NUCLEAR SPECTROSCOPY

IN THE 2s-1d SHELL

by

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PREFACE

The work described in this thesis was carried out in the Department of Nuclear Physics of the Australian National University under the supervision of Dr. R.H. Spear.

The experiment to measure lifetimes of excited states in $^{27}$Al was undertaken as part of a research program in progress at the time of my joining the department. The experimental work was carried out jointly by Dr. R.H. Spear, Dr. C.J. Piluso and myself. The Doppler shift analysis of the data was done by myself under the guidance of Dr. Piluso.

The gamma-ray polarisation studies of $^{27}$Al were initiated by Dr. Spear and carried out jointly by him, Dr. A.M. Baxter, Dr. R.A.I. Bell, Dr. L.E. Carlson, Dr. D.C. Kean and myself. The data reduction was performed by myself with considerable assistance from Dr. T.R. Ophel. The light nucleus group as a whole gave much advice on problems associated with the analysis.

The experiments to study levels in $^{32}$S arose out of discussions between Dr. Spear, Dr. Carlson and Professor D.C. Peaslee. The study of the 5.80 MeV level was performed by Dr. Spear, Dr. Carlson, Dr. C.E. Moss and myself with the bulk of the data analysis being undertaken by myself.

The measurement of gamma-ray decay schemes of levels of $^{32}$S involved Dr. Spear, Dr. Moss, Dr. Baxter, Dr. Carlson and myself with some assistance from Mr. F. Ahmad. The data from this experiment was analysed jointly by Dr. Moss and myself with Dr. Moss taking the major role.

The third experiment involving $^{32}$S was to determine spin-parity combinations for known levels and the properties of the newly discovered 6.58 MeV level. Those who took part in this were Dr. Spear, Dr. Kean, Dr. Baxter, Dr. Carlson and myself with assistance from Mr. T. Esat in the final stages. Apart from that of the data concerning the decay scheme
of the 6.58 MeV level which was handled by Dr. Kean, all the analyses were performed by myself.

The work described in this thesis has appeared or will appear in the following publications:

(1) Lifetimes of Excited States in $^{27}$Al via the Reaction $^{12}$C($^{16}$O,p)$^{27}$Al,
P.R. Gardner, C.J. Piluso and R.H. Spear,

(2) Polarisation Studies of Ground-State Radiation from the 2.21 and 2.73 MeV Levels of $^{27}$Al,
P.R. Gardner, A.M. Baxter, R.A.I. Bell, L.E. Carlson,
D.C. Kean, T.R. Ophel and R.H. Spear,

(3) The 5.80 MeV State of $^{32}$S,
P.R. Gardner, C.E. Moss, R.H. Spear and L.E. Carlson,

(4) Gamma-Ray Decay Schemes of Levels at Intermediate Energies in $^{32}$S,
C.E. Moss, R.H. Spear, F. Ahmad, A.M. Baxter, L.E. Carlson and P.R. Gardner,

(5) Spin-Parity Combinations in $^{32}$S,
P.R. Gardner, D.C. Kean, R.H. Spear, A.M. Baxter, R.A.I. Bell and L.E. Carlson,

In the course of analysis of data taken during the above experiments, many computer programs were used. Most of these were already available in the laboratory, notably the peak area and centroid programs
of Dr. Ophel, Dr. Bell and Dr. Carlson, the lineshape fitting program of Dr. Ophel, the Doppler shift programs of Dr. Piluso and Dr. Carlson, the gamma-ray angular distribution program of Dr. Bell, the transmission coefficient program of Dr. D.J. Baugh and the "MANDY" program of Dr. E. Sheldon. Some small routines were written by Dr. Piluso, Dr. Carlson, Dr. Moss and myself, for specific calculations, but no major programs were required to be written by myself.

I have received considerable assistance and guidance from all of the past and present members of the light nucleus group, whose names have appeared above and I gratefully acknowledge this. Special thanks must go to Dr. Spear for his guidance and able supervision. I also thank Dr. Carlson and Dr. Kean for the constructive criticism they have given me during the writing of this thesis. I am indebted to my wife, Vanessa, for the many hours she has spent in preparing the diagrams and to Mr. M. Doobov for undertaking the proof reading.

Finally, I would like to thank Professor J.O. Newton and Professor Sir Ernest W. Titterton for the opportunity of working in this department, and the Australian National University for the award of a Postgraduate Research Scholarship.

No part of this thesis has been submitted for a degree at any other University.

P.R. Gardner

P.R. GARDNER.

Canberra,

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ABSTRACT

The properties of excited states of $^{27}$Al and $^{32}$S have been investigated by various techniques of gamma-ray and charged particle spectroscopy.

Chapter 1 describes several nuclear models which have had some success in describing the known properties of nuclei in the 2s-ld shell. Brief reviews of attempts to describe $^{27}$Al and $^{32}$S in terms of nuclear models are given in chapters 3 and 4.

The principles and theory behind the Doppler shift attenuation method (DSAM) and gamma-ray angular distribution and polarisation experiments are outlined in chapter 2, together with a brief description of the 61 cm double focusing magnetic spectrometer.

Lifetimes of the 1.01, 2.21, 2.73, 3.00, 4.51 and 5.50 MeV levels of $^{27}$Al, populated via the reaction $^{12}$C($^{16}$O,p)$^{27}$Al, have been measured using the DSAM. The experiments are described in chapter 3 and the results are in good agreement with previous ones obtained with the DSAM. The present result ($\lesssim 7$ fs) for the 5.50 MeV level and the previous result ($\lesssim 10$ fs, Vo 71) for the 7.44 MeV level are discussed in terms of the rotational model.

The (E2/M1) mixing ratios of the ground-state gamma-ray transitions from the 2.21 and 2.73 MeV levels of $^{27}$Al have been measured using a three-crystal Compton polarimeter in order to resolve discrepancies between previous experimental results for the 2.73 MeV transition. The results are discussed in terms of the nuclear models that have been applied to low-lying levels of $^{27}$Al.

Chapter 3 contains descriptions of experiments to measure properties of excited states of $^{32}$S. Studies of the reaction $^{29}$Si($\alpha$,n$\gamma$)$^{32}$S indicate that the 5.80 MeV state of $^{32}$S has an excitation
energy of 5798.2 ± 1.0 keV, a mean lifetime of 14 ± 7 fs, and a spin of 1. Particle-gamma coincidence studies of the reaction $^{32}\text{S}(p,p'\gamma)^{32}\text{S}$ indicate that the state decays 100% to the $0^+$ ground state, with all other possible transitions having intensities ≤ 2% of the ground-state transition.

The reaction $^{32}\text{S}(p,p'\gamma)^{32}\text{S}$ has been studied with a 12.7 × 10.2 cm NaI(Tl) gamma-ray detector in conjunction with a 61 cm double focussing magnetic spectrometer to determine the gamma-ray decay schemes of all known levels in $^{32}\text{S}$ between the excitation of 5.40 and 7.15 MeV.

Inelastically scattered alpha particles from the reaction $^{32}\text{S}(\alpha,\alpha')^{32}\text{S}$ have been studied with solid state counters at extreme backward angles in order to determine spin-parity combinations for levels in $^{32}\text{S}$ at excitation energies $E_x$ up to 7.15 MeV. The results are consistent with those spins and parities which are well established, show that the 5.80 MeV spin 1 state has negative parity, and reduce the range of possible values of spin and parity for the 6.41, 6.67, 6.76 and 6.85 MeV levels. A previously unreported natural parity level was found at $E_x = 6.58$ MeV. Magnetic analysis of the reaction $^{32}\text{S}(p,p')^{32}\text{S}$ confirmed the existence of this level, and established its excitation energy as 6.581 ± 0.003 MeV; particle-gamma coincidence studies showed that the level decays predominantly by gamma-ray transitions to the 2.23 MeV $2^+$ state.

The results obtained from the above experiments involving $^{32}\text{S}$ are discussed in terms of the nuclear models that have been applied to this nucleus.

A brief discussion is presented in chapter 5 of the nuclear shapes of $^{27}\text{Al}$ and $^{32}\text{S}$ in relation to their known properties and the predictions of nuclear models.
CHAPTER 1.

NUCLEAR MODELS

The nuclei studied in the work described in this thesis lie in the 2s-1d shell. The purpose of this chapter is to describe various nuclear models and their degrees of success in accounting for the properties of the nuclei in various regions throughout the 2s-1d shell.

1-1 APPLICATION OF THE SHELL MODEL TO THE 2s-1d SHELL

1-1.1 Nuclei near closed shells

Nuclei near closed shells, i.e., with one particle in a $d_{5/2}$ orbit outside a doubly-magic $^{16}O$ core or one "hole" in the 2s-1d shell, are most likely to be amenable to description by the extreme single-particle model (espm). In this model it is assumed that the effect of each particle interacting with all the others in the nucleus can be represented by a spherically symmetric potential acting on each particle individually. Including the effects of a strong inverted spin-orbit interaction accounts for the closing of shells at the so-called "magic numbers". The neutrons and protons in the nucleus are assumed to independently occupy quantum states $|n\ell j>$ in the potential according to the Pauli Principle in a manner analogous with the filling of electron states in an atom. The individual nucleons are paired off in such a way that the properties of the nucleus are attributed to the last unpaired nucleon.

As examples of the espm applied to the 2s-1d shell, fig. 1.1 shows energy level schemes for $^{17}O$ and $^{17}F$, which can be thought of as a single neutron and proton respectively moving in a $1d_{5/2}$ orbit outside
Fig. 1.1. Low-lying energy levels of $^{17}_F$ and $^{17}_O$ (from Ajzenberg-Selove, Aj 71).
an inert $^{160}$ doubly-magic core. The ground states both have $J^\pi = 5/2^+$ as predicted by the simple model. The $1/2^+$ and $3/2^+$ levels can be attributed to the excitation of the odd nucleon from the $1d_{5/2}$ to the $2s_{1/2}$ and $1d_{3/2}$ orbits respectively. Their excitation energies yield the magnitudes of the $1d_{5/2} - 1d_{3/2}$ splittings and the $1d_{5/2} - 2s_{1/2}$ separations for a single neutron and proton outside an $^{160}$ core. This information is useful when using a more sophisticated model to consider more than one particle in the $2s$-$1d$ shell, as will be shown in the next section. The negative parity states can be thought of as arising from collective excitation of the core ($Bi$ 68) or excitation of a core nucleon to the $1d_{5/2}$ or $2s_{1/2}$ orbits leaving a $1p_{1/2}$ "hole" with particle-like properties ($So$ 66).

The possibility of core excitation in $^{170}$ or $^{17F}$ demonstrates that the $^{160}$ core may not necessarily be completely inert. The magnetic dipole moment of $^{170}$ is $-1.89$ nm which is close to the value $-1.91$ nm for a single neutron as predicted by the espm. However, the electric quadrupole moment, instead of being negligibly small as expected for a single neutron and a spherical core, takes on the finite value of $-0.027$ barns. This shows that even the doubly-magic $^{160}$ core becomes non-spherical under the influence of a single extra-core nucleon. This will be discussed further in the next section.

In nuclei with more than one particle in the $2s$-$1d$ shell the espm fails to explain dynamical properties other than the ground-state angular momenta. A more sophisticated model is needed which allows all particles outside a closed shell to contribute to the nuclear properties and which includes the effects of residual particle-particle interactions.

1-1.2 Nuclei with several nucleons in the $2s$-$1d$ shell.

In order to describe nuclei with more than one nucleon in the
2s-ld shell it is necessary to assume that the nuclear field experienced by a particle in the nucleus consists of three parts, namely,

\[ V_k = V_0(r) + V_1(r)(\vec{s} \cdot \vec{r}) + \sum_{i} V_{ik}(r_{ik}) \]  \hspace{1cm} (1.1)

The first term is the central potential, the second represents the spin-orbit interaction while the third term describes the residual interaction of the two-body type among \( k \) particles outside a closed shell.

The nuclear wave functions are then written in terms of products of the espm wave functions for the \( k \) extra-core nucleons. The Pauli Principle demands the wave functions to be antisymmetric on exchange of space or spin co-ordinates for any pair of identical nucleons. For example, the product wave function describing a state consisting of two particles (1) and (2) with equal \( j \) is

\[ \psi(r_1)\psi(r_2) = u_n(r_1)u_n(r_2)\sum_{m} (jjmM-m|JM)\phi_{jm}(1)\phi_{jM-m}(2) \]  \hspace{1cm} (1.2)

where \( u_n(r) \) and \( \phi_{jM} \) are the radial and spin wave functions respectively. From the symmetry property of the Clebsch-Gordan coefficients,

\[ (jjmM-m|JM) = (-)^{j+j-J}(jjM-mm|JM), \]

one can see that antisymmetric wave functions exist only for even \( J \), the total angular momentum of the nucleus. For \( k \) particles the wave functions can be represented by Slater determinants corresponding to a particular configuration, e.g.,

\[
\psi = |nj>| = \left| \begin{array}{c}
\psi_1(r_1) & \psi_1(r_2) & \ldots & \psi_1(r_k) \\
\psi_2(r_1) & & & \\
& & & \\
& & & \\
& & & \\
\psi_k(r_1) & \psi_k(r_2) & \ldots & \psi_k(r_k)
\end{array} \right| \]

(1.3)
The possible degeneracy between different states $\psi$ of the $k$ particles will be removed by the effect of the residual interaction $V_{ik}(r_{ik})$. The final states $\psi$ of the nucleus produced by diagonalization of the nuclear matrix elements of $V_{ik}(r_{ik})$ are mixtures of the various configurations $\psi$. The residual interaction tends to couple the orbital and spin angular momenta together in an L-S coupling scheme while the spin-orbit interaction tends to result in $jj$ coupling so that inclusion of both interactions may lead to wave functions intermediate between L-S and $jj$ coupling. Generally, $jj$ coupling predominates in heavy nuclei where the $ls$ force becomes relatively stronger and is generally used in considering the $2s-1d$ shell.

Consider now examples of such calculations on nuclei in the $2s-1d$ shell. Fig. 1.2 (Mc 68) compares the low-lying experimental levels of $^{170}$ and $^{180}$. The zero of energy has been adjusted in each case so that the two sets of energy levels correspond. This was done by taking into account the contribution of the $^{160}$ ground-state binding energy and the single-particle energy to the ground state. The levels in $^{180}$ were computed by Pandya (Pa 63) from configuration mixtures of the $1d_{5/2}$ and $2s_{1/2}$ states present in $^{170}$ from which the single-particle information was obtained. These give rise to states with the following total isospin $T$ and angular momentum $J$ values:

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<tr>
<th>Configuration</th>
<th>$T$</th>
<th>$J$</th>
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<tbody>
<tr>
<td>$(d_{5/2})^2$</td>
<td>1</td>
<td>0,2,4</td>
</tr>
<tr>
<td>$(s_{1/2})^2$</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$(d_{5/2},s_{1/2})$</td>
<td>1</td>
<td>2,3</td>
</tr>
</tbody>
</table>

Five of the above states were identified from the levels in $^{180}$ known at the time of the calculation. The $3^+$ state was subsequently found
Fig. 1.2. Comparison of low-lying levels of $^{17}_{0}$ and $^{18}_{0}$; to make the two sets of levels correspond, the contribution of the $^{16}_{0}$ ground-state binding energy and the single-particle energy to the ground state have been taken into account in adjusting the zero of energy (from Pandya, Pa 63).
at 1.5 MeV relative excitation energy on the scale shown in fig. 1.2. In the limited configuration space assumed, eight two-body matrix elements arise from which can be constructed three and four-body matrix elements for $^{190}$ and $^{200}$.

Arima et al. (Ar 68) have carried out similar calculations in terms of the $(2s_{1/2}, 1d_{5/2})^N$ configuration where $n = 2-4$ for the isotopes $^{18,19,20}$O, $^{18,19,20}$F and $^{20}$Ne. The residual nucleon-nucleon interaction was parameterized in terms of the two-body matrix elements which were varied until a best fit to the experimental levels was obtained. The resulting wave functions were used to compute static and dynamic nuclear properties.

Wildenthal et al. (Wi 68) treat nuclei with $20 \leq A \leq 28$ by the same procedure, allowing the particles outside the $^{16}$O core to occupy $1d_{5/2}$ and $2s_{1/2}$ orbits. The $1d_{3/2}$ orbit was excluded because of the difficulty in handling the matrix elements. The upper region of the $2s$-$1d$ shell has been considered by Glaudemans et al. (G1 64) by including configurations of particles in the $2s_{1/2}$ and $1d_{3/2}$ orbits outside an inert $^{28}$Si core in order to limit the complexity of the calculations. Such a truncation of the configuration space gave rise to discrepancies with observed moments and transition rates. Erné (Er 66) treated nuclei in the range $^{33}$S to $^{41}$Ca by assuming an inert $^{32}$S core. Goldstein and Talmi (Go 56) and Pandya (Pa 56) have derived levels in $^{38}$Cl in terms of those of $^{40}$K by calculating a linear relationship between the energy levels of the two configurations $(d_{3/2})^{3}f_{7/2}$ and $d_{3/2}f_{7/2}$.

Wildenthal et al. (Wi 71) have also performed shell model calculations on the properties of positive parity states of nuclei with $A = 30 - 35$. For these calculations they have used configurations $(1s)^4(1p)^{12}(1d_{5/2})^{n_1}(2s_{1/2})^{n_2}(1d_{3/2})^{n_3}$ for which $n_1 \geq 10$ and claim good
agreement between calculated energy levels and spectroscopic factors and experimental data. These authors also claim satisfactory agreement with many available data on electric quadrupole observables if effective charges of 0.5e are added to the proton and neutron. Reference to this work is made in chapter 4 with respect to $^{32}\text{S}$.

Shell model calculations have been carried out recently by de Voigt et al. (Vo 72) and Wildenthal and McGrory (Wi 72) for masses $A = 27, 28$ and 29 using the truncated $1d_{5/2}^22s_{1/2}^31d_{3/2}$ configuration space. De Voigt et al. calculated electromagnetic transition rates and multipole moments, while Wildenthal and McGrory calculated energy levels, single-nucleon spectroscopic factors and electromagnetic transition rates. The generally good agreement between the results and experimental data suggest that shell model techniques are capable of providing a comprehensive and quantitative explanation of nuclear structure in this region of the 2s-1d shell. Further reference is made to these works in chapter 3 in connection with $^{27}\text{Al}$.

In general, however, although reasonable agreement can be achieved using the shell model between calculated and experimental energy levels for nuclei in the 2s-1d shell, the agreement between predicted and observed electromagnetic transition rates and multipole moments is often poor. This is usually because of the necessity to limit the configuration spaces used in the calculations in order that they may be reasonably well handled when the number of active particles becomes large. Whitehead (Wh 72) has attempted to get round this problem by making use of the so-called Lanczos iterative procedure to diagonalize matrices that consider all possible particles in the 2s-1d shell. Such large scale shell model calculations give good agreement with experiment for the case of low-lying levels in $^{24}\text{Mg}$, but $^{28}\text{Si}$ is not so well handled. It is apparent at this stage
that a simpler model which considers the collective motions of many nucleons may be useful in treating nuclei well away from closed shells.

1-2 COLLECTIVE BEHAVIOUR IN THE 2s-1d SHELL

As mentioned previously, moving away from closed shells one encounters deformations of nuclear shapes in the 2s-1d shell. This is illustrated in fig. 1.3 which shows the trend of the observed (En 66) ground-state quadrupole moments Q. These are given (Pr 62) in terms of the intrinsic quadrupole moment $Q_0$ as

$$Q = Q_0 \frac{3K^2 - J(J+1)}{(J+1)(2J+3)}$$

(1.4)

where $K$ is the projection of $J$ along the symmetry axis $z'$,

$$Q = \frac{1}{e} \int \rho \left(3z'^2 - r^2\right) dV, \quad r^2 = x'^2 + y'^2 + z'^2,$$

$\rho$ is the nuclear charge density and $dV$ is the volume element. Eqn. 1.4 demonstrates that a prolate deformation ($z'^2 > x'^2 + y'^2$) results in a positive value for $Q_0$ while an oblate deformation produces a negative $Q_0$. Thus the variation of nuclear deformation is loosely coupled to that of the ground-state quadrupole moments. The trend of the deformation throughout the 2s-1d shell from spherical, through prolate, reaching a maximum at $A = 23$, and oblate at $A \approx 28$ to spherical again was first ascribed by Rainwater (Ra 50) to the polarizing effect of the "outer" nucleons on the core. This tends to deform the core and was later discussed by Mottelson (Mo 60) in terms of the long-range component of the nucleon-nucleon force tending to align the extra-core nucleons in opposition to the short-range component producing pairing effects and tending to maintain a spherical shape. The systematic
Fig. 1.3. Trend of observed ground-state quadrupole moments in light to medium weight nuclei (from Enge, En 66). The scale factor $1/ZR^2$ takes into account the increasing size of the nuclei; $Z$ is the nuclear charge and $R$ is the average nuclear radius.
variation of nuclear shape in the 2s-ld shell has also been discussed by Hirko (Hi 69, Hi 69a), Mermaz et al. (Me 69) and Nakai et al. (Na 70). In these regions of large deformation it is to be expected that a model considering collective motions of many nucleons may be more successful in explaining observed nuclear properties than the spherical shell model with limited configuration spaces.

In even-even nuclei near closed shells or regions of sphericity, surface vibrational modes give rise to many of the low-lying levels. Away from closed shells where nuclei possess permanent deformations rotational motions become important, producing smaller excitations than vibrational modes. This is especially true for large deformations where the moments of inertia become large.

An odd A nucleus can be described in terms of the collective model as either a single particle strongly coupled to a deformed core (Nilsson model) where, in contrast to the shell model, both core and single particle can undergo excitations, or as a single particle weakly coupled to spherical core (weak coupling model).

1-2.1 Rotational behaviour

Bands of rotational energy levels in an even-even nucleus are produced by successive increments in its rotational angular momentum while preserving the nuclear shape. Such levels belonging to a band based on the ground state, have the following characteristics:

(1) the excitation energies are proportional to J(J+1)

(2) the spins and parities Jπ have the sequence 0+, 2+, 4+....

(3) enhanced E2 transition rates occur in gamma decay.

Rotational behaviour was first ascribed to heavy even-even nuclei by Bohr and Mottelson (Bo 52, Bo 53) but has also been applied to nuclei
in the 2s-1d shell, as in the case of $^{24}\text{Mg}$ (Br 68) and $^{20}\text{Ne}$ (Ku 67).

In an odd $A$ deformed nucleus, the extra particle moves in the deformed potential of the core whose rotations are slow compared to the particle motion. If the nucleus is strongly deformed than its rotational excitations will be small and can be regarded as perturbations of the single-particle states. The angular momentum coupling scheme is shown in fig. 1.4 for a nucleon with total angular momentum $\underline{j}$ coupled to an even-even spheroidal core with rotational angular momentum $\underline{R}$ and an axis of symmetry $z'$. $\underline{J}$ is the total resultant angular momentum and $\Omega$ and $K$ are the projections of $\underline{j}$ and $\underline{J}$ respectively on the $z'$ axis. $M$ is the projection of $\underline{J}$ on the space fixed axis $z$. Rotation about a symmetry axis is meaningless in quantum mechanics since it cannot be detected. Hence $\underline{R}$ is perpendicular to $z'$ and $K = \Omega$. The Hamiltonian for the above system consists of three parts:

$$H = H_{\text{rot}} + T_{\text{p}} + V(r) \quad (1.5)$$

where $H_{\text{rot}}$ is the rotational Hamiltonian, $T_{\text{p}}$ is the single-particle kinetic energy and $V(r)$ is the deformed potential experienced by the particle. The rotational Hamiltonian is

$$H_{\text{rot}} = \frac{\mathbf{K}^2}{2\phi} \underline{R}^2 = \frac{\mathbf{K}^2}{2\phi} (\underline{J} - \underline{j})^2 \quad (1.6)$$

where $\phi$ is the moment of inertia about the axis of rotation.

Expanding $H_{\text{rot}}$ gives

$$H_{\text{rot}} = \frac{\mathbf{K}^2}{2\phi} (J^2 + j^2 - 2K^2 - J_+j_- - J_-j_+) \quad (1.7)$$

where $K = J_{z'}$, $J_+ = J_{x'} + iJ_{y'}$ and $j_+ = j_{x'} + ij_{y'}$.

This enables the total Hamiltonian $H$ to be rewritten
Fig. 1.4. Schematic diagram showing the coupling of a particle with total angular momentum $j$ to a spheroidal core with total angular momentum $R$. 
\[ H = \frac{\hbar^2}{2\phi} (J(J+1) - 2K^2) + H_0 + H_{rpc} \] (1.8)

where
\[ H_0 = T_p + V(r) + \frac{\hbar^2}{2\phi} J^2 \] (1.9)
relates to the motion of the particle

and
\[ H_{rpc} = -\frac{\hbar^2}{2\phi} (J_+ j_- + J_- j_+) \] (1.10)
is analogous to the classical Coriolis force, coupling particle and core motions. Because the operators \( J^\pm \) have non-vanishing matrix elements between states with \( K \) differing by one unit the \( rpc \) term mixes rotational bands of different \( K \). However, in the limit of strong coupling the \( rpc \) term is small and can be treated by perturbation theory and \( K \) remains a good quantum number.

The eigenfunctions of \( H \) are the \( D \) functions which transform the spherical harmonics under finite rotations. Thus the nuclear wave functions will have the form (Mc 68)

\[ |JMK> = \left( \frac{2J+1}{8\pi^2} \right)^{\frac{3}{4}} D_{MK}^{J}(\theta,\phi,\psi) \chi_{\Omega}^{\phi}(r) \] (1.11)

where \( \theta, \phi \) and \( \psi \) are the Euler angles specifying the orientation of the principal axes of the deformed nucleus and \( \chi_{\Omega}^{\phi}(r) \) is the wave function describing the configuration of the extra particle relative to the principal axes. The invariance of a spheroidal shape under rotation through 180° means that eqn. 1.11 must be rewritten as

\[ |JMK> = \left( \frac{2J+1}{16\pi^2} \right)^{\frac{3}{4}} \left[ D_{MK}^{J} \chi_{\Omega}^{\phi}(r) + (-)^{J+\frac{3}{2}} D_{M-K}^{J} \chi_{-\Omega}^{\phi}(r) \right] \] (1.12)

The eigenvalues of (1.12) in the limit of a tightly bound particle and a small Coriolis force are then given for an odd \( A \) nucleus by
where \( a \), the decoupling parameter, corresponds to a partial decoupling of the particle motion from the rotator and is given by

\[
a = (-)^{\frac{J}{2}} <\chi_\Omega|j_\Omega = \frac{J}{2}> \quad (1.14).
\]

\( J \) takes on the values in (1.13) of \( K, K+1, K+2 \)...... States with energies \( E_J \) form a rotational band with a first member having \( J = K \).

The assumption that the Coriolis mixing may be small enough to be treated by perturbation theory breaks down in the 2s-1d shell where the magnitude of the rpc term approaches that of the separation of the intrinsic levels. Malik and Scholz (Ma 67) have approached this problem by including the Coriolis term in the Hamiltonian and diagonalizing it exactly. They include in their calculations mixtures of all six single-particle and single-hole bands in the 2s-1d shell and obtain good fits to low-lying energy levels.

The wave function for an even-even nucleus is that for the deformed rotating core and, in a manner similar to eqn. 1.12 for an odd \( A \) nucleus may be written

\[
|JMK> = \left[ \frac{2J+1}{16\pi^2} \right]^{\frac{1}{2}} \left[ D^J_{MK} \chi_K(r) + (-)^{J+K} D^J_{M-K} \chi_{-K}(r) \right] \quad (1.15).
\]

If the nucleus possesses axial symmetry then \( K = 0 \) because the rotation \( R \) is about an axis perpendicular to the symmetry axis. The energy eigenvalues of eqn. 1.15 are

\[
E_{JK} = \frac{\hbar^2}{2\phi} (J(J+1) - K^2) + \frac{\hbar^2}{2\phi} \cdot K^2 \quad (1.16)
\]
The ground-state rotational band with $K = 0$ will have the familiar level scheme given by

$$E_J = \frac{\hbar^2}{2\phi} J(J+1)$$

(1.16').

For $K = 0$ the wave function (eqn. 1.15) vanishes for $J$ odd and has positive parity for $J$ even. Thus the spins and parities of the ground-state rotational band have the sequence $J = 0^+, 2^+, 4^+, 6^+\ldots$. For $K \neq 0$, corresponding to rotational bands built on core excited states, $J$ may take on all integer values.

