INTERFEROMETRIC STUDY OF DENSITY FLUCTUATIONS IN A TOKAMAK PLASMA

by

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Declaration

I declare that, except where explicitly stated, this thesis is my own original work and that no part of it has been previously accepted or presented for the award of any other degree or diploma, and that no material previously published or written by another person is included. Chapter 2 of the thesis presents a description of the LT-4 tokamak which is largely a synopsis of research reports and publications produced by the laboratory and constitutes the work of a large number of people. Chapter 3, on the theory of scattering from plasma inhomogeneities, is a distillation of important results produced in collaboration with Dr. John Howard and Dr. L. E. Sharp towards the production of a book titled “Forward Angle Collective Scattering on Fusion Plasmas” which will be published as part of the Adam Hilger series on plasma physics.

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In a large laboratory like this one, there are lots of people who in some way contributed to the successful completion of this thesis and who created an interesting and enjoyable environment in which to work.

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Abstract

Density fluctuations in the LT-4 tokamak plasma are investigated using a Phase Scintillation Interferometer operating at 10.6 μm which is sensitive to density fluctuations of $\delta n_e/n_e \geq 10^{-4}$. The plasma is imaged across a linear detector array which can be rotated to record projections in any direction, from toroidal to poloidal.

The theory of forward scattering from plasmas is developed from the Rytov approximation and aspects of the Fourier diffraction projection theorem relevant to plasma scattering. The result is a clear conceptual picture of diffraction from arbitrary extended refractive media, from which important analytical tools are developed.

The Phase Scintillation Interferometer is used to image density perturbations produced by large scale magnetohydrodynamic (MHD) modes in the plasma associated with Mirnov oscillations. Structural characteristics are determined, and a comparison between experimental and computed projections of the Dubois model is made which shows that the density fluctuations are consistent with a model of rotating magnetic islands. Island widths and local magnetic field fluctuations are determined and are found to compare well with measured poloidal magnetic field fluctuations.

The interferometer is used in conjunction with other diagnostics to investigate minor and major disruptions in LT-4. The time frequency distribution is introduced as an important analytical tool in the characterization of the various regimes of MHD activity. Frequency and amplitude variations of an $m = 3$ mode during current rise appear correlated with variations in toroidal loop voltage. The mode is also found to persist throughout the whole discharge and to play a part in mode locking which precedes major disruptions. Mode frequencies are found to vary in a regular way with the safety factor $q(a)$. Precursor oscillations before
minor and major disruptions are identified. A strong \( m = 1 \) type of internal relaxation is found to follow rapid growth and locking of an \( m = 2 \) mode during minor disruptions.

The interferometer is also applied to the measurement of fine scale density fluctuations in the LT-4 tokamak during periods of low level MHD activity. Line integral measurements indicate an edge fluctuation level of about 10% and broad band spectra typical of strong turbulence. Anisotropy in the spectrum of fluctuations perpendicular to the magnetic field is observed. This observation runs counter to reported measurements of isotropic fluctuations made on other tokamaks using small angle scattering techniques. Very long correlation lengths along the field lines are observed, which are consistent with nearly all models of turbulence in tokamak plasmas. The images are numerically filtered so as to isolate and display counter-propagating structures in the turbulent flow.
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Chapter 1

INTRODUCTION

Several decades after the tokamak’s favourable confinement properties were demonstrated[1] and after the initial optimism for the machine, fundamental problems still persist with MHD instabilities and anomalous transport\(^1\) of particles and energy from the plasma.

Although the role instabilities play in the density and current limits of tokamak plasmas are fairly well understood[2, 3], the role of instabilities in anomalous transport remains poorly understood. The problem is not least compounded by the experimental difficulties in determining the nature of fluctuations in the plasma responsible for the enhanced transport. The hope of finding conclusive evidence for turbulence driven transport has motivated the development of scattering diagnostics for tokamaks for at least a decade[4, 5]. Such techniques, to this day, utilize the angular scattering of radiation from refractive index variations to directly measure the spectrum of plasma fluctuations within a scattering volume. Unfortunately, to achieve reasonable spatial resolution whilst trying to observe larger scale length structures expected in large tokamaks, the beam wavelength must also be made longer. However, high density fusion type plasmas presents a limit to the maximum wavelength one can use due to operation near the critical density limit. A compromise seems to be in the far infrared (FIR) region of the spectrum applied most successfully on the Text tokamak[6], however, even by using FIR sources, the small scattering angles involved for large scale structures would provide insufficient spatial resolution for large machines like TFTR and JET.

\(^1\)The anomaly refers to the discrepancy between theoretically calculated transport coefficients and those determined experimentally.
Imaging diagnostics can in theory produce high resolution images of plasmas using tomographic techniques, however, in practice, tomography of structures small compared with the plasma minor radius would be exceedingly costly to implement and technically very challenging to say the least. The application of imaging techniques towards the analysis of microfluctuations in tokamak plasmas using a single probe beam has been demonstrated as a useful diagnostic on the TCA and LT-4 tokamaks [7, 8]. Although tomographic recovery of local fluctuation levels are not possible with such systems without a large amount of a priori information, still the methods are very useful in determining the structure of large scale MHD modes as well as to some extent the spatial distribution of small scale structures.

Curiously, the measurement of large scale MHD modes relevant to the macroscopic stability of plasmas has been left to arrays of X-ray detectors and Mirnov coils[9, 10], whilst interferometers have been used principally for measuring mean line integral densities. Early attempts at using interferometers to obtain spatially resolved plasma perturbations used an array of independent beams which could only be used for resolving structures of the scale size of the plasma and effectively filtered out small scale density fluctuations[11]. On the other hand, scattering diagnostics, of use in determining power spectra of plasma fluctuations were of little use in the analysis of large scale MHD activity.

The development and application of an interferometer capable of imaging both small scale density fluctuations characteristic of plasma turbulence, as well as narrow band, large scale MHD activity constitutes the scope of this thesis. The diagnostic developed for the purpose is a “Phase Scintillation Interferometer” operating at 10.6\(\mu\)m with a phase sensitivity of 10\(^{-9}\) radians. The interferometer was first proposed by Sharp [12] as an extension to work already performed on Cleo using microwaves to scatter from plasma fluctuations. The choice of the probing wavelength and the method of detection, (imaging instead of far field scattering) follows naturally from the theory of interplanetary scintillation[13], originally developed to study refractive index variations in the earths ionosphere, using radio telescopes to record intensity fluctuations in the emissions from known
stellar radio sources. The radio wavelengths were short enough that the ground stations were effectively in the optical near field of the ionosphere, where density variations in the ionosphere modulated the intensity of the transmitted radiation. The modulations were observed to drift across the ground and the drift velocity of ionospheric irregularities were determined using multiple observer points and correlation techniques[14].

The main problem with imaging devices is their inability to provide localized measurements along the line of sight. The problem is common to many fields of physics grouped under the very general field of inverse scattering and tomography. Some related fields include,

- Interstellar and interplanetary scintillations,
- Ultrasonic tomography,
- Seismology,
- and X-ray crystallography

just to name a few. In some cases a known source is used to probe the medium in a non-perturbing way such as in seismology or plasma scattering, whilst in others, emissions from extended objects are used such as in X-ray tomography of plasmas.

Plasma scattering in the laboratory is only one aspect of a subclass of inverse scattering problems where the scattered field conforms to solutions of the inhomogeneous Helmholtz equation[15]. Although the central problem of scattering theory is finding solutions to the inverse scattering problem, this in general is too difficult to solve except by the use of many a priori conditions, and by performing multiple simultaneous projections of the plasma. Such problems are called illconditioned as inversions have to be produced with typically insufficient data for a complete determination of the source. Instead, we adopt the typical, and much simpler approach of restricting ourselves to the fully deterministic problem of obtaining the form of the scattered radiation from a known distribution, and then using this knowledge to infer properties of the scatterer which are conveyed unambiguously.
In this thesis, we present a general formalism in which the diffracted field from an arbitrary, extended medium can be determined in a simple way by use of the Fourier diffraction projection theorem (FDPT) given that the scatterer satisfies certain weak constraints. This formalism provides insight into how tomography may be performed on tokamak plasmas without the requirement that line integral projections of the medium be formed.

The foundations of optical scattering from waves in transparent media may be traced back to the early 1930’s when Brillouin first proposed that light waves would scatter from transparent waves in the same way that X-rays were observed to scatter from crystals. However the foundational work on the diffraction of light from waves in transparent media is due to Raman and Nath where they correctly noted that waves in transparent media acted as phase gratings to the incident radiation, and produced a theory which for the most part explained the complex scattered fields which were observed experimentally even from strong phase perturbing screens. An excellent review of the history of this era and many references may be found in Bergmann[16]. The representation of the scattering medium as a transparent thin phase grating also forms the basis of modern investigations into far forward scattering from plasmas by very high frequency laser beams[17, 18, 19], where the plasma is represented as a thin refractive screen.

The construction of a phase scintillation interferometer on the LT-4 Tokamak was motivated by a need to spatially resolve the high order modes thought responsible for anomalous transport. A high priority was to observe turbulent fluctuations present in the plasma with the hope of identifying non-sinusoidal ‘coherent’ structures[20, 21] not readily determined by power spectral techniques such as microwave scattering.

Chapter 2 provides an overview of the LT-4 Tokamak. The principal diagnostics on LT-4 are presented and the limitations of the diagnostics in determining the nature of plasma instabilities and fluctuations are discussed. There is also an overview of the characteristic regimes of MHD activity in LT-4. The LT-4 tokamak, like many other smaller tokamaks, tends to disrupt when the $q$ at the edge is a little less than 3. The tokamak displays a great deal of MHD activity.
which is used to characterize the various plasma regimes.

The theory of forward angle scattering from plasmas and its relation to the Fourier diffraction projection theorem is discussed in chapter 3. The concept of the Rytov phase is introduced, and the conditions under which the Rytov approximation can be considered a good approximation to the scattered field is discussed. It is shown that the solution in the Rytov approximation is more general than the solution for the scattered field in the Born approximation, and describes both large phase shift interferometry as well as large angle scattering within the one formalism. The expression for the Rytov phase is then used to characterize the performance of a wide range of optical diagnostics used on plasmas. These methods and their application to fusion plasmas are reviewed.

Chapter 4 presents a detailed discussion of the phase scintillation interferometer and its operation on the LT-4 tokamak. Certain anomalies appeared in the diffracted field from sound waves which lead to the identification of adverse effects due to non-Gaussian beam profiles, and a theoretical framework is provided for the analysis of truncated gaussian beams.

Chapter 5 centers on the interferometric study of low order MHD activity in LT-4. An analysis of MHD mode activity is presented and an \( m = 2 \) island is identified from its projections. The projections are shown to conform with a modified form of the Dubois model, extended to model \( m \geq 2 \) tearing modes. The assumption made in this work is that the density isobars correspond closely to magnetic flux surfaces. For the proper tomographic analysis of the MHD modes in LT-4, a knowledge of their instantaneous frequencies are required. Although MHD activity in tokamak plasmas are spectrally non-stationary, little in the way of applying joint time frequency domain (TFD) analysis of MHD signals has taken place. It is found that the TFD analysis of the MHD data allows a convenient categorization of the various regimes of plasma activity.

Finally in chapter 6, fine scale density fluctuations are examined. The main aim is to produce images of small scale density perturbations. The motivation for imaging is that it may be possible to resolve the nature of the instabilities by directly observing characteristic structures of the turbulent flow. A general
discussion of scattering from random media is presented, and a new method is introduced whereby the location of thin random phase screens may be obtained using line integral imaging techniques.
Bibliography


Chapter 2

THE LT-4 TOKAMAK

The LT-4 tokamak is one of many tokamaks currently operating around the world designed to study the properties of toroidally confined plasmas. The operational parameters of LT-4 allows for the investigation of equilibrium, stability and transport processes in fully ionized hot plasmas. These studies are of major relevance to the thermonuclear fusion program.

A brief description of the LT-4 Tokamak is presented followed by an outline of some important diagnostics on the machine. An overview of the operational regimes of LT-4 together with some characteristic MHD activity is presented and the chapter finishes with a discussion of phase scintillation interferometry on LT-4.

2.1 Description of Apparatus

The main dimensions and operating conditions of the LT-4 tokamak are listed in table 1.

As a thin high resistivity vacuum vessel was used on LT-4, the location of the plasma column was stabilized during normal operation to within 1mm vertical or horizontal displacement by use of external windings which automatically feedback on plasma position shifts[1].

The vacuum vessel’s cutoff frequency is about 30kHz whilst most Mirnov oscillations are below this, so that the signals remain unattenuated.

Fig. 2.1 shows a cut away view of the tokamak, exposing some of the 128 TF coils, the location of diagnostic ports, and the iron core. A large number of coils were used to minimize field ripple which can be a problem for a system of discrete coils when the resistive diffusion time of the field into the vacuum vessel
Table 1. Main dimensions and operating parameters of the LT-4 tokamak

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean major radius</td>
<td>$R_0 = 0.5$ m</td>
</tr>
<tr>
<td>Minor radius of vacuum vessel</td>
<td>$b = 0.14$ m</td>
</tr>
<tr>
<td>Minor plasma radius (normal operation)</td>
<td>$a = 0.10$ m</td>
</tr>
<tr>
<td>Toroidal field (max.)</td>
<td>$B_0 = 3$ T</td>
</tr>
<tr>
<td>Plasma current (max.)</td>
<td>$I_p = 70$ kA</td>
</tr>
<tr>
<td>Available flux swing of transformer</td>
<td>0.3 V/s</td>
</tr>
<tr>
<td>Plasma duration (max.)</td>
<td>160 ms</td>
</tr>
<tr>
<td>Interval between pulses</td>
<td>4–5 min</td>
</tr>
<tr>
<td>Typical mean plasma density</td>
<td>$\bar{n} = 3 \times 10^{19}$ m$^{-3}$</td>
</tr>
<tr>
<td>Electron temp.: peak on minor axis</td>
<td>$T_0 = 500$ eV</td>
</tr>
<tr>
<td>Typical energy confinement time</td>
<td>$\tau_E \sim 0.5$ ms</td>
</tr>
</tbody>
</table>

Table 2.1: Main dimensions and operating conditions of the LT-4 tokamak

approaches the lifetime of the plasma. Diagnostic access to the plasma is via four sets of 200x20mm rectangular vertical viewing ports with 18 9.5mm diameter ports providing tangential views of the plasma. The plasma minor radius is confined to 10cm by a stainless steel limiter with an adjustable vertical limiter which can be used to modify the minor radius of the plasma.

A maximum operating toroidal field of 3T was supplied by the Canberra homopolar generator operating with a cycle time of about 5 minutes. The plasma current is the secondary of a transformer whose primary current is supplied by capacitor banks and a mercury arc mains rectifier power supply [2]. The plasma current attains its maximum value within 15 ms and the pulse length (about 100 ms) is set by the saturation of the iron core.

2.2 Principal Diagnostics

The diagnostics on LT-4 can be divided into two groups. The first and simplest, both in terms of construction and analysis, are various magnetic pick up loops distributed around the vacuum vessel. These are used for measuring loop volt-
Figure 2.1: Plan view of LT-4 showing the iron core, toroidal field coils, restraint frame and vacuum chamber.
Figure 2.2: Arrangement of diagnostics on LT-4.

age, plasma current and plasma position. Loop voltage is obtained by measuring the poloidal flux changes using four toroidal coils located at R=39, 61 cm and z=±11 cm. Plasma current is obtained by numerical integration of the voltage signal from a Rogovskii coil located outside the vacuum vessel. The plasma’s position is determined by suitably located sine and co-sine coils on the vacuum vessel. A description of the various pickup coils used on tokamaks can be found in Hutchinson’s thesis[3].

Various arrays of small Mirnov coils (100 turns, 3.2mm radius) are arranged azimuthally and poloidally outside the vacuum vessel to detect local variations in the poloidal magnetic field ($\dot{B}_\theta$) at a minor radius of 14.5 cm, well outside the plasma. Two sets of coils are arranged poloidally, called M16 (16 individually monitored equispaced coils) and M8 at $\phi = 45^\circ$ and $191^\circ$ respectively (see Fig. 2.2). Another array (N8) consists of 8 equispaced toroidal coils. A single magnetic probe is also mounted inside the vacuum vessel to pick up any high frequency oscillations which may be missed by the external Mirnov coils.
The only diagnostic available which measures rapid internal variations in the core of the plasma is the 18 channel soft X-ray (SXR) detector array. Fig. 2.3 provides a schematic view of the soft X-ray diagnostic.

Unfortunately, as the X-ray emissivity depends on $T_e$, $n_e$, $Z_{eff}$, and also on the recombination of ions and impurity radiation, it is difficult to deduce absolute parameters within the plasma[4].

The only density measuring device routinely used on LT-4 is a rotating grating HCN Mach-Zehnder interferometer, operating at $\lambda = 337\mu m$, [5] providing single point, line integral density measurements within a band width of 1 kHz and able to resolve line integral density fluctuations greater than $10^{13} \text{ cm}^{-2}$. Typical line integral densities for LT-4 are $5 \times 10^{14} \text{ cm}^{-2}$.

Electron temperature is measured by the doppler broadening of scattered laser
light from the plasma using a Q-switched ruby laser operating at $\lambda = 694.3$ nm, where the scattered light is detected at 90° to the beam direction. The laser beam is focussed at the plasma mid plane and can be located at any one of three major radial positions. The scattered spectrum is measured using a series of prisms and photomultiplier tubes. The thermal velocities of electrons in LT-4 are low enough that they can be considered non-relativistic to the extent that the relativistic blue shift in the spectrum is entirely negligible.

Hard X-rays (>100keV) are detected using a NaI scintillation photomultiplier. A fuller description of the LT-4 diagnostics including ECE spectroscopic measurements can be found in the literature[2]. Fig. 2.2 gives the location of most diagnostics on LT-4.

2.3 MHD Activity and Fluctuation Measurements

In LT-4 the plasma current is controlled from an external power supply whilst density is controlled by adjusting a programmable rate of gas puffing during the pulse. It can be seen from the Hugill diagram (Fig. 2.4), that for LT-4 (or any other tokamak) to achieve high currents and densities, it is necessary for the Hugill trajectory of the discharge to pass through a very narrow window. This window is unfortunately a very strong function of the impurity level in the plasma and impurities are a major problem for tokamaks. A discussion of impurities and plasma confinement can be found in a review by Hugill[6]. In Fig. 2.4, $q(a)$ represents the safety factor at the plasma's edge and refers to the average helicity of field lines in the toroidal geometry where $q(r) = rB_T/\rho B_\theta$ and where $B_T$ and $B_\theta$ refer to the toroidal and poloidal field components at a minor radius $r$. The motivation for achieving low $q$ operation in ohmic plasmas is to obtain higher temperatures and hence higher $\beta$. In scanning the various safety factors and density ranges in LT-4, the toroidal current was held fixed to maintain a roughly constant power input to the plasma whilst the toroidal magnetic field was varied by adjusting the rotor speed of the homopolar generator. From scans in the toroidal field and plasma density, four major operating regimes in LT-4 have been identified[1].

2–6
Figure 2.4: Hugill diagram of LT-4 showing the trajectory of a plasma discharge and boundary curves within which stable operation can be maintained.

The four basic regimes and their location in the Hugill diagram are clearly marked in Fig. 2.4. The regimes are identified by their distinct MHD activity as can be seen from shot data in Fig. 2.5 taken for the discharge in the Hugill diagram of Fig. 2.4.

- Regime I: $3.6 > q(a) > 2.9$

This regime is characterized by little detectable activity on the magnetic coils except for weak irregular signals. We will return to a discussion of these 'irregular' signals in chapter 5 where non-stationary spectral analysis techniques are used to identify coherent structures in the Mirnov coil data.

The soft X-ray signals show strong sawtooth activity at $\sim 5kHz$ accompanied by weak precursor oscillations at $\sim 20kHz$. Sawtooth oscillations in the plasma centre are indicative of a $q$ on axis of about one and a peaked current profile\cite{7}. The $q = 1$ surface is located at about $r = 3cm$, and is inferred from the inversion radius of the soft X-ray signals.
Figure 2.5: Signals from typical shot passing through all four MHD regimes.
Figure 2.6: Maximum percentage poloidal field variation in the four regimes mapped against \( q(a) \)

- Regime II: \( 2.9 > q(a) > 2.8 \)

This regime is characterized by small amplitude \( m=3 \) oscillations on the Mirnov coils with \( \tilde{B}_\theta/B_\theta \approx 0.05\% \) and frequencies in the range of 40 kHz. The X-ray data shows stronger and more regular oscillations around 20kHz superimposed on the sawtooth signals which are diminished in amplitude compared to regime I sawteeth.

- Regime III: \( 2.8 > q(a) > 2.5 \)

This regime is by far the most active, exhibiting pronounced MHD activity with \( \tilde{B}_\theta/B_\theta \approx 0.3\% \) with a significant \( m=2 \) component. This regime is characterized by a sudden growth and saturation of \( \sim 20 \text{kHz} \) oscillations observed both on mirnov coils and SXR detectors. Sawtooth oscillations typically cease in this regime. Fig. 2.6 shows the variation in the amplitude of the magnetic fluctuations in the different regimes.

The SXR fluctuations and Mirnov oscillation are phase locked in this regime
Figure 2.7: Temporal evolution of the SXR profile showing a substantial drop during regime III activity.

and the sudden growth in the amplitude of the fluctuations (presumably extending right across the minor radius) typically occurs simultaneously with a sharp drop in the X-ray emission profile (see Fig. 2.7).

The internal oscillation recorded on the soft X-ray array demonstrates clear \( m=1 \) (odd) parity and the absence of sawtooth activity is generally indicative of poor confinement and a flattened current profile.

- **Regime IV:** \( q(\alpha) < 2.5 \)

Below a \( q(\alpha) \) of 2.5, and operating at low densities and low impurity levels, the plasma may again enter a quiet MHD regime which is denoted as regime IV, where the amplitude of magnetic fluctuations drops below 0.01%. Surprisingly, strong sawtooth oscillations reappear (similar in amplitude to regime I sawteeth) which are indicative of the recovery of a peaked current profile. The low level of MHD activity during regime IV operation is also supportive of the possibility that the...
plasma has regained stable operation. The X-ray emission profile remains rather flat however. Any further decrease in $q(a)$ may results in a major disruption.

2.4 Need for a Density Imaging Diagnostic

The measurement of MHD fluctuations and their correlations with various bulk plasma properties forms the large part of the scientific work on the LT-4 tokamak. Despite this, inherent limitations in the available diagnostics severely limits the ability to resolve the structure of these fluctuations.

Knowledge of the magnetic perturbations outside the plasma cannot provide a unique description of the perturbed currents in the plasma without considerable a priori knowledge[8]. Although the presence of coherent magnetic perturbations provides strong circumstantial evidence for the existence of tearing modes, still no direct determination of their structure is possible on LT-4 except by X-ray tomography from emissions near the plasma center.

The measurement of line integral density variations with very good resolution across the line of sight can provide a direct measure of internal mode structure. Further, as an absolute measurement of the density fluctuation level is possible, estimates of local magnetic field fluctuations can be made provided island widths can be determined from the line of sight data and the magnetic shear scale length can be estimated.

It should be noted that although micro-fluctuations have not been observed in LT-4 by use of existing diagnostics, their observation constituted the main motivation for building a phase scintillation interferometer. Thus, its design was planned to allow for the detection of structures having a wide range of scale sizes. It was also necessary for the interferometer to have a wide band width ($\sim 1\text{MHz}$) and high sensitivity to very small changes in line integral densities.
Bibliography


Chapter 3

THEORY OF PLASMA SCATTERING

The following is a review, together with new material, on the scattering of radiation from inhomogeneous refractive media. The representation of the scattered field in terms of the Fourier diffraction projection theorem (FDPT) is established, which provides a unified treatment of all coherent optical methods used to investigate plasmas. Certain limiting properties of the scattered field are determined, and the applicability of frequency and/or real space representations of the scattered field are discussed for each of the limiting cases.

The Rytov approximation to the scattered field is introduced and the Born approximation is shown to be contained within the Rytov approximation. The Rytov approximation provides a unified description of both large phase shift interferometry (which violates the Born approximation) and far field scattering. The limitations of the Rytov approximation are discussed. Different forms of the FDPT for the Rytov and Born approximations are determined. Optical systems and mixing techniques used to image plasmas are discussed.

3.1 SCATTERING FROM PLASMA REFRACTIVE INDEX VARIATIONS

The microscopic treatment of plasma scattering theory is based on the classical electrodynamic interaction between an incident plane wave and a collection of essentially free plasma electrons. Though scattering from ions can be neglected (for reasons of their greater mass), it turns out that for scattering from inhomogeneities much larger than the Debye length, and fluctuations much slower than the plasma frequency, the scattered spectrum will be dominated by the electron density fluctuations following the plasma density variations[1].
As we will be dealing with the properties of the scattered radiation from the near field to the far field diffraction limits by use of the inhomogeneous Helmholtz equation, it is instructive to derive the expression for the scattered field directly from the wave equation, instead of from a consideration of the scattering from single electrons [2, 3].

3.1.1 The Helmholtz Wave Equation

We begin by considering the propagation of an arbitrary incident wave (Fig. 3.1) through a weakly scattering medium (in this case the plasma). The propagation of the incident wave is governed by the relative permittivity $\varepsilon_r$, which is a complex tensor quantity, however, under the assumption that the wave frequency of the

Arbitrary Incident Wave

Figure 3.1: Geometry for diffraction from macroscopic plasma irregularities. Note that the form of the incident illumination is not specified.
incident monochromatic field is very high[1], \((\omega_0 \gg \omega_p + \omega_\infty)\) where \(\omega_0\), \(\omega_p\) and \(\omega_\infty\) are the beam, plasma and electron cyclotron frequencies respectively) then we may write

\[
\varepsilon_r = \varepsilon_r(r,t) = 1 - \frac{n_e(r,t)}{n_\infty}
\]  

(3.1)

where \(n_e(r,t)\) is the plasma density and the cutoff density is

\[
n_\infty = \frac{k_0^2}{4\pi e_c^2},
\]  

(3.2)

where \(k_0\) is the incident wavenumber. As long as \(n_e/n_\infty \ll 1\) and \(\varepsilon_r \approx 1\) then the plasma may be treated as a weakly scattering medium.

