EXPERIMENTAL STUDIES OF PLASMA
CONFINED
IN A TOROIDAL HELIAC

by

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This thesis represents my own original research. All authorities and sources consulted have been acknowledged.

Xuehua Shi
Xuehua Shi
September 1989
Abstract

This thesis describes in detail the properties of and experimental results obtained on the first toroidal heliac (helical axis stellarator) SHEILA.

The machine construction, operation and the Langmuir probe diagnostics used in these experiments are described. Computational results for the wide range of vacuum configurations made accessible by the addition of a helical winding to the basic heliac coil configuration are presented and discussed.

The main part of this thesis is dedicated to the study of a low beta argon plasma ($\tilde{\beta} \sim 10^{-4}$) confined in various heliac configurations. The existence of plasma equilibrium surfaces is verified experimentally. Observations of phenomena associated with magnetic surface resonances, e.g. the deterioration of plasma confinement when the rotational transform profile includes certain low-order rational surfaces, are described. Some features associated with the surface destruction and the magnetic island formation appear in both plasma pressure and floating potential profiles. A known symmetry-breaking error in coil position is shown to have significant effect on the vacuum magnetic configurations.

Low frequency coherent fluctuations in plasma density, floating potential and magnetic field are studied for three typical heliac configurations. Spatial mode structures are analyzed in a magnetic coordinate system. The measured frequencies and wavenumbers of the fluctuations, together with their parametric dependences on the magnetic field and the electron collision frequency, are compared with a collisional drift wave model. The agreement is excellent. The model is derived from a linearized two-fluid theory and related through an appropriate magnetic coordinate transformation to the heliac geometry. The influence of these fluctuations on the observed particle confinement is briefly discussed.
Author's Contributions

Prof. S.M. Hamberger and Dr. L.E. Sharp were responsible for the scientific design of SHEILA with mechanical design supplied by Mr. R. Gresham of the School's Mechanical Workshop. The various Langmuir probes were designed jointly by Dr. B.D. Blackwell and the author and made by Mr. E.K. Wedhorn. The mechanical design of the 2-D probe was by Mr. A.D. Campbell. The magnetic probe was designed by the author and made in the School's Precision Workshop. The microwave interferometer system was prepared by Dr. Sharp. The RF electrical circuit was devised by Dr. Blackwell and the decoupling transformer designed and made jointly by Dr. Blackwell and the author.

The BLINE code was written by Dr. Blackwell and all computational results presented in this thesis have been obtained by the author with this code. The preliminary experimental results in the standard heliac configuration were obtained jointly by Dr. Blackwell and the author. The detailed studies of equilibrium configurations and the fluctuations and their interpretations and dispersion calculations were made by the author.

Prof. Hamberger suggested the possibility of observing features associated with island and ergodic regions in the probe floating potential profile, and the possibility of analyzing the mode spatial structure of the fluctuations in heliac geometry by approximating it as a periodic cylinder in magnetic coordinates.
Author’s Publications

‘First Studies of Plasma Confined in a Toroidal Heliac’
Blackwell, B.D., Hamberger, S.M., Sharp, L.E., and Shi, X.H.,

‘Experimental and Theoretical Studies of Toroidal Heliac’

‘Experimental Investigation of Different Configuration in a Flexible Heliac’
Shi, X.H., Blackwell, B.D., Hamberger, S.M.,

‘Experimental Results from a Flexible Heliac’
Shi, X.H., Blackwell, B.D., Hamberger, S.M.,

‘Experimental Studies of Plasma Confined in a Toroidal Heliac.’
Blackwell, B.D., Hamberger, S.M., Sharp, L.E., and Shi, X.H.,

‘Drift Wave Study in a Toroidal Heliac’
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Chapter 1

Introduction

In recent years, plasma physicists have shown renewed interest in the stellarator confinement geometry following the substantial progress made during the last decade in plasma parameters, physics understanding, and improvement of the stellarator concept.

Research into stellarator plasmas has a long history, starting in the early 1950's. These toroidal magnetic plasma confinement systems differ from another and more common type of machine, the tokamak, by having their appropriate magnetic configurations produced entirely by currents flowing in external coils. This essential feature allows nominally current-free plasma operation, which brings many attractive advantages. These include the possibility of steady state operation; eliminating the source of current-driven major disruptions; providing a good confinement geometry even before plasma formation; and permitting independent control of the magnetic configuration and the plasma profiles, which allows optimization of the magnetohydrodynamic (MHD) and transport properties as the plasma pressure increases (Carreras et al., 1988).

However, owing to the lack of axisymmetry in stellarators, the full three-dimensional MHD equilibria have to be considered. Thus, even though ohmic current-free plasma operation can be achieved, the diamagnetic and Pfirsch-
Schluter currents driven by the plasma pressure will play an important role in the 3-D equilibria. These currents may produce magnetic field components which can resonate with the magnetic field lines on certain flux surfaces, resulting in phenomena such as the appearance of magnetic islands and the destruction of some flux surfaces. Such resonance due to the equilibrium currents is unique to 3-D equilibria (Reiman and Boozer, 1984). In general, as the plasma pressure increases, the widths of these islands increase. When the islands are sufficiently large and overlap, the flux surfaces are destroyed and the equilibrium may no longer exist. This leads to the concept of an equilibrium $\beta$ limit for a stellarator, where $\beta$ is the ratio between the plasma pressure and the total magnetic pressure.

The study of the optimization of the magnetic configuration for stellarators to obtain higher $\beta$ values has always been an important subject. To investigate the effect of the various quantities used to characterize the configuration on the plasma confinement properties, four main types of stellarator configurations are currently being actively studied in the world (Carreras et al., 1988, Uo, 1985, Lyon et al., 1983, Brossman et al., 1983, Herrnegger et al., 1986, Boozer et al., 1983). They differ from each other by emphasizing different field characteristics, such as the rotational transform, the shear of the magnetic field lines between the magnetic surfaces, and the sign and depth of any magnetic well etc., (see Chapter 3).

The helical axis stellarator (heliac) is one of these four types. The coil configuration of a heliac consists essentially of a toroidally directed central conductor and a set of toroidal field (TF) coils whose centres follow a closed helical path about the central conductor as shown in Fig. 1.1. The strong fundamental helical component of the vacuum field generated by the current flowing in the displaced TF coils, in combination with the tokamak-like poloidal field generated by the toroidally flowing current in the central conductor, produces nested toroidal flux tubes with a helical axis which rotates about the ring. Such a configuration has an
inherently large rotational transform per field period (and so a shorter connection length between regions of good and bad curvature), a low global shear (although the local shear may be strong), and may have a substantial mean magnetic well. These distinguishing features are mostly due to the torsion in the magnetic axis. In general, these features are properties favourable for plasma equilibrium and stability. Helical equilibria, which are stable to all ideal fixed-boundary modes and resistive interchanges, have been found numerically with $\beta$ more than 20% for a straight heliac with helical symmetry (a two-dimensional problem) (Boozer et al., 1983; Monticello et al., 1984; Merkel et al., 1983). Gardner extended these results to free-boundary equilibria and showed that any zero-pressure configuration with a uniform magnetic well across the plasma will always satisfy the Mercier stability criterion, a necessary condition for linear ideal MHD stability, as $\beta$ is increased (Blackwell et al., 1987).

Figure 1.1: *Heliac configuration (from Boozer et al., 1983).*
However, in a toroidal heliac the physical picture can be quite different and the flux surfaces may break-up at lower $\beta$ values because of the existence of the resonant equilibrium currents which are unique to the 3-D equilibria. Numerical studies of the 3-D MHD equilibrium of H-1, a large toroidal heliac currently under construction in this Laboratory, have been carried out at Oak Ridge National Laboratory (Carreras et al., 1987) and at this Laboratory (Blackwell and Takeuchi, 1987). In well-chosen configurations, the 3-D equilibrium codes, such as the ‘VMEC’ code (Hirshman et al., 1983), the ‘NEAR’ code (Hender et al., 1985) and the ‘BETA’ code (Bauer et al., 1984), all have convergence problems at similar $\beta$ values about 7%, below which the flux surface deformation, axis shift and pressure-induced island formation are unlikely to be serious. The calculations also indicate that the surface destruction and the formation of the magnetic islands are obviously associated with those ‘weak’ surfaces (low-order rational surfaces, see Chapter 3). A study of the Mercier criterion in Oak Ridge, for which the VMEC equilibrium results are used as input, has also demonstrated that the dominant contribution to the instability comes from the Pfirsch-Schluter current, in particular those contributions associated with the low-order rational surfaces included.

These low-order rational surfaces are a consequence of the high rotational transform per field period in the heliac-type configurations. It is important, therefore, to find ways to be able to carefully control the rotational transform profile to avoid those potentially dangerous surfaces; this is possible because of the relatively low shear in the heliac configurations. One such concept for a flexible heliac configuration was proposed by Harris et al., (1985). They showed theoretically that an additional helical winding, closely wound around the central conductor and having the same helicity as the magnetic axis, would add a significant degree of control of the rotational transform profile.

The heliac concept is related to the original ‘figure-8’ devices proposed by
Spitzer (1958). The concept of stabilization by a mean magnetic well in closed configurations was first discussed by Furth and Rosenbluth in 1964, shortly followed by some theoretical studies of helical magnetic fields possessing mean magnetic wells (Furth et al., 1966, McNamara et al., 1966). However, the idea was not pursued until 1983, when the basic toroidal heliac scheme was proposed by the Princeton Plasma Physics Laboratory and followed by some preliminary design and computational studies (Boozer et al., 1983). Research on this type of stellarator is still at an early stage. The first toroidal heliac device, SHEILA (Blackwell et al., 1985), which forms the topic of this thesis was built in this Laboratory in 1984, where, as mentioned above, a much larger heliac H-1 (Hamberger et al., 1987 and 1989) is under construction. Two other toroidal heliacs, TJ-II in Spain and Asperator H in Japan, are respectively under design and in the early stages of operation. The parameters and current status of these machines are listed in Table 1.1.

This thesis concerns the first research work on the prototype heliac SHEILA. Because of the size and the small plasma input power the $\beta$ value of SHEILA plasma is very low ($\sim 10^{-5} - 10^{-4}$), so that any finite $\beta$ corrections, i.e. adding the magnetic fields produced by the pressure driven currents to the vacuum magnetic fields, can be safely ignored. SHEILA was originally designed to conduct simple experiments aimed at comparing the magnetic configuration actually obtained with low $\beta$ plasma to the computed vacuum configuration from the coil geometry and to provide operating experience for H-1. A helical control winding was added in 1986 following the theoretical proposal by Harris et al., allowed the concept of a flexible heliac to be first tested in this machine. As will be described here, it has been experimentally verified that the magnetic configuration and its rotational transform profile can be easily controlled and widely varied by adjusting the current in the additional helical winding.

Since the first operation of SHEILA in late 1984 a great deal of experimental
data have been obtained. This thesis discusses in detail some of the experimental results which include the main plasma parameters obtained, data for the heliac configuration studies obtained from measurements made by Langmuir probes, studies of plasma confinement in various heliac configurations, the effect on the vacuum configurations of a small error in conductor geometry which introduces a symmetry-breaking error field, and those relating to the fluctuations of plasma density, potential and magnetic field (Blackwell et al., 1985 and 1989; Shi et al., 1988 and 1989). Examples of various vacuum magnetic configurations which have been numerically calculated are also presented.

The outline of this thesis is as follows: The apparatus and diagnostic methods are described in Chapter 2, and the computed vacuum magnetic configurations in Chapter 3. The experimental investigation of different heliac configurations and the effect of a small error in coil positions on the vacuum magnetic configuration are presented in Chapter 4. Chapter 5 contains details of the study of the plasma fluctuations in this machine, and a comparison is made with a linear theory of collisional drift waves. The work is summarized in Chapter 6.
<table>
<thead>
<tr>
<th>Name</th>
<th>Location</th>
<th>$R_0$ (m)</th>
<th>$\bar{a}$ (m)</th>
<th>$B_{max}$ (T)</th>
<th>$N$</th>
<th>Helical Winding?</th>
<th>$I_r/I_h$ (kA)</th>
<th>Status</th>
</tr>
</thead>
<tbody>
<tr>
<td>HX-1</td>
<td>Princeton</td>
<td>1.5</td>
<td>0.25</td>
<td>1.0</td>
<td>3</td>
<td>No</td>
<td>--</td>
<td>design study</td>
</tr>
<tr>
<td>SHEILA</td>
<td>Canberra</td>
<td>0.2</td>
<td>0.03</td>
<td>0.25</td>
<td>3</td>
<td>Yes</td>
<td>25/5</td>
<td>first operation in 1984</td>
</tr>
<tr>
<td>H-1</td>
<td>Canberra</td>
<td>1.0</td>
<td>0.22</td>
<td>1.0</td>
<td>3</td>
<td>Yes</td>
<td>500/50</td>
<td>under construction</td>
</tr>
<tr>
<td>TJ-II</td>
<td>Madrid</td>
<td>1.5</td>
<td>0.2</td>
<td>1.0</td>
<td>4</td>
<td>Yes</td>
<td>219/95</td>
<td>under design</td>
</tr>
<tr>
<td>Asperator H</td>
<td>Sendai</td>
<td>0.5</td>
<td>0.05</td>
<td>0.3</td>
<td>4</td>
<td>No</td>
<td>100 / --</td>
<td>first operation in 1989</td>
</tr>
</tbody>
</table>

Table 1.1: World list of heliacs. $I_r$ and $I_h$ are the quasi-steady currents in the central conductor and the helical winding respectively. $N$ is the number of field periods.
Chapter 2

SHEILA Apparatus and Diagnostics

2.1 Coil configurations and vacuum system

The construction of the SHEILA apparatus is shown in Fig. 2.1. The main magnetic field coils are located inside a 0.65 m diameter, 0.6 m high and 3.25 mm thick stainless-steel vacuum tank with 25 mm thick lids. The coils inside the vacuum vessel are fed through coaxial connectors underneath the tank. An alternative transparent polycarbonate lid can be fitted to the tank for visual observation purposes, but at the expense of vacuum quality.

The coil configuration is shown in Fig. 2.2, which consists of a set of 24 toroidal field coil pairs, a toroidally directed central conductor (also referred to as the 'poloidal field ring' or 'ring'), a helical winding, and two sets of vertical field coils.

Each toroidal field coil pair is made from radially split, 0.15 m square, 6 mm thick copper plates with 0.11 m circular apertures (see Fig. 2.3) and is mounted inside insulated box section extensions made from 1.2 mm thick stainless steel sheet. These extensions are supported vertically between upper and lower radial
Figure 2.1: The SHEILA heliac, showing the magnetic field coils as labelled, the support structure and vacuum vessel.
Figure 2.2: Coil configuration of SHEILA.
arms spaced at 15° intervals around the azimuth so that the mid-plane of each
coil pair is radial. The vertical and radial position of each TF coil pair can be
adjusted so that its centre lies on a specified toroidal helix about the central
conductor, displaced from it by a distance \( \rho_s \leq 0.05 \text{ m} \). Various helices may be
generated according to the general law \( \theta = N\phi + \alpha \sin N\phi \), where \( \theta, \phi \) are the
usual poloidal and toroidal angles and \( N \) is the number of periods of the helix,
which in this case is the same as the number of field periods defined in Chapter
1. This number can, for 24 pairs of toroidal field coils, be chosen to have values
of 1, 2, 3, 4, 6 or 8. Also, \( \alpha \) is a constant which is adjusted to optimize the heliac
configuration. For the SHEILA device the helix has been fixed with parameters
\( N = 3, \rho_s = \text{const} = 0.025 \text{ m}, \alpha = 0.1 \), which approximates to the form of the first
Princeton reference design (Ellis, Jr. et al., 1984).

Figure 2.3: Toroidal field coil pair, showing coaxial current feeds with others joined
by a connecting block. Stray fields are reduced by the planar and coaxial current
paths, and by the antisymmetry of the slots in each pair.
The central conductor consists of four turns of square-section copper (7 mm × 7 mm) machined from plate which are insulated from each other by 0.4 mm thick vacuum quality epoxy resin (Torseal). As shown in Fig. 2.4, it is supported by three radial arms and four legs from below. This four-turn ring forms the minor geometric axis of the machine with a mean radius of 0.1875 m. The helical winding is a single copper conductor of 6 mm diameter circular cross-section closely wound three times around the central conductor in phase with the helix followed by the toroidal coil centres. The displacement of the helix conductor with respect to the central conductor is 14.3 ± 1 mm.

![Diagram of the four-turn central conductor, its supports and the coaxial current feed.](image)

**Figure 2.4:** *The four-turn central conductor, its supports and the coaxial current feed. The current crossover is enlarged on the top, where the sequence of the four turns is numbered.*
The configuration is completed by two sets of vertical field coils. One set consists of two single-turn coils of radius 0.313 m located 0.192 m above and 0.145 m below the median plane inside the vacuum vessel. The other set is made from two single-turn coils of radius 0.334 m located at 0.199 m above and 0.127 m below the median plane, this time outside the vacuum tank. This second pair of vertical field coils is used for a fine adjustment of the vertical field. The machine parameters of SHEILA are summarized in Table 2.1.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of field periods</td>
<td>3</td>
</tr>
<tr>
<td>Number of turns of TF coil set</td>
<td>48</td>
</tr>
<tr>
<td>Aperture of TF coil (m)</td>
<td>0.11</td>
</tr>
<tr>
<td>Mean radius of ring (m)</td>
<td>0.1875</td>
</tr>
<tr>
<td>Swing radius of TF coil (m)</td>
<td>0.025</td>
</tr>
<tr>
<td>Swing radius of helical winding (m)</td>
<td>0.0143</td>
</tr>
<tr>
<td>Maximum magnetic field (T)</td>
<td>≤ 0.25</td>
</tr>
<tr>
<td>Pulse duration (ms)</td>
<td>≤ 55</td>
</tr>
</tbody>
</table>

Table 2.1: Machine parameters

Considerable care has been taken in the design of the current feeds for these main coil sets and inter-coil connections in order to minimize the effects of the stray magnetic fields on the designed vacuum magnetic surfaces. Coaxial conductors are used to feed each coil set and each turn of each toroidal field coil (see Fig. 2.3). The stray fields produced by the current flowing in those connectors between toroidal field coil pairs are cancelled by a parallel plate transmission return line (see Fig. 2.1).

The machine and coil system were manufactured and assembled in the Re-
search School's Workshop. The accuracy of the coil positions was checked after assembly. The error in the vertical and in the radial positioning of the toroidal field coils was less than 0.4 mm, and the toroidal placement of the mid-plane of each coil-pair was accurate to within ±1 minute of arc. The circularity of the central conductor was better than ±0.15 mm and the positional error of the central conductor and the vertical field coils in the vertical direction was less than ±1 mm. During subsequent operation, especially after removal of the 4-turn poloidal field ring for repairs to the insulation between turns, new errors in the ring position were introduced. The significance of these small errors on the vacuum magnetic field configuration will be discussed in Chapter 4.

The vacuum vessel is evacuated by a single stage rotary pump with a pumping speed 29.5 m³/hour and an oil ('Santovac V') diffusion pump fitted with a refrigerated baffle with a pumping speed 350 l/s. The vacuum base pressure is 2 × 10⁻⁴ Pa. During operation various gases, either argon, helium or hydrogen, are introduced at constant rate through a needle valve on a radial port to provide filling pressures in the range 10⁻³ – 10⁻¹ Pa. The pressure is monitored by both Pirani and ionization gauges with ranges from 10⁻¹ – 10⁵ Pa and 10⁻² – 10⁻⁷ Pa respectively. As the ionization gauge is calibrated for nitrogen the correction factors used for the gases are 1.1 for argon, 0.43 for hydrogen and 0.15 for helium (Summers, 1969).

