THE DYNAMICS OF GALACTIC BULGES

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DOCTOR OF PHILOSOPHY

by

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To my parents
The work reported in this thesis is that of the candidate alone, except where acknowledged in the text.

Brian Jarvis
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ABSTRACT

A detailed theoretical and observational study has been made of the structure and dynamics of the bulges of disc galaxies. Self-consistent, axisymmetric rotating models with two integrals of motion, energy and angular momentum, have been constructed for the bulges of the edge-on disc galaxies NGC 7814, 4594 and 7123. Accurate surface photometry and kinematical observations have been combined to show that the bulges of these disc galaxies are consistent with oblate, rotationally flattened models (unlike the brighter elliptical galaxies). The determination of $V_m/\sigma$ values for NGC 4762, 4179 and Ham I, also confirm this finding. Under the assumption of constant $M/L$, the models accurately reproduce the observed two-dimensional surface brightness distributions and the observed kinematics, to the limits of the data. $M/L$ values are derived for the three galaxies for which models were constructed, and are found to be consistent with recently observed $M/L$ values for elliptical galaxies ($M/L_\nu = 7$ to 10).

There are two extremes of bulge morphology in the sample. At one extreme are the peanut- and box-shaped bulges, which rotate cylindrically (i.e., the rotational velocity depends only on cylindrical radius): the type example is NGC 128. At the other extreme are the spheroidal bulges (like NGC 7814) which rotate non-cylindrically, and which have been successfully modelled. We discuss the dynamical differences between these two extreme bulge morphologies, and their different evolutionary histories. Stronger radial colour gradients are predicted for the more spheroidal bulges, and this is observed. There also appears to be a difference in the globular cluster populations of spheroidal and peanut bulges; the peanut
bulges may be deficient in globular clusters. The implications of this difference, for globular cluster formation, are discussed briefly.

The thick disc components of disc galaxies, first identified by Burstein, have been modelled as the response of the bulge component to the flat potential of the disc. For the SO Galaxy NGC 4762, the models give an excellent representation of the detailed photometric data.
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CHAPTER I

INTRODUCTION AND THESIS OUTLINE

1.1 INTRODUCTION

Galaxies are the fundamental building blocks of the universe and hence have long been objects of great astronomical interest. The diversity of their forms is great, from elliptical galaxies with a largely symmetrical smooth distribution of luminosity, to highly irregular galaxies with no apparent order. Valuable clues to their evolutionary history can be gleaned from an understanding of their current dynamical state. For simplicity, it has been natural to look firstly at those systems which exhibit apparent axial symmetry, i.e. elliptical galaxies and the bulges of disc galaxies. Detailed observations over the last few years have allowed us to begin to understand what elliptical galaxies really are. However, it is only relatively recently that similar observations have been made for bulges. It is this area which is the principal aim of this thesis; to gain an understanding of the current dynamical state of the bulges of disc galaxies. An attempt will be made to achieve this by a suitable comparison between numerical models and real observational data. However, before describing the procedures to be followed in this task, a discussion of previous work in this area is presented.

1.2 DISCUSSION OF PREVIOUS WORK

The conventional picture of a galactic bulge has been that of an elliptical galaxy co-existing in the middle of a disc. In view of this and the pitfalls inherent from the lack of suitable observational data
in formulating an accurate dynamical model capable of reproducing all the available data, it is instructive briefly to review the history of our understanding of elliptical galaxies.

The history of the study of elliptical galaxies can be naturally divided into two epochs. The first, prior to 1975 was hampered by lack of suitable observational material adequately to restrict the range of possible models. The most easily measured observable parameter was the apparent light distribution. The only kinematical data available were a small sample of central velocity dispersions (Minkowski, 1962; Burbidge et al., 1961a,b,c) which were subsequently shown to be systematically high (Richstone and Sargent, 1972; Morton and Thuan, 1973; Faber and Jackson, 1976). This data alone was unable to discriminate between oblate, prolate or triaxial systems. Nevertheless, models were constructed assuming oblate spheroids with an isotropic velocity dispersion. This was prompted by their flattened appearance, and it was believed that collapse of the rotating protocloud proceeded most rapidly along the axis of rotation (minor axis). Simple models built on this sparse observational background were constructed by Lynden-Bell (1962), Prendergast and Tomer (1970) and Wilson (1975). These models gave the illusion that the morphology and dynamical properties of ellipticals were well understood. However, a clue to the incorrectness of these rotationally flattened models was found by Wilson, although he explained this in terms of the inadequacy of the distribution function he chose.

Wilson found that not only could he not make models flatter than E4 with radial intensity profiles which matched those of real galaxies, but in addition these models flatter than E4 violated the Ostriker-Peebles (1973) stability criteria against non-axisymmetric perturbations.
The dynamical basis of these models used by Prendergast and Tomer and Wilson was expressed in terms of the distribution function of the stellar integrals of motion, with the solution for the model spatial characteristics being well established from the work of many authors (eg. Chandrasekhar, 1943, 1960; Woolley, 1954; Woolley and Robertson, 1956; Woolley and Dickens, 1961; Spitzer and Härm, 1958; Oort and von Herk, 1959; Michie, 1963a,b; Michie and Bodenheimer, 1963; King, 1966).

Other models for elliptical galaxies were also devised, notably those of Gott (1975), involving the dissipationless collapse of the galactic protocloud, and the hydrodynamical models of Larson (1975) with dissipation. However, all of the authors constructing models for elliptical galaxies did not take account of some important observations which already existed at the time. The most enlightening of these, with hindsight, was the change in position angle with radius of the elliptical isophotes (eg. Evans 1951, and Liller 1960, 1966). This phenomenon has only recently received attention again, by Williams and Schwarzschild (1978), Bertola and Galletta (1980) among others, and has two interpretations: either the galaxy has isodensity surfaces which are spheroids with axes of different orientations, or more probably, these surfaces are triaxial ellipsoids which have the same orientation, but whose axes have different ratios. The isophote twisting is then a projection effect.

The second epoch in the understanding of elliptical galaxies was marked by the observations of Bertola and Capaccioli (1975), Illingworth (1977) and Schechter and Gunn (1979). They were able to show that the bright ellipticals have rotational velocities typically only about one-third of those required by the rotationally flattened models. Further, Schechter and Gunn (1979) and Bertola and Galletta (1978) subsequently
showed non-vanishing mean rotation on the projected minor axis of some ellipticals. All of these observations have led to the discarding of the classical model in which the flattening is due to rotation alone. The conclusion is that, if the elliptical galaxies are oblate, their flattening is due to a global velocity anisotropy, or, if they do have isotropic velocity dispersions, they must be prolate and rotating end over end. The general case of triaxial ellipsoids is also consistent with this picture.

Given then the long standing view of the similarity in morphology (Sandage, 1961), radial profile shape (de Vaucouleurs, 1959), stellar content, including the $L \propto \sigma_0^4$ relationship between luminosity $L$ and central velocity dispersion $\sigma_0$, and other basic properties (Freeman, 1975; Faber and Jackson, 1976; Sargent et al. 1977 and Whitmore, Kirshner and Schechter, 1979) between ellipticals and bulges of disc galaxies, it was logical to enquire whether the bulges are also supported in the same way as ellipticals. However, relatively recent data (Kormendy and Bruzual, 1978; Kormendy, 1980b,c) have permitted these similarities to be examined in more detail, with the result that some small but significant differences do exist.

(i) The first of these is the shape of the minor axis luminosity profile. Whereas ellipticals (except possibly for the core regions) are well approximated by a de Vaucouleurs $r_k^4$ law (eg. de Vaucouleurs and Capaccioli, 1979) to the limits of available photometry, deviations from the $r_k^4$ law are not unknown on the minor axes of galactic bulges. Notable among these is NGC 4565 (Kormendy and Bruzual, 1978; Jensen and Thuan, 1981; Spinrad et al., 1978; Hegyi and Gerber, 1977). All these authors have found an excess over the $r_k^4$ law in the outer parts; the profile consists of two distinct components, the inner being much
steeper than the outer. However, before one concludes that the bulge is showing deviations from the $r^{-k}$ law, it must be determined whether or not this outer segment is the same dynamical component. In the case of NGC 4565, there is some doubt because of an abrupt increase in colour at the position where the slopes of the luminosity profiles change (from $B-V = 1.0$ for $r < 60''$ to monotonically increasing for $r > 60''$, Jensen and Thuan, 1981). In addition, the change in slope occurs where the bulge isophotes are most box-shaped. Comparison of M/L values between the inner and outer profiles would be very useful in disentangling this problem.

Tsikoudi (1977) noted deviations of a slightly different kind in a program of surface photometry of the SO galaxies NGC 3115, 4111 and 4762. As for NGC 4565, she also found an excess over the $r^{-k}$ law on the minor axis of NGC 4762. However, here at least, this is due to the presence of the thick disc (Burstein, 1979), shown in Chapter VI to arise naturally where a bulge potential field co-exists with a flat disc potential. In contrast, NGC 4111 shows a deficit of light below the $r^{-k}$ law, beyond $r \sim 15''$. This may be caused by the rapid rotation of the bulge which is distinctly peanut-shaped. Finally, it should be noted that there are indeed many bulges which do follow the $r^{-k}$ law closely; there is a real need to understand how and when the deviations occur.

(ii) There is also an increasing body of evidence suggesting that there are differences in the shapes of elliptical galaxies and the bulges of disc galaxies. No elliptical galaxy is known which shows a clear box or peanut-shape morphology. The best example to date of the latter kind is the edge-on SO galaxy NGC 128, studied in this work (see Figure 5.1). Kinematical observations reject the possibility that the bulge of this galaxy is akin to an elliptical (see Chapter V); no elliptical galaxy
approaches the kinematics of the box-shaped bulges (e.g. NGC 1381). These types of bulge are now believed to be dynamically distinct from elliptical galaxies and even from some bulges of disc galaxies (see Chapter V). Several authors (de Vaucouleurs, 1974; Kormendy and Illingworth, 1980; Hamabe et al. 1980a) have also suggested that in the mean, bulges are flatter than ellipticals, which have a mean shape \( \sim E^4 \) (Sandage, Freeman and Stokes, 1970). This finding was supported by de Vaucouleurs (1974) who found from a study of bulges in the range Sa - Sc that they were typically \( E^4 - E^5 \) compared to \( E^3.5 \) for ellipticals.

(iii) Without doubt, the most significant evidence against the similarity of elliptical galaxies and the bulges of disc galaxies comes from recent kinematical observations (Illingworth, 1977; Kormendy, 1980a; Kormendy and Illingworth, 1981, and this work). From observations of rotation rates in the bulges of edge-on galaxies away from the disc, it is now clear that they are rotating much more rapidly than the bright elliptical galaxies. Moreover, they are qualitatively consistent with the rotating oblate-spheroid models of Binney (1978). Even before these recent observations, it was known that some bulges rotate rapidly, notably the central region of M 31 (Babcock, 1939; Rubin et al. 1973; Pellet, 1976 and Richstone and Schectman, 1980). Bertola and Capaccioli (1977) had also observed significant rotation in the bulge of NGC 128.

Evidence for bulge rotation also comes from rotation curves on the major axis of SO galaxies, from the inner parts where the bulge dominates the disc (NGC 4762, Bertola and Capaccioli, 1978; NGC 3115, Williams, 1975; Rubin, Peterson and Ford, 1980, and others). For example, Tsikoudi's (1977) decomposition of the major axis luminosity profile of NGC 3115 shows that the bulge light dominates to at least 6"
from the nucleus by which the rotation curve as observed by Williams has risen to 160 km.s\(^{-1}\); this is well in excess of the rotation for most elliptical galaxies. Similarly, for NGC 4762 the major axis is bulge-dominated to a distance of about 10" (Bertola and Capaccioli, 1978), by which the rotation curve has risen to about 80 km.s\(^{-1}\). In summary, even at the very small distances where the disc starts to dominate the luminosity profile on the major axis, the rotation curve velocity measured from the bulge has risen in most cases to values above those commonly seen in ellipticals.

1.3 THESIS OUTLINE

It is now clear that recent data have shown significant photometric and kinematic differences between elliptical galaxies and the bulges of disc galaxies. In particular, there is much evidence for a high degree of rotational support in bulges. Can the observed two-dimensional photometric and kinematic properties of galactic bulges be adequately modelled via rotationally flattened oblate models? This is the subject of this thesis. A small sample of edge-on disc galaxies with large apparent bulges was chosen for detailed study and comparison with models. A wide range of bulge morphologies were selected in an attempt to understand the origin of this range. The data were used as a direct test for the models.

Chapter II of this thesis gives a detailed description of the theoretical models; they are based on a lowered Maxwellian velocity distribution function, with an angular momentum term. The models are axisymmetric, time-independent and self-consistent (each star moves in the smooth potential field of all other stars). The effect on the bulge of an imposed flat disc potential is also included. This was invoked to explore the origin of thick discs, recently observed in edge-on SO
and spiral galaxies (e.g., Burstein, 1979; van der Kruit and Searle, 1981). The chapter concludes with an outline of the numerical techniques used in constructing a model of a particular bulge. The necessary dynamical checks on the integrity of the models are also described.

Chapter III describes the observational and data reduction techniques used to acquire the data in a form suitable for comparison with the models. Comparison is also made with standard photometric and kinematic galaxies to test these techniques.

In Chapter IV, models are constructed for three of the six program galaxies with spheroidal bulges, NGC 7814, 4594 and 7123. In general, the agreement between the model predictions and the observations is excellent for both the two-dimensional surface brightness and the kinematics. The dynamical significance of the final adopted models is interpreted with the aid of the $V_m(o)/\sigma o \sim \epsilon_{\text{max}}$ diagram, where $V_m(o), \sigma o$ and $\epsilon_{\text{max}}$ are the maximum rotation velocity of the bulge on the major axis, the central velocity dispersion and the maximum ellipticity of the bulge isophotes respectively. The remaining three galaxies in the sample, NGC 4762, 4179 and Ham I are also discussed here. The mass-to-light ratios are computed for those galaxies with a fitted model.

Chapter V contains a discussion of the cylindrically rotating bulges. There is a growing body of evidence to suggest that, kinematically, bulges fall into two distinct classes, the cylindrically rotating and non-cylindrically rotating bulges. It is suggested that the nature of the rotation is revealed by the morphological form of the bulge; the box and peanut-shaped bulges are all cylindrically rotating and the spheroidal bulges are all non-cylindrically rotating. The implications
for galaxy formation are also discussed in terms of the density distribution of stars in the energy-angular momentum plane. Finally, a possible correlation between the presence of a halo globular cluster system and the bulge morphology is also discussed.

In Chapter VI, the dynamical origin of thick discs is investigated, from the assumption that they result from the response of the bulge material to the flatter potential field of the disc (Freeman, 1977). The success of the models is demonstrated by constructing a model for the thick disc of NGC 4762 based on Tsikoudi's (1977) photometry.
DYNAMICAL MODELS OF GALACTIC BULGES

CHAPTER I I

2.1 INTRODUCTION

In this chapter a set of dynamical models for the bulges of disc galaxies are described. The models are axisymmetric and are based on a distribution function of the two isolated integrals of motion, energy and angular momentum. The bulge models have non-zero total angular momentum, and are spatially truncated by an energy cutoff.

The theory of the models and the assumptions on which they are based are discussed in section 2.2. The numerical schemes for constructing the models are detailed in section 2.3. In section 2.4, two numerical checks are described for testing the models and section 2.5 briefly discusses the projection of the models on to the plane of the sky for comparison with observation. Finally, in section 2.6, the assumptions on which the models are based are reiterated and the observational data required to investigate their validity discussed.

2.2 THE MODELS

Recent observations (Bertola and Capaccioli 1975; Illingworth 1977; Schechter and Gunn 1979) have shown that elliptical galaxies are probably not simple oblate systems flattened by rotation alone (see Chapter I). However, it is becoming clear from the work of Illingworth and Schechter (1981) and Kormendy (1981) that the bulges of disc galaxies are rapidly rotating. In view of this, it would seem worthwhile to examine the application of the rotationally flattened oblate stellar dynamical models (Prendergast and Tomer 1970, hereafter PT; Wilson 1975) to these systems. This kind of model is self-consistent and time independent: the construction
begins with our assumed functional dependence of the distribution function on the isolating integrals of the motion and then solves for the corresponding stationary density distribution and gravitational potential fields. (This is not the only way that one could proceed. Lynden-Bell (1962) demonstrated that one can in principle find a distribution function for any axially symmetric system, given its density distribution. This procedure to date has been not much used, because of technical difficulties.)

The disc of a disc galaxy provides a significant contribution to the total (i.e. bulge + disc) potential, and should not be ignored. In this study the disc potential is represented as a static external potential. The bulge is modelled in a self-consistent manner with every bulge star moving in the collective potential field set up by the stars of the bulge itself plus imposed potential of the disc. The inclusion of the disc was found to model more realistically the potential in which the bulge stars move (see Chapter VI). A further refinement to the models would be to include the potential field generated by the massive halo of the type invoked by Ostriker, Peebles and Yahil (1974) to account for the flat rotation curves of spiral galaxies. Massive haloes were not included here, although their effect would be very easy to include at a later stage. They were omitted because the main interest of this thesis is the question of rotational flattening. In the observable parts of the bulge, the bulge potential almost certainly dominates the halo potential field; the effect of the halo field would just be to deepen slightly the potential well in which the bulge exists, necessitating only a small adjustment of the fitted bulge models. For most of the galaxies in the sample, the rotation data does not extend far enough to define the halo potential adequately. Inclusion of the halo at this stage would simply add to the number of parameters, without contributing significantly to the
essential dynamics of the situation.

(\textit{i}) \textit{The Bulge}

In the absence of a formation theory for galaxy formation rigorous enough to predict reliably the expected distribution function, the choice remains largely arbitrary. This leads to a large number of plausible distribution functions satisfying the collisionless Boltzmann equation. In view of this and the success in applying King models (King 1966) to globular clusters and some ellipticals, it is assumed that the distribution function for the bulges of disc galaxies has the separable form:

\begin{equation}
\begin{split}
    f(E,J) &= a [\exp(-\beta E_0) - \exp(-\beta E)] \exp(\gamma J) \\
    \text{where } E &< E_0 \text{ is constant and } \alpha, \beta \text{ and } \gamma \text{ are constants.}
\end{split}
\end{equation}

The two integrals of motion, \( E \) and \( J \), are the energy per unit mass and the component of angular momentum per unit mass parallel to the axis of symmetry. In spherical polar co-ordinates \((r, \theta, \phi)\) with corresponding velocities \( v_r, v_\theta, v_\phi \), these integrals are:

\begin{equation}
\begin{split}
    E &= \frac{1}{2} (v_r^2 + v_\theta^2 + v_\phi^2) + U(r, \theta), \\
    J &= r \sin \theta \ v_\phi.
\end{split}
\end{equation}

\( U(r, \theta) \) is the total gravitational potential at \((r, \theta)\). Lowered Maxwellian distribution function systems with no mean rotation (i.e. \( \gamma = 0 \)) are King models.

The density at any point in the model is found by integrating \( f(E,J) \) over velocity space, i.e.

\begin{equation}
\begin{split}
    \rho(r, \theta, U) &= \iiint f(E,J) \, dv_r \, dv_\theta \, dv_\phi. \\
    \text{(b)}
\end{split}
\end{equation}

Then a self-consistent model in which each star moves in the smoothed collective gravitational field of all the other stars is constructed by solving the (non-linear) Poisson's equation:
\begin{equation}
\n\nabla^2 U = 4\pi G\rho(r, \theta, U).
\end{equation}

Most of the numerical effort goes into finding the solution to this equation.

Substituting equation (a) into equation (b) and performing the \( v_r \) and \( v_\theta \) integrations, one obtains:

\begin{equation}
\rho(r, \theta, U) = \frac{2\pi a}{\beta} \int_{-v_c}^{v_c} \left[ \exp(-\beta \xi) + \beta \xi \exp(-\beta E) - 1 \right] \exp(\gamma \nu \phi \sin \theta) d\phi
\end{equation}

where \( \xi = \frac{1}{2} v^2 + U \) and \( v_c = (-2U)^{\frac{1}{2}} \) is the local cutoff velocity.

Thus \( U = 0 \) defines the boundary of the model and externally to this, \( \rho(r, \theta, U) = 0 \) for all \( U(r, \theta) > 0 \). The constants in equation (d) are now eliminated by introducing dimensionless variables with suitable scale factors.

Aside from those cases where there is no chance of confusion, let an asterisk subscript distinguish dimensional quantities from their dimensionless equivalents. In this notation, equation (c) becomes:

\begin{equation}
\nabla^2 U^*_*(r^*_*, \theta^*_*) = 4\pi G\rho^*_*(r^*_*, \theta^*_*, U^*_*)\).
\end{equation}

Now make the change of variables: \( W = -\beta U^*_* \rho = \left( \frac{\rho^*_*}{\rho^*_0} \right) \rho^*_* \)

and \( r = r^*_*/\ell^*_* \), so \( \nabla^2 = \ell^*_*^2 \nabla^2 \). Poisson's equation can then be rewritten as:

\begin{equation}
\nabla^2 W(r, \theta) = -\lambda \rho(r, \theta, W)
\end{equation}

where \( \lambda = (4\pi G\ell^*_*^2 \rho^*_0^*)/\rho^*_0 \). The zero subscript refers to the origin.

The scale length factor \( \ell^*_* \) can be made equivalent to the core radius, \( r_c \), as defined by King (1966) if

\begin{equation}
\lambda = 9/\rho^*_0.
\end{equation}

Thus only relative values of the density are needed and Poisson's equation in final form becomes:

\begin{equation}
\nabla^2 W(r, \theta) = -9\rho^*_*/\rho^*_0^*.
\end{equation}
The solution of this equation is discussed in section 2.3. From equation (d), the RHS of equation (e) is,

\[
\int_{-\nu_c}^{\nu_c} [\exp(-\beta \xi) + \beta \xi \exp(-\beta E_0) - 1] \exp (\gamma \nu \sin \phi) \, d\nu \phi
\]

\[
\int_{-\nu_c}^{\nu_c} [\exp(-\beta \xi_0) + \beta \xi_0 \exp(-\beta E_0) - 1] \, d\nu \phi
\]

Let also \( \eta = \left( \frac{\beta}{2} \right)^{\frac{1}{2}} \nu \phi \) and \( \gamma = r \left( \frac{2}{\beta} \right)^{\frac{1}{2}} \gamma * \); then in its fully expanded dimensionless form, equation (e) becomes,

\[
\nabla^2 \tilde{W}(r, \theta) = -9 \int_{-W}^{W} [\exp(-\eta^2 + \tilde{W}) + (\eta^2 - \tilde{W}) - 1] \exp(\gamma \nu \sin \theta) \, d\eta
\]

\[
\int_{-W}^{W} [\exp(-\eta^2 + \tilde{W}_0) + (\eta^2 - \tilde{W}_0) - 1] \, d\eta
\]

where \( \tilde{W}, \gamma, \eta \) and \( r \) are all dimensionless.

Two parameters are thus required to specify uniquely the dimensionless bulge model; they are the central bulge potential \( \tilde{W}_0 \) and the rotation parameter \( \gamma \). In this form, the dimensionless potential is the same as that used by King, i.e. King models are produced when \( \gamma = 0 \) (non-rotating). This was one of the properties used to test the models - see section 2.5.

The following observables and moments of the velocity dispersion were calculated from the converged model.

(1) velocity dispersion:

\[
\frac{<v^2>}{r} = \frac{1}{\rho} \int_{-\tilde{W}_0}^{\tilde{W}_0} \left[ \exp(-\eta^2 + \tilde{W}) + (\eta^2 - \tilde{W}) - \frac{(\eta^2 - \tilde{W})^2}{2} - 1 \right] \exp(\gamma \nu \sin \theta) \, d\eta
\]

\[
\frac{<v^2>}{\phi} = \frac{2}{\rho} \int_{-\tilde{W}_0}^{\tilde{W}_0} [\exp(-\eta^2 + \tilde{W}) + (\eta^2 - \tilde{W}) - 1] \exp(\gamma \nu \sin \theta) \, d\eta
\]

Following King, these have been normalised to the respective velocity dispersion of an isothermal sphere.
(2) mean rotation velocity:

\[
\langle v_\phi \rangle = \frac{2}{3} \bar{\rho} \int_{-W}^{W} \left[ \exp(-n^2 + w) + (n^2 - w) - 1 \right] \exp(\gamma \sin \theta) \cdot n \cdot d \eta
\]

(3) projected rotational velocity:

\[
V = \frac{1}{\Sigma} \int_{-\infty}^{\infty} v \langle v_\phi \rangle \rho \, dx
\]

where \( \Sigma \) is the projected surface brightness and \( v \) is the direction cosine, and \( x \) is along the line-of-sight.

