TAPERED OPTICAL WAVEGUIDES

and

METAL-CLAD POLARISERS

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July, 1988
For the Very Best Parents,

Mine.

For your Love and Support

and being there whenever

I needed you.

"Love endures long and is patient and kind"

PREFACE

This dissertation is an account of work carried out in the Department of Applied Mathematics and later in the newly formed Optical Sciences Centre, in the Institute of Advanced Studies at the Australian National University between February, 1985 and July, 1988 under the supervision of Drs. John Love and Adrian Ankiewicz.

While I have benefitted immensely from discussions with members of these departments and other colleagues, unless specifically disowned, the material presented herein is my own.

None of this work has been submitted to any other institution of learning for any degree.

Wanda M. Henry.
PUBLICATIONS


ACKNOWLEDGEMENTS

"The first ten million years, they were the worst. And the second ten million, they were the worst too. The third ten million, I didn't enjoy at all. After that I went into a decline." - Marvin, the android robot. Hitch-Hiker's Guide to the Galaxy, Douglas Adams.

Thankfully, my time here in the Applied Maths Department and later, the Optical Sciences Centre, has mostly, been quite the opposite to this and a most enjoyable period of my life has passed too quickly. I first joined the Applied Maths Department in the long vacation in the 1983/84 summer as a Vacation Scholar. The enthusiasm of the head, Barry Ninham and other members made it such a great place to work that I had no choice about joining the group for my PhD. I felt that I belonged even more when I was given a desk of my own, rather than occupying the desks of absentee members.

In particular, I would like to thank those members who were easy to get tea money from. They made my 'job' as person in charge of the tea and coffee supplies, bearable. Over nearly three years I finally got most of the others trained. I hope those who follow in my steps appreciate the mammoth effort this took.

In particular, I would like to sincerely thank my supervisors, Drs. John Love and Adrian Ankiewicz. Words are not sufficient to describe my gratitude for their advice, encouragement, enthusiasm and experience which they tried to pass on. John was very understanding, and many times when I was feeling depressed, even to the stage of giving up, he managed to lift my spirit and make me see that I was not unique in this respect. He has put a lot of effort into helping me whenever possible, and I am very grateful. Adrian, with his great sense of humour has also been a worthy supervisor. He has made an effort to ensure that I was my cheery self as often as possible, and helped with my problems with great patience. Both John and Adrian have been good friends as well as colleagues. I am thankful to them for proof-reading this thesis.
My fellow students have also earned my thanks. Zheng has been an admirable office-mate. Derek and Andrew have done their share of cheering me up while being good sounding boards, and Andrei has been a valued friend, both academically and personally.

My most grateful thanks to Diana Wallace who has helped with the layout of this thesis and who took care of 'administrative details' throughout my time in the department.

I would also like to thank Lawrie Brown and Roland Kjellander, who helped out in times of dire need, when my Macintosh was playing up, and the staff of the School Computer Unit, particularly Julie, who helped solve some of the many computing problems I encountered, before I managed to tear all of my hair out.

I would like to thank the Australian Government for financial support and finally, I would like to sincerely thank the Australian Telecommunication and Electronics Research Board for their generous financial support through a scholarship and equipment and conference grants.
ABSTRACT

"""We want you to tell us.... the answer."

"The Answer? The answer to what?"

"Life. The Universe. Everything."

"Tricky."

"But can you do it?"


This thesis presents the theoretical analysis of tapered waveguides and polarising devices fabricated by the addition of a metal layer or region to a waveguide. In each case the waveguides are single-mode and a modal rather than a ray approach is therefore appropriate.

Chapter 1 introduces the basic theory and defines the waveguide and modal parameters which will be used throughout the thesis. The vector wave equations that govern the electromagnetic properties of propagation are introduced. For slab geometries, these reduce to a single equation for the longitudinal field components, while for fibres which are weakly-guiding, the vector equations simplify to the scalar wave equation for the transverse electric field components.

Chapter 2 describes the local-mode approach for analysing tapered waveguides and makes a comparison between this approach and the exact analysis for the slab infinite parabolic waveguide with a parabolic taper shape. The Stewart-Love criterion which provides a delineation between adiabatic and highly lossy tapers is derived together with an alternative criterion based on the local-mode equations.
Chapter 3 considers tapered step-profile waveguides. The slab waveguide is considered first because the local modes can be determined exactly, without the need for any approximations. In the case of fibres, the weak-guidance approximation is used to simplify the analysis. Theoretical results for the excess loss of several tapers as a function of wavelength, which are qualitatively the same as experimental results, are presented, together with the variation of the excess loss as the taper is drawn, with the wavelength fixed. The Stewart-Love criterion for each taper is discussed, together with a taper shape for minimal excess loss based on this criterion. The spot-size of the field along the taper is also considered.

Chapter 4 examines tapered W-fibres. These fibres suffer much larger losses than the matched-clad fibres considered in chapter 3. The new delineation criterion based on the local-mode equations is compared with the Stewart-Love criterion for both W-fibres and matched-clad fibres. For W-fibres, the latter does not accurately delineate between adiabatic and non-adiabatic taper shapes. In the last section of this chapter, a physical mechanism for the large absorption peaks near the cutoff wavelength of straight W-fibres is proposed. It is based on the slight tapering that occurs because of the non-uniformities in the core radius, created during drawing process.

Chapter 5 considers the excess loss from fused taper couplers. The fibre coupler is modelled by the corresponding slab waveguide.

Chapter 6 examines polarisers which are fabricated by the addition of a metal layer or region to a single mode fibre. The device is modelled by a slab waveguide and we propose the addition of a buffer layer between the cladding and metal regions to improve the polarising ability.
Chapter 7 offers an explanation for the experimentally observed limitation on the polarising ability of these devices. We propose that the excitation of the higher-order modes of the polariser is responsible for the limitation on the extinction ratio obtainable. The power flow within the device is also examined.

Chapter 8 considers slab polarising couplers which could be adopted in integrated optics and describes the corresponding fibre devices.
NOTES ON TEXT

References are labelled within each chapter by superscript, ie. reference\textsuperscript{9}. A list of references is found at the end of each chapter.

Figures are labelled within each chapter according to the chapter and figure number, ie. Figure 8-3 means the third figure in chapter 8.

Equations are labelled from 1 within each chapter. Back-references are labelled according to chapter also, ie. equation 3-6 means equation 6 in chapter 3.

S.I. units are used throughout unless otherwise stated.
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CHAPTER 1
Basic Theory and
the Wave Equations

1-1. Introduction

This thesis is going to examine devices belonging to two categories - tapered structures and polarisers fabricated by the addition of a metal layer to fibres or slab waveguides. In each case the device is fabricated from standard, isotropic single-mode waveguides. It is therefore necessary to follow a modal rather than a ray approach to the analysis, since the latter is inaccurate when diffraction is significant. This involves solving the vector wave equations which are an alternative form of Maxwell's equations. The geometries involved, however, make the analysis of even the simplest multi-layered, step-profile fibres rather complicated. Fortunately, the slight variation between core and cladding refractive indices enables us to use the weak guidance approximation\(^1,2,3\) which reduces the analysis to the solution of the scalar wave equation.

In the case of slab geometries, the modes can only be transverse electric (TE) or transverse magnetic (TM) and the vector equations decouple into simple scalar equations for the non-zero field components\(^4\).

In this chapter, the co-ordinate systems used to describe the variation of the refractive index and the electro-magnetic fields are given, together with the governing equations of the modal fields. The waveguide and modal parameters which characterise the waveguide and modes, respectively, will be defined\(^4\).

1-2. Co-ordinate Systems

The co-ordinate systems are shown in Figure 1-1. The z axis is chosen to be along the
waveguide axis. For fibres with circular symmetry in the cross-section, as in Figure 1-1a), there are two transverse co-ordinates. These can be chosen as cartesian (x,y), or more conveniently, polar (r,θ). For slab geometries, as in Figure 1-1b), where there is no variation in the y direction, only the x co-ordinate is required.

For many waveguides, the refractive index profile, n, is a step function, as shown in Figure 1-2a). The refractive index changes abruptly at r=ρ, the core radius or halfwidth, from n_\text{co} in the core, to n_\text{cl} in the cladding. The surface defined by r=ρ is the core-cladding interface. For guidance, it is necessary that n_\text{co} > n_\text{cl}. For graded-index waveguides, there is a continuous variation in index in the core while the cladding is uniform, as shown in Figure 1-2b). In the case of the infinite parabolic profile, used in chapters 2 and 3, where n decreases monotonically away from the axis, the characteristic radius ρ, is not obvious. By convention, we choose a typical value for n_\text{cl} and define the characteristic radius ρ, to be the radius at which the index takes this value.
Figure 1-2. The variation in refractive index as a function of radius $r$ for fibres, or transverse displacement $x$ for slab geometries, for a) step-profile waveguides and b) graded-index waveguides.

1-3. Electro-Magnetic Fields

Light propagating down a waveguide can be expressed as the sum of two parts. The first represents light which propagates without loss in a non-absorbing waveguide, and can be represented as a superposition of the bound modes of the waveguide. The second part represents light which is lost with propagation through radiation. In this thesis, only the bound part will be considered and the spatial dependence of the electric field, $E$, and magnetic field, $H$, is of the form

$$E(x,y,z) = \sum_j a_j E_j(x,y,z) \quad (1a)$$

$$H(x,y,z) = \sum_j a_j H_j(x,y,z) \quad (1b)$$

where $a_j$ is the modal amplitude representing the contribution of the $j$th modal field to the total field, and the summation is over all bound modes. Assuming the light source to be
monochromatic, the time dependence of the field is of the form \( \exp(-i \omega t) \) where \( \omega = 2\pi c/\lambda \) is the angular frequency of the radiation, \( c \) is the speed of light in a vacuum and \( \lambda \) is the wavelength of the source. This time dependence will be assumed implicitly henceforth.

The electric and magnetic fields are related through Maxwell's equations so that only the electric field need be considered in the following description. For waveguides with cylindrical symmetry, the electric field, \( E_j \), of the \( j \)th bound mode is of separable form

\[
E_j(x,y,z) = e_j(x,y) \exp(i\beta_j z)
\]

(2)

The propagation constant of the \( j \)th mode, \( \beta_j \), is determined from an eigen-equation which arises from solving Maxwell's equations in the core and cladding regions and applying the boundary conditions at the core-cladding interface. The vector dependence of equation 2 can be resolved into transverse and longitudinal components

\[
e_j(x,y) = e_{tj}(x,y) + e_{zj}(x,y) \hat{z}
\]

where \( e_{tj} \) and \( e_{zj} \) are the transverse and longitudinal components, respectively, and \( \hat{z} \) is a unit vector in the longitudinal direction. Then the total electric field becomes

\[
E(x,y,z) = \sum_j a_j [ e_{tj}(x,y) + e_{zj}(x,y) \hat{z} ] \exp(i\beta_j z)
\]

(3)

There is a similar expression for the total magnetic field.

**1-4. Waveguide and Modal Parameters**

**1-4-1. Waveguide Parameters**

As is common in many types of analysis, it is useful to parameterise in terms of
non-dimensional quantities. There are two important parameters which characterise the waveguide, namely, the profile height parameter, \( \Delta \), and the waveguide parameter, \( V \), defined respectively as

\[
\Delta = \frac{n_{co}^2 - n_{cl}^2}{2 n_{co}^2}
\]

(4)

\[
V = \frac{1}{\rho k n_{co} (2\Delta)^\frac{1}{2}}
\]

(5)

where \( k = 2\pi/\lambda \). The number of bound modes\(^4\) which can propagate along a particular waveguide depends primarily on the value of \( V \). When \( V \gg 1 \), the waveguide supports many bound modes and is said to be multi-moded. Throughout this thesis, only waveguides which are single-moded under normal operating conditions will be considered. For step-profile fibres this occurs when \( V < 2.405 \), while for the corresponding slab waveguide, \( V < \pi/2 \).

1.4.2. Core and Cladding Modes

When single-mode waveguides are used in devices such as couplers, tapers or polarisers, the outer protective jacket is normally removed during fabrication, thereby exposing the glass to air, which has refractive index \( n = 1 \). In this situation the core and cladding regions have indices which are approximately the same, e.g. \( n_{co} = n_{cl} = 1.45 \) for silica glass, while there is a large index difference between glass and air. The latter, when combined with the large cladding radius produces a correspondingly large \( V \)-value associated with the cladding-air interface. Therefore the glass-air structure is multi-moded and higher-order modes, which would normally be cutoff, are supported. These are commonly called cladding modes because most of the power is in the cladding. Modes with most of their power in the core are commonly called core modes. There is no well-defined difference
between core and cladding modes, since the former gradually evolve into the latter.
However, it is convenient to introduce a delineation condition based on their effective
indices. The effective index of a mode is defined by \( n_e = \beta / k \). Core modes will be
associated with \( n_e > n_{cl} \) and cladding modes with \( n_e < n_{cl} \).

1-4-3. Modal Parameters

There are two dimensionless parameters \( U_j \) and \( W_j \), which characterise each mode of the
waveguide in the core and cladding regions, respectively, and are defined in terms of the
propagation constant by

\[
U_j = \rho \left[ k^2 n_{co}^2 - \beta_j^2 \right]^{1/2} \\
W_j = \rho \left[ \beta_j^2 - k^2 n_{cl}^2 \right]^{1/2}
\]

Clearly \( V^2 = U_j^2 + W_j^2 \). More generally, if a waveguide is multilayered with regions of
refractive index \( n_r \), between the core and cladding, such as the W-fibre, then there will be a
parameter \( W_{jr} \) or \( Q_{jr} \) associated with each region. If \( n_e > n_r \), then \( W_{jr} \) is defined as in
equation 7 with \( n_r \) replacing \( n_{cl} \). If this is not the case, i.e. \( n_e < n_r \), \( W_{jr} \) is replaced by \( Q_{jr} \)
defined by

\[
Q_{jr} = \rho \left[ k^2 n_r^2 - \beta_j^2 \right]^{1/2}
\]

For cladding modes of multi-layered structures, \( n_{cl} > n_e > n_{min} \), where \( n_{min} \) is the
minimum refractive index of the waveguide.

1-5. Vector and Scalar Wave Equations

We are now in a position to consider the equations that govern the behaviour of light in the
waveguide. The vector wave equations, which are derived from Maxwell's equations for
source-free media$^4$, are

$$\begin{align*}
[\nabla_t^2 + k^2 n^2 - \beta^2] e_t &= -\nabla_t (e_t \cdot \nabla_t \ln n^2) \\
[\nabla_t^2 + k^2 n^2 - \beta^2] e_z &= -i \beta e_t \cdot \nabla_t \ln n^2 \\
[\nabla_t^2 + k^2 n^2 - \beta^2] h_t &= - (\nabla_t \times h_t) \times \nabla_t \ln n^2 \\
[\nabla_t^2 + k^2 n^2 - \beta^2] h_z &= [\nabla_t h_z - i\beta h_t] \cdot \nabla_t \ln n^2
\end{align*}$$

(9)

The operators $\nabla_t$ and $\nabla_t^2$ are defined, in rectangular coordinates, by

$$\nabla_t = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y}$$

$$\nabla_t^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$$

(10)

where $\hat{x}$ and $\hat{y}$ are unit vectors parallel to the x and y axes, respectively.

These equations show that it is the terms involving $\ln n^2$ that couple the longitudinal and transverse components and make an analytical approach complicated in general. However, there are cases where the equations can be decoupled. Step-profile waveguides are an example. The refractive index has uniform values $n_{co}$ in the core and $n_{cl}$ in the cladding, with a discontinuity at the core-cladding interface. Thus the right-hand sides of the equations vanish and they decouple except at the interface. The decoupled equations can then be solved separately in the core and the cladding. Imposing the conditions that the longitudinal fields and transverse magnetic fields are continuous at the interfaces leads to an eigen-equation$^4$ for discrete values of $\beta_j$. 
1-5.1. Step-Profile Slab Waveguides

For step-profile slab waveguides, the vector equations reduce to a single equation for the longitudinal components given by

\[
\left[ \frac{d^2}{dX^2} + p \right] \psi = 0 \quad (11)
\]

where \( \psi \) denotes \( e_Z \) for TM modes, or \( h_Z \) for TE modes, \( X = x/\rho \) is the normalised transverse co-ordinate and

\[
P = \begin{bmatrix}
U^2 & \text{core} \\
-W^2 & \text{r th region when } \beta > k n_r \\
Q^2 & \text{r th region when } \beta < k n_r
\end{bmatrix}
\quad (12)
\]

The non-zero transverse components for TE modes are given by

\[
e_y = -i \frac{1}{p} \left( \frac{\mu_0}{\varepsilon_0} \right)^{1/2} \rho k \frac{dh_z}{dX}, \quad h_x = i \rho \beta \frac{dh_z}{dX} \quad (13a)
\]

The corresponding components for the TM modes are

\[
h_y = i \frac{1}{p} \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} \rho \beta \frac{de_z}{dX}, \quad e_x = i \rho \beta \frac{de_z}{dX} \quad (13b)
\]

where \( \varepsilon_0 \) and \( \mu_0 \) are the free-space dielectric constant and permeability, respectively. The conditions that lead to the eigen-equation are: \( e_Z \) or \( h_Z \) and \( h_x \) or \( h_y \) should be continuous, bounded and tend to zero as \( |X| \to \infty \).
1-5-2. Weak Guidance and the Scalar Wave Equation

The analysis is not quite as simple for fibres, particular those that are multilayered, because there are both magnetic and electric longitudinal field components in the fundamental mode and higher-order modes with the same symmetry. However, when the variation in the refractive index is small, \( \Delta \ll 1 \), the weak guidance approximation\(^1\) can be invoked. Under this restriction, the modes are approximately TEM with \( e_z = h_z = 0 \) and the transverse electric and magnetic modal fields are approximately plane polarised in orthogonal directions. They are related by

\[
h_t = \left( \frac{\varepsilon_0}{\mu_0} \right)^{\frac{1}{2}} n \hat{z} \times e_t
\]

where \( \hat{z} \) is the unit vector in the z direction and \( n = n_{co} \) is the refractive index. The variation in refractive index is small so that the effective index \( n_e = n_{co} \) and the propagation constant is then \( \beta = k n_{co} \). Since this equation relates the electric and magnetic fields, only the electric field need be considered in the analysis. The electric field can be expressed in terms of two orthogonal components

\[
e_t(r,\theta) = A_x e_x(r,\theta) \hat{x} + A_y e_y(r,\theta) \hat{y}
\]  

(14)

where \( \hat{x} \) and \( \hat{y} \) are unit vectors in the x and y directions, respectively, \( A_x \) and \( A_y \) are constants and \( e_x \) \( \hat{x} \) and \( e_y \) \( \hat{y} \) are the polarisation states. For waveguides with circular symmetry, the radial dependences of \( e_x \) \( \hat{x} \) and \( e_y \) \( \hat{y} \) are identical. Within the weak guidance approximation, the vector wave equations decouple and simplify to the scalar wave equation

\[
[ \nabla_t^2 + P ] \psi = 0
\]  

(15)
where $\psi$ denotes either $e_x$ or $e_y$, $P$ is defined in equation 12 and $\nabla_t^2$, expressed in normalised co-ordinates $(R, \theta)$, is given by

$$\nabla_t^2 \psi = \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \psi}{\partial R} \right) + \frac{1}{R^2} \frac{\partial^2 \psi}{\partial \theta^2}$$

where $R=r/\rho$, is the normalised radial co-ordinate.

To determine the modal fields, we solve the scalar wave equation in each region and apply boundary conditions to determine the eigen-equation which can be solved for $U$ or $\beta$ for specific values of $V$ and $\Delta$. These conditions are:

1. $\psi$ is continuous across every interface.
2. Likewise, $\psi'$ (with respect to the radial co-ordinate) is continuous.
3. $\psi$ and $\psi'$ are bounded and vanish as $R \to \infty$.

1-6. References


CHAPTER 2
The Analysis of Tapered Waveguides
The Local-Mode Equations

2-1. Introduction

Tapered single-mode waveguides have received much attention recently. Horn shaped tapers were first considered as a means of connecting fibres of different core radii\textsuperscript{1,2}. More recently, they have been used as beam expanders. In some of these devices, the field spreads out as the cross-sectional area of the core increases\textsuperscript{3}, whereas in other devices the core radius decreases so that the field is not as strongly confined to the core region and also spreads out\textsuperscript{4}.

Tapered structures have also been utilised in the fabrication of spectral filters. Experimentally, it is found that the output power in a spectral scan has a modulation which is sinusoidal-like\textsuperscript{5-8}. When the wavelength is fixed and measurements are made as the fibre is drawn, there is also a modulation\textsuperscript{6-8}. The loss in power, or excess loss, responsible for this modulation, is caused by coupling to cladding modes in the taper.

Excess loss also occurs in fused taper couplers. To achieve the desired splitting ratio, the beatlength between the odd and even modes of the coupler, which determines the length required for power transfer from the incoming fibre to the second one, needs to be relatively small. This can be achieved in two ways. The first is by physically making the cores closer, which can be done by chemically etching or polishing the cladding and then abutting the two fibres. Alternatively, two parallel fibres are fused and drawn, creating a tapered structure in which the fields spread into the cladding. It is the latter type for which the effects of tapering are important\textsuperscript{9-16}. 
2-2. Methods for the Analysis of Tapers

2-2-1. Exact Solutions

There are several ways of analysing propagation through tapered waveguides. One approach is to determine the exact modes of the tapered region. The incoming fundamental mode will then excite the exact normal modes of the complete taper. At the end of the tapered section, the modes of the outgoing straight fibre will be excited. Any power not going into the fundamental mode will excite the cladding modes, from which power is absorbed by the outer protective jacket, and accounts for the excess loss.

The tapering of the waveguide destroys the translational invariance and introduces a $z$ dependence into the longitudinal variation of the field components which cannot be separated with the usual $\exp(i\beta z)$ factor. Physically, this means that tapering bends the phase front of a mode so that it is no longer planar. Unfortunately, this deviation prevents the simplification of the wave equations by separation of variables so that, in general, a numerical solution is required. However, there are certain structures for which exact solutions exist, for example the slab infinite parabolic profile taper\textsuperscript{17}, which will be discussed in the latter part of this chapter, and the Marcatili tapers\textsuperscript{18}. If the actual taper is a section of one of these specific shapes, then the incoming field will excite the exact modes of the tapered region. The excitation coefficient for each mode is defined in terms of an overlap integral of the incoming field and the field of the exact mode. The same procedure applies at the end of the taper where the outgoing modes are excited. Power not going into the outgoing fundamental mode will be responsible for the excess loss. The number of waveguides and taper shapes for which an exact analysis is possible is limited, thus alternative analytical approaches are required.
2-2-2. Local-Mode Solution

For slowly-varying taper shapes, the bending of the phase front of the modes is minimal and the field in the tapered region is very much like that of the fundamental local mode. By definition, a local mode is the mode that would exist on a straight fibre having the local parameters. The excess loss can be explained by modal coupling to higher-order modes from which power is lost from the fibre beyond the tapered region. These higher-order modes are essentially cladding modes which are supported in the tapered region because of the finite cladding. The outer jacket of the fibre, usually responsible for eliminating any power in the cladding modes by absorption, must be removed during the heating and drawing process to prevent contamination of the glass, and thereby exposing the cladding to a region of lower refractive index, normally air. As the tapering occurs, the field of the fundamental mode spreads into the cladding and some of its power is coupled into higher-order modes due to the change in the cross-section. Conversely, there is power coupling from the higher-modes back into the fundamental mode. It is this coupling mechanism that is responsible for the modulation in the excess loss.

The power in each mode along the taper is determined by a set of local-mode equations\textsuperscript{19}. As will be shown in the following chapter, for slowly-varying tapers when only the fundamental mode and one other are involved, negligible excess loss at certain wavelengths is possible because power coupled to the first higher-order mode as the fibre is tapered down, is totally recaptured as the fibre is tapered up\textsuperscript{20}. This approach has also been applied to horn shaped tapers where the core radius increases monotonically\textsuperscript{21}.
2-2-3. Abrupt Tapers

A similar approach, which also uses local modes, is the sudden approximation method\(^7\). It is more appropriate when the tapering is sufficiently severe resulting in a large fraction of power in the fundamental local mode being coupled to many higher-order modes. This involves modelling the taper by one in which the severely tapered regions are replaced by an abrupt transition between the beginning and end of each region. These regions are determined by the Stewart-Love delineation curves\(^2^2\) which will be discussed later in this chapter. This produces a shorter taper with discontinuities where segments have been removed. In the analysis of the taper, the incoming field excites the local modes in the first segment. These propagate to the next discontinuity and excite the local modes of the next segment, and so on until the end of the taper. Power not going into the fundamental mode here produces the excess loss. This model gives good agreement with experimental measurements\(^7\).

There are other methods which are more appropriate to the horn devices where strong coupling occurs between the fundamental and cladding modes because of steep tapering, and the fundamental local mode is no longer a good approximation to the fields within the taper. Two examples are the beam propagation method\(^2^3\) and the clinging beam method\(^2^4\), which predict the spot-size, or spread, of the field.

2-3. Local-Mode Equations

In this thesis, the local mode approach will be taken. The propagation of light through different tapers will be determined through the solutions to the local-mode equations. To apply local-mode theory, we make the assumption that in the tapered region, the field can be expressed as a summation over the local forward and backward propagating
orthonormal modes and the radiation field.

2-3-1. Backward-Propagating Modes

The backward-propagating modes can, in fact, be neglected. This can be seen from a simple argument in terms of the Fresnel reflection coefficient. The taper angle is less than 90°. Thus, the worst possible case of a 90° taper angle where light is incident from a region with refractive index $n_{CO}$ onto a region with refractive index $n_{cl}$, i.e. reflection from a planar interface, gives an upper limit on the fraction of reflected light. The reflection coefficient in this case is

$$R = \left( \frac{n_{co} - n_{cl}}{n_{co} + n_{cl}} \right)^2$$

For the index values involved, $n_{CO} = n_{cl} = 1.5$, this gives $R \approx \Delta^2/4$. Since $\Delta \leq 10^{-2}$, for practical fibres $R$ is negligible, which justifies neglecting the backward-propagating modes.

2-3-2. Effect of the Finite Cladding

Tapering bends the phase front of a mode. In a waveguide with an infinite cladding this results in radiation because the distortion of the planar phase front causes the local phase velocity to exceed the speed of light, $c/n_{cl}$, a finite distance into the cladding. For a waveguide with a finite cladding, this mechanism is replaced by coupling to cladding modes. Thus, unless the tapering is so severe that there is radiation from the cladding to air, the radiation field can be ignored and the total field can be expressed as a summation over the local forward-propagating orthonormal core and cladding modes.
\[ E(r, \phi, z) = \sum_j b_j(z) e^{i r, \phi} \]  \hspace{1cm} (1)

where \( z=0 \) corresponds to the start of the taper. The coefficients \( b_j(z) \), containing phase and amplitude information, are determined by the local-mode equations\(^\text{19}\)

\[ \frac{db_j}{dz} = i \beta_j b_j + \sum_{l \neq j} C_{jl} b_l \]  \hspace{1cm} (2)

where \( C_{jl}(z) \) is the coupling coefficient, to be defined below, which determines the rate of constant mode coupling, \( \beta_j(z) \) is the local propagation of the \( j \)th mode and \( i = \sqrt{-1} \).

Provided the tapering is not too severe we can restrict the coupled mode equations to a finite set and still obtain the desired accuracy. In the case of severe tapering the sudden approximation approach is more appropriate\(^7\). If there was no power coupling between the modes, the phase dependence of the fields for \( z>0 \), would be given by \( \exp[i B_j(z) z] \), where

\[ B_j = \frac{1}{z} \int_0^z \beta_j(t) \, dt \]  \hspace{1cm} (3)

and \( \beta_j(z) \) is the local value of the propagation constant of the \( j \)th local mode. This suggests the transformation

\[ b_j(z) = \exp(i B_j z) \, g_j(z) \]  \hspace{1cm} (4)

to remove the implicit phase dependence of each mode. At the start of the taper \( g_j(0)=0 \) for higher-order modes and \( g_0(0)=1 \) for the fundamental mode.
The power in each mode is determined by $|g_j|^2$. The $g_j$ are complex quantities due to the phase difference between power that is being coupled from other modes and power already in the mode. For weak power transfer, the $g_j$ are negligible for higher-order modes while for the fundamental mode, $g_0=1$. Differentiating equation 4 with respect to $z$ gives

$$b_j'(z) = i\beta_j b_j + \exp(iB_j z) g_j'$$

where $'$ denotes the derivative. Substituting into the local-mode equations 2 gives

$$g_j'(z) = \sum_{l \neq j} C_{jl} g_l \exp[ i (B_l - B_j) z]$$

(5)

The relative phase of the power from other modes at each point along the taper depends on the last factor of each term in the summation which is why, as previously mentioned, the functions $g_j$ are complex. The local-mode equations cannot easily be solved analytically, so for computational purposes we separate the equations into real and imaginary parts

$$g_j^{\text{real}} = \sum_{l \neq j} \{ g_l^{\text{real}} C_{jl} \cos((B_l - B_j)z) - g_l^{\text{imag}} C_{jl} \sin((B_l - B_j)z) \}$$

$$g_j^{\text{imag}} = \sum_{l \neq j} \{ g_l^{\text{imag}} C_{jl} \cos((B_l - B_j)z) + g_l^{\text{real}} C_{jl} \sin((B_l - B_j)z) \}$$

(6)

which are solved numerically. This was done for each taper structure using a Runge-Kutta-Merson method which involves an automatic step size calculated at each point along the taper so as to keep the error in the new value within suitable limits. The limits were chosen so that the values of $g_j$ were accurate to a few significant figures. A minimum step size had to be set also. As the step size is progressively decreased, the values of $g_j$ converge, so, for a given accuracy in the $g_j$, the minimum step size can be determined.
The coupling coefficient is defined in terms of a weighted overlap integral\(^1\)

\[
C_{jl} = \frac{k}{4} \left( \frac{e_0}{\mu_0} \right)^{\frac{1}{2}} \left( \frac{1}{\beta_j - \beta_1} \right) \int_{A_\infty} e_j \cdot e_1 \frac{\partial n^2}{\partial z} \, dA
\]  

(7)

where \(A_\infty\) denotes the infinite cross section and \(dA\), the area element, denotes \(r \, dr \, d\phi\) for fibres, and is replaced by the linear element \(dx\) for slab structures. Clearly, \(C_{jl} = -C_{lj}\).