### 2.2 Electrical circuit

The electrical circuit is shown schematically in Fig. 2.5. The parameters of the major components in this circuit are shown in Table 2.2, where the inductances of the major field coils were measured in the absence of the vacuum vessel. The inductance of the external vertical field coil was not measured independently. The magnetic field is produced by a slowly varying current from two capacitor banks
Figure 2.5: Electrical circuit of SHEILA.
of 2 mF and 5 mF with 10 kV max, which could be fired individually, together, or sequentially. The low impedance load presented by the coils is matched by a 15:1 iron-cored step-down transformer biased to provide a maximum flux swing of 0.2 Wb. Typical current waveforms, with the two banks fired separately, are shown in Fig. 2.6 (a). The maximum magnetic fields available are 0.18 T (at 3.5 kA) for the 5 mF bank and 0.25 T (at 5 kA) for the 2 mF bank, which are essentially limited by the saturation of the iron core of the transformer as shown in Fig. 2.6 (b).

<table>
<thead>
<tr>
<th>Component</th>
<th>Resistance (mΩ)</th>
<th>Inductance (µH)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Toroidal field coil</td>
<td>7.1</td>
<td>28.4</td>
</tr>
<tr>
<td>Ring</td>
<td>2.4</td>
<td>12.0</td>
</tr>
<tr>
<td>VF inner</td>
<td>1.2</td>
<td>3.2</td>
</tr>
<tr>
<td>VF external</td>
<td>1.6</td>
<td></td>
</tr>
<tr>
<td>Helical winding</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>L1</td>
<td>0.8</td>
<td>12.3</td>
</tr>
<tr>
<td>L2</td>
<td>0.9</td>
<td>12.7</td>
</tr>
<tr>
<td>D1</td>
<td>0.7</td>
<td>1.4</td>
</tr>
<tr>
<td>D2</td>
<td>0.7</td>
<td>1.4</td>
</tr>
<tr>
<td>D3</td>
<td>1.2</td>
<td>2.9</td>
</tr>
<tr>
<td>Helix shunt</td>
<td>12 (max)</td>
<td>1.1 (max)</td>
</tr>
<tr>
<td>Vertical field shunt</td>
<td>0.52</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 2.2: Circuit parameters.
Figure 2.6: (a) Current waveforms (1.17 kA/div) in the main circuit with the 2 mF and the 5 mF banks fired separately. (b) Current waveform affected by the saturation of the iron core, indicated by the earlier drop of the current, compared with the normal one, 5 mF bank.

To maintain the magnetic geometry as constant as possible during the current pulse, the coils are connected in series. A shunt is used to adjust the helical winding current relative to the current in the central conductor in order to vary the configuration, and another shunt for the external vertical field coil is used for fine adjustment. Since these shunts do not have exactly matched time constants there is a phase shift between the currents in the coil being shunted and in the main circuit. Another important cause of this phase shift is the additional current induced in these shunted circuits by the mutual coupling with the four-turn central conductor. A air-cored decoupling transformer is included in this circuit to compensate for the induced currents (both induced slow and RF currents, see below) in the shunted helical winding circuit. The winding arrangement of the transformer is shown in Fig. 2.7. The stray fields of the primary and the secondary are reduced by the opposing directions of the current in the two matched
halves. The phase difference between the currents in the helical winding and in the main circuit before and after connection of this decoupling transformer are compared in Fig. 2.8.

Figure 2.7: Structure of the decoupling transformer.

Figure 2.8: Phase comparison of the current in the main circuit (top trace, 1.17 kA/div) and the current in the helical winding (lower trace, 0.47 kA/div) with (a) and without (b) the anti-transformer in the circuit.
The vacuum vessel also carries eddy currents mostly due to its coupling with the vertical field coils. The penetration time of the vertical field and the time for the current in the inner vertical field coil to establish its field were measured. A square pulse from a signal generator, through a power amplifier, was applied to the external (or internal) vertical field coil in series with a 4Ω resistor to limit the current to less than 2 A. The voltage induced in the four-turn central conductor inside the vacuum chamber was recorded on an oscilloscope with a low pass filter of 3 kHz cutoff frequency. The results are shown in Fig. 2.9, where the decay time of the differential signals in the ring indicates that the time constant of the vacuum vessel for the vertical field is about 1 ms.

Figure 2.9: Top trace: the square pulse from the signal generator to (a) the external vertical field coil, and (b) the inner vertical field coil, 50 mV/div, lower trace: the induced signals in the central conductor, 500 µV/div.

In Fig. 2.5 RF current circuit is marked by shaded lines. An oscillatory current at 96 kHz is applied to the central conductor (ring) (≤ 100 A – turns) to induce an oscillatory electric field in the toroidal direction to break down the gas and maintain the plasma with a weak alternating ohmic current. The RF current from a signal generator is driven by a solid-state amplifier with a
maximum output power 400 W, and is supplied through a 12:1 transformer to a
series resonant circuit formed mainly by appropriate capacitors (C1, C2 and C3)
and the ring (see Fig. 2.5). L1 and L2 function as RF isolating reactances and the
rest of the main circuit is shorted by the large capacitor C4. By positioning the
two matched halves of the RF choke (L1, L2) and the primary of the decoupling
transformer (D1, D2) on either side of the central ring the average RF potential
difference between the TF coils and the four turns of the ring is reduced. The
presence of an RF potential difference may adversely affect the gas breakdown.

2.3 Diagnostics

With the limited RF input power available, only modest plasma parameters can
be obtained in SHEILA, i.e. typical electron density $n_e \sim 10^{17} - 10^{18} \text{m}^{-3}$ and
electron temperature $T_e \sim 5-10 \text{eV}$ (see Chapter 4). These conditions are ideally
suited to the use of Langmuir probes as a diagnostic technique. In addition, a
visible light detector, a microwave interferometer for line integral density mea-
surement and magnetic probes are used as supplementary diagnostics. Rogowski
coils followed by active integrators are used to measure the currents in the main
electrical circuit and in the shunts (see Fig. 2.5). These integrators have an
integration time constant 1 ms and a decay time constant 1 s. In the standard
heliac configuration (i.e. $I_h = 0$, see Chapter 4) the helical winding can be used
to measure the loop voltage.

Around the side of the vacuum vessel there are fourteen 25 mm diameter ports
and two 125 mm diameter ports, which are located at different heights to allow
access to various parts of the plasma column. These are shown in Fig. 2.10. Most
of these ports are used for probe measurements. The detailed arrangement of the
diagnostics is changed according to the particular needs of each study, and this
will be discussed in sections describing the experiments.
Figure 2.10: Plan view of SHEILA showing the side ports, the locations of some diagnostics, the ring with the helical winding, and the periodically positioned TF coils.

### 2.3.1 Langmuir probe details

Both single and double probes are used. The construction of the single probe is shown in Fig. 2.11. Each single probe has a 1 mm (or 0.3 mm) diameter, 2 mm
long tungsten wire electrode exposed to the plasma. In order to minimize the disturbance when scanning across the plasma, the rest of the wire is sheathed in thin-walled silica tubing (1.5 mm diameter). The end of the silica tube may become contaminated with sputtered metal particles during the discharges, which would cause uncertainty in the current collecting area of the electrode. To avoid this problem the end of the silica tube is designed in a funnel shape to make a small gap between the electrode and the wall of the tube. The double probes have basically the same construction with the two electrodes separated by approximately 3 mm.

![Image of probe structure]

Figure 2.11: Structure of a single langmuir probe.

The probes can be moved in and out radially through the small (25 mm diameter) ports to make chordal scans, or mounted in a probe manipulator (Fig. 2.12) at one of the large ports (φ = 120° or 150°). The manipulator allows the probe tip to be accurately located in two-dimensions at any point within a large area of the plasma cross-section, its position being controlled from outside the vacuum vessel by means of a mechanical linkage. The motion of the probe is guided by a map cross-hair moving over the surface of a cross-section map, which
acts as a template of the magnetic flux surfaces. The maps are calculated for different heliac configurations and plotted by a computer program according to the position of the fixed pivot point and the distance of the probe tip to the intersection point of the probe tube and the bar which connects the map cross-hair. Fig. 2.13 shows cross-sections of the vacuum magnetic surfaces for a particular heliac configuration at $\phi = 120^\circ$ (calculated by BLINE field line tracing code, see Chapter 3) and the computed template, where each surface is represented by a fixed number of points.

Figure 2.12: 2-D probe manipulator. A pivot and lever arm system allows independent control over probe insertion depth and orientation.
Figure 2.13: Vacuum magnetic surfaces (left) and the template (right), the horizontal axis is the distance from the central conductor in each graph.
Two poloidal arrays and one toroidal array of Langmuir probes are used in the fluctuation studies. Each of the two poloidal arrays consists of seven probes which can be pre-set to sample a given magnetic surface, while the toroidal array consists of four probes positioned at designated toroidal locations (see Chapter 5). The two poloidal probe arrays and the 2-D probe are shown schematically in Fig. 2.14.

Figure 2.14: Poloidal probe arrays at $\phi = 120^\circ$ (top), $\phi = 150^\circ$ (middle) and the 2-D probe at $\phi = 120^\circ$ cross-sections (below).

The positional accuracy of the probes is within ±1 mm, while the probe with the manipulator can be positioned with respect to the external map to an accuracy of ±0.5 mm. A systematic error, however, of at least 1 mm remains for this probe.
2.3.2 Temperature and floating potential measurements

The electron temperature is measured by conventional probe method. It can be derived from the characteristic curves of either the double probe or the single probe. The electrical circuits for the probe current measurements are shown in Fig. 2.15 (a) and (b) for the double probe and the single probe respectively. The value of the resistance $R$ in both circuits is chosen to be much smaller than the probe output impedance $R_p$, which is of the order of several kΩ.

![Diagram of probe current measurements](image)

Figure 2.15: The electrical circuits measuring (a) double probe current, (b) single probe current, and (c) the probe floating potential, $R = 100\,\Omega$, $R_1 = 1\,M\Omega$, $R_2 = 10\,k\Omega$ and $C_s$ representing the stray capacitance.

The data points of the probe characteristic curves are obtained by changing the applied voltage (derived from batteries) shot by shot. As an example, Fig. 2.16 shows the signals of the current drawn by the double and single probes for some applied (bias) voltages. The isolation amplifiers employed have a 20 kHz
frequency response (-3 dB). The probe currents have an AC component due to local density fluctuations, so the signals are time averaged either graphically or electronically through a two-stage low-pass RC filter \((R = 10 \, k\Omega, C = 2 \, nF)\) to obtain the values of the DC current for the characteristic curves. Typical probe characteristic curves measured at a fixed moment during each shot are shown in Fig. 2.17.

Figure 2.16: *Currents drawn by the double probe (left, 0.5 mA/div) and the single probe (right, 0.25 mA/div) for various applied voltages as indicated (in volts) on the left of the photos, both probes 1 cm from the centre of the plasma. Probe diameter: 0.3 mm, argon filling pressure: \(3.2 \times 10^{-2} \, \text{Pa}\), \(B = 0.14 \, \text{T}\), RF amplifier power: 150 W. Top trace: the RF potential across the four-turn ring, 200 V/div.*

The following equations are used to obtain the electron temperature:

\[
\frac{T_e}{e} = \frac{\Delta V}{\ln I_{e1} - \ln I_{e2}} \tag{2.1}
\]

and

\[
\frac{T_e}{e} = - \frac{I_{1\text{sat}} - I_{2\text{sat}} - 1.705 \Delta V}{2(\frac{dI}{dV} + S)}. \tag{2.2}
\]

Eq. 2.1 is used for the single probe method, where the quantities are obtained as shown in Fig. 2.17 (a). Eq. 2.2 is used for the double probe method. The
Figure 2.17: The characteristic curves of (a) the single probe, and (b) the double probe, with the conditions as for Fig. 2.16.

Ion currents to the two electrodes are assumed to vary linearly with plasma-probe potential fall and the proportionality constant $S$ is found from the slope of the ion saturation regions AB or CD. $dI_s/dV$ is the slope of the double probe characteristic at the origin. The other quantities in Eq. 2.2 are obtained as shown in Fig. 2.17 (b) (Swift and Schwar, 1970). The temperatures derived from the
single and the double probe methods agree with each other within experimental uncertainty (±15%).

It is found in our experiment that the surface condition of the probe greatly affects the shape of the characteristic curve and value of the ion saturation current. Fig. 2.18 shows two characteristic curves measured by a double probe with (a) dirty and (b) clean surface conditions. When the probe surface is contaminated, a insulating layer forms and the current to the probe is greatly reduced for small applied potentials, so that the shape of the probe characteristic is distorted. A glow discharge is used to clean the probe, especially for new probes or probes which have been exposed to air for several days. This circuit is shown in Fig. 2.19: a voltage – 400V with respect to the vacuum vessel is applied to the probe and a 11 kΩ resistor used to limit the current. A suitable pressure range for a glow discharge in argon is 50 – 60 Pa, and the ion bombardment current drawn by the probe is about 0.5 mA. Usually, a glow discharge for 3-5 minutes is enough to clean the probe.
Figure 2.18: (a) dirty and (b) clean probe characteristics, probe diameter 1 mm, argon filling pressure $2.0 \times 10^{-2}$ Pa, $B = 0.2$ T, RF amplifier power: 110 W.

Figure 2.19: Glow discharge circuit for cleaning the probe.
The probe floating potential $V_f$ (i.e. the potential at which the probe draws zero current), which is closely related to the plasma potential, is measured by the single probe with a large resistance $(R_1 + R_2 \gg R_p)$ in the circuit, as shown in Fig. 2.15 (c). A typical waveform of the measured floating potential in SHEILA is shown in Fig. 2.20, where the same low frequency, coherent oscillations appear as those in the probe current signal.

![Figure 2.20: Floating potential signal (left), 5 V/div, with oscillations shown in expanded time scale (right), probe location 1 cm from the plasma centre, 1 mm diameter, argon filling pressure: $3.2 \times 10^{-2}$ Pa, $B = 0.14$ T, RF amplifier power: 380 W.](image)

2.3.3 Density measurements

When a single probe is biased to a large negative voltage (or a large voltage, say 90 V, is applied between the two electrodes of a double probe) it draws the ion saturation current. The actual amount of ion current drawn by the negatively biased electrode is influenced by the surrounding ion sheath, as determined by the ratio $\xi_p = r_p/\lambda_d$, where $r_p$ is the radius of the probe and $\lambda_d$ is the Debye length. The upper limit of this current is found by assuming that no undisturbed plasma particles (at infinity), which are energetically capable of reaching the
probe, will be excluded by intervening effective potential barriers (Chung et al., 1975). This condition corresponds to \( \xi_p \to 0 \). Taking into account potential barriers, the normalized ion current, \( j_i^* \), to a cylindrical probe in the limit \( T_i/T_e = 0 \) is shown in Fig. 2.21 for various ratios \( \xi_p \) (Laframboise, 1966), where \( T_i \) is the ion temperature. \( j_i^* \) is defined as:

\[
j_i^* = I_i / (A_p n_e Z_i e (T_e / (2\pi M_i))^{1/2})
\]  

(2.3)

where \( I_i \) is the ion current drawn by the probe, \( A_p \) is the surface area of the electrode, \( n_e \) is the undisturbed plasma density, \( Z_i \) is the charge number of the ions (\( Z_i = 1 \) for singly charged ions) and \( M_i \) is the ion mass. \( \chi_p^* \) in Fig. 2.21 is the probe potential relative to the plasma potential normalized by the electron temperature, \( \chi_p^* = Z_i e \phi_p / T_e \).

![Figure 2.21: Normalized ion current \( j_i^* \) versus normalized probe potential for various ratios of probe radius to electron Debye length; ion-attracting cylindrical probe; \( T_i/T_e = 0 \). (From Laframboise, 1966)](image)

Since the ions are much colder than the electrons in the SHEILA plasma,
we have \( \frac{T_i}{T_e} \approx 0 \) and the graph in Fig. 2.21 then can be used to find the approximate value of \( j^*_i \). In the plasma parameter range we study the estimated values of \( \xi_p \) for a 1 mm diameter probe lie between 20 and 40, which correspond to \( j^*_i = 1.5 - 2.0 \) (\( \sim 2.5 - 3.5 \) for a 0.3 mm diameter probe). Using the estimated value of \( j^*_i \), the measured ion saturation current and the electron temperature the electron density can be calculated from Eq. 2.3.

![Diagram of microwave interferometer](attachment:image_url)

**Figure 2.22:** *Microwave interferometer for measuring the line integral plasma density.*

The line integral density is measured by a double-pass Michelson microwave interferometer, shown schematically in Fig. 2.22. The 2 mm wavelength microwaves are generated by a carcinotron oscillator and launched from a horn antenna via an isolator and a vacuum window. The waves traverse the plasma and are reflected by a mirror. The reflected signal is collected by the same antenna and received by a diode detector through a directional coupler. An inherent reference signal is generated at the detector from the mismatch at the directional coupler input. The signal returned from the plasma and the reference signal have comparable amplitude and therefore interfere. Their relative phases can be adjusted in the absence of the plasma by moving the position of the reflecting mirror. The presence of the plasma introduces an extra phase shift \( \alpha_p \), which can be related to the electron line integral density (along the chord scanned) \( \int_{d_p} n_e dl \) by (Miyamoto, 33)
\[ \int_0^{d_p} n \sigma dl = \frac{1.11 \times 10^{15} \rho_p}{\lambda} \frac{\sigma}{2\pi} \text{ m}^{-2} \]  

(2.4)

where \( \lambda \) is the wavelength of the microwaves.

To calibrate the interferometer and set the phase shift without the presence of plasma the following steps are followed before each shot: firstly, the mirror is moved over several wavelengths and the output signal of the detector is recorded (Fig. 2.23 (a)). The detector signal can be assumed to be sinusoidally related to the phase shift. The signal during the calibration procedure varies irregularly with the hand movement of the mirror, however, it is apparent that the positive and negative extreme limits of the signal correspond to phase shift of \( \pi/2 \) and \( 3\pi/2 \) respectively and thus provide the necessary calibration. The output of the detector is then adjusted to zero or to an appropriate negative value to give a large range where the output of the detector monotonically increases with the phase shift. Fig. 2.23 (b) shows the output signal for a typical plasma discharge, where the phase shift before plasma presents is adjusted to zero. Comparing with Fig. 2.23 (a), the average phase shift for the shot is approximately of \( 0.12 \times 2\pi \), according to Eq. (2.4), which corresponds to a line integral density of \( 6.7 \times 10^{16} \text{ m}^{-2} \).

### 2.3.4 Fluctuation Measurements

As already mentioned before, low frequency fluctuations (~10 kHz) normally appear on both the ion saturation current and on the floating potential signals. For fluctuation measurements the frequency response of the probe circuit is limited by the stray capacitance \( C_s \), which is mainly the capacitance of the cable connecting the probe to the amplifier. The measured value of \( C_s \) is of the order of 300 pF in these circuits, and the upper frequency responses are about \( 1/(\text{RC}_s) \) for the two circuits in Fig. 2.15 (a) and (b), and about \( 1/(R_2 C_s) \) for the circuit
Figure 2.23: Output signal from microwave interferometer, (a) calibration signal before a shot, 100 mV/div and scan time 2 s/div, (b) line integral density signal during a shot, 100 mV/div and scan time 5 ms/div, the fluctuation in the base line is caused by the vibration of the reflecting mirror due to the discharge. Top trace: RF potential across the central conductor, 200 V/div

in Fig. 2.15 (c), which are both much higher than the frequencies we study.

