory, though the uncertainties are obviously much greater at higher \( E/p \) and correspondingly higher temperature.
Fig 4.13. Comparison of the electron temperature with other determinations.
Heuristically the fall off in the electron temperature, $T_e$, in Fig 4.12 as $n_e/n_N$ increases is due to the reduction in ohmic heating input arising from the increasing influence of electron-electron and electron-ion collisions. Hence arises the characteristic shape of the conductivity temperature, $T_\sigma$. At low $n_e/n_N$, $T_\sigma$ is proportional to $n_e$; then as ionisation increases, $T_\sigma$ eventually reaches a maximum and thereafter falls once more with the decreasing $T_e$.

4.33 Plasma Profiles

In order to use the data of Fig 4.12 to obtain the conductivity temperature profile we need the profiles of $n_e$ and $E/n_N$. $E$ deviates from a constant value by an amount dependent upon the rate of current rise but $\leq 50\%$ for most times in our experiment. Meanwhile $n_N$ is expected to be uniform until appreciable degrees of ionisation are reached. Thereafter the deviation from uniformity is limited by the free transport of the neutrals across the discharge. The mean free path for neutral-neutral collisions is of the order of the chamber dimensions so we ignore these collisions. Then a 1 eV neutral crosses the discharge in $\sim 10\mu$sec. The ionisation time is also $\sim 10\mu$sec so that the m.f.p. for ionisation is again of the order of the discharge dimensions. For this reason we make the approximation $E/n_N = \text{const.}$.

We take $n_e$ as peaked on axis, reflecting our expectations concerning the effect of decay of preionisation density and particle loss during ionisation. For definiteness let

$$\frac{n_e}{n_{eo}} = \frac{1}{1 + \left(\frac{r}{r_0}\right)^2};$$

$r_0$ is chosen to be 2 cm to approximate most closely the experimental profile at 200$\mu$sec (Fig 4.8).

Fig 4.14 then shows the calculated conductivity temperature profile
Fig 4.14. The conductivity temperature profile evolution for $E/p = 20 \text{ Vm}^{-1} \text{ mtorr}^{-1}$.
for $E/p = 20$ with various degrees of ionisation. As ionisation increases the initial centrally peaked distribution develops into a hollow profile.

4.4 CONSISTENCY

Our model of the discharge during the ionisation stages leads us to the prediction of the evolution of a hollow conductivity profile with increasing ionisation which is strikingly similar to the experimental results in its general form. In attempting to discover how closely the experimental results conform to our model we consider the comparison in more detail.

4.41 The Peak Conductivity Temperature

Taking $E/p = 20$ in the theory represents something of an approximation since, assuming the neutral density is correctly given by the filling pressure (0.5 mtorr), the variation of $E$ causes $E/p$ to fall to as low as ~10 early on. This merely exacerbates the most outstanding discrepancy between theory and experiment which regards the peak conductivity temperature reached at a given radius. The model gives this as ~7 eV whereas the experimental results give ~12 eV. A peak $T_\sigma$ of 12 eV could arise theoretically (c.f. Fig 4.12) only if $E/p \geq 40$ which is outside the probable experimental uncertainty limits of $E$ or $p$. This discrepancy would also appear to exceed that allowed by the acknowledged approximations of the theory. It seems, therefore, that we must seek an explanation in factors outside those considered hitherto.

The theory we have considered ignores all cross-field transport processes. In particular we have assumed the thermal conduction to be negligible. At first sight it might be supposed that thermal conduction could not cause our discrepancy since it would normally tend to reduce the peak temperature. However, we must not confuse the conductivity temperature with the true electron
temperature for whilst $T_\sigma$ may peak on axis owing to the peaked electron density, the actual temperature, $T_e$, being a monotonically decreasing function of $n_e$, will in fact be minimum on axis. Thermal conduction is thus able to raise the minimum $T_e$ which has the effect of increasing the peak $T_\sigma$.

Naturally $T_e$ cannot be increased above its appropriate maximum value for the given $E/p$. This is $\sim 14$ eV for $E/p = 20$; if we allow for uncertainty in the collision cross section this might be increased to $\sim 16$ eV which might then allow our peak $T_\sigma$ to be $\sim 12$ eV.

A different factor of which the theoretical model takes no account is the possibility of an appreciable current being carried by runaway electrons. If this were the case then the conductivity would be increased in proportion to the current carried by the runaways. The number density of runaways required is $10^{-3} n_N$ and so their effect upon the discharge other than increasing the current would be small. The extreme sensitivity of the runaway rate to changes in $E/p$ makes it impossible to say more than that such a level of runaways is not inconsistent with theoretical runaway rates, but then neither would be a level far smaller.

When using the multicoil magnetic probe runaways may be discounted since their formation is suppressed by the presence of the probe. Therefore we show for comparison in Fig 4.15 the evolution of $T_\sigma$ derived from multicoil data. The peak $T_\sigma$ is $\sim 7$ eV. Unfortunately, we may not immediately attribute this agreement with theory entirely to the suppression of runaways because the presence of the probe may cause real cooling of the plasma. Nevertheless it is a suggestive coincidence.

The electron density at which $T_\sigma$ is a maximum is predicted to be (c.f. Fig 4.12)
EVOLUTION OF ELECTRON TEMPERATURE

Fig 4.15. Multicoil Probe Results.
The Langmuir probe results indicate that both at 100 μsec, \( r = 0 \) (the point of maximum \( T_\sigma \)) and at 200 μsec, \( r = 4 \) (Fig 4.8) the density is \( \sim 0.6 \times 10^{19} \). This agrees within the error limits with the theoretical value, especially since we are near the lowest density measurable with this Langmuir probe where the experimental uncertainty is greatest.

Finally as regards Fig 4.14 we note that the high conductivities predicted at large radii in contradiction of the experimental results reflect merely the inadequacy of the chosen density profile to represent the observed profile at these radii. The true density profile falls off more rapidly at large radii causing the observed conductivity to fall correspondingly.

4.42 \( H_\alpha \) Emission

A further reflection upon the consistency of the results concerns the radial profile of \( H_\alpha \) emission. We will suppose that Hydrogen atoms diffuse rapidly so that their density is approximately uniform. The rate coefficient for excitation of the \( H_\alpha \) line we may take as having a similar form to that for ionisation then the \( H_\alpha \) emission is predicted to be proportional to \( n_e T_e^3 \).

Using the \( n_e \) profile from the Langmuir probe results and \( T_e \) defined as \( T_e = T_\sigma \) for \( r \leq 4 \) cm and extrapolated to rise to 14 eV at \( r = 6 \) cm (in an attempt to model the effects of hypothetical thermal conduction) we derive the predicted \( H_\alpha \) profile at time 300 μsec. Fig 4.16 compares the result with the observed profile. The agreement appears to be fairly satisfactory considering the experimental and theoretical uncertainty.

A profile of this form has immediate implications for energy balance since the total energy loss due to inelastic collisions is approximately proportional
Fig 4.16. The predicted and observed $H_\alpha$ emission profiles.
to the $H_\alpha$ emission. If, therefore, these losses were balanced by ohmic heating input the current profile would of necessity take the same form as the $H_\alpha$. This is not the case. The outer peak in the $H_\alpha$ emission may be balanced by the hollow current profile but the pronounced peak on axis has no corresponding peak in current density. The uncertainty in current measurement is greatest near the centre but would not be sufficient to allow such a discrepancy; and although the accuracy of the abelised $H_\alpha$ measurement is poor near the centre the central peak is felt to be significant. It would seem, therefore, that a mechanism supplying extra heat to the plasma centre is indicated. Thermal conduction would appear to be the most likely candidate.

Non-uniformity of the Hydrogen atom density could also account for this discrepancy since a centrally peaked distribution could cause the $H_\alpha$ emission peak while allowing the major energy loss, which is by collisions with molecules to conform to the ohmic heating input. The free diffusion of atoms would be expected to limit any build-up of atom density on axis, although the possibility cannot be discounted. Such a variation in atom density might also improve agreement between the theoretical and experimental $H_\alpha$ emission profile.

4.5 DISCUSSION

It has been shown that we can understand the evolution of the discharge during the ionisation stages primarily in terms of the energy balance between ohmic heating input and losses due to inelastic collisions with neutral particles.

The results appear to have very wide applicability in Tokamak-type devices. It is highly probable that in most Tokamaks the loss processes during
preionisation and early ionisation will lead to an electron density which is peaked on axis as is observed on LT-3. The density should grow exponentially, thus preserving its shape, at least until a degree of ionisation of about 1 is approached. Again observations on LT-3 confirm this and show that a peaked density profile persists until the ionisation is practically complete.

In the presence of such a centrally peaked density profile the theory predicts that the current density, initially peaked on axis, evolves into a hollow profile. This effect resembles the usual skin-effect for a conducting medium but occurs even when the electric field is uniform throughout the plasma. It is a manifestation of a true fall in electron temperature near the discharge centre.

The experimental results conform very closely, in general, to the theoretical model. The most outstanding discrepancy concerns the peak conductivity which is experimentally higher than allowable in the model. It is possible that this discrepancy may reflect an appreciable current carried by runaway electrons. However another possible explanation may be that electron heat conduction is not negligible. This factor might also explain the observed \( H_\alpha \) emission profile.

Thermal conduction might theoretically reach the required level owing to the toroidal enhancement by the factor \( q^2 \) in the 'Pfirsch-Schluter' regime\(^{48}\). In LT-3 \( q \) can reach \( \sim 10 \) in the early stages and so only a fairly small enhancement would be required to give conductivities large enough. If electron thermal conduction is appreciable, though, then the electron diffusion coefficient should lead to particle diffusion at least as important as the heat conduction. The diffusion is ambipolar so provided the ion diffusion coefficient is much smaller than the electron diffusion coefficient diffusion may be kept
down to the low level consistent with the experimental results. However, even in this stage when the ion temperature is low it is difficult to see how such a comparatively small ion diffusion could arise.

The electron density flattens after about 300 μsec, presumably owing to the increased ionisation rate at the radii where the current peaks. By about 400μsec it reaches a measurable level near the wall. At this time the OI signal begins to rise, indicating the influx of impurities from the walls. We attribute this influx to sputtering due to wall bombardment by thermal particles. The introduction of a probe, quenching all runaways changes the OI signal by less than 20%. This confirms that supra-thermal runaway electrons are not responsible for the main impurity influx.

The experiments show that the hollow current profile persists after ionisation is completed (at ~400μsec). This is probably due to the positive feedback provided by the excess heat input at the current-density maximum, which must be balanced by thermal conduction once the neutral density falls. In other words a 'skin effect' due to overheating takes over as ionisation is completed.

It thus appears probable that most Tokamaks will enter the fully ionised current-rise stage with an already hollow current profile which will considerably increase the importance of the skin effect slowing the current penetration into the plasma.
CHAPTER 5.

