

THE THEORY AND DESIGN OF ZOOM LENSES

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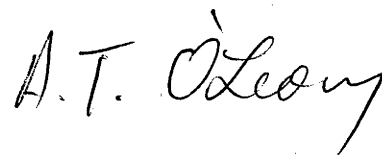
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Except where due acknowledgement has been made in the text,
the work described in this thesis is the candidate's own.

A handwritten signature in cursive script, reading "A. T. O'Leary". The signature is written in dark ink and is positioned above the printed name.

A. T. O'LEARY

ABSTRACT

This thesis contains a new approach to the paraxial and third order design and theory of zoom lenses, especially those consisting of thin elements. Equations describing the paraxial and third order behaviour of zoom lenses are derived and are used to design a series of three element zoom lenses.

In the paraxial theory the derivatives, with respect to the axial separations, of the paraxial coefficients are required. Optimization processes for the automatic design of lenses require the availability of the derivatives, with respect to axial separations and curvatures, of the third and fifth order aberration coefficients. These derivatives can be calculated from formulae given herein and these form the basis of a FORTRAN programme for calculating such derivatives.

The derivatives, with respect to axial separations, of the paraxial coefficients, occur in the formulae describing the paraxial properties as functions of the zoom parameter. These formulae may be solved to obtain the powers and initial ($z = 0$) separations of the elements of the zoom lens. For two and three element zoom lenses the equations are solved analytically.

Formulae are obtained for the third order aberration coefficients as functions of the zoom parameter. Using the thin lens approximation the shapes of the individual elements are introduced and from the resulting expressions sets of simultaneous equations are obtained in the particular case of the three (thin) element zoom lens. The solutions of these sets of equations give zoom systems for which

selected third order aberrations are substantially corrected over the whole zoom range. Whereas optimization processes give one solution to a particular problem, the present method gives all possible solutions by a simple analytic or semi-analytic approach.

INTRODUCTION

This thesis is divided into two parts. Part I deals with the aberration coefficients and the derivatives of aberration coefficients for symmetric optical systems (§2). Part II, which deals with the theory and design of zoom lenses, uses many of the results of Part I. A zoom lens is an optical system able to produce, in a fixed plane, a corrected image with variable magnification. This effect is produced by the controlled movement of the constituent elements relative to each other (§14).

Essentially the material of Part I is taken from the work of Buchdahl[†] [1], but some new material is presented. The notation (§2 and 4) used throughout this thesis is basically that of L.

The equations of paraxial optics (§2) are linear and the paraxial coefficients occurring in these equations can be used to build up expressions for the contributions of surfaces to the aberration coefficients (§5). With the correct choice of ray co-ordinates (§2) the 'a' aberration coefficients (§5) describe the aberrations of an optical system. However when calculating the derivatives of aberration coefficients (and higher order aberration coefficients), the 'b' coefficients (§5) are also required. Although formulae can be written down for these, it is simpler in practice to employ the so-called 'principle of duality' (§6).

[†] The theory presented in this work will be referred to frequently. The entire work will be referred to as L (it presents a Lagrangian optical theory) and any section or equation preceded by L refers to a section or equation in L.

The paraxial, third and fifth order 'a' and 'b' coefficients of a system of surfaces can be obtained from an essentially simple computer programme employing the principle of duality. Such a programme, COEFF, has been written (Appendix 2) and is used as a sub-programme in programme DERIVATIVES (§12 and Appendix 2).

In the design of optical systems a knowledge of the effects of variations of design parameters (chapter 2) is of great importance. In zoom lens theory and design this knowledge is especially useful since the axial separations between lens elements are continuously variable. The derivatives of the final paraxial coefficients (§8), with respect to axial separations, can be used to describe the variation of the first order (paraxial) properties of zoom lenses (§§17 and 18). Equations (8.11-13), though derived from the theory of L Part III, do not appear in L.

In Part II of this thesis an approach to the design of zoom lenses is presented. In this we allocate the initial axial separations, powers and curvatures of thin elements in such a way that the paraxial and third order terms in the expansion of $\hat{e}'$ are controlled. The results of this design procedure can be viewed only as a first approximation to be used, say, as the starting point in an optimization process for obtaining a final system, consisting of real (thick) lenses, with balanced aberrations throughout the whole zoom range.

Optimization processes generally require a knowledge of at least the first derivatives, with respect to thicknesses, curvatures, etc., of the third and fifth order aberration coefficients. These derivatives can be obtained by the method of finite differences, but

are given more accurately by the exact formulae presented in chapter 2. The first derivatives of the third order aberration coefficients (§10) and the fifth order aberration coefficients (§11) are given in a form which may be used directly in computing schemes. The derivatives of the third order coefficients, in a slightly different form, are given in L. The derivatives of the fifth order aberration coefficients are not given in L, but the method of their production is given.

In view of the importance of the first derivatives of aberration coefficients, a computer programme, called DERIVATIVES, was written and is summarized in §12. The basis of this programme is the formulae contained in §§10 and 11.

The calculation of derivatives of fifth order aberration coefficients with respect to the first extra-axial curvatures requires certain formulae for the derivatives of the intrinsic contributions of order five. These formulae are given at the end of §12 and are obtained directly from the theory of L §156, specifically equation L (156.3).

Part II of this thesis discusses the paraxial and third order aberration theory of zoom lenses, especially those consisting of thin elements (§30), from a new point of view. The notation used is essentially that of L, although modifications have been made in special circumstances.

As previously stated the elements of zoom lenses are movable. The form of this movement can be used to classify zoom lenses, which in the past have been classified as either 'mechanically' or 'linearly' ('optically') compensated systems. The meanings of these terms are explained in §14. These classifications are purely

artificial, linear compensation being merely a special case of mechanical (or non-linear) compensation.

The theory set out in chapters 4 and 5 holds for linear compensation, but it must be emphasized that this restriction is not necessary. However, if we are seeking analytic solutions of equations (19.5), (20.4), (20.5), it appears we must restrict ourselves to linear compensation. It must be pointed out that non-linear compensation will allow better control over the optical properties of zoom lenses. This is because if the separation between a particular pair of elements is written as a power series in the zoom parameter (§17) then the coefficients of this series may be regarded as degrees of freedom to be allocated by the designer. The usual degrees of freedom are the powers and the initial separations of the elements (§15). Allowing non-linear compensation, even of only a quadratic nature, greatly increases the number of degrees of freedom available.

The paraxial theory and design of zoom lenses may be considered independently of the aberrational theory and design if the coefficients in the series for the separations are known numbers. In optical compensation (§14) all coefficients are zero except those of the linear terms in the zoom parameter. The coefficients of the linear terms in the separations are chosen so that each separation varies at the same rate.

The final paraxial coefficients can be written as polynomials in the zoom parameter (§18). From these polynomials conditions can be obtained which, if satisfied, ensure that the system has the desired properties. It is found that the height of the final image can be made equal to the initial ideal image height at n points in the zoom

range (§§17,18). n is the number of elements in the zoom part ② of the zoom lens (§15). The variation of the image height can be controlled by a judicious choice of these n points which are called the points of full compensation.

The equations obtained for the paraxial design of zoom lenses (§20) can only be solved in certain cases. If we restrict ourselves to the consideration of thin elements (§22) we find that the design parameters (powers and initial separations) can be allocated analytically for the two and three element zoom lenses. The work of Bergstein and Motz [8] indicates that it should be possible to solve the four element problem analytically.

For systems consisting of any number of thick or thin elements numerical solutions are possible (§25). The method of numerical solution utilizes the theory of §§9,18.

Once the powers and initial separations of the elements have been chosen using the theory of first order design, it remains only to allocate the shapes (i.e. curvatures) of the elements. Extra axial powers (§40), if allowed, must also be allocated.

The aberrations of zoom lenses can be written in terms of what it is proposed to call fixed aperture aberration coefficients (§26). These are aberration coefficients calculated for an entrance pupil with fixed diameter and position. The actual entrance pupil varies in both diameter and position as the elements move (§28). The dependence of the third order fixed aperture aberration coefficients on z can be obtained explicitly through the use of certain polynomials related to the paraxial coefficient polynomials before each element (§27). The actual aberration coefficients can then be written in

terms of the fixed aperture coefficients, the pupil magnification and the entrance pupil position, all of which are functions of z , the zoom parameter (§28). It is found that the actual aberration coefficients of order three are ratios of polynomials in z .

These aberration coefficients depend on the curvatures of the elements of the zoom lens. Once powers are allocated to the elements, only one set of parameters, the shapes, has to be specified to obtain the aberrations at a particular value of z . It is possible to leave the shapes as variables (§30) and then the actual aberration coefficients can be written in terms of the shapes of the elements and z (§§28,31). Control of the aberrations is exercised through the allocation of the shapes of the elements (§32).

Whichever method of control of aberrations is used, the designer obtains certain equations, involving the shapes as variables, which have to be solved. We find that in the case of the three element zoom lens these equations can always be solved, either analytically (§36) or semi-analytically (§§37,38). In certain cases the solutions are only approximate, but one must realize that thin lens design is only intended as a starting point for further, more realistic, work.

The calculations for §§36, 37 and 38 were carried out on a programmable desk computer. However, if necessary, the work could be done with a machine of lesser capabilities, but to perform all the calculations by hand (using, say, a slide rule) would be extremely time consuming, if at all possible.

I note in §40 that all third order aberration coefficients can be controlled if first extra axial curvatures are allowed. The calculations in this case would be similar to those involved in §38.

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PART I

ABERRATION COEFFICIENTS AND THEIR DERIVATIVES

CHAPTER 1

ABERRATION THEORY OF OPTICAL SYSTEMS

1. Introduction. Aberration coefficients and their derivatives

The theory of optical aberration coefficients and their derivatives is contained in L. The purpose of chapters 1 and 2 of this thesis is to give a summary of the basic results of L and to present certain formulae not appearing in L.

In the notation of L (L §§1 and 7) the aberration of a ray is

$$\epsilon'_k = H'_k - m'_k H_1.$$

The notation is explained in §3 below. ϵ'_k may be expanded in series of ascending powers of suitable ray co-ordinates. These series will be the sums of homogeneous polynomials of degree 3, 5, 7, ..., $2n+1$. The coefficients occurring in the polynomials of degree $2n+1$ are the monochromatic n^{th} order aberration coefficients. Formulae for the primary and secondary aberration coefficients are given in L. The formulae used in calculating the primary aberration coefficients are reproduced below in §5.

The aberration coefficients of a system depend on the axial separations of surfaces, the axial and extra-axial curvatures of surfaces, the refractive indices of glasses, etc., of the system.

These quantities we call the parameters of the system. Any change in the constitution of the system we speak of as a 'variation of parameters'. When such variations are small we have as a first approximation for the change of any aberration coefficients G'

$$\delta G' = \sum \frac{\partial G'}{\partial \tau} \delta \tau ,$$

the summation being extended over the varied parameters τ .

In L equations for the first derivatives of the primary and some secondary aberration coefficients are given. The full set of derivatives of the secondary coefficients appears in §11 below.

2. Notation. Paraxial optics

In Part II of this thesis many results obtained in L will be utilized. In order to familiarize the reader with the notation and basic results contained in L, I attempt, in this chapter and the next, to summarize Parts I and III of L. L deals, in the main, with monochromatic light propagating through symmetric optical systems. A symmetric optical system is a system which is axially symmetric and which is invariant under reflection in any plane containing the axis of symmetry.

The basic notation of L can be explained as follows. We consider a single ray, with direction cosines $(\alpha, -\beta, -\gamma)$, intersecting a symmetric surface. A primed symbol (e.g. α') denotes the quantity represented after refraction at the surface. We will use a set of co-ordinates (x, y, z) such that the origin coincides with the axial point A (the pole) of the surface and the x-axis lies along the axis

of symmetry. The plane $x = 0$ is tangent to the surface at the pole A and is called the polar tangent plane. Any ray intersecting the surface may be specified by variables Y, Z, V, W where Y and Z are the y - and z - co-ordinates of the point of intersection of the ray with the polar tangent plane. V and W are defined[†] by the relations

$$V := \beta/\alpha, \quad W := \gamma/\beta, \quad (2.1)$$

so that $\alpha = (1 + V^2 + W^2)^{-\frac{1}{2}}$.

Any symbol with a tilde underneath it will stand for both the y - and z -components of the quantity represented. For example

$$\underline{Y} = (Y, Z), \quad \underline{V} = (V, W). \quad (2.2)$$

The equation of a ray, before refraction, can then be written as

$$\bar{x} = t \quad \text{and} \quad \bar{y} = \underline{Y} - \underline{V}t, \quad (2.3)$$

where $(\bar{x}, \bar{y}, \bar{z})$ are current co-ordinates on the ray and t is a variable parameter. If we use d' to denote the axial separation (or simply separation) between two consecutive surfaces then

$$\left. \begin{aligned} \underline{Y} &= \underline{Y}' - d' \underline{V}' \\ + \\ \underline{V} &= \underline{V}' \end{aligned} \right\} . \quad (2.4)$$

$\underline{Y}, \underline{V}$ are $\underline{Y}$ and $\underline{V}$ at the second surface of the pair.
+ +

The shape of a spherical surface (we shall generally be dealing with spherical surfaces) can be described by its radius of curvature $c := 1/r$. The quantity $\underline{l}$, which is related to the angle

[†] The equations $A := B$ and $A =: B$ are defining relations for A and B respectively.

of incidence of a ray, is defined by

$$\underline{I} = c\underline{Y} - \underline{V}. \quad (2.5)$$

Paraxial optics deals with rays which lie in an infinitesimal neighbourhood of the optical axis. This means that the variables specifying a paraxial ray are of infinitesimal magnitude. If $\underline{y}$, $\underline{v}$ and $\underline{i}$ are the paraxial equivalents of $\underline{Y}$, $\underline{V}$ and $\underline{I}$, it is shown in L §§4 and 5 that the basic relations of paraxial optics are

$$\left. \begin{aligned} \Delta(N\underline{i}) &= 0 \\ c\Delta\underline{y} &= \Delta(\underline{i} + \underline{v}) = 0 \end{aligned} \right\}. \quad (2.6)$$

Combining (2.6) with (2.4) gives the recurrence relations

$$\left. \begin{aligned} \underline{y}_{+} &= [1 - (1 - k) cd'] \underline{y} - kd' \underline{v} \\ \underline{v}_{+} &= (1 - k) c\underline{y} + k\underline{v} \end{aligned} \right\}, \quad (2.7)$$

where k , the refractance, is defined by $k := N/N'$. The surfaces of an optical system can be labelled $1, \dots, j, \dots, k$. Relations (2.7) must then hold for surfaces j and $j+1$ (since $\underline{y}_j \equiv \underline{y}_{j+1}$) and so we see that $\underline{y}_{j+1}, \underline{v}_{j+1}$ are linearly related to $\underline{y}_j, \underline{v}_j$, from which it follows that $\underline{y}_j, \underline{v}_j$ are linearly related to $\underline{y}_1, \underline{v}_1$. This is indicated by writing

$$\left. \begin{aligned} \underline{y}_j &= y_{pj} \underline{y}_1 + y_{qj} \underline{v}_1 \\ \underline{v}_j &= v_{pj} \underline{y}_1 + v_{qj} \underline{v}_1 \end{aligned} \right\}. \quad (2.8)$$

The coefficients y_{pj} etc. in (2.8) are called the 'canonical' paraxial coefficients of the system. The term 'canonical' refers to the initial ray co-ordinates used to prescribe rays entering the system

(L §§3,6). Canonical co-ordinates are a special case of the more general 'paracanonial' co-ordinates (L §12). For paracanonial co-ordinates the relations equivalent to (2.8) are

$$\left. \begin{aligned} \underline{y}_j &= y_{aj} \underline{s} + y_{bj} \underline{t} \\ \underline{v}_j &= v_{aj} \underline{s} + v_{bj} \underline{t} \end{aligned} \right\}, \quad (2.9)$$

where $\underline{s}$, $\underline{t}$ are the paraxial equivalents of the paracanonial co-ordinates

$$\left. \begin{aligned} \underline{s} &= \sigma \underline{y}_1 + \bar{\sigma} \underline{v}_1 \\ \underline{t} &= \tau \underline{y}_1 + \bar{\tau} \underline{v}_1 \end{aligned} \right\}. \quad (2.10)$$

The choice of σ , $\bar{\sigma}$, τ , $\bar{\tau}$ determines the type of paracanonial co-ordinates. Generally subscripts p or q indicate canonical co-ordinates while subscripts a or b indicate paracanonial co-ordinates.

Although the paraxial variables are infinitesimal, the canonical (or paracanonial) paraxial coefficients are finite. For example, with canonical co-ordinates the initial paraxial coefficients are, from (2.8), $y_{p1} = 1$, $y_{q1} = 0$, $v_{p1} = 0$, $v_{q1} = 1$.

Consider two arbitrary paraxial rays, one of which is meridional (distinguished by a bar). Let

$$\underline{\lambda} := N(\bar{y} \underline{v} - \bar{v} \underline{y}). \quad (2.11)$$

As the two rays propagate through the system $\underline{\lambda}$ is invariant. This invariance is demonstrated in L §5. $\underline{\lambda}$ is called the paraxial invariant. This invariance is of great importance in the theory of aberration coefficients.

3. The aberration of a ray

It is shown in L §5 that paraxial optics is stigmatic, that is, all rays issuing from a particular object point intersect at one image point. If $\underline{h}_1$ is the initial object height then the j^{th} object height is, according to L (5.101),

$$\underline{h}_j = m_j \underline{h}_1, \quad (3.1)$$

where the paraxial magnification associated with the conjugate object planes is

$$m_j = N_1 v_{01} / N_j v_{0j}. \quad (3.2)$$

The subscript 0 indicates that v_{01} , v_{0j} are angles associated with an axial object point. If we consider non-paraxial rays, a ray from a particular object point will not, in general, pass through the ideal image point. Instead it will pass through some other point whose co-ordinates $\underline{H}_j \neq m_j \underline{H}_1$, where $\underline{H}_1$ represents the initial co-ordinates of a non-paraxial ray. The aberration of a ray, for a given object height $\underline{H}_1$ is

$$\underline{\epsilon}_j := \underline{H}_j - m_j \underline{H}_1. \quad (3.3)$$

It is shown in L §5 that

$$\underline{\Delta}_j = N_j v_{0j} \underline{h}_j, \quad (3.4)$$

where $\underline{h}_j$ is given by (3.1). Thus, from (3.3) we have, since $\underline{h}_1 \equiv \underline{H}_1$, that

$$\underline{H}_j = \underline{h}_j + \underline{\epsilon}_j. \quad (3.5)$$

If (3.5) is multiplied by $N_j v_{0j}$ then we may write

$$\underline{\Delta}_j = \underline{\lambda}_j + \underline{\hat{\epsilon}}_j, \quad (3.6)$$

where $\underline{\Delta} := N_j v_{0j} \underline{H}_j$, $\underline{\hat{\epsilon}}_j = N_j v_{0j} \underline{\epsilon}_j$. $\underline{\hat{\epsilon}}_j$ is called the 'augmented aberration' and $N_j v_{0j}$ is called the 'augmenting factor'. Since $\underline{\lambda}$ is invariant it follows that

$$\Delta \underline{\hat{\epsilon}}_j = \Delta \underline{\Delta}_j. \quad (3.7)$$

The symbol Δ is defined in L §2. $\Delta X (\equiv X' - X)$ denotes the change in a quantity X , associated with a ray, due to refraction at a surface.

Summing over all surfaces we have

$$\underline{\hat{\epsilon}}'_k = \sum_{j=1}^k \Delta \underline{\Delta}_j. \quad (3.8)$$

Eq. (3.8) is the basic equation of the theory of aberrations of symmetric optical systems as presented in L, and is used to obtain formulae for the aberration coefficients of such systems (see L §§8,9,10,11 and L chs.II and VIII).

4. Notation for aberration coefficients

The following notation will always be employed.[†]

(i) The third order coefficients are denoted by $a, b, c; A, B, C$ as follows

$$\begin{aligned} g_{00}^{(1)} &= a, & g_{10}^{(1)} &= b, & g_{11}^{(1)} &= c, \\ \bar{g}_{00}^{(1)} &= \bar{a}, & \bar{g}_{10}^{(1)} &= \bar{b}, & \bar{g}_{11}^{(1)} &= \bar{c}, \\ G_{00}^{(1)} &= A, & G_{10}^{(1)} &= B, & G_{11}^{(1)} &= C, \\ \bar{G}_{00}^{(1)} &= \bar{A}, & \bar{G}_{10}^{(1)} &= \bar{B}, & \bar{G}_{11}^{(1)} &= \bar{C}. \end{aligned}$$

[†] See Appendix 3.

(ii) The fifth order coefficients are denoted by $s_1, s_2, s_3 \dots s_6$; $\bar{s}_1, \bar{s}_2, \bar{s}_3 \dots \bar{s}_6$ as follows

$$\begin{aligned} g_{00}^{(2)} &= s_1, & g_{10}^{(2)} &= s_2, & g_{11}^{(2)} &= s_3, \text{ etc.}, \\ \bar{g}_{00}^{(2)} &= \bar{s}_1, & \bar{g}_{10}^{(2)} &= \bar{s}_2, & \bar{g}_{11}^{(2)} &= \bar{s}_3, \text{ etc.}, \\ G_{00}^{(2)} &= s_1, & G_{10}^{(2)} &= s_2, & G_{11}^{(2)} &= s_3, \text{ etc.}, \\ \bar{G}_{00}^{(2)} &= \bar{s}_1, & \bar{G}_{10}^{(2)} &= \bar{s}_2, & \bar{G}_{11}^{(2)} &= \bar{s}_3, \text{ etc.} \end{aligned}$$

(iii) Intrinsic contributions will be represented by German characters (here written $\underline{X}$).

5. The third order aberration coefficients

The third order aberration coefficients are the sums of the third order contributions. In terms of the intrinsic contributions of order three these are (see L §10)

$$g_{\mu\nu}^{(1)} = \underline{g}_{\mu\nu}^{(1)}, \quad \bar{g}_{\mu\nu}^{(1)} = \underline{\bar{g}}_{\mu\nu}^{(1)}. \quad (5.1)$$

Each contribution may be written as

$$g = g_a s + g_b t,$$

as usual. The intrinsic contributions may be written as (see L ch.IV)

$$\begin{aligned} \underline{a}_a &= \frac{1}{2}N(1-k) y_a i_a^2 (i'_a - v_a), & \underline{a}_b &= q \underline{a}_a - \frac{1}{2}N_1 v_a^2,^\dagger \\ \underline{\bar{a}}_a &= q \underline{a}_a, & \underline{\bar{a}}_b &= q \underline{a}_b, \\ \underline{b}_a &= 2 \underline{\bar{a}}_a, & \underline{b}_b &= 2 \underline{\bar{a}}_b + N_1^2 \tilde{\omega}, \\ \underline{\bar{b}}_a &= q \underline{b}_a, & \underline{\bar{b}}_b &= q \underline{b}_b, \end{aligned}$$

[†] For interpretation of N_1 when using paracanonial co-ordinates, see L §12.

$$\begin{aligned}
 \underline{c}_a &= \frac{1}{2}(\underline{b}_a - N_1^2 \tilde{\omega}) , & \underline{c}_b &= \frac{1}{2}\underline{b}_b , \\
 \overline{\underline{c}}_a &= q \underline{c}_a , & \overline{\underline{c}}_b &= q \underline{c}_b ,
 \end{aligned} \tag{5.2}$$

where

$$\tilde{\omega} := c\Delta(1/N) , \quad q := i_b/i_a . \tag{5.3}$$

6. The principle of duality

Let u be any linear paraxial variable, and let u_a, u_b be the paraxial coefficients associated with it. Then the 'dual' of u_a is defined to be u_b while the dual of u_b shall be u_a . Buchdahl [2] discusses the duals of the various aberration coefficients. Specifically $\overline{g}_{n-\nu, n-\mu b}^{(n)}$ is the dual of $g_{\mu\nu a}^{(n)}$.

Consider the coefficients of $s \underline{S} \underline{\xi}^{n-\mu} \eta^{\mu-\nu} \rho^\nu$ and of $t \underline{T} \underline{\xi}^\nu \eta^{\mu-\nu} \rho^{n-\mu}$ in the expansion of $\Delta \underline{\Lambda}$. The second combination of variables arises from the first by the formal interchange of s with t and of $\underline{S}$ with $\underline{T}$. The coefficients referred to are $g_{\mu\nu a}^{(n)}$ and $\overline{g}_{\mu-\nu, n-\mu b}^{(n)}$, i.e. $g_{\mu\nu a}^{(n)}$ and its dual. We see that whatever function $g_{\mu\nu a}^{(n)}$ is of the paraxial coefficients at the surface in question and those preceding it, $\overline{g}_{\mu-\nu, n-\mu b}^{(n)}$ must be the same function of the dual coefficients. Thus we have the principle of duality:

'If any function of the paraxial and aberration coefficients vanishes, the dual function also vanishes.'

The principle of duality may be used to calculate 'b' coefficients if a method for calculating 'a' coefficients is available. If a formula is available for a particular 'a' coefficient, one replaces each quantity in the formula by its dual. The formula then

gives the dual of the 'a' coefficient in question. This will be one of the 'b' coefficients required. If a computing scheme is available for computing 'a' coefficients, to calculate 'b' coefficients one merely uses the duals of the original data. For example if the original data are the initial paraxial coefficients, one replaces y_{a1} by y_{b1} , v_{a1} by v_{b1} and vice versa. The 'b' coefficients are obtained in what is called dual order. For example the usual order for the 'b' surface contributions of order three is $(\underline{a}_b, \bar{\underline{a}}_b, \underline{b}_b, \bar{\underline{b}}_b, \underline{c}_b, \bar{\underline{c}}_b)$. The dual order is $(\bar{\underline{c}}_b, \underline{c}_b, \bar{\underline{b}}_b, \underline{b}_b, \bar{\underline{a}}_b, \underline{a}_b)$. Thus the formula which gives $\underline{a}_a$ in the original calculation gives $\bar{\underline{c}}_b$ when the dual calculation is performed. The dual order for the fifth order surface contributions is given in L §11.

CHAPTER 2

THEORY OF VARIATION OF DESIGN PARAMETERS

7. Equivalent variation of co-ordinates

The design parameters of an optical system are the separations d'_j between adjacent surfaces, the axial curvatures c_{0j} of the surfaces and the refractive indices N_j of the media of which the system is constructed. The extra axial curvatures (see L §56) c_{1j} , c_{2j} , etc. of surfaces and the dispersions of glasses are other design parameters.

Let τ_j be a design parameter at the j^{th} surface of a system of surfaces. If τ_j is changed by an amount $\partial\tau_j$, y'_{0j} , v'_{0j} , $\underline{y}'_j$ and $\underline{v}'_j$ will change by $\partial y'_{0j}$, $\partial v'_{0j}$, $\partial \underline{y}'_j$ and $\partial \underline{v}'_j$ respectively, the initial co-ordinates being kept constant. We ask what changes, ∂s , ∂t , $\partial \underline{s}$, $\partial \underline{t}$, in the initial co-ordinates must be made to produce the same change in y'_{0j} , v'_{0j} , $\underline{y}'_j$, $\underline{v}'_j$ as the change in τ_j produced. The required changes $\partial \underline{s}$ and $\partial \underline{t}$ in the paracanonical co-ordinates are called the equivalent variations of the co-ordinates. These may be written as

$$\left. \begin{aligned} \partial \underline{s} &= (\Gamma_s \underline{s} + \bar{\Gamma}_s \underline{t}) \partial \tau \\ \partial \underline{t} &= (\Gamma_t \underline{s} + \bar{\Gamma}_t \underline{t}) \partial \tau \end{aligned} \right\} . \quad (7.1)$$

The $\Gamma_s \dots \bar{\Gamma}_t$ are functions of the paracanonical co-ordinates (see L §121).

It is possible to write, in terms of the equivalent variations, an expression for the change in $\hat{\underline{e}}'_k$ when τ_j is changed. If we write

$$\underline{\Xi} := (S, T) \quad (7.2)$$

then the derivative $(\partial \hat{\underline{\epsilon}}'_k / \partial \underline{\Xi})(\partial \underline{\Xi} / \partial \tau_j)$ is given by equation L (121.6)

This is very lengthy and so is not reproduced here.

8. The derivatives of the paraxial coefficients

In L §§125, 135, 146 it is shown that the algebraic equations for the equivalent co-ordinate variations have the general form[†]

$$\left. \begin{aligned} \frac{\partial \underline{S}}{\partial \tau_j} &= - \frac{\partial \Delta \cdot \underline{\Lambda}_{bj}}{\partial \tau_j} - a_{bj} \underline{U}_j + \frac{\partial \hat{\underline{\epsilon}}_{bj}}{\partial \underline{\Xi}} \frac{\partial \underline{\Xi}}{\partial \tau_j} \\ \frac{\partial \underline{T}}{\partial \tau_j} &= \frac{\partial \Delta \cdot \underline{\Lambda}_{aj}}{\partial \tau_j} + a_{aj} \underline{U}_j - \frac{\partial \hat{\underline{\epsilon}}_{aj}}{\partial \underline{\Xi}} \frac{\partial \underline{\Xi}}{\partial \tau_j} \end{aligned} \right\} \quad (8.1)$$

$a, \underline{U}$ are quantities such as, for example, $\cdot Nv, \underline{V}$. When $\tau_j = d_j$ the first term in each of (8.1) does not appear, i.e. we take $\partial / \partial d_j \equiv 0$ for these terms. From (8.1) we see that

$$\left. \begin{aligned} \frac{\partial \underline{S}}{\partial \tau_j} &= - a_{bj} (u_{aj} \underline{S} + u_{bj} \underline{T}) + 0(3) \\ \frac{\partial \underline{T}}{\partial \tau_j} &= a_{aj} (u_{aj} \underline{S} + u_{bj} \underline{T}) + 0(3) \end{aligned} \right\} \quad (8.2)$$

Comparing (7.1) and (8.2) we have

$$\left. \begin{aligned} \theta_s &= - a_b u_a, & \bar{\theta}_s &= - a_b u_b \\ \theta_t &= a_a u_a, & \bar{\theta}_t &= a_a u_b \end{aligned} \right\} \quad (8.3)$$

[†] The antepriem in (8.1) indicates division by the paraxial invariant $\lambda = N(y_a v_b - v_a y_b)$.

where the $\theta_s, \dots, \bar{\theta}_t$ are coefficients in the power series expansions of the $\Gamma_s \dots \bar{\Gamma}_t$ occurring in (7.1) (see L 121.2)).

In the paraxial region

$$\frac{\partial s}{\partial \tau} = \theta_s s + \theta_t t, \quad \frac{\partial t}{\partial \tau} = \theta_t s + \bar{\theta}_t t \quad (8.4)$$

or

$$\begin{aligned} \frac{\partial s}{\partial \tau} &= -a_b(u_a s + u_b t), & \frac{\partial t}{\partial \tau} &= a_a(u_a s + u_b t), \\ \frac{\partial s}{\partial \tau} &= -a_b u, & \frac{\partial t}{\partial \tau} &= a_a u. \end{aligned} \quad (8.5)$$

Thus we obtain

$$\begin{aligned} \frac{\partial y_i}{\partial \tau_j} &= y_{ai} \frac{\partial s}{\partial \tau_j} + y_{bi} \frac{\partial t}{\partial \tau_j}, \quad i \geq j \\ &= (a_{aj} y_{bi} - a_{bj} y_{ai}) u_j. \end{aligned}$$

In L §5 the notation

$$(f|g) = f_a g_b - f_b g_a,$$

where f and g are more or less complex symbols, is introduced. Using this we have

$$\frac{\partial y_i}{\partial \tau_j} = (a_j | y_i) u_j, \quad i \geq j. \quad (8.6)$$

In a similar manner

$$\frac{\partial v'_i}{\partial \tau_j} = (a_j | b_i) u_j, \quad i \geq j. \quad (8.7)$$

In general the derivatives of y_i may be obtained from those for v'_i by writing y_i wherever v'_i appears. It is therefore necessary to consider only the derivatives of v'_i .

L §§125,135,146 gives the form of a , $\underline{u}$ for $\tau = d, c_0, k$.

The results are summarized in Table I. In Table I $\bar{N} := \Delta N$.

Table I

τ	a	$\underline{u}$
d	$\backslash Nv$	$\underline{v}$
c_0	$\backslash \bar{N}y$	$\underline{y}$
k	$\backslash N'y$	$\underline{I}'/k$

We now proceed to form higher derivatives of paraxial coefficients. We have

$$\frac{\partial v_k}{\partial \tau_j} = (a_j | v_k) u_j, \quad k \geq j, \quad (8.8)$$

and so

$$\frac{\partial^2 v'_k}{\partial \tau_i \partial \tau_j} = \frac{\partial}{\partial \tau_i} (a_j | v_k) u_j.$$

For $i \geq j$, $\frac{\partial a_j}{\partial \tau_i} = 0$ and $\frac{\partial u_j}{\partial \tau_i} = 0$, so that

$$\frac{\partial^2 v'_k}{\partial \tau_i \partial \tau_j} = \left(a_j | \frac{\partial v_k}{\partial \tau_i} \right) u_j. \quad (8.9)$$

However

$$\frac{\partial v'_{bk}}{\partial \tau_i} = \frac{u_{bi}}{u_{ai}} \frac{\partial v'_{ak}}{\partial \tau_i}. \quad (8.10)$$

Substituting (8.10) into (8.9) we obtain

$$\frac{\partial^2 v'_k}{\partial \tau_i \partial \tau_j} = (a_j | u_i) \frac{\partial v'_{ak}}{\partial \tau_i} \frac{u_j}{u_{ai}}, \quad \text{for } i \geq j. \quad (8.11)$$

Furthermore, for $\ell \geq i$

$$\frac{\partial^2 v'_{bk}}{\partial \tau_\ell \partial \tau_i} = \frac{u_{bi}}{u_{ai}} \frac{\partial^2 v'_{ak}}{\partial \tau_\ell \partial \tau_i},$$

and so, by a process similar to that above

$$\frac{\partial^3 v'_k}{\partial \tau_\ell \partial \tau_i \partial \tau_j} = (a_j | u_i) \frac{\partial^2 v'_{ak}}{\partial \tau_\ell \partial \tau_i} \frac{u_j}{u_{ai}}, \quad \text{for } \ell \geq i \geq j. \quad (8.12)$$

If $i_s \geq i_{s-1} \geq \dots \geq i_2 \geq i_1$, then

$$\frac{\partial^s v'_k}{\partial \tau_{i_s} \dots \partial \tau_{i_2} \partial \tau_{i_1}} = (a_{i_1} | u_{i_2}) \frac{\partial^{s-1} v'_{ak}}{\partial \tau_{i_s} \dots \partial \tau_{i_3} \partial \tau_{i_2}} \frac{u_{i_1}}{u_{ai_2}}. \quad (8.13)$$

For $\tau = d$, $a = \backslash Nv$ and $\underline{u} = \underline{v}$ which means that if any subscript is repeated, i.e. if $i_\ell = i_{\ell+1}$ then

$$(a_{i_\ell} | u_{i_\ell}) = \backslash N(v|v) = 0,$$

and so if any subscript is repeated the derivative is zero. A similar situation occurs for $\tau = c_0$.

