INTERNATIONAL BORROWING FOR RESOURCE EXTRACTION

by

Ngo Van Long

In compliance with the requirements relating to Admission
to Examination, and Submission of Theses, for the Degree of Doctor
of Philosophy of the Australian National University, it is affirmed
that, unless otherwise stated, the work that follows is my own.

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ABSTRACT

This is a theoretical study of some basic aspects of foreign investment (and foreign borrowing) and natural resource exploitation. Our emphasis is on optimal planning and regulation but the consequences of the lack of social interventions are also pointed out.

Whether a community should plan for more or less consumption for future generations depends not only on its rate of time preference but also the cost of foreign borrowing. It also is warned that even for a small country, the Fisher Separation Theorem - separating production decisions from financing decisions - may not apply when natural resources have aesthetic values not reflected in market prices.

The problems of efficient allocation of resources over time and distribution of gains between the home country and foreign investors received particular attention. Optimal fiscal mixes (in particular, tax on foreign capital and royalty) are derived under various assumptions about the behaviour of firms. Conditions under which the usual static optimal tax on foreign capital must be modified are identified.

It is shown that foreign investors may react adversely to the risk of nationalization and that resources may be exhausted too quickly as a consequence.

There is also another argument for more conservation : the country may obtain better information as events are observed, causing a revision of expectations. The irreversibility of resource depletion

calls for more caution in this case. The optimal behaviour of a risk neutral community exhibits properties akin to risk aversion under these circumstances.

Under uncertainty, the optimal allocation of natural resources between the domestic and the foreign sectors depends on the community's attitudes to risk. Not only primary characteristics of the community's utility function - such as risk aversion or risk preference - but also secondary properties such as the measures of relative and absolute risk aversion play important roles in the determination of the optimal amount of foreign investment in natural resources.

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CHAPTER I

INTRODUCTION

The problem of optimal exploitation of natural resources has recently gained a great deal of interest. While the adequacy or otherwise of the market mechanism in its role of allocating resources over time has been a perennial topic of discussion, there seems to be more concern when natural resources are explicitly considered. This is perhaps partly because of the striking irreversibility of resource depletion. In addition to economic efficiency, the normative problem of the distribution between generations - what Solow (1974a) termed "intergenerational equity" - becomes more pronounced.

There is another aspect of the distributive and allocative problems of resource exploitation when we are concerned with the "welfare" of a single national economy operating in an international market. In particular, the welfare aspects of foreign investment in natural resources seem to be a matter of real concern for the host countries. The authors of the official document "Overseas Investment in Australia"¹ (1972) observed that :

"For various reasons, some of them non-economic, foreign control and exploitation of Australia's natural resources undoubtedly appear to give rise to widespread resentment. Australians are not unique in this respect - many countries have legislated to reserve mining rights for the State in order to exclude, or at least control, foreign enterprises" (p.90).

¹ Commonwealth Treasury, Treasury Economic Paper No.1, Australian Government Publishing Service, Canberra 1972.

They addressed to themselves the following questions :

"Are the economic benefits of foreign capital sufficient to outweigh the gains it reaps from Australia? ... Should we be content to have our natural resources developed by foreign companies? ... Could we finance more of our own development from local savings?" (p.1)

The document provided some statistics but contained little theoretical analysis of aspects of foreign investment in natural resources. An important issue raised was the extracting of "rent", "for the benefit of Australians rather than the foreign investor" (p.92). The authors were concerned that

"the value to Australia of foreign investment in the mining sector can be seriously reduced if taxation is avoided or royalties set too low". (p.91).

This seems to suggest that there are "optimal" royalty rates but the analysis was not developed any further.²

In view of interests shown in the topic, it seems desirable that formal models be developed in order to analyse some of the issues involved. In particular, one may inquire into what should be the proper role of royalty and its relationship with the "optimal tax" on foreign capital considered in MacDougall (1960), Kemp (1962a), (1962b) (and further analysed in Kemp (1966) and Jones (1967)). Does the familiar tax rule need qualifications when natural resources are introduced? In other words, should a country equate the marginal cost of borrowing with the marginal product of capital (in a one-sector economy)?

² The document referred to a paper by Fitzgibbons and Hendrikx (1971) entitled "A Methodology for the Analysis of Social Gains from the Extractive Industries". In spite of its title, natural resources do not appear in the model - implying that supplies may be regarded as infinite.

These considerations suggest further questions : What are the optimal "fiscal mixes" when foreign companies are competitive and maximize short-run (instantaneous) profits?³ or when foreign companies feel the insecurity of tenure? Are the time paths of exploitation by foreign companies less satisfactory than local companies because they react adversely to the risk of nationalization?

One might also consider related questions of interest : For a given rate of time preference should a country owning natural resources plan for more (or less) consumption for more distant generations if it has access to an international capital (loan) market? How different is the planned consumption path of a country under autarky from that of a country engaging in intertemporal trade?

When uncertainty is introduced, there are additional factors affecting the choices of the community. How should a country allocate its natural resources between the domestic and foreign sectors if the resource industry is risky? How does the optimal allocation depend on the community's attitudes to risk taking? If expectations are subject to revision as events are observed, what are the implications of learning, given the irreversibility of resource depletion, when the host country and foreign investors expect to benefit from learning? Should the host country proceed more cautiously, even if it is risk neutral?

³ This might be the case when resources are exploited as common properties (petroleum exploitation is a typical example).

The purpose of this thesis is to provide some partial answers to the questions listed above. Obviously, there are other problems associated with foreign investment in natural resources. The operations of multi-national corporations, the effects of capital flows on short-term stabilization, the monetary aspects of international transactions etc. are by themselves topics of considerable interest. We choose to ignore these complicating factors, mainly because of limitations on time and space. For the same reason, we do not allow international trade nor capital accumulation. These are important omissions and constitute topics for further research.

Our study is purely theoretical. As a possible justification, we quote Burmeister and Dobell (1970) :

"We are interested here primarily in theories - abstract and possibly artificial mathematical models of economic structures - and we shall stick narrowly to that interest. We elect to do so not because there are no important issues of empirical fact or econometric techniques bearing upon our topic, nor because there are no vital questions of development in particular economies. We do so because the study of theoretical models ... is clearly more than can be discussed in one book, and we have had to concentrate our efforts." (pp.2-3)

It is believed that the study of such abstract models as follow has something to contribute to the solution of pressing contemporary problems : Theories can point to the relevant issues and problems and can clarify the choices available to real communities.⁴

We shall begin our study with a survey of the theories of international capital movements. The Appendices to this survey are extensions and clarifications of the theories.

⁴ See Burmeister and Dobell (1970), p.3.

Chapter III presents a survey of theories of resource exploitations.

Optimal international borrowing for resource exploitation is the subject of Chapters IV and V. - Chapter IV for non-replenishable resources and Chapter V for replenishable resources. In both Chapters, we assume that the economy operates in a certainty framework. There we are concerned with both the cases of "perfect" and "imperfect" capital market. In the case of a "perfect" capital market, the optimizing country may borrow or lend as much as it wishes, provided that an intertemporal budget constraint is satisfied. In the case of an "imperfect" capital market, borrowing is restricted to productive uses. This is essentially the case considered by Bardhan (1967) and Hamada (1966) and (1969). In a deterministic (i.e. non-stochastic) framework, with individual foreign investors being perfectly competitive, there is no fundamental difference between the debt and equity forms of foreign borrowing (foreign investment). The Kemp-MacDougall simple optimal tax on foreign capital is considered and modified in the context of resource extraction, and optimal mixes of profit tax and royalty are derived. The roles of royalties (under different assumptions about the behaviour of firms) are clarified. We also consider the cases of common property resources, externalities due to the non-commercial (e.g. aesthetic) aspects of natural resources, variations in the time horizon and the rate of discount etc.

Chapter VI concentrates on the analysis of the behaviour of firms facing the threat of nationalization. The insecurity of tenure, represented by the probability distribution of the date of nationalization, is shown to affect the time path of resource

exploitation. The actual path under uncertainty is compared with the socially optimal path. We also examine the behaviour of firms which believe that the likelihood of nationalization depends on accumulated profits.

While the uncertainty considered in Chapter VI represents private risks with no social counterpart, Chapter VII introduces uncertainty at a more basic level : the uncertainty about the profitability of resource exploitation. As time goes by and actual profits are observed, the uncertainty is "reduced". If both sides - the host country and the foreign investors - expect to learn, and current plans take into account future learning, the problem is quite complicated. Our emphasis here is on the effects of the irreversibility of resource depletion on the optimal plan. The optimal tax on foreign capital should take into account the costs and benefits of learning about the uncertainty (which is possible only with experimentation, and which may depend on the scale of exploitation).

Chapter VIII turns to an examination of attitudes to risk and their effects on the optimal allocation of resources between the domestic and foreign sectors. The central conclusions of the analysis are presented in Chapter IX.

It will be observed that our emphasis is on optimal planning and regulation. We are well aware of that central planning has externalities and imperfections of its own, but we do not touch on this problem because of our limited aims.

CHAPTER II

THEORIES OF INTERNATIONAL CAPITAL

MOVEMENTS : A SURVEY

1. INTRODUCTION :

Broadly speaking, foreign investment theory is concerned with the movement of real resources across national boundaries in order to earn present and future returns. Thus foreign investment models may range from the study of pure consumption loan in a world without production to the other extreme case where borrowing for consumption is not allowed.

We will restrict our attention to non-monetary models, that is, to models in which contracts are specified essentially in terms of real commodities. This is an enormous simplification. Our only justification is that it may pay to study simpler models first, as monetary factors may obscure the real character of intertemporal choice.

There are a large number of models on foreign investment and confusion is bound to arise since these models are based on different assumptions about market institution. For example Connolly and Ross (1970) assume the existence of complete markets for all goods - present and future - so that borrowing against human capital is allowed in their model; Hamada (1966) puts an upper limit on the ratio of a country's indebtedness to its physical capital stock, while Bardhan (1967) rules out any borrowing for consumption so that a country's debt is identifiable with a fraction of the physical capital

stock it uses.¹ The common feature of these models is of course the fact that real resources move internationally to earn present and future returns. It seems useful to begin our survey by examining the nature of international capital movements under different market assumptions. This will be followed by a survey of some representative models.

2. THE NATURE OF FOREIGN INVESTMENT UNDER DIFFERENT MARKET ASSUMPTIONS :

It is useful to start off with an idealized market system in which the prices of all goods are known to all economic agents. Producers and consumers take prices as given. Producers maximize intertemporal profits and consumers maximize intertemporal utility subject to their (intertemporal) budget constraints. We shall refer to this idealized world as the *Arrow-Debreu world* - after a paper by Arrow and Debreu (1954). Assuming a finite time horizon, the existence of an equilibrium set of prices can be proved under certain assumptions about preferences, technology and endowments. Note that while the initial endowments are crucial in the determination of the equilibrium price system, once the equilibrium price system is established, the ownership of assets is a matter of indifference : an individual's wealth is unaltered whether he sells or rents his assets. "Capital flow" can be defined trivially in this model : given any definition of "current income" and "current expenditure" for each

¹ Bardhan (1967) requires that domestic saving is non-negative. Hamada (1969) - as distinct from the 1966 paper - allows negative saving, but imposes a lower limit.

period, the excess of current income over current expenditure may be called "net lending". While such definitions are unnecessary in the Arrow-Debreu world, they seem useful for our understanding of "less perfect" market systems. Thus one may imagine that Arrow-Debreu economic agents issue (or buy) securities in order to finance their current deficit (or surplus). If the set of economic agents is partitioned into "countries", international capital movements can then be defined. As Laing (1964) put it,

"The movement of capital into a country involves commodities being imported by that country in exchange for claims by foreigners on its assets. The repayments at later date of its foreign debts with interest likewise requires the export of commodities. The two sets of transactions can be regarded as one act of intertemporal trade". (Laing (1964), p.2).

Note that the word "capital" in the above quotation does not refer to capital goods (except when the "commodities being imported" happen to be capital goods) and the word "assets" need not refer to tangible commodities : in the Arrow-Debreu framework, borrowing against human capital is equally acceptable. Models of international capital movements in the Arrow-Debreu framework include Connolly and Ross (1970) and Wan (1971) - the latter considering the case of continuous time. Both papers are based on the existence of the idealized capital market just described which is implicit in the Arrow-Debreu model. This capital market is often referred to as the *Fisherian capital market* - in honour of Irving Fisher, author of "The Theory of Interest" (1930). Given this ideal Arrow-Debreu-Fisher capital market, the optimal policies for a *small* country involve two steps : First, maximize the present value of the net output stream (net of payments to foreign factor inputs), then maximize utility over time, subject to the constraint that the present value

of consumption does not exceed the present value of net output. This result - that production and consumption decisions are separable for economic agents facing given prices - is known as the *Fisher Separation Theorem*. While the theorem is normally stated and proved for finite horizon models, it also applies to infinite horizon models, provided that a maximum exists for each of the two steps. It seems justifiable to argue that any price system for which such problems have no maximum is not qualified as an equilibrium price system.

A basic assumption underlying the Arrow-Debreu-Fisher idealized market is that individuals are "honest" in the sense that *all contracts are honoured*. The possibility of dishonesty (referred to as "moral hazard" in the insurance circle) and non-convex transaction costs including information costs seem to account partially at least for the lack of a "perfect" capital market in the Arrow-Debreu-Fisher sense. Uncertainty about natural events by itself - as distinct from the dishonesty problem - does not create any problem in the Arrow-Debreu world, as long as there are markets for all contingent commodities, i.e. if all natural risks are insurable.

While the Arrow-Debreu complete market system seems a useful starting point for the study of international capital movements, it is essential to introduce "market imperfection" at some stage. Realism dictates this. We know, for example, that borrowing against human capital - without collateral - is virtually non-existent. It is however difficult to model an "imperfect" capital market in a satisfactory manner. Hamada (1966), for example, assumes that a small country can borrow at a constant rate of interest, but the ratio of its

debt to its physical capital stock must be within arbitrarily specified limits. In another paper, Hamada (1969) assumes that a small country has to pay higher interest rate as its stock of debt rises (this, he explains, reflects the risk of default), and that additional borrowing must not exceed a certain fraction of national income. Bardhan (1967) rules out any borrowing for consumption so that a country's debt is identifiable with a fraction of the physical capital stock it uses. Some feature of a perfect market remains in Bardhan's model : it is a matter of indifference as to whether the home country rents foreign-owned capital goods or borrows to purchase capital goods (the debt being tied to the purchase of capital goods) : in one case it pays rental, in the other case it pays interest. If capital goods are subject to depreciation, the terms of repayments are such that the outstanding debt always has a physical counter-part of equal value. The same interpretation is applicable to various models of foreign investment which, at first sight, might seem to refer only to the movements of *physical capital goods*. Thus it would seem that the apparent difference between "trade-in-debt" models and "capital goods movements" models in many cases disappears under our interpretation : The second category may be obtained from the first category by adding the restriction that borrowing is tied to the purchase of capital goods, or alternatively, by assuming that saving behaviour is such that people do not borrow for consumption. For descriptive models with non-negative savings such as Hamada (1966), Kemp (1968), Fisher and Frenkel (1972b), foreign debt always has its physical counterpart.

While Hamada (1969) mentions the default risk facing foreign lenders, his model is deterministic. There is no uncertainty from the point of view of the borrowing country in the model. This remark

applies equally to all models previously mentioned. One may take the view that Connolly and Ross' model (1970), being cast in the Arrow-Debreu framework, formally takes into account uncertainty, simply by interpreting commodities as contingent commodities (i.e. conditional on the "states of nature"). For simplicity, consider a two-period model.² There is uncertainty about the "state of nature" which will occur in period two. Suppose there are n mutually exclusive states of nature $\theta_1, \theta_2, \dots, \theta_n$. Only one physical commodity (corn) will be available in period two. Thus there are n contingent commodities in period two. Any "bundle" $(x_1, x_2, \dots, x_n)$ of these contingent commodities may be regarded as a "security" promising to pay x_j units of "corn" if the state of nature θ_j occurs. Thus a security is characterized by a "pattern of returns" across states of nature. If there exist as many linearly independent patterns of returns (securities) as the number of the states of nature, the securities market is said to be *complete*. In the real world we may expect the number of linearly independent securities to be less than the number of states of nature, because of the moral hazard (dishonesty) problem and because of transaction and information costs. The securities market is then *incomplete*. In either case, the net exporter (in value terms) of securities may be called the borrowing country. An optimizing country which "borrows" will not only face the problem of deciding on how much to borrow but also on the composition of its debt : e.g. what proportion of the

² This is a discrete time version of Diamond (1967).

foreign capital inflow will be "equity investment" (as opposed to foreign purchases of "bonds"). If foreign investors are competitive, the problem facing the home country is simply a variation of the classic "optimal tariffs" problem, once its attitude to risk and evaluation of risks (i.e. probability assignment) are specified. Thus there are optimal taxes on foreign purchases and sales of securities (these "taxes" are of course equivalent to "taxes" on the returns payable to foreigners in the case of foreign purchase of domestic securities and taxes on the returns payable to domestic investors who buy foreign securities). Note that some of the "taxes" may be negative.

So far we have been concerned with the case of "portfolios foreign investment" and assumed that individual foreign investors are competitive and take patterns of returns and their prices as given. While differential information is formally allowed to some extent by allowing differences in the evaluation of risks, the competitive portfolios choice approach is not adequate for explaining "direct foreign investment" which involves the control of business enterprises by foreigners. We must admit that the existing formal literature is inadequate in this respect. Models of imperfect competition in general equilibrium such as Negishi (1961) and Arrow and Hahn (1971) are still in their infancy. Further research in this direction seems essential. For the time being we restrict ourselves to the survey of models in which foreign investors are competitive.

3. DESCRIPTIVE MODELS OF INTERNATIONAL CAPITAL MOVEMENTS :

Most descriptive models of international capital movements either do not consider savings or assume that gross saving is always non-negative, so that in both cases foreign debt is always identifiable

with some physical capital stock. The first type of models concentrates on the competitive allocation of a *given* world capital stock while models with savings are rather more concerned with the relationship between saving behaviour and the ownership pattern of the world's wealth. Most models with savings assume some simple saving behaviour. Wan (1971) however, considers the case where competitive individuals "save" (or rather consume) in such a way so as to maximize their intertemporal utility.

3.1 Models with a given capital stock :

The one-good, two-country model used by Hobson (1914), MacDougall (1960) and Kemp (1962a) seems to be the simplest model for the analysis of international capital movements. Assuming a fixed world capital stock, the competitive allocation of world capital is that which equalizes the marginal products of capital in the two countries. Figure 1 illustrates this. Let OH be the stock of capital owned by country A and HO' that owned by country B. MP is the marginal product schedule for country A, $M'P'$ for country B. HC is the amount of capital owned by country B and lent to country A since at C marginal products of capital are equalized.

Without capital movements, the marginal products of capital in A and B would be HT and HT' respectively. Thus the marginal product of capital in B rises as a result of its investment abroad. The pre-investment income of B is the area $M'T'HO'$ and its post-investment income is the area $M'QLHO'$, being the sum of its output $M'QCO'$ and its foreign investment income $QLHC$. The

Figure 1

Figure 2

national income of country A also rises. In fact the free flow of capital maximizes world's income.

While the competitive allocation of world's capital is unique in the one-good model, it is in general indeterminate in a two-good model with constant return to scale. Consider figure 2. Let OO' be the world's capital stock, OL_A and DL_B be the labour endowments of country A and B respectively. Suppose that C_1 is a competitive allocation of the world's capital stock. The internal allocation of capital between the two industries is given by M for country A and N for country B. If there is no international difference in technology, factor proportions in both countries are the same for each industry. In this case, the point C_2 is another competitive allocation of the world's capital. Internal allocations are M' and N' in this case. Since factor proportions remain unchanged in each industry, factor incomes and hence national incomes are not changed.

The indeterminacy is due to the fact that an increase in the amount of capital used in a country can be absorbed by the re-allocation of labour between the two industries. This indeterminacy is the basic factor underlying Mundell's Theorem of the redundant tariff (1957) : Suppose country A imposes a tariff on its imports (good X, say). If X is capital intensive in country A, rental would tend to rise in A if there was no equilibrating international capital movement. An equilibrium with no commodity trade can be reached by a capital movement into country A such that A becomes self-sufficient in X, with factor proportions being kept unchanged.

Domestic prices and international prices are identical and equal to the pre-tariff prices. A similar analysis applies if X is labour intensive : capital moves out of A , provided that rental income can be brought back in the form of X , without tax or tariff. The Mundell Theorem of tariff redundancy can be proved mathematically, using results of the factor prices equalization theorem, as in Pitchford (1969).

The effects of capital mobility on the equilibrium prices and rental are thoroughly explored in Rowan and Pearce (1966). The effects of both capital and labour mobility are studied in details in Amano (1966). For a concise exposition, see Takayama (1972), chapter 13.

Mundell's model can be complicated further by introducing tax on capital earnings or tariff imposed by the second country etc. Most papers incorporating these extensions have been mainly concerned with normative aspects of the problems (e.g. optimal tax on foreign earnings and optimal tariff) and hence will be surveyed in section 3.

The result that free trade allocation of world's capital is not unique is independent of the assumption of identical production function. Chipman (1971) shows that, under certain conditions, the world production possibility frontier will contain a flat segment of efficient points corresponding to diversification in both countries and that movements along the flat segment correspond to movements of capital between the two countries. We note that if there is no international difference in production functions and if one good is always more capital intensive, there is no such flat segment. Ethier and Ross (1971) give a diagrammatic exposition of Chipman's results and

argue that although there may be only one price ratio allowing diversification, that price is compatible with wide variations in world output and in world demands. Chipman's sufficient conditions have been weakened by Uekawa (1972).

3.2 Dynamic Descriptive Models with Simple Saving Rules :

Hamada (1966) analyses a series of descriptive growth models with international capital movements. There is only one output which can be consumed, invested at home or lent abroad. Production functions are identical, and are linear homogeneous, concave and twice differentiable. Let $\bar{K}_i(t)$ be the stock of capital owned by country i , ($i = 1, 2$), and $K_i(t)$ be the amount of capital used in country i . Net borrowing by country 1 is defined as :

$$B(t) = K_1(t) - \bar{K}_1(t) = \bar{K}_2(t) - K_2(t) \quad (1)$$

At each instant, capital mobility ensures that marginal products of capital are equalized, and hence :

$$K_1(t)/L_1(t) = K_2(t)/L_2(t). \quad (2)$$

From (1) and (2), the momentary equilibrium $B(t)$ is :

$$B(t) = (L_1\bar{K}_2 - L_2\bar{K}_1)/(L_1 + L_2). \quad (3)$$

By making different assumptions about $\dot{\bar{K}}_i$ and $\dot{L}_i$, a series of models can be generated. Hamada considers five models. First, consider his Model One :

Let

$$\dot{L}_1/L_1 = \dot{L}_2/L_2 = \lambda; \quad s_1 = s_2 = s \quad (4)$$

$$\dot{\bar{K}}_1 = s[F_1(\bar{K}_1+B, L_1) - rB] \quad (5)$$

$$\dot{\bar{K}}_2 = s[F_2(\bar{K}_2-B, L_2) + rB] \quad (6)$$

where B is given by (3) and r is the marginal product of capital which is the same in both countries, due to (3); s_i is the marginal propensity to save out of national income of country i .

It is easily seen that the capital-labour ratio (2) in each country approaches the equilibrium value k^* defined by $k^*/f(k^*) = s/\lambda$ provided that the Inada *derivative* conditions are satisfied. Also from (3), $B(t)$ is a differentiable function of time, and

$$\begin{aligned} \dot{B} &= [L_1(0)\dot{\bar{K}}_2 - L_2(0)\dot{\bar{K}}_1]/[L_1(0) + L_2(0)] \\ &= srB \end{aligned}$$

hence

$$\dot{B}/B = \dot{K}/K = sr - s(F_1+F_2)/K < 0$$

because $rK_i < F_i$, by concavity. Hence international debt will increase, but the ratio debt/world capital will decrease continuously and approaches zero.

Hamada's Models Two, Four and Five are more general than Model One, by allowing differences in the saving ratios and/or the labour growth rates. When saving ratios differ, there exists the possibility of multiple equilibria, and the debt-capital ratio does not necessarily converge to zero. Model Three is based on a quite different assumption: all profit income is saved and all wages income is consumed. The debt-

capital ratio converges to a value which is uniquely determined by the initial endowments of capital. This reflects the extreme assumption about saving behaviour.

Gale (1974) generalized some of Hamada's results, by allowing the difference between savings out of wage income and savings out of profits.

It is worth noting that in Hamada's models momentary equilibrium is unique : at any point of time, given the stocks of capital owned by the two countries and their labour forces, the values taken by all variables are uniquely determined. This determinacy is due to the assumption that *there is only one output*. For reasons already discussed in connection with the indeterminacy of capital allocation in Mundell's model, we would expect that in a *two-sector* growth model with international capital movement, such momentary indeterminacy (about the allocation of world's capital and outputs in each country, for *given* relative prices) persists. In addition to this, there is a second kind of momentary indeterminacy, namely factor prices (and hence goods prices and factor proportions) in each industry may not be uniquely determined : this is essentially similar to the causal indeterminacy in the closed-economy two-sector growth model. In order to gain understanding of this second kind of indeterminacy, let us consider Kemp's "International Trade and Investment in a Context of Growth" (1968).

Kemp (1968) assumed that technology is the same in both countries. Labour is internationally immobile and grow exponentially at a constant rate n . Capital is internationally mobile and hence earns the same rate of return everywhere. Each country saves a constant

fraction of its national income, but the two (average) propensities to save are allowed to differ. There are two commodities, a consumption good and an investment good. Momentary equilibrium requires that total savings (in terms of the investment good) equals the world's output of the investment good.

It can be shown that the conditions for the uniqueness of momentary equilibrium in this model are formally identical to those of the closed-economy two-sector growth model. Defining s^L as the labour-weighted average of the two propensities to save (the weights being the proportions of the world labour force resident in each country) and s^K as the capital-weighted average of the two propensities to save, (the weights being the proportion of the world capital stock *owned* by each country), Kemp (1968) showed that any one of the following five conditions is sufficient for the uniqueness of momentary equilibrium goods price ratio, factor prices and world output of each good :

- (i) $s^L = s^K$
- (ii) $s^L < s^K$ and the consumption good is more capital intensive.
- (iii) $s^L > s^K$ and the consumption good is less capital intensive.
- (iv) equal factor intensities.
- (v) the sum of the two elasticities of factor substitution is not less than unity.