Isolation amplifiers (input impedance 1 MΩ, bandwidth from DC to 1 MHz at -3 dB) are used to transfer these fluctuation signals to the oscilloscope or the data acquisition system. Lower frequency isolation amplifiers with the same input impedance and a 3 dB bandwidth from DC to 20 kHz are also used in the local plasma temperature and density measurements and in the phase correlation measurements.

The magnetic field fluctuations are monitored by magnetic probes inside a 4 mm I.D. silica tube which can be inserted into the plasma. Each probe coil has 233 turns wound in 8 layers around a 1 mm diameter, 2.6 mm long PTFE core. The high frequency cut-off for the probe is about 100 kHz. These signals are band-pass filtered in the range 1 kHz \( \leq f \leq 25 kHz \) and amplified 100 times before being recorded by the data acquisition system.
2.3.5 Validity of the Langmuir probe results

So far, only general probe methods have been described. However, the existence of a magnetic field and a RF field in these experiments requires care in the interpretation of the probe characteristic curves.

In the presence of a magnetic field the electrons are much freer to move along the magnetic field lines than to move across them. If the Larmor radius of the electrons is much smaller than the probe dimensions then the electron population in those flux tubes which intercept the probe can be replenished only by electron diffusion across the field lines. The electron current to the probe will then be reduced below that when there is no magnetic field and, therefore, the shape of the single probe characteristic curve will be modified. The double probe characteristic curve is much less affected because it draws a much lower electron current than the single probe. The reduction of the electron current to the single probe is characterised by a reduction factor \( r_c \) (Tagle et al., 1987; Stangeby, 1982):

\[
    r_c = 2 \frac{\lambda_{mf}}{A_p^{1/2}} (1 + \frac{T_i}{T_e}) \left( \frac{D_\perp}{D_\parallel} \right)^{1/2}
\]

where \( \lambda_{mf} \) is the electron mean free path, and \( D_\perp \) and \( D_\parallel \) are the perpendicular and parallel diffusion coefficients. The electron current collected by the probe for the applied voltage \( V \) greater than \( V_f \) are reduced by an amount depending on \( r_c \) and \( V \), and the electron saturation current is reduced to

\[
    I_{sat} = \frac{1}{4} n_e c_e \frac{r_c}{1 + r_c}
\]

where \( c_e \) = the random electron velocity = \((8T_e/\pi m_e)^{1/2}\). For typical SHEILA conditions, i.e. \( n_e \sim 7 \times 10^{17} \text{m}^{-3}, T_e \sim 7 \text{eV}, T_i \sim 0 \), the electron mean free path is about 0.7 m, \( D_\perp \sim 0.2 \text{m}^2\text{s}^{-1} \) (see Chapter 5), \( D_\parallel = \lambda_{mf} c_e / 4 \sim 3.4 \times 10^5 \text{m}^2\text{s}^{-1} \) (Stangeby, 1982; Bohm et al., 1949), and with \( A_p^{1/2} \sim 2 \text{mm} \) the reduction factor \( r_c \) is around 0.5 – 0.6. According to Tagle et al., the reduction in the electron current corresponding to \( r_c \) in the above range will introduce a relative error of
30% or less in the measured electron temperature under the condition that the slope of the single probe characteristic curve is taken in the range where the applied voltage \( V \) is not much higher than the floating potential \( V_f \) (within one or two times the electron temperature).

The shape of the single probe characteristic curve can also be affected if the measurements are made in the presence of an RF field and the time-averaged data used. The reason is as follows. In an RF field the plasma potential may oscillate at the RF frequency with an amplitude \( V_{RF} \) and the single probe characteristic in Fig. 2.17 (a) will shift back and forth along the \( V \) axis. Since the characteristic curve is nonlinear, the average current at an average voltage \( V \) will not be the same as the instantaneous current at voltage \( V \). In SHEILA experiments, an RF frequency signal, caused by both the pick-up and the oscillations in the plasma potential induced by the applied RF field, is superimposed on the probe current signal. This is time-averaged together with the low frequency oscillations arising from the local density fluctuations by the two-stage low-pass filter (see section 2.3.2) before plotting the probe characteristic curve. Since the directly measured \( V_{RF} \) under typical experimental conditions on SHEILA is only about 3 \( V_{p-p} \) or less, the errors introduced by the filtering are not considered to be serious. This assumption is supported by the fact that the electron temperature and density derived from both single- and double-probes are self-consistent and the derived densities from probe method (notice that \( T_e \) is involved in the deriving procedure) are also in reasonable agreement (within 20%) with the microwave interferometer results. Detailed comparison will be given in Chapter 4.

### 2.4 Data acquisition and analysis

The data acquisition system, previously used for the LT-4 tokamak (Bell et al., 1984), is also employed for SHEILA. The diagnostic outputs are connected to
CAMAC controlled ADC's (Analogue to digital converter) with 12 bit resolution at 5 kHz sampling rate for slow signals, and 8 bit resolution at 200 kHz sampling rate for the fluctuation signals. The CAMAC system is controlled by a network of small mini computers. This network (Vance 1982; Gorbould and How 1984) consists of a host PDP-11/04 computer plus three satellites. One satellite handles the CAMAC crates, while the others, each with its own processor, 60 kbyte of memory and graphics terminal, are used for analysis. The data is transferred to a hard disc within 30 seconds, after which it is accessible to any satellite for processing.

The program ‘NPLOT’ (How, 1985), which is able to plot or print any raw, derived or processed signal (diagnostic) as a function of time, is used for the SHEILA data analyses. Basic signal processing such as smoothing, integration, differentiation, plotting of envelopes, or Fourier analysis can also be carried out. The fast Fourier transforms give both the linear (or logarithmic) RMS amplitude and the phase graphically or in tabular form, analyzed in any desired time window.
In this Chapter, the computed vacuum magnetic configurations for SHEILA will be presented. Some relevant concepts are briefly reviewed first, followed by a description of the numerical field line tracing code and the detailed arrangement of the current in the coils. Finally, the results of various heliac configurations are given and their basic features are discussed.

3.1 Vacuum magnetic configuration

As mentioned in Chapter 1, magnetic configurations with good confinement geometry can be provided in stellarators by currents in the external coils before plasma starts. These configurations are called vacuum magnetic configurations, and are often used as bases of studies of plasma equilibrium, stability and transport.

A vacuum configuration in a stellarator usually consists of a set of nested
toroidal surfaces, which are formed either by one field line after an infinite number of toroidal transits (irrational surfaces) or by an infinite number of lines which close upon themselves after a finite number of toroidal turns (rational surfaces). Each of these surfaces encloses some toroidal volume $V_t$ and contains a definite toroidal magnetic flux $\psi$, so these surfaces are also called flux surfaces. The one with vanishingly small cross-section ($\psi = 0$) forms the magnetic axis. The magnetic surfaces can be labelled by any parameter which is unique to each surface, such as the toroidal magnetic flux $\psi$. In this thesis we use, as convenient, two other labels as flux surface parameters: $X$, when dealing with the study of the heliac geometry, and $r$, an effective mean radius in the fluctuation studies. The parameter $X$, unlike $r$ or $\psi$, requires no apriori knowledge of the magnetic field strength or the flux surface cross-sectional area.

In addition to the existence of the closed magnetic surfaces, several important quantities are often used to characterize the confinement properties of a configuration. These are described below.

### 3.1.1 Rotational transform and magnetic shear

The rotational transform $\tau$ is the magnetic surface average of the 'twist' of the field lines. The 'safety factor' $q$ used in tokamaks is the reciprocal of $\tau$. The rotational transform in each surface can be calculated as

$$\tau \equiv \lim_{N_t \to \infty} \frac{1}{2\pi N_t} \int \frac{d\theta}{d\phi} d\phi$$

(3.1)

where the integration is along a field line, $N_t$ is the number of toroidal transits made by the field line. In rational surfaces the rotational transform is the ratio of two integers. The surfaces with low-order rational numbers are regarded as 'weak' surfaces, because spatially resonant field harmonics can cause them to break up into sizable magnetic islands, and the confinement of the configuration is degraded.
It is often desirable to have the rotational transform defined in the helical frame, i.e. the frame rotating with the vector from the central conductor to the magnetic axis. Such defined rotational transform is denoted as $t_h$, which is related to the usual $t$ defined in the laboratory frame by:

$$t_h = N - t$$  \hspace{1cm} (3.2)

The variation of rotational transform with magnetic surface is called magnetic shear. If the rotational transform increases with the surface parameter (either $X$, $\psi$ or $r$) the shear is positive. Shear is generally considered as a favourable property for MHD stability and can also limit the size of the magnetic islands produced by resonant field perturbations. On the other hand, in a configuration with high rotational transform, such as in a heliac, large shear tends to introduce more low-order rational surfaces into the confined region.

### 3.1.2 Magnetic well

Having a magnetic well in a configuration means that the field $B$ in this configuration decreases monotonically towards the magnetic axis from all directions. Strictly speaking, this can only occur in a linear (i.e. open) system.

In a closed system, a local (flux-surface-averaged) magnetic well or hill may exist, which depends on the sign of the derivative $V_t''(\psi)$ (Furth et al., 1966; Kulsrud, 1966). The negative sign corresponds to the existence of a local magnetic well, which is in favour of MHD stability.

The first derivative of $V_t(\psi)$ is known as the specific volume, which can be calculated on a flux surface by an integral along the field line

$$V_t'(\psi) = dV_t/d\psi = \lim_{N_t \to \infty} \frac{1}{N_t} \int \frac{dl}{B}$$  \hspace{1cm} (3.3)

If a global magnetic well exists, its depth $W(\psi)$ is given by

$$W(\psi) = \frac{V_t'(0) - V_t'(\psi)}{V_t'(0)}$$  \hspace{1cm} (3.4)
3.1.3 Vacuum field spectrum

The set of harmonic amplitudes of the Fourier decomposition of the vacuum magnetic field $B$ (or $1/B^2$) on each flux surface with respect to the poloidal angle $\theta$ and the toroidal angle $\phi$ is important in determining the MHD equilibrium, stability and transport. The dominant non-resonant harmonics in a heliac are $(n, m) = (0, 1), (N, 0)$ and $(N, 1)$ (where $n$ and $m$ are toroidal and poloidal mode numbers in laboratory frame). The $(N, 0)$ ripple component arises from the helical excursion of the magnetic axis sampling the $1/R$ variation of the toroidal magnetic field, and the $(0, 1)$ and $(N, 1)$ ripple components are related to toroidal and helical curvatures (Reiman and Boozer, 1984). These field ripples affect the single particle orbits of particles with pitch angles large enough that they can be trapped in local ripple wells. Since the orbit widths of these trapped particles are much wider than those of the untrapped particles, the ratio of the trapped particles to the untrapped ones is important in determining particle and energy transport coefficients.

The other harmonics in the spectrum can resonate with the corresponding rational surfaces and cause them to break up into magnetic islands. The island width on a rational surface depends on the amplitude of the resonant field harmonic. In a low $\beta$ plasma the island width $W_i$ can be related to the amplitude of the resonant component $\delta_{n,m}$ by $W_i \propto \delta_{n,m}^{1/2}$, where $\delta_{n,m}$ is obtained from the Fourier decomposition of $1/B^2$ in the vacuum field (Reiman and Boozer, 1984). Since the widths of these islands generally increase when the mode numbers decrease, the low-order resonant harmonics are more critical.
3.2 Computation details

3.2.1 BLINE field line tracing code

This computer code numerically traces the trajectory of magnetic field lines in vacuum in a toroidal geometry using well-established procedures (Gibson, 1967; Ehrhardt et al., 1985; Blackwell, 1988). The coil set can be either approximated by a sufficient number of circular filamentary conductors, or made up of small linear Biot-Savart current elements. The resultant magnetic field vectors generated by the currents in the coil set are calculated. After a proper starting point is chosen, a field line is traced in steps of constant path length for many toroidal rotations to outline a surface. A set of starting points and the maximum path length required can be specified beforehand. The criterion for an unsuccessful trace is that the trace leaves the machine, or hits a coil, or both before reaching the maximum path length required.

A simple iterative procedure is used to find an approximate magnetic axis. The method is to start the field line tracing at an initial guess point in the \( \phi = 0 \) plane, after an appropriate number of toroidal turns, find all the punctures in this plane and take the new starting point as the mean of these. If \( N \) fold symmetry of the field exists, this process can be accelerated by taking the new starting point as the mean of all the punctures in the \( N \) symmetry planes. Under normal conditions, the error typically reduces by at least a factor of 3 for each iteration.

The last closed surface can also be found in an iterative manner, by directing the code to explore starting points beyond a known closed surface. Once an unsuccessful field line is found, new starting points are always tried midway between the outermost successful trace and the innermost unsuccessful trace.

Most computations have been performed on an IBM-PC with an 8087 numeric co-processor running at 4.77 MHz. For SHEILA with a typical step size of 2 mm, 2 hours is required for a 50 metre trace, or about 9 hours for a magnetic axis.
and 4 surfaces. However, the code was later installed on the Fujitsu VP-100 at the A.N.U. Supercomputer Facility and ran about 600 times faster. This greatly helps to explore the detailed structure of the magnetic islands in the vacuum field and the various configurations affected by an external error field.

In addition to the field line tracing, the code also calculates the rotational transform and the specific volume $V'_\psi(\psi)$ for each magnetic surface according to Eqs. (3.1) and (3.3). To obtain high accuracy, the integral of $dl/B$ in Eq. (3.3) is accumulated at the same time as the trajectory. The required accuracy of the rotational transform and the specific volume can be achieved when the field line is followed sufficiently far. Convergence is accelerated by the use of a Blackman weighting function in the averaging procedure. Vacuum field harmonics on each surface are obtained by the discrete Fourier transform of $1/B^2$ in uniform steps in $\chi (\chi \equiv \int Bdl)$ performed over the length of the traced field line (Boozer, 1982).

### 3.2.2 Coil details

This code described above is used to calculate the vacuum configuration of SHEILA. The coil currents, except the current in the helical winding, are approximated by circular current filaments distributed as follows: three equal filamentary currents (at radii 58.2, 64.6 and 71.0 mm) in each of the 48 TF coils, one filament at the centre of each of the four turns of the central conductor, and one filament at the centre of each vertical field coil. The ampere-turns of the set of toroidal field coils, the central conductor, and the two sets of the vertical field coils (inner and external), are in the fixed ratio 48:4:2:0.42. These ratios were carefully chosen in the original design to produce an optimized magnetic configuration (referred to here as the standard configuration) before the helical winding was added. The helical winding is approximated by a single filament of winding law $R = R_0 + \rho_h \cos N\phi$, $Z = \rho_h \sin N\phi$, where $R_0$ is the mean radius of the central conductor (0.1875 m) and $\rho_h$ is the swing radius of the helical winding.
The helical filament is then represented by many small linear Biot-Savart current elements. For the results presented below, the typical field line tracing length is about 200 m with a constant step size of 2 mm.

### 3.3 Results

#### 3.3.1 Standard configuration

Fig. 3.1 shows a toroidal surfaces with indented bean-shaped cross-sections calculated in the standard condition (without the helical winding). The magnetic axis rotates three times around the central conductor in one toroidal turn and each poloidal rotation represents one of three nominally identical field periods. The surface cross-sections at four phases \( \phi = 0, 30^\circ, 60^\circ, 90^\circ \) within one field period are shown in Fig. 3.2. The last closed surface, which encloses the largest volume between the central conductor and the toroidal field coils, has an average radius about 0.035 m.

The radial profile of the rotational transform \( \tau \) is shown in Fig. 3.3(a), where the flux surface parameter \( X \) is taken as the horizontal distance (in centimetres) measured radially outwards from the magnetic axis \( X = 0 \) in the plane \( \phi = 0 \). The rotational transform, as shown in Fig. 3.3(a), varies from 1.18 on the axis to 1.35 on the last surface, corresponding to a low shear \( (\tau(a) - \tau(0))/\tau(0) \sim 0.14 \), where \( \tau(a) \) and \( \tau(0) \) are the rotational transforms on the last closed surface and the magnetic axis respectively. The average value of rotational transform per field period \( \tau/N \) is around 0.4, which is high in comparison to most other types of stellarator. The low-order rational surfaces introduced by the shear in this configuration are \( \tau = 4/3, 5/4, 6/5, \) and \( 9/7 \).

In Fig. 3.3(b) the depth of the magnetic well, \( W \), is plotted as a function of the flux surface parameter \( X \), where it shows a significant global well with a depth about 3% on the last closed magnetic surface.
Figure 3.1: A typical vacuum magnetic surface with field lines, calculated for standard coil setting (without helical winding). The bean-shaped cross section at $\phi = 220^\circ$ is shown below.
Figure 3.2 (a): Cross-sections (in actual size) of vacuum magnetic surfaces at $\phi = 0$ and $\phi = 60^\circ$ in one field period for the standard configuration. The apertures of the toroidal field coils are shown for comparison.
Figure 3.2 (b): As for Fig. 3.2(a), $\phi = 30^\circ$ and $\phi = 60^\circ$. 
Figure 3.3: Profiles of rotational transform r and magnetic well for the standard configuration. The locations of the low-order rational surfaces are shown on the top graph.
Typical spectra of $1/B^2$ in the vacuum (in the laboratory frame) on four surfaces, $X = 0.5, 1.0, 1.5, \text{ and } 2.0$, are shown in Fig. 3.4 (a) and (b). The lowest 10th of the spectra are shown on the right of Fig. 3.4, and the lowest 100th of the spectra are displayed on the left on an expanded scale. The amplitudes of these harmonics are normalized to the zero field component and plotted versus their spatial frequencies, at which the harmonics vary along the field lines. This frequency for each harmonic is given by:

$$f_{n,m} = \frac{1}{\oint B dl} \left| n - im \right|,$$

(3.5)

where the integration is made along the magnetic axis for one toroidal turn. According to Eq. (3.5), the resonant condition on a flux surface for a given harmonic is that $n = \pm m$.

The amplitude of the largest field component $(n, m) = (3, 0)$ in Fig. 3.4 is 41% on the first surface and is reduced to 32% on the fourth surface, while the amplitudes of the other two ripple components $(0, 1)$ and $(3, 1)$ monotonically increase from 9% and 7% on the first surface to 32% and 22% on the fourth surface. The dominant low-order resonant harmonics in SHEILA are $(3, 2)$ and $(3, 3)$, which have their largest amplitudes on the fourth surface of 8.5% and 2.9% respectively. The existence of these low-order harmonics in SHEILA suggests that the rational surfaces $\ell = 1.5$ and 1 should be deliberately removed from the configuration.
Figure 3.4 (a): The vacuum spectra of $1/B^2$ in the low frequency range for $X = 0.5$ and 1.0 flux surfaces in the standard configuration. Left: 1/100 of the spectrum, right: 1/10 of the spectrum. The mode numbers $(n, m)$ of the important harmonics are labeled.
Figure 3.4 (b): As for Fig.3.4(a), but for $X = 1.5$ and $2.0$ flux surfaces.
3.3.2 Configuration scans

This section presents the computational results of the greatly extended range of possible heliac configurations made available by incorporating the helical winding. When the current ratios in all main coils (except in the helical winding coil) are kept fixed as given in section 3.2.2, the current ratio $I_h/I_r$ is varied to obtain different heliac configurations, where $I_r$ is the total current in the four turns of the central conductor and $I_h$ is the current in the helical winding as defined in Chapter 1.