(4) projected velocity dispersion:

\[
\sigma_v^2 = \frac{1}{\Sigma} \int_{-\infty}^{\infty} \frac{1}{r} \langle v_1^2 \rangle \rho \, dx - v^2
\]

where the summation convention is intended.

Of particular interest to the degree of rotational support in a galactic bulge is the run of \( V(r)/\sigma_0 \) with radius along the major axis, where \( \sigma_0 \) is the central velocity dispersion. (see Chapter IV)

From equations (g) and (h) evaluated on the major axis, this quantity can be found from,

\[
\frac{V(r)}{\sigma_0} = \left[ \frac{1}{\Sigma(r)} \int_{-\infty}^{\infty} v \langle v_\phi \rangle \rho \, dx \right] \cdot \left[ \frac{1}{\Sigma(r=0)} \int_{-\infty}^{\infty} \frac{1}{r} \langle v_1^2 \rangle \rho \, dx \right]^{-1/2}
\]

(ii) The Disc

As mentioned earlier, the disc of a disc galaxy contributes significantly to the potential and its effect has been included in these models as an externally imposed potential. As will be discussed in Chapter VI, the addition of a disc potential field in the models is a more realistic approximation to the dynamical environment in which the bulge co-exists, and as a result of this, the final models were found to be in better agreement with the observational data.

Another motivation for the inclusion of a static disc potential field into the models was to examine the dynamical origin
of "thick discs" observed in S0 galaxies by Burstein (1979), see Chapter VI.

Let $U_b$ and $U_d$ be the potential of the bulge and disc respectively at any point in the model. Then

$$V^2(U_b + U_d) = 4\pi G \int f(\frac{1}{2}c^2 + U_b, U_d, J)d^3c + 4\pi G\rho_d$$

where $\rho_b$ and $\rho_d$ are the corresponding bulge and disc densities at this point and $c$ are the velocity components. The disc potential field satisfies Laplace's equation outside the disc and Poisson's equation within, i.e.

$$V^2U_d = 0 \quad \text{outside disc}$$

and

$$V^2U_d = 4\pi G\rho_d \quad \text{inside disc}.$$  

Thus equation (h) may be written as:

$$V^2U_b = 4\pi G \int f(\frac{1}{2}c^2 + U_b, U_d, J)d^3c$$

This means that the bulge model, in the external potential field of the disc, can still be solved self-consistently; the only modification required is that the energy integral of the bulge include the total (bulge + disc) potential. In view of their analytic simplicity, the disc models of Miyamoto and Nagai (1975; hereafter MN) were used. The disc potential field $U_d(r, z)$ is given in cylindrical co-ordinates ($r, z$) as:

$$U_d(r, z) = \frac{G M_d}{\{r^2 + [a+(z^2+b^2)^{1/2}]^2\}^{3/2}}$$

where $M_d$ is the mass of the disc, and $a$ and $b$ are scale lengths in the equatorial and vertical directions respectively. This axisymmetric potential field is free from singularities, differentiable, and tends to the Keplerian potential for large $r$ and $z$. From Poisson's equation using this potential field, MN showed that the corresponding density distribution is given by,
\[
\rho(r, z) = \frac{b^2 M_d}{4 \pi} \cdot \frac{a r^2 + [a + 3(z^2 + b^2)^{1/2}] [a + (z^2 + b^2)^{1/2}]^2}{[r^2 + (a + (z^2 + b^2)^{1/2})^2]^{3/2} (z^2 + b^2)^{3/2}}.
\]

We now require equation (i) in the same dimensionless units as the bulge. Using the same notation as before, the dimensional potential at the origin is,

\[
U_d^0 = U_d(0, 0) = \frac{G M_d}{(a+b)}
\]

\[
= \frac{G}{(a+b)} \cdot \frac{M_d}{M_b} \cdot \rho_b^0 r_c^3 \mu_b^b
\]

where \(M_b = \rho_b^0 r_c^3 \mu_b^b\).

The quantity \(\mu_b\) is defined as:

\[
\mu_b = \int_0^T \frac{\rho_b}{\rho_b^0} 4 \pi s^2 ds.
\]

where \(r_c\) is the tidal (boundary) radius of the model. As before and following King, the core radius \(r_c\) is defined by,

\[
4 \pi G r_c^2 \rho_b^0 = 9 \quad (\beta = 2 j^2 \text{ in Kings notation})
\]

i.e. \(\beta = \frac{9}{4 \pi G r_c^2 \rho_b^0}\).

In the same dimensionless units as the bulge, the dimensionless disc potential is \(W_d = -\beta U_d\) and therefore,

\[
W_d^0 = \frac{-9 G}{4 \pi G r_c^2 \rho_b^0} \cdot \frac{M_d}{(a+b)} \cdot \frac{M_d}{M_b} \cdot \rho_b^0 r_c^3 \mu_b^b
\]

\[
= \frac{-9 \mu_b r_c}{4 \pi (a+b)} \cdot \frac{M_d}{M_b}
\]

where \(a\) and \(b\) have been rewritten in dimensionless units as \(a/r_c\) and \(b/r_c\) respectively.

The final form for the dimensionless central disc potential is then:

\[
W_d^0 = \frac{-9 Q}{4 \pi (a+b)} \quad \text{where} \quad Q = \frac{\mu_b}{M_b}.
\]
The mass ratio $M_d/M_b$ cannot be specified a priori but must be determined after the model has been constructed. This is because a redistribution of the mass in the bulge due to the presence of the disc will alter $\mu_b$. Instead, a value is assigned to $Q$, $W^0_b$ and $\gamma$, the model computed, $\mu_b$ calculated a posteriori and $M_d/M_b$ retrieved from $M_d/M_b = Q/\mu_b$. In practice, the values given to $\mu_b$ for a suitable input value of $Q$ are similar to those derived by King as a function of central potential for his globular cluster models. If this is done, then the ratio of the input to output mass ratio does not vary by more than a factor of two.

With the addition of the disc to the bulge models, the number of free parameters now required to uniquely specify the model has increased to five i.e. $W^0_b$, $\gamma$, $a$, $b$ and $Q$. The first two parameters define the bulge and the remaining three the disc. The disc to bulge mass ratio enters implicitly through $Q$.

To summarize, the models presented above are intended to represent the present dynamical state of the bulges of disc galaxies. The models are tidally truncated and are flattened mainly by rotation: the externally imposed disc potential also contributes to their flattening. To test these models they must be compared against observed two dimensional light distributions and kinematical properties of real galaxies. Before this is done, the numerical techniques used in calculating the models are detailed.

2.3 NUMERICAL TECHNIQUES

The main thrust of the numerical techniques was to develop an algorithm for solving the dimensionless Poisson equation (e). To this end, it has been popular to express the density and potential functions in terms of Legendre series (cf PT and Wilson) i.e.

$$W(r,\theta) = \sum_{n=0}^{2N-2} a_n(r) P_n(\cos \theta)$$
and \( \rho(r,\theta) = \sum_{n=0}^{2N-2} b_n(r)P_n(\cos \theta) \).

The summations need only extend over even values of \( n \) due to the equatorial symmetry of the models. The value of \( N \) is chosen in accordance with the desired accuracy of the numerical solution. Clearly as the models become flatter (\( \gamma \) increasing), a greater number of terms are required in the expansion.

Using these expansions and the boundary conditions

\[
W(r=0,\theta) = W_0, \quad 3W/3r \bigg|_{r=0} = 0 \quad \text{and} \quad 3W/3r \bigg|_{r=\infty} = 0,
\]

PT showed that the solution to equation (e) is,

\[
W(r, \theta) = W_0 + \frac{1}{\lambda} \left[ \int_0^r s b_0(s)ds - \frac{1}{r} \int_0^r s^2 b_0(s)ds \right] - \frac{\lambda}{r} \frac{P_n(\cos \theta)}{2n+1} \left[ \int_0^r s^{1-r} b_n(s)ds + \frac{1}{r^{n+1}} \int_0^r s^{n+2} b_n(s)ds \right] \quad (j)
\]

This expression was evaluated on a polar grid in \( r \) and \( \theta \). The divisions in \( r \) were variable so that a fine grid was used near the centre where the density gradient is large. To avoid unnecessary interpolation in the \( \theta \) direction, the rays were chosen to coincide with the \( \theta_k \) values defined by the Gauss-Legendre integration (see below). This differs from the otherwise similar procedure of van Albada and van Gorkom (1977; hereafter AG) who used an evenly spaced grid in \( \theta \).

The two integrals in the summation of equation (j) were evaluated using the recurrence relations derived by AG. At any point \((r_1, \theta_1)\) in the \((r, \theta)\) grid they write,

\[
\int_r^\infty s^{1-r} b_n(s)ds + \frac{1}{r^{n+1}} \int_0^r s^{n+2} b_n(s)ds = F_n(r_i) + G_n(r_i)
\]

Hence the value of \( F_n \) at \( r_{i-1} \) is related to the value at \( r_i \) by,

\[
F_n(r_{i-1}) = r^{n}_{i-1} \int_{r_{i-1}}^{r_i} s^{1-n} b_n(s)ds + \left( \frac{r_{i-1}}{r_i} \right)^n F_n(r_i)
\]
A trapezoidal approximation to the integral gives,

\[ F_n(r_{i-1}) = \left( \frac{r_{i-1}}{r_i} \right)^n \left[ \frac{2}{1+(r_{i-1}/r_i)} \cdot \frac{r_i^2-r_{i-1}^2}{2} \right] b_n(r_{i-2}) + F_n(r_i) \]

where \( r_{i-2} = \frac{r_{i-1} + r_i}{2} \). A starting condition of \( F_n(r_i) = 0 \) enables \( F_n \) to be evaluated for all values of \( r \) and \( n \).

A similar recurrence relation also exists for \( G \), i.e.

\[ G_n(r_{i+1}) = R^{1+n} \left[ \frac{1+R}{2R} \cdot \frac{(r_{i+1}^2-r_i^2)}{2} \right] b_n(r_{i+2}) + G_n(r_i) \]

where \( R = r_i/r_{i+1} \). The starting condition is \( G_n(r_i) = G_n(0) = 0 \).

All that remains to calculate are the \( b_n \). These are evaluated using either a 12 or 24 point Gauss-Legendre quadrature over \( \cos \theta \), i.e.

\[ b_n(r) = \frac{1}{2} (2n+1) \sum_{k=1}^{m} w_k \rho(r_k, \theta_k) P_n(\cos \theta_k), \]

where \( m = 12 \) or 24. The weights \( w_k \) have been taken from Davis and Polonsky (1964). In terms of the \( F_n \) and \( G_n \) recurrence functions, the \( n=0 \) term of equation (j) is,

\[ \lambda \left[ F_0(0) - F_0(r) - G_0(r) \right]. \]

The numerical scheme for computing a self-consistent stationary model is now clear. An initial guess is made for the bulge density distribution at every point in the grid. This allows the calculation of the density expansion coefficients \( b_n(r) \). Next, the recurrence relations are used to compute the potential field corresponding to this density field, from Poisson's equation. The contribution from the disc potential at each point is added, and a new density distribution calculated by integrating the distribution function. This process was repeated until convergence occurred. Convergence was assumed to occur when the difference in density at every grid point in the model between successive iterations was less than 0.1 percent.

Generally, ten to twenty iterations were required for convergence.
using a grid of about 450 shells and 14 rays (including the major and minor axis). The total CPU time required using the Australian National University Univac 1100/82A computer was typically less than five minutes. Longer times were required for rapidly rotating models.

2.4 NUMERICAL CHECKS

Several numerical checks were made to confirm the accuracy and dynamical integrity of the models. The simplest of these was afforded by comparing spherical (King) models without a disc, (computed in the generalised three-dimensional grid) with those published by King. The models were indistinguishable within the range of central potentials consistent with the observed range of central concentrations, from the least concentrated globular clusters to the elliptical galaxies i.e. \( W_b^0 \propto -3 \) to -13.

A second more stringent test was the virial theorem, \( T/\Omega = -\frac{1}{2} \), where \( T \) and \( \Omega \) are the total internal kinetic and potential energies respectively. These are easily shown to be,

\[
\Omega = \frac{1}{2} \int_0^\infty \frac{a_n(s)b_n(s)}{\rho_o (2k+1)} \cdot 4\pi s^2 ds - \frac{1}{2} \int_0^\infty \frac{b_n(s)}{\rho_o} \cdot 4\pi s^2 ds
\]

and

\[
T = \frac{1}{2} \int_0^1 \frac{\rho}{\rho_o} (2\langle v_r^2 \rangle + \langle v_\phi^2 \rangle) d\cos \theta_2 \pi s^2 ds
\]

in dimensionless units. The (non zero shifted) potential at infinity, \( W_\infty \) is

\[
W_\infty = W_o + \lambda \int_0^{r_t} s b_0(s) ds
\]

where \( r_t \) is the maximum radius of the boundary.

For the rotating models without a disc, the virial theorem was found to be satisfied to better than 0.1 percent in all cases. However, a disadvantage in using the virial theorem as a dynamical check is that it is heavily weighted to those regions of the model where most of
the mass resides i.e. near the centre. For this reason, little
confidence may be placed in the virial theorem as a test of the outer
parts of the model. An independent point to point check in the model
is the most desirable, so the stellar hydrodynamical equations (HEs)
were used as a check.

In spherical polar co-ordinates the \( r \) component of the HEs,
from Ogorodnikov (1965) is

\[
\frac{3<v_r>}{\partial t} + \frac{<v_r> \partial <v_r>}{\partial r} + \frac{1}{r} \frac{<v_\theta> \partial <v_r>}{\partial \theta} + \frac{1}{r \sin \theta} \frac{<v_\phi> \partial <v_r>}{\partial \phi} \\
- \frac{1}{r} (<v_\theta>^2 + <v_\phi>^2) + \frac{2}{r} \frac{<v_r>^2}{r} + \frac{<v_r> <v_\theta>}{r} \cot \theta + \frac{1}{\rho} \frac{3 (\rho \sigma_{rr})}{\partial r} \\
+ \frac{1}{r} \frac{\partial (\rho \sigma_{r\theta})}{\partial \theta} + \frac{1}{r \sin \theta} \frac{\partial (\rho \sigma_{r\phi})}{\partial \phi} - \frac{1}{r} (\sigma_{\theta \theta} + \sigma_{\phi \phi}) + \frac{2 \sigma_{rr}}{r} \\
+ \frac{\sigma_{r\theta}}{r} \cot \theta = \frac{3W}{\partial r}
\]

where \( \sigma_{ij} = \frac{1}{\rho} \int (v_i - <v_i>) (v_j - <v_j>) f(E, J) d^3c \).

It is easy to show that \( \sigma_{ij} = \delta_{ij} \sigma_{ii} \) where \( \delta_{ij} \) is the Kronecker
delta. The models are stationary \( (\partial / \partial t = 0) \) and axisymmetric
\( (\partial / \partial \phi = 0) \). Also \( <v_r> \) and \( <v_\theta> \) are zero and therefore the \( r \) component
of the HEs is reduced to

\[
\frac{1}{\rho} \left[ \frac{3 \rho <v_r^2>}{\partial r} + \frac{\rho <v_r^2>}{r} - \frac{\rho <v_\phi^2>}{r} \right] - \frac{<v_\phi^2>}{r} = \frac{3W}{\partial r} \tag{k}
\]

since \( \sigma_{ii} = \frac{1}{\rho} \int (v_i - <v_i>)^2 f(E, J) d^3c = \frac{1}{\rho} \int v_i^2 f(E, J) d^3c \equiv <v_i^2> \).

Similarly the \( \theta \) component of the HEs is.

\[
\frac{3<v_\theta>}{\partial t} + \frac{<v_\theta> \partial <v_\theta>}{\partial r} + \frac{1}{r} \frac{<v_\theta> \partial <v_\theta>}{\partial \theta} + \frac{1}{r \sin \theta} \frac{<v_\phi> \partial <v_\theta>}{\partial \phi} - \frac{<v_\phi^2> \cot \theta}{r} \\
+ \frac{<v_r> <v_\theta>}{r} + \frac{1}{r} \left[ \frac{3 \rho \sigma_{r\theta}}{\partial r} + \frac{\rho \sigma_{r\theta}}{r \partial \theta} + \frac{\rho \sigma_{r\phi}}{r \sin \theta \partial \phi} - \frac{\rho \sigma_{\phi \phi} \cot \theta}{r} \right] \\
+ \frac{3}{2} \frac{\rho \sigma_{r\theta}}{r \partial \theta} + \frac{\rho \sigma_{r\theta} \cot \theta}{r} = \frac{3W}{r \partial \theta} .
\]
This reduces in a similar way to,

\[
\frac{1}{\rho} \left[ \frac{\tan \theta}{r} \frac{\partial \langle v_r^2 \rangle}{\partial \theta} - \frac{\rho \langle v_\phi^2 \rangle}{r} + \frac{\rho \langle v_r^2 \rangle}{r} \right] = \frac{\langle v_\phi^2 \rangle}{r} = \frac{\tan \theta}{r} \frac{\partial \omega}{\partial \theta}. \tag{1}
\]

The \( \phi \) component vanishes in an axisymmetric model. Equations (k) and (l) were applied at every grid point along the major and minor axes of the model. This is a better check for the outer parts of centrally concentrated models than the virial theorem.

To illustrate the accuracy of a model, a moderately concentrated rotating system was constructed with \((w_b^0, \gamma) = (-8, 0.3)\). Figure 2.1 shows the radial distribution of error in the HE's, normalised to the potential gradient along the major and minor axes. To a radius of \(0.9r_o\), the error does not exceed 0.2 percent. The morphology of the radial error distribution also illustrates well where the greatest need is required for a finer radial grid. This in fact could be used to force the radial grid to follow the density gradient in the model. This would ensure a more constant error, independent of central concentration and position in the model. For all the computed models, with or without discs, the error derived from the HEs along the major and minor axes did not exceed one percent.

In summary, the models are believed to be dynamically accurate within the domain of input parameters commensurate with real physical systems.

2.5 PROJECTION

To compare the models with real observations, the models must first be projected on to the plane of the sky. In doing this the models were assumed to be transparent and have a constant mass to light ratio. The projected surface density \( \Sigma \), was computed from the integral

\[
\Sigma = \int_{-\infty}^{\infty} \rho \, dx \tag{m}
\]
Figure 2.1 The percentage error in equation (k) (see text) normalised to the potential gradient i.e.

\[ \% \text{ error} = 100 \left[ 1 - \frac{\partial}{\partial r} \frac{1}{\rho} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial r} \right) - \frac{\rho}{r} \frac{\partial v_r}{r} - \frac{v_\phi}{r} \right] - \frac{v_\phi^2}{r} \]

where \( A = \frac{1}{\rho} \left[ \frac{\partial}{\partial r} \frac{1}{\rho} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial r} \right) + \frac{\rho}{r} \frac{\partial v_r}{r} - \frac{v_\phi}{r} \right] - \frac{v_\phi^2}{r} \)

The two labelled curves show the distribution and magnitude of this error along the major and minor axes. The local increase in error near \( r/r_t \sim 0.1 \) is due to the grid in \( r \) increasing from \( \Delta r = 6 \times 10^{-4} r_t \) to \( \Delta r = 3 \times 10^{-3} r_t \). The error is less than 0.1 percent over most of the model.
where $x$ is along the line of sight in a right-handed rectangular co-ordinate system centred on the galaxy. The trade-off when using a fine grid in the models came in the projection in that equation (m) could be used directly. In other projection schemes (e.g. Wilson 1975), the above integral could not be applied directly due to a much coarser grid in regions of high spatial density gradient. Another advantage of using equation (m) directly in a fine grid is the speed of the projection code. The average CPU time per model was about 15 seconds. As a numerical check on the accuracy of the projection, triaxial ellipsoids were projected and compared with their theoretically projected values. The agreements were excellent.

### 2.6 SOME MODEL PROPERTIES

Many of the model properties may be conveniently described in the central potential - rotation parameter $(W^0, \gamma)$ plane. The first problem to be encountered when constructing a model for a particular galaxy is the initial choice of $W^0_b$ and $\gamma$. One method by which these quantities may be estimated is to construct a variable which will parametrize the "concentration" of the model and galaxy. Then, by measuring this quantity from the data, a starting $(W^0_b, \gamma)$ pair may be interpolated from the $(W^0_b, \gamma)$ plane. The effectiveness of this parameter is greatest if it can be computed quickly and accurately. It must also be sufficiently sensitive to changes in concentration over the range of interest in astrophysical situations. Consider a concentration index $K$ defined by

$$K = \frac{<r^2>}{<r>^2}$$

where $<r> = \int \rho dr / \int \rho dr$

and $<r^2> = \int \rho r^2 dr / \int \rho dr$.

This particular concentration index lies within the range $K \leq \frac{3}{4}$. The upper limit is realised with a uniform sphere; $K$ increases with
decreasing model concentration. Table 2.1 tabulates the concentration index $K$ for some King models ($\gamma = 0$).

### TABLE 2.1

**K INDEX FOR SOME KING MODELS**

| $|W_0^0|$ | $\log c$ | $K$  |
|--------|----------|------|
| 2.5    | 0.590    | 0.576|
| 3.0    | 0.672    | 0.563|
| 4.0    | 0.840    | 0.532|
| 5.0    | 1.029    | 0.490|
| 6.0    | 1.255    | 0.437|
| 7.0    | 1.528    | 0.367|
| 8.0    | 1.833    | 0.283|
| 9.0    | 2.119    | 0.199|
| 10.0   | 2.350    | 0.135|
| 12.0   | 2.739    | 0.059|

It can be seen that $K$ has the desirable property of being monotonic with concentration and reasonably linear over the above range of central potentials.

Using this definition for $K$, a grid of models was then constructed with different values of $W_0^0$ and $\gamma$. These allowed loci of constant concentration to be drawn in the $(W_0^0, \gamma)$ plane, shown in the lower panel of Figure 2.2. A series of models with constant central potential are shown in the $(K, \gamma)$ plane of the upper panel of Figure 2.2. The most striking result from this figure is that the concentration of the models is nearly independent of their initial $W_0^0$ values for $\gamma$ greater than about 0.6 (corresponding to a flattening of nearly E6). However, the bulges of most disc galaxies in the $(K, \gamma)$ plane do not lie in this region of ambiguity, but have $0 \leq \gamma \leq 0.6$.

Once the concentration index has been computed from the observational data, several possible models result from Figure 2.2. In practice, each of these models is then constructed and the best model chosen by comparing the minor axis profiles of the model and
Figure 2.2

**Upper Panel** Contours formed by three models with different central potentials in the \((K, \gamma)\) plane showing the dependence of model mass concentration with increasing net rotation. Net rotation and flattening increase with increasing \(\gamma\). Models become more concentrated with decreasing \(K\) (see text). \(\gamma = 0\) corresponds to King models.

**Lower Panel** Contours of equal model mass concentration in the \((\omega_b, \gamma)\) plane. Contours are labelled with their concentration index \(K\) (see text).
data (see Chapter IV).

2.7 SUMMARY

The models presented above were constructed to represent the current stationary dynamical state of the bulges of disc galaxies. They assume the bulges are oblate and flattened mainly by rotation. The distribution function is truncated by an energy cutoff, and has an angular momentum term which gives non-zero net rotation. The only way to test the assumptions on which this model is based is to compare the structure and internal kinematics of the resultant models with the real data. To do this the following observational material is required:

(i) accurate two-dimensional surface brightness maps of the bulges of edge-on disc galaxies,

(ii) spatial mean rotation profiles in the bulge and

(iii) spatial velocity dispersion profiles in the bulge.

The methods used to obtain this observational data for the candidate objects chosen in this study are described in the next chapter.
CHAPTER III

OBSERVATIONAL TECHNIQUES AND DATA REDUCTION

3.1 INTRODUCTION

This chapter describes the observational and data reduction techniques used to test and constrain the bulge dynamical models of Chapter II. A program of two-dimensional surface photometry for four suitable candidate edge-on SO and early type disc galaxies was a major part of the observational work. Section 3.2 describes the acquisition and reduction of the photoelectric photometry and in section 3.3 the photographic photometry is discussed. An external check on the surface photometry was made by inclusion in the sample of galaxies, the luminosity distribution standard NGC 3379. This galaxy was selected for this purpose by the Working Group on Galaxy Photometry of IAU commission 28 (IAU Trans., XIB, 304). The photometry is compared in section 3.4.

