MULTIDIMENSIONAL DECISION ANALYSIS IN PUBLIC INVESTMENT ANALYSIS: THEORY AND PRACTICE

by

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"I believe that the major advances in the next decade will not be in the direction of adding to the stockpile of techniques that universities can teach, but insights into how to make the analysis procedure simple. If we can make it simple enough for any intelligent, logical person to understand and use for himself or herself, the products of our research will be easy to sell." (18, p.6)

"Modelling systems, identifying objectives and solving programs are tough enough, but the awesome problem in public investment is the decision making process. We know so little about decision makers, about how the decision making process works, about the institutions which control decisions and about how these institutions react. If the analyst is really going to affect decision making he must have more knowledge about the process." (25, p.15)

CONTENTS

1. INTRODUCTION

Most important investment decisions involve several criteria or dimensions, e.g. a weapon system could be judged by cost, portability, reliability and firepower. Moreover, the values of these dimensions that alternative courses of action will produce is rarely known for certain, because they will occur in the future or because analysis to obtain the information would be too expensive or too time-consuming. These comments apply to public sector investment particularly, because the market does not provide a price that incorporates several dimensions for public goods, and because much public investment is one-off, having rarely been done before, e.g., Medibank.

1.1 Aim

The aim of this thesis is to investigate the application of multidimensional decision analysis (MDA) in public investment analysis, based on a study of its theory and procedures.

1.2 Outline

The outline of the thesis is as follows. Chapter 1 is an introduction, with a summary of the terms and notation used. Chapter 2 notes that the thesis is limited to the analysis of decisions which have dimensions that can be traded off against each other. Any decision analysis should start with a consideration of all alternatives and so this investigation of decision

analysis then reviews various alternatives to MDA for public investment analysis and finds that MDA is apparently worthy of further study (Chapter 3). Chapter 4 then reviews the theory and practice of MDA when dimension levels for each alternative are known with certainty, or more succintly, MDA under certainty. Next, Chapter 5 treats MDA under uncertainty by outlining the basics of utility theory and its application in MDA. The next six chapters elaborate on Chapter 5 by treating probability and the various kinds of utility models that have been developed.

Chapter 12 discusses some published applications of the theory, concentrating on MDA under uncertainty, and then a case study of public investment analysis, that of deciding types of truck for a military fleet, is presented. Based on the theory and practice of MDA in public investment analysis covered up to this point, Chapter 13 then makes a preliminary evaluation of the usefulness of MDA and finds that simple models appear to be the more useful for most public investment analysis. The thesis concludes with some suggestions on how to simplify and hence improve the applicability of MDA.

1.3 Terms and Notation

Some of the terms used in this thesis should be clarified at this stage. The most important are underlined in the following brief, introductory description of the elements of MDA.

MDA decomposes a decision into its dimensions and considers each dimension separately. Then the units on each dimension, e.g., miles/gallon, are transformed into value units if the MDA is done with the value of each alternative known for each dimension known with certainty or into utility units if the MDA is done under uncertainty. This transformation allows for non-linear preferences of the decision-maker, as in Figure 1.1. These transformations are value and utility functions, respectively.

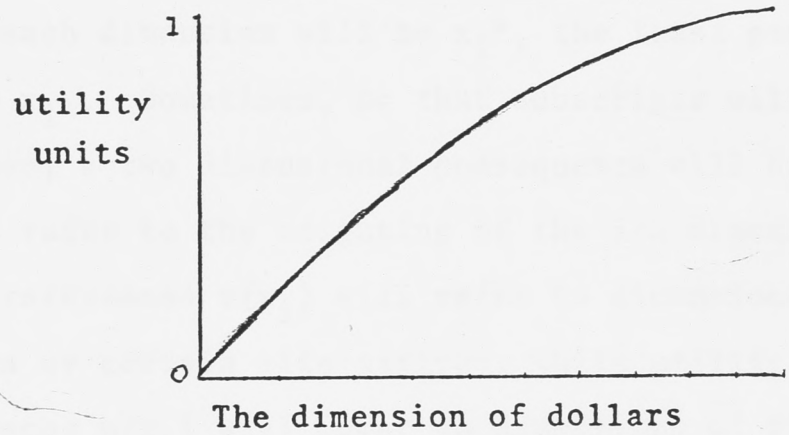


Fig. 1.1 An example of a utility function

After each dimension has been transformed, levels attained by an alternative course of action on each of the dimensions are combined into a total value or expected utility figure for the alternative by using a model; often an additive model is used but the model used should be justified for each decision. The alternative with the highest total value or utility figure indicates the most preferred alternative.

The notation used in the thesis refers to the consequences of alternative courses of action. Each consequence is measured on criteria, or dimensions, or attributes. Let $X = X_1 \times X_2 \times X_3 \ldots X_n$ be the consequence space, where X_i is the ith dimension. A specific alternative x will be referred to as $(x_1, x_2, \ldots x_n)$ where x_i, $i = 1, \ldots n$ refers to the level of the ith dimension. An uncertain alternative will sometimes be referred to as $\tilde{x}$. $x_{\bar{i}}$ refers to non-i dimensions and $\hat{x}_i$ refers to a certainty equivalent of the ith dimension. The most preferred level of the x_is on each dimension will be $x_i{}^*$, the least preferred will be x_{i*}. Sometimes, so that subscripts will not be needed, a two dimensional consequence will be (y, z). w_i will refer to the weighting of the ith dimension. Value preferences $v(x_i)$ will refer to dimensions of riskless or certain alternatives, while utility preferences $u(x_i)$ will refer to dimensions of risky or uncertain alternatives. Preference relations are: is preferred to (>), is less preferred than (<), and is indifferent to ($\sim$).

2. THE TWO MAJOR APPROACHES TO MULTIDIMENSIONAL DECISIONS

There are two major approaches to multi-dimensional problems, the non-compensatory and the compensatory, which can be put (a little imprecisely) in the more familiar terms of satisficing and optimising. Ackoff (1) suggests another approach, that of adaptivising, but others (30; 54) have pointed out that this third approach is really just an approach to the quality of decision making, rather than an alternative to the two major approaches above.

The first approach is the (apparently) non-compensatory approach, where levels in one dimension are not compensated for by levels in another dimension, that is, there are no rates of substitution or indifference curves. One example of this approach is linear programming, where one dimension alone is optimised subject to constraints on others. Another example is lexicographic ordering, where alternatives are judged on dimensions in priority orderings, and lower dimensions are considered only if alternatives are tied on prior dimensions. A third example is satisficing where a "cut-off" level is nominated on priority ordered dimensions, and further dimensions are considered only if two or more alternatives are acceptable in prior dimensions.

However, this non-compensatory approach does imply a comparison of dimensions. For instance, shadow prices indicate the marginal value of the constraints in linear programming, which are implicitly traded off if the linear programming solution is accepted. Lexicographic ordering implies that the rate of substitution is zero, not that it does not exist. Satisficing implies the rate of substitution goes from zero to unity at the "cut-off" point, not that it does not exist. That lexicographic ordering does exist is shown by the example of martyrs who would not trade the dimension of faith against life, and satisficing is apparently evident in some subsistence farming where alternatives must meet a "survival" test before they will be assessed further. Certainly, these approaches are valid if they reflect the decision maker's beliefs. But their assumption that the rate of substitution is zero is too often made for convenience rather than realism and the added benefits from exploring the decision space by relaxing this assumption are overlooked when deciding that the expected cost in divergence from the compensatory optimum is less than the cost of the analysis required to reach the compensatory optimum. If decision analysis could be facilitated, the necessity for the non-compensatory approach might decline.

The second approach to multidimensional decisions is the compensatory one. Here the rates of substitution are made explicit, and the levels attained by alterntiives on each dimension are combined in one way or another.

The simplest way of combining them is the additive model (this and other models are discussed later). The additive model is used by benefit-cost analysts, for instance, when they add the benefits of time-saving and increased production, each of these having been determined from market values.

The compensatory approach is the one usually taken in micro-economics (which is based on either indifference or "utility" curves). Psychological testing has verified that it is a realistic one (61). Because of this, and because of its widespread use, this thesis will concentrate on compensatory decision analysis. If the rate of substitution is zero for the range of dimension values in a decision problem so that a non-compensatory analysis is required, then lexicographic ordering or satisficing can certainly be carried out. The procedures are simple, and can even be carried out when outcomes are uncertain, as MacCrimmon has shown. (140) (However, MacCrimmon's procedure is based solely on endpoints of a subjective probability distribution which is assumed to be uniform, so that not even the modal value of the PERT formula is used.) This thesis takes up the decision problem when the simple non-compensatory procedures have been used, if they are applicable, and more than one alternative remains.

3. DIFFERENT APPROACHES TO PUBLIC INVESTMENT ANALYSIS

There are approaches to public investment analysis which are different to MDA. Before investigating MDA further, it would be wise to investigate whether MDA is different to these other approaches, and whether these differences are important. A brief review of most of the possible approaches to project evaluation is in Perry and Dillon's article (169); here only the major approaches of cost-benefit analysis (CBA), cost-effectiveness analysis (CEA), mathematical programming, simulation and unaided intuition are covered. The aim is merely to discover differences to MDA and evaluate their possible importance, it is not to dismiss the other approaches.

3.1 Cost-benefit Analysis and Cost-Effectiveness Analysis

CBA and CEA are methods of project appraisal which are very widely used. Their differences to MDA can be grouped under the headings of generality, treatment of risk, and coverage. It will be argued that MDA does not replace CBA/CEA, but it does appear to be a useful development of them.

3.1.1 Generality

The usual goal of CBA/CEA is a potential Pareto gain in efficiency. Underlying this goal are seven, often unstated, assumptions or value judgments:

(i) Individual preferences count. However, public decisionmaking is in reality the result of bargaining by a few informed groups who have stakes in the area involved (126). "Under the specialized structure of contemporary societies most individuals are not directly or actively engaged in policy making, rather they delegate their rights in this regard to representatives and abide by consensus policy decisions." (179, p.438) Without this apparent irrationality, the "rational" individualistic bases of CBA would make it difficult to provide a social welfare function, as Arrow has shown (10).

(ii) The market measures individual preferences. However, this assumes that the market is perfectly competitive, which is probably not the case in Australia, with its quite high concentration ratios. Nor is this assumption valid if the project is not a marginal one.

(iii) Market prices are derived from an optimal income distribution - even if the CBA/CEA analyst assigns no weights to market prices, the absence of weights merely means that the present income distribution is considered optimal.

(iv) A potential Pareto gain is satisfactory, or will become actual by redistribution measures of the government. However, this is uncertain.

(v) Public goods and intangibles with which CBA/CEA often deals and which are not traded on the market can be priced by some "objective" method. However, this is dubious.

(vi) Value functions are linear with money.

(vii) The additive model can be used to aggregate benefits and costs. The additive model assumes independence of value functions, which has been quite controversial in CBA literature, e.g. (93; 146).

In brief, the basis of CBA and CEA is not free of value judgments. If these judgments did characterise the decisionmaker, then MDA would produce the same results as CBA and CEA. But if other value judgments applied, then CBA and CEA would be largely irrelevant but MDA could still be used, incorporating the decisionmakers own value judgments. That is, MDA appears to be more general than CBA.

3.1.2 Treatment of Uncertainty

CBA and CEA usually have an objective function of maximising discounted benefits or minimizing discounted costs. However, this objective function relies on necessarily uncertain forecasts of the discount rate, the benefits and costs, and the duration of the project (165). Each of these can be important.

For instance, the selection of a discount rate can be crucial. If Corps of Engineers' projects had been discounted at 4, 6 or 8 percent, instead of 2.75 percent, then 9, 64 and 80 percent of them would have had a benefit-cost ratio of less than unity (218). An initial Australian study (166) found that discount rates could vary from 3.2 to 10.5 percent, depending on which controversial basis the discount rate was derived. Indeed, the adequacy of a discount rate rather than a discount algorithm is arguable (60).

Moreover, procedures to incorporate uncertainty about benefits, costs and project duration into CBA and CEA are crude. One method is to add a risk premium to the (uncertain) discount rate or to costs. Dasgupta and Pearce (33) aver that this is inadequate, because this only allows for overestimates of benefits, while estimates may be under as well as over. However, they are confusing sensitivity testing with allowing for uncertainty. The principle of a risk premium is to reduce an uncertain amount to a certainty equivalent, which can then be compared to other certain amounts. The calculation of certainty equivalents requires two things: the probability distribution of uncertain outcomes, and the attitude of the decisionmaker to this uncertainty. That is, a risk premium to the discount rate or to costs cannot be calculated without the decisionmaker's attitude being assessed. Perhaps this attitude could be linear, if the project being considered is one of many, but there may be public projects when this will not be so, e.g., a large project in a small region.

Thus, "it seems generally agreed that any valuation of net benefits in which uncertainty plays a major part can only proceed if information is available on the attitude of the political decisionmaker to uncertain outcomes." (165, p.21)

Another method of allowing for risk in CBA/CEA is to cut off analysis at an early date, which amounts to changing the discount rate arbitrarily at the cut-off date, and hence suffers from the criticism made in the previous paragraph.

A third reaction to the existence of uncertainty is sensitivity testing. However, it is often a piecemeal approach, with the sensitivity testing of an uncertain variable being done with other variables held at their most likely levels with little thought given to probabilistic dependence among variables. Full joint distributions of all variables would provide a more complete picture. Moreover, if testing does show results are sensitive to an uncertain variable, CBA/CEA offers no good guide for further analysis.

There have been attempts to incorporate probability distributions into CEA, e.g. (73; 176), but there is no doubt that such methods have hardly ever been used (176)

In brief, the treatment of uncertainty in CBA/CEA is not "objective" or developed. On the other hand, MDA purports to handle uncertainty in a thorough manner by making explicit the subjective probability distributions of the decisionmaker and his attitudes to the risks they show.

3.1.3 Completeness

The third area where MDA and CBA/CEA differ is in completeness. The objectives in a micro-economic policy decision can be far more numerous than the single efficiency one of most CBA/CEA. For instance, the objectives or benefits from a dam could be (based on (84)),

1. irrigation for increased agricultural output;
2. municipal needs;
3. industrial needs;
4. fishing (professional);
5. recreation;
6. improved navigation;
7. improved flood control;
8. hydro-electric power;
9. maintain water quality;
10. a buffer for drought years and groundwater recharge;
11. prevention of run-off damage;
12. enhance regional development;
13. travel time alterations from changing roads;
14. political effect; and
15. implementation effects, e.g. on the bureaucracy involved, which are themselves multidimensional (30).

Items 5, 9, 13, 14 and 15 will be difficult to measure on a monetary, market-derived scale, and items 7, 10 12 and 15 will be very uncertain. CBA/CEA would tend to ignore these objectives, and yet these eight objectives are more than half of those listed and could be valid objectives of the investment decision.

But it must be admitted that some CBA/CEA does attempt to incorporate multiple objectives. The objective of distribution can be added to the objective of equity by constraining one and optimising the other (139; 145). This can be extended to several objectives and a mathematical programming formulation used (41). The problems with this non-compensatory approach have been mentioned in Chapter 2.

Another development in CBA/CEA to handle multiple objectives is to display scores (or ranges of scores) on each objective for each alternative, and assist the decisionmaker to come to an informed decision in a discussion. This approach can include many objectives, even non-quantifiable ones, and allows the decisionmaker to use an (implicit) compensatory or non-compensatory approach as he feels is appropriate. It is undoubtedly a very successful approach, e.g. (138; 177). However, all it does is provide the information to the decisionmaker, it does not help him process it. It will be shown in Section 3.4 that relying on unaided intuition in this manner is not wise, because unaided intuition cannot process multidimensional and uncertain information in an adequate fashion. Moreover, the scoring approach evades the issue of uncertainty.

A third development in CBA/CEA for handling multiple objectives is a weighted model to add scores on objectives. This has been recommended for the simple equity-distribution problem, with various methods of finding weights proposed (72; 82; 139; 214). And it has also been suggested for more than two objectives (91). This development is actually just an application of MDA under certainty using an additive model, without the rigorous treatment of assumptions, value functions, weights and models that occurs in MDA. This lack of rigour has led many writers to express reservations about this development, e.g. (138; 157; 176), which may not have been expressed if more had been known about MDA.

In brief, MDA appears to provide a more thorough method of incorporating all objectives into public investment analysis than CBA/CEA.

3.1.4 Examples

Two examples will illuminate how MDA is different to CBA/CEA, and how it offers more promise in public investment analysis. The first example refers to a CEA done by the writer for the Department of Defence which foundered because the conventional CEA structure was not able to handle multiple objectives and uncertainty. The second example is a comparison of CBA and MDA in a decision about a second airport at a major city. It bears repeating that the aim of this discussion is not

to dismiss CBA/CEA, but merely to demonstrate that MDA is a development that may be used in conjunction with some CBA/CEA principles to assist public sector decisionmakers.

The CEA example was of two alternative weapons; details of the analysis are classified but its basic principles are not. CEA usually fixes cost or effectiveness at some level and selects the alternative with the highest level of the unfixed variable. At first, the decision makers in this case arbitrarily fixed the level of effectiveness at, say, 66.7 percent damage to a target, and this was initially used in the study. But it soon became apparent that the validity of this level of 66.7 percent dependedon the total number of targets, the lethality of the weapons, and the enemy's repair capability; moreover, the cost of changing the effectiveness level from 66.7 to 80 percent, say, could have differed for each weapon and may have been important to the decisionmakers. That is, "evidently cost-effectiveness is not a simple parameter but a function relationship" (12, p.176). Furthermore, the debate over this correct criterion for effectiveness was then clouded by the other criteria or objectives which became obvious. In addition to cost, these were: the number of sorties required to damage 66.7 percent of the target, with fewer sorties presenting the attacking aircraft fewer times to anti-aircraft defences; the difference in vulnerability to defences on each pass caused by different release heights for the two weapons; and where the weapons were manufactured. To

add to the complexity the valuation of vulnerability is not linear with the number of sorties, for a bomb which requires one sortie to achieve a fixed level of effectiveness is more than twice as valuable as one which requires two sorties, because the element of surprise is lacking in the second sortie. Nor is the valuation of cost linear, for it turned out to be kinked because a budget level cut-off for costs soon surfaced, but this budget level was for reserve stocks, not for lifetime usage, and so was not completely binding on the CEA (if it had been binding, fixing costs rather than effectiveness could have been attempted). Thus non-linear valuations for cost existed, as they did for the many other dimensions in the decision, and conventional CEA was inadequate in this situation.

Moreover, it was difficult to incorporate the uncertainty existing in the decision into the CEA. The complex model used to calculate sortie numbers required to produce the fixed level of effectiveness did not provide a variance about average values; the costs were uncertain for a weapon which had not been produced; and estimates of vulnerability in a future air defence environment were difficult. Of course, some sensitivity testing of cost and vulnerability was carried out, but interactions of uncertainties are difficult in such sensitivity analyses, and attitudes to risk are not incorporated.

In brief, while the cost-effectiveness analysis of the two alternative bombs did help to clarify some of the issues involved, the analysis was not general, or complete, nor did it treat uncertainty well. As proof of this, the decision makers' judgment had to supplement the analytical results quite heavily, and the decision makers considered the analysis "inconclusive".

The second example of the inadequacy of CBA is the study of the second Sydney airport (130). It found the discounted added social costs of 15 different sites with various runway layouts, with costs including costs of travel time, noise and urbanisation. Dynamic programming was used to find the optimum site development and air traffic management sequence over thirty years. Sensitivity testing of various uncertain parameters was carried out - these parameters included traffic forecasts, political constraints on noise at the existing Sydney airport, and the cost of noise. The first five sites selected were found to be reasonably robust to these tests. The whole study took 24 professionals one year to complete.

This CBA was without doubt a well done analysis and achieved the requirements of a CBA in illuminating the usual costs and benefits involved. And yet the government did not accept any of the recommendations. The government did not develop the first Sydney airport further, and its choice of the second airport was not one of the five most preferred in the study. Obviously political factors

were more important than the factors involved in the CBA (as they were in the third London Airport study (71)). This does not mean that the CBA was valueless, for the economic aspects had to be known; however, it does indicate that a more complete aid to public investment analysis is required.

Compare the second Sydney airport CBA with a MDA of the decision on the development of a second Mexico City airport (113; 159). This analysis was requested after two previous studies had arrived at opposite conclusions. The site of the second airport had been determined, but the phasing of its development was not - about 100 alternative sequences of development were available. Probability distributions of six dimensions were elicited, as were attitudes to uncertainty, and the conditions for a multiplicative utility model verified, all with the staff of the department involved in the decisions. Dimensions of safety and capacity were added to those of the second Sydney airport CBA above. Then a further analysis was carried out, which included flexibility, political effects, externalities, and effectiveness. The total time spent by consultants on the project was 50 man-days. Unlike the CBA, the MDA appears to have had an important influence on the final decision (159).

3.1.5 Conclusions

CBA is based on several value judgments while MDA's explicit recognition of the decisionmaker's values

makes it more general than CBA. MDA's treatment of uncertainty is more systematic than CBA's. And MDA's coverage of relevant dimensions is greater than CBA's. Not that CBA in unnecessary. When its implicit value judgments agree with the decisionmaker's, CBA is identical to MDA. However, MDA does appear to go beyond CBA in its applicability. It is a procedure for those who agree that "Economists would serve their profession better by seeing themselves as efficient management consultants rather than by deluding themselves that they will be publicly acclaimed as 'philosopher kings'" (165, p.29), and so offers a structure for those who see the analyst/decisionmaker relationship as an interactive one (30).

3.2 Mathematical Programming

It has been found that mathematical programming enables a decisionmaker to explore in depth a far greater range of alternatives than he would using conventional project analysis, e.g. (130; 182). There are two simple methods of using mathematical programming for the multiple objective decisions that concern this thesis. The first method is goal programming, e.g. (133), where weighted deviations from stated goals are minimized, and lexicographic ordering of some dimensions can be achieved through pre-emptive priority weightings. The second method is to adjust linear programming for multiple goals (21). In this method weights are given

as coefficients to the appropriate dimensions in the objective function. After an initial run, the decision maker may vary the weights to feel his way towards an optimal solution.

However, there are four important assumptions underlying most of these mathematical programming approaches:

(i) valuations of the dimensions are linear;

(ii) the additive model is applicable (and hence it is used in the objective function);

(iii) the dimensions are continuous, not discrete (so that alternatives can be at any level on any dimension); and

(iv) certainty exists (so that alternatives will reach levels on each dimension for sure).

The first assumption, that of linear valuations, is evaded in several forms of inter-active goal programming (43;76;155). Here the decision maker is asked to provide weights to each dimension, i.e. trade-offs of their importance, and the program run. Results of this are plotted and the decision maker chooses a step size where weights are closer to what he really feels they are. This is repeated until the solution is satisfactory to the decision maker. Essentially, this procedure uses local linear approximations to the curved value function until the slope of these approximations equals that of the value function.

Once goal programming becomes inter-active it is essentially (although not computationally) the same as inter-active "linear" programming approaches to multidimensional decisions. These approaches also evade the first assumption above of linear valuations, by considering trade-offs between dimensions at the margin of non-inferior solutions. Because these trade-offs are made at the margin they incorporate the decision maker's preferences over a small interval, and hence linearity is assumed only over that interval. Dyer (44) computerised the process so that the decision maker need only make ordinal choices between holistic, multidimensional choices, and found that his subjects (who were students) had more confidence in the method than in their own ad hoc, often unstructured search. But another researcher (212) has found that unstructured searches were more popular than Dyer's program or a similar one.

Another process somewhat similar to Dyer's is Haimes's surrogate worth trade-off method (84) where decision makers assess ordinally the relative worth of small changes on just two dimensions. Yet another similar approach by Wallenius and Zionts (213; 227) has been tested, though not actually used, with operational managers in a company. Mention should also be made of STEM (16), in which the decision maker interacts with the program until a "best compromise" result is obtained.

Insofar as all the above approaches require ordinal rather than cardinal answers, they are both simple and accurate (44, p.204). However, the answers require an evaluation of trade-offs between several dimensions at the same time (except for the surrogate worth trade-off method which requires trade-offs between two dimensions only) and this can be difficult if there are many dimensions in the decision problem, nor does it provide the concentration on one or two dimensions at a time allowed in MDA. Moreover, decision makers often desire to answer a previous question differently in the light of new knowledge aroused in the decision process (25; 213) and this can be difficult to incorporate in a program.

All the above extensions of multi-objective mathematical programming may have overcome the first restrictive assumption above - linear valuations - but the others - of continuity, additivity and certainty - still remain. One of these assumptions, of continuity, can be overcome by a mixed integer program, but this is still very much an undeveloped research area (155). This is a severe restriction; for instance, investment in a truck is restricted to the discrete levels of payload which manufacturers provide, and it is significant that in the only application of multi-objective mathematical programming which has actually influenced decision makers - the Rio Colorado study (25) - a mixed integer program was necessary.

Because none of these assumptions are in theory necessary for MDA, it seemed worthwhile to investigate MDA further in this thesis. The investigation will show the assumptions in the better mathematical programming approaches - the surrogate worth trade-off method for instance - are not restrictive, but this could not be foreseen when this research began.

3.3 Simulation

Faced with multiple goals, integer dimensions and constraints, complex interrelationships, and uncertainty, many researchers have preferred simulation to mathematical programming. For example, Johnsen (103), after reviewing most approaches to multidimensional decisions, fell back on a compl-x simulation model. However, the model had not been completed when his book was written, and so had not proved to be valid, or more useful than other approaches. Legasto (134) used simulation to explore the multiple objective results of economic policies in the Phillipines, but concluded that the optimum policy had to be found by other means. And Fromm and Taubman (75) used the Brookings Institute economietric model of the U.S. economy to derive multidimensional results of policy alternatives, with the results combined in various models using weights that had been almost arbitrarily chosen, e.g., where dimensions were concerned with expenditure their weights were simply their proportion of GNP. No guide to which model was appropriate was given, nor were weights varied in a sensitivity analysis. That is, Fromm and Taubman's research, although it did use some optimising heuristics, did little to find the "best" policy alternative.

These words sum up simulation as an aid to multidimensional decision making:

> "The Monte Carlo algorithm can handle large numbers of objectives with little extra effort, so that its role becomes less that of an optimum seeker and more a portrayer of the significant relationships present in the system under study. The emphasis on an optimum is thus replaced by emphasis on a "road map" from which a decision maker may judge the consequences of different courses of action." (206, p.241)

This is excellent if that is all the decision maker requires, and sometimes that is all he requires (212). But, as noted when discussing the score card development of CBA/CEA, man's unaided intuition is an inadequate processor of multidimensional, uncertain, information, and so something more may be required. If it is, MDA appears to offer thorough optimising heuristics with which to process the results of simulation studies.

3.4 Unaided Intuition

The above methods, then, appear to be somewhat inadequate. Why not use unaided managerial judgment or intuition? The answer is that unaided intuition is also inadequate for multidimensional decisions, especially when made under uncertainty, as several psychological studies have found.

Studies of human perception in multidimensional situations have found that people usually collapse every dimension onto one scale, and that this collapsing is not done in an optimising fashion. This also applies to human judgment as well as perception. For instance,

Lindblom (135) found that public service administrators avoided taking all dimensions into account by using a "method of successive limited comparisons". In this, only alternatives which differ slightly in a few dimensions from existing policies are considered, and the full range of available alternatives is ignored. This would explain why "budgeting is incremental, not comprehensive" (217, p.15) with the last year's expenditure being the most important determinant of this year's. And Slovic and Lichtenstein (190, pp.49-57), in a review of psychological research on human judgment, list several studies that indicate that the use of a dimension varies with the method of its presentation (e.g. the scale used, its relationship to other dimensions, the order in which it is presented, and the total number of dimensions). Examples of this inadequate processing of multiple dimensions abound. For example, sixteen professional financial services which made 7,500 recommendations on individual stocks from 1928 to 1932 had an average return 1.4 percent less than the market average (192). And experienced analysts' predictions of yearly earnings are no better than simple projections of past earnings. (192)

The reasons for this inadequate processing of multiple dimensions are provided in Simon and Newell's "Human problem solving: the state of the theory in 1970" (187). After reviewing computer simulations of problem solving, they conclude that the human mind,

faced with the complexity of solving a multidimensional problem, for instance, uses simple heuristic methods to carry out highly selective searches of the decision space, with simple serial processing of information and a short-term memory.

This inadequate processing of multidimensional information is reflected in the results of research into how people weight dimensions. This research shows that humans usually weight only a few dimensions when they make holistic, intuitive judgments and it is only when each dimension is weighted separately and explicitly that weights are more uniform. For example, Hoepl and Huber (94), using regression analysis, found weights implicitly used in holistic judgments of teachers by students. When the students then provided direct weights for each of the dimensions, the weights were found to be more uniform. Slovic (189) found stock-brokers did the same when rating stocks on eleven dimensions. O'Connor (163) and Slovic and Lichtenstein (190) review nearly all such studies, and the conclusion is virtually uncontested: humans make intuitive holistic judgments of multidimensional problems in a manner which produces results which do an injustice to their actual beliefs.