The variation in refractive index along the taper can be expressed in terms of the variation of the core radius. Thus

\[
\frac{\partial n^2}{\partial z} = \frac{\partial n^2}{\partial \rho} \frac{d\rho}{dz}
\]

and the coupling coefficients become

\[
C_{jl} = \frac{k}{4} \left( \frac{e_0}{\mu_0} \right)^{\frac{1}{2}} \left( \frac{1}{\beta_j - \beta_1} \right) \frac{d\rho}{dz} \int_{A_\infty} e_j \cdot e_1 \frac{\partial n^2}{\partial \rho} \, dA
\]  

(8)

For fibres where the weak guidance approximation is applicable, the electric fields are either x or y polarised and the inner product of the two orthonormal electric fields becomes \(\psi_j \psi_1 / \sqrt{N_j N_1}\) where \(\psi\) is a solution to the scalar wave equation and \(N\) is the normalisation constant, defined by\(^1\)
\[ N = \frac{n_{co}}{2} \left( \frac{\varepsilon}{\mu_0} \right) \frac{1}{2} \int_{\Lambda_{\omega}} \psi^2 \, dA \]

since in the weak guidance approximation $\beta = kn_{co}$. For the higher-order cladding modes this relationship is not as accurate, so to be more precise, we back track one step and replace $n_{co}$ by $\beta/k$. The coupling coefficients then become

\[ C_{jl} = \frac{1}{2} \frac{k^2}{(\beta_j \beta_l)^2} \frac{1}{\beta_j - \beta_l} \frac{d\rho}{dz} \left[ \int_{\Lambda_{\omega}} \psi_j \psi_l \frac{\partial n^2}{\partial \rho} \, dA \right]^{1/2} \left[ \int_{\Lambda_{\omega}} \psi_j^2 \, dA \int_{\Lambda_{\omega}} \psi_l^2 \, dA \right] \]  

(9)

For future reference, we define a quantity $I$, in terms of the normalisation integrals in the denominator.

\[ I = \left[ \int_{\Lambda_{\omega}} \psi^2 \, dA \int_{\Lambda_{\omega}} \psi_1^2 \, dA \right] \]  

(10)

In the following chapters, the local-mode equations will be solved for different waveguides and taper shapes and the theoretical results will be compared to those obtained experimentally.

2-4. Delineation Criterion

2-4-1. Qualitative Derivation

A delineation criterion has been established by Stewart and Love\textsuperscript{22} to provide a guide as to
whether a particular taper shape will be approximately adiabatic at each position along its length and therefore have a low excess loss. The criterion is derived from the physical argument that, for low loss, the local taper length scale must be much larger than the beat length for coupling between the fundamental mode and the next higher-order cladding mode with the same symmetry. Conversely, if the taper length scale is short compared with the beat length scale, then the loss will be high. This suggests that a delineation criterion between approximately adiabatic and lossy tapering can be derived by equating these two lengths.

In the case of a single-mode fibre taper, the taper length scale, \( z_t(z) \), is the distance to the apex of the right triangle formed by the local radius \( \rho(z) \), and the waveguide axis with the local taper angle \( \Omega_t(z) \), at the apex. The loss length scale is the beat length \( z_b(z) \), between the local fundamental \( \text{HE}_{11} \) mode, with propagation constant \( \beta_1(z) \) and the lowest-order local cladding mode to which power can couple, \( \text{HE}_{12} \), which has a propagation constant \( \beta_2(z) \). Thus

\[
\frac{z_t}{\tan \Omega_t} = \frac{\rho}{\tan \Omega_t} \quad \text{and} \quad z_b = \frac{2\pi}{\beta_1 - \beta_2}
\]

For an adiabatic taper, \( z_t \gg z_b \), which leads to

\[
\tan \Omega_t \ll \rho \frac{\beta_1 - \beta_2}{2\pi}
\]

This inequality must hold at each point along the taper where the propagation constants take on the local values, i.e. \( \beta_j = \beta_j(z) \). The delineation curve is defined by

\[
\Omega_d = \tan \Omega_d = \rho \frac{\beta_1 - \beta_2}{2\pi} \quad (11)
\]
Since practical angles are normally small, typically ≤1°, this approximation is valid. If the local taper angle $\Omega_t$ is well below the local value at each point along the taper, then the taper will be approximately adiabatic. If the converse is true, then the taper is expected to be lossy. This is the situation when the sudden approximation method is more applicable than the approach here where a restricted set of local-mode equations is solved.

2-4-2. Quantitative Derivation

A similar criterion can be derived from the local-mode equations. When there is coupling only between the fundamental mode and one other, and the amount of power coupled is small, the local-mode equations can be solved analytically. To first order the solutions are

\[
\begin{align*}
   b_1(z) &= b_1(0) \exp\int_0^z \beta_1(x) dx \\
   b_2(z) &= b_1(0) \exp[i\int_0^z \beta_2(x) dx] \int_0^z C_{12}(x) \exp[i\int_0^z (\beta_1(t) - \beta_2(t)) dt] dx
\end{align*}
\]

The second expression for $b_2$, shows that the fraction of power in this mode is negligible at the end of the taper, $z=L$, provided

\[
\left| \int_0^L \int_0^x C_{12}(x) \exp[i\int_0^z (\beta_1(t) - \beta_2(t)) dt] dx \right| \ll 1
\]

The exponential factor causes the integral to oscillate in phase. If the taper is slowly varying, maximum power transfer occurs over half a beatlength, i.e. $\pi/(\beta_1 - \beta_2)$. The integrand in the phase integral and the coupling coefficient are approximately constant over this distance, and the inequality reduces to the local constraint.
\[ \frac{C_{12} z_b}{\pi} \ll 1 \]

An upper limit to the fraction of power lost to the second mode, \( P_2 \), is given approximately by the square of the left hand side.

\[ P_2 = \left( \frac{C_{12} z_b}{\pi} \right)^2 \]

When \( C_{12} z_b = 1 \) this is approximately 10\%. We choose this to define the new delineating angle

\[ \Omega_n = \tan \Omega_n = \frac{1}{C_{w_0, 12} z_b} \quad (12) \]

where \( C_{w_0, 12} \) denotes the coupling coefficient without the taper angle factor \( dp/dz \). Substituting from equation 9 gives the new delineating angle in terms of the Stewart-Love delineation angle.

\[ \Omega_n = \frac{2(\beta_1 - \beta_2) (\beta_1 \beta_2)^2}{I k^2 \Omega_d} \]

where \( I \) and \( \Omega_d \) are defined in equations 10 and 11, respectively.

The Stewart-Love delineation criterion tells us only that a taper will be approximately adiabatic provided the taper angle is much less than the value defined by equation 11. It also indicates that a taper with a larger taper angle will be more lossy. However, to properly quantify how loss relates to the delineation criterion, it is necessary to solve the local-mode equations. This is carried out in chapters 3 to 5 for single step-profile
waveguides, W-fibres and couplers, respectively. The new delineation criterion suggests that provided the taper angle curves of the tapers are below the delineating angles, the excess loss should be less than 10%. This will be discussed in more detail in chapter 4.

2-5. Infinite Parabolic Slab Waveguide Taper

In this section we consider a waveguide and taper shape for which there is an exact solution of the scalar wave equation for the modal fields. This is the slab infinite parabolic waveguide with a parabolic taper shape. The refractive index is illustrated in Figure 2-1, which shows parabolic contours of constant refractive index in the x-z plane. D is the distance from the x axis to the point where the contour lines converge.

![Figure 2-1. Constant refractive index contours for the infinite parabolic profile waveguide and taper. Index values are ordered n₀ > n₁ > n₂ > n₃.](image)

The refractive index is thus a function of the transverse x and longitudinal z co-ordinates given by

\[
n^2(x, z) = n_0^2 \left( 1 - 2\Delta \frac{D^4}{(D + z)^4} \frac{x^2}{\rho_0^2} \right) \tag{13}\n\]
where $\rho_0$ is the characteristic length in the transverse direction at $z=0$, $n_0$ is the refractive index along the $z$ axis and $\Delta = (n_0^2 - n(\rho_0)^2) / 2n_0^2$, is the profile height parameter. If we express equation 13 in the form

$$n^2(x,z) = n_0^2 \left( 1 - 2\Delta \frac{x^2}{\rho^2(z)} \right)$$

(14)

where

$$\rho(z) = \frac{(D + z)^2}{D^2} \rho_0$$

then equation 14 can be regarded as the refractive index profile for a uniform slab infinite parabolic waveguide with characteristic length scale $\rho(z)$. To apply local-mode theory, we first determine the local modes of the slab infinite parabolic waveguide and determine the coupling coefficients. After choosing suitable values of the parameters, the local-mode equations can be solved. This allows the field, which is the superposition of all the local modes, to be determined at any point along the taper. The field can then be compared with the exact field.

2-5-1. Local Modes

The local mode fields are TE or TM modes. Assuming weak guidance, the differences between the polarisations can be neglected and it is sufficient to consider TE modes only. The local mode fields are determined by the scalar wave equation 1-15. After substituting for the refractive index from equation 14 and changing to the normalised $x$ co-ordinate $X = x/\rho(z)$, the scalar wave equation becomes
\[
\left[ \frac{d^2}{dX^2} + U^2 - V^2 X^2 \right] \chi(X) = 0
\]

where the modal parameters \( U \) and \( V \) are now \( z \) dependent through \( \rho(z) \). Making the transformation \( Y = \sqrt{V} X \) leads to a standard form

\[
\left[ \frac{d^2}{dY^2} + \frac{U^2}{V} - Y^2 \right] \chi = 0 \tag{15}
\]

which gives solutions

\[
\chi_n(X) = \exp \left[ -\frac{V}{2} \frac{X^2}{2} \right] H_n \left( \frac{1}{\sqrt{V} X} \right) \tag{16}
\]

where \( H_n \) is the Hermite polynomial of order \( n \). The eigen-equation is

\[
U^2 = (2n + 1) \sqrt{2} \tag{17}
\]

Rearranging equation 17 gives the propagation constant explicitly in terms of \( V \)

\[
\beta_n = \frac{1}{\rho} \left[ \frac{V^2}{2 \Delta} - \sqrt{V (2n + 1)} \right]^{\frac{1}{2}} \tag{18}
\]

where \( n = 0, 1, \ldots \) and \( n=0 \) corresponds to the fundamental TE\(_0\) mode of the slab waveguide.
Coupling Coefficient

The taper shape is contained in the characteristic transverse length, \( \rho \), which varies with \( z \) as

\[
\rho(z) = \frac{(D + z)^2}{D^2} \rho_0 \quad (19)
\]

where \( \rho_0 \) is the initial value at \( z=0 \) and \( D \) is a constant. Differentiating equation 19 with respect to \( z \) and equation 14 with respect to \( \rho \) gives the variation in refractive index along the taper

\[
\frac{\partial n^2}{\partial z} = \frac{8\Delta n_0^2 X^2}{(D + z)} \quad (20)
\]

The taper angle is \( \Omega_t = 2(D+z)\rho_0/D^2 \) which increases monotonically with \( z \). To determine the coupling coefficient we need to evaluate the normalisation integral for each mode. Making the transformation from \( X \) to \( Y \) and using the standard form of equation 15 and integrating, gives\(^{26}\)

\[
N_n = \int_{-\infty}^{\infty} \chi_n^2 \ dX = \frac{1}{V^2 \sqrt{\pi} 2^n n!} \quad (21)
\]

We can now solve the local mode equations numerically.
2-5-2. Exact Solution

The "exact" results\(^2\) for this profile are in fact only an approximation based on the angle \(\theta\), one of the polar co-ordinates \((R, \theta)\) illustrated in Figure 2-2, being sufficiently small so that \(\sin \theta / \cos^2 \theta = \theta\).

![Figure 2-2. Co-ordinate system. The origin of the \((R, \theta)\) plane is at the point \(x=0, z=-D\).](image)

In this approximation, the modal fields for an up-taper, where the characteristic length \(\rho\) increases with \(z\), are given by

\[
E_n(R, \theta) = \exp\left(-\frac{\alpha \theta^2}{2}\right) H_n(\sqrt{\alpha \theta}) \frac{\mathcal{H}^{(1)}_n(k_n R)}{\sqrt{\alpha(2n + 1)}}
\]  

(22)

where

\[
\alpha = \frac{k_n D^2}{\rho_0} \sqrt{2\Delta}
\]

\(n=0,1,...\) is the modal number, \(H_n\) is the Hermite Polynomial of order \(n\) and \(\mathcal{H}^{(1)}_n\) is the Hankel function of the first kind. Using the definition of \(\alpha\) and the approximation \(\theta = x/(z+D)\) gives
\[ \alpha \theta^2 = \frac{\kappa \rho D^2}{\rho_0} \sqrt{2 \Delta \left( \frac{x}{z+D} \right)^2} \]

Substituting for \( \rho_0 \) from equation 19 and rearranging gives \( \alpha \theta^2 = VX^2 \). Therefore the exact solution is virtually identical to the corresponding local mode solution of equation 16 apart from the the Hankel function factor. This function which depends on \( R \) and \( \theta \) and hence, \( x \) and \( z \), introduces a variation in the phase in the transverse \( x \) direction. Physically, this is because the phase fronts of the exact solutions are in fact curved and not parallel to the \( x \) axis as they are for the local mode solutions. Because of the restriction on \( \theta \), the curvature is small and, as has been shown, the local mode is a good approximation to the power distribution\(^1\).

2-5-3. Finite Length Taper

We now consider a finite length \( t_L \), of the taper of Figure 2-1 sandwiched between two uniform slab waveguides whose cross-sections and profiles match the ends of the taper. The situation is illustrated in Figure 2-3. If the fundamental mode of the uniform waveguide in \( z<0 \), travelling from left to right, is incident on the taper, the excess loss due to the taper can be analysed in two ways.

![Figure 2-3. A finite section of the taper illustrated in Figure 2-1 sandwiched between two uniform slab waveguides.](image-url)
The first approach is to use the exact modes of the taper, as given by equation 22. At $z=0$, there is a slight mismatch between the uniform slab waveguide and the taper due to the finite taper angle. Thus the incident fundamental mode, whilst predominantly exciting the fundamental mode of the taper, will also excite higher-order taper modes to a much lesser extent. In other words, this occurs because the incident fundamental mode is plane polarised while the phase fronts of the modes of the taper are curved. Along the length of the taper, the taper modes are orthogonal and do not couple. At the end of the taper, $z=tL$, there is another mismatch with the outgoing slab waveguide for the reasons given above. Thus the incident fundamental taper mode suffers a slight power loss to radiation modes of the slab waveguide, which is partially compensated by the excitation of the fundamental mode of the slab waveguide by the higher-order taper modes.

The second approach uses the local modes. In this case, by definition, all the power of the fundamental mode of the slab waveguide incident on the taper excites the fundamental local mode of the taper. Because the latter is not an exact mode of the taper, it couples power to higher-order local modes as it propagates the length of the taper. At the end of the taper, all of the power in the fundamental local mode excites the fundamental mode of the outgoing waveguide.

2-5-4. Low Excess Loss

Whichever method we employ, the excess loss must be the same. However, if the excess loss is small, we would expect the fundamental local mode to closely resemble the fundamental exact mode at each $z$ value. This is illustrated in Figure 2-4 which shows the fields of the exact and local fundamental modes at the start of the taper for the case $D=20$ mm, $\Delta=0.003$ and $\rho_0=3.68$ $\mu$m. The exact solution has been multiplied by a complex constant so that the exact and fundamental local mode fields are purely real, ie. in phase, at
Figure 2-4. Real a) and imaginary b) parts of the transverse field at the start of the taper. Solid and dashed curves are for the fundamental local and exact modes, respectively.
Real part of field are indistinguishable

Figure 2-5. Real a) and imaginary b) parts of the transverse field at the end of the taper. Solid and dashed curves are for the fundamental local and exact modes, respectively.
x=z=0. Figure 2-4a) shows the real part of the electric field of the exact solution, denoted by a dashed curve, and the fundamental local mode, denoted by the solid curve. The two curves are virtually identical and cannot be distinguished. The exact solution has a very small imaginary part as shown in Figure 2-4b), due to the curvature of the phase fronts.

To quantify the local mode approach, the local-mode equations were solved numerically. Propagation from left to right in Figure 2-3 was analysed for a taper with $t_L=8\text{mm}$, $D=20\text{mm}$, $\Delta=0.003$ and $\rho_0=3.68\mu\text{m}$. Plots of the real and imaginary parts of the field of the fundamental exact mode and the total field at the end of the up-taper are illustrated in Figure 2-5. As can be seen, the exact and approximate fields again differ only slightly.

2.5.5. Delineation Criterion

The values of the taper parameters used in the example above resulted in low excess loss, ie. less than $10^{-3}\%$. The delineation curve between the $\text{TE}_0$ and the $\text{TE}_2$ modes is calculated from equation 11 using the values of the propagation constants of the local modes from equation 18, and is shown as a function of $V$ by the dashed line in Figure 2-6. Since $V$ is proportional to $\rho(z)$, tapering up corresponds to increasing values of $V$.

The solid curve is a plot of the taper angle of the up taper considered above. The taper shape is defined by equation 19. Since it lies well below the delineating curve, we would expect there to be low loss, confirmed by the above analysis.
Figure 2-6. Delineation curve for the TE$_0$ to TE$_2$ modes is shown by the dashed curve. The taper angle curve for the up taper is shown by the solid curve. ($\lambda = 1.3\mu m$)

2-6. Conclusion

Local-mode theory has been used to derive coupled equations which determine the power in the local modes of a waveguide which is tapered. The coupling coefficients have been derived for slab structures and for fibres for which the weak guidance approximation applies. In the following chapters the coupling coefficients will be evaluated and the local mode equations will be solved numerically for specific waveguides and taper shapes.

A more general form of the delineation criterion has been derived using the coupling
coefficient and the weak guidance solution to the local-mode equations. In chapters 4 and 5, a comparison between this and the Stewart-Love criterion will be made.

In the last section, we have seen that the local-mode approach for tapers which are sections of taper shapes with known solutions, can be as valuable as the exact approach, and is easier to interpret physically. The local mode approach avoids the overlap integrals at the beginning and end of the taper which are necessary in the exact analysis, but introduces coupling along the length of the taper. In the exact approach, there is no coupling between the modes. The local mode approach is applicable to arbitrary taper shapes, provided the tapering is not severe, while the exact approach is limited to specific taper shapes.

2-7. References


22. W. J. Stewart and J. D. Love. Design limitations on tapers and couplers in single


CHAPTER 3
Tapered Single-Mode Waveguides

3-1. Introduction

3-1-1. Loss Due to the Tapering of Single-Mode Waveguides

This chapter examines the excess loss that occurs when a single-mode step-profile waveguide is tapered. To fabricate the taper, the waveguide is heated and drawn. The outer jacket, which normally absorbs any power present in higher-order modes, must be removed to prevent contamination of the waveguide, thereby exposing it to an outside medium, usually air. The large refractive index difference at the outer cladding interface permits the support of higher-order modes over the tapered region. These are cladding modes which exist because of the large cladding radius and large refractive index difference between the cladding and air giving rise to a large cladding V-value or equivalently, a multi-mode waveguide.

The large refractive index variation means that, on first inspection, the weak-guidance approximation, which depends on slight variations in refractive index, may be inaccurate and a full vector analysis is required to determine the exact modes and their propagation constants. However, because the cladding V-value is large, the field on the cladding-air interface is negligible and within the cladding, the field is, to a good approximation, plane polarized\(^1\). Taking advantage of these points, we make the assumption that the field is zero on the cladding-air interface and then use the weak-guidance approximation within.

The modal analysis for the slab waveguide is straight forward without the need for the above approximations, and can be repeated with the approximation that the field vanishes on the cladding-air interface, allowing a comparison between the two results. This will
give some indication as to how the results for the fibre are affected by this approximation. Results for the excess loss of power as a function of wavelength will be presented, together with the dependence on fibre-elongation at a fixed wavelength. As will be shown, the theoretical results for the fibre\(^2\) are qualitatively the same as the experimental results\(^3\text{-}^6\).

This chapter concentrates on matched-cladding waveguides. Depressed-cladding and W-fibres will be examined in the following chapter.

3-1-2. Propagation Along Single-Mode Tapers

As the radius of the taper decreases, the spot size of the fundamental mode changes as the field spreads into the cladding. For a fibre, it initially decreases down to V=2 and then increases until it reaches a maximum, close to where the effective and cladding indices are equal\(^7\). Below the V-value where this maximum occurs, there is significant power in the cladding and the tapering of the cladding-air interface becomes important, causing the spot size to again decrease. In an adiabatic, linear down-taper where there is negligible excess loss, the spot size of the total field is identical to that of the fundamental local mode. For a non-adiabatic taper, the spot size of the total field is identical to that of the fundamental mode until there is significant power in the higher-order cladding modes. As the power in the higher-order modes increases, the spot size of the total field differs considerably from that of the fundamental local mode. This will be discussed in the last section of the chapter.

3-2. Coupling Coefficients for Step-Profile Waveguides

In this section, we consider the refractive index for step-profile waveguides with a finite cladding and determine the coupling coefficients. The square of the refractive index distribution for the fibre profile illustrated in Figure 3-1, is given by

\[
n^2(R) = n_{co}^2 \left[ 1 - 2\Delta H(R - 1) \right] - n_{cl}^2 2 \Delta_{\infty} H(R - D) \quad (1)
\]
where $\Delta_{\infty} = (n_{cl}^2 - n_{air}^2) / 2n_{cl}^2$ with $n_{air} = 1$, is the relative index difference between the cladding and air, $R = r/p$ is normalised radial co-ordinate, $p$ is the core radius, $D = d/p$ is the normalised cladding radius, $n_{co}$ is the refractive index of the core, $n_{cl}$ is the refractive index of the cladding and $H$ is the Heaviside step function.

![Refractive index profile for a finite-cladding step-profile fibre.](image)

The derivative with respect to core radius, $p$, is thus

$$\frac{\partial n^2}{\partial p} = \frac{R}{\rho} \left[ n_{co}^2 2\Delta \delta(R - 1) + n_{cl}^2 2\Delta_{\infty} \delta(R - D) \right]$$

(2)

where $\delta$ is the Dirac delta function. This is substituted into equation 2-10 for the coupling coefficient.

The coupling coefficient involves an integration with respect to angle. Unless the angular dependence of the two modes is identical, the integral vanishes and there is no coupling between them. Thus the fundamental $HE_{11}$ mode will couple only to higher-order modes with the same symmetry, i.e. the $HE_{1m}$ cladding modes ($m = 2, 3, ...$). Within the weak-guidance approximation, the $HE_{1m}$ modes are approximately plane polarized which means that the fields involved have no angular dependence. Using this to reduce the integral to one dimension and transforming to the normalised radial co-ordinate $R = r/p$, the coupling coefficients become
\[
C_{jl} = \frac{1}{2} \frac{k^2}{(\beta_j \beta_l)^{1/2}} \frac{1}{\beta_j - \beta_l} \frac{1}{\rho} \frac{d\rho}{dz} \frac{\psi_j(\rho) \psi_1(\rho) n_{\infty}^2 2\Delta + \psi_j(d) \psi_1(d) n_{cl}^2 2\Delta D}{\int_{A_-} \psi_j^2 R dR \int_{A_-} \psi_1^2 R dR}^{1/2}
\]

where \( A_\infty \) denotes \( R \geq 0 \).

It is appropriate at this point to note that the coupling coefficient has the same units as the propagation constant, metres\(^{-1} \). The corresponding equation 2-8 for the coupling coefficients of the slab structure with the same refractive index profile as in Figure 3-1, is

\[
C_{jl} = \frac{1}{2} \frac{k^2}{(\beta_j \beta_l)^{1/2}} \frac{1}{\beta_j - \beta_l} \frac{1}{\rho} \frac{d\rho}{dz} \frac{e_j(\rho) \cdot e_1(\rho) n_{\infty}^2 2\Delta + e_j(d) \cdot e_1(d) n_{cl}^2 2\Delta D}{\int_{A_-} e_j^2 dX \int_{A_-} e_1^2 dX}^{1/2}
\]

where \( e_j \) in the integrals denotes the transverse component of the electric field of the jth mode, and \( A_\infty \) denotes \( X \geq 0 \). The integrals in the denominator are included because the electric fields in the numerator are not assumed to be orthonormal in this expression. The difference in the geometries results in a factor of D in the last term of the numerator in the equation for the slab waveguide, rather than \( D^2 \) as in the equation for the fibre. This term only becomes important when there is significant power in both modes at the cladding-air interface. As will be seen, because of the large cladding radius and hence large \( V \)-value, this only occurs when the core \( V \)-value is quite small.

3-3. Slab Waveguide

Consider a tapered slab waveguide, one of the simplest structures we can analyse. The
analysis can be done exactly and repeated with the approximation of zero field on the outer boundary, to see how the results vary. On a slab waveguide, the fundamental mode is the TE\(_0\) mode, for which the \(x\) dependence of the modal fields is identical to that of the TM\(_0\) mode in the weak-guidance approximation. This mode is symmetric about the line \(X=0\), i.e. the middle of the waveguide. As will be shown, coupling due to the taper is mainly to the TE\(_2\) mode, which is the nearest mode of the same symmetry.

3-3-1. Eigen-Equation

The transverse electric field of a symmetric, or even, TE mode of an untapered slab waveguide with a finite cladding in the \(X\geq 0\), is given by

\[
e_y = \begin{cases} 
\cos(UX) & X \leq 1 \\
A \cosh[W(X-1)] + B\sinh[W(X-1)] & 1 \leq X \leq D, \ U < V \\
A \cos[Q(X-1)] + B \sin[Q(X-1)] & 1 \leq X \leq D, \ U > V \\
E \exp[W_\infty(X-D)] & X \geq D
\end{cases}
\]

where \(U, Q, W\) and \(W_\infty\) are defined in equations 1-6, 7 and 8. The expressions involving \(W\) or \(Q\) are chosen according to whether the effective index, \(n_e>n_{c1}\) or \(n_e<n_{c1}\), respectively. The transverse electric field is related to the longitudinal magnetic field, \(h_z\), by

\[
e_y = -\frac{i}{P} \left(\frac{\mu_0}{\varepsilon_0}\right)^{\frac{1}{2}} \rho k \frac{\partial h_z}{\partial X}
\]

where \(P\) is defined by equation 1-12, \(k = 2\pi/\lambda\) is the wavenumber and \(i = \sqrt{-1}\). Integrating equation 6 gives the longitudinal magnetic field in terms of the transverse electric field.
Since the longitudinal magnetic field vanishes as $x$ tends to infinity, the constant from the indefinite integration must be zero. The transverse magnetic field, $h_y$, is proportional to the transverse electric field and is given by:

\[ h_y = -\frac{\omega}{k} \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} e_y \quad (8) \]

Equations 7 and 8 show that the boundary conditions that the transverse and longitudinal magnetic field components must be continuous is equivalent to the transverse electric field and its integral with respect to $x$, being continuous. This leads to the eigen-equation. In matrix form, when $U < V$, i.e. $n_e > n_{ci}$, it is given by

\[
\begin{bmatrix}
\cos U & -1 & 0 & 0 \\
\sin U & 0 & 1 & 0 \\
0 & \cosh[W(D - 1)] & \sinh[W(D - 1)] & -1 \\
0 & \sinh[W(D - 1)] & \cosh[W(D - 1)] & \frac{W_{\infty}}{W}
\end{bmatrix}
\begin{bmatrix}
U \\
W \\
0 \\
0
\end{bmatrix}
= 0 \quad (9)
\]

When $U > V$, i.e. $n_e < n_{ci}$, it is
These equations are solved by searching for an interval where the determinant changes sign and then restricting the interval until the eigen-value is sufficiently accurate. Care needs to be taken near U=V as the matrices in equations 9 and 10 become singular.

Results

The values of the modal parameter, U, as a function of V for the first four symmetric TE modes are shown in Figure 3-2 for the normalised cladding radius D=11.

Figure 3-2. Modal parameter U as a function of V for the first four TE modes of the slab waveguide. The curves are labelled according to the modal number. (symmetric)
For V-values above normal cutoff, ie. W=0, the U-values are virtually the same as those of the corresponding modes of a structure with an infinite cladding. For V-values between V=1.4 and the normal cutoff of the mth mode, V=mπ/2 where m is even, the curves for the higher-order cladding modes lie very close together just above the cutoff line U=V. At smaller V-values, the curves deviate from the line forming a curve concave down. Here, where the modes are essentially cladding modes, the modal parameter, U, can be determined approximately from the modal parameters of a cladding-air, step-profile waveguide, ie. by ignoring the core. The cladding V-value, V_{cl}, is related to the core V-value by

\[
V_{cl} = V_{co} \left( \frac{\rho_{cl}}{\rho_{co}} \right) \frac{n_{cl}}{n_{co}} \left( \frac{\Delta_m}{\Delta} \right)^{\frac{1}{2}} = V_{co} \frac{n_{cl}}{n_{co}} \left( \frac{\Delta_m}{\Delta} \right)^{\frac{1}{2}}
\]  

(11)

When n_{co}=1.45, \( \Delta=0.003 \) and D=11, this becomes \( V_{cl}=100 \ V_{co} \), which is large. For large V-values, \( U_{cl} = (m - 1/2)\pi \) where m is the modal number. \( U_{cl} \) is related to the core Q-value by \( Q = U_{cl}/D \) which gives for the core U-value

\[
U = \left( \frac{V_{co}^2 + U_{cl}^2}{D^2} \right)^{\frac{1}{2}} = \left( \frac{V_{co}^2 + (m - 1/2)^2 \pi^2}{D^2} \right)^{\frac{1}{2}} = V_{co} + \frac{(m - 1/2)^2 \pi^2}{2 D^2 V_{co}}
\]  

(12)

We can understand, from the last expression, why the curves for the higher-order modes lie close to the line U=V, since \( D^2 >> 1 \). When the cladding V-value becomes very small, eg. \( V_{cl} = 3\pi/2 \) for the TE₂ mode, this approximation no longer applies. However, the corresponding core V-value is approximately 0.03, which is much smaller than values occurring experimentally and so this range in \( V_{cl} \)-value is of no practical importance.
3-3-2. Transition V-values

When the V-value of a higher-order mode is well above the corresponding normal cutoff value, as occurs in multi-mode waveguides, the mode will be a core-mode. When the converse is true, the cladding is the important guiding region as significant power has spread from the core to the cladding, and the mode is a cladding-mode. There is a similar relation for the fundamental mode. When the V-value is above 0.3, \( n_e > n_{cl} \) and the mode is a core mode. However, below 0.3 the cladding becomes more important in guiding the mode and even the fundamental mode becomes more like a cladding mode with \( n_e < n_{cl} \), and a large fraction of power is in the cladding. We can discriminate between core and cladding modes by defining a transition V-value, \( V_t \), where \( n_e = n_{cl} \) and hence, \( U = V_t \). For the above parameter values, this occurs at \( V=0.3 \), as shown by Figure 3-2.