RELAXATION OF THE CURRENT PROFILE

Ecclesiastes 3:3

5.1 INTRODUCTION

Several numerical simulations\textsuperscript{14-17} have shown that neoclassical transport coefficients should lead to a pronounced and prolonged skin effect in present generation Tokamaks. The electron temperature and current density evolve into profiles sharply peaked towards the outside of the plasma and current penetration is severely limited.

Düchs et al.\textsuperscript{15} have made a fairly extensive survey of conditions and heat conductivities relevant to the parameters of the ST Tokamak. In most cases their simulation shows a persistent skin effect. This is in contrast to the experimentally observed electron temperature profiles\textsuperscript{49} which indicate a moderately hollow distribution of current only prior to about 10 msec from the start of the discharge. For this reason there was included in their code a 'skin limiter' rule designed to simulate enhanced transport due to instabilities; this enabled profiles consistent with experiment to be obtained.

Düchs et al.\textsuperscript{16} have also investigated the parameters relevant to the larger PLT Tokamak. Again their code predicts an appreciable skin effect leading to excessive heat loss to the limiter.

A recent study (McBride et al.\textsuperscript{17}) presents scaling arguments that the skin effect is important in lower density faster rise-time devices but not in
higher density slower rise-time devices. In the latter, equilibration of energy between electrons and ions occurs and ion heat conduction then prevents a skin forming. Numerical simulation results confirm the scaling arguments and show adequate current penetration in a PLT size device. This apparent contradiction of the results of Düchs et al, using the same physics input, arises because of the more realistic electron density employed by the latter.

McBride et al. also review various micro-instabilities which have been proposed as a means for anomalous current penetration. They conclude that in general other mechanisms are required to provide the full penetration experimentally observed in present generation Tokamaks.

These simulations concern essentially fully ionised plasmas and start with a uniform current distribution. However the previous chapter has shown that it is probable that a Tokamak enters the fully ionised state with an already hollow current profile. Thus the tendency toward the formation of a skin profile is enhanced still more.

The explanation of the discrepancy between theory and experiment would appear to lie in the consideration of some sort of instability which enhances diffusion. Düchs et al. argue that MHD disturbances, most likely tearing modes, are the probable cause of the current penetration. This is based on the demonstration that when a minimum of the safety factor, q, occurs within the plasma (as is the case for a hollow current profile) resistive modes of all 'm' values can become unstable. Moreover the immediate consequence of a relatively small enhancement of resistivity or electron viscosity due to postulated microturbulence is to accelerate tearing mode growth.

The concern of the present chapter is the mechanism of current penetration. The hollow current profile which forms in LT-3 is observed to
relax very rapidly after about 500 \mu\text{sec} to a state of approximately uniform current. Simultaneously we observe a helical perturbation of the plasma of the mode predicted by resistive MHD theory (m = 4, n = 1) for the observed q-profile. During this period runaway electrons diffuse across the magnetic field very rapidly, indicating considerable disruption of the magnetic surfaces, as would be expected in the presence of a tearing mode of large amplitude.

The evidence is very strong that, in our case at least, resistive instabilities are responsible for current penetration.

5.2 MAGNETIC PROBES

The gross properties for the period of interest have been discussed in the previous chapter and are shown in Fig 4.1 which we repeat here for ease of reference. Likewise the magnetic probe method and interpretation has been discussed so we concentrate here upon the results.

Fig 5.1 shows the poloidal magnetic field evolution from 500 to 700 \mu\text{sec}, measured by the single coil probe, and Fig 5.2 the derived toroidal current density. Evidently the discharge undergoes a rapid relaxation during this period after having maintained a fairly constant shape for the previous \sim 200 \mu\text{sec} (c.f. Fig 4.3).

A most revealing reflection of this is in the safety factor profile (Fig 5.3). We see very clearly the minimum in q caused by the hollow current profile. The minimum value remains very constant in time at just below 4. Such a profile of q is theoretically unstable to tearing modes of the approximate form

\begin{equation}
\exp \left[ i(m\theta + n\phi) \right]
\end{equation}

with m = 4 and n = 1.
Fig. 4.1. The Bulk Properties of the Plasma
Fig 5.1  EVOLUTION OF POLOIDAL MAGNETIC FIELD
Fig 5.2 EVOLUTION OF CURRENT DENSITY
Fig 5.3. The Safety Factor Profile.
In an attempt to determine the significance of the minimum value of \( q \) for the relaxation of the current profile a scan has been made, altering the magnitude of the toroidal magnetic field. This was performed using the multi-coil magnetic probe which gives the complete profile from one shot. The resulting safety factor profile evolution is shown in Fig 5.4. No maintenance of a constant minimum \( q \)-value is observed with the multiprobe even when the toroidal field is the same as in Fig 5.3. Rather it may be observed that almost as soon as the minimum \( q \) falls below an integer value a relaxation of the profile takes place. This sometimes occurs at more than one integer value (e.g. Fig 5.4(a)) as the minimum \( q \) falls.

5.3 **PLASMA DEFORMATION MODE STRUCTURE**

Internal magnetic probes lead us to suspect the presence of MHD instability associated with integer \( q \) surfaces. In order to ascertain the mode structure of any deformation of the plasma column a series of magnetic coils was introduced at a minor radius of 9 cm with their axes aligned parallel to the \( \theta \)-co-ordinate. The signals from the probes were integrated by passive, 100\( \mu \)sec time constant, R-C integrators. In this way slow changes due to the rise in total current were minimised whilst rapid changes due to helical perturbation were retained.

The probes were placed at various values of \( \theta \) and \( \phi \). The estimated accuracy in positioning the probes was to within \( \pm 2^\circ \) and \( \pm 2 \text{ mm} \): the relative accuracy of response was limited by differences in coil dimensions and alignment to within \( \pm 10\% \). Some extraneous pick-up, particularly during the start of the discharge, affecting both signals and base lines, contributed errors of up to \( 10\% \).
Fig 5.4. The Safety Factor Profile from multicoil probe results. 
$B_\phi$ is respectively (a) 1.2, (b) 1.1, (c) 1.0, (d) 0.9, 
(e) 0.8 times that shown in Fig 4.1.
The general appearance of the signals during the first 350\,\mu\text{s} of the discharge was consistent with the rising poloidal field determined from the rate of rise of the gas current. From 350\,\mu\text{s} to about 700\,\mu\text{s} all signals indicated a stationary mode structure and from about 750\,\mu\text{s} to 1 ms a time varying structure.

In Fig 5.5 the structure of the perturbation signals from 350 - 700\,\mu\text{s} is shown for the conditions of Fig 4.1. In Fig 5.5(a) the open circles show the $\phi$ variation at $\theta = 0$ at the times indicated, the values at 350\,\mu\text{s} being taken to define the zeros. The solid curves at 450 and 600\,\mu\text{s} are of the form $\sin (\phi - \phi_0)$, with $\phi_0 = 300^\circ$. Such a curve gave the best fit to all data in the time range 350 -700\,\mu\text{s}, indicating an $n = 1$ perturbation of varying amplitude and constant phase.

Assuming the poloidal field is approximately constant for constant radius (as would be expected from pressure balance considerations) but taking into account the major radial variation of the toroidal field, the co-ordinates of a magnetic field line in the vicinity of an integer $q$ surface may be expressed as,

\begin{equation}
\phi (\theta) = \phi (0) + q (\theta - 2\frac{r}{R} \sin \theta)
\end{equation}

Equation (5.2) may be used to relate the $\theta$ -variation of a perturbation at constant $\phi$ to its $\phi$-variation at constant $\theta$. Thus the solid curve of Fig 5.5(b) shows the $\theta$-variation at $\phi = 90^\circ$ (the position of the $\theta$-probes) derived from the $\sin (\phi - 300)$ curve of Fig 5.5(a). In applying equation (5.2) $q$ was taken as 4, $r$ as 6 cm and $R$ as 40 cm. The open circles in Fig 5.5(b) are normalised experimental results at $t = 500 \, \mu\text{s}$. It is to be noted that the $\phi$ and $\theta$ measurements were recorded during different discharges. The agreement between the derived curve and the experimental points indicates the presence of
Fig 5.5. The external magnetic probe perturbation signals.
(a) $\phi$-variation at various times  (b) $\theta$-variation at time 500 $\mu$sec (c) Perturbation amplitude.
helical \( n = 1, \ m = 4 \) perturbation.

In order to obtain information about the growth of the perturbation, the difference between the values for \( \phi = 0 \) and \( \phi = 191 \) (Fig 5.5(a) was taken as indicative of the perturbation amplitude. This difference is plotted as a function of time in Fig 5.5(c). The signals may be interpreted as indicating slow growth of the perturbation prior to 500 \( \mu \)sec. Between 500 and 600 \( \mu \)sec more rapid growth occurs; some relaxation then takes place followed by another period of slightly faster growth.

The magnitude of the observed signals corresponds to a maximum helical perturbation of a few per cent of the total current.

In Fig 5.6 the signals from the \( \theta \) -probes, with slowly varying components subtracted, are plotted as a function of time. The ordinate shows the probe positions in \( \theta \). At the bottom of the figure the volts per turn signal is shown, again with the slowly varying component subtracted. The skew lines, separated by 45 \( \mu \)sec, were drawn parallel to one another to cut the base lines of the upper signals as near as possible on average to the positive going peaks. Allowing for some irregularities in the \( m = 4 \) perturbation, the figure is consistent with a rotation of the mode structure in a direction of increasing \( \theta \) at an angular frequency of \( \omega = 3.5 \times 10^4 \) \( \text{sec}^{-1} \).

If a rotation of the plasma is present due to drifts perpendicular to the toroidal field, the angular frequency of rotation, \( \omega_d \), may be expressed as

\[
(5.3) \quad \omega_d = \frac{1}{r_q} \left| \left( \frac{\nabla P_e}{e n e} \right) + \left( \frac{B}{B^2} \right) \right|
\]

where \( r \) is the radius of the integer \( q \) surface and the second bracketed term is the electron diamagnetic drift contribution. We expect rotation to approximate to the diamagnetic drift frequency. Writing
Fig 5.6. The time variation of the perturbation showing the mode rotation.
where $\Lambda$ is a scale length for pressure variation, the rotation observed in LT-3, for $r_q = 6$ cm, $B_\phi = 0.5$ T (in the negative $\phi$-direction) and $T_e \approx 20$ eV (appropriate at this radius) leads to $\Lambda \approx 2$ cm which is consistent with the observed profiles.

