Distribution of Matter in Clusters of Galaxies

A thesis submitted for the degree of Doctor of Philosophy of the Australian National University

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Declaration of Authorship

The work presented in this thesis is entirely that of the candidate alone, except where indicated below, and where indicated in the main text.

Chapter 2: The original design of the observing strategy was obtained through discussion with Dr. Ravi Subrahmanyan and Dr. Ron Ekers.

Chapter 5: The AAT prime focus imaging for MS2137-23 was carried out under the guidance of Dr. Warrick Couch and Dr. Tom Broadhurst.

Haida Liang

[Signature]

January 1995
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To my family
Abstract

We present observations of the Sunyaev-Zel'dovich effect in 5 clusters of galaxies with a Fourier synthesis telescope – the Australia Telescope. A synthesis telescope has the advantage of simultaneously imaging the discrete radio sources and the Sunyaev-Zel'dovich effect. Observations were made at 8.8 GHz and in an ultra-compact 122.4m array configuration. Extended emission either due to radio halo sources or a blend of weak radio sources below the detection threshold, limited our ability to detect the SZ effect. However, upper limits to the SZ effect obtained for the cluster MS2137-23 was useful in constraining the cluster gas temperature when combined with the X-ray measurements and gravitational lensing constraints.

The advantages of a multi-wavelength analysis of cluster mass density distributions and the properties of the intra-cluster medium were demonstrated through the examples of MS2137-23 and Abell 2218. Both clusters possess a giant arc that constrains the cluster central mass. In the case of Abell 2218 where there is a large amount of good data available in various wave-bands, we combined gravitational lensing constraints, optical photometry, measurements of the Sunyaev-Zel'dovich effect, X-ray surface brightness and temperature to constrain the cluster potential, $H_0$ and test the usual assumptions of mass-follows-light and hydrostatic equilibrium.
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Chapter 1

Introduction

1.1 Sunyaev-Zel’dovich Effect

The Sunyaev-Zel’dovich effect (SZ effect) is the change in brightness temperature of the cosmic microwave background (CMB) radiation along the line of sight towards a dense cluster of galaxies (Sunyaev and Zel’dovich 1972). Clusters contain hot intracluster gas which emits X-rays by thermal bremsstrahlung radiation with luminosities of $10^{43-45}$ ergs/s. The temperature of this hot diffuse gas is typically $\sim 10^8$K and the electron number density is of order $\sim 10^{-3}$ cm$^{-3}$ (Sarazin 1986). The photons from the cosmic microwave background (CMB) radiation are scattered by the hot gas in a cluster which results in a change in brightness temperature of the CMB radiation. There are two forms of SZ effect resulting from the scattering: the thermal effect and the kinematic effect.

1.1.1 Thermal SZ effect

The Doppler effect due to the random thermal motion of the hot electrons in the intra-cluster medium (ICM) causes the CMB photons to change their frequencies after scattering off the electrons. For $h\nu \ll kT_e$, the scattering
process on average increases the photon energy (or frequency) and thus distorts the blackbody spectrum. If the thermal plasma is optically thin then multiple scattering can be neglected and the problem is reduced to the case of single scattering. Since scattering conserves the number of photons along the line of sight to us, on average the lower frequency photons are shifted to higher frequencies and the result is a decrease in intensity in the background radiation spectrum at low frequencies and an increase in intensity at high frequencies. Thus one should detect a ‘dip’ in brightness temperature in the Rayleigh-Jeans part of the CMB radiation spectrum in the direction of a cluster and an increase in brightness temperature in the Wien part of the spectrum (Figure 1.1). The expected change in brightness temperature is of order of one part in $10^4$ of the background radiation temperature. The inverse Compton scattering of a Planckian radiation field with that of a non-relativistic Maxwellian electron gas can be approximated by a Kompaneets equation\(^1\) that neglects the change in photon frequency due to recoil effect, if $h\nu \lesssim 10kT_r$ (Syunyaev 1980). The change in CMB intensity due to scattering by electrons is given by

$$\frac{\Delta I_\nu}{I_\nu} = y \frac{x e^x}{e^x - 1} \left( \frac{e^x + 1}{e^x - 1} - 4 \right)$$

(1.1)

where $x = h\nu/kT_r$, the comptonization factor $y$ is given by

$$y = \int \frac{kT_g(r)}{m_e c^2} \sigma_T n_e(r) dl$$

(1.2)

where $T_g$ is the gas temperature, $\sigma_T$ is the Thompson cross section, $l$ is the distance along the line of sight and $n_e$ is the electron number density (Syunyaev & Zel’dovich 1981). The change in intensity in the CMB radiation has a minimum at $\nu = 129$ GHz, it changes sign at $\nu = 218$ GHz and it has a maximum at $\nu = 371$ GHz (c.f. the unperturbed CMB spectrum peaks at $\nu = 161$ GHz). Figure 1.2 shows the change in CMB intensity along the line of sight to a cluster due to the thermal SZ effect. The corresponding increment in the radiation temperature is given by

$$\frac{\Delta T_r}{T_r} = \frac{\Delta I_\nu}{I_\nu} \frac{d \ln T_r}{d \ln I_\nu} = y \left( \frac{e^x + 1}{e^x - 1} - 4 \right)$$

(1.3)

\(^1\)Strictly speaking, it is only valid to apply the Kompaneets equation for an infinite, homogeneous medium that produces a large number of scatterings. This is not true in an intracluster medium where many photons go through unscattered. However, it was shown in Syunyaev (1980) that the Kompaneets equation is still a good approximation for $h\nu \lesssim 10kT_r$. 2
Figure 1.1: The CMB spectrum before (solid curve) and after scattering by a hot gas of $T_g = 7 \times 10^7$, optical depth of $\tau_T = 3.3$ to exaggerate the difference and peculiar velocity $V_r = 0 \text{ km s}^{-1}$ (dotted curve), $V_r = 3000\text{ km s}^{-1}$ (dashed curve).
where $T_r = 2.735 \pm 0.06$ K is CMB radiation temperature (Mather et al. 1990). Figure 1.3 gives $\Delta T_r$ as a function of frequency. The SZ effect is independent of redshift of the cluster since the CMB energy density is $\propto (1 + z)^4$ which cancels out with the usual factor of $(1 + z)^{-4}$ in surface brightness. In the Rayleigh-Jeans part of the spectrum, $x \ll 1$, and thus in the centimeter wave band, the decrement in brightness temperature tends to the limit of

$$\frac{\Delta T_r}{T_r} = 2y = 2 \int \frac{kT_x(r)}{m_ec^2} \sigma_T n_e(r) dl$$

Thus $\Delta T_r/T_r$ is independent of frequency in the Rayleigh-Jean part of the spectrum.

Measurement of the thermal SZ effect has a number of astrophysical implications:

- **Origin of CMB.** A detection would unequivocally demonstrate that the microwave radiation comes from behind the clusters; providing strong support for the cosmological origin of the microwave background radiation.

- **As a probe of cluster evolution.** The SZ effect is proportional to the pressure in the ICM integrated along the line of sight and is independent of distance. The distance independent nature of the SZ effect leads to the possibility of obtaining information on cluster gas density and temperature for clusters at any distance.

- **To determine $H_0$.** If we have data on X-ray temperature and surface brightness distributions of the hot gas as well as the profile of the SZ effect, then we can deduce the distance to the cluster directly, independent of any model assumptions about the cluster other than spherical symmetry and that the ICM is homogeneous on small scales (Silk and White 1978). The method goes as follows. The X-ray surface brightness distribution is given by

$$S_x(\theta) = (1 + z)^{-4} \int \Lambda_x(T_g(r)) n_e^2(r) dl$$

where $\Lambda_x(T_g)$ is the temperature dependent emissivity, $n_e(r)$ is the electron density and $l = \sqrt{r^2 - \theta^2 D_g^2}$ is the linear distance along the line of
Figure 1.2: Surface brightness as a function of frequency of the thermal SZ effect (solid curve) for an optical depth of $\tau_T = 0.03$ and $T_g = 7 \times 10^7$K, and kinematic SZ effect (dotted curve) for $V_r = 3000\text{km s}^{-1}$. The blackbody curve (dashed curve) of the CMB radiation scaled down 200 times is also plotted for comparison.
Figure 1.3: The thermal SZ effect (solid curve) for $\tau_T = 0.03$ and $T_s = 7 \times 10^7$ K, and the kinematic SZ effect for $V_r = 3000$ km s$^{-1}$ (dotted curve) and $V_r = -3000$ km s$^{-1}$ (dashed curve) in brightness temperature as a function of frequency.
sight with \( r \) and \( D_a \) being the radial distance from the cluster centre and the angular diameter distance (Weinberg 1972). The SZ effect written in a similar form gives

\[
\Delta T_\nu(\theta) = \int \Lambda_{sz}(T_g(r))n_e(r)dl
\]

(1.6)

where \( \Lambda_{sz}(T_g(r)) \propto T_g \). Since \( S_x \), \( \Delta T_\nu \) and \( T_g \) can be measured, there are only two unknowns in equations 1.5 and 1.6, \( n_e \) and the cluster linear size \( l \). Thus Hubble's constant \( H_0 \) can be deduced by assuming spherical symmetry (i.e. the cluster linear size along the line of sight is the same as that across the plane of the sky) and measuring the angular size and redshift of a nearby cluster. The uncertainties caused by the non-sphericity of a cluster can be overcome by observing a sample of clusters so that the non-sphericity of the clusters can be averaged out. Since \( S_x \propto <n_e^2> \) and \( \Delta T_\nu^2 \propto <n_e(r)^2> \), the main theoretical limitation to the method is the assumption that the gas is not clumped on small scales, i.e. \( C(r) = \frac{\sigma^2(r)}{\sigma^2(r)|r}| = 1 \). Apart from the intrinsic limitations in the assumption, the uncertainties in the \( H_0 \) measurement provided by this method is still large compared with the conventional methods owing to the difficulties of the observations involved. While the conventional determination of \( H_0 \) in the local universe using methods such as the Tully-Fisher relation yields a measurement of \( H_0 \) with a greater accuracy, the present method allows us to measure \( H_0 \) using clusters at large distances rather than just from the local universe, and thus eliminating systematic effects such as a local deviation from the universal expansion that affects the conventional estimates. Furthermore, the present method measures \( H_0 \) directly without any presumption about the universality of the "standard candles". The Hubble constant has recently been measured for the clusters Abell 2218 and Abell 665 by various groups using this method and the results are \( H_0 = 40 - 50 \pm 12 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Abell 665; Birkinshaw et al. 1991), \( H_0 = 65 \pm 25 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Abell 2218; Birkinshaw & Hughes 1994), \( H_0 = 24^{+13}_{-10} \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Abell 2218; McHardy et al. 1990) and \( H_0 = 38^{+18}_{-16} \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Abell 2218; Jones 1994).

- To determine \( q_0 \). If we have measurements of the S-Z decrement and
X-ray surface brightness and temperature for a sample of high redshift clusters \((z > 0.5)\), then the deceleration parameter \(q_0\) can be deduced using the method described above (Silk and White 1978).

- **To determine the distribution of the intra-cluster medium.** The SZ effect is dependent on the electron density \(n_e\) and the X-ray surface brightness is dependent on \(n_e^2\), so the SZ effect is more sensitive to the gas distribution outside the core of a cluster than the X-ray surface brightness. Thus the SZ effect can serve as an important complement to the X-ray measurements. For instance, efforts have been made to combine optical and X-ray data to calculate the mass fraction in gas for some well studied clusters like Coma. It was found that from 16% to 44% of the total cluster virial mass is in the form of gas, depending strongly on how the gas and total mass distribution varies with radius beyond 1-2 Mpc (Briel *et al.* 1992). This is important in terms of not only understanding the distribution of the ICM itself but also the dark matter content and the baryonic fraction in a cluster. It is found that the baryonic fraction within a 5 Mpc radius of the Coma cluster is uncomfortably large compared with the baryonic fraction deduced from primordial nucleosynthesis models for \(\Omega_0 = 1\) (White *et al.* 1993).

### 1.1.2 Kinematic SZ effect

If the cluster has a peculiar velocity (i.e. deviations from the Hubble flow) that has a radial component then the scattering of the CMB photons by the electrons, through the Doppler effect due to the bulk motion of the electrons in the ICM, would also cause a change in the brightness temperature of the CMB (Syunyaev & Zel'dovich 1981; Rephaeli & Lahav 1991). The kinematic SZ effect is given by

\[
\Delta T_r/T_r = -\frac{V_r}{c} \int \sigma_T n_e(r) dl
\]  

(1.7)

where \(V_r\) is the receding radial component of the peculiar velocity. Kinematic SZ effect \(\Delta T_r\) is independent of frequency (see Figure 1.3). Unlike the thermal effect, the sign of the kinematic effect is independent of frequency and is solely
dependent on the direction of the radial component of the peculiar velocity. Thus we can measure the radial component of the peculiar velocity of a cluster at any redshift provided that we can separate the kinematic and thermal effects. Note in Figure 1.2 that the intensity of the kinematic SZ effect is a maximum when the thermal effect is zero. This occurs at a frequency of 218 GHz. Thus by observing near 218 GHz we can hope to measure the radial component of the peculiar velocity of a cluster. This measurement can in turn be used to separate out the kinematic effects from the thermal effects at other frequencies. The kinematic SZ effect is significant compared with the thermal effect in the Rayleigh-Jeans limit if $V_r$ is of order $10^3$ km s$^{-1}$ or larger. Peculiar velocities of such magnitudes have been claimed for some nearby clusters using the Tully-Fisher (for spirals) or Faber-Jackson (for ellipticals) relations (Tully & Fisher 1977; Faber & Jackson 1976; Faber et al. 1989;). The advantage of the present method over the conventional methods, is the distance independent nature of the SZ effect which offers the possibility of measuring the peculiar velocity of a distant cluster. Furthermore, unlike the conventional methods the present method avoids any errors caused by the assumption of the existence of "standard candles". The disadvantages of the method are the possible contamination by a high velocity, gas rich galaxy in the cluster which could produce the same kinematic SZ effect, and the difficulty in detecting the effect for low peculiar velocity ($\lesssim 10^3$ km s$^{-1}$) clusters.

It is also possible to measure the tangential component of the peculiar velocity through the degree of linear polarization of the CMB caused by the transverse motion of the cluster (Syunyaev & Zel'dovich 1981). Another method for measuring the transverse motion of a cluster is the "moving lens" method where the cluster acts as a moving gravitational lens to the CMB radiation field and produces a characteristic two-sided brightness pattern (Birkinshaw & Gull 1983; Gurvits & Mitrofanov 1986).

The peculiar velocities of clusters are important probes of the large scale matter distribution. Bulk flows on scales $\sim 100$ Mpc provide an important test of the galaxy formation models such as the cold dark matter model when compared with the COBE results on CMB anisotropies (Dekel 1994).
1.2 Determination of the Cluster Mass Distributions

Extensive studies have been conducted to determine the mass distribution and dark matter content of individual galaxies. Despite long being suspected of harbouring vast quantities of hidden mass (Zwicky 1933), rich clusters of galaxies are still poorly understood objects in terms of their mass distributions. Only recently have detailed and independent estimates of their mass distribution become available; the mass-tracers used and the observational techniques employed can be summarised as follows:

- **Cluster Galaxies**: there have been a long tradition of using individual galaxies as probes to provide mass estimates via application of the Virial Theorem to the observed dispersion in their radial velocities. The method rests upon the assumption that all matter in the cluster is distributed like the galaxies and the galaxies themselves are in dynamical equilibrium.

- **Hot Intracluster Gas**: as well as being an important mass component of clusters, its X-ray emission provides an ideal tracer – through the hydrostatic equation – of the total underlying mass. The assumption that the gas is in hydrostatic equilibrium with the cluster’s gravitational potential is thought to be reasonably secure for the central few Mpc, and the gas density and temperature profiles required to solve the hydrostatic equation are readily available from the X-ray data. Furthermore, the properties of the gas can be even more tightly constrained from measurements of the SZ effect, in particular its clumpiness.

- **Gravitational Lensing**: here the lensing action of the cluster on background sources, as revealed in deep high resolution imagery (Tyson et al. 1990), is used to provide a direct measure of the shape and depth of the cluster potential and hence the projected mass distribution (Kaiser & Squires 1993, Broadhurst et al. 1994). Unlike the first 2 methods, this approach is not reliant upon assumptions of hydrostatic or dynamical
equilibrium.

From the small number of clusters analysed so far, there have been some interesting discrepancies found between these different methods. The central mass for some clusters (Abell 2218 and Abell 1689) deduced from gravitational lensing was a factor of 2-2.5 too large for the gas at the observed temperature to be in hydrostatic equilibrium (Miralda-Escude & Babul 1994). Additionally, the total mass-to-light ratios provided by the lensing method appear to be a factor of ~3 higher than those derived from virial analysis (Fahlman et al. 1994). On the other hand, Tyson et al. (1990) found in the case of Abell 1689 that the velocity dispersion inferred from modelling the lensing-induced distortions of objects behind the cluster was consistent with that measured for the galaxy population.

Rich clusters of galaxies are found to have ~ 30% of their mass in the form of intra-cluster gas and ~ 10% in galaxies. For most of the rich clusters, $M_{gal} < M_{gas} < M_{dark\text{matter}}$ (Sarazin 1992). Combined X-ray and optical studies of some rich clusters of galaxies, e.g. Perseus cluster (Eyles et al. 1991), Coma cluster (Hughes 1989; Watt et al. 1992; White et al. 1993; Briel et al. 1994), Abell 665 (Hughes & Tanaka 1992), Abell 85 and Abell 2199 (Gerbal et al. 1992), have reached the conclusion that the dark matter distributions are more centrally concentrated than the galaxies, which are in turn more concentrated than the gas distribution. This has led to the suggestion that the dark matter in clusters is dissipational and thus it is most likely to be baryons and unlikely to be weakly interacting particles (Sarazin 1992; Eyles et al. 1991; Sciama 1993). However, this suggestion has been challenge by Tsai (1993) who found that the increase of the ratio of baryons to dark matter in clusters was consistent with cold dark matter (CDM) simulations and thus with the dark matter being dissipationless, if processes of tidal stripping, merging and dynamical friction were taken into consideration.

The mass fraction of the luminous matter in a cluster and thus the baryonic fraction on scales $\lesssim 1\text{Mpc}$ has been found to be uncomfortably large compared with the baryonic density $\Omega_b$ predicted from the standard nucleosynthesis model if the Universe is at its closure density $\Omega_0 = 1$. For example, in the
Coma cluster, the baryonic fraction was found to exceed the value predicted from the nucleosynthesis model and the observed light element abundance by a factor $> (0.5h_0^2 + 3h_0^{1/2})\Omega_0$, where $h = H_0/50$ (White et al. 1993). While White et al. (1993) found that cooling and other dissipative effects cannot sufficiently enhance the baryonic fraction in clusters above the universal mean to eliminate the discrepancy, Babul & Katz (1993) found from their simulations that cluster total masses are underestimated and that baryons are more concentrated in clusters. Other numerical simulations based on hierarchical galaxy formation such as West & Richstone (1988) and Carlberg (1994) also found that the virial mass of clusters underestimates the cluster masses. However, recent simulations of Bromley et al. (1994) found that the virial mass can be an overestimate of the cluster mass.

The question of whether the high baryonic fraction in clusters poses a serious problem for the standard cosmological models of the Universe is, thus, still open until we have detailed mass estimates of many clusters and better understanding of the formation and evolution of structures. In fact, the study of cluster total mass function along with the cluster gas temperature has important implications for cosmological models of the formation of structure in the universe. For example, the very existence of a massive system like A2163 has already posed strong contraints on the bias parameter in the context of cold dark matter models of the universe with biased galaxy formation (Arnaud et al. 1992). Detailed multi-wavelength studies of a large sample of clusters will help us to resolve the discrepancies in the different mass estimates and to understand the nature of the dark matter in clusters.

### 1.3 Thesis Overview

Chapter 2 & 3 deals with our search for the SZ effect using the Australia Telescope (AT). Chapter 2 describes the observing strategy, cluster selection and data reduction techniques. Chapter 3 discusses the effects of radio source confusion and gives the results of the search for SZ effect. Self-consistent cluster mass models and methods to constrain the intracluster gas parameters
are discussed in Chapter 4. Chapter 5 gives an example of a multi-wavelength analysis of a cluster MS2137-23 using optical data from the Anglo-Australia Telescope, radio data on the SZ effect from the AT and X-ray data from the Einstein archive. This data is combined to constrain the parameters of the ICM and to estimate the masses of the various matter components. In Chapter 6, we analyse a well studied cluster Abell 2218 using published data in various wave-bands to address the problems of the discrepancy between the analysis of the cluster potential and total mass using different methods. Conclusions and future prospects are given in Chapter 7.

We assume $H_0 = 50\text{km s}^{-1}\text{Mpc}^{-1}$ and $q_0 = 0.5$ throughout the thesis, unless otherwise specified.

1.4 References


Chapter 2

The Search for the 
Sunyaev-Zel'dovich Effect I: 
Observing Strategy

2.1 Review of past observations of the SZ effect

Ever since the first suggestion of the SZ effect (Syunyaev & Zel'dovich 1972), attempts have been made to confirm it. The first "detection" was claimed by Parijskij (1972) in the Coma cluster but this was generally considered unlikely given the radio halo source in the centre of the cluster. Many attempts have been made in the 80's to detect this effect mainly using single dish radiometry (eg. Birkinshaw and Gull 1984, Birkinshaw 1986, Uson 1986). The earlier results were questionable, and there were inconsistencies between different experiments. Enormous amounts of observing time have been required (eg. 1 year observing for the OVRO group). The last couple of years have seen some break-throughs in obtaining detections of the effect in a much shorter observing time. For example, the Cambridge group has detected the effect in 5 clusters using the Ryle telescope (Jones 1994), the Berkeley group has detected the effect in A2163 at millimetre wavelengths with SUZIE and
the Caltech group has detected in the Coma cluster using the 5.5m telescope at Owens Valley (Herbig et al. 1995). Table 2.1 gives a summary of the observations of the SZ effect up to 1994. It is difficult to compare the measured values of the central decrement because of the way different observing methods, conditions and assumptions used affect the central SZ decrement (see section 2.3.1). Therefore, we will not give the results of the measurements in Table 2.1. However, a summary of the SZ temperature decrement measured by some of the groups is listed in Birkinshaw (1990). In the following section we will examine the various techniques and observing methods used for the detection of the SZ effect.

2.1.1 Single dish observations

Radiometers

Most single dish observations of the SZ effect were made using radiometers rather than bolometers. Beam-switching techniques are often used with the 2 beams symmetrically offset from the axis of the telescope. The “main” beam is pointed towards the cluster centre and the reference beam at a “blank” reference position. Ideally the difference in the antenna temperature between the two beams gives the SZ effect. There are, however, a number of problems associated with this technique. Firstly, at centimetre wavelengths, radio source confusion is significant at the low resolution and high sensitivity levels needed to detect the SZ effect. Any radio source or combination of sources falling in the reference beam would mimic the SZ effect, while sources in the main beam would diminish the SZ effect. High resolution imaging with an array is necessary to avoid the radio sources or to determine the positions and flux densities of the sources to subtract from the data. Secondly, the detection efficiency is strongly dependent on the apparent angular size of the cluster gas distribution. If the angular radius of the cluster is larger than the separation between the two beams then the reference beam is still inside the cluster and thus the SZ effect is underestimated. For example, in Figure 2.3 if the beam-throw is ~ 7' as in Birkinshaw et al. (1987) then the measured $\Delta T_r$ would only be 65% of the peak value. On the other hand, sensitivity is degraded
Table 2.1: Summary of Observations of the SZ effect

<table>
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<tr>
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<th>$\nu$</th>
<th>Method</th>
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<td>10.7</td>
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Col. (1) gives the cluster name; Col. (2) gives the frequency of the observations; Col. (3) gives the method: I - interferometry, B - bolometer, R - radiometer; Col. (4) gives the reference.
for clusters with an angular size much smaller than the beam size due to beam dilution. Finally, there are also systematic effects such as the reflection of the spillover signals from the ground, spillover of signals from one beam to the other and atmospheric effects that may not cancel between the two beams. Some of these systematic effects can be reduced by position-switching, i.e. alternating the pointing position of the main and reference beams. For example, position-switching in azimuth keeps the ground spillover constant. Despite all the problems, some of the first convincing detections of the SZ effect were eventually made with single dish radiometers (Birkinshaw et al. 1984; Uson 1987).

**Bolometers**

For observations at frequencies \( \geq 100 \) GHz, bolometers are used instead of radiometers since radiometers have high noise temperature at such high frequencies. The same beam-switching techniques are used, except in this case the beam separation has to be significantly reduced because of the limitation in the telescope optics and the scale of the atmospheric turbulence. The major difficulty in observations at high frequencies is the high atmospheric noise. Thus it is crucial to observe at a site where the seeing is good and the atmospheric transparency is high for as many days of the year as possible. While the radio source confusion is very much reduced at such high frequencies, the contribution from thermal galactic gas and dust becomes important. As it has been mentioned in the previous chapter, measurement of the SZ effect on the Wien side of the blackbody curve is important both in confirming the reality of the effect through its unique spectral feature and also in disentangling the kinematic and thermal SZ effect. Recently, the Berkeley group had the first detection of the effect both in decrement (at 2.2 mm) and increment (at 1.2 mm) for the cluster A2163, using SUZIE mounted on the 10m telescope of Caltech Submillimeter Observatory (Wilbanks et al. 1994).

### 2.1.2 Interferometry

Synthesis arrays measure components of the Fourier transform of the sky image corresponding to each spacing between pairs of antennas. Images are
reconstructed from these discretely sampled Fourier components. The most obvious advantage of an array is that it makes an image of the cluster field such that both the confusion sources and the SZ effect are imaged simultaneously. By exploiting the difference in the angular size of the discrete radio sources and the SZ effect, the radio sources measured using the long spacings in an array can be subtracted from the short spacing data which is most sensitive to the SZ effect. Since the flux densities of the radio sources are measured concurrently with the SZ effect, the source subtraction procedure in synthesis observations is not limited by the intrinsic variation of the radio source flux and the difference in flux calibration between, for example, an imaging array and a single dish. Furthermore, synthesis telescopes register only correlated signals, thus they are not sensitive to ground spillovers or atmospheric emissivity fluctuations and man-made interference is much reduced. The VLA has been used to observe SZ effect in A2218 at 5 GHz but without success (Partridge et al. 1987), but recently the Cambridge Ryle Telescope has detected the SZ effect in 5 clusters (Jones 1994) at 15.4 GHz. The major draw back in detecting the SZ effect with the currently available interferometers is the low efficiency (< 20% as compared to ~ 65% for the Owen's valley 40m telescope) due to the high resolution or long baseline. An ideal instrument for the detection of the SZ effect would be an array of many small antennas with low noise receivers that could be packed close together. As shown in Figure 2.6 the measured SZ effect in flux densities decreases rapidly as the baseline length increases for a given observing frequency.

2.2 The Australia Telescope

The Australia Telescope (AT) is a synthesis array consisting of 6 antennas with 5 of the antennas on a continuous 3 km east-west rail track (see The Australia Telescope 1992) and the 6th antenna on a short rail track 3 km away. In the ultra-compact configuration, the 5 antennas can be closely packed into a array with a spacing of 30.6 m between adjacent antennas. The AT achieves its optimum brightness sensitivity in such a configuration, which is a necessity for observations of the SZ effect. Continuum observations can be performed
simultaneously in the 2 linear polarizations with 32 frequency channels in a pair of 128 MHz wide observing bands. The AT operates at 21 cm, 13 cm, 6 cm and 3 cm. The highest observing frequency currently available at the AT of 8.8 GHz ($\lambda \sim 3$ cm) was selected for observations of the SZ effect, so that the effects from discrete non-thermal radio sources are minimised. Our first observation on the SZ effect was in July 1991 when the telescope had only one observing band available, but in the subsequent observations we had a pair of 128 MHz observing bands available.