Observations of rotation and velocity dispersion in a sample of SO's were made using the Image Photon Counting System (IPCS) of the 4m Anglo Australian Telescope (AAT) at Siding Spring Observatory. The procedure for velocity measurement is described in section 3.6 and in section 3.7 I discuss the technique used for the derivation of velocity dispersion.

3.2 PHOTOELECTRIC PHOTOMETRY

(i) Acquisition

All of the photoelectric photometry was done with the 0.6m f/18 telescope also at Siding Spring Observatory. Johnson's UBV
system was used with the bandpasses defined by the following filters:

V : 2 mm Schott GG495
B : 2 mm Schott BG12 + 1 mm Schott GG385
and U : 1 mm Schott UG1.

The detector was a dry-ice cooled 1P21 photomultiplier tube with an SSR pulse amplifier and the Mt Stromlo General Purpose Scaler (GPS). The GPS is a computer controlled photon counting system; the user defines the sequence of instructions to be executed through a BASIC control language on a PDP 11/34 computer, and outputs the data to data cassette or floppy disc.

The photoelectric observations consisted of a series of long strings of short integrations as the photoelectric aperture drifted across the galaxy in an east-west line. Each of these strings was stored in memory as an array and co-added to similarly repeated strings. All observations were made using an aperture of 1.36 mm diameter which corresponds to 29.0 seconds of arc. The aperture was centred visually on the nucleus of the galaxy to be observed and the telescope slewed to the west a distance of at least 5D_{25}. The D_{25} values were taken from the Second Reference Catalogue of Bright Galaxies (RC2) (de Vaucouleurs, de Vaucouleurs and Corwin 1976). For galaxies with no available D_{25} value (NGC 7123), a visual estimate was made from a deep B plate. Once a suitably bright reference star was found by moving the eyepiece of the Johnson offset guider, the telescope was advanced slightly further to the west before the tracking rate was offset, allowing the telescope to drift east. When the reference star crossed the cross-hair in the offset guider, a series of either one or two second integrations (depending on the galaxy observed) was commenced with the GPS. The constant drift rate east was preset with a
variable frequency oscillator to suit the integration time per channel and the total distance to be traversed by the drift scan. At the completion of each drift scan the data was displayed on a Tektronix 4010 terminal, with the observer having the option of either accepting or rejecting the last scan. If accepted, the scan was added to the previous scans and the average displayed. A completed observation was typically the average of 10 scans each of 600 channels, which was then saved on data cassette for later evaluation and reduction. This gave a total of approximately 1500 counts per channel per second integration in B. The drift rate was also adjusted to give about five channel integrations per aperture. Approximately every two hours throughout the night, four to six standard stars were observed: these were used for the determination of the transformation coefficients. Each standard star was observed in the U, B and V passbands with an integration period of ten seconds. This sequence was repeated after a similar observation of nearby sky.

(ii) Reduction

The reduction of the drift scans involved finding and subtracting the background sky. This was achieved using the following steps. The total average drift scan was firstly "cleaned" by applying a 3-σ iterative clipping routine outside a user-defined window enclosing the galaxy. Careful attention was made to ensure that this window included a sufficiently large section of the drift scan so as to include all of the galaxy plus a large allowance for safety. Upon convergence, this process was repeated with the
rejection criteria lowered to 2-σ. A linear least squares fit was then applied to the clipped data to determine the sky count level. This fitted sky was then subtracted from the cleaned data to produce a sky subtracted scan in preparation for transformation on to the appropriate magnitude scale for the filter used. In all cases the sky was fit to better than 0.5%. The accuracy of the fit was limited by sky photon noise and variable extinction between each averaged scan. A linear fit was also assumed. The transformation coefficients obtained from the standard stars measured before and after each completed set of drift scans were then computed using mean extinction coefficients. The adopted extinction coefficients were:

\[ K_V = 0.16 \text{ magnitude.airmass}^{-1} \]
\[ K_{B-V} = 0.12 - 0.04 (B-V) \text{ magnitude.airmass}^{-1} \]
and \[ K_{U-B} = 0.36 \text{ magnitude.airmass}^{-1}. \]

The coefficients adopted for the drift scan transformations were taken as the average of the standard star coefficients obtained before and after each drift scan set. Once the sky subtracted drift scans were transformed on to a magnitude scale, the drift scans were ready to define the magnitude zero-point for the photographic photometry.

3.3 PHOTOGRAPHIC PHOTOMETRY

(i) Acquisition

All the photographic plates were obtained using the 1.0 m f/8 Ritchey-Chretien telescope at Siding Spring Observatory. The telescope in this configuration has a plate scale of 25.0 arc seconds.mm\(^{-1}\) and a 45 arc minute field of good definition. The
plate-filter combinations used to define the B and V passbands were:

\[
B = IIa-O + GG385, \\
V = IIa-D + GG495.
\]

All B and V plates were hypersensitized before exposure. After an initial soaking for 15 hours in nitrogen at room temperature, the plates were flushed with hydrogen every hour for a total of seven hours. Tests were carried out prior to exposure to check for plate fog and speed. Calibration for the plates was provided by use of a spot sensitometer, housed in the 1.0 m telescope building. The sensitometer was constructed by G. de Vaucouleurs and consists of a stack of 15 tubes, each with an entrance aperture of well determined size. All exit apertures are of identical size. This produced a set of three rows of five spots covering a range in surface brightness of 3.85 magnitudes. Uniform illumination at the base of the tubes is provided by an opal glass diffuser, illuminated by a tungsten lamp light source located approximately 50 cm below the diffuser. The calibration spots were exposed at the same time as the telescope exposure, on a separate plate from the same box and hypering batch as the telescope plate. Spot exposure times were always within 20% of the telescope exposure times and in as similar temperature and humidity conditions as possible. The sensitometer room was well ventilated to the outside air, so conditions were similar to those in the dome at the time of exposure. Both plates were hand processed simultaneously in D19 and 20°C for five minutes, within 18 hours of exposure.

(ii) Reduction

(a) Microphotometry

The photographic plates were digitised using the Mt Stromlo Perkin-Elmer Photometric Data Systems (PDS) micro-densitometer.
This instrument has a 12 bit analog-to-digital converter with density increments of 0.00125 density units. A large degree of flexibility in the measuring procedure was possible using the PDS control program FORNAX written at Mt Stromlo for the PDS. The raw PDS data was written directly to nine track magnetic tape for further reduction.

To ensure that the PDS measuring runs were as similar as possible, each run was treated as a photometric session. The 12-bit configuration of the PDS enabled all runs to be performed in the density mode. Constancy in the set-up procedure was ensured by:

(i) allowing a warmup period of at least two hours,
(ii) using a plate of clear glass, the same thickness as the exposed plates to set the PDS voltage and density controls, and,
(iii) using an offset of 0.20 in density above clear plate.

Two scanning apertures were used for the 1.0 m plates. The high resolution "galaxy map" was performed with a square 25 \( \mu m \) aperture, each scan row having 1200 pixels spaced 25 \( \mu m \) apart (ie the galaxy map was 30 mm x 30 mm). The low resolution "sky map" with the same centre as the galaxy map, was scanned with a square pixel size of 125 \( \mu m \), each pixel separated by 125 \( \mu m \). The sky scan was 688 x 688 pixels in size (ie 86 mm x 86 mm). Both of these aperture sizes could be obtained with the X4 objectives. Using these scan areas, the sky map fell totally within the unvignetted field of the plate ie 35x35 minutes of arc or 86x86 mm. The galaxy and sky maps were scanned at speeds of 25 and 35 PDS speed units respectively. These corresponded to physical plate speeds of 5 and 7 mm.s\(^{-1}\) respectively. The speeds were sufficiently slow to enable the logarithmic amplifier with its finite time constant to
respond to rapid changes in plate density and hence input signal. Each PDS session involved the following set of measurements; the repeated set-up for different apertures being necessary since the PDS "sees" the plate with different f ratios for different apertures. For this reason, the two aperture sets were tied together by scanning the calibration spots with both apertures.

(i) set up PDS with the 25 $\mu$m$^2$ aperture,
(ii) scan the calibration spot plate,
(iii) scan two adjacent areas (61x61 pixels) of fog in each corner of the galaxy plate,
(iv) scan the galaxy frame,
(v) rescan the fog spots in each corner,
(vi) set up PDS with the 125 $\mu$m$^2$ aperture,
(vii) scan the same fog spots (13x13 pixels) in each corner of the galaxy plate,
(viii) scan the sky frame,
(ix) rescan the fog spots in each corner,
(x) scan the calibration spot plate.

All the above data was written on to magnetic tape for later reduction. The drift in the PDS photoelectric system observed over several hours of measurement was less than four PDS units (0.001 density units). Constancy between sessions was excellent.

(b) Spot Calibration

The data reduction was performed on the Mt Stromlo VAX 11/780 computer using the PANDORA picture processing software package. The characteristic curve (CC) was fitted in the log (intensity) - opacitance domain (de Vaucouleurs 1968) ie
\[ \log I = \sum_{i=0}^{n} a_i (\log \omega)^i \]

where \( \omega = 10^D - 1 \) and \( D \) is the plate spot density above fog. \( n \) is typically three or four. This method has the advantage over fitting in the log I-density plane in that it results in a large degree of linearization of the CC, especially at small log I. All the CCs could be fitted using a third order polynomial with a standard error of less than 0.02 magnitudes. Independent fits of the CCs were made for the calibration spots corresponding to the sky map and galaxy map. The corresponding functional fits were then used to transform the galaxy and sky maps into log I.

(a) Sky Fitting

The next stage in the reduction of the photographic photometry was the most important and most difficult - the sky fitting. As is well known, the largest errors are systematic and every effort was made to minimise these using both internal and external checks. Most of the techniques used here are standard and well tried. The basis of the sky background determination using multi-order polynomial fits is similar to that of Jones et al (1967). The sky map was firstly segmented into square areas each with a side length of 15 or 20 pixels. A 2-\( \sigma \) iterative clip was then performed in each box to clip out stars and large noise deviations. With each of these boxes, a single mean value was adopted as representative of the sky intensity in this box. The very faintest isophotes of the galaxy were then determined using the sky map. With a generous allowance for additional isophotes beyond the faintest detected, an inner radius \( R_1 \) was chosen defining an inner exclusion area which was to be excluded from the sky fit. An outer
radius R2 was also chosen to lie completely within the unvignetted field of the plate (see Duus 1979). Figure 3.1 summarises the plate acquisition and reduction areas described above. Using the interatively clipped log I sky map, a linear least squares quadratic surface was then fitted to the annulus between R1 and R2. Subsequent subtraction of the sky allowed an examination of the quality of the fit. This was most readily assessed by plotting a histogram of residuals in magnitudes between the observed sky and the fitted sky. The standard error of the residuals was typically 0.01 magnitudes for the 1.0 m plates. Little difference was seen between fitting a plane or a quadratic surface, indicating that the unvignetted area on the 1.0 m plates is very flat.

Using the coefficients saved from the fitted surface, a new sky intensity map was created with the same physical and pixel size as the galaxy map. Subsequent subtraction in linear intensity units gave the final sky subtracted galaxy map. Luminosity profiles at any position angle and through any point in the map could then be determined on an arbitrary magnitude scale.

The final step was to use the photoelectric drift scans to place the zeropoint of the photographic profiles on to the standard Johnson magnitude zeropoint. Allowance for the finite size of the scanning aperture and the "smearing" effect of the aperture drifting during each integration had to be made when calibrating the photographic data. This was achieved by constructing a computer code which simulated a drift scan through the photographic data. The code allowed for any aperture size (seconds of arc) and drift rate (seconds of arc second of time⁻¹). With these parameters fixed by the drift scan, a "phase factor" allowed for small differences in starting position, permitting direct comparison
Figure 3.1 Schematic diagram of the galaxy and sky map scan areas on the 1.0m plates. The radius R2 was chosen to be less than the radius of the unvignetted area (outer dashed circle). R1 was chosen to include all the galaxy light. The scan areas used to measure the fog are also shown.
between the photographic and photoelectric data. This defined the zero point.

The first galaxy reduced in this way was NGC 3379, which acted as a check against systematic errors—see Section 3.4. The procedure of Jones et al. uses a much higher order two-dimensional polynomial for the sky fit. This is primarily to account for sky (or plate) variations on a smaller scale. However, high order polynomial fitting can run into difficulty due to the behaviour of the fitted function in the unconstrained exclusion area around the galaxy. This problem is particularly true when $R_1/R_2$ approaches unity and so should be avoided. Segmenting and iterative clipping can reduce the order of the fitting polynomial significantly. Examination of the sky subtracted residuals on a television screen showed no spatial systematic behaviour.

3.4 PHOTOMETRY OF NGC 3379

The El galaxy NGC 3379 was included in the surface photometry program as an external check on the photometric accuracy of the reduction procedure. This galaxy has been extensively observed by many authors eg Miller and Prendergast (1962), Burkhead and Kalinowski (1974) and de Vaucouleurs and Capaccioli (1979). Together with their own observational material, de Vaucouleurs and Capaccioli collated all reliable existing data to produce a “mean” east-west luminosity profile in the Johnson B photoelectric passband. It is this combined luminosity profile from their table 2B, column 7 which is used for comparison with the work presented here. Figure 3.2 shows the E-W luminosity profile of NGC 3379 in
TABLE 3.1

E-W LUMINOSITY PROFILE OF NGC 3379 IN B

<table>
<thead>
<tr>
<th>log r</th>
<th>( \mu_B )</th>
<th>log r</th>
<th>( \mu_B )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.59</td>
<td>18.05</td>
<td>1.61</td>
<td>21.57</td>
</tr>
<tr>
<td>0.64</td>
<td>18.24</td>
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</tr>
<tr>
<td>0.70</td>
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<td>22.89</td>
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</tr>
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<td>1.04</td>
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<td>1.16</td>
<td>19.81</td>
<td>2.18</td>
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</tr>
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<td>1.21</td>
<td>20.19</td>
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<tr>
<td>1.55</td>
<td>21.34</td>
<td>2.57</td>
<td>27.80</td>
</tr>
</tbody>
</table>

\( r \) in arc seconds

\( \mu_B \) in magnitudes per square arc second
Figure 3.2  

Upper panel  The E-W luminosity profile of NGC 3379 in the Johnson B band. The filled circles are from the present work. The solid line is taken from the composite profile of de Vaucouleurs and Capaccioli (1979). The sky brightness is also shown for the plate from which the profile was derived.

Lower panel  Plot of the residuals between the present study and de Vaucouleurs and Capaccioli as a function of de Vaucouleurs magnitude and log r.
NGC 3379

![Graph showing 
\( \mu_B \) vs. \( \log r \) with BJ and deV data points.

\( \Delta \mu_B(BJ-deV) \) vs. \( \log r \) with values at 19, 21, 23, 25, 27.

The graph displays the brightness \( \mu_B \) as a function of \( \log r \) for the objects BJ and deV, with the sky line indicated.

The differential brightness \( \Delta \mu_B(BJ-deV) \) is shown with data points at different \( \log r \) values.
the B passband folded about the photometric centre. The profile was derived from a single plate of 90 minutes exposure (plate number 3883) using a IIa-0 + GG385 plate-filter combination. The profile is tabulated in Table 3.1. The plate was kindly loaned by E.B. Newell. The luminosity profile between $2.38 < \log r < 2.56$ is represented by the west side only because the east side overlaps with NGC 3384. The solid line represents the observed surface brightness distribution in B from de Vaucouleurs and Capaccioli. The sky brightness on plate 3883 is also shown. The lower panel of Figure 3.2 shows the residuals between the present work and that of de Vaucouleurs and Capaccioli. As can be seen, the residuals are less than 0.1 magnitudes at a surface brightness of about $25 \mu_B$ increasing at the 1% sky brightness level ($\sim 27.5 \mu_B$) to 0.25 magnitudes. However, the standard errors of these residuals ($\log r > 2.38$) are larger than those to $\mu_B = 25$ by a factor of 2 since only the west side of the luminosity profile is used in this overlap region. Again, it is stressed that this profile is based on one plate only.

To summarise: the E-W luminosity profile of NGC 3379 derived here in the Johnson B band is in good agreement ($\Delta \mu_B \leq 0.25 \mu_B$) with the best combined profile available to 1% of the sky surface brightness.

3.5 KINEMATICAL OBSERVATIONS

(i) IPCS Data Acquisition

The University College London, Image Photon Counting System (IPCS) on the RGO spectrograph of the AAT was used for the deter-
mination of velocity dispersion profiles and rotation curves. This system has been described previously by Boksenberg (1978) and is briefly described here.

The IPCS is a photon counting device with a two-dimensional image formed by a high gain EMI 4-stage image intensifier (S20 photocathode), optically coupled to a continuously scanning television camera (Philips Plumbicon). The incident photons are centroided by the event-centre-detection logic and located in a 512 k pixel digital memory. When used for these observations on the RGO spectrograph, the pixel size is 15 μm (in the direction of dispersion) by 55 μm (along the slit). The long slit spectra were obtained using the 1200 lines.mm⁻¹ grating blazed in the first order at 5000 Å. The spectra were centred near 5000 Å, close to the Mgb I triplet. The two-dimensional format was chosen to give 2000 increments in the dispersion direction and 100 increments, each of 2.54 arc seconds in the spatial direction. The total wavelength covered was about 1000 Å with a dispersion of 33 Å.mm⁻¹ and a resolution of 1Å. The kinematical data presented in this thesis was acquired over several observing runs. Some AAT IPCS runs were done with an image format of 1600x100 pixels in the dispersion and spatial directions respectively. This was mainly due to instrument constraints at the time.

Each observing run began by observing at least three different standard stars as templates for the measurement of the galaxy velocity dispersion and rotation. For most but not all observing runs (see later) the pixel to pixel sensitivity variations in the detector were mapped out using a tungsten lamp as a uniform source of light. This enabled flatfielding of the raw spectral
data. The system vignetting along the slit was determined by observing the twilight sky at the end of each night. Dark sky would have been better in retrospect, as twilight sky is polarised, but pressure on time was extreme. An observation started with a short arc exposure (60s) followed by a 1000s exposure of the galaxy. This sequence was repeated typically three to six times until the required signal-to-noise ratio in the galaxy spectrum was obtained. Each arc and galaxy spectrum was written to nine track magnetic tape for later reduction. The orientation and position of the spectrograph slit with respect to the nucleus of the galaxy were also recorded.

(ii) Flatfielding and Wavelength Calibration

For those observations where a flatfield and vignetting frame were available, the pixel to pixel variations in the galaxy spectra were firstly removed by dividing by the frame of the uniform tungsten light source. The large scale vignetting profile was removed using the twilight exposure. Each frame was then ready to correct for drifts in the zero point of wavelength with time and telescope position, and for geometric distortions in the spectrograph and IPCS. The wavelength calibration is the single most important step in the reduction of IPCS spectra, since the measurement of small redshifts and broadening of the absorption features relies on the establishment of a consistent and accurate wavelength scale over the duration of the observations. After checking for irregular large wavelength shifts between two consecutive arc spectra (separated by a galaxy spectrum), the arc spectra were added to form a single arc spectrum representative of
the galaxy spectrum. A third order polynomial was fitted to the positions of the arc lines in this composite spectra. Typically 20 lines were included in the fit with an rms residual of less than 0.15 Å. This fitted calibrating polynomial was then used to rebin the galaxy spectrum into both linear wavelength and log wavelength maps. The increments per channel were $\Delta \lambda = 0.5$ Å and $\Delta \ln \lambda = 0.00004$ respectively. The rebinning gave a channel to channel velocity difference of 27.6 km.s$^{-1}$ for the log wavelength map. A check of the calibration was made by rebinning the arc spectra with its own fitted polynomial and comparing this map with the known positions of the spectral lines. This procedure was continued for each arc pair until all of the galaxy frames were rebinned with the same starting and increment value. A final composite frame was formed by the addition of all rebinned frames. The last step necessary before the computation of velocity and velocity dispersions could be made was sky subtraction. Typically five to ten rows parallel to the dispersion direction were chosen from the vignetting corrected spectra on both sides of the galaxy spectrum, well away from the centre of the galaxy. These were averaged and the mean per pixel subtracted from every row in the galaxy frame in the same column. The sky lines, in particular the 5577 Å line, were used to monitor the accuracy of the sky subtraction. The residual flux, after the subtraction of the 5577 Å sky line, expressed as a percentage of the total flux in that line, indicated that the subtraction was accurate to four to seven per cent.
3.6 VELOCITY DETERMINATION

Fourier techniques were used for the determination of both velocities and velocity dispersions. The technique for determining the Doppler shifts is similar to that of Simkin (1974). Before a discrete fast fourier transform (DFFT) could be applied to the spectra, some data preparation was necessary due to the spectra being finite and discretely sampled. A window of 1024 channels was chosen to include all the strong absorption features in the spectra. The mean in this window was subtracted from all channels and the resultant data string normalised to rms unity power. The final step was the application of a cosine-bell function to the first and last 100 channels to avoid the well known problems of "ringing" and "aliasing" (see Brault and White 1971 for a full discussion). A suitable template star spectrum of spectral type similar to that of the galaxy ie KO III was prepared in exactly the same way as above. This acted as a velocity and zero velocity dispersion standard.

A DFFT was then applied to each row in the galaxy frame as well as the template spectrum. Calculation of the cross-correlation from the DFFTs of the two spectra preceded backward transformation and normalisation to allow the determination of the cross-correlation peaks from their derivative. Normalisation resulted in a cross-correlation value of unity being obtained for a perfect correlation. The above procedure enabled the shift in channels of every row in the galaxy spectrum to be determined relative to the template spectrum. The internal consistancy obtained when cross-correlating the same galaxy spectrum with different standard stars
from G0 to K5 was about 0.2 channels (\( \sim 6 \text{ km.s}^{-1} \)). Standard corrections were made to correct the observed velocities for solar motion with respect to the velocity centroid of the Local Group of galaxies.

As for the surface photometry, an important check on the accuracy of the reduction algorithms was afforded by comparison with the work of other observers. To check the redshift determination procedure, the major axis of NGC 128 was chosen for several reasons. Firstly it has been extensively observed by Bertola and Capaccioli (1977, hereafter BC) and secondly, NGC 128 is a member of the sample of program galaxies studied in this thesis. Extreme pressure on observing time with the AAT made the choice of a program object necessary to the exclusion of a more widely observed galaxy. The upper panel of Figure 6.3 shows the rotation curve at a position angle of 0° through the nucleus of NGC 128. The open boxes are from the 30 min exposures on plates Q743 and Q744 taken by BC at the same position angle as the present work (filled circles). The shorter 10 min exposure by BC has not been included here. Following BC, a systemic heliocentric velocity of 4250 km.s\(^{-1}\) was chosen to plot their points. It is evident from Figure 6.3 that there is no true systematic difference between the rotation curve derived by BC and that of this study. The standard deviations of each velocity determination are also similar between both sets of data. From this result it is concluded that the rotation curve determination procedure successfully reproduces the results of these authors at least.
3.7 VELOCITY DISPERSION

(i) Measurement Procedure

The determination of accurate velocity dispersions of galaxies is in principle no more difficult than accurate redshifts. However, large differences between observers using apparently similar techniques have been common place in the literature. These differences have in part been due to early measurements; nevertheless significant systematic disagreements still appear (see Terlevich et al 1981 for comparison of observers). In view of these disagreements, a procedure was sought which would be simple but accurate to apply. To this end, the power spectrum technique used by Illingworth and Freeman (1974) was adopted. This method worked very well for globular clusters; velocity dispersions from the integrated light have been checked by velocity measurement of individual cluster stars (K.C. Freeman: private communication). The various Fourier procedures that are now in use (see later) are fairly much equivalent; this procedure was chosen because of its immediate attraction of simplicity and transparency of application. The method involves comparing the slope of the galaxy power spectrum with the slope of a template star convolved with a Gaussian velocity distribution of known width.

Let \( I(\lambda) \) be the spectral intensity distribution of the unbroadened galaxy. Assume also that the stars contributing to the observed velocity broadening have a Maxwellian velocity distribution along the line of sight ie

\[
f(v_r) = \left(2\pi \langle v_r^2 \rangle \right)^{-\frac{1}{2}} \exp \left(-\frac{v_r^2}{2 \langle v_r^2 \rangle}\right)
\]  

(a)
Then the observed spectral intensity distribution of the broadened galaxy is:

\[ G(\lambda) = \frac{c}{\lambda_0} \left( \frac{1}{2\pi \sigma^2} \right)^{\frac{1}{2}} \int_{-\infty}^{\infty} \frac{1}{\exp \left( -\frac{c^2(\lambda - \lambda_0)^2}{2\sigma^2} \right) \sigma} \exp \left( -\frac{c^2(\lambda - \lambda_0)^2}{2\sigma^2} \right) d\lambda \]  

(b)

where \((\lambda - \lambda_0)/\lambda_0\) has been substituted for \(v_r/c\). Now if \(\lambda_0^2 < \sigma^2\) \(r\) in equation (b) is constant over the wavelength window of interest, the integral in (b) is a convolution integral ie

\[ G(\lambda) = I(\lambda) * B(\lambda) \]

where * denotes convolution and \(B(\lambda)\) is the Gaussian broadening function. From the convolution theorem, it follows that

\[ \hat{G}(\nu) = \hat{I}(\lambda) \cdot \hat{B}(\lambda) \]

where \(\hat{\cdot}\) denotes the Fourier transform.