As an example of possible rotational bands in even-even nuclei in the 2s-1d shell, fig. 1.5 shows the energy levels of $^{24}$Mg and fig. 1.6 shows plots of the energies of selected levels against $J(J+1)$. It is apparent that there is strong evidence for two rotational bands in $^{24}$Mg based on $K = 0$ and $K = 2$. This nucleus is known to be in a region of large prolate deformation from the signs and magnitudes of quadrupole moments in the $A = 24$ region. Fitting eqn. 1.16' to experimental energy levels yields the value of $\phi$, the moment of inertia. For the case of $^{24}$Mg the moments of inertia are found to lie close to those of a rigid body. In heavier nuclei $\phi$ is found to be intermediate in value between that of a rigid body and that for a drop of irrotational fluid.

The electromagnetic properties predicted by the rotational model are a critical test of its validity and, in particular, must be able to account for the large E2 transition rates seen in even-even nuclei. The transition rate $T$ for gamma decay is proportional to the square of the matrix element between the initial and final states $|J_1 M_1\rangle$ and $|J_2 M_2\rangle$ and the multipole operator $Q^\sigma_{LM}$. The superscript $\sigma$ distinguishes between electric and magnetic radiation. If the
Fig. 1.5. Rotational energy levels belonging to $K = 0$ and $K = 2$ bands in $^{24}\text{Mg}$ (from Branford, Br 68).
Fig. 1.6. Plot of excitation energy versus $J(J + 1)$ for the levels shown in fig. 1.5 (from Branford, Br 68).
emitted photon has angular momentum $L$ and projection $m$ then conservation of angular momentum requires $|J_1 - J_2| \leq L \leq J_1 + J_2$ and $m = M_1 - M_2$. The parity change is $\pi_1 \pi_2 = (-)^L$. The transition rate $T$ can be expressed in terms of the so-called reduced matrix element $B(\sigma L, J_1 \rightarrow J_2)$ given (Pr 62) by

$$B(\sigma L, J_1 \rightarrow J_2) = (2J_1 + 1)^{-1} \sum_{M_1 M_2} |<J_2 M_2|Q_{LM}|J_1 M_1>|^2$$

(1.17).

Then

$$T_{LM}^\sigma = \frac{8\pi (L+1)}{L((2L+1)!!)} \frac{\Delta E}{2L+1} B(\sigma L)$$

(1.18)

where $\Delta E = E_1 - E_2$, the energy change involved in the transition.

Transition rates are usually expressed in terms of the values, in Weisskopf units (W.u.), established using the extreme single-particle model and assuming an infinite square-well potential with a radius $R = r_0 A^{1/3}$. These are given (En 66) for electric and magnetic transitions as

$$T(E, L) = \frac{4.4(L+1)}{L((2L+1)!!)^2} \left( \frac{3}{L+3} \right)^2 \left( \frac{E_Y}{197} \right)^{2L+1} R^{2L} \times 10^{21} \text{ sec}^{-1}$$

(1.19)

$$T(M, L) = \frac{1.9(L+1)}{L((2L+1)!!)^2} \left( \frac{3}{L+3} \right)^2 \left( \frac{E_Y}{197} \right)^{2L+1} R^{2L-2} \times 10^{21} \text{ sec}^{-1}$$

The within-band E2 transition rates are found to be simply related by Clebsch-Gordan vector addition coefficients because the intrinsic nuclear structure is the same for all members of a rotational band. The cross-band transition rates are similarly related as they involve the same change in intrinsic structure. The results can be formally
expressed as follows (for within-band transitions)

\[ B(E2,J \rightarrow J',K \rightarrow K) = \frac{5}{16\pi} e^2 Q^2 \langle J'K0 | J'K \rangle^2 \]  

(1.20)

where \( Q \), the intrinsic quadrupole moment, is defined in eqn. 1.4. Eqn. 1.20 shows that, for nuclei in regions of large deformation in the 2s-1d shell, within-band E2 transition rates will be considerably enhanced over the single-particle estimate. Cross-band transitions involve a change in the intrinsic structure of the nucleus and will be characteristically of the order of one single-particle unit.

Cross-band transitions are governed by the so-called K selection rule, i.e.,

\[ L \geq |K' - K| = \Delta K \]

This rule may be understood by considering transitions between a \( K = 2 \) and \( K = 0 \) band. At least two units of angular momentum are involved in such a transition in order to change the intrinsic nuclear structure. Thus, even if the spins of the initial and final states only differ by 0 or 1 the transition cannot occur by dipole radiation.

1-2.2 Nilsson states

The Nilsson model is analogous to the spherical shell model but assumes an axially symmetric deformed potential in which single-particle states are filled. Such a description has been successful for many odd A nuclei in the 2s-1d shell.

Single-particle states in the Nilsson model are derived using a Hamiltonian of the form
where $H$ is the harmonic oscillator Hamiltonian

$$H = \frac{p^2}{2m} + \sum_{m} (\omega_{x}^2 x^2 + \omega_{y}^2 y^2 + \omega_{z}^2 z^2),$$

$C$ is the spin-orbit interaction term and $D\delta^2$ is a term producing a potential intermediate between that of a square well and a harmonic oscillator. In the case of axial symmetry

$$\omega_{x}^2 = \omega_{o}^2 (1 + 2\delta^2) = \omega_{y}^2, \quad \omega_{z}^2 = \omega_{o}^2 (1 - 4\delta^2)$$

where $\omega_{o} = \omega_{o}(1 - 4\delta^2 - 16\delta^4 - 16\delta^3 - \delta^5)$. The approach adopted by Nilsson (Ni 55) was to diagonalize the matrix whose elements are

$$H_{nm} = \int \psi_n H_{nil} \psi_m \, d\tau$$

where the $\psi$s constitute a set of harmonic oscillator functions. The quantum numbers associated with the final states are $N$, the total number of oscillator quanta, the particle orbital angular momentum $\ell$, $\Lambda$ the projection of $\ell$ on the symmetry axis and $\Sigma$, the projection of the particle spins $s$. The parameters $C$ and $D$ in eqn. 1.24 were adjusted so that for $\delta = 0$ the shell model ordering of levels was reproduced. For $\delta \neq 0$, $\ell$ and $j$ are not conserved although $\Omega$, the projection of $j$ is conserved. This causes mixing of the states.
labelled by $|N\Lambda\Sigma\rangle$. The new states were labelled by the triad $|Nn_z\Lambda\rangle$ where $N$ and $\Lambda$ are as above and $n_z$ is the oscillator number corresponding to the number of nodes along the axis of symmetry. Fig. 1.7 shows Nilsson single-particle levels plotted against the deformation parameter $\delta$. Each level is labelled by $K^\pi$, $|Nn_z\Lambda\rangle$ and a Nilsson orbit number.

The Nilsson model has been successfully applied to odd $A$ nuclei throughout the region of large deformation from $^{19}$F to $^{31}$P. The general applicability of the model in the $2s$-$1d$ shell has been discussed by Bhatt (Bh 62). As illustrative examples, Pilt (Pi 72) has reported experimental evidence for a $K^\pi = 1/2^-$ band in $^{21}$Ne based on a $1p_{1/2}$ hole in Nilsson orbit number 4; Poletti et al. (Po 69) and Maier et al. (Ma 70) discuss experimental results in terms of rotational levels in $^{23}$Na, i.e., a $K^\pi = 1/2^+$ band based on Nilsson orbit nine and a $K^\pi = 3/2^+$ band based on Nilsson orbit seven. However, the agreement between experimental and theoretical electromagnetic properties is only fair.

The Nilsson model provides a good description of properties of excited states in the mirror nuclei $^{25}$Mg and $^{25}$Al. Kean and Ollerhead (Ke 72) show that while the known properties of the ground-state and $K^\pi = 1/2^+$ band in $^{25}$Mg are in good agreement with the rotational model, poorer agreement is achieved for higher $K^\pi = 1/2^+$ and $1/2^-$ bands whose cross-band transitions appear to violate the $K$ selection rule. $^{27}$Al is difficult to describe in terms of a simple model but much of the available data suggests that the level scheme of $^{29}$Al may be interpreted in terms of the Nilsson model assuming a prolate deformation (Hi 69).
Fig. 1.7. Energy levels predicted by the Nilsson Model as a function of deformation $\delta$ (from Enge, En 66).
1-2.3 Vibrational behaviour

As mentioned previously, nuclei lying between regions of large deformation and closed shells can be expected to exhibit low-lying levels arising from vibrations of the equilibrium shape. Evidence for such behaviour is found in the region around $A \approx 32$ of the 2s-1d shell.

The vibrational model likens the nucleus to a drop of incompressible liquid whose surface can be described by

$$R = R_0 (1 + \sum_{\lambda \mu} a_{\lambda \mu} Y_{\lambda \mu}(\theta, \phi)) \quad (1.21)$$

The $Y_{\lambda \mu}$ are spherical harmonics and $\lambda \mu$ represent the collective degrees of freedom. The Hamiltonian of vibrations about this nuclear surface is

$$H_{\text{vib}} = \sum_{\lambda \mu} \frac{B_\lambda}{2} |\hat{A}_{\lambda \mu}|^2 + \frac{C_\lambda}{2} |a_{\lambda \mu}|^2 \quad (1.22)$$

$B_\lambda$ plays the part of the moment of inertia with respect to a change in deformation while $C_\lambda$ represents the degree of resistance to deformation. The phonons have angular momentum $\lambda$ with component $\mu$ along some space fixed axis, parity $(-)^\lambda$ and frequency $\omega = \sqrt{C_\lambda / B_\lambda}$.

The energy eigenvalues of $H_{\text{vib}}$ are the harmonic oscillator energies $E = E_0 + \sum_{\lambda \mu} (n_{\lambda \mu} + 1/2)\hbar \omega_\lambda$, where $n_{\lambda \mu}$ is the number of phonons in the $\lambda \mu$ mode of vibration. Dipole ($\lambda = 1$) vibrations are not likely to occur because they would involve a displacement of the centre of gravity which would only happen under the action of external forces.

The lowest mode of vibration is normally quadrupole ($\lambda = 2$), thus the first excited states of vibrational nuclei are expected to have $J^\pi = 2^+$. Higher energy states can have $n_{2\mu}$ phonons associated with them.
Fig. 1.8 shows the predicted energy level diagram of even-even nuclei likely to display vibrational behaviour. Two quadrupole phonons, for instance, may couple to give levels with $J^\pi = 0^+, 2^+, \text{ and } 4^+$. The centre of gravity of this triplet would be at an energy approximately twice that of the one-phonon $2^+$ state. This kind of behaviour has been seen in many even-even nuclei (Ei 64).

In the 2s-1d shell rotational and vibrational excitations are expected to be of similar magnitude because the small moments of inertia increase the rotational level separation (see eqn. 1.17). Thus rotation-vibration coupling can lead to band mixing and rotational bands based on vibrational states will be appreciably different from those of the symmetric rotator.

Enhanced electric transition rates are expected to occur in vibrational nuclei since the whole nuclear charge is involved in the vibration. The electric multipole operator $Q^E_{\lambda \mu}$ can annihilate one phonon of order $\lambda$. Thus the $2^+$ member of the two-quadrupole phonon triplet with $J^\pi = 0^+, 2^+, \text{ and } 4^+$ would be expected to decay strongly to the one-quadrupole phonon $2^+$ state rather than to the ground zero-phonon state directly. The ratio of $B(E2)$ values for the transitions between the second $2^+$ state ($2^+_2$) and first $2^+$ state ($2^+_1$) and between the first $2^+$ and ground states is given by

$$\frac{B(E2: 2^+_2 \rightarrow 2^+_1)}{B(E2: 2^+ \rightarrow 0^+)} = 2 \quad (1.23).$$

The transition ($2^+_1 \rightarrow 0^+$) is usually strongly enhanced over the single-particle estimate.

Chapter 4 discusses the properties of $^{32}$S and demonstrates that many of them are consistent with the vibrational model picture.
Fig. 1.8. Schematic vibrational energy level schemes for even-even nuclei (from Eichler, Ei 64).
1-2.4 Weak and intermediate coupling

As mentioned at the beginning of this section, nuclei possessing small deformations can be regarded as a single particle weakly coupled to an even-even core whose collective excitations are predominantly vibrational. The Hamiltonian of such a system is

$$H = H_c + H_p + H_{\text{int}}$$  \hspace{1cm} (1.28)

where $H_c$ is the Hamiltonian for the vibrating core, $H_p$ is the spherical shell model single-particle Hamiltonian and $H_{\text{int}}$ represents the particle-core interaction. Neglecting particle-surface interactions enables the nuclear wave function to be written as a product of single-particle and vibrating core wave functions.

In the weak coupling model levels are constructed in an odd $A$ nucleus either by exciting the odd nucleon to higher single-particle states or by exciting the core. The latter kind will include a low-lying multiplet with angular momentum $J$ given by $|j-2| \leq J \leq j+2$ arising from quadrupole core vibrations. Attempts have been made to apply the weak coupling model to the $A \approx 28$ region of the $2s-1d$ shell where the core deformations are known to pass from prolate to oblate. Thankappan (Th 66) has discussed $^{27}$Al in terms of a $d_{5/2}$ proton hole weakly coupled to a $^{28}$Si core. Good agreement is obtained between calculated and experimental E2 transition probabilities but the treatment is less successful for M1 transition rates.

In the upper region of the $2s-1d$ shell the particle-core coupling strength may be intermediate between those assumed in the weak coupling and Nilsson model pictures. This occurs when the separations of the excited core and single-particle levels are of similar magnitudes. In such cases the single-particle states may be coupled to several
core-vibrational states and a diagonalization procedure is required. Such a procedure has been carried out by Castel and Stewart (Ca 70) for $^{29}$Si, $^{31}$P and $^{35}$Cl with moderate success, and by Castel et al. (Ca 71) for $^{32}$S in which vibrational modes are coupled to one particle-one hole and two particle-two hole states. This is discussed further in Chapter 4.

1-3 OTHER MODELS APPLIED TO THE 2s-1d SHELL

1-3.1 The Davydov-Filippov model

In the rotational models described in section 1-2 it was universally assumed that the even-even nuclear core possessed axial symmetry. In the Davydov-Filippov model this assumption is relaxed and a Hamiltonian is used (Da 58) with three unequal moments of inertia with respect to the three space-fixed axes. The Hamiltonian for an asymmetric rotator may be written (Ro 67).

$$H_{ar} = \sum_{\lambda=1}^{3} \frac{\alpha^{2} J_{\lambda}^{2}}{2 \phi_{\lambda}}$$

$$= \sum_{\lambda=1}^{3} \frac{AJ_{\lambda}^{2}}{2 \sin^{2}(y - \frac{2\pi\lambda}{3})}$$

(1.29)

where $\phi_{\lambda}$ are the moments of inertia, $\alpha$ is a variable parameter with the dimensions of energy and $y$ measures the deviation from axial symmetry taking values between 0 and $\frac{\pi}{3}$. These extreme limits correspond to axial symmetry. Application of the Pauli Principle to the eigenfunctions of $H_{ar}$ means that, for an even-even nucleus, no states are allowed for $J = 1$, two arise for $J = 2$, one for $J = 3$, three for $J = 4$,
two for $J = 5$, etc. In particular, it should be noted that this model, like the vibrational model, predicts two low-lying $2^+$ levels in an even-even nucleus. Thibaud et al. (Th 69) have applied the model in preference to the vibrational model to describe the level scheme of $^{32}\text{S}$ (see chapter 4).

1-3.2 The Hartree-Fock method

The Hartree-Fock method is a variational technique in which the nuclear ground-state wave function is described as a determinant of single-particle wave functions computed in a self-consistent potential. The derivation of such a potential is performed in a manner analogous to the Hartree method for computing atomic fields; a given particle is assumed to move in the average spherically symmetric field arising from all the other particles in the system. In the case of a nucleus these could be all the nucleons or just those outside a supposedly inert core. The ground-state wave function is expressed as a superposition of basis states $|\alpha>$

$$|\psi> = \sum_{\alpha} c_{\alpha} |\alpha>$$

where the superposition coefficients are normalised to unity and are able to be varied to achieve a minimum in the expectation value $E$ of the many-body Hamiltonian. This gives rise to a determinant of Hartree-Fock states $|\psi>$. Such calculations were first carried out in the 2s-1d shell by Kelson (Ke 63) who considered axially symmetric solutions for nucleons outside an inert spherical closed (1s-1p) shell. Maintaining $j_z'$ as a good quantum number preserved axial symmetry for the nuclear shape. Later calculations by Bassichis et al. (Ba 67) were less restricted in
the sense that all nucleons were included in the calculations. Their results produced solutions (prolate, oblate and spherical) for each of the three closed subshell nuclei $^{12}$C, $^{28}$Si and $^{32}$S. The minimum-energy solutions corresponded to oblate shapes for $^{12}$C and $^{28}$Si but prolate for $^{32}$S. Banerjee et al. (Ba 69) have performed Hartree-Fock calculations which predict the following shapes for nuclei in the 2s-1d shell: $^{20}$Ne(prolate spheroid), $^{24}$Mg(triaxial ellipsoid), $^{28}$Si(oblate spheroid), $^{32}$S(triaxial ellipsoid) and $^{36}$Ar(prolate spheroid). The Hartree-Fock calculations of Zofka and Ripka (Zo 71) on N = Z nuclei between $^{12}$C and $^{40}$Ca favour deformed solutions for $^{20}$Ne, $^{24}$Mg and $^{36}$Ar but spherical ones for $^{12}$C, $^{28}$Si and $^{32}$S.

1-4 SUMMARY

Because of the many-body nature of the nuclear system, it is necessary to resort to models of the nucleus in order to be able to establish a quantitative explanation of the available experimental data and predict properties yet to be measured. Several such models have been briefly described in this chapter, together with the degrees of success achieved in their application to nuclei in the 2s-1d shell.

The shell model is naturally most successful in considering a few particles outside spherical closed $^{16}$O or $^{40}$Ca shells, but modern computing techniques have enabled sufficient numbers of active particles to be considered that shell model calculations have been successfully applied throughout the 2s-1d shell, particularly by B.H. Wildenthal and his collaborators. Indeed, many of the collective properties described by the rotational and vibrational models have been reproduced by these calculations.

The rotational model is most applicable to nuclei in regions
of large permanent deformation, while the vibrational model is most relevant to spherical nuclei. In transition regions of nuclear shape, nuclear properties may be successfully described in terms of rotation-vibration coupling. The weak coupling model may be applied to even-odd nuclei with a spherical even-even neighbour. In this regard, Hartree-Fock calculations have been very useful in understanding the trend of nuclear shapes throughout the 2s-1d shell.

Chapters 3 and 4 describe experiments to measure properties of $^{27}\text{Al}$ and $^{32}\text{S}$ and the interpretation of the results in terms of the models outlined in this chapter.
CHAPTER 2.

EXPERIMENTAL TECHNIQUES

This chapter contains brief descriptions of the techniques used in experiments discussed in chapters 3 and 4, together with some appropriate theoretical background.

2-1 DOPPLER SHIFT ATTENUATION METHOD (DSAM) FOR NUCLEAR LIFETIME DETERMINATION

2-1.1 Introduction

It can be shown that, if a source of gamma rays is moving with velocity \( v \), the gamma rays will be observed to have the energy

\[
E(\theta) = E_0(1 + \frac{v}{c}\cos\theta), \quad \frac{v}{c} \ll 1
\]  

(2.1)

where \( \theta \) is the angle between the source velocity and the direction of gamma-ray emission and \( E_0 \) is the (unshifted) energy of the gamma ray as seen by a detector at rest relative to the source. If the source of radiation is travelling in a medium, it will be slowed down by collisions within the medium and the Doppler shift will become dependent on time, since the instantaneous velocity is time dependent. Hence

\[
E(\theta, t) = E_0(1 + \frac{v(t)}{c}\cos\theta)
\]  

(2.2)

where \( v(t) \) is the velocity of the source at the time of gamma-ray emission.

Suppose now, the source of gamma rays is a nucleus excited
to a state with lifetime $\tau$ following a nuclear reaction. At $t = 0$ the recoil velocity is $v(0)$. If $\tau$ is much less than the time taken for the recoiling ion to be stopped in the medium, gamma decay will occur before the nucleus has had a chance to slow down and the full Doppler shift will be observed. If, however, $\tau$ is much greater than the slowing down time, gamma decay will occur after the nucleus has stopped and a zero Doppler shift will be observed. For nuclear lifetimes between these two extremes, the Doppler shift will be attenuated by a factor $F(\tau)$ ranging in value between zero and unity.

The probability $p(t)$ for decay of the nucleus at time $t$ is

$$p(t) = \frac{1}{\tau} e^{-\frac{t}{\tau}},$$

at which time the velocity is $v(t)$. The mean energy of gamma rays emitted by many such nuclei is

$$\langle E(\theta) \rangle = \frac{\int_0^\infty \frac{1}{\tau} e^{-\frac{t}{\tau}} E(\theta,t) dt}{\int_0^\infty \frac{1}{\tau} e^{-\frac{t}{\tau}} dt}$$

$$= E_0 (1 + \frac{v(0)}{c} \cos \theta F(\tau))$$

(2.3)

(2.4)

where

$$F(\tau) = \frac{1}{\tau} \int_0^\infty e^{-\frac{t}{\tau}} \frac{v(t)}{v(0)} dt$$

(2.5).

It has been shown by Blaugrund (Bl 66) that, if one takes into account the fact that, as the excited nuclei slow in the stopping medium, there is a net change in direction $\phi$, which is dependent on the recoil velocity (and hence on time), eqn. 2.5 is modified by the insertion of the mean multiple scattering factor $\overline{v.\cos \phi(t)}$. Blaugrund approximates this factor by $\overline{v.\cos \phi(t)}$ and eqn. 2.5 thus becomes
The basis of the DSAM is, therefore, to determine nuclear lifetimes through a measurement of $F(\tau)$. Eqn. 2.5 demonstrates that the solution of this problem requires a knowledge of the processes involved in the slowing down of heavy ions in a medium; these are collisions between the moving ion and atoms in the stopping medium (unfortunately termed nuclear stopping) and collisions between the moving ion and atomic electrons in the stopping medium (electronic stopping). These two mechanisms are, for certain ranges of energy, both significant contributors to the slowing down process and are described in more detail in the following sections.

2-1.2 Electronic stopping

If an atom enters a stopping medium at velocities much greater than the orbital velocities of its electrons, these will be quickly stripped off and the bare nucleus will proceed through the medium. Slowing down will then occur by collisions with the atomic electrons of the stopping medium. Because the nuclear mass is very much greater than the electronic mass, there will be very little change in the direction of motion. Energy will be transferred to the electrons in the form of excitation to higher energy states or ejection out of the atomic orbitals leaving an ionised atom. Simple arguments can show that the stopping power due to such a mechanism has the form

$$\frac{dE}{dx} = \frac{Z_1^2 e^4 \cdot N B}{m_e v^2}$$  \hspace{1cm} (2.6)
volume in the stopping medium. The dimensionless constant $B$ contains the more complicated dependence on $Z_1$, $m_e$ and $v$ and is usually a slowly varying logarithmic expression; Bohr (Bo 13), Bethe (Be 30) and Bloch (Bl 33) have calculated slightly differing forms for $B$ using different starting points and assumptions.

When the nucleus has slowed to a velocity comparable to the highest electron orbital velocity ($v \approx \frac{e^2}{\hbar} Z_1$), the probability for electron capture becomes finite. As the moving ion further slows down, the processes of electron loss and capture will continuously occur with the probability of electron loss decreasing and the probability of electron capture increasing as the ion velocity decreases to match those of successive electron orbitals.

The theoretical treatment of the electronic stopping mechanisms is complicated and has been described by Lindhard et al. in a series of papers (Li 53, 54, 61, 63 and 68) and Blaugrund (Bl 66). To make the formulae less cumbersome, they introduce the following dimensionless variables which, ignoring the proportionality constants, have the following correspondence:

\[
\begin{align*}
t & = \theta, \\
E & = \epsilon, \\
x & = \rho, \\
m & = \gamma_1. 
\end{align*}
\]

Using the Thomas-Fermi model for the atom, they found that, in the velocity range $v < v_2 = \frac{e^2}{\hbar} Z_1$, the electronic stopping power is proportional to velocity and is given by

\[
\left( \frac{\partial \epsilon}{\partial \rho} \right)_e = k_e \frac{\lambda}{2}
\]  

(2.7)
where
\[
\kappa_e = Z_1 \frac{1}{\beta} \frac{0.0793 Z_1^{\gamma_2} Z_2^{-\gamma_2} (A_1 + A_2)^{\gamma_2}}{(Z_1^{\gamma_3} + Z_2^{\gamma_3})^{1/2} A_1^{\gamma_2} A_2^{\gamma_2}} \tag{2.7'}
\]

and the subscript 2 refers to the stopping medium.

For velocities less than \( v_2 \), experimental data are shown by Northcliffe (No 63) to yield values of the constant \( \kappa_e \) differing by up to 20\% from those predicted by eqn. 2.7'. In view of this, many workers, e.g., Blaugrund et al. (Bl 67), employ a multiplying factor \( f_e \) for the electronic stopping power such that

\[
\left( \frac{\delta \varepsilon}{\delta \rho} \right)_e = f_e \kappa_e^{\gamma_2} \tag{2.7''}
\]

An average of many measurements gave \( f_e = 1.16 \).

For velocities \( v > v_2 \), the stopping power approaches a maximum and then decreases with the \( 1/v^2 \) dependence described earlier. An example for carbon ions stopping in gold is shown in fig. 2.1. Northcliffe and Schilling (No 70) have made compilations of heavy ion stopping powers for various stopping media using experimental values of heavy ion stopping in aluminium as a basis and then extrapolating to other stoppers using the theory of Lindhard et al. For velocities in the region corresponding to the maximum in the electronic stopping power, a satisfactory fit can be made to the data of Northcliffe and Schilling by expressing the electronic stopping power in the form

\[
\left( \frac{\delta \varepsilon}{\delta \rho} \right)_e = \kappa_e^{\gamma_2} - k_3^{\gamma_2} \tag{2.8}
\]

and finding values of \( \kappa_e \) and \( k_3 \) that yield a best fit. The error associated with \( \left( \frac{\delta \varepsilon}{\delta \rho} \right)_e \) obtained in this way for 20 MeV \(^{27}\text{Al} \) ions stopping in nickel is estimated to be about 10\% (see chapter 3).
Fig. 2.1. Electronic stopping power measured for carbon ions stopping in gold as a function of ion energy. The dashed line represents the estimate derived from eqns. 2.7 and 2.7* (from Northcliffe, No 63).
Ormrod et al. (Or 65), Fastrup et al. (Fa 66) and Hvelplund and Fastrup (Hv 68) have investigated the dependence of \( \frac{\partial e}{\partial \rho} \) on \( Z_1 \) for a given ion velocity and find that there is oscillatory behaviour about the Lindhard estimate given by eqn. 2.7 (the so-called "Z_1 oscillations"). An example of this for \( Z_2 = 6 \) (carbon) is shown in fig. 2.2. Several authors using the DSAM for measuring nuclear lifetimes, e.g., Bell et al. (Be 69), have taken this into account when estimating \( \frac{\partial e}{\partial \rho} \).

### 2-1.3 Nuclear stopping

As the moving ion slows down as a result of the electronic stopping mechanisms described above, eventually the velocity will decrease to a value corresponding to that of the orbital velocity of the least tightly bound of its electrons. The ion will then become a neutral atom and further energy loss will occur by elastic collisions with the atoms in the stopping medium (nuclear stopping). Because the masses of the moving and stationary atoms are comparable, scattering through large angles may occur as mentioned in subsect. 2-1.1.

Lindhard et al. (Li 63) have calculated the atom/atom differential scattering cross section using a screened Thomas-Fermi potential. This has the form shown in fig. 2.3 and is given by

\[
\frac{d\sigma}{dt} = \frac{\pi a^2}{2t^{3/2}} f(t^{1/2}) \tag{2.9}
\]

where \( t = e^2 \sin^2 \frac{\theta}{2} \), \( \theta \) being the deflection angle in the centre of mass system, and

\[
a = 0.8853 \left( \frac{\hbar^2}{m_1 e^2} \right) (Z_1^{2/3} + Z_2^{2/3})^{-1/2}
\]
STOPPING IN CARBON ($Z_2 = 6$)

$V = 9 \times 10^7$ cm/s

Fig. 2.2. Measured electronic stopping powers versus $Z_1$ for a common ion velocity $v$ and
$Z_2 = 6$ (carbon), showing the "$Z_1$ oscillations" (from Ormrod, Or 65).
The universal atom/atom scattering cross section (from Lindhard et al., Li 63). The various symbols are explained in the text.
is the parameter describing the screening of the nuclear fields by the inner electrons.

The function $f$ was calculated by Lindhard et al. from the Fermi function belonging to a single Thomas-Fermi atom and it describes the scattering at all energies and at all scattering angles for all atom-ion combinations. Fig. 2.3 shows that, for high values of $\varepsilon$, the cross section approaches that for Rutherford scattering. From eqn. 2.9 the specific energy loss due to nuclear stopping can be calculated, giving

$$\left(\frac{\text{d}E}{\text{d}\rho}\right)_n = \frac{1}{\varepsilon} \int_0^\varepsilon f(x)dx$$

(2.10).

