Essays on Dynamic Macroeconomics and Monetary Policy

Jiao Wang

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This thesis is my original research work under the direction of my supervisory committee. The first research paper is conducted jointly with Dr Ran Li from Peking University, to which I am the corresponding author and responsible for framing the research question, checking the mathematical work done by Ran, writing and revising the paper at different stages, which is equivalent to more than 7,400 words. The second research paper is conducted jointly with Professor Ippei Fujiwara from Keio University and ANU, to which I am the corresponding author and responsible for conducting the modeling and mathematical work, writing and revising the paper at different stages, which is equivalent to 8,800 words. The third research paper is conducted solely by me.

Signature: Jian Wang

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Abstract

This thesis investigates monetary policy within the New Keynesian framework in dynamic macroeconomics. It includes three original research papers. The first paper examines the rules and transmission mechanisms of monetary policy in one of the fast growing economies in the 21st century, China, by extending a standard New Keynesian dynamic stochastic general equilibrium model with financial frictions and investment-specific shocks in order to capture some of the Chinese characteristics and applying a Bayesian estimation strategy to real-time data. It offers a new way of empirically examining the rule of China’s monetary policy and indicates a structural break of the neutral technology development that may have caused the slowing down of GDP growth since 2010.

The second paper revisits optimal monetary policy in open economies, in particular, focusing on the noncooperative policy game under local currency pricing in a theoretical two-country dynamic stochastic general equilibrium model. Quadratic loss functions of noncooperative policy makers and welfare gains from cooperation are obtained in the paper. The results show that noncooperative policy makers face extra trade-offs regarding stabilizing the real marginal costs induced by deviations from the law of one price under local currency pricing. As a result of the increased number of stabilizing objectives, welfare gains from cooperation emerge even when two countries face only technology shocks, which usually leads to equivalence between cooperation and noncooperation. Still, gains from cooperation are not large, implying that frictions other than nominal rigidities are necessary to strongly recommend cooperation as an important policy framework to increase global welfare.

The third paper focuses on the noncooperative policy game specified by choice of policy instrument for implementing optimal monetary policy in a two-country open-economy model similar to the one in the second paper. It examines four options of policy instruments including the producer price index inflation rate, the consumer price index inflation rate, the import price inflation rate and the nominal interest rate. It shows that choosing different policy instruments generally leads to different equilibria and,
in particular, choosing the nominal interest rate results in equilibrium indeterminacy. In addition, the welfare ranking of these policy instruments depends on a country’s degree of openness which is measured as the weight assigned to imported goods in the consumers’ utility function. In less open countries, domestically produced goods carry a relatively higher weight in the consumers’ utility function. For these less open countries, choosing the producer price index inflation rate induces a larger welfare cost from noncooperation than choosing the consumer price index inflation rate would. Choosing the consumer price index inflation rate in turn causes a larger welfare cost than choosing the import price inflation rate. Conversely, the reverse is true when countries are more open. This result sheds light on the important role that policy instrument choice plays in determining the equilibrium outcomes, to which policy makers should pay special attention when implementing optimal monetary policy under noncooperation.
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Chapter 1

Introduction

Jordi Galí (2015) offers a nice overview of current monetary policy analysis in his book, *Monetary Policy, Inflation and the Business Cycle*: Over the past two decades, monetary economics has been among the most fruitful research areas within macroeconomics. The efforts of many researchers to understand the relationship among monetary policy, inflation and the business cycle have led to the development of a framework—the so-called New Keynesian model—that is widely used for monetary policy analysis. The central mission of monetary policy analysis is to understand the links between monetary policy and the aggregate performance of an economy, that is, the transmission mechanisms of monetary policy, and to determine the objectives of monetary policy and how the latter should be conducted in order to attain those objectives. The present thesis contributes to the New Keynesian literature with a deeper understanding of both the transmission mechanisms and the objectives of monetary policy.

In a standard market economy, a central bank conducts its monetary policy by adjusting its policy interest rate in response to aggregate conditions of the economy according to its policy function.\(^1\) Changes in the interest rate then have an influence on the valuation of financial assets and their expected return, as well as on the consumption and investment decisions of households and firms. Those decisions can in turn have consequences for GDP growth, employment and inflation. The transmission mechanisms of monetary policy shocks in such a standard economy are well established in the New Keynesian literature. What is not so clear, on the other hand, are the transmission mechanisms of monetary policy in a non-standard market economy with various distortions and unidentified monetary policy rules resulting from unfinished market reform and state intervention. These conditions are all observable in one of the

\(^1\) A standard market economy under the New Keynesian framework is an economy where there are no distortions other than monopolistic competition in goods market and short-term nominal rigidities.
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fastest growing emerging market economies (hereafter, EMEs): China. Understanding the links between monetary policy and the aggregate conditions in China thus calls for extensions of the standard New Keynesian models to incorporate these factors. This is exactly what Chapter 2 does.

Specifically, Chapter 2 looks into the transmission mechanisms of China’s monetary policy and the main source of fluctuations in its business cycle, provided that the monetary policy rule is known. To this end, a standard New Keynesian dynamic stochastic general equilibrium (hereafter, DSGE) model is extended with financial frictions and investment-specific technology shocks to capture some of the characteristics in China’s macroeconomy. Moreover, since there is no consensus on the form of the policy rules that the People’s Bank of China (hereafter, PBoC) has been employing, a hybrid form of monetary policy function is proposed in this chapter and coefficients of the variables in the function are estimated by applying a Bayesian estimation strategy to Chinese data.\(^2\) This offers a new way of examining the rule of China’s monetary policy for future researchers. The results show that the PBoC indeed has been employing a hybrid monetary policy rule over the sample period from 2001 to 2017 by adjusting the policy rate in response to inflation, output growth and real money growth. This finding suggests that assuming a pure Taylor-type rule or a pure quantity-type rule for the Chinese monetary policy, as a number of past studies did, may induce misspecification problems. It also finds that there is a structural change in the policy rule before and after the onset of the Global Financial Crisis, with a larger response of the PBoC to real money growth in the post crisis period, everything else equal. Given the estimated monetary policy function, variance decompositions show that neutral technology shocks and preference shocks are the main drivers of fluctuations in output and consumption, while neutral technology shocks, investment-specific technology shocks, markup shocks and marginal efficiency of investment shocks are the key sources of the variation in investment in China. Finally, historical decomposition indicates that a structural break in the neutral technology development may have caused the slowing down of GDP growth in China since 2010, to which policy makers should pay special attention.\(^3\)

Central banks in the world, either the Federal Reserve Bank or the PBoC, do not conduct their monetary policy in an arbitrary or whimsical manner. Their decisions

\(^2\) The proposed monetary policy function is hybrid in the sense that it is in form of a combination of the pure Taylor-type rule and the pure quantity-type rule. The latter two policy rules are used extensively in the literature for standard New Keynesian model economies but not directly applicable to the Chinese economy.

\(^3\) The neutral technology refers to the technology applied to production of consumption goods or final goods, differentiating from the investment-specific technology, which is applied to investment goods sector.
are meant to be purposeful, that is, they seek to attain certain objectives, while taking as given the constraints posed by the workings of domestic as well as foreign private agents. What should be the objectives of monetary policy and how the latter should be conducted in order to attain those objectives? The New Keynesian literature is advantageous in answering these questions due to the inclusion of microfoundations of private agents, which makes households’ welfare a natural objective. In normative dimensions, the objectives of monetary policy are thus derived from the maximization of households’ welfare and central banks as Ramsey social planners conduct optimal monetary policy.

Over the years there have been many studies investigating optimal monetary policy in the context of both closed and open economies under different settings, such as under cooperation or noncooperation, producer currency pricing (hereafter, PCP) or local currency pricing (hereafter, LCP), and with or without home bias. As a result, our understanding of how monetary policy should be conducted, especially in an interconnected world, is deepened. There is, however, one last missing piece, which has not yet been analyzed in a theoretical two-country DSGE model. That is, how the optimal noncooperative monetary policy under LCP should be conducted, or whether there is any gain from cooperation under LCP. Chapter 3 is dedicated to addressing these questions.

By solving the equilibrium conditions under monopolistic competition and applying second-order approximation to the households’ welfare function, quadratic loss functions for both cooperative and noncooperative policy makers are obtained in Chapter 3. The loss functions show that under LCP noncooperative policy makers naturally aim to stabilize variables whose fluctuations are to be minimized by cooperative policy makers, and that they also seek to stabilize fluctuations in the real marginal costs that firms face when setting prices in both domestic and export markets. These additional objectives are unique to the noncooperative game and are therefore the sources for potential gains from cooperation. The welfare gain from cooperation is then computed by solving the nonlinear Ramsey problem. Within the reasonable range of parameter calibration and with only technology shocks, the welfare gain from cooperation is in general nonzero but the size is very small. This finding implies that frictions other than nominal rigidities are necessary to strongly recommend cooperation as an important policy framework to increase global welfare.

In the welfare comparison between cooperation and noncooperation in Chapter 3,

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4Prior to the New Keynesian model, the traditional literature based on the Mundell-Fleming-Dornbusch model assigned ad hoc objectives to monetary authorities for conducting monetary policy which, as Obstfeld and Rogoff (1995) argue, may lead to misleading policy implications.
the noncooperative game is defined as one in which the domestic policy maker conducts its optimal monetary policy while taking as given the entire path of the foreign policy maker’s producer price index (hereafter, PPI) inflation rate. This is an open-loop Nash equilibrium.

A natural question that follows is whether a different Nash game, that is, a different choice of policy instrument, would affect the equilibrium outcome and welfare under noncooperation. Chapter 4 addresses this research question in a two-country DSGE model similar to the one in Chapter 3 by considering four options of policy instruments, including the PPI inflation rate, the consumer price index (hereafter, CPI) inflation rate, the import price inflation rate and the nominal interest rate. Impulse response results show that choosing different policy instruments indeed leads to different equilibria and, in particular, choosing the nominal interest rate induces equilibrium indeterminacy. In addition, the welfare ranking of these policy instruments depends on the parameter measuring a country’s degree of openness, which is the weight assigned to imported goods in consumers’ utility function. Less open countries are defined as those where domestically produced goods carry a relatively higher weight in the consumers’ utility function. For these less open countries, selecting the PPI inflation rate as policy instrument induces a larger welfare cost from noncooperation than choosing the CPI inflation rate would. Choosing the CPI inflation rate in turn causes a larger welfare cost than choosing the import price inflation rate. Conversely, when countries are more open, choosing the PPI inflation rate as policy instrument leads to a smaller welfare cost from noncooperation than choosing the CPI inflation rate does, which in turn generates a smaller welfare cost than the noncooperative regime specified by the choice of the import price inflation rate. This result sheds light on the important role that policy instrument choice plays in determining the equilibrium outcomes, to which policy makers should pay special attention when implementing optimal monetary policy under noncooperation.
Chapter 2

A Structural Investigation of the Chinese Economy with A Hybrid Monetary Policy Rule

2.1 Introduction

Since around 2010, China has been experiencing a gradual slow-down of GDP growth from an average of 10 per cent over the thirty years to 2010 to 6.7 per cent in 2016.\(^1\) Chinese President Xi Jinping described this as the ‘new normal’ of the Chinese economy in May 2014.\(^2\) The slowing down of China’s economic growth has attracted a great deal of attention among policymakers and scholars but so far there has been no consensus on its sources. This motivates us to conduct a structural investigation of the Chinese economy to better understand the sources of business fluctuations in China, especially fluctuations in output.

There is one puzzle that needs to be solved before we can proceed with the structural investigation. China’s monetary policy is the puzzle. On the one hand, it has been assigned too many objectives—maintaining price stability, promoting economic growth, supporting employment and achieving balance of payments equilibrium. On the other hand, there is no consensus on the form of the policy rules that the PBoC has been employing, let alone whether such policy rules are able to achieve all the said objectives. Based on the limited communications between the PBoC and the public, some suspect

\(^1\)The GDP growth rate in 2011, 2012 and 2013 are 9.5 per cent, 7.7 per cent and 7.7 per cent, respectively. Source: the National Bureau of Statistics of China.

\(^2\)See, for example, the Xinhua news report titled Xi’s ‘new normal’ theory. http://news.xinhuanet.com/english/china/2014-11/09/c_133776839.htm
a money rule, by which the PBoC manages aggregate money supply, has been the major monetary policy rule, while some speculate an analogous Taylor-type price rule may be in action. Without a well defined monetary policy rule, it will be difficult to accurately model China’s macroeconomy. The transmission mechanism of a monetary policy shock to the economy is uncertain and the effects will be difficult to predict for the central bank.\(^3\)

What is the monetary policy rule of the PBoC? Has the rule changed over time? What do the data say about the actual monetary policy rules at work? What are the main sources of business fluctuations of the Chinese economy given that the monetary policy rules are known? These are the questions the paper aims to address.

To this end, we extend a standard New Keynesian DSGE model with financial frictions and investment-specific (hereafter, IS) technology shocks. The financial friction mechanism was first introduced by Bernanke et al. (1999) to model market imperfection of the financial sector. The IS technology shock was suggested and developed by Greenwood et al. (1988, 1997) as a viable alternative to neutral technology shocks as sources of business cycles. Studies by Kaihatsu and Kurozumi (2014), and Justiniano et al. (2011) find that the financial friction shock and the IS technology shock are important sources of business fluctuations in the United States. There are a number of studies applying DSGE models to the Chinese economy. See, for example, Xu and Chen (2009), Mehrotra et al. (2013), Yuan and Feng (2014), and Zhang et al. (2014). None of these studies have explicitly taken into account financial frictions or shocks to investment.\(^4\) It is reasonable to expect that they are significant drivers of China’s business fluctuations.

We propose a hybrid form of monetary policy rule for the extended model. Past studies on China’s monetary policy tend to make a choice between Taylor-type rules and quantity rules that have been used in studies of advanced economies. For example, Zhang (2009) argues that a Taylor-type rule is likely to be more effective than a quantity-type rule in managing the economy. Liu and Zhang (2010) show that using

\(^3\)To be fair, the puzzle that is China’s monetary policy has many facets. The unknown policy rules that is examined in the chapter is one of them. Another important facet of the puzzle is related to the expectations of private agents towards the monetary policy conduct. For example, instead of assuming rational expectations, people may believe that the central bank follows, say, a Taylor-type rule when the reality is the central bank is following a money rule. Those types of equilibria where beliefs do not agree with reality are studied in Kulish and Pagan (2017). This is beyond the scope of the current chapter and will be left for future research.

\(^4\)Yuan et al. (2011) and Kang and Gong (2014) incorporate financial frictions, but no IS technology shocks, in their models.
both rules outperform a single rule in a four-equation New Keynesian model. Since there is no consensus on the specific form of the policy rules, we incorporate a general form of monetary policy rule that encompasses the pure Taylor-type rules or quantity-type rules for estimations.

The main findings of the paper are as follows. Firstly, the central bank of China has been employing a hybrid monetary policy rule during 2001-2017 where the PBoC conducted monetary policy by adjusting the policy rate in response to the inflation rate, output, output growth as well as real money growth in the economy. It is evident that there is a structural change in the monetary policy rule after the onset of the Global Financial Crisis, with a larger response of the policy rate to real money growth in the post crisis period, \textit{ceteris paribus}. Secondly, the main sources of business fluctuations in output and consumption growth are neutral technology shocks and preference shocks, and exogenous demand shocks and intermediate-good price markup shocks play lesser but still important roles. The fluctuations in investment are driven by shocks to IS technology, neutral technology, intermediate-good price markup, and marginal efficiency of investment (hereafter, MEI), while the fluctuations in loans activities are primarily contributed by IS technology shocks and net worth shocks. Thirdly, the negative neutral technology shocks have been the main contributor to the slowing down of China’s GDP growth since around 2010 while the on-average positive investment growth has been contributed by both technology shocks and price markup shocks, although the individual contribution of those shocks is quite volatile over the full sample period.

The remainder of the paper is organized as follows. Section 2.2 constructs the model. Section 2.3 proceeds with the estimation. Section 2.4 reports and discusses the results, followed by the concluding remarks in Section 2.5.

### 2.2 The Model

The model is very close to that of Kaihatsu and Kurozumi (2014 hereafter, KK), except for the central bank’s behavior. There are households that consist of worker and entrepreneur members, financial intermediaries, intermediate-good firms, consumption-good firms, investment-good firms, capital-good firms and a central bank in the economy. The financial accelerator mechanism of Bernanke et al. (1999) is employed in the financial sector. The economy is subject to both technology shocks and financial shocks.

\footnote{Note that Liu and Zhang (2010) use the concept of a ‘hybrid rule’ in their study which actually means that the central bank uses both the quantity rule and the Taylor rule to conduct monetary policy. Because of the small scale of their model, this is mathematically solvable.}
Each agent’s behavior is described in details as follows.

2.2.1 Households

The representative household consists of a continuum of members normalized to unity. A proportion of members are workers, denoted by $m \in [0, 1]$, and the rest are entrepreneurs. All members are assumed to pool consumption and make joint consumption-saving decisions. The representative household maximizes

$$
E_0 \sum_{t=0}^{\infty} \beta^t \exp(z^b_t) \left[ \frac{(C_t - \theta C_{t-1})^{1-\sigma}}{1-\sigma} + \exp(z^m_t) \frac{(M_t/P_t)^{1-\sigma}}{1-\sigma} - (Z^*_t)^{1-\sigma} \exp(z^h_t) \int_0^1 \frac{(h_t(m))^{1+\chi}}{1+\chi} dm \right]
$$

subject to the budget constraint

$$
P_tC_t + M_t + D_t = r^n_{t-1} D_{t-1} + M_{t-1} + P_t \int_0^1 W_t(m) h_t(m) dm + T_t
$$

(2.2.2)

where $E_0$ is the rational expectation operator, $\beta \in (0, 1)$ is the discount factor, $\sigma > 0$ and $\theta \in [0, 1]$ are the degrees of relative risk aversion and internal consumption habit persistence, respectively, $\chi > 0$ is the inverse of the elasticity of labor supply, $z^b_t$ is the intertemporal preference shock, $z^h_t$ and $z^m_t$ represent the labor supply shock and money demand shock, respectively, $C_t$ is the consumption level, $M_t/P_t$ is the real money balance the household is holding, $h_t(m)$ is the labor supply of worker $m$ to the intermediate-good firms $f \in [0, 1]$ and $h_t(m) = \int_0^1 h_t(m, f) df$, $Z^*_t$ is the composite technological level (which will be explained below), $P_t$ is the price of consumption goods, $D_t$ is the deposit saved in financial intermediaries, $r^n_t$ is the gross deposit rate which is assumed to be the policy rate, $W_t(m)$ is worker $m$’s real wage, and $T_t$ consists of profits received from firms and a lump-sum public transfer.

The first-order conditions with respect to consumption and deposits are

$$
\Lambda_t = \exp(z^b_t)(C_t - \theta C_{t-1})^{-\sigma} - \beta \theta E_t \exp(z^b_{t+1})(C_{t+1} - \theta C_t)^{-\sigma}
$$

(2.2.3)

$$
1 = E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} \frac{r^n_t}{\pi_{t+1}}
$$

(2.2.4)
where $\Lambda_t$ is the marginal utility of consumption and $\pi_t = P_t/P_{t-1}$ is the gross inflation rate of the consumption-good price. We assume that households in the economy cannot make optimal decisions over money holdings due to the underlying frictions as a shortcut of modeling the distortions in China. The detailed discussion is given in Section 2.2.4 when introducing monetary policy rule of the central bank.

### 2.2.1.1 Workers

The labor market is monopolistically competitive. Demand for worker $m$’s labor services is given by

$$h_t(m) = h_t\left(\frac{W_t(m)}{W_t}\right)^{-\theta^w_t},$$

where $h_t = [\int_0^1 (h_t(m))^{\theta^w_t/(\theta^w_t-1)} dm]^{\theta^w_t/(\theta^w_t-1)}$ is the aggregate labor service with substitution elasticity $\theta^w_t > 1$ and $W_t = \int_0^1 (W_t(m))^{1-\theta^w_t} dm$ is the aggregate wage.

The nominal wage is adjusted according to a Calvo (1983) pricing mechanism. In each period a fraction of $1 - \xi_w \in (0, 1)$ of workers gets to reoptimize their wages while the remaining fraction $\xi_w$ of workers’ wages is set by indexation to both the gross steady-state balanced growth rate $z^*$ and a weighted average of past and steady-state inflation $\pi^{\gamma_w}_{t-1}\pi^{1-\gamma_w}$, where $\gamma_w \in (0, 1)$ is the relative weight on past inflation ($z^*$ will be explained later). Each worker that gets to reset their wage at time $t$ chooses $P_tW_t(m)$ to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left[ \Lambda_{t+j} h_{t+j}(m) \frac{P_tW_t(m)}{P_{t+j}} \prod_{k=1}^{j} (z^* \pi_{t+k-1} \pi^{1-\gamma_w}) \frac{\exp(z^*_t)\pi_t^{1-\gamma_w} \exp(z^*_{t+j})h_{t+j}(m)}{1+\chi} \right]$$

subject to

$$h_{t+j}(m) = h_{t+j} \left[ \frac{P_tW_t(m)}{P_{t+j}W_{t+j}} \prod_{k=1}^{j} (z^* \pi_{t+k-1} \pi^{1-\gamma_w}) \right]^{-\theta^w_{t+j}}$$

(2.2.6)

The first-order conditions for reoptimized wage $W^0_t$ are given by

$$1 = \frac{E_t \sum_{j=0}^{\infty} \beta^j \left[ (1+\lambda^w_{t+j}) \exp(z^*_{t+j}) \exp(z^*_{t+j})(Z^*_{t+j})^{1-\gamma} \right] \left[ \Lambda_{t+j} h_{t+j} \left( \frac{W^0_t(z^*_{t+j})}{W^0_t(z^*_{t+k})} \prod_{k=1}^{j} (\frac{\pi_{t+k-1} \pi^{1-\gamma_w}}{\pi_{t+k}}) \right) \right]^{-1+\lambda^w_{t+j}}}{E_t \sum_{j=0}^{\infty} \beta^j \left[ (1+\lambda^w_{t+j}) \exp(z^*_{t+j}) \exp(z^*_{t+j})(Z^*_{t+j})^{1-\gamma} \right] \left[ \Lambda_{t+j} h_{t+j} \left( \frac{W^0_t(z^*_{t+j})}{W^0_t(z^*_{t+k})} \prod_{k=1}^{j} (\frac{\pi_{t+k-1} \pi^{1-\gamma_w}}{\pi_{t+k}}) \right) \right]^{-1+\lambda^w_{t+j}}}$$
where $\lambda^*_w = 1/(\theta^*_w - 1) > 0$ is the wage markup.

The aggregate wage in equation (2.2.5) is reduced to

\[
1 = (1 - \xi_w) \left( \left( \frac{W_0^t}{W_t} \right) - \frac{1}{\lambda^*_w} \right) + \sum_{j=1}^{\infty} \left\{ \left( \frac{(z^*_j)^j W_0^t}{W_t} \prod_{k=1}^{j} \left[ \left( \frac{\pi_t-k}{\pi} \right)^{\gamma_w} \frac{\pi}{\pi_t-k+1} \right] \right)^{-\frac{1}{\lambda^*_w}} \right\} \tag{2.2.8}
\]

### 2.2.1.2 Entrepreneurs and Financial Intermediaries

At the end of period $t - 1$, entrepreneurs hold real net worth $N_{t-1}$ left from this period and obtain a loan $L_{t-1}$ from financial intermediaries at gross real loan rate $E_{t-1}r^E_t$. They optimally purchase capital $K_{t-1}$ from capital-good firms at price $Q_{t-1}$, and choose the capital utilization rate $u_t$. Then they provide capital service $u_t K_{t-1}$ to intermediate-good firms at rental rate $R^k_t$, and sell the rest of their capital $(1 - u_t)K_{t-1}$ back to capital-good firms at price $Q_t$. After paying back their loan to the financial intermediaries, a fraction $1 - \eta_t \in (0, 1)$ of entrepreneurs becomes workers, while the remaining $\eta_t$ survives into the next period.

It is assumed that a higher utilization rate will lead to a higher depreciation rate $\delta(u_t)$ during intermediate-good firms’ production. $\delta(.)$ satisfies $\delta' > 0$, $\delta'' > 0$, $\delta(1) = \delta \in (0, 1)$, and $\delta'(1)/\delta''(1) = \tau > 0$. With higher utilization rate, entrepreneurs can provide more capital services but the resultant higher depreciation rate will result in a lower rental rate.

The first-order conditions for optimal decisions on utilization rate and purchasing capital can be derived as

\[
R^k_t = Q_t \delta'(u_t) \tag{2.2.9}
\]

\[
E_t \Lambda_{t+1} r^E_{t+1} = E_t \Lambda_{t+1} \frac{u_{t+1} R^k_{t+1} + Q_{t+1}(1 - \delta(u_{t+1}))}{Q_t} \tag{2.2.10}
\]

where the EF premium function $F(.)$ depends on entrepreneurs’ leverage ratio $Q_t K_t/N_t$ and satisfies $F' > 0$ and $\mu = (QK/N)F'(QK/N)/F(QK/N) \geq 0$ as in regular DSGE models with a financial accelerator mechanism, such as in Hirose (2008). $z^*_t$ denotes
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a shock to the EF premium. The gross real loan rate \( E_t r^E_{t+1} \) consists of deposit rate \( E_t (r^p_t / \pi_{t+1}) \) and the EF premium

\[
E_t r^E_{t+1} = E_t \frac{r^p_t}{\pi_{t+1}} F \left( \frac{Q_t K_t}{N_t} \right) \exp(\tilde{z}^r_t) \tag{2.2.11}
\]

Evolution of net worth \( N_t \) is

\[
N_t = \eta_t \left[ r^E_t Q_{t-1} K_{t-1} - (E_{t-1} r^E_{t-1}) L_{t-1} \right] + (1 - \eta_t) \chi Z^*_t \tag{2.2.12}
\]

where \( \chi \) is a constant, \( \chi Z^*_t \) represents the transfer from entrepreneurs who become workers to surviving entrepreneurs, \( \eta_t \) is the probability of surviving and given by

\[
\eta_t = \eta \exp(\tilde{z}^n_t) / (1 - \eta + \eta \exp(\tilde{z}^n_t)), \quad \text{where } \tilde{z}^n_t \text{ is a shock to net worth, and } r^E_t \text{ is the ex-post marginal return on capital and given by}
\]

\[
r^E_t = \frac{u_t R^k_t + Q_t (1 - \delta(u_t))}{Q_{t-1}}. \tag{2.2.13}
\]

2.2.2 Intermediate-good Firms and Consumption-good Firms

Each intermediate-good firm \( f \in [0, 1] \) produces output \( Y_t(f) \) according to the production function

\[
Y_t(f) = \left( Z_t h_t(f) \right)^{1-\alpha} \left( K_t(f) \right)^{\alpha} - \phi y Z^*_t \tag{2.2.14}
\]

where \( h_t(f) \) is the labor input from workers at real wage \( W_t \), \( K_t(f) \) is the capital input from entrepreneurs at real rental rates \( R_t \), \( Z_t \) is the neutral technology and evolves according to a stochastic process

\[
\log Z_t = \log z + \log Z_{t-1} + z^z_t
\]

\( z > 1 \) is the gross steady-state rate of neutral technology change and \( z^z_t \) represents a non-stationary neutral technology shock. \( h_t(f) = \int_0^1 (h_t(m, f))^{(\theta^p_r - 1)/\theta^p_r} dm \) denotes the labor input, and \( \alpha \in (0, 1) \) is the capital elasticity of output. \( \phi \in [0, 1] \) in the fixed cost term \(-\phi y Z^*_t \) is chosen to ensure that the zero profit condition holds at
the steady state, and \( y \) is the steady-state value of the detrended output \( y_t = Y_t / Z^* \). \( Z^*_t \) denotes the composite technological level following \( Z^*_t = Z_t(\Psi_t)^{\alpha/(1-\alpha)} \) where \( \Psi_t \) is the level of IS technological level. \( Z^*_t / Z^*_{t-1} \) is the gross rate of balanced growth with steady-state rate \( z^* = z\psi^{\alpha/(1-\alpha)} \), derived by equation (2.2.14), and \( \psi \) is the steady-state rate of \( \Psi_t \).

From the first-order conditions for optimal labor and capital inputs we obtain

\[
\frac{1 - \alpha}{\alpha} = \frac{W_t h_t}{R_t^k u_t K_{t-1}}
\]

and the real marginal cost is given by

\[
m_{ct} = \left( \frac{W_t}{(1 - \alpha)Z_t} \right)^{1-\alpha} \left( \frac{R_t^k}{\alpha} \right)^\alpha
\]

where \( h_t = \int_0^1 h_t(f) df \) and \( u_t K_{t-1} = \int_0^1 K_t(f) df \). Aggregating function (2.2.14) over intermediate-good firms yields

\[
Y_t d_t = (Z_t h_t)^{1-\alpha} (u_t K_{t-1})^\alpha - \phi y Z^*_t
\]

where \( d_t = \int_0^1 (P_t(f) / P_t)^{-\theta^p_t} df \) is intermediate-good price dispersion.

Each consumption-good firm chooses a combination of intermediate goods \( \{Y_t(f)\} \) at price \( P_t(f) \) and produces consumption goods \( Y_t \), subject to the production function

\[
Y_t = (\int_0^1 Y_t(f)^{(\theta^p_t - 1)/\theta^p_t} df)^{\theta^p_t/(\theta^p_t - 1)}, \text{ where } \theta^p_t > 1 \text{ represents elasticity of substitution between intermediate goods. Profit maximization of consumption-good firms yields demand for intermediate-good } f \text{ as } Y_t(f) = Y_t(P_t(f)/P_t)^{-\theta^p_t}.
\]

It is assumed that consumption-good firms operate under perfect competition, while intermediate-goods firms face monopolistic competitive market. Hence, the price of consumption-good \( Y_t \) is given by

\[
P_t = \left( \int_0^1 P_t(f)^{1-\theta^p_t} df \right)^{1/\theta^p_t}
\]

Intermediate-good firms set price under the Calvo (1983) pricing mechanism, which assumes a fraction of \( 1 - \xi^p \in (0, 1) \) of intermediate-good firms reoptimizes price in each period, while price of the rest is set by indexation to a weighted average of past inflation and steady-state inflation, with \( \gamma^p \in [0, 1] \) the relative weight on past inflation,
i.e., $\pi_{t-1}^{\gamma_p} \pi^{1-\gamma_p}$. Price is reoptimized in the current period so as to maximize

$$E_t \sum_{j=0}^{\infty} \beta^j \left( \frac{\Lambda_{t+j}}{\Lambda_t} \right) \left[ \frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left( \pi_t^{\gamma_p} \pi_{t+k-1}^{1-\gamma_p} \right) - mc_{t+j} \right] Y_{t+j}(f)$$

subject to

$$Y_{t+j}(f) = Y_{t+j} \left[ \frac{P_t(f)}{P_{t+j}} \prod_{k=1}^{j} \left( \pi_t^{\gamma_p} \pi_{t+k-1}^{1-\gamma_p} \right) \right]^{-\theta_{t+j}}$$

where $\beta^j \frac{\Lambda_{t+j}}{\Lambda_t}$ shows the stochastic discount factor between period $t$ and $t + j$. Solving the above problem, reoptimized price $P_0^t$ is given by

$$1 = \frac{E_t \sum_{j=0}^{\infty} \beta^j \xi^p_j (1+\lambda_{t+j}^p) \prod_{k=1}^{j} \left( \pi_t^{\gamma_p} \pi_{t+k-1}^{1-\gamma_p} \right) - mc_{t+j}}{E_t \sum_{j=0}^{\infty} \beta^j \Lambda_{t+j} Y_{t+j} \left( \frac{P_0^t}{P_{t+j}} \prod_{k=1}^{j} \left( \pi_t^{\gamma_p} \pi_{t+k-1}^{1-\gamma_p} \right) \right) - \lambda_{t+j}^p}$$

(2.2.19)

equation (2.2.18) can be further reduced to

$$1 = (1 - \xi_p) \left( \frac{P_0^t}{P_t} \right)^{-\frac{1}{\lambda_t^p}} + \sum_{j=1}^{\infty} \beta^j \xi^p_j \prod_{k=1}^{j} \left( \frac{\pi_t^{\gamma_p} \pi_{t+k-1}^{1-\gamma_p}}{\pi_t^{\gamma_p} \pi_{t+k}^{1-\gamma_p}} \right)^{-\frac{1}{\lambda_t^p}}$$

(2.2.20)

where $\lambda_t^p = 1/(\theta_t^p - 1)$ denotes the intermediate-good price markup.

### 2.2.3 Investment-good Firms and Capital-good Firms

The investment-good firm $f_i$ converts one unit of consumption goods into differentiated investment goods equal to $\Psi_t$ units and supply them to capital-good firms. Capital-good firms accumulate capital $K_t$ by choosing an optimal combination of investment goods $\{I_t(f_i)\}$ to make further investment $I_t$ and purchasing $(1 - \delta(u_t))K_{t-1}$ capital goods back from entrepreneurs. The accumulated capital $K_t$ is again sold to entrepreneurs. Here, the level of IS technology $\Psi_t$ is identical across investment-good firms and follows the process

$$\log(\Psi_t) = \log(\Psi) + \log(\Psi_{t-1}) + z_t^\Psi$$
where $z_t^\psi$ is a non-stationary IS technology shock.

Under monopolistic competition, the investment-good firm $f_i$ faces demand

$$I_t(f_i) = I_t\left(\frac{P^i_t(f_i)}{P_t}\right)^{-\theta_t^i} \quad (2.2.21)$$

and corresponding aggregate price of investment good price

$$P^i_t = \left(\int_0^1 P^i_t(f_i)^{1-\theta_t^i} df_i\right)^{1/(1-\theta_t^i)} \quad (2.2.22)$$

where $I_t = (\int_0^1 I_t(f_i)^{\theta_t^i-1}/\theta_t^i df_i)^{\theta_t^i/(\theta_t^i-1)}$, where $\theta_t^i > 1$ is the substitution elasticity, and $P^i_t(f_i)$ is the price of investment goods produced by firm $f_i$ set by maximizing profit $(P^i_t(f_i)/P_t - 1/\Psi_t) I_t(f_i)$.

The corresponding first-order conditions give

$$P^i_t = P^i_t(f_i) = (1 + \lambda_t^i) P_t / \Psi_t, \quad (2.2.23)$$

where $\lambda_t^i \equiv 1/(\theta_t^i - 1) > 0$ is the investment-good markup. Combining optimal choice of $P^i_t(f_i)$ with (21) and (22) leads to $P^i_t = P^i_t(f_i)$ and $I_t(f_i) = I_t$. Hence, the gross rate of change in the relative price of investment goods to consumption goods is given by

$$r^i_t = \frac{P^i_t/P_t}{P^i_{t-1}/P_{t-1}} = \frac{1 + \lambda_t^i \Psi_t}{1 + \lambda_{t-1}^i \Psi_{t-1}}$$

The capital-good firms’ problem is to choose an optimal combination of investment goods $\{I_t(f_i)\}$ and maximize profit

$$E_t \sum_{j=0}^{\infty} \beta^j \frac{\Lambda_t}{\Lambda_{t-1}} \left\{ Q_{t+j} \left[ K_{t+j} - (1 - \delta(u_{t+j})) K_{t+j-1} \right] - \frac{P_{t+j}^i}{P^i_{t+j}} I_{t+j} \right\}$$

subject to

$$K_t = (1 - \delta(u_{t})) K_{t-1} + \exp(z_t^\nu) \left(1 - S\left(\frac{I_t/I_{t-1}}{(z^* \psi)}\right)\right) I_t \quad (2.2.24)$$

Here $S((I_t/I_{t-1})/(z^* \psi)) = (\zeta/2)[(I_t/I_{t-1})/(z^* \psi) - 1]^2$ is the adjustment cost with $\zeta > 0$,.
and $z_t^\nu$ represents an MEI technology shock that affects the transformation of investment goods into capital goods.

The optimal decision is determined by equation (2.2.21) and the first-order conditions:

$$\frac{P^i_t}{P_t} = Q_t \exp(z_t^\nu) \left[ 1 - S\left(\frac{I_t}{I_{t-1}}\right) z^* \psi - S'\left(\frac{I_t}{I_{t-1}}\right) \frac{I_t}{I_{t-1}} z^* \psi \right]$$

$$+ E_t \beta \frac{\Lambda_{t+1}}{\Lambda_t} z^* \psi Q_{t+1} \exp(z_{t+1}^\nu) S'(\frac{I_{t+1}}{I_t}) (\frac{I_{t+1}}{I_t})^2$$

(2.2.25)

2.2.4 Central Bank

The central bank is assumed to do two things in the model economy. First, it controls the money supply in the market according to a quantity rule, and second, it adjusts the interest rate according to a policy rule. Both rules are defined as follows.

The quantity rule regarding the money supply is defined as in Christiano, Eichenbaum, and Evans (2005)

$$M_t^s = \mu_t M_{t-1}^s$$

and

$$\log \mu_t = \log \mu_0 + z_{t}^{mg}$$

(2.2.26)

where $M_t^s$ is the nominal money supply at time $t$ and $M_t^s = M_t$ when the market clears, $\mu_t$ is the gross growth rate of the money supply, $\mu_0$ is the gross steady state rate, and $z_{t}^{mg}$ represents a money supply shock and follows an AR(1) stochastic process.