In fact, it is immediate from definitions that s^K is the proportion of world capital income saved and s^L is the proportion of

world wage income saved. In general $s^K \neq s^L$ even though in each country the proportion of wage income saved is the same as the proportion of capital income saved. Because of the assumptions of identical technology and perfect mobility of capital, at any point of time the international economy behaves like a closed economy. Interpretations of the five conditions (i) - (v) are identical to interpretations of the corresponding conditions for the closed-economy two-sector growth model. Appendix II.C presents a set of interpretations of these conditions. Note, however, that while any of the conditions (i) - (v) ensures the uniqueness of prices and world outputs, the Mundell indeterminacy remains : i.e. outputs in individual countries and the international allocation of world capital is indeterminate. This kind of indeterminacy is irrelevant if we are interested only in the evolution of the stock of world capital and the stocks of capital owned by individual countries.

We just stated that *at any point of time* our international economy behaves like a closed economy. Momentarily, Kemp's s^L and s^K are the counterparts of the familiar s_w and s_r in, say, Drandakis (1963). The dynamic behaviour of the two models are remarkably different, however, because s^K is *not* a constant over time *unless* the two propensities to save (out of national incomes) are identical. If these differ, s^K will evolve due to the evolution of the proportions of the world capital stock owned by each country. For this reason, Kemp (1968) was not able to give simple sufficiency conditions for the uniqueness and stability of long run equilibrium for the case of a variable s^K .

To conclude our discussion of Kemp's model, we note that while Kemp's assumptions (1968) differ from Oniki and Uzawa (1965) only

be allowing capital movements, the results are remarkably different. For example, Oniki and Uzawa's model is causal while Kemp's model is not.

The same indeterminacy would appear in a small, open economy producing two goods and facing given international prices. In order to eliminate this indeterminacy, Frenkel and Fischer (1972a) assume that capital goods are physically immobile but their ownership claims can be traded. Thus a country can only transport its consumption goods to a foreign country in exchange for the ownership of capital goods (i.e. for securities). Only consumption goods can move across national boundaries. This is sufficient to ensure that the domestic rate of interest (in terms of the consumption goods) is equal to the international rate of interest, i . Let r be the domestic rental rate and p the domestic price of capital goods (all in terms of the consumption good), then an interior equilibrium requires $i = r/p$. If the capital goods sector is labour intensive, then $dr/dp < 0$ and hence r and p are uniquely determined given i . Using this assumption on factor intensity, Frenkel and Fischer (1972a) show that the equilibrium growth path is unique and is globally stable, provided that non-specialization holds.

In another paper, Fischer and Frenkel (1972b) relax the assumptions of (i) immobility of newly produced capital goods, (ii) a labour intensive investment sector and (iii) non-specialization. On the other hand, they add the assumption that gross investment disturbs the regular production process. Without this compensating assumption, the international allocation of newly produced capital goods would be indeterminate. While this "adjustment costs" assumption seems to be a step in the right direction Fischer and

Frenkel's specification of the effects of the adjustment is quite unsatisfactory : According to their specification, the *proportion* of total output lost as a result of a given amount of gross investment I is a constant. In this model, the momentary equilibrium level of investment in the home country is that level which equates the domestic rate of return on capital with the given world rate of return.

Another approach to international capital movements is to assume that the market adjusts sluggishly. For example, the flow of capital goods may be a linear function of the rental differential. This approach is taken by Donaldson and Neher (1969). It is unfortunate that, for reasons of mathematical convenience, they assume that saving is a constant fraction of domestic output, not domestic income (the difference being interest payable abroad). Moreover, it would seem more desirable to incorporate the source of the sluggish adjustment in the model.

3.3 Descriptive Models with a Fisherian Capital Market :

The models surveyed in section 3.2 are all based on simple saving rules. According to Arrow and Kurz (1970), this may be due to an "imperfect" capital market (p.128). An opposite saving hypothesis would be that the consumer chooses his consumption-savings program so as to maximize his utility over time. This, of course, requires that consumers have perfect foresight. The basic conceptual framework for the study of consumer behaviour under this assumption is from Fisher (1930), and is further formalized by Arrow and Debreu (1954). The Arrow-Debreu model (1954) is quite general and formally allows for international borrowing and lending. For further insight, additional assumptions must be made. For example, Wan (1971) assumes a two-country world, each comprising identical consumers. There is only one

good, which can be consumed or invested. The capital stock can be consumed. The world's capital stock is perfectly mobile, so that marginal products of capital are equalized at each instant. Thus, given $k(t)$, the rate of interest at time t $i(t)$ is uniquely determined by $f'_1(k_1) = f'_2(k-k_1) = i - \delta$ where δ is rate of depreciation. If ρ_j stands for the utility discount rate of consumers in country j , their rate of change in consumption is

$$\dot{c}_j = (\rho_j - i) U'_j(c_j) / U''_j(c_j).$$

If sufficient restrictions are placed on the utility and production functions, the time paths of variables such as debt, commodity flow across national boundaries etc. can be characterized.

Wan (1971) also extends his analysis to the infinite horizon case. He assumes that $\rho_j = 0$ ($j = 1, 2$) and shows that no equilibrium growth path exists. This is simply another way of stating the well-known proposition that the Ramsey's optimal stationary state cannot be supported by a competitive price system if the utility rate of discount is zero : No individual is willing to replace his capital stock if the net rate of return is zero.

4. NORMATIVE MODELS :

4.1 Free Trade and Investment, Pareto Optimality and the Core :

If commodities are properly defined, the proposition that "a competitive free trade equilibrium is from the world's point of view, Pareto optimal" is easily seen to be applicable to general models which include international investment. A precise characterization of the relationship between a competitive free trade equilibrium and Pareto optimality is in Debreu (1959). By free trade, we now mean free trade

and investment. Debreu's model should be interpreted as a multi-period model, in which commodities available at different dates are different commodities. Debreu's statement of the theorem that a laissez-faire competitive equilibrium is a Pareto Optimum is quite general. It only requires (i) non-satiation at the equilibrium (ii) convexity of preferences and (iii) convex consumption sets (which represent non-economic restraints on consumers' choice). Externalities are of course, assumed away in Debreu's model. *Internal* economies (such as increasing returns to scale, internal learning by doing), whether static or dynamic, are allowed for in the theorem. While internal economies may prevent the existence of a laissez-faire competitive equilibrium, they do not destroy the property that such an equilibrium, if it exists, is a Pareto Optimum. There are, however, some authors who seem to suggest that the competitive allocation of resources may not be Pareto optimal due to dynamic internal economies. This suggestion is of course, fallacious unless it is given generous interpretations. This will be discussed in more detail in Appendix II.B.

Another property usually attributed to free trade competitive equilibria is that they are "in the core".³ Broadly speaking, an allocation is said to be "in the core" if no group of individuals can make themselves better off by refusing to trade with non-members. In the Arrow-Debreu framework it is easy to prove that laissez-faire equilibria are in the core. As a corollary, we obtain the proposition that from the point of view of a single country, *free trade is no worse than autarky*. In other words, no distribution of an autarky commodity bundle can be a *Pareto improvement* over a free trade situation. The general validity of this proposition has been recently questioned by

³ For a precise definition, see Arrow and Hahn (1971) p.184.

Gale (1971). Gale showed that, in a model with *infinitely* many generations of individuals (with each generation consisting of a finite number of individuals living in only two periods), free trade (and investment) may be worse than no trade. In Gale's model, an individual in country A, in his youth, can invest in country B by transporting consumer goods to B in exchange for the ownership of investment goods produced and installed in B (investment goods themselves are immobile internationally). The return from this investment (i.e. its rental) will flow back to the investor (in the form of consumer goods) in the following period. The investment goods survive only one period. Assuming zero population growth, the steady state is characterized by two constant flows of consumer goods in each period between A and B - the purchase of investment goods in B and the rental payments to the old people in the investing country (A). Gale showed that, under certain conditions, the *net* flow from A to B will be *positive* in each period: The investing country constantly exports its consumer goods to the other country (this implies that the rate of interest is *negative*).⁴ Clearly every individual in A can be made better off under autarky. Thus a free trade (and investment) allocation is not in the core. Green (1972), however, rescued the proposition that a free trade is in the core by arguing that for every individual in A to take concerted action to withdraw from trade so as to be better off under autarky, agreements

⁴ If there is population growth, the important variable is not the rate of interest, but the difference between the rate of interest and the rate of population growth.

must be reached between those living and those as yet unborn. Thus, at least semantically, one might uphold the proposition. One cannot deny, however, that a central government can improve the welfare of individuals in A by refusing to trade and by acting as an intermediary between generations in A. Moreover, if the equilibrium rate of interest is more negative than a certain negative number, both countries can be made better off under autarky, provided that suitable financial intermediation is arranged for generations in each country. This result, established by Green (1972) for Gale's model, is not surprising, if one is reminded of earlier works by Samuelson (1958) and Cass and Yaari (1966) : If, in a stationary equilibrium, the rate of interest is negative, then consumer's marginal rate of substitution of present for future consumption is less than unity, i.e.

$$-\left. \frac{dc(t+1)}{dc(t)} \right|_{u \text{ constant}} < 1$$

hence each will be better off if he is told to give up an infinitesimal amount of consumption now in exchange for an equal amount of consumption in the next period. If the number of generations is *infinite*, everyone can be made better off under this scheme, since there is no end to the process.

If the equilibrium rate of interest is positive, it is not possible for any country to improve its position by refusing to trade. A free trade equilibrium is Pareto optimal and is in the core in this case. Furthermore, in this case Gale's model shows that a Pareto optimum is consistent with some country being a *permanent debtor*.

The possibility that some free trade situation is inferior to some redistribution of an autarky bundle does not imply that free

trade is not gainful : As Gale (1974) pointed out, for any distribution of an autarky bundle there is a superior situation achievable by free trade *with redistribution*. It must be obvious - although Gale (1974) did not point out - that such a free trade situation with redistribution must be associated with a positive interest rate in balance growth in order to be a Pareto Optimum.

We note that in Gale's model (1971) no country uses its monopoly power to exploit other countries. It is well known, however, that from the point of view of a single country which can influence the international terms of trade, some interference with trade (and capital movements) is "better" than no trade. To this subject we now turn.

4.2 Optimal interference with capital movements in models with a given capital stock :

The benefits of international capital movements from the point of view of the investing country, is analysed in some detail in Hobson (1914). Hobson noted a lending country should restrict its investment abroad so as not to depress the returns on its capital already existing in the foreign country. Thus a lending country is likened to a monopolist firm. MacDougall (1960) made the same remark about the lending country, but failed to notice that the borrowing country also has its monopsonistic position.⁵ The issue of optimal borrowing was later taken up by Kemp (1962a).

⁵ Another, related, problem not pursued here is the removal of the assumption of passive reaction. Hamada (1966) considered the case in which capital flows are subject to interference by both countries.

Assuming a given stock of world capital which is internationally mobile, Kemp showed that the free movements of capital is not optimal from the point of view of the borrowing country. Since the borrowing country faces an upward sloping foreign supply curve of capital service it must behave as a monopsonist by equating the marginal return from borrowing (the marginal product of foreign capital in the home country) with the marginal cost of borrowing - not the average cost, which is the supply price of foreign capital service. The following diagram (Figure 3) illustrates the argument : Let OO' be the world capital stock, OH of which is owned by the home country. Capital used in the home country is measured from left to right, and that in the foreign country from right to left. The two marginal productivity schedules are MP and $M'P'$. From the point of view of the borrowing country, TP is the schedule of marginal return from borrowing, and $T'M'$ is the supply curve (average cost curve) of foreign capital service. The marginal cost curve is $T'G$. TP intersects $T'G$ at J which determines the optimal foreign borrowing HI . The gross rate of return on capital in the lending country is JI . A tax on foreign earnings at the rate JN per unit ensures that foreign capital earns the same net rate of return in both countries.

A similar analysis applies to the case of a monopolistic lending country. Kemp (1962b) showed that in this one-good model, a lending country wishing to maximize its national income should restrict its lending by *taxing* the earnings of its capital invested abroad.

It should be noted that in Kemp's analysis (1962a and 1962b) company (income) tax was abstracted away. Pitchford (1974) showed that if company tax is the only instrument (i.e. it is not allowed to have discriminatory tax on the earnings of foreign investment), then

the borrowing country could achieve the same optimal result with a positive company tax. The lending country, however, should impose a negative company tax so as to restrict its investment to the level of the zero-tax free trade level. If company taxes are positive and equal, it is not clear whether the borrowing country should tax or subsidize the earnings of foreign capital. In any case, the following result holds: the optimal tax rule from the borrowing country's point of view, restricts foreign investment to the level which would be obtained if the borrowing country imposed no tax on profit. A similar proposition applies to the optimal lending case.

Now, (1982) also considered the case where there are two goods

and showed that if the borrowing country is another policy instrument. The model was first analyzed in Kemp (1961) and Jones (1967). It was shown that the borrowing country should impose a positive company tax to restrict its investment to the level which would be obtained if the borrowing country imposed no tax on profit. The lending country, on the other hand, should impose a negative company tax to restrict its investment to the level which would be obtained if the lending country imposed no tax on profit. The optimal tax rule from the borrowing country's point of view, restricts foreign investment to the level which would be obtained if the borrowing country imposed no tax on profit. A similar proposition applies to the optimal lending case.

Figure 3

the borrowing country could achieve the same optimal result with a positive company tax. The lending country, however, should impose a *negative* company tax so as to restrict its investment abroad below the zero-tax free trade level. If company taxes are positive and *given*, it is not clear whether the borrowing country should tax or subsidize the earnings of foreign capital. In any case, the following basic result holds : the optimal tax mix, from the borrowing country's point of view, restricts foreign investment below the level which would be obtained if the borrowing country imposed no tax of any kind. A similar proposition applies to the optimal lending case.

Kemp (1962b) also considered the case where there are two goods and where tariff (or subsidy) is another policy instrument. The model was further analysed in Kemp (1966) and Jones (1967). It was shown that if the foreign country completely specialized in one good, it always pays the home country to levy a positive tax on capital flows. The situation is more complicated if the foreign country is incompletely specialized. Suppose the home country is the borrower and exports commodity two. If commodity two is capital intensive in the foreign country, the possibility of conflict arises, because the foreign terms of trade p^* and rental r^* move in the same direction, so that "an improvement in the commodity terms of trade can be obtained only at the expense of an increase in the interest burden on foreign capital invested at home" (Jones (1967), p.11) - and vice versa. In this case, if the capital inflow into the home country is relatively "*small*", it would be optimal to *subsidize* foreign investment at home and to tax exports; if the capital inflow is "*large*", it would be optimal to *tax* foreign investment at home and to subsidize exports. If the home country's export good is labour intensive abroad, it should

tax foreign investment at home and levy a positive tariff on its imports. Similar results hold when the home country is a capital lender. Jones (1967) showed that "although a tax rate or a tariff can be negative, they can never both be negative" (p.15). Using the "*magnification effect*", Jones (1967) showed that the region in which Kemp (1966) found both tax and tariff negative is empty.

The Kemp-Jones model was further explored by Rakowski (1970) who looked at the combinations of tariff and tax which are compatible with a general international equilibrium. Further analysis can be pursued : for example, the problem of the multiplicity of the optimal combination of tariff and tax, the second order conditions etc. Other complications can be introduced, such as the existence of foreign tariff and taxes, additional factors of production etc. See, for example, Gehrels (1971) and Nadel (1971). Another direction of generalization is to take into account capital accumulation. This will be discussed below.

4.3 Optimization Models with Variable Capital Stocks :

The first formal attempt to introduce savings in models of foreign investment seems to be Negishi (1965). Negishi's world comprised two countries, both producing a single (and identical) output, which can be either consumed or invested. The foreign country behaves competitively, so that the net rates of return on its capital at home and abroad are equated. Two alternative assumptions about saving behaviour in both countries were adopted : (i) a constant fraction of national income is saved, and (ii) a constant fraction of capital income is saved. The home country wishes to maximize its

consumption *in balanced growth equilibrium*⁶ by interfering with international capital movements. Negishi showed that, under the assumption that *a fixed proportion of capital income is saved* in each country, the optimal policy in the steady state for a capital lending country is to *subsidize* its investment abroad. This result contrasts with Kemp (1962b) and seems rather "paradoxical". Various attempts have been made to explain this "paradox". Kemp (1965) explained it in terms of the optimal balance between the "expansion effect" of the subsidy (which increases home capitalists' income and hence savings) and its (bad) "allocation effect". Jones (1967) attributed the curious result to the "second best" nature of the problem (p.38). Appendix II.A. will look at the "first best" solution and show that such curious result disappears when domestic savings are totally controllable. Francis (1970) contradicted Negishi's result by using an alternative, but unusual, assumption about capital earnings (he assumed that when the home country borrows, the *net* rate of return on foreign capital in the home country is different from the net rate of return in the foreign country).

Negishi's "paradoxical" result is in fact due to his assumption that the optimizing country has only one policy instrument : to tax (or subsidize) the earnings of foreign capital (if it borrows) or the earnings of its capital invested abroad (if it lends). Domestic savings can only be influenced by this single policy instrument.

⁶ The model is rather restrictive because attention is focused on balanced growth alone.

Thus Negishi's world is a "second-best" world. If we allow domestic savings to be directly and totally controllable, Negishi's paradox will disappear : a lending country should always *tax* its investment abroad. The tax rule is in general, different from Kemp's static tax rule, since the foreign capital stocks is no longer given.

Even if we retain Negishi's assumption that domestic savings are not directly controllable, Negishi's result - that the lending country must subsidize its investment abroad - can be shown to depend crucially on an implicit assumption : that all the tax-subsidy proceeds go to domestic capitalists. If only a fraction of the proceeds go to domestic capitalists, Negishi's result no longer holds.

Negishi's analysis is concerned with optimal policies along balanced growth paths. Obviously there is a need to go further and to construct models capable of producing policy prescriptions not only in a steady state but also in transient states.

Hamada's 1966 model seems to be the first attempt to study the problem of optimal borrowing in a dynamic framework. Hamada considers the case of a small country facing a given international rate of interest, i , which is constant over time. The country's objective is to maximize an integral of discounted utility over an infinite time horizon. Hamada imposes the restriction that at all times the ratio of the country's debt, B , to its net wealth, $W \equiv K - B$, must be within limits where K is the capital stock currently employed in the country. These arbitrary limits, together with a constant rate of interest, seem artificial. Hamada justifies his limits by stating that without this restriction, one would obtain the trivial solution of "borrowing as much as possible and consuming as much as possible" (p.53).

This trivial solution, however, would not arise if an intertemporal budget constraint is added to the country's optimization problem. This problem may thus be formulated as follows :

$$\text{Maximize } \int_0^{\infty} U(c) e^{-\rho t} dt \quad (7)$$

subject to :

$$\int_0^{\infty} c(t) e^{-it} dt = \int_0^{\infty} [f(k) - v(t)] e^{-it} dt - D(0) \quad (8)$$

$$k(0) = k_0 + D(0) \quad (9)$$

$$\dot{k}(t) = -\delta k(t) + v(t) \quad (10)$$

where $v(t)$ is investment, k_0 is the country's initial endowment of capital, $D(0)$ is the initial borrowing (lending).

Obviously, with constraint (8), a country cannot "borrow as much as possible and consume as much as possible".

One way of solving (7) subject to (8), (9) and (10) is : First, maximize the right-hand-side of (8), i.e. maximize the (present value of) "net worth", subject to (9) and (10). If a maximum exists, denote it by $A(0)$ - total asset. Then maximize (7) subject to

$$\dot{A}(t) = -c(t) + iA(t) \quad (11)$$

$$A(t) \geq 0 \quad (12)$$

Condition (12) corresponds to Arrow and Kurz's condition that "total asset" be positive if consumption is to be positive indefinitely (Arrow and Kurz (1970), pp.156-153).

The solution to problem (7) is trivial : First, select $D(0)$ such that $f'[k(0)] = \delta + i$. Then set $v(t) = \delta k(t)$. Thus $A(0) = -D(0) + [f[k(0)] - \delta k(0)]/i$. The consumption plan is given by :

$$\dot{c}(t) = (\rho - i)U''/U' \quad (13)$$

A maximum exists for (7) if $\dot{c}(t)/c(t) < i$ in view of (8).

We have thus at one extreme Hamada's model with rigid restrictions on the debt/net wealth ratio, and at another extreme, a Fisherian market extending to the indefinite future. It seems that some compromise may yield "more reasonable" results. One such compromise is Bardhan's model (1967).

Bardhan (1967) avoided the difficulties discussed above by (i) ruling out the possibility of borrowing for consumption, (ii) postulating that the rate of interest is an increasing function of the stock of foreign capital borrowed (for a given domestic labour force) and (iii) assuming that the existing world capital stock is internationally mobile. Bardhan also assumed that the country never lends abroad and that foreign capital yields disutility. These two assumptions can be relaxed without creating any problem. A constant population growth rate was postulated so that the stock of domestic capital *per head* depreciated at a constant rate, α . Denoting the amount of foreign capital borrowed per head by k_f ($k_f = K_f/L_d$), Bardhan assumed the following relationship :

$$r = r(k_f), \quad r' > 0, \quad r' \cdot k_f + r > 0$$

where r is the rate of interest. As Bardhan pointed out, this implies

some rate of growth in the availability of capital in the international capital market. The social welfare function is given by :

$$V(c, k_f) = U(c) - D(k_f)$$

where $U(c)$ is the utility of per capita consumption and $D(k_f)$ is the social disutility of having foreign capital in one's country. The social objective is to minimize over the infinite horizon the shortfall of the actual welfare from the maximum sustainable level of such welfare. The subjective rate of discount δ is zero.

The optimal choice of k_f at any instant is characterized by :

$$f'(k) - r(k_f) = D'(k_f)/U'(c) + r'(k_f) \cdot k_f$$

which is identical to the static optimal condition if $D'(k_f) = 0$.

Assuming $D'(0) = 0$, Bardhan showed that the optimal amount of k_f in the golden-rule balanced growth path is positive if $\alpha > r(0) > 0$ and zero if $0 < \alpha \leq r(0)$. The golden-rule capital stock used, $k = k_f + k_d$, is such that $f'(k) = \alpha$. If $D(k_f) = 0$ identically and if lending ($k_f < 0$) is allowed, the following diagram (figure 4) adapted from Pitchford (1970), is suggestive :

Figure 4 illustrates the case where $r(0) < \alpha$ (point A is to the right of B). The intersection C is the optimal long-run growth path. If $k_d(0)$ is to the left of C, domestic capital is accumulated and $k_f(t)$ is chosen such that $f' = r(k_f) + r'(k_f)k_f$. If $k_d(0)$ is to the right of C, decumulation of domestic capital, k_d , takes place until C is reached.

Figure 4

It is a simple matter to allow lending ($k_f < 0$) and to introduce a social discount rate δ . If $D(k_f) = 0$ identically, it is easily seen that if $r(0) < \alpha + \delta$, the home country would choose a time path such that $f'(k) = r(k_f) + r'(k_f)k_f$ at all times and the optimal time path converges to an equilibrium point at which the home country is a permanent debtor - simply because capital is "cheap" to borrow. If $r(0) > \alpha + \delta$, the country would lend in the long run. Again, this shows that there is nothing wrong with being a permanent debtor or creditor : "trade" is gainful.

If the world existing capital stock is not mobile so that countries can increase their capital stock only by investing newly produced goods, then it is not always possible or desirable to follow the paths AC or DC in figure 4. Bade (1972) analysed this problem of imperfect mobility by imposing arbitrary limits on additional borrowing ($\dot{k}_f - \alpha k_f$). As mentioned earlier, adjustment costs would seem more acceptable than arbitrary limits, but the analysis would also become more complicated. Hamada (1969) also assumed imperfect mobility - a departure from his earlier model (1966). The assumption that newly borrowed goods can be consumed was retained in Hamada (1969), in contrast to Bardhan (1967) and Bade (1972) where borrowing for consumption was not allowed. In order to avoid the "infinite rate of borrowing" problem Hamada postulated arbitrary limits on the ratio of new borrowing to national income.

Hamada (1969), Bardhan (1967), Pitchford (1970) and Bade (1972) shared a common assumption : the cost of borrowing schedule is upward sloping and is *exogenously* given. It is therefore not surprising that their formula for taxing foreign capital is identical to Kemp's static

tax rule. Manning (1972) argued that the cost of borrowing schedule has the foreign capital stock as a parameter. If foreign savings can be influenced by domestic policies, the foreign capital stock becomes an endogenous variable. In this case, Kemp's static tax rule no longer applies, not even in the long-run. Manning's analysis is based on the simplifying assumption that the home country does not accumulate capital. If this assumption is relaxed, not much can be said about the optimal policies in the transient states.

4.4 Optimal Interference with a Fisherian International Capital Market :

A common feature of optimization models mentioned in section 4.3 is that borrowing for consumption is either restricted - as in Hamada's models (1966) and (1969) - or completely ruled out. In a Fisherian model, intertemporal trade is unrestricted, apart from an intertemporal budget constraint. As we have argued, any Debreu-type general equilibrium model may be interpreted so as to include intertemporal trade among countries, and the optimal tariffs, applied to intertemporal trade, are optimal interference with a Fisherian international capital market. Connolly and Ross (1970) considered a simple version of such models, in a finite horizon framework. Miller (1968) and Webb (1970) considered the infinite horizon case, but their consumers behave in a rather myopic fashion. These models do not seem to have enough structure to analyse problems such as the optimal accumulation paths, the pattern of indebtedness etc. Variational models such as that of Wan (1971) might be more appropriate for these purposes, but Wan (1971) only described a competitive world, in which no country used its monopolistic power. Extensions of Wan's analysis, while difficult, are interesting problems for further research.

APPENDIX IIA

ON A PARADOX IN THE THEORY OF INTERNATIONAL CAPITAL MOVEMENTS¹

1. INTRODUCTION :

It is now well known (see Kemp (1962a) and (1962b)) that in a one-sector, two country model with *given* capital endowments a country should tax the earnings of its capital invested abroad in order to exploit its monopolistic advantage. If it borrows capital it should tax the earnings of foreign capital.

In a model which explicitly allows for saving and capital depreciation, Negishi (1965) shows that if saving in each country is a fixed fraction of its capital earnings the optimal policy in the steady state for a capital lending country is to *subsidize* its investment abroad (assuming an interior maximum). Various attempts have been made to explain this 'paradox'. Kemp (1965) explains this in terms of the optimal balance between the 'expansion effect' of the subsidy and its (bad) 'allocation effect'. Jones (1967) attributes this curious result to the 'second best' nature of the problem.² Francis (1970) contradicts Negishi's result by using an alternative, but (as will be seen later) unusual assumption about capital earnings.

