HIGH ENTHALPY HYPERSONIC BOUNDARY LAYER FLOW

by

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The contents of this thesis, except as described in the
Acknowledgement, and where credit is indicated by a reference,
are entirely my own work.

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(Gilbert Yanow)

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ABSTRACT

The aim of this project has been to carry out a theoretical and experimental study of the properties of a laminar boundary layer formed by very high enthalpy flows (in excess of 12eV or 7000 cal/gm) in an ionizing monatomic gas. An ionizing monatomic gas, Argon, has been used because it is possible to achieve a wider range of the transport properties and a better determination of the thermodynamic relations.

Experiments have been made with a Free Piston Shock Tunnel, and has required a detailed study of its nozzle operation, including the effects of low levels of Helium driver gas contamination.

The investigation has shown that the use of variable transport properties and their respective derivatives is very important in the solution of equilibrium boundary layer equations of high enthalpy flow. When calculations are made with the transport properties set constant at the free stream values, significant errors result in the velocity profile, enthalpy profile, and surface heat transfer rate predictions. In addition, the effect of low level Helium contamination on the surface heat transfer rate is minimal.

It was found that the variation of the ionization was much smaller in the chemically frozen boundary layer solutions than in the equilibrium boundary layer calculations. Consequently, the variation of the transport properties, in the case of the former, was not essential in the integration.

It has been demonstrated that neither of the extreme solutions of an equilibrium nor of a frozen boundary layer will adequately

predict the surface heat transfer rate in very high enthalpy flows. This has been attributed to non-equilibrium gas relaxation processes in the boundary layer. However, a satisfactory approximation can be obtained with what has been called a composite boundary layer calculation. That is, a level in the boundary layer is defined where the gas is assumed to change from primarily chemically frozen in nature to primarily equilibrium in nature. The position of this changeover is determined by a simple criterion based on the recombination rate in the gas stream tubes that are taken to form the boundary layers. The solutions are joined by matching the velocities and shear stress.

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CHAPTER 1

INTRODUCTION

1.1 Purpose

The objective of this investigation is the study of the boundary layer structure and surface heat transfer in a very high enthalpy hypersonic flow. The work has included theoretical and experimental studies. An important subsidiary study was made of the nozzle operation of a reflected shock tunnel - the device used to produce high enthalpy hypersonic flow in the laboratory - with special attention being given to the effects of Helium driver gas contamination upon the flow characteristics.

For the purpose of this study, the term "very high enthalpy" will be taken to refer to free stream stagnation enthalpies in excess of 12eV (approximately 7000 cal/gm).

1.2 Boundary Layers

A thin region known as the "boundary layer" develops on a vehicle when it travels through an atmosphere. This very important layer is brought about by the viscous nature of the gas, through which the velocity of flow changes from the free stream value to zero at the body surface¹.

Boundary layer problems, in the context of this study, are classified as follows:

(a) Subsonic - At subsonic speeds the heat dissipated through the viscous forces will be small and there will be no chemical reactions occurring in the gas. The fluid and transport properties remain essentially constant throughout the boundary layer, and the solution of the problem is greatly simplified.

¹In low density flows, movement or "slip" may occur at the surface.

(b) Supersonic - At supersonic speeds sufficient kinetic energy will be dissipated for substantial temperature changes to occur within the boundary layer, and this will influence the thermodynamic and transport properties of the fluid. Fortunately, the changes that do occur are such that they can be approximated in a relatively simple manner (e.g.: $P_r = 1$, $\rho\mu = \text{constant}$) and the boundary layer problem is readily solved.

(c) Hypersonic - At hypersonic speeds, the range of interest in this study, the dissipated kinetic energy increases to the point where ionization of the gas occurs. The presence of free electrons greatly modifies the viscosity and thermal conductivity of the gas and the solution of the boundary layer problem becomes much more difficult. This is illustrated by the equation of motion for the boundary layer. Taking the y-axis normal to the surface, the equations for a reacting gas boundary layer (neglecting thermal diffusion) are¹, in two dimensions,

Conservation of Mass:

$$\frac{\partial \rho u r^j}{\partial x} + \frac{\partial \rho v r^j}{\partial y} = 0, \quad (1-1)$$

Conservation of Momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{\partial P}{\partial x} + \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right), \quad (1-2)$$

Conservation of Species:

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial}{\partial y} \left(\rho D_i \frac{\partial C_i}{\partial y} \right) + \dot{w}_i, \quad (1-3)$$

¹See Dorrance (1962), Chapter 2.

Conservation of Energy:

$$\rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y} \left[\frac{\mu}{Pr} \frac{\partial H}{\partial y} + \mu \left(1 - \frac{1}{Pr} \right) \frac{1}{2} \frac{\partial u^2}{\partial y} \right] - \frac{\partial}{\partial y} \left[\left(\frac{1}{Le} - 1 \right) \rho D_i \sum_{i=1}^n h_i \frac{\partial C_i}{\partial y} \right] \quad (1-4)$$

where

$j = 0$ for two dimensional flow, $j = 1$ for axisymmetric flow,

ρ = density of the total mixture,

P = pressure of the total mixture,

μ = viscosity of the total mixture,

C_i = the mass fraction of species i ,

D_i = coefficient of diffusion of species i through the mixture,

$\dot{w}_i$ = mass rate of change of species i per unit volume (reaction rate),

$$H = \frac{1}{2} u^2 + h$$

$$h = \sum_i C_i h_i,$$

$$h_i = \int c_{pi} dT + h_i^0,$$

h_i^0 = heat of formation of species i ,

$$Pr = \text{Prandtl number} = \mu c_p / k,$$

k = thermal conductivity of the total mixture,

$$c_p = \sum_i C_i c_{pi},$$

$$Le = \text{Lewis number} = \rho D_i c_p / k,$$

These equations are nonlinear partial differential equations and difficult to solve. Fay and Riddell (1958), using integral transformations suggested by Lees (1956), were able to separate the variables and reduce the relations to ordinary differential equations. The equations of Fay and Riddell are applicable to diatomic dissociating gas boundary layers formed over axisymmetric bodies or flat plates.

If the gas is in thermochemical equilibrium in the boundary layer, i.e. $\dot{w}_i = 0$, only the transformed versions of equations (1-2) and (1-4) need be solved. Solution of the former will give the velocity

profile, while solution of the latter will give the enthalpy profile. The criterion of chemical equilibrium will then permit the calculation of the temperature, density, and species profiles.

If the gas is assumed chemically frozen, i.e. $\dot{w}_1 = 0$, Equation (1-3) must be used in addition to the two mentioned. Equation (1-1) is used in the transformation of the other equations.

In the case of flow over a flat plate, there is no pressure gradient along the x-axis since the stream lines remain parallel to the surface.

In the equations of Fay and Riddell, the transport properties of thermal conductivity, viscosity, and diffusion appear in dimensionless groupings in the form of the Prandtl number and the Lewis number. Another dimensionless parameter is introduced - the density-viscosity product - defined by

$$\frac{\rho \mu}{\rho_w \mu_w}, \quad (1-5)$$

where the subscript refers to conditions at the surface or wall. The Prandtl number, Lewis number, and density-viscosity product appear in the transformed equations of motion as parts of derivatives.

Numerical methods are used to integrate the transformed equations. Fay and Riddell obtained solutions at the stagnation point. They allowed the density-viscosity product to vary throughout the boundary layer but assumed the Prandtl number was equal to a constant value of 0.71. The Lewis number was used with constant values of either 1.0, 1.4, or 2.0. The viscosity was calculated by using Sutherland's equation,

$$\mu = \frac{K T^{\frac{1}{2}}}{C + (1/T)}, \quad (1-6)$$

where K and C are empirical constants and T is temperature.

Fay and Kemp (1963) extended the work of Fay and Riddell to an ionizing diatomic gas. The same basic boundary layer equations were used, but with different integral transformation. The treatment of the thermodynamic and transport properties also had to be revised.

The density-viscosity product in the transformed equations of motion was replaced by a density-thermal conductivity product,

$$\frac{\rho k}{\rho_w k_w}, \quad (1-7)$$

In modern boundary layer studies, the prime interest is in the heat transfer to the surface of the body. Fay and Kemp felt that with this in mind, the density-thermal conductivity product was more appropriate than the density-viscosity product.

The thermal conductivity adopted by Fay and Kemp was assumed to be the sum of the thermal conductivity of the electrons, as determined by Spitzer (1956) for a singly ionized plasma, and the thermal conductivity of an atom-molecule mixture, as calculated by Yun (1962). Fay and Kemp considered the viscosity of the mixture of molecules, atoms, ions and electrons to be relatively unimportant insofar as calculating the stagnation point heat transfer was concerned. Accordingly, they estimated the viscosity in a simple manner. Only an atom-molecule mixture was considered, and the viscosity was calculated with the relation,

$$\mu = \frac{0.82 + 1.18\beta}{1 + \beta} \left(\frac{1 + \beta - 2\alpha\beta}{1 + \beta + 2\alpha\beta} \right) \mu_A, \quad (1-8)$$

where β is the dissociation fraction, α is the ionization fraction, and μ_A is the viscosity of a pure atomic gas. The effect of the free electrons on viscosity was not considered.

The boundary layer equations were integrated allowing the density-thermal conductivity product and the Prandtl number to vary throughout the boundary layer, but the Lewis number was set to constant values between 0.3 and 1.0.

Finson and Kemp (1965) analysed the problem of stagnation point heat transfer in ionized monatomic gases. This work was a further development of the equations used by Fay and Kemp, and the density-thermal conductivity product was again used. The pure ion viscosity was estimated from the pure ion thermal conductivity using perfect gas relations.

Finson and Kemp obtained good agreement between their theory and measurements at low Mach numbers when compared to the data of Rutowski and Bershader (1964). However, the measurements of Reilly (1964) at a Mach number of 12 (a stagnation enthalpy of 2000 cal/gm) were about 40% lower than the theory of Finson and Kemp.

Back (1967) made a study of laminar boundary layer heat transfer from a partially ionized monatomic gas. He modified the original equations of Fay and Riddell (1958) by basically changing the dissociation fraction to an ionization fraction. The thermodynamic relations were changed from those of a dissociating diatomic gas to those of an ionizing monatomic gas. Back integrated the boundary layer equations by assuming constant values for the Prandtl number of 0.1 to 0.67, and values for the Lewis number of 0.25 to 2.0. He set the density-viscosity product equal to 1 at all times.

Knöös (1968) investigated the equilibrium boundary layer structure in a 1eV (approximately 600 cal/gm) shock generated plasma flow. He did not use the equations of Fay and Riddell, but rather the equations for a Rayleigh boundary layer, which were applied to the problem of a side wall boundary layer in a shock tube. Knöös also

had quite different representations of the specific heat and Prandtl number. This was because the use of the Rayleigh equations required that the specific heat of the mixture of atoms, ions, and electrons be calculated under "equilibrium" conditions, whereas the other investigators discussed above assumed the specific heat was the sum of the chemically frozen specific heats of the mixture components - i.e. Knöös had $c_p = (\partial h / \partial T)_p$ for the mixture and the others had $c_p = \sum c_i c_{pi}$.

In his analysis, Knöös demonstrated that the viscosity, thermal conductivity, and density-viscosity product could all vary greatly throughout the boundary layer. He showed that setting the density-viscosity product equal to a constant was a very poor assumption.

A large section of Knöös' paper was devoted to an examination of where the assumption of thermochemical equilibrium would break down in an Argon Rayleigh boundary layer. The boundary layer was taken to be in local near chemical equilibrium when the available recombination rate exceeded the time dependent net recombination rate needed for equilibrium. Knöös concluded that a large portion of the Rayleigh boundary layer would not be in chemical equilibrium. Using a similar technique, Knöös also showed that the same statement would be true in the case of temperature equilibrium.

1.3 Shock Tunnel Operation and Contamination

High enthalpy gas flows are produced in the laboratory by allowing the reflected shock region of a shock tube to act as a reservoir for a nozzle. This is illustrated in Figure 1.1. The driver gas (usually Hydrogen or Helium) pressure is raised and the diaphragm separating the high pressure and low pressure test gas section bursts. The consequent shock front followed by a driver-test gas contact surface travels down the tube and reflects at the end.

The Free Piston Shock Tunnel developed by Stalker (1967) differs from others in the method used to raise the pressure of the driver gas. This is illustrated in Figure 1.2. The piston compresses the driver gas until the pressure is raised to the point where the main metal diaphragm is burst. The principal performance difference is in the flow stagnation enthalpies that are produced. In high enthalpy shock tunnels, the driver gas is ignited to produce high pressure and stagnation flow enthalpies of approximately 3000 cal/gm. The Free Piston Shock Tunnel operates normally with flow stagnation enthalpies at, or in excess of, 7000 cal/gm.

A major problem arises in shock tunnels when the reflected shock front interacts with the shock tube boundary layer. Kaegi and Muntz (1964) investigated the test flow duration of a hypersonic reflected shock tunnel, using a combustion driver. Their results indicated that driver-test gas mixing significantly reduced test times at high enthalpy conditions. Davies (1965) and more recently Davies and Wilson (1969) have theoretically investigated this problem using the bifurcation model of Mark (1957). They suggest that the reflected shock boundary layer interaction will permit early contamination of the test gas by the driver gas.

Bull and Edwards (1968) experimentally studied the reflected shock interaction process in a shock tube and found that the driver gas appeared much earlier than predicted on the basis of a simple shock tube theory. Their measurements were in close agreement with the predictions of Davies (1965).

Slade (1970) experimentally studied this problem using a quadrupole mass spectrometer and confirmed that low level contamination may occur quite early in the test run.

1.4 Points for Investigation

In the very high enthalpy flow of this investigation, the

transport properties of viscosity and thermal conductivity will vary greatly across the boundary layer. In order to determine the importance of these variations in relation to the overall boundary layer solution, it was decided to integrate the equations of motion, taking into account not only the differing values of viscosity, thermal conductivity, Prandtl number, Lewis number, and specific heat at each level in the boundary layer, but also the gradients or derivatives of these quantities throughout the region.

Since there is the possibility of the presence of Helium in the boundary layer, it was decided that the equations of motion must be integrated in such a way as to allow the estimation of the effects of contamination.

The solution of the boundary layer problem is completely dependent upon the free stream conditions that are assumed to exist above it. The Free Piston Shock Tunnel is a relatively new device and therefore its characteristics are not as completely understood as with the older forms of shock tunnels. Consequently, it was deemed necessary to carefully investigate the nozzle flow. Special attention was given to any changes in the flow characteristics that might occur due to the Helium contamination.

The transport properties are changed primarily by the presence of free electrons. It seemed logical to carry out the work using a monatomic gas that would go directly to the ionized state. The thermodynamic relations for a monatomic gas are also more straightforward and convenient to use than a dissociating gas, since only the translational and ionized modes of energy need be considered.

1.5 Procedure

In Chapter 2, the theoretical solutions are developed for a very high stagnation enthalpy boundary layer, first in a condition of thermochemical equilibrium and then in a chemically frozen state.

Full consideration is given to the variation of the transport properties. The boundary layer equations will be integrated in a fully coupled manner, using a "multiple shooting" technique. The results of these solutions will be used in a discussion of the physical processes that could occur in a boundary layer.

In Chapter 3, the problem of Helium contamination is investigated. Firstly, the pure Argon nozzle flow is calculated, and secondly the calculations are repeated with various amounts of Helium assumed to be present. Boundary layer solutions for both the equilibrium and frozen cases are shown with Helium contamination. A relatively straightforward theory will be developed to facilitate clearer understanding of the physical processes that will be altered, and to what extent, when Helium is present in the flow.

The experimental results are described and discussed in Chapter 4. Laboratory investigations include time resolved pitot pressure of the on axis nozzle flow, a Mach-Zehnder interferometer study of the flow over an inclined flat plate, and surface heat transfer measurements at several stations on the plate. It will be shown that neither the equilibrium nor frozen boundary layer solutions can adequately predict the surface heat transfer rate in a very high enthalpy flow. A crude theoretical model will be developed that will give satisfactory predictions.

Chapter 5 is a summary and discussion of the overall project. The methods and mixtures rules used to calculate the transport properties of viscosity and thermal conductivity for the ionized Argon with and without Helium contamination are given in Appendix A. The experimental techniques and data reduction methods are discussed in Appendix B.

CHAPTER 2

BOUNDARY LAYERS

2.1 Introduction

The equations of motion for a reacting gas boundary layer will be examined in this chapter. They will be transformed from partial differential equations to ordinary differential equations, using the concept of "similarity".

The reaction rates in the boundary layer are not known to any great degree of accuracy nor reliability. A non-equilibrium solution of the boundary equations is difficult, and it seems wise to examine two special cases.

If the reaction rates are zero, the gas may be taken to be "chemically frozen". The variation of species throughout the boundary layer will be controlled entirely by diffusion processes.

If the reaction rates are infinite, the gas may be assumed to be in "thermochemical equilibrium". In this case, the species concentrations in the boundary layer will be according to the predictions of equilibrium chemistry, e.g. the Saha equation for an ionizing monatomic gas boundary layer.

The boundary layer equations for an equilibrium and for a frozen monatomic gas will be written in this chapter in such a way as to take into full account the variation of the transport properties throughout the boundary layer. Numerical integrations will be carried out for several conditions.

2.2 Equations of Motion

The boundary layer equations of motion for flat plate and axisymmetrical bodies are well known¹. However, before preceding

¹See for example Dorrance (1962) Chapter 2.

and presenting the boundary layer equations, it is important to briefly discuss the problem of possible thermal diffusion in the boundary layer. This transport process will depend on the temperature gradients that are present. As will be shown later, the viscous forces do not cause a large temperature gradient until low in the boundary layer where, it should be noted, the stagnation enthalpy will be smaller by a factor of three or four than the free stream value.

In their work, Fay and Riddell (1958) indicated that in the case of the equilibrium boundary layer thermal diffusion was unimportant at stagnation temperatures at, or less than, $10,000^{\circ}\text{K}$, but not necessarily in the case of the frozen boundary layer. This value of stagnation temperature is roughly the same order as might be expected at the point in the boundary layer where the temperature gradient would be significant. As will be discussed later, at this point the boundary layer will be most apt to be approximated by a state of equilibrium. If it is assumed that the thermal diffusion of Argon will be the same order as that of air, it seems acceptable to ignore this quantity in these calculations. This is specially true considering the small part thermal diffusion would appear to play and the poor accuracy with which one can determine the thermal diffusion coefficient.

Neglecting the effects of thermal diffusion, the equations for the case of zero pressure gradient, i.e. flat plate flow, may be written as,

mass conservation:

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0 \quad ; \quad (2-1)$$

species conservation:

$$\rho u \frac{\partial C_i}{\partial x} + \rho v \frac{\partial C_i}{\partial y} = \frac{\partial}{\partial y} \left(D_i \rho \frac{\partial C_i}{\partial y} \right) + \dot{w}_i \quad , \quad (2-2)$$

momentum:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y} \right) \quad , \quad (2-3)$$

energy:

$$\begin{aligned} \rho u \frac{\partial H}{\partial x} + \rho v \frac{\partial H}{\partial y} = \frac{\partial}{\partial y} \left(\frac{k}{c_p} \frac{\partial H}{\partial y} \right) + \frac{\partial}{\partial y} \left[\frac{1}{2} \mu \left(1 - \frac{1}{Pr} \right) \frac{\partial u^2}{\partial y} \right] \\ + \frac{\partial}{\partial y} \left[\sum_{i=1}^n \frac{k}{c_p} (Le-1) h_i \frac{\partial C_i}{\partial y} \right] \quad , \end{aligned} \quad (2-4)$$

where

$$H = h + \frac{u^2}{2} \quad ,$$

$$Pr = \frac{\mu c_p}{k} = \text{Prandtl Number}, \quad (2-5)$$

$$Le = \frac{D_i \rho c_p}{k} = \text{Lewis Number},$$

Solutions can readily be found for the flat plate problem only at extreme values of the reaction rate, i.e. $\dot{w}_1 = 0$ for a "frozen" boundary layer and $\dot{w}_1 = \infty$ for an "equilibrium" boundary layer.

Fay and Riddell (1958) studied the problem of stagnation point heat transfer in dissociated air. Back (1967) applied this theory in a survey study of heat transfer in ionized monatomic gases. Back assumed the Blasius solution¹ for his velocity profile and held the transport properties constant at the free stream values. The results of Knöös (1968) showed that the transport properties could in fact vary considerably throughout an ionizing boundary layer and therefore the assumption by Back of constant transport properties in the boundary layer was unacceptable in this study.

2.3 Transformed Equations of Motion

Flows of different fluids, with identical initial flow conditions about geometrically similar bodies, are considered mathematically "similar"² when the streamlines are also similar. This means that a functional relationship must exist between the x and y axis in a two dimensional flow. Using this relation, it is possible to reduce the equations of motion from two dimensions to one dimension.

To invoke the concept of "similarity", it is necessary to find a suitable method of integral transformation to move the problem from the x, y plane to a new plane where the partial differential equations will reduce to ordinary differential equations, the transformations used were the standard relations by Lees(1956),

¹That is, the velocity profile of a non-ionized ideal gas.

²See Schlichting (1968) Chapter 1.

$$\eta = \frac{u_e}{(2\xi)^{\frac{1}{2}}} \int_0^y \rho dy, \quad (2-6)$$

and

$$\xi = \int_0^x \rho_w \mu_w u_e dx. \quad (2-7)$$

These equations assume a constant relationship between the x and y co-ordinates (i.e. the concept of similarity) over the whole plate. If this is not the case, equations (2-6) and (2-7) may be applied over smaller, localized regions and these solutions can then be "patched" together. This method is the concept of "local similarity".

In modern high enthalpy boundary layer problems, the major interest is in heat transfer. With this in mind, Fay and Kemp (1963) suggested that the use of a density-thermal conductivity product is more appropriately used in the transformed equations of motion than the density-viscosity product.

For the purposes of this study, the use of viscosity in Equation (2-7) offered some conceptual disadvantage. In dealing with ionized monatomic gases, the effects of ionization upon viscosity are less obvious than the effects upon thermal conductivity and specific heat¹. Thus, where possible, it seemed desirable to use the latter quantities in the boundary layer equations.

¹See Appendix A.

Parametric studies can be made of boundary layers assuming different values of Prandtl Number, e.g. Back's work. In these cases, if the viscosity has been replaced with the other parameters mentioned, the problem is simplified significantly.