The formulae above hold for mixed derivatives; care must then be taken if any subscript is repeated. For example, in the first order design of zoom lenses, it is necessary to calculate derivatives such as

$$\begin{aligned} \text{(i)} \quad & \frac{\partial^{s+1} v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_1} \partial c_{0j}} & j < i_1 \\ \text{(ii)} \quad & \frac{\partial^{s+1} v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_2} \partial c_{0j} \partial d_{i_1}} & j \geq i_1 \\ \text{(iii)} \quad & \frac{\partial^{s+1} v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_{s-t}} \partial c_{0j} \partial d_{i_{s-t-1}} \dots \partial d_{i_1}}, & i_{s-t} > j \geq i_{s-t-1}. \end{aligned}$$

These are the three possibilities which occur when calculating the derivative

$$\frac{\partial}{\partial c_{0j}} \frac{\partial^s v_{ak}}{\partial d_{i_s} \dots \partial d_{i_1}} .$$

Care must be taken to ensure that all possibilities have been accounted for. In the present case the three derivatives (i)-(iii) cover all possibilities. Using (8.13) these derivatives may be obtained.

9. The derivative of the augmented aberration

It is shown in L §§128,142,152 that in general,

$$\frac{\partial \hat{\varepsilon}'_k}{\partial \tau_j} = a_j(u_j | \hat{\varepsilon}) + \frac{\partial \hat{\varepsilon}}{\partial \varepsilon} \frac{\partial \varepsilon}{\partial \tau_j} + \frac{\partial \Delta}{\partial \tau_j} . \quad (9.1)$$

Certain additions of subscripts etc., must be made on the right hand side for different τ_j . These additions are

- (i) $\tau = d$: $\hat{\varepsilon}$ takes a subscript (kj). This notation is defined by $G_{(kj)} := G'_k - G_j$. The last term on the right must be omitted since Δ is explicitly independent of d_j .
- (ii) $\tau = c_0$: $\hat{\varepsilon}$ takes a subscript [kj]. This notation is defined by $G_{[kj]} := G'_k - G'_j$.
- (iii) $\tau = k$: $\hat{\varepsilon}$ takes a subscript [kj] and an antepime, and Δ an antepime.

Once the k derivatives have been calculated the derivatives with respect to refractive indices are obtained as follows. We write $J \equiv j+1$. Then we have

$$\begin{aligned}
\frac{\partial G'}{\partial N_J} &= \frac{\partial G'}{\partial k_j} \frac{\partial k_j}{\partial N_J} + \frac{\partial G'}{\partial k_{j+1}} \frac{\partial k_{j+1}}{\partial N_J} \\
&= \frac{1}{N_J} \left(-k_j \frac{\partial G'}{\partial k_j} + k_{j+1} \frac{\partial G'}{\partial k_{j+1}} \right) .
\end{aligned} \tag{9.2}$$

10. Derivatives of the third order aberration coefficients

From (9.1) we obtain, by equating coefficients of $\underline{S}_j \dots \underline{T}_j$, the derivatives of the third order aberration coefficients. Usually only the derivatives of 'a' coefficients are required. These are given by (see L §§128,142,152)

$$\begin{aligned}
\frac{\partial A'_{ak}}{\partial \tau} &= (3\theta_s - \bar{\theta}_t) A_a + \theta_t (A_b + \bar{A}_a + B_a) + \frac{\partial a_{=a}}{\partial \tau} \\
\frac{\partial \bar{A}'_{ak}}{\partial \tau} &= \bar{\theta}_s A_a + 2\theta_s \bar{A}_a + \theta_t (\bar{A}_b + \bar{B}_a) + \frac{\partial \bar{a}_{=a}}{\partial \tau} \\
\frac{\partial B'_{ak}}{\partial \tau} &= 2\bar{\theta}_s A_a + 2\theta_s B_a + \theta_t (B_b + \bar{B}_a + 2C_a) + \frac{\partial b_{=a}}{\partial \tau} \\
\frac{\partial \bar{B}'_{ak}}{\partial \tau} &= \bar{\theta}_s (2\bar{A}_a + B_a) + (\theta_s + \bar{\theta}_t) \bar{B}_a + \theta_t (\bar{B}_b + 2\bar{C}_a) + \frac{\partial \bar{b}_{=a}}{\partial \tau} \\
\frac{\partial C'_{ak}}{\partial \tau} &= \bar{\theta}_s B_a + (\theta_s + \bar{\theta}_t) C_a + \theta_t (C_b + \bar{C}_a) + \frac{\partial c_{=a}}{\partial \tau} \\
\frac{\partial \bar{C}'_{ak}}{\partial \tau} &= \bar{\theta}_s (\bar{B}_a + C_a) + 2\bar{\theta}_t \bar{C}_a + \theta_t \bar{C}_b + \frac{\partial \bar{c}_{=a}}{\partial \tau} .
\end{aligned} \tag{10.1}$$

As in §9 certain modifications must be made for the different τ_j . These are exactly the same as in §11. The formulae for

$$\frac{\partial g_{=a}^{(1)}}{\partial c_0} , \quad \frac{\partial g_{=a}^{(1)}}{\partial k}$$

are given in L §§137,147.

11. Derivatives of the fifth order aberration coefficients

(a) The derivatives of the fifth order aberration coefficients are not given in L. In the expression for these derivatives certain coefficients, which occur in the expressions of $\Gamma_s \dots \bar{\Gamma}_t$, are present. These coefficients $\alpha_s, \bar{\alpha}_s \dots \gamma_t, \bar{\gamma}_t$ are given by equation L (122.5). The derivatives of the fifth order contributions to the aberration coefficients also appear. These may be calculated by taking the derivatives of equations L (11.3) and equations L (218.7).

(b) The derivatives of the fifth order 'a' coefficients are then given by (see (9.2) and L §§129,143,153)

$$\begin{aligned} \frac{\partial s'_{1ak}}{\partial \tau} &= (5\theta_s - \bar{\theta}_t) s_{1a} + \theta_t(s_{1b} + \bar{s}_{1a} + s_{2a}) + 3\alpha_s A_a + \\ &+ \alpha_t(\bar{A}_a + B_a) + \frac{\partial s_{1a}}{\partial \tau} \end{aligned}$$

$$\begin{aligned} \frac{\partial \bar{s}'_{1ak}}{\partial \tau} &= \bar{\theta}_s s_{1a} + 4\theta_s \bar{s}_{1a} + (\bar{s}_{1b} + \bar{s}_{2a}) \theta_t + \bar{\alpha}_s A_a + \\ &+ (2\alpha_s + \bar{\alpha}_t) \bar{A}_a + \alpha_t \bar{B}_a + \frac{\partial \bar{s}_{1a}}{\partial \tau} \end{aligned}$$

$$\begin{aligned} \frac{\partial s'_{2ak}}{\partial \tau} &= 4\bar{\theta}_s s_{1a} + 4\theta_s s_{2a} + \theta_t(s_{2b} + \bar{s}_{2a} + 2s_{3a} + 2s_{4a}) + \\ &+ (2\bar{\alpha}_s + 3\beta_s) A_a + \beta_t \bar{A}_a + (2\alpha_s + \bar{\alpha}_t + \beta_t) B_a + \\ &+ \alpha_t(\bar{B}_a + 2C_a) + \frac{\partial s_{2a}}{\partial \tau} \end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{s}'_{2ak}}{\partial \tau} = & \bar{\theta}_s (4\bar{s}_{1a} + s_{2a}) + (3\theta_s + \bar{\theta}_t) \bar{s}_{2a} + \theta_t (\bar{s}_{2b} + 2\bar{s}_{3a} + 2\bar{s}_{4a}) + \\
& + \bar{\beta}_s A_a + (2\bar{\alpha}_s + 2\beta_s + \bar{\beta}_t) \bar{A}_a + \bar{\alpha}_s B_a + (\alpha_s + 2\bar{\alpha}_t + \beta_t) \bar{B}_a + \\
& + 2\alpha_t \bar{C}_a + \frac{\partial \bar{s}_{2a}}{\partial \tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{s}'_{3ak}}{\partial \tau} = & \bar{\theta}_s s_{2a} + (3\theta_s + \bar{\theta}_t) s_{3a} + \theta_t (s_{3b} + \bar{s}_{3a} + s_{5a}) + 3\gamma_s A_a + \\
& + \gamma_t \bar{A}_a + (\bar{\alpha}_s + \gamma_t) B_a + (\alpha_s + 2\bar{\alpha}_t) C_a + \alpha_t \bar{C}_a + \frac{\partial s_{3a}}{\partial \tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{s}'_{3ak}}{\partial \tau} = & \bar{\theta}_s (\bar{s}_{2a} + s_{3a}) + 2(\theta_s + \bar{\theta}_t) \bar{s}_{3a} + \theta_t (\bar{s}_{3b} + \bar{s}_{5a}) + \bar{\gamma}_s A_a + \\
& + (2\gamma_s + \bar{\gamma}_t) \bar{A}_a + (\bar{\alpha}_s + \gamma_t) \bar{B}_a + \bar{\alpha}_s C_a + 3\bar{\alpha}_t \bar{C}_a + \frac{\partial \bar{s}_{3a}}{\partial \tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{s}'_{4ak}}{\partial \tau} = & 2\bar{\theta}_s s_{2a} + (3\theta_s + \bar{\theta}_t) s_{4a} + \theta_t (s_{4b} + \bar{s}_{4a} + 2s_{5a}) + 2\bar{\beta}_s A_a + \\
& + (2\beta_s + \bar{\beta}_t) B_a + \beta_t (\bar{B}_a + 2C_a) + \frac{\partial s_{4a}}{\partial \tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{s}'_{4ak}}{\partial \tau} = & \bar{\theta}_s (2\bar{s}_{2a} + s_{4a}) + 2(\theta_s + \bar{\theta}_t) \bar{s}_{4a} + \theta_t (\bar{s}_{4b} + 2\bar{s}_{5a}) + \\
& + \bar{\beta}_s (2\bar{A}_a + B_a) + (\beta_s + 2\bar{\beta}_t) \bar{B}_a + 2\beta_t \bar{C}_a + \frac{\partial \bar{s}_{4a}}{\partial \tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{s}'_{5ak}}{\partial \tau} = & 2\bar{\theta}_s (s_{3a} + s_{4a}) + 2(\theta_s + \bar{\theta}_t) s_{5a} + \theta_t (s_{5b} + \bar{s}_{5a} + 4s_{6a}) + \\
& + 2\bar{\gamma}_s A_a + (\bar{\beta}_s + 2\gamma_s + \bar{\gamma}_t) B_a + \gamma_t \bar{B}_a + (\beta_s + 2\bar{\beta}_t + 2\gamma_t) C_a + \\
& + \beta_t \bar{C}_a + \frac{\partial s_{5a}}{\partial \tau}
\end{aligned}$$

$$\begin{aligned}
\frac{\partial \bar{s}'_{5ak}}{\partial \tau} &= \bar{\theta}_s (2\bar{s}_{3a} + 2\bar{s}_{4a} + s_{5a}) + (\theta_s + 3\bar{\theta}_t) \bar{s}_{5a} + \theta_t (\bar{s}_{5b} + 4\bar{s}_{6a}) + \\
&+ \bar{\gamma}_s (2\bar{A}_a + B_a) + (\bar{\beta}_s + \gamma_s + 2\bar{\gamma}_t) \bar{B}_a + \bar{\beta}_s C_a + (3\bar{\beta}_t + 2\bar{\gamma}_t) \bar{C}_a + \\
&+ \frac{\partial \bar{s}_{5a}}{\partial \tau} \\
\frac{\partial \bar{s}'_{6ak}}{\partial \tau} &= \bar{\theta}_s s_{5a} + (\theta_s + 3\bar{\theta}_t) s_{6a} + \theta_t (s_{6b} + \bar{s}_{6a}) + \bar{\gamma}_s B_a + \\
&+ (\gamma_s + 2\bar{\gamma}_t) C_a + \gamma_t \bar{C}_a + \frac{\partial s_{6a}}{\partial \tau} \\
\frac{\partial \bar{s}'_{6ak}}{\partial \tau} &= \bar{\theta}_s (\bar{s}_{5a} + s_{6a}) + 4\bar{\theta}_t \bar{s}_{6a} + \theta_t \bar{s}_{6b} + \bar{\gamma}_s (\bar{B}_a + C_a) + 3\bar{\gamma}_t \bar{C}_a + \\
&+ \frac{\partial \bar{s}_{6a}}{\partial \tau} .
\end{aligned} \tag{11.1}$$

Once again modifications, as in §9, must be made according to whether τ is d , c_0 or k .

12. The DERIVATIVES programme

A computer programme, written in FORTRAN, and called DERIVATIVES, has been prepared to calculate the derivatives of the third and fifth order monochromatic coefficients of orders one and two. The derivatives are calculated with respect to the parameters d_j , c_{0j} , k_j , N_j , c_{1j} and c_{2j} at each surface of an optical system. The programme utilizes the results obtained in Part III of L and summarized in §§7-11 above. A listing of programme DERIVATIVES is contained in Appendix 2.

DERIVATIVES consists of a MAIN programme and several subprogrammes. These subprogrammes are listed below, along with their functions.

(i) COEFF. This subroutine calculates the 'a' and 'b' aberration coefficients of orders three and five for a system of spherical surfaces. COEFF itself calls several subprogrammes.

(a) PARAX. This subroutine uses the linear relations of paraxial optics to calculate the paraxial coefficients at the j^{th} surface given the paraxial coefficients at the $j-1^{\text{th}}$ surface.

(b) PRIM. The primary intrinsic contributions at the j^{th} surface are calculated in PRIM using equations (5.2) for the 'a' coefficients. Input to PRIM is the paraxial coefficients at the j^{th} surface.

(c) SECIN. In this subroutine the fifth order intrinsic contributions to the aberration coefficients are calculated. The formulae used are those of L §218. Only formulae for the 'a' coefficients are used.

(d) EXTRA. Once the intrinsic contributions are calculated, it is necessary to use equations L (11.3) to calculate the actual contributions to the fifth order aberration coefficients. The extra terms needed are calculated in subroutine EXTRA.

Only the formulae for the 'a' coefficients (paraxial, third and fifth orders) are programmed. The 'b' coefficients are calculated by entering dual data into the various subroutines, as explained in §6. The results of the calculations of COEFF are stored and returned to MAIN.

(ii) COLOUR. This subroutine uses equations L (93.2) and L (98.2 - 4) to calculate the chromatic paraxial aberration coefficients

of chromatic order one and two. There is an option to exclude all calculations related to chromatic coefficients and their derivatives.

(iii) THETA. Equations (8.3) are executed and values of $\theta_s \dots \bar{\theta}_t$ are returned to MAIN.

(iv) PDERIV. Data to PDERIV are $\theta_s \dots \bar{\theta}_t$ which have been calculated in THETA. These are used in PDERIV to calculate the derivatives of the primary monochromatic coefficients using equations (10.1). The derivatives of the intrinsic contributions $\partial_{\underline{a}}/\partial\tau$, etc., are calculated in MAIN and added to the output of PDERIV.

(v) ALFA. This calculates the coefficients $\alpha_s \dots \bar{\gamma}_t$ mentioned in §11 and needed for calculating the derivatives of the fifth order aberration coefficients. Equations L (122.5) are used. The derivatives of the primary intrinsic coefficients needed are added in MAIN.

(vi) SDERIV. This executes equations (11.1) except for the derivatives of the contributions to the fifth order aberration coefficients. These are added in MAIN.

(vii) BITS. The derivatives of the fifth order intrinsic contributions are calculated in MAIN. To these we must add other terms calculated by taking the derivatives of equations L (11.3). This is done in BITS and the results are returned to MAIN.

(viii) CDERIV. This calculates the derivatives of the chromatic paraxial aberration coefficients of chromatic orders one and two with respect to separations, curvatures, refractive indices and dispersions. This subroutine calls several other subroutines.

(a) THETA1. This calculates the coefficients $\theta_s \dots \bar{\theta}_t$ given by equations L (164.5) contained in Part II of L. The $\theta_s \dots \bar{\theta}_t$ are $\theta_{1s} \dots \bar{\theta}_{1t}$ are $\theta_{0s} \dots \bar{\theta}_{0t}$

the $\theta_s \dots \bar{\theta}_t$ calculated for monochromatic light by subroutine THETA.

(b) COLDER. Equations L (166.4) are executed. These can be used for derivatives with respect of d_j, c_j, k_j if the correct surface contribution derivatives are added in CDERIV.

The derivatives with respect to refractive indices and dispersions are calculated in CDERIV.

(ix) PRINT. When all derivatives (for chromatic and monochromatic aberration coefficients) have been calculated they are transferred to PRINT which controls the printing of results.

The calculation of derivatives with respect to refractive indices is performed in MAIN using refractance derivatives as explained in §9. The calculation of the derivatives of the monochromatic aberration coefficients with respect to the extra-axial curvatures c_{1j}, c_{2j} is performed largely in MAIN using equations L (157.2) and L (161.1). The coefficients $\alpha_s \dots \bar{\gamma}_t$ needed in $\partial s'_{1ak} / \partial c_{ij}$, etc., are calculated using L (156.1). These are then used as data for SDERIV. The derivatives $\partial s_{1a} / \partial c_1$, etc., are calculated in MAIN using the following equations for the derivatives of the fifth order intrinsic contributions.

$$\frac{\partial s_{1a}}{\partial c_1} = \frac{1}{2}k_1 + \frac{1}{4}k_2 [c_0 y_a (7 - 8k) - v'_a] + \frac{1}{2}k_3 (3k + 1)$$

$$\begin{aligned} \frac{\partial \bar{s}_{1a}}{\partial c_1} = & \frac{1}{2}\tilde{q} k_1 + k_2 [c_0 y_a (1 - k) \tilde{q} + \{(3 - 4k) c_0 y_a - v'_a\} q] + \\ & + k_3 [kq + \frac{1}{2}(1 + k) \tilde{q}] \end{aligned}$$

$$\frac{\partial \bar{s}_{=2a}}{\partial c_1} = 2\tilde{q} k_1 + k_2 [c_0 y_a (1 - k) q + \{(6 - 7k) c_0 y_a - v'_a\} \tilde{q}] +$$

$$+ k_3 [(4k + 1) \tilde{q} + (2k + 1) q]$$

$$\frac{\partial \bar{s}_{=2a}}{\partial c_1} = 2\tilde{q}^2 k_1 + \tilde{q} k_2 [3c_0 y_a (1 - k) \tilde{q} + \{(4 - 5k) c_0 y_a - v'_a\} q] +$$

$$+ k_3 [(1 + k) \tilde{q}^2 + (4k + 1) q\tilde{q} + k q^2]$$

$$\frac{\partial \bar{s}_{=3a}}{\partial c_1} = \tilde{q}^2 k_1 + \tilde{q} k_2 [c_0 y_a (1 - k) q + \frac{1}{2}\{(5 - 6k) c_0 y_a - v'_a\} \tilde{q}] +$$

$$+ \frac{1}{2} k_3 [(3k + 1) \tilde{q}^2 + 2kq\tilde{q} + (1 + k) q^2]$$

$$\frac{\partial \bar{s}_{=3a}}{\partial c_1} = \tilde{q}^3 k_1 + \tilde{q}^2 k_2 [c_0 y_a (1 - k) \tilde{q} + \frac{1}{2}\{(5 - 6k) c_0 y_a - v'_a\} q] +$$

$$+ \frac{1}{2}\tilde{q} k_3 [(1 + k) \tilde{q}^2 + 2kq\tilde{q} + (3k + 1) q^2]$$

$$\frac{\partial \bar{s}_{=4a}}{\partial c_1} = 2\tilde{q}^2 k_1 + \tilde{q} k_2 [2c_0 y_a (1 - k) q + \{(5 - 6k) c_0 y_a - v'_a\} \tilde{q}] +$$

$$+ 2\tilde{q} k_3 [k\tilde{q} + (2k + 1) q]$$

$$\frac{\partial \bar{s}_{=4a}}{\partial c_1} = 2\tilde{q}^3 k_1 + \tilde{q}^2 k_2 [2c_0 y_a (1 - k) \tilde{q} + \{(5 - 6k) c_0 y_a - v'_a\} q] +$$

$$+ 2q\tilde{q} k_3 [(2k + 1) \tilde{q} + kq]$$

$$\frac{\partial \bar{s}_{=5a}}{\partial c_1} = 2\tilde{q}^3 k_1 + \tilde{q}^2 k_2 [3c_0 y_a (1 - k) q + \{(4 - 5k) c_0 y_a - v'_a\} \tilde{q}] +$$

$$+ \tilde{q} k_3 [k\tilde{q}^2 + (4k + 1) q\tilde{q} + (k + 1) q^2]$$

$$\frac{\partial \bar{s}_{=5a}}{\partial c_1} = 2\tilde{q}^4 k_1 + \tilde{q}^3 k_2 [c_0 y_a (1 - k) \tilde{q} + \{(6 - 7k) c_0 y_a - v'_a\} q] +$$

$$+ q\tilde{q}^2 [(2k + 1) \tilde{q} + (4k + 1) q]$$

$$\frac{\partial s_{6a}}{\partial c_1} = \frac{1}{2}\tilde{q}^4 k_1 + \tilde{q}^3 k_2 [c_0 y_a (1 - k) q + \frac{1}{4}[(3 - 4k) c_0 y_a - v'_a] \tilde{q}] + \\ + \tilde{q}^3 k_3 [\frac{1}{2}(k + 1) q + k \tilde{q}]$$

$$\frac{\partial \bar{s}_{6a}}{\partial c_1} = \frac{1}{2}\tilde{q}^5 k_1 + \frac{1}{4}q\tilde{q}^4 k_2 [c_0 y_a (7 - 8k) - v'_a] + \frac{1}{2}q^2\tilde{q}^3 k_3 (3k + 1) , \quad (12.1)$$

where

$$k_1 = \bar{N} c_0 i'_a y_a^5$$

$$k_2 = \bar{N} y_a^4 i_a$$

$$k_3 = \bar{N} k y_a^4 i_a^2$$

and $\tilde{q} = y_b/y_a$, $q = i_b/i_a$.

Equations (12.1) are derived from equation L (156.3) but are not given in L. The derivatives of the contributions to the fifth order aberration coefficients are obtained using equations (12.1) and the derivatives of equations L (11.3).

Instructions for the use of DERIVATIVES are contained in the listing in Appendix 2.

PART II

THE THEORY AND DESIGN OF ZOOM LENSES

CHAPTER 3

THE ZOOM LENS

13. Introduction. The theory and design of zoom lenses

In chapters 3-6 below we study the paraxial and third order theory of zoom lenses and apply this theory to the problem of the three element zoom lens.

A zoom lens is an optical system able to produce a corrected image, with a variable magnification, in a fixed plane. This is achieved by moving the constituent elements relative to each other and the image plane.

The paraxial theory is based on the axial separation derivatives of the final paraxial coefficients v'_{ak} and y_{ak} . We can use this approach because the derivatives of all orders of these quantities can be written down in a simple form. This is not so for the derivatives of the primary aberration coefficients so we therefore consider formulae for the 'addition' of optical systems and consider zoom lenses as systems of movable thin elements.

This approach gives the primary aberration coefficients as functions of a simple zoom parameter. These functions are built up from certain simple polynomials in the zoom parameter. These polynomials, which are related to the intermediate paraxial

coefficients can be calculated quite easily, especially for the three element zoom lens.

From the formulae for the primary aberration coefficients we can obtain simultaneous equations which, in the case of the three element zoom lens, may be solved without recourse to optimization. All possible zoom lenses, with the characteristics demanded by the designer, can therefore be obtained.

14. Properties of the zoom lens

A zoom lens is an optical system which produces, in a fixed plane, a corrected image whose magnification is continuously variable in a chosen range. Conventionally this variation of magnification is produced by the movement of the elements of the system relative to one another. In this work 'element' will mean a single component of a system, whereas 'lens' will, in general, refer to the whole system. Lohmann [3] has described a new class of zoom lens which utilizes lateral movements of non-rotationally symmetric elements, but in this work zoom lens will be taken to refer to the conventional type of system.

Zoom lenses have been placed in one of two categories depending on how the first order behaviour of the image is controlled. The ideal image produced by a zoom lens is not stationary, that is, either the ideal image plane moves, or the image height varies (in a fixed plane) as the zooming process occurs. The control of the image is called image compensation. There are two types of image compensation, namely

- (i) mechanical image compensation
- (ii) linear image compensation.

In mechanical compensation the movements of the elements are controlled by a system of cams in such a way that the elements move non-linearly with respect to each other, that is, the positions of the elements are not linearly related to each other. A more descriptive name for this type of image compensation would be non-linear image compensation.

In linear compensation the movements of the elements are linearly related to each other. This means that if one particular element is in a position characterized by a parameter z_1 , then the position of a second element would be characterized by

$$z_2 = a_2 + b_2 z_1, \quad (14.1)$$

where a_2 and b_2 are constants. A particularly simple case of linear compensation is called optical compensation. In this alternate elements are fixed relative to one another and move between the remaining elements which are fixed relative to the image plane. In this case (14.1) becomes

$$z_2 = a \pm z_1. \quad (14.2)$$

It is linear compensation, and specifically optical compensation, that this work deals with. It must be emphasized, however, that the methods are completely general and may be applied to non-linear compensation.

A complete zoom lens may be considered to consist of two parts:

(i) That part whose elements move. This part produces the zoom effect and we shall denote it by $\textcircled{Z}$. The terms zoom system or zoom lens will generally refer to $\textcircled{Z}$. The aberrations of $\textcircled{Z}$ will vary during the zooming process. One of the designer's aims will be to control this variation.

(ii) A part whose elements are fixed relative to each other. This we denote by $\textcircled{K}$. $\textcircled{K}$ is designed to have aberrations which balance the average values of the aberrations of $\textcircled{Z}$. The properties of $\textcircled{K}$ are chosen so that the whole lens will have a focal length which varies between two specified values. There will be an aperture stop somewhere between $\textcircled{Z}$ and $\textcircled{K}$ (or in $\textcircled{K}$). This ensures that the brightness in the final image is constant as zooming occurs. We will not deal further with $\textcircled{K}$.

15. Design considerations for zoom lenses

In designing a zoom lens one must take into account several factors other than those considered in designing lenses of fixed focal length.

In an ordinary design one aims to produce an optical system with a specified focal length and image distance and with a particular state of correction of aberrations at the numerical aperture and field angle for which the system is to work. Cost and the size of the lens must be taken into account.

For a zoom lens the magnification, and hence the focal length, is variable. Although the aperture of the stop is constant, the entrance pupil aperture and the field angle vary. Taking these

variations into account, the designer must produce a lens with a satisfactory state of correction of aberrations for all positions of the movable elements. The ratio of the maximum magnification to the minimum magnification must have a particular chosen value. The maximum (or minimum) focal length must have a particular chosen value. The change in the length of (Z) must be as small as possible since a large change in the length of (Z) will increase the length of the whole lens. Cost must be taken into account and this will determine the number of elements to be used and whether aspheric surfaces may be employed.

16. Usual methods of design

Several approaches to first order design are described in the literature. The first zoom lenses used mechanical compensation. Yamaji [4] describes first order design by discussing several classes of zoom lenses. The more primitive types employ two movable elements; one to vary magnification and one to compensate for the resulting movement of the image plane. If the first movement is a simple displacement then it is necessary for the second movement to be quite complicated. This necessitates use of cams to control the movements of the elements. Wear in these cams causes deterioration of the performance of the lens.

A small number of elements allows control over relatively few aberrations. Each element added to control aberrations must be controlled by a cam thus adding to the complexity of the lens. Apart from difficulties in producing cams with sufficient wear reducing properties, the problem with previous methods for design of

mechanically compensated systems is that there is no one design method applicable for all problems. Yamaji also considers linear and optical compensation, but better discussions of these are contained elsewhere.

Bergstein [5] and Bergstein and Motz [6,7,8] consider the problem of optical compensation in general with emphasis on certain specific cases, namely the two, three and four element zoom lenses.

Writing F_v for the focal length of the v^{th} element and X_v , X'_v for the v^{th} object and image distances measured from the respective focal points, Bergstein uses the Newtonian image equation

$$X'_v = - F_v^2 / X_v, \quad v = 1 \dots n,$$

to produce a terminating continued fraction which expresses the final image distance in terms of the focal lengths and separations of the system. He shows that if n is the number of elements, the deviation of the image plane from the ideal image is zero for n positions of the elements. Bergstein produces a set of 'varifocal equations' which, when solved, give the values of the focal lengths and separations necessary to produce a lens with the chosen properties.

Jamieson [9] considers optical compensation but points out that the proposed method of design is applicable to linear compensation. The arrangement of the elements at a particular position during the zoom process may be specified by a zoom parameter z . Jamieson considers a ratio of polynomials in z (the numerator polynomial being of degree n) for the image position. If this ratio of polynomials is decomposed into a terminating continued fraction, then, taking into account subsidiary equations involving the ratio of the maximum and minimum magnifications, the possible first order solutions of the problem are obtained.

Other discussions of this subject are presented by Pegis and Peck [10], Wooters and Silvertooth [11] and Back and Lowen [12].

Discussions of the problem of control of aberrations are given by Yamaji [4], Bergstein and Motz [13] and Jamieson [9]. Jamieson uses the aberration formulae of Wynne [14] and by a simple change of variables shows that each aberration may be expressed as a quadratic form in the shape factors

$$X_i = c_{1i} + c_{2i}, \quad i = 1 \dots n,$$

where

$$c_{1i} = \text{first curvature of the } i^{\text{th}} \text{ lens}$$

$$c_{2i} = \text{second curvature of the } i^{\text{th}} \text{ lens}.$$

The coefficients of X_i and X_i^2 depend only on the first order design and the zoom parameter z . By setting each third order aberration equal to a desired value at chosen values of z , it is possible to control the third order performance of the lens. The points at which the aberrations have the chosen values are called the points of full compensation for the aberration in question. Jamieson applies this method to the design of three and four element systems. The systems of non-linear simultaneous equations obtained are solved using a least squares optimization method with a computer.

17. Proposed theory of zoom lenses

The design of a zoom system entails the assignment of values to the parameters describing the system. For the first order design these parameters are the powers (or focal lengths) of the elements, the initial separations of the elements and the initial distance of

the ideal image plane from the last element. For the control of the aberrations the available parameters are the shapes of the elements, the refractive indices of glasses and any extra-axial curvatures allowed.

The rates of change of the spacings of the elements (the coefficients b_i in (14.1)) can also be variables for use in design. If this is so the first and higher order design procedures cannot be carried out independently. Although this poses no problem in computer design, it does present difficulties when analytic solutions are sought. Because of this rates of change of spacings will not be considered as variables.

The variables mentioned must satisfy certain constraining relations. For example, it has been stated that the ratio of the maximum and minimum magnifications must be a chosen number. If we define the relative magnification operating range as

$$R := \frac{m_{\max}}{m_{\min}}, \quad (17.1)$$

where $m_{\max}$ and $m_{\min}$ are the maximum and minimum magnifications respectively, then this is one constraint. The points at which the displacement of the ideal image plane from the chosen image is zero are called the points of full image shift compensation, or simply the points of full compensation. There are n of these points for a system consisting of n elements. The zoom range is the range of values of z which is encompassed by the zoom process. The requirement that all n points of full compensation lie in the zoom range gives rise to n constraints. Finally, it is usually required that the length of the lens be a minimum. This requires that each separation be a minimum. From this we obtain a further $n-1$ constraints. The minimum length

system is thus described by $2n$ equations in $2n$ variables (n powers, $n-1$ separations and the initial image distance).

The shape of a thin lens is

$$S := \frac{c_1 + c_2}{c_1 - c_2} . \quad (17.2)$$

Specification of power and shape gives the curvatures of the thin lens.

These are

$$c_1 = \frac{(S+1)\phi}{2(N-1)} , \quad c_2 = \frac{(S-1)\phi}{2(N-1)} , \quad (17.3)$$

where ϕ is the power of the lens and

$$\phi = (N-1)(c_1 - c_2) , \quad (17.4)$$

for a thin lens.

The refractive indices of the elements of a zoom lens can always be chosen so that the Petzval curvature is zero or some small residual. This leaves only the shapes to be allocated (if we are considering elements consisting of spherical surfaces). There are n shapes and so n conditions may be satisfied. For instance a particular aberration might be set to a chosen value at n points or the aberration and its derivative may be set equal to chosen values at $n/2$ points. Obviously there are many different conditions the designer might wish his system to satisfy.

The theory of zoom lenses will be described as follows. The criterion of first order stability will be the variation of the intersection height of an 'a' ray with the initial ideal image plane. This is simpler than considering the variation of the position of the ideal image plane.

As we shall always be dealing with lens elements rather than surfaces, a subscript i will denote the i^{th} element rather than the i^{th} surface as in L. A subscript n will denote the last element. A primed symbol will denote the quantity represented after the element in question. As in L intersection heights will be denoted by y and ray angles by v . The $n-1$ initial separations between the elements will be denoted by $d_1, d_2, \dots, d_{n-1}$ and the initial image distance will be denoted by d_n . The ideal image plane will mean the initial ideal image plane and the intersection height of a ray in this plane will be simply y' . In this notation the intersection height of an 'a' ray with the ideal image plane is initially

$$y'_a = y_{an} - d_n v'_{an} . \quad (17.5)$$

We shall use a zoom parameter z to specify the configuration of the zoom lens at any particular time. If Z is the displacement of a chosen lens from its initial position and if Z_{max} is the largest allowed value of Z then

$$z := Z/Z_{\text{max}} . \quad (17.6)$$

All distances will be scaled in this manner, i.e. by division by Z_{max} . These distances are said to be normalized and the resulting system is called the normal system. All the theory contained herein relates to the normal system.

$g(z)$ will denote the quantity g when the system is in the configuration specified by z . In general, (17.5) may thus be written as

$$y'_a(z) = y_{an}(z) - d_n(z) v'_{an}(z) . \quad (17.7)$$

For a system employing linear compensation we write

$$d_i(z) = d_i + a_i z, \quad i = 1, \dots, n. \quad (17.8)$$

Since $y_{an}(z)$ and $v'_{an}(z)$ depend linearly on each of the $d_i(z)$ ($i = 1, \dots, n-1$), both of these are $(n-1)^{\text{th}}$ degree polynomials in z . Because of this we find that $y'_a(z)$ is an n^{th} degree polynomial in z . It thus has n zeros and by suitable choice of the parameters of the zoom lens we may constrain all of these zeros to lie in the zoom range. This is the basis of the first order design.