Figs. 3.5-3.12 show the cross-sections at $\phi = 0$ and $\phi = 60^\circ$ of the closed flux surfaces for some typical values of $I_h/I_r$ in the range of $-0.15 \leq I_h/I_r \leq 0.25$; when below $-0.15$ the closed flux surfaces no longer exist in the computations, and the upper limit 0.25 is the maximum ratio we can obtain. It can be seen that as the helical winding current increases in either direction with respect to the ring current, the volume contained by the last closed flux surface decreases. Well defined surfaces do not exist when $I_h/I_r$ is around 0.1 (see next section). However, the flux surfaces gradually reappear when the ratio $I_h/I_r$ further increases from 0.1; this time the magnetic axes of those configurations have larger helical excursions and the toroidal effect makes the cross-sections at the two phases much less symmetric around the ring compared with those in the standard configuration.
Figure 3.5: Cross-sections of vacuum magnetic surfaces (in actual size) at $\phi = 0$ and $\phi = 60^\circ$ in one field period for $I_h/I_r = -0.05$. 
Figure 3.6: As for Fig. 3.5, $I_h/I_r = -0.1$. 
Figure 3.7: As for Fig. 3.5, $I_h/I_r = -0.15$. 


Figure 3.8: As for Fig.3.5, $I_h/I_r = 0.05$. 

$\phi = 60^\circ$ 

$\phi = 0^\circ$
Figure 3.9: As for Fig. 3.5, $I_h/I_r = 0.075$. 
Figure 3.10: As for Fig. 3.5, $I_h/I_r = 0.15$. 

59
Figure 3.11: As for Fig.3.5, $I_h/I_r = 0.2$. 

60
Figure 3.12: *As for Fig.3.5, $I_h/I_r = 0.25$.)*
The rotational transform profiles of these configurations are shown in Fig. 3.13, where \( \tau(0) \), the rotational transform on the magnetic axis, is varied from 0.56 to 1.88 which is more than a factor of three. The magnetic shear changes sign in this range, it nearly disappears when \( \tau(0) \) approaches 1.5.

Figure 3.13: Rotational transform profiles for various values of \( I_h/I_r \).
The profiles of the magnetic well depth $W$ for various $I_h/I_r$ ratios are shown in Fig. 3.14, where it shows that the changing of the current ratio also results in different depth wells, and even local magnetic hills in some cases. The deepest well on the last closed surface is about 4.5% at $I_h/I_r = 0.05$.

Figure 3.14: Profiles of the magnetic well for various values of $I_h/I_r$. 

![Graph showing profiles of magnetic well depth for various $I_h/I_r$ ratios.](image-url)
3.3.3 Resonances and surface destruction

The success of obtaining the wide range of configurations makes it possible to explore the dangerous resonances when the \( \tau \) profile passes through low-order rational surfaces. A typical example is presented below to demonstrate the destruction of surfaces as a consequence of resonance.

Fig. 3.15 shows the changing of the configuration when the \( \tau = 1.5 \) rational surface gradually moves into the confinement region, corresponding to the ratio \( I_h/I_r \) varying in the range between 0.05 and 0.11. In Fig. 3.15(a), since the \( \tau = 1.5 \) surface is near the outer region of the configuration, it resonates with the field harmonic \((n, m)=(3, 2)\) inherent in SHEILA and results in two magnetic islands located on either side of the main confinement region. As the ratio \( I_h/I_r \) increases, the two islands move further in following the changing location of the \( \tau = 1.5 \) surface, and the volume inside the last closed flux surface becomes smaller and smaller (see Fig. 3.15(b)), until finally, because of the nearly disappearing magnetic shear in the rotational transform profile, the resonance becomes so strong that the whole configuration bifurcates into two islands surrounded by a large ergodic field region (Fig. 3.15(c)).
Figure 3.15: Cross-sections in $\phi = 0$ plane, (a) $I_h/I_r = 0.05$, (b) $I_h/I_r = 0.075$, (c) $I_h/I_r = 0.11$. 
Magnetic islands resulting from the resonance between the \((n, m) = (3, 3)\) field harmonic and the \(\iota = 1\) surface have also been seen in the computations. For instance, in Fig. 3.5 three islands appear near the centre of the configuration, where the rotational transform is around unity, and in Fig. 3.16, following the changing location of the \(\iota = 1\) flux surface the three islands move to the mid-radius of the configuration. These islands are sizable, however, they are still localized because of the existence of a magnetic shear.

Figure 3.16: Cross-section at \(\phi = 0\) for \(I_h/I_r = -0.075\), where the three islands are resulted from the resonance between the \((n, m) = (3, 3)\) field harmonic and the \(\iota = 1\) rational surface.
Chapter 4

Experimental Studies of Equilibrium Configurations

In this Chapter, the experimental studies of equilibrium plasmas obtained for the first time in a toroidal heliac will be presented. The locations of equilibrium surfaces are verified by chordal scans of the plasma pressure made by Langmuir probes. As will be shown, in most cases the results are consistent with points of equal pressure lying on the same surface as computed for the vacuum configurations. However, clear discrepancies appear in configurations involving a \( \tau = 1 \) surface. The reason for this became clear after an error field, caused by positional errors in the central conductor and its associated helical winding, was found.

The experiments are mostly conducted in argon plasma with the typical parameters given in Table 4.1. Despite the low plasma temperatures, the low densities and the large rotational transforms in this machine still enable the electrons to sample the toroidal and helical field variations between collisions sufficiently to make the experimental observations relevant to fusion physics. For example, the electron mean free path with the typical argon plasma parameters of Table 4.1 lies in the range 0.5 – 1.0 m, while the toroidal and helical curvature connection lengths, which can be roughly estimated as \( R_0/\tau \) and \( R_0/\tau_h \) respectively, are
about 0.16 m and 0.10 m (in standard configuration).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RF amplifier output power (W)</td>
<td>≤ 400</td>
</tr>
<tr>
<td>Plasma current (A)</td>
<td>&lt; 100</td>
</tr>
<tr>
<td>Electron temperature (eV)</td>
<td>6 – 10</td>
</tr>
<tr>
<td>Average electron density (m(^{-3}))</td>
<td>5 \times 10^{17} – 1 \times 10^{18}</td>
</tr>
<tr>
<td>Electron mean free path (m)</td>
<td>~ 0.5 – 1</td>
</tr>
<tr>
<td>Energy confinement time ((\mu s))</td>
<td>&gt; 30</td>
</tr>
<tr>
<td>Central plasma beta</td>
<td>~ 10^{-4}</td>
</tr>
</tbody>
</table>

Table 4.1: Argon plasma parameters

The studies in this Chapter include the experimental verification of the wide range of heliac configurations computed in Chapter 3, the dependence of the gross plasma confinement properties on the magnetic configuration, and the critical effects of a symmetry-breaking error field on the heliac configurations. The detailed results will be presented in the following sections.

4.1 Results in standard configuration

Plasma was first obtained in SHEILA by an argon gas breakdown in the standard configuration. The existence of a confined plasma column in the predicted region was evidenced, at the simplest level, by a photograph taken from above through the transparent polycarbonate lid. This is shown in Fig. 4.1, where the luminous helical column encircles the central ring three times and is clearly restricted to
Figure 4.1: Photograph taken from above through a transparent polycarbonate lid, showing the luminous argon plasma encircling the central conductor three times. The toroidal coil current feeder transmission line system blocks the plasma light along a path that closely follows the plasma axis.

that region where the computed closed magnetic surfaces lie.

A set of waveforms is recorded routinely for each shot. These are: the mean magnetic field derived from the current measured in the main circuit by a Rogowski coil; the DC power input to the RF amplifier; the RF voltage on the four-turn ring measured by a 100:1 voltage divider across the ring; the visible light signal; the microwave interferometer signal; and the ion current signals
Figure 4.2: Typical waveforms: (a) 20 ms RF pulse: (i) mean magnetic field strength; (ii) envelope of RF voltage at the four-turn ring; (iii) visible light emitted from chord near the plasma mid radius; (iv) ion current to double probe biased to 15 V at plasma centre. (b) 12 ms RF pulse: (i) DC power to RF amplifier; (ii) interferometer signal with (solid line) and without (dashed line) plasma; (iii) as for (a)(iv), but with 90 V bias and 3 kHz low-pass filter.

from Langmuir probes. These waveforms are shown in Fig. 4.2 for two typical RF pulses.
The RF current in the ring can be obtained by dividing the RF voltage around the ring by the impedance of the ring given in table 2.2. For instance, the 250 V\textsubscript{p-p} RF voltage (at 96 kHz) before gas breakdown shown in Fig. 4.2(a) corresponds to a total RF current with an amplitude of about 70 A in the ring. The output power from the amplifier has been calibrated with a 50Ω load, and is about 80% of the DC input power to the amplifier.

It was not found possible to measure the plasma current directly because of the difficulty of putting a sufficiently sensitive Rogowski coil only around the plasma in the very limited space. However, the upper limit of the plasma current is given by the total RF current in the four-turn ring, which is usually less than 100 A. A rough estimation has been made of the loop voltage induced by the changing flux generated by the slowly varying current in the ring at times around the peak magnetic field, which gives a small loop voltage about 0.1 V/turn. If we take conductivity appropriate to a 7 eV plasma this will only cause a slow current about 10 A to flow in the conducting plasma; this is small enough both in its contribution to the magnetic field and the plasma heating, that it can be ignored. Attempts to measure this slow plasma current have been made using one circular Rogowski coil encircling the plasma and the four-turn ring, in series with a second identical Rogowski coil monitoring four turns of the current in the main circuit to subtract the contribution from the ring current; the observations put an upper limit on this current of 20 A.

4.1.1 Pressure profile measurements

The plasma density and temperature profiles are measured by double Langmuir probe chordal scans in two cross-sections at \( \phi = 0 \) and \( \phi = 30^\circ \). The experimental conditions applied are: \( B = 0.2 \) T, base pressure \( 1.3 \times 10^{-3} \) Pa, argon filling pressure \( 2.7 \times 10^{-2} \) Pa, and 150 W DC power to the RF amplifier. These profiles are shown in Figs.4.3 and 4.4. Because of the highly reproducible discharges in argon plasma,
Figure 4.3: Profiles of the electron density, temperature and plasma pressure scanned along the midplane \((z = 0)\) at \(\phi = 0\). Corresponding computed flux surfaces are shown reduced in the inset, with line indicating probe path.

The profiles shown in Figs. 4.3 and 4.4 are obtained shot by shot. In Fig. 4.3 the density and the plasma pressure are peaked inside the region where the computed closed magnetic surfaces are, while the radial variation of the temperature in that region is relatively small.

In Fig. 4.4 the obvious feature is the two humps in the curve which result from the two lobes of the bean-shaped cross-section intercepted by the probe scan path at \(\phi = 30^\circ\).

The pressure data in Figs. 4.3 and 4.4 can be more generally plotted as
functions only of the flux surface parameter $X$ if the equilibrium surfaces in the plasma exist. $X$ is itself derived by computing the magnetic surface which passes through the point of the probe scan. Such a plot is given in Fig. 4.5, where it clearly shows that most of the data points lie within experimental uncertainty on the same smooth curve. A few points scattered, marked by open triangles, were taken when the probe had its greatest perturbing effect during the scan.

Figure 4.4: As for Fig. 4.3, but for chord above midplane ($z = +0.025\,\text{m}$) at $\phi = 30^\circ$. 
Figure 4.5: Pressure data of Figs. 4.3 and 4.4 plotted against the flux surface parameter $X$. 'Outside' implies $R > 0.2275 \text{ m}$ for $\phi = 0$, and $R > 0.1875 \text{ m}$ for $\phi = 30^\circ$.

4.1.2 Parametric dependence of the density

In appropriate gas pressure ranges, plasmas have been obtained in three different gases. Those ranges are $4.0 \times 10^{-3} - 6.7 \times 10^{-2} \text{ Pa}$ for argon, $3.5 \times 10^{-2} - 2.7 \times 10^{-1} \text{ Pa}$ for helium and $9.3 \times 10^{-3} - 6.7 \times 10^{-2} \text{ Pa}$ for hydrogen. The dependences of the central electron density measured by a Langmuir probe on the gas filling pressure, under various applied magnetic fields and RF powers are shown in Fig. 4.6 for the three gases, where the fully ionized plasma is achieved in argon under low gas filling pressure and high applied magnetic field and RF power. The highest central densities achieved are about $3 \times 10^{18} \text{ m}^{-3}$ in argon, and about $8 \times 10^{17} \text{ m}^{-3}$,
Figure 4.6: Range of discharges obtained in SHEILA for various filling gases (A, H, He), filling pressures (expressed as atom number density), magnetic field strengths, and the DC powers to the RF amplifier.

2 × 10^{17} \text{m}^{-3} in helium and hydrogen respectively.

The densities measured by probes have been checked with the microwave interferometer results. For instance, the line integral density obtained by probe chordal scan at \( \phi = 30^\circ \) (Fig. 4.4) has been compared with the microwave interferometer data measured through a similar chord at \( \phi = 330^\circ \) (see Fig. 4.7). The probe method gave a line integral density about \( 0.56 \times 10^{17} \text{m}^{-2} \), and the microwave interferometer measurement showed a similar result of \( 0.63 \times 10^{17} \text{m}^{-2} \), the difference between them was less than 15%. 

75
Figure 4.7: (a) The probe scan path at $\phi = 30^\circ$, (b) the microwave path at $\phi = 330^\circ$.

### 4.1.3 Energy confinement time

Based on the above results, the energy confinement time in argon plasma can be roughly estimated, in order of magnitude, for this small toroidal heliac. Although the RF power actually absorbed by the plasma has not been measured, it is unlikely to exceed two thirds of the total output power from the amplifier. With the following parameters under high RF power: $\bar{n}_e \sim 1.5 \times 10^{18} \text{ m}^{-3}$, $T_e = 7 \text{ eV}$, 430 W DC power to RF amplifier and a total plasma volume of about $4 \times 10^{-3} \text{ m}^3$, the global energy confinement time $\tau_e$ is

$$
\tau_e = \frac{\bar{n}_e T_e V_p}{\text{Ohmic input power}} = \frac{1.5 \times 10^{18} \times 7 \times 1.6 \times 10^{-19} \times 4 \times 10^{-3}}{430 \times 0.8 \times (2/3)} \sim 30 \mu s,
$$

where $V_p$ is the plasma volume. If we take into account that much of the Ohmic input power must go into inelastic atomic processes in such a low temperature argon plasma, this must be regarded as a significant underestimate of the loss by transport processes.
Figure 4.8: Pressure profiles on the midplane at $\phi = 120^\circ$ for various current ratios, derived from measured temperature and density profiles.

4.2 Heliac configuration scan

In the following sections, the favourable results obtained in the standard case are extended to the wide range of heliac configurations computed and described in Chapter 3. The resonance features appearing when the rotational transform profile passes the low-order rational surfaces will also be discussed.

4.2.1 Identification of configurations

The wide range of heliac configurations has been obtained by changing the current ratio $I_h/I_r$. These configurations are then identified by probe measurements in a similar manner as in section 4.1.1.
The density and temperature profiles along the midplane chord have been measured by the 2-D Langmuir probe at $\phi = 120^\circ$ for various current ratios. Experimental conditions are set up as follows: $B=0.15$ T, base pressure $2.7 \times 10^{-4}$ Pa, argon filling pressure $1.3 \times 10^{-2}$ Pa, and 380 W DC power to the RF amplifier. A group of pressure profiles, obtained from the product of the measured $n_e$ and $T_e$ profiles, are shown in Fig. 4.8, where the plasmas are seen to gradually move away from the central conductor as the current ratio increases. Fig. 4.9 shows the peak positions of these pressure profiles in Fig. 4.8 compared with the computed positions of the magnetic axes of those configurations. General agreement has been achieved for all cases, except for the case of $I_h/I_r = -0.05$ where an 8 mm difference between the measured peak pressure position and the computed position for the magnetic axis has been found. It is also noticeable that no computed data are available at $I_h/I_r$ around 0.1 in Fig. 4.9: this is due to the resonance between the inherent field harmonic $(n, m) = (3, 2)$ and the rational surface $\tau = 1.5$ which occurs around $I_h/I_r \sim 0.1$, and this results in the configurations not having a single magnetic axis in that range.

Further detailed measurements of the pressure profiles in two different cross-sections have been made to verify the existence and geometry of the equilibrium surfaces. Three examples are given here to illustrate sets of well defined equilibrium surfaces and their coincidence with the corresponding computed vacuum surfaces. Fig. 4.10 shows the pressure profiles measured along a midplane chord by the 2-D Langmuir probe at $\phi = 120^\circ$ and along a horizontal chord 16 mm above the midplane by another single Langmuir probe at $\phi = 60^\circ$ in three typical configurations. These are configurations with $I_h/I_r = 0$, 0.15 and 0.25, corresponding again to the standard configuration (this time the helical winding is in place), a highly asymmetric configuration (see Fig. 3.10) and one having the highest attainable rotational transform (see Fig. 3.12).

The pressure data in Fig. 4.10 are then plotted as functions of the flux surface
parameter $X$. Considering the possible systematic errors in both probes, the peak of each measured pressure profile is matched to the computed magnetic axis at $\phi = 120^\circ$ (or to the point of intersection of the chord with the computed magnetic surface closest to the axis at $\phi = 60^\circ$), and then all other probe positions along the chord scanned can be related to their appropriate magnetic surfaces. This is shown in Fig. 4.11 where it shows that in each case the pressures measured in different poloidal and toroidal locations but on the same computed flux surface are seen to be very close to each other, and the dependence of the pressure on the different flux surfaces lies within the experimental uncertainty on a single smooth curve.

Figure 4.9: Positions of peak pressure of the profiles in Fig. 4.8 compared with the computed magnetic axis (dashed line).
Figure 4.10: Pressure profiles for three cases, obtained from horizontal scans at \( \phi = 120^\circ \) along a midplane chord, and at \( \phi = 60^\circ \) along a chord 16 mm above the midplane.
Figure 4.11: *Pressure data of Fig. 4.10, plotted as functions of the flux surface parameter X.* 'Inside/outside' refers to the position of the plasma with respect to the magnetic axis.
4.2.2 Effect of low-order resonances on plasma properties

Strong resonance effects and accompanying degradation of the plasma confinement would be expected to occur in those configurations which include low-order rational surfaces. Such resonance features have been studied earlier in classical stellarators, most notably in the shear-free $\ell = 2$ Wendelstein devices (Grieger et al., 1985). In this section, the experimental observations of the effects of low-order resonances in SHEILA are described.

Two related quantities, namely the minimum electric field required for breakdown and the plasma density obtained in the discharge, are found to be sensitive to the change of configuration. Detailed argon gas breakdown measurements have been made in the standard configuration. The experimental results of the dependence of the minimum electric field intensity for breakdown on gas pressure for different frequencies of the applied electric fields and magnetic field strengths show many similarities to the early results for microwave breakdown at higher gas pressure and are in good qualitative agreement with the theory of diffusion controlled breakdown (Brown, 1959). This suggests that the gas breakdown field in SHEILA has a strong dependence on the characteristic diffusion length of the system, which is in turn related to the confinement properties of a given magnetic configuration. This work is described in detail by Blackwell et al., (1988). The peak plasma density is also a good measure of the confinement properties of a magnetic configuration. The density obtained in steady state is directly related to the ionization rate and the particle diffusion loss in a diffusion controlled discharge. When all the conditions, such as the gas pressure, magnetic field strength and the Ohmic heating power, are maintained approximately constant, the variation of the plasma density can be regarded as a rough measure of the variation of the particle confinement time in the system.
The dependence of the above two quantities on the magnetic configuration has been measured experimentally. During the configuration scan, all experimental conditions are kept constant, except for the helical winding current, whose value is incremented using the variable shunt. The experimental conditions in this experiment are the same as those in the pressure profile scans in the previous section, except that for the measurements of the minimum electric field for breakdown, the applied magnetic field has been increased to $B = 0.2 \, \text{T}$ in order to achieve gas breakdown over a wide range using weak electric fields.