Consequently, equation (2-7) has been modified by use of the definition of the Prandtl Number to the form

$$\xi \equiv \int_0^x \frac{\rho_w k_w Pr_w}{c_{pw}} u_e dx \quad . \quad (2-7a)$$

2.4 Equilibrium Boundary Layer Equations

Application of equations (2-6) and (2-7a) to the Equations of Motion, equations (2-1) to (2-5), gave rise to the following results in the case of an equilibrium boundary layer:

momentum:

$$\frac{c_{pw}}{Pr_w} \frac{d}{d\eta} \left(\frac{Y Pr}{c_p} \frac{d^2 f}{d\eta^2} \right) + f \frac{d^2 f}{d\eta^2} = 0 \quad , \quad (2-8)$$

energy:

$$\begin{aligned} \frac{c_{pw}}{Pr_w} \frac{d}{d\eta} \left(\frac{1+A}{c_p} \frac{dg}{d\eta} \right) + \frac{f}{Y} \frac{dg}{d\eta} \\ - \frac{c_{pw}}{Pr_w} \frac{u_e^2}{H_s} \frac{d}{d\eta} \left(\frac{1-Pr}{c_p} \frac{df}{d\eta} \frac{d^2 f}{d\eta^2} \right) = 0 \quad , \end{aligned} \quad (2-9)$$

where

$$\begin{aligned} f' &= u/u_e \quad , \quad f = \int_0^\infty df/d\eta \quad & f(0) &= f'(0) = 0, \\ g &= (u^2/2 + h)/H_s \quad , \quad & g(0) &= g_w, \\ Y &= \rho k / \rho_w k_w \quad , \quad & f'(\infty) &= g(\infty) = 1, \\ A &= (Le - 1) h_i \left(\frac{d\alpha}{dh} \right)_P \quad . \end{aligned}$$

The rate of change of ionization with the specific enthalpy at constant pressure is directly determined using the equation

for specific enthalpy and Saha's equation (see Chapter 3, equation (3-4) and (3-6)),

Pure Argon:

$$\left(\frac{d\alpha}{dh}\right)_P = \left\{ \frac{5}{2} R_A (1+\alpha) \frac{dT}{d\alpha} + \frac{5}{2} R_A T + I \right\}^{-1}, \quad (2-10a)$$

Helium-Argon Mixtures:

$$\left(\frac{d\alpha}{dh}\right)_P = \left\{ x_{A_2} \frac{5}{2} R_A (1+\alpha) \frac{dT}{d\alpha} + x_A \frac{5}{2} R_A T + x_A I + x_H \frac{5}{2} R_H \frac{dT}{d\alpha} \right\}^{-1}, \quad (2-10b)$$

where

$$\frac{dT}{d\alpha} = \frac{\frac{1}{2} \left(\frac{3.8208}{P_A} \right) T^{3/2} \left(\frac{5}{2} + \frac{\epsilon}{kT} \right) \exp \left(-\frac{\epsilon}{kT} \right)}{\left(\frac{\alpha^2}{1-\alpha^2} \right)^{1/2} \left(\frac{1}{1-\alpha^2} \right)^{3/2}} \quad (2-11)$$

The number 3.8208 was obtained from the evaluation of the constant terms in Saha's equation, in c.g.s. units. It should also be emphasised that the pressure is assumed to remain constant throughout the boundary layer.

2.5 Frozen Boundary Layer Equations

If there are no chemical reactions in the boundary layer, the gas is considered chemically "frozen". The value of ionization at any point in the boundary layer will be determined solely by diffusion.

The momentum equation, equation (2-8) is still valid. The role played by the Saha equation in the equilibrium case is now filled by an equation of species concentration. The energy equation is also rewritten in a different form.

conservation of species:

$$Le \frac{c_{pw}}{Pr_w} \frac{d}{d\eta} \left[\frac{Y}{c_p} \frac{dS_i}{d\eta} \right] + f \frac{dS_i}{d\eta} = 0, \quad (2-12)$$

energy:

$$\begin{aligned} \frac{d^2 \Theta}{d\eta^2} + \left[\frac{c_p}{c_{pw}} \frac{Pr_w}{Y} f + \frac{c_{pw}}{c_p} Le \alpha_e \frac{dS_i}{d\eta} \right] \frac{d\Theta}{d\eta} \\ + \frac{u_e^2 Pr}{T_e c_p} \left(\frac{d^2 f}{d\eta^2} \right)^2 = 0, \end{aligned} \quad (2-13)$$

where

$$\begin{aligned} S_i &= \alpha / \alpha_e, & S_i(0) &= 0 \text{ or } S_{iw}, \\ \Theta &= T / T_e, & \Theta(0) &= \Theta_w, \\ & & S_i(\infty) &= \Theta_w(\infty) = 1. \end{aligned}$$

2.6 Boundary Layer Equations with Variable Transport Parameters

If a solution is desired which will include the effects of the transport property changes through the boundary layer, the quantities within the brackets in the above equations must be differentiated.

In the case of the equilibrium boundary layer, in this investigation, the following new parameters are defined, using the frozen specific heat,

$$\begin{aligned} C_1 &= Y Pr / c_p, \\ C_2 &= (1+A) / c_p, \end{aligned} \quad (2-14)$$

and

$$C_3 = (1-Pr) / c_p.$$

Equations (2-8) and (2-9) are then rewritten in the form

$$f'''' + (K_1 f + C_1' / C_1) f'' = 0, \quad (2-8a)$$

and

$$g'' + (c_2'/c_2 + K_2 f) g' - K_3 \left[(f'')^2 + f' f'' \right] - \frac{u_e^2}{c_2 H_s} c_3' f' f'' = 0, \quad (2-9a)$$

where

$$K_1 = (Pr_w / c_{pw}) / c_1,$$

$$K_2 = (Pr_w / c_{pw}) / (Y c_2), \quad (2-15)$$

and

$$K_3 = (u_e^2 c_3) / (H_s c_2).$$

In the solution of a frozen boundary layer with variable transport parameters, it is found that the momentum equation, in the form of equation (2-8), remains valid. One additional transport parameter is defined,

$$c_4 = Y / c_p \quad (2-14a)$$

and then equation (2-12) and (2-13) are modified to the following:

$$s_i'' + \left[c_1'/c_1 + K_4 f/Le \right] s_i' = 0, \quad (2-12a)$$

and

$$\theta'' + \left[c_4'/c_4 + K_4 f + K_L s_i' \right] \theta' + K_T (f'')^2 = 0, \quad (2-13a)$$

where

$$K_4 = (Pr_w / c_{pw}) / C_4 ,$$

$$K_L = (c_{pw} / c_p) Le \alpha_e , \quad (2-15a)$$

and

$$K_T = (Pr u_e^2) / (T_e c_p) .$$

2.7 Integration of the Boundary Layer Equations

Back (1967) assumed equation (2-8) to be of the form

$$f'''' + f f'' = 0,$$

and obtained solutions for the momentum equation by curve fitting the original Blasius solution with the proper scaling. Back then solved his equivalent equation to equation (2-9) via a Runge-Kutta integration. This procedure is valid only if all the transport properties are assumed constant in value - otherwise equations (2-8) and (2-9) are coupled. (It should be noted that in their paper, Fay and Riddell did integrate the equations in a coupled manner.)

In this work, the equations are coupled and integrated using the multiple shooting technique of Osborne (1969). This procedure is a modification of the standard initial value problem. In the latter, one assumes the boundary conditions at the surface, making a guess of the unknown values, and then integrates the problem to an upper limit of $\eta \rightarrow \infty$. The final asymptotic values calculated (usually a η value of 3 to 5 gives a sufficiently good approximation to $\eta \rightarrow \infty$) are compared with the upper boundary values that are known. The initial numbers used may be modified and the problem redone. The method is repeated until the final predictions and the upper boundary values are matched to the desired accuracy.

Two major problems with this type of solution are:

- (1) that there is sometimes instability in the differential equations (i.e. two possible solutions are close to one another) and
- (2) one must usually start with "good" guesses.

Osborne overcame these problems by dividing the boundary layer into a number of intermediate levels and then carrying out an initial value type solution from the i^{th} to the $(i^{\text{th}} + 1)$ interface. He also developed a sophisticated method to ensure quick convergence of the problem. The starting "guesses" at each level need only be the free stream values.

A slightly modified Newton method was used in the overall calculation. Using the free stream values, the equations and their respective derivatives would be evaluated at each interface. Correction factors based on these derivatives would then be algebraically added to the first solution estimates, and the sum of the squares of the difference between the new values and the old was calculated. The procedure would be repeated and a new sum of squares calculated. Subsequent values of these sums would be compared with the preceding values, and if the quantity had not been reduced, the solution would be recalculated using only half of the Newton correction. This procedure of bisection could be repeated many times. Final solutions were obtained when the sum of the squares fell below a certain tolerance, or one of several other criteria were met.

Calculations were done on an IBM 360-50 computer, and the basic integration subroutine was the IBM supplied Runge-Kutta procedure, RKGS, which has a built in step size determination procedure. Appropriate substitutions were made to convert the problem to a system of first order, non-linear equations. Consequently, the

equilibrium cases required solution of a set of five equations, and the frozen solutions were obtained by integrating a set of seven equations. As many as 100 intermediate levels were used, but generally the division of the boundary layer into 50 layers proved more than adequate for the multiple shooting.

It was necessary to obtain an accurate estimate of the derivatives of the transport parameters¹. In this study, all these values were calculated numerically from data of each individual solution.

Initially, the problem was solved with the transport parameters held constant at the free stream values. Based on this solution, the transport parameters were recalculated throughout the boundary layer, and the new table of values stored in the computer. The program then looped, and the second time around the required derivatives were estimated using the first differences from the tabulated data. The looping could be repeated as many times as desired.

The major changes occurred in the solution at the second loop. The convergence time of subsequent loops increased as smaller and smaller corrections were applied. The solution was normally terminated after the second looping was completed.

2.8 Displacement Thickness

The effective physical thickness which the boundary layer exhibits in changing or displacing the flow of the free stream gas, is called the "displacement thickness" and is defined by

$$\delta^* = \int_0^{\infty} \left(1 - \frac{\rho u}{\rho_e u_e} \right) dy \quad (2-16)$$

In terms of the transformation parameters this equation becomes

$$\delta^* = \frac{(2\xi)^{\frac{1}{2}}}{\rho_e u_e} \int_0^{\infty} \left(\frac{\rho_e}{\rho} - f' \right) d\eta \quad (2-17)$$

¹Transport "parameters" will be taken as the newly defined quantities of equation (2-14) and (2-14a). Transport "properties" will refer to viscosity, thermal conductivity and diffusion.

Using the results of the boundary layer integrations, equation (2-17) was in turn integrated using a $\frac{3}{8}$ Simpson Rule. The value of δ^* in terms of y was obtained using a quadrature formula derived from equation (2-6).

2.9 Heat Transfer

The heat transfer to the surface of the flat plate, or in the boundary layer proper, is taken to be due to conduction and to diffusion of ion-electron pairs,

$$-q = k \frac{\partial T}{\partial y} + D_i \rho h_i \frac{\partial \alpha}{\partial y}, \quad (2-18)$$

where the negative sign is shown to indicate heat transfer to the surface.

now

$$\frac{\partial T}{\partial y} = \frac{d\eta}{dy} \frac{dT}{d\eta} = \frac{\rho u_e}{(2\xi)^{\frac{1}{2}}} \frac{dT}{d\eta} = \frac{\rho u_e}{(2\xi)^{\frac{1}{2}}} T_e \frac{d\theta}{d\eta}, \quad (2-19a)$$

and similarly

$$\frac{\partial \alpha}{\partial y} = \frac{\rho u_e}{(2\xi)^{\frac{1}{2}}} \alpha_e \frac{dS_i}{d\eta}. \quad (2-19b)$$

Using in turn equations (2-19a) and (2-19b), two relations for the heat transfer in transformed variables may be obtained,

$$-q = \frac{k \rho u_e T_e}{(2\xi)^{\frac{1}{2}}} \left[\theta' + \frac{Le}{c_p T_e} \alpha_e h_i S_i' \right], \quad (2-20a)$$

and

$$-q = \frac{k \rho u_e H_s}{(2\xi)^{\frac{1}{2}} c_p} \left[(1 + A) g' - f' f'' \frac{u_e^2}{H_s} \right]. \quad (2-20b)$$

Equation (2-20a) was used in frozen boundary layer calculations and equation (2-20b) was employed in the equilibrium boundary layer solutions for both constant and variable transport parameters.

2.10 Transport Properties

The equations estimating the basic transport properties of thermal conductivity and viscosity for pure ionized Argon and mixtures of ionized Argon and neutral Helium, have been developed in Appendix A. Those equations were employed in the above calculations.

The diffusion term was calculated according to an equation given by Camac, et al (1963) for the Lewis Number,

$$Le = 0.255 (T/10^4)^{-0.16} \quad (2-21)$$

This formula was based on use of ambipolar¹ diffusion in the Argon. As Back (1967) has shown, the Lewis Number is fairly constant about a figure of 0.25 under this condition.

It should be noted that Fay and Kemp(1963) indicated that whenever the thickness of the diffusing layer is large compared to the debye distance, ambipolar diffusion will always occur in a plasma. Since, as it will be shown, the electron numbers will usually be high (even if the ionization fraction is small) when there is a large density gradient, it follows that the debye distance will be small in comparison to the boundary layer thickness. Consequently, ambipolar diffusion is a good assumption.

In the still lower levels of the boundary layer the density gradient will continue to be large, but the electron numbers quickly fall to such a level that the gas can be considered not to be a plasma and the Lewis number will simply be set equal to one.

¹That is, diffusion of electron-ion pairs.

The free stream conditions used in the boundary layer calculations are those that exist after the oblique shock which is formed at the leading edge of the inclined flat plate. Consequently, a range of conditions was obtained by adjusting the tilt of the plate. As the inclination angle is increased (rear of the plate raised), the boundary layer free stream Mach number and flow velocity will decrease while the pressure and density will increase. The range of conditions could be further expanded by using both the 7.5° half angle nozzle with an exit area ratio of 147 and the 15° half angle unit with an exit area ratio of 1204¹.

In general, it was desired to maintain a high free stream velocity and Mach number while using different free stream pressures. These basic parameters would ensure a very high stagnation enthalpy hypersonic flow with different values of free stream density, temperature, ionization, and transport properties.

There are many complicating factors which had to be taken into account when making the final choices of plate inclination-nozzle conditions. These, and the calculations that were carried out, are discussed in detail in Chapter 3. For the purposes of this immediate discussion, let it simply be stated that two conditions will be shown; a 9.5° plate inclination with the small exit nozzle, and a 19.5° plate inclination with the large exit nozzle. Table 2.1 is an abridged version of Tables 3.8a and 3.8b, and illustrates typical free stream conditions at a distance of 2 cm. up the plate from the leading edge. Table 2.1 also shows the calculated free stream conditions in the small exit nozzle at a distance of 12.88 cm. from the throat. This later calculation is useful for illustrative purposes, to avoid the complications brought about by the oblique shock.

¹The former will be referred to as the "small exit" nozzle, the latter as the "large exit" nozzle.

Nozzle Area Ratio	% He By Mass	Stag. Enthalpy (cal/gm)	Plate Angle (deg.)	Shock Angle (deg.)	Pressure ₂ (dyne/cm ²)	Density (gm/cc)	Temperature (degree-K)	Velocity (cm/sec)	Mach No.
147	0.0	7,635	9.5	18.9	1.44 E+5	7.46 E-6	9,274	7.18 E+5	4.00
	0.99	7,522		20.1	1.04 E+5	5.77 E-6	7,984	7.29 E+5	4.20
	4.76	7,268		16.8	9.08 E+4	7.93 E-6	3,853	7.41 E+5	5.36
1204	0.0	7,635	19.5	28.2	4.68 E+4	1.65 E-6	13,637	7.10 E+5	3.26
136	0.0	7,635	—	—	1.14 E+5	6.35 E-6	8,518	7.36 E+5	4.83
(12.88 cm. small exit)									

TABLE 2.1

Typical free stream boundary layer conditions for a distance up the plate from the leading edge of 2 cm. and 12.88 cm. downstream of throat in small exit nozzle.

Locally similar solutions were calculated for distances up the plate from the leading edge of 1, 2, and 3 cm. These were "pieced" together to form the total theoretical curves shown. Flow divergence was taken into account in the manner described in the next Chapter. Experimental points will be discussed in Chapter 4.

However, before looking at the flat plate results, let us establish some basic concepts and determine what changes result in solutions when the transport properties are allowed to vary throughout the boundary layer.

If the transport parameters, as defined above, are held constant, the parameter K_1 as given by equation (2-15) also becomes a constant. It has been found useful to consider K_1 a direct function of the velocity profile, or simply a "profile factor". This concept is illustrated in Figure 2.1. The thickness of the boundary layer is inversely proportioned to K_1 .

K_1 is the inverse of the classic density-viscosity product looked at in a different light (i.e. $(\rho \mu / \rho_w \mu_w)^{-1}$). Over the years, this parameter has been set equal to one so often that its physical significance has been overlooked. It has been called the profile factor in this study to emphasise its significance.

The profile factor is a valuable concept when dealing with variable transport parameters throughout the boundary layer. One can then consider the effects that the transport parameters have on the velocity profile at various points in the boundary layer. This is illustrated in Figure 2.2, where the results are plotted for the small exit nozzle wall boundary layer 12.88 cm. downstream.

At the surface K_1 , by definition, equals unity. Coming

off the plate, the profile factor increases but remains below the free stream value for a period. Eventually, the free flow value of K_1 is not only reached, but greatly surpassed. After peaking in value, K_1 quickly falls back to its final free stream figure. The effect of this behavior is to make the boundary layer "think" it is very thick at the base.

A detailed explanation of the change in velocity profile is given in equation (2-8a). The temperature increases as it ascends from the surface, with a consequent increase in the thermal conductivity. However, simultaneously the density drops off at a quicker rate, as is shown in Figure 2.3. The result is that the parameter Y decreases in value. This means that the derivative of C_1 is negative and in fact is larger in absolute value than C_1 itself. When C_1'/C_1 , a negative number greater in absolute value than one, is fed into equation (2-8a), this has a "braking" action on the integration with the observed effect on the velocity profile.

The inclusion of the variable transport parameters also has a marked effect on the enthalpy in the boundary layer. The new shape of the velocity profile produces a hot region well above the surface, but decreases the temperature gradient near and on the surface. These results are illustrated in Figures 2.4 and 2.5, the temperature and ionization profiles of the same boundary layer calculation of Figure 2.2.

The lowering of the wall temperature gradient will in turn produce a lower surface heat transfer rate. It should be realized that the above developments are greater the higher the flow enthalpy, since the density drop off will be determined by the temperature gradient.

Figures 2.6 and 2.7 show two items that are of general interest in boundary layer studies. Figure 2.6 is the shear stress, defined by $\mu(du/dy)$, and it can be seen that there is

a non-zero value at the surface. Although the velocity profile of the boundary layer with variable transport parameters appears as if it were suffering an adverse pressure gradient¹, there should not be any separation at the surface. Figure 2.7 shows stagnation enthalpy as a function of velocity. As can be seen, the constant parameter calculation can be approximated with a linear relationship. This procedure will not be as good in the case of the variable calculation.

With regard to the inclined flat plate in the test section, calculations and measurements will be shown for a 19.5° positive inclination with the large exit nozzle, and a 9.5° positive inclination with the small exit unit. These results will amply display the physics of the situation.

Figure 2.8 depicts the equilibrium velocity profiles. Figures 2.9, 2.10, and 2.11 show the calculated temperature, ionization, and density profiles for the 19.5° plate inclination, large exit case with variable transport parameters. Results for the distances up the plate from the leading edge of one and three cm. have been drawn to show the effects of flow divergence upon the internal structure of the boundary layer in an equilibrium condition.

The small exit 9.5° plate inclination results are alike, as might be expected from the similarity shown in the velocity profiles.

The flow divergence tends to lower the temperature and density throughout the boundary layer. In the case of ionization, the free stream value is lowered. At a distance of three cm., the curve is displaced upward. This last point can be seen by referral to Figure 2.12, which shows the conversion from the η axis to the y axis.

Figures 2.13 and 2.14 demonstrate the roles played by the transport properties in the equilibrium boundary layer. Figure

¹The velocity profile with variable transport parameters also appears similar to that of a boundary layer with mass transfer (blowing).

2.13 indicates the profiles of the thermal conductivity and viscosity. The thermal conductivity suffers a short term lowering of its value at about $\eta=0.5$. This is attributed, as discussed in Appendix A, to the fact that the ion-atom and ion-ion collision cross-sections are greater than the atom-atom. At the onset of ionization, the electron numbers are still very small; However, this number quickly builds up, and the electrons soon dominate with their much larger thermal conductivity. The viscosity falls as the electron numbers build up. This is again a function of the ion-atom and ion-ion cross-sections¹. The thermal conductivity peaks and the viscosity hits its minimum value at the point of maximum ionization.

Figure 2.14 discloses the profiles of the Prandtl Number and the newly defined parameter C_1 , as given in equation (2-14). The shape of the two is similar except in the very important lower regions where the Prandtl Number goes to a constant limit, but C_1 varies right into the plate surface. This behavior of C_1 is what plays such a major role in the variable transport parameter calculations.

Figure 2.15 shows the calculated surface heat transfer rates. The measurements will be discussed in Chapter 4. It is to be noted that a calculation which considers fully the variation of the transport properties gives a far lower heat transfer rate. This is due to the smaller temperature gradient at the surface that is predicted with the variable transport parameter calculation.

2.13 Frozen Boundary Layer Calculations

In the case of equilibrium, the boundary conditions of velocity and total enthalpy (along with pressure) were specified, but the temperature, ionization, and density take on different values, depending on the prediction of the Saha equation.

¹See Knöös(1968).

With a frozen boundary layer, the boundary conditions of velocity, temperature, and ionization, together with the pressure, are specified. The range of values which the thermodynamic and transport properties can take is much more limited, since the quantities are controlled by diffusion processes. Consequently, little difference was found between constant and variable transport parameter frozen solutions. For example, the ionization does not obtain a value greater than the free stream figure. This is in contrast to the equilibrium calculation where the ionization reaches a peak value much bigger than the free stream ionization.

From a computer standpoint, the frozen solutions usually converged at a slow rate. After a series of survey calculations were made, it was decided that on a basis of computer economics the variable transport parameter frozen solutions could not be justified.

The free stream conditions used in a frozen boundary layer calculation pose a problem - if the free stream chemistry is frozen, the only ionization that would be in the flow is that which was present before the oblique shock. This would be zero in the case of the large exit nozzle, and about 1% in the case of the small exit. In order to allow a better comparison between the equilibrium and frozen boundary layer calculations, it was assumed that the free stream flow went to a state of thermochemical equilibrium after the oblique shock, and that only the boundary layer chemistry was frozen.

The results shown below will be for the same conditions, namely the small exit nozzle, 9.5° plate inclination, with a fully catalytic surface, with a fully non-catalytic surface, and at a

distance up the plate of 1 cm. from the leading edge.

Figure 2.16 displays the velocity profiles. The surface recombination rate does have an effect upon the velocity in the boundary layer. If there is no recombination at the surface, the change of velocity in the boundary layer is more gradual and commences at a higher level than with the catalytic surface.

Figures 2.17, 2.18, and 2.19 exhibit the calculated temperature, density, and ionization profiles. The free stream values for these calculations are slightly different than the typical values shown in Table 2.1. For the work described below, these values are $T_\infty = 9703^\circ\text{K}$, $\rho_\infty = 9.69 \times 10^{-6} \text{ gm/cc}$, and $\alpha_\infty = 0.034$. The calculated differences between a catalytic and non-catalytic surface boundary layer¹ will, of course, be dependent on the amount of ionization that is present. It can be seen that free stream ionization levels as low as 3% will produce some change in the boundary layer structure. The ionization profile, with regard to the 100% catalytic surface, is determined by diffusion processes.