The nuclear stopping power derived from the atom/atom differential scattering cross section is shown in fig. 2.4. By using the dimensionless variables mentioned in the previous section, all nuclear stopping powers can be described by the one curve shown in fig. 2.4. It can be seen from this diagram that nuclear stopping predominates over electronic stopping for velocities less than $v_1 \approx 0.015v_2$.

At low velocities and for heavy scatterers, the effects of direction changes due to atomic collisions on the slowing down process become more important than energy loss in DSAM experiments. This point has been treated by Blaugrund (Bl 66) who has calculated expressions for $\cos\phi(t)$ (defined in subsect. 2-1.1) in terms of the dimensionless parameters described in the previous section.

2-1.4. Qualitative features of $F(\tau)$

In this subsection we restrict the discussion to the region of recoil velocities where only electronic stopping is important and
Fig. 2.4. Nuclear stopping powers \( \left( \frac{d\epsilon}{d\rho} \right)_n \) derived from the universal scattering cross section shown in fig. 2.3. The electronic stopping power curve is also shown for comparison (from Lindhard et al., Li 63).
\(\cos \phi(t)\) can be neglected \((\frac{V}{c} \approx 1-2\%)\). Then, both experiment and the Lindhard et al. theory show that \(\frac{dE}{dx}\) is proportional to \(v\). Thus

\[
\frac{dE}{dx} = \frac{1}{v} \frac{dE}{dv} \frac{dv}{dt} = m \frac{dv}{dt} = \text{constant} \times v \tag{2.11}
\]

Defining a constant \(\alpha\) by \(\frac{dv}{dt} = -\frac{v}{\alpha}\) gives

\[
\frac{dv}{v} = -\frac{dt}{\alpha} \quad \text{or} \quad v(t) = v(0)e^{-\frac{t}{\alpha}} \tag{2.12}
\]

Hence the constant \(\alpha\) is a characteristic slowing down time whose magnitude depends on the details of the stopping medium and the recoiling ion; calculated and measured values are typically of the order of 50 fs. Clearly, this stopping time determines the range of lifetimes measurable with a specific experimental design.

From eqns. 2.12 and 2.5 one can write

\[
F(\tau) = \frac{1}{\tau} \int_{0}^{\infty} e^{-\frac{t}{\alpha}} e^{-\frac{t}{\tau}} dt = \frac{\alpha}{\alpha + \tau} \tag{2.13}
\]

The form of eqn. (2.13) is shown in fig. 2.5 and is the general shape of \(F(\tau)\) even when nuclear stopping is included. The fact that the curve becomes flat at both short and long values of \(\tau\) means that comparing such a curve with experimental values of \(F(\tau)\) results in large errors for the \(\tau_s\) deduced for these extremes; this is the practical limitation of the DSAM.

2-1.5 Calculation of \(F(\tau)\)

In the general case, where nuclear stopping and \(\cos \phi(t)\) cannot be neglected, eqn. 2.5' must be solved numerically - or a
Doppler shift attenuation factor $F(\tau) = a/(a + \tau)$ for $a = 50$ fs calculated assuming pure electronic stopping of the form $\left( \frac{d\epsilon}{dp} \right)_e = k_e \epsilon^{1/2}$. 

Fig. 2.5.
suitable functional form derived for \( v(t) \). The quantity calculated by Lindhard et al. is not \( v(t) \) but \( t(E) \) which is related to \( v(t) \) through its derivative as follows:

\[
v(t) = \frac{dx}{dt} = \frac{dx}{dE} \frac{dE}{dt} = \frac{dE}{dt} \frac{1}{dx}.
\]

But

\[
v(t) = \sqrt{\frac{2E(t)}{M}},
\]

thus

\[
dt = \sqrt{\frac{M}{2}} \frac{dE}{E \sqrt{\frac{dE}{dx}}}
\]

which, on integrating, gives

\[
t(E) = \sqrt{\frac{M}{2}} \int_E^{E_0} \frac{dE}{E \sqrt{\frac{dE}{dx}}} \tag{2.14}.
\]

In terms of the dimensionless parameters described in subsect. 2-1.2, eqn. 2.14 becomes

\[
\theta(\varepsilon) = \sqrt{\frac{m}{2}} \int_{\varepsilon}^{\varepsilon_0} \frac{d\varepsilon}{\varepsilon \sqrt{\frac{d\varepsilon}{dp}}} \tag{2.14'}.
\]

The formulae for \( \cos \phi(t) \) quoted by Blaugrund (Bl 66) involves the calculation of another integral

\[
I = \int_{\varepsilon}^{\varepsilon_0} \frac{d\varepsilon}{\varepsilon \left( \frac{d\varepsilon}{dp} \right)_n} \frac{d\varepsilon}{\varepsilon \left( \frac{d\varepsilon}{dp} \right)_e} \tag{2.15}
\]

where \( \left( \frac{d\varepsilon}{dp} \right)_n \) = nuclear stopping power, \( \left( \frac{d\varepsilon}{dp} \right)_e \) = electronic stopping power and

\[
\left( \frac{d\varepsilon}{dp} \right) = \left( \frac{d\varepsilon}{dp} \right)_n + \left( \frac{d\varepsilon}{dp} \right)_e
\]
One could solve eqns. 2.14' and 2.15 if one had functional forms for 
\( \left( \frac{d\epsilon}{dp} \right)_e \) and \( \left( \frac{d\epsilon}{dp} \right)_n \). Unfortunately, this is not the case; what one does 
have is

\[
\left( \frac{d\epsilon}{dp} \right)_e = k_e \epsilon^{1/2}
\]

and \( \left( \frac{d\epsilon}{dp} \right)_n \) in terms of a graph (fig. 2.4). It is, therefore, necessary 
to produce a function to fit the \( \left( \frac{d\epsilon}{dp} \right)_n \) curve (which is difficult over 
a wide range of \( \epsilon \), or approximate the curve numerically. In the 
computer code written by Dr. C.J. Piluso, this was done by approximating 
the function \( \left( \frac{d\epsilon}{dp} \right)_n \) by a series of straight line segments. In the similar 
code written by Dr. L.E. Carlson, analytical expressions are used to fit 
appropriate regions of the curve.

2-1.6. Experimental configurations for DSAM experiments

DSAM experiments can be conducted using a single gamma-ray 
detector, as in the case described in chapter 4, where an experiment 
to measure the lifetime of the 5.80 MeV level of \(^{32}\text{S}\) is described. 
This experiment involved the detection of gamma rays from the reaction 
\(^{29}\text{Si}(\alpha,n)^{32}\text{S}\) with a Ge(Li) detector. In such cases, where the light 
outgoing particle is not detected, the centroids of the gamma-ray lines 
will depend on the average component of the recoil velocity along 
the beam direction. If the light particles (and hence the recoil 
nuclei) have an isotropic distribution in the centre of mass (c.m.) 
system, then eqn. 2.4 becomes

\[
\langle E(\theta) \rangle = E_0 (1 + \frac{\nu_{cm}}{c} \cdot F(\tau) \cos \theta) \quad (2.4')
\]

where \( \nu_{cm} \) is the velocity of the centre of mass. If an anisotropic
distribution in the c.m. system is assumed of the form

\[ W(\theta_R) = 1 + a \cos \theta_R \]

for the recoil nuclei, then eqn. 2.4 becomes

\[ < E(\theta) > = E_0 \left( 1 + F(\tau) \left( \frac{v_{\text{cm}}}{c} + \frac{a v_R}{3c} \right) \cos \theta \right) \]

(2.4')

where \( v_R \) is the velocity of the recoil in the c.m. frame (and is dependent on excitation energy of the residual nucleus). For the extreme case (\( a = 1/2 \)), the ratio \( \frac{v_R}{6v_{\text{cm}}} \) is rarely greater than 5% and the effects of asymmetry can often be neglected. This is especially true if reactions are studied with bombarding energies just above threshold, when the outgoing light particles will have isotropic distributions in the c.m. system, corresponding to \( L = 0 \). If such is the case, then one \( F(\tau) \) curve is valid for all excitation energies as eqn. 2.4' contains no dependence on excitation energy.

These single detector DSAM experiments are primarily used when the spectra are sufficiently simple to enable the lines of interest to be resolved. It is also necessary that the recoil velocity is not so high that the Doppler broadening arising from the finite detector solid angle and recoil distribution renders the gamma lines unduly broad. The usual procedure is that described in chapter 4, where a target is used with a backing thick enough to stop the recoil nucleus. The energies of the gamma rays of interest are then determined as a function of angle \( \theta \). A fit of eqn. 2.4' to the data then yields the product \( \frac{v_{\text{cm}}}{c} F(\tau) \). The quantity \( \frac{v_{\text{cm}}}{c} \) can be calculated from the kinematics of the reaction, thus enabling \( F(\tau) \) to be determined.

When Doppler-shifted gamma rays are detected in coincidence
with the light outgoing particles, the direction of recoil is limited by the reaction kinematics. Such an arrangement was used to measure lifetimes of excited states of $^{27}$Al, via the reaction $^{12}$C$(^{16}$O,p)$^{27}$Al, in the experiment described in chapter 3. This procedure is useful when the gamma ray spectrum is complicated and where the large recoil associated with a heavy ion reaction produces considerable Doppler broadening of the gamma-ray lineshapes. This broadening can arise from the finite detector solid angles and the finite lifetime of the state of interest which, for intermediate values of $\tau$, significantly smears the lineshape over a region extending from zero to full shift. It is common practice in such experiments to set the gamma-ray detector at $0^\circ$ with respect to the beam direction, where the Doppler broadening is a minimum. However, the finite detector solid angles then cause a shift in the centroid of the lineshape from the point detector case. This can be taken into account by expressing eqn. 2.4 as

$$<E(\theta_\gamma)> = E_0(1 + F(\tau)Q_1 \frac{\bar{V}}{c} (\cos\theta_\gamma \cos\theta_R - \sin\theta_\gamma \sin\theta_R)) \quad (2.4'')$$

where $\theta_\gamma$ is the mean angle of gamma-ray detection with respect to the beam direction, $\theta_R$ is the angle of recoil, the barred quantities are the mean values over the range of $\theta_R$ allowed by the light particle detector solid angle and $Q_1$ is the gamma-ray detector solid-angle correction factor described in subsect. 2-2.4. For $\theta_\gamma = 0^\circ$, eqn. 2.4 becomes

$$<E(\theta_\gamma = 0^\circ)> = E_0(1 + F(\tau)Q_1 \frac{\bar{V}}{c} \cos\theta_R) \quad (2.4'''').$$

$F(\tau)$ is then usually determined by observing the full Doppler shift from
nuclei recoiling from a thin target into vacuum and the attenuated Doppler shift from recoils into a thick backing.

When heavy ion reactions are used to populate states of interest, the recoil velocities can become very high; in the case described in chapter 3, $\frac{V}{c} \approx 4\%$. Such high recoil velocities mean that second order terms in $\frac{V}{c}$ are significant in the Doppler shift formula (eqn. 2.4). The relativistic form of this equation is

$$< E(\theta) > = \frac{E_0 (1 - \frac{V^2}{c^2})^{1/2}}{1 - \frac{V}{c}\cos \theta} $$

(2.16).

Expanding the numerator and denominator of the right hand side of eqn. 2.16 gives

$$< E(\theta) > = E_0 (1 + \frac{V}{c}\cos \theta + \frac{V^2}{c^2}\cos^2 \theta - \frac{1}{2}\frac{V^2}{c^2}) $$

(2.16').

The term $\frac{1}{2}\frac{V^2}{c^2}$ is the so-called "transverse Doppler shift", that is, the residual Doppler shift that remains at $\theta = 90^\circ$.

The use of such heavy ion reactions in DSAM experiments has several advantages; the large shifts enable the determination of short lifetimes in the range 1 to 5 fs, lineshape fitting may be used to determine $\tau$ directly from the gamma-ray spectra and nuclear scattering and stopping may be neglected. However, a particle-gamma ray coincidence experiment using heavy ion beams always suffers from low count rate.

2-2 GAMMA RAY ANGULAR DISTRIBUTIONS

2-2.1 Introduction

A brief outline is given here of the theory of gamma-ray
angular distributions in an axially symmetric system. Detailed discussions of the theory have been presented by several authors (e.g., Li 61a, Po 65, Li 64, Fe 65) but the definitive work is probably that of Rose and Brink (Ro 67a).

As outlined in subsect. 1-2.1, the transition rate T for gamma decay between states \( |J_{1M_1}\rangle \) and \( |J_{2M_2}\rangle \) is proportional to
\[
P(M_1)|<J_{2M_2}|Q_L^\sigma|J_{1M_1}|^2 \text{ where } P(M_1) \text{ is the population of states in the } \pm M_1 \text{ substate, such that } \sum_{M_1} P(M_1) = 1. \]
Application of the Wigner-Eckart theorem separates the geometrical and dynamical aspects of the transition, such that
\[
<J_{2M_2}|Q_L^\sigma|J_{1M_1}| = (J_{2M_2Lm}|J_{1M_1}|<J_2||Q_L^\sigma||J_1> \text{ (2.17),}
\]
\[
<J_2||Q_L^\sigma||J_1> \text{ being the so-called reduced matrix element of } Q_L^\sigma \text{ and } (J_{2M_2Lm}|J_{1M_1}| \text{ a Clebsch-Gordan coefficient. Thus, the angular distribution of the gamma radiation summed over all magnetic substates is given by}
\]
\[
W_{L}(\theta) = \sum_{M_1M_2} P(M_1) (J_{2M_2Lm}|J_{1M_1}|)^2 |<J_2||Q_L^\sigma||J_1>|^2 \times F_{Lm}(\theta) \text{ (2.18)}
\]
where \( F_{Lm}(\theta) \) is the radiation pattern for multipolarity L and change m in magnetic quantum number. \( \theta \) is the angle between the direction of radiation and the z space-fixed quantization axis. From eqn. 2.18, it can be seen that \( W(\theta) \) depends only on the population parameters \( P(M_1) \) and the angular momentum quantum numbers. Hence knowing \( P(M_1) \), it should be possible to derive information regarding the angular momenta involved in the transition from a measurement of the angular distribution of gamma radiation. In the case where the magnetic substates \( M_1 \) are equally populated, the incoherent superposition of distributions from the different substates will always render the
distribution isotropic. In other words, an anisotropic gamma-ray angular distribution can only result from a system of nuclei that have a degree of alignment with respect to the z axis. This is the case if $P(M'_1) \neq P(M_1)$. Complete alignment corresponds to the situation where only one initial magnetic substate contributes to the angular distribution.

Radiation of more than one multipolarity may contribute to the transition but normally only the two lowest orders given by $|J_1 - J_2| \leq L \leq J_1 + J_2$ will be significant; this is because of the rapid decrease in gamma-ray transition probability with increasing $L$. Multipole mixing may be allowed for in the expression for $W(\theta)$ by introducing the multipole amplitude mixing ratio $\delta$ which is defined by Rose and Brink as

$$
\delta = \frac{\langle J_1 ||q^l_{L嬖}||J_2\rangle(2L' + 1)^l_2}{\langle J_1 ||q^l_{L嬖}||J_2\rangle(2L + 1)^l_2}
$$

where $L$ represents the higher order multipolarity occurring in the transition. The role of the mixing ratio $\delta$ in gamma-ray angular distributions is discussed further in subsect. 2-2.3.

2-2.2 Angular distributions of gamma rays following nuclear reactions

If the state $|JM\rangle$ is formed as a result of a nuclear reaction, the beam direction defines an axis of axial symmetry which can be taken as the z quantization axis. The degree of alignment of the state can be maximised in two ways; the first involves the use of a nuclear reaction proceeding through the compound nucleus of the type $X(h_1, h_2)^*\ast$, where $h_2$ is undetected. The incoming particles possess zero orbital angular momentum component along the z axis. If the
bombarding energy is just above threshold for the reaction, the outgoing light particles will have low energy (and orbital angular momentum) and consequently will be predominantly s-wave. In such cases, the magnetic substates $M$ that may be populated of final states $J$ are restricted to

$$M_J \leq |J_0 + s_1 + s_2|$$

where $J_0$ is the spin of the target nucleus and $s_1$ and $s_2$ are the spins of the projectile and outgoing light particles, respectively. An example of the use of such a method for producing alignment of nuclear states is given in chapter 4, where a measurement of the spin $J$ of the 5.80 MeV level of $^{32}$S populated by the reaction $^{29}$Si(α,n)$^{32}$S is described. Obviously, the degree of alignment becomes large only if $J \geq |J_0 + s_1 + s_2|$ and thus the method is particularly favourable for reactions where $J_0$ or $s_1$ are zero, as in the above example.

When the bombarding energy is well above threshold for populating the state, outgoing light particles with non-zero orbital angular momentum (p and d wave, etc.) may be produced, reducing the degree of nuclear alignment. A measurement of the gamma-ray angular distribution may then produce ambiguous results for $J$ because different combinations of possible magnetic substates may give rise to the same distribution for different $J$ values. The situation can, however, be saved by detecting in coincidence the outgoing particles on or near the beam axis as in the so-called Method II type experiment (Li 61a). In the point particle detector case, only magnetic substates populated by outgoing particles with component $\ell_z = 0$ will contribute to the gamma-ray angular distribution, thus physically restricting the population parameters and enhancing the degree of alignment. The finite solid angle of a particle detector may relax this restriction by detecting
particles off the beam axis; the populations of higher magnetic substates may be estimated using expressions given by Litherland and Ferguson (Li 61a).

2-2.3. The Rose and Brink angular distribution formulae

The general form of the angular distribution of gamma rays as measured by a polarisation-insensitive detector in an axially symmetric system is

\[
W(\theta) = \sum_k a_k P_k(\cos \theta)
\]  \hspace{1cm} (2.20)

where \(P_k(\cos \theta)\) is a Legendre polynomial. The coefficients \(a_k\) have the form (Ro 67a)

\[
a_k = B_k(J_1) \frac{R_k(L'L'J_1J_2) + 2\delta R_k(L'JJ_1J_2) + \delta^2 R_k(LLJ_1J_2)}{(1 + \delta^2)}
\]  \hspace{1cm} (2.21).

\(L'\) and \(L\) represent the two multipolarities involved in the gamma ray transition \((L = L' + 1)\) between states of definite parity. The coefficients \(B_k(J_1)\) describe the degree of alignment of the initial state and are given by

\[
B_k(J_1) = \sum_{M_1 = 0}^{J_1} \frac{P(M_1)}{\sqrt{2}} \rho_k(J_1 M_1)
\]  \hspace{1cm} (2.22).

The terms \(\rho_k(J_1 M_1)\) are statistical tensors expressed as

\[
\rho_k(J_1 M_1) = (2 - \delta_{M_1,0}) \left((-1)^{J_1-M_1} 2^{J_1+1} \sqrt{2} (J_1 J_1 M_1 - M_1) |k0\right)\]  \hspace{1cm} (2.23)

and are zero for \(k\) odd. \(\delta_{M_1,0}\) is a Kronecker delta function.
From eqns. 2.22 and 2.23 it can be seen that $B_0(J_1) = 1$.

The geometrical factors, depending on the initial and final states and the multipolarities involved in the transition are contained in the second term of eqn. 2.21. The $R_k$ coefficients are given by

$$R_k(LL'J_1J_2) = (-)^{1+J_1-J_2+L'-L-k} \times (2J_1 + 1)(2L + 1)(2L' + 1) \times (LL'1-1|k0)W(J_1J_1LL';kJ_2)$$

(2.24)

where $W(J_1J_1LL';kJ_2)$ is a Racah coefficient. Rose and Brink (Ro 67a) give extensive tabulations of $\rho_k(J_1)$ and $R_k(LL'J_1J_2)$. The sum over $k$ in eqn. 2.20 will be restricted by the Clebsch-Gordan and Racah coefficients contained in eqn. 2.24, that is, $k \leq 2J_1$ and $|L'-L| \geq k \leq L+L'$. The factor $(1 + \delta^2)$ in the denominator of eqn. 2.21 normalizes the coefficient $a_0$ to 1.

2-2.4. Solid angle attenuation coefficients

The angular distribution given by eqn. 2.20 corresponds to an ideal point gamma-ray detector. In practice, the detector subtends a finite solid angle at the target spot and eqn. 2.20 should be integrated over the acceptance angles of the detector, taking into account the efficiency of the detector material for gamma-ray absorption. This will cause a smoothing out of any anisotropy in the distribution such that (Ro 53)

$$W(\theta) = \sum_k a_kQ_kP_k(\cos\theta)$$

(2.20')

where the $Q_k$ are the solid angle correction coefficients given by
\( Q_k = \frac{J_k}{J} \); for a cylindrical detector with its axis passing through the target spot.

\[
J_k = \int_{\text{detector}} \varepsilon(\beta) P_k(\cos\beta) \sin\beta \, d\beta \tag{2.25}
\]

where \( \varepsilon(\beta) \) is the efficiency for detection of a gamma ray entering the crystal at an angle \( \beta \) to the detector axis. Similar expressions can be evaluated for detectors with more complicated geometries. For the cases involving co-axial Ge(Li) detectors, the computer program written by Dr. F.C. Huang was used to calculate the \( Q_k \) coefficients.

2-2.5. Gamma ray linear polarisation

Photons are said to be linearly polarised if the electric vectors possess some preferential orientation in the plane normal to the direction of motion. In subsect. 2-2.3, the formulae of Rose and Brink were presented for the angular distribution of gamma rays as measured by a polarisation-insensitive detector. The direction-polarisation correlation is given by

\[
W(\theta, \gamma) = \left[ \sum_{k M_1 LL'} P(M_1) P_{k}(J_1 M_1) \left[ R_k(L'L'J_1 J_2) + 2 \delta R_k(L'LJ_1 J_2) \right] \right] \frac{\delta^2 R_k(LLJ_1 J_2)}{1 + \delta^2} \times \left[ P_k(\cos\theta) + (-)^{\pi'} \cos(2\gamma) K_k(LL') P_k^2(\cos\theta) \right] \tag{2.26}
\]

where the newly introduced quantities are; \( \gamma \), the angle between the reaction plane and the electric vector of the emitted gamma ray; \( K_k(LL') \), the coefficients listed by Fagg and Hanna (Fa 59); \( \pi' \), characterising the \( L' \) radiation as electric (\( \pi' = 0 \)) or
magnetic \((\pi' = 1)\) and \(P_k^2(\cos \theta)\), the associated Legendre polynomials.

In this work, the theoretical linear polarisation is defined as

\[
P_T(\theta) = \frac{W(\theta, \gamma = 0^\circ) - W(\theta, \gamma = 90^\circ)}{W(\theta, \gamma = 0^\circ) + W(\theta, \gamma = 90^\circ)}
\]

(2.27)

which limits \(P_T\) to values between -1 and +1, with \(P_T = 0\) corresponding to an unpolarised gamma ray. If only \(E2/M1\) multipole mixtures are considered, eqn. 2.27 becomes, for \(\theta = 90^\circ\),

\[
P_T = \pm \sqrt{\frac{-8\delta R_2(12J_1J_2) + 1.25 \sum_{M_1} P(M_1) \rho_2(J_1M_1) \delta^2 R_4(22J_1J_2) - 2 \sum_{M_1} P(M_1) \rho_1(J_1M_1)(1 + \delta^2) - \sum_{M_1} P(M_1) \rho_2(J_1M_1) [R_2(11J_1J_2)] + 2 \delta R_2(12J_1J_2) + \delta^2 R_2(22J_1J_2)] + 0.75 \sum_{M_1} P(M_1) \rho_4(J_1M_1) \delta^2 \times R_4(22J_1J_2)}{1 + \delta^2}
\]

(2.28)

with the positive sign indicating \(E2/M1\) mixing and the negative sign \(M2/E1\) mixing. The term

\[
\sum_{M_1} P(M_1) \rho_o(J_1M_1) = 1
\]

and the terms

\[
\sum_{M_1} P(M_1) \rho_2(J_1M_1) [R_2(11J_1J_2) + 2 \delta R_2(12J_1J_2) + \delta^2 R_2(22J_1J_2)]
\]

\[1 + \delta^2\]
in eqn. 2.28 are the \( a_k \) angular distribution coefficients defined by eqn. 2.20. Thus, eqn. 2.26 reduces to

\[
P_T = \pm \frac{8 \sum_{M_1} P(M_1) \rho_2(J_1M_1)p_2(J_1J_2)\delta}{3a_2 - \frac{1 + \delta^2}{1 + \delta^2} + 1.25a_4}
\]

\[
P_T = \pm \frac{3a_2}{2 - a_2} + \frac{1.25a_4}{2 - a_2 + 0.75a_4}
\]

(2.29)

For pure dipole radiation (\( \delta = 0 \)) eqn. 2.29 further simplifies to

\[
P_T = \pm \frac{3a_2}{2 - a_2} + \text{magnetic}
\]

(2.30)

and for pure quadrupole radiation (\( \delta = \infty \))

\[
P_T = \pm \frac{3a_2 + 1.25a_4}{2 - a_2 + 0.75a_4} + \text{electric}
\]

(2.30')

Eqn. 2.29 demonstrates that a measurement of the linear polarisation \( P \) of a gamma ray combined with a measurement of the angular distribution coefficients \( a_k \) may yield a value for the quadrupole/dipole mixing ratio \( \delta \). Chapter 4 contains a description of the use of a Compton polarimeter to measure the \( E2/M1 \) mixing ratios of the 2.73 and 2.21 MeV ground-state transitions in \( ^{27}\text{Al} \).

2-3 THE A.N.U. DOUBLE FOCUSSING MAGNETIC SPECTROMETER

The A.N.U. double focussing spectrometer has been described in detail by Elliott (El 68) and Carter (Ca 70a). What follows here is a brief outline of the major properties of the instrument.
2-3.1. General properties

The spectrometer is of the double focussing type with a non-homogeneous magnetic field produced by coils wound in a kidney shaped geometry around the pole pieces. It can deflect particles of magnetic rigidity up to that of 27 MeV alpha particles through 180° on a 61 cm radius. A summary of the main properties is given in Table 2.1. The spectrometer assembly rotates about a vertical axis on a circular track. An angular scale enables it to be set at a position reproducible to ±0.05° within the range -10° to 155°.

Vertical and horizontal slits define the entrance aperture and can be adjusted to subtend angles at the target over the range ±5° and ±3° respectively. Two types of detector mounts are used with the spectrometer. The first enables a single solid-state counter to be mounted on the focal plane. Provision is made through sliding seal mechanisms for five different slits with widths between 0.8 and 12.7 mm and six different absorbing foils to be placed in front of the detector. The second is designed to allow mounting of up to three 5 cm long position-sensitive detectors along the focal plane. Three different foil thicknesses may be placed in front of the detectors by operating flexible cables from outside the vacuum.

2-3.2. Magnetic field production and measurement

The magnet power supply is based on a 100 hp AC/DC converter set with a maximum output of 600 A. The current stabilization is obtained by amplifying the voltage difference between a standard water-cooled load resistor and mercury reference cell. This signal is then fed to a phase-sensitive rectifier controlling the exciting field of an amplidyne generator-amplifier, which in turn controls the
### TABLE 2.1

Summary of spectrometer properties

<table>
<thead>
<tr>
<th>Property</th>
<th>Measured value (El 68)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Solid angle (msr)</td>
<td>13.0 ± 0.3</td>
</tr>
<tr>
<td>Radial focal length (cm)</td>
<td>24.2 ± 0.3</td>
</tr>
<tr>
<td>Axial focal length (cm)</td>
<td>19.9 ± 0.3</td>
</tr>
<tr>
<td>Angle between focal plane and normal to mean ray (degrees)</td>
<td>48.0 ± 1.0</td>
</tr>
<tr>
<td>Magnification = image size/object size</td>
<td>0.81 ± 0.02</td>
</tr>
<tr>
<td>Dispersion = ( \frac{\text{fractional change in image position}}{\text{fractional change in momentum}} )</td>
<td>3.58 ± 0.07</td>
</tr>
<tr>
<td>Resolving power = ( \frac{\text{particle energy}}{\text{energy resolution}} )</td>
<td></td>
</tr>
<tr>
<td>At full solid angle</td>
<td>760 ± 40</td>
</tr>
<tr>
<td>At 1/8 solid angle</td>
<td>1110 ± 85</td>
</tr>
<tr>
<td>Hysteresis = reproducibility of position of group when field is taken</td>
<td></td>
</tr>
<tr>
<td>(a) to max field and back</td>
<td>1/2500</td>
</tr>
<tr>
<td>(b) to zero field and back</td>
<td>1/25000</td>
</tr>
</tbody>
</table>
exciting field of the main generator.

The magnetic field is measured at a point 18 cm from the entrance pole tip by a nmr probe. Because the double focussing field is non-uniform, compensating coils are mounted to cancel the transverse field gradient in the region of the nmr sample. The rf power is supplied to the probe by a Harvey-Wells oscillator tunable over the frequency range 7.8 to 48.0 MHz. This corresponds to proton energies of from 0.7 to 26.7 MeV.