In this case, the Fourier transform of the broadening function, \(B(\lambda)\), is a real function ie

\[ B(\lambda) \rightarrow B(\nu) = \exp \left( -2\pi^2 \lambda_0^2 <\sigma^2> r \nu^2 / c^2 \right) \]

Finally the power spectrum \(P(\nu)\), is,

\[ \log P(\nu) = \log \left[ \frac{\nu^2}{\sigma^2} \left( \Re \{G(\nu)\} + \Im \{G(\nu)\} \right) \right] \]

\[ = -\frac{4\pi^2 \lambda_0^2 <\sigma^2> r}{2.303 c^2} \nu^2 + \log \left[ \frac{\nu}{\lambda_0} \frac{\nu^*}{\lambda_0} \right] \]  

(c)

The first term on the RHS is the contribution from the broadening function, while the second is the power spectrum of the unbroadened star alone.

To obtain the unknown velocity dispersion of a galaxy, a grid of power spectra were constructed by convolving a standard star with a set of Gaussian functions of known velocity dispersions. A well exposed spectrum of the GO star HD219709 was chosen as the grid standard. Before the convolution was performed, the raw data was
divided by a third order polynomial fit to the continuum to reduce any low frequency spikes in the resultant power spectrum. The wavelength window from which the power spectrum was calculated was 5005 Å to 5517 Å. For $\lambda_0 = 5261$ Å, the change in $\lambda_0 <v^2>^\frac{1}{2}$ over this window is less than 5 percent, small compared with the internal error of the fit. Once the power spectrum had been computed, a running mean over ± 11 points (± 0.02 cycles. Å⁻¹) was applied to reduce the large amplitude fluctuations inherent in power spectra of this type. Tests to examine the effect of smoothing on the derived slope were made by smoothing lines of known slope upon which noise of various amplitudes was superimposed. Smoothing in the frequency domain had no effect on the derived velocity dispersion (using the above running mean). The power spectrum rapidly becomes dominated by noise with increasing frequency. This noise component must be accurately determined and subtracted in order to derive the slope of the power spectrum in the absence of noise. A linear least squares fit to the noise was assumed in the log (power) - frequency domain and subtracted. The resultant smoothed, noise subtracted power spectrum was convolved according to equation (c) with a range of velocity dispersions $<v^2>^\frac{1}{2}$ up to 300 km.s⁻¹ in 50 km.s⁻¹ increments. This enabled visual interpolation between the grid for intermediate values of velocity dispersion without difficulty. Figure 3.3 shows the grid of convolved star spectra of HD219709 using the above procedure. This same method was repeated with the K5 star HD196983 to show the independence with spectral type of the overall slopes in the convolved star power spectra. These two stars represent the extremes in spectral type observed for the bulges of disc galaxies.
Figure 3.3 The grid template of power spectra formed by convolving the G0 star HD219709 with a set of Gaussian functions of known broadening. The smoothed power spectra are computed from a spectral window 512 Å wide, centred at 5261 Å. Each power spectrum is labelled with the velocity dispersion (km.s$^{-1}$) used in the broadening function.
Apart from unimportant small scale high frequency differences between the two grids of curves, no overall systematic differences of slope were seen. Since the determination of the velocity dispersion depends only on the gross overall difference in slope of the power spectra, the standard star chosen is not critical.

Again, for an external check of the accuracy of this method when applied to galaxies, a determination of the spatial distribution of velocity dispersion in NGC 3379 was made. This is compared with the results of Davies (1981). Three integrations each of 1000s were made with the IPCS slit along the major axis at a position angle of 65°. Unfortunately, for technical reasons, a flatfield for this sequence was not available. The lack of a flatfield produced a distinct change in the slope of the smoothed power spectrum of NGC 3379 (see $r=0''$ panel of Figure 3.4). This resulted in an instructive assessment of the noise subtraction procedure in the absence of a flatfield.

The power spectrum of a single row (ie spectrum) from a flatfield belonging to another observation was calculated. This is shown in the upper left panel of Figure 3.5. It can be seen that the "flatfield noise" spectrum is highly non-white, ie flatfield noise contributes a large amount of power that decreases with increasing frequency up to more than half the Nyquist frequency. The mean photon count per pixel in this row was 1600. Clearly, the power spectra of non-flatfielded galaxy spectra with high photon counts per pixel (eg for galaxy centres) will be significantly effected by the power spectrum of the flatfield. Care must then be taken to determine correctly the noise power spectrum for subtraction. In an attempt to determine the total number of photon
Figure 3.4 Plots of the smoothed power spectra of the central ten rows in the 3000s observation of NGC 3379. The solid straight lines indicate the adopted fit of the noise component in each spectrum. Note the domination by the flatfield noise in the central three rows. The mean counts per pixel per row from the centre are approximately 2000, 1000, 500, 300, 200 and 150 respectively. The frequency units are cycles Å⁻¹ with a Nyquist frequency (≡1.0) corresponding to 1Å per channel.
Figure 3.5 Plots of the mean power spectra when 20 different sets of white noise were introduced into the same row of a flatfield. The flatfield row had an average of 1600 counts per pixel. Each panel is labelled with the standard deviation of each set of noise. The introduced noise with standard deviations of 40, 80, 160 and 320 correspond to intrinsic counts per channel of 800, 320, 94 and 25 respectively. The sloping solid line shows the fit to the flatfield noise. The horizontal solid line shows the fit to the photon noise. The lower right hand panel shows the mean of 20 sets of noise alone, typical of the introduced noise. The frequency is in units of cycles. $\AA^{-1}$ with a Nyquist frequency (=1) of $1\AA$ per channel.
Figure 3.6  Same as Figure 3.5 except that only one set of noise is introduced. The dashed lines show the flatfield and photon noise components from Figure 3.5. The solid line indicates the noise component that would be chosen in the absence of knowledge of the dashed lines.
counts below which the photon noise dominates the flatfield noise, noise of known standard deviation was introduced into the flatfield. Using always the same row of the flatfield, white noise with a standard deviation, \( \sigma = 40 \) was added to this row before its power spectrum was again computed. This was repeated with 20 sets of independent noise to derive the average of the 20 resulting power spectra. Mean power spectra were also constructed for sets of noise with standard deviations of 80, 160 and 320. These are illustrated in Figure 3.5. The lower right hand panel shows the average power spectrum of 20 sets of the white noise itself. As expected, the noise generation algorithm produces noise well approximated by white noise. With the total counts per pixel of 1600 in the raw flatfield row, introducing noise of 40, 80, 160 and 320 standard deviations is equivalent in the power spectrum plane to having intrinsic counts per pixel of 800, 320, 94 and 25 counts per pixel respectively. From Figure 3.4 it can be seen that for total counts per pixel of less than about 100, the photon noise dominates the flatfield noise. For total counts per pixel greater than this, the slope of the power spectrum at low frequencies will have a significant added contribution from the flatfield noise. Figure 3.6 illustrates the flatfield power spectra likely to be seen with real data. Each panel shows the power spectrum of the same row of the original flatfield when computed now with only one set of white noise introduced. As expected, the flatfield noise is almost totally masked by random (photon) noise at a much lower standard deviation than when the average of many sets of noise is taken. The dashed line represents the "true" slopes of the low frequency flatfield noise and the high frequency photon noise taken from Figure 3.5.
The solid line is the noise component of the power spectrum that would be adopted in the absence of previous knowledge of the flatfield's power spectrum properties. Again, for total counts of less than 100 per pixel, there is little difference between the "true" noise components and that adopted.

To conclude, in the absence of a suitable flatfield exposure for flatfielding, contribution to the power spectrum from "flatfield noise" becomes significant when the total photon counts per pixel in the galaxy exposures exceed about 100 counts.

3.8 COMPARISON WITH NGC 3379

With some understanding of the effect of photon and flatfield noise on the power spectra, the spatial velocity dispersion along the major axis of NGC 3379 was determined. Using the reduction procedure described above, power spectra of the central 10 rows in the galaxy frame were computed. These are shown in Figure 3.4. The solid line in each panel shows the noise spectrum adopted for each power spectra, following what was learned from Figure 3.6 about the behaviour of the flatfield contribution to the noise power. Guided by Figure 3.6, the fits to the noise power are fairly unambiguous. The central three rows show clear evidence of flatfield noise dominating the photon noise (cf Figure 3.6).

When using the convolved star grid, arbitrary shifts in log power were made in matching the star power spectra to the galaxy power spectra since the velocity dispersion is determined only by its slope. The most reliable region in which the fit was made was between 0.02 and 0.09 cycles. Å⁻¹. In spite of a third order
continuum fit divided out of the galaxy spectrum before computation of the power spectrum, low frequency spikes can still occur when the continuum is of a more complex form than a third order polynomial. At the high frequency end, the accuracy of the noise subtraction becomes important. This is analogous in the two-dimensional sense to fitting the sky background on a photographic plate. Noise peaks above the mean noise level may also be troublesome at the high frequency end. However, in most cases an accurate determination of the velocity dispersion was possible. Figure 3.7 shows the best fit from the template grid to each row power spectrum for NGC 3379. The heavy solid line indicates the smoothed power spectrum of the appropriate row in the galaxy spectrum. The thin solid line shows the best fit to this power spectrum. An estimate of the standard deviation of the fit was made from the dashed lines which represent velocity dispersions chosen to be typically 50 km.s⁻¹ either side of the adopted value. The derived velocity dispersions have been folded about the photometric centre and plotted in Figure 3.8 together with those of Davies. Although Davies' velocity dispersion data is not directly comparable with the present work due to a difference in observed position angle (Davies: p.a. = 49°, this work: p.a. = 65°), both show similar dispersion values and trends of decreasing velocity dispersion with distance from the centre of the galaxy. Until recently, the velocity dispersion most often quoted in the literature has been the central velocity dispersion. Since this value has been much more extensively observed than the spatial distribution of values, more weight has been given to it. Table 3.2 shows a summary of previous determinations of the central velocity dispersion in NGC 3379.
Figure 3.7 Plots of the fit from the template grid of a star convolved with various Gaussian functions of known broadening to the power spectra of each of the central ten rows in the IPCS spectrum of NGC 3379. The thick solid line shows the smoothed noise subtracted power spectrum of the galaxy. The thin solid line is the best fitting power spectrum chosen from the template grid. The dashed lines show the slope of the power spectra typically 50 km.s\(^{-1}\) either side of the adopted value for comparison.
Figure 3.8 The velocity dispersion profile on NGC 3379 folded about the photometric centre of the galaxy. The open circles show the profile from the present work (p.a. = 65°). The filled circles show the profile derived by Davies (1981) at a position angle of 49°.
NGC 3379

○ Jarvis p.a. = 65°
• Davies p.a. = 49°
TABLE 3.2
MEASUREMENTS OF THE CENTRAL VELOCITY DISPERSION OF NGC 3379

<table>
<thead>
<tr>
<th>BBF</th>
<th>deV</th>
<th>FJ</th>
<th>W</th>
<th>S</th>
<th>WKS</th>
<th>D</th>
<th>This work</th>
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<tbody>
<tr>
<td>187</td>
<td>125</td>
<td>240</td>
<td>133</td>
<td>233</td>
<td>246</td>
<td>253</td>
<td>230 ± 20</td>
</tr>
</tbody>
</table>

Type V V F V F F F F

BBF, Burbidge, Burbidge and Fish (1961); deV, de Vaucouleurs (1974); FJ, Faber and Jackson (1976); W, Williams (1977); S, Sargent et al. (1978); WKS, Whitmore, Kirschner and Schechter (1979); D, Davies (1981); V = visual determination; F = Fourier determination.

The WKS value used here is an average of the values they determined from their high and low dispersion spectra. It is interesting to note that when the methods for the determination of velocity dispersion are broadly classified as based on a visual fitting procedure or a Fourier technique, a clear distinction in the derived values is evident for NGC 3379. The three lowest values obtained by BBF, deV and W are all obtained from largely visual fits while the Fourier determinations systematically give a considerably larger value. The reason for this is unclear. The central velocity dispersion derived here is in good agreement with the scatter of values recently obtained by other Fourier methods. This suggests that the procedure for velocity dispersion determination by power spectra comparison is a sound one. As a final comment, both this work and (especially) Davies' work found that the velocity
dispersion profile is quite peaked at the centre of the galaxy \((r < 2'')\). If this is true, then it is possible that small centering errors may contribute to some of the disagreement between authors.

### 3.9 OTHER EXTERNAL CHECKS

As another check on the accuracy of the power spectrum method when applied to galaxies, a comparison was made with the central velocity dispersion of the E2 galaxy NGC 1404 determined by A.J. Pickles (private communication) using the Sargent \textit{et al} (1977) Fourier quotient method. The same spectral region was chosen for each determination. Figure 3.9 shows the fit of the stellar spectrum (thin line) broadened to a dispersion determined by the Sargent Fourier quotient method to the galaxy spectrum (thick line). The broadened stellar spectrum has been normalised to the intensity of the Mgb I lines. The velocity dispersion determined from the power spectrum method described above is shown in Figure 3.11. The agreement is fortuitous in view of the errors associated with each method. The Sargent \textit{et al} method involves fitting a parabola in the log power-frequency plane. The error is based on the quality of this fit. The fit is illustrated in Figure 3.10.

To conclude, the power spectrum method described above gives results in good agreement with results obtained by other authors using a variety of different Fourier techniques. We prefer this method for two reasons. Firstly, the method is simple in concept and very straightforward to apply, with little subjectivity. Secondly, reliable dispersions were obtainable for spectra having a low signal to noise ratio (20 counts per pixel above sky) and also for spectra with high counts but no flatfield.
Figure 3.9  The spectrum of the nucleus of the E2 galaxy NGC 1404 (thick line) compared to the spectrum of a broadened star of similar spectral type (thin line). The star was broadened by a Gaussian profile with a velocity dispersion of 268 km.s$^{-1}$ as determined by the Sargent et al. (1977) Fourier method. The broadened star has been normalised to the intensity of the Mgb I triplet.
NGC 1404

σ = 268 km.s⁻¹
Figure 3.10 Parabolic fit (dashed line) to the power spectrum of the centre of NGC 1404 determined using the Sargent et al. (1977) method.
NGC 1404 $\sigma = 268$ km.s$^{-1}$
Figure 3.11 Fit of the power spectrum of the velocity broadened template star (thin solid line) to the smoothed power spectrum of the centre of NGC 1404 (thick solid line). The units of velocity dispersion are km.s\(^{-1}\). The dashed lines show for comparison, fits 40 and 60 km.s\(^{-1}\) above and below the adopted value of 260 km.s\(^{-1}\).
Log Power

NGC 1404 centre

Frequency (cycles Å⁻¹)
CHAPTER IV

COMPARISON: MODELS WITH OBSERVATIONS

4.1 INTRODUCTION

In Chapter II, self-consistent models of the bulges of disc galaxies were constructed. The models assumed a distribution function having two isolating integrals of motion; they are rotationally flattened. Models so constructed are oblate and axisymmetric. The validity of these models can be tested by comparing them against the observed photometric and kinematic properties of real galaxies. For simplicity, the observed mass-to-light ratio was initially assumed constant. Seven suitable galaxies were observed, of which three (NGC 7814, 7123 and 4594) were selected for modelling. The remaining four galaxies (NGC 4179, 4762, 128 and Ham I) were used to augment the growing number of observational data supporting the view that bulges are rotationally flattened. Section 4.2 describes the selection criteria for the galaxy sample and in section 4.3 the procedure for fitting the models to the observational data is described. Each galaxy, for which a model was constructed, is treated in a separate section. All the model and observational data relevant to NGC 7814, 4594 and 7123 are given in sections 4.4, 4.5 and 4.6 respectively. The remaining galaxies in the sample are discussed in section 4.7 together with the $V_m(0)/\sigma_o$ vs $\epsilon_{max}$ diagram. Section 4.8 discusses the degree of success of the models and summarizes the conclusions.
4.2 SELECTION OF GALAXIES

Suitable program galaxies, observable from Mt Stromlo, were selected according to the following criteria:

(i) the galaxies were to be very nearly edge-on \((i \sim 90^\circ)\) to facilitate the identifying, with a minimum of ambiguity, the observed photometric components of the galaxy, namely the bulge, lens and disc.

(ii) the galaxies were to have a large bulge-to-disc luminosity ratio, with the bulge large and bright enough for detailed surface photometry. This biased the sample towards SO and early type spiral galaxies. The SO galaxies in some cases (NGC 128, 4762 and 4179) have the added advantage for our purpose of no measurable internal absorption in the disc.

(iii) the bulges of the galaxies were to cover as wide a range as possible in morphological form, from almost spherical (eg NGC 4594) through box-shape (eg NGC 1381) to peanut-shape (eg NGC 128), in order to gain some insight into how these different bulge morphologies arise. For example, are the square-shaped bulges dynamically intermediate between the spheroidal and peanut-shaped bulges?

(iv) the galaxies should appear "normal". (However, in spite of the clear tidal interaction with NGC 127, NGC 128 was included because it is an extreme example of the peanut-shaped bulges). NGC 3379 was also included in the sample to check the accuracy of the surface photometry reduction technique (see Chapter III). The selected final list of galaxies is given in Table 4.1.
### TABLE 4.1
PARAMETERS OF GALAXIES STUDIED

<table>
<thead>
<tr>
<th>NGC</th>
<th>R.A.(1950)</th>
<th>Dec.(1950)</th>
<th>Type</th>
<th>Dist(Mpc)</th>
<th>m_9</th>
<th>log D_25</th>
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<td></td>
<td>h m s</td>
<td>o' ''</td>
<td></td>
<td></td>
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<td>E1</td>
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<td>10.0\textsuperscript{a}</td>
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<td>7814</td>
<td>0 0 14</td>
<td>+15 52 0</td>
<td>SA\textsuperscript{a}</td>
<td>25.0\textsuperscript{e}</td>
<td>11.4</td>
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<td>-</td>
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<td>12 37 23</td>
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<td>4762</td>
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<td>+11 30 5</td>
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<td>12.3\textsuperscript{a}</td>
<td>11.1</td>
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<td>16.7\textsuperscript{a}</td>
<td>11.8</td>
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<tr>
<td>128</td>
<td>0 26 40</td>
<td>+2 35 20</td>
<td>SA\textsuperscript{a}</td>
<td>55.4\textsuperscript{a}</td>
<td>12.6</td>
<td>1.53</td>
</tr>
<tr>
<td>Ham I</td>
<td>1 6 28</td>
<td>-80 34 28</td>
<td>SA\textsuperscript{a}</td>
<td>56.7\textsuperscript{a}</td>
<td>12.5\textsuperscript{a}</td>
<td>-</td>
</tr>
</tbody>
</table>

* RC2
a this work
b de Vaucouleurs and Capaccioli (1979)
c Williams (1977)
d \( H_o = 75 \text{ km.s}^{-1}. \text{ Mpc}^{-1} \)
e Kormendy and Illingworth (1981)
4.3 FITTING THE MODELS

Using the observed two-dimensional surface brightness distribution for each of the sample galaxies, our aim was to find a model from the model parameter space, capable of accurately reproducing the observed two-dimensional light distribution and the observed kinematical properties of the bulge. It was found empirically that the search for the best fitting model was expedited by first examining the minor axis profile. The observed minor axis profile was plotted as log surface brightness versus log radius and fitted by eye to a series of similar model plots until a satisfactory fit was obtained. Within the domain of realistic disc scale lengths (a and b) and disc-to-bulge mass ratio, Q (= \( \mu_{bd} M_d / M_b \)) for the particular galaxy under study, it was found that the parameters a, b and Q have only a second order effect on the projected two-dimensional model surface brightness distribution. The central dimensionless potential \( W^0_b \), and the rotation parameter \( \gamma \), have the dominant effect, and together determine the gross overall photometric and kinematic properties of the models. In view of this, the models were iterated around the choice of a \( (W^0_b, \gamma) \) pair after a, b and Q were carefully chosen but kept constant. Using this scheme, very satisfactory fits were obtained for all the galaxies modelled in this sample. In attempting to fit the models to the data, it was found that the minor axis profile constrained the models to a small area of \( (W^0_b, \gamma) \) space. In other words, once the minor axis was successfully fitted, the major axis and intermediate profiles were largely determined. This means that the minor axis profile is
sensitive to the rotation in a directly measurable way. Once the minor axis had been successfully fitted, small adjustments to $W_0$ and $\gamma$ were made by examining the detailed flattening of the isophotes to bring the model and observation into final agreement. However, as stated above, only small adjustments could be made if the quality of the minor axis fit was to be retained.

From the final adopted model, the spatial variation of the projected rotation velocity and velocity dispersion were computed and compared with observation. The quantity $V_m(0)/\sigma_0$, of particular dynamical importance was also calculated, where $V_m(0)$ is the maximum projected rotation velocity on the major axis and $\sigma_0$ is the central velocity dispersion.

4.4 NGC 7814

(i) Photometry

The SAO edge-on galaxy NGC 7814 was the first galaxy in the sample to be fitted. The bulge is of intermediate flattening and has the largest apparent size of the sample galaxies ($D_{25} = 6.3 \text{ min arc; RC2}$). The bulge is cut by a very distinct dust lane on the major axis, extending to about 7" above the galactic plane. This is clearly seen in Figure 4.1. The first extensive surface photometry of NGC 7814 was undertaken by van Houten (1961), in B of the UBV system. The deepest plate available for this study was a two hour V plate (plate 4119, IIa-D + GG495) taken at the f/8 focus of the 1.0 m telescope at Siding Spring Observatory. Using this plate, the galaxy surface brightness distribution in V was mapped, using the surface photometry techniques described in Chapter III.
Figure 4.1  **Upper Panel**  The galaxy NGC 7814 from a two hour V band (IIa-D+GG495) exposure taken at the f/8 focus of the 1.0 m telescope. South is up and east to the left.

**Lower Panel**  Pseudo isodensity contours of the above plate to accentuate the outer bulge. The disc is slightly warped due to distortions in the television display.
<table>
<thead>
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<th>r (arc secs)</th>
<th>( \mu_V^{(SW)} )</th>
<th>( \mu_V ) (NE)</th>
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<th>( \mu_V )</th>
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</thead>
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Note: The mean of the reflected minor axis surface brightness is used for \( r > 10 \) arc sec.

* magnitudes per square arc second.
Figure 4.2 Minor axis profile of NGC 7814 in V. The units of surface brightness are mag per square arc second. The observations, shown as solid dots, have been reflected and averaged about the major axis. The solid line shows the best fitting model (see text) to the data. Note the strong absorption from the dust lane for log $r > 0.8$ sec.
NGC 7814 minor axis

- obs NE + SW
- model
Within the accuracy and limit of the photometry ($\mu_\nu \approx 26$ $\mu_\nu$), the surface brightness distribution was found to be very symmetric about the minor axis for $r > 10''$. Hence the adopted minor axis profile tabulated in Table 4.2 and shown in Figure 4.2 is the mean of the NE and SW profiles. The de Vaucouleurs $r^{-1/4}$ law was found to accurately fit the minor axis over the entire surface brightness range observed, excluding the central 7'' obscured by dust. The linear least squares fit was 

$$
\mu_\nu = 3.75 \, r^{-1/4} + 13.08
$$

where $r$ is in arc seconds.

We now come to the construction of a suitable model. In the absence of any quantitative data concerning the scale lengths $a$ and $b$ of the disc, a scale height ratio of $b/a = 0.1$ was chosen as reasonable from examination of a direct plate. Similarly, the mass ratio $M_d/M_b$ was adopted to be 0.25. As mentioned above, $a$, $b$ and $M_d/M_b$ are "soft" parameters for the overall properties of the bulge; the properties are dominated by changes in $W_0^b$ and $\gamma$. The central potential of the adopted model for NGC 7814 was deepened from $-8.5$ to $-9.51$ by the introduction of the disc. The main contribution from the disc to the total potential field for the best model, depends only on the mass ratio and not on the disc scale lengths: if the adopted mass ratio was a little in error then this would result in a slightly different central bulge potential being chosen to compensate.