3-3-3. Delineation Curves

The delineation curve for coupling between the TE\( q \) and TE\( 2 \) modes calculated from equation 2-11 using the modal parameters above with \( \Delta=0.003 \) and \( \lambda=1.3\mu m \), is shown in Figure 3-3. As discussed in the previous chapter, this curve gives some indication as to whether a taper is expected to be approximately adiabatic. The curve exhibits a characteristic minimum at \( V=0.5 \), which corresponds to the value where the U-V curves for the two modes are closest and, hence, where the fields are more closely matched. This is close to the effective index transition \( n_e = n_{cl} \), for the TE\( 0 \) mode from core to cladding guidance.
3-3-4. Normalisation and the Coupling Coefficient

The normalisation integrals are all that remain to be evaluated to determine the coupling coefficients.

The transverse electric fields from equation 5 are substituted into the normalisation integrals in equation 4 and evaluated in each region\(^9\). When \( U < V \), the integral becomes

\[
\int_{0}^{\infty} e_j^2 \, dX = \left[ 1 \right. \\
+ \left. \frac{\sin (2U)}{4U} \right] + \left. \frac{E_j^2}{2W} \right] + \left. \frac{AB}{W} \sinh^2[W(D - 1)] \right]
\]

\[
+ A^2 \left[ \frac{D - 1}{2} + \frac{\sinh[2W(D - 1)]}{4W} \right]
\]

\[
+ B^2 \left[ \frac{\sinh[2W(D - 1)]}{4W} - \frac{D - 1}{2} \right]
\]

\( (13) \)
where \( A = \cos U, \ B = -U/W \sin U \) and \( E = A \cosh[W(D - 1)] + B \sinh[W(D - 1)]. \)

When \( U > V \), we use the field expressions involving \( Q \) rather than \( W \) and the integral becomes

\[
\int_{0}^{\infty} e_y^2 dX = \left[ \frac{1}{2} + \frac{\sin(2U)}{4U} + \frac{E^2}{2W_\infty} + \frac{AB}{Q} \sin^2[Q(D - 1)] \right]
\]

\[+ A^2 \left[ \frac{D - 1}{2} + \frac{\sin[2Q(D - 1)]}{4Q} \right] \]

\[- B^2 \left[ \frac{\sin[2Q(D - 1)]}{4Q} - \frac{D - 1}{2} \right] \]

(14)

where \( A = \cos U, \ B = -U/Q \sin U \) and \( E = A \cos[Q(D - 1)] + B \sin[Q(D - 1)] \). For the case \( U=V \), we consider the limit as \( W \) or \( Q \) tend to 0.

Using these integrals and equation 5 to evaluate the fields at the core-cladding and cladding-air interfaces, we can evaluate the coupling coefficients as a function of \( V \).

3.3.5. Results and Discussion

The sinusoidal taper illustrated in Figure 3-4 was chosen as a typical taper based on measurements of the outer diameter of tapers used in experiments\(^3\). The core radius is given by

\[
\rho(z) = \frac{\rho_{\text{max}} - \rho_{\text{min}}}{2} \cos \left( \frac{2\pi z}{\ell_1} \right) + \frac{\rho_{\text{max}} + \rho_{\text{min}}}{2} \]

(15)

where \( \rho_{\text{max}} \) and \( \rho_{\text{min}} \) are the maximum and minimum core radii, respectively, \( \ell_1 \) is the length of the taper and \( 0 \leq z \leq \ell_1 \).
Figure 3-4. Sinusoidal taper shape.

Differentiating gives

$$\frac{dp}{dz} = -\pi \frac{\rho_{\text{max}} - \rho_{\text{min}}}{t_1} \sin \left( \frac{2\pi z}{t_1} \right)$$

(16)

The dashed curve in Figure 3-3 shows the local taper angle when $\rho_{\text{max}} = 2.6\mu\text{m}$, $\rho_{\text{min}} = 1.5\mu\text{m}$ and $t_1 = 1\text{mm}$. This curve just crosses the delineation curve, which suggests that the excess loss can be expected to be small but not negligible.

The power in the modes at the end of the taper was determined by solving the local-mode equations 2-6 numerically using four modes. Assuming unit input power, results for the power in the fundamental mode as a function of wavelength at the end of the taper are shown by the solid curve in Figure 3-5. The slight oscillations in the curve for wavelengths below 1.0\mu\text{m} occur because some of the power that is lost from the
Figure 3-5. Power in the fundamental mode at the end of the taper. The solid curve shows the results for the exact analysis and the dashed curve shows the results when the field on the cladding-air interface is set to zero.

The slight oscillations in the curve for wavelengths below 1.0μm occur because some of the power that is lost from the fundamental mode is actually coupled back from the higher-order modes back to the fundamental mode.

If mode conversion is predominantly between only the fundamental mode and the first higher-order mode, then we would expect periodic oscillations with negligible excess loss at certain wavelengths because of phase matching between the two modes over the length of the taper. Over a wavelength range where the coupling coefficients are approximately independent of wavelength, the oscillations are sinusoidal-like. This is the case for the
step-profile fibre which will be discussed in the next section. Figure 3-5 shows that the excess loss increases with wavelength. This increase in loss has two possible explanations. The first is that, with increasing wavelength, the V-values decrease, causing a sideways displacement to lower V-value of the taper angle curves. Figure 3-3 shows that if the dashed taper angle curve is shifted to lower V-values, more of the curve would lie above the solid delineation curve, and, therefore, the excess loss over the total length is expected to be greater. The other explanation is that at the longer wavelengths shown in Figure 3-5, the relative phases of the modes are such that not as much power is coupled back into the fundamental mode.

3-3-6. Effect of the Cladding-Air Interface

We can now use the slab waveguide to determine the effect of assuming the field on the cladding-air interface is zero, which is used to simplify the analysis of the fibre. This can be done by repeating the slab analysis after setting the field on the interface to zero so that it makes no contribution to the coupling coefficient. These results for the previous taper shape are shown by the dashed curve in Figure 3-5. A comparison shows that the qualitative behaviour of the curves is similar. The curve from the exact analysis lies below the curve from the analysis using the zero field approximation. They differ by less than an extra 1% loss in power from the fundamental mode until the wavelength is greater than 1.1 µm. As the wavelength increases, the extra loss increases and the curves diverge.

As the wavelength increases and the V-value decreases, the modal fields spread out more so that at the cladding-air interface they become more significant and the contribution to the coupling coefficient from this interface increases. This results in stronger coupling in the exact analysis and there is more loss in power from the fundamental mode.

The results above show that the field on the outer cladding-air interface has little affect on the qualitative results. However, quantitatively, the loss is lower if the field here is set to
zero. We expect, therefore, the results for the fibre obtained by assuming the field is zero on the outer boundary, to be qualitatively the same as the results an exact analysis would, give, and, for the purpose of modelling the behaviour of practical tapers, sufficiently accurate. Fibres typically have a normalised cladding radius which is larger than $D=11$ used in the slab analysis. Furthermore, the field decreases faster in the fibre cladding than in the slab waveguide because of the cylindrical geometry. Thus, we expect the effect of neglecting the field at the outer cladding-air interface to be even smaller than for the slab waveguide.

3-4. Tapered Fibres

3-4-1. Weak-Guidance and Field Approximations

For the single-mode fibres that will be considered in this thesis, the difference between the refractive indices of the core and cladding regions is very small, typically $\Delta<0.01$. However, the difference between the cladding and air indices is large, so that the weak-guidance approximation would not appear to apply. But by considering a step-profile fibre consisting of an inner region with refractive index equal to that of the cladding and surrounded by air, we can show that the field lines within the cladding-air interface are, to a very good approximation, plane polarized\(^1\). Furthermore, because the $V$-value for such a fibre, $V_{cl}$, is so large, the field at this interface is negligibly small. Thus, the weak-guidance approximation within the cladding-air interface, assuming zero field at the interface, will be a accurate approximation.

Figure 3-6 shows the transverse electric and magnetic field lines for the fundamental mode over the cross-section of a cladding-air step-profile fibre with $V_{cl} = 14$ and $\Delta=0.262$. These results were obtained numerically \(^1\). $V_{cl} =14$ is the value defined by equation 12 with $D=15.625$, $\Delta_{oo}=0.262$, $\Delta=0.003$ and $n_{co}=1.45$, for a core radius slightly less than the minimum of the tapered fibre which will be considered in this section.
Figure 3-6. Transverse field lines of a) the electric and b) the magnetic fields of a cladding-air step-profile fibre with \( V_{cl} = 14 \) and \( \Delta = 0.262 \).

As can be seen from the figure, the electric field is virtually plane polarised everywhere, while the magnetic field only has significant bending outside the cladding. The fields of the local modes in the taper will exhibit similar features and, therefore, the weak-guidance approximation will be an adequate approximation for the exact local modes within the cladding-air interface.

With the large index difference between the cladding and air, the field is strongly contained within the interface. For example, when \( V = 0.1 \), which is well below the minimum value in typical tapers, the \( V_{cl} \) value associated with the cladding-air structure is 14.5 and there is less than 0.13% of the total power in the fundamental mode outside the cladding. This percentage is much smaller for larger values of \( V \). Thus, to a good approximation, the field of the fundamental mode can be set to zero outside the cladding-air interface. This will also be a good approximation for the lower-order cladding modes provided the core radius is not too small. Of course, with higher-order modes more power will be outside the cladding and the field is non-zero on the cladding-air interface. However, the
coupling coefficient depends on the product of the electric fields of two modes at the interface so this approximation will be adequate provided we do not have a steep taper angle for which there is considerable coupling to higher-order modes.

Thus, we conclude that the vector analysis of the fibre can be simplified to a scalar analysis within the weak-guidance approximation with the assumption that the fields on the cladding-air interface vanish.

An identical local-mode approach for fibres tapering from radii of single-mode up to multimode values has been published recently. To determine the modal fields, the weak-guidance approximation was used within the outer cladding-air interface, but without the approximation that the field vanishes on the interface for the higher-order modes. This was because there is significant coupling between the higher-order modes for this taper and the contribution to the coupling coefficient from the fields on the interface can be significant. An approximation for the field here was determined by considering the field in the cladding as a radial standing wave which can be decomposed into a radially outgoing and ingoing wave. The law of Fresnel reflection at a dielectric boundary was used to determine the transmitted wave that is excited by the outgoing wave component impinging on the interface. This was used as the value of field at the interface.

3-4-2. Eigen-Equation

The weak-guidance approach gives the solution to the scalar wave equation for the HE_{1m} modes in terms of Bessel functions and modified Bessel functions.
\[
\psi(R) = \begin{cases} 
J_0(UR) & 0 \leq R \leq 1 \\
A J_0(WR) + B K_0(WR) & 1 \leq R \leq D, \ U < V \\
A J_0(QR) + B Y_0(QR) & 1 \leq R \leq D, \ U > V \\
0 & R \geq D 
\end{cases}
\] (17)

The expressions in terms of \( W \) or \( Q \) are again chosen according to \( n_e > n_{cl} \) or \( n_e < n_{cl} \), respectively. Applying the boundary conditions that \( \psi \) and its first derivative with respect to \( R \) are continuous at both interfaces leads to the eigen-equation, which is expressible in matrix form. When \( U < V \) this is

\[
\begin{bmatrix}
I_0(W) & K_0(W) & -J_0(U) \\
I_1(W) & -K_1(W) & \frac{U}{W} J_1(U) \\
I_0(WD) & K_0(WD) & 0
\end{bmatrix}
\begin{bmatrix}
I_0(W) & K_0(W) & -J_0(U) \\
I_1(W) & -K_1(W) & \frac{U}{W} J_1(U) \\
I_0(WD) & K_0(WD) & 0
\end{bmatrix} = 0
\]

When \( U > V \) the electric field, \( \psi \), is in terms of Bessel functions with argument \( Q \) rather than \( W \) and the eigen-equation becomes

\[
\begin{bmatrix}
J_0(Q) & Y_0(Q) & -J_0(U) \\
J_1(Q) & Y_1(Q) & -\frac{U}{Q} J_1(U) \\
J_0(QD) & Y_0(QD) & 0
\end{bmatrix}
\begin{bmatrix}
J_0(Q) & Y_0(Q) & -J_0(U) \\
J_1(Q) & Y_1(Q) & -\frac{U}{Q} J_1(U) \\
J_0(QD) & Y_0(QD) & 0
\end{bmatrix} = 0
\]

These determinants were solved numerically, as described for the slab waveguide, for \( D=15.625 \). The modal parameter, \( U \), as a function of \( V \) for the first four \( HE_{1m} \) modes, \( m=1, 2, 3, 4 \), is shown in Figure 3-7. As with the slab waveguide, the curves for the higher-order modes, which would normally be cutoff on an infinite cladding fibre, lie above, but close to, the cutoff line \( U = V \). The upturn in all four curves for smaller \( V \) is due to the finite cladding radius, i.e. guidance is provided by the cladding-air guide rather than the core-cladding guide. The exact analysis for small \( V \) shows the curves to be
concave down as are those in Figure 3-2 for the slab waveguide.

![Figure 3-7](image)

**Figure 3-7.** Modal parameter $U$ as a function of $V$ for the first four HE$_{1m}$ modes of a finite-cladding step-profile fibre.

### 3-4-3. Delineation Criterion

The delineation curve between the HE$_{11}$ and HE$_{12}$ modes, defined by equation 2-11, is shown by the dashed curve in Figure 3-8 for the parameters above, which are typical of single-mode step-profile fibres. There is a dip around $V=1.2$ which is attributable to the $V$-value where the modal parameters are closest, as shown in Figure 3-7. This curve suggests that tapers with local taper angle curves which cross the delineation curve, such as the solid curves a) and b), will be quite lossy. Results for the taper shapes in this chapter will show that the Stewart-Love delineation criterion for fibres is useful as a guide as to whether a given taper shape will result in significant excess loss.
Angle

Figure 3-7. The delineation curve for the HE_{11} and HE_{12} modes is shown by the dashed curve. The curves marked a), b) and c) are the taper angle curves for 4, 8 and 16 mm sinusoidal tapers, respectively.

The transition V-value where W=0, is \( V_t = 0.8 \), while the dip in the delineation curve occurs around \( V = 1.2 \). These values are consistent with those of the slab waveguide where \( V_t = 0.3 \) and the dip occurs around \( V = 0.5 \), i.e. approximately 50% difference.

3-4-4. Normalisation and the Coupling Coefficient

Substituting for the field from equation 17 into the normalisation integrals in equation 3 and using known integrals and relations between Bessel functions\(^{11} \), we have for \( U < V \)
\[ \int_0^\infty \psi^2 \text{d}R = \left[ \frac{1}{2} J_1^2(U) + \frac{J_0^2(U)}{2 \left[ I_0(W) K_0(W) - I_0(WD) K_0(WD) \right]} \right]^x \left[ -\frac{1}{W^2} + \frac{1}{[ I_1(W) K_0(WD) + I_0(WD) K_1(W)]^2} \right] \]  \hspace{1cm} (20)

When \( U > V \) this becomes

\[ \int_0^\infty \psi^2 \text{d}R = \left[ \frac{1}{2} J_1^2(U) + \frac{J_0^2(U)}{2 \left[ J_0(Q) Y_0(QD) - J_0(QD) Y_0(Q) \right]} \right]^x \left[ -\frac{4}{\pi^2 Q^2} + \frac{1}{[ J_1(Q) Y_0(QD) - J_0(QD) Y_1(Q)]^2} \right] \]  \hspace{1cm} (21)

Substituting for the field at the core-cladding interface from equation 17, equation 3 for the coupling coefficient becomes

\[ C_{jl} = \frac{1}{\rho} \frac{\text{d}p}{\text{d}z} \frac{k^2}{(\beta_j \beta_l)^2} \left( \frac{1}{\beta_j - \beta_l} \right) \frac{\Delta n_{co}^2 J_0(U_j) J_0(U_l)}{J_0^2(U_1)} \int_0^\infty \psi_j^2 \text{d}R \int_0^\infty \psi_l^2 \text{d}R \]  \hspace{1cm} (22)

where the integrals are given by equations 20 and 21.

3-5. Wavelength Dependence of Excess Loss for a Sinusoidal Taper Shape

The taper shape chosen for this fibre is sinusoidal and given by equation 15. Taper lengths of 4, 8 and 16 mm with \( \rho_{max} = 3.1 \mu m \) and \( \rho_{min} = 0.16 \mu m \) were considered. The taper angle curves are the solid ones shown in Figure 3-8 and marked a), b) and c), respectively. Results, which were again determined numerically, showing the wavelength dependence of
the power in the fundamental mode, assuming unit input power, are illustrated in Figure 3-9. Like the experimental results\textsuperscript{3-6}, the curves are sinusoidal-like. As the taper length decreases, the taper angle steepens and the maximum excess loss increases. Simultaneously, the oscillation period in wavelength increases, and is approximately inversely proportional to the taper length.

![Figure 3-9. Power in the fundamental mode at the end of a) 4mm, b) 8mm and c) 16mm sinusoidal tapers.](image)

3-6. Minimum and Maximum of Excess Loss

The excess loss for the two longer tapers in Figure 3-9, is virtually zero at certain wavelengths, indicative of coupling between the HE\textsubscript{11} and HE\textsubscript{12} modes alone, whereas the power in the fundamental mode for the 4 mm taper is less than one for all wavelengths due to additional coupling with other higher-order modes, predominately the HE\textsubscript{13} mode.
This coupling is not strong enough though, to distort the sinusoidal nature of the curve. Note the simple correlation with the plots of taper angle in Figure 3-8. Each solid curve is actually for one half of the taper, running from the beginning to the waist, ie. narrowest part. As the curves for the shorter tapers move up into the shaded lossy region, the maximum excess loss in Figure 3-9 increases rapidly.

It is intuitive that the maxima in the power correspond to phase matching between the HE_{11} and HE_{12} modes and the minima correspond to the two modes being out of phase at the end of the taper, ie.

$$\int_{0}^{1} (\beta_1(z) - \beta_2(z)) \, dz = m\pi$$

(23)

where m is an even or odd integer, respectively.

The spatial variation in power of the fundamental mode along the length of the 8mm taper is illustrated in Figure 3-10. The two plots correspond approximately to wavelengths for adjacent minimum and maximum in Figure 3-9. There is a symmetry about the midpoint of the taper for the curve corresponding to a maximum, while the curve corresponding to a minimum is approximately antisymmetric. The shaded regions denote the parts which lie above the delineating curve in Figure 3-8. It is clear that the coupling occurs predominantly in these regions, justifying the delineating curve as a reasonable criterion between significant and negligible coupling at each position along single-mode step-profile fibres.
3-6-1. Interpretation in Terms of Two-Mode Coupling

To understand the spectral dependence of the taper loss, consider coupling in a two mode system. The coupled mode equations are

$$\frac{db_j}{dz} = i\beta_j b_j + C_{ji} b_i$$

for j=1, l=2 and j=2, l=1. Setting

$$b_j = g_j \exp[i(f_j + B_j)]$$ with 0 ≤ f_j < 2π and $$B_j(z) = \int_0^z \beta_j(t) \, dt$$

where the f_j(z) and g_j(z) are real, denoting phase and amplitude variation respectively. The
subscript $j$ denotes either mode. Substituting into the differential equation gives

$$\frac{dg_j}{dz} + ig_j \frac{df_j}{dz} = C_{jl} g_l \exp[i (f_j - f_l + B_1 - B_2)]$$

We can evaluate this equation for $j = 2$ and $l = 1$ at the start and end of the taper, i.e. $z = 0, t_1$.

$$\frac{dg_2}{dz} (0) + i g_2(0) \frac{df_2}{dz} (0) = C_{21}(0) g_1(0) \quad (24)$$

$$\frac{dg_2}{dz} (t_1) + i g_2(t_1) \frac{df_2}{dz} (t_1) = C_{21}(t_1) g_1(t_1) \exp[i \omega(t_1)] \quad (25)$$

where $\omega = f_1 - f_2 + B_1 - B_2$ is the total relative phase between the two modes. For a taper which is symmetric, $C_{21}(t_1) = -C_{21}(0)$. Since there is no power in the second mode at the start and end of the taper with no excess loss, $g_2(0) = g_2(t_1) = 0$. As can be seen in Figure 3-10, the rate of change of power in the second mode is antisymmetric about the midpoint of the taper, i.e. $g'(0) = -g'(t_1)$ where ' denotes differentiation with respect to $z$. Using these facts, adding equations 24 and 25 and cancelling leads to $\exp[i \omega(t_1)] = 1$ which gives

$$f_1 - f_2 + \int_0^{t_1} (\beta_1 - \beta_2) \, dz = 2n\pi \quad (26)$$

Making the substitution to a normalised longitudinal coordinate, $Z = z / t_1$, so that the integral depends on the taper shape and not the taper length, this becomes

$$f_1 - f_2 + t_1 \int_0^1 (\beta_1 - \beta_2) \, dZ = 2n\pi \quad (27)$$
For a given taper shape, the integral only depends on the wavelength of the source. If we assume that \( f_1 - f_2 \) is approximately constant for successive peaks in Figure 3-9, we can intuitively understand why the period of oscillation, or spacing of the peaks, is approximately inversely proportional to the taper length.

3-6-2. Optimal Taper Shape for Minimum Excess Loss

The nature of the delineating curve suggests that a taper with a taper angle curve coincident with the delineation curve should have a maximum excess loss that is minimal for a given taper length. The taper shape, shown in the inset in Figure 3-11, is determined from

\[
\zeta = \int_{\rho_{\text{max}}}^{\rho} \left( \frac{d\rho}{dz} \right)^{-1} d\rho
\]

which was determined numerically from the delineation taper angle curve. We see that the taper angle is greatest at the ends and waist of the taper. For the taper ratio of the sinusoidal tapers considered above, 19.42, the taper length in this case is 7.15 mm. The maximum excess loss is only 7% (0.32 dB) compared with 45% (2.60 dB) for the 8 mm sinusoidal taper.

Similarly, if we consider taper shapes obtained by translating the delineating curve vertically, as in Figure 3.11, so that the local taper angle is \( f \) times the corresponding value of the delineating curve, then the taper length is \( 7.15/f \) mm, assuming the taper ratio is fixed. The maximum loss in dB is indicated in Figure 3-11, ranging from 0.03 dB (0.7%) for the longest taper to 0.9 dB (18.7%) for the shortest taper. The shortest taper is only 3.58 mm. This maximum excess loss is much less than that of the 4 mm sinusoidal taper where the maximum excess loss is nearly 7 dB (80%).
Consider a fixed length of the fibre which is heated to allow the fibre to be drawn into a taper. Experimentally, it is found that if the fundamental mode of the fibre is excited at a fixed wavelength, there is a modulation in output power that depends on the elongation of the fibre due to drawing. We can model this behaviour by considering a sinusoidal taper shape and choosing the minimum cladding radius so that the amount of glass in the
taper is equal to that initially in the heated region. This can be determined by evaluating the solid of revolution formed by rotating the cladding taper shape about the fibre axis. The cladding radius is equal to the core radius multiplied by $D$, the ratio of the cladding and core radii. Thus, if $V$ denotes the volume of glass in the initial length $L$ of the fibre, then

$$V = \pi \rho_{\text{max}}^2 D^2 L = D^2 \pi \int_0^{\frac{L}{2}} \rho^2(z) dz$$

where $\rho_{\text{max}}$ is the initial core radius and $\rho(z)$ is radius in the taper. Assuming the sinusoidal taper shape defined by equation 15, we have

$$\pi L \rho_{\text{max}}^2 = \frac{r_1 \pi}{8} [3\rho_{\text{max}}^2 + 3\rho_{\text{min}}^2 + 2\rho_{\text{max}} \rho_{\text{min}}]$$

which reduces to a quadratic in $\rho_{\text{min}}/\rho_{\text{max}}$, the ratio of minimum to maximum core radii. This is solved and we choose the root between 0 and 1 which is given by

$$\frac{\rho_{\text{min}}}{\rho_{\text{max}}} = \frac{2}{3} \left[ \sqrt{\frac{L}{t_L}} - 2 - \frac{1}{2} \right]$$

(28)

The elongation of the taper is $t_L - L$, where $L \leq t_L \leq 8L/3$. When $t_L = L$ there is no tapered region. The upper limit is where the minimum core radius is zero.

The local-mode equations are solved as described above. Figure 3-12 shows the theoretical output power in the fundamental mode normalised to input power, as a function of elongation for the sinusoidal taper with a source wavelength of 1.32 $\mu$m and $L=3.11$mm. The characteristic oscillation in the curve is qualitatively the same as experimental results\textsuperscript{3-5} with an oscillation in which the period decreases and the amplitude increases as the elongation increases. The former agrees with the results for the 4, 8 and 16 mm sinusoidal tapers in Figure 3-9, where the period of oscillation in the excess loss is approximately inversely proportional to the length of the taper. The amplitude increases because the taper angle is increasing.
Figure 3-12. Power in the fundamental mode at the end of the taper as it is drawn.

3-8. Spot Size of a Tapered Single-Mode Fibre

3-8-1. Adiabatic Tapers

Consider a finite-cladding, single-mode fibre which is gently tapered so that it remains approximately adiabatic along its entire length. There is virtually no excess loss and the fields everywhere in the taper are accurately described by the local fundamental mode.

The spot size, $\omega(z)$, of the fundamental mode along the taper can be defined in a variety of ways. For our purpose, we adopt the Petermann definition and set
where $\psi$ is the solution of the scalar wave equation for the local fundamental mode and $r$ is the radius.

Infinite Parabolic Profile Fibre

A simple analytical expression can be obtained for the infinite parabolic profile fibre. The refractive index profile is defined by

$$n^2(r) = n_{co}^2 \left[ 1 - 2\Delta \left( \frac{r}{\rho} \right)^2 \right]$$  \hspace{1cm} (30)

for $r \geq 0$. The local mode field is given by

$$\psi(r,z) = \exp \left[ -\frac{1}{2} V(z) \left( \frac{r}{\rho(z)} \right)^2 \right]$$  \hspace{1cm} (31)

where $V(z) = \rho(z) \frac{\sqrt{2} \Delta}{n_{co}}$ and $\rho(z)$ is the characteristic radius at which the refractive index, $n(r) = n_{cl}$. Substitution into equation 29 gives the spot-size, $\omega = \rho(z) \sqrt{2}/\sqrt{V(z)}$, which means that $\omega$ varies with $\rho(z)^{1/2}$. In other words, the spot-size decreases monotonically with decreasing taper cross-section.
Finite-Cladding Step-Profile Fibre

The spot size for a step-profile fibre with finite-cladding is obtained by substituting the scalar fields of equation 17 into equation 29. A closed form expression can be obtained in terms of Bessel functions and modified Bessel functions, but this does not readily reveal the variation of $\omega(z)$ along the taper, and it is more straightforward to evaluate the integrals numerically.

For a fibre with $\lambda=1.3\mu m$, $n_{co}=1.447$, $\Delta=0.003$ and initial core radius $\rho(0)=4\mu m$, the initial $V$-value is $V(0)=2.17$. The ratio of cladding-core radii is $D=15.625$. We assume a linear taper shape for simplicity, with

$$p(z) = p_0 - \Omega z$$

where $\Omega$ is the taper angle and the $z$ is the distance along the taper. If the fibre is tapered to zero core radius over 2mm, $\Omega = 2.0 \times 10^{-3}$. The spot size of the fundamental local mode is shown by the dashed curve in Figure 3-13.

3-8-2. Physical Interpretation

Initially, the spot size decreases because the effect of tapering dominates the diffraction effects due to the finite core size. The minimum occurs at a distance along the taper corresponding to $V=2$ and increases thereafter to the peak value of 12$\mu m$. At the peak, the effect of the tapered cladding-air interface exactly matches the diffraction effect of the finite core and this occurs when

$$V = \left[ \frac{2}{\ln D} \right]^2$$

For $D=15.625$ this gives $V=0.85$. This $V$-value occurs when $z=1.21mm$. From Figure 3-13, we see that the peak occurs near $z=1.26mm$ which is in good agreement.
Figure 3-13. Spot size along a 2mm linear taper. The solid curve is for the total field and the dashed curve is for the fundamental mode alone.

It is now clear that the monotonic decrease in spot-size for the infinite parabolic profile fibre is a consequence of the domination of the taper effect over diffraction. As this profile tapers, the difference in refractive index values at the z axis, r=0, and at a fixed radius, \( r=r_f \), increases and contracts the field accordingly.

3-8-3. Non-Adiabatic Tapers

The spot-size of the field of a taper can be measured experimentally. For an adiabatic taper, the spot-size is virtually that of the local fundamental mode. However, when the tapering is too rapid for the propagation to be adiabatic, it is found that the spot-size is smaller than
that of the local fundamental mode\textsuperscript{13}. The delineation curve in Figure 3-8 suggests that the linear taper considered above, with taper angle, $\Omega = 2.0 \times 10^{-3}$, is non-adiabatic. The reason the spot-size differs from that of the local mode is coupling of power to the higher-order cladding modes. The spot-size depends on all the modes excited and $\psi$ in equation 29 is replaced by a summation over all local modes

$$\psi = \sum_{j=0}^{\infty} g_j \psi_j \exp(i B_j z) \quad (32)$$

where $g_j$ are the solutions to the local mode equations and $B_j$ is defined by equation 2-3. This can be illustrated by considering the infinite parabolic profile fibre.

**Infinite Parabolic Fibre**

Consider a taper such that coupling is predominantly from the fundamental mode to the first higher-order mode with the same symmetry, i.e. the HE\textsubscript{12}. For this mode

$$\psi(r) = \left[ 1 - \sqrt{\frac{r}{\rho}} \right] \exp \left[ -\frac{1}{2} \sqrt{\frac{r}{\rho}} \right]$$

Substitution into equation 29 gives $\omega = \rho \sqrt{6/\sqrt{V}}$. The total field is given by the superposition of the two modes, $\psi = a(z)\psi_0 + b(z)\psi_1$, where $|a|$ and $|b|$ are the amplitudes of the fundamental HE\textsubscript{11} and HE\textsubscript{12} modes, respectively. The z-dependent phase of each mode is contained in the complex coefficients $a$ and $b$, which are determined from the local mode equations. The total field is therefore complex, and the square of the field in the integrals in equation 29 is replaced by the square of the magnitude of the field. Evaluating equation 29 for the spot-size of the total field gives

$$\omega = \rho \left( \frac{2}{V} \right)^{\frac{1}{2}} \left[ 1 - \frac{2 \left( b (a - b) \right)_{\text{real}}}{|a|^2 + |b|^2} \right]^\frac{1}{2} \quad (33)$$
In the adiabatic limit, \( b = 0 \), and this reduces to the spot-size of the fundamental mode.