Given these restrictions on \(\varepsilon_r\), we investigate the solution to the vector wave equation for the propagation of electromagnetic waves in an idealized source free medium. The wave equation can be expressed in terms of the electric displacement \(D = \varepsilon_0 \varepsilon_r E\) [4]:

\[
(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2})D = -\nabla \times \nabla \times (D - \varepsilon_0 E).
\]  

(3.3)

Given

\[
E = [E_0(r) + E_s(r,t)] \exp(-j\omega_0 t),
\]  

(3.4)

where \(E_0\) is the incident field and \(E_s\) is the scattered field and where the first order solution (setting \(E_s = 0\)) is just the equation for free space propagation of the incident field,

\[
(\nabla^2 + k_0^2)E_0 = 0.
\]  

(3.5)

This is of course the homogeneous Helmholtz equation for free space propagation which, in order to distinguish it from the total field (incident and scattered), we now write as \(E_0(r,t) = E_0(r) \exp(-j\omega_0 t)\). The general solution of Eqn. 3.5 is a linear superposition of plane waves. The inhomogeneous wave equation for \(E_s\) is given by

\[
(\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2})E_s = \nabla \nabla \cdot \left( \frac{n_e}{n_\infty} E \right) - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \left( \frac{n_e}{n_\infty} E \right),
\]  

(3.6)

where use has been made of the vector identity \(\nabla \times \nabla \times \equiv \nabla \nabla - \nabla^2\). Eqn. 3.6 is an inhomogeneous Helmholtz equation. The source term on the right hand
side is often called the scattering potential or forcing function and is denoted by \( f(r, t) \). The time derivatives can be eliminated using a Fourier representation together with the low temperature (nearly monochromatic or quasi-stationary) approximation. Noting that \( \nabla \cdot E = 0 \) in the plasma, the scattering potential becomes

\[
f(r, t) = \frac{1}{n_e} \left[ \nabla (E \cdot \nabla n_e) + k_0^2 n_e E \right].
\]

Let us consider a single component of the vector wave equation. If we take the scalar scattering potential to be impulsive,

\[
f(r, t) = \delta(r - r') \delta(t - t'),
\]

then the inhomogeneous wave equation has the Green’s function solutions:

\[
g^{(\pm)}(r; r', t') = -\frac{\delta(t' - [t \mp |r - r'| / c])}{4\pi |r - r'|}.
\]

In experiments, one usually considers the scattering potential as making no contribution to the total field at remote times \( t \rightarrow -\infty \) before the radiation source is activated. In this case it is appropriate to use the retarded Green’s function \( g^{(+)} \) to construct the particular solution for the wave equation \([4]\). The first order vector field at the observation point \( P = P(r) \) outside the plasma is constructed from the superposition of wave fields scattered by the collection of point elements constituting the plasma:

\[
E(r, t) = \int_{-\infty}^{\infty} dt' \int_{V_r} dr' f(r', t') g^{(+)}(r, t; r', t')
= -\int_{V_r} dr' \left[ \frac{f(r', t)}{4\pi |r - r'|} \right] g(r, r')
\]

where the numerator of the integrand is evaluated at the retarded time \( t' = t - q/c \).

Substituting Eqn. 3.7 for the scattering potential, and making use of the Fourier integral representation for \( n_e \), we obtain

\[
E_s(r, t) = \frac{1}{n_e} \int_{V_r} dr' \left[ k_0^2 n_e E + \nabla' (E \cdot \nabla' n_e) \right] g(r, r')
\]

where

\[
g(r, r') = \frac{-\exp(jk_0|r - r'|)}{4\pi |r - r'|}
\]
and where the scattering potential has been evaluated at time $t$ by virtue of the quasi-stationary approximation. Notice the natural appearance of the time independent Green's function kernel $g$. The two terms in the integrand are discussed by TATARKI[5] who notes that in the radiation zone of the scattered field ($k_0 | r - r' | \gg 1$), the longitudinally polarized second term ensures that the scattered field $E_s$ is transverse to the direction of propagation.

To show this, an integration by parts may be performed in the radiation zone giving,

$$\nabla g = jk_0 g \hat{\sigma}$$

(3.13)

and given the identity,

$$E = \hat{\sigma} (\hat{E}_0 \cdot \hat{E} - \hat{\sigma} \times (\hat{\sigma} \times \hat{E})),$$

(3.14)

then

$$E_s(r, t) = -4\pi r_e \exp(-j\omega_0 t) \int_{V_r} dr' n_e(r', t) [\hat{\sigma} \times (\hat{\sigma} \times \hat{E}(r'))] g(r, r').$$

(3.15)

The classical electron radius is introduced through the relation

$$r_e = k^2_0/(4\pi n_e).$$

(3.16)

Unfortunately the wave equation, which serves to describe the interaction between the radiation field and the transparent medium, does not yield an exact solution for the diffracted field in closed form. One approximation valid for scattering in the forward direction is where the double cross product in Eqn. 3.14 can be replaced by $-E$. This reduces to the scalar diffraction theory, which is exact for scalar fields such as sound, and is approximate for vector field forward scattering. This means that the polarization remains unchanged so that we may introduce the complex field $u(r, t)$ defined as

$$E(r, t) = \hat{E}_0 u(r, t).$$

(3.17)

The solution to the scattered scalar field is then

$$u_s(r, t) = \int_{V_r} dr' f(r', t) g(r, r')$$

(3.18)
where the scattering potential is

\[ f(r,t) = 4\pi r e_n(r,t)u(r) \]  

(3.19)

and where

\[ u(r,t) = u_0(r) + u_n(r,t). \]  

(3.20)

Eqn. 3.18 is the solution to the inhomogeneous Helmholtz equation

\[ (\nabla^2 + k_0^2)u = f \]  

(3.21)

where, for \( f(r) = 0 \), Eqn. 3.20 reduces to the homogeneous Helmholtz equation

\[ (\nabla^2 + k_0^2)u_0 = 0 \]  

(3.22)

valid for free space propagation. Comparing the expression for the scalar scattering potential with the vector scattering potential in Eqn. 3.7, it can be seen that the scalar approximation is valid provided the first term in Eqn. 3.7 can be neglected. The conditions under which this simplification is valid are discussed in the literature[6].

3.1.2 Approximations to the Scattered Field

Unfortunately, the solution (Eqn. 3.18) to the inhomogeneous wave equation is not obtainable in general as the scattered field appears on both sides of the equation. A survey of approaches to this problem is given by STROHBEHN[6]. However the simplest approximations most relevant to plasma scattering are when the scattered field can be expanded in an perturbation series in amplitude or in phase. The amplitude perturbation series leads to first order to the Born approximation, whilst the first order phase expansion leads to the Rytov approximation. A detailed analysis of these approximations is presented by GOODMAN[7].

The Born series is a series expansion of the light amplitude,

\[ u = u_0 + u_1 + u_2 + ... \]  

(3.23)

whilst the Rytov series is an expansion of the complex phase,

\[ u = \exp(\psi_0 + \psi_1 + \psi_2 + ...) \].  

(3.24)
It will be shown that the first order Rytov solution, known as the Rytov approximation, can be constructed from the first order Born solution in cases where the Born approximation is not valid. Consequently, results derived in the Born approximation can be transformed into equivalent expressions valid in the Rytov approximation. It is known that in many circumstances the Rytov approximation yields more general solutions to the scattered field than the Born approximation, not least because the validity of the first order Rytov approximation is not restricted to the weak scattering limit[6].

Substituting the Born expansion of Eqn. 3.23 and \( f(r,t) \) of Eqn. 3.19 into the wave equation 3.21 and taking only first order terms gives[5],

\[
\left( \nabla^2 + k_0^2 \right) u_1 = 4\pi \rho_e n_e u_0. \tag{3.25}
\]

Eqn. 3.25 is valid provided

\[
|u_1/u_0| \ll 1. \tag{3.26}
\]

As a necessary, (though not sufficient) condition for a small scattered component is that \( |\Delta \phi| \ll \pi \), where \( \Delta \phi \sim \rho_e \lambda_0 n_e L \) is the extra phase shift on propagating through the plasma, and where \( L \) is the thickness of the plasma slab. Then we arrive at the condition for the Born approximation to hold,

\[
|n_e/n_\infty| \ll \Delta K_z/k_0 \tag{3.27}
\]

where \( \Delta K_z = 2\pi/L \) is the wavenumber uncertainty in the z-direction. Given the first order approximation to be valid, then

\[
u_s = \int_{-\infty}^{\infty} dr' f_1(r',t) g(r,r') \tag{3.28}
\]

where

\[
f_1(r,t) = 4\pi \rho_e n_e(r,t) u_0(r) \tau(r) \tag{3.29}
\]

and where \( \tau(r) \) is a window function which restricts the volume of integration:

\[
\tau(r) = \begin{cases} 1 & r \in V_P \\ 0 & \text{otherwise} \end{cases} \tag{3.30}
\]
For the Rytov approximation, we write the light amplitude in the form,

$$u(r,t) = \exp[\psi(r,t)]$$

(3.31)

where $\psi$ is a complex phase. Inserting this expression into Eqn. 3.21 for the inhomogeneous Helmholtz equation yields the Riccati equation for the complex phase:

$$\nabla^2 \psi + \nabla \psi \cdot \nabla \psi + k_0^2(1 - \frac{n_e}{n_{cr}}) = 0.$$  

(3.32)

Then, expanding $\psi$ out only to first order ($\psi = \psi_0 + \psi_1$) we obtain,

$$\nabla^2 \psi_0 + \nabla \psi_0 \cdot \nabla \psi_0 + k_0^2 = 0$$

(3.33)

for free space propagation and

$$\nabla^2 \psi_1 + 2\nabla \psi_0 \cdot \nabla \psi_1 = 4\pi r_e n_e$$

(3.34)

where we neglect the $(\nabla \psi_1)^2$ term, valid provided[6]

$$(\nabla \psi_1)^2 \ll 4\pi r_e n_e = k_0^2 n_e/n_{cr}. \quad (3.35)$$

From dimensional arguments, $|\nabla \psi_1| \sim K\Delta \phi$ so that the above condition becomes, $n_e/n_{cr} \ll (\Delta K_e/K)^2$. This condition is then dependant on the scale size of the inhomogeneity. Note that the condition for the validity of the Rytov approximation depends on the gradient of the phase and not the absolute phase and that provided the phase gradient is not too strong, then the approximation is valid for arbitrarily large phase shifts. In particular, for forward scattering where $|K/k_0| \ll 1$ then the Rytov approximation is generally far superior to the Born approximation. For large angle scattering however, $|K/k_0| \sim 1$, so that the Rytov approximation is roughly equivalent to the Born approximation. The attractiveness of the Rytov approximation is in its ability to accommodate interferometry, where $K/k_0 \sim 0$. Second order correction to $\psi$ are discussed by STROHBEHN[6], however we will ignore higher order approximations which is equivalent to ignoring the effects of strong refraction. It now remains to identify the nature of the solutions to Eqn. 3.34.
It turns out that the solution to $\psi_1$ in Eqn. 3.33 is given by

$$\psi_1 = \frac{u_1}{u_0}$$  \hspace{1cm} (3.36)

where $u_1$ is the solution for the scattered field in the Born approximation in Eqn. 3.25 which can be verified by direct substitution. Hence,

$$\psi_1(r, t) = \frac{1}{u_0(r)} \int_{-\infty}^{\infty} dr' f_1(r', t) g(r, r').$$  \hspace{1cm} (3.37)

As we will not consider higher order corrections to be necessary, we set $\psi_s = \psi_1$ and $u_s = u_1$, and $f = f_1$ so that

$$u(r, t) = u_0(r) \exp (\psi_s(r, t))$$  \hspace{1cm} (3.38)

Because of the relative simplicity in evaluating scattered fields for $u_s$, we will develop the theory in the Born approximation, and where necessary, extend the solution to the Rytov approximation by inserting $u_s$ and $u_0$ into Eqn. 3.36 after they are determined.

### 3.1.3 Free Space Propagation

Assume an incident field propagates in the positive $z$ direction with wavenumber $k_0$. For arbitrary $u$, the wave field in the plane $z = 0$ can be decomposed into its incident “angular spectrum”[8]:

$$A(k_x, k_y; 0) = \iiint_{-\infty}^{\infty} u(x', y'; 0) \exp [-j(k_x x' + k_y y')] dx' dy'$$  \hspace{1cm} (3.39)

$$\triangleq A_i(k_x, k_y)$$  \hspace{1cm} (3.40)

where $x'$ and $y'$ denote spatial coordinates on the incident plane (here taken at $z = 0$ and denoted by subscript “i”), and where $(k_x, k_y)$ are wavenumbers on the incident plane.

Satisfaction of the homogeneous wave equation requires that the Fourier amplitudes $A$ in some plane $z$ be expressed in terms of the angular components $A_i$ on the incident plane by the relation

$$A(k_x, k_y; z) = H(k_x, k_y; z) A_i(k_x, k_y)$$  \hspace{1cm} (3.41)
where

\[ H(k_x, k_y; z) = \exp \left[ j(k_0^2 - k_x^2 - k_y^2)^{1/2}z \right]. \]  

(3.42)

This result is easily obtained upon Fourier transformation of the homogeneous Helmholtz equation (c.f. Eqn. 3.22). The transfer function \( H \) describes free space propagation and may be regarded as a linear, dispersive, finite bandwidth spatial filter [9]. Observe that \( H \) is of unit modulus and exhibits phase dispersion for spatial frequencies less than the bandlimit. This implies that the angular spectrum is essentially unchanged during propagation in free space so that in principle one should be able to obtain the amplitudes \( A_i(k_x, k_y) \) at any observer plane \( z \). Propagation over a distance \( z \) introduces only a phase delay between the spectral amplitudes, depending on the angle of propagation.

The filter transmission is zero for spatial frequencies greater than \( k_0 \) radians per unit length. In other words, for \( k_x^2 + k_y^2 > k_0^2 \), \( k_z \) is imaginary and the wave propagation is evanescent. In the remainder of this chapter we implicitly ignore the evanescent wave fields as these contributions diminish exponentially away from the plasma and are insignificant at distances greater than a few wavelengths for scattering in the forward direction.

One can determine the light amplitude on any plane \( z \) from the angular spectrum at the plane \( z = 0 \) by first applying the propagation operator to \( A_i(k_x, k_y) \) and then performing an inverse Fourier transform. Doing this we obtain,

\[ u(x, y; z) = \int \int_{-\infty}^{\infty} [H(k_x, k_y, z)A_i(k_x, k_y)] \exp(jk_x x) \frac{dk_x dk_y}{(2\pi)^2}. \]  

(3.43)

Supposing we have a single plane wave \( k = (k_x, k_y, k_z) \) and assuming the radiation is monochromatic then \( k = (k_x, k_y, (k_0^2 - k_x^2 - k_y^2)^{1/2}) \). Notice that the angular spectrum of any monochromatic source will then lie on a hemisphere in Fourier space with radius \( k_0 \) and centre \((0, 0, -k_0)\). This rather innocuous result is of central importance in diffraction tomography and its implications will be discussed in the next sections where the attractiveness of working in Fourier space over the real space representation of the scattered field will be made clearer.
3.1.4 Approximations to Free Space Propagation

Several important approximations to free space propagation are introduced. Later, similar approximations will be considered for the scattered field in terms of the properties of the scatterer. The parabolic approximation in frequency space allows the free space propagator $H$ to be expressed as,

$$H_F = \exp(jk_0z)\exp\left[-j\left(\frac{\kappa_x^2 + \kappa_y^2}{2k_0}\right)z\right]$$ \hspace{1cm} (3.44)

where $\kappa_x$ and $\kappa_y$ are taken as the displacements from the mean wavenumber which is assumed to be $k_0 = (0, 0, k_0)$. The inverse fourier transform of $H_F$ is just the well known Fresnel kernel $h_F$,

$$h_F(x, y; z) = \frac{\exp(jk_0z)}{j\lambda_0z} \exp\left[\frac{jk_0}{2z}(x^2 + y^2)\right].$$ \hspace{1cm} (3.45)

By the fourier convolution theorem and from Eqn. 3.41, we obtain

$$u(r, t) = u_i \ast h_F$$

$$= \int_{-\infty}^{\infty} u(x', y'; 0)h_F(x - x', y - y'; z)dx'dy'. \hspace{1cm} (3.46)$$

However, in relating the amplitude $u$ to the angular spectrum $A_i$, it is instructive to write the total field in terms of $A_i$. From GOODMAN[8], we may express the total field $u$ as a fourier transform over $u_i$ with extra quadratic phase factors. By simply expanding out $h_F$ and applying the Fourier convolution theorem we obtain,

$$u(r, t) = h_F(x, y; z)(A_i \ast \Gamma_F)(k_0^x \frac{x}{z}, k_0^y \frac{y}{z}) \hspace{1cm} (3.47)$$

where

$$\Gamma_F(\kappa_x, \kappa_y; z) = j\lambda_0z \exp(-j2\pi z/z_F)$$ \hspace{1cm} (3.48)

and where

$$z_F/(2\pi) = (2k_0)/((\kappa_x^2 + \kappa_y^2)^{1/2}).$$ \hspace{1cm} (3.49)

Note that $z_F$ is the Fresnel length for diffraction from perturbations of wavenumber $(\kappa_x^2 + \kappa_y^2)^{1/2}$. 

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This is a very important result as it indicates not only the precise relationship between the field and the angular spectrum, but also the likely experimental degree of difficulty in extracting the spectrum from the measured field. In general, performing numerical deconvolutions of recorded data in the presence of noise is an illconditioned process which can generate significant errors, so that various limits of the diffracted field are sought which provide a simpler relationship between the field and the angular spectrum.

In the *ultra near field*, \( z/z_{F} \ll 1 \), the second order terms in \( H_{F} \) can be neglected. The propagated spectrum is then identical, apart from a constant phase factor, to the initial angular spectrum. All near field techniques rely on minimal phase dispersion to recover images and angular spectra.

On the other hand, when second order terms can be neglected in the exponent of \( h_{F} \), (i.e. \( z\lambda_{0} \gg (x^{2} + y^{2}) \)) then \( \Gamma_{F} \) becomes a simple delta function in which case,

\[
u(r) = h_{F}(x, y, z)A_{4}(k_{0} \frac{x}{z}, k_{0} \frac{y}{z}).
\]

This is of coarse the Fraunhofer limit, however, unlike the near field limit where their is minimal phase dispersion, in the fraunhofer limit, \( h_{F} \) behaves as a phase dispersive filter. The phase curvature in the far field is so strong that relative phase information between spectral components is lost, and only the power spectrum may be extracted from the angular spectrum.

Whilst the two most important limits for recovering the angular spectrum are the ultra near field and the far field, it seems that the phase as well as the amplitude of the angular spectrum can only be recovered in the near field and that it is effectively impossible to recover the spatial relative phase in the far field due to strong phase dispersion. Thus, far field techniques can only be used to determine power spectra. An exception to this is when lenses are used, and where the parabolic approximation still holds on propagation through the optical system.
3.1.5 The Lens as a Fresnel Transforming Device

It will be shown that the effect of propagation through a lens is to Fresnel transform the light amplitude and the parabolic approximation. Given an analytic expression for \( u(r) \) in the absence of a lens and under conditions where the Fresnel approximation is valid, then the field beyond the lens can be expressed simply as a coordinate transform on \( u(r) \), i.e. \( (x, y; z) \rightarrow (x', y'; z') \). This follows naturally from the Fresnel transforming properties of lenses and the fact that an analytic Fresnel transform of \( u_i(x, y) \) exist. Various limits of optical systems are investigated in terms of the angular spectrum of the incident field. For small angle scattering in the paraxial approximation and for large enough optical elements, we may assume apertures to be effectively infinite. Then the optical field through a series of thin lenses can be obtained from multiple coordinate transformations of our scalar field \( u(r) \).

Suppose we have a light amplitude function \( u_0 \) specified at a distance \( d_0 \) in front of a thin lens, and observe the field \( u_1 \) at a distance \( d_1 \) in front of the lens. Then,

\[
\begin{align*}
    u_1(x_1, y_1; d_0 + d_1) &= \int_{\infty}^{-\infty} dx_0 \ dy_0 h_L(x_0, y_0; x_1, y_1; d_0, d_1)u_0(x_0, y_0; 0) \quad (3.51)
\end{align*}
\]

where the lens kernel \( h_L \) is given by[8]

\[
\begin{align*}
    h_L &= \frac{-1}{\lambda d_0 d_1} \exp\left[jk_0(d_0 + d_1)\right] \exp\left[j\frac{k_0}{2d_1} (x_1^2 + y_1^2)\right] \exp\left[j\frac{k_0}{2d_0} (x_0^2 + y_0^2)\right] \\
    &\times \int_{\infty}^{-\infty} \int_{\infty}^{-\infty} \sigma_L(x, y) \exp\left[j\frac{k_0}{2} \left( \frac{1}{d_0} + \frac{1}{d_1} - \frac{1}{f} \right) (x^2 + y^2)\right] \\
    &\times \exp\left\{-jk_0 \left[ \left( \frac{x_0}{d_0} + \frac{x_1}{d_1} \right) x + \left( \frac{y_0}{d_0} + \frac{y_1}{d_1} \right) y \right] \right\} dx \ dy \\
    &\quad (3.52)
\end{align*}
\]

and where the integration is over the plane containing the lens with pupil function \( \sigma_L(x, y) \). This development follows Goodman's very closely[8], however, where he proceeds to find approximations to \( h_L \) in order to derive the lens law for imaging, we instead show the correspondence between \( h_L \) and the Fresnel kernel \( h_F \). Assuming that \( \sigma_L = 1 \) then the correspondence is greatly simplified. Performing the elementary integral over the lens coordinates and completing the square we
obtain the expression,

\[ h_L = \alpha \exp (j\phi_L) h_F(x_0, y_0; \alpha x_1, \alpha y_1 : z = d_0 + \alpha d_1). \]  

(3.53)

where

\[ \alpha = 1/(1 - d_1/f), \]

(3.54)

\[ \phi_L(x_1, y_1; d_1) = -\frac{k_0 \alpha}{2f} (x_1^2 + y_1^2 + 2d_1^2) \]

(3.55)

and where \( h_F \) is the Fresnel kernel given by Eqn. 3.45.

Given an analytic expression for \( u(x, y; z) \), (call it \( u_F(x, y; z) \)), then (to within an arbitrary phase)

\[ u_1(x_1, y_1; z = d_0 + d_1) = \alpha \exp (j\phi_L) u_F(\alpha x_1, \alpha y_1; z = d_0 + \alpha d_1) \]  

(3.56)

where \( u_F \) is the Fresnel transform of \( u_0 \). Note that in the limit as \( f \to \infty \), then \( \alpha \to 1 \) and we recover the light amplitude for the no lens solution (to within an arbitrary phase),

\[ u_1(x_1, y_1; z = d_0 + d_1) = u_F(x_1, y_1; z = d_0 + d_1). \]  

(3.57)

The lens transform can be applied a number of times for each lens in an optical system. To determine if there are any planes other than \( d_0, d_1 = 0 \) on which the incident field can be recovered, we set \( z = 0 \) in \( u_F \) in Eqn. 3.56 which is satisfied for \( \alpha = -d_0/d_1 \) and we recover the lens law, \( 1/d_0 + 1/d_1 = 1/f \). Apart from a quadratic phase factor we then obtain

\[ u_1(x_1, y_1; d_0, d_1) = -\frac{1}{M} u_0\left(\frac{x_1}{M}, -\frac{y_1}{M}; 0\right) \]  

(3.58)

and so a direct fourier transform relation between \( A_i \) and \( u_1 \) can be obtained at a location remote from the \((x_0, y_0)\) plane. Note however that for \( z \to \infty \) in \( u_F \), (whilst keeping \( d_0, d_1 \) finite) implies that \( \alpha \to \infty \) and so,

\[ \frac{\alpha x_1}{d_0 + \alpha d_1} \to \frac{x_1}{f}. \]  

(3.59)

and this yields (again to within an arbitrary phase),

\[ u_1(x_1, y_1; z_1 = d_0 + d_1) = \frac{1}{j\lambda_0 f} \exp \left[ -\frac{k_0 d_0}{2f} \left( \frac{1}{d_0} - \frac{1}{f} \right) (x_1^2 + y_1^2) \right] A_i(k_0 \frac{x_1}{f}, k_0 \frac{y_1}{f}) \]  

(3.60)

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and when \( d_0 = f \), the quadratic phase term cancels leaving a 1:1 correspondence between the light amplitude and the angular spectrum with no apparent phase dispersion, unlike the far field limit for free space propagation (Eqn. 3.50). Thus, one may record the angular spectrum at the focal plane of a lens and numerically fourier transform it to obtain the image, or record the image and numerically transform it to obtain the angular spectrum.

### 3.1.6 Diffraction From Thin Phase Screens

Before introducing the fourier diffraction projection theorem (FDPT), a simple scattering problem such as the diffraction from a thin sinusoidal phase screen is analysed, and a simple analytic expression for the diffraction of an arbitrary beam from such a screen is derived. The solution to the scattered field \( u_s \) in the Born approximation is shown to be equivalent to a superposition of incident fields \( u_i \) propagating outwards at various angles determined by the spectrum of the scatterer. The usefulness of such a representation is in its intuitive simplicity and computationally efficiency for thin coherent phase screens and arbitrary incident beams. The idea is borrowed from the Abbey theory of diffraction, where beams similar to the incident field emanate from the medium and interfere with each other. This approach is in contrast to the method of performing a Fresnel integral to determine the total field where the simplicity of this physical picture is often lost in the algebra.