CHAPTER 4
THE FIRST ORDER THEORY OF ZOOM LENSES
EMPLOYING LINEAR IMAGE COMPENSATION

18. The variation of first order image height

The intersection height of an 'a' ray with the ideal image plane is given by (17.7) as

$$y'_a(z) = y_{an}(z) - d_n(z) v'_{an}(z) . \quad (18.1)$$

$y_{an}(z)$, $v'_{an}(z)$ are $(n-1)^{th}$ degree polynomials in z and so

$$\left. \begin{aligned} y_{an}(z) &= \sum_{j=0}^{n-1} \frac{1}{j!} \left(\frac{\partial^j y_{an}(z)}{\partial z^j} \right)_{z=0} z^j \\ v'_{an}(z) &= \sum_{j=0}^{n-1} \frac{1}{j!} \left(\frac{\partial^j v'_{an}(z)}{\partial z^j} \right)_{z=0} z^j \end{aligned} \right\} \quad (18.2)$$

From (18.1) it follows that

$$y'_a(z) = \sum_{j=0}^{n-1} \frac{1}{j!} \left[\left(\frac{\partial^j y_{an}(z)}{\partial z^j} \right)_{z=0} - d_n(z) \left(\frac{\partial^j v'_{an}(z)}{\partial z^j} \right)_{z=0} \right] z^j . \quad (18.3)$$

Write

$$\left. \begin{aligned} \frac{1}{j!} \left(\frac{\partial^j y_{an}(z)}{\partial z^j} \right)_{z=0} &= y_{an}^{(j)} \\ \frac{1}{j!} \left(\frac{\partial^j v'_{an}(z)}{\partial z^j} \right)_{z=0} &= v'_{an}{}^{(j)} \end{aligned} \right\} , \quad (18.4)$$

and in this notation (18.3) becomes

$$y'_a(a) = \sum_{j=0}^{n-1} \left[y_{an}^{(j)} - d_n(z) v'_{an}{}^{(j)} \right] z^j . \quad (18.5)$$

Once the quantities $y_{an}^{(j)}$, $v_{an}^{(j)}$ ($j = 0, \dots, n-1$) are known $y_a'(z)$ is determined. These quantities can be obtained as follows. We consider only $v_{an}'(z)$. Since there are $n-1$ separations we have

$$\frac{\partial v_{an}'(z)}{\partial z} = \sum_{i=1}^{n-1} \frac{\partial d_i(z)}{\partial z} \frac{\partial v_{an}'(z)}{\partial d_i(z)}, \quad (18.6)$$

where, from (17.8), $\left(\frac{\partial d_i(z)}{\partial z} \right)_{z=0} = a_i$. Therefore

$$\left(\frac{\partial v_{an}'(z)}{\partial z} \right)_{z=0} = \sum_{i=0}^{n-1} a_i \frac{\partial v_{an}'}{\partial d_i}, \quad (18.7)$$

since $d_i = d_i(0)$ and $v_{an}' = v_{an}'(0)$. Assuming linear compensation, taking the derivative of (18.6) gives

$$\frac{\partial^2 v_{an}'(z)}{\partial z^2} = \sum_{\ell=1}^{n-2} \sum_{i=\ell+1}^{n-1} \frac{\partial d_i(z)}{\partial z} \frac{\partial d_\ell(z)}{\partial z} \frac{\partial^2 v_{an}'(z)}{\partial d_i(z) \partial d_\ell(z)}$$

or
$$\left(\frac{\partial^2 v_{an}'(z)}{\partial z^2} \right)_{z=0} = \sum_{\ell=1}^{n-2} \sum_{i=\ell+1}^{n-1} a_i a_\ell \frac{\partial^2 v_{an}'}{\partial d_i \partial d_\ell}.$$

This process may be continued to give

$$\begin{aligned} \left(\frac{\partial^j v_{an}'(z)}{\partial z^j} \right)_{z=0} &= \sum_{i_1=1}^{n-j} \sum_{i_2=i_1+1}^{n-j+1} \dots \sum_{i_j=i_{j-1}+1}^{n-1} a_{i_1} a_{i_2} \dots a_{i_j} \times \\ &\times \frac{\partial^j v_{an}'}{\partial d_{i_1} \partial d_{i_2} \dots \partial d_{i_j}}. \end{aligned} \quad (18.8)$$

If (18.8) is written out in full it is seen that it is merely the weighted sum of all separation derivatives of differential order j . These separation derivatives are readily calculated using (9.13), while making sure that only air space separations are considered. Given a paraxial ray trace through the zoom lens in its initial

configuration it is possible to write $y'_a(z)$ as a polynomial with known coefficients.

19. The magnification range of a zoom lens

According to L (5.103) the magnification of a system of n elements is

$$m = \frac{N_1 v_{p1}}{N'_n v'_{pn}}, \quad (19.1)$$

and so $m_{\max}$ corresponds to the minimum value of $v'_{pn}(z)$. Therefore

$$R = \frac{(v'_{pn})_{\max}}{(v'_{pn})_{\min}}. \quad (19.2)$$

The number r is defined by

$$r := \frac{v'_{pn}(1)}{v'_{pn}(0)}. \quad (19.3)$$

If the maximum value of $v'_{pn}(z)$ occurs at $z = 1$ then

$$r = \frac{v'_{pn}(1)}{v'_{pn}(0)} = \frac{(v'_{pn})_{\max}}{(v'_{pn})_{\min}} = R. \quad (19.4a)$$

If the maximum value of $v'_{pn}(z)$ occurs at $z = 0$ then

$$r = \frac{v'_{pn}(1)}{v'_{pn}(0)} = \frac{(v'_{pn})_{\min}}{(v'_{pn})_{\max}} = \frac{1}{R}. \quad (19.4b)$$

To obtain a lens with a particular value of R we may have either $r = R$ or $r = \frac{1}{R}$.

From (18.4), r is given by

$$r = \frac{v'_{an}(0) + v'_{an}(1) + \dots + v'_{an}(n-1)}{v'_{an}(0)} . \quad (19.5)$$

20. Control of first order image height

We are going to place constraints on the variables ϕ_i , d_i in such a way that all n zeros of $y'_a(z)$ are real and lie in the interval $0 \leq z \leq 1$. To do this we write

$$y'_a(z) = \delta(z - z_1)(z - z_2) \dots (z - z_n) . \quad (20.1)$$

δ is a scale factor chosen so that the left hand side of (20.1) equals its right hand side. The numbers z_i ($i = 1, \dots, n$) are chosen by the designer in the range $0 \leq z_i \leq 1$. (20.1) may be rewritten as

$$y'_a(z) = \delta \sum_{k=0}^n (-1)^k \gamma_{n-k}(n) z^k . \quad (20.2)$$

The function $\gamma_k(n)$ of the zeros z_i is defined as the sum of all cross products of k roots, e.g. for $n = 4$

$$\begin{aligned} \gamma_1(4) &= z_1 + z_2 + z_3 + z_4 \\ \gamma_2(4) &= z_1 z_2 + z_1 z_3 + z_1 z_4 + z_2 z_3 + z_2 z_4 + z_3 z_4 \\ \gamma_3(4) &= z_1 z_2 z_3 + z_1 z_2 z_4 + z_1 z_3 z_4 + z_2 z_3 z_4 \\ \gamma_4(4) &= z_1 z_2 z_3 z_4 . \end{aligned} \quad (20.3)$$

$\gamma_0(n)$ is defined to be unity. (20.2) is now the desired form of $y'_a(z)$ whilst (18.5) is the actual form. Equating (20.2) to (18.5) we obtain

$$y_{an}^{(0)} - d_n v_{an}'^{(0)} = \delta \gamma_n(n) , \quad (20.4)$$

$$y_{an}^{(j)} - d_n v_{an}'^{(j)} - a_n v_{an}'^{(j-1)} = (-1)^j \delta \gamma_{n-j}(n) , \quad (20.5)$$

for $j = 1, 2, \dots, n-1$, while for $j = n$

$$\delta = (-1)^{n+1} a_n v_{an}'^{(n-1)} . \quad (20.6)$$

At $z = 0$ we have that $y_a'(0) = 0$ and so $\gamma_n(n) = 0$. (20.4)

then becomes

$$y_{an}^{(0)} - d_n v_{an}'^{(0)} = 0 .$$

(20.5) and (20.4) constitute n constraints which must be satisfied by the variables ϕ_i and d_i . Equation (19.5) is one further constraint. When the powers and separations are chosen in such a way that (19.5), (20.4) and (20.5) are satisfied, the lens will have a relative operating range of R and the variation of the image height will have zeros at the chosen values of z .

If the conditions

$$d_i(0) = (d_i)_{\min} , \quad i = 1, \dots, n-1 , \quad (20.7)$$

are also imposed on the variables then a minimum length system will be obtained.

21. Choice of the points of full compensation

The variation of the image height as given by (20.1) assumes that the radius of the entrance pupil is constant. In general this is

not the case, and it will be shown later than the radius of the entrance pupil is given by a polynomial of degree n . For the present, however, we shall assume that the radius of the entrance pupil can be approximated by a linear function of z . It is shown in §27 that the pupil magnification range is the same as the image magnification range and so if the radius of the entrance pupil is h_0 when $z = 0$, then the radius of the entrance pupil may be approximated by

$$h(z) = h_0 \left[1 + \left(\frac{1}{r} - 1 \right) z \right]. \quad (21.1)$$

Using this approximation, the actual variation of the image height is

$$y'_a(z) = \frac{r\delta}{h_0} \frac{(z-z_1)(z-z_2) \dots (z-z_n)}{[r + (1-r)z]}. \quad (21.2)$$

If $z_1 \dots z_n$ are chosen, initially, to be equally spaced, (21.2) may be graphed (ignoring the scale factor $\frac{r\delta}{h_0}$) and from this graph it is easily seen how the compensation points should be shifted to obtain balanced variation of the image height. The ideal result is to have the same value for the image height variation at each extremum of $y'_a(z)$.

22. The two element zoom lens

Cruickshank [15] shows that for thin lenses (i.e. lenses of zero thickness)

$$\begin{aligned} v_{ai} &= \phi_i y_{ai} + v_{ai} \\ y_{ai+1} &= y_{ai} - d_i v'_{ai}. \end{aligned} \quad (22.1)$$

ϕ_i is the power of the i^{th} lens and d_i is the separation between the

i^{th} and $i+1^{\text{th}}$ lenses. These transfer equations may be used to obtain the coefficients in the polynomials for $y_{a2}(z)$ and $v'_{a2}(z)$. If we consider a lens with object at infinity, we may set $y_{a1} = 1$ and $v_{a1} = 0$. Repeated use of (22.1) gives

$$\begin{aligned} y_{a2}^{(0)} &= 1 - d_1 \phi_1 \\ y_{a2}^{(1)} &= -a_1 \phi_1 \\ v_{a2}^{(0)} &= \phi_1 + \phi_2 - d_1 \phi_1 \phi_2 \\ v_{a2}^{(1)} &= -a_1 \phi_1 \phi_2 \end{aligned} \quad (22.2)$$

Substituting from (22.2) into (20.4), (20.5) and (19.5) gives the following results. From (20.4)

$$(1 - d_1 \phi_1) - d_2(\phi_1 + \phi_2 - d_1 \phi_1 \phi_2) = 0. \quad (22.3)$$

From (20.5)

$$a_1 \phi_1 + d_2 a_1 \phi_1 \phi_2 - d_2(\phi_1 + \phi_2 - d_1 \phi_1 \phi_2) = -\delta \gamma_1, \quad (22.4)$$

where $\delta = a_1 a_2 \phi_1 \phi_2$ and $\gamma_1 = \gamma_1(2)$. From (19.5)

$$-a_1 \phi_1 \phi_2 = (r - 1)(\phi_1 + \phi_2 - d_1 \phi_1 \phi_2). \quad (22.5)$$

For a minimum length system $d_1 = 0$. In practice we require $d_1 > 0$, for example, $d_1 = 0.1$.

It is possible to solve (22.3-5) directly for ϕ_1 , ϕ_2 and d_2 . The problem becomes simpler if we make the substitutions

$$\begin{aligned} d_1 &= s_1 + \frac{1}{\phi_1} + \frac{1}{\phi_2} \\ d_2 &= s_2 + \frac{1}{\phi_2} \end{aligned}$$

Equations (21.2) become

$$\begin{aligned}
 y_{a2}^{(0)} &= -\phi_1 \left(s_1 + \frac{1}{\phi_2} \right) \\
 y_{a2}^{(1)} &= -a_1 \phi_1 \\
 v_{a2}^{(0)} &= -s_1 \phi_1 \phi_2 \\
 v_{a2}^{(1)} &= -a_1 \phi_1 \phi_2 ,
 \end{aligned} \tag{22.6}$$

and substitution into (20.4), (20.5) and (19.5) gives

$$s_1 s_2 - \phi_1 / \phi_2 = 0 , \tag{22.7}$$

$$a_1 (s_2 - s_1) \phi_1 \phi_2 = -\delta \gamma_1 , \tag{22.8}$$

$$s_1 (1 - r) + a_1 = 0 . \tag{22.9}$$

The minimum length condition gives the further equation

$$s_1 + \frac{1}{\phi_1} + \frac{1}{\phi_2} = (d_1)_{\min} \tag{22.10}$$

where $(d_1)_{\min}$ is the chosen minimum value of d_1 (for example $d_1 = 0.1$).

The solution of equations (21.7-10) is

$$\begin{aligned}
 s_1 &= a_1 / (r - 1) \\
 s_2 &= a_1 / (r - 1) - a_2 \gamma_1 \\
 \phi_1 &= \frac{[a_1 - a_2 (r - 1) \gamma_1 + (r - 1)^2]}{(r - 1) [(r - 1) d_1 - a_1]} \\
 \phi_2 &= \frac{(r - 1) [a_1 - a_2 (r - 1) \gamma_1 + (r - 1)^2]}{[(r - 1) d_1 - a_1] [a_1 - a_2 (r - 1) \gamma_1]} .
 \end{aligned} \tag{22.11}$$

The values of the separations d_1 and d_2 are $d_1 + (d_1)_{\min}$ and $d_2 + s_2 + \frac{1}{\phi_2}$. If we substitute for s_2 , ϕ_1 and ϕ_2 then the solution

to the minimum length two element zoom lens is

$$\begin{aligned}
 d_1 &= (d_1)_{\min} \\
 d_2 &= -a_1 \gamma_1 + \frac{(r-1)(d_1 - a_2 d_1 \gamma_1) + a_1 d_1}{\Phi} \\
 \phi_1 &= \frac{\Phi}{(r-1)[(r-1)d_1 - a_1]} \\
 \phi_2 &= \frac{(r-1)\Phi}{[(r-1)d_1 - a_1][a_1 - a_2(r-1)\gamma_1]} . \quad (22.12)
 \end{aligned}$$

where $\Phi = a_1 - a_2(r-1)\gamma_1 + (r-1)^2$.

For a particular value of $(d_1)_{\min}$, this is the only solution to the problem of the two element zoom lens.

23. The three element zoom lens

For the three element zoom lens, solution of equations (19.5), (20.4) and (20.5) directly becomes rather involved. If we make the changes of variables

$$\begin{aligned}
 d_3 &= s_3 + \frac{1}{\phi_3} \\
 d_2 &= s_2 + \frac{1}{\phi_2} + \frac{1}{\phi_3} \\
 d_1 &= s_1 + \frac{1}{\phi_1} + \frac{1}{\phi_2} , \quad (23.1)
 \end{aligned}$$

the problem becomes easier to solve. Using (22.1) it is found that

$$\begin{aligned}
 y_{a3}^{(0)} &= -\phi_1/\phi_2 + s_1 \phi_1 \phi_2/\phi_3 + s_1 s_2 \phi_1 \phi_2 \\
 y_{a3}^{(1)} &= a_1 s_2 \phi_1 \phi_2 + a_2 s_1 \phi_1 \phi_2 + a_1 \phi_1 \phi_2/\phi_3
 \end{aligned}$$

$$\begin{aligned}
y_{a3}^{(2)} &= a_1 a_2 \phi_1 \phi_2 \\
v_{a3}^{(0)} &= -\phi_1 \phi_3 / \phi_2 + s_1 s_2 \phi_1 \phi_2 \phi_3 \\
v_{a3}^{(1)} &= (a_1 s_2 + a_2 s_1) \phi_1 \phi_2 \phi_3 \\
v_{a3}^{(2)} &= a_1 a_2 \phi_1 \phi_2 \phi_3, \quad (23.2)
\end{aligned}$$

and $\delta = a_1 a_2 a_3 \phi_1 \phi_2 \phi_3$. Substitution into (20.4), (20.5) and (19.5) gives

$$\begin{aligned}
\frac{s_1}{\phi_3^2} + \frac{s_3}{\phi_2^2} - s_1 s_2 s_3 &= 0 \\
a_1 \frac{\phi_1 \phi_2}{\phi_3} + a_3 \frac{\phi_1 \phi_3}{\phi_2} - (a_1 s_2 s_3 + a_2 s_1 s_3 + a_3 s_1 s_2) \phi_1 \phi_2 \phi_3 &= \\
&= -\delta \gamma_2 \\
(a_1 a_2 s_3 + a_1 a_3 s_2 + a_2 a_3 s_1) \phi_1 \phi_2 \phi_3 &= \delta \gamma_1 \\
\phi_2^2 &= \frac{1-r}{(1-r) s_1 s_2 + a_1 s_2 + a_2 s_1 + a_1 a_2}. \quad (23.3)
\end{aligned}$$

In equations (23.3) $\gamma_1 = \gamma_1(3)$ and $\gamma_2 = \gamma_2(3)$. For the minimum length system we require

$$d_1 = (d_1)_{\min}$$

$$d_2 = (d_2)_{\min}.$$

For the moment we do not consider the minimum length system. It will be seen that if we substitute for δ in equations (23.3), ϕ_1 does not appear. The value of ϕ_1 is determined solely by the choice of d_2 . (23.3) is a set of four equations in the five variables ϕ_2 ,

ϕ_3, s_1, s_2, s_3 . For the moment take s_3 to have a fixed value.

Equations (23.3) can now be solved for ϕ_2, ϕ_3, s_1, s_2 in terms of s_3 . The solution of (23.3) for general a_1, a_2 and a_3 can be written down, but it is rather difficult to obtain. Substituting specific values for a_1, a_2 and a_3 makes the problem very much simpler. For optical compensation $a_1 = -1, a_2 = 1$ and $a_3 = -1$. The solution of equations (23.3) is

$$s_2 = (s_3 - \gamma_1) + \frac{s_3(s_3 - \gamma_1 + 1)}{(1 - r) \left[s_3^2 - s_3 \gamma_1 + \gamma_2 + \frac{s_3 - \gamma_1 + 1}{1 - r} \right]}, \quad (23.4)$$

$$s_1 = s_2 - s_3 + \gamma_1, \quad (23.5)$$

$$\phi_2 = \pm \left(\frac{(1 - r)}{(1 - r) s_1 s_2 - s_2 + s_1 - 1} \right)^{\frac{1}{2}}, \quad (23.6)$$

$$\phi_3 = \pm \left(\frac{s_1}{s_3(s_1 s_2 \phi_2^2 - 1)} \right)^{\frac{1}{2}}. \quad (23.7)$$

ϕ_1 is determined as follows. The last of equations (23.1) is

$$d_1 = s_1 + \frac{1}{\phi_1} + \frac{1}{\phi_2}.$$

From this, choosing a particular value for d_1 , we have

$$\phi_1 = \frac{\phi_2}{(d_1 - s_1) \phi_2 - 1}. \quad (23.8)$$

The minimum value of d_1 is 1.0 so a realistic choice would be

$d_1 = 1.1$. The separation d_2 is obtained from the second of equations (23.1). It is

$$d_2 = s_2 + \frac{1}{\phi_2} + \frac{1}{\phi_3}. \quad (23.9)$$

We may obtain the system with $d_2 = (d_2)_{\min}$ as follows. Calculate d_2 for several values of s_3 . By graphing d_2 versus s_3 it is possible to find where $d_2 = (d_2)_{\min}$ (see figure 1).

There will be several solutions for each choice of s_3 , corresponding to the choice of sign of ϕ_2 and ϕ_3 . The different solutions will be named according to the signs of ϕ_2 and ϕ_3 . The solution which takes the + sign for both ϕ_2 and ϕ_3 , for example, will be called the (+, +) system. Each solution must be investigated when searching for minimum length solutions.

24. Two three element zoom lenses

To obtain a particular value of R we may have either $r = R$ or $r = \frac{1}{R}$. We consider both of these solutions. Jamieson [9] has considered solutions of the three element zoom lens when $R = 3$. We reproduce his results using the theory of §23.

(i) First consider the $r = 3$ solution. Jamieson has chosen the points of full compensation to be at $z = 0, 0.42, 1$. This gives $\gamma_1 = 1.42$ and $\gamma_2 = 0.42$.

It is found that values of s_3 less than 1.5 give rise to imaginary powers. Figure 1a shows a graph of d_2 versus s_3 for the four possible solutions. From this it is seen that only the (+, +) and (-, +) solutions are physically possible. Only the (-, +) solution gives a minimum length solution. This occurs when $s_3 = 2.46$. If we consider $d_2 = 0.1$ rather than $d_2 = 0.0$, $s_3 = 2.485$. Choosing $d_1 = 1.1$ we find that the only minimum length solution is

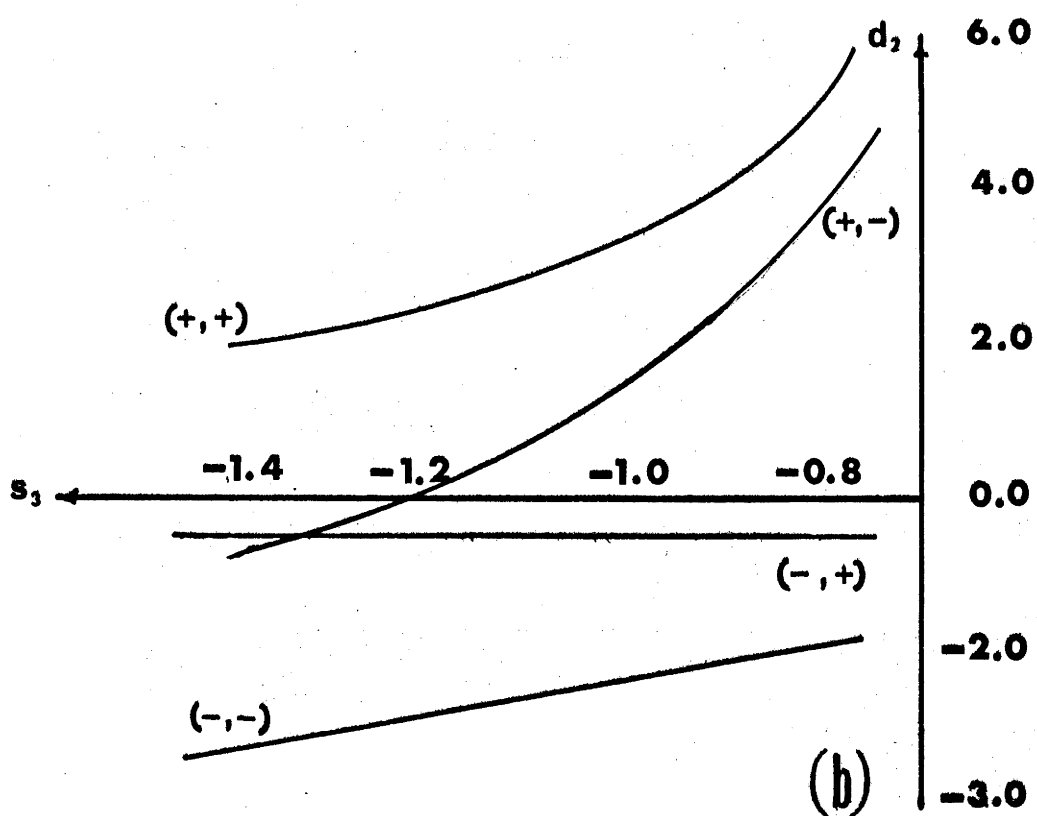
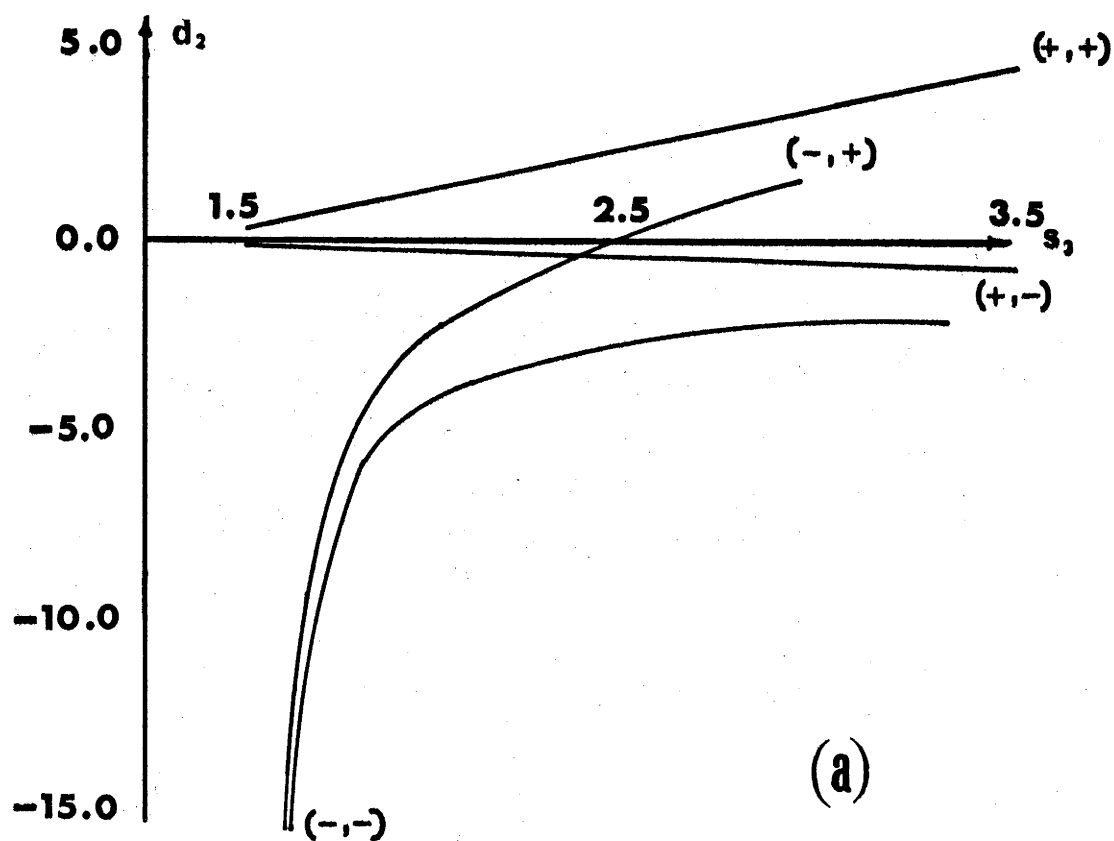


Figure 1. d_2 versus s_3 for (a) $r = 3.0$

(b) $r = 0.333$.

ϕ_1	ϕ_2	ϕ_3	d_1	d_2	d_3
0.286426	-0.884442	0.700642	1.10	0.10	3.913

This is the same as the system obtained by Jamieson.

(ii) The other way of obtaining $R = 3$ is to put $r = \frac{1}{3}$. Jamieson has chosen the points of full compensation to be at $z = 0, 0.58, 1$ and so $\gamma_1 = 1.58, \gamma_2 = 0.58$.

Values of s_3 greater than -0.5 give rise to imaginary solutions. The four possible solutions are graphed versus s_3 in figure 1b. The only realizable solutions are the $(+, +)$ and $(+, -)$ solutions. Once again it is the alternating sign solution which has minimum length. This occurs at $s_3 = -1.23$. If, instead of $d_2 = 0$ we choose a small positive value we find, for $s_3 = -1.21$ that:

ϕ_1	ϕ_2	ϕ_3	d_1	d_2	d_3
-0.344683	0.692099	-0.887040	1.10	0.083	-2.337344

The negative image distance indicates that the image is virtual.

These results agree with those of Jamieson.

25. The n element zoom lens

The n element zoom lens is described by $2n$ equations in $2n$ variables, and so, even for relatively small n , it soon becomes difficult to solve the equations analytically. It is, however, possible to solve the equations numerically. For this purpose a computer programme, utilizing damped least squares optimization, was written. The programme minimizes a function of the form

$$F = \sum_{i=1}^{2n} (\gamma_i - \gamma_i^{(\infty)}) . \quad (25.1)$$

The γ_i are functions of the parameters of the zoom lens. The $\gamma_i^{(\infty)}$ are the required final values of these functions. Each of (20.4), (20.5) and (19.5), as well as (20.7) is used to obtain the function γ_i and the corresponding $\gamma_i^{(\infty)}$.

In the optimization process the derivatives of F with respect to separations and curvatures are required. The γ_i are functions of the paraxial coefficients and their derivatives with respect to z . It is thus necessary to calculate the derivatives $\frac{\partial \gamma_i}{\partial d}$ and $\frac{\partial \gamma_i}{\partial c_0}$. Whether the γ_i depend on paraxial coefficients or the derivatives of paraxial coefficients it is necessary to evaluate expressions such as

$$\frac{\partial}{\partial d_j} \frac{\partial^s v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_1}} \quad \text{and} \quad \frac{\partial}{\partial c_0} \frac{\partial^s v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_1}} .$$

The former can be calculated directly using (8.13); with the latter more care must be taken. As pointed out in §8 there are three distinct cases to be considered.

$$(i) \quad \frac{\partial^{s+1} v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_1} \partial c_{0j}} , \quad j < i_1 .$$

We consider

$$\frac{\partial^s}{\partial d_{i_s} \dots \partial d_{i_1}} \frac{\partial v'_{ak}}{\partial c_{0j}} = \frac{\partial^s}{\partial d_{i_s} \dots \partial d_{i_1}} \cdot \bar{N}_j (y_j / v'_k) y_j .$$

The result is

$$\frac{\partial^{s+1}_{v'_{ak}}}{\partial d_{i_s} \dots \partial d_{i_1} \partial c_{0j}} = \bar{N}_j(y_j/v_{i_1}) \frac{\partial^s_{v'_{ak}}}{\partial d_{i_s} \dots \partial d_{i_1}} \frac{y_{aj}}{v_{ai_1}}, \quad (25.2)$$

for $j > i_1$.

$$(ii) \quad \frac{\partial^{s+1}_{v'_{ak}}}{\partial d_{i_s} \dots \partial d_{i_2} \partial c_{0j} \partial d_{i_1}}, \quad j \geq i.$$

We consider

$$\frac{\partial^s}{\partial d_{i_s} \dots \partial d_{i_2} \partial c_{0j}} \frac{\partial v'_{ak}}{\partial d_{i_1}} = \frac{\partial^s}{\partial d_{i_s} \dots \partial d_{i_2} \partial c_{0j}} \bar{N}_{i_1}(v_{i_1}/v'_k) v_{ai_1}$$

and the final result is

$$\frac{\partial^{s+1}_{v'_{ak}}}{\partial d_{i_s} \dots \partial d_{i_2} \partial c_{0j} \partial d_{i_1}} = \bar{N}_{i_1}(v_{i_1}/y_j) \frac{\partial^s_{v'_{ak}}}{\partial d_{i_s} \dots \partial d_{i_2} \partial c_{0j}} \frac{v_{ai_1}}{y_{aj}} \quad (25.3)$$

for $j \geq i$. The derivative on the right of (25.3) can be calculated using (25.2).

$$(iii) \quad \frac{\partial^{s+1}_{v'_{ak}}}{\partial d_{i_s} \dots \partial d_{i_{s-t}} \partial c_{0j} \partial d_{i_{s-t-1}} \dots \partial d_{i_1}}, \quad i_{s-t} > j > i_{s-t-1}.$$

Here we consider the expression

$$\begin{aligned} \frac{\partial^{t+1}}{\partial d_{i_s} \dots \partial d_{i_{s-t}} \partial c_{0j}} \frac{\partial^{s-t}_{v'_{ak}}}{\partial d_{i_{s-t-1}} \dots \partial d_{i_1}} &= \frac{\partial^{t+1}}{\partial d_{i_s} \dots \partial d_{i_{s-t}} \partial c_{0j}} \times \\ &\times \bar{N}_{i_1}(v_{i_1}/v_{i_2}) \frac{\partial^{s-t-1}_{v'_{ak}}}{\partial d_{i_{s-t-1}} \dots \partial d_{i_2}} \frac{v_{ai_1}}{v_{ai_2}}. \end{aligned}$$

The final result of this is

$$\begin{aligned}
 & \frac{\partial^{s+1} v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_{s-t}} \partial c_{0j} \partial d_{i_{s-t-1}} \dots \partial d_{i_1}} = v_{i_1} (v_{i_1} / v_{i_2}) \times \\
 & \times \frac{\partial^s v'_{ak}}{\partial d_{i_s} \dots \partial d_{i_1} \partial c_{0j} \partial d_{i_{s-t-1}} \dots \partial d_{i_2}} \frac{v_{ai_1}}{v_{ai_2}} \quad (25.4)
 \end{aligned}$$

for $i_{s-t} \geq j > i_{s-t-1}$.

By repeated application of (25.4) we obtain an expression on the right which resembles the left of (25.3). The left of (25.4) can then be evaluated.

CHAPTER 5

THE ABERRATIONS OF ZOOM LENSES

26. Fixed aperture aberrations

Aberrations of zoom lenses may be considered firstly from the point of view of what it is proposed to call fixed aperture aberrations. In zoom lenses the stop is generally placed somewhere after the last element of (Z). As the elements of (Z) are moved the pupil magnification, and hence the diameter of the entrance pupil, varies. At the same time the position of the entrance pupil, relative to the first element, changes. Fixed aperture aberrations ignore the changes in the position and diameter of the entrance pupil.

Consider two optical systems placed with respect to one another in such a way that they possess a common axis of symmetry. Let the final paracanonical coefficients of the two systems be $y'_{p1}, y'_{q1}, \dots, A'_{p1}, \dots$; and $y'_{p2}, y'_{q2}, \dots, A'_{p2}, \dots$; respectively.