This Appendix draws attention to a special feature of Negishi's model which seems to have been unnoticed so far : Negishi implicitly

¹ A version of this Appendix appeared in the Economic Record, September 1973.

² See Jones (1967), p.38.

assumes that while foreign capitalists save a fraction (s^*) of their *net* capital earnings, domestic capitalists save a fraction (s) of their *gross* capital earnings, or equivalently, *all* tax revenue is distributed to home capitalists (who save a fraction s of it) and all subsidy is financed by taxing home capitalists. Negishi's model can be generalized so that a fraction $0 \leq \gamma \leq 1$ of the tax-subsidy proceeds goes to domestic capitalists who save a fraction $0 \leq \beta \leq 1$ of it³. β is allowed to be different from s as we make the distinction between capital earnings and capitalists' transfer income. The proportion of the tax-subsidy proceeds saved is therefore $0 \leq \beta\gamma \leq 1$, the rest being consumed. Negishi's model is a special case for it implies $\beta\gamma = s$ (e.g. $\gamma = 1$, $\beta = s$). In our more general model an interior maximum may be consistent with tax or subsidy on borrowing or lending, depending on the relationships among parameters.

A second task of this paper is to examine boundary solutions. It will be demonstrated that the free flow of capital is optimal if the maximum is a point at which the home country owns no capital. In particular when $\beta\gamma = s$ (Negishi's case) one obtains a boundary solution if $s > s^*$ or if $s = 1$, or $s^* = 1$. When $\beta\gamma = 0$ (all tax-subsidy proceeds are consumed) one *always* obtains a boundary solution and both tax and subsidy are possible.

Finally it will be shown that in the first-best problem (i.e. when domestic saving is totally controllable) the Negishi paradox disappears. The optimal solution is that the home country exploits

³ A more general interpretation is possible : $0 \leq \beta\gamma \leq 1$ is the fraction of the tax-subsidy proceeds saved no matter by whom.

its monopoly power by setting marginal revenue equal to replacement cost and at home marginal product equals the depreciation rate. The general results are reported in Sections 2, 3 and 4, and tedious proofs are provided in Sections 5, 6 and 7.

2. NEGISHI'S MODEL RE-EXAMINED AND GENERALIZED :

Following Negishi let us denote :

k : capital owned by the home country.

z : capital lent (borrowed if z negative) by the home country.

f : home production function.

s : proportion of home capital earnings saved ($0 \leq s \leq 1$)

α : rate of depreciation of capital

k^* , f^* , s^* are the foreign counterparts of k , f and s respectively. We do not rule out the case where $s = s^*$ or $f = f^*$ identically. The Inada conditions are assumed for the production functions. This will ensure that production takes place in both countries.

In addition we generalize Negishi's model by assuming that a fraction $0 \leq \beta \leq 1$ of the tax-subsidy proceeds goes to domestic capitalists who save a proportion $0 \leq \beta \leq 1$ of it.

In the foreign country the net (= gross) rate of return to foreign capital is f^* . By perfect competition, domestic capital lent abroad also earns the gross rate of return f^* , while capital employed at home earns f' . The difference $f^* - f'$ is the amount of tax (subsidy if negative) per unit of capital lent abroad. If the

home country borrows⁴, $f^* - f'$ is the per unit subsidy (tax if negative) on capital borrowed. $(f^* - f')z$ represents the total tax-subsidy proceeds. Income of domestic owners of capital is composed of their capital earnings kf' and their transfer income $\gamma(f^* - f')z$. Their saving is assumed to be :

$$skf' + \beta\gamma(f^* - f')z = s(k - z\beta\gamma/s)f' + z\beta\gamma f^* \quad (1)$$

Negishi's expression for domestic saving is :

$$s(k - z)f' + szf^* \quad (2)$$

implying $\beta\gamma/s = 1$. Perhaps he had in mind $\gamma = 1$ and $\beta = s$.

Alternatively (2) corresponds to the case where domestic capitalists save out of *gross* capital earnings.

One wishes to maximize domestic consumption (or net income) in the steady state where saving equals capital depreciation for both countries :

$$\text{Maximize } y = f(k - z) + zf^* - \alpha k \quad (3)$$

Subject to :

$$s(k - z\beta\gamma/s)f' + z\beta\gamma f^* = \alpha k \quad (4)$$

$$s^*k^*f^* = \alpha k^* \quad (5)$$

⁴

Francis (1970) argues that when the home country *borrow*s, interest payments to foreign capital should be $|z|f'$ (and not $|z|f^*$). In the foreign country, however, foreign capital earns f^* per unit. This implies some sort of imperfection in the competitive process. We adopt Negishi's more reasonable assumption that interest payments to foreign capital is $|z|f^*$.

Assuming a maximum exists with $k^* > 0$ and $k > 0$, one obtains⁵ from (5) :

$$f^{*'} = \alpha/s^* \quad (6)$$

At an interior maximum, from (3), (4) and (6), there exists a constant λ such that :

$$f' - \alpha = \lambda[(k - z\beta\gamma/s)f'' + (f' - \alpha/s)] \quad (7a)$$

$$f' = \alpha/s^* \lambda[(k - z\beta\gamma/s)f'' + (f' - \alpha/s^*)\beta\gamma/s] \quad (7b)$$

Hence, provided $f' \neq \alpha/s^*$:

$$\frac{f' - \alpha}{f' - \alpha/s^*} = \frac{(k - z\beta\gamma/s)f'' + (f' - \alpha/s) - (f' - \alpha)\beta\gamma/s}{(k - z\beta\gamma/s)f''} \quad (8)$$

(8) is obtained from (7) using the algebraic property that $a/b = c/d$ implies $a/b = (c - \delta a)/(d - \delta b)$, provided $d - \delta b \neq 0$.

Negishi's implicit assumption $\beta\gamma = s$ reduces (8) to :

$$\frac{f' - \alpha}{f' - \alpha/s^*} = \frac{(k - z)f'' + \alpha - \alpha/s}{(k - z)f''} \quad (9)$$

For $s < 1$, the right hand side of (9) is positive and greater than unity, implying :

$$f' > \alpha/s^* = f^{*'} \quad (10)$$

hence the conclusion that the home country must *tax* foreign capital (if it borrows) but it must *subsidize* its capital invested abroad (if it lends). The latter policy prescription is rather paradoxical.

⁵ The Inada conditions ensure that $(k - z)$ and $(k^* - z)$ are positive but *do not* ensure that $k^* > 0$, as asserted wrongly by Francis (1970) p.255.

Note that for $s = 1$ (9) holds if and only if $s^* = 1$ but for this case it can be shown⁶ that the global maximum is at $k^* = 0$, $k > 0$ and $z > 0$. One may conclude therefore that for Negishi's case ($\beta\gamma = s$) an interior maximum, if it exists, is characterized by (10).

For the more general model (10) needs not be true at an *interior* maximum. We shall consider three cases $s = s^*$, $s > s^*$ and $s < s^*$.

Case 1 : $s = s^*$. No interior maximum.

At an interior maximum one would obtain from (4) and (6)

$$f^{*'} = \alpha/s^* = \alpha/s = f' \quad (11)$$

In view of (7a) and (7b) this is true (at an interior maximum) only if $s^* = 1$. It can be shown⁷ however that when $s^* = s = 1$ the global maximum occurs on the *boundary* with $k^* = 0$. Therefore for $s = s^*$, the global maximum is a boundary one.

Case 2 : $s > s^*$.

It will be shown that if an interior maximum exists it is characterized by domestic borrowing and a *subsidy* on capital borrowed.

From (4) and (6), and noting that $s > s^*$:

$$\begin{aligned} (k - z\beta\gamma/s)f' &= k\alpha/s - (z\beta\gamma/s)\alpha/s^* \\ &< (k - z\beta\gamma s)\alpha/s^* \end{aligned} \quad (12)$$

Hence : $f' > \alpha/s^* = f^{*'}$ if and only if $(k - z\beta\gamma/s) > 0$
and $f' > \alpha/s^* = f^{*'}$ if and only if $(k - z\beta\gamma/s) < 0$, which is possible

⁶ See Section 5, this Appendix.

⁷ See Sections 5 and 6, this Appendix.

only when $\beta\gamma > s$. The latter situation ($f' > f^* = \alpha/s$) can be dismissed easily, for it implies the left-hand side of (8) is positive and greater than unity, and hence requires $(f' - \alpha/s) - (f' - \alpha)\beta\gamma/s$ to be positive, i.e. $\beta\gamma/s < (f' - \alpha/s)/(f' - \alpha) \leq 1$, contradicting $\beta\gamma > s$.

We are left with $f' < \alpha/s^* = f^*$ and $(k - z\beta\gamma/s) > 0$. Lending ($z > 0$) can be ruled out by showing that $f' > \alpha$. Suppose $f' \leq \alpha \leq \alpha/s$. The left hand side of (8) is then non-negative and less than unity, and this requires that $(f' - \alpha/s) - (f' - \alpha)\beta\gamma/s > 0$, i.e. $\beta\gamma/s > (f' - \alpha/s)/(f' - \alpha) \geq 1/s$. But $\beta\gamma \leq 1$, hence a contradiction. Therefore $f' > \alpha$. The left-hand side of (8) is then negative. For this to hold it is necessary that $(f' - \alpha/s) - (f' - \alpha)\beta\gamma/s > 0$, hence $f' > \alpha/s$. Using this result together with (4) and (6) :

$$\begin{aligned} (z\beta\gamma/s)\alpha/s^* &= k\alpha/s - (k - z\beta\gamma/s)f' \\ &< (z\beta\gamma/s)\alpha/s \end{aligned} \quad (13)$$

which is consistent with $s > s^*$ if and only if $z < 0$.

Finally note that $(f' - \alpha/s) - (f' - \alpha)\beta\gamma/s > 0$ and $f' > \alpha$ imply $\beta\gamma/s < (f' - \alpha/s)/(f' - \alpha) \leq 1$. Negishi's implicit assumption ($\beta\gamma/s = 1$) therefore rules out the existence of an interior maximum when $s > s^*$. In our general model an interior maximum is possible when $s > s^*$, but it is only consistent with domestic borrowing ($z < 0$) and a *subsidy* on capital borrowed. This 'paradoxical' result is not present in Negishi's model.

Case 3 : $s < s^*$

In this case an interior maximum may be consistent with a tax on capital borrowed, a subsidy on capital lent (Negishi's paradox) or a *tax* on capital lent by the home country. The latter situation is possible only when $\beta\gamma/s > 1$ and is therefore ruled out by Negishi's assumption.

As $s < s^*$, from (4) and (6) :

$$\begin{aligned}(k - z\beta\gamma/s)f' &= k\alpha/s - z(\beta\gamma/s)\alpha/s^* \\ &> (k - z\beta\gamma/s)\alpha/s^*\end{aligned}$$

Hence $f' > \alpha/s^* = f^*$ if and only if $(k - z\beta\gamma/s) > 0$ and $f' < \alpha/s^* = f^*$ if and only if $(k - z\beta\gamma/s) < 0$.

When $(k - z\beta\gamma/s) > 0$, $f' > \alpha/s^* \geq \alpha$. The left hand side of (8) is the positive and not less than unity. Hence $(f' - \alpha/s) - (f' - \alpha)\beta\gamma/s \leq 0$ i.e. $\beta\gamma \geq (f' - \alpha/s)/(f' - \alpha)$. This condition is certainly met if $f' < \alpha/s$, in which case the home country borrows⁸ and *taxes* foreign capital, or if $\beta\gamma/s \geq 1$, which includes Negishi's case.

When $(k - z\beta\gamma/s) < 0$ (which is true only if $z > 0$ and $\beta\gamma/s > 1$), $f' < f^* = \alpha/s^* < \alpha/s$, implying a *tax* on a capital invested abroad.

The results obtained for the generalized Negishi model are complex but instructive. In this second-best model (i.e. where domestic saving is not totally controllable) one must expect that policy implications in general differ from those of a first best model.

⁸ This is established by an argument similar to (13).

Various possibilities can arise, depending on the relationships among parameters. It is possible to have tax or subsidy on borrowing or lending. Negishi's implicit restriction $\beta\gamma = s$ rules out the possibility that $f' < f^*$ at an interior maximum. It is interesting to note that even in the generalized model the free flow of capital ($f'=f^*$) is inconsistent with an interior maximum.

We now turn our attention to boundary solutions.

3. BOUNDARY SOLUTIONS :

A sufficient condition for boundary solutions is that (7a), (7b), (6) and (4) are inconsistent. Thus, as shown in section 2, when $s = s^* \neq 1$ these four equations are inconsistent. When $s = s^* = 1$ they are consistent but the global maximum can be shown to be a boundary one, with $k^* = 0$. The proof is contained in Section 5. Two 'polar' cases deserve special attention :

(a) Negishi's case ($\beta\gamma/s=1$) which rules out an interior maximum when $s > s^*$ (as shown in case 2, section 2) or when $s = 1$ or $s^* = 1$ (as shown in section 2) and

(b) $\beta\gamma = 0$ (all the tax-subsidy proceeds are consumed), in which case no interior maximum exists. For the latter case if $s \geq s^*$ it is optimal for the home country to be the sole supplier of capital and exploit its monopolistic advantage by setting marginal revenue equal to the rate of depreciation.

In order to achieve this the home country may have to subsidize or tax capital lent, depending on α, s and on the two production functions. The full analysis for the case $\beta\gamma = 0$ is in Section 6.

For the general case the following results are obtained for boundary solutions :

If $k = 0$ at the maximum, net income is obtained by maximizing

$$y = f(-z) + z\alpha/s^*$$

which gives $f' = \alpha/s^* = f^*$, implying *no tax or subsidy* on capital borrowing.

If $k^* = 0$ at the maximum, there exists λ such that :

$$f' - \alpha = \lambda[(k - \beta\gamma z/s)f'' + (f' - \alpha/s)]$$

$$f' - zf^{*''} - f^* = \lambda[(k - \beta\gamma z/s)f'' + (f' - zf^{*''} - f^*)\beta\gamma/s]$$

Not many results can be gleaned about the general case. If $\beta\gamma > 0$ and $s = 1 = s^*$ then $f^{*'} > \alpha > f'$, implying a *tax* on capital invested abroad (see Section 5). If, in addition $\beta\gamma = s$, then $zf^{*''} + f^* = \alpha$ (marginal revenue from lending equals replacement cost). When $s = 1$ and $\beta\gamma = 0$, $f' = \alpha = zf^{*''} + f^*$, which is the familiar *taxation* rule found in Kemp (1962a). If $\beta\gamma = 0$ and s is 'sufficiently' small, the home country should *subsidize* capital lending. These results (concerning the case $\beta\gamma = 0$) are derived in Section 6.

While it is unambiguous that the free flow of capital is optimal if the home country owns no capital, both tax and subsidy are possible when the home country owns all the world capital stock and lends capital abroad. A subsidy on capital lending is never optimal, however, if domestic saving is totally controllable. The solution of this first-best model is contained in Section 7.

4. FURTHER COMMENTS :

By generalizing Negishi's model we have shown that subsidizing capital invested abroad is a possibility, not a necessity. This possibility does not arise, however, in a first-best optimization model, where the optimal relation is $zf^{*''} + f^{*'} = f' = \alpha$. The first equality is the familiar tax rule found in Kemp (1962a). The second equality reminds us of the golden rule for a closed economy (see, for example, Phelps (1966)). The simple, unambiguous result of the first-best model sharply contrasts with complicated results of the second-best model. This paper can be regarded partly as an exercise in the theory of the second-best.

Two further points deserve attention : First, the possibility of a boundary solution is due to the assumption that only capitalists save. Boundary solutions cannot happen if a constant fraction of national income is saved. Second, there is no presumption that it is optimal to try to reach the steady state where home consumption is maximized, for even when it is possible to reach it, the adjustment cost may be enormous.

5. PROOF THAT WHEN $s = s^* = 1$ AND $\beta\gamma > 0$ THE GLOBAL MAXIMUM OCCURS ON THE BOUNDARY WITH $k^* = 0$ AND $f^{*'} > \alpha > f'$.⁹

If $k^* > 0$, one obtains from (4) and (6) :

$$f^{*'} = \alpha = f' \quad (14)$$

⁹ This Section and Sections 6 and 7 contain proofs of some assertions made in Sections 2-4.

Net income is therefore

$$y = f(\underline{k-z}) - \alpha(\underline{k-z}) \quad (15)$$

where $f'(\underline{k-z}) = \alpha$

If $k^* = 0$ ($z > 0$), maximizing (3) subject to (4) gives :

$$f' - \alpha = \lambda[(k-z\beta\gamma)f'' + f' - \alpha] \quad (16)$$

$$f' - zf^{*''} - f^{*'} = \lambda[(k-z\beta\gamma)f'' + (f' - zf^{*''} - f^{*'})\beta\gamma] \quad (17)$$

It is clear that $f' \neq \alpha$, because otherwise we would have $\lambda = 0$, by (16), which implies $f' = zf^{*''} + f^{*'} = \alpha$, and hence $f^{*'} > \alpha$. But this, together with $f' = \alpha$ and $s = 1$ violates (4). Hence $f' \neq \alpha$.

Consider two cases :

(i) $\underline{f' > \alpha}$: This implies $f^{*'} < \alpha$, by (4) and noting that $s = 1$ and $z > 0$. Net income in this case is less than (15) for :

$$\begin{aligned} y &= f(k-z) + zf^{*'} - \alpha k \\ &= f(k-z) - \alpha(k-z) + z(f^{*'} - \alpha) \\ &< f(k-z) - \alpha(k-z) \\ &\leq f(\underline{k-z}) - \alpha(\underline{k-z}) \text{ which is (15)} \end{aligned}$$

Thus $f' > \alpha$ is non optimal.

(ii) $\underline{f' < \alpha}$: This implies, by (4), $f^{*'} > \alpha$.

Net income is, for some $(k-z)$ such that $f'(k-z) < \alpha$:

$$y = f(k-z) - \alpha(k-z) + z(f^{*'} - \alpha) \quad (18)$$

We now show that (18) is greater than (15) :

$$\begin{aligned}
& [f(k-z) - \alpha(k-z)] - [f(\underline{k-z}) - \alpha(\underline{k-z})] + z(f^{*'} - \alpha) \\
& > [f'(k-z) - \alpha][(k-z) - (\underline{k-z})] + z(f^{*'} - \alpha) \\
& > [f'(k-z) - \alpha](k-z) + z(f^{*'} - \alpha) \\
& \geq [f'(k-z) - \alpha](k - \beta\gamma z) + \beta\gamma z(f^{*'} - \alpha) \\
& = 0, \text{ by (4).}
\end{aligned}$$

6. ANALYSIS OF THE CASE $\beta\gamma = 0$.

As $\beta\gamma = 0$, (4) reduces to :

$$skf' = \alpha k \quad (19)$$

Consider the three regions $(k > 0, k^* > 0)$, $(k = 0, k^* > 0)$ and $(k > 0, k^* = 0)$ to characterize local maximum and then compare them.

Region 1 : $k > 0, k^* > 0$.

From (5) and (19) :

$$f'(k-z) = \alpha/s \quad (20)$$

$$f^{*'}(k^*+z) = \alpha/s^* \quad (21)$$

(20) determines $k-z$ uniquely. Let $(\underline{k-z})$ be the solution of equation (20). Maximizing (3) subject to (20) and (21) is then equivalent to :

$$\text{Max } z\alpha/s^* - \alpha k \quad (22)$$

Subject to :

$$k-z = \underline{k-z} \quad (\text{a constant})$$

$$k > 0$$

$$z < e \quad (e \text{ is a constant determined by } f^{*'}(e) = \alpha/s^*).$$

Clearly, for $s^* = 1$ maximum net income in this region is :

$$y = f(\underline{k-z}) - \alpha(\underline{k-z}) \quad (23)$$

where $f'(\underline{k-z}) = \alpha/s$.

If $s^* < 1$ any increase in z (coupled with an equivalent increase in k to keep $k - z$ constant, will improve the objective function. But $z < c$. A maximum does not exist for y . Its supremum is :

$$f(\underline{k-z}) + \underline{z}\alpha/s^* - \alpha \underline{k} \quad (24)$$

with $\underline{z} = e$ and $\underline{k-z} = \underline{k} - \underline{z}$.

It will be shown that a global maximum can *never* be in region 1.

The economic interpretation of the results in this region is intuitively plausible : the home country can borrow or lend capital at the rental rate α/s^* as long as $k^* > 0$. The per unit cost of maintaining any given capital stock is α . Thus if $s^* = 1$, the home country is indifferent as to how much it should borrow or lend given that $k^* > 0$ and $k > 0$. However, if $s^* < 1$, the lending rate exceeds the cost of maintaining capital, hence it pays to lend as much as possible. But this contradicts $k^* > 0$. Hence a maximum does not exist in this region if $s^* < 1$.

Region 2 : $k = 0, k^* > 0$

The maximum in this region is obtained by maximizing (3) with respect to z , bearing in mind $k = 0$ and $f^* = \alpha/s^*$. Net income is :

$$y = f(-z_0) + z_0\alpha/s^* \quad (25)$$

where $f'(-z_0) = \alpha/s^* (= f^*)$. No tax subsidy is required.

Region 3 : $k > 0, k^* = 0$

As in region 1, $k - z$ is uniquely determined by (20). The problem reduces to :

$$\text{Maximizes } zf^{*'}(z) - k \quad (26)$$

Subject to :

$$k - z = \underline{k - z} \quad (\text{as in region 1})$$

$$k > 0$$

This gives :

$$\alpha = f^{*'}(z) + zf^{*''}(z) \quad (27)$$

Equation (27) has the interpretation that the marginal replacement cost of lending capital equals its marginal revenue. However marginal product at home is constrained to be $f' = \alpha/s \geq \alpha$. This reveals the second best nature of the problem.

The home country's maximum income in this region is :

$$y = f(\bar{k} - \bar{z}) + \bar{z}f^{*'}(\bar{z}) - \alpha\bar{k} \quad (28)$$

where $f'(\bar{k} - \bar{z}) = \alpha/s$ and $\bar{z}$ is the solution of (27). Of course $\bar{k} - \bar{z} = \underline{k - z}$, by definition.

If $s = 1$, $f' = \alpha = f^{*'}(z) + zf^{*''}(z)$, which is the usual formula for *taxing* capital invested abroad. If $s < 1$ whether the tax or subsidy is required depends on the values of α, s , and on the production functions.

Having analysed individual cases we now attempt to determine where the global maximum occurs. We first show that it cannot be in region 1.

From (27) $f^* > \alpha$. Hence (28) exceeds (23). To see that (28) is not less than (24) rewrite (28) as :

$$y = f(\bar{k} - \bar{z}) + \bar{z}[f^*(\bar{z}) - \alpha] - \alpha(\bar{k} - \bar{z})$$

and (24) is equivalent to :

$$y < f(k - z) + z[f^*(z) - \alpha] - \alpha(k - z)$$

But $(\bar{k} - \bar{z}) = (k - z)$ and $z[f^*(z) - \alpha]$ attains its maximum at z because $\bar{z}$ is the solution of (27).

It remains to see under what conditions the global maximum occurs in region 2 and not in region 3, and vice-versa.

For $s \geq s^*$ (28) exceeds (25). That is, lending is optimal. This is seen by noting that

$$\begin{aligned} f(\bar{k} - \bar{z}) + \bar{z}f^*(\bar{z}) - \alpha\bar{k} &> f(\bar{k} - \bar{z}) - \alpha(\bar{k} - \bar{z}) \\ &\geq f(-z_0) - \alpha(-z_0) \geq f(-z_0) - \alpha(-z_0)/s^*. \end{aligned}$$

(The first inequality follows from $f^*(\bar{z}) > \alpha$, the second from the fact that $f'(-z_0) = \alpha/s^* \geq \alpha/s = f'(\bar{k} - \bar{z}) \geq \alpha$ and the third because $s^* \leq 1$).

For $s < s^*$ the relation between (28) and (25) is ambiguous. If s is sufficiently small (25) will exceed (28). The ambiguity is due to the fact that there are two factors working in opposite directions : if the home country does not own capital, domestic production can be profitably carried to the point where $f' = \alpha/s^*$ (beyond the point at which $f' = \alpha/s$ as dictated by constraint (19) when $k > 0$). On the other hand there is monopolistic gain if the foreign country does not possess capital.

7. ANALYSIS OF THE FIRST-BEST PROBLEM, WHERE DOMESTIC SAVING IS TOTALLY CONTROLLABLE.

In the first-best problem, the constraint (4) is dropped. The methods used are similar to those used in Section 6.

Consider the three regions $(k^* > 0, k > 0)$, $(k^* = 0, k > 0)$ and $(k^* > 0, k = 0)$.

Region 1 : $k^* > 0, k > 0$.

From (5),

$$f'(k^* + z) = \alpha/s^* \quad (29)$$

The problem is to maximize with respect to k and z .

$$f(k-z) + z\alpha/s^* - \alpha k \quad (30)$$

Subject to :

$$k > 0$$

$$z > c \quad (c \text{ is the constant determined by } f'(c) = \alpha/s^*).$$

This gives $f' = \alpha$ and $f' = \alpha/s^*$, which is true if and only if $s^* = 1$, in which case maximum net income is :

$$y = f(\underline{k-z}) - \alpha(\underline{k-z}) \quad (31)$$

where $f'(\underline{k-z}) = \alpha$.

If $s^* < 1$, any increase in z accompanied by an equivalent decrease in k will improve the objective function (30). But $z < c$. Therefore a maximum does not exist for y . Its supremum is :

$$f(\underline{k-z}) + \underline{z}\alpha/s^* - \alpha k \quad (32)$$

where $\underline{z} = c$ and $f'(k-\underline{z}) = \alpha$.

It will be shown however, that a global maximum can never be in region 1.

The economic interpretation of the result in this region is the same as that given for region 1 in Section 6.

Region 2 : $k^* > 0, k = 0$.

Net income is maximized by maximizing (3) with respect to z , noting that $f^* = \alpha/s^*$. Its maximum value in this region is :

$$y = f(-z_0) + z_0 \alpha/s^* \quad (33)$$

where $f'(-z_0) = \alpha/s^* = f^*$.

Region 3 : $k^* = 0, k > 0$.

Maximizing (4), and noting $k^* = 0$, gives :

$$f' = \alpha = f^*(z) + zf^{*'}(z) \quad (34)$$

This implies a tax on capital invested abroad.

Maximum net income in that region is :

$$f(\bar{k}-\bar{z}) + \bar{z}f^*(\bar{z}) - \alpha\bar{k} \quad (35)$$

where $f'(\bar{k}-\bar{z}) = \alpha$ and $\bar{z}$ is the solution of (34).