We have as our starting point the scattered field \( u_s \) (Eqn. 3.18) with the scalar scattering potential \( f \), (Eqn. 3.19). Our aim is to obtain an expression for \( u_s \) in terms of the spectrum of \( n_e \). We first need to determine the spectral representation of the greens function \( g(r, r') \). This is given by[10]:

\[
g(r, r') = -\frac{j}{2} \int_{-\infty}^{\infty} \frac{dk_x}{(2\pi)^2k_z} \frac{dk_y}{k_y} \exp j[k_x(x-x') + k_y(y-y') + |z-z'|] \tag{3.61}
\]

where \( k_z = (k_0^2 - k_x^2 - k_y^2)^{1/2} \), and an origin is chosen such that \( |z-z'| = z - z' \) within the medium so that for forward scattering only,

\[
g(r, r') = -\frac{j}{2} \int_{-\infty}^{\infty} \frac{dk_x}{(2\pi)^2k_z} \frac{dk_y}{k_y} \exp (jk.r) \exp (-jk.r'). \tag{3.62}
\]
Introducing this expression for \( g \) into Eqn. 3.18 and after some rearrangement, we easily obtain

\[
u_s(r) = \frac{-j}{2} \int_{-\infty}^{\infty} \frac{dk_x}{(2\pi)^2} \frac{dk_y}{(2\pi)^2} \exp(jk_x r) \int_{-\infty}^{\infty} dr' f(r') \exp(-jk_x r'). \tag{3.63}\]

The rightmost integral is recognized as the Fourier transform of the scattering potential. For now we simply state the basic result,

\[
u_s(r) = \frac{-j}{2} \int_{-\infty}^{\infty} \frac{dk_x}{(2\pi)^2} \frac{dk_y}{(2\pi)^2} \exp(jk_x r) F(k) \tag{3.64}\]

where \( F(k) \) is the Fourier transform of \( f(r) \). We investigate the implications of this more fully in the next section. From the Fourier convolution theorem and by the Born approximation,

\[
F(k) = \frac{1}{(2\pi)^3} U_0 * \hat{N}_e \tag{3.65}
\]

where \( U_0 \) is the Fourier transform of the incident field \( u_0 \):

\[
U_0(k) = \int_{-\infty}^{\infty} dr \exp(-jk \cdot r) u_0(r) \tag{3.66}
\]

and

\[
\hat{N}_e = \frac{1}{(2\pi)^3} N_e \ast T. \tag{3.67}
\]

Here, \( \hat{N}_e \) is the three dimensional convolution of the transform \( \hat{N}_e \) of the electron density distribution with the Fourier transform \( T \) of the plasma volume \( \tau(r) \) (c.f. Eqn. 3.30). Thus we have,

\[
F(k) = \int_{-\infty}^{\infty} \frac{dK}{(2\pi)^3} \hat{N}_e(K) U_0(k-K). \tag{3.68}
\]

Inserting into Eqn. 3.64 gives

\[
u_s(r) = -j2\pi \rho_c \int_{-\infty}^{\infty} \frac{dK}{(2\pi)^3} \hat{N}_e(K) \int_{-\infty}^{\infty} \frac{dk_x dk_y}{(2\pi)^2} \exp(jk_x r) U_0(k-K). \tag{3.69}\]

Introducing the parabolic approximation (equivalent to the Fresnel approximation) will allow us to integrate over \( k_x, k_y \) in Eqn. 3.69 as,

\[
k_z \approx k_0 - \frac{k_x^2 + k_y^2}{2k_0} \tag{3.70}
\]
provided \( k_0 = (0,0,k_0) \), and the denominator \( k_z \) in Eqn. 3.68 can be set to \( k_0 \). We would like to transform \((x,y,z)\) to \((x'(Kx, z, z'), y'(Ky, z, z'), z)\) where the coordinates \((x',y',z)\) map the trajectory of a diffraction order propagating from the scatterer in the direction,

\[
k = (K_x, K_y, k_0 - \frac{(K_x^2 + K_y^2)}{2k_0})
\]  

(3.71)

from a spectral component \( K = (K_x, K_y, (K_x^2 + K_y^2)/2k_0) \) in the plasma. Note that the condition \(|k| = k_0\) implies that only wavenumbers \( K \) in the medium for which \(|K + k_0| = k_0\) contribute to the scatterer. Our new coordinates are,

\[
(x',y',z') = \left( x - \frac{K_x}{k_0}(z - z'), y - \frac{K_y}{k_0}(z - z'), z \right)
\]  

(3.72)

correct to within the Fresnel approximation. The new coordinates \((x',y')\) are natural to the problem as they describe, in the Fresnel approximation, the trajectory of radiation scattered from a feature having wave vector \( K \). Expanding out \( U(k - K) \) in its fourier integral representation and then substituting in

\[
(x,y,z) = (x' + \frac{K_x}{k_0}(z - z'), y' + \frac{K_y}{k_0}(z - z'), z)
\]  

(3.73)

and integrating, we obtain the useful relation

\[
\begin{align*}
  u_s(r) &= -j\tau_{\lambda}f_{\lambda} \int \int_{-\infty}^{\infty} \frac{dK_x dK_y}{(2\pi)^2} \exp[j(K_x x + K_y y)] \\
  &\quad \times \int_{0}^{L} \exp[jK_F(z - z')]\tilde{N}_s(K_x, K_y; z')u_0(x', y'; z).
\end{align*}
\]  

(3.74)

The integral limits between \( z = 0 \) and \( z = L \) indicate that the plasma is confined within a slab, and \( K_F = 2\pi/Z_F \) where \( Z_F \) is the Fresnel length defined in Eqn. 3.49.

Eqn. 3.74 indicates that for each spectral component \((K_x, K_y)\), there is an associated diffracted beam \( u_0(x', y'; z) \) and where the solution \( u(r) \) is a superposition of such beams, each equivalent to \( u_0 \) but emanating out of the screen at various angles depending on the spectrum of the scatterer. Note that the locus of points \((x', y') = \text{const} \) labels one of the diffracted beams. This expression is of greatest value in studying the field diffracted from a thin phase grating. Assume the phase screen is sinusoidal and thin, i.e.

\[
n_s(x, y, t) = \Delta n_s \cos(K_x x - \Omega t)\delta(z - z')
\]  

(3.75)
then its fourier transform in the \((x, y)\) plane is

\[
\hat{N}_e = \frac{\Delta n_e}{2}(2\pi)^2[\delta(K_x - K') \exp(j\Omega t) + \delta(K_x + K') \exp(j\Omega t)]\delta(K_y)\delta(z - z')
\]

(3.76)

and for an arbitrary field \(u_i(x, y)\) incident on the screen,

\[
u(x, y; z) = u_0(r) - j\pi\lambda_0 \frac{\Delta n_e}{2} \exp(jK_F(x - x')) \times \]
\[
\times(\exp[j(K_x x - \Omega t)]u_0(r_+) + \exp[-j(K_x x - \Omega t)]u_0(r_-))
\]

(3.77)

where

\[
r_+ = (x - \frac{K_x}{k_0}(z - z'), y - \frac{K_y}{k_0}(z - z'), z)
\]

(3.78)

\[
r_- = (x + \frac{K_x}{k_0}(z - z'), y + \frac{K_y}{k_0}(z - z'), z)
\]

(3.79)

\(K_F = K_x^2/2k_0\) and \(z > z'\). Given an expression for \(u(x, y; z)\) for a sinusoidal phase screen at \(z'\), we may now introduce lenses which only require further coordinate transforms on \((x, y)\) (c.f. Eqn. 3.56). Note that for diffraction from a thin phase screen, all that need be determined is the fresnel transform of the incident field in the absence of the phase screen. Then the final solution is merely a superposition of the incident field taken to propagate at various angles. Note also that this approach is both intuitively appealing with respect to understanding the diffraction processes as well as being computationally efficient for studying arbitrary incident fields on thin phase screens. This representation will be of most use when studying the effects of non-gaussian beam profiles on a range of diagnostic methods currently in use on tokamaks.
3.2 THE DIFFRACTION PROJECTION THEOREM

The Fourier diffraction projection theorem (FDPT) first stated by WOLF[11, 12] relates the three dimensional transform of a “weakly” scattering semi-transparent medium to the properties of the diffracted wave. This relationship allows the extraction of some three dimensional information about the refractive index distribution though, in general, many directions of illumination are required to accurately characterize the scattering medium. We extend the applicability of the FDPT beyond the weak scattering limit by showing that the results are also relevant to the Rytov approximation.

The statement of the diffraction projection theorem for a weakly scattering medium and for conditions under which the scalar wave equation (Eqn. 3.21) is valid, is:

**The two dimensional Fourier transform of the forward scattered field**

*u* in some plane *z* outside the medium, gives the values of the three-dimensional Fourier transform of the scattering potential on a hemispherical surface in the spatial frequency domain.

The content of this theorem is encapsulated in the frequency domain representation of the solution to the inhomogeneous wave equation (Eqn. 3.64), and restated here,

\[
A_s(\kappa_x, \kappa_y, \omega; z) = G(\kappa_x, \kappa_y; z) F(\kappa, \omega)
\]  
(3.81)

where

\[
G(\kappa_x, \kappa_y; z) = \frac{-j \exp(j\kappa_z z)}{2 \kappa_z} 
\]  
(3.82)

and

\[
H(\kappa_x, \kappa_y; z) = \frac{-j}{2\kappa_z} 
\]  
(3.83)
Scattered Amplitude

Figure 3.2: Diffraction geometry for incident plane wave illumination of the plasma. The scattered wave field is measured in some arbitrary plane z.

is the two dimensional Fourier transform of the Green's function \( g(r, r') \) and where \( \kappa = (\kappa_x, \kappa_y, \kappa_z) \), \( |\kappa| = k_0 \) and \( \kappa_z = (k_0^2 - \kappa_x^2 - \kappa_y^2)^{1/2} \). Note that the transfer function \( G \) is similar to the free space propagation function \( H \) except for an imaginary factor. From here on we denote the scattered wave vector \( \kappa \) by

\[
\kappa = k_0 + \mathbf{K}. \tag{3.84}
\]

Thus a Fourier component in the total field \( A(\kappa_x, \kappa_y, \omega; z) \) can be expressed as

\[
A(\kappa_x, \kappa_y, \omega; z) = H(\kappa_x, \kappa_y; z) \left( A_0(\kappa_x, \kappa_y, \omega) - \frac{j}{2\kappa_x} F(\kappa, \omega) \right). \tag{3.85}
\]

As \( \kappa_x \) and \( \kappa_y \) vary from \(-k_0\) to \(k_0\), the vector \( \kappa \) is confined to the hemispherical surface in the transform space \((\kappa_x, \kappa_y, \kappa_z)\) that is centred on the origin and has radius \(k_0\). This result is depicted schematically in the \((\kappa_x, \kappa_y)\) plane in Fig. 3.3.
Figure 3.3: Schematic diagram showing the semi-circular slice in the Fourier transform plane $\kappa_x \kappa_y$ along which the value of the transform of the scattering potential is determined by the angular spectrum $A_\chi$ of the scattered radiation.

Note that the expression for the angular spectrum of the scattered field is derived for an arbitrary 3-dimensional density distribution provided the Born or the Rytov approximations hold. The Fourier diffraction projection theorem then allows for a quantitative description of diffraction effects upon the recovery of structural information using model scattering potentials. Alternatively, it gives insight into tomographic reconstruction of plasma fluctuations, where the concept of a projection of the plasma is generalized to include the diffraction of the probing and scattered radiation.
3.2.1 Plane Wave Illumination

It turns out that the expressions for the scattered field become very simple when considering plane waves.

Let our incident field be a plane wave of the form

$$u_0(r, t) = a_0 \exp j(k_0 \cdot r - \omega_0 t)$$

(3.86)

where without loss of generality, we consider the wave to propagate in the $z$ direction, i.e. $k_0 = (0, 0, k_0)$. The scattering potential (Eqn. 3.68) is now given by

$$F_0(\kappa - k_0, \omega) = 4\pi e a_0 \tilde{N}_e(\kappa - k_0, \omega - \omega_0).$$

(3.87)

The density fluctuation wave vector $K$ is given by

$$K = \kappa - k_0$$

(3.88)

where

$$\kappa = (\kappa_x, \kappa_y, (k_0^2 - \kappa_x^2 - \kappa_y^2)^{1/2})$$

(3.89)

and when the plasma volume is large compared to the typical scale sizes of the inhomogeneities in the plasma we may write

$$A_s(\kappa_x, \kappa_y, \omega; z) = G(\kappa_x, \kappa_y; z) [4\pi e a_0 N_e(K, \Omega)]$$

(3.90)

where $\omega = \Omega + \omega_0$. This result shows that in the Born approximation, the scattered field in a single 'view' of the plasma is linearly proportional to the spectrum of the density fluctuations measured along a semi circular cut through the spatial Fourier transform of the plasma density. The result is depicted schematically in Fig. 3.4 for the $K_xK_z$ plane.

From this expression and Fig. 3.4, measurement of the scattered angular spectrum at various incident beam direction can be used to map the spectrum of the plasma, slice by hemispherical slice, until the spectrum of the scattering potential is sufficiently mapped that it can be inverted to obtain a reconstruction of the scattering potential. This is a remarkable result insofar as the Rytov approximation needs to be satisfied to reconstruct the scattering potential. This result does
Figure 3.4: Schematic diagram showing the semi-circular slice in the Fourier transform plane $K_xK_y$ of the plasma density fluctuations along which the value of the transform of the plasma density distribution is determined by the angular spectrum $A_\alpha$ of the scattered radiation.

not depend on the geometric optics limit of forming path integrals, 'projections', of the object as is the case with the "Fourier slice theorem" which forms the basis for X-Ray tomography.

The forward scattered wave vector $\kappa$ lies on a hemisphere which passes through the origin and has centre $(-k_0x, 0, -k_0z)$ in general. The axis of the hemisphere is in a direction normal to the measurement plane. The equation of the hemispherical surface is a restatement of the conservation of momentum relation together with the low temperature condition $|k_x| = |k_0|$. The resulting vector relation is illustrated in the Fig. 3.4. The scattering angle $\theta_s$ is given by the familiar Bragg
equation for elastic scattering:

\[
\sin \left( \frac{\theta_s}{2} \right) = \frac{K}{2k_0}
\]  

(3.91)

which, in the forward scattering limit, gives \( \theta_s \approx K/k_0 \) where \( K = |K| \).

In principle, the angular spectrum of the forward scattered wave can yield information about the electron density distribution to a wavenumber bandlimit of \( 2k_0 \) (fixed by the propagation condition for homogeneous waves). By changing the angle of illumination (or detection), the entire transform up to this bandlimit is accessible. However, the conditions under which the FDPT is valid requires that the scattering angles be small.

It will now be shown that with an incident plane wave, a very similar result can be derived for the spectrum of the Rytov phase. Starting with the expression for \( u_s \), we have

\[
\psi_s(r, t) = u_s(r, t)/u_0(r, t)
\]

(3.92)

(3.93)

From Eqn. 3.63 we then obtain

\[
\psi_s(r, t) = -j2\pi r_e \int \int_{-\infty}^{\infty} \frac{dk_x dk_y}{(2\pi)^2 k_e} \exp \left( j(k - k_0).r \right) \int \int_{-\infty}^{\infty} dr'n_e(r', t) \exp (-j(k - k_0).r').
\]

(3.94)

Fourier transforming \( \psi_s \) leads to the particularly simple relation

\[
\Psi_s(K_x, K_y; z) = G(K_x, K_y; z) [4\pi r_e N_e(K, \Omega)].
\]

(3.95)

The scattering volume is taken to be much larger than the scale size of the irregularities in the medium, (c.f. Eqn. 3.67). The correspondence with \( A_s \) in the Born approximation is quite clear, with the total complex phase being

\[
\psi(r, t) = j(k_0 . r - \omega_0 t) + \psi_s(r, t).
\]

(3.96)

### 3.2.2 Limiting Forms of the Scattered Field

In Sec. 3.1.4, various limits to free space propagation were considered. It was found that the parabolic approximation in frequency space was equivalent to the
parabolic approximation (Fresnel limit) in real space. Also, the ultra near field and far field limits were investigated. The analysis of the validity of these limits to scattering from extended media are now considered.

The parabolic approximation

If the parabolic approximation (c.f. Eqn 3.44) holds then the next highest order correction to the argument of $G$ needs to be much less than a radian to maintain correct relative phase information: This requires

$$k_0 z \left( \frac{(K_x^2 + K_y^2)}{(2k_0)^2} \right)^2 \ll 1 \quad (3.97)$$

which reduces to

$$k_0 z \ll (\Delta \Omega_K)^{-2} \quad (3.98)$$

where $\Delta \Omega_K$ is the solid angle of contribution of scattered radiation to a point $r$ as depicted in Fig. 3.5. Provided this condition is satisfied, then

$$G(k_z; z) \approx \frac{-j}{2k_0} \exp(jk_0 z) \exp(-jK_F z) = G_F(k_F; z) \quad (3.99)$$

where $K_F = 2\pi/z_F$ (c.f. Eqn. 3.49). In this limit we obtain for the Rytov phase spectrum $\Psi_s$ and scattered angular spectrum $A_s$ (Eqn. 3.90, 3.95) (assuming plane wave illumination)

$$\Psi_s(K_x, K_y, \Omega; z) = -jr_\varepsilon \lambda_0 \exp(jK_F z)N_s(K_F, \Omega) \quad (3.100)$$

$$A_s(K_x, K_y, \omega; z) = -jr_\varepsilon a_0 \lambda_0 \exp(jK_F z)N_s(K_F, \Omega) \quad (3.101)$$

where $K_F = (K_x, K_y, K_F)$. The phase and amplitude perturbations can be efficiently computed from $\Psi_s$ by use of fast Fourier transform (FFT) techniques.

The ultra near field

As in Sec. 3.1.4, we define the ultra near field limit as where phase dispersion between spectral components are negligible. Starting from the parabolic approximation, and introducing the condition that

$$K_F z \ll 1 \quad (3.102)$$

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Figure 3.5: Cone of forward scattered rays received at the observation position $P$.

implies that

$$k_0 z \ll \Delta \Omega_K^{-1}. \quad (3.103)$$

Note that the near field condition is considerably more restrictive than the parabolic approximation. In this limit,

$$\Psi_s(K_x, K_y, \Omega; z) = -j \pi \lambda_0 N_e(K_F, \Omega) \quad (3.104)$$

$$A_s(K_x, K_y, \omega_0 + \Omega; z) = -j \pi \lambda_0 a_0 \exp(j k_0 z) N_e(K_F, \Omega). \quad (3.105)$$

For the ultra near field condition to hold, then

$$K_F L \approx \frac{K_F}{\Delta K_z} \leq K_F z \ll 1, \quad (3.106)$$

where $L$ is the depth of the plasma slab, and $\Delta K_z \sim 2\pi/L$ is the wave number spread over which significant spectral variation in $K_z$ can occur. Thus as long as
Then, taking the Fourier transform of $\Psi_s$ in $(K_x, K_y, \Omega)$ gives

$$\psi_s(x, y; L) = -j\lambda_0 r e^{j k_0 z} \int_{-\infty}^{\infty} dz' n_e(x, y, z').$$

Thus, in the ultra near field, directly measuring the scattered phase via some interferometric technique and numerically Fourier transforming the recorded phase gives effectively a slice through the spectrum of the density distribution. Again, if enough of these slices can be determined, then the plasma can be tomographically reconstructed by an inverse Fourier transform performed on the computed spectrum.

**The far field limit**

To arrive at a far field representation of the scattered radiation and its relation to the spectrum of the scattering potential, it is necessary that the spatial extent of the scattering volume $V_P$ be bounded. Starting with the expression for the scattered field in Eqn. 3.28, and approximating the Green's function (Eqn. 3.12) for an observer point $P$ well away from the scattering volume $V_P$ leads to the expansion (see Fig. 3.1),

$$|r - r'| = r - \hat{r} r + [r^2 - (\hat{r} r')^2] / 2 r + ....$$

The Far field limit (taking only the first order expansion) to the scattered field from an extended source is valid provided

$$k_0 \left[ r^2 - (\hat{r} r')^2 \right] / 2 r \ll 1$$

3-27
where \(|r - r'| \approx r\) in the denominator. Note, this condition must now hold for a source distributed through the whole volume \(V_p\). The factor \((r'^2 - (\hat{r}.r')^2)^{1/2}\) is just the projection of \(r'\) in the direction of \(\hat{r}\) and so has maximum dimension of length \(R\) where \(R\) is the projection of the scattering volume on a plane perpendicular to the direction \(\hat{r}\). Thus,

\[
\frac{k_0 R^2}{2r} \simeq k_0 r (\Delta \Omega_s) \tag{3.113}
\]

so that

\[
k_0 r \ll (\Delta \Omega_s)^{-1} \tag{3.114}
\]

where \(\Delta \Omega_s\) is the solid angle subtended by the scattering volume in the direction \(\hat{r}\). Thus, for a thin beam propagating through a long plasma, the far field condition is more easily satisfied for forward scattering directions where the projected area of the scattering volume is given by the cross section of the beam. For large angle scattering however, the projected area can also become very large, and so detectors are constructed which have restricted acceptance angles, ensuring the far field condition is retained, which also provides for spatial resolution.

Introducing the approximation into the Green’s function kernel of the inhomogeneous wave equation gives,

\[
u_s(r) = \frac{-1}{4\pi} \exp \left(\frac{jk_0 z}{r}\right) \int_{-\infty}^{\infty} \mathrm{d}r' f(r') \exp \left(-\frac{jk_0 \hat{r}.r'}{r}\right). \tag{3.115}
\]

For small angles such as in forward scattering, \(r \approx z\) (known as the Fraunhofer approximation) and

\[
u_s(r) = g_F F(\kappa) \tag{3.116}
\]

where, \(\kappa \approx k_0 \hat{r}\), and

\[
g_F(r) = \frac{-1}{4\pi z} \exp \left(\frac{jk_0 z}{r}\right) \exp \left(\frac{k_0}{2z} (x^2 + y^2)\right) \tag{3.117}
\]

where

\[
\hat{r} \approx (x/z, y/z, 1 - (x^2 + y^2)/2z). \tag{3.118}
\]
Thus the total scattered field in the Fraunhofer limit of the Born approximation is given by

\[ u(r) = h_F \left( A_i - \frac{j}{2k_0} F \right) \] (3.119)

For forward scattering, if \( K_F \) is small compared to \( \Delta K_z = 2\pi/L \) then \( F(\kappa_F) \approx F(K_x, K_y, k_0) \). Thus if the screen is thin enough we may consider the scattered field detected at some plane \( z \) to be a 2-dimensional Fourier transform of the scattering potential.

**Summary**

It has been demonstrated that the scattered field bears a particularly simple relation to the distribution and structure of the density fluctuations for measurements made in either the ultra near field or in the far field of the scattering volume. If, for example, it is desired to directly map plasma density profiles or the structure of low order tearing modes in tokamak plasmas then a diagnostic technique which images or directly observes the ultra near field phase variations is preferred. Tomographic techniques can then be used to extract the density profile from the phase information. These methods rely mainly on extraction of phase information since intensity variations are small and are not so simply related to the properties of the scattering medium.

On the other hand, when studying the statistical properties of random fluctuations, measurements are best performed in the far field of \( V_P \) where \( u_* \) is proportional to \( \tilde{N}_e \) and it can then be shown that the angular distribution of the scattered intensity is related to the instantaneous three dimensional spatial Fourier transform of the plasma density along a particular slice through the fourier transform plane. Application of the FDPT to Raman-Nath and Bragg scattering will be treated in the book to be published.
3.3 OPTICAL DETECTION METHODS

In experiments, a direct measure of the electric field is not possible for high frequency radiation due to the limited band width of solid state detectors. In any case, even if detectors could measure the instantaneous electric field, the extraction of information from the side bands of the carrier frequency $\omega_0$ would require some form of optical mixing for $\Omega \ll \omega_0$. Also, as scattered fields are generally very weak (as they must be for the Born approximation to hold) then optical mixing provides for convenient low noise amplification of the weak signals which share similar coherence properties to the local oscillator. First developed for radar and microwave applications, mixing techniques have, since the advent of lasers and masers, also found extensive application in optical spectroscopy[13].

Optical fields are typically recorded using nonlinear (square law) detectors. The resulting photo-current is proportional to the local intensity $I = uu^*$ of the total field. We confine attention to the detection of phase and amplitude variations neglecting polarization changes. Most optical detection techniques assume that the plasma within the scattering volume satisfies the Rytov approximation (a notable exception being reflectometry). In this section we investigate optical techniques used to determine the scattering potential in different plasma scattering regimes within the Rytov approximation.

As already noted, it is the local intensity $I = uu^*$ of the field at $P$ which is typically sensed by a detecting element. The resulting photocurrent, in the notation of HOLZHAUER and MASSIG[14] is given by

$$i(t) = 2C\frac{1}{T} \int_{t-T}^{t} dt' \int A dx dy uu^*$$

(3.120)

$$C = \frac{\eta e}{h\nu_0} \frac{1}{2\sqrt{\epsilon_0/\mu_0}}$$

(3.121)

where

$\eta$ = detector quantum efficiency;

$h\nu_0$ = photon energy;

$\sqrt{\epsilon_0/\mu_0}$ = impedance of free space;

$A$ = detector area;

3-30
\[ u = \text{total field amplitude at the detector.} \]  

(3.122)

We will neglect spatial incoherence of the field \( u \) across the detecting surface. We assume \( A \) is small, in which case \( i(t) \) is simply proportional to \( uu^* \) at \( P \). The total field at \( P \) in the absence of any external fields may be expressed in the form

\[ u = u_0 \exp(\psi_s). \]

The nature of the scattered radiation determines which of the various mixing techniques are applicable for a retrieval of the complex phase \( \psi_s \) of the scattered radiation. We will assume that

\[ | \partial \psi_s / \partial t | \ll \omega_0 \]  

(3.123)

and that \( \omega_0 \) greatly exceeds the band width of the detection system. We will only consider single point measurements in this section and so for convenience, drop the explicit spatial dependence of \( \psi_s \).

Three general forms for \( \psi_s(t) \) relevant to interferometry or scattering are,

- (i) \( \psi_s(t) \) exhibits large amplitude (> \( \pi \)) slow variations. This characterizes phase variations observed in the interferometric limit. Interferometers measuring line integral plasmas density profiles usually operate in this regime.

- (ii) \( | \psi_s(t) | \ll 1 \). This regime is typified by the small amplitude broad band phase fluctuations impressed on a wave front in the ultra near field of diffraction from small scale plasma fluctuations.

- (iii) \( \dot{\psi}_s = j \Omega \). This case corresponds to doppler shifted radiation scattered at large angles out of the path of the undiffracted beam.

For simplicity, we assume that no intensity modulations occur within the scattering volume of the plasma, and so consider the scattered phase as being purely imaginary \( (\psi_s(t, L) = j\varphi_s(t, L)) \) just outside the scattering volume which would be so provided the scatterer satisfied the Rytov approximation and the depth of the medium was significantly less than its fresnel length \( z_F \). Then at an arbitrary
plane \( z \) in front of the scatterer we may express the scattered phase as a complex quantity, \( \psi_s = \chi_s + j\phi_s \).

If we assume for simplicity that the complex phase immediately beyond the scatterer is purely imaginary, then from Eqn. 3.100 it is clear that \( K_{Pz} \ll 1 \) must also hold for an arbitrary scatterer up to a particular band pass, in which case Eqn. 3.110 must be true in general so that the scattered phase immediately beyond the plasma forms a projection of the plasma density fluctuations. It is this direct correspondence between phase and line integral density in the limit of no intensity modulations which greatly simplifies the interpretation of the experimentally measured phase and hence both for analytic and experimental convenience it is preferable that the scattered phase immediately beyond the scatterer satisfy the ultra-near field condition.