The hybrid rule for setting the interest rate is given as

$$\log(r_t^n) = W \left( - \log \left( 1 - \frac{1}{\lambda_t} \exp(z_t^\nu) \exp(z_t^m) \right) \right)$$

$$+ (1 - W) \left( \phi_r \log(r_{t-1}^n) + (1 - \phi_r) (\log r^n + \frac{\phi_x}{4} \sum_{j=0}^{3} \log(\frac{\pi_{t-j}}{\pi}) + \phi_y \log \frac{Y_t}{Z_t^*} + \phi_{\Delta y} \log \frac{Y_t}{Y_{t-1}} \right)$$

$$+ z_t^\nu$$

(2.2.27)

where $0 \leq W \leq 1$, $r^n$ is the gross steady-state policy rate, $\lambda_t$ is the detrended marginal utility of consumption, defined later in Section 2.3, $m_t = M_t/P_t$ is the real money balance, $z_t^m$ is the real money balance shock, $\phi_r \in [0, 1]$ represents the degree of policy rate smoothing, $\phi_x, \phi_y, \phi_{\Delta y} \geq 0$ represents the degrees of policy responses to inflation,
output, and output growth, and \( z^r \) represents a policy rate shock and follows an AR(1) stochastic process. The hybrid rule is essentially a linear combination of a component of a Taylor-type rule (the part multiplied by \( 1 - W \)) and a component of a money demand in real term (the part multiplied by \( W \)). The first component borrows from a Taylor rule proposed by Kaihatsu and Kurozumi (2014) while the second component would be the right hand side of the optimal money demand of households, were there no frictions preventing households from optimally choosing \( m_t \). The hybrid rule is proposed and constructed in a way that, together with the quantity rule above, encompasses the pure Taylor rule and the pure quantity rule.

**Discussion:**

Equations (2.2.27) and (2.2.26) fully describe the central bank’s behavior in the model economy in three scenarios.

If \( 0 < W < 1 \), both components in equation (2.2.27) show up as the central bank’s monetary policy objectives. The central bank conducts monetary policy by not only responding to inflation and output conditions like an advanced economy’s authority according to a pure Taylor rule, but also taking into account the real money demand of households. Note that we assume that due to the underlying frictions, households cannot make optimal choices over how much money to hold in the economy. In this case, the central bank recognizes that and includes the money demand component in its policy function. This is a shortcut of modeling the constraint the Chinese citizens are subject to and the policy rule that takes that constraint into account. Subsequently, equation (2.2.26) pins down the nominal money when the market clears and completes the model.

If \( W = 0 \), only the Taylor-type rule component shows up in equation (2.2.27), and it reduces to a pure Taylor-type rule as in Kaihatsu and Kurozumi (2014). In this scenario, households are still assumed to be constrained from making optimal choice of real money holdings but the central bank does not include the real money demand component in the policy rule. Equation (2.2.26) again clears the market for the quantity of money and the model is complete.

If \( W = 1 \), only the real money demand component shows up in equation (2.2.27) and it reduces to be identical to the optimal money demand condition of households in the absence of underlying frictions. In this case, the central bank sets the policy rate following the same rule describing households’ optimal choices over money holdings.

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\(^6\)Recall that in the households’ utility maximization problem in unconstrained equilibrium, the optimality condition with respect to \( m_t \) at time \( t \) is given by \( \log r^m_t = -\log \left( 1 - \frac{1}{
abla_2 \exp (z^b_t) \exp (z^m_t) (m_t)^{-\sigma}} \right. \).
The quantity of money is determined by equation (2.2.26) and the interest rate is set by equation (2.2.27). The model is complete.

In a nutshell, equations (2.2.27) and (2.2.26) describe the central bank’s behavior in the model economy in a generalized form that encompasses the pure Taylor rule and the quantity rule without imposing \textit{ex ante} model restrictions. Answers to the question which policy rule is the central bank using can be easily obtained by statistical readings of the posterior mean estimates of parameters.

### 2.2.5 System of Equations

The consumption-good market clearing condition is

\[
Y_t = C_t + \int_0^1 \frac{I_t(f_i)}{\Psi_t} df_i + gZ^*_t\exp(\tilde{z}^o_t) = C_t + \frac{I_t}{\Psi_t} + gZ^*_t\exp(\tilde{z}^o_t). \tag{2.2.28}
\]

The system of equations consists of equations (2.2.3), (2.2.4), (2.2.7) – (2.2.13), (2.2.15) – (2.2.17), (2.2.19) – (2.2.20), (2.2.23) – (2.2.25), (2.2.27) – (2.2.28), together with the stochastic processes for the twelve exogenous shocks \(z^x_t\), \(x \in \{b, g, w, p, i, r, z, \psi, \nu, \mu, \eta, mg\}\), where \(z^b_t\) is the preference shock, \(z^g_t = (g/y)\tilde{z}^g_t\) is the exogenous demand shock which is a shock to demand for the consumption-good excluding that for consumption \(C_t\) and investment \(I_t/\Psi_t\), \(z^w_t\) is a composite shock to the labor disutility disturbance \(z^h_t\) and the wage markup \(\lambda^w_t\), \(z^p_t\) and \(z^i_t\) are shocks associated with the intermediate-good price markup \(\lambda^p_t\) and the investment-good price markup \(\lambda^p_t\), \(z^r_t\) is a shock to the monetary policy rate, \(z^\psi_t\) and \(z^\nu_t\) are neutral and IS technology shocks, respectively, \(z^\mu_t\) is the MEI shock, \(z^\eta_t\) is a shock to the external finance premium, \(z^\nu_t\) is a shock to the net worth of entrepreneurs, with \(z^\nu_t = \eta(r^E/z^* - 1)\tilde{z}^\nu_t\). Each of the twelve exogenous shocks follows an AR(1) stationary stochastic process

\[
z^x_t = \rho_x z^x_{t-1} + \varepsilon^x_t, \quad \varepsilon^x_t \sim \text{i.i.d. } N(0, \sigma^2_x), \quad x \in \{b, g, w, p, i, r, z, \psi, \nu, \mu, \eta, mg\}.
\]
2.3 Estimation

2.3.1 Estimation Methodology

We adopt a Bayesian likelihood approach from KK with twelve China quarterly time series: output $Y_t$, consumption $C_t$, investment $I_t$, labor (hours worked) $h_t$, the real wage $W_t$, the price of consumption goods $P_t$, the relative price of investment goods $P^i_t/P_t$, the monetary policy rate $r^n_t$, the loan rate $E_t(r^E_{t+1}\pi_{t+1})$, real loans $L_t$, real net worth $N_t$, and real money balances $M_t/P_t$.\footnote{There are studies in the literature that have employed Bayesian estimation strategies for estimating the Chinese economy. Most of the data series are small-scale. \textit{Wang and Tian} (2014) apply a Bayesian estimation approach using four data series while \textit{Sun and Sen} (2011) use seven data series.}

Before estimation, the equilibrium conditions presented in the previous section are rewritten in terms of detrended variables. As mentioned previously, the model economy consists of a non-stationary stochastic technology trend $Z_t^*$ and variables are detrended as $y_t = Y_t/Z_t^*$, $c_t = C_t/Z_t^*$, $w_t = W_t/Z_t^*$, $\lambda_t = \Lambda_t(Z_t^*)^\sigma$, $i_t = I_t/(Z_t^*\Psi_t)$, $k_t = K_t/(Z_t^*\Psi_t)$, $r_k^t = R_k^t\Psi_t^*$, $q_t = Q_t\Psi_t^*$, $n_t = N_t/Z_t^*$, $l_t = L_t/Z_t^*$ and $m_t = M_t/(Z_t^*P_t)$. The stationarized system is then log-linearized around its deterministic steady sate with a capital utilization rate of unity (i.e. $u_{ss} = 1$). Details are reported in the Appendix.

Following \textit{Smets and Wouters} (2007) and KK, we use the Kalman filter to evaluate the likelihood function for the log-linearized system and apply the Metropolis–Hastings algorithm to generate draws from the posterior distribution of model parameters.\footnote{Our estimation is done using Dynare \textit{Adjemian et al.} (2011). In each estimation, 200,000 draws were generated and the first half of these draws was discarded. The scale factor for the jumping distribution in the Metropolis–Hastings algorithm was adjusted so that an acceptance rate of around 24 per cent was obtained.}

2.3.2 The Data

The data are obtained from the CEIC China Premium Database and the sample period is 2001:Q1 to 2017:Q2. Data on prices is from the CPI. The relative price of investment $P^i_t/P_t$ is proxied with the PPI divided by CPI. Data on nominal GDP, consumption, investment and wages is deflated with the CPI. Data on real loans is CPI-deflated. Real net worth is proxied by data on the Shanghai Stock Exchange Composite Index deflated by CPI. The inverse of the City Labor Market Demand-Supply Ratio is used as a proxy for labor and normalized to be equal to zero as in \textit{Smets and Wouters} (2007). SHIBOR is used as a proxy for the loan rate, and the policy interest rate is the household deposit saving rate. The aggregate money supply is $M_2$. All series are
seasonally adjusted. Corresponding observation equations are

\[
\begin{bmatrix}
100 \Delta \log Y_t \\
100 \Delta \log C_t \\
100 \Delta \log I_t \\
100 \Delta \log h_t \\
100 \Delta \log W_t \\
100 \Delta \log P_t \\
100 \Delta \log \left( P^*_t / P_t \right) \\
100 \Delta \log \left( r^E_t \right) Y_t \\
100 \Delta \log L_t \\
100 \Delta \log N_t \\
100 \Delta \log M_t
\end{bmatrix}
= 
\begin{bmatrix}
\bar{z}^* \\
\bar{z}^* \\
\bar{z}^* + \bar{\psi} \\
\bar{h} \\
\bar{z}^* \\
\bar{\pi} \\
\bar{r^m} \\
\bar{r^E} + \bar{\pi} \\
\bar{z}^* \\
\bar{z}^* + \bar{\pi}
\end{bmatrix}
+ 
\begin{bmatrix}
z_t^* + \hat{y}_t - \hat{y}_{t-1} \\
z_t^* + \hat{c}_t - \hat{c}_{t-1} \\
z_t^* + z_t^i + \hat{i}_t - \hat{i}_{t-1} \\
\hat{h}_t \\
z_t^* + \hat{w}_t - \hat{w}_{t-1} \\
\hat{\pi}_t \\
\hat{r}_t^n \\
E_t \bar{r}^E_{t+1} + E_t \bar{\pi}_{t+1} \\
z_t^* + \hat{l}_t - \hat{l}_{t-1} \\
z_t^* + \hat{n}_t - \hat{n}_{t-1} \\
z_t^* + \hat{m}_t - \hat{m}_{t-1}
\end{bmatrix}
\]

where \(\bar{z}^* = 100(z^* - 1)\), \(\bar{\psi} = 100(\psi - 1)\), \(\bar{\pi} = 100(\pi - 1)\), \(\bar{r^m} = 100(r^m - 1)\), \(\bar{r^E} = 100(r^E - 1)\), and hatted variables represent log-deviations from their respective steady-state values.

### 2.3.3 Fixed Parameters and Prior Distributions

There are two sets of parameters: one to be estimated while the other is fixed to avoid any identification issue. The fixed parameters are the depreciation rate \(\delta\), the wage markup \(\lambda_w\), the steady state investment-good price markup \(\lambda_i\), and the steady-state ratio of exogenous demand to output \(g/y\). \(\delta\) is set to 0.025 per quarter, implying an annual depreciation rate of 0.10 which is consistent with most empirical studies on the Chinese economy. \(\lambda_w\) and \(\lambda_i\) are taken from KK: \(\lambda_w = 0.2\), \(\lambda_i = 0.2\). \(g/y\) is set at the sample mean 0.212.

The prior distributions of the 49 parameters to be estimated are listed in Table 2.1. The prior distributions of the steady-state rates of balanced growth, IS technological change, inflation, the real loan rate and the policy rate (i.e., \(z^*\), \(\bar{\psi}\), \(\bar{\pi}\), \(\bar{r^E}\), \(\bar{r^m}\)) are set to be Gamma distributions with a standard deviation of 0.1 and the mean given by their respective sample mean. The prior distributions of the inter-temporal elasticity of substitution \(\sigma\) and the output elasticity of capital \(\alpha\) are identical to those in KK. The prior means of \(\sigma\) and \(\alpha\) are assumed to be 2 and 0.6, respectively, following Zhang (2009). The prior distribution of \(W\) is set to be a Uniform distribution with domain \([0,1]\), imposing no prior restriction on the hybrid policy rule. For the parameters of shocks, we choose the Beta distribution with a mean of 0.5 and a standard deviation of 0.2 for the persis-
tence of each shock (i.e., $\rho_x, x \in \{b, g, w, p, i, r, z, \psi, \nu, \mu, \eta, mg\}$) and an Inverse Gamma distribution with a mean of 0.5 and a standard deviation of infinity for the standard deviation of each innovation (i.e., $\sigma_x, x \in \{b, g, w, p, i, r, z, \psi, \nu, \mu, \eta, mg\}$). The rest of the parameters have the same prior distribution as in the KK model.

2.4 Results

In this section we present the results in three main parts. The first part reports the statistics of the posterior mean estimates of parameters over the sample period 2001Q1-2017Q2. Prior and posterior plots of distributions are provided for showing which parameters are unidentified. See Figure 2.1 for details. Possible changes of policy rule are also considered in this part. The second part of the section presents variance decompositions of output, consumption, investment and loans based on the estimated model. Both forecast error variance decompositions and historical decompositions are reported. Through this exercise we are able to answer some fundamental questions about the main sources of economic fluctuations in China. The final part presents the impulse responses to technology shocks and financial shocks.

2.4.1 Estimates of $W$

The first row of Table 2.1 reports the posterior mean of $W$ and the 90% confidence interval. On the full sample period, the weight of the quantity-rule component in the monetary policy rule function, $W$, is estimated to be 0.09 and is statistically different from zero. Equation (2.2.27) is in its general form. It is a hybrid monetary policy rule with quantity of money component taking up a small share of the rule. This result shows that over the past decade or so, the PBoC conducted monetary policy by adjusting the policy rate according to the real money growth, inflation rate, output level and output growth in the economy with assigned weights. Other macroeconomic conditions were subsequently pinned down through the interest rate channel in equilibrium. This finding could serve as a benchmark approach for estimating China’s monetary policy rules as macro and financial conditions in China evolve over time.

We also conduct subsample estimations searching for possible policy rule changes. During the sample period, there was a global breakdown of the financial system which might have caused some policy changes to the PBoC. We set 2009Q1 as the potential change point and estimate the model over the two subsamples, 2001Q1-2008Q4 and 2009Q1-2017Q2. Results are reported in Tables 2.2 and 2.3.

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9I thank one of the external anonymous examiners for the suggestion.
As shown in Tables 2.2 and 2.3, the mean estimates of $W$ over the two subsamples are considerably different from each other. $W$ is estimated to be 0.08 over the subsample from 2001Q1 to 2008Q4 and 0.24 over the subsample from 2009Q1 to 2017Q2, both of which are statistically significant from zero. China’s monetary policy rule included a small component of real money growth before 2009 while the weight of real money growth was significantly increased after 2009. This could indicate that the Global Financial Crisis may have had impact on the monetary policy practice of the PBoC. The PBoC has had a larger adjustment of its policy rate in responding to changes in the real money growth of the economy in the post-crisis time, everything else equal. We have also tested other possible structural change points: 2007Q1 and 2008Q1, and the results are very similar to those using 2009Q1 as the structural change point.

2.4.2 Variance Decompositions

This section reports the forecast error decompositions of the variances of output growth, consumption growth, investment growth and loans growth in Table 2.4 over the full sample period, and in Tables 2.5 and 2.6 over the two subsample periods. Historical decompositions of output growth and investment growth are reported in Figures 2.2 and 2.3 based on the estimated model.

Table 2.4 shows the relative contribution of each shock to the variations of output growth, consumption growth, investment growth and real loans growth at forecast horizons $T = 8$, 32 quarters, evaluated at the posterior mean estimates of parameters using data over the full sample period. The main source of the output fluctuation is the neutral technology shock. The second important source is the preference shock. The exogenous demand shock and the intermediate-good price markup shock play lesser but still important roles in contributing to the output fluctuation, and the latter’s role increases mildly from short-term (14 per cent) to long-term (16 per cent) horizons. The rest of the shocks are negligible. The four shocks which explain the most of the fluctuations in output growth also contribute the most of the fluctuations in consumption growth. The relative contribution of each of the four shocks to the variation of consumption growth is also similar to that to the output growth in the short-term horizon. Noticeably, the preference shock becomes the dominant source of the consumption fluctuation in the long-term, making up 45 per cent of the variation. The intermediate-good price markup shock, IS technology shock, neutral technology shock, and MEI shock all play significant roles in explaining the fluctuations of investment growth in the short run (around 20 per cent each). The IS technology shock becomes even more prominent in the long run (41 per cent). Almost half of the variation in real
loans growth is explained by the IS technology shock and 28 per cent by the net worth shock in the short-run. The intermediate-good markup shock also plays a sizable role in affecting loans activities.

Tables 2.5 and 2.6 report the forecast error variance decompositions of output growth, consumption growth, investment growth and real loans growth in the two subsamples: 2001Q1-2008Q4 and 2009Q1-2017Q2, respectively. Compared to the relative contribution of each of the shocks over the full sample period, the external demand shock plays an even larger role and the IS technology shock becomes non-negligible in explaining the variation in output growth over the subsample period before the financial crisis. Shocks to the neutral technology and to the external demand become more prominent in contributing to the variation in output growth in the period from 2009 to 2017. More than two thirds of the variation in consumption growth are explained by the preference shock in the period before 2009 (compared to 46 per cent over the full sample) while half of that variation is contributed to by the neutral technology shock in the period after 2009 (compared to 34 per cent over the full sample). The most significant change in the relative contribution of each of the shocks in explaining the fluctuations in investment growth is the MEI shock. Its contribution reduces to be minor in the subsample period before the crisis, accounting only for 4 per cent of the variation of investment growth while increases to be the second important source in the subsample period after the crisis, contributing to 24 per cent of the fluctuations in investment activities. In the subsample before the financial crisis, the net worth shock becomes the dominant contributor to fluctuations in real loans growth while in the subsample after the crisis, the most noticeable change in explaining the variation in real loans activities is that the neutral technology shock becomes sizable, taking up almost 10 per cent of the variation (relative to only about 5 per cent over the full sample period).

The results above demonstrate the main sources of business fluctuations in China. The real sectors, that is, consumption-good sectors, are primarily driven by shocks to the neutral technology, preference, price markup, and external demand while the financial sectors are dominated by the IS technology and net worth shocks.

To get a closer look at the fundamentals of business fluctuations in China, we present the historical decompositions of the percentage point deviations of output growth and investment growth from their respective steady states in Figures 2.2 and 2.3. Figure 2.2 shows a steady decreasing trend of output growth from around 2011 and the neutral technology shock is the main negative contributor: It was consistently positively contributing to the output growth in 2001-2007 and mid-2009 - early 2011, and then
consistently negatively contributing to the output growth since mid-2011. This implies that negative developments of the neutral technology may have been one of the primary drivers of the slowing down of China’s GDP growth that we discussed at the beginning of the paper. There is a drastic fall in output growth from around mid-2008 to early 2009 in Figure 2.2. This corresponds to the onset of the global financial crisis. A sudden global meltdown of the financial system and then of the real economy overseas may affect technology and production through trade and financial channels.

Figure 2.3 shows that investment growth is on average positive over the full sample period, and its key drivers include shocks to the neutral technology, IS technology, investment-good price markup, intermediate-good price markup and MEI, although their contributions are all volatile. The neutral technology shock is a positive contributor to the investment growth from 2001 to 2005, and a consistently negative contributor since around 2012, while the MEI shock is a prominent and negative contributor to the investment growth in 2006-2008 and a persistently positive factor since 2010. Shocks to the IS technology, investment-good and intermediate-good price markups are all quite volatile over the whole sample period. Noticeably, there is a significant fall in the investment growth in 2016 and the investment-good price markup seems to be the main driver of the drop.

Figures 2.2 and 2.3 together bring us another perspective on China’s growth story: investment was steadily growing, at least until 2016, while economic growth showed clear signs of slowing down over the past decade.

2.4.3 Impulse Responses

Section 2.4.1 has discussed monetary policy rules in China. Section 2.4.2 has taken a variance decomposition approach to examine the main sources of business-cycle fluctuations in key macroeconomic variables. In this section, we present the impulse responses to shocks to the monetary policy rate, neutral technology and net worth. The variables of interest are the growth rates of output, consumption, labor, investment, real loans, net worth, the deposit rate (policy rate), the loan rate, and the inflation rate. All shocks are positive and within one standard deviation. All figures are plotted at the posterior mean estimates of the respective variables and over 40 periods.

As shown in Figure 2.4, a positive shock to the monetary policy rate leads to a decrease in output, consumption, labor (hours worked), investment, real loans, net worth and inflation. The loan rate increases due to the increase of the deposit rate (policy rate). These are textbook responses to a monetary policy tightening.

Figure 2.5 shows the impulse responses to a production technology improvement
shock. Output rises, so does consumption. Investment and loans increase to support production boom, and firms’ net worth increase subsequently. Labor services fall due to improved productivity: workers do not have to work as much. The policy rate rises to prevent the economy from overheating. The loan rate rises and prices fall as a result.

Figure 2.6 shows that a positive shock to net worth increases investment activities. Output is increasing by less than investment. Consumption falls. Labor services increase to meet the higher production level. The price level increases. Real loans decrease due to rising net worth: firms do not have to borrow as much. The loan rate falls. The deposit rate rises in response to rising output and inflation.

2.5 Concluding Remarks

Policymakers and scholars are increasingly concerned with the recent economic slowdown in China. Our findings show that it is the negative neutral technology development that has caused this output fluctuation. After over thirty years of driving high-speed economic development, the growth potential of neutral technological advancement has shown a clear sign of slowing down. This has important policy implication of encouraging technological innovations and industrial upgrading in China.

We construct a rich DSGE model in this paper for the structural investigation of the Chinese economy. The results show that it captures important features of the economy that have not been found in previous studies using a simple model. For example, we find that China’s monetary policy rule is a hybrid rule. The central bank of China conducts monetary policy by adjusting the policy rate in response to inflation, output conditions as well as real money growth, which accounts for the constrained households of the economy. The paper also finds clear evidence for changes of monetary policy rules around the time of the onset of Global Financial Crisis. With everything else equal, the PBoC responds more to the development of real money in the post crisis time. Financial friction shocks and investment-specific technology shocks are indispensable sources of investment fluctuations. Neutral technology development was a consistently positive contributor to output growth during the period 2001-2007 and became a negative contributor after 2010. Future work on sources of business fluctuations in China and China’s monetary policy rule can draw on these results in this paper.
Table 2.1: Prior and posterior distributions of estimated parameters - full sample

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Type</th>
<th>Mean</th>
<th>S.D.</th>
<th>Posterior Mean</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W$ Weight on quantity rule</td>
<td>U</td>
<td>0.0911</td>
<td>0.179</td>
<td>[0.0179, 0.1686]</td>
<td></td>
</tr>
<tr>
<td>$\sigma$ Risk aversion</td>
<td>G</td>
<td>2</td>
<td>0.75</td>
<td>[0.5647, 1.1324]</td>
<td></td>
</tr>
<tr>
<td>$\theta$ Habit persistence</td>
<td>B</td>
<td>0.7</td>
<td>0.1</td>
<td>[0.4176, 0.6862]</td>
<td></td>
</tr>
<tr>
<td>$\chi$ Inverse of elasticity of labor supply</td>
<td>G</td>
<td>2</td>
<td>0.75</td>
<td>[1.5509, 3.7217]</td>
<td></td>
</tr>
<tr>
<td>$\zeta$ Elasticity of investment adjustment</td>
<td>G</td>
<td>4</td>
<td>1.5</td>
<td>[2.2265, 4.5573]</td>
<td></td>
</tr>
<tr>
<td>$\tau$ Inverse of elasticity of utilization rate adjustment cost</td>
<td>G</td>
<td>0.22</td>
<td>0.1</td>
<td>[0.108, 0.5033]</td>
<td></td>
</tr>
<tr>
<td>$\phi$ Output share of fixed production cost</td>
<td>B</td>
<td>0.25</td>
<td>0.125</td>
<td>[0.0052, 0.0986]</td>
<td></td>
</tr>
<tr>
<td>$\alpha$ Capital elasticity of output</td>
<td>B</td>
<td>0.6</td>
<td>0.1</td>
<td>[0.086, 0.202]</td>
<td></td>
</tr>
<tr>
<td>$\gamma_w$ Wage indexation</td>
<td>B</td>
<td>0.5</td>
<td>0.15</td>
<td>[0.1017, 0.3878]</td>
<td></td>
</tr>
<tr>
<td>$\xi_w$ Wage stickiness</td>
<td>B</td>
<td>0.5</td>
<td>0.1</td>
<td>[0.5621, 0.7186]</td>
<td></td>
</tr>
<tr>
<td>$\gamma_p$ Intermediate-good price indexation</td>
<td>B</td>
<td>0.5</td>
<td>0.15</td>
<td>[0.1461, 0.4882]</td>
<td></td>
</tr>
<tr>
<td>$\xi_p$ Intermediate-good price stickiness</td>
<td>B</td>
<td>0.5</td>
<td>0.1</td>
<td>[0.7412, 0.8664]</td>
<td></td>
</tr>
<tr>
<td>$\phi_r$ Monetary policy rate smoothing</td>
<td>B</td>
<td>0.75</td>
<td>0.1</td>
<td>[0.6873, 0.8412]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{\pi}$ Monetary policy response to inflation</td>
<td>G</td>
<td>1.5</td>
<td>0.25</td>
<td>[1.3955, 2.1307]</td>
<td></td>
</tr>
<tr>
<td>$\phi_y$ Monetary policy response to output</td>
<td>G</td>
<td>0.125</td>
<td>0.05</td>
<td>[0.0201, 0.0681]</td>
<td></td>
</tr>
<tr>
<td>$\phi_{\Delta_y}$ Monetary policy response to output growth</td>
<td>G</td>
<td>0.125</td>
<td>0.05</td>
<td>[0.0267, 0.0869]</td>
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</tr>
<tr>
<td>$\pi^*$ Steady-state rate of balanced growth</td>
<td>G</td>
<td>1.163</td>
<td>0.1</td>
<td>[1.079, 1.3613]</td>
<td></td>
</tr>
<tr>
<td>$\psi^*$ Steady-state rate of IS technological change</td>
<td>G</td>
<td>0.077</td>
<td>0.04</td>
<td>[0.0144, 0.1195]</td>
<td></td>
</tr>
<tr>
<td>$\bar{n}$ Normalized steady-state hours worked</td>
<td>N</td>
<td>0</td>
<td>2</td>
<td>[-2.4195, 2.3909]</td>
<td></td>
</tr>
<tr>
<td>$\pi^*$ Steady-state inflation rate</td>
<td>G</td>
<td>0.272</td>
<td>0.1</td>
<td>[0.1459, 0.3955]</td>
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</tr>
<tr>
<td>$\pi_{\pi}$ Steady-state policy rate</td>
<td>G</td>
<td>1.03</td>
<td>0.1</td>
<td>[0.9655, 1.2442]</td>
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<tr>
<td>$\eta$ Entrepreneur survival probability</td>
<td>B</td>
<td>0.973</td>
<td>0.02</td>
<td>[0.9276, 0.9838]</td>
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<tr>
<td>$n/k$ Steady-state net worth-capital ratio</td>
<td>B</td>
<td>0.5</td>
<td>0.07</td>
<td>[0.5089, 0.7027]</td>
<td></td>
</tr>
<tr>
<td>$\mu$ Elasticity of EF premium</td>
<td>G</td>
<td>0.07</td>
<td>0.02</td>
<td>[0.0141, 0.0271]</td>
<td></td>
</tr>
<tr>
<td>$\bar{r}_E$ Steady-state real loan rate</td>
<td>G</td>
<td>1.242</td>
<td>0.05</td>
<td>[1.1374, 1.2929]</td>
<td></td>
</tr>
<tr>
<td>$\rho_b$ Persistence of preference shock</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>[0.606, 0.9558]</td>
<td></td>
</tr>
<tr>
<td>Parameter</td>
<td>Type</td>
<td>Mean</td>
<td>S.D.</td>
<td>Prior Mean</td>
<td>90% interval</td>
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<tr>
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</tr>
<tr>
<td>$\rho_g$</td>
<td>B</td>
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<td>0.2</td>
<td>0.9707</td>
<td>[0.947, 0.9944]</td>
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<tr>
<td>Persistence of exogenous demand shock</td>
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<td></td>
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<tr>
<td>$\rho_w$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2055</td>
<td>[0.0349, 0.3726]</td>
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<tr>
<td>Persistence of wage shock</td>
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<td></td>
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<tr>
<td>$\rho_p$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9039</td>
<td>[0.8407, 0.9774]</td>
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<tr>
<td>Persistence of intermediate-good price markup shock</td>
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</tr>
<tr>
<td>$\rho_i$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.8897</td>
<td>[0.8323, 0.9448]</td>
</tr>
<tr>
<td>Persistence of investment-good price markup shock</td>
<td></td>
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<tr>
<td>$\rho_r$</td>
<td>B</td>
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<td>0.2</td>
<td>0.2256</td>
<td>[0.0632, 0.3813]</td>
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<tr>
<td>Persistence of monetary policy shock in hybrid rule</td>
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<tr>
<td>$\rho_z$</td>
<td>B</td>
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<td>0.2</td>
<td>0.1307</td>
<td>[0.0295, 0.2226]</td>
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<tr>
<td>Persistence of neutral technology shock</td>
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</tr>
<tr>
<td>$\rho_\psi$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9568</td>
<td>[0.9249, 0.9909]</td>
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<tr>
<td>Persistence of IS technology shock</td>
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<tr>
<td>$\rho_\nu$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.985</td>
<td>[0.9774, 0.9924]</td>
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<tr>
<td>Persistence of MEI shock</td>
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<td></td>
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<tr>
<td>$\rho_\mu$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5413</td>
<td>[0.4254, 0.6784]</td>
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<td>Persistence of EF premium shock</td>
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<tr>
<td>$\rho_\eta$</td>
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<td>0.2</td>
<td>0.8246</td>
<td>[0.7015, 0.9634]</td>
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<tr>
<td>Persistence of net worth shock</td>
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<tr>
<td>$\rho_{mqg}$</td>
<td>B</td>
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<td>0.2</td>
<td>0.3782</td>
<td>[0.1867, 0.5541]</td>
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<td>Persistence of monetary shock in quantity rule</td>
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<tr>
<td>$\sigma_b$</td>
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<td>Inf</td>
<td>2.8286</td>
<td>[1.5204, 3.8269]</td>
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<td>S.D. of preference shock innovation</td>
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<tr>
<td>$\sigma_g$</td>
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<td>Inf</td>
<td>0.8964</td>
<td>[0.7581, 1.0319]</td>
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<td>S.D. of exogenous demand shock innovation</td>
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<tr>
<td>$\sigma_w$</td>
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<td>Inf</td>
<td>0.4458</td>
<td>[0.3527, 0.5515]</td>
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<tr>
<td>S.D. of wage shock innovation</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>$\sigma_p$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.1624</td>
<td>[0.1037, 0.2165]</td>
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<tr>
<td>S.D. of intermediate-good price markup shock innovation</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>1.1037</td>
<td>[0.9333, 1.2754]</td>
</tr>
<tr>
<td>S.D. of investment-good price markup shock innovation</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>$\sigma_r$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.1287</td>
<td>[0.1088, 0.1467]</td>
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<tr>
<td>S.D. of monetary policy shock innovation in hybrid rule</td>
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<tr>
<td>$\sigma_z$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>1.7636</td>
<td>[1.4635, 2.0584]</td>
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<td>S.D. of neutral technology shock innovation</td>
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<td></td>
<td></td>
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</tr>
<tr>
<td>$\sigma_\psi$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.3763</td>
<td>[0.2735, 0.4741]</td>
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<tr>
<td>S.D. of IS technology shock innovation</td>
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<tr>
<td>$\sigma_\nu$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>4.1778</td>
<td>[3.3739, 4.9908]</td>
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<tr>
<td>S.D. of MEI shock innovation</td>
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<td>$\sigma_\mu$</td>
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<td>Inf</td>
<td>0.2854</td>
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<tr>
<td>S.D. of EF premium shock innovation</td>
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<tr>
<td>$\sigma_\eta$</td>
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<td>0.5</td>
<td>Inf</td>
<td>0.8999</td>
<td>[0.5881, 1.2119]</td>
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<tr>
<td>S.D. of net worth shock innovation</td>
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</table>
Table 2.2: Prior and posterior distributions of estimated parameters - subsample: 2001Q1-2008Q4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type</td>
<td>Mean</td>
</tr>
<tr>
<td>$\sigma_{mg}$ S.D. of monetary policy shock</td>
<td>IG</td>
<td>0.5</td>
</tr>
<tr>
<td>innovation in quantity rule</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Note: In the type of prior distributions, U, B, G, IG, and N stand for Uniform, Beta, Gamma, Inverse Gamma, and Normal distributions, respectively.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type</td>
<td>Mean</td>
</tr>
<tr>
<td>$W$ Weight on quantity rule</td>
<td>U</td>
<td>0.0859</td>
</tr>
<tr>
<td>$\sigma$ Risk aversion</td>
<td>G</td>
<td>0.375</td>
</tr>
<tr>
<td>$\theta$ Habit persistence</td>
<td>B</td>
<td>0.5852</td>
</tr>
<tr>
<td>$\chi$ Inverse of elasticity of labor supply</td>
<td>G</td>
<td>2.4886</td>
</tr>
<tr>
<td>$\zeta$ Elasticity of investment adjustment cost</td>
<td>G</td>
<td>1.9989</td>
</tr>
<tr>
<td>$\tau$ Inverse of elasticity of utilization rate adjustment cost</td>
<td>G</td>
<td>0.22</td>
</tr>
<tr>
<td>$\phi$ Output share of fixed production cost</td>
<td>B</td>
<td>0.471</td>
</tr>
<tr>
<td>$\alpha$ Capital elasticity of output</td>
<td>B</td>
<td>0.3066</td>
</tr>
<tr>
<td>$\gamma_w$ Wage indexation</td>
<td>B</td>
<td>0.2939</td>
</tr>
<tr>
<td>$\xi_w$ Wage stickiness</td>
<td>B</td>
<td>0.6172</td>
</tr>
<tr>
<td>$\gamma_p$ Intermediate-good price indexation</td>
<td>B</td>
<td>0.3015</td>
</tr>
<tr>
<td>$\xi_p$ Intermediate-good price stickiness</td>
<td>B</td>
<td>0.7693</td>
</tr>
<tr>
<td>$\phi_r$ Monetary policy rate smoothing</td>
<td>B</td>
<td>0.6478</td>
</tr>
<tr>
<td>$\phi_\pi$ Monetary policy response to inflation</td>
<td>G</td>
<td>1.9313</td>
</tr>
<tr>
<td>$\phi_y$ Monetary policy response to output</td>
<td>G</td>
<td>0.043</td>
</tr>
<tr>
<td>$\phi_{\Delta y}$ Monetary policy response to output growth</td>
<td>G</td>
<td>0.0963</td>
</tr>
<tr>
<td>$\bar{\tau}$ Steady-state rate of balanced growth</td>
<td>G</td>
<td>1.2358</td>
</tr>
<tr>
<td>$\bar{\psi}$ Steady-state rate of IS technological change</td>
<td>G</td>
<td>0.073</td>
</tr>
<tr>
<td>$\bar{h}$ Normalized steady-state hours worked</td>
<td>N</td>
<td>1.409</td>
</tr>
<tr>
<td>Parameter</td>
<td>Prior Type</td>
<td>Prior Mean</td>
</tr>
<tr>
<td>-----------</td>
<td>------------</td>
<td>------------</td>
</tr>
<tr>
<td>( \pi ) Steady-state inflation rate</td>
<td>G</td>
<td>0.272</td>
</tr>
<tr>
<td>( \overline{r} ) Steady-state policy rate</td>
<td>G</td>
<td>1.03</td>
</tr>
<tr>
<td>( \eta ) Entrepreneur survival probability</td>
<td>B</td>
<td>0.973</td>
</tr>
<tr>
<td>( n/k ) Steady-state net worth-capital ratio</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \mu ) Elasticity of EF premium</td>
<td>G</td>
<td>0.07</td>
</tr>
<tr>
<td>( \overline{r} E ) Steady-state real loan rate</td>
<td>G</td>
<td>1.242</td>
</tr>
<tr>
<td>( \rho_b ) Persistence of preference shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_g ) Persistence of exogenous demand shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_w ) Persistence of wage shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_p ) Persistence of intermediate-good price markup shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_i ) Persistence of investment-good price markup shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_r ) Persistence of monetary policy shock in hybrid rule</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_z ) Persistence of neutral technology shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_\phi ) Persistence of IS technology shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_\nu ) Persistence of MEI shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_\mu ) Persistence of EF premium shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_\eta ) Persistence of net worth shock</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \rho_{m,g} ) Persistence of monetary shock in quantity rule</td>
<td>B</td>
<td>0.5</td>
</tr>
<tr>
<td>( \sigma_b ) S.D. of preference shock innovation</td>
<td>IG</td>
<td>0.5</td>
</tr>
<tr>
<td>( \sigma_g ) S.D. of exogenous demand shock innovation</td>
<td>IG</td>
<td>0.5</td>
</tr>
<tr>
<td>( \sigma_w ) S.D. of wage shock innovation</td>
<td>IG</td>
<td>0.5</td>
</tr>
<tr>
<td>( \sigma_p ) S.D. of intermediate-good price markup shock innovation</td>
<td>IG</td>
<td>0.5</td>
</tr>
<tr>
<td>( \sigma_i ) S.D. of investment-good price markup shock innovation</td>
<td>IG</td>
<td>0.5</td>
</tr>
<tr>
<td>( \sigma_r ) S.D. of monetary policy shock innovation in hybrid rule</td>
<td>IG</td>
<td>0.5</td>
</tr>
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</table>
### Table 2.3: Prior and posterior distributions of estimated parameters - subsample: 2009Q1-2017Q2