If the modes are in phase and \( |b| < |a| \), the spot-size of the total field is less than that of the fundamental mode. However, if the modes are completely out of phase the reverse is true. Therefore, as the relative phase of the two modes changes we expect the spot-size to oscillate about the fundamental local mode result. This will also occur as the power in each mode changes as the field propagates down the taper.

**Step-Profile Fibre**

For the step-profile fibre taper, the total field is given by equation 32. Substituting this into equation 29 with \( \psi^2 \) replaced by \( |\psi|^2 \) gives

\[
\omega^2 = 2\rho^2 \frac{\sum_{j=0}^{\infty} \sum_{l=0}^{\infty} g_j g^*_l \exp[i(B_j - B_l)z] \Theta_{jl}}{\sum_{j=0}^{\infty} |g_j|^2}
\]

where \( * \) denotes complex conjugate and

\[
\Theta_{jl} = \frac{k}{(\beta_j \beta_l)^2} \left[ \frac{\int_0^\infty |\psi_j|^2 R^3 dR}{\int_0^\infty |\psi_1|^2 R^3 dR} \right]^{1/2} \left[ \frac{\int_0^\infty |\psi_j|^2 R^3 dR}{\int_0^\infty |\psi_1|^2 R^3 dR} \right]^{1/2}
\]

The integrals in the denominator are given by equations 20 and 21. The integral in the numerator, for reasons given above, is determined numerically along the linear fibre taper described above, in which the core radius decreases from 4\( \mu \)m to zero in a distance of 2mm. The values of \( g_j \) along the taper were determined numerically, as for the previous
taper shapes. The results for a wavelength of 1.3μm are shown by the solid curve in Figure 3-13. 

Clearly, the spot size for the total field is smaller than that of the fundamental local mode until after the peak in the fundamental mode. As we have seen above, the latter would give the spot size of the field if the taper were adiabatic. Both curves reach a peak but the curves for the total field peak at a distance farther along the taper at z=1.5mm compared to z=1.26mm for the fundamental local mode. At this point, ρ = 1.4μm and V=0.76. For z values up to 1.5mm, i.e. before the peaks, these curves are qualitatively the same as those obtained experimentally where it was found that the measured spot size is less than that predicted theoretically by assuming the fundamental local mode propagates adiabatically. The experimental results are not given for core sufficiently small radii to show the peak.

The oscillations in Figure 3-13 toward the end of the taper are due to the beating, or changing relative phase, between the fundamental and higher-order local modes.

3-9. Conclusion

This chapter has shown theoretical results for single-mode fibre tapers which agree qualitatively with those obtained experimentally. These are:

1. The wavelength dependence of excess loss in tapered single-mode fibres is approximately periodic with a sinusoidal-like dependence.

2. The excess loss at fixed wavelength oscillates with increasing amplitude and decreasing period as a taper is drawn.

3. The spot size of the field within a tapered region differs from that of the fundamental local mode because of coupling to high-order modes caused by the tapering.
4. For fixed maximum and minimum core radii, the spacing of the excess loss minima is approximately inversely proportional to the taperlength which explains the increased period of oscillation measured as a taper is drawn.

Using the slab waveguide to determine the effect of setting the field on the cladding-air interface to zero has shown that the analysis of the fibre using this approximation together with weak guidance, is appropriate provided the minimum core radius is not too small.

3-10. References


4-1. Introduction

Depressed-clad or W-fibres were first considered in the early 1970's when they showed great promise for reducing pulse dispersion in the single-mode fibre area\textsuperscript{1,2,3}. The characteristic refractive index profile of these fibres is shown in Figure 4-1.

![Figure 4-1. Refractive index profile of the finite-cladding W-fibre which shows the dimensions of the W shape. The curves labelled a), b) and c) illustrate the three possible cases of the effective index.](image)

Between the core and outer cladding regions there is a depressed region of lower refractive index. This depression leads to three advantages over a matched-cladding single-mode fibre. Firstly, the W-fibre is single-mode for much larger core radius values\textsuperscript{1,4,5}. This is
because, depending on the width and depth of the depression, the second mode is cut off at larger $V$ values. Secondly, the large index difference at the core-depression interface serves to confine more of the field within the core and thereby reduces attenuation due to irregularities at the core-depression interface. Thirdly, the depression, if deep or wide enough, introduces a cutoff wavelength of the fundamental mode at finite $V$-values, because the depressed index lowers the effective index. The most important feature, however, is the possibility of zero dispersion. In a matched-cladding fibre, waveguide dispersion due to modal group velocity spread decreases as wavelength increases, and has the same sign as the material dispersion of the glass, so the two add. In a W-fibre, on the other hand, group velocity increases with wavelength, resulting in a negative waveguide dispersion in opposition to the glass dispersion. Thus at the near infra-red wavelengths, which are used in optical fibre systems, the overall dispersion can be reduced.

Other fibres have depressed regions in their cross-sections to serve different purposes from the W-fibre. Hi-Bi (high-birefringence) fibres such as the Panda, Bow-Tie and elliptic-cladding fibres, owe their short beat lengths between the two polarisations of the fundamental mode to stress-optic effects caused by asymmetric regions introduced into the cross-section. These regions are normally of lower index than the cladding. In the case of single-mode, single-polarisation fibres, one polarisation state of the fundamental mode is cut off, which is achieved by a combination of asymmetry and depressed regions.

Unfortunately, if these fibres are tapered to make wavelength filters, couplers, etc, there is a much larger excess loss than for matched-cladding fibres. The explanation for this enhanced loss for all these fibres can be expressed in terms of the behaviour of the modes of the W-fibre alone as it is tapered.

As has been shown, the fundamental mode of the W-fibre is cut off at finite core radius values, or equivalently, at finite $V$-values. As for the matched-cladding fibre, the finite cladding without the absorbing jacket allows cladding modes to be supported, including the
fundamental mode which makes the transition from being a core mode to a cladding mode at a higher $V$-value than the fundamental mode of the matched-cladding fibre. At core radii or $V$-values below the normal fundamental cutoff value, it is found that the values of the modal parameters $U$, of the $HE_{11}$ and $HE_{12}$ modes, become very close together, which results in a sharp dip in the delineation curve. At this point, the coupling coefficient becomes very large and there is substantial coupling of power from the fundamental mode to the $HE_{12}$ mode, which can be several orders of magnitude greater than for the matched-cladding fibre. This chapter will analyse the excess loss when W-fibres are tapered.

The behaviour of the $HE_{11}$ and $HE_{12}$ modal parameters beyond the "cutoff" wavelength of the fundamental mode can also help explain the experimentally observed spectral absorption peaks on a straight, untapered W-fibre$^9$. The secondary peak, found experimentally, can also be attributed to this, as will be explained in the last section of this chapter.

4-2. Modes of W-fibres

The refractive index profile for a W-fibre illustrated in Figure 4-1, shows the 'W' shape. The normalised outer radius of the depression is $T$ and the normalised cladding radius is $G$. We introduce a parameter, $\chi$, which quantifies the depth of the depressed region relative to the outer cladding and core regions and is defined by

\[ \chi = \frac{n_{cl}^2 - n_d^2}{n_{co}^2 - n_{cl}^2} \]  

(1)

where $n_{co}$, $n_{cl}$ and $n_d$ are the refractive indices of the core, cladding and depressed regions, respectively. When $\chi$ is small, the depression is small and the term
depressed-clad fibre, rather than W-fibre, is usually used.

Experimental results, at a fixed wavelength, for the excess loss of a slightly depressed cladding fibre as it is drawn\textsuperscript{7} exhibit oscillations which are very similar to that of a matched-cladding fibre but the onset of the oscillation occurs sooner. In this case, the cutoff $V$-value of the fundamental mode is quite small and the delineation curves are very similar to those of a matched-cladding fibre, being slightly displaced downwards to smaller angles as $V$ decreases\textsuperscript{8}. We shall, therefore, restrict this chapter to W-fibres where the differences between the cladding-depression and core-cladding indices are comparable, in which case $\chi$ is relatively large.

The particular fibre parameters chosen are for an experimental fibre, SMAK 71D, because of the availability of experimental results for the last section of this chapter\textsuperscript{9}. The refractive indices are $n_{c0} = 1.448$, $n_{c1} = 1.445$ and $n_d = 1.442$, and the associated relative index differences are $\Delta_{c0-c1} = 0.00207$, $\Delta_{c0-d} = 0.00383$ and $\Delta_{c1-d} = 0.00177$ for the core-cladding, core-dip and cladding-dip regions, respectively. The value of $\chi$ is 0.84933. The normalised radial dimensions shown in Figure 4-1 are $T = 8.5$ and $G = 15.432099$.

4-2-1. Modal Fields and the Eigen-Equation

The scalar and vector analysis of the W-fibre with an infinite cladding have been derived\textsuperscript{10}. The derivation for a W-fibre with a finite cladding parallels that for the matched-cladding fibre described in section 3-4 and, as before, it is assumed that the field at the cladding-air interface vanishes. The latter approximation will be valid provided the $V$-value does not become too small.

In terms of the normalised radial co-ordinate, $R$, the field of the $HE_{1m}$ modes is given by
where $U, W_j$ and $Q_j$ are the modal parameters defined in equations 1-6, 7 and 8. The parameters $W$ and $Q$ with subscript $d$ relate to the depressed region, while those without subscript, as usual, relate to the cladding. The expressions in each region are in terms of modified Bessel functions when $n_e > n_r$, where $n_r$ is the refractive index of the region, whereas the expressions in terms of Bessel functions are used when the converse is true.

Matching $\psi$ and its first derivative with respect to $R$ at the boundaries leads to the eigen-equation, which in matrix form, is:

\[
\begin{bmatrix}
-J_0(U) & I_0(W_d) & K_0(W_d) & 0 & 0 \\
J_0(Q_d) & Y_0(Q_d) & -W_dI_1(W_d) & -W_dK_1(W_d) & 0 \\
 W_dI_1(W_d) & -W_dK_1(W_d) & -I_0(W_dT) & -K_0(W_dT) & 0 \\
-J_0(Q_dT) & Y_0(Q_dT) & -J_0(Q_dT) & -Y_0(Q_dT) & 0 \\
W_dI_1(W_dT) & -W_dK_1(W_dT) & -W_1I(W_dT) & W_1K(W_dT) & 0 \\
-Q_dI_1(Q_dT) & -Q_dK_1(Q_dT) & Q_1I(Q_dT) & Q_1Y(Q_dT) & 0 \\
0 & 0 & 0 & I_0(WG) & K_0(WG) \\
0 & 0 & 0 & J_0(QG) & Y_0(QG)
\end{bmatrix}
\]

\[
\begin{bmatrix}
-J_0(U) & I_0(W_d) & K_0(W_d) & 0 & 0 \\
J_0(Q_d) & Y_0(Q_d) & -W_dI_1(W_d) & -W_dK_1(W_d) & 0 \\
 W_dI_1(W_d) & -W_dK_1(W_d) & -I_0(W_dT) & -K_0(W_dT) & 0 \\
-J_0(Q_dT) & Y_0(Q_dT) & -J_0(Q_dT) & -Y_0(Q_dT) & 0 \\
W_dI_1(W_dT) & -W_dK_1(W_dT) & -W_1I(W_dT) & W_1K(W_dT) & 0 \\
-Q_dI_1(Q_dT) & -Q_dK_1(Q_dT) & Q_1I(Q_dT) & Q_1Y(Q_dT) & 0 \\
0 & 0 & 0 & I_0(WG) & K_0(WG) \\
0 & 0 & 0 & J_0(QG) & Y_0(QG)
\end{bmatrix}
= 0
\]

This matrix has three forms according to the effective index. When $n_e > n_{cl}$, the components in terms of $U, W_d$ and $W$ are used. When the effective index falls below the
cladding index and \( n_{c1} > n_e > n_d \), the components in terms of \( U \), \( W_d \) and \( Q \) apply. In a finite-cladding W-fibre this would be below cutoff. If \( n_e < n_d \), then the components in \( U \), \( Q_d \) and \( Q \) apply. This condition does not occur in the tapers considered below, but is included for completeness. These three ranges of refractive index are shown by the dashed lines marked a), b) and c), respectively, in Figure 4-1.

The eigen-equation was solved numerically as in chapter 3. The curves for the modal parameter, \( U \), as a function of \( V \) for the first four \( HE_{1m} \) modes are shown in Figure 4-2a). The fundamental \( HE_{11} \) and first higher-order \( HE_{12} \) modes appear to cross near \( V = 1.48 \). However, the detail in Figure 4-2b) shows this is not the case. This \( V \)-value is approximately the value for which the fundamental mode of the corresponding infinite-cladding W-fibre becomes cut off.

### 4-2-2. Transition from Core to Cladding Modes

The fundamental mode of the corresponding W-fibre with an infinite cladding is cut off when \( U = V \) and hence, \( W = 0 \), i.e. \( n_e = n_{c1} \). At this point the cladding field is a solution of Laplace's equation rather than the scalar wave equation. The two independent solutions which are axisymmetric are a constant and \( \ln R \). Since the solution must be bounded as \( R \to \infty \), only the constant is valid. Setting the field equal to a constant in the cladding region, the eigen-equation at cutoff becomes

\[
\begin{bmatrix}
  -I_0(V_c) & I_0(W_d) & K_0(W_d) & 0 \\
  V_cJ_1(V_c) & W_dI_1(W_d) & -W_dK_1(W_d) & 0 \\
  0 & I_0(W_dT) & K_0(W_dT) & 1 \\
  0 & W_dI_1(W_dT) & -W_dK_1(W_dT) & 0 \\
\end{bmatrix}
= 0
\]

where \( V_c \) is the cutoff \( V \)-value and \( W_d^2 = V_c^2 (\chi - 1) \). This equation can be solved to determine the cutoff \( V \)-value as a function of \( \chi \) and \( T \).
Figure 4-2. Modal parameter $U$ as a function of $V$ for a) the first four HE$_{l,m}$ modes and b) the HE$_{11}$ and HE$_{12}$ modes in the region where the curves are closest.
For the SMAK 71D fibre with an infinite cladding, cutoff occurs at $V = 1.419$. With the finite-cladding fibre the transition point, where $W = 0$, occurs at $V = 1.424$. Thus, the effect of the finite cladding is to increase the $V$-value, or equivalently reduce the wavelength, at which the effective index equals that of the cladding. This occurs because the lower index of air causes the effective index of the finite-cladding $W$-fibre to be slightly less than the $W$-fibre with an infinite cladding.

4-2-3. Relationship to Coaxial Coupling

The modal analysis of all the waveguides discussed in this thesis uses the normal modes of each waveguide. Each normal mode is a state of electromagnetic resonance in the waveguide cross-section, regardless of the complexity of its cross-sectional geometry and refractive index profile, and can be excited using an appropriate source at the end of the waveguide. The attractiveness of the normal mode analysis is that it reveals the physical behaviour of the waveguide in terms of natural states of the system. For this reason, we use normal modes to analyse waveguides with quite complex cross sections as in the finite-cladding $W$-fibre, couplers and metal-clad waveguide polarisers.

However, in the case of these more complicated structures, there is an alternative approach to the analysis of propagation which is commonly adopted. This involves identifying modes of component parts of the waveguide, eg. the fundamental mode of each core of a single mode coupler, and then describing propagation along the composite structure in terms of coupling of these two modes governed by coupled-mode equations \(^\text{11}\). Clearly, these are not the normal modes of the complete waveguide. The solution to the coupled-mode equations shows that as these sub-modes propagate, power swaps between them. This corresponds to a superposition of the normal modes of the complete structure which results in interference or beating.

Consider the finite-cladding $W$-fibre. The modal parameter curves in Figure 4-2 are very
similar to those obtained by considering the fibre as a coaxial coupler consisting of a matched-cladding, step-profile fibre with cladding index equal to the depression index of the W-fibre and a ring-shaped, step-profile fibre with three index regions. These are illustrated by the refractive index profiles shown in the insets in Figure 4-3. The modal parameter, $U$, is plotted as a function of $V$ for these two structures in Figure 4-3. The curves correspond to the "fundamental" modes of each waveguide and are essentially identical to those of the fundamental $HE_{11}$ and $HE_{12}$ modes of the whole W-fibre except where the former cross. At this $V$-value the curves of the normal modes are very close, but do not cross.

Figure 4-3. Modal parameter $U$ as a function of $V$ for the fundamental modes of a) an infinite-cladding core-depression step-profile fibre and b) a ring-shaped profile fibre.
This behaviour of the curves in Figure 4-3 can be understood by considering the fields of the various modes. The modal field of the core-depression fibre is simply the fundamental mode field of a step-profile fibre with an infinite cladding. Most of its power is in the core, whereas the field of the depression-cladding-air fibre has most of its power in the region of largest index, i.e., the cladding. The electric fields of the exact HE_{11} and HE_{12} normal modes at \( V=2.0 \) are shown in Figure 4-4. The field of the fundamental HE_{11} mode is virtually the same as that of the fundamental mode of the core-depression fibre while the higher-order HE_{12} mode is more like the fundamental mode of the depression-cladding-air fibre. Figure 4-5 shows the field for the fundamental HE_{11} mode for \( V=1.4 \) which is below the usual cutoff of the fundamental mode of the W-fibre. In this case the power in the fundamental mode has moved out to the cladding region and is virtually a cladding mode like the higher-order modes.

The transition from the power of the fundamental mode being mainly in the core to the cladding is quite abrupt, as shown by the fields in Figure 4-6 for a) \( V=1.48 \) and b) \( V=1.47 \). In the coaxial coupler approach, this is where the curves for the modal parameter, \( U \), for the step-profile fibre and the first mode of the ring-shaped profile fibre cross. In the exact approach, the fundamental mode changes from being like the core mode to become more like the first mode of the ring-shaped fibre. The HE_{12} mode is always a cladding mode in this range of \( V \)-values and its \( U \)-value always lies above that of the fundamental mode.

4-3. Tapered W-Fibres

4-3-1. Delineation Criterion

The delineation curves between the HE_{11} and the first two HE_{1m} modes are shown in Figure 4-7. The curve for the fundamental HE_{11} and HE_{12} modes has a very prominent dip between \( V \)-values 1.4 and 1.55, coincident with the transition of the fundamental mode from a core to a cladding mode. The curves for coupling between the fundamental HE_{11}
Figure 4-4. Electric field for a) the HE_{11} and b) the HE_{12} modes of the W-fibre for V=2.0. The units along the vertical axis are arbitrary.
Figure 4-5. Electric field for the fundamental mode at V=1.4 a) across the entire cross-section and b) in the core and part of the depression.

The units along the vertical axis are arbitrary.
Figure 4-6. Electric field for the W fibre $HE_{11}$ mode a) $V = 1.48$ and b) $V = 1.47$. The units along the vertical axis are arbitrary.
and the HE$_{13}$ modes also exhibits this dip, but to a much lesser extent. These delineation curves suggest that the W-fibre, if tapered so that the taper angle curve passes through the dip, will be highly lossy. For W-fibres with smaller depressions, the dip is not so prominent and the delineation curves resemble those of a matched-cladding fibre$^8$.

![Delineation curves](image)

Figure 4-7. Delineation curves for the HE$_{11}$ - HE$_{12}$ and HE$_{13}$ modes of a finite-cladding W-fibre. The dashed curves are the taper angle curves for four taper shapes. $\lambda = 1.3\mu m$

4-3-2. Coupling Coefficient

The coupling coefficient between the various local modes of the W-fibre is determined as for the matched-cladding fibre, resulting in a form similar to equation 3-3. The overlap integral is identical to that of a matched-cladding fibre, as given by equations 3-19 and 20, except for the extra contribution from the depression-cladding interface. Assuming that the field on the cladding-air interface is zero, the coupling coefficient is
\[
C_{ij} = \frac{1}{2} \frac{k^2}{(\beta_j \beta_i)^2} \frac{1}{\beta_j - \beta_i} \frac{1}{\rho} \frac{\partial}{\partial z} \left[ \int_{A_m} \psi_j^2 R dR \int_{A_m} \psi_i^2 R dR \right]^{-\frac{1}{2}}
\]

\[
J_0(U_j) J_0(U_i) n_c^2 2\Delta - \psi_j(T) \psi_i(T) n_d^2 2\Delta_d T^2
\]

where \(\psi_j\) is defined in equation 2. The normalisation integrals are a little more complicated because of the presence of the depressed region but can be determined analytically.

4-3-3. Solution of the Local Mode Equations

Again, the local mode equations are solved using the same numerical techniques as for the matched-cladding fibre. However, because of the more rapid onset of mode coupling and the automatic step-shortening in the numerical routine, the number of local-mode equations was restricted to three, so as not to exceed the available CPU time to complete the solution for each taper shape.

4-3-4. Results

Four sinusoidal taper shapes, defined by equation 3-14, were considered to determine the effectiveness of the delineation criterion, defined by equation 2-11. The core taper angles for each are shown by the dashed curves in Figure 4-7. The curve marked a) is for an 8 mm taper with a wavelength of 1.3\(\mu\)m and maximum and minimum core radii of 3.67 and 2.81 \(\mu\)m respectively, corresponding to an initial V-value of approximately 1.7 at the beginning of the taper. A significant part of the taper angle curve is well above the delineation curve for the \(HE_{11} - HE_{12}\) modes. Therefore, we expect to observe large amounts of coupling, not only to the first higher-order mode, but to other higher-order modes as well as coupling between the higher-order cladding modes. In addition, the excess loss curves are not expected to be sinusoidal-like because this is no longer a two
mode system, as was the case for the matched-cladding fibre. Unfortunately, the results that would support this were unobtainable due to numerical problems. The coupling coefficients become so large around V-values corresponding to the dip region that an extremely small step size was required, which effectively made the CPU time required too large. Nevertheless, the spatial dependence of the power in the modes along the taper, plotted in Figure 4-8, is useful because it shows that large power loss does not occur until 1.85 mm along the taper, a position corresponding to $V=1.53$, which is very close to the dip in the delineation curve. Beyond this, extremely large loss commences, including strong coupling to the second higher-order mode, the HE$_{13}$. In other words, there is essentially a complete loss of power from the fundamental mode.

Figure 4-8. Power in the fundamental mode along the 8 mm taper, marked a) in Fig. 4-7. The second taper shape corresponding to curve b) in Figure 4-7 is 10 mm long with maximum and minimum core radii of 3.55 and 3.44 $\mu$m, respectively. As the taper angle
curve is close to the delineation curve between the HE_{11} and HE_{12} modes but well below the delineation curve between the HE_{11} and HE_{13} modes, we expect significant coupling only to the first higher-order mode. Results for the power variation in the fundamental mode along the taper are shown in Figure 4-9 for \( \lambda = 1.3\mu m \). Figure 4-9a) shows that there is a maximum of almost 3% power loss from the fundamental mode. Figure 4-9b) shows that this power is lost to both the HE_{12} and HE_{13} modes, which suggests that the power loss might be greater if more higher-order modes were included in the analysis. However, the loss of power from the fundamental mode is not as large as we would have expected from the delineation curves. This point is discussed again later in the chapter.

The third taper, marked c) in Figure 4-7 is well below the delineation curves. The taper length is 14mm with maximum and minimum core radii of 3.77 and 2.88\( \mu m \), respectively. We therefore expect this taper to show negligible loss which is borne out by the numerical results which show the loss is approximately 0.0002%. The spatial dependence of power in the fundamental and first two higher-order modes is shown in Figure 4-10. Figure 4-10a) shows that it is basically a two mode system with small oscillations superimposed due to a third mode having some effect. Figures 4-10b) and c) show that it is the second higher-order mode, the HE_{13} that most of the power is coupled to, with a fraction then being coupled to the HE_{12}. This is contrary to what we expect from the delineation curves. However, as we show later in this chapter, by considering the coupling coefficients for which \( C_{13} > C_{12} \), we can understand why this is so. Thus we have found an example where the delineation criterion does not accurately predict the correct mode to which power is being coupled. Nevertheless it does predict correctly that negligible power is lost. Since the numerical analysis was carried out with only the first two higher-order modes, the accuracy could be improved if more higher-order modes are included.
Figure 4-9. Power in a) the fundamental mode and b) the first two higher-order modes for the 10mm taper, marked b) in Fig. 4-7.
a) Power in the fundamental mode

Distance along the taper in mm

b) Power in the second mode

Distance along the taper in mm
Figure 4-10. Power in a) the $HE_{11}$, b) the $HE_{12}$ and c) the $HE_{13}$ modes along the 14mm taper, marked c) in Fig. 4-7.
Power in fundamental mode

Power in second mode

distance along the taper in mm
Figure 4-11. Power in a) the $HE_{11}$, b) the $HE_{12}$ and c) the $HE_{13}$ modes along the 1mm taper, marked d) in Fig. 4-7.
The final taper shape has the same core radii as the previous one but the taper length is much shorter, being only 1 mm. Thus the taper angle curve, marked d) in Figure 4-6, is a translation of curve c). Since the taper angle curve cuts the delineation curve for the \( HE_{11} \) and \( HE_{12} \) modes, we would expect significant coupling. This is not found to be the case. This can be seen in Figure 4-11a) which shows the power in the \( HE_{11} \) mode along the taper. Clearly, the loss is less than 0.3% which, although small, is three orders of magnitude greater than for the third taper. Figure 4-11b) shows that it is to the \( HE_{13} \) and not the \( HE_{12} \) that most of the power goes. So again, the delineation curves do not correctly indicate to which modes power is coupled.

### 4-4. Delineation Curve and Coupling Coefficients

In chapter 2, the Stewart-Love delineation criterion was defined and a new delineation criterion was derived from the local-mode equations. The Stewart-Love curves are defined by equation 2-11 and the new delineation curves are defined in equation 2-12. In this section, we compare the two using matched-cladding and W-fibres.

#### 4-4-1. Matched-Cladding Fibres

The two sets of delineation curves for the \( HE_{11} \) to \( HE_{12} \) and \( HE_{13} \) modes of the matched-cladding fibre discussed in section 3-4 are illustrated in Figure 4-12. Clearly, the corresponding curves are very similar, which is why the Stewart-Love delineation criterion is most useful. The corresponding minima, which are approximately equal, occur around the same V-value. However, the curves for the coupling coefficient criterion are somewhat sharper. This suggests that the Stewart-Love criterion will predict taper shapes with V-values greater than 1.5 to be more lossy if their taper angle curves just cross the delineation curve. This region is not really of much importance, however, because curves for practical tapers do not cut the delineation curve in this region. The loss from the 16mm matched-cladding fibre sinusoidal taper considered in section 3-3, has a taper angle curve...
which lies just below the minima of both delineation curves. The maximum excess loss of approximately 10% is consistent with the value of $\pi^2$ discussed in section 2-4-2. The 7.15mm taper considered in section 3-4, for which the taper angle curve was the same as the Stewart-Love delineation curve has a maximum excess loss of 7.2% which is also consistent.

![Figure 4-12](image)

Figure 4-12. Delineation angle for the HE$_{11}$ to HE$_{12}$ and HE$_{13}$ modes of a matched-cladding fibre as a function of $V$. The solid curves are from the new delineation criterion and the dashed curves are from the Stewart-Love delineation criterion.

4-4-2. W-Fibres

A more pronounced difference between the two criteria is evident in Figure 4-13 which shows the corresponding curves for the W-fibres discussed above in section 4-3. The prominent dips in the delineation curves for the HE$_{11}$ and HE$_{12}$ modes occur at the same $V$-value, but the minimum of the dip in the coupling coefficient curve is much lower and
on both sides of the dip, the two curves differ. The major difference is that on either side of the dip the coupling coefficient criterion curves are well above those of the Stewart-Love criterion. The vertical asymptotes for the coupling coefficient delineation curves are due to the vanishing of the coupling coefficient when the contributions from fields at the core-depression and depression-cladding interfaces to the overlap integral cancel. The delineation curves suggest there would be little loss for a taper shape with a taper angle curve between the two delineation curves, whereas the Stewart-Love curve in this case would incorrectly predict significant excess loss. These curves show that the Stewart-Love criterion is not as accurate for predicting losses from tapered W-fibres. However, when the taper shape passes through the dip region, they do correctly predict large loss.

Figure 4-13. Delineation angle for the HE_{11} to HE_{12} and HE_{13} modes of a W-fibre as a function of V. The solid curves are from the new delineation criterion and the dashed curves are from the Stewart-Love delineation criterion.
4-4-3. Discussion

The reason for the difference between the two sets of delineation curves is clear from their definitions. The Stewart-Love delineation criterion requires knowledge only of the modal propagation constants, whereas the coupling-coefficient delineation criterion involves the modal field distribution as well. In the case of the matched-cladding fibres, the similarity of the two sets of curves in Figure 4-12 is a consequence of the relatively simple profile, whereas the divergence in Figure 4-13 arises because of the more complex profile of the W-fibre. Thus, the Stewart-Love delineation curves will not, in general, be as accurate as the coupling coefficient delineation curves in predicting the maximum local taper angles for which the total low excess loss is small. However, for practical tapers, they have the advantage of ease in evaluation.

4-5. Absorption Peaks in Straight W-fibres

The severe attenuation of the fundamental mode of a strongly-tapered W-fibre is a consequence of the virtually identical propagation constants of the HE\textsubscript{11} and HE\textsubscript{12} modes in the vicinity of the normal cutoff of the fundamental mode. This explanation of loss can also account for the large absorption peaks observed in the spectral analysis of non-tapered, straight W-fibres\textsuperscript{9}.

These measurements are reproduced in Figure 4-14, for each of the three fibres used in the experiment. The parameters are for the SMAK 71D fibre which have been used throughout this chapter. Each curve has the same characteristic behaviour. The initial decreasing loss with increasing wavelength occurs because this is below the normal cutoff wavelength of the second mode of the fibre. The loss is then negligible until beyond the normal cutoff wavelength of the fundamental mode where there is a gradual increase in attenuation as the field of the fundamental mode spreads farther into the cladding and is absorbed by the surrounding jacket. Superimposed onto this increase are two spikes, the larger of which is
quite close to the cutoff wavelength.

Figure 4-14. Plots of experimentally measured loss from the fundamental mode as a function of wavelength for three W-fibres\(^9\).

4-5-1. Physical Explanation of the Loss Peaks

The major spike has been attributed to mode coupling between the "fundamental" modes of the core-depression fibre and the depression-cladding-air fibre, which were discussed earlier in section 4-2-3. However, no physical mechanism has been put forward\(^9\).