3.3.1 Homodyne Detection

Substituting in \( u = u_0 \exp(\psi_s)(u_0 = a_0 \exp(jk_0r)) \) into Eqn. 3.117 and assuming that the detector bandwidth is greater than the highest significant fluctuation frequency then for

\[
I(t) = I_0 + \tilde{I}(t),
\]

we obtain to first order

\[
\tilde{I}(t) \approx 2I_0\chi_s
\]

where

\[
I_0 = a_0^2.
\]

This simple measurement technique is here referred to as *homodyne* detection for consistency with the definition used by SURKO and SLUSHER[15]. In this method, the measured photo-current is derived from the self beating of the undiffracted beam \( (u_0) \) and the diffracted components of the transmitted radiation.

We immediately note that the complex part of \( \psi_s \) can not be recovered using such direct *homodyne* detection techniques. In order then to obtain the phase
modulations at the object or image plane it is necessary that spatial filtering techniques be used\cite{16}. All homodyne methods then either rely on spatial filtering through free space propagation\cite{17}, or spatial filtering at the focal plane of lenses to introduce phase and amplitude filters in order to obtain a measurable signal.

### 3.3.2 Heterodyne Detection

The simplest mixing technique is to select a local oscillator derived from the same source used for the probe beam. As long as the plasma induced frequency shifts are small compared to $\omega_0$ then the two beams have very similar spectral properties allowing for a coherent superposition of the fields.

![Figure 3.6: Geometry for heterodyne optical mixing experiment.](image)

For simplicity it is assumed that the local oscillator and scattered waves are parallel. We also assume both the scattered and local oscillator fields to be well...
described by plane waves at least over the area of the detector over which the
field is assumed to express little variation in amplitude or in phase. The total
scattered Rytov phase is

\[ u(t) = a(t) \exp \{-j[\omega_0 t - \phi(t)]\} \]  \hspace{1cm} (3.127)

where

\[ a(t) = a_0 \exp (\chi_s) \]  \hspace{1cm} (3.128)
\[ \phi(t) = \phi_0(t) + \phi_s(t) \]  \hspace{1cm} (3.129)

and \( \phi_0(t) \) is the phase of the incident beam. Likewise, the field of the external
local oscillator at \( P \) is expressed as

\[ u_{LO}(t) = a_{LO} \exp (-j\omega_{LO} t). \]  \hspace{1cm} (3.130)

Here, \( \phi_0(t) \) is possibly mechanically introduced relative phase between the refer­
ence and probe beams. The control of this phase variation is the central technical
problem of heterodyne interferometry. In frequency shift heterodyne detection,
the local oscillator is offset in frequency by an amount \( \omega_{IF} = \omega_{LO} - \omega_0 \). Though
technically more difficult to implement, this approach has a number of advantages
which will be explored in the next section. For the moment we take \( \omega_{LO} = \omega_0 \).

Combining \( u \) and \( u_{LO} \) at the detector gives the local intensity:

\[ I(t) = |u(t) + u_{LO}|^2 \]
\[ = I_0(t) + I_{LO} + I_B(t) \]  \hspace{1cm} (3.131)

where \( I_0(t) \) represents here the homodyne intensity given by \( a_0^2(t) \) and the inter­
ference or beat signal is given by

\[ I_B(t) = 2a_{LO}a(t) \cos [\phi_0(t) - \phi_s(t)]. \]  \hspace{1cm} (3.132)

When \( \phi_0(t) \) is time independent, and neglecting terms of first order in \( \chi_s(t) \) the
expression for \( I_B(t) \) reduces to

\[ I_B(t) = I_{B0} \cos [\phi_0 + \phi_s(t)] \]  \hspace{1cm} (3.133)

where \( I_{B0} = 2a_{LO}a_0 \).
For $\phi_* \ll 1$ and $\phi_0 = \pm \pi/2$ we obtain

$$I_B/I_{B0} = \phi_*(t)$$

and the detector current is proportional to the first order Rytov phase. However, the main problem with zero frequency shift heterodyne detection is that the sign of the phase change may be ambiguous for large phase shifts as is the case with scattering experiments where $\phi_* = \pm \Omega$. None-the-less, the total single sided spectra can still be measured by such techniques[18]. Similar ambiguities arise when $\phi_*$ exceeds $\pi$ radians as is often the case with interferometry, however, with slow phase changes, assumptions on the differential smoothness of plasma density variations can overcome the ambiguity.

For slow, large amplitude phase variations, the ambiguity may be resolved by use of two heterodyne detectors set in phase quadrature, where

$$\phi_0^{(1)} - \phi_0^{(2)} = \pm \pi/2.$$  

(3.135)

The Rytov phase may then be extracted from the ratio of the intensities $I_{B1}$ and $I_{B2}$ as:

$$\phi_*(t) = \arctan \left( I_{B1}/I_{B2} \right).$$

(3.136)

This expression, together with the signs of $I_{B1}$ and $I_{B2}$ allows unambiguous determination of $\phi_*(t)$. This method is often employed for interferometric plasma density measurements. An interesting variant of the method requires only a single heterodyne detector but alternates the relative phase by $\pi/2$ radians at a frequency much greater than the characteristic rate of the bulk plasma density variations[19].

A spectral approach is required to determine the sign of the Doppler shifted Fourier components scattered from high frequency plasma fluctuations. Consider a single fourier component of the scattered Rytov phase, $\phi_*= \Omega t$. Given two recorded signals in phase quadrature ($I_{B1}$ and $I_{B2}$) then we may form from the separate signals a complex function

$$z(t) = I_1 + jI_2$$

$$= \exp \left[ j(\Omega t + \delta) \right].$$

(3.137)  (3.138)
Numerical Fourier transformation of the complex signal \( z(t) \) then yields

\[
|Z(\omega)| = 2\pi \delta(\omega - \Omega)
\]  

(3.139)

which unambiguously resolves the sign of \( \Omega \). In practice the wavenumber of the plasma wave is known so that the sign of the Doppler shift gives the direction of propagation of the disturbance. The method, known as "homodyne spectroscopy"[20] has been successfully demonstrated on the TEXT tokamak[21].

A further variant on the heterodyne technique is of use in interferometry where a combination of low frequency large amplitude and high frequency low amplitude phase fluctuations may be observed as in the case of interferometric imaging of the plasma[22]. To achieve high phase sensitivity for the low amplitude high frequency density fluctuations in the presence of the more slowly varying possibly large amplitude bulk plasma and mechanically induced phase shifts requires appropriate frequency discrimination. As large phase changes modulate the responsivity of the interferometer, (c.f. Eqn. 3.133) the slow variations need to be compensated within some feedback loop. By introducing a small constant amplitude phase perturbation in the reference arm of the interferometer, the slowly varying amplitude modulations of the mixed signal from that reference can be used as part of a feedback loop to automatically compensate for the slow, large amplitude phase variations and maintain a fixed phase separation of \( \pm \pi/2 \) radians. This condition ensures linearity between the detected signal from a square law detector and the phase perturbations on the wavefront for \( \phi_s \ll 1 \).

3.3.3 Frequency Shift (Super) Heterodyne Detection

The major advantage in using an intermediate frequency is to observe the relevant plasma signals shifted out of the \( 1/f \) noise range of the detectors. This in principle should give better signal to noise. Also of importance is the avoidance of the sign ambiguity in the rate of change of phase, and avoidance of the need to maintain a precise phase separation between mixed beams in interferometry which is technically difficult to achieve in a tokamak environment.

For frequency shift heterodyne detection the local oscillator frequency is cho-
sen such that \( \omega_{IF} = \omega_0 - \omega_{LO} \) is somewhat greater than the bandwidth of the Doppler shifted scattered spectrum: \( |\Omega_{max}| \ll |\omega_{IF}| \ll \omega_0 \). The frequency offset or intermediate frequency (IF), can be achieved by Doppler shifting part of the source radiation mechanical or acousto-optically [23, 24]. Alternatively, another commonly used technique in the FIR range is the mixing of two independent lasers tuned to sightly different frequencies [25]. The time varying beat intensity is

\[
I_B(t) = 2I_{B0}\cos[\phi_*(t) - \phi_0(t) - \omega_{IF}t]
\]

(3.140)

where the signals have been filtered about the IF frequency. Given \( \phi_*(t) = \Omega t \) then Fourier transformation of \( I_B \) yields

\[
|I(\omega)| = \pi\delta[\omega - (\omega_{IF} - \Omega)] + \delta[\omega + (\omega_{IF} - \Omega)]
\]

(3.141)

where \( I(\omega) \) is the Fourier transform of \( I(t) \). The frequency ambiguity has been removed by the shift, so separating the "red" and "blue" sidebands.

There are a number of means, both electronic and numerical, for extracting the plasma induced phase shifts \( \phi_* \) from the mixed signal \( I_B(t) \). One way to extract phase variations is by measuring the advancement or delay in the zero crossings of the IF frequency against that of the mixed signal from the scattered radiation. In the case of acousto-optic modulation schemes, the reference signal can be obtained from the acoustic driver. For mechanical Doppler shift devices, a second mixed signal must be provided using a detector which samples another beam coming off the grating. The IF signal may even be digitized and recorded if it can be brought down to a low enough frequency consistent with being still much greater than plasma fluctuations. The recorded signal can then be numerically analysed to extract the information in the sidebands.

A useful alternative to the provision of a separate reference is to employ a time lagged version of the phase modulated carrier [26], as the phase reference and form the mixed signal,

\[
I(t, \tau) = [I_B(t) + I_B(t + \tau)]^2.
\]

(3.142)

Choosing \( \tau = \pi(N + 1/4)/\omega_0 \) subject to \( |\partial \phi_*(t)/\partial t| \ll 2\pi/\tau \) and removing the DC and high frequency content leaves

\[
\bar{I}/I_0 \simeq (\partial \phi_*/\partial t)\tau.
\]

(3.143)
Then by integration of $I$ one can recover $\phi_s(t)$ up to some frequency depending on the time constant of the integrator.

When acoustic and mechanical isolation cannot be achieved, more sophisticated approaches are required for the extraction of $\phi_s$ in the presence of significant phase noise. The most common approach is to install an additional compensating interferometer traversing largely the same optical path but operating at a much shorter wavelength so as to be relatively insensitive to the plasma phase shifts compared with the mechanically induced noise[27].

Imaging and Spatial Resolution

Having discussed the extraction of the temporal properties of scattered radiation in the previous section, we simply state that the spatial properties of the distribution can be determined at some appropriate image plane of an optical system by detector arrays, making use of one of the heterodyne techniques illustrated in the previous section. The only alteration is to make the phase a function of position. As with the temporal spectrum extracted from the Fourier transform of the recorded fluctuations, so too the spatial power spectrum can be extracted by spatial Fourier transforms across the detector array(s). This method is in principle equivalent to spectra obtained from multiple Langmuir probe measurements of plasma fluctuations[28]. A complete account of near-field and far-field techniques using the full paraphernalia of analytical tools developed in this chapter, will appear in a book to be published\(^1\).

\(^1\)"Forward Angle Collective Scattering on Fusion Plasmas", by J. Howard, R. Nazikian and L. E. Sharp in the Adam-Hilger series on plasma physics

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Bibliography


Chapter 4

THE PHASE SCINTILLATION INTERFEROMETER

The design of a high sensitivity Mach-Zehnder interferometer operating at 10.6\(\mu\)m presents very severe constraints on the interferometer's construction with regards to mechanical stability, laser beam uniformity and detector performance in the hostile environment of a tokamak laboratory. The construction and performance of the Mach-Zehnder interferometer is discussed in this chapter. Separate sections will focus on mechanical stability, laser performance, image quality, and detector sensitivity.

Diffraction from sharp edges and apertures are found to have a detrimental effect on the phase imaging quality of the interferometer. The diffraction of a gaussian beam from a rectangular aperture is derived and later used to determine if the irregularities observed experimentally can be modelled by diffraction effects from apertures in the interferometer and the tokamak.

4.1 Mach-Zehnder Interferometer: A General Description

The phase modulations imposed on a beam traversing a plasma can be determined using a wide range of optical techniques as discussed in chapter 3. In LT-4, the scintillation interferometer is capable of imaging a broad spectrum of plasma perturbations with a flat spectral response over the range 0 \(\leq K \leq 30\) radians.cm\(^{-1}\) where \(K\) is the wave number in the medium in the direction perpendicular to the propagation of the laser beam. Restriction of the beam width to a third of the minor radius of LT-4 required that an interferometric technique be used to avoid the low wavenumber rolloff associated with homodyne techniques[1, 2]. Also, to resolve the smallest scale size structures present in the plasma, it was necessary to use a single broad beam to image the plasma across a linear detector array (as
proposed by Sharp[3]), rather than to use a more conventional multiple beam diagnostic.

Eventually, a Mach-Zehnder interferometer was selected over other varieties of interferometers for the ease with which the laser could be isolated from back reflections without the need for extra optical isolators. A range of relevant interferometers are discussed in Born & Wolf[4] and Steel[5].

The main difficulty in the use a two path interferometer on tokamaks, is the long (many meters) optical path lengths involved and the need to stabilize such large structures to within a fraction of a wave length of the source (at least in zero-frequency shift heterodyne methods). The high sensitivity of these instruments to plasma phase shifts makes them especially sensitive to airborne sound waves and mechanical noise. The pulsed operation of these machines (tokamaks) also presents considerable problems from a structural point of view.

A block diagram of the Mach-Zehnder interferometer is shown in Fig. 4.1. A 10W CO$_2$ laser operating predominantly in the fundamental TEM$_{00}$ mode and tuned to the P20 line of the 10.6$\mu$m band is used. An active feedback system is used to stabilize the laser cavity length by monitoring laser power fluctuations using a line tuned confocal etalon which feeds back into the laser with a pyro-electric detector sensing the output from the etalon. The output power is modulated by an intra-cavity mirror, and the etalon output is integrated using a lock-in-amplifier, which is used to adjust the laser cavity length by adjusting the location of the piezo driven mirror in a negative feedback arrangement.

Any higher transverse modes are diffracted out of the beam during its 2m travel from the laser output window to the ZnSe beam expander. The expander increases the beam diameter to 70mm and its exit output aperture of 68mm sets the diffraction limit of the instrument for measurements in the direction of the major radius of the LT-4 tokamak. However the diffraction limit in the toroidal direction of the plasma is set at 40mm by the width of the window on the tokamak. A 50% reflective ZnSe beam splitter divides the signal equally into the two arms of the interferometer. These consist of a reference beam which is phase modulated at 20 to 40kHz by a sound wave launched by a piezoelectric speaker, and a signal
Figure 4.1: Schematic diagram of the Mach-Zehnder interferometer highlighting important design features.

beam which propagates through the plasma.

These beams recombine at a ZnSe beam splitter with a reflectance/transmittance ratio for the reference/signal arms of 2/1 respectively. The stronger reference signal minimizes homodyne intensity fluctuations compared to the interference term.
which carries the phase information of the imaged plasma. The Mach-Zehnder in­
terferometer produces two output beams, one of which is incident on the detector array, whilst the other is incident on a single room temperature HgCdTe detector. The signals from the HgCdTe room temperature detector is used to maintain the average relative phase between the reference and signal arms of the interferometer to $\pi/2$ radian by modulating the position of a mirror mounted at 45° in the signal arm. The frequency of the feedback control system is restricted to $1\text{kHz}$ so as to respond only to the large amplitude low frequency mechanically induced phase shifts $f < 100\text{Hz}$.

The interferometer optics are mounted on two horizontal optical benches which for thermal stability are constructed out of 30mm diameter fused silica tubing. The two benches are vertically spaced by 3 bundles of ten 30mm diameter fused silica tubes. If required, the silica tubing can be temperature controlled using a water jacket for additional thermal stability. The main structural material used in the interferometer is 35mm thick 'Asbestoslux' sheet making up the upper and lower optical benches of the C-frame. This material is used for its vibration damping characteristics and, being an insulator, does not suffer from electromagnetically induced vibrations generated by the pulsed magnetic fields of the tokamak.

All optical paths are enclosed to minimize pickup from airborne sound waves. In addition, all exposed surfaces are covered with a 1.5mm thick 'SoundCoat', a visco-elastic material which dramatically reduces structural resonances. This is covered with a coating of 12mm thick 'SoundMat', a sheet foam which has a high absorption efficiency for airborne sound waves.

To isolate the structure from floorborne vibrations, the instrument is kinematic­ally mounted (through load spreading 250mm diameter brass disks) on three composite supports (Fig. 4.1), each of which consists of three layers of vibration absorbing material interleaved with 12mm thick plywood sheeting. The top layer is 12mm thick high hysteresis rubber 'Sorbathane' and the lower two layers are made up of 30mm thick compressed polyurethane foam whose density has been chosen by experiment to give a resonant frequency of around 3Hz. A further sig-
significant reduction of the natural frequency of the interferometer's structure would require sophisticated commercial isolators. The 3Hz resonant frequency presented little problems owing to the short plasma lifetime (100ms).

The range of fluctuation scale sizes which can be directly measured by the array is set at the lower end by the dimensions of the detector elements of (300μm x 300μm) and by their spacing of 1mm, and at the upper end by the 15mm separation of the two extreme detector elements. This can be effectively increased by a factor of two or three through the use of an imaging lens to demagnify the plasma and project it onto the detector array. Optical constraints set both by the dimensions of the plasma viewing window as well as the 70mm diameter lenses limit the maximum correlation length which can be determined in the plasma to around 70mm in the major radius and 40mm in the toroidal directions.

The imaging lens allows remote location of the detector array whilst still forming a phase image of the plasma at the detector plane [6].

To minimize electromagnetic pickup, the detector array with its preamplifiers are enclosed in a copper canister for shielding against the high frequency magnetic fluctuations produced by MHD instabilities. The detector-preamplifier container is surrounded by a 'Mu-Metal' can with an outer casing of glassy metal magnetic material to attenuate slowly varying (< 100Hz) magnetic fields associated with the pulsed magnetic fields. The detected signals pass through analogue bandpass filters which filter out low frequency residual mechanical noise as well as restricting the high frequency response to half that of the digital sampling rate so as to prevent aliasing. Two digital converter configurations are used to sample and store the data. In one, the central detector (channel 8) was recorded at 2 MHz whilst the other 15 channels were recorded at 200kHz with each channel being able to store 16k data samples at 12bits resolution. In the other configuration, all 15 channels were recorded simultaneously at a rate of 2MHz with each channel being able to store 4k 10bit data points. The overall frequency response is 5kHz to 1MHz, at a maximum digital sampling rate of 2MHz. The instrumental band pass was sufficient to observe all density fluctuations of interest in LT-4, from sawteeth oscillations to small scale turbulence. The digitized signals are then computer
analysed using a variety of techniques including digital filtering and the use of complex Fourier transforms to produce power spectra and filtered images[7].

4.1.1 Mechanical stability and feedback control

Side on and top views of the interferometer are shown in Fig. 4.2 and its location on the interferometer is shown in Fig. 4.3. These illustrate the C-frame structure commonly used for such interferometers. The optical mounts and quartz benches of the interferometer also appears in the figures.

For the scintillation interferometer to produce a signal proportional to the line integral density, the reference and probe beams need to be kept at $\pi/2 \pm 0.4$ radians, which for 10.6$\mu$m corresponds to an allowable phase excursion of 0.6 $\mu$m. The level of acceptable phase excursion corresponds to an upper limit of 10% variations in the calibration of the detectors.

The quartz optical benches are made of pre-stressed quartz legs allowing a maximum relative path length variation of $\sim 20\mu$m. This is in spite of the fact that the underlying asbestos cement structure has poor mechanical stability. The most significant source of relative motion arises from low frequency oscillations of the cantilever overhanging the tokamak. The cantilever motion is compensated by the active adjustment of the interferometer arm lengths using the feedback system.

Apart from a mean phase shifts, phase shear due to flexing of the structure can also occur. However, with a maximum amplitude of 20$\mu$m bending distributed over 1m (the length of the interferometer section), the relative phase across our 3cm beam would be no greater than the acceptable phase error of 0.4 radians. This method of achieving stable operation was adequate for LT-4, however for larger tokamaks rigidity must be designed into the support structure such as with the construction of the space frame for the 2mm interferometer on JET [8].

Phase uniformity across the beam is not absolutely essential for the operation of the interferometer as only the phase difference between the two beams are relevant. However, optical elements within the interferometer (mirrors and beam splitters) need to be flat to within $\lambda/40$ and such optics are readily available in the
infra red. Beam splitters are also required to be either plane parallel or wedged to avoid problems due to secondary reflections (wedged optics are preferable).

The optical mounts are also a major source of noise. The standard 3 point kinematic mounts housing the mirrors and beam splitters are excessively prone to vibration, since they constitute little more than spring-loaded weights with a
Figure 4.3: Top view of the interferometer structure and its location on LT4. The soft X-ray detectors view the same poloidal slice of the plasma as the scintillation interferometer.

natural frequency of around 100Hz. This problem was overcome by adding lead blocks to their corners which reduces their natural frequency and amplitude of vibrations to acceptable levels.

The principle of the interferometer’s feedback stabilization system is described
in section 3.3. In practice, feedback stabilization proved successful in cancelling vibrationally induced phase shifts. The modulated signal from the room temperature detector (effectively monitoring the interferometers cavity length) is filtered, integrated via a lock-in-amplifier, inverted, amplified again and then used to drive a piezo-controlled mirror in the reference arm of the interferometer. The gain of the feedback loop is increased to just below the onset of unstable operation. Fig. 4.4 shows the effect with and without feedback on the phase stability of the interferometer. Successful operation of the feedback system depended on very stable laser performance.

The additional advantage of phase locking the interferometer to \( \pi/2 \) radians is that the low frequency large amplitude phase shifts can be read directly from the feedback signal driving the piezo mirror and without the \( \pi \) radians phase ambiguity associated with more standard fringe counting interferometers. Although this is in principle an ideal technique for measuring mean plasma density, in practice the plasma phase shifts could not be resolved from the mechanical noise. The mean phase shift for a typical line integral density of \( 5 \times 10^{14} \text{cm}^2 \) is around 0.15 radians.

An in-line HeNe interferometer was initially used to monitor the mechanically induced phase shifts. However, maintaining accurate interferometric alignment for both the HeNe and CO\(_2\) interferometers proved too difficult. Since LT-4 had an independent HCN interferometer to monitor the mean plasma density, it was not a high priority for the phase scintillation interferometer to produce similar measurements using a new technique. However, in future, this technique may prove suitable as a means of measuring all density fluctuations of interest using a single diagnostic.

4.1.2 Laser stability and mode purity

Phase scintillation interferometry requires beam uniformity (in phase and amplitude) over the whole detection surface for proper operation. This allows for accurate calibration, effective feedback control, and minimization of diffraction effects due to irregular beam profiles.
Figure 4.4: Feedback switched off (a) showing variable sensitivity to large phase shifts. In (b), feedback is switched on, showing stabilization of interferometer. Feedback signal has half the period of the modulations in (a) indicating the lack of phase ambiguity.

The laser consists of a 1.2m water-cooled glass cavity with a plane mirror at one end (instead of a line selector grating) and a 5 m curvature ZnSe 20% transmissive output coupler. The active medium is a standard CO₂ laser mix of
N₂, He, and CO₂.

Line selection is achieved using an extra-cavity etalon, the output of which is fed back to the laser via a beam-splitter in front of the output coupler. The extra-cavity etalon proved more stable than an intra-cavity grating and allowed for servo-stabilization of the laser cavity. Brewster windows were not used due to their lack of mechanical stability and sensitivity of the laser beam to even small movement.

Removing the Brewster window had no detectable effect on the diffraction experiments as in forward scattering experiments, scattering is independent of polarization.

The gas discharge is maintained by a 12 kV power supply. The optimal operating condition for near-saturated stable laser performance is a gas pressure of 20 mTorr (continuous gas flow) and a 40 mA discharge current[9].

Beam uniformity is achieved by minimizing transverse cavity modes. This is done by suitable design of the cavity dimensions and with the provision of an intra-cavity iris diaphragm. The use of the diaphragm to cut out higher order modes was offset however by increased diffraction losses. Deterioration of mirrors and output couplers are a major cause of irregular beam profiles and gas discharge lasers destroy optics rather quickly. A clear and thorough discussion of cavity design, stability and diffraction enhanced losses is given by Kogelnik[10].

Spatial filtering of the beam outside the cavity is necessary to achieve a reasonably smooth beam profile. The technique is to take advantage of the greater divergence of the higher order modes in free space propagation, so that all that was needed was to allow a greater propagation path for the beam before expansion and entry into the interferometer. The spatial filter consists of several mirrors creating a folded beam path which allowed for 2m extra propagation before entry into the beam expander. The result is a sufficiently smooth beam profile for interferometry (Fig. 4.5), and an operating beam power of 6W.
4.1.3 Image Quality and Resolution

To record an image using a linear array of detectors it is necessary to ensure that signal recording is unaliassed. Aliasing is a potential problem when one discretely samples a continuous object. It is necessary to sample the image at either the Nyquist frequency of the optical system (i.e. twice the pass band of the imaging lens) or at the Nyquist frequency of the plasma fluctuations, whichever is the lesser. Using 10.6μm beams, and from theoretical estimates of the typical mode numbers expected in the plasma \( \frac{m}{n} \approx 30/10 \), it is clear that the optical systems band width is well in excess of the expected band width of plasma fluctuations and that the detector spacing of 1mm is adequate to avoid spatial aliasing.

One implication of the theory in chapter 3 is that in order to form reliable...
images of plasmas it is necessary to select a wavelength rather shorter than is commonly used for either single chord density measurements or that used for large angle scattering experiments. For thin parabolic lenses, images of the projections of transparent objects can be obtained provided the parabolic approximation is satisfied (implying not too large scattering angles) and the medium is considerably thinner than the Fresnel length of its shortest resolvable structures. Fortunately, these two conditions are not mutually exclusive and going to shorter wavelengths tends to satisfy them both.