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior</th>
<th>Posterior</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Type Mean S.D.</td>
<td>Mean 90% interval</td>
</tr>
<tr>
<td>$\sigma_z$ S.D. of neutral technology shock innovation</td>
<td>IG 0.5 Inf</td>
<td>2.1435 [1.6746, 2.6424]</td>
</tr>
<tr>
<td>$\sigma_\psi$ S.D. of IS technology shock innovation</td>
<td>IG 0.5 Inf</td>
<td>0.4347 [0.2928, 0.5719]</td>
</tr>
<tr>
<td>$\sigma_\nu$ S.D. of MEI shock innovation</td>
<td>IG 0.5 Inf</td>
<td>3.4362 [2.1762, 4.4342]</td>
</tr>
<tr>
<td>$\sigma_\mu$ S.D. of EF premium shock innovation</td>
<td>IG 0.5 Inf</td>
<td>0.1793 [0.1365, 0.2183]</td>
</tr>
<tr>
<td>$\sigma_\eta$ S.D. of net worth shock innovation</td>
<td>IG 0.5 Inf</td>
<td>1.2113 [0.7844, 1.6362]</td>
</tr>
<tr>
<td>$\sigma_{mg}$ S.D. of monetary policy shock innovation in quantity rule</td>
<td>IG 0.5 Inf</td>
<td>0.4336 [0.3271, 0.5388]</td>
</tr>
<tr>
<td>$W$ Weight on quantity rule</td>
<td>U 0.2466</td>
<td>0.2466 [0.0046, 0.5121]</td>
</tr>
<tr>
<td>$\sigma$ Risk aversion</td>
<td>G 2 0.375</td>
<td>1.0819 [0.7355, 1.397]</td>
</tr>
<tr>
<td>$\theta$ Habit persistence</td>
<td>B 0.7 0.1</td>
<td>0.6755 [0.5693, 0.7927]</td>
</tr>
<tr>
<td>$\chi$ Inverse of elasticity of labor supply</td>
<td>G 2 0.75</td>
<td>2.2995 [1.3164, 3.3768]</td>
</tr>
<tr>
<td>$\zeta$ Elasticity of investment adjustment cost</td>
<td>G 4 1.5</td>
<td>4.839 [3.2622, 6.4291]</td>
</tr>
<tr>
<td>$\tau$ Inverse of elasticity of utilization rate adjustment cost</td>
<td>G 0.22 0.1</td>
<td>0.2482 [0.0801, 0.4042]</td>
</tr>
<tr>
<td>$\phi$ Output share of fixed production cost</td>
<td>B 0.25 0.125</td>
<td>0.124 [0.0181, 0.2228]</td>
</tr>
<tr>
<td>$\alpha$ Capital elasticity of output</td>
<td>B 0.6 0.1</td>
<td>0.1712 [0.1142, 0.2296]</td>
</tr>
<tr>
<td>$\gamma_w$ Wage indexation</td>
<td>B 0.5 0.15</td>
<td>0.4677 [0.2475, 0.6837]</td>
</tr>
<tr>
<td>$\xi_w$ Wage stickiness</td>
<td>B 0.5 0.1</td>
<td>0.6025 [0.4935, 0.7156]</td>
</tr>
<tr>
<td>$\gamma_p$ Intermediate-good price indexation</td>
<td>B 0.5 0.15</td>
<td>0.5818 [0.3744, 0.7944]</td>
</tr>
<tr>
<td>$\xi_p$ Intermediate-good price stickiness</td>
<td>B 0.5 0.1</td>
<td>0.8019 [0.7269, 0.8755]</td>
</tr>
<tr>
<td>$\phi_r$ Monetary policy rate smoothing</td>
<td>B 0.75 0.1</td>
<td>0.801 [0.6807, 0.9109]</td>
</tr>
<tr>
<td>$\phi_\pi$ Monetary policy response to inflation</td>
<td>G 1.5 0.25</td>
<td>1.6664 [1.2879, 2.0263]</td>
</tr>
<tr>
<td>$\phi_y$ Monetary policy response to output</td>
<td>G 0.125 0.05</td>
<td>0.0794 [0.0267, 0.1376]</td>
</tr>
</tbody>
</table>

*Note: In the type of prior distributions, U, B, G, IG, and N stand for Uniform, Beta, Gamma, Inverse Gamma, and Normal distributions, respectively.*
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Type</th>
<th>Mean</th>
<th>S.D.</th>
<th>Posterior Mean</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{\Delta y}$</td>
<td>G</td>
<td>0.125</td>
<td>0.05</td>
<td>0.1161</td>
<td>[0.0576, 0.1709]</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>G</td>
<td>1.163</td>
<td>0.1</td>
<td>1.1451</td>
<td>[1.0167, 1.2853]</td>
</tr>
<tr>
<td>$\bar{\psi}$</td>
<td>G</td>
<td>0.077</td>
<td>0.04</td>
<td>0.0763</td>
<td>[0.0176, 0.1353]</td>
</tr>
<tr>
<td>$\bar{h}$</td>
<td>N</td>
<td>0</td>
<td>2</td>
<td>-2.3959</td>
<td>[-4.5247, -0.1487]</td>
</tr>
<tr>
<td>$\bar{\pi}$</td>
<td>G</td>
<td>0.272</td>
<td>0.1</td>
<td>0.2334</td>
<td>[0.1023, 0.3559]</td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>G</td>
<td>1.03</td>
<td>1.1051</td>
<td>[0.9671, 1.2318]</td>
<td></td>
</tr>
<tr>
<td>$\eta$</td>
<td>B</td>
<td>0.973</td>
<td>0.02</td>
<td>0.9292</td>
<td>[0.8976, 0.9599]</td>
</tr>
<tr>
<td>$n/k$</td>
<td>B</td>
<td>0.5</td>
<td>0.07</td>
<td>0.5809</td>
<td>[0.4934, 0.6743]</td>
</tr>
<tr>
<td>$\mu$</td>
<td>G</td>
<td>0.07</td>
<td>0.02</td>
<td>0.0336</td>
<td>[0.0223, 0.0446]</td>
</tr>
<tr>
<td>$\bar{r}^E$</td>
<td>G</td>
<td>1.242</td>
<td>0.05</td>
<td>1.2259</td>
<td>[1.1388, 1.3026]</td>
</tr>
<tr>
<td>$\rho_b$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3179</td>
<td>[0.0456, 0.5699]</td>
</tr>
<tr>
<td>$\rho_g$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9493</td>
<td>[0.9213, 0.9821]</td>
</tr>
<tr>
<td>$\rho_w$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2446</td>
<td>[0.0481, 0.4495]</td>
</tr>
<tr>
<td>$\rho_p$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.8293</td>
<td>[0.6927, 0.9755]</td>
</tr>
<tr>
<td>$\rho_i$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.8467</td>
<td>[0.7585, 0.944]</td>
</tr>
<tr>
<td>$\rho_r$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.4587</td>
<td>[0.1, 0.8045]</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.1655</td>
<td>[0.0381, 0.2872]</td>
</tr>
<tr>
<td>$\rho_{\phi}$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9619</td>
<td>[0.9374, 0.99]</td>
</tr>
<tr>
<td>$\rho_{\eta}$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.9659</td>
<td>[0.9472, 0.9853]</td>
</tr>
<tr>
<td>$\rho_{\mu}$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.2073</td>
<td>[0.0559, 0.3554]</td>
</tr>
<tr>
<td>$\rho_{\eta}$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.5218</td>
<td>[0.285, 0.7456]</td>
</tr>
<tr>
<td>$\rho_{mg}$</td>
<td>B</td>
<td>0.5</td>
<td>0.2</td>
<td>0.3737</td>
<td>[0.1227, 0.65]</td>
</tr>
<tr>
<td>$\sigma_b$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>1.8163</td>
<td>[0.8817, 2.7888]</td>
</tr>
<tr>
<td>$\sigma_g$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.7046</td>
<td>[0.551, 0.8704]</td>
</tr>
<tr>
<td>$\sigma_w$</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.3571</td>
<td>[0.2584, 0.4449]</td>
</tr>
</tbody>
</table>
### Table 2.4: Forecast error variance decompositions - full sample

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Prior Type</th>
<th>Mean</th>
<th>S.D.</th>
<th>Posterior Mean</th>
<th>90% interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_p$</td>
<td>S.D. of intermediate-good price markup shock innovation</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.1899</td>
</tr>
<tr>
<td>$\sigma_i$</td>
<td>S.D. of investment-good price markup shock innovation</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>1.2825</td>
</tr>
<tr>
<td>$\sigma_r$</td>
<td>S.D. of monetary policy shock innovation in hybrid rule</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.1086</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>S.D. of neutral technology shock innovation</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>1.7497</td>
</tr>
<tr>
<td>$\sigma_\phi$</td>
<td>S.D. of IS technology shock innovation</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.2778</td>
</tr>
<tr>
<td>$\sigma_\mu$</td>
<td>S.D. of MEI shock innovation</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>3.4748</td>
</tr>
<tr>
<td>$\sigma_\eta$</td>
<td>S.D. of net worth shock innovation</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.3267</td>
</tr>
<tr>
<td>$\sigma_{m_g}$</td>
<td>S.D. of monetary policy shock innovation in quantity rule</td>
<td>IG</td>
<td>0.5</td>
<td>Inf</td>
<td>0.7029</td>
</tr>
</tbody>
</table>

Note: In the type of prior distributions, U, B, G, IG, and N stand for Uniform, Beta, Gamma, Inverse Gamma, and Normal distributions, respectively.
### Table 2.5: Forecast error variance decompositions - subsample: 2001Q1-2008Q4

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Loans</th>
</tr>
</thead>
<tbody>
<tr>
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<td>T=8 T=32</td>
<td>T=8 T=32</td>
<td>T=8 T=32</td>
<td>T=8 T=32</td>
</tr>
<tr>
<td>$z^b$ Preference</td>
<td>14 13.19</td>
<td>69.14 67.35</td>
<td>2.53 2.4</td>
<td>2.75 2.21</td>
</tr>
<tr>
<td>$z^g$ Exogenous demand</td>
<td>22.94 21.56</td>
<td>2.03 2.06</td>
<td>1.06 1.03</td>
<td>1.42 1.19</td>
</tr>
<tr>
<td>$z^w$ Wage</td>
<td>0.82 0.9</td>
<td>0.44 0.45</td>
<td>1.26 1.19</td>
<td>1.5 1.16</td>
</tr>
<tr>
<td>$z^i$ Investment-good price markup</td>
<td>0.52 0.51</td>
<td>0.02 0.03</td>
<td>2.33 1.81</td>
<td>0.25 0.25</td>
</tr>
<tr>
<td>$z^r$ Hybrid monetary policy</td>
<td>0.9 0.85</td>
<td>0.26 0.25</td>
<td>1.96 1.47</td>
<td>1.38 1.13</td>
</tr>
<tr>
<td>$z^z$ Neutral technology</td>
<td>40.2 37.83</td>
<td>21.23 20.72</td>
<td>20.91 15.78</td>
<td>4.97 5.29</td>
</tr>
<tr>
<td>$z^v$ IS technology</td>
<td>4.23 8.08</td>
<td>2 3.53</td>
<td>25.32 38.75</td>
<td>34.75 41.56</td>
</tr>
<tr>
<td>$z^v$ MEI</td>
<td>1.68 1.59</td>
<td>0.32 0.61</td>
<td>4.2 3.32</td>
<td>1.5 3.24</td>
</tr>
<tr>
<td>$z^\mu$ EF premium</td>
<td>0.55 0.54</td>
<td>0.18 0.22</td>
<td>3.39 2.68</td>
<td>1.22 1.04</td>
</tr>
<tr>
<td>$z^\eta$ Net worth</td>
<td>0.44 0.44</td>
<td>1.53 1.93</td>
<td>5.8 4.72</td>
<td>40.16 35.32</td>
</tr>
<tr>
<td>$z^{mg}$ Quantitative monetary policy</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
<td>0 0</td>
</tr>
</tbody>
</table>

### Table 2.6: Forecast error variance decompositions - subsample: 2009Q1-2017Q2

<table>
<thead>
<tr>
<th></th>
<th>Output</th>
<th>Consumption</th>
<th>Investment</th>
<th>Loans</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T=8 T=32</td>
<td>T=8 T=32</td>
<td>T=8 T=32</td>
<td>T=8 T=32</td>
</tr>
<tr>
<td>$z^b$ Preference</td>
<td>8.02 7.63</td>
<td>29.61 28.26</td>
<td>0.01 0.01</td>
<td>0.07 0.07</td>
</tr>
<tr>
<td>$z^g$ Exogenous demand</td>
<td>24.82 23.74</td>
<td>2.1 2.16</td>
<td>0.22 0.15</td>
<td>0.23 0.19</td>
</tr>
<tr>
<td>$z^w$ Wage</td>
<td>0.14 0.17</td>
<td>0.28 0.33</td>
<td>0.13 0.14</td>
<td>0.58 0.53</td>
</tr>
<tr>
<td>$z^p$ Intermediate-good price markup</td>
<td>8.84 10.96</td>
<td>11.04 12.76</td>
<td>20.4 17.97</td>
<td>21.13 18.66</td>
</tr>
<tr>
<td>$z^i$ Investment-good price markup</td>
<td>0.48 0.53</td>
<td>0.05 0.09</td>
<td>8.3 6.15</td>
<td>0.4 0.59</td>
</tr>
<tr>
<td>$z^r$ Hybrid monetary policy</td>
<td>0.57 0.56</td>
<td>0.85 0.82</td>
<td>1.08 0.71</td>
<td>0.78 0.66</td>
</tr>
<tr>
<td>$z^z$ Neutral technology</td>
<td>54.62 51.98</td>
<td>53.21 50.9</td>
<td>28.87 18.32</td>
<td>9.69 8.98</td>
</tr>
<tr>
<td>$z^v$ IS technology</td>
<td>0.13 2</td>
<td>1.48 2.4</td>
<td>12.98 35.08</td>
<td>34.89 38.26</td>
</tr>
<tr>
<td>$z^v$ MEI</td>
<td>1.67 1.71</td>
<td>0.47 1.35</td>
<td>24.47 18.86</td>
<td>3.83 7.5</td>
</tr>
<tr>
<td>$z^\mu$ EF premium</td>
<td>0.02 0.02</td>
<td>0 0</td>
<td>0.3 0.19</td>
<td>3.76 3.07</td>
</tr>
<tr>
<td>$z^\eta$ Net worth</td>
<td>0.1 0.11</td>
<td>0.04 0.07</td>
<td>2.19 1.67</td>
<td>24.36 21.25</td>
</tr>
<tr>
<td>$z^{mg}$ Quantitative monetary policy</td>
<td>0.58 0.59</td>
<td>0.87 0.85</td>
<td>1.07 0.75</td>
<td>0.28 0.24</td>
</tr>
</tbody>
</table>
Figure 2.1: Prior-posterior distributions of estimated parameters
CHAPTER 2. CHINA’S MONETARY POLICY AND BUSINESS CYCLE

Figure 2.2: Historical decomposition of output growth

Figure 2.3: Historical decomposition of investment growth
Figure 2.4: Impulse responses to monetary policy rate shock $e_r$ (+1 s.d.)

Figure 2.5: Impulse responses to neutral technology shock $e_z$ (+1 s.d.)
Figure 2.6: Impulse responses to net worth shock $e_\eta$ (+1 s.d.)
Chapter 3

Optimal Monetary Policy in Open Economies Revisited

3.1 Introduction

In a world of integrated trade in goods and assets, sovereign nations become more and more interdependent. The prolonged recession after the Global Financial Crisis again reminds policy makers in major economies of the depth and scope of such interrelations. Understanding the nature of cross-country spillovers of shocks and policy impacts comes back to center stage in policy discussions. Should central banks cooperate in order to internalize the possible externality from policy reactions? Is there any gain from such cooperation? And if so, how large might it be?

The desirability of policy cooperation, namely whether there exist gains from cooperation, has been one of the central issues in macroeconomics. The root of the discussion can be traced way back to Hume (1752), who first noticed possible policy spillovers among countries. Since then, there have been vast studies investigating the nature of policy games in open economies. Recently, many have studied optimal monetary policy in open economies using the microfounded, open-economy sticky-price models based

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1Spillovers originated from other countries are as important for interdependent large open economies as for small open economies. The difference is that in the case of two large open economies, there is a dimension of interaction between policy makers, which is absent for the case of one large and one small open economies. Policy makers in small economies take policies in large economies as given and should only care about the movements of macroeconomic and financial variables of the large economies that can have direct impact on the small open economies, such as output or consumption of the large economy, monetary policy rate, and CPI inflation rate. A useful example can be found in Davis et al. (2017). Their research questions are different from the analysis in Chapter 3 but the model can be regarded as a limiting case of the two-country model when the home country becomes a small open economy with country size approaching to zero.
on the so-called New Open Economy Macroeconomics (hereafter, NOEM) initiated by Obstfeld and Rogoff (1995) and Svensson and van Wijnbergen (1989). Contrary to the traditional studies using the Mundell-Fleming model, correct welfare can be computed with the NOEM models. Thus, comparison of different policies becomes possible without resort to ad hoc criteria.

As will be discussed in detail in the next subsection, optimal monetary policy in open economies has been investigated under many different settings in the NOEM, such as under cooperation or noncooperation, PCP or LCP, and with or without home bias. Consequently, our understanding of how monetary policy should be conducted in an interconnected world is deepened. There is, however, one last missing piece, which has not yet been analyzed in a theoretical DSGE model. That is, how the optimal noncooperative monetary policy under LCP should be conducted, or whether there is any gain from cooperation under LCP. These are the questions to which we aim to give answers in this paper.

For this purpose, we first solve the equilibrium conditions under monopolistic competition and sticky prices in a two-country model. Also, the Ramsey (deterministic) steady states under both cooperative and noncooperative regimes are computed. In both schemes, the deterministic steady state turns out to be identical to that under the flexible-price equilibrium. Thus, the exact welfare comparison between cooperation and noncooperation becomes possible. Then, we approximate welfare around this deterministic steady state up to the second order. In a noncooperative regime, even if the steady state is efficient thanks to the optimal subsidy, linear terms cannot be eliminated. Following Sutherland (2002), Benigno and Woodford (2005) and Benigno and Benigno (2006), we take a second-order approximation to the structural equations to substitute out the linear terms by only second-order terms. Correct welfare metrics up to the second-order approximation are thus obtained.

Our loss functions show that noncooperative policy makers naturally aim to stabilize variables whose fluctuations are to be minimized by cooperative policy makers as analyzed in Engel (2011), including output, PPI inflation rates, import price inflation rates, and deviations from the law of one price. In addition, they also seek to stabi-
lize fluctuations in the real marginal costs that firms face when setting prices in both domestic and export markets. These additional objectives are unique to the noncooperative game and therefore the sources for potential gains from cooperation, which are absent in the previous studies on optimal monetary policy in open economies.\footnote{Technically, these additional objectives arise from the linear terms in the second-order approximated welfare, that are eventually substituted by second-order approximated aggregate supply conditions.}

Then, in order to clarify the nature of optimal monetary policy in open economies, we compare impulse responses under optimal monetary policies among three cases: (1) PCP; (2) cooperative regime and LCP; (3) noncooperative regime and LCP. Note that in our setting with only technology shocks, optimal cooperative as well as noncooperative policies result in identical allocations and prices under PCP.\footnote{Note that our model assumes a Cobb-Douglas aggregator for domestic and foreign goods. For the cases where cooperative and noncooperative policies produce different outcomes, see Benigno and Benigno (2003).}

Fluctuations in CPI inflation rates become smaller under LCP than under PCP. This is because the violation of the law of one price induces inefficient price dispersions within producer as well as export prices, as emphasized by Engel (2011). As a result, the ‘inward-looking’ policy that focuses on stabilization of PPI inflation rates is no more optimal under LCP. In addition, under LCP, noncooperative policy makers stabilize CPI inflation rates more than cooperative central banks do. This larger stabilization motive arises from the unique objectives in the loss functions under noncooperation. Inability to cooperate constrains the dynamics toward more efficient outcomes. Reactions of domestic output to a domestic technology shock become smaller under noncooperation. Without any frictions, the global welfare increases when the production of the country with favorable efficiency shocks increases. This difference in the responses of output creates room for cooperative policies to improve global welfare.

We also compute the welfare gain from cooperation under LCP by solving the nonlinear Ramsey problem. Welfare gains from cooperation become largest with log utility even though both countries become insular in structural equations under PCP. Still, welfare gains computed from nonlinear Ramsey problems are not sizable with only technology shocks. Within the reasonable range of parameter calibration, the welfare cost stemming from the inability to cooperate can only be, at most, 0.04 per cent in consumption units, in response to one standard deviation of technology shocks. Corsetti (2008) remarks that in early leading studies, the quantitative assessment of the welfare gains from cooperation is found far from sufficient to be in favor of cooperation, and whether it still holds in richer models is a critical research question. Our paper finds that given only price rigidities, sizable welfare gains may not arise from cooperation.
3.1.1 Literature Review

Let us first classify the previous studies on optimal monetary policy in open economies by three dimensions.\(^6\) The first dimension regards the assumption on nominal rigidities, that is, either the one-period ahead price setting or the staggered price setting à la Calvo (1983).\(^7\) In early studies using the NOEM framework, nominal rigidities are introduced as the one-period ahead price setting. With money supply as the control variable of monetary policy, analytical solutions can be obtained. Thus, no approximation is necessary for optimal policy analysis. With more relevance to the price setting behavior and monetary policy in practice, the staggered price contract together with nominal interest rates controlled by the central bank becomes the major assumption about nominal frictions, in particular after Clarida, Galí, and Gertler (2002) in the open-economy context.\(^8\) In contrast to the one-period ahead price setting, optimal monetary policy is analyzed in a linear-quadratic framework following Rotemberg and Woodford (1997), Woodford (2003) and Benigno and Woodford (2005). Central banks maximize the correctly approximated social welfare up to the second order subject to the linearly approximated structural equations. The second dimension is about export price setting, namely PCP or LCP.\(^9\) In the former, export prices fully reflect exchange rate fluctuations, while not at all in the latter. The third dimension is whether monetary policy in open economies is conducted in a cooperative or noncooperative manner.

Table 3.1 offers a taxonomy of previous studies on optimal monetary policy in open economies. Regarded as the beginning of the NOEM framework for monetary policy analysis in open economies, Obstfeld and Rogoff (1995) develop a micro-founded two-country model with PCP and a one-period in advance price setting rule to analyze the dynamics of exchange rates and other variables in response to money supply shocks. Their investigation of the (log-linearized) global welfare appraises the first-order welfare effect of monetary expansion on raising global demand and world output. It also suggests that the conventional Mundell-Fleming paradigm may overstate the importance of

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\(^6\)Corsetti, Dedola, and Leduc (2010) offer a comprehensive survey of optimal monetary policy in open economies including other aspects such as financial market imperfections.

\(^7\)Rotemberg (1982) proposes a different type of price stickiness due to a staggered cost adjustment process.

\(^8\)According to the Calvo rule, firms reset prices in a forward-looking rational expectation manner. This raises the question of how to affect and/or manage the private sector’s rational expectation for monetary policy practice. Related theoretical discussions include conducting monetary policy under commitment versus under discretion, delegation problem, credibility of cooperation, targeting rules versus instrument rules, etc.. Investigation of these issues are beyond the scope of this paper. As an incomplete list, see, for example, Persson and Tabellini (1995), Benigno (2002), Bilbiie (2002), Jensen (2002), Woodford (2003) and Svensson (2002, 2003, 2004).

\(^9\)Corsetti and Pesenti (2005b) briefly analyze a third and less used pricing behavior: dollar-pricing.
the *beggar-thy-neighbor* effects that a currency-depreciating country inflicts on trading partners since the induced terms-of-trade and current-account effects are only of second-order welfare importance. Corsetti and Pesenti (2001) extend the model of Obstfeld and Rogoff (1995) to highlight the international dimension of distortions stemming from a country’s monopoly power on its terms of trade by assuming different elasticities of substitution within and across goods categories.\(^\text{10}\) They keep the assumptions of PCP, one-period ahead price resetting rule and money supply shocks but focus on national welfare. A domestic monetary expansion can be either *beggar-thy-neighbor* or *beggar-thyself* depending on the elasticity of substitution, giving rise to national policy makers’ incentives to manipulate the terms of trade in favor of their own welfare. Obstfeld and Rogoff (2002) assume the one-period ahead rule for nominal wage setting (prices for goods are flexible) and the existence of the non-tradable sector for their examination of international cooperation under PCP. Utility of each country is expressed in terms of covariances of logs of endogenous variables, and monetary rules are assumed as explicit (log-linear) functions of exogenous productivity shocks in a stochastic environment.\(^\text{11}\) When nominal stickiness has little interaction with real distortions, welfare gains from cooperation (in percentage output) are relatively small.\(^\text{12}\)

Devereux and Engel (2003) assume LCP and both strategic games in a two-country model while keeping the price setting in the period-by-period basis.\(^\text{13}\) They derive and compare optimal monetary rules (as log-linear functions of productivity and velocity shocks) to examine the desirability of flexible exchange rates as advocated in Friedman (1953). The flexible exchange rate regime is no longer optimal under LCP. Distortions stemming from the violation of the law of one price should be corrected by restricting the fluctuations of nominal exchange rates. Thus, optimal policy under LCP fully stabilizes fluctuations in exchange rates in both games. Corsetti and Pesenti (2005a) propose a unifying approach to model the exchange rate pass-through in which PCP and LCP are two extreme cases of the parameterization.\(^\text{14}\) No welfare gains from cooperation are found under either complete or no exchange rate pass-through. In general cases with partial exchange rate pass-through they argue that a country can do better than

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\(^\text{10}\) For a detailed discussion on this issue, see Tille (2001).

\(^\text{11}\) Productivity shocks, along with cost-push shocks, become the main sources of exogenous disturbances that optimal monetary policy responds to in later studies.

\(^\text{12}\) Note that Obstfeld and Rogoff (2002) define the cooperation gain as the gain from deviating from flex-wage equilibrium to cooperative equilibrium. Noncooperative equilibrium lies between the flex-wage and cooperative policy responses and thus the estimation is an upper bound on the gains from moving from noncooperation to cooperation.

\(^\text{13}\) Much empirical evidence points to the possibility of LCP, see, for example, Engel (1999), Engel and Rogers (2001), Parsley and Wei (2001), and Atkeson and Burstein (2008).

\(^\text{14}\) Corsetti and Pesenti (2005a) also offer insights from the staggered price setting.
‘keeping one’s house in order’ but whether the gains are sizable is left for future studies with more realistic model settings.

Clarida, Galí, and Gertler (2002), Benigno and Benigno (2003, 2006) all assume the staggered price adjustment rule à la Calvo (1983) and obtain quadratic loss functions under cooperation as well as noncooperation for their respective optimal policy analysis under PCP. By contrast, Clarida, Galí, and Gertler (2002) choose output as policy variables under noncooperative regime and set the first derivative of national utility function to zero by assuming an appropriate subsidy to eliminate the linear terms in the second-order approximation of the utility function. As a result, Ramsey steady states become different between cooperation and noncooperation. Benigno and Benigno (2003) set up a ratio of the notional price over the average actual price as noncooperative policy instruments, and obtain a zero first derivative of the national utility function from households’ price setting condition as monopolists selling goods to achieve the elimination of the linear terms. The quadratic loss function under noncooperation can be derived since price stability turns out to be optimal monetary policy. Benigno and Benigno (2006) choose PPI inflation rates for their noncooperative games and make use of second-order approximations of some of the structural equations to substitute out those linear terms following Sutherland (2002) and Benigno and Woodford (2005). Besides the methodological differences, these three studies also take on different focuses on the implications of optimal policy analysis. Specifically, Clarida, Galí, and Gertler (2002) appraise the potential gains from cooperation arising from internalizing the terms-of-trade externalities, in the context of (inefficient) cost-push shocks and discretionary optimal policy. Benigno and Benigno (2003) explore the theoretical conditions under which flexible-price allocations are optimal, and cooperative and noncooperative allocations coincide under PCP. Finally, Benigno and Benigno (2006) show how to design simple rules for noncooperative policy makers to achieve cooperative allocations in the linear-quadratic framework with technology shocks, markup shocks and government spending shocks.

Engel (2011) incorporates the staggered price setting rule for optimal monetary analysis under LCP and the cooperative regime. Home bias in consumption preferences is also assumed. With home bias, central banks face the trade-off between the costs of currency misalignment and stabilization of asymmetric output fluctuations. The derived quadratic global loss function highlights international relative price misalign-

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15 Benigno and Benigno (2003) also assume a one-period ahead price setting rule for policies under commitment and the Calvo rule for policies under discretion.

16 Faia and Monacelli (2008) and Duarte and Obstfeld (2008) also consider different consumption preferences for a similar purpose.
ments stemming from the violation of the law of one price under LCP. Thus, optimal cooperative policy under LCP should trade off these misalignments with inflation and output goals, and should target CPI inflation rates rather than just PPI inflation rates. Our paper is an extension from Engel (2011) to the noncooperative game, providing the final block of the class of the NOEM literature as summarized in Table 1.17

Table 3.1: Taxonomy of optimal monetary policy in open economies

<table>
<thead>
<tr>
<th>Games</th>
<th>Pricing</th>
<th>PCP</th>
<th>LCP</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ahead</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ahead</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: OR denotes Obstfeld and Rogoff, CP denotes Corsetti and Pesenti, CGG denotes Clarida, Galí and Gertler, BB denotes Benigno and Benigno, and DE denotes Devereux and Engel.

The rest of the paper is organized as follows. Section 3.2 specifies the model and derives equilibrium conditions. Section 3.3 sets up optimal policy problems in both nonlinear and linear-quadratic frameworks. Quadratic loss functions under LCP and noncooperation are derived. Section 3.4 compares impulse responses under both games and computes welfare cost stemming from noncooperation. Section 3.5 concludes.

3.2 The Model

The model is close to the one considered in Engel (2011). There are two countries of equal size, Home and Foreign, each populated with a continuum of households with population size normalized to unity. Agents in the two countries consume both home goods and foreign goods but have a symmetric home bias. Households supply labor services to firms within their own country via a competitive labor market. Households are also the owner of domestic firms. Firms maximize profits in a monopolistically

17 For the sake of completeness of this literature review, we note that there are other studies that also incorporate the key features of the models in the class of the NOEM. Obstfeld and Rogoff (1998, 2000) and de Paoli (2009) assume PCP in their monetary policy analysis. Bacchetta and van Wincoop (2000), Betts and Devereux (2000) and Engel (2003) consider LCP and one-period ahead price setting. Sutherland (2006) assumes LCP for monetary policy analysis in a small open economy.
CHAPTER 3. OPTIMAL MONETARY POLICY

A competitive market using labor as the only input according to aggregate technology. Governments levy a lump-sum tax on households and subsidize firms so that the deterministic steady-state output level becomes efficient. Central banks are benevolent and aim to maximize social welfare through either cooperation or noncooperation.

3.2.1 Households

A representative household in the home country maximizes welfare:

$$W_{H,t_0} \equiv \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [u(C_t) - v(h_t)]$$ (3.2.1)

subject to the budget constraint:

$$\mathbb{E}_t [m_{t,t+1} A_{t+1}] + B_{t+1} + P_t C_t \leq A_t + (1 + i_{t-1}) B_t + W_t h_t + \Pi_t + T_t,$$

for $t \geq t_0$, where the consumption aggregator $C_t$, the aggregate consumption of locally produced goods $C_{H,t}$, and the aggregate consumption of imported goods $C_{F,t}$ is given by

$$C_t = C_{H,t}^{\frac{\nu}{2}} C_{F,t}^{1-\frac{\nu}{2}},$$ (3.2.2)

$$C_{H,t} = \left[ \int_0^1 C_{H,t}(j)^{1-\frac{1}{\epsilon}} \, dj \right]^{\frac{1}{1-\frac{1}{\epsilon}}},$$ (3.2.3)

$$C_{F,t} = \left[ \int_0^1 C_{F,t}(j^*)^{1-\frac{1}{\epsilon}} \, dj^* \right]^{\frac{1}{1-\frac{1}{\epsilon}}},$$ (3.2.4)

respectively. $u(.)$ is the period utility function, increasing and concave in consumption. $v(.)$ is the period disutility function, increasing and convex in labor $h_t$ (measured by working hours). $W_t$ denotes the nominal wage. $A_{t+1}$ denotes the holdings of the state contingent (Arrow) securities at the end of period $t$ denominated in the domestic currency, which equates the marginal rates of substitutions of two countries even ex post. $m_{t,t+1}$ denotes the price of the Arrow securities in period $t$ which gives an unitary return in period $t + 1$. $B_t$ is the amount of one-period risk-free nominal bonds held at the beginning of period $t$ with net rate of return $i_{t-1}$. $\Pi_t$ represents the dividend from the ownership of firms. $T_t$ represents the lump-sum tax levied by the government. $\beta$ is the discount factor. $\epsilon$ denotes the elasticity of substitution among differentiated varieties within each country. $\nu \in [0, 2]$ determines the (symmetric) home bias. When $\nu$ is larger (smaller) than unity, consumer preference exhibits home (foreign) bias. There is no
CHAPTER 3. OPTIMAL MONETARY POLICY

home bias when \( \nu \) equals unity. \( C_{H,t}(j) \) and \( C_{F,t}(j^*) \) denote the home representative household’s consumption of the goods produced by the home firm \( j \) and the foreign firm \( j^* \), respectively. Note that Lagrange multipliers on the constraints in equations (3.2.2) to (3.2.4) represent CPI \( P_t \), PPI \( P_{H,t} \), and the import price index \( P_{F,t} \). A representative household in the foreign country solves a similar optimization problem on the welfare:

\[
W_{F,t} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [ u (C_t^*) - v (h_t^*) ] .
\] (3.2.5)

3.2.2 Firms

Firm \( j \) in the home country sets prices in a monopolistically competitive market to maximize the present discounted value of profits:

\[
E_{t_0} \sum_{t=t_0}^{\infty} \theta^{t-t_0} m_{t_0,t} \Pi_t (j) ,
\]

where

\[
\Pi_t (j) = (1 + \tau) P_{H,t}(j) C_{H,t}(j) + (1 + \tau) S_t P_{H,t}^*(j) C_{H,t}^*(j) - W_t h_t (j)
\]

subject to the production function:

\[
Y_t (j) = \exp (z_t) h_t (j) ,
\] (3.2.6)

and the resource constraint:

\[
Y_t (j) = C_{H,t}(j) + C_{H,t}^*(j).
\] (3.2.7)

\( S_t \) denotes the nominal exchange rate of the foreign currency in units of the home currency. \( \tau \) represents the government subsidy rate. Firm \( j \) produces \( Y_t (j) \) of the product by hiring \( h_t (j) \) of labor service from the domestic households according to aggregate production technology \( \exp (z_t) \), where \( z_t \) follows an AR(1) exogenous process. Firms set their optimal prices in a staggered manner à la Calvo (1983) rule. Each time, only with probability \( 1 - \theta \), can they re-optimize their prices. Note that the Lagrange multiplier on a constraint where the production function in equation (3.2.6) and the resource constraint in equation (3.2.7) are combined represents nominal marginal costs:

\[
NMC_t = \frac{W_t}{\exp (z_t)}.
\]
There is no firm specificity in marginal costs.

Regarding the export price, there are two types of price setting. Under PCP, firms fully reflect changes in exchange rates in export prices. Thus, the law of one price holds:

\[ P_{H,t}(j) = S_t P_{H,t}^*(j). \]

On the other hand, under LCP, firms faces the same Calvo (1983) friction even when setting export prices. As a result, firm \( j \) reoptimizes both \( P_{H,t}(j) \) and \( P_{H,t}^*(j) \) in order to maximize profits.\(^\text{18}\)

Firm \( j^* \) in the foreign country solves a similar profit maximization problem.

### 3.2.3 Governments and Central Banks

The government in each country collects a lump sum tax from households and subsidizes firms to eliminate steady state distortions stemming from monopolistic competition.\(^\text{19}\) Thus, the subsidy rate is given by

\[ \tau = \frac{1}{\epsilon - 1}. \]

Governments’ budget constraints are

\[
\begin{align*}
T_t &= \tau \int_0^1 \left[ P_{H,t}(j) C_{H,t}(j) + S_t P_{H,t}^*(j) C_{H,t}^*(j) \right] \, dj, \\
T^*_t &= \tau \int_0^1 \left[ \frac{P_{F,t}(j^*)}{S_t} C_{F,t}(j^*) + P_{F,t}^*(j^*) C_{F,t}^*(j^*) \right] \, dj^*.
\end{align*}
\]

Balanced budgets are always achieved for the two governments.

Benevolent central banks aim to maximize social welfare as Ramsey planners. We consider two cases: both central banks cooperate to maximize global welfare; each maximizes social welfare of its own country in a noncooperative game. Details of such optimal policies will be discussed later.

\(^{18}\)We do not consider interim cases as in Monacelli (2005).

\(^{19}\)There is no strategic interaction between the government and the central bank.
3.2.4 Aggregate Conditions

Taking the integral of equation (3.2.6) over $j$ gives the aggregate production function of the home country:

$$Y_t = \exp(z_t) h_t.$$ 

Taking the integral of the resource constraint equation (3.2.7) over $j$ and making use of the Hicksian demand functions for good $j$ by consumers in both countries gives the aggregate resource constraint of the home country:

$$Y_t = C_{H,t} \Delta_{H,t} + C_{H,t}^* \Delta_{H,t}^*,$$

where $\Delta_{H,t} \equiv \int_1^0 \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} dj$ and $\Delta_{H,t}^* \equiv \int_1^0 \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} dj$ are the price dispersion terms. (Derivation of the Hicksian demand functions is in Appendix A.) The foreign country has an analogous production function and resource constraint.