Not only is multidimensionality inadequately handled by the human mind, but so is uncertainty. There are several cognitive biases or heuristics which affect subjective probability (95; 190; 196; 209). One such bias is anchoring, which is the tendency people have to anchor their judgment to the first value they think of. For example a businessman forecasting sales may use this

year's sales as a starting point, even if this year is atypical, and adjustments to this first estimate are often insufficient or are made in constant amounts as new information is received. Some of the factors which affect which value will be the anchor are availability and representativeness. Availability refers to the ease with which a first estimate of an uncertain variable can be made. For instance, the presentation of information affects judgments, even when the information is equally likely (163). Representativeness refers to the similarity between some information and the population to which it belongs, and this has too large an effect on probability judgments. For instance, intuition apparently works on "a law of small numbers", so that a small sample is incorrectly thought to be as representative of a population's characteristics, such as its correlation or its average, as is a large sample.

Indeed, the most common reaction to uncertainty is to ignore it (227), and most managers appear to make their decisions on the assumption that a value near the average of a parameter will occur (95) and the difficult task of assessing the full distribution is not attempted. For example, Burton and Kates (228) show how too simple attitudes towards uncertainty of natural hazards in resource management can lead to serious losses.

At the heart of the processing of uncertainty is Bayes Theorem, which is the optimal method of amending previously held probabilistic beliefs in the light of new information. If humans are intuitive Bayesians, then their assessments of uncertainty in the process of working through a decision problem should

be logical. However, numerous studies, e.g. (52), have found that humans are apparently conservative, and do not change their prior probabilities with new evidence as much as they should if they were using a Bayesian model correctly. These results might occur just because probabilistic data in the real world is conditionally dependent and requires different aggregation procedures than the simple Bayesian procedures for conditionally independent data (221). Another consideration which makes one doubt the relevance of the results of these studies is the presence of cognitive bias of availability, referred to above, which suggests that sample results would often overwhelm the prior judgments, which is the reverse of conservatism. These two contradictory considerations would suggest that humans are simply not Bayesians, conservative or otherwise, and a study by Lathrop (132) reinforces this view. He found that most studies suggesting conservatism averaged subjects' results. This is valid if subjects are using the same mental processes, with random variations around the average. However, Lathrop plotted each individuals's result, and found their revised probability estimates (supposedly using Bayes Theorem) were "all over the scatterplot". Conjoint measurement testing for independence was negative, showing that subjects "were not obeying the formal rules of probability at all" (163, p.8) - that is, they were not even conservative Bayesians, they were not handling uncertainty in any logical consistent, manner at all!

3.5 Conclusion

This section briefly reviewed several of the most common approaches to multidimensional policy analysis under uncertainty. It appeared that MDA was different to CBA/CEA in its generality, treatment of uncertainty and completeness; that MDA did not share some of the restrictive assumptions of mathematical programming; that MDA could help if a decision maker required simulation to be more than a road map; and that intuition needed an aid such as MDA in multidimensional decision making under uncertainty. When this thesis was begun there appeared to be no "alternative" to these approaches other than MDA which deserved further study. A more recent review of some decision approaches (30) appeared to confirm this need to investigate further methods of public investment analysis. But "alternative" is not the precise word for MDA - this section has shown that MDA's value may be just as much in complementing the other approaches by offering structure and rigour, as in offering a completely alternative approach. Certainly a review of the theory and practice of MDA in public investment analysis might be fruitful.

4. MULTIDIMENSIONAL DECISION ANALYSIS UNDER CERTAINTY

MDA can be done when the outcomes of each alternative are known with certainty, or when these outcomes are uncertain. The crucial difference between the two situations is that, theoretically, value functions for dimensions can be used under uncertainty.

MDA under certainty is important for two reasons. The first reason is that MDA has most often been carried out under certainty for, as Huber noted in 1973, there are "hardly any field or field-like studies of situations where multiplicity of attributes and uncertainty were both formally considered." (99, p.1394) The second reason is that later chapters of this thesis will consider using models for certainty when outcomes of alternative courses of action are uncertain. In particular, it is hoped to demonstrate that these three aspects of MDA under certainty can and should be used in MDA under uncertainty:

(i) the weighted additive model;

(ii) value functions; and

(iii) weights elicited under certainty.

Thus this chapter is crucial to the theme of the thesis. However, the area of MDA under certainty is an established one, and so the treatment will not be more detailed than necessary.

4.1 The Underlying Assumptions of MDA Under Certainty

For unidimensional decisionmaking under certainty there are three basic assumptions - the decision maker can order the alternatives, do so transitively, and he does not have infinite value for one alternative (this is the Archimaedian axiom - if

infinite value existed, there would be no difficulty in deciding among alternatives). The presence of multiple dimensions does not remove the need for these axioms, for instance, the values of alternatives, the v(x) s, still require ordinality only, not cardinality, and the Archimaedian axiom is still required for trade-offs to occur between dimensions. However the presence of multiple dimensions makes necessary a fourth axiom. This is the axiom of value independence: the decision maker's (ordinal) values on each dimension are independent of levels at which other dimensions are set.

There are other names for this fourth axiom of value independence:

(i) preferential independence (178, p.26); but this is sometimes confused with Debreu-independence to be discussed later, and is not specific to certainty;

(ii) weak conditional utility independence (178, p.26) - but this uses "utility" when "value" applies (and when it is applied to a set of dimensions it implies Debreu-independence within that set from dimensions outside the set (178, p.27);

(iii) monotonicity (26, p.A-43) - but this can also refer to Debreu-independence (62, p.7); and

(iv) single cancellation (216, p.A-43) - but this means little and apparently only refers to the two dimension situation (62, p.7).

To evade all this confusion the term "value independence" appears appropriate, for it simply and clearly emphasizes the aspect of independence under certainty in the axiom, and it also is complementary to the term "utility independence" widely used for independence under uncertainty. There is no need to refer to conditions or singleness, as this is clearly implicit in the term "independence". (It should be noted that Keeney (114) has used the term value independence for Fishburn-independence, but Fishburn-independence refers to uncertainty, and so "value" seems somewhat inappropriate.)

The first three axioms above simply mean that the decision maker is rational and logical. Although many decision makers may not base their decision on the first two axioms, some studies have shown that decision makers were surprised that they did not act upon the first two axioms (e.g. 141) and were willing to amend their decisions to coincide with them, and in one sample of 27 businessmen the three who did base decisions on the axioms were the most successful businessmen (184).

The fourth axiom, value independence, is not so obvious. There are occasions when it may not apply. For instance (216, p.A-44), there may be three dimensions in a choice between cars. A big car is preferred to a small one if both cost the same amount, whether this be a large amount or a small amount. But a big car may not be preferred to a small car if the third dimension is the existence of power steering. If

both cars have power steering, the big car is preferred. But if both cars do not have power steering, the small car may be preferred to the big car, for its easier handling. Thus the ordering of car size is not independent of the level at which the third dimension is set, that is, value independence does not occur. Other examples (69) occur when the value for a high "probability of death" reverses if "pain" is set at too high a level, and when government satisfaction at a class's increase in wealth becomes dissatisfaction if other classes remain extremely poor.

Such value dependencies could be handled in three ways. One way is to reframe the dimension so that reversal does not occur. Another way is to break a dimension into two so that, for example, "pain" became "pain below excruciating" and "pain above excruciating". A third way (69) is to introduce another dimension so that, for example, "equity" is added to the income distribution problem.

Value dependence does occur sometimes (e.g. with some time streams of money) but overall it is not too common - there do not appear to be any other examples of it in the literature, for one thing. Moreover, simple questioning or the analyst's common sense would show if it did not exist. Formally, value independence occurs if $(x_i, x_j, x_{\overline{ij}})$ for any values of $x_{\overline{ij}}$. This implies that indifference curves over x_i and x_j do not depend on the values of $x_{\overline{ij}}$. This property can be tested by finding a value of x_i, x_i^o say, which sets

$(x_i^o, x_j^o) \sim (x_i^1, x_j^1)$ with $x_{\overline{ij}}$ set at their most preferred levels. One then checks whether the value of x_i^o has to be changed so that $(x_i^o, x_j^o) \sim (x_i^1, x_j^1)$ still, with $x_{\overline{ij}}$ set at their least preferred levels. Alternatively, and more simply, the decision maker could be asked if his trade-offs between x_i and x_j, that is, the extra amount of x_i which makes up for a given decrease in x_j, vary as $x_{\overline{ij}}$ do.

The four axioms are implied in most aids to decision making, e.g. CBA/CEA. If all the axioms are not agreed to by the decision maker, MDA has little to offer him. He can only make intuitive, holistic rankings of the alternatives. That is, MDA is not descriptive, but is conditionally prescriptive (118) - if the axioms are agreed with, MDA offers a guide to decision making.

The following discussion assumes that the four axioms are valid for the decision maker.

4.2 The Benchmark Model

There are two approaches to MDA under certainty. The first is Raiffa's (178) "benchmark" model. Consider a two-dimensional decision with dimensions y and z. A benchmark level of y is set at y^b; this y^b must be easy to imagine - it could be y_* or y^* or some most likely value of y, and y could be a dimension in dollar terms. (If there is a dimension of which one or more dimensions is not value independent, then that dimension should be the benchmark dimension, and hence the possible randomness produced by value dependence will be eliminated

by making preferences for dependent dimension(s) with the other fixed. If there are several dimensions which are value dependent, all these will be benchmark dimensions while the others are varied. (6)) Then, by varying z, values of z^1 are found such that $(z^1, y^b) \sim (z, y)$ for each alternative (z, y). If indifference curves for z and y have been established, perhaps by MacCrimmon and Toda's (142) method, then (z^1, y^b) equivalent to each alternative (z,y) can be read off an indifference diagram, by moving along the indifference curve from (z,y) to where the value of y is y^b, and the corresponding value of z^1 read off its axis. If there are more than the two dimensions y and z, the combinations of levels for each alternative are equated to a combination $(x_i, \ldots, x_{\bar{i}}^b)$, where one dimension, x_i, varies to achieve equivalent preference with the alternatives, and the $x_{\bar{i}}^b$, which are benchmark values of the other dimensions, are fixed.

The procedure is a sequential one (5). For instance, consider an alternative with three dimensions, (x_1, x_2, x_3). Benchmark values of x_2 and x_3 are set at x_2^b and x_3^b. Then by varying x_1 an hypothetical alternative $(x_1^1, x_2^b, x_3) \sim (x_1, x_2, x_3)$ is found, and then another hypothetical alternative $(x_1^1, x_2^b, x_3^b) \sim (x_1^1, x_2^b, x_3)$ is found.

The preferred alternative is the one with the most preferred of the x^1s , for with the other dimensions held at benchmark levels, the x^1s are in effect one-dimensional proxies for each alternative, with the other dimensions standardised at benchmark values. If

the rate of substitution between x and y is constant for all their alternative values, then any (z,y) pair can be expressed as $(z + \lambda y, o)$ where λ is the rate of substitution (178). One then only needs to rank the $(z + \lambda y)$ value.

One apparent advantage of Raiffa's "benchmark" model is that it is general, and does not assume particular relationships between the dimensions. Fischer (61, p21) avers that it does assume that trade-offs between any two dimensions do not depend upon the level of other dimensions. However, the use of benchmark values for all but the one varying dimension ensures that trade-offs are made with these benchmark values in mind, whether they affect the trade-off or not. (In a later paper, Fischer corrects his earlier statement. (216, pA-47)) That is, the benchmark model does not assume trade-offs are independent of other dimensions. Later paragraphs on the robustness of the additive model (which does assume this independence) indicate that this generality of the benchmark model is not essential , and may even be undesirable. Moreover, the method requires decision makers to make preferences between combinations of dimensions, and literature on unaided intuition (see Section 3.4) indicates that this will be difficult and probably inconsistent. Moreover, when there are more than two dimensions, the method is tedious and somewhat confusing. If there are D dimensions and A alternatives, there are (D - 1).A separate preferentially equivalent judgments to make, varying levels of one dimension each time to find equivalence. In short, the benchmark

method is tedious, and its generality does not appear to compensate for this.

4.3 An Introduction to the Weighted Additive Model

The second approach to MDA under certainty is to use models to combine values on each individual dimension into the one measure. The level of each alternative on each dimension is valued, $v(x_i)$, and weights for each dimension, w_i, are found. This procedure is simpler than the benchmark method, for the $v(x_i)$ can be derived without trading off among dimensions, and the w_i can be derived by trading off only two dimensions at a time.

Discussion of this second approach will be in four parts: an introduction with some emphasis on the assumptions of additive and multiplicative models; the additive model and its applicability; the derivation of value functions for the model; and the derivation of weights for the model.

The additive value model is the one most commonly used in MDA under certainty:

$$V(x) = \sum_{i=1}^{n} w_i . v(x_i) \qquad (4.1)$$

The important assumption (62), of the additive value model is that trade-offs between levels of any two dimensions are independent of the levels at which other dimensions are held; if there are only two dimensions, the assumption is that trade-offs are independent of the levels at thich the two dimensions are combined. Generalising, this assumption means that indifference curves for any two dimensions are the same whatever the levels of other dimensions. This assumption will be called Debreu-independence, after the first person

to prove its necessity for additive MDA under certainty (39; 178). Naming it after Debreu limits it to MDA under certainty. Other names for it, such as joint independence, or the term often used of preferential independence, do not do this. Note that Debreu-independence implies that the value functions are cardinal, not just ordinal as before. If Debreu-independence exists (and the four axioms of Section 4.1) then the addition of weighted values will presevve the ordinal ranking of the decisionmaker's preferences among alternatives, which is all that is required under certainty. There have been many proofs of the additivity proposition, based at the minimum on the axioms and assumptions given above (64; 65; 66).

Ordinal measurements are unique up to a monotonic transformation, so that the additivity proposition implies the multiplicative model

$$\log V(x) = \log \prod_{i=1}^{n} w_i \cdot v(x_i) = \sum_{i=1}^{n} \log(w \cdot v(x_i)) \qquad (4.2)$$

for the logarithmic transformation is a monotonic one of V(x) in (4.1). For the same reason, the multiplicative model implies a quasi-additive one (216). The importance of this is that a multiplicative model is no more or less assumption-free than the additive model, under certainty, if the same weights (which could all equal unity if no explicit weights are used) are used in both, as they are in (4.1) and (4.2).

For instance, this assumption applies to conjoint measurement models. "Conjoint measurement theory constructs the overall and single attribute (value) functions simultaneously" (216, p.A-27), so that its value functions have weights implied in them, and these can vary, unlike the weights in the weighted additive model which are constant and imply constant trade-offs and indifference curves. However, these value functions are extremely difficult to derive for more than two dimensions, for they involve simultaneous trade-offs between all dimensions, over the full range of the dimensions. These conjoint measurement value functions are more difficult to derive than value functions derived for one dimension at a time, as required for the weighted additive model. An example is a conjoint measurement application (81) of housewives' preferences for discount cards defined on three dimensions - the size of the discount, number of stores accepting the discount card, and the cost of the discount card. Simplifying the demensions to just three points still required judgments for $3^3 = 27$ three-dimensional combinations. That is, conjoint measurement is limited in the number of dimensions it can handle. It is also limited in how much can be incorporated in the implied value functions by the number of points on each of the dimensions that can be included, e.g., the three points of the dimensions in the above housewife example produced straight line value functions with just one kink. (81, p.292)

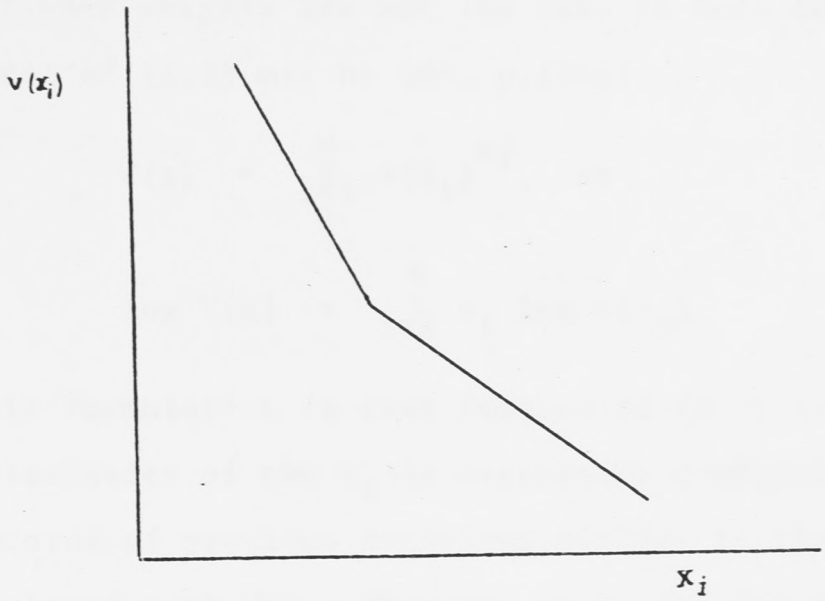


Fig. 4.1 Value Function from Conjoint Measurement

The value functions for the weighted additive model can be more flexible and so incorporate more of the decision maker's preferences.

In addition to these two limitations, conjoint measurement does not allow the decomposition of the decision into its constituent parts encouraged by MDA. Finally, conjoint measurement cannot be extended to decision making under uncertainty, while the value functions and concepts of the weighted additive model can be. Apparently these four reasons are why the conjoint measurement model is rarely used outside of psychological research; they are the reasons why it will not be discussed further here.

It is now appropriate to return to the weighted additive model. As noted earlier the additive and the multiplicative are both, in effect, additive models. But when weights are not the same in both the multiplicative model of (4.2) may be (99, p.1398).

$$V(x) = \prod_{i=1}^{n} . v(x_i)^{w_i}, \quad \text{or}$$

$$\log V(x) = \sum_{i=1}^{n} w_i \log v(x_i) \qquad (4.3)$$

This formulation is used instead of (4.2) to allow calculation of the w_i as regression coefficients from records of previous decisions similar to the one being analysed with MDA. This may be useful for psychologists investigating how humans make decisions, but it is not for managers making one-off decisions with MDA, as will be the usual case. Nor is it useful in such one-off cases to ask managers to choose among a set of alternatives similar to the set to be evaluated with the model to derive regression coefficients, as suggested in (99), for one may as well ask the decision maker to choose among the set of alternatives involved in the decision. Moreover, such holistic judgments do not allow the decomposition of the problem which studies of unaided intuition suggest is necessary (e.g. (190; 163)). The important result of (4.3) for practical MDA is that the logarithmic transformation of $v(x_i)$ makes meaningful weights w_i to be almost impossible to

elicit, for instance, are the logarithms to base e or 10? That is, the multiplicative model (4.3) is less intuitively meaningful than the additive model.

The real difference between the two models is in the implications for indifferncce curves between dimensions. The weights in the additive model (4.1) imply that the indifference curve for each pair of dimensions, i and j, is constant with a slope of $-w_j/w_i$. The multiplicative model (4.3) on the other hand has an indifference curve with a slope of $(-w_j/w_i \cdot x_i/x_j)$, that is, the slope varies with levels of x_i and x_j. The constant slope of the additive model may at first seem contrary to economic theory, which usually assumes that indifference curves are convex to the origin. However, it must be remembered that the axes for the indifference curves are not linear functions of quantities of dimensions, as assumed in economic theory, but value functions of these dimensions, which incorporate any diminishing marginal valuation of a dimension into them, and by the fourth axiom of value independence, these value functions will not change for differing levels of other dimensions. That is, indifference curves convex to the origin are implied in the value function axes, and so the linear slope of the weighted additive model is not contrary to economic theory. This oversight of the difference between linear functions and value functions as indifference curve axes is common, which is unfortunate for it is an important

defence of the weighted additive value model. Sharpe (185, pp.287-288) does not mention the difference between the two. And Easton (48) does not make explicit the existing non-linearity of the axes when discussing various models.

But Easton (48) does clarify the other assumptions made when a model is chosen. He derives the following general model of which the additive and multiplicative models are forms.

$$V(x) = \{\frac{1}{n} \sum_{i=1}^{n} (w_i \cdot v(x_i))^{(1-p)}\}^{1/(1-p)}, \quad p \neq 1 \qquad (4.4)$$

$$V(x) = \Pi w \cdot v(x_i)^{\frac{1}{n}}, \quad p = 1 \qquad (4.5)$$

The parameter p determines the shape of the indifference curves. If p = 0, then equation (4.4) reduces to the arithmetic mean, implying the additive model. If p = +1, equation (4.4) reduces to the geometric mean, implying the multiplicative model (4.2). In general, if p is positive, increasing its value increases the importance of high valuations relative to low valuations, that is the indifference curve is more convex. If p is negative, increasing its magnitude does the reverse. Thus the choice of the model is a value judgment, depending on the choice of p. But no method of eliciting p is available! As value functions have already incorporated the relative valuations of high and low scores, the case for assuming p = 0, implying the additive model, is stronger than any other. For any

other choice of p is counting relative valuations more than once.

One last point about the additive model is that a zero valuation of an alternative on one dimension does not give a zero valuation for the alternative. If the decision maker requires this, the particular alternative with a zero level on a dimension could be removed from consideration before the additive model is used to compare alternatives, or the multiplicative model (4.2 and 4.5) could be used with linear value functions, despite its arbitrary p = 1.

4.4 The Applicability of the Additive Model

Now, how useful is the weighted model? Does its Debreu-independence assumption limit its usefulness? Edwards (53) has suggested that it is difficult to imagine circumstances under which Debreu-independence does not exist, and Fischer (62, p.9) agrees. But is there proof? The question is important and will be answered at length, because the applicability of the additive model is crucial to the recommended procedures developed in this thesis. The answer to the question will discuss three topics:

i the correlation of additive and other models' results based on the same data;

ii the correlation of additive model results with holistic judgments, i.e. the acceptability of the additive model; and

iii the correlation of additive model results with the external environment, i.e., the accuracy of the additive model.

First, the correlation of additive and other model results will be discussed. Yntema and Torgerson (224) approached the additive model from the viewpoint of analysis of variance, in which there are main effects from each dimension separately, and interaction effects, e.g., for three dimensions

$$\begin{aligned} V(x) &= M + v(x_1) + v(x_2) + v(x_3) + \\ &= v(x_1)v(x_2) + v(x_1)v(x_3) + \\ &\quad v(x_2)v(x_3) + v(x_1)v(x_2)v(x_3) \end{aligned} \tag{4.5}$$

where M is a constant - the grand mean of the values for each alternative - and the weights have been deleted for clarity. They suggest that if the value functions are monotonically increasing or decreasing, then using the main effects without the interaction terms will adequately approximate V(x) and any error in the approximation should be in the same direction for all alternatives considered. This "is not the sort of statement that can be proven rigorously" (224, p.22) but they give this example of an interactive model,

$$V(x) = v(x_1).v(x_2) + v(x_1).v(x_3) + v(x_2).v(x_3) \tag{4.6}$$

If each $v(x_i)$ can vary from 1 to 7 there are 7^3 combinations likely, and V(x) can vary from 3 to 147. "If two alternatives differ in total value by more than 6 (they will 86 percent of the time), then the probability of a wrong decision (by using a main effects additive approximation rather than the interaction model) is only

0.005. If the difference in worth is more than 12, the approximation never makes a mistake" (224, p.22). Obviously the additive model approximation is very good.

Fischer (62, pp. 16-19) carried Yntema and Torgerson's work further by testing additive approximation to two other interaction models,

$$V(x) = v(x_i) + \sum_{i=1}^{N} \sum_{j=1}^{N} v(x_i).v(x_j), \quad i \neq j \tag{4.7}$$

$$V(x) = x_i + \sum_{i=1}^{n} x_i \tag{4.8}$$

Unfortunately for this discussion of models under certainty, only the Pearson product moment correlation coefficient, which is interval-scaled, was used, while a rank correlation coefficient would have been more appropriate. Nevertheless, the correlation coefficients for 1000 vectors of random dimension values were generated for n = 3, 6 and 9 dimensions. For equation (4.7) the coefficients increased from 0.964 to 0.990, as dimensions increased from three to nine, which corroborates Yntema and Torgerson's conclusions. But for equation (4.8), the coefficients fell from 0.858 to 0.480 with increasing dimension numbers, which is disappointing, and shows that when interactions are very extensive, the additive model may be inadequate if there are many dimensions.

Another comparison of models can be done on some econometric results. Fromm and Taubman (75) used the Brookings Institute model of the U.S. economy

to provide total values of fourteen economic policy alternatives, based on six dimensions such as real personal consumption expenditure. They used four models, the additive, the Cobb Douglas multiplicative, and two constant elasticity of substitution (CES) models. The additive and the multiplicative model are forms of the general CES model. The fourteen policy alternatives were rated 1, 4, 7 and 10 quarters after their implementation. Using ranks calculated from the cardinal values given in (75, Table 5.5), the Spearman rank correlation coefficients (all significant at below 0.01) between the additive model and the other three models were

Model	Cobb Douglas				CES 1				CES 2			
Quarter	1	4	7	10	1	4	7	10	1	4	7	10
r_s	.89	1.0	.99	1.0	.86	.99	1.0	1.0	.86	.99	.99	1.0

The coefficients show that the correlation is below .90 only in the first quarter after the policy implementation, and after that the models are virtually the same. Again the strength of the additive model as an approximation of others is demonstrated.

Yet another approach is just to compare an additive and a multiplicative model. Sums and products of random variables are highly correlated, even if the variables are independent. "For example, for independent random variables X_i uniformly distributed over (0.0, 1.0), $(1/10) \sum_{i=1}^{10} X_i$ and $(\prod_{i=1}^{10} X_i)^{1/10}$ are correlated with a

Pearson correlation coefficient of 0.89" (47, p.235). Furthemore, agreement among nine different additive and multiplicative models with 32 dimensions in a real world problem, measured by Kendall's coefficient of concordance, was a high W = 0.96, with the same three alternatives in the first three positions for all models (47).

Thus Yntema and Torgerson's claim for the generality of the additive model appears justified except for cases when interactions are severe. So the question about the applicability of the additive model becomes: are interactions very extensive when actual decisions are made? Or more precisely there are two questions: do humans consider interactions when they make intuitive decisions, and should they consider interactions?

Whether humans consider interactions when they make decisions is primarily a question for psychologists to answer. For if humans make accurate intuitive decisions, using an additive or a non-additive model, there seems to be little reason for MDA or for any other decision aid for that matter. But, as noted in Section 3.4, unaided intuition is inaccurate. Nevertheless, if humans use a model which an additive MDA model approximates, then at least MDA results should be acceptable to decision makers for the results will be similar to their intuitive judgments.

Innumerable empirical studies have found that the additive model was an accurate predictor of the decision maker's actual holistic intuitive preferences, with no other models any better (extensively reviewed in (61; 99; 163; 190)). One technique used in these studies was analysis of variance, in which the amount of interaction could be measured. Even when interaction between dimensions existed in the intuitive judgment the additive model approximation provides correlations with intuitive decisions in the high .90s (61; 163). In some examples of these instances, sixteen of twenty-nine clinicians judging mentally ill patients could have used interactions, and stockbrokers used interactions in their judgment (163). But these interactions were either not important enough to outweight the results of an additive main effects approximation, or were handled in an inappropriate manner in the intuitive decision.

Because most analysis of variance research showed no clear evidence that interactions were important for the additive model, some researchers have investigated different types of model, such as the lexicographic, which might be expected to correlate more highly with intuitive judgments. Tversky (208) has twice found evidence that such models may be used. Einhorn (55) has, too, but in a comparison of two such models used by decision makers with an additive model he found reasonable

correlations of 0.85 and 0.81. Moreover, some of Einhorn's experimental procedures have been questioned, and so his finding that non-additive models are used is suspect, although his finding of their similarity to an additive model's results is not. One recent study of physicians' judgments of illnesses based on multiple cues (184) found a linear model predicted the holistic diagnosis better than conjunctive or disjunctive models. Moreover, another study to discover the existence of such models found no evidence for them (163). It appears then, that the additive model is a useful one, and that interactions are not important in most, if not all, cases.

In addition to analysis of variance, multiple regression has been used to compare the linear additive model with intuition; here interactions cannot be isolated, but the technique can be used when the number of dimensions increases above about two, for analysis of variance requires too many subject responses in these situations (61). Even though multiple regression usually assumes linear value functions, the results of its use have shown overwhelming support for the adequacy of the additive model. Apparently, the only such research where correlation fell below the mid .90s was a study of job preferences of prospective school teachers (cited in (61)) - average correlations were in the .60s and .70s for students with and without prior teaching experience respectively. However, there appears to have been "noise" in this study (61, p.28).