5.4 RUNAWAY ELECTRONS

The velocity of runaway electrons is mainly directed along the magnetic field. There is also a component of drift across the magnetic field as noted in Chapter 3. In general the effect of the combination of these motions is to constrain the electrons to move on 'runaway surfaces' which have approximately the same form as the magnetic surfaces but are shifted outward by an amount dependent upon the runaway energy. If the magnetic surfaces are perturbed in such a way as to produce stochastic wandering of the magnetic field lines, however, corresponding diffusion of the runaways will occur as they move around the torus.

Because of the high velocity of the runaways, even very slight disruption of the magnetic surfaces can lead to rapid runaway diffusion. The runaway electrons are therefore a sensitive monitor of the integrity of the magnetic surfaces and their diffusion can be used to estimate the degree of perturbation of the surfaces.

5.4.1 Measurements

The runaway probe employed here consists of a 1 mm diameter molybdenum wire to which is fixed a target consisting of a short cylinder of molybdenum. This is cut at a $45^\circ$ angle, thus presenting both to the runaways...
and to the X-ray detector an effectively circular detection area. Two such probes are used with target diameters 2 mm and 4 mm respectively. The target serves the purpose of providing a well defined collection area since runaways preferentially strike the target rather than the wire owing to its larger diameter and its proximity to the discharge centre; moreover, X-rays emitted in the direction of the detector by electrons striking the wire support should be absorbed by the target, thus preventing their detection.

A runaway moving on a runaway surface which is intercepted by the target or its wire support is able to make a number of circuits around the torus before colliding with the probe. A runaway actually formed on such a surface, being accelerated from thermal energies, would reach energies no more than a few keV before being 'absorbed' by the probe; it would thus be undetected in the present case. The runaways observed must, in general, therefore be created within the radius to which the probe is introduced and diffuse outward till they collide with the probe.

The raw signal from the X-ray detector tends to be very noisy owing to the erratic flux of runaways onto the target; we therefore integrate the output to obtain a measure of the total flux to the probe. Typical integrated signals are illustrated in Fig 5.7. It is evident that the main runaway flux to the probe occurs between about 250 and 850 μsec and that thereafter the signal is negligible. This almost certainly reflects the fact that after ~850μsec practically no runaways remain in the discharge.

The total relative integrated X-ray signal observed to time 1 msec is shown in Fig 5.8 as a function of probe position for the different size targets. Each point is the mean of five shots with the error bars indicating the standard error of the mean.
Fig 5.7. Typical Runaway Electron Signals.
Fig 5.8. Total integrated x-ray signal, radial variation.
The time of detection of runaways is illustrated in Fig 5.9 where we plot the median and lower and upper quartiles of the X-ray signal. (These are defined as the mean over different shots of the times when the integrated signal reaches 1/2, 1/4 and 3/4, respectively, of its maximum value.)

5.42 Interpretation

The flux of runaway electrons to the probe is largely determined by their diffusion rate which is itself a reflection of perturbations in the magnetic surfaces. Owing to the complexity of the processes involved and the statistical fluctuations from shot to shot it is not possible to perform for the observations a complete analysis in terms of the diffusion rate as a function of time and space. It is possible, however, to interpret the broad features of the results and to obtain an estimate of the radial variation of an average diffusion rate during this period.

The effect of the curvature of the magnetic field lines leads to a displacement outward of the runaway surfaces from the magnetic surfaces by a distance $\Delta_r$ whose approximate magnitude is most conveniently expressed as:

$$\Delta_r \approx \frac{q p}{e B \phi}$$

where $q$ is the safety factor at the radius of interest, $p$ the (relativistic) longitudinal momentum of the electron and $e$ the electronic charge. Mean runaway energies up to $\sim 600$ keV have been estimated by the absorption method in this case which would give rise to a shift $\Delta_r \sim 3$ cm.

At large $z$-values ($\sim 6$ cm) runaway surfaces intersect the vacuum vessel owing to their outward shift, $\Delta_r$. We should therefore expect to detect no runaways unless there is very rapid diffusion. This is reflected in the small magnitude of signal detected and, more revealingly, in the late appearance
Fig 5.9. Time of occurrence of x-ray signals; median and lower and upper quartiles.
of what signal is detected. Practically no signal is detected prior to about 750 nsec at large radii, indicating that the very rapid diffusion necessary occurs only at these late times.

As the probe is moved in, the magnitude of the signal observed rapidly increases and the time of its observation becomes earlier. This reflects the increasing numbers of runaways which strike the probe rather than the walls and, presumably, the smaller diffusion rate necessary to do so.

An interesting effect is observed for \(4.5 \gtrsim z \gtrsim 3\) where the ratio of the signal for different targets, \(n\), undergoes a transition. At large radii where we observe only rapidly diffusing runaways only a small proportion of them will be absorbed and we expect \(n\) to be constant, equal to the ratio of the effective detection areas. (We find \(n \approx 3.2\); whereas the nominal ratio of dimensions would lead to \(n = 4\); however this discrepancy may be explained as due to an increase in the effective collection area of the small target because of bending due to thermal shock and incomplete shielding of the detector from X-rays emitted from the wire.) On the other hand at small radii, where we observe slowly diffusing runaways, all runaways which reach the target radius will be absorbed by the targets so we expect the signals to be the same: \(n = 1\).

In the transition region it is possible to obtain an estimate of the absolute value of the diffusion coefficient. This is presented in Fig 5.10 in terms of the RMS step length, \(\delta\), per transit around the torus major circumference. The analysis used to obtain this is given in appendix 4. The error bars indicate an estimate of the total uncertainty arising from random errors in the data and inaccuracy of the interpretative model.
As a verification of this determination we have performed a quenching experiment by simultaneously introducing a 1 mm tungsten wire vertically into the torus at a $\phi$-value of $-80^\circ$ from the target. The effect of the quenching probe on the X-ray signal observed at 6 cm is shown in Fig 5.11 together with the predicted quenching effect deduced from the data of Fig 5.8. The agreement obtained is satisfactory within the overall error limits, which are the same order of magnitude for the calculated curve as for the observed data. The effect of the probe on diffusing runaways is small and the reduction in signal is due largely to the inhibition of runaway formation.

The points on Fig 5.10 at $r \geq 6$ cm are estimated assuming the effective radius of the liner for runaways is 9 cm (owing to the presence of inhomogeneities in $B_\phi$, etc.) then the maximum $z$-value which runaways can freely intersect is $\sqrt{(9 - 3)^2 - 3^2} \approx 5.2$ cm. Runaways appreciably outside this surface will intersect the walls within approximately 2 transits of the torus, so to reach $r$ cm unabsorbed requires the step size to be $\frac{(r - 5.2)}{2}$.

We note that this step size at large radii may not be representative of all runaways since our observations select only the rapidly diffusing runaways. It remains clear, however, that some runaways do diffuse at this rate.

As the target is moved inwards beyond $\sim 3$ cm the X-ray signal falls again. This is due to the inhibition of runaway formation because of absorption by the probe. The signal observed with the larger target is actually smaller than that with the small target owing to faster absorption by the former. At the same time it is noticeable that the runaways are observed later. This may be a reflection of such low diffusion rates at early times that very little transport to the probe occurs until a more general perturbation of the magnetic surface occurs.
Fig 5.11. The x-ray signal observed at 6 cm with the introduction of a quencher.
Finally with the target on axis or thereabouts the only signal observed occurs at about 150 \( \mu \text{sec} \). The explanation of this phenomenon lies in the shift of the magnetic surfaces. During the period while the current peaks on axis there is a shift of the magnetic axis from the vacuum vessel centre and runaways can be created. When the hollow current profile forms, however, the magnetic axis returns to the vacuum vessel axis and the runaway surfaces are centred. (\( \Delta r \) is small since energies are small). The runaways are then absorbed by the probe and an X-ray burst is observed. Thereafter no further runaways can form because all magnetic surfaces are intersected by the probe.

In summary then, the overall picture is fairly clear: the runaways observed are created (mainly early in the discharge) within a radius of \( \sim 4 \) cm and diffuse outward. This diffusion reflects a perturbation of the magnetic surfaces which increases in magnitude until, at about 800 \( \mu \text{sec} \), it is so severe that practically all runaways are lost. Diffusion step sizes up to \( \sim 1 \) cm are observed.

5.5 DISCUSSION

During the early stages of the discharge, the Tokamak LT-3 develops a hollow current profile. The safety-factor \( q(r) \) exhibits a local minimum within the discharge at a value which would lead one to expect the development of a resistive MHD (tearing mode) perturbation with \( m = 4 \). Observations of magnetic perturbations during this period confirm the presence of a slowly growing \( m = 4, n = 1 \) mode. It may well be that the minimum value of \( q \) is maintained at its constant value by the growth of this mode; in other words, the mode grows until transport is enhanced sufficiently to prevent further development of the hollow current-profile. The mode appears to be stationary, not rotating with the diamagnetic drift. This may be a consequence of some sort of 'locking into'
inhomogeneities in the toroidal magnetic field.

Only small quantities of runaway electrons are observed prior to 500 \( \mu \text{sec} \), corresponding to comparatively small diffusion rates.

Between 500 and 700 \( \mu \text{sec} \) the hollow current distribution relaxes to a substantially flat profile. Simultaneously, substantial diffusion of runaway electrons occurs. The diffusion step length is indicative of overall disruption of the magnetic surfaces. The gross energy confinement time, \( \tau_E \) (Fig 4.1) ceases to grow at about this time, reflecting a deterioration in the overall confinement properties of the discharge. By 700 \( \mu \text{sec} \) the Langmuir probe data (Fig 4.8) show the electron density to be substantially uniform over most of the plasma.

The deformation mode-structure data (Fig 5.5(c)) suggest that the relaxation process may be by stages. The growth of the helical perturbation apparently suffers a set-back at about one hundred microsecond intervals at \( \sim 500, 600 \) and 700 \( \mu \text{sec} \). The magnetic probe measurements (e.g. Fig 5.3) and the runaway measurements (Fig 5.7) are not inconsistent with this suggestion although the correlation is rather tenuous.

The helical perturbation continues after 700 \( \mu \text{sec} \), as illustrated by the mode structure measurements, and is observed to rotate at a rate consistent with the diamagnetic drift.

There is little doubt that the relaxation process must be attributed to the growth of the helical perturbation (almost certainly a tearing mode) to such a level that disruption of the magnetic surfaces occurs. Such disruption can occur as a result of the presence of two incommensurable helical perturbations or simply by the interaction of one mode with the asymmetry arising from toroidality. The calculated level of perturbation for disruption
to occur is not inconsistent with our observations.

Variation of the toroidal magnetic field confirms the importance of integer q-values for the relaxation process. Relaxation corresponding to q = 5 and possibly 3 is observed though the concomitant helical mode structure has not been determined.