2-3.3. Advantages and uses of the double focussing type spectrometer

The A.N.U. double focussing spectrometer has the following advantages over solid-state particle detectors:

(1) High resolving power ($E/\delta E$), with a maximum value of the order of 1100. For 10 MeV alpha particles, this means a resolution $\delta E$ of 10 keV. Factors such as kinematic broadening and target thickness effects will be additional to this in any actual experiment.

(2) Ability to detect particles at 0° for certain reactions, without damage to the detectors.

(3) Low background from X-rays and secondary electrons because of the remoteness of the detectors from the target.

(4) Lower ADC dead-time because only a specified region of the spectrum is detected.

(5) Ability to provide particle discrimination when used with a solid-state detector at the focal plane. However,
protons and alpha particles of the same energy cannot be distinguished without degrading the energy of the latter with a foil.
CHAPTER 3

GAMMA-RAY STUDIES IN $^{27}$Al

The first section of this chapter contains a brief review of the attempts to describe $^{27}$Al in terms of nuclear models. This is followed by a description of an experiment to measure lifetimes of excited states of $^{27}$Al by the DSAM. The final section contains an account of an experiment to determine the E2/M1 mixing ratios of the 2.21 and 2.73 MeV ground-state transitions by the use of a gamma-ray polarimeter.

3-1 APPLICATION OF NUCLEAR MODELS TO $^{27}$Al

The nucleus $^{27}$Al, whose energy level scheme is shown in fig. 3.1, lies in the transition region of the 2s-1d shell where the nuclear shapes are changing from prolate to oblate. It is a particularly interesting nucleus because, to date, it has resisted complete description by the collective model which has proved successful for neighbouring nuclei such as $^{25}$Mg.

3-1.1. Strong coupling model

The Nilsson model requires that the spectra of nuclei having the same odd-nucleon number be similar. This means that $^{27}$Al should be compared with $^{25}$Mg, $^{25}$Al and $^{27}$Si. The ground-state quadrupole moment of $^{27}$Al has a value 0.15 barns (Fu 69), which corresponds to a deformation parameter $\delta \sim 0.23$, and is only slightly less than the value $\sim 0.22$ barns for $^{25}$Mg (Lu 62). In view of the success of the Nilsson model in describing the mirror nuclei $^{25}$Mg and $^{25}$Al, it is reasonable to expect
Fig. 3.1. Energy levels of $^{27}$Al that are studied in this work. The excitation energies, spins, parities and gamma decay schemes are taken from Smulders et al. (Sm 68), Häusser et al. (Ha 68), Carter (Ca 70a) and Endt and Van der Leun (En 73).
this model to be capable of accounting for the properties of $^{27}$Al (Bh 62). The $J^\pi = 5/2^+$ ground state would, on the basis of the Nilsson model, be the head of a $K^\pi = 5/2^+$ rotational band based on Nilsson orbit 5 containing the $J^\pi = 7/2^+$, $9/2^+$ and $11/2^+$ levels at excitation energies $E_x = 2.21$, $3.00$ and $4.51$ MeV respectively. A second rotational band based on the $0.84$ MeV $1/2^+$ first excited state would be a $K^\pi = 1/2^+$ rotational band based on Nilsson orbit 9 and contain the $1.01$ MeV $3/2^+$ and $2.73$ MeV $5/2^+$ levels. Ophel and Lawergren (Op 64) have measured the E2/M1 mixing ratios for the $2.73$ MeV to $1.01$ MeV and $1.01$ to $0.84$ MeV transitions in the $K = 1/2$ band and found them consistent with values calculated using the Nilsson model.

However, many of the measured properties of the low-lying levels are inconsistent with the strong coupling picture. For instance, Ophel and Lawergren show that while the ground-state transition from the $2.21$ MeV level has an E2 width in accordance with the model, the M1 width is 5 times smaller than the predicted value. The inelastic proton scattering data of Crawley and Garvey (Cr 68) are also inconsistent with rotational behaviour in $^{27}$Al; they obtained cross sections for excitation of the $0.84$ and $1.01$ MeV levels which were higher than expected on the basis of the rotational model, while that for the $2.21$ MeV level was lower.

Smulders et al. (Sm 68) show that the following transition strengths in $^{27}$Al are inconsistent with rotational behaviour in this nucleus: (i) the K selection rule forbids M1 transitions between the $K = 1/2$ and $5/2$ bands ($\Delta K = 2$) yet the M1 strengths to the ground state from the $J^\pi = 3/2^+$ and $5/2^+$ members of the $K = 1/2$ band are a factor 10 greater than the same strengths in $^{25}$Mg where the collective model gives quite a good description of the same bands; (ii) if the levels
were purely rotational, and the E2 enhancement of the 9/2 to 5/2 transition is 7.4 W.u., then the E2 enhancement of the in-band 9/2 to 7/2, K = 5/2 transition should be 22 W.u. instead of the measured ≤ 0.6 W.u.; (iii) the cross-band ground-state transitions from the J^π = 1/2^+ and 3/2^+ members of the K = 1/2 band have strong E2 enhancements.

Häusser et al. (Ha 68) have studied the 4.51 MeV level by triple angular correlations; they confirm the assignment of J^π = 11/2^+ suggested by Smulders et al. (Sm 68) and show that the level exhibits strong E2 transitions to the 3.00 MeV 9/2^+ state and the 2.21 MeV 7/2^+ state, in accordance with membership of the ground-state rotational band. Häusser et al. suggest that the electromagnetic properties of the ground-state rotational band require some admixture from the K = 1/2 band based on the 0.84 MeV level. Such Coriolis mixing, however, should only occur to second order as ΔK = 2 (see subsect. 1-2.1).

Recently, Carter et al. (Ca 71a) have reported the results of a search undertaken in this laboratory for high-spin states in ^{27}Al. Among other results, they found that the 5.50 MeV state has a spin of 5/2, 7/2 or 11/2 with a J^π = 11/2^+ assignment being most likely, and that the 7.44 MeV state has a spin of 5/2, 7/2, 9/2, 11/2 or 13/2 with a J^π = 13/2^+ assignment being most likely. They consider the possibility that the two states might be the 11/2^+ and 13/2^+ members of the ground-state rotational band, and emphasize the need for further information on the properties of the states, especially their lifetimes.

3-1.2. Weak coupling model

In contrast to the strong coupling rotational model of ^{27}Al, Thankappan (Th 66) has described ^{27}Al in terms of a model where the levels are assumed to be given by the coupling of a proton hole in the
$1d_{5/2}$ subshell to the $0^+$ ground state and 1.78 MeV, $2^+$ first excited state of $^{28}\text{Si}$. Fig. 3.2 shows schematically the multiplet of levels expected to result from such a model.

Generally good agreement was obtained by Thankappan between calculated and experimental excitation energies below 3.00 MeV. However, two serious discrepancies appeared: (i) the $3/2^+$ 2.98 MeV level was unaccounted for by the model; (ii) the $5/2^+$ level was predicted to be at 1.9 MeV instead of at the known energy of 2.73 MeV.

The situation regarding electromagnetic transition rates was more serious; although agreement between theory and experiment was generally good for the $E2$ transitions and branching ratios, poor agreement was obtained for $M1$ transitions. For instance, the strength of the $M1$ 1.01 MeV ground-state transition was about six times the experimental value.

Evers et al. (Ev 67) performed similar calculations for $^{27}\text{Al}$ and obtained good agreement between calculated and experimental values of $E2$ and $M1$ transition strengths and $E2/M1$ mixing ratios. In particular, improved agreement was obtained for the $M1$ transitions of the two $3/2^+$ states to the ground state by allowing a small single particle admixture for the 1.01 MeV level. The two marked discrepancies concerned: (i) the $M1$ transition strength between the $3/2^+$ and $1/2^+$ members of the multiplet; (ii) the $E2/M1$ mixing ratio of the 2.73 MeV to ground state transition. This was reported by Ophel and Lawergren (Op 64) to be $\delta = -0.04 \pm 0.06$ compared to the value $\delta = -0.821$ calculated by Evers et al., who suggested an error in the experimental value. It is not clear whether the sign convention used by Evers et al. is in accordance with that of Rose and Brink (Ro 67a).
Fig. 3.2. Schematic energy level scheme for a $1d_{5/2}$ particle coupled to the ground state and first excited state of $^{28}\text{Si}$ (from Thankappan, Th 66).
De Voigt et al. (Vo 72) have performed shell model calculations for nuclei with $A = 27, 28$ and $29$. They assumed a truncated $(1d_{5/2})$
$(2s_{1/2})(1d_{3/2})$ configuration space with a maximum of four holes in the $1d_{5/2}$ subshell. Electric quadrupole moments and transition strengths were calculated by performing least squares fits to 74 experimental data with one adjustable parameter, the isoscalar effective charge, yielding the values $e_p = 1.6e$ for the proton and $e_n = 0.6e$ for the neutron. The results for magnetic dipole transitions were obtained from separate fits to 17 experimental data in $A = 27$ nuclei with two adjustable effective single particle matrix elements.

In general, the authors claimed good agreement between the results of their calculations for $^{27}$Al, those predicted by the previously mentioned models and experimental data. In particular, they claimed somewhat better reproducibility of the experimental properties of the 1.01 MeV $3/2^+$ level which were poorly reproduced by the rotational model and weak coupling model of Thankappan (Th 66). However, the E2/M1 mixing ratio of the 2.73 MeV ground-state transition calculated by de Voigt et al. is still in serious disagreement with the result of Ophel and Lawergren (Op 64). Also, the properties of the 4.51 MeV level and those of levels above 4.8 MeV excitation were poorly reproduced by the calculations.

Similar calculations have been carried out by Wildenthal and McGrory and published in a related paper (Wi 72). The results for level energies, single nucleon transfer properties and electromagnetic decays are in reasonable accord with experimental data and indicate that the shell model can provide a fairly comprehensive account of $^{27}$Al and neighbouring nuclei. It is interesting to note that little
indication of well-defined rotational bands arises from the calculated B(E2) values.

3-1.4 Other approaches to the $^{27}$Al problem

It can be seen from subsects. 3-1.1 and 3-1.2 that neither the strong nor weak coupling pictures of $^{27}$Al is completely successful in accounting for its properties (although the shell model shows some promise). Röpke et al. (Ro 70) have adopted an intermediate approach to the problem by proposing the existence of two low-lying rotational bands in $^{27}$Al which are built on top of a γ-vibrational state (i.e., a state where the vibration involves a loss of the nuclear symmetry axis) in $^{26}$Mg. Strong rotation-vibration coupling gives rise to the simultaneous appearance of both strong and weak coupling features. The model was based on a comparison with the nearby nuclei $^{25}$Mg and $^{25}$Al. Calculated excitation energies were fairly well reproduced by experimental values, as were E2 transition strengths apart from that of the 1.01 MeV ground-state transition. However, difficulties arose in the interpretation of calculated spectroscopic factors.

Dehnhard (De 72) has suggested that a possible solution to the $^{27}$Al puzzle lies in the choice of shape for the $^{27}$Al nucleus, which lies in the transition region between prolate ($^{25}$Mg) and oblate ($^{28}$Si) shapes. The strong coupling calculations of Bhatt (Bh 62) and Malik and Scholz (Ma 67) suggest a prolate shape gives the best fit to the observed data; however, Dehnhard points out that much of the experimental data can be understood in terms of a Coriolis band mixing model if an oblate shape is assumed for $^{27}$Al. This suggestion is supported by the Hartree-Fock calculations of Ripka (Ri 68). Dehnhard has calculated E2 transition rates and spectroscopic factors on this basis and has achieved generally good agreement with the experimental results. In particular, he was able
to explain the strong E2 transitions from the 1/2\(^+\) and 3/2\(^+\) states at 0.84 and 1.01 MeV to the ground state. However, his calculations somewhat overestimate the E2 strengths of the 2.73 MeV, 5/2\(^+\) to 5/2\(^+\) ground-state transition and the 9/2\(^+\) to 7/2\(^+\), 3.00 to 2.21 MeV transition.

3-1.5. Motivation for the present work

It can be appreciated from the previous subsections that extensive experimental data are required before meaningful comparisons can be made with present and future model predictions. Also, discrepancies between the experimental results of different authors for nuclear properties need to be resolved. In the following sections experiments are described where lifetimes of excited states and the E2/M1 mixing ratios of the 2.21 and 2.73 MeV ground-state transitions in \(^{27}\)Al are measured and the results compared with those previously reported. The lifetime measurements enabled those of other authors to be confirmed while the experiments to measure the mixing ratio of the 2.73 MeV ground-state transition resolved a serious discrepancy between previously reported values.

3-2 LIFETIMES OF EXCITED STATES IN \(^{27}\)Al VIA THE REACTION \(^{12}\)C(\(^{16}\)O, p)\(^{27}\)Al

3-2.1. Introduction

As mentioned in the previous section, Carter et al. (Ca 71a) have reported the results of a search for high-spin states in \(^{27}\)Al using the reactions \(^{12}\)C(\(^{16}\)O, p)\(^{27}\)Al and \(^{24}\)Mg(\(\alpha\), p)\(^{27}\)Al. They suggest that the 5.50 and 7.44 MeV levels of \(^{27}\)Al may be the 11/2\(^+\) and 13/2\(^+\) members of the ground-state rotational band but emphasize the need for further information on the properties of these states, particularly their lifetimes, before further interpretation in terms of nuclear models can
be made.

The work described in this section was undertaken to measure the lifetimes of the 5.50 and 7.44 MeV states using the reaction $^{12}_{\text{C}}(^{16}_{\text{O}},p)^{27}_{\text{Al}}$; it was also hoped to obtain information on the lifetimes of other states in $^{27}_{\text{Al}}$. This reaction should preferentially populate high-spin states because of the large angular momentum transfer. In the past, various methods have been used for lifetime measurements in this nucleus; see, for example, Endt and Van der Leun (En 67). Doppler shift attenuation method (DSAM) experiments have been confined to the study of de-excitation gamma rays in singles mode following inelastic scattering and capture reactions such as $^{27}_{\text{Al}}(p,p')^{27}_{\text{Al}}$ (Smulders et al., Sm 68) and $^{26}_{\text{Mg}}(p,\gamma)^{27}_{\text{Al}}$ (Röpke and Lam, Ro 68). Such reactions have the advantage of experimental simplicity and high count rate but give rise to the problems of multiple feeding and an increase in Doppler broadening of the gamma rays as a result of the large spread in recoil direction of the excited ion, as discussed in sect. 2.1. However, it can be shown (Sm 68) that the centroid of the lineshape of the shifted gamma ray is not affected by the latter, only the lineshape itself. The present experiment uses a particle-gamma ray coincidence technique to reduce errors arising from multiple feeding of levels and Doppler broadening. The large initial recoil speeds associated with the reaction used also result in reduced uncertainties in the calculated heavy-ion stopping powers.

3-2.2. Experimental procedure

Two runs were made at bombarding energies of 25.5 and 26.0 MeV with beams of $^{16}_{\text{O}}$ from the A.N.U. EN tandem accelerator. Gamma rays were detected in a 40 cm$^3$ Ge(Li) detector set at 0° to the beam direction
at distances of 5.0 and 2.2 cm respectively from the target. Fully Doppler-shifted gamma rays were observed from ions recoiling from a thin ($\approx 10 \mu g/cm^2$) $^{12}$C foil target into vacuum, and attenuated shifts were observed from targets consisting of approximately 10 $\mu g/cm^2$ $^{12}$C on thick tantalum and thick nickel backings for the 25.5 and 26.0 MeV runs respectively. Uncertainties in the target thicknesses were estimated to be 2 $\mu g/cm^2$. Carbon build-up was minimised by changing the target position slightly when the target type was changed every few hours. Beam currents were limited to 200 nA in order to keep the neutron flux at the Ge(Li) detector within acceptable limits. The sources of the neutrons appeared to be associated with both the target and the collimators and were probably caused, in part, by carbon build-up. The data collection rate was seriously impaired by the need to limit the beam current; the poor statistics resulting from this and the low reaction yield proved the major limitation of the experiment. Protons were observed in a 150 mm$^2$, 1000 $\mu$m thick annular surface barrier detector set about 2 cm from the target at 180° to the beam direction. Fig. 3.3 shows the coincidence proton spectrum obtained at 26.0 MeV bombarding energy.

Proton-gamma ray coincidences were collected on magnetic disc in 256 x 4096 channels by address recording in an IBM 1800 computer. Fig. 3.4 shows the block diagram of the electronics used; a standard fast-slow coincidence system was employed with cross-over timing with Ortec 420 single-channel analysers (SCAs). The FWHM time resolution was about 60 ns. A gamma ray calibration spectrum using the 1173 and 1332 keV lines from a $^{60}$Co source was simultaneously collected using a parallel circuit for gamma-gamma coincidences between the Ge(Li) detector and a 12.7 x 10.2 cm NaI(Tl) crystal. The logic pulses from the time-to-amplitude converter (TAC) in this circuit were mixed in with the proton pulses being collected in the computer; the peak position was
Fig. 3.3. Coincidence proton spectrum obtained from the reaction $^{12}\text{C}(^{16}\text{O},p\gamma)^{27}\text{Al}$, showing the digital windows (shaded areas) used for the DSAM lifetime measurements.
Fig. 3.4. Block diagram of the electronic system used in the DSAM lifetime measurements.
adjusted to fall in a region of the proton spectrum where there were no
groups of interest. The $^{60}$Co calibration spectrum was then collected in
the computer in coincidence with the pulses in this peak.

During subsequent play-back of the data, digital windows were
set on the proton spectrum in order to examine gamma rays in coincidence
with proton groups of interest, as indicated in fig. 3.3. No useful
gamma-ray spectra were obtained from peaks other than those indicated in
the diagram.

3-2.3. Data analysis

Fig. 3.5 shows the full and attenuated photopeak shifts of
the gamma rays of interest from $^{27}$Al. The rather poor statistics,
combined with the considerable Doppler broadening, gives rise to the pro­
blem of deciding where a peak ends and the background begins. Consequently,
centroid determinations were made after subtracting linear backgrounds de­
termined from the regions of the spectra flanking the peaks which were
outside the zero to full shift range. It should be noted that, despite
the rather poor statistics, meaningful centroid determinations can still
be made.

Because of the large momentum transfer associated with the
reaction used, high recoil speeds result for the $^{27}$Al ions ($v/c = 0.039$).
As discussed in sect. 2-1, at such high speeds the contribution to the
heavy-ion stopping power by nuclear effects can be neglected. For recoil
energies above 13 MeV, the Lindhard, Scharff and Schiott (Li 63) theory
for stopping by electronic effects, with its simple velocity proportionality,
predicts stopping powers significantly higher than those interpolated
from experimental data (Northcliffe and Schilling, No 70). The results
of Northcliffe and Schilling were fitted by the expression
Fig. 3.5. Full and attenuated photopeak Doppler shifts from gamma rays detected in coincidence with protons corresponding to the digital windows shown in fig. 3.3.
The values obtained for $^{27}$Al ions (in units of MeV/mg/cm$^2$) are

\[
-k = 6.74, \quad k_3 = 0.12 \text{ for stopping in carbon}
\]

\[
-k = 1.22, \quad k_3 = 0.0099 \text{ for stopping in tantalum}
\]

\[
-k = 2.63, \quad k_3 = 0.033 \text{ for stopping in nickel.}
\]

These values are subject to an uncertainty estimated to be 10%.

Using the above relationship for the stopping power, values of $F$, the ratio of the attenuated to full shift, were calculated as a function of the mean lifetime $\tau$ for each level of interest in $^{27}$Al, taking into account second order terms in $\nu/c$ and reduction of the full shift by Doppler broadening arising from the finite detector solid angles. The computer program "DSAM" written by Dr. C.J. Piluso for the IBM 1800 computer was used for these calculations. This program has provision for generating Doppler lineshapes as a function of $\tau$, but the generally poor statistics of the data precluded significant fits by these lineshapes which served merely as useful checks on the values of $F$ obtained from centroid determinations.

3-2.4. Results and discussion

Values of $F$ and $\tau$ are set out in Table 3.1 and compared with previous work, including results received during the course of the present work from de Voigt et al. (Vo 71), who studied levels up to 7.81 MeV excitation via the capture reaction $^{23}$Na($\alpha$,y)$^{27}$Al. The poor statistics precluded measurements on levels above the 5.50 MeV state. Both measurements on the 3.00 MeV level and one on the 1.01 MeV level were made using gamma rays in cascade from higher states; appropriate allowances were
made for the uncertainties in the lifetimes of the initial states.

It can be seen that the results are in satisfactory agreement with others obtained by the DSAM. The errors assigned are derived from:
(i) errors in the centroid determination; (ii) an additional uncertainty estimated to be 10% to take into account uncertainties in the stopping powers; (iii) uncertainties amounting to up to 2 fs arising from uncertainties in the target density and thickness (these are lower here than with previous measurements because of the high initial recoil velocity).

Assuming a lifetime $\tau \leq 7$ fs for the 5.50 MeV level, and using the results of Carter et al. (Ca 71a) for the branching ratio and multipole mixing ratios for the transition to the 3.00 MeV $9/2^+$ level, the following spin and parity combinations for the 5.50 MeV level may be rejected with a high degree of confidence:

$$
5/2^+ (|M|^2 (E2) \geq 198 \text{ W.u.}); \\
5/2^- (|M|^2 (E2) \geq 5.8 \times 10^3 \text{ W.u.}); \\
7/2^- (|M|^2 (M2) \geq 58 \text{ W.u.}); \text{ and} \\
11/2^- (|M|^2 (M2) \geq 130 \text{ W.u.}),
$$

where $|M|^2$ is the strength of the transition in the single-particle units defined in subsect. 1-2.1. In addition, an $11/2^-$ assignment would imply $M2$ and $E3$ enhancements of at least 376 and $2.7 \times 10^5$ W.u., respectively for the transition to the 2.21 MeV $(7/2^+)$ state. Hence, we conclude that $J^\pi(5.50) = 7/2^+$ or $11/2^+$.

De Voigt et al. (Vo 71) give an upper limit of 10 fs for the lifetime of the 7.44 MeV level. Again, assuming the branching ratio and multipole mixing ratios given by Carter et al. (Ca 71a) for the transition to the 3.003 MeV $9/2^+$ level, the following spin and parity
TABLE 3.1
LIFETIMES OF EXCITED STATES IN $^{27}$A1 BY DSAM AND OTHER TECHNIQUES

<table>
<thead>
<tr>
<th>Level (keV)</th>
<th>Present work</th>
<th>Previous work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Bombarding Energy (MeV)</td>
<td>Transition (keV)</td>
</tr>
<tr>
<td>1014</td>
<td>25.5</td>
<td>1014-0$^a$</td>
</tr>
<tr>
<td></td>
<td>26.0</td>
<td>1014-0</td>
</tr>
<tr>
<td>2211</td>
<td>25.5</td>
<td>2211-0</td>
</tr>
<tr>
<td>2734</td>
<td>25.5</td>
<td>2734-1014</td>
</tr>
<tr>
<td>3003</td>
<td>26.0</td>
<td>3003-0$^b$</td>
</tr>
<tr>
<td></td>
<td>26.0</td>
<td>3003-0$^c$</td>
</tr>
<tr>
<td>4509</td>
<td>26.0</td>
<td>4509-3003</td>
</tr>
<tr>
<td>5499</td>
<td>25.5</td>
<td>5499-3003</td>
</tr>
</tbody>
</table>

a) Fed from the 2.73 MeV level.
b) Fed from the 7.44 MeV level ($\tau \leq 10$ fs, de Voigt et al., Vo 71).
c) Fed from the 5.50 MeV level.
d) Smulders et al. (Sm 68).
e) Average of the values obtained by Booth and Wright (Bo 62), Evers et al. (Ev 67), Metzger et al. (Me 60) and Hough and Mouton (Ho 66).
f) Average of the values obtained by Schaller and Miller (Sc 64), Vanhuysse and Vanpraet (Va 63), Booth et al. (Bo 64) and Metzger et al. (Me 60).
g) de Voigt et al. (Vo 71).
h) Schaller and Miller (Sc 64).
i) Robinson et al. (Ro 68a).
j) Röpke and Lam (Ro 68).
k) McDonald et al. (Mc 71).
assignments for the 7.44 MeV level appear very unlikely:

\[ \frac{5}{2}^+ \ (|M|^2 (M3) \geq 3.9 \times 10^4 \ W.u.); \]
\[ \frac{5}{2}^- \ (|M|^2 (M2) \geq 196 \ W.u. \text{ and } |M|^2 (E3) \geq 1.3 \times 10^3 \ W.u.); \]
\[ \frac{7}{2}^- \ (|M|^2 (M2) \geq 21 \ W.u.); \]
\[ \frac{11}{2}^- \ (|M|^2 (M2) \geq 21 \ W.u.); \text{ and} \]
\[ \frac{13}{2}^- \ (|M|^2 (M2) \geq 220 \ W.u.). \]

These results suggest that \( J^\pi(7.44) \) may be restricted to \( \frac{7}{2}^+, \frac{9}{2}^+, \frac{11}{2}^+, \) or \( \frac{13}{2}^+ \).

Arguments given by Carter et al. indicate that, of the possible spin and parity assignments for the 5.50 and 7.44 MeV levels, the most probable values are \( \frac{11}{2}^+ \) and \( \frac{13}{2}^+ \) respectively. Enhancements for E2 transitions calculated assuming these assignments are listed in Table 3.2, together with some other relevant E2 enhancements. It is noteworthy that the 7.44 MeV level has a substantial branch to the 5.50 MeV level (Ca 71a), but as the mixing ratio for the transition is not known, the E2 enhancements cannot be estimated. These results support the suggestion that states at 0 \( (\frac{5}{2}^+) \), 2.21 \( (\frac{7}{2}^+) \), 3.00 \( (\frac{9}{2}^+) \), 5.50 \( (\frac{11}{2}^+) \) and 7.44 \( (\frac{13}{2}^+) \) MeV may form a rotational band. However, the weak 3.00 to 2.21 MeV E2 transition remains anomalous for any simple rotational model, and it is stressed that the spin-parity assignments are not unique.

3-3 POLARIZATION STUDIES OF GROUND-STATE RADIATION FROM THE 2.21 AND 2.73 MEV LEVELS OF \( ^{27}\text{Al} \)

3-3.1. Introduction

Among the properties of states below 3.00 MeV excitation, only the E2/M1 mixing ratio of the 2.73 MeV to ground-state transition remains seriously uncertain. Ophel and Lawergren (Op 64) report a value of
### TABLE 3.2

E2 ENHANCEMENTS (IN W.u.) FOR A POSSIBLE ROTATIONAL BAND IN $^{27}$Al

<table>
<thead>
<tr>
<th>Initial State</th>
<th>2211 (7/2$^+$)</th>
<th>3003 (9/2$^+$)</th>
<th>5499 (11/2$^+$)</th>
<th>7441 (13/2$^+$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 (5/2$^+$)</td>
<td>7.9±1.2$^a$</td>
<td>7.4±0.7$^a$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2211 (7/2$^+$)</td>
<td></td>
<td>≤ 0.6$^a$</td>
<td>≥ 13</td>
<td></td>
</tr>
<tr>
<td>3003 (9/2$^+$)</td>
<td></td>
<td></td>
<td>≥ 4.4</td>
<td>≥ 7.5</td>
</tr>
</tbody>
</table>

$^a$ Smulders et al. (Sm 68).

(The 11/2$^+$ and 13/2$^+$ assignments for the 5499 and 7441 keV levels are probable only).

All energies in keV.
-0.04±0.06 from a triple correlation measurement of the 2.73→1.01→0 MeV cascade, combined with a measurement of the linear polarization of the 2.73 MeV transition using the reaction $^{27}\text{Al}(p,p')^{27}\text{Al}$. The latter measurement was made using a sum-coincidence Compton polarimeter (see subsect. 3-3.2) consisting of two NaI(Tl) crystals. Van der Leun et al. (Va 67) give a value of -0.09±0.03 from angular correlation studies of the reaction $^{26}\text{Mg}(p,\gamma)^{27}\text{Al}$, in agreement with the result of Ophel and Lawergren (Op. 64). However, Lumpkin et al. (Lu 71) have recently reported a value of +0.22±0.09 obtained from linear polarization studies of the reaction $^{27}\text{Al}(p,p')^{27}\text{Al}$ with a Compton Polarimeter comprising two Ge(Li) crystals. This value is in serious disagreement with the previous measurements. All the values have signs in accordance with the convention of Rose and Brink (Ro 67a). The above discrepancy is important with respect to the application of the various models discussed in sect. 3-1 to the nucleus $^{27}\text{Al}$. The mixing ratios for the ground-state transitions from the 2.21 and 2.73 MeV levels have been measured in an attempt to resolve the discrepancy for the 2.73 MeV transition and thus make comparisons with present and future model predictions more meaningful. The experiments are described in the following subsections.