A study not reviewed in the above literature has relevance to management. Dean and Roepcke (37) developed a 23 dimensional, weighted additive model for managing U.S. Army R and D resources. To check the validity of additivity they had estimates made of the value of the main effects and of the interaction between two dimensions in 252 cases, covering all the dimensions. In only 30 of the 252 cases was the interaction more than 25 percent of the total value, indeed, in only 43 of the 252 cases did a non zero valuation of interactions occur. They therefore continued to ignore interactions and to use their additive model.

Why is the additive model so useful even when interactions occur? Yntema and Torgerson's suggestion above that, if functions are monotonically increasing or decreasing, additivity is useful, has been tested in computer simulation studies and a "high degree of fit" between non-linear models and linear approximations was found (36, p.98). Two other reasons for the usefulness of the additive approximation are that its weights are not effected by imprecision in the multidimensional value, and slight errors in a value function makes the additive model more appropriate (36). As the decision context is usually one where monotone value functions are measured with some error, the additive model is a good approximation.

In brief, then, the results of an additive model will be acceptable. But will they be accurate - how important are interactions in the real world? To answer this, the results of the additive model must be compared with the correct result or the results of more complex models, and not with human holistic preferences.

The studies of the validity of the additive model are reviewed in Dawes (35; 36). When there have been enough previous similar decisions to produce regression coefficients for use in an additive model, the additive model almost always makes better decisions about a given situation than a human decision maker. For instance, the weights to be given to the eleven dimensions ofthe Minnesota Multiphasic Personality Inventory (MMPI) were found by regressing MMPI results with the results of later, more extensive, diagnoses of whether a patient was psychotic or neurotic. In a plethora of studies, there has not been one case where a clinical psychologist who presumably could include interactions made a better diagnosis based on MMPI results than an additive model. In similar studies, predicted GPA was taken to be a function of 10 dimensions such as self-ratings of conscientiousness, or of three dimensions such as previous GPA. Correlations between final actual GPA and human predictions ranged from 0.19 to 0.37; correlations between final actual GPA and an additive model's predictions ranged from 0.38 to 0.57. That is, the additive model was again the more accurate.

Moreover, the additive model is better than human decision makers even if regression coefficients are not available. By using subjective weights in a model, results have shown with only one exception in 172 cases that the additive model was a better predictor of psychosis or GPA than humans were. Indeed, on the average, an additive model with equal weights is even better than an additive model with subjective weights! (35; 36; 56) (However, this does not mean even weighting is better than subjective weighting in particular cases rather than average ones - see (35, p.15, footnote 1;28, p.44).

Despite this undoubted validity of the additive model, there is a reluctance among some analysts to use it. For example, Starr and Stein (197), after discovering that each dimension must be transformed by a value function before it is used in an additive model (or else the scale of measurement affects the result) suggest a multiplicative model using untransformed dimensions so that two alternatives A and B can be compared by

$$\frac{\text{Index A}}{\text{Index B}}$$

where, for example,

$$\text{Index A} = y_A^{w_y} \cdot z_A^{w_z} \quad \text{and}$$

$$\text{Index B} = y_B^{w_y} \cdot z_B^{w_z}$$

and w_y and w_z are the appropriate weights for two dimensions y and z. However, Index A = $w_y.\log y_A + w_z.\log z_A$. That is, Starr and Stein are using an additive model with a value function such that $v(y) = \log y$ and $v(z) = \log z$. Whether

this value function is in fact the value function of the decision maker is not tested for or justified. MDA under certainty using elicited value functions and an additive model is thus superior to this procedure, and if value functions are not available, this procedure is simply an example of MDA. Moreover, Starr and Stein do not explain how the weights should be derived. In brief, the additive value model need not be dismissed.

It is time to summarise and conclude the discussion of the applicability of the weighted additive model. If dimensions are monotonically increasing or decreasing, the main effects captured in an additive model should usually outweigh any neglected interaction effects. The additive model provides results which correlate as highly or more highly with human decision makers' intuitive, holistic, decisions than any other model. This suggests that interaction terms are either unimportant or are being processed incorrectly by humans. Moreover, human decision makers make poorer judgments than an additive model, which indicates that even if interaction terms are important, they will be processed incorrectly by humans. The concusion one is compelled to come to is that the weighted additive model of MDA under certainty is valid, even when Debreu-independence does not strictly apply.

4.5 Procedures for Deriving Value Functions

Procedures for finding the value functions for each dimension, and the weights for each dimension are almost similar, for in each case the objective is to map values onto one scale which varies from x_{i*} to x_i^* for each dimension i, or

from w_{i*} to w_i^* with one w_i for each dimension. However, exposition will be clarified if the procedures for deriving value functions and those for deriving weights are dealt with separately. Deriving value functions will be considered first.

Discussion for mapping value functions will be in terms of just one dimension y, for by the axiom of value independence levels of other dimensions do not affect a value function. For normalisation, the least preferred level of the dimension y_*, is set to zero and the most preferred, y^*, to 1. These two values of y can be actual values achieved by alternatives under consideration, or they can be easily imagined "benchmark" values of y. The next step is usually to rank the y's from the smallest to the largest, to produce an ordinal scale. While such a scale is adequate under certainty for unidimensional decisions, it is not adequate for multidimensional problems with Debreu-independence, forseveral scales need to be compared and so a cardinal scale is required. Thus the next step is to achieve a cardinal scale (which is unique up to a positive linear transformation, that is, $u(x_i) = a + b.u(x_i)$ where a and b are constants, $b > o$). This is commonly done by rating the values of y. Some alternatives to rating are the Churchman - Ackoff (24) successive comparison method (found to be so complex as to be virtually unusable by one researcher (50), although it has been used elsewhere (199)), and methods based on metrics (time-consuming). A more practicable alternative to rating is magnitude estimation (106; 198). Here the values of y are presented to the decision maker in a random sequence

and he has to indicate the ratio of the strength of his preference for each y in relation to his preference for the first value of y presented. The concept of ratios has to be explained to some decision makers beforehand. This has been used to assess preferences for occupations, odors, politics, crimes, etc. (198). Magnitude estimation has two characteristics, in addition to its simplicity. The first is that it is a ratio scale, which is unique up to multiplication by a positive constant. The second is that a group preference could be found by taking the geometric mean of the responses, which dampens the extreme responses from biased or aberrant respondents. The arithmetic mean of values of y expressed on a ratio scale does not do this as effectively. (Of course, the median of responses could also be used as the response of the group, if desired.)

Which method of deriving value functions is best? There has been one test (107) which showed that the magnitude estimation method may be inferior to simple rating, although if an unidentified source of error in the ratio scales could have been eliminated, the magnitude estimation method would have proved better. Another study (83), of valuations of the severity of burns, found that magnitude estimation provided better results than rating, as measured by survival rates of patients. However, this study is an exception and most research has shown that usually "data on judgments are unaffected by the method used to collect them", as Eckenrode noted (50, p.189).

For instance, a recent study (164) compared three methods of deriving value functions - eliciting graphs, magnitude estimation, and using comparisons between dimensions.

It found that the first two methods produced similar results, but that the third method (which was not suggested in the above review of procedures) was significantly different.

Three other recent studies confirms Eckenrode's findings of no difference between methods. von Winterfeld (215) compared simple rating and a variation of the magnitude estimation method to elicit value functions and found no significant differences in results. Fischer (62) compared direct rating with direct dollar judgment of levels of a dimension, to find correlations quite high. A third study which confirms this result is (28), in which seven methods of evaluating weights (including magnitude estimation) were tested and the result was that "methods did not differ" (28, p.41). Thus Eckenrode appears correct.

In any case, even if one method did produce value functions different to another, the difference to the choice among alternatives would be negligible. Fischer (62) took four variously curved value functions over a value range of 0 to 100, and compared the results of equally-weighted additive models using these value functions with results from three approximations to the value function. The first approximation was simply a straight line from 0 to 100; the second was two straight lines, one from 0 to the real value function at 50, then another from there to 100; the third was straight line approximations from 0 to 25 to 50 to 75 to 100 on the real value function. The lowest Pearson product moment correlation coefficient for the gross linear approximation was .691, for the second approximation it was .851, for the third approximation it was .969. Varying the number of

dimensions only made slight changes to the correlation coefficients. Considering the extreme non-linearity of the value functions in the study, the results show that the value function need not be precisely accurate; that is, even if simple methods of deriving them produce only approximations to the actually held values, the additive model's accuracy will not be affected. Between three to five points on the function other than the endpoints should be all that is required on which to fair in the function. Indeed, if the value function is close to being linear, perhaps one point is adequate - O'Connor (163) found correlations of 0.98 when he tried this in a real world application of a weighted additive model.

Given these findings of similar functions from various methods of elicitation and the unimportance of small differences in value functions, one may as well use the simplest methods available, rather than a complex one. Ranking and rating appear to be the easiest to use, in that order (50) but magnitude estimation could be used as an alternative to rating, if the decision maker preferred it.

4.6 Procedures for Deriving Weights

The question is often asked, "What does it mean to say that one attribute is twice as important as another?" e.g. (140, p.29). Essentially, weights are used to stretch or shrink scales on each dimension which have been normalised to measure from 0 to 1, say, so that the relative importance of each of these scales is incorporated into the decision. That is, w_is are the relative values of raising each x_{i*} to x_i^* (with other x_i held at their x_{i*} levels). That is, <u>weights are scale - or problem - dependent</u>. For instance, the importance

of the range dimension in a choice between three missiles is different if its x_{i*} to x_i^* distance is 10,000 miles to 20,000 miles rather than 10,000 miles to 100,000 miles. That is, weights do not depend solely on the level of x_{i*} or x_i^* alone.

However, if there are only two levels for each dimension (i.e. two alternatives) with each value of x_{i*} set to 0 and the other value of x_i, x_i^*, set to unity then Fishburn (65) has proved that one only needs to elicit the relative values of each x_i^* as weights, not the relative values of raising each x_{i*} to x_i^*, which are more complex judgments to make. The essence of Fishburn's proof is that setting $x_{i*} = 0$ eliminates the importance of the lower part of the normalised value function scales when the x_{i*} of zero is multiplied by the weight in the additive model.

The following discussion of how to evaluate weights will assume a choice between two alternatives, so that only the x_i^* need to be evaluated. For more than two alternatives, read "raising x_{i*} to x_i^*" for "x_i^*".

What procedures should be used to evaluate weights? The previous finding that different methods of eliciting judgments did not produce significantly different results, still seems to apply. Some studies which investigated different methods of deriving weights, as against different methods of deriving value functions, are Eckenrode's (50) and those studies he referenced. Eckenrode tested six methods - ranking, rating, three versions of paired comparisons of metrics, and the successive comparison procedure. The six

methods provided consistent results in three different types of problem, with Kendall coefficients of concordance and Pearson product-moment correlation coefficients in the mid to high 90s. Eckenrode lists many other published experiments confirming this degree of consistency among methods.

That the derivation of weights is as consistent as the derivation of value functions is to be expected, for both are simply mapping values onto a scale.

However, it is significant that Eckenrode did not include magnitude estimation among the methods he investigated. For a most important fact is that no weight can be zero in a weighted additive model, for if it were, that dimension would not be incorporated into the decision. Thus rating is not usable unless some method of deriving a "base" of zero if found, for rating produces a cardinal scale which is unique up to a positive linear transformation and so permits some weights to vary quite legitimately. For example, if the w_i for two dimensions were 1 and 0.5, a legitimate transformation could change these to 1 and 0.9 or to 1 and 0.1. Fischer (62) calculated correlations between additive model results when the ratio of the highest w_i to the lowest w_i varied from 81 : 1 to 9:1 to 1:1. The Pearson product moment correlations were only in the mid .70s for the 81:1 and 1:1 results, but correlations were higher for the 81:1 and 9:1 and for the 9:1 and 1:1 results. Thus the amount of error using the cardinal scale of rating to find w_is could be serious, although

it will usually not be. The important point is that there is no way of assessing the error because a cardinal scale has no true zero unless one is established as a base.

Only the ratio scale of magnitude estimation has a true zero (34, p.29) and so it evades the problem of cardinal scales referred to in the previous paragraph. Hence one correct procedure would be to rank the x_i^*s, then set the most preferred of the x_i^*s to be a w_i of 1, say, then use magnitude estimation to find each other w_i, $1 > w_i > 0$. However, rating could also be used if a base of zero was set at the relative preference for all dimensions set at x_{i*}. This would be a natural base from which to compare raising each dimension, but appears to be rarely used in rating schemes, which is unfortunate considering rating is often easier than magnitude estimation.

Research into the derivation of weights has shown that humans usually weight only a few dimensions when they make holistic judgments, and that when weights are derived for each dimension separately the weights are far more uniform. For example, Hoepl and Huber (94) found weights used in holistic judgments of teachers by students, using regression analysis. When the students provided direct weights for each of the dimensions, the weights were more uniform than those found by the regression analysis. Slovic (189) found stockbrokers did

the same when rating stocks on eleven dimensions. O'Connor (162; 163) and Slovic and Lichtenstein (190) review such studies, which are virtually conclusive on this point. Research has also shown that weights for dimensions at the bottom of a hierarchy of dimensions calculated from weights given to higher dimensions do not agree with the weights that subjects give directly to the bottom dimensions. The lesson to be learnt from this psychological research is that the derivation of weights can be difficult and the weights derived can be uncertain. For instance, O'Connor (163) found that water pollution experts gave water purity a weighting only 1.7 times that given to the relatively unimportant dimension of water colour. Later discussion produced a greater divergence between the two. Indeed, O'Connor found that gaining agreement about weights was more difficult than gaining agreement about value functions.

Fortunately, this uncertainty about elicited weights is not very important. For instance, O'Connor used each of the widely varying weights for each of his subjects with the agreed upon value functions in an additive model. Weights which were averages of the widely varying ones were used with the value functions in an additive model to provide a benchmark. Correlations of the individual results with the benchmark result were all above 0.92, most were above 0.95. Sharpe (185) also cites a computer selection study which had conclusions "almost completely insensitive to even severe changes in goal weights". von Winderfeldt (215) had a similar

experience. As well, the study of Fischer (62), varying the ratio of highest to lowest weights, has already been mentioned - it found correlations were in the mid 0.80s or above if the difference in ratios was not too large. Thus results appear to be quite stable as the weights in an additive model vary.

However, it would be appropriate to investigate more closely the factors which affect the stability of results with varying weights. As noted, Fischer (62) found that, when weights rose linearly from the lowest to the highest weight, it was only when the ratio of the highest to the lowest weight rose from 1:1 to extremely unlikely values of 91:1 or so that the stability of results was affected, and that the number of dimensions involved in the additive value model appeared to have no effect on the stability of results. But what if the weights did not rise linearly from the lowest to the highest weight, and what if the number of alternatives as well as the number of dimensions in the additive value model was varied - would the results still be stable? It was to answer this question that the following experiment was carried out. A simulation program of an additive value model was written that varied:

(i) the number of alternatives;

(ii) the relative height of the highest weight to the lowest weight;

(iii) the number of dimensions;

(iv) the pattern of the rise of weights from the highest to the lowest.

As regards (iv) there were four patterns of weights used to represent the various patterns that weights could be. The four patterns were, for n = 1, 2...N dimensions,

(i) level (i.e. level weights)

(ii) exponential ($w_n = 1 - 1/e^{((n/N) \times 5)}$)

(iii) arithmetic progression ($w_n = n$) (i.e. linear weights)

(iv) logarithmic ($w_n = -\ln(1 - n/N)$)

Because, in an additive model, the order of dimensions and their weights is immaterial, it is possible to assume weights are all level or rising from the lowest to the highest in this fashion. Graphically, the four patterns of weights are shown in Figure 4.1.

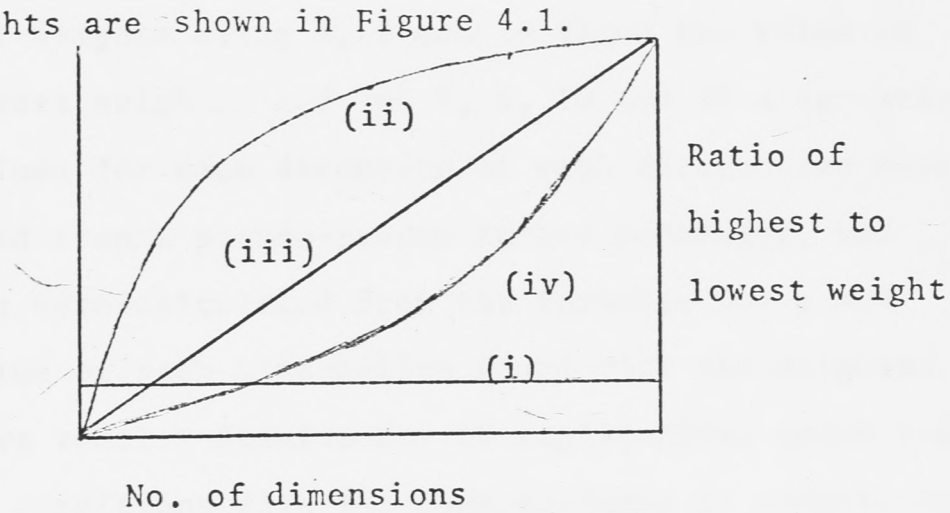


Figure 4.1 Patterns of Weights

In the long run, $E(v(x)) = E(\Sigma w_i . x_i) = \Sigma w_i . E(\Sigma x_i)$, with Σw_i constant for any particular weighting pattern. Thus one expects the ranking of alternatives to be unaltered whichever weighting scheme is used, for Σw_i will be a constant for each scheme and $E(\Sigma x_i)$ will be the same for each alternative.

Nevertheless, the variance of rankings around these expected values may be quite large, and will depend on variables such as the height of the largest weight compared to the lowest, the number of dimensions, and the number of alternatives involved. Using the four representative patterns, the resulting rankings of alternatives can be compared using the Friedman Multi-sample Test which is similar to the Kendall Coefficient of Concordance for agreement in rankings (26). Essentially, the test will discover if the rankings of alternatives differ with the various weighting patterns used. A computer program was run for 2, 5, 8, 11 and 14 dimensions in each alternative; and for the highest weights being 2, 9 and 16 times the value of the lowest weight; and for 3, 6, 10 and 20 alternatives. The values for each dimension of each alternative were obtained from a pseudo-random number generator, the weights were calculated from the formulae above and the value of each alternative found with the weighted additive vodel. Results for 50 replications under each set of conditions with the four patterns of weights are shown in Table 4.1. The test statistic is the χ^2 and values for Type I error levels are shown in the table. Table 4.2 shows the results when only the three rising, non-linear, patterns of weights - patterns (ii), (iii) and (iv) ~~about~~ ABOVE - were used. The results are similar to those in Table 4.1 except that χ^2 values are smaller in Table 4.2 indicating that there is a closer agreement in the rankings, as one would intuitively expect when the three similar, rising, patterns of weights were

The precise Type I error levels associated with the χ^2 values are not important, for the weighting patterns used are representative of extreme conditions, not actual ones, and in any case it is the Type II error level which is pertinent - the probability of erroneously accepting that the rankings are the same. Unfortunately, the Type II error levels are not available, and would in any case depend upon the discrimination between the ranks which a particular decision maker required for a particular decision. However, the probability of a Type II error, for a given set of conditions, becomes larger when the probability of a Type I error becomes smaller. Thus in the tables, when the Type I error level becomes small, there is a strong likelihood that the rankings are not the same.

The tables indicate that the changes in Type II error level are insignificant when the relative height of the highest weights changes, which is consistent with the results of Fischer (62), although he worked only with a linear pattern of weights, fewer alternatives, and a larger change in height. The tables also indicate that the Type II error level changes slightly when the number of dimensions and the number of alternatives change. The Type I error level appears highest for two dimensions, then drops for five dimensions and then only declines slightly for more dimensions. The tables indicate that the Type II error level increases with the number of alternatives.

TABLE 4.1

Average Values of χ^2, the Friedman Multi-Sample Test Statistic for Agreement in Rankings Using the Four Patterns of Weights in an Additive Value Model. (Type I Error Levels and χ^2 Values in Parentheses)

No. of Alternatives	Relative Height of Highest Weight	No. of Dimensions 2	5	8	11	14
3	(0.10-4.61;	0.05-5.99;		0.01-9.21)*		
	2	7.10	5.74	4.62	5.03	4.97
	9	6.69	5.44	6.24	5.24	5.15
	16	5.93	5.71	5.16	5.64	5.16
6	(0.10-9.24;	0.05-11.07;		0.01-15.09)*		
	2	17.59	14.93	14.10	14.33	12.98
	9	17.17	15.19	15.15	13.57	13.56
	16	16.95	14.62	14.45	13.92	13.99
10	(0.10-13.36;	0.05-15.51;		0.01-20.09)*		
	2	31.87	28.38	26.52	26.79	25.42
	9	31.59	27.56	27.15	25.00	24.92
	16	31.21	27.48	27.39	25.61	25.67
20	(0.10-27.20;	0.05-30.14;		0.01-36.19)*		
	2	68.33	60.65	57.10	56.68	55.76
	9	68.29	61.29	57.01	55.51	57.42
	16	68.26	60.72	57.52	57.66	56.83

* These are Type I error levels and corresponding X^2 values.

TABLE 4.2

Average Values of X^2, the Friedman Multi-Sample Test Statistic for Agreement in Rankings Using the Three Non-Linear Patterns of Weights in an Additive Value Model (Type I Error Levels and χ^2 Values in Parentheses)

No. of Alternatives	Relative Height of Highest Weight	No. of Dimensions 2	5	8	11	14
3	(0.10-4.61; 0.05-5.99; 0.01-9.21)*					
	2	5.40	4.73	4.21	4.55	4.47
	9	5.17	4.52	5.35	4.87	4.73
	16	4.55	4.56	4.36	4.76	4.73
6	(0.10-9.24; 0.05-11.07; 0.01-15.09)*					
	2	13.50	12.51	12.22	12.15	12.16
	9	13.02	12.66	11.88	12.10	12.70
	16	13.45	12.33	12.31	12.22	12.45
10	(0.10-13.36; 0.05-15.51; 0.01-20.09)*					
	2	24.37	23.25	22.34	23.10	22.38
	9	24.23	22.44	23.14	21.87	22.23
	16	23.94	22.66	22.37	22.28	22.56
20	(0.10-27.20; 0.05-30.14; 0.01-36.19)*					
	2	52.16	49.42	48.69	48.83	48.32
	9	52.17	49.27	48.17	48.12	49.47
	16	52.14	49.59	48.65	49.17	48.77

* These are Type I error levels and corresponding X^2 values.

In practical terms, the results of the computer simulation mean that time need not be spent on getting precise values of weights, and that the time could more profitably (in terms of a more robust solution) be spent in ensuring that the number of alternatives was no larger than necessary. Moreover, the addition of more and more dimensions should not decrease the robustness of the solution from an additive value model.

4.7 Conclusion

This chapter has found the additive value model for use in MDA under certainty is both acceptable to decision makers and, judging from the occasions when human and additive value models' decisions can be validated against an objective standard, that it will also be more accurate than them. Rating or magnitude estimation was found bo be quite simple and adequate for eliciting value functions and weights for dimensions. Considerable evidence was presented to show that the results of an additive value model were quite robust to changes in value functions and weights. Thus the model appears to be an excellent method of MDA under certainty. The following chapters will recommend its use for MDA under uncertainty as well.

5. UTILITY THEORY

5.1 The Elements of Utility Theory

MDA under certainty has been covered. Decision making under uncertainty is a far more complex and difficult task, for the alternatives do not have a point value, but rather a probability distribution of values. Choosing among the alternatives on the basis of one or more moments of this distribution of values is unsatisfactory, because third or fourth order moments may have to be included to satisfactorily describe such distributions, and preferences for these moments are difficult to assess.

Utility theory approaches this difficulty by mapping the distribution onto a utility scale, and the alternative with just the highest expected or first moment of utility is to be preferred, without regard to higher moments. The utility transformation rests on axioms, similar to the four underlying MDA under certainty. Unidimensional utility theory posits that the decision maker can assign utilities to uncertain alternatives on a monotonically increasing (or decreasing) cardinal scale. An Archimaedian axiom limits utility to finite values and, analogous to transitivity, a sure-thing principle ensures that utilities for a dimension are independent of levels of that dimension in other alternatives. Note that monotonicity is explicitly incorporated, while it is not in MDA under certainty, although Yntema and Torgerson (224) consider it a condition for value additivity.

The presence of multiple dimensions sometimes requires another axiom, analogous to the certainty axiom of value independence. This axiom is utility independence: utilities for each dimension are independent of the levels at which other dimensions are held. This axiom can be stated more formally: $u(x_i \mid x_{\bar{i}}^o)$, the utility of x_i conditional on the value of $x_{\bar{i}}$ being held at x_i^o - is a positive linear transformation of $u(x_i \mid x_{\bar{i}})$ for all values of $x_{\bar{i}}$. Utility independence is sometimes referred to as strong conditional utility independence (178) (when this applies to a set of dimensions, strong conditional utility independence becomes the uncertainty analogue of weak conditional utility independence for the set of dimensions, i.e., value independence plus Debreu-independence from dimensions outside the set (216).) However, the term utility independence is suggested as a clear uncertainty analogue to value independence. Note that, unlike value independence, utility independence is not crucial to the use of MDA, and models have been developed which do not require its existence. As all the axioms apply to utility, they are usually expressed in probabilistic terms, e.g. (42; 178).

Again, the unidimensional axioms appear rational and acceptable. Fortunately, the multidimensional axiom of utility independence is fairly frequently applicable, because more is usually preferred to less, whatever else is happening. Moreover, utility independence is

simple to test for. It can be tested for while deriving the utility functions for each dimension by asking the decision maker if his certainty equivalents $\sim$ 50/50 gambles at various values of x_i would be different if levels of $x_{\bar{i}}$ were different. For instance, the certainty equivalent $\hat{x}_1^2 \sim (x_1^*, x_1^1)$ should remain at the same value if the values of theother dimensions $(x_{\bar{i}})$ were changed from the least preferred to the most preferred levels, say. Or, more generally, utility independence can be tested for by simply asking the decision maker if he thought about the levels of $x_{\bar{i}}$ when considering what the certainty equivalent $\sim$ 50/50 gambles at two values of x_i was (117). If the decision maker answers no, it can be assumed that the certainty equivalent is not influenced by what level of $x_{\bar{i}}$ was, and hence utility independence exists.

5.2 Deriving Utility Functions

The mapping of dimensional values onto a utility scale should be done with questions involving lotteries to be a strictly legitimate measure of utility for uncertain outcomes. There are three major procedures for deriving a utility function for a dimension, y, say. The first, the Basic Reference Lottery Ticket (BRLT) method, asks the decision maker to nominate a probability p, such that $p.(y^*) + (1 - p).(y_*) \sim (y^1)$ whence $u(y^1) = p$, if $u(y_*) = 0$ and $u(y^*) = 1$. This method is applicable if an attribute cannot be put on a continuous scale (101), e.g. y_*, y^1 and y^* are owning 0, 1 and 2

cars respectively. This property of the BRLT method will be especially useful in multidimensional studies, in contrast to unidimensional studies which commonly use the continuous dimension of money. However, a decision maker may have preferences for some particular probabilities, thus biasing his answers to BRLT questions. Moreover, this first method has been found to be sometimes more difficult than the next one to be discussed (100; 101) because decision makers find it difficult to distinguish between low probabilities (e.g., .2, .1 and .05) and between high probabilities, and consider them all just very unlikely or very likely respectively.

The second method of eliciting utilities is the Equally Likely Gamble's Certainty Equivalent (ELCE) method. In this, the p is set at a neutral 0.5, and for each dimension the decision maker is asked to choose a certain value, $\hat{y}^1 \sim$ to a 50/50 lottery involving y_* and y^*. These two extreme values of y must be levels relevant both to the decision maker's experience and to the problem at hand. This certainty equivalent has a utility of 0.5 on a scale $0 \geq u(y) \geq 1$. A similar value $\hat{y}_i^2 \sim$ a 50/50 lottery involving y_* and y^1 has a utility of 0.25 and so on to y^5, or perhaps to y^7, after which a line is faired through these points. It is easier if the questioning is indirect (5). That is, y values are nominated and the decision maker asked if equivalence to the 50/50 lottery occurs, until it does, rather than asking what is the y value at which equivalence occurs. Figure 5.1 illustrates this method.