In this chapter subscripts 'p' and 'q' refer to the paracanonical co-ordinates of component optical systems on their own. Subscripts 'a' and 'b' refer to paracanonical coefficients of composite systems. Let the separation between systems 1 and 2 be d_1 . Let the canonical coefficients of system 2, on its own, be $\sigma_2, \bar{\sigma}_2, \tau_2, \bar{\tau}_2$. According to L (48.4) the coefficient of spherical aberration for the composite system is

$$\begin{aligned}
 A'_a = & A'_{p1} + \pi_2^4 A'_{p2} + \pi_2^3 \rho_2 (A'_{q2} + \bar{A}'_{p2} + B'_{p2}) + \\
 & + \pi_2^2 \rho_2^2 (\bar{A}'_{q2} + B'_{q2} + \bar{B}'_{p2} + C'_{p2}) + \pi_2 \rho_2^3 (\bar{B}'_{q2} + \bar{C}'_{p2} + C'_{q2}) + \\
 & + \rho_2^4 \bar{C}'_{q2} .
 \end{aligned} \tag{26.1}$$

The coefficients π_2 and ρ_2 are

$$\left. \begin{aligned} \pi_2 &= \sigma_2 y'_{p1} + (\bar{\sigma}_2 - d_1 \sigma_2) v'_{p1} \\ \rho_2 &= \tau_2 y'_{p2} + (\bar{\tau}_2 - d_1 \tau_2) v'_{p1} \end{aligned} \right\} . \quad (26.2)$$

If we choose $\pi_1 = 1$ and $\rho_1 = 0$ then by analogy with (26.1) we may write, for an n element system

$$\begin{aligned} A'_{an} &= \sum_{i=1}^n \left\{ \pi_i^4 A'_{pi} + \pi_i^3 \rho_i (A'_{qi} + \bar{A}'_{pi} + B'_{pi}) \right. \\ &\quad + \pi_i^2 \rho_i^2 (\bar{A}'_{qi} + B'_{qi} + \bar{B}'_{pi} + C'_{pi}) + \pi_i \rho_i^3 (\bar{B}'_{qi} + \bar{C}'_{pi} + C'_{qi}) + \\ &\quad \left. + \rho_i^4 \bar{C}'_{qi} \right\} , \end{aligned} \quad (26.3)$$

where

$$\left. \begin{aligned} \pi_i &= \sigma_i y'_{ai-1} + (\bar{\sigma}_i - d_{i-1} \sigma_i) v'_{ai-1} \\ \rho_i &= \tau_i y'_{ai-1} + (\bar{\tau}_i - d_{i-1} \tau_i) v'_{ai-1} \end{aligned} \right\} . \quad (26.4)$$

d_{i-1} is the separation between elements $i-1$ and i . $\sigma_i, \bar{\sigma}_i, \tau_i, \bar{\tau}_i$ are the paracanonical co-ordinates used to obtain the paracanonical coefficients of element i on its own. The elements of the composite system may be either individual lenses or may be groups of lenses. We will take the elements to be individual thin lenses. If we write

$$\begin{aligned} \bar{\pi}_i &= \sigma_i y'_{bi-1} + (\bar{\sigma}_i - d_{i-1} \sigma_i) v'_{bi-1} \\ \bar{\rho}_i &= \tau_i y'_{ni-1} + (\bar{\tau}_i - d_{i-1} \tau_i) v'_{bi-1} \end{aligned} , \quad (26.5)$$

the third order aberration coefficients of the total system may be written as

$$\begin{aligned}\bar{A}'_{an} = & \sum_{i=1}^n \left\{ \pi_i^3 \bar{\pi}_i A'_{pi} + \pi_i^2 \bar{\pi}_i \rho_i (A'_{qi} + B'_{pi}) + \pi_i^3 \bar{\rho}_i \bar{A}'_{pi} + \right. \\ & + \pi_i^2 \rho_i \bar{\rho}_i (\bar{A}'_{qi} + \bar{B}'_{pi}) + \pi_i \bar{\pi}_i^2 \rho_i^2 (B'_{qi} + C'_{pi}) + \\ & \left. + \pi_i \rho_i^2 \bar{\rho}_i (\bar{B}'_{qi} + \bar{C}'_{pi}) + \bar{\pi}_i \rho_i^3 C'_{qi} + \rho_i^3 \bar{\rho}_i \bar{C}'_{qi} \right\},\end{aligned}$$

$$\begin{aligned}\bar{B}'_{an} = & \sum_{i=1}^n \left\{ 2\pi_i^2 \bar{\pi}_i^2 A'_{pi} + \pi_i \bar{\pi}_i^2 \rho_i (2A'_{qi} + B'_{pi} + \bar{\pi}_i^2 \rho_i^2 B'_{qi} + \right. \\ & + \pi_i^2 \bar{\pi}_i \rho_i (2\bar{A}'_{pi} + B'_{pi}) + \pi_i \bar{\pi}_i \rho_i \bar{\rho}_i (2\bar{A}'_{qi} + B'_{qi} + \bar{B}'_{pi} + \\ & + 2C'_{pi}) + \pi_i^2 \bar{\rho}_i^2 \bar{B}'_{pi} + \pi_i \rho_i \bar{\rho}_i^2 (\bar{B}'_{qi} + 2\bar{C}'_{pi}) + \\ & \left. + \bar{\pi}_i \rho_i^2 \bar{\rho}_i (\bar{B}'_{qi} + 2C'_{qi}) + 2\rho_i^2 \bar{\rho}_i^2 \bar{C}'_{qi} \right\},\end{aligned}$$

$$\begin{aligned}C'_{an} = & \sum_{i=1}^n \left\{ \pi_i^2 \bar{\pi}_i^2 A'_{pi} + \pi_i \bar{\pi}_i^2 \rho_i (A'_{qi} + \bar{A}'_{pi}) + \bar{\pi}_i^2 \rho_i^2 \bar{A}'_{qi} + \right. \\ & + \pi_i^2 \bar{\pi}_i \bar{\rho}_i B'_{pi} + \pi_i \bar{\pi}_i \rho_i \bar{\rho}_i (\bar{B}'_{pi} + B'_{qi}) + \bar{\pi}_i \rho_i^2 \bar{\rho}_i \bar{B}'_{qi} + \\ & \left. + \pi_i^2 \bar{\rho}_i^2 C'_{pi} + \pi_i \rho_i \bar{\rho}_i^2 (\bar{C}'_{pi} + C'_{qi}) + \rho_i^2 \bar{\rho}_i^2 \bar{C}'_{qi} \right\},\end{aligned}$$

$$\begin{aligned}\bar{C}'_{an} = & \sum_{i=1}^n \left\{ \pi_i \bar{\pi}_i^3 A'_{pi} + \bar{\pi}_i^3 \rho_i A'_{qi} + \pi_i \bar{\pi}_i^2 \bar{\rho}_i (\bar{A}'_{pi} + B'_{pi}) + \right. \\ & + \pi_i \bar{\pi}_i \bar{\rho}_i^2 (\bar{B}'_{pi} + C'_{pi}) + \bar{\pi}_i \rho_i \bar{\rho}_i^2 (\bar{B}'_{qi} + C'_{qi}) + \pi_i \bar{\rho}_i^3 \bar{C}'_{pi} + \\ & \left. + \rho_i \bar{\rho}_i^3 \bar{C}'_{qi} \right\}.\end{aligned}\tag{26.6}$$

27. $\pi, \bar{\pi}, \rho, \bar{\rho}$ as functions of the zoom parameter

The functions $\pi_i, \bar{\pi}_i, \rho_i, \bar{\rho}_i$ depend on the paracanonical co-ordinates of the element i , and those of all elements preceding it. They also depend upon the paraxial paracanonical coefficients after element i . According to L (32.5), using the convention established in §26, we have

$$\left. \begin{aligned} y'_{ai} &= \pi_i y'_{pi} + \rho_i y'_{qi} \\ v'_{ai} &= \pi_i v'_{pi} + \rho_i v'_{qi} \\ y'_{bi} &= \bar{\pi}_i y'_{pi} + \bar{\rho}_i y'_{qi} \\ v'_{bi} &= \bar{\pi}_i v'_{pi} + \bar{\rho}_i v'_{qi} \end{aligned} \right\} . \quad (27.1)$$

Substituting these relations in (26.4), (26.5) gives

$$\begin{aligned} \pi_i &= (\sigma_i y'_{pi-1} + [\bar{\sigma}_i - d_{i-1} \sigma_i] v'_{pi-1}) \pi_{i-1} + \\ &\quad + (\sigma_i y'_{qi-1} + [\bar{\sigma}_i - d_{i-1} \sigma_i] v'_{qi-1}) \rho_{i-1} \\ \rho_i &= (\tau_i y'_{pi-1} + [\bar{\tau}_i - d_{i-1} \tau_i] v'_{pi-1}) \pi_{i-1} + \\ &\quad + (\tau_i y'_{qi-1} + [\bar{\tau}_i - d_{i-1} \tau_i] v'_{qi-1}) \rho_{i-1} \\ \bar{\pi}_i &= (\sigma_i y'_{pi-1} + [\bar{\sigma}_i - d_{i-1} \sigma_i] v'_{pi-1}) \bar{\pi}_{i-1} + \\ &\quad + (\sigma_i y'_{qi-1} + [\bar{\sigma}_i - d_{i-1} \sigma_i] v'_{qi-1}) \bar{\rho}_{i-1} \\ \bar{\rho}_i &= (\tau_i y'_{pi-1} + [\bar{\tau}_i - d_{i-1} \tau_i] v'_{pi-1}) \bar{\pi}_{i-1} + \\ &\quad + (\tau_i y'_{qi-1} + [\bar{\tau}_i - d_{i-1} \tau_i] v'_{qi-1}) \bar{\rho}_{i-1} . \end{aligned} \quad (27.2)$$

In a zoom lens, each separation d_i is a function of the zoom parameter z . If we substitute $d_i(z) = d_i + a_i z$ whenever a separation occurs, we find

$$\begin{aligned}
\pi_i &= (b_{ai} z + c_{ai}) \pi_{i-1} + (b_{bi} z + c_{bi}) \rho_{i-1} \\
\rho_i &= (g_{ai} z + h_{ai}) \pi_{i-1} + (g_{bi} z + h_{bi}) \rho_{i-1} \\
\bar{\pi}_i &= (b_{ai} z + c_{ai}) \bar{\pi}_{i-1} + (b_{bi} z + c_{bi}) \bar{\rho}_{i-1} \\
\bar{\rho}_i &= (g_{ai} z + h_{ai}) \bar{\pi}_{i-1} + (g_{bi} z + h_{bi}) \bar{\rho}_{i-1} . \quad (27.3)
\end{aligned}$$

We wish to find π_i , $\bar{\pi}_i$, ρ_i , $\bar{\rho}_i$ as power series in z . There are $i-1$ separations preceding the i^{th} element and so each of π_i , $\bar{\pi}_i$, ρ_i , $\bar{\rho}_i$ will be a polynomial of degree $i-1$ at most. Write

$$\begin{aligned}
\pi_i &= \sum_{j=0}^{i-1} p_{ij} z^j, & \bar{\pi}_i &= \sum_{j=0}^{i-1} \bar{p}_{ij} z^j, \\
\rho_i &= \sum_{j=0}^{i-1} r_{ij} z^j, & \bar{\rho}_i &= \sum_{j=0}^{i-1} \bar{r}_{ij} z^j . \quad (27.4)
\end{aligned}$$

Substitution of these series into (27.3) gives, for example,

$$\sum_{j=0}^{i-1} p_{ij} z^j = (b_{ai} z + c_{ai}) \sum_{j=0}^{i-2} p_{i-1,j} z^j + (b_{bi} z + c_{bi}) \sum_{j=0}^{i-1} r_{i-1,j} z^j .$$

Equating coefficients of z^j gives

$$p_{ij} = p_{i-1,j} c_{ai} + r_{i-1,j} c_{bi} + p_{i-1,j-1} b_{ai} + r_{i-1,j-1} b_{bi} . \quad (27.5)$$

This expression holds true for $j = 0, 1, \dots, i-1$ if we put

$$p_{i-1,-1} = r_{i-1,-1} = p_{i-1,-1} = r_{i-1,-1} = 0 . \quad (27.6)$$

In a similar manner we find

$$r_{ij} = p_{i-1,j} h_{ai} + r_{i-1,j} h_{bi} + p_{i-1,j-1} g_{ai} + r_{i-1,j-1} g_{bi} . \quad (27.7)$$

Once again this holds for $j = 0, 1, \dots, i-1$ if (27.6) is taken into account. The expressions for the barred coefficients are

$$\begin{aligned}\bar{p}_{ij} &= \bar{p}_{i-1,j} c_{ai} + \bar{r}_{i-1,j} c_{bi} + \bar{p}_{i-1,j-1} b_{ai} + \bar{r}_{i-1,j-1} b_{bi} \\ \bar{r}_{ij} &= \bar{p}_{i-1,j} h_{ai} + \bar{r}_{i-1,j} h_{bi} + \bar{p}_{i-1,j-1} g_{ai} + \bar{r}_{i-1,j-1} g_{bi}\end{aligned}\quad (27.8)$$

with the understanding that

$$\bar{p}_{i-1,-1} = \bar{r}_{i-1,-1} = \bar{p}_{i-1,i-1} = \bar{r}_{i-1,i-1} = 0. \quad (27.9)$$

From (27.5), (27.7) and (27.8) it is possible, given the paracanonical co-ordinates with which the aberration coefficients of each element have been calculated, to obtain $\pi_i, \bar{\pi}_i, \rho_i, \bar{\rho}_i$ as functions of z .

28. Effect of entrance pupil variation

Once the full set of polynomials $\pi_i, \bar{\pi}_i, \rho_i, \bar{\rho}_i$ ($i = 1, \dots, n$) are known, the fixed aperture aberration coefficients may be calculated through the use of (26.3) and (26.6) (we are assuming the aberration coefficients of the individual elements are known). If canonical co-ordinates have been used in calculating the aberration coefficients of each element, the entrance pupil for the whole system will have unit radius and will be in contact with the first element.

According to L (33.1), the actual position of the entrance pupil will be

$$p = - \frac{(y_{bi} - v'_{bi} d)}{(y_{ai} - v'_{ai} d)}, \quad (28.1)$$

where d is the distance of the stop behind the i^{th} refracting surface. In zoom lenses the stop is generally placed after the last movable element and so

$$p = - \frac{(y_{bn} - v'_{bn} d)}{(y_{an} - v'_{an} d)}, \quad (28.2)$$

where we have suppressed the dependence upon z of y_{bn} , ..., v'_{an} , d ($\equiv d(0) - z$). We do this whenever no confusion is likely to be caused. L (33.2) gives the radius of the entrance pupil. If e is the radius of the stop, the radius of the entrance pupil will be

$$h = \frac{e}{(y_{an} - v'_{an} d)}. \quad (28.3)$$

Since y_{bn} , etc., are functions of z , it follows that p and h are functions of z .

The paraxial invariant must be constant during the zoom process. For an object at infinity $v_{a1} = 0$ and so the paraxial invariant is

$$\lambda = h v_{b1}. \quad (28.4)$$

To keep λ constant, v_{b1} must be a function of z . For the system with unit aperture and stop in contact with the first surface, we have, taking co-ordinates in the object plane and the first tangential plane

$$\left. \begin{aligned} \underline{S} &= \underline{Y}_1 \\ \underline{T} &= \underline{V}_1 \end{aligned} \right\} \quad (28.5)$$

for an object at infinity. For the actual system, which has aperture

h and whose stop is at a distance p from the first element,

$$\left. \begin{aligned} *S &= h \underline{S} + \frac{p\lambda}{h} \underline{T} \\ *T &= \frac{\lambda}{h} \underline{T} \end{aligned} \right\} . \quad (28.6)$$

Using L (32.1) to make the transformation from co-ordinates (28.5) to co-ordinates (28.6) we obtain

$$\begin{aligned} *A'_{an} &= h^4 A'_{an} \\ *\bar{A}'_{an} &= \lambda h^2 (\bar{A}'_{an} + p A'_{an}) \\ *\bar{B}'_{an} &= \lambda^2 (\bar{B}'_{an} + 4p \bar{A}'_{an} + 2p^2 A'_{an}) \\ *C'_{an} &= \lambda^2 (C'_{an} + 2p \bar{A}'_{an} + p^2 A'_{an}) \\ *\bar{C}'_{an} &= \frac{\lambda^3}{h^2} (\bar{C}'_{an} + p [\bar{B}'_{an} + C'_{an}] + 3p^2 \bar{A}'_{an} + p^3 A'_{an}) . \end{aligned} \quad (28.7)$$

The $*G_{\mu\nu}^{(1)}$ are the actual aberrations of the zoom lens for an object at infinity.

29. A theorem concerning pupil magnification

We prove the following theorem.

'If, for a zoom lens, there is full image compensation at both ends of the zoom range (i.e. at $z = 0$ and $z = 1$), then the pupil magnification range is the same as the image magnification range.'

The pupil magnification of a zoom lens is obtained from (28.3). Written in full it is

$$\begin{aligned}
 \frac{h(z)}{e} &= \frac{1}{y_{an}(z) - d(z) v'_{an}(z)} \\
 &= \frac{1}{y_{an}(z) - (d - a_n z) v'_{an}(z)} .
 \end{aligned}
 \tag{29.1}$$

Because we have full compensation at both ends of the zoom range, it follows that at $z = 0$

$$\frac{y_{an}(0)}{v'_{an}(0)} = d_n , \tag{29.2}$$

and at $z = 1$

$$\frac{y_{an}(1)}{v'_{an}(1)} = d_n + a_n . \tag{29.3}$$

From equation (28.4)

$$\frac{h(0)}{e} = \frac{1}{y_{an}(0) - d v'_{an}(0)}$$

and

$$\frac{h(1)}{e} = \frac{1}{y_{an}(1) - (d + a_n) v'_{an}(1)} .$$

This means that

$$\frac{h(1)}{h(0)} = \frac{v'_{an}(0)}{v'_{an}(1)} = \frac{1}{r} . \tag{29.4}$$

If $m(z)$ is the image magnification, we also have, from the definition of r

$$\frac{m(1)}{m(0)} = \frac{v'_{an}(0)}{v'_{an}(1)} = \frac{1}{r} . \tag{29.5}$$

From these results we see that the theorem stated above is proved.

30. The third order aberration coefficients of a thin lens in air

When canonical co-ordinates are used, the third order aberration coefficients of a thin lens in air are particularly simple. Use of canonical co-ordinates does not imply any loss of generality. As in [15] we use the shape factor S to specify the curvatures of lenses. S is defined by

$$S := \frac{c_1 + c_2}{c_1 - c_2} . \quad (30.1)$$

The contribution to the coefficient of primary spherical aberration from a single surface is, according to (5.2)

$$\underline{a}_a = \frac{1}{2}N(1 - k) y_a i_a^2 (i'_a - v_a) . \quad (30.2)$$

For canonical co-ordinates, applying this formula to a thin lens in air gives

$$\begin{aligned} \underline{a}_{p1} &= \frac{1}{2}c_1^3 \frac{N - 1}{N^2} \\ \underline{a}_{p2} &= -\frac{1}{2}N(N - 1) \left(c_2 - c_1 + \frac{c_1}{N} \right)^2 \left([c_2 - c_1] N + \frac{c_1}{N} \right) , \end{aligned} \quad (30.3)$$

where N is the refractive index of the lens and c_1 and c_2 are the curvatures of the first and second surfaces respectively. Using (30.3) and substituting from (17.3), the coefficient of primary spherical aberration is

$$\begin{aligned}
 A'_p &= a_{p1} + a_{p2} \\
 &= \frac{\phi^3}{8} \left[\frac{N+2}{N(N-1)^2} s^2 - \frac{4(N+1)}{N(N-1)} s + \frac{4N^3 - 4N^2 - N + 2}{N(N-1)^2} \right] .
 \end{aligned} \tag{30.4}$$

In a similar manner, using (5.2) and (30.3) the other aberration coefficients are found to be

$$\bar{A}'_p = \frac{\phi^2}{4} \left[-\frac{(N+1)}{N(N-1)} s + \frac{2N+1}{N} \right] \tag{30.5}$$

$$B'_p = \frac{\phi^2}{2} \left[-\frac{(N+1)}{N(N-1)} s + \frac{2N+1}{N} \right] \tag{30.6}$$

$$\bar{B}'_p = \phi$$

$$C'_p = \frac{\phi}{2} \frac{(N+1)}{N} \tag{30.7}$$

$$\bar{C}'_p = 0 . \tag{30.8}$$

Using the identities L (20.41-47) and the equation L (26.4), the 'q' coefficients are found to be

$$A'_q = \frac{\phi^2}{4} \left[-\frac{(N+1)}{N(N-1)} s + \frac{1}{N} \right] \tag{30.9}$$

$$\bar{A}'_q = \frac{\phi}{2N} \tag{30.10}$$

$$B'_q = \bar{B}'_q = C'_q = \bar{C}'_q = 0 . \tag{30.11}$$

31. Fixed aperture aberration coefficients as functions of shape

If, in calculating the aberration coefficients appearing in (26.3), (26.6), we use canonical co-ordinates then we may write the third order fixed aperture aberration coefficients directly as functions of the shapes of the individual elements. Substituting from (30.4-11) in (26.3), (26.6) we obtain

$$A'_{an} = \sum_{i=1}^n \left\{ \pi^4 \frac{\phi^3}{8} K_1 S^2 + \pi^3 \frac{\phi^2}{4} K_2 \left(\pi \frac{\phi}{2} + \rho \right) S + \right. \\ \left. + \pi^2 \frac{\phi}{2} \left(\pi^2 \frac{\phi^2}{4} K_3 + \pi \rho \phi K_4 + \rho^2 K_4 \right) \right\}$$

$$\bar{A}'_{an} = \sum_{i=1}^n \left\{ \pi^3 \bar{\pi} \frac{\phi^3}{8} K_1 S^2 + \pi^2 \frac{\phi^2}{8} K_2 \left(\pi \bar{\pi} \phi + \frac{3}{2} \bar{\pi} \rho + \frac{1}{2} \pi \bar{\rho} \right) S + \right. \\ \left. + \pi \frac{\phi}{2} \left(\pi^2 \bar{\pi} \frac{\phi^2}{4} K_3 + \pi \bar{\pi} \rho \frac{\phi}{2} K_5 + \pi^2 \bar{\rho} \frac{\phi}{2} K_6 + \bar{\pi} \rho^2 K_7 \right) \right\}$$

$$\bar{B}'_{an} = \sum_{i=1}^n \left\{ \pi^2 \bar{\pi}^2 \frac{\phi^3}{4} K_1 S^2 + \pi \bar{\pi} \frac{\phi^2}{4} K_2 \left(\pi \bar{\pi} \phi + \bar{\pi} \rho + \pi \bar{\rho} \right) S + \right. \\ \left. + \pi \phi \left(\pi \bar{\pi}^2 \frac{\phi^2}{4} K_3 + \bar{\pi}^2 \rho \phi K_7 + \pi \bar{\pi} \rho \phi K_6 + 2 \bar{\pi} \rho \bar{\rho} K_7 + \pi \rho^2 \right) \right\}$$

$$C'_{an} = \sum_{i=1}^n \left\{ \pi^2 \bar{\pi}^2 \frac{\phi^3}{8} K_1 S^2 + \frac{\phi^2}{8} \pi K_2 \left(\pi \bar{\pi}^2 \phi + \bar{\pi}^2 \rho + \pi \bar{\pi} \bar{\rho} \right) S + \right. \\ \left. + \frac{\phi}{2} \left(\pi^2 \bar{\pi}^2 \frac{\phi^2}{4} K_3 + \pi \bar{\pi}^2 \rho \phi K_7 + \pi^2 \bar{\pi} \bar{\rho} \phi K_6 + \bar{\pi}^2 \rho^2 K_8 + \right. \right. \\ \left. \left. + \pi \bar{\pi} \rho \bar{\rho} \phi + \pi^2 \bar{\rho}^2 \frac{\phi}{2} K_7 \right) \right\}$$

$$\begin{aligned}
\bar{C}'_{an} = \sum_{i=1}^n \left\{ \pi \bar{\pi}^3 \frac{\phi^3}{8} K_1 S^2 + \bar{\pi}^2 \frac{\phi^2}{8} (\pi \bar{\pi} \phi + \frac{\bar{\pi} \rho}{2} + \frac{3}{4} \pi \bar{\rho}) S + \right. \\
+ \bar{\pi} \frac{\phi}{2} (\pi \bar{\pi} \frac{\phi^2}{4} K_3 + \bar{\pi}^2 \rho \frac{\phi}{2} K_8 + 3 \pi \bar{\pi} \rho \frac{\phi}{2} K_6 + \bar{\pi} \rho \bar{\rho} K_8 + \\
\left. + \pi \bar{\rho}^2 [K_4 - K_8]) \right\}, \quad (31.1)
\end{aligned}$$

where

$$\begin{aligned}
K_1 &= \frac{N+2}{N(N-1)^2} & K_2 &= -\frac{4(N+1)}{N(N-1)} \\
K_3 &= \frac{3N+2}{N} + \left(\frac{N}{N-1} \right)^2 & K_4 &= \frac{3N+2}{N} \\
K_5 &= \frac{4N+3}{N} & K_6 &= \frac{2N+1}{N} \\
K_7 &= \frac{N+1}{N} & K_8 &= \frac{1}{N}. \quad (31.2)
\end{aligned}$$

In (31.1) a subscript i has been dropped from every symbol.

We see that each fixed aperture aberration coefficient is a quadratic form in each shape factor. The coefficients of S_i^2 , S_i^1 , S_i^0 are all polynomials of degree $4(i-1)$ at most. If the refractive indices have been chosen, we can, using (31.1) and (28.7), write down the actual aberration coefficients as functions of the shapes and z .

32. Control of the third order aberrations

In a zoom system consisting of thin elements with spherical surfaces, the variables available for control of the third (and higher) order aberrations are the shapes and refractive indices of the thin

elements. It is always possible to choose the values of the refractive indices in such a way that the Petzval curvature (i.e. $*C'_{an} - \frac{1}{2}*\bar{B}'_{an}$) is either zero or some chosen small residual. This done, the only variables available to control the remaining primary aberrations are the shapes. Increasing the number of elements in a zoom lens will, aside from allowing an increased magnification range, allow better control of the aberrations.

It is necessary for each aberration of the zoom lens ($\textcircled{Z} + \textcircled{K}$) to be substantially corrected throughout the zoom range. This may be achieved by requiring that the value of each aberration be the same at several chosen points in the zoom range. System $\textcircled{K}$ is then designed to balance this level of the aberration. $\textcircled{K}$ must be sufficiently complex that the value of each of the primary aberrations can be chosen independently.

The points for which a particular aberration has the same value are called the points of full compensation for the aberration in question. The number of these points, for all aberrations, depends on the number of elements in the zoom lens (in $\textcircled{Z}$). If the levels of aberrations are to be chosen by the designer then the total number of points of full compensation for all aberrations will be n . Thus if each of T aberrations is to be controlled at P_K ($K = 1, \dots, T$) points in the zoom range, we succeed if (see [9])

$$\sum_{K=1}^T P_K = n . \quad (32.1)$$

If the levels of each aberration are allowed to be variable, we succeed if

$$\sum_{K=1}^T (P_K - 1) = n . \quad (32.2)$$

It is possible to control the aberrations of $\textcircled{Z}$ in other ways. We might, for example, require that the first derivative, with respect to z , of each aberration be zero at certain points in the zoom range. Choice of these points would be dictated by experience. Another possibility is that the first and higher derivatives (if possible) of aberrations be made zero at one point in the zoom range. Obviously many different conditions can be imposed. The best method of control will only be determined through experience or a deeper theoretical understanding of the aberrations of zoom lenses.

CHAPTER 6

CONTROL OF ABERRATIONS IN A THREE ELEMENT ZOOM LENS

33. Use of fixed aperture aberration coefficients

Equations (28.7), which give the actual aberration coefficients in terms of the fixed aperture aberration coefficients, may be solved to give the fixed aperture coefficients. The results are

$$A'_a = \frac{1}{h^4} *A'_a, \quad (33.1)$$

$$\bar{A}'_a = \frac{*A'_a}{\lambda h^2} - \frac{p}{h^4} *A'_a, \quad (33.2)$$

$$\bar{B}'_a = \frac{*B'_a}{\lambda^2} - \frac{4p}{\lambda h^2} *A'_a + \frac{2p^2}{h^4} *A'_a, \quad (33.3)$$

$$C'_a = \frac{*C'_a}{\lambda^2} - \frac{2p}{\lambda h^2} *A'_a + \frac{p^2}{h^4} *A'_a, \quad (33.4)$$

$$\bar{C}'_a = \frac{h^2}{\lambda^3} *C'_a - \frac{p}{\lambda^2} (*B'_a + *C'_a) + \frac{3p^2}{\lambda h^4} *A'_a - \frac{p^3}{h^4} *A'_a. \quad (33.5)$$

The fixed aperture aberrations are known functions of the shapes and z . They may thus be used to obtain equations for the shapes. For example, for a three element zoom lens, there are three shape factors to be allocated. Choosing a value for $*A'_a$ and three values of z , we have three equations involving the three shape factors. By allowing $*A'_a$, $*A'_a$, etc., to be variable it is possible to control more than one aberration.

34. Polynomials for a three element zoom lens

We obtain the functions $\pi_i, \bar{\pi}_i, \rho_i, \bar{\rho}_i$, ($i = 1, 2, 3$) assuming canonical co-ordinates and optical compensation. Once these polynomials are available it is possible to obtain the fixed aperture aberration coefficients as functions of shape and z .

The initial separations between the elements will be denoted by d_1 and d_2 . For optical compensation the rates of change of these separations are $a_1 = -1$ and $a_2 = 1$. For canonical co-ordinates $\sigma = 1$, $\bar{\sigma} = 0$, $\tau = 0$ and $\bar{\tau} = 1$. Using these values we find, comparing (27.2) and (27.3), that

$$\begin{aligned}
 c_{ai} &= y'_{pi-1} - d_{i-1} v'_{pi-1} \\
 c_{bi} &= y'_{qi-1} - d_{i-1} v'_{qi-1} \\
 b_{ai} &= -a_{i-1} v'_{pi-1} \\
 b_{bi} &= -a_{i-1} v'_{qi-1} \\
 h_{ai} &= v'_{pi-1} \\
 h_{bi} &= v'_{qi-1} \\
 g_{ai} &= g_{bi} = 0 \quad \text{and} \quad i = 2, 3. \quad (34.1)
 \end{aligned}$$

For canonical co-ordinates the final paraxial coefficients are, using (22.1)

$$\begin{aligned}
 v'_{pi} &= \phi_i & y'_{bi} &= 1 \\
 v'_{qi} &= 1 & y'_{qi} &= 0. \quad (34.2)
 \end{aligned}$$

If we agree that y_{a0} etc. are the paraxial coefficients before the first element, then for canonical co-ordinates $\pi_1, \bar{\pi}_1, \rho_1, \bar{\rho}_1$ are given by (26.4) and (26.5). We find $\pi_1 = 1, \bar{\pi}_1 = 0, \rho_1 = 0, \bar{\rho}_1 = 1$, and so we must have

$$p_{10} = 1, \quad \bar{p}_{10} = 0, \quad r_{10} = 0, \quad \bar{r}_{10} = 1. \quad (34.3)$$

We may now use (27.5), (27.7) and (27.8) to calculate π_2, π_3 etc.

With the aid of (33.1-3) it is found that

$$\begin{aligned} p_{20} &= (1 - d_1 \phi_1) & \bar{p}_{20} &= -d_1 \\ p_{21} &= \phi_1 & \bar{p}_{21} &= 1 \\ r_{20} &= \phi_1 & \bar{r}_{20} &= 1 \\ r_{21} &= \bar{r}_{21} = 0, & & \end{aligned} \quad (34.4)$$

$$\begin{aligned} p_{30} &= (1 - d_1 \phi_1)(1 - d_2 \phi_2) - d_2 \phi_1 \\ p_{31} &= \phi_1(1 - d_2 \phi_2) - \phi_2(1 - d_1 \phi_1) - \phi_2 \\ p_{32} &= -\phi_1 \phi_2 \\ \bar{p}_{30} &= -[d_1(1 - d_2 \phi_2) + d_2] \\ \bar{p}_{31} &= \phi_2(d_1 - d_2) \\ \bar{p}_{32} &= -\phi_2 \\ r_{30} &= (1 - d_1 \phi_1) \phi_2 + \phi_1 \\ r_{31} &= \phi_1 \phi_2 \\ r_{32} &= 0 \\ \bar{r}_{30} &= 1 - d_1 \phi_1 \\ \bar{r}_{31} &= \phi_2 \\ \bar{r}_{32} &= 0. \end{aligned} \quad (34.5)$$

From these coefficients the required polynomials are

$$\begin{aligned}
 \pi_2 &= (1 - d_1 \phi_1) + \phi_1 z, \\
 \bar{\pi}_2 &= -d_1 + z, \\
 \rho_2 &= \phi_1, \\
 \bar{\rho}_2 &= 1, \\
 \pi_3 &= (1 - d_1 \phi_1)(1 - d_2 \phi_2) - d_2 \phi_1 + [\phi_1(1 - d_2 \phi_2) - \\
 &\quad - \phi_2(1 - d_1 \phi_1) - \phi_2] z - \phi_1 \phi_2 z^2, \\
 \bar{\pi}_3 &= -[d_1(1 - d_2 \phi_2) + d_2] + \phi_2(d_1 - d_2) z - \phi_2 z^2, \\
 \rho_3 &= (1 - d_1 \phi_1) \phi_2 + \phi_1 + \phi_1 \phi_2 z, \\
 \bar{\rho}_3 &= (1 - d_1 \phi_1) + \phi_2 z.
 \end{aligned} \tag{34.6}$$

Using these results in equations (27.1) we can write down the final canonical paraxial coefficients of the optically compensated three element zoom lens. They are

$$\begin{aligned}
 y'_{a3} &= \pi_3 & v'_{a3} &= \pi_3 \phi_3 + \rho_3 \\
 y'_{b3} &= \bar{\pi}_3 & v'_{b3} &= \bar{\pi}_3 \phi_3 + \bar{\rho}_3.
 \end{aligned} \tag{34.7}$$

We see that using canonical co-ordinates the polynomials associated with the third element are just the paraxial coefficients (as functions of z) at the first surface of the third element. The method applied above can be used to find the final paraxial coefficients for any zoom lens. The method is not restricted to the special case of thin elements and can be used instead of the method discussed in §18.

Given a first order design, for example the one obtained in §24 (i), the polynomials (34.6) can be calculated. These polynomials, when substituted into equations (31.1) give the fixed aperture

aberration coefficients as functions of the shapes and $\bar{z}$. For even as few as three elements the substitution is quite lengthy, numerous multiplications of pairs of quadratic and quartic expressions being necessary. To facilitate this lengthy, but essentially simple, process a FORTRAN programme can be prepared. Such a programme carries out the procedure set out in this section and then performs the multiplications necessary to produce the coefficient polynomials appearing in (31.1). Input to the programme consists of the results of a first order design, namely the powers and separations, as well as the diameter and position of the stop. The output is the coefficient polynomials mentioned above, the pupil radius polynomial and the pupil position polynomial. Once these polynomials are available it is possible to proceed with the control of the third order aberrations.