Using the above disc parameters, a model was constructed which is in excellent agreement with the observed projected brightness distribution in V; its parameters are given in Table 4.3.
TABLE 4.3
MODEL PARAMETERS FOR NGC 7814

<table>
<thead>
<tr>
<th>$v_b^0$</th>
<th>$\gamma$</th>
<th>$M_d/M_b$</th>
<th>a</th>
<th>b</th>
<th>$r_t$</th>
<th>$v_d^0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-8.5</td>
<td>0.2</td>
<td>0.25</td>
<td>10</td>
<td>1</td>
<td>45.4</td>
<td>-1.01</td>
</tr>
</tbody>
</table>

Here $r_t$ is the tidal radius of the model, in units of core radii. Figure 4.2 illustrates the quality of the model fit to the observed minor axis. The two-dimensional fit of the projected model to the observed surface brightness distribution is illustrated in Figure 4.3. Due to the observational and model symmetry of the bulge, the data has been reflected and averaged about the minor axis to improve the signal-to-noise ratio of the data. The upper panel shows the model fit to the unsmoothed reduced data. The increased noise from the plate for the fainter isophotes, limits the fit in the outer parts. To extend the range of the fit, the lower panel shows the same data after the outer regions have been smoothed with a two-dimensional Gaussian filter. The half-widths of the smoothing beam ranged from 1"x1" for the fifth faintest isophote ($\mu_V = 21.91$) to 3"x3" for the faintest isophote. ($\mu_V = 23.91$). The isophotes are drawn in intervals of 0.50 $\mu_V$ in V with the faintest (outermost) isophote in the lower panel of Figure 4.3 at a surface brightness of 23.9 $\mu_V$. The faintest isophote in the upper panel is then 22.9 $\mu_V$. For the sake of clarity in the inner region of the galaxy, the model fit has only been shown to $\mu_V = 23.9$ ($\sim 24.9 \mu_B$). The minor axis fit illustrates the quality of the fit at fainter surface brightnesses. It can be seen from Figures 4.2 and 4.3 that the model is in excellent agreement with the observed
Figure 4.3 **Upper Panel** Observed surface brightness distribution of NGC 7814 in V. The isophotes are in 0.5 mag sq arc sec$^{-1}$ intervals with the faintest isophote at 22.91 \( \mu_V \). The data has been folded and averaged about the minor axis. The solid lines show the adopted model (see text). Note the prominent dust lane and the luminous disc above.

**Lower Panel** Same as above except that the range of the fit has been extended by smoothing the outermost five isophotes with a two-dimensional Gaussian filter. The smoothing beam ranged from 1"x1" for the fifth faintest isophote (\( \mu_V = 21.91 \)) to 3"x3" for the faintest isophote (\( \mu_V = 23.91 \)).
two-dimensional light distribution in V to the limit of the photometry.

(ii) Kinematic Data

It has been shown above that the models are capable of successfully reproducing the observed light distribution of NGC 7814 in V. However, this alone is insufficient to establish the models as being truly representative of the current dynamical state of galactic bulges. For example, in the earlier attempts (e.g., Prendergast and Tomer 1970, Wilson 1975) to fit rotating models to elliptical galaxies, the model fits to the observed brightness distributions were fairly successful. However, subsequent kinematical observations (e.g., Illingworth 1977) indicated that the models possessed at least twice as much net rotation as observed. Clearly then, the models must also be consistent with the observed kinematical data, since the kinematical data places such severe constraints on the models.

All the kinematical data on NGC 7814 is taken from Kormendy and Illingworth (1981); hereafter KI). They have mapped the spatial distribution of velocity dispersion and rotation velocity in the bulge along cuts parallel and perpendicular to the major axis. The cuts parallel to the major axis were made at heights of 4", 12" and 18" above the disc, while the cuts perpendicular to the disc were at 20" and 40" from the minor axis. The minor axis was also observed.

(iii) Model Discussion

A direct kinematical check on the models is to compute the model's spatial variation of velocity and velocity dispersion, and
compare this directly with the observations of Kl. Using the model length scale (1 dimensionless model unit = 3.8") determined from the fitted model to the observed light distribution, simulated cuts at 4", 12" and 18" parallel to the major axis were made. Similar cuts were also constructed at 20" and 40" parallel to the minor axis, as well as along the minor axis. To match the model data, there is only one degree of scaling allowed (which is equivalent to choosing $M/L$ eg the central velocity dispersion $\sigma_o$ or the maximum rotation velocity $V_m(0)$, but not both independently. There is no freedom to choose a lengthscale, since this is fixed when the model is fitted to the surface photometry. In view of the difficulty in reliably estimating $V_m(0)$ for the major axis from the offset cuts, it was decided to use the central velocity dispersion as the free parameter. Instead of adopting the observed point central value from the minor axis cut, the value used was that obtained by extrapolating a linear fit over all the points back to $z=0"$. (For NGC 7814 this was the same value as the observed point central dispersion; 167 km.s$^{-1}$). This method of choosing $\sigma_o$ has the advantage of utilising all of the minor axis dispersion data and is not dependent on errors associated with individual point measurements. Also, this $\sigma_o$ reflects to a large extent the global change in $\sigma(r)$. With this value of $\sigma_o$, $V_m(0)$ for the major axis is 123 km.s$^{-1}$, since $(V_m(0)/\sigma_o)_{\text{model}} = 0.74$. Figure 4.4 compares the model kinematical distributions with the observed kinematical distributions. It is clear that the agreement is excellent within the observational errors. The only systematic deviations of the models from the data appear in the outermost determination of velocity for the 4", 12" and 18" parallel cuts. In
Figure 4.4 Rotation velocities $V$ and velocity dispersions for NGC 7814 from Kormendy and Illingworth (1981), shown as filled circles and squares. The slit positions for each data set are given in the title; $\perp$ indicates slits perpendicular to the major axis while $\parallel$ indicates slits parallel to the major axis. The offsets are in seconds of arc and the velocities are in km.s$^{-1}$. All data are folded about the nucleus, with different sides represented by the filled circles and filled squares. The crosses are derived from intersecting slit positions and serve as internal consistency checks. The dashed lines indicate the fit of the adopted model from this work, scaled to $\sigma_0 = 167$ km.s$^{-1}$. 
all cases, the observed velocity of the last point is about 80 km.s\(^{-1}\) above that predicted by the model, but it is not yet clear how seriously these deviations should be taken (see for comparison the second outermost point for the minor axis \(V(r)\) data in Figure 4.4).

The quantity \(\frac{V_m(0)}{\sigma_o}\) is of global dynamical significance when comparing the bulges of disc galaxies. This quantity is a measure of the fraction of dynamical support offered by ordered motion (rotation) to random motion (dispersion) i.e \((\frac{V_m(0)}{k \sigma_o})^2\) measures the ratio of rotational to random kinetic energy. \(k\) is a constant depending on the amount of velocity anisotropy (2 \(< k \leq 3\)). This follows the practice of Illingworth (1977) who also used the central velocity dispersion. However KI use a mean global velocity dispersion. The central velocity dispersion is used here for two reasons. Firstly, it is a more easily observable quantity (only recently have spatial dispersions been obtainable) and hence less likely to suffer from observational error. Secondly, it is not clear exactly how a mean global dispersion \(<\sigma>_o\) should be defined, analogous to where or how the observed ellipticity \(e\) should be measured. The quantity \(\frac{V_m(0)}{\sigma_o}\) is unambiguous. However, Binney (1980a,b) noted that this difference is minimal. KI use a mean dispersion to minimise any population differences (if they exist) which determine \(V_m(0)\) and \(\sigma\). Since the rotation curve on the major axis is not directly observable because of the disc, KI computed a value \(V_m\) corrected to \(z=0''\) i.e \(V_m(0)\). They found \(V_m(0) = 123 \pm 12\) km.s\(^{-1}\). Their central velocity dispersion determined from the minor axis cut was \(167 \pm 18\) km.s\(^{-1}\). This gives \((\frac{V_m(0)}{\sigma_o})_{obs} = 0.74 \pm 0.11\). The error is formal. This is to be compared with the same quantity
derived from the adopted model; \( \frac{V_m(0)}{\sigma_0}_{\text{mod}} = 0.74 \), in excellent agreement with observation. The maximum ellipticity \( \epsilon_{\text{max}} = 1 - b/a \) derived from the model was 0.44.

We can also calculate the observed mass-to-light ratio \((M/L)_v\), from the observed velocity dispersion, surface photometry and the dynamical model assumptions. From the model assumptions of Chapter II, the observed mass-to-light ratio in \( V \) is (cf Wilson 1975)

\[
(M/L)_V = 0.0109. \frac{\lambda \Sigma_0}{\sigma_0^2} \frac{\sigma_0^{*2}}{I_v^o r_c} \cdot \frac{H_o}{V_r} \frac{M_0}{L_v^o}
\]  

(A)

where \( \Sigma_0 \) is the dimensionless model central surface brightness,

\( \sigma_o^* \) is the observed central velocity dispersion in \( \text{km.s}^{-1} \),

\( \sigma_0 \) is the dimensionless model central velocity dispersion,

\( r_c \) is the observed core radius in arc seconds,

\( I_v^o \) is the central surface brightness measured in units, of one 20th mag star per square arcsec,

\( H_o \) is the Hubble constant \((75 \text{ km.s}^{-1}\text{Mpc}^{-1})\),

\( V_r \) is the observed radial velocity of the galaxy, and

\( \lambda \) is a model dependent parameter.

The term \( \frac{\lambda \Sigma_0}{\sigma_0^2} \) is model dependent, but never differs much from 18. For the NGC 7814 model this term has a value of 18.87.

The corresponding relation for the blue mass-to-light ratio is

\[
(M/L)_B = 0.0062. \frac{\lambda \Sigma_0}{\sigma_o^2} \frac{\sigma_0^{*2}}{I_B^o r_c} \cdot \frac{H_o}{V_r} \frac{M_0}{L_B^o}
\]  

(B)

Assuming a distance of 25 Mpc (KI), a core radius of 4" (485 pc) and \( \sigma_o = 167 \text{ km.s}^{-1} \), the model gives a mass-to-light ratio in \( V \) of
7.0 using equation (A). This is consistent with recent observations by Faber and Jackson (1976) who find a mean of 7 for the observed mass-to-light ratio in the nucleus regions of elliptical galaxies. It must be noted however that this value is derived from observations near the centre of the galaxy and may not be the same as the ratio of total mass to total luminosity. On the other hand, it has been seen that a very satisfactory fit to the surface photometry and kinematics was possible using a constant M/L. The constancy of M/L in bulges is supported by observations of NGC 4594 by Faber et al. (1977) who reported no discernible increase in M/L over the easily visible halo. They conclude that if NGC 4594 is typical of disc galaxy bulges, increases in M/L if they occur, are to be found only at very low surface brightness where they will be very difficult to measure directly. Their findings are at odds with those of Ostriker, Peebles and Yahil (1974) who argue that M/L increases dramatically with radius.

4.5 NGC 4594

(i) Photometry

The second galaxy from the sample to be modelled was the "Sombrero" galaxy NGC 4594. This SA nearly edge-on system possesses a large spheroidal bulge totally enveloping at large radii a fairly prominent disc (see Figure 4.5). Because of its large angular size \(D_{25} = 8.9 \text{ min arc}; \) RC2) and brightness \(M_B \sim 9.3; \) RC2), NGC 4594 has been extensively observed. The most comprehensive surface photometry to date is by van Houten (1961) and Boroson (1980). It was also previously observed by Williams and Hiltner (1943), Oort
Figure 4.5  **Upper Panel** The galaxy NGC 4594 from a $2^{h}33^{m}$ V band (103a-D+GG495) exposure taken at the f/8 focus of the 1.0m telescope. North is up and west is to the right.

**Lower Panel** Pseudo isodensity contours of the above plate to accentuate the bulge. The horizontal lines are from the television screen.
# Table 4.4

Observed Minor Axis Profile of NGC 4594 in V

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* magnitudes per square arc second.
Figure 4.6 The minor axis profile of NGC 4594 in V. The crosses show the observed minor axis profile to the south of the major axis and the solid dots to the north. The solid line shows the best model fit to the average minor axis profile. The units of surface brightness are magnitudes per square arc second in V and r is in seconds of arc. Note the strong disc absorption in the southern minor axis profile.
Figure 4.7  **Upper Panel** Observed surface brightness distribution of NGC 4594 in V. The isophotes are in 0.5 mag per square arc second intervals with the faintest isophote at 23.20 \( \mu_V \). The southern half of the galaxy has been reflected about the major axis. The solid curved lines show the adopted best fitting model (see text) to the data.

**Lower Panel** Same as above except that an average of the northern and southern halves of the galaxy are shown, reflected and averaged about the major axis. The fit has also been extended by smoothing the outermost isophote with a two-dimensional Gaussian filter of 6"x6". The faintest isophote is at a surface brightness of 23.70 \( \mu_V \).
(1946) and de Vaucouleurs (1961) among others. For this study, the surface photometry was derived from a \(2^\text{h}33^\text{m}\) exposure using a 103a-D+GG495 plate-filter combination (plate 2480). Since no photoelectric drift scans were available for this galaxy, the photographic photometry was calibrated using van Houten's central surface brightness in B and a central colour of \(B-V=1.05\) (David Griersmith; private communication). The minor axis profile is shown in Figure 4.6 and tabulated in Table 4.4. Aside from the prominent dust lane, the minor axis profile in \(V\) is asymmetric for \(r \gtrsim 55''\), with differences up to 0.2 \(\mu \) \(V\). For the purposes of fitting the equatorially symmetric model, a mean minor axis profile was used by reflecting the minor axis about the major axis, and averaging. The mean minor axis profile was again found to be well represented by an \(r^{1/4}\) law to the limit of the photometry (\(\gtrsim 26 \mu \) \(V\)). A linear least squares fit gave

\[
\mu \text{ }_V = 2.74 \text{ } r^{1/4} + 13.09,
\]

where \(r\) is in arc seconds.

Again, using the same disc parameters as before and the fitting techniques detailed in section 4.3, a model was constructed to fit the observed light distribution of NGC 4594 in \(V\); the parameters are given in Table 4.5.

### Table 4.5

**Model Parameters for NGC 4594**

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<tr>
<th>(W_b^0)</th>
<th>(\gamma)</th>
<th>(M_d/M_b)</th>
<th>(a)</th>
<th>(b)</th>
<th>(r_t)</th>
<th>(W_d^0)</th>
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</thead>
<tbody>
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<td>0.25</td>
<td>10</td>
<td>1</td>
<td>67.7</td>
<td>-1.01</td>
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</table>
Figure 4.6 shows the minor axis model fit (solid line) to the mean profile data. Apart from the central region of the data effected by the seeing point spread function and the disc, the model is in good agreement over the entire range of surface brightness measured. The two-dimensional fit is shown in Figure 4.7. The upper panel shows the model (solid curved lines) fitted to the southern half of the galaxy, reflected about the major axis. The isophotes are at 0.5 magnitude spacing with the faintest isophote at 23.20 μ_v. The lower panel shows the model fit to NGC 4594 where the northern and southern halves have been averaged and reflected about the major axis. The faintest isophote (23.70 μ_v) has been plotted from a region smoothed with a two-dimensional Gaussian filter of halfwidth 6". The isophotes are drawn to only 23.70 μ_v for the same reasons as before. Apart from the disc light near the major axis, the overall two-dimensional fit is in excellent agreement with observation.

(ii) Kinematic Data and Model Discussion

As before, once a suitable model had been found to fit the observed light distribution, a comparison of the model kinematical properties with those observed was made. Kinematical data for NGC 4594 was available from several investigators. HI observations have been made by Faber et al (1977), Gallagher, Faber and Balick (1975) among others. Velocity dispersions have been measured by Williams (1977), Whitmore, Kirshner and Schechter (1979; hereafter WKS), Whitmore and Kirshner (1981) and KI. Optical rotation curves have been observed in the disc by Faber et al (1977) and in the bulge by KI. Again, it is the data from KI that is used for comparison.
here; their estimates of velocity dispersion are in good agreement with other authors, and more importantly, they have the only spatial measurements of dispersion and rotation in the bulge.

Extrapolating the run of velocity dispersion obtained by KI along the major axis back to the centre, one finds \( \sigma_0 = 225 \pm 15 \text{ km.s}^{-1} \). For \( r < 30" \) on the major axis, the measured velocity dispersion should be that of the bulge alone due to its total dominance close to the centre. By comparison, Williams finds \( \sigma_0 = 215 \pm 35 \text{ km.s}^{-1} \) while WKS find \( 263 \pm 32 \text{ km.s}^{-1} \). In view of these two independent measurements, it would seem that the above adopted value is a fair compromise. So, using \( V_m(0) = 130 \pm 24 \text{ km.s}^{-1} \) from KI, the observed value of \( V_m(0)/\sigma_0 \) is \( 0.58 \pm 0.11 \). The corresponding model value is 0.56, again in excellent agreement and well within the quoted observational error of 0.11 for \( V_m(0)/\sigma_0 \).

With freedom again to scale either \( \sigma_0 \) or \( V_m(0) \), while preserving their ratio, Figure 4.8 shows the model kinematical distributions compared to those observed by KI. The dashed lines show the model predictions scaled to \( \sigma_0 = 225 \text{ km.s}^{-1} \). The agreement between the model and observation kinematics is excellent. Apart from the expected difference between the model and observation on the major axis (where the disc velocities are measured), the model rotation is about 20 per cent below that observed for the 30" parallel cut. However, some or all of this discrepancy can be understood by inspection of the last panel in Figure 4.8, indicating detectable contribution from the disc even at a height of 30". The models are in even better agreement at larger displacements from the major axis where there is no doubt that the
Figure 4.8 Rotation velocities $V$ and velocity dispersions for NGC 4594 from Kormendy and Illingworth (1981), shown as filled circles and squares. The slit positions for each data set are given in the title; $\perp$ indicates slits perpendicular to the major axis, while $\parallel$ indicates slits parallel to the major axis. The offsets are in seconds of arc and the velocities are in km.s$^{-1}$. All data are folded about the nucleus, with different sides represented by the filled circles and filled squares. The crosses are derived from intersecting slit positions and serve as internal consistency checks. The dashed lines indicate the fit of the adopted model from this work, scaled to $\sigma_0 = 225$ km.s$^{-1}$. 
bulge alone is being measured. It is interesting to note that the model data is in slightly better agreement with the crosses, derived from intersecting slit positions, especially for the dispersion profiles.

A determination of M/L can also be made for NGC 4594 from the photometry and the adopted model. The Whitmore-Kirshner relation between absolute magnitude and velocity dispersion gives a distance of 15.8 Mpc for \( \sigma_0 = 225 \, \text{km.s}^{-1} \) and \( M_B = 9.3 \) (RC2). Application of the Hubble law yields a distance of 12.4 (\( H_0 = 75 \, \text{km.s}^{-1}. \, \text{Mpc}^{-1} \)) for an assumed velocity of 930 km.s\(^{-1}\) (Faber et al 1977). From these two values a distance of 14.1 Mpc is adopted, in good agreement with Williams, who derived a distance of 13.3 Mpc. Using this value and a core radius of 6" (387 pc), the mass-to-light ratio in V from equation (A) is 7.6. This compares favourably with 8.4 in the photographic band determined by Williams (corrected to an assumed \( H_0 = 75 \, \text{km.s}^{-1}. \, \text{Mpc}^{-1} \)). Faber et al find M/L = 3 to 4 for the spheroidal component alone in the B passband from rotational motions of HI in the disc and optical absorption line velocities.

4.6 NGC 7123

(i) Photometry

The third and last galaxy from the sample to have a model constructed was the SAO galaxy NGC 7123. This galaxy is morphologically similar to NGC 7814, characterised by a large moderately flattened bulge cut by a distinct dust lane (see Figure
Figure 4.9 Upper Panel The galaxy NGC 7123 from a six hour B band (IIa-O+GG385) exposure taken at the f/8 focus of the 1.0 m telescope. North is up, east to the right.

Lower Panel Pseudo isodensity contours of the above plate to accentuate the outer bulge. Note the luminous disc.
<table>
<thead>
<tr>
<th>$r$ (arc secs)</th>
<th>$\mu_B$ (SW)*</th>
<th>$\mu_B$ (NE)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21.77</td>
<td>21.50</td>
</tr>
<tr>
<td>4</td>
<td>20.47</td>
<td>20.86</td>
</tr>
<tr>
<td>6</td>
<td>20.80</td>
<td>21.37</td>
</tr>
<tr>
<td>7</td>
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<td>21.58</td>
</tr>
<tr>
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</tr>
<tr>
<td>83</td>
<td>26.75</td>
<td>26.10</td>
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</table>

* magnitudes per square arc second
Figure 4.10 Minor axis profile of NGC 7123 in B. The units of surface brightness are magnitudes per square arc second. The NE and SW profiles are identified by their respective symbols. The solid line shows the best fitting model (see text) to the data. Note the strong absorption near the major axis.
NGC 7123 minor axis

- obs SW
- obs NE
- model
Figure 4.11 **Upper Panel** Observed surface brightness distribution of NGC 7123 in B. The isophotes are in 0.5 mag per square arc second intervals with the faintest isophote at 24.2 B. The data has been folded and averaged about the minor axis. The solid curved lines show the adopted best fitting model (see text) to the data.

**Lower Panel** Same as above except that the range of the fit has been extended by smoothing the outermost four isophotes with a two-dimensional Gaussian filter of halfwidth 3"x3". The faintest isophote is at a surface brightness of 24.70 B. The dashed line shows the position of the spectrograph slit, displaced 4" SW of the major axis. The isophotes indicate that the slit was positioned along the disc and therefore not measuring the bulge alone.
NGC 7123

NGC 7123
4.9). However, it is much more distant (47.5 Mpc; \( H_0 = 75 \) km.s\(^{-1}\). Mpc\(^{-1}\)) than NGC 7814, and hence fainter. No previously published photometry was available. The surface brightness distribution in B was mapped using a IIa-O + GG385 plate-filter combination (plate 4113) exposed for six hours with the 1.0 m telescope. Figure 4.10 shows the minor axis profiles in B. The data are tabulated in Table 4.6. Accurate surface photometry was difficult below \( \mu_B = 25 \), due to two nearby bright stars; to this surface brightness, the photometry was considered reliable. NGC 7123 is not exactly edge-on as evidenced by asymmetry in the minor axis profiles of Figure 4.10. This is also evident from a direct photograph (see Figure 4.9). The \( r^{1/4} \) law was again found to represent well the minor axis luminosity profile to \( \mu_B = 26 \). A linear least squares fit to the mean minor axis profile gave,

\[
\mu_B = 4.01r^{1/4} + 14.86
\]

where \( r \) is in seconds of arc.

The same model parameters as NGC 7814 were also found to accurately model the observed surface brightness distribution of NGC 7123 in B. This is however not surprising in view of their similar morphological appearance. The model parameters are listed again in Table 4.7.

<table>
<thead>
<tr>
<th>TABLE 4.7</th>
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</thead>
<tbody>
<tr>
<td>MODEL PARAMETERS FOR NGC 7123</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>( W_0^o )</th>
<th>( \gamma )</th>
<th>( M_d/M_b )</th>
<th>( a )</th>
<th>( b )</th>
<th>( r_t )</th>
<th>( W_0^o )</th>
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</thead>
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<td>10</td>
<td>1</td>
<td>45.4</td>
<td>-1.01</td>
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</table>
The uniqueness of the model was more difficult to verify in this case however, due to the minor axis profiles being less tightly defined. There may be models close in parameter space which would better model the light distribution if better photometry were available. The two-dimensional fit of the model is illustrated in Figure 4.11. The figure panels follow the same convention as those for NGC 7814; the data has been reflected and averaged about the minor axis. In the lower panel, the four outermost isophotes have been smoothed in the way described above. The isophotes are spaced 0.5 magnitudes per square arc second apart, with the faintest isophotes at $24.20 \mu_B$ and $24.70 \mu_B$ for the upper and lower panels respectively. Again, the observed light distribution is well modelled using the same model as NGC 7814. On this basis alone, it would be expected that the bulges are also kinematically similar.