We assume a complete assets market, and thus trades in the Arrow securities equate the marginal rates of substitution between two countries even \textit{ex post}:

$$\frac{u'(C_{t+1})}{u'(C_t)} \frac{P_t}{P_{t+1}} = \frac{u'(C_{t+1}^*)}{u'(C_t^*)} \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*}.$$ 

With the assumption of the symmetric initial conditions of wealth, the standard risk sharing condition is obtained as follows:

$$u'(C_t^*) = e_t u'(C_t),$$

where we define the real exchange rate:

$$e_t \equiv \frac{S_t P_t^*}{P_t}.$$ 

Note that $e_t$ is unity only when purchasing power parity (PPP) holds (i.e. identical consumption preferences and under PCP). Otherwise it is time-varying either because of the non-identical consumption preferences under PCP, or due to the imperfect pass-through under LCP.
### 3.2.4.1 Gains from Price Stability

Under PCP,

\[
\Delta^*_H,t \equiv \int_0^1 \left[ \frac{S_t P_{H,t}^*(j)}{S_t P_{H,t}} \right]^{-\epsilon} dj = \Delta_{H,t}.
\]

Resource constraint and production function becomes

\[
C_{H,t} + C_{H,t}^* = \Delta^{-1}_{H,t} Y_t = \Delta^{-1}_{H,t} \exp (z_t) h_t.
\]

Price dispersion stemming from the staggered price contracts becomes distortionary and works as if it were a negative technology shock. Thus, welfare can be enhanced by achieving price stability, namely \( P_{H,t}(j) = P_{H,t} \), \( P_{H,t}^*(j) = P_{H,t}^* \) or \( \Delta_{H,t} = \Delta^*_{H,t} = 1 \).

### 3.2.5 Equilibrium Conditions

The home representative household’s period utility is specified as

\[
u(C_t) \equiv \frac{C_t^{1-\sigma} - 1}{1 - \sigma},
\]

\[
u(h_t) \equiv \frac{\chi h_t^{1+\omega}}{1 + \omega}.
\]

The system of equations consists of the first-order necessary conditions from solving households’ as well as firms’ optimization problem together with market clearing conditions. All nominal variables are detrended as follows: \( p_{H,t} = P_{H,t}/P_t, \ p_{H,t}^* = P_{H,t}^*/P_t^* \), \( p_{F,t} = P_{F,t}/P_t, \ p_{F,t}^* = P_{F,t}^*/P_t^* \), \( \pi_t = P_t/P_{t-1}, \ \pi_t^* = P_t^*/P_{t-1}^* \), \( \pi_{H,t} = P_{H,t}/P_{H,t-1}, \ \pi_{H,t}^* = P_{H,t}^*/P_{H,t-1}^* \), \( \pi_t = P_{F,t}/P_{F,t-1}, \ \pi_{F,t}^* = P_{F,t}^*/P_{F,t-1}^* \), \( MC_t = NMC_t/P_t, \ MC_t^* = NMC_t^*/P_t^* \), \( w_t = W_t/P_t \) and \( w_t^* = W_t/P_t^* \). Thus the system of equilibrium conditions is summarized as follows:

<table>
<thead>
<tr>
<th>Table 3.2: Equilibrium conditions</th>
</tr>
</thead>
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<table>
<thead>
<tr>
<th>Home</th>
<th>Foreign</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) ( C_t^{-\sigma} = \beta E_t \frac{1+\mu}{\pi_{t+1}} C_{t+1}^{-\sigma} )</td>
<td>(xix) ( (C_t^<em>)^{-\sigma} = \beta E_t \frac{1+\mu}{\pi_{t+1}} (C_{t+1}^</em>)^{-\sigma} )</td>
</tr>
<tr>
<td>(ii) ( \frac{\chi h_t^{1+\omega}}{C_t} w_t ) = ( w_t )</td>
<td>(xx) ( \frac{\chi (h_t^{1+\omega})}{(C_t^<em>)} ) = ( w_t^</em> )</td>
</tr>
<tr>
<td>(iii) ( C_{H,t} = \frac{\nu}{2} P_{F,t}^{-1} C_t )</td>
<td>(xxi) ( C_{H,t}^* = (1 - \frac{\nu}{2}) P_{F,t}^{-1} C_t )</td>
</tr>
<tr>
<td>(iv) ( C_{F,t} = (1 - \frac{\nu}{2}) p_{F,t}^{-1} C_t )</td>
<td>(xxii) ( C_{F,t}^* = \frac{\nu}{2} p_{F,t}^{-1} C_t )</td>
</tr>
</tbody>
</table>
(v) \( p_{H,t}^{\frac{1}{2}} p_{F,t}^{\frac{1}{2}} = \left( \frac{v}{2} \right)^{\frac{1}{2}} (1 - \frac{v}{2})^{1-\frac{1}{2}}, \)

(vi) \( MC_t = \frac{w_p}{\exp(z_t)}, \)

(vii) \( \exp(z_t) h_t = C_{H,t} \Delta H_t + C_{H,t} \Delta H_t, \)

(viii) \( Y_t = \exp(z_t) h_t, \)

(ix) \( \Delta H_t = (1 - \theta) \left( \frac{1 - \theta \pi_{t+1}^{H,t}}{1 - \theta} \right)^{\frac{s}{\nu + 1}}, \)

\[ + \theta \pi_{H,t}^{s} \Delta H_{t-1}, \]

(x) \( \Delta_{H,t}^* = (1 - \theta) \left( \frac{1 - \theta \pi_{t+1}^{H,t}}{1 - \theta} \right)^{\frac{s}{\nu + 1}}, \)

\[ + \theta \left( \pi_{H,t}^{s} \right)^{\epsilon} \Delta_{H,t-1}^*, \]

(xi) \( K_{H,t} = F_{H,t} \left[ \frac{1 - \theta (\pi_{H,t}^{s})^{1-1}}{1 - \theta} \right]^{\frac{1}{s}}, \)

(xii) \( K_{H,t} = \frac{C_{H,t} MC_t}{\epsilon_t} + \beta \theta \epsilon_t C_{t+1}^{\epsilon+1} e_t \pi_{H,t+1}^{s} K_{H,t+1}, \)

(xiii) \( F_{H,t} = \frac{C_{H,t} p_{H,t}}{e_t} + \beta \theta \epsilon_t C_{t+1}^{\epsilon+1} e_t \pi_{H,t+1}^{s} F_{H,t+1}, \)

(xiv) \( K_{F,t}^* = F_{H,t} \left[ \frac{1 - \theta (\pi_{F,t}^{s})^{1-1}}{1 - \theta} \right]^{\frac{1}{s}}, \)

(xv) \( K_{F,t}^* = \frac{C_{F,t} MC_t}{\epsilon_t} + \beta \theta \epsilon_t C_{t+1}^{\epsilon+1} e_t \left( \pi_{F,t+1}^{s} \right)^{\epsilon} K_{F,t+1}^{s}, \)

(xvi) \( F_{H,t}^* = C_{H,t} p_{H,t} + \beta \theta \epsilon_t C_{t+1}^{\epsilon+1} e_t \left( \pi_{H,t+1}^{s} \right)^{\epsilon-1} F_{H,t+1}, \)

(xvii) \( \pi_{H,t} = \pi_{t} \frac{p_{H,t}}{p_{H,t+1}}, \)

(xviii) \( \pi_{H,t} = \pi_{t} \frac{p_{H,t}}{p_{H,t-1}}. \)

(xxvii) \( (C_t^s)^{-\sigma} = \epsilon_t C_t^{-\sigma}. \)

These equations together with monetary policy rules solve the rational expectations equilibrium. Equations (xi) to (xiii), (xiv) to (xvi), (xxix) to (xxx) and (xxxii) to (xxxiv), which are derived from firms’ profit maximization problems, represent the new Keynesian Phillips curves for \( p_H, p_H^*, p_F \) and \( p_F^*, \) respectively. \( Ks \) and \( Fs \) are auxiliary
variables, the details of which are shown in Appendix A.

Under PCP, equations (xiv) to (xvi) and (xxix) to (xxxi) collapse to

\[(xxxviii) \quad p_{H,t}^* = \frac{p_{H,t}}{e_t},\]
\[(xxxix) \quad p_{F,t} = e_t p_{F,t}^* ,\]

and equations (x) and (xxvii) are replaced by

\[(xxxx) \quad \Delta_{H,t} = \Delta_{H,t},\]
\[(xxxxi) \quad \Delta_{F,t}^* = \Delta_{F,t}^*.\]

### 3.2.6 Log-Linearized Equations

We approximate the above structural equations around the deterministic steady state up to the first order. Note that the deterministic steady state is efficient as monopolistic distortion in production is effectively eliminated by an appropriate subsidy. Thus, this deterministic steady state coincides with the Ramsey steady state, which will be discussed in the following section.\(^{20}\) Details of the derivation of the steady state are also shown in Appendix A. Below, the circumflex \(\hat{\cdot}\) indicates the log-deviation of a variable from its respective steady state.

Linear approximation to equations (xi) to (xiii), (xxxii) to (xxxiv), (xxix) to (xxxi) and (xiv) to (xvi) leads to the New Keynesian Phillips curves:

\[
\pi_{H,t} = \beta E_{t} \pi_{H,t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta} (\hat{mc}_t - \hat{p}_{H,t}) , \tag{3.2.8} \\
\pi_{F,t}^* = \beta E_{t} \pi_{F,t+1}^* + \frac{(1-\beta\theta)(1-\theta)}{\theta} (\hat{mc}_t^* - \hat{p}_{F,t}^* ) , \tag{3.2.9} \\
\pi_{F,t} = \beta E_{t} \pi_{F,t+1} + \frac{(1-\beta\theta)(1-\theta)}{\theta} (\hat{mc}_t - \hat{p}_{F,t} + \hat{e}_t) , \tag{3.2.10} \\
\pi_{H,t}^* = \beta E_{t} \pi_{H,t+1}^* + \frac{(1-\beta\theta)(1-\theta)}{\theta} (\hat{mc}_t - \hat{p}_{H,t} + \hat{e}_t) . \tag{3.2.11} 
\]

As in the closed-economy model of Galí and Gertler (1999) or the open-economy model under PCP of Benigno and Benigno (2006), in equations (3.2.8) and (3.2.9), PPI inflation rates depend on the real marginal costs that producers face when setting prices for the domestic market. Equations (3.2.10) and (3.2.11), appearing specifically in the open-economy model under LCP, show that import price inflation rates depend on the

\(^{20}\)As Woodford (2003), Chapter 6 argues, this type of steady state is the one that is appropriate for ranking alternative policies. See also Benigno and Woodford (2005) and Khan, King, and Wolman (2003).
real marginal costs that producers face when setting prices for the importing country’s market.\footnote{Note that $\frac{MC_t}{p_{H,t}} = \frac{NMC_t}{p_{H,t}}$ is the marginal cost evaluated at output price level while $MC_t = \frac{NMC_t}{p_t}$ is the marginal cost evaluated at consumer price level. The former is relevant to firms’ pricing decisions.}

First-order approximation to equations (ix) to (x) and (xvii) to (xviii) results in

$$\hat{\Delta}_{H,t} = \hat{\Delta}^*_{H,t} = \hat{\Delta}^*_{F,t} = \hat{\Delta}_{F,t} = 0.$$ Together with linearly approximated equations (ii) to (viii), (xx) to (xxvi), and (xxxvii), we have

$$\hat{mc}_t - \hat{p}_{H,t} = (\sigma + \omega) \hat{y}_t - (1 + \omega) z_t + \frac{(2 - \nu) (1 - \sigma)}{2} (\hat{q}_t + \hat{e}_t) + \frac{2 - \nu}{2} \hat{d}_t,$$ \hfill (3.2.12)

$$\hat{mc}^*_t - \hat{p}^*_{F,t} = (\sigma + \omega) \hat{y}^*_t - (1 + \omega) z^*_t + \frac{(2 - \nu) (1 - \sigma)}{2} (\hat{q}^*_t - \hat{e}_t) + \frac{2 - \nu}{2} \hat{d}^*_t,$$ \hfill (3.2.13)

$$\hat{mc}^*_t - \hat{p}^*_{F,t} + \hat{e}_t = (\sigma + \omega) \hat{y}^*_t - (1 + \omega) z^*_t + \frac{(2 - \nu) (1 - \sigma)}{2} (\hat{q}^*_t - \hat{e}_t) - \frac{\nu}{2} \hat{d}^*_t,$$ \hfill (3.2.14)

$$\hat{mc}_t - \hat{p}^*_{H,t} - \hat{e}_t = (\sigma + \omega) \hat{y}_t - (1 + \omega) z_t + \frac{(2 - \nu) (1 - \sigma)}{2} (\hat{q}_t + \hat{e}_t) - \frac{\nu}{2} \hat{d}_t,$$ \hfill (3.2.15)

where $\hat{q}_t$ and $\hat{q}^*_t$ denote log deviations of the domestic and foreign terms of trade from their steady states:

$$Q_t \equiv \frac{P_{F,t}}{S_t P_{H,t}^*} = \frac{p_{F,t}}{e_t p_{H,t}^*},$$ \hfill (3.2.16)

$$Q^*_t \equiv \frac{S_t P_{H,t}^*}{P_{F,t}} = \frac{e_t p_{H,t}^*}{p_{F,t}},$$ \hfill (3.2.17)

and $\hat{d}_t$ and $\hat{d}^*_t$ denote those of the deviations from the law of one price:

$$D_t \equiv \frac{S_t p_{H,t}^*}{P_{H,t}} = \frac{e_t p_{H,t}^*}{p_{H,t}},$$ \hfill (3.2.18)

$$D^*_t \equiv \frac{P_{F,t}}{S_t p_{F,t}^*} = \frac{p_{F,t}}{e_t p_{F,t}^*}.$$ \hfill (3.2.19)

Equations (3.2.12) to (3.2.15) show that, in open economies, deviations from steady state of the real marginal costs are not only proportional to deviations from steady state of output, but also depend on relative prices. The first and the second terms are those also included in the New Keynesian models in the closed economy. The third
and the fourth terms appear only in open economies. Specifically, the third terms capture the interdependence: economic activities abroad affect the domestic economy via international relative prices. The qualitative impacts depend on $\sigma$. When $\sigma > 1$ ($\sigma < 1$), positive changes in the international relative prices exert negative (positive) impacts on the real marginal costs. When $\sigma = 1$, the spillovers are zero. Note that the transmission mechanism of such spillovers differs under PCP and LCP. Under PCP, the real exchange rate moves in proportion to the terms of trade of the home country. A deterioration of the terms of trade, associated with a real exchange rate depreciation, has two opposing effects: it increases the consumption through the global assets market and therefore increases the marginal costs; it decreases the consumption due to higher import prices and therefore decreases the marginal costs. According to the terminologies by Clarida, Galí, and Gertler (2002) for PCP, the former is called the risk-sharing effect while the latter is called the terms-of-trade effect. When $\sigma > 1$ ($\sigma < 1$), the latter (former) dominates or, in other words, the home and foreign goods are Edgeworth substitutes (complements). When $\sigma = 1$, the two effects are cancelled out and thus two countries become insular. Under LCP, on the other hand, consumer prices of the imported goods are inelastic to movements in exchange rates and thus changes in the terms of trade do not entail the expenditure-switching effect as under PCP. Consumption and the real marginal costs are less responsive to the international relative prices represented by the third terms.\footnote{\textsuperscript{22}See also Corsetti and Pesenti (2005b) for a discussion in a one-period ahead price adjustment model under LCP and Corsetti, Dedola, and Leduc (2010) for a discussion focusing on effects of international relative prices on consumption.} A depreciation of the real exchange rate leads to an improvement of the home terms of trade under LCP due to the increases in the home-currency denominated revenues from export sales. It is deviations from the law of one price that affect the real marginal costs under LCP, which are the fourth terms. Equations (3.2.12) and (3.2.15) illustrate that deviations from the law of one price for the home goods increase (decrease) the real marginal costs that firms face when selling the home goods domestically (abroad), \textit{ceteris paribus}. The changes in the marginal costs in turn lead to PPI inflation at home (import price deflation abroad), via the New Keynesian Phillips curves in equations (3.2.8) and (3.2.11). As will be shown later, these terms are also objectives to be minimized by noncooperative policy makers under LCP. Note that the spillovers on the marginal costs represented by the fourth terms exist independently of goods’ substitutability or complementarity, that is whether $\sigma$ is greater, smaller or equal to 1.

Log-linearization to the aggregate resource constraints in equations (vii) and (xxv),
and the risk sharing condition in equation (xxxvii) gives
\[ \hat{y}_t - \hat{y}_t^* + \frac{\nu}{2} \hat{p}_{H,t} + \frac{2 - \nu}{2} \hat{p}_{H,t}^* - \frac{\nu - 1}{\sigma} \hat{e}_t - \frac{\nu}{2} \hat{p}_{F,t}^* - \frac{2 - \nu}{2} \hat{p}_{F,t} = 0. \] (3.2.20)

Also, we have log exact deviations for the definitions of inflation rates in equations (xvii), (xviii), (xxxv) and (xxxvi):
\[
\begin{align*}
\pi_{H,t} &= \pi_t + \hat{p}_{H,t} - \hat{p}_{H,t-1}, \quad (3.2.21) \\
\pi_{H,t}^* &= \pi_t^* + \hat{p}_{H,t}^* - \hat{p}_{H,t-1}, \quad (3.2.22) \\
\pi_{F,t} &= \pi_t + \hat{p}_{F,t} - \hat{p}_{F,t-1}, \quad (3.2.23) \\
\pi_{F,t}^* &= \pi_t^* + \hat{p}_{F,t}^* - \hat{p}_{F,t-1}. \quad (3.2.24)
\end{align*}
\]

Note that under PCP, the law of one price holds, thus
\[ \hat{d}_t = \hat{d}_t^* = 0. \]

Consequently,
\[
\begin{align*}
\hat{p}_{H,t} &= \hat{e}_t + \hat{p}_{H,t}^*, \\
\hat{p}_{F,t} &= \hat{e}_t + \hat{p}_{F,t}^*.
\end{align*}
\]

\section*{3.3 Optimal Monetary Policy in Open Economies}

In this section, we first set up the Ramsey problem. Optimal monetary policy under noncooperation is derived in an open-loop Nash equilibrium. Then, we derive the quadratic loss functions which central banks aim to minimize by the second-order approximation to social welfare around the Ramsey steady state.

\subsection*{3.3.1 Ramsey Policy Problems}

Central banks under cooperation maximize global welfare:
\[ W_{W,t_0} = W_{H,t_0} + W_{F,t_0}, \]
subject to the nonlinear equilibrium conditions in equations (i) to (xxxvii).

On the other hand, under noncooperation, the domestic central bank maximizes equation (3.2.1) subject to equations (i) to (xxxvii) given \( \{\pi_{F,t}\}_{t=t_0}^\infty \), while the foreign central bank maximizes equation (3.2.5) subject to equations (i) to (xxxvii) given
The equilibrium conditions of the Ramsey policy under both cooperation and noncooperation are shown in Appendix B. The choice of the policy variables taken as given in a noncooperative game is crucial in determining the equilibrium. We follow Benigno and Benigno (2006) and choose PPI inflation rates as the policy variables for the noncooperative game.

The aims of computing the Ramsey policy in this paper are twofold. First, we need to obtain the Ramsey steady state around which the equilibrium conditions are approximated. It turns out that irrespective of cooperation or noncooperation, the Ramsey steady state is that under the flexible price equilibrium, or the equilibrium under the constant aggregate price levels. Second, we compute the welfare cost stemming from the inability to cooperate. The welfare cost is computed in the next section in a conventional manner following Lucas (1992) in a consumption unit.

### 3.3.2 Linear-Quadratic Framework

As Appendix B shows, the characteristics of the optimal noncooperative monetary policy under LCP is not easy to be understood from the optimality conditions from the Ramsey policy. In this subsection, we derive the quadratic objective functions which the central banks aim to minimize under LCP in a noncooperative game.

The domestic welfare can be approximated up to the second order as

\[
W_{H,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( C_t^{1-\sigma} - \frac{1}{1-\sigma} \frac{\hat{h}_t^{1+\omega}}{1+\omega} \right) + \text{t.i.p} + \text{h.o.t},
\]

where \( C \) is steady-state value of \( C_t \), t.i.p and h.o.t denote the terms independent of policy and higher order term than the second order, respectively. As shown by Kim and Kim (2003) with a simple example, existence of the linear terms in the loss functions leads to spurious welfare evaluation. Thus, these must be substituted out by the second-order terms.

---

23Wang (2015) examines a set of choices as policy variables including PPI inflation rates, import price inflation rates, CPI inflation rates, outputs and nominal interest rates in a two-country model with LCP. When nominal interest rates are chosen to be the policy variables, equilibrium indeterminacy occurs. This repeats the findings in Blake (2012), de Fiore and Liu (2002) and Coenen et al. (2010) although they use different models with nominal rigidities from Wang (2015).
3.3.2.1 Closed Economy

In a closed economy, the log exact form of the resource constraint is given by

\[ z_t + \hat{h}_t = \hat{c}_t + \hat{\Delta}_{H,t}. \]

Thus, as shown by Woodford (2003), the linear terms \( \hat{c}_t - \hat{h}_t \) are replaced by the price dispersion terms \( -\hat{\Delta}_{H,t} \), which is of the second order and eventually replaced by the quadratic term of inflation rates (see Appendix C).²⁴

3.3.2.2 Open Economies

In open economies, linear terms cannot be easily substituted out as in the closed economy.

- PCP and Cooperation
  
  For example, under PCP with a logarithmic utility function, as shown in Fujiwara, Kam, and Sunakawa (2015), the log exact form of the home resource constraint is given by

  \[
  z_t + \hat{h}_t = -\hat{p}_{H,t} + \hat{c}_t + \hat{\Delta}_{H,t} = 2\hat{q}_t + \hat{c}_t + \hat{\Delta}_{H,t}.
  \]

  The linear terms \( \hat{c}_t - \hat{h}_t \) are now replaced by not only the price dispersion terms \( -\hat{\Delta}_{H,t} \) but also the terms of trade \( -\hat{q}_t \) which is absent in the closed economy.

  Note that under cooperative regime, the sum of the linear terms of the global welfare \( \hat{c}_t - \hat{h}_t + \hat{c}_t^* - \hat{h}_t^* \) leads to the cancellation of the terms-of-trade term by using the log exact form of the foreign resource constraint:

  \[
  z_t^* + \hat{h}_t^* = -2\hat{q}_t + \hat{c}_t^* + \hat{\Delta}_{F,t}^*.
  \]

  The terms of trade externality is internalized, by definition, under cooperation.

- PCP and Noncooperation
  
  Under noncooperative regime, each central bank in an open economy is incentivized to strategically manipulate the terms of trade in its favor. This indeed represents the terms-of-trade externality as analyzed in Corsetti and Pesenti (2001), Benigno (2002) and Benigno and Benigno (2006). Sutherland (2002),

²⁴Note that \( z_t \) is independent of policy.
Benigno and Woodford (2005) and Benigno and Benigno (2006) substitute out the linear terms by the quadratic terms by using the second-order approximation to the structural equations for correct welfare evaluation.

- **LCP and Cooperation** Like the case under PCP, social welfare under cooperation under LCP can be approximated up to the second order without resort to the second-order approximation to the equilibrium conditions, as shown by Engel (2011). Log-linear approximation to the resource constraints in equations (vii) and (xxv) results in

\[
\begin{align*}
\dot{z}_t + \dot{h}_t &= c_t + \frac{\nu}{2} (-p_{H,t} + \dot{\Delta}_{H,t}) + \frac{2 - \nu}{2} \left( -p_{H,t}^* - \frac{1}{\sigma} \hat{\epsilon}_t + \dot{\Delta}_{H,t}^* \right), \\
\dot{z}_t^* + \dot{h}_t^* &= c_t^* + \frac{\nu}{2} (-p_{F,t}^* + \dot{\Delta}_{F,t}^*) + \frac{2 - \nu}{2} \left( -p_{F,t} - \frac{1}{\sigma} \hat{\epsilon}_t + \dot{\Delta}_{F,t} \right),
\end{align*}
\]

where the log exact forms of the demands in equations (iii), (xxi), (iv), (xxii) and the risk sharing condition in equation (xxxvii) are substituted. Together with the log exact forms of equations (v) and (xxiii), we can derive

\[
\hat{c}_t - \dot{h}_t + \dot{c}_t^* - \dot{h}_t^* = -\frac{\nu}{2} \dot{\Delta}_{H,t} - \frac{2 - \nu}{2} \dot{\Delta}_{H,t}^* - \frac{\nu}{2} \dot{\Delta}_{F,t} - \frac{2 - \nu}{2} \dot{\Delta}_{F,t}. 
\]

Thus, central banks under cooperation aim to stabilize fluctuations in four inflation rates: \(\pi_{H,t}, \pi_{H,t}^*, \pi_{F,t}^*, \) and \(\pi_{F,t}^*\). Appendix C shows how to transform price dispersions into inflation rates.

- **LCP and Noncooperation** Under the noncooperative regime and LCP, linear terms for the terms of trade cannot be eliminated. Thus, they need to be substituted out by the second-order approximation to AS equations under the assumption of commitment, resource constraints and price dispersions. Details are shown in Appendix C. In particular, equations (4.5.118) and (4.5.119) in Appendix C show how linear terms can be replaced by quadratic terms.

Upon obtaining the quadratic expressions for the linear terms, the loss function that
the home central bank aims to minimize is then given by

\[ L_t = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \begin{array}{l}
\frac{1 + \omega}{2} (\hat{y}_t - z_t)^2 \\
+ \frac{\nu (2 - \nu)}{16} \left(1 + \frac{\sigma + \omega (\nu - 1)}{\gamma}\right) \left(\hat{p}_{H,t}^* \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t}\right)^2 \\
+ \frac{\nu (2 - \nu)}{16} \left(1 - \frac{\sigma + \omega (\nu - 1)}{\gamma}\right) \left(\hat{p}_{F,t} - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}^*\right)^2 \\
+ \frac{\nu (2 - \nu)}{16} \left(1 + \frac{\alpha}{\gamma}\right) (\pi_{H,t})^2 + \frac{\epsilon (2 - \nu)}{8\delta} \left(1 + \frac{\alpha}{\gamma}\right) (\pi_{F,t})^2 \\
+ \frac{\nu (2 - \nu)}{16} \left(1 - \frac{\alpha}{\gamma}\right) (\pi_{F,t})^2 + \frac{\epsilon (2 - \nu)}{8\delta} \left(1 - \frac{\alpha}{\gamma}\right) (\pi_{H,t})^2 \\
+ \frac{\nu (2 - \nu)}{16} \left(1 + \frac{\alpha}{\gamma}\right) (\pi_{H,t})^2 + \frac{\epsilon (2 - \nu)}{8\delta} \left(1 - \frac{\alpha}{\gamma}\right) (\pi_{F,t})^2 \\
\end{array} \right\}, \]

where

\[ L_t = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \begin{array}{l}
\frac{\sigma - 1}{2} - \frac{(\sigma - 1)^2 (2 - \nu) (\omega \nu + 1)}{4\gamma} \left(\hat{y}_t - \frac{2 - \nu}{2} \hat{q}_t + \frac{(2 - \nu) (1 - \sigma)}{2\sigma} \hat{e}_t\right)^2 \\
+ \frac{\nu (2 - \nu)}{16} \left(1 + \frac{\alpha}{\gamma}\right) (\pi_{H,t})^2 + \frac{\epsilon (2 - \nu)}{8\delta} \left(1 + \frac{\alpha}{\gamma}\right) (\pi_{F,t})^2 \\
+ \frac{\nu (2 - \nu)}{16} \left(1 - \frac{\alpha}{\gamma}\right) (\pi_{F,t})^2 + \frac{\epsilon (2 - \nu)}{8\delta} \left(1 - \frac{\alpha}{\gamma}\right) (\pi_{H,t})^2 \\
\end{array} \right\}, \]
and the loss function that the foreign central bank aims to minimize is given by

\[
L^*_t = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1+\omega}{2} (\hat{y}_t^*-z_t^*)^2 + \frac{\nu(2-\nu)}{16} \left( 1 - \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) (\hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t})^2 + \frac{\nu(2-\nu)}{16} \left( 1 + \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) (\hat{p}_{F,t} - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}^*)^2 + \frac{\nu(2-\nu)}{8\delta} \left( 1 - \frac{\alpha}{\gamma} \right) (\pi_{H,t}^*)^2 + \frac{\nu(2-\nu)}{8\delta} \left( 1 + \frac{\alpha}{\gamma} \right) (\pi_{F,t}^*)^2 + \frac{(\sigma - 1)^2}{4\gamma} (2-\nu) (\omega \nu + 1) (\hat{y}_t - \frac{\nu}{2} \hat{q}_t + (2-\nu) (1-\sigma) \hat{e}_t)^2 + \frac{\nu(2-\nu)}{8\gamma} \left( \frac{-\sigma + 1 + \omega}{2} \right) \left[ (1+\omega) (\hat{y}_t^* - z_t) + \frac{2-\nu}{2} \left( \hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t} \right) \right]^2 \right. 
\]

\[+ \frac{\nu(2-\nu)}{8\gamma} \left( \frac{-\sigma + 1 + \omega}{2} \right) \left[ (1+\omega) (\hat{y}_t - z_t) - \frac{\nu}{2} \left( \hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t} \right) \right] \]

\[+ \frac{\nu(2-\nu)}{8\gamma} \left( \frac{-\sigma + 1 + \omega}{2} \right) \left[ (1+\omega) (\hat{y}_t^* - z_t^*) + \frac{2-\nu}{2} \left( \hat{p}_{F,t} - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}^* \right) \right]^2 \]

\[+ \frac{\nu(2-\nu)}{8\gamma} \left( \frac{-\sigma + 1 + \omega}{2} \right) \left[ (1+\omega) (\hat{y}_t^* - z_t^*) - \frac{\nu}{2} \left( \hat{p}_{H,t} + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t} \right) \right]^2 \]
minimize is given by

\[
L_{t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1 + \omega}{2} (\hat{y}_t - z_t)^2 + \frac{\epsilon \nu}{4\delta} (\pi_{H,t})^2 + \frac{\epsilon (2 - \nu)}{4\delta} (\pi_{F,t})^2 \\
+ \frac{\nu (2 - \nu) \Omega}{8} (\hat{d}_t)^2 + \frac{\nu (2 - \nu) (1 - \Omega)}{8} (\hat{d}_t^*)^2 \\
+ \frac{\nu (1 - \Omega)}{4} \left( (1 + \omega) (\hat{y}_t - z_t) + \frac{2 - \nu}{2} \hat{d}_t \right)^2 \\
+ \frac{(2 - \nu) \Omega}{4} \left( (1 + \omega) (\hat{y}_t^* - z_t^*) - \frac{\nu}{2} \hat{d}_t^* \right)^2 \\
- \frac{\nu (1 - \Omega)}{4} \left( (1 + \omega) (\hat{y}_t^* - z_t^*) + \frac{2 - \nu}{2} \hat{d}_t^* \right)^2 \\
- \frac{(2 - \nu) \Omega}{4} \left( (1 + \omega) (\hat{y}_t^* - z_t^*) - \frac{\nu}{2} \hat{d}_t^* \right)^2 \right\},
\]

while that for the foreign central bank is

\[
L_{t_0}^* = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1 + \omega}{2} (\hat{y}_t^* - z_t^*)^2 + \frac{\epsilon \nu}{4\delta} (\pi_{F,t}^*)^2 + \frac{\epsilon (2 - \nu)}{4\delta} (\pi_{H,t}^*)^2 \\
+ \frac{\nu (2 - \nu) \Omega}{8} (\hat{d}_t^*)^2 + \frac{\nu (2 - \nu) (1 - \Omega)}{8} (\hat{d}_t^*)^2 \\
+ \frac{\nu (1 - \Omega)}{4} \left( (1 + \omega) (\hat{y}_t^* - z_t^*) + \frac{2 - \nu}{2} \hat{d}_t^* \right)^2 \\
+ \frac{(2 - \nu) \Omega}{4} \left( (1 + \omega) (\hat{y}_t - z_t) - \frac{\nu}{2} \hat{d}_t \right)^2 \\
- \frac{\nu (1 - \Omega)}{4} \left( (1 + \omega) (\hat{y}_t - z_t) + \frac{2 - \nu}{2} \hat{d}_t \right)^2 \\
- \frac{(2 - \nu) \Omega}{4} \left( (1 + \omega) (\hat{y}_t^* - z_t^*) - \frac{\nu}{2} \hat{d}_t^* \right)^2 \right\},
\]

where \(\Omega \equiv \frac{1 + \omega^2}{1 + \omega} \) and \(0 < \Omega \leq 1\).

Equations (3.3.2) and (3.3.3) show that the noncooperative loss function of each policy maker under LCP consists of nine quadratic terms. The first terms, quadratic deviations from steady state of output (employment), represent the inefficient fluctuations in output and therefore consumption stemming from markup fluctuations in the realization of productivity shocks, which hinder consumption smoothing; the second and third terms, squared inflation rates of local as well as imported products, arise from the staggered price contracts, which create price dispersions; the fourth and fifth
terms are the direct consequences from the breakdown of the law of one price; the final
four terms, as explained in Section 3.2.6, represent inefficient fluctuations in the real
marginal costs, which leads to fluctuations in both PPI and import price inflation rates. The signs associated with those terms represent the national central bank’s incentives to simultaneously stabilize the inflation rates relevant to its own country and destabilize those relevant to the counterpart country.

Table 3.3 offers comparison of the loss functions under LCP and noncooperation to those under (1) PCP and cooperation, (2) PCP and noncooperation, and (3) LCP and cooperation. We start the comparison given LCP (Table 3.3, column 2). The first five terms in the noncooperative loss functions, in equations (3.3.2) and (3.3.3), are also those in the cooperative loss functions. The last four terms regarding fluctuations in the real marginal costs, representing the terms-of-trade externality, are unique to the noncooperative policy makers. The existence of the additional terms indicates national policy makers’ additional concern for stabilization of inflation rates in both goods categories. Under LCP, that means gains from stabilization of CPI inflation rates.

Then, we compare column 2 to column 1. The number of objectives (trade-offs) that policy makers aim to minimize is substantially reduced from LCP to PCP, regardless of the nature of strategic games. The key to understand this difference is the law of one price, which holds only under PCP, renders (a) price dispersions within export goods identical to those within locally produced and consumed goods; (b) \( \hat{d}_t = \hat{d}_t^* = 0 \) by definitions; and (c) stabilization of the real marginal costs is in line with stabilization of output fluctuations. Therefore, the additional trade-offs regarding fluctuations in the real marginal costs that separate the noncooperative loss functions away from the cooperative ones under LCP no longer exist under PCP. Allocations and prices under both games coincide under PCP.
### Chapter 3. Optimal Monetary Policy

Table 3.3: Quadratic loss functions under PCP / LCP and under cooperation / noncooperation

<table>
<thead>
<tr>
<th></th>
<th>PCP</th>
<th>LCP</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cooperation</strong></td>
<td>$\frac{1 + \omega}{2} (\hat{y}_t - z_t)^2$</td>
<td>$\frac{1 + \omega}{2} (\hat{y}_t - z_t)^2$</td>
</tr>
<tr>
<td></td>
<td>$+ \frac{1 + \omega}{2} (\hat{y}_t^* - z_t^*)^2$</td>
<td>$+ \frac{1 + \omega}{2} (\hat{y}_t^* - z_t^*)^2$</td>
</tr>
<tr>
<td></td>
<td>$+ \frac{c}{2\delta} (\pi_{H,t})^2 + \frac{c}{2\delta} (\pi_{F,t})^2$</td>
<td>$+ \frac{c}{2\delta} (\pi_{H,t})^2 + \frac{c}{2\delta} (\pi_{F,t})^2$</td>
</tr>
<tr>
<td><strong>Noncooperation</strong></td>
<td>$(1 + \omega)\nu (\hat{y}_t - z_t)^2$</td>
<td>$(1 + \omega)\nu (\hat{y}_t - z_t)^2$</td>
</tr>
<tr>
<td>(Home)</td>
<td>$+ \frac{(1 + \omega)(2 - \nu)}{4} (\hat{y}_t^* - z_t^*)^2$</td>
<td>$+ \frac{(1 + \omega)(2 - \nu)}{4} (\hat{y}_t^* - z_t^*)^2$</td>
</tr>
<tr>
<td></td>
<td>$+ \frac{c}{2\delta} (\pi_{H,t})^2 + \frac{c}{2\delta} (\pi_{F,t})^2$</td>
<td>$+ \frac{c}{2\delta} (\pi_{H,t})^2 + \frac{c}{2\delta} (\pi_{F,t})^2$</td>
</tr>
<tr>
<td><strong>Noncooperation</strong></td>
<td>$(1 + \omega)\nu (\hat{y}_t^* - z_t^*)^2$</td>
<td>$(1 + \omega)\nu (\hat{y}_t^* - z_t^*)^2$</td>
</tr>
<tr>
<td>(Foreign)</td>
<td>$+ \frac{(1 + \omega)(2 - \nu)}{4} (\hat{y}_t - z_t)^2$</td>
<td>$+ \frac{(1 + \omega)(2 - \nu)}{4} (\hat{y}_t - z_t)^2$</td>
</tr>
<tr>
<td></td>
<td>$+ \frac{c}{2\delta} (\pi_{F,t})^2 + \frac{c}{2\delta} (\pi_{H,t})^2$</td>
<td>$+ \frac{c}{2\delta} (\pi_{F,t})^2 + \frac{c}{2\delta} (\pi_{H,t})^2$</td>
</tr>
</tbody>
</table>

Note: we present the period loss functions in the Table. The loss function of each policy maker is the present discounted value of the sum of current and expected future period loss functions.