Figure 2.20 is to allow conversion from the η to the y axis, and Figure 2.21 is the calculated surface heat transfer rates for the catalytic and non-catalytic wall. The fully catalytic wall calculation, gives for all practical purposes the same surface heat transfer rates as with the constant transport property equilibrium boundary layer solution. This means that the heat energy transferred to the surface by conduction and diffusion processes will add up to give the same result in both cases. The non-catalytic surface has a lower heat transfer rate - if the parameter S , defined as a/a_e , is always equal to 1, then S' must always be equal to zero. Therefore, the heat transfer component brought down to the surface by diffusion of¹
 i.e. either complete ion-electron recombination or no recombination at the surface, respectively.

electron-ion pairs is missing (reference to equation (2-20a) will verify). Dorrance (1962)¹ explains this physically by a "pile up" of a diffusion-inhibiting blanket of unrecombined particles which reduces the heat transfer. The larger the free stream ionization, the larger will be the difference between the catalytic and non-catalytic surface heat transfer rates.

The above arguments also hold if there is some, but not complete, recombination on the surface. In this case, the derivative S' is not zero but is nevertheless diminished in value.

2.14 Summary of Chapter 2

In this Chapter, boundary layer equations have been developed in which the transport property of viscosity does not appear directly. New transport parameters have been defined and these have been fully differentiated to form boundary layer equations that take into complete account the variation of the transport properties throughout the boundary layer.

These relations have been integrated in a fully coupled manner. The importance of using variable transport parameters, in the case of the equilibrium condition, has been demonstrated. Their use results in lower predicted surface heat transfer rates for pure Argon flows.

At the same time, the work has shown that the added computing time for a full variational transport parameter frozen calculation is not warranted, and the setting of the transport properties to their constant free stream values is sufficient. In the case of a frozen boundary layer, the surface heat transfer rate is lowered if there is either no recombination or slow recombination on the surface of the plate.

¹See Page 91 of reference.

CHAPTER 3

FREE STREAM CONDITIONS AND EFFECTS OF HELIUM CONTAMINATION

3.1 Introduction

The gas conditions in the test section are determined by the reflected shock heated gas acting as a reservoir for the nozzle flow. It is important to have a knowledge of the processes that produce the high enthalpy gas and that occur during the subsequent nozzle expansion. It is also important to have an understanding of the conditions that will alter these processes, and to what extent flow characteristics will be altered.

In this Chapter, theoretical investigations are made of the reflected shock region, of the nozzle flow, and of the changes produced in the gas by passage across the oblique shock front produced by the inclined flat plate in the test section. As discussed in Chapter 1, previous work suggests that there may be a low level of Helium driver gas contamination early in the flow. Consequently, the above investigations take into account the possible changes produced by the presence of varying amounts of Helium in the test gas. In the case of contamination, possible changes in the boundary layer structure, or surface heat transfer, are also examined.

3.2 Pure Argon

3.2.1 Development of the Reflected Shock Region

The conditions across the initial shock front, generated at the time of the metal diaphragm rupture, will be governed by the normal shock jump conservation relations, in shock co-ordinates

$$\text{mass: } \rho_1 u_1 = \rho_2 u_2 , \quad (3-1)$$

$$\text{momentum: } P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 , \quad (3-2)$$

$$\text{energy: } h_1 + u_1^2/2 = h_2 + u_2^2/2 , \quad (3-3)$$

where the subscripts 1 and 2 refer to the regions before and after the shock passage, respectively.

Additionally, there are the thermodynamic equations of specific enthalpy, state, and, if equilibrium is assumed, the Saha equation, i.e.

$$h = \frac{5}{2} (1 + \alpha) R T + \alpha I , \quad (3-4)$$

$$P = (1 + \alpha) R T , \quad (3-5)$$

$$\frac{\alpha^2}{(1 - \alpha)} = \frac{(2 m_e k)}{h^2} \frac{k T^{3/2}}{\rho} 2 \frac{Z_1}{Z_0} \exp \left(- \frac{\epsilon}{kT} \right) . \quad (3-6)$$

Equations (3-1) through (3-6) have been solved in an iterative manner for the six unknowns of ρ_2 , u_2 , h_2 , P_2 , α , and T . The results obtained, based on an initial shock tube pressure of 6.77×10^5 dyne/cm² (2" hg) Argon and a measured shock velocity of 5.52×10^5 cm/sec., are shown in Table 3.1.

The normal shock will reflect at the nozzle end of the shock tube. In laboratory co-ordinates, the reflected shock front will move back up the tube with a velocity W_R , and the gas behind the front is brought to rest in the laboratory system.

In the shock system, the gas velocity after the reflected shock is given by,

u_2	-	4.74 E+5 cm/sec
P_2	-	2.84 E+7 dyne/cm ²
ρ_2	-	7.74 E-4 gm/cc
h_2	-	3,600 cal/gm
T_2	-	15,280 degree-K
α_2	-	0.156 .

TABLE 3.1CALCULATED FIRST NORMAL SHOCK CONDITIONS

u_3	-	1.19 E+5 cm/sec
P_3	-	2.46 E+8 dyne/cm ²
ρ_3	-	3.86 E-3 gm/cc
h_3	-	7,635 cal/gm
T_3	-	21,535 degree-K
α_3	-	0.424
a_3	-	2.82 E+5 cm/sec

TABLE 3.2CALCULATED REFLECTED SHOCK CONDITIONS

a_*	-	2.65 E+5 cm/sec
T_*	-	19,940 degree-K
ρ_*	-	2.40 E-3 gm/cc
P_*	-	1.37 E+8 dyne/cm ²
h_*	-	6,800 cal/gm
α_*	-	0.375

TABLE 3.3CALCULATED CONDITIONS IN THE THROAT OF THE NOZZLE

$$u_3 = W_R - v_3 = W_R,$$

where

v_3 = velocity in laboratory frame of reference.

A pocket of very high enthalpy gas is formed, and then in turn this gas forms a reservoir for the nozzle. Equations (3-1) through (3-6) can be rearranged using relations (3-7) to the forms

$$\rho_3 = \rho_2 (u_3 + v_2) / u_3, \quad (3-1R)$$

$$u_3 = (1/v_2) [(P_3/\rho_2) - (P_2/\rho_2)] - v_2, \quad (3-2R)$$

$$h_3 = h_2 + v_2 (v_2 + 2u_3)/2, \quad (3-3R)$$

$$h_3 = 5/2 (P_3/\rho_3) + \alpha I, \quad (3-4R)$$

with equations (3-5) and (3-6) remaining the same. Iteration over this set of equations, using the results shown in Table 3.1, gave rise to the values shown in Table 3.2.

The values of Table 3.2 may be modified further, depending on how far the shock tube operation is from the "tailored" mode. That is, the pressures on either side of the Argon-Helium contact surface are such that this interface remains stationary. The Australian National University Free Piston Shock Tunnel, "T2", design is such that when an initial shock tube pressure of 2" hg of Argon is used, the measured reflected shock region pressure is within 5% of that shown in Table 3.2, and therefore the shock tube operation was considered, for all practical purposes, tailored.

Without specifying what mechanisms may occur, a parametric investigation was carried out of the effects of enthalpy loss and/or freezing in the reflected shock heated region¹. The results of these calculations were carried through to the test section in the form of a sensitivity study and will be discussed in more detail below.

The prime assumptions made were that regardless of the loss the pressure of the reflected shock region remained constant, and the gas always returned to a state of thermodynamic equilibrium. That is, any loss of enthalpy was followed by a lowering of the temperature, loss of ionization, and a density change, with the new values satisfying the Saha Equation.

The speed of sound in a gas is given by

$$a^2 = \left(\frac{\partial P}{\partial \rho} \right)_S \quad (3-8)$$

For an ideal gas equation (3-8) becomes

$$a^2 = \gamma (P/\rho) \quad (3-9)$$

By comparison of equation (3-8) and (3-9), one may define an "effective γ " for isentropic conditions, namely

$$\gamma_{\text{eff}} = \frac{dP}{P} \frac{\rho}{d\rho} \quad (3-10)$$

Stalker (1961), using Lighthill's (1957) concept of an "ideal dissociating gas", derived an expression for the effective gamma of a dissociating gas. A similar solution can be obtained in the case of an ionizing gas.

¹ e.g. there may be loss via radiation or loss via heat conduction to the walls of the shock tube.

Using equations (3-4) through (3-6) with the standard thermodynamic result obtained by a combination of the First and Second Law of Thermodynamics,

$$T dS = dh - dP/\rho \quad (3-11)$$

it can be shown that the differential change of entropy of a monatomic gas, dS , is given by

$$dS = \frac{5}{2} R d\alpha + I \frac{d\alpha}{T} - (1 + \alpha) I \frac{dT}{T^2} + 2 R \frac{d\alpha}{\alpha} + 2 R \frac{d\alpha}{(1 - \alpha)} \quad (3-12)$$

Under isentropic conditions $dS = 0$ and therefore equation (3-12) may be solved for the rate of change of ionization with temperature. The result is

$$\frac{d\alpha}{dT} = \frac{(1 + \alpha) I}{5/2 R T^2 + I T + 2 R T^2 [\alpha(1 - \alpha)]^{-1}} \quad (3-13)$$

Equation (3-10) may also be rewritten for isentropic conditions in the form

$$\gamma_{\text{eff}} = \frac{dP}{P} \frac{\rho}{d\rho} \quad (3-14)$$

The quantity dP/P can be obtained from the equation of state, equation (3-5), and a value for $\rho/d\rho$ can be calculated from Saha's Equation, equation (3-6). The final result is

$$\gamma_{\text{eff}} = \frac{5/2 T + I/(RT^2) - 2 [\alpha(1-\alpha^2)]^{-1} \frac{d\alpha}{dT}}{3/2 T + (I/R) T^2 - (2-\alpha) [\alpha(1-\alpha)]^{-1} \frac{d\alpha}{dT}} \quad (3-15)$$

and the speed of sound is then evaluated from the relation

$$a^2 = \gamma_{\text{eff}} \frac{P}{\rho} \quad (3-16)$$

3.2.2 Nozzle Conditions

The mass conservation equation, equation (3-1), is modified for the problem of nozzle flow to

$$\rho_1 u_1 A_1 = \rho_2 u_2 A_2 = \rho_* u_* A_* \quad (3-17)$$

Equation (3-2) does not apply to nozzle flow, but equations (3-3) through (3-6) remain the same, except now the conditions of region 2 refer to those in the throat of the nozzle, while region 1 refers to the reflected shock conditions¹. Additionally, there are now equations (3-13), (3-15), (3-16), and an integrated form of equation (3-12), i.e.

$$S = \frac{5}{2} R \alpha + 2 R \ln \left\{ \frac{\alpha}{(1-\alpha)} \right\} + \frac{(1+\alpha) I}{T} + S_0 \quad (3-18)$$

where the entropy change is referenced to the unspecified value S_0 . All of these equations were solved by iteration for the conditions in the throat of the nozzle. A value of the ionization fraction

¹Note that in equation (3-3) the energy of the reflected shock region equals the stagnation enthalpy.

lower than that in the reflected shock region would be chosen. Using equation (3-18), a corresponding value of the temperature would be found. A calculation of the throat flow velocity could then be made via equations (3-6), (3-4), and (3-3). An independent determination of the velocity was also obtained through equations (3-13), (3-15), and (3-16). These two results were compared and the iteration continued until their difference fell within a specified tolerance. The results obtained, based on the values shown in Table 3.2, are given in Table 3.3.

Logan (1971) has examined the possibility of non-equilibrium nozzle flow at these conditions. His results indicate that equilibrium isentropic flow is a good first approximation in the case of Argon, and this condition has been adopted in this work.

Nozzle flow calculations were carried out for two conical configurations, each with the same throat radius of 0.159 cm. The first nozzle had a 15° half angle and an on axis length of 20 cm. The second nozzle had a 7.5° half angle and an on axis length of 13.4 cm. Figure 3.1 shows the relationship between on axis position and area ratio for the two configurations.

As a first approximation, it has been assumed that the nozzle wall boundary layer is sufficiently thin to not significantly displace the flow.

3.2.3 Results and Discussion for Pure Argon

3.2.2.1 Enthalpy Loss in the Reflected Shock Heated Gas

As stated, a sensitivity study was carried out on the effect of enthalpy loss in the reflected shock heated gas, and the results of this work are illustrated in Figure 3.2. The most sensitive parameter to enthalpy loss, under isobaric equilibrium conditions, is the ionization fraction. The density also showed a sensitivity

to enthalpy loss with almost a unity ratio between the percentage of enthalpy loss and the percentage of density gain. The temperature and the speed of sound displayed about the same characteristic of a much lesser response to enthalpy losses.

3.2.2.2 Exit Conditions

Mach No.: The maximum stagnation enthalpy was taken at 7,635 cal/gm, as shown in Table 3.2. Nozzle calculations have been carried out for this figure and for possible stagnation enthalpy losses of up to about 25%. Figure 3.3 illustrates the calculated exit Mach No. for the two nozzles as a function of the stagnation enthalpy. The 15° half angle nozzle exit Mach No. exhibited a linear behavior and actually increases in value as enthalpy was lost. This interesting feature is explained by the fact that while both the exit gas velocity and speed of sound decrease as enthalpy is lost, the speed of sound decreases at a slightly more rapid rate.

The 7.5° half angle nozzle displayed a non-linear behavior in contrast to the larger exit nozzle. This is probably due to the fact that while the ionization fraction is zero for all practical purposes in regards to the 15° half angle nozzle, this is not the case with the 7.5° half angle unit. For the values of 100%, 94%, 87%, 80%, and 75% of the maximum possible stagnation enthalpy, there were ionization fractions of 6.7×10^{-3} , 1.3×10^{-3} , and 4×10^{-5} , respectively, for the first three, and the last two had ionization fractions of less than 10^{-6} .

Any ionization fractions as small as 10^{-3} must be called "significant"; for example, the value of the effective gamma is slightly lowered from 1.67 to 1.66 at the ionization level of 4×10^{-5} ; at the level of 1.3×10^{-3} this lowering has increased

to the point where the effective gamma is 1.53.

3.2.2.3 Pitot Pressure

Figure 3.4 shows the calculated pitot pressures as a function of the stagnation enthalpy. The most striking feature is the almost complete insensitivity of this parameter to enthalpy loss. The experimental points shown will be discussed in Chapter 4.

3.3 Argon-Helium Mixtures

3.3.1 Theory

In this phase of the investigation, it is assumed that the final effects of adding an arbitrary amount of Helium to the pure Argon test slug are independent of the manner in which the contamination is added. There is one qualification in that the characteristic time taken for contamination must be short enough so that the properties of the pure Argon alone would not have significantly changed.

Consequently, the following model has been adopted: At some arbitrary time, a specified amount of cold Helium is added to the known amount of pure Argon at reflected shock conditions, in the volume defined by the reflected shock region. The gases interact. The temperature of the reflected shock region is the order of 20,000 °K at high pressures and therefore as a first approximation it is justified to assume that no ionization of the Helium takes place. Hence, the mixture interacts thermally but not chemically. That is, the Helium affects the rate of ionization of the Argon by absorbing some of the enthalpy of the system but not by direct chemical reaction such as increasing the three body recombination rate.

The total enthalpy of a mixture of gases is given by

$$h = \sum_i x_i h_i \quad , \quad (3-19)$$

where x_i is the mass fraction of component i . Specifically, in the case at hand

$$h = x_A h_A + x_H h_H , \quad (3-20)$$

or

$$h = x_A \left[\frac{5}{2} (1+\alpha) R_A T + \alpha I \right] + \frac{5}{2} x_H R_H T . \quad (3-21)$$

Since no further mass is added or taken from the system, (the later nozzle flow will be considered as an expansion of the system and not a loss of mass) the mass fractions are taken to remain constant. In this context, the mass fraction of the Argon includes the mass of Argon ions.

The total pressure of ionized Argon and neutral Helium mixture has four components,

$$P = P_H + P_{AN} + P_{AI} + P_e , \quad (3-22)$$

where the subscripts AN, AI, and e refer to the neutral Argon atoms, the first ionized Argon ions, and the electrons resulting from the Argon ionization. The density ratio between the Argon total density (atoms and ions) and the Helium density can be obtained through the respective equations of state,

$$\frac{\rho_A}{\rho_H} = \frac{P_A}{1 + \alpha} \frac{1}{P_H} \frac{R_H}{R_A} , \quad (3-23)$$

Equations (3-22) may be written as

$$P = \frac{R_o T}{V} (n_{AN} + n_{AI} + n_e + n_H) , \quad (3-24)$$

where the n terms are the numbers of moles of each component in the total volume V . R_o is the universal gas constant. The

definition of the ionization fraction, α , is given by

$$\alpha = \frac{n_{AI}}{n_{AN} + n_{AI}}, \quad (3-25)$$

and from charge neutrality, $n_{AI} = n_e$. The first three terms in the bracket of equation (3-24) can be factored. Then,

$$P = \frac{R_o T}{V} \left[(n_{AN} + n_{AI}) (1 + \alpha) + n_H \right]. \quad (3-26)$$

The Argon partial pressure is given by the Argon mole fraction and the total pressure,

$$P_A = \left[\frac{(n_{AN} + n_{AI}) (1 + \alpha)}{(n_{AN} + n_{AI}) (1 + \alpha) + n_H} \right] P. \quad (3-27)$$

Similarly, the Helium partial pressure is given by

$$P_H = \left[\frac{n_H}{(n_{AN} + n_{AI}) (1 + \alpha) + n_H} \right] P, \quad (3-28)$$

and using these two relations, equation (3-23) becomes,

$$\frac{\rho_A}{\rho_H} = \frac{(n_{AN} + n_{AI})}{n_H} \frac{R_H}{R_A}. \quad (3-29)$$

The right hand side of equation (3-29) is constant in value, so the Helium density may be defined by

$$\rho_H = F \rho_A, \quad (3-30)$$

where F is some arbitrary constant of value less than one.

This same result is obtained for the case of no Argon ionization. Thus, in all cases there will be differential changes of pressure between the two gases, but the density will change in a like manner for both gases.

The entropy of a gas mixture is given by the general relation

$$S = \sum_i x_i S_i \quad (3-31)$$

or specifically in the case at hand by

$$S = x_A \left[\frac{5}{2} R_A \alpha + \frac{I}{T} + \frac{(1+\alpha) I}{T} + 2 R_A \ln \frac{\alpha}{1+\alpha} + S_o \right] \\ + x_H R_H \left[\frac{3}{2} \ln \frac{T}{T_o} - \ln \frac{\rho_H}{\rho_{Ho}} \right], \quad (3-32)$$

where S_o , T_o , and ρ_{Ho} are all referenced to some arbitrary datum level. In this study, to avoid any complications from entropy changes resulting from diffusion of one gas into the other, this reference level has been set at the final reflected shock conditions of the completely mixed system after any possible isentropic expansions.

By differentiating equation (3-32), considering the mixture to behave in an isentropic manner, and differentiating equation (3-29), a value of the rate of ionization change with temperature can be determined,

$$\frac{d\alpha}{dT} = \frac{(1+\alpha) + \frac{x_H R_H}{x_A R_A} I}{\frac{5}{2} R_A T^2 + I T + \frac{2 R_A T^2}{\alpha(1-\alpha)} + \frac{x_H}{x_A} \frac{(2-\alpha)}{\alpha(1-\alpha)} R_H T^2} \quad (3-33)$$

The effective value of gamma as defined in equation (3-10) can also be determined for the mixture. In this event

$$\frac{dP}{P} = \frac{dP_A}{P} + \frac{dP_H}{P}, \quad (3-34a)$$

and

$$\frac{d\rho}{\rho} = \frac{d\rho_A}{\rho_A}, \quad (3-34b)$$

and therefore

$$\gamma_{\text{eff}} = \frac{\frac{5}{2T} + \frac{I}{R_A T^2} - \left[\frac{P_A}{P} \frac{2}{\alpha(1-\alpha^2)} + \frac{P_H}{P} \frac{(2-\alpha)}{\alpha(1-\alpha)} \right] \frac{d\alpha}{dT}}{\frac{3}{2T} + \frac{I}{R_A T^2} - \frac{(2-\alpha)}{\alpha(1-\alpha)} \frac{d\alpha}{dT}} \quad (3-35)$$

where the term $\frac{d\alpha}{dT}$ is defined by equation (3-13).

If P_H/P is set equal to zero and P_A/P set equal to one, equation (3-35) simply becomes equation (3-15). However, it should be noted that for a non-zero Helium pressure component, both of these equations tend to a value of 5/3 as the ionization fraction goes to zero.

The final values of T , α , and P of the various mixture ratios can be solved by using equations (3-21), (3-6), (3-30), and the equations of state for the pure gases. The nozzle flow conditions can be determined with the added use of equation (3-35).

3.3.2 Argon-Helium Mixtures Calculations

3.3.2.1 Reflected Shock Region

Table 3.4 illustrates the effects of Helium contamination

% Mass He	Pressure (dyne/cm ²)	Density (gm/cc)	Temperature (°K)	Ionization Fraction	Sonic Velocity (cm/sec)
0	2.46 E+8	3.86 E-3	21,537	0.42	2.82 E+5
0.99	2.59 E+8	3.90 E-3	21,351	0.41	2.88 E+5
4.76	3.08 E+8	4.05 E-3	20,641	0.36	3.09 E+5
8.26	3.55 E+8	4.21 E-3	19,974	0.31	3.27 E+5
H _s = 7,635 (cal/gm) stagnation enthalpy					

TABLE 3.4

Calculated Effects of Helium Contamination Upon Original Argon
Reflected Gas System

in specified amounts on the original pure Argon reflected shock conditions.

The addition of Helium cooled the test gas with a corresponding drop in temperature and ionization. There was a rise in the density and pressure, but the enthalpy was assumed to remain unchanged. Fast response stagnation pressure readings of the reflected shock region taken by Stalker (1967) show no significant changes during the first few hundreds of micro-seconds after shock reflection, other than from a pressure "dip" at about 20 micro-seconds. Accordingly, if Helium is added to the test slug, a corresponding expansion of the mixture must occur to maintain a constant pressure. It has been assumed that this expansion is isentropic, and the reflected shock conditions following such a process are shown in Table 3.5.

The gas mixture must do work on its surroundings to expand, therefore the enthalpy of the gas mixture drops, as shown in Table 3.5. The expansion reverses the trend of the density, and rather than increasing as contamination level is increased, it drops. The temperature and ionization fraction are further lowered. The speed of sound, while still increasing as more Helium is added, does so at a lesser rate after expansion.

Using the conditions of Table 3.5 as the final reservoir values, the gas mixtures were further expanded to obtain the gas conditions in the throat of the nozzle. These results are shown in Table 3.6.