35. Characteristics of the fixed aperture aberration coefficients

Certain important features of the fixed aperture aberrations can be deduced from the constancy of π_1 , $\bar{\pi}_1$, ρ_1 , $\bar{\rho}_1$. The actual values of these when canonical co-ordinates are used give rise to other features.

Because π_1 , $\bar{\pi}_1$, etc., are constant it immediately follows that the coefficients of S_1^2 and S_1 are constant. This is important because in any simultaneous equations obtained by sampling the fixed aperture aberration coefficients, the coefficients of S_1^2 , S_1 are the same in each equation.

For canonical co-ordinates we have

$$\pi_1 = 1, \quad \bar{\pi}_1 = 0, \quad \rho_1 = 0, \quad \bar{\rho}_1 = 1.$$

A look at the equation for $\bar{A}'_{an}$ shows that the coefficient of S_1^2 is zero since $\bar{\pi}_1 = 0$. Similarly we find that the coefficients of S_1^2 and S_1 are zero in the expressions for $\bar{B}'_{an}$, C'_{an} and $\bar{C}'_{an}$.

These features simplify the solution of simultaneous equations obtained from equations (31.1).

36. Control of spherical aberrations

As pointed out in §32, there are many possible ways to control aberrations. Here we consider what is perhaps the simplest method. We do not control the amplitude of the variation of spherical aberration, but merely require that the value of spherical aberration be the same at certain chosen points in the zoom range. If we specify a value for the coefficient of spherical aberration then we may exercise control at three points in the zoom range. If we relax control on the level of this coefficient we may, theoretically, control it at four points.

As a test system, we take the one studied by Jamieson and reproduced in §24 (i). The first order design parameters of this system are

ϕ_1	ϕ_2	ϕ_3	d_1	d_2
0.286426	-0.884442	0.700642	1.100	0.102

The zoom lens will be designed to work at a numerical aperture of $f/8$. The full field will be 5° at the maximum focal length.

The focal length of a lens is, according to L (5.104)

$$f = \frac{N_1}{N'_n v'_{pn}}, \quad (36.1)$$

where v'_{pn} is calculated using canonical co-ordinates. Using (33.7) and the data for the system under consideration, it is found that

$$v'_{p3} = 0.18330 + 0.18923 z + 0.17748 z^2.$$

The maximum focal length of the lens is thus $f = 5.45554$ units. A full field of 5° corresponds to $v_{b1} = 0.04363$. With these values for the focal length and field angle it is found that a numerical aperture of $f/8$ corresponds to an initial entrance pupil radius of 0.3195 units. The stop is placed at an initial distance of 1.1 units [16], this being the minimum realistic value. It is found, using (29.1), that for $z = 0$ the pupil magnification is 1.93843 and so it follows that the radius of the stop must be 0.16482 units. The paraxial invariant is $\lambda = 0.0140$.

We will duplicate the results obtained by Jamieson[†] for his system 6d (in [9]). Jamieson has chosen a value of 0.001 for his coefficient of spherical aberration. Because of a difference in formulae we aim for $*A'_{a3} = 0.0005$. Following Jamieson we choose the points of full compensation for spherical aberration to be $z = 0, 0.3, 1.0$. With these points it is found that the simultaneous equations for S_1 , S_2 and S_3 , obtained from (33.1), are

$$\begin{aligned} 0.012353 S_1^2 - 0.025912 S_1 - 0.15009 S_2^2 + 0.012668 S_2 + \\ + 0.047929 S_3^2 + 0.027218 S_3 = 0.105225, \end{aligned}$$

[†] Systems in [9] will be referred to by J followed by the number given in [9], e.g. System 6d in [9] is J6d.

$$\begin{aligned}
&0.012353 S_1^2 - 0.025912 S_1 - 0.240798 S_2^2 + 0.059652 S_2 + \\
&\quad + 0.135159 S_3^2 + 0.060647 S_3 = 0.219556 , \\
&0.012353 S_1^2 - 0.025912 S_1 - 0.607084 S_2^2 + 0.313535 S_2 + \\
&\quad + 1.192477 S_3^2 + 0.050612 S_3 = 2.893702 . \quad (36.2a,b,c)
\end{aligned}$$

These equations may be solved analytically. The result of elimination of S_1 and S_2 from (36.2c) using (36.2a,b) is a quartic equation in S_3 . The roots of this quartic are possible values of S_3 . The roots are

$$\begin{array}{ll}
1.831789 & -1.692984 \\
1.919399 & -1.645075 .
\end{array} \quad (36.3)$$

Similarly elimination of S_1 and S_3 gives a quartic in S_2 . The roots of this, in corresponding order to (36.3) are

$$\begin{array}{ll}
1.904647 & -1.489476 \\
-0.709821 & 1.155313 .
\end{array} \quad (36.4)$$

Substitution of a value of S_3 , and the corresponding value of S_2 , into (36.2a) gives a quadratic equation for S_1 . Both roots are possible values of S_1 . Because of this there are eight possible solutions to equations (36.2a,b,c). The solutions occur in pairs (conjugate pairs) which differ only in the value of S_1 . Both solutions in such a pair have exactly the same primary spherical aberration at every point in the zoom range. This occurs because the two shapes give rise to the same contribution to the fixed aperture spherical aberration. The full set of solutions to equations (36.2a,b,c) is contained in table II. The curvatures of the various solutions are given in table III. Solution number 4 in table III is the solution obtained by

Table II

s_3	s_2	s_1
1.831789	1.904647	6.938346
1.831789	1.904647	-4.840718
1.919399	-1.489476	5.473511
1.919399	-1.489476	-3.375883
-1.692984	-0.709821	4.061690
-1.692984	-0.709821	-1.964062
-1.645075	1.155313	5.264882
-1.645075	1.155313	-3.167254

Table III

	Lens 1		Lens 2		Lens 3	
	c_1	c_2	c_1	c_2	c_1	c_2
System 1	1.584802	1.185524	-2.378871	-0.740894	1.382897	0.406201
System 2	-0.766256	-1.166634	-2.378871	-0.740894	1.382897	0.406201
System 3	1.292364	0.893086	0.400875	2.038851	1.425680	0.448985
System 4	-0.474318	-0.873596	0.400875	2.038851	1.425680	0.448985
System 5	1.010510	0.611232	-0.237652	1.400323	-0.338419	-1.315111
System 6	-0.192464	-0.591741	-0.237652	1.400323	-0.338419	-1.315111
System 7	1.250714	0.851436	-1.765176	-0.127199	-0.315020	-1.291715
System 8	-0.432667	-0.831945	-1.765176	-0.127199	-0.315020	-1.291715

Jamieson. The advantage of the present method is that all possible solutions to the particular problem are obtained.

If we set $*A'_{a3} = 0.0$ at $z = 0, 0.3$ and 1.0 , it is found that there are no real solutions.

Relaxing control on the level of $*A'_{a3}$ we may demand that spherical aberration be equal at say $z = 0, 0.3, 0.6, 1.0$. The resulting simultaneous equations are

$$0.012353 S_1^2 - 0.025912 S_1 - 0.150090 S_2^2 + 0.012668 S_2 + 0.047929 S_3^2 + \\ + 0.027218 S_3 - 95.965814 *A'_{a3} = 0.057243$$

$$0.012353 S_1^2 - 0.025912 S_1 - 0.240798 S_2^2 + 0.059652 S_2 + 0.135159 S_3^2 + \\ + 0.060647 S_3 - 374.542056 *A'_{a3} = 0.032431$$

$$0.012353 S_1^2 - 0.025912 S_1 - 0.367480 S_2^2 - 0.139017 S_2 + 0.359830 S_3^2 + \\ + 0.100378 S_3 - 1394.700100 *A'_{a3} = -0.134601$$

$$0.012353 S_1^2 - 0.025912 S_1 - 0.607084 S_2^2 + 0.313535 S_2 + 1.192477 S_3^2 + \\ + 0.050612 S_3 - 7773.230910 *A'_{a3} = -0.991301$$

If $*A'_{a3}$ is eliminated from these equations we are left with three equations similar to (36.2a,b,c). These may be solved as before and we find that in this case there are no real solutions.

37. Control of spherical aberration and coma

If we allow the values of spherical aberration and coma, at the points of full compensation, as variables, then five conditions may be satisfied. We may, for example, demand that spherical aberration and coma have three and two points of full compensation respectively. Jamieson has chosen this approach and has used $z = 0, 0.3, 1.0$ and $z = 0, 1$ as the points of full compensation for spherical aberration and coma respectively. The five equations controlling spherical aberration and coma at these points are

$$0.012353 S_1^2 - 0.025912 S_1 - 0.150090 S_2^2 + 0.012668 S_2 + 0.047929 S_3^2 + \\ + 0.027218 S_3 - 95.965814 *A'_{a3} = 0.057243$$

$$0.012353 S_1^2 - 0.025912 S_1 - 0.240798 S_2^2 + 0.059652 S_2 + 0.131590 S_3^2 + \\ + 0.060647 S_3 - 374.542056 *A'_{a3} = 0.032431$$

$$0.012353 S_1^2 - 0.025912 S_1 - 0.607084 S_2^2 + 0.313535 S_2 + 1.192477 S_3^2 + \\ + 0.050612 S_3 - 7773.230910 *A'_{a3} = -0.991301$$

$$-0.045235 S_1 + 0.241040 S_2^2 - 0.300620 S_2 - 0.086920 S_3^2 - 0.188720 S_3 - \\ - 669.729589 *\bar{A}'_{a3} + 459.205441 *A'_{a3} = -0.146348$$

$$-0.045235 S_1 + 0.062494 S_2^2 - 0.595951 S_2 - 0.966947 S_3^2 - 0.785230 S_3 - \\ - 6297.566305 *\bar{A}'_{a3} + 6616.527511 *A'_{a3} = 1.050266$$

(37.1a,b,c,d,e)

These equations are obtained by substituting $z = 0, 0.3, 1.0$ in (33.1) and $z = 0, 1$ in (33.2).

These equations cannot be solved analytically, but we can, as a first step eliminate $*A'_{a3}$ and $*\bar{A}'_{a3}$. The result is three equations in S_1, S_2 and S_3 . The three equations are

$$0.012353 S_1^2 - 0.025912 S_1 - 0.118842 S_2^2 - 0.003517 S_2 + 0.017879 S_3^2 + \\ + 0.015702 S_3 = 0.065790$$

$$0.012353 S_1^2 - 0.025912 S_1 - 0.144378 S_2^2 + 0.008907 S_2 + 0.033622 S_3^2 + \\ + 0.026926 S_3 = 0.070350$$

$$0.017877 S_1^2 - 0.016266 S_1 - 0.335070 S_2^2 + 0.136318 S_2 + 0.059043 S_3^2 + \\ + 0.090472 S_3 = 0.215252$$

(37.2a,b,c)

We use the fact that the coefficients of S_1^2 and S_1 are the same in (37.2a,b) to develop a graphical method to find solutions.

Elimination of S_1 and S_1^2 between (37.2a,b) provides one quadratic equation in S_2 and S_3 , namely

$$S_2^2 - 0.486529 S_2 - (0.616502 S_3^2 + 0.439536 S_3 - 0.178571) = 0 \quad (37.3)$$

S_2 will be real if

$$S_3^2 + 0.712951 S_3 - 0.193663 \geq 0 \quad (37.4)$$

Thus S_2 is real when

$$S_3 \geq 0.209862 \quad \text{or} \quad S_3 \leq -0.922813 \quad .$$

We seek solutions of equations (37.2a,b,c) in the following manner.

Choose three values of S_3 in one of the regions above (i.e.

$S_3 \geq 0.209862$ or $S_3 \leq -0.922813$).

Now carry out the following steps for each value of S_3 .

(1) Solve (37.3). There are two solutions; call them S_{2+} and S_{2-} .

For example, choosing $S_3 = 0.3, 1.0, 1.5$ we obtain the results exhibited in table IV.

Table IV

S_3	S_{2+}	S_{2-}
0.3	0.5039416	-0.017413
1.0	1.211069	-0.724540
1.5	1.631442	-1.144914

(2) Solution S_{2+} and solution S_{2-} must be investigated. By way of example we consider the S_{2+} solution. Values of S_3 and the corresponding values of S_{2+} are substituted into equation (37.2a). Since this is quadratic in S_1 there are again two solutions, S_{1+} and

S_1^- . Results of solution of (37.2a) are contained in table V.

(3) We now use equation (37.2c). From this we obtain the function

$$Y = 0.017877 S_1^2 - 0.017266 S_1 - 0.335070 S_2^2 + 0.136318 S_2 + 0.059043 S_3^2 + 0.090472 S_3 \quad (37.5)$$

Values of S_3 and the corresponding values of S_2 and S_1 are substituted into this and values of Y obtained. The S_3 , S_2+ , S_1+ solution gives values of Y which are shown in the fifth column of table V.

Table V

S_3	S_2+	S_1+	S_1^-	$Y (S_1+)$
0.3	0.503942	3.964441	-1.866813	0.228576
1.0	1.211069	5.310544	-3.212916	0.235633
1.5	1.631442	6.276910	-4.179282	0.195094

(4) A graph may now be drawn showing Y versus S_3 . Alternatively a parabola may be fitted through the points shown in table V. The quadratic approximation to Y is

$$Y = -0.257004 S_3^2 + 0.344186 S_3 + 0.148450 \quad (37.6)$$

For (37.2c) to be satisfied requires $Y = 0.215252$. Putting $Y = 0.215252$ in (37.6) gives a quadratic equation, the roots of which give values of S_3 for which $Y = 0.215252$ (if the quadratic approximation is good). One of these roots may be in a region of S_3 which is forbidden. If this is so it does not give rise to a solution of the original equations. For each root in the chosen region of S_3 we may calculate S_2+ and then S_1+ . Substitution of these values into the original equations (37.1a,b,c,d,e) gives the corresponding levels of

spherical aberration and coma.

(5) A check should be carried out to see if the quadratic approximation holds. Substitution of an allowed root of (37.6), and the corresponding values of S_2^+ and S_1^+ , into (37.5) will show whether the approximation gives a value of Y close to the required value. A comparison of the values of the levels of spherical aberration calculated from (37.1b,c) and a comparison of the values of the levels of coma calculated from (37.1d,e) will show how good the approximation is. If the approximation is not good, three points chosen in the neighbourhood of the approximate solution should be used to repeat steps (1)-(5).

In the $S_3 \geq 0.209862$ region there are four possible search combinations which we label

S_3	S_2^+	S_1^+
S_3	S_2^+	S_1^-
S_3	S_2^-	S_1^+
S_3	S_2^-	S_1^-

There are another four combinations in the $S_3 \leq -0.922813$ region. To find all real solutions each of these combinations must be considered. In the present case only two real solutions were found. These solutions are shown in Table VI. The curvatures given by these solutions are shown in Table VII.

Table VI

S_3	S_2	S_1	$*A_{a3}$	$\bar{*}A_{a3}$
0.222943	0.339383	3.746070	1.51×10^{-4}	-1.06×10^{-4}
1.302119	1.467281	5.890979	3.22×10^{-4}	-4.10×10^{-4}

Table VII

	Lens 1		Lens 2		Lens 3	
	c_1	c_2	c_1	c_2	c_1	c_2
System 9	0.947499	0.548221	-1.096938	0.541037	0.597721	-0.379473
System 10	1.375707	0.976429	-2.020673	-0.382696	1.124233	0.147538

The solutions shown are, of course, approximate. However it would be possible to refine these solutions even further by repeating steps (1)-(5) for points closer and closer to the actual solution.

System 9 agrees (to within 5%) with the solution given by Jamieson (system J7).

38. Control of spherical aberration, coma and astigmatism

If we allow the levels of spherical aberration, coma and astigmatism, at their points of full compensation, to be variables, six conditions may be satisfied. That is, we may control each aberration at two points (we choose the end points) in the zoom range. The six equations are obtained by putting $z = 0, 1$ in (33.1,2,3). Thus

$$\begin{aligned}
& 0.012353 S_1^2 - 0.025912 S_1 - 0.150090 S_2^2 + 0.012668 S_2 + 0.047929 S_3^2 + \\
& \quad + 0.027218 - 95.965814 *A'_{a3} = 0.057243 \\
& 0.012353 S_1^2 - 0.025912 S_1 - 0.607084 S_2^2 + 0.313535 S_2 + 1.192477 S_3^2 + \\
& \quad + 0.050612 S_3 - 7773.230910 *A'_{a3} = -0.991301 \\
& -0.045235 S_1 + 0.241040 S_2^2 - 0.300620 S_2 - 0.086920 S_3^2 - 0.188720 S_3 - \\
& \quad - 699.729589 *\bar{A}'_{a3} + 459.205441 *A'_{a3} = -0.146348 \\
& -0.045235 S_1 + 0.062494 S_2^2 - 0.595951 S_2 - 0.966974 S_3^2 - 0.785230 S_3 - \\
& \quad - 6297.566305 *\bar{A}'_{a3} + 6616.527511 *A'_{a3} = 1.050266 \\
& -0.774210 S_2^2 + 1.865800 S_2 + 0.315260 S_3^2 + 1.189900 S_3 - \\
& \quad - 5102.040785 *\bar{B}'_{a3} + 13393.087436 *A'_{a3} - \\
& \quad - 4394.682338 *A'_{a3} = 0.009590 \\
& -0.012870 S_2^2 + 0.238779 S_2 + 1.568234 S_3^2 + 2.321167 S_3 - \\
& \quad - 5102.040785 *\bar{B}'_{a3} + 21441.802544 *\bar{A}'_{a3} - \\
& \quad - 11263.892694 *A'_{a3} = -2.212227 \quad (38.1a,b,c,d,e,f)
\end{aligned}$$

Once again the equations controlling the aberrations cannot be solved analytically. However, because of certain features of the equations, a graphical method, similar to that employed in §37, may be used. As in §37 the coefficients of S_1 and S_1^2 in (38.1a,b) and the coefficients of S_1 are the same in (38.1c,d). S_1 does not appear explicitly in (38.1e,f), but we note that the coefficient of $*\bar{B}'_{a3}$ is the same in these equations. We may therefore eliminate S_1 and S_1^2 between (38.1a,b), S_1 between (38.1c,d) and $*\bar{B}'_{a3}$ between (38.1e,f). This leaves three equations in S_2 , S_3 , $*A'_{a3}$ and $*\bar{A}'_{a3}$. These are

$$\begin{aligned}
& 0.456994 S_2^2 + 0.300867 S_2 + 1.144548 S_3^2 + 0.0233940 S_3 - \\
& - 7677.265096 *A'_{a3} = -1.048544 \\
& -0.178546 S_2^2 - 0.295331 S_2 - 0.880054 S_3^2 - 0.596510 S_3 - \\
& - 5597.836716 *\bar{A}'_{a3} + 6157.322060 *A'_{a3} = 1.196614 \\
& 0.761340 S_2^2 - 1.627021 S_2 + 1.252974 S_3^2 + 1.131267 S_3 + \\
& + 8048.715108 *\bar{A}'_{a3} - 6869.210356 *A'_{a3} = -2.221817 .
\end{aligned}
\tag{38.2a,b,c}$$

From (38.2b,c) we can eliminate $*\bar{A}'_{a3}$ and this gives

$$\begin{aligned}
& 0.206996 S_2^2 - 0.841592 S_2 - 0.005082 S_3^2 + 0.112226 S_3 + \\
& + 813.818272 *A'_{a3} = -0.205631 .
\end{aligned}
\tag{38.3}$$

$*A'_{a3}$ may now be eliminated between (38.2a) and (38.3). The result of this is a single equation quadratic in S_2 and S_3 . This is

$$S_2^2 - 5.106824 S_2 + (0.733160 S_3 + 0.723450 S_3^2 + 1.997956) = 0. \tag{38.4}$$

Equation (38.4) has real roots if

$$S_3^2 + 0.986755 S_3 - 6.167761 \leq 0 .$$

The last condition holds if S_3 lies in the region

$$-3.025408 \leq S_3 \leq 2.038654. \text{ We confine our search for solutions to } (38.1a,b,c,d,e,f) \text{ to this region alone.}$$

The procedure for finding solutions is as follows. First choose three values of S_3 in the allowed region and repeat each of the following steps for these values.

- (1) Substitute S_3 into (38.4) and solve for S_2 . Call the two solutions S_{2+} and S_{2-} .

(2) Substitute S_3 and S_2 (either S_{2+} or S_{2-}) into (38.3) and thence obtain $*A'_{a3}$.

(3) Now use the results obtained so far to solve (38.2c). This gives $*A'_{a3}$.

(4) S_1 is now calculated by substituting known values of S_3 , S_2 , $*A'_{a3}$, $*A'_{a3}$ into equation (38.1c).

(5) From (38.1a) we obtain the expression

$$Y = 0.012353 S_1^2 - 0.025912 S_1 - 0.150090 S_2^2 + 0.012668 S_2 + 0.047929 S_3^2 + 0.027218 S_3 - 95.965814 *A'_{a3} \quad (38.5)$$

Using the values of the variables obtained in steps (1)-(4) a value of Y can be obtained.

(6) A graph of Y versus S_3 can be plotted for the chosen values of S_3 . Alternatively we may fit a quadratic through the three points. From (38.1a) we see that we require $Y = 0.057243$. Putting this into (38.5) gives a quadratic equation the roots of which are the values of S_3 for which $Y = 0.057243$. Using these values of S_3 , the values of the other variables can be obtained by repeating steps (1)-(4).

As in §37 we must investigate several possible solutions. In this case there are only two possible combinations, namely

$$\begin{array}{ccc} S_3 & S_{2+} & S_1 \\ S_3 & S_{2-} & S_1 \end{array} .$$

However these must be checked throughout the whole of the allowed region. One combination may give rise to several solutions.

Taking $S_3 = 0, 1, 2$ we find that only the S_2^- combination gives real solutions. Table VIII contains values of the various variables and Y for three values of S_3 .

Table VIII

S_3	S_2^-	$*A'_{a3}$	$*\bar{A}'_{a3}$	S_1	Y
0.0	0.426923	1.42×10^{-4}	-0.85×10^{-4}	4.125632	0.067777
1.0	0.802600	2.81×10^{-4}	-2.31×10^{-4}	1.685884	-0.046910
2.0	2.176032	5.42×10^{-4}	-7.25×10^{-4}	14.692406	1.796901

(7) The solution obtained by fitting a quadratic to the values for S_3 and solving for $Y = 0.057243$ must be checked by substitution in (38.1a,b,c,d,e,f). If the approximation is not sufficiently accurate then the steps outlined above must be repeated for three points in the neighbourhood of the solution. In the example above (table VIII) we see that there must be one root near zero and another between $S_3 = 1$ and $S_3 = 2$. The quadratic fitted through $S_3 = 0, 1, 2$ is not satisfactory but does give an indication of where the solutions lie.

After a few repetitions of steps (1)-(6), for each solution, accurate results are obtained.

It is found that in all of the allowed region of S_3 there are only two real solutions. These are set out in table IX and table X.

Table IX

S_3	S_2	S_1	$*A'_{a3}$	$*\bar{A}'_{a3}$	$*\bar{B}'_{a3}$
0.049787	0.435838	4.003185	1.42×10^{-4}	-0.92×10^{-4}	-2.2×10^{-4}
1.792207	1.620422	5.872532	5.28×10^{-4}	-4.98×10^{-4}	-9.3×10^{-4}

Table X

	Lens 1		Lens 2		Lens 3	
	c_1	c_2	c_1	c_2	c_1	c_2
System 11	0.998830	0.599552	-1.175934	0.462041	0.512660	-0.464034
System 12	1.327705	0.928427	-2.130221	-0.492244	1.358230	0.381535

Although only two points of full compensation were specified for spherical aberration, it is found that there are in fact three. The extra one occurs at $z = 0.5$. It will be noticed that system 11 is similar to system 9 and system 12 is similar to system 10. Neither system 11 or 12 is the same as system J8. An analysis of system J8 using a computer programme for aberrations (see §39) indicates that J8 is in fact not a solution of the problem.

39. Aberrations of systems 1 - 12

The primary aberrations of systems 1 - 12 can be obtained by substituting values of S_1 , S_2 and S_3 into equations (31.1). Through the use of equations (28.7) graphs of the primary aberrations can be drawn. In each of the three cases considered (i.e. in §§36,37,38) there is more than one solution with the required properties. It is possible to choose between alternative solution on the basis of the behaviour of the primary aberrations, but it is more satisfactory to take into account the secondary aberrations.

To do this a computer programme, which was written to calculate aberration coefficients (up to the ninth order), was modified for use with zoom lenses. At a chosen number of points,

equally spaced in the zoom range, the programme calculates the diameter and position of the entrance pupil and uses these to calculate the actual aberration coefficients of orders one to nine. Using this programme graphs of certain of the fifth order aberration coefficients were obtained for systems 1 - 12.

We now consider each of the sets of solutions 1 - 8, 9 and 10 and 11 and 12, separately.

(i) Figure 2 shows the primary spherical aberration of the conjugate pairs of systems 1 and 2, and 5 and 6. These are, respectively, the 'worst' and 'best' of the eight solutions (considering primary spherical aberration as the only criterion of quality). On the figures the primary aberrations are denoted by σ_1 , σ_2 , etc., and the secondary aberrations by μ_1 , μ_2 , etc.

Graphs of primary spherical aberration for all eight systems show that system 4, which is the solution given by Jamieson, is far from being best.

Let us assume that for the zoom lens being designed only spherical aberration is important (we have corrected only primary spherical aberration). We can then choose between the different systems on the basis of third and fifth order spherical aberration. The fifth order spherical aberration for each of systems 1, 2, 5 and 6 is shown in figure 3. Average (or d.c.) levels of aberrations are not important because we can design $(\mathbb{K})$ to compensate for these. To obtain a balance between third and fifth order spherical aberration requires that the overall $(\mathbb{Z} + \mathbb{K})$ third order coefficient be negative when the overall fifth order coefficient is positive and vice versa, i.e. the third and fifth order coefficients of spherical

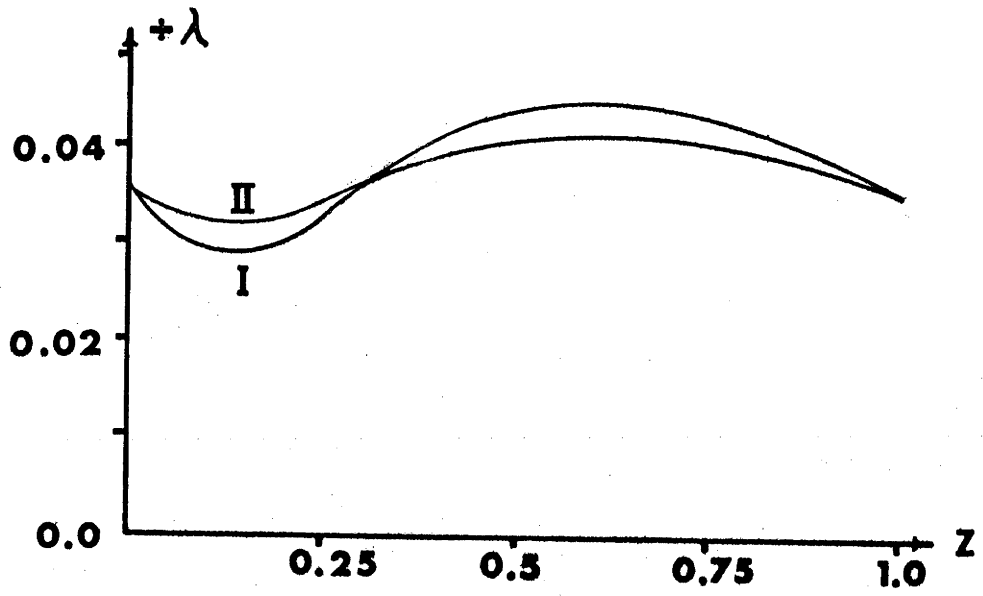


Figure 2. Primary spherical aberration for (I) systems 1 and 2, and (II) systems 5 and 6.

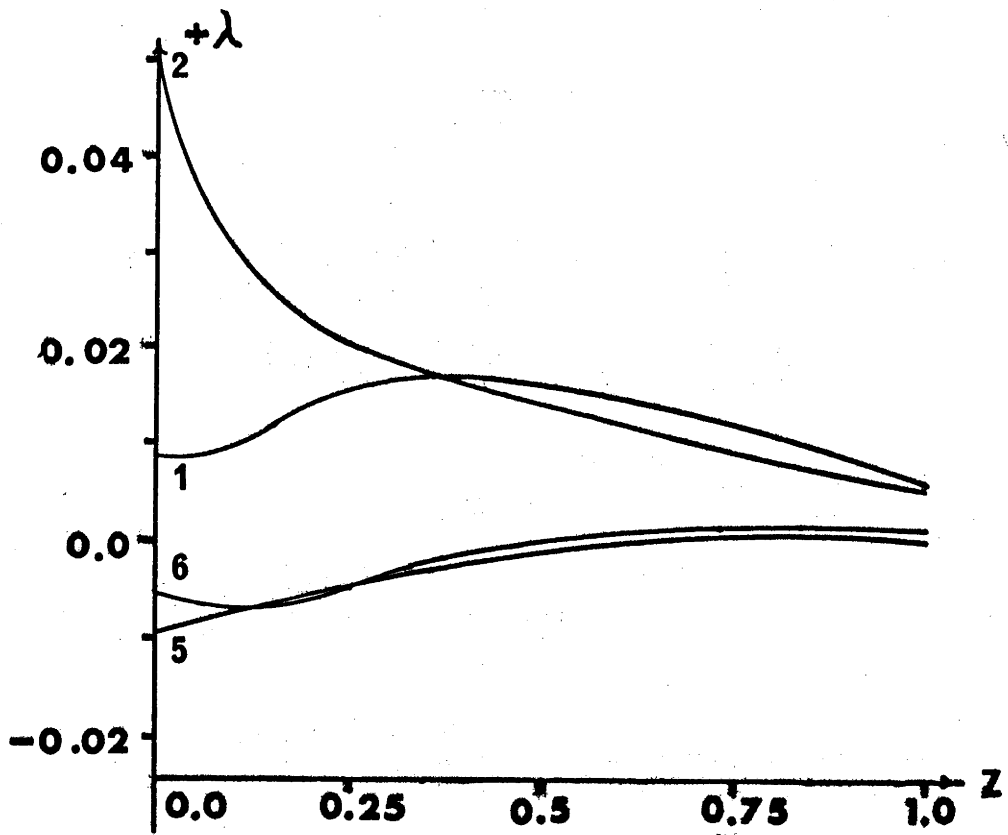


Figure 3. Secondary spherical aberration for systems 1, 2, 5 and 6.

aberration should be 180° out of phase. In none of the systems obtained is this true or even approximately true and so we must find a system in which neither the third nor the fifth order coefficients vary wildly. System 6 is one such system and system 8 is another (not shown).

(ii) Figure 4 shows spherical aberration and coma for systems 9 and 10. A glance at this shows that, from the viewpoint of primary aberrations, the system obtained by Jamieson, system 9, is the better. The fifth order spherical aberration and coma of these systems are shown in figure 5. These confirm the view that system 9 is the better of the two.

(iii) Figures 6a and b show spherical aberration, coma and astigmatism for systems 11 and 12 respectively. Because of the similarity between systems 9 and 11 and 10 and 12 we expect that system 11 will be the better of the pair. Consideration of both the third and fifth order aberrations confirms this. The fifth order aberrations are shown in figures 7a and b.

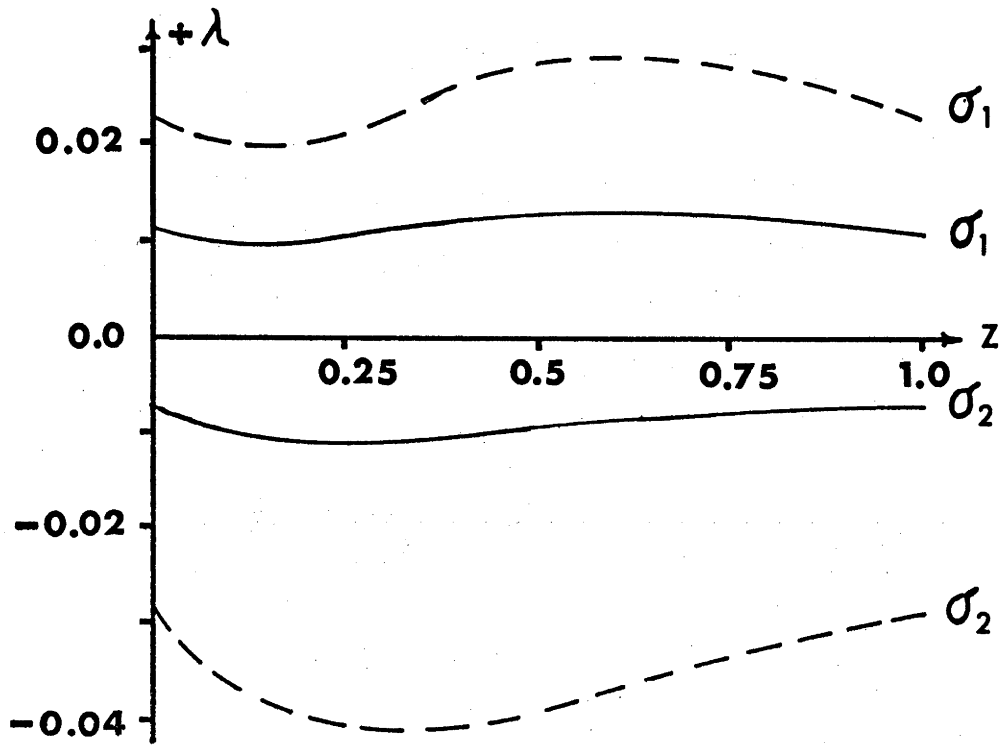


Figure 4. Primary spherical aberration (σ_1) and coma (σ_2) for system 9 (full) and system 10 (broken).

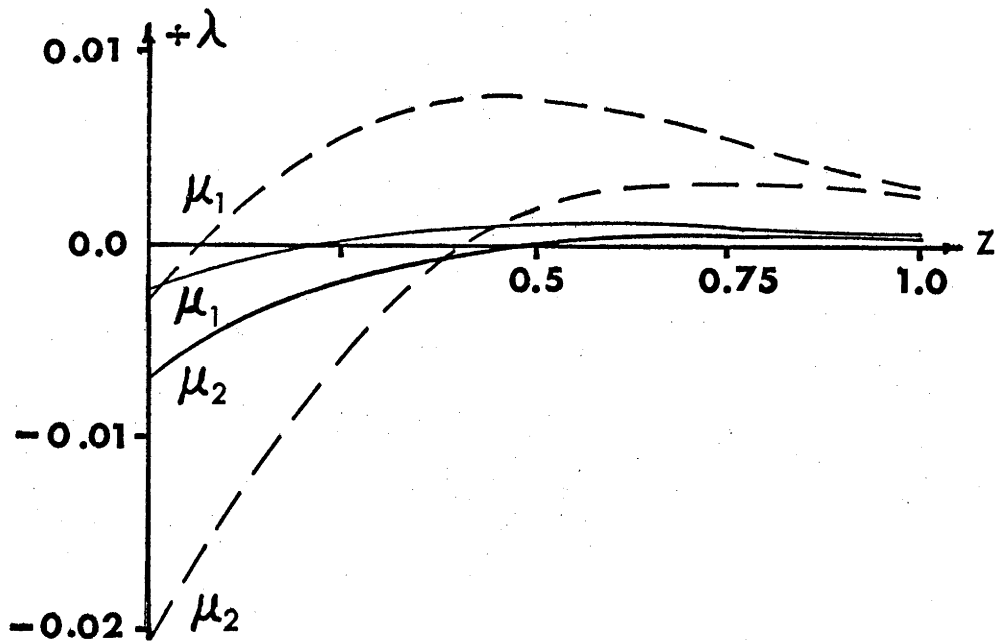


Figure 5. Secondary spherical aberration (μ_1) and coma (μ_2) for system 9 (full) and system 10 (broken).