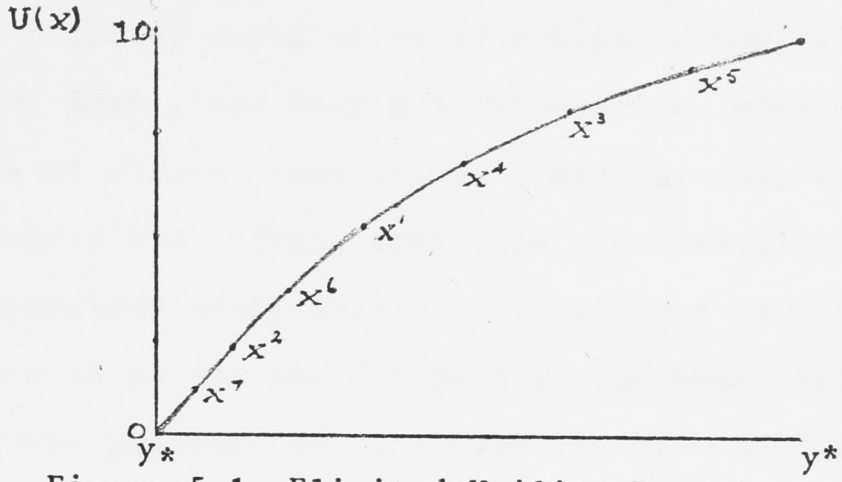


Figure 5.1 Elicited Utility Function

The third method of deriving utility functions is the Ramsey method. The decision maker is given two lotteries, each with two outcomes. He has to select a level of one of these outcomes if the probability in both lotteries is 0.5. In view of the need to consider two lotteries at once, it is not surprising that the Ramsey method has not been often used or recommended (101).

If a utility curve is fitted to obtain an algebraic formulation of the utility function it should be of a particular type, depending on the risk aversion of the decision maker. Risk aversion ususlly prevails, as elicited utility functions have shown, e.g. (195; 203), because bureaucratic managers believe that good decisions are normally expected but bad decisions are reprehensible - forcing risk aversion (85). This risk aversion will also exist for their politician masters,

for "at each election, individual voters compare their assessment of past government performance with their own personal expectation of how government should perform" (126, p.10), and thus politicians are judged on their term of office, just as, in a similar way, bureaucratic managers are. (Note that this risk aversion is not so commonplace among mainly self-employed farmers (101), which is as one would expect if the above explanation for the presence of risk aversion was correct.) Thus risk aversion should be expected in public investment decision, even though "the law of large numbers" would suggest that the risk of income loss in any one single public investment project would be diversified away by the returns from other, uncorrelated, projects. For the return on public projects are not only efficiency returns but also include political effect, and political gains may not compensate for political losses. For example, effects on a particular region or a particular voting bloc may not be compensated (180).

Risk aversion exists if the certainty equivalent for a 50/50 lottery between two values of x_i is less than the average of these values. (For instance, on the graph of the utility function, Figure 5.1, at u = 0.5, the y value of the utility function is less than that of a linear function, so risk aversion exists.) If the difference between the two decreases (increases, is constant) as the values of y increase, then the risk aversion decreases (increases, is constant). This can

be found by tediously questioning the decision maker for pairs of y (65), but can also be gauged from the points on the graph of the utility function. To do this, divide the y range into three or more equal sized segments. The points on the vertical utility scale halfway between the utilities of the bounds of each of these segments correspond to certainty equivalent values of y. Then if the horizontal axis differences between the upper bounds of each segment and its certainty equivalent decreases (increases, is constant) as the values of y increases, then the risk aversion decreases (increases, is constant).

If the risk aversion is decreasing, as it should reasonably be for some dimensions (195) - why be more averse to a gamble if you can better afford it? - then the utility function might be of the form $u(x) = \log x$ or $x^c (0 < c < 1)$. If the risk aversion is increasing, the utility function might be quadratic, $u(x) = x + bx^2 (b < 0)$ or some other polynomial of appropriate form. If the risk aversion is constant, then the utility function is linear or of the form $u(x) = 1 - e^{-cx}$ $(c > 0)$ (5). This last exponential form is often used for its simplicity, for all that is required to specify it is x_{i*}, x_i^* and one $\tilde{x}_i \sim 50/50$ gamble at (x_{i*}, x_i^*), e.g. (90; 117; 124).

5.3 Procedure of Application

These basics of utility theory are applied in this sequence for multi-attribute problems:

(i) formulate the decision into alternative courses of action, measurable on dimensions that define the preferences of the decision maker;

(ii) assess the probability distributions of the results of each course of action, on each dimension (Chapter 6);

(iii) derive the utility function of the decision maker for the range of results on each dimension (this Chapter);

(iv) use the appropriate utility model to find the expected utility of each course of action (Chapters 7 to 10);

(v) adopt the alternative with the highest expected utility.

5.4 Differences between Value and Utility

The differences between value and utility can be summarized in a table.

TABLE 5.1

DIFFERENCES BETWEEN VALUE AND UTILITY FUNCTIONS

Condition	Value	Utility
Alternatives	certain	uncertain
Scale of function	ordinal	cardinal
Is scale necessarily monotonically increasing or decreasing	no	yes
Independence of preferences on each dimension from levels of other dimensions	value independence	utility independence
Independence strictly required for the additive model	Debreu-independence	Fishburn-independence (see Ch. 7)

"Scale of function" in the above table needs elaboration, for not all functions dealing with certain multidimensional alternatives are ordinal, and not all functions dealing with uncertain multidimensional alternatives are cardinal (as suggested in (61, pp.45-46), for instance). For example, while the value of a unidimensional alternative, $v(x)$, requires only an ordinal scale, the value functions on each dimension, $v(x_i)$, are cardinal in the additive value model used with multidimensional alternatives. And while the utility of a unidimensional alternative, $u(x)$, needs to be cardinal if there is probabilistic dependence of levels on each alternative, if probabilistic independence exists then $u(x)$ need only be ordinal.

The appropriateness of the terminology adopted in this thesis for the various forms of independence should be clear from Table 5.1. For example, the connection between value and utility independence is brought out. Moreover, alternatives under certainty can be seen as 'degenerate' gambles, i.e., having a probability of one. Thus if utility independence exists, so does value independence. If utility independence exists for a set $(x_i, x_{i+1}, \ldots,)$ from another set $(x_j, x_{j+1}, \ldots)$, then value independence exists for that set, and if value independence exists for a set, then Debreu-independence of pairs of dimensions in that set from outside dimensions also exists. Finally, if Fishburn-independence exists allowing an additive utility model, then

Debreu-independence allowing an additive value model also exists (178). However, these relationships are not reversible, e.g., value independence does not imply utility independence.

6. PROBABILITY

6.1 Eliciting Subjective Probability Distributions

Utility theory deals with preferences for uncertain consequences for which some probability distributions are required. These could be derived from historical data or from a model based on the relative frequency concept of probability. Perhaps Schlaifer's "sparse data rule" could be used to produce an approximate cumulative distribution function from N observations (8), where the kth ranked observation is an estimate of the $k/(N + 1)$ fractile.

However, as decisions often deal with unique events objective probability distributions of variables involved are usually not available. So the decision maker's subjective probability should be used, that is, the probability distribution he believes in and wishes to act on because it is based on whatever information and experience he possesses. That is, probability becomes a measure of one's belief in the outcome of a particular event. Indeed, even if objective probability distributions are available, they should not be used unless the decision maker believes in them - when they are subjective probability distributions anyhow.

However, the use of subjective probability is not a simple straightforward matter. Human incapacity to correctly process uncertainty has already been noted;

for instance, humans are not Bayesians, and use incorrect heuristics. This incapicity can also be observed in elicited subjective probability distributions which are too "tight" when compared to available objective probability distributions. For instance, Alpert and Raiffa (4) asked for the 1 and 99 percentiles of subjective probability distribution of quantities which were easily verified objectively, such as yearly foreign car imports. But the true objective value was not within the subjective percentiles for 40 to 50 percent of the time. Even when this was pointed out to subjects, with urgings to widen their distributions, subjective probability distributions were too tight.

This tightness has been observed in many other experiments e.g. with managers in classes at the London Graduate School of Business Studies (156). This together with the other biases in subjective probability noted in Section 3.4 would indicate that "objective" distributions should be used wherever possible, at least as a starting point for deriving subjective distributions.

But what if there is no objective distribution available, or if the objective distribution is based on such sparse data that the decision maker does not believe in it, from a knowledge that there is no law of small numbers corresponding to the law of large numbers? Then the subjective probability distribution one must fall back on must be "correct" because there is no criterion other than belief to judge whether bias exists or in which direction it operates if it does exist.

(Of course, the subjective distribution should be internally consistent, that is, probabilities sum to one for mutually exclusive and exhaustive events, as well as being consistent with the beliefs of the individual.) However, there are several subjective distributions to be believed in! A review of the psychological bases of subjective probability by Hogarth (96) comes to these pessimistic conclusions. Because the human mind does not handle uncertainty well, and usually operates in deterministic terms, there is no distribution in the mind waiting to be elicited; thus elicitation will be difficult, for different elicitation methods elicit different distributions of the same variable by the same person. This is confirmed by another review of subjective probability (209) and is well illustrated by graphs of four such distributions elicited by four methods in (219). Moreover, "in reality, subjective probabilities determine preferences among bets and are not derived from them, as in the axiomatic theory of rational decision" (209, p.1130). Indeed, some experiments (95) have shown that deterministic point estimates were more accurate than elicited distributions, or at least the modes of these distributions were. Hogarth concludes, "From the evidence available, I would hypothesize that for the statistically naive (that is, many managers or those who provide them with information) probabilistic forecasting will depress predictive ability in the short run" (95, p.20). As training programs are impractical in many cases, these are pessimistic findings indeed.

In view of these findings, should MDA under uncertainty be limited to those few decisions where there are objective probability distributions or where trained estimators of subjective probability are available? No, because, as noted, the test of a subjective probability distribution is the decision maker's belief in it, and he could believe in his own subjective distribution more than he does in one elicited from a trained statistician. MDA under uncertainty has to use the subjective probability distributions it has available. Indeed, the desire to include all possible outcomes in decision making (if only to evade later criticism that some possibilities were overlooked) is one of the reasons put forward for decision analysis, e.g. (222), and is why distributions were used in the Mexico City airport decision analysis, for example (113). Given that the decision maker admits uncertainty, that is, has a subjective probability distribution which he believes in and wants to incorporate into his decision making, then decision analysis can assist, even if the decision maker is statistically naive, for the distribution is going to be incorporated into the decision one way or another anyhow, if the manager believes in it. What can be done is to elicit a subjective probability distribution in a way that will minimize the effect of human biases, but which will be convincing to the decision maker.

There are several methods of eliciting distributions available (42; 88; 101; 156; 219). Indirect methods, which have the decision maker answering set questions which assume a normative stochastic model in the decision maker's mind (e.g. Bayes Theroem) or assume his subjective distribution to be of the beta (96) or any other form, would obviously be unreliable, given Hogarth's findings mentioned above. Moreover, indirect methods assume a linear utility function for money, if money is involved in the questioning, or ask hypothetical questions which can be difficult to answer (101). The indirect approach is the basis of the probability wheel technique (196) in which a pointer on a two-coloured wheel is spun, and the decision maker asked when he considers the likelihood of an event to equal the probability that the pointer will stop in a varying colour. This obviously requires an awareness and belief in the relative frequency concept of probability and in randomness, and it has been found that executives feel uncomfortable about the implication that business risk is like gambling at a casino and find the method tiring (156).

There are several simple direct approaches. Fractile assessments of the cumulative distribution function can be made. Assume x is likely to fall in the range x^o to x^1; what is a value of $x^{0.5}$ such that it is equally likely that x will be less than or greater than $x^{0.5}$? $x^{0.5}$ will be the 50 percentile? Similarly find $x^{0.75}$ if x is sure to be between $x^{0.5}$ and x^1, $x^{0.25}$ between x^0 and $x^{0.5}$, and so on until 5, or perhaps 7, percentiles

are elicited and a curve faired through them. This approach is favoured by many writers, e.g., (4; 88; 156; 219). Another and far simpler procedure is to elicit the mode ("most likely value") and the 5 and 95 percentiles ("1/20 chance of being less/more than") and use approximation formulae given in (167) to find expected utility. But all these direct and indirect procedures use lottery-type questions which are unfamiliar to many people and need to use hypothetical situations which prompt hypothetical answers in many cases. Thus these procedures are often difficult to apply, and even if they can be applied the decision maker has the impression (137, p.403):

> *"Look, I feel that I know something about the probability distribution for this attribute, but (I am being forced)... to think in terms which make me uncertain as to whether I am correctly translating this information."*

These procedures with lottery-type questions also put the decision maker in relatively passive role compared to the analyst, and so do not closely involve the decision maker in building up his distribution. Smith (193) has suggested a method in which the decision maker does not answer lottery questions and need only rank differences in probability between pairs of discrete intervals of x and between differences between intervals. These rankings are manipulated by rank correlation techniques to produce a probability distribution function. However, the number of rankings is too large if the

number of intervals (howsoever they are chosen) is larger than say 3 or 4, and Hampton et al. (88, p.27) believe the method "will be psychologically and intuitively meaningless to the decision maker".

There are two very simple approaches to the elicitation of subjective probability distributions which do not suffer from any of the defects mentioned, but which appear to have been overlooked in most decision analysis literature; one approach is suitable for continuous variables, or a discrete variable with several possible outcomes, the other approach is suitable for a discrete variable with only a few possible outcomes.

The first approach is that developed at the University of New England (5) and by some World Bank staff (173). The World Bank staff had first asked subjects to draw continuous subjective distributions, but had abandoned this because the decision makers appeared to be "aesthetically influenced by the deceptively attractive appearance of the smooth curve and impressed by the complicated formulas. His judgment seems to lose its sharpness ..." (173, p.52-53). The World Bank staff then developed a second method in which the decision maker himself divides the possible range of x into i = 1, 2, ...n intervals and allots probability p_i to each interval with $\sum_{i=1}^{n} p_i = 1$. (The intervals could be just a straightforward "high, "medium" and "low" if necessary.)

The decision maker varies the size and the number of intervals and their probabilities until he is satisfied that the histogram he is working with adequately represents his beliefs. This author suggests that the decision maker can be asked to spread 100 percent over the intervals in proportion to their beliefs, and researchers at the University of New England sometimes give decision makers matches to spread over the intervals. The great advantage of this approach is that it does not require answers to lottery-type questions, allows the decision maker/analyst relationship to be flexible, allows quick revisions to early assessments (which could have been effected by anchoring, for instance), and involves the decision maker in a way which he can easily understand, thus encourgaing a belief in the elicited subjective probability distribution. Appropriately enough, it is sometimes called the visual impact method (5).

The second very simple approach to probability elicitation is useful for discrete distributions. Here a statement of an outcome is made, and the decision maker asked to tick a box indicating his subjective estimate of its likelihood. For instance (adapted from (202; 210), the statement or event might be:

Event 2: Public opinion will demend a heavier emphasis on preventive health and general welfare services.

0.0	.1	.2	.3	.4	.5	.6	.7	.8	.9	1.0
Won't occur	very unlikely		unlikely		fifty-fifty		likely		very likely	cer -ai

The numerical probabilities which fit the verbal descriptions can be changed, but the use of both helps the decision maker. Certainly this approach should be easy to implement, but it is limited to discrete variables with two, perhaps three, values, and it has no inherent tests for consistency like the 100 percentage points in the visual impact method.

Both these simple approaches elicit in an easily understood way the information which the decision maker can provide and no more. The decision maker is involved in the building of the distribution and hence believes in it and can change it easily as his awareness of uncertainty increases in the process of elicitation and interaction with the analyst.

Whichever method of elicitation the analyst uses, there are some essential first steps (based on (196) and Section 12.2). First, motivate the decision maker by noting the importance of his judgments about a variable to the decision. Second, ensure that the decision maker knows he should not allow his preferences for outcomes to affect his judgment of their likelihood. Perhaps this unscrambling of probability and desirability could be done formally, with scales for probability and desirability to be filled in. This might be useful for the discrete variables, but would be difficult for continuous variables, and anyhow may complicate the assessment without increasing accuracy.

Third, define the variable very carefully in terms the decision maker is familiar with - the clairvoyant test is useful here: could a clairvoyant reveal the answer in a single number without requesting clarification of terms? Fourth, lead the decision maker into an awareness of uncertainty, without allowing the usual biases to cloud the elicitation. This author has found that the deterministic prejudice of the human mind means that elicitation must start with a single point estimate which can then be demonstrated to be not absolutely certain to occur. Decision makers then quickly grasp what is required. If the interview begins with the limits of uncertain outcomes, decision makers can be uneasy with the whole process, and, moreover, these limits become two anchors. However, it must be admitted that most modern analysts advocate beginning with the limits; e.g. (74; 196). Whichever approach is used, the decision maker should be prodded to try to ensure that his distribution is not too tight or other biases are not affecting his judgment. The visual impact method is useful at this stage, for it allows changes to be quickly made, and the interrelationships of judgments to be seen. The fifth step arises out of the fourth, and that is testing for probabilistic independence. If discussion shows that some elicited probabilities will be dependent on another stochastic variable, then this other will have to be held fixed while distributions are elicited (this problem of probabilistic dependence is discussed in the next section).

The sixth and last step is verification, that is, testing to see if the decision maker really believes in his judgments. This can be done by drawing inferences e.g., if the probability of a value occurring is really twice that of another.

Finally, some mention should be made of scoring rules which have been developed to try to ensure elicited probabilities are really what the decision maker believes they are. However, they complicate the assessment procedure and their effectiveness has not been proven (88; 95). Indeed, unconscious biases can still occur even though decision makers are rewarded for correct answers (209, p.1130), so that using scoring rules may be akin to worrying about ants while elephants are rampaging through the campsite. If the assessor is aware that the correctness of an important decision depends on his honesty, there should be no need for scoring rules.

6.2 Dealing with Probabilistic Dependence

A problem which may arise in multidimensional decision analysis is that two or more dimensions may be probabilistically dependent, in which case a joint distribution will be required. Not only is assessing joint distributions hard work, but using them in utility analysis is tedious - a computer program will probably be required. Thus probabilistic dependence should be evaded where possible. Such dependence can be tested for by

introspection and checked by assessing several unconditional probabilities, p(y) and p(z), and then comparing their products p(y).p(z) to a separate assessment of the joint probability (5). If the two agree reasonably closely, independence can be assumed.

However, it is important that probabilistic independence is not assumed to exist when it does not. There has been negligible research into probability models in decision analysis, and probabilistic independence has usually been assumed for simplicity, e.g. (46; 113). But the problem of probabilistic dependence does arise in simulation models for risk analysis, and there at least the existence of dependence can seriously affect results. World Bank staff found that overlooking dependence (they call it correlation) "may lead to a completely wrong interpretation in the analysis" (173, p.45), and they "believe that the influence of correlations on the outcome of the analysis is more important than the influence of the shape of any particular distribution" (173, p.47). These conclusions were based on results from several project evaluations. Now, this dependence will also be as critical in a multidimensional utility model as in a simulation model, for in each it is the relationships between the variables that affects whether a low value on one variable can be compensated by a high value on another (with independence) or not (with dependence). This was strikingly demonstrated in perhaps the only

investigation of dependence in multidimensional decision analysis, in Dillon and Perry's review in (169). A hypothetical but realistic example with four dimensions was used, three of them binomial and stochastic, and two of these were made probabilistically dependent. The expected utilities of four alternatives were calculated using the multiplicative model, incorporating the dependencies. It was found that the results of an additive model incorporating the dependencies had a product-moment correlation coefficient of virtual unity with the results of the multiplicative model. But then probabilistic independence was assumed, and the additive model used with the four possible combinations of independent probabilities. Here the product-moment correlation coefficient varied from 0.98 (significant at 2%) to 0.79 (insignificant at 10%). That is, probabilistic dependence could quite seriously affect the results of a decision analysis if it is overlooked, depending on the severity of the dependence and the number of dimensions involved.

If tests show that probabilistic dependence exists, what can be done? There are four procedures available (173; 174). The first is to transform the dimensions until independence is achieved. The most practicable way of doing this is to combine the dependent dimensions into one. But doing this means that one of the benefits of multidimensional decision

analysis, the clarity obtained from disaggregation, is missed. So a trade-off may have to be made. The second method of handling dependence is to relate jointly-dependent variables to a hierarchical system of variables to achieve conditional independence. This would be done after analysing the causes of uncertainty and grouping dimensions with similar causes. However, this procedure would require the decision maker to provide conditional probabilities for groups of dimensions - not an easy task, for correlations of just two dimensions are difficult enough to think about. The third method of handling dependence is more promising, and involves using certainty equivalents in an additive value model (see Chapter 15). Certainty equivalents, by definition, will not suffer from probabilistic dependence, and using them in a decision analysis will probably be easier than a utility analysis of a full joint distribution. (Note, however, that certainty equivalents for independent dimensions will have to be elicited first to allow elicitation of certainty equivalents for the dependent dimensions.)

If neither of the three procedures above are used, and the decision maker still wants probabilistic dependence to be incorporated into the decision analysis, then a full joint distribution will be necessary. This is found by assessing the distribution of one variable (say y), and then, for each interval of this variable, the conditional distribution of z is assessed. (In other words, keep the number of values of y small.)

The joint probabilities are $p(y, z) = (p(y).p(y|z)$. Equations to use these probabilities in utility models will be given as the utility models are discussed.

Conditional probabilities are used in cross-impact analysis to compute scenarios and this deserves mention here (230 to 236). Most of the research has been on the mathematical aspects of combining probabilities of events with such uncertain results that one writer has remarked, "There seems peculiarly little point in applying a method which is designed to provide a rigorous mathematical answer to the implications of data stemming from human judgement if human judgement is to be used to select a particular answer from the range of those supplied by the rigour of mathematics." (232, p.342). The conclusion seems to be despair over the use of conditional probabilities at all because of the difficulty of manipulating them and because "psychological research by Kahremann and Trevesky has already shown that man is not Bayesian at all." (231, p.87) While cross-impact manipulations of probabilities are more complicated as those of MDA, these conclusions should be borne in mind by a decision analyst who attempts to allow for conditional probabilities. The point being made here is that MDA must be an approximation for precise calculation of actual probabilities is unlikely, and cross-impact experience confirms this.

Finally, it could be noted that decision theory uses Bayes Theorem to incorporate objective test data into prior subjective probability distributions. This will not be covered here, because of space constraints. However, two

points might be noted. Bayesian preposterior analysis could be used to see if further study to refine some of the estimates used in a decision analysis would be worthwhile. And if the likelihoods used in Bayesian analysis have to be subjective, they will suffer from all the imperfections of elicited subjective probabilities.

7. THE ADDITIVE UTILITY MODEL

As noted in Chapter 4, the weighted additive value model appears to be adequate for most decisions made under certainty. However, there is no consensus on the applicability of an additive utility model for decisions made under uncertainty. Thus several other types of utility models must be discussed along with the additive. The discussion will cover models in the order of decreasing number of assumptions in them. The following Table 7.1 lists the models and their assumptions.

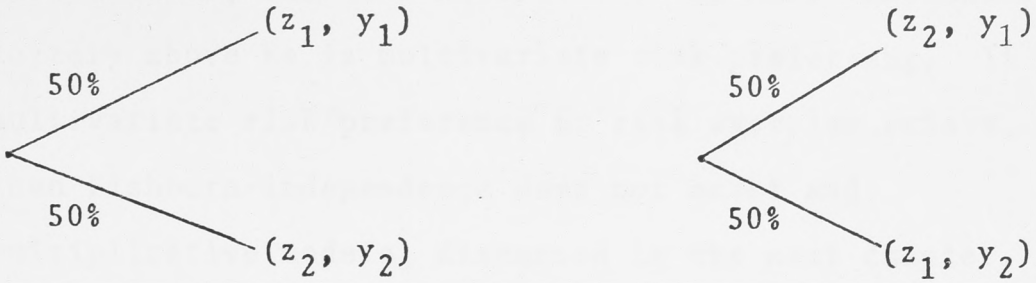
The additive utility model assumes that Fishburn-independence exists (sometimes called marginality (178) or marginal equivalence (65)). The assumption is here called Fishburn-independence after the first person to prove it, and thus to limit it to decisions under uncertainty and to highlight that it is the analogue of Debreu-independence. Proofs of Fishburn-independence have been given by Fishburn and others (64), with a very simple proof being given by Raiffa (178), which is based on utilities of base levels of (y, z), that is (y_*, z_*), not affecting the result when the levels are associated with other levels of y and z.

The assumption of Fishburn-independence is that the utility of a level of a dimension depends on the marginal probability distribution of that dimension, and not on the joint probability distribution of it with any other dimension. Take two levels of each of two dimensions: y_1 and y_2, and z_1 and z_2. Suppose lotteries gives a 50/50 chance at these y_1 and y_2 and z_1 and z_2.

TABLE 7.1

UTILITY MODELS AND THEIR MAJOR ASSUMPTIONS

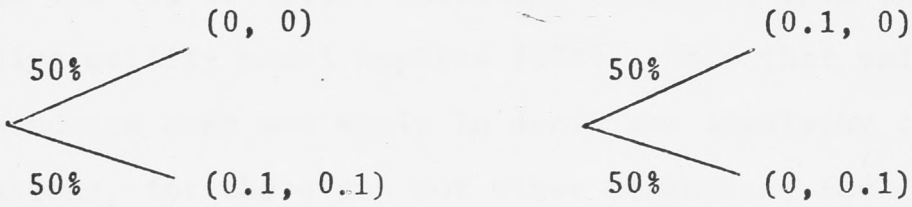
Utility Models	Assumptions for each dimension of:			
	Fishburn-independence	Utility Independence	Debreu-independence	Value independen
Additive, with Fishburn-independence for all levels of each dimension	X			
Additive, with Fish-burn-independence for any two levels of each dimension	X	X	X	
Multiplicative		X	X	
Quasi-additive with utility independence		X	X	
Quasi-additive without utility independence				X
R(V), with V found using the additive value model			X	X
R(V), with V found using the bench-mark model				X



Lottery 1 Lottery 2

Fishburn-independence implies that the decision maker would not pay anything to have either of the y's combined with either of the z's before the lottery is taken.

Keeney (108) gives an actual example of a test for Fishburn-independence. The decision maker had to choose between alternative telephone equipments which meant different delays to customers in two geographical areas in America. Consider the probabilities of delay of 0 and 0.1. Let the lotteries in the above figure be



Then the first lottery is preferred to the second because the combinations in the first mean equity between the customers in each area. Thus the decision maker prefers the first to the second lottery, and so the test for Fishburn-independence is negative.

Richard has investigated Fishburn-independence in some detail (81). When a decision maker opts for the more "equitable" lottery above he is considered multivariate risk averse, when he is indifferent between the two lotteries he is

risk neutral, and when he opts for the more "inequitable" lottery above he is multivariate risk preferring. If multivariate risk preference or risk aversion exists, then Fishburn-independence does not exist and multiplicative models, discussed in the next chapter, may be applicable. Actually, one study (40) has found that multivariate risk preference or risk aversion usually exists, but as this thesis plans to demonstrate, this doesnot invalidate the additive model in practice, even though it does in theory.

Notice that strictly each of the possible values of x_i should be tested with each of possible values of x_j, $j \neq i$. If this would be too tiresome, because there were too many dimensions, a procedure using the concepts of utility independence and value independence is available. If utility independence and value independence exist, and Fishburn-independence holds for any two levels of each of the dimensions (as in the two lotteries discussed earlier), then the additive utility model applies (178). Note that value independence does not apply in decisions involving two dimensions, for there are not other dimensions for trade-offs to be independent from, so in two-dimensional situations only utility independence needs to be tested for before testing for Fishburn-independence.

The additive utility model is used if Fishburn-independence exists (113):

$$u(x) = \sum_{i=1}^{n} w_i \cdot u(x_i) \qquad (7.1)$$

where $u(x_i) = \sum_{j=1}^{m} p_j \cdot u(x_{ij})$, i.e., the expected utility of dimension x_i, and w_i = the weight for dimension x_i, found by the methods outlines in Chapter 4, or by methods to be discussed in the next chapter. The model can also be presented as (from (5))

$$u(x) = \sum_{j=1}^{m} p_j \cdot \sum_{i=1}^{n} w_i \cdot u(x_{ij}) \qquad (7.2)$$

The second model is more difficult to assess, as the probabilities of combination of levels of dimensions, rather than of the values of each dimension, must be elicited. It need only be used if the dimensions are probabilistically dependent.

Fishburn-independence implies Debreu-independence (7; 8) because decision making under certainty simply deals wthh degenerate gambles. This means that $u(x) = v(x)$ up to a positive linear transformation; it also means that, if Fishburn-independence exists, then $v(x) = u(x)$ up to a positive linear transformation. Thus value functions (which are easy to elicit) can be used in place of utility functions (which require lottery-type questions to elicit) if Fishburn-independence exists (46).

The additive utility model is a simple one, and it is often used whether its assumptions do strictly apply or not (see Chapter 12). However, as noted, there is no consensus that the additive model is a general one, so other models will have to be discussed.