2.3 Simulated observations with the AT

In this section, we discuss the feasibility of using the AT as an instrument for such a measurement. To find out how sensitive the AT would be for the detection of the SZ effect, we will simulate an observation of a real cluster with the AT.

2.3.1 Method

For our purposes, it is much easier to work in the visibility domain than the image domain for comparison of data with models and error analysis, since each visibility is independent and has well defined noise associated with it. In the image, each point has noise which depends on the spacing, gridding and weighting methods used in the image formation process and the pixels in the image are correlated, thus any error analysis in the image domain is complicated. The Fourier Transform is performed on the model to produce the noise free simulated data. It is then compared with the raw data. The same principle applies to convolutions and other processes. For this reason, in the case of single dish observations, it is difficult to compare the measured $\Delta T_r$ quoted in literature for various observations with each other or with a model because the "raw" measured $\Delta T_r$ is a convolution of the "true" decrement with the telescope beam. It is always best to perform as much mathematical operations on the noise free model than on the noisy data when making comparison of model with data. Synthesis telescopes collect data in the form of visibilities. Thus
if we can calculate the visibilities versus (u,v) spacings (or projected baselines in units of wavelengths) for the AT by assuming a certain cluster model then we can simulate observations with a certain AT configuration. In this section, the visibility versus spacings will be calculated for an observation with the AT under ideal conditions, ie. without noise. These visibilities are then compared with the expected noise for the AT given a certain amount of integration time.

The complex visibility of a source for any antenna spacing is in units of Janskys and it is defined to be the Fourier Transform of the brightness distribution multiplied by the primary beam of the antenna elements. The procedure to calculate the visibility function is described as follows:

- Calculate the intensity of the SZ effect $\Delta I_r$ from a model.
- Grid the intensity (or brightness) vs angular distance profile into a 2-D array ($512 \times 512$). In principle there is no need to work in 2-D if all clusters are spherically symmetric since it is simple to perform a Hankel transform on the 1-D profile. However, the programs developed are designed to be general such that it can be applied to asymmetric clusters.
- Multiply the grided brightness distribution by the antenna primary beam.
- Perform 2-D FFT on the resulting function. The resulting visibility function is circularly symmetric and represented by real numbers for clusters which are spherically symmetric.

The next step is to express the visibility in terms of (u,v) spacing in units which can be directly compared with observations. Visibility in mJy is related to spacings $s = \sqrt{u^2 + v^2}$ measured in wavelengths, via

$$V(i) = \Delta^2 \text{FT}^{(2)}[B_p \Delta I_r]$$  \hspace{1cm} (2.1)

and

$$s(i) = \frac{i - 1}{n\Delta}$$  \hspace{1cm} (2.2)

where $\text{FT}^{(2)}$ is a 2-D discrete Fourier transform, $B_p$ is the primary beam, $\Delta$ is the grid width expressed in radians, and $\Delta I_r$ is in units of mJy ster$^{-1}$. A
synthesis telescope like the AT can not sample the full \((u,v)\) plane, it can only sample the part of the \((u,v)\) plane corresponding to the spacings available. In order to simulate an observation with the AT, we need to sample the visibility function in the same way as the AT. The AT offers the highest brightness sensitivity at the shortest spacings (30.6m) for which the projected spacing changes with the hour angle (see Figure 2.1). For clusters with \(\delta > -50^\circ\), any data obtained with projected spacing < 22m are affected by the shadowing of a 22m antenna by its adjacent neighbour. As a result of "shadowing", the incident radiation onto the shadowed dish is reduced but at the same time spurious correlations may be produced because of the leakage of the radiation from one antenna to its shadowed neighbour. The errors in the data collected when shadowing occurs is complicated and difficult to correct, therefore it is best to exclude the data collected during "shadowing". In some cases, such spurious correlations (or cross-talk) may occur when the dishes are in "near-shadowed" positions. Some tests were done to find out if and when cross-talk occurs and it was found that cross-talk does not occur for the AT at 4.5 GHz and 8.7 GHz until the antennas are heavily shadowed (R. Subrahmanyan private communication). We can sample the \((u,v)\) plane as the hour angle changes in an observation, simulating how the AT collects the visibility data (see Figure 2.2). It is easiest to demonstrate the above procedure by considering a ‘real’ case.

### 2.3.2 A Simulated Observation of A2218

The cluster A2218 has been observed many times over the years for the SZ effect by a number of groups (e.g. Birkinshaw et al. 1984; Uson 1985; Partridge et al. 1987). The SZ effect has recently been reliably detected in this cluster by the Ryle telescope (Jones et al. 1993). Here we will take results from the Ryle Telescope which is an interferometer similar to the AT and simulate an observation with the AT. We will ignore the effects of radio source confusion in our simulation, but this important point will be addressed in section 2.7.

The profile of the SZ decrement on the sky shown in Figure 2.3 agrees well with the Ryle data for the cluster after subtraction of radio sources. The
Figure 2.1: Flux density of the SZ decrement versus hour angle for a cluster at $\delta = 0^\circ$ (solid curve), $-20^\circ$ (dotted curve), $-40^\circ$ (dashed curve), $-60^\circ$ (dot-dashed curve).
Figure 2.2: An example of the (u,v) tracks for an AT 122m array at 8.8 GHz. The tracks represent the parts of the (u,v) plane sampled by the AT for a source at $\delta \sim -23^\circ$. The missing parts of the ellipses in the north-south direction corresponds to the hour angles when the antennas are shadowed.
solid curve in Figure 2.4 is the Fourier transform of the surface brightness of the SZ decrement in Figure 2.3 multiplied by the primary beam of the Ryle telescope (13m antennas) at 15.4 GHz. The Ryle Telescope observations are shown by the crosses with error bars. We can now simulate an AT observation of a SZ decrement given in Figure 2.3 using the above methods for a cluster at a declination of −66°. The simulated AT data (open circles) is plotted in Figure 2.4 along with the Ryle results (crosses). Note the big difference between the AT simulated data and the Ryle results is mainly due to the difference in observing frequency. For comparison, the 1σ error bars plotted are for an observing time of 27×12hrs for both the Ryle data and the simulated AT data. Thus for the same amount of observing time, the AT can achieve a S/N of ~ 14 at 8.8 GHz in the shortest baseline as compared to a S/N of ~ 5 for the Ryle at 15.4 GHz. This is mainly because of the higher sensitivity of the AT receivers at 8.8 GHz compared to the Ryle receivers at 15.4 GHz. Also the AT at 8.8 GHz and 30.6m baseline is slightly lower in resolution than the Ryle at 15.4 GHz at its shortest baseline of 18m.

2.4 Observing Strategy

In this section, we will discuss in detail the optimum angular size or redshift of a cluster for the highest detection efficiency with the AT. Figure 2.5 shows the profile of the SZ decrement for 3 model clusters with the same gas properties as A2218 but at redshifts $z = 0.05, 0.3, 1.0$ assuming a Friedman-Lemaitre universe with $H_0 = 50$ and $q_0 = 0.5$. The visibilities in mJy versus projected baselines in number of wavelengths, calculated using the procedure described in section 2.3.1 for the same 3 redshifts is shown in Figure 2.6. The AT 30.6m spacings collect data in the range of projected baseline lengths between the two vertical lines shown in Figure 2.6. Thus the flux density of the SZ effect sampled by the shortest spacings of the AT is largest for $z = 0.3$ among the 3 different redshifts. The signal to noise ratio (SNR) in 60hrs of observing with the AT is plotted against the redshift for a cluster with the gas properties of

1A2218 is at $\delta \sim +66^\circ$
Figure 2.3: A model profile of the SZ decrement for A2218
Figure 2.4: Simulated and observed visibilities in mJy for A2218 versus interferometer baselines in wavelengths. The cross points are the flux densities measured by the Ryle. The solid curve is a model fit to the Ryle data observed at 15.4 GHz; the dotted curve is the simulated visibilities if it were observed at 8.8 GHz by the AT. (Note the difference between the solid and dotted curve is mainly due the difference in the observing frequency.) The circles are the simulated AT data at 30.6m, 61.2m, 91.8m and 122.4m baselines. The dot-dashed curve is the same as the dotted curve except that it takes into account the effect of the radio halo source at the centre of A2218. The error bars are all 1σ. Confusion noise due to weak radio sources has not been taken into account in the simulations.
A2218 in Figure 2.7. The S/N increases rapidly with redshift (since nearby clusters are too heavily resolved with the 30.6m spacing) and then becomes almost independent of redshift. From Figure 2.6, we can see the importance of observing at short baselines because of the steep rise of the visibility curve with baseline length. The optimum redshift or angular size of a cluster for the detection of SZ effect has a slight dependence on the shape of the SZ profile. The shape of the SZ effect is dependent on the gas temperature $T_g$ and the underlying gravitational potential. The higher the gas temperature, the broader the SZ profile.

2.5 Sources of Radio Emission

Radio sources in the field are one of the greatest annoyances to studies of the CMB. An incorrect subtraction of the radio sources can either mask or mimic the SZ effect. Thus only clusters that do not contain strong radio sources are suitable for the search of the SZ effect.

These radio sources consist of foreground and background sources and cluster radio sources. In order to separate the effects of these radio sources from the SZ effect, it is important to understand the nature and properties of these radio sources such as spectral indices and angular size. Summarised below are the properties of the different types of radio sources of concern here.

Radio Galaxies & Quasars
Radio galaxies and quasars dominate the high end of the radio luminosity function. Strong radio sources with luminosities of $10^{41} - 10^{46}$ ergs s$^{-1}$ identified with a galaxy, usually a giant elliptical galaxies, are generally referred to as radio galaxies. Radio galaxy sizes range from less than 10 pc to a few hundred kpc. Giant radio galaxies can have radio lobes stretching out to megaparsecs. The spectral indices of these radio galaxies are in the range $-1.3$ to $-0.5$ with a median of $\sim -0.8$. Compact radio sources ($\sim 1 - 100$pc) such as quasars have flat spectra ($> -0.5$; Kellerman & Owen 1988).

$^2$The spectral index $\alpha$ is defined as $S \propto \nu^\alpha$, where $S$ is the flux density and $\nu$ is the frequency. This definition will be used throughout the thesis.
Figure 2.5: SZ decrement versus the angular radius from cluster centre for 3 clusters of exactly the same gas properties but at different redshifts $z = 0.05$ (solid curve), $z = 0.3$ (dotted curve) and $z = 1.0$ (dashed curve).
Figure 2.6: SZ effect in flux densities versus the projected antenna spacings in wavelengths for the 3 clusters at $z = 0.05$ (solid curve), $z = 0.3$ (dotted curve) and $z = 1.0$ (dashed curve). The projected baseline of the shortest antenna spacings (30.6 m) of the AT at 8.8 GHz spans the range between the two vertical lines (dot-dashed lines).
Figure 2.7: Estimated SNR in 60hrs with the AT 30.6m baseline data versus redshift for clusters with the same intracluster gas properties.
Cluster Radio Sources

Clusters of galaxies have an abundance of elliptical galaxies, thus we should detect an excess of radio galaxies above the background towards the centre. This has been seen in a sample of nearby Abell clusters studied by Unewisse (1994), where a 33% excess radio sources above the background was found within a radius of 1 Mpc (<z> ~ 0.05) and above a flux limit of 5-10 mJy at 843 MHz. When we translate these numbers to a cluster at z ~ 0.3, we expect to find a total of 2–3 (including both cluster and field sources) sources within a 3' radius above 120μJy at 8.8 GHz. As we will see later in Chapter 3 that in our observations we find on average ~ 4 such sources in each cluster field. The clusters we have selected are at moderate redshifts (z ~ 0.2 – 0.5). What would the flux density of a modest radio galaxy be in such distant clusters? As an example, the flux density of a radio galaxy M87 at 8.8 GHz would be ~ 1mJy if it were at z ~ 0.3. M87 is 2 kpc in extent and thus it would have an angular size < 1" at a redshift of z ~ 0.3. The clusters we have selected for the observation of the SZ effect have no radio source above 3 mJy, thus we have excluded clusters that contain any high luminosity, giant radio galaxy and it is unlikely that any radio galaxy we find in these clusters would be extended on arcmin scales.

Radio Halo Sources

Diffuse non-thermal radio halo sources have been found in some clusters of galaxies such as A2256, A2255 and A2319 and notably the well known halo Coma C in the Coma cluster (Willson 1970; Kim et al. 1990; Harris et al. 1980; Bridle et al. 1976; and Harris et al. 1978). The characteristics of these diffuse cluster halo sources are their large size (typically cluster scale sizes, up to 1Mpc) and steep spectrum (typically α < -1; Giovannini et al. 1993; Sarazin 1986). These clusters with non-thermal halo emission are quite rare. A number of surveys to search for cluster halo sources have failed to find them (Jaffe et al. 1979; Cane et al. 1981; Andernach et al. 1981; Hanisch 1982). These halo sources tend to be associated with clusters of high X-ray luminosity and temperature (Vestrand 1982). The emission mechanism of these radio halos are thought to be synchrotron emission because of the power law spectrum and some indication of polarisation. The origin of these halo sources are still
uncertain and there are many different models proposed to explain the origin of these halo sources (Sarazin 1986; Jaffe 1991; Hanisch 1982b), but currently there is still no consensus. Because of their steep non-thermal radio spectra, they are easily distinguished from the SZ effect; but because the angular scales are similar they will be hard to separate accurately.

**Thermal Radio Emission**

Thermal Bremsstrahlung emission becomes significant if the cluster gas is sufficiently cold and dense (Tarter 1978; Schlickeiser 1991). It would be most significant in the centre of cooling flow clusters. For example, in Figure 2.8 we plot the thermal radio emission on top of the SZ decrement for a cluster with the central electron density of $n_e = 0.22 \text{ cm}^{-3}$ (this is denser than the average number of $10^{-2} - 10^{-3} \text{ cm}^{-3}$ but a likely value in the cores of cooling flow clusters) and temperature $T_g = 6 \times 10^7 \text{K}$. The thermal radio emission falls off rapidly with radius compared with the SZ effect since free-free emission is $\propto n_e^2$ and the SZ effect is $\propto n_e$. Since the angular scale of the radio emission is small ($\sim 15''$) compared with the SZ effect ($\sim 1'$), we can separate it using high resolution data. In the example given in Figure 2.8 the total flux of the thermal emission is $\sim 50 \mu\text{Jy}$ and the effect it has on the measurement of the decrement is shown in Figure 2.9.

**Field Radio Sources**

The clusters we have chosen for observations of the SZ effect contain no radio source with flux density $> 3\text{mJy}$ (see section 2.6), thus the field radio source population along the line of sight to a cluster is similar to those in the micro-Jansky source counts at 8.44 GHz by Windhorst et al. (1993) and at 5 GHz by Fomalont et al. (1991). Source counts at flux densities of Jansky or milli-Jansky levels are dominated by radio galaxies and quasars. However, below a few milli-Janskys, only a small fraction of the sources are radio galaxies or quasars. Most of these weak radio sources are identified with faint blue galaxies (Fomalont et al. 1991; Windhorst et al. 1987). They are thought to be low luminosity starburst galaxies, normal spirals or low luminosity elliptical galaxies.

Fomalont et al. (1991) found that the micro-Jansky sources ($16 - 1000\mu\text{Jy}$)
Figure 2.8: The profile of the SZ decrement and the thermal radio emission for a cluster with $n_e = 0.22 \text{ cm}^{-3}$ and $T_g = 6 \times 10^7 \text{K}$.
Figure 2.9: The flux density versus baseline length in wavelength units for just the SZ effect without taking into consideration of the thermal emission (solid curve) and SZ decrement plus the thermal emission (dotted curve). A 30.6m baseline corresponds to an average projected baseline of \(~ 800\) wavelength at 8.8 GHz.
at 5 GHz have a median spectral index of $\alpha \sim -0.38$ with 60% of the sources having $\alpha > -0.5$. The average angular size of these sources increase with decreasing flux density. For example, above 60$\mu$Jy most radio sources have angular size $< 1.5''$, but between 16 and 60$\mu$Jy the median angular size is 4''. Similarly, Windhorst et al. (1993) found that sources with flux densities in the range 14.5 to 1000$\mu$Jy at 8.44 GHz have spectral indices spanning the range $-1.3$ to 2 with a median of $\alpha \sim -0.35$. The median angular size of these sources is $\sim 2.6''$ with 40% of the sources with angular size $> 5''$.

### 2.6 Selection of Candidates

We attempt to select clusters that are most likely to produce a strong SZ effect that can be detected with the AT. The optimum selection criteria are:

(i) **high X-ray luminosity**: Since $L_x \propto n_e^2$ and $\Delta T_r \propto n_e$, a high $L_x$ implies a large $n_e$ which in turn gives a high $\Delta T_r$.

(ii) **absence of "strong" (> 3 mJy) radio sources** in the field at 3cm so that the error introduced due to subtracting sources are reduced.

(iii) **declination $\lesssim -50^\circ$** so that we have complete uv-coverage; The average projected baseline length decreases as the declination increases towards $0^\circ$, thus the brightness sensitivity increases for declinations closer to $0^\circ$ (see Figure 2.1). However, at the same time the uv-coverage and hence the image sidelobes become worse as the declination approaches $0^\circ$ because of "shadowing". Thus the optimum declination for the AT is $\delta \sim -50^\circ$.

(iv) **angular extent of the cluster gas is best matched to the AT synthesized beam**, i.e. the cluster needs to be relatively distant. For example, in the previous section it is shown that if all the clusters in the universe are like A2218 then the optimum cluster to observe would be one at $z \gtrsim 0.15$ (Figure 2.7).

Cluster candidates were selected from the Extended Medium Sensitivity Survey (EMSS) (Henry et al. 1992), Abell Catalogue (Abell et al. 1989) and the AAT Deep Cluster Catalogue (Couch et al 1991). Firstly, we checked for the presence of any radio source in the Parkes Catalogue and the Molonglo
Table 2.2: List of Clusters

<table>
<thead>
<tr>
<th>Cluster</th>
<th>RA</th>
<th>DEC</th>
<th>z</th>
<th>$L_x \times 10^{44}$ ergs/s</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS2137-23</td>
<td>21 40 12.8</td>
<td>−23 39 27</td>
<td>0.313</td>
<td>15.58 [0.3-3.5]keV</td>
</tr>
<tr>
<td>A2163</td>
<td>16 15 46.2</td>
<td>−06 08 46</td>
<td>0.201</td>
<td>60 [2-10]keV</td>
</tr>
<tr>
<td>A370</td>
<td>02 39 52.9</td>
<td>−01 34 38</td>
<td>0.373</td>
<td>9.7 [0.5-4.5]keV</td>
</tr>
<tr>
<td>A3444</td>
<td>10 23 50.4</td>
<td>−27 15 25</td>
<td>0.254</td>
<td>28.3 [0.1-2.4]keV</td>
</tr>
<tr>
<td>J1780.5BL</td>
<td>03 23 13.0</td>
<td>−51 05 24</td>
<td>0.49</td>
<td>&lt; 2 [0.1-2.4]keV</td>
</tr>
</tbody>
</table>

†Einstein IPC position
††Optical centre between the 2 D galaxies
†††ROSAT PSPC survey position
††††AAT plate position (Couch et al., 1991)

† This upper limit to $L_x$ was obtained from ROSAT all sky survey after the observation for the SZ effect had been made.

4. M. Pierre private communication

Radio Catalogue and select the clusters that have no known radio sources in the field. Next we observed the suitable candidates at the AT in snapshot mode at 6 and 3 cm for 1—2 hours and select the clusters that satisfy criterion (ii). Unfortunately, we could not find any cluster that satisfied all the above criteria since at the time there were no strong X-ray clusters known at $\delta < -50^\circ$ with reasonably large $z$. We decided to drop criterion (iii) and just adopt any cluster with $\delta < 0^\circ$. This decision had a number of unforeseen disadvantages as will be discussed later.

The clusters we selected to search for the SZ effect are MS2137-23, A2163, A370, A3444 and J1780.5BL. Some known properties of the candidate clusters considered are summarised in Table 2.2. Column 1 gives the cluster name: Column 2 & 3 gives the centre of the cluster in J2000.0 coordinates: Column 4 gives the redshift: Column 5 & 6 gives the X-ray luminosity and the references.
2.7 Observations and Techniques

2.7.1 DC Offsets

The detection of the SZ effect is also dependent on our ability to exclude any instrumental effects. Instrumental effects such as DC offsets manifest themselves as signals at the phase centre. To avoid any instrumental effect, the pointing centre for the primary beam was fixed at the cluster centre where we expect the SZ effect to be strongest, but the phase centre was shifted away from the pointing centre by 0.5 – 1° so that any instrumental effect will be shifted well outside the primary beam (5 to 10 primary beam width away from the field of interest) and thus attenuated and far from the position of interest. Such a large shift of the phase centre is made possible because the two 128 MHz continuum bands are divided into 32 channels, eliminating the off-axis bandwidth smearing. This procedure was tested by observing a calibrator source with the phase centre off and then on the pointing centre. The data for the “off” position was then processed with the AIPS task UVFIX to shift the phase centre back to the pointing centre, and compared with the data from the “on” position. We found after the shift the phase was behaving as expected and that with a shift of 1° on the sky at 8.7 GHz, and the flux density of the source at the pointing centre was only attenuated by ~ 4%. Any instrumental error occurring at the phase centre is shifted well away from the cluster and would be easily recognised. We checked for any errors at this position in the data processing stage and found ~ 3 – 4σ instrumental error at this position for some baselines.

2.7.2 Observations

Observations for the SZ effect at the AT are summarized in Table 2.3. Time was always shared between this project and the CMB anisotropy experiment (Subrahmanyan et al. 1993) on the AT 122.4m array. Because of the time sharing, we only obtained 8 hrs coverage instead of a full 12 hrs for J1780.5BL (July 1991 run) even though it can be observed for 12 hrs without shadowing.
Table 2.3: Summary of SZ effect Observations with the AT

<table>
<thead>
<tr>
<th>Date</th>
<th>Cluster</th>
<th>RA</th>
<th>DEC</th>
<th>Phase offset</th>
<th>Freq. MHz</th>
<th>Eff. time hrs</th>
<th>HA hrs</th>
</tr>
</thead>
<tbody>
<tr>
<td>July 91</td>
<td>J1780.5BL</td>
<td>03 23 12.95</td>
<td>-51 05 23.8</td>
<td>(-0.7,0.35)</td>
<td>8640</td>
<td>29</td>
<td>-4 to +4</td>
</tr>
<tr>
<td>Dec. 91</td>
<td>J1780.5BL</td>
<td>03 23 12.95</td>
<td>-51 05 23.8</td>
<td>(3.0,1.4)</td>
<td>(4800,4928)</td>
<td>10</td>
<td>-6 to +6</td>
</tr>
<tr>
<td></td>
<td>A2163</td>
<td>16 15 46.16</td>
<td>-06 08 45.5</td>
<td>(1.0)</td>
<td>(8640,8768)</td>
<td>35</td>
<td>-6 to +6</td>
</tr>
<tr>
<td>Nov. 92</td>
<td>MS2137-23</td>
<td>21 40 12.79</td>
<td>-23 39 26.9</td>
<td>(0,1)</td>
<td>(4544,4416)</td>
<td>10</td>
<td>-3 to +3</td>
</tr>
<tr>
<td></td>
<td>MS2137-23</td>
<td>21 40 12.79</td>
<td>-23 39 26.9</td>
<td>(0.0,5)</td>
<td>(8896,8768)</td>
<td>40</td>
<td>-3 to +3</td>
</tr>
<tr>
<td></td>
<td>A370</td>
<td>02 39 52.9</td>
<td>-01 34 38.3</td>
<td>(0,0.5)</td>
<td>(8896,8768)</td>
<td>28</td>
<td>-2 to +2</td>
</tr>
<tr>
<td></td>
<td>A3444</td>
<td>10 23 50.3</td>
<td>-27 15 27.0</td>
<td>(0,1)</td>
<td>(4544,4416)</td>
<td>9</td>
<td>-3 to +3</td>
</tr>
<tr>
<td></td>
<td>A3444</td>
<td>10 23 50.3</td>
<td>-27 15 27.0</td>
<td>(0.0,5)</td>
<td>(8896,8768)</td>
<td>37</td>
<td>-3 to +3</td>
</tr>
<tr>
<td></td>
<td>A2163</td>
<td>16 15 40.0</td>
<td>-06 08 45.5</td>
<td>(0,0.5)</td>
<td>(8896,8768)</td>
<td>0</td>
<td>-3 to +2</td>
</tr>
</tbody>
</table>

Col.(1) gives the date of the observation; Col.(2) gives the cluster name; Col.(3) & (4) gives the coordinates in J2000.0; Col.(5) gives the phase centre offset from the pointing centre in degrees; Col.(6) gives the observing frequencies; Col.(7) gives the effective integration time; Col.(8) gives the coverage in hour angle range.

of the antennas. For the same reason, we only obtained 5 hrs coverage instead of 6 hrs for A2163 and 4 hrs coverage instead of 6hrs for A370 in December 1991. We lost at least 20hrs observing time because of thunderstorms and correlator failures during the December 1991 run. The November 1992 observation of A2163 was badly affected by the Sun and was discarded. The source PKS1934-638 was used as a flux calibrator using the flux densities calculated from Kesteven (1990). The flux densities assumed for PKS1934-638 in the present analysis are 2.52 Jy, 2.46 Jy, 6.93 Jy and 6.68 Jy for 8.768, 8.896, 4.544 and 4.416 GHz respectively. Recently, Reynolds (private communication 1993) found that those flux densities for PKS1934-638 from Kesteven (1990) has been underestimated by ~ 10% at 8.8 GHz but overestimated by ~ 10% at 4.5 GHz. All the flux densities quoted in this thesis should be corrected accordingly, once the precise flux calibration for PKS1934-638 has been determined. A phase calibrator from either the AT catalogue or the VLA catalogue was observed every 20 to 30 mins for 5 mins. The phase calibrators used for J1780.5BL, A2163, MS2137-23, A370 and A3444 are 0308-61 (AT catalogue), 1615+02, 2135-20, 0238-08 and 1034-29 (VLA catalogue) respectively.
2.7.3 Point Source Subtraction

In order to achieve high brightness sensitivity, the AT dishes were arranged in the most compact configuration where the separation between the adjacent dishes was 30.6 m and the maximum baseline was 122.4 m. One of the major difficulties in detecting the SZ effect is the contamination of the signals by radio sources in the field. We observe the clusters at the highest available frequency of \( \sim 8.8 \) GHz at the AT in order to minimize the flux contribution from any confusing sources, since radio sources tend to have steep spectra and the primary beam is smaller at higher frequencies than lower frequencies. The median spectral index for micro-Jansky radio sources is \(-0.35\) at 8.4 GHz (Windhorst et al. 1993). To detect the SZ effect, we need to subtract all the confusing sources in the field. The effects of confusion from isolated point sources are relatively easy to deal with for an interferometer such as the AT which is capable of mapping both the radio sources and the SZ effect simultaneously. The SZ effect is expected to have a large angular extent and has the strongest signal at the shortest baselines. At longer baselines, the signal from the SZ effect decreases rapidly and thus the contribution from the SZ effect signal is expected to be negligible compared with the unresolved radio sources. We measure a larger fraction of the total decrement at a spacing of 30.6 m than at 61.2 m, 91.8 m and 122.4 m (Figure 2.6), whereas any unresolved radio source would have the same flux density at 30.6 m as 122.4 m. The radio source flux and position can be determined from an image synthesized with all the baselines in the 122.4 m array except the 30.6 m baseline to avoid any coupling between the SZ effect and the discrete sources.