The specific enthalpy drop is accounted for by the kinetic motion which the gas has obtained. This velocity amounts to, in enthalpy units, about 1000 cal/gm. The throat pressure changes very little as Helium is added, while the density, temperature

% Mass He	Pressure (dyne/cm ²)	Density (gm/cc)	Temperature (°K)	Ionization Fraction	Specific Enthalpy (cal/gm)	Sonic Velocity (cm/sec)
0	2.46 E+8	3.86 E-3	21,537	0.42	7,635	2.82 E+5
0.99	2.41 E+8 [@]	3.67 E-3	21,138	0.40	7,522	2.86 E+5
4.76	2.41 E+8	3.43 E-3	20,022	0.34	7,268	3.02 E+5
8.26	2.41 E+8	3.07 E-3	18,776	0.27	6,876	3.13 E+5

@ Value measured by Stalker (1967)

TABLE 3.5

Calculated Final Conditions with Helium After Isentropic expansion
in the Reflected Shock Region

% Mass He	Pressure (dyne/cm ²)	Density (gm/cc)	Temperature (°K)	Ionization Fraction	Specific Enthalpy (cal/gm)	Sonic Velocity (cm/sec)
0	1.37 E+8	2.40 E-3	19,940	0.375	6,800	2.65 E+5
0.99	1.34 E+8	2.28 E-3	19,548	0.35	6,660	2.69 E+5
4.76	1.39 E+8	2.14 E-3	18,413	0.28	6,305	2.84 E+5
8.26	1.32 E+8	1.91 E-3	17,137	0.21	5,844	2.94 E+5

TABLE 3.6

Calculated Throat Conditions with Helium Contamination

and ionization fraction drop.

3.3.2.2 Nozzle Flow

The presence of the Helium contamination modifies the real gas thermodynamics during the expansion of the test gas down the nozzle. Calculations for both nozzle configurations were carried out. The only parameter that is different between the two units, as far as the solution is concerned, is the area ratio, and therefore the physics is fully demonstrated by the results for the 7.5° half angle nozzle alone.

3.3.2.3 Temperature

Figure 3.5 shows how the calculated temperature varies down the small exit nozzle for different levels of contamination. Additionally, a curve for a pure Helium flow is included, but note should be taken of the different stagnation enthalpy for this case, 11,670 cal/gm, as compared to a maximum of 7,635 cal/gm for the Argon system. This former value is obtained if the shock tube is filled initially with 20" hg of Helium¹, and is included only for a relative comparison with the other results.

The most interesting result is the change of slope or general curve character from that of a pure Argon flow as Helium is added. Curve "B", 1% mass Helium, shows an "Argon character" until a distance down the nozzle of approximately 10 cm (area ratio of about 86), and then there is a change of slope. This changeover point moves progressively up the nozzle towards the throat as more Helium is added to the system.

3.3.2.4 Velocity, Density, Pitot Pressure, and Static Pressure

No curves are shown for these parameters since they proved to be very insensitive to contamination. For example, in the case of a Helium content of 4.76% by mass, the exit velocity

¹This condition is dynamically equivalent to filling the shock tube with 2" Hg of Argon, if the Argon acted as a "perfect gas".

was affected by an amount less than 5% of the pure Argon value. Similar results were also true for the density and Pitot pressure calculations.

The static pressure in the nozzle and at its exit dropped by relatively large amounts, as high as an order of magnitude when approximately 10% by mass of Helium was present in the mixture. This pressure drop compensated the fall in temperature so the density remained fairly constant in value.

3.3.2.5 Reynold's Number

The predicted Reynold's Number, as given by $\rho u x / \mu$, is of quite different character as the contamination level is increased. In Figure 3.6, it can be seen that with only 1% mass Helium, curve "B", the slope shows a change at a position of about 10 cm. It may be noted that this is about the same point at which the corresponding temperature curve showed modification.

As the level of Helium is raised, the changes become more pronounced, and the slope changes from negative to positive.

3.4 Test Section Conditions

3.4.1 Nozzle Exit Values

The gas conditions at the nozzle exit for the various cases considered are summarized in Table 3.7. The stagnation enthalpies have already been presented in Table 3.5. It should be noted that as the Helium level goes beyond about 1% by mass, the Mach Number increases. At a concentration of 8.26% by mass, the exit Mach Number of the 7.5° half angle nozzle is within about 30% of the value obtained with the pure Helium flow shown.

3.4.2 Flow Divergence

A conical nozzle configuration has been used throughout this investigation. The emerging flow can, therefore, be considered a source-type (that is, as if the flow were produced at a point source in the nozzle throat), if the nozzle wall boundary layer is

Nozzle	% Mass He	Pressure (dyne/cm ²)	Density (gm/cc)	Temperature (degree-K)	Ionization Fraction	Velocity (cm/sec)	Mach No.
15° half angle	0.0	3.97 E+3	6.76 E-7	2816.0	0.0	7.81 E+5	7.90
	100.0	1.36 E+2	1.35 E-7	48.0	0.0	1.09 E+6	26.75
	(10" Hg)						
7.5° half angle	0.0	1.02 E+5	5.86 E-6	8335.0	8.41 E-3	7.38 E+5	4.87
	0.99	8.08 E+4	5.58 E-6	6381.0	2.47 E-4	7.46 E+5	4.86
	4.76	3.60 E+4	5.45 E-6	2221.0	0.0	7.59 E+5	7.24
	8.26	1.69 E+4	5.44 E-6	857.0	0.0	7.91 E+5	11.0
	100.0	7.25 E+3	2.19 E-6	159.0	0.0	9.77 E+5	13.15
	(20" Hg)						

TABLE 3.7

Nozzle Exit Conditions

thin as assumed. Consequently, the velocity vector of the gas will diverge as one proceeds from the on axis position toward the nozzle wall.

This divergence means that the density and pressure of the flow continually decrease after the gas leaves the nozzle. Since these parameters are important in the boundary layer calculations, it is necessary to estimate the effects of flow divergence upon the gas conditions.

Hall (1963) has investigated divergence effects in flow over hypersonic test bodies. Hall's analysis included centrifugal effects. For flat plate flow, these are not present since the stream lines after the oblique shock are assumed to remain parallel to the surface. The original problem, as defined by Hall for two dimensional flow over a sharp wedge, is solved here in a simple geometrical manner. The problem is illustrated in Figure 3.7.

The relationship between the ambient density, velocity, and pressure changes in high Mach Number diverging flow was given by Hall as

$$\frac{d\rho}{\rho} \approx -\frac{2x}{R}, \quad (3-36)$$

and

$$\frac{dP}{P} \approx -\frac{2\gamma x}{R}. \quad (3-37)$$

3.4.3 Conditions Across the Oblique Shock Front

To simplify the analysis, the shock front is considered to remain straight. Therefore, the complications brought about by divergence of the flow are (1) the ambient pressure (or density) has been lowered and

(2) the stream line is not parallel to the axis of the nozzle.

The first problem is handled by using equations (3-36) and (3-37) to correct the nozzle exit values. For the second item, the mass conservation equation (3-1) is taken as valid, and examination of the velocity vector components after the oblique shock (they will add to form a flow parallel to the surface of the plate) will show that from the geometry

$$\frac{u_{2n}}{u_{1n}} = \frac{\rho_1}{\rho_2} = \frac{\tan(\beta - \theta)}{\tan(\beta - \delta)}, \quad (3-38)$$

where the subscript n denotes normal to the shock wave and

where δ is the flow divergence angle, and its value is obtained from the relation, in radians,

$$\delta \approx \frac{x_p \cos \theta \tan \beta}{R + x_p \cos \theta}, \quad (3-39)$$

where θ and β are in radians. The term x_p is the point where the stream line contacts the shock front, given in terms of distance parallel to the surface of the flat plate measured from the leading edge.

The work of Petschek and Byron (1957); Wong and Bershader (1966); and Oettinger and Bershader (1967), has indicated that the position of the shock front (the angle β) is determined by ideal gas or frozen conditions. This is because the finite relaxation time of the gas flow is long enough to prevent immediate production of ionization after passage of the shock layer.

The shock angle and inclination angle have been taken to be related by the standard result

$$\frac{\tan (\beta-\Theta)}{\tan \beta} = \frac{(\gamma_N - 1) M_N^2 \sin^2 \beta + 2}{(\gamma_N + 1) M_N^2 \sin^2 \beta}, \quad (3-40)$$

where M_N and γ_N are the free stream nozzle exit Mach Number and effective γ .

Equations (3-36), (3-37), (3-38) and (3-39) were used to calculate the frozen after shock conditions for various x_p distances.

The results of the above calculations are shown in Tables 3.8a and 3.8b. Table 3.8a is for a pure Argon flow and Table 3.8b is for an Argon-Helium flow. As can be seen, the flow divergence can produce lowering of the pressure and density in excess of 100% of the non-divergent values.

Suggested modification of the shock angle with the addition of Helium to the flow is quite interesting. A maximum value, as shown in Table 3.8b, was obtained with a contamination level of about 1%. This is due to the fact that with this amount the calculated exit Mach Number is approximately unchanged from the pure flow value, but the calculated effective value of the quantity γ is higher. At greater levels of contamination, the exit Mach Number, as calculated, increased with corresponding lowering of the shock angle.

	θ	β	x_p (cm)	Pressure (dyne/cm ²)	Density (gm/cc)	Temperature (°K)	u_2 (cm/sec)	M_2
15° half angle	9.5	16.2	0.0	2.31 E+4	1.67 E-6	6,634.0	7.55 E+5	4.98
			1.0	1.85 E+4	1.44 E-6	6,181.0	7.58 E+5	5.18
			2.0	1.46 E+4	1.22 E-6	5,731.0	7.61 E+5	5.39
			3.0	1.13 E+4	1.03 E-6	5,270.0	7.63 E+5	5.64
	19.5	28.2	0.0	7.16 E+4	2.25 E-6	15,316.0	6.93 E+5	3.00
			1.0	5.81 E+4	1.93 E-6	14,469.0	7.02 E+5	3.13
			2.0	4.68 E+4	1.65 E-6	13,637.0	7.10 E+5	3.26
			3.0	3.73 E+4	1.40 E-6	12,815.0	7.17 E+5	3.40
7.5° half angle	9.5	18.9	0.0	2.76 E+5	1.21 E-5	10,942.0	7.08 E+5	3.63
			1.0	2.02 E+5	9.61 E-6	10,124.0	7.14 E+5	3.81
			2.0	1.44 E+5	7.46 E-6	9,274.0	7.18 E+5	4.00
			3.0	9.64 E+4	5.59 E-6	8,280.0	7.22 E+5	4.26
	19.5	28.7	0.0	6.21 E+5	1.98 E-5	15,060.0	6.56 E+5	2.87
			1.0	4.63 E+5	1.58 E-5	14,086.0	6.69 E+5	3.03
			2.0	3.37 E+5	1.24 E-5	13,109.0	6.79 E+5	3.18
			3.0	2.37 E+5	9.43 E-6	12,084.0	6.87 E+5	3.36

TABLE 3.8a

Calculated Conditions After the Oblique Shock in the Test Section
for Pure Argon (Ideal Gas Shock Angle Determination), no Enthalpy Losses

	% He	θ	β	x_p (cm)	Pressure (dyne/cm ²)	Density (gm/cc)	Temperature (°K)	u_2 (cm/sec)	M_2
7.5° half angle	0.99	9.5	20.1	0.0	2.19 E+5	9.36 E-6	10,325.0	7.17 E+5	3.63
				1.0	1.55 E+5	7.43 E-6	9,193.0	7.24 E+5	3.88
				2.0	1.04 E+5	5.77 E-6	7,984.0	7.29 E+5	4.20
				3.0	6.43 E+4	4.34 E-6	6,544.0	7.33 E+5	4.66
		19.5	32.0	0.0	5.52 E+5	1.35 E-5	18,008.0	6.52 E+5	2.50
				1.0	4.02 E+5	1.08 E-5	16,491.0	6.69 E+5	2.68
				2.0	2.86 E+5	8.44 E-6	14,935.0	6.82 E+5	2.87
				3.0	1.95 E+5	6.48 E-6	13,281.0	6.93 E+5	3.09
	4.76	9.5	16.8	0.0	1.87 E+5	1.29 E-5	4,872.0	7.33 E+5	4.72
				1.0	1.33 E+5	1.02 E-5	4,369.0	7.37 E+5	5.01
				2.0	9.08 E+4	7.93 E-6	3,853.0	7.41 E+5	5.36
				3.0	5.76 E+4	5.93 E-6	3,267.0	7.44 E+5	5.85
15° half angle (10" Hg He)		19.5	29.4	0.0	5.59 E+5	1.76 E-5	10,698.0	6.71 E+5	2.92
				1.0	4.08 E+5	1.40 E-5	9,813.0	6.85 E+5	3.11
				2.0	2.91 E+5	1.10 E-5	8,940.0	6.96 E+5	3.31
				3.0	2.01 E+5	8.38 E-6	8,054.0	7.05 E+5	3.53
	100.0	9.5	13.1	0.0	6.20 E+5	4.98 E-7	599.0	1.07 E+6	7.41
				1.0	4.99 E+5	4.27 E-7	562.0	1.07 E+6	7.67
				2.0	3.98 E+5	3.63 E-7	527.0	1.07 E+6	7.94
				3.0	3.13 E+5	3.05 E-7	495.0	1.07 E+6	8.21

TABLE 3.8b

Calculated Conditions After the Oblique Shock in Test Section for Ar-He Mixtures
(Ideal Gas Shock Angle Determination), No Enthalpy Losses

3.5 Contamination Effects on Boundary Layer Calculations

The effects of Helium contamination on the boundary layer calculations will manifest themselves in changes to the thermodynamics (e.g. equation of state, determination of specific enthalpy, and specific heat) and to the transport properties. The new thermodynamic relation has been discussed previously, and the new transport property equations are given in Appendix A.

3.5.1 Nozzle Calculations

Initially, interest was focused on what effects, if any, the nozzle throat and nozzle wall boundary layers might have on the free stream flow. These problem areas are far more complicated than the equations of Chapter 2 were designed to handle, but work was carried out to get a relative comparison between pure Argon, Argon-Helium, and pure Helium produced boundary layers.

To enable any use of the flat plate equations, it was necessary to assume that the radius of curvature of the nozzle region was large in comparison to the boundary layer thickness, and then limit the integration to a very localized section - so localized that the pressure could be taken to remain constant over it. Calculations were carried out for the small exit nozzle configuration with use of the sonic conditions in the throat, the conditions 2.24 cm. down stream, and 12.88 cm. down stream. The pure Argon flow was based on an initial shock tube pressure of two inches Hg; the Argon-Helium mixture assumed 4.76% mass Helium; and the pure Helium flow was based on an initial shock tube pressure of 20 inches Hg.

The integrations in the case of the first two Argon flows

used a variable transport parameter technique, while the pure Helium flow assumed constant parameters. The results of the displacement thickness are shown in Table 3.9. The displacement thickness parameter is given in terms of δ^*/R_x^{+1} .

As can be seen in Table 3.9, pure Argon displacement thickness is thin at all times.

Helium in the flow causes the displacement thickness to increase, but the value is still very small. As one goes closer to the nozzle exit, this effect is more pronounced, but still not significant in relation to changing the effective area ratio.

Consequently, it would appear that some major change in boundary layer displacement of the flow when Helium is homogeneously mixed into the test gas does not occur.

3.5.2 Flat Plate Calculations

3.5.2.1 Equilibrium Boundary Layer

Figures 3.8, 3.9, and 3.10 show the changes that occur to the temperature, ionization fraction, and density respectively, while Figure 3.11 allows conversion from the η axis to the y axis. All the information shown in these curves is for the small exit, 9.5° plate inclination, two cm. up the plate, and with variable transport parameters.

The Helium will cause the temperature to drop throughout the boundary layer. The density is interesting, in that with 1% mass Helium the value is lowered throughout the boundary layer, but with the larger 4.76% mass contamination the density regains some of its lost value. In the latter case, the minimum density peak is also flattened. The lowering of the specific enthalpy of the Argon is best illustrated by the major changes

in the ionization fraction. The peak ionization with a contamination¹ where R_x^* is the radius of curvature of the nozzle at the distance x down the nozzle from the throat.

level of one percent is not much below the pure Argon value, but occurs slightly higher in the boundary layer; the free stream value for the one percent level is considerably lowered.

The 4.76% by mass Helium content resulted in a drop in the peak ionization of almost two orders of magnitude. The actual thickness of the boundary layer, however, was changed very little by the Helium contamination.

The changes in the transport properties due to the presence of Helium are shown in Figures 3.12, 3.13, 3.14; Figures 3.12 and 3.13 are profiles of the thermal conductivity and viscosity, respectively, for the same calculations as the preceding drawings. Notice how a small amount of Helium, in this instance 1% by mass, does not destroy the basic shape of the curves, but only changes the positions of maxima and minima. At a level of 4.76% mass Helium, the situation is quite different, and the influence of ionization upon the results are greatly abated.

Figure 3.14 shows how the modifications of the transport properties feeds through to the transport parameters of Prandtl Number and C_1 . The same basic comments made previously hold here also. With 4.76 % Helium by mass, the Prandtl Number is practically a constant throughout the whole boundary layer. C_1 is also drastically changed. The most important feature here is the modification to the value of the gradient of C_1 at the surface. It is most interesting that these changes are such to maintain the surface heat transfer rate at almost the same value regardless of the level of Helium for the figures shown.

Figure 3.15, the calculated surface heat transfer rates, shows the importance of using a variable transport parameter

calculation. With a constant parameter solution, it would appear that a substantial lowering of the heat transfer would occur with contamination. However, it turned out in each case that the transport parameter derivatives were of such a value to compensate and the final heat transfer rates predicted varied very little, regardless of the contamination level up to at least 5% by mass.

3.5.2.2 Frozen Boundary Layer

If there are no chemical reactions in the boundary layer during the period of interest, the chemistry is considered "frozen".

All results shown will be for the small exit nozzle, 9.5° plate inclination, and at a distance up the plate from the leading edge of 2 cm.

Figure 3.16 shows the velocity profiles for a frozen boundary layer with a fully catalytic wall. A contamination level of 1% Helium by mass modifies the velocity profile.

Figures 3.17 and 3.18 are the temperature and density profiles for the above conditions. 1% Helium by mass lowers the temperature throughout the boundary layer, by partition of the total available enthalpy. The density was also lowered, and the maximum decrease correlates with the maximum difference between the velocity profiles.

Figure 3.19 is the ionization profile. As can be seen, the level of ionization is reduced considerably by introduction of 1% mass Helium. The smaller number of electrons in the gas will produce an increase in the viscosity. Consequently, it is not surprising that the viscous effects cause the velocity to start to drop off at a higher level in the boundary layer.

<u>Region</u>	<u>Gas</u>	<u>Displacement Thickness</u> <u>Parameter (δ^*/R_x^*)</u>
Throat	100% Ar	assumed 0
	95.24% Ar + 4.76% He	assumed 0
	100% He	assumed 0
2.24 cm down stream	100% Ar	less than 3.0 E-04
	95.24% Ar + 4.76% He	less than 3.0 E-04
	100% He	less than 3.0 E-04
12.88 cm down stream	100% Ar	less than 2.0 E-04
	95.24% Ar + 4.76% He	order of 8.0 E-04
	100% He	order of 3.0 E-03

TABLE 3.9

Relative Nozzle Displacement Thickness

Figure 3.20 shows the difference that Helium contamination causes in the conversion from the η to the y axis.

A level of 1% mass Helium in the boundary layer caused essentially no change to the surface heat transfer rate for the fully catalytic plate and therefore no graph is shown. This result is explained in the following manner. The free stream ionization is lower with contamination and consequently the diffusion of ion-electron pairs to the surface contributes less to the overall heat transfer. However, the temperature gradient at the surface, as shown in Figure 3.17, is larger and compensates by producing a larger conduction of heat flux to the surface. The thermal conductivity at the surface is made larger with the presence of Helium. These results mean that the lowering of the surface heat transfer rate with a non-catalytic surface is not as great with Helium contamination.

3.6 Summary of Chapter 3

This study has included the effects of enthalpy loss and Helium driver gas contamination. The results of the work have been used to predict the nozzle exit conditions for a pure Argon test gas flow and with various levels of Helium contamination. The Helium increases the exit Mach Number of the nozzle, while lowering the temperature. Calculations were then made of the changes to these values when the gas flow traverses an oblique shock front.

The presence of Helium will alter the displacement thickness of the boundary layer in the throat and on the walls of the nozzle. However, this parameter remains small in all cases and would not be expected to cause any changes to the effective area ratio of the nozzle.

The flat plate boundary layers are also affected by the presence of Helium. The Helium lowers the temperature, density, and ionization in the boundary layer. If contamination levels of 5% by mass are reached the general profiles of the thermal conductivity and viscosity will be greatly altered.

With regard to calculations of equilibrium boundary layers with full variation of the transport properties, and frozen boundary layers with fully catalytic surfaces, the Helium contamination up to about 5% by mass will cause practically no change to the surface heat transfer rates. In the case of a frozen, non-catalytic surface calculation with contamination, the lowering of the heat transfer by the absence or diminishing of the diffusion of electron-ion pairs is lessened due to the lower level of free stream ionization.

CHAPTER 4

EXPERIMENTS AND DATA CORRELATION WITH THEORY

4.1 Introduction

The Free Piston Shock Tunnel is unique in its ability to produce very high stagnation enthalpy flows. In this study, conditions were used which gave rise to a calculated stagnation enthalpy of 7,635 cal/gm for a pure Argon flow. This figure would be in excess of 100% greater than that obtainable with combustion driven shock tunnels.

The Free Piston Shock Tunnel is a comparatively new device, and it is important to carefully check the flow behavior to determine test times for various experiments. In this Chapter, experimental data will be presented and discussed for test section pitot pressure, test section density via Mach Zehnder Interferometry, and surface heat transfer rates on the flat plate.

In all the data shown, the point is the mean average of many determinations, and the flags are the standard deviations of the data.

Details of the equipment and data reduction methods are discussed in Appendix B.

4.2 Properties of the Test Section Flow

4.2.1 Calculated Conditions

The test section has previously been discussed in Chapter 3. However, for purposes of clarity, the calculated pure Argon flow nozzle exit conditions are repeated below in Table 4.1 with some additional information.

4.2.2 Pitot Pressure Measurements

The reader is referenced back to Figure 3.4, the calculated

	<u>Large Exit Nozzle</u>	<u>Small Exit Nozzle</u>
Area Ratio	1204	147
Density (gm/cc)	6.76 E-7	5.86 E-6
Ionization Fraction	assumed 0	0.008
Temperature ($^{\circ}$ K)	2,816	8,335
Pressure (dyne/cm ²)	3.97 E+3	1.02 E+5
Velocity (cm/sec)	7.81 E+5	7.38 E+5
Speed of Sound (cm/sec)	9.89 E+4	1.52 E+5
Mach No.	7.90 ✓	4.87
Effective Gamma	1.67	1.32
Pitot Pressure (dyne/cm ²)	4.13 E+5	3.19 E+6
Thermal Conductivity (cal/cm-sec- $^{\circ}$ K)	2.24 E-4	4.08 E-4
Viscosity (gm/cm-sec)	1.16 E-3	1.95 E-3
Specific Heat (cal/gm- $^{\circ}$ K)	0.1247	0.1257
Prandtl Number	0.64	0.60
$\rho k / \rho_w k_w$	0.571	0.348
Local Reynold's Number	9.14 E+3	2.99 E+4

TABLE 4.1

Calculated Conditions at the Exit of the Nozzles

test section pitot pressures.

The first experimental points are based on the Pitot pressure measurements taken 85 micro-seconds $\pm$ 15 micro-seconds from the time of shock reflection. The experimental technique used is discussed fully in Appendix B. There is good agreement between theory and experiment, validating the assumption of isentropic nozzle flow with insignificant boundary layer effects at this time.