8. THE MULTIPLICATIVE UTILITY MODEL

If utility independence and Debreu-independence exist, but Fishburn-independence does not, then the multiplicative model is applicable. If there are only two dimensions, then the quasi-additive model is also applicable in this situation, but the multiplicative model is more practicable (see Chapter 9).

The proofs of the multiplicative model are due to Keeney and are complex; they are based on utility independence and Debreu-independence (called preferential independence by Keeney) of sets of dimensions, and the cardinal utility function property of being unique up to positive linear transformations. Proofs for more than three dimensions are in (110), for two dimensions in (111).

The conditions for the multiplicative model are that for some dimension, x_j say, Debreu-independence with each other dimension exists, and just x_j is utility independent of other dimensions. Of course, if all pairs of dimensions are Debreu-independent, then just any one of the dimensions needs to be utility independent of others. The multiplicative model is (113)

$$K.u(x) + 1 = \prod_{i=1}^{N} (K.k_i.u(x_i) + 1) \qquad (8.1)$$

where $u(x_i)$ is the utility of x_i scaled from 0 to 1, k_i are scale factors ranging from 0 to 1 for each $u(x_i)$ and K is another scaling constant. The formula is solved for u(x).

The $u(x_i)$ are found by one of the methods in Chapter 5. If probabilistic independence exists the $u(x_i)$ refer to the expected utility of an alternative on dimension x_i; if probabilistic dependence exists, then the $u(x_i)$ refer to the utilities of the particular levels of each dimension in a joint outcome x, the expected utility of an alternative is then the sum of the utilities of each such outcome multiplied by its probability of occurrence.

The procedure to find the k_i and K is given by Keeney (110), and is based on the BRLT method of eliciting utilities. The decision maker is presented with three hypothesized consequences, C1, C2 and C3, involving all the attributes, and asked to choose a probability such that $p.C1 + (1 - p).\ C2 \sim C3$ for sure. The two consequences the decision maker must choose between probabilistically are combinations of the most preferred and least preferred levels of all attributes,

$$C_1 = (x_1^*, x_2^*, \ldots x_n^*) = 1, \text{ and}$$

$$C_2 = (x_1^*, x_{2*}, \ldots x_{n*}) = 0$$

The consequence for sure is, when finding k_i,

$$C_3 = (x_{1*}, \ldots, x_{i*}, x_{i+1*}, \ldots x_{n*})$$

Once a value of p is found which makes $p.C_1 + (1-p).C_2 \sim C_3$ k_i is set to p. This is done for each x_i. The k_i can be understood as the relative preferences for raising a particular x_i from x_{i*} to x_{i*}, with all other attributes remaining at x_{i*}.

Another method of finding the k_is is based on the Ramsey method of eliciting utilities (118). The end levels of two dimensions are taken (with other dimensions held constant) and other levels of the dimensions are found so that, for x_1 and x_2 say,

$$u(x_1, x_{2*}) \sim u(x_{1*}, x_2) \tag{8.2}$$

Using basic algebra and the multiplicative or additive utility models, the utility of these two consequences can be equated to yield

$$k_1.u(x_1) = k_2.u(x_2) \tag{8.3}$$

As $u(x_1)$ and $u(x_2)$ can be read off the utility functions for these dimensions, k_1 can be expressed as a linear function of k_2, or vice versa. This is done for all dimensions. With all k_is then expressed relative to each other, all that remains to be done is to find the absolute value of one of the k_is. This has to be done by the first of Keeney's two methods outlined above, with the p selected set equal to a k_i. With this absolute value, all the other k_is can be scaled to it using their linear relationships. Two clear examples of this second method are in (117; 120).

The two advantages of this second method are that, except for one dimension, it does not require the extreme x_i s to be considered in evaluating the k_is, and it does not make the decision maker consider simultaneously all dimensions set at x_i* and x_{i*}. Keeney has used this second method in some recent tests but it should be noted that its Ramsey method has been found to be the more

difficult than the first, BRLT, method in many unidimensional situations (see Chapter 5).

Once the k_i are found K can be found by solving

$$K + 1 = \prod_{i=1}^{n} (Kk_i + 1) \qquad (8.4)$$

This can be done by iteration (107), with a first value of K of $-1 < K < 0$ if $\sum_{i=1}^{n} k_i > 1$, and $K > 0$ if $\sum_{i=1}^{n} k_i < 1$. After an iteration, if the r.h.s. > l.h.s. of (8.4) then the first value of K must be increased and vice versa. The total utility of each outcome is then found from the multiplicative model (8.1) above. Note that if $\Sigma k_i = 1$ then $K = 0$ and the multiplicative model becomes the additive model. Thus checking for $\Sigma k_i = 1$ is in effect a check for Fishburn-independence.

Finding the k_i with the above two methods involved lottery questions. However, it might be asked whether lottery questions are required, for each dimensions is known for sure to be pertinent to the decision, or it would not have been included. Moreover, the meanings of the k_i and the w_i used in the weighted additive value model are the same, that is, the relative value of raising each x_{i*} to x_i*, with $x_{\bar{i}}$ held at $x_{\bar{i}*}$. Thus as these x_{i*} to x_i* scales are known to exist with certainty, the technique to derive them could more easily be to simply set C1 to 1 and C2 to 0 and use rating to set each C3 a k_i value within this range.

If rating is used in this way to find the weights k_i, the information required for the multiplicative model is almost the same as that for the additive model. Utility functions, weights (k_i in the multiplicative model, w_i in the additive), and utility and Debreu-independence are all found similarly. One advantage of the multiplicative model in these circumstances is that the sum of the weights equalling one is the test for Fishburn-independence rather than the more difficult lottery questions used to test for Fishburn-independence in the additive model. Another advantage of the multiplicative model is that it does not require Fishburn-independence. On the other hand, the additive model has two advantages. The first is that the weights do not have to be assessed relative to C_2 as the upper limit of unity, and thus their derivation does not require a consideration of all dimensions raised to their most desirable levels simultaneously. (This is because only the values of the w_is are used directly in the additive model, not the value of C_2, and setting the w_i of the most desired dimension to unity and scaling other w_is relative to it produces the same rank ordering of consequences as setting C_2 to unity and scaling the w_is relative to it.) The second advantage of the additive model over the multiplicative model is that it is intuitively understandable by most decision makers. Weighing the advantages ofeach of the two models is a matter for the decision maker and the analyst. Perhaps

the acceptability of the additive model outweighs the advantages of the multiplicative model, especially as the results of the two are extremely closely correlated, even when there is no Fishburn-independence. Chapter 11 discusses this correlation in detail.

In short, the multiplicative model might be a useful model for the analyst to know and use, although it is more difficult for a decision maker to understand.

9. THE QUASI-ADDITIVE AND OTHER UTILITY MODELS

There are two quasi-additive utility models, both developed by Keeney before his multiplicative model. One assumes utility independence in all dimensions, and the other does not. Proofs of the quasi-additive models are relatively simple (109; 111), depending on the property of a utility function of being unique up to a positive linear transformation, which allows utility functions to be scaled one on another. Proofs of theother models are complex, and these models will be discussed briefly after the quasi-additive models.

9.1 Utility Independence

The first quasi-additive model requires only that dimensions be utility independent. Diccussion of the two-dimensional model will precede that for more dimensions. The model is (178)

$$u(y,z) = k_1.u(y) + k_2.u(z) + (1 - k_1 - k_2).u(y).u(z) \quad (9.1)$$

where k_1 and k_2 are constants and u(y) and u(z) are utilities of an alternative on dimensions y and z. The utilities are found as usual, with tests for utility independence being carried out while they are being derived. Finding the constants uses the ELCE method (108). After the utility functions have been found, find which of y* or z* is most preferred. Say y* is most preferred. Then a value $\hat{y}$ of y is found such that

$u(y^1) \sim u(z^*)$. Thus $k_1.u(y^1) = k_2$, because $u(z^*) = k_2$ by definition. From the utility function, the value $u(y^1)$ can be found, and hence k_1 (or k_2) can be expressed in terms of the other constant. Then a certainty equivalent is found, $u(\hat{y}, \hat{z}) \sim$ a 50/50 lottery at (y_*, z_*) and (y^*, z^*). This certainty equivalent has a utility value of 0.5, and so setting this on the left hand side of (9.1) with y and z equal to $\hat{y}$ and $\hat{z}$, and solving, provides a value of k_1 (or k_2). Then the other constant can be found from their known relationship. Using these constants, u(x,y) can be found for any alternative. Note that if $(1 - k_1 - k_2) = 0$, the additive model applies.

It should be clear that this method of finding constants is more complicated than that which could be used to find weights in the additive and multiplicative models, that is, rating the values of raising y_* to y^* and z_* to z^*. The multiplicative model could be used, in place of this quasi-additive model, for when there are only two dimensions there is no need to test for Debreu-independence, which is an additional condition for the multiplicative model but which is not relevant in two-dimensional decisions. Note that when the multiplicative weights k_1 and k_2 sum to unity the additive model applies. This is the same condition as that for the quasi-additive model, which indicates that the quasi-additive and multiplicative weights k_1 and k_2 are similar, despite their different derivations.

Procedures are more complex when there are $n > 3$ dimensions (111). Again, utility functions for each attribute are found. However, in addition, 2^n scaling constants are necessary, and finding these is tedious. These scaling constants range from

$$u(x_1^*, x_2^*, \ldots, x_n^*) = 1 \quad \text{to}$$

$$u(x_{1*}, x_{2*}, \ldots, x_{n*}) = 0$$

These constants rate the utility of hypothetical consequences containing each possible combination of x_i^* and x_{i*}, $i = 1, 2,,,n$. Raiffa (178) gives one probabilistic method of determining these constants, but the method is tedious. Apparently the model has only been used once, but then it was used in a 3-dimensional problem under certainty where simple magnitude estimation procedures were used to find weights and the value functions (225).

9.2 Utility Dependence

The question now arises - what if utility independence does not exist? Keeney (109) has developed models forthis situation, but they are complicated, so the first thing to do might be to attempt to transform the dependent attributes into one that is independent. There are two ways of doing this (2). The first, the a priori way, finds a new x_i by analysis. An example occurs when the problem is minimizing costs at a counter (x_1) while minimizing the waiting time of customers at the counter (x_2). Here, waiting time itself can be

expressed as a cost (from lost sales, ill will). The objective transformation, then, is the cost to avoid the delay. A similar transformation would be to transform intensity and frequency into the one attribute of loudness. The second method of transformation is a posteriori. Here the efficiency curve, or the curve of most efficient transformation of x_1 into x_2, or vice versa, is drawn, and the decision maker is then asked his utility for these combinations. Some ingenuity here will pay dividends in simpler decision analysis.

But if mutual utility independence is still not attainable, then a modified version of the quasi-additive model is applicable. Keeney has developed this only for n = 2 or 3 dimension problems, but utility dependence will probably only apply for 2 or 3 dimensions. Strictly speaking, the model applies only when n - 1 dimensions are utility dependent of the others, but the model should be a good approximation when all dimensions are utility dependent, because the model gives five degrees of freedom, more than previous models (109).

For the 2 dimension case, with z utility indepent of y, but not vice versa, then

$$(9.2) \quad u(y_1 z) = (1.u(y|z^o).(1-u(z|y^o)) + k_2.u(y|z^1)u(z\ y^o)$$

where $u(y/z^o)$, for instance, means the utility of the value of y conditional on z held constant at z^o while the utility function is being elicited. Thus for 2 attributes, three conditional utility functions are required, each scaled from 0 to 1:

$$y/z^o,\ y/z^1,\ z/y^o$$

In the application of the model, the utility function on the utility independent attribute, $u(z/y^o)$, is set to a standard scale of 0 to 1; however, weights, k_1 and k_2 are required to scale the other utility functions to this one. To do this, Keeney suggests that the analyst first find consequences $(y^o,z^1) \sim (y^1,z^o)$. Thus z_* to z^1 on the z/y^o utility scale equals y_o to y^1 on the utility scale, so $k_1 = u(y^1|z^o)/u(z^1|y^o)$ as both scales have a zero origin. Then find consequences $(y^o,z^1) \sim (y^1,z^1)$, and then $k_2 = u(y^1|z^1)/u(z^1|y^o)$, similarly. Alternatively, and more simply, one could use the methods suggested in Chapter 4 to find weights for $(y^*|z_*)$, $(y^*|z^*)$ and $(z^*|y^*)$, with a base of $y_*,z_*)$

The 3 dimension problem with utility dependence is similar in spirit, but requires six conditional utility functions and five weights for them which would be difficult to elicit. The full model is in (109), where Keeney also describes how indifference (or iso-preference) curves can be used. However, the derivation of indifference curves requires the decision maker to vary two attributes at once, while assessing his preference for consequences containing them, and so are probably as difficult to elicit as conditional utility functions, even though questions involving lotteries are not involved. This was confirmed with university administrators (166).

9.3 Other Utility Models

There are other utility models, but there have been no atemmpts to apply them. They are developments in the theory of utility which are mentioned here solely for completeness.

The concept of generalized utility independence (67; 70) arose to account for cases of value dependence using the terms of this thesis. It is a generalisation of quasi-additivity. It in effect allows for a transformation of the value dependent dimension so that, for example if x_1 is generalized utility independent of x_2,

$$u(x) = u(x_1) + u(x_2) \cdot f(x_1) \qquad (9.3)$$

It can be shown that either an additive or multiplicative model can be used if certain generalized utility independence conditions and some "essentiality" requirements hold for x_1. The models require 2^n constants to be evaluated as the utilities of extreme alternatives plus two conditional utility functions for each affected dimension, which would certainty be tedious and difficult. An analyst would be advised to attempt the corrections for value dependence outlined in the previous Section before attempting this model.

Bilateral independence (68) is an attempt to allow for utility dependence on both dimensions rather than just one as in the quasi-additive model (9.2) above.

The model requires two conditional utility functions for each dimensions, and a symmetrical attitude to risk in each dimension involved. Again, application would be even more difficult than the quasi-additive model (9.2).

The most general development of utility theory is Farquhar's fractional independence scheme (58; 59). Using the mathematical concept of fractional hypercubes, he has developed decompositions that include all models discussed so far, and which can, moreover, allow for more and different dependence conditions on each dimension. The "fractional hypercube methodology uses an extraordinary amount of special notation and terminology" (59, p.257) and is primarily developed for interaction terms between and among dimensions and requires conditional two dimensional utility functions, in addition to the conditional one dimensional utility functions required in previous models. As the theme of this thesis is that, in practice, these interaction terms are unimporannt, there seems little point in hacking a path into the lush mathematical jungle of fractional hypercubes. No attempt has been made to apply the fractional hypercube models that extend beyond the quasi-additive models.

Fishburn (68) has attempted to approximate situations with complex interactions among dimensions with a mixed additive-multiplicative model requiring only conditional one dimensional utility functions. Using mathematical approximation theory and making

the assumption of multivariate risk aversion on all pairs of dimensions, he shows that a simple additive utility model can be a good approximation providing Δ is small, where

$$\Delta = \{u(y^*, z_*) + u(y_*, z^*)\} - \{u(y_*, z_*) + u(y^*, z^*)\} \qquad (9.4)$$

This appears to be another way of saying that the additive utility model is adequate if multivariate risk aversion is small, for (9.3) is merely the difference between the utility for the two gambles in the test for Fishburn-independence, and is zero if independence exists. However, if the (difficult) bilateral independence model is used, the model is a close fit no matter the degree of multivariate risk aversion. This is to be expected, as the bilateral independence model even has one more degree of freedom than the quasi-additive model assuming utility dependence of one dimension (9.2).

9.4 The Treatment of Time

Most investment decisions involve a time sequence of outcomes. In private investment decisions, this means discounting in one or another of its various forms must be used. But public investment decisions have time effects that are more complex because non-financial attributes are involved which are difficult to discount (if only because there is no marekt cost of capital to assist in establishing a discount rate); because

horizons are infinite or more difficult to forecast than the life of a product or a mine, say; because valuations of a level of a dimension may depend on that dimension's level in the past or future; and because valuations of a level of a dimension may depend on the level of another dimension in the past, present or future. For all these reasons, it is appropriate to consider the treatment of time here.

The time sequence of outcomes can be handled by treating each outcome as a dimension. When each outcome or dimension, x_y, is known with certainty, then Meyer (149) has shown that, if each x_t is value independent of x_{t+1}, then either the additive or multiplicative value model is appropriate. The additive model is appropriate if Debreu-independence of each (x_t, x_{t+1}) pair exists, as noted in (149). Koopmans has shown that the additive model is appropriate if just x_1 and x_2 are Debreu-independent and value independent of each other, and a stationarity assumption exists so that two time streams are ordered similarly if the x_1 of each stream are removed and the remaining outcomes advanced one period.

In decisions involving uncertainty, the additive or multiplicative utility models can be used if each x_t is utility independent of x_{t-1}, $t = 2, 3...n$, and x_{n-1} is utility independent of x_n with Fishburn-

independence existing for any two x_ts permitting the additive model (149).

Now it could be argued that utility independence for each outcomes on the same dimension is not likely in decision situations involving time. For example, the utility for a high level of unemployment will depend on whether there was a high level of unemployment the year before. One way of evading the complexity involved when such utility dependence exists is to recast the outcomes. For instance, the time period could be extended from one year to five years say, when the interdependencies between years could be averaged out.

If this recasting of outcomes to produce utility independence is not possible, there are several approaches available. Meyer (151) provides one. Rather than each outcome x_t being utility independent, if it can be shown that each subset of dimensions $\{x_1, \ldots x_t\}$ is utility independent of $\{x_{t+1}, \ldots x_n\}$, and vice versa, then either

$$u(x) = \sum_{t=1}^{n} a_t . u(x_t), \qquad a_t > 0 \qquad (9.5)$$

or $$u(x) = \prod_{t=1}^{n} (b_t + c_t . u(x_t))$$

Testing for the assumptions for these would be tiresome, as each subset must be considered. Moreover, if utility

independence for each x_t did not exist, it is unlikely to exist for all subsets.

Fishburn (63) produced a model that did allow for some interdependency between attributes in neighbouring periods, by using an assumption called Markovian dependence to produce the model

$$u(x) = \sum_{t=1}^{n} u(x_t, x_{t+1}) - \sum_{t=2}^{n-1} u(x_{t+1}) \qquad (9.6)$$

The utility functions are conditional on all other dimensions being held at a set level. Markovian dependence demands that the decision maker is indifferent between sets of uncertain time streams if and only if the probabilities of each set of streams are the same. An example is given in (15, p.24). Unfortunately, Markovian dependence was "not felt to be sufficiently intuitively meaningful to be used" (14, p.3) in the only decision analysis for which it has been considered, which considering its complexity is not surprising.

Notice that Fishburn required a two "dimensional" (i.e. consequences at different times) utility function in model (9.6), whereas Meyer required only the usual one dimensional utility functions. Bell also requires two-dimensional utility functions in his more generalised model. Bell (14; 15) uses the concept of mutually conditional utility independence. For a three dimensional situation (each dimension could be a set of outcomes at different times), X_1 and X_2 are mutually conditionally utility independent if each is utility independent of the

other, given that X_3 is at a fixed level. If each subset $X_1, \ldots X_{t-1}$ is mutually conditionally utility independent of $X_{i+1}, \ldots X_n$, and vice versa, and $n \geq 4$, then

$$u(x) = \sum_{t=1}^{n-1} u(x_t, x_{t+1}) - \sum_{t=2}^{n-1} (x_t) \qquad (9.7)$$

or

$$u(x) = \prod_{t=1}^{n-1} (\beta + \mu(x_t, x_{t+1}))/\prod_{t=2}^{n-1} (\beta + \mu(x_t | x_{t+1}^0)) - \beta$$

where β is a constant and all two dimensional utility functions are conditional on a fixed level of all the other dimensions.

In a detailed application of MDA involving time, Bell (15) did use this model for two of the four decision dimensions involved, with, furthermore, two of the decision dimensions being mixed in the time dimensions, so that, for instance, the two dimensional utility function was of the form $u(y_t, z_t)$ and the one dimensional function was $(z_t | z_{t+1}^0)$. Despite this complexity, the decision analysis did provide usable utilities although it has not been used in an actual decision. Bell (15, pp.43-44) is open minded about whether the complex utility model would provide better management policies than a simple additive model. But

he is convinced that the MDA approach which involves some consideration of utility independence concepts is a useful one.

One argument against the complexity of Bell's model is that the time and effort expended on it may mean that consideration is not given the problem of the decision horizon (simply assumed to be 50 years in Bell's application). Meyer (149) considers the situations where the horizon is variable or uncertain, and his approach requires joint utilities for an uncertain stream of consequences and uncertain horizons. Models can be built up depending on the dependencies between these.

There is much other complexity in all decision analysis involving time. For instance, there is the large amount of uncertainty involved, and analysts must consider all the other factors which may be involved and their uncertainty. As well (149), the rate of resolution of this uncertainty about other factors will affect the period of uncertainty about the project, so that utility functions will include utility for risk and attitude to risk, which in principle it should not. Finally, making each year's outcome a dimension can make a MDA extremely cumbersome, e.g., if there are just four decision dimensions and 20 years of outcomes for each, the MDA involves 4 x 20 = 80 "dimensions", and if there several alternatives involved, eliciting the subjective probability distributions will also be a daunting task. The obvious conclusion, then, is that

some simplifying assumptions in the data used in the model will be necessary, suggesting that the model should be a simple and robust one and not the more complex ones mentioned in this section.

9.5 Conclusion

Several non-additive and non-multiplicative models have been reviewed in this chapter. The quasi-additive model with utility independence was found to be similar to the multiplicative model, and its constants were more difficult to elicit than those in the multiplicative model, especially when $n > 2$. The other models discussed were developed for situations when utility or value independence did not exist, and require more information to be elicited from the decision maker than for models based on utility or value independence. These other models have rarely been used. The quasi-additive model with utility dependence on one dimension has been used (in two dimensional decisions only (90; 108)) but as Chapter 12 will demonstrate, the additive model may have been adequate. The more advanced models have not been used, and the difficulty of eliciting the information required may see them remain so.

All these comments also apply to the complex models developed to MDA under uncertainty involving time sequences of outcomes. Hence the more advanced models will not be considered further in this thesis.

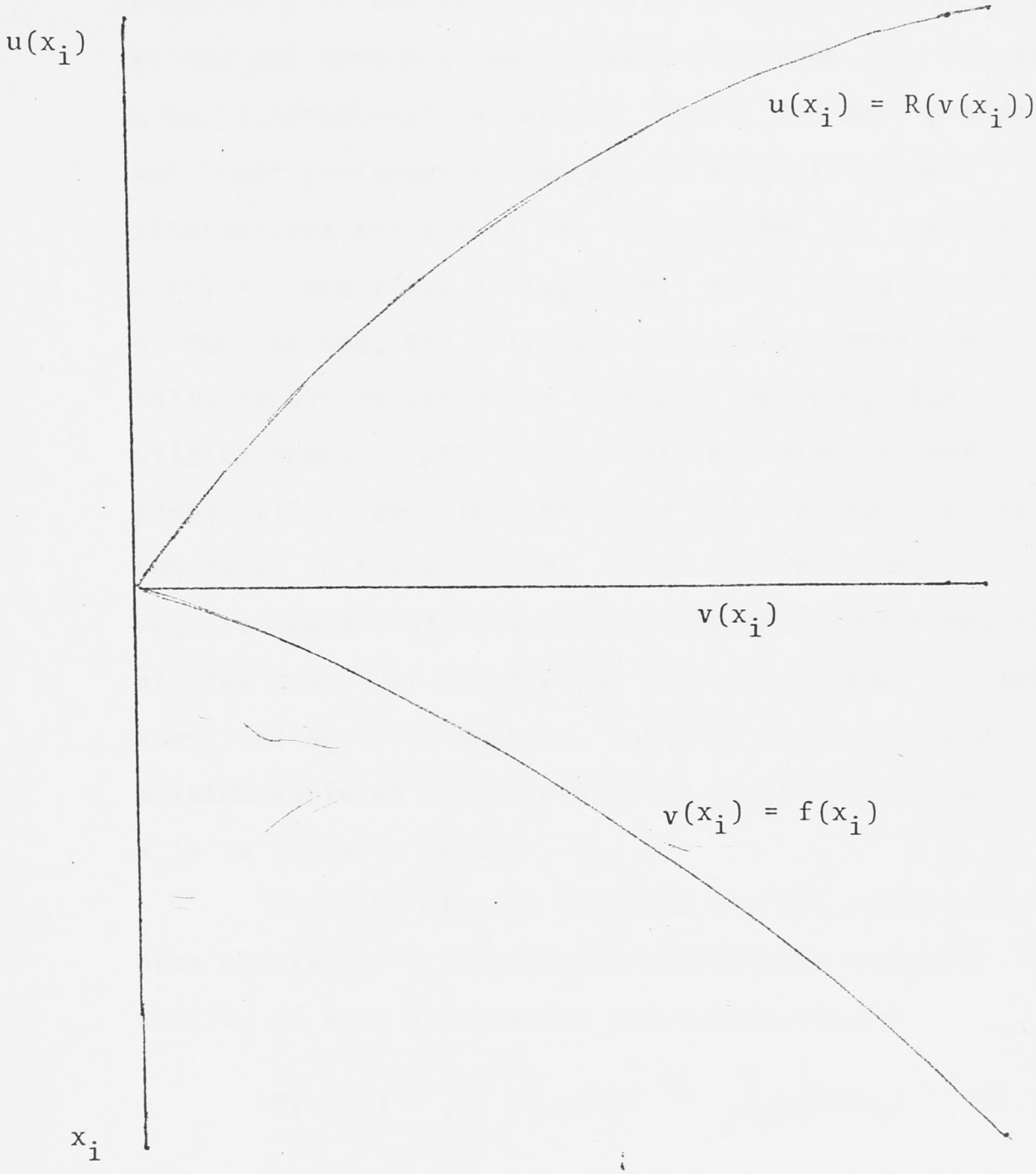
10. THE R(V) MODEL

10.1 The Basic Model

It can be shown (67; 178) that if $v(x)$ and $u(x)$ exist, then a monotonic transform R exists such that $u(x) = R(v(x))$. That is, if alternatives are evaluated using value functions, then the resulting evaluations can be transformed into utility evaluations that can be used in decision making under certainty. This is the basis of the R(V) model, developed by Raiffa (178), which might be used if the utility independence assumptions of other utility models in previous chapters do not apply.

The model first requires $v(x)$ for each alternative x. This can be found by the benchmark model or by the additive model. If the benchmark model is used, then each alternative will have been evaluated on a varying standard dimension, $v(x_i^1)$, with other dimensions held constant at a benchmark level. Then R can be obtained by finding a utility function for the values on the one standard dimension, using either the BRLT or ELCE method. That is, R is a transformation that captures non-linearities between value and utility functions due to utility dependence. This is shown in Figure 10.1, where x_i is transformed to $v(x_i)$, and then to $u(x_i)$.

Figure 10.1: Elements of the R(V) Model



If, because of its generality and ease of application, the additive value model is used to find $v(x_i)$ for each dimension of each alternative, finding R can be a little more difficult, for the utility function is derived from not one standard dimension, x_i^1, but from a multidimensional valuation. First, the most preferred and least preferred values of the multidimensional alternatives are set at the limits, that is, $v(x^*) = u(x^*) = 1$ and $v(x_*) = u(x_*) = 0$. Then, using the BRLT or ELCE method, the utilities of some intermediate holistic alternatives are derived. Deriving this utility function will be the only occasion for the probabilistic questions strictly required for utility functions in the decision. Other utility models require these utility functions for each dimension, with n(= the number of dimensions) times the number of lottery questions as a consequence, and they also require a multidimensional holistic utility function for the k_i's in Keeney's models.

Once the utility function for the v(x)s have been obtained, it is possible to find the expected utility of each alternative under uncertainty:

$$E(u(x)) = \sum_{j=1}^{m} p_j . u(x_j) = \sum_{j=1}^{m} p_j . R(u_j) \qquad (10.1)$$

where p_j refers to the probability that the jth outcome will occur. Note that the R(V) model is thus apparently limited to probabilistically dependent situations.

The advantages of the R(V) model are analogous to those of the benchmark model in MDA under certainty. It makes no assumptions about utility independent dimensions, because utility functions are derived with other dimensions held constant (if the benchmark model is used to find the v(x)s), or over the whole set of dimensions in the decision (if the additive value model is used to find the v(x)s). Nor does the R(V) model assume any of the other independencies of other utility models.

However, it does have disadvantages. The tediousness of the benchmark model in finding the v(x)s has already been mentioned (Section 4.2) If the additive value model is used, deriving the utility function requires answers to lotteries involving multidimensional alternatives. And the difficulty of assessing the joint probability dimensions required for equation (10.1) has been noted. Nevertheless, the R(V) model does offer a solution to decision analysis when utility independence does not exist which would be more easy to use than the quasi-additive utility model.