Quadratic loss functions are minimized by the central banks subject to the co-
constraint relating to cross country output difference, equation (3.2.20), the familiar New Keynesian Phillips curves with equations (3.2.12)-(3.2.15) substituted into equations (3.2.8)-(3.2.11):

$$\pi_{H,t} = \beta E_t E_{t+1} \left[ (1 + \omega) z_t + \frac{(2 - \nu)(1 - \sigma)}{2} (\hat{q}_t + \hat{e}_t) + \frac{2 - \nu}{2} \hat{d}_t \right], \quad (3.3.4)$$

$$\pi_{F,t} = \beta E_t E_{t+1} \left[ (1 + \omega) z^*_t + \frac{(2 - \nu)(1 - \sigma)}{2} (\hat{q}^*_t - \hat{e}_t) - \frac{\nu}{2} \hat{d}^*_t \right], \quad (3.3.5)$$

$$\pi^*_t = \beta E_t E_{t+1} \left[ (1 + \omega) z^*_t + \frac{(2 - \nu)(1 - \sigma)}{2} (\hat{q}^*_t - \hat{e}_t) + \frac{2 - \nu}{2} \hat{d}^*_t \right], \quad (3.3.6)$$

$$\pi_{H,t}^* = \beta E_t E_{t+1} \left[ (1 + \omega) z_t + \frac{(2 - \nu)(1 - \sigma)}{2} (\hat{q}_t + \hat{e}_t) - \frac{\nu}{2} \hat{d}_t \right], \quad (3.3.7)$$

where $\hat{q}_t = \hat{p}_{F,t} - \hat{e}_t - \hat{p}_{H,t}$, $\hat{q}^*_t = \hat{e}_t + \hat{p}^*_H - \hat{p}_{F,t}$, $\hat{d}_t = \hat{p}^*_H + \hat{e}_t - \hat{p}_{H,t}$, and $\hat{d}^*_t = \hat{p}_{F,t} - \hat{e}_t - \hat{p}^*_F$, as well as the relations between inflation rates and relative prices from detrending the system, equations (3.2.21)-(3.2.24), and definitions of aggregate price indexes:

$$\frac{\nu}{2} \hat{p}_{H,t} + \frac{2 - \nu}{2} \hat{p}_{F,t} = 0, \quad (3.3.8)$$

$$\frac{2 - \nu}{2} \hat{p}_{H,t}^* + \frac{\nu}{2} \hat{p}_{F,t}^* = 0. \quad (3.3.9)$$

Under noncooperation, the domestic central bank minimizes (3.3.2) subject to equations (3.2.20), (3.2.21)-(3.2.24), (3.3.4)-(3.3.7), and (3.3.8)-(3.3.9), given foreign PPI inflation rates $\{\pi_{F,t}^*\}$ for all $t \geq t_0$. Similarly, the foreign central bank minimizes (3.3.3) subject to equations (3.2.20), (3.2.21)-(3.2.24), (3.3.4)-(3.3.7), and (3.3.8)-(3.3.9), given domestic PPI inflation rates $\{\pi_{H,t}^*\}$ for all $t \geq t_0$. Each central bank conducts optimal commitment policy from the timeless perspective as in Woodford (2003).

### 3.4 Results

In this section, we first draw impulse responses of the two countries to a positive technology shock to the home country. The dynamics are obtained under the optimal monetary policy in Section 3.3.2. We consider cooperative and noncooperative games under both PCP and LCP. As discussed in previous section, cooperative and noncooperative allocations and prices coincide under PCP. We then compute welfare gains from cooperation using the Ramsey policy problem presented in Section 3.3.1.
3.4.1 Impulse Responses

The baseline parameters are calibrated as in Table 3.4. $\beta$, $\chi$ and the probability of not being able to reset prices $\theta$ are set at the conventional values. $\nu$ is set at 1.5 as in Engel (2011) which means that households put 3/4 of the weight on consumption of domestic goods in utility. $\sigma$ usually takes the range from 1 to 5. We set it to 1, consistent with our derivation of simplified loss functions in previous section. The elasticity of substitution among different varieties within goods category is set at 7.66, implying a degree of market power that results in prices being at a level 15% higher than marginal costs on average. Empirical data show that the range of the inverse of the Frisch elasticity $1/\omega$ is 0.05-0.3 so we set $\omega$ at 4.71 in the range. Note that Engel (2011) assumes a linear disutility of labor, $\omega = 0$, which later we will show to be a special case in which welfare gains from cooperation are zero. In addition, the log-technology follows an AR(1) stochastic process with serial correlation $\rho$ set at 0.856 and standard deviation at 0.0064.25

Table 3.4: Parameter values (Baseline)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.99</td>
<td>Subjective discount factor</td>
</tr>
<tr>
<td>$\theta$</td>
<td>0.75</td>
<td>Probability of a firm not being chosen to reset its prices at each period</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>7.66</td>
<td>Elasticity of substitution among different products within goods category</td>
</tr>
<tr>
<td>$\nu$</td>
<td>1.5</td>
<td>Weight that households put on consumption of domestic goods in utility ($\nu/2$)</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>1</td>
<td>Inverse of the intertemporal elasticity of substitution of consumption</td>
</tr>
<tr>
<td>$\chi$</td>
<td>1</td>
<td>Coefficient associated with disutility of labor</td>
</tr>
<tr>
<td>$\omega$</td>
<td>4.71</td>
<td>Inverse of the Frisch elasticity</td>
</tr>
</tbody>
</table>

Figure 3.1 depicts the impulse responses under PCP and under LCP to one standard deviation of a positive technology shock to the home country (we scale up the impulse responses by 100 so the dynamics in Figure 3.1 are measured in per cent). In response to technology improvement shocks, optimal policy is always expansionary in the country experiencing such shocks and contractionary in the country without shocks. Specific to results in Figure 3.1, it means a (nominal and) real exchange rate depreciation for the home country.

25For the value of $\epsilon$, see Rotemberg and Woodford (1997). For the range of $\sigma$, see Benigno and Benigno (2006), for the range of $\omega$, see Erceg, Gust, and Lopez-Salido (2007), and for technology calibration, see Schmitt-Grohé and Uribe (2007).
Under PCP, optimal policy brings in efficient responses of output and fully stabilizes PPI inflation rates, in response to efficient shocks. A one standard deviation of the home technology shocks leads to an increase of home output by 0.64 per cent. With the efficient responses of output, optimal policy is able to fully stabilize PPI inflation rates of the two countries. Imported goods prices then fluctuate with exchange rates and changes in CPI inflation rates reflect changes in import price inflation rates proportionately (the proportion is equal to the weight of imported goods in the consumption basket, i.e. 25 per cent). The home terms of trade weakens with the real depreciation. Foreign output stays unchanged when $\sigma = 1$ because there are no spillovers.

Under LCP and cooperation, optimal policy trades off output responses with stabilization of CPI inflation rates. Specifically, a one standard deviation of the home productivity improvement shock now leads to an increase of home output by less than 0.64 per cent, which translates into a fall in PPI inflation rates of the home country. The real exchange rate depreciation under LCP leads to an improvement of the home terms of trade, raising the real purchasing power of the home country at any given price level. Thus demand for both goods rises and foreign output increases to meet the higher demand. CPI inflation rates of both countries are stabilized to a much larger extent by optimal policy under LCP than under PCP.

Under LCP and noncooperation, optimal policy seeks to stabilize CPI inflation rates more so than it does under cooperation, as demonstrated by the additional terms in the noncooperative loss functions in Section 3.3.2. As a trade-off, home output increases less than it does under cooperation and home PPI inflation rates fall further. Optimal policy is less expansionary in the home country and thus the real exchange rate depreciates less under noncooperation than under cooperation. The home terms of trade deteriorates and the foreign terms of trade improves, compared to their respective cooperative positions. Given any price level, foreign consumers’ demand for the foreign goods increases and foreign output rises further accordingly.
Figure 3.1: Impulse responses under PCP and LCP to a positive technology shock to the home country by one S.D.

3.4.2 Welfare Cost

The welfare cost from noncooperation is measured in consumption units by Lucas (1992). Specifically, the welfare cost measures the proportion of aggregate consumption that a representative household has to give up so that it is as well-off under the cooperative regime as under the noncooperative regime. Denote ‘c’ and ‘n’ as superscript for the cooperative game and noncooperative game, respectively. Following Schmitt-Grohé and Uribe (2007), denote $\lambda^c$ as the welfare cost from noncooperation for the home
representative household and we have

\[ W^n_{H,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [u ((1 - \lambda^c) C_t^c) - v (h_t^c)] . \]

When \( \sigma = 1 \),

\[ W^n_{H,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \log \left( (1 - \lambda^c) C_t^c \right) - \chi \left( h_t^c \right) \right) , \]

thus \( \lambda^c \) is given by

\[ \lambda^c = 1 - \exp \left( (1 - \beta) \left( W^n_{H,t_0} - W^c_{H,t_0} \right) \right) , \]

where \( W^c_{H,t_0} \) and \( W^n_{H,t_0} \) are the present discounted value of the lifetime utility of the home representative household under cooperation and noncooperation, respectively, as defined in equation (3.2.1). When \( \sigma \neq 1 \),

\[ W^n_{H,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \frac{((1 - \lambda^c) C_t^c)^{1-\sigma}}{1-\sigma} - \chi \left( h_t^c \right)^{1+\omega} \right) , \]

thus \( \lambda^c \) is given by

\[ \lambda^c = 1 - \left( \frac{W^n_{H,t_0} + \mathbb{E}_{t_0}^{c}}{\mathbb{E}_{t_0}^{c}} \right)^{-\frac{1}{\sigma}} , \]

where \( \mathbb{E}_{t_0}^{c} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( C_t^c \right)^{1-\sigma} \) and \( \mathbb{E}_{t_0}^{c} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \chi \left( h_t^c \right)^{1+\omega} \) are the present discounted value of the home representative household’s lifetime stream of consumption and working hours under cooperative policy, respectively, and \( W^n_{H,t_0} \) is the present discounted value of the lifetime utility of the home representative household under noncooperative policy.

We apply the perturbation method to the nonlinear model in Section 3.3.1 to compute \( W^c_{H,t_0} \) and \( W^n_{H,t_0} \). Figure 3.2 depicts the welfare cost from noncooperation of the home country, the foreign country and the world economy as functions of \( \nu \) and \( \sigma \) for \( 0 \leq \nu \leq 2 \) and \( \sigma = 1, 3, 5 \) when \( \omega = 4.71 \). Figure 3.3 depicts the three-dimension figures of the welfare cost from noncooperation as functions of \( \nu \) and \( \sigma \) for \( 0 \leq \nu \leq 2 \) and \( 1 \leq \sigma \leq 5 \) when \( \omega = 4.71 \). The remaining parameters are calibrated as in Table 3.4.

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26We develop our code in Dynare and execute it in MATLAB. Code is available upon request.
In the baseline parameterization as shown by the line of $\sigma = 1$ in Figure 3.2, the estimated mean welfare cost from noncooperation is $\lambda^c = 0.037\%$ in response to a positive home technology shock of one standard deviation. It means that the home households under the cooperative optimal policy have to give up 0.037 per cent of their consumption to be as well-off as under the noncooperative regime. Figure 3.3 shows that in general there exist nonzero gains from cooperation under LCP. The welfare gains from cooperation are largest under $\sigma = 1$ even though two countries are insular in structural equations under PCP. Overall, the size of the gain is relatively small, though not negligible. These results imply that in order to have a large welfare gain from cooperation, frictions other than nominal rigidities or other shocks must be considered.

There are two special cases in which gains from cooperation under LCP become zero: 1) consumption preferences exhibit no home bias, $\nu = 1$ and closed economy, $\nu = 0$ or 2; and 2) disutility of labor becomes linear, i.e. $\omega = 0$. The former makes the two countries identical in every aspect or reduce to closed economies. In particular, when there is no home bias, as mentioned in Engel (2011), there exists no trade-off between eliminating distortions from the breakdown of the law of one price and the inefficient output fluctuations. The latter eliminates the costs stemming from fluctuating labor and therefore output, which are the sources of the deviations from the law of one price as a determinant of the real marginal costs.
Figure 3.2: Welfare costs from noncooperation as functions of $\nu$ under $\sigma = 1, 3, 5$, in percentage

Welfare cost from noncooperation in Home country, $\omega = 4.71$, in percentage

Welfare cost from noncooperation in Foreign country, $\omega = 4.71$, in percentage

Welfare cost from noncooperation in the world economy, $\omega = 4.71$, in percentage
Figure 3.3: Welfare costs from noncooperation as functions of $\nu$ and $\sigma$ in three-dimension, in percentage

- Welfare cost from noncooperation in Home country, $\omega = 4.71$, in percentage
- Welfare cost from noncooperation in Foreign country, $\omega = 4.71$, in percentage
- Welfare cost from noncooperation in the world country, $\omega = 4.71$, in percentage
3.5 Conclusion

This paper finds that there exist gains from cooperation with optimal monetary policy under LCP in response to technology shocks. A two-country DSGE model is developed in the paper and a linear-quadratic approach is adopted to obtain the quadratic loss functions of noncooperative policy makers. The paper shows that noncooperative policy makers under LCP face extra trade-offs regarding stabilizing the real marginal costs induced by deviations from the law of one price. Optimal monetary policy seeks to stabilize CPI inflation rates more so than it does under cooperation. Also, our study suggests that as long as nominal rigidities are the sole distortions in the economy, gains from cooperation are not sizable.

This paper follows Engel (2011) in the optimal monetary policy analysis. One of the strong assumptions of the model is a complete assets market. Corsetti, Dedola, and Leduc (2010) review the development in the NOEM literature and point out that a complete assets market is a highly restrictive assumption which prohibits investigations of inefficiencies other than nominal rigidities. Given the findings in this paper, it would be interesting to investigate the welfare implication of optimal monetary policy under LCP and the incomplete assets market.
Chapter 4

Choice of Policy Instrument and Optimal Monetary Policy in Open Economies

4.1 Introduction

In the context of open economies, the interaction between policy makers when implementing optimal monetary policy, that is whether they should conduct their monetary policy in a cooperative or noncooperative way, is one of the key issues in monetary policy analysis. While the definition of cooperation between two policy makers in the literature is straightforward, researchers differ on the specification of monetary policy under noncooperation, which takes form of a choice of policy instrument.¹

Policy instrument choice for noncooperative games is a small but important part of the monetary policy literature, and yet to be examined extensively. The question at stake is whether the strategic interaction specified by the choice of instrument matters for the outcome of implementing optimal monetary policy under noncooperation. And if so, to what extent it matters?

This question was first discussed in an early literature starting with Poole (1970) and later Sargent and Wallace (1975). The literature is based on the Hicksian IS-LM models and compares two classical monetary policy instruments that monetary authorities operate through–interest rate changes or money stock changes–in closed economies and, as international extensions, in small open economies. It does not consider the

¹The cooperative policy is defined as one in which policy makers jointly maximize the weighted sum of the aggregate welfare of both countries, where the weights are equal to each country’s respective size.
impact of domestic monetary policy on the rest of the world. It typically argues that
the choice of instrument could have a significant effect on the volatility of macrovari-
ables and that the welfare ranking of instruments depends on the particular values of
parameters. Later studies by Canzoneri and Henderson (1989), Henderson and Zhu
(1990), and Turnovsky and d’Orey (1989), among others, start to examine monetary
policy instrument choice with two-country models. They still work with the two clas-
sical monetary policy instruments but have strategic game considerations, inspired by
the work of Hamada (1976). The conclusion drawn from these works is that the non-
cooperative equilibrium that emerges from the strategic game played by central banks
depends crucially on the instrument chosen by the two players. Since the mid-1990s,
the NOEM framework has become the workhorse for monetary policy analysis.2 Under
this framework, Obstfeld and Rogoff (1998, 2002), Devereux and Engel (2003), and
Sutherland (2004) consider the traditional choice of the money supply as policy in-
strument, Clarida, Galí, and Gertler (2002) and Corsetti and Pesenti (2005a) specify
monetary policy regime in terms of output and the nominal interest rate, respectively,
while Benigno and Benigno (2006) choose the inflation rate of the producer goods for
their noncooperative strategic space. Yet none of these studies has explicitly addressed
the issue of whether choosing a different policy instrument would change the equilibrium
outcome of their respective noncooperative monetary policy.

This paper contributes to the NOEM literature on policy instrument choice by ex-
amining four options of policy instrument and their impact on equilibrium outcomes
and welfare in a noncooperative optimal monetary policy environment. The impact is
examined in a microfounded, sticky-price, two-country DSGE model under LCP. The
four policy instrument options are the nominal interest rate, the PPI inflation rate, the
CPI inflation rate, and the inflation rate of price for imported goods.3 The noncoop-
erative game in the paper is defined as one in which each policy maker maximizes the
aggregate welfare function of its own country, taking as given the entire path of the
foreign policy makers’ instrument, as in Blake and Westaway (1995), i.e. an open-loop

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2 The literature was initiated by Obstfeld and Rogoff (1995) and Svensson and van Wijnbergen
(1989). Chapter 3 provides a detailed taxonomy of this literature. The NOEM literature assumes
explicit microfoundation of private sector, providing a natural criterion for evaluating welfare implica-
tions of alternative noncooperative monetary policies.

3 Under PCP, the law of one price holds and the only appropriate option for policy makers is the
PPI inflation rate, because fluctuations of import price inflation rates fully reflect fluctuations of the
nominal exchange rate, and the CPI inflation rate moves in proportion to the PPI inflation rate.
Nash equilibrium.\(^4\) Lombardo and Sutherland (2006) and Coenen et al. (2010) also explicitly evaluate the impact of different policy instruments on the equilibria and welfare under the NOEM framework, although they only consider the money supply and the nominal interest rate as policy instruments and adopt the closed-loop equilibrium, which makes their results not directly comparable to this paper.

Two key findings are obtained. First, the choice of policy instrument does matter for the equilibrium under noncooperative games. Choosing different policy instruments leads to different equilibria. In particular, choosing the nominal interest rate as policy instrument leads to equilibrium indeterminacy. This repeats the findings in Blake (2012) and de Fiore and Liu (2002), although they use different models.\(^5\) Second, the welfare ranking of the policy instruments, excluding the nominal interest rate, depends on the degree of openness of the economy, which is measured by the weight assigned to imported goods in consumers’ utility function. When countries are less open, that is, the domestically produced goods carry a higher weight in the consumption basket than the imported goods, choosing the inflation rate of the domestically produced goods (the PPI inflation rate) as policy instrument generates a larger welfare cost from noncooperation than choosing the CPI inflation rate or the inflation rate of price for the imported goods does. Conversely, the reverse is true when countries are more open: selecting the PPI inflation rate, among the three choices, for the noncooperative game then induces the smallest deviation of the welfare from its cooperative level.

In the paper, there are special cases in which the choice of policy instrument is irrelevant to the equilibrium outcome. They are (1) consumers put equal weight on both types of goods, or (2) the disutility function is linear in labor. Under LCP, these are also the special cases in which gains from cooperation become zero.

The rest of the paper is organized as follows. Section 4.2 sets up the model and derives equilibrium conditions. Section 4.3 specifies the cooperative policy and the noncooperative policy under each of the four strategic games. Section 4.4 compares impulse responses under cooperation and each of the four noncooperative games and

---

\(^4\)The alternative Nash equilibrium would be a closed-loop equilibrium for which the sequence of foreign instruments is known to be dependent on some of the other system variables. See, for example, Coenen et al. (2010) for a discussion on the distinction between the open- and closed-loop Nash game and a list of relevant studies adopting the closed-loop Nash equilibrium into different models.

\(^5\)Blake (2012) uses several canonical New Keynesian models to examine the indeterminacy of fixed nominal interest rate rules in finite horizons, while de Fiore and Liu (2002) examine the conditions on equilibrium determinacy with a feedback interest rate rule in a small open economy model under PCP. They find that a passive interest rate always ensures equilibrium determinacy while an active interest rate can lead to determinacy only if the degree of openness exceeds a certain threshold and a certain critical level, both of which are determined by fundamental parameters in preferences and in technologies.
computes welfare costs from noncooperation given each of the games. Section 4.5 contains some concluding remarks.

4.2 The Model

The model is nearly identical to the one in Chapter 3. There are two countries of equal size, Home and Foreign, each populated with a continuum of households with population size normalized to unity. Agents in the two countries consume both home goods and foreign goods but put different weights on the two categories of goods for utility. This is a popular assumption in the open-economy macroeconomics literature, and can be regarded as a short-cut way of modeling the openness of a country. A less open economy puts less weight on consumption of imported goods.\footnote{In other words, consumers exhibit home bias.} Households supply labor services to firms within their own country via a competitive labor market. Households are also the owners of domestic firms. Firms maximize profits in a monopolistically competitive market using labor as the only input according to aggregate technology. Firms choose domestic prices and export prices separately under LCP. The law of one price does not hold. Governments levy a lump-sum tax on households and subsidize firms so that the deterministic steady-state output level becomes efficient. Central banks as policy makers are benevolent and aim to maximize social welfare through either cooperation or noncooperation. In the cooperative equilibrium, both central banks conduct optimal monetary policies that maximize joint welfare which is defined here as the population-weighted sum of the utility of the representative households in both economies; In the noncooperative equilibrium, each central bank maximizes the welfare of its own country, taking as given the entire path of the other central bank’s instrument. The candidates for policy instruments include the PPI inflation rate, the CPI inflation rate, the inflation rate of price for imported goods, and the nominal interest rate. Correspondingly, we define the Nash game given each of the four policy specifications as Game 1, 2, 3 and 4, respectively.

The structure of the home country is briefly described below. The full details are available in Chapter 3. The foreign country has an identical structure. Where appropriate, foreign variables are denoted with an asterisk.
4.2.1 Households

A representative household in the home country maximizes welfare

$$W_{H,t_0} \equiv \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [u(C_t) - v(h_t)]$$ (4.2.1)

subject to the budget constraint:

$$\mathbb{E}_t [m_{t,t+1} A_{t+1} + B_{t+1} + P_t C_t] \leq A_t + (1 + i_{t-1}) B_t + W_t h_t + \Pi_t + T_t,$$

for $t \geq t_0$, where the consumption aggregator $C_t$, the aggregate consumption of locally produced goods $C_{H,t}$, and the aggregate consumption of imported goods $C_{F,t}$ is given by

$$C_t = C_{H,t}^{\nu} C_{F,t}^{1-\nu},$$ (4.2.2)

$$C_{H,t} = \left[ \int_0^1 C_{H,t}^j (1-\frac{1}{\epsilon}) \, dj \right]^{1-1},$$ (4.2.3)

$$C_{F,t} = \left[ \int_0^1 C_{F,t}^j^{*} (1-\frac{1}{\epsilon}) \, dj^{*} \right]^{1-1},$$ (4.2.4)

respectively. $u(.)$ is the period utility function, increasing and concave in consumption. $v(.)$ is the period disutility function, increasing and convex in labor $h_t$ (measured by working hours). $W_t$ denotes the nominal wage. $A_{t+1}$ denotes the holdings of the state contingent (Arrow) securities at the end of period $t$ denominated in the domestic currency, which equates the marginal rates of substitutions of two countries even ex post. $m_{t,t+1}$ denotes the price of the Arrow securities in period $t$ which gives an unitary return in period $t+1$. $B_t$ is the amount of one-period risk-free nominal bonds held at the beginning of period $t$ with net rate of return $i_{t-1}$. $\Pi_t$ represents the dividend from the ownership of firms. $T_t$ represents the lump-sum tax levied by the government. $\beta$ is the discount factor. $\epsilon$ denotes the elasticity of substitution among differentiated varieties within each country. $\nu \in [0,2]$ determines the (symmetric) home bias. When $\nu$ is larger (smaller) than unity, consumer preference exhibits home (foreign) bias. There is no home bias when $\nu$ equals unity. $C_{H,t}^j$ and $C_{F,t}^{*}$ denote the home representative household’s consumption of the goods produced by the home firm $j$ and the foreign firm $j^{*}$, respectively. Note that Lagrange multipliers on the constraints in equations (4.2.2) to (4.2.4) represent CPI $P_t$, PPI $P_{H,t}$, and the import price index $P_{F,t}$.
4.2.2 Firms

Firm $j$ in the home country sets prices in a monopolistically competitive market to maximize the present discounted value of profits:

$$
E_{t_0} \sum_{t=t_0}^{\infty} \theta^{t-t_0} m_{t_0,t} \Pi_t(j),
$$

where

$$
\Pi_t(j) = (1 + \tau) P_{H,t}(j) C_{H,t}(j) + (1 + \tau) S_t P^*_H(j) C^*_H(j) - W_t h_t(j)
$$

subject to the production function:

$$
Y_t(j) = \exp(z_t) h_t(j),
$$

and the resource constraint:

$$
Y_t(j) = C_{H,t}(j) + C^*_H(j).
$$

$m_{t_0,t}$ is the stochastic discount factor by which firms value profits for their owner, $S_t$ denotes the nominal exchange rate of the foreign currency in units of the home currency. $\tau$ represents the government subsidy rate. Firm $j$ produces $Y_t(j)$ of the product by hiring $h_t(j)$ of labor service from the domestic households according to aggregate production technology $\exp(z_t)$, where $z_t$ follows an AR(1) exogenous process. Firms set their optimal prices, $P_{H,t}(j)$ and $P^*_H(j)$, in a staggered manner à la Calvo (1983) rule. Each time, only with probability $1 - \theta$, can they re-optimize their prices. Note that the Lagrange multiplier on a constraint where the production function in equation (4.2.5) and the resource constraint in equation (4.2.6) are combined represents nominal marginal costs:

$$
NMC_t = \frac{W_t}{\exp(z_t)}.
$$

There is no firm specificity in marginal costs.

4.2.3 Governments and Central Banks

The government in the home country collects a lump sum tax from households and subsidizes firms to eliminate steady state distortions stemming from monopolistic com-
petition. Thus, the subsidy rate is given by

\[ \tau = \frac{1}{\epsilon - 1}. \]

The government always achieves a balanced budget constraint:

\[ T_t = \tau \int_0^1 \left[ P_{H,t}(j)C_{H,t}(j) + S_t P_{H,t}^*(j)C_{H,t}^*(j) \right] \, dj. \]

Benevolent central banks aim to maximize social welfare as Ramsey planners. We consider two cases: both central banks cooperate to maximize global welfare; each maximizes social welfare of its own country in an open-loop Nash (noncooperative) game. Details of such optimal policies will be discussed later.

### 4.2.4 Aggregate Conditions

Taking the integral of equation (4.2.5) over \( j \) gives the aggregate production function of the home country

\[ Y_t = \exp (z_t) h_t. \]

Taking the integral of the resource constraint equation (4.2.6) over \( j \) and making use of the Hicksian demand functions for good \( j \) by consumers in both countries gives the aggregate resource constraint of the home country

\[ Y_t = C_{H,t} \Delta_{H,t} + C_{H,t}^* \Delta_{H,t}^*, \]

where \( \Delta_{H,t} \equiv \int_0^1 \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} \, dj \) and \( \Delta_{H,t}^* \equiv \int_0^1 \left[ \frac{P_{H,t}^*(j)}{P_{H,t}} \right]^{-\epsilon} \, dj \) are the price dispersion terms.\(^8\)

The global assets market is assumed complete, and thus trades in the Arrow securities equate the marginal rates of substitution between two countries even \( \textit{ex post} \). With the assumption of the symmetric initial conditions of wealth, the standard risk sharing condition is obtained as follows:

\[ u' (C_t^*) = e_t u' (C_t), \]

\(^7\)There is no strategic interaction between the government and the central bank.

\(^8\)See Chapter 3 for the details of deriving the Hicksian demand functions.
where the real exchange rate is defined as
\[ e_t \equiv \frac{S_t P_t^*}{P_t}. \]

### 4.2.5 Equilibrium Conditions

The home representative household’s period utility is specified as

\[ u(C_t) \equiv \frac{C_t^{1-\sigma} - 1}{1-\sigma}, \]
\[ v(h_t) \equiv \frac{h_t^{1+\omega}}{1+\omega}. \]

The system of equations consists of the first-order necessary conditions from solving households’ as well as firms’ optimization problem together with market clearing conditions. All nominal variables are detrended by the aggregate price indexes, \( P_t \) and \( P_t^* \), and inflation rates are defined as follows:

\[ \pi_t = \frac{P_t}{P_{t-1}} - 1, \quad \pi_t^* = \frac{P_t^*}{P_{t-1}} - 1, \]
\[ \pi_{H,t} = \frac{P_{H,t}}{P_{H,t-1}} - 1, \quad \pi_{H,t}^* = \frac{P_{H,t}^*}{P_{H,t-1}} - 1, \]
\[ \pi_{F,t} = \frac{P_{F,t}}{P_{F,t-1}} - 1, \quad \pi_{F,t}^* = \frac{P_{F,t}^*}{P_{F,t-1}} - 1. \]

The detailed system of equilibrium conditions is summarized in Table 3.2 in Chapter 3. These equations together with monetary policy rules solve the rational expectations equilibrium.

### 4.3 Optimal Monetary Policies

In this section, we first set up the Ramsey problem under both cooperative and noncooperative games.

- **Cooperative Policy**

Central banks under cooperation maximize global welfare:

\[ W_{W,t_0} = W_{H,t_0} + W_{F,t_0}, \]

where

\[ W_{H,t_0} \equiv \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [u(C_t) - v(h_t)] \]
as in equation (4.2.1) and
\[
W_{F,t_0} \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [u(C^*_t) - v(h^*_t)], \tag{4.3.1}
\]
subject to the nonlinear equilibrium conditions summarized in Table 2 in Chapter 3.

- **Noncooperative Policy**
  - **Game 1**
    In this noncooperative case, the domestic central bank maximizes equation (4.2.1) given \(\{\pi^{*}_{F, t}\}_{t=t_0}^{\infty}\) while the foreign central bank maximizes equation (4.3.1) given \(\{\pi^{*}_{H, t}\}_{t=t_0}^{\infty}\), both central banks are subject to the same nonlinear equilibrium conditions summarized in Table 3.2 in Chapter 3.
  - **Game 2**
    In this noncooperative case, the domestic central bank maximizes equation (4.2.1) given \(\{\pi^{*}_{t}\}_{t=t_0}^{\infty}\) while the foreign central bank maximizes equation (4.3.1) given \(\{\pi^{*}_{t}\}_{t=t_0}^{\infty}\), both central banks are subject to the same nonlinear equilibrium conditions as above.
  - **Game 3**
    In this noncooperative case, the domestic central bank maximizes equation (4.2.1) given \(\{\pi^{*}_{H, t}\}_{t=t_0}^{\infty}\) while the foreign central bank maximizes equation (4.3.1) given \(\{\pi^{*}_{F, t}\}_{t=t_0}^{\infty}\), both central banks are subject to the same nonlinear equilibrium conditions as above.
  - **Game 4**
    In this noncooperative case, the domestic central bank maximizes equation (4.2.1) given \(\{i^{*}_{t}\}\) while the foreign central bank maximizes equation (4.3.1) given \(\{i^{*}_{t}\}_{t=t_0}^{\infty}\), both central banks are subject to the same nonlinear equilibrium conditions as above.

We apply the perturbation method to solve the above Ramsey problems. Deterministic steady states around which the system is locally approximated are obtained and reported in Appendix A in Chapter 3.\(^9\) Results are presented and discussed in the next section.\(^10\)

---

\(^9\)As discussed in Chapter 3, the deterministic steady state turns out to be irrespective of cooperation or noncooperation. It is that under the flexible price equilibrium thanks to the appropriate fiscal subsidy rate that eliminates the monopolistic distortion in output at the steady state. This allows me to make meaningful comparison of the welfare under alternative policies.

\(^10\)We develop the code in Dynare and execute it in MATLAB. Code is available upon request.
4.4 Results

In this section, we first depict the impulse responses of the two countries in response to a technology shock to the home country under both cooperative policy and noncooperative policies as defined in Section 4.3. We then compute welfare costs from noncooperation specified by each of the games above. Note that under Game 4, where a central bank chooses the optimal allocation taking as given the foreign interest rate, a locally indeterminate (explosive) equilibrium emerges. The rationale behind the indeterminacy is that the domestic central bank would choose a best response to the exogenously given foreign interest rate such that a saddle-path equilibrium is reestablished, and when two such strategies are combined together, they would produce too many unstable roots, as Coenen et al. (2010) argue. Below we report dynamics and welfare costs from noncooperation under cooperation and Games 1-3.

4.4.1 Impulse Responses Results

The baseline calibration for parameter values is identical to that in Table 3.4 in Chapter 3. Briefly, $\beta$, $\chi$ and the probability of not being able to reset prices $\theta$ are set at the conventional values. $\nu$ is set at 1.5 which means consumers put $3/4$ of the weight on domestic goods in utility, that is, consumption preference exhibits home bias or equivalently, countries are less open. The inverse of intertemporal elasticity of substitution in consumption, $\sigma$, is set to 1 and the elasticity of substitution among different varieties within goods category, $\epsilon$, is set at 7.66. The inverse of the Frisch elasticity $\omega$ is set to 4.71. And finally the log-technology follows an AR(1) stochastic process with serial correlation $\rho$ set at 0.856 and standard deviation at 0.0064.

Figure 4.1 depicts the impulse responses of the key macrovariables under both cooperation and noncooperative games specified by Game 1-3 in response to a positive technology shock of one standard deviation to the home country (we scale up the impulse responses by 100 so the dynamics in Figure 4.1 are measured in per cent). Figure 4.1 shows that optimal monetary policies under different noncooperative games achieve different equilibria under LCP in response to the same technology shock. Specifically, choosing the PPI inflation rate as policy instrument for implementing the optimal policy (Game 1) gives rise to the largest deviation of the real variables from their respective cooperative allocations, while taking the import price inflation rate in the counterpart country as given (Game 3) generates the smallest deviation. Game 2 with the CPI inflation rate as policy instrument sees a deviation of a degree that lies in between the first two.
To understand the dynamics behind these noncooperative regimes, it is important to first recognize that, under LCP, policy makers under noncooperation seek to stabilize inflation rates that are relevant to their own country and de-stabilize those relevant to the foreign country more so than it does under cooperation, as demonstrated by the additional terms in the quadratic loss functions under noncooperation in Section 3.3.2 in Chapter 3.\textsuperscript{11} With $\nu > 1$, the two countries are less open and the producer goods weigh more than the imported goods in the consumption basket for utility. The stabilizing/destabilizing incentive at home then becomes strongest under Game 1 where the PPI inflation rate of the foreign country is taken as given. As a result, the CPI inflation rate of the two countries is stabilized to the greatest extent under Game 1, and to a lesser extent under Game 2. Game 3 sees the smallest deviation of the CPI inflation rate from its allocation under cooperation.

Bearing this in mind, under LCP with $\nu > 1$, optimal policy also trades off output responses with stabilization of the CPI inflation rate. As a trade-off, home output increases the least under Game 1, which translates into the farthest fall in the home PPI inflation rate and corresponds to the biggest cut back in home working hours. In response to a technology improvement shock, the home (foreign) monetary policy thus becomes the least expansionary (contractionary) under Game 1, evidenced by the movements in nominal interest rates in both countries. The resultant (nominal and) real exchange rate depreciation and terms-of-trade improvement of the home country is of the smallest degree again under Game 1.\textsuperscript{12} This means the real purchasing power of the home consumers is raised by the smallest amount at any given price level, thus the home aggregate consumption (demand for both goods) rises the least under Game 1. Foreign output rises the most under Game 1 because the smallest improvement of the home terms of trade mirrors the smallest deterioration of the foreign terms of trade, giving foreign consumers stronger purchasing power under Game 1, relative to that under Game 2 or 3, which raises foreign demand for the foreign goods at any given price level. Foreign aggregate consumption is compensated and working hours are extended the most under Game 1 accordingly.

\textsuperscript{11}See equations (3.3.2) and (3.3.3) in Chapter 3 for quadratic loss function of the home and foreign policy maker, respectively.
\textsuperscript{12}Recall that under LCP a depreciation of the home currency leads to an improvement of the home terms of trade as households receive more revenues from export sales denominated in the home currency, leading to a stronger real purchasing power of the home consumers.
Figure 4.1: Impulse responses of both countries to home technology improvement shocks by one SD.
4.4.2 Welfare Cost from Noncooperation

Table 4.1 reports welfare costs from noncooperation under each of the three strategic games. The welfare cost is measured in consumption units, following Lucas (1992). The first two rows report the welfare costs from noncooperation for the home and foreign households, respectively, in the baseline parameterization as in Section 4.4.1. It shows that when $\nu > 1$ in the baseline model, welfare cost from the inability to cooperate is largest under Game 1 where the PPI inflation rate is selected as policy instrument. The welfare cost in this case amounts to 0.0297% in consumption units for each country, meaning that households in each country have to give up about 0.03 per cent of their aggregate consumption to be as well off under cooperation as under noncooperative Game 1. Welfare cost from noncooperation reduces to $\lambda^H = \lambda^F = 0.0219\%$ under Game 2 and $\lambda^H = \lambda^F = 0.0114\%$ under Game 3 for both countries. Note that the gain from cooperation is relatively small in absolute values with only technology shocks (Chapter 3 computes the welfare gain from cooperation over the reasonable range of parameter calibration and in general the size of the gain with only technology shocks is small).