Given that only small scattering angles are involved ($\theta_s \leq 10^{-2}\text{rad}$) and modest magnifications of the order of unity are required, an accurate image can be produced using only a single lens slightly wider than the beam and appropriately located between the plasma and the detector plane to obtain a typical magnification of $1/2$ or $1$. The distance from the plasma midplane to the detectors plane is 115cm. The 5cm diameter lens has a focal length of 25cm. At a location of 38cm in front of the detectors, the magnification is $1/2$ and the effective object plane of the lens/detector system is displaced only 4cm from the plasma midplane, (note that the plasma diameter is approximately 20cm). The effective object plane is determined by setting the image plane at the detectors and using the lens law to determine the corresponding object plane. For a 1:1 magnification, the displacement of the effective object plane from the plasma midplane is less than the plasma thickness, and where the Fresnel length $z_F$ for scale lengths as small as 2mm are of the dimensions of the plasma diameter. Thus, providing the effective object plane remains within the plasma, the Fresnel lengths of imaged objects are long enough that a true image of the plasma is formed. In the study of low order MHD activity, the Fresnel lengths are so long that the lens acts as a condenser rather than as an imaging optic, where, except at the Fourier transform plane of the lens, (i.e. at the focal plane) diffraction effects become negligible.

Although the lenses used are marginally wider than the probing beam, the problem of the vignetting of the object is avoided by the very small scattering angles involved[11]. The aperture of the lens then acts as a spatial filter. The lens can be thought of as a linear shift invariant filter, and so transfer functions, point
spread functions and other useful quantities in linear systems theory can be used to describe the properties of the optical system[12].

Abberations are another source of systematic error which can contribute to the degradation of image quality. As only a single imaging lens is used, the potential problem of aberations needs to be considered. From linear optics we know that, by the parabolic approximation for lenses, and under the paraxial approximation (appropriate for small angle scattering), aberations simply do not occur[13]. The thin parabolic lens approximation is commonly used in Fourier optics as it models the characteristics of a perfect lens.

The only aberation of possible significance can arise from image defocus which is unavoidable for transparent objects of finite extent. However, provided the depth of the object is less than a quarter Fresnel length of the smallest resolved structures then the defocus aberation is negligible provided the object plane of the imaging system is within the plasma. The small relative aperture of the lens ensures that the parabolic thin lens approximation is valid. With an imaging lens of focal length $f = 25$cm, the f-number of the imaging system is $\sim 8$.

Finally, the spherical phase curvature introduced by the lens has no noticeable effect on image quality. This is because both reference and probe beams traverse the same optical path after recombination at the output coupler, and so the wave fronts are matched and image intensity is independent of the phase curvature. For an optical system with a large f-number, image distortion due to field curvature is also negligible.

4.1.4 Detection and Phase Sensitivity

The biasing and amplifier circuits for the liquid nitrogen cooled HgCdTe detector array are shown schematically in Fig. 4.6. Photoconductive detectors require an appropriate bias current to operate. The detectors use a 10:1 voltage divider to provide the necessary bias. Detector conductivity responds linearly to changes in beam intensity, producing voltage changes which are amplified by low noise, 2MHz band width, 60db AC coupled preamplifiers. The preamplifiers are powered by two 12V car batteries in series, which operate at very low noise levels provided
The output signals from the preamplifiers are fed into buffer amplifiers which lower the output impedance for transmission of the signal to other filters and amplifiers. Shielding the detectors required considerable care as only a few microvolts pickup on the detector cables could saturate the preamplifiers. After amplification and filtering, the signals are transmitted along 30m co-axial shielded cables (enclosed within a 25m unearthed copper pipe) to the ADC’s. The detector/amplifier, tokamak and data acquisition systems use a single earth point to avoid earth loops resulting from environmental noise as well as transients from the machine.

Fig. 4.7 displays three images of the reference sound wave recorded at three points of time which clearly indicates the wavelength and phase velocity of the sound wave. These are determined by inspection, and conform with expected
Figure 4.7: Recorded reference signal across the detector array at three times.

values for sound waves in air. Note that given the sound wave frequency and the velocity of propagation, the alignment of the detector array can be accurately determined as $K = K_0 f$ where $K$ is the observed wavenumber, and $f$ is the direction of the detector array.

In optical heterodyne detection, optimal sensitivity is theoretically poorer than for frequency shift heterodyne detection. Low frequency $1/f$ noise (typically less
than 1MHz), is universally observed in solid state devices and gas lasers. In DC current gas lasers, 1/f noise is highly correlated with current and density fluctuations[14], implicating plasma instabilities as the source of power modulation. Acousto-optic modulators (for CO$_2$ lasers) operating at 40MHz are commonly used to avoid the 1/f detector noise[15], however phase sensitive detection is a complex and costly process and in any case zero frequency shift heterodyne detection proved more than adequate for the detection of the lowest levels of density fluctuations of interest in the plasma [16].

In determining the interferometers sensitivity to line integral density variations a specific detectivity ($D_\lambda^*$) is determined for the entire detection system. The value obtained is equal to the $D_\lambda^*$ of the HgCdTe detectors, indicating detector noise limited operation which is typical for infrared and sub-infrared detection systems.

From $D_\lambda^*$ optimum phase sensitivity is determined, and for typical line integral densities a minimum detectable $\delta n_e/n_e$ is derived for both narrow band and broad band signals.

For a signal gain of $10^5$, the noise level (with the laser off) is 80mV for a band width of 1MHz. Assuming for a moment that the pre-amplifier and amplifiers have zero noise, this gives an effective detector shot noise of $8 \times 10^{-7}V$. The response of the detectors is calibrated using a known infrared source. A typical value of 250V/W is obtained. This gives an effective noise equivalent power of $3 \times 10^{-12}W.Hz^{-1/2}$ or $3 \times 10^{-9}W$ for a band width of 1MHz for detector shot noise alone. A detector area of $300\mu m \times 300\mu m$ then gives $D_\lambda^* = 9 \times 10^9 cm.Hz^{1/2}/W$. This conforms with the manufacturers calibration of the detectors. From this figure the ultimate sensitivity of the interferometer can be estimated[17]. Most DC discharge gas lasers demonstrate noise levels several orders of magnitude higher than the shot noise limit derived from the photon statistics. This is principally due to discharge noise in the frequency range of interest. The laser noise for a beam power of 0.5W.cm$^{-2}$ (the intensity of our laser beam at the detectors) matched the detector noise and corresponded to a laser power of 3W (as about half the power in a M-Z interferometer is lost at the output coupler).

For a $S/N=1$ over a 1MHz bandwidth, the signal power required is the NEP,
equivalent to $10^{-9} V$ at the detector output and corresponds to a minimum detectable phase shift of $2 \times 10^{-5}$ radians, given that for our interferometer, \( \delta V/V_0 \simeq 0.6 \delta \phi \), where \( V_0 \) is the mean voltage produced by the beam. This expression is easily calculated by using the fact that \( I_R = 2 I_S \) where \( I_S \) is the signal beam intensity and \( I_R \) is the reference beam intensity on the detector array and by the formula

\[
\frac{\tilde{V}}{V_0} = \frac{\tilde{I}}{I} = 2 \frac{\sqrt{I_R I_S}}{I_R + I_S} \delta \phi.
\] (4.1)

For typical line integral densities of $5 \times 10^{14} \text{cm}^2$, we obtain a mean phase shift through the plasma of \( \phi_0 \simeq 0.15 \) radians so that the minimal detectable line integral density over a bandwidth of 1 MHz is

\[
\frac{\bar{n}_e}{n_e} \geq 2 \times 10^{-4}
\] (4.2)

or even better when operating at higher densities.

For narrow band plasma oscillations (such as Mirnov fluctuations) with instantaneous bandwidth of $\sim 2 \text{kHz}$, we have

\[
\frac{\bar{n}_e}{n_e} \geq 10^{-5}.
\] (4.3)

Further more, for coherent detection of a reference sound wave, a bandwidth of 1 Hz is possible which allows for a detectable phase shift as small as $2 \times 10^{-8}$ radians.
4.2 FINITE APERTURES AND NON-GAUSSIAN BEAM PROFILES

During a period where the linearity the interferometer was tested against sound wavelength produced by a piezo-electric speaker, it was found that small deviations away from strict linearity occurred even when the sound waves were very thin and at the object plane of the optical system. As the individual detectors had been tested and found linear against frequency outside of the interferometer, the only possibility was that non-ideal beam profiles due to apertures and diffraction from edges were responsible for the modulations.

The assumption that our illuminating field is a uniform Gaussian beam is inadequate to describe the phenomena observed. Amplitude modulations in the response of the detector to the reference sound wave indicated a period similar to the period of intensity modulations of the diffraction pattern from the apertures.

An intuitive appreciation of just how such irregularities can modulate the detected signals can be obtained by noting that diffraction from edges can be described by the generation of edge waves which emanate from the aperture and interfere with the through beam[18]. This was at least the implication of a rigorous solution to the diffraction problem of a plane wave from a semi-infinite conducting plane. The interference between the edge wave (a cylindrical wave apparently emanating from the edge), and the through beam varies with their relative phase and produces modulations which we observe as the intensity ripples in the Fresnel diffraction pattern. Modulations to the detected fluctuations occurs through the interference of these extra edge waves with the through beam. Fig. 4.8 shows schematically how the edge wave acts as an extra local oscillator. One would expect that the peak modulation to the detected signal to occur when the relative phase between the through beam and the edge wave is \( \pi/2 \) radians and that the effect is most significant at shorter sound wavelengths. The edge waves produce a kind of homodyne optical mixing where for a very long sound wavelength, the modulations should disappear. In fact, taking a very simple model (motivated by its physical simplicity), of a cylindrical wave interfering with an apertured plane wave and then passing through a sound wave, the full complexity of the
experimental observations are easily predicted. However, in the more general case of Gaussian beam illumination, the straightforward solution to the Fresnel integral is preferable, yet the physical picture of edge waves still provides the appropriate intuitive model for understanding the equations and their solutions.

The effect of such irregularities on the detected signals and in particular their effect on the extraction of reliable information from observed data can be investigated by considering the simplified case of the diffraction of a Gaussian beam from a slit placed before the sound field.
4.2.1 Finite apertures and the Complex Error Function

In our treatment of the diffraction problem, we will be relying on an important result derived in chapter 3,

- Given a thin small amplitude sinusoidal phase screen at position \( z_2 \) propagating in the \( x \) direction and an incident field which in the Fresnel approximation is denoted by \( u_0(x, y; z) \), then for \( z > z_2 \),

\[
\begin{align*}
   u(x, y; z) &= u_0(x, y; z) + j \Phi \exp(j(\kappa_+ r' - \omega_+ t))u_0(x_+, y; z) \\
            &\quad - j \Phi \exp(-j(\kappa_- r' - \omega_- t))u_0(x_-, y; z) \quad (4.4)
\end{align*}
\]

where \( r' = (x, y, z - z_2) \), \( \kappa_\pm = (\pm K_z, 0, K_F) \) and \( r_\pm \) are as defined in sec. 3.1.6.

We now concern ourselves only with apertured gaussian beams and set the beam waist at the co-ordinate origin, place an aperture at the location \( z = z_1 \) and a sound wave at \( z = z_2 \) where \( z_2 > z_1 > 0 \). Using a rectangular aperture \( P(x, y) \) at \( z_1 \) we have

\[
P(x, y) = \begin{cases} 
1, & \text{for } -a < x < b \text{ and } -c < y < d \quad (a, b, c, d > 0) \\
0 & \text{otherwise.} 
\end{cases} \quad (4.5)
\]

Given a gaussian beam waist at \( z = 0 \) and a radius \( w_0 \) defined as the 1/e point of the intensity profile, we have,

\[
u_0(x, y; z) = a_0 \gamma(z) \xi(x, z) \xi(y, z) \quad (4.6)
\]

where

\[
\xi(x, z) = \exp \left( \frac{-\gamma(z)x^2}{2w_0^2} \right) \quad (4.7)
\]

and

\[
\gamma(z) = \frac{1 - jz/z_R}{1 + (z/z_R)^2} \quad (4.8)
\]

and where \( z_R = k_0 w_0^2 \) is the Rayleigh length of the beam[19]. Using Eqn. 4.7 as the field incident on the aperture at \( z = z_1 \), the field beyond \( z_1 \) is given by,

\[
u_1(x, y; z) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x, y) h_F(x, y; x', y'; z - z_1)u_0(x', y'; z_1) dx' dy' \quad (4.9)
\]

4-21
where \( h_F \) is the fresnel kernel. Solving the integral we obtain for \( z > z_i \) (after some algebra),

\[
\begin{align*}
u_0(x, y; z) &= \frac{a_0}{4} \gamma(z) \xi(x, z) \xi(y, z) \\
&\times \left\{ \Phi \left( \eta(x, z) \sqrt{\zeta} + \frac{b}{2\sqrt{\zeta}} \right) - \Phi \left( \eta(x, z) \sqrt{\zeta} + \frac{-a}{2\sqrt{\zeta}} \right) \right\} \\
&\times \left\{ \Phi \left( \eta(y, z) \sqrt{\zeta} + \frac{d}{2\sqrt{\zeta}} \right) - \Phi \left( \eta(y, z) \sqrt{\zeta} + \frac{-c}{2\sqrt{\zeta}} \right) \right\}
\end{align*}
\]

(4.10)

where \( \Phi(z) \) is the complex error function

\[
\Phi(z) = \frac{2}{\sqrt{\pi}} \int_0^z \exp(-t^2) dt
\]

(4.11)

with

\[
\zeta = \frac{u_0^2}{2} \left( \gamma(z_1) - j \frac{z_R}{(z - z_1)} \right)^{-1}
\]

(4.12)

and

\[
\eta(x, z) = \frac{j k_0 x}{(z - z_1)}
\]

(4.13)

It is clear from the above expression that in the limit of an infinite aperture, the solution reduces to the standard case of an ideal Gaussian beams. Also of interest is the compliment of \( P(x, y) \) for the study of homodyne scattering techniques which use opaque screens[20]. The analytical expression can be used to determine the optimal dimensions of an opaque screen for maximizing the scattered power whilst achieving the required bandwidth. In this case,

\[
P_c(x, y) = \begin{cases} 
0, & \text{for } -a < x < b \text{ and } -c < y < d \ (a, b, c, d > 0) \\
1, & \text{otherwise.}
\end{cases}
\]

(4.14)

and the solution is simply the compliment of Eqn. 4.11,

\[
u_{1c}(x, y; z) = \frac{a_0}{4} \gamma(z) \xi(x, z) \xi(y, z) \\
\times \left\{ 2 - \Phi \left( \eta(x, z) \sqrt{\zeta} + \frac{b}{2\sqrt{\zeta}} \right) + \Phi \left( \eta(x, z) \sqrt{\zeta} + \frac{-a}{2\sqrt{\zeta}} \right) \right\} \\
\times \left\{ 2 - \Phi \left( \eta(y, z) \sqrt{\zeta} + \frac{d}{2\sqrt{\zeta}} \right) + \Phi \left( \eta(y, z) \sqrt{\zeta} + \frac{-c}{2\sqrt{\zeta}} \right) \right\}.
\]

(4.15)
In the limit as $w_0 \to \infty$, Eqn. (4.10) reduces to the solution for plane wave illumination of a rectangular aperture. To see this we note that as $w_0 \to \infty$ then

$$
\eta(x', z) \sqrt{\zeta} + \frac{b}{2 \sqrt{\zeta}} \to \exp \left(-\frac{j \pi}{4}\right) \left\{ \sqrt{\frac{2k_0}{z - z_1}} (b - x') \right\}
$$

(4.16)

and

$$
\Phi \left( \exp \left(-\frac{j \pi}{4}\right)x \right) = \sqrt{2} \exp \left(-\frac{j \pi}{4}\right) \left[ C \left( \sqrt{\frac{2}{\pi}} x \right) + j S \left( \sqrt{\frac{2}{\pi}} x \right) \right],
$$

(4.17)

where $C(x), S(x)$ are the cosine and sine Fresnel integrals. Thus we have in that limit,

$$
u_0(x, y; z) = -j \frac{a_0}{2} \left\{ \left[ C \left( \sqrt{\frac{k_0}{\pi(z - z_1)}} (b - x) \right) - C \left( -\sqrt{\frac{k_0}{\pi(z - z_1)}} (a + x) \right) \right] \\
+ j \left[ S \left( \sqrt{\frac{k_0}{\pi(z - z_1)}} (b - x) \right) - S \left( -\sqrt{\frac{k_0}{\pi(z - z_1)}} (a + x) \right) \right] \\
\times \left\{ \left[ C \left( \sqrt{\frac{k_0}{\pi(z - z_1)}} (d - x) \right) - C \left( -\sqrt{\frac{k_0}{\pi(z - z_1)}} (c + x) \right) \right] \\
+ j \left[ S \left( \sqrt{\frac{k_0}{\pi(z - z_1)}} (d - x) \right) - S \left( -\sqrt{\frac{k_0}{\pi(z - z_1)}} (c + x) \right) \right] \right\}
$$

(4.18)

which is precisely the diffracted field expected from a rectangular aperture for plane wave illumination[13]. The complex error function is easily evaluated iteratively using a series expansion in three regions of the complex plain. The results of the numerical analysis illuminate various aspects of the diffraction of non-Gaussian beams from phase screens, and the representation of the diffracted field as a superposition of the incident field in various directions of propagation allows a clear picture of the effect of apertures on the interference between the diffraction orders which produces the observed irregularities. The results will be presented in a future publication where a large variety of typical diagnostics are investigated for their sensitivity to non-ideal beam profiles.
Bibliography


Chapter 5

MHD ACTIVITY IN THE LT-4 TOKAMAK

In this chapter, MHD phenomena in LT-4 are characterized using a number of diagnostics. The principal diagnostics in the analysis of MHD modes are the scintillation interferometer, the soft X-ray detector array and the poloidal/toroidal array of Mirnov coils.

An overview of the various MHD regimes in LT-4 was presented in Chapter 2. In this chapter, the analysis of mode activity will be arranged along the same lines as in chapter 2, starting with the relatively quiet high \( q(a) \) regime I discharges and proceeding to low \( q(a) \) discharges which exhibit strong MHD activity and often terminate in disruptions. An essential tool in the analysis of quasi-stationary spectra – the time frequency distribution (TFD) – will be introduced. The feasibility of tomographically inverting the scintillation data given the interferometers limited access to the plasma will be investigated. The data is then compared with projections of models for tearing modes, internal kink and external kink modes [1, 2]. These modes were selected as they are likely to constitute the dominant mode activity in LT-4 [3]. Large amplitude \( m=1 \) and \( m=2 \) modes are identified, and island widths and locations of rational surfaces are determined from the projections.

5.1 TIME FREQUENCY DISTRIBUTIONS AND QUASI STATIONARY SPECTRA

It is well known that MHD fluctuations, as observed by magnetic pick-up coils, appear locally coherent yet demonstrate considerable frequency variation over the duration of the discharge. It is also clear that the power spectrum of such non-stationary processes is neither unique to the recorded data, nor of much use in
characterizing the measured signals. Clearly, a time dependent representation of
the signal spectrum is a more appropriate means of displaying such data. One of
the first references to such a joint time-frequency distribution (TFD) was made
by Gabor in 1946 [4], and with the advent of computers, various forms of TFDs
have found limited use in electrical engineering and speech analysis [5, 6].

A TFD is of value in the analysis of signals for which spectral variations form a
useful characterization of the process. Hence, application to ergodic processes such
as turbulence or speech analysis where there may be no clearly defined spectral
peaks or where the dominant frequencies jump discontinuously, produces little use­
ful information. However, provided the signal can be considered quasi-stationary
then TFDs may have an application. By quasi-stationary we mean that the rate
of change of spectral components is sufficiently slow so that an instantaneous fre­
quency can be defined for the signal which then varies in a regular or slow way.
Of the various possible approaches to a joint TFD, the simplest and perhaps most
useful for MHD analysis is a sliding Gaussian apodizing function. Given a signal
\( x(t) \), and a window function

\[
W(t - t_0; \tau) = \exp \left( -\frac{(t - t_0)^2}{\tau^2} \right)
\]

we define the joint time-frequency distribution as

\[
F(\omega, t_0; \tau) = \int_{-\infty}^{\infty} x(t)W(t - t_0; \tau)\exp(-j\omega t)dt.
\]

Thus, it seems straightforward to choose some time window \( \tau \) and a step length
as some fraction of \( \tau \) to obtain our TFD. However, the optimal time window for
both spectral and temporal resolution may need to be determined iteratively as it
will depend on the time rate of change of the spectral components. The condition
for both optimal spectral and temporal resolution is that

\[
\tau(t_0) \simeq |\dot{\omega}(t_0)|^{-1/2}
\]

where \( \dot{\omega}(t) \) is defined as the instantaneous rate of change of frequency of the
signal. To illustrate this condition, let us assume that we have a purely FM
modulated signal as depicted in Fig. 5.1. We see from the figure that for a time
Figure 5.1: Optimal time window for an FM signal.

window of $2\tau$, the frequency spread of the signal $x(t)$ at $t_0$ is approximately

$$\Delta \omega_{x(t)} \approx 2\tau \frac{d\omega}{dt} \bigg|_{t_0}$$  \hspace{1cm} (5.4)

whereas the spectral point spread varies as the inverse width of the time window,

$$\Delta \omega_{2\tau} \approx \frac{\pi}{\tau}.$$  \hspace{1cm} (5.5)

The optimal window width occurs when $\Delta \omega_{2\tau}$ is set equal to $\Delta \omega_{x(t)}$ as the uncertainties tend to add in quadrature.

It would be advantageous to determine the optimal width for each time point without the need to use an iterative scheme which can be computationally involved. This can be done provided an instantaneous angular frequency $\omega(t)$ and $\dot{\omega}(t)$ can be defined. To do this, it is necessary to construct the associated complex

5-3
analytic function \( z(t) \) from \( x(t) \) where

\[
z(t) = x(t) - jH(x(t))
\]  \( (5.6) \)

and where \( H(x(t)) \) is the Hilbert transform of the real signal \([7]\) defined as,

\[
H(x(t)) = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{x(t')dt'}{t' - t} .
\]  \( (5.7) \)

The relationship between \( x(t) \) and \( z(t) \) can be seen by taking the Fourier transform of \( z(t) \) giving,

\[
Z(\omega) = \mathcal{X}(\omega) - \frac{i}{\pi} \int_{-\infty}^{\infty} dt'x(t') \int_{-\infty}^{\infty} dt \frac{\exp(-j\omega t)}{t' - t} 
\]

\[
= \mathcal{X}(\omega) + \frac{i}{\pi} \int_{-\infty}^{\infty} dt'x(t') \exp(-j\omega t') \int_{-\infty}^{\infty} dt'' \frac{\exp(-j\omega t'')}{t''} .
\]  \( (5.8) \)

where \( Z(\omega) \) and \( \mathcal{X}(\omega) \) are the Fourier transforms of \( z(t) \) and \( x(t) \) respectively and \( t'' = -(t' - t) \). Integrating over \( t'' \),\( t' \) and noting that the integral over \( t' \) is just the Fourier transform of \( x(t) \) gives,

\[
Z(\omega) = \begin{cases} 
2\mathcal{X}(\omega) & \text{if } \omega \geq 0 \\
0 & \text{otherwise}
\end{cases}
\]  \( (5.9) \)

The analytic function is most useful in characterizing signals which are not strictly monochromatic but whose amplitude modulations are slowly varying relative to the phase variations of the signal. For \( x(t) \) real, the single sided power spectrum of \( z(t) \) and \( x(t) \) are equivalent, and the transformation from \( x(t) \) to \( z(t) \) is easiest to perform in frequency space where the transform is computed, filtered, and inverted. We may then write

\[
z(t) = \exp\left(\theta(t)\right)
\]  \( (5.10) \)

where \( \theta \) is a complex phase and \( \dot{\theta} = \omega(t) \). Hence \( \dot{\omega} \; t \) = \( \dot{\theta} \). Note that as long as \( \theta(t) \) remains bounded, then \( |z(t)| \) is non-singular and non-zero everywhere. From this expression we easily derive the result

\[
|\ddot{\theta}(t)| = \left| \frac{\int_{0}^{\infty} \omega^2 Z(\omega) \exp(j\omega t) d\omega}{z(t)} + \left( \int_{0}^{\infty} j\omega Z(\omega) \exp(j\omega t) d\omega \right)^2 \right|
\]  \( (5.11) \)

so that, by the use of FFTs, the rate of change of the instantaneous frequency can be determined and then used to obtain the appropriate time window \( \tau(t) \) of integration. In practice, \( \dot{\theta}(t) \) can jump about rapidly and so \( \tau(t) \) can be smoothed by appropriate filtering before application.
5.2 INTERFEROMETRY AND RELIABLE INFORMATION

We now address the problem of just how much information can be extracted on the structure of plasma modes from a limited set of line integrals which do not constitute a complete view of the plasma. (We define a “complete view” as a single projection of the entire object). To restate the problem, the phase scintillation interferometer produces up to a 3cm projection of a 10cm minor radius plasma. Even though 16 line integrals are formed, they do not span the entire plasma and so present some difficulties in extracting tomographic inversions if no *a priori* constraints are imposed on the nature of the projections outside the field of view. In practice this implies that the projections of the mode must be sufficiently localized.

It is shown in the following section that no typical MHD modes have projections so localized that the scintillation interferometer can reconstruct them with no *a priori* knowledge of the toroidal mode numbers. Certainly in those cases where sufficient information is available for the unambiguous interpretation of the phase scintillations, then the interferometer provides unprecedented spatial resolution of these modes.

The next major deficiency of the interferometer for tomographic applications is its limited number of independent views of the plasma. Even with a single complete view the amount of reliable information recoverable is very small, and would in fact be insufficient to reconstruct the angular harmonics of mode(s). It was pointed out by Howard[8] that for $M$ complete views of the plasma, $M^2 + M$ unique numbers which characterize the object can be extracted from the $M$ projections. These numbers are the generalized moments of the object and hence specify the coefficients of its radial and angular harmonics. In order to extract unambiguously each recoverable moment, sufficient detectors are necessary to avoid aliasing of the projections.

At first sight our experiment produces only a single projection which would be completely inadequate for recovering the object. A single projection only indicates unambiguously the centre of mass of the plasma. However, it is possible to make
use of the assumption – reasonable for large aspect ratio tokamaks – that these modes rigidly rotate past the detectors so that successive digitized projections constitute independent views of the plasma.

The time frequency distribution would then become an essential part of the reconstruction procedure. MHD oscillations are typically locally coherent and the dominant frequencies vary slowly with time.

In cases where one or more of the above conditions for performing reliable inversions are not satisfied, then further constraints, based on a priori knowledge of mode structure needs to be introduced. In particular, when strong oscillations and several modes appear throughout the plasma, then there is no hope of reconstructing the modes from partial projections and parametric models are then used to simulate the projections as a means of determining the internal mode structure. Other diagnostics such as the soft X-ray detectors and Mirnov coils provide independent information on the dominant modes in the plasma which help to constrain the models to the most likely set of possible configurations. Note that there is in general a great deal of degeneracy in the number of possible structures whose projections would fit the recorded signals so it is important that one knows essentially what to look for.