The pitot pressure at a flow time of 160 micro-seconds $\pm$ 15 micro-seconds from shock reflection is a different matter. The measured value dropped by a very significant amount. Three typical oscillograms illustrating this pressure fall are shown in Figures 4.1a, 4.1b, and 4.1c. The data of Figure 4.1a has a time base of 50 micro-seconds per division, while that of Figure 4.1b is 20 micro-seconds per division. Both traces are for the small exit nozzle; Figure 4.1c has a 20 micro-second base and is large exit data.

The initial 30-40 micro-seconds of the traces can be attributed to nozzle starting time. The rise time of the pitot pressure gauge was a function of the density of the gas flow. In the instance of the small exit unit, this latter time was estimated at about 30 micro-seconds, and when using the 15° half angle nozzle this period lengthened to about 40-50 micro-seconds.

At first glance, these observations look analogous to the pressure dip in the reflected shock region, observed by Dunn (1969a, 1969b) and others. Dunn explained this dip as resulting from an expansion wave generated at the initial interaction of the reflected shock and the driver-test gas interface. Dunn also defined the usable test time as the period required for this

expansion wave to reach the shock tube end wall, and obtained typical values for usable test time of about 200 micro-seconds. Stalker (1967) has observed a dip in reflected shock pressure measurements of the Australian National University device at approximately 20-30 micro-seconds after shock reflection. Logan (1971) using streak photography has correlated this time interval with the period required for the expansion wave to reach the end wall. Thus the pressure dip arising from the reflected shock expansion occurs on a time scale which is too short to be clearly evident in the pitot pressure traces, indicating that the observed pitot pressure drop must be due to some other source.

Davies and Wilson (1969) have investigated the problem of interaction of the reflected shock front and shock tube boundary layer. As the reflected shock moves back up the shock tube, it comes in contact with the wall boundary layer formed after the initial forward passage of the front. The stagnation pressure of the majority of this boundary layer is less than that of the free stream, and therefore the boundary layer gas may not penetrate the reflected shock front. Because the boundary layer cannot negotiate this pressure jump, it separates ahead of the shock. An oblique shock forms at this point, and eventually combines with the normal front at some distance from the surface. This is illustrated in Figure 4.2.

Boundary layer gas is trapped under the separated region beneath the shock, and is carried along with the front. This system is known as a "bifurcated foot". Some of the gas immediately above the boundary layer traverses the double oblique shock system ahead of the reflected shock and into the region

behind it. A small amount of the boundary layer gas may also get passed across this shock configuration, and if so it will move along the wall with the other penetrating gas toward the end of the shock tube at a velocity higher than that of the reflected shock free stream.

It is possible, according to Davies and Wilson, that when the reflected shock passes through the Helium-Argon contact surface a small amount of Helium driver may leak into the test gas slug, move along the side wall, gather at the nozzle end of the shock tube, and finally move toward the center in front of the entrance to the throat of the nozzle.

Based on the discussion in Chapter 3, the drop in test section Pitot pressure is not likely attributed to energy loss mechanisms, Helium mixing with the Argon in the reflected shock region, or the Helium altering the throat or nozzle wall boundary layer.

When Helium was used as a test gas and a driver gas, no Pitot pressure drop was observed. This suggests that the fall of Pitot pressure is due to some difference between the flow characteristics of the Argon and Helium. Consequently, a plausible explanation for the observed Pitot pressure is that the contaminating, relatively cold Helium gathers at the nozzle end of the reflected shock region and moves toward the center of the shock tube. When this gas mass reaches the area of the throat, it feeds into the periphery of the Argon flow. The Helium may occupy a greater relative area in the throat than further down the nozzle, as shown in Figure 4.3. This would effectively increase the area ratio as far as the Argon was concerned and thereby cause a Pitot pressure drop. The possibility of this happening can be substantiated using a very simple argument.

Let it be assumed that the gases are not mixed in the reflected shock region, and the Argon and Helium are at the same approximate pressure in the general area of the throat. Let it also be assumed that when the Helium feeds into the throat, as described above, it occupies a certain percentage of the total area. From the Argon real gas nozzle calculations, at an area ratio of 100 (that is, the area at the point in question in the nozzle is 100 times greater than the throat area occupied by the Argon), the gas pressure of the Argon would have dropped by a factor of about 800 of that in the throat. It would seem logical that the Helium would suffer approximately the same relative pressure drop. However, to obtain this the Helium would need only an area increase of about 20. This value was obtained using ideal gas relations. Therefore, the Helium would occupy a much smaller percentage of the total area.

This particular phenomenon of Pitot pressure drop could then be attributed to the fact that the Argon acts as a real ionizing gas in the nozzle flow, while the Helium acts as an ideal gas with no ionization.

4.2.3 Attempted Measurements of the Density Field

The Mach-Zehnder interferometer is widely used in studies of flow visualization, and it has proven a very valuable tool in providing a quantitative picture of density fields. Figure 4.4 shows the fringe shifts that could be expected for a 10 cm. path length, the width of the flat plate, for various densities of neutral Argon. The results are based on the data of Alpher and White (1965). With regard to the large nozzle, the exit diameter was also approximately 10 cm., but for the small nozzle the exit flow was only approximately 5 cm. wide. Therefore, the numbers given

from Figure 4.4 should be divided by two in this latter case.

The expected fringe shifts after the oblique shocks in this study would range between approximately 0.1 to 0.40 for wavelengths of 4000 to 6000 $\text{\AA}$. To enable use of the Mach-Zehnder¹, it would be necessary to (1) make each fringe shift measurement very accurate, and (2) to make a sufficient number of measurements to improve the overall accuracy by statistical averaging.

A computer method of data reduction, "CIDRS" (computerized interferometric data reduction system), was developed in an attempt to satisfy the above requirements. This system, and the method of data acquisition, are discussed in Appendix B. From controlled calibration tests, it was determined that "CIDRS" would read interferograms with an overall practical accuracy of ± 0.04 of a fringe², or in terms of density, about $1.5 \text{ E-}6 \text{ gm/cc}$ or $2.2 \text{ E+}16 \text{ atoms/cc}$.

A two wave-length interferometric system was used, and photographs were taken just before the shock tunnel was fired and again during the time of the flow. A 19.5° plate inclination was used at all times. The results of this phase of the investigation were not as conclusive as hoped for. In the case of the large exit nozzle, data was reduced from 72 photographs³ and the standard deviation was about $3.0\text{E-}6 \text{ gm/cc}$ in terms of density. This error was approximately the value of the expected density, and therefore this data could not be used. The additional error would be due to non-uniform drifts of the Mach-Zehnder system during the period between the taking of the two pictures (approximately 30 seconds), and the shot to shot variation of

¹The unit at the Australian National University was manufactured by the Zeiss Company.

²This was based on 1600 fringe shift measurements.

³200 readings were made on each interferogram.

the shock tunnel itself.

In reference to the small exit data, an average density reading behind the oblique shock was about $8.0\text{E}-6 \pm 4.0\text{E}-6$ gm/cc. There were only 26 photographs analysed in this instance.

On the basis of these results, it would seem wise to carry out a program to develop a suitable light source that would allow time resolved interferometry over periods in excess of 100 micro-seconds, thereby eliminating the drift errors. The true shot to shot variation of the shock tunnel operation could then be properly estimated.

While "CIDRS" did not provide the desired results in this investigation, it is believed that the method will be of value in a number of other problem areas, e.g. interpretation of time resolved shock tube data, reduction of spectroscopic data, and in general with a wide variety of data that is recorded on film.

4.3 Boundary Layers

4.3.1 Leading Edge Effects

It is not the aim of this project to carry out an investigation of the leading edge effects associated with rarefied hypersonic flow. Indeed, the object of using an inclined flat plate was to maintain a very high stagnation enthalpy while avoiding this problem area. However, it is important that a sufficient study be made to be sure of this point.

The generally accepted flow picture is that at, and very near to, the leading edge, the gas flow is described by kinetic theory. This area is followed downstream by a region of transition to what is known as the "merged region". Pan and Probst (1966) characterize the merged region by continuum flow where the viscous boundary layer and oblique shock wave are merged. There may be wall slip. Further downstream there are

the regions of strong interaction and then weak interaction. These are depicted by the fact that the boundary layer and shock front separate and a region of inviscid flow forms between them. However, they are in close enough proximity for the boundary layer to affect the position of the shock front. The resultant curvature of the shock produces a pressure gradient.

Heat transfer measurements made in the merged region, as indicated by Vidal, Golian, and Bartz (1963), will fall below the predictions made using the equations derived in Chapter 2. Measurements made in the strong interaction region must also be rejected because of the existence of a significant pressure gradient.

McCroskey, Bogdonoff, and McDougall (1966) have shown that the relevant merged region interaction parameter is given by

$$\bar{V} = M_2 (C/Re_2)^{\frac{1}{2}}, \quad (4-1)$$

where C is the constant in a linear viscosity law and M_2 and Re_2 are the post oblique shock Mach Number and local Reynold's Number, respectively. McCroskey, et al, experimentally demonstrated that when $\bar{V}$ has a value between 0.15 to 0.20, merged region effects, such as lowering of the surface density and heat transfer, could be expected.

Dorrance (1962) has shown that the relevant strong interaction parameter is

$$\bar{X} = M_2^3 (C/Re_2)^{\frac{1}{2}}. \quad (4-2)$$

From the work of Hayes and Probst (1959), it appears that strong interaction would be present for values of $\bar{X}$ greater than 3 or 4, and weak interaction would be the criteria below these values.

Table 4.2 shows the calculated values of $\bar{V}$ and $\bar{X}$ at 1, 2, and 3 cm. up the plate from the leading edge for the small exit, 9.5° plate inclination, and, for the large exit, 19.5° plate inclination data, with a pure Argon flow. The values of the Mach Number and the Reynold's Number at each point have been calculated with flow divergence considered in the manner described in Chapter 3. The only measurement that might suffer some merged region influence is at 1 cm. with the large exit nozzle, at a plate inclination of 19.5° . It would appear that the strong interaction region is very limited in extent and consequently the assumption of a zero pressure gradient is valid to a good approximation.

It has been assumed that the flat plate induced pressure gradient can come about only through some form of leading edge effect and not by changes in the free stream conditions. In his discussion Dorrance (1962) (Chapter 4) shows that if g_w is small and the pressure gradient parameter varies "slowly", the integrated boundary layer solutions are insensitive to the variation of the pressure gradient. Further, the concept of "local similarity", e.g. the integration of the boundary layer equations at a specific location using the free stream conditions present there, has been proven to be acceptable. Accordingly, in this work the

DATA	DISTANCE UP PLATE (cm)	$\bar{V}$	$\bar{X}$
Small exit, 9.5° plate inclination	1 2 3	0.09 0.08 0.07	1.3 1.3 1.3
Large exit, 19.5° plate inclination	1 2 3	0.14 0.11 0.10	1.4 1.2 1.2

TABLE 4.2

Calculated Merged Region and Strong Interaction Parameters¹

¹It should be noted that Rudman and Rubin(1968) concluded in their work that at a free stream Mach No. equal to or less than 8, the merged region went directly into the weak interaction region.

two dimensional flat plate boundary layer equations have been solved at different locations up the plate using the free stream conditions calculated at the respective positions.

It should additionally be noted that referral to Figures 2.6 and 2.12, typical shear stress and η to y axis conversion, respectively, indicate that in the lower regions of the boundary layer the $\partial/\partial y (\mu \partial u/\partial y)$ term in the momentum equation is about an order of magnitude larger than the $\partial p/\partial x$ term (due to flow divergence). This substantiates the use of the zero pressure gradient flat plate equations.

4.3.2 Heat Transfer

Figure 4.5a shows typical Argon heat transfer traces. The upper and lower traces were recorded by gauges 1.75 and 2.25 cm. up the plate from the leading edge, respectively. There is a dramatic change in the character at approximately the same time that the Pitot pressure drops. The initial noise is mainly attributed to the presence of electrons in the flow. After the Pitot pressure drops, the heat transfer also falls, and at the same time the noise for the most part disappeared. It is suggested that this last point is due to the quenching of the flow ionization by the injection of Helium into the test gas.

Figure 4.5b indicates typical heat transfer traces obtained when Helium was used both as the driver and test gas. This data is shown for comparison purposes with Figure 4.5a. The upper and lower traces were made by gauges 1.1 and 2.66 cm. up the plate from the leading edge. Note that there is no change in slope, and that the curves are for all practical purposes noise free. The former point is important, and it correlates with the fact that no Pitot pressure drop was observed with a Helium driver Helium test gas arrangement.¹ The noise free traces are again attributed to the fact that there was no ionization of the Helium.

Referring back to Figure 2.15, it shows the calculated and measured surface heat transfer rates for boundary layers in thermo-chemical equilibrium. Theoretical predictions are shown for both the cases of constant and variable transport parameters. The measured results are for a period approximately 85 micro-seconds after shock reflection. It is reiterated that in the figures the point is the mean value of many determinations, and the flags are the standard deviations of the data.

The measurement at 1.1 cm. distance up the plate has probably suffered merged region effects, as suggested in Table 4.2. The variable transport parameters markedly lowered the surface heat transfer rate. The large exit prediction and measurements are close to a factor of two from one another, the limit set as acceptable, but the correlation of the small exit measurements and theory is bad.

It was deemed wise at this point, based on the above

¹Pitot pressure measurements with air as the test gas showed the same large scale fall as with Argon.

results, to carry out a limited series of calculations and measurements for a pure Helium flow. The Helium would be divorced of real gas effects and thereby act as a check on the basic theory. The result, using an initial shock tube pressure of 10 inches Hg is shown in Figure 4.6, again for a time of about 85 micro-seconds after shock reflection. The large exit nozzle was used with a plate inclination of 9.5° and data is only shown for a gauge position 2.66 cm. up the plate from the leading edge. (The gauges closer to the leading edge could have possibly suffered merged region effects.)

The experimental and calculated data showed reasonable agreement. If, with pure Helium, a fairly thick nozzle turbulent boundary layer did develop towards the exit, the flow divergence would be lessened and the flow pressure would be increased. As a result, the measured heat transfer rate would be expected to be larger than the theoretical prediction.

4.3.3 Composite Boundary Layer Calculations

As recall of Figure 2.21 will show, the chemically frozen boundary layer heat transfer predictions, with or without a catalytic surface, were higher than for a boundary layer in thermochemical equilibrium with variable transport properties. The results of Chapter 3 showed that Helium contamination would not lower the predicted heat transfer rates to correlate with the data, and some other mechanism must be found to account for the low heat transfer rates.

Wong and Bershader (1966) argued that after a shock the gas goes to a high temperature, given by the ideal gas equations. Atom-atom collisions occur which tend to produce a low level of ionization. Eventually, the electron numbers are sufficiently

high to cause atom-electron reactions to predominate, with a subsequent rapid pick up of the ionization level. Finally, the gas reaches near equilibrium as the rate of recombination comes into balance with that of ionization. Any impurities will cause the above chain to proceed at a faster rate.

In the situation at hand, the same processes are at work. However, as outlined below, it is probable that the gas relaxation processes may be such as to promote a close approach to local equilibrium conditions near the wall.

Blottner (1964) has carried out a series of chemical non-equilibrium boundary layer calculations for air. He assumed the Prandtl Number was either 1 or 0.7, and the Lewis Number was either 1.0 or 1.4. His results showed that the temperature and composition profiles of the non-equilibrium boundary layer could be considerably different from either equilibrium or frozen results. However, Blottner's constant transport property calculations did not suggest a major change in surface heat transfer rates.

Pallone, Moore, and Erdos (1964) carried out similar calculations in air. They, however, used variable transport properties. The maximum temperature in their boundary layers was about 5000°K and consequently the amount of ionization was small. This meant that the Prandtl Number remained essentially constant in value at 0.75.

The inclusion of variable transport properties to the extent that occur in this work into a non-equilibrium calculation would be most difficult, and computer time consuming. This, coupled with the poor knowledge of the rate coefficients needed in the calculation, demanded that some easier approach to the

problem of non-equilibrium be found. A simpler first approximation to the problem could be the defining of a level in a boundary layer, at which the gas would change from that of essentially frozen in character to that of essentially equilibrium in character. The question is how to establish the position of the point where the two solutions (i.e. chemically frozen and thermochemically equilibrium calculations) could be put together.

Continuing the practice adopted in the calculation of frozen boundary layers, let it be assumed that the free stream is in thermochemical equilibrium but that the upper portion of the boundary layer is in a chemically frozen state.

The equilibrium constant for chemical reactions is given by the ratio of the ionizing reaction rate coefficients over the recombination rate coefficients. Wong and Bershader (1966) give expressions for the reaction rate coefficient for Argon ionization and for the recombination constant. They are, respectively,

$$\begin{aligned}
 K_I = 10^{-6} & \left\{ \frac{(2 k T)^3}{\pi m_e} \right\} \left\{ 4.4 \left[(\beta_E/T) + 2 \right] \exp (4 \beta_E/T) \right. \\
 & + 9.5 \left[(\Theta_I/T) + 2 \right] \exp (-\Theta_I/T) \\
 & \left. + 4.4 (\beta_E/T) \left[1 + (\Theta_I/T) \right] \left[(\Theta_I/\beta_E) - 1 \right] \exp(-\Theta_I/T) \right\} \left[\frac{\text{cm}^3}{\text{sec}} \right],
 \end{aligned} \tag{4-3}$$

and

$$K_E = \frac{12 (2 \pi m_e)^{3/2}}{h^3} (kT)^{3/2} \exp(-\Theta_I/T), \tag{4-4}$$

where

$$\begin{aligned} \theta_I &= \text{ionization temperature, } ^\circ\text{K } (1.828 \times 10^5), \\ \beta_E &= \text{excitation temperature, } ^\circ\text{K } (1.335 \times 10^5). \end{aligned}$$

Wong and Bershader gave the rate coefficient for ionization, K_I , as a function of electron temperature, but the assumption will be made here of thermal equilibrium.

The first term in the second bracket of equation (4-3) is approximately 1000 times greater than the other terms, and to a good first approximation the latter terms may be ignored. Taking the recombination rate coefficient, K_R , as the ratio K_I/K_E and calculating the numerical constants, it is found

$$K_R = 1.3035 \times 10^{-32} \left[\frac{13.35 \times 10^4}{T} + 2 \right] \exp \left[\frac{4.93 \times 10^4}{T} \right] \left\{ \frac{\text{cm}^6}{\text{sec}} \right\}. \quad (4-5)$$

Figure 4.7 shows the value of the recombination rate coefficient given by equation (4-5). The recombination rate increases as the temperature falls, in agreement with a similar equation given by Kn88s (1968). Notice how the value increases very rapidly below temperatures of about 7000°K , and therefore approach to the surface must promote an approach to equilibrium. Consequently, the recombination rate, that is the number of recombinations occurring per unit time, appears to be a reasonable criterion for determining the position of the matching layer in the composite boundary layer.

Wong and Bershader assumed only a three body recombination process, i.e.

and therefore the rate of change of the electron numbers is given by

$$\frac{dN_e}{dt} = N_e N_A K_I - N_e^3 K_R, \quad (4-7)$$

where the term $N_e^3 K_R$ is the recombination rate.

The integration of the boundary layer equations essentially defined 50 stream tubes within the boundary layer. The characteristic time in each stream tube for particles to remain in the region of interest was taken to be the period required for the gas particles to travel three cm. from the leading edge up the plate.

The criterion where the solution was switched from a frozen integration to an equilibrium integration was then taken to be given by

$$I_c < \frac{N_e}{N_e^3 K_R t_c}, \quad (4-8)$$

where t_c is the characteristic time and I_c is some arbitrary constant to be determined.

Equation (4-8) can be interpreted as meaning that if the number of electrons is greater than some factor times, the possible number that could be recombined within the characteristic time, the gas is defined as frozen.

Figure 4.8 shows how rapidly the value of this criterion changes with temperature. Referring back to Figure 2.17, the temperature profile of the frozen boundary layer, it can be appreciated how suddenly the changeover of the two conditions might indeed be.

Changeover stream tubes were defined based on choices of I_c nearest the values of 10 and 100. Table 4.3 shows the choices of stream tube¹, the corresponding values of temperature, the constant I_c , and the height above the surface.

The problem of matching the lower equilibrium and upper frozen calculations in a composite boundary layer bears some comment. To be correct, not only should the actual values of enthalpy and velocity be matched, but also their derivatives. The enthalpy gradient, g' , and velocity gradient, f'' , respectively, are measures of any heat flux or shear stress passed across the change-over layer from the frozen portion to the equilibrium portion of the composite boundary layer.

The boundary layer equations, however, were not designed to accommodate non-zero values of g' and f'' as $\eta \rightarrow \infty$. That is, it was assumed that once the values of g and f' went to unity, they would remain at this value and therefore the gradients of these quantities would go to zero. In the composite boundary layer, it is taken that g and f' may only be unity at a single level and consequently gradients may be present.

In addition to the intrinsic difficulty with the boundary layer equations, there is also a problem area with the integration of the relations when gradients are present at the upper boundary of the solution. Osborne (1969) assumed that the differential equations to be integrated were subject to the general boundary conditions given by

$$B_1 x(a) + B_2 x(b) = c \quad , \quad (4-9)$$

where B_1 and B_2 are $n \times n$ matrices, $x(a)$ and $x(b)$ are the values of x at the lower and upper boundary, respectively, and c is a vector of the values of the boundary conditions. B_1 and B_2 must

¹These values are based on integrations at two cm. up the plate.

η	$T(^{\circ}\text{K})$	I_c	$y(\text{cm})$	MODEL
0.330	7269	17.8	0.018	small exit
0.440	8823	132	0.029	9.5° plate inclination
0.275	5850	7.3	0.022	large exit
0.385	7415	115	0.039	19.5° plate inclination

TABLE 4.3

Matching Levels in Composite Boundary Layers

remain uncoupled and consequently the maximum number of boundary conditions that can be stated is 5 in the case of an equilibrium boundary layer ($f_w = f'_w = 0$, $g_w = \text{constant}$, $f'_e = 1$, and $g_e = 1$) and 7 in the case of the frozen boundary layer ($f_w = f'_w = 0$, $\theta_w = \text{constant}$, $S_{iw} = 0$ or a constant, $f'_e = 1$, $\theta_e = 1$, and $S_{ie} = 1$). To include the additional boundary conditions of f''_e and g'_e would require the defining of two dummy variables to make B_1 and B_2 in the equilibrium solution 7 X 7 matrices. Numerical experimentation showed this procedure to yield unsatisfactory results. Therefore, if a direct integration from the surface to the matching layer is done, the effects of heat flux and shear stress at the matching level must be neglected with the present method.

Figure 4.9 shows a partial profile of the heat flux for both a complete equilibrium and frozen boundary layer solution based on the same free stream conditions. As can be seen, in the frozen solution, the heat flux in the region of the changeover is about a factor of 4 less than at the wall, while in the equilibrium case it is a factor of 2 less than at the wall. This suggests that the heat flux from the frozen portion of the composite boundary layer may not play a dominant role in determining the surface heat transfer rate.

Figure 4.10 shows a partial profile for the shear stress of both a complete equilibrium and frozen calculation for the same free stream conditions. This graph would indicate that the shear stress in the region of the changeover is large. If the gradient of velocity f''_e feeds through the integration and affected the value of the enthalpy gradient at the wall, the surface heat transfer rate would be altered considerably.