Observation of MHD mode structure in T-3\textsuperscript{53} has shown the presence of perturbations with m = 4-6 associated with the early stages when the current is rising. This has been related\textsuperscript{54} to the poor confinement properties of the discharge at these times. It seems probable that these observations are related to the phenomena reported here. Indeed it is likely that the processes observed on LT-3 may occur quite widely in the early stages of Tokamak discharges and provide an explanation for the absence of prolonged skin-effect in terms of magnetic surface disruption by tearing modes.

The actual disruption of surfaces is probably detrimental as far as reactor research is concerned since it leads to energy loss and probably impurity influx due to wall bombardment. Whether, by suitable choice of current rise rate, it is possible to achieve suppression of skin current formation by tearing modes without excessive surface disruption remains to be discovered.
CHAPTER 6.

THE DISRUPTIVE INSTABILITY

Ecclesiastes 3:7

6.1 INTRODUCTION

The most characteristic feature of the Disruptive Instability in Tokamaks is the negative-going spike on the externally measured toroidal voltage. Its immediate cause is the rapid expansion of the plasma minor radius. The redistribution of the toroidal current from a centrally peaked to a flatter profile induces a variation in the electric field, $E_\phi$, across the discharge. Lenz's law states that the variation must be in such a direction as to oppose the change in current; thus the electric field will be peaked in the positive direction on the minor axis. The natural result of this variation is that $E_\phi$ at the outside of the discharge will fall: the negative voltage spike. The precise form of the voltage spike will be determined by the primary circuit. In some machines the disruption can lead to a complete extinguishing of the plasma, while in others such as LT-3, where the form of the primary current is strongly constrained, the discharge recovers and many cycles, terminated by disruptive instabilities, may occur during one discharge.

The duration of the instability, and hence the voltage spike, is typically about 10 μsec. Changes are therefore occurring about three orders of magnitude faster than in the stable discharge. This fact, combined with the erratic nature of the instability, renders the experimental investigation of
the instability itself extremely difficult. In particular pulsed techniques such as laser scattering face almost insurmountable problems.

Investigations of the internal magnetic field on LT-1 using magnetic probes have revealed clearly the minor radial expansion of the current channel and the corresponding reduction in major radius. These relied upon the shot to shot reproducibility of the instability, however, and thus could give only an overall picture. High-speed photography on ATC has also shown the plasma expansion, and in addition, helical surface structure was visible prior to the disruption. Such structure has been studied using external magnetic probes which show predominantly $m = 2, n = 1$ modes leading to the instability. Runaway electrons have been shown to leave the discharge with escape velocities consistent with magnetic surface break-up. Further information upon the turbulent mixing of the plasma is provided by the observation of bremsstrahlung from impurity ions which are redistributed at the instability.

Recently on LT-3 quite appreciable ion heating at the disruptive instability has been observed by measurements of the doppler broadening of impurity ion line radiation. Charge exchange fast neutral emission measurements, too, show that plasma ions can reach very high energies ($\sim 500$ eV). Estimates of the mean electric field at the discharge centre during the disruption on T-4 have shown that the resistivity is anomalously high, up to ten times classical. These facts point to a very high degree of turbulence during the instability.

In view of the importance of the safety factor for stability, and the observation of helical perturbations, theoretical attempts to explain the nature of the disruptive instability have centred upon MHD mechanisms. The nonlinear
development of helical MHD modes fails to predict the observed voltage spike except in the extreme case where vacuum 'bubbles' form in the plasma. This latter model requires interaction with the limiter in order to cause a negative voltage spike but the spike has been observed in the absence of such interaction. Minardi has proposed a model based on the bifurcation of MHD equilibria while Ohkawa considers anomalous transport at a rational q surface to be a possible explanation. Perhaps the most plausible suggestion to date postulates the existence of two incommensurable helical perturbations; these may lead to an increased instability growth rate or, more importantly, to gross disruption of the magnetic surfaces.

In the present chapter we discuss new measurements on the disruptive instability in LT-3. These are primarily of two types. Firstly the multicoil magnetic probe is used to obtain the poloidal magnetic field profile. The importance here lies in the fact that the complete profile is obtained simultaneously during a single instability, thus avoiding the uncertainties involved in shot to shot measurements. Secondly, employing an electrostatic (Langmuir) probe we investigate high frequency oscillations of the plasma during the instability. These cast light upon the instabilities involved in the disruption.

The results confirm the triggering of the instability to be associated with MHD perturbations but suggest that an important role is played by microinstabilities.

6.2 MAGNETIC PROBES

6.21 Measurements

In the early stages of the discharge it has proved possible to
introduce magnetic probes right on to the minor axis without undue perturbation of the plasma. This is not the case during the hotter stages of the discharge where the disruptive instability occurs. In fact it is found that introducing a probe closer than about 3 cm from the centre suppresses the disruption completely. For this reason it is possible to obtain the poloidal magnetic field evolution during the disruptive instability only outside about 3.5 cm.

Even with the probe at or outside this radius, some perturbation of the plasma inevitably occurs. Nevertheless the use of the multicoil magnetic probe provides an accurate measurement of the magnetic field profile of the plasma in the presence of the probe. The general appearance of the instability in the presence of the probe is little different from that observed with the plasma unperturbed. The main tendency appears to be that the probe reduces the violence of the disruption, for example, as indicated by a smaller voltage spike. We may therefore assume that there is little difference in character between the perturbed and unperturbed discharges and therefore our conclusions will apply equally to either, though possibly with some quantitative modification.

The time resolution possible was limited by the writing speed of the oscilloscope/camera system employed. In order to obtain the requisite number of simultaneous traces a 'multiscope' system was used by which the cathode ray tubes of nine dual-beam scopes were photographed together. The maximum speed attainable was $10 \mu$ sec/div. A high frequency ($\sim 500$ k Hz) oscillation was observed on the signals, whose magnitude was up to 25% of the major perturbation at the disruption. In the subsequent digitisation process no attempt was made to follow this oscillation since it could not be properly resolved on all traces. A visual average was therefore taken corresponding to
an effective time resolution of about 2 μsec.

In order to extrapolate the measurements to the minor axis there was included in the least-squares fitting procedure for the radial profile of the point (0, 0). This corresponds to the most reasonable assumption that there was no vertical shift of the plasma column.

The results obtained for the measured magnetic field and their extrapolated least-squares fit at a disruptive instability are illustrated in Fig 6.1. The innermost coil in this case is at z = 3.5 cm. The time indicated is measured simply from the beginning of the traces on the oscilloscopes, which could be triggered by the variation in \( V_\phi \) as the instability was approached. Note that time is increasing towards the 'front' of the figure. The minimum of \( V_\phi \) for this shot was at time 46 μsec on the scopes.

Using equation (4.3) we obtain the toroidal current density as shown in Fig 6.2. The error introduced by ignoring the outward shift of the plasma is discussed in appendix 2 where it is shown that the identification of z with r (the radius from the true magnetic axis) leads, in our case, to errors in \( j(0) \) less than ~15%, only the same order of magnitude as the uncertainty in extrapolation.

The direction of z in Fig 6.2 has been reversed so as to give a more favourable perspective. We can see very vividly the rapid relaxation of the peaked current profile, in the space of a few microseconds, to a much flatter distribution.

As mentioned earlier, this relaxation leads to large inductive electric fields. Fig 6.3 shows the evolution through the disruption of the toroidal electric field. It should be emphasised that this is derived from equation (4.4) which is exact in the \( \Phi \)-independent case and is not affected by
Fig 6.1. EVOLUTION OF POLOIDAL MAGNETIC FIELD
Fig 6.2. EVOLUTION OF CURRENT DENSITY
Fig 6.3. EVOLUTION OF ELECTRIC FIELD
questions of plasma displacement. Peak electric fields greater than ten times the quiescent value are observed.

The profile of the safety factor, \( q \), is illustrated for various times in Fig 6.4. Prior to the disruption, \( q \) varies from a value ~ 3 at the aperture to less than 1 on axis. As the instability develops \( q \) rises, corresponding to the displacement of current towards larger minor radii, first in the region between about 4 cm and 7 cm and later throughout the entire plasma.

6.22 Discussion

The present results constitute a considerable advance upon the magnetic probe investigations on LT-1, since it is possible to follow the expansion of the current in a single disruption with a time resolution of about 2 \( \mu \)sec. Indeed even this speed limitation arises only from the data recording system and could be improved (at a cost) quite simply. Nevertheless, several features of the instability are clarified by the present data.

Prior to the instability the current profile has a form peaked on axis and falling off to a noticeable 'pedestal'. The instability shown in Fig 6.2 appears to illustrate a small precursor fall in the peak current followed by a slight recovery; this is peculiar to the event shown and does not appear in all disruptions. The main negative voltage spike then occurs as the current rapidly spreads, filling most of the vacuum vessel.

Perhaps one of the most notable facts concerning the instability is that at the time when the central current density is falling the toroidal electric field on axis is actually rising to very large values. This phenomenon of decreasing current with increasing electric field is quite often observed in plasma turbulence experiments and is referred to as 'current inhibition'. It is perhaps conceivable that it could be caused in a Tokamak by MHD turbulence.
Fig 6.4. Safety Factor Profile at the indicated times.
though it is more usually associated with velocity-space microinstabilities.

In LT-3 the magnitude of the resistivity anomaly and the observed ion heating lend the latter greater probability.

The redistribution of the current reduces the poloidal magnetic field energy by an amount which, if it were contained in the plasma, would raise the poloidal beta by several hundred per cent. Such an increase is not observed (c.f. appendix 1) and since the negative voltage spike is insufficient to return the energy to the primary circuit it must presumably pass into the plasma and out again to the walls very rapidly.

The energy input into the plasma is evidently through the high electric field which heats the plasma via whatever turbulent mechanism is present. If this energy were distributed through all the plasma particles, considerable recycling of the particles would be necessary to lose the energy to the walls. However if, as is suggested by neutral particle measurements, a small proportion of the ions were heated to very high energies, the loss of these could more easily account for the lack of energy containment. In fact a mechanism for the immediate escape of high energy ions exists in the deviation of their drift surfaces from the magnetic surfaces. If the ion energy were preferentially transverse to the magnetic field, as would be expected if cyclotron instabilities were responsible for heating, then the ions would be trapped particles. As such they would move on very large 'banana' orbits which would in most cases intersect the vacuum walls. Of course the magnetic surface disruption may allow particles to move to the walls along the magnetic field lines and thus cause the rapid energy loss.

The profiles of the electric field (Fig 6.3) show that $E_\phi$ actually reverses near the outside of the plasma. This is a reflection of the negative $V_\phi$
spike. The current remains positive at these radii throughout the spike. This fact may be understood by considering that the effective electric field in the plasma frame is \( E + v B \) where \( v \) is the bulk plasma velocity. If therefore the plasma has an outward radial velocity of expansion \( v_r \), there is a positive contribution to the effective field of magnitude \( v_r B \). There almost certainly is a bulk motion of the plasma at the disruption. The value of \( v_r \) is uncertain, but if we make the reasonable assumption that the bulk expansion is at approximately the same rate as the expansion of current \( (v_r \approx 10^3 \text{ msec}^{-1}) \), then this contribution causes the electric field effective in driving \( j_\phi \) to be as high in the outer regions as it is on axis. Thus the resistivity anomaly exists throughout the plasma not only at the centre.