By working with the modes of the complete W-fibre, we postulate that the very slight variation in diameter which occurs during the drawing process\(^\text{12}\), equivalent to slight tapering, is responsible. Our justification is that this peak occurs exactly at the minimum of the prominent dip in the delineation curves in Figure 4-13, where the fibre is most sensitive to tapering. Even a representative figure for the core taper angle as low as 10\(^{-6}\) radians is large enough to lie above the minimum in the coupling delineation curve in Figure 4-13. This means that at and close to this minimum there will be significant coupling between the HE\(_{11}\) and HE\(_{12}\) modes. The power in the latter is readily absorbed by the jacket because the modal field amplitude is much greater at the cladding-jacket interface.
For the SMAK 71D considered in the first section in this chapter, the dip occurs at \( V = 1.48 \). With the parameters of this fibre\(^ 1 \), this \( V \)-value corresponds to a wavelength of 1.583\( \mu \)m. The experimental absorption peak for this fibre is observed at a wavelength of 1.58\( \mu \)m\(^ 9 \).

It has been suggested that the secondary peak in Figure 4-14 is due to mode coupling between the "fundamental" mode of the core-depression fibre and a higher-order mode of the depression-cladding-air fibre. However, an examination of the the coupling coefficients for the fundamental and several lower-order modes of the complete W-fibre does not support this hypothesis.

In order to appreciate our explanation, it is necessary to understand the physical arrangement of the fibres during the experiment\(^ 9 \). A 30m length of each fibre was wound around two 15cm diameter spools 5m apart, ie. about 95% of the fibre was straight and 5% bent. The effect of the curvature is twofold. Firstly, it shifts the fundamental-mode field outward in the jacket, but secondly, and more importantly, it increases the wavelength at which the \( HE_{11} \) and \( HE_{12} \) modes couple most strongly. The wavelength shift is consistent with the observed vanishing of the major spike when all of the fibre is wound around the spool.

4-6. Conclusion

In this chapter we have seen that W-fibres are much more sensitive to tapering than matched-cladding fibres. This is because near the cutoff of the fundamental mode, the propagation constants and modal fields of the fundamental \( HE_{11} \) and \( HE_{12} \) modes of the W-fibre are very similar, resulting in large coupling coefficients and hence strong coupling. The last section shows that even the slight irregularities along a straight W-fibre are sufficient to cause significant coupling of power from the fundamental mode near the cutoff wavelength resulting in the observed absorption peaks.
The new delineation criterion derived in chapter 2 was considered in more detail to show that in general it is more accurate than the Stewart-Love criterion. This was done by considering both criteria for matched clad and W-fibres. However, for practical tapers the latter has the advantage of ease in evaluation.

4-7. References


CHAPTER 5
Tapered Couplers

5-1. Introduction

Single-mode fibre couplers are important passive devices for combining and splitting signals in optical communication systems, particularly LAN's (local area networks) and CAN's (customer access networks). Accordingly, they have received much attention in the literature, e.g. references 1-7.

As is well known, the coupling of power between the cores can be described either by the beating between the fields of the fundamental symmetric and lowest order antisymmetric modes of the coupler or by considering the modes of the component fibres and using coupled-mode theory to determine the coupling between them. The former approach will be followed in this chapter. In order to produce practical couplers with beat lengths of the order of millimetres, it is necessary to increase the overlap of the fundamental-mode field of one fibre over the core of the second fibre. There are essentially two methods of fabricating couplers which satisfy this requirement. The first consists of partially removing the cladding of each fibre by etching or polishing and abutting the exposed surfaces. In the second, the two fibres are fused together and drawn producing a tapered structure. As mentioned in the previous chapter, tapering provided part of the motivation for studying tapered single-mode fibres. We now take a step further and consider tapered couplers where excess loss occurs due to the coupling of power from the two lowest order modes of the coupler to the higher-order modes.

The cross-section of a well-fused fibre coupler is illustrated in Figure 5-1. Even if the fibres have the simple step-profile and are analysed within the weak-guidance approximation with zero field on the cladding-air interface, as used in earlier chapters, the determination of the (local) modes and eigen-values for this geometry is not
straightforward. Of course, methods to determine the eigen-values do exist\textsuperscript{8-11}, but the corresponding fields may not be sufficiently accurate or readily evaluated to allow the determination of the coupling coefficients.

Figure 5-1. Cross-section of a well-fused fibre coupler.

A simple alternative that provides insight into the effects of tapering and yet remains analytically tractable is the single-mode, step-profile slab coupler illustrated in Figure 5-2.

Figure 5-2. Cross-section of a step-profile slab waveguide coupler.

By calculating excess loss for specific taper shapes, we can give a quantitative meaning to
the delineation curves. This, in turn, provides confidence in using the delineation criterion for the fibre coupler to predict whether a given tapered coupler is expected to be highly lossy.\textsuperscript{12,13}

5-2. Electric Fields and the Eigen-Equation

As we saw in chapter 3, the fundamental mode of the single slab waveguide is the $TE_{q}$ mode. Likewise, for the single-mode coupler we consider the first two $TE$ modes, which are symmetric and antisymmetric about the axis $X=0$. These closely resemble the sum and difference, respectively, of the $TE_{q}$ modes of the component waveguides.

5-2-1. Symmetric Modes

We start by considering the symmetric $TE$ local modes of the coupler. The analysis for the antisymmetric $TE$ modes is similar. We use an exact vector analysis as was done for the slab waveguide in section 3-3. Thus we do not assume zero field on the cladding-air interface. All higher-order $TE$ modes propagate in the cladding with effective indices below the cladding index. The longitudinal component of the electric field is zero and the symmetric transverse component is $y$ directed and given, in terms of the normalised transverse co-ordinate $X=x/\rho$, in the region $X \geq 0$ by

$$e_y(X) = \begin{cases} 
\cosh WX & X \leq G, U \leq V \\
\cos QX & X < G, U > V \\
A \cos(U(X-G-1)) + B \sin(U(X-G-1)) & G \leq X \leq G+2 \\
E \cosh(W(X-G-2)) + F \sinh(W(X-G-2)) & G+2 \leq X \leq D, U < V \\
E \cos(Q(X-G-2)) + F \sin(Q(X-G-2)) & G+2 \leq X \leq D, U > V \\
H \exp[-W_{\infty}(X-D)] & X \geq D
\end{cases}$$

(1)
where $G$ and $D$ are the distances normalised to core half width, from the centre of the coupler to the inner core-cladding and cladding-air interfaces, respectively, illustrated in Figure 5-2 and $U$, $W$, $Q$ and $W_\infty$ are defined in section 3-3.

In a fibre coupler, the normalised core-to-core separation, $2G+2$, changes as the component fibres are twisted and fused together\textsuperscript{3,6,7}. This could be incorporated into the model by having $G$ and $D$ as functions of $z$. However, as accurate measurements of the taper shapes, fused cross-sections and the refractive index profiles of fibre couplers are not readily available and we are looking for the qualitative behaviour of the slab coupler, this would only complicate the solution of the local mode equations unnecessarily. Thus in the analysis, we assume that both $G$ and $D$ are fixed and that the coupler tapers uniformly.

Continuity conditions discussed in section 3-3 lead to the eigen-equation

\[
\begin{bmatrix}
\cosh WG & -\cos U & \sin U & 0 & 0 & 0 \\
\cos QG & 0 & 0 & 0 & 0 & 0 \\
-W\sinh WG & U\sin U & U\cos U & 0 & 0 & 0 \\
Q\sin QG & 0 & \cos U & \sin U & -1 & 0 & 0 \\
0 & U\sin U & U\cos U & 0 & W & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & = 0 \\
0 & 0 & 0 & \cosh WT & \sinh WT & -1 \\
0 & 0 & 0 & \cos QT & \sin QT \\
0 & 0 & 0 & W\sinh WT & W\cos WT & W_\infty \\
0 & 0 & 0 & -Q\sin QT & Q\cos QT \\
\end{bmatrix}
\]

where $T=D-G-2$ is the normalised thickness of the outer cladding. The components in terms of $W$ or $Q$ are chosen according to the effective index being above or below the
cladding index, respectively.

5-2-2. Antisymmetric Modes

The corresponding antisymmetric modes are obtained by replacing \( \cosh WX \) by \( \sinh WX \) and \( \cos QX \) by \( \sin QX \) only in the region \( 0 \leq X < G \) in equation 1. The field in the \( X < 0 \) region is given by \( e_y(X) = -e_y(X) \). This leads to a similar eigen-equation

\[
\begin{bmatrix}
\sinh WG & -\cos U & \sin U & 0 & 0 & 0 \\
\sin QG & 0 & 0 & 0 & 0 & 0 \\
-W \cosh WG & \sin U & \cosh U & 0 & 0 & 0 \\
-Q \cos QG & 0 & 0 & 0 & 0 & 0 \\
0 & \cos U & \sin U & -1 & 0 & 0 \\
0 & U \sin U & U \cos U & 0 & W & 0 \\
0 & 0 & 0 & \text{cosh WT} & \sinh WT & \text{cosh WT} \\
0 & 0 & 0 & \cos QT & \sin QT & \text{sin QT} \\
0 & 0 & 0 & W \sinh WT & W \cosh WT & W_{\infty} \\
0 & 0 & 0 & -Q \sin QT & Q \cos QT & \text{det}
\end{bmatrix} = 0
\]

The eigen-equations were solved with parameter values \( \Delta = 0.003 \), \( n_{co} = 1.45 \), \( G = 2 \) and \( T = 10 \). This gives a cladding-to-core radius ratio of 11, which is comparable to that of a typical fibre. Unfortunately, numerical problems were encountered for larger values of \( T \). With smaller values of \( T \), we expect the field at the air-cladding interface to be larger and therefore make a larger contribution to the coupling coefficient. However, this will only be important if a long wavelength is used or the coupler is tapered to a small core radius so that the \( V \)-value becomes small enough for the field to spread sufficiently far into the cladding. If we consider the modes of the coupler as the superposition of the fundamental modes of the component waveguides, the beatlength will be sufficiently small only if the
overlap between the two component fields over either of the cores is large enough, in which case the field at the outer cladding-air may be significant.

5-2-3. Results

The eigen-values for the first four symmetric TE modes are illustrated in Figure 5-3a) with those of the corresponding antisymmetric TE modes in Figure 5-3b). Throughout this chapter the modes are labelled $TE_{m} \pm$ where $m=0,1,2,3$ is the modal number and + and - denote symmetric and antisymmetric modes, respectively. Again, the higher-order modes lie close to the cutoff line $U=V$ until $V$ becomes sufficiently small. The first symmetric and antisymmetric modes correspond to the solid curves and $U \rightarrow \pi/2$ as $V \rightarrow \infty$. Both second modes lie above the $U=V$ line up to approximately the cutoff value of the second mode on the single slab waveguide, ie. $V=\pi/2$. The curves for the symmetric and antisymmetric modes show similar features, and, as is expected, the modal parameters of the symmetric modes are below those of the corresponding antisymmetric modes, with the difference increasing with decreasing $V$.

5-2-4. Mode Coupling

For the single slab waveguide, coupling of power can only occur between modes with the same symmetry about the centre of the core, ie. the $TE_{m}$ modes where $m$ is even. The same is true for the coupler and transfer of power from the first symmetric and antisymmetric modes occurs only to higher-order modes of the same symmetry. These higher-order modes correspond to the modes of the component single slab waveguides with the same modal number which are symmetric or antisymmetric about the centre of the core. However, the fields of the higher-order coupler modes are modes of the entire cladding and as a result exhibit no symmetry properties with respect to the centre of either core.
Figure 5-3. Modal parameters $U$, for the first four a) symmetric and b) antisymmetric TE modes, labelled 0 to 3.
Figure 5-4. Transverse electric field of a) the $\text{TE}_0$ and b) the $\text{TE}_1$ antisymmetric modes of the coupler in the region $X > 0$. The units along the vertical axis are arbitrary.
The modal fields for the first two antisymmetric coupler modes are shown in Figure 5-4. As can be seen, the fields of the higher-order modes are predominantly in the cladding and are more like the modes of the cladding-air waveguide in which the cores are replaced by cladding material. The symmetric and antisymmetric modes correspond to the $\text{TE}_2m$ and $\text{TE}_{2m+1}$ modes of the cladding-air waveguide, respectively. The total cladding thickness is larger than the corresponding thickness of a single slab waveguide, and, as a result, there are more modes to which power from the first antisymmetric mode can couple than is the case for the tapered single slab waveguide. The same is true for the symmetric coupler modes.

5-3. Delineation Curves

The Stewart-Love delineation curves for the symmetric and antisymmetric modes of the coupler, calculated from equation 2-11, are shown by the solid curves in Figure 5-5, assuming a wavelength of 1.3$\mu$m.
modes. The dashed curves are the taper angles of a 4mm taper at wavelengths of a) 1.3 and b) 1.6µm.

The curves for the antisymmetric modes are slightly lower for 0.4 <\textit{V}<1.2, and so we expect the excess losses of the symmetric and antisymmetric modes to be similar.

For small \textit{V}-values, when the core radius become very small the fundamental modal fields have spread into the cladding and all modes of the coupler are cladding modes. As shown in section 3-3-2, provided \textit{V} is not too small, the modal parameters of cladding modes are given approximately by

\[ U = V + \frac{(j - \frac{1}{2})^2 \pi^2}{D^2 V} \]

where \( j = 2m \) for symmetric modes and \( j=2m+1 \) for antisymmetric modes, is the modal number of the cladding-air structure when the cores of the coupler are ignored, and \( D=14 \) is the normalised distance from the centre of the coupler to the cladding-air interface. By considering \( j=0 \) and 2 for symmetric modes and \( j=1 \) and 3 for antisymmetric modes, we find that the difference in the \( U \)-values and hence, the difference between the corresponding propagation constants, is larger for the antisymmetric modes. As a result, the delineation curve for the antisymmetric modes must lie above that of the symmetric modes. By a similar argument the same is true for the \( \text{TE}_0^{±} \) and \( \text{TE}_2^{±} \) modes.

For large \textit{V}-values, the modal parameters of the coupler modes converge to the value of the corresponding mode of the single slab waveguide. However, the lower the value of \( m \), the closer the values of the modal parameters and hence, the propagation constants of the coupler modes. As a result, for large \( V \), the delineation curves for the symmetric modes are slightly lower for each pair of modes.

The delineation curve for the \( \text{TE}_0 \) and \( \text{TE}_2 \) modes of the single slab waveguide lies between the two pairs of curves for the coupler modes. This suggests that the overall loss
of the coupler will be much greater than that of the component waveguide.

5-4. Coupling Coefficients

The coupling coefficients are determined as for the component waveguide, resulting in an equation similar to equation 3-4. The overlap integral is also similar to that of the component waveguide given by equation in chapter 3, but with extra contributions from the additional interfaces. The coupling coefficient is

$$C_{jl} = \sqrt{\frac{k^2}{(\beta_j \beta_l)^2}} \frac{1}{\beta_j - \beta_l} \frac{1}{\rho} \frac{dp}{dz} \frac{F_{\text{contributions}}}{\left[ \int_{A_1} \psi_j^2 R \, dR \int_{A_2} \psi_1^2 R \, dR \right]^\frac{1}{2}}$$

where

$$F_{\text{contributions}} = -n_{co}^2 2\Delta G E_j(G)E_l(G) + n_{co}^2 2\Delta (G+2) E_j(G+2)E_l(G+2)$$

$$+ n_{cl}^2 2\Delta G E_j(D)E_l(D)$$

The normalisation integral, while straightforward to evaluate, is slightly more complicated because of the additional regions. It can be derived analytically, but is readily evaluated numerically.

5-5. Results

The local mode equations 2-6 were solved separately for the symmetric and antisymmetric modes, since coupling only occurs between modes of the same symmetry. The number of modes in each case was restricted to four, to minimize the CPU time required. Again, the sinusoidal taper shape defined in equation 3-15, with maximum and minimum core radii of
2.6 and 1.0 μm, respectively, was considered. The local taper angle curve for a 4 mm taper is shown by the dashed curves in Figure 5-5 for wavelengths of a) 1.3 and b) 1.6 μm. Both curves lie below the two sets of delineation curves suggesting that the excess loss for both the symmetric and antisymmetric modes will be small.

The variation in the power in the symmetric and antisymmetric TE_0 modes with wavelength is shown in Figure 5-6, assuming unit power initially in each. The wavelength ranges from 0.8 to 1.7 μm. The curve for the fundamental symmetric mode shows that the excess loss is indeed small. The curves are not periodic, like those of the fibre, because coupling involves several higher-order modes. The curve for the antisymmetric mode shows that the excess loss is quite significant. For example, at a wavelength of 1.42 μm the power lost is 23%. This is not consistent with the Stewart-Love delineation criterion and is explained below.

Figure 5-6. Power in the symmetric and antisymmetric TE_0 modes as a function of wavelength, assuming initial input power to each.
5-5-1. Coupling Coefficient Delineation Criterion

To understand the losses in Figure 5-6, we consider the alternative delineation criterion based on the appropriate coupling coefficient, as discussed in section 4-4. The delineation curves between pairs of the first four modes are plotted as a function of $V$ in Figures 5-7a) and b) for the symmetric and antisymmetric modes, respectively.

The curves in Figure 5-7a) for the $TE_0$, $TE_1$, $TE_2$, and $TE_3$ modes lie below the corresponding curves in the Stewart-Love curves in Figure 5-5 for $V$-values above 0.6, and are therefore more restrictive on the taper angle. Similarly, the corresponding curves for the antisymmetric modes in Figure 5-7b) are also below the Stewart-Love delineation curves, except in the neighbourhood of $V$=0.65, where there are vertical asymptotes.

The vertical asymptotes in both Figures 5-7a) and b) are a consequence of a zero coupling coefficient due to cancellation of the contributions in equation 5 from the various interfaces. A similar phenomenon was also observed with the corresponding W-fibre delineation curves in Figure 4-13.

5-5-2. Spatial Dependence Along The Taper

Consideration of the spatial dependence of the power in both the first four symmetric and antisymmetric modes at a particular wavelength provides an insight into the coupling between the modes. We choose a wavelength of 1.3μm, corresponding to strong coupling between the antisymmetric modes and, apparently, negligible coupling between the symmetric modes, as shown by Figure 5-6.
Figure 5-7. Coupling coefficient delineation curves between the first four a) symmetric and b) antisymmetric TE modes as a function of $V$. Superimposed is the taper angle curve.
Figure 5-8. Spatial dependence of power along the taper for a) the fundamental mode and b) the higher-order symmetric modes.
Figure 5-9. Spatial dependence of power along the taper for a) the fundamental mode and b) the higher-order symmetric modes.
Symmetric Modes

The spatial dependence of power in the fundamental symmetric mode is shown in Figure 5-8a) while Figure 5-8b) shows the dependence for the higher-order modes. The taper angle curve is superimposed on the delineation curves in Figure 5-7a).

We first notice that, although the symmetric fundamental-mode curve in Figure 5-6 indicates virtually no excess loss over the complete length of the coupler, the curves in Figure 5-8b) show quite significant coupling to the higher-order modes along the steepest parts of the taper, with maxima of 2.4, 4.4 and 0.6% in the $TE_m$ modes for $m=1,2,3$, respectively. Note that the largest power coupling is to the $TE_2$ mode which is consistent with the $TE_0$ - $TE_2$ modes delineation curve lying below the curve for the $TE_0$ - $TE_1$ modes in Figure 5-7a) in the region of $V$-values corresponding to the peak in the taper angle curve.

It is remarkable that the accumulated phase differences between the $TE_0$ and the higher-order modes in the middle of the taper, where most of the power has coupled back into the fundamental mode, is such that the condition for minimal excess loss described in section 3-6, is satisfied. Clearly they must all be in phase which accounts for the symmetry of the curves about the centre of the coupler.

Antisymmetric Modes

The spatial dependence of the power in the antisymmetric $TE_0$ mode is shown in Figure 5-9a), while Figure 5-9b) shows the dependence for the higher-order modes. The taper angle curve is superimposed on the delineation curves in Figure 5-7b).

The behaviour of coupling in Figure 5-9b) is quite different from that of the symmetric modes, but is consistent with the delineation curves. Along the initial downtaper, coupling from the antisymmetric $TE_0$ mode is mainly to the $TE_2$ and $TE_3$ modes. This is consistent
with in Figure 5-7b), where the taper angle curve is closest to the delineation curves between the TE\(_0\) mode and these two modes. In this region, the curves for coupling between the higher-order modes are much lower than the curves involving the TE\(_0\) mode, suggesting that coupling between the higher-order modes is stronger than coupling back to the TE\(_0\) mode. This accounts for the monotonic decrease in the power in the fundamental mode for z<0.7mm and the overall decrease in the first half of the taper as shown in Figure 5-9a).

When 0.4<V<0.8, coupling from the fundamental mode TE\(_0\) to the TE\(_1\) and TE\(_2\) modes is negligible because the coupling coefficient becomes small. This corresponds to the regions near the vertical asymptotes in Figure 5-7b). However, coupling to the TE\(_3\) still occurs. The delineation curves for the TE\(_3\) to TE\(_2\) and TE\(_1\) to TE\(_2\) modes and to a lesser extent the TE\(_3\) to TE\(_1\) modes, suggests substantial coupling and we expect that most of the power coupled into the TE\(_3\) couples into the TE\(_1\) and does not couple back into the fundamental TE\(_0\) mode.

The region 0.4<V<0.8 corresponds to 1.4<z<2.6mm along the taper. As can be seen in Figure 5-9b), these are the regions where the power in the TE\(_1\) mode increases substantially. In the region between 1.6 and 2.4mm, the curves in Figure 5-9 are approximately constant. This is because the taper angle becomes very small and hence there is little coupling. Between z=2.4 and 2.6mm, the power in the higher-order modes increases because of coupling through the TE\(_3\) mode as described above. In the last section of the taper beyond 2.6mm, coupling from the TE\(_0\) to the TE\(_1\) and TE\(_2\) modes occurs, but there is still much inter-modal coupling. The power in the fundamental mode reaches a minimum at z=3.0mm and then regains some power in the last section of the taper.

We have seen above that the coupling coefficient delineation curves help us to understand why there is such a large loss of power in the antisymmetric TE\(_0\) mode. The spatial dependence of the modes suggests that the excess loss would be even greater if more
modes were included in the analysis.

5-6. Total Excess Loss of the Tapered Coupler

Of major interest is the effect the excess loss has on the power in the output ports. At points beyond the coupling region, where the cores are again well separated, the field in the output ports is given by the superposition of the symmetric and antisymmetric TE0 modal fields. These modes have a phase difference dependent on the wavelength of the source and the length of the coupler, which accounts for the beating phenomenon and the transfer of power between the cores. Excess loss occurs in both output ports and can be readily quantified.

Assuming unit input power into one of the ports of the coupler, the power in the output ports is given by

$$P_{\pm} = |g_e \exp (iB_{\text{dif}} t_f) \pm g_o \exp (-iB_{\text{dif}} t_f)|^2$$  \hspace{1cm} (6)

where \( \pm \) denotes the two output ports, \( e \) and \( o \) denote the symmetric and antisymmetric modes respectively and

$$B_{\text{dif}} = \frac{1}{2t_f} \int_0^{t_f} (\beta_e - \beta_o) \, dz$$  \hspace{1cm} (7)

In the case of a coupler where there is negligible excess loss, equation 6 reduces to \( \cos^2(\pi B_{\text{dif}} t_f) \) in one output port and \( \sin^2(\pi B_{\text{dif}} t_f) \) in the other.

The power in each output port of the tapered slab coupler, together with the total output power are plotted in Figure 5-10 as a function of wavelength. These curves were
determined assuming an initial unit power entering one input port, which is equivalent to half in each of the fundamental symmetric and antisymmetric modes. The curve for total power shows that an excess loss of more than 10% occurs at longer wavelengths when the modal fields are well spread over the cross-section. This excess loss modifies the normally sinusoidal-like curves for the power in each output port.

Figure 5-10. Power in a) and b) each output port and c) the total power at the end of the coupler assuming unit power entering one input port.

Figure 5-11 shows the corresponding curves for a 2mm taper with the same maximum and minimum core radii as the 4mm taper above. In this case, the total excess loss is as large as 60% and the curves for the power in each output port are so distorted that they are no longer sinusoidal-like.
Figure 5-11. Power in a) and b) each output port and c) the total power at the end of the coupler assuming unit power entering one input port of a 2mm tapered coupler.

5-7. Conclusion

From the results for the slab coupler, we expect that for the fibre coupler there will be a modulation of power in the output ports in addition to the one due to the beating of the symmetric and antisymmetric modes. This is exactly what is found experimentally. We also expect the excess loss of the coupler to be greater than that of the component fibres because of stronger intermodal coupling. The Stewart-Love delineation criterion can be used as a guide to predict a taper shape which is not lossy, provided the taper shape lies well below the curves for coupling between all lower order modes$^{12,13}$.\"
5-8. References


CHAPTER 6
Optical Fibre-Metal Polarisers

6-1. Introduction

With the increasing use of phase modulation, instead of traditional amplitude modulation, in optical systems, sensors and even integrated optics, polarising devices have become more important. Before 1980, in-line polarising was not possible because of the lack of technology to control the polarisation state in conventional single-mode fibres\(^1,2\). However, with the development of polarisation-maintaining fibres, coherent optical-transmission systems have been developed. These, together with high grade fibre sensors such as optical fibre gyroscopes\(^3\), have created the need for control of the polarisation states, ie. polarisers.

Polarisation-maintaining fibres can be classified into two categories - Hi-Bi and Low-Bi (Bi meaning birefringence)\(^4\). The large birefringence between the two polarisation states of the fundamental mode is achieved through the introduction of stress around the core as in PANDA\(^5\), bow-tie\(^6\) and flat depressed cladding fibres\(^7\), or through geometrical differences between the refractive indices in two orthogonal directions as in elliptical\(^8\) or dumbell\(^9\) core fibres, side-pit fibres\(^10\), four-section fibres\(^11\) and side-tunnelling fibres\(^12\). Low-bi fibres can be used in long-span coherent optical-transmission systems, eg. submarine cables, provided a polariser is attached at the output end.

Polarisers can be fabricated in several ways. The first is to take a stress-bifringent, polarisation-maintaining fibre, eg. PANDA, bow-tie or flat-depressed cladding fibre, and bend it so that the axis of the fast mode, ie. the one whose effective index closest to the cladding index,\(n_c\), is in the plane of the bend. This mode is more susceptible to bending loss than its orthogonally polarised counterpart, and hence differential attenuation is produced\(^4\). These fibres can also be fabricated into couplers to achieve a polarising coupler.
so that light input in one port splits into two orthogonal polarisations in the output ports. Other similar polarising couplers using birefringent material in either the cores or cladding have also been proposed.

Another way in which polarisers can be made is to incorporate a metal layer. This can be achieved by removing part of the cladding and adding a metal layer, or alternatively, a birefringent crystal to a single-mode fibre. Polarisers have also been fabricated by including a D-shaped region of liquid metal close to the core. Polarising fibres and couplers incorporating metal layers will be discussed in this and the following chapters.

In this chapter we model the polariser using a slab waveguide and show how the addition of a buffer layer between the metal and cladding can improve the polarising ability. Chapter 7 investigates and offers a possible explanation for an experimentally observed limitation on the polarising ability of these devices, while chapter 8 considers two polarising couplers using metal layers which could be adopted in integrated optics. The fibre coupler analogue was first considered in 1984 with a measured extinction ratio of 15dB.

6-2. Optical Fibre Polarisers Incorporating Metal

The fundamental mode of a single-mode optical fibre has two orthogonal polarisation states, designated X and Y according to the direction of the transverse electric field. An ideal polariser would totally extinguish one polarisation state leaving the orthogonal one unattenuated. In a practical polariser both polarisation states are attenuated. This is because they are not exactly plane polarised but have major and minor components in orthogonal directions. The state which loses more power does so through the major field component while the other state suffers slight attenuation because of its minor field component.

As is well known, the absorption is considerably larger for the polarisation state with the
major component of the transverse electric field aligned in a direction normal to the metal-dielectric interface. This creates an extinction ratio because one polarisation state is absorbed substantially more than the other.

Examples of devices incorporating metal are illustrated in Figure 6-1. The first is a D-shaped fibre with a metal layer on the flat side\textsuperscript{20}. The second is a fibre with a D-shaped region of liquid metal close to the core\textsuperscript{18}. The third is a polariser with a buffer layer between the cladding and metal layer\textsuperscript{19}. This chapter will show how the use of a dielectric buffer layer between the cladding and metal layer can improve the polarising ability of these devices.

Figure 6-1. Examples of fibre polarisers. a) Metal-clad D-shaped fibre. b) Fibre with D-shaped liquid metal region. c) Metal-clad fibre with buffer region.
6-3. Background

6-3-1. Surface Modes

Consider the simple structure consisting of a semi-infinite planar metal layer abutting a semi-infinite uniform dielectric layer, as illustrated in Figure 6-2. This structure will support just one mode propagating parallel to the interface and with TM polarisation relative to the direction of propagation\(^\text{21}\). There is no corresponding TE mode. We can justify these claims through simple analysis.

Figure 6-2. Cross-section of a dielectric-metal structure with metal in the region \(x>0\) and dielectric in the region \(x<0\). The dielectric-metal interface is along the \(y\) axis.

TM Modes

We introduce cartesian axes orientated as shown in Figure 6-2, with the interface in the \(y\)-\(z\) plane. The modal electric field propagating in the \(z\) direction depends only on the \(x\) and \(z\) co-ordinates, with separable form

\[
E(x,z) = e(x) \exp(i\beta z)
\]
where $\beta$ is the propagation constant. For TM modes the transverse component is $x$ directed. The non-zero longitudinal component is a solution of the wave equation 1-12 away from the interface. Since the field must vanish at large distances from the interface, the appropriate forms are

$$e_z = \begin{cases} 
    i \exp(-w_m x) & x \geq 0, \text{ metal} \\
    i \exp(w_d x) & x \leq 0, \text{ dielectric} 
\end{cases}$$ (1)

where

$$w_j^2 = \beta^2 - k_j^2 n_j^2$$

and subscript $j$ denotes (d) dielectric or (m) metal.

At optical frequencies, the refractive index of metal is complex, ie. $n_m = n_{m,\text{real}} + i n_{m,\text{imag}}$. For most metals, $n_{m,\text{imag}} \gg n_{m,\text{real}}$, so that the dielectric constant, $\varepsilon = n^2$, is approximately real, large and negative.