From the above results, it would seem that as a first approximation, the solutions could be mated in the composite boundary

layer by matching the values of velocity, the shear stress, and the specific enthalpy, as well as possible, and by ignoring matching of the heat flux.

As an alternative to the direct integration of the problem, lower portions of the previous equilibrium calculations were matched to the upper portions of frozen calculations, where both the solutions were based on the same free stream conditions. The concept of similarity has been exploited. It has been assumed that when the velocity, shear stress, and specific enthalpy are matched, the character of the gas in the lower portion of the boundary layer may better be described by the conditions of an equilibrium solution at some other position on the plate, but which are still similar mathematically.

In terms of the transformed variables, the shear stress is by $\mu f''$.

Now,

$$\frac{\partial}{\partial y} = \frac{d\eta}{dy} \frac{\partial}{\partial \eta}, \quad (4-10)$$

so that from equations (2-5) and (2-7a) the shear stress x, y plane will be given by

$$\mu \frac{\partial u}{\partial y} = u_e^{3/2} (\rho_w k_w Pr_w / c_{pw})^{-1/2} \rho x^{-1/2} \mu f'', \quad (4-11)$$

At the matching level, the ratio between the frozen and equilibrium shear stress will be given by

$$\frac{(\text{shear stress})_{Fr}}{(\text{shear stress})_{Eq}} = \frac{(\mu f'')_{Fr}}{(\mu f'')_{Eq}} \frac{x_{Eq}^{1/2} \rho_{Fr}}{x_{Fr}^{1/2} \rho_{Eq}}. \quad (4-12)$$

Using equation (4-8), the choice of the matching level in the frozen boundary layer solution can be made. The stream tube in the equilibrium calculation having the same velocity as that of the matching level is located, and the corresponding shear stresses in terms of the transformed variables and densities are put into equation (4-12). The ratio of the $x^{\frac{1}{2}}$ distances is adjusted so that the ratio of the shear stress in the x, y co-ordinates will be unity. Table 4.4 shows the results of these calculations for distances at 2 cm. up the plate for the large and small exit problems.

Any resultant increase in $x_{Eq}^{\frac{1}{2}}$, assuming $x_{Fr}^{\frac{1}{2}}$ to remain fixed, will decrease the surface heat transfer rate by the same amount, since through similarity these quantities are coupled (i.e. $Q \propto \sqrt{x}$).

Figures 4.11 and 4.12 show the predicted surface heat transfer rates based on these choices of I_c for the small exit and large exit problems. If the choice of I_c is made at approximately 10, it can be seen that in no instance is the difference between the predicted and measured heat transfer rates greater than a factor of two. Indeed the accuracy is much better than this generally, with the normalized standard deviation of all measurements from the theory being less than 40%.

The lower the values of I_c , the greater is the tendency of the gas to depart from the frozen condition. It is clear that the best correlation between theory and data is obtained when a matching layer is chosen where there is a strong tendency to depart from the frozen state.

The large exit data, with a 19.5° plate inclination gave the best correlation with theory when I_c was set equal to 10. It is interesting to note that this case also provided the best overall match of shear stress, velocity, and specific enthalpy at the

Case	Calculation	η	I_c	f'	$\mu f''$	h (cal/gm)	ρ (gm/cc)	$(X_{Eq}/X_{Fr})^{-1/2}$
small exit	frozen	0.330	17.8	0.284	0.798 E-3	1198	0.924 E-5	- -
9.5° plate		0.440	132	0.376	0.106 E-2	1397	0.761 E-5	- -
inclination								
	equilibrium	0.422	- -	0.284	0.193 E-2	926	0.951 E-5	0.402
		0.468	- -	0.376	0.194 E-2	1098	0.838 E-5	0.496
large exit	frozen	0.275	7.3	0.262	0.432 E-3	1162	0.368 E-5	- -
9.5° plate		0.385	115	0.365	0.639 E-3	1365	0.290 E-5	- -
inclination								
	equilibrium	0.553	- -	0.365	0.112 E-2	1623	0.231 E-5	0.716

TABLE 4.4

MATCHING LEVEL DATA

matching level. This would suggest that further efforts should be made in developing a method of integration of the boundary layer equations that will allow the inclusion of the velocity and enthalpy gradient as boundary conditions. The composite boundary layer would seem to offer a reasonable alternative to the much more difficult non-equilibrium calculations in very high enthalpy flows, and may be particularly useful for engineering calculations.

The work of Kn88s (1968) predicted that much of the lower portion of an ionizing boundary layer may not be in either chemical or thermal equilibrium. It is to be emphasized that the concept of a composite boundary layer does not need to contradict the work of Kn88s. The basic assumption in the composite calculation is that at a certain point in the boundary layer the gas conditions are better approximated by the extreme case of thermochemical equilibrium, rather than the extreme case of a chemically frozen boundary layer. It is considered that this condition will be true when there is significant recombination, regardless of whether the gas is in a state of equilibrium or not.

4.4 Summary of Chapter 4

In this Chapter, the experimental data and its correlation with theory have been examined. There was good agreement between the measurement of test section Pitot pressure and an isentropic, real gas nozzle expansion theory until a time of flow of approximately 100 micro-seconds after shock reflection. After this period, the measured Pitot pressure dropped by approximately a factor of two. It has been suggested that this drop may be attributed to the injection of Helium driver gas into the periphery of the Argon flow in the area of the throat. The Argon expands down the nozzle as a real, ionizing gas, while the Helium expands.

as an ideal gas. This difference in flow character causes an increase in the effective area ratio in relation to the Argon with the observed fall in Pitot pressure.

Attempts at obtaining a detailed flow visualization behind the oblique shock, via Mach-Zehnder interferometry, were not up to expectation. However, a computerized interferometric data reduction system ("CIDRS") was developed, which should prove of value to many researchers.

The surface heat transfer rates showed a change of slope and general character at approximately 100 micro-seconds after shock reflection, correlating with the Pitot pressure data. The surface heat transfer rate is not adequately predicted with either an equilibrium or a frozen boundary layer assumption. However, improved prediction can be obtained by treating the boundary layer in two regions (a composite B/L), allowing the gas to pass from a chemically frozen region to one that is approximated by local thermochemical equilibrium. The position of the matching layer between the two regions is chosen by use of a simple criterion based on the recombination rate, and the parameters of velocity and shear stress are matched across it.

CHAPTER 5

CONCLUSIONS

The main aim of this project has been to study the boundary layer formed in very high enthalpy flow of a monatomic gas. A theoretical treatment has been developed that appears to correlate with surface heat transfer measurements made on an inclined flat plate.

The boundary layer investigation showed the importance of using a full variational transport parameters solution in the case of an equilibrium boundary layer. When dealing with very high enthalpy flows and cold walls there is an extreme fall of density coming off the surface. This manifests itself in an increase in certain transport parameter derivatives, which in turn significantly modify the integration of the boundary layer equations. The velocity profile takes on the classic shape given by an adverse pressure gradient or mass transfer (blowing), with a subsequent lowering of the surface heat transfer rate.

The importance of using a full variational transport parameters calculation was further emphasized in the investigation of Helium contamination in the boundary layer. If constant parameters are used, the results indicate large scale differences between the predictions of the pure Argon boundary layer and Argon-Helium boundary layers. In fact, results appeared to be very sensitive to the level of contamination. However, the variable transport parameters calculation showed that the transport parameter derivatives took on compensating values, depending on the contamination level, with the final result being that Helium

presence up to 5% by mass made very little difference in the surface heat transfer rates that might be expected.

When dealing with chemically frozen boundary layers, it was found that the extra effort and computing time needed for a calculation using variable transport parameters was not justified. This was attributed to the more arbitrary nature of the boundary conditions of temperature and ionization in comparison to the equilibrium boundary layer calculations, and the fact that the ionization profile was controlled entirely by diffusion processes.

There was a very poor correlation between both the equilibrium and frozen boundary layer predictions of surface heat transfer rates and the measured values. However, the theoretical values could be improved to an acceptable tolerance by assuming a level in the boundary layer where the gas conditions rapidly changed from a primarily frozen character to a primarily equilibrium character. At this juncture, a frozen boundary solution and an equilibrium solution with variable transport parameters were mated by matching of velocities and shear stress to form what was called a "composite boundary layer".

The placement of the matching level was determined by using a simple criterion based on the recombination rate in the respective stream tubes of the boundary layer.

The composite boundary layer calculations suggest that the chemical non-equilibrium in a very high stagnation enthalpy boundary layer will give rise to temperature, ionization, and density profiles that are different from either thermochemical equilibrium or chemically frozen solutions. These changes in internal structure produce changes in the profiles of the thermal conductivity and viscosity. These modifications to the transport

properties are sufficient to produce predictions of the surface heat transfer rate that are considerably less than those of either the equilibrium or frozen calculations. This aspect of the work may have very important engineering applications in the design of hyper-velocity vehicles. At present, the conclusions must be regarded as tentative, but there clearly is strong justification for further study of this effect.

There are certain other conditions that should be mentioned in connection with the discrepancy between the theoretical and measured heat transfer rates. The present day methods of calculation of gas mixture transport properties when one of the gases is ionized are not particularly accurate. However, it is the author's opinion that since many terms in the calculation of the transport properties used in this work were ignored, it would seem that the results might have been underestimated rather than overestimated.

The general results of the other quoted researchers would seem to substantiate that diffusion was not nearly as important as conduction in the transport of heat in a high enthalpy boundary layer. Consequently, the use of only ambipolar diffusion and the neglect of thermal diffusion would seem not to have caused a major error. Again, if anything, the results would probably been underestimated rather than overestimated with the difference between theory and experiment likewise minimized.

Since the approximations made tended to decrease the difference between prediction and measurement, it is felt that the reason for the discrepancy can not be traced to these simplifications, and non-equilibrium would still seem the most likely cause.

Starting with first principles, a study of the Free Piston Shock Tunnel has also been carried out from the initial rupture of the main compression tube diaphragm to the conditions after passage across an oblique shock in the test section. This information was used as the free stream conditions for the boundary layer calculations.

Experimentally, it was found that there was a major drop in the test section Pitot pressure of the order of 50%, approximately 100 micro-seconds after shock reflection. Careful theoretical investigation revealed that this change in flow character could not be attributed to enthalpy freezing and/or loss in the reflected shock region; could not be attributed to a homogeneous Helium contamination of the test Argon in the reflected shock region at levels as high as 9% by mass; nor could it be attributed to changes in the wall boundary layer in the throat or nozzle due to the presence of Helium.

The most plausible remaining explanation to the Pitot pressure drop is that the contaminating Helium moves along the shock tube side walls and gathers at the nozzle end wall. This gas then slowly moves in toward the center and eventually is injected into the periphery of the Argon flow in the area of the throat. The Argon expands down the nozzle as a real ionizing gas, while the Helium acts as a perfect gas. This difference in flow characteristics produces an increase in the effective area ratio of the nozzle with the subsequent drop in Pitot pressure.

Good agreement was obtained between prediction and experiment for the test section Pitot pressure up to the 100 micro-second time, using a pure, real gas, isentropic Argon flow theory. It was additionally assumed that the effects of nozzle wall boundary layer displacement could be ignored.

Figure 1.1. Drawing of standard reflected shock tunnel.

Figure 1.2. Drawing of Free Piston Shock Tunnel, "T2", of the A.N.U. Aerophysics Laboratory.

Figure 2.1. Velocity profiles for various values of the Profile Constant, equilibrium boundary layer.

Figure 2.2. Velocity profiles for constant and variable transport parameters. Arrows are K_1 at different levels.

Figure 2.3. Density profiles for constant and variable transport parameter calculations, equilibrium boundary layer.

Figure 2.4. Temperature profile for constant and variable transport parameter calculations, equilibrium boundary layer.

Figure 2.5. Ionization profile for constant and variable transport parameter calculations, equilibrium boundary layer.

Figure 2.6. Shear stress for constant and variable transport parameter calculations, equilibrium boundary layer.

Figure 2.7. Stagnation enthalpy-velocity relationship for constant and variable transport parameters, equilibrium boundary layer, large exit, $x_p = 1$ cm.

Figure 2.8. Equilibrium flat plate velocity profiles. Dotted lines are small exit nozzle, 9.5° plate inclination. Solid lines are large exit nozzle, 19.5° plate inclination, $x_p = 1$ cm.

Figure 2.9. Temperature profile for variable transport parameter calculation, large exit nozzle, equilibrium boundary layer.

Figure 2.10. Ionization profile for variable transport parameter calculation, large exit nozzle, equilibrium boundary layer.

Figure 2.11. Density profile for variable transport parameter calculation, large exit nozzle, equilibrium boundary layer.

Figure 2.12. Conversion from η to y -axis for variable transport parameter calculation, large exit, equilibrium boundary layer.

Figure 2.13. Thermal conductivity and viscosity for variable transport parameter calculation, large exit nozzle equilibrium.

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Figure 2.14. Prandtl No. and C_f for variable transport parameter calculation, large exit nozzle, equilibrium.

Figure 2.15. Heat transfer rates. Dotted lines ($\diamond$ data) are small exit nozzle, 9.5° plate inclination. Solid lines ($\circ$ data) are large exit nozzle, 19.5° plate inclination.

Figure 2.16. Velocity profile for Frozen Boundary Layer. Transport properties set constant at free stream values. Solid line 100% catalytic surface; Dotted line 100% non-catalytic.

Figure 2.17. Temperature profile for frozen boundary layer. Transport properties set constant at free stream values. Solid line 100% catalytic surface; dotted 100% non-catalytic.

Figure 2.18. Density profile for frozen boundary layer. Transport properties set constant at free stream values. Solid line 100% catalytic surface; dotted 100% non-catalytic.

Figure 2.19. Ionization profile for frozen boundary layer. Transport properties set constant at free stream values. Solid line 100% non-catalytic; dotted 100% catalytic.

Figure 2.20. Conversion from η to y axis for frozen boundary layer. Transport properties set constant at free stream values. Solid line 100% catalytic surface; dotted 100% non-catalytic.

Figure 2.21. Surface heat transfer rate for frozen boundary layer. Transport properties set constant at free stream values. Solid line 100% catalytic; dotted 100% non-catalytic.

Figure 3.1. Nozzle area ratio as function of on axis distance from throat.

Figure 3.2. Reflected shock gas changes due to enthalpy loss.

Figure 3.3. Exit Mach no. as function of stagnation enthalpy.

Figure 3.4. Exit Pitot pressure as function of stagnation enthalpy.

Figure 3.5. Gas temperature down small exit nozzle as function of Helium contamination.

Figure 3.6. Reynold's number down small exit nozzle as function of Helium contamination.

Figure 3.7. Two dimensional diverging flow over sharp leading edge inclined flat plate (after wedge problem of Hall (1963)).

Figure 3.8. Temperature profile for variable transport parameter calculation, small exit nozzle with contamination at $x_p = 2.0$ cm.

Figure 3.9: Ionization profile for variable transport parameter calculation, small exit nozzle with contamination at $x_p = 2.0$ cm.

Figure 3.10. Density profile for variable transport parameter calculation, small exit nozzle with contamination at $x_p = 2.0$ cm.

Figure 3.11. Conversion from η to y -axis for variable transport parameter calculation, small exit nozzle with contamination at $x_p = 2.0$ cm.

Figure 3.12. Thermal conductivity for variable transport parameter calculation with contamination at $x_p = 2.0$ cm, equilibrium boundary layer.

Figure 3.13. Viscosity for variable transport parameter calculation with contamination at $x_p = 2.0$ cm, equilibrium boundary layer.

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Figure 3.14. Transport parameters for variable transport parameter calculation with contamination at $x_p = 2.0$ cm, equilibrium boundary layer.

Figure 3.15. Calculated surface heat transfer rates for small exit nozzle with contamination, equilibrium boundary layer.

Figure 3.16. Frozen boundary layer velocity profile for small exit nozzle, 9.5° plate inclination, $x_p = 2$ cm., 100% catalytic surface, with Helium contamination.

Figure 3.17. Frozen boundary layer temperature profile for small exit nozzle, 9.5° plate inclination, $x_p = 2$ cm., 100% catalytic surface, with Helium contamination.

Figure 3.18. Frozen boundary layer density profile for small exit nozzle, 9.5° plate inclination, 100% catalytic surface, $x_p = 2$ cm., with Helium contamination.

Figure 3.19. Frozen boundary layer ionization profile for small exit nozzle, 9.5° plate inclination, $x_p = 2$ cm., with Helium contamination. Solid line 100% non-catalytic surface; dotted line 100% catalytic surface.

Figure 3.20. Conversion from the x axis to the y axis for the frozen boundary layer, small exit, 9.5° plate inclination, $x_p = 2$ cm., 100% catalytic surface.

Figure 4.1a. Pitot Pressure trace, small exit nozzle, time base 50 micro-seconds/division.

Figure 4.1b. Pitot pressure trace, small exit nozzle, time base 20 micro-seconds/division.

Figure 4.1c. Pitot pressure trace, large exit nozzle, time base 20 micro-seconds/division.

Figure 4.2. Bifurcated shock.

Figure 4.3. Change of effective nozzle area ratio by Helium feeding into Argon flow.

Figure 4.4. Possible fringe shifts for 10 cm. path length with Mach-Zehnder interferometer.

Figure 4.5a. Argon surface temperature history, time base 20 micro-seconds/division.

Figure 4.5b. Helium surface temperature history, time base 20 micro-seconds/division.

Figure 4.6. Helium heat transfer rate at approximately 85 micro-seconds after shock reflection, large exit nozzle, and 9.5° plate inclination. Constant transport parameters were assumed.

Figure 4.7. Recombination rate coefficient for Argon.

Figure 4.8. Matching criterion for composite boundary layer calculation. Small and large exit nozzles.

Figure 4.9. Heat flux for small exit nozzle, plate inclination of 9.5° , $x_p = 2$ cm.

Figure 4.10. Shear stress for small exit nozzle, 9.5° plate inclination, $x_p = 2$ cm.

Figure 4.11. Composite boundary layer calculation of surface heat transfer rates, small exit nozzle, 9.5° plate inclination.

Figure 4.12. Composite boundary layer calculation of surface heat transfer rates, large exit nozzle, 19.5° plate inclination.

APPENDIX A

TRANSPORT PROPERTIES

A.1 Pure Gases

Amdur and Mason (1958) have calculated the unionized viscosity of the rare gases using the approximate relation for a monatomic gas of Hirschfelder, Curtiss, and Bird (1954),

$$\mu \times 10^4 \text{ (gm/cm-sec)} = \frac{0.2669(MT)^{\frac{1}{2}}}{\Omega(2,2)}, \quad (\text{A-1})$$

where the values of the collision integral $\Omega(2,2)$ have also been given by Hirschfelder, Curtiss and Bird.

The tables given by Amdur and Mason are of the general form $\mu \propto T^\omega$. Curve Fitting, the formulae for pure Argon and Helium can be approximated by

$$\mu_A = 2.55 \times 10^{-6} T^{0.77} \text{ (gm/cm-sec)}, \quad (\text{A-2})$$

and

$$\mu_H = 1.29 \times 10^{-6} T^{0.84} \text{ (gm/cm-sec)}. \quad (\text{A-3})$$

The thermal conductivity of a pure gas is easily obtained once the viscosity is determined from the standard relation

$$k = \frac{15}{4} \frac{R}{M} \mu. \quad (\text{A-4})$$

Once again, the values of Amdur and Mason have been fitted to obtain the formulae,

$$k_A = 5.80 \times 10^{-7} T^{0.75} \text{ (cal/cm-sec-}^\circ\text{K)} \quad , \quad (\text{A-5})$$

and

$$k_H = 2.39 \times 10^{-6} T^{0.84} \text{ (cal/cm-sec-}^\circ\text{K)} \quad . \quad (\text{A-6})$$

A.2 Gas Mixtures

Cowling, Gray and Wright (1963) have presented an essay on the physical significance of the mixture formulae for viscosity and thermal conductivity, and it is considered worthwhile as an introduction to the subject to briefly review this work.

Say, for example, one is interested in the thermal conductivity of a binary mixture of pure, nonreacting monatomic gases. Let the total thermal conductivity be k , and the partial thermal conductivities¹ be k_1' and k_2' . Then,

$$k = \sum_i k_i' = k_1' + k_2'$$

Each partial value will be a function of the number of molecules of both gases transporting heat energy and the number of molecules of both gases which impede the transport of heat energy. Mathematically, this may be expressed in the form

$$k_1' = \frac{n_1 k_1 + k_2' B_{12} n_1}{n_1 + A_{12} n_2} \quad . \quad (\text{A-7})$$

¹The partial thermal conductivity is taken to be the thermal conductivity of the i th component, taking into consideration the presence of the other components.

That is, the transport of the heat flux of gas 1 is given by the number of molecules, n_1 , times the "pure" thermal conductivity of gas 1, plus a certain portion of the heat flux of gas 2 which has been picked up and subsequently transferred to the molecules of gas 1. The constant B_{12} is a measure of the efficiency of the transport of energy between the gases.

On the other hand, the molecules, n_1 , may also impede the transfer of heat of gas 1, as well as the molecules n_2 . The constant A_{12} is a measure of how efficient the molecules n_2 are in impeding of the heat flux of gas 1.

There will be an equation of the type shown in (A-7) for each partial thermal conductivity. After some algebraic rearranging, one obtains the following system of equations,

$$\begin{aligned} -k_1' A_1 + k_2' B_{12} &= -k_1, \\ k_1' B_{12} - k_2' A_2 &= -k_2, \end{aligned} \quad (A-8)$$

where

$$A_i = 1 + \sum_{\substack{j=1 \\ i \neq j}} \frac{n_j A_{ij}}{n_i}. \quad (A-9)$$

If k_1' , and k_2' are solved, via determinants, one obtains

$$k_1' = \frac{\begin{vmatrix} -k_1 & B_{12} \\ -k_2 & -A_2 \end{vmatrix}}{\begin{vmatrix} -A_1 & B_{12} \\ B_{21} & -A_2 \end{vmatrix}} \quad \text{and} \quad k_2' = \frac{\begin{vmatrix} -A_1 & -k_1 \\ B_{21} & -k_2 \end{vmatrix}}{\begin{vmatrix} -A_1 & B_{12} \\ B_{21} & -A_2 \end{vmatrix}}. \quad (A-10)$$

since $k = k'_1 + k'_2$, one obtains finally

$$k = \frac{\begin{vmatrix} 0 & 1 & 1 \\ k_1 & -A_1 & B_{12} \\ k_2 & B_{21} & -A_2 \end{vmatrix}}{\begin{vmatrix} -A_1 & B_{12} \\ B_{21} & -A_2 \end{vmatrix}} \quad (A-11)$$

This result can be extended beyond a binary system, and in general

$$k = \frac{\begin{vmatrix} 0 & 1 & 1 & 1 & \dots \\ k_1 & -A_1 & B_{12} & B_{13} & \dots \\ k_2 & B_{21} & -A_2 & B_{23} & \dots \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \end{vmatrix}}{\begin{vmatrix} -A_1 & B_{12} & B_{13} & \dots \\ B_{21} & -A_2 & \cdot & \dots \\ B_{31} & B_{32} & -A_3 & \dots \\ \cdot & \cdot & \cdot & \cdot \end{vmatrix}} \quad (A-12)$$

For most cases, the diagonal elements are much larger than the off diagonal and consequently a legitimate first approximation to k may be found by ignoring these elements. The following mixture rule is thus obtained:

$$k = \sum_{i=1}^n k_i \left[1 + \sum_{\substack{j=1 \\ i \neq j}}^k G_{ij} \frac{x_j}{x_i} \right]^{-1}, \quad (A-13)$$

where the x 's are the mole fractions. A rule for viscosity is similarly determined, i.e.

$$\mu = \sum_{i=1}^n \mu_i \left[1 + \sum_{\substack{j=1 \\ i \neq j}}^k G_{ij} \frac{x_j}{x_i} \right]^{-1} \quad (A-14)$$

The remaining problem is to determine values for the coefficient. Wilke (1950) defined G_{ij} by

$$G_{ij} = \sum_{i=1}^n \frac{1.385 \mu_i}{x_i \rho_i} \sum_{\substack{j=1 \\ j \neq i}}^n \frac{x_j}{D_{ij}}, \quad (A-15)$$

where D_{ij} are diffusion coefficients. He then demonstrated that for ideal gas behavior, the G_{ij} may be given by,

$$G_{ij} = \frac{1}{2\sqrt{2}} (1 + M_i/M_j)^{-\frac{1}{2}} \left[1 + (\mu_i/\mu_j)^{\frac{1}{2}} (M_j/M_i)^{1/4} \right]^2. \quad (A-16)$$

Mason and Saxena (1958) used a more rigorous argument, but eventually validated the use of equation (A-13) and they gave a very similar definition for the G_{ij} as that of Wilke,

$$G_{ij} = \frac{1.065}{2\sqrt{2}} (1 + M_i/M_j)^{-\frac{1}{2}} \left[1 + (k_i/k_j)^{\frac{1}{2}} (M_i/M_j)^{1/4} \right]^2. \quad (A-17)$$

Thus, if the transport properties of individual pure gases are known, an approximation to the properties of these gases in a mixture can be determined.