(ii) Kinematic Data

Kinematical data was obtained with the IPCS on the AAT using the acquisition and reduction techniques described in Chapter III. A total exposure of 6000s was made with the spectrograph slit displaced 4" south-west but parallel to the major axis. The corrected systemic velocity (for solar motion with respect to the velocity centroid of the local group of galaxies - see RC2) was found to be $3568 \pm 30 \text{ km.s}^{-1}$, placing NGC 7123 at a distance of 47.5 Mpc ($H_0 = 75 \text{ km.s}^{-1}. \text{ Mpc}^{-1}$). The observed rotation curve is tabulated in Table 4.8 and shown in the upper panel of Figure 4.12. The velocity dispersion profile, reflected and averaged about the minor axis is tabulated in Table 4.9 and shown in the lower panel of Figure 4.12. Figure 4.13 shows the best fit of the
### TABLE 4.8

OBSERVED ROTATION CURVE OF NGC 7123 4"SW \(\|^a\)

<table>
<thead>
<tr>
<th>(r) (arc secs)</th>
<th>(V) (NW)(|^b)</th>
<th>(V) (SE)</th>
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<td>0 (\pm 10)</td>
</tr>
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<td>-234</td>
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<tr>
<td>33</td>
<td>232 (\pm 50)</td>
<td>-278 (\pm 50)</td>
</tr>
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<tr>
<td>46</td>
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\(\|^a\) relative to a corrected systemic velocity of 3568 km.s\(^{-1}\).

\(\|^b\) units in km.s\(^{-1}\).

### TABLE 4.9

OBSERVED VELOCITY DISPERSION PROFILE OF NGC 7123 4"SW \(\|^*\)

<table>
<thead>
<tr>
<th>(r) (arc sec)</th>
<th>(\text{(km.s}^{-1}))</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>170 (\pm 25)</td>
</tr>
<tr>
<td>2.5</td>
<td>150 (\pm 25)</td>
</tr>
<tr>
<td>5</td>
<td>150 (\pm 25)</td>
</tr>
<tr>
<td>7.5</td>
<td>125 (\pm 25)</td>
</tr>
<tr>
<td>10</td>
<td>125 (\pm 40)</td>
</tr>
</tbody>
</table>

\(\|^*\) reflected and averaged about the minor axis.
Figure 4.12 **Upper Panel** Observed optical rotation curve of NGC 7123 4" SW and parallel to the major axis. The data has been plotted relative to a corrected systemic velocity of $3568 \text{ km}\cdot\text{s}^{-1}$. The error associated with each observation is also shown. The dashed line shows the model rotation curve scaled to $\sigma_o = 175 \text{ km}\cdot\text{s}^{-1}$. The large discrepancy is due to the observation measuring the disc rotation and not the bulge rotation. This is clearly shown by examination of the slit position from the lower panel of Figure 4.11.

**Lower Panel** The observed velocity dispersion profile of NGC 7123 4" SW and parallel to the major axis, shown as open circles. The data has been reflected and averaged about the minor axis. The dashed line shows the model dispersion profile scaled to $\sigma_o = 175 \text{ km}\cdot\text{s}^{-1}$. 

Figure 4.13 Velocity dispersions derived from the power spectra of NGC 7123. Each panel shows the best fit from the velocity broadened template star grid (see Figure 3.3) to the power spectrum of each row in the galaxy spectrum. The slit was positioned 4" SW and parallel to the major axis. Each panel is labelled with its corresponding distance from the minor axis. The thick solid line is the smoothed power spectrum of each spectral row and the thin solid line is the adopted best fit, in units of km.s$^{-1}$. The dashed lines show for comparison the slope of the template power spectra for dispersions typically 50 km.s$^{-1}$ above and below the adopted value.
convolved template power spectrum to the smoothed power spectra of each row in the galaxy spectrum. The thick solid line indicates the galaxy power spectrum and the thin solid line is the best fitting convolved standard star power spectrum taken from Figure 3.3. The units of $\sigma$ are $\text{km.s}^{-1}$.

(iii) Model Discussion

The rotation curve shown in Figure 4.12 extends to the turnover point ($r \sim 20''$), enabling a fairly unambiguous determination of $V_m$ of $250 \pm 30$ $\text{km.s}^{-1}$ at $z=4''$. The velocity dispersion on the minor axis, found by extrapolating the run of $\sigma$ back to $r=0''(z=4'')$ was $\sigma = 170 \pm 20$ $\text{km.s}^{-1}$. In the absence of additional parallel and perpendicular cuts from which $\sigma_o$ could be calculated, the model for NGC 7123 was used to estimate $\sigma_o$. From the scale length determined from the model fit to the surface brightness distribution (1 dimensionless model unit = 3.17''), a cut was made at an offset equivalent to 4'' parallel to the major axis. Comparing the run of $\sigma$ along this cut with the model major axis, enabled a central velocity dispersion of $\sigma_o = 175 \pm 10$ $\text{km.s}^{-1}$ to be estimated. Hence an observed value of $(V_m/\sigma_o)_{\text{obs}} = 1.43 \pm 0.24$ was found which appears very large for the observed flattening. The model value, $(V_m(0)/\sigma_o)_{\text{mod}} = 0.74$, is almost a factor of two smaller. However, examination of the position of NGC 7123 in the $V_m(0)/\sigma_o - \nu_{\text{max}}$ plane (see section 4.7) and the position of the slit from Figure 4.11, strongly suggests that this large discrepancy can be understood; the rotation measured is that of the disc and not the bulge. It is well known that the disc rotates significantly faster than the bulge (Faber et al. 1977,
Kormendy and Illingworth 1981) – see also Figure 4.8 from KI. The model predicts the true bulge should have a $V_m(0) = 130 \text{ km.s}^{-1}$ on the major axis, similar to that observed for NGC 7814.

It was unfortunate that the slit was not positioned further from the major axis than 4", as this would have permitted a valid comparison of the observed and model kinematical properties. The spectroscopic observations were made before the photometry was complete: in the usual compromise between placing the slit far enough from the disc to avoid it and yet not so far that the bulge surface brightness is too low to measure, we erred. It would be a worthwhile investment to observe this galaxy again.

Assuming a distance of 47.5 Mpc to NGC 7123, $\sigma_0^* = 175$ km.s$^{-1}$, $r_c = 3.2"$ and $A_B = 0.38$ (RC2) the blue mass-to-light ratio is 10.8, slightly larger than recently observed mass-to-light ratios in B for elliptical and disc galaxy bulges. However, some or all of this slightly larger value could be attributed to an error in $r_c$ since the uniqueness of the model is questionable in view of the noisy data.

4.7 THE $V_m(0)/\sigma_0 \sim \epsilon_{\text{max}}$ PLANE AND OTHER SAMPLE GALAXIES

(i) The $V_m(0)/\sigma_0 \sim \epsilon_{\text{max}}$ plane

It has been shown by Binney (1980a), Kormendy (1980), Illingworth and Schechter (1981), and KI, that if one kinematical quantity characterises the rotation of a galaxy, it is $V_m(0)/\sigma_0$, the ratio of the peak rotation velocity on the major axis to the central line-of-sight velocity dispersion. In the interest of demonstrating conclusively that all bulges of disc galaxies are
rotationally flattened, it is useful to measure this quantity in as many disc galaxies as possible. Pursuant to this aim, the kinematical properties of a further four galaxies were observed, NGC 128, 4762, 4179 and Ham I.

However, before these quantities were determined, it was necessary to have an understanding of which easily observed kinematical quantity best describes the degree of rotational support for the bulge. The most popular method to date of achieving this is to determine the domain (or line) in the $V_m(0)/\sigma_0$ vs maximum ellipticity ($e_{\text{max}}$) plane, where a bulge with known ellipticity, $V_m(0)/\sigma_0$, and flattened only by rotation would lie. It is not clear however, how these dynamical quantities should be measured. For example, there has been some confusion in the literature as to whether the central velocity dispersion $\sigma_0$ or a "mean dispersion", $<\sigma>$ should be used. Binney (1978 and 1980a) computed theoretical lines in the $<V>/<\sigma>\sim\epsilon$ plane where $<V>$ and $<\sigma>$ are the mean rotation velocity and velocity dispersions respectively for models having different amounts of velocity anisotropy. However, these same predictions have been used when $\epsilon$ (where to define this is largely arbitrary) has been plotted against $V_m(0)/\sigma_0$ (eg Capaccioli, 1979) and $V_m(0)/<\sigma>$ (eg KI, 1981). KI used $<\sigma>$ and found their points lie close to the theoretical line of Binney. If $\sigma_0$ was adopted instead, most of their points fall below this line (see also Kormendy 1980). This is interesting in view of Binney (1979) finding that for bulges of constant ellipticity and isotropic velocity distribution, $<V>/<\sigma>\approx V_m(0)/\sigma_0$ to good accuracy.

In order partially to resolve this situation, the bulge dynamical models were used to construct their own domain in this
plane. The quantity used was the ratio of the maximum rotation velocity on the major axis, $V_m(0)$ to the central velocity dispersion $\sigma_0$. The observed value of $V_m(0)/\sigma_0$ could then be unambiguously compared with the theoretical value. Observationally, $\sigma_0$ is best defined by extrapolating the run of $\sigma$ on the minor axis back to the centre. Some ambiguity also exists as to where the ellipticity $\varepsilon$ should be measured. For example, should $\varepsilon$ be taken to be the maximum observed ellipticity, the value at the radius where $V_m(0)$ occurs, or mean ellipticity to some radius? For this work, the maximum ellipticity $\varepsilon_{\text{max}}$ is used. This should be a suitable quantity because it is readily measured and for bright galaxies occurs at a surface brightness well above the threshold of the photometry. In practice, it should not matter too much how $V_m$, $\sigma$ and $\varepsilon$ are defined, except that when comparing theory with observation, the same definition should be used.

Using the above definitions for $V_m$, $\sigma$ and $\varepsilon$, two sets of models were constructed. The first set consisted of a grid of models in which the bulge was rotationally flattened with no externally applied potential field. Models so constructed inhabit the horizontally hatched area (Region 1) of Figure 4.24. The upper and lower bounds defining this area are not intended to suggest strict limits to $V_m(0)/\sigma_0$, but merely to indicate the region in which most models fall for edge-on models. The vertically hatched area (Region 2) is defined by the locus of models with a disc included. The disc-to-bulge mass ratio was kept constant with $M_d/M_b = 0.25$. Also, the disc scale lengths were held constant at $a=10$ and $b=1$. Both hatched regions were produced by changing $V_0^b$ and $\gamma$ only. Region 2 is the relevant region to use here.
since all the models fitted above included a disc with the same parameters. The models were found to have the property that the radius at which $V_m(0)$ occurred was always less than the radius at which $\epsilon_{\text{max}}$ occurred. Moreover, a systematic equalising of the radii at which these occur, resulted when the models were more highly rotated. With the generation of regions 1 and 2, the data could then be plotted in the $V_m(0)/\sigma_0 \sim \epsilon_{\text{max}}$ plane and compared with the theoretical predictions to gauge the importance of rotational support and flattening in real bulges.

(iii) NGC 4762

NGC 4762 is an edge-on galaxy classified as S0$^1$ by Sandage (1961) and S(r)B0$^0$ by de Vaucouleurs and de Vaucouleurs (1964) — see Figure 4.14. Surface photometry has been performed by van Houten (1961), Tsikoudi (1977) and Burstein (1979) among others. It is this galaxy in which Burstein first identified the thick disc component (see Chapter VI). The purpose here is to derive an observed $V_m(0)/\sigma_0$ value for the bulge. Previous determinations of $\sigma_0$ have been made by Minkowski (1962) who found $\sigma_0 = 195$ km.s$^{-1}$, and Faber and Jackson (1976) who give an upper limit less than 150 km.s$^{-1}$. No direct measurements of rotation in the bulge alone have been made; Bertola and Capaccioli (1978) have measured the rotation along the major axis and find $V_m(0) = 165$ km.s$^{-1}$, although this is likely to be effected by the disc.

A total exposure of 4000s was made with the IPCS slit displaced 6'' north-west and parallel with the major axis. The rotation curve is tabulated in Table 4.10 and plotted in Figure 4.15. The systemic radial velocity corrected for solar motion with
Figure 4.14  **Upper Panel**  The galaxy NGC 4762 from a five hour J band (IIIa-J+GG385) exposure taken at the f/8 focus of the 1.0 m telescope. The major axis is at a position angle of 30.5°. The white line is a plate flaw.

**Lower Panel**  Pseudo isodensity contours of the above plate to accentuate the structure of the galaxy. Note the faint warped disc.
### TABLE 4.10
OBSERVED ROTATION CURVE OF NGC 4762 6"NW

<table>
<thead>
<tr>
<th>r (arc sec)</th>
<th>V (NE)b</th>
<th>r (arc sec)</th>
<th>V (SW)</th>
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<tr>
<td>38</td>
<td>100</td>
<td>48.7</td>
<td>-85</td>
</tr>
<tr>
<td>40</td>
<td>199</td>
<td></td>
<td></td>
</tr>
<tr>
<td>43</td>
<td>107</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

---

a  relative to corrected systemic velocity of 925 km.s\(^{-1}\)

b  units in km.s\(^{-1}\).
The data have been plotted relative to a corrected systemic velocity of $925 \text{ km.s}^{-1}$. The error associated with each observation is also shown. The open circle shows the measured velocity dispersion and its error on the minor axis at $z=6''$, taken from Figure 4.16.
Figure 4.16 The fit of the best fitting power spectrum from the velocity broadened template star grid (thin solid line) to the smoothed power spectrum of NGC 4762 (thick solid line) at z=6″. The units of velocity dispersion are km.s\(^{-1}\). The dashed lines show for comparison, fits 50 km.s\(^{-1}\) above and below the adopted value of 100 ± 10 km.s\(^{-1}\).
NGC 4762

Log$_{10}$ Power

Frequency (cycles Å$^{-1}$)

150
100
50
respect to the velocity centroid of the local group of galaxies was found to be $925 \pm 30 \text{ km.s}^{-1}$, in good agreement with $907 \text{ km.s}^{-1}$ found by Bertola and Capaccioli. The observed rotation curve gives a peak rotation velocity of $V_m = 120 \pm 25 \text{ km.s}^{-1}$ for $z=6''$. The error is estimated. The velocity dispersion on the minor axis computed from the power spectrum was found to be $100 \pm 10 \text{ km.s}^{-1}$.

Figure 4.16 shows the fit of the convolved template star (thin solid line) to the smoothed power spectrum of the galaxy (thick solid line). The match is of high quality. With the lack of other parallel and perpendicular cuts in the bulge, it is not possible to extrapolate these values to the major axis. With this in mind, a value of $V_m/\sigma (z=6'') = 1.2 \pm 0.28$ was adopted. The error is formal. The maximum bulge ellipticity is taken from Tsikoudi's (1977) photometry in B. A value of $\varepsilon_{\text{max}} = 0.55 \pm 0.1$ was used. A more accurate determination of $\varepsilon_{\text{max}}$ was difficult because of the strong contribution from the disc at significant bulge latitudes. A decomposition of the bulge and disc would be necessary before a suitable model could be fitted.

(iii) NGC 4179

The classification of NGC 4179 was listed as uncertain, but possibly E7 by de Vaucouleurs and de Vaucoulerus (1964). In RC2 it is classified as an edge-on lenticular. Large scale 1.0 m plates from this work show that it is an SBO galaxy, not quite edge-on ($i \approx 80^\circ$) - see Figure 4.17. No useful photometric or kinematical data was available. Therefore, to obtain a measurement of $V_m/\sigma$, a 5000s exposure was taken with the IPCS slit positioned 5'' north-east and parallel to the major axis to avoid the disc as much as possible.
Figure 4.17  **Upper Panel** The inner region of NGC 4179 from a 45′′ B band (IIa-O+GG385) exposure taken at the f/8 focus of the 1.0m telescope. East is up and north is to the right.

**Lower Panel** The outer region of NGC 4179 from a 6′′ J band (IIIa-J+GG385) exposure taken at the f/8 focus of the 1.0m telescope.
### TABLE 4.11

**OBSERVED ROTATION CURVE OF NGC 4179 5° NE**

<table>
<thead>
<tr>
<th>r (arc sec)</th>
<th>V (SE)b</th>
<th>V (NW)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.3</td>
<td>- 8.3</td>
<td>- 8.3</td>
</tr>
<tr>
<td>3.8</td>
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<td>- 38</td>
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<tr>
<td>11</td>
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<td>- 66</td>
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<td>14</td>
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<td>- 85</td>
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<td>- 121</td>
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<tr>
<td>19</td>
<td>182</td>
<td>- 104</td>
</tr>
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<td>22</td>
<td>157</td>
<td>- 97</td>
</tr>
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<td>- 96</td>
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<td>- 127</td>
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<td>37</td>
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<td>- 128</td>
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<tr>
<td>39</td>
<td>138</td>
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<td>- 173</td>
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<td>44</td>
<td>-</td>
<td>- 154</td>
</tr>
<tr>
<td>49</td>
<td>-</td>
<td>- 173</td>
</tr>
</tbody>
</table>

a relative to corrected systemic velocity of 1251 km.s\(^{-1}\)

b units in km.s\(^{-1}\).

### TABLE 4.12

**OBSERVED ROTATION CURVE OF HAM I 3° E**

<table>
<thead>
<tr>
<th>r (arc sec)</th>
<th>V (S)b</th>
<th>V (N)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>121</td>
<td>121</td>
</tr>
<tr>
<td>2.5</td>
<td>99</td>
<td>- 102</td>
</tr>
<tr>
<td>5.1</td>
<td>50</td>
<td>- 102</td>
</tr>
<tr>
<td>7.6</td>
<td>33</td>
<td>- 74</td>
</tr>
<tr>
<td>10</td>
<td>132</td>
<td>- 86</td>
</tr>
<tr>
<td>13</td>
<td>179</td>
<td>66</td>
</tr>
<tr>
<td>15</td>
<td>135</td>
<td>- 138</td>
</tr>
<tr>
<td>18</td>
<td>143</td>
<td>- 138</td>
</tr>
<tr>
<td>20</td>
<td>143</td>
<td>- 165</td>
</tr>
</tbody>
</table>

a relative to corrected systemic velocity of 4050 km.s\(^{-1}\)

b units in km.s\(^{-1}\).
Figure 4.18 Observed rotation curve of NGC 4179 with the slit displaced 5" NE and parallel to the major axis. The data have been plotted relative to a corrected systemic velocity of 1251 km.s\(^{-1}\). The error associated with each observation is also shown. The open circle shows the measured velocity dispersion on the minor axis 5" from the centre, taken from Figure 4.19.
NGC 4179

$V \text{ (km.s}^{-1}\text{)}$

<table>
<thead>
<tr>
<th>SE</th>
<th>r (sec)</th>
<th>NW</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>20</td>
<td>60</td>
</tr>
</tbody>
</table>

$5'' \text{NE} \parallel$
Figure 4.19 Fit of the best fitting power spectrum from the velocity broadened template star grid (thin solid line) to the smoothed power spectrum of NGC 4179 (thick solid line), at z=5′. The units of velocity dispersion are km.s$^{-1}$. The dashed lines show for comparison, fits 50 km.s$^{-1}$ above and below the adopted value of 150 ± 20 km.s$^{-1}$. 
The corrected systemic radial velocity was found to be \(1251 \pm 20\) km.s\(^{-1}\) in good agreement with \(1279\) km.s\(^{-1}\) from RC2. The rotation curve, plotted relative to the systemic velocity is shown in Figure 4.18 and tabulated in Table 4.11. The rotation curve gives an observed \(V_m\) of \(160 \pm 20\) km.s\(^{-1}\). The best estimate of the velocity dispersion \(5''\) from the centre along the minor axis was found to be \(150 \pm 20\) km.s\(^{-1}\). Figure 4.19 shows the fit of the velocity broadened star power spectrum (thin solid line) to the smoothed galaxy power spectrum (thick solid line). Combining these two datum gives \(V_m/\sigma (z=5'') = 1.07 \pm 0.02\) where the error is formal. No corrections for inclination have been made. The surface photometry gave a value of \(\epsilon_{\text{max}} = 0.57 \pm 0.05\) for the maximum ellipticity in B. However, NGC 4179 is not quite edge on so this value may be smaller than its intrinsic value.

(iv) Ham I

Ham I (named due to its similarity to a edge-on hamburger), situated only 9° from the south celestial pole, is an unusual and interesting object. Morphologically it resembles both a highly flattened elliptical galaxy with a very distinct dust lane along the major axis and a late type SO galaxy with little evidence of a luminous disc — see Figure 4.20. This at first seems contradictory, since by definition an SO galaxy has a luminous disc. The kinematics reveal the latter interpretation is probably correct. The rotation curve and velocity dispersion were derived from a 3000s exposure with the slit displaced 3'' in a perpendicular direction on the east side from the major axis and parallel to the major axis to avoid the dust lane. The rotation curve is shown in Figure 4.21 and
Figure 4.20 Upper Panel The galaxy Ham I from a $4^{h}30^{m}$ B band (IIa-0+GG385) exposure taken at the f/8 focus of the 1.0m telescope. North is up and east is to the right.

Lower Panel Pseudo isodensity contours of the above plate to accentuate the outer bulge.
Figure 4.21 Observed rotation curve of Ham I with the slit displaced 3" E and parallel to the major axis. The data have been plotted relative to a corrected systemic velocity of 4050 km.s$^{-1}$. The error associated with each observation is also shown. The open circle shows the measured velocity dispersion on the minor axis 3" from the centre, taken from Figure 4.22.
Figure 4.22 Fit of the best fitting power spectrum from the velocity broadened template star grid (thin solid line) to the smoothed power spectrum of Ham I (thick solid line) at $z=3''$. The units of velocity dispersion are km s$^{-1}$. The dashed lines show for comparison, fits 50 km s$^{-1}$ above and below the adopted value of 150 ± 20 km s$^{-1}$. 
Figure 4.23 Observed surface brightness distribution in B of Ham I. The isophotes are in arbitrary units of log I spaced 0.5 mag apart. The faintest isophote is 1.0 mag below the next brightest. The data was smoothed with a two-dimensional Gaussian beam of halfwidth 2"x2" before the isophotes were plotted. Note the strong absorption on the major axis.
tabulated in Table 4.12. Unfortunately, the rotation curve is noisy due to the apparent faintness of the galaxy and the relatively short exposure. However, there is still clear indication of significant rotation with \( V_m (z=3") \) of about \( 200 \pm 50 \text{ km.s}^{-1} \). Ham I is the second most distant object in the sample, with a corrected systemic velocity of \( 4050 \pm 30 \text{ km.s}^{-1} \). This places the galaxy at a distance of \( 54 \text{ Mpc} \) \( (H_0 = 75 \text{ km.s}^{-1}. \text{Mpc}^{-1}) \), similar to NGC 128 (55.4 Mpc; this work). The velocity dispersion from Figure 4.22 was found to be \( 150 \pm 20 \text{ km.s}^{-1} \), giving an observed \( V_m/\sigma(z=3") = 1.33 \pm 0.38 \), the most highly rotating galaxy in the sample. Most of the formal error results from the uncertainty in \( V_m \).

Figure 4.23 shows the isophotes of Ham I in B on an arbitrary log I scale, with an isophote spacing of 0.5 mag. The faintest isophote is 1.0 mag below the next brightest. The data has been taken from a 4.5 hour exposure using a IIa-0 + GG385 plate-filter combination (plate 4029), and reduced using the techniques described in Chapter III. The map was firstly smoothed with a two-dimensional Gaussian filter of halfwidth 3" before plotting the isophotes. The distribution of ellipticities give \( \varepsilon_{\text{max}} = 0.67 \pm 0.05 \).

Ham I is interesting because it is morphologically like a dustlane elliptical galaxy but kinematically similar to an SO galaxy. The former type however would seem less likely since we now know that most ellipticals have an insufficiently large \( V_m(0)/\sigma_0 \) to be consistent with their observed flattening in the absence of significant velocity anisotropy (eg Illingworth 1977). Davies et al have found that fainter ellipticals \( (> -20 M_B) \), do rotate sufficiently fast to be consistent with rotational flattening.
However, concentric aperture photometry using the magnitude-aperture relations as function of morphological type from RC2 show that the absolute magnitude of Ham I is about -20.5, precluding Ham I as a likely faint dust lane elliptical. The observed $\frac{V_m}{\sigma}(z=3")$ for Ham I is at least a factor of three larger than that observed in most ellipticals. At the same time however, there is no clear evidence from the surface photometry alone of a disc (or lens), characteristic of an SO galaxy. Putting all this information together it seems that we are observing a rapidly rotating bulge without a disc. From a dynamical standpoint, the models suggest that there is no reason why such systems cannot exist in dynamical equilibrium. No dynamical connection has yet been seen (or modelled) which requires a disc to be present in order for a bulge to rotate rapidly. If this is true, then it is natural to ask why such systems are not more commonly seen.