The electron density in this experiment is measured by the 2-D probe at $\phi = 120^\circ$ positioned at the centre of the plasma, while the minimum electric field for breakdown $E_{\text{BD}}$ (in r.m.s.) is derived from the RF voltage $V_r$ (peak to peak) applied to the four-turn ring by

$$E_{\text{BD}} = \frac{V_r}{2 \times 4 \times \sqrt{2} \times L_h} \quad (4.2)$$

where $L_h$ is the length of the helical magnetic axis, $L_h \sim 1.4 \, \text{m}$. The induced voltage in the secondary was also checked with the directly measured loop voltage in the standard configuration using the helical winding, which gives an induced electric field in the secondary about 80% of the value derived from Eq. 4.2. This is consistent with a numerical calculation, which gives a flux linkage about 76% between the ring and the helical winding (Blackwell et al., 1988). However, in this study only the relative values of the electric field are needed, so that this correction is not important.

The results are shown in Fig. 4.12, where the central electron density and the minimum electric field required for breakdown are plotted versus the current ratio $I_h/I_r$. The most obvious features in this graph are that the sharp increases in breakdown field and the rapid decreases in the central electron density occur around the same $I_h/I_r$ values of 0.1 and $-0.075$, which correspond to configurations involving an $\tau = 1.5$ surface and an $\tau = 1$ surface respectively. Reference
Figure 4.12: Central plasma density (a) and the minimum electric field required for breakdown (b), as functions of $I_h/I_r$. The vacuum field configurations shown correspond (from left to right) to $I_h/I_r = -0.075$, 0, 0.11 and 0.25.
to the computed vacuum configurations illustrated on the top of the graph shows
the bifurcation of the configuration into two islands surrounded by large ergodic
field regions at \( I_h/I_r \sim 0.1 \). This would be expected to result in a greatly increased
diffusion of particles out of the system. However, the more pronounced resonance
around \( I_h/I_r \sim -0.075 \) can not be simply explained by the computed vacuum
magnetic configuration shown on the top, where the surfaces are only locally
disturbed by the three magnetic islands.

No breakdown can be achieved with the maximum RF power available when
\( I_h/I_r < -0.14 \). This is obviously caused by the greatly reduced volume inside
the closed magnetic surfaces when the opposing current in the helical winding
becomes too large. Clearly, the best configurations for confinement, as charac­
terized both by ease of breakdown and by highest density, are those in the range
\( 0 < I_h/I_r < 0.05 \); according to the computation in Chapter 3, these configura­
tions have the largest and most symmetric surfaces with respect to the central
conductor.

4.2.3 Experimental evidence of island formation

Although the poor confinement and breakdown properties in some ranges shown
in Fig. 4.12 indirectly give evidence of the surface destruction resulting from
island formation, a more direct manifestation of islands can be obtained by mea­
suring the local plasma properties with probes, in those regions indicated by the
vacuum field calculations to include islands. Two examples are given in Figs.
4.13 and 4.14, where a series of chordal pressure scans are made by the 2-D probe
in \( \phi = 120^\circ \) plane in configurations with \( I_h/I_r = 0.075 \) and 0.11 under the same
experimental conditions as those in the density scan in Fig. 4.12.

In Fig. 4.13 the scan paths are along the midplane and 20 mm above and below
the midplane. Comparison of the computed vacuum configuration displayed on
the right with these profiles show that the pressure gradient is generally larger

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inside the closed regions, including the small intersected islands, and the profiles are significantly flattened in the ergodic regions indicated by the heavy lines with arrow heads marking the boundaries between the ergodic regions and the closed surface regions. Another example is shown in Fig. 4.14, where the pressure profile is measured along the midplane in the configuration with $I_h/I_r = 0.11$; reference to the computed vacuum magnetic configuration on the right shows the scan path is through a region filled with ergodic field lines, and resulting in a pressure profile which is low and flat.
Figure 4.13: Three chordal pressure scans for $I_h/I_r = 0.075$, along the midplane and 20 mm above and below the midplane, at $\phi = 120^\circ$. The appropriate vacuum configuration is shown on the right. The heavy lines indicate the ergodic regions and the arrow heads indicate the boundaries between the closed-surface regions and the ergodic regions.
Figure 4.14: Pressure profile at $\phi = 120^\circ$ for $I_h/I_r = 0.11$, compared with the vacuum field configuration. The heavy lines and arrows have the same meaning as in Fig. 4.13.
A more detailed computation of field lines in those ergodic regions has been carried out in order to find a better explanation for their plasma confinement properties. It is found that in those regions the field lines can be followed for typically between 15 m and 100 m before encountering any physical obstacle. These distances are much longer than the electron mean free path, which is of the order of 1 m for both electron-ion and electron-neutral collisions in the outer plasma region with low electron density. Consequently, under these conditions the ergodic regions behave as quasi-confinement regions and produce the lower level, flat pressure profile.

Some features associated with the island structures and the ergodic regions also appear in the floating potential profiles. The floating potential, which is closely related to the plasma potential, depends on the local confinement properties and thus its changes at the boundaries between 'closed' and 'open' field line regions would be expected. A chordal scan of floating potential is made by the 2-D probe along the midplane at $\phi = 120^\circ$; the profile is shown in the central plot of Fig. 4.13, where the pronounced changes in floating potential coincide quite well with the boundaries of those different regions. This suggests a possible technique, which is worthy of further exploitation, to locate the regions with the island and ergodic structures in a magnetic configuration.

### 4.3 Error field study

#### 4.3.1 Effect of error field on the 3-fold symmetry

The obvious discrepancy shown in Fig. 4.9 between the calculated magnetic axis and the peak pressure position for the configuration with $I_h/I_r = -0.05$ ($t(0) \sim 1$) stimulated a more detailed experimental investigation of the plasma equilibrium geometry, in particular of the pressure profiles at the nominally identical symmetry planes. A group of pressure profiles is then measured along the midplane
Figure 4.15: pressure profiles along the midplane at $\phi = 0$ and $\phi = 120^\circ$ for (a) $I_h/I_r = -0.05$ and (b) $I_h/I_r = 0$. The unperturbed magnetic axes positions calculated under conditions of three-fold symmetry are shown for comparison.

chord by a single Langmuir probe in the $\phi = 0$ plane to compare with those made in the $\phi = 120^\circ$ plane.
As an example, Fig. 4.15 shows the comparison of the profiles measured in the two nominally symmetric planes for the configuration with $I_h/I_r = -0.05$ and the standard configuration. The former case clearly shows significant difference between the two pressure profiles, with their positions displaced from the nominal magnetic axis by about 13 mm outwards at $\phi = 0$ and by about 8 mm inwards at $\phi = 120^\circ$. For the standard case, the departures from symmetry are much smaller, although they are in the same sense.

These results suggest that certain types of error which break the 3-fold symmetry of the field may exist. A careful check of the positions of each coil set was made again. While the errors in the toroidal field coils and vertical field coils remained approximately the same as at the time of original installation, some previously unknown errors in the position of the ring together with its closely-fitting helical winding were found. Fig. 4.16 shows the measured radius of the ring relative to the design value ($R_0 = 0.1875\text{ m}$) as a function of toroidal position: this indicates a mean horizontal displacement of about 1.5 mm of the centre of the ring from the major axis of the machine, as illustrated above the graph in Fig. 4.16. The height of the ring as function of the toroidal position $\phi$ was also measured. The deviation $d_h$ of the ring height from its designed value ($h_{ring}=260\text{ mm}$) versus $\phi$ is plotted in Fig. 4.17. The errors in the ring height are smaller (\(\leq 1\text{ mm}\)) and change more frequently with the toroidal position, and are therefore regarded as less important than the obvious $n=1$ error in the horizontal position of the ring.
Figure 4.16: Deviation $d$ ($d = R - R_0$) of the radial position of the central conductor from its design value ($R_0 = 0.1875\,\text{m}$) as function of the toroidal angle $\phi$. The horizontal displacement of the centre of the central conductor, inferred from the graph below, is schematically shown above.
The vacuum magnetic configurations in the range studied have been recalculated with the small horizontal displacement of the ring centre (together with the helical winding) from the true major axis being taken into account. It is then found that the good agreement between the measured and computed magnetic axis positions in the \( \phi = 0 \) and \( \phi = 120^\circ \) planes is recovered for all cases, as illustrated in Fig. 4.18. This time the disparities in the case of \( I_h/I_r = -0.05 \) are much smaller than before, and if a systematic error of 2 mm in the 2-D probe radial position is assumed the measured peak pressure positions all agree well with the theoretical dependence of the computed magnetic axes in the \( \phi = 120^\circ \) plane.

It should also be noticed that because of the breaking of the symmetry, the midplane chord does not necessarily intersect the magnetic axis (see Fig. 4.20 below). This fact should be considered in the comparison between the peak pressure positions measured along the midplane chord and the calculated radial positions of the magnetic axes. However, in most cases this correction is small (\( \leq 2 \text{ mm} \)).

![Figure 4.17: Variation of the ring height around its design value versus the toroidal angle \( \phi \).](image)

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Figure 4.18: *Positions of peak pressures measured along the midplane chords at* \( \phi = 0 \) *and* \( \phi = 120^\circ \) *for various values of* \( I_h/I_r \), *compared with the magnetic axes computed with and without error field (outer and middle curves respectively).*

### 4.3.2 Effect of error field on configurations including a \( \tau = 1 \) surface

The more pronounced resonance feature around \( I_h/I_r = -0.075 \) in Fig. 4.12, where the configurations include a \( \tau = 1 \) rational surface in the middle, can be explained after the error field is introduced. The vacuum magnetic configuration at \( I_h/I_r = -0.075 \), calculated with and without taking the horizontal position errors in the ring and in the helical winding into account, are compared in Fig. 4.19. Fig. 4.19(b) shows the significant effect of the small position errors in coils on the configuration, where most magnetic surfaces are destroyed and the volume inside closed magnetic surfaces is greatly reduced. This explains the great difficulty in gas breakdown.
Figure 4.19: Computed vacuum field configurations for \( I_h/I_r = -0.075 \) at \( \phi = 0 \), (a) without error field and (b) with error field taken into account.

Another typical example is given in Fig. 4.20, which shows the large excursion of the magnetic axis in three nominally symmetry planes (\( \phi = 0^\circ, 120^\circ \) and \( 240^\circ \)) and surface distortions due to the presence of the error field for the configuration with \( I_h/I_r = -0.05 \) (\( \epsilon(0) \sim 1 \)).
Figure 4.20: Vacuum field configurations for $I_h/I_r = -0.05$ in three nominally symmetry planes ($\phi = 0, 120^\circ$ and $240^\circ$), computed with the inclusion of error field.
Figure 4.21: Computed vacuum field configurations for standard case at $\phi = 0$, without error field (a) and with error field (b) taken into account.

4.3.3 Effect of error field on configurations with $\varepsilon(0) > 1$

The computation results show that for the configurations with $\varepsilon(0)$ greater than unity the effect of the error field on the displacements of the magnetic axis in the symmetry planes from the nominal positions are much smaller and the distortion or vertical displacement of the surfaces are not obvious. Therefore, the curves in Fig. 4.11 of the pressure data versus the flux surface parameter $X$, where $X$ is derived under symmetric condition, are very little affected by the error field.

However, the error field can significantly affect the quality of the vacuum magnetic surfaces. For instance, more detailed results computed for the standard case with and without error field taken into account are shown in Fig. 4.21. For the symmetrical case, it shows only some fairly small high-order magnetic islands (e.g. $n=9, m=7$) near the edge, while with the error field taken into account larger additional high-order islands (e.g. $n=6, m=5$ and $n=5, m=4$) appear. At the same time, some of the outermost surfaces are destroyed.

A similar comparison has been made for a configuration with $I_h/I_r = 0.15$, which is shown in Fig. 4.22. So far, such higher order islands caused by the error
field have not been observed experimentally because of their relatively small size compared with those discussed in section 4.2.3.

4.3.4 Vacuum field spectra

Vacuum field spectra have been recalculated with the error field taken into account. The effect of the error field on the magnetic configurations described above can then be related to the additional harmonics appearing in the spectra introduced by the error field. A comparison is made between two spectra (with and without error field taken into account) for a flux surface in the standard configuration. This is shown in Fig. 4.23. It shows that there are many more harmonics appearing in the spectrum once the error field is introduced; these additional peaks result from the breaking of the 3-fold field symmetry in the machine. Among these harmonics the lowest-order resonant harmonic \((n, m) = (1, 1)\) should be responsible for the large-scale surface destruction in Fig. 4.19(b). The other additional higher-order resonant harmonics in the spectrum, such as \((5, 4)\) and \((4, 3)\) (the \((6, 5)\) harmonic may be too small to see, but is very near the resonant condition), can be responsible for the appearance of the \(m=4, n=5\)
Figure 4.23: The lowest 100th and 10th spectra of $1/B^2$ on two vacuum flux surfaces with similar rotational transforms ($\iota \sim 1.22$) in standard configuration, (a) with error field and (b) without error field taken into account.

islands and the surface destruction in the outer region, where $\iota = 1.33 \sim 4/3$ in the standard configuration, as shown in Fig. 4.21(b).
4.4 Summary

The plasma produced in a toroidal heliac has been studied experimentally. From the preliminary results obtained in the standard configuration, it can be concluded that the simple arrangement of circular coils proposed by the Princeton group does indeed provide a closed confinement system with a helical magnetic axis. The plasma properties and the estimated energy confinement time indicate that the confinement for this machine, at least at low plasma $\beta$, is very favourable.

The existence of the computed heliac vacuum magnetic surfaces for a wide range of rotational transforms (see Chapter 3) has been experimentally confirmed. During the configuration scan noticeable resonance phenomena were observed when low-order rational surfaces were present in the confinement region. The formation of magnetic islands and the accompanying surface destruction are consistent with features observed in both the plasma pressure and the probe floating potential profiles.

A positional error in the central conductor and the helical winding has been found. The discrepancies between the experiments and the theory in some cases are then successfully explained. The experimental and theoretical demonstration of the effects of such a small (0.8% of the major radius) error in coil positions on the vacuum magnetic configuration provides valuable experience for future operation of the much larger heliac H-1 under construction in this laboratory.
Chapter 5

Studies of Low Frequency Fluctuations

5.1 Introduction

Density fluctuations have been widely observed in various types of magnetic confinement devices. For hydrogen plasma in typical toroidal confinement devices, the wavenumber and frequency spectra of the fluctuations in the edge region usually have a turbulent nature, with dominant components in the low frequency and small wavenumber ranges, \( f \leq 200\, \text{kHz} \) and \( k_y < 300\, \text{m}^{-1} \), where \( k_y \) is the poloidal wavenumber (see for example Levinson et al., 1984; Zweben and Gould, 1985; Schmitz et al., 1985). In the case of heavier ion plasma, when appropriate magnetic fields are applied, very low frequency and highly coherent oscillations in the density, potential and magnetic field may appear.

These fluctuations are generally attributed to instabilities of the drift wave type. In the case of heavier gas plasma the wavenumber and frequency spectra are much simpler and analytic dispersion relations for drift waves can be used to compare with the experimental results (Hendel et al., 1968; Kawahata and Fujiwara, 1976). Because of the low electron temperature, but fairly high density in
the plasma edge, resistive MHD instabilities are other possible candidates for the underlying cause of fluctuations in that region. Among them a promising model is the rippling instability, which is driven by the unstable convection of current in the presence of a resistivity gradient in a sheared magnetic field (Carreras et al., 1982; Harris et al., 1984).

In this Chapter the experimental studies of fluctuations observed on SHEILA, where the plasma parameters are similar to those in the plasma edge region of the large fusion machines, will be reported. The experiments are conducted in argon plasma under conditions where the fluctuations are coherent.

It is found that these fluctuations are global with radial wavelength of the same order as the minor radius and that changing the configuration results in very clear changes in mode structure. The mode numbers have been analyzed in a straight field line coordinate system. The experimental results fit well to a drift wave model, derived from a linearized two-fluid theory in cylindrical geometry (Fredrickson and Bellan, 1985), but adapted to the magnetic coordinate system. An examination of the effect of the fluctuations on the plasma confinement properties is also presented in this Chapter.

In the following sections, the experimental study of the plasma density fluctuations will be presented first, followed by a brief discussion of the linear drift wave model, its applicability to our experiment and a detailed comparison between the experimental results and the predictions of this theory. Finally a rough estimate is made of the particle loss due to the fluctuations in SHEILA plasma.

5.2 Experimental results

In this section experimental details concerning the conditions set for this study and the probe method for measuring the density fluctuations are discussed first. The measured equilibrium profiles are given for those configurations in which the
density fluctuations will be studied. In the main part of this section the detailed measurements of the general dependence of these fluctuations and their spectra and spatial mode structure in three typical heliac configurations will be presented.

5.2.1 Experimental details

The detailed studies of the coherent fluctuations in argon plasma have been made in three heliac configurations which encompass a wide range of rotational transforms. For convenience, these are named as configurations I, II and III, corresponding to $I_h/I_r = 0.15$, 0 and $-0.05$. In these studies, in order to simplify the heliac configuration to apply theory based on cylindrical geometry (see below) the magnetic surfaces are labeled by the effective mean radius $r$, where $r$ is defined by equating $\pi r^2$ to the cross-sectional area of the surface averaged over $\phi = 0$, $30^\circ$, $60^\circ$ and $90^\circ$, and $r = 0$ corresponds to the magnetic axis. The computed $i$ profiles for the three configurations (with error field being taken into account) versus $r$ are shown in Fig. 5.1, where the central $i$ value of the profile with $I_h/I_r = -0.05$ is shifted up from unity due to the resonance.

Again, Langmuir probes are used as the main diagnostic in this study. Most of the probe details and the data acquisition and analysis have been previously outlined in Chapter 2. The fluctuations in the floating potential and in the magnetic field are measured by the single floating probes and the magnetic probes respectively, While care is required when the density fluctuations are measured by the Langmuir probe ion saturation current signals. In general, the fluctuations in the ion saturation current can be caused by several factors, such as the fluctuations in the plasma density, in the electron temperature or in the floating potential, etc. A thorough discussion of this matter was given by Zweben and Gould (1983) for the probe measurements of edge-plasma turbulence in the Caltech research tokamak, and they concluded that the fluctuations in the probe ion saturation current were essentially due to fluctuations in the local plasma density.
in their experiment. For the radiation-dominated low temperature argon plasma on SHEILA, the contributions from the electron temperature fluctuations may have to be considered. However, in an experiment with similar argon plasma conditions in the Encore tokamak (Fredrickson and Bellan, 1985), the measured fluctuations in electron temperature were about three times smaller than those in plasma density, although such fluctuation level in the electron temperature showed some stabilizing effect on the mode. Therefore, for simplicity in our experiment the fluctuations in the temperature are assumed to be zero, so that the fluctuations in the ion saturation current are purely caused by the local plasma density fluctuations. The fluctuations with the same frequency spectrum are also clearly seen in the 2 mm microwave interferometer signals.
Conventional probe array methods are used for the fluctuation phase correlation measurements on the outer flux surfaces. The correlation technique gives the poloidal and toroidal mode number spectra of the fluctuations and their direction of propagation. Since it is not always convenient to pre-set the probe arrays on the smaller inner magnetic surfaces and also since for these surfaces with bean-shaped cross-section more sampling points are needed in the mode analysis, another method is mostly used to measure the phase correlation of the fluctuation signals. This method is based on the fact that the discharges in argon plasma on SHEILA are highly reproducible, and the density fluctuations so coherent (when an appropriate magnetic field is applied) that the phase difference between the signals at any two given locations is repeatable from shot to shot to within ±10°. By virtue of this the fluctuation phase correlation on a flux surface can be measured using only two probes: the first one is the 2-D probe at $\phi = 120^\circ$, which is used to record the density fluctuations at any point on a given flux surface, while a second reference probe is located at a fixed position on the same flux surface at $\phi = 0$. In this way, many more points can be sampled on a flux surface.