Now note that (35) is identical to (28) with $s = 1 \geq s^*$. Using argument employed in Section 6, (35) is easily seen to dominate (31), (32) and (33). Hence the global maximum occurs in region 3.

APPENDIX IIB

ON THE OPTIMALITY OF FREE TRADE AND INVESTMENT IN THE PRESENCE OF DYNAMIC INTERNAL ECONOMIES¹

I

That an international competitive equilibrium (with no restrictions on trade and investment) is a Pareto Optimum is a well-known proposition.² The proposition relies on the convexity of preferences and of the consumption sets, and non-satiation at the equilibrium,³ and, of course, the absence of externalities.

It is important to note that the definition of a competitive equilibrium involves the existence of all futures market and the intertemporal maximization of utilities by consumers and of profits by producers, and that the conclusion reached by Hahn (1966) and Shell and Stiglitz (1967) - that a "competitive" system is in general unstable and non-optimal - applies only to a rather imperfect system characterized by myopia.

Recent works by Negishi (1968), Johnson (1970) and Ohyama (1972) seem to cast some doubt on the classic welfare proposition. Negishi (1968) argues that "there are cases in which ... interference with the competitive allocation ... is desirable even if no external effects are present. The reason lies in the existence of dynamic internal

¹ This Appendix is a version of Long (1974d). Helpful comments by Professor Kemp, F. Milne and E. Seiper are gratefully acknowledged.

² The proposition is implicit in Debreu (1959). For an explicit treatment see Takayama (1972), Chapter 7.

³ See Debreu (1959), p.94.

economies ..." (p.57-58). According to Negishi, "the common sense of this possibility may be found in the increments to consumers' surplus generated by the price changes which result from the growth of an infant industry. These increments may be foreseen by private producers, but since they do not accrue to the producers the increments fail to induce private producers spontaneously to create the industry" (p.59). Negishi's proposition is supported by Ohyama (1972). Johnson (1970) refers approvingly to Negishi's paper and presents a partial equilibrium, diagrammatic argument for protection on the ground of "uncaptured increases in consumers' and producers' surpluses" (p.74).

The purposes of this appendix are :

- (i) to re-affirm that, in a fully competitive international economy, protection is unnecessary if the learning process is internal to the firm.

and

- (ii) to point out that the case for intervention in the presence of dynamic internal economies can be upheld only if there are market imperfections - such as lack of foresight on the part of producers.

II.

Johnson's argument is based on a partial equilibrium model, with diagrams portraying costs and gains resulting from the protection of an infant industry behaving as a monopolist in its country. Negishi's paper (1968) also gives, in an Appendix, a "heuristic partial equilibrium version of the infant industry argument ... in terms of consumers' surplus" (p.66). Negishi's paper itself (1968), presents the arguments

in a general equilibrium framework, and analyses both the cases of a competitive and monopolistic infant industry. Ohyama's approach is similar to Negishi's, although he is mainly concerned with the case of a small country facing given prices of traded goods. We chose to argue using Negishi's general equilibrium model, since, in Negishi's own words, "partial equilibrium analysis gets a first approximation ... by assuming that other things are equal ... No one can deny its significance as a heuristic method. But there is the danger of a sweeping generalization if one forgets its basic assumption that other things are equal. The role of general equilibrium analysis is ... to check the general applicability and relevance of the results obtained under the assumption that other things are equal" (Negishi (1972), p.2). In the present context, we only need to point out that when demand and/or cost curves of the infant industry shift, other demand and cost curves cannot remain stationary. Notwithstanding our disagreement with Negishi's argument for protection, we believe that Negishi's approach - the use of a general equilibrium framework - is itself an important contribution to the infant industry debate.

We note that while Johnson and Ohyama seem to be mainly concerned with welfare comparisons from the point of view of a single country, Negishi looks at the infant industry problem from the point of view of the world's welfare. However, as Negishi (1968) points out, his model "can be applied to the analysis from the point of view of a single country, provided the foreign offer curve is interpreted as a kind of input-output relation" (p.60).

We adopt the following strategy : We first present a well-known, well-proven theorem which says that, even in the presence of dynamic internal economies, a competitive equilibrium is a Pareto optimum. This

will be followed by a re-examination of Negishi's crucial test 2. We will show that this test, while useful and relevant for some purposes, does not demonstrate that a socially intervened equilibrium may be "superior" to a competitive equilibrium. In fact, a competitive equilibrium can never be inferior.

III.

Negishi (1968) considers a two-period model of the world economy. Let x and x^* denote the world consumption vector of commodities in the first period (the present) and the second period (the future) respectively, the asterisk denoting variables in the second period. The initial resources are c and c^* . Consumers maximize intertemporal utility by choosing (x, x^*) , subject to their intertemporal budget constraints. Indifference surfaces are convex. For the moment, we need not specify whether individual indifference maps are identical and whether Engel curves are straight lines through the origin, although both these assumptions are explicitly stated in Part VIII of Negishi's paper (1968). The input-output vector for all industries, except the infant industry, is (y, y^*) . The set of all technologically feasible (y, y^*) is Y , which is closed and convex. The infant industry produces the first commodity, and its input-output vector is (q, q^*) . Of course $q_1 \geq 0$ and $q_1^* \geq 0$, where the subscript 1 denotes the first commodity. The set of possible q is Q and that of possible q^* is $Q^*(q_1)$, implying the learning-by-doing possibility. The set of possible (q, q^*) need not be convex. It is assumed that "external effects, static or dynamic, do not exist at all. Therefore, there is no need to distinguish the infant industry and the firm" (Negishi (1968), p.61).

Negishi's approach is to perform welfare comparisons between alternative social states, using (equilibrium) price and quantity data.⁴ Given the present framework, these social states must, of course, be equilibrium states. Equilibria are thus assumed to exist.

A close inspection reveals that Negishi's model is just a special case of Debreu (1959). Dynamic internal economies are allowed for in Debreu's concept of the production set,⁵ where commodities at different dates are differentiated.

The well-known theorem of welfare economics that, in the absence of externalities, a laissez-faire competitive equilibrium is a Pareto optimum, is precisely stated and proved in Debreu (1959), with a minimum number of assumptions.⁶ These are convexity of preferences, non-satiation at the equilibrium, and convex consumption sets (which represent non-economic restraints on consumers' choice). Debreu's definition of a competitive equilibrium of course involves the assumption that producers know their production sets and maximize intertemporal profits (properly discounted). The theorem contradicts Negishi's assertions such as "the competitive allocation of resources may be imperfect even though the learning process is internal to the firm" (p.59) and "the protective tariff, though not optimal, turns out to be better than laissez-faire" (p.65) - unless the term "competitive allocation" used by Negishi is not to be understood in its usual sense

⁴ The technique is well-known. See for example, Samuelson's 1950 paper.

⁵ Debreu (1959) does not assume everywhere that production sets are convex. In particular, the convexity of production sets is not assumed for the theorem we shall invoke. Admittedly, an equilibrium may not exist due to the lack of convexity. But for a meaningful comparison between social states, equilibria must of course be assumed.

⁶ Debreu (1959), p.94.

and his assumption of perfect foresight (p.59) is dropped. For example, if firms are myopic, a "myopic competitive allocation" - a concept as yet not well defined - may be inferior to an equilibrium with protection.

It is interesting to note that the theorem of welfare economics we just invoked can be used to derive Negishi's valid test 1 (which is arrived at by Negishi using a different approach). For the purpose of test 1, Negishi postulates a (hypothetical) competitive equilibrium, supposing the infant industry *does not exist*. This competitive equilibrium is characterized by $(\bar{p}, \bar{p}^*)$, (x, x^*) and (y, y^*) . Perfect foresight is of course assumed here. If there is *no* technologically feasible (q, q^*) such that at the price $(\bar{p}, \bar{p}^*)$ the infant industry's profit $(\bar{p}'q + \bar{p}^*'q^*)$ is positive, then the infant industry should not be protected. This is Negishi's test 1 (p.61), which can be derived straightforwardly by noting that if there is *no* (q, q^*) such that $\bar{p}'q + \bar{p}^*'q^* > 0$, then $(\bar{p}, \bar{p}^*)$, (x, x^*) , (y, y^*) with $(q, q^*) = (0, 0)$ constitute a laissez-faire competitive equilibrium at which the infant industry's competitive output is $(0, 0)$ since it cannot do any better. By virtue of the theorem we invoked, this equilibrium is a Pareto optimum and there is no need for any social intervention.

IV.

In the preceding section, the argument for infant industry protection based on dynamic internal economies alone were shown to be contradicted by a well-known theorem in welfare economics. We now take a closer look at Negishi's test 2 which supposedly provides a sufficient condition for the superiority of an intervened equilibrium

over a competitive allocation. If the term "competitive allocation" is understood in its usual sense (so that firms know their production sets and maximize profits over time), this test (and hence its variants, tests 3 and 4) will be shown to be invalid. The test, however, can be useful for making welfare comparisons between a socially intervened situation and any other allocation which is not a competitive equilibrium (in its usual sense).

In parts VI - VII of his paper (1968) Negishi considers a situation in which world resources are optimally allocated *under the restriction* that the infant industry's output of the first commodity in the first period, q_1 , is not less than a certain threshold level $\bar{q}_1 > 0$. He assumes that such a (conditionally) optimal situation is achieved by a competitive price system (p, p^*) at which the "profit" of the infant industry, $p'q + p^*q^*$, is maximized under the restriction that $q_1 \geq \bar{q}_1$. Due to this restriction the maximum profit achievable by the infant industry may well be negative.⁷ In this conditionally optimal situation, the profit of the other industries is maximized at (y, y^*) and consumers' chosen bundle is (x, x^*) .

Note that the (x, x^*) and (y, y^*) in this situation are not the same as those consumption and production vectors when the infant industry does not exist, and the (q, q^*) in this situation need not be

⁷ Although Negishi does not explicitly specify how the infant industry is to be protected (if necessary), his assumption that all industries maximize profits at the price (p, p^*) is consistent with the following method of protection: the government instructs the infant industry to take market prices as given and to maximize its profit subject to the restriction $q_1 \geq \bar{q}_1$. Any loss will be covered by a subsidy. See also p.63 where it is stated: "since (p, p^*) is a system of optimal prices, protection needs to be carried out in such a way that no price is changed by protection".

the same as that which maximizes profit at the price $(\bar{p}, \bar{p}^*)$, in spite of the fact that Negishi uses the same notation.

Negishi then proposes to compare the socially intervened equilibrium with a *competitive allocation* with no infant industry. He denotes "the consumption vector when there were no infant industry ... by $(\bar{x}, \bar{x}^*)$ and that available when the infant industry is allowed to grow by (x, x^*) " (p.63). For a *meaningful welfare comparison* between a laissez-faire situation and a socially intervened situation, one must interpret a situation in which "there were no infant industry" (p.63) as one in which the infant industry *was not competitively viable* rather than as one in which the infant industry was somehow banned or not conceived of by private producers. This interpretation seems to be supported by the following quotation : "the competitive allocation of resources may be imperfect ... The increments to consumers' surplus ... may be foreseen by private producers, but since they do not accrue to the producers the increments fail to induce private producers ... to create the industry" (p.59).⁸ With this interpretation in mind, we shall denote by $(\bar{p}_c, \bar{p}_c^*)$ the competitive price system⁹ associated with $(\bar{x}, \bar{x}^*)$, $(\bar{y}, \bar{y}^*)$ and $(\bar{q}, \bar{q}^*)$, (the latter is, by assumption, $(0,0)$ identically).

⁸ Incidentally, this "common sense explanation" can be quite misleading. We know, for example, that consumers' surplus is not captured by firms in a competitive equilibrium - with or without dynamic internal economies. Can we then conclude that, for this non-appropriability, any competitive equilibrium may be sub-optimal?

⁹ The subscript c is used since this price system is not, or need not be, the same as the price system $(\bar{p}, \bar{p}^*)$ postulated by Negishi in his Section III.

Let U be the set of consumption vectors preferred or indifferent to (x, x^*) . Negishi observes that a sufficient condition for $(\bar{x}, \bar{x}^*) \notin U$ is $p'x + p^*x^* > p'\bar{x} + p^*\bar{x}^*$ or equivalently, $p'(y - \bar{y}) + p^*(y^* - \bar{y}^*) > -p'q - p^*q^*$. In words, the "loss" of the infant industry is more than compensated for by the "gain" accrued to other industries. This is Negishi's test 2 (p.63). He concludes that "the infant industry should be promoted by protection if its loss is less than $p'(y - \bar{y}) + p^*(y^* - \bar{y}^*)$ " (p.63) and that, even if the infant industry enjoys a monopolistic profit, "such an economic system is ... superior to the system without the infant industry" (p.63).

We will argue that Negishi's test 2 is not valid for the purpose of establishing the superiority of a socially intervened equilibrium over a laissez-faire competitive allocation. It seems clear that Negishi assumes a *consistent* world indifference map for his four sections III - VII (pp.60-63) when he writes : "the world indifference map ... is assumed to exist" (p.60, section III). This assumption is explicitly stated later on (p.64, section VIII). For the sake of completeness only, we will show that Negishi's test is not valid whether the Scitovsky world indifference surfaces intersect or not.

- (i) Suppose that the Scitovsky world indifference surfaces may intersect. In this case $(\bar{x}, \bar{x}^*) \notin U$ does not imply that $(\bar{x}, \bar{x}^*)$ is Pareto inferior to (x, x^*) .¹⁰ The passing of test 2 thus does not mean that the protection situation is "superior" to a laissez-faire competitive equilibrium, nor does it imply that "the competitive allocation of resources may be imperfect".

¹⁰ For a precise statement of this point, see Negishi (1972), p.49.

(ii) Suppose that the world indifference map is consistent. In this case, if $(\bar{x}, \bar{x}^*) \notin U$, then $(\bar{x}, \bar{x}^*)$ is Pareto inferior to (x, x^*) , provided that compensation is paid. It will be demonstrated, however, that $(\bar{x}, \bar{x}^*)$ *always* belongs to U . Suppose that, on the contrary, $(\bar{x}, \bar{x}^*) \notin U$. This must imply that $(\bar{x}, \bar{x}^*)$ is on a lower indifference surface than (x, x^*) , since we are considering the case of a consistent indifference map. Recall that $(\bar{p}_c, \bar{p}_c^*)$ is the competitive price system associated with $(\bar{x}, \bar{x}^*)$, $(\bar{y}, \bar{y}^*)$ and $(\bar{q}, \bar{q}^*) (= (0, 0))$. Now $(\bar{x}, \bar{x}^*)$ is on a lower indifference surface only if $\bar{p}_c' \bar{x} + \bar{p}_c^* \bar{x}^* < \bar{p}_c' x + \bar{p}_c^* x^*$. This cannot be true, however, since from the market clearing conditions,¹¹ we obtain

$$\bar{p}_c' \bar{x} + \bar{p}_c^* \bar{x}^* = \bar{p}_c' \bar{y} + \bar{p}_c^* \bar{y}^* + \bar{p}_c' \bar{q} + \bar{p}_c^* \bar{q}^* + \bar{p}_c' c + \bar{p}_c^* c^*$$

$$\bar{p}_c' x + \bar{p}_c^* x^* = \bar{p}_c' y + \bar{p}_c^* y^* + \bar{p}_c' q + \bar{p}_c^* q^* + \bar{p}_c' c + \bar{p}_c^* c^*$$

But $(\bar{y}, \bar{y}^*)$ and $(\bar{q}, \bar{q}^*)$ maximize profits at the price $(\bar{p}_c, \bar{p}_c^*)$. Hence $(\bar{x}, \bar{x}^*) \notin U$ implies a contradiction and *cannot be true*. Negishi's test 2, if used to compare an intervened equilibrium with a competitive allocation, is therefore not valid, for it provides a sufficient condition for something which cannot be true. The same error is made by Ohyama.

Let us turn our attention to Negishi's comparison between a situation with a tariff protected infant industry and a laissez-faire competitive equilibrium (Negishi's section VIII). Here, Negishi

¹¹ These conditions are :

$$x - y - q - c = 0, \quad x^* - y^* - q^* - c^* = 0$$

$$\bar{x} - \bar{y} - \bar{q} - c = 0, \quad \bar{x}^* - \bar{y}^* - \bar{q}^* - c^* = 0$$

explicitly assumes a consistent world indifference map, and he considers the sufficient condition for $(\bar{x}, \bar{x}^*) \notin U$ (his test 4). Again, the test is not valid since it can be shown easily, by an argument similar to that used in the preceding paragraph¹² that $(\bar{x}, \bar{x}^*)$ always belongs to U .

Finally, we wish to emphasize once again that Negishi's test 2 can be useful for other purposes. More precisely, if $(\bar{x}, \bar{x}^*)$ and $(\bar{y}, \bar{y}^*)$ used in Negishi's test 2 are consumption and production vectors of some arbitrary allocation (which is not a competitive equilibrium), then test 2 is meaningful. It provides, in this case, a useful sufficient condition for the superiority of a socially intervened situation over a (non-competitive) allocation with no infant industry. After all, it is not totally unrealistic to assume that producers lack imagination and foresight, or that they may be local maximizers. In defending test 2 along these lines, we must not forget our basic conclusion that in a world with perfect foresight, the line of argument based on dynamic internal economies for interfering with a competitive world market is not tenable.

¹² Formally, after a proper reinterpretation to incorporate commodity 0 (according to Negishi's trick), the two equations in the preceding paragraph remain valid, since $\bar{p}'\bar{m} = \bar{p}^*\bar{m}^* = \bar{p}_c'\bar{m} = \bar{p}_c^*\bar{m}^* = 0$. (Note that, by definition, $m_0 = -m_1$ etc. and the first two components of the price vector $\bar{p}_c$, and of course $\bar{p}_c^*$, are identical, reflecting free trade).

APPENDIX IIC

CAUSALITY IN KEMP'S MODEL OF TRADE AND CAPITAL MOVEMENTS

1. INTRODUCTION :

Assuming identical technology and perfect capital mobility, Kemp (1968) showed that any one of the following five conditions is sufficient for the uniqueness of momentary equilibrium goods price ratio, factor prices and world output of each good :

(i) $s^L = s^K$

(ii) $s^L < s^K$ and the consumption good is more capital intensive.

(iii) $s^L > s^K$ and the consumption good is less capital intensive

(iv) equal factor intensities

(v) the sum of the two elasticities of factor substitution is not less than unity.

Note that s^L and s^K are, at any point of time, identical to the familiar s_w and s_r in Drandakis (1963). Technically, Drandakis sufficient conditions may be related to the invertibility of the function $k(\omega)$. Economic derivation and interpretation of these conditions are possible, in terms of demand and supply responses. To this purpose we will derive "the world transformation surface".

Since capital is perfectly mobile internationally, and production functions are identical, it is always possible to assume that each country diversifies in production, and hence the international

economy behaves as a closed economy. We can then derive the world's transformation surface relating output per head of good 2 (x_2) to output per head of good 1 (x_1) and the world's stock of capital per head, k . Thus we wish to characterize the function $x_2 = g(k, x_1)$.

2. THE WORLD TRANSFORMATION SURFACE WHEN CAPITAL IS MOBILE :

Under our assumptions, the world transformation surface is the same as that of a two-sector closed economy. This we will derive below.

Let k_i be the capital/labour ratio in sector : ($k_i = K_i/L_i$), and $y_i = X_i/L_i$. y_i is related to k_i by

$$y_i = f_i(k_i). \quad (1)$$

The marginal rate of substitution is given by :

$$\omega_i(k_i) = [f_i(k_i)/f'_i(k_i)] - k_i \quad (2)$$

It will be assumed that $\omega_i \rightarrow 0$ as $k_i \rightarrow 0$ and $\omega_i \rightarrow \infty$ as $k_i \rightarrow \infty$. These conditions are weaker than the Inada conditions. All C.E.S., including Cobb-Douglas, production functions satisfy these properties. It does not necessarily imply that $f'(0) = \infty$: consider for example, $f(k) = \log(k+1)$. Our assumption ensures that even when it is possible to produce a good using one production factor alone, it is never efficient to do so if the overall capital/labour ratio is positive and finite.

Let $\rho = L_1/L$. Consider the problem of choosing k_1, k_2 and ρ so as to :

$$\text{Maximize } x_2 = (1-\rho)f_2(k_2) \quad (3)$$

subject to :

$$\rho k_1 + (1-\rho)k_2 = k \quad (4)$$

$$\rho f_1(k_1) = x_1 \quad (5)$$

where k and x_1 are given.

From the Lagrangean :

$$L = (1-\rho)f_2(k_2) + \lambda_1[x_1 - \rho f_1(k_1)] + \lambda_2[k - \rho k_1 - (1-\rho)k_2] \quad (6)$$

The necessary conditions are (4), (5) and

$$\frac{\partial L}{\partial \rho} = -f_2 - \lambda_2 k_1 + \lambda_2 k_2 - \lambda_1 f_1 = 0 \quad (7)$$

$$\frac{\partial L}{\partial k_1} = -\lambda_2 \rho - \lambda_1 \rho f'_1 = 0 \quad (8)$$

$$\frac{\partial L}{\partial k_2} = (1-\rho)(f'_2 - \lambda_2) = 0 \quad (9)$$

Hence :

$$\lambda_2 = f'_2 \quad (10)$$

$$\lambda_1 = f'_2 / f'_1 \quad (11)$$

Substitute (10) and (11) into (7) :

$$f_2 / f'_2 - k_2 = f_1 / f'_1 - k_1 \quad (12)$$

It is a well-known result in modern optimization theory that

$\lambda_1 = \partial x_2 / \partial x_1$ ($\equiv \partial g / \partial x_1$) and $\lambda_2 = \partial x_2 / \partial k$ ($\equiv \partial g / \partial k$). Hence we obtain :

Proposition 1

$$(i) \quad g_k = f'_2 \quad (13)$$

$$(ii) \quad g_{x_1} = -f'_2/f'_1 \quad (14)$$

Result (i) is remarkable : keeping x_1 constant (but allowing ρ and k_1 to vary optimally), the effect of a marginal increase in k on x_2 is equal to the marginal product of capital in good 2.

Properties of the second derivatives of $g(k, x_1)$ are given by :

Proposition 2

$$(i) \quad g_{kk} < 0$$

$$(ii) \quad g_{x_1 x_1} \leq 0 \quad (\text{with equality only if the factor intensities are equal for both sectors - i.e. at overall capital/labour ratios where factor intensity reversals occur})$$

$$(iii) \quad g_{x_1 k} = g_{k x_1} \geq 0 \quad \text{as} \quad k_1 \geq k_2$$

$$(iv) \quad g_{kk} g_{x_1 x_1} - (g_{x_1 k})^2 = 0 \quad (\text{i.e. the determinant of the Hessian is identically zero}).$$

More precisely, let :

$$A = -\rho(f_2 f_2''/f_2'^2)(f_1' \cdot k_1 - f_1 - f_1' \cdot k_2) + (1-\rho)f_1^2 f_1''/(f_1')^2 < 0 \quad (15)$$

Then :

$$(i') \quad g_{kk} = A^{-1} f_2'' f_1^2 f_1''/(f_1')^2 \quad (16)$$

$$(ii') \quad g_{x_1 x_1} = A^{-1} (k_1 - k_2)^2 (f_2'' f_1''/f_1'^2) \quad (17)$$

$$(iii') \quad g_{x_1 k} = A^{-1} (k_1 - k_2) (-f_2'' f_1 f_1''/f_1'^2) \quad (18)$$

Before proving proposition 2, we need the following lemma, which is of interest in itself :

Lemma : Let $A < 0$ be defined by (15). The three equations (4), (5) and (12) give ρ , k_1 and k_2 as implicit functions of k and x . The partial derivatives of these implicit functions are :

$$(i) \quad \partial \rho / \partial x_1 = A^{-1} f_1 [(\rho f_2 f_2'' / f_2'^2) + (1-\rho) f_1 f_1'' / f_1'^2]$$

$$(ii) \quad \partial \rho / \partial k = - A^{-1} \rho f_1' f_2 f_2'' / (f_2')^2$$

$$(iii) \quad \partial k_1 / \partial x_1 = A^{-1} (k_1 - k_2) (-f_2 f_2'' / f_2'^2)$$

$$(iv) \quad \partial k_1 / \partial k = A^{-1} f_1 f_2 f_2'' / f_2'^2$$

$$(v) \quad \partial k_2 / \partial x_1 = A^{-1} (k_1 - k_2) (-f_1 f_1'' / f_1'^2)$$

$$(vi) \quad \partial k_2 / \partial k = A^{-1} f_1^2 f_1'' / f_1'^2$$

Proof of the Lemma :

Differentiate the system (4) (5) and (12) with respect to x_1 (taking ρ , k_1 and k_2 as functions of x_1) :

$$\begin{bmatrix} k_1 - k_2 & \rho & 1 - \rho \\ f_1 & \rho f_1' & 0 \\ 0 & f_1 f_1'' / f_1'^2 & -f_2 f_2'' / f_2'^2 \end{bmatrix} \begin{bmatrix} \partial \rho / \partial x_1 \\ \partial k_1 / \partial x_1 \\ \partial k_2 / \partial x_1 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (19)$$

(i), (iii) and (v) are obtained immediately by solving (19) using Cramer's rule. Repeat the operations, with respect to k , and (ii), (iv) and (vi) are obtained. "A" is the determinant of the matrix of coefficients in (19).

Q.E.D.

Proof of proposition 2

Using (3.13) and (3.14) :

$$g_{kk} = f_2'' \partial k_2 / \partial k \quad (20)$$

$$g_{x_1 x_1} = (-f_2''/f_1') \partial k_2 / \partial x_1 + (f_2' f_1'' / f_1'^2) \partial k_1 / \partial x_1 \quad (21)$$

$$g_{x_1 k} = g_{k x_1} = f_2'' \partial k_2 / \partial x_1 \quad (22)$$

Using results (vi), (v), (iii) of the lemma to substitute in (20) - (22), we obtain all the results of propositions 2 by direct calculation.

Q.E.D.

Note that (13) and (14) can also be obtained by direct calculation and substitution

$$\partial x_2 / \partial k = (1-\rho) f_2' \partial k_2 / \partial k - f_2 \partial \rho / \partial k = f_2'$$

$$\partial x_2 / \partial x_1 = (1-\rho) f_2' \partial k_2 / \partial x_1 - f_2 \partial \rho / \partial x_1 = -f_2' / f_1'.$$

It is also interesting to note that (18) is an alternative proof of Rybczynski's theorem.