Images constructed from these spacings between 61.2 and 122.4 m are still of very low resolution (\( \sim 40'' \) at 8.7 GHz) and also have high side-lobe levels for clusters with \( \delta > -50^\circ \) where shadowing limits the hour angle coverage. Tests conducted with the 122.4 m configuration AT data indicated two problems with this procedure: i) the very high sidelobes in the observations of clusters with \( \delta > -30^\circ \) give position ambiguities for the sources near the detection limit; ii) the low resolution beam still contained blends of weak sources. After this problem was recognized, we made additional proposals to obtain higher
resolution data at the same frequency for each cluster either at the AT or VLA (see Table 2.5). In principle, we need the high resolution data to be of higher point source sensitivity than the 61.2-122.4m baseline data, since we need to detect any underlying weak point source that might merge into one source in the low resolution data. The VLA has a much higher point source sensitivity than the AT even though the AT 122m array has an advantage over the VLA in surface brightness sensitivity. We observed some of the clusters with the VLA D-array and the others with the AT 1.5 km or 3 km array to detect and measure positions of the weak radio sources. By combining observations with the VLA which has high point source sensitivity with the AT 122m which has high brightness sensitivity, we can adopt the following procedure for the estimate of the radio source positions and flux densities.

Since the VLA has a higher point source sensitivity than the AT, it can be used to image the cluster point sources and determine the position of the sources in a relatively short amount of time. Accurate positions of the radio sources were then determined from the VLA image at 8.7 GHz. To avoid any error caused by the difference in flux calibration and image resolution, the variability of radio sources, we did not use the flux densities of the sources determined from the VLA data but instead used the source positions to constrain fits of the flux densities of the sources in the AT data with spacings in the range 61.2–122.4m. For the clusters A3444 and J1780.5BL where VLA data was not available, radio source positions were taken from observations with the AT at 4.5 GHz with a 3km or a 1.5km array and again the flux densities of the sources were fitted using all the data from the AT with 61.2–122.4m baselines. Fitting of point source models in the visibility domain is preferred to fitting sources in the deconvolved image, since it is less time consuming and avoids any error in the deconvolution process. Deconvolution algorithm such as the CLEAN algorithm (Högboom 1974) is a non-linear operation where it replaces a source and its side-lobe response with a “clean” source without any side-lobes. Simulations show that the CLEAN algorithm is particularly non-linear in images with high side-lobe responses when the noise level is approached (Formalont et al. 1993). The CLEAN algorithm also modifies the noise statistics in an image. For example, the $rms$ noise in a simulated Gaussian noise image
is decreased after the image has been deconvolved with \textit{CLEAN}, presumably because \textit{CLEAN} was able to redistribute the noise into the sources.

The \textit{AIPS UVFIT} program was made to fit up to 10 sources simultaneously by modifying the original Fortran code. The modified program was tested by fitting the source flux densities of a simulated field with 6 sources and random noise which closely resembles the flux densities and noise levels in our observations. Table 2.4 shows the simulated source flux densities (Col.1) and the flux densities found by \textit{UVFIT} (Col.2). The random noise added to the simulated field was 40\mu Jy and the positions of the sources were known and fixed in \textit{UVFIT}. The results of this test are consistent with the variation expected from the noise.

For all the clusters observed, the \textit{UVFIT} program was applied to the 61.2–122.4m spacing data with the positions of the radio sources obtained from the high resolution images as fixed parameters and the flux densities of the sources as free parameters. The sources are then subtracted from the full data set (all the 30.6-122.4m spacing data) using the AIPS \textit{UVSUB} program. The resulting image constructed from the the 30.6m spacing data alone should contain the SZ effect, thermal noise, and the residuals of source subtraction and confusion noise from any weak undetected sources.

The success of the point source subtraction procedure can be tested by examining the long baseline images, i.e. images constructed with all baselines in the 122.4m array excepted the shortest. As an example we will show the

<table>
<thead>
<tr>
<th>$S_{in}$ (mJy)</th>
<th>$S_{fit}$ (mJy)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.778</td>
<td>0.740±0.040</td>
</tr>
<tr>
<td>0.452</td>
<td>0.389±0.040</td>
</tr>
<tr>
<td>0.700</td>
<td>0.629±0.035</td>
</tr>
<tr>
<td>0.401</td>
<td>0.399±0.042</td>
</tr>
<tr>
<td>0.076</td>
<td>0.124±0.032</td>
</tr>
<tr>
<td>0.207</td>
<td>0.264±0.036</td>
</tr>
</tbody>
</table>
Figure 2.10: AT image of MS2137-23 at 8.8 GHz made with 61.2m, 91.8m & 122.4m baselines. The first contour is at 3σ. The image has not been deconvolved.

Image of MS2137-23 constructed with the AT data from the 61.2m, 91.8m and 122.4m baselines before and after the subtraction of point sources (see Figure 2.10 and 2.11). The rms noise of the resulting image (σ₀) is then compared with the thermal noise of the system (σₜₕ) and the confusion noise (σₑ) expected from the source counts. The thermal noise of the receiver system is estimated by taking the variance of the difference image of the 2 orthogonal linear polarizations. In all the cases, the thermal noise thus determined were
Figure 2.11: AT image of MS2137-23 at 8.8 GHz made with 61.2m, 91.8m & 122.4m baselines after subtraction of the point sources. The first contour is at 1σ. The image has not been deconvolved.
Table 2.5: Summary of High Resolution Observations

<table>
<thead>
<tr>
<th>Date</th>
<th>Cluster</th>
<th>RA</th>
<th>DEC</th>
<th>Config.</th>
<th>Freq.</th>
<th>(\sigma)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>h m s</td>
<td>° ′ ″</td>
<td></td>
<td>GHz</td>
<td>(\mu)Jy/beam</td>
</tr>
<tr>
<td>Aug. 92</td>
<td>A2163</td>
<td>16 15 46.16</td>
<td>-06 08 38.4</td>
<td>VLA D-array</td>
<td>8.4</td>
<td>70</td>
</tr>
<tr>
<td>Aug. 92</td>
<td>A370</td>
<td>02 39 52.9</td>
<td>-01 34 38.3</td>
<td>VLA D-array</td>
<td>8.7</td>
<td>20</td>
</tr>
<tr>
<td>Aug. 92</td>
<td>MS2137-23</td>
<td>21 40 08.8</td>
<td>-23 39 22.1</td>
<td>VLA D-array</td>
<td>8.7</td>
<td>23</td>
</tr>
<tr>
<td>May 93</td>
<td>MS2137-23</td>
<td>21 40 12.8</td>
<td>-23 39 26.9</td>
<td>AT 1.5km</td>
<td>4.5</td>
<td>80</td>
</tr>
<tr>
<td>Dec. 92</td>
<td>A3444</td>
<td>10 23 50.3</td>
<td>-27 15 27.0</td>
<td>AT 6km</td>
<td>4.8</td>
<td>30</td>
</tr>
<tr>
<td>May 93</td>
<td>J1780.5BL</td>
<td>03 23 12.8</td>
<td>-51 05 23.8</td>
<td>AT 1.5km</td>
<td>4.5</td>
<td>60</td>
</tr>
</tbody>
</table>

Col. (1) gives the date of observation; Col. (2) gives the cluster name; Col. (3) & (4) gives the RA and DEC of the pointing centre of the telescope in J2000.0 coordinates; Col. (5) gives the telescope and configuration; Col. (6) gives the frequency of observation and Col. (7) gives the \(\text{rms}\) noise in the map after deconvolution.

consistent with the expected instrumental noise estimated from the system temperature of the receivers. If the source subtraction has been successful then \(\sigma \sim \sqrt{\sigma_{th}^2 + \sigma_{c}^2}\). Table 2.6 lists \(\sigma\), \(\sigma_{th}\) and \(\sigma_{c}\) for both the image constructed from shortest baseline data and the image made with the 61.2–122.4m baseline data.

### 2.7.4 VLA and Higher Resolution AT Observations

VLA observations and “high resolution” AT observations used to determine the accurate positions of the point radio sources are summarised in Table 2.5. We observed MS2137-23 and A370 for 2 hrs using the VLA D-array. The VLA data for A2163 was obtained and calibrated by M. Jones et al.. The data on A2163 was badly affected by the weather. For the high resolution AT observations to achieve the same point source sensitivity as the 122m array observations, we needed the same amount of observing time which was not possible. Instead the high resolution AT data were taken in the 6cm band since the micro-Jansky radio sources at 4.8 and 8.4 GHz have a median spectral index of \(\sim -0.35\) and thus the detection efficiency is on average higher at 6cm than 3cm. This procedure is more risky because it would miss a weak inverted spectrum source.
2.7.5 Extended Sources

Diffuse Halo Sources

Diffuse non-thermal radio sources are known to exist in some clusters, thus we also need to confirm the absence of such sources in the clusters. We also made a short AT observation in the ultra-compact configuration at \( \sim 4.5 \) GHz to search for the presence of any diffuse non-thermal sources. Since diffuse non-thermal radio sources have very steep spectra (\( \alpha < -1 \) where \( S \propto \nu^\alpha \)) (Giovannini et al. 1993), whereas the SZ effect has an inverted spectra (\( \alpha = 2 \)), an observation at a lower frequency is much more sensitive to the halo source than the SZ effect. Since the size of the synthesized beam of the 30.6m baseline data at 8.8 GHz is roughly the same as that of 61.2m baseline data at 4.5 GHz, any non-thermal halo source will be much stronger at 4.5 GHz in a map made with 61.2m baseline data than that at 8.8 GHz with 30.6m baseline data; the SZ effect is, however, much stronger at 8.8 GHz with 30.6m baseline data than at 4.5 GHz with 61.2m spacings. Thus by observing the clusters at both 8.8 GHz and 4.5 GHz in the 122.4m array, we can hope to identify any contaminating halo sources.

Let us consider here the effect of possible non-thermal radio halo sources on our observations of the SZ effect at the AT. The best studied radio halo source “Coma C” has been imaged at frequencies from 0.031 GHz to 4.8 GHz (Giovannini et al. 1993 and references there in). The spectral index of the source was \(-0.8\) in the central 8′ region and \(-1.8\) in the surrounding areas. If we take a spectral index of \(-0.8\) for “Coma C” and extrapolate the flux densities to 8.8 GHz then the total flux of “Coma C” would be \( \lesssim 0.1 \) mJy if it were at a redshift of \( z \sim 0.3 \). The angular size of the halo would be \( \sim 3′ \) at \( z \sim 0.3 \), thus it would not pose a serious problem to the detection of the SZ effect. Furthermore, given the rarity of the detected non-thermal halo sources in clusters, radio halo sources were not originally considered to be of serious threat to our AT observation for the SZ effect. On the other hand, it has been speculated that the presence of a radio halo source is correlated with the existence of a strong SZ effect (Moffet et al. 1989) since all the 3 clusters A665, 0016+16 and A2218 for which there are confident detections of the SZ
effect possess an extended radio source near the cluster centre. The total flux densities of these halo sources are 7.06 mJy, 2.69 mJy and 2.79 mJy at 1.4 GHz for A665, 0016+16 and A2218 respectively. As an example, we have estimated the effect of the extended source in A2218 has on the detection of the SZ effect in A2218, if it were observed with the AT at 8.7 GHz (see Figure 2.4). The flux density of the extended source at 8.7 GHz was extrapolated from its flux density at 1.4 GHz and 4.8 GHz in Table III of Moffet et al. (1989; source # 17). We modelled the halo source as a Gaussian of 80'' FWHM with a total flux density of 0.13 mJy at 8.7 GHz. As we can see from Figure 2.4, the extended source is too weak to affect the detection of the SZ effect at 8.7 GHz.

Extended sources were found in every cluster that we had data at both 4.5 GHz and 8.8 GHz in a compact configuration with the AT. However, it is not always easy to distinguish halo emission from a blend of weak, discrete radio sources. We need not only low resolution data but also high resolution data of sufficient sensitivity to be certain that the emission is diffuse.

Weak Radio Sources

Discrete radio sources with a flux density below the sensitivity of the VLA image and the AT long baseline (longer than 30.6m baselines) images can blend into the large beam of a map made with 30.6m spacing data and have a significant effect on the detection of the SZ effect. The contribution from those radio sources too weak to be detected in any of the images, can be estimated from the field radio source counts at 8.44 GHz given by Windhorst et al. 1993. Extrapolation of the source counts down to 1.5 μJy was found to be consistent with the statistical analysis of noise fluctuations in their 10'' resolution image below the 14.5 μJy limit (see Fig. 3 in Windhorst et al. 1993). Contribution of the radio sources below 1.5 μJy to the rms noise in our maps is negligible since the slope of the source counts below 1 μJy must turn over and converge so as not to distort the CMB spectrum. Thus after subtraction of the point sources, the variance\(^3\) should be a combination of the thermal variance \(\sigma_{th}^2\)

\(^3\)All the rms noise quoted for the images are in fact \(\sqrt{\text{variance}}\). Software packages such as AIPS quotes rms noise when in fact it is \(\sqrt{\text{variance}}\) that is calculated. The difference,
from the instrument and the confusion variance $\sigma_c^2$ from the radio sources in
the field. The confusion noise is composed of any residual noise from the source
subtraction process and the noise contribution from the weak sources too weak
to be detected individually.

We estimate the confusion noise in an image using the method described
in Subrahmanyan et al. (1993). The noise contribution due to radio sources
varies over the primary beam. The expected confusion noise at an angular
distance of $(\theta_0, \phi_0)$ from the centre of the primary beam is given by

$$\sigma_c^2(\theta_0, \phi_0) = \int_S \int_\theta \int_\phi S^2 B_{sp}(\theta, \phi) n(S) dS d\theta d\phi$$  \hspace{1cm} (2.3)

where $B_{sp}(\theta, \phi)$ is the flux contribution of a source with unit flux at $(\theta, \phi)$ to
a pixel at the position $(\theta_0, \phi_0)$ in the image. It is given by

$$B_{sp}(\theta, \phi) = B_s(\theta - \theta_0, \phi - \phi_0) \times B_p(\theta, \phi)$$  \hspace{1cm} (2.4)

where $B_s$ is the synthesized beam and $B_p$ is the primary beam. The differential
source counts $n(S)$ at 8.44 GHz and over the flux density range of 14.5 to 1000
$\mu$Jy is given by

$$n(S) = dN(S)/dS = (24.6 \pm 3.7) S_{\mu Jy}^{-2.3\pm0.2} \text{arcmin}^{-2}$$  \hspace{1cm} (2.5)

(Windhorst et al. 1993) and the differential source counts at 4.86 GHz over
the flux range of 16 to 1000 $\mu$Jy is given by

$$dN(S)/dS = (27.4 \pm 3.3) S_{\mu Jy}^{-2.18\pm0.19} \text{arcmin}^{-2}$$  \hspace{1cm} (2.6)

We take the detection threshold of sources in the VLA and AT “high reso-
lution” images to be $4\sigma$, where $\sigma$ is the $rms$ noise in the deconvolved image
(see Table 2.5). Thus we take this flux threshold as the upper limit in flux
densities for the calculation of $\sigma_s^2$. The confusion noise $\sigma_s(\theta_0, \phi_0)$ varies with
the position in the image relative to the primary beam centre. The confusion
noise $\sigma_c$ and the $rms$ noise in the actual image $\sigma_o$ are the average over the
primary beam.

\begin{hyp}{n}
however, is very small in a synthesis image. The mean of an infinitely large image from a
synthesis telescope should be zero since synthesis telescopes do not collect zero spacing data.
\end{hyp}
Table 2.6: Summary of Results of Radio Source Subtraction

<table>
<thead>
<tr>
<th>Cluster</th>
<th>$\sigma_{61-122m\mu Jy/beam}$</th>
<th>$B_t$</th>
<th>$\sigma_{30m\mu Jy/beam}$</th>
<th>$B_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\sigma_h$ $\sigma_c$ $\sigma_f$ $\sigma_o$</td>
<td>$\sigma_h$ $\sigma_c$ $\sigma_f$ $\sigma_o$</td>
<td>$\sigma_h$ $\sigma_c$ $\sigma_f$ $\sigma_o$</td>
<td>$\sigma_h$ $\sigma_c$ $\sigma_f$ $\sigma_o$</td>
</tr>
<tr>
<td>J1780.5BL</td>
<td>34 28 44 43 ± 5</td>
<td>44 $''$ × 40 $''$</td>
<td>37 68 77 146 ± 45</td>
<td>115 $''$ × 104 $''$</td>
</tr>
<tr>
<td>MS2137-23</td>
<td>31 42 52 35 ± 7</td>
<td>168 $''$ × 27 $''$</td>
<td>35 87 88 96 ± 44</td>
<td>414 $''$ × 65 $''$</td>
</tr>
<tr>
<td>A370</td>
<td>39 58 70 101 ± 28</td>
<td>3721 $''$ × 27 $''$</td>
<td>49 94 106 165 ± 73</td>
<td>8780 $''$ × 71 $''$</td>
</tr>
<tr>
<td>A3444</td>
<td>39 49 63 79 ± 14</td>
<td>156 $''$ × 27 $''$</td>
<td>39 103 110 159 ± 72</td>
<td>367 $''$ × 72 $''$</td>
</tr>
<tr>
<td>A2163</td>
<td>49 81 95 68 ± 18</td>
<td>733 $''$ × 26 $''$</td>
<td>59 141 153 700 ± 310</td>
<td>1646 $''$ × 71 $''$</td>
</tr>
</tbody>
</table>

Note in the above estimate of confusion noise due to weak radio sources below the 4$\sigma$ detection limit, we have not taken into account of the increase of the number of radio sources due to the presence of a cluster and it was also assumed that the subtraction of the detected discrete radio sources is perfect. Furthermore, this estimate has not included the contribution from weak diffuse radio sources. More radio sources are found in the VLA images which were of similar resolution to the images of Windhorst et al. than that predicted by the field source counts, indicating the presence of a cluster. The contribution from the weak radio sources has to be adjusted accordingly to reflect the presence of a cluster. From the studies of Unewisse (1994), we estimate a 33% excess of radio sources within a 3$'$ radius and above a flux limit of 120$\mu$Jy at 8.8 GHz for a cluster at $z \sim 0.3$. We modify the above field counts by 33% in the normalisation factor when calculating $\sigma_c$ in Table 2.6 since $\sigma_c$ was estimated within the primary beam (i.e. 3$'$ radius).

2.8 References


Chapter 3

The Search for the Sunyaev-Zel'dovich Effect II: Results

In this chapter, we present the results on the observations of the SZ effect for each individual cluster.

3.1 MS2137-23

The cluster MS2137-23 was first discovered in the Einstein Extended Medium Sensitivity Survey (Stocke et al. 1991; Henry et al. 1992). It has a high X-ray luminosity and was later discovered to have both a giant arc and a radial arc which shows that the cluster is massive and has a sharp core. We observed this cluster for 10 days at 8.8 GHz with the ultra-compact AT 122m array. Due to shadowing of the antennas at large hour angles at the declination of this cluster, the hour angle coverage was from $-3$ to $3$ hrs each day. We also observed this cluster with the VLA D-array at 8.7 GHz for 2 hrs in August 1992 and the rms noise in the deconvolved image was $23\mu$Jy at a resolution of $8.1'' \times 17.3''$. The positions of the sources 1, 2, 3, 4 and 5 were determined from the VLA image in Figure 3.1 and used to fit the flux densities in the AT data with 61 - 122m baselines (see Table 3.1 for results of the fit). The cluster was
Table 3.1: List of Radio Sources in MS2137-23

<table>
<thead>
<tr>
<th>Source</th>
<th>RA</th>
<th>DEC</th>
<th>Δα</th>
<th>Δδ</th>
<th>S_{8832}</th>
<th>S_{4480}</th>
<th>S_{8832}^\prime</th>
<th>S_{4480}^\prime</th>
<th>α_{8832-4480}</th>
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<tr>
<td>1</td>
<td>21</td>
<td>40</td>
<td>15.2</td>
<td>-23</td>
<td>39 40</td>
<td>33</td>
<td>0.81</td>
<td>0.88</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>21</td>
<td>40</td>
<td>12.4</td>
<td>-23</td>
<td>39 40</td>
<td>-6</td>
<td>-13</td>
<td>0.45</td>
<td>0.45</td>
</tr>
<tr>
<td>3</td>
<td>21</td>
<td>40</td>
<td>13.2</td>
<td>-23</td>
<td>42 14</td>
<td>6</td>
<td>-167</td>
<td>0.12</td>
<td>0.22</td>
</tr>
<tr>
<td>4</td>
<td>21</td>
<td>40</td>
<td>17.9</td>
<td>-23</td>
<td>36 31</td>
<td>71</td>
<td>176</td>
<td>0.39</td>
<td>0.88</td>
</tr>
<tr>
<td>5</td>
<td>21</td>
<td>40</td>
<td>02.3</td>
<td>-23</td>
<td>39 17</td>
<td>-145</td>
<td>190</td>
<td>0.26</td>
<td>1.0</td>
</tr>
<tr>
<td>C1</td>
<td>21</td>
<td>40</td>
<td>24.6</td>
<td>-23</td>
<td>42 39</td>
<td>162</td>
<td>-192</td>
<td>0.52</td>
<td>2.3</td>
</tr>
<tr>
<td>C2</td>
<td>21</td>
<td>40</td>
<td>24.3</td>
<td>-23</td>
<td>44 13</td>
<td>158</td>
<td>-286</td>
<td>0.11</td>
<td>1.7</td>
</tr>
<tr>
<td>C3</td>
<td>21</td>
<td>40</td>
<td>23.6</td>
<td>-23</td>
<td>44 10</td>
<td>148</td>
<td>-283</td>
<td>0.96</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Col.(1) gives the sequence number of the sources; Col.(2) & (3) gives the RA and DEC of the sources in J2000.0 coordinates; Col.(4) & (5) gives the angular distance from the source to the pointing centre of the AT 122m array observations (also the X-ray centre of the cluster); Col.(6) & (7) gives the flux densities at 8.8 GHz before and after the primary beam correction (i.e. apparent and true flux densities); Col.(8) & (9) gives the apparent and true flux densities at 4.5 GHz; Col.(10) gives the spectral index between 8.8 GHz and 4.5 GHz.

Table 3.1 also observed at 4.5 GHz with the AT in a 1.5km configuration to check for any radio source just outside the VLA 8.7 GHz primary beam (see Figure 3.2). This was important for MS2137-23 where the VLA observation was centred (−54.9", 4.8") away from the pointing centre of the AT image which leaves sources C1, C2 and C3 well outside the VLA primary beam and were thus undetected in the VLA image. The contour plot of the AT image is overlayed on top of an optical I-band image in Figure 5.3. The point sources were then subtracted from the AT 122m data at 8.8 GHz. The rms noise in the final images $\sigma_o$, with all the point sources listed in Table 3.1 subtracted, is compared with the expected noise $\sigma_f$ given in Table 2.6 (from both the thermal noise $\sigma_{th}$ from the instruments and the confusion noise $\sigma_c$). It shows that $\sigma_o$ and $\sigma_f$ in the 61-122m baseline images agree well with each other and thus the point source subtraction process has been successful. There was some residual signal in the 30m baseline image, i.e. (u,v) spacing range of 500 to 950 wavelengths, as shown in Figure 3.3. We need extra information to find out whether the excess emission is due to some extended emission not detected in the higher

---

1The VLA observation was centred on the X-ray position quoted in Henry et al. 1992, and the AT observation was centred on the position quoted in Fort et al. 1992.
Figure 3.1: A VLA image of MS2137-23 at 8.7 GHz in the D-array configuration. The first contour is $3\sigma$. The cross gives the cluster centre.
Figure 3.2: AT image of MS2137-23 at 4.5 GHz in a 1.5km configuration. The first contour is $3\sigma$. The cross gives the cluster centre.
resolution images or the SZ effect.

We also observed the cluster at 4.416 GHz/4.544 GHz in the 122m configuration to check for the presence of any diffuse radio source. The flux densities of the point sources were fitted using the data of 91-122m baselines (see Table 3.1). The 4.5 GHz image synthesized with the data in the (u,v) spacing range of 500 to 950 wavelengths is shown in Figure 3.4 on the same scale as the 8.8 GHz image (Figure 3.3). The structures in both images agree with each other, indicating the presence of some extended radio emission at 21 40 23.7 – 23 40 57 (J2000) not detected in the high resolution images. The superposition of a 4.5 GHz image at a lower resolution (see Figure 3.5) using data with only the 30m baselines, i.e. (u,v) spacing < 500 wavelength, with the above images (Figure 3.3 and 3.4) show that the structure seen in the image was mainly due to some extended radio emission rather than the SZ effect since we see positive flux at the cluster centre in the low resolution image where we would expect a decrement due to SZ effect. This radio emission can be either a diffuse radio source or a blend of a few discrete weak radio sources. The flux density of the extended source quoted in Table 3.7 is a maximum since the position of the source is such that its first negative sidelobe falls right at the centre of the cluster where one expects the SZ effect to be strongest (see Figure 3.3), and by fitting just the radio source the fitting routine would inevitably maximise the flux of the radio source and minimising any SZ effect present. The problem of separating the flux densities of the extended source from that of the SZ effect is difficult because of the high sidelobes of the synthesized beam due to limited (u,v) coverage. Thus we can only place an upper limit on the SZ effect at the cluster centre. Figure 3.6 shows the visibility in mJy at the cluster centre as a function of (u,v) spacings or interferometer baselines at 8.8 GHz after the subtraction of the point sources listed in Table 3.1 (see Chapter 5 for more detailed analysis).
Figure 3.3: A contour map of MS2137-23 imaged with the AT 122m array at 8.8 GHz with only data from the 30m baseline (i.e. (u,v) spacing from 500 to 950 wavelengths). The cross marks the cluster centre.
Figure 3.4: A contour map of MS2137-23 imaged with the AT 122m array at 4.5 GHz with only data from the 61m baseline (i.e. (u,v) spacing from 500 to 950 wavelengths). The cross marks the cluster centre.
Figure 3.5: A contour map of MS2137-23 imaged with the AT 122m array at 4.5 GHz with only data from the 30m baseline (i.e. (u,v) spacing < 500 wavelengths). The cross marks the cluster centre.
Figure 3.6: The flux density versus baselines for MS2137-23 at the cluster centre (position of the cD galaxy) at 8.8 GHz after the subtraction of discrete point sources. The error bars are 1\sigma.
Table 3.2: List of Radio Sources in A370

<table>
<thead>
<tr>
<th>Source</th>
<th>RA</th>
<th>DEC</th>
<th>$\Delta \alpha$</th>
<th>$\Delta \delta$</th>
<th>$S_{8832}$</th>
<th>$S_{8832}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>02 39 55.4</td>
<td>-01 34 07</td>
<td>37</td>
<td>31</td>
<td>1.7</td>
<td>1.8</td>
</tr>
<tr>
<td>2</td>
<td>02 39 56.4</td>
<td>-01 34 29</td>
<td>53</td>
<td>9</td>
<td>0.50</td>
<td>0.54</td>
</tr>
<tr>
<td>3</td>
<td>02 39 56.9</td>
<td>-01 35 43</td>
<td>-30</td>
<td>-65</td>
<td>0.16</td>
<td>0.19</td>
</tr>
<tr>
<td>4</td>
<td>02 40 00.5</td>
<td>-01 36 28</td>
<td>114</td>
<td>-109</td>
<td>0.25</td>
<td>0.45</td>
</tr>
<tr>
<td>5</td>
<td>02 39 56.3</td>
<td>-01 31 37</td>
<td>51</td>
<td>181</td>
<td>0.15</td>
<td>0.32</td>
</tr>
<tr>
<td>6</td>
<td>02 39 56.3</td>
<td>-01 35 00</td>
<td>3</td>
<td>-22</td>
<td>0.19</td>
<td>0.19</td>
</tr>
<tr>
<td>7</td>
<td>02 40 25.4</td>
<td>-01 30 26</td>
<td>488</td>
<td>255</td>
<td>0.13</td>
<td>7.7</td>
</tr>
</tbody>
</table>

3.2 A370

A370 was the first cluster to have a giant arc detected and associated with gravitational lensing effects (Soucail et al. 1987; Kovner 1988). It has 2 central dominant galaxies and the galaxy distribution clearly shows two clumps. It has a high X-ray luminosity (see Table 2.2) and temperature ($T_g \sim 10^8$K; Bautz et al. 1994) and recent ROSAT observations also show a double structure like that found in the optical image (Böhringer et al. 1994). We observed this cluster for 10 days at 8.8 GHz with the AT in the 122m array configuration. The hour angle coverage on this cluster was $-2$ to 2 hrs. We again observed this cluster with the VLA D-array at 8.7 GHz for 2 hrs and the $rms$ noise in the deconvolved image was 19$\mu$Jy at a resolution of $9.9'' \times 8.6''$ (see Figure 3.7). The radio source positions from the VLA image and the 8.8 GHz flux densities from the AT 61-122m baselines are shown in Table 3.2. The $rms$ noise in the image made with all the baselines between 61-122m after the subtraction of the discrete point sources was consistent with the expected noise $\sigma_f$ (see Table 2.6). Thus the source subtraction procedure has been successful. This cluster is at $\delta \sim -02^\circ$ hence the side-lobe levels are very high ($\sim 99\%$) and it is thus difficult to distinguish a main lobe of a source from its sidelobes. We took 4.5 GHz data on the 122m array but the data was corrupted due to some correlator problems.