A standard program in the NAG Fortran library (NAG, 1987), which computes the discrete Fourier transform of a sequence of M complex data values, has been used to analyse the spatial mode structure and the mode propagation directions for the two probe method.

5.2.2 Equilibrium profiles

The equilibrium quantities, such as the electron density $n_e$, the electron temperature $T_e$, and the probe floating potential $V_f$, are measured as functions of the flux surface parameter $r$ by the 2-D probe along the midplane chord at $\phi = 120^\circ$ in each of the three chosen configurations studied. These profiles are shown in Fig. 5.2. The experimental conditions for these measurements are as follows: argon
filling pressure $2.7 \times 10^{-2}$ Pa, 380 W DC power to the RF amplifier, while the magnetic fields applied were 0.17 T for configuration I, 0.13 T for configuration II and 0.1 T for configuration III. This variation is necessary because the coherent fluctuations only exist in a preferred magnetic field range, which depends on configuration (see section 5.2.3 below).

The density profiles all show central maxima consistent with the particles being well confined inside closed flux surfaces. Each of these profiles can be approximated by a Gaussian form: 

$$n_e = n_e(0) \exp\left[-\frac{1}{2}\left(\frac{r}{b}\right)^2\right],$$

where $n_e(0)$ is the density at the magnetic axis and $b$ is a constant. The floating potential profiles are relatively flat over the whole plasma region. The temperature profiles measured under higher RF power are flatter than those measured under lower RF power in the standard configuration (Figs. 4.3 and 4.4 in Chapter 4). Similar flat and slightly hollow electron temperature profiles have also been observed in low temperature argon plasma discharges in other devices (Vojtsenya et al., 1977; Fredrickson and Bellan, 1985), where the radiation was the dominant energy loss process.
Figure 5.2: The electron density, temperature and the floating potential measured as functions of the effective radius $r$ by the 2-D Langmuir probe along the midplane chord at $\phi = 120^\circ$ in (a) configuration I, (b) configuration II, and (c) configuration III. The dashed lines are Gaussian approximations.
5.2.3 Dependence of fluctuations on the magnetic field strength

It is found that the density fluctuation level has a strong dependence on the magnetic field, while its frequency spectrum is much less sensitive to the magnetic field. When the applied magnetic field exceeds some critical value, which varies with the configuration and the ion mass (Blackwell et al., 1989), coherent oscillations appear in both the ion saturation current and the floating potential. These signals are recorded digitally at $2 \times 10^5$ samples/s for subsequent Fourier analysis. Typical signals and their frequency spectra averaged over 1 ms are shown in Fig. 5.3. The dominant frequencies are 11 kHz, 13 kHz, and 7 kHz for configurations I, II, and III respectively.

Fig. 5.4 shows the measured amplitude of the density fluctuations versus magnetic field. The threshold of the magnetic field for the onset of the fluctuations, $B_t$, is indicated on the graph for each configuration. As the magnetic field is increased above this critical value, the fluctuation amplitude increases rapidly to a maximum, then slowly decreases. The theoretical curves are also shown on the same graph for comparison, and will be discussed later.

The same frequency spectra can also be seen in the poloidal magnetic field signals, but the magnetic field fluctuations are much weaker. As an example, Fig. 5.5 shows the correlated oscillations in the magnetic probe signal and in the Langmuir probe ion saturation current signal in configuration II. Both probes are positioned near the plasma edge. The measured values of the magnetic field fluctuations near the plasma edge are typically

$$\frac{\tilde{B}_\theta}{B} \sim 10^{-5} \frac{\tilde{n}_e}{n_e},$$

(5.1)

where $B$ is the total magnetic field and $B_\theta$ is its poloidal component.
Figure 5.3: Fluctuations in the ion saturation current signals and their spectra. 
(a), (b) and (c) as for Fig.5.2.
Figure 5.4: Density fluctuation amplitude and the computed linear growth rate for the dominant modes in the three configurations versus the magnetic field. The experimentally measured threshold value of magnetic field for the onset of the fluctuations, $B_t$, is indicated for each case. (a), (b) and (c) correspond to configurations I, II and III.
Figure 5.5: The correlated signals of the magnetic field fluctuations $\dot{B}_0$ (middle trace) and the fluctuations on probe ion saturation current (bottom trace). Note that the oscillation persists, decreasing in frequency and amplitude, after the termination of the RF pulse (top trace: four turn ring voltage 200 V/div).
5.2.4 Dependence on the electron collision frequency

At a given magnetic field the frequency and the amplitude of the density fluctuations are observed to vary with both the gas filling pressure and the applied RF power. However, the experimental data can be more generally related to the total electron collision frequency $\nu$ (electron-ion and electron-neutral) which can be derived from the knowledge of the filling argon pressure and the electron temperature and density at a given RF power. As an example, Fig. 5.6 shows the fluctuation frequency and amplitude as functions of the total electron collision frequency in configuration II. In the low collision frequency range ($\nu \leq 6$ MHz) the fluctuation frequency decreases and its level increases with the collision frequency. At higher collision frequencies the fluctuation amplitude decreases markedly.
Figure 5.6: Measured fluctuation frequency and amplitude as functions of the total electron collision frequency $\nu = \nu_{ei} + \nu_{en}$ in configuration II, $B = 0.13\, \text{T}$. Lines are theoretical results calculated from the analytic dispersion relation for the dominant mode $m = 3$, $k_\parallel = 0.6\, \text{m}^{-1}$. 
5.2.5 Mode structure

5.2.5.1 Magnetic coordinates

The inherently three-dimensional geometry of the heliac configuration, with its non-planar axis, irregularly shaped flux surfaces of toroidally variable cross-section, and considerable departure from any symmetries in laboratory space, suggests that it would be more useful to relate all measured quantities to their location in a coordinate system directly related to the magnetic field. Of several possible straight field line systems, we have chosen for convenience a variant of the coordinate system used in the PEST equilibrium and stability code (Dewar et al., 1984).

The coordinates \( r, \vartheta, \phi \) are defined as follows: \( \phi \) is taken as the usual toroidal angle in the laboratory frame; the flux ('minor radius') coordinate \( r \) is defined previously in section 5.2.1; while the poloidal coordinate \( \vartheta \) is chosen so that \( d\vartheta/d\phi = \text{const} = n_h \) along each field line. This coordinate system is illustrated in Fig. 5.7, which shows lines of constant \( \vartheta \) and \( \phi \) on a flux surface \( r = \text{const} \) and the unequally spaced \( \vartheta \) coordinate in a cross-section of the flux surface. The density fluctuations can then be written in the form \( \tilde{n}_e = \sum_{m,n_h} \tilde{n}_e(r) \exp i(m\vartheta - n_h\phi) \), where \( n_h \) is a toroidal mode number measured in the helical frame, i.e. the mode number upon a closed loop at constant \( \vartheta \), and \( m \) is a poloidal mode number measured along the short loop at constant \( \phi \), which is the same as in the laboratory frame.
Figure 5.7: Helical magnetic surface with constant $\vartheta$ and $\phi$ lines, the intervals of equal $\vartheta$ are shown in a cross-section of the flux surface.
5.2.5.2 Mode number measurements

The variations of phase and amplitude of the density fluctuations around the periphery of the flux surface cross-section are derived by comparing the ion saturation current signals simultaneously measured by the 2-D probe located at $\phi = 120^\circ$ and the reference probe at $\phi = 0$. Fourteen points equally spaced in $\psi$, but not equally spaced in real space (see Fig. 5.8 below), are sampled for each flux surface in these configurations. The measured fluctuation patterns (on one typical surface for each configuration) are shown on the left in Fig. 5.8, where the dashed lines represent the density fluctuations on the surface shown, with the relative amplitudes and their phases indicated by the length and direction of the arrows. The discrete Fourier transform in $\psi$ space are computed by the program described in section 5.2.1. The spectra, which are displayed on the right in Fig. 5.8, show that in every case the modes propagate poloidally in the electron diamagnetic drift direction. For both configurations I and II, two major modes, $m = 2$ and $m = 1$ in configuration I, and $m = 3$ and $m = 1$ in configuration II, appear to co-exist at the same frequency, while in configuration III, a single mode, $m = 1$, is dominant. The radial profiles of these poloidal modes can be obtained from the computed spectra for different flux surfaces. As shown in Fig. 5.9, the modes appear to peak at different radial positions.
Figure 5.8: Typical density fluctuation patterns (dashed lines) along the magnetic surfaces (solid lines): (a) $r = 18$ mm, configuration I, (b) $r = 23.9$ mm, configuration II, (c) $r = 20.1$ mm, configuration III. Their corresponding $\theta$ spectra are shown on the right.
Figure 5.9: Measured radial profiles of the poloidal modes and the calculated eigenfunctions. (a), (b) and (c) correspond to configurations I, II and III respectively.
The probe array method has been used to measure the mode structure on an outer flux surface in configuration II. The relative phase of the signal from each probe in the two poloidal arrays at $\phi = 120^\circ$ and $\phi = 150^\circ$ (both of which are set on the same surface $r = 0.03 \text{ m}$ near the outermost flux surface) is measured as a function of its poloidal coordinate $\theta$. This is plotted in Fig. 5.10, where the straight lines correspond to single modes $m=2, 3, $ and $4$ propagating in the electron diamagnetic drift direction. The observation is clearly most consistent with the presence of a dominant $m=3$ mode propagating in the same direction, as found using the two-probe technique.

The toroidal probe array consists of four probes which are positioned at approximately the same $\theta$ ($\pm 5^\circ$) on the same flux surface as the poloidal probe arrays, at $\phi = 0, 90^\circ, 120^\circ$ and $150^\circ$. The phase difference between these probe

![Figure 5.10: Phase of the fluctuation signal versus the poloidal coordinate $\theta$. Probe arrays are positioned on $r = 0.03 \text{ m}$ flux surface.](image-url)
Phase of the fluctuation signal versus the toroidal angle $\phi$. Probe array are positioned on $r = 0.03$ m flux surface.

signals, shown in Fig. 5.11, corresponds to the presence of a dominant $n_h=5$ mode. The approximate matching between the measured mode numbers and the computed rotational transform (in the helical frame) on that surface, i.e. $\ell_h = 1.7 \approx 5/3$, implies that the fluctuations are close to resonance with the magnetic field lines, i.e. the helicities of the fluctuations and the magnetic field lines are very similar.

The magnitude of $k_\parallel$, the wavenumber along the magnetic field lines, is a measure of how close the fluctuations are to this resonant condition. The parallel wavelength $k_\parallel$ can be expressed approximately as:

$$k_\parallel = \frac{2\pi(n_h - \ell_h m)}{L_h}.$$  \hspace{1cm} (5.2)

When the inferred mode numbers ($n_h=5$, $m=3$) are inserted the value for $k_\parallel$ on the measured surface is about $0.5$ m$^{-1}$, corresponding to a parallel wavelength.
about 13 m, or 9 toroidal rotations. This small value of $k_{||}$ can be resolved because of the well-defined magnetic geometry of plasma-current-free stellarators. This will be discussed in the next section.

5.2.5.3 Direct measurement of $k_{||}$

Given that $k_{||}$ is sufficiently small, the value of $k_{||}$ for each $m$ mode can be directly measured on each flux surface in the following way. The fluctuation amplitude and relative phase is measured (again using the 2-D and reference probes) at those locations corresponding to consecutive interceptions of a given field line with the plane $\phi = 120^\circ$. This is equivalent to having a toroidal probe array with a probe separation equal to the length of the helical plasma column in one toroidal turn, and so is subject to aliasing errors for parallel wavelengths shorter than one toroidal turn (approximately 1.4 m). Once a field line is followed far enough both the measured amplitude and phase appear to be periodic functions along the field line for each configuration studied. As an example, Figs. 5.12 and 5.13 show the fluctuation amplitudes and relative phases along field lines on two different flux surfaces in configuration II. The amplitude and phase repeat every nine turns for the inner surface ($r=15.7$ mm) and every eleven turns for the second surface ($r=23.9$ mm). The possible values of $k_{||}$ are then obtained for each of the two flux surfaces by the discrete Fourier transform (as in section 5.2.5.2) performed over one period.

Similar measurements and analyses are made in the other two configurations and the measured radial profiles of $k_{||}$ for each of these poloidal modes are shown in Fig. 5.14.

As a further check, the phase lag between two probes separated toroidally by $120^\circ$ along a field line on a same flux surface ($r=15$ mm) in configuration III has been measured and gives $k_{||} \approx -0.4$ m$^{-1}$, which is consistent with the result given in Fig. 5.14(c) by the above method, and eliminates the possibility
Figure 5.12: The variations of the fluctuation amplitude and phase along a field line on a flux surface with $r = 15.7$ mm in configuration II.

of aliasing. Fig. 5.14(b) also shows that the value of $k_\parallel$, derived from the mode number measurement in the previous section, lies approximately on the smoothly extended line formed by the directly measured $k_\parallel$ data for the $m=3$ mode in configuration II.
Figure 5.13: As for Fig. 5.12, but $r = 23.9$ mm.
Figure 5.14: $k_\parallel$ profiles obtained from direct measurement of $k_\parallel$. The horizontal bars indicate the ranges where the modes have largest amplitude. (a), (b) and (c) correspond to configurations I, II and III.
5.3 Theory and comparison with experiment

5.3.1 Linear drift wave theory

Any stability study in a stellarator is greatly complicated by the three-dimensional nature of its configuration. This is particularly the case in the heliac, and to date no drift wave theory has been derived for such a configuration. For this reason we shall adapt the theoretical model of Fredrickson and Bellan (1985) for drift-Alfven waves in a cylindrical geometry and use it to compare with our experimental results. The model is based on the linearized versions of the Braginski two-fluid equations (Braginski, 1965) with the following assumptions: (i) the fluctuation frequency $\omega$ is much below the ion cyclotron frequency $\omega_{ci}$, (ii) the plasma $\beta$ is in the range $m_e/M_i \ll \beta \ll 1$, where $m_e$ and $M_i$ are the electron and ion mass respectively, and (iii) the ions are cold. A set of reduced equations for $\tilde{n}_e$, $\tilde{\Phi}$ and $\tilde{A}$, which are respectively the fluctuation amplitudes in the electron density, the space potential and the parallel component of the vector potential, is derived in the same paper under several further assumptions, namely: (i) the density profile is Gaussian, (ii) the electron temperature is uniform in space and constant in time, (iii) the magnetic shear is zero, i.e. $k_\parallel = \text{const.}$, and (iv) the dependence of the Alfven velocity $V_A$ and the electron-ion collision frequency $\nu_{ei}$ on the minor radius can be neglected. The equations are:

$$\rho_i^2 \nabla^2_\perp (\tilde{\Phi} - \frac{\omega}{k_\parallel} \tilde{A}) + \Omega f [1 - (\frac{\omega}{k_\parallel V_A})^2](\tilde{\Phi} - \frac{\omega}{k_\parallel} \tilde{A}) = 0,$$  \hspace{1cm} (5.3)

$$\left(\Omega f \right)^{-1} \rho_i^2 \nabla^2_\perp (-\frac{\omega}{k_\parallel V_A^2} \tilde{\Phi} + \tilde{A}) = 0,$$  \hspace{1cm} (5.4)

$$-\Psi = f \left(\frac{\omega \Omega}{k_\parallel} \tilde{A} - i\omega \tau_\parallel \tilde{\Phi} \right),$$  \hspace{1cm} (5.5)

where

$$-\Psi = \frac{T_e}{e n_e} \tilde{n}_e - \tilde{\Phi},$$  \hspace{1cm} (5.6)

$$f = \frac{1}{1 - i\omega \tau_\parallel},$$  \hspace{1cm} (5.7)
\[ V_A = \frac{B}{(\mu_0 n_e M_i)^{1/2}}, \]  
\[ \Omega = \frac{\omega_*}{\omega} - 1, \]  
\( \omega_* \) is the electron diamagnetic drift frequency:  
\[ \omega_* = -(m/r)(T_e \frac{dn_e}{dr})/n_e e B, \]  
\( m \) is the poloidal mode number, \( \rho_i \) is the ion Larmor radius evaluated at the electron temperature:  
\[ \rho_i^2 = T_e/(M_i \omega_\text{ci}^2), \]  
and  
\[ \tau_\| = m_e \nu_{ei}/(k_\| T_e), \]  
where \( \nu_{ei} \) is the electron and ion collision frequency.

Equation (5.3) has solutions of the form \( e^{im\phi} J_m(k_r r) \), and the dispersion relation is then obtained as:  
\[ (1 - i\omega \tau_\|)w_{kl}^2 \rho_i^2 = (\omega_* - \omega)[1 - (\omega / k_\| V_A)^2], \]  
where  
\[ k_r = X_j^m / r_a, \]  
\( X_j^m \) is the jth zero of the mth Bessel function and \( r_a \) is the radius of the plasma. If the lowest radial mode is considered, \( k_r = X_1^m / r_a \).

In this simple model the radial eigen-functions of the variables, \( \tilde{n}_e/n_e, \tilde{\Phi} \) and \( A \), will all have the simple form \( J_m(k_r r) \). This can be proved as follows. If the modes described by the dispersion relation (Eq.(5.13)) exist, i.e. \( \omega \neq 0 \), then the only way to satisfy Eq.(5.4) is to set  
\[ \tilde{\Phi} = \frac{\omega}{k_\| V_A} \tilde{\Phi}. \]  
Using Eqs. (5.15) and Eq. (5.6) to substitute for \( \tilde{A} \) and \( \Psi \) respectively in Eq. (5.5) gives  
\[ \frac{\tilde{n}_e}{n_e} = [1 + f(\frac{\omega^2 \Omega}{k_\|^2 V_A^2} - i\omega \Omega \eta)] \frac{e^{\tilde{\Phi}}}{T_e}. \]
Eqs. (5.15) and (5.16) give the linear relations between the variables $n_e/n_e$, $\Phi$ and $\tilde{A}$. The radial dependence of each of these variables can then be independently solved from Eq. (5.3), which yields the Bessel function form.

### 5.3.2 Validity of the theory

Before this model is used to compare with our experimental results, the above noted assumptions must be justified for conditions on SHEILA and some approximations and modifications have to be made. From preceding results it can be seen that most assumptions are approximately satisfied in our experiment. However, there are a few exceptions. Firstly, the value of $k_\parallel$ varies slightly with the minor radius as shown in Fig. 5.14. Secondly, since the Alfven velocity $V_A$ and the electron collision frequency $\nu_{ei}$ both depend on the electron density $n_e$, it is not reasonable to assume them to be constant with the minor radius in a real experiment. Therefore, the values of $k_\parallel$, $V_A$ and $\nu_{ei}$ used in this model are taken from the radial region where the modes have their maximum amplitude. In addition, $\nu_{ei}$ in the dispersion relation is replaced by the total electron collision frequency $\nu$ (see section 5.2.4). This is because under the conditions prevailing during this study the degree of ionization of the argon is about 20% or less, so that collisions between the electrons and neutrals, which occur at a comparable rate to those between the electron and ions near the plasma edge, must be taken into account in the momentum balance. Finally, we note that the condition $\beta \gg m_e/M_i$ is barely satisfied ($\bar{\beta} \sim 10^{-4}$, cf. $m_e/M_i$(argon) $\sim 10^{-5}$), so that it is expected that the coupling between the drift wave and the Alfven wave (see below) must be weak.
5.3.3 Details of solving the dispersion relation

When solving the dispersion equation in the cylindrical geometry the heliac configuration is approximated as a cylinder by defining an effective radius \( r \) for each flux surface. The toroidal field coils will be considered as the effective plasma boundary, giving an average maximum plasma radius \( r_a \) of about 0.04 m. This is because experimentally there are still significant density fluctuations on the last closed flux surface in all three configurations as shown in Fig. 5.9.