5.3 MODELS OF MHD MODES AND LINE INTEGRAL MEASUREMENTS

The models considered are of three mode structures thought to be important in LT-4; the $m/n = 1/1$ internal kink or resistive kink mode, the $m/n = 2/1$ tearing mode, and the $m/n = 3/1$ external kink mode. Here $m$ denotes the dominant poloidal mode number. In Chapter 2 we noted that strong sawteeth are observed during Regime I with little or no mode activity observed on the Mirnov coils. The sawtooth oscillations are considered indicative of the growth and subsequent collapse of an $m = 1$ island [10, 11]. The detailed mechanism is still a matter of some controversy [12], however, recent X-ray data from JET should help to resolve the mystery[13]. Far infrared interferometry has also been used to produce projections of sawtooth oscillations on TEXT with a resolution
of several centimeters [14].

A phenomenological model (known as the Dubois model) of the $m = 1$ resistive kink was first developed to simulate soft X-ray precursor oscillations, based on the assumption that magnetic surfaces are isothermal and equi-density [15].

A modification of the Dubois model suitable for application to higher mode numbers ($m \geq 2$ tearing modes) is introduced and applied to the $m = 2$ tearing mode. In particular, the $m = 2$ tearing mode is considered to play a central role in the onset of the disruptive instability [16] and so the identification of this mode from its projections is of major importance.

In Chapter 2, we noted that the magnetic fluctuations at the plasma edge as seen by the Mirnov coils are dominated by an $m = 3$ structure when $q(a) \leq 3$. The modified Dubois model can easily handle external kinks by locating the resonant surface outside the plasma in the vacuum region. A mode can appear as either a tearing or as an external kink depending on whether the resonant surface is inside or outside the plasma. The correspondence between the external kink and tearing modes is discussed very clearly in a review by White [17].

5.3.1 The Dubois Model

The Dubois model is a parametric description of the perturbed magnetic surfaces in tokamak plasmas. Magnetic surfaces are labelled by a parameter $\xi$ and for a given $\xi$ the following polar equation maps out two surfaces (Fig. 5.2),

$$\rho_{1,2}(\theta, \xi) = \delta \cos(\theta) + \left( r_0^2 - \delta^2 \sin^2(\theta) \right)^{1/2} - w_{1,2} \left[ \cos^2(\theta/2) + \xi - 1 \right], \quad (5.12)$$

where $\rho_{1,2}$ and $\theta$ are measured from the centre of the inner most flux surface, and $\delta = a\delta_1/2$ where $a$ is an arbitrary parameter between 0 and 1 and $\delta$ is some fraction of the island width which represents the displacement of the inner flux surfaces from the origin ($r = 0$) of the unperturbed surfaces. The two surfaces mapped by $\rho$ and $\theta$ for each $\xi$ correspond to $w_1 = \delta_1$ in region 1 for $r \leq r_0$ and $w_2 = -\delta_2$ in region 2 ($r > r_0$) where $r_0$ is the radius of the mode rational surface as measured again from the centre of rotation of the unperturbed surfaces. The full island width $\Delta = \delta_1 + \delta_2$ where $\delta_1$ and $\delta_2$ need not be equal. The centre of rotation
of the perturbed structure ($\Delta = 0$) is the co-ordinate origin. For convenience however, it need not coincide with the centre of the kinked inner flux surfaces or to the origin of the unperturbed surfaces. The centre of rotation is selected as the point which maps the outer perturbed flux surfaces onto themselves through an arbitrary rotation angle. The polar co-ordinates $\rho$ and $\theta$ are measured in the co-ordinate system $r'$ and $\chi$ from the origin of rotation shown as point A in Fig. 5.2. For a given $(r', \chi)$, the surface labelling parameter $\xi$ is determined. Assuming the density is constant on each magnetic surface, it follows that the density at any point $(r', \chi)$ depends only on $\xi$. Given a monotonic equilibrium density profile $n_{e0}(r)$ for the unperturbed cylindrically symmetric initial configuration, we can
replace the independent variable $r$ by a dependent parameter $\zeta(\xi)$ which is only a function of the flux surface labeling parameter $\xi$. Here $\xi$ is a single-valued function of $r$ in each of the regions $r \leq r_0$ and $r > r_0$. Within the island we have $0 \leq \xi \leq 1$, and outside the island $\xi > 1$. Although a Gaussian function is used by Dubois [15] to describe the unperturbed X-ray emissivity profile, the technique works equally well for any arbitrary monotonic unperturbed density profile. In what follows we assume an unperturbed profile of the form

$$n_{e0}(r) = n_e(0)\{1 - (r/a)^2\}^{1/\nu}$$

(5.13)

where $\nu$ is a profile peaking parameter. Inside the island, we choose to set $n_e$ constant so that we have,

$$\zeta = r_0 - \delta - \delta_1(\xi - 1) \quad \text{for} \quad r \leq r_0,$n
e$$

$$= r_0 - \delta + \delta_2(\xi - 1) \quad \text{for} \quad r > r_0,$n
e$$

$$= r_0 - \delta \quad \text{for} \quad 0 \leq \xi \leq 1.$$  

(5.14)

Minor modifications must be introduced into the model to handle higher order tearing modes. The model was not originally intended for application to higher order modes however with minor modifications it produces a reasonable representation of $m \geq 2$ tearing modes. The first modification is to replace $\theta$ by $m\theta$ in where $m$ is the dominant poloidal mode number. One problem is that the model introduces a constant amplitude $\cos(m\theta)$ perturbation on the magnetic surfaces which is independent of the distance from the rational surface. This is not of great concern for the $m = 1$ mode as the centre of rotation is conveniently chosen to coincide with the centre of the external magnetic surfaces. To remedy this problem for higher order modes, the angularly dependent terms in the equation for $\rho_{1,2}$ are multiplied by a Gaussian roll-off function $F(r - r_0; w, m)$. Furthermore, both $\delta$ as well as the displacement of the centre of rotation from the centre of the inner flux surfaces are set to zero. Thus the centre of rotation remains at co-ordinate $O$ and,

$$\rho_{1,2} = r_0 - w_{1,2} \left(\frac{1 + F\cos(m\theta)}{2}\right) - w_{1,2}(\xi - 1)$$

(5.15)
where

\[ F(r - r_0; w_{1,2}) = \exp \left( - \frac{(r - r_0)^2}{2w_{1,2}} \right). \]  \hspace{1cm} (5.16)

As discussed previously, placing \( r_0 > 1 \) allows external kink modes to be modelled.

### 5.3.2 Projections Of The Dubois Model

Projections of the Dubois model are obtained for comparison with the scintillations data. The projections can be computed using the Fourier diffraction projection theorem (described in Chapter 3), or to a very good approximation, one can assume diffraction effects to be negligible so that the simpler Fourier slice theorem can be used. The perturbed density profile is Fourier transformed, and the transform is then evaluated along a line through its origin and the one dimensional inverse transform is computed to obtain the projection. By rotating the line through the origin, rotation of the mode about the plasma axis is simulated.

Fig. 5.3 shows the equi-density contours of the \( m = 1 \) mode and below it are the contours of constant projected density variation (i.e. excluding the \( m = 0 \) contribution) as a function of mode rotation angle. Thus, each detector sees a horizontal slice through the projection contours, and dashed curves indicate negative density change. As the detectors only span a limited field of view, the whole array would only sample a horizontal strip of the projections. The impact parameter (or impact radius) is defined as the minimum distance a beam or chord approaches the plasma centre and is a convenient means of labeling the detector channels. From the projections of the \( m = 1 \) mode in the Dubois model, it is clear that the mode has only one inversion point located at the plasma centre, due to the fact that an \( m = 1 \) mode has odd parity. In fact, the minimum number of inversions a projection of a structure of poloidal mode number \( m \) is in fact \( m \) zeros and the mode is either odd or even symmetric about the origin if \( m \) is odd or even respectively[18].

From the model, it is clear that the determination of the mode from a partial projection is feasible if its projection is well localized in the field of view of the interferometer. For the \( m = 1 \), an identifiable feature should be a clear inversion
through the plasma centre and nowhere else. The location of the $q = 1$ surface is approximately determined by the location of the peak fluctuation amplitude.

For the $m = 2$ tearing mode, a contour plot of the modified Dubois model and its projections are shown in Fig. 5.4. The contours demonstrate even parity about the plasma origin and (unlike the $m = 1$ mode in Fig. 5.3) there is a strong inversion of the projections about the $q = 2$ rational surface. From numerical experiments, the impact radius for the inversion of the projections is very close to the radius of the $q = 2$ surface for a variety of initial profiles and island widths. Also, near the inversion layer, the signals never quite go to zero, but a second harmonic develops which dominates the fundamental and does not invert. The peak of the second harmonic is a good indicator of the location of the $q = 2$ rational surface and the distance to the peak fluctuation amplitude provides approximately the island half width. Thus, the island width and rational surface of an $m = 2$ tearing mode can be readily extracted from its projections. Note also that the mode has even parity about the origin. However, the projections display four, not two inversions, which is due to the extra inversion in the radial perturbed density profile. Unfortunately, as the interferometer has only a limited view of the plasma, it cannot independently determine the mode number if limited to just observing the rational surface, hence Mirnov coil data is necessary in determining the mode number. Fig. 5.5 displays the contour plot and projections of an $m = 3$ external kink mode where the rational surface has been placed outside the plasma. Note from the projections that an inversion at large radii is still observed due to the $m = 3$ structure even though the perturbed radial density profile has no inversion point within the plasma. From a partial projection about the inversion layer it is not clear whether the inversion is due to an island within the plasma or not. One aspect of the inversion which may be used to determine it as an external kink mode is the absence of a second harmonic about the inversion layer and the broadness of the island width which is incommensurate with the very low level of fluctuations assuming the density must be more or less flat across such islands. In such cases however, information from Mirnov coils is usually necessary to have as a guide.
Figure 5.3: Plot of (a) equi-density contours of the $m = 1$ mode with $r_0 = 0.3$, $\delta_1 = 0.1$ and $\delta_2 = 0.09$, and for a parabolic density profile and in (b), contour plot of projections against rotation angle where negative contours are denoted by broken lines.
Figure 5.4: Plot of (a) equi-density contours for an $m = 2$ tearing mode from the Dubois model with $r_0 = 0.6$, $\delta_1 = 0.1$ and $\delta_2 = 0.09$, and for a parabolic density profile and in (b) a contour plot of the projections against rotation angle.
Figure 5.5: Plot of (a) equi-density contours of the $m = 3$ external kink mode with $r_0 = 1.1$, $\delta_1 = 0.1$ and $\delta_2 = 0.09$, and for a parabolic density profile and in (b) the contour plot of projections against rotation angle.
5.4 STRUCTURE OF LARGE AMPLITUDE M=2 AND M=1 MODES

From the previous discussion on reliable reconstructions from incomplete views, the optimal condition for the reliable extraction of MHD mode structure is when there is a single dominant mode of large amplitude in the plasma. The other necessary condition is that sufficient views of the mode are available. The latter condition requires rigid body toroidal and/or poloidal rotation of the plasma. In practice, large amplitude single modes typically occur when operating at high density.

Characteristic of high density discharges in LT-4 is the appearance of large amplitude single modes, typically \( m = 1 \) or \( m = 2 \). Projections are obtained of a large amplitude \( m=2 \) mode and results compared with signatures of an \( m = 2 \) island generated using the modified Dubois model.

5.4.1 A Large Amplitude \( m=2 \) Island

From Fig. 5.6, a dominant \( m = 2 \) mode is observed on the Poloidal Mirnov coil array. The \( m = 2 \) structure is present throughout the discharge with no detectable \( m = 3 \) activity. The plot of \( \langle \bar{I}^2 \rangle^{1/2}/I_0 \) (where \( I \) is the chordal X-ray emissivity) in Fig. 5.7 shows the fluctuations peaking towards the plasma edge which is consistent with the presence of a large amplitude \( m = 2 \) island and a quiescent plasma interior.

Fig. 5.8 shows signatures obtained from the Dubois model compared with the line integral density measurements obtained using the scintillation interferometer. Note the similarity between the signatures in the raw data and the characteristic signatures obtained using the Dubois model, indicating an island half width of 2cm and a mode rational surface at \( r_0 = 6.5 \)cm. Although the Dubois model contains many higher order angular harmonics, in practice as the harmonic content of the scintillation data is low, the model is filtered of harmonics not observed in the signals.

The harmonic content of the signals can be estimated from the angular harmonics obtained from the time frequency distribution of the density fluctua-
This information is used to filter the harmonic content of the Dubois model. The second harmonic in Fig. 5.9 corresponds to an \( \frac{m}{n} = \frac{4}{2} \) component in the island. The third harmonic, barely visible in the density fluctuations, corresponds to an \( \frac{m}{n} = \frac{6}{3} \) component of the island. Variations in the harmonic content between the soft X-ray and the scintillation TFDs is due to the different regions of the plasma being sampled relative to the proximity of the mode rational surface. The scintillation chords pass predominantly through the island where the higher angular harmonics are strongest whilst the X-ray channel passes through the plasma center. Because the higher harmonic components of the spectrum are changing very rapidly, a standard power spectral analysis of the data reveals little harmonic content.
Figure 5.6: Poloidal Mirnov coil data (a) of a large amplitude $m=2$ rotating island and (b) a plot of $1/q(a)$ during the discharge.
Figure 5.7: Ratio of the RMS X-ray fluctuations $\bar{I}$ to the DC X-ray emissivity $I_0$ for each X-ray channel during a period of strong m=2 activity.
Figure 5.8: Signatures of model $m=2$ island projections compared with experiment. Scintillations raw data (b) taken during strong $m=2$ activity is consistent with a half width of 2cm and rational surface at $r_0 = 6.5$cm.
TFD of a $m=2$ Mode Rotation

Figure 5.9: Time frequency distribution of density and soft X-ray fluctuations during strong $m=2$ mode activity. The soft X-ray channel samples mostly the plasma centre and so possesses fewer harmonics than the scintillations channel predominantly sampling the island interior.
5.4.2 Radial field fluctuations at the Rational Surface

It is generally agreed that toroidally periodic structures observed in tokamak plasmas are indicative of the formation of magnetic islands as a result of finite resistive effects. The development of radial magnetic field fluctuations (say by the growth of a resistive tearing mode) should significantly enhance heat transport by rapid conduction of electrons along field lines which develop a radial component. A means then of determining directly the radial magnetic field fluctuations at the rational surface is certainly desirable.

Both the plasma density and temperature are thought to be fairly uniform on magnetic flux surfaces (at least for large islands) and so diagnostics which measure plasma density and/or temperature can be used as a qualitative indication of the changing magnetic field geometry. Before proceeding to discuss the amplitude of radial magnetic field fluctuations, it is worth noting that in general, density is not as uniform as temperature on flux surfaces. The critical island width \( w_* \) for density flattening to occur across a magnetic island is given by [19]

\[
w_* \simeq \frac{L_s}{L_n}
\]

where \( \rho_* \) is the ion Larmor radius calculated at the electron temperature, and \( L_s , L_n \) are the magnetic shear and density scale lengths respectively. For an electron temperature of a few hundred electron volts, a \( q=2 \) surface at \( r_* = 6.5\text{cm} \) (derived from the interferometer projections) and \( q(a) \simeq 3 \) at \( r=10\text{cm} \), the critical island width is about 1cm. Beyond this width the density should flatten across the island.

For an island in the outer region of the plasma, the magnetic shear length \( L_s \) at the rational surface \( r_* \) can be estimated by knowing \( q(a) \) at the edge and the mode rational surface \( r_* \) from the scintillation data so that,

\[
L_s^{-1} = \frac{r_*}{Rq_*^2} q'
\]

where

\[
q' \sim \frac{q_a - q_r}{a - r_*}.
\]
From this we obtain

\[ w \approx 4 \left( \frac{r_L \tilde{B}_r}{L} \right)^{1/2} \]  \hspace{1cm} (5.20)

where the simplifying assumption is made that the radial field perturbations consist of only a single Fourier component \[ [20] \] of poloidal mode number \( m \) and where \( w \) is the full width of the magnetic island and \( \tilde{B}_r \) is the amplitude of radial field fluctuations at \( r_* \). For \( m = 2 \), \( r_* = 6.5 \text{cm} \) and \( w = 4 \text{cm} \) then \( L_* \approx 90 \text{cm} \) which leads to a relative field perturbation of \( \tilde{B}_r/B \approx 3.4 \times 10^{-3} \). For a toroidal field of \( 1.5 \text{T} \) then \( \tilde{B}_r \approx 5 \times 10^{-3} \text{T} \).

Given the location of the rational surface, it is instructive to estimate the island width from the magnetic probe measurements. The half width \( \delta \) of an island can be estimated from the relative poloidal magnetic field fluctuations where \[ [21] \]

\[ \delta \approx \left( \frac{2\pi^2 - m \pi r_{\text{coil}} \tilde{B}_\theta}{m B_\theta} \right)^{1/2} \]  \hspace{1cm} (5.21)

and where \( r_{\text{coil}} \) is the location of the pick-up coil. The magnetic probe is located just behind the limiter where \( q \approx q(a) \) so that \( B_\theta \approx 0.1 \text{T} \). The amplitude of the pickup signal is \( 0.3 \text{V} \), at a frequency (given by the time frequency distribution) of \( 18 \text{kHz} \) at \( 40 \text{ms} \) into the discharge. At \( 18 \text{kHz} \), \( \tilde{B}_\theta(T) = .01 v \) where \( v \) is the signal amplitude in volts. At \( 40 \text{ms} \), we obtain a halfwidth of \( \delta = 1.9 \text{cm} \). This is in unexpectedly good agreement with the half width of \( 2 \text{cm} \) obtained from the density projections. Also, from the mean line integral density measured by the HCN interferometer compared with the amplitude of density fluctuations measured by the phase scintillation interferometer we arrive at the ratio

\[ \frac{n_e}{n_{e0}} \big| \sim 8 \times 10^{-2} \]

whereas \( \tilde{B}_\theta/B_\theta \sim 3 \times 10^{-2} \) as measured by the magnetic probe, making them of the same order as would be expected for MHD modes.

### 5.4.3 A Large Amplitude \( m=1 \) Mode

The formation of a regular \( m = 1 \) structure in the absence of sawtooth oscillations is investigated. The line integral signatures obtained are quite consistent with the
simple harmonic form of an \( m=1 \) internal kink assuming a small island width [22, 23]. Data taken of this mode across a 2cm projection through the plasma center is shown in Fig. 5.10.

What is clear from the line integral data is the verification of the \( m=1 \) nature of the mode by its odd parity across the plasma centre. Below the scintillation data in Fig. 5.10 is the corresponding X-ray data showing the localized nature of the mode. The gradual phase change across the center of the X-ray projections is due to the large volume of integration of each channel through the plasma of a fanning array as opposed to the parallel array of the interferometer channels.
5.5 Plasma Regimes in LT-4

In this section, operational regimes of LT-4 are examined by the use of time frequency distributions (TFDs) of X-ray, density, and magnetic field fluctuations. The TFDs provide a convenient means of characterizing the various plasma regimes. In particular, they are found to be most useful for identifying and tracking modes whose frequency and/or amplitude changes rapidly in time. The various plasma regimes of MHD activity are labeled I to IV in order of reducing values of $q(a)$ typical of each regime. The relationship between plasma regimes and $q(a)$ is only an approximate one as the radius of the current channel may vary considerably, as also may the current profile for a given $q(a)$[24]. Thus plasma regimes are more easily identified by the nature of their TFDs.

5.5.1 Regime I $3.6 > q(a) > 3.2$

It was considered for some time, that apart from transient oscillations observed on the mode coils during current startup, regime I discharges displayed otherwise weak and irregular mode activity as recorded on the Mirnov coils and strong sawtooth oscillation in the plasma interior. This result is quite understandable if one uses routine power spectral techniques to identify regular mode behaviour in the plasma. However, with the application of the TFD to the mode coil data (Fig. 5.11), there appears an $m/n = 3/1$ mode whose frequency varies in a regular way throughout the discharge.

The characteristic feature of the TFD is that the mode frequency follows variations in $1/q(a)$ after the initial current rise phase. During current rise, the mode frequency rapidly ramps down from 45kHz to 30kHz in 5ms with an instantaneous bandwidth of ~ 3kHz and a fluctuation amplitude of $\bar{B}_\theta/B_\theta \sim 0.02\%$. After the soft X-ray emission profile peaked and for $q(a) > 3.2$, the $m/n = 3/1$ mode drops in amplitude with an increase in its spectral width to ~8kHz. The mode amplitude varies erratically with a mean level of $\bar{B}_\theta/B_\theta \sim 0.005\%$ however the mode frequency variations are very regular. At $q(a) \simeq 3.5$, a transient $m/n = 7/2$ mode (observed on the Mirnov coils) rapidly ramps down in frequency and disappears.
The frequency of sawteeth are totally unrelated to the weak oscillations observed on the mode coils. It is found that this previously thought transient $m/n = 3/1$ mode during current start up actually persists throughout whole discharge and appears in every plasma regime, as does the transient $m/n = 7/2$ mode.

Looking back at the very beginning of a typical plasma discharge, we see that the $m/n = 3/1$ mode appears to go through a regular cycle of frequency and amplitude ramping before settling down to more regular mode activity (see Fig. 5.12). The density and mode coil TFDs look almost identical, apart from the sound wave reference signal marking a horizontal line across the density TFD at 30kHz. As can be seen from Fig. 5.11, the reference sound wave rarely interferes with the mode frequencies if chosen carefully.

Fig. 5.13 is a display of some basic diagnostic data of the discharge. It appears that the variations in the $m = 3$ mode frequency during the current startup phase seems correlated with variations in the loop voltage.
Figure 5.10: Plot of (a) density projections of an m=1 mode displaying odd parity across the center, and (b) the interpolated X-ray projections clearly showing the localized nature of the mode.
Regime I Discharge: TFD

Figure 5.11: TFD of (a) poloidal magnetic field fluctuations during a regime I discharge and (b) is the $1/q(a)$ variations over the same discharge. A weak though regular $m=3$ oscillation is observed to persist throughout the discharge.
CURRENT STARTUP: $n_e$ & $B_\theta$ TFDs.

Figure 5.12: Plot of (a) TFD of poloidal field fluctuations during the current rise phase and (b) shows the corresponding TFD of the density fluctuations recorded by the scintillation interferometer. The reference sound wave can be clearly seen on the density TFD.
CURRENT STARTUP: OHMIC HEATING

LT4 Shot number 552737, taken on 22-NOV-85 at 12:27:08

Figure 5.13: Basic diagnostic data during current rise phase in LT-4.
5.5.2 Regime II $3.2 > q(a) > 3.1$

A regime II discharge evolves in precisely the same way as a regime I discharge until $q(a) \approx 3.2$. At this point, (see Fig. 5.14) a sudden change in the gradient of the $m = 3$ mode frequency and in the rate of change of the mode amplitude develops at 33ms. Of course, no observable changes occur to the amplitude or frequency until several milliseconds after the turning point, however the TFD allows for the accurate pinpointing of a transition by observing changes in the frequency gradients at times prior to any noticeable change using other techniques.

It is plausible that the sudden increase in mode activity and downward dive in mode frequency is due to the $q = 3$ rational surface moving out into a more resistive edge plasma and also interacting with a resistive wall which would both have the effect of reducing the rate of mode rotation [25, 26]. The second harmonic in the TFD during regime II is an $m/n = 6/2$ component of the mode, and harmonics of the mode are expected to develop expected as the amplitude increases. For $q(a) < 3.2$, the mode frequency appears to vary inversely with changes in $1/q(a)$, and opposite to the sense of variation of the mode frequency in regime I. The combined effect of extra resistivity at the edge together with some wall interaction looks especially plausible as the process is reversed when the $q = 3$ surface starts to move back into the plasma. At $t=33\text{ms}$ the field fluctuations rise about an order of magnitude above the level in regime I ($\bar{B}_\theta/B_\theta \sim 0.05\%$).

In the plasma interior, no dramatic change in the sawtooth oscillations occurs however their amplitude is diminished and precursor oscillations become more prominent before the sawtooth crash.

As $q(a)$ rises above 3.2, the $m = 3$ mode reverts back to its low amplitude form typically found in regime I.

Fig. 5.15 shows the TFD of the scintillations data together with the density projections of the $m = 3$ mode (with the sound field removed) taken at an impact radius of 6.0cm. The spectrum shows a high background of low frequency random plasma fluctuation, especially below the reference sound wave frequency. The lack of any phase change across the detector array is due to the inversion layer of the
$m = 3$ mode being so close to the edge of the plasma, and hence inaccessible to the interferometer channels.
Regime II Discharge: TFD

Figure 5.14: TFD of poloidal field fluctuations during regime II and (b) the variations in $1/q(a)$ during the discharge. The transition from regime I to regime II appears quite reversible.
Figure 5.15: (a) TFD of scintillations during regime II activity and (b) filtered data of the $m/n = 3/1$ mode at an impact radius of 6cm. The uniform phase indicates the inversion radius must be near the plasma edge as expected for a mode resonant on the $q = 3$ surface.
5.5.3 Regime III $3.1 > q(a) > 2.9$

A typical regime III discharge is presented in Fig. 5.16. The TFD of the poloidal field fluctuations again shows a similar mode evolution to that of regime II discharges until $q(a) \approx 3.1$ when again a rapid downward shift in the mode frequency together with a rapid rise in the fluctuations amplitude occurs. The poloidal field fluctuations in regime III are around 0.1% to 0.3% and the transition into regime III is accompanied by a complete cessation of sawtooth oscillations and are replaced by the precursors oscillations which grow in amplitude until they become the dominant mode [3]. The poloidal field fluctuations display significant $m = 2$ and $m = 3$ mode activity, unlike the regime II and regime I discharges which are dominated by $m = 3$ activity alone at the plasmas edge. The soft X-ray emissions drop sharply in the plasma centre and a regular $m = 2$ or combined $m = 2$ and $m = 1$ mode appears upon the disappearance of the sawteeth oscillations. The absence of sawtooth oscillations is generally indicative of poor confinement and is thought to coincide with a flattening of the current profile[25].

Fig. 5.17 displays spectra of $m = 3$ and $m = 2$ modes indicating that they are phase locked. The TFD of the X-ray data in Fig. 5.18 clearly shows the rise in regular MHD activity on the X-rays at $\sim 20$kHz commensurate with the cessation of sawtooth oscillations at the lower frequency of 3kHz. The 20kHz signal on the X-ray channels are predominantly due to an $m = 2$ oscillation determined from the even parity of the fluctuations.