A.3 Ionized Argon Plasma

A.3.1 Thermal Conductivity

A.3.1.1. Fully First Ionized

Consider the problem of a fully first ionized Argon gas, and let it be assumed that the plasma behaves as a Lorentz gas. That is, the electrons do not interact with one another and the ions are at rest. Spitzer (1956) has determined the thermal conductivity for this binary mixture of electrons and ions, namely

$$k_L = 20 \left(\frac{2}{\pi} \right)^{3/2} \frac{(kT)^{5/2} k}{M_e^{1/2} e^4 Z \ln \Lambda} \quad , \quad (A-18)$$

or in c.g.s. units

$$k_L = 4.67 \times 10^{-12} \frac{T^{3/2}}{Z \ln \Lambda} \quad (\text{cal/cm-sec-}^\circ\text{K}) \quad , \quad (A-19)$$

where

Z = charge of ion and

$$\ln \Lambda = \frac{3}{2} \frac{1}{Z Z_1 e^3} \left(\frac{k^3 T^3}{\pi n_e} \right)^{1/2} \quad (A-20)$$

In the case of singly ionized Argon $Z=Z_1=1$ and

$$\ln \Lambda = 1.24 \times 10^4 T^{3/2} n_e^{-1/2} \quad , \quad (A-21)$$

where

n_e = number of electrons per cc.

The thermal conductivity of an actual gas may be found by applying a multiplying conversion factor δ_L to k_L however, there is one more correction to k which must be made. The fact that there is a transport of thermal energy requires that there be a movement of electrons in the gas. This electron flow will in turn set up electric fields which will feed back and change the velocities of the charged particles. The net effect is that the thermal conductivity is reduced. If we call this second correction ϵ , as Spitzer did, the thermal conductivity of fully first ionized Argon plasma may be represented by

$$k_s = \delta_L \epsilon k_L \quad . \quad (A-22)$$

From the information of Spitzer, the values of

$$\delta_L = 0.225$$

and

$$\epsilon = 0.419 ,$$

have been used which give

$$k_s = \frac{4.40 \times 10^{-13} T^{5/2}}{\ln \Lambda} \quad (\text{cal/cm-sec-}^\circ\text{K}) \quad . \quad (\text{A-23})$$

A.3.1.2 Partially First Ionized

If the Argon is not fully ionized, the mixture becomes ternary rather than binary with a composition of ions, electrons, and neutral atoms. Firstly, however, let us consider the binary mixture of electrons and ions and apply the mixture rule of equations (A-13) and (A-17) to equation (A-23).

Then,

$$k_s = \frac{k_I}{1 + G_{IE} (x_E/x_I)} + \frac{k_E}{1 + G_{EI} (x_I/x_E)} \quad . \quad (\text{A-24})$$

Chapman (1954) has shown that

$$k_I/k_E = (M_E/M_I)^{1/2} , \quad (\text{A-25})$$

where M_E and M_I refer to the masses of the electron and Argon ions; it can be assumed that $M_A = M_I$ where M_A is the mass of the Argon atom. Using relation (A-25), the coefficients defined by equation (A-17) are found to be approximately

$$G_{IE} = (1.065/2) (M_E/M_A)^{1/2} ,$$

$$G_{EI} = 2.13/\sqrt{2} ,$$

(A-26)

which, when applied to equation (A-24) give

$$k_s = \frac{k_I}{1 + (1.065/2) (M_E/M_A)^{1/2} (x_E/x_I)} + \frac{k_E}{1 + (2.13/\sqrt{2}) (x_I/x_E)} \quad (A-27)$$

using the definition of mole fraction,

$$x_i = \frac{m_i / (\text{gm-mole-wt})_i}{\sum_{j=1}^n m_j / (\text{gm-mole-wt})_j}, \quad (A-28)$$

where m_i and m_j are the total masses of component i and j respectively. Because of charge neutrality

$$x_E/x_I = x_I/x_E = 1 \quad (A-29)$$

Using relations (A-29) and (A-25), equation (A-27) becomes,

$$k_s = \frac{k_I}{(M_I/M_E)^{1/2}} + \frac{k_E}{1 + 2.13/\sqrt{2}},$$

or more simply

$$k_s \approx k_E/2.51 \quad (A-30)$$

It is seen that k_E and k_I may be defined in terms of k_s by,

$$k_E = 2.51 k_s \quad (A-31)$$

and

$$k_I = 2.51 (M_E/M_A)^{1/2} k_s.$$

Application of equations (A-13) and (A-17) to a three component gas composed of Argon atoms, ions, and electrons with

the employment of relation (A-31) gives, in units of (cal/cm-sec-°K)

$$k = k_A / \left\{ 1 + (1.065/4) \left[1 + 0.663 (k_A/k_S)^{1/2} (M_A/M_E)^{1/4} \right]^2 \alpha (1-\alpha)^{-1} \right\} \\ + \left[2.51 (M_E/M_A)^{1/2} k_S \right] / \left\{ 1 + (1.065/4) \left[1 + 1.58 (k_S/k_A)^{1/2} (M_E/M_A)^{1/4} \right]^2 \right. \\ \left. (1-\alpha) \alpha^{-1} + (2.13/\sqrt{2}) (M_E/M_A)^{1/2} \right\} \\ + \left[2.51 k_S \right] / \left\{ 1 + (1.065/4) \left[1 + 1.58 (k_S/k_A)^{1/2} (M_E/M_A)^{1/4} \right]^2 (1-\alpha) \alpha^{-1} \right. \\ \left. + 2.13/\sqrt{2} \right\} \quad (A-32)$$

It is appropriate at this point to briefly discuss the work of Camac, Fay, Feinberg, and Kemp (1963). In their paper, they used the mixture rule for the thermal conductivity

$$k = \sum_{j=1}^n \frac{x_j k_j}{x_k G_{jk}} \quad (A-33)$$

where $G_{jj}=1$. This is the same as equation (A-13); however, the

G_{jk} coefficients were defined by

$$G_{jk} = \left[2 / \left\{ 1 + (m_j/m_k) \right\} \right]^{1/2} (Q_{jk}/Q_{jj}) \quad , \quad (A-34)$$

with Q_{jk} and Q_{jj} being defined as hard sphere cross sections. After algebraic manipulation, the following equation was presented.

$$k = k_S \left\{ 1 + 2\sqrt{2} (M_E/M_A)^{1/2} (k_S/k_A) (Q_{AE}/Q_{AA}) x_A (1-x_A)^{-1} \right\}^{-1} \\ + k_A \left\{ 1 + (Q_{AI}/Q_{AA}) (1-x_A) (2 x_A)^{-1} \right\}^{-1} \quad (A-35)$$

Using the value of Q_{AI} and Q_{AA} given, a numerical comparison was made of (A-33) and (A-35). There appeared to be a discrepancy of a factor of two between the equations; equation (A-35) gave the lower value. However, even the smaller figure is larger than that predicted by equation (A-32) or than the available experimental evidence might seem to indicate.

In Figure A.1, curves "A" and "B" were calculated using equations (A-32) and (A-35), respectively, for the same Argon plasma. In this instance, the overall density was taken so that there was about 4% ionization at $10,000^{\circ}\text{K}$ (i.e. the pressure was approximately 2×10^5 dyne/cm²).

The experimental segment "C" was determined by Fay and Arnoldi (1968) from end wall heat transfer rates in a shock tube. This value should include the effect of ionization because the result was taken at a time when the temperature history of the gauge was at an asymptotic value, about 100 micro-seconds after shock reflection. The quoted value was $(8.4 \pm 2.0) \times 10^{-7} T^{3/4}$ (cal/cm-sec- $^{\circ}\text{K}$) over the range of $10,000 - 12,000^{\circ}\text{K}$.

The segment "D" is taken from work by Bunting, Devoto, and Bershader (1969). This result came from interferometric study of the gas behind a reflected shock. The quoted result was 4.25×10^{-5} , over the temperature range of about $3000 - 7000^{\circ}\text{K}$.

The point "E" was given by Lauver (1964), and he apparently estimated it from analysis of viscosity and molecular beam data. The last curve "F" is the "frozen" thermal conductivity as given by Amundur and Mason (1958).

The results of this study seem to suggest that a small amount of ionization has apparently its greatest effect in impeding the transport of thermal energy. This initial slight lowering of the thermal conductivity is probably a function of the larger collision cross-section for the ions; however, as soon as a reasonable number of electrons are present, the very large electronic thermal conductivity becomes the controlling element.

A.3.2 Viscosity

A.3.2.1 Fully First Ionized

A similar approach to that used for the deviation of equation (A-32) has been taken in the development of the viscosity relationship. According to Chapman (1954), the viscosity of a gas composed solely of singly ionized ions is given

by,
$$\mu_i = \frac{5}{8} \frac{4 M_i^{\frac{1}{2}} k^{5/2} T^{5/2}}{\pi^{\frac{1}{2}} e^4 A} \quad , \quad (A-36)$$

where

$$A = \ln 2 \left\{ 1 + x^2 - x^2(1+x^2)^{-1} \right\}$$

and

$$x = \frac{4kT}{n^{1/3} e^2} \quad . \quad (A-37)$$

Numerically
$$\mu_i = 4.85 \times 10^{-14} (T^{5/2}/A) \text{ (gm/cm-sec)} \quad (A-36a)$$

and

$$x = 2.39 \times 10^3 (T/n^{1/3}) \quad . \quad (A-37a)$$

It should be noted that the symbol "n" refers to the total particle number per cc, and is used in equation (A-37b) to give an estimate of the inter-molecular distance.

Again, from Chapman's paper, for a singly ionized Argon atom,

it can be shown that

$$\mu_e / \mu_i = (M_e / M_i)^{\frac{1}{2}}, \quad (A-38)$$

and

$$\mu_s \approx \mu_i$$

A.3.2.2. Partially First Ionized

Application of equations (A-14) and (A-16) to the previous relations lead to an equation very similar to equation (A-30), namely

$$\mu = \mu_A / \left\{ 1 + \frac{1}{4} \left[1 + (\mu_A / \mu_s)^{\frac{1}{2}} \right]^2 \alpha (1-\alpha)^{-1} \right\} + \mu_s / \left\{ 1 + \frac{1}{4} \left[1 + (\mu_s / \mu_A)^{\frac{1}{2}} \right]^2 (1-\alpha) \alpha^{-1} \right\}, \quad (A-39)$$

where terms that are very small have been dropped

and $\mu_A = 2.55 \times 10^{-6} T^{0.77} \text{ (gm/cm-sec)}$.

This last relation (A-40) was obtained by curve fitting the data of Amdur and Mason (1958).

Figure A.2, Curve "A", clearly illustrates the definite effect ionization appears to have on viscosity. The presence of charged particles in the gas apparently greatly inhibits the transfer of momentum with a consequent severe drop in the viscosity. The curve "B" was obtained from equation (A-40). Curve "C" will be discussed later.

A check of both the equations (A-30) and (A-39), the mixture values of thermal conductivity and viscosity, respectively, can be obtained by investigation of the Prandtl Number,

$$\text{Pr} = \mu c_p / k \quad , \quad (\text{A-40})$$

where the specific heat at constant pressure for the plasma may be defined by

$$c_p = (1 + \alpha) c_{pA} \quad . \quad (\text{A-41})$$

Figure A.3 illustrates the value calculated for Pr using equations (A-32), (A-39), and (A-41) and has been shown in contrast to the values obtained by Finson and Kemp (1965).

At low temperature, that is temperatures where no ionization is significant, the value of Pr for a pure monatomic gas should be $2/3$. The results of this study tend to a quite constant value of about 0.65 at this limit; the accuracy being dictated by how well the data of Amdur and Mason (1958) was fitted. The value of Finson and Kemp (1965) seems to oscillate about 0.77. As the temperature increases, the latter values seem to be low; at 6000°K there is practically no ionization, yet a figure of $\text{Pr} = 0.60$ is indicated. The results of this study suggest that the Prandtl Number is not lowered until a temperature of about 7500°K , or in the case of this particular study an ionization of 0.5%. The figure of Finson and Kemp is probably explained by the use of the mixing rule of Camac et al, which gave too large a value of the thermal conductivity.

In the temperature region, where the ionization is over a few percent, the two approaches to the Prandtl Number give about the same result. Consequently, the suggestion presents itself that the transport properties of Camac et al and Finson and Kemp are

sufficiently accurate if the temperature is above 12,000°K, but lose accuracy below this value.

A.4 Mixture of Ionized Argon and Neutral Helium

A.4.1 Viscosity

Following the general procedure established in the case of the three component gas, it is an easy matter to extend the solution to four constituents, i.e. Argon atoms, Argon ions, electrons and Helium atoms. Again from the work of Amdur and Mason (1958)

$$k_H = 2.39 \times 10^{-6} T^{0.84} \quad (\text{cal/cm-sec-}^\circ\text{K}) \quad (\text{A-6})$$

and

$$\mu_H = 1.29 \times 10^{-6} T^{0.84} \quad (\text{gm/cm-sec}) \quad (\text{A-3})$$

Application of the mixture rules (A-13) and (A-16) leads to the following equation in units of (gm/cm-sec)

$$\begin{aligned} \mu = & \mu_A / \left\{ 1 + \frac{1}{2} \left[1 + (\mu_A / \mu_s)^{\frac{1}{2}} \right]^2 \alpha (1-\alpha)^{-1} + 0.354 (1 + M_A / M_H)^{-\frac{1}{2}} \right. \\ & \left. \left\{ 1 + (\mu_A / \mu_H)^{\frac{1}{2}} (M_H / M_A)^{1/4} \right\}^2 (\rho_H M_A) [(1-\alpha) \rho_A M_H]^{-1} \right\} \\ & + \mu_s / \left\{ 1 + \frac{1}{2} \left[1 + (\mu_s / \mu_A)^{\frac{1}{2}} \right]^2 \alpha (1-\alpha)^{-1} + 0.354 (1 + M_A / M_H)^{-\frac{1}{2}} \right. \\ & \left. \left\{ 1 + (\mu_s / \mu_H)^{\frac{1}{2}} (M_H / M_A)^{1/4} \right\}^2 (\rho_H M_A) (\alpha \rho_A M_H)^{-1} \right\} \\ & + \mu_H / \left\{ 1 + 0.354 (1 + M_H / M_A)^{-\frac{1}{2}} \left[1 + (\mu_H / \mu_A)^{\frac{1}{2}} (M_A / M_H)^{1/4} \right]^2 \right. \\ & \left. (1-\alpha) \rho_A M_H (\rho_H M_A)^{-1} \right. \\ & \left. + 0.354 (1 + M_H / M_A)^{-\frac{1}{2}} \left\{ 1 + (\mu_H / \mu_s)^{\frac{1}{2}} (M_A / M_H)^{1/4} \right\}^2 (\alpha \rho_A M_H) (\rho_H M_A)^{-1} \right\} \end{aligned} \quad (\text{A-42})$$

At this point, it is important to briefly discuss the model used to describe the manner in which the Argon becomes contaminated. The reflected shock, in the shock tube, produces a pocket of hot test gas. If there are no losses, the system is adiabatic, and therefore if cold Helium is injected into the volume there will be a redistribution of energy until an equilibrium system temperature is reached. That is, the temperature and ionization of the Argon will be lowered while the temperature of the injected Helium will rise. After this new equilibrium condition is obtained, it has been assumed that the gas mixture can then undergo enthalpy loss with a corresponding new final equilibrium state and/or the overall system may undergo isentropic expansion or compression. However, the important point is the initial thermal interaction of the gases.

The unionized viscosity of both Argon and Helium is very similar, e.g. at $10,000^{\circ}\text{K}$ $\mu_{\text{A}} = 31.0 \times 10^{-4} \text{ (gm/cm-sec)}$ and $\mu_{\text{H}} = 29.2 \times 10^{-4} \text{ (gm/cm-sec)}$ and at 3000°K $\mu_{\text{A}} = 12.1 \times 10^{-4} \text{ (gm/cm-sec)}$ and $\mu_{\text{H}} = 10.7 \times 10^{-4} \text{ (gm/cm-sec)}$. Consequently, the main effect of the presence of Helium is to lower the ionization level. Curve "C" will account for different levels of Helium. The curve, as shown, is for 1% mass Helium. The 4.76% mass Helium would be exactly the same, only end between the 1% ionization and 5% ionization levels.

A.4.2 Thermal Conductivity

Application of the mixture rules (A-14) and (A-17) gave rise to the following estimate of the four component thermal conductivity:

$$\begin{aligned}
k = k_A / & \left(1 + 0.266 \left[1 + 0.633 (k_A/k_S)^{\frac{1}{2}} (M_A/M_E)^{\frac{1}{4}} \right]^2 \alpha (1-\alpha)^{-1} + 0.376 \left[1 + 0.633 (k_A/k_S)^{\frac{1}{2}} (M_A/M_E)^{\frac{1}{4}} \right]^2 (M_E/M_A)^{\frac{1}{2}} \alpha (1-\alpha)^{-1} \right. \\
& \left. + 0.376 (1 + M_A/M_H)^{-\frac{1}{2}} \left[1 + (k_A/k_H)^{\frac{1}{2}} (M_A/M_H)^{\frac{1}{4}} \right]^2 (\rho_H M_A) \left\{ (1-\alpha) \rho_A M_H \right\}^{-1} \right) \\
& + \left\{ 2.51 (M_E/M_A)^{\frac{1}{2}} k_S \right\} / \left(1 + 0.266 \left[1 + 1.58 (k_S/k_A)^{\frac{1}{2}} (M_E/M_A)^{\frac{1}{4}} \right]^2 (1-\alpha) \alpha^{-1} + 0.376 (1 + M_A/M_H)^{-\frac{1}{2}} \left[1 + 1.58 (k_S/k_H)^{\frac{1}{2}} (M_E/M_H)^{\frac{1}{4}} \right]^2 \right. \\
& \left. (\rho_H M_A) (\alpha \rho_A M_H)^{-1} \right) \\
& + (2.51 k_S) / \left(1 + 0.376 \left[1 + 1.58 (k_S/k_A)^{\frac{1}{2}} (M_E/M_A)^{\frac{1}{4}} \right]^2 (1-\alpha) \alpha^{-1} + 0.376 \left[1 + 1.58 (k_S/k_H)^{\frac{1}{2}} (M_E/M_H)^{\frac{1}{4}} \right]^2 \right. \\
& \left. (\rho_H M_A 1.37 \times 10^{-4}) (\alpha \rho_A M_E)^{-1} + 1.50 \right) \\
& + k_H / \left(1 + 0.376 \left[1 + (k_H/k_A)^{\frac{1}{2}} (M_H/M_A)^{\frac{1}{4}} \right]^2 (1-\alpha) \rho_A M_H (\rho_H M_A)^{-1} + 0.376 \left[1 + 0.633 (k_H/k_S)^{\frac{1}{2}} (M_H/M_E)^{\frac{1}{4}} \right]^2 \right. \\
& \left. (\alpha \rho_A M_H) (\rho_H M_A)^{-1} + 0.376 \left[1 + 0.633 (k_H/k_S)^{\frac{1}{2}} (M_H/M_E)^{\frac{1}{4}} \right]^2 (M_E/M_H)^{\frac{1}{2}} (\alpha \rho_A M_E 7300) (\rho_H M_A)^{-1} \right).
\end{aligned}$$

(A-43)

In contrast to viscosity, the unionized thermal conductivities of Argon and Helium are quite different, as reference to curves "A" and "B" of Figure A.4 will quickly illustrate. The interaction of Helium and Argon, as far as the transport of thermal energy is concerned, is therefore more complicated. The Helium lowers the temperature and ionization of the Argon, and also the thermal conductivity. However, this loss is more than compensated for by the increased transport of the heat flux by the Helium itself, and the final end result is an overall increase in the system thermal conductivity.

Reference to Figure A.4 shows that the Helium changes the slope of the curve and decreases the "sharp corner" that appears at about $10,000^{\circ}\text{K}$ of the pure ionized Argon thermal conductivity. Therefore, the presence of Helium, in regards to thermal conductivity, increases the value of this transport property and produces a more even transition from the state with no ionization to the state with ionization.

A.4.3 Prandtl Number

The most interesting and perhaps unexpected consequence of Helium contamination is exhibited in the Prandtl Number. As discussed earlier, the presence of ionization in a pure gas lowers the value of the Prandtl Number. However, as shown in Figure A.5, the presence of Helium gas in the system, appears to have the same effect.

The lowering of the Prandtl Number may be explained by a physical argument, again based on the difference between specific heat and transport properties.

The specific heat is a measure of the thermal energy which an atom or molecule can absorb. This capacity is independent of

the presence of other atoms or molecules. That is, the specific heat of an Argon atom is unaffected by the presence of Helium atoms. This is not the case with the transport properties. These are a function not only of the atom itself, but also the interaction of the atom with other particles. Consequently, the thermal conductivity, especially of Argon, is strongly affected by the presence of the Helium, and the result is a lowering of the value of the Prandtl Number.

A.5 Summary

It has been shown that the transport properties of a pure gas are affected by the presence of ionization. In the case of viscosity, the ionization lowers the overall value, while in the case of thermal conductivity, the value of the overall transport property is increased.

The contamination of the ionized Argon by Helium atoms additionally affects the transport properties of the mixture. The presence of Helium will lower the ionization of the Argon. The Helium produces the greatest increase in the thermal conductivity.