The rows in $\nu = 0.5$ in Table 1 show that the ranking of the welfare costs is reversed when the two countries become more open with $\nu < 1$. Welfare cost from noncooperation is computed to be the smallest under Game 1 while the largest under Game 3. As a robustness check, the rows in $\sigma = 2$ report welfare costs from noncooperation when the intertemporal elasticity of substitution of consumption is not unitary. They show that given the degree of openness in the baseline calibration, the welfare ranking of the strategic games stays unchanged.

There are two special cases in which choice of policy instrument is irrelevant to the equilibrium outcome under LCP, as shown in rows of $\nu = 1$ and $\omega = 0$ in Table 4.1. They are the exact special cases in which welfare costs from noncooperation are zero found in Chapter 3.

---

13See Section 4.2 in Chapter 3 for details in derivation of expressions for consumption units.
Table 4.1: Welfare costs from noncooperation under different games.
Baseline parameter values: $\nu = 1.5$, $\omega = 4.71$, $\sigma = 1$.

<table>
<thead>
<tr>
<th></th>
<th>Game 1</th>
<th>Game 2</th>
<th>Game 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>baseline</td>
<td>$\lambda_H$</td>
<td>.0297</td>
<td>.0219</td>
</tr>
<tr>
<td></td>
<td>$\lambda_F^*$</td>
<td>.0297</td>
<td>.0219</td>
</tr>
<tr>
<td>$\nu = 0.5$</td>
<td>$\lambda_H$</td>
<td>.0114</td>
<td>.0219</td>
</tr>
<tr>
<td></td>
<td>$\lambda_F^*$</td>
<td>.0114</td>
<td>.0219</td>
</tr>
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<td>$\sigma = 2$</td>
<td>$\lambda_H$</td>
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<td>.0037</td>
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<tr>
<td></td>
<td>$\lambda_F^*$</td>
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<td>.0041</td>
</tr>
<tr>
<td>$\nu = 1$</td>
<td>$\lambda_H$</td>
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<td>0</td>
</tr>
<tr>
<td></td>
<td>$\lambda_F^*$</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$\omega = 0$</td>
<td>$\lambda_H$</td>
<td>0</td>
<td>0</td>
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<tr>
<td></td>
<td>$\lambda_F^*$</td>
<td>0</td>
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</tbody>
</table>

### 4.5 Conclusions

This paper adds to the discussion on policy instrument choice in the NOEM literature by explicitly considering four options of policy instruments—the nominal interest rate, the PPI inflation rate, the CPI inflation rate and the import price inflation rate—for implementing noncooperative optimal monetary policy. It examines the impact of selecting different policy instruments on the equilibrium and welfare of the noncooperative policy in a two-country DSGE model under LCP. It shows that, in general, the choice of policy instrument does matter for the equilibrium outcome and affects the size of welfare cost from noncooperation. Excluding the choice of taking as given the nominal interest rate of the foreign country, which leads to equilibrium indeterminacy, the welfare ranking of the other three policy instruments depends crucially on the degree of openness of a country. As consistent with the previous chapter, this paper assumes the presence of only technology shocks. As an extension of the current paper, it would be interesting to re-examine the equilibrium and welfare implications of the noncooperative regimes if other exogenous shocks, such as markup shocks, hit the economy.
Appendix

Technical Appendix for Chapter 2

- log-linearized equilibrium conditions

\[
\hat{\lambda}_t = -\frac{1}{1-\beta\theta(z^*)^{-\sigma}} \left\{ \frac{\sigma}{1-\theta/z^*} \left[ \hat{c}_t - \frac{\theta}{z^*} (\hat{c}_{t-1} - z_t^*) \right] - z_t^b \right\} \\
+ \frac{\beta\theta(z^*)^{-\sigma}}{1-\beta\theta(z^*)^{-\sigma}} \left[ \frac{\sigma}{1-\theta/z^*} \left( E_t^\hat{c}_{t+1} + E_t^z_{t+1} - \frac{\theta}{z^*} \hat{c}_t \right) - E_t^z_{t+1} \right]
\]

\[
\hat{\lambda}_t = E_t^\hat{\lambda}_{t+1} - \sigma E_t^z_{t+1} + \hat{r}_t^n - E_t\hat{\pi}_{t+1}
\]

\[
\hat{m}_t = -\frac{1}{\sigma} \left( \hat{\lambda}_t + \frac{1}{\hat{r}_t^m} \hat{r}_t^n - z_t^m - z_t^b \right)
\]

\[
\hat{w}_t = \hat{w}_{t-1} - \hat{\pi}_t + \gamma_w \hat{\pi}_{t-1} - z_t^* + \beta(z^*)^{1-\sigma} \left( E_t^\hat{w}_{t+1} - \hat{w}_t + E_t\hat{\pi}_{t+1} - \gamma_w \hat{\pi}_t + E_tz_{t+1}^* \right)
\]

\[
+ \frac{(1 - \xi_w)}{\xi_w} \frac{(1 - \beta(z^*)^{1-\sigma}\xi_w)}{1 + \chi(1 + \lambda_w)/\lambda_w} \left( \chi \hat{h}_t - \hat{\lambda}_t - \hat{w}_t + z_t^b \right) + z_t^w
\]

\[
\hat{l}_t = \frac{1 + \lambda^i}{1 + \lambda^i - n/k} (\hat{q}_t + \hat{k}_t) + \left( 1 - \frac{1 + \lambda^i}{1 + \lambda^i - n/k} \right) \hat{n}_t
\]
\[
E_t \hat{r}_t^{E+1} = \left(1 - \frac{1 - \delta}{r E \psi}\right) E_t \hat{r}_t^k + \frac{1 - \delta}{r E \psi} E_t \hat{q}_t + \hat{q}_t - E_t \hat{z}_t^{\psi+1} \\
E_t \hat{r}_t^{E+1} = \hat{r}_t^a - E_t \hat{r}_t^{E+1} - \mu (\hat{n}_t - \hat{q}_t - \hat{k}_t) + z_t^\mu \\
\hat{n}_t = \frac{\eta^{E}}{z^*} \left\{ \frac{1 + \lambda^i}{n/k} \left[ (1 - \frac{1 - \delta}{r E \psi} \right) \hat{r}_t^k + \frac{1 - \delta}{r E \psi} \hat{q}_t - \hat{q}_t - z_t^{\psi} \left\} - \left( \frac{1 + \lambda^i}{n/k} - 1 \right) E_t \hat{n}_t^{E+1} + \hat{n}_t - z_t^{\psi} \right\} + z_t^\eta \\
0 = \hat{w}_t + \hat{h}_t - (\hat{r}_t^k + \hat{u}_t + \hat{k}_t - z_t^* - z_t^{\psi}) \\
\hat{u}_t = \tau (\hat{r}_t^k - \hat{q}_t) \\
\hat{m}c_t = (1 - \alpha) \hat{w}_t + \alpha \hat{r}_t^k \\
\hat{n}_t = \gamma_p \hat{n}_t - \beta (z^*)^{1-\sigma} (E_t \hat{n}_t - \gamma_p \hat{n}_t) + \frac{(1 - \xi_p)(1 - \beta (z^*)^{1-\sigma} \xi_p)}{\xi_p} \hat{m}c_t + z_t^p \\
\hat{y}_t = (1 + \phi) [(1 - \alpha) \hat{h}_t + \alpha (\hat{u}_t + \hat{k}_t - z_t^* - z_t^{\psi})] \\
\hat{y}_t = \frac{c}{y} \hat{c}_t + \frac{i}{y} \hat{i}_t + z_t^\eta \\
\hat{k}_t = \frac{1 - \delta - r E \psi}{z^* \psi} \hat{u}_t + \frac{1 - \delta}{z^* \psi} (\hat{k}_t - z_t^* - z_t^{\psi}) + \left(1 - \frac{1 - \delta}{z^* \psi} \right) (\hat{i}_t + z_t^\nu) \\
\hat{q}_t = \zeta (\hat{i}_t - \hat{i}_{t+1} + z_t^* + z_t^{\psi}) - \beta (z^*)^{1-\sigma} \zeta (E_t \hat{i}_t + \hat{i}_t + z_t^* + z_t^{\psi}) - z_t^\nu + z_t^i 
\]
\[
\begin{align*}
\hat{r}_t^n &= (1 - W) (r^n - 1) \left( -\hat{\lambda}_t - \sigma \hat{m}_t + z_t^b + z_t^m \right) \\
&\quad + W \left[ \phi_r \hat{r}_{t-1}^n + (1 - \phi_r) \left( \frac{\phi_r}{4} \sum_{j=0}^{3} \hat{\pi}_{t-j} + \phi_{\pi} \hat{y}_t \right) + \phi_{\Delta y} (\hat{y}_t - \hat{y}_{t-1} + z_t^*) \right] + z_t^r \\
\hat{r}_t^n &= (1 - W) (r^n - 1) \left( -\hat{\lambda}_t - \sigma \hat{m}_t + z_t^b + z_t^m \right)
\end{align*}
\]

if \( W \in (0, 1) \)

where hatted variables represent log-deviations from steady state values and \( z_t^* = z_t^\pi + \alpha/(1 - \alpha) z_t^\psi \).

- Steady-state conditions used in estimations:

\[
\begin{align*}
\beta &= \frac{(z^\pi)^{\sigma \pi}}{r^n}, \\
\lambda^p &= \phi, \\
h &= \frac{1 - \alpha u}{\alpha z^\pi w}, \\
i &= \frac{1 - \delta}{z^\pi}, \\
c &= 1 - \frac{g}{y} - \frac{i}{y}, \\
y &= \frac{i}{k}, \\
w &= (1 - \alpha) \left[ \frac{1}{\frac{\alpha}{w^k}} \right]^{\frac{1}{1 - \alpha}}, \\
k &= \frac{(1 + \phi)(z^\pi w^k)^\alpha \left( \frac{h}{k} \right)^{1 - \alpha}}, \\
i &= \frac{i}{k}, \\
y &= \frac{k}{y}, \\
r^k &= \frac{1 + \lambda^i}{u} \left( r^{E, \psi} - 1 + \delta \right), \\
w &= (1 - \alpha) \left[ \frac{1}{\frac{\alpha}{w^k}} \right]^{\frac{1}{1 - \alpha}}, \\
k &= \frac{(1 + \phi)(z^\pi w^k)^\alpha \left( \frac{h}{k} \right)^{1 - \alpha}}, \\
i &= \frac{i}{k}, \\
y &= \frac{k}{y}, 
\end{align*}
\]
Technical Appendix for Chapter 3

Appendix A. Structural equations

A.1 Structural Equations of Private Agents

In this section we show the derivation of the first-order conditions listed in Table 3.2 in the text. First, equations (i)-(ii), (xix)-(xx) are derived from the representative household’s optimization problem with respect to consumption, labor and nominal bond holdings in the home and foreign country, respectively. Next, equations (iii)-(iv), (xxi)-(xxii) are from cost minimization problem of the two representative households. The home representative household, for example, chooses \( C_{H,t} \) and \( C_{F,t} \) to minimize

\[
P_{H,t}C_{H,t} + P_{F,t}C_{F,t}
\]

subject to the aggregate consumption

\[
C_t = C_{H,t}^\nu C_{F,t}^{1-\nu},
\]

(4.5.1)

taking as given the price indexes \( P_{H,t} \) and \( P_{F,t} \). The first-order conditions give (iii)-(iv). Similarly the foreign consumers’ cost minimization problem gives (xxi)-(xxii). Substituting the Hicksian demand functions (iii)-(iv) into equation (4.5.1) gives price index equation (v). Analogously, substitution of the foreign Hicksian demand functions into foreign consumption aggregator \( C_t^* \) gives (xxiii). Equations (vi)-(viii) for the home country and (xxiv)-(xxvi) for the foreign country are derived in the text.

Next, we derive firms’ price optimizing conditions under LCP. Specifically, home firm \( j \) takes into account the probability that it will not get to reset prices consecutively for certain periods of time and chooses \( P_{H,t_0}(j) \) and \( P_{H,t_0}^*(j) \) to maximize its present discounted value of profits

\[
\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \theta^{t-t_0} m_{t_0,t} \{ (1 + \tau) P_{H,t_0}(j)C_{H,t}(j) + (1 + \tau) S_t P_{H,t_0}^*(j)C_{H,t}^*(j) - W_i h_t(j) \}
\]
subject to the demand functions

\[ \begin{align*}
C_{H,t} (j) &= \left( \frac{P_{H,t_0} (j)}{P_{H,t}} \right) ^{-\epsilon} C_{H,t} \\
C^*_t (j) &= \left( \frac{P^*_{H,t_0} (j)}{P^*_{H,t}} \right) ^{-\epsilon} C^*_t
\end{align*} \]

and the resource constraint

\[ Y_t (j) = \exp (z_t) h_t = C_{H,t} (j) + C^*_t (j), \]

taking as given the aggregate price indexes \( P_{H,t}, P^*_{H,t} \) and consumption levels \( C_{H,t}, C^*_t \).

Since all the domestic firms face the same optimizing problem, they eventually set the same price for the same market. Denote the optimal price as \( \tilde{P}_{H,t_0} \) and \( \tilde{P}^*_{H,t_0} \). The first-order conditions with respect to \( \tilde{P}_{H,t_0} = \tilde{P}_{H,t_0} (j) \) are given by

\[ \left( \frac{\tilde{P}_{H,t_0}}{P_{H,t_0}} \right) = \frac{\epsilon}{(\epsilon - 1) (1 + \tau)} \frac{\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{t-t_0} C_{t}^{-\sigma} C_{H,t} M C_{t} \left( \frac{P_{H,t_0}}{P_{H,t}} \right)^{-\epsilon}}{\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{t-t_0} C_{t}^{-\sigma} C_{H,t} P_{H,t} \left( \frac{P_{H,t_0}}{P_{H,t}} \right)^{1-\epsilon}}. \]

In addition, the price index \( P_{H,t_0} \) evolves according to

\[ P_{H,t_0}^{1-\epsilon} = \theta P_{H,t_0-1}^{1-\epsilon} + (1 - \theta) \tilde{P}_{H,t_0}^{1-\epsilon}, \]

that is

\[ \frac{\tilde{P}_{H,t_0}}{P_{H,t_0}} = \left[ 1 - \theta (\pi_{H,t_0})^{\epsilon-1} \right] \frac{1}{1 - \theta}, \]

where we define \( \pi_{H,t_0} = \frac{P_{H,t_0}}{P_{H,t_0-1}} \).

Combining the above two equations regarding \( \frac{\tilde{P}_{H,t_0}}{P_{H,t_0}} \) and define

\[ \begin{align*}
K_{H,t_0} &\equiv \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{t-t_0} C_{t}^{-\sigma} C_{H,t} M C_{t} \left( \frac{P_{H,t_0}}{P_{H,t}} \right)^{-\epsilon} \\
F_{H,t_0} &\equiv \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{t-t_0} C_{t}^{-\sigma} C_{H,t} P_{H,t} \left( \frac{P_{H,t_0}}{P_{H,t}} \right)^{1-\epsilon}
\end{align*} \]

and we obtain equation (xi) in Table 3.2. Write \( K_{H,t_0} \) and \( F_{H,t_0} \) for any \( t \geq t_0 \) in a
recursive way and we have equations (xii)-(xiii). Note that we have imposed the subsidy condition \( \frac{\epsilon}{(\epsilon - 1)(1 + \tau)} = 1 \) in the text.

The first-order conditions with respect to \( \tilde{P}_{H,t}^* = \tilde{P}_{H,t}^*(j) \) are given by

\[
\left( \frac{\tilde{P}_{H,t_0}^*}{P_{H,t_0}^*} \right) = \frac{\epsilon}{(\epsilon - 1)(1 + \tau)} \left( \beta \theta \right) \left( \tilde{P}_{H,t_0}^* \right) \left( \tilde{P}_{H,t_0}^* \right)^{-\epsilon} \]

and the evolution of \( P_{H,t_0}^* \) is given by

\[
\frac{\tilde{P}_{H,t_0}^*}{P_{H,t_0}^*} = \left[ \frac{1 - \theta \left( \pi_{H,t_0}^* \right)^{\epsilon - 1}}{1 - \theta} \right]^{\frac{1}{\epsilon}} \]

Define \( K_{H,t_0}^* \) and \( F_{H,t_0}^* \) in an analogous way and combine the two above equations, and we obtain equation (xiv). Equations (xv) and (xvi) are the recursive expressions of \( K_{H,t_0}^* \) and \( F_{H,t_0}^* \) for any \( t \geq t_0 \). Repeat the proceeding process for foreign firm \( j^* \)'s optimization problem and we have equations (xxix)-(xxxiv).

Next, equations regarding price dispersion, (ix)-(x) and (xxvii)-(xxviii) are derived as follows: Take, for example, the definition of price dispersion within home goods sold in the domestic market,

\[
\Delta_{H,t} = \int_0^1 \left[ \frac{P_{H,t}(j)}{P_{H,t}} \right]^{-\epsilon} \, dj.
\]

By the law of large number, it can be written as

\[
\Delta_{H,t} = \int_0^\theta \left[ \frac{P_{H,t-1}(j)}{P_{H,t}} \right]^{-\epsilon} \, dj + \int_\theta^1 \left[ \frac{\tilde{P}_{H,t}}{P_{H,t}} \right]^{-\epsilon} \, dj
\]

\[
= \theta \int_0^1 \left( \frac{P_{H,t-1}}{P_{H,t}} \right)^{-\epsilon} \left[ \frac{P_{H,t-1}(j)}{P_{H,t-1}} \right]^{-\epsilon} \, dj + (1 - \theta) \int_0^1 \left( \frac{\tilde{P}_{H,t}}{P_{H,t}} \right)^{-\epsilon} \, dj
\]

\[
= \theta \left( \frac{P_{H,t-1}}{P_{H,t}} \right)^{-\epsilon} \Delta_{H,t-1} + (1 - \theta) \left[ 1 - \theta \left( \frac{P_{H,t-1}}{P_{H,t}} \right)^{1-\epsilon} \right] \cdot \frac{1 - \theta}{1 - \theta} \frac{1}{\epsilon} \Delta_{H,t-1} + (1 - \theta) \left[ 1 - \theta \left( \frac{P_{H,t-1}}{P_{H,t}} \right)^{1-\epsilon} \right]^{\frac{1}{\epsilon}} \Delta_{H,t-1}.
\]

In the last step we make use of the definition for period \( t - 1 \) and the price evolution process of \( P_{H,t} \) as shown above. The remaining three price dispersion equations can be
Finally, equations (xvii)-(xviii) and (xxxv)-(xxxvi) are from the detrending definitions. Equation (xxxvii) is the risk sharing condition from the assumption of complete assets market in the text.

A.2 Deterministic Steady State

In this section we derive the deterministic steady-state values of endogenous variables in Table 3.2. At this steady state, log-technology is at its zero mean, that is \( z = 0 \). Prices are stable, that is \( \Delta_H = \Delta_H^* = \Delta_F = \Delta_F^* = 1 \); \( \pi = \pi^* = \pi_H = \pi_H^* = \pi_F = \pi_F^* = 1 \); \( K_H = F_H \); \( K_F = F_F^* \); \( K_H^* = F_H^* \); \( K_F = F_F \). Given these relations, the steady-state system in Table 3.2 can be solved as follows:

\[
p_H = p_F = p_H^* = p_F^* = MC = MC^* = w = w^* = k
\]

\[
e = 1
\]

\[
i = i^* = \frac{1}{\beta} - 1
\]

\[
C^* = C
\]

\[
K_H = F_H = K_F^* = F_F^* = \frac{\nu}{2(1 - \beta \theta)} C
\]

\[
K_H^* = F_H^* = K_F = F_F = \frac{2 - \nu}{2(1 - \beta \theta)} C
\]

\[
C_H = C_F^* = \frac{\nu}{2} k^{-1} C
\]

\[
C_H^* = C_F = \left(1 - \frac{\nu}{2}\right) k^{-1} C
\]

\[
Y = h = Y^* = h^* = k^{-1} C,
\]

where steady-state aggregate consumption \( C \) is given by

\[
C = \left(\frac{k^{1+\omega}}{\chi}\right)^{1/(\omega+\sigma)}
\]
and \( k \equiv \left( \frac{\nu}{2} \right)^{\frac{2}{\nu}} \left( 1 - \frac{\nu}{2} \right)^{1 - \frac{2}{\nu}} \). Note that the steady-state equations equating the real wage to marginal rate of substitution between consumption and leisure, and the steady-state equation regarding the resource constraint are given as

\[
\chi h^\omega C^\sigma = k
\]

\[
h = k^{-1} C.
\]

They are useful for the second-order approximation to the utility functions that we will show later.
Appendix B. Ramsey Policy

In this section, we first set up the Ramsey problem for the cooperative global policy maker and derive the necessary optimality conditions. We then solve for the deterministic steady state of this system. We repeat the proceeding process for the noncooperative policy makers.

The structural equations describing decentralized decisions of private agents and aggregate equilibrium conditions are given as follows (they are listed in Table 3.2 in the text and \( k = \left( \frac{\nu}{2} \right)^{\beta} (1 - \frac{\nu}{2})^{1-\beta} \) in the following equations):

\[
C_t^{-\sigma} w_t - \chi h_t^\omega = 0 \tag{4.5.2}
\]

\[
\beta E_t \left( \frac{1 + i_t}{\pi_{t+1}} C_{t+1}^{-\sigma} \right) - C_t^{-\sigma} = 0 \tag{4.5.3}
\]

\[
\frac{\nu}{2} p_{H,t}^{-1} C_t - C_{H,t} = 0 \tag{4.5.4}
\]

\[
\left( 1 - \frac{\nu}{2} \right) p_{F,t}^{-1} C_t - C_{F,t} = 0 \tag{4.5.5}
\]

\[
1 - k^{-1} (p_{H,t})^{\nu/2} (p_{F,t})^{1-\nu/2} = 0 \tag{4.5.6}
\]

\[
(C_t^*)^{-\sigma} w_t^* - \chi (h_t^*)^\omega = 0 \tag{4.5.7}
\]

\[
\beta E_t \left( \frac{1 + i_t^*}{\pi_{t+1}^*} (C_{t+1}^*)^{-\sigma} \right) - (C_t^*)^{-\sigma} = 0 \tag{4.5.8}
\]

\[
(1 - \frac{\nu}{2}) p_{H,t}^{-1} C_t^* - C_{H,t}^* = 0 \tag{4.5.9}
\]
\[
\frac{\nu}{2} p^{*-1}_{F,t} C^*_t - C^*_F,t = 0 \tag{4.5.10}
\]

\[
1 - k^{-1} \left( p^{*}_{H,t} \right)^{1-\nu/2} \left( p^{*}_{F,t} \right)^{\nu/2} = 0 \tag{4.5.11}
\]

\[
C^*_t e_t - (C^*_t)^{-\sigma} = 0 \tag{4.5.12}
\]

\[
\frac{w_t}{\exp(z_t)} - MC_t = 0 \tag{4.5.13}
\]

\[
C_{H,t} \Delta_{H,t} + C^*_{H,t} \Delta^*_{H,t} - \exp(z_t) h_t = 0 \tag{4.5.14}
\]

\[
\exp(z_t) h_t - Y_t = 0 \tag{4.5.15}
\]

\[
(1 - \theta) \left[ \frac{1 - \theta (\pi^{*}_{H,t})^{\epsilon-1}}{1 - \theta} \right]^{\frac{\epsilon}{\epsilon-1}} + \theta (\pi^{*}_{H,t})^{\epsilon} \Delta_{H,t-1} - \Delta_{H,t} = 0 \tag{4.5.16}
\]

\[
(1 - \theta) \left[ \frac{1 - \theta (\pi^{*}_{H,t})^{\epsilon-1}}{1 - \theta} \right]^{\frac{\epsilon}{\epsilon-1}} + \theta (\pi^{*}_{H,t})^{\epsilon} \Delta^{*}_{H,t-1} - \Delta^{*}_{H,t} = 0 \tag{4.5.17}
\]

\[
F_{H,t} \left[ \frac{1 - \theta (\pi^{*}_{H,t})^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{\epsilon-1}} - K_{H,t} = 0 \tag{4.5.18}
\]
\[-F_{H,t} + \frac{C_{H,t} \rho_{H,t}}{e_t} + \beta \theta e_t \frac{C_{t+1}^{-\sigma} e_t}{C_t^{-\sigma}} \pi_{H,t+1}^{\epsilon - 1} F_{H,t+1} = 0 \]  \hspace{1cm} (4.5.19)

\[-K_{H,t} + \frac{C_{H,t} MC_t}{e_t} + \beta \theta e_t \frac{C_{t+1}^{-\sigma} e_t}{C_t^{-\sigma}} \pi_{H,t+1}^{\epsilon} K_{H,t+1} = 0 \]  \hspace{1cm} (4.5.20)

\[F_{H,t}^* \left[ \frac{1 - \theta (\pi_{H,t}^*)^{\epsilon - 1}}{1 - \theta} \right] = -K_{H,t}^* = 0 \]  \hspace{1cm} (4.5.21)

\[-F_{H,t}^* + C_{H,t}^* p_{H,t} + \beta \theta e_t \frac{C_{t+1}^{-\sigma} e_t}{C_t^{-\sigma}} (\pi_{H,t+1}^*)^{\epsilon - 1} F_{H,t+1}^* = 0 \]  \hspace{1cm} (4.5.22)

\[-K_{H,t}^* + \frac{C_{H,t}^* MC_t}{e_t} + \beta \theta e_t \frac{C_{t+1}^{-\sigma} e_t}{C_t^{-\sigma}} (\pi_{H,t+1}^*)^{\epsilon} K_{H,t+1}^* = 0 \]  \hspace{1cm} (4.5.23)

\[\frac{w_t^*}{\exp (z_t^*)} - MC_t^* = 0 \]  \hspace{1cm} (4.5.24)

\[C_{F,t} \Delta_{F,t} + C_{F,t}^* \Delta_{F,t}^* - \exp (z_t^*) h_t^* = 0 \]  \hspace{1cm} (4.5.25)

\[\exp (z_t^*) h_t^* - Y_t^* = 0 \]  \hspace{1cm} (4.5.26)

\[(1 - \theta) \left[ \frac{1 - \theta (\pi_{F,t})^{\epsilon - 1}}{1 - \theta} \right] h_t^* + \theta (\pi_{F,t})^\epsilon \Delta_{F,t-1} - \Delta_{F,t} = 0 \]  \hspace{1cm} (4.5.27)
\[(1 - \theta) \left[ \frac{1 - \theta (\pi_{F,t}^*)^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{\epsilon-1}} + \theta (\pi_{F,t}^*)^\epsilon \Delta_{F,t-1}^* - \Delta_{F,t}^* = 0 \tag{4.5.28} \]

\[F_{F,t}^* \left[ \frac{1 - \theta (\pi_{F,t}^*)^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{\epsilon-1}} - K_{F,t}^* = 0 \tag{4.5.29} \]

\[-F_{F,t}^* + e_t C_{F,t}^* p_{F,t}^* + \beta \theta E_t \left( \frac{(C_{t+1}^*)^{-\sigma} e_t}{(C_{t}^*)^{-\sigma} e_{t+1}} \right) (\pi_{F,t+1}^*)^{\epsilon-1} F_{F,t+1}^* = 0 \tag{4.5.30} \]

\[-K_{F,t}^* + e_t C_{F,t}^* MC_t^* + \beta \theta E_t \left( \frac{(C_{t+1}^*)^{-\sigma} e_t}{(C_{t}^*)^{-\sigma} e_{t+1}} \right) (\pi_{F,t+1}^*)^\epsilon \pi_{F,t+1}^* = 0 \tag{4.5.31} \]

\[F_{F,t} \left[ \frac{1 - \theta (\pi_{F,t})^{\epsilon-1}}{1 - \theta} \right]^{\frac{1}{\epsilon-1}} - K_{F,t} = 0 \tag{4.5.32} \]

\[-F_{F,t} + C_{F,t} p_{F,t} + \beta \theta E_t \left( \frac{(C_{t+1}^*)^{-\sigma} e_t}{(C_{t}^*)^{-\sigma} e_{t+1}} \right) \pi_{F,t+1} F_{F,t+1} = 0 \tag{4.5.33} \]

\[-K_{F,t} + e_t C_{F,t} MC_t^* + \beta \theta E_t \left( \frac{(C_{t+1}^*)^{-\sigma} e_t}{(C_{t}^*)^{-\sigma} e_{t+1}} \right) \pi_{F,t+1} K_{F,t+1} = 0 \tag{4.5.34} \]

\[\pi_t \frac{p_{H,t}}{p_{H,t-1}} - \pi_{H,t} = 0 \tag{4.5.35} \]

\[\pi_t^* \frac{p_{H,t}^*}{p_{H,t-1}} - \pi_{H,t}^* = 0 \tag{4.5.36} \]
\[
\pi_t^* \frac{p_{F,t}^*}{p_{F,t-1}^*} - \pi_{F,t}^* = 0
\] (4.5.37)

\[
\pi_t \frac{p_{F,t}}{p_{F,t-1}} - \pi_{F,t} = 0
\] (4.5.38)

### B.1 Cooperation

A global policy maker maximizes welfare of both countries

\[
W_{W,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \frac{C_t^{1-\sigma}}{1-\sigma} + \frac{C_{t+1}^{1-\sigma}}{1-\sigma} - \chi \frac{h_t^{1+\omega}}{1+\omega} - \chi \frac{h_{t+1}^{1+\omega}}{1+\omega} \right)
\]

with respect to 39 endogenous variables \( \{i_t, i_t^*, C_t, C_t^*, C_{H,t}, C_{F,t}, C_{H,t}^*, C_{F,t}^*, h_t, h_t^*, \pi_t, \pi_{H,t}, \pi_{H,t}^*, \pi_{F,t}, \pi_{F,t}^*, \pi_{H,t}, \pi_{H,t}^*, p_{H,t}, p_{F,t}, p_{H,t}^*, p_{F,t}^*, w_t, w_t^*, e_t, MC_t, MC_t^*, Y_t, Y_t^*, \Delta_{H,t}, \Delta_{H,t}^*, \Delta_{F,t}, \Delta_{F,t}^*, K_{H,t}, K_{H,t}^*, F_{H,t}, F_{H,t}^*, K_{F,t}, K_{F,t}^*, F_{F,t}, F_{F,t}^* \} \) for all \( t \geq t_0 \), subject to the above 37 structural constraints equations (4.5.2) ~ (4.5.38) associated with Lagrangian multipliers \( \lambda_{1,t} \sim \lambda_{37,t} \) in sequence.

The 39 first-order conditions for all \( t \geq t_0 \) are as follows (we use an itemized list to keep track of the endogenous variable with respect to which the particular first-order condition is derived).

- \( i_t \):
  \[
  \lambda_{2,t} \beta \mathbb{E}_t \frac{C_t^{1-\sigma}}{\pi_{t+1}} = 0
  \]
  that is
  \[
  \lambda_{2,t} = 0
  \] (4.5.39)

- \( i_t^* \):
  \[
  \lambda_{7,t} \beta \mathbb{E}_t \frac{C_t^{1-\sigma}}{\pi_{t+1}} = 0
  \]
  that is
  \[
  \lambda_{7,t} = 0
  \] (4.5.40)
\[ C_t^{\sigma} + \lambda_{1,t}(\sigma)C_t^{\sigma-1}w_t + \lambda_{3,t}(1 - \frac{\nu}{2})p_{H,t}^{\sigma-1} + \lambda_{4,t}(1 - \frac{\nu}{2})p_{F,t}^{\sigma-1} + \lambda_{11,t}(\sigma)C_t^{\sigma-1}e_t \]

\[ + \lambda_{18,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-1}}{C_t^{\sigma-1+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{H,t+1}^{\epsilon} H_{t+1} + \lambda_{18,t-1}\theta(\sigma) \frac{C_t^{\sigma-1}}{C_t^{\sigma-1}} \frac{e_{t}}{e_{t-1}} \pi_{H,t}^{\epsilon} F_{H,t} \]

\[ + \lambda_{19,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-1}}{C_t^{\sigma-1+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{H,t+1}^{\epsilon} K_{H,t+1} + \lambda_{19,t-1}\theta(\sigma) \frac{C_t^{\sigma-1}}{C_t^{\sigma-1}} \frac{e_{t}}{e_{t-1}} \pi_{H,t}^{\epsilon} K_{H,t} \]

\[ + \lambda_{21,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-1}}{C_t^{\sigma-1+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{H,t+1}^{\epsilon} F_{H,t}^{*} + \lambda_{21,t-1}\theta(\sigma) \frac{C_t^{\sigma-1}}{C_t^{\sigma-1}} \frac{e_{t}}{e_{t-1}} \pi_{H,t}^{\epsilon} F_{H,t}^{*} \]

\[ + \lambda_{22,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-1}}{C_t^{\sigma-1+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{H,t+1}^{\epsilon} K_{H,t}^{*} + \lambda_{22,t-1}\theta(\sigma (4.5.41) 

\[ C_t^{\sigma-\sigma} + \lambda_{6,t}(\sigma)C_t^{\sigma-\sigma-1}w_t^* + \lambda_{8,t}(1 - \frac{\nu}{2})p_{H,t}^{\sigma-1} + \lambda_{9,t}(1 - \frac{\nu}{2})p_{F,t}^{\sigma-1} - \lambda_{11,t}(\sigma)C_t^{\sigma-\sigma-1} \]

\[ + \lambda_{29,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{F,t+1}^{\epsilon} F_{F,t+1}^{*} - \lambda_{29,t-1}\theta(\sigma) \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma}} \frac{e_{t-1}^{\pi}}{e_t} \pi_{F,t}^{\epsilon} F_{F,t}^{*} \]

\[ + \lambda_{30,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{F,t+1}^{\epsilon} K_{F,t+1}^{*} - \lambda_{30,t-1}\theta(\sigma) \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma}} \frac{e_{t-1}^{\pi}}{e_t} \pi_{F,t}^{\epsilon} K_{F,t}^{*} \]

\[ + \lambda_{32,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{F,t+1}^{\epsilon} F_{F,t+1} - \lambda_{32,t-1}\theta(\sigma) \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma}} \frac{e_{t-1}^{\pi}}{e_t} \pi_{F,t}^{\epsilon} F_{F,t} \]

\[ + \lambda_{33,t}\theta\sigma\mathcal{E}_{t}^{\lambda} \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma+1}} \frac{e_{t+1}^{\pi}}{e_t} \pi_{F,t+1}^{\epsilon} K_{F,t+1} - \lambda_{33,t-1}\theta(\sigma) \frac{C_t^{\sigma-\sigma}}{C_t^{\sigma-\sigma}} \frac{e_{t-1}^{\pi}}{e_t} \pi_{F,t}^{\epsilon} K_{F,t} = 0 \]

\[ (4.5.42) \]

\[ C_{H,t} : \]

\[ -\lambda_{3,t} + \lambda_{13,t}\Delta_{H,t} + \lambda_{18,t}p_{H,t}e_{t-1} + \lambda_{19,t}MC_t e_{t-1} = 0 \]

\[ (4.5.43) \]

\[ C_{H,t}^* : \]

\[ -\lambda_{8,t} + \lambda_{13,t}\Delta_{H,t}^* + \lambda_{21,t}p_{H,t}^* e_{t-1} + \lambda_{22,t}MC_t e_{t-1} = 0 \]

\[ (4.5.44) \]
• $C_{F,t}$:

$$-\lambda_{4,t} + \lambda_{24,t} \Delta_{F,t} + \lambda_{32,t} p_{F,t} + \lambda_{33,t} MC_t^* e_t = 0 \quad (4.5.45)$$

• $C_{F,t}^*$:

$$-\lambda_{9,t} + \lambda_{24,t} \Delta_{F,t}^* + \lambda_{29,t} p_{F,t}^* e_t + \lambda_{30,t} MC_t^* e_t = 0 \quad (4.5.46)$$

• $h_t$:

$$-\chi h_t^\omega - \lambda_{1,t} \omega h_t^{\omega - 1} - \lambda_{13,t} \exp(z_t) + \lambda_{14,t} \exp(z_t) = 0 \quad (4.5.47)$$

• $h_t^*$:

$$-\chi h_t^{\omega^*} - \lambda_{6,t} \omega h_t^{\omega^* - 1} - \lambda_{24,t} \exp(z_t^*) + \lambda_{25,t} \exp(z_t^*) = 0 \quad (4.5.48)$$

• $\pi_t$:

$$+\lambda_{34,t} \frac{p_{H,t}}{p_{H,t-1}} + \lambda_{37,t} \frac{p_{F,t}}{p_{F,t-1}} = 0 \quad (4.5.49)$$