The expansion of the current channel is observed to begin towards the outside of the plasma, resulting initially in a rise in the outer portion of the current pedestal. This effect is most clearly seen in Fig 6.4. The initial rise in \( q \) is localised between the \( q = 1 \) and the \( q = 2 \) surfaces. Only later as the disruption fully develops does the perturbation appear to affect the plasma inside the \( q = 1 \) surface and outside the \( q = 2 \) surface.

It is now widely agreed that MHD effects are responsible at least for the triggering of the disruptive instability. We may suppose that this initial \( q \)-perturbation is the triggering phase. Measurements show that MHD perturbations of the form \( m = 1, n = 1 \) and \( m = 2, n = 1 \) exist in LT-3 at this stage, corresponding to the integer \( q \) surfaces present in the plasma. The localisation of the initial perturbation between these surfaces supports the theory that an interaction between the different helical perturbations triggers the instability possibly through 'magnetic braiding'.
6.3 HIGH FREQUENCY FLUCTUATIONS

In view of the observed highly anomalous resistivity during the disruptive instability and the appreciable ion heating, we are led to investigate the possibility of plasma instabilities additional to the acknowledged helical MHD perturbations. Such microinstabilities may have frequencies between the ion cyclotron frequency, $\omega_{ci}$, which for our plasma is $\sim 5 \times 10^7 \text{s}^{-1}$ (i.e. ~10 MHz) and the electron plasma frequency $\omega_{pe} \sim 5 \times 10^{11} \text{s}^{-1}$ (~100 GHz). However, since it is mainly ion heating that is observed, we may expect ion- rather than electron-waves which reduces the upper limit to the order of the ion plasma frequency $\omega_{pi} \sim 10^{10} \text{s}^{-1}$ (~2 GHz).

Broadly, we may distinguish two possible types of ion-instability: those of the ion-acoustic type whose frequency spectrum peaks around $\omega_{pi}$ and which depends for its growth on an electron temperature higher than the ion temperature, and ion-cyclotron waves having a frequency around the cyclotron frequency or its harmonics with no such limitation on growth.

6.3.1 Electric Probe Measurements

In order to study the high frequency fluctuations in the plasma we employ an electrostatic probe of the construction described previously (section 2.2). In this case the probe is capacitively coupled to a balanced, terminated 78Ω line giving a low frequency cut-off around 1 MHz. The line feeds differentially a fast, wide band, oscilloscope (Tektronix 7904) and traces are recorded photographically. Writing speeds up to 50 nsec/cm prove possible. The scope is triggered appropriately during the disruptive instability in an attempt to observe high-frequency fluctuations of the plasma in the vicinity of the probe tip.
Fig 6.5(a) shows a typical trace. Bursts of high frequency oscillations are visible; the period of observation is indicated in Fig 6.5(b) by the $V_p$ signal, showing the spike, and the gate of the fast scope. In Fig 6.6 we show similar traces at a faster sweep rate. The fluctuations are now resolved.

It is unfortunately not possible to use the fast traces such as Fig 6.6 to obtain a realistic frequency spectrum of the oscillations because the frequency response of the probe and lines at these frequencies was not flat. Experiments are proceeding in an attempt to rectify this. However we may say that fluctuations are observed in the frequency range $\sim 20 - 100$ MHz.

The measured fluctuation amplitude ($\sim 1$V) corresponds to a plasma potential fluctuation amplitude attenuated by the ratio of the line impedance to the probe-plasma sheath impedance. This attenuation is $\sim 10^{-2}$ so the observed signals correspond to plasma potential variations between the probe electrodes of $\sim 100$ V and hence local fields $\sim 5$ kV/m.

### 6.32 Discussion

A degree of ambiguity exists regarding the source of the observed fluctuations. It is possible that these arise not from the plasma potential variation associated with instabilities but from the interference of runaway electrons.

A highly relativistic electron makes transits around the torus at a frequency given by

\[(6.1) \quad v_r = \frac{c}{2\pi R}\]

where $c$ is the velocity of light. For LT-3 we find $v_r \sim 100$ MHz. An electron of smaller energy will travel slower and have a correspondingly smaller transit frequency. If, therefore, bunching of the runaway electrons
Fig 6.5  
(a) The fluctuation probe signal.  
(b) Upper: $V_0$ (105 V/turn per div); Lower: Fast scope gate; 2.5 $\mu$sec per division.
Fig 6.6  (a) The fluctuations, resolved, at 50 nsec per div.  
(b) As Fig 6.5.
occurs so that their density is modulated in $\phi$, it is possible that they might strike the probe and induce oscillations in the frequency band which we are observing. Experiments to resolve this ambiguity are still in process.

If the observed fluctuations are truly a reflection of plasma instabilities, as appears more probable, the frequency band in which they appear points to the existence of very large amplitude ion-cyclotron waves.

Current inhibition and anomalous ion heating have been observed in experiments on ion-cyclotron drift waves in a Q-machine. Conditions at the LT-3 disruptive instability appear to be favourable for their generation. These waves are driven by the longitudinal electric field and constitute a likely candidate for the efficient transfer of energy to the ions. Of course ion-acoustic type instabilities, simultaneously excited, are not excluded by the present observations.

6.4 CONCLUSION

We have seen that the disruptive instability begins as a perturbation of the current channel localised between the $q = 1$ and $q = 2$ surfaces, supporting the theory that an interaction of two helical MHD modes is responsible for triggering the disruption. As the perturbation spreads to affect the whole plasma very large inductive electric fields are observed within the discharge. Current inhibition occurs, however, and the peak current density falls; anomalous resistivities $\geq 10$ times the quiescent value are observed which probably extend throughout the plasma.

It appears probable that microinstabilities play an important role in causing the resistivity anomaly and also the observed ion heating. In this case it is impossible to understand the disruptive instability purely in MHD.
terms. We must suppose that the expansion of the current channel triggered by helical modes gives rise to microinstabilities driven by the induced electric field. These cause a further fall in peak current density and the disruption occurs.

Identification of such microinstabilities is then an important aspect of any discussion of the disruptive instability. Preliminary investigations suggest the presence of ion-cyclotron waves during the instability though these observations cannot be claimed to be unambiguous. Further experiments must be performed before positive identification can be made.*

The prospect of utilising the disruptive instability as an additional ion-heating mechanism is of considerable interest. Ion-acoustic instabilities would be unable to raise the ion temperature above the electron temperature but ion-cyclotron heating would not be limited in this way. In reactor-size Tokamaks where the ion and electron temperatures are expected to be nearly equal this would be an important difference. Ion-cyclotron heating would then be the more useful mechanism.

On the other hand if the disruptive instability depends for its development upon acoustic waves then the near equality of temperatures could lead to the stabilisation of the microinstability and suppression of the disruption. This would be a most favourable result in itself.

* Note added in proof. Experiments have now demonstrated that runaway electrons are not directly responsible for the observed high frequency fluctuations.
CHAPTER 7.

CONCLUSION

7.1 DIAGNOSTICS

The temperature and energy confinement time of the plasma in LT-3 are orders of magnitude below those of the most 'successful' present generation Tokamaks. This may be regarded as an advantage, however. For, as the present thesis has shown, the possibility of introducing into the plasma probes of various kinds allows experimental measurements to be made which are either extremely difficult or very restricted by other means. For example, we have used magnetic probes extensively to derive the current density distribution across the plasma. There is as yet no sufficiently accurate, alternative, non-perturbing means of making this measurement directly, despite the considerable effort expended in this direction. Even those methods which have been proposed and shown some degree of success such as heavy ion-beam probing cannot be expected to provide the time resolution possible with the magnetic probe.

The chief drawback of the use of probes is that they perturb the plasma. Therefore, although probe measurements are comparatively easy to perform, they can be extremely difficult to interpret, with any confidence, in terms of plasma parameters. During the early stages of the discharge, with which the majority of the present work has been concerned, the lower temperatures lead to a minimisation of the plasma perturbation and, as has been
shown, probes can provide results of considerable significance.

The implementation of the 'diamagnetic measurement' of the plasma poloidal beta has demonstrated the viability and the difficulty of using a single $V_\theta$ loop. Moreover it has considerably enhanced the diagnostic capabilities of the device. The total kinetic energy of the plasma and the energy containment time are immediately derivable and in conjunction with the mean conductivity temperature, the diamagnetic measurement provides a considerably more convincing estimate of the electron density, albeit a mean value, than was previously available.

7.2 THE PLASMA EVOLUTION

We have followed the plasma through its various stages of evolution seizing upon particular characteristics, revealed by the experiments, for more detailed investigation. The results have been interpreted, with varying degrees of completeness, in terms which have, where possible, eschewed the esoteric complexity of some plasma theory. This has been achieved by two major devices: firstly by simplification, perhaps sometimes oversimplification, of the theoretical model and secondly by appealing either to the broad generalisations of theory or to well established particular results. By this means it has proved possible on a number of occasions to obtain fair quantitative agreement between theory and experiment without recourse to the now ubiquitous computer simulation. No doubt more complete models, requiring numerical solution, might improve agreement and highlight discrepancies where they exist. Nonetheless the present approach has contributed considerably to the theoretical understanding of the processes at work as well as to the
phenomenological description of the discharge.

It has been shown that the behaviour of the plasma in the pre-breakdown phase is dominated by the drift of electrons across the magnetic field. In particular we find that a certain critical electron line density, dependent only upon machine dimensions, is necessary before these drifts are compensated by the plasma current. Only after this condition is fulfilled does the ionisation of the total filling gas proceed. This has significance for Tokamak preionisation design as well as for an understanding of the effects of preionisation upon the discharge development.

During the ionisation stages of the discharge the energy balance between ohmic heating and inelastic collisional losses proves to be of the utmost importance. We find that an electron density peaked on axis, plausibly the most likely profile in any Tokamak, leads to the formation of a hollow current profile purely as a result of electron energy balance. The plasma therefore enters the fully ionised state with an already skin-like current density which would then be expected to be maintained or enhanced by the overheating mechanism.