10.2 Implications Arising from the R(V) Model

The R(V) model (10.1) which links utility and value functions has some implications for decision situations in which utility and probabilistic independence exist , as well as for those discussed in the previous section in which they do not. These implications will

be discussed for the additive utility model first, and then for the multiplicative utility model.

Chapter 7 noted that, because Fishburn-independence of utilities implies Debreu-independence of values, the additive utility model can legitimately use value functions in place of the more difficult to elicit utility functions. That is, in this situation the same result is obtained from a decision analysis using value functions as is obtained from a decision analysis using utility functions. This means that the R transform is linear, and hence there is no need to try to elicit it. It also means that the additive value model is usable with probabilistic independence, so that

$$u(x) = \Sigma E(w_i . v(x_i)) \qquad (10.2)$$

and the R(V) model, with R being linear in this situation, is not limited to probabilistic dependence, as (10.1) is, if Fishburn-independence exists.

Now, actually value functions found by rating and utility functions found by lottery questions are not dissimilar. Fischer (62) found that results from multidimensional utility functions under certainty correlated highly with results of multidimensional value functions - product-moment correlation coefficients were in the mid to high 0.90s - indicating that the two functions are quite similar. In a unidimensional decision analysis

with doctors (78), there was "good correspondence" (163, p.36) between their utility and value functions. The only other test of similarity between the two functions (13) was somewhat unsatisfactory because two-thirds of the subjects who gave value functions did not provide consistent utility functions when answering lottery questions - a not insignificant finding in itself as regards applicability of utility functions. This similarity of the two functions should not be surprising when one considers both the research outlined in Sections 4.5 and 4.6 which Eckenrode summarises in his finding that "data on judgments are unaffected by the method used to collect them" (50, p.189) and the research outlined in Sections 3.3 and 6.1 which shows that answers to probabilistic questions are not precise measures of attitudes to uncertainty. Indeed, Tversky (229) shows how risk aversion can switch to risk preference over a part of the utility function if the type of questions are changed. Bearing in mind this imprecision inherent in functions derived from probabilistic, lottery-type, questioning there is no evidence that value functions cannot be used instead of utility functions in MDA under uncertainty.

This similarity of utility and value functions has implications for the use of the R(V) model (10.1) when a multiplicative utility model is strictly appropriate because, although utility independence exists, Fishburn-independence does not exist. The additive value model can be used as before in the R(V) model, but there

is no need to use lottery questions to elicit the utility function and the transform R, for the simple rating used for value functions will be sufficient as it gives approximately the same answer. In brief, (10.1) is usable for all utility dependence or independence situations under probabilistic dependence and does not require lottery questions.

It would be helpful if the previous discussion had implications for utility independence without Fishburn-independence (i.e. the multiplicative model (8.1)) under the more usual situation of probabilistic independence. And in fact it appears to do so. The $u(x_i)$ in the multiplicative model (8.1) refer to the expected utility of an alternative on each dimension under probabilistic independence. Now, if value and utility functions are similar, the model could refer to the expected value of an alternative on each dimension . That is, more easily-elicited value functions could be used in place of utility functions right at the start of a decision anlaysis, even if probabilistic independence exists. It has already been noted in Chapter 8 that the constants in the multiplicative model (8.1) can quite legitimately be found using a value function also. So there is no need to use lottery questions at all for the multiplicative model with probabilistic independence!

But the similarity of utility and value functions, together with the R(V) model, has an even more important implication. Instead of using the multiplicative model with probabilistic independence, why not use the additive value model with a R transform? This R transform derived from holistic, multidimensional, questions will capture the non-additive interactions between dimensions as well as utility dependencies. That is, the probabilistic independence additive model (10.2) becomes the more general model

$$u(x) = R(E(w_i.v(x_i))) \qquad (10.3)$$

10.3 Conclusion

In conclusion the R(V) model appears to be a very general one. Not only does it offer a simple method of analysis whenever utility and probabilistic dependence exists but together with the observed similarity of elicited utility and value functions, it provides a mechanism for MDA under uncertainty in conditions of utility and probabilistic independence, that does not require utility functions.

A diagram summarizes this discussion of the R(V) model succintly.

	Probabilistic Dependence	Probabilistic Independence
Utility dependence	(10.1)	(10.3)
Utility independence with Fishburn-independence	(10.1)	(10.3)
Utility independence with Fishburn-independence	(10.1) with R linear	(10.2)

Figure 10.2: R(V) Models

11. CORRELATIONS BETWEEN UTILITY MODELS

The high correlation between the additive value model and other value models has been noted. Whether the additive utility model correlates as well with other models has not been investigated as thoroughly; indeed, the data is sparse.

von Winterfeldt (215) studied students' utilities for apartments measured on 14 dimensions. The mean correlation between holistic preferences and the results of an additive utility model was 0.84, which is high considering the large number of dimensions and the fact that tests showed that although Debreu-independence existed, Fishburn-independence did not (216). The only other test of the additive utility model is by Fischer (62). In the first of two experiments, he tested five students' holistic preferences with additive value and utility models for choices among hypothetical three dimensional cars, and for choices among hypothetical nine dimensional cars, each with probabilistic dependence. In this experiment, Fischer found that the additive value and the additive utility models correlated in the mid to high 0.90s, but that the additive value model correlated a little better with holistic value judgments than did the additive utility model with holistic utility judgments. This suggested that Fishburn-independence may be an important assumption affecting the additive utility model's applicability. So

in a second experiment with ten students, Fischer tested correlations between utility holistic judgments, the additive utility model and the R(V) model (which does not assume Fishburn-independence) for a choice among hypothetical three dimensional jobs under uncertainty with probabilistic dependence. An analysis of variance showed that, despite significant interaction between the dimensions, interactions in the utility holistic choices were only 1.2% of the sums of squares, and so it is not surprising that the R(V) and additive utility models produced identical product-moment correlation coefficients of 0.925 with the holistic judgments, and that the two models themselves had a product-moment correlation coefficient of 0.95, and produced almost similar absolute interval-scale deviations from the holistic judgments - revealing "that the degree of prediction afforded by (all) the decomposition models was about as high as possible given the degree of reliability inherent in the holistic utility judgments" (62, p.80).

In the case study of a non-hypothetical decision to be described in the next chapter, the results of the additive utility and the multiplicative utility models had a product-moment correlation coefficient of 0.96, again quite high, even though the k_is in the multiplicative model did not sum to one, which indicates Fishburn-independence did not exist.

No other studies of multidimensional utility models appear to be available. However, Keeney has found the constants for a six dimensional multiplicative model and for three separate two dimensional quasi-additive utility models. These were derived from actual decision makers or those close to them in their area of expertise and experience, although hypothetical decisions were the basis of the quasi-additive models. By making some assumptions, the constants can be used to indicate whether an additive model could have been used instead of the multiplicative and quasi-additive models which were used because Fishburn independence was shown not to exist. Assume that there is probabilistic independence, so that expected utility can be used in all the models (178). Then using possible combinations of $E(u(x_i))$, for i = 1 to 6, or i = 1 and 2, as they vary in equal intervals from 0 to 1, one can find the total utility for all these possible outcomes, using the values of k_is given in (106, p.119 and 130-131) and (113, p.113). The product-moment correlation coefficient between results of the multiplicative model and of the additive model was a high 0.98, which was also the coefficient for the additive and the two quasi-additive models in which utility independence applied.

One cannot apply this procedure to a fourth case, a quasi-additive model with utility dependence, given in (199) because of the dependence between the utility

functions on the dependent variable, and hence between the expected utilities, However, if combinations of values of x and y, each from 0 to 1, are inserted into the full quasi-additive model, and into two separate additive models each using one of the two dependent variable utility functions, the results are interesting. If the interval scale correlations are close, then expected utilities of uncertain consequences will also be close, which would indicate that the second conditionally dependent utility function for the quasi additive model is unnecessary, and that an additive model would suffice. When the linear dependent function is used in the additive model, the Pearson coefficient for results from the quasi-additive and the additive models is 0.98. When the non-linear dependent function is used, the coefficient is somewhat lower at 0.87.

This scattered evidence suggests two conclusions. First, if utility independence exists, then the additive utility model correlates highly with other models, expecially if probabilistic independence exists. Second, the additive utility model provides answers that will be almost as acceptable (because they agree with holistic judgments) as the additive value model under certainty. Unfortunately, there is no method of assessing whether the additive utility model is as accurate as the additive value model was found to be in Section 4.4, because there is no method of assessing what the result of the model should have been by later information, as in GPA's under certainty for example, for utility analysis does not produce the correct decision under uncertainty, but merely the wise one.

Why do the utility models produce results which are so similar? The first explanation would be that the interaction terms are so insignificant. And this would be reinforced by a second explanation, which is based on the fact that Debreu-independence is the certainty analogue of Fishburn-independence. As Debreu-independence is unimportant in decisions under certainty and thus permits the additive value model to be so general, one would also suspect that Fishburn-independence is, (although the relationship is not a necessary one, even though the reverse relationship is). For expected utilities on each dimension used with probabilistic independence are certainty equivalents and so it is little wonder that the additive utility model correlates so highly with other models under probabilistic independence. Models are then, in effect, models under certainty and Debreu-independence has been shown not to be a constraint to the applicability of the additive value model in this situation. However, this explanation does not apply to probabilistic dependence, for then there are no expected utilities on each dimension which are certainty equivalents. Hence the reason for the robustness of the additive model with probabilistic dependence is not known.

The implication of this evidence is that the additive utility model appears to be almost as good as any other with utility independence, and that if this is so, Fishburn-independence can usually be assumed and so

the simple additive value model using simply-elicited value functions can be used under uncertainty just as well as under certainty. The full implications of this finding depend on the aims of a decision analysis and will be elaborated on in Chapter 13.

However, three reservations are pertinent. First, the above sample of applications of multidimensional utility models is a small one. Second, the boundaries of the conditions under which the model similarities occur are not known, nor are the reasons for the similarities. Third, some decisionmakers may require a model to be a precise representation of reality, which should include interactions, however unimportant.

12. APPLICATIONS OF MULTIDIMENSIONAL UTILITY THEORY

Huber (99) has given a thorough review of field and field-like applications of MDA under certainty, with results that have been summarized in Chapter 4. Jackson (101) has reviewed agricultural and non-agricultural applications of unidimensional utility theory used under uncertainty. He found that in only a few cases had MDA under uncertainty been actually used in real decisions and then a linear utility function was usually assumed. A few other examples of utility theory had used real world data, but the examples had not been used in helping to make a decision. The important findings from the few real-world applications were that, although utility functions can be elicited, even for groups (29; 195 especially), there is concern about utility functions on both practical (128) and theoretical grounds (100); that satisfactory probability data is sometimes difficult to obtain (23); and that a great difficulty is convincing managers to use decision analysis, (20; 23) especially sophisticated models (27). After his survey of U.S. firms that actually used or trained in decision analysis found no drastic change in general decision making procedures and only occasional changes in individual decisions, Brown (20, p.88) concluded:

> *"My own feeling is that (decision analysis's) contribution to the quality of decision making often seems to come more from forcing meaningful structure on informal reasoning than from supplementing it by formal analysis."*

Although it is rarely mentioned (one exception is (79)), one other reason why unidimensional decision analysis under uncertainty is difficult to have adopted is that most decision problems are multidimensional.

However, there is very little application of multi-dimensional decision analysis under uncertainty in real world situations either. Fischer's and von Winterfeldt's experiments with students have been mentioned, but a test of theory is its actual applicaton in a real world decision. The rest of this chapter will briefly review the few applications of MDA under uncertainty, and then present a case study of MDA under uncertainty used in a real public investment analysis.

Some field-like applications are due to Keeney (108; 115). He elicited quasi-additive utility functions for a realistic but hypothetical decision with each of four decision makers or those close to them in their field of expertise and experience. One was a telephone company executive, choosing among telephone installations; one was a chief nurse, choosing between blood bank levels; one was an oceanographer choosing between surveys; and the fourth was a deputy chief of the New York Fire Department. Keeney found that two dimensional utility functions were elicitable for a decision maker with statistics or other quantitative training or for one who compensated for a lack of this training with enthusiasm, but that the utility functions were somewhat unsatisfactory if the decision maker was uninterested.

Moreover, for more than two dimensions, the BRLT method of eliciting utility functions and k_is for quasi-additive models was too time consuming and too complex (115, p.221), at least without an interactive computer program.

Keeney and others have continued these exploratory, field-like studies at IIASA e.g. (17; 120; 123; 124; 125). These have involved both utility functions and probability distributions (usually obtained from a simulation model) for various multidimensional decisions (using proxy decision makers) involving, for example, spruce budworm, energy policy, salmon fishing. And an interactive computer program (207) has been developed to assist in MDA. However, these studies have not been intended to assist an actual decision maker but rather to show how MDA could be used. (Another such example is (38).) Nevertheless, the research is valuable for its concern with the continuing complications of impacts over time and of differing, multiple, decision makers.

An almost real world application of MDA under uncertainty (45; 46) dealt with choosing between various educational curriculums, with the stochastic results of these being measured on 41 dimensions which were results in various tests. Because school principals found lottery questions meaningless, thus making utility functions out of the question, value functions were found by the "direct ordered metric" method of Fishburn (65), which entailed answering 50 questions for each of the 41 dimensions - this was used because the principals had doubts about their ability to use rating

and magnitude estimation. If they had realized they were going to have to answer 50 x 41 = 2050 questions about two differences between test percentile scores as an alternative, they might have chosen rating; indeed, there were complaints of "over-sophistication". A linear programming formulation was used to derive a group utility function from the results of the "direct ordered metric" questionnaires. Ranking and rating was used to find weights (which were not problem-dependent however) although how group weights were derived is not obvious. An additive model was used, with subjective probability distributions which were assessed with a method essentially similar to the histogram method. Although no tests for Fishburn- or Debreu-independence were made (or for utility or value independence), the results have met with "generally favourable" reactions from school principals, and the model has been published and distributed to many of them as a "guide" to decision making, to be verified by intuition.

Another application of MDA under uncertainty involved the timing and siting of a second airport at Mexico City (this has been discussed briefly in Chapter 3) (113; 159). Utility functions for the six dimensions were derived using the ELCE method, and subjective probability distributions were derived by a direct method assuming a continuous distribution. These were elicited in committee meetings with the Director of Airports and his staff, where views converged on a single

answer quite quickly. Complete testing for utility independence and Debreu-independence gave positive results, and so the multiplicative model using k_i found with probabilistic questions was employed to "indicate effective strategies" out of 100 possible alternatives. Sensitivity testing showed that modal point estimates rather than complete probability distributions of the dimensions would not have altered the results. This suggests an additive value model may have been adequate with the modal points as certainties, but the decision makers wanted full distributions to be included.

This would appear to have been a classic, thorough, multidimensional decision analysis, the success of which would indicate no requirement for simplification or approximation. However, some approximations were made: three points in time represented the thirty years time span in the decision, no discounting was done, and probabilistic independence was assumed even though it was admitted to be an over-simplification. Moreover, the actual decision involved dimensions not included in the multiplicative model, such as externalities, political effectiveness and flexibility! In the final recommendation to the Mexican President, these dimensions were handled by simply ranking alternatives on them (which assumes certainty equivalents - see Chapter 15) and using dominance to arrive at three final alternatives to choose from. Nevertheless, the results of the complete decision

analysis had a profound effect on the direction of the Departmental advice to the President, and "it would appear, from all the evidence that the participants were able to assemble, that the essential recommendations of the study were followed." (116, p.367)

Another real-world application of MDA under uncertainty involved the chief executive of a mining company deciding whether and how to bid for rights to extensive ore deposits and how to extend production capacity (90). The two dimensions used were NPV and production capacity and a quasi-additive utility function was used. Lottery-type questions were successfully used to elicit probabilities and utilities, and the utility independence requirement for the quasi-additive utility function was verified. The problem of probabilistic dependence of prices in succeeding time periods was evaded in a very rough fashion by eliciting unconditional distributions and then using one random number to sample from all the distributions. However, the decision maker "was satisfied that his views and values were properly represented and, hence, he had no conflict with accepting the optimal strategy" (90, p.46). But it was fortunate that a third dimension, shortness of capital, which the decision maker thought of right at the completion of the analysis, did not affect the optimal strategy, or the whole complex analysis would have had to be repeated. It should be noted that

the third term in the quasi-additive utility function which differentiates it from an additive function (90, p.44) had an extremely low value compared to the other two terms, suggesting that an additive model would have been adequate. Nevertheless, this was a successful application which had a direct influence on a decision.

Litchfield et al (137) claim MDA under uncertainty has been "useful" in an R and D environment and demonstrate using a hypothetical example. They have found that lottery-type questions are difficult to apply to elicit probabilities and utilities, and so use the visual impact method for probabilities and value ratings for utilities. They have found that utility independence (judged by their value ratings) does not always exist, yet an additive model is always adequate. Weights are found by a form of magnitude estimation. But despite their multidimensional utility function, they do not use expected utility as their criterion, as von Neumann-Morgenstein theory has demonstrated should be the one to use! They present the decision maker with complex patterns of the whole distribution of utility, without a completely satisfactory method of deriving the optimum alternative from these patterns having yet been developed. Nevertheless, as noted, the MDA has been useful.

There appear to be no other published applications of MDA under uncertainty. Overall, these four real-world applications show that it can be used effectively in real-world situations, even when groups of people are the decision maker, that the simple models appear to be quite robust to possible violations of their assumptions, and that these violations or any approximations in the model construction do not make the application ineffective in their rather general aims of problem understanding rather than problem solving.

A Case Study

An application of MDA under uncertainty was carried out for a real decision as part of this thesis for three reasons. First, to give the writer first-hand experience of the techniques. Second, to add another application to the very small sample of real-world applications. Third, to verify the strengths and weaknesses of MDA under uncertainty apparent from the earlier applications.

The decision maker was a comparatively "brainy" Lieutenant-Colonel in the Australian Army who was expected by others above and below him in rank to make a decision on the basic type of truck required for first-line transport support of an army division even if the minor adjustments to his decision might be made in later reviews. A friend of the writer's, he had some months previously said that the decision was an extraordinarily difficult one for him, because it depended on so many considerations (dimensions) about which he had very little data (uncertainty).

The problem at this stage was not to decide on a particular make of truck, but rather to decide on the size and the mobility requirements of the truck, so that the field of investigation could be narrowed and preliminary position papers begun to be written. Previously, an expensive cost-effectiveness study had been conducted, with the officer considering its recommendations "contentious" because it used "hard" peace-time data when his requirement was for war-time, because some of the costs included were of questionable relevance and because it did not include other dimensions important to him. The cost-effectiveness study concluded that very small and medium vehicles were appropriate. In another study, ad hoc procedures linked with intuition indicated that the required truck was in the medium range.

The steps followed in the MDA were those listed in Section 5.3. Firstly, in discussion with the analyst, the decision maker said that the dimensions and the range of alternatives for the decision were those shown in Figure 12.1. The first dimension, cost per equivalent truck unit, was based on a 5 ton truck, with the number of other sized trucks to carry equivalent payload being based on the decision maker's and some U.S. Army experience. Then (necessarily) subjective probability distributions were found. Preliminary truck costs were extracted from a trade magazine and from a specialist staff officer; and repair, petrol, oil, lubricants, tyre and driver costs were obtained from aggregated Army historical

records. The decision maker began by putting most likely, and upper and lower bounds on each of these components of cost, and then a distribution of cost for each vehicle was assessed using the visual impact method.

The next two dimensions of availability and mobility were also stochastic. As a basis for the decision maker's subjective probability distributions, distributions for these were obtained from officers in several different Army Corps, as well as a public servant experienced in both military and civilian truck fleets. In the first two interviews, the analyst soon discovered that it was unwise to begin with a request for probability distributions, as the interviewee was not used to them, and considered them either completely unreliable or impossible to elicit.

Figure 12.1 is an example of the form given to the later interviewees, with a request to simply tick the appropriate category for each type of truck. When this was done, the analyst would indicate a tick the position of which had taken some time to decide, and ask if the interviewee would like to spread his judgment over two categories in perhaps a 60/40 ratio. With this nudge, subjective probability distributions were soon filled in for as many types of truck as the interviewees felt knowledgeable about. It was found that the dimensions of mobility and availability had to be precisely defined, and that assessments in percentage form, rather than fractions or decimal figures, were the easiest. All interviewees felt happy with their assessments and only one declined to make the percentages

sum to 100, but nearly all voluntarily emphasized that they were based on experience and may not apply in the future. The results of these interviews were given to the decision maker and he made final probability distributions based on them.

The other two criteria, cross-loading ability and troop-carrying capability, were known with virtual certainty for each type of truck by the decision maker, and value functions were elicited for them from the decision maker by asking him to rate them on a percentage scale.

The third step in the study was to find utility functions for each dimension. Utility functions for the stochastic dimensions were derived using the BRLT method for the continuous cost scales and for the non-continuous mobility and availability scales. The ELCE method was found to be incomprehensible by the decision maker. The value and utility functions are shown in Figure 12.3 and 12.4. Weights were found by rating the raising of each dimension from x_i^* to x_{i*}^*, and also by Keeney's probabilistic method of finding k_is, with no important difference in results between the two methods. The decision maker preferred the probabilistic method, because he said it made him consider all the dimensions in relation, although why the rating method did not do this is not known. By the time the k_is were elicited, the decision maker was used to answering probabilistic questions.

The fourth step of the MDA was to find the appropriate utility model. At this stage it became obvious why most published applications of MDA have evaded the issue and adopted the additive model, for there was no time left for the compli-

cated questioning required for a theoretically correct examination for utility and Debreu-independence. However, in discussion, the decison maker verified that in his opinion utility and Debrue-independence did exist, even though the strict tests for it were not possible. (As the decision maker would be using the model to make the decision, this could perhaps appear to be sufficient validity). For example, utility independence was verified by asking if his utility for the dimension of availability would change if the mobility of a truck changed. When the decision maker replied no, he was asked if his utility for availability would change if the costs per equivalent truck unit changed. Again the answer was no. The decision maker soon understood what was being discussed, and verified utility independence existed for each dimension from all other dimensions. A similar type of discussion verified the existence of Debreu-independence.

With utility and Debreu-independence verified it was possible to use the multiplicative model. The k_is added to 0.64, implying non-additivity, and K was found by iteration to be 2.0. Results of the multiplicative model are shown in Figure 12.5 which indicate that a 5 ton truck is preferred to a 2½ ton and a 7½ ton truck. (It might be noted that the R(V) model was not used because this writer had not fully realized the implications of the R(V) model (Section 10.2) when the case study was done).

The total analysis took about 10 hours of the decision maker's time and 2 to 3 weeks of the analyst's time. Even so, the analysis was somewhat rushed towards the end; for example, re-questioning of the decision maker about some utility functions was necessary because they did not exhibit the risk-

aversion one would expect. Much time was spent in recouching the analyst's questions about hypothetical alternatives into realistic terms. For instance, chances of a truck having either very good availability or very bad availability were only credible if spare parts were assumed to be obtainable solely from a politically unstable foreign country. This realism was important for the accuracy and acceptability of the decision analysis, and required more preparation and imagination than the analyst at first realized. Questioning for an application of MDA under uncertainty is time-consuming and tiring. It would be a rare decision maker holding real executive responsibility, either an individual or a committee, who would have the time and patience to undergo the complete and theoretically proper interrogation. However, despite the rush in this application, both the decision maker and the analyst were at first satisfied that the results of the questioning were accurate enough. With more experience on the part of both the analyst and decision maker the time scale might be shortened, but not considerably.

At the end of the analysis, the decision maker was presented with the results in the form shown in Figure 12.5. He was surprised at the differences between the preferred vehicles, and would have liked to do some sensitivity testing with varying weights and preferences. He was especially concerned about the weight given to cost, and was searching for more realistic and comprehensible questions to find the k_i for cost that would compare the full cost over ten years of, say, a cheap 2½ ton Japanese truck with drivers with the equivalent unit cost of a large very expensive truck with its drivers over ten year. He also started considering criteria which had not

been included, to differentiate between the better trucks. In short, the decision analysis was a part of the decision making process, not its culmination. But the decision maker did not regret the time, for he thought the way the analysis forced a consideration of all the issues involved was worthwhile, and he certainly understood the problem better. However, his comments indicated that he did not fully understand the analysis, most especially the utility aspects, despite the analysts attempt to explain it at the start of the case study. Utility is certainly difficult to comprehend quickly. His general comments on decision analysis, as well as those of others involved in the case study, are incorporated in the next chapter.

Postscript to the Case Study

As the decision maker could still not arrive at a decision, the analyst suggested that a survey be made of how truck types were selected by successful Australian truck fleet managers. The analyst conducted this survey of eleven loading fleet operators, three in the public sector, (168) and found that their approach was both similar to and different from the decision analysis approach he had adopted. There were two main similarities. First, the investment decision was always a multidimensional one, with the number of dimensions ranging from three to six. The most important dimension was the payload of a truck, which should be as large as possible within constraints on loading, etc. Second, the major source of information was their own experience. But there were differences too. First, the operators all used lexicographic ordering for the first few factors considered, although trade offs must have been made intuitively to settle on the required

levels. The only explicit trade-offs made were expressed in money values, such as maintenance costs and initial cost. Second, no probability distributions were used, and the operators carried out trials of a vehicle until they could obtain a benchmark or certainty equivalent measure of it on the dimension about which they were uncertain. If there was too much uncertainty the vehicle was not even considered (that is, its certainty equivalent was of negligible value). Indeed, operators invariably said that when an investment decision was made no uncertainty existed. Third, there was usually very little objective analysis of the decision, and unaided intuition and judgment were used.

The survey found that as large a truck as possible should be bought. This had a direct influence on the decision maker. He recommended that trucks of about 7½ tons payload be bought - the upper limit of those considered feasible in the decision analysis. The basic procedures discovered in the survey will probably be used in the selection of a make of truck later.

It is not easy to compare the effect of the two studies on the decision maker. Admittedly, he adopted the procedures of the second, but not of the first. The second also provided more information than the first, e.g. about reliabilities of various makes of trucks. And the second was more easy for him to understand.

But the first study using MDA need not be dismissed out of hand. It took two to three weeks instead of four months; was limited to the decision maker's experience, while the second expanded it by surveying experiences of others; and the first did provide results that suggested the 7½ ton truck was among the better trucks. The point is that the two studies were different types and so their effects should be also. Nevertheless it must be admitted that the first study using decision analysis was difficult to comprehend, and that its use of explicit trade-offs rather than lexicographic ordering and its use of explicit probability distributions rather than certainty equivalents made it less convenient to executives in the Army. These aspects will be discussed in the next chapter.