3. THE SUFFICIENT CONDITIONS FOR CAUSALITY :

Let x_1 be the output of the *consumption goods* sector. Apart from the usual efficiency conditions subsumed in our transformation function, momentary equilibrium requires that the excess demand for investment goods be zero. The excess demand for investment goods is :

$$N(x_1) = s_r g_k \cdot k + s_w [g(k, x_1) + (-g_{x_1}) x_1 - g_k \cdot k] - g(k, x_1) \quad (23)$$

The term in the square brackets is wage income in terms of the investment goods - being the difference between the value of output and capital income, both in terms of the investment good; $(-g_{x_1})$ is the price of good 1 in terms of good 2.

A momentary equilibrium *exists* since $N(x_1)$ is continuous, with $N(0) < 0$ (investment good in excess supply if no consumption good is produced), $N(\bar{x}_1) > 0$, where $\bar{x}_1$ is defined by $g(k, \bar{x}_1) = 0$: Thus there exists some x_1 such that $N(x_1) = 0$.

The uniqueness of momentary equilibrium is ensured if $N'(x_1) > 0$ at $N(x_1) = 0$.

$$N'(x_1) = k(s_r - s_w)g_{kx_1} - s_w x_1 g_{x_1 x_1} - g_{x_1} \quad (24)$$

It is immediate that *any one* of the following conditions are sufficient for the uniqueness of momentary equilibrium:

- (i) $s_r = s_w$
- (ii) $k_1 = k_2$ (so that $g_{kx_1} = 0$)
- (iii) $k_1 > k_2$ and $s_r > s_w$, because $k_1 \geq k_2$ implies that $g_{kx_1} \geq 0$.
- (iv) $k_1 < k_2$ and $s_r < s_w$.

With some tedious manipulation, another sufficient condition can be derived:

- (v) $\sigma_1 + \sigma_2 \geq 1$, where σ_i is the elasticity of substitution in sector i .

The interpretation of condition (iii) is obvious : if the consumption good is more capital intensive ($k_1 > k_2$), an increase in the production of the consumption good (i.e. decrease in the production of the investment good) will increase capital income and reduce wage income. The fall in wage income is equal to the rise in the value of output (in terms of the investment good) *minus* the rise in capital income. The total saving (in terms of the investment good) will increase if $s_r > s_w$. Condition (iv) has a similar interpretation. Condition (i) implies that total saving is a constant fraction of total income : hence a decrease in the output of the investment good is accompanied by an increase in saving due to an increase in the value of output (total income) in terms of the investment good. Condition (ii) implies that a decrease in the output of the investment good does not affect income and hence saving. In all cases, saving does not move in the same direction as investment supply.

Thus the sufficient conditions for the uniqueness of momentary equilibrium (i.e. for causality) can be explained in terms of demand and supply responses rather than the invertibility of a certain function $k(\omega)$.

CHAPTER III

ECONOMIC THEORIES OF RESOURCE EXPLOITATION : A SURVEY

1. INTRODUCTION :

Are we consuming our resources too fast? Too wastefully? If so, what is wrong with the market mechanism in its role of allocating resources over time? How should we use our natural resources? These questions cannot be answered without reference to a code of ethics and without a proper understanding of the working of the market mechanism. While economic theory has little to say about ethical premises, it can offer considerable insights into problems such as how to attain certain desired economic objectives - given a set of ethical premises - and how to interfere with the market mechanism if its invisible hand fails to fulfil the desired objectives. Moreover, social choice cannot be made without knowing the consequences of such choice. Economists can contribute by characterizing the economic consequences of alternative courses of actions - including the consequences of laissez-faire. In particular, economic theory can describe these consequences qualitatively, while the unknown parameters - which vary with time and location - to be estimated are left to the applied economist.

The theories of resource exploitation in a market economy will be surveyed in section 2, while section 3 looks at planning models of resource uses.

2. THEORIES OF RESOURCE EXPLOITATION IN A MARKET ECONOMY :

2.1 Some Classical Contributions¹ :

To the classical economist, land for cultivation seemed to be the most important natural resource. Ricardo developed the theory of land rent and touched the subject of mines only for completeness. Malthus presented a pessimistic view of the world by emphasising the finitude of land. Malthus' dismal picture of ultimate misery is very sketchy. He was perhaps more concerned with the final stage and ignored the optimizing behaviour of economic agents. The converse applies to Ricardo, who never mentioned a final stage but was mainly concerned with the process of land use. According to Ricardo, better grades of land are used first. As demand grows, these better quality parcels of land are cultivated more intensively up to a point where it becomes more profitable to use lower grades of land rather than applying more capital and labour to the better quality parcels.

While Malthus' and Ricardo's assertions are not self-evident, they are at least very suggestive. The inadequacy of their models however points to the need of a micro-economic foundation : how do firms exploit natural resources? What are the equilibrium conditions for an extractive industry?

2.2 The Theory of the Mining Firm and the Mining Industry :

The first important contribution to the theory of the mining firm is that of Gray (1914). Using verbal arguments and numerical

¹ For an exposition of the classical theories, see Barnett and Morse (1963).

illustrations, Gray studied the optimal extraction path of a competitive mining firm facing a given price which is *constant over time*.

Assuming that the average cost of extraction $AC(E)$ is U-shaped, Gray showed that :

- i) if the rate of interest is zero, the firm will minimize its instantaneous average cost.
- ii) if the rate of interest is positive, production would be tilted towards the present.
- iii) marginal cost is always kept below price.

These results are due to the exhaustibility of the resource stock : if more is produced now, less will be available for the future. Gray contrasted his results with the Marshallian results for the non-mining competitive firm, which equates its marginal cost with price. Gray's results are illustrated diagrammatically in Scott (1967). Scott (1967) interpreted Gray's conclusions, using the concept of "user cost" - the opportunity cost of future profit foregone by additional present extraction. He also extended Gray's analysis to cover the case in which cumulative output affects cost, but no firm conclusion was reached : the complexity of the problem seems to defy intuitive arguments. In his analysis, Scott (1967) postulated a "user cost" curve. This does not seem helpful, since user cost, while implicitly present in the solution of dynamic optimization problem, usually comes out as part of the solution rather than helps the problem solving process. Gray's results can be proved quite simply by diagrams and some elementary algebra, and can be generalized to cover the case of a monopolistic mining firm. The more general result, that a mining firm will maximize its instantaneous average profit if the rate of interest is zero, is

proved diagrammatically - Appendix III.A. An interesting corollary is that a monopolist will produce at some point where its *average cost is falling*.

Gray's analysis was based on the assumption of stationary prices. It was Hotelling (1931) who clearly enunciated the condition for the equilibrium movement of "net price" : the market is in equilibrium only if the "net price" - or marginal profit - for each mining firm grows at the rate of interest. For a competitive industry, marginal profit is the market price minus the marginal extraction cost. As Solow rightly pointed out, while net price (i.e. marginal profit) grows exponentially, "the market price can fall or stay constant ... if extraction costs are falling through time" (Solow (1974b) p.3). Hotelling's analysis (1931) was, however, based on the assumption of static demand and cost curves. By writing the quantity demanded as a function of net price alone, Hotelling seemed to assume that the average extraction cost is a constant. Thus, in his model, the market price rises and the rate of production falls along the demand curve. Hotelling's model, although very simple, offers considerable insights into the problem of resource exhaustion. Quite contrary to the authors of "The Limits of Growth" (1972) who seemed to stress the suddenness with which the car of civilization hits the wall, Hotelling's model tends to support the view that "if a wall exists it will act as a brake, give signs of warning and slow the car to equilibrium" - as expressed by Scott Gordon (1973) : Presumably as the price of the resource good rises, there is more incentive to develop and use substitutes.

Hotelling's model has been explored further by Herfindhal (1967). Assuming a competitive industry with constant average cost of extraction

for any grade of a resource, Herfindhal showed that given two grades, the better quality grades will be exhausted before the other grade is exploited.² The proof is simple : Let C_a and C_b be the average (= marginal) extraction cost of the two grades a and b . Suppose the two grades were exploited simultaneously in the time interval $[0, t]$. The principle that "net price" grows at the rate of interest implies that

$$P_t = (P_0 - C_a)e^{rt} - C_a = (P_0 - C_b)e^{rt} - C_b$$

hence

$$(C_b - C_a)e^{rt} = (C_b - C_a)$$

which is impossible if r is positive. A similar proof by contradiction shows that the better grade is exhausted before the lower grade is used (See Herfindhal (1967), p.73 ; for an intuitive verbal exposition of this result, see Solow (1974b) p.4). Thus Herfindhal was able to justify the Ricardian assertion that resources are used in order of declining economic quality. Note, however, that Herfindhal's analysis is based on the constant cost assumption. In Appendix IIIB, we will show that if the marginal costs of extraction is of the form

$$MC_i(q_i) = \delta + \theta_i q_i$$

for the deposit i , and δ is the same for all i , then for any pair (k, j) , there exists some common time interval such that $q_k(t)$ and $q_j(t)$ are positive simultaneously.

² In what follows, quality differences are assumed to be *wholly* reflected in differences in extraction costs, so that the outputs are homogeneous.

It is important to note that Hotelling's equilibrium condition for the movement of net price - that marginal profit grows at the rate of interest - depends *crucially* on the assumption that the demand or cost functions are independent of the resource stock $R(t)$ or accumulated output $R_0 - R(t)$. If, for example, the cost of extraction increases as the amount of resource remaining in the ground becomes smaller, then "net price" grows at a rate exceeding the market rate of interest. More precisely, let $\psi(t)$ be the marginal profit (net price) at time t . Then it is necessary that

$$\dot{\psi}(t)/\psi(t) = i(t) + \frac{1}{\psi(t)} \partial C(E, R) / \partial R(t)$$

where $i(t)$ is the rate of interest and $\partial C / \partial R(t)$ measures the "shift" in the total cost of extracting E as the resource, R , is depleted. Solow (1974b), Nordhaus (1973) and others ignored this added complication, perhaps because they considered it to be relatively unimportant. We also note that even if extraction cost is independent of $R(t)$, Hotelling's equilibrium condition for the movement of "net price" does not say much about the movement of the market price of the resource good if the average extraction cost is not constant. Gordon (1967) developed a model of a competitive mining industry with rising average and marginal costs. He correctly pointed out that among firms with different cost functions and/or different deposits, their marginal costs will not be equalized in general. Gordon concluded that this implies social inefficiency, since - according to Gordon - a socially optimal programme would require the "equalization of marginal costs among operating mines". In Appendix IIIB we will show that Gordon's conclusion is not valid.

The studies we have surveyed so far assume that firms do not share a common deposit or reservoir. Common property, however, is a characteristic of petroleum exploitation. Under the "rule of capture", whatever a land-owner can withdraw from an underground oil pool (which transcends surface property lines) is his. Hence "if there is more than one producer from a common pool, then adjacent producers are in danger of losing all "their" oil to their neighbours" (Davidson (1963), p.94). Davidson (1963) rightly pointed out that this common property characteristic tends to induce the over-exploitation of the resource. Davidson's analysis, was, however, deficient in several respects. He arbitrarily postulated a "user cost due to the rule of capture" and two other user cost functions - "user cost inherent in all raw materials" and "user cost of ultimate recovery" - and claimed that each firm would, in each period, equate its instantaneous marginal profit with the combined (or net) marginal user cost he postulated. Using Davidson's model, Khoury (1969) deduced that "net marginal user costs under competitive extraction are likely to be negative" (Khoury (1969), p.446). As Watkins (1970) pointed out, this conclusion does not make sense, for it implies that a firm may wish to produce at a point where its marginal profit is negative. Watkins did not seem to realize that the basic error is not Khoury's deduction, but rather Davidson's arbitrary user cost. Essentially as noted above, user costs are implicit in and should be derived from the solution of an optimization problem. While the concept of user cost may be useful for an intuitive explanation of certain results, it would not help the problem solving process.

The dynamics of the mining industry also received some attention in a paper by Smith (1968). Smith assumed a differential equation describing the rate of change in the real capital stock used in the industry instead of deriving it from the optimizing behaviour of firms. Letting Π denote pure profit, Smith assumed that the flow of capital is proportional to pure profit, or

$$\dot{K} = \delta \Pi$$

where $\delta > 0$ is "a behavioural constant for the industry" (p.415). This formulation seems to oversimplify the dynamic behaviour of optimizing economic agents. Smith also assumed that firms maximize instantaneous profit. This was criticized by Cummings and Burt (1969), who argued that firms do not maximize instantaneous profit unless the mine is a common property - such as the case of ground water or petroleum. They proposed an alternative analysis, using dynamic programming. Their model is so general that no insights seem to be obtainable - apart from the familiar conclusion that marginal profit is not equated to zero, but is equated to the marginal user cost - which is a "shadow price" representing future profit foregone. The authors did not prove that the marginal user cost is positive.

Finally, we must note important contributions to the theory of exhaustible resources by Gale (1967) and Vousden (1973). Their models, though cast in a macro-economic framework, can be re-interpreted (with some adaptation) as micro-economic models. Despite this, we choose to discuss their results in the section devoted to (macro-economic) planning models.

2.3 The Micro-economic Approaches to the Theory of Exploitation of Replenishable Resources :

While land for cultivation, timber and fisheries are common examples of replenishable resources, the fisheries have been particularly singled out for economic analysis - perhaps because of common property characteristics of the fishing industry.

As Scott Gordon (1954) observed, the great bulk of research that had been done on the fishing industry (prior to Gordon's work) had been in the field of biology. It was Gordon (1954) who first introduced a systematic and in-depth economic analysis of the industry. Gordon's main conclusion was that the competitive exploitation of common fishing grounds is socially wasteful - mainly because each firm ignores the effects of its exploitation on the fish population. Gordon's analysis seemed to be concerned with the long run equilibrium, though he did not state this explicitly.

Scott (1955) criticized Gordon (1954) for his failure to consider the short-run and to contrast short-run with long-run exploitation decisions. He correctly pointed out that if a sole owner were taking over a fishing ground *for a season only*, he would operate in exactly the same way as the competitive fishermen had (assuming, of course, that he faced a given price and that there was no economies of scale nor static externality due to over-crowding). Scott also argued that if the sole owner had a permanent right to exploit the resource, he would not maximize his instantaneous profit but rather equate the marginal instantaneous profit with the "marginal user cost". While Scott's conclusions are essentially correct, his assumption that the

marginal user cost is positive is not always valid. It would seem more desirable to derive the marginal user cost from an optimization problem and to find conditions under which it is positive (and not zero).

While Scott (1955), following Gordon (1954), advocated for the sole ownership of individual fishing grounds, Turvey (1964) argued that this may not be practicable and in such cases one must be contented with second-best measures (such as the regulation of mesh size). Turvey's analysis was mainly concerned with long-run equilibrium in much the same way as those of Gordon (1954) and Scott (1955). These authors did not specify the natural growth function and thus even their long-run analyses are incomplete, their analyses being mostly verbal, supplemented by diagrams. These defects were partially rectified by Vernon Smith, (1968) and (1969).

Smith (1968) assumed that, without human exploitation, the resource mass grows according to the law :

$$\dot{R} = g(R) \quad (1)$$

where $g(R)$ has an inverted U shape.³ The exploitation is affected by "K homogeneous firms or units of capital, each producing an output rate, x " (p.411). Hence, with exploitation

$$\dot{R} = g(R) - Kx \quad (2)$$

For each firm, the cost of output x is

$$C(x, R, K) = \phi(x, R, K) + \hat{\Pi} \quad (3)$$

³ For a discussion of the growth function, see Beverton and Holt (1957).

where $\hat{\Pi}$ is the normal return on a unit of capital employed elsewhere. The industry's revenue is a function of its output

$$\rho = \rho(Kx) \quad (4)$$

Each firm takes as given price, $\rho(Kx)/Kx$, the resource stock, R , and the "size" of the industry, K , and maximizes *instantaneous* profit, so that at each instant :

$$\rho(Kx)/Kx = \partial\phi/\partial x = C_x \quad (5)$$

$K(0)$ is given, and

$$\dot{K}(t) = \alpha[\rho(Kx)/K - C] \quad (6)$$

Thus Smith (1968) formalized Gordon's model (1954) and added two transition equations for $\dot{R}(t)$ and $\dot{K}(t)$. The constant α is a "behavioural constant" (p.415). The cost function (3) reflects "crowding externalities" ($\phi_K \geq 0$) and "stock externalities" ($\phi_R \leq 0$).

Assuming that the momentary equilibrium condition (5) yields a unique x for any given pair (R, K) , the two equations (2) and (6) become an autonomous system of differential equations. Any rest point of this system is a steady state equilibrium for the competitive fishermen. Assuming that the fishing ground is small so that $\rho(Kx) = pKx$ (p constant) Smith considered the "optimal" steady state if the fishing ground is under a centralized management. Smith assumed that the central management maximizes, with respect to K, x and R , the *long-run* profit

$$\Pi = pKx - KC(x, R, K) \quad (7)$$

subject to

$$\dot{R} = g(R) - Kx = 0 \quad (8)$$

At an interior maximum of problem (7) subject to (8),

$$p = C_x + \lambda = C/x^* - K^*C_K/x^* + \lambda \quad (9)$$

where

$$\lambda = K^*C_R/g'(R^*) > 0 \quad (10)$$

and R^* is the optimal resource stock in the steady state.

Comparing (9) with (5), Smith concluded that the social optimum can be decentralized *in the steady state equilibrium* by an extraction fee $U = K^*C_R/g'(R^*)$ and an "annual" licence fee $L = K^*C_K$ on each "vessel", for then each firm (vessel) will choose x so as to maximize

$$\Pi^* = px - C(x, R, K) - L - Ux \quad (11)$$

(9) and (10) imply that $\Pi^* = 0$, hence there is no incentive for entry or exit. The maximization of (11) by each firm is consistent with a socially optimal steady state. Smith's optimum is restricted to steady state considerations. The problem of optimal resource exploitation in transient states was left unsolved. Furthermore, Smith's definition of an optimal steady state implies that society does not discount future utility.

In another paper, Smith (1969) generalized his model of commercial fishing by adding another variable (the mesh size) and modifying the transition equation for $\dot{R}(t)$ to allow more complex interaction. His analysis was however mainly restricted to the case of a quadratic natural

growth function $g(R)$. Smith (1969) compared a socially optimal steady state with competitive steady states. He showed that while they differ, there is no indication that the competitive steady states correspond to larger outputs and smaller fish populations, as asserted by Gordon (1954) (p.141), or smaller outputs and larger capital requirements, as asserted by Christy and Scott (1965) (p.9). These authors reached different conclusions not because their models are different, but rather because Gordon (1954) and Christy and Scott (1965) did not specify their models fully and their assertions are conjectural in nature.

In both his (1968) and (1969) papers, Smith did not attempt to characterize socially optimal exploitation paths for any given initial resource stock. He was concerned with steady states only. The optimal exploitation in transient states when the fishing ground is under a central management was considered in Quirk and Smith (1969) and Brown (1974). Assuming a constant price, Brown showed that when the resource stock is low, it is optimal not to exploit the resource initially. On the other hand, if the social rate of discount is sufficiently high relative to the biological growth rate, it may be optimal to exhaust the resource (Quirk and Smith (1969), pp.20-21).

While contributors to the theory of economic exploitation of common property replenishable resources stress the importance of regulation or sole ownership, it must be remembered that some resources are not scarce relative to wants and/or exploitation technology and costs. In this case, there may be no need for any social intervention in the competitive exploitation of common property resources. The possibility of artificial replenishment is also neglected in the models surveyed above.

Finally, we must note the planning models studied by Plourde (1970) and (1971) and Burt and Cummings (1970). These will be examined later.

2.4 Resource Exploitation in General Equilibrium and Optimality :

The studies surveyed so far are essentially partial equilibrium models. It must be obvious however that the familiar results concerning the existence and optimality of a competitive equilibrium also hold when resources - exhaustible and renewable - are introduced. The Arrow-Debreu (finite horizon) model of competitive equilibrium (1954) is quite general and formally allows for natural resources, when it is remembered that commodities at different dates are differentiated. Thus, for example, the initial endowment vector ω may include initial resource stocks ω_i^0 ($i = 1, 2, \dots, m$). The decision to extract q_i^1 units of resource i in period one involves the joint production of q_i^1 and of inventories $x_i^1 (= \omega_i^0 - q_i^1)$ to be left under the ground - essentially in the same way as the use of a new car produces jointly transport services and a one-year-old car, resource theory being just a special case of capital theory.

The Pareto Optimality of the Arrow-Debreu model of competitive equilibrium is however based on the assumption that all future markets exist and the preferences of generations as yet unborn are reflected in market prices. These assumptions are rather extreme. In the real world, while current decisions are based partially on expectations about future preferences and technology, it is obvious that future generations are not fully represented. This is not to suggest that for this reason the market allocation is, in some sense, inefficient or non-optimal :

one cannot blame the market for not having information which even central planners do not possess.

While the Arrow-Debreu model assumes that economic agents have perfect foresight, there is another class of model which is based on the assumption of myopia : economic agents are only guided by current prices and their expected current rates of change. This sort of process has been studied in Hahn (1966), and Shell and Stiglitz (1967) and has been shown to be non-optimal in general, since the process does not guarantee that the transversality condition at infinity (or even, eventually, feasibility conditions) would be met. An example of this process, in the resource extraction context, is provided by Solow (1974b) : price may rise according to the (correct) Hotelling rule, but the initial price may be too low, leading to exhaustion before the demand is choked off by a high price. In this case, any person perceiving this will make money by clever arbitrage.⁴

Other possible sources of market failure, especially in the context of resource exploitation, are summarized in Solow (1974b). The most obvious sources of failure are common property resource exploitation and exploration, and the monopolistic or oligopolistic element in the extractive industries. Secondly, some risks are individual risks without social counterparts : For example, the insecurity of tenure is a special form of uncertainty which is particularly relevant for the extractive industries. To these we must

⁴ Nordhaus (1973) expressed the view that very few planners, or private economic agents have the ability, or perhaps even the desire, to check consistency for several decades.

add institutional distortions such as short leases which lead to overexploitation, and the "public goods" aspect of natural resources : the resources may yield aesthetic or recreational values not appropriated by the market mechanism.

Another usual claim by "interventionists" is that private discount rates are "too high". As Gordon (1966) pointed out, two arguments are frequently advanced to support this view : Firstly, society can pool risks more effectively than individual firms, and secondly, it is "unethical" to discount future utilities. The first argument is not very convincing : the stock market and the insurance industry allow risk sharing and it is doubtful whether a central authority can do better.⁵ As to the second argument, a distinction must be made between utility discount and consumption discount : even if a society does not discount future utilities (i.e. the rate of time preference is zero), the rate of interest (in terms of the consumption good, say) is positive if future consumption is higher than present consumption⁶ (in Ramsey's model, along the path of accumulation, the rate of interest is positive - being equal to the marginal product of capital less the depreciation rate). Nordhaus (1973) noted that tax on capital income may raise the private rate of discount above the social rate.

Finally we note that many critics of the market mechanism seem to feel that, to use Solow's words, "the future may be too important to

⁵ See Houthakker's comment on Nordhaus (1973).

⁶ This point is also made by Solow (1974b).

be left to the accident of mistaken expectations and the ups and downs of the Protestant ethic" (Solow (1974b), p.9). This consideration leads us to the study of planning models of resource uses.

3. PLANNING MODELS OF RESOURCE USES :

In its simplest form, the non-replenishable resource planning problem is the problem of a "cake eater" : how to consume a cake (of a known size R_0) in such a way that total (intertemporal) utility is maximized? Gale (1967) considered this problem briefly in a discrete time model as an example of the non-existence of an optimal solution. He obtained the interesting result that if the time horizon is infinite and the utility function $U(E)$ is strictly concave, then with a zero rate of utility discount, the problem has no solution. This is because the "resource" would be spread as "thinly" as possible, due to the diminishing marginal utility hypothesis. Thus for any consumption program there is a superior consumption program which spreads the resource more thinly. The non-existence of an optimal program is not peculiar to the discrete time formulation. In fact the same conclusion is reached in a continuous time model, with an *infinite time horizon*, a *zero discount rate*, and a *strictly concave utility function*.

Vousden (1973) considered the case of a *free time horizon*. The free time horizon formulation implies that whatever happens beyond the terminal date (which is to be chosen endogenously) does not count. Obviously, in the resource extraction context, the free time horizon formulation and the infinite time horizon formulation are *equivalent* if the integrand, $U[E(t)]e^{-\delta t}$, takes a non-negative value $\bar{U}$ when

$E(t) = 0$. If $U(0) < 0$, the two formulations are *not equivalent*, since the infinite time horizon formulation then implies that if exhaustion takes place at some finite time T^* , the value of the objective integral is *reduced* as time proceeds beyond T^* , while the free time horizon formulation does not register this "disutility". This is precisely why Vousden (1973), using the free time formulation, obtained different results for different classes of *strictly concave* utility function $U(E)$: If $U(0) \geq 0$, (class I utility function), no solution exists for the zero discount case - a result which would also be obtained if the time horizon is fixed at infinity. If $U(E) > 0$ for all E exceeding some E^0 (class II utility function), $U(0) < 0$ a solution exists for the zero discount case, and the optimal extraction path is given by

$$E(t) = E^* \quad \text{for } t \leq T^* = R_0/E^*$$

where E^* is the extraction rate which maximizes average utility. Note that if the time horizon is not free but fixed at infinity, then with a zero discount rate, *no solution* exists for class II utility function, as well as for class I utility function.

One wonders which time horizon formulation is more "reasonable". If $U(0) \geq 0$, there is no need for such a choice, since both formulations yield the same solution. If $U(0) < 0$, the free time horizon formulation would appear to neglect the welfare of future generations, when the resource is exhausted.

As we have noted, the simple planning model analysed by Gale (1967) and Vousden (1973) applies to the theory of the mining firm as well provided that a modification is made : the function $U(E)$, now

interpreted as a profit function, might have a maximum at some E_M . In this case, it is easily shown that the optimal extraction rate is always less than E_M . This is because the resource commands a *positive* imputed price.

Plourde, (1970) and (1971), reported a similar result for the case of replenishable resource. He showed that the resource always commands a positive imputed price - indicating that restraint on exploitation is required. This result is however dependent on Plourde's assumptions that $U(E)$ has no maximum (1970) or that the resource stock yields positive marginal product in the exploitation activity (1971) ; in fact, a Cobb-Douglas production function was used in Plourde (1971). While Plourde's assumptions seem rather special, there are models with assumptions which seem too general. For example, Burt and Cummings (1970) did not seem to make enough restrictions on their functions and simply interpreted the Lagrange multipliers and the marginal conditions involving these multipliers. Furthermore, their assertion that either a steady state is reached or the resource is exhausted in finite time was not supported by any proof and no argument was used to rule out the possibility of exhaustion at infinity.