(v) Discussion

A determination of $\frac{V_m(0)}{\sigma_0}$ has been made for seven of the eight galaxies in the sample of Table 4.1. Table 4.13 summarises all of the data from the previous sections. Column 2 lists the value of $V_m$ derived from a spectrum offset in distance from the major axis. $V_m(0)$ in column 3, gives the corrected value of $V_m$ which would be observed on the major axis in the absence of a disc. $\sigma_{\text{minor}}$ in column 4, gives the velocity dispersion on the minor axis found by extrapolating the run of velocity dispersion parallel to the major axis back on to the minor axis. This was used instead of the central velocity dispersion $\sigma_0$ when no estimate of $\sigma_0$ was available. Column 5 lists the central velocity dispersion
### TABLE 4.13

**ROTATION-ELLIPTICITY RESULTS**

<table>
<thead>
<tr>
<th>Galaxy</th>
<th>$V_m^{(a)}$</th>
<th>$V_m(0)^{(b)}$</th>
<th>$q_{\text{minor}}^{(b)}$</th>
<th>$\sigma_o^{(c)}$</th>
<th>$V_m(0)/\sigma_o^{(d)}$</th>
<th>$V_m/\sigma_{\text{minor}}^{(e)}$</th>
<th>$\epsilon_{\text{max}}^{(f)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NGC 7814</td>
<td>83 ± 14</td>
<td>123 ± 12</td>
<td>-</td>
<td>167 ± 18</td>
<td>0.74 ± 0.11</td>
<td>-</td>
<td>0.44</td>
</tr>
<tr>
<td>NGC 4594</td>
<td>108 ± 19</td>
<td>130 ± 24</td>
<td>-</td>
<td>225 ± 15</td>
<td>0.58 ± 0.12</td>
<td>-</td>
<td>0.32</td>
</tr>
<tr>
<td>NGC 7123</td>
<td>250 ± 30</td>
<td>-</td>
<td>170 ± 20</td>
<td>175 ± 10</td>
<td>-</td>
<td>1.47 ± 0.24</td>
<td>0.44</td>
</tr>
<tr>
<td>NGC 4762</td>
<td>120 ± 25</td>
<td>-</td>
<td>100 ± 10</td>
<td>-</td>
<td>-</td>
<td>1.20 ± 0.28</td>
<td>0.54 ± 0.05</td>
</tr>
<tr>
<td>NGC 4179</td>
<td>160 ± 20</td>
<td>-</td>
<td>150 ± 20</td>
<td>-</td>
<td>-</td>
<td>1.07 ± 0.20</td>
<td>0.57 ± 0.05</td>
</tr>
<tr>
<td>Ham I</td>
<td>200 ± 50</td>
<td>-</td>
<td>150 ± 20</td>
<td>-</td>
<td>-</td>
<td>1.33 ± 0.38</td>
<td>0.65 ± 0.03</td>
</tr>
<tr>
<td>NGC 128</td>
<td>200 ± 20</td>
<td>200 ± 20</td>
<td>210 ± 20</td>
<td>210 ± 20</td>
<td>0.95 ± 0.13</td>
<td>0.95 ± 0.13</td>
<td>0.50 ± 0.05</td>
</tr>
</tbody>
</table>

---

**Notes:**

- (a) Velocity in km.s\(^{-1}\)
- (b) Velocity dispersion in km.s\(^{-1}\)
- (c) $\epsilon_{\text{max}} = 1 - (b/a)_{\text{min}}$
- (d) Tsikoudi (1977)
- (e) Hodge and Merchant (1966)
- (f) Kormendy and Illingworth (1981)
The $V_m(0)/\sigma_0$ vs $\varepsilon_{\text{max}}$ diagram for the sample of galaxies listed in Table 4.1. $V_m(0)$ is the maximum rotation velocity on the major axis, $\sigma_0$ is the central velocity dispersion and $\varepsilon_{\text{max}}$ is the maximum ellipticity of the isophotes. The horizontally hatched area (Region 1), shows the region defined by the rotating bulge models without a disc. The upper and lower limits serve only to indicate where most models lie. The solid lines labelled oblate and prolate are from the theoretical models of Binney (1978). The vertically hatched area shows the region defined by rotating bulge models with a disc (Region 2). The disc-to-bulge mass ratio has been held constant at 0.25. All disc models have $b/a=0.1$, with $b=1$. The filled circles show the observed value of $V_m(0)/\sigma_0$. The errors are formal. The open circles show the model value. The point at $(V_m(0)/\sigma_0, \varepsilon_{\text{max}}) = (0.74, 0.44)$ is the superposition of three points. It is the model point for NGC 7814 and NGC 7123 and the observed point for NGC 7814.
where it has been measured directly or inferred from intersecting slit data. The ratio \( V_m(0)/\sigma_o \) from columns 3 and 5 is formed in column 6. Column 7 gives \( V_m/\sigma_{\text{minor}} \) along the observed cut parallel to the major axis. For the three galaxies for which models were constructed (NGC 7814, 7123 and 4594), the maximum ellipticity \( \epsilon_{\text{max}} \) in column 8 was taken from the model but is almost identical to that for the isophotometry (see Figures 4.3, 4.7 and 4.11). All the kinematical data is from this work except for NGC 7814 and NGC 4594 where the data has been taken from KI.

The data from Table 4.13 is plotted in the \( V_m(0)/\sigma_o \sim \epsilon_{\text{max}} \) plane in Figure 4.24. The first point to note is the excellent agreement between the rotating bulge models in Region 1 and the oblate model predictions of Binney, even though our Region 1 is defined by the central velocity dispersion and not a global value. This is supportive of Binney's statement that little difference should be seen when global values are used. However, observationally this may not be true since significant velocity dispersion gradients have been seen (e.g., NGC 7123; this work). In way of illustration, KI quoted a mean dispersion of \( 127 \pm 9 \, \text{km.s}^{-1} \) for NGC 7814 compared with a central dispersion of \( 167 \pm 18 \, \text{km.s}^{-1} \). If their central dispersion is adopted, then \( V_m(0)/\sigma_o = 0.74 \), 24 per cent smaller than \( V_m(0)/\langle \sigma \rangle = 0.97 \). This places NGC 7814 in Region 2, defined by the set of rotating models with a disc-to-bulge mass ratio of 0.25. The same effect, although weaker only because the dispersion profile is much flatter, is illustrated by NGC 4594. Nevertheless, the value of \( V_m(0)/\sigma_o \) derived from the NGC 4594 model (open circles) is in good agreement with the observed value (filled circles). The model value of \( V_m(0)/\sigma_o \)
falls well within the error associated with the observed value. The agreement is even more impressive for NGC 7814; the model and observations are in exact agreement. As noted earlier, the large discrepancy for NGC 7123 should be attributed to the rotation of the disc, rather than the bulge.

The prototype peanut-shape bulge galaxy NGC 128 was also included in Figure 4.24. Even though \( V_m(0) \) was not observed directly on the major axis, no correction to \( V_m \) was made because the bulge rotates cylindrically; \( V_m(0) \) was set equal to \( V_m \). Because of the special nature of its rotation, NGC 128 is discussed separately in Chapter V.

**4.8 CONCLUSIONS**

It has been shown observationally that all the bulges studied in this and KI's sample rotate at least as rapidly as required by the oblate-spheriod models in which their flattening is due to rotation. More importantly, it has been demonstrated that the observed light distributions and kinematical properties are very well approximated for some of them at least by a distribution function with only two isolating integrals of motion, the total energy and the angular momentum in the z-direction. The models so constructed were oblate and flattened via rotation, unlike the brighter elliptical galaxies. The models were shown at the same time to accurately reproduce the observed internal kinematics within the observational errors, over the entire spatial range observed. An important assumption used in comparing the models and having much current interest is the local mass-to-light ratio, \( M/L \). The models were projected and compared with real data under the assumption that
the bulges were transparent and have a constant M/L. To a first approximation this would seem to be true to the surface brightnesses observed. However, galaxies like NGC 3115 in which a fairly large colour gradient is present may be indicating a changing M/L and require an improvement over a one-population model (Miller and Prendergast 1968, Strom et al. 1977), because a more blue colour leads to a decreasing M/L at first sight (i.e., a change from an elliptical galaxy population towards a globular cluster population). Faber et al. has reported that M/L_B for NGC 4594, although low, does not change significantly away from the nucleus out to a distance of 3'.

The M/L values for the three galaxies modelled above are consistent with the recent determination for the nuclear regions of elliptical galaxies (Faber and Jackson 1976; Williams 1977; Efstathiou, Ellis and Carter, 1980), averaging around 7. Thus there appears to be no systematic difference between the M/L values for spiral (bulges) and elliptical galaxies. The most straightforward interpretation of this is that the inner parts of the bulge component of disc galaxies have the same stellar population mix as elliptical galaxies.

The three program galaxies modelled were all morphologically fairly similar and dynamically representative of all the other galaxies in the sample except for NGC 128. This galaxy is kinematically different from the rest (see Chapter V) and has not been modelled yet. However, from the discussion in Chapter V, it seems clear enough what to do.

The success of the models also has important implications for classical collapse models of disc galaxy formation. These are discussed in the next Chapter.
CHAPTER V

THE CYLINDRICAL ROTATORS

5.1 INTRODUCTION

The models described in Chapter II, and the bulges they were used to fit in Chapter IV, are spheroidal in shape and rotate non-cylindrically i.e. the projected rotational velocity decreases monotonically with distance perpendicular to the equatorial plane. There is another class of bulges, which have a box to peanut-shaped appearance and rotate cylindrically i.e. the projected rotational velocity depends only on the cylindrical radius R. This class of cylindrical rotators is the subject of this chapter. The prototype of this class is the edge-on SO galaxy NGC 128. In Section 5.2, the kinematics of its bulge are discussed, together with the available surface photometry. Other bulges, observed by other authors and exhibiting the same phenomena are discussed in Section 5.3. The dynamical difference and origin of the two classes of bulge are discussed in Section 5.4 in the context of the stellar energy - angular momentum plane. Finally, the main conclusions of the chapter are collected in Section 5.6.

5.2 NGC 128

NGC 128 has been classified SO, pec by Sandage (1961) and SO pec by de Vaucouleurs (1963). It is the dominant member of a group of at least four (NGC 126, 127 and 130) and possibly five (NGC 125) galaxies (Humason et al 1956, Hodge and Merchant 1966, and Bertola and Capaccioli 1977). This object was included in the sample list because it represented
the extreme of an apparent morphological sequence in bulges of disc
galaxies. Burbidge and Burbidge (1959) were first to call attention
to the peculiarities of its bulge. Figure 5.1 shows these peculiarit-
ies well. The photographs are taken from a five hour B exposure at the
f/8 focus of the 1.0 m telescope at Siding Spring Observatory using a
IIa-0+GG385 plate-filter combination (plate 4031). The bulge is
distinctly peanut-shaped, formed by four luminous extensions that
emanate from the nucleus at angles of approximately 45° from the major
and minor axes. Hodge and Merchant concluded from their photometry
that the luminosity and ellipticity profiles were consistent with the
view that NGC 128 is a disc-shaped object seen edge-on. This is supported
by the kinematical observations below, and by Bertola and Capaccioli (1977).
A luminous bridge also connects NGC 127 with NGC 128, an almost certain
indication of tidal interaction. This is also supported by the apparent
warping of the disc toward the west of the major axis.

In a direct attempt to understand the dynamical difference that
gave rise to the peanut (and box) shaped bulges, the bulge velocity field
of NGC 128 was mapped. The completeness and accuracy of the mapping
were hampered by the small apparent size of the bulge and its relatively
low brightness; the integrated magnitude is only 12.6 μB (RC2).

Long slit spectra were taken with the RGO spectrograph and the
IPCS on the AAT, at three positions in the bulge. These are given in
Table 5.1.
Figure 5.1  **Upper Panel** The galaxy NGC 128 from a $5^h$ B band (IIa-0+GG385) exposure taken at the f/8 focus of the 1.0m telescope. Note the well defined peanut shaped bulge. West is up and north is to the left.

**Lower Panel** Pseudo isodensity contours of the above plate to accentuate the outer bulge.
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TABLE 5.1
BULGE SLIT POSITIONS IN NGC 128

<table>
<thead>
<tr>
<th>Position angle</th>
<th>Offset (sec arc)*</th>
<th>Exposure (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>0°</td>
<td>4</td>
<td></td>
</tr>
<tr>
<td>90°</td>
<td>5 ⊥ S</td>
<td>5000</td>
</tr>
</tbody>
</table>

* Offsets are relative to the nucleus. || denotes parallel to major axis. ⊥ denotes perpendicular to major axis.

The rotation and velocity dispersion measurements from these spectra were derived using the techniques described in Chapter III. The rotation and dispersion profiles on the major axis are tabulated in Tables 5.2 and 5.3 respectively and shown in Figure 5.2. The filled circles show the results from this work and the open squares are from Bertola and Capaccioli (1977; hereafter BC). These serve as a check on the accuracy of the redshift determination algorithm from Chapter III. It is clear that there is good agreement within the observational error of both sets of data. The turnover velocity has been reached and is estimated at $200 \pm 20 \text{ km.s}^{-1}$. The corrected systemic velocity $V_0$, was found to be $4310 \pm 40 \text{ km.s}^{-1}$, in good agreement with $4384 \pm 50 \text{ km.s}^{-1}$, obtained by BC. For $H_0 = 75 \text{ km.s}^{-1}. \text{Mpc}^{-1}$, this places NGC 128 at a distance of 57.5 Mpc.

The rotation of the bulge alone was measured by displacing the slit 4" east and parallel with the major axis (p.a.=0°). The resultant rotation and dispersion profiles are shown in Figure 5.4 and tabulated in Tables 5.4 and 5.5 respectively. A measure of the internal consistency of the data comes from comparison of the systemic velocities of the
TABLE 5.2

OBSERVED ROTATION CURVE OF NGC 128 - MAJOR AXIS

<table>
<thead>
<tr>
<th>r (arc sec)</th>
<th>V(N)*</th>
<th>V(S)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>41</td>
<td>41</td>
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<td>232</td>
<td>-152</td>
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* relative to a corrected systemic velocity of 4310 km.s⁻¹
Units in km.s⁻¹

TABLE 5.3

OBSERVED VELOCITY DISPERSION PROFILE OF NGC 128 - MAJOR AXIS

<table>
<thead>
<tr>
<th>r (arc sec)</th>
<th>σ (km.s⁻¹)</th>
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Figure 5.2

Upper Panel Observed rotation curve of NGC 128 along the major axis. The data has been plotted relative to a corrected systemic velocity of 4310 km s$^{-1}$. The filled circles show the rotation curve derived from this work. The open squares show for comparison, the rotation curve along the major axis obtained by Bertola and Capaccioli (1977). The agreement between both rotation curves is well within observational error.

Lower Panel Observed velocity dispersion profile of NGC 128 along the major axis. The data has been folded and averaged about the photometric centre.
The top graph shows the radial velocity ($V$) as a function of time ($r$) for NGC 128. The graph includes data points labeled 'BJ' and 'BC'.

The bottom graph shows the radial velocity dispersion ($\sigma$) as a function of time ($r$) for NGC 128.
Figure 5.3 The fit of the power spectrum of the velocity broadened template star (thin solid line) from Figure 3.3 to the smoothed power spectra of each row in the spectrum of NGC 128 (thick solid line), displaced 4" | | E. Each panel is labelled with the displacement in seconds of arc from the minor axis.
### TABLE 5.4
OBSERVED ROTATION CURVE OF NGC 128 4"|E

<table>
<thead>
<tr>
<th>r (arc sec)</th>
<th>V(N)*</th>
<th>V(S)</th>
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</table>

* relative to a corrected velocity of 4310 km.s\(^{-1}\).
Units in km.s\(^{-1}\)

### TABLE 5.5
OBSERVED VELOCITY DISPERSION PROFILE OF NGC 128 4"|E

<table>
<thead>
<tr>
<th>r (arc sec)</th>
<th>(\sigma) (km.s(^{-1}))</th>
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<tr>
<td>10</td>
<td>180 ± 20</td>
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</tbody>
</table>
Figure 5.4  **Upper Panel** Observed rotation curve of NGC 128 4" east and parallel to the major axis. The data has been plotted relative to a systemic velocity of 4310 km.s$^{-1}$. The error associated with each observation is also shown.

*Lower Panel* Observed velocity dispersion profile of NGC 128 4" east and parallel to the major axis. The data has been folded and averaged about the photometric axis.
Figure 5.5 The fit of the power spectrum of the velocity broadened template star (thin solid line) from Figure 3.3 to the smoothed power spectra of each row in the spectrum of NGC 128 (thick solid line), along the major axis. Each panel is labelled with the displacement in seconds of arc from the minor axis.
TABLE 5.6

OBSERVED ROTATION CURVE OF NGC 128 5" S

<table>
<thead>
<tr>
<th>r(arc sec)</th>
<th>$V_W - V(z=0'')^a$</th>
<th>$V_E - V(z=0'')^b$</th>
</tr>
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</tr>
<tr>
<td>12.5</td>
<td>19</td>
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</tr>
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</table>

a $V_W$ = velocity west of major axis, km.s$^{-1}$
b $V_E$ = velocity east of major axis, km.s$^{-1}$

Note: all measures have an error of $\pm$ 20 km.s$^{-1}$
Figure 5.6  Observed rotation curve of NGC 128 5'' south and perpendicular to the major axis. The data has been plotted relative to the rotation velocity on the major axis (z=0''). Note the independence of rotation velocity with height above the disc (cylindrical rotation).
spectra parallel to the major axis. The corrected systemic velocity from the spectra offset 4" east of the major axis was $4330 \pm 40 \text{ km.s}^{-1}$, consistent within observational error with $4310 \pm 40 \text{ km.s}^{-1}$ obtained from the major axis. The most interesting feature of Figure 5.3 is that the bulge is rotating just as rapidly as on the major axis. Examination of the surface photometry from Hodge and Merchant, indicates that the major axis luminosity profile is dominated by the bulge light to at least $r = 15"$ and probably to $r = 20"$. Hence, the upper panel of Figure 5.2 is showing the rotation of the bulge alone to about $r = 20"$. This cylindrical type of rotation is undoubtedly a unique feature among the list of candidate galaxies chosen for this study. It is then possible to predict the form of the rotation curve perpendicular to the major axis for cuts displaced from the minor axis. The results of the cut 5" south of the nucleus and perpendicular to the major axis are shown in Figure 5.6 and tabulated in Table 5.6. As expected, the rotational velocity is constant up to 12" (3.3 kpc; $H_0 = 75 \text{ km.s}^{-1} \text{ Mpc}^{-1}$) above the disc with no indication of a decrease to the limit of the data. The bulge of NGC 128 is rotating like a differentially rotating cylinder to a large distance above the plane.

5.3 OTHER CYLINDRICAL ROTATORS

Recent observations have shown that this is not a unique property among bulges. Davies and Illingworth (1981) have reported cylindrical rotation in the box-shaped bulge of NGC 1381, an edge-on SO galaxy in the Fornax cluster, although in this case less net rotation than for NGC 128 was observed. The bulge of this galaxy is markedly box-shaped. The same effect has also been observed in NGC 4565 by KI. They found that for a 27" perpendicular cut, the velocity is almost constant up to 30" (4.5 kpc) above the disc. Only at $z \geq 30"$ is a smooth decrease
in velocity observed. The bulge of this galaxy is also clearly peanut-shaped, but to a lesser extent than NGC 128. (Extensive surface photometry of NGC 4565 has been done by Jensen and Thuan (1981; hereafter JT) and van der Kruit and Searle (1980)).

To date all galaxies with box or peanut-shaped bulges which have been observed kinematically (NGC 128, 4565 and 1381), show cylindrical rotation. This would indicate that one can predict the gross overall kinematical properties of the bulge from its morphology i.e. if a bulge is peanut or box-shaped, then it is a cylindrical rotator; if it shows a spheroidal bulge then it is a non-cylindrical rotator. Clearly, kinematical data from a large sample of galactic bulges with mixed morphologies is required to verify this prediction.

5.4 THE ENERGY-ANGULAR MOMENTUM (E,J) PLANE

It has been shown in Chapter IV that the structure and kinematics of the spheroidal bulges are well represented by models with a distribution function of the form

\[ f(E,J) = a \left[ \exp (-\beta E) - \exp (-\beta E_0) \right] \exp (\gamma J) \]

where \(E\) is the energy of a star and \(J = J_z\) is its angular momentum about the rotation axis. This functional form alone was unable to produce peanut or even box-shaped bulges; as the rotation parameter \(\gamma\) increased, the models just continued to flatten. This raises the obvious question; what is the dynamical difference between cylindrically rotating (peanut or box-shaped) bulges and non-cylindrically rotating (spheroidal shaped) bulges. A possible answer to this question lies in an examination of the energy-angular momentum \((E,J)\) plane for the stars of the bulge. Figure 5.7 shows this plane in schematic form. The area of the \((E,J)\) plane, accessible to bulge stars, is symmetric about the \(J = 0\) line (radial orbits)
Figure 5.7  The energy angular momentum (E,J) plane. The plane is bounded to the left and right by curved lines indicating the maximum angular momentum a test particle can attain for a given energy i.e. a circular orbit. The third side (top) is bounded by a flat line corresponding to the distribution function energy cutoff (see text). The sloping dashed lines show contours of constant f (equally spaced in intervals of f) for the distribution function used here. They have a greater slope at more bound energies. These model the spheroidal (non-cylindrically rotating) bulges well. The thick solid lines roughly parallel to the direct circular orbit line show the likely form of the constant f contours which would model the box and peanut-shaped (cylindrically rotating) bulges.
$f(E, J) = \alpha (e^{-\beta E} - e^{-\beta E_0}) e^{\gamma J}$
with test particles moving in retrograde orbits \((J < 0)\) to the left, and test particles moving in direct orbits \((J > 0)\) to the right. The plane is bounded to the left and right by curved lines indicating the maximum angular momentum a test particle can have for a given energy, corresponding to the circular orbit at that energy. The third side (top) is bounded by a straight line at constant \(E\), representing the lowered Maxwellian energy cutoff \(E_o\). The difference between the cylindrical and non-cylindrical rotators will lie in the way the \((E,J)\) plane is populated.

As illustrated, the straight but sloping dashed lines in Figure 5.7 indicate contours of constant space density \(f = f(E,J) \rho^3 \, \mathrm{d}^3v\) for the distribution function used to model the non-cylindrical rotators. These contours have a greater slope at more bound energies, resulting in more net rotation in the inner parts of the model. The important point to note from these contours is that there is a significant proportion of low angular momentum matter at all energy values, from the central potential up to the cutoff energy. It is these low angular momentum stars that predominate near the minor axis of the bulge. To produce the peanut and square-shaped bulges, these stars near the minor axis need to be depleted, particularly at large distances from the nucleus: these are the stars of low angular momentum and intermediate energy.

In order to depopulate the region near the minor axis, it seems intuitively clear that unless we invoke third integrals we should include only those stars with relatively low orbital eccentricities. Such orbits are confined to a torus-shaped region with the \(z\)-direction as its axis of symmetry. Although such orbits can rise to significant heights above the equatorial plane of the bulge, they are of course excluded from the true minor axis of the bulge (although they will contribute to the minor axis profile in projection).
Figure 5.7 shows contours of a distribution function that would have this property. The contours are the curves roughly parallel to the direct circular orbit line in the (E,J) plane. For the potential of the r^1 surface density law (Young, 1976), the two contours shown correspond approximately to orbital eccentricities e = 0.33 and 0.50, for regions away from the bottom of the potential well. (Here e is defined as (R_{max} - R_{min})/(R_{max} + R_{min}), where R_{max}, R_{min} are the apo- and peri-galactic cylindrical radii for the orbit: the two curves then correspond to R_{max}/R_{min} = 2 and 3). A bulge whose distribution function has all the stars concentrated near these low eccentricity curves would very likely have a square or peanut-shaped appearance in projection, and would appear to be in cylindrical rotation.

To summarise: in the spheroidal non-cylindrically rotating bulges, the distribution function includes stars with (E,J) values covering almost the entire accessible area. For the peanut and box-shaped cylindrical rotators, it seems likely that (if f depends only on E and J) most stars have (E,J) values close to the direct circular velocity curve, as shown in Figure 5.7.

5.5 IMPLICATIONS FOR BULGE FORMATION

How does this all fit in with the present view that dissipation plays an important part in the formation of disc galaxies (see for example Larson (1976), Gott (1973)). Only in dissipative systems can flat components such as galactic discs be produced. The presence of colour gradients in some bulges (Faber, 1977), and their high central densities, argue strongly for dissipation. Also KI have pointed out how the internal kinematics of bulges are consistent with dissipative formation
pictures.

We need now to consider the dissipative evolution of a gaseous protosystem in the \((E,J)\) plane. Gas clouds will lose energy through encounters; these encounters will also drive the angular momenta of the clouds towards the local mean angular momentum. So, for a protosystem with positive net angular momentum, the gas clouds will, in the mean, migrate towards the lower right of the \((E,J)\) plane shown in Figure 5.7.