The dispersion equation is then numerically solved by a program called 'disp1' (see appendix A) for each \( m \) mode. This program calculates the coefficients of the dispersion equation from a set of input data and then finds all the roots (real and imaginary frequencies for a given \( k_\parallel \)) of this complex polynomial equation. The input data are supplied from the experimental results, which include the electron density taken from the region where the mode has its maximum amplitude, the ion mass for argon, the constant \( b \) of the Gaussian density distribution from the measured density profiles, the average electron temperature, the magnetic field strength, the poloidal mode number and the value of \( k_r \) obtained from Eq. 5.14. The range of \( k_\parallel \) is given as \( 0 - 3 \text{ m}^{-1} \). The program and the input data for the various poloidal modes in the three configurations are given in appendix A.

5.3.4 Comparison between theory and experiment

The output of 'disp1' gives three roots of the dispersion equation for each observed \( m \) mode, two of which correspond to waves with negative growth rate over the parameter range of our experiment. In the following, only the unstable drift wave branch will be considered.

Fig. 5.15 shows the unstable wave branches for the observed mode number
Figure 5.15: Experimental results compared with the solutions of the dispersion Equation. The symbol ▲ is for the $m = 1$ mode in each case. (a), (b) and (c) correspond to configurations I, II and III.
m in the three configurations. Also shown on the same plots for the real part of the frequency are the experimentally measured frequency and \( k_{||} \) for each mode, the value of \( k_{||} \) being taken as that where the mode has largest amplitude. In every case experiment and theory agree closely. The dispersion curves on the right in Fig. 5.15 show that each mode has its largest theoretical growth rate at some small but finite value of \( k_{||} \). The parallel wavelength of each observed saturated mode is also shown for comparison; in each case the dominant mode has its \( k_{||} \) close to the value corresponding to the maximum linear growth rate.

The experimentally measured angular frequencies for the dominant modes in the three configurations compared with the electron diamagnetic frequencies and the ion cyclotron frequencies under the conditions we study are shown in Table 5.1. The mode frequency observed are typically about half order or one order smaller than both the electron diamagnetic and the ion cyclotron frequencies in each configuration. This is consistent with earlier assumptions.

<table>
<thead>
<tr>
<th>angular frequency</th>
<th>configuration I</th>
<th>configuration II</th>
<th>configuration III</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \omega ) (rad/s)</td>
<td>( 7.2 \times 10^4 )</td>
<td>( 8.4 \times 10^4 )</td>
<td>( 4.2 \times 10^4 )</td>
</tr>
<tr>
<td>( \omega_* ) (rad/s)</td>
<td>( 2.7 \times 10^5 ) (for ( m = 2 ))</td>
<td>( 6.7 \times 10^5 ) (for ( m = 3 ))</td>
<td>( 2.8 \times 10^5 ) (for ( m = 1 ))</td>
</tr>
<tr>
<td>( \omega_{ci} ) (rad/s)</td>
<td>( 3.0 \times 10^5 )</td>
<td>( 4.0 \times 10^5 )</td>
<td>( 2.3 \times 10^5 )</td>
</tr>
</tbody>
</table>

Table 5.1: Measured fluctuation frequencies in the three configurations compared with the electron diamagnetic and the ion cyclotron frequencies.

Comparisons are also made between the predicted radial dependence of the poloidal modes with those measured ones, which are shown in Fig. 5.9, where the calculated radial eigen-functions of those \( m \) modes are shown by solid lines and their maxima are normalized to those of the measured profiles. Although the absolute peak positions of the measured profiles are not the same as those of the theoretical ones, the general behaviour is very similar.
The real frequency and growth rate (imaginary part of the frequency) for each \( m \) mode as a function of the total electron collision frequency \( \nu \) can be obtained from the dispersion equation by varying the electron density and the neutral density (this time \( k_\parallel \) is fixed). This has been done by a program called 'disp2', where for simplicity the total electron collision frequency is varied by only varying the electron density (appendix A). As an example, for the dominant mode \( (m = 3) \) in configuration II, the variation of the mode frequency and growth rate with collision frequency are plotted and compared with the experimental data in Fig. 5.6. Fig. 5.6(a) shows very good agreement between the measured and calculated fluctuation frequencies, especially in the lower collision frequency range. In Fig. 5.6(b), the increase in growth rate with \( \nu \) for \( \nu < 6 \, \text{MHz} \) qualitatively matches the experimental amplitude data. However, at higher collision frequencies, the observed fluctuation amplitude decreases much more rapidly than the predicted growth rate.

The dependence of the linear growth rate of each \( m \) mode on the magnetic field can also be obtained from the dispersion equation with \( k_\parallel \) fixed (program 'disp3' is used, see appendix A). The results for the dominant modes in the three configurations are compared with the experimental results in Fig. 5.4. The theoretical curves show no threshold behaviour, but have their maximum growth rates at fields very close to those where maximum amplitudes are seen in configurations I and III. In configuration II there is a factor of about 1.7 disagreement between them.

The phase difference between the density and the potential fluctuations calculated from Eq. (5.16) is about 40° to 60° in each configuration at those surfaces near the last closed flux surface, with the density fluctuations leading the potential fluctuations. However, the experimentally measured phase difference between them for these modes in the saturated state is much smaller (~ 10°), with the density fluctuations leading the potential fluctuations, resulting in an outward
transport of plasma particles.

The magnetic perturbations, arising from the predicted coupling of the drift wave to the Alfven wave can be expressed by (Fredrickson and Bellan, 1985)

\[
\frac{\tilde{B}}{B} \approx \frac{i}{2} \frac{k_{\perp}}{k_{\parallel}} \frac{\omega}{\omega_{ci}} \beta \left( \frac{e\Phi}{T_e} \right),
\]

(5.17)

where \( k_{\perp} \) is the perpendicular wavenumber. Using experimentally derived values, e.g. \( \omega/\omega_{ci} \sim 0.1, k_{\perp}/k_{\parallel} \sim 100, \beta \sim 10^{-5} \) (near the plasma edge) and \( e\Phi/T_e \sim \tilde{n}_e/n_e \) we obtain

\[
\frac{\tilde{B}}{B} \approx 5 \times 10^{-5} \frac{\tilde{n}_e}{n_e},
\]

(5.18)

which, in order of magnitude, is consistent with the experimental results given in section 5.2.3. A more intuitive way of looking at this result is to regard the local magnetic field oscillations as arising only from the effect of a density perturbation on the local pressure balance, i.e.

\[
\frac{B^2}{2\mu_0} + n_e T_e \sim \text{const},
\]

(5.19)

\[
\frac{\tilde{B}}{B} \sim \frac{1}{2} \beta \frac{\tilde{n}_e}{n_e},
\]

(5.20)

and is consistent with the fluctuations being very nearly electrostatic in nature.

5.3.5 Summary and discussion

Low frequency fluctuations in electron density, floating potential and magnetic field signals in an argon plasma have been studied in a heliac geometry. These fluctuations are essentially electrostatic. The measured parametric dependence of these fluctuations on the magnetic field and the electron collision frequency suggests that these are collisional drift waves as described by Hendel, et al., 1968; and Venema, 1986. Detailed studies of the mode spatial structure in several typical configurations clearly show the close correspondence between the dominant mode numbers \((n_h, m)\) and the rotational transform in the helical frame.
\( z_h \), i.e. \( z_h \approx n_h/m \). For the heliac configuration, with its non-planar magnetic axis and azimuthally variable bean-shaped cross-sections, the use of the magnetic coordinates has simplified the physics and the interpretation of the experimental results.

These experimental results are supported by a simple linear collisional drift wave theory (Fredrickson, and Bellan, 1985). Although this model, derived in cylindrical geometry, has been adapted to the heliac configuration by straightening out the field lines and assigning an average radius for each irregularly shaped flux surface, the agreement of the measured wave frequencies, amplitudes and even their radial eigen-functions with this theory is surprisingly good. The dependence of the fluctuations on the total electron collision frequency in the low frequency range also agrees very well with the theory. However, it is noted that in these comparisons, the possibility of bulk plasma rotation is ignored. There is not enough data to confirm this assumption, although the strongly asymmetric heliac geometry suggests that such rotation is certainly unlikely.

This good agreement may be a consequence of the fact that the unperturbed plasma quantities depend only on the flux surface label (Blackwell et al., 1985). This we have equated to the mean radius, just as in a circular cross-section cylinder. Another possible reason for this could be that the large scale (long wavelengths in both toroidal and poloidal directions) fluctuations are affected only in an average sense by the irregular geometry.

By the same token, the agreement is interesting given the exclusion of other features such as shear, temperature gradient and toroidicity considered in more complete drift wave treatments (e.g. Cordey et al., 1979; Hasselberg and Rogister, 1980; Venema, 1986). This may be due to the low shear and the observed absence of mode localization at rational surfaces in this experiment.

Although the linear collisional drift wave theory can clearly account for the main experimental results, there are still some remaining discrepancies. These
can possibly be explained by the various mode damping mechanisms and the non-linear mode coupling which are not included in this simple linear model.

For example, the second mode which appears in both configurations I and II does not correspond to a large linear growth rate (see Fig. 5.15). However, the linear theory predicts that this second mode would have a very similar frequency to the dominant one, so that it could readily be non-linearly excited by the main mode.

In this model the ions are assumed cold, so any viscous damping mechanism for drift waves caused by the ion-ion or ion-neutral collisions (Venema, 1986; Timofeev, 1964) is not included. This exclusion might explain the discrepancy in the observed variation of amplitude and linear growth rate at the higher electron collision frequencies (see Fig. 5.6), as well as in the threshold dependence on magnetic field strength (Fig. 5.4). However, it is found experimentally that, in all the cases studied, when the ion Larmor radius reaches a critical size, i.e. $k_d \rho_i \sim 1$, the fluctuations are stabilized; this agrees qualitatively with a stability criterion derived from a linear slab model theory (Hendel et al., 1968).

Finally, because of the fairly constant temperature profile measured in every configuration studied and the very small plasma current (< 100 A at 96 kHz), the current driven rippling instability in a sheared magnetic field with the gradient of the resistivity $\eta$ ($\eta \propto T_e^{2/3}$) is unlikely to occur. The rippling modes were predicted in their linear and single-helicity quasi-linear phases as highly localized and purely growing modes (Carreras et al., 1982). However, neither of these features is consistent with our experimental observations of the modes on SHEILA. In addition, the fluctuations are observed in our experiment to persist in the absence of ohmic heating current, i.e. in the afterglow (see Fig. 5.5).
5.4 Influence of fluctuations on particle confinement

In the previous Chapter it was shown that changes in the magnetic configuration can have a large effect on the particle confinement. By comparison the presence or absence of fluctuations has a less obvious but still significant effect. For instance, Fig. 5.16 shows, for each configuration, the dependence on magnetic field strength $B$ of both the mean density and the fluctuation amplitude (for configuration III the density is somewhat lower because the probe is on an outer flux surface) for otherwise constant conditions. It is clear that the general monotonic increase in density with $B$ is reduced when the fluctuations appear, i.e. when $B > B_t$, and, at least qualitatively, the effect (as indicated by the reduction in $n_e$ in comparison with its extrapolated value) increases with the fluctuation amplitude. Very roughly, the maximum particle loss due to the fluctuations appears to be about one third of the total.

Although no precise measurement of particle confinement time has been made, we can make an order of magnitude estimate from the density decay observed immediately following the end of the RF pulse. Fig. 5.17 shows a typical waveform taken in configuration I. In order to reduce the uncertainty in the density decay slope, the magnetic field is deliberately chosen to be just above the threshold value so that the fluctuation level is very low both during the pulse and in the decay phase. The density slope in Fig. 5.17 indicates a total loss rate $S_{total} = n_e^{-1}(-dn_e/dt) \approx 4 \times 10^3 \text{ s}^{-1}$.

We may estimate the time-averaged cross-field particle flux $\Gamma$ caused by a simple coherent electrostatic mode of amplitude $\bar{\Phi}$ from the readily-derived result (Powers, 1974)

$$\Gamma = \frac{k_\phi}{2B} \bar{n}_e \bar{\Phi} \sin \delta,$$

where $\delta$ is the phase difference between the density and potential fluctuations. If
Figure 5.16: Density and its relative fluctuation level as functions of the applied maximum magnetic field. (a), (b) and (c) correspond to configurations I, II and III.

we take as typical the quantities measured in configuration I on the flux surface \( r = 0.022 \text{ m} \), the values \( k_\theta = m/r \approx 90 \text{ m}^{-1}, \tilde{n}_e \approx 1 \times 10^{17} \text{ m}^{-3}, \tilde{\Phi} = \tilde{V}_f \approx 2 \text{ V} \),
Figure 5.17: Ion saturation current signal in configuration I. The straight line shows the decay rate immediately after the RF is switched off.

$B \approx 0.16T$, and the measured value of $\delta$ of approximately $10^9$, Eq. (5.21) predicts an outwards fluctuation-driven particle flux across the $r = 0.022\ m$ surface of $\Gamma \approx 1 \times 10^{19}\ m^{-2}\ s^{-1}$. The corresponding contribution to the particle loss rate is $\dot{S} = 2\Gamma/n_e \approx 1 \times 10^3\ s^{-1}$, for $n_e \approx 1 \times 10^{18}\ m^{-3}$. The fluctuations would seem to contribute about one quarter of the total particle loss rate. This is consistent with the results shown in Fig. 5.16.
Chapter 6

Summary and Discussion

6.1 Object of the research

The basic concept of a helical axis stellarator, the various computed vacuum heliac configurations and the first experimental results on a toroidal heliac have been presented in this thesis. The aims of this work on the very low plasma $\beta$ prototype heliac SHEILA can be summarized as follows:

(i) To experimentally investigate the existence of an equilibrium plasma column confined in this new configuration.

(ii) To demonstrate the practical utility of the proposal of Harris et al. (from Oak Ridge National Laboratory) to gain flexibility in an experimental heliac device by adding a helical winding to the basic heliac coil configuration.

(iii) To study the plasma confinement in various heliac configurations, which are made available by the additional helical winding.

(iv) To investigate the nature of the low frequency fluctuations appearing in the plasma density, floating potential and the magnetic field and their effect if any on the plasma particle confinement.
6.2 Summary of results

Plasmas are obtained in various gases in the standard heliac configuration using low RF input power (typically between 150 W and 400 W). Since the plasma beta is very low, the position and shape of the surfaces with equal plasma pressure may be directly compared with the computed vacuum surfaces. The existence of the equilibrium magnetic surfaces in the plasma has been evidenced by the results of the Langmuir probe chordal scans, which show the clear dependence of the measured plasma pressures on the computed vacuum surface parameter. The high reproducibility of the plasma with typical parameters $n_e \approx 1.0 \times 10^{18} \text{m}^{-3}$ and $T_e \approx 6 - 10 \text{eV}$ (in argon) and the very low power required for its production suggest that, at least at low beta, the heliac configuration has potentially good confinement properties. The estimated global energy confinement time, even when no account is taken of inelastic losses, is about $30 \mu$s. For comparison, the confinement time to be expected for a tokamak of comparable size and density, using the empirical scaling law in Alcator tokamak (Blackwell, et al., 1983), is only about the order $2 \mu$s.

The proposal of Harris et al. (1985) for a flexible heliac configuration has been practiced on SHEILA for the first time. By varying the current in the helical winding, the range of heliac configurations can be greatly extended, and the possibility that rotational transforms can be varied by a factor of two has been experimentally demonstrated. The various heliac configurations are identified using Langmuir probes and show excellent geometric agreement with the computed vacuum magnetic surfaces in most cases. Obvious deterioration of the plasma confinement caused by the flux surface becoming broken due to the major resonances, has been observed in those configurations, which, as predicted by the computed results, contain very low order rational surfaces, and the optimum plasma confinement region has been found between these major resonances. Evi-
dence of the formation of large magnetic islands has been shown in the measured
density and floating potential profiles. The locations of the island structures
inferred from the data are consistent with the calculated vacuum field.

The apparent discrepancy between theory and experiment in those configura-
tions with their rotational transform near unity suggested the existence of some
source of error fields in this machine and stimulated a search for the possible
cause. A small (about 0.8 % of the major radius) horizontal displacement of
the central conductor (together with the helical winding) from its true position
was found. The effect of the symmetry-breaking error field produced by the er-
ror positions in both the central conductor and helical winding on the vacuum
configurations has been studied both theoretically and experimentally. Results
showed a significant influence of such a small error in coil positions on those con-
figurations which originally contain a $\tau = 1$ surface. The agreement between the
measured and computed magnetic configurations is then recovered for all cases
when this error field is taken into account.

Low frequency fluctuations are observed in the plasma density, the floating
potential and the magnetic field. In argon plasma in the appropriate magnetic
field range, these fluctuations are highly coherent. The influence of these low
frequency fluctuations on the plasma confinement seems to be a less significant
factor than the choice of the magnetic configuration. However, a reduction in
mean plasma density which follows the onset of the fluctuations is clearly ob-
served and can be roughly explained by the particle cross-field diffusion caused
by the plasma density and potential fluctuations measured near the edge. The
spatial mode spectrum of these fluctuations, which is analyzed in a magnetic coor-
dinate system, shows significant changes with varying rotational transform and a
close matching between the helicity of the dominant mode and the magnetic field
lines is found in every case studied. The measured frequencies and wavenumbers
of the fluctuations in three typical configurations, together with their paramet-
ric dependences on the magnetic field and the electron collision frequency, are supported by a collisional drift wave model, derived from the Braginski two-fluid equations (Braginski, 1965; Fredrickson and Bellan, 1985). These fluctuations are thus most likely to be collisional drift waves and are essentially electrostatic in nature.

6.3 Discussion

Although in this work the verification of the equilibrium surfaces is based on the limited plasma pressure data made by Langmuir probe chordal scans, the existence of closed surfaces is further supported by the recent electron beam measurement performed on SHEILA, in which a low energy electron beam also traced out the magnetic surfaces similar to those computed vacuum flux surfaces (Blackwell and Tou, 1989). As a more direct and accurate method to measure the vacuum flux surfaces the electron beam measurement indicates some additional discrepancies between the measured electron drift surfaces, which should be very close to the vacuum flux surfaces if the electron beam energy is low enough, and the computed vacuum flux surfaces under higher helical winding currents ($I_h/I_r > 0.15$). This suggests other possible error fields may exist and further investigation is needed.

In the experiments described in this thesis, argon is used as the main working gas. For the configuration studies, this is convenient because of its ease of breakdown and the highly reproducible plasma obtained in argon discharges. The use of heavier gas also results in the much simpler fluctuation spectrum of only a few discrete modes which can be well described in terms of a simple linear drift wave model. Based on these results obtained in argon plasma, it is important to increase the input RF power on SHEILA in order to carry on further research in fully ionized hydrogen plasma, and a more accurate drift wave model derived in

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heliac geometry may also be necessary for further study of fluctuations.