An important aspect of planning is the choice of a discount rate. While economists perhaps have little to say about the 'proper' discount rate, their models can instruct a great deal on the effects of discounting. Solow (1974b) reported a dramatic result in some models with resource extraction and capital accumulation : "it is possible that the optimal path with a positive discount rate should lead to consumption per head going asymptotically to zero, whereas a zero discount rate leads to perpetually rising consumption per head" (p.10).

These possibilities are not just of academic interest : if one believes that non-replenishable resources matter and that planning is essential, then "the choice of a discount rate can be pretty important and one ought not to be too casual about it" (Solow (1974b), p.10).

There are important aspects of natural resources not captured by competitive models. For example, resources may have aesthetic or recreational values which are not reflected in market prices. Vousden (1973) introduced a "conservation motive" in the social utility function in order to capture this aspect of natural resources, and showed that, under certain conditions, the resources may be conserved instead of being exhausted.

In this connection, we note that it is important to develop planning models of resource uses in a wider context. Pitchford (1973) developed a model of full optimization with respect to population, capital, and natural resources - both replenishable and non-replenishable.⁷ Optimal long run steady states⁸ were examined under various assumptions about the community's preferences. It was shown that the optimal quantities of capital and labour used in normal production activities should be less than the levels which would have been optimal had natural resources been non-depletable. The relevant marginal products are those corrected for their effects on natural resources. Optimal paths toward the long-run steady states were also considered. Pitchford showed that under certain conditions, no steady state may exist.

⁷ Pitchford's emphasis is on optimal population, however.

⁸ Non-replenishable resources are, of course, excluded from steady states considerations.

The non-existence of a steady state received a dramatic emphasis by D'Arge and Kogicu (1973) in a model of resource use and pollution. According to the authors, a doomsday must come and an "optimal" plan must involve the choice of the doomsday. Their model neglects the role of technological progress - which is a central theme of Barnett and Morse (1963) - and substitutability.⁹ Some sufficient conditions - in terms of various elasticities of substitutions - for a steady state with positive consumption were reported in Solow (1974b).

It is important to note that most optimization models of resource use need not be interpreted as models of central planning : under idealized conditions, competitive behaviour by atomistic producers and consumers may produce the same results - except in important cases of externalities, where one may expect that the market fails. Centralized decision making is likely to have externalities and imperfections of its own, however.

4. CONCLUSION :

Theoretical models of resource uses have provided us with considerable insights into the aspects of resource uses. Resource theory is in fact just a special case of the general theory of capital.

⁹ In addition to substitution, the problems of resource depletion can be alleviated by recycling. Recyclable resources have recently received attention in Weinstein and Zeckhauser (1974).

Lastly, one must mention the problem of optimal resource use under uncertainty. Kemp (1974) studied a simple "cake eating" problem, where the size of the cake is unknown. He showed that consumption need not be monotone decreasing as in the certainty case considered by Gale (1967) and Vousden (1973). Dasgupta and Heal (1974) considered the uncertainty about the date at which a substitute becomes available. They assumed that the probability of the discovery date of the substitute is exogenous : there was no "research activity" in the model.

Since depletion - rather than accumulation - is the main concern, the problem of "intergenerational equity" - to use a term by Solow (1974a) - is more pronounced.

The problems of optimal resource uses have been mainly studied in the context of a closed economy. In view of the considerable amounts of international trade and investment connected with natural resources, it seems desirable to make some steps toward the studies of resource planning in an international context. Some aspects of international trade and resource extraction for a small country has been studied in Vousden (1974). Foreign borrowing and lending and direct foreign investment may also be studied in conjunction with resource extraction, for models without explicit recognition of natural resources might be thought of as based on the assumption that resources are non-depletable or in infinite supply. We shall not dwell on this now, since the reasons for this extension have been stated in Chapter I, and these matters will also come up subsequently in the Chapters which follow.

Using optimal control techniques, the approach adopted here has the advantage of being accessible to a large number of economists.

However for reasonably simple problems, dynamic programming.

Consider a fixed stock of resource, S_0 , which is accessible to only one firm. An abstract conclusion that $S_0 = S_0$ holds at the present moment and T denotes that point in time when the firm will cause its extraction. T is a decision variable for the firm and is not exogenously determined.

A version of this Appendix appeared in the *Australian Economic Papers*, December 1974.

See appendix IIIA.

APPENDIX IIIA

THE THEORY OF THE MINING FIRM AND THE OPTIMAL EXTRACTION OF A NON-RENEWABLE RESOURCE :

A DIAGRAMMATIC ANALYSIS¹

It is well-known that the traditional static theory of the firm cannot be applied without modification to the analysis of the dynamic problems of a mining firm. The concept of (marginal) "user cost" has been used in the literature² to represent the discounted 'future' marginal profit foregone by an incremental extraction now. Although this concept is useful for an intuitive understanding of the issues involved, the actual user cost itself, like many other shadow prices in optimization models, in general comes out of the solution of the problem rather than helps to obtain the solution. This paper presents a diagrammatic approach to a simple optimal extraction problem without recourse to an arbitrary construction of a user cost curve. Although our results can be obtained quite simply and definitely using optimal control techniques, the approach adopted here has the advantage of being accessible to a larger section of economists. Moreover for reasonably simple problems, diagrams help intuition.

Consider a fixed stock of resource, R_0 , which is accessible to only one firm. We assume continuous time. Let $t = 0$ denote the present moment and T denote that point in time when the firm will cease its extraction. T is a decision variable for the firm and is not exogenously determined.

¹ A version of this Appendix appeared in the Australian Economic Papers, December 1974.

² See Appendix IIIB.

Let $E(t)$ denote the rate of extraction at time t . The (instantaneous) average cost of extraction, $C(E)$ is assumed to be a function of the extraction rate alone. For simplicity we assume away the possibility of storage so that all output $E(t)$ is sold. The instantaneous demand curve $P(E)$ facing the firm is assumed to be the same over time. The instantaneous average profit curve is obtained by subtracting the instantaneous average cost curve $C(E)$ from the instantaneous average revenue curve (i.e. demand curve) $P(E)$. When $P(E)$ is downward sloping we call the firm a monopolist and when $P(E)$ is horizontal we call it a competitive firm. In either case, we assume that the instantaneous average profit curve, denoted by $Y(E)/E$, increases with E initially, reaches a maximum at E^* and then decreases. In order to avoid the trivial case when extraction is uneconomic, we assume that $Y(E)/E$ is non-negative for some E . This is depicted in figure 1.

From the familiar average-marginal relationship, the instantaneous marginal profit curve $Y'(E)$ crosses the average profit curve from above at E^* and declines to zero at E_m , the point where the instantaneous profit is maximized.

The firm's objective is to maximize its (properly discounted) profit *overtime*, and not its instantaneous profit. Since cost and demand conditions are assumed to be the same over time, it is obvious that if the firm's rate of discount is zero, its optimal extraction rate is a *constant* over time until time T , for there is no reason for it to be otherwise. For simplicity we shall assume that the firm's rate of discount is zero, and note that a positive discount rate only tilts the optimal path of extraction toward the present.

Figure 1

Figure 2

We shall consider two cases : (i) There is no constraint on T (the point of time when the firm will cease its extraction) and (ii) T is constrained not to exceed some $\bar{T}$ which is exogenously determined.

Case 1 : T is unconstrained.

Given our assumptions of continuous time and of stationary demand and cost conditions it is easily seen that the resource will be exhausted in this case. In order to prove this, suppose the firm ceases its extraction at time T and leaves some residual stock of resource R_f . Noting that the optimal rate of extraction is a constant, the firm's total extraction is³ :

$$E.T = R_o - R_f < R_o \quad (1)$$

and hence its profit over time is :

$$V = Y(E).T \quad (2)$$

But from (1), T can be increased, keeping E constant and hence a larger profit over time is possible. Therefore it is optimal to exhaust the resource. In other words when E and T are optimally chosen we have :

$$E.T = R_o \quad (3)$$

Our problem is to determine E (and hence T). Consider figure 2, which is a reproduction of figure 1, with an additional axis to measure time.

³ By our convention of denoting the present time by $t = 0$, T is also the time *interval* during which the firm operates.

The graph of $E.T = R_0$ is a rectangular hyperbola on the plane TOE. For any constant rate of extraction E' , say, this curve determines the time T' it takes to exhaust the resource. The areas of rectangles such as $E'M'T'0$ and $E^*M^*T^*0$ are equal to R_0 , the resource stock. For any constant rate of extraction E , the total profit over time is, from (2) and (3) :

$$V = Y(E).T = \frac{Y(E)}{E} \cdot R_0 \quad (4)$$

which is the volume of a box with the area of its base equal to R_0 and its height equal to $Y(E)/E$. Among boxes with equal areas of the base ($= R_0$), the one with maximum height must have the largest volume. The biggest box is clearly the one with base $E^*M^*T^*0$ and height $OH^* = Y(E^*)/E^*$. In other words the rate of extraction which maximizes profit over time is E^* (that rate which maximizes instantaneous *average* profit). This result is intuitively obvious : since the resource stock is fixed it is optimal to make the most profit out of each unit of the resource good.

We summarize our results in the following theorem :

Theorem 1 : Given our assumptions, it is optimal to exhaust the resource. The optimal rate of extraction is a constant, E^* . It is that rate which maximizes instantaneous average profit.

Corollary 1 : The optimal rate of extraction is less than the rate which maximizes instantaneous profit (except when the maximum profit is zero, in which case $E^* = E_m$). See figure 1.

Corollary 2 : A monopolist mining firm will produce at a point where its average cost is falling.

Proof : At the point where instantaneous average profit is maximized, the slope of the (instantaneous) average cost curve must equal the slope of the (instantaneous) average revenue (i.e. demand) curve, which is negatively sloped by assumption.

Corollary 3 : A competitive mining firm (i.e. its demand curve is horizontal) will produce at the point where its instantaneous average cost is at its minimum.

The last result is due to Gray (1914), and is a trivial corollary of our theorem 1.

We have so far assumed that the instantaneous average profit curve has a maximum at some point E^* which is positive. When this is not true, for example when the instantaneous average profit *always* declines as E increases, there exists *no* optimal extraction rate⁴ : For any constant positive extraction rate, a smaller extraction rate will increase the profit over time, but zero extraction is non-optimal. This non-existence of an optimal extraction path is not present when the firm's rate of discount is positive, in which case it can be seen intuitively (and can be shown mathematically) that the firm will start at some extraction rate E' and reduces it toward zero as the resource declines. Even when the rate of discount is zero, this non-existence of an optimal rate is not present if one assumes further than the extraction rate must be not less than a certain positive rate $\bar{E}$ (in which case of course $\bar{E}$ is the optimal extraction rate).

⁴ For essentially the same results, see Gale (1967) and Vousden (1973).

Case 2 : Constrained time horizon $T \leq \bar{T}$.

If the firm's decision variable T is constrained to be not greater than a certain $\bar{T}$, its optimal extraction rate will in general be different from E^* . We shall consider three subcases :

(a) $\bar{T} \geq T^*$ where T^* is the optimal exhaustion time when T is unconstrained.

(b) $\bar{T} < T^*$ and $\bar{T} \geq T_m$, where T_m is the time it takes to exhaust the resource if the constant extraction rate is E_m (the rate which maximizes instantaneous profit), and

(c) $\bar{T} < T^*$ and $\bar{T} \leq T_m$.

Case (a) is trivial. E^* is the optimal extraction rate and T^* is the exhaustion time as before, for the constraint on T is not 'tight'.

Case (b) implies that at the constant extraction rate E_m the resource will be exhausted before the time $\bar{T}$. We shall show that it is optimal to exhaust the resource in this case : For any time path of extraction which does *not* exhaust the resource by the time $\bar{T}$, it follows from the above observation that in some time interval the extraction rate is less than E_m . By increasing the rate of extraction toward E_m , more profit will be earned. This shows that the non-exhaustion policy is non-optimal.

We now show that, for case (b), the optimal exhaustion time is $\bar{T}$ itself. In other words, the optimal extraction rate is $\bar{E}$, where $\bar{E} \cdot \bar{T} = R_0$. This can be seen by noting that any constant extraction rate less than $\bar{E}$ will not exhaust the resource by the time $\bar{T}$, and hence is not optimal. On the other hand, any constant

extraction rate greater than $\bar{E}$ will generate a 'profit box' which is smaller than the 'profit box' generated by $\bar{E}$ and $\bar{T}$, for the areas of the bases of these two boxes are the same ($= R_0$), but the former box has height $\Pi(E)/E < \Pi(\bar{E})/\bar{E}$ as $E > \bar{E}$ ($> E^*$, by definition).

For case (c) we shall show that *instantaneous* profit maximization is optimal. We first show that except when $\bar{T} = T_m$ it is *non-optimal* to exhaust the resource. For resource exhaustion implies that the extraction rate exceeds E_m for some time interval (as $R_0 = E_m T_m \geq E_m \bar{T}$ by definition), and total profit over time can be increased by reducing the extraction rate in this time interval to E_m . This argument also establishes the non-optimality of any extraction rate exceeding E_m . On the other hand any extraction rate below E_m is non-optimal, for over-all profit can be increased by increasing the extraction rate to E_m (this does not reduce the profit in any period). Thus for 'sufficiently short' planning horizon (i.e. $\bar{T} < T_m$), instantaneous profit maximization is optimal.

We have been able to prove some intuitive results concerning the optimal extraction program for a mining firm. Rather restrictive assumptions were made so that the problem is manageable using diagrammatic techniques alone. Various extensions can be made at the cost of mathematical complexity. It is hoped that this Appendix serves as an intuitive approach to the vastly complicated problems of resource exploitation.

APPENDIX IIIB

PRIVATE VERSUS SOCIAL MANAGEMENT OF THE EXTRACTIVE INDUSTRY : THE MULTIPLE DEPOSITS CASE

1. INTRODUCTION

It is well known that a market economy may fail to achieve a social optimum. Considerable works have been done to identify various sources of market failure. For the extractive industry, several sources of failure have been identified. Spatially common property resources were considered by H. Gordon (1954), Scott (1955), Davidson (1963), Smith (1963) and others. Long (1974a) showed that short-leased mines are examples of "dynamic common properties" : if a mining firm is faced with *fixed* time horizon which is sufficiently short, it will maximize *instantaneous* profit, thereby causing a divergence between social and private optimal paths.¹ Private risks which have no social counterparts - such as the risk of nationalization considered in Long (1974f) - constitute another source of divergence.

R.L. Gordon (1968) believed that there is another source of market failure in the extractive industry. According to him, "the socialist managers can be expected to minimize production cost in a given period by equating marginal production costs among operating mines... Nothing will insure that similar equalities prevail among independently operated mines". (p.282)

¹ Under certain plausible assumptions, an optimizing mining firm will not maximize its instantaneous profits if the time horizon is free. See Gray (1914) Scott (1967) and Appendix IIIA. There are, of course, instances where instantaneous profit maximization is optimal. For example, if a superior substitute is expected to be available in the near future, there is no reason to conserve the resources.

We will show that Gordon's presumption expressed in the first statement quoted above is incorrect. Using Gordon's model, we will show that the equalization of marginal production costs among operating mines can violate the necessary conditions for a *social* optimum. An intuitive explanation of our result is that each mine has an imputed user cost, and optimality requires the equalization of the sums of marginal production cost and marginal user cost, not the equalization of marginal production costs. Thus Gordon's suspicion of market failure is unwarranted.

2. GORDON'S MODEL RE-EXAMINED

2.1 The Assumptions :

Gordon (1968) assumed that there are a number of mines, say $n \geq 2$, with initial deposits K_i^0 ($i = 1, 2, \dots, n$). Let $q_i(t)$ denote the rate of extraction at the i th mine at time t . The extraction is subject to :

$$q_i(t) \geq 0 \quad (1)$$

$$\int_{t_1}^{t_2} q_i(t) dt \leq K_i^0 \quad (2)$$

The extraction cost is a function of q_i alone. The (total) cost function is quadratic :

$$C_i(q_i) = \delta q_i + \theta_i q_i^2 \quad (3)$$

$\delta > 0$, $\theta_i > 0$. The marginal cost is :

$$MC_i(q_i) = \delta + \theta_i q_i \quad (4)$$

Note that the constant δ in (3) and (4) has no subscript, since Gordon assumed "the equality of minimum average costs among deposits". (p.284)

Gordon (1968) asserted that "in the case of social management, production costs at any point of time will be minimized so that the social cost curve is generated exactly as is a supply curve in conventional competitive theory". (p.284) Thus Gordon took it as self-evident that the socialist managers should be "equating marginal production costs among operating mines". (p.282) This is where Gordon's analysis went wrong. In what follows we will show that if $\theta_i K_i^0 \neq \theta_j K_j^0$ then along a *socially optimal* extraction path, marginal production costs among operating mines are never equalized.

2.2 The Social Objective :

Gordon postulated a demand function which is independent of time :

$$P(t) = \lambda - \alpha q(t) \quad (5)$$

where $\alpha \geq 0$

$$\lambda > \delta > 0 \quad (6)$$

$$q(t) = \sum q_i(t) \quad (7)$$

Gordon did not specify the social objective explicitly, but it is apparent that, following Hotelling (1931), he wished to maximize the discounted stream of social benefit minus extraction costs, where the benefit is defined as the area under the demand curve :

$$B[q(t)] = \int_0^{q(t)} P(q') dq' \quad (8)$$

$$= \lambda q(t) - \alpha [q(t)]^2/2 \quad (9)$$

so that :

$$B'[0] = \lambda > \delta \quad (10)$$

(10) insures that *all deposits will be eventually exhausted along a socially optimal path.*

For those who do not like the idea of "the area under the demand curve", we note that our analysis which follows is in fact independent of the form of the benefit function. Any other function $B[q(t)]$ with $B'[0] > \delta$ will suffice.

To summarize, the social objective is to find functions $q_i(t)$ ($i = 1, 2, \dots, n$) so as to maximize

$$\int_{t_1}^{t_2} S[q_1(t), \dots, q_n(t)] e^{-rt} dt \quad (11)$$

where :

$$\begin{aligned} S \equiv & \lambda \sum q_i(t) - \alpha [\sum q_i(t)]^2 / 2 \\ & - \sum [\delta q_i(t) + \theta_i q_i(t)^2] \end{aligned} \quad (12)$$

subject to (1) and (2). Note that we can set $t_1 = 0$ and that t_2 is *not fixed*.

2.3 Analysis :

Our problem is a special case of a more general problem for which the necessary conditions are given by Hestenes' theorem 11.1 (1966). We will make use of the following necessary conditions :

There exist *constant* multipliers $\beta_0 \geq 0, \mu_i \geq 0$ ($i = 1, 2, \dots, n$) and multipliers $\gamma_i(t)$ ($i = 1, 2, \dots, n$) such that :

i) $\gamma_i(t)$ is piecewise continuous, $\gamma_i(t) \geq 0$ and $\gamma_i(t) = 0$ if $q_i(t) > 0$.

ii) $\mu_i \geq 0$, and $\mu_i = 0$ if the integral constraint (2) holds with strict inequality for K_i^0 .

iii) not all multipliers vanish simultaneously.

iv) the Hamiltonian function

$$H = \beta_0 S[q_1(t), \dots, q_n(t)] e^{-rt} - \sum \mu_i q_i(t) + \sum \gamma_i(t) q_i(t)$$

is a continuous function of time, and for $j = 1, 2, \dots, n$:

$$\partial H / \partial q_j(t) = \beta_0 e^{-rt} [\lambda - \alpha \sum q_i(t) - \delta - \theta_j q_j(t)] - \mu_j + \gamma_j(t) = 0 \quad (13)$$

We note that $\beta_0 \neq 0$. For if $\beta_0 = 0$, then (13) implies that $\mu_j = 0$, because $\gamma_j(t) = 0$ for at least some t ($q_j(t) > 0$ for some t). But $\beta_0 = 0$ and $\mu_j = 0$ implies, by (13), that $\gamma_j(t) = 0$ for all t . This applies to all j . Hence condition (iii) is violated.

Since we wish to know whether marginal production costs are equalized among operating mines, we must be sure that there exist at least two mines with indexes k, j ($k \neq j$) such that $q_k(t) > 0$ and $q_j(t) > 0$ for some common time interval. We can prove the following result which is stronger than we need: For any pair (k, j) , there exists some common time interval such that $q_k(t) > 0$ and $q_j(t) > 0$.

Proof: From (13), for all t ,

$$\begin{aligned} \partial H / \partial q_k(t) - \partial H / \partial q_j(t) &= \beta_0 e^{-rt} [\theta_j q_j(t) - \theta_k q_k(t)] \\ &+ [\mu_j - \mu_k] + [\gamma_k(t) - \gamma_j(t)] = 0 \end{aligned} \quad (14)$$

If there is no common time interval such that $q_k(t) \cdot q_j(t) > 0$, then there exist t' and t'' such that $q_k(t') > 0$, $q_j(t') = 0$, $q_k(t'') = 0$ and $q_j(t'') > 0$. Consequently $\theta_j q_j(t') - \theta_k q_k(t') < 0$ and $[\gamma_k(t') - \gamma_j(t')] \leq 0$. Hence, by (14), $\mu_j - \mu_k > 0$. But $\theta_j q_j(t'') - \theta_k q_k(t'') > 0$ and $[\gamma_k(t'') - \gamma_j(t'')] \geq 0$, implying $\mu_j - \mu_k < 0$. Hence we obtain a contradiction, μ_j and μ_k being constants.

Q.E.D.

We are now in the position to prove our assertion : For any pair (k, j) such that $\theta_k K_k^0 \neq \theta_j K_j^0$, then for any t^* such that $q_k(t^*) > 0$ and $q_j(t^*) > 0$, their marginal production costs are *not* equalized at t^* .

Proof : The proof will be established by contradiction.

Suppose the marginal costs are equalized at t^* . Then :

$$\theta_j q_j(t^*) - \theta_k q_k(t^*) = 0 \quad (15)$$

Now $q_k(t^*)$ and $q_j(t^*)$ are positive, hence $\gamma_k(t^*) = \gamma_j(t^*) = 0$.

Thus, by (14),

$$\mu_j = \mu_k \quad (16)$$

(16) implies that for all t , $q_k(t) > 0$ if and only if $q_j(t) > 0$.

For suppose $q_k(t) > 0$ while $q_j(t) = 0$. Then $\gamma_k(t) = 0$ and $\gamma_j(t) \geq 0$, thus violating (14), in view of (16).

(14), (16) and the result that $q_k(t) > 0$ if and only if $q_j(t) > 0$ imply that (15) holds for all t .

Since we are considering the case where $\theta_k K_k^0 \neq \theta_j K_j^0$, we may assume, without loss of generality, that $\theta_k K_k^0 < \theta_j K_j^0$. We then

obtain the result that if the k th deposit is eventually exhausted, then the j th deposit is not, for :

$$\begin{aligned} \int_{t_1}^{t_2} q_j(t) dt &= \int_{t_1}^{t_2} q_k(t) (\theta_k / \theta_j) dt \\ &= (\theta_k / \theta_j) K_k^0 < K_j^0 \end{aligned} \quad (17)$$

Thus, the equalization of marginal production costs leads us, through the necessary conditions, to the sub-optimal result that some resource is not exhausted, and hence contradicts the hypothesis that the path considered is optimal.

Q.E.D.

2.4 Some heuristic explanations :

We have shown that marginal production costs are, in general, not equalized among operating mines. If we define $\mu_j e^{rt}$ as the (marginal) user cost of deposit j at time t , then (13) says that the sums of marginal production cost and marginal user cost are equalized among operating mines. Marginal user cost measures the future "profit" foregone by a marginal increase in extraction now. This imputed cost, however, cannot be derived before the complete solution is obtained.

Our conclusion that marginal production costs are not equalized if $\theta_k K_k^0 \neq \theta_j K_j^0$ implies that even for deposits with *identical cost functions* ($\theta_k = \theta_j$), the equalization of marginal cost does not occur if the deposits are of different sizes. Does this contradict the efficiency condition, for static models, that the cost of producing any given output is minimized? The answer is no. The extraction rates $q_i(t)$ not only determine total output $q(t)$ but

also the *life* of each "deposit". Suppose there are two separate petroleum reservoirs, each with a pump permanently installed and no other pump can be added. Suppose the reservoirs are of different sizes, but the pumps are identical in the sense that they require identical operating cost for a given rate of extraction. The equalization of marginal production costs may imply that the economic life of one pump is terminated too soon. If, however, the two pumps share a *common* reservoir, optimality naturally requires the equalization of marginal costs.

2.5 Decentralized production :

It is obvious that the socially optimal extraction path for the problem considered can be achieved by a competitive mechanism. Let $q_i^*(t)$ be the optimal social choice ($i = 1, 2, \dots, n$). The planning board can set the price in each period as

$$P^*(t) = \lambda - \alpha \sum q_i^*(t)$$

Each individual producer then takes $P^*(t)$ as given, and maximizes the discounted stream of profit. For the j th firm, the necessary condition at each t is identical to (13), with $\lambda - \alpha \sum q_i^*(t)$ given ($= P^*(t)$). For the j th firm, the discounted marginal profits for all operating periods are the same, being equal to μ_j . Note that these are different for different firms.

3. FINAL REMARKS

Our conclusions remain unchanged for more general costs and benefit functions. Gordon's model was used only for ease of comparison. This Appendix serves as a warning against drawing conclusions for dynamic models using results in static theories.

APPENDIX IIIC

USER COST AND ITS APPLICATION TO RESOURCE EXTRACTION PROBLEMS.

1. INTRODUCTION :

The concept of user cost, attributed to Keynes (1944), and described by Scott (1953) as "widely known, little understood and almost never used" (p.368), has recently gained some attention in several studies of resource extraction problems.¹ Broadly speaking, it is used to represent the effects of present activities on future returns. Keynes introduced the concept primarily for his definition of income which he thought should be net of depreciation due to use. As a digression, Keynes also discussed the relevance of the user cost concept to intertemporal optimization problems and hence to the determination of short and long run supply prices. It is the optimization aspect of the user cost concept that has raised interest and disagreements among some economists.

The purposes of this Appendix are :

- (i) to determine the role of the user cost concept in intertemporal optimization problems and its implicit place in modern optimization techniques, and
- (ii) to examine some uses and misuses of the concept, with particular references to resource extraction problems.

¹ See, for example, Davidson (1963), Scott (1967), Khoury (1969) and Watkins (1970).