Comparison with the TFD of the scintillations data shows that the external $m = 3$ mode locks into the growing $m = 2$ mode. The result is that the scintillations and Mirnov coil data ramp rapidly down in frequency as the sawtooth oscillations disappear and the large amplitude $m = 2$ mode starts to grow. Sometimes a strong $m = 1$ mode is observed on the X-ray channels during regime III activity indicative of $q(0) \approx 1$ on axis. Fig. 5.19 displays line integral densities through a typical regime III discharge in which it is clear that a number of modes are superposed and that the partial view of the interferometer is insufficient to resolve the mode activity[27].
Regime III Discharge: TFD

Figure 5.16: (a) TFD of poloidal field fluctuations in regime III showing a further downward drop in frequency with increased mode activity and a significant m/n = 2/1 component. In (b) the variation in 1/q throughout the discharge is presented.
Figure 5.17: Amplitude and phase of spatial Fourier components v's time on the Mirnov coils for the $m = 3$ and $m = 2$ modes. The figure clearly shows the modes to be phase locked in regime III.
Figure 5.18: Plot of (a) TFD of scintillations data and (b) corresponding TFD of the soft X-ray data. The growth of regular ~20kHz oscillations coincides with a reduction in the amplitude of sawtooth oscillations seen in the Soft X-rays TFD.
Regime III: Line Integral Density

Figure 5.19: Raw data of phase scintillations taken during strong MHD activity in regime III. The data indicates a superposition of several large amplitude modes and together with the limited view of the interferometer, the mode structures can not be unambiguously recovered from the partial projections.
5.5.4 Regime IV $q(a) < 2.9$

Lowering $q(a)$ even further typically results in a major disruption unless special care is taken to minimize the impurity concentration in the plasma. A regime IV discharge is difficult to achieve in LT-4 yet as the plasma tends to disrupt in regime III. Regime IV is of great interest as it appears to regain mode activity indicative of good confinement after passing through regime III which has generally very poor confinement properties. In fact, the recovery of strong sawtooth oscillations with very low levels of external magnetic field fluctuations are very similar to the characteristics of regime I plasmas.

One characteristic of regime IV plasmas is that the transition through regime III and into regime IV shows little correlation between mode frequency variations and changes in $1/q(a)$, typical of the other regimes. Another is that in transition into regime IV, magnetic and density fluctuations indicate mode activity at a fixed frequency, but of similar amplitude as the poloidal field oscillations in regime I. Also, strong sawtooth oscillations are recovered as well.

Fig. 5.20 displays the TFD of a type I regime IV discharge. Type I discharges are those which pass through regime III both on the way into and out of regime IV where as type II regime IV discharges are those which pass in through regime III but exit through regime II (see Fig. 5.22).

Type I is the most interesting for its similarity with those discharges which normally end in major disruptions (discussed in the next section). The transition into or out of regime IV appears to be triggered by the plasma crossing $q(a) = 3$ without mode locking into the vacuum vessel wall. A closer examination of type I plasmas indicates that the X-ray fluctuations during regime III activity are considerably lower than compared with the normal regime III discharges. Fig. 5.21 shows the recovery of internal sawtooth activity upon entry into regime IV. The phase and amplitude mode spectra of the Mirnov oscillations indicates that the mode amplitudes decrease gradually to very low levels but without reducing in their frequency of rotation. This cannot be explained by any model of mode locking and must instead be due to the recovery of a stable current profile in the plasma[28, 29]. This proposition is supported by the reoccurrence of sawtooth
oscillations which provides fairly strong evidence that a peaked current profile has reformed in the plasma.

Type II regime IV discharges are not as quiescent as type I discharges (see Fig. 5.22) however the mode amplitudes are observed to decrease gradually upon entry into regime IV and with no appreciable drop in frequency (Fig. 5.23). The minimum $q(a)$ of these discharges are a little below three where as type I discharges can go as low as $q(a) = 2.4$. From the X-ray fluctuations in Fig. 5.23, it can be seen that the regime III phase with its small amplitude sawteeth oscillations is quite untypical of standard regime III discharge conditions.

As the presence of sawtooth oscillations are indicative of a peaked current profile, then a necessary condition for entry into regime IV seems to be that the current profile remains sufficiently peaked (or at least not hollow) as $q(a)$ drops below 3. That this condition should (by experience) require very low impurity levels in the plasma for low $q(a)$ operation is hardly surprising given the sensitivity of plasma profiles to impurities concentrations[24].
Figure 5.20: TFD of poloidal field fluctuations in a type I regime IV plasma showing weak fixed frequency signal in the quiescent region indicative of regime one mode activity, and (b) shows the variation in $1/q(a)$ during the discharge.
Figure 5.21: (a) Spectrum of poloidal mode number amplitude and phase upon entry into regime IV showing gradual decrease in amplitude but not frequency, and (b) the regeneration of strong sawtooth activity upon entry into regime IV.
Figure 5.22: TFD of poloidal field fluctuations in a type II regime IV plasmas showing stronger MHD activity during the quiescent period and (b) shows the variation in $1/q(a)$ during the discharge.
Regime IV-II Transition

(a)

LT-4 shot #552724  
FILTER: 11 POINTS HI-PASS  
INTEGRATION: 30.00 to 43.00 ms  
COIL SET: M-16 COILS

Mode Spectrum

AMPLITUDE

PHASE

(b)

Soft X-rays

Figure 5.23: (a) Spectrum of poloidal mode number amplitude and phase upon entry into regime IV type II, shows very gradual decrease in amplitude, and (b) regeneration of strong sawtooth activity upon entry into regime IV. Note sawteeth never completely disappeared from the regime III phase.
5.6 MAJOR DISRUPTIONS

Major disruptions presents perhaps the single greatest obstacle to the achievement of a viable fusion reactor based on tokamaks. Major disruptions not only limit the maximum attainable current for a given toroidal field ($B_t$) and the maximum density to approximately $n < CB_t/Rq_a$, but also presents considerable risk of material damage to the machine.

The difficulty in understanding disruptions is two-fold. Firstly, it is experimentally difficult to determine the $q$-profile in the lead-up to a major disruption. Information on the $q$-profile would provide an invaluable guide as to how unstable equilibria evolve to trigger the disruptive instability. None-the-less multi-channel soft X-ray diagnostics and bolometer arrays can be used to determine general features of the plasma temperature profile which then provides indirect information on the form of the current distribution. Secondly, attempts to numerically simulate major disruptions are hampered by the complexity of the phenomenon which requires that both the development of instabilities and plasma transport be treated self-consistently. Fully three dimensional numerical simulations [16] of the evolution and coupling of unstable 2/1 and 3/2 tearing modes have identified at least one mechanism for the rapid loss of thermal energy in tokamaks when large amplitude islands of differing helicities overlap to destroy nested flux surfaces. These broken surfaces can greatly enhance thermal transport by redirecting a small portion of the electrons high thermal conductivity along field lines into the radial direction.

One of the first attempts at modelling self-consistently the evolution of plasma profiles, taking into account instabilities and transport, was by Turner and Wesson [30] using a one-dimensional model in which the unstable modes interacted only by their effect on the background current profile. Later, attempts by Bondeson [31] to extend these simulations to three-dimensions for low $\beta$ cylindrical plasmas has produced results in good agreement with experiment for the minor disruption but not for the major disruption. Bondeson speculates that toroidal instabilities, unaccounted for by the cylindrical model, are responsible for the rapid termination.
of the plasma.

Further work on mode stabilization by resistive walls indicates an alternative mechanism where by explosive growth of an \( m = 2 \) island can occur if the mode locks to the vacuum vessel[26]. The stabilizing effect of the resistive vacuum vessel for rotating modes has been understood for some time[25], however it is only recently that the rapid growth and locking of the mode to the wall within a single cycle has been computationally modelled.

An experimental analysis of major disruptions in LT-4 is discussed in the chronological order of events leading up to the disruption. Typical features of LT-4 disruptions are:

- The termination of regime III mode activity for \( q(a) \leq 2.9 \) followed by a quiescent period lasting about 3-4ms before the major disruption.
- A steady drop and broadening of the X-ray emission profile during the quiescent period.
- The appearance of minor disruptions, sometimes in a series of regular bursts, before the major disruption.
- Rapid growth of a mode locked \( m = 2 \) precursor 50\( \mu \)s before the major disruptions.
- The \( m = 2 \) perturbation rapidly breaks into a more complex fluctuations pattern accompanied by a negative voltage spike.

Fig. 5.24 displays typical Mirnov coil signals prior to the major disruption. As is common with most discharges the plasma passes through mode activity characteristic of regimes II & III however, unlike the transition into regime IV, no low level activity is observed during the quiescent period as \( q(a) \) falls below three and there is no reappearance of sawteeth or any other mode activity in the plasma centre (see Fig. 5.25). From Fig. 5.24, mode rotation ceases within a single cycle, consistent with recent simulations of mode locking. If one looks carefully into the quiescent region however, there appears some indication that the previously rotating modes are still present but stationary. The difference between
the gradual decrease in mode amplitude upon entry into regime IV (see Fig. 5.21) and the sudden cessation of mode rotation in the lead up to the major disruption is quite clear. Fig. 5.25 shows the mode amplitude drop to nearly zero within a single rotation of the phase locked modes. The measured X-ray flux through a central chord in the plasma shows a 40% decline in emissivity over a 2.5ms time period after locking takes place. The degraded confinement in the absence of any observed mode rotation is indicative of the presence of large amplitude locked modes.

Typically in the lead up to the major disruption is the onset of one or several minor disruptions. The burst of mode activity at 44.7ms in Fig. 5.25 is due to a minor disruption, and a second one a little later on is displayed in Fig. 5.26.

Fig. 5.26 and Fig. 5.27 shows respectively the minor disruption on the integrated poloidal Mirnov coil data and the line integral density measurements between an impact radii of 6cm and 4cm. The scintillation data (Fig. 5.27) indicates that the density fluctuations associated with the minor disruption occur about the the q=2 surface at \( r_s \approx 0.6a \). Detailed mode analysis of the scintillation data during such transient phenomena proved difficult due to the limited view of the plasma and the irregularity of the fluctuations. However, the integrated coil data shows clearly the mode activity during a minor disruption, which conforms with Bondeson's simulations surprisingly well. Fig. 5.26 illustrates the rapid slowing down and increasing amplitude of an \( m = 2 \) mode which appears to saturate and to trigger the rapid growth of a stationary \( m = 1 \) mode. The appearance of the \( m = 1 \) is coincident with the disappearance of the \( m = 2 \) mode. The slow down of the \( m = 2 \) island before triggering the \( m = 1 \) relaxation of the plasma column is characteristic of mode locking to the vacuum vessel. The accepted explanation for the triggering of an \( m = 1 \) is that the unstable \( m = 2 \) mode constricts the current channel around the \( q = 1 \) surface leading to an internal sawtooth like relaxation. The rapid drop and flattening of the X-ray emission profile after the minor disruption is indicative of a further loss of confinement due to the sawtooth like relaxation of the plasma core [32].

The major disruption is again initiated by the growth of a very large amplitude
m=2 island as can be seen on the poloidal coil data in Fig. 5.28. The m=2 mode is already locked when it appears on the Mirnov coils.

Despite the fact that minor disruptions or "predisruptions" are fairly well understood through numerical simulations of cylindrical plasmas, a corresponding simulation of the major disruption is yet to be performed but it is speculated by Bondeson [31] that its simulation will be possible provided the full toroidal geometry is taken into account.

When the major disruption does not lead to a complete termination of the plasma then it is possible for the flattened current profile to peak up again, producing a peaked X-ray emission profile and leading to the appearance of a succession of modes, typically from m/n=4/1 to m/n=1/1 within a millisecond after the major disruption. A typical TFD of the post disruptive plasma is shown in Fig. 5.29. The initial mode is an m = 4 oscillation which can be observed on all TFDs of density, X-rays and Magnetic probe data. Following this transient phase, a burst of m=3 mode activity is observed on the scintillation signals and magnetic probe data but not on the X-ray signals, suggesting that the m=3 transient is an external mode. The frequency curve associated with the m=3 mode ramps up from 20kHz to 30kHz before the mode suddenly terminates and is followed by an m=2 structure which displays a similar time frequency signature. However, the m=2 appears to be located deeper into the plasma than the m=3, the magnetic probe signals are relatively weak compared to the density fluctuations. When the m=2 mode disappears, an m=1 structure appears on the scintillation channel and soft X-rays but not on the magnetic probe, which clearly indicates the internally localized nature of the m = 1 mode.
Figure 5.24: Plot of (a) TFD of magnetic fluctuations on transition from regime II to regime III and from there to mode locking with no noticeable change in frequency, and (b) shows integrated poloidal coil data displaying waves in the poloidal direction at the time of mode locking.
Figure 5.25: (a) Amplitude and phase spectrum of poloidal modes during mode locking showing that locking occurs within a single cycle, and (b) displays central soft X-ray emissions showing degraded confinement after mode locking.
Figure 5.26: Integrated signals from the poloidal array of Mirnov coils during a minor disruption showing transient growth and locking of an $m = 2$ mode then triggering the growth of a stationary $m = 1$ mode.
Density Modulations During a Minor Disruption

Figure 5.27: Projections of density fluctuations across the scintillation detectors between impact radii of 6cm and 4cm during a minor disruption.
Low q Disruption in LT-4

(a) Poloidal Array
Mirnov Coils (m16)

(b) Soft X-ray
Projections

Figure 5.28: (a) High current disruption in LT-4 showing rapid growth of a precursor $m = 2$ mode and (b) Soft X-ray signals showing rapid decline of the emission profile during mode locking and rapid peaking of the profile after the major disruption.
TFD of a Disruption in LT-4

Figure 5.29: The TFDs of density, X-rays and magnetic field fluctuations in the post disruptive plasma showing mode activity during current repenetration. The variation in the amplitude of the various components indicates the relative location of the modes.
Bibliography


SMALL SCALE DENSITY FLUCTUATIONS IN LT-4

Ever since low-frequency ($\omega \ll \omega_a$) small scale ($k_\perp a \gg 1$) density fluctuations were identified in the interior of tokamak plasmas[1], they have been the focus of an intense research effort directed at identifying their cause and determining their effect on plasma confinement. Characteristics of these fluctuations common to most tokamak plasmas are,

- broadband power spectra ($\Delta \omega/\omega$, $\Delta k/k \sim 1$),
- a broad wavenumber spectrum for each frequency $\omega$,
- density fluctuations varying from less than 1% in the plasma interior to 50% at the plasma edge,
- $K_\parallel \ll K_\perp$,
- the observed frequencies are of the order of the electron diamagnetic drift frequency

$$\omega_{ee} = k_\perp v_{ee} \quad (6.1)$$

where

$$v_{ee} = \frac{c}{eB} \frac{T_e}{n} \nabla_\perp n \approx \frac{c}{eB} \frac{T_e}{L_n} \frac{1}{L_n} \quad (6.2)$$

is the diamagnetic drift velocity,

- propagation is predominantly in the electron diamagnetic drift direction,
fluctuations typically peak for $k \rho_s \sim 0.1 - 0.5$, where $\rho_s$ is the ion Larmor radius calculated at the electron temperature,

$$\rho_s (\text{cm}) = 10^2 \sqrt{\frac{M_i T_e^{1/2} (\text{eV})}{2Z B(g)}}. \quad (6.3)$$

In the absence of any other universally observed phenomena likely to explain anomalous transport, turbulent convection of particles and energy is thought to be the major cause of poor confinement in tokamaks. Although there are a multitude of theoretically possible mechanisms which could lead to turbulence and which can be made to conform with available data[2] in reality, progress in identifying cause(s) of anomalous transport are limited by the inadequacy of present diagnostics. Given the inability to determine causative factors of transport from measurements of density fluctuations alone [3], only a phenomenological description of density fluctuations in LT-4 is presented.

A brief introduction to fluctuation induced radial transport follows. The statistical properties of coherent optical fields propagating through random quasi-homogeneous refractive media are discussed and experimental results from LT-4 are presented.

6.1 Transport Induced by Random Fluctuations

This discussion is restricted to the effects of random fluctuations on transport. Although narrow-band fluctuations (important during periods of strong MHD activity) does enhance transport[4, 5], poor confinement still persists even in the absence of these well defined low order modes. As small scale broad-band density fluctuations are generally observed in tokamak plasmas, it is natural to consider these responsible for the generally observed non-classical confinement.

Random fluctuations in magneto-plasmas induce radial transport of particles and energy only when perturbed quantities such as temperature, density, electric field, magnetic field, and current are correlated. Losses may occur through radial guide centre drifts of charged particles or through electrons diffusing along braided magnetic field lines[6]. Thus, the radial drift velocity induced by turbulence in a
hydrogen plasma (for $B \simeq B_T$ and $\omega \ll \omega_{ci}$) can be written as[7]

$$
\vec{v}_r = \frac{c\vec{E}_{pol}}{B_T} + v \frac{\vec{B}_r}{B}.
$$

(6.4)

The total turbulent particle flux $\Gamma_p$ is given by

$$
\Gamma_p = \langle \vec{n} \vec{v} \rangle
$$

(6.5)

where $n$ is the particle density. Inserting Eqn. 6.4 into Eqn. 6.5, and neglecting magnetic fluctuations for now, we obtain

$$
\Gamma_p = \frac{c\langle \vec{E}_{pol} \vec{n} \rangle}{B_T}.
$$

(6.6)

Radial heat flux has both a conductive and convective contribution. Although there is some disagreement on the numerical factor, the total electron thermal heat flux due to $E \times B$ drifts is[8]

$$
Q_r = \frac{5}{2} \frac{c}{B} \langle \vec{p} \vec{E}_{pol} \rangle
$$

(6.7)

where $\vec{p}$ is the fluctuating pressure, with the conductive part of the electron heat flux given by

$$
q_r = \frac{5}{2} \frac{n}{B} c \langle \vec{T} \vec{E}_{pol} \rangle.
$$

(6.8)

The expressions for particle and heat flux indicates the degree of difficulty involved in extracting local relative phases in the plasmas interior. Even if the local level of fluctuations in $E$, $T$, $B$ and $n$ can be determined by some tomographic technique, extracting the local relative phases between these quantities is most unlikely using present day diagnostics. In the absence of a directly measurable local electric field, the radial flux may be estimated by rather rough mixing length arguments[9]. If the nature of the plasma inhomogeneity driving the turbulence is known, then the saturation mechanism would tend to smooth out the inhomogeneity. For instance, if the density gradient is the source of free energy (as with electrostatic drift waves[9]) then assuming strong turbulence has developed,

$$
\frac{\vec{n}}{n} \simeq \frac{1}{k L_n}.
$$

(6.9)
This relation is in general consistent with experiment[10]. In the absence of a
direct measurement, the velocity fluctuations may be estimated from Boltzmanns
relation,
\[ \frac{e\ddot{\phi}}{T_e} = \frac{\ddot{n}}{n} \] (6.10)
where \( \ddot{E}_\text{pol} \sim k_\perp \ddot{\phi} \). Combining these expressions, an estimate of \( v_r \) may be ob-
tained:
\[ v_r \simeq \frac{cT_e}{eB} \left( \frac{\ddot{n}}{n} \right) k_\perp. \] (6.11)
Thus, by the use of simplifying assumptions on the nature of the turbulence,
estimates of local energy and particle flux may be obtained from the amplitude
and scale size of the density perturbations.

Quantities of interest such as the plasma diffusivity, \( \kappa(r) \) may be estimated in
the same way
\[ \kappa_\perp(r) = \alpha v_r \delta r \] (6.12)
\[ \simeq \alpha \frac{cT_e}{eB} \left( \frac{\ddot{n}}{n} \right). \] (6.13)
where \( \delta r \) is the correlation length in the radial direction and \( \alpha \) is the correlation
between the plasma displacement and velocity. The diffusivity is related to the
local particle flux via,
\[ \Gamma_p = \kappa_\perp(r)(dn/dr). \] (6.14)
The estimated particle diffusivity from the measured density fluctuations can be
compared to the global diffusivity \( \chi_\perp = a^2/\tau_B \).

Of particular interest is to determine whether the turbulent convective heat
flux is comparable with the heat flux \( q_r(\vec{B}) \) due to the magnetic braiding of field
lines where,
\[ q_r(\vec{B}) = n\chi_e(\vec{B}) \frac{dT_e}{dr} \] (6.15)
and where in a collisionless plasma[11],
\[ \chi_e(\vec{B}) = qR\nu_e(\vec{B}/B)^2. \] (6.16)
Here, $q$ is the safety factor, $R$ the major radius and $v_e$ the electron thermal velocity along the field lines. Thus, the ratio $q_r(\vec{B})/q_r(\vec{E})$ can be estimated and hence their relative significance determined.
6.2 Scattering From Random Plasma Fluctuations

In chapter 3, a precise relationship between the scattered angular spectrum and the Fourier transform of the scattering potential, via the Fourier diffraction projection theorem was established. Although such a description of the scattered radiation is of use in the analysis of coherent or regular structures, the analysis of random (stochastic) fields requires instead a statistical approach. The field scattered from a spatially and temporally stochastic medium can be characterized statistically by measures such as power spectra, correlation, covariance, and coherence functions. Further, certain random fields may possess properties such as homogeneity and isotropy which determine the applicability of various statistical measures to the characterization of the random processes. In particular, the extraction of a power spectrum from the correlation function measured at a point in the medium requires that the correlation function, (or at least the coherence function) be shift invariant. As will be seen later, the coherence function is simply the normalized correlation function and where the weakest possible inhomogeneity is when the coherence function (but not the correlation function) is shift invariant. Much of what follows is drawn from discussions by Papoulis[12], Tatarski [13], and Goodman[14].

The most important quantity which can be derived from a random process is its power spectrum. We take as our random process the scattering potential \( f(r, t) \) within some volume \( V_p \) where the scattering potential in the Born approximation is given by

\[
f(r, t) = u_0(r) \tau(r, t) n_e(r, t)
\]

(6.17)

and where \( \tau(r, t) \) is a step function equal to unity within the space-time window recorded. The definition of the power spectral density \( S(K, \Omega) \) of \( f(r, t) \) is

\[
S_f(K, \Omega) = \frac{|F(K, \Omega)|^2}{V_p T}
\]

(6.18)

where \( F(K, \Omega) \) is the Fourier transform of \( f(r, t) \),

\[
F(K, \Omega) = \int_{-\infty}^{\infty} dr dt f(r, t) \exp (-jK \cdot r - j\Omega t).
\]

(6.19)
Although any signal \( f(r, t) \) possesses a power spectral density, the interpretation of such a measure depends largely on the stationarity of the signal within the recorded space-time window.

The statistical correlation function \( \Gamma_f(r, t) \) is defined as

\[
\Gamma_f(r, t) = \langle f(r_1, t_1) f^*(r_1 + r, t_1 + t) \rangle. \tag{6.20}
\]

This expression denotes an ensemble average over the whole process \( f(r, t) \). The correlation function \( \Gamma_f(r, t) \) is given by

\[
\Gamma_f(r, t) = \frac{1}{V_p T} \int_{V_p} \int_{-T/2}^{T/2} dr_1 dt_1 \ f(r_1, t_1) f^*(r_1 + r, t_1 + t). \tag{6.21}
\]

To establish whether a medium is homogeneous, we form only the temporal correlation function \( \Gamma_f(r_1, r_1 + r; \tau) \) at a fixed point in space where

\[
\Gamma_f(r_1, r_1 + r; \tau) = \frac{1}{T} \int_{-T/2}^{T/2} dt_1 \ f(r_1, t_1) f^*(r_1 + r, t_1 + \tau). \tag{6.22}
\]

When \( r = 0 \) we recover the standard autocorrelation function. If \( \Gamma_f(r_1, r_1 + r; \tau) \) is independent of \( r_1 \) then we have the trivial yet very important result that \( \Gamma_f(r_1, r_1 + r; \tau) = \Gamma_f(r, \tau) \) for any \( r_1 \) within \( V_p \). Clearly, a spatially homogeneous correlation function allows the determination of \( \Gamma_f(r, \tau) \) from a twin probe measurement at a single point \( r_1 \) within \( V_p \) without having to performing spatial averages over the entire volume as suggested by Eqn. 6.21.

This simplification in determining the statistical correlation function of the medium is of central importance in being able to extract the power spectral density of a random process from the the spatial correlation function by use of a fixed and movable probe[15, 16]. Unfortunately, the strict condition of homogeneity is rarely tested in such two probe experiments and results are obtained which could deviate significantly from the true power spectral density in the spatial window being sampled. To clarify this point, using Eqn. 6.21,6.18 and from the Fourier convolution theorem, the power spectral density \( S_f \) and the autocorrelation \( \Gamma_f \) of \( f(r, t) \) are easily shown to be Fourier transform pairs,

\[
S_f(K, \Omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dK \ dt \ \Gamma_f(r, t) \exp[-j(K, r + \Omega t)]. \tag{6.23}
\]
Note that as $S_f$ is real symmetric, i.e. $S_f(K, \Omega) = S_f(-K, -\Omega)$, then by the inverse transform of $S_f$ (which is $\Gamma_f$) is a real function, as expected. Rewriting $\Gamma_f(r, \tau)$, making use of (Eqn. ? and Eqn. ?) gives,

$$
\Gamma_f(r, \tau) = \overline{\Gamma_f(r_1, r_1 + r; \tau)}, \quad (6.24)
$$

where the average is taken over $r_1$. Thus,

$$
S_f(K, \Omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dK dt \overline{\Gamma_f(r', r' + r; \tau)} \exp[-j(K.r + \Omega t)]. \quad (6.25)
$$

In two probe measurements however, $\Gamma_f(r_1, r_1 + r; \tau)$ is determined usually for a fixed probe at $r_1$, and so its Fourier transform need not necessarily equal the power spectral density unless $\Gamma_f(r_1, r_1 + r; \tau) = \overline{\Gamma_f(r', r' + r; \tau)}$ within the volume $V_p$.

A further consequence which trivially follows from the shift invariance of the correlation function is that the power spectrum of the complete random process and the temporal correlation function at a fixed point $r_1$ form a Fourier transform pair,

$$
S_f(K, \Omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dK dt \Gamma_f(r_1, r_1 + r; \tau) \exp[-j(K.r + \Omega t)]. \quad (6.26)
$$

This result is otherwise known as the Wiener-Khinchin theorem [14].