Figure 3.8 shows the flux density at the centre of the cluster after the subtraction of the discrete sources versus baseline lengths in units of wavelengths.
Figure 3.7: A VLA image of A370 at 8.7 GHz in the D-array configuration. The first contour corresponds to $3\sigma$ level. The cross gives the cluster centre.
There is excess emission in the 30m baseline image after the subtraction of the point sources. Since $\sigma_o$ from the image made with 30m baseline agrees well with $\sigma_f$ in Table 2.6, the excess emission may just be due to the weak sources in the very large beam near $\delta \sim 0^\circ$. A plot of the flux densities versus baseline length at the optical centre of the cluster is shown in Figure 3.8.
Table 3.3: List of Radio Sources in A2163

<table>
<thead>
<tr>
<th>Source</th>
<th>RA</th>
<th>DEC</th>
<th>$\Delta \alpha$</th>
<th>$\Delta \delta$</th>
<th>$S_{8704}$</th>
<th>$S_{8704}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h : m : s</td>
<td>° : ' : '&quot;</td>
<td>° : ' : '&quot;</td>
<td>° : ' : '&quot;</td>
<td>mJy</td>
<td>mJy</td>
</tr>
<tr>
<td>1</td>
<td>16 : 15 : 43.3</td>
<td>-06 : 08 : 44</td>
<td>-44</td>
<td>4</td>
<td>1.5</td>
<td>1.6</td>
</tr>
<tr>
<td>2</td>
<td>16 : 15 : 45.7</td>
<td>-06 : 08 : 02</td>
<td>-7</td>
<td>46</td>
<td>0.13</td>
<td>0.13</td>
</tr>
<tr>
<td>3</td>
<td>16 : 15 : 43.8</td>
<td>-06 : 06 : 30</td>
<td>-35</td>
<td>-138</td>
<td>0.91</td>
<td>1.4</td>
</tr>
<tr>
<td>4</td>
<td>16 : 15 : 41.2</td>
<td>-06 : 09 : 09</td>
<td>-74</td>
<td>-21</td>
<td>0.074</td>
<td>0.085</td>
</tr>
<tr>
<td>5</td>
<td>16 : 15 : 48.2</td>
<td>-06 : 11 : 02</td>
<td>30</td>
<td>-135</td>
<td>0.46</td>
<td>0.71</td>
</tr>
</tbody>
</table>

3.3 A2163

This is the hottest cluster known with X-ray temperature of $T_g \sim 1.7 \times 10^8$ K and X-ray luminosity of $L_x[2-10keV] \sim 6 \times 10^{45}$ ergs s$^{-1}$ (Arnaud et al. 1992). The AT observation was centred on the Einstein IPC position. We observed the cluster with the AT 122m array in December 1991 and November 1992. The November 1992 data was affected by the Sun and therefore not included here. This cluster was observed by M. Jones et al. with the VLA D-array at 8.4 GHz in August 1992. The data was kindly provided by Mike Jones. The VLA image is shown in Figure 3.9 where the $rms$ noise in the image was 70$\mu$Jy at a resolution of 11.6" x 8.7". The VLA observation was centred 7.1" north of the AT pointing centre. The positions of the radio sources from the VLA image and the 8.8 GHz flux density from the AT image with data from the 61-122m baselines are listed in Table 3.3. From Table 2.6, we see that $\sigma_0$ and $\sigma_f$ agree well in an image made with data from 61-122m baselines after the subtraction of point sources. However, there was a large amount of residual flux left in the 30m spacing image at 8.8 GHz after source subtraction (see Figure 3.12). Further observations at 1.3 and 2.3 GHz in a 1.5km array revealed an extended halo source at the centre of the cluster (see Figure 3.10 and 3.11). The primary beam corrected flux densities and radio luminosities of the halo source at various frequencies are summarised in Table 3.4. The radio luminosity of this halo source is much stronger than Coma C ($1.2 \times 10^{25}$ W Hz$^{-1}$ m$^{-2}$ as compared to $1.2 \times 10^{24}$ W Hz$^{-1}$ m$^{-2}$ (Giovannini et al. 1993) for Coma C). The halo source totally masks out any SZ effect present in the cluster. Since the centroid of the halo source agrees well with the cluster X-ray
Figure 3.9: A VLA image of A2163 at 8.4 GHz in the D-array configuration. The first contour corresponds to $3\sigma$ level.

Table 3.4: Properties of the Radio Halo Source in A2163

<table>
<thead>
<tr>
<th></th>
<th>8.8 GHz</th>
<th>2.3 GHz</th>
<th>1.3 GHz</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S$ (mJy)</td>
<td>1.1</td>
<td>27.2</td>
<td>65.5</td>
</tr>
<tr>
<td>size (arcmin$^2$)</td>
<td>32</td>
<td>68</td>
<td>79</td>
</tr>
<tr>
<td>$L_{\text{radio}}$ ($\times 10^{23}$ W Hz$^{-1}$ m$^2$)</td>
<td>1.9</td>
<td>52</td>
<td>124</td>
</tr>
</tbody>
</table>
Figure 3.10: An image of A2163 at 2.3 GHz observed with the AT in a 1.5km configuration.
Figure 3.11: An image of A2163 at 1.3 GHz observed with the AT in a 1.5km configuration.
Figure 3.12: The flux density versus baselines for A2163 at the cluster centre (X-ray centre from *Einstein* IPC data) at 8.8 GHz after the subtraction of discrete point sources. The error bars are $1\sigma$. 
centre (see Table 3.7), the flux density of the halo source quoted in Table 3.7 is a lower limit to the true source flux because of the SZ effect. We can estimate the expected SZ effect at 8.8 GHz from the detection at 136 GHz by Wilbanks et al. (1994). They found a comptonisation parameter of $y = 3.78^{+0.74}_{-0.65} \times 10^{-4}$ at the centre of the cluster. If we assume that all the decrement is due to the thermal SZ effect and take the shape of the gas density distribution from Arnaud et al. (1992), then we expect a $\sim -0.4$ mJy decrement at 8.8 GHz in the 30m baseline data at the cluster centre. If we take into account of the SZ effect, the listed halo flux density at 8.8 GHz would have to be increased by 30%.

Unfortunately, most of our observations with the 122m array were scheduled in November and December when the Sun is close to this cluster and thus badly affect any observation at 4.5 GHz. We could, however, estimate the spectral index of the halo emission from the 1.3 and 2.3 GHz observations shown in Figure 3.11 and 3.10. The spectral index of the diffuse emission between 1.3 and 2.3 GHz was found to be $\alpha \sim -1.6$. The strong halo source in the cluster made it impossible to detect the SZ effect at 8.8 GHz.

3.4 A3444

This cluster was suggested by M. Pierre as a high X-ray luminosity cluster shortly before our scheduled 122m array observations at the AT but after our VLA observations. Consequently, we were not able to obtain the VLA observations for this cluster. We observed the cluster with the AT 122m array at 8.8 GHz for 10 days with hour angle coverage of $-3$ to $3$ hrs each day. We also observed this cluster with the AT at 4.8 GHz and 8.8 GHz in a 6km array configuration. To achieve the same point source sensitivity at 8.8 GHz in a 6km array as that of the 122m array observations at the AT, it requires the same amount of observing time on the 6km array as it does on the 122m array. To save observing time, we observed the cluster at 4.8 GHz, since observations at 4.8 GHz should enable us to achieve a equivalent point source sensitivity at 8.8 GHz in a shorter time than an observation at 8.8 GHz for steep spectra.
Table 3.5: List of Radio Sources in A3444

<table>
<thead>
<tr>
<th>Source</th>
<th>RA</th>
<th>DEC</th>
<th>Δα</th>
<th>Δδ</th>
<th>S_{8832}^mJy</th>
<th>S_{8832}^mJy</th>
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</thead>
<tbody>
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<td>243</td>
<td>52</td>
<td>2.4</td>
<td>12</td>
</tr>
<tr>
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<td>10 23 50.1</td>
<td>-27 15 23</td>
<td>-2</td>
<td>4</td>
<td>1.2</td>
<td>1.2</td>
</tr>
<tr>
<td>3</td>
<td>10 23 57.3</td>
<td>-27 16 52</td>
<td>94</td>
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<td>0.69</td>
<td>1.6</td>
</tr>
<tr>
<td>4</td>
<td>10 23 52.2</td>
<td>-27 17 05</td>
<td>25</td>
<td>-98</td>
<td>0.60</td>
<td>0.76</td>
</tr>
<tr>
<td>5</td>
<td>10 23 54.3</td>
<td>-27 18 13</td>
<td>54</td>
<td>-166</td>
<td>0.72</td>
<td>1.5</td>
</tr>
<tr>
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<td>10 23 59.7</td>
<td>-27 19 02</td>
<td>126</td>
<td>-215</td>
<td>0.27</td>
<td>1.4</td>
</tr>
<tr>
<td>7</td>
<td>10 23 44.6</td>
<td>-27 29 37</td>
<td>-76</td>
<td>-310</td>
<td>0.30</td>
<td>7.0</td>
</tr>
<tr>
<td>8</td>
<td>10 24 06.1</td>
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<td>210</td>
<td>-267</td>
<td>0.089</td>
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</tr>
<tr>
<td>9</td>
<td>10 23 46.3</td>
<td>-27 14 01</td>
<td>-53</td>
<td>86</td>
<td>0.045</td>
<td>0.057</td>
</tr>
</tbody>
</table>

radio sources. We also observed at 8.8 GHz simultaneously using the second observing band available to check for any possible inverted spectra sources. This strategy is justified since the majority of radio source have steep spectra (Windhorst et al. 1993) at this resolution and flux limit (~ 30μJy) as we can see from Table 3.1. The only problems with this method are if there were some weak flat spectra radio sources with flux densities just below the flux limit of the 61-122m baseline data, and if the centroid positions of the radio sources are different at the two frequencies. Figure 3.13 and 3.14 show the AT 6km images of the cluster at 4.8 GHz and 8.8 GHz respectively. As it is shown in the images, all the sources that were present in the 8.8 GHz image were found in the 4.8 GHz image. The positions of the radio sources from the 4.8 GHz image and the flux densities of the sources fitted with the AT 61-122m baseline data at 8.8 GHz are in Table 3.5. The flux density versus baselines at the cluster X-ray centre after subtraction of the point sources is shown in Figure 3.15.

After subtraction of the sources in Table 3.5, the rms noise σ_o in the images made from the 61-122m baselines and just the 30m baselines agrees with the expected noise σ_f (see Table 2.6). An image of A3444 at 4.5 GHz was made by combining the 6 km data with the 122m data (see Figure 3.16). It showed some very diffuse, low brightness emission. To investigate the reality of the diffuse emission, we also made 1.3 and 2.3 GHz images of the cluster with an AT 1.5 km configuration since diffuse halo sources are known to have very

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Figure 3.13: A 4.8 GHz image of A3444 observed with the AT in a 6km array. The cross indicates the centre of the cluster. The first contour corresponds to a $3\sigma$ level.
Figure 3.14: A 8.8 GHz image of A3444 observed with the AT in a 6km array. The first contour corresponds to a $3\sigma$ level.
Figure 3.15: The flux density versus baselines for A3444 at the cluster centre (X-ray centre from the ROSAT all sky survey) at 8.8 GHz after the subtraction of discrete point sources. The error bars are $1\sigma$. 
Figure 3.16: An AT image of A3444 at 4.5 GHz with a 6km configuration plus the 122m configuration.

steep spectra ($\alpha \lesssim -1$). However, we did not find much diffuse emission in the 1.3 GHz image shown in Figure 3.17, so the structure seen in Figure 3.16 is unlikely to be a cluster halo.

3.5 J1780.5BL

This cluster was one of the higher redshift clusters $z \sim 0.5$ at declination $\delta < -50\degree$ found in the AAT Deep Cluster Catalogue (Couch et al. 1991). At
Figure 3.17: An image of A3444 at 1.3 GHz with a 1.5km AT configuration.
Table 3.6: List of Radio Sources in J1780.5BL

<table>
<thead>
<tr>
<th>Source</th>
<th>RA (h m s)</th>
<th>DEC (° ' '')</th>
<th>Δα ''</th>
<th>Δδ ''</th>
<th>S4864 mJy</th>
<th>S8704 mJy</th>
<th>S24704 mJy</th>
<th>α8704-4864</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>03 23 38.6</td>
<td>-51 00 00</td>
<td>242</td>
<td>324</td>
<td>3.0</td>
<td>11</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>03 23 43.8</td>
<td>-51 07 47</td>
<td>290</td>
<td>-144</td>
<td>0.86</td>
<td>1.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>03 23 15.5</td>
<td>-51 12 24</td>
<td>24</td>
<td>-420</td>
<td>0.62</td>
<td>2.5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>03 23 03.6</td>
<td>-51 07 30</td>
<td>-88</td>
<td>-126</td>
<td>0.22</td>
<td>0.39</td>
<td>0.34</td>
<td>0.40</td>
</tr>
</tbody>
</table>

The time of the radio observations there were no X-ray data available on this cluster, or any other high redshift clusters at such southern declinations. So we included it on the basis of its optimum declination. Later we found that it was not detected in the ROSAT all sky survey (M. Pierre private communication) and would have been excluded from our sample. We observed this cluster at 8.7 GHz with the AT 122m array both in July 1991 and December 1991. This cluster is too far south to be observed by the VLA and we were not able to obtain a full 12 hrs synthesis at 6 cm with the AT in a long baseline array as in the case of A3444. However, we were able to obtain a short ~ 6.5 hrs observation at 4.544/4.416 GHz with the AT in a 1.5km configuration with 10 hrs coverage in hour angle (see Figure 3.18) and we found 4 sources (source #1 to 4 in Table 3.1). The \textit{rms} noise of the image after deconvolution was 59\mu Jy/beam. We then fit for the point source flux densities in the data from 61-122m baselines at both 4.8 GHz and 8.8 GHz by fixing the source positions obtained with the 1.5 km array (see Table 3.6). The result of the fit shows that sources #1,2,3 are too weak to be detect at 8.8 GHz. Thus we only subtract source #4 from the 8.8 GHz data. The \textit{rms} noise in the image made from the 61-122m baseline data after source subtraction is consistent with the expected thermal and confusion noise (see Table 2.6). Thus the point source subtraction procedure has been successful. The flux density at the cluster centre after the subtraction of point sources is plotted against the (u,v) spacings in Figure 3.19.

The lowest resolution images made of data with baselines between 500 and 1000 wavelengths at both 8.7 GHz (30m baseline data) and 4.8 GHz (60m baseline data) had residual sources left which indicates that the sources are either truely extended or blends of sources (see Figure 3.20 and 3.21). The 2
Figure 3.18: A 4.5 GHz image of J1780.5BL observed with the AT in a 1.5km array. The first contour corresponds with the $3\sigma$ level.
Figure 3.19: The flux density versus baselines for J1780.5BL at the cluster centre (optical centre from Couch et al. 1991) at 8.8 GHz after the subtraction of discrete point sources. The error bars are $1\sigma$. 
extended sources had flux densities at 8.7 GHz of 0.23±0.04 mJy and 0.29±0.04
at an angular distance of (40", -260") and (40", 20") away from the cluster
centre. Their 4.8 GHz flux at a similar resolution suggest that both sources
have flat spectra (see Table 3.7), but the S/N is too low to draw any firm
conclusions.

3.6 Discussion

3.6.1 Extended Emission

The properties of the extended radio emission found in each cluster are
summarised in Table 3.7. Moffet et al. (1989) speculated that the presence
of radio halo source in a cluster is correlated with the existence of a strong
SZ effect. However, we find from the present observations that extended radio
sources also exist in clusters with weak SZ effect and even clusters with low X-
ray luminosity (J1780.5BL). Therefore, it appears that the number of extended
radio sources we find at such low resolutions and flux limits, is due to confusion.
On the other hand, observations of one “blank” field by Subrahmanyan et
al. (1993) using the same observing technique and parameters with the AT
did not find any apparently extended emission. One would expect that the
increased galaxy density in a cluster would result in an increased radio source
density and thus an increased probability of having a number of discrete radio
sources blending into an extended source. However, from Unewisse’s (1994)
studies, there should only be a 33% excess in the number of sources which
would not increases $\sigma_c$ enough to explain the difference. Alternatively, we can
speculate that non-thermal radio halo sources are more common in clusters at
intermediate redshifts ($z \sim 0.3$). The flux densities of these extended sources
we have found in the cluster fields are of higher radio luminosities than Coma
C, if they are indeed cluster sources (c.f. the total flux of Coma C at 8.8 GHz
would be $\lesssim 0.1$ mJy if it were at $z \sim 0.3$ and $\sim 0.8$ mJy for M87.).

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Figure 3.20: A contour map of J1780.5BL imaged with the AT 122m array at 8.8 GHz with only data from the 30m baseline (i.e. (u,v) spacing from 500 to 950 wavelengths). The thermal noise for this observation is 37\mu Jy. The cross marks the cluster centre.
Figure 3.21: A contour map of J1780.5BL imaged with the AT 122m array at 4.8 GHz with only data from the 61m baseline (i.e. (u,v) spacing from 500 to 950 wavelengths). The thermal noise for this observation is 109μJy. The cross marks the cluster centre.
Table 3.7: List of Extended Radio Sources

<table>
<thead>
<tr>
<th>Cluster</th>
<th>RA</th>
<th>DEC</th>
<th>Δα</th>
<th>Δδ</th>
<th>S_{15cm}^6</th>
<th>S_{6cm}^6</th>
<th>S_{6cm}^3</th>
<th>S_{6cm}^1</th>
<th>α</th>
</tr>
</thead>
<tbody>
<tr>
<td>MS2137-23</td>
<td>21 40 23.7</td>
<td>-23 40 57</td>
<td>150</td>
<td>-90</td>
<td>0.18 ± 0.03</td>
<td>0.38</td>
<td>0.77 ± 0.09</td>
<td>0.93</td>
<td>-1.3</td>
</tr>
<tr>
<td>J1780.5BL</td>
<td>03 23 17.3</td>
<td>-51 05 14</td>
<td>40</td>
<td>20</td>
<td>0.29 ± 0.04</td>
<td>0.3</td>
<td>0.26 ± 0.11</td>
<td>0.26</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>03 23 17.3</td>
<td>-51 09 44</td>
<td>40</td>
<td>-20</td>
<td>0.23 ± 0.04</td>
<td>1.4</td>
<td>0.92 ± 0.11</td>
<td>1.5</td>
<td>-0.1</td>
</tr>
<tr>
<td>A2163</td>
<td>16 15 45.5</td>
<td>-06 08 46</td>
<td>-10</td>
<td>0</td>
<td>1.1 ± 0.06</td>
<td>1.1</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.6.2 The SZ effect

The simulations for A2218 (see Figure 2.4) show that the AT at 8.7 GHz is 
~ 2.5 times more sensitive than the Ryle at 15.4 GHz in detecting the SZ effect 
in a cluster like A2218. Then why is it that the AT has not been successful 
in detecting the effect whereas the Ryle has had 5 detections so far? We will 
address the following possibilities:

- *Observing time*: Unlike the Ryle, the AT is not a dedicated SZ telescope. 
  Typically at the AT we can realistically obtain only 40 to 100 hrs effective 
  integration time (or up to 10 days of observing time). For an integration 
  time of 50hrs, the AT can achieve a S/N of 5 at the shortest baseline 
  which is the same as the S/N achieved by the Ryle in 27 days. Thus 
  the relatively short observing time does not explain why we have not 
detected the SZ effect.

- *Dynamic range limits*: In order to avoid errors from the inaccurate sub-
  traction of stronger sources in the clusters, we have excluded clusters 
  with sources stronger than 3 mJy (see section 2.6). Since the majority of 
  radio sources are stronger at lower frequencies, they are stronger for the 
  AT (8.7 GHz) than the Ryle (15.4 GHz). Thus for the same limit on the 
  strength of the radio sources in the cluster, there are more clusters that 
  would be suitable for observations of the SZ effect with the Ryle than 
  with the AT, which means that the Ryle has a better chance of finding a 
good candidate for observations. The subtraction of the “strong” 
  sources in the clusters have been successful and the residuals from the
subtraction process were not significant, in other words the images were not dynamic range limited. It appears that for future observations, we can relax criterion (ii) in section 2.6, where the strongest source allowed in the field was limited to 3 mJy.

- **Halo sources**: A2163 has a extended halo source at the cluster centre. The halo emission is ~ 3 times larger than the SZ effect and thus completely masks out the decrement. Effects of halo radio sources diminishes rapidly as the frequency increases since it is known to have very steep spectra $\alpha \lesssim -1$ (Giovannini et al. 1993).

- **Weak radio source confusion**: Weak extended sources have been detected in almost every cluster that we have observed with the AT. The effects of weak discrete radio sources below our detection threshold decreases with increasing frequency, since the median spectral index is $\sim -0.35$ for micro-Jansky sources at 8.4 GHz (Windhorst et al. 1993). Thus the Ryle observations suffer less from blends of weak discrete sources. On the other hand, the flux density of the SZ decrement is $\propto \nu^2$, thus given the same resolution we gain an advantage of $\nu^{2.35}$ by observing at a higher frequency. It is shown in Table 2.6 that it is the confusion noise from the weak radio sources that limits our ability to detect the SZ effect.

- **Incomplete hour angle coverage**: Ideally we would like to have a full 12 hrs synthesis for each cluster that we observe, so that we can readily identify the position, extension and thus the flux density of the sources in the field. Because of the insufficient information on X-ray clusters in the South, our choice of candidate clusters have been limited and the clusters we observed were all $> -30^\circ$ in declination except for J1780.5BL. Shadowing of one antenna by a neighbouring antenna in the 122m configuration for clusters north of $-30^\circ$, reduces the hour angle coverage to 6 hrs. Apart from the degree of freedom introduced because of the missing information, an incomplete coverage increases the sidelobe levels and the size of the synthesized beam which in turn increases the effects of radio source confusion. We have shown, however, that we can successfully measure the point source flux densities even with such poor hour angle
coverage given the accurate source positions determined from relatively high resolution data. However, we need to detect and measure the point source to a fainter flux level when the (u,v) coverage is incomplete.

- **Cluster selection:** It is possible that we have not selected the clusters that have strong SZ effect. As we have already mentioned above that there are less X-ray clusters available as candidates in the South than the North. The most sensitive X-ray satellite prior to ROSAT was *Einstein* which did not conduct a all-sky survey but pointed at well studied clusters. Since the majority of the optically well studied clusters were in the north, we don't have a large sample of southern clusters to choose from. Furthermore, a great deal of optical followup work is still needed for clusters found in the ROSAT all-sky survey to ascertain that the X-ray emission is indeed from the intracluster gas rather than an AGN. Thus currently the best source of cluster candidates is still the EMSS catalogue from *Einstein*.

In summary, observations at higher frequency than 8.7 GHz and better coverage in the Fourier plane could improve our chances of detecting the SZ effect. We need high frequency to reduce the effect of the extended radio sources and weak radio source confusion, however as we have shown above in some of the clusters the extended radio source is not strong enough to mask out a SZ effect as strong as that detected in A2218. We also need more southern X-ray clusters than what we have at the present to select candidates strong in the SZ effect. The ROSAT all-sky survey will no doubt produce many southern X-ray cluster candidates suitable for observations of the SZ effect. It is also important to improve the coverage of the Fourier plane so that we have less degrees of freedom in fitting flux densities of the radio source than a partial coverage. One way of improving the (u,v) coverage is to observe a cluster south of $-50^\circ$ in declination, so that we can have a full 12 hrs synthesis without shadowing of the antennas in the 122m array configuration. We can also observe the cluster in another short baseline array, preferably with 45m baselines, or a 244m array (i.e. 60m spacing between antennas) to avoid shadowing at large hour angles of the antennas for a cluster closer to the equator than $-50^\circ$. Finally,
observations at higher resolution with a higher point source sensitivity than what the 122m array offers is as important as the observations of the SZ effect itself. The higher resolution is for accurate determination of the source position and these observations need to have a higher point source sensitivity than the observations of the SZ effect itself to detect and subtract any weak discrete source that may blend together in the low resolution beam used for the detection of the SZ effect. For clusters that are north of $-45^\circ$, they can be observed by the VLA in the D-array which could achieve a higher point source sensitivity than a 50hrs observation with the AT 122m array within a few hours. For the clusters south of $-45^\circ$, we have better (u,v) coverage in the 122m array for SZ detection and thus less need for a high resolution observation to help determine the positions and flux densities of discrete sources. However, it is still important to have data on any weak discrete radio sources that would blend together in an image constructed with the shortest baseline alone and become significant compared with the SZ effect. We can achieve this in a reasonable amount of time by observing the cluster with the AT in a 375m or 750m configuration at 5 GHz since on average the radio sources are stronger at lower frequencies. Note it is important that the high resolution observations for source detection should be of sufficient resolution to estimate the individual source positions accurately but still close to the resolution of the observations for the SZ effect so that the sources are not resolved.

3.7 Conclusions

We have searched for the SZ effect in 5 southern clusters and have obtained upper limits for some of the clusters. The sensitivity of the AT is sufficient to detect the SZ effect for clusters with SZ effect as strong as those already detected by the Ryle telescope. Extended radio sources were found in almost all the clusters. Except for A2163 where the extended emission is due to a radio halo source, the extended emission in the clusters were consistent with a blend of weak discrete radio sources from source counts estimates. The AT observations at 8.8 GHz is largely limited by the confusion noise due to the weak discrete sources below $\sim 100\mu$Jy. Future observations need to be
conducted at a higher frequency or subtract sources down to a lower flux limit.