2. USER COST AND INTERTEMPORAL OPTIMIZATION TECHNIQUES :

Consider the following discrete time, intertemporal optimization problem : An economic agent (e.g. a firm) is endowed with a stock of assets A_0 at time $t = 0$. A_0 may be a scalar, representing a single asset, or a vector, representing a bundle of assets, in which case $A_0 = (A_0^1, A_0^2, \dots, A_0^n)$. In any period, the economic agent can engage in a number of activities e.g. hiring labour, purchasing materials and equipments etc. These activities can be represented by a vector of "control variables" $q_t = (q_t^1, q_t^2, \dots, q_t^h)$. The stock of assets is transformed as a result of these activities and the mere passage of time :

$$A_{t+1} = F(q_t, A_t, t) \quad (1)$$

A return is earned in each period as a result of the agent's economic activities. The return function is denoted by $Y(q_t, A_t, t)$. The economic agent wishes to maximize the present value of the stream of returns :

$$\underset{q_t}{\text{Maximize } W} = \sum_{t=0}^T Y(q_t, A_t, t) \quad (2)$$

subject to (1) and possibly other constraints. Note that the discount factor appears implicitly in the return function Y . A scrap value function $S(A_T, T)$ may also be added and the time horizon may be extended to infinity, provided that the sum converges.

It is obvious that the optimal q_0 cannot be determined without knowing how A_1 (which is a function of q_0) will be optimally used for all relevant values of A_1 . Suppose that it is possible to compute

the maximum present value of the return stream realizable from time $t = t_1$, given any relevant value of A_{t_1} . Denote this "value function" by

$$V(A_{t_1}, t_1) \equiv \text{maximum} \sum_{t=t_1}^T Y(q_t, A_t, t) \quad (3)$$

If the return is monetary and if there are perfectly competitive markets for all goods, the market value of A_t will be $V(A_t, t)$.

Suppose that the economic agent knows $V(A_1, 1)$. He can then determine the optimal q_0 by choosing q_0 so as to :

$$\text{Maximize } Y(q_0, A_0, 0) + V[F(q_0, A_0, 0), 1] \quad (4)$$

Expression (4) of course corresponds to the final stage of the "backward recursion" method² of solution for a dynamic programming problem. In principle, $V(A_1, 1)$ can be obtained by proceeding step by step from the easily calculable function $V(A_T, T)$.

Although the solution of problem (2) can be obtained without the explicit use of the user cost concept, it will be clear that the concept is present implicitly. Let us rewrite (4) as :

$$\text{Maximize}_{q_0} Y(q_0, A_0, 0) - \{V[F(0, A_0, 0), 1] - V[F(q_0, A_0, 0), 1]\} \quad (5)$$

(5) is a measure of "income" in period 0.

Problems (4) and (5) are equivalent since they differ only by a constant (a term not involving the control variable q_0). The expression in the curly brackets is a possible definition of the *user cost* function of the current activity q_0 . It is the difference between

² See, for example, Arrow and Kurz (1970) and Intrilligator (1971).

the value of the assets at the end of period 0 had the economic agent refrained from engaging in activity q_0 , and the actual value of the assets if activity q_0 is carried out. From the point of view of intertemporal optimization (as distinct from income definition) it is *immaterial* whether the first term in our definition of the user cost function is modified to allow for maintenance cost etc.³ The economic agent's behaviour (e.g. his demand and supply responses) is independent of any such modification.

Given that there is no need for an explicit use of a user cost concept in solving intertemporal optimization problems and that the introduction of the concept does not make the solution any easier to obtain, the only justification for such a concept is that, apart from its use in the definition of income for any given period, it can be a useful heuristic expositional device. Using this device, one can explain for example, how the optimal output may depart from "myopic" output. For instance, Keynes (1944) argued that it is illegitimate, in general, to "equate the short-period supply price to the marginal factor cost alone" (p.67). Consider a special case of (4). Let q_0, A_0 be scalars representing output and capital. Assume that :

$$Y = pq_0 - wC(q_0, A_0) \quad (6)$$

where p, w are given price and wage rate, and C is the labour requirement function. An interior maximum of (4) is characterized by

³ Keynes (1944), Lerner (1943), Lutz and Lutz (1951) and Scott (1953) present arguments on what should and what should not be included in user cost. From the optimization point of view, this is immaterial as long as the first term of our user cost, however modified, does not contain q_0 as a variable.

$$p = w\partial C/\partial q_0 + (-\partial V/\partial A_1) \cdot (\partial A_1/\partial q_0) \quad (7)$$

Thus, price is equal to the "sum of the marginal factor cost and the marginal user cost" (Keynes (1944), p.67).

It should be noted that the value function V , and hence the user cost function (defined by (5) above), cannot be arbitrarily specified using intuitive reasoning. The function V , by definition, is the outcome of an optimization process. The sign of its partial derivatives, for example, cannot be assumed. In order to obtain properties of V , the complete model has to be specified. As will be pointed out, errors of these kinds have been made, giving rise to incorrect conclusions.

While the user cost function can be easily seen as being implicitly present in a discrete time model, this may not seem so obvious in a continuous time model since more technical arguments need to be used for this case. Let $A(t)$, $q(t)$ be the stock of assets and the control variables as before. Assets are transformed by the transition equations :

$$\dot{A}^j(t) = dA^j(t)/dt = \phi^j[q(t), A(t), t] \quad j = 1, 2, \dots, n \quad (8)$$

The objective function is :

$$\text{Maximize } \int_0^T Y(q, A, t) dt \quad (9)$$

subject to (8) and possibly other constraints. If an optimal path exists then at each point of time $q(t)$ must maximize the Hamiltonian function :

$$H \equiv Y[q(t), A(t), t] + \psi(t) \dot{A}(t) \quad (10)$$

where $\psi(t)$ is a vector of "shadow prices". The shadow prices associated with the optimal path of course come out as part of the solution of the problem (9). Assuming differentiability, the maximization of H gives

$$\partial Y / \partial q^i(t) = - \sum_j [\psi^j(t) \partial \dot{A}^j(t) / \partial q^i(t)] \quad (11)$$

Define, analogously to (3), the "value function" :

$$V[A(t_1), t_1] = \text{maximum} \int_{t_1}^T Y(q, A, t) dt \quad (12)$$

It can be shown⁴ that the shadow prices are marginal values of assets

$$\psi^j(t) = \partial V / \partial A^j(t) \quad (13)$$

The right-hand-side of (11) may therefore be interpreted as "marginal user cost".

3. APPLICATIONS TO RESOURCE EXTRACTION PROBLEMS :

We have shown that user cost or marginal user cost is implicitly present in intertemporal optimization problems since the effects of current activities on future returns must be taken into account. The necessity of allowing for user cost is obvious. The concept, however, must be used with caution. It is useful as an interpretative tool but may be misleading as an analytical tool because it is quite often forgotten that user cost is not exogenously given but is a result of an optimization process.

⁴ See, for example, Arrow and Kurz (1970), chapter 2, Dorfman (1969) and Intrilligator (1971), chapter 14.

3.1 User Cost in a Simple Model of Resource Extraction :

Consider the simple model studied in Appendix IIIA. It can be formulated mathematically as

$$\text{Maximize } \int_0^T Y[E(t)] dt$$

subject to :

$$\dot{R}(t) = -E(t)$$

$$E(t) \geq 0$$

$$R(0) = R_0$$

$$R(T) \geq 0$$

Assuming that the average profit function $Y(E)/E$ has a maximum at some $E^* > 0$, the general proof of section 4 shows that it is optimal to extract at E^* until exhaustion, and

$$Y'(E^*) = \psi > 0 \quad (14)$$

where the shadow price ψ has the interpretation of "marginal user cost". In this context, user cost is "the opportunity cost over time of using raw material" (Scott (1967), p.35).

It has been sometimes believed that marginal user cost of raw material is zero if current price is expected to continue (Davidson (1963), p.91 and 95). Our analysis shows that this is wrong. The error was made because Davidson simply postulated a user cost function without making sure that it is the outcome of an optimization process.

3.2 User Cost when Cumulative Output Affects Production :

"Even when reserves are homogeneous the remaining material will be more remote from the shaft and mill. Thus costs will rise" (Scott (1967), p.41). The instantaneous profit function should now be written as

$$Y(E,R) = P(E).E - C(E,R)$$

R is the remaining material. The effects of cumulative production just described imply $\partial C/\partial R < 0$. The cost function and hence the profit function have R as a parameter. As R falls, the cost function "shifts up" and hence the profit function "shifts down" — by this we simply mean that $Y(E,R_1) > Y(E,R_2)$ if $R_2 < R_1$. It is interesting to compare the optimal time path of extraction when cumulative production affects costs with the optimal path when costs are not affected by cumulative production. For a meaningful comparison, we assume that at zero cumulative production, extraction costs are the same for both cases :

$$C(E,R_0) = C^0(E)$$

where $C^0(E)$ is the cost function for the case of "no depletion effect" (i.e. cumulative production does not raise cost).

Assume that the rate of discount is zero. We know that the optimal extraction path for the "no depletion effect" case is $E(t) = E^*$ where E^* maximizes average instantaneous profit $[P(E)E - C^0(E)]/E$ — assuming, of course, that average instantaneous profit has a maximum. Does the optimal extraction path for the case with "depletion effect" lie above the path $E(t) = E^*$ for some initial time interval? Scott

(1967) argues convincingly that this is not true in general : "It is sometimes argued ... that expected deterioration will occur in a future period, therefore user costs are less than if there were no expected deterioration, therefore it pays to increase the present rate of output. The argument is clearly wrong : the manager cannot prevent the last units extracted from having higher costs by anticipating the date of their removal. This error results from forgetting that the rise in costs experienced through time is an invariable function of cumulated production but not of time" (p.41). But Scott goes on to argue that "the deterioration probably leads to a stretching-out of the removal process in excess of the time that would be required if the same amount of material were available under constant conditions" (p.42). This is *not true* in general. In fact Section 4 shows that the optimal extraction rate under depletion effect may even *exceed* E^* , thereby shortening the life of the mine, or it may be *identical* to E^* until extraction ceases, or it may be below E^* for all time t — depending on how the cost function is shifted up by cumulative production. Briefly, for the zero discount case, we can show that for all time t , average instantaneous profit is maximized along the optimal path. That is $E(t)$ is chosen so as to maximize :

$$Y(E,R)/E = P(E) - [C(E,R)/E]$$

If $P(E) = P$ (the firm is a price taker), the optimal $E(t)$ minimizes average instantaneous cost. While total (and hence average) instantaneous cost curves "shift up" with cumulative production, the rate of extraction which minimizes average instantaneous cost may shift to the left or to the right, or may remain stationary - this last case occurs if $C(E,R)$ is multiplicatively separable.

3.3 User Cost when Extraction Rates Affect Ultimate Recovery :

If the resource in question is crude oil, then "given the technology and the costs of production, only a fraction of the total volume of crude oil trapped in an underground pool can be brought to the surface during the productive life of the reservoir. Moreover ... the quantity of crude oil that can ultimately be recovered from the pool will depend on the rate at which the oil is brought to the surface. In general after some point, the faster the rate of production, the lower the volume of crude oil that can ultimately be recovered" (Davidson (1963), pp.91-92). The geologist has therefore developed the concept of the Maximum Efficient Rate of Production (MER) which is a rate of output which will maximize the ultimate recovery from the pool. Davidson pointed out that the economist's optimal rate of production in general differs from MER. The presence of MER however does affect the optimal rate of production $E(t)$. Davidson postulated a "user cost of ultimate recovery" function :

$$U_u = \gamma[S_0 - S_r(E)] \quad (15)$$

where γ is a constant representing "the discounted net value of a barrel of oil which could have been produced in the future" (p.94), S_0 is the maximum ultimate recovery if the well is always operated at MER and $S_r(E)$ is the amount that can ultimately be recovered for any given production rate E , $S_r = S_0$ if $E \leq \text{MER}$, and S_r decreases as E increases above MER.

While it is clear that (15) seems to yield a plausible result (i.e. smaller production rate due to positive "marginal user ultimate recovery"), the cost of use of the constant γ does not seem justified since Davidson's average cost curve is not horizontal.

The geological fact of a MER can be accommodated in our model. Let $R(t)$ stand for the remaining quantity of resource that can be recovered, we allow for wastage due to production in excess of MER by writing :

$$\dot{R}(t) = - E(t) - W[E(t)] \quad (16)$$

where W is the "wastage function", $W = 0$ for $E \leq \text{MER}$ and $W' > 0$ and is increasing as E exceeds MER. If the rate of discount is zero, it can be shown that the optimal extraction rate is a constant such that :

$$Y/E = Y'(1+W/E)/(1+W') \leq Y' \quad (17)$$

with strict inequality if and only if MER is *less* than E^* , the rate of extraction which maximizes average instantaneous profit (in which case it is easily seen that the optimal extraction rate is somewhere between MER and E^*). If the rate of discount is *positive*, (17) characterizes the terminal rate of extraction. Rates of extraction for less distant future "periods" must be higher so that discounted marginal profits are equalized. The marginal user cost implicit in (17) is higher than that in (14) if MER is present and is less than E^* .

3.4 User Cost and Common Property Resources :

Some resources exhibit common property characteristics. In the case of petroleum reservoir, under the "rule of capture" whatever a land-owner can withdraw from an underground oil pool (which transcends property lines) is his. Hence if there is more than one producer from a common pool, then adjacent producers are in danger of losing all "their" oil to their neighbours. The "optimal" extraction policy, from

the point of view of an individual producer is difficult to characterize for two main reasons :

- (i) "the impact of differential production rates between wells located on neighbouring tracts on the volume of oil eventually recovered from a well is a function of the ease of migration of the oil in the reservoir, which in turn is a function of the physical nature of the reservoir and well spacing pattern" (Watkins (1970), p.513),

and

- (ii) without agreements among producers, an individual producer's "optimal" extraction policy depends on what he expects others will do, but other producers' actions depend on their expectations of their rivals' actions etc.

We may, for simplicity, assume away the first source of difficulty. For example, we may assume that the *total* stock of recoverable resource is fixed (MER is infinite) and that the fluid moves "freely". The rate of change in the remaining resource stock is given by :

$$\dot{R}(t) = - \sum_{i=1}^n E_i(t) \quad (18)$$

where $E_i(t)$ is the rate of extraction by the i th producer, ($i = 1, 2, \dots, n$). If each producer simply assumes that other producers just maximize their instantaneous profits and all producers of the reservoir in question *take prices as given*, then each producer will attempt to maximize his intertemporal profit, subject to :

$$\dot{R} = - E(t) - \bar{W}(t) \quad (19)$$

where $E(t)$ is his own extraction rate and $\bar{W}(t)$ is the sum of other producers' instantaneous profit maximizing outputs. If the rate of discount is zero, the individual producer's "optimal" extraction rate is characterized by⁵ :

$$Y(E) = Y' \cdot [E + \bar{W}] > Y'(E) \cdot E \quad (20)$$

so that $E(t)$ exceeds the rate of extraction which maximizes his average instantaneous profit, E^* . If $\bar{W}$ becomes very large, Y' approaches zero, implying that he maximizes his (total) instantaneous profit. Our analysis seems to justify the commonly made assumption in the theory of common property resources : If economic agents are atomistic competitors, they will maximize instantaneous returns. Fiscal intervention may be necessary. Suppose that a small country, faced with given prices for its resource goods, possesses a stock of resource R_0 which is exploited by atomistic competitive firms that equate their marginal instantaneous profit to zero. The social optimum is however achieved only by equating marginal profit to the shadow price in (14) - the marginal user cost of the resource. The optimal tax per unit of extraction, i.e. the *optimal royalty*, is therefore equal to the *marginal user cost* of the resource. Note that the optimal royalty here is not a mean for confiscating pure profits.

Let us return to our analysis of an individual producer's "optimal" extraction policy. We did not use any "user cost curve", but the user cost is of course implicit in the solution. If other producers do not exist, the optimal extraction rate for our producer is E^* (at which marginal instantaneous profit equals average instantaneous

⁵ See Section 4.

profit). Given that other producers also exploit the resource, our producer now extracts more - indicating that the (implicit) marginal user cost is "shifted down".

Davidson arrived at a similar result by postulating a "user cost due to the rule of capture" :

$$U_c = \gamma [S_r(E) - E] \quad (19)$$

where $(S_r - E)$ "represents the total possible barrels that could be lost to the neighbour" (Davidson (1963), p.94). Note the words "could be lost". Davidson did not specify how each producer assumes others behave. His function U_c seems arbitrary in spite of the fact that it yields the "right" sign for the marginal user cost of the rule of capture. Davidson's arbitrary user cost functions are

- (i) User cost inherent in all raw material $U_m = \beta E$ where $\beta > 0$ if expected price movements are favourable, $\beta < 0$ otherwise and $\beta = 0$ if current prices are expected to continue. (This is wrong, as we have shown in Section 3.1).
- (ii) User cost of ultimate recovery U_u in (15).
- (iii) U_c above.

Davidson then added his three marginal user cost to obtain a "total marginal user cost" which is $\beta - \gamma$. When his β is zero, this yields the contradictory result that a firm produces at a point where its marginal instantaneous profit is negative.

3.5 User Cost for Growing or Decaying Resources :

So far we have restricted our attention to resources which are non-renewable and are not subject to natural growth or decay. For

those types of resources, we concluded that marginal user cost cannot be negative, for there is no reason why a firm should produce at rates where (current) marginal revenues are insufficient to cover (current) marginal operating costs (including royalties). If a resource is subject to growth or decay, the implicit marginal user cost *may* well be negative. For example, if a forrest needs trimming, then current production has a dual nature : as an output and as an input (investment for future growth). It may pay in this case to "produce" at rates where current marginal costs exceed current marginal revenues.

For the case of decaying resources, Scott (1967) argued that "the user-cost curve will be pushed below its constant conditions position ... This flattening-out of the curve induces the manager to move upward along his current profit curve toward the maximum-profit-per-period rate. When future calendar periods have little profit to offer, make hay while the sun shines" (p.47). This argument was not supported by any formal analysis. In fact the effect of natural decay on present production seems *ambiguous* even for simple models. Consider the following *two-period* model : The initial resource stock is R_1 . Assume

$$R_2 = (1-\alpha)(R_1 - Q_1) \quad (20)$$

Q_1 is output in period one, and α is the rate of decay. Suppose that R_1 is sufficiently large so that it never pays to exhaust R_1 in the first period (i.e. If Q_1^* maximizes $Y_1(Q)$, then $R_1 > Q_1^*$. Assume also that $(1-\alpha)(R_1 - Q_1)$ is sufficiently small so that $Q_2 = (1-\alpha) \times (R_1 - Q_1)$. The necessary condition for an optimal extraction policy is :

$$dY_1/dQ_1 - (dY_2/dQ_2)(dQ_2/dQ_1) = 0$$

i.e. $Y'_1 - (1-\alpha)Y'_2 = 0$

Hence

$$\frac{dQ_1}{d\alpha} = \frac{-(1-\alpha)Y''_2(R_1 - Q_1) - Y'_2}{Y''_1 + (1-\alpha)^2 Y''_2}$$

which is ambiguous in sign - assuming $Y''_t < 0$.

Instead of a proportional decay function as (20), let us consider a "loss" function. Suppose that the firm knows that at the beginning of period two, an amount of the resource will be taken away from it by the State, say. The amount taken away is R_L or $(R_1 - Q_1)$, whichever is the less. The amount left over for the firm to extract in period two is

$$R_2 = \text{minimum } (0, R_1 - Q_1 - R_L) \quad (21)$$

A diagrammatic solution is presented in figures 1 and 2. Two marginal profit curves Y'_1 and Y'_2 are drawn, with origins O_1 and O_2 respectively. If R_L is sufficiently small, it pays to produce in both periods and marginal profits are equalized (the discount factor is implicit in Y_2 so that all values are implicitly discounted). As long as the triangle $CO_2Q_1^*$ is less than CBA, it pays to produce "at A". A "small" increase in R_L will shift A to the left and hence less is produced in period one. This contrasts with Scott's result quoted above. If the increase in R_L is "large", however, $CO_2Q_1^*$ is larger than CBA (figure 2), the firm will produce at $Q_1 = Q_1^*$ and, of course $Q_2 = 0$.

Figure 1

Figure 2

4. THE NECESSARY CONDITIONS FOR AN OPTIMAL EXTRACTION PATH

The following general formulation covers several individual cases discussed in this appendix.

$$\text{Maximize}_{E,T} \int_0^T e^{-\delta t} Y(E,R) dt$$

subject to

$$\dot{R} = -E - W(E) - \bar{W}, \quad E \geq 0, \quad R(T) \geq 0.$$

Following Hestenes (1966), theorem 11.1, form the Hamiltonian $H(t, R, E, \Psi, \lambda) = e^{-\delta t} Y(E, R) + \Psi \dot{R} + \lambda E$.

The following conditions are necessary for an optimal path :

$E(t)$ maximizes H for given R, t and Ψ :

$$\frac{\partial H}{\partial E} = e^{-\delta t} Y_E - \Psi(1+W') + \lambda = 0 \quad (22)$$

Ψ is continuous with

$$\dot{\Psi} = -\frac{\partial H}{\partial R} = -e^{-\delta t} Y_R \quad (23)$$

H is a continuous function of time, with

$$\dot{H} = \frac{\partial H}{\partial t} = -\delta e^{-\delta t} Y(E, R) \quad (24)$$

$$H(T) = 0 \quad (25)$$

$$\Psi(T)R(T) = 0 \quad \text{for } T < \infty \quad (26)$$

λ if piece-wise continuous, with possible discontinuities when

$$E = 0 ; \quad \lambda \geq 0 \quad \text{and} \quad \lambda E = 0.$$

Conditions (25) and (26) are obtained using Hestenes' *control parameter* approach. Let b_1 and b_2 be two control parameters.

Write

$$R(T) = X(b_1) \equiv b_1^2 \geq 0$$

$$T = Z(b_2) \equiv b_2^2 \geq 0$$

The transversality conditions are from Hestenes (1966),

$$H(T) \frac{dZ}{db_2} = 0 \Rightarrow H(T) = 0$$

$$\Psi(T) \frac{dX}{db_1} = 0 \Rightarrow \Psi(T)R(T) = 0.$$

For ease of interpretation, define

$$p = \psi e^{\delta t} \quad (27)$$

$$\gamma = \lambda e^{\delta t} \quad (28)$$

For the zero discount ($\delta = 0$) case, (24) and (25) imply $H(t) = 0$ for all t :

$$Y(E, R) - \Psi[E + W(E) + \bar{W}] = 0 \quad (29)$$

When $E > 0$, $\Psi = Y_E / (1+W')$, by (22). Hence, using (27)

$$Y(E, R) = Y_E [E + W(E) + \bar{W}] / [1+W'] \quad (30)$$

which gives the results of sections 3.1, 3.2, 3.3 (in particular, equation 17) and 3.4 (in particular (20)) for the *zero discount* case.

If $\delta > 0$ and $Y(E, R) = Y(E)$, (22) - (28) give :

$$Y' - p(1+W') + \gamma = 0 \quad (22')$$

$$\dot{p} = \delta p \quad (23')$$

$$\lim_{t \rightarrow T} e^{-\delta t} \{Y[E(t)] - p(t)[E(t) + W(E(t)) + W]\} = 0 \quad (25')$$

$$p(T)R(T) = 0 \quad \text{for } T < \infty \quad (26')$$

The resource is exhausted in finite time because

(i) if $p(0) \leq 0$ then $Y' \leq 0$, implying $E(t) \geq E_m$ and hence exhaustion in finite time.

(ii) if $p(0) > 0$ then $p(t) \rightarrow \infty$ as $t \rightarrow \infty$, implying $Y' \rightarrow \infty$ as $t \rightarrow \infty$, but $Y'(0)$ is finite, hence $E(t)$ cannot approach zero asymptotically. Finite exhaustion implies

$$\lim_{t \rightarrow T} \{Y[E(t)] - p(t)[E(t) + W(E(t)) + \bar{W}]\} = 0 \quad (31)$$

(A.1') and (A.10) give :

$$\lim_{t \rightarrow T} \{Y[E(t)] - \frac{Y'[E(t) + W(E(t)) + \bar{W}]}{(1+W')}\} = 0 \quad (32)$$

which is the counterpart of (30) for $\delta > 0$.

For $\delta > 0$, and if $Y(E, R) = Y(E)$, (22') and (23') give

$$\dot{E} = \frac{\delta Y'}{Y'' - pW''} \quad (33)$$

which is negative because

(i) $Y'' - pW'' < 0$ is the second order condition for maximizing the Hamiltonian function, and

(ii) $\lim_{t \rightarrow T} Y'[E(t)] > 0$ by (32) hence $p(T) > 0$, therefore $p(t) > 0$ by (23'), which implies $Y'[E(t)] > 0$ by (22').

CHAPTER IV

INTERNATIONAL BORROWING FOR RESOURCE EXPLOITATION :

NON-REPLENISHABLE RESOURCES.¹

1. INTRODUCTION :

As we have noted, it is obvious that the welfare potential of a country is improved when it has access to an international capital market, simply because its set of feasible actions is enlarged. The question of interest is how such welfare potential can be realized. What are the optimal time paths of borrowing, extraction and consumption? Can such time paths be achieved by a decentralized competitive economy, possibly with taxes and subsidies? What are the possible sources of market failure? Answers to these questions depend, naturally, on the nature of the society's preference structure, on the international environment, on the nature of the uncertainty facing the economy, and on various other factors.

The uncertainty problem will be looked at in a later chapter. This chapter will be concerned with the optimal borrowing, extraction and consumption policies under two different set of assumptions about the international capital market : A Bardhan-type of borrowing possibilities and a Fisherian capital market. The differences caused by the presence or absence of a pure conservation motive or by different discount rates will be emphasized - particularly in Sections 2.4 and 3.2.

¹ Part 2 of this chapter is a revised version of Long (1974a).

2. THE CASE OF AN IMPERFECT WORLD CAPITAL MARKET :

Important studies of optimal foreign borrowing such as Bardhan (1967), Hamada (1966) and (1969), Pitchford (1970), Manning (1972) and Bade (1972) are concerned with the case where the borrowing country is *not* faced with an intertemporal budget constraint, and is required to use most or all of its borrowing in *productive* activity. Borrowing for consumption is either not allowed or severely restricted.

For direct comparison with models mentioned above, we first consider the case of borrowing for extraction in an imperfect world capital market.