3-3.2. The Compton polarimeter

The operation of this type of instrument is based on the fact that the Compton scattering cross section for gamma rays is a maximum in the plane normal to the electric vector. The polarimeter used in the experiments discussed in this section is of the type described by Twin et al. (Tw 70). It consists of a 60 cm$^3$ Ge(Li) crystal which serves as a Compton scatterer and two 7.62 × 7.62 cm NaI(Tl) crystals mounted to detect gamma rays Compton scattered out of the Ge(Li) crystals. The three crystals are arranged so that the mean angle of Compton scattering
Fig. 3.6. Schematic arrangement of the detectors used in the three-crystal Compton polarimeter. The angle 75° is the mean Compton scattering angle, γ is the angle between the electric vector and the reaction plane while θ is the angle between the beam and gamma ray emission directions.
from the Ge(Li) to the NaI(Tl) crystals is about 75°. This is shown in fig. 3.6 which gives the schematic arrangement of the detectors. One NaI(Tl) crystal is mounted to detect gamma rays Compton scattered in a plane normal to the reaction plane while the other crystal is mounted to detect gamma rays scattered in a plane parallel to the reaction plane. Both NaI(Tl) crystals are shielded from direct gamma radiation from the target by 12 cm thick lead shielding. Fig. 3.7 shows a photograph of the arrangement of the equipment. The whole polarimeter assembly is mounted on a rotatable table so that the instrumental asymmetry can be measured at 0° where the gamma rays are unpolarized, as can be seen from the form of eqn. 2.26.

The definition of polarization P given in sect. 2-2 is

\[ P(\Theta) = \frac{W(\Theta, \gamma = 0^\circ) - W(\Theta, \gamma = 90^\circ)}{W(\Theta, \gamma = 0^\circ) + W(\Theta, \gamma = 90^\circ)} \]

where \( \Theta \) is the angle between the direction of gamma ray emission and the beam direction and \( \gamma \) is the angle between the reaction plane and the electric vector of the emitted gamma ray. From this definition it can be seen that the relative numbers of gamma rays scattered from the Ge(Li) crystal to the two NaI(Tl) crystals are related to the polarization of the incident gamma radiation. Defining the measured polarimeter asymmetry as

\[ A = \frac{N(0^\circ) - N(90^\circ)}{N(0^\circ) + N(90^\circ)} \]

where \( N(0^\circ) \) and \( N(90^\circ) \) are the numbers of counts in the full energy peak in the spectra for gamma rays scattered parallel and perpendicular to the reaction plane respectively, enables one to define a sensitivity factor \( R \) as

\[ R = -A/P. \]
Fig. 3.7. Photograph of the arrangement of the detectors and lead shielding of the three-crystal Compton polarimeter.
Subsect. 3-3.4 describes measurements to determine R as a function of energy using gamma rays of known polarization.

Taras (Ta 68) has used a computer program to do a systematic study of the factors affecting the sensitivity R for Compton polarimeters. A figure of merit is the minimization of the ratio $\Delta P/P$, where $\Delta P$ is the uncertainty in a measurement of P. This can be achieved by maximizing R and obtaining the best possible counting statistics. However, these two requirements are largely incompatible and a compromise has to be reached. For example, R is a maximum for the point detector case which corresponds to a zero count rate. Taras concludes that $\Delta P/P$ is a minimum for the conditions that apply for maximum count rate, provided the source to scatterer distance is set at about 15 cm, the mean scattering angle is set as large as allowed by the experimental arrangement and the scatterer dimensions are about 5 cm. That is to say, the absorber crystals should be as large as possible and set close to the scatterer. In the detector arrangement used here, the distance between the Ge(Li) and NaI(T1) detector surfaces was set about 1 cm and the mean scattering angle was about 75°. The source to scatterer distance was 16.5 cm.

3-3.3. The polarimeter electronic system.

Pulses from the two Ge(Li)/NaI(T1) detector combinations were gated and summed by the electronic setup shown in block diagram form in fig. 3.8. This consisted of two parallel fast-slow coincidence circuits with extrapolated zero strobe timing for the Ge(Li) detector and cross-over timing with SCAs for the NaI(T1) detectors. The lower level discriminators of the NaI(T1) detector SCAs were set at levels corresponding to energies of about 300 keV being deposited in the NaI(T1) crystals in order that gamma rays below 2 MeV could be studied with the polarimeter.
Fig. 3.8. Block diagram of the polarimeter electronic system.
This meant that pair production events were also included in the sum-coincidence spectra. No pulse height window was set for the Ge(Li) detector apart from a lower level set above the preamplifier noise.

The Ge(Li) and NaI(Tl) detector linear amplifiers were adjusted to give the same pulse height versus gamma ray energy calibration by matching the position of the 1.28 MeV photopeak in the singles spectra from the three detectors using a $^{22}\text{Na}$ source.

3-3.4. Calibration of the polarimeter

The polarimeter was calibrated using pure E2 gamma rays whose polarizations can be readily calculated in terms of the $a_2$ and $a_4$ angular distribution coefficients, as discussed in sect. 2-2. These gamma rays were obtained from the decay of the first excited states of even-even nuclei populated by inelastic proton scattering, namely $^{28}\text{Si}(E_\gamma = 1.78 \text{ MeV})$, $^{32}\text{S}(2.23 \text{ MeV})$ and $^{12}\text{C}(4.43 \text{ MeV})$. Beams of protons were obtained from the A.N.U. EN tandem accelerator. Bombarding energies were chosen from brief excitation functions to coincide with resonances at 3.10, 4.10 and 5.37 MeV, respectively.

Sum coincidence spectra for the two Ge(Li)/NaI(Tl) detector combinations were collected simultaneously in $2 \times 1024$ channels in an IBM 1800 computer. Relevant regions of the spectra obtained from the above reaction are shown in fig. 3.9. The polarimeter resolution can be seen to be almost entirely determined by that of a NaI(Tl) detector for the Compton scattered gamma rays, i.e., about 40 keV for a 3 MeV incident gamma ray. The polarimeter was set with the Ge(Li) detector axis alternatingly at angles of 90° and 0° to the proton beam direction; two runs were made at each angle. Angular distributions were also measured for the calibration gamma rays using the Ge(Li) detector alone. Care was taken to shield the
Fig. 3.9. Relevant regions of the sum-coincidence spectra for the polarimeter calibration with pure E2 gamma rays scattered in directions: (a) parallel and (b) perpendicular to the reaction plane with the polarimeter set at 90° and 0° to the proton beam direction. (I) denotes the first escape peaks.
detector from gamma radiation from the nearest collimator, about 50 cm upstream from the target. The instrumental anisotropy was determined using the isotropic 3.56 MeV ground-state gamma rays from the first $J^\pi = 0^+$ state of $^6$Li populated via the reaction $^9$Be(p,$\alpha)^6$Li at a bombarding energy of 4.5 MeV. Reaction yields were monitored by a 40 cm$^3$ Ge(Li) detector set at 90° to the beam direction.

The expression $W(\theta) = 1 + a_2 P_2(\cos\theta) + a_4 P_4(\cos\theta)$ was least squares fitted to the data. Table 3.3 shows the angular distribution coefficients, corrected for finite detector solid angle, for the 1.78, 2.23 and 4.43 MeV calibration gamma rays. The resulting polarizations $P$ measured at 90° to the beam direction are then given by the formula (see sect. 2-2)

$$P(E2) = \frac{3a_2 + (5/4)a_4}{2 - a_2 + (3/4)a_4}.$$

The asymmetry $A$, after correction for instrumental effects, is then given in terms of $N(0^\circ)$ and $N(90^\circ)$, the numbers of counts in the full energy peaks of the sum-coincidence spectra for gamma rays scattered parallel and perpendicular to the reaction plane, respectively.

Fig. 3.10 shows the polarimeter sensitivities thus determined for the 1.78, 2.23 and 4.43 MeV gamma rays. The areas of the full energy peaks were determined using a simple exponential background subtraction. The solid curves were calculated for mean Compton scattering angles of 70°, 75° and 80° using a computer program adapted from that described by Taras (Ta 68) for the particular detector setup used here. The calculation uses the Klein-Nishina formula for Compton scattering and divides the detectors up into small radial and axial elements to determine the probability of absorption by a NaI(Tl) detector of a gamma ray scattered
### TABLE 3.3

ANGULAR DISTRIBUTION COEFFICIENTS, CALCULATED POLARIZATIONS, MEASURED ASYMMETRIES AND RESULTING POLARIMETER EFFICIENCIES FOR PURE E2 GAMMA RAYS

<table>
<thead>
<tr>
<th>Target</th>
<th>$E_p$ (MeV)</th>
<th>$E_\gamma$ (MeV)</th>
<th>Angular distribution coefficients</th>
<th>Polarization $P$(E2)</th>
<th>Asymmetry $A^*$</th>
<th>Polarimeter sensitivity $R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{28}$Si</td>
<td>3.10</td>
<td>1.78</td>
<td>$0.45 \pm 0.02$</td>
<td>$-0.12 \pm 0.02$</td>
<td>$0.81 \pm 0.04$</td>
<td>$-0.17 \pm 0.01$</td>
</tr>
<tr>
<td>$^{32}$S</td>
<td>4.10</td>
<td>2.23</td>
<td>$0.10 \pm 0.01$</td>
<td>$0.28 \pm 0.01$</td>
<td>$0.31 \pm 0.01$</td>
<td>$-0.06 \pm 0.01$</td>
</tr>
<tr>
<td>$^{12}$C</td>
<td>5.37</td>
<td>4.43</td>
<td>$0.50 \pm 0.02$</td>
<td>$-0.15 \pm 0.02$</td>
<td>$0.93 \pm 0.04$</td>
<td>$-0.06 \pm 0.01$</td>
</tr>
</tbody>
</table>

* Corrected for instrumental effects using measurements taken at 0° to the beam direction.
Fig. 3.10. Polarimeter sensitivities \( R \) determined for the 1.78, 2.23 and 4.43 MeV pure E2 calibration gamma rays. The solid curves were calculated for mean scattering angles of 70°, 75° and 80° using a computer program adapted from that of Taras (Ta 68).
from the Ge(Li) detector. The fact that the 4.43 MeV point falls slightly below the curves can be explained by a contribution of about 25% to be expected from pair production events (Marion, Ma 68) in the full energy peak. Because of the generally good agreement between the calculated curve and experimental points, polarimeter sensitivities for the 2.21 and 2.73 MeV gamma rays of $^{27}$Al were determined from the curves assuming a mean scattering angle of $(75\pm5^\circ)$.

3-3.5. Measurement of the polarization of the 2.21 and 2.73 MeV ground-state transition gamma rays of $^{27}$Al

The states of interest in $^{27}$Al were populated via the reaction $^{27}$Al(p,p')$^{27}$Al using a beam of 4.41 MeV protons. The bombarding energy was chosen from an excitation function for the 2.73 MeV gamma rays measured over a limited range. The target was 0.6 μm aluminium foil. The polarization measurements were as described in the previous section. Three runs were made at the two positions 90° and 0°. Beam currents were limited to 40 nA to render contributions from random coincidences negligible.

Results of angular distribution and polarization measurements for the 2.21 and 2.73 MeV transitions are given in Table 3.4; the probable errors quoted arise from the statistical uncertainties in the data. Sum-coincidence spectra are shown in fig. 3.11. Because the polarimeter resolution was about 40 keV, the 2.98 and 3.00 MeV peaks were not resolved. The 2.21 MeV peak is, however, well isolated; its area was determined using a simple exponential background subtraction. Because the 2.73 MeV peak is superposed on a complex background arising from the 3 MeV gamma rays, several different background subtraction techniques were investigated. The results presented in Table 3.4 were obtained by subtracting a cubic polynomial background; other techniques gave results which differed from
### TABLE 3.4
SUMMARY OF RESULTS FOR THE 2.21 and 2.73 MEV TRANSITIONS

<table>
<thead>
<tr>
<th>Transition</th>
<th>Asymmetry A *</th>
<th>Polarimeter sensitivity R †</th>
<th>Polarization P</th>
<th>Angular distribution coefficients a₂</th>
<th>a₄</th>
<th>Mixing ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.21 → 0.0 MeV</td>
<td>0.057 ± 0.002</td>
<td>0.170 ± 0.005</td>
<td>-0.33 ± 0.01</td>
<td>0.19 ± 0.02</td>
<td>0.00 ± 0.02</td>
<td>-0.46 ± 0.02</td>
</tr>
<tr>
<td>(7/2⁺ → 5/2⁺)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.73 → 0.0 MeV</td>
<td>-0.036 ± 0.011</td>
<td>0.140 ± 0.005</td>
<td>0.26 ± 0.08</td>
<td>0.20 ± 0.03</td>
<td>0.04 ± 0.04</td>
<td>-0.13 ± 0.17</td>
</tr>
<tr>
<td>(5/2⁺ → 5/2⁺)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>-0.26</td>
</tr>
</tbody>
</table>

* Corrected for instrumental effects using measurements taken at 0° to the beam direction.

† Deduced from fig. 3.10.
Fig. 3.11. Relevant regions of the sum-coincidence spectra obtained from the reaction $^{27}\text{Al}(p,p'\gamma)^{27}\text{Al}$ showing the 2.21, 2.73 and 3.00 MeV full energy and first escape peaks (I) for gamma rays scattered in directions: (a) parallel and (b) perpendicular to the reaction plane with the polarimeter set at 90° and 0° to the proton beam direction.
these by amounts which were insignificant when compared to the statistical errors. Attempts to perform lineshape fits to the data were unsuccessful because of the lack of accurately measured polarimeter lineshapes for gamma rays near 3 MeV.

Figs. 3.12 and 3.13 show plots of polarization versus the E2/M1 mixing ratio for the 2.21 and 2.73 MeV transitions calculated using the formulae of Rose and Brink (Ro 67a). The shaded areas show the values consistent with the angular distribution data while the solid areas represent the measured polarizations. The positive solution for the 2.21 MeV transition mixing ratio has been excluded on the basis of angular correlation work by Van der Leun et al. (Va 67). The positive solution for the 2.73 MeV transition has been excluded by the angular correlation results of Ophel and Lawergren (Op 64). The negative solutions are in agreement within the stated errors with those of the above authors for both transitions.

The dotted area on fig. 3.13 corresponds to the range of values of polarization for the 2.73 MeV transition required to reproduce the value for the mixing ratio given by Lumpkin et al. (Lu 71). The value of polarization obtained in the present work is significantly different from this range. The error associated with the present result for the mixing ratio of the 2.73 MeV transition is larger than those given by Ophel and Lawergren (Op 64) and Van der Leun (Va 67) who used angular correlation techniques. The result of Lumpkin et al. also has a smaller error than the present result but corresponds to a value of polarization where the mixing ratio does not vary rapidly with polarization.

3-3.6. Discussion

Table 3.5 summarises the available experimental values for the mixing ratios of the 2.21 and 2.73 MeV transitions in $^{27}$Al together with
Fig. 3.12. Plot of polarization $P$ versus the $(E2/M1)$ mixing ratio $\delta$ for the 2.21 MeV transition in $^{27}$Al. The shaded areas represent the values consistent with the angular distribution data while the solid areas represent the measured polarization and errors.
Fig. 3.13. Plot of polarization $P$ versus the $\langle E2/M1 \rangle$ mixing ratio $\delta$ for the 2.73 MeV transition in $^{27}$Al. The shaded areas represent the values consistent with the angular distribution data. The solid areas represent the measured polarization and errors while the dotted area represents the range of values required by the value of mixing ratio given by Lumpkin et al. (Lu 71).
TABLE 3.5
COMPARISON OF PRESENT AND PREVIOUS MIXING RATIO RESULTS WITH THEORETICAL
CALCULATIONS USING THE WEAK COUPLING AND SHELL MODELS

<table>
<thead>
<tr>
<th>Transition</th>
<th>Present Work</th>
<th>Experiment</th>
<th>Theory</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
</tr>
<tr>
<td>$2.21 \rightarrow 0.0 \text{ MeV}$</td>
<td>-0.46 ± 0.02</td>
<td>-0.46 ± 0.04</td>
<td>-0.40 ± 0.02</td>
</tr>
<tr>
<td>$2.73 \rightarrow 0.0 \text{ MeV}$</td>
<td>$-0.13 \pm 0.17$ $-0.26$</td>
<td>-0.04 ± 0.06</td>
<td>-0.09 ± 0.03</td>
</tr>
</tbody>
</table>

(a) Ophel and Lawergren (Op 64)  
(b) Van der Leun et al. (Va 67)  
(c) Lumpkin et al. (Lu 71)  
(d) Evers et al. (Ev 67)  
(e) Thankappan (Th 66)  
(f) De Voigt et al. (Vo 72): shell model
values predicted from calculations using the weak coupling model (Thankappan, Th 66 and Evers et al., Ev 67) and the shell model (de Voigt et al., Vo 72). The present results confirm all previous measurements of the E2/M1 mixing ratio for the 2.21 MeV transition. In the case of the 2.73 MeV transition, the present result is in agreement with those of Ophel and Lawergren (Op 64) and Van der Leun (Va 67) within the stated errors but the polarization obtained in the present work is several standard deviations from that required by the mixing ratio of Lumpkin et al. (Lu 71).

The shell model calculations of de Voigt et al. (Vo 72) predict the correct sign of the mixing ratio for both transitions and the correct magnitude for the 2.21 MeV transition. The magnitude predicted for the 2.73 MeV transition is outside the stated errors of all authors. The predictions of Thankappan (Th 66) and Evers et al. (Ev 67) using the weak coupling model reproduce the magnitude of the mixing ratio of the 2.21 MeV transition; both authors produce the correct sign for the 2.73 MeV transition but overestimate the magnitude by factors of up to 20 compared to the experimental results. However, it is not clear whether the sign convention used by these two authors is in accordance with that of Rose and Brink (Ro 67a).

Excluding the result of Lumpkin et al. (Lu 71), the weighted mean of all experimental results for the E2/M1 mixing ratio of the 2.73 MeV ($5/2^+ \rightarrow 5/2^+$) transition in $^{27}$Al is $-0.081 \pm 0.027$. The result obtained from the DSAM experiments described in sect. 3-2 for the lifetime of the 2.73 MeV level is $9 \pm 6$ fs. Previous results are $16 \pm 7$ fs (Sm 68) and $11 \pm 4$ fs (Sc 64). The weighted mean of these three results is $11 \pm 3$ fs. Table 3.6 shows the E2 and M1 enhancements of the 2.73 MeV transition calculated using the weighted mean values for the mixing ratio of this transition and the lifetime of the 2.73 MeV level together with the
### Table 3.6

E2 and M1 enhancements of the 2.73 MeV to ground state \((5/2^+ \rightarrow 5/2^+)\) transition in W.u.

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Shell model (a)</th>
<th>Coriolis band mixing model (b)</th>
<th>Weak coupling model (c)</th>
<th>Weak coupling model (d)</th>
<th>Rotation-vibration interaction model (e)</th>
</tr>
</thead>
<tbody>
<tr>
<td>E2</td>
<td>0.14 ± 0.08</td>
<td>1.6</td>
<td>1.5</td>
<td>3.6</td>
<td>2.6</td>
</tr>
<tr>
<td>M1</td>
<td>3.0 ± 0.8 × 10^{-2}</td>
<td>0.6 × 10^{-2}</td>
<td>2.0 × 10^{-2}</td>
<td>0.6 × 10^{-2}</td>
<td></td>
</tr>
</tbody>
</table>

(a) de Voigt et al. (Vo 72)  
(b) Dehnhard (De 72)  
(c) Thankappan (Th 66)  
(d) Evers et al. (Ev 67)  
(e) Röpke et al. (Ro 70)
branching ratio results of Smulders et al. (Sm 68). The enhancements predicted by the models discussed in sect. 3-1 are also included in table 3.6. It can be seen that all models except the rotation-vibration model of Röpke et al. (Ro 70) overestimate the E2 strength of the 2.73 MeV transition by factors of 10 or more. The M1 strength is more closely reproduced although the shell model calculations of de Voigt et al. (Vo 72) and the weak coupling calculations of Evers et al. (Ev 67) underestimate this by a factor of 5. These figures illustrate how the weak coupling and shell models fail to reproduce the magnitude of the mixing ratio of the 2.73 MeV ground-state transition.

The weighted mean value for the mixing ratio for the 2.73 MeV transition differs in magnitude from that of the corresponding transition in $^{27}$Si ($+0.40 \pm 0.07$, Main et al. Ma 71). Glaudemans and Van der Leun (Gl 71) have shown that many E2/M1 mixing ratios of corresponding transitions in mirror nuclei in the 2s-1d shell are equal in magnitude but opposite in sign. This trend is consistent with the assumptions that (i) for $\Delta T = 0$ transitions of reasonable strength, the E2 matrix elements are largely isoscalar in character while M1 matrix elements are largely isovector; (ii) isoscalar and isovector matrix elements are identical for corresponding transitions in mirror nuclei if Coulomb effects are ignored.

The difference in the E2/M1 mixing ratios of the $5/2^+ \rightarrow 5/2^+$ transitions in $^{27}$Al and $^{27}$Si has been discussed by Main et al. (Ma 71) who point out that the E2 strength of the transition in $^{27}$Si is $14 \pm 11$ times that in $^{27}$Al. They estimate that the isoscalar and isovector contents of the E2 matrix elements must be nearly equal in order to give rise to such a difference.
In this chapter experiments have been described to measure (i) the lifetimes of the 1.01, 2.21, 3.00, 4.51 and 5.50 MeV levels of $^{27}$Al and (ii) the E2/M1 mixing ratio of the ground-state transitions from the 2.21 and 2.73 MeV levels.

The results obtained in the present work for lifetimes of levels up to 4.51 MeV excitation confirmed previous measurements. The result for the 5.50 MeV level, taken in conjunction with that of de Voigt et al. (Vo 71) for the 7.44 MeV level, is consistent with the suggestion made by Carter et al. (Ca 71a) that these two levels are the $11/2^+$ and $13/2^+$ members of the $K^\pi = 5/2^+$ ground-state rotational band. However, the spin and parity assignments for these levels are not unique and await determination. Such a simple Nilsson model interpretation raises several difficulties. For example, the 4.51 MeV level originally assigned as the $11/2^+$ member of the $K^\pi = 5/2^+$ band, is difficult to interpret in terms of this picture. Also, the E2 strength of the transition between the 3.00 MeV $9/2^+$ and 2.21 MeV $7/2^+$ members of the band is anomalously low.

The measured value of the E2/M1 mixing ratio for the 2.21 MeV ground-state transition is in agreement with those of all previous authors. That obtained for the 2.73 MeV ground-state transition enabled the discrepancy between the result of Lumpkin et al. (Lu 71) and previous authors to be resolved. Calculations using the weak coupling model (Th 66, Ev 67) reproduce the magnitude of the mixing ratio for the 2.21 MeV transition, as do the shell model calculations of de Voigt et al. (Vo 72) which also produce the correct sign. This degree of success is not repeated in the case of the 2.73 MeV transition where the calculated values of the mixing ratio using the shell and weak coupling models are considerably greater.
than the measured values, although the correct sign is produced. The overestimation of the E2 strength of this transition by the shell model is surprising in view of the success of the model for neighbouring nuclei and is the major discrepancy for levels of $^{27}$Al below 4.5 MeV excitation.
CHAPTER 4

SPECTROSCOPIC STUDIES IN $^{32}$S

This chapter contains a brief review of theoretical calculations on $^{32}$S using various nuclear models, followed by descriptions of experimental work to measure properties of states of $^{32}$S up to 7.12 MeV excitation.

4-1 APPLICATION OF NUCLEAR MODELS TO $^{32}$S

4-1.1. Introduction

The purpose of the first section of this chapter is to discuss briefly the various approaches taken to date to account for the properties of $^{32}$S; its energy level scheme is shown in fig. 4.1. The spins and parities of levels shown in the diagram are as known prior to the commencement of the work described in the following sections. The gamma-ray decay schemes are given in fig. 4.9. In order to simplify the discussion, when levels with identical spin and parity are involved, low-lying levels with spin and parity $J^\pi$ are labelled as $J^\pi_1$, $J^\pi_2$ etc. in ascending order of excitation.

4-1.2. The inverted coexistence model

One of the first nuclear models to successfully explain some of the properties of low-lying states in $^{32}$S was the inverted coexistence model of Bar-Touv and Goswami (Ba 69a). In this model, the low-lying $0^+_2$ levels in light even-even nuclei such as $^{12}$C, $^{28}$Si, and $^{32}$S are assumed to correspond to the spherical solutions of the Hartree-Fock Hamiltonians for these nuclei. The $0^+_1$ ground states are then assigned as the axially
Fig. 4.1. Energy levels of $^{32}{\text{S}}$; the spins and parities shown are as known prior to the commencement of the present work.
deformed solutions which are the heads of rotational bands containing the \(2^+_1\) and \(4^+_1\) levels. The term "inverted coexistence" was used to distinguish the effect from the reverse situation in doubly closed shell nuclei such as \(^{16}\text{O}\) and \(^{40}\text{Ca}\). A difficulty with this model, when applied to \(^{32}\text{S}\), is the fact that the Hartree-Fock calculations of Zofka and Ripka (Zo 71) favour spherical ground-state solutions.

The anomalously large spacing between the \(0^+\) and \(2^+\) members of the ground-state rotational band is explained by Bar-Touv and Goswami in terms of mixing between the deformed \(0^+_1\) state and spherical \(0^+_2\) state. The \(4^+\) member of the band is then correctly predicted at excitation energy \(E_x = 4.46\) MeV. For \(^{32}\text{S}\), the ratio \(R\) of the reduced E2 transition rates for the \(2^+_1 \rightarrow 0^+_1\) and \(0^+_2 \rightarrow 2^+_1\) gamma ray transitions is, on the basis of the model, given by

\[
R = \frac{B(E2; 2^+_1 \rightarrow 0^+_1)}{B(E2; 0^+_2 \rightarrow 2^+_1)} = 0.51 .
\]

Using lifetime measurements of Piluso et al. (Pi 69) for the \(0^+_2\) state of \(0.52^{+0.30}_{-0.10}\) ps and the value \(0.34 \pm 0.04\) ps (En 67) for the \(2^+_1\) state, Bar-Touv and Goswami obtained \(R = 0.52 \pm 0.19\), in excellent agreement with the predicted value. However, the corresponding agreement for the ratio \(B(E2; 4^+_1 \rightarrow 2^+_1)/B(E2; 2^+_1 \rightarrow 0^+_1)\) is poor. They predict the \(6^+\) member of the ground-state rotational band to be at \(E_x \sim 8\) MeV.

4.1.3. The asymmetric rotator model

Thibaud et al. (Th 69) have measured lifetimes in \(^{32}\text{S}\), between 2.23 and 6.62 MeV excitation, via the reaction \(^{31}\text{P}(p,\gamma)^{32}\text{S}\) and interpret their results in terms of the asymmetric rotator model of Davyдов and Filippov (Da 58). They found good agreement between the value for the
ratio \( R = \frac{B(E2; \, ^2_2 - ^2_1)}{B(E2; \, ^2_1 - ^0_1)} \) calculated using their measured values of the lifetimes for the \( ^2_1 \) and \( ^2_2 \) levels and that predicted by the Davydov-Filippov model using \( \gamma = 20^\circ \) (see subsect. 1-3.1 for a definition of gamma).

However, the corresponding agreements for the ratios

\[
R = \frac{B(E2; \, ^2_2 - ^2_1)}{B(E2; \, ^2_1 - ^0_1)} \quad \text{and} \quad \frac{B(E2; \, ^2_2 - ^0_1)}{B(E2; \, ^2_1 - ^0_1)}
\]

are much poorer.

4-1.4. The vibrational model

More recently, there have been attempts to explain the features of low-lying states of \(^{32}\)S using the vibrational model. In this model, discussed by Garvey et al. (Ga 71) and Ingebretsen et al. (In 71), the \(^2_1 \) state is a one-phonon quadrupole vibrational state, while the \(^0_2, ^2_2 \) and \(^4_1 \) levels are members of the two-phonon quadrupole vibrational triplet (as shown in fig. 1.7). The "centre of gravity" of this triplet should correspond to roughly twice the excitation energy of the one-quadrupole phonon state. Reference to fig. 4.1 shows that the low-lying levels of \(^{32}\)S are strongly suggestive of such a scheme.