The non-zero transverse magnetic field component is\textsuperscript{22}

$$h_y = \begin{cases} 
    -\frac{k n_m^2}{w_m} \left( \frac{\varepsilon_0}{\mu_0} \right)^{\frac{1}{2}} \exp(-w_m x) & x \geq 0 \\
    k n_d^2 \left( \frac{\varepsilon_0}{\mu_0} \right)^{\frac{1}{2}} \exp(w_d x) & x \leq 0 
\end{cases}$$ (2)
Continuity of $e_z$ and $h_y$ at the interface leads to the eigen-equation

$$\frac{n_d^2}{n_m^2} w_m = - w_d \quad (3)$$

This is a complex equation which can be rearranged to explicitly define the propagation constant $\beta = \beta_{\text{real}} + i \beta_{\text{imag}}$, in terms of the wavenumber $k$, and the refractive indices of the dielectric and the metal.

$$\beta = \frac{k n_m n_d}{\pm \sqrt{n_d^2 + n_m^2}} \quad (4)$$

The negative sign is chosen so that $\beta_{\text{real}}$ and $\beta_{\text{imag}}$ are positive, corresponding to propagation in the positive $z$ direction. For the mode to be non-radiative, the real part of the effective index should be larger than the refractive index of the dielectric. It follows from equation 4 that this condition is satisfied if the real part of $n_m^2$ is sufficiently negative. The imaginary part of the propagation constant results in a power attenuation factor $\exp(-2 \beta_{\text{imag}} z)$, as the mode propagates. The attenuation is substantial. For example, in the case of a silica-copper interface at a wavelength of 1.3\,\mu m, $n_d=1.4354$ and $n_m=0.505 + i \, 6.92^{23}$, equation 4 gives $\beta = 7.089 \times 10^6 + i0.230 \times 10^5$ m$^{-1}$ and the attenuation is almost 2000dB/cm.

**TE Modes**

For possible TE modes where the electric field is $y$ directed, we replace $e_z$ by $h_z$ in equation 1 and the non-zero transverse magnetic field component is
Both $h_z$ and $h_x$ are continuous at the interface which leads to the eigen-equation

$$w_m = -w_d$$  \hspace{1cm} (6)$$

Substituting into equation 5 shows that all field components are proportional to $\exp(-w_m x) = \exp(-w_{m,\text{real}} x) \exp(-iw_{m,\text{imag}} x)$. $w_{m,\text{real}}$ must be zero, otherwise the field components will not vanish as $x \to -\infty$. Unless $w_{m,\text{imag}}$ is also zero, the fields are oscillatory, and again, do not satisfy this condition. Thus there are no non-radiative TE modes and the interface supports only one TM mode for which the transverse magnetic and electric fields are aligned along the $y$ and $x$ axes, respectively.

The cross-sectional power distribution, $(e_xh_y^*)\text{real}/2$, of the TM mode discussed above, is illustrated in Figure 6-3. For TM modes, $e_x \propto h_y/n^2$ in each region and is therefore discontinuous across the interface. As a result, the power distribution is also discontinuous and the intensity, shown in arbitrary units, falls off exponentially away from the interface and is much steeper in the metal than the dielectric. This is basically an example of the skin effect in metals.
6-3-2. Metal-clad Fibre Polarisers

The existence of the solitary TM-polarised mode on a dielectric-metal interface - often referred to as a surface wave - can be used as the basis for a polariser, i.e. a device which preferentially eliminates the power in one polarisation state of the fundamental mode of a single-mode waveguide. This can be achieved by incorporating a metal layer into a waveguide close to the core, resulting in cross-sections of the type illustrated in Figure 6-1.

To help understand the operation of the polariser, we regard the composite core-cladding-metal layer waveguide as the combination of two isolated waveguides, as we did for the W-fibre in chapter 4. These are: firstly, the step-profile waveguide consisting of core and cladding regions, and secondly, the cladding-metal structure. By an analogy with fibre couplers, we expect the modes of the polariser to resemble a combination of the modes of the component waveguides, designated as core and surface modes.

If the component fibres of a coupler are very "different", each mode of the composite
structure closely resembles one of the fundamental-modes of the component fibres. When the isolated fibres are "identical", the exact modes can be approximated by the sum and difference of the component modes. Whether fibres are "identical" or "different" is determined by a comparison of their effective indices. The exact modes can be determined by solving the relevant wave equation for the total structure. Alternately, coupled-mode theory can be used to approximate the exact fields by a linear combination of the fields of the component fibres. In keeping with our approach in earlier chapters, we adopt the former approach in this and the remaining chapters.

Keeping the above in mind, consider the polarisation of the modal fields of the polariser in terms of the direction of the electric field. With reference to Figure 6-1, the transverse field of one polarisation state of the fundamental mode is x-directed while the other is y-directed. The field of the TM surface mode is x-directed, so there are two lowest-order X-polarisation states which are analogous to the composite modes of the fibre coupler, i.e. combinations of the X-polarisation state of the fundamental core mode and the cladding-metal TM surface mode. As there is no TE surface mode, we expect only one Y-polarisation state which closely resembles that of the fundamental core mode. Both polarisations of the fundamental mode will suffer attenuation because the exponential tails of their cladding fields extend into the metal where they are absorbed, but we expect significantly greater absorption for the X-polarisation because of resonant coupling from the core mode to the TM surface mode. A simple mechanical anologue to this situation would be the resonance between two coupled simple pendula when one is placed in a damping medium. This suggests that the two composite X-polarisation states have the potential to be much lossier than the solitary Y-polarisation state, resulting in strong differential absorption and hence polarising action.

6-3-3. Buffer Layer

It follows from the above discussion that we expect the loss of the two lowest-order
X-polarisation states to be significantly greater than that of the Y-polarisation state when the effective indices of the component core and surface modes are closely matched. For typical fibre refractive index values and almost all metals examined, the effective index of the surface mode was much higher than the refractive index of the core. For example, the real part of the effective index of the surface mode discussed in section 6-3 is 1.467 while the refractive index of the core is typically 1.45.

One way to reduce the effective index of the surface mode is to use a cladding with a lower refractive index. This has been investigated for slab structures by choosing a lower cladding index value between the core and metal\textsuperscript{24-27}. Clearly this is not a practical solution for the circular cross-section of fibres. Instead, a buffer layer with an index lower than that of the cladding can be introduced between the metal and cladding, as shown in Figure 6-1c), to lower the effective index of the surface mode so that its real part approximately matches that of the core-cladding mode. As will be shown, the use of the buffer layer can increase the absorption of the X-polarisation state by approximately an order of magnitude without greatly affecting the attenuation of the Y-polarisation state.

6-4. Model for Metal Fibre Polarisers

The polarisers illustrated in Figure 6-1 could be analysed for their propagation characteristics by solving the vector wave equations 1-9. The weak-guidance approximation, which was used to simplify the analysis of tapered fibres, is no longer applicable because of the large difference between the dielectric constants of the cladding and metal. As the fields in the dielectric tapers were negligible at the cladding-air interface, we were able to neglect the large difference in the indices of the cladding and air, and use the weak guidance approximation within the interface. However, the propagation along tapers is only marginally affected by the cladding-air interface, whereas the absorption in the polariser depends critically on the fields at the dielectric-metal interface.
In the polariser, the dielectric-metal interface must be close to the core to achieve a short device length and, for significant absorption to occur, the fields here should be relatively large. Thus, a full vector analysis is required. Because of the complex, multilayered, cross-sectional shapes illustrated in Figure 6-1, this would require extensive numerical analysis, thereby masking the dominant analytical features. Furthermore, the lack of available information on the precise shape and profile variation of these devices reduces the value of such an approach. As will be discussed in the following section, the minor field components of the X and Y-polarisation states causes bending of the field lines in the cross-section. However, this is very small and for the purpose of giving a qualitative description, we can model the fibre polariser by the corresponding slab polariser.

6-4-1. Polarisation States of the Fibre Polariser

As shown in section 3-4, the fields of a fibre are approximately plane polarised with bending of the field lines occurring principally close to interfaces. Here we quantify the extent to which the fields are plane polarised in the polariser.

For the polariser in Figure 6-4 we assume $\lambda = 0.83 \mu m$, $n_{co} = 1.47165$ and $n_{cl} = 1.458$, giving $\Delta = 0.013618$. The dimensions shown in Figure 6-4 are $t = 0.67 \rho$ and $s = 6 \rho$ and cladding radius is $60 \mu m$, respectively. We expect the field lines of the polariser to be similar to those of the dielectric fibre, except close to and within the D-shaped metal region. We want to determine the angle of the cross-sectional field lines in this fibre at the region corresponding to the metal in the polariser. The maximum bend in the field lines occurs at the co-ordinates $(r_D, \theta_D)$. The cladding radius is relatively large and, as was shown in section 3-4, the bending due to the cladding-air interface occurs principally outside the interface. Thus, it is sufficient to consider an infinite-cladding fibre, for which the exact fields are known, with parameters identical to the polariser. Thus we choose a $V$-value of 1.5, giving $p = 0.82$ and $U = 1.3206$.22.
Figure 6-4. A practical polariser with liquid metal in the hatched region showing the point $(r_D, \theta_D)$. The radial co-ordinate is measured from the centre of the core and the angle co-ordinate is measured from the horizontal line shown.

The angle of the field line for either polarisation state is

$$\theta_p = \arctan \left[ \frac{e_y}{e_x} \right]$$

The co-ordinates $(r_D, \theta_D)$ are 6.228° and 74.43°. At this point the polarisation angles are 0.107° and 90.107° for the X and Y-polarisation states of the fundamental mode, respectively. This means that the ratio of minor to major components is $1.87 \times 10^{-3}$. Thus the fields of the two polarisation states are, to a good approximation, plane polarised, which is consistent with weak-guidance within the cladding-air interface. Furthermore, because of the 'skin effect' of the metal, i.e. zero tangential electric field in a perfect conductor, this will modify the direction of the field lines locally to be more orthogonal to the interface for the X-polarisation state and reduce the electric field amplitude for the Y-polarisation state.
6-4-2. Slab Waveguide Model

The slab model of the polariser is shown in Figure 6-5. From the figure it can be seen that if the axes are chosen so that the buffer-metal interface is parallel to the y axis, the X and Y-polarisations of the fibre fundamental mode correspond, respectively, to the TM and TE modes of the slab waveguide. This model has the advantage that the analysis is tractable and yet still displays all the features of the practical structure.

![Slab Waveguide Model](image)

Figure 6-5. Slab waveguide model for the metal-clad fibre polariser with buffer layer. The metal layer and outer cladding extend indefinitely in the y and z directions. The core, cladding and buffer thicknesses are $2\rho$, $t_{cl}$ and $t_b$ respectively.

It is assumed that the outer cladding on the opposite side to the metal is infinitely thick. It is, in practice, of finite width, and the effect of this will be discussed in the following chapter. The metal layer is also assumed to be infinitely thick to simplify the analysis. As will be shown in the following section, this assumption is valid provided the metal layer is
sufficiently thick for the field to be negligible at the outer metal-air interface. If the metal layer is too thin, the field can penetrate into the air. This will occur first with the TE mode, with its electric field parallel to the interface. This is the principle used in some polarisers which rely on this mode being cutoff because of the exposure to air.

6-4-3. Effect of a Finite Metal Layer

To quantify the effect of assuming an infinite metal layer, consider a dielectric-metal-air three layer structure. There will be two surface modes, one associated with each interface. We are concerned only with the surface mode on the dielectric-metal interface. The eigen-equation for the TM modes of this structure is obtained by applying the appropriate boundary conditions to the fields at each interface using a simple generalisation of the analysis used for the dielectric-metal interface above, and is given by

\[
\frac{n_m^2}{n_{air}^2} \frac{w_{air}}{w_m} \frac{\tanh (w_m t_m)}{1 + \frac{w_m}{w_d} \frac{n_d^2}{n_m^2} \tanh (w_m t_m)} + \frac{w_m}{w_d} \frac{n_d^2}{n_m^2} = -1
\]

where \( t_m \) is the thickness of the metal layer and the \( w_j \) are defined in equation 1.

The propagation constants of the surface mode on the dielectric-metal interface for various metal layer thicknesses with \( n_{air} = 1.0 \), \( n_d = 1.43544268 \) and \( n_m = 0.505 + i 6.92 \) (copper at a wavelength of 1.3\( \mu \)m), are given in Table 6-1. These show that, provided the metal layer is thicker than 0.05\( \mu \)m, the propagation constant is virtually the same as that of dielectric-metal surface mode. At this thickness the propagation constant differs from that of a dielectric-metal structure by 7.5 \( \times \) 10\(^{-3} \) % and 1.668 % in the real and imaginary
parts, respectively. Thus the approximation of an infinite metal layer is justified.

Table 6-1. Propagation constant of a dielectric-metal-air three layer structure for various thicknesses of the metal layer.

<table>
<thead>
<tr>
<th>t_m in μm</th>
<th>β_real m⁻¹ x 10⁶</th>
<th>β_imag m⁻¹ x 10⁵</th>
</tr>
</thead>
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<tr>
<td>1.0</td>
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<td>0.22999330022</td>
</tr>
<tr>
<td>0.8</td>
<td>7.0893512611997</td>
<td>0.22999330022</td>
</tr>
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<td>0.22999330022</td>
</tr>
<tr>
<td>0.4</td>
<td>7.0893512611997</td>
<td>0.22999330023</td>
</tr>
<tr>
<td>0.2</td>
<td>7.0893515771518</td>
<td>0.22999955061</td>
</tr>
<tr>
<td>0.1</td>
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<td>0.23382908253</td>
</tr>
<tr>
<td>0.05</td>
<td>7.1088641766317</td>
<td>0.31320609494</td>
</tr>
<tr>
<td>0.02</td>
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<td>1.04592609860</td>
</tr>
<tr>
<td>0.01</td>
<td>8.2001721467459</td>
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</tr>
<tr>
<td>0.001</td>
<td>42.939400242433</td>
<td>61.4481292381</td>
</tr>
</tbody>
</table>

6-5. Analysis

6-5-1. Derivation of the Eigen-Equation

The longitudinal field components of the TE and TM modes satisfy the wave equation 1-11, and the non-zero transverse field components are then given by equations 1-13 a) and b)³². The propagation constant, β, is determined by applying the boundary conditions that e_z or h_z and h_x or h_y must be continuous at each interface. An alternative description of the boundary conditions is that ψ and Sψ/P are continuous, where ψ is the non-zero longitudinal field component, ' denotes differentiation with respect to the transverse X
co-ordinate, $P$ is defined in equation 1-12 and $S=n^2$ for TM modes and 1 for TE modes.

For the TM modes of the five layer structure in Figure 6-5, this leads to the eigen-equation

\[
\frac{1 + K_0 \tan (2U)}{K_0 - \tan (2U)} = K_{bc1} \frac{W_{cl}}{U} \frac{n_{co}^2}{n_{cl}^2}
\]

where

\[
K_0 = -\frac{n_{cl}^2}{n_{co}^2} \frac{U}{W_{cl}}
\]  

(10a)

\[
K_{bc1} = \frac{K_{mb} + \tanh (W_{cl}T_{cl})}{\tanh (W_{cl}T_{cl}) K_{mb} + 1}
\]

(10b)

\[
K_{mb} = \frac{W_m n_b^2}{W_b n_m^2} + \frac{\tanh (W_bT_b)}{\tanh (W_bT_b)} \frac{W_m n_b^2}{W_b n_m^2}
\]

(10c)

where $U$ and $W_j$ are defined by equation 1-6 and 7, $T_j = t_j/\rho$ is the normalised thickness of the $j$th layer and subscripts $co$, $cl$, $b$ and $m$ denote the core, cladding, buffer and metal, respectively. The eigen-equation for TE modes is obtained by setting $n_j = 1$ for each region in equations 9 and 10.

For reasons given above, the propagation constant is complex, ie. $\beta = \beta_{real} + i\beta_{imag}$.
which results in a power attenuation factor in the field components of $\exp(-2\beta_{\text{imag}}z)$, or alternatively, an absorption factor in the field of $\exp(-\beta_{\text{imag}}z)$. This is equivalent to a loss in dB/cm of $8.686 \times 10^{-2} \beta_{\text{imag}}$. Physically, absorption occurs only in the metal, thus as the mode propagates, there must be a redistribution of power in the cross-section to retain the modal field shape, i.e. transverse power flow. This will be discussed in more detail in the following chapter. The complex eigen-equation was solved numerically using a two-dimensional Newton's method.

6-6. Results and Discussion

A value of 1.3 microns was chosen for the wavelength, with $n_{\text{co}}=1.45$ and $n_{\text{cl}}=1.4354$. This gives $\Delta = (n_{\text{co}}^2 - n_{\text{cl}}^2)/2n_{\text{co}}^2 = 0.01$, which are all typical values for optical waveguides. The choice of the metal does not affect the results significantly, so copper, $n_m=0.505 +i 6.92$ at $\lambda =1.3\mu m$, was chosen as representative. Clearly, the imaginary part of the refractive index is much larger than the real part, giving the dielectric constant $\varepsilon \propto n_m^2$ a large, negative real part.

Throughout this chapter, the two TM modes will be labelled by subscripts 0 and -1 according to convention. The mode labelled 0 denotes the one which is similar to the fundamental TM$_0$ mode of a step-profile waveguide when the metal and core regions are well separated, while the mode labelled -1 is more like the surface mode. The choice -1 is made because the real part of the propagation constant for this mode is usually greater than for the mode labelled 0 and by convention, should have a smaller mode number.

6-6-1. Effect of Cladding Thicknesses

If we keep the buffer thickness fixed, Figure 6-6 shows the imaginary part of the propagation constant as a function of cladding thickness with $p=1.0 \mu m$, $T_b = 0.5$, $n_b = 1.4137$ and other parameters as above. In other words, only the thickness of the cladding is varying.
Figure 6-6. Imaginary part of the propagation constant $\beta$, plotted against normalised cladding thickness, $T_{cl}$, with $T_{b}=0.5$.

For sufficiently large thickness, the $TE_0$ and $TM_0$ modal fields are very small at the buffer-metal interface and there is negligible attenuation. As the cladding thickness is reduced, the attenuations of the $TE_0$ and $TM_0$ modes at first increase as their fields penetrate farther into the metal, whereas the $TM_1$ mode remains virtually identical to the surface mode of a buffer-metal interface.

As the cladding thickness decreases further, the curve for the $TM_0$ mode passes through a peak. This resonant peak corresponds approximately to "matching" of the real parts of the propagation constants of the isolated $TM_0$ core and surface modes. There is very little effect on the $TM_1$ mode. The TE mode loss increases monotonically and flattens out with increasing cladding thickness, as the metal surface is closer to the core causing more absorption. As the cladding thickness decreases to zero, the curves asymptotically approach the values obtained for a structure with cladding-core-buffer-metal layers$^{24-27}$.

We conclude from these curves, that to maximise the differential absorption of the slab waveguide, the cladding must be of comparable size to the core radius, ie. $T_{cl} = 1.0$. 
6-6-2. Effect of Buffer Thickness

If instead, we fix the cladding thickness $T_{cl} = 1.5$, Figure 6-7 shows the dependence of loss on the buffer layer thickness. These curves are similar to those showing the dependence on the cladding thickness in Figure 6-6 and the behaviour is physically the same. Note that the peak in the TM$_0$ curve occurs near $T_b = 1.0$. As the buffer thickness decreases, the loss curves tend asymptotically to values for the structure without buffer, ie. a cladding-core-cladding-metal layer waveguide.

![Figure 6-7](image)

Figure 6-7. Imaginary part of the propagation constant $\beta$ plotted against normalised buffer thickness, $T_b$ for fixed $T_{cl} = 1.5$.

6-6-3. Effect of Combined Cladding and Buffer Thicknesses

The two curves above show that the loss of the TM$_0$ mode is greatest when the core, cladding and buffer thickness are comparable. Figure 6-8 shows the dependence of the loss of the TE$_0$, TM$_0$ and TM$_{-1}$ modes when the combined cladding and buffer thickness is fixed at $T_{cl} + T_b = 2.0$, ie. equal to the width of the core, but the fraction of each layer varies. For this buffer index, the loss of the TM$_0$ mode is two orders of magnitude greater.
than that of the TE mode when $T_{c1} = 2.0$ and $T_b = 0.0$. As the cladding thickness decreases, the relative loss increases to a maximum of approximately three orders of magnitude. Again, note the maximum at $T_{c1} = 1$ for the TM$_0$ mode.

This curve suggests that it is best to use a buffer thickness which is slightly larger than the cladding thickness. However, this may cause mismatch problems because when the polariser is placed in-line, as the incoming and outgoing waveguides do not have a buffer region. For the following section we use $T_{c1} = 1.5$ and $T_b = 0.5$.

Figure 6-8. Imaginary part of the propagation constant $\beta$ plotted against $T_{c1}$, with $T_{c1} + T_b = 2.0$, fixed.

6-6-4. Effect of the Buffer Index Value

Description of Results

In Figures 6-9 to 6-13, plots a) and b) are for the real and imaginary parts, respectively, of the propagation constant $\beta$ as a function of core radius for the TE$_0$, TM$_0$ and TM$_{-1}$ modes. Plots c) are a combination of a) and b) for the two TM modes and show the path of the
propagation constants in the complex $\beta$ plane as the core radius changes. The arrows indicate the direction for decreasing core radius. The normalised cladding and buffer thicknesses are $T_{c1}=1.5$ and $T_{b}=0.5$, respectively. Since these values are fixed, this means that as the core radius decreases, the cladding and buffer thicknesses decrease in the same proportion, as if the waveguide were tapered.

Figures 6-9 a) and b), respectively, show the real and imaginary parts of the propagation constant when the buffer and cladding indices are matched, i.e. effectively no buffer. Consider the $TE_0$ and the $TM_0$ modes first. The real parts are virtually the same as those of the corresponding modes of a core-cladding step-profile waveguide. The imaginary parts first increase with decreasing core radius until they peak and decrease. The increase is because the modal fields are spreading out into the cladding, and hence the amplitude of the field near the metal increases. Below the peak, the fields spread into the outer cladding on the opposite side to the metal and the amplitude of the field near the metal layer decreases. The real and imaginary parts of the propagation constant of the $TM_1$ mode at all core radii are virtually identical to those of the surface mode of a cladding-metal structure. As a result, the curve for the $TM_1$ mode in Figure 6-9c) is virtually a single point while the curve for the $TM_0$ mode is an inverted U-shape below.

Figure 6-10 shows the same results for a buffer index $n_b=1.418$, which are similar to those in Figure 6-9. Figure 6-10a) shows that this buffer index has little effect on the real part of the propagation constant of all three modes or the imaginary part of the propagation constant of the $TM_1$ mode. However, the imaginary part of the $TE_0$ mode is smaller while that of the $TM_0$ mode is larger than the corresponding values shown in Figure 6-9b). The former results from the cladding-buffer interface reducing the amplitude of the $TE_0$ mode at the metal, while the effective index of the core $TM_0$ mode is better matched to the surface mode, thereby enhancing absorption. The curve for the $TM_0$ mode in Figure 6-10c) differs slightly from the corresponding curve in Figure 6-9c), being displaced vertically to higher imaginary values. The curve for the $TM_1$ mode is still virtually a
single point.

a)

\[ \beta_{\text{real}} \]

\[ \begin{array}{c}
\text{core radius in microns} \\
0 & 1 & 2
\end{array} \]

\[ \begin{array}{c}
6.90e+6 & 7.00e+6 & 7.10e+6
\end{array} \]

\[ \begin{array}{c}
\text{TM}_1 \\
\text{TM}_0 \\
\text{TE}_0
\end{array} \]

b)

\[ \beta_{\text{imag}} \]

\[ \begin{array}{c}
\text{core radius in microns} \\
0 & 1 & 2
\end{array} \]

\[ \begin{array}{c}
10^0 & 10^1 & 10^2 & 10^3 & 10^4 & 10^5
\end{array} \]
Figure 6-9. a) real and b) imaginary parts of the propagation plotted against core radius $\rho$, for the TE$_0$, TM$_0$ and TM$_{-1}$ modes for a polariser without a buffer and a cladding thickness $T_{cl}=2.0$. c) shows the propagation constants of the TM modes in the complex plane. $n_b = n_{cl} = 1.4354$

Figure 6-11 shows the results for a smaller buffer index of 1.4137. The real parts of the propagation constants in Figure 6-11a) again show little difference from the curves for the TE$_0$ and TM$_0$ modes in the previous Figure 6-10a). However, the real part of the propagation constant of the TM$_{-1}$ mode is much lower for large core radii so that the three curves appear to converge near $\rho=1.8\mu m$. As can be seen from the curves below for lower buffer indices, the curve for the TM$_{-1}$ mode actually cuts and lies beneath those of the TE$_0$ and TM$_0$ modes at large core radii. This will be discussed below. The curves for the imaginary part of the propagation constant shown in Figure 6-11b) are similar to those in Figure 6-10b). The curve for the TE$_0$ mode is again slightly lower while that of the TM$_0$ mode is slightly higher for the reasons given above. In Figure 6-11c), the curve for the TM$_{-1}$ mode is virtually a line parallel to the real axis rather than a single point. This is because the imaginary part of the propagation constant is approximately constant while the real part lies between the limiting values for large and small radii. The curve for the TM$_0$
mode is similar to the corresponding curve in Figure 6-10c) and does not touch the curve for the TM\(_{1}\) mode.

Figure 6-12 shows the results for a buffer index of 1.408. Figure 6-12a), the curve for the real part of the propagation constant of the TM\(_{1}\) mode intersects those of the TE\(_0\) and TM\(_0\) modes, and, at large core radii, lies below them. Figure 6-12b) shows an interesting feature in the imaginary parts of the propagation constants. The curve for the TM\(_0\) mode reaches a peak near \(\rho=1.3\mu m\) and then decreases gradually with \(\rho\) until \(\rho=0.7\mu m\). At this radius the curve suddenly decreases to zero and the mode is cutoff. This will be discussed in more detail below. The corresponding curve for the TM\(_{1}\) mode has a slight dip near \(\rho=1.3\mu m\) where the TM\(_0\) curve peaks. Figure 6-12c) shows that the curves for both TM modes differ from the corresponding curves in Figure 6-11c). The curve for the TM\(_{1}\) mode now has a distinct minimum while the maximum in the curve for the TM\(_0\) mode has been slightly distorted.
Figure 6-10. Results for a buffer index value $n_b = 1.418$. 

\[ \beta_{\text{imag}} \]

Core radius in microns

- $\Delta$ TM$_{-1}$
- $\blacksquare$ TM$_{0}$
- $\times$ TE$_{0}$
a) 

\[ \beta_{\text{real}} \]

\[ 7.10 \times 10^6 \]

\[ 7.00 \times 10^6 \]

\[ 6.90 \times 10^6 \]

core radius in microns

b) 

\[ \beta_{\text{imag}} \]

\[ 10^5 \]

\[ 10^4 \]

\[ 10^3 \]

\[ 10^2 \]

\[ 10^1 \]

\[ 10^0 \]

core radius in microns

\( \triangle \) TM\_1

\( \square \) TM\_0

\( + \) TE\_0
Figure 6-11. Results for a buffer index $n_b = 1.4137$.

- $\Delta$ TM$_{-1}$
- $\blacksquare$ TM$_{0}$
- $\times$ TE$_{0}$
b) Figure 6-12. Results for a buffer index $n_B=1.408$.

c) Figure 6-13 shows the results for a buffer index of 1.406. This is a reduction of only 0.002 from the previous index but the curves show dramatic changes. The curves for the real part of the propagation constants in Figure 6-13a) do not now cross. One curve has a maximum while the other has a minimum at the same core radius of approximately 1.3μm. For larger radii, one mode is virtually the same as the $TM_0$ core mode while the other is...
more like the surface mode. For smaller radii, this is reversed which makes the labelling of a TM mode according to the component mode it most resembles, inconsistent. We follow the convention given in section 6-6 and use the large radius behaviour to label. Figure 6-13b) for the imaginary part of the propagation constant shows the curves for the TM₀ and TM₋₁ modes crossing at a radius corresponding to where the real parts of the propagation constants are closest. For this buffer index, it is the TM₋₁ mode, which is the core-like mode at small radii, that is cutoff below ρ=0.6μm. The curves in Figure 6-13c) again do not cross, but they show a dramatic change from those in Figure 6-12c). The branches corresponding to small core radii seem to have been interchanged. The transition between Figures 6-12c) and 6-13c) occurs continuously as the buffer index decreases. In other words, the curves must approach one another, touch and separate with the branches exchanged. This need not occur at the same core radii, i.e. the two modes are not necessarily degenerate. This changeover of the TM₀ and TM₋₁ is a necessary consequence of the monotonic reduction in buffer index, and can be likened to propagation along the asymmetric coupler described below.

a)
Asymmetric Coupler Analogy

Consider the evanescent coupler shown in Figure 6-14, which consists of two cores with parallel axes, one of which has a uniform cross-section while the other is tapered such that the only position along the taper where the two cross-sections match is at the line CD. The tapering is assumed sufficiently slow that the device is adiabatic and the local modes totally
describe the propagation. The mode of the uniform fibre can be likened to the TM$_0$ mode above and the mode of the tapered fibre to the TM$_1$ above. At the bottom of the coupler, the two modes of the coupler are the fundamental modes of each core in isolation because the two cores are so different. As these two modes propagate up the coupler, the propagation constant of fibre A remains constant and that of fibre B increases with the cross-section until the two cores become quite similar.

Figure 6-14. An asymmetric coupler consisting of two parallel dissimilar fibres. The first has a constant core radius whereas the radius of the second is tapered between radii that are smaller and larger than that of the first fibre. The line CD shows where the radii of the two fibres are equal.

Just below CD, the similarity between the cores is sufficient for the two modes to evolve into the two modes of the composite two-core coupler. At CD, the modes are the even and
odd modes of a symmetric coupler, the former having the slightly greater propagation constant. Just below CD, the fundamental mode of fibre B has a smaller propagation constant than the mode of fibre A and so we expect it to become the odd coupler mode on CD, and similarly, the mode of fibre A becomes the even mode.

The evolution of the modes above CD follows a similar argument. Thus, the even mode on CD becomes the fundamental mode of fibre B, while the odd mode mode becomes the fundamental mode of fibre A. In other words, the modes of the two fibres 'swap' over in the middle of the coupler, the fundamental mode of fibre A becoming that of fibre B, and vice versa. In terms of the polariser, this is equivalent to the $\text{TM}_q$ and $\text{TM}_{-1}$ modes exchanging roles as the buffer index moves through the 'resonance' value where the two component modes evolve into the composite modes of the polariser.

**TE Mode Summary**

The results above show that the value of the buffer index has little effect on the imaginary part of the propagation constant, or the loss, of the TE modes. The magnitude of the maximum loss suffers a slight decrease as the index value is decreased. This is because the field is more confined within the core and cladding by the buffer of lower index, and therefore there is a smaller fraction of the total field near the metal where it can be absorbed. This is also why the real part of the propagation constant is insensitive to the buffer index.