3.8 References


Chapter 4

Modelling the Mass Distribution and Properties of the ICM

In this Chapter, we will summarised the different methods that have been used in literature to model the cluster total mass distribution and the properties of the ICM, and describe the modelling procedure used in this thesis for clusters to predict the observable consequences of the S-Z effect and the X-ray surface brightness profile for a cluster. Specifically, the diminution in CMB and X-ray surface brightness are deduced by assuming a model for the cluster total mass distribution, an equation of state for the gas and hydrostatic equilibrium. Finally the SZ effect deduced from the models were used to simulate observations with the Australia Telescope (AT).

4.1 Review of the Various Modelling Methods

First of all, we will review the different methods that have been used for modelling the matter distribution in clusters and the properties of the ICM.

4.1.1 Cluster Dynamics

Traditionally, cluster masses are estimated from virial analysis and the mass distributions are often assumed to follow that of the light (e.g. Kent and
Once there is sufficiently good data on optical photometry and spectroscopy (i.e. \( \geq 300 \) member galaxies with velocity measurements), the cluster mass distribution can be determined using a generalised virial analysis (Merritt 1987) as follows. The "stellar hydrostatic" equation, or Jean's equation, is given by

\[
\frac{d(n(r)\sigma_r^2)}{dr} + \frac{2n(r)}{r}(\sigma_r^2 - \sigma_t^2) = -\frac{Gn(r)M_{\text{tot}}(r)}{r^2} \tag{4.1}
\]

where \( r \) is the 3-D radius, \( n(r) \) is the galaxy space density, \( \sigma_r \) and \( \sigma_t \) are the radial and tangential velocity dispersions, and \( M_{\text{tot}} \) is the total binding mass. The galaxy space density can be determined from the measured projected galaxy density profile through Abel integral inversion:

\[
n(r) = \frac{1}{\pi} \int_0^r \frac{d\Sigma(R)}{\sqrt{R^2 - r^2}} \tag{4.2}
\]

The line of sight velocity dispersion can be measured but there is no simple inverse between the projected velocity dispersion with a 3-D velocity dispersion. From simple geometry, \( \sigma_\phi \) is related to the radial and tangential velocity dispersion, \( \sigma_r \) and \( \sigma_t \) as follows:

\[
\Sigma(R)\sigma_\phi^2(R) = 2 \int_0^\infty \left[ \sigma_r^2 - \frac{R^2}{r^2}(\sigma_r^2 - \sigma_t^2) \right] \frac{n(r)rdr}{\sqrt{r^2 - R^2}} \tag{4.3}
\]

where \( R \) is the projected radius. In the above 3 equations, the observable quantities are \( \Sigma, \sigma_\phi^2 \) and the unknown quantities are \( n, \sigma_r^2, \sigma_t^2 \) and \( M_{\text{tot}} \). The range of cluster total mass distribution is constrained by assuming a functional form for \( M_{\text{tot}} \) and reject the ones that give negative solutions for \( \sigma_r^2 \) and \( \sigma_t^2 \). In addition, the line of sight velocity distribution can also help to restrict the range of models since the distribution varies according to the type of galaxy orbits. More recently, Dejonghe and Merritt (1992) have developed an algorithm that can be successfully applied to mass estimates for globular clusters, where the potential is strongly constrained by the joint distribution of positions and line of sight velocities. Unfortunately, this method is not as successful when applied to galaxy clusters because galaxy clusters are not as relaxed as globular clusters and it is difficult to distinguish the effects of anisotropy and substructure in a joint distribution of velocities and positions.
4.1.2 X-ray Methods

Since the arrival of X-ray astronomy and the X-ray high resolution imagery and spectroscopy, it has been possible to estimate the cluster mass distribution using X-ray data by assuming that the cluster gas is in hydrostatic equilibrium. Unlike the galaxies, the cluster gas is collisional and thus guaranteed to have isotropic orbits which makes the analysis simpler in comparison to the method described in the last section.

The assumption of hydrostatic equilibrium is justified if the sound crossing time is short compared to the cluster age and the heating and cooling times are much longer than the sound crossing time. The sound crossing time in a cluster is given by

$$t_s \sim 6.6 \times 10^8 \text{yr} \left( \frac{T_g}{10^8 \text{K}} \right)^{-1/2} \left( \frac{D}{\text{Mpc}} \right)$$

(4.4)

where for $T_g = 10^8 \text{K}$ and diameter of cluster of $D \sim$ a few Mpc, $t_s \sim 10^9$ yrs which is much less than the probable age of clusters $10^{10} \text{ yrs}$ (Sarazin 1986). The conventional heating and cooling models in general have heating and cooling times much longer than sound crossing time (Sarazin 1986). For example, the cooling time for thermal bremsstralung X-ray emission is given by

$$t_{cool} = (d \ln T_g/dt)^{-1} = 8.5 \times 10^{10} \text{yr} \left( \frac{n_e}{10^{-3} \text{ cm}^{-3}} \right)^{-1} \left( \frac{T_g}{10^8 \text{K}} \right)^{1/2}$$

(4.5)

where for $T_g = 10^8 \text{K}$ and electron density of $n_e = 10^{-3} \text{ cm}^{-3}$, $t_{cool} = 8.5 \times 10^{10} \text{ yrs} \gg t_s$.

The equation for hydrostatic equilibrium is given by

$$\nabla P = -\rho_g \nabla \phi(r)$$

(4.6)

where $P$ is the pressure, $\rho_g$ is the gas density and $\phi$ is the cluster potential. If we assume spherical symmetry then the above equation can be reduced to

$$\frac{dP_g}{dr} = -\rho_g \frac{d\phi}{dr}$$

(4.7)

where $r$ is the radius from the cluster centre.
Since the intracluster gas is very low in density, we can apply the ideal gas law to it:

\[ P = \rho_g kT_g / \mu m_p \]  
(4.8)

where \( \mu \) is the mean molecular weight (\( \approx 0.6 \) for cosmic abundance).

If we assume that the gas is isothermal, then substituting equation 4.8 into 4.7 we get

\[ \frac{d \ln \rho_g}{dr} = -\frac{\mu m_p}{kT_g} \frac{d \phi(r)}{dr} \]  
(4.9)

The X-ray surface brightness distribution can be deduced from the gas density distribution:

\[ S_x(R) = 2 \int_{R}^{R_t} \frac{\epsilon(r) r dr}{\sqrt{r^2 - R^2}} \]  
(4.10)

where \( \epsilon \) is the emissivity for thermal bremsstrahlung radiation, it is given by

\[ \epsilon(r) = \int_{\nu} \epsilon_{\nu} d\nu \]  
(4.11)

where \( \epsilon_{\nu} \) is the emissivity per unit frequency, given by

\[ \epsilon_{\nu} = 5.443 \times 10^{-39} Z^2 g(\nu, T) \exp(-h\nu/kT)^{T^{-1/2} n_e n_i} \]  
(4.12)

with units of ergs cm\(^{-3}\) s\(^{-1}\) Hz\(^{-1}\) ster\(^{-1}\), where \( Z \) is the charge of the ion, \( g \) is the gaunt factor and \( n_e \) and \( n_i \) are electron and ion density.

Thus from the observed X-ray surface brightness distribution, we can deduce the mass distribution of the X-ray gas. By substituting Poisson's equation for a spherically symmetric system

\[ \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d \phi(r)}{dr} \right) = -4 \pi G \rho_{tot} \]  
(4.13)

and equation 4.8 into equation 4.7, we obtain the total binding mass as a function of the gas temperature and density

\[ M_{tot}(r) = -\frac{kT_g(r)}{\mu m_p G} \frac{d \ln \rho_g}{dr} - \frac{d \ln T_g}{dr} \]  
(4.14)

Note that \( M_{tot} \) has a stronger dependence on \( T_g \) than \( \rho_g \) (Fabricant et al. 1984).
Analytic $\beta$ model

The $\beta$-model is often used in the literature for its simplicity to calculate the parameters for the intracluster medium and the total mass (e.g. Sarazin 1986; Cavaliere & Fusco-Femiano 1976). Cluster potentials are usually assumed to be self-gravitating isothermal spheres. This assumption comes naturally if one assumes mass follows light since galaxy count distributions are well fitted by self-gravitating isothermal spheres in general. For a simple analytic solution, the analytic King model (1962) is usually used to approximate the isothermal sphere model. The total mass density distribution for an analytic King model (1962) is given by

$$p(r) = \rho_0[1 + \left(\frac{r}{r_c}\right)^2]^{-3/2}$$

(4.15)

where $r_c$ is the core radius and $\rho_0$ is the central density. A further simplification to the problem can be made by noting that for galaxies we also have the relation:

$$\frac{dP_{\text{gal}}}{dr} = -\rho_{\text{gal}} \frac{d\phi}{dr}$$

(4.16)

If the velocity dispersion of cluster galaxies are isotropic, then we have

$$P_{\text{gal}} = \rho_{\text{gal}} \sigma_p^2$$

(4.17)

where $\sigma_p$ is the line of sight velocity dispersion. Thus by comparing equations 4.7 and 4.16, we get

$$\rho_g(r)/\rho_{g0} = (\rho_{\text{gal}}(r)/\rho_{\text{gal}0})^\beta$$

(4.18)

where $\beta = \frac{\mu m_p \sigma_g^2}{k T_g}$ is the ratio of kinetic energy per unit mass in galaxies to gas (Cavaliere & Fusco-Femiano 1976). If we approximate the galaxy density distribution by the analytic King model we get

$$\rho_g = \rho_{g0}[1 + \left(\frac{r}{r_c}\right)^2]^{-3\beta/2}$$

(4.19)

for the gas density distribution.

In this case the X-ray surface brightness is simply

$$S_x(R) = S_x(0)(1 + (R/r_c)^2)^{-3\beta + 0.5}$$

(4.20)
The SZ effect derived from equation 1.4 is given by

$$\Delta T_r(r)/T_r = -\frac{2\sqrt{\pi} \sigma_T k T_g}{m_e c^2} n_e(0) r_c \Gamma(3\beta - 1/2) \frac{1}{\Gamma 3\beta/2} [1 + (r/r_c)^2]^{-1/2} \quad (4.21)$$

where $n_e(0)$ is the central electron density.

This approach in modelling the gas density in clusters and X-ray surface brightness is frequently used in literature because of its simplicity (Sarazin 1986) and the fact that it is a good empirical function to fit the X-ray surface brightness distribution. However, it has been pointed out in the case of elliptical galaxies by Killeen & Bicknell (1988) that this model is not self-consistent. The analytical King model is only a good approximation to the isothermal sphere model within a few core radii of a cluster. Thus the gas density derived is only a good approximation to the gas within a few core radii. The inconsistency in the above model lies in the fact that the galaxy distribution was assumed to be isothermal (i.e. velocity dispersion $\sigma_r$ is constant) whereas analytic King model (which does not give a $\sigma_r$ constant w.r.t the radius except at small radius) was also used to described the galaxy distribution. In fact there is no need to assume both King and isothermal models for the galaxy distribution and it will be shown in section 4.2.1 that it is still fairly simple to deduce the gas distribution and hence $S_x$ by just assuming that the galaxy distribution is either isothermal or follows the King model but not both. The differences between the results deduced from the above simple approximation and the self-consistent models will be shown later in Figure 4.4 and 4.5.

**Numerical Methods**

Hughes (1989) derived a more general and self-consistent method for modelling the distribution of mass in clusters. It was noted in equation 4.14 that the total mass has a strong dependence on $T_g$, thus the following approach was adopted by Hughes. Equation 4.14 can be rearranged to give

$$\frac{dT_g}{dR} = -\frac{1}{\rho_g} \frac{d\rho_g}{dR} T_g - \frac{4\pi \mu m_p G}{k R^2} \int_0^R \rho_{tot}(r)r^2dr \quad (4.22)$$

Note that equation 4.22 is the same as the Jean's equation (equation 4.1) with $T_g$ in place of $\sigma^2$ for an isotropic system in equilibrium. Firstly, the gas
distribution was parametrized as

\[ \rho_g(R) = \rho_g(0)(1 + \frac{R^2}{R_c^2})^{-3\beta/2} \]  \hspace{1cm} (4.23)

and the parameters \( R_c \) and \( \beta \) are fitted using the observed X-ray surface brightness distribution. Secondly, the total mass distribution were parametrized either as King profiles or of the form

\[ \rho_{\text{tot}}(r) = \rho_{\text{tot}}(0)[1 + \left(\frac{r}{r_c}\right)^2]^{-n/2} \]  \hspace{1cm} (4.24)

Finally, trial forms of \( T_g(r) \) were generated from equation 4.22 and compared with the X-ray spectral data. Given a temperature profile \( T_g(r) \), the X-ray spectrum is fully described by the average gas temperature, metal abundance, emission measure (i.e. \( \int n_e n_p dl \)) and the neutral hydrogen column density along the line of sight to the cluster.

The range of viable models are strongly constrained by the extent of the observed X-ray emission, i.e. \( T_g(r) \) should not fall to zero at a radius smaller than the observed maximum radius of the X-ray emission. On the other hand, if \( T_g \) increases with radius without limit than the pressure at infinity would be greater than zero and the cluster would not be gravitationally bound.

Recently, M. Arnaud & D. Elbaz (Elbaz 1994) have devised an alternative method based on the above method to estimate the cluster mass distribution. The equation of hydrostatic equilibrium can be rearranged to give

\[ P(r) = P_0 - G \int_0^r \frac{M_{\text{tot}}(r)\rho_g(r)}{r^2} dr \]  \hspace{1cm} (4.25)

and a critical pressure is defined as

\[ P_0^{\text{crit}} = G \int_0^\infty \frac{M_{\text{tot}}(r)\rho_g(r)}{r^2} dr \]  \hspace{1cm} (4.26)

If \( P_0 > P_0^{\text{crit}} \) then the pressure of the gas is always positive and the cluster is, thus, not gravitationally bound and such solutions are rejected.

If \( P_0 < P_0^{\text{crit}} \) then the pressure drops to zero at a finite radius and the cluster is gravitationally bound.
4.1.3 Gravitational Lensing

Mass distributions of clusters inferred from the gravitational lensing effects on the background galaxies are free from many of the assumptions employed in the methods discussed above. There is no need for any assumptions on the dynamical state of the cluster or the isotropy of the orbits. Since the first association of a giant arc in A370 (Soucail et al. 1987; Lynds & Petrosian 1986; Kovner 1988) with the gravitational deflection of light of background sources by the cluster, such arcs have been found in ~ 30 clusters (Fort and Mellier 1994) and they provide a good measure of the mass enclosed within the radius of the arc (see review of Fort and Mellier 1994 and references therein). The thin lens approximation can be applied to clusters of galaxies for small bending angles. From simple geometry, the lens equation is given by

$$\vartheta = \theta - \frac{D_{ds}}{D_s} \alpha(\xi)$$

(4.27)

where $\vartheta$ is the angular separation between the source and the lens, $\theta$ is the angular separation between the image and the lens, $\alpha$ is the deflection angle, $\xi$ is the impact parameter (i.e. proximity of the light ray to the deflector), $D_{ds}$ is the angular diameter distance between the deflector and the source and $D_s$ is the angular diameter distance from the observer to the source (Young 1981). The deflection angle is directly related to the projected cluster mass density or the 2-D potential $\psi$, where

$$\alpha(\xi) = \frac{1}{c^2} \nabla \psi(\xi)$$

(4.28)

(Fort and Mellier 1994). In the case of a spherically symmetric surface density, we have

$$\alpha(\theta) = \frac{M_p(\theta)}{\pi \Sigma_{crit} D_d \theta D_d D_s}$$

(4.29)

where $M_p$ is the projected mass. Giant arcs are produced when the source is almost directly behind the deflector, i.e. when $\vartheta \sim 0$. If the the source is exactly along the line of sight behind the cluster, then an Einstein ring would be produced. Critical lines in the image plane coincides with the position of the Einstein rings which corresponds to the caustic lines in the source plane. These are the positions where the amplification is infinite if the source is a
true point source\footnote{Since no source is a true point source, the actual amplification does not diverge.}. In the limit of $\vartheta \to 0$, the projected mass enclosed within the critical circle is given by

$$M_p(b) = \Sigma_{\text{crit}} \pi b^2 D_d^2$$  \hspace{1cm} (4.30)

where $b$ is the radius of the critical circle which is roughly the radius of a giant arc and $\Sigma_{\text{crit}}$ is the critical surface density. The critical surface density is given by

$$\Sigma(b) = \Sigma_{\text{crit}} = \frac{c^2}{4\pi G D_d D_{ds}}$$  \hspace{1cm} (4.31)

Inside the critical line is the region of “strong lensing” effects where giant arcs and multiple images occur. The position of the giant arc provides a strong constraint ($\sim 6\%$ accuracy) on the mass enclosed within the radius of the arc (Kochanek 1992), however this central region is rather small $\sim 20''$ (Fort and Mellier 1994) and provides only a rough estimate of the core radius and little constraint on the global distribution of the cluster potential.

Arclets can form just outside the critical circle and the number of arclets provide a rough measure of the line of sight cluster velocity dispersion, where the number of arclets is $\propto \sigma_p^4$ (Fort 1990). However, arclets alone can not constrain the core radius (Miralda-Escude 1991).

In the “weak lensing” regime, i.e. in the outer regions of a cluster away from the critical line, the background galaxies are only slightly distorted and the distortions are hardly discernable individually. However, collectively such distortions can be used to measure the shear and thus the cluster potential by mapping the average ellipticities in regions around the cluster and assuming that the orientations of the major axes of the galaxies are random (Tyson \textit{et al.} 1990; Kaiser and Squire 1993). Recently, preliminary analyses of weak gravitational distortion of background galaxies by clusters of galaxies have proved these methods to be viable for the determination of cluster potentials for the outer parts of clusters $\sim 1$ Mpc (Tyson \textit{et al.} 1990; Kaiser and Squire 1993; Smail \textit{et al.} 1994; Breimer 1994; Bonnet \textit{et al.} 1994).

An alternative weak lensing method has been proposed by Broadhurst \textit{et al.} (1994) where the joint distribution in redshift and magnitude of background
galaxies is compared with that of the field galaxies and the difference in the slopes of the distributions can be used to determine the mass of the deflector. This method measures the surface density of the cluster directly, and unlike the method above, it is sensitive to a constant density lens which does not produce shear. However, in practical terms this method is much more time consuming than the above method since it is much harder to obtain spectroscopy data than photometry data.

4.2 Modelling of the SZ effect and X-ray surface brightness

In this section we combine the models for the mass density distribution for the total cluster mass (i.e. gas + galaxies + dark matter) and equations of state for the cluster gas, to model both the SZ decrement and the X-ray surface brightness. We will examine the inter-dependencies between the various quantities such as the shape of the mass distribution and the gas temperature profiles. We will incorporate the SZ effect in our modelling of the properties of the ICM. The general expression for the gas density distribution as a function of the cluster potential and gas temperature is given by

$$\rho_g(r)/\rho_g(0) = (T_g(r)/T_g(0))^{-1} \exp\left[-\frac{\mu m_p}{kT_g(0)} \int_0^r (T_g(r)/T_g(0))^{-1} \frac{d\phi(r)}{dr} dr\right]$$

(4.32)

The cluster potential $\phi$ can be derived from equation 4.13, given a cluster mass density distribution. The gas density distribution can then be substituted into equation 4.10 and 1.1 to obtain the X-ray surface brightness and SZ effect.

4.2.1 Isothermal Models

In the following section, we will examine a number of different types of total mass distributions with the gas being isothermal. Most of the observational data so far suggest that clusters of galaxies are roughly isothermal on the large scales (e.g. Mushotzky 1994; Henry et al. 1993; Durret et al. 1994). Thus as far as the available data is concerned, it is reasonable to assume that the gas
is isothermal. Spherical symmetry and hydrostatic equilibrium are assumed in the following models.

Model A

In this model, King’s self-consistent truncated isothermal model (King 1966) is used to describe the cluster density distribution. (The analytic King model described above is an approximation to this model.) Note however the King model used here and the singular isothermal model in Model C are derived from distribution functions that are dependent on energy only rather than both energy and angular momentum, in other words isotropic velocity dispersion was assumed (Binney and Tremaine 1987). On the other hand, just because a density distribution can be described by a King model or isothermal model, does not necessarily imply that velocity dispersion is isotropic. The method described here to calculate $\Delta T_r/T$ and $S_x$ is not restricted to isotropic cases. From the assumption of hydrostatic equilibrium and isothermal equation of state for the cluster gas, we have equation 4.32 which can be reduced to

$$\rho_g(r)/\rho_g(0) = \exp(\beta[\omega(r) - \omega_0])$$

(4.33)

where $\beta = \frac{\mu m_p \sigma^2}{k T_g}$ and $\omega = -\phi/\sigma^2$ which is given by King’s truncated isothermal model. A program written by Mark Winsall which calculates the potential for the King Model as a function of $r/r_c$ via the equation

$$\frac{d^2 \omega}{dr^2} + \frac{2 d\omega}{r dr} = -9 \frac{\rho}{\rho_0}$$

(4.34)

was used to obtain $\omega(r)$. The density $\rho$ is also related directly to the potential $\omega$ by

$$\rho \propto e^\omega \int_0^\omega e^{-\eta \eta^{3/2}} d\eta$$

(4.35)

Figure 4.1 shows the X-ray surface brightness profiles for different $T_g$ that has the same potential and total X-ray luminosity.

Similarly, the SZ profile in the Rayleigh-Jeans limit is hence given by

$$\Delta T(R/r_c)/T = -(4\sigma T k/m_c c^2) n_e(0) T_g e^{-\omega \beta} \int_{R/r_c}^{R/r_c} \exp(\omega(r)\beta) \frac{x dx}{\sqrt{x^2 - (R/r_c)^2}}$$

(4.36)
Figure 4.1: The X-ray surface brightness profile for a King potential with $\omega_0 = 8.5$ and total X-ray luminosity of $7 \times 10^{44}$ ergs/s and $\beta = 0.62$. The dotted curve is for $T_g = 3 \times 10^7$ K and $n_e(0) = 3 \times 10^{-3}$ cm$^{-3}$; the solid curve is for $T_g = 1 \times 10^8$ K and $n_e(0) = 2 \times 10^{-3}$ cm$^{-3}$.
where \( R \) is the projected distance from the centre of the cluster, \( x = r/r_c \), and King's central potential \( \omega_0 = \phi_0/\sigma^2 \). The above integral can be numerically integrated to give a SZ profile. Figure 4.2 shows a collection of the SZ profiles with different \( T_g \) for the same cluster potential and total X-ray luminosity. Figure 4.3 shows the SZ effect that would be observed by the AT at 8.8 GHz in the 122m configuration (see section 2.3.1 for details on the method).

**Model B**

We will consider a family of cluster mass density profiles given by

\[
\rho_{tot}(r) = \frac{\rho_{tot}(0)}{[1 + (r/r_c)^2]^{n/2}}
\]

(4.37)

In the case of \( n = 3 \), we have the analytic King model used in the \( \beta \) model. The corresponding potential can be derived from Poisson’s equation and is thus given by

\[
\phi(r) = -4\pi G \rho_{tot}(0)r_0^2 \ln[r/r_0 + \sqrt{1 + (r/r_0)^2}] - \frac{4\pi G \rho_0 r_0^2 \mu m_p}{k T_g} \left( \frac{\ln(x + \sqrt{1 + x^2}) - 1}{r/r_0} \right)
\]

(4.38)

Thus from equation 4.32 for an isothermal gas, the gas density distribution is given by

\[
\rho_g/\rho_g(0) = \exp\left[\frac{4\pi G \rho_0 r_0^2 \mu m_p}{k T_g} \left( \frac{\ln(x + \sqrt{1 + x^2}) - 1}{r/r_0} \right) \right]
\]

(4.39)

We can compare the X-ray surface brightness distribution calculated from the \( \beta \) model with the self-consistent analysis of a total mass density distribution described by the analytic King model. Figure 4.4 and 4.5 shows the difference between \( \Delta T \) and \( S_x \) calculated here and using the analytic \( \beta \) model described in the last section. As we can see the difference in the SZ profile for the \( \beta \) model and the self-consistent model with the same cluster potential is significant, whereas the difference in the X-ray surface brightness is negligible in the central parts. It is important to realise that the analytic \( \beta \) model is not a very good approximation to this model in calculating \( \Delta T \) and the outer parts of \( S_x \).

We investigate the effect of \( n \) on the X-ray surface brightness distribution (Figure 4.6) and the simulated AT observation of the SZ decrement (Figure 4.7). The curves in both figures are for the same total mass, X-ray luminosity and temperature. The central mass density \( \rho_{tot}(0) \) and the central
Figure 4.2: The SZ decrement $\Delta T$ at 8.8 GHz profile for a King potential with $\omega_0 = 8.5$, total X-ray luminosity of $7 \times 10^{44}$ ergs/s and $\beta = 0.62$. The dotted curve is for $T_g = 3 \times 10^7$K and $n_e(0) = 3 \times 10^{-3}$ cm$^{-3}$; the solid curve is for $T_g = 1 \times 10^8$K and $n_e(0) = 2 \times 10^{-3}$ cm$^{-3}$
Figure 4.3: The flux density of the SZ decrement versus baseline length in wavelength units for a King potential with \( \omega_0 = 8.5 \), total X-ray luminosity of \( 7 \times 10^{44} \text{ ergs/s} \) and \( \beta = 0.62 \). The dotted curve is for \( T_g = 3 \times 10^7 \text{K} \) and \( n_e(0) = 3 \times 10^{-3} \text{ cm}^{-3} \); the solid curve is for \( T_g = 1 \times 10^8 \text{K} \) and \( n_e(0) = 2 \times 10^{-3} \text{ cm}^{-3} \).
Figure 4.4: SZ temperature decrement versus radius for central electron density of $10^{-3}\text{cm}^{-3}$, $T_g = 10^8\text{K}$, $r_c = 0.25\text{ Mpc}$ and $\beta = 2/3$. The solid curve gives the self-consistent analysis of the analytic King model (equation 4.15); the dotted curve gives the result from the $\beta$ model.
Figure 4.5: The X-ray surface brightness distribution for central electron density of $10^{-3}\text{cm}^{-3}$, $T_g = 10^8\text{K}$, $r_c = 0.25\text{ Mpc}$ and $\beta = 2/3$. The solid curve gives the self-consistent analysis of the analytic King model (equation 4.15); the dotted curve gives the result from the $\beta$ model.
electron density $n_e(0)$ were left as free parameters. Thus the steeper the mass profile, the higher the central X-ray surface brightness and the greater the SZ flux density at the AT spacings.

Model C

Beers and Tonry (1986) found that by carefully choosing the cluster centre, namely using X-ray centres and positions of cD/D galaxies cluster, density distributions are found to have a cusp at the centre. In fact they found that the projected density distribution in the central $0.02h^{-1}\text{kpc}$ to $1h^{-1}\text{Mpc}$ obtained by stacking a set of clusters with dominant central galaxy is consistent with a singular isothermal model, or de Vaucouleurs law with effective radius $r_e \approx 3h^{-1}\text{Mpc}$. This prompts us to model the cluster density distribution using a singular isothermal model. It was pointed out in their paper, however, that such galaxy density distribution implies that mass does not follow light into the centre of clusters, because if the total mass distribution is as concentrated as the galaxy distribution then the tidal stress from such a potential would probably disrupt any galaxy that comes close to it.

Let us suppose that the total mass distribution is described by the singular isothermal model and find out how such an extreme model for the mass distribution affects the SZ profile.