What is the difference between the evolutionary history of the spheroidal and the peanut-shaped bulges? In our picture, each kind of galaxy populates some interval of energy in the \((E,J)\) plane. First we need to know how these intervals compare for the two kinds of bulges: for example, are the peanut-shaped bulges more or less bound? To this end, the minor axis surface brightness profiles of NGC 7814 and NGC 128 were compared (these two systems are prototypes of the two bulge morphologies and have comparable integrated magnitudes and lengthscales). It turned out that the shapes of the minor axis profiles, the lengthscales and the central surface brightnesses were fairly similar. The implication is that the central bulge potential and the energy interval populated must be similar for these two systems, although the total angular momentum of NGC 128 will of course be larger.

If our views on the \((E,J)\) structure of the peanut bulges are correct, then the similarity of the present energy range for NGC 7814 and NGC 128 makes it unlikely that their initial protosystems were similar in all three of size, density and total angular momentum. To obtain the NGC 128 morphology and kinematics, most of the stars in the bulge now have \((E,J)\) values that lie close to the circular orbit line in the \((E,J)\) plane. This distribution must result from a combination of
dissipative evolution and the initial state of the protosystem. The NGC 128 type bulge covers a similar energy range to the spheroidal bulge of NGC 7814, so its potential energy is similar; however its kinetic energy is largely in the form of ordered (i.e. rotational) motion. This contrasts with NGC 7814, whose kinetic energy is mainly in random stellar motions: although NGC 7814 is rotationally flattened, its main support comes from random motions. So we need to find a formation picture for these two kinds of bulges which results in similar binding energy but different random internal energy.

The dissipative evolution of a protosystem will proceed until the conditions become right for large scale conversion of gas to stars. At this point, dissipation ceases and the binding energy of the system is defined. From the well known L-V^4 law for bulge systems, this point seems to occur at a roughly constant value of the surface density for the system.

The discussion of disc galaxy formation by Fall and Efstathiou (1980) is also helpful here. They consider the evolution of a protosystem (gas plus dark halo matter) which has acquired its angular momentum by tidal torques before condensing out of the general expansion. They quantify the angular momentum content by the parameter $\lambda = J |E|^{1/2} G^{-1} M^{-5/2}$ where $J$, $E$ and $M$ are now the total angular momentum, energy and mass of the protosystem. The $\lambda$-values predicted by the cosmological N-body experiments of Efstathiou and Jones (1979) are $\lambda = 0.07 \pm 0.04$ (rms spread). So, in the mean, the least bound systems at a given mass will require the most angular momentum.

This suggests two obvious possible differences between the protosystems for peanut and spheroidal bulges. (i) the proto-peanut is a more
diffuse system of somewhat higher angular momentum, in the mean, as suggested in the previous paragraph. Its dissipative life, until it reaches the critical surface density for gross star formation, is relatively long: with its higher angular momentum, this would lead to a system with relatively little random energy, as in the peanut bulges. We cannot push this picture too far; if the protocloud is too diffuse, then its collapse time will become too long. (ii) alternatively, the proto-peanut is a system of similar size but larger angular momentum, as allowed by the significant spread in $\lambda$ found in the experiments of Efstathiou and Jones. Its internal motions are then already significantly ordered, even before significant dissipation takes place. In either case (i) or (ii), the high angular momentum means that there will not be extensive migration of particles towards very bound energies, so the inner parts of the bulge will not be much polluted by material dissipating inwards from the outer parts of the protosystem.

If this picture is correct so far, then it has implications for the presence of observable colour gradients in the bulges of disc galaxies. We recall Hartwick's (1976) theory for the metal enrichment of spheroidal systems: star formation in a region is arrested before all the gas is converted to stars (say by a supernova), and this enriched gas is then lost to the region, to form stars again elsewhere. This model has predicted successfully the form of the abundance distribution in the galactic halo (e.g. Searle and Zinn, 1978). In our picture for the formation of a spheroidal bulge, this lost enriched gas is of relatively low angular momentum in the mean, so will gradually diffuse inwards towards the bulge centre; as it suffers successive generations of star formation, this gas will gradually enrich the inner parts of the bulge. On the other hand, the enriched lost gas in a forming peanut
bulge is of high angular momentum in the mean, and is thereby excluded from the inner parts of the bulge. The obvious prediction is that spheroidal bulges will have significantly stronger colour gradients than the peanut-shaped bulges.

This prediction is easy to test from available photometry. The first example is the large edge-on spiral NGC 4565, which has a peanut-shaped bulge. Detailed photometry has been done by several authors: we will use the data of Jensen and Thuan (1981: JT) which is the most recent and in general agreement with most other authors. They find no colour gradient on the minor axis to z = 60", where the bulge light dominates. The colour is constant at B-r \sim 1.2. The work of Frankston and Schild (1970) and Hamabe et al (1980) also confirms the absence of colour gradient for z \leq 60" on the minor axis.

JT argue that the apparent bulge of NGC 4565 is actually a bar seen end-on. Their main evidence for this view is (i) the closeness of the fit of the minor axis profile to an exponential, and (ii) the similar colour of the bulge (bar) and the disc. If they are correct, then the observations of the bulge of NGC 4565 are not directly relevant here. However, it will be shown in Chapter VI that an exponential profile, perpendicular to the major axis, arises naturally when the bulge is in the presence of the much flatter potential field of the disc. This seems plausible for NGC 4565 since the disc is large relative to the bulge, and would certainly affect the motions and distribution of the stars in the bulge. Point (ii) can probably be understood, for a system with a peanut-shaped bulge, from the relatively close dynamical similarity of the bulge and the disc (see Figure 5.7). The evolutionary history of the peanut bulge and the disc are fairly similar in our picture, so a similar
chemical composition is not surprising. (This is of course not so for
the spheroidal bulges which are dynamically quite unlike the disc).
For example, for NGC 128, Hodge and Merchant (1966) have measured the
photoelectric B-V colour along the major axis and within the observat­
ional error find a fairly constant value of B-V = 1.0 in both bulge and
disc to r = 70" from the nucleus. In addition, KI have obtained
kinematic data on the bulge of NGC 4565, with the result that the V/θ
data is certainly consistent with a rotationally flattened bulge. This
is a coincidence if the bulge is in fact a bar seen end-on.

Photoelectric observations in UBV of the two SO galaxies NGC 128
and 1381 (both have peanut or box-shaped bulges) also show little or no
colour gradients in the bulge (K.C. Freeman, personal communication).
On the other hand, the SO galaxy NGC 3115, which has a large spheroidal
bulge, has a clear colour gradient (Strom et al 1977). Also, there is a
well established abundance gradient in our galactic bulge, which appears
to be spheroidal (see below). To summarise, it seems that the data is
consistent with our prediction that spheroidal bulges have significantly
stronger colour gradients than peanut-shaped bulges.

We should also attempt to discuss the formation of globular
clusters within this picture of the differences between spheroidal and
peanut-shaped bulges. Galaxies with spheroidal bulges appear to have
well developed globular cluster systems. Our galaxy has an extensive
halo globular cluster system and the bulge is probably spheroidal.
Eggen, Lynden-Bell and Sandage (1962) have shown that there are stars with
high energy and low angular momentum in the solar neighbourhood, so our
galactic bulge is probably of the spheroidal (NGC 7814) type (see
Figure 5.7). M31 also appears to have a spheroidal bulge (Ruiz, 1976),
and is known to have globular clusters. Both the globular clusters in our galaxy and M31 appear to have a wide range of orbital eccentricities (Freeman and Seitzer, 1981), as one would expect for objects in a spheroidal bulge. NGC 3115 has a spheroidal bulge and kinematics similar to NGC 7814 (Illingworth and Schechter, 1981), and again has an easily observed halo globular cluster system (Strom et al. 1979). In contrast, the box and peanut-shaped bulges may be deficient in globular clusters. NGC 4111 seems to have none; its bulge is peanut-shaped over a wide range of surface brightness (c.f. Tsikoudi, 1977). More impressively, JT also noted a complete lack of globular clusters surrounding NGC 4565. From the work of Harris and Racine (1979), relating the expected number of globular clusters in a galaxy with the absolute mag. of its spheroidal component, they showed that NGC 4565 should have about 300 clusters. So, unless the clusters have a very different luminosity function from those observed in other systems, they are probably not present. Again, we recall that the bulge of NGC 4565 is peanut-shaped. It seems then, that there is an apparent association of globular clusters with spheroidal type bulges and not the box or peanut-shaped bulges.

What is the main difference between the spheroidal and box-shaped bulges that could lead to globular cluster formation in the spheroidal type only, if all this is correct? One possibility lies in differences in the amplitude of the energy loss suffered by individual protoclouds of the two systems. Some of the high energy, low angular momentum clouds in the proto-spheroidal bulges will suffer large net changes in energy before they are finally converted to stars. These large net changes may include large single-event energy changes associated with encounters with other clouds. The shock compression resulting from these encounters may provide favourable conditions for cluster formation (Gunn, 1978). In contrast, in the pro-peanut bulges, this dissipation
will be much less dramatic, particularly if the main difference between the protoclouds is just the initial rotation rate of the protoclouds. For example, in a dissipative system where most material is in ordered motion, no single large energy losses will occur, and cluster formation may be inhibited.

5.6 CONCLUSIONS

The main conclusions based on the presently available data from this work and others are:

(1) The bulges of disc galaxies fall into two natural classes: the cylindrically rotating bulges and the non-cylindrically rotating bulges,

(2) the non-cylindrical rotators are invariably spheroidal shaped; their isophotes are roughly elliptical, although of greatly varying ellipticity from bulge to bulge,

(3) the non-cylindrically rotating bulges are well modelled with a distribution function of energy and angular momentum only, including stars over the whole permitted region in $E$ and $J$.

The cylindrical rotators can probably be modelled with a distribution function that is again a function of $E$ and $J$ only, but includes only stars of low orbital eccentricity,

(4) the presence of colour gradients in spheroidal bulges and their relative absence in peanut bulges fits well with the picture of dissipative collapse of the galactic protosystems,

(5) the colours of the bulge and disc are expected to be more similar for systems with peanut bulges than for galaxies with spheroidal bulges. The disc and bulge for the peanut-shaped bulge systems
are fairly similar dynamically, differing only in their degree of internal energy, so their evolutionary and chemical histories were also probably fairly similar,

(6) it appears that galaxies with peanut-shaped bulges may be deficient in globular clusters. The well-developed globular cluster populations are found in disc galaxies with spheroidal bulges, and this must be of importance for understanding globular cluster systems.

(7) elliptical galaxies and the bulges of disc galaxies are dynamically distinct, in spite of their first order photometric similarities. The bulges are rotationally flattened, in contrast to the brighter ellipticals whose flattening probably results from an anisotropic velocity dispersion. Hence any formation scenario for the formation of galaxies must make clear distinction between elliptical and disc galaxies.
6.1 INTRODUCTION

Burstein's (1979) photometry of a selection of edge-on S0 galaxies showed that the luminosity profiles perpendicular to the major axes at various galactocentric distances R, along the disc, were nearly exponential. In addition, these profiles showed a near independence of scale height with R. Observationally, this is seen as a thickening of the disc in the Z direction at low surface brightness, where the thin disc would normally have vanished (see Figure 6.1). Burstein named this structure the thick disc. Tsikoudi (1977) had already discovered this exponential component in NGC 4762, but in a slightly different way. She found that the luminosity profile along the minor axis followed the \( r^{1/4} \) law in the inner parts, but further out there was an excess of light above this law. This excess itself was exponential with height above the plane (see Figure 6.2). It is this exponential component which Burstein identifies as the thick disc, and its presence in NGC 4762 was confirmed by his photometry. Recent observations by van der Kruit and Searle (1981) have shown similar features in two edge-on spiral galaxies with significant bulges, notably NGC 4344 and 5907. Jensen and Thuan (1981) have also identified a thick disc in the edge-on spiral NGC 4565.

What are the thick discs dynamically? Freeman (1977) suggested that they may result from a response of the bulge to the much flatter potential field of the disc. Burstein (1979) suggested that they may be a dynamically separate component or an enhanced high velocity tail of the thin disc. We choose here to examine the first possibility since
Figure 6.1  **Upper Panel** The inner parts of the galaxy NGC 4762 from a 20 minute B band (IIa - 0 + GG 385) exposure. The central bulge and the thin disc are illustrated.

**Lower Panel** The outer parts of the galaxy NGC 4762 from a 4.5 hour B band (IIa - 0 + GG 385) exposure. The vertical lines indicate the positions of the perpendicular profiles made by Tsikoudi (40", 52" and 80") from the galactic centre. The horizontal line indicates the beginning of the thick disc observed by Burstein.
Figure 6.2 Observed minor axis profile of NGC 4762 in the B band from Tsikoudi. Note the exponential excess of light over the $r^{1/n}$ law for $z > 60''$ (cf Figure 6.3).
bulges are nearly isothermal and tend to adopt a roughly exponential distribution perpendicular to the disc in a flat disc-like potential. We will show that the thick disc in NGC 4762 can be formed by the response of the bulge to the flat potential of the disc. Section 6.2 describes the models and in section 6.3 a model is constructed for NGC 4762. Section 6.4 concludes with a discussion of the success of the models.

6.2 THE THICK DISC MODEL

Our purpose here is to illustrate the point that a roughly isothermal bulge (e.g. a King model) in the potential field of a disc can form a thick disc structure very much like those observed. So to illustrate the important features, we will construct self-consistent, non-rotating models in the presence of a disc-like potential. Chapter II describes the details of the adopted model. Here the models are non-rotating, so $\gamma = 0$, and the distribution function for the bulge is the same as that for a King model. The bulge was chosen to have no net rotation to illustrate, as unambiguously as possible, that the dominant factor producing the thick disc is the presence of the disc potential alone. Rotation is a refinement which is omitted here for clarity (although it is easy to include: see Chapter II).

One approach to the investigation of a King model in the presence of the nearly flat potential field of a disc is to examine what changes occur when the disc is flattened from a much more spherically symmetric configuration. Three models were computed, all with the same bulge-to-disc mass ratio (see section 6.3), but with the disc itself having different amounts of flattening. The model parameters are tabulated in Table 6.1 (see Chapter II for an explanation of the quantities).
TABLE 6.1
PARAMETERS OF THICK DISC MODELS

<table>
<thead>
<tr>
<th>Model</th>
<th>$W_b^0$</th>
<th>$W_d^0$</th>
<th>a</th>
<th>b</th>
<th>Q</th>
<th>$M_d/M_b$</th>
<th>$r_t/r_c^{(e)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-7.5</td>
<td>-2.3</td>
<td>10</td>
<td>5</td>
<td>48.1</td>
<td>1.31</td>
<td>78.0</td>
</tr>
<tr>
<td>II</td>
<td>-7.5</td>
<td>-2.7</td>
<td>10</td>
<td>2.5</td>
<td>48.1</td>
<td>1.31</td>
<td>66.0</td>
</tr>
<tr>
<td>III</td>
<td>-7.5</td>
<td>-3.4</td>
<td>10</td>
<td>1</td>
<td>52.7</td>
<td>1.31</td>
<td>64.0</td>
</tr>
</tbody>
</table>

The projected surface density distribution of the disc as seen from the Z direction was kept nearly constant for all three models, differing by less than 0.2 magnitudes in the mean. The upper panel of Figure 6.3 shows the projected profiles of the bulge alone for the three models along the minor axis. For comparison, all three models have been scaled to have the same tidal radius. In each of the models, the $r^4$ law was fitted over the four innermost points (excluding the central point). Note that all three models closely follow the $r^4$ law near the centre despite differences in the total (bulge + disc) potential at the centre. The change in slope of the $r^4$ law between models I to III is expected since the slope is largely determined by the total central potential (bulge + disc). The most spherical model (I), was chosen to be closely $r^4$ over approximately six magnitudes in surface brightness: the bulge's central potential was then $W_b^0 = 7.5$. It is immediately apparent that as the disc is flattened (models II and III), the mass distribution within the bulge changes significantly, resulting in an excess of light over the $r^4$ law in the outer parts. The greater the flattening, the greater the excess. This excess itself decreases exponentially with height along the minor axis, as shown from model III in the lower panel of Figure 6.3 (cf. Figure 6.2). Qualitatively like Tsikoudi's, it is this component of the total
Figure 6.3 **Upper Panel** The projected minor axis profiles of models I, II and III on the same magnitude and distance scale except for model I which has been plotted one magnitude brighter for clarity. The solid straight line shows the least squares $r^4$ fit to the innermost four points (excluding the central point). Only three of these four points are shown for models I and II.

**Lower Panel** The projected minor axis profile of model III in the magnitude $-Z$ plane. The magnitude scale is arbitrary but Z is scaled to Tsikoudi's photometry of NGC 4762. The difference (model - $r^4$) shown as crosses, is plotted four magnitudes fainter for clarity.
light distribution which is identified with Burstein's disc.

6.3 COMPARISON WITH NGC 4762

For quantitative comparison with observation, the best example to date of a galaxy with a prominent thick disc and extensive surface photometry is NGC 4762 (Burstein, 1979; Tsikoudi, 1977). Tsikoudi's photometry in B was used here from which she derived a disc-to-bulge luminosity ratio of 1.3. Model III was taken to be the best model for this galaxy. The model was scaled to Tsikoudi's photometry by fitting the $r^4$ law luminosity distribution in the inner parts (see lower panel of Figure 6.3). Comparison between the lower panel model of Figure 6.3 and the photometry of Figure 6.2 shows a good overall agreement. In both cases the exponential component becomes significant at a height $Z \sim 0.9'$ and survives to about $Z \sim 2.0'$ (the limit of Tsikoudi's photometry). Note again that the scaling of the models is made via the $r^4$ law fit to the central regions only of the model, and hence does not explicitly define the value for the beginning of the exponential disc, or its scale height. In both cases, the scale heights are very similar, about $0.51'$ which, assuming the distance to NGC 4762 is 15 Mpc (Bertola and Capaccioli, 1975), corresponds to a physical scale height of 2.2 kpc.

Tsikoudi also observed the luminosity distribution along cuts perpendicular to the major axis at various galactocentric distances, shown in Figure 6.4. Apart from the innermost 10'' in $Z$, dominated by the disc, her profiles are exponential to the limits of the photometry. In particular, the scale heights are at most weakly dependent on the galactocentric distance at which the cuts are made. This same effect has also been observed in NGC 4244 and NGC 5807 by van der Kruit and Searle (1981). Figure 6.5 shows the profiles of model III at the same
galactocentric distances as Tsikoudi's photometry i.e. 40", 52" and 80" from the galactic centre. Again, the model has been able to reproduce quite well the overall exponential nature of the perpendicular profiles as well as the scale height being nearly independent of the galactocentric distance, at least for the R = 40" and 52" cuts. We do not regard the poor representation of the 80" perpendicular profile seriously: the model is mainly illustrative, and the fit could undoubtedly be improved by including bulge rotation, which will be significant in this region (see Figure 4.14).

Burstein also noted that thick discs show a shallow luminosity gradient along lines parallel with the major axis. Model III exhibits the same behaviour; the profiles are flatter and become more so as the cut distance above the plane increases. This decrease in slope will give the isophotes a box shape appearance at low surface brightness (cf Figure 6.1).

Once a satisfactory model had been constructed for NGC 4762, it was natural to look for further examples of galaxies with possible thick discs. Tsikoudi found a similar excess over the $r^2$ law in the minor axis profile of NGC 3115. However, whereas the excess was found in both B and V for NGC 4762, she found it only in B for NGC 3115 and not in V. This seemed unusual and in fact an examination of the B-V colour profile from her data indicated a likely systematic error in the outer parts of the B minor axis profile. In addition, neither the B photometry of van Houten (1961) nor Miller and Prendergast (1968) on the minor axis were able to confirm Tsikoudi's data.

Tsikoudi also observed NGC 4111 and found that the observed minor axis profile fell below the $r^2$ law fitted in the central region. How
Figure 6.4  Observed perpendicular profiles of NGC 4762 in B at distances of 40", 52" and 80" from the galactic centre from Tsikoudi. Note the near independence of scale height with increasing galactocentric distance.
NGC 4762

- observed
  (bulge + disk)

Magnitude vs. Z (min arc) for NGC 4762 at different angular separations: 40", 52", 80".
Figure 6.5 Projected perpendicular profiles of model III at galactocentric distances of 40", 52" and 80" along the major axis on an arbitrary magnitude scale. The plus signs show the total light, bulge + disc, and the solid line shows the contribution from the bulge alone. Each profile has been plotted two magnitudes fainter than that above for clarity.
can this be reconciled in view of the excess observed in NGC 4762? The simplest interpretation lies in the rotation of the bulge. Of a sample of 56 disc galaxies observed by Mayall and Lindblad (1970), NGC 4111 was found to be the fourth most rapidly rotating, with an angular velocity of 240 km s\(^{-1}\) kpc\(^{-1}\). From Tsikoudi's deconvolution of the luminosity profile on the major axis of NGC 4111, it is clear that Mayall and Lindblad were measuring the bulge rotation and not the disc i.e. the bulge is dominant and rapidly rotating. The bulge is also peanut-shaped, which again suggests rapid rotation (see Chapter V).

Some models with discs were constructed with a moderate to large amount of rotation. It was found that the material in the outer parts of the galaxy \((r > r_t/2)\) responded to this rotation by a more rapid decrease in projected surface density on the minor axis. This tended to drive any excess, formed by the disc potential, back towards the \(r^{1/4}\) distribution; with sufficient rotation the minor axis profile actually fell below the \(r^{1/4}\) law.

6.4 CONCLUSIONS

From the above discussion it is concluded that thick discs may be successfully explained as the response of the bulge population to the potential field of the flat disc. For NGC 4762, the perpendicular profiles and thick disc are well reproduced by this model. On the basis of this model, thick discs cannot be precluded from any disc galaxy in not too rapid rotation, and so should not be a characteristic of SO galaxies only as suggested by Burstein (1979). In fact, thick discs have been identified in the spiral galaxies NGC 4344, 5907 and 4565 as noted earlier on. A fairly strong bias probably exists in favouring their detection more readily in SO galaxies, since SO galaxies have
systematically larger bulge-to-disc mass ratios than spirals. However, if thick discs had not been observed in any spiral galaxies, then our picture for the origin of thick discs would be seriously in doubt. The examples of thick discs in NGC 4344, 5907 and 4565 give us some confidence in its validity.
Several aspects of interesting future work arise from the results summarised in Section 5.6.

(i) The most important of these is to construct self-consistent models with contours of constant $f$ close and parallel to the circular orbit line of the $(E,J)$ diagram and to see whether our views on the structure of peanut bulges is correct. This will hopefully lead to bulge models which can easily be made box and peanut-shaped. At the same time, this will indicate whether the peanut-shaped bulges are much more rapidly rotating box-shaped bulges.

(ii) A systematic plate search should be undertaken to ascertain quantitatively, the relative frequency of occurrence of box and peanut-shaped bulges. As de Vaucouleurs (1974) pointed out, this may be a common phenomenon. A casual examination reveals many such examples, e.g. IC 2531, 4767, NGC 5746, 1886, 4111, 5907, 7332, 2683 etc.

(iii) It seems very important to verify the validity of the correlation between the presence of a halo globular cluster system and the box or peanut-shaped bulges.

(iv) It is also of great interest to derive accurate mass-to-light ratios of galaxies. With accurate models this should now be possible, at least for the spheroidal bulges. Not only will this enable total masses to be calculated, but also spatial variations in local $M/L$ values to be more fully understood.

(v) The present models form an excellent basis from which to gain a better insight into the $L_{IR} \propto V_{max}^4$ law for disc galaxies. The form of this law has been shown (e.g. Aaronson et al 1979) to arise in a natural way assuming constancy between galaxies of their central mass
surface density, mass profiles and mean M/L ratios. The power law has been found to be best defined in the infrared (\(\lambda \approx 1.6\mu\)) where the last of the above three assumptions is expected to be most nearly correct. The \(L_{IR} \propto V_{\text{max}}^4\) relationship gives a tool to probe the relationship between the luminous and dark components of galaxies, since this is a relation between the luminous component which provides the luminosity \(L\) and the dark component which determines the maximum rotational velocity \(V_{\text{max}}\) of the rotation curve. Construction of models including a dark and a luminous component will be essential for this investigation.

(vi) An observational program aimed at measuring more of the box and peanut-shaped bulges to confirm that they all rotate cylindrically. As noted above, there is no shortage of such objects and the kinematical properties of these bulges place very useful constraints on the models as has been shown.
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