These quadrupole vibrational states would be expected to gamma decay by enhanced E2 transitions, corresponding to the annihilation of one quadrupole phonon. That is to say, the \(^0_2, ^2_2 \) and \(^4_1 \) two-phonon states should strongly decay to the \(^2_1 \) one-phonon state, with no ground-state branches. The observed ground-state transition from the \(^2_2 \) level can be explained by allowing mixing of the one and two-phonon \(^2_+ \) states (Ga 71). However, such mixing causes a shift from the unperturbed energies such that, if the \(^2_2 \) state is required to consist of 94% two-phonon + 6% one-phonon amplitude admixture in order to reproduce the observed cross-over transition, the ratio \( E_x(2^+_2)/E_x(2^+_1) = 2.5 \), while...
the observed ratio is 1.9. The $0^+_2$ and $4^+_1$ members of the two-phonon triplet exhibit $B(E2)$ values for the transitions to the one-phonon $2^+_1$ state in accordance with those predicted by the above vibrational picture.

The vibrational model may also be applied to negative parity states in $^{32}\text{S}$. Enhanced E2 transition rates from negative parity levels in the 6 - 8 MeV region are observed in $^{32}\text{S}$ (L.E. Carlson, personal communication) and $^{36}\text{Ar}$ (B.W. Sargent, personal communication); for example, the E2 transition from the 6.62 MeV $4^{-}$ state of $^{32}\text{S}$ has a strength of approximately 20 W.u. These enhanced E2 transitions may be explained by an extension of the vibrational model. The coupling of one quadrupole phonon and one octupole phonon produces five negative parity states of spins, 1, 2, 3, 4 and 5 (Pr 62). Each of these states will decay to the one-octupole phonon state (the lowest $3^{-}$ state) by an E2 transition, and to the one-quadrupole phonon state (the lowest $2^+$ state) by an E3 transition. The E2 transition should have the same enhancement as the E2 transition from the one-quadrupole phonon state to the $0^+$ ground state, and the E3 transition should have the same enhancement as the E3 transition from the one-octupole phonon state to the ground state. For $^{32}\text{S}$, the single-octupole phonon state is identified with the level at 5.01 MeV ($J^\pi = 3^-$); the observed enhancements for the relevant transitions to the ground state are approximately 9 W.u. (E2) (Ingebretsen et al., In 71) and 20 W.u. (E3) (Ollerhead et al., O1 70).

At the time of commencement of the work described in the following sections, there was insufficient information available to permit identification of all possible members of the quadrupole-octupole vibrational quintuplet in $^{32}\text{S}$; possible candidates were the $2^{-}$ state at 6.23 MeV (Endt and Van der Leun, En 67), the $4^{-}$ state at 6.62 MeV (Dorum, Do 68) and a state listed by Endt and Van der Leun at $E_x = 5.80$ MeV with $J = (1,2)$. Poletti and Grace (Po 66) assigned a spin 1 to this level from
a study of particle-gamma angular correlations with the reactions \(^{32}\text{S}(p,p'\gamma)^{32}\text{S}\). This is in disagreement with the earlier result of \(J = 2\) obtained by Lombard et al. (Lo 64) from electron scattering measurements.

4-1.5. Particle-hole calculations in \(^{32}\text{S}\).

Castel et al. (Ca 71) have performed calculations in which the vibrations of an even-even core are coupled to \((2s_{1/2}^{-1})(1d_{3/2}^{-1})\) particle-hole excitations. In addition to reproducing the energy level ordering in \(^{32}\text{S}\) up to about 6 MeV excitation, they were able to produce good agreement between calculated and experimental E2 transition strengths in the decay of low-lying one-phonon and two-phonon states. They tentatively identify a predicted 1\(^+\) level at \(E_x = 5.8\) MeV with the level known, prior to the commencement of the work described in this chapter, to exist at 5.80 MeV and to have a spin of 1 or 2 (En 67).

4-1.6. Shell model calculations in \(^{32}\text{S}\).

Shell model calculations have been performed for positive parity levels in the mass region \(A = 30 - 35\) by Wildenthal et al. (Wi 71), who considered basis states with particles in the \(1d_{5/2}, 2s_{1/2}\) and \(1d_{3/2}\) orbits with a minimum number of 10 particles in the \(1d_{5/2}\) orbit. They were able to reasonably well reproduce the level ordering and spacing for the eight positive parity levels definitely assigned in \(^{32}\text{S}\) up to \(E_x = 5.55\) MeV. The one notable discrepancy concerned the level at \(E_x = 4.46\) MeV, which was predicted to be 1 MeV higher in excitation. The calculations gave good agreement with experimentally measured spectroscopic factors (Gr 68) for the reaction \(^{31}\text{P}(^{3}\text{He},d)^{32}\text{S}\) populating \(J = 0, 1\) and 2 states.

The magnitude of the quadrupole moment for the \(2^+_1\) first excited
state, as measured by Nakai et al. (Na 70) \((-20 \pm 6 \text{ eF}^2\) \), was slightly larger than the value \((-13.7 \text{ eF}^2\) calculated by Wildenthal et al. The experimental B(E2) values for the \(2^+_2 \rightarrow 0^+_1\) and \(2^+_1 \rightarrow 0^+_1\) transitions are in good agreement with those of the calculations but the B(E2)'s calculated for the transitions from the \(2^+_2\), \(0^+_2\) and \(4^+_1\) states to the \(2^+_1\) state are weaker by a factor \(\sim 2\) compared to the experimental values.

Similar calculations have been performed by Glaudemans et al. (G1 71a) for the \(A = 30 - 34\) mass region. They produce transition strengths in \(^{32}\text{S}\) in fair agreement with experiment, although there are several discrepancies for both M1 and E2 transitions.

Negative parity states in \(^{32}\text{S}\) could also be interpreted in terms of the shell model by considering excitations to the \(1f_{7/2}\) subshell or out of the \(1p\) shell. Recently, Grawe et al. (Gr 73) have reported the results of calculations assuming a configuration space consisting of particles in the \(2s_{1/2}, 1d_{3/2}, 1f_{7/2}\), and \(2p_{3/2}\) orbits outside an inert \(^{28}\text{Si}\) core. Only one particle was allowed in the fp shell. The \(1d_{3/2} - 2s_{1/2}\) and \(2p_{3/2} - 1f_{7/2}\) separation energies were treated as variable parameters to obtain a best fit to the experimental energy levels of \(^{32}\text{S}\). The purpose of the calculations was to reproduce M1 transition strengths in \(^{32}\text{S}\) and reasonable agreement between experiment and theory was claimed. Discrepancies that arose were ascribed partly to the assumption of an inert \(^{28}\text{Si}\) core.

4-1.7. Motivation for the present work.

The previous subsections show that several different approaches can be used, with varying degrees of success, to account for the properties of \(^{32}\text{S}\) levels below \(E_x = 5.01\text{ MeV}\), where there are extensive experimental data. For levels of higher excitation, comparisons with the theoretical
calculations are more difficult because of the lack of experimental information. In particular, prior to the commencement of the work described in the following sections, no definite spin-parity assignments had been made for the levels at $E_x = 5.80, 6.41, 6.67, 6.76, 6.85$ and 7.12 MeV. The purpose of this work was to investigate the properties of $^{32}$S with emphasis on those levels between 5.80 and 7.12 MeV excitation. Of particular interest, was the location of possible $J^\pi = 1^-, 3^-$ and $5^-$ members of the proposed quadrupole-octupole vibrational quintuplet discussed in subsect. 4-1.4.

Section 4-2 describes experiments to determine the lifetime, spin and gamma decay modes of the 5.80 MeV level, in order that its possible identification as the $1^-$ member of the quadrupole-octupole quintuplet might be investigated. Sect. 4-3 describes experiments to measure the branching ratios of all known levels of $^{32}$S between $E_x = 5.40$ and 7.15 MeV, in order to resolve notable uncertainties in the published values. Experiments to determine spin-parity combinations of all known levels in $^{32}$S up to 7.15 MeV are described in sect. 4-4. During the course of this work, a new level was discovered at $E_x = 6.58$ MeV.

4-2 A STUDY OF THE 5.80 MEV LEVEL OF $^{32}$S

4-2.1. Introduction

As mentioned in subsect. 4-1.4, a possible $1^-$ member of the proposed quadrupole-octupole vibrational quintuplet in $^{32}$S is a state listed by Endt and Van der Leun (En 67) as having an excitation energy of $5799 \pm 8$ keV and a spin of 1 or 2. A mean lifetime $\tau$ of $14 \pm 2$ fs has been reported by Lombard et al. (Lo 64) from measurements of inelastic electron scattering. The state was believed to decay 100% to the ground state. Since then it has been reported by Morrison (Mo 70) and by Graue
(Gr 68) that the state is populated in the $^{31}$P($^3$He,d)$^{32}$S reaction via an $l_p = 1$ transition, indicating $J^\pi = 0^-, 1^-$ or $2^-$. This information, taken in conjunction with the gamma decay characteristics of the state, suggest a $J^\pi = 1^-$ assignment, in agreement with the result of Poletti and Grace (Po 66).

The work described in this section was undertaken with the specific aim of using gamma-ray spectroscopic techniques to obtain more definite values for the parameters of the 5.80 MeV state, in order that its identification as a member of the proposed quadrupole-octupole quintuplet might be more rigorously evaluated. DSAM measurements via the reaction $^{29}$Si($\alpha$,n$_\gamma$)$^{32}$S were used to determine the mean lifetime (subsect. 4-2.2), angular distribution studies of gamma rays from the same reaction were used to obtain a spin assignment (subsect. 4-2.3) and particle-gamma coincidence studies of the reaction $^{32}$S(p,p'$\gamma$)$^{32}$S were used to determine the decay scheme of the state (subsect. 4-2.4).

4-2.2. Lifetime measurement

Experimental procedure

The 5.80 MeV state was populated via the reaction $^{29}$Si($\alpha$,n$_\gamma$)$^{32}$S using a 250 nA beam of 9.8 MeV $^4$He$^{++}$ ions from the A.N.U. EN tandem accelerator. Targets of thickness approximately 200 $\mu$g/cm$^2$ were prepared by vacuum evaporation of SiO$_2$ from a tantalum boat onto a 0.051 cm thick tantalum backing; the Si of the target material was enriched to 92% in $^{29}$Si. Gamma-ray spectra were taken at angles $\theta$ to the beam direction of 0°, 55°, 90° and 135° with a 40 cm$^3$ Ortec Ge(Li) detector 9 cm from the target. The detector resolution was 3.5 keV. At the bombarding energy used, the 6131 keV state of $^{16}$O was populated by inelastic alpha particle scattering; the full energy and escape peaks of the gamma rays resulting from the ground-
state transition from this level were used in conjunction with the 2614 keV gamma rays from $^{208}$Tl to provide an energy calibration.

Data Analysis

Centroids of the full energy and escape peaks of the 5.80 MeV gamma radiation were determined after subtraction of an exponential background using the subroutines MARK 1 and MARK 2 written by Dr. L.E. Carlson for the IBM 1800 computer. Fig. 4.2 shows sections of the spectra containing the 5.80 MeV gamma-ray peaks for the four angles involved. The excitation energy of the state was determined from the unshifted $90^\circ$ data as $5798 \pm 1$ keV; this agrees well with the value $5797.6 \pm 1.0$ keV reported recently by Coetzee et al. (Co 72), who studied the reaction $^{31}$P(p,$\gamma$)$^{32}$S.

The centroid positions were fitted with the expression (see sect. 2-1)

$$E(\theta) = E_o \left[1 + F(\tau) \frac{v_{cm}}{c} \cos(\theta)\right]$$

where $E(\theta)$ is the energy of the gamma ray emitted at angle $\theta$, $E_o$ is the unshifted gamma-ray energy, $v_{cm}$ is the velocity of the centre of mass in the laboratory frame, and $F(\tau)$ is the ratio of attenuated to full Doppler shift. At 9.8 MeV bombarding energy, $v_{cm}/c = 0.0088$. This formula assumes that the angular distribution of the recoiling nuclei is symmetrical about $90^\circ$ in the centre of mass system (see subsect. 2-1.6). The effect of any asymmetry has been investigated by Ingebretsen et al. (In 69). For their work using the reaction $^{32}$S(a,p)$^{35}$Cl, they estimate that the effective forward velocity may be increased over $v_{cm}$ by up to 5%. The effect is estimated to be less than 1% for the present measurements using the $^{29}$Si(a,n)$^{32}$S reaction, and has therefore been neglected.
Fig. 4.2. Section of spectra containing the 5.80 MeV gamma-ray peaks taken at 0°, 55°, 90° and 135° at 9.8 MeV bombarding energy.
The smaller value arises because it is possible to work closer to the threshold energy for the state under study with the \((a,n)\) reaction than with the \((a,p)\) reaction. Fig. 4.3 shows a least squares fit to the data of the above expression for \(E(\Theta)\), together with the calculated line corresponding to full shifts. For the sake of simplicity, the diagram shows a combination of the data for the full energy and escape peaks; in the analysis, each peak was treated separately. The weighted mean value of \(F(\tau)\) derived from this analysis is \(0.980 \pm 0.011\).

The relationship between \(F(\tau)\) and \(\tau\) for the experimental situation involved was evaluated using the stopping theory of Lindhard et al. (Li 63) and including the effects of nuclear scattering as discussed by Blaugrund (Bl 66). The computer code written by Dr. L.E. Carlson was used for these calculations and the results were checked against those produced using the similar code written by Dr. C.J. Piluso. Uncertainties in calculated values of \(F(\tau)\) arising from uncertainties in the stopping power \((\frac{d\varepsilon}{dp})\) thus derived were estimated by writing (see subsect. 2-1).

\[
\left(\frac{d\varepsilon}{dp}\right) = f_e \left(\frac{d\varepsilon}{dp}\right)_e + f_n \left(\frac{d\varepsilon}{dp}\right)_n
\]

where the multiplying factors \(f_e\) and \(f_n\) for the electronic stopping power \((\frac{d\varepsilon}{dp})_e\) and nuclear stopping power \((\frac{d\varepsilon}{dp})_n\) were given by \(f_e = f_n = 1.00 \pm 0.15\).

**Results**

Table 4.1 shows the results for the mean lifetime of the 5.80 MeV state together with those for the 4.28 and 5.01 MeV states, which were derived as checks on the experimental procedure and method of analysis. Table 4.2 compares the present and previous DSAM results and shows that the lifetimes obtained in the present work for the 4.28 and 5.01 MeV levels of \(^{32}\text{S}\) are in agreement with many of the previously reported values.
Fig. 4.3. A least squares fit of the expression
\[ E(\theta) = E_0 [1 + F(\tau) (v_{cm}/c) \cos \theta] \]
to the energies of the 5.80 MeV gamma-ray peaks shown in fig. 4.2. For the sake of simplicity, the diagram shows a combination of the data for the full-energy and escape peaks; in the analysis each peak was treated separately. The dashed line represents the calculated full shifts.
### TABLE 4.1
RESULTS OF PRESENT LIFETIME MEASUREMENTS IN $^{32}\text{S}$

<table>
<thead>
<tr>
<th>Level (keV)</th>
<th>Transition (keV)</th>
<th>F($\tau$)</th>
<th>$\tau$ (fs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4277</td>
<td>4277 $\rightarrow$ 0</td>
<td>$0.873 \pm 0.016$</td>
<td>$59 \pm 9$</td>
</tr>
<tr>
<td>5006</td>
<td>5006 $\rightarrow$ 2230</td>
<td>$0.195 \pm 0.046$</td>
<td>$850 \pm 300$</td>
</tr>
<tr>
<td>5799</td>
<td>5799 $\rightarrow$ 0</td>
<td>$0.980 \pm 0.011$</td>
<td>$14 \pm 7$</td>
</tr>
<tr>
<td>Level (keV)</td>
<td>Present work (fs)</td>
<td>Previous work (fs)</td>
<td>(a)</td>
</tr>
<tr>
<td>------------</td>
<td>------------------</td>
<td>--------------------</td>
<td>-----</td>
</tr>
<tr>
<td>4277</td>
<td>59±9</td>
<td>29±2</td>
<td>74±6</td>
</tr>
<tr>
<td>5006</td>
<td>850±300 -200</td>
<td>1500±2500 -700</td>
<td>750±50</td>
</tr>
<tr>
<td>5799</td>
<td>14±7</td>
<td>14±2*</td>
<td></td>
</tr>
</tbody>
</table>

(a) Lombard et al. (Lo 64).
(b) Piluso et al. (Pi 69).
(c) Ollerhead et al. (Ol 70).
(d) Evans et al. (Ev 68).
(e) Thibaud et al. (Th 69).

(F) Garvey et al. (Ga 71).
(g) Carlson (personal communication).
(h) Ingebritsen et al. (In 71).
(i) Coetzee et al. (Co 72).
(j) Carr et al. (Ca 73).

* Non-DSAM
However, it should be noted that the scatter in these values is somewhat greater than the individual errors assigned to each value. This may be an indication that the errors in the calculated stopping powers are underestimated. Broude et al. (Br 73) have investigated the effect of different stopping materials on the measured value of the lifetime of the 1.01 MeV level in $^{27}$Al and found it to vary with atomic number of the backing by amounts considerably greater than the experimental errors. Highest values, in agreement with those obtained from the recoil distance method, were obtained with Ca, Ti and Ba backings, while Cu, Ta and Au backings gave significantly lower results. These conclusions cannot be extended to explain the variation in the published values shown in table 4.2 for the lifetimes of the 4.28 and 5.01 MeV levels of $^{32}$S, because all of the recorded backings are Au, including those used to obtain the extreme values of $29 \pm 2$ fs (Pi 69) and $80 \pm 10$ fs (In 71) for the 4.28 MeV level. Furthermore, these results were both obtained with the same reaction, $^{31}$P(p,$\gamma$)$^{32}$S.

The value of the mean lifetime obtained in the present work for the 5.80 MeV state is $14 \pm 7$ fs, where the quoted error arises predominantly from the uncertainty in the experimental value for $F(\tau)$. Errors due to uncertainties in the electronic and nuclear stopping powers derived from the theory of Lindhard et al. (Li 63) are about 15% and those due to the uncertainty in the density of the target material are about 5%. In the process of vacuum evaporation from tantalum, the SiO$_2$ is reduced to some degree, but the exact proportions of Si, SiO and SiO$_2$ in the target are not known. A thin ($\approx 5 \ \mu g/cm^2$) target was prepared by evaporating $^{29}$SiO$_2$ onto thin carbon foil by the same procedure as for the targets used in the DSAM experiments. Measurements of elastic scattering of 1.5 MeV alpha particles from the A.N.U. AK accelerator confirmed that this target was not completely SiO$_2$. In the analysis it was assumed the target contained
equal proportions of Si, SiO and SiO₂, and the 5% uncertainty used in the error analysis corresponds to an 8% variation in density, which is the maximum that could arise from uncertainty in the target composition.

The present result of 14 ± 7 fs for the lifetime of the 5.80 MeV level is in good agreement with the earlier result of 14 ± 2 fs obtained by Lombard (Lo 64) from inelastic electron scattering. However, this experiment suffered from poor resolution and resulted in a spin assignment (J = 2) for the 5.80 MeV level in disagreement with all subsequent measurements. The value of 8 ± 5 fs obtained by Coetzee et al. (Co 72) from DSAM studies of the \(^{31}\text{P}(p,\gamma)^{32}\text{S}\) reaction and the recent result of < 10 fs of Carr et al. (Ca 73), who studied the reaction \(^{29}\text{Si}(\alpha,n)^{32}\text{S}\), are also consistent with the present result.

If the 5.80 MeV state is assumed to decay 100% to the 0\(^+\) ground state, a lifetime of 14 ± 7 fs implies gamma-ray transition strengths which are unreasonably large for all spin-parity combinations except 1\(^+\), 1\(^-\) and 2\(^+\); for example, J\(^\pi\) = 2\(^-\) would require an M2 enhancement of \(47^{+50}_{-15}\) W.u.

4-2.3. Angular distribution measurements

Experimental procedure.

Angular distributions of 5.80 MeV gamma rays from the reaction \(^{29}\text{Si}(\alpha,\gamma)^{32}\text{S}\) were measured at bombarding energies of 8.7 and 8.8 MeV using \(^4\text{He}^{++}\) beams of approximately 250 nA. The threshold for population of the 5.80 MeV state is 8.34 MeV. Hence, at the bombarding energies used the emission of s-wave neutrons will predominate and interpretation of the angular distributions will be correspondingly simplified. The target was the same one used for the lifetime measurements described in the previous section. Gamma-ray spectra were recorded at angles θ = 0°, 15°, 30°, 45°, 60°, 75° and 90° (in random order) with a 40 cm\(^3\) Ortec Ge(Li)
detector located 10 cm from the target. Repeat measurements were made at 0°, 45° and 90°. A 40 cm³ Nuclear Diodes Ge(Li) detector set 6 cm from the target at θ = 270° served as a monitor.

The anisotropy of the moving detector system was measured using the isotropic 2.31 MeV gamma rays resulting from the decay of the first excited state of $^{14}$N ($J^\pi = 0^+$). This state was populated by bombarding a 100 μg/cm² $^{11}$B target on a 0.051 cm thick tantalum backing with 8 MeV $^4$He++ ions.

**Analysis of data and results**

At each angle, the number of counts observed in the second escape peak of the 5.80 MeV gamma ray was normalised to the number recorded simultaneously in the second escape peak of the 5.80 MeV gamma ray in the monitor detector spectrum. Corrections were applied for instrumental asymmetry (~ 4%) and for absorption in the target backing (~ 2%). The angular distributions so obtained (fig. 4.4) were fitted with the expression $W(\theta) = 1 + a_2P_2(\cos \theta) + a_4P_4(\cos \theta)$. The normalised coefficients, corrected for solid angle effects, appear in table 4.3.

An attempt was made to distinguish between spin assignments of 1 and 2 for the 5.80 MeV state by fitting the formulae of Rose and Brink (Ro 67a) to the data. This was done using the computer program written by Dr. R.A.I. Bell (Be 73) for the IBM 1800 computer. For J = 1, a best fit (well within the 0.1% confidence limit) was obtained with the physically reasonable population parameters $P(0) = 0.56$, $P(\pm 1) = 0.44$ at 8.8 MeV bombarding energy and $P(0) = 0.58$, $P(\pm 1) = 0.42$ at 8.7 MeV bombarding energy (see fig. 4.4). Population parameters to be expected for J = 2 were estimated by using outgoing neutron penetrabilities calculated from the formulae of Marion and Young (Ma 68) and the appropriate Clebsch-Gordan coefficients for weighting the production of the final magnetic
### TABLE 4.3
NORMALIZED EXPERIMENTAL ANGULAR DISTRIBUTION COEFFICIENTS

<table>
<thead>
<tr>
<th>$E_\alpha$(MeV)</th>
<th>$a_2$</th>
<th>$a_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.8</td>
<td>-0.38 ± 0.06</td>
<td>0.06 ± 0.06</td>
</tr>
<tr>
<td>8.7</td>
<td>-0.31 ± 0.06</td>
<td>-0.10 ± 0.06</td>
</tr>
</tbody>
</table>

### TABLE 4.4
NORMALIZED ANGULAR DISTRIBUTION COEFFICIENTS CALCULATED USING MANDY FOR $E_\alpha = 8.8$ MEV

<table>
<thead>
<tr>
<th>$J^\pi$</th>
<th>$a_2$</th>
<th>$a_4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$2^+$</td>
<td>0.43</td>
<td>-0.21</td>
</tr>
<tr>
<td>$2^-$</td>
<td>0.39</td>
<td>-0.13</td>
</tr>
<tr>
<td>$1^+$</td>
<td>-0.20</td>
<td></td>
</tr>
<tr>
<td>$1^-$</td>
<td>-0.20</td>
<td></td>
</tr>
</tbody>
</table>
Fig. 4.4. Fits of the angular distribution formulae of Rose and Brink (Ro 67a) to the data taken at 8.7 and 8.8 MeV. For both $J = 1$ and $J = 2$, the population parameters $P(0)$ and $P(\pm 1)$ were allowed to vary freely between 0 and 1, and for $J = 2$ the population parameter $P(\pm 2)$ was constrained to be 0.05.
substates by $\ell = 0, 1$ and 2 neutrons. The results obtained, essentially
the same for both bombarding energies, indicated that $P(\pm 2) = 0.05$. The
best fit to the data for $J = 2$ using this value of $P(\pm 2)$ yielded a value
of $\chi^2$ well outside the 0.1% confidence limit (see fig. 4.4). In fact,
fits to the data for $J = 2$ produced values of $\chi^2$ below the 0.1% confidence
limit only for values of $P(\pm 2)$ greater than 0.50 (see fig. 4.5). Assuming
that this is physically unreasonable, it is concluded that $J(5.80) = 1$.

The angular distribution data were also compared to calculations
made with the statistical compound nucleus program MANDY of Sheldon and
Van Patter (Sh 66). The results for a bombarding energy of 8.8 MeV are
shown in table 4.4. Comparison with the experimentally determined co-
efficients (table 4.3) shows that a $J = 1$ assignment is very strongly
favoured. An assignment of $J = 1$ is consistent with the conclusion reached
by Poletti and Grace (Po 66).

4-2.4. Measurement of the decay scheme

The decay scheme of the 5.80 MeV level was measured as part of
an experiment described in detail in sect. 4-3 to determine the gamma-ray
decay schemes of all known levels in $^{32}$S between the excitation energies
of 5.40 and 7.15 MeV. This involved a study of the reaction $^{32}$S(p,p'\gamma)$^{32}$S
with a 12.7 x 10.2 cm NaI(Tl) gamma-ray detector in conjunction with a 61 cm
double focussing magnetic spectrometer.

The coincident gamma-ray spectrum for the 5.80 MeV level was
analysed using the lineshape fitting program described by Elliott et al.
(El 68a) (and in sect. 4-3). The result obtained by fitting a 5.80 MeV
lineshape to the data is shown in fig. 4.6 and indicates a 100% decay to
the ground state. Upper limits for the intensities of possible weak
transitions were estimated by adding lineshapes of the appropriate energies
to the fitted spectrum and noting the intensities at which these extra
Fig. 4.5. Plots of $\chi^2$ versus $P(\pm 2)$ for the best fits with $J = 2$ to the data shown in fig. 4.4.
Fig. 4.6. The gamma spectrum taken in coincidence with protons populating the 5.80 MeV level of $^{32}$S. A 5% random coincidence background has been subtracted. The full line shows the 5.80 MeV lineshape fit to the data.
The value reported by Lombard et al. mistakenly assumes $J^\pi(5.80) = 2^-$. If we therefore ignore this result in averaging procedure, the lifetime becomes $(11 \pm 5) \text{ fs}$. This makes no difference to subsequent discussion.
components changed the fit significantly. The results are summarized in table 4.5. The only previous attempt to set limits on the strengths of weak transitions is that of Coetzee et al. (Co 72), who reported an upper limit of 5% for a branch to the 2.23 MeV level.

4-2.5. Summary and discussion

The work described in this section indicates that the 5.80 MeV state of $^{32}$S has an excitation energy of $5798 \pm 1$ keV, a mean lifetime of $14 \pm 7$ fs, a spin of 1, and a 100% decay to the $0^+$ ground state, with all other possible transitions having intensities less than 2% of the ground-state transition. It is not possible to determine the parity of the level from these results alone; negative parity implies an E1 transition of approximately $4 \times 10^{-4}$ W.u., and positive parity implies an M1 transition of approximately 0.01 W.u., neither of which values is physically unreasonable. However, experiments described in sect. 4-4 indicate that the 5.80 MeV level has natural parity. Hence, for the purposes of the present discussion it will be assumed that the parity of the 5.80 MeV state is negative. Evidence for this also comes from DWBA analyses of angular distribution data from the reaction $^{31}$P($^3$He,d)$^{32}$S; both Morrison (Mo 70) and Graue et al. (Gr 68) report $l_p = 1$, and hence negative parity.

There are now four measurements of the lifetime of the 5.80 MeV level (see table 4.2), i.e., $14 \pm 2$ fs (Lombard et al., Lo 64), $8 \pm 5$ fs (Coetzee et al., Co 72), $14 \pm 7$ fs (present work), and < 10 fs, recently reported by Carr et al. (Ca 73). On the basis of the first three results it will be assumed that the lifetime of the level is $12 \pm 3$ fs. This implies that the E1 transition to the ground state has a strength of $(4.1 \pm 1.0) \times 10^{-4}$ W.u.; the weakness of this $\Delta T = 0$ transition is consistent with the operation of the isospin selection rule for E1 transitions in self-conjugate nuclei. The same is true for the E1 transitions to the
TABLE 4.5

DECAY SCHEME OF THE 5.80 MEV STATE OF $^{32}\text{S}$

The spin and parity assignments listed are taken from Endt and Van der Leun (En 67) and Ollerhead et al. (Ol 70).