In fact, however, the hollow current profile persists only for a very short period once ionisation is complete. A rapid relaxation of the profile then occurs which is attributable to the growth, in accordance with theoretical predictions, of a resistive MHD instability which ultimately leads to disruption of the magnetic surfaces and anomalous current penetration and particle and heat diffusion. This appears to be the mechanism which enables the anomalously rapid evolution to a centrally peaked current distribution near maximum current to occur in most Tokamaks.
Finally we turn our attention to the characteristic disruptive instability, arguably one of the most important areas of Tokamak study. Even though this occurs at the hotter stages of the discharge magnetic probes are shown to have considerable usefulness in following the instability. The current density evolution is clearly illustrated in its catastrophic redistribution at the disruption and the very large positive electric field on the plasma axis is derived. The early development of the perturbation of the safety factor profile suggests that the instability is initially related to the $q = 1$ and $q = 2$ surfaces and it may therefore be supposed to be MHD in origin. However the extremely high anomalous resistivity observed during the negative voltage spike suggests the presence of microinstabilities. Preliminary investigations confirm the existence of high frequency plasma fluctuations at a frequency which indicates the waves to be ion-cyclotron in nature. These probably play a very important role in the disruptive instability.

7.3 FURTHER STUDY

The investigations of this thesis have, in many cases, broken new ground in Tokamak research. Naturally, therefore, the present work is far from being the final word on the various aspects of Tokamak evolution studied. This applies both to the theoretical and the experimental. Fuller and possibly more rigorous theoretical studies will doubtless clarify many of the issues still outstanding. Meanwhile the applicability of the present results to other Tokamaks can only finally be established by experiments elsewhere. This is most important in order to distinguish any machine-dependent features of the results. The way in which scaling in size will change the behaviour of the
discharge may also be determined in this way.

The understanding of preionisation and prebreakdown in Tokamaks is really still very restricted. However, it will probably remain a rather low priority as long as it is possible to achieve the onset of bulk ionisation by the toroidal current by ad hoc means. The ionisation and current rise stages, on the other hand, are of considerable importance for energy efficiency and plasma purity. It is desirable to reach the fully ionised state as quickly as possible in order to minimise radiative energy loss. We have shown, however, that the current profile is of importance in this process. Only further experiments can show whether, by judicious choice of the rate of current rise, it is possible to achieve full current penetration without excessive perturbation of the magnetic surfaces and unacceptable interaction with the vacuum-chamber walls.

The investigations of the disruptive instability have raised the important question of microinstabilities. A great deal more experimental research will be necessary to determine their nature and role. To this end the repertoire of techniques for fluctuation observation such as microwave scattering and small angle laser scattering will be necessary. Probes have not outlived their usefulness, however, and can still supplement the other techniques in study of both fluctuations and MHD behaviour. One immediate possibility is the use of internal magnetic probes to investigate magnetic island structure.
In addition to the incentive of the prospect of controlled fusion the Tokamak offers an inherent fascination of its own. Its symmetries make possible the analytic treatment of much of the theory at least to a reasonable approximation; while the toroidal geometry gives rise to phenomena such as runaway electrons and neoclassical transport which have no true counterpart in linear devices. For this reason it constitutes in many ways an ideal plasma research device providing stimulation for theorist and experimentalist alike.

Perhaps the Tokamak will not, in the end, prove to be the most satisfactory approach to controlled fusion but its fascination tempts one to hope that it will.
APPENDICES ON DIAGNOSTICS
APPENDIX 1: THE DIAMAGNETIC MEASUREMENT

A1.1 Introduction

The measurement of the poloidal beta of a Tokamak plasma by the so-called diamagnetic effect has several attractive features. In principle it provides a continuous measure of the total transverse plasma energy throughout a single discharge in contrast to pulsed techniques, such as laser scattering, which give a measurement at a single instant and require many shots to give a complete profile. This makes the diamagnetic measurement particularly attractive for studying transient, and sometimes irreproducible, phenomena such as the disruptive instability. Its results have been shown to be consistent with other methods in stable discharges.

The measurement is based on the fact that the zero order pressure balance equation may be written in the integrated form

\[ 1 - \beta_I = \frac{8 \pi B_\phi \delta \phi}{\mu_0 I^2} \]

where

\[ \beta_I \equiv \frac{\pi a^2}{\mu_0} \frac{8 \pi}{I^2} \]

is the poloidal beta, \( I \) the plasma current, \( B_\phi \) the toroidal magnetic field, \( p \) the transverse plasma kinetic pressure (bar denoting mean within radius \( a \)), \( a \) some minor radius outside which \( p = 0 \), and \( \delta \phi \) (\( \equiv \pi a^2 (B_\phi - B_\phi(a)) \)) the change in toroidal flux due to the plasma.

The difficulty in determining \( \delta \phi \) in Tokamaks resides mainly in the fact that the large toroidal magnetic fields require that we measure relative flux variations of the order of \( 10^{-3} \), and other variations in \( B_\phi \) must
therefore be accurately compensated.

Under most conditions in Tokamaks the pressure is isotropic. However, even in conditions where strong anisotropy may be expected, for example in the "runaway regime", measurement of the transverse energy remains an important diagnostic.

A1.2 Experimental Measurements

If the liner of the Tokamak has minor radius \( w \), thickness \( \lambda \ (<< w) \) and conductivity \( \sigma \) and we have a loop of radius \( \rho \ (> w) \) measuring the induced poloidal e.m.f. \( (V_\theta) \) then it may be shown that:

\[
(A1.3) \delta \Phi = - \left[ \int V_\theta \, dt + \pi \rho^2 \mu_0 I_B + \frac{\mu_0 \sigma w \lambda}{2} \cdot V_\theta + \frac{\mu_0 \sigma \omega w \pi (\rho^2 - w^2)}{2} \cdot \frac{\partial B}{\partial t} \right]
\]

where \( I_B \) is the current per unit length in the \( B_\phi \) winding and \( R \) the torus major radius. This is a straightforward generalisation of the equation obtained by Rothman and is strictly true only in cylindrical geometry; however, the toroidal corrections are negligible. The first two terms of this expression are very much larger than the others and the diamagnetic signal is essentially the difference of their magnitudes (they have opposite signs). The last two terms are corrections for the finite conductivity of the vacuum chamber wall.

Two methods of obtaining the very small difference between the terms of this equation in Tokamaks are reported. The first on T-5, measures the voltage across the \( B_\phi \) winding as well as the current in it, thus effectively using the winding as the \( V_0 \) loop. The second on T-3, utilises the long rise-time of \( B_\phi \) relative to the duration of the discharge; this is impossible, however, when the \( B_\phi \) rise-time is of the same order as the discharge duration.
It was decided to attempt to implement the diamagnetic measurement on LT-3 using a single turn V_θ loop, despite the difficulties in this method reported by Razumova. The major problem consists in the fact that the derivation of Eq. (A1.3) assumes toroidal symmetry. No difficulty would arise if a mean value round the torus of the flux were used but a single V_θ loop measures flux at only one \( \phi \) value. Again if \( B_\phi \) were always proportional to \( I_B \), inhomogeneities in \( B_\phi \), arising for example from uneven distribution of windings, would require only a slight alteration in the coefficients of Eq. (A1.3). However, inhomogeneities penetrate the copper shell only over times of the order of \( \tau \approx 10^{-2} \) sec so that there is a small additional component of \( B_\phi \):

\[
\Delta B_\phi = \frac{1}{\tau} \exp\left(-\frac{t}{\tau}\right) \int \exp\left(\frac{t'}{\tau}\right) \Delta B_{\phi_0} \, dt'
\]

where \( \Delta B_{\phi_0} \) is the inhomogeneous component in the absence of the copper shell (\( \Delta B_{\phi_0} \propto B_\phi \)). This causes a stray pick-up in \( V_\theta \) proportional to \( \Delta B_\phi \).

We find that provided this effect is compensated it is possible to perform the measurement. We employ an analogue circuit as shown in Fig. (A1.1) for the summation and integration, with a single operation amplifier. In this way the coefficients of Eq. (A1.3) are determined solely by passive components. \( I_B \) is measured by a rogowski coil around the current feed to the \( B_\phi \) winding. The compensating input from \( V_\theta \) provides a term of the form of \( \Delta B_\phi \) with \( \tau = C_3 R_3 R_4 / (R_3 + R_4) \), \( R_5 \ll R_4 \).

The trimming resistors and capacitors are finely adjusted to give minimum signal in the absence of plasma. A small ripple remains but this
Fig A1.1. The Analogue Summation/Integration Circuit. Each branch is labelled according to the term it provides.
defines the baseline, relative to which the measurement of $\beta$ is made. By this means we ensure the absence of any extraneous pick-up, of whatever nature, caused by the $B_\phi$ winding or circuit.

In the presence of the plasma we obtain a signal, $\delta \phi$, which may be used together with the measured values of $B_\phi$ and $I$ to obtain $1 - \beta_1$ from Eq. (A1.1). The calibration of rogowski coils used and the effective radius of the $V_\phi$ loop may be checked by obtaining a direct relative calibration of the diamagnetic measurement by seeking conditions such that $\beta = 0$. These occur at high current and low pressure. We find that our calibration is very well confirmed and the overall calibration of the measurement of $\beta_1$ is believed to be correct to better than $4/\beta_1 \%$.

It is important that any pick-up from the primary windings, iron core or plasma current should be compensated. There is a spurious signal proportional to the flux in the core, probably due to stray fields. This is easily compensated provided no core-saturation occurs and has been taken into account in the calibration procedure described above. That no other pick-ups (such as vertical field pick-up due to coil misalignment) exist due to the toroidal current circuit is indicated by the good agreement of the absolute and relative calibrations, since any spurious signal should show up as a discrepancy here. In addition we reversed the direction of primary and plasma currents, which should leave the true diamagnetic signal unchanged while reversing any electrical pick-up. The results obtained for calibration and other discharges were unchanged to within experimental accuracy. This demonstrates the absence of any such pick-up.

There exists a systematic error owing to mains hum in the
operational amplifier employed. This is important only at low signal levels corresponding to very small plasma currents.

The overall accuracy of the measurement of $\beta_I$ under most conditions is $(5 \text{ to } 10)/\beta_I \%$.

The frequency response of the circuit measuring $\delta \phi$ is flat to about 500 kHz where the self inductance of the rogowski coil measuring $I_B$ becomes important. Similar limitations are obtained by consideration of skin effects in the liner and the assumption of MHD equilibrium inherent in Eq. (A1.1). However, it is found convenient to filter the inputs to remove pick-up of the 2 MHz R. F. used to preionise the plasma. For this purpose we employ low-pass filters whose 3 dB point is at 270 kHz. These then define the overall frequency response.

A1.3 Results

Typical results for the time evolution of $\beta_I$ in LT-3 are shown in Fig. (A1.2) for a stable discharge, (a), and an unstable discharge, (b) obtained by lowering $R_\phi$ by 15%, the other conditions being the same. The error bars on the $\beta$ curve show the estimated systematic error. The gross energy confinement time, $\tau_E$, is calculated from the total kinetic energy, obtained from $\beta_I$, and the ohmic heating input. This may be written in the form:

$$\tau_E = \frac{3}{8} \bar{\mu}_0 R \frac{\beta_I}{R_p}$$

where $R_p$ is the resistance of the plasma. $R_p$ was calculated ignoring any inductive component of $V_\phi$ due to contraction of the plasma column. This may introduce an additional error in $\tau_E$ of up to about 15%.