The singular isothermal model is the simplest solution derived from the Maxwellian distribution function with isotropic velocity dispersion. It is given by

$$\rho(r) = \frac{\sigma^2}{2\pi G r^2}$$

(4.40)

where $\sigma$ can be identified with the line of sight velocity dispersion for the case of isotropic velocity dispersion. In this case we know that $\sigma_r$ is constant w.r.t $r$, thus we can simply deduce the gas density distribution from equation 4.18. But the gas density is infinite at the centre, which implies that the $\Delta T/T$ diverges at the centre. This is clearly not physical. As it was stated in Beers and Tonry (1986) that the galaxy density distribution they found is equally well consistent with de Vaucouleur's law with effective radius $\sim 3h^{-1}\text{Mpc}$. The de Vaucouleur's law has infinite density gradient but finite
Figure 4.6: X-ray surface brightness profile for $n = 1, 5$ in Model B and de Vaucouleurs mass density distribution. All the curves shown have the same total mass ($M_{\text{tot}} = 1.65 \times 10^{14} M_\odot$) and X-ray luminosity ($L_x = 7 \times 10^{44}$ ergs/s) and temperature ($T_g = 10^8$). The solid curve is for $n = 1$ and $n_e = 0.05 \times 10^{-3}$ cm$^{-3}$; the dotted curve is for $n = 5$ and $n_e = 0.24 \times 10^{-3}$ cm$^{-3}$; and the dashed curve is for de Vaucouleurs law with $n_e = 0.07 \times 10^{-3}$ cm$^{-3}$. 
Figure 4.7: Simulated flux density versus baseline plot for the SZ effect observed with the AT at 8.8 GHz for $n = 1, 5$ in Model B and de Vaucouleurs mass density distribution. All the curves shown have the same total mass ($M_{tot} = 1.65 \times 10^{14} M_{\odot}$) and X-ray luminosity ($L_x = 7 \times 10^{44}$ ergs/s) and temperature ($T_g = 10^8$). The solid curve is for $n = 1$ and $n_e = 0.05 \times 10^{-3}$ cm$^{-3}$; the dotted curve is for $n = 5$ and $n_e = 0.24 \times 10^{-3}$ cm$^{-3}$; and the dashed curve is for de Vaucouleurs law with $n_e = 0.07 \times 10^{-3}$ cm$^{-3}$.
density at the centre. It is an empirical law derived from surface brightness distribution of elliptical galaxies but it also has been known to fit regular clusters of galaxies. Surface density distribution for de Vaucouleur’s law is given by \( \Sigma(b) = \Sigma_0 \exp[-7.67(b/r_e)^{1/4}] \). The corresponding density distribution is not analytical and has to be solved numerically (Young 1976). If the mass distribution is described by de Vaucouleur’s law then \( \Delta T/T \) will not diverge at the centre. By assuming cluster gas is isothermal and using the cluster potential corresponding to the de Vaucouleur’s law (Young 1976), SZ profile and X-ray surface brightness distribution are derived for the same total mass, X-ray luminosity and temperature as the total mass density models in Model B shown in Figure 4.7, 4.6.

4.2.2 Non-isothermal Models

The most commonly used non-isothermal models are the polytropic model, i.e. the equation of state is given by \( P \propto \rho^\gamma \). By substituting this equation into the ideal gas law, we get \( \rho_g \propto \left( \frac{kT_g}{\mu m_p} \right)^{1/(\gamma-1)} \). The following equation can thus be deduced from the equation of hydrostatic equilibrium:

\[
\frac{1}{\rho_g} \frac{dP}{dr} = \frac{k}{\mu m_p} \frac{\gamma}{\gamma - 1} \frac{dT_g}{dr} = -\frac{d\phi}{dr} \tag{4.41}
\]

which in turn gives

\[
\frac{T_g}{T_{g0}} = 1 + \frac{\alpha - 1}{\gamma - 1} \frac{\phi(r)}{\phi_0} \tag{4.42}
\]

where \( \alpha = 1 + \frac{\mu m_p \phi_0 (\gamma - 1)}{kT_{g0} \gamma} \). For bound systems \( \phi_0 < 0 \) which implies \( \alpha > 1 \) and \( 0 < \gamma < 1 \), \( \alpha < 1 \) and \( \gamma > 1 \), or \( \alpha < 1 \) and \( \gamma < 0 \). Let us look at the 3 cases separately:

- \( \alpha > 1 \) and \( 0 < \gamma < 1 \). Temperature increases monotonically up to the tidal radius; gas density decreases monotonically to \( \rho_g (r_t)/\rho_{g0} = \alpha^{1/(\gamma-1)} \) at the tidal radius.

- \( \alpha < 1 \) and \( \gamma > 1 \). Both temperature and gas density decreases monotonically. Note that for \( \gamma > 5/3 \) it is convectively unstable (Sarazin 1986). Thus we will only consider cases where \( \gamma < 5/3 \). If \( \alpha < 0 \) then the gas
distribution terminates at a radius \( r_g < r_t \), where \( \phi(r_g)/\phi_0 = \alpha/(\alpha - 1) \). If \( 1 > \alpha > 0 \) then the gas density reaches a value of \( \alpha^{1/(\gamma-1)} \) at the tidal radius.

- \( \alpha < 1 \) and \( \gamma < 0 \). Temperature decreases monotonically and reaches zero at \( r_g < r_t \), but gas density increases monotonically and tends to infinity at \( r_g \). This is not physical.

The SZ profile is therefore given by

\[
\Delta T(b)/T = \alpha \int_{b/rc}^{R_t/rc} \frac{x dx}{\sqrt{x^2 - (b/rc)^2}} (1 + (\alpha - 1)(1 - \phi(x)/\phi_0))^{\gamma/(\gamma-1)} \]

Similarly, the X-ray surface brightness profile can be calculated.

The polytropic equation of state restricts the possible temperature profiles in the sense that it forces the strong coupling between temperature and gas density, which does not necessarily have any physical significance. Polytropic equations of state do not have any physical significance except in the limiting cases of isothermal and adiabatic states. Note that we do not need to be restricted to specific equations of state, nor do we have to be restricted to specific functional forms for the cluster total mass density. With better quality X-ray data from ROSAT and ASCA, it is necessary to investigate a more general form of the cluster potential and gas temperature, in the sense that we should not be restricted to the parameter space given by the specific parametric functional forms for the cluster mass and temperature distribution when we apply the modelling methods to real clusters. We should model the cluster potential and gas temperature profiles non-parametrically.

### 4.3 Multi-Wavelength Analysis

In this section, we will discuss the strategy in modelling the matter distribution and properties of intracluster medium using all the probes discussed in section 4.1. Consider an ideal case of a cluster that possesses a giant arc and the following observations have been obtained:

- Deep wide-field CCD image in sub-arcsec seeing conditions.
• High resolution X-ray surface brightness profile.

• X-ray spectrometry to obtain the gas temperature profiles.

• Measurements of the SZ effect.

• Optical spectroscopy of member galaxies.

From the deep optical images we can derive the cluster potential out to 1 Mpc from the cluster centre through analysis of weak gravitational lensing distortions of the background galaxies (Fahlman et al. 1994; Smail et al. 1994). The position of the giant arc can be used to normalise the cluster potential derived from the weak lensing analysis. This potential can then be used with the gas temperature profile to derive the X-ray surface brightness profile and compare that with the observed profile to test the validity of the assumption of hydrostatic equilibrium. If the assumption of hydrostatic equilibrium is valid then we can combine the lensing constraints with the X-ray data and SZ effect to further constrain the cluster potential and derive $H_0$ or the clumpiness of the ICM. Optical spectroscopy of member galaxies provide the line of sight velocity of the galaxies. With the information on the cluster potential, the galaxy velocity and position, we can derive some information on the galaxy orbits. Furthermore, once we have accurate estimates of the gas mass, stellar mass and the cluster total mass, we can calculate the baryonic fraction of the universe on cluster scales which has important cosmological implications (White et al. 1993).

We shall apply some of the less ambitious multi-wavelength analysis procedures to 2 clusters in the following two chapters.

4.4 References


Chapter 5

A Multi-Wavelength Study of MS2137-23

5.1 Introduction

For the cluster MS2137-23, observational data are available for us to apply more than one of the methods described in Chapter 4 and we will present new constraints on the cluster potential and parameters that fit all the data in various wavebands. The cluster MS2137-23 was first discovered in the *Einstein* Extended Medium Sensitivity Survey (EMSS) (Stocke *et al.* 1991; Henry *et al.* 1992; Gioia *et al.* 1990). It is a strong X-ray source with a luminosity $L_x \sim 10^{45}$ ergs s$^{-1}$ and has a redshift of $z = 0.313$ with a central dominant cD galaxy. A giant gravitational ‘arc’ (15.5” from its centre), a radial arc (5” from its centre) and a number of smaller ‘arclets’ have been found by Fort *et al.* (1992). This cluster is unique in that it has both a giant tangential arc and a radial arc which enabled Mellier *et al.* (1993) to model the cluster potential and successfully predict the positions of the arclets. The presence of the radial arc enables the cluster potential in the centre of the cluster to be well constrained, since the position of the radial arc depends on the size of the core radius (Miralda-Escude 1993). The cluster velocity dispersion and core radius
thus deduced were $950 - 1250 \text{ km s}^{-1}$ and $35 - 55 \text{ kpc}$ respectively. The cluster field is also devoid of “strong” (> 1mJy at 8.8GHz) radio sources which made it a good candidate for observations of the Sunyaev-Zel’dovich effect (Sunyaev & Zel’dovich 1972) at 8.8GHz with the Australia Telescope. We have also obtained deep CCD image of this cluster at the AAT in an attempt to measure the large scale cluster potential from the weak gravitational lensing effects using the method of Broadhurst et al. (1994). Given the unique properties of this cluster and the data available in the optical, radio and X-rays, the cluster can be used as an excellent example to demonstrate how a self-consistent mass model can be constructed from all the data in different wavebands by using the different methods to model the mass distribution.

In this chapter, we present new observations of the galaxy distribution and the SZ effect and then determine which sets of cluster parameters, if any, would simultaneously fit the optical, X-ray and radio data, given the cluster potentials determined from modelling the gravitational arcs. In 5.2 we describe the method of analysis; details of the new results from optical and SZ effect observations are presented in 5.3 and 5.5. In 5.4 we reanalyse the existing X-ray data from the Einstein archive. The results of the analysis is given in 5.6.

5.2 Modelling of Cluster Parameters

The cluster potential has been extensively modelled using information on the gravitational arcs first studied by Mellier et al. (1993) and then by Miralda-Escude (1994). Mellier et al. showed that the ellipticity and orientation of the projected dark matter potential follows that of the cD envelop and thus the light distribution. The best model for the projected cluster potential gives the ellipticity of the potential as $\epsilon = 0.08 \pm 0.03$, core radius $\theta_0 = \theta_{\text{c}}^{\text{arc}} + 0.5$ (i.e. $r_c \sim 0.045\text{Mpc}$) and the line of sight velocity dispersion $\sigma_{\text{los}}$ in the range 1250 to 950 km/s for a background lensed source redshift in the range 0.5 to 3. Since the ellipticity of the projected potential is small, we can approximate it by a circular potential and deproject it back to the three-dimensional potential,
Figure 5.1: A B-band close up image of the cluster MS2137-23 showing the giant arc and the radial arc and some arclets (AAT prime-focus image). The giant arc is labeled as “A1” and the radial arc is labeled as “A2”.

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given by
\[ \Phi(r) = \frac{4}{3} \sigma_{\text{los}}^2(0) \ln[1 + \left(\frac{r}{r_c}\right)^2] \] (5.1)
where \( \sigma_{\text{los}} \) is the line of sight velocity dispersion and \( r_c \) is the core radius. We will only consider the spherically symmetric cases.

Miralda-Escude (1994) examined a wider range of profiles for cluster mass density distribution constrained by the position of the radial and tangential arcs. We will just consider the density profiles allowed by the positions of the lensing arcs from Miralda-Escude (1994).

Model 1:
\[ \rho_{\text{tot}}(r) = \frac{\rho_0}{1 + r^2/r_c^2} \] (5.2)
where the core radius \( r_c = 3'' \) (i.e. 0.017 Mpc) if we do not consider the cD as a separate mass component on top of the dark matter distribution. This was derived from the ratio of the distance of the radial arc from the centre to the distance of the other image of the radial arc from the centre.

Model 2:
\[ \rho_{\text{tot}}(r) = \frac{\rho_0}{(1 + r/r_c)^2} \] (5.3)
with \( r_c = 2.5'' \) (i.e. 0.014 Mpc).

Model 3:
\[ \rho_{\text{tot}} = \frac{\rho_0}{(r/r_c)[1 + (r/r_c)^2]^{3/4}} \] (5.4)
with \( r_c = 10'' \) (i.e. 0.057 Mpc).

Model 4:
\[ \rho_{\text{tot}} = \rho_0 \frac{3 + (r/r_c)^2}{(1 + (r/r_c)^2)^2} \] (5.5)
with \( r_c = 5'' \) (i.e. 0.028 Mpc).

Model 4 is just the same as that considered in Mellier et al. (1993), however, Miralda-Escude claimed a core radius of 5'' rather than 8'' as given by Mellier
et al. As stated in Miralda-Escude (1994), the discrepancy is because the Mellier et al. model does not correctly reproduce the distance of the radial arc from the centre. This distance is, however, difficult to determine accurately because part of the radial arc is embedded in the halo of the central cD galaxy.

We can now deduce the X-ray surface brightness profiles and SZ effect profiles from these density profiles assuming that the cluster gas is in hydrostatic equilibrium. From the equation of hydrostatic equilibrium and ideal gas law, we have

$$\frac{d \ln n_e(r)}{dr} + \frac{d \ln T_g(r)}{dr} = - \frac{\mu m_p}{kT_g(r)} \frac{d \phi(r)}{dr}$$

where $n_e$ is the electron density, $T_g$ is the gas temperature, $\phi$ is the cluster potential and $\mu$ and $m_p$ are the mean molecular weight and proton mass respectively. Given the above density profiles, we can solve Poisson’s equation and obtain the corresponding potentials which would in turn give us the gas density distribution from the above equation. In the case of isothermal gas, the gas density is simply given by

$$\frac{n_e(r)}{n_e(0)} = \exp\left(-\frac{\mu m_p}{kT_g} [\phi(r) - \phi_0]\right)$$

Recent observations with ASCA (e.g. Mushotzky in “New Horizon of X-ray Astronomy – first results from ASCA”) and ROSAT PSPC spatially resolved temperature observations (Henry et al. 1993) have shown that the cluster gas temperature is mostly isothermal on the large scales for various clusters. Also studies by Durret et al. (1994) of a number of clusters using the Einstein IPC data have shown that isothermality is the preferred solution. So we will concentrate on the isothermal cases.

The X-ray surface brightness distribution is given by

$$S_x = 2 \int_b^{R_t} \frac{\epsilon(r)rdr}{\sqrt{r^2 - b^2}}$$

where $b$ is the projected radius and $\epsilon$ is the X-ray emissivity of mainly thermal bremsstrahlung radiation. The emissivity is proportional to $n_e^2$ and can be calculated using a Raymond-Smith code (Raymond & Smith 1977). We then model the observed X-ray surface brightness distribution by convolving the model X-ray surface brightness profile with the point spread function of the instrument.
The SZ effect is given by

$$\frac{\Delta T_r}{T_r} \propto \int n_e(r)T_g(r) \frac{rdr}{\sqrt{r^2 - b^2}}$$

(5.9)

where $b$ is the projected radius, and $T_r$ is the radiation temperature of the cosmic microwave background. We have used the Australia Telescope which is an interferometric array to measure the SZ effect. We can model the observed visibilities by Fourier transforming the SZ brightness profile and sampling the data in the same way as a radio synthesis telescope samples the data in the uv-plane (see Chapter 2 & 3).

The densities and thus the potentials are only well constrained by the gravitational lensing constraints within the radius of the giant arc. To effectively compare the cluster potential with the observed X-ray and SZ effect data, it is necessary to extend the potential somewhat further especially since the resolution of the SZ effect and X-ray data available are much lower than the radius of the giant arc. Weak lensing methods provide one of the best model independent means of obtaining the profile of the potential in the outer regions of the cluster (Kaiser & Squire 1994, Smail et al. 1994, Broadhurst et al. 1994). Unfortunately, at the present stage we do not have sufficiently good optical data to measure the cluster potential using the weak lensing methods. However, one other way of constraining the outer parts of the potential is to measure the distribution of galaxy counts in the cluster. Since it has been shown by Mellier et al. (1993) that the cluster mass distribution follows light in the central parts of the cluster, we can assume that this applies to the outer parts of the mass distribution. Recent studies by Durret et al. (1994) found that for the 12 clusters they studied the total binding mass distribution follows that of the light distribution.

5.3 Optical Photometry

5.3.1 Observations

The cluster MS2137-23 was observed with the Anglo-Australian Telescope (AAT) in prime-focus with a 1K Tektronix CCD (0.39" pixel\(^{-1}\)) in August
1993. The cluster was observed at 4 pointings around the cluster centre so that the final mosaiced image is $\sim 10' \times 10'$. A number of exposures were taken around each pointing centre by dithering $\sim 10''$ between exposures. It was observed for $3 \times 600$ sec in the B-band and $2 \times 600$ sec in the I-band at each pointing, so that the central region of the cluster accumulates a total exposure time of 7,200 sec in B and 4,800 sec in I. A $6.3' \times 5.8'$ offset field $\sim 25'$ to the east of the cluster was also observed for $6 \times 600$ sec in B and $4 \times 600$ sec in I. The exposures were taken in turn by rotating around the 4 fields around the cluster centre and the offset field to minimize any systematic differences between the cluster fields and the offset field caused by any effect that drifts with time. Standard stars from Landolt (1992) were observed each night. The KPNO interference filters were used. The average seeing was 1.6''.

5.3.2 Image processing

The basic image processing was done using the IRAF package. Since a $\sim 10''$ dithering was applied between exposures of the same field, we have of order 40 and 25 non-coincident B and I-band frames per night respectively. A flatfield was obtained by stacking all the exposures with the same filter in one night and taking the median with a sigma-clipping (3$\sigma$) algorithm applied. The sigma-clipping algorithm ensures that pixels affected by comic rays or bright objects do not bias the median calculation. The bias correction for each frame was estimated by fitting a first order polynomial to the overscan region. Each frame was then bias subtracted and divided by the flatfield; the overscan region was trimmed off and the vignetted corners were masked out. The contribution from the dark current is negligible since the night sky contributes a few $\times$ 1000 ADU/pix (Astronomical Data Units) in 600 sec whereas the dark current contributes $\sim 12$ ADU/pix. All the frames in each colour taken towards the cluster were adjusted to a common reference coordinate (but only adjusted to the nearest pixel), stacked together and co-added by taking the median. Similarly, all the frames towards the offset field were co-added. The final combined B and I images of the cluster field and the offset field are shown in Figure 5.2, 5.3, 5.4.
Figure 5.2: B-band image of the cluster field MS2137-23. The circle is centred on the cluster.
Figure 5.3: I-band image of the cluster field MS2137-23 overlayed with contour image of a AT 4.5 GHz observation. The contours are $80\mu$Jy $(5,10,20,30)$. Note that the North-South elongation of the radio sources is not real, it just reflects the shape of the beam.
Figure 5.4: B-band image of the offset field.
5.3.3 Photometric Calibration

The standard stars in the regions T Phe and PG 2213-006 from Landolt (1992) were used for flux calibration. The *apphot* task in IRAF was used to carry out aperture photometry on the standard stars. Corrections for the atmospheric extinction of 0.335 mag/airmass in B and 0.144 mag/airmass in I (Schroeder and Bessell private communications) were applied. The AAT filter plus detector system was then transformed to the standard Cousins photometric system (1976). It is known that there is a mismatch between the Landolt and the Cousins system (E-region standards). The standard magnitudes were first transferred to the Cousins system using the corrections given in Menzies et al. (1991). The transformation between the instrumental magnitudes and that of the standard Cousins system was found to be

\[
B = b + 0.065118 \times [1.0874 \times (b - i) + 0.73854] + 25.061 \quad (5.10)
\]

\[
I = B - [1.0874 \times (b - i) + 0.73854] \quad (5.11)
\]

where B and I are the magnitudes on the Cousins system, b and i are the instrumental magnitudes.

5.3.4 Object detection

The Faint Object Classification and Analysis System (FOCAS) version 3.3 was used for object detection (Jarvis and Tyson 1981, Valdes 1982) on the final co-added images. Each of the 600 sec exposures were airmass corrected before co-adding. The FOCAS package produces a list of parameters for the objects detected, such as the position, shape and photometric information. The detection threshold was set to be 2.5 times the rms sky noise and an object is considered real if it has more than 6 connected pixels above the threshold. Merged objects are then split into individual fragments. The sky noise is different in the various regions of the final mosaiced cluster images because of the differing depth of total exposure. The rms sky noise in the cluster B and I frames varies from 6 to 14 $e^-/pix$ and from 18 to 39 $e^-/pix$ respectively; and the offset B and I frames have a rms sky noise of 8 and
22.5 $e^-/pix$ respectively. Although FOCAS calculates the sky noise around each individual object independent of the user input sky noise, the detection threshold is set to be 2.5 times the user input sky noise which is assumed to be constant across the image. We set the sky noise to be that of the offset frames for both the B and I cluster and offset frames. The detected objects are then classified into galaxies, stars and noise using the resolution classifier (Valdes 1982) in FOCAS.

Figure 5.5 and 5.6 show the number counts in I and B, respectively, of all the objects detected (except those classified as noise) in the offset fields along with the expected star counts in the direction of 0014+16 which is at a similar galactic latitude to the cluster. The object detection is complete down to $I = 22.5$ and $B = 24.5$. As shown in Figure 5.6 and 5.5, the contribution from galactic stars is negligible at $B > 23$ and $I > 21$. To check if the classification has worked properly, we compare the counts of stars as classified by FOCAS for our offset field with the predicted star counts in the direction 0014+16 (i.e. $b' \sim 46^\circ$) from the Bahcall & Soneira Galaxy model (Bachall and Soneira 1981). Figure 5.7 shows that the FOCAS determined star counts for the cluster field and the offset field agrees well with the predicted counts at a similar galactic latitude for $I < 21.5$. Thus the star-galaxy separation has worked very well in the magnitude range where the contamination from the stars is not negligible, since the contribution from stars is insignificant at $I > 21$ (see Figure 5.5).

Similarly, we compare the galaxy counts (by taking all the objects that were not classified as either noise or stars) obtained in the offset frame with of the field galaxy counts determined by Hall & Mackay (1984). Figure 5.8 shows that the galaxy counts in the offset frame agrees reasonably well with the field galaxy counts determined by Hall & Mackay and there is a clear evidence of an overdensity of galaxies in the cluster frame (cf. Figure 5.7 which shows that there is no significant difference between the star counts in the cluster frame and the offset frame). Is the offset field located in a local void? We have compared the number counts in both B and I for our offset field with that

---

1 The Hall & Mackay (1981) I counts were obtained from a region 20% smaller than our offset field, thus the uncertainties in their counts are larger compared with our offset field.
Figure 5.5: I-band counts for galaxies and stars in 2 offset "blank" fields towards 2139-24 ($b' \sim -47^{\circ}$; filled circles) and 2258-35 (open circles) along with the predicted star counts towards 0014+16 ($b' \sim -46^{\circ}$; stars) from Bahcall & Soneira (1981) models.
Figure 5.6: B-band counts for galaxies and stars in 2 offset “blank” fields towards 2139-24 (filled circles) and 2258-35 (open circles) along with the predicted star counts towards 0014+16 (stars) from Bahcall & Soneira (1981) models.
Figure 5.7: Star counts in I for the cluster (filled circles) and offset (open circles) frames compared to the predicted counts (stars) towards 0014+16 ($b'' \sim -46^\circ$) by Bahcall and Soneira (1981). The cluster and the offset field are at a galactic latitude of $b'' \sim -47^\circ$. 
Figure 5.8: I-band galaxy counts for both the cluster (filled circles) and offset (open circles) frames along with the I-band counts of field galaxies (stars) from Hall & Mackey (1981).

of another offset field $\sim 10^\circ$ away taken during the same nights. Figure 5.6 and 5.5 show that the number counts from the two fields agree very well and thus our offset field is not likely to be located in a local void. The cluster frame has a significant overdensity of galaxies.

The photometric transformation shown in the above section was applied to the FOCAS produced magnitudes with an extra $2.5 \log(600)$ added to the zero point for the integration time. The FOCAS aperture magnitude was checked
against that produced by the task `apphot` in IRAF for the standard stars and they agree to within ±0.006 mag. Extinction from the Galaxy in the direction of MS2137-23 was $A_B \sim -0.03$ from Burstein and Heiles (1984) and we have not included this correction in our final magnitudes shown in the catalogue. The objects detected in B and I frames were matched and considered to be the same objects if the X and Y position difference is $< \pm 1.5"$.

We examine the accuracy of the photometry by comparing our B and I aperture magnitudes in the central region of the cluster with that determined by Fort et al. (1992). The B and I magnitudes of the galaxies shown in Table 1 of Fort et al. (1992) are plotted in Figure 5.9 against the aperture magnitude determined in our present study. For both B and I magnitudes there is a systematic difference of ~0.3 mag between our measurement and that of Fort et al. (1992), with ours being systematically fainter. This discrepancy is most likely due to the difference in the seeing conditions and the size of the aperture used for the estimate of the magnitude.

### 5.3.5 Galaxy Number Density Distribution

The task of estimating the distribution of the stellar mass in a cluster of galaxies is a difficult one without the detailed knowledge of the cluster membership through spectroscopy. Since we do not have spectroscopy of enough galaxies, we have obtained observations in two colours for both the cluster and the offset field to separate out the most probable cluster members. Figure 5.10 and 5.11 shows the B-I colour versus the I magnitude for the cluster field and the offset fields respectively. By comparing the colour-magnitude (CM) diagrams, we can see that there is an over-density of galaxies with B-I colour between 3 and 4 in the cluster field which is where the E/S0 sequence galaxies in the cluster lie. We can pick out the E/S0 sequence galaxies from the CM diagram and count the number of galaxies as a function of radius. By selecting galaxies within this colour range, we are maximising the fraction of cluster members amongst all the galaxies selected. Thus the galaxies brighter than $I=22.5$ (above the completeness limit for the photometry) with B-I colour in the range 3 to 4 were counted within equal area annuli (and background
Figure 5.9: The B and I aperture magnitudes determined by FOCAS in our present study is plotted against that of Fort et al. (1992) for the same galaxies. The open circles are the I magnitudes and the filled circles are the B magnitudes. The solid curve is where the points would lie if the magnitudes determined by us and that of Fort et al. 1992 are the same. The dotted curve is shifted up from the solid curve by 0.3 mag.
Figure 5.10: B-I colour versus the I magnitude (aperture magnitude) for the cluster field towards MS2137-23
Figure 5.11: B-I colour versus the I magnitude (aperture magnitude) for the offset field.
subtracted) and plotted against an effective average radius (Couch & Newell 1984) \( r_{\text{eff}} = \frac{2(r_2^3 - r_1^3)}{3(r_2^2 - r_1^2)} \) at which the number of objects per pixel is equal to the average in the annulus assuming that the counts versus radius is linear over the annuli. A background galaxy number density was determined from the offset field 25′ away from the cluster centre and subtracted from the galaxy density profile. This background galaxy number density determined from our offset field agrees with that from another “blank” field 10° away (see Figure 5.5). We can now compare the shape of the projected cluster total mass distribution extrapolated from the central region determined from the lensing arcs with the galaxy density distribution. Figure 5.12 shows the galaxy density profile plotted with the projected cluster total mass distribution from the various models described in the previous section. The galaxy distribution is rather unusual, it has a sharp core on top of a very extended flat distribution. The projected cluster mass distributions have been scaled so that it matches the galaxy count distribution at a radius of 150″ (or 0.85 Mpc) from the centre. For consistency, the model profiles were “background” subtracted by its value at 25′ from the cluster centre. The scaled projected cluster mass profile agrees with the galaxy distribution. Thus we cannot exclude any of the models of binding mass profile given in section 5.2 by assuming that mass-follows-light.