<table>
<thead>
<tr>
<th>Final state $E_x$(MeV)</th>
<th>$J^\pi$</th>
<th>Intensity of transition (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0$^+$</td>
<td>100</td>
</tr>
<tr>
<td>2.23</td>
<td>2$^+$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>3.78</td>
<td>0$^+$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>4.28</td>
<td>2$^+$</td>
<td>&lt; 2</td>
</tr>
<tr>
<td>4.46</td>
<td>4$^+$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>4.69</td>
<td>1$^+$</td>
<td>&lt; 2</td>
</tr>
<tr>
<td>5.01</td>
<td>3$^-$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>5.41</td>
<td>3$^+$</td>
<td>&lt; 1</td>
</tr>
<tr>
<td>5.54</td>
<td>2$^+$</td>
<td>&lt; 2</td>
</tr>
</tbody>
</table>
2.23 \( (2^+_1) \), 3.78 \( (0^+_2) \) and 4.28 \( (2^+_2) \) MeV states, which have strengths of less than \( 2.3 \times 10^{-5} \), \( 1.3 \times 10^{-4} \) and \( 5.4 \times 10^{-4} \) W.u., respectively.

4-3 GAMMA RAY DECAY SCHEMES OF LEVELS AT INTERMEDIATE ENERGIES IN \(^{32}\text{S}\)

4-3.1. Introduction

Because of the high level density in the intermediate excitation energy range from 5 to 9 MeV, the nuclear models discussed in sect. 4-1 require the accumulation of much information before they can be tested effectively. At the beginning of the work described in this section, there were some notable uncertainties about gamma-ray branching ratios of levels in this region (see sect. 4-3.4 and table 4.6). In this section, measurements of the gamma-ray decay schemes of all known levels in \(^{32}\text{S}\) between \( E_x = 5.40 \) and 7.15 MeV are reported. The technique employed is one previously used to study \(^{27}\text{Al}\) (Elliott et al., El 68b) and \(^{27}\text{Al}\) (Kean et al., Ke 69).

4-3.2. Experimental procedure

Beams of protons were used to populate states in \(^{32}\text{S}\) via the reaction \(^{32}\text{S}(p,p'\gamma)^{32}\text{S}\). A separate particle-gamma coincidence measurement was made for each level of interest. The beam energies ranged from 8.03 to 9.17 MeV (see fig. 4.7); they were selected by measuring excitation functions over a limited energy range for each proton group. The target consisted of approximately 100 \( \mu \text{g/cm}^2 \) of CdS (containing natural sulphur) evaporated onto 100 \( \mu \text{g/cm}^2 \) gold foil. Protons were detected in a 5 cm long, 6 mm wide position-sensitive detector at the focal plane of the 61 cm double focussing spectrometer described in sect. 2-3. The detector surface was covered with a layer of 6 \( \mu \text{m} \) aluminium foil to absorb any alpha particles which might also be present. Gamma rays were detected at 90° to the beam.
direction in a 12.7 cm (diameter) × 10.2 cm (length) NaI(Tl) crystal mounted with its front face 4.1 cm from the target spot; a 1.6 mm thick steel plate supported the crystal in the top of the target chamber. The proton beam was stopped in a beam dump 3 m from the target. The structure of the target chamber required that the magnetic spectrometer be set at 90° to the beam direction. It was necessary to limit the beam current to below 50 nA in order to reduce the beam-induced background in the NaI(Tl) detector to an acceptable level. Accumulation times for the coincidence spectra shown in fig. 4.7 ranged from 4 to 60 hours.

A block diagram of the electronic system used for the particle-gamma coincidence measurements is shown in fig. 4.8. Standard cross-over timing circuits were employed in this arrangement. A TAC spectrum was stored in 256 channels of a pulse-height analyser for use in calculating the ratio of real-to-random coincidences; the width of the time peak was approximately 80 ns (FWHM). The coincidence gamma-ray spectrum, gated by a window on the position spectrum and by a 400 ns wide window on the time spectrum, was stored in 256 channels of a second pulse-height analyser.

4-3.3. Analysis of the data

The coincidence gamma-ray spectra were analysed on a Univac 1108 computer using a lineshape fitting program previously described by Elliott et al. (El 68a) and Ophel (Op 71). This program calculates lineshapes for gamma rays of interest by interpolating between members of a set of measured responses of the detector to monoenergetic gamma rays from sources or suitable nuclear reactions. For the NaI(Tl) detector used in the experiments described here, measured lineshapes are available for gamma-ray energies of 0.66, 1.17, 2.37, 4.43 and 6.13 MeV. Fig. 4.7 shows the fits obtained, using lineshapes interpolated for the components specified in the diagram, to the coincidence gamma-ray spectra. Where
Fig. 4.7. Coincidence gamma-ray spectra for various levels of $^{32}$S populated via the reaction $^{32}$S(p,p'γ)$^{32}$S. For the spectra from all of the levels except the 5798 and the 6854 keV levels, a calculated sum spectrum has been subtracted from the data and the randoms have been treated as a component in the fit, as described in the text. For the spectra from the 5798 and 6854 keV levels, no sum correction was made, and a singles lineshape of intensity calculated from the ratio of reals-to-randoms in the TAC spectrum was subtracted from the data. For the 5413, 6224, 6410 and 6762 keV levels, the ground-state component shown in the fit was used only to estimate an upper limit.
Fig. 4.8. Block diagram of the electronic system used in the experiments to measure gamma decay schemes of 32S levels.
appropriate, corrections were made for the summing of cascade gamma rays, assuming isotropic angular correlations. The uncertainties in the summing corrections resulted in considerable uncertainties in the intensities of weak ground-state branches. For a triple cascade, such as the one from the 6.76 MeV level, the most important were the three possible double-sum corrections; the triple sum correction is very small and has been ignored. Each spectrum was also corrected for random coincidences (~10%) either by treating the singles spectrum as a component in the lineshape fitting program, or by subtracting a singles spectrum with an intensity normalization deduced from the measured ratio of real-to-random coincidences.

Except for the ground-state transitions noted below, upper limits for the intensities of unobserved branches were determined by adding lineshapes of the appropriate energies to the fitted spectrum and noting by visual inspection the intensities at which these extra components changed the fit significantly. For the ground-state transitions from the 5.41, 6.22, 6.41 and 6.76 MeV levels, lineshapes for the ground-state gamma rays were specified as components in the fits and the resulting intensities, plus their errors as calculated by the program, were used as upper limits.

4-3.4. Results

Coincidence gamma-ray spectra for all levels studied are shown in fig. 4.7. The data for the 5.80 MeV level have been shown in subsect. 4-2.4, but are included here for the sake of completeness. The full curves represent the fits to the data obtained from the lineshape analysis. The magnetic spectrometer made the resolution of the various levels and the identification of the particles as protons very easy, so that contributions to each spectrum from $^{32}$S levels other than the one of interest are negligible. No evidence was found for contributions from contaminant reactions. The inclusion in the spectra of ground-state components for
the 5.41, 6.62, 6.41 and 6.76 MeV levels improved the fits slightly; however, the intensities of these components were used only to determine upper limits because it was considered that adding these components only compensated for inaccuracies in the correction for summing and randoms.

The branching ratios obtained are summarized in table 4.6, and upper limits for unobserved transitions are shown in fig. 4.9. The errors indicated are due to statistical uncertainties in the data and to uncertainties inherent in the lineshape analysis. No attempt has been made to correct for errors arising from anisotropies in the gamma-ray angular distributions. For a 2 MeV gamma ray, the attenuation factors for the geometry used are $Q_2 = 0.56$ and $Q_4 = 0.06$, and therefore the effects of anisotropies are considerably reduced; Elliott et al. (El 68a) estimate 15% as a safe upper limit on the consequent error in gamma-ray intensities.

4-3.5. Discussion

The branching ratios obtained are compared with previous results in table 4.6.

The 5.41 MeV level

The present data are consistent with a 100% branch to the 2.23 MeV $2^+$ level. Coetzee et al. (Co 72) indicate that this transition accounts for only $(90 \pm 5)$% of the decay of the level and that therefore $(10 \pm 5)$% of the decay must consist of transitions to other levels. The present limits, combined with the limit of 1% set by Piluso et al. (Pi 69) for a ground-state branch, are just consistent with this.

The 5.50 MeV Level.

The decay scheme obtained for this level agrees with all previous measurements except those of Piluso et al. (Pi 69), who found a greater percentage of the decay to be to the 2.23 MeV level than reported by other
Fig. 4.9. Gamma-ray branching of $^{32}$S levels. The branching ratios of levels above $E_x = 5.1$ MeV are from the present work. The branching ratios of levels below $E_x = 5.1$ MeV, as well as all of the spins, parities and excitation energies, are as listed by Coetzee et al. (Co 72). Parentheses have been added to the $J^\pi$, T assignments for the 6762 and 7117 keV levels to indicate that these assignments are not as well established as the others.
TABLE 4.6

GAMMA-RAY BRANCHING RATIOS (%) IN $^{32}$S

The excitation energies are taken from Coetzee et al. (Co 72).

<table>
<thead>
<tr>
<th>$E_x$(keV) (Initial state)</th>
<th>$E_x$(keV) (Final state)</th>
<th>Present work</th>
<th>Previous work</th>
<th>Subsequent work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(a)</td>
<td>(b)</td>
<td>(c)</td>
</tr>
<tr>
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<td>0</td>
<td>&lt; 6</td>
<td>&lt; 10</td>
<td></td>
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<td></td>
<td>100</td>
<td>90±5</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>3778</td>
<td>&lt; 4</td>
<td>&lt; 20</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4282</td>
<td>&lt; 3</td>
<td>&lt; 10</td>
<td></td>
</tr>
<tr>
<td></td>
<td>unknown</td>
<td></td>
<td>&lt; 10</td>
<td></td>
</tr>
<tr>
<td>5549</td>
<td>0</td>
<td>40±2</td>
<td>41±5</td>
<td>40±10</td>
</tr>
<tr>
<td></td>
<td>2230</td>
<td>60±2</td>
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<td>60±10</td>
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<td>5798</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>100</td>
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<td></td>
<td>2230</td>
<td>&lt; 1</td>
<td>&lt; 5</td>
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</tr>
<tr>
<td>6224</td>
<td>0</td>
<td>&lt; 6</td>
<td>&lt; 2</td>
<td>&lt;0.5</td>
</tr>
<tr>
<td></td>
<td>2230</td>
<td>100</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td></td>
<td>5007</td>
<td>&lt; 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6410</td>
<td>0</td>
<td>&lt; 4</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2230</td>
<td>100</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>6621</td>
<td>2230</td>
<td>9±3</td>
<td>3±2</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>4459</td>
<td>10±3</td>
<td>28±5</td>
<td>15±10</td>
</tr>
<tr>
<td></td>
<td>5007</td>
<td>81±5</td>
<td>66±5</td>
<td>85±10</td>
</tr>
<tr>
<td></td>
<td>5413</td>
<td>&lt; 3</td>
<td>(3±2)</td>
<td></td>
</tr>
<tr>
<td>6666</td>
<td>0</td>
<td>&lt; 6</td>
<td>&lt; 6</td>
<td></td>
</tr>
<tr>
<td></td>
<td>2230</td>
<td>53±5</td>
<td>(50)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>3778</td>
<td>47±5</td>
<td>(50)</td>
<td></td>
</tr>
</tbody>
</table>

(Continued)
### TABLE 4.6 (cont.)

<table>
<thead>
<tr>
<th>$E_x$(keV) (Initial state)</th>
<th>$E_x$(keV) (final state)</th>
<th>Present work</th>
<th>Previous work</th>
<th>Subsequent work</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(a) (b) (c) (d) (e)</td>
<td>(f)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6762</td>
<td>0 &lt; 5 3±2 &lt; 2 2230 &lt; 5 &lt;25 4459 &lt;26</td>
<td>5007 100 (97) 20 80</td>
<td>6854 0 &lt; 7 &lt;10 2230 &lt;16 &lt;20</td>
<td>4282 74±8 100 70±10</td>
</tr>
</tbody>
</table>

(a) Coetzee et al. (Co 72).
(b) Forsblom et al. (Fo 70).
(c) Viitasalo and Fant (Vi 70).
(d) Piluso et al. (Pi 69).
(e) Endt and Van der Leun (En 67).
(f) Grawe et al. (Gr 73).
authors.

The 5.80 MeV level

The present results, indicating a 100% decay to the ground state, are in good agreement with previous and subsequent work.

The 6.22 MeV level

The present results, which indicate a 100% decay to the 2.23 MeV level, are in good agreement with all previous work. The upper limit of 4% set by the present work for a branch to the 5.01 MeV level is not inconsistent with the value of (3 ± 2)% recently reported by Grawe et al. (Gr 73).

The 6.41 MeV level

The results obtained for this level agree with all previous results.

The 6.62 MeV level

There is considerable disagreement among the various measurements of the branching ratios of this level. Coetzee et al. (Co 72) report a tentative new branch of (3 ± 2)% to the 5.41 MeV 3^+ level, in agreement with the recently received result of 2% from Grawe et al. (Gr 73). The upper limit reported here (< 3%) is not inconsistent with these results. The level was found to be strongly resonant at a bombarding energy of 8.97 MeV.

The 6.67 MeV level

All measurements agree that this level decays with approximately equal probability to the 2.23 MeV 2^+_1 state and the 3.78 MeV 0^+ 2 state. It is remarkable that there is no evidence for a ground-state transition even though there is a strong branch to the 3.78 MeV 0^+ level.

The 6.76 MeV level

Coetzee et al. (Co 72) report a (3 ± 2)% branch to the ground state,
based upon observation at one of the two $^{31}$P$(p,\gamma)^{32}$S resonances at which they found the level to be populated. They tentatively concluded that the remainder of the decay consisted of a 97% branch to the 5.01 MeV 3$^-$ level. The present results indicate a 100% branch to the 5.01 MeV level; however, the upper limit of 5% for a ground-state branch is not inconsistent with the results of Coetzee et al. Grawe et al. (Gr 73) set an upper limit of 2% for this branch. They also report an additional 20% branch to the 4.46 MeV 4$^+$ level, seen at two resonances in the $^{31}$P$(p,\gamma)^{32}$S reaction, which is not inconsistent with the present result of < 26% for this branch. Coetzee et al. argue that this level has a spin and parity of 2$^-$ of 3$^-$, based largely on their observation of a ground-state branch, which has not been confirmed by subsequent measurements.

The 6.85 MeV level

The present results for this level confirm the branches to the 4.46 MeV 4$^+$ level and the 5.41 MeV 3$^+$ level which were reported by Forsblom et al. (Fo 70) but not by Coetzee et al. (Co 72) and Grawe et al. (Gr 73), whose results indicate the decay to be 100% to the 4.28 MeV level.

The 7.00 MeV level

The major decay of this level is to the 2.23 MeV 2$^+_1$ state. There is no evidence in the present work for the 25% ground-state branch reported by Viitasalo and Fant (Vi 70). The present results are not inconsistent with a total intensity of (10 ± 5)% to levels other than the 2.23 MeV level (Co 72).

The 7.12 MeV level

The decay scheme obtained in the present work disagrees with that reported by Coetzee et al. (Co 72); the (8 ± 4)% branch to the 4.46 MeV level, which is seen in the present measurement, is not apparent in the results of Coetzee et al.
4-3.6. Conclusion

By means of a particle-gamma coincidence study of the reaction $^{32}\text{S}(p,p'\gamma)^{32}\text{S}$, the decay schemes for all known levels in $^{32}\text{S}$ between $E_x = 5.40$ and 7.15 MeV have been determined. The results verify some previous work and provide additional evidence concerning controversial branching ratios for the 5.41, 6.62, 6.76, 6.85, 7.00 and 7.12 MeV levels. Most of the previous information has been obtained from study of the reaction $^{31}\text{P}(p,\gamma)^{32}\text{S}$ with Ge(Li) detectors; since the resulting spectra are usually complex, ambiguities sometimes occur and it is not always easy to assign useful upper limits to unobserved branches. However, the technique used here is highly selective in that each spectrum contains only the gamma rays associated with the decay scheme of the level being studied. Thus, in spite of the inferior resolution of the NaI(Tl) detector, ambiguities rarely arise and upper limits on the relative intensities of unobserved branches may be readily assigned.

4-4 SPIN-PARITY COMBINATIONS IN $^{32}\text{S}$

4-4.1. Introduction

The first section of this chapter discussed those levels of $^{32}\text{S}$ where there are insufficient experimental data to permit a thorough comparison with the predictions of nuclear models. In particular, the spins and parities of the 6.41, 6.67, 6.76, 6.85 and 7.12 MeV levels were unknown at the commencement of the work described herein. The location and study of $3^-$ and $5^-$ states in the region of $E_x = 7$ MeV was of considerable interest with respect to the possible existence of a vibrational quintuplet of negative parity states (see subsect. 4-1.4).

This section describes the measurement of spin-parity combinations for levels in $^{32}\text{S}$ up to $E_x = 7.15$ MeV, with the main interest centering on
the region above 5 MeV. The technique used involves the study at extreme backward angles of inelastically scattered alpha particles from the reaction $^{32}\text{S}(\alpha,\alpha')^{32}\text{S}$. Since the target and projectile in this reaction both have $J^\pi = 0^+$, alpha particles scattered at $0^\circ$ or $180^\circ$ to the beam direction can populate only those states in the residual nucleus which have natural parity ($\pi = (-)^J$) (Litherland, Li 61b). In cases where the level spin is known, this technique provides a model-independent determination of the parity.

4-4.2. A study of the reaction $^{32}\text{S}(\alpha,\alpha')^{32}\text{S}$

Experimental procedure

A difficulty which arises in studying inelastic alpha particle scattering at $0^\circ$ or $180^\circ$ from $J^\pi = 0^+$ targets is that it is frequently not possible to place the detector exactly at $0^\circ$ or $180^\circ$; furthermore, the finite solid angle subtended by the detector means that alpha particles scattered at nearby angles are inevitably included. Consequently, if one uses only an annular counter encircling the incident beam (e.g., Ollerhead et al., 01 71), it may be difficult to decide whether the observation of a small yield for the inelastic scattering to a particular state means that the state has natural parity but the integrated cross section is low, or that the state has unnatural parity but the detector is not precisely at $180^\circ$. In the present work, this problem is avoided by measuring angular distributions of the scattered alpha particles and noting the manner in which the cross section varies as the detection angle approaches $0^\circ$ or $180^\circ$. If the cross section does not approach zero at $180^\circ$, the state has natural parity. If the cross section does tend to zero at $180^\circ$, then either the state has unnatural parity or the state has natural parity but the reaction mechanism at the chosen bombarding energy produces an angular distribution with a small cross section at $180^\circ$. In order to remove the latter possibility, data were obtained at a number of bombarding energies. A detailed excitation
function over a small energy region showed structure of typical width ~ 40 keV. Thus by obtaining data at approximately 100 keV intervals and using energy-averaged angular distributions, firm spin-parity assignments could be made.

Inelastically scattered alpha particles were detected at four angles. A 115 μm thick annular surface barrier detector was mounted 16 cm from the target so that it subtended detection angles in the laboratory coordinate system ranging from 177.5 to 178.9° relative to the incident beam direction. An energy resolution of 30 keV for scattered alpha particles was achieved by cooling the detector and by mounting a permanent magnet near the detector to suppress secondary electrons from the target. The background from alpha particles backscattered from the beam stop was minimized by constructing this from a thick block of carbon placed 3 m from the target. In addition to the annular detector, 100 μm thick surface barrier detectors were mounted 18 cm from the target at mean laboratory angles of 160°, 140° and 120°. The dimensions of collimators on these three detectors were chosen to maintain an energy resolution of about 30 keV in each case. To permit comparison of yields, the relative acceptance solid angles of the four detectors were measured using an alpha-particle source mounted in place of the target. Much care was taken to maintain good transmission through the annular detector in order to reduce X-ray background. Background due to backscattering of alpha particles from the metal surfaces of the target chamber walls and the target frame was minimized by lining these surfaces with polythene sheet.

The target consisted of approximately 60 μg/cm² of natural CdS evaporated onto a thin carbon film. Spectra from the four detectors were collected simultaneously in 2 x 2048 channels of a pulse-height analyser and 2 x 2048 channels of an IBM 1800 computer for 500 μC of integrated beam. Bombarding energies $E_\alpha$ ranged from 16.8 MeV in steps of 0.1 MeV to 17.6 MeV
and thence in steps of 0.2 MeV to 18.2 MeV. Typical spectra obtained at $E_\alpha = 17.5$ MeV are shown in fig. 4.10. The statistical quality of the data was considerably better for the annular counter than for the other counters, corresponding to the difference in acceptance solid angles. The excitation energies of $^{32}\text{S}$ levels adopted for fig. 4.10, and for other figures in this section, are as listed by Coetzee et al. (Co 72), except for the 6410 keV level for which the excitation energy is taken from the work of Forsblom et al. (Fo 70). In order to monitor possible contributions to the spectra from $^{34}\text{S}$ (natural abundance 4.2%), parallel spectra were taken at each bombarding energy using a target which was made in identical fashion to the natural CdS target, except that the sulphur was enriched to 85.6% $^{34}\text{S}$.

Results

After subtraction of linear backgrounds, peak areas were extracted for alpha-particle groups corresponding to particular levels in $^{32}\text{S}$ and were then summed over all bombarding energies. At some energies and angles, certain groups were obscured by contaminant peaks from $^{12}\text{C}$, $^{13}\text{C}$, $^{14}\text{N}$ and $^{16}\text{O}$ in the target. In these cases, allowance was made in the summation over all incident energies by normalizing all the summed yields to the same total integrated beam. Contributions from $^{34}\text{S}$ were found to be negligible for all groups of interest. After correction for the different detector solid angles, the data thus yielded energy-averaged four-point "angular distributions" for the inelastic alpha particles leading to each of the $^{32}\text{S}$ states studied. These distributions are shown in fig. 4.11; spins and parities are indicated on the diagram in cases where they are well established (Coetzee et al., Co 72). It can be clearly seen that levels known to have unnatural parity exhibit minima close to zero at $180^\circ$, while those with known natural parity do not. Fig. 4.12 shows the $180^\circ$ yield normalized to the sum of the yields at the other three angles; the known natural and unnatural parity levels can be seen to form two distinct bands. It is concluded from the results shown
Fig. 4.10. Alpha-particle energy spectra obtained at angles of 178° (annular counter) and 160° by bombarding a target of natural CdS with 17.5 MeV $^4$He$^{++}$ ions. The numbers associated with the arrows indicate excitation energies in keV.
Fig. 4.11. Energy-averaged four-point angular distributions from the reaction $^{32}\text{S}(\alpha,\alpha')^{32}\text{S}$. The full curves are drawn as a guide to the eye.
in figs. 4.11 and 4.12 that the 5.80, 6.41, 6.67, 6.76, 6.85 and 7.12 MeV levels all have natural parity.

In addition to the observed alpha-particle groups leading to the previously known levels of $^{32}$S, there was strong and consistent evidence in the 180° data for a group corresponding to a previously unreported level at $E_x = 6.58 \pm 0.01$ MeV. The group concerned could not be ascribed to any possible contaminant, and its energy varied with bombarding energy in such a way that it could correspond only to an atomic mass $A = 32 \pm 1$. This evidence prompted a study of the reaction $^{32}$S($p$,$p'$)$^{32}$S with a magnetic spectrometer in order that the possible existence of this new level in $^{32}$S might be investigated further. This work is described in the next subsection.

4-4.3. Magnetic analysis of the reaction $^{32}$S($p$,$p'$)$^{32}$S

Energy spectra of protons inelastically scattered from $^{32}$S were studied with the high-resolution magnetic spectrometer described in sect. 2-3 in order to confirm the existence of the 6.58 MeV level observed in the $^{32}$S($\alpha$,$\alpha'$)$^{32}$S data, to measure the excitation energy of the 6.58 MeV level more accurately, and to investigate the possible existence of other previously unreported levels in the region $6.4 < E_x < 7.1$ MeV.

The target used consisted of approximately 100 $\mu$g/cm$^2$ of PbS (sulphur enriched to 99.9% $^{32}$S). It was mounted in the target chamber described in the previous section (4-3), and bombarded with protons of energy $E_p = 11.2$ MeV. This bombarding energy corresponded to a relatively strong population of the 6.58 MeV level. The entrance slits of the spectrometer were set to produce acceptance angles of $\pm 1^\circ$ in the horizontal plane and $\pm 3^\circ$ in the vertical plane. Inelastically scattered protons were detected by a 500 $\mu$m thick surface barrier detector mounted behind a 1.6 mm slit at the focal plane of the spectrometer. Aluminium foil of thickness 0.001 cm was placed in front of the detector to separate protons from alpha
4.12. Energy-averaged ratio of 180° yield to the sum of the yields at other angles for alpha particles scattered from $^{32}$S. The symbols used for the 4695 and 6621 keV levels indicate upper limits.
particles.

Fig. 4.13 shows proton spectra obtained by varying the magnetic field of the spectrometer at laboratory angles of 30° and 150°. The background between peaks appeared to be due to protons scattered from the target ladder and was minimized by suitable collimation of the beam. The arrows indicate previously known levels of \(^{32}\)S. In each case an excitation energy scale is shown; this was obtained by least squares fitting to the data of an expression \(E_x = a + bf + cf^2\), where \(f\) is the NMR frequency corresponding to the half-height of the high-energy edge (estimated by manual fitting) of the group from a \(^{32}\)S level of excitation energy \(E_x\), and \(a\), \(b\) and \(c\) are fitted parameters. All \(^{32}\)S groups shown in fig. 4.13 were used in the fitting procedure. At both angles the 6.58 MeV level is clearly evident. The least squares analysis gives its excitation energy as \((6582 \pm 4)\) keV at 30° and \((6579 \pm 6)\) keV at 150°; consequently its excitation energy is taken as \((6581 \pm 3)\) keV. Restricting the fits to the 6.41, 6.62 and 6.67 MeV levels, gives the value \(E_x(6.58) = (6579 \pm 3)\) keV. The variation of group energy with angle is consistent only with \(A = 32\). There is no indication in either spectrum of any other previously unreported levels in \(^{32}\)S.

4.4.4. Gamma-ray decay scheme of the 6.58 MeV level of \(^{32}\)S

Experimental procedure

The previous section (4-3) described a study of the gamma-ray decay schemes of \(^{32}\)S levels in the region of excitation under consideration here. It was considered desirable to determine the decay scheme of the newly discovered 6.58 MeV level. This was done using the technique described in sect. 4-3.

A target consisting of approximately 200 \(\mu g/cm^2\) of PbS (sulphur enriched to 99.9% \(^{32}\)S) evaporated onto a thin carbon backing was bombarded
Fig. 4.13. Spectra obtained by magnetic analysis of protons scattered at 30° and 150° from a target of Pb\textsuperscript{32}S. The bombarding energy was 11.2 MeV.
with 9.16 MeV protons, and inelastically scattered protons populating the 6.58 MeV level were detected at the focal plane of the double-focussing spectrometer with a surface barrier detector as described in the previous subsection (4-4.3). The entrance slits of the spectrometer were set at ±2.5° (horizontal) and ±5.0° (vertical) and the focal plane slit was 3.2 mm wide. Sheets of 2.5 mm lead and 0.4 mm copper were inserted between the NaI(Tl) detector face and the steel support plate in order to attenuate low-energy background radiation. The beam energy of 9.16 MeV was chosen to optimize the yield to the 6.58 MeV level while keeping the energy low enough that beam-induced background radiation remained tolerable. Beam currents were limited to 40 nA so that pile-up in the gamma ray spectrum remained within acceptable limits.

Due to the weak population of the 6.58 MeV level, the coincidence count rate was extremely low. The spectrum shown in fig. 4.14 was obtained after running for 43 hours; the insert of the diagram shows the particle spectrum obtained by sweeping the magnetic field of the spectrometer, and the effective window imposed by the 3.2 mm detector slit.

Results

The coincidence gamma-ray spectrum was analysed using the lineshape fitting program described in sect. 4-3. The spectra were corrected for random coincidences (13%) by subtracting a singles spectrum with an intensity normalization deduced from the measured ratio of real-to-random coincidences. The results of the lineshape analysis are consistent with there being no decay mode of the 6.58 MeV level other than via the 2.23 MeV first excited state. The counts in the high-energy region of the spectrum can be wholly accounted for by summing of the cascade gamma rays, assuming that these have an isotropic angular correlation. An upper limit of 3% can be placed on the direct branching to the ground state, if it is assumed that the summing
Fig. 4.14. Gamma-ray spectrum taken in coincidence with protons populating the 6.58 MeV level of $^{32}$S. Random coincidences have been subtracted from the data. The full line shows a lineshape fit obtained by assuming that the level decays solely to the 2.23 MeV level, and including a calculated contribution from summing. The insert shows the proton spectrum obtained by sweeping the magnetic field of the spectrometer, and the effective window imposed by the 3.2 mm detector slit.
correction may be overestimated by a factor of two due to possible anisotropy of the angular correlation of the gamma rays. It is noteworthy that the coincidence gamma-ray spectrum shown in fig. 4.14 is characteristic of $^{32}$S, which further supports the validity of the assignment of peaks in the charged particle spectrum to a level at $E_x = 6.58$ MeV in $^{32}$S.