The rapid rise in $\beta_I$ between about 0.3 and 0.5 msec corresponds
Fig A1.2. Typical Results; filling pressure 0.5 mtorr.
(a) Stable discharge $q_{\text{aperture}} = 4.1$ at peak current.
(b) Unstable discharge $q = 3.5$. 

(a) Stable discharge

(b) Unstable discharge
to the bulk ionisation of the (hydrogen) gas, as is confirmed by observations of $H$ emission. Subsequently $\beta_1$ is approximately constant at about 0.6 until, as peak current is reached, $\beta_1$ and consequently $\tau_e$ rise; a similar effect to that reported in T-3.

In the unstable case a clear drop in $\beta_1$ occurs at the disruptive instability indicating a loss of plasma energy. The fall in energy confinement time at the instability is even more marked. However, the evolution of $\beta_1$ and $\tau_e$ right up to the first instability is strikingly similar to that in the stable discharge.

Although values of $\beta_1$ between 0.2 and about 1.0 have been reported in Tokamaks, the present results confirm the general trend of $\beta_1 = \frac{1}{4}$, and do so at low temperatures ($T_e \approx 40$ eV) and at lower toroidal magnetic fields ($B_\phi = 0.5$ T) than previously reported.

The abrupt change in $\beta_1$ at the disruptive instability is not always a fall. In fact a very systematic trend is observed depending upon the ratio of toroidal magnetic field to total plasma current or, equivalently, on the value of $q$ at the aperture. We find that for the range of magnetic fields available ($B_\phi = 0.4 - 0.8$ T) the poloidal beta (and hence the plasma kinetic energy) falls at the disruption for $q_{\text{aperture}} < 4.0$ and rises for $q_{\text{aperture}} > 4.0$. This changeover point is defined quite accurately: to within about 10%. The maximum rise or fall observed is about 30\% of the beta value. In other words the plasma energy changes by at most 30\%.

It is tempting to suppose that the significance of the change from rise to fall is that the presence of the $q = 4$ surface within the plasma allows an increased energy loss mechanism to lower the energy. The value of $q$ at
the aperture may not be the determining factor, however. Processes occurring at the inner regions of the plasma may be the cause of the observed effect. Further experiments are necessary to clarify this point.
A2.1 Introduction

Langmuir probe characteristics in a pulsed discharge may be obtained either by applying a constant bias voltage which is varied from shot to shot or by applying a voltage rapidly varying during a single shot. The latter technique allows the characteristic to be displayed directly on an oscilloscope by applying the probe voltage to the x-plates and the probe current to the y-plates.

The bias voltage must be swept at a rate whose lower limit is obviously set by the time during which the discharge conditions may be considered constant. The maximum sweep rate is limited essentially by the parasitic capacitances of the probe. At high frequencies the current passed by these can become an appreciable proportion of the observed probe current and hence distort the measurement.

Despite these limitations the swept technique offers considerable advantages. It avoids shot-to-shot variations, giving a complete characteristic in one shot. It reduces the likelihood of the development of unipolar arcs. It can reduce the problems of probe contamination and heating. Especially, it removes much of the (sometimes considerable) tedium of data reduction. A major drawback to the technique, however, is that in its usual implementation a single sweep is employed which gives data on the plasma only at a single brief instant of the discharge. In contrast, the static technique provides information for the whole discharge with a time resolution limited only by shot-to-shot variations and probe response.

A multiple sweeping technique has been developed for use on LT-3
in an attempt to combine the advantages of a swept probe with the facility of obtaining information at more than one instant of the discharge. This is achieved by the display of a number of characteristics on the oscilloscope each translated by an amount proportional to the time at which it is taken.

A2.2 Implementation

The probe is continuously swept with a triangular wave. The probe circuit is shown in Fig. A2.1(a). The bias voltage is obtained from a function generator, stepped up by the 10 : 1 transformer to provide adequate voltage. The current provided by the generator used is sufficient for the present purposes but for larger probe currents an amplifier would be necessary. The probe current is measured by a second isolating transformer in parallel with a ten ohm resistance. This acts as a current transformer whose impedance is negligible compared to the effective probe impedance (~ 5 kΩ).

In order to display the characteristics a Tektronix 555 oscilloscope is used. This has fully independent dual beams thus enabling the upper beam to be used to display the characteristics while the lower beam displays the time evolution of another plasma parameter, in our case \( V_\phi \). The probe current signal is applied via the scope plug-in amplifier to the y-plates. The horizontal displacement of the upper beam is controlled by the external-x input which is driven by the circuit of Fig. A2.1(b). The saw-tooth output from the lower beam time base is added to the probe voltage signal and applied to the external-x. The x-gain is adjusted so that for zero probe voltage the upper beam sweeps at just the same rate as the lower beam, as if it operated from the same time base. Then when the oscillating probe voltage is applied the centre point moves so as to define the time. In other words the characteristic is translated by an amount
Fig A2.1. (a) Langmuir probe circuit
(b) Scope horizontal input circuit.
which is proportional to the time elapsed. The 'bright-up' is controlled by a pulse generator which is gated on by the gate of the lower beam time base. The pulse length is adjusted to last for just one cycle of the probe sweeping signal, to provide a number of separated characteristics.

The result is illustrated in Fig. A2.2. The sweeping signal frequency used here is 50 kHz and a characteristic is taken every 50 μsec. In this early stage of the discharge the density is rising and the saturation current therefore also is observed to rise from practically zero. The characteristics are quite symmetric, as expected for this probe. Fluctuations on the characteristics caused by plasma variation faster than the sweep rate are not too serious in this period. Naturally the characteristic curves do not close after the full cycle since they have been translated by 20 μsec (0.4 div). This effect can be easily compensated in analysing the curves. In addition to this factor there is a visible 'hysteresis' effect. For example, even for the 'baseline' where no plasma is present and no true probe current flows the characteristic curve is approximately a flat rectangle.

The hysteresis is due to the parasitic capacitance mentioned earlier. If the probe has effectively a capacitance in parallel with it then there is an extraneous current passed by this capacitance proportional to the derivative of the probe voltage. Even though this capacitance is kept as small as possible (~ 18 pF in our case) at these high frequencies the effect is noticeable. Herein, however, lies the motivation for using a triangular wave sweeping signal rather than any other form such as sine-wave or saw-tooth. The derivative of the triangular wave has a particularly simple shape: the square wave; hence the rectangular shape of the 'base line' signal. For the upward or the downward
Fig A2. 2. Upper: The swept Langmuir probe characteristics, 50 V per division horizontal, 4.4 mA per division vertical. Lower: $V_A$, 21 V/turn per division; 50 $\mu$sec per division.
sweeping halves of the characteristic, therefore, it is a simple matter to correct for the constant upward or downward shift. In order that this correction should be manageable and that sufficient current should be available from the generator we are still limited in the present implementation to frequencies \( \leq 100 \text{ kHz} \). The capacitance may be compensated for by subtracting an appropriate current from the observed current, for example by using a simulated probe without plasma\(^8\). This would then allow an increase in frequency up to that where the probe-plasma-sheath impedance becomes important (perhaps \( \sim 500 \text{ kHz} \)).

The present results are quite adequate for our needs, however, especially in view of the uncertainty of interpretation of characteristics in high magnetic fields, and the comparative ease and speed with which the characteristics may be analysed constitute a considerable advantage over static techniques.
APPENDIX 3: MAGNETIC PROBE DATA ANALYSIS

A very important question arises in the interpretation of magnetic probe measurements concerning the perturbation of the plasma by the probe. If such perturbation is extreme, little significance may be attached to the straightforward interpretation of the results described previously. The possible ways in which our analysis might be invalidated may be considered in three categories:

(a) The presence of the probe may so alter the overall character of the discharge as to provide no information upon behaviour of the plasma in the absence of the probe. (An example of this is the suppression of the disruptive instability by the introduction of a probe to less than ~3.5 cm from axis.)

(b) The progressive introduction of the single coil probe may perturb the plasma in a systematic way which gives rise to spurious effects in the profile because of the progressive perturbation of the magnetic field.

(c) The probe may perturb the plasma locally in such a way that the measured magnetic field is not representative of the overall field at that radius.

The main effect of (c) is to restrict the spatial resolution of the measurement to greater than the radius of the non-conducting 'hole' in the plasma caused by the probe. This is almost certainly negligible for our profiles. In any case the spatial resolution cannot be worse than the finest structure actually demonstrated. We may therefore safely discount (c).

To some extent the effects (a) and (b) can be monitored through the other parameters of the discharge. We find, for example, that inserting the
probe right in to the vacuum vessel axis \((z = 0)\) causes the toroidal voltage in the early stages to increase by about 20\% (the total current is unchanged, being determined largely by the external circuit). There evidently is some perturbation. The extent of the effect upon the profiles (e.g. Fig. 4.3) may be estimated by comparison of the results obtained from single coil and multicoil measurements.

Errors of the type (b) are completely averted when using the multicoil probe since all measurements are for a single shot. Whatever else may obtain therefore, we can be assured that the multicoil probe does give a true measurement of the \(B_\phi\) profile for a discharge in the presence of the probe.

There is some justification a priori for expecting single coil measurements to approximate quite closely to the profile in the absence of the probe. This is based upon the realisation that \(B_\phi\) depends only upon the total current inside the probe radius whereas the main perturbation is at or outside that radius. However, if the probe acts to reduce the total current outside the radius to which it is inserted, rather than simply redistributing it, then a corresponding increase in current within the radius must occur. This would result in a deduced current profile which was too peaked on axis. In other words we should expect to be underestimating the hollowness of the profile.

Fig. A3.1 allows a comparison of the results obtained with the two probes. The current density, which requires differentiation of \(B_0\), is the most sensitive indicator. It can be seen that the main difference observed is the depression of the current peak in the multicoil case. This deviation is characteristic of the entire period of observation; the multicoil probe results reflect the overall pattern of Fig 4.3 but with a generally broader and flatter
Fig A3.1. Comparison of Results of Single- and Multi-coil Magnetic Probes.
current distribution. The similarity of the profiles allows us greater confidence that the single coil probe measurements realistically reflect the profiles in the absence of the probe.

A further possible source of systematic error in the interpretation of the measurements lies in the possibility of a shift of the magnetic axis. A vertical shift is adequately accounted for by taking the zero of \( r \) at the \( B_R \) zero crossing provided we can introduce the probe far enough. However, no allowance has been made for any outward shift.