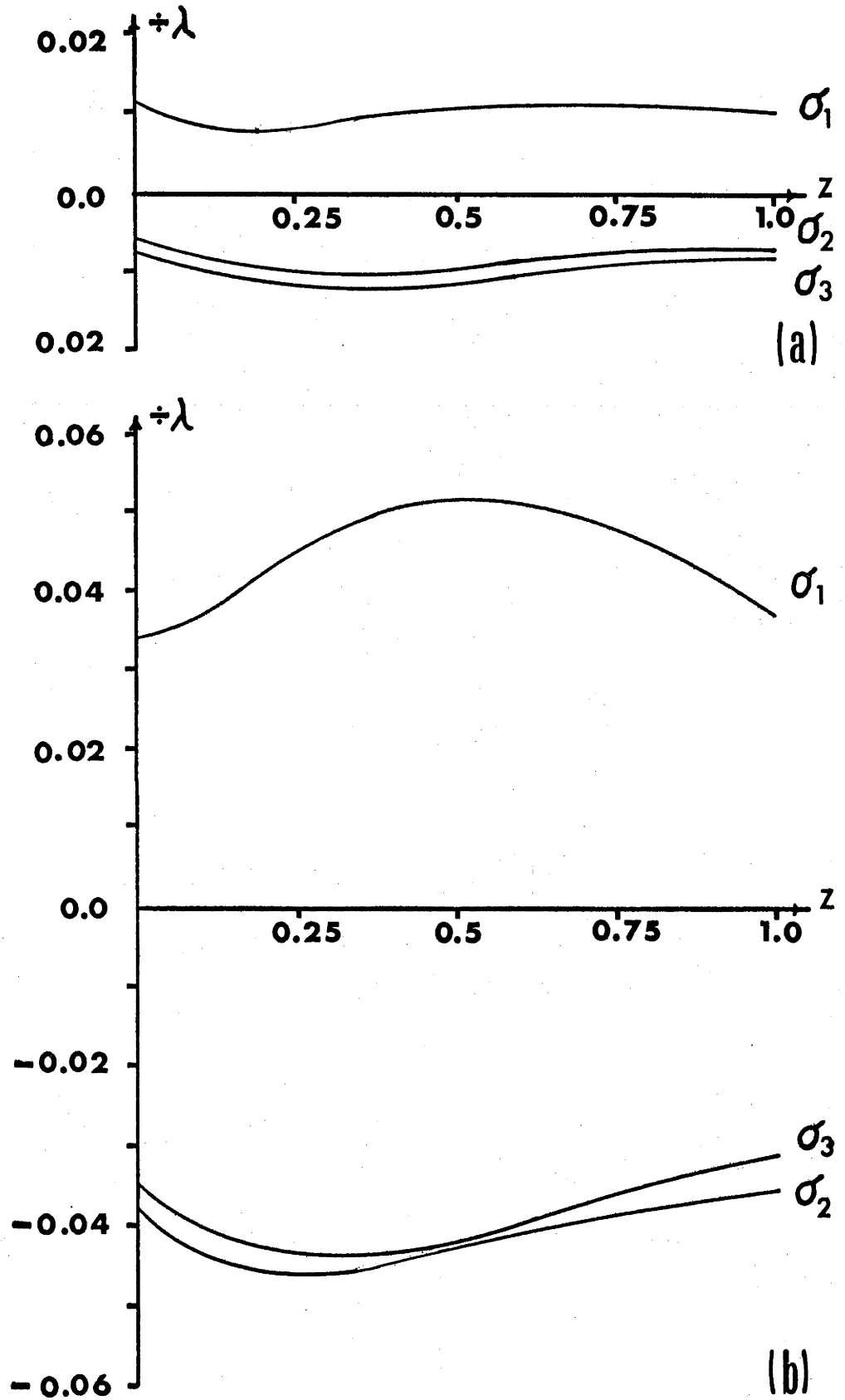


Figure 6. Primary spherical aberration (σ_1), coma (σ_2) and astigmatism (σ_3) for (a) system 11, and (b) system 12.

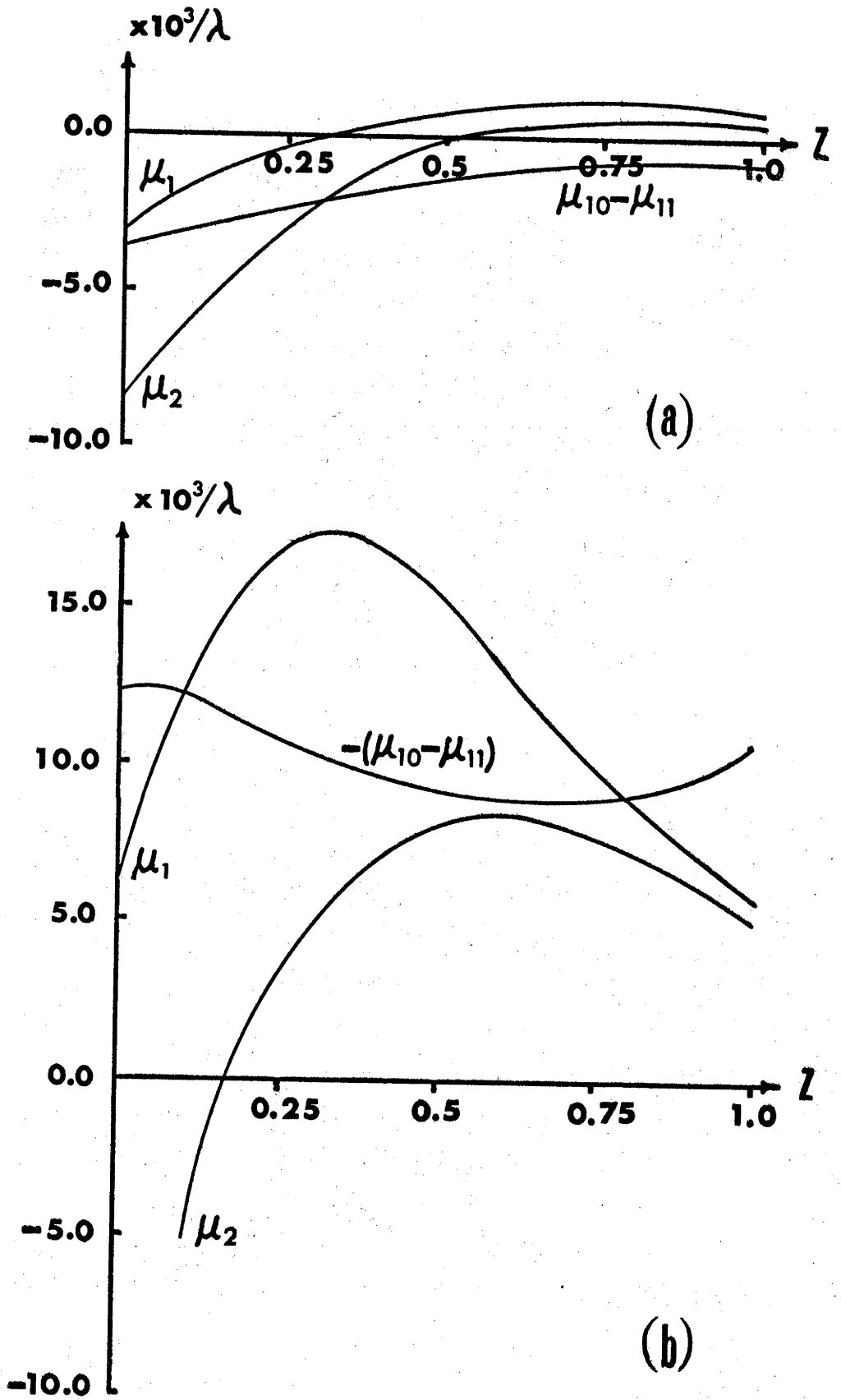


Figure 7. Secondary spherical aberration (μ_1), coma (μ_2) and astigmatism ($\mu_{10}-\mu_{11}$) for (a) system 11, and (b) system 12.

APPENDIX 1

40. Use of aspheric surfaces for controlling aberrations

The contributions to the third order aberration coefficients from an aspheric surface are, according to L (67.1)

$$\begin{aligned}
 \underline{a}_a &= \overset{o}{\underline{a}}_a + \alpha & \overline{\underline{b}}_a &= \overset{o}{\overline{\underline{b}}}_a + 2\tilde{q}^2\alpha \\
 \overline{\underline{a}}_a &= \overset{o}{\overline{\underline{a}}}_a + \tilde{q}\alpha & \underline{c}_a &= \overset{o}{\underline{c}}_a + \tilde{q}^2\alpha \\
 \underline{b}_a &= \overset{o}{\underline{b}}_a + 2\tilde{q}\alpha & \overline{\underline{c}}_a &= \overset{o}{\overline{\underline{c}}}_a + \tilde{q}^3\alpha .
 \end{aligned} \quad (40.1)$$

In (40.1) $\alpha = c_1(\Delta N) y_a^4$ and $q = y_b/y_a$. The terms such as $\overset{o}{\underline{a}}_a$ are the contributions calculated for a spherical surface. These contributions are given by equations (5.2) and (5.3). c_1 is the first extra axial curvature of the surface in question. The meaning of this is explained in L §56. It suffices to say here that c_1 is a measure of the departure from sphericity of a surface.

For a single thin lens in air, using canonical co-ordinates, we have (using p and q to indicate canonical co-ordinates) $y_p = 1$ and $y_q = 0$ at both surfaces. This means that $\tilde{q} = 0$ at both surfaces and so the contributions to all third order aberration coefficients, except that of spherical aberration, are the same as for spherical surfaces. Let c_{11} and c_{12} be the first extra axial curvatures of the first and second surfaces respectively. The coefficient of spherical aberration for the lens is

$$\begin{aligned}
 A'_p &= a_{p1} + a_{p2} \\
 &= \overset{o}{A}'_p - (N - 1)(c_{11} - c_{12}) \\
 &= \overset{o}{A}'_p - \phi_1 .
 \end{aligned} \quad (40.2)$$

$\phi_1 = (N - 1)(c_{11} - c_{12})$ is the first extra axial power of the thin lens.

Equations (26.3, (26.6) give the fixed aperture aberrations for a system of thin lenses and we find, substituting (40.2) for A'_{pi} wherever this occurs, that the fixed aperture aberration coefficients are

$$\begin{aligned} A'_{an} &= {}^o A'_{an} - \sum_{i=1}^n \pi^4 \phi_1 \\ \bar{A}'_{an} &= {}^o \bar{A}'_{an} - \sum_{i=1}^n \pi^3 \phi_1 \\ \bar{B}'_{an} &= {}^o \bar{B}'_{an} - \sum_{i=1}^n 2\pi^2 \phi_1 \\ C'_{an} &= {}^o C'_{an} - \sum_{i=1}^n \pi^2 \phi_1 \\ \bar{C}'_{an} &= {}^o \bar{C}'_{an} - \sum_{i=1}^n \pi \phi_1 \end{aligned} \quad (40.3)$$

The terms such as ${}^o A'_{an}$ are the fixed aperture aberration coefficients calculated for spherical surfaces, i.e. using equations (31.1). If equations (42.3) are used in (33.1), instead of equations (31.1), we see that three extra variables, occurring linearly, have been introduced. Considering the levels of the third order aberrations to be variables, this gives us ten degrees of freedom, i.e. three shapes, three first extra axial powers and four aberrational levels. Using these we can control the four third order aberrations (the five Seidel aberrations except σ_4 which is determined by our choice of refractive indices) at ten points. For example we might require that the number of points of full compensation for each aberration be

spherical aberration:	3 ,
coma:	3 ,
astigmatism:	2 ,
distortion:	2 .

Substituting the values of z at the chosen points of full compensation, we obtain ten equations in ten unknowns. Because seven of these occur linearly, and because of the characteristics remarked upon in §35, we may use a process similar to that described in §38 to find solutions. Simplification occur because the coefficient of ϕ_{11} behaves similarly to the coefficient of S_1^2 .

APPENDIX 2

41. The programme DERIVATIVES

A programme, called DERIVATIVES, was described in §12. This programme, which calculates the derivatives, with respect to design parameters, of chromatic paraxial and monochromatic third and fifth order aberration coefficients.

The theory used in the programme is contained in Part I of this thesis and is discussed in §12. Instructions for the use of the programme are contained in the comments at the start of the programme itself. Comments throughout the programme explain the purpose of the different sections and subroutines. A listing of DERIVATIVES follows.

```

C**** PROGRAMME DERIVATIVES
C THIS PROGRAMME CALCULATES CHROMATIC PARAXIAL, THIRD AND FIFTH ORDER
C ABERRATIONS AND THEIR FIRST DERIVATIVES WITH RESPECT TO THE FOLLOWING
C PARAMETERS -----SEPARATIONS---CURVATURES---REFRACTANCES---REFRACTIVE
C INDICES ----- ALSO FIRST AND SECOND EXTRA-AXIAL CURVATURES WHEN THESE
C ARE ZERO.
C PRODUCTION OF CHROMATIC ABERRATIONS AND DERIVATIVES IS OPTIONAL.
C SPECIFY NOCOL = ANY NUMBER OTHER THAN 2 TO PRODUCE THESE QUANTITIES.
C THE THEORY USED IN THIS PROGRAMME IS CONTAINED IN 'OPTICAL ABERRATION
C COEFFICIENTS' BY H.A. BUCHDAHL, DOVER PUBLICATIONS 1968 LIBRARY OF
C CONGRESS CATALOGUE NUMBER 68-16165
C THE THESIS MENTIONED BELOW IS THE THESIS ENTITLED 'THEORY AND DESIGN
C OF ZOOM LENSES' BY A.T. O'LEARY A.N.U. 1972
C ENTRY OF DATA. 1) CARD SPECIFYING NUMBER OF SURFACES ( 10 OR LESS ).
C 2) CARD SPECIFYING YA1, VA1 AND REFRACTIVE INDEX OF IMAGE SPACE MEDIUM.
C 3) CARD SPECIFYING YB1, VB1, SURFACE 4) CARDS SPECIFYING SEPARATION, CURVATURE
C REFRACTIVE INDEX OF J TH SURFACE. FIRST SEPARATION MUST BE ZERO.
C 5) CARD SPECIFYING VALUES OF OPTION PARAMETERS. THESE ARE NOCOL AND FULL.
C IF NOCOL = 2 NO CHROMATIC QUANTITIES ARE PRODUCED. IF FULL = 2 THEN
C AUXILIARY QUANTITIES ( THETAS AND ALFAS ) ARE PRODUCED.
C INTEGER*4 FULL
C DIMENSION HW(10), W1(10), W2(10), W3(10), W4(10), W5(10), XJ(10)
C DIMENSION FGA(11,4), FGB(11,4)
C DIMENSION YA(10), YB(10), VA(11), VB(11), XIA(10), XIB(10), D(10)
C DIMENSION P2BA(10), P3A(10), P3BA(10), P1BA(10), P1BB(10), P2B(10)
C DIMENSION C(10), XN(10), Q(10), W(10), P1A(10), P1B(10), P2A(10)
C DIMENSION S5A(10), S6A(10)
C DIMENSION PSUMA(11,6), PSUMB(11,6), S1A(10), S2A(10), S3A(10), S4A(10)
C DIMENSION P2BB(10), P3B(10), P3BB(10), SA(10,6), SB(10,6), XK(10)
C DIMENSION SSUMA(11,12), SSUMB(11,12)
C DIMENSION ST(12,10), PT(6,10)
C COMMON/B19/FSUMA,FSUMB
C COMMON/B21/FGA,FGB
C DIMENSION FSUMA(11,4), FSUMB(11,4)
C DIMENSION PGA(11,6), PGB(11,6), A(12)
C DIMENSION SGA(11,12), SGB(11,12)
C COMMON/INT/KOUT,KINP,NN,KN,NOCOL,FULL
C COMMON/RESULT/DERP(10,6), CERS(10,12), CERPI(10,6),
C 1,XKERPI(10,6), XKERS(10,12), PDN(10,6), SDN(10,12),
C 1,CERS1(10,12), CERS2(10,12)
C COMMON/B1/PGA,PGB
C COMMON/MISC/J,QA,QTILD
C COMMON/THET/TV,TBY,TV,TBV,TTY,TTBY,TTV,TTBV,TTTBY,TTTV,TTTBV
C COMMON/BD1/TH1,TH2,TH3,TH4
C COMMON/BD2/ZZ1,ZZ2,ZZ3,ZZ4,ZZ5,ZZ6
C COMMON/B3/A
C COMMON/B4/PSUMA,PSUMB
C COMMON/B2/SGA,SGB
C COMMON/BC3/8TA,8TB,8TC,8TD,8TE,8TF,8TG,8TH,8TI,8TJ,8TK,8TL
C COMMON/BC2/G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12

```

```
COMMON/DATA/ C,D,XN,XK,XJ,Q,XIA,XIB,YA,YB,VA,VB,W,XLA
COMMON/COEFS/SSUMA,SSUMB,S1A,S2A,S3A,S4A,S5A,S6A
COMMON/POEFS/PIA,PIBA,P2A,P2BA,P3A,P3BA,PIB,P1BB,P2B,P2BB,
1 P3B,P3BB
COMMON/TERMS/W,W1,W2,W3,W4,W5
COMMON/BC1/WA,WB,WC,WD,WE,WF,WG,WH,WI,WJ,WK,WL
KINP=1
KOUT=3
CALL COEFF
PSUMB,SSUMB ARE IN DUAL ORDER
WE PUT THEM IN ORDINARY ORDER FOR CLARITY
DO 1111 J=1,KN
PRIMARY
TEMA=PSUMB(J,6)
TEMB=PSUMB(J,5)
TEMC=PSUMB(J,4)
TEMF=PSUMB(J,2)
TEMG=PSUMB(J,1)
PSUMB(J,1)=TEMA
PSUMB(J,2)=TEMB
PSUMB(J,3)=TEMC
PSUMB(J,4)=TEMD
PSUMB(J,5)=TEMF
PSUMB(J,6)=TEMG
SECONDARY
TEMH=SSUMB(J,12)
TEMI=SSUMB(J,11)
TEMJ=SSUMB(J,10)
TEMK=SSUMB(J,9)
TEML=SSUMB(J,8)
TEMN=SSUMB(J,7)
TEMN=SSUMB(J,6)
TEMQ=SSUMB(J,5)
TEMQ=SSUMB(J,4)
TEMR=SSUMB(J,3)
TEMS=SSUMB(J,2)
TEMT=SSUMB(J,1)
SSUMB(J,1)=TEMH
SSUMB(J,2)=TEMI
SSUMB(J,3)=TEMJ
SSUMB(J,4)=TEMK
SSUMB(J,5)=TEML
SSUMB(J,6)=TEMN
SSUMB(J,7)=TEMN
SSUMB(J,8)=TEML
SSUMB(J,9)=TEMQ
SSUMB(J,10)=TEMR
SSUMB(J,11)=TEMS
SSUMB(J,12)=TEMT
1111
```

```

C      THIS SECTION CALCULATES DERIVATIVES OF THE PRIMARY ABERRATION COEFFS
C      WITH RESPECT TO DJ, COJ, KJ, CIJ WHERE J IS THE SURFACE NUMBER.
C      REMEMBER THIS SECTION REQUIRES ASPHERICITIES
C      C1=0.
C      WE HAVE ASSUMED A SPHERICAL SURFACE
C      THE PRIMARY COEFF DIFFERENCES G1(K,J) ARE CALCULATED NEXT
C      AND ARE STORED IN PGA, PGB
C      DO20I=1,6
C      DO20L=1,KN
C      PGA(L,I)=PSUMA(NN+1,I)-PSUMA(L,I)
C      PGB(L,I)=PSUMB(NN+1,I)-PSUMB(L,I)
C      WRITE(KOUT,95951)
20      FORMAT(' THE FOLLOWING COEFFICIENT DIFFERENCES ARE THE DIFFERENCES
95951 1 BETWEEN THE FINAL AND JTH ABERRATION COEFFICIENTS')
C      WRITE(KOUT,95952)
95952  FORMAT(' PRIMARY COEFFICIENT DIFFERENCES G1(K,J)')
C      DO57J=1,KN
C      WRITE(KOUT,2008)
C      WRITE(KOUT,300)(PGA(J,I),I=1,6)
C      WRITE(KOUT,2009)
C      WRITE(KOUT,300)(PGB(J,I),I=1,6)
C      CONTINUE
57      NOW CALCULATE SECONDARY COEFF DIFFERENCES G2(K,J) AND STORE IN SGA, SGB
C      DO51I=1,12
C      DO51L=1,KN
C      SGA(L,I)=SSUMA(NN+1,I)-SSUMA(L,I)
C      SGB(L,I)=SSUMB(NN+1,I)-SSUMB(L,I)
C      WRITE(KOUT,95953)
95953  FORMAT(' SECONDARY COEFFICIENT DIFFERENCES G2(K,J)')
C      DO58J=1,KN
C      WRITE(KOUT,2008)
C      WRITE(KOUT,300)(SGA(J,I),I=1,12)
C      WRITE(KOUT,2009)
C      WRITE(KOUT,300)(SGB(J,I),I=1,12)
C      CONTINUE
58      IF(NOCOL.EQ.2)GOTO11112
C      CALL COLOUR
C      CALCULATE CHROMATIC COEFF DIFFERENCES AND STORE IN FGA, FGB
C      DO54I=1,4
C      DO54L=1,KN
C      FGA(L,I)=FSUMA(KN,I)-FSUMA(L,I)
C      FGB(L,I)=FSUMB(KN,I)-FSUMB(L,I)
C      WRITE(KOUT,95954)
95954  FORMAT(' PARAXIAL CHROMATIC COEFFICIENT DIFFERENCES')
C      DO400IJ=1,KN
C      WRITE(KOUT,2008)
C      WRITE(KOUT,440)(FGA(J,I),I=1,4)
C      WRITE(KOUT,440)(FGB(J,I),I=1,4)
4001  CONTINUE
11112 CONTINUE

```

```

C THIS IS J DEFINED O.A.C. P323
  XJ(1)=XLA*C(1)/(XN(1)*XIA(1))
  DO678JA=1,NN
C J=JA SECTION DEALS WITH DJ
  QA=Q(J)
  IF(J.EQ.1)GOTO21
C CARD 21 STARTS SECTION FOR COJ
  YN=XN(J)/XLA
  CALL THETA(VA(J),VB(J),VA(J),VB(J),YN)
  TY=TH1
  TBY=TH2
  TV=TH3
  TBV=TH4
  T=TY+TBV
M=J
CALL PDERIV(TY,TBY,TV,TBV,T,M)
C DERP STORES PRIMARY D DERIVATIVES (I.E. DERIVATIVES OF PRIMARY COEFFS )
  DERP(J,1)=ZZ1
  DERP(J,2)=ZZ2
  DERP(J,3)=ZZ3
  DERP(J,4)=ZZ4
  DERP(J,5)=ZZ5
  DERP(J,6)=ZZ6
C DERIVATIVES W.R.T. COJ
21 YN=(XN(J+1)-XN(J))/XLA
  CALL THETA(YA(J),YB(J),YA(J),YB(J),YN)
  TTY=TH1
  TTBY=TH2
  TTV=TH3
  TTBV=TH4
  TT=TTY+TTBV
C WE NOW NEED THE DERIVATIVES OF THE PRIMARY INTRINSIC CONTRIBS (A COEFFS )
  DINTC=.5*XN(J)*(1.-XK(J))*XIA(J)*(3.*XK(J)*XIA(J)-2.*VA(J))
  DINTC=DINTC*YA(J)*#2
  QSTAR=XLA/(XN(J)*XIA(J)*#2)
  DBA=QA*DINTC+QSTAR*PIA(J)
  DB=2.*DBA
  DBB=DB*QA+QSTAR*P2A(J)
  DC=.5*(DBB-(W(J)*XLA**2)/C(J))
  DBC=QA*DC+QSTAR*P3A(J)
M=J+1
CALL PDERIV(TTY,TTBY,TTV,TTBV,TT,M)
C DERP STORES PRIMARY C DERIVATIVES
  CERP(J,1)=ZZ1+DINTC
  CERP(J,2)=ZZ2+DBA
  CERP(J,3)=ZZ3+DB
  CERP(J,4)=ZZ4+DBB
  CERP(J,5)=ZZ5+DC

```

CERP(J,6)=ZZ6+DBC

THIS SECTION CALCULATES DERIVATIVES W.R.T. C1J

QTILD=YB(J)/YA(J)

DAKCI=(XN(J+1)-XN(J))*YA(J)**4

CERP STORES PRIMARY C1 DERIVATIVES

THESE ARE CALCULATED FROM O.A.C. P237

CERP1(J,1)=DAKCI

CERP1(J,2)=QTILD*DAKCI

CERP1(J,3)=QTILD*CERP1(J,2)

CERP1(J,4)=QTILD*CERP1(J,3)

CERP1(J,5)=QTILD*CERP1(J,4)

CERP1(J,6)=QTILD*CERP1(J,5)

CONTINUE

THIS SECTION CALCULATES DERIVATIVES OF SECONDARY ABERRATION COEFFS

W.R.T. DJ, COJ, KJ, C1J

THIS SECTION DEALS WITH DJ

THERE ARE NO SURFACE CONTRIBS

IF(JA.EQ.1)GOTO4041

CALL ALFA(TY,TBY,TJ,XLA)

A(1)...12) ARE ALFA.....GAMMA GIVEN ON P198 OF O.A.C.

A(1)=WA

A(2)=WB

A(3)=WC

A(4)=WD

A(5)=WE

A(6)=WF

A(7)=WG

A(8)=WH

A(9)=WI

A(10)=WJ

A(11)=WK

A(12)=WL

JJ=J

CALL SDERIV(TY,TBY,TJ,XLA)

SDD IS SECONDARY DERIVATIVE W.R.T. DJ

SDBD IS SECONDARY DERIVATIVE W.R.T. DJ (BARRED COEFF)

DER SDBD STORES SECONDARY D DERIVATIVES(I.E. DERIVATIVES OF SECONDARY COEFFS)

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DER SDBD STORES

DERS(J,10)=G10
DERS(J,11)=G11
DERS(J,12)=G12

THIS SECTION DEALS WITH COJ
THERE ARE SURFACE CONTRIBS TO ALFA'S AND DERIVATIVES
ALFA'S REQUIRE DERIVATIVES OF PRIMARY B CONTRIBS
CONTINUE
DINQC=QA*DINTC+QSTAR*PIA(J)-(1.-XK(J))*XLA *YA(J)*VA(J+1)
REMEMBER DUAL ORDER FOR B COEFFS
DQBA=QA*DINQC+QSTAR*P3BB(J)
DQB=2.*DQBA+W(J)*((XLA**2)/C(J)
DQBB=QA*DQB+QSTAR*P2BB(J)
DQC=.5*DQBB
DQBC=QA*DQC+QSTAR*P1BB(J)

N=J+1
CALL ALFA(TTY,TTY,TTV,TTV,TT,N,XLA)
A(1)=WA-DINQC/XLA
A(2)=WB-DQBA/XLA
A(3)=WC-DQB/XLA
A(4)=WD-DQBB/XLA
A(5)=WE-DQC/XLA
A(6)=WF-DQBC/XLA
A(7)=WG-DINTC/XLA
A(8)=WH-DBA/XLA
A(9)=WI-DB/XLA
A(10)=WJ+DQB/XLA
A(11)=WK+DC/XLA
A(12)=WL+DBC/XLA

WE NOW NEED DERIVATIVES OF FULL CONTRIBS. THESE ARE MADE UP OF BITS
PLUS DERIVATIVES OF THE PRIMARY INTRINSIC CONTRIBS.
WE FIRST CALCULATE THE DERIVATIVES OF THE INTRINSIC CONTRIBS (SPHERICAL).

X=XK(J)
XX=XJ(J)
XXX=1./(C(J)*XIA(J))
DD=VA(J+1)-3.*VA(J)
REMEMBER TO DEFINE WW(J) ETC IN PROG 1.
DE=VA(J+1)**2-VA(J)**2
DW1=XX*(DD*DINTC+((1.-X)*YA(J)-VA(J)*DD*XXX)*PIA(J))
DSIC=3.*(WW(J)*DINTC+.25*YA(J)*((1.+X**2)*XIA(J)+(1.-X)*VA(J+1))*P
11A(J))
DP3=4.*QSTAR*SIA(J)+DW1
DS2C=4.*QA*DSIC+DP3
DW2=(.5*DINTC-VA(J))*XXX*PIA(J))*XX**2
DW3=.25*XX*XLA*(YA(J)*(3.*VA(J+1)**2-VA(J)**2)*(1.-X)-VA(J)
1*VA(J+1)*DE*XXX)
DS3C=QA*(2.*QA*DSIC+4.*QSTAR*SIA(J)+DW1)+QSTAR*W1(J)+DW2+DW3
DS4C=DS3C-(2.*DW2+DW3)
DS4C=DS4C+DS4C

```

DW4=XLA*(XLA*W(J))*2/C(J)
DP4=3.*QSTAR*W1(J)-DW2+DW3
DP5=-(QSTAR*2.*(W2(J)-W3(J))-DW4)
DW5=.125*W(J)*XX*(XIA(J)-2.*VA(J))*(XLA*XX)**2
DP6=QSTAR*W4(J)+DW5
DS5C=QA*(2.*DP4+QA*(3.*DP3+QA*4.*DS1C))+DP5
DS6C=DP6+QA*(DP5+QA*(DP4+QA*(DP3+QA*DS1C)))
WE COULD NOW ADD THE ASPHERICAL PART. THIS IS NOT DONE FOR THE MOMENT.
WE CALCULATE THE DERIVATIVE OF THE FULL CONTRIB.
CALL BITS(DINTC,DBA,DBB,DC,DBC, J,XLA)
DSF1C=DS1C+BTA
DSFB1C=QA*DS1C+QSTAR*S1A(J)+BTB
DSFB2C=QA*DS2C+QSTAR*S2A(J)+BTD
DSFB3C=QA*DS3C+QSTAR*S3A(J)+BTF
DSFB4C=QA*DS4C+QSTAR*S4A(J)+BTI
DSFB5C=QA*DS5C+QSTAR*S5A(J)+BTJ
DSFB6C=QA*DS6C+QSTAR*S6A(J)+BTL
DSF2C=DS2C+BTC
DSF3C=DS3C+BTE
DSF4C=DS4C+BTG
DSF5C=DS5C+BTH
DSF6C=DS6C+BTK
JJ=J+1
CALL SDERIV(ITTY,TTBY,TTV,TTBV,TTJJ,JJ)
CERS STORES SECONDARY C DERIVATIVES
CERS(J,1)=G1+DSF1C
CERS(J,3)=G3+DSF2C
CERS(J,5)=G5+DSF3C
CERS(J,7)=G7+DSF4C
CERS(J,9)=G9+DSF5C
CERS(J,11)=G11+DSF6C
CERS(J,2)=G2+DSFB1C
CERS(J,4)=G4+DSFB2C
CERS(J,6)=G6+DSFB3C
CERS(J,8)=G8+DSFB4C
CERS(J,10)=G10+DSFB5C
CERS(J,12)=G12+DSFB6C
NOW CONSIDER KJ
WE NEED SURFACE CONTRIBS FOR BOTH ALFAS AND DERIVATIVE
DERIVATIVES W.R.T. KJ
J=J-1
IF(J.EQ.0)J=1
6006 YN=XN(J+1)/XLA*(-1.)
QA=Q(J)
THE RESULTS OF THIS SECTION ARE THE DERIVATIVES OF ANTEPRIMED
ABERRATION COEFFS
CALL THETA(YA(J),YB(J),XIA(J),XIB(J),YN)
ITTY=TH1

```

```

C
TTTBV=TH2
TTTV=TH3
TTTBV=TH4
TTT=TTTT+TTTBV
CALCULATE DERIVATIVES OF INTRINSIC CONTRIBS
X=XK(J)
XX=XJ(J)
DD=VA(J+1)-3.*VA(J)
XM=.5*XN(J)*YA(J)*(XIA(J)*(1.-2.*X)+ VA(J))*(XIA(J)**2)/XLA
DINTK=XM
DBAK=QA*XM
DBAK=DBAK+DBAK
DBBK=DBBK+QA
DCK=.5*(DBBK-XLA*C(J)/XN(J))
DBCK=QA*DCK
M=J+1
CALL PDERIV(TTTY, TTBV, TTV, TTBV, TTT, M)
WE NOW DIVIDE THE OUTPUT OF PDERIV BY THE PARAXIAL INVARIANT
XKERP STORES PRIMARY K DERIVATIVES
XKERP(J,1)=ZZ1/XLA+DINTK
XKERP(J,2)=ZZ2/XLA+DBAK
XKERP(J,3)=ZZ3/XLA+DBK
XKERP(J,4)=ZZ4/XLA+DBBK
XKERP(J,5)=ZZ5/XLA+DCK
XKERP(J,6)=ZZ6/XLA+DBCK
XLK=XLA*X
PT IS USED IN CALCULATING THE PRIMARY N DERIVATIVES
PT(1,J)=XLK*XKERP(J,1)
PT(2,J)=XLK*XKERP(J,2)
PT(3,J)=XLK*XKERP(J,3)
PT(4,J)=XLK*XKERP(J,4)
PT(5,J)=XLK*XKERP(J,5)
PT(6,J)=XLK*XKERP(J,6)
DINQK ETC. ARE DERIVATIVES OF PRIMARY INTRINSIC Q CONTRIBS
DINQK=QA*DINTK+VA(J+1)*XIA(J)
DBAKQ=QA*DINQK
DBBKQ=2.*DBAKQ+C(J)*XLA/XN(J)
DBBKQ=QA*DBBKQ
DCKQ=.5*DBBKQ
DCBKQ=QA*DCKQ
CALL ALFA(TTTY, TTBV, TTV, TTBV, TTT, M, XLA)
A(1)=WA-DINQK
A(2)=WB-DBAKQ
A(3)=WC-DBKQ
A(4)=WD-DBBKQ
A(5)=WE-DCKQ
A(6)=WF-DCBKQ
A(7)=WG+DINTK
A(8)=WH+DBAK
A(9)=WI+DBK