In the case of simultaneous measurements using a regular array of probes with a spacing dense enough to avoid aliasing, the modulo square of the Fourier transform of the probe array data can be computed using FFT’s to extract the power spectral density. The Phase Scintillation interferometer measures line integral densities in the optical near field, and the 16 element linear detector array allows for a direct computation of the power spectral density without having to perform crosscorrelations as with twin probe measurements. Consequently, a possible lack of homogeneity within the space-time window sampled by the interferometer does not affect the reliability of the power spectral density obtained.

Another statistical measure of considerable importance is the statistical coherence spectrum. The temporal coherence function is related to the temporal correlation function by the expression

$$
\mu(r_1, r_1 + r; \tau) = \frac{\Gamma(r_1, r_1 + r; \tau)}{|\Gamma(r_1, r_1; \tau)|^{1/2}|\Gamma(r_2, r_2; \tau)|^{1/2}}. \quad (6.27)
$$
With probe measurements, this quantity is easily determined before Fourier transformation, however, the coherence function of the medium is properly determined by the statistical correlation function of the $\Gamma_f(r, t)$, where

$$\mu(r; t) = \frac{\Gamma(r; t)}{|\Gamma(0; t)|}. \quad (6.28)$$

In this form we see that for a random process, the coherence function is only a normalized correlation function and hence is of little importance when using direct Fourier transforming methods to obtain the power spectrum. However, if temporal correlation functions are being used, say from two probe measurements to obtain the power spectrum, then the coherence function is of use only if there are weak inhomogeneities in the correlation function.

Finally, a relatively simple test of homogeneity is possible without testing directly for the shift invariance of the correlation function. The degree of homogeneity in the signal is still of importance in the interpretation of the power spectral density. As $S_f(K, \Omega)$ and $\Gamma_f(r, t)$ form a Fourier transform pair, we can obtain $\Gamma_f(r, t)$ by inverse Fourier transforming $S_f(K, \Omega)$ which we obtain directly by taking the modulo square of the Fourier transform of $f(r, t)$ (c.f. Eqn. 6.18). That is,

$$\Gamma_f(r, t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dK d\Omega \ S_f(K, \Omega) \exp [j(K \cdot r + \Omega t)]. \quad (6.29)$$

Also from the array of detectors, we may form a temporal correlation function about the central detector in the array. Fourier transforming this would give a spectrum which could be compared with the power spectral density obtained by Fourier transforming the whole array data. If they are in reasonable conformity, then the signal can be assumed homogeneous. If not, then some qualitative degree of homogeneity may be estimated.

In determining the power spectral density of the scattering potential from a knowledge of the radiation scattering by the plasma, we make use of a simple relationship in linear systems theory. For a random process $f(r, t)$ and a shift invariant impulse response function $h$, the output $u(r, t)$ of the linear system is

$$u = f \ast h \quad (6.30)$$
whilst the Fourier transform of \( u \) is

\[
U(K, \Omega) = F(K, \Omega).H(K, \Omega).
\]  
(6.31)

From this expression, the power spectral density \( S_u \) of the output of the linear system is given by

\[
S_u(K, \Omega) = U(K, \Omega)U^*(K, \Omega) = |H(K, \Omega)|^2 S_f(K, \Omega).
\]  
(6.32)

Although a filter function \( H \) has been defined over all the space-time variables, the result still holds for Fourier transforms in any one or more of the independent variables.

This expression implies is that the power spectrum of the scattered radiation is linearly related to the power spectral density of the plasma, but modulated by the square of the linear filter function. This greatly simplifies the theoretical analysis of scattering from turbulent or random media as the precise nature of the random field \( f(r,t) \) need not be known and only its statistical properties are required.

6.2.1 Angular spectrum and the Rytov phase of scattered light

In Chapter 3, the linear relation between the scattered angular spectrum \( A(\kappa_x, \kappa_y, \omega) \) and the incident angular spectrum \( A_i(k_0, \omega_0) \) was determined in terms of the free space propagator \( H(\kappa_z; z) \), the transform of the scattering potential \( F(\kappa, \omega_0) \), and the scattering transfer function \( G(\kappa_z; z) \) by the relation,

\[
A_s(\kappa_x, \kappa_y, \omega; z) = G(\kappa_z; z)F(\kappa, \omega_0)
\]  
(6.33)

where

\[
F(\kappa, \omega_0) = \int_{-\infty}^{\infty} \frac{dK}{(2\pi)^3} 4\pi r e \tilde{N}_e(K, \Omega)U_0(\kappa - K, \omega_0 - \Omega)
\]  
(6.34)

and where \( \omega_s = \omega_0 + \Omega \). The free space component of the optical field is given by

\[
A_0(\kappa_x, \kappa_y, \omega_0; z) = H(\kappa_z; z)A_i(\kappa_x, \kappa_y, \omega_0; 0).
\]  
(6.35)
Taking the square of $A_s$, we obtain,

$$\langle |A_s(\kappa_x, \kappa_y, \omega; z)|^2 \rangle = \frac{\langle |F(\kappa - k_0, \omega)|^2 \rangle}{4\kappa^2}. \quad (6.36)$$

Then, from Eqn. 6.34, and in the case of plane wave illumination,

$$F(\kappa - k_0, \omega_0 + \Omega) = 4\pi r_0 a_0 \hat{N}(K_F, \Omega) \quad (6.37)$$

where, as previously defined, $K_F = (\kappa_x, \kappa_y, \kappa_z - k_0)$ for a plane wave propagating in the $z$ direction. Hence,

$$\langle |A_s(\kappa_x, \kappa_y, \omega_0 + \Omega; z)|^2 \rangle = (4\pi)^2 r_0^2 a_0^2 \frac{\langle |\hat{N}_e(K_F, \Omega)|^2 \rangle}{4\kappa^2}$$

$$= \frac{4\pi^2 r_0^2}{\kappa^2} a_0^2 \frac{\pi_e}{V_p} S(K_F, \Omega) \quad (6.38)$$

where

$$S(K, \Omega) = \frac{\langle |\hat{N}_e(K, \Omega)|^2 \rangle}{\pi_e V_p}. \quad (6.39)$$

The expression $S(K, \Omega)$ is called the plasma form factor, which for the scattered field, is evaluated at $K = K_F$. Note that the scattered spectrum $A_s$ is independent of $z$. This is because both the free space propagator $H$ and the scattering propagator $G$ are purely phase dispersive in $z$, and the scattered power spectrum does not retain relative phase information between the various wavenumber components.

The phase scintillation interferometer measures the Rytov phase from which the power spectrum as well as images of the line integral density fluctuations can be obtained. Given that one has a measure of the Rytov phase $\psi(x, y, t; z)$ across an evenly spaced array of probes, then the Fourier transform $\Psi(\kappa_x, \kappa_y, \Omega; z)$ is easily obtained by an FFT. From Chapter 3, the expression for $\Psi$ is given by,

$$\Psi(K_x, K_y, \Omega; z) = -j r_e \lambda_0 \exp(jK_F z) \hat{N}(K_F, \Omega). \quad (6.40)$$

Squaring the Rytov phase we obtain,

$$\langle |\Psi(K_x, K_y, \Omega; z)|^2 \rangle = r_e^2 \lambda_0^2 \pi_e V_p S(K_F, \Omega) \quad (6.41)$$

where $S(K_F, \Omega)$ is the plasma form factor. Again, the power spectrum of the Rytov phase in the optical near field is independent of $z$ and proportional to the plasma power spectrum.
In practice the complex quantity $\psi$ is not measured but rather what is measured is a real signal which is some combination of the real and imaginary parts of the Rytov phase. Assuming the power of the local oscillator is some fraction of the power of the probe beam, then the measured signal $I(r, t)$ can be expressed as

$$I(x, y, t; z) = I_0[\alpha \chi_s(x, y, t; z) + \phi_s(x, y, t; z)]$$  \hspace{1cm} (6.42)

where $0 < \alpha^2 < 1$, $\chi_s = \Re \psi$, $\phi_s = \Im \psi$ and $I_0$ is a constant for plane wave illumination. This result is only valid for $|\chi|, |\phi| \ll 1$. Again from Chapter 3, the real and imaginary parts of the Rytov phase satisfy the following relations,

$$\Psi(K_z, K_y, \Omega; z) = -j r c \alpha_0 \exp(j K_F z) \tilde{N}(K_F, \Omega)$$  \hspace{1cm} (6.43)

and

$$\chi(K_F, \Omega; z) = H_\chi(K_z, K_y; z) \tilde{N}_e(K_F, \Omega)$$  \hspace{1cm} (6.44)

$$\phi(K_F, \Omega; z) = H_\phi(K_z, K_y; z) \tilde{N}_e(K_F, \Omega)$$  \hspace{1cm} (6.45)

where

$$H_\chi(K_z, K_y; z) = \sin(K_F z)$$  \hspace{1cm} (6.46)

$$H_\phi(K_z, K_y; z) = -\cos(K_F z)$$  \hspace{1cm} (6.47)

are the transfer functions valid for a thin screen. Eqn. 6.44, 6.45 are easily determined by noting that $N(K)$ is hermitian and $\chi(K), \phi(K)$ as defined above are hermitian and anti-hermitian respectively so that their inverse Fourier transforms are respectively, real and imaginary. Hence, taking the Fourier transform of $I$ in Eqn. 6.42 we obtain,

$$\langle |I(K_z, K_y, \Omega; z)|^2 \rangle = I_0^2 V_{\Omega} \tilde{N}_e |\alpha H_\chi + H_\phi|^2 S(K, \Omega).$$  \hspace{1cm} (6.48)

Note that a linear relationship is maintained between the spectrum of the Rytov phase and the plasma form factor. For simplicity, taking $\alpha = 1$, gives a power spectral transfer function,

$$|H_\chi + H_\phi|^2 = 2 \sin^2(2K_F z - \pi/4)$$

$$\approx 1 \quad \text{for} \quad K_F z \ll 1.$$  \hspace{1cm} (6.49)
The extension of this result to finite depth random phase screens is treated by Goodman[14] however, the thin phase screen result is sufficient for demonstrating that a thin random phase screen can be located by observing the Rytov phase using an imaging diagnostic. It is well known that the location of a coherent phase screen can be determined by scattering experiments to only a multiple of its Fresnel length. Restricting the discussion to the real part of the Rytov phase (detected as the homodyne contribution), we obtain (for a single wavenumber $K_F = (K_0, 0, K_0^2/2k_0))$,

$$\chi(K_0, 0; z) = \sin (zK_0^2/2k_0)\hat{A}(K_F)$$  \hspace{1cm} (6.50)

which is clearly sinusoidal in $z$ with the period equal to the Fresnel length.

However, let us make the assumption that the spectrum $S(K)$ is white up to some cutoff, i.e.

$$S(K) = 1 \text{ for } |K_x|, |K_y| \leq K_0$$
$$= 0 \text{ otherwise.} \hspace{1cm} (6.51)$$

Consider that the measurement is being performed by a single detector scanning the beam to determine the RMS amplitude of fluctuations or that the thin random screen is scanning across the beam. Then the measured mean square fluctuation $\chi_{MS}^2(z)$ for a homogeneous medium is

$$\chi_{MS}^2(z) = \Gamma \chi(0; z)$$
$$= \int \int_{-K_0}^{K_0} dK_x dK_y |\chi(K_x, K_y; z)|^2$$
$$= \int \int_{-K_0}^{K_0} dK_x dK_y \sin^2 (K_F z).$$  \hspace{1cm} (6.52)

Without compromising on the essential physical result, assume the random phase screen is non-uniform only in the $x$-direction so that $K_y = 0$. Then the integral in Eqn. 6.53 reduces to a simple cosine Fresnel integral. We finally come to normalized expressions for $\chi_{MS}$ (and $\phi_{MS}$),

$$\chi_{MS} = \frac{1}{2} - \frac{1}{2K_0} \sqrt{\frac{\pi k_0}{z}} C \left( K_0 \sqrt{\frac{z}{\pi k_0}} \right)$$  \hspace{1cm} (6.53)

$$\phi_{MS} = \frac{1}{2} + \frac{1}{2K_0} \sqrt{\frac{\pi k_0}{z}} C \left( K_0 \sqrt{\frac{z}{\pi k_0}} \right).$$  \hspace{1cm} (6.54)
and $C$ is the cosine Fresnel integral where asymptotically,

$$\lim_{z \to 0} \chi_{MS}(z) = 0$$

$$\lim_{z \to 0} \phi_{MS}(z) = 1$$

$$\lim_{z \to \infty} \chi_{MS}(z) = 1/2$$

$$\lim_{z \to \infty} \phi_{MS}(z) = 1/2$$

(6.55)

Fig. 6.1 displays the RMS amplitude of the real and imaginary parts of the Rytov phase against distance for the case of a coherent and a broad band phase screen. Note that for a broad band thin screen, imaging diagnostics can be used to precisely locate its position along the line of sight. This can have possible applications to tokamak plasmas as a means of resolving whether the fluctuations are localized on the edge plasma by using a single beam probing a central chord through the plasma. As the image plane of the lens is scanned through the random phase screen the phase modulations will peak and the homodyne modulations will go to zero. In the far field, both the real and imaginary parts of the Rytov phase approach the same limiting value. In practice, the presence of more than one thin screen can produce poor spatial resolution unless they are well separated.

The minimal separation of two random screens for each to be resolved is about $10Z_F(\Lambda_{\min})$ where $Z_F(\Lambda_{\min})$ is the fresnel length of the smallest resolved structure in the random medium assuming a white spectrum. Thus, given $K_0$ (assuming it is limited by the medium) and the separation of the top and bottom of the plasma, then a maximum wavenumber $k_0$ can be selected for an adequate resolution of the two random screens at the edge plasma. The results are easily extended to gaussian spectra discussed elsewhere.
Figure 6.1: (a) Amplitude of real and imaginary Rytov phase variation with $z$ for a coherent phase screen of wavenumber $K_0$. The dashed line is the imaginary part of the Rytov phase. In (b), the phase screen is taken to have a white spectrum with cutoff at $K = K_0$. Note that the random screen can be located to within a Fresnel length of the smallest structure within it.
6.3 Experimental Observations

In this section we investigate conditions for which strong MHD activity is absent. Such conditions typically occur in regime 1 discharges where \( q(a) \geq 3.5 \). However, some residual level of MHD activity is always present as can be seen in the time frequency diagram of a typical regime 1 discharge in Chapter 5.

The corresponding macroscopic plasma conditions are typically \( I_p \sim 50kA, B_t \sim 2T, n \sim 3 \times 10^{13} cm^{-3}, T_e \sim 200eV, \) and \( 3.5 \leq q(a) \leq 4.0 \).

For these experiments the plasma was imaged with unity magnification, the effective detector spacing being 1mm. The experimental arrangement allowed the detector array to be rotated for alignment at various angles to the magnetic field lines. As well, the beam can be scanned across the minor cross section of the plasma, however because of the limited size of the windows on the vacuum vessel, complete radial scans cannot be taken. Fig. 1 shows typical signals recorded radially along a 1.5cm projection of the plasma at an impact radius of \( r_0 = 6cm \). The small scale fluctuations are characterized by a lack of obvious correlations across the detector array. Fig. 6.2 on the other hand displays data recorded along the direction of the magnetic field, demonstrating the long correlation lengths along the field lines. The window width sets the correlation length in the toroidal direction, and is thus insufficient to resolve the toroidal correlation length and wavenumber of the fluctuations. The level of fluctuations for \( r_0 < 0.3a \) is from a 1% to 2% of the line integral density. If we assume that the beam propagates through many uncorrelated structures \( p \), then the phase fluctuations will appear as a Gaussian random variable where,

\[
\left\langle \left( \frac{\delta n_e}{n_e} \right)^2 \right\rangle_{\text{local}} \sim \frac{1}{\sqrt{p}} \left\langle \left( \frac{\delta n_e}{n_e} \right)^2 \right\rangle_{\text{int}}. \tag{6.56}
\]

Fig. 6.3 shows correlations, both radially and toroidally and in various frequency bands. Given a typical correlation length in the radial direction of 1cm, and assuming the medium to be isotropic and homogeneous, then \( p \sim 20 \) in LT-4 and

\[
\left\langle \left( \frac{\delta n_e}{n_e} \right)^2 \right\rangle_{\text{local}} \sim 0.4\%.
\]
Figure 6.2: Recorded signals for array aligned perpendicular to the magnetic field at an impact radius of 6cm. Note that the detector array is longer than the apparent correlation length of the fluctuations.

Of course, the assumption of homogeneity may not be correct, however only radial scans across the plasma can help decide on the radial distribution of fluctuations. If we assume the fluctuations to be concentrated within a band of thickness $0.2a$
Figure 6.3: Recorded density for detectors aligned parallel to the field at an impact radius of 6cm. Note that the array length is insufficient to determine correlation lengths of the structures.

at the outer edge of the plasma where the mean density is approximately a tenth of the average, then $p \sim 4$, and

$$\left\langle \left( \frac{\delta n_e}{n_e} \right)^2 \right\rangle_{\text{local}}^{1/2} \sim 10\%.$$
Figure 6.4: Cross correlations of data recorded (a) across magnetic field lines in three different frequency bands and (b) along field lines integrated over all frequencies.

Power spectra may be obtained by squaring the space-time Fourier transform of the raw data. Again at an impact parameter of 6cm, two power spectra of the raw data in Fig. 6.1 and 6.2 are shown. Fig. 6.4 shows the power spectra of fluctuations in the toroidal direction($B_\parallel$), showing peaking at $K_\parallel = 0$ where $K_\parallel$
denotes the toroidal wavenumber. The band width of the fluctuations is about 4 rad.cm⁻¹ which is about the spectral resolution of the detector array. Hence all we may say is that the correlation lengths of the perturbations in the toroidal...
direction are considerably longer than 1.5cm and that the mean wavenumber must be very close to zero along the field lines. Although the scattered power rapidly falls away beyond 200kHz, there is still significant integrated power in the signal at higher frequency to make a significant contribution to the image produced of these structures.

The power spectrum taken in the radial direction (Fig.6.5) shows clear dispersion bands for both positive and negative wavenumbers and is reasonably symmetric. The spectrum is consistent with the possibility that the laser beam is passing through regions of counter propagating structures. For instance, as the beam passes through both upper and lower midplane of the plasma, and if turbulent structures have some component of their velocity in the poloidal direction, then the projection through the plasma will consist of counter propagating structures. The band width of the spectra is approximately 15rad.cm\(^{-1}\) which indicates a plasma correlation length \(L_r \sim 1\text{cm}\) which is approximately equal to the wavelength of the fluctuations, inferred from peak wavenumber of the spectrum. Such conditions are also characteristic of strong turbulence. The phase velocity of the fluctuations is of the order of \(2 \times 10^5\text{cm.s}^{-1}\) which is of the same order as the diamagnetic drift velocity for our plasma. The Width of the spectra is however so broad that no clear dispersion is identified aside from two distinct bands.

If the images produced are indeed of two random phase screens at the top and bottom edges of the plasma, then the spectra can be filtered for either the positive or negative wavenumbers and inverted to obtain an image with the counter propagating structures separated out. Note that since the line integral density is measured simultaneously across an array of detectors, a Fourier transform of the density can be computed, filtered and inverted to obtain the image of any particular spectral component of the signal. Fig.6.6 shows an image of a plasma projection along the radial direction at \(r_0 = 6\text{cm}\) and over an interval of \(50\mu\text{s}\). The light and dark bands indicate positive and negative density perturbations. The top figure is the raw data before filtering. The bottom two figures are respectively the images of forward and backward moving structures, which, if due to poloidal rotation, would correspond to the upper and lower density perturbations of the
Figure 6.6: Spectrum of fluctuations across magnetic field lines at impact radius $r_0 = 6\text{cm}$, showing $\langle K_\perp \rangle \sim 15\text{rad.cm}^{-1}$. The lack of clear dispersion is indicative of strong turbulence.

plasma. The filtering of such data is an important step in identifying coherent structures in the signals by use of line of sight imaging diagnostics where uncorrelated structures are superposed. In this case, use was made of the clear separation of the positive and negative wavenumber components in the spectrum. The fil-
Figure 6.7: Fourier filtering of (a) line integral density isolating counter propagating fluctuations in (b) and (c).

The resulting filtered image in Fig. 6.6(b) appears considerably more coherent than the image in Fig. 6.6(c), indicating that the positive and negative wavenumber components of the projections are uncorrelated, lending
support to the contention that they represent fluctuations in separate regions of the plasma. \(^1\)

The assumption of isotropy made previously in the estimate of the local fluctuations level turns out not to be so accurate. Spectra taken at both \(r_0 = 0\) and \(r_0 = 0.6a\) (Fig. 6.7) shows clear differences, indicating that the medium is anisotropic. The increased level of fluctuations at \(r_0 = 0.6a\) is also clear from the figure. Both these observations indicate that the fluctuations probably concentrate towards the plasma edge. The projection through the centre of the plasma would only sample the poloidal wavenumber component \(K_\theta\) of the fluctuations where \(K_\theta \sim 5 \text{rad.cm}^{-1}\). On the other hand the projections at an impact radius of \(0.6a\) would provide both information on the poloidal as well as the radial wavenumbers. The increasing wavenumber of the projections at larger impact radii are consistent with both radially propagating and/or poloidally propagating structures.

6.4 Discussion And Comparison With Other Tokamaks

It is universally observed in tokamak plasmas that small scale fluctuations have a broad frequency spectrum, a frequency range of the order of the electron diamagnetic drift frequency \(\omega_\text{e}^*\) and a peaking of the density fluctuations at the plasma edge with an amplitude of \(\bar{n}/n \sim 0.1 - 0.5\) and much less in the plasma interior. As is also commonly observed, the wavelengths are small relative to the minor radius in the direction perpendicular to the field lines. For LT-4, the poloidal wavenumber (determined from the centre of the plasma, and the spectral widths agrees well with the rough mixing length arguments as do most tokamaks. In the analysis of turbulence in general (hydrodynamic included), it is noted that the density fluctuations are typically nearly gaussian, but that the difference, though small, is a crucial property of the dynamics of turbulence[18]. However, if only the "effects" of turbulence say on transport need to be evaluated then the Gaussian

\(^1\)As an aside, it is obvious that attempts to determine the chaotic dimension of plasma fluctuations from forward scattering, invariably produce effectively infinite dimensions because of the superposition of completely uncorrelated structures along the line of sight.
distribution approximation is adequate. Within the uncertainties associated with the finite space time window for obtaining the spectra, the observations on LT-4 are certainly consistent with the general form of Gaussian fluctuations spectra. An interesting result obtained on other machines[19] is that the spectral width $\Delta K$ appears independent of plasma temperature. However, one must take care with such measurements to properly account for the effect of the scattering volume on the spectral width, as clearly this would be independent of plasma parameters as well.

In general, typical features of turbulent fluctuations on other machines are reproduced in LT-4 and in particular, the phase scintillation interferometer has been used to provide single shot complete power spectra up to a maximum resolved wavenumber of 30 rad.cm$^{-1}$. Although single shot complete power spectra can also be obtained on Text using the FIR multichannel scattering diagnostic[20], the Phase Scintillation interferometer on LT-4 has been the first to also produce resolved images of these fluctuations[21] and to apply velocity space filtering techniques to try and separate independent structures along the line of sight.

It should be noted that the filtered data in the previous section bears a resemblance to poloidal probe array data obtained by Zweben[22] in searching for coherent structures in turbulent plasmas. As with the linear poloidal array of Zweben's, the identification of characteristic structures in the flow is very difficult and probably requires unjustified $a$ priori constraints such as the frozen flow hypothesis. Further investigation of coherent structures will not be perused here, however, as experiments by Zweben demonstrates [23], 2-dimensional arrays are essential when searching for coherent structures in strongly turbulent media.

6.5 Are Power Spectra Useful?

Spectral methods are useful in predicting the response of a linear system to random fluctuations. The basic result for the analysis of linear systems is that the power spectrum of the output is linearly related to the power spectrum of the input, and the precise nature of the distribution need not be known to compute its response.
On the other hand, as to whether a random process is adequately characterized by its power spectral density depends on whether coherent (repeated) structures exist in the medium. Consider for instance hydrodynamic turbulence, which is characterized by vortices appearing on many different scale sizes [18]. In this case, space time segments comparable to the size of the vortices may demonstrate a phase relationship between various spectral components indicative of the coherent structures. Such structures need not be sinusoidal and hence a Fourier decomposition (as is produced by scattering experiments) provides an inadequate basis for their identification.

A knowledge of the power spectrum but not of the phase spectrum is insufficient to recover the initial distribution function. Further, instantaneous spatially localized measurements of plasma density are necessary to identify such structures as they are typically randomly distributed in space and time. An imaging diagnostic would seem ideal to resolve these structures, however, the problem with both imaging and small angle scattering is the line integration possibly through many uncorrelated structures which also degrades information by making the measurement non-local.

By the central limit theorem, given any local distribution function, the distribution of a sum of these processes is approximately Gaussian provided certain reasonable conditions are met [14]. This means that line integrals through a thick enough turbulent field will produce a gaussian spectrum in both space and time. In my view, this is a partial explanation as to why small angle scattering experiments [24, 25] typically observe gaussian spectra. At the other extreme, by limiting the scattering volume so that one can obtain localized spectral information, the spectral resolution limit cuts. This only serves to emphasize the inadequacy of spectral techniques in characterizing turbulent media. Unfortunately, other techniques in signal analysis designed to identify randomly distributed characteristic structures in a medium are still in their infancy and generally require enormous amounts of computer time and a great deal of careful interpretation. Judging by the almost insurmountable problems and technological limitations experimentalists face today in extracting local information from fusion plasmas, the whole field of tomography
and the technology of remote sensing and data analysis still has enormous scope for improvement. In recent times, major advances have occurred in the study of hydrodynamic turbulence by focusing on the characteristic structures of the flow and not on its statistical (spectral) properties and diagnostics methods and analysis techniques have evolved together with that shift in emphasis[26]. The same will be true of plasma turbulence studies and even now, novel diagnostic methods are being developed to visualize the plasma flows.

A question which remains undecided is whether techniques which unambiguously observe a finite region of the plasma will be preferred over systems which extract local information by some tomographic procedure. In my opinion, at the present time, the computational overheads involved in tomographically reconstructing the plasma provides a very strong motivation for the development of techniques which instead produce spatially local measurements of the plasma interior.
Figure 6.8: Power spectra (taken from a single shot) recorded across the magnetic field lines at two radii (a) $r_0 = 0$ and (b) at $r_0 = 0.6a$ showing anisotropy in the measured spectrum.
Bibliography