During the most of the first millisecond the plasma inductance is small ('hollow' current profile) and so the outward shift, \( \Delta \), expected is small. In fact the Shafranov formula predicts \( \Delta \approx 0 \). The applied vertical field almost exactly compensates the tendency towards outward shift. In addition a magnetic probe was introduced horizontally from the outside providing an experimental measurement of \( \Delta \) from the zero crossing. This was found to be \( \Delta = 0.0 \pm 0.5 \) cm except during the very early period while the current peaks on axis. A shift of this magnitude has no appreciable effect upon the profiles.

For the measurements of Chapter 6, however, there is certainly an appreciable outward shift. We make the approximation (valid in the regions near axis where there is any necessity to consider \( \Delta \)) that the magnetic surfaces are concentric circles in cross section, of radius \( \rho \), whose centres are shifted by \( \Delta \). When the magnetic probe is at radius \( r \) from the vessel axis, therefore, the true radius from the magnetic axis is \( \rho = (r^2 + \Delta^2)^{\frac{1}{2}} \). In addition the direction of the true poloidal magnetic field \( B_p \) is such that we measure:
The analysis program ignoring \( \Delta \) utilises as the total apparent current within \( r \)

\[
I_{ap}(r) = \frac{2 \pi r B_R(r)}{\mu_0}
\]

and then the apparent current density is

\[
j_{ap}(r) = \frac{1}{2\pi r} \frac{d I_{ap}}{dr}
\]

Whereas the true total current is

\[
I(\rho) = \frac{2\pi \rho B_p(\rho)}{\mu_0}
\]

and the current density

\[
j(\rho) = \frac{1}{2\pi \rho} \frac{dI}{d\rho}
\]

Combining equations (A3.1) (A3.2) and (A3.4) gives:

\[
\bar{j}_{ap} \equiv \frac{I_{ap}(r)}{\pi r^2} = \frac{I(\rho)}{\pi \rho^2} \equiv \bar{j}
\]

In other words the mean apparent current density within a given (apparent) radius is equal to the mean actual current density within the corresponding actual radius. Thus if the current density were constant no error would occur if we ignore \( \Delta \) and identify \( r \) with \( \rho \).

In the case when \( j(\rho) \) is not constant we approximate it near the axis by the first two non-zero terms of its Taylor expansion:

\[
j(\rho) \approx j(0) \left( 1 - \frac{\rho^2}{\lambda^2} \right)
\]

where \( \lambda^2 \) is determined by \( \frac{d^2 j}{d\rho^2} \) at \( \rho = 0 \). A little manipulation then discovers the corresponding apparent current density to be:
The apparent current density near the centre is thus equal to the true current density less a constant $j(o). \Delta^2/(2\lambda^2)$. The magnitude of this correction may be determined from the apparent current density profile by observing that it is just that amount which $j_{ap}$ has fallen at a radius $\Delta/\sqrt{2}$, i.e.

$$j_{ap}(r) = j(o) \left( 1 - \frac{r^2}{\lambda^2} - \frac{\Delta^2}{2\lambda^2} \right)$$

The applicability of this equation is restricted by the validity of the approximate expansion (Eq. A3.7) but since the correction $(j - j_{ap})$ is important only at radii $r \leq \Delta$ it will in general provide a good approximation. In particular for the profiles of Fig 6.2 with the expected shift $\Delta \leq 2$ cm the apparent peak current density underestimates the true value by less than $\sim 10\%$. This is less than the uncertainty introduced by the extrapolation and we are thus justified in ignoring it.

Our assumptions of cylindrical symmetry take no account of any toroidal corrections. However, introducing the probe vertically avoids spurious effects due to variation of magnetic field with major radius. The analysis should therefore accurately reflect the profile in so far as it may be described by a cylindrical model; that is, to zero order in $a/R$.

Finally, the effect of the choice of the number of terms, $N_{fn}$, of the least squares polynomial should be mentioned. This is illustrated in Fig A3.2. It may be seen that the minimum in $j$ at $\sim 7$ cm for $N_{fn} = 8$ is exaggerated by the fitting procedure, as is suggested also by an examination of Fig 4.2. Although this short-coming may be removed for the profile illustrated by different choice of the fitting polynomial it is felt desirable to maintain $N_{fn}$ fixed for all
Fig A3.2. The effect of the choice of fitting polynomial.
the profiles throughout the whole period of interest. This avoids the somewhat arbitrary change of $N_{fn}$ from one profile to the next but it means that we must accept a compromise in order to fit all the profiles satisfactorily. If in doubt we can return to the actual data (Fig 4.2) to determine the significance of any feature. It should be emphasised that the significance of the overall hollowness of the current profile is in no way brought into doubt by these questions. The magnitude of the probable error in $j$ may be estimated from Fig A3.2. This estimate has been included in Fig 4.3.

For the curves of Fig 6.1 the number of terms was chosen so as to optimise the fitting of the points available, provided no spurious oscillations were introduced within the minimum radius at which measurements existed. Six terms was usually most satisfactory.
APPENDIX 4: RUNAWAY DIFFUSION

We consider a one dimensional slab model in which the equations governing the diffusion are:

(A4.1) \[ j = - D \frac{\partial \rho}{\partial x} \]

(A4.2) \[ \frac{\partial j}{\partial x} = - s \rho - \frac{\partial \rho}{\partial t} \]

where \( \rho \) is the density of runaways, \( j \) the runaway flux, \( D \) the diffusion coefficient, \( x \) the spatial co-ordinate and \( s \) the proportion of runaways absorbed per unit time, \( t \). We ignore the energy spectrum of the runaways.

In order to render the equations tractable we approximate the presence of the target by taking:

(A4.3) \[ s = \text{constant} \quad 0 \leq x \leq w \]

\[ = 0 \quad w < x \]

which is equivalent to assuming we have a rectangular target whose end is at \( x = 0 \) and whose length is \( w \), mounted on an infinitesimally thin wire. This approximation gives an adequate representation of the experimental situation considering the many other uncertainties involved. We also assume \( D \) is effectively constant over the target length.

We wish to determine the proportion of runaways which strike the probe whilst diffusing outwards. For this purpose it is unnecessary to attempt to model the time-dependent experimental situation. Instead we treat the time-independent case, prescribing the flux at \( x = 0 \):

(A4.4) \[ j (0) = j_0 \]
The second boundary condition consists of the requirement that \( p = 0 \) at the boundary \( (x = b > w) \).

In the region \( x > w \),

\[
(A4.5) \quad j = \text{constant} = j_1 \quad \text{(say)}
\]

leading to:

\[
(A4.6) \quad \rho(x) = j_1 \int_{x}^{b} \frac{dx'}{D}
\]

We have, therefore, at \( x = w \):

\[
(A4.7) \quad \rho(w) = j_1 K = \rho_1 \quad \text{(say)}
\]

where

\[
(A4.8) \quad K = \int_{w}^{b} \frac{dx}{D}
\]

In the region \( 0 < x < w \)

\[
(A4.9) \quad \frac{d^2 j}{dx^2} = k^2 j
\]

and an identical equation governs \( \rho \). (We have defined \( k^2 \equiv s/D \).) This may be solved, prescribing \( j(0) = j_0 \) and \( \rho(w) = \rho_1 \) to obtain:

\[
(4.10) \quad j(w) = -Dk \left( -\frac{j_0}{Dk} \sech kw + \rho_1 \tanh kw \right) = j_1
\]

Substituting from (A4.7) we find finally:

\[
(A4.11) \quad j_1 = \frac{1}{\cosh kw + KkD \sinh kw}
\]

The total flux to the probe is then

\[
(A4.12) \quad F = j_0 - j_1 = j_0 \left( 1 - \frac{1}{\cosh kw + KkD \sinh kw} \right)
\]
Consider now a different probe, denoted by primed quantities,
whose linear dimensions are z times the first, then:

\[ k' = z^3 k, \quad w' = zw \]  \hspace{1cm} (A4.13)

and so the ratio of the fluxes to the probe is:

\[
\eta = \frac{F'}{F} = \frac{1}{1 - \frac{1}{\cosh (kwz^{3/2}) + KkDz^{1/2} \sinh (kwz^{3/2})}}
\]

\[
= \frac{1}{1 - \frac{1}{\cosh (kw) + KkD \sinh (kw)}}
\]  \hspace{1cm} (A4.14)

Note that in the limit \( kw \gg 1 \) equation (A4.14) implies \( F'/F = 1 \); the slow diffusion limit. Whilst if \( kw \ll 1 \) and \( KD(w) \gg w \) then \( F'/F = z^2 \); the fast diffusion limit.

\( KD(w) \) is a measure of the scale length of the increase in \( D \) outside \( w \) and may be estimated from the fall off of the runaway measurements as follows: From (A4.8)

\[
KD(w) = -K \frac{dK}{dw}
\]  \hspace{1cm} (A4.15)

But \( K \propto \rho(w) \), from(A4.7)and in the region of rapid diffusion \( \rho(w) \propto F \) (the observed flux) so

\[
KD(w) = -F \frac{dF}{dw}
\]  \hspace{1cm} (A4.16)

Then knowing \( w \) and \( z \) a measurement of \( \eta \) allows us to obtain \( k \) by solving equation(A4.14). For a target of width \( d \) the absorption coefficient, \( s \), is

\[
s = \frac{d}{2 \pi r} N_t
\]  \hspace{1cm} (A4.15)

where \( r \) is the radius of the corresponding runaway orbit and \( N_t \) is the number of transits around the torus made by the runaways per unit time. From \( k \) and
s we may deduce \( D \), the diffusion coefficient, or, more directly, the RMS diffusion step length which is:

\[
\delta = \left( \frac{d}{2 \pi r} \right)^{\frac{1}{2}} \frac{1}{k}
\]

(A4.16)

As a confirmation of the \( k \)-value obtained we substitute back in equation (A4.12) to obtain \( F/j_0 \). Combining this with the relative magnitude of runaway signal observed as a function of \( Z \) we can deduce the reduction of runaway population due to the suppression of runaways at radii intersected by the target probe. This gives the predicted effect of quenching by a wire quencher (Fig 5.11), the absorption of diffusing runaways by the quencher being negligible at these radii.
REFERENCES

2. G. Francis et al Plasma Physics and Controlled Nuclear Fusion Research (Culham 1965), 1, 53.
20. B. S. Liley et al Proc. IREE 29 (1968) 221.
23. M. G. Bell to be published.
24. E. L. Bydder to be published.
26. A. Eberhagen, ibid, p. 43.

45. R. L. Freeman, E. M. Jones Culham Laboratory Rep. CLM-R 137.


61. E. L. Bydder to be published.


68. e.g. M. Yameda et al Phys. Rev. Lett. 34 (1975) 650.


## SYMBOLS

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