FIGURE 12.1 - THE DECISION MATRIX

Basic Fleet Truck Type / Criteria	Costs per Equivalent Truck Unit*	Availability* (i.e. repairability, reliability, etc.)	Mobility*	Cross-loading capability	Passenger-carrying possibilities	
3/4 ton						
1 ton GS (General Service)						
1 ton CL (Commercial)						
2½ ton GS						
2½ ton CL						
5 ton GS						
5 ton CL						
7-8 ton GS						
7-8 ton CL						

* Stochastic criteria

FIGURE 12.2 - AN EXAMPLE OF A COMPLETED INTERVIEW FORM

Mobility / Truck Type	3/4 Ton	1 ton GS	1 ton CL	2½ ton GS	2½ ton CL	5 ton GS	5 ton CL	7-8 ton GS	7-8 ton CL	
1. Very Good		40	40	25			40			
2. Good		50 ✓	60 ✓	50 ✓	40	40	60 ✓			
3. Average		10		25	40 ✓	60 ✓				
4. Poor					20					
5. Very Poor										

FIGURE 12.3 - VALUE FUNCTIONS

Dimension	3	5
Alternative	Cross-loading capability	Passenger-carrying possibilities
3/4 ton	.2	.6
1 ton GS	.3	.6
1 ton CL	.3	.6
2½ ton GS	.6	.9
2½ ton CL	.6	.9
5 ton GS	.8	.8
5 ton CL	.8	.8
8 ton GS	.85	.6
8 ton CL	.85	.6

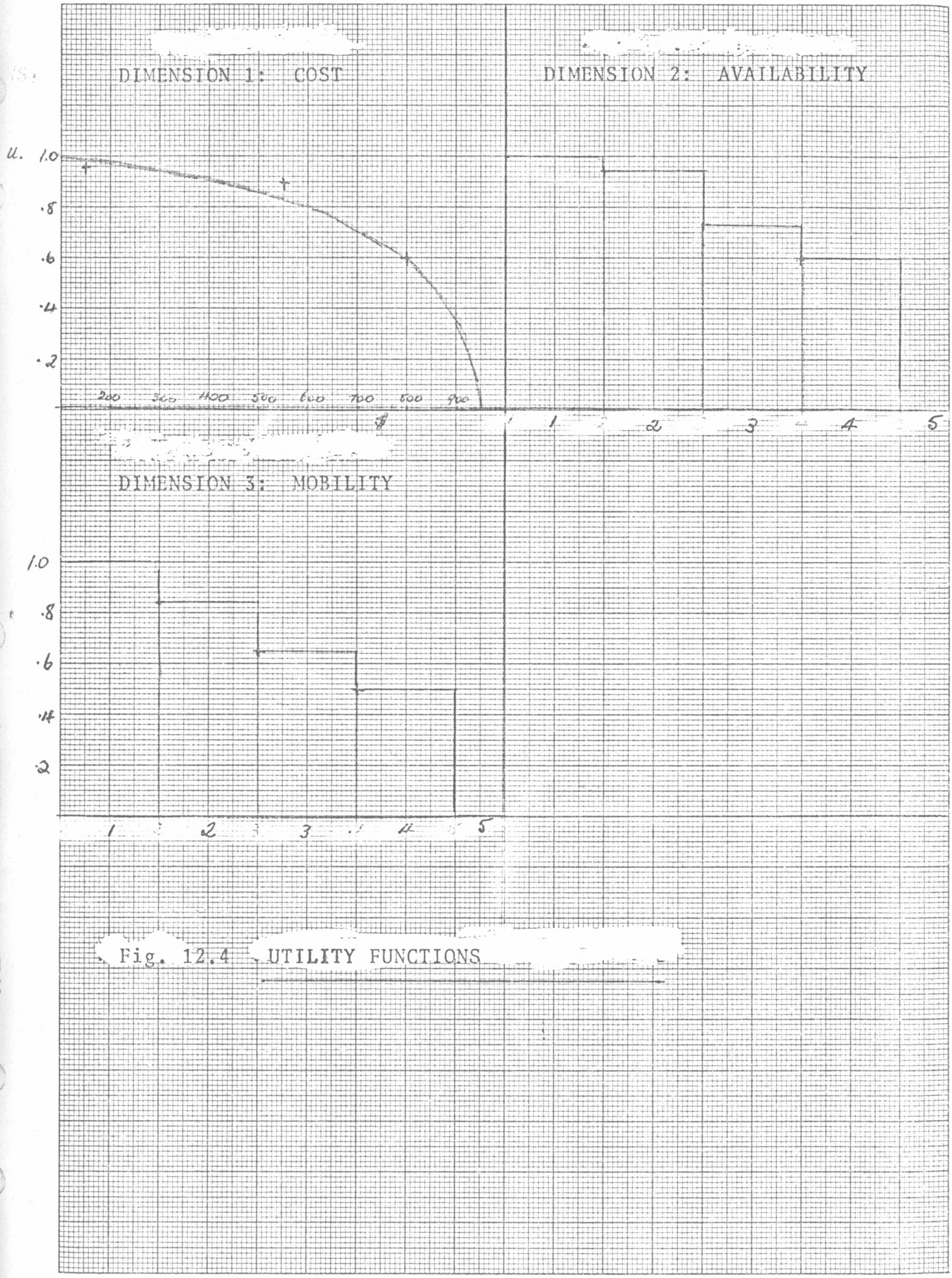
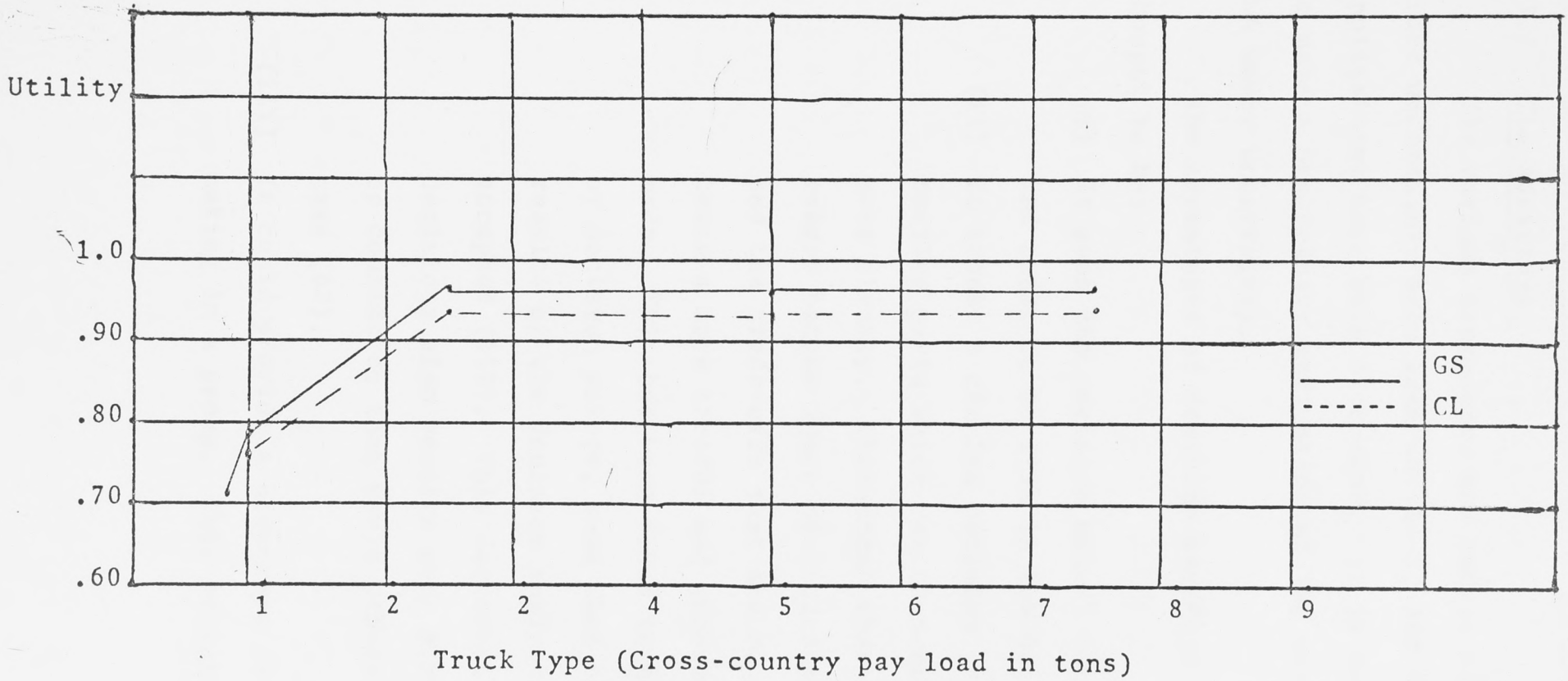


Fig. 12.4 UTILITY FUNCTIONS

FIGURE 12.5 - TOTAL EXPECTED UTILITIES FOR VARIOUS TRUCKS



13. A PRELIMINARY EVALUATION OF MULTIDIMENSIONAL DECISION ANALYSIS UNDER UNCERTAINTY

13.1 The Advantages

The various procedures and models of MDA under uncertainty have been surveyed, and its few applications have been discussed. It is now appropriate to make a preliminary evaluation of the usefulness of MDA under uncertainty.

The advantages of decision analysis have been thought to be:

(i) It uses the decision maker's own preferences and attitude to risk in the decision.

(ii) It breaks a complex decision problem into smaller parts which can then be examined more clearly. This means that decision makers become aware of issues involved and the trade-offs that are required - hence a more careful and informed decision is made. This has been found to be appreciated by decision makers, even when the precise results of the decision analysis are not accepted (152). This decomposition of decisions also permits more dimensions to be considered than would otherwise be the case (62).

(iii) It could provide a structure for decision making in a group. This structure provides

a basis for communication within the group; facilitates the development of alternatives and goals; isolates and identifies real sources of conflict that need to be resolved (17); and allows various areas of expertise to be incorporated into the decision making (53).

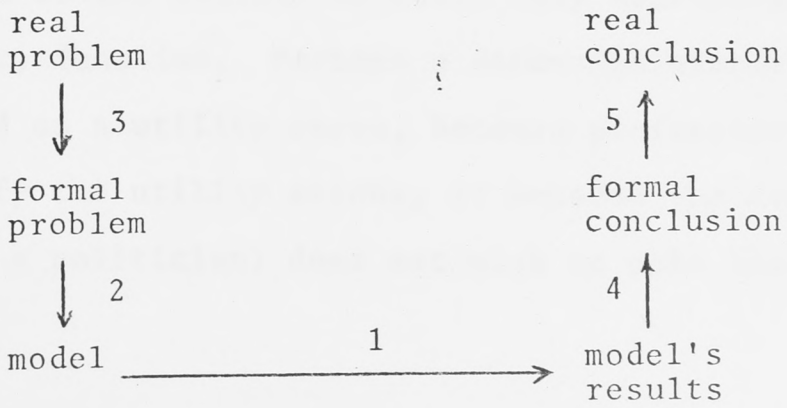
(iv) Once the variables in a decision analysis algotithm have been established, the decision maker could simply concentrate on the values of these variables in similar, later, decisions (224). This also provides consistency in the solution of problems.

Whether all these advantages are achieved in practice has to be discussed, and requires a close examination of MDA theory. There are two aspects of any theory: the internal consistency of its model, and the model's "fit" to the real world.

13.2 Model Formulation

The situation in which decision models are used can be shown in this figure, from (201).

Figure 13.1 - The Decision Model and Reality



The bottom line 1, where a model is logically analyzed to produce numerical results, is dependent on the internal consistency of the model. The internal consistency of MDA utility models is undoubted, for the mathematical proof of each has been established. It is this internal consistency where most research effort in MDA has been expended.

But what about the "fit" of the model to the real problem? This depends on the "fit" of the formal problem to both the real problem and to the formal model. Consider line 2, which concerns the fit of the formal problem to the model. In MDA under uncertainty, the formal problem refers in theory to utility functions and models derived through lottery questions. However, the formal problem is not as simple as that. A utility function can vary with the methods used to elicit it (209). Moreover, the BRLT method of elicitation which must be used for the discrete dimensions often found in MDA suffers from the problem of bias for particular probabilities and if the decision maker has a love of gambling per se, which is never tested for, then the utility function elicited with lottery questions is biased. Furthermore, the curve-fitting used on elicited points of the utility function only approximates the utility function. Perhaps a dimension simply cannot be scaled on a utility curve, because preferences do not satisfy the utility axioms, or because the decision maker (e.g. a politician) does not wish to make his utility

function known or be made explicit. As well, subjective probability distributions also vary with elicitation methods, and rely on the decision maker successfully abstracting preferences for payoffs from his beliefs about their likelihood. In brief, the inputs to the model from the formal problem are approximations, not the clear unfuzzy facts that the numbers associated with them make them appear.

But the really crucial link in the "fit" of the model to the real problem is line 3, which is the "fit" of the real problem to the formal problem. This is problem formulation. One question to arise here is whether the dimensions included in the formal problem really do include all the factors involved and whether they are defined properly, e.g., does service frequency adequately define customer satisfaction? But more importantly, one might ask whether all the possible dimensions, all the possible alternatives, and all the possible outcomes have been included. This latter is especially critical for MDA under uncertainty, for unexpected outcomes could occur. For instance, in an analysis of a decision about whether a farmer should spread superphosphate or not, the farmer did not include in the formal problem the outcome of rain after the superphosphate was bought and before it could be spread, and this actually occurred and ruined the superphosphate (101). Moreover, there will be assumptions of discreteness (e.g., high, medium, low prices) or of

probabilistic independence in almost every model, because the full model would be impossible to build or be too difficult to structure.

Under what conditions in public investment analysis would lines 2 and 3 in Figure 13.1 be as "noise"-free or "pure" as decision theorists seem to suppose? This depends on five factors of the real problem (suggested by those in (30; 98; 160; 194)).

(i) the nature of key tasks carried out by the public body involved;

(ii) the level within the body at which the analysis is made;

(iii) the environment in which the investment project will be developed and operated;

(iv) the dependencies among the organizations involved;

(v) the number of decision makers involved.

Each of these factors will be discussed in turn, with the understanding that other factors are held at some fixed level so that just one factor can be concentrated on. The aim of the following discussion is to demonstrate that the situations where MDA under uncertainty is used will probably be those where lines 2 and 3 will be impure.

The nature of the key tasks carried out by the public body involved affect the purity of lines 2 and 3. If the tasks are simple, not variable, and/or have been carried out often so that experience has been built up

(e.g., many engineering tasks in Telecom) then MDA could be straightforward. Because of the known structure of the real problem,the ease of evaluating an analysis, and the known cause and effect relationship, the model should be reasonably valid because line 3 at least will be pure. These straightforward tasks do exist in the public sector, but Weberian bureaucratic rules have been developed for decisions concerning this type of task and because of the certainty surrounding this type of task, the simple additive value model could be also used.

So it is suggested that MDA under uncertainty would more often be considered for tasks which are not straightforward (e.g. new types of health centres, location of new dams), and hence line 3 would be more or less impure.

The level within the body at which the analysis is made is also a factor affecting problem and model formulation. If the analysis is purely a technical one, then the formal problem would be expected to be similar to the real problem, for the analysis deals with specialist, core, technologies which are, at this technical level, sealed off from outside influences. However, at the managerial level of analysis, the complexities of co-ordination within the organization and of interaction with the environment of the organization would mean that the formal problem and the static model are likely to be very different to the real, changing,

multi-faceted, and probably unique real problem. As most public investment analyses will be managerial, with technical analysis being limited to the implementation of an investment decision, MDA again is faced with an impure line 3.

The environment in which the investment project will be developed and operated also affects line 3. The most uncertain variable in public investment is public reaction. President Lyndon Johnson, architect of his Great Society programs, was surprised at the rioting in the ghettoes after all the things he had done for the poor and black. But other uncertainties involve economic conditions, demand for services (e.g. for air travel), climatic conditions, bureaucratic abilities (e.g. the PPBS disappointment), and later legislative changes. These uncertainties can indeed be incorporated into the model, albeit with somewhat uncertain probability distributions (line 2), but probabilistic dependencies may exist if the environment is very much uncertain and interrelated and this may have to be ignored in the model. In any case, even if the interrelationships are incorporated by various forms of simulation (e.g. subjective information modelling (30)) there will be a requirement for simplification in the simulation models. In brief, the complex uncertain environment means line 3 will be impure to some extent in public investment analysis.

Dependencies among the organizations also affects line 3. If dependencies are few or serial, then standard rules or schedules can co-ordinate organizations. But if dependencies are complex or reciprocal, then co-ordination becomes more difficult. An example of dependencies in public investment analysis is redistribution of income between classes (organizations). These dependencies raise the problem of multiple decision makers, to be discussed in the next paragraph. But it should be noted that the "bureaucratic" model of interacting organizations deciding public investment is a powerful one for which there is some empirical evidence (3; 11). This "bureaucratic" model posits that public organizations have predictable positions on most issues and that these are limited to the narrow world-view of the organization, commonly called "empire-building". As public investment analysis involves several organizations and lobby groups, this suggests complexity in the real problem's goals because it is "still not understood how hard an agency will pull on a given issue, and what particular tactics will be employed" (11, p.31). Hence line 3 will be impure.

Finally, there is the problem of multiple decision makers. This situation is an outstanding characteristic of public investment analysis (223), and all the models discussed so far assume a single

decision maker. Of course, there are some multiple decision models but, as the following discussion of them will illustrate, they are of extremely limited usefulness.

Firstly, developments in multiple dicision maker models should be reviewed. (The review must necessarily be brief, as the field is large and complex, and the point is only a small part of the thesis' argument). Recent multiple decision maker models have had to overcome Arrow's Impossibility Theorem (10). This showed that if the decision makers each ranked the alternatives, then there is no social welfare function combining this information which satisfies five "reasonable" assumptions. The reasonableness of these assumptions has been questioned, and it is possible to have an ordinal social welfare function provided some strict conditions of anchoring are met (80). But it is by postulating a cardinal function for each decision maker that a social welfare function for certain alternatives is best constructed, and an additive model is usually sufficient for this (47). As regards uncertain alternatives, Harsanyi (89) proved that if the group is indifferent between two uncertain alternatives whenever the individuals are indifferent, then the social welfare function is additive. (This is akin to the test for Fishburn-independence).

Keeney (131) has also examined conditions very similar to Arrow's for an additive cardinal social welfare function which is additive. Nash (47; 158) examined a model with utilities measured from a "status quo" position which is the alternative that the decision makers receive if they cannot achieve a mutually

acceptable bargain. This model can be expressed as a multiplicative model. In one application of it (47) finding the status quo alternative was extremely difficult. Of course, if each decision maker is taken to be a dimension, then any of the various models discussed previously in this thesis can be applied if the relationships between the decision makers are appropriate, e.g., the additive or multiplicative (122). Using this last approach, social welfare functions for three easily defined groups have been found - users of a computer system, voters and families relocated due to highway construction (127). Nevertheless there remain some perplexing and disturbing problems about additive and multiplicative functions (122, p.436).

How useful are these multiple decision maker models in decreasing "noise" in lines 2 and 3 (which is our main concern here)? Firstly, they all require that each decision maker has already done a MDA before the preferences are combined. This would mean trained analysts would be required with each decision maker. The difficulty of implementing PPBS is insignificant compared to the difficulty of implementing this. Secondly, these models ignore the realities of multiple decision making with bargaining, deception, and interpersonal tensions coalescing with the requirement for information from various sources before a decision can be made by an single decision maker. In this regard, "research findings indicate that interacting groups are generally superior to other groups (i.e., synthetic,

nominal, Delphi) for evaluation or decision tasks" (160, p.7). Indeed, if there is goal conflict among groups then decision analysis is an inappropriate mechanism for decision making because analysis accentuates the conflicts! (30) For MDA requires measurable, precise, goals, but if an analyst tries to elicit these precise goals rather than allowing the initial discussion to centre on diffuse goals and alternatives which can have many interpretations, he is likely to increase conflict. Alternatively, the analyst may settle for "harmless" goals for the sake of the model (267). In the same vein, a model decided by external or senior authority may disguise conflict. Nutt cites the MDA of the TFX purchase done by the American Department of Defence as an example. "Policy makers and operating personnel (i.e. air force, navy) placed radically different weights on evaluation criteria used to select General Dynamics as the TFX contractor. Conflict was hidden in analytical procedures and not fully recognized until the U.S. Navy refused delivery of the F-111", (160, p.11). In brief, in the initial non-analytical system of multiple decision makers in public investment analysis, social welfare function models have little place. Not that group goals cannot be achieved. Jorgenson outlines several examples of it being done, but the "formulation activity results in complex and heterogeneous information (which) is changed all the time through the group process" (104, p.142). Only by very simple, interactive, dynamic models (not those of classical MDA) will group decision making be assisted (30).

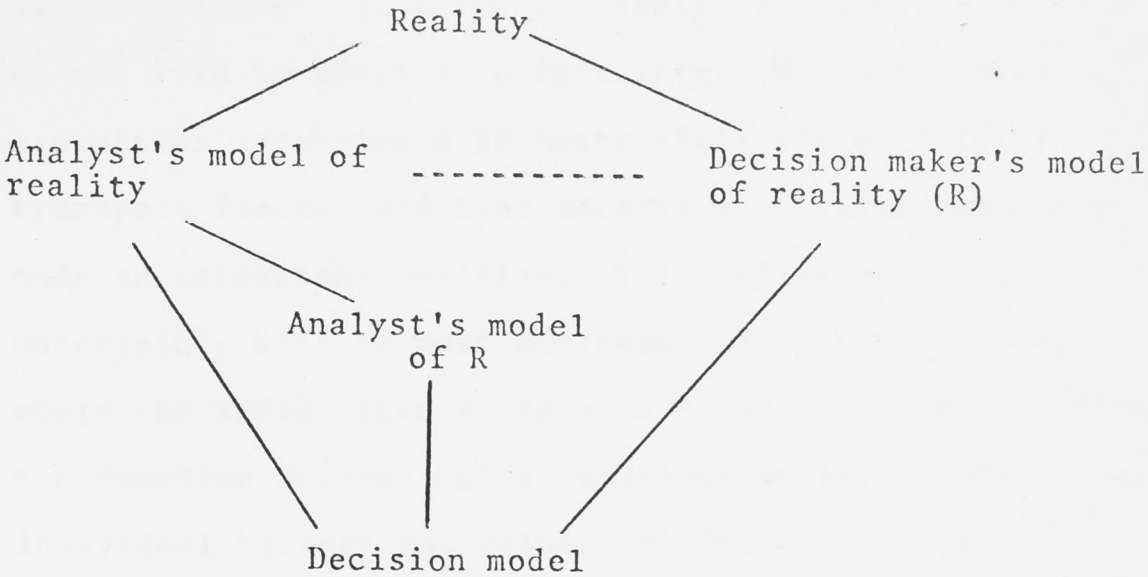
So it should be obvious that MDA will be a difficult process, not assisted in the real world by complex social welfare function models, and that public investment analysis using MDA will thus have an impure line 3.

The conclusion one is drawn to from this discussion of problem implementation is that when MDA under uncertainty is used in public investment analysis the model will very rarely have a good "fit" to the real problem. The model will caricature the world, so that the model's output will caricature the real decision to be made.

13.3 Implementation

Not only is the formulation phase of a MDA likely to have "noise", but so is the implementation phase, which is lines 4 and 5 of Figure 13.1. For instance, there will be changes in utility functions through time, so that the formal conclusion is not related to the input to the model, that is, line 4 will be impure. It is at line 5 relating the formal conclusion and the real conclusion where the greatest barrier to the successful implementation of MDA under uncertainty occurs, and this barrier refers to the perception of differences between analyst and decision maker . The relationships between the model, the world, the decision maker and the analyst are summarized in this diagram based on Jackson (101).

Figure 13.2 - Interrelationships of Perceptions



The unbroken lines indicate perception, the dashed line indicates communication. Thus the decision model depends on how the analyst perceives the world, how he perceives how the decision maker perceives it, and how he perceives how these relate to the decision model. Any of these perception or communication links could introduce "noise" into the model as far as the decision maker is concerned, without the analyst even being aware of its existence. This is especially critical in MDA because the framing of questions to elicit utility functions and k_is for dimensions other than money in real terms that the decision maker will accept and respond to is extremely difficult.

Even a dimension of cost can be difficult to elicit, as the problems in finding the k_i for cost in the truck investment case study demonstrated. These perception

links are most noticeable in MDA's explicit treatment of uncertainty. As noted in Chapter 3, managers usually do not like to admit to uncertainty. Not one of the executives interviewed in Australia's eleven leading transport fleets said that uncertainty existed when they made an investment decision. This reliance on managed uncertainty will be most noticeable in bureaucracies, where the ideal often appears to be an organization which can function whoever holds positions in it, so that individual beliefs and values are not as important as unassailable "facts". Moreover, bureaucracies take longer to arrive at decisions than private companies (223), and so there is more opportunity to manage the uncertainty involved in a decision.

Now, it is the difference in perceptions of analysts and decision makers that makes possible, on the one hand, a statement that decision analysis offers an approach to complex decisions that is rational and incorporates manager's beliefs and attitudes, and on the other hand, the extremely scant and continuing use of decision analysis by practising managers. For instance, Jackson (101) interviewed farmers who had "a good deal of exposure" (101, p.152) to decision analysis over years and found they were not using it. The six farmers he used to analyse real-world decisions "were not really convinced of the merits of the approach" (101, p.152). And a consideration of the applications in Chapter 12 is disheartening. It is unknown how many

school principals continue to use the MDA model to select curriculums. The Mexican government does not appear to be using MDA on other real-world problems after the departure of the airport decision analysts. It is not reported that the chief executive of the mining company asked for more decisions to be analysed. Litchfield et al are not explicit about how "useful" MDA is and how often it has been used in their company. And the Army officer in the case study will almost certainly not be using the technique again.

What has gone wrong? It is not possible, after years and years of preaching and experimentation, to ignore the unacceptability of decision analysis to practising managers. Only two writers appear to have tried to understand why this unacceptability occurs (as distinct from being concerned with the application of decision theory in practice). Dillon (42) addressed himself to acceptance among agricultural economists, and so the points he raised were not meant to apply to public investment analysis. However, it may be useful to discuss two of his points here. Firstly, is decision analysis unacceptable because other decision techniques are available to handle uncertainty? The discussion in Chapter 3 would indicate that this is not so for public investment analysis. Secondly, is decision theory personalistic when general techniques are required? There may be relevance here for public investment analysis,

for the problem of multiple decision makers has been already noted.

Moreover, the personalistic nature of decision analysis leads into Jackson's (101) reason for its unacceptability. In an original hypothesis, arrived at after real-world decision analysis with several farmers, he emphasises the philosophic committment inherent in decision analysis - man is free to choose between alternatives as he perceives them using his own perceptions of and attitudes to uncertainty. The decision maker uses his own view of the problem to make a choice.

> *"This implies the existential notion of reality being, in some sense, subjective. By encouraging the decision maker to include his feelings in his problem decision theory recognizes the importance of considering the individual as a whole, both his intellect and his emotion .... He acts on the basis of the truth as he believes it, not on the tuuth based on objective criteria. This is the existentialist notion of truth." (101, p.164)*

Jackson suggests that the one dimensional, technocratic, nature of Western society makes this decision theory approach inappropriate, and that managers look to techniques and gadgets to make decisions for them, rather than facing up to their situation. Hence, managers do not use decision theory.

However, there is another interpretation of how people view decision theory. Jackson's farmers who did not use decision theory preferred intuitive decision making based on experience - which is even more

existential and subjective. And although the farmers did not like to use probability distributions, the point estimates of probability they preferred to use can also be viewed as certainty equivalents, which incorporate both the decision maker's uncertainty and his attitude towards it that is uncovered in decision analysis. Just because people sometimes find it hard to express utilities and probabilities, it does not necessarily mean that reason and objectivity have almost destroyed the subjective dimension of man, for it could simply mean that this subjective dimension of man is difficult to express in the terms required by decision analysis. It is this other interpretation of why decision analysis is unacceptable which will now be explored.

There is much evidence to suggest that practising managers, in both the public and private sectors, use very subjective decision making processes. Some of this evidence is the following items. First, the highly rational techniques of operations research have had only a very limited usefulness in public investment analysis, as Chapter 3 showed. Second, a survey of 496 managers (183) showed that "rationality" was ranked only eighth out of ten characteristics of importance to a decision maker, while "perception, or the ability to formulate problems" was ranked first by 82% of the managers. Third, most of the data used in managerial decisions is soft and speculative which must be "synthesized" rather than "analyzed" (154, p.255).

Fourth, the exact roles of leadership which is so important in management are so subjective that the social sciences cannot agree on what constitutes them (154). Finally, the decision making process is almost invariably a dynamic process which the ordered steps of decision analysis do not capture (144).

The reason for the subjectivity that this evidence illustrates may be in the abilities of the two hemispheres of the brain (154; 201). The left hand hemisphere operates linearly, and sequentially, as language does. The right hemisphere operates in a holistic relational way,as pictures do. From the previous paragraph, most decisions are predominantly processes of the right hemisphere. Most importantly, the ambiguities, and irrationality of the right hemisphere "by their very mode of operation, (are) not readily accessible to causal explanation or even to linguistic exploration" (154, p.52), and so elicited models, utility functions and probability distributions are not precise, and are not perceived by decision makers to be so. Of course, some people do have a well-developed linear, left hemisphere, and this can be measured by tests used in Decision Perception Analysis (105). Decision Perception Analysis research has found that managers can be placed on a quantitative/qualitative scale, and so can jobs. Jobs having a high quantitative rating would be those in engineering, for instance, or those in the public sector which are unlike public investment analysis as measured on the five factors

discussed earlier. Those managers who most match their jobs on the quantitative/qualitative scale are the ones who are happiest in their work, because, in the terms used here, the models used in the job "fit" the real problems and solutions as perceived by the managers, and hence acceptability occurs.

In brief, decision analysis, rather than being too "existential", may not be existential enough, because the models used do not fit the decision maker's perception of the real problem and solution.

13.4 Conclusion

The conclusion that one is drawn to by the above discussion of model formulation and of implementation is that decisions and decision makers are bound together, and that decision analysis must recognize this. In short, decisions are grown, not made by rational decision makers (144), so that models cannot be developed or recommended without regard to their acceptability and purpose. That is, a utility model cannot provide the optimal course of action, but, as Keeney puts it, rather just "indicate effective strategies" (113, p.116), or be just "a method for decision analysis rather than decision making" (116, p.368), in an open rather than a closed loop of decision making. What evidence there is in applications of utility theory (Chapter 12) reinforces this. If this more limited aim had been emphasized as the goal of MDA, rather than the optimising aim, there may have been more applications. In brief, the decision model cannot be a surrogate for the decision-space, a replacement for it, as physical models can be for physical phenomena, but the decision model can be used to provide perspectives on the decision-space (201).