5.4 X-ray Data

The cluster MS2137-23 was first discovered in the EMSS to have a high X-ray luminosity (Stocke et al. 1991, Gioia et al. 1990, Henry et al. 1992). The EMSS source flux were calculated in a 2.4′ × 2.4′ detection cell. The counts in the detection cells were corrected for vignetting, mirror scattering, and absorption by the Galaxy and converted to flux in the detection cell by assuming a 6 keV Raymond-Smith spectrum. Henry et al. then corrected for the extended flux that fell outside the detection cell and calculated the total luminosity for each of the EMSS clusters by assuming that all the clusters have a X-ray surface brightness that can be described by a “\( \beta \)-model”, i.e.

\[
S_x(\theta) = S_x(0)[1 + (\theta/\theta_0)^2]^{(-3\beta+1/2)}
\] (5.12)
Figure 5.12: Galaxy density distribution for MS2137-23: galaxy counts in equal area bins versus effective radius in arcsec from the central cD galaxy. The filled circles are the background subtracted galaxy counts in each radial bin that are above the I completeness limit of 22.5 with B-I colour between 3 and 4. The curves are the projected total mass distribution for each of the model scaled to the observed galaxy counts at a radius of 150". The solid curve is for Model 4; the dotted curve is for Model 1; the dashed curve is for Model 2; the dot-dashed curve is for Model 3.
with $\beta = 2/3$ and core radius $\theta_0$ corresponding to 0.25 Mpc. The total X-ray luminosity thus deduced was $15.58 \times 10^{44}$ ergs/s in the 0.3-3.5kev band for MS2137-23. The assumed core radius of 0.25 Mpc is clearly in contradiction to that deduced from the positions of the gravitational lensing arcs (Mellier et al. 1993). We will first examine whether Henry et al.'s assumption of a universal core radius of 0.25 Mpc is valid in the case of MS2137-23 by reanalysing the unbinned and unsmoothed *Einstein* IPC data. This cluster falls near the edge of the IPC detector where the point spread function is relatively large. The point spread function was obtained from a bore-sight calibration image of Cyg X-3 at a similar position on the IPC detector to the cluster. Figure 5.13 shows the radially averaged observed surface brightness of both the Cyg X-3 and MS2137-23 image as well as the expected surface brightness assuming the "$\beta$ model" above convolved with the Cyg X-3 point spread function (PSF). The slight difference between the normalised X-ray profile of Cyg X-3 and MS2137-23 is because of the energy difference between the two sources, with Cyg X-3 being harder in the X-ray. The PSF of a X-ray detector changes with the energy of the incoming photons.

To correctly simulate the observations we convolved a circularly symmetric beta model with the PSF and add a constant background to each pixel. The background was determined from the regions of the IPC detector at similar positions relative to the detector centre as the cluster itself but devoid of any obvious X-ray sources. Regions on the detector affected by the "ribs" were masked out from both the observed and simulated images.

A Kolmogorov-Smirnov test rejected the hypothesis that the two images were the same at a 99.98% confidence level for the "$\beta$ model" described above with a core radius of 0.25 Mpc. The test was run for a series of core radii and we can place a conservative upper limit of 0.12 Mpc for the core radius of the cluster in the "$\beta$ model" to be consistent with the X-ray data. This upper limit is still consistent with the core radius of 0.045 Mpc deduced from the gravitational lensing methods. Thus the total X-ray luminosity quoted for the cluster in Henry et al. (1992) is not correct. We will in the following analysis,

---

1 This data was obtained from the SAO archives.

2 This data was obtained from the SAO archives.
Figure 5.13: Normalised, radially averaged and background subtracted photon counts versus radius from centre of the X-ray emission for MS2137-23 (open circles) and Cyg X-3 (filled circles). The solid curve corresponds to the normalised X-ray surface brightness from a “$\beta$ model” with $\beta = 2/3$ and core radius 0.06 Mpc (or 11’’); the dotted curve gives the normalised X-ray surface brightness with core radius 0.25 Mpc (or 44’’).
normalise the X-ray surface brightness profile using the flux of the cluster in the 2.4′ × 2.4′ detection cell and deduce the total X-ray luminosity for each model cluster potential (see Table 5.2 & Figure 5.14) considered in section 5.2.

5.5 Observations of the SZ Effect

We searched for the SZ effect in the cluster MS2137-23 with the Australia Telescope at 8.8 GHz in the most compact configuration. Details of the observations and data reduction procedures have been discussed in Chapter 2 & 3. There were 8 discrete radio sources in the field with a peak apparent flux density in the 8.8 GHz image of \( \sim 0.9 \) mJy. The flux densities and spectral indices of the radio sources, I band magnitude and B-I colours of the possible optical identification of the radio sources are shown in Table 5.1. The radio source flux densities were determined from all the data from the 122m array except the shortest baseline (30m). These discrete sources were then subtracted from the data and the resulting flux density at the centre of the cluster is plotted against the baseline length in Figure 5.15. We also observed the cluster at 4.5 GHz and found that there was a weak extended source close to the centre of the cluster. The maximum apparent flux density of the source was 0.180 ± 0.033mJy at 8.8 GHz. The flux density of the source has been quoted as a maximum since the position of the source is such that its first negative sidelobe falls right at the centre of the cluster where one expects the SZ effect to be strongest. Because of the presence of this weak extended source and the sidelobes induced weighting between the extended source flux and the SZ decrement, the interpretation of the SZ effect is less clear but we can consider two extremes: i) Fitting the maximum possible flux in the extended radio source; By fitting the radio source only, the fitting routine would inevitably maximise the flux of the radio source and minimise any SZ decrement. The minimum SZ decrement thus determined is 0.

ii) Fitting the minimum possible flux in the extended source; By assuming that there is zero flux in the extended source, we measure a maximum SZ decrement of \(-0.13 \pm 0.04\) mJy at the cluster centre as shown in Figure 5.15. It is difficult to separate the flux densities of the extended source from the SZ
Table 5.1: Opt. ID of Radio Sources in MS2137-23

<table>
<thead>
<tr>
<th>Source</th>
<th>RA</th>
<th>DEC</th>
<th>(\Delta \alpha_{R-O})</th>
<th>(\Delta \delta_{R-O})</th>
<th>(S_{8832})</th>
<th>(S_{4480})</th>
<th>(\alpha)</th>
<th>(m_I)</th>
<th>B-I</th>
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<tbody>
<tr>
<td>1</td>
<td>21 40 15.2</td>
<td>-23 39 40</td>
<td>1.4</td>
<td>0.6</td>
<td>0.88</td>
<td>1.5</td>
<td>-0.78</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>2</td>
<td>21 40 12.4</td>
<td>-23 39 40</td>
<td>4.4</td>
<td>3.3</td>
<td>0.45</td>
<td>0.72</td>
<td>-0.69</td>
<td>24.30</td>
<td>2.20</td>
</tr>
<tr>
<td>3</td>
<td>21 40 13.2</td>
<td>-23 42 14</td>
<td>0.9</td>
<td>-3.7</td>
<td>0.22</td>
<td>1.1</td>
<td>-2.3</td>
<td>20.15</td>
<td>4.63</td>
</tr>
<tr>
<td>4</td>
<td>21 40 17.9</td>
<td>-23 36 31</td>
<td>1.0</td>
<td>-1.4</td>
<td>0.88</td>
<td>0.64</td>
<td>0.48</td>
<td>18.34</td>
<td>3.42</td>
</tr>
<tr>
<td>5</td>
<td>21 40 02.3</td>
<td>-23 36 17</td>
<td>-2.1</td>
<td>0.9</td>
<td>1.0</td>
<td>2.1</td>
<td>-1.1</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>C1</td>
<td>21 40 24.6</td>
<td>-23 42 39</td>
<td>2.0</td>
<td>-1.4</td>
<td>2.3</td>
<td>5.1</td>
<td>-1.2</td>
<td>19.13</td>
<td>4.42</td>
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<td>21 40 24.3</td>
<td>-23 44 13</td>
<td>-1.9</td>
<td>-4.5</td>
<td>1.7</td>
<td>8.7</td>
<td>-2.4</td>
<td>21.42</td>
<td>3.77</td>
</tr>
<tr>
<td>C3</td>
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<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
</tbody>
</table>

Col. (1) gives the source number; Col. (2) & (3) gives the RA and DEC of the radio source in J2000.0 coordinate; Col. (4) & (5) gives the difference between the radio position and its optical ID in arcsec; Col. (6) & (7) gives the 8.8 GHz and 4.5 GHz flux densities; Col. (8) gives the I-band magnitude of the optical IDs; Col. (9) gives the B-I colours.

5.6 Results and Discussion

5.6.1 Cluster Parameters

We will now examine the range of parameters for the cluster gas that fits the X-ray data given the model cluster total mass distributions defined in section 5.2. Since the gravitational lensing arcs constrain the core radius and the central mass density, the relevant free parameters are the cluster temperature and central electron density. For a given cluster gas temperature, the central electron density of the ICM is determined from the X-ray flux within the 2.4' × 2.4' detection cell. The cluster gas temperature can in turn be constrained by the X-ray surface brightness profile and the measurements of the SZ effect which is a measure of the pressure along the line of sight. Figure 5.14 shows the model X-ray surface brightness after convolution with the PSF for Model 1 in section 5.2 along with the observed X-ray surface bright-
ness. As is shown in Figure 5.14, the X-ray surface brightness data constrains the temperature of the gas to be $T_g \leq 1 \times 10^8 \text{K}$.

If all the X-ray emission is produced by the hot intracluster gas, then the strong X-ray luminosity and the rather low SZ effect implies that the gas must have relatively high density and low temperature. In the case of a spherically symmetric cluster with isothermal gas, the constraints on the cluster temperature and central electron density by the observed upper limits on the absolute value of the flux density of the SZ effect at the shortest baseline are similar for the various models that we have considered. The temperature is constrained to be $\leq 7 \times 10^7 \text{K}$ and the central electron density $\geq 0.2 \text{cm}^{-3}$ (see Table 5.2 for detail). Figure 5.15 shows the SZ signal deduced from Model 1 described in section 5.2 for different temperatures along with the observed upper limits. Table 5.2 shows the cluster parameters such as the gas temperature $T_g$ and the central electron density $n_e(0)$ for various models that satisfy the constraints from the gravitational lensing, X-ray surface brightness and SZ effect upper limits. We have tabulated for each model the highest allowed temperature and calculated the expected total X-ray luminosity, central decrement of the SZ effect, the flux density of the SZ effect at 8.8 GHz at the shortest baseline and the central cooling time. In these models we have assumed that the gas extends to $r_t = 10' \text{ (or 3.4 Mpc)}$ and the total X-ray luminosity was integrated out to $10'$. For the 4 models considered in section 5.2, the X-ray surface brightness is rather peaked for $T_g < 7 \times 10^7 \text{K}$ and thus has negligible dependence on the choice of $r_t$. For example, for Model 4 with $T_g = 7 \times 10^7 \text{K}$, the total X-ray luminosity $L_x = 1.9 \times 10^{45} \text{ergs/s}$ if the gas extends to $1'$ (c.f. $L_x = 2 \times 10^{45} \text{ergs/s}$ out to $10'$).

The high density and relatively low temperature of the cluster central gas needed to satisfy the constraints set by the optical, X-ray and radio data means that the cooling time at the centre of the cluster is much smaller than a Hubble time (see Table 5.2), thus it is likely that there is cooling flow present at the centre of the cluster if there is no sufficient heating. Note that the cooling time at the centre of this cluster is smaller than the typical values of a few $\times 10^8 \text{yrs}$ (Arnaud 1988).
Figure 5.14: X-ray surface brightness profile. The filled circles are the measured surface brightness. The curves are for model 1 with various temperatures: $T_g = 2 \times 10^8$ K (dot-dashed); $T_g = 1 \times 10^8$ K (solid); $T_g = 7 \times 10^7$K (dashed); $T_g = 6 \times 10^7$K (dotted). Note the small scale wiggles are due to edge effects in the numerical computation of the Hankel transform.
Figure 5.15: Flux densities versus the projected baselines. The open circles are the flux densities at the cluster centre after the subtraction of discrete radio sources. The value at the shortest baseline is an upper limit to the absolute flux density of the SZ effect at the cluster centre. The 3 curves give the simulated flux density of the SZ effect from Model 1 with 3 different $T_g$ and the corresponding electron central density that fits the observed X-ray surface brightness profile: $T_g = 1 \times 10^8$K (dotted); $T_g = 7 \times 10^7$K (dot-dashed); $T_g = 6 \times 10^7$K; (solid).
Table 5.2: Cluster Parameters

<table>
<thead>
<tr>
<th>Model</th>
<th>$r_c$ (kpc)</th>
<th>$T_g$ ($\times 10^7$K)</th>
<th>$L_x(0.3-3.5)_{keV}$ ($\times 10^{45}$ergs/s)</th>
<th>$n_e(0)$ (cm$^{-3}$)</th>
<th>$\Delta T_g(0)$ (mK)</th>
<th>$S_{3cm}$ (mJy)</th>
<th>$t_{cool}(0)$ ($\times 10^8$ yrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>17</td>
<td>7</td>
<td>1.9</td>
<td>0.16</td>
<td>-2.2</td>
<td>-0.13</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>14</td>
<td>7</td>
<td>2.0</td>
<td>0.14</td>
<td>-2.0</td>
<td>-0.14</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>57</td>
<td>7</td>
<td>2.0</td>
<td>0.23</td>
<td>-2.3</td>
<td>-0.12</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>28</td>
<td>7</td>
<td>2.0</td>
<td>0.19</td>
<td>-2.6</td>
<td>-0.12</td>
<td>4</td>
</tr>
</tbody>
</table>

Col. (1) gives the model number; Col. (2) gives the core radius; Col. (3) gives the gas temperature; Col. (4) gives the total X-ray luminosity deduced; Col. (5) gives the central electron density; Col. (6) gives the expected central decrement (SZ effect); Col. (7) gives the flux density of the SZ effect from 30m baseline at 8.8 GHz; Col. (8) gives the central cooling time.

Miralda-Escude & Babul (1994) found that the X-ray surface brightness profiles are broader than that deduced from gravitational lensing models with gas in hydrostatic equilibrium for some lensing clusters. Gravitational lensing models require the gas to have temperature higher than the observed value for the gas to be in equilibrium. Consider one of the compact clusters examined in Miralda-Escude & Babul (1994), A2218, if it were observed by the *Einstein* in the same way as MS2137-23 was observed then its X-ray profile would be resolved. Thus the X-ray emission from MS2137-23 is narrower than that of A2218. Optically, MS2137-23 is a much poorer cluster than A2218 with a rather peaked galaxy density distribution on top of a very broad distribution of galaxies.

Since our analysis shows that the X-ray emission is unresolved in the *Einstein* image, it is still possible that the X-ray emission is from a point source such as an AGN. If the emission is partly due to an AGN then the result of our limit on the SZ effect would be consistent with there being very little hot gas in the cluster.

The radio source close to the central galaxy of the cluster has a flux density of 0.9 mJy at 8.8 GHz which at the cluster redshift corresponds to a radio luminosity similar to M87 in Virgo. The X-ray luminosity of emission

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3This radio source is closest to the galaxy "G3" in Fort et al. (1992), it is more than 5" from the centre of the cD.
from M87 is 70% of the total Virgo X-ray luminosity (Böhringer et al. 1994). Böhringer et al. found that the X-ray component associated with M87 has a temperature of $2.4^{+0.3}_{-0.2}$ keV (i.e. $2.8 \times 10^7$K) and the overall cluster temperature is $\sim 3$ keV. It is possible that the cluster MS2137-23 has two components contributing to the X-ray emission similar to the M87/Virgo system, where the X-ray emission consists of a central cool and dense component due to the central galaxy and a hot and sparse component due to the cluster gas. The central galaxy in MS2137-23 has an extended halo in the optical similar to M87.

### 5.6.2 Matter Distribution

In this section, we will consider the mass distribution of the 3 different components of matter in the cluster – the total mass of galaxies, the intracluster gas and the dark matter, and the mass fraction of the baryons in the cluster.

**Stellar mass**

We will estimate the total stellar mass in the galaxies by estimating the total luminosity and assuming an average $M/L$. To estimate the total B-band luminosity, we first convert the apparent magnitude to the absolute luminosity for all the galaxies in the cluster field and the offset field brighter than $m_B = 24$ by assuming that all the galaxies are at the cluster redshift. The absolute luminosity is given by

$$L/L_\odot = 10^{0.4[M_\odot - (m_B - 5 \log D_L - 25 - K_B - A_B)]}$$

where $M_\odot = 5.48$ is the absolute B magnitude of the Sun, $m_B$ is the apparent B magnitude, $D_L$ is the luminosity distance, $K_B$ is the mean K-correction and $A_B$ is the extinction from the galaxy. The K-correction is the magnitude difference between the redshifted and the rest frame spectral energy distribution when observed through a fixed spectral band (Pence 1976). We have adopted the mean $K_B = 0.8$ for all galaxy types at $z = 0.313$ from Pence (1976). The difference between the total luminosity in the cluster frame and the offset frame for a given area gives the integrated B-band luminosity of the stellar light from the cluster. To obtain the total integrated stellar luminosity, we
use a B-band luminosity function to estimate the contribution to the total luminosity of the galaxies below the completeness limit of our detection. A pure Schechter form (Schechter 1976) for the luminosity function was adopted:

$$n(L)dL = n^*(L/L_*)^\alpha \exp(-L/L_*)d(L/L_*)$$

(5.14)

where $L_*$ = $3.9 \times 10^{10} L_\odot$ (or $M_B = -21.0$) and $\alpha = -1.15$ for all galaxy types (Lilly 1993). To avoid overestimating the total luminosity of the cluster because of any bright foreground galaxies, we have excluded any galaxy that is brighter in apparent magnitude than the central cD galaxy. The total stellar mass is then estimated by assuming a $M/L_B \sim 11$ for E/SO type galaxies.

Table 5.3 shows the projected stellar mass, gas mass and total binding mass of the cluster estimated within 0.088, 0.57 and 1.36 Mpc. The projected masses are given instead of the 3-D masses to avoid any uncertainties resulting from the deprojection process.

**Intracluster gas mass**

The projected intracluster gas mass can be estimated by integrating equation 5.7 along the line of sight for each of the models in section 5.2 that fits all the available data. Table 5.3 shows the projected gas mass for $T_g = 10^7K$, the highest allowed temperature for all the models considered. A lower gas temperature would imply a higher concentration of gas mass towards the centre and on average a higher gas density to maintain the same observed X-ray luminosity. Thus the gas mass given in Table 5.3 is a lower limit.

**Total cluster mass**

The projected total cluster mass is estimated by integrating along the line of sight the total mass densities for each model given in section 5.2.

We have also listed the ratio of the projected mass of the luminous matter to that of the total cluster binding mass in Table 5.3 for $T_g = 7 \times 10^7$K. At lower temperatures, the gas distribution becomes more peaked, thus we expect the gas distribution to be at least as peaked as it is shown in Table 5.3 since $T_g = 7 \times 10^7$K is an upper limit. The luminous matter consists of the stellar mass and the intracluster gas mass. The stellar mass is found to be up to $\sim 10$

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4This is converted from a $M/L_V = 8$ assumed for E/SO galaxies in David et al. (1990).
Table 5.3: Projected Mass of Various Components of Matter

<table>
<thead>
<tr>
<th></th>
<th>Model 1</th>
<th></th>
<th>Model 2</th>
<th></th>
<th>Model 3</th>
<th></th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>15.6''</td>
<td>100''</td>
<td>240''</td>
<td>15.6''</td>
<td>100''</td>
<td>240''</td>
<td>15.6''</td>
</tr>
<tr>
<td>$M_{\text{gal}}$</td>
<td>0.16</td>
<td>0.56</td>
<td>0.91</td>
<td>0.16</td>
<td>0.56</td>
<td>0.91</td>
<td>0.16</td>
</tr>
<tr>
<td>$M_{\text{gas}}$</td>
<td>0.50</td>
<td>3.0</td>
<td>5.3</td>
<td>0.53</td>
<td>3.0</td>
<td>4.5</td>
<td>0.5</td>
</tr>
<tr>
<td>$M_{\text{tot}}$</td>
<td>6.2</td>
<td>45</td>
<td>100</td>
<td>6.2</td>
<td>53</td>
<td>120</td>
<td>6.2</td>
</tr>
<tr>
<td>$M_{\text{bary}}/M_{\text{tot}}$</td>
<td>0.11</td>
<td>0.08</td>
<td>0.06</td>
<td>0.11</td>
<td>0.07</td>
<td>0.05</td>
<td>0.11</td>
</tr>
</tbody>
</table>

The masses within a radius of 15.6'', 100'' and 240'' are given in $10^{13} M_\odot$ units.

The masses within a radius of 15.6'', 100'' and 240'' are given in $10^{13} M_\odot$ units.

times smaller than the gas mass. The $M_{\text{bary}}/M_{\text{tot}}$ decreases with increasing radius for all the models except for Model 3 where the $M_{\text{bary}}/M_{\text{tot}}$ increases with radius. Recent studies of Coma cluster (White et al. 1993; Briel et al. 1994), Perseus cluster (Eyles et al. 1991), Abell 85 and Abell 2199 (Gerbal et al. 1992) and Abell 665 (Hughes & Tanaka 1992) claimed an increase of the fraction of luminous to dark matter with radius. This fraction of $M_{\text{bary}}/M_{\text{tot}}$ is also a lower limit to the baryonic mass fraction. MS2137-23 seems to have a more peaked gas distribution than these typical rich clusters. It has been pointed out in Sciama et al. (1993) that the mass fraction of luminous to dark matter decreases as a function of radius for galaxies but seems to increase for clusters of galaxies. Specifically, Sciama et al. compared the spiral galaxy NGC 3198 studied in detail by van Albada et al. (1985) with Abell 665 (Hughes & Tanaka 1992) and found that at the edge of the optical structure the dark-to-luminous mass ratio in NGC 3198 is $\sim 10$ times larger than in Abell 665 but at the centre it is $\sim 200$ times smaller. Our example shows that given the currently available data, MS2137-23 appears to be consistent with a dark matter distribution that is broader than the luminous matter unlike the other rich clusters that have been studied, instead it resembles the dark matter distribution in a galaxy. Better quality data is needed to be certain of the distribution of the binding mass and the luminous mass at the outer parts of the cluster.
5.7 Conclusions

We have demonstrated that the cluster properties and matter distribution can be constrained by a self-consistent multi-wavelength analysis. If the cluster gas is isothermal and in hydrostatic equilibrium then the cluster gas has to be colder than $\sim 7 \times 10^7$K to satisfy the constraints from the requirements from gravitational lensing, X-ray surface brightness and SZ effect upper limits. We find that the X-ray core radius of the cluster assumed in Henry et al. (1992), and thus the total X-ray luminosity calculated, is inconsistent with the Einstein IPC data. We found the total X-ray luminosity to be $\sim 2 \times 10^{45}$ ergs/s and the core radius to be $< 0.1$ Mpc, consistent with the small core radius found in the gravitational lensing analysis. The cluster is not very rich optically (Abell richness 0-1). It has a rather peaked galaxy distribution on top of a plateau of sparsely distributed galaxies.

We cannot rule out any of the functional forms for the cluster total mass density given in Miralda-Escude (1994) (see section 5.2), given the available data. The shapes of these model density distributions are consistent with the galaxy number density distribution. Many improvements can be made to constrain models of cluster potential and gas properties, by obtaining high resolution X-ray images and temperature measurements and observations of the SZ effect at higher frequency than 8.8 GHz (to reduce confusion from weak extended radio sources). Rosat HRI imaging observations and ASCA spectral imaging observations have already been made for MS2137-23 by Gioia et al.. It is important to constrain the shape of the cluster potential at large radius in order to determine the cluster mass within a large radius and the cluster gas properties to a high accuracy. In future it would be best not to assume any specific functional forms for the cluster potential or gas distribution but rather develop general non-parametric models from the multi-wavelengths data. This is not possible with the poor quality of X-ray data and the lack of temperature data in the present analysis. With deep optical images taken in good seeing, the cluster potential at large radius can be deduced using weak lensing analysis. This cluster is unique in that the cluster potential profile can be "over-constrained" by the radial and the giant tangential arcs together with
weak lensing analysis and cluster galaxy velocity dispersion measurements. Weak lensing analysis provides a good constraint on the shape of the cluster density profile but the normalisation factor of the density profile is usually not as well constrained, (Smail et al. 1994) whereas the giant arcs give a very good estimate of the total mass enclosed within the radius of the arc. Thus by combining the strong and weak lensing analysis we can constrain both the normalisation factor and the shape of the cluster potential and compare the total mass deduced out to a large enough radius, where there are enough galaxies, with velocity measurements available to deduce a velocity dispersion and estimate the total mass using virial analysis. Up to now, clusters of galaxies have been assumed to be virialised without any direct evidence, here is an opportunity for us to test this hypothesis and gain new insight into the discrepancy between masses deduced from the weak lensing analysis only, with that from the virial analysis found in some clusters (Fahlman et al. 1994). With a knowledge of the shape of the cluster potential at a radius comparable to the size of the X-ray images, a direct test of the assumption of hydrostatic equilibrium is possible, if we have high resolution X-ray image and temperature measurements of the cluster.

5.8 References


Chapter 6

A Multi-wavelength study of A2218

6.1 The Cluster A2218

A2218 is an Abell richness class 4 cluster at a redshift of $z=0.175$ with a velocity dispersion of $1370^{+160}_{-120}$ km$s^{-1}$ (Le Borgne et al. 1992). It has a X-ray luminosity of $L_x = 1.01 \times 10^{45}$ ergs/s in the 2-10 keV energy range and a temperature of 6.7 keV (i.e. $7.8 \times 10^7$K) measured by the Ginga satellite\(^1\) (McHardy et al. 1990, David et al. 1993). The heavy element abundance is 0.2 of solar. There are gravitational arcs found in this cluster, both around the central cD and the bright galaxy of a smaller clump (Figure 6.1). The Sunyaev-Zel’dovich decrement has been successfully measured in this cluster (Jones et al. 1994). Given the wealth of data available on this cluster, we can hope to obtain a better understanding of the dynamical state and/or properties of the intra-cluster medium, as well as the matter distribution and $H_0$ than most other clusters available.

\(^1\)Preliminary results from ASCA also show that the gas temperature is $\sim 6.8$keV and that the gas clearly extends to 4 Mpc (M. Bautz 1994)
Figure 6.1: An r-band optical image of A2218 showing the arcs and arclets from Fig. 4 in Pelló et al. (1992). The scale is $10''/7$mm, north is to the left and east is down.

6.2 The Multi-wavelength Data

There has been a large amount of data collected for the cluster in the optical, X-ray and radio wavelengths by various groups.

6.2.1 Optical Data

The optical data relevant to this study are the deep imaging data taken in sub-arcsecond seeings that revealed the giant arc and the arclets (see Figure 6.1; Pello et al. 1988; Pello et al. 1992), the photometry data over a large field centred around the cluster to provide the galaxy density distribution (Dressler 1976; Butcher et al. 1983; Le Borgne et al. 1992) and the spectroscopy data (Le Borgne et al. 1992) for calculating the velocity dispersion of the cluster and verifying the cluster members.