In summary, the results obtained on SHEILA are very encouraging for the further research on the heliac type of stellarator. However, for making access to the important physics topics, such as the finite plasma beta effect on both the equilibrium and stability, the effect of shear and magnetic well on stability and the effect of magnetic geometry on transport of particles and heat etc., a significant increase in both the machine size and operating field strength is required. It is therefore important to extend the studies in the prototype machine SHEILA to the larger heliac H-1 (Hamberger, et al., 1989) in the near future.
Appendix A

Programs For Solving the Equation of the Dispersion Relation

The programs 'disp1', 'disp2' and 'disp3' are all written in Fortran. The quantities in these programs are expressed in mixed units as shown in the input data tables. The program 'disp1' is used to calculate the dispersion relation (Eq. 5.13, in Chapter 5), i.e. the real and the imaginary parts of the frequency, $\omega_r$ and $\omega_i$, versus the parallel wavenumber $k_\parallel$ for a given poloidal mode number. The programs 'disp2' and 'disp3' are used to calculate the dependence of $\omega_r$ and $\omega_i$ on the electron total collision frequency $\nu$ and on the magnetic field strength respectively when all the other parameters in the dispersion relation are fixed.

The input data to 'disp1' are listed in Tables A1 to A3 for the three configurations studied. The results are shown in Fig. 5.15 in Chapter 5.

The input data to 'disp2' are listed in Table A4 for the $m = 3$ mode in configuration II. The results are shown in Fig. 5.6 in Chapter 5.

The input data to 'disp3' are listed in Tables A5 to A7 for the dominant poloidal modes in the three configurations, and the results are shown in Fig. 5.4.
in Chapter 5.

The experimental results have been used as input data in these tables, where the values of the parallel wavenumber $k_\parallel$ and the electron density $n_e$ are taken from the radial region where the modes have their maximum amplitude.

<table>
<thead>
<tr>
<th>Content</th>
<th>For $m = 2$ mode</th>
<th>For $m = 1$ mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of $k_\parallel$ (cm$^{-1}$)</td>
<td>0 - 0.03</td>
<td>0 - 0.03</td>
</tr>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$7 \times 10^{11}$</td>
<td>$9 \times 10^{11}$</td>
</tr>
<tr>
<td>$M_i$ for argon (g)</td>
<td>$6.66 \times 10^{-23}$</td>
<td>$6.66 \times 10^{-23}$</td>
</tr>
<tr>
<td>Constant in Gaussian distribution $b$</td>
<td>1.7</td>
<td>1.7</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>6.5</td>
<td>6.5</td>
</tr>
<tr>
<td>$B$ (G)</td>
<td>1680</td>
<td>1680</td>
</tr>
<tr>
<td>Poloidal mode number $m$</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>$k_r$</td>
<td>1.26</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table A.1: Input data to ‘disp1’ for configuration I

<table>
<thead>
<tr>
<th>Content</th>
<th>For $m = 3$ mode</th>
<th>For $m = 1$ mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of $k_\parallel$ (cm$^{-1}$)</td>
<td>0 - 0.03</td>
<td>0 - 0.03</td>
</tr>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$5 \times 10^{11}$</td>
<td>$1 \times 10^{12}$</td>
</tr>
<tr>
<td>$M_i$ for argon (g)</td>
<td>$6.66 \times 10^{-23}$</td>
<td>$6.66 \times 10^{-23}$</td>
</tr>
<tr>
<td>Constant in Gaussian distribution $b$</td>
<td>1.5</td>
<td>1.5</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>6.3</td>
<td>6.3</td>
</tr>
<tr>
<td>$B$ (G)</td>
<td>1250</td>
<td>1250</td>
</tr>
<tr>
<td>Poloidal mode number $m$</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>$k_r$</td>
<td>1.53</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table A.2: Input data to ‘disp1’ for configuration II
### Table A.3: Input data to ‘disp1’ for configuration III

<table>
<thead>
<tr>
<th>Content</th>
<th>For m = 1 mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of (k_\parallel) (cm(^{-1}))</td>
<td>0 – 0.03</td>
</tr>
<tr>
<td>(n_e) (cm(^{-3}))</td>
<td>6 \times 10^{11}</td>
</tr>
<tr>
<td>(M_i) for argon (g)</td>
<td>6.66 \times 10^{-23}</td>
</tr>
<tr>
<td>Constant in Gaussian distribution (b)</td>
<td>2.0</td>
</tr>
<tr>
<td>(T_e) (eV)</td>
<td>10.5</td>
</tr>
<tr>
<td>(B) (G)</td>
<td>950</td>
</tr>
<tr>
<td>Poloidal mode number (m)</td>
<td>1</td>
</tr>
<tr>
<td>(k_r)</td>
<td>0.94</td>
</tr>
</tbody>
</table>

### Table A.4: Input data to ‘disp2’ for \(m = 3\) mode in configuration II

<table>
<thead>
<tr>
<th>Content</th>
<th>For m = 3 mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of (n_e) (cm(^{-3}))</td>
<td>(0.13 – 7.5) \times 10^{12}</td>
</tr>
<tr>
<td>(k_\parallel) (cm(^{-1}))</td>
<td>0.006</td>
</tr>
<tr>
<td>(M_i) for argon (g)</td>
<td>6.66 \times 10^{-23}</td>
</tr>
<tr>
<td>Constant in Gaussian distribution (b)</td>
<td>1.5</td>
</tr>
<tr>
<td>(T_e) (eV)</td>
<td>6.3</td>
</tr>
<tr>
<td>(B) (G)</td>
<td>1250</td>
</tr>
<tr>
<td>Poloidal mode number (m)</td>
<td>3</td>
</tr>
<tr>
<td>(k_r)</td>
<td>1.53</td>
</tr>
</tbody>
</table>

### Table A.5: Input data to ‘disp3’ for \(m = 2\) mode in configuration I

<table>
<thead>
<tr>
<th>Content</th>
<th>For m = 2 mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of (B) (G)</td>
<td>100 – 3000</td>
</tr>
<tr>
<td>(k_\parallel) (cm(^{-1}))</td>
<td>0.006</td>
</tr>
<tr>
<td>(n_e) (cm(^{-3}))</td>
<td>7 \times 10^{11}</td>
</tr>
<tr>
<td>(M_i) for argon (g)</td>
<td>6.66 \times 10^{-23}</td>
</tr>
<tr>
<td>Constant in Gaussian distribution (b)</td>
<td>1.7</td>
</tr>
<tr>
<td>(T_e) (eV)</td>
<td>6.5</td>
</tr>
<tr>
<td>Poloidal mode number (m)</td>
<td>2</td>
</tr>
<tr>
<td>(k_r)</td>
<td>1.26</td>
</tr>
<tr>
<td>Content</td>
<td>For $m = 3$ mode</td>
</tr>
<tr>
<td>---------------------------------------------</td>
<td>------------------------</td>
</tr>
<tr>
<td>Range of $B$ (G)</td>
<td>100 - 3000</td>
</tr>
<tr>
<td>$k_{</td>
<td></td>
</tr>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$5 \times 10^{11}$</td>
</tr>
<tr>
<td>$M_i$ for argon (g)</td>
<td>$6.66 \times 10^{-23}$</td>
</tr>
<tr>
<td>Constant in Gaussian distribution $b$</td>
<td>1.5</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>6.3</td>
</tr>
<tr>
<td>Poloidal mode number $m$</td>
<td>3</td>
</tr>
<tr>
<td>$k_r$</td>
<td>1.53</td>
</tr>
</tbody>
</table>

Table A.6: Input data to ‘disp3’ for $m = 3$ mode in configuration II

<table>
<thead>
<tr>
<th>Content</th>
<th>For $m = 1$ mode</th>
</tr>
</thead>
<tbody>
<tr>
<td>Range of $B$ (G)</td>
<td>100 - 3000</td>
</tr>
<tr>
<td>$k_{</td>
<td></td>
</tr>
<tr>
<td>$n_e$ (cm$^{-3}$)</td>
<td>$6 \times 10^{11}$</td>
</tr>
<tr>
<td>$M_i$ for argon (g)</td>
<td>$6.66 \times 10^{-23}$</td>
</tr>
<tr>
<td>Constant in Gaussian distribution $b$</td>
<td>2.0</td>
</tr>
<tr>
<td>$T_e$ (eV)</td>
<td>10.5</td>
</tr>
<tr>
<td>Poloidal mode number $m$</td>
<td>1</td>
</tr>
<tr>
<td>$k_r$</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table A.7: Input data to ‘disp3’ for $m = 1$ mode in configuration III
PROGRAM DISPl

REAL*8 WR(384), WI(384)
REAL*8 TE, SIGMA, MI, N, KII(128), VTS
REAL*8 VEi, V, B, PM, KR, VAS, WS, ROS
REAL*8 AR(4), AI(4), ZR(4), ZI(4), TOL, X02AJF, KMIN, KMAX
INTEGER IFAIL, M
IFAIL=0
M=4
WRITE(*,*) 'ENTER DENSITY N'
READ (*, *) N
WRITE(*,*) 'ENTER ION MASS MI'
READ(*,*)MI
WRITE(*,*) 'ENTER PARALLEL WAVENUMBER KII MIN, KII MAX'
READ(*,*)KMIN,KMAX
WRITE(*,*) 'ENTER GAUSSIAN DISTRIBUTION CONSTANT SIGMA'
READ(*,*) SIGMA
WRITE(*,*) 'ENTER ELECTRON TEMPERATURE TE'
READ(*,*)TE
WRITE(*,*) 'ENTER MAGNETIC FIELD B'
READ(*,*)B
WRITE(*,*) 'ENTER POLOIDAL MODE NUMBER PM'
READ(*,*)PM
WRITE(*,*) 'ENTER Kr'
READ(*,*)Kr
DO 10 I=1,128
KII(I)=KMIN+DFLOAT(I-1)*(KMAX-KMIN)/128.
10 CONTINUE
VAS=B*B/(4*3.14159*N*MI)
WS=1.E+8*PM*TE/(SIGMA*SIGMA*B)
ROS=0.625E+28*TE*MI/(B*B)

V=VEi+Ven, lh/Ir=0Ven-1.5E+6, Ih/Ir-0.15 Ven-1.2E+6, Ih/Ir— 0.05
V=Vei+2.8e+6
VTS=1.758E+15*TE

as vt in the paper is (KTE/ME)**(1./2.) not (2KTE/ME)**(1./2.)
WRITE(*,*)'Kr=',Kr
TOL=X02AJF(.1)
WRITE(*,*) 'TOL', TOL
WRITE(*,*)'VAS=',VAS,'WS=',WS,'ROS=',ROS,'VTS=',VTS,'V=',V
DO 40 I=1,128
AR(1)=1.
AI(1)=0.
AI(2)=Kr*Kr*ROS*VAS*V/VTS
AR(2)=-WS
AI(3)=0.
AR(3)=-KII(I)*KII(I)*VAS*(1.+Kr*Kr*ROS)
AR(4)=WS*VAS*KII(I)*KII(I)
AI(4)=0.
IFAIL=0
M=4
CALL C02ADF(AR, AI, M, ZR, ZI, TOL, IFAIL)
WRITE(*,*)I
WR(3*(I-1)+1)=ZR(1)
WR(3*(I-1)+2)=ZR(2)
WR(3*(I-1)+3)=ZR(3)
WI(3*(I-1)+1)=ZI(1)
WI(3*(I-1)+2)=ZI(2)
WI(3*(I-1)+3)=ZI(3)

40 CONTINUE

C C02ADF, a routine to find all the roots of a complex polynomial
C equation (NAG, 1987)

OPEN(UNIT=1,NAME=’DISP.DAT1’,TYPE=’NEW’)
DO 50 I=1,128
WRITE(1,*)KII(I),WR(3*(I-1)+1)/(2*3.1416),
* WI(3*(I-1)+1)
50 CONTINUE
CLOSE(1)

OPEN(UNIT=2,NAME=’DISP.DAT2’,TYPE=’NEW’)
DO 60 I=1,128
WRITE(2,*)KII(I),WR(3*(I-1)+2)/(2*3.1416),
* WI(3*(I-1)+2)
60 CONTINUE
CLOSE(2)

OPEN(UNIT=3,NAME=’DISP.DAT3’,TYPE=’NEW’)
DO 70 I=1,128
WRITE(3,*)KII(I),WR(3*(I-1)+3)/(2*3.1416),
* WI(3*(I-1)+3)
70 CONTINUE
CLOSE(3)

STOP
END
PROGRAM DISP2

REAL*8 WR(384),WI(384)
REAL*8 TE,SIGMA,MI,N(128),KII,VTS
REAL*8 Vei(128),V (128),Ven,B,PM,KR,VAS(128),WS,ROS
REAL*8 AR(4),AI(4),ZR(4),ZI(4),TOL,XO2AJF,NMIN,NMAX
INTEGER IFAIL,M

IFAIL=0
M=4
WRITE(*,*)'ENTER DENSITY N MIN, N MAX'
READ(*,*)NMIN,NMAX
WRITE(*,*)'ENTER ION MASS MI'
READ(*,*)MI
WRITE(*,*)'ENTER PARALLEL WAVENUMBER KII'
READ(*,*)KII
WRITE(*,*)'ENTER GAUSSIAN DISTRIBUTION CONSTANT SIGMA'
READ(*,*)SIGMA
WRITE(*,*)'ENTER ELECTRON TEMPERATURE TE'
READ(*,*)TE
WRITE(*,*)'ENTER MAGNETIC FIELD B'
READ(*,*)B
WRITE(*,*)'ENTER POLOIDAL MODE NUMBER PM'
READ(*,*)PM
WRITE(*,*)'ENTER Kr'
READ(*,*)Kr

V=Vei+Ven, lh/Ir=0, Ven=1.5e+6, Ih/Ir=0.15, Ven=1.2e+6, Ih/Ir=0.05
Ven=2.8e+6

DO 10 I=1,128
N(I)=NMIN+DFLOAT(I-1)*(NMAX-NMIN)/128.
VAS(I)=B*B/(4*3.14159*N(I)*MI)
Vei(I)=2*(6.3E-5)*(TE**(-3./2.))*N(I)
V(I)=Vei(I)+Ven
10 CONTINUE

WS=1.E+8*PM*TE/(SIGMA*SIGMA*B)
ROS=0.625E+28*TE*MI/(B*B)
VTS=1.758E+15*TE

WRITE(*,*)'Kr=',Kr
TOL=XO2AJF(.1)
WRITE(*,*)'TOL=',TOL
WRITE(*,*)'WS=',WS,' ROS=',ROS
DO 40 I=1,128
AR(1)=1.
AI(1)=0.
AI(2)=Kr*Kr*ROS*VAS(I)*V(I)/VTS
AR(2)=WS
AI(3)=0.
AR(3)=-KII*KII*VAS(I)*(1.+Kr*Kr*ROS)
AR(4)=WS*VAS(I)*KII*KII
AI(4)=0.

WRITE(*,*)"as vt in the paper is (KTE/ME)**(1./2.) not (2KTE/ME)**(1./2.)"
WRITE(*,*)'Kr='", Kr

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C02ADF, a routine to find all the roots of a complex polynomial equation (NAG, 1987)

OPEN(UNIT=1, NAME='DISP.DAT1', TYPE='NEW')
DO 50 I=1, 128
   WRITE(1,*) V(I), WR(3*(I-1)+1)/(2*3.1416), WI(3*(I-1)+1)
   * CONTINUE
CLOSE(1)

OPEN(UNIT=2, NAME='DISP.DAT2', TYPE='NEW')
DO 60 I=1, 128
   WRITE(2,*) V(I), WR(3*(I-1)+2)/(2*3.1416), WI(3*(I-1)+2)
   * CONTINUE
CLOSE(2)

OPEN(UNIT=3, NAME='DISP.DAT3', TYPE='NEW')
DO 70 I=1, 128
   WRITE(3,*) V(I), WR(3*(I-1)+3)/(2*3.1416), WI(3*(I-1)+3)
   * CONTINUE
CLOSE(3)

STOP
END
PROGRAM DISP3

REAL*8 WR(384),WI(384)
REAL*8 TE,SIGMA,MI,N,B(128),VTS
REAL*8 VeI,V,KII,PM,KR,VAS(128),WS(128),ROS(128)
REAL*8 AR(4),AI(4),ZR(4),ZI(4),TOL,X02AJF,BMIN,BMAX
INTEGER IFAIL,M

IFAIL=0
M=4
WRITE(*,*)'ENTER DENSITY N'
READ(*,*)N
WRITE(*,*)'ENTER ION MASS MI'
READ(*,*)MI
WRITE(*,*)'ENTER MAGNETIC FIELD B MIN, B MAX'
READ(*,*)BMIN,BMAX
WRITE(*,*)'ENTER GAUSSIAN DISTRIBUTION CONSTANT SIGMA'
READ(*,*)SIGMA
WRITE(*,*)'ENTER ELECTRON TEMPERATURE TE'
READ(*,*)TE
WRITE(*,*)'ENTER PARALLEL WAVENUMBER KII'
READ(*,*)KII
WRITE(*,*)'ENTER POLOIDAL MODE NUMBER PM'
READ(*,*)PM
WRITE(*,*)'ENTER Kr'
READ(*,*)Kr

DO 10 I=1,128
B(I)=BMIN+DFLOAT(I-1)*(BMAX-BMIN)/128.
VAS(I)=B(I)*B(I)/(4*3.14159*N*MI)
WS(I)=1.E+8*PM*TE/(SIGMA*SIGMA*B(I))
ROS(I)=0.625E+28*TE*MI/(B(I)*B(I))
CONTINUE

V=VeI+1.5e+6
VT=3.48E+15*TE/2
WRITE(*,*)'Kr=',Kr
WRITE(*,*)'B (1) =' , B (1)
TOL=X02AJF(.1)
DO 40 I=1,128
AR(1)=1.
AI (1)=0.
AI(2)=Kr*Kr*ROS(I)*VAS(I)*V/VTS
AR(2)=-WS(I)
AI(3)=0.
AR(3)=KII*KII*VAS(I)*(1.+Kr*Kr*ROS(I))
AR(4)=WS(I)*VAS(I)*KII*KII
AI(4)=0.
IFAIL=0
M=4
CALL C02ADF(AR, AI, M, ZR, ZI, TOL, IFAIL)
WRITE(*,*)I
WR(3*(I-1)+1)=ZR(1)
WR(3*(I-1)+2)=ZR(2)
WR(3*(I-1)+3)=ZR(3)
WI(3*(I-1)+1)=ZI(1)
WI(3*(I-1)+2)=ZI(2)
WI(3*(I-1)+3)=ZI(3)

C02ADF, a routine to find all the roots of a complex polynomial equation (NAG, 1987)

OPEN(UNIT=1,NAME='DISP.DAT1',TYPE='NEW')
DO 50 I=1,128
   WRITE(1,*)(B(I),WR(3*(I-1)+1)/(2*3.1416),
   WI(3*(I-1)+1)
50 CONTINUE
CLOSE(1)

OPEN(UNIT=2,NAME='DISP.DAT2',TYPE='NEW')
DO 60 I=1,128
   WRITE(2,*)(B(I),WR(3*(I-1)+2)/(2*3.1416),
   WI(3*(I-1)+2)
60 CONTINUE
CLOSE(2)

OPEN(UNIT=3,NAME='DISP.DAT3',TYPE='NEW')
DO 70 I=1,128
   WRITE(3,*)(B(I),WR(3*(I-1)+3)/(2*3.1416),
   WI(3*(I-1)+3)
70 CONTINUE
CLOSE(3)

STOP
END
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