**TM Mode Summary**

The buffer has considerably more effect on the TM modes. A buffer index below that of the cladding results in an increase and decrease in the imaginary part of the propagation constants of the $\text{TM}_0$ and $\text{TM}_{-1}$ modes, respectively, and lowers the real part of the propagation constant of the $\text{TM}_{-1}$ mode at large core radii. If the buffer index is sufficiently low, the mode which is core-like at large radii becomes more like a surface
mode at small radii, and vice versa. The mode with the lower real part of the propagation constant can also become cut off at a finite core radius.

![Diagram of TM modes](image)

Figure 6-15. Power distribution of the first two TM modes of the slab waveguide polariser when the buffer is matched to the cladding. The $TM_0$ mode closely resembles the $TM_0$ mode of a core-cladding waveguide while the $TM_{-1}$ is more like the surface mode.

Figure 6-9 shows that, without a buffer, the real part of the propagation constant of the $TM_0$ mode is only slightly different from that of the $TM_0$ mode of a core-cladding waveguide, while the propagation constant of the $TM_{-1}$ mode is virtually identical to that of a cladding-metal surface mode. This is analogous to a fibre coupler where the component fibres are very dissimilar. The modal fields are virtually those of a core-cladding waveguide and a surface mode as shown in Figure 6-15.

As the buffer index is lowered below that of the cladding, by the analogy with the asymmetric coupler discussed above, the modes of the polariser are more like linear combinations of the component ones but each more closely resembles one of the
component core or surface modes. With the buffer, the real part of the propagation constant of the TM\textsubscript{1} mode lies between two limiting values. The limit for large core radius is the real part of the propagation constant for the surface mode of a metal-buffer structure and the limit for small core radius is the real part of the propagation constant for the surface-mode of a metal-cladding structure. This is illustrated in Figure 6-16, which shows the real part of the propagation constant of a cladding-buffer-metal structure for various buffer indices, as the buffer thickness changes such that it is always half the core radius, ie. $T_b=0.5$. The propagation constant of the TM\textsubscript{0} core mode is also shown.

![Figure 6-16. Propagation constant of the fundamental TM\textsubscript{0} mode of a step profile core-cladding waveguide and the real part of the propagation constant of the surface mode of a cladding-buffer-metal three layer structure for various buffer indices, as a function of core radius. The buffer thickness is half the core radius, ie. $T_b=0.5$.](image-url)

There is a similar feature for the imaginary part of the propagation constant of the TM\textsubscript{1} mode. Since $T_{\text{cl}}$ and $T_b$ have been fixed, for large radius the core and metal-dielectric interface are well-separated and the TM\textsubscript{1} mode is like an isolated surface mode of a metal-buffer structure. As the radius decreases to zero, the core and buffer regions effectively vanish, leaving a metal-cladding structure. If the buffer index is chosen so that the lower limit of the real part of the propagation constant is below the propagation constant
of the TM$_0$ core mode at a large radius, then there is a value of the core radius, as shown in Figure 6-16, for which the real parts of the propagation constants of the component modes are equal. The radius at which the cross-over of the curves occurs decreases with the buffer index value. If the cross-over radius is low enough for the core-cladding mode to have spread far into the cladding, the component fields will 'interact' significantly. This means that if the buffer index is chosen to be sufficiently low, then the TM modes of the polariser will be more like linear combinations of the component modes. Figures 6-11 and 6-12 show that at large core radii the TM$_0$ is more like a core mode while the TM$_{-1}$ is more like a surface mode. As the core radius decreases they become more like a combination of the two component modes, and the real parts of the propagation constants are more closely matched. As the radius is decreased further, the modes again become more like the component modes.

If the buffer index is sufficiently low, as illustrated in Figure 6-13, the modes behave as described above except at small radii. Here, the modes have swapped, as explained in terms of the asymmetric coupler above. At intermediate radius values the modes are like neither the core nor surface modes alone but are similar to their sum and difference.

At large core radii, an incoming TM$_0$ mode would excite the TM$_0$ mode of the polariser. If the polariser could be tapered adiabatically, the power would remain in this mode which becomes very lossy. The power in the core region at the start of the taper, where the radius is large, would seem to spread out into the cladding and across to the buffer-metal interface as the radius decreases.

6-6-5. Modal Cutoff

In the curves for the TE$_0$ and one of the TM modes in Figures 6-9 to 6-13, there is a peak in the imaginary part of the propagation constant below which each curve decreases to zero. This peak occurs at larger radius values for the TM mode. As the core radius decreases,
there is a value below which the structure only supports the TE\textsubscript{0} mode and one TM mode, viz. the one is which more like a surface mode, ie. a cutoff radius exists. There is a second, smaller cutoff radius below which it can support only the TM mode. At these radii the respective modes are cutoff and become radiative\textsuperscript{32}.

Cutoff occurs when the real part of the effective index, $\beta_{\text{real}}/k$, is no longer above the cladding index, or as will be shown below, $W=0$. The transverse dependence of the field in the outer cladding is proportional to

$$
\exp(W X) = \exp(W_{\text{real}} X) \left( \cos (W_{\text{imag}} X) + i \sin (W_{\text{imag}} X) \right), \quad X < 0
$$

where $W=W_{\text{real}} + iW_{\text{imag}}$. Unless $W_{\text{real}} \geq 0$, the field does not decrease exponentially with decreasing $X$ and therefore, is not bound as $X \to -\infty$. At cutoff, $W_{\text{real}} = 0$. From the definition of $W$

$$
\frac{W^2}{\rho^2} = \beta^2 - k^2 n_{\text{cl}}^2
$$

and equating the real and imaginary parts, it follows that at cutoff $\beta_{\text{real}} = k n_{\text{cl}}$, $\beta_{\text{imag}} = 0$ and $W_{\text{imag}} = 0$. It is this condition that is responsible for the zeros in the curves for the imaginary part of the propagation constant in Figures 6-10 to 6-13.

The behaviour above can be characterised physically as follows. As the core radius initially decreases, the curves for the TE\textsubscript{0} and TM\textsubscript{0} modes in Figures 6-9 to 6-12 at first increase due to a larger fraction of the field being in the metal. However, as the core radius decreases further, the field spreads farther into the cladding on the opposite side of the core to the metal, while the metal layer prevents the field from spreading out on this side. The unbounded cladding allows the modal fields to spread indefinitely, until, at the cutoff radius, there is virtually no field at the metal-buffer interface and no absorption. Thus, the
curves for the imaginary part of the propagation constant pass through a maximum and then decrease with decreasing radius.

Of course, in a practical polariser the cladding has a finite thickness. This would cause the cutoffs to occur at smaller core radii and, as will be shown in the following chapter, higher-order modes can then be supported.

6-6-6. Extinction Ratio and Buffer Index

In a polariser, we are interested in maximising the extinction ratio between the two polarisations. This depends predominantly on the relative size of the imaginary part of the propagation constants of the TM_0 and TE_0 modes. Table 6-2 shows the maximum value of the ratio of the two for various buffer indices, and the core radius values where the maximum ratio in each case occurs. As shown, quite substantial extinction ratios can be achieved by using a buffer. For the first two indices, the maximum occurs where the waveguide is not single-moded.

The absence of the buffer reduces the maximum considerably, and there is a very shallow maximum at a radius greater than 2.0μm. At this radius the component core-cladding waveguide supports higher-order modes. At radius values where only the TE_0 and TM_0 modes are supported by this waveguide the ratio is less than 70.
Table 6-2. Maximum value of the ratio of imaginary parts of the propagation constant of the TM\textsubscript{0} and TE\textsubscript{0} modes for the various buffer index values, and the core radius where the maximum occurs.

<table>
<thead>
<tr>
<th>(n_b)</th>
<th>(\beta_{TM imag} / \beta_{TE imag})</th>
<th>(\rho) in (\mu m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.418</td>
<td>(4.5 \times 10^3)</td>
<td>2.5</td>
</tr>
<tr>
<td>1.4137</td>
<td>(5.0 \times 10^3)</td>
<td>1.9</td>
</tr>
<tr>
<td>1.406</td>
<td>(7.0 \times 10^3)</td>
<td>1.2</td>
</tr>
<tr>
<td>1.38</td>
<td>(5.0 \times 10^3)</td>
<td>0.55</td>
</tr>
</tbody>
</table>

As can be seen from Figure 6-16, the core radius value where the maximum ratio occurs for each buffer index value is approximately equal to the radius where the propagation constant of the TM\textsubscript{0} mode of the core-cladding waveguide is equal to the real part of the propagation constant of the surface mode of the buffer-metal structure. If the buffer index is too low, the polarising effect is reduced because the field is confined within the core and cladding regions because of the large index difference between the cladding and buffer indices.

In summary, by using a buffer, the maximum loss of the TM\textsubscript{0} mode can be increased by an order of magnitude, and the ratio of the imaginary part of the propagation constants of the TM\textsubscript{0} and TE\textsubscript{0} modes can be increased by almost two orders of magnitude.

6-7. Discussion and Conclusion

The results for the slab waveguide given above show how the use of a buffer layer between the cladding and metal can significantly improve the polarising ability of the device. We can use these results to predict a way of improving the results for the fibre polariser. Firstly, consider the fundamental core mode of the fibre in isolation from the metal layer,
and the mode of the structure consisting of cladding-buffer-metal layers. The buffer is chosen so that the real part of the propagation constant of the surface mode crosses the propagation constant of the fundamental mode at a core radius where the overlap of the isolated modal fields is significant. This should be chosen to be below the cutoff value of the next order mode of the fibre. It may be advantageous to taper the cladding by polishing, deposit the buffer layer and then taper the entire structure to a core radius where maximum loss is expected to occur, so that the incoming field changes adiabatically to the field of the polariser. The buffer could then be etched to add the metal.

The results shown in Figure 6-17 show the real part of the propagation constant of the surface mode of a cladding-buffer-metal layer structure and the propagation constant of the fundamental mode of a step-profile fibre with $n_{c0}=1.45$ and $\Delta =0.01$, giving $n_{cl}=1.435$, and a buffer index $n_b=1.406$. A fibre with these parameters at a wavelength of 1.3\,\mu m is single moded below $\rho=2.4266$\,\mu m, and, below $\rho=2.0$\,\mu m less than 70\% of the power lies within the core. Since the curves cross near $\rho=1.8$\,\mu m, this suggests that a buffer index of 1.406 should improve the polarising ability of the fibre polariser with these parameters. Of course, this is not the only choice and may not even be the best.

Figure 6-17. Propagation constant of the fundamental mode of a fibre without metal and real part of the propagation constant of the surface mode of a three-layer structure.
6-8. References


7-1. Introduction

An ideal single-mode fibre polariser would totally extinguish one polarisation state of the fundamental mode, leaving the orthogonal polarisation unattenuated. In a practical polariser, such as the metal-clad fibre polariser, both polarisations are attenuated, but there is a relative attenuation, or extinction ratio, because of the different attenuation of the two polarisations in the metal. Thus, although one polarisation could be virtually extinguished, the orthogonal polarisation would still suffer significant attenuation.

In principle, the power loss from both polarisations increases with increasing length of the polariser. However, if the polarising fibre is spliced into a regular fibre, it has been observed experimentally\(^1,2,3\) that the extinction ratio does not increase linearly with the device length but falls below the expected value. This is illustrated by one set of experimental results\(^2\) in Figure 7-1.

![Figure 7-1. Experimental attenuation curves for the two polarisation states of the fibre polariser as a function of device length\(^2\).](image)
One explanation for this phenomenon attributes it to coupling between the polarisations because of imperfections in the polarising fibre due to surface roughness at the metal-cladding interface. A simple model incorporating a coefficient to quantify the coupling produces theoretical results which exhibit the qualitative features of the experimental measurements. However, whilst, in principle, polarisation coupling could be reduced with improvements in the fabrication of polarisers, the observed change in extinction ratio could not be completely overcome because of a more fundamental limitation.

7-1-1. Splicing Mismatch

Splicing the polarising fibre between lengths of regular fibre introduces a mismatch at each of the two splices. The situation is illustrated in Figure 7-2, where, for simplicity, the regular and polarising fibres are assumed to be identical apart from the metal layer in the polarising fibre.

Now consider a fundamental mode propagating along the regular fibre from \( z<0 \). At splice A it will excite, predominantly, the fundamental mode of the polariser, but a small fraction of power will excite higher-order cladding modes of the polariser. These modes exist because of the finite cladding. At splice B, virtually all the remaining power in the fundamental mode of the polariser excites the fundamental mode of the regular fibre. However, a small fraction of the power in the higher-order modes also excites the same mode. Hence we can identify two flows of power within the polariser and examine their respective attenuations.
Figure 7-2. Longitudinal cross-section showing a polariser spliced between the incoming and outgoing fibres. Propagation is in the positive z direction.

The power in the fundamental mode is attenuated because of absorption in the metal layer. As we showed in the previous chapter, the rate of absorption depends on polarisation and is considerably larger for the X-polarisation state, where the transverse electric field is normal to the cladding-metal interface as shown in Figure 6-1. The higher-order cladding modes propagate with a large fraction of power in the cladding, and this fraction increases with increasing order. Thus the attenuation of these modes by the metal layer in the cladding should also increase with order, as will be demonstrated.

In the case of the Y-polarisation state, this means that the small fraction of incident power exciting the higher-order modes with the same orientation is attenuated much more quickly than the fundamental-mode power. Thus the coupling from the higher-order modes back to the fundamental mode is negligible and will be ignored.
However, in the case of X-polarisation state, the power in the fundamental mode is attenuated more rapidly than the power in the first few higher-order modes. Thus, beyond a critical polariser length, the ratio of power in the higher-order modes to power in the fundamental mode will increase, and, at the end of the polariser, more power will couple into the fundamental mode from the higher-order modes than from the fundamental mode of the polariser. It is this effect which reduces the extinction ratio of the polariser. Thus, for relatively short devices, the power in this polarisation of the fundamental mode of the outgoing regular fibre depends predominantly on the excited fundamental mode of the polariser. For relatively long devices, only higher-order modes which are not strongly absorbed will determine the power.

Cladding modes in standard single-mode fibres are normally absorbed by the lossy jacket surrounding the core. The length scale for this effect is large compared with the length scale for absorption by the metal layer. Accordingly, the effect of the jacket can be ignored and the cladding will be assumed to be surrounded by air. This is especially true for polarisers where the jacket has been removed in order to polish the cladding and add a metal layer.

7-2. Model

For reasons described in chapter 6, we model the fibre polariser by the slab polariser illustrated in Figure 7-3. The metal thickness can be assumed to be infinite as we showed in section 6-4-3. Our analysis covers a more general situation than that in the experiment by the inclusion of a buffer region between the cladding and the metal since this significantly increases the extinction ratio, as shown in section 6-6-4. The outer cladding on the opposite side to the metal is of finite thickness $d$, with air on the outside.
Figure 7-3. Transverse cross-section of the slab waveguide model with finite cladding. The notation is identical to that in Figure 6-5, with the origin of the co-ordinates at the centre of the core.

7-3. Eigen-Equation

The derivation of the eigen-equation for this six layer structure parallels that of the five layer structure in section 6-5, and is readily shown to be given by

\[
\frac{1 + K_0 \tan (2U)}{K_0 - \tan (2U)} = K_{bcl} \frac{W_{cl}}{U} \frac{n_{co}^2}{n_{cl}^2} \tag{1}
\]

where

\[
K_0 = -\frac{n_{cl}^2}{n_{co}^2} \frac{U}{W_{cl}} \left[ 1 + \frac{n_{cl}^2 W_{air}}{n_{air}^2 W_{cl}} \tanh (W_{cl}D) \right] \tag{2a}
\]
and \( D = d / \rho \) is the normalised thickness of the finite cladding. All other parameters are as defined in section 6-5. The eigen-equation for the TE modes is obtained by replacing \( n_{CO} = n_{CL} = n_{b} = n_{m} = 1 \).

The eigen-equation was solved as before, using a two dimensional Newton's method. Values of propagation constant for the first nine TM modes are given in Table 7-1. Parameters chosen here are representative ones of \( \lambda = 1.3 \ \mu m, n_{CO} = 1.45, n_{CL} = 1.435, \ n_{b} = 1.406, \ \rho = 1.35 \ \mu m, \ T_{CL} = t_{CL} / \rho = 1.5, \ T_{b} = t_{b} / \rho = 0.5, \ D = 10.0 \) and \( n_{m} = 0.505 + i \ 6.92 \) (copper at \( \lambda = 1.3 \ \mu m \)). For comparison, the imaginary part of the propagation constant of the \( TE_0 \) mode is \( 4.614 \times 10^{-1} \text{m}^{-1} \).

Note the relatively high attenuation of the \( TM_{-1} \) and the \( TM_0 \) modes compared to the higher-order modes. It is the relatively low attenuation of the latter which accounts for the extinction ratio limitation.
Table 7-1.

<table>
<thead>
<tr>
<th>Mode</th>
<th>$\beta_{\text{real}} \ (\text{m}^{-1})$</th>
<th>$\beta_{\text{imag}} \ (\text{m}^{-1})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM$_1$</td>
<td>$6.96637 \times 10^6$</td>
<td>$1.46266 \times 10^4$</td>
</tr>
<tr>
<td>TM$_0$</td>
<td>$6.97917 \times 10^6$</td>
<td>$1.75713 \times 10^3$</td>
</tr>
<tr>
<td>TM$_1$</td>
<td>$6.93424 \times 10^6$</td>
<td>$6.45198 \times 10^1$</td>
</tr>
<tr>
<td>TM$_2$</td>
<td>$6.92446 \times 10^6$</td>
<td>$2.50584 \times 10^2$</td>
</tr>
<tr>
<td>TM$_3$</td>
<td>$6.91025 \times 10^6$</td>
<td>$5.34527 \times 10^2$</td>
</tr>
<tr>
<td>TM$_4$</td>
<td>$6.89255 \times 10^6$</td>
<td>$6.43303 \times 10^2$</td>
</tr>
<tr>
<td>TM$_5$</td>
<td>$6.86934 \times 10^6$</td>
<td>$5.55341 \times 10^2$</td>
</tr>
<tr>
<td>TM$_6$</td>
<td>$6.84022 \times 10^6$</td>
<td>$5.90559 \times 10^2$</td>
</tr>
<tr>
<td>TM$_7$</td>
<td>$6.80761 \times 10^6$</td>
<td>$8.45734 \times 10^2$</td>
</tr>
</tbody>
</table>

7-3-1. Effect of a Finite Cladding

Figure 7-4 shows the imaginary part of the propagation constant for the first five TM modes as a function of core radius. Other parameter values are given above Table 7-1. Note that the thicknesses of all layers scale with $\rho$. The corresponding curves for the TE$_0$, TM$_0$ and TM$_1$ modes of the polariser with an infinite cladding are shown in Figure 6-13b).

The finite cladding enables the waveguide to support higher-order modes, each of which has an effective index whose real part is below the cladding index. The imaginary part of the propagation constant of the higher-order modes does not decrease to zero near the normal cutoff radius, but goes through a minimum and starts to increase as $\rho$ decreases. As can be seen from the curves and table, the lowest of the higher-order modes (TM$_1$ and TM$_2$) are the least lossy, with loss increasing with order. This can be understood by considering the way in which the modal fields spread out from the core as the radius
decreases.

Figure 7-4. Imaginary part of the propagation constant plotted against $p$ for the $TE_0$ mode and first five TM modes for the polariser with a finite cladding.

At core radii well above the cutoff radius of each higher-order mode, the propagation constants of the modes show a similar behaviour to the two lower-order TM and the $TE_0$ modes for a waveguide with an infinite cladding as shown in, for example, Figure 6-11. As the core radius decreases, the real part of the propagation constant decreases while the imaginary part increases as the field spreads from the core and the fraction of the field at the buffer-metal interface increases. As the core radius decreases further, the field spreads more into the cladding on the opposite side to the metal, while the metal prevents the field from spreading on its side. As a result, the amplitude of the field at the interface decreases and its loss decreases as the radius approaches the normal cutoff value. However, the finite cladding prevents the mode from being cut off and simultaneously prevents the spread of the field on this side. The amplitude of the field at the buffer-metal interface therefore increases again. Hence, the imaginary part of the propagation constant passes through a minimum and again starts to increase. This minimum can be seen for the higher-order modes for $1.0<p<1.5 \mu m$ and for the $TM_{-1}$ mode, which is the core-like mode near $p=0.5 \mu m$. As the radius decreases further, the field gradually spreads into the air.
region beyond the cladding and at very small radius values, corresponding to cladding V-values approaching cutoff, the curves for the imaginary part of the propagation constant peak again and the field at the buffer-metal interface starts to decrease.

The variation of the TE\textsubscript{0} mode is not as pronounced. As the core radius decreases, the imaginary part of the propagation constant goes through a point of inflection rather than a maximum and minimum, because of the reduced effect of the metal on TE modes, as discussed in section 6-3-2.

7-4. Power Flow and Absorption

7-4-1. Longitudinal and Transverse Power Flow

The modes of the polariser are attenuated as they propagate along the device because of absorption in the metal, where the refractive index is complex. In order to preserve the modal field pattern along the polariser, there must be a redistribution of power in the cross-section. Thus, while most of the power flow is parallel to the waveguide axis, there must also be transverse power flow towards the metal. The incoming waveguide is non-absorbing so the power flow in the transverse direction is zero. By continuity, the transverse power flow at the start of the polarizer must also be zero. This means that all modes, including the backward propagating ones, excited at the interface must superimpose in such a way as to cancel transverse power flow within individual modes.

An example of the converse effect occurs in the fibre coupler. The first two modes of the composite structure each propagate power parallel to the axis of the coupler, but the superposition of them gives a flow of power in the coupler cross-section, which is responsible for the beating phenomenon of these devices.
For the dielectric-metal waveguides, the longitudinal power flow density is given by the real part of the Poynting vector

\[ \frac{1}{2} \left( e_x h_y^* \right)_{\text{real}} \]  

(3)

where \( * \) denotes complex conjugate. Similarly, the transverse power flow density is

\[ -\frac{1}{2} \left( e_z h_y^* \right)_{\text{real}} \]  

(4)

Hence, the local direction of power flow relative to the transverse x-axis is given by the angle \( \Omega \) where

\[ \Omega = \arctan \left( \frac{\left( e_x h_y^* \right)_{\text{real}}}{\left( e_z h_y^* \right)_{\text{real}}} \right) \]

For a polarizer with \( \rho = 1.3 \mu m, T_b = 0.5, T_{c1} = 1.5 \) and \( n_b = 1.406 \), the direction of power flow in the dielectric at the buffer-metal interface is at an angle of 88.8° to the X axis, i.e. is very close to purely longitudinal, while in the metal the angle is -21.2°. In other words, power is flowing in the opposite direction to the mode, i.e. longitudinal power flow in the dielectric region is always in the positive z direction, whereas in the metal it is in the negative direction. Physically, power flow behaves as if the metal is not only drawing power from the upstream transverse direction, but also from the modal field farther downstream.

The average power flow density in the dielectric is much greater than in the metal, and the total longitudinal power flow given by
This change of direction in the longitudinal power flow at the interface can be anticipated just from the boundary conditions that $e_z$ and $h_y$ are continuous. For slab structures

$$e_x = \frac{\beta}{\frac{1}{2} h_y}$$

in each region, i.e. $e_x \propto h_y / n^2$. Thus, at the interface, $e_z$ and $h_y$ are continuous but $e_x$ changes sign since $n_m^2$ is essentially real and negative in the metal.

7-4-2. Equivalent Snell’s Law for Dielectric-Metal Interfaces

The unexpected refraction behaviour at the interface can be further understood by introducing complex angles of incidence and refraction and considering a modified Snell’s Law. Power flow is determined by the real part of the Poynting vector and is associated with the real angle $\Omega$ defined above. To determine the complex angles we use the complete Poynting vector.

$$\mathbf{P} = \frac{1}{2} \mathbf{e} \times \mathbf{h}^* = -\frac{1}{2} e_z h_y^* x + \frac{1}{2} e_x h_y^* z$$
Figure 7-5. A dielectric-metal interface showing the real part of the complex angle of incidence and refraction in the plane of incidence.

Consider the dielectric-metal interface shown in Figure 7-5. The complex angle of incidence is defined by

\[
\sin \theta_d = \frac{p_x}{\sqrt{p_x^2 + p_z^2}} = \frac{e_x}{\pm \sqrt{e_x^2 + e_z^2}}
\]

(8a)

and

\[
\cos \theta_d = \frac{-p_x}{\sqrt{p_x^2 + p_z^2}} = \frac{-e_z}{\pm \sqrt{e_x^2 + e_z^2}}
\]

(8b)

\(\pm\) is chosen so as to make the real part of \(\theta_d > 0\). The complex angle of refraction is defined likewise, choosing the same sign as for the angle of incidence.
If we assume that Snell's Law can be generalised, then the angles of incidence and refraction would be related by

\[ \sin \theta_d = \frac{n_m}{n_d} \sin \theta_m \]  

(9)

For interfaces between non-absorbing dielectric media in a waveguide, Snell's law is equivalent to the invariance of the propagation constant \( \beta \) over the cross-section\(^5\), ie.

\[ \beta = k n \cos \theta_Z \]

where \( \theta_Z \) is the angle between the wave vector and the z axis in a region of refractive index \( n \). We can regard equation 9 as a generalisation of this relationship to complex refractive indices if we replace \( \theta_Z \) by the complementary angles in Figure 7-5.

The complex angle of refraction is also given by the Poynting vector in the metal through equation 8. Table 7-2 shows the values of \( \sin \theta_d \) for the first three TM modes of the polariser. The values in the first column were obtained by determining the angle of refraction from the Poynting vector in the metal region. Equation 9 was then used to determine the angle of incidence. The values in the second column were determined from the Poynting vector in the dielectric region (buffer).

Table 7-2

<table>
<thead>
<tr>
<th>TM(_j) modal number, j</th>
<th>( \sin \theta_D ) from Snell's Law</th>
<th>( \sin \theta_D ) from Poynting vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>0.015699 - i 0.20625</td>
<td>0.015555 - i 0.20715</td>
</tr>
<tr>
<td>0</td>
<td>0.015711 - i 0.20623</td>
<td>0.015130 - i 0.20756</td>
</tr>
<tr>
<td>1</td>
<td>0.015725 - i 0.20629</td>
<td>0.015052 - i 0.20629</td>
</tr>
</tbody>
</table>
The good agreement between the two methods may not be fortuitous. It is well known that the modes on non-absorbing slab waveguides can be regarded as the superposition of two plane waves propagating at the same angle to the axis, in which case the invariance of the propagation constant and Snell's Law are identical. The extension to absorbing media through the use of plane wave attenuation is straightforward, and the interpretation in terms of complex angles is consistent with this approach. We now consider an example where the relationship is exact.

7.4.3. Surface Modes

We can repeat the procedure above for the surface mode of a dielectric-metal structure. As with the TM modes of the polariser, power flow parallel to the interface is positive in the dielectric and negative in the metal, with total power flow positive. Power flows towards the metal in the transverse direction and the \( x \) component of the real part of the Poynting vector is given by

\[
\frac{1}{2} \frac{k n_d^2}{|w_d|^2} \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} w_d \exp \left( \frac{1}{2} w_d \exp (-2 w_d \\text{real } x) \right) \quad x < 0, \text{ dielectric} \quad (10a)
\]

\[
\frac{1}{2} \frac{k n_d^2}{|w_d|^2} \left( \frac{\varepsilon_0}{\mu_0} \right)^{1/2} w_d \exp \left( -2 w_m \\text{real } x \right) \quad x > 0, \text{ metal} \quad (10b)
\]

where the \( w_j \) are defined in equation 6-1. The angles calculated from Snell's Law and the Poynting vector are identical and so Snell's Law in terms of the complex angles given by the Poynting vector, holds for a surface mode.
7-4-4. Derivation from the Eigen-Equation

Snell's law can be derived directly from the eigen-equation for a surface mode and the boundary conditions across the interface. The eigen-equation 6-4 can be written as

\[ \beta^2 = \frac{k^2 n_d^2 n_m^2}{n_d^2 + n_m^2} \]  

(11)

Rearranging and substituting for \( w_d \) gives

\[ \frac{n_d^2}{n_m^2} \frac{\beta^2}{w_d^2} = -1 \]  

(12)

At the dielectric side of the interface, the components of the electric field are \( e_d z = -i \) and \( e_d x = \beta/w_d \). Substituting this into equation 12, multiplying both sides by \( n_d^2 - n_m^2 \) and rearranging gives

\[ \frac{n_d^2 e_d^2}{e_d^2 x + e_d^2 z} = \frac{n_m^4 e_d^2 x}{n_d^4 e_d^2 x + n_m^4 e_d^2 z} \]  

(13)

The longitudinal field components and \( h_y \) are continuous across the interface, giving

\[ \frac{n_d^2 e_d x}{n_m^2} = e_m x \]

Substitution into equation 13 leads to
\[
\frac{n_d^2 e_d^2}{e_d^2 x + e_d^2 z} = \frac{n_m^2 e_m^2}{e_m^2 x + e_m^2 z}
\]  \quad (14)

Taking the square root of both sides and substituting the components of the complex Poynting vector for the field components reduces this equation to Snell's Law

\[
n_d \sin \theta_d = n_m \sin \theta_m
\]

The eigen-equation for a mode of the polariser is different from that of a surface mode because of the multiplicity of layers. However, the boundary conditions at each interface are the same which explains why Snell's Law holds approximately for the polariser.

**7.5. Modal Excitation Due to the Mismatch**

We are now in a position to examine the excitation of the higher-order modes because of the mismatch at the start of the polarising device and the re-excitation of the fundamental mode of the outgoing waveguide because of the mismatch at the end of the polariser.

At splice A in Figure 7-2 between the regular fibre and the polariser, there is a mismatch because of the metal layer. Thus the incident fundamental mode will excite both the fundamental core mode and higher-order cladding modes of the polariser. If the incident mode is a TE mode, the excited modes will be TE modes, and similarly for an incident TM mode. Because the only differences in the cross-sections of the two waveguides are the metal and buffer layers, it is intuitive that virtually all the incident power will go into the fundamental mode and only a small fraction will excite the cladding modes.

To quantify the excitation, we express the electric and magnetic fields of the polariser as a sum of orthonormal forward-propagating core and cladding modes.
\[ E_{\text{pol}} = \sum_{j=-1}^{N} a_j e_j \exp(\imath \beta_j z) \quad (15a) \]

\[ H_{\text{pol}} = \sum_{j=-1}^{N} a_j h_j \exp(\imath \beta_j z) \quad (15b) \]

where \( N+2 \) is the total number of bound modes, and \( a_j \) is the amplitude coefficient of the \( j \)th mode and is to be determined. The summation from \( j=-1,0,1,... \) is adopted for consistency with previously established notation. Coupling to the radiation field or to backward-propagating modes has been omitted. Whilst in practice there would be coupling to these, the fraction of incident power scattered and reflected would be minute, and would affect both the core and cladding modes. Since it is the relative power in the core and all cladding modes that is of prime concern, the slight error in ignoring power in the radiation field and backward-propagating modes can be safely neglected.