Gravitational Lensing

The thin lens approximation (Blandford & Kochanek 1988) can be applied to clusters of galaxies, since it only requires that the angular deviation of light rays be small and the size of the deflector be much smaller than the
propagation medium (Fort & Mellier 1994). If we consider clusters as uniform sheets of matter then the radius at which the surface mass density reaches the critical density is given by

\[ \Sigma(b) = \Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_s}{D_l D_{ls}} \]  

(6.1)

Inside this radius of the critical circle, “strong” lensing effects such as multiple images of sources occur. The giant arcs appear close to this radius. Thus judging by the position of the giant arc from the cluster centre, we can estimate the projected mass within the radius of the giant arc.

In the case of A2218, Pello et al. (1992) found 32 arclets in their deep images (see Figure 6.1). The arclets have colours compatible with high redshift galaxies \((z > 0.6)\) and one of the arcs (object number 359 in Pello et al. 1992) has a spectroscopic redshift of \(z = 0.702\), thus favouring the gravitational lensing hypothesis for the arcs and arclets. As pointed out in Miralda-Escude & Babul (1994; MB here after), there is another object in the field (object number 328 in Le Borgne et al. 1992) which is very likely another image of the arc (number 359). The critical line should thus intersect the arc which puts the critical radius at \(20.8''\).

**Cluster galaxy density distribution**

Dressler (1976) measured the galaxy density distribution out to \(11.6'\). There was little evidence for morphological segregation. Thus, apart from the central region near the cD galaxy, the galaxy number density should be a fair representation of light distribution in the cluster. We expect the light distribution to be more peaked in the centre than the galaxy number density distribution due to the central cD galaxy. In the optical image from Pello et al. (1988, 1992; or Fig. 14 in Fort & Mellier 1994), the cluster appears to have 2 clumps with the massive clump centred around the cD. The second clump is centred around a bright galaxy with 2 arcs around it. The location of these arcs show that the second clump is of much lower mass (MB).
6.2.2 X-ray Data

The cluster was observed by the *Einstein Observatory* with both the Imaging Proportional Counter (IPC) and High Resolution Imager (HRI) (Boynton *et al.* 1982). Recently, it has also been observed by ROSAT with PSPC (MB). The background subtracted PSPC profile from MB is shown in Figure 6.4. The X-ray temperature of the cluster was measured with the *Ginga* satellite (McHardy *et al.* 1990) and found to be well fitted by an isothermal model with a temperature $6.7^{+0.5}_{-0.4}$ keV. In the following discussions we will assume that the gas is isothermal.

6.2.3 Radio Data

The SZ effect has successfully been detected in this cluster by the Ryle Telescope (Jones *et al.* 1993). After subtracting the discrete radio sources in the field, a negative source was found at $16^h35^m35^s +66^\circ 18'50''$ (c.f. ROSAT X-ray centroid position of $16^h35^m42^s +66^\circ 18'46''$). The flux density of the SZ signal at various baselines are shown in Figure 6.5. Moffet *et al.* (1989) had mapped extensively a region $20' \times 20'$ around the centre of the cluster at 1440, 4860 and 14940 MHz with the VLA. From their survey the sources that would contribute significantly in the field of view of the Ryle observations are given in Table 6.1.

Jones *et al.* deconvolved the image using the *CLEAN* algorithm and subtracted the *CLEAN* components using a point source model from the visibility data. We have also analysed the original calibrated visibility data using our analysis methods (see Chapter 2 for detail; the data was kindly provided by M. Jones), i.e. fitting the sources using the *UVFIT* routine in *AIPS* and using the source positions found by Moffet *et al.* (1989). We found that the VLA position for source #3 in Table 6.1 was significantly different from the source positions in the Ryle map such that point source subtractions were unsuccessful using the VLA positions. Source #4 was too weak to be seen in the Ryle map. We constructed an image using only the Ryle data from baselines longer than 36m and deconvolved the image using the *CLEAN* algorithm in *AIPS*.
Table 6.1: Radio sources in the Ryle field

<table>
<thead>
<tr>
<th>Src</th>
<th>RA</th>
<th>DEC</th>
<th>δx1</th>
<th>δx2</th>
<th>δx3</th>
<th>δy1</th>
<th>δy2</th>
<th>δy3</th>
<th>S1</th>
<th>S2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h m s</td>
<td>o '</td>
<td>''</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>mJy</td>
<td>mJy</td>
</tr>
<tr>
<td>1</td>
<td>16 35 09.90</td>
<td>+66 19 21.73</td>
<td>-193</td>
<td>-204</td>
<td>-191</td>
<td>+36</td>
<td>+36</td>
<td>+34</td>
<td>1.69</td>
<td>1.99 ± 0.049</td>
</tr>
<tr>
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<td>16 35 35.43</td>
<td>+66 20 45.08</td>
<td>-40</td>
<td>-42</td>
<td>-40</td>
<td>+119</td>
<td>+120</td>
<td>+117</td>
<td>0.91</td>
<td>1.25 ± 0.049</td>
</tr>
<tr>
<td>3</td>
<td>16 36 03.75</td>
<td>+66 22 22.87</td>
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<td>+127</td>
<td>+97</td>
<td>+100</td>
<td>+96</td>
<td>1.83</td>
<td>2.07 ± 0.049</td>
</tr>
<tr>
<td>4</td>
<td>16 35 29.12</td>
<td>+66 20 00.20</td>
<td>-78</td>
<td></td>
<td></td>
<td>+74</td>
<td></td>
<td></td>
<td></td>
<td>0.095 ± 0.049</td>
</tr>
</tbody>
</table>

Col. (1) gives the source number; Col. (2) & (3) gives the position of the radio sources in B1950.0 coordinates as listed in Moffet et al. (1989); δx1, δy1, δx2, δy2, δx3 and δy3 are the offset of source positions found by the Moffet et al. (1989), Jones et al. (1993) and our analysis from that of the X-ray position of the cluster. Col. (10) & (11) are the flux densities given in Jones et al. (1993) and our flux densities deduced from UVFIT.

by restricting the region of emission to the 3 visible sources. The centroid of the source positions thus obtained were used along with the VLA position of the 4th source as inputs to the UVFIT program in AIPS to obtain the flux densities of the 4 sources. After subtracting the 4 sources, we found a 'dip' at the position of 16h35m34s +66°18'57'' in the lowest resolution image (i.e. image made with only data from the 18m baseline) which agrees well with the position given in Jones et al. (1993) of 16h35m35s +66°18'50''. The flux densities after source subtraction at the position of the 'dip' for both analyses are plotted against baselines in metres in Figure 6.5. The results of the analyses are in fairly good agreement.

6.3 Previous Studies

The first extensive multi-wavelength study of the cluster was done by Boynston et al. (1982) where they examined the X-ray surface brightness, the SZ measurements and cluster profile from optical galaxy counts. The X-ray images of A2218 from the IPC and HRI detectors on the Einstein Observatory were analysed in their paper. The gas and galaxy distributions were modelled by assuming that the gas is in hydrostatic equilibrium and follows the polytropic relations, and the cluster potential is traced by the galaxy surface
density which is described by an analytic King model out to many core radii. Their best-fit model to the HRI brightness distribution and galaxy surface density shows that the gas is nearly isothermal with gas temperature in the range 10 keV to 30 keV (i.e. $1.2 \times 10^8$K to $3.5 \times 10^8$K). The X-ray temperature were not known at the time of their analysis.

In their recent analysis of A2218, Miralda-Escude and Babul (1994) pointed out that the position of the giant arc in A2218 is in conflict with the X-ray surface brightness distribution measured by the PSPC and the X-ray temperature measured by the *Ginga* satellite. MB assumed that the clusters are spherically symmetric and in hydrostatic equilibrium. They examined two families of parametrized density profiles:

**Model 1**: 
$$\rho(r) = \frac{\rho_0}{[1 + (r/r_c)^2]^{\gamma/2}}$$  \hspace{1cm} (6.2)

**Model 2**: 
$$\rho(r) = \frac{\rho_0}{(r/r_c)^\alpha (1 + r/r_c)^{\gamma-\alpha}}$$  \hspace{1cm} (6.3)

The density profiles are constrained by the position of the arc. The projected mass within the critical circle is given by

$$M_p = \Sigma_{crit} \pi b^2 D_t^2$$  \hspace{1cm} (6.4)

where $b$ is the angular radius of the critical circle. The X-ray surface brightness distribution was then modelled by assuming that the gas is in hydrostatic equilibrium. They found that the central mass implied by lensing is a factor of 2–2.5 too large for the gas at the observed ASCA temperature to be in hydrostatic equilibrium solely due to thermal pressure support given the above family of cluster density profiles. The best fit to the X-ray surface brightness for Model 1 was $\gamma = 3.3$ and $r_c = 14''$ given the observed gas temperature (Figure 6.2). They found that if the gas temperature is raised to $2 \times 10^8$K (i.e. 2.5 times larger than the observed temperature) then a cluster density profile with $\gamma = 1.82$ and $r_c = 17''$ as described by equation 6.2 yields a good fit to the PSPC X-ray surface brightness profile (Figure 6.2).
Figure 6.2: X-ray surface brightness distribution. The filled circles are the observed ROSAT PSPC data. The dotted curve corresponds to the best fit for Model 1 (with $\gamma = 3.3$ and $r_c = 14''$) given the observed $T_g = 8 \times 10^7$ K (MB). The solid curve gives the best fit for Model 1 (with $\gamma = 1.82$ and $r_c = 17''$) given $T_g = 2 \times 10^8$ K (MB).
6.4 Present Analysis

In this section we examine the mass distribution and the assumptions of hydrostatic equilibrium in the light of the above analysis by MB, with the addition of the galaxy density distribution data from Dressler (1976) and the measurements of the SZ effect by Jones et al. (1993). We will also consider some other forms of total mass density distribution than those used in MB.

Firstly, we will assume that mass-follows-light and use the galaxy density profile as the projected mass profile. We deproject to obtain a 3-D mass profile and normalise the mass density using the gravitational lensing constraint on the mass within the giant arc. Using assumptions of hydrostatic equilibrium, we can then model the X-ray surface brightness distribution. Dressler (1976) found that his galaxy count distribution is well fitted by the analytic King profile

$$S(r) \propto (1 + (r/r_c)^2)^{-1}$$

with $r_c = 0.40\text{Mpc}$. Dressler (1976) found that there were only marginal effects of morphological segregation in the cluster which means that the distribution of light is not very different from the galaxy number density distribution except for the very centre of the cluster because of the cD galaxy. The 3-D cluster mass density corresponding to the above surface density profile is

$$\rho(r) \propto (1 + (r/r_c)^2)^{-3/2}$$

if the cluster extends to infinity. However, a real cluster has a finite radius, thus we will assume that the cluster has a mass density profile described by the above equation, but that the density cuts off at a finite radius $r_t$. The resulting projected surface density is plotted against the galaxy density profile measured by Dressler (1976) with a suitable scaling factor in Figure 6.3. The surface density agrees reasonably well with the data. The resulting X-ray surface brightness is given by

$$S_x(R) = 2n_0^2 \int_0^R \exp\left(-\frac{2\mu m_p}{kT_g}(\phi(r) - \phi_0)\right) \Lambda(T_g) \frac{r \, dr}{\sqrt{r^2 - R^2}}$$

where $\Lambda(T)$ is the temperature dependent emissivity, $n_0$ is the central electron density and the cluster potential $\phi$ is obtained from Poisson's equation.
The X-ray surface brightness thus calculated is then convolved with the point spread function (PSF) of the detector. The observed and simulated X-ray surface brightness profiles are plotted in Figure 6.4, which shows that if mass follows light and the gas is in hydrostatic equilibrium, then the X-ray surface brightness profile implied by the lensing constraint is much narrower than the observed profile given the measured gas temperature of $8 \times 10^7$ K.

We can also compare the expected SZ effect with the observed decrement measured by the Ryle (Figure 6.5). The expected SZ effect assuming mass-
Figure 6.4: X-ray surface brightness distribution. The filled circles are the background subtracted ROSAT PSPC data from MB. The solid curve is the best fit of the X-ray data with the cluster potential described by equation 6.2 and $T_g = 2 \times 10^8$ K (MB). The dotted curve gives the X-ray surface brightness after convolution with the detector PSF if mass-follows-light and $T_g = 8 \times 10^7$ K (measured X-ray temperature); the dashed curve gives the expected X-ray surface brightness with $T_g = 2.5 \times 10^8$ K.
follows-light and $T_g = 8 \times 10^7$ K (dotted curve) is slightly larger than the observed values, but it is better than the fit to the X-ray surface brightness (dotted curve). If we increase the gas temperature to $T_g = 2.5 \times 10^8$ K and still assume that mass-follows-light, then we can improve the fit to the X-ray surface brightness data (dashed curve in Figure 6.4) but get a rather poor fit for the SZ effect (dashed curve in Figure 6.5). Similarly, we find that the best fit to the X-ray surface brightness in MB for Model 1 with $T_g = 2 \times 10^8$ K (solid curve in Figure 6.4) gives a rather poor fit to the SZ effect (solid curve in Figure 6.5).

We have shown that the X-ray data do not agree with the constraints set by gravitational lensing for the observed gas temperature if we assume hydrostatic equilibrium and mass-follows-light. Furthermore, we can not find a solution to the gas temperature that gives a satisfactory fit to both the observed X-ray surface brightness and the SZ effect with the assumptions of hydrostatic equilibrium and mass-follows-light (Figure 6.4 and 6.5). As we know from Chapter 1, $H_0$ can be deduced if X-ray surface brightness, gas temperature and the SZ effect are known. Can the discrepancy between lensing, X-ray and the SZ effect be resolved by choosing a different $H_0$? From the observed data, the following quantities are directly measured:

- 1) gas temperature;
- 2) angular distribution of galaxy number density;
- 3) total mass within the radius of the giant arc;
- 4) angular distribution of X-ray surface brightness;
- 5) the SZ effect in flux densities as a function of baseline length.

Assuming that mass follows light, the observed quantities 2) & 3) above uniquely determines the mass density distribution for a given value of $H_0$. The cluster potential can be deduced from the total mass density distribution from Poisson’s equation. From equation 6.7, we can see that $S_x/n_0^2$ is uniquely determined given the cluster potential and gas temperature, which can then be
Figure 6.5: The SZ effect at 15.4 GHz in flux densities versus baseline length in wavelength units. The filled circles are the Ryle results from Jones et al. (1993) and the open circles are from the same Ryle data but analysed using our own procedures. The error bars are 1σ. The position difference in the x-direction between the filled and open circles are due to the deliberate difference in averaging the data for clarity. The solid curve is the expected SZ effect from the best fit of the X-ray data with the cluster potential described by equation 6.2 and $T_g = 2 \times 10^8$ K (MB). The dotted curve gives the expected SZ effect if mass-follows-light and $T_g = 8 \times 10^7$ K (measured X-ray temperature); the dashed curve gives the expected SZ effect for $T_g = 2.5 \times 10^8$ K.
compared with the measured X-ray surface brightness distribution after convolution of the detector PSF. By comparing the $S_x/n_0^2$ distribution obtained from equation 6.7 with that of the measured X-ray surface brightness, we can obtain 2 pieces of information: the central electron density $n_0$ and the consistency in the shape of the 2 profiles. The SZ decrement for a spherically symmetric cluster with isothermal gas in hydrostatic equilibrium is given by

$$
\Delta T_e(R) \propto \int_0^R \exp\left(-\frac{\mu m_p}{kT_g}\left[\phi(r) - \phi_0\right]\right)T_g \frac{rdr}{\sqrt{r^2 - R^2}}
$$

(6.8)

Thus given the central electron density, the gas temperature and the cluster potential, the SZ effect is uniquely determined as well, which can also be compared with 5).

We find in Figure 6.6 that the shape of the X-ray surface brightness favours a low $H_0$ ($\lesssim 20$), whereas the data on the SZ effect favours a high $H_0$ ($\sim 100$). Thus we can not find a $H_0$ that is consistent with all the data available, if mass-follows-light, the gas is smoothly distributed and in hydrostatic equilibrium and the cluster is spherically symmetric. If, however, the gas is clumped, then for the same X-ray surface brightness, the absolute value of the SZ effect is reduced by $\sqrt{\frac{\Omega_{\rm cl}}{\Omega_{\rm X}}}$, Thus it is possible to fit both the X-ray surface brightness an the SZ effect if $H_0 \sim 20$ and the gas is clumped with a large clumpiness factor $\frac{\Omega_{\rm cl}}{\Omega_{\rm X}} \sim 4$.

From the analysis of MB and our analysis so far, it appears that the lensing constraints and the X-ray data are in conflict if the gas is assumed to be in hydrostatic equilibrium. Furthermore, we have found in the present analysis that the X-ray data disagrees with the data on the SZ effect. This leads us to the suggestion that the functional forms given in equation 6.2 and 6.3 do not reflect the true profile of the cluster total mass density. The position of the giant arc only constrains the innermost part ($0.08$ Mpc) of the cluster potential. Ideally, the outer parts of the cluster potential can be determined from weak lensing analysis, but such analysis requires deep optical data taken in sub-arcsec seeing conditions and is thus not available for the present studies. Given the available data, the cluster potential outside 0.08 Mpc is free to vary and we should be able to vary the outer parts of the cluster potential until it fits the X-ray surface brightness profile, keeping the gas temperature fixed.
Figure 6.6: The X-ray surface brightness after convolution with the detector PSF assuming mass-follows-light and $T_0 = 8 \times 10^7$ K for $H_0 = 20$ (dashed curve), 50 (dotted curve) and 100 (solid curve). The filled circles are the measured X-ray surface brightness from ROSAT PSPC observations.
Figure 6.7: The simulated SZ effect expected at 15.4 GHz with the Ryle Telescope assuming mass-follows-light and $T_g = 8 \times 10^7$ K for $H_0 = 20$ (dashed curve), 50 (dotted curve) and 100 (solid curve). The open circles are the actual SZ effect observed with the Ryle.
to the observed value. Figure 6.8 shows an example of such a potential (solid curve). Since the cluster potential within the radius of the arc (i.e. 0.08 Mpc or 20.8") has only negligible influence on the global X-ray surface brightness distribution, the potential can be obtained by trial and error until it fits the X-ray surface brightness distribution and the central parts of the potential can then be adjusted to fit the lensing constraint while keeping the function smooth. The X-ray surface brightness and the SZ effect corresponding to such a potential with $T_g = 8 \times 10^8$ K and electron central density $n_e(0) = 7.3 \times 10^{-3}$ cm$^{-3}$ are shown in Figure 6.9 and 6.10 respectively. Both the X-ray surface brightness and the SZ effect thus deduced give reasonable fits to the observed data for $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$. We conclude that there is no real conflict between the gravitational lensing constraints from the position of the giant arc and that of the observed X-ray surface brightness and temperature, under the assumption of hydrostatic equilibrium. The discrepancy noted by MB in their studies results from the limited range of parametric functions for the cluster potential rather than the assumption of hydrostatic equilibrium.

6.5 Discussions and Conclusions

We have tested the validity of our method of processing the data on the SZ effect, specifically the radio source subtraction procedure, by analysing the Ryle data on A2218 and comparing the results with that of Jones et al. (1993) and obtained good agreement with their results.

We found that the lensing constraints are in conflict with the observed X-ray surface brightness profile and gas temperature if the galaxy surface density traces mass and the intracluster gas is in hydrostatic equilibrium. The X-ray surface brightness profile derived from the mass-follows-light assumption is narrower than the observed profile (Figure 6.4), suggesting that the true underlying total mass distribution is broader than the galaxy distribution at least in the outer parts of the cluster. Furthermore, the X-ray surface brightness profile does not agree with the data on the SZ effect for any value of $H_0$ if mass follows light and the gas is not severely clumped. Thus the problem lies in the
Figure 6.8: Profiles of cluster potential in units of \((m \text{ s}^{-1})^2\). The dotted curve shows the profile of the gravitational potential if mass follows light. The solid curve shows an empirical form of the cluster potential that satisfies both the lensing and X-ray surface brightness constraints. The dashed curve shows the potential corresponding to the best fit for Model 2 with the observed gas temperature (i.e. \(\gamma = 5.3, \alpha = 0\) and \(r_e = 31''\)) and the dot-dashed curve is the best fit for Model 1 (i.e. \(\gamma = 3.3\) and \(r_e = 14''\)) from MB.
Figure 6.9: X-ray surface brightness profile. The filled circle is the observed surface brightness from ROSAT PSPC. The curve shows that the X-ray surface brightness for a cluster potential given in Figure 6.8 (solid curve) with $T_g = 8 \times 10^7$ K.
Figure 6.10: The SZ effect in flux densities versus baseline length. The filled circles are the Ryle data at 15.4 GHz analysed by Jones et al. (1993) and the open circles are the same data analysed with our own methods. The curve gives the SZ effect expected for a cluster potential given in Figure 6.8 (solid curve) with $T_g = 8 \times 10^7$ K.
assumption that mass follows light rather than the assumption of hydrostatic equilibrium, since the consistency between the X-ray surface brightness distribution and the SZ effect has little dependence on the assumption of hydrostatic equilibrium.

Similarly, the discrepancy between the gravitational lensing constraints and the data on X-ray surface brightness and temperature noted by MB is the result of the restricted range of functional forms used for the cluster potential. As we have noticed in Figure 6.4 and 6.5 that while increasing the gas temperature resolves the discrepancy between the lensing constraint and the X-ray surface brightness distribution, the difference between the expected and the observed SZ effect still remains.

We found a cluster potential that fits the available data on the SZ effect, the X-ray surface brightness and temperature with $H_0 = 50$ km s$^{-1}$ Mpc$^{-1}$ under the assumption of hydrostatic equilibrium. The amount of information that is available on this cluster has reached a point where we should not be restricted to specific functional forms for the cluster potential. We have demonstrated the need for non-parametric modelling of the cluster potential. In other words, we have enough information for this cluster to simultaneously determine the shape of the cluster potential and the value of $H_0$. With the new X-ray spectral data from ASCA, it is possible to obtain a X-ray temperature distribution directly and thus we can drop the assumption of isothermality. With the addition of a temperature profile, and possibly a weak lensing analysis of the cluster potential out to at least $\sim 1$ Mpc, we can allow even more free parameters in the analysis of the cluster matter distribution, dynamical state and the properties of the ICM. In particular, we can obtain a quantitative measure of the difference in dark and luminous matter density as a function of radius and directly test consistency of the potential deduced from lensing with that of the X-ray surface brightness and temperature under the assumption of hydrostatic equilibrium. The SZ effect then adds extra information on the value of $H_0$ and the clumpiness factor. It is difficult to disentangle the two parameters, but it is possible to set an upper limit to $H_0$ since a low $H_0$ with a high clumpiness factor can produce the same SZ effect as a high $H_0$ but low clumpiness factor, given the X-ray surface brightness and temperature.
We have seen the advantages of studying clusters at intermediate redshifts which have clear signatures of gravitational lensing effects. While clusters at intermediate redshifts do not have the S/N advantage of the nearby clusters in optical and X-ray studies, they have the possibility of strongly distorting images of background galaxies which is not possible with a nearby cluster because of the geometry. Furthermore, given the current technology the size of an intermediate cluster is such that it can be fitted almost entirely within the field of view of an optical or X-ray detector, whereas mosaicing of a number of fields is required both in the optical and X-rays for nearby clusters. We also gain a S/N advantage in detecting the SZ effect in intermediate redshift clusters as compared with nearby ones because the size of the cluster hot gas matches the resolution of a synthesis telescope (or the size of the beam throw for single dish telescopes) much better than the nearby clusters.

6.6 References

Bautz M., 1994, in New Horizon of X-ray Astronomy – First Results from ASCA, eds. F. Makino & T. Ohashi, Universal Academy Press, Inc.


Chapter 7

Conclusions

We searched for the Sunyaev-Zel'dovich effect in 5 clusters of galaxies. Upper limits to the SZ effect were obtained for some of the clusters. We were able to constrain the intracluster gas temperature from these upper limits to the SZ effect, when combined with measurements of the X-ray surface brightness. We have demonstrated that the AT is sensitive enough for the detection of the SZ effect, however, the confusion noise from the weak radio sources below our detection threshold limits our ability in detecting the SZ effect at 8.8 GHz with the AT. We make the following suggestions for future observations of the SZ effect with the AT:

• observe at a higher frequency; Although this will reduce confusion (radio source flux $\propto \nu^{-0.35}$) and increase the intensity of the SZ effect ($\Delta I_\nu \propto \nu^2$), there is a disadvantage in observing at a higher frequency because the brightness sensitivity decreases with increasing frequency for a given baseline length. The brightness sensitivity increases with the beam area which is $\propto \nu^{-2}$. Overall it is better to observe at a higher frequency if we ignore atmospheric effects.

• observe southern clusters; clusters south of $-50^\circ$ in declination with high X-ray luminosity should be chosen to avoid the effects of shadowing of the antennas at large hour angles that gives high side-lobe responses. Complete 12 hrs coverage of the $(u,v)$ plane reduces the side-lobe levels and thus reducing the effects of confusion. By avoiding declinations close
to the equator, we obtain a rounder beam which is better matched to the cluster shape and thus more efficient in detecting the SZ effect and reducing the effects of confusion.

- **deep high resolution observations**: sensitive high resolution observations of the cluster radio sources are as important as the observations for the SZ effect itself. Ideally we should make deeper high resolution observations than low resolution observations of the SZ effect, so that we can detect the weak radio sources just below the detection threshold in the low resolution observations. Blending of these weak radio sources in a low resolution image is a major set back for our detection of the SZ effect. The “high” resolution observations should be high enough to avoid confusion but not so high that the field sources are resolved.

Extended radio emission was found in almost every cluster we observed, some of which turned out to be radio halo sources and the rest were most likely to be the blend of weak radio sources that were too weak to be detected individually.

We have demonstrated the usefulness of multi-wavelength analysis of the properties of the intra-cluster medium and the cluster mass distribution through the analysis of 2 clusters MS2137-23 and A2218 which have giant gravitational lensing arcs present in the cores of the clusters. In the case of MS2137-23, the combination of the X-ray, radio and optical data constrains the gas temperature to be $\lesssim 7 \times 10^7$ K. We have shown that mass does not follow the distribution of light in A2218 and that it is premature to conclude that there is a discrepancy between the lensing constraint from the giant arc and X-ray surface brightness and temperature data as claimed by Miralda and Babul (1994). We need a direct measure of the cluster potential in the outer regions of the cluster from weak lensing analysis before we are able to test out the validity of the assumption of hydrostatic equilibrium.

We should place high priority on studies in various wavelength bands of clusters of galaxies with giant gravitational arcs, since giant arcs provide unique constraints on the cluster central mass. We should try to obtain the following data in various wave-bands:
• Deep wide-field CCD image in sub-arcsec seeing conditions. This is necessary in the analysis of weak gravitational lensing distortions of the background galaxies. It has been shown that the Kaiser & Squire (1993) method for weak lensing analysis can be successfully applied to deduce the cluster potential out to 1 Mpc from the cluster centre.

• High resolution X-ray surface brightness profile.

• X-ray spectrometry to obtain the gas temperature profiles.

• Measurements of the SZ effect. We have seen in Chapter 5 that even upper limits can place strong constraints when combined with all the other information from various wave-bands.

• Optical spectroscopy of member galaxies.

Firstly, by comparing the cluster potential from the weak lensing analysis normalised by the central cluster mass deduced from the giant arcs, with the X-ray surface brightness and temperature, we can directly test the validity of the assumption of hydrostatic equilibrium. Secondly, a combined analysis of the X-ray surface brightness, temperature and the SZ effect can provide an estimate of $H_0$. Thirdly, by comparing the cluster mass estimates from the lensing analysis with that of the virial mass estimates from optical spectroscopy, we can also test the validity of the assumption of virial equilibrium. Finally, if the assumption of hydrostatic equilibrium was found to be valid then we can combine all the above information to derive a best fit cluster potential (or mass density distribution) and $H_0$ simultaneously.
Bibliography