Finally, it is noted that the Prandtl Number is reduced due to the ionization effects on the viscosity and thermal conductivity and the difference in the way in which specific heat and the transport properties are affected by the mixture ratios.

Figure A.1. Thermal conductivity estimates of ionized Argon.

Figure A.2. Viscosity estimates.

Figure A.3. Estimates of Prandtl No. for Argon.

Figure A.4. Thermal conductivity for Argon-Helium mixtures.

Figure A.5. Estimates of Prandtl No. for Argon-Helium mixtures.

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APPENDIX B

EXPERIMENTAL TECHNIQUES

B.1 Introduction

Experimental investigations were carried out in the test section covering Pitot pressures, surface heat transfer rates on the inclined flat plate, and attempts were made to obtain detailed flow visualization with a Mach-Zehnder interferometer. In this Appendix, the measurement techniques that were used, and the data reduction methods employed will be described and evaluated.¹

Figure B.1 shows the general layout of the laboratory. The rear of the dump tank is seen in the center of the photograph. A beam splitter and cameras are on the left, and differential inputs and oscilloscopes are shown at the right.

B.2 Pitot Pressures

Pitot pressures were measured using the same basic technique that Stalker (1967) had employed in the initial testing of this particular shock tunnel. However, special care was taken to try and improve as much as possible the time resolution of the measurements.

The probe used employed a pressure transducer manufactured in the laboratory using a PZT-5 piezo electric element 3 mm. in diameter and 3 mm. thick, manufactured by Brush-Clevite Corp. It was mounted at the rear of a small cavity of approximately 1.27 mm. diameter and 0.5 mm. deep. The entrance to this cavity, which faced directly into the on axis flow, was about 0.9 mm. in diameter.

¹It should be mentioned that as a standard procedure the shock velocity and reflected shock pressure were measured each run.

The cavity was filled with about 10 milligrams of steel wool to act as a heat exchanger to suppress spurious voltages that were developed when the hot gas came into direct contact with the transducer.

The output of the gauge was fed to a cathode follower with an input impedance of approximately 10^7 ohms, and then to a type 2A63 differential input of a Tektronix 561 oscilloscope. Care was taken to ensure that all cables were of the low noise type and were as short as possible. Typical output records have already been illustrated in Chapter 3.

The gauge calibration was checked by suddenly exposing the transducer under vacuum to a pressure of one atmosphere. This was accomplished by placing a thin diaphragm over a hole in the dump tank system, with the input cavity of the transducer directly behind. The diaphragm was ruptured when the system was pumped down. This calibration yielded a figure of 2.4 ± 0.1 volts/atmosphere for the probe.

The initial response time of the probe system was estimated to be between 30-50 microseconds - this being the time for the small cavity to fill and come up to pressure. (The minimum time was realized with the small exit nozzle, while, because of the lower output density, the longer time was experienced with the large exit configuration.) After this, it was estimated that the probe could follow events within a time of about 10 to 20 microseconds.

B.2 Mach-Zehnder Interferometer

The interferometer used in the Australian National University Aerospace Laboratory is a Mach-Zehnder type manufactured by Zeiss. This unit was used in a two wave-length technique. That is, the

combined beam coming out of the interferometer optics was "split" via a partially silvered optical flat and then each of the two resulting beams were passed through different monochromatic filters and interferograms were then recorded on HP-4 sheet film.

The beam splitter, filter holders, and camera backs are shown in Figure B.2.

The light source for the interferometer was an Argon flash lamp. The time duration of a typical light pulse was nominally 10 to 15 microseconds. The lamp was triggered, after a certain time delay, from the reflected shock region pressure transducer. The accuracy of the lamp triggering was normally better than five microseconds.

In practice, photographs were taken just before the shock tunnel was fired and then again during the flow. Fringe shifts were calculated by comparing the "before" and "during" flow records. The time delay between these interferograms was normally about 30 seconds.

The theoretical work of Chapter 3 indicated possible nozzle exit densities of about 10^{16} to 10^{17} atoms/cc, depending on the nozzle configuration used. Using the Argon refractivity given by Alpher and White (1965), expected recorded fringe shifts over a path length of ten cm. could be the order of 0.03 to 0.4 in the optical frequencies. It should be noted that to add to the complications of measuring such small fringe movements are the problems of shot to shot reproducibility and any possible system drifts that may occur between the before and during flow records.

To achieve a satisfactory degree of accuracy, it was necessary to (1) put some sort of clear datum line on each photograph with which to reference all fringes, (2) to develop

some method of large scale data handling so that statistical methods could be applied to the data, and (3) to develop some method of estimating any possible optical drifts.

Requirements (1) and (3) were answered with two pieces of special equipment. The first was a rotatable straight edge which was placed at the rear of the field of view (rear of the plate). This unit was attached to the Mach-Zehnder and was completely independent of the Shock Tunnel system. The recorded image of this marker was used as the datum line for the fringe shifts.

The second item was an expanding channel with a small leak which sealed at each end against the windows of the test section. The interior of the pipe would pump down to the ambient dump tank pressure, and it would continue to remain at this level for a few hundred microseconds after the shock tunnel was fired. By comparing the position of the fringes in the channel on the before and during records, one could estimate the homogeneous optical drifts that might have occurred over the whole field of view. This method was not 100% effective (i.e. the drift was certainly not always homogeneous over the entire field of view), but it was normally easy to verify if a non-homogeneous drifting had occurred by inspection of the photographs.

Figure B.3 is a photograph of the straight edge and the channel. Figure B.4 is a typical interferogram.

Requirement (2) was the most difficult problem, and to solve it a computerized interferometric data reduction system ("CIDRS") was developed¹.

¹See Yanow (1970).

There are many approaches that could have been taken in the development of a computerized data reduction method. However, there is an inherent danger in complete automation; one can lose the "feel" of the data, and if the results obtained are unexpected, one may be at a loss to determine what, if anything, may be wrong. Accordingly, "CIDRS" was developed with the philosophy of keeping the experimenter at least a part of the system, insofar as some decision making.

"CIDRS" is composed of eight programs; two of these are considered optional and are used as tools for decision making. Briefly these programs are:

- (1) PICSCN digitizes the picture by scanning parallel to the surface of the plate from the rear (where the marker straight is located) to the leading edge, and downward toward the plate surface. Scan separation was 0.025 cm., and the reading separation was 0.0125 cm. Output is recorded on tape.
- (2) SCNPLT simulates microdensitometer plotting. This program (as well as all the others of "CIDRS") has the option for repeated three point data smoothing to filter out noise.
- (3) SCANRD "reads" the position of each maxima and minima of every densitometer scan. Output is recorded on tape.
- (4) SCNDIF studies the area about the region of the reference line (called STPNT) and is used in decision making.
- (5) PLTSCN studies the area just over the surface of the plate. This information is used in decision making when the reading co-ordinate system is reordered to have the x-axis origin at the leading edge and the y-axis origin at the plate surface.
- (6) STPNT determines the position of the datum line for each scan. Output is recorded in hexadecimal numbers on punched cards.

(7) ARRAYN determines the actual fringe movements, the fringe and scan co-ordinate system is reordered, and fringe movement data along the x-axis (along each scan) is averaged into five regions of 0.5 cm. length. Any optical drift is taken into account, and checks on the data are carried out for possible skipped readings, film scratches, etc. Output is recorded in hexadecimal numbers onto punched cards.

(8) GASSOL takes the data of ARRAYN and using a matrix least square solution (IBM SSP subroutine LLSQ) determines the gas composition of the flow. The fringe shift data is also examined and statistically studied to determine such quantities as standard deviation.

To get an estimate of the overall practical accuracy of "CIDRS", the following test was carried out: The dump tank system was pumped down and a "before" flow interferogram was taken. A specified amount of Argon was quickly leaked into the system, and another interferogram was recorded. This process was repeated many times with different amounts of Argon and using different wave lengths.

The fringe movements were averaged on each photograph into 200 different sub-regions, where each of these would have been built up from three to five fringe movement readings. If "CIDRS" was perfect, each of the sub-regions for each photograph would have exactly the same reading. In this way, the overall accuracy of determining the position of the datum line, fringe movement, and all other needed information was determined as a whole.

From the rough readings of the manometer used to read the input Argon, upper and lower limits could be set on the expected fringe shifts. If the test readings fell outside of these limits, due to film imperfections or some other cause, they were excluded

from the statistical analysis. Table B.1 is a summary of eight such tests.

Taking averages of Table B.1, it was determined that just under 6% of the total data could be expected to be bad and rejected. Of the other 94% of the data, an average reading accuracy of ± 0.04 fringe could be expected. This limit represents Argon densities from 1.4×10^{16} atoms/cc at $4000 \text{ \AA}^0$ to 2.7×10^{16} atoms/cc at $7000 \text{ \AA}^0$. This limit is slightly larger than the lowest expected densities, which would come from the large exit nozzle (15° half angle), but this figure is below the smallest expected density after passing through the oblique shock produced from the tip of the inclined flat plate.

B.3 Flat Plate Surface Heat Transfer Rates

One of the most successful and best documented methods of determining heat transfer rates is with the use of thin film gauges. A strip of very thin (the order of one micron) platinum is deposited on some backing material, usually pyrex glass. The heat capacity of the film is very small, so in a very short time, approximately a microsecond, the gauge will assume the surface temperature of the glass. Consequently, by using the film as a resistance thermometer¹ a very accurate surface temperature history can be recorded.

In this work, the thin film was obtained by painting a narrow line of Hanovia O5 solution across a pyrex glass insert, which was nominally one cm. wide. After firing at approximately the annealing temperature of the glass (600 to 700°C) for about thirty minutes, the strip of solution was converted into a thin platinum gauge. In practice, to help produce greater durability

¹A small current is passed through the film and the voltage changes which occur as the gauge resistance alters are directly proportional to the surface temperature.

RUN	SUB-REGIONS EXCLUDED	MEAN READING	STANDARD DEVIATION
1	0	0.625	0.060
2	22	1.121	0.035
3	17	1.201	0.052
4	10	0.115	0.035
5	4	0.894	0.028
6	6	0.683	0.036
7	26	0.821	0.038
8	5	0.730	0.043

TABLE B.1

TEST OF "CIDRS" PRACTICAL ACCURACY

in light of the high erosion of the shock tunnel flows, the thin film was built up by overpainting. Three coats produced a gauge that would last for up to ten shots while still having a fast response.

It has become common practice to coat the gauges with a thin layer of some dielectric material to protect the gauge from possible electrical shorting by the free electrons in the flow. A vacuum coating of SiO , which is converted to SiO_2 has proved adequate. Alternatively, the gauges may be insulated with another Hanovia product, Hanovia Blue Luster solution. This material is fired into the glass in the same manner as used in the original formation of the thin film. Both methods have been employed in this investigation. The Hanovia Blue Luster has the advantage of ease of application. Once more, to secure durability, three coats of the solution were in turn fired into the glass.

Silver conducting paint was used to effect a conducting path going from the ends of the gauges (a small length at the end of each thin film was left uncoated) to the sides of the glass insert. Leads were then silver soldered to this paint, and then these were attached to a set of "vacuum terminal connections" at the back of the dump tank.

The output from each end of the respective gauges was fed to a type RM122 differential Tektronix input, and the temperature changes of the gauge occurring during the flow were recorded on an oscillogram. A sweep of twenty microseconds/division was used. Figure B.5 shows the gauge circuit.

In an attempt to minimize any systematic errors, gauges with resistances of approximately 10 to 150 ohms were constructed and used. Gauge currents were varied from about 1.5 to 9 milliamps.

Figure B.6 is a picture of the flat plate with a gauge system inserted. The glass was held in place using a silicon rubber potting material.

A statistical approach was also employed in obtaining the heat transfer data. Each glass backing was composed of four gauges situated at distances from one to three centimeters back up the plate from the leading edge. Normally, at least ten oscillograms were recorded at each gauge position, using at least two different gauge system inserts. Such readings were repeated at plate inclinations of 9.5° and 19.5° for each of the two nozzle configurations. The oscilloscopes were again triggered by the reflected region pressure transducer, and all times were referenced to the start of this pressure reading.

The oscillograms were read at 71 points from the start of each temperature trace. (Readings were made every two microseconds.) This could read between 150 to 160 microseconds after the start of the reflected shock region pressure, depending on the nozzle starting processes. Typical examples of heat transfer oscillograms are shown in Chapter 4.

To facilitate the reading of the large number of traces, the oscillograms were projected onto a calibrated screen and the readings were recorded via voice on a tape recorder. The information was later transferred to punched computer cards. Nominal time to read each oscillogram was only a few minutes using this technique.

The gauges were calibrated (i.e. the parameter β as defined below, was evaluated) using a standard pulsing technique employing a constant current source¹. That is, the gauge itself

¹See Hartunian and Varwig (1961).

was used as a surface heating element with a known I^2R heat output. As the temperature of the surface glass increased, so did the resistance of the thin film, and the change in voltage across the gauge was monitored.

The circuit used in the calibration is shown in Figure B.7. Resistors R_1 and R_2 were used as current limiters. C_1 was composed of five 850 micro-farad electrolytic capacitors in parallel, and S_1 was a polar relay. C_2 was chosen so that when S_2 was closed, the polar relay in turn closed, but would reopen in 20 milliseconds. The differential voltage change across the thin film was monitored and recorded. The current output of the $C_1 R_2$ system was also recorded. There was a slight "droop" in the current pulse due to the finite capacity of the condenser bank, and this small correction was applied to the calibration calculations.

It was found that the calibration of the gauges could change during the operation of the shock tunnel, attributed to the high erosion of the gas flow. Consequently, gauges were normally used for from four to six shots, and then they were removed and re-calibrated. The value of β used was based on an average of the two calibrations.

The resistance and current of each gauge was monitored just before each shot. When the gauge resistance had increased by about 20%, it was generally found that the signal became rather noisy. At this time, a new gauge system would be installed and the old one re-calibrated. Calibration figures for β were in the range of 0.05 to 0.07 ($\text{cal/cm}^2\text{-}^\circ\text{K-sec}$).

Carslaw and Jaeger (1959) studied the problem of heat transfer into a semi-infinite body. It is quite reasonable,

considering the time scales involved, to assume the glass backing is such a body, and thus the heat transfer rate, q , at time t is given in terms of electrical parameters by (e.g. Meyer (1960))

$$q(t) = \frac{\beta}{\pi \alpha E_f} \left[\frac{E(t)}{t^{1/2}} + \frac{1}{2} \int_0^t \frac{E(t) - E(\tau)}{(t - \tau)^{3/2}} d\tau \right], \quad (B-1)$$

where

E_f = initial gauge DC voltage drop,

α = temperature coefficient of resistance of film,

based on calibration at low and high temperature within range of interest.

$$\beta = (\pi k \rho c_p)^{1/2},$$

$E(\tau)$ = voltage reading across the gauge at time τ ,

with τ falling in the interval 0 to t .

Cook and Felderman (1966) have put equation (B-2) into a very convenient form for numerical integration on digital computers, namely

$$q_n(t) = \frac{\beta}{\pi \alpha E_f} \left[\frac{E(t_n)}{t_n^{1/2}} + \sum_{i=1}^{n-1} \left\{ \frac{E(t_n) - E(t_i)}{(t_n - t_i)^{3/2}} - \frac{E(t_n) - E(t_{i-1})}{(t_n - t_{i-1})^{3/2}} + 2 \frac{E(t_i) - E(t_{i-1})}{(t_n - t_i)^{1/2} + (t_n - t_{i-1})^{1/2}} \right\} + \frac{E(t_n) - E(t_{n-1})}{(\Delta t)^{1/2}} \right], \quad (B-2)$$

where the interval 0 to t is divided into n equal divisions, separated by time steps of Δt .

If the heat transfer flux to the surface was a step function, the history of the voltage (or temperature) change across the gauge would go as time to the one-half power. In practice, a step input does not usually seem to be the case, and accordingly higher order terms have been included - i.e. the voltage history has been taken to go as the following:

$$E(t) = a_1 t^{\frac{1}{2}} + a_2 t + a_3 t^{3/2} + a_4 t^2 + \dots \quad (B-4)$$

The data reduction of the heat transfer records was carried out in two steps. Initially, all the sets of 71 readings for a particular gauge position, plate, inclination, and nozzle configuration, were read into a program called QRTFIT. This program would take each set of points and systematically fit the information to firstly equation (B-3) to the $t^{\frac{1}{2}}$ power, then secondly fit the same data to a polynomial to the t power, up to the t^4 power (eight different fittings). Additionally, for each fitting the statistical estimators of standard deviation and standard error, along with a statistical "F" test, were calculated. This information was stored on magnetic tape.

The second step of the data reduction was carried out by program QRATE. This program would examine the statistical data as calculated above, and use this information to pick the best order polynomial fit. This could range in practice from anything between an occasional $t^{\frac{1}{2}}$ to $t^{7/2}$ power. Then using the smoothed results of each best fit in turn, the time resolved heat transfer rates, via equation (B-2) were calculated. Standard

deviations for each time increment were also determined, and finally the results were averaged into overlapping time steps of twelve micro-seconds with the inclusion of statistical information to determine how constant the heat transfer rate had been over the averaged time step.

Using the above technique, it was possible to ascertain how accurately one could, in practice, obtain heat transfer information, including all the errors introduced by shot to shot variations of flow, differences in gauges, and any other cause. It was also possible to determine the most common time of most constant heat transfer rates and many other bits of statistical information which aided in the total evaluation of the data. It was found that over a series of shots, one usually obtained a total standard deviation for the heat transfer data of about 25%.

Figure B.1. Photo of Laboratory

Figure B.2. Beam splitter, filter holders, and camera backs used in two wave-length interferometry.

Figure B.3. Straight edge and drift movement channel used in conjunction with "CIDRS".

Figure B.4. Typical interferogram, $6000 \text{ \AA}^\circ$, magnification of print is approximately 2.

R_g -gauge; capacitance in micro-farads; resistance in ohms.

Figure B.5. Thin film gauge circuit used in the heat transfer measurements.

Figure B.6. Flat plate with thin film gauge system inserted.

Figure B.7. Circuit used in thin film gauge calibration.

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SYMBOLS AND NOMENCLATURE

The prime definitions of letter and Greek symbols, subscripts, and special terms used repeatedly in this work are listed below. In certain instances, the item may be given a different meaning in the text. When this does occur, it will be clearly indicated. When the symbol represents a mathematical expression, the defining equation in the text will be shown.

LETTER SYMBOLS

A	$(Le-1)h_1 \left(\frac{d\alpha}{dh} \right)_p$, see equation (2-9); also area
a	Speed of sound
C	Constant in linear viscosity relation
C_1	$Y Pr/c_p$, see equation (2-14)
C_2	$(1+A)/c_p$, see equation (2-14)
C_3	$(1-Pr)/c_p$, see equation (2-14)
C_4	Y/c_p , see equation (2-14a)
C_i	ρ_i/ρ , species mass fraction
c_p	$\sum_i C_i c_{pi}$, frozen specific heat at constant pressure
D_i	Coefficient of diffusion of species i through mixture of species
E	Symbol for power of ten in computer type print out, e.g. $10^x = Ex$
F	See equation (3-30)
f^*	u/u_e , see equation (2-9), $f = \int_0^\infty df/dy$
g	$(u^2/2 + h)/H_s$, see equation (2-9)
H	$u^2/2 + h$, total enthalpy
h_i	Specific enthalpy of species i, $\int c_{pi} dT + h_i^0$
h_i^0	Heat of formation of species i
I	Ionization energy of Argon, 9026 cal/gm
I_c	Special constant in composite boundary layer calculation, see equation (4-8)

k	Thermal Conductivity: Boltzmann's Constant
K_1	$(Pr_w/c_{pw})/C_1$, see equation (2-15)
K_2	$(Pr_w/c_{pw})/(Y C_2)$, see equation (2-15)
K_3	$(Pr_w/c_{pw})/(H_S C_2)$, see equation (2-15)
K_4	$(Pr_w/c_{pw})/C_4$, see equation (2-15a)
K_E	Equilibrium constant for Argon, see equation (4-4)
K_I	Argon reaction rate coefficient for ionization, see equation (4-3)
K_L	$(c_{pw}/c_p) Le \alpha_e$, see equation (2-15a)
K_R	Argon recombination rate coefficient, see equation (4-5)
K_T	$(Pr u_e^2)/T_e c_p$, see equation (2-15a)
Le	$D_i \rho c_p/k$, Lewis Number
M	Mach number; molecular mass
m	Mass
n	Number of moles
N_e	Electron density
P	Pressure
Pr	$\mu c_p/k$, Prandtl Number
$Q, q, -q$	Heat flux
R	R_o/M
R_o	Universal Gas Constant
Re	$\rho u x/\mu$, Reynold's Number
S	Entropy
S_i	α/α_e , see equation (2-13)
T	Temperature
t	time
t_c	Characteristic time, see equation (4-8)
u	Velocity along x axis in shock co-ordinate system
V	Volume
$\bar{V}$	$M (C/Re)^{1/2}$, rarefaction parameter

v	Velocity along y axis in shock coordinate system
$\dot{w}_i$	Mass rate of change of species i per unit volume
x_i	ρ_i/ρ , mass fraction of species i
x_p	Distance along flat plate measured from leading edge
Y	$\rho k/\rho_w k_w$, see equation (2-9)
Z_1	Partition function for first ionization level of Argon
Z_0	Partition function for ground level of Argon

GREEK SYMBOLS

α	Ionization fraction
β	Angle between plate surface and nozzle axis
β_E	Temperature of excitation of Argon, see equation (4-3)
γ	c_p/c_v , ratio of specific heats
γ_{eff}	Effective value of γ , see equation (3-10)
δ	Divergence angle of nozzle flow
δ^*	Displacement thickness, see equation (2-16)
ϵ	Energy of Argon ionization, 2.524×10^{-11} ergs/molecule
η	See equation (2-6)
θ	Angle of plate inclination
θ_I	Temperature of Argon ionization, see equation (4-3)
μ	Viscosity
ξ	See equation (2-7)
ρ	Density
$\bar{X}$	$M^3 (C/Re)^{\frac{1}{2}}$, leading edge interaction parameter

SUBSCRIPTS

A	Refers to Argon
AI	Refers to Argon ions
AN	Refers to neutral Argon atoms
e	Boundary layer free stream conditions
H	Refers to Helium
N	Conditions at nozzle exit
S	Stagnation conditions
s	Stagnation conditions; Spitzer component of thermal conductivity.
w	Conditions at the surface of the flat plate or wall
*	Sonic conditions

SPECIAL TERMS

composite boundary layer	A mating of an equilibrium and frozen boundary layer solutions to estimate the non-equilibrium case.
equilibrium	A gas in which the temperature and ionization follow the Saha equation. Also thermochemical equilibrium.
frozen	A gas in which no chemical reactions occur during the time of interest.
large exit	Refers to flow with nozzle having an exit area ratio of 1204 and with a 19.5° plate inclination.
small exit	Refers to flow with nozzle having an exit area ratio of 147 and with a 9.5° plate inclination.
transport parameters	The special terms defined in equations (2-14) and (2-14a).
transport properties	Thermal conductivity, viscosity, and diffusion (ambipolar).
very high enthalpy	Flows with stagnation enthalpies in excess of 12 eV or approximately 7000 cal/gm.

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