• $\pi_t^*$:

$$+\lambda_{35,t} \frac{p_{H,t}^*}{p_{H,t-1}^*} + \lambda_{36,t} \frac{p_{F,t}^*}{p_{F,t-1}^*} = 0 \quad (4.5.50)$$
\[ -\lambda_{15,t} \left[ 1 - \theta (\pi_{H,t})^{\epsilon - 1} \right] \frac{\theta}{1 - \theta} \pi_{H,t}^{\epsilon - 2} + \lambda_{15,t} \theta \epsilon \pi_{H,t}^{\epsilon - 1} \Delta_{H,t-1} + \lambda_{17,t} F_{H,t} \left[ 1 - \theta (\pi_{H,t})^{\epsilon - 1} \right] \frac{\theta}{1 - \theta} \left( \frac{\theta}{1 - \theta} \right) (\pi_{H,t})^{\epsilon - 2} + \lambda_{18,t-1} \theta (\epsilon - 1) \frac{C_t^{\sigma} e_t}{C_{t-1}^{\sigma} e_{t-1}} \pi_{H,t}^{\epsilon - 2} F_{H,t} + \lambda_{19,t-1} \theta (\epsilon) \frac{C_t^{\sigma} e_t}{C_{t-1}^{\sigma} e_{t-1}} \pi_{H,t}^{\epsilon - 1} K_{H,t} - \lambda_{34,t} = 0 \quad (4.5.51) \]

\[ -\lambda_{16,t} \left[ 1 - \theta (\pi_{H,t}^*)^{\epsilon - 1} \right] \frac{\theta}{1 - \theta} \theta \epsilon (\pi_{H,t}^*)^{\epsilon - 2} + \lambda_{16,t} \theta \epsilon (\pi_{H,t}^*)^{\epsilon - 1} \Delta_{H,t-1}^* + \lambda_{20,t} F_{H,t}^* \left[ 1 - \theta (\pi_{H,t}^*)^{\epsilon - 1} \right] \frac{\theta}{1 - \theta} \left( \frac{\theta}{1 - \theta} \right) (\pi_{H,t}^*)^{\epsilon - 2} + \lambda_{21,t-1} \theta (\epsilon - 1) \frac{C_t^{\sigma} e_t}{C_{t-1}^{\sigma} e_{t-1}} \pi_{H,t}^{\epsilon - 2} F_{H,t}^* + \lambda_{22,t-1} \theta (\epsilon) \frac{C_t^{\sigma} e_t}{C_{t-1}^{\sigma} e_{t-1}} \pi_{H,t}^{\epsilon - 1} K_{H,t}^* - \lambda_{35,t} = 0 \quad (4.5.52) \]
\[ \pi_{F,t} : \]
\[ -\lambda_{26,t} \left[ \frac{1 - \theta \left( \pi_{F,t}^{\epsilon-1} \right)}{1 - \theta} \right] \frac{1}{\epsilon} \theta \epsilon \left( \pi_{F,t} \right)^{\epsilon-2} \]
\[ + \lambda_{26,t} \theta \epsilon \left( \pi_{F,t} \right)^{\epsilon-1} \Delta_{F,t-1} \]
\[ + \lambda_{31,t} F_{F,t} \left[ \frac{1 - \theta \left( \pi_{F,t}^{\epsilon-1} \right)}{1 - \theta} \right] \frac{1}{\epsilon} \left( \frac{\theta}{1 - \theta} \left( \pi_{F,t} \right)^{\epsilon-2} \right) \]
\[ + \lambda_{32,t-1} \theta (\epsilon - 1) \frac{C_{t-\sigma}^{\epsilon-\sigma} e_{t-1}}{C_{t-1}^{\epsilon-\sigma}} F_{F,t} \]
\[ + \lambda_{33,t-1} \theta (\epsilon) \frac{C_{t-\sigma}^{\epsilon-\sigma} e_{t-1}}{C_{t-1}^{\epsilon-\sigma}} K_{F,t} \]
\[ - \lambda_{37,t} = 0 \quad (4.5.53) \]

\[ \pi_{F,t}^{*} : \]
\[ -\lambda_{27,t} \left[ \frac{1 - \theta \left( \pi_{F,t}^{\epsilon-1} \right)}{1 - \theta} \right] \frac{1}{\epsilon} \theta \epsilon \left( \pi_{F,t}^{*} \right)^{\epsilon-2} \]
\[ + \lambda_{27,t} \theta \epsilon \left( \pi_{F,t}^{*} \right)^{\epsilon-1} \Delta_{F,t-1}^{*} \]
\[ + \lambda_{28,t} F_{F,t}^{*} \left[ \frac{1 - \theta \left( \pi_{F,t}^{\epsilon-1} \right)}{1 - \theta} \right] \frac{1}{\epsilon} \left( \frac{\theta}{1 - \theta} \left( \pi_{F,t}^{*} \right)^{\epsilon-2} \right) \]
\[ + \lambda_{29,t-1} \theta (\epsilon - 1) \frac{C_{t-\sigma}^{\epsilon-\sigma} e_{t-1}}{C_{t-1}^{\epsilon-\sigma}} F_{F,t}^{*} \]
\[ + \lambda_{30,t-1} \theta (\epsilon) \frac{C_{t-\sigma}^{\epsilon-\sigma} e_{t-1}}{C_{t-1}^{\epsilon-\sigma}} K_{F,t}^{*} \]
\[ - \lambda_{36,t} = 0 \quad (4.5.54) \]

\[ p_{H,t} : \]
\[ -\lambda_{3,t} \frac{\nu}{2} \frac{C_{t}}{p_{H,t}^{2}} - \lambda_{5,t} \frac{\nu}{2} k^{-1} (p_{H,t})^{-1+\nu/2} (p_{F,t})^{1-\nu/2} \]
\[ + \lambda_{18,t} \frac{C_{H,t}}{e_{t}} + \lambda_{34,t} \pi_{t} \frac{1}{p_{H,t-1}} - \lambda_{34,t+1} \beta E_{t}(\pi_{t+1}) \frac{p_{H,t+1}}{p_{H,t}^{2}} \]
\[ = 0 \quad (4.5.55) \]
• $p_{H,t}^*$:

$$-\lambda_{8,t}\left(1 - \frac{\nu}{2}\right)\frac{C_t^*}{p_{H,t}^2} - \lambda_{10,t}\left(1 - \frac{\nu}{2}\right)k^{-1}(p_{H,t}^*)^{-\nu/2}(p_{F,t}^*)^{\nu/2}$$

$$+\lambda_{21,t}C_{H,t}^* + \lambda_{35,t}\pi_t^*\frac{1}{p_{H,t-1}^*} - \lambda_{35,t+1}\beta\mathcal{E}_t(\pi_{t+1}^*)\frac{p_{H,t+1}^*}{p_{H,t}^2} = 0 \quad (4.5.56)$$

• $p_{F,t}^*$:

$$-\lambda_{4,t}\left(1 - \frac{\nu}{2}\right)\frac{C_t^*}{p_{F,t}^2} - \lambda_{5,t}\left(1 - \frac{\nu}{2}\right)k^{-1}(p_{H,t}^*)^{\nu/2}(p_{F,t}^*)^{-\nu/2}$$

$$+\lambda_{32,t}C_{F,t}^* + \lambda_{37,t}\pi_t^*\frac{1}{p_{F,t-1}^*} - \lambda_{37,t+1}\beta\mathcal{E}_t(\pi_{t+1}^*)\frac{p_{F,t+1}^*}{p_{F,t}^2} = 0 \quad (4.5.57)$$

• $p_{F,t}^*$:

$$-\lambda_{9,t}\frac{\nu}{2}\frac{C_t^*}{p_{F,t}^2} - \lambda_{10,t}\frac{\nu}{2}k^{-1}(p_{H,t}^*)^{1-\nu/2}(p_{F,t}^*)^{-1+\nu/2}$$

$$+\lambda_{29,t}\mathcal{E}_tC_{F,t}^* + \lambda_{36,t}\pi_t^*\frac{1}{p_{F,t-1}^*} - \lambda_{36,t+1}\beta\mathcal{E}_t(\pi_{t+1}^*)\frac{p_{F,t+1}^*}{p_{F,t}^2} = 0 \quad (4.5.58)$$

• $w_t$:

$$\lambda_{1,t}C_t^{-\sigma} + \lambda_{12,t}\frac{1}{\exp(z_t)} = 0 \quad (4.5.59)$$

• $w_t^*$:

$$\lambda_{6,t}C_t^{*-\sigma} + \lambda_{23,t}\frac{1}{\exp(z_t^*)} = 0 \quad (4.5.60)$$
• $e_t$:

$$\lambda_{11,t} c_t^{-\sigma} - \lambda_{18,t} \frac{C_{H,t} P_{H,t}}{e_t^2} - \lambda_{19,t} \frac{C_{H,t} M C_t}{e_t^2} - \lambda_{22,t} \frac{C_{H,t}^* M C_t}{e_t^2} + \lambda_{29,t} C_{F,t}^* p_{F,t} + \lambda_{30,t} C_{F,t}^* M C_t^* + \lambda_{33,t} C_{F,t}^* M C_t^* - \lambda_{18,t} \beta \theta E_t \frac{C_{t-1}^* e_{t-1}}{C_{t-1}^*} \pi_{H,t+1} e_{t+1} F_{H,t+1} + \lambda_{18,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{H,t}^* e_{t-1} F_{H,t}$$

$$- \lambda_{19,t} \beta \theta E_t \frac{C_{t-1}^* e_{t+1}}{C_{t-1}^* e_{t}^2} \pi_{H,t}^* K_{H,t+1} + \lambda_{19,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{H,t} e_{t-1} K_{H,t}$$

$$- \lambda_{21,t} \beta \theta E_t \frac{C_{t-1}^* e_{t+1}}{C_{t-1}^* e_{t}^2} \pi_{H,t} e_{t-1} F_{F,t+1}^* + \lambda_{21,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{H,t} e_{t-1} F_{H,t}$$

$$- \lambda_{22,t} \beta \theta E_t \frac{C_{t-1}^* e_{t+1}}{C_{t-1}^* e_{t}^2} \pi_{H,t} e_{t-1} K_{H,t+1} + \lambda_{22,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{H,t} e_{t-1} K_{H,t}$$

$$+ \lambda_{29,t} \beta \theta E_t \frac{C_{t-1}^* e_{t+1}}{C_{t-1}^* e_{t}^2} \pi_{H,t} e_{t-1} F_{F,t+1}^* - \lambda_{29,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{H,t} e_{t-1} F_{H,t}$$

$$+ \lambda_{30,t} \beta \theta E_t \frac{C_{t-1}^* e_{t+1}}{C_{t-1}^* e_{t}^2} \pi_{F,t+1} e_{t-1} K_{F,t+1} - \lambda_{30,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{F,t} e_{t-1} K_{F,t}$$

$$+ \lambda_{32,t} \beta \theta E_t \frac{C_{t-1}^* e_{t+1}}{C_{t-1}^* e_{t}^2} \pi_{F,t+1} e_{t-1} F_{F,t} - \lambda_{32,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{F,t} e_{t-1} F_{F,t}$$

$$+ \lambda_{33,t} \beta \theta E_t \frac{C_{t-1}^* e_{t+1}}{C_{t-1}^* e_{t}^2} \pi_{F,t+1} e_{t-1} K_{F,t+1} - \lambda_{33,t-1} \theta \frac{C_{t-1}^*}{C_{t-1}^* e_{t-1}} \pi_{F,t} e_{t-1} K_{F,t} = 0$$

(4.5.61)

• $MC_t$:

$$-\lambda_{12,t} + \lambda_{19,t} \frac{C_{H,t}}{e_t} + \lambda_{22,t} \frac{C_{H,t}^*}{e_t} = 0$$

(4.5.62)

• $MC_t^*$:

$$-\lambda_{23,t} + \lambda_{30,t} e_t C_{F,t}^* + \lambda_{33,t} e_t C_{F,t} = 0$$

(4.5.63)

• $Y_t$:

$$\lambda_{14,t} = 0$$

(4.5.64)
• \( Y_t^* \):

\[
\lambda_{25,t} = 0 \quad (4.5.65)
\]

• \( \Delta_{H,t} \):

\[
\lambda_{13,t} C_{H,t} - \lambda_{15,t} + \lambda_{15,t+1} \beta \theta E_t (\pi_{H,t+1})^\epsilon = 0 \quad (4.5.66)
\]

• \( \Delta_{H,t}^* \):

\[
\lambda_{13,t} C_{H,t}^* - \lambda_{16,t} + \lambda_{16,t+1} \beta \theta E_t (\pi_{H,t+1})^\epsilon = 0 \quad (4.5.67)
\]

• \( \Delta_{F,t} \):

\[
\lambda_{24,t} C_{F,t} - \lambda_{26,t} + \lambda_{26,t+1} \beta \theta E_t (\pi_{F,t+1})^\epsilon = 0 \quad (4.5.68)
\]

• \( \Delta_{F,t}^* \):

\[
\lambda_{24,t} C_{F,t}^* - \lambda_{27,t} + \lambda_{27,t+1} \beta \theta E_t (\pi_{F,t+1})^\epsilon = 0 \quad (4.5.69)
\]

• \( F_{H,t} \):

\[
\lambda_{17,t} \left[ 1 - \theta \left( \frac{1}{\pi_{H,t}} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} - \lambda_{18,t} + \lambda_{18,t-1} \theta \frac{C_{t-\sigma}^t}{C_{t-1}^{t-\sigma}} \frac{e_t}{e_{t-1}} \pi_{H,t}^{t-1} = 0 \quad (4.5.70)
\]

• \( K_{H,t} \):

\[
-\lambda_{17,t} - \lambda_{19,t} + \lambda_{19,t-1} \theta \frac{C_{t-\sigma}^t}{C_{t-1}^{t-\sigma}} \frac{e_t}{e_{t-1}} \pi_{H,t}^t = 0 \quad (4.5.71)
\]

• \( F_{H,t}^* \):

\[
\lambda_{20,t} \left[ 1 - \theta \left( \frac{1}{\pi_{H,t}} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} - \lambda_{21,t} + \lambda_{21,t-1} \theta \frac{C_{t-\sigma}^t}{C_{t-1}^{t-\sigma}} \frac{e_t}{e_{t-1}} \pi_{H,t}^{t-1} = 0 \quad (4.5.72)
\]
The cooperative Ramsey problem thus consists of the above 76 equations for 76 unknowns.

The deterministic steady state is derived in two steps: First, steady-state values of the 39 endogenous variables are derived in Appendix A.2. Given these steady-state endogenous variables, the above optimality conditions (dropping time subscripts) can solve for the steady-state values of the 37 Lagrange multipliers.

**B.2 Noncooperation**

The home policy maker maximizes
APPENDIX

\[ W_{H,t_0} = E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \frac{C_{t}^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{h_{t}^{1+\omega}}{1 + \omega} \right) \]

with respect to 38 endogenous variables \( \{i_{t}, i_{t}^{*}, C_{t}, C_{t}^{*}, H_{t}, C_{F,t}, C_{F,t}^{*}, h_{t}, h_{t}^{*}, \pi_{t}, \pi_{t}^{*}, \pi_{H,t}, \pi_{F,t}, p_{H,t}, p_{F,t}, p_{F,t}^{*}, w_{t}, w_{t}^{*}, e_{t}, MC_{t}, MC_{t}^{*}, Y_{t}, Y_{t}^{*}, \Delta_{H,t}, \Delta_{H,t}^{*}, \Delta_{F,t}, \Delta_{F,t}^{*}, K_{H,t}, K_{H,t}^{*}, F_{H,t}, K_{F,t}, F_{F,t}, K_{F,t}^{*}, F_{F,t}^{*}\}, \) taking as given \( \{\pi_{F,t}^{*}\} \) for all \( t \geq t_0 \), subject to 37 structural constraints equations (4.5.2) \( \sim \) (4.5.38) associated with Lagrangian multipliers \( \lambda_{1,t} \sim \lambda_{37,t} \) in sequence.

The 38 first-order conditions are as follows:

- \( i_{t} \): equation (4.5.39)
- \( i_{t}^{*} \): equation (4.5.40)
- \( C_{t} \): equation (4.5.41)
- \( C_{t}^{*} \):
  \[
  \lambda_{6,t} (-\sigma) C_{t}^{*-\sigma-1} w_{t}^{*} + \lambda_{8,t} \left( 1 - \frac{\nu}{2} \right) p_{H,t}^{*-1} + \lambda_{9,t} \frac{\nu}{2} p_{F,t}^{*-1} - \lambda_{11,t} (-\sigma) C_{t}^{\sigma-\sigma-1} \]
  \[
  + \lambda_{29,t} \beta \theta \sigma E_{t} \frac{C_{t+1}^{*-\sigma} e_{t}}{C_{t}^{*-\sigma+1} e_{t+1}} \left( \pi_{F,t+1}^{*} \right)^{\epsilon-1} F_{F,t+1}^{*} - \lambda_{29,t-1} \theta \sigma \frac{C_{t}^{*-\sigma-1} e_{t-1}}{C_{t-1}^{*-\sigma} e_{t}} \left( \pi_{F,t}^{*} \right)^{\epsilon-1} F_{F,t}^{*} \]
  \[
  + \lambda_{30,t} \beta \theta \sigma E_{t} \frac{C_{t+1}^{*-\sigma} e_{t}}{C_{t}^{*-\sigma+1} e_{t+1}} \left( \pi_{F,t+1}^{*} \right)^{\epsilon} K_{F,t+1}^{*} - \lambda_{30,t-1} \theta \sigma \frac{C_{t}^{*-\sigma-1} e_{t-1}}{C_{t-1}^{*-\sigma} e_{t}} \left( \pi_{F,t}^{*} \right)^{\epsilon} K_{F,t}^{*} \]
  \[
  + \lambda_{32,t} \beta \theta \sigma E_{t} \frac{C_{t+1}^{*-\sigma} e_{t}}{C_{t}^{*-\sigma+1} e_{t+1}} \left( \pi_{F,t+1}^{*} \right)^{\epsilon} F_{F,t+1}^{*} - \lambda_{32,t-1} \theta \sigma \frac{C_{t}^{*-\sigma-1} e_{t-1}}{C_{t-1}^{*-\sigma} e_{t}} \left( \pi_{F,t}^{*} \right)^{\epsilon} F_{F,t}^{*} \]
  \[
  + \lambda_{33,t} \beta \theta \sigma E_{t} \frac{C_{t+1}^{*-\sigma} e_{t}}{C_{t}^{*-\sigma+1} e_{t+1}} \left( \pi_{F,t+1}^{*} \right)^{\epsilon} K_{F,t+1}^{*} - \lambda_{33,t-1} \theta \sigma \frac{C_{t}^{*-\sigma-1} e_{t-1}}{C_{t-1}^{*-\sigma} e_{t}} \left( \pi_{F,t}^{*} \right)^{\epsilon} K_{F,t}^{*} \]
  \[
  = 0 \]
  \[(4.5.78)\]
- \( C_{H,t} \): equation (4.5.43)
- \( C_{F,t} \): equation (4.5.45)
- \( C_{H,t}^{*} \): equation (4.5.44)
- \( C_{F,t}^{*} \): equation (4.5.46)
- \( h_{t} \): equation (4.5.47)
• $h_t^*$:

\[-\lambda_{6,t} \omega \chi h_t^{\omega-1} - \lambda_{24,t} \exp (z_t^*) + \lambda_{25,t} \exp (z_t^*) = 0\]  \hspace{1cm} (4.5.79)

• $\pi_t$: equation (4.5.49)
• $\pi_t^*$: equation (4.5.50)
• $\pi_{H,t}$: equation (4.5.51)
• $\pi_{H,t}^*$: equation (4.5.52)
• $\pi_{F,t}$: equation (4.5.53)
• $p_{H,t}$: equation (4.5.55)
• $p_{H,t}^*$: equation (4.5.56)
• $p_{F,t}$: equation (4.5.57)
• $p_{F,t}^*$: equation (4.5.58)
• $w_t$: equation (4.5.59)
• $w_t^*$: equation (4.5.60)
• $e_t$: equation (4.5.61)
• $MC_t$: equation (4.5.62)
• $MC_t^*$: equation (4.5.63)
• $Y_t$: equation (4.5.64)
• $Y_t^*$: equation (4.5.65)
• $\Delta_{H,t}$: equation (4.5.66)
• $\Delta_{H,t}^*$: equation (4.5.67)
• $\Delta_{F,t}$: equation (4.5.68)
• $\Delta_{F,t}^*$: equation (4.5.69)
• $F_{H,t}$: equation (4.5.70)
• $K_{H,t}$: equation (4.5.71)
• $F_{H,t}^*$: equation (4.5.72)
• $K_{H,t}^*$: equation (4.5.73)
• $F_{F,t}$: equation (4.5.74)
• $K_{F,t}$: equation (4.5.75)
• $F_{F,t}^*$: equation (4.5.76)
• $K_{F,t}^*$: equation (4.5.77).

The foreign policy maker maximizes

$$W_{F,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \chi \frac{h_t^{1+\omega}}{1+\omega} \right)$$

with respect to 38 endogenous variables $\{i_t, i_t^*, C_t, C_t^*, C_{H,t}, C_{H,t}^*, C_{F,t}, C_{F,t}^*, h_t, h_t^*, \pi_t, \pi_t^*, p_{H,t}, p_{F,t}, p_{H,t}^*, p_{F,t}^*, w_t, w_t^*, \varepsilon_t, MC_t, MC_t^*, Y_t, Y_t^*, \Delta_{H,t}, \Delta_{H,t}^*, \Delta_{F,t}, \Delta_{F,t}^*, K_{H,t}, K_{H,t}^*, F_{H,t}, F_{H,t}^*, K_{F,t}, K_{F,t}^*, F_{F,t}, F_{F,t}^* \}$, taking as given $\{\pi_{H,t} \}$ for all $t \geq t_0$, subject to 37 structural constraint equations (4.5.2) $\sim$ (4.5.38) associated with Lagrangian multipliers $\lambda_{i,t}^* \sim \lambda_{37,t}^*$ in sequence.

Note that we use lambda with asterisk as Lagrangian multipliers for the foreign maximization problem. While most of the first-order conditions in this optimization problem are the same with those in the cooperative optimization above, and we indeed make references to them to save space, one needs to keep in mind that all $\lambda_{i,t}$ in the following first-order conditions should be replaced to $\lambda_{i,t}^*$ for $i = \{1,...37\}$.

The 38 first-order conditions are

• $i_t$: equation (4.5.39)
• $i_t^*$: equation (4.5.40)
• $C_t$:

\[
\lambda^{*}_{1,t}(\sigma)C_{t}^{-\sigma-1}w_t + \lambda^{*}_{3,t}\frac{\nu}{2}p_{H,t}^{-1} + \lambda^{*}_{4,t}(1 - \frac{\nu}{2})p_{F,t}^{-1} + \lambda^{*}_{11,t}(\sigma)C_{t}^{-\sigma-1}e_t \\
+ \lambda^{*}_{18,t}\beta\theta\sigma\mathbb{E}_t \frac{C_{t+1}^\sigma}{C_t^{-\sigma+1}} \frac{e_{t+1}}{e_t} \pi_{H,t+1}^\epsilon F_{H,t+1} + \lambda^{*}_{18,t-1}\theta(-\sigma) \frac{C_{t}^{-\sigma-1}}{C_{t-1}^{-\sigma}} \frac{e_t}{e_{t-1}} \pi_{H,t}^\epsilon F_{H,t} \\
+ \lambda^{*}_{19,t}\beta\theta\sigma\mathbb{E}_t \frac{C_{t+1}^\sigma}{C_t^{-\sigma+1}} \frac{e_{t+1}}{e_t} \pi_{H,t+1}^\epsilon K_{H,t+1} + \lambda^{*}_{19,t-1}\theta(-\sigma) \frac{C_{t}^{-\sigma-1}}{C_{t-1}^{-\sigma}} \frac{e_t}{e_{t-1}} \pi_{H,t}^\epsilon K_{H,t} \\
+ \lambda^{*}_{21,t}\beta\theta\sigma\mathbb{E}_t \frac{C_{t+1}^\sigma}{C_t^{-\sigma+1}} \frac{e_{t+1}}{e_t} \pi_{H,t+1}^\epsilon F_{H,t+1}^* + \lambda^{*}_{21,t-1}\theta(-\sigma) \frac{C_{t}^{-\sigma-1}}{C_{t-1}^{-\sigma}} \frac{e_t}{e_{t-1}} \pi_{H,t}^\epsilon F_{H,t}^* \\
+ \lambda^{*}_{22,t}\beta\theta\sigma\mathbb{E}_t \frac{C_{t+1}^\sigma}{C_t^{-\sigma+1}} \frac{e_{t+1}}{e_t} \pi_{H,t+1}^\epsilon K_{H,t+1}^* + \lambda^{*}_{22,t-1}\theta(-\sigma) \frac{C_{t}^{-\sigma-1}}{C_{t-1}^{-\sigma}} \frac{e_t}{e_{t-1}} \pi_{H,t}^\epsilon K_{H,t}^* \\
= 0
\]

\[(4.5.80)\]

• $C_t^*$: equation (4.5.42)

• $C_{H,t}$: equation (4.5.43)

• $C_{F,t}$: equation (4.5.45)

• $C_{H,t}^*$: equation (4.5.44)

• $C_{F,t}^*$: equation (4.5.46)

• $h_t$:

\[-\lambda^{*}_{1,t}\omega h_t^{\omega-1} - \lambda^{*}_{13,t}\exp (z_t) + \lambda^{*}_{14,t}\exp (z_t) = 0\]

\[(4.5.81)\]

• $h_t^*$: equation (4.5.48)

• $\pi_t$: equation (4.5.49)

• $\pi_t^*$: equation (4.5.50)

• $\pi_{H,t}^*$: equation (4.5.52)

• $\pi_{F,t}^*$: equation (4.5.53)

• $\pi_{H,t}^*$: equation (4.5.54)

• $p_{H,t}$: equation (4.5.55)
• $p_{H,t}$: equation (4.5.56)
• $p_{F,t}$: equation (4.5.57)
• $p_{H,t}^*$: equation (4.5.58)
• $w_t$: equation (4.5.59)
• $w_t^*$: equation (4.5.60)
• $e_t$: equation (4.5.61)
• $MC_t$: equation (4.5.62)
• $MC_t^*$: equation (4.5.63)
• $Y_t$: equation (4.5.64)
• $Y_t^*$: equation (4.5.65)
• $\Delta_{H,t}$: equation (4.5.66)
• $\Delta_{H,t}^*$: equation (4.5.67)
• $\Delta_{F,t}$: equation (4.5.68)
• $\Delta_{F,t}^*$: equation (4.5.69)
• $F_{H,t}$: equation (4.5.70)
• $K_{H,t}$: equation (4.5.71)
• $F_{H,t}^*$: equation (4.5.72)
• $K_{H,t}^*$: equation (4.5.73)
• $F_{F,t}$: equation (4.5.74)
• $K_{F,t}$: equation (4.5.75)
• $F_{F,t}^*$: equation (4.5.76)
• $K_{F,t}^*$: equation (4.5.77)
The noncooperative Ramsey problem thus consists of 113 equations for 113 unknowns.

The deterministic steady state is derived in two steps: First, steady-state values of the 39 endogenous variables are derived in Appendix A.2. Given these values, the above optimality conditions (dropping time subscripts) can solve for the steady-state values of 74 Lagrange multipliers.

We note that thanks to the efficient subsidy, the steady-state values of endogenous variables are always given by Appendix A.2 regardless of strategic games, which allows for meaningful welfare comparison.
Appendix C. Second-order Approximation

Section (C.1) to (C.4) show how to substitute linear terms $\hat{c}_t - \hat{h}_t$ and $\hat{c}^*_t - \hat{h}^*_t$ with second-order terms by using the second-order approximation to some of the structural equations. Specifically, C.1 shows the second-order approximation to the price dispersion terms which comes in useful later. C.2 shows the second-order approximation to the price setting conditions to obtain AS equations. C.3 shows the second-order approximation to the resource constraints. C.4 solves for the linear terms using results obtained in C.1-3.

Section C.5 presents the general form and simplified form of the quadratic loss functions for noncooperative policy makers under LCP, making use of the results in C.4.

C.1

Price dispersion terms are shown in equations (ix)-(x) and (xxvii)-(xxviii) in the text.

Take a second-order approximation to equation (ix) around its deterministic steady state $\Delta H = 1$ and we have

$$\hat{\Delta}_{H,t} = \frac{\theta \epsilon}{2 (1 - \theta)} \pi^2_{H,t} + \theta \hat{\Delta}_{H,t-1}.$$  

Taking a summation on both sides from initial time $t_0$ to infinity gives

$$\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_{H,t} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\theta \epsilon}{2 (1 - \theta)} \pi^2_{H,t} + \theta \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_{H,t},$$

that is

$$\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_{H,t} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\epsilon \theta}{2 (1 - \beta \theta) (1 - \theta)} \pi^2_{H,t},$$  \hspace{1cm} (4.5.82)

where $\hat{\Delta}_{H,t_0-1} = 0$. 
Analogously, equations (x), (xxvii) and (xxviii) are approximated to be

$$
\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_{H,t}^* = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\epsilon}{2(1-\beta\theta)(1-\theta)} \pi_{H,t}^* \tag{4.5.83}
$$

$$
\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_{F,t} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{\epsilon}{2(1-\beta\theta)(1-\theta)} \pi_{F,t}^2 \tag{4.5.84}
$$

$$
\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \hat{\Delta}_{F,t}^* = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \frac{2(1-\beta\theta)(1-\theta)}{(1-\theta)} \pi_{F,t}^* \tag{4.5.85}
$$

C.2

We consider home firms’ optimal pricing conditions first. Consider an arbitrary period of time in the infinite time horizon, $t$, where $t \geq t_0$, as shown in equations (xi)-(xvi) in the text, the optimality conditions with LCP can be written as

$$
\tilde{P}_{H,t} = \frac{K_{H,t}}{F_{H,t}} \tag{4.5.86}
$$

$$
\tilde{P}_{H,t} = \left[ 1 - \theta \left( \frac{P_{H,t-1}}{P_{H,t}} \right)^{1-\epsilon} \right]^{1/\epsilon} \tag{4.5.87}
$$

where

$$
K_{H,t} \equiv \mathbb{E}_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_T^{-\sigma} C_{H,T} M C_T \left( \frac{P_{H,t}}{P_{H,T}} \right)^{-\epsilon} \tag{4.5.88}
$$

$$
F_{H,t} \equiv \mathbb{E}_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_T^{-\sigma} C_{H,T} P_{H,T} \left( \frac{P_{H,t}}{P_{H,T}} \right)^{1-\epsilon} \tag{4.5.89}
$$

for choosing $\tilde{P}_{H,t}$ for the domestic market, and

$$
\tilde{P}_{H,t}^* = \frac{K_{H,t}^*}{F_{H,t}^*} \tag{4.5.90}
$$

$$
\tilde{P}_{H,t} = \left[ 1 - \theta \left( \frac{P_{H,t-1}}{P_{H,t}} \right)^{1-\epsilon} \right]^{1/\epsilon} \tag{4.5.91}
$$
where

\[ K_{H,t}^* \equiv E_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_T^{-\sigma} C_{H,T}^* MC_T \left( \frac{P_{H,t}^*}{P_{H,T}^*} \right)^{-\epsilon} \]  \hspace{1cm} (4.5.92)

\[ F_{H,t}^* \equiv E_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_T^{-\sigma} C_{H,T}^* \epsilon T P_{H,T}^* \left( \frac{P_{H,t}^*}{P_{H,T}^*} \right)^{1-\epsilon} . \]  \hspace{1cm} (4.5.93)

for choosing \( \tilde{P}_{H,t}^* \) for the export destination market. Note that we use a slightly different notation for summation as we are now considering the path from an arbitrary time \( t \) onward.

Following Benigno and Woodford (2005) we take a second-order approximation to (4.5.86) and (4.5.87) and make use of the second-order approximation to equations (4.5.88) and (4.5.89). After a few messy steps we obtain the second-order home AS equation

\[ E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\tilde{m}_{c_t} - \tilde{p}_{H,t}) \]  \hspace{1cm} (4.5.94)

\[ = - E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} (\tilde{m}_{c_t} - \tilde{p}_{H,t}) \left( 2 (-\sigma \hat{c}_t + \hat{c}_{H,t}) + \hat{p}_{H,t} + \tilde{m}_{c_t} + \frac{\epsilon}{2\delta} \pi_{H,t}^2 \right) \right] + \text{t.i.p} \]

where we have used equation (4.5.82), and the t.i.p term contains terms that are independent of policy and determined by parameters and predetermined initial conditions only:

\[ V_{H,t_0} = \pi_{H,t_0} - \frac{(1 - 2\epsilon + \epsilon \theta)}{2 (1 - \theta)} \pi_{H,t_0}^2 + \frac{(1 - \beta \theta)}{2} \pi_{H,t_0} \tilde{Z}_{H,t_0}, \]  \hspace{1cm} (4.5.95)

where \( \tilde{Z}_{H,t_0} = E_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{-t} \left( 2 (-\sigma \hat{c}_t + \hat{c}_{H,t}) + \hat{p}_{H,t} + \tilde{m}_{c_t} - \frac{\beta \theta (1 - 2\epsilon)}{1 - \beta \theta} \pi_{H,t+1} \right) \).

Similarly we approximate equations (4.5.90) and (4.5.91) along with (4.5.92) and (4.5.93) up to the second order and obtain

\[ E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} (\tilde{m}_{c_t} - \tilde{p}_{H,t}^* - \hat{e}_t) \]  \hspace{1cm} (4.5.96)

\[ = - E_t \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} (\tilde{m}_{c_t} - \tilde{p}_{H,t}^* - \hat{e}_t) \left( 2 (-\sigma \hat{c}_t + \hat{c}_{H,t}) + \hat{p}_{H,t}^* + \hat{e}_t + \tilde{m}_{c_t} + \frac{\epsilon}{2\delta} \pi_{H,t}^2 \right) \right] + \text{t.i.p}, \]

where we have used equation (4.5.83) and the t.i.p term contains

\[ V_{H,t_0}^* = \pi_{H,t_0}^* - \frac{(1 - 2\epsilon + \epsilon \theta)}{2 (1 - \theta)} \pi_{H,t_0}^* + \frac{(1 - \beta \theta)}{2} \pi_{H,t_0}^* \tilde{Z}_{H,t_0}^*, \]  \hspace{1cm} (4.5.97)
where $\bar{Z}_{H,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{t-t_0} \left( 2 \left( -\sigma \hat{\epsilon}_t + \hat{c}_{H,t}^* \right) + \bar{m} c_t + \bar{p}_{H,t}^* + \hat{\epsilon}_t - \frac{\beta \theta (1-2\epsilon)}{(1-\beta \theta)} \pi_{H,t+1}^* \right)$.

We then look at foreign firms’ optimal pricing conditions. As shown in equations (xxix)-(xxxiv) in the text, the optimality conditions can be written as

$$\frac{\hat{P}_{F,t}^*}{P_{F,t}^*} = \frac{K_{F,t}^*}{F_{F,t}^*} \quad (4.5.98)$$

$$\frac{\hat{P}_{F,t}^*}{P_{F,t}^*} = \left[ 1 - \theta \left( \frac{P_{F,t}^{*,-1}}{P_{F,t}} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad (4.5.99)$$

where

$$K_{F,t}^* = \mathbb{E}_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_{T}^{*^{-\sigma}} C_{F,T}^* MC_T^* \left( \frac{P_{F,t}^*}{P_{F,T}^*} \right)^{-\epsilon} \quad (4.5.100)$$

$$F_{F,t}^* = \mathbb{E}_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_{T}^{*^{-\sigma}} C_{F,T}^* P_{F,T}^* \left( \frac{P_{F,t}^*}{P_{F,T}^*} \right)^{1-\epsilon} \quad (4.5.101)$$

for choosing $\hat{P}_{F,t}^*$ for the domestic market and

$$\frac{\hat{P}_{F,t}}{P_{F,t}} = \frac{K_{F,t}}{F_{F,t}} \quad (4.5.102)$$

$$\frac{\hat{P}_{F,t}}{P_{F,t}} = \left[ 1 - \theta \left( \frac{P_{F,t}^{-1}}{P_{F,t}} \right)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \quad (4.5.103)$$

where

$$K_{F,t} = \mathbb{E}_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_{T}^{*^{-\sigma}} C_{F,T} MC_T \left( \frac{P_{F,t}}{P_{F,T}} \right)^{-\epsilon} \quad (4.5.104)$$

$$F_{F,t} = \mathbb{E}_t \sum_{T=t}^{\infty} (\beta \theta)^{T-t} C_{T}^{*^{-\sigma}} C_{F,T} P_{F,T} \left( \frac{P_{F,t}}{P_{F,T}} \right)^{1-\epsilon} \quad (4.5.105)$$

for choosing $\hat{P}_{F,t}$ for the export destination market.