```

```

C
C
A(10)=WJ+DBBK
A(11)=WK+DCK
A(12)=WL+DBCK
NOW WE NEED DERIVATIVES OF THE SECONDARY INTRINSIC CONTRIBS
DWK ETC. ARE DERIVATIVES OF W, WI ETC. DEFINED O.A.C. P324
DWK=.25*XIA(J)*(X*XIA(J)-VA(J+1))
DW1K=XX*(XM *DD-XIA(J)*PIA(J)/XLA)
DW2K=.5*XM *XX**2
DW3K=XX*XIA(J)*(VA(J)**2-3.*VA(J+1)**2)
DW4K=W(J)*C(J)*XLA**3/XN(J)
DW5K=.125*XLA*C(J)*XX**2/XN(J)*XLA
DS1K ETC. ARE DERIVATIVES OF SECONDARY INTRINSIC CONTRIBS
DS1K=3.*(PIA(J)*DWK/XLA+WW(J)*DINTK)
DS2K=4.*QA*DS1K+DWK
DS3K=QA*(DW1K+2.*QA*DS1K)+DW2K+DW3K
DS4K=DS3K-2.*DW2K-DW3K
DS4K=DS4K+DS4K
XD=DW3K-DW2K
DS5K=DW4K+QA*(2.*XD+QA*(3.*DW1K+QA*4.*DS1K))
DS6K=DW5K+QA*(DW4K+QA*(XD+QA*(DW1K+QA*DS1K)))
THE ABOVE ARE ALREADY DIVIDED BY THE PARAXIAL INVARIANT
CALL BITS(DINTK,DBAK,DBBK,DCK,DBCK,J,XLA)
DSF1K=DS1K+BTB
DSF2K=DS2K+BTB
DSF3K=DS3K+BTB
DSF4K=DS4K+BTB
DSF5K=DS5K+BTB
DSF6K=DS6K+BTB
DSFB1K=QA*DS1K+BTB
DSFB2K=QA*DS2K+BTB
DSFB3K=QA*DS3K+BTB
DSFB4K=QA*DS4K+BTB
DSFB5K=QA*DS5K+BTB
DSFB6K=QA*DS6K+BTB
C
C
JJ=J+1
CALL SDERIV(TTIT,TTTB,TTTV,TTTBV,TTT,JJ,JJ)
CALL STORES SECONDARY K DERIVATIVES
XKERS(J,1)=G1/XLA+DSF1K
XKERS(J,3)=G3/XLA+DSF2K
XKERS(J,5)=G5/XLA+DSF3K
XKERS(J,7)=G7/XLA+DSF4K
XKERS(J,9)=G9/XLA+DSF5K
XKERS(J,11)=G11/XLA+DSF6K
XKERS(J,12)=G12/XLA+DSF6K
XKERS(J,2)=G2/XLA+DSF81K
XKERS(J,4)=G4/XLA+DSF82K
XKERS(J,6)=G6/XLA+DSF83K
XKERS(J,8)=G8/XLA+DSF84K
XKERS(J,10)=G10/XLA+DSF85K
XKERS(J,12)=G12/XLA+DSF86K

```

C ST IS NEEDED FOR CALCULATING SECONDARY N DERIVATIVES

```
ST(1,J)=XLK* XKERS(J,1)
ST(2,J)=XLK* XKERS(J,2)
ST(3,J)=XLK* XKERS(J,3)
ST(4,J)=XLK* XKERS(J,4)
ST(5,J)=XLK* XKERS(J,5)
ST(6,J)=XLK* XKERS(J,6)
ST(7,J)=XLK* XKERS(J,7)
ST(8,J)=XLK* XKERS(J,8)
ST(9,J)=XLK* XKERS(J,9)
ST(10,J)=XLK* XKERS(J,10)
ST(11,J)=XLK* XKERS(J,11)
ST(12,J)=XLK* XKERS(J,12)
IF(J.EQ. JA)GOTO6001
J=J+1
```

6001 GOTO6006

CONTINUE

```
DO6008I=1,6
IF(JA.EQ.1)GOTO87
```

GOTO6007

C 87 PDN STORES PRIMARY N DERIVATIVES

```
PDN(J,I)=PT(I,J)/XN(J)
```

GOTO6008

6007 PDN(J,I)=(PT(I,J)-PT(I,J-1))/XN(J)

6008 CONTINUE

```
DO6009I=1,12
```

```
IF(JA.EQ.1)GOTO88
```

GOTO6002

C SDN STORES SECONDARY N DERIVATIVES

```
SDN(J,I)=ST(I,J)/XN(J)
```

GOTO6009

6002 SDN(J,I)=(ST(I,J)-ST(I,J-1))/XN(J)

6009 CONTINUE

C

FINALLY WE CALCULATE DERIVATIVES W.R.T. C1J,C2J
THIS WE DO NOT CALL ALFA. USE P.235 INSTEAD (O.A.C.)

```
A(7)=(XN(J+1)-XN(J))*YA(J)**4/XLA
```

```
A(8)=QTILD*A(7)
```

```
A(1)=-A(8)
```

```
A(9)=A(8)+A(8)
```

```
A(11)=QTILD*A(8)
```

```
A(2)=-A(11)
```

```
A(10)=A(11)+A(11)
```

```
A(3)=-A(10)
```

```
A(4)=A(5)+A(5)
```

```
A(6)=QTILD*A(5)
```

```
WE NOW CALCULATE
```

THE DERIVATIVES OF THE INTRINSIC CONTRIBS FOR C1J

USES THE EQUATIONS (12.1) IN THESIS

WE NEED THE FOLLOWING QUANTITIES

C

C

C

C

```

ZKB=DAKCL*YA(J)*XIA(J)
ZKA=ZKB*(J)*X
ZKC=DAKCL*X*XIA(J)**2
RAT=VA(J+1)/YA(J)
CQT=C(J)*(1.-X)*QA
CQT=C(J)*((1.-X)*QTILD
CCK7=C(J)*((7.-8.*X)-RAT
CCK3=C(J)*((3.-4.*X)-RAT
CCK6=C(J)*((6.-7.*X)-RAT
CCK4=C(J)*((4.-5.*X)-RAT
CCK5=C(J)*((5.-6.*X)-RAT
CC1=1.+X
CC2=CC1+X
CC3=CC2+X
CC4=CC3+X

```

C

```

THE FOLLOWING SET ARE THE DERIVATIVES OF THE INTRINSIC CONTRIBS
DS1C1=.5*ZKA+.25*ZKB*CK7+.5*ZKC*CC3
DSB1C1=.5*QTILD*ZKA+ZKB*(CQT+.25*CK3*QA)+ZKC*(X*QA+.5*CC1*QTILD)
DS2C1=2.*QTILD*ZKA+ZKB*(CQ+CK6*QTILD)+ZKC*(CC4*QTILD+CC2*QA)
DSB2C1=QTILD*(2.*QTILD*ZKA+ZKB*(3.*CQT+CK4*QA))+ZKC*(CC1*QTILD**2
1+QA*(CC4*QTILD+X*QA))
DS3C1=QTILD*(QTILD*ZKA+ZKB*(CQ+.5*QTILD*CK5))+.5*ZKC*(CC1*QA**2+
1QTILD*(2.*X*QA+CC3*QTILD))
DSB3C1=QTILD*(QTILD*(QTILD*ZKA+ZKB*(CQT+.5*CK5*QA))+.5*ZKC*(CC1*QT
1ILJ**2+QA*(2.*X*QTILD+CC3*QA)))
DS4C1=QTILD*(2.*(QTILD*ZKA+ZKC*(X*QTILD+CC2*QA))+ZKB*(CQ+CQ+QTILD*
1CK5))

```

C

```

DSB4C1=QTILD*(QTILD*(QTILD*(2.*ZKA+ZKB*(2.*CQT+CK5*QA))+2.*QA*ZKC*
1(CC2*QTILD+X*QA))
DS5C1=QTILD*(ZKC*(X*QTILD**2+QA*(CC4*QTILD+CC1*QA))+QTILD*(ZKB*(3.
1*2*QTILD*CK4)+2.*QTILD*ZKA))
DSB5C1=(QA*ZKC*(CC2*QTILD+CC4*QA)+QTILD*(ZKB*(CQT+CK6*QA))+2.*ZKA*Q
1TILD)*QTILD**2
DS6C1=(ZKC*(.5*CC1*QA+QTILD*X)+ZKB*(CQ+.25*QTILD*CK3)+QTILD*.5*ZKA
1)*QTILD**3
DSB6C1=.5*(QA*(QA*ZKC*CC3+.5*QTILD*ZKB*CK7)+QTILD**2*ZKA)*QTILD**2

```

C

```

CALL BITS(DAKCL, DAKBC1, DBKBC1, DCKC1, DCKBC1, J, XLA)
DSF1C1=DS1C1+BTB
DSF2C1=DS2C1+BTE
DSF3C1=DS3C1+BTE
DSF4C1=DS4C1+BTG
DSF5C1=DS5C1+BTE
DSF6C1=DS6C1+BTB
DSF7C1=DS7C1+BTB
DSF8C1=DS8C1+BTB
DSF9C1=DS9C1+BTB
DSF10C1=DS10C1+BTB
DSF11C1=DS11C1+BTB
DSF12C1=DS12C1+BTB
DSF13C1=DS13C1+BTB
DSF14C1=DS14C1+BTB
DSF15C1=DS15C1+BTB
DSF16C1=DS16C1+BTB
DSF17C1=DS17C1+BTB
DSF18C1=DS18C1+BTB
DSF19C1=DS19C1+BTB
DSF20C1=DS20C1+BTB
DSF21C1=DS21C1+BTB
DSF22C1=DS22C1+BTB
DSF23C1=DS23C1+BTB
DSF24C1=DS24C1+BTB
DSF25C1=DS25C1+BTB
DSF26C1=DS26C1+BTB
DSF27C1=DS27C1+BTB
DSF28C1=DS28C1+BTB
DSF29C1=DS29C1+BTB
DSF30C1=DS30C1+BTB
DSF31C1=DS31C1+BTB
DSF32C1=DS32C1+BTB
DSF33C1=DS33C1+BTB
DSF34C1=DS34C1+BTB
DSF35C1=DS35C1+BTB
DSF36C1=DS36C1+BTB
DSF37C1=DS37C1+BTB
DSF38C1=DS38C1+BTB
DSF39C1=DS39C1+BTB
DSF40C1=DS40C1+BTB
DSF41C1=DS41C1+BTB
DSF42C1=DS42C1+BTB
DSF43C1=DS43C1+BTB
DSF44C1=DS44C1+BTB
DSF45C1=DS45C1+BTB
DSF46C1=DS46C1+BTB
DSF47C1=DS47C1+BTB
DSF48C1=DS48C1+BTB
DSF49C1=DS49C1+BTB
DSF50C1=DS50C1+BTB
DSF51C1=DS51C1+BTB
DSF52C1=DS52C1+BTB
DSF53C1=DS53C1+BTB
DSF54C1=DS54C1+BTB
DSF55C1=DS55C1+BTB
DSF56C1=DS56C1+BTB
DSF57C1=DS57C1+BTB
DSF58C1=DS58C1+BTB
DSF59C1=DS59C1+BTB
DSF60C1=DS60C1+BTB
DSF61C1=DS61C1+BTB
DSF62C1=DS62C1+BTB
DSF63C1=DS63C1+BTB
DSF64C1=DS64C1+BTB
DSF65C1=DS65C1+BTB
DSF66C1=DS66C1+BTB
DSF67C1=DS67C1+BTB
DSF68C1=DS68C1+BTB
DSF69C1=DS69C1+BTB
DSF70C1=DS70C1+BTB
DSF71C1=DS71C1+BTB
DSF72C1=DS72C1+BTB
DSF73C1=DS73C1+BTB
DSF74C1=DS74C1+BTB
DSF75C1=DS75C1+BTB
DSF76C1=DS76C1+BTB
DSF77C1=DS77C1+BTB
DSF78C1=DS78C1+BTB
DSF79C1=DS79C1+BTB
DSF80C1=DS80C1+BTB
DSF81C1=DS81C1+BTB
DSF82C1=DS82C1+BTB
DSF83C1=DS83C1+BTB
DSF84C1=DS84C1+BTB
DSF85C1=DS85C1+BTB
DSF86C1=DS86C1+BTB
DSF87C1=DS87C1+BTB
DSF88C1=DS88C1+BTB
DSF89C1=DS89C1+BTB
DSF90C1=DS90C1+BTB
DSF91C1=DS91C1+BTB
DSF92C1=DS92C1+BTB
DSF93C1=DS93C1+BTB
DSF94C1=DS94C1+BTB
DSF95C1=DS95C1+BTB
DSF96C1=DS96C1+BTB
DSF97C1=DS97C1+BTB
DSF98C1=DS98C1+BTB
DSF99C1=DS99C1+BTB
DSF100C1=DS100C1+BTB

```

[illegible]

```

ONLY C2J LEFT AND THESE ARE TRIVIAL
DO S1C2=(XN(J+1)-XN(J))*YA(J)*#6
CERS2=STORES SECONDARY C2 DERIVATIVES
CERS2(J,1)=DS1C2
CERS2(J,2)=QTILD*CERS2(J,1)
CERS2(J,3)=4.0*CERS2(J,2)
CERS2(J,4)=QTILD*CERS2(J,3)
CERS2(J,5)=0.5*CERS2(J,4)
CERS2(J,6)=QTILD*CERS2(J,5)
CERS2(J,7)=CERS2(J,4)
CERS2(J,8)=2.0*CERS2(J,7)
CERS2(J,9)=CERS2(J,8)
CERS2(J,10)=QTILD*CERS2(J,9)
CERS2(J,11)=0.25*CERS2(J,10)
CERS2(J,12)=QTILD*CERS2(J,11)
IF(NOCOL.EQ.2)GOTO1111
CALL CDERIV

```

```

111111
CONTINUE
IF(FULL.NE.2.)GOTO11113
C
THEIRAS AND ALFAS ARE OPTIONAL PRINTOUT
WRITE(KOUT,300)TY,TBY,TV,TBV
WRITE(KOUT,3501)
WRITE(KOUT,300)(A(N),N=1,12)
WRITE(KOUT,2011)
WRITE(KOUT,300)TTY,TTBY,TTV,TTBV
WRITE(KOUT,3502)
WRITE(KOUT,300)(A(N),N=1,12)
WRITE(KOUT,2016)
WRITE(KOUT,300)TTY,TTBY,TTV,TTTBV
WRITE(KOUT,3503)
WRITE(KOUT,300)(A(N),N=1,12)
WRITE(KOUT,3504)

```

```

11113 WRITE(KOUT,300 )(A(N),N=1,12)
      CONTINUE
      WRITE(KOUT,679)JA
678  CONTINUE
      THIS ENDS MAIN PROGRAMME
      CALL PRINT
C
      FORMAT(4E13.5)
440  FORMAT('OEND DERIVATIVES FOR SURFACE ',I2)
679  FORMAT(3E13.5)
300  FORMAT(28H0THETAS FOR K IN USUAL ORDER)
2016 FORMAT('OALFAS FOR CO',)
3502 FORMAT('OALFAS FOR D',)
3501 FORMAT('OALFAS FOR K',)
3503 FORMAT('OALFAS FOR C1',)
3504 FORMAT(29H0THETAS FOR CO IN USUAL ORDER)
2011 FORMAT(I2)
100  FORMAT(3E13.5,I2)
600  FORMAT(28H0THETAS FOR D IN USUAL ORDER)
2002 FORMAT(9H0A COEFFS)
2008 FORMAT(9H0B COEFFS)
2009 NO EXTRA DATA, C2 IS NOT NEEDED AND C1=0 FOR ALL J
C
      CONTINUE
      END

```

```

SUBROUTINE PRINT
COMMON/CESULT/ FDN(10,4), DSF(10,8), DF(10,4), XKF(10,4), CF(10,4)
COMMON/RESULT/ DERP(10,6), DERS(10,12), CERP(10,6), CERS(10,12)
COMMON/INT/KOUT,KINP,NN,KN,NOCOL,FULL
COMMON/INT/KOUT,KINP,NN,KN,NOCOL,FULL
WRITE(KOUT,19999)
FORMAT(1,MONOCHROMATIC SECTION')
20000 1 TO SEPARATION J')
FORMAT(1,DERIVATIVES OF FINAL ABERRATION COEFFICIENTS WITH RESPECT
TO SEPARATION J')
WRITE(KOUT,20002)
WRITE(KOUT,20001)
20002 1
FORMAT(1,
-')
P1A P2A P2A
20001 1 P3A J')
DD20003 J=2, NN
WRITE(KOUT,20004) (DERP(J,I), I=1,6), J
20004 1
FORMAT(1,6E13.5,12)
WRITE(KOUT,20002)
WRITE(KOUT,20005)
20005 1
FORMAT(1,
S1A S2A S2A
S3A J')
DD20006 J=2, NN
WRITE(KOUT,20004) (DERS(J,I), I=1,6), J
20006 1
WRITE(KOUT,20002)
WRITE(KOUT,20007)
20007 1
FORMAT(1,
S4A S5A S5A
S6A J')
DD20008 J=2, NN
WRITE(KOUT,20004) (DERS(J,I), I=7,12), J
20008 1
END SEPARATION DERIVATIVES
C***
20010 1 TO AXIAL CURVATURE J')
FORMAT(1,DERIVATIVES OF FINAL ABERRATION COEFFICIENTS WITH RESPECT
TO AXIAL CURVATURE J')
WRITE(KOUT,20002)
WRITE(KOUT,20001)
20011 1
DD20011 J=1, NN
WRITE(KOUT,20004) (CERP(J,I), I=1,6), J
20011 1
WRITE(KOUT,20002)
WRITE(KOUT,20005)
20012 1
DD20012 J=1, 6
WRITE(KOUT,20004) (CERS(J,I), I=1,6), J
20012 1
WRITE(KOUT,20002)
WRITE(KOUT,20007)
20013 1
DD20013 J=1, 6
WRITE(KOUT,20004) (CERS(J,I), I=7,12), J
20013 1
END CURVATURE DERIVATIVES
C***
WRITE(KOUT,20020)

```

```

20020 FORMAT('1 DERIVATIVES OF FINAL ABERRATION COEFFICIENTS WITH RESPECT
1 TO REFRACTANCE J'),
WRITE(KOUT,20002)
DO20021 J=1,NN
WRITE(KOUT,20001)
20021 WRITE(KOUT,20004) (XKERP(J,I),I=1,6),J
WRITE(KOUT,20002)
WRITE(KOUT,20005)
DO20022 J=1,NN
WRITE(KOUT,20004) (XKERS(J,I),I=1,6),J
WRITE(KOUT,20002)
WRITE(KOUT,20007)
DO20023 J=1,NN
WRITE(KOUT,20004) (XKERS(J,I),I=7,12),J
C**** END REFRACTANCES DERIVS
20030 FORMAT('1 DERIVATIVES OF FINAL ABERRATION COEFFICIENTS WITH RESPECT
1 TO REFRACTIVE INDEX J'),
WRITE(KOUT,20002)
WRITE(KOUT,20001)
DO20031 J=1,NN
WRITE(KOUT,20004) (PDN(J,I),I=1,6),J
WRITE(KOUT,20002)
WRITE(KOUT,20005)
DO20032 J=1,NN
WRITE(KOUT,20004) (SDN(J,I),I=1,6),J
WRITE(KOUT,20002)
WRITE(KOUT,20007)
DO20033 J=1,NN
WRITE(KOUT,20004) (SDN(J,I),I=7,12),J
C**** END REFRACTIVE INDEX DERIVS
20040 FORMAT('1 DERIVATIVES OF FINAL ABERRATION COEFFICIENTS WITH RESPECT
1 TO FIRST EXTRA AXIAL CURVATURE WHEN THIS IS ZERO'),
WRITE(KOUT,20002)
WRITE(KOUT,20007)
DO20041 J=1,NN
WRITE(KOUT,20004) (CERP1(J,I),I=1,6),J
WRITE(KOUT,20002)
WRITE(KOUT,20005)
DO20042 J=1,NN
WRITE(KOUT,20004) (CERS1(J,I),I=1,6),J
WRITE(KOUT,20002)
WRITE(KOUT,20007)
DO20043 J=1,NN
WRITE(KOUT,20004) (CERS1(J,I),I=7,12),J
C**** END FIRST EXTRA AXIAL CURVATURE DERIVS
20050 FORMAT('1 DERIVATIVES OF FINAL ABERRATION COEFFICIENTS WITH RESPECT
1 TO SECOND EXTRA AXIAL CURVATURE WHEN THIS IS ZERO'),

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```

20051 WRITE(KOUT,20051)
      FORMAT('DERIVATIVES OF ALL THIRD ORDER COEFFICIENTS ARE ZERO')
      WRITE(KOUT,20002)
      WRITE(KOUT,20005)
      DO20052J=1,NN
20052 WRITE(KOUT,20004)(CERS2(J,I),I=1,6),J
      WRITE(KOUT,20002)
      WRITE(KOUT,20007)
      DO20053J=1,NN
20053 WRITE(KOUT,20004)(CERS2(J,I),I=7,12),J
      IF(NDCOL.EQ.2)GOTO12233
20060 WRITE(KOUT,20060)
      FORMAT('CHROMATIC DERIVATIVES SECTION')
20061 WRITE(KOUT,20061)
      FORMAT('DERIVATIVES ARE THOSE OF PARAXIAL COEFFICIENTS OF CHROMAT
      IC ORDER ONE AND TWO')
20062 FORMAT('DERIVATIVES OF CHROMATIC COEFFICIENTS WITH RESPECT TO SEP
      ARATION J')
      WRITE(KOUT,20063)
20063 FORMAT('O
      -')
20064 WRITE(KOUT,20064)
      FORMAT('F1A F2A J')
      DO20065J=2,NN
20065 WRITE(KOUT,20066)(DF(J,I),I=1,4),J
20066 FORMAT(4E13.5,12)
20070 WRITE(KOUT,20070)
      FORMAT('DERIVATIVES OF CHROMATIC COEFFICIENTS WITH RESPECT TO CUR
      VATURE J')
      WRITE(KOUT,20063)
      WRITE(KOUT,20064)
      DO20071J=1,NN
20071 WRITE(KOUT,20066)(CF(J,I),I=1,4),J
20080 WRITE(KOUT,20080)
      FORMAT('DERIVATIVES OF CHROMATIC COEFFICIENTS WITH RESPECT TO REF
      RACTIVE J')
      WRITE(KOUT,20063)
      WRITE(KOUT,20064)
      DO20081J=1,NN
20081 WRITE(KOUT,20066)(XKF(J,I),I=1,4),J
20090 WRITE(KOUT,20090)
      FORMAT('DERIVATIVES OF CHROMATIC COEFFICIENTS WITH RESPECT TO REF
      RACTIVE INDEX J')
      WRITE(KOUT,20063)
      WRITE(KOUT,20064)
      DO20091J=1,NN
20091 WRITE(KOUT,20066)(FDN(J,I),I=1,4),J
20100 WRITE(KOUT,20100)
      FORMAT('DERIVATIVES OF CHROMATIC COEFFICIENTS WITH RESPECT TO FIR
      ST DISPERSION OF MEDIUM J')

```

```

WRITE(KOUT,20063)
WRITE(KOUT,20064)
NZ=NN-1
DO20101 J=1,NZ,2
20101 WRITE(KOUT,20066) (DSF(J,I),I=1,4),JL
20102 WRITE(KOUT,20102)
20102 FORMAT(10DERIVATIVES OF CHROMATIC COEFFICIENTS WITH RESPECT TO SEC
10ND DISPERSION OF MEDIUM J')
WRITE(KOUT,20063)
WRITE(KOUT,20064)
DO20103 J=1,NZ,2
20103 WRITE(KOUT,20066) (DSF(J,I),I=5,8),JL
12233 CONTINUE
      RETURN
      END

```

```

SUBROUTINE BITS(BA,BB,BC,BD,BE,I,AL,
COMMON/BC3/BTA,BTB,BTC,BTD,BTE,BTF,BTG,BTH,BTI,BTJ,BTK,BTL
DIMENSION UA(11,6), UC(11,6), AU(6), BU(6)
COMMON/B4/UA,UC
DO 1 JX=1,6
  AU(JX)=UA(I,JX)
  BU(JX)=UC(I,JX)
  BTA=3.*(AU(1)*BB-BU(1)*BA)/AL
  BTB=(-BU(2)*BA+(AU(2)-2.*BU(1))*BB+AU(1)*BC)/AL
  BTC=(-(2.*BU(2)+BU(3))*3.)*BA+(AU(3)+2.)*(AU(2)-2.*BU(1)+AU(3))
  1*BBB+AU(1))*((2.*BD+BC))/AL
  BTD=(-BU(4)*BA-(4.*BU(2)-AU(4)+2.*BU(3))*BB+(2.*AU(2)-BU(1)+AU(3))
  1*BC+2.*AU(1)*BE)/AL
  1*BTE=(-3.*BU(5)*BA+(3.*AU(5)-2.*BU(2))*BB+(2.*AU(2)-BU(1))*BD+AU(1)
  1*BE)/AL
  BTF=(-BU(6)*BA+(AU(6)-2.*BU(5))*BB+(AU(5)-BU(2))*BC-BU(2)*BD
  1+3.*AU(2)*BE)/AL
  BTG=(-(2.)*(BU(4)*BA-(AU(4)-2.*BU(3))*BB)+AU(3))*((2.*BD+BC))/AL
  3TH=((2.*AU(4)-BU(3))*BC+2.)*(-BU(4)*BB+2.*AU(3)*BE))/AL
  BTI=-2.)*(BU(6)*BA+(BU(4)-AU(6)+2.*BU(5))*BB)+AU(5)*BC+AU(3)*BE
  BTJ=(BTI+(2.)*(AU(4)+AU(5))-BU(3))*BD)/AL
  1AU(5))*BE)/AL
  BTK=(-(2.*BU(6))*BB+(2.*AU(6)-BU(5))*BD+AU(5)*BE)/AL
  BTL=(-BU(6))*((BD+BC)+3.*AU(6)*BE)/AL
RETURN
END

```

C

```

SUBROUTINE COEFF
INTEGER#4 FULL
DIMENSION WW(10), W1(10), W2(10), W3(10), W4(10), W5(10), XJ(10)
DIMENSION YA(10), YB(10), VA(10), VB(10), XIA(10), XIB(10), D(10)
DIMENSION P2BA(10), P3A(10), P3BA(10), P1BA(10), P1BB(10), P2B(10)
DIMENSION C(10), XN(10), Q(10), W(10), P1A(10), P1B(10), P2A(10)
DIMENSION S5A(10), S6A(10), S1B(10), S2B(10), S3B(10), S4B(10), S5B(10)
DIMENSION S6B(10), SF1A(10), SF2A(10), SF3A(10), SF4A(10), SF5A(10)
DIMENSION SF6A(10), SF1BA(10), SF2BA(10), SF3BA(10), SF4BA(10)
DIMENSION SF5BA(10), SF6BA(10), SF1BB(10), SF2BB(10), SF3BB(10)
DIMENSION SF5BB(10), SF6BB(10), SF1B(10), SF2B(10), SF3B(10)
DIMENSION PSUMA(11,6), PSUMB(11,6), S1A(10), S2A(10), S3A(10), S4A(10)
DIMENSION P2BB(10), P3B(10), P3BB(10), SA(10,6), SB(10,6), XK(10)
DIMENSION SSUMA(11,12), SSUMB(11,12)
COMMON/INT/KINP, NN, KN, NOCOL, FULL
COMMON/B4/ PSUMA, PSUMB
COMMON/TERMS/WW, W1, W2, W3, W4, W5
COMMON/POEFS/P1A, P1BA, P2A, P2BA, P3A, P3BA, P1B, P1BB, P2B, P2BB,
1 P3B, P3BB
COMMON/COEFS/SSUMA, SSUMB, S1A, S2A, S3A, S4A, S5A, S6A
COMMON/DATA/ C, D, XN, XK, XJ, Q, XIA, XIB, YA, YB, VA, VB, W, XLA
COMMON/BB1/YI, YX, YY
COMMON/BB3/PP1, PP1B, PP2, PP2B, PP3, PP3B
COMMON/BB4/ZW1, ZW2, ZW3, ZW4, ZW5
COMMON/BB5/SS1, SS2, SS3, SS4, SS5, SS6
COMMON/BB6/X1, X2, X3, X4, X5, X6
COMMON/BB7/XX1, XX2, XX3, XX4, XX5, XX6
C
EQUIVALENCE(P1A(1), SA(1,1)), (P1BA(1), SA(1,2)), (P2A(1), SA(1,3))
EQUIVALENCE(P1B(1), SB(1,1)), (P1BB(1), SB(1,2)), (P2B(1), SB(1,3))
EQUIVALENCE(P2BA(1), SA(1,4)), (P3B(1), SB(1,5)), (P3BB(1), SB(1,6))
EQUIVALENCE(SF1A(1), TA(1,1)), (SF1BA(1), TA(1,2)), (SF2A(1), TA(1,3)),
1(SF2BA(1), TA(1,4)), (SF3A(1), TA(1,5)), (SF3BA(1), TA(1,6)), (SF4A(1),
2(TA(1,7)), (SF4BA(1), TA(1,8)), (SF5A(1), TA(1,9)), (SF5BA(1), TA(1,10)),
3(SF6A(1), TA(1,11)), (SF6BA(1), TA(1,12)), (SF1BB(1), TB(1,3)),
EQUIVALENCE(SF1B(1), TB(1,1)), (SF3B(1), TB(1,5)), (SF4B(1),
1(TB(1,7)), (SF4BB(1), TB(1,4)), (SF5B(1), TB(1,8)), (SF5BB(1), TB(1,10)),
2(SF6B(1), TB(1,11)), (SF6BB(1), TB(1,12))
READ(KINP, 100) NN
WRITE(KOUT, 150) NN
KN=NN+1
READ(KINP, 200) Y, V, XN(KN)
READ(KINP, 200) YY, VV
WRITE(KOUT, 251) Y, V, YY, VV
FORMAT('OYA=', E13.5, ' VA=', E13.5, ' YB=', E13.5, ' VB=', E13.5)
251 REMEMBER TO PUT D(1)=0
C
READ(KINP, 200) (D(I), C(I), XN(I), I=1, NN)