7.5.1. First Splice

Continuity at the interface at splice A requires the polariser fields to match the incident electric field \( E_{0i}=e_{0i}\exp(\imath \beta z) \), and magnetic field \( H_{0i}=h_{0i}\exp(\imath \beta z) \). Hence, if the interface corresponds to \( z=0 \), then equation 15 gives

\[ e_{0i} = \sum_{j=-1}^{N} a_j e_j \quad (16a) \]

and

\[ h_{0i} = \sum_{j=-1}^{N} a_j h_j \quad (16b) \]
where \( N+2 \) is the number of bound TM modes of the device. The number of TE modes is \( N+1 \) and the lowest value of \( j \) is 0. To determine the \( a_j \), we need an orthogonality condition. The usual conjugated form of orthogonality for orthonormal modes on non-absorbing waveguides is

\[
\frac{1}{2} \int_{A_{\infty}} e_j X h_k \cdot z \, dA = \begin{cases} 1 & j = k \\ 0 & j \not= k \end{cases} \quad (17a)
\]

where \( A_{\infty} \) denotes the entire cross-section. This is inapplicable to an absorbing waveguide and must be replaced by the unconjugated form

\[
\frac{1}{2} \int_{A_{\infty}} e_j X h_k \cdot z \, dA = \begin{cases} 1 & j = k \\ 0 & j \not= k \end{cases} \quad (17b)
\]

Multiplying equation 16a) by \( 1/2 \) \( h_k \) and applying the orthogonality condition of equation 17b) determines the coefficients

\[
a_k = \frac{1}{2} \int_{A_{\infty}} e_{0i} X h_k \cdot z \, dA \quad (18)
\]

The values of the coefficients, \( a_j \), for the first nine TM modes are calculated from equation 18 using the parameters in section 7-3 and a core radius of 1.35\( \mu \)m. Clearly, it is the core-like TM\(_0\) which is predominantly excited while the higher-order modes are excited to a much lesser extent.
Table 7-3

<table>
<thead>
<tr>
<th>Mode</th>
<th>$a_j$ real</th>
<th>$a_j$ imag</th>
</tr>
</thead>
<tbody>
<tr>
<td>TM-1</td>
<td>0.15168</td>
<td>0.270216</td>
</tr>
<tr>
<td>TM0</td>
<td>1.02390</td>
<td>-0.040031</td>
</tr>
<tr>
<td>TM1</td>
<td>0.08528</td>
<td>-0.168844</td>
</tr>
<tr>
<td>TM2</td>
<td>0.14246</td>
<td>-0.898544</td>
</tr>
<tr>
<td>TM3</td>
<td>0.14282</td>
<td>-0.019093</td>
</tr>
<tr>
<td>TM4</td>
<td>-0.07003</td>
<td>-0.099299</td>
</tr>
<tr>
<td>TM5</td>
<td>0.016905</td>
<td>-0.102757</td>
</tr>
<tr>
<td>TM6</td>
<td>0.066979</td>
<td>-0.056379</td>
</tr>
<tr>
<td>TM7</td>
<td>0.070258</td>
<td>0.0110873</td>
</tr>
</tbody>
</table>

In non-absorbing waveguides, the power in each orthonormal mode is $|a_k|^2$ and the total power flow is just the sum of these terms. However, with absorbing waveguides, more care is required. The power in each mode is given by

\[
\frac{1}{2} \left| a_k \right|^2 \left[ \int e_k \times h_k \cdot z \, dA \right]_{\text{real}}
\]

Hence the total power, $P_{pol}$, in the device will contain cross terms because the conjugated orthogonality condition does not apply.

\[
P = \left[ \sum_{j=1}^{N} \sum_{k=1}^{N} a_j \ast a_k \phi_{jk} \right]_{\text{real}}
\]

\[
= \sum_{k=1}^{N} |a_k|^2 \phi_{kk} \text{ real} + \sum_{j=1}^{N} \sum_{k>j} \left[ a_j a_k^\ast \phi_{jk} + a_j^\ast a_k \phi_{kj} \right]_{\text{real}}
\]
\[\phi_{jk} = \frac{1}{2} \int_{A_e} e_j(X) h_k^* \ast dA \quad (20b)\]

\[E(X,L) = \sum_{j=1}^{N} a_j e_j(X) \exp(-\beta_j \text{imag} L) \exp(i \beta_j \text{real} L) \quad (21)\]

The total electric field at the end of the device, \(z=L\), is

\[a_{out} = \sum_{j=1}^{N} b_{j0} \exp(i \beta_j L) \quad (22)\]

The coefficients \(b_{j0}\) are determined in a similar way to the coefficients \(a_j\). The electric field of the \(j\)th device mode can be expressed as the sum over all modes of the outgoing waveguide.
\[ a_j e_j = \sum_{l=0}^{M} b_{jl} e_{10} \quad (23a) \]
\[ a_j h_j = \sum_{l=0}^{M} b_{jl} h_{l0} \quad (23b) \]

where \( b_{jl} \) represents the contribution of the \( j \)th device mode to the \( l \)th outgoing mode and \( M+1 \) is the total number of core and cladding outgoing modes. To determine \( b_{j0} \), the contribution to the fundamental mode, we take the complex conjugate of equation 23b), cross multiply by \( e_{10} \) and apply the orthogonality condition of equation 17a) to give

\[ b_{j0} = b_j = a_j^* \phi_{0j} \]

where \( \phi_{0j} \) is defined in equation 20b). Equation 18 gives \( b_j = |a_j|^2 \), and equation 22 becomes

\[ a_{\text{out}} = \sum_{j=-1}^{N} |a_j|^2 \exp(i \beta_j L) \quad (24) \]

The power, \( P_{\text{out}} \), in the fundamental mode is given by

\[ P_{\text{out}} = \sum_{j=-1}^{N} \sum_{k=-1}^{N} b_{jk} \exp(i (\beta_j \text{real} - \beta_k \text{real}) L) \text{real} \quad (25) \]

The last factor represents the phase differences between the pairs of modes.

7-6. Results and Discussion

The theoretical attenuation of power, calculated from equation 25 and normalised to input
power, is plotted in Figure 7-6 in dB as a function of device length in centimetres for the TE and TM modes.

Figure 7-6. The solid curves show the theoretical attenuation of the modes of the slab polariser and the corresponding y-polarisation of the fibre as a function of device length. The dashed curve averages out the oscillations in the upper curve. The lower dashed curve shows the anticipated result for the Y-polarisation state of the corresponding fibre taking into account the circular geometry.

Diamonds and squares denote TM and TE modes, respectively. The oscillations in the upper solid curve arise because of accumulated phase differences between higher-order TM modes over the length of the polariser, and the dashed line averages out these oscillations.

For relatively short devices, less than a few millimetres, the power in the TM fundamental mode beyond the polariser depends predominantly on the attenuation of the corresponding device TM₀ mode. For devices longer than a few millimetres, all modes except the TM₁ and TM₂ modes are absorbed, and these alone determine the power beyond the device. Thus the slope of the curve for relatively short and long devices is determined essentially by these three modes. Between these two extremes, the determination of power loss
involves all the TM modes.

The characteristic behaviour in Figure 7-6 is qualitatively very similar to the experimental curves in Figure 7-1. The latter, however, do not display the fine-scale oscillations of Figure 7-6, which may be due to the limited number of measurements\(^2\). The attenuation of the TM mode in Figure 7-6 is approximately an order of magnitude greater than that of the corresponding X-polarisation in Figure 7-1. This can be accounted for by the matching of the core-cladding and dielectric-metal effective indices through the buffer layer. If the buffer layer is replaced by cladding material, the imaginary part of the propagation constant of the TE mode suffers only a relatively small change from 0.82 to 0.46.

In Figure 7-6, the loss of the TE mode is approximately 0.5 dB/cm, compared to the experimental measurement of around 3 dB/cm for the corresponding Y-polarisation in Figure 7-1. This difference can be attributed to the difference in geometry between the present slab model and the circular fibre. In the slab polariser cross-section, the electric field is plane polarised, whereas the field in the fibre cross-section is slightly curved\(^6\). This means that, at the cladding-metal interface in Figure 7-2, the nominally y-polarisation of the fundamental-mode field has both X and Y, or TM and TE components. The TM component is of order \(\Delta\) smaller than the TE component, but, according to Table 7-1, suffers an attenuation of around 150 dB/cm compared with 0.5 dB/cm for the TE component. Since \(\Delta = 0.01\), the TM component effectively adds 1.5 dB/cm to the attenuation, resulting in a total attenuation of about 2 dB/cm for the Y-polarised fundamental mode. The lower dashed line in Figure 7-6 denotes this attenuation, which is more in keeping with the experimental measurement.

There is also the possibility that if the X-polarisation state within the polariser is attenuated so that virtually only the Y-polarisation state is present at the end, the X-polarisation state of the fundamental mode in the outgoing fibre might be excited through the minor field components.
This can be illustrated by the polariser with a D-shaped region of liquid metal shown in Figure 6-3. Assuming the metal is perfectly conducting, there is no field over the D-shaped region. Since the field is continuous across the mismatch at splice B, initially, the total field in the outgoing fibre must be zero over the region abutting the metal. The excitation of the X-polarisation of the fundamental mode is given by the overlap integral

\[ a_0^{(x)} = \frac{1}{2} \int_{A_0} e_x^{(pol)} h_y^{(x)} \, dA \]

where subscripts denote the field component and superscripts pol and x denote the total field at the end of the polariser and the X-polarisation state of the fundamental mode of the outgoing fibre, respectively. From symmetry properties, \( e_x^{(pol)} \) is antisymmetric with respect to the x axis while \( h_y^{x} \) is symmetric. Thus the overlap integral vanishes, showing that the Y-polarisation states of modes within the polariser cannot excite the X-polarisation states of the modes of the outgoing fibre.

7-7. Conclusion

We have shown that the mismatch at the ends of a metal polariser, where it has been spliced inline, causes excitation of higher-order modes in the device. Those TM modes that are not highly attenuated re-excite the fundamental mode of the outgoing fibre, resulting in a limitation on the polarising ability. The polarising effect can be enhanced by the insertion of a buffer layer, and the effect of the circular geometry can be estimated from the planar model.

7-8. References


CHAPTER 8
Polarising Couplers

8-1. Introduction

Each time a light signal passes through an optical device, there is a loss of power. This is due to such things as tapering, bending, surface roughness, etc. Collectively, the effect of all these nonuniformities is to introduce excess loss, i.e., the difference in power between the light entering and leaving the device. For devices in series, the excess loss is cumulative, so that if two devices can be combined into a single device, the total excess loss could be reduced.

This chapter will examine two such devices which combine a coupler and a polariser into a polarising coupler. With a signal in one input port, polarised light from the output ports can be obtained. The first device consists of two metal-clad polarisers joined so that the metal layer forms a common interface. As will be shown, it is possible to obtain equal intensities of polarised light from the two output ports irrespective of the source wavelength. However, the total loss of power from the device will depend on the device length. The second device consists of two metal-clad polarisers joined so the metal layers are symmetrically placed on the outside of the device. In this case the TM-polarised input power is absorbed, causing at least a 50% loss in total power. The ratio of the remaining power in each output port has the usual sinusoidal wavelength and device length dependence. Once again a slab waveguide model will be considered. Both devices have potential application in integrated optics, and there are corresponding fibre devices.

8-2. Polarising Coupler With a Central Metal Layer

This type of polarising coupler consists of a buffer-metal-buffer layer sandwiched between the cores of a typical single-mode coupler as shown schematically in Figure 8-1. In
principle, this device can function as a 3dB splitter that produces equal amounts of power in the output ports, with the added advantage that the fields are polarised and out of phase.

![Cross-section of a polarising coupler](image)

Figure 8-1. Cross-section of a polarising coupler fabricated by inserting a buffer-metal-buffer structure between the cores of a typical coupler. The outer cladding layers are assumed to be infinitely thick.

As we know, provided the metal layer is sufficiently thin, then the symmetric TE₀ mode becomes much lossier than the antisymmetric mode. If the device is long enough, power in the TM modes together with power in the symmetric TE₀ mode is lost through absorption and only power in the antisymmetric TE₀ mode is present in the output ports. This mode has equal amounts of power in each core region, but the fields are of opposite sign. This is equivalent to the fields being out of phase.

8-2-1. Analysis

The analysis of the symmetrical coupler can be approached in two ways. The first is to
consider the normal modes of the complete structure and examine the superposition, or beating, of those modes that are excited. The second is to look at the component waveguides and consider coupling of power between them by solving a set of coupled mode equations. The latter approach is not appropriate because of the large variation in refractive index between the dielectric and metal, so the former approach will be followed here.

8-2-2. Modal Fields and the Eigen-Equation

For reasons expressed in chapter 6, we analyse the corresponding slab waveguide to reveal the features of the polarising coupler. The transverse fields of the modes of the polarising coupler will be symmetric and antisymmetric (even and odd) because of the symmetry of the structure. If we choose the origin on the symmetry axis and let $\psi = e_z$ for TM modes, i.e. the longitudinal components, and $\psi = h_z$ for TE modes, then for $X > 0$

$$\psi = \begin{cases} 
\sinh (MX) & \text{even modes} \\
\cosh (MX) & \text{odd modes} \\
A \sinh (B(X - \frac{T_m}{2})) + C \cosh (B(X - \frac{T_m}{2})) & \text{buffer} \\
E \sinh (W(X - G)) + F \cosh (W(X - G)) & \text{cladding} \\
H \sin (U(X - D)) + J \cos (U(X - D)) & \text{core} \\
K \exp (-W(X - D - 2)) & \text{outer cladding}
\end{cases}$$

(1)

where $G = T_m/2 + T_b$ and $D = G + T_{cl}$ are in terms of the normalised thickness of the cladding, buffer and metal layers, i.e. $T_{cl} = t_{cl}/\rho$, $T_b = t_b/\rho$ and $T_m = t_m/\rho$. $A$, $C$, $E$, $F$, $H$, $J$ and $K$ are constants to be determined and the modal parameters $M = W_m$, $B = W_b$, $W$ and $U$
are defined in equation 1-6 and 7. The non-zero transverse field components are given in terms of $e_z$ or $h_z$ by equation 1-13a).

The symmetry conditions of the transverse field components lead to $\psi(0) = 0$ for symmetric modes and $\psi'(0) = 0$ for antisymmetric modes - these are satisfied by the above. The remaining boundary conditions described in chapters 1 and 6 determine the constants together with the eigen-equation.

For symmetric TM modes the eigen-equation is given by

$$\frac{-\frac{n_{co}^2}{n_{cl}^2} W}{1 + \frac{n_{co}^2}{n_{cl}^2} W \tan(2U)} + \tan(2U) = \frac{L \tanh(W T_{cl}) + 1}{\frac{n_{cl}^2}{n_{co}^2} U \left[ L + \tanh(W T_{cl}) \right]}$$

where

$$L = \frac{W \frac{n_{m}^2}{n_{cl}^2}}{\tanh(M \frac{T_{m}}{2}) + \frac{B}{M} \frac{n_{b}^2}{n_{cl}^2} \tanh(B T_{b})} + \frac{W \frac{n_{b}^2}{n_{cl}^2} \tanh(M \frac{T_{m}}{2}) \tanh(B T_{b})}{\tanh(M \frac{T_{m}}{2}) + \frac{B}{M} \frac{n_{m}^2}{n_{b}^2} \tanh(B T_{b})}$$

The eigen-equation for the antisymmetric TM modes is obtained by replacing $\tanh(M T_{m}/2)$ by $\coth(M T_{m}/2)$. The eigen-equations for the corresponding TE modes are obtained by setting $n_{co} = n_{cl} = n_{b} = n_{m} = 1$. As for the single polarisers, a two dimensional Newton's method is used to determine the solution of equation 2 for the propagation constant.
8-2-3. Results

The real and imaginary parts of the propagation constant are plotted as a function of metal layer thickness, $t_m$, for $p=1.5$, 1.0 and 0.7 $\mu$m in Figures 8-2, 8-3 and 8-4, respectively, for the TE$_0^\pm$, TM$_0^\pm$ and TM$_1^\pm$, where + denotes the symmetric and - denotes the antisymmetric modes. In each case the buffer index is $n_b =1.4137$, and the normalised thicknesses are $T_{cl}=1.5$ and $T_b=0.5$. 
Figure 8-2. a) real and b) imaginary parts of the propagation constant as a function of the thickness of the metal layer for $\rho=1.5\mu$m.
Figure 8-3. a) real and b) imaginary parts of the propagation constant as a function of the thickness of the metal layer for $\rho=1.0\mu m$. 
Figure 8-4. a) real and b) imaginary parts of the propagation constant as a function of the thickness of the metal layer for $\rho=0.7\mu m$.

8-2-4. Discussion

It is clear from the plots of both the real and imaginary parts of the propagation constants in
the figures above, that as the thickness of the metal layer increases, the propagation constants of each pair of symmetric and antisymmetric TE and TM modes approach a common value, equal to that of the corresponding mode of the polariser in chapter 6. In other words, provided the metal is sufficiently thick, the propagation constants of the symmetric and antisymmetric modes are virtually identical to those of the corresponding modes of the component polarisers. Thus, in this limit, the device operates as a polariser in the same manner as the component single polariser.

The propagation constants of the symmetric and antisymmetric modes are virtually identical, resulting in a very large beat length and so the attenuation will effectively account for all the power in the device before any coupling can occur. For example, for the values of $n_b$, $T_{cl}$ and $T_b$ given above, $\rho=0.7\mu m$ and $t_m=0.11\mu m$, the propagation constants of the symmetric and antisymmetric TE modes are given by $\beta^+ = 6.952977 \times 10^6 + i 14.31524$ and $\beta^- = 6.952963 \times 10^6 + i 9.26583$, respectively. The units are $m^{-1}$. The real parts of the propagation constants give a beatlength of 44.9 cm. Over this distance, the power in each of the modes is attenuated to a fraction of $2.6 \times 10^{-6}$ and $2.43 \times 10^{-4}$, respectively, of the initial power, i.e. effectively total loss of power.

The above features can be understood by considering the power distribution of the modes in the cross-section. The field of each mode decreases exponentially in the metal so that for a sufficiently thick metal layer, the modal fields will be virtually identical to the sum and difference of the fields of the modes of the component waveguides. This is illustrated schematically, since the fields are complex, in Figure 8-5.

A comparison can be made with a regular fibre coupler where the composite fields are only approximately the sum and difference of the fields of the component waveguides, as shown by Figure 8-6. As a result, the propagation constants of the composite modes differ significantly from that of the component fibres, resulting in a beatlength which is much smaller than that of the polarising coupler.
Figure 8-5. Components of the fields of the symmetric and antisymmetric TE modes of the polarising coupler (schematic). The peak at the dielectric-metal interface has been purposely exaggerated.

Figure 8-6. Modal fields of a regular fibre coupler (schematic).
Physically, in an evanescent coupler, the coupling phenomenon can be interpreted as the tail of the field in the one fibre exciting the field of the other fibre, resulting in a transfer of power between the cores\textsuperscript{2}. However, the thick metal layer in the polarising coupler effectively isolates the cores of the component waveguides so that there is no interaction, and as a result, the device cannot function as a coupler.

At the other limit, $t_m \to 0$, i.e. vanishing metal thickness, the values of $\beta_{\text{real}}$ for the TE\textsubscript{0} modes and one each of the symmetric and antisymmetric TM modes, approach the propagation constants of the corresponding lowest order modes of a non-absorbing coupler. For $\rho=1.0$ and 0.7$\mu$m, the symmetric TM\textsubscript{1} mode reduces to the symmetric TM\textsubscript{0} mode, and the antisymmetric TM\textsubscript{1} ceases to propagate as $\beta_{\text{real}} \to 0$, while the symmetric TM\textsubscript{0} mode is cut off.

Between these two limiting cases, both coupling and absorption occur to a varying extent. As the core radius decreases, the fields spread into the cladding and the losses of the TE modes and the core-like TM modes increase.

As the metal thickness decreases, the losses of both the symmetric TE\textsubscript{0} and TM\textsubscript{0} modes increase. This is because, as the metal thickness decreases, there is a larger fraction of the field of the symmetric modes present in the metal than for the corresponding antisymmetric mode. As the thickness decreases further, a larger fraction of the total power is retained closer to the core so that less is present at the metal interface, and the loss decreases to zero as the metal vanishes. The latter is also true for the antisymmetric TM\textsubscript{0} mode, while for the corresponding TM\textsubscript{1} mode, the power distribution peaks at the two metal-dielectric interfaces resulting in extremely large losses as the metal thickness decreases.

8-2-5. Polarising Coupler

As shown above, it is not possible to operate this device as both a polariser and a coupler in
which power transfers between the two cores. However, at sufficiently thin metal layers, it is possible to eliminate all but the antisymmetric TE\(_0\) mode. This suggests a polarising coupler that is wavelength independent. Over a short distance, the TM modes will be attenuated so we will only consider the TE modes. At the beginning of the coupler, the TE component of the incoming field excites the symmetric and antisymmetric modes equally since the coupler is symmetrical. The power in the even mode will be absorbed leaving only power in the odd mode. At the end of the coupler, the power in this mode is split equally between the two cores, and furthermore, since the absorption in the coupler is relatively insensitive to wavelength, the split is likewise wavelength independent. Such a device would constitute a polarising 3dB splitter. Unfortunately, the loss in total power is high, i.e. at least 75%. However, there is the advantage that the fields in the output ports are completely out of phase.

For example, for the values of \(n_b\), \(T_{cl}\) and \(T_b\) given above, \(\rho=0.7\mu m\) and \(t_m=0.035\mu m\), the propagation constants of the symmetric and antisymmetric TE modes are given by \(\beta^+=6.953175\times10^6 + i\, 39.375\) and \(\beta^-=6.9529443\times10^6 + i\, 1.3968\), respectively. For a coupler length of 3cm this gives losses of 8% and 91% from the antisymmetric and symmetric TE\(_0\) modes, respectively, while the core-like TM modes are virtually totally attenuated. For a slightly longer length of 4cm, the corresponding losses of the TE modes are 10.5% and 96%.

8.3. Polarising Coupler With External Metal Layers.

We now consider a polarising coupler which is identical to the one considered above, except that the central metal and buffer layers have been replaced by symmetric buffer and metal layers on the outside of the device, as shown in Figure 8-7.
The thickness of the metal layer is effectively infinite. With the metal on the outside instead of between the cores, the component waveguides cannot be isolated electromagnetically, and the modes are more similar to those of a regular fibre coupler.

8-3-1. Modal Analysis and the Eigen-Equation

The symmetry of the device requires the transverse fields of the modes, i.e. $\psi'$, to be symmetric or antisymmetric (even and odd) about the axis of symmetry of the structure. If we set $\psi = e_z$ for TM modes and $\psi = h_z$ for TE modes, then for $X \geq 0$
\[ \psi = \begin{cases} 
\sinh(WX) & \text{even modes} \\
\cosh(WX) & \text{odd modes} 
\end{cases} \]

\[
\begin{align*}
A \sin(U(X - D)) + C \cos(U(X - D)) & \text{ core} \\
E \sinh(W(X - D - 2)) + F \cosh(W(X - D - 2)) & \text{ cladding} \\
H \sinh(B(X - D - 2 - T_{cl})) + J \cosh(B(X - D - 2 - T_{cl})) & \text{ buffer} \\
K \exp(-M(X - D - 2 - T_{cl} - T_b)) & \text{ metal} 
\end{align*}
\]

(3)

where \( D = d/\rho \) is the normalised distance from the axis of symmetry of the device to the inner core-cladding interface, and \( T_{cl} \) and \( T_b \) are the normalised thicknesses of the cladding and buffer layers respectively. Remaining parameters are defined below equation 1.

Applying boundary conditions discussed in the previous section leads to the eigen-equation, which, for symmetric TM modes, is given by

\[
\begin{align*}
\frac{n_{cl}^2 B}{n_b^2 W} \left[ L + \tanh(WT_{cl}) \right] & = - \left[ 1 + \frac{n_b^2}{n_m^2} \frac{M}{B} \tanh(BT_{tb}) \right] \\
& \left[ \tanh(BT_{tb}) + \frac{n_b^2}{n_m^2} M \frac{B}{B} \right]
\end{align*}
\]

where
The corresponding eigen-equation for the antisymmetric TM modes is obtained by replacing \( \tanh(WD) \) by \( \coth(WD) \). The eigen-equations for the symmetric and antisymmetric TE modes are obtained by setting \( n_{co} = n_{cl} = n_b = n_m = 1 \) in the corresponding eigen-equations for the TM modes. Each of the four equations was solved using a two dimensional Newton's method.

8-3-2. Results

The real and imaginary parts of the propagation constant as a function of \( D \) are shown in Figure 8-8 with \( \rho = 1.2 \mu m \), \( n_b = 1.406 \), \( T_{cl} = 1.5 \) and \( T_b = 0.5 \), for each pair of \( TM_i \) and \( TE_q \) modes. As shown by Figure 6-13, at large core radii the \( TM_0 \) and \( TM_1 \) modes of the single polariser are more like the modes of the component slab waveguide and dielectric-metal structure, respectively. At small core radii this is reversed. For intermediate values, \( 1.0 \mu m < \rho < 1.5 \mu m \), both modes are approximately a linear combination of the component modes. However, at \( \rho = 1.2 \mu m \) the \( TM_1 \) is more like the core mode. For this reason, only the results for the \( TM_1 \) modes of the coupler will be presented in this section. The results for the \( TM_0 \) modes are similar.

Figure 8-8a) shows the real parts of the propagation constant of the symmetric and antisymmetric modes separating as the core-to-core separation decreases. Thus, the beat length for both polarisations decreases, and the device operates like a regular coupler. Plots of the imaginary part of the propagation constant in Figure 8-8b) show a similar behaviour with reducing core separation.
Figure 8-8. Real a) and imaginary b) parts of the propagation constant as a function of D, the normalised distance from the axis of symmetry to the inner core-cladding interface, for the symmetric and antisymmetric TM$_{1}$ and TE$_{0}$ modes of the polarising coupler with metal on the outer cladding.

However, compared with the corresponding curves for the coupler with the central metal layer in Figure 8-4, it is the antisymmetric and not the symmetric mode of each pair that is lossier. This is to be expected, as the fields of the antisymmetric modes are relatively more
spread out than the corresponding symmetric modes and are therefore more sensitive to the metal layers on the edge of the coupler.

The polarising action of this coupler is stronger than that of the central-metal coupler by two orders of magnitude. For example, at $D=1$, $\beta_{TM imag}/\beta_{TE imag} = 7 \times 10^3$ and the beat length associated with the symmetric and antisymmetric TE modes is thus

$$z_b = \frac{2\pi}{(\beta^+ - \beta^-)_\text{real}} = 8.73 \times 10^{-4} \text{ m}$$

This is much less than the length of a normal coupler. Over this beat length, the loss of power from the TE modes is less than 0.2%, while the power remaining in the TM modes is approximately $5.0 \times 10^{-4}$ %, ie. it is virtually completely lost.

8.3.3. Higher-Order Modes

An ordinary evanescent single-mode coupler consists of two cores, each of which in isolation and surrounded by an infinite cladding, is single-moded. The composite "single-mode" structure supports two modes - the first symmetric, or fundamental mode, and the first odd mode. All other modes are cut off.

The introduction of the metal layers which bound the cladding makes the coupler multi-moded because of the very large differences between the dielectric constant of the cladding, $n_{cl}^2=2$ for silica glass, and the real part of the dielectric constant of the metal, typically $(n_m^2)_{\text{real}}=-50$.

Apart from the TE$_0$, TM$_{-1}$ and TM$_0$ modes, all other higher-order modes are cladding modes with the real part of the effective index below the cladding index. The propagation constant for the symmetric and antisymmetric TM$_1$ modes, the least lossy of the
higher-order modes, is shown in Figure 8-9 as a function of core radius for D=1.5. Figure 8-9a) shows that the real part of the propagation constant is less than the cutoff value \( k_{nc1} = 6.987 \times 10^6 \), while Figure 8-9b) shows that the losses are greater than that of the corresponding mode of the single polariser with a finite cladding, discussed in chapter 7.

a)

![Graph showing \( \beta_{\text{real}} \) as a function of core radius.]

b)

![Graph showing \( \beta_{\text{imag}} \) as a function of core radius.]

Figure 8-9. Propagation constant for the symmetric and antisymmetric TM\(_1\) modes as a function of core radius. a) shows the real part and b) the imaginary part. The normalised thickness of the cladding between the cores is fixed at D=1.5 with all other parameter values given in Figure 8-8.
However, as expected, they are below the loss values for the $TM_{11}$ modes in Figure 8-8b). For fixed core radius, the field near the metal interface increases as $D$ decreases and the loss will be greater.

8-4. Conclusion

In one method of fabricating fibre couplers, two single-mode fibres are twisted, heated and drawn as they fuse. The first type of polarising coupler considered, which has the metal layer between the component waveguides, could not be fabricated using this process, but, in principle, could be fabricated from a dual-core fibre with a rectangular slot filled with liquid metal, by analogy with the polariser illustrated in Figure 6-1b). However, it may be of more value as a device for integrated optics. The second device, with metal layers on the outside, could also be appropriate for integrated optics. The fibre analogue could also be fabricated by carefully adding metal layers to a regular coupler. The cross-section of the corresponding fibre polarising coupler is illustrated in Figure 8-10.

![Figure 8-10. A fibre polarising coupler.](image)

This device would have similar loss curves to those in Figure 8-8b). The difference
between the curves for the X and Y-polarisation states would be less than those of the TM and TE modes because of absorption through the minor field component of the Y-polarisation state, as discussed in chapter 7. Since there is a significantly large dielectric-air interface, the higher-order modes could cause a limitation on the extinction ratio for reasons discussed in chapter 7.

A fibre polarising coupler similar to Figure 8-10 has been fabricated. In this device four parallel fibres, with the outer two coreless, were fused and tapered to form a coupler, with an approximately elliptic cross-section. A metal layer was then deposited around the cladding. The fabrication process is shown schematically in Figure 8-11. The two outer "dummy" fibres were present to facilitate differential absorption. Both polarisation states suffer absorption by the metal layer mainly through their major field components. As the metal is further from the cores along the X axis, it is the X-polarisation state which suffers less absorption. The extinction ratio in this case was 15dB at the end of the device while the excess loss of the X-polarisation state was 2.8dB.

Figure 8-11. The drawing process of the four-fibre polarising coupler.
8-5. References.


"It is finished" - John 19:30, The Modern Language Bible.