Repeat the exact same procedure as in approximating home firms’ price setting.
equations and we obtain the two second-order foreign AS equations

\[ E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \hat{mc}_t^* - \hat{p}_{F,t}^* \right) \]

\[ = - E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} \left( \hat{mc}_t^* - \hat{p}_{F,t}^* \right) \left( 2 \left( -\sigma \hat{c}_t^* + \hat{c}_{F,t}^* \right) + \hat{p}_{F,t}^* + \hat{mc}_t^* \right) + \frac{\epsilon}{2\delta} \pi_{F,t}^2 \right] + \text{t.i.p.} \]

and

\[ E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left( \hat{mc}_t^* - \hat{p}_{F,t}^* + \hat{e}_t \right) \]

\[ = - E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{1}{2} \left( \hat{mc}_t^* - \hat{p}_{F,t}^* + \hat{e}_t \right) \left( 2 \left( -\sigma \hat{c}_t^* + \hat{c}_{F,t}^* \right) + \hat{p}_{F,t}^* - \hat{e}_t + \hat{mc}_t^* \right) + \frac{\epsilon}{2\delta} \pi_{F,t}^2 \right] + \text{t.i.p.} \]

where we have used equations (4.5.85) and (4.5.84), and the first t.i.p contains

\[ V_{F,t_0}^* = \pi_{F,t_0}^* - \frac{1 - 2\epsilon + \epsilon\theta}{2(1-\theta)} \pi_{F,t_0}^{*2} + \frac{(1 - \beta\theta)}{2} \pi_{F,t_0}^* \bar{Z}_{F,t_0}^*, \]

where \( \bar{Z}_{F,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{t-t_0} \left( 2 \left( -\sigma \hat{c}_t^* + \hat{c}_{F,t}^* \right) + \hat{p}_{F,t}^* - \hat{e}_t + \hat{mc}_t^* - \frac{\beta\theta(1-2\epsilon)}{(1-\beta\theta)} \pi_{F,t+1}^* \right) \), and the second t.i.p contains

\[ V_{F,t_0} = \pi_{F,t_0} - \frac{1 - 2\epsilon + \epsilon\theta}{2(1-\theta)} \pi_{F,t_0}^{2} + \frac{(1 - \beta\theta)}{2} \pi_{F,t_0} \bar{Z}_{F,t_0}, \]

where \( \bar{Z}_{F,t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} (\beta \theta)^{t-t_0} \left( 2 \left( -\sigma \hat{c}_t^* + \hat{c}_{F,t}^* \right) + \hat{p}_{F,t} - \hat{e}_t + \hat{mc}_t - \frac{\beta\theta(1-2\epsilon)}{(1-\beta\theta)} \pi_{F,t+1} \right) \)

Look at the four AS equations. The real marginal costs and aggregate consumption of each type of goods can be substituted out by using the exact and approximated log-linear forms of structural equations (ii)-(viii) and (xx)-(xxvi).\(^{14}\) They are now expressed

\(^{14}\)Although one needs to be careful in the substitution: exact log-linear equations can be used to substitute both linear terms and squared terms while log-linearized equations can be used to substitute the squared terms only.
as
\[ \omega \hat{h}_t + \sigma \hat{c}_t - \hat{p}_{H,t} = \frac{-1}{2} \left[ (\omega + 1) (\hat{y}_t - z_t) + \frac{2 - \nu}{2} \left( \hat{p}_{H,t} + \frac{1}{\sigma} \hat{c}_t - \hat{p}_{H,t} \right) \right]^2 \]
\[ + \frac{1}{2} (1 - \sigma)^2 \hat{c}_t^2 - \frac{\epsilon}{2 \delta} \pi_{H,t}^2 \tag{4.5.110} \]
\[ \omega \hat{h}_t^* + \sigma \hat{c}_t^* - \hat{p}_{F,t}^* = \frac{-1}{2} \left[ (\omega + 1) (\hat{y}_t^* - z_t^*) + \frac{2 - \nu}{2} \left( \hat{p}_{F,t}^* - \frac{1}{\sigma} \hat{c}_t^* - \hat{p}_{F,t}^* \right) \right]^2 \]
\[ + \frac{1}{2} (1 - \sigma)^2 \hat{c}_t^*^2 - \frac{\epsilon}{2 \delta} \pi_{F,t}^2 \tag{4.5.111} \]
\[ \omega \hat{h}_t^* + \sigma \hat{c}_t^* - \hat{p}_{F,t}^* = \frac{-1}{2} \left[ (\omega + 1) (\hat{y}_t^* - z_t^*) + \frac{2 - \nu}{2} \left( \hat{p}_{F,t}^* - \frac{1}{\sigma} \hat{c}_t^* - \hat{p}_{F,t}^* \right) \right]^2 \]
\[ + \frac{1}{2} (1 - \sigma)^2 \hat{c}_t^*^2 - \frac{\epsilon}{2 \delta} \pi_{F,t}^2 \tag{4.5.112} \]
\[ \omega \hat{p}_H^* + \sigma \hat{c}_t^* + \frac{\nu}{2 - \nu} \hat{p}_{H,t} \]
\[ = \frac{-1}{2} \left[ (\omega + 1) (\hat{y}_t^* - z_t^*) + \frac{2 - \nu}{2} \left( \hat{p}_{H,t}^* - \frac{1}{\sigma} \hat{c}_t^* - \hat{p}_{H,t}^* \right) \right]^2 \]
\[ + \frac{1}{2} (1 - \sigma)^2 \hat{c}_t^*^2 - \frac{\epsilon}{2 \delta} \pi_{H,t}^2 \tag{4.5.113} \]

C.3

Now we approximate resource constraints, equations (vii) and (xxv), in the text.

Make use of equations (iii) and (xxi) to write equation (vii) as
\[ \exp \left( z_t \right) h_t = \frac{\nu}{2} \hat{p}_{H,t}^{-1} C_t \Delta_{H,t} + \frac{2 - \nu}{2} \hat{p}_{H,t}^{-1} C_t^* \Delta_{H,t}^* \]

Take a second-order approximation around its deterministic steady state and we obtain
\[ z_t + \hat{h}_t - \frac{\nu}{2} \hat{c}_t + \frac{2 - \nu}{2} \hat{p}_{H,t} - \frac{2 - \nu}{2} \hat{p}_{H,t}^* \]
\[ = \frac{-1}{2} \left( \hat{h}_t + z_t \right)^2 + \frac{\nu}{4} \left( \hat{p}_{H,t} - \hat{c}_t \right)^2 + \frac{2 - \nu}{4} \left( \hat{p}_{H,t}^* - \hat{c}_t^* \right)^2 \]
\[ + \frac{\nu}{2} \hat{\Delta}_{H,t} + \frac{2 - \nu}{2} \hat{\Delta}_{H,t}^* \]

Note that up to the first order, the above equation is approximated to
\[ z_t + \hat{h}_t = \frac{\nu}{2} \hat{c}_t - \frac{\nu}{2} \hat{p}_{H,t} + \frac{2 - \nu}{2} \hat{c}_t^* - \frac{2 - \nu}{2} \hat{p}_{H,t}^* \]

which is sufficient to substitute \( \left( \hat{h}_t + z_t \right)^2 \). As Benigno and Woodford (2005) argue, linear technology terms are independent of policy when as part of quadratic policy objective, so we can drop out \( z_t \). Then we make use of exact log-linear relations of price index (xxiii), risk sharing condition (xxxvii) and quadratic price dispersion equations.
After a few steps of algebra we obtain

$$\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \left( \hat{h}_t - \hat{c}_t \right) + \frac{2 - \nu}{2} \left( \hat{c}_t - \hat{c}_0 \right) + \frac{\nu}{2} \left( \hat{p}_{H,t} - \hat{p}_{F,t}^* \right) \right]$$

$$= \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{\nu (2 - \nu)}{8} \left( \hat{p}_{H,t} + \frac{1}{\sigma} \hat{c}_t - \hat{p}_{H,t} \right)^2 + \frac{\nu}{2} \frac{\epsilon \theta}{2 (1 - \beta \theta) (1 - \theta)} \pi_{H,t}^2 + \frac{2 - \nu}{2} \frac{\epsilon \theta}{2 (1 - \beta \theta) (1 - \theta)} \pi_{H,t}^2 \right].$$

Similarly foreign resource constraint, equation (xxv) is approximated as

$$\mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \left( \hat{h}_t^* - \hat{c}_t^* \right) - \frac{2 - \nu}{2} \left( \hat{c}_t - \hat{c}_0 \right) - \frac{\nu}{2} \left( \hat{p}_{H,t} - \hat{p}_{F,t}^* \right) \right]$$

$$= \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[ \frac{\nu (2 - \nu)}{8} \left( \hat{p}_{F,t} - \frac{1}{\sigma} \hat{c}_t - \hat{p}_{F,t} \right)^2 + \frac{\nu}{2} \frac{\epsilon \theta}{2 (1 - \beta \theta) (1 - \theta)} \pi_{F,t}^2 + \frac{2 - \nu}{2} \frac{\epsilon \theta}{2 (1 - \beta \theta) (1 - \theta)} \pi_{F,t}^2 \right].$$

C. 4

Given the results in C.1-3, we now have enough second-order equations to find pure quadratic expressions for the linear terms $\hat{c}_t - \hat{h}_t$ and $\hat{c}_t^* - \hat{h}_t^*$. Note that the linear terms of interest appear explicitly in resource constraints (4.5.114) and (4.5.115) only. So our indirect goal is to find the appropriate linear combination for $\hat{c}_t - \hat{c}_t^*$ and $\hat{p}_{H,t} - \hat{p}_{F,t}^*$ first.

Write equations (4.5.110)-(4.5.113) and (4.5.114)-(4.5.115) as a group and denote the (pure quadratic) expressions on the RHS of each of the equations as $f_1$, $f_2$, $f_3$, $f_4$, $f_5$ and $f_6$:

$$\omega \hat{h}_t + \sigma \hat{c}_t - \hat{p}_{H,t} = f_1$$
$$\omega \hat{h}_t + \sigma \hat{c}_t^* + \frac{\nu}{2 - \nu} \hat{p}_{F,t}^* = f_2$$
$$\omega \hat{h}_t^* + \sigma \hat{c}_t^* - \hat{p}_{F,t}^* = f_3$$
$$\omega \hat{h}_t^* + \sigma \hat{c}_t + \frac{\nu}{2 - \nu} \hat{p}_{H,t} = f_4$$
$$\left( \hat{h}_t - \hat{c}_t \right) + \frac{2 - \nu}{2} \left( \hat{c}_t - \hat{c}_0 \right) + \frac{\nu}{2} \left( \hat{p}_{H,t} - \hat{p}_{F,t}^* \right) = f_5$$
$$\left( \hat{h}_t^* - \hat{c}_t^* \right) - \frac{2 - \nu}{2} \left( \hat{c}_t - \hat{c}_0 \right) - \frac{\nu}{2} \left( \hat{p}_{H,t} - \hat{p}_{F,t}^* \right) = f_6$$

After a few steps of algebra we obtain

$$\hat{p}_{H,t} - \hat{p}_{F,t}^* = -\frac{2 - \nu}{2} \left( \frac{\sigma + \omega (1 - \nu)}{\gamma} \right) (f_1 - f_3)$$
$$-\frac{2 - \nu}{2} \left( \frac{\sigma - \omega (1 - \nu)}{\gamma} \right) (f_2 - f_4)$$
$$+ \frac{\sigma \omega (2 - \nu)}{\gamma} (f_5 - f_6)$$
\[
\hat{c}_t - \hat{c}_t^* = \frac{\nu}{2} \left( \frac{1 + \omega (2 - \nu)}{\gamma} \right) (f_1 - f_3) - \frac{2 - \nu}{2} \left( \frac{1 + \nu \omega}{\gamma} \right) (f_2 - f_4) + \frac{\omega (1 - \nu)}{\gamma} (f_5 - f_6)
\]

where \(\gamma = \sigma \nu \omega (2 - \nu) + \sigma + \omega (1 - \nu)^2\). Substitute the two expressions in equations (4.5.114) and (4.5.115) and we finally obtain

\[
\hat{c}_t - \hat{h}_t = -\frac{\nu}{2} \left( \frac{\sigma - 1 - \omega}{\gamma} \right) (f_1 - f_3) - \frac{\nu}{2} \left( \frac{\sigma - 1 + \omega + \frac{2}{\nu}}{\gamma} \right) (f_2 - f_4) - \frac{1}{2} \left( 1 + \frac{\sigma - \omega (1 - \nu)}{\gamma} \right) f_5 - \frac{1}{2} \left( 1 - \frac{\sigma - \omega (1 - \nu)}{\gamma} \right) f_6 \tag{4.5.116}
\]

and

\[
\hat{c}_t^* - \hat{h}_t^* = \frac{\nu}{2} \left( \frac{\sigma - 1 - \omega}{\gamma} \right) (f_1 - f_3) + \frac{\nu}{2} \left( \frac{\sigma - 1 + \omega + \frac{2}{\nu}}{\gamma} \right) (f_2 - f_4) - \frac{1}{2} \left( 1 - \frac{\sigma - \omega (1 - \nu)}{\gamma} \right) f_5 - \frac{1}{2} \left( 1 + \frac{\sigma - \omega (1 - \nu)}{\gamma} \right) f_6. \tag{4.5.117}
\]
Plug in the denoted expressions of $f_1$ to $f_6$ and we obtain

$$
-(\hat{c}_t - \hat{h}_t) = + \frac{\nu (2 - \nu)}{16} (1 - \frac{\sigma - \omega (1 - \nu)}{\gamma}) \left( \hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{c}_t - \hat{p}_{H,t} \right)^2 \\
+ \frac{\nu (2 - \nu)}{16} (1 - \frac{\sigma - \omega (1 - \nu)}{\gamma}) \left( \hat{p}_{F,t}^* - \frac{1}{\sigma} \hat{c}_t - \hat{p}_{F,t} \right)^2 \\
+ \frac{\epsilon}{4 \delta} \left[ \frac{\nu}{2} \left( 1 + \frac{\alpha}{\gamma} \right) \pi_{H,t}^2 + \frac{1 - \nu}{2} \left( 1 + \frac{\alpha}{\gamma} \right) \pi_{F,t}^2 \right] \\
+ \frac{\epsilon}{4 \delta} \left[ \frac{\nu}{2} \left( 1 - \frac{\alpha}{\gamma} \right) \pi_{F,t}^2 + \frac{2 - \nu}{2} \left( 1 - \frac{\alpha}{\gamma} \right) \pi_{H,t}^2 \right] \\
- \frac{(\sigma - 1)^2}{2} \left( \frac{2 - \nu}{2} \right) \left( \frac{\nu \nu + 1}{\gamma} \right) (\hat{c}_t)^2 \\
+ \frac{(\sigma - 1)^2}{2} \left( \frac{2 - \nu}{2} \right) \left( \frac{\nu \nu + 1}{\gamma} \right) (\hat{c}_t^*)^2 \\
+ \frac{\nu (2 - \nu)}{8} \left( -\frac{\sigma + 1 + \omega}{\gamma} \right) \left[ (\omega + 1) (\hat{y}_t - z_t) + \frac{2 - \nu}{2} \left( \hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{c}_t - \hat{p}_{H,t} \right) \right]^2 \\
+ \frac{\nu (2 - \nu)}{8} \left( \frac{\sigma - 1 + \omega + 2}{\nu} \right) \left[ (\omega + 1) (\hat{y}_t^* - z_t^*) - \frac{\nu}{2} \left( \hat{p}_{F,t}^* - \frac{1}{\sigma} \hat{c}_t - \hat{p}_{F,t}^* \right) \right]^2 \\
- \frac{\nu (2 - \nu)}{8} \left( -\frac{\sigma + 1 + \omega}{\gamma} \right) \left[ (\omega + 1) (\hat{y}_t^* - z_t^*) + \frac{2 - \nu}{2} \left( \hat{p}_{F,t}^* - \frac{1}{\sigma} \hat{c}_t - \hat{p}_{F,t}^* \right) \right]^2 \\
- \frac{\nu (2 - \nu)}{8} \left( \frac{\sigma - 1 + \omega + 2}{\nu} \right) \left[ (\omega + 1) (\hat{y}_t - z_t) - \frac{\nu}{2} \left( \hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{c}_t - \hat{p}_{H,t} \right) \right]^2.
$$

(4.5.118)
and

\[- (\hat{c}_t^* - \hat{h}_t^*) = \frac{\nu (2 - \nu)}{16} \left(1 - \frac{\sigma - \omega (1 - \nu)}{\gamma}\right) \left(\hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t}\right)^2 \]

\[+ \frac{\nu (2 - \nu)}{16} \left(1 + \frac{\sigma - \omega (1 - \nu)}{\gamma}\right) \left(\hat{p}_{F,t}^* - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}\right)^2 \]

\[+ \frac{\epsilon}{4\delta} \left[\frac{\nu}{2} \left(1 - \frac{\alpha}{\gamma}\right) \pi_{H,t}^2 + \frac{2 - \nu}{2} \left(1 - \frac{\alpha}{\gamma}\right) \pi_{F,t}^2\right] \]

\[+ \frac{\epsilon}{4\delta} \left[\frac{\nu}{2} \left(1 + \frac{\alpha}{\gamma}\right) \pi_{F,t}^2 + \frac{2 - \nu}{2} \left(1 + \frac{\alpha}{\gamma}\right) \pi_{H,t}^2\right] \]

\[- \frac{(\sigma - 1)^2}{2} \left(\frac{2 - \nu}{2}\right) \left(\omega \nu + 1\right) \left(\hat{c}_t^*\right)^2 \]

\[+ \frac{(\sigma - 1)^2}{2} \left(\frac{2 - \nu}{2}\right) \left(\omega \nu + 1\right) \left(\hat{c}_t\right)^2 \]

\[- \frac{\nu (2 - \nu)}{8} \left(-\sigma + 1 + \omega\right) \left(\omega + 1\right) \left(\hat{y}_t - z_t\right) + \frac{2 - \nu}{2} \left(\hat{p}_{H,t}^* + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t}\right)^2 \]

\[- \frac{\nu (2 - \nu)}{8} \left(\sigma - 1 + \omega + \frac{\nu}{2}\right) \left(\omega + 1\right) \left(\hat{y}_t^* - z_t^*\right) - \frac{\nu}{2} \left(\hat{p}_{F,t}^* - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}\right)^2 \]

\[+ \frac{\nu (2 - \nu)}{8} \left(-\sigma + 1 + \omega\right) \left(\omega + 1\right) \left(\hat{y}_t^* - z_t^*\right) + \frac{2 - \nu}{2} \left(\hat{p}_{F,t} - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}^*\right)^2 \]

\[+ \frac{\nu (2 - \nu)}{8} \left(\sigma - 1 + \omega + \frac{\nu}{2}\right) \left(\omega + 1\right) \left(\hat{y}_t - z_t\right) - \frac{\nu}{2} \left(\hat{p}_{H,t} + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t}\right)^2 , \]

(4.5.119)

where we denote

\[\alpha = \omega + 1 + (1 - \sigma) (1 - \nu)\]

\[\gamma = \sigma \nu \omega (2 - \nu) + \sigma + \omega (1 - \nu)^2\]

\[\delta = \frac{(1 - \theta) (1 - \beta \theta)}{\theta} .\]
In the special case imposing $\sigma = 1$, the above expressions reduce to

$$
- \left( \hat{c}_t - \hat{h}_t \right) = \frac{\epsilon \nu}{4\delta} (\pi_{H,t})^2 + \frac{\epsilon (2 - \nu)}{4\delta} (\pi_{F,t})^2 \\
+ \frac{\nu (2 - \nu) \Omega}{8} \left( \hat{d}_t \right)^2 + \frac{\nu (2 - \nu) (1 - \Omega)}{8} \left( \hat{d}_t^* \right)^2 \\
+ \frac{\nu (1 - \Omega)}{4} \left( (\omega + 1) (\hat{y}_t - z_t) + \frac{2 - \nu}{2} \hat{d}_t \right)^2 \\
+ \frac{(2 - \nu) \Omega}{4} \left( (\omega + 1) (\hat{y}^*_t - z^*_t) - \frac{\nu}{2} \hat{d}_t^* \right)^2 \\
- \frac{\nu (1 - \Omega)}{4} \left( (\omega + 1) (\hat{y}^*_t - z^*_t) + \frac{2 - \nu}{2} \hat{d}_t^* \right)^2 \\
- \frac{(2 - \nu) \Omega}{4} \left( (\omega + 1) (\hat{y}_t - z_t) - \frac{\nu}{2} \hat{d}_t \right)^2,
$$

(4.5.120)

and

$$
- \left( \hat{c}^*_t - \hat{h}^*_t \right) = \frac{\epsilon \nu}{4\delta} (\pi^*_{F,t})^2 + \frac{\epsilon (2 - \nu)}{4\delta} (\pi^*_{H,t})^2 \\
+ \frac{\nu (2 - \nu) \Omega}{8} \left( \hat{d}_t^* \right)^2 + \frac{\nu (2 - \nu) (1 - \Omega)}{8} \left( \hat{d}_t^* \right)^2 \\
- \frac{\nu (1 - \Omega)}{4} \left( (\omega + 1) (\hat{y}_t - z_t) + \frac{2 - \nu}{2} \hat{d}_t \right)^2 \\
- \frac{(2 - \nu) \Omega}{4} \left( (\omega + 1) (\hat{y}^*_t - z^*_t) - \frac{\nu}{2} \hat{d}_t^* \right)^2 \\
+ \frac{\nu (1 - \Omega)}{4} \left( (\omega + 1) (\hat{y}^*_t - z^*_t) + \frac{2 - \nu}{2} \hat{d}_t^* \right)^2 \\
+ \frac{(2 - \nu) \Omega}{4} \left( (\omega + 1) (\hat{y}_t - z_t) - \frac{\nu}{2} \hat{d}_t \right)^2.
$$

(4.5.121)

where we denote $\Omega = \frac{1 + \omega^2}{1 + \omega}, 0 < \Omega \leq 1, \hat{d}_t = \hat{p}_{H,t} + \hat{c}_t - \hat{p}_{H,t}$ and $\hat{d}_t^* = \hat{p}_{F,t} - \hat{c}_t - \hat{p}_{F,t}^*$.

C.5

Given the second-order expressions of the linear terms in the utility functions, equations (4.5.118) and (4.5.119), the quadratic loss function of the home noncooperative policy
maker under LCP is given by

\[
L_{t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \frac{1 + \omega}{2} (\dot{y}_t - z_t)^2 + \frac{\nu (2 - \nu)}{16} \left( 1 + \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \hat{p}^*_H, t + \frac{1}{\sigma} \epsilon_t - \hat{p}^*_{H, t} \right)^2 \right. \\
+ \frac{\nu (2 - \nu)}{16} \left( 1 - \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \hat{p}^*_F, t - \frac{1}{\sigma} \epsilon_t - \hat{p}^*_{F, t} \right)^2 \\
+ \frac{\nu (2 - \nu)}{16} \left( 1 - \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \frac{1 + \alpha}{\gamma} \right) \left( \pi^*_F, t \right)^2 + \frac{\epsilon (2 - \nu)}{8\delta} \left( 1 + \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \pi^*_F, t \right)^2 \\
+ \frac{\nu (2 - \nu)}{16} \left( 1 - \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \frac{1 + \alpha}{\gamma} \right) \left( \pi^*_H, t \right)^2 + \frac{\epsilon (2 - \nu)}{8\delta} \left( 1 + \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \pi^*_H, t \right)^2 \\
\left. + \frac{\nu (2 - \nu)}{8\gamma} \left( 1 + \omega \right) \left( \dot{y}_t - z_t \right)^2 \frac{(\sigma - 1)^2 (2 - \nu) (\omega \nu + 1)}{16 \gamma} \left( \dot{y}_t - \frac{2 - \nu}{2} \dot{q}_t + \frac{(2 - \nu) (1 - \sigma)}{2\sigma} \epsilon_t \right)^2 \right. \\
+ \frac{\nu (2 - \nu)}{8\gamma} \frac{(-\sigma + 1 + \omega)}{2\gamma} \left[ (1 + \omega) (\dot{y}_t - z_t) + \frac{2 - \nu}{2} \left( \hat{p}^*_H, t + \frac{1}{\sigma} \epsilon_t - \hat{p}^*_{H, t} \right)^2 \right. \\
+ \frac{(2 - \nu)}{8\gamma} \frac{(\sigma - 1 + \omega)}{2\gamma} \left[ (1 + \omega) (\dot{y}_t - z_t) - \frac{2 - \nu}{2} \left( \hat{p}^*_F, t - \frac{1}{\sigma} \epsilon_t - \hat{p}^*_{F, t} \right)^2 \right. \\
- \frac{\nu (2 - \nu)}{8\gamma} \frac{(-\sigma + 1 + \omega)}{2\gamma} \left[ (1 + \omega) (\dot{y}_t - z_t) - \frac{2 - \nu}{2} \left( \hat{p}^*_H, t + \frac{1}{\sigma} \epsilon_t - \hat{p}^*_{H, t} \right)^2 \right. \\
- \frac{(2 - \nu)}{8\gamma} \frac{(\sigma - 1 + \omega)}{2\gamma} \left[ (1 + \omega) (\dot{y}_t - z_t) - \frac{2 - \nu}{2} \left( \hat{p}^*_F, t - \frac{1}{\sigma} \epsilon_t - \hat{p}^*_{F, t} \right)^2 \right. \\
\right. \right\} \right. \\
(4.5.122)
\]
while the quadratic loss function of the foreign noncooperative policy maker under LCP is given by

\[
L_t^\ast = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \begin{array}{l}
\frac{1 + \omega}{2} (\hat{y}_t^\ast - \hat{z}_t^\ast)^2 \\
\frac{\nu (2 - \nu)}{16} \left( 1 - \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \hat{p}_{H,t}^\ast + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t} \right)^2 \\
\frac{\nu (2 - \nu)}{16} \left( 1 + \frac{\sigma + \omega (\nu - 1)}{\gamma} \right) \left( \hat{p}_{F,t} - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}^\ast \right)^2 \\
+ \frac{\epsilon \nu}{8 \delta} \left( 1 - \frac{\alpha}{\gamma} \right) (\pi_{H,t})^2 + \frac{\epsilon (2 - \nu)}{8 \delta} \left( 1 - \frac{\alpha}{\gamma} \right) (\pi_{F,t})^2 \\
+ \frac{\epsilon \nu}{8 \delta} \left( 1 + \frac{\alpha}{\gamma} \right) (\pi_{F,t}^\ast)^2 + \frac{\epsilon (2 - \nu)}{8 \delta} \left( 1 + \frac{\alpha}{\gamma} \right) (\pi_{H,t}^\ast)^2 \\
+ \left( \frac{\sigma - 1}{2} \right)^2 (2 - \nu) (\omega \nu + 1) \left( \hat{y}_t - \frac{2 - \nu}{2} \hat{q}_t + \frac{(2 - \nu)(1 - \sigma)}{2\sigma} \hat{e}_t \right)^2 \\
\end{array} \right. \\
+ \left( \frac{\sigma - 1}{2} - \frac{(\sigma - 1)^2 (2 - \nu)(\omega \nu + 1)}{4 \gamma} \right) \left( \hat{y}_t^\ast - \frac{2 - \nu}{2} \hat{q}_t^\ast - \frac{(2 - \nu)(1 - \sigma)}{2\sigma} \hat{e}_t \right)^2 \\
- \frac{\nu (2 - \nu)}{8 \gamma} \left( -\sigma + 1 + \omega \right) \left( 1 + \omega \right) (\hat{y}_t - \hat{z}_t) + \frac{2 - \nu}{2} \left( \hat{p}_{H,t}^\ast + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t} \right)^2 \\
- \frac{\nu (2 - \nu)}{8 \gamma} \left( \sigma - 1 + \omega + \frac{2}{\nu} \right) \left( 1 + \omega \right) (\hat{y}_t^\ast - \hat{z}_t^\ast) - \frac{\nu}{2} \left( \hat{p}_{F,t} - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}^\ast \right)^2 \\
+ \frac{\nu (2 - \nu)}{8 \gamma} \left( -\sigma + 1 + \omega \right) \left( 1 + \omega \right) (\hat{y}_t^\ast - \hat{z}_t^\ast) + \frac{2 - \nu}{2} \left( \hat{p}_{F,t} - \frac{1}{\sigma} \hat{e}_t - \hat{p}_{F,t}^\ast \right)^2 \\
+ \frac{\nu (2 - \nu)}{8 \gamma} \left( \sigma - 1 + \omega + \frac{2}{\nu} \right) \left( 1 + \omega \right) (\hat{y}_t - \hat{z}_t) - \frac{\nu}{2} \left( \hat{p}_{H,t} + \frac{1}{\sigma} \hat{e}_t - \hat{p}_{H,t} \right)^2 \\
\right) \\
(4.5.123) 
\]
of the home policy maker under LCP and noncooperation is given by

\[ L_{t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \begin{array}{c}
\frac{1+\omega}{2} (\hat{y}_t - z_t)^2 \\
+ \frac{\epsilon \nu}{8} (\pi_{H,t})^2 + \frac{\epsilon (2-\nu)}{8} (\pi_{F,t})^2 \\
+ \frac{\nu (2-\nu) \Omega}{d_t} + \frac{\nu (2-\nu) (1-\Omega)}{8} (d_t)^2 \\
+ \frac{(2-\nu) \Omega}{4} \left[ (1+\omega) (\hat{y}_t - z_t) - \frac{\nu}{2} \hat{d}_t \right]^2 \\
- \frac{\nu (1-\Omega) + (1+\omega) (\hat{y}_t - z_t) + \frac{\nu}{2} \hat{d}_t \right]^2 \\
- \frac{(2-\nu) \Omega}{4} \left[ (1+\omega) (\hat{y}_t - z_t) - \frac{\nu}{2} \hat{d}_t \right]^2 \\
\end{array} \right\}, \] (4.5.124)

and the quadratic loss function of the foreign policy maker is given by

\[ L^*_{t_0} = \mathbb{E}_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left\{ \begin{array}{c}
\frac{1+\omega}{2} (\hat{y}_t^* - z_t^*)^2 \\
+ \frac{\epsilon \nu}{8} (\pi_{H,t}^*)^2 + \frac{\epsilon (2-\nu)}{8} (\pi_{F,t}^*)^2 \\
+ \frac{\nu (2-\nu) \Omega}{d_t^*} + \frac{\nu (2-\nu) (1-\Omega)}{8} (d_t^*)^2 \\
+ \frac{(2-\nu) \Omega}{4} \left[ (1+\omega) (\hat{y}_t^* - z_t^*) - \frac{\nu}{2} \hat{d}_t^* \right]^2 \\
- \frac{\nu (1-\Omega) + (1+\omega) (\hat{y}_t^* - z_t^*) + \frac{\nu}{2} \hat{d}_t^* \right]^2 \\
- \frac{(2-\nu) \Omega}{4} \left[ (1+\omega) (\hat{y}_t^* - z_t^*) - \frac{\nu}{2} \hat{d}_t^* \right]^2 \\
\end{array} \right\}. \] (4.5.125)

They are equations (3.3.2) and (3.3.3) in the text.
Appendix D. Log-linearization

In this section, we show the complete set of exact or approximated log-linear form of the structural equations (as in Table 3.2 in the text).

Exact log-linear form of deviations from the law of one price:

\[ \hat{d}_t = \hat{p}^*_H,t + \hat{e}_t - \hat{p}_{H,t} \]  \hspace{1cm} (4.5.126)
\[ \hat{d}^*_t = \hat{p}^*_F,t + \hat{e}_t - \hat{p}^*_H,t. \]  \hspace{1cm} (4.5.127)

Log deviations of the terms of trade from the steady state are

\[ \hat{q}_t = \hat{p}_{F,t} - \hat{e}_t - \hat{p}^*_H,t, \]
\[ \hat{q}^*_t = -\hat{q}_t, \]

and exact log-linear form of price indexes equations (v) and (xxiii):

\[ \frac{\nu}{2} \hat{p}_{H,t} + \frac{2 - \nu}{2} \hat{p}_{F,t} = 0, \]  \hspace{1cm} (4.5.128)
\[ \frac{\nu}{2} \hat{p}^*_F,t + \frac{2 - \nu}{2} \hat{p}^*_H,t = 0. \]  \hspace{1cm} (4.5.129)

Exact log-linear form of definitions of detrending, equations (xvii), (xviii), (xxxv) and (xxxvi):

\[ \pi_{H,t} = \pi_t + \hat{p}_{H,t} - \hat{p}_{H,t-1} \]
\[ \pi_{F,t} = \pi_t + \hat{p}_{F,t} - \hat{p}_{F,t-1} \]
\[ \pi^*_{H,t} = \pi^*_t + \hat{p}^*_H,t - \hat{p}^*_{H,t-1} \]
\[ \pi^*_{F,t} = \pi^*_t + \hat{p}^*_F,t - \hat{p}^*_{F,t-1}. \]

Subtracting the first equation by the second and using (4.5.128) gives

\[ \hat{p}_{H,t} - \hat{p}_{H,t-1} = \frac{2 - \nu}{2} (\pi_{H,t} - \pi_{F,t}); \]  \hspace{1cm} (4.5.130)

Subtracting the fourth equation by the third and using (4.5.129) gives

\[ \hat{p}^*_F,t - \hat{p}^*_{F,t-1} = \frac{2 - \nu}{2} (\pi^*_F,t - \pi^*_H,t). \]  \hspace{1cm} (4.5.131)

Exact log-linear form of equation (xxxvii):

\[ \hat{c}_t - \hat{c}^*_t = \frac{1}{\sigma} \hat{e}_t. \]  \hspace{1cm} (4.5.132)
Exact log-linear form of equations (iii), (iv), (xxi) and (xxii):

\[
\hat{c}_{H,t} = -\hat{p}_{H,t} + \hat{c}_t \quad (4.5.133)
\]

\[
\hat{c}_{F,t} = -\hat{p}_{F,t} + \hat{c}_t \quad (4.5.134)
\]

\[
\hat{c}^*_{H,t} = -\hat{p}^*_{H,t} + \hat{c}^*_t \quad (4.5.135)
\]

\[
\hat{c}^*_{F,t} = -\hat{p}^*_{F,t} + \hat{c}^*_t \quad (4.5.136)
\]

Exact log-linear form of equations (ii) and (xx):

\[
\hat{w}_t = \omega \hat{h}_t + \sigma \hat{c}_t \quad (4.5.137)
\]

\[
\hat{w}^*_t = \omega \hat{h}^*_t + \sigma \hat{c}^*_t \quad (4.5.138)
\]

Exact log-linear form of equations (vi) and (xxiv):

\[
\hat{m}_c = \hat{w}_t - z_t \quad (4.5.139)
\]

\[
\hat{m}_c^* = \hat{w}_t - z^*_t \quad (4.5.140)
\]

Exact log-linear form of equations (viii) and (xxvi):

\[
\hat{y}_t = z_t + \hat{h}_t \quad (4.5.141)
\]

\[
\hat{y}^*_t = z^*_t + \hat{h}^*_t \quad (4.5.142)
\]

so from the above conditions (4.5.137)-(4.5.142) we obtain deviations from steady state of the real marginal costs: (equations (3.2.12)-(3.2.15) in the text.)

\[
\hat{m}_c - \hat{p}_{H,t} = (\sigma + \omega) \hat{y}_t - (1 + \omega) z_t + \frac{2 - \nu}{2} (1 - \sigma) (\hat{q}_t + \hat{e}_t) + \frac{2 - \nu}{\sigma} \hat{d}_t,
\]

\[
\hat{m}_c^* - \hat{p}_{F,t} = (\sigma + \omega) \hat{y}^*_t - (1 + \omega) z^*_t + \frac{2 - \nu}{2} (1 - \sigma) (\hat{q}^*_t + \hat{e}^*_t) + \frac{2 - \nu}{\sigma} \hat{d}^*_t,
\]

\[
\hat{m}_c^* - \hat{p}_{F,t} + \hat{e}_t = (\sigma + \omega) \hat{y}^*_t - (1 + \omega) z^*_t + \frac{2 - \nu}{2} (1 - \sigma) (\hat{q}^*_t + \hat{e}^*_t) - \frac{\nu}{2} \hat{d}^*_t,
\]

\[
\hat{m}_c - \hat{p}_{H,t} + \hat{e}_t = (\sigma + \omega) \hat{y}_t - (1 + \omega) z_t + \frac{2 - \nu}{2} (1 - \sigma) (\hat{q}_t + \hat{e}_t) - \frac{\nu}{2} \hat{d}_t.
\]

Log-linearized resource constraints, equations (vii) and(xxv):

\[
\hat{y}_t = \hat{c}_t - \frac{\nu}{2} \hat{p}_{H,t} - \frac{2 - \nu}{2} \hat{p}^*_{H,t} - \frac{2 - \nu}{\sigma} \hat{1} \hat{e}_t, \quad (4.5.143)
\]

\[
\hat{y}^*_t = \hat{c}^*_t - \frac{\nu}{2} \hat{p}^*_{F,t} - \frac{2 - \nu}{2} \hat{p}^*_{F,t} + \frac{2 - \nu}{\sigma} \hat{1} \hat{e}_t, \quad (4.5.144)
\]

so subtracting equation (4.5.144) from (4.5.143) gives
\[ \hat{y}_t - \hat{y}^*_t = -\frac{\nu}{2} \hat{p}_{H,t} - \frac{2 - \nu}{2} \hat{p}^*_{H,t} + \frac{(\nu - 1)}{\sigma} \hat{e}_t + \frac{\nu}{2} \hat{p}^*_{F,t} + \frac{2 - \nu}{2} \hat{p}_{F,t}, \]

where we have used equation (4.5.132). This is equation (3.2.20) in the text.

New Keynesian Phillips curves are derived as follows. Write the second-order approximation of the AS equation (4.5.95) in Appendix C in a recursive way:

\[ V_{H,t} = \delta \hat{z}_{H,t} + \frac{\delta}{2} \hat{z}_{H,t} \bar{x}_{H,t} + \frac{\epsilon}{2} \pi^2_{H,t} + \beta E_t (V_{H,t+1}). \]

Up to the first order, it reduces to

\[ V_{H,t} = \delta \hat{z}_{H,t} + \beta E_t \pi_{H,t+1}. \]

where we have defined \( \hat{z}_{H,t} = \hat{m}_t - \hat{p}_{H,t} \) and \( \delta = \frac{(1-\beta\theta)(1-\theta)}{\theta} \). Up to the first order, equation (4.5.95) itself reduces to

\[ V_{H,t} = \pi_{H,t}, \]

so combining the above two equations, we obtain the corresponding log-linear NKPC

\[ \pi_{H,t} = \beta E_t \pi_{H,t+1} + \frac{(1 - \beta\theta)(1 - \theta)}{\theta} (\hat{m}_t - \hat{p}_{H,t}). \] (4.5.145)

Similarly, we can obtain the other three New Keynesian Phillips curves as follows:

\[ \pi^*_{F,t} = \beta E_t \pi^*_{F,t+1} + \frac{(1 - \beta\theta)(1 - \theta)}{\theta} (\hat{m}^*_t - \hat{p}^*_{F,t}), \] (4.5.146)

\[ \pi_{F,t} = \beta E_t \pi_{F,t+1} + \frac{(1 - \beta\theta)(1 - \theta)}{\theta} (\hat{m}^*_t - \hat{p}^*_{F,t} + \hat{e}_t), \] (4.5.147)

\[ \pi^*_{H,t} = \beta E_t \pi^*_{H,t+1} + \frac{(1 - \beta\theta)(1 - \theta)}{\theta} (\hat{m}_t - \hat{p}^*_{H,t} - \hat{e}_t). \] (4.5.148)

These are equations (3.2.8)-(3.2.11) in the text.
Bibliography