Now, if MDA is to provide perspectives on effective strategies rather than provide the optimum solution, there are several inferences to be drawn. They all concern the acceptability of MDA. The sophistication and precision of decision models which are unnecessary for this limited aim could be discarded. Greater comprehensibility and understanding would result, so that the decision maker would use the conclusions to increase his understanding of the issues involved to allow him to make a better final decision. There is evidence to support this claim. Bertie Tell (204) tested four different models on a real-life evaluation problem of the Swedish Air Force. Subjects were senior air force staff officers who were motivated by knowing that the results would be used by the Swedish Department of Defence. The dimensions used had been structured by the Air Staff a short time before. After the thirty subjects had given the functions and weights required for four different types of models, they were asked to rank them on these attributes:

(i) time requirement at construction;

(ii) believed precision;

(iii) ease of understanding the questions posed by the model;

(iv) ease of finding the information required by the model;

(v) which method the subject wants the Air Force to use.

For all of these attributes, a simple additive value model was superior to Keeney's multiplicative utility model, with a statistically significant difference occurring in almost every situation. As well, the two simple additive models tested were always ranked superior to the two relatively complicated models tested. This experiment shows that simplicity aids acceptability. As well, this simplicity would mean that the decision analysis model would be a vehicle for communication among the many people involved in the decision, for simplicity aids understanding. Furthermore, simplicity encourages the decision maker to break down the problems into its constituent parts, so that they can be considered in isolation. Thus, simple models used in a decision analysis rather than a decision making role reinforce the advantages of MDA outlined in Section 13.1, rather than detract from them. Furthermore, simplicity permits sensitivity analysis to be easily done, for the data required for new weights and value functions, or even new dimensions, can be quickly obtained and used. This is especially important for the dynamic circumstances of much decision making.

In terms of techniques, simplification would indicate that the easily understood additive model should be used for MDA under uncertainty, with expected value rather than expected utility being used as the criterion. This is the same as using the R(V) model with an additive value model and a R transformation that is linear. If the

analyst is uncertain whether R may affect results if it is non-linear, he could derive two or three points on the R function. That is, the assumptions of the additive utility model should not be ignored. If Fishburn-independence obviously does not exist, the dimensions involved might be joined or transformed so that it could be assumed to exist, of course.

The following table summarises the simple models that should be used in place of more complex ones, based on the discussions in this thesis. The first, complex, model is separated from the second, simple, model by a "/", and when two equations for a model are referred to, they refer to probabilistic independence and dependence situations respectively.

TABLE 13.1

THEORETICALLY CORRECT MODELS AND THE PREFERRED SIMPLE MODELS FOR VARIOUS INDEPENDENCE CONDITIONS

Condition	Model
Value dependence of a dimension	Generalised utility independence model (9.3)/transformation of the dimension (Section 4.1) and then the additive value model (4.1).
Value independence without Debreu-independence	Non-additive value models/additive value model (4.1)
Value independence with Debreu-independence	Additive value model (4.1)/ additive value model (4.1)
Utility independence without Fishburn-independence	Multiplicative utility model (8.1)/ additive value model with an R transformation if the transformation is strongly non-linear (10.1) or (10.3)
Utility independence with Fishburn-independence	Additive utility model (7.1) or (7.2)/ additive value model (7.1) or (7.2) using value functions.
Utility dependence of dimension(s)	Quasi-additive (9.2) or Fishburn or Farqhuar's dependence model/additive value model with an R transformation if the transformation is strongly non-linear (10.1) or (10.3).

The next three chapters discuss areas where MDA could be simplified, while not diminishing its ability to achieve its limited, "perspectives", aim; indeed, these simplifications may improve its ability to achieve this aim.

14. DECOMPOSITION

The first of three concepts which can be used to simplify the application of MDA deals with decomposition. If there are only a few dimensions to be considered, say $n \leq 10$, then all the models discussed above could conceivably be used. But if there are more dimensions than this, the number of functions and the number of weights to be elicited is quite large, raising the tediousness and the length of the questioning of the decision maker to inordinate heights. When this occurs, various types of dimensional decomposition can be used to simplify the decision analysis. Of course, whether decomposition is required and the extent of it ultimately depends on the desires of the decision maker.

One of the most commonly used methods of decomposition assists decision makers in deciding on weights for many goals by using a hierarchy of goals. This would be especially useful in public investment analysis, for government departments often have higher-level goals, such as "to improve health", which are difficult to express as concrete working goals, but which must be borne in mind if working goals are not to become an "end" themselves rather than a "means" (188). The weights of the additive value or additive utility model are especially amenable to this decomposition. An example concerns high-level goals which are defined and weighted as follows:

	Higher-level Goal	Weight
1.	Speed	.5
2.	Reliability	.2
3.	Flexibility	.1
4.	Prestige	.2
		1.0

Then the effects or relevance of each lower-level dimension to these higher-level goals are defined in a matrix, with each column summing to 1. Note that if the problem-dependency of weights is to be incorporated into the decision, this relevance matrix should refer to the relevance of raising each x_{i*} to x_i^* with $x_{\bar{i}}$ held at $x_{\bar{i}*}$.

Dimension \ Higher-level Goal	1	2	3	4
Low cost	.6	.1	.4	.2
Modularity	.1	.2	.2	.2
Good supplier support	.1	.6	.1	.2
Low risk	.2	.1	.3	.4
	1.0	1.0	1.0	1.0

This second matrix is then multiplied by the vector of higher-level goal weights to give final dimension weights.

Dimension	Weight
Low cost	.40
Modularity	.15
Good supplier support	.22
Low risk	.23
	1.00

This basic principle of hierarchical additive decomposition has been widely used. For instance, four computer systems were assessed on 41 dimensions after the above decomposition had been twice done on six higher-level goals. Sensitivity testing showed changes in goal weights did not severely affect the final computer system rankings (185). A similar procedure is followed by the widely-used relevance tree model, where the alternatives, as well as the dimensions, can be decomposed. Such a procedure is used by NASA, USN, Honeywell, and other organizations (12).

Another form of decomposition is based on the goal-fabric concept. Here dimensions are broken down into lower level sub-dimensions which can be analysed as a sub-decision. Thus performance could be broken down into maneouvrability, range, speed, etc. This was proposed in an additive model for transport planning (13), and Keeney (113) provides a real world example of this approach. In his decision analysis for the Mexico City airport study, the dimension of airport capacity was broken down into three dimensions of capacity in each of 1975, 1985 and 1995, because utility functions were

different for each of these because demand would be different. The utility for the alternatives on these sub-dimensions was found using the multiplicative model and then inserted into the full decision analysis.

Some recent decomposition research has centred on the requirements of utility and Debreu-independence needed for the multiplicative-model, and on simpler methods of verifying their existence than the tests outlined in Chapters 7 and 8. The core of Keeney's (112) research is to combine dimensions and then test for utility and/or Debreu-independence of these sets of dimensions from other sets. Heretofore in this thesis, Debreu-independence has referred to the independence of trade-offs between two dimensions from other dimensions, and utility independence to the independence of utility on one dimension from levels of other dimensions. These are their usual applications in a particular decision analysis. However, if value independence refers to a set of dimensions, then trade-offs within that set are independent from the levels of dimensions outside the set, that is, the set is also Debreu-independent of outside dimensions. Thus, in referring to a set of dimensions, one can use the term Debreu-independence or value independence; writers on decomposition concentrate on trade-offs, and so the term Debreu-independence will be used. It is the certainty analogue of utility independence among sets of dimensions and is implied in the existence of the latter. Basing his proofs on this, Keeney finds several relationships between sets of

dimensions, the two most important being:

(i) If two sets of dimensions are utility independent, then their union and intersection are utility independent, too - so that if a chain of intersecting utility independent sets of dimensions is constructed, then the sets of dimensions and their unions are utility independent of their complements in the chain, which could mean that some particular dimensions need not be tested for utility independence if they are involved in utility independent chains.

(ii) If (x_1, x_2, x_3) is a set of dimensions, and if (x_2, x_3) is Debreu-independent of x_1, and x_2 is utility independent of (x_1, x_3), then (x_2, x_3) is utility independent of x_1 - thus tests for Debreu-independence might be used to supplant some more difficult, lottery-question testing for utility independence if sets of dimensions are being considered. However, an analyst needs only to test for utility independence on one dimension if Debreu-independence exists for all pairs of dimensions anyhow (123) so this second finding may not have much practical relevance.

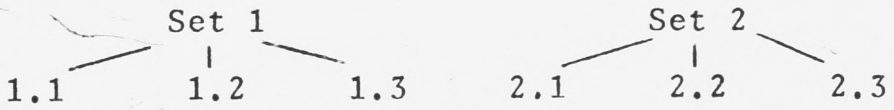
Ting's (207) research in decomposition is similar to Keeney's, but he concentrates more on Debreu-independence. He develops relationships among Debreu-independent sets of dimensions which can be used to evade testing each pair of dimensions for properties of utility or Debreu-independence. The most relevant of these relationships are:

(i) If two of the pairs in three dimensions or sets of dimensions are Debreu-independent, then the third pair is also - and so need not be tested. This applies even if there are more than three dimensions involved in the analysis (117).

(ii) If the union of two sets of dimensions is Debreu-independent of another set or union of sets, then the union and the set or the two unions are also utility independent if one attribute in the union is utility independent of an attribute in the other set or union of sets - so utility independence need not be tested for each set in this situation.

It is now appropriate to make an assessment of Keeney's and Ting's procedures, First, the procedures require an enormous amount of concentration for even the analyst to keep track of the interrelationships among the

sets of interconnected attributes, with the decision maker being unlikely to be able to do so. Second, the time and concentration required for testing sets for these relationships will be lengthy. As Keeney notes, "Decision makers find it very difficult to think about lotteries (required for utility independence tests) involving more than one attribute, because they must consider simultaneously both trade-offs between different levels of the attributes and the probabilities that the various consequences will occur" (112, p.13). Third, whether sets of dimensions can often be constructed that will have non-empty intersections is debatable. Judging from the applications mentioned in Chapter 13, in most situations the intersections would be empty, with a goal-fabric hierarchy of dimensions like this:



with each set of dimensions describing an aspect of the decision space. This would suggest that Ting's approach which emphasises non-empty intersections may be more often used. In the example above, if the union of sets 1 and 2 is Debreu-independent from a third, then sets 1 and 2 are also utility independent from the third.

In brief, decomposition of goals and the goal-fabric approach can be very useful if there are many dimensions, but much of the tedious testing for independencies in most decision analysis appears to be unavoidable, unless the analyst relies on general

discussion with the decision maker to verify the independencies - a course both might prefer in real applications, given the robustness of the models to violations of independence and the limited aim of practical decision analysis.

15. CERTAINTY EQUIVALENTS

The second of the three points of MDA simplification is the certainty equivalent. Certainty equivalents offer a promising way of easing the difficulty of decision analysis. A certainty equivalent ($\hat{x}_i$), is an amount of x_i, such that $\hat{x}_i$ for sure $\sim$ ($\tilde{x}_i$) where $\tilde{x}_i$ refers to the full uncertain outcome of x_i for a consequence. Conditional certainty equivalents, ($\hat{x}_i/x_{\bar{i}}$), are those where $x_{\bar{i}}$ are held at some constant levels. A certainty equivalent combines both the decision maker's utility andthe probability distribution of a dimension, for $u(x_i) = u(\sum_{j=1}^{m} p_j . x_{ij}) = E(u(x_i))$. Thus, certainty equivalents could be elicited rather than utility functions and probability distributions, by discussing with the decision maker the full uncertainty involved in an alternative, and asking him which of various amounts of x_i would be indifferent to the uncertain alternative. Because managers have been found to operate as though certainty exists this must be how they operate, intuitively. It could be argued that the use of certainty equivalents means that the full benefits of decomposing a decision into its constituent parts of risk and attitudes to risk are not obtained. This may be so, but one could also argue that if the full uncertainty of an outcome is discussed with a decision maker, even though a precise distribution function is not obtained, and if indirect questioning is used to find the

point where $u(\hat{x}_i) \sim u(\tilde{x}_i)$, then risk and attitudes to risk are being elicited. Considering the variability of distributions and utility functions with methods used to elicit them, it is not certain that certainty equivalents are less "accurate" than values derived from probability distributions and utility functions.

There are many advantages to the use of certainty equivalents. One is that the uncertain outcomes which are indifferent to the certainty equivalent are not hypothetical, as they often must be in the elicitation of utility functions, but are the actual alternatives facing the decision maker. This leads to another advantage of certainty equivalents, which is the increased acceptability of the analysis to the decision maker when using certainty equivalents, for managers appear to intuitively operate using them and the sophistication of distribution and utility functions are not usually explicitly involved. The final advantage of using certainty equivalents is that it saves time. If a full decision analysis is done, each alternative on each dimension must have a distribution function, and each dimension must have a utility function whereas if certainty equivalents are used, a certainty equivalent replaces each distribution function and a value function replaces each utility function, and both replacements are more easily elicited. This saving of time assists a dynamic consideration of the problem.

One disadvantage to the use of certainty equivalents is that subordinates cannot be entrusted to make decisions based on the decision maker's previously derived utility function (5). However, this disadvantage can be worried about after decision analysts have first convinced decision makers to use MDA under uncertainty themselves. The pressing problem is to convince decision makers to use decision analysis to supplement their intuition, let alone convincing them that someone else could be trusted to use it too.

Thus far, discussion has centred on certainty equivalents for each dimension, and not the models that combine them. If Fishburn-independence exists, so that the additive utility model applies, then $u(x) = \sum_{i=1}^{N} u(x_i)_N = \sum_{i=1}^{N} v(\hat{x}_i)$, even if there is probabilistic dependence between the dimensions (178), for Fishburn-independence means that any probabilistic dependence would not affect the utilities for each x. This presents another method of evading the difficulties of probabilistic dependence if Fishburn-independence exists or can be assumed to exist (Section 6.2).

For non-additive utility models, there is strictly a requirement for probabilistic independence before certainty equivalents can be used in the model (178; 110; 111) with the expected utility of an alternative being expressed in terms of the certainty equivalents of one-dimensional outcomes. However, the importance of this

use of certainty equivalents is that if certainty equivalents are used one is dealing with MDA under certainty. In this situation, the additive value model has been found to be quite adequate, so that MDA under uncertainty with probabilistic independence can be decomposed into a simple additive value model if certainty equivalents are used, no matter whether the additive or multiplicative utility model may have applied. Moreover, with probabilistic independence, results of the additive value model will be unique up to a monotonic transformation, so that the monotonic R transformation forthe R(V) utility model for MDA under uncertainty will not change the ordering of alternatives, and hence need not be elicited from the decision maker. That is, and this is an important conclusion, there is no need for utility functions or uncertainty models of any sort if probabilistic independence exists, for an additive value model used with certainty equivalents will be adequate. The one minor proviso to this concerns the quasi-additive utility model with utility dependence. The additive and multiplicative utility models assume utility independence and hence imply that value independence exists, so that the value additive model can be used. But even if utility independence does not exist then value independence may still exist. So if utility independence does not exist, then the quasi-additive utility model with utility dependence might still be applicable, using certainty equivalents y_i conditional on two standard levels of z.

That is,

$$E(u(y,z)) = k_2 u(\hat{y},z^0).(1 - u(\hat{z}|y^0) + k_2 u(\hat{y}|z^1)u(\hat{z}|y^0)$$

where z^0 and z^1 are the two standard levels of z and two scaling constants are required. However, this model with more than two attributes is an exceedingly difficult one to apply, as Chapter 9 notes.

In conclusion, the use of carefully elicited certainty equivalents has many benefits for the acceptability and tractability of MDA under uncertainty, especially if probabilistic independence exists, with no proven degradation in the accuracy of its measurement of the decision maker's assessment of and attitude to risk resulting from their use.

16. DOMINANCE

16.1 Dominance Under Certainty

Two concepts - decomposition and certainty equivalents - which can be used to simplify decision analysis have been discussed. The third and final concept to be discussed is that of dominance.

Under certainty, all that is required to test for dominance is to rank each alternative on each dimension. Then an alternative is dominated if it is bettered or equalled on all dimensions (with at least one dimension bettered) by at least one other alternative. It may be thought that this dominance is unlikely to occur. However, in Keeney's MDA for the second Mexico City airport, for example, four out of eight alternatives were dominated. The four dominant alternatives were then redefined into five more precise alternatives, and two of these were dominated, which left only three to consider in more detail, and one of these dominated the others in all but one dimension, so that the trade-off for this one dimension from the others was all that was finally required.

If uncertainty exists, this procedure could also be used with certainty equivalents. If uncertain alternatives are ranked in terms of simple preference for each dimension, then this implies that the certainty equivalents are in that preference order. Careful discussion of the uncertainty involved in alternatives

is required to assure that the decision maker is aware of risk and that his attitude to this is incorporated in the ranking. This ranking of certainty equivalents would be easier than specifying the utilities of the x_is. Of course, the certainty equivalents could be derived from utility functions and probability distributions, if they exist. Indeed, Keeney remarks that the certainty equivalents calculated from the probabiiity distributions and utility functions elicited in the first, static, part of his analysis were "a particularly useful feature". They permitted "an analysis of dominance and (gave) insight into how much of attribute x_i it would be necessary to trade-off for a specified amount of attribute x_j for any alternative to be preferred to another." (159, p.511). In the truck investment case study of Chapter 12, the certainty equivalents calculated from the probability distributions and utility functions ranked thus:

TABLE 16.1: TRUCK CASE STUDY CERTAINTY EQUIVALENTS

Dimension / Alternative	Cost	Availability	Mobility	Cross loading	Passenge carrying possibil ity
3/4 ton *	9	1	5	9	5
1 ton GS	8	1	4	7	5
1 ton CL*	7	9	6	7	5
2½ ton GS	6	5	1	5	1
2½ ton CL	4	6	7	5	1
5 ton GS	5	3	2	3	3
5 ton CL	2	8	7	3	3
8 ton GS	2	3	2	1	5
8 ton CL	1	6	7	1	5

Of the nine alternatives, only two were dominated. The 3/4 ton truck was dominated or equalled by the 1 ton GS truck on all dimensions, and the 1 ton CL was dominated by the 2½ ton GS on all dimensions. This indicates that dominance may not always be as useful as hoped, but that it still may help to narrow the field a bit.

16.2 Stochastic Dominance

Stochastic dominance refers to dominance under uncertainty. It is especially useful if probability distributions for alternatives are available, but utility functions are not. Perhaps a researcher's model could have provided the first, but the actual decision maker is not available to provide the second. On the other hand, both must be available, at least implicitly, for certainty equivalents to be elicited. However, it is doubtful whether stochastic dominance will be as useful in multidimensional situations as it is in the unidimensional situations to which it has been applied, for the dominance must exist over all dimensions for an alternative to be deleted from consideration.

There are three degrees of stochastic dominance, first (FSD), second (SSD) and third (TSD). When $U_i(x)$ is the ith derivative with respect to x of the utility of x; $F_o(x)$ and $G_o(x)$ are the cumulative distribution functions of alternatives f and g which are within (a, b); and $F_n(R) = \int_a^R F_{n-1}(x)dx$, for R varying from a to b, and analogously for $G_n(R)$; then

the assumptions about the utility function and the corresponding rules for stochastic dominance are:

FSD $U_1(x) > 0$ $F_1(x) \leq G_1(x)$

(that is, no risk aversion or preference)

SSD $U_1(x) > 0$ $F_2(x) \leq G_2(x)$

$U_2(x) < 0$

(that is, risk aversion)

TSD $U_1(x) > 0$ $F_3(x) \leq G_3(x)$

$U_2(x) < 0$ $F_2(b) \leq G_2(b)$

$U_3(x) > 0$

(that is, decreasing risk aversion)

A complete description of these conditions, with proofs, is given in (9). Essentially, as more assumptions are made about the utility function, more and more alternatives will be dominated. Risk aversion is a reasonable assumption to make about a public investment decision maker, as noted in Chapter 5, so that SSD could be used. However, whether TSD could be used would depend on the decision maker, for although decreasing risk aversion is reasonable for a dimension of wealth, it may not be for the dimensions in a MDA.

The calculations of the $F_n(x)$s in stochastic dominance can be quite tiresome, and computer programs are required (9). However, another form of dominance called E-V dominance requires only the mean (E) and the variance (V) of the alternatives distributions - f dominates g if $E(f) \geq E(g)$ and $V(f) \leq V(g)$ with at least one of the strict inequalities holding. Although E-V

dominance assumes a quadratic utility function, with increasing risk aversion, the results of E-V dominance and TSD are remarkably alike, at least when risk aversion is not strong (171; 172). It seems that the complexity of finding stochastic dominance is not worth the extra cost, compared to E-V dominance.

The case study of truck investment was used to test for E-V dominance. The figures derived for mean/standard deviation were:

TABLE 16.2: TRUCK CASE STUDY E-V½ VALUES

Alternative / Dimension	Cost	Availability	Mobility	Cross Loading	Passenger carrying possibilities
3/4 ton	76.3/6.86	2.45/0.59	2.45/0.59	9	5
1 ton GS	65.39/7.06	2.45/0.59	2.35/0.58	7	5
1 ton CL*	56.75/6.41	2.70/0.64	2.80/0.60	7	5
2½ ton GS	30.0/2.20	2.30/0.64	2.20/0.60	5	1
2½ ton CL	24.15/1.94	2.65/0.73	2.75/0.70	5	1
5 ton GS	23.62/1.56	2.25/0.63	2.15/0.58	3	3
5 ton CL	17.9/0.75	2.5/0.60	2.75/0,70	3	3
8 ton GS	17.8/1.44	2.25/0.63	2.15/0.58	1	5
8 ton CL	14.6/1.34	2.65/0.73	2.75/0.70	1	5

In each case, the lowest mean value is to be preferred, for a cardinal scale was assumed for availability and mobility, and for the non-stochastic alternatives of cross loading and passenger-carrying possibilities the rank orderings are from Table 16.1 The 1 ton CL truck is

is again dominated by the 2½ ton GS truck as it was when certainty equivalents were ranked. But the 3/4 ton truck is not again dominated by the 1 ton GS truck - for instance, the 1 ton's mean cost is lower but its standard deviation is also slightly higher than the 3/4 ton's, so the 1 ton is not dominant on cost. The difference to the situation when certainty equivalents were used would be due to the utility function of the decision maker being different to that assumed in the E-V analysis.

In brief, stochastic and E-V dominance is likely to have a limited place in MDA under uncertainty as compared to the place of dominance with certainty equivalents. It would only be useful where distribution functions were available, but utility functions were not.

16.3 Conclusion

Dominance may be useful to a decision analyst in cutting down the number of alternatives to be considered. Thus it needs to be done early in the analysis, and ranking of uncertain alternatives by their intuitive certainty equivalents appears to be useful for this. If any alternttives are dominated they can be removed from consideration.

It has been argued (e.g. (18; 170)) that this is as far as decision analysis should go, for going further is trying to derive a univariate figure of merit for a multivariate problem and makes far too many assumptions,

e.g. about the model. Indeed, this may be so, especially if only two or three alternatives remain, and the decision between them relies only on the trade-off of one dimension against the others, as in the Mexico City airport decision analysis.

However, as shown in the truck selection case study, the use of dominance may leave several alternatives to be considered. Moreover, when a final decision is made, the decision maker will decide which alternative is "best", implying a univariate final selection. Thus a decision analyst, attempting to aid a decision maker by indicating "effective strategies", would do well to usually continue the analysis past the test for dominance, especially as a decision maker's intuitive steps towards a univariate solution may be suspect.

17. SUMMARY AND CONCLUSION

17.1 Summary

This paper began with a review of various guides to public investment analysis, and found MDA to be worthy of study because it appeared to require fewer assumptions and attempted to allow for uncertainty. Then an investigation of the additive value model for MDA under certainty found that the model was acceptable and accurate, that it was robust to variations in the value functions and the weights, and that rating was a simple and effective method of eliciting value functions and weights.

Then MDA under uncertainty was addressed. The elements of utility theory were reviewed, and the differences between utility and value functions outlined, and the terms used for independence relationships in each were justified. A chapter on probability concluded that precision in subjective distribution functions was specious, and the simple visual impact method appeared the best way of assessing them. Then several utility models were outlined, with emphasis on the additive, the multiplicative, two quasi-additive and the R(V) models. Based on scattered and preliminary evidence, the results of all models which assume utility independence appear to be highly correlated. Thus the straightforward additive model could be used. Moreover, because value

functions are more easy to elicit and comprehend, and because value functions are similar to utility functions, there appeared to be a case for using value functions in preference to utility functions.

The few published applications of MDA under uncertainty were then reviewed, to find that applications were satisfactory even when the full, theoretically correct, analysis was not done, as it never was because of time constraints. A case study of investment in Army trucks corroborated the finding that time was a constraint on a MDA, but it also highlighted the point about acceptability of the results of a decision analysis - the most that a MDA can hope for is "to indicate effective strategies", rather than to find the optimum one. Given this limited objective and the difficulty of acceptability, a preliminary evaluation of MDA under uncertainty concluded that techniques should be simplified as much as possible, e.g., by using the additive value model.

Three concepts which could simplify MDA were then covered. First, decomposition of dimensions by a goal-fabric would be useful when there are many dimensions, and relevance matrices can aid in finding weights for dimensions. Second, the use of certainty equivalents can greatly simplify MDA under uncertainty, especially if probabilistic independence exists, by allowing the additive value model to be used, without any apparent loss of accuracy in measuring the decision maker's assessment of and attitude towards risk. Third,

dominance may permit some alternative to be dropped early in an analysis, but stochastic dominance would appear to have little usefulness in multidimensional situations.

17.2 Conclusion

Chapter 3 outlined several approaches to public investment analysis. Regarding the best developed of these approaches, mathematical programming, it might be argued that if certainty equivalents and an additive model are used, then MDA is little different to Haimes' (54) multiple worth surrogate trade-off method of linear programming in its assumptions. This is undoubtedly true in many cases but it could not have been known until the research in this thesis has been done. In any case, it is not true if dimensions are not continuous, and it was suggested that in the real world all the dimensions will not be continuous. The other approaches of investment analysis investigated in Chapter 3 all remain inferior to MDA, even if the simplified techniques of MDA suggested are used.

That is, multidimensional decision analysis would appear to have some advantages over all other methods of public investement analysis, for it uses the decision maker's own preferences, does not use the additive model without being aware of its foundations, and incorporates risk in a systematic fashion. Nevertheless, theoretically correct models have not been satisfactorily used for a full decision because of the time required of busy decision makers. Moreover, only rarely have the results of utility models been accepted in toto by decision makers because, we suggest,

its approach is different to how most managers make decisions, and the elicitation of model parameters require answers to hypothetical questions. What seems to be required for MDA to be applied in public investment analysis is an awareness that it can only aid decisions, not make them, and that this will be achieved more easily if its techniques are simplified as much as possible. If this argument is accepted, the steps for a MDA would appear to be not those listed in Section 5.4, but rather a dynamic, iterative, set of explorations:

1. Discuss the decision with the decision maker(s) and decide on a decision matrix of alternatives and dimensions. This should not be more detailed or precise than is necessary for the particular stage of the decision making process; for instance, carefully defined objectives at an early part of the process may be counter-productive (Section 13.2).
2. Assess whether value and utility independence, monotonicity, and probabilistic independence exist. This should not be done tiresomely, for the additive model appears to be robust to all but major absences. Rather, the assessment should be done by common sense, discussion with the decision maker, and an occasional test question, e.g., whether certainty equivalents for x_i change as $x_{\bar{i}}$ does for a test of utility independence. If the conditions do not obviously exist, then serious consideration should be

given to altering the decision matrix so that they do. The usefulness of the additive model which can be used when they do exist for exploring the decision space, will probably outweigh any loss from changing the first-cut decision matrix.

3. After the decision maker is fully aware of any uncertainty in the decision space, ask him to rank the alternatives on each dimension, and test for dominance. (With luck, the decision analysis may be able to be stopped at this stage, with only two, perhaps three, alternatives undominated and easy to choose among intuitively).

4. For the undominated alternatives elicit certainty equivalents more carefully than was done in step 3. Alternatively, if the decision maker wishes to provide probability distributions, these could be used with value functions to derive expected value certainty equivalents. The visual impact method should be used to elicit probability distributions and rating should be used to elicit value functions as these methods appear to be as accurate as any other and are more easy to use than others. Finally, the value of each alternative should be calculated using a weighted additive value model, with a R transformation if necessary. This model is easily understood, and apparently robust enough for the necessarily limited aim of MDA which is achieved in the next step.

5. Use a computer to carry out sensitivity testing of the results of step 4, altering the variables which the decision maker wants to, to enhance the decision maker's understanding.

6. Expect and permit the decision maker to make the final decision, not the model.

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NOTE: IIASA in (14) and subsequent references means International Institute for Applied Systems Analysis, Laxenburg, Austria.

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