4-4.5. Discussion

The results of the present work are summarized in table.4.7. The spin-parity combinations obtained are consistent with those spins and parities which are well established. Table 4.7 also contains results recently received from Grawe et al. (Gr 73), which can be seen to be in agreement with the present work. The 7.00 MeV level was not observed in any of the inelastic alpha particle scattering data; this is consistent with a $T = 1$ assignment for the level and the effective operation of the isospin selection rule. The unambiguous assignment of natural parity to the $J = 1$ state at $E_x = 5.80$ MeV provides a model-independent confirmation of the negative parity assignment previously made from DWBA analysis of angular distributions from the $^{31}$P($^3$He,d)$^{32}$S reaction (Graue et al., Gr 68 and Morrison, Mo 70).

4-5 SUMMARY AND DISCUSSION

This chapter has discussed the various models that have been applied to $^{32}$S, and has given descriptions of experiments undertaken to measure properties of states in the intermediate excitation energy range of from 5 to 7 MeV, where there was a considerable lack of information, particularly with regard to spins and parities.

Sect. 4-2 contained accounts of experiments to measure the lifetime, spin and decay scheme of the 5.80 MeV level, in order to evaluate its possible membership of the quintuplet of negative parity states arising
**TABLE 4.7**

SPIN-PARITY COMBINATIONS IN $^{32}$S

<table>
<thead>
<tr>
<th>$E_x$(keV)†</th>
<th>$J^\pi$ *</th>
<th>Parity; natural (N) or unnatural (U)</th>
</tr>
</thead>
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<tr>
<td></td>
<td></td>
<td>Present work</td>
</tr>
<tr>
<td>0</td>
<td>0$^+$</td>
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</tr>
<tr>
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<td>2$^+$</td>
<td>N</td>
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<tr>
<td>3778</td>
<td>0$^+$</td>
<td>N</td>
</tr>
<tr>
<td>4282</td>
<td>2$^+$</td>
<td>N</td>
</tr>
<tr>
<td>4459</td>
<td>4$^+$</td>
<td>N</td>
</tr>
<tr>
<td>4695</td>
<td>1$^+$</td>
<td>U</td>
</tr>
<tr>
<td>5007</td>
<td>3$^-$</td>
<td>N</td>
</tr>
<tr>
<td>5413</td>
<td>3$^+$</td>
<td>U</td>
</tr>
<tr>
<td>5549</td>
<td>2$^-$</td>
<td>N</td>
</tr>
<tr>
<td>5798</td>
<td>1$^-$</td>
<td>N</td>
</tr>
<tr>
<td>6224</td>
<td>2$^-$</td>
<td>U</td>
</tr>
<tr>
<td>6410</td>
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</tr>
<tr>
<td>6581</td>
<td></td>
<td>N</td>
</tr>
<tr>
<td>6621</td>
<td>4$^-$</td>
<td>U</td>
</tr>
<tr>
<td>6666</td>
<td></td>
<td>N</td>
</tr>
<tr>
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<td>(2$^-$,3)</td>
<td>N</td>
</tr>
<tr>
<td>6854</td>
<td></td>
<td>N</td>
</tr>
<tr>
<td>7004</td>
<td>1$^+,{T=1}$</td>
<td>-</td>
</tr>
<tr>
<td>7117</td>
<td>(2$^+,{T=1}$)</td>
<td>N</td>
</tr>
</tbody>
</table>

† The excitation energies are taken from Coetzee et al. (Co 72), Forsblom et al. (Fo 70) and the present work.

* The $J^\pi$ assignments are as listed by Coetzee et al. (Co 72); the parentheses for the 6762 and 7117 keV levels indicate that these assignments are not as well established as the others.
The result for the lifetime of $14 \pm 7 \text{ fs}$ is consistent with previous and subsequent measurements. The spin was found to be 1, in agreement with previous results obtained from stripping reactions combined with those of gamma decay and lifetime measurements, and with previous particle-gamma angular correlation studies. The present work confirmed that the level decays 100% to the ground state.

Sect. 4-3 described an experiment to measure the gamma ray decay schemes of all known levels of $^{32}\text{S}$ in the intermediate excitation energy region. The results generally agree well with previous ones for strong branches. Controversial weak branches have, in some cases, been confirmed by the present results, but in other cases are not apparent in the present data.

The experiments, discussed in sect. 4-4, to determine spin-parity combinations for all levels of $^{32}\text{S}$ up to 7 MeV excitation show that the 5.80, 6.41, 6.67, 6.76, 6.85 and 7.12 MeV levels have natural parity. In particular, the negative parity of the 5.80 MeV level was confirmed by this model-independent method. The data obtained from this experiment revealed a possible new natural parity level of $^{32}\text{S}$ at 6.58 MeV. This was confirmed by magnetic analysis of inelastic proton scattering. The gamma decay of this new level was found to be predominantly to the 2.23 MeV level.

4-5.1. **Summary of present information on spins and parities of $^{32}\text{S}$ levels between 5.40 and 7.12 MeV**

The spins and parities of the 5.55 MeV $2^+$ and 6.22 MeV $2^-$ levels are well established (En 67) and require no further discussion.

The 5.41 MeV level

Poletti and Grace (Po 66) assigned $J = 3$ to this level from particle-gamma angular correlation studies of the reaction $^{32}\text{S}(p,p'\gamma)^{32}\text{S}$. 
The lifetime of between 100 and 200 fs (Pi 69; O1 70; Ga 71) indicates positive parity.

The 5.80 MeV level

The results of the present experiments determine this level to have $J^\pi = 1^-$. This is consistent with the results of Poletti and Grace (Po 66), Graue et al. (Gr 68) and Morrison (Mo 70).

The 6.41 MeV level

The spin and parity of this level are not known. The present results indicate a 100% decay to the 2.23 MeV $2^+_1$ state; if it is assumed that this transition has as its lowest order multipole either $E1$, $M1$ or $E2$, then it follows that $J^\pi = 0^+, 1^+, 2^+, 3^+$ or $4^+$. The present result of natural parity for this level reduces this range of possibilities to $J^\pi = 0^+, 1^-, 2^+, 3^-$ or $4^+$.

The 6.58 MeV level

The present results indicate that this previously unknown level decays predominantly to the 2.23 MeV $2^+_1$ level and has natural parity. Arguments similar to those applied to the 6.41 MeV level then suggest $J^\pi = 0^+, 1^-, 2^+, 3^-$ or $4^+$.

The 6.62 MeV level

This level has a spin and parity $4^-$ (Dorum, Do 68; Coetzee et al., Co 72; Cairns et al., Ca 72; Graue et al., Gr 73). This is consistent with the present result of unnatural parity for this level.

The 6.67 MeV level

The spin and parity of this level are also unknown. The present and previous results indicate that the level decays with approximately equal probability to the 2.23 MeV $2^+_1$ state and the 3.78 MeV $0^+_2$ state. If it is assumed that for each of these two decay modes the lowest order multipole is either $E1$, $M1$ or $E2$, then the decay scheme requires $J^\pi = 1^+$.
or $2^+$. The present work indicates natural parity for this level, hence $J^{\pi} = 1^-$ or $2^+$.

The 6.76 MeV level

Coetzee et al. (Co 72) argue that this level has a spin and parity of $2^-$ or $3^+$, based largely on their observation of a $(3 \pm 2)\%$ ground-state branch. The existence of this branch has not been confirmed in the present work or that of Grawe et al. (Gr 73). Angular distribution studies of the reaction $^{31}P(p,\gamma)^{32}S$ by these last authors give $J = 3, 4$ or $5$. The present results and those of Grawe et al. indicate natural parity; thus if the ground-state branch is established then $J^{\pi}$ is restricted to $3^-$. However, there is strong evidence that the level has spin and parity $5^-$. Källne and Sundberg (Ka 71) propose $J^{\pi} = 5^-$ on the basis of inelastic proton scattering at 185 MeV. Also, the systematic variation of excitation energies of the lowest $5^-$ states in even nuclei in the mass region $A = 28 - 40$ strongly suggest that the first $5^-$ state in $^{32}S$ should occur at an excitation energy within about 500 keV of 6.2 MeV (Greene et al., Gr 72). The 6.41, 6.58, 6.67 and 7.12 MeV levels, whose spins and parities are all unknown, exhibit strong gamma-ray transitions to $2^+$ states, which renders a $J^{\pi} = 5^-$ assignment most unlikely for any of them. The remaining level in this region with unknown spin and parity is the 6.76 MeV level. The decay scheme results and the natural parity assignment given in the present work are consistent with a $5^-$ assignment for this level.

The 6.85 MeV level

The spin and parity of this level are not known. If it is assumed that for each of the three decay modes found in the present work, the lowest order multipole is either $E1$, $M1$ or $E2$, then the decay scheme requires $J^{\pi} = 2^+, 3^\pm$ or $4^+$. The present assignment of natural parity then restricts $J^{\pi}$ to $2^+$, $3^-$ or $4^+$. This agrees with the conclusions reached by Grawe et
al. (Gr 73) from observation of feeding of this level by a $J = 4$ resonance in the $^{31}\text{P}(p,\gamma)^{32}\text{S}$ reaction.

The 7.00 MeV level.

Armini et al. (Ar 68a) assigned $J^\pi = 1^+$, $T = 1$ to this level on the basis of an observation of a super-allowed $\beta^+$ decay to this level from the ground state of $^{32}\text{Cl}$.

The 7.12 MeV level

Graue et al. (Gr 68) tentatively assigned $J^\pi = 2^+$, $T = 1$ to this level from DWBA analysis of angular distributions from the reaction $^{31}\text{P}(^3\text{He},d)^{32}\text{S}$.

4-5.2. Discussion in terms of the intermediate coupling model

As previously discussed in subsect. 4-1.5, Castel et al. (Ca 71) have considered the coupling of the $J^\pi = 2^+$ one-phonon state in $^{32}\text{S}$ to particle-hole excitations for energies up to about 8 MeV. The assignment in the present work of $J^\pi = 1^-$ to the 5.80 MeV level is in disagreement with their calculations which require that this level have $J^\pi = 1^+$.

4-5.3. The shell model

Subsect. 4-1.6 discussed the shell model calculations of Wildenthal et al. (Wi 71) and Glaudemans et al. (Gl 71a) who consider excitations in the 2s-1d shell. These calculations are relevant only to positive parity levels and predict, among other things, a second $4^+$ level for $^{32}\text{S}$ at about 5.6 MeV. The 6.41 and 6.58 MeV levels are possible candidates for this assignment.

Negative parity states in $^{32}\text{S}$ need to be considered in terms of excitation to the $1f_{7/2}$ subshell or out of the 1p shell. Fig. 4.15 shows the experimentally determined negative parity levels of $^{32}\text{S}$ compared with
Fig. 4.15. Comparison of experimentally determined negative parity levels in $^{32}$S with the results of shell model calculations by Grawe et al. (Gr 73) with a $2s_{1/2}$, $1d_{5/2}$, $1f_{7/2}$, $2p_{3/2}$ configuration space. The results shown here correspond to a set of parameters for the two-body residual interaction and single-particle binding energies chosen to give best agreement with the positions of experimental negative parity levels and known binding energy of $^{32}$S.
the predictions of the calculations of Grawe et al. (Gr 73) discussed in subsect. 4-1.6. These authors consider a model space including the $2s_{1/2}$, $1d_{3/2}$, $1f_{7/2}$ and $2p_{3/2}$ orbits outside an inert $^{28}\text{Si}$ core. The $3_1^-$ and $1^-$ levels are well reproduced by the calculations, while the positions of the $2^-$, $4^-$ and $5^-$ levels represent fair agreement between theory and experiment. It is interesting to note that these shell model calculations predict no second $3^-$ state in the region $E_x = 5 - 7 \text{ MeV}$.

Shell model calculations by Erne (Er 66) and Greene et al. (Gr 72) for $^{34}\text{S}$, $^{36}\text{Ar}$, $^{38}\text{Ar}$ and $^{40}\text{Ca}$ assuming a pure $(d_{3/2})^{-1}f_{7/2}$ configuration indicate that the properties of low-lying negative parity states in these nuclei can be interpreted in terms of simple shell model structure. It is possible, therefore, that a similar situation exists for $^{32}\text{S}$.

4-5.4. The vibrational model

The $J^T = 1^-$ assignment to the 5.80 MeV level is consistent with the identification of this level as a possible member of the proposed quintuplet of negative parity states predicted by the coupled quadrupole-octupole vibrator model (see subsect. 4-1.4). This model predicts an enhanced E2 transition to the 5.01 MeV $3^-$ state and an enhanced E3 transition to the 2.23 MeV $2^+$ state. The upper limits imposed by the results given in sect. 4-2 for the 5.80 MeV level are 470 and 4200 W.u., respectively, so that the failure to observe branches to these states is not inconsistent with the model. In order to explain the observed E1 transition to the ground state ($|M|^2 \sim 4 \times 10^{-4}$ W.u.), it is necessary to introduce a non-collective component into the wavefunction. This is also true for the 5.01 MeV $3^-$ one-phonon state which has an E1 transition of approximately $10^{-4}$ W.u. to the 2.23 MeV $2^+$ state in addition to the E3 transition of approximately 20 W.u. to the ground state (Ollerhead et al., 01 70). Thus the non-collective component required for the 5.80 MeV state
is similar to that required for the $3^-$ state itself.

The branching ratios and lifetimes of the five two-phonon states may be calculated from the decay properties of the two single-phonon states. Results for the 5.80 MeV $1^-$ state, the 6.22 MeV $2^-$ state and the 6.62 MeV $4^-$ state are presented in table 4.8. It was assumed that each member of the quintuplet decays in the following way: by an E2 transition of 9 W.u. to the $3^-$ one-phonon state at 5.01 MeV, by an E3 transition of 20 W.u. to the $2^+$ one-phonon state at 2.23 MeV and by dipole transitions of strengths $1 \times 10^{-4}$ W.u. (both for E1 and M1) to all states of appropriate angular momentum. These transition strengths were chosen to be approximately equal to those observed for the E2, E3 and E1 decay of the two single-phonon states (O1 70, In 71).

The degree of agreement between calculations and experiment is encouraging in view of the very simple nature of the calculations. The major decay of the 5.80 MeV level is reproduced by the calculations but the one-octupole phonon decay to the $2^+_1$ level is overestimated. The predicted lifetime of 38 fs is about three times greater than the measured value of $12 \pm 3$ fs. The major decay of the 6.22 MeV level is also reproduced by the calculations, but the branching ratios to the $2^+_{2}$, $1^+$ and $3^-_1$ levels are overestimated. There is reasonable agreement between the predicted lifetime of 125 fs for this level and the experimental value of $80 \pm 20$ fs. In the case of the 6.62 MeV $4^-$ level, the calculations predict the major branches to be to the 4.46 MeV $4^+$ level (52%) and to the 5.01 MeV $3^-$ level (38%). This is borne out by experiment, but the observed magnitudes (10% and 81%, respectively) are not in agreement with the predictions. Also, the calculations predict a 9% branch to the 5.41 MeV $3^+$ level which is not seen in the present data. The observed 9% branch to the 2.23 MeV $2^+$ level is underestimated by the calculations which predict only a 1% branch.

Subsect. 4-5.2 shows that the most likely candidate for the $5^-$
The calculated values were obtained assuming the states to be members of the two-phonon quintuplet predicted by the vibrational model. Experimental values were derived from the present work and from results listed by Piluso et al. (Pi 69), Evans et al. (Ev 68), Coetzee et al. (Co 72) and Ingebretsen et al. (In 71).

<table>
<thead>
<tr>
<th>Initial state $E_x$(MeV),$J^\pi$</th>
<th>Final state $E_x$(MeV),$J^\pi$</th>
<th>Calculated width (eV)</th>
<th>Branching (%)</th>
<th>Lifetime (fs) of initial state</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.80, 1$^-,$</td>
<td>0,0$^+$</td>
<td>$1.3 \times 10^{-2}$</td>
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<tr>
<td>2.23, 2$^+$</td>
<td>$3.1 \times 10^{-3}$</td>
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</tr>
<tr>
<td>4.69, 1$^+$</td>
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</tr>
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<td>5.01, 3$^-$</td>
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<td>6.21, 2$^-,$</td>
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<td>$1.2 \times 10^{-4}$</td>
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</table>
membership of the quintuplet is the 6.76 MeV level. Table 4.9 shows the gamma-ray transition strengths, branching ratios and lifetimes calculated on the basis that the 6.76 MeV level is the 5⁻ member. The major decays are predicted to be to the 4.46 MeV 4⁺ level (53%) and to the 5.01 MeV 3⁻ level (46%). The present results give < 26% and 100%, respectively, for these branches, while Grawe et al. (Gr 73) report (20 ± 12)% and (80 ± 12)%, respectively, in reasonable agreement with the prediction. The measured lifetime value of >300 fs (Gr 73) is in accord with the predicted value of 415 fs. Thus it can be concluded that the measured properties of the 6.76 MeV level are consistent with its being the 5⁻ member of the proposed quadrupole-octupole vibrational quintuplet.

Possible candidates for 3⁻ membership are the levels at \( E_x = 6.41, 6.58 \) and 6.85 MeV. Table 4.10 compares the measured properties of these levels with those predicted by the model for the 3⁻ member. It can be seen that the measured branching ratios for the 6.41 and 6.58 MeV levels are in considerably better agreement with the theory than are those of the 6.85 MeV level, although the measured lifetime for this level is in fair agreement with the predicted value. It is concluded from these results that the 6.41 and 6.58 MeV levels are the most likely candidates for the 3⁻ membership of the vibrational quintuplet.

An attempt was made to investigate possible systematics of the \( J^\pi = (1 - 5)^- \) quintuplet of states in even-even nuclei throughout the periodic table from \(^{28}\text{Si}\) to \(^{200}\text{Hg}\). However, lack of experimental information on low-lying negative parity states in the mass region \( A = 60 - 200 \) precluded any definite conclusions from this search.

4-5.5. Outstanding experimental information required

The outstanding experimental information required to permit evaluation of all of the models which have been applied to the region of intermediate
The calculated values were obtained assuming the state to be the $J^\pi = 5^-$ member of the two-phonon quintuplet predicted by the vibrational model. Experimental values were derived from the present work and from results listed by Grawe et al. (Gr 73).

<table>
<thead>
<tr>
<th>Initial state $E_x$ (MeV), $J^\pi$</th>
<th>Final state $E_x$ (MeV), $J^\pi$</th>
<th>Calculated width (eV)</th>
<th>Branching (%)</th>
<th>Lifetime (fs) of initial state</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.76, (5^-)</td>
<td>2.23, 2^+</td>
<td></td>
<td>8.4 x 10^-4</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.46, 4^+</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.01, 3^-</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.62, 4^-</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE 4.10  
BRANCHING RATIOS AND LIFETIMES OF THE 6.41, 6.58 AND 6.85 MeV STATES OF $^{32}$S 

The calculated values were obtained assuming the states to be the $3^{-}$ member of the two-phonon quintuplet predicted by the vibrational model. Experimental values were derived from the present work and from results listed by Coetzee et al. (Co 72).

<table>
<thead>
<tr>
<th>Initial state $E_x$ (MeV), $J^\pi$</th>
<th>Final state $E_x$ (MeV), $J^\pi$</th>
<th>Calculated width (eV)</th>
<th>Branching (%)</th>
<th>Lifetime (fs) of initial state</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.41, ($3^-$)</td>
<td>2.23, $2^+$</td>
<td>$5.0 \times 10^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.28, $2^+$</td>
<td>$6.6 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.46, $4^+$</td>
<td>$5.1 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.01, $3^-$</td>
<td></td>
<td></td>
<td>$5.8 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>5.41, $3^+$</td>
<td></td>
<td></td>
<td>$6.9 \times 10^{-5}$</td>
</tr>
<tr>
<td></td>
<td>5.55, $2^+$</td>
<td>$4.4 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.22, $2^-$</td>
<td></td>
<td></td>
<td>$1.4 \times 10^{-8}$</td>
</tr>
<tr>
<td>6.58, ($3^-$)</td>
<td>2.23, $2^+$</td>
<td>$5.7 \times 10^{-3}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.28, $2^+$</td>
<td>$8.3 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4.46, $4^+$</td>
<td>$6.6 \times 10^{-4}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5.01, $3^-$</td>
<td></td>
<td></td>
<td>$8.2 \times 10^{-6}$</td>
</tr>
<tr>
<td></td>
<td>5.41, $3^+$</td>
<td></td>
<td></td>
<td>$1.1 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>5.55, $2^+$</td>
<td>$7.5 \times 10^{-5}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6.22, $2^-$</td>
<td></td>
<td></td>
<td>$9.6 \times 10^{-8}$</td>
</tr>
</tbody>
</table>

(Continued)
<table>
<thead>
<tr>
<th>Initial state (E_x, J^m)</th>
<th>Final state (E_x, J^m)</th>
<th>Lifetimes (fs) of initial state</th>
<th>Branching (%)</th>
<th>Calculated</th>
<th>Experimental</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6.85, 3^-)</td>
<td>(2.23, 2^+)</td>
<td>(6.8 \times 10^{-3})</td>
<td>66</td>
<td>(2.1 \times 10^{-5})</td>
<td>&lt;16</td>
</tr>
<tr>
<td></td>
<td>(4.28, 2^+)</td>
<td>(1.2 \times 10^{-3})</td>
<td>11</td>
<td>9</td>
<td>74</td>
</tr>
<tr>
<td></td>
<td>(4.46, 4^+)</td>
<td>(9.4 \times 10^{-4})</td>
<td>2</td>
<td>9</td>
<td>2</td>
</tr>
<tr>
<td></td>
<td>(5.01, 3^-)</td>
<td>(1.1 \times 10^{-5})</td>
<td>10</td>
<td>2</td>
<td>&lt;2</td>
</tr>
<tr>
<td></td>
<td>(5.41, 3^-)</td>
<td>(2.1 \times 10^{-4})</td>
<td>2</td>
<td>3</td>
<td>&lt;3</td>
</tr>
<tr>
<td></td>
<td>(5.55, 2^+)</td>
<td>(1.5 \times 10^{-4})</td>
<td>2</td>
<td>3</td>
<td>&lt;3</td>
</tr>
<tr>
<td></td>
<td>(6.22, 2^-)</td>
<td>(5.3 \times 10^{-7})</td>
<td>0</td>
<td>11</td>
<td>&lt;11</td>
</tr>
</tbody>
</table>

Note: \(M_L\) and \(E2\) columns are not shown in the table.
excitation in $^{32}$S is the following: further investigation of the existence of a ground-state branch from the 6.76 MeV level, firm spin assignments for the levels at 6.41, 6.58, 6.67, 6.76, 6.85 and 7.12 MeV, and lifetime determinations for the levels at 6.41, 6.58 and 6.76 MeV. Existing lifetime data for other states in this region are given in tables 4.8, 4.9 and 4.10.
CHAPTER 5

CONCLUSION

Chapters 3 and 4 of this thesis have outlined the various attempts to interpret experimentally measured properties of $^{27}$Al and $^{32}$S in terms of nuclear models. Experiments to measure some of the properties of excited states of these two nuclei have been described and the results obtained have been compared to the predictions of the models. No single model is, at present, capable of providing a complete description of either $^{27}$Al or $^{32}$S; several may be used with varying degrees of success to account for the different properties such as spins, parities and excitation energies of these two nuclei.

It is interesting to consider the nuclear shapes of $^{27}$Al and $^{32}$S in relation to their positions in the 2s-1d shell and to the predictions of nuclear models. $^{27}$Al lies in the transition region of the 2s-1d shell where the nuclear shapes are changing from a strong prolate deformation around mass $A \approx 23-25$ to the oblate deformation found at $A \approx 28$. In the upper region of the shell, there is a tendency for the nuclei to assume oblate or prolate shapes with smaller deformations; sphericity is achieved at the shell closure at $^{40}$Ca. It is not certain whether the deformation of $^{27}$Al ($Q = 0.15$ barns, Fu 69) is prolate or oblate as this is model-dependent on the assignment of $K = 1/2$ or $5/2$ for the ground state. Hartree-Fock calculations (Ri 68) suggest an oblate shape for $^{27}$Al and the Coriolis band-mixing calculations of Dehnhard (De 72) assuming an oblate deformation close to that of $^{28}$Si, give better agreement with experimental $B(E2)$ values than do similar calculations performed by Malik and Scholz (Ma 67) with a prolate shape. The measured value of $Q_0 = 0.6$ barns (Na 70) for $^{32}$S indicates a prolate deformation for the $2^+_1$ state. The magnitude of this
quadrupole moment is surprisingly large in view of the proposed vibrational character of $^{32}\text{S}^+$; it is comparable to that of known rotators (e.g. $^{24}\text{Mg}$) in the 2s-1d shell. This result is consistent with the Hartree-Fock calculations of Bassichis et al. (Ba 67) but not with those of Banerjee et al. (Ba 69) and Zofka and Ripka (Zo 71) for the ground state of $^{32}\text{S}$. However, as pointed out by Bar-Touv and Goswami (Ba 69a), the spherical and deformed solutions of the Hartree-Fock Hamiltonians for light closed sub-shell nuclei are close in energy and may coexist as two distinct states in the same nucleus.

The non-zero deformation of $^{27}\text{Al}$ suggests possible rotational behaviour; however, the rotational model has been shown to have limited success in accounting for the properties of this nucleus. This is somewhat surprising in view of the successful description of neighbouring nuclei, most recently $^{29}\text{Si}$ (Vi 73), in terms of the rotational model. This model has also been applied to the low-lying $0^+_1$, $2^+_1$, $0^+_2$ and $4^+_1$ levels of $^{32}\text{S}$ (Ba 69a) with some success on the assumption of a deformed ground state, but no other possible members of the ground-state rotational band have yet been located.

The proposed spherical shape (from Hartree-Fock calculations) for the ground state of $^{32}\text{S}$ and the observed level scheme suggested that this nucleus is a possible vibrator; the harmonic vibrator model has been quite successful in explaining many of the properties of low-lying positive parity states. The extension of the model to negative parity states around 5-7 MeV is limited by lack of experimental information on spins, parities and gamma decay rates. There is, however, a serious difficulty in reconciling such vibrational behaviour with the large quadrupole moment for the $2^+_1$ state;

\[ + \text{Olin et al. (O1 73) have recently given a preliminary report of a new measurement of } Q_o(2^+_1) \text{ for } ^{32}\text{S which yields a result } \sim 0.15 \text{ barns, i.e., in better agreement with the vibrational picture.} \]
the traditional harmonic vibrator model assumes a spherical equilibrium shape. Such a difficulty also exists with $^{114}$Cd and other nuclei around $A \sim 110$, which display vibrational features not explicable by the rotational model. A large number of measurements of $Q(2^+_1)$ for $^{114}$Cd yield values ranging from near zero, as expected for a harmonic vibrator, to the rigid rotator value. A recent, more precise measurement (Be 71) (using the reorientation effect) indicates $Q(2^+_1)$ is intermediate between these extremes (-0.3 barns) but is still inconsistent with the picture of a spherical harmonic vibrator.

Extensive theoretical effort has been expended, with some success, to reconcile the supposed vibrational behaviour with non-spherical shapes and it is necessary to introduce anharmonic effects into the calculations. For example, Tamura and Udagawa (Ta 66) propose that the magnitude of $Q(2^+_1)$ for $^{114}$Cd can be reproduced by allowing mixing of the $2^+_1$ and $2^+_2$ states. Sørenson (Sø 67) has considered anharmonic vibrations in $^{114}$Cd in a calculation which replaces pairs of fermions in an even fermion system by an expansion in terms of boson operators. Non-linear terms up to third order were included in the calculation. It is possible that similar calculations along these lines for $^{32}$S will account for the large quadrupole moment and apparent vibrational character. Mixing of the $2^+_1$ and $2^+_2$ states of $^{32}$S is already required (Ga 71, Ha 71) to account for the cross-over transition from the $2^+_2$ state. However, difficulties then arise with reproducing the ratio $E_x(2^+_2)/E_x(2^+_1)$.

The large scale shell model calculations that have been carried out on $^{27}$Al and $^{32}$S, particularly by Wildenthal and his collaborators, show that this model may be capable of giving a comprehensive account of their properties. In particular, the magnitude and sign for $Q(2^+_1)$ for $^{32}$S was fairly well reproduced by the model. Present deficiencies may be partly due to the adoption of truncated configuration spaces for the calculations. For example, ignoring particle excitation to the $1f_{7/2}$ orbits in $^{32}$S restricts the relevance of the calculations to positive parity levels only.
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<tr>
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<th>Author(s)</th>
<th>Title/Publication Details</th>
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<tr>
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<td>Nucl. Phys. 57 (1964) 403.</td>
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<td>(Br 73)</td>
<td>C. Broude, F.A. Beck and P. Engelstein</td>
<td>Preprint received from Centre de Recherches Nucleaires, Strasbourg (1973).</td>
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<tr>
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<td>D. Dehnhard</td>
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</tr>
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</table>

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