```



```
C
C
C
C
C
NEXT LINE GIVES FINAL VALUES OF V
VA(NN+1)=C(NN)*YA(NN)-XK(NN)*XIA(NN)
VB(NN+1)=C(NN)*YB(NN)-XK(NN)*XIB(NN)
DO3K=1,NN
PRIM CALCULATES PRIMARY INTRINSIC CONTRIBS
CALL PRIM(XIA(K),YA(K),VA(K),Q(K),XN(K),WK(K),XLA)
PNA(K) ARE PRIMARY INTRINSIC CONTRIBS FROM SURFACE K FOR A RAY. SIMILARLY
PNBA(K) ARE BARRED CONTRIBS
PIA(K)=PP1
PIBA(K)=PP1B
P2A(K)=PP2
P2BA(K)=PP2B
P3A(K)=PP3
P3BA(K)=PP3C
QB=1./Q(K)
C
C
CALL PRIM(XIB(K),YB(K),VB(K),QB,XN(K),XK(K),W(K),XLB)
AS IN PREVIOUS COMMENT BUT FOR B RAY
PIB(K)=PP1
PIBB(K)=PP1B
P2B(K)=PP2
P2BB(K)=PP2B
P3B(K)=PP3
P3BB(K)=PP3B
WRITE(KOUT,752)
752 FORMAT('13RD ORDER SURFACE CONTRIBUTIONS FROM SURFACE K')
WRITE(KOUT,725)
WRITE(KOUT,600)(PIA(K),PIBA(K),P2A(K),K,K=1,NN)
WRITE(KOUT,825)
WRITE(KOUT,600)(P2BA(K),P3A(K),P3BA(K),K,K=1,NN)
REMEMBER DUAL COEFFS MUST BE WRITTEN IN REVERSE ORDER.
WRITE(KOUT,925)
WRITE(KOUT,600)(P3BB(K),P2B(K),P2BB(K),K,K=1,NN)
WRITE(KOUT,950)
WRITE(KOUT,600)(P2B(K),PIBB(K),P1B(K),K,K=1,NN)
WRITE(KOUT,600)
PLA ETC ARE STORED AS SA, SB IN PREPARATION FOR SUMMATION
WRITE(KOUT,851)
851 FORMAT('13RD ORDER ABERRATION COEFFICIENTS BEFORE SURFACE K')
DO5I=1,6
INITIAL ABERRATION COEFFS ARE ZERO
PSUMA(1,I)=0.0
PSUMB(1,I)=0.0
DO6K=2,KN
PSUMA(K,I) CONTAIN PRIMARY ABERRATION COEFFS AT K TH SURFACE.
I=1 GIVES A, I=2 GIVES A BAR I=3 GIVES B, ETC
PSUMA(K,I)=PSUMA(K-1,I)+SA(K-1,I)
PSUMB(K,I)=PSUMB(K-1,I)+SB(K-1,I)
CONTINUE
6
5
```

```

WRITE(KOUT,1000) (PSUMA(K,1),PSUMA(K,2),PSUMA(K,3),K,K=1,KN)
WRITE(KOUT,600) (PSUMA(K,1),PSUMA(K,2),PSUMA(K,3),K,K=1,KN)
WRITE(KOUT,1100) (PSUMA(K,4),PSUMA(K,5),PSUMA(K,6),K,K=1,KN)
WRITE(KOUT,600) (PSUMA(K,4),PSUMA(K,5),PSUMA(K,6),K,K=1,KN)
WRITE(KOUT,1200) (PSUMB(K,6),PSUMB(K,5),PSUMB(K,4),K,K=1,KN)
WRITE(KOUT,600) (PSUMB(K,6),PSUMB(K,5),PSUMB(K,4),K,K=1,KN)
WRITE(KOUT,1300) (PSUMB(K,3),PSUMB(K,2),PSUMB(K,1),K,K=1,KN)
WRITE(KOUT,600) (PSUMB(K,3),PSUMB(K,2),PSUMB(K,1),K,K=1,KN)
J24K=1,NN
C SECIN CALCULATES SECONDARY INTRINSIC CONTRIBS ( UNBARRED )
CALL SECIN(PIA(K),Q(K),VA(K+1),VA(K),XIA(K),W(K),XLA,XK(K),C(K),
1XN(K))
C WW IS W DEFINED P. 324 O.A.C. EQN 218.8 W1-W5 ARE ALSO DEFINED THERE
W5(K)=ZW5
W4(K)=ZW4
W3(K)=ZW3
W2(K)=ZW2
W1(K)=ZW1
WW(K)=ZWW
SNA(K) ARE
S1A(K)=SS1
S2A(K)=SS2
S3A(K)=SS3
S4A(K)=SS4
S5A(K)=SS5
S6A(K)=SS6
QB=1./QA
CALL SECIN(PIB(K),QB ,VB(K+1),VB(K),XIB(K),W(K),XLB,XK(K),C(K),
1XV(K))
C AS PREVIOUS COMMENT BUT FOR B RAY
S1B(K)=SS1
S2B(K)=SS2
S3B(K)=SS3
S4B(K)=SS4
S5B(K)=SS5
S6B(K)=SS6
C
C EXTRA CALCULATES THE TERM TO BE ADDED TO THE INTRINSIC CONTRIBS
C TO GIVE THE FULL CONTRIBS
CALL EXTRA(PSUMA(K,1),PSUMA(K,2),PSUMA(K,3),PSUMA(K,4),PSUMA(K,5)
1,PSUMA(K,6),PSUMB(K,6),PSUMB(K,5),PSUMB(K,4),PSUMB(K,3),
2PSUMB(K,2),PSUMB(K,1),PIA(K),P2A(K),P3A(K),QA,XLA)
SFNA(K) ARE SECONDARY FULL CONTRIBS FROM K TH SURFACE N=1 S1
N=2 S2 ETC A RAY
SF1A(K)=S1A(K)+X1 /XLA
SF2A(K)=S2A(K)+X2 /XLA
SF3A(K)=S3A(K)+X3 /XLA
SF4A(K)=S4A(K)+X4 /XLA
SF5A(K)=S5A(K)+X5 /XLA

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```

SF6A(K)=S6A(K)+X6 /XLA
SFNBA(K) ARE BARRED FULL CONTRIBS. SIMILAR COMMENTS APPLY TO
SFNB(K), SFNBB(K) FOR B RAY
SF1BA(K)=QA*SF1A(K)+XX1 /XLA
SF2BA(K)=QA*SF2A(K)+XX2/XLA
SF3BA(K)=QA*SF3A(K)+XX3 /XLA
SF4BA(K)=QA*SF4A(K)+XX4 /XLA
SF5BA(K)=QA*SF5A(K)+XX5 /XLA
SF6BA(K)=QA*SF6A(K)+XX6 /XLA

CALL EXTRA(PSUMB(K,1),PSUMB(K,2),PSUMB(K,3),PSUMB(K,4),PSUMB(K,5),
1,PSUM3(K,6),PSUMA(K,6),PSUMA(K,5),PSUMA(K,4),PSUMA(K,3),
2,PSUMA(K,2),PSUMA(K,1),PIB(K),P2B(K),P3B(K),QB,XLB),
SF1B(K)=S1B(K)+X1 /XLB
SF2B(K)=S2B(K)+X2 /XLB
SF3B(K)=S3B(K)+X3 /XLB
SF4B(K)=S4B(K)+X4 /XLB
SF5B(K)=S5B(K)+X5 /XLB
SF6B(K)=S6B(K)+X6 /XLB
SF1BB(K)=QB*SF1B(K)+XX1 /XLB
SF2BB(K)=QB*SF2B(K)+XX2 /XLB
SF3BB(K)=QB*SF3B(K)+XX3 /XLB
SF4BB(K)=QB*SF4B(K)+XX4 /XLB
SF5BB(K)=QB*SF5B(K)+XX5 /XLB
SF6BB(K)=QB*SF6B(K)+XX6 /XLB
4 WRITE(KOUT,195)
195 FORMAT('15TH ORDER SURFACE CONTRIBUTIONS FROM SURFACE K')
196 WRITE(KOUT,196)
196 FORMAT('O',S1P,S3PB,K')
196 S1P S3PB S1PB S2P S2PB
WRITE(KOUT,601)(SF1A(K),SF1BA(K),SF2A(K),SF2BA(K),SF3A(K),SF3BA(K),
1,K,K=1,NN)
WRITE(KOUT,197)
197 FORMAT('O',S4P,S6PB,K')
197 S4P S6PB S3PB S5P S5PB
601 FORMAT(6E13.5,I2)
WRITE(KOUT,601)(SF4A(K),SF4BA(K),SF5A(K),SF5BA(K),SF6A(K),SF6BA(K),
1,K,K=1,NN)
C****REMEMBER DUAL ORDER FOR B COEFFICIENTS
WRITE(KOUT,198)
198 FORMAT('O',S1Q,S3QB,K')
198 S1Q S3QB S1QB S2Q S2QB
WRITE(KOUT,601)(SF6BB(K),SF6B(K),SF5BB(K),SF5B(K),SF3BB(K),SF3B(K),
1,K,K=1,NN)
WRITE(KOUT,199)
199 FORMAT('O',S4Q,S6QB,K')
199 S4Q S6QB S4QB S5Q S5QB
WRITE(KOUT,601)(SF4BB(K),SF4B(K),SF2BB(K),SF2B(K),SF1BB(K),SF1B(K),
1,K,K=1,NN)

```



```
1800 FORMAT(54H0 SUM S3Q K)
1900 FORMAT(54H0 SUM S5Q K)
1400 FORMAT(54H0 SUM S1P K)
660 FORMAT(4E13.5,I2) AP
725 FORMAT(4H0.5,I2) BP
600 FORMAT(3E13.5,I2)
500 FORMAT(4E13.5,I2)
400 FORMAT(F10.6)
300 FORMAT(3E13.5)
200 FORMAT(3F10.6)
      RETURN
      END
```

SUM S3QB
SUM S5QB
SUM S1PB
APB

SUM S4Q
SUM S6Q
SUM S2P
BP

SUM S4QB
SUM S6QB
SUM S2PB
K)

```

SUBROUTINE THETA(R1,R2,R3,R4,R5)
THIS SUBROUTINE CALCULATES THETAS GIVEN IN O.A.C. P198
COMMON/BD1/TH1,TH2,TH3,TH4
XH1=-R5*R2
XH2=R5*R1
TH1=XH1*R3
TH2=XH1*R4
TH3=XH2*R3
TH4=XH2*R4
RETURN
END

```

C

C
SUBROUTINE PRIM(X1,X2,X3,X4,X5,X6,X7,X8)
THIS SUBROUTINE CALCULATES THE PRIMARY SURFACE CONTRIBUTIONS
COMMON/BB3/PP1,PP1B,PP2,PP2B,PP3,PP3B
X=(X6*X1-X3)*X1**2
XX=1.-X6
PP1=.5*X5*XX*X2*X
PP1B=X4*PP1
PP2=PP1B+PP1B
PP2B=PP2*X4
PP3=.5*(PP2B-X7*X8**2)
PP3B=X4*PP3
RETURN
END

```
C
C
SUBROUTINE PARAX(Y1,Y2,Y3,Y4,Y5,Y6)
THIS SUBROUTINE CALCULATES THE PARAXIAL COEFFICIENTS
COMMON/BB1/Y1,YX,YYY
YY=Y3+(1.-Y5)*Y1
YX=Y2-Y4*YYY
YI=Y6*YX-YYY
RETURN
END
```

```

C
SUBROUTINE SECIN(Z1,Z2,Z3,Z4,Z5,Z6,Z7,Z8,Z9,Z10)
THIS SUBROUTINE CALCULATES THE SECONDARY INTRINSIC CONTRIBS O.A.C. P324
COMMON/BB4/ZWW,ZW1,ZW2,ZW3,ZW4,ZW5
COMMON/BB5/SS1,SS2,SS3,SS4,SS5,SS6
ZWW=(1.+Z8**2)*Z5**2+Z3**2-3.*Z4**2
ZJ=Z9*(Z7/(Z5*Z10)
ZW1=ZJ*(Z3-3.*Z4)*Z1
ZW2=.5*Z1*ZJ**2
ZW3=.25*Z7*ZJ*Z3*(Z3**2-Z4**2)
ZW4=.5*Z7*(Z6*Z7)**2
ZW5=.125*Z6*(Z7*ZJ)**2
SS1=3.*Z1*ZWW
SS2=4.*Z2*SS1+ZW1
SS3=ZW2+ZW3+Z2*(ZW1+2.*Z2*SS1)
X=ZW3-ZW2
SS4=SS3-ZW3-ZW2
SS5=SS4+SS4
SS5=ZW4+Z2*(X+X+Z2*(3.*ZW1+Z2*SS1*4.))
SS6=ZW5+Z2*(ZW4+Z2*(X+Z2*(ZW1+Z2*SS1)))
RETURN
END

```

```

C
C
C
SUBROUTINE EXTRA(A1,A2,A3,A4,A5,A6,A7,A8,A9,A10,A11,A12,A13,A14,
1A15,A16,A17)
THIS SUBROUTINE CALCULATES THE ITERATIVE PART OF THE SECONDARY
SURFACE CONTRIBS USING O.A.C. P14
COMMON/BB6/X1,X2,X3,X4,X5,X6
COMMON/BB7/XX1,XX2,XX3,XX4,XX5,XX6
AI=(A16*A1-A7)
AIB=A16*A2-A8
BI=(A16*A3-A9)
BIB=(A16*A4-A10)
CIB=(A16*A5-A11)
X1=(A1-A7-A7)*A13+A1*A14
X2=(BI-2*(A8+A9))*A13+(A1+A2-A7+A3)*A14+(A1+A1)*A15
X3=(CI-A11-A11)*A13+(A5-A8)*A14+(A1+A2+A2)*A15
X4=(A10+A10)*(-A13)+(BI+A4-A9)*A14+(A3+A3)*A15
X5=-((A12+A12)*A13+(CI-A10+A6-A11)*A14+(BI+(A4+A5)*2.)*A15
X6=-A12*A14+(CI+A6+A6)*A15
XY1=A1B-A16*AI
XY2=BIB-A16*BI
XY3=CIB-A16*CI
XX1=XY1*A13
XX2=XY2*A13+XY1*A14
XX3=XY3*A13+XY1*A15
XX4=XY2*A14
XX5=XY3*A14+XY2*A15
XX6=XY3*A15
RETURN
END

```

```

C
C
SUBROUTINE PDERIV(E1,E2,E3,E4,E5,I)
THIS SUBROUTINE CALCULATES PRIMARY DERIVATIVES EXCEPT FOR SURFACE CONTRIBS
DIMENSION PGA(11,6),PGB(11,6)
COMMON/BD2/ZZ1,ZZ2,ZZ3,ZZ4,ZZ5,ZZ6
COMMON/BI/PGA,PGB
ZZ1=(3.*E1-E4)*PGA(I,1)+E3*(PGB(I,1)+PGA(I,2))+PGA(I,3))
ZZ2=E2*PGA(I,1)+(E1+E1)*PGA(I,2)+E3*(PGB(I,2)+PGA(I,4))
ZZ3=(E2+E2)*PGA(I,1)+(E1+E1)*PGA(I,3)+E3*(PGB(I,3)+PGA(I,4))+2.*PGA
  1(I,5))
ZZ4=E2*(PGA(I,2)+PGA(I,2)+PGA(I,3))+E5*PGA(I,4)+E3*(PGB(I,4)+PGA(I
  1,6))+PGA(I,6))
ZZ5=E2*PGA(I,3)+E5*PGA(I,5)+E3*(PGB(I,5)+PGA(I,6))
ZZ6=E2*(PGA(I,4)+PGA(I,5))+(E4+E4)*PGA(I,6)+E3*PGB(I,6)
RETURN
END

```

```

C
C
SUBROUTINE ALFA(U1,U2,U3,U4,U5,I,UX)
THIS SUBROUTINE CALCULATES ALFAS ETC.
EXCEPT FOR SURFACE CONTRIBS
DIMENSION U(11,6),UB(11,6)
COMMON/B4/U,UB
COMMON/BC1/WA,WB,WC,WD,WE,WF,WG,WH,WI,WJ,WK,WL
WA=(U2*U(I,1))+(U1+U1)*UB(I,1)+U3*(UB(I,2))+UB(I,3))/UX
WB=(U2*(U(I,2))+UB(I,1))+U5*UB(I,2)+U3*UB(I,4))/UX
WC=(U2*(UB(I,1))+UB(I,1))+U(I,3))+U5*UB(I,3)+U3*(UB(I,4))+UB(I,5)
  1+UB(I,5))/UX
WD=(U2*(UB(I,2))+UB(I,3))+U(I,4))+(U4+U4)*UB(I,4)+(U3+U3)*UB
  1(I,6))/UX
WE=(U2*(UB(I,3))+U(I,5))+(U4+U4)*UB(I,5)+U3*UB(I,6))/UX
WF=(U2*(UB(I,4))+UB(I,5))+U(I,6))+U3*UB(I,6))/UX
WG=((U4-3.*U1)*U(I,1))-U3*(UB(I,1))+U(I,2))+UB(I,3))/UX
WH=-((U2*U(I,1))+(U1+U1)*U(I,2))+U3*(U(I,4))+UB(I,2))/UX
WI=-((U2+U2)*U(I,1))+(U1+U1)*U(I,3))+U3*(U(I,4))+U(I,5))+UB(I,3)
  1))/UX
WJ=-((U2*(U(I,2))+U(I,2))+U(I,3))+U5*U(I,4))+U3*(U(I,6))+UB(I,4)
  1))/UX
WK=-((U2*U(I,3))+U5*U(I,5))+U3*(U(I,6))+UB(I,5))/UX
WL=-((U2*(U(I,4))+U(I,5))+(U4+U4)*U(I,6))+U3*UB(I,6))/UX
RETURN
END

```

```

C
C
C
SUBROUTINE SDERIV(V1,V2,V3,V4,V5,I,II)
THIS SUBROUTINE CALCULATES THE SECONDARY DERIVATIVES EXCEPT
FOR THE SURFACE CONTRIBS
COMMON/BC2/G1,G2,G3,G4,G5,G6,G7,G8,G9,G10,G11,G12
DIMENSION SGA(11,12),SGB(11,12),A(12),PGA(11,6)
COMMON/B1/PGA
COMMON/B2/SGA,SGB
COMMON/B3/A
WE ARE GOING TO WORK WITH UNDIMENSIONED QUANTITIES
YY1=SGA(11,1)
YY2=SGA(11,2)
YY3=SGA(11,3)
YY4=SGA(11,4)
YY5=SGA(11,5)
YY6=SGA(11,6)
YY7=SGA(11,7)
YY8=SGA(11,8)
YY9=SGA(11,9)
YY10=SGA(11,10)
YY11=SGA(11,11)
YY12=SGA(11,12)
C
YA1=A(1)
YA2=A(2)
YA3=A(3)
YA4=A(4)
YA5=A(5)
YA6=A(6)
YA7=A(7)
YA8=A(8)
YA9=A(9)
YA10=A(10)
YA11=A(11)
YA12=A(12)
C
YX1=PGA(11,1)
YX2=PGA(11,2)
YX3=PGA(11,3)
YX4=PGA(11,4)
YX5=PGA(11,5)
YX6=PGA(11,6)
THE FOLLOWING COMBINATIONS OCCUR MORE THAN ONCE
F1=V5
F2=3.*V1+V4
F3=V1+3.*V4
F4=YA1+YA1+YA8
F5=YA8+YA8+YA1
F6=YA2+YA11
F7=YA3+YA10+YA10
C

```

```

F8=YA3+YA3+YA10
F9=YA5+YA12+YA12
F10=YA5+YA5+YA12
F=F2+F2-F3
XF=V2+V2
FX=4.*V1
FF=FI+FI
XFF=XF+XF

G1=F*YY1+V3*(SGB(I,1))+YY2+YY3)+3.*YA1*YX1+YA7*(YX2+YX3)
G2=V2*YY1+FX*YY2+(SGB(I,2))+YY4)*V3+YA2*YX1+F4*YX2+YA7*YX4
G3=XFF*YY1+FX*YY3+V3*(SGB(I,3))+YY4+(YY5+YY7)*2.)+(YA2+YA2+3.*YA3)*
1 YX1+YA9*YX2+(F4+YA9)*YX3+YA7*(YX4+YX5+YX5)
G4=V2*(4.*YY2+YY3)+F2*YY4+V3*(SGB(I,4))+2.*(YY6+YY8))+YA4*YX1
G4=G4+(F8+YA2+YA2)*YX2+YA2*YX3+(F5+YA9)*YX4+(YA7+YA7)*YX6
G5=V2*YY3+F2*YY5+V3*(SGB(I,5))+YY6+YY9)+3.*YA5*YX1+YA11*YX2
G5=G5+F6*YX3+F5*YX5+YA7*YX6
G6=V2*(YY4+YY5)+FF*YY6+V3*(SGB(I,6))+YY10)+YA6*YX1+F10*YX2
G6=36+F6*YX4+YA2*YX5+3.*YA8*YX6
G7=XF*YY3+F2*YY7+V3*(SGB(I,7))+YY8+YY9+YY9)+(YA4+YA4)*YX1
G7=G7+F8*YX3+YA9*(YX4+YX5+YX5)
G8=V2*(YY4+YY4+YY7)+FF*YY8+V3*(SGB(I,8))+YY10+YY10)+YA4*(YX2+YX2+YX
13)+F7*YX4+(YA9+YA9)*YX6
G9=XF*(YY5+YY7)+FF*YY9+V3*(SGB(I,9))+YY10+4.*YY11)+(YA6+YA6)*YX1
G9=G9+(YA4+F10)*YX3+YA11*YX4+(F7+YA11+YA11)*YX5+YA9*YX6
G10=V2*(2.*(YY6+YY8))+YY9)+F3*YY10+V3*(SGB(I,10))+4.*YY12)
G10=G10+YA6*(YX2+YX2+YX3)+(F9+YA4)*YX4+YA4*YX5+(3.*YA10+YA11+YA11)
1 *YX6
G11=V2*YY9+F3*YY11+V3*(SGB(I,11))+YY12)+YA6*YX3+F9*YX5+YA11*YX6
G12=V2*(YY10+YY11)+4.*V4*YY12+V3*SGB(I,12))+YA6*(YX4+YX5)+3.*YA12*Y
1 X6
1 RETURN
END

```

```

C
C
SUBROUTINE CDERIV
THIS SECTION CALCULATES DERIVATIVES OF THE PARAXIAL CHROMATIC COEFFS
CHROMATIC DERIVATIVES REQUIRE THETA1. SEEP. 249 O.A.C.
COMMON/CESULT/ FDN(10,4), DSF(10,8), DF(10,4), XKF(10,4), CF(10,4)
DIMENSION CUL(11), CU2(11)
DIMENSION C(10), D(10), XN(10), XK(10), XJ(10), Q(10)
DIMENSION FT(4,10)
DIMENSION XIA(10), XIB(10), YB(10), VA(11), VB(11), W(10)
DIMENSION FN(10), FNB(10), FNQB(10), F2N(10), F2NB(10)
DIMENSION F2NQ(10), F2NQB(10), FSUMA(11,4), FSUMB(11,4)
COMMON/CU/ CUL, CU2
COMMON/DATA/ C, D, XN, XK, XJ, Q, XIA, XIB, YA, YB, VA, VB, W, XLA
COMMON/B23/ FN, FNB, FNQ, FNQB, F2N, F2NQ, F2NB, F2NQB
COMMON/B22/ QL, QM, QN, QO
COMMON/B20/ XH1, XH2, XH3, XH4
COMMON/B19/ FSUMA, FSUMB
COMMON/MISC/ J, QA, QTILD
COMMON/INT/ KOUT, KINP, NN, KN
COMMON/THET/ TY, TBY, TV, TTV, TTY, TTBY, TTBV, ITTY, ITTBY, ITTV, ITTBV
FIRSTLY DERIVATIVES W.R.T. D
JA=J
IF(J.EQ.1) GOTO22
THETA1 HAS NO DERIVATIVES OF INT CONTRIBS IN CASE OF D
CALL THETA1(TY, TBY, TV, TTBV, J, XLA)
TTY=XH1
TBY=XH2
TV=XH3
TTBV=XH4
M=J
CALL COLDER(TY, TBY, TV, TTBV, TTY, TTBY, TTV, TTBV, M)
DIFD=QL
DIFBD=QM
D2FD=QN
D2FBD=QO
DERIVATIVES W.R.T. CO. THESE REQUIRE SURFACE CONTRIB DERIVATIVES.
DERIVATIVES OF INTRINSIC FI ETC W.R.T. CO
YIA=YA(J)/XIA(J)
DFC=FN(J)*YIA
DFBC=QTILD*DFC
DFQC=FNQ(J)*YIA
DFQBC=DFQC*QTILD
M=J+1
CALL THETA1(TTY, TTBY, TTV, TTBV, M, XLA)
TTY=XH1-DFQC/XLA
TTBY=XH2-DFQBC/XLA
TTTV=XH3+DFC/XLA
TTTBV=XH4+DFBC/XLA
QN=1./((XN(J)*XIA(J))*XIB(J))
DF2C=YIA*F2N(J)+ON*FSUMA(J,1)*FNB(J)
YIB=YB(J)/XIB(J)

```

```

DF2BC=YIB*F2NB(J)+ON*FSUMB(J,2)*FN(J)
CALL COLDER(TTY,TIBY,TTV,TTBV,TTLY,TTIBY,TTIV,TTIBV,M)
D1FC=QL+DFC
D1FBC=QM+DFBC
D2FC=QN+DF2C
D2FBC=QQ+DF2BC
DERIVATIVES W.R.T. KO
XLX=XLA*XK(J)
DFNK=-FN(J)/(XLA*XK(J))
DFNQK=-FNQ(J)/(XLA*XK(J))
DFNBK=QA*DFNK
DFNQBK=QA*DFNQK
M=J+1
CALL THETA1(TTTY,TTTBY,TTTV,TTTBV,M,XLA)
TTTTBY=XH2-DFNBK
TTTTIV=XH3+DFNK
TTTTBV=XH4+DFNBK
TTTTLY=XH1-DFNQK
CALL COLDER(TTTY,TTTBY,TTTV,TTTBV,TTTY,TTTIBY,TTTIV,TTTIBV,M)
DF2K=-F2N(J)/(XLA*XK(J))
DF2BK=-F2NB(J)/(XLA*XK(J))
D1FK=QL+DFNK
D1FBK=QM+DFNBK
D2FK=QN+DF2K
D2FBK=QQ+DF2BK
FT(1,J)=D1FK*XLX
FT(2,J)=D1FBK*XLX
FT(3,J)=D2FK*XLX
FT(4,J)=D2FBK*XLX
D020031=1.4
IF(JA.EQ.1)GOTO8
GOTO2004
FDN(J,I)=FT(I,J)/XN(J)
GOTO9
FDN(J,I)=(FT(I,J)-FT(I,J-1))/XN(J)
CONTINUE
CONTINUE
THIS SECTION REQUIRES DERIVATIVES W.R.T. N.
THIS SECTION CALCULATES DERIVATIVES W.R.T. DISPERSIONS 1 AND 2.
THIS ASSUMES DERIVED GENERAL DISPERSION DERIVATIVES ARE CORRECT
CERTAINLY ARE CORRECT FOR SIMPLE LENSES
X=XK(J)
JL=J+1
IF(CUL(JL).EQ.0.)GOTO11
WJJ=YA(J)*X*XIA(J)-YA(JL)*XIA(JL)
WBJ=YA(J)*X*XIB(J)-YA(JL)*XIB(JL)
DF1DS1=WJJ
DFBDS1=WBJ
DF1DN=FDN(J,1)
DF1BDN=FDN(J,2)

```

```

DF2DS1=DF1DN+FSUMA(KN,1)*WBJ-FSUMB(KN,1)*WJJ
DBFDIS=DF1BDN+FSUMA(KN,2)*WBJ-FSUMB(KN,2)*WJJ
DF2DS2=WJJ
DFBDS2=WBJ
CONTINUE
RETURN
END

```

11

```

SUBROUTINE THETA1(AY1,AY2,AY3,AY4,I,XAL)
DIMENSION FSUMA(11,4),FSUMB(11,4)
COMMON/B19/FSUMA,FSUMB
COMMON/B20/XH1,XH2,XH3,XH4
XH=AY2*FSUMA(I,1)
XH2=AY3*FSUMB(I,2)
AY=AY4-AY1
FHH=FSUMA(I,2)+FSUMB(I,1)
XH1=(XH+XH2)/XAL
XH2=(AY2*FHH+AY*FSUMB(I,2))/XAL
XH3=(-AY3*FHH+AY*FSUMA(I,1))/XAL
XH4=-XH1
RETURN
END

```

```

SUBROUTINE COLOUR
DIMENSION XK(10), W(10), XJ(10), Q(10), XIA(10), XIB(10)
DIMENSION YA(10), VB(10), VA(11), VB(11), C(10), D(10), XN(10)
DIMENSION F2NQ(10), F2NQB(10), FSUMA(11,4), FSUMB(11,4)
DIMENSION S5A(10), S6A(10), P1A(10), P2A(10), P1BA(10), P2BA(10)
DIMENSION XXN(11,3), CUL(11), CU2(11), XK1(11), XK2(11)
DIMENSION FN(10), FNB(10), FNQ(10), FNQB(10), F2N(10), F2NB(10)
COMMON/CU/CU1, CU2
COMMON/B23/FN, FNB, FNQ, FNQB, F2N, F2NQ, F2NB, F2NQB
COMMON/INI/KOUT, KINP, NN, KN
COMMON/DATA/ C, D, XN, XK, XJ, Q, XIA, XIB, YA, YB, VA, VB, W, XLA
COMMON/B19/FSUMA, FSUMB
READ(KINP, 200) XL1, XL2, XL3
WRITE(KOUT, 210) XL1, XL2, XL3
DO6000 I=1, KN
READ(KINP, 200) (XXN(I, J), J=1, 3)
WRITE(KOUT, 220) (XXN(I, J), J=1, 3)
WRITE(KOUT, 240)
DO6001 K=1, KN
D1=XXN(K, 1)-XXN(K, 3)
D2=XXN(K, 2)-XXN(K, 3)
DL=XL1-XL3
YWI=DL/(1.+2.5*DL)
YU2(K)=(YW2*D1-YW1*D2)/(YWI*YW2*(YWI-YW2))
YU2=DL/(1.+2.5*DL)
CU1(K)=D1/YWI-CU2(K)*YWI
DL=XL2-XL3
WRITE(KOUT, 230) CU1(K), CU2(K), K
CONTINUE
DO6002 K=1, KN
DO6002 I=1, 4
FSUMA(1, I)=0.
FSUMB(1, I)=0.
FSUMA CONTAINS 1ST ORDER CHROMATIC ABERR COEFFS. SIMILARLY FSUMB
1 3 GIVES N=1, UNBARRED. 2 GIVES N=1, BARRED. SIMILARLY FSUMB
3 GIVES N=2, UNBARRED. 4 GIVES N=2, BARRED.
DO6003 J=1, NN
X=CU1(J)/XN(J)-CU1(J+1)/XN(J+1)
XK1(J)=XK(J)*X
XX=CU2(J)/XN(J)-CU2(J+1)/XN(J+1)-X*CU1(J+1)/XN(J+1)
XK2(J)=XK(J)*XX
FN(J)=XN(J+1)*YA(J)*XIA(J)*XK1(J) *(-1.)
FNB(J)=Q(J)*FN(J)
TILD=FB(J)/YA(J)
WRITE(KOUT, 1)
WRITE(KOUT, 2) TILD
WRITE(KOUT, 6503) X, XK1(J), XX, XK2(J), J
WRITE(KOUT, 1) * FN(J)
FNQ(J)=TILD * FN(J)
FNQB(J)=TILD*FNB(J)

```

```

K=J+1
FSUMA(K,1)=FSUMA(J,1)+FNB(J)
FSUMA(K,2)=FSUMA(J,2)+FNB(J)
FSUMB(K,1)=FSUMB(J,1)+FNQ(J)
FSUMB(K,2)=FSUMB(J,2)+FNQ(B(J))
F2M=-XN(J+1)*YA(J)*XK2(J)
F2MB=Q(J)*F2M
F2MQB=TILD*F2M
F2MQB=TILD*F2MB
WRITE(KOUT,3)
WRITE(KOUT,3)
F2N(J)=F2M+(FSUMA(J,1)*FNB(J)-FSUMB(J,1)*FN(Q(J))/XLA
F2NQ(J)=F2MQ+(FSUMA(J,2)*FNB(J)-FSUMB(J,2)*FN(Q(J))/XLA
F2NQ(B(J)=F2MQB+(FSUMA(J,2)*FNBQ(B(J)-FSUMB(J,2)*FNQ(B(J))/XLA
F2NQMA(K,3)=F2N(J)
FSUMA(K,4)=FSUMA(J,4)+F2NB(J)
FSUMB(K,3)=FSUMB(J,3)+F2NQ(J)
FSUMB(K,4)=FSUMB(J,4)+F2NQ(B(J)
WRITE(KOUT,6506)
WRITE(KOUT,6501)
WRITE(KOUT,6507)
WRITE(KOUT,6504)
WRITE(KOUT,6508)
WRITE(KOUT,6503)
WRITE(KOUT,6500)
WRITE(KOUT,6501)
WRITE(KOUT,6502)
WRITE(KOUT,6503)
1,KNI)
WRITE(KOUT,6504)
WRITE(KOUT,6505)
WRITE(KOUT,6503)
(FSUMA(K,3),FSUMA(K,4),FSUMB(K,3),FSUMB(K,4),K,K=1)
1,KNI)
FORMAT('TILD=',E13.5)
FORMAT('O*****')
FORMAT('ORDER TWO')
SUM FIP
FORMAT('4E13.5,I2)
FORMAT('ORDER ONE')
FORMAT('1PARAXIAL ABERRATION COEFFICIENTS')
F2PB
FIPB
SUM F2P
SUM F2Q
FORMAT('SURFACE CONTRIBUTIONS')
CNUL='E13.5,CNU2='E13.5,2X,'K='I2)
FORMAT('3F10.6)

```

```
240 FORMAT('OFIRST TWO COEFFS IN SERIES FOR DELTA N')  
210 FORMAT('REFRACTIVE INDICES AT WAVELENGTHS',3E13.5)  
220 FORMAT(33X,3E13.5)  
    RETURN  
    END
```

```

SUBROUTINE COLDER(QC,QD,QE,QF,QH,QI,QJ,QK,I)
DIMENSION FGA(11,4),FGB(11,4)
COMMON/B21/FGA,FGB
COMMON/B22/QL,QM,QN,QO
QL=QE*(FGB(I,1))+FGA(I,2))+(QC-QF)*FGA(I,1)
QM=QE*(FGB(I,2))+QD*FGA(I,1)
QN=QE*(FGB(I,4))+FGB(I,3))+FGA(I,3)*(QC-QF)+QH*FGA(I,1)+QJ*FGA(I,2)
QO=QD*FGA(I,3)+QE*FGB(I,4)+QI*FGA(I,1)+QK*FGA(I,2)
RETURN
END

```

APPENDIX 3

42. The expansion of $\hat{\underline{\epsilon}}'$

The augmented aberration, $\hat{\underline{\epsilon}}'$, of a ray can be expanded as a power series in the ray co-ordinates $\underline{Y}$, $\underline{V}$.

If we write

$$\xi := Y^2 + Z^2, \quad \eta := YV + ZW, \quad \zeta := V^2 + W^2,$$

then in terms of canonical co-ordinates $\underline{\Delta}$ may be written

$$\underline{\Delta} = \sum_{n=1}^{\infty} \sum_{\mu=0}^n \sum_{\nu=0}^{\mu} (g_{\mu\nu}^{(n)} \underline{Y}_1 + \bar{g}_{\mu\nu}^{(n)} \underline{V}_1) \xi_1^{n-\mu} \eta_1^{\mu-\nu} \zeta_1^{\nu}.$$

The $g_{\mu\nu}^{(n)}$, $\bar{g}_{\mu\nu}^{(n)}$ are called the contributions to the aberration from a particular surface. From equation (3.8) the augmented aberration is thus

$$\hat{\underline{\epsilon}}'_k = \sum_{n=1}^{\infty} \sum_{\mu=0}^n \sum_{\nu=0}^{\mu} (G_{\mu\nu k}^{(n)} \underline{Y}_1 + \bar{G}_{\mu\nu k}^{(n)} \underline{V}_1) \xi_1^{n-\mu} \eta_1^{\mu-\nu} \zeta_1^{\nu}$$

where

$$G_{\mu\nu j}^{(n)} = \sum_{i=1}^{j-1} g_{\mu\nu i}^{(n)}, \quad \bar{G}_{\mu\nu j}^{(n)} = \sum_{i=1}^{j-1} \bar{g}_{\mu\nu i}^{(n)}.$$

The $G_{\mu\nu}^{(n)}$, $\bar{G}_{\mu\nu}^{(n)}$ are called the aberration coefficients of the system.

Similar equations may be written for paracanonical co-ordinates.

APPENDIX 4

43. Table of symbols

The following table lists those symbols commonly appearing in this thesis and which are not contained in L. The number in brackets, after the meaning of the symbol, is the section in which the symbol is first used.

<u>Symbol</u>	<u>Meaning</u>
a	rate of change of a separation. (14)
b	coefficient occurring in formulae for π , $\bar{\pi}$. (27)
c	coefficient occurring in formulae for π , $\bar{\pi}$. (27)
d	separation between two surfaces. (2)
δ	scale factor in image height equation. (20)
e	radius of aperture stop. (28)
ϕ	power of a thin lens. (17)
ϕ_1	first extra axial power of a thin lens. (40)
g	coefficient occurring in formulae for ρ , $\bar{\rho}$. (27)
$\gamma_k(n)$	functions of roots of a polynomial. (20)
$h(z)$	radius of entrance pupil. (21)
h_0	radius of entrance pupil at $z = 0$. (21)
$K_1, K_2, \dots, K_8$	functions of refractive index occurring in formulae for the fixed aperture aberration coefficients. (31)
$\textcircled{K}$	the auxiliary system in a zoom lens. (14)
λ	the paraxial invariant. (8)

<u>Symbol</u>	<u>Meaning</u>
$p(z)$	the distance of the stop from the first surface. (28)
$p, \bar{p}$	coefficients in the expansions of $\pi, \bar{\pi}$. (27)
$\pi, \bar{\pi}$	polynomials occurring in formulae for the fixed aperture aberration coefficients. (26)
R	the magnification range of a zoom lens. (19)
r	$v'_{pn}(1)/v'_{pn}(0)$. (19)
$r, \bar{r}$	coefficients in the expansions of $\rho, \bar{\rho}$. (27)
$\rho, \bar{\rho}$	polynomials occurring in the formulae for the fixed aperture aberration coefficients. (26)
S	the shape of a thin lens. (17)
$v_{an}^{(j)}$	the derivative of order j of v'_{an} . (18)
$y_{an}^{(j)}$	the derivative of order j of y'_{an} . (18)
y'	the image height in the ideal image plane. (17)
z	the zoom parameter. (14)
Z	distance of an element from its initial position. (17)
$Z_{\max}$	maximum value of Z . (17)
$\textcircled{Z}$	the zoom part of a zoom lens. (14)

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