2.1 Summary of Results :

Static analysis of optimum foreign borrowing gives the rule that marginal product of capital should be equated to marginal cost of borrowing. (See e.g. Kemp (1962a)). This implies a tax on foreign capital income in the case where the supply curve of foreign capital is upward sloping. The same conclusion is obtained in a dynamic framework where the home country accumulates capital. (See Pitchford (1970) and Hamada (1969)). Are there exceptions to this rule?

Apart from rather trivial exceptions such as the inclusion of psychic costs in the cost of borrowing e.g. hatred of foreign capital, as in Bardhan (1967) there are important cases to be considered. Manning (1972) analyses the situation where the supply curve of foreign capital is dependent on the foreign capital stock which accumulates or decumulates according to domestic policy. He shows that the usual formula does not hold and that in some cases it is optimal to subsidize foreign capital income in some time interval. Another exception is the situation where capital depletes resources. This problem will be studied in the following pages.

Our main conclusion is that when capital is optimally employed in the extractive industry, its marginal product must exceed the marginal cost of borrowing (with the possible exception of a learning-by-doing process). This is because the resource commands an imputed cost, its supply being fixed so that more extraction now means less is available for the future. The implication for a private enterprise economy is clear : if the resource is exploited by a single firm, the imputed cost is properly taken account of, provided the firm is not shortsighted. If, however, the resource is a 'common property' spatially and/or intertemporally, firms are likely to maximise instantaneous profit and over-exploit the resource. Examples of common property resources are petroleum reservoirs and short-leased mines.² In this case taxation measures are required to achieve the socially optimal extraction path. The imputed cost of the resource will serve as a guideline for setting an 'optimal royalty' or an 'optimal tax' on factor inputs or optimal combinations of these two.

Foreign capital introduces additional complications. Without price discrimination, the marginal return on capital required by foreign capital lenders is the average cost of borrowing, from the home country's point of view. Also, in the case of direct foreign investment, pure profit accrued to foreigners cannot be counted as a social gain and should be confiscated. In general, the royalty is not the best mean to

² Short-leased mines are examples of 'dynamic common properties'. If a firm is faced with a *fixed* time horizon which is 'sufficiently' short, it will maximize instantaneous profit (see Section 2.4). This will create a divergence between social and private optimal paths.

confiscate rent because it affects production plans. In a deterministic framework, the competitive bidding for the mining rights is the natural solution (or alternatively a 100 percent tax on pure profit).

A model of foreign capital and exhaustible resources has been developed by Fitzgibbons and Hendrikx (1971) but their framework, being static, omits the important aspect of resource depletion. Since their social cost measure does not include the imputed cost of the resource, their net social gain is inflated. We must also note that the imputed cost of the resource is of a complicated nature. It comes out of the optimal solution and in general cannot be determined before the complete solution is obtained.

Section 2.2 presents the basic production and cost assumptions. The characterization of an optimal path is contained in Section 2.3, which also discusses fiscal implications. Effects of changes in the rate of discount, the planning horizon and non-linearity of the utility function are analysed in Section 2.4. Implications for decentralization are discussed as the analysis is developed. The concluding section points out possible extensions and modifications.

2.2 The Basic Production and Cost Assumptions :

The nation possesses a stock of homogeneous resource R_0 . Output (extraction) is a function of capital used :

$$E(t) = F[K(t)].$$

Two types of production function will be considered. First, the usual concave production function with $F(K) > 0$ for $K > 0$ and $F(0) = 0$. The second type of production function requires a fixed, positive amount $\bar{K}$ of capital (in addition to variable capital input) for positive

extraction level. This reflects set-up or overhead costs. F is concave for $K \geq \bar{K}$ and $F(K) = 0$ for $K \leq \bar{K}$. Figure 1 illustrates type 2 production function.

Another type of production function exhibits increasing marginal product of capital for $K \leq \bar{K}$ for some $\bar{K}$ and diminishing marginal product for $K \geq \bar{K}$. In many respects it is similar to type 2 production function. In particular they both give rise to a U-shaped average cost curve. It will be seen that this type of average cost curve is essential for the existence of an optimal program when a zero discount rate is assumed.

We could have allowed the production function to have an extra argument : the resource stock that remains. This would complicate the analysis without adding much to the basic points we wish to make.

For simplicity we abstract away the problem of capital accumulation. This simplification may also be justified on the ground that the loss of realism is negligible for small resource stock or short planning horizon.

The home country is assumed to own a *fixed* stock of capital, K_d , which can be lent in the world market at rental rate r_0 . Let K be the stock of capital employed in the extractive industry, which is assumed to be the only capital using sector.

Define $K_f = K - K_d$. K_f stands for net borrowing from abroad. (If $K_f < 0$, this means capital is lent). Define the 'average borrowing cost' function $r(K_f)$, with $r(K_f) = r_0$ for $K_f \leq 0$. For $K_f > 0$ two cases will be considered :

- (i) $r = r_0$ for all K_f
- (ii) $r' > 0$ for $K_f > 0$, with $\lim_{K_f \rightarrow 0} r(K_f) = r_0 > 0$
- and $\lim_{K_f \rightarrow 0} r'(K_f) = 0$.

The rising cost of borrowing may reflect, for example, increasing risk of default, as perceived by foreign capitalists. It does not necessarily imply that the home country is 'big'. This point was made by Hamada (1969).

Total *social* cost of capital used is defined as

$$\phi(K) \equiv r_0 K_d + r(K_f) \cdot K_f.$$

It is assumed that $\phi(K)$ is convex (see figure 1).

As is well-known, atomistic (competitive) domestic borrowers will take $r(K_f)$ as given. Individual borrowers tend to neglect the effect of their borrowing on average cost $r(K_f)$. (Hence the usual argument for taxing foreign capital income when $r' > 0$).

On the other hand, if there is only one private domestic borrower, his total cost of borrowing is $\Pi(K) = r(K_f) \cdot K$ (assuming he cannot discriminate against domestic capitalists). If $K_d = 0$, $\phi(K) = \Pi(K)$ identically. Hence there is no need to tax his borrowing. However, if $K_d > 0$ he would borrow *less* than socially optimal because his private marginal cost Π_K is higher than the social marginal cost ϕ_K (assuming $r' > 0$).

The above assumptions about production and cost can be summarized in the familiar (social) cost of extraction diagrams below. Figure 2 corresponds to type 1 production function and Figure 3, type 2. As

Figure 1

Figure 2

Figure 3

$E \equiv F(K)$ one can write $K = F^{-1}(E)$ (for type 2 production function the inverse exists for $E > 0$). Of course $(F^{-1})' = \frac{dK}{dE} = \frac{1}{F'}$.

2.3 The Model and Its Implications :

a) The Model :

It is convenient to start off with a model with public decision making. Results can be reinterpreted or compared with the case where production decisions are made by the private sector.

We assume that there is no international trade. The resource good is directly consumable. Relaxation of these assumptions will complicate the analysis without changing any fundamental results.

The country maximizes the discounted stream of consumption :

$$\text{Maximize} \quad \int_0^T [F(K) - \phi(K)] e^{-\delta t} dt$$

subject to :

$$\dot{R} = -F(K)$$

$$R(0) = R_0$$

$$R(T) \geq 0$$

$$K(t) \geq 0$$

$$T \geq 0$$

Complications such as non-linear utility will be introduced later. Note that T is *not* fixed exogeneously. The case of fixed T (possibly $T = \infty$) will be considered in Section 2.4.

Let us write :

$$H(t) = e^{-\delta t} [F(K) - \phi(K)] + p[-F(K)] \quad (1a)$$

$$L = F(K) - \phi(K) - \psi F(K) + \lambda_1 \dot{R} + \lambda_2 K, \quad \text{where } \psi(t) = p(t)e^{\delta t} \quad (1b)$$

The necessary conditions are given by Arrow and Kurz, (1970) proposition 5.

$$\frac{dL}{dK} = (1-\psi-\lambda_1)F' - \phi' + \lambda_2 = 0 \quad (2)$$

$$\dot{\psi} = \delta\psi \quad (3)$$

$$\dot{R} = -F(K) \quad (4)$$

$$\text{Max } H(T) = e^{-\delta T} [F(K(T)) - \phi(K(T)) - \psi(T)F(K(T))] = 0 \quad (5)$$

$$e^{-\delta T} \psi(T)R(T) = 0, \quad e^{-\delta T} \psi(T) \geq 0 \quad \text{for } T < \infty \quad (6)$$

$$\lambda_1 \geq 0, \quad \lambda_1 \dot{R} = \lambda_1 R = 0 \quad (7)$$

$$\lambda_2 \geq 0, \quad K \geq 0, \quad \lambda_2 K = 0 \quad (8)$$

$$\dot{H}(t) = \frac{\partial H}{\partial t} = -\delta e^{-\delta t} [F(K) - \phi(K)] \quad (9)$$

b) The zero discount case ($\delta=0$)

We will be concerned mainly with the non-trivial case where there exists some $K > 0$ such that $F(K) - \phi(K) > 0$.

Since $\delta = 0$, ψ is a constant, implying a constant rate of extraction until the resource is exhausted. From (9), $\dot{H}(t) = 0$ for $\delta = 0$. Together with (5), this implies $H(t) = 0$ for all t .

Hence

$$\psi = \frac{F(K) - \phi(K)}{F(K)} = \frac{F' - \phi'}{F'} \quad (10)$$

($\psi > 0$ if $F(K) - \phi(K) > 0$ for some $K > 0$. This will be assumed throughout).

Is there some $K > 0$ such that (10) holds? To answer this question it is convenient to write :

$E \equiv F(K)$. For $E > 0$, define the inverse function

$K \equiv F^{-1}(E)$. Rewrite (10) as :

$$1 - \psi = \frac{\phi(K)}{F(K)} = \frac{\phi'}{F'} \Leftrightarrow \frac{\phi[F^{-1}(E)]}{E} = \frac{d\phi/dK}{dE/dK}$$

or, equivalently :

$$\frac{\phi[F^{-1}(E)]}{E} = \phi_E (= 1 - \psi) \quad (11)$$

In words, average cost of extraction equals marginal cost of extraction, or equivalently, average cost of extraction is at its minimum.³

Referring to Figures 2 and 3 one finds that no positive E or $F(K)$ exists which satisfies (11) for type 1 production function. An optimal program does not exist for this class of production function : Since average cost rises with the rate of extraction, total cost of exhausting the resource stock can always be reduced by reducing the rate of extraction towards zero. However, as will be seen, with a positive discount rate an optimal extraction program exists for both types of production function.

For type 2 production function, extraction is at level E^* where average extraction cost is at its minimum. The interpretation of the 'shadow price' ψ^* is interesting. From (11) :

$$\psi^* = 1 - \text{Minimum average cost of extraction.}$$

³ This result was stated by Scott (1967). Professor L. Roy Webb pointed out to me that Ramsey's saving rule $U(Y-S)-B = U'(Y-S).K$ has a similar interpretation if $\dot{K} = S$ (no capital depreciation) and if the divergence between actual utility and Bliss (B) level is regarded as a cost.

Note that cost is in terms of the resource good itself so that ψ^* is the *maximum profit* (or net output) *per unit extracted*. Let V be the present value of net output stream. Using (10) :

$$V = \int_0^T [F(K^*) - \phi(K^*)] dt = \psi^* \int_0^T F(K^*) dt = \psi^* R_0 \quad (12)$$

Equation (12) well justifies the name 'shadow price' of ψ^* . If there is a competitive market for the resource stock its market value would be $\psi^* R_0$.⁴ In this simple model $\psi^* R_0$ is the dynamic analogue of the Ricardian concept of rent.

c) The positive discount case :

With the assumption that $F(K) - \phi(K) > 0$ for some $K > 0$, it is easily seen that resource is eventually exhausted if the program is to be optimal. To show that the resource will be exhausted in finite time, consider two mutually exhaustive cases (the first of these will be ruled out later) :

(i) $\psi(0) = 0$. From (3) this implies $\psi(t) = 0$ for all t .

By (2), output is a constant until the exhaustion time, T^* say.

(ii) $\psi(0) \neq 0$. If $\delta = 0$, output is a constant. If $\delta > 0$ and $\psi(0) < 0$, $\psi(t)$ declines over time, implying output increases over time (by (2)) until the resource is exhausted. If $\delta > 0$ and $\psi(0) > 0$, $\psi(t) = 1$ in finite time, implying that $K(t)$ declines to zero in finite time, T^* .

⁴ Assuming, of course, that $\phi(K)$ is also the private cost of borrowing either because of appropriate fiscal measures or because $r' = 0$.

Obviously $T^* \leq T$, the terminal date of the free time program. Now, for $t > T^*$

$$\dot{H}(t) = -\delta e^{-\delta t} [0] = 0$$

But

$$H(T) = 0$$

$$\therefore H(t) = 0 \text{ for } t \geq T^*$$

By the continuity of $H(t)$, consider the left-hand limit :

$$\lim_{t \rightarrow T^*} [F(K(t)) - \phi(K(t)) - \psi(t)F(K(t))]e^{-\delta t} = 0$$

$$\lim_{t \rightarrow T^*} [F(K(t)) - \phi(K(t)) - \left(\frac{F' - \phi'}{F'} \right) F(K(t))]e^{-\delta t} = 0$$

i.e. $F(K^*)$ minimizes average cost of extraction (see 11).

For type 1 production function $K^* = 0$ and $F(K^*) = 0$. The extraction path is shown in Figure 4.

For type 2 production function, condition (2) (that K maximizes $(1 - \psi)F(K) - \phi(K)$) implies $K \notin (0, \bar{K}]$. Therefore K cannot tend to zero. $F(K)$ must tend to $F(K^*) = E^*$, as shown in Figure 5.

Since we assumed $F(K) - \phi(K) > 0$ for some K , the minimum average cost of extraction is below 1 (i.e. $\text{Min. } \phi(K)/F(K) < 1$).

Since

$$\lim_{t \rightarrow T^*} \psi(t) = \lim_{t \rightarrow T^*} \frac{F(K(t)) - \phi(K(t))}{F(K(t))}$$

$$\lim_{t \rightarrow T^*} \psi(t) = 1 - \text{Minimum average cost} > 0.$$

Hence $\psi(t)$ is positive and increasing, implying a declining rate of extraction.

Figure 4

Figure 5

The initial extraction level, E_0 , is below the myopic profit maximizing level, E_m , for $\frac{\phi'(K(0))}{F'(K(0))} = (1 - \psi(0)) < 1$ (see Figures 4 and 5).

d) Fiscal implications :

We have seen that along the optimal extraction path the marginal product of capital must exceed its marginal cost (i.e. instantaneous profit is *not* maximized). The optimal rate of extraction is characterized by the relation :

$$(1 - \psi(t))F' = \phi'$$

where $1 > \psi(t) > 0$. This is different from the usual optimal relation ($F' = \phi'$) obtained in static optimization models where land or resources are not included.

If production decisions are made by private firms, the socially optimal extraction program can be achieved by appropriate fiscal measures. There are two separate issues :

- (a) the regulation of the extraction rate, and
- (b) the confiscation of pure profit or rent.

We shall consider two types of market institutions :

- (i) the resource is exploited by a single firm which maximizes profit over time, with the price of its product fixed in the world market, and
- (ii) the resource is exploited by a number of firms,⁵ n , which

⁵ It is assumed that n is fixed. Alternatively, if variables are interpreted on the 'per-head' basis, one can assume that production exhibits constant returns to scale, and n is indeterminate.

maximize their instantaneous profit, because the resource is a common property.

For the first case the private optimal extraction path coincides with the social optimal path, assuming no divergence between the private cost of capital $\Pi(K)$ and the social cost $\phi(K)$. Rent can be confiscated by the competitive auctioning of mining rights or 100 percent tax on pure profit. If the rate of discount $\delta = 0$, rent can also be confiscated by an optimal royalty or an optimal tax on inputs, or an optimal combination of the two. This is seen quite simply for the firm will

$$\text{Max}_{K(t), T} \int_0^T [(1 - \rho)F(K) - \phi(K)] dt \quad (13)$$

(subject to the usual conditions), where ρ is the royalty rate. Setting $\rho = \psi^* = 1 - \text{Minimum average social cost of extraction (as determined by (11))}$ will effectively reduce the price received by the private producer to $(1 - \psi^*)$. Extraction will be at E^* and no (pure) profit is earned by the private firm. The shadow price of problem (13) turns out to be zero. Figure 6 illustrates the solution.

As remarked earlier, a royalty has the same effect as a uniform tax on all inputs. For simplicity consider the case where $\phi(K) = r_0 K$. The supply price of capital service is r_0 . If a tax rate θ is imposed on capital earning, the supply curve of capital service, as seen by the producer, shifts upwards. The producer has to pay the gross rental rate g where $(1 - \theta)g = r_0$.

The producer's problem is :

$$\text{Max}_{K(t), T} \int_0^T [F(K) - \frac{r_0 K}{(1-\theta)}] dt = \text{Max}_{K(t), T} \frac{1}{(1-\theta)} \int_0^T [(1-\theta)F(K) - r_0 K] dt$$

Figure 6

Figure 7

which is equivalent to (13). Thus the optimal tax on capital income, given zero royalty rate, is $\theta^* = \psi^*$. In general, given any royalty rate, the optimal tax rate can be determined and vice versa, according to the formula :

$$(1-\theta^*)(1-\rho^*) = 1 - \psi^* = \text{minimum social average cost of extraction.} \quad (14)$$

If one rate is too high, the other rate should be negative. The government has one degree of freedom in determining ρ and θ .

If firms are instantaneous profit maximizers the rate of extraction should be restricted by an optimal royalty and/or tax on capital earnings. The socially optimal extraction rate is given by :

$$(1 - \psi(t))F' = \phi' = r'(K_f) \cdot K_f + r(K_f) . \quad (15)$$

Without any fiscal intervention, the competitive behaviour results in :

$$F' = r$$

(because atomistic firms take r as given). Figure 7 depicts the competitive level of capital used K_c , the statically optimal level K_s obtained in models where resources are not included, and the truly optimal level $K^*(t)$.

$K^*(t)$ is determined at the intersection of $(1-\psi(t))F'$ and ϕ' . Note that $\psi(t) (= 1 - \phi_E)$ is the marginal profit of extraction whose discounted value is the same for all time in the interval $[0, T^*]$. Hence $\psi(t)$ also has the interpretation of opportunity cost (or user cost) of the resource.

In order to induce the competitive industry to select the optimal level $K^*(t)$, an optimal combination of tax on capital earning and royalty must be devised. Let ρ be the royalty per unity of output and θ the proportional tax rate on capital earning. Firms will solve the problem :

$$\text{Max } (1 - \rho)F(K) - g.K$$

(where g is the gross rate of returns to capital defined by $(1 - \theta)g = r$). This gives the amount of capital employed as the solution of the "derived demand" equation :

$$(1 - \rho)F' = r/(1 - \theta) \quad (16)$$

From (15) and (16) for any given ρ , one can solve for θ :

$$\theta = \frac{K_f \cdot r'(K_f) + r[1 - (1-\psi)/(1-\rho)]}{K_f \cdot r'(K_f) + r(K_f)} \quad (17)$$

where r and r' are evaluated at $K^*(t)$. This, in general, differs from the familiar static tax formula where the second term of the numerator of the right-hand-side of (17) is zero, which is the case if the royalty is $\rho = \psi$.

Note that formula (17) applies also to the case where $r' = 0$ for all K_f . Thus if $r' = 0$ and $\rho = \psi$, no tax on capital income is required. If $r' = 0$ and $\rho = 0$, the tax rate on capital income is ψ . In general, for $r' = 0$, $(1 - \theta)(1 - \rho) = (1 - \psi)$.

Of course ρ and ψ are not the best means of confiscating rent or pure profit. Profit taxation or the competitive bidding for the mining rights are the correct measures.

This section is partly an analysis of a special type of externality. The market fails to achieve the best intertemporal allocation when the resource is a common property of a static and/or dynamic kind. Section 2.4.b will clarify the concept of a dynamic (or intertemporal) common property by showing that an intertemporal profit maximizing firm will behave as if it were myopic (instantaneous) profit maximizer if it is faced with a fixed and sufficiently short planning horizon.

2.4 Changes in the Rate of Discount, the Planning Horizon and Non-linear Utility Function.

(a) Changes in the rate of discount :

The difference between the optimal policies for the zero discount and positive discount rate was analysed in Section 2.3. We are now concerned with the problem of determining how the extraction paths differ if the (positive) discount rates differ.

Let $\delta' > \delta > 0$. Variables corresponding to the program with the discount rate δ' are denoted by a prime. Let T' and T denote the exhaustion time for the two programs respectively. From Section 2.3 $\psi'(T') = \psi(T) = \text{Inf} \frac{F(K) - \phi(K)}{F(K)}$.

Hence

$$\psi'_0 e^{\delta' T'} = \psi_0 e^{\delta T} \quad (18)$$

We first show that $\psi'_0 < \psi_0$. Suppose otherwise, i.e. $\psi'_0 \geq \psi_0$. Then, by (18), $T > T'$ and $\psi'(t) > \psi(t)$ for $t > 0$, implying $F(K'(t)) < F(K(t))$. Thus

$$R_0 = \int_0^{T'} F(K'(t)) dt < \int_0^{T'} F(K(t)) dt < \int_0^T F(K(t)) dt = R_0 .$$

Hence there is a contradiction. It is now obvious that the path associated with a lower discount rate starts at a lower level of extraction and takes longer to exhaust the resource. Note that (18) implies $F(K'(T')) = F(K(T))$.

If the private rate of discount δ_1 , say, is different from the social rate δ_2 , the optimal fiscal measure is a profit tax (or subsidy) rate $\gamma(t)$ such that $(1 - \gamma)\exp(-\delta_1 t) = \exp(-\delta_2 t)$.

(b) Changes in the planning horizon :

In Section 2.3, the terminal time was left free (to be determined endogenously). It was shown that instantaneous profit maximization was non-optimal. When the terminal time is fixed exogenously this may not be true : we will show that if the fixed time is 'sufficiently' short, it is optimal to behave myopically. Thus a firm with a 'short' fixed time horizon will maximize instantaneous profit, causing a divergence between private and social optimal extraction paths. This situation will be analysed below.

We retain the basic model of Section 2.3. Let T^* be the time at which the resource is exhausted for the free time problem in Section 2.3. Let $\bar{T}$ be a fixed time. Let $F(K_m) \equiv E_m$ be the level of output which satisfies $F'(K_m) = \phi'(K_m)$. Consider three mutually exclusive cases :

- (i) $\bar{T} \geq T^*$
- (ii) $\bar{T} < T^*$ and $\bar{T} > R_0/E_m$
- (iii) $\bar{T} < T^*$ and $\bar{T} \leq R_0/E_m$

Literally, case (ii) applies when the constant extraction rate E_m (the 'instantaneous profit maximizing' level) would exhaust the resource before $\bar{T}$, and case (iii) applies when $\bar{T}$ is so short

that at the constant extraction rate E_m the resource will not be exhausted before $\bar{T}$.

Case (i) is trivial. The optimal path for the free time problem is followed until T^* and output is zero for $T^* \leq t \leq \bar{T}$.

For case (ii) one can prove that (a) exhaustion is optimal, (b) the resource is just exhausted at time $\bar{T}$ and hence (c) the optimal extraction rate $E(t)$ is less than E_m .

Case (iii) is of particular interest : instantaneous profit maximization is optimal and hence the resource is not exhausted except for the borderline case $\bar{T} = R_0/E_m$.

Proofs : Note that all the necessary conditions except (5) in Section 2.3 are applicable for the fixed time case.

From condition (2), $\psi(t) = 0$ implies $\phi' = F'$ or equivalently $E(t) = E_m$. $\psi(t) > 0$ implies $E(t) < E_m$ and $\psi(t) < 0$ implies $E(t) > E_m$.

For case (ii) we shall prove assertions (a), (b) and (c) above, using a sufficiency proof. Let the asterisk (*) denote values taken by variables along the path which satisfies the necessary conditions and which just exhausts the resource at time $\bar{T}$. This path must be associated with $\psi^*(0) > 0$, because $\psi^*(0) \leq 0$ implies $E(t) \geq E_m$, as already observed, and this would exhaust the resource before $\bar{T}$. Therefore $\psi^*(t) > 0$ for all t . Compare the path to be proved as optimal with any other path :

$$\begin{aligned}
& \int_0^{\bar{T}} e^{-\delta t} [(F^* - \phi^*) - (F - \phi)] dt \\
&= \int_0^{\bar{T}} e^{-\delta t} [(F^* - \phi^*) - (F - \phi)] dt - \int_0^{\bar{T}} e^{-\delta t} (\dot{\psi}^* - \delta \psi^*) (R^* - R) dt \\
&= \text{the first integral} - (R^*(t) - R(t)) \psi(t) e^{-\delta t} \Big|_0^{\bar{T}} + \int_0^{\bar{T}} e^{-\delta t} \psi^* (\dot{R}^* - R) dt \\
&\quad (\text{by integration by parts}) \\
&\geq \int_0^{\bar{T}} e^{-\delta t} [(1 - \psi^*) F^* - \phi^* - (1 - \psi^*) F + \phi] dt \\
&> 0 \quad \text{by condition (2) : } K^* \text{ maximizes } (1 - \psi^*) F(K) - \phi(K).
\end{aligned}$$

This establishes (b) and hence (a) and (c).

For case (iii), we shall prove that $E^*(t) = E_m$. By definition of case (iii) $E_m \bar{T} \leq R_0$, hence the value of the objective integral if $E(t) = E_m$ for $t \leq \bar{T}$ is :

$$\int_0^{\bar{T}} e^{-\delta t} [F(K_m) - \phi(K_m)] dt > \int_0^{\bar{T}} e^{-\delta t} [F(K(t)) - \phi(K(t))] dt$$

for any $K(t) \neq K_m$, because K_m maximizes $F - \phi$ by definition. Note that the integral on the left-hand-side is meaningful only for case (iii). For cases (i) and (ii) output cannot be at the rate $F(K_m)$ for all time $t \leq \bar{T}$.

(c) Changes in the utility function :

Let $C = F(K) - \phi(K)$, the objective assumed so far is equivalent to

$$\text{Max}_{C(t), T} \int_0^T e^{-\delta t} C(t) dt .$$

If we define a utility function $U(C)$ with $U(0) = 0$, $U' > 0$, $U'' < 0$ it may be expected that the extraction path will differ.

For simplicity we consider the case of zero discount. The strict concavity of $U(C)$ may be due to the usual "diminishing marginal utility" assumption, or one may interpret this as representing (or including) the cost of transforming the resource good into a directly consumable good. In writing $C = E - \phi(F^{-1}(E))$ the implicit assumption is made that net output cannot be stored or exchanged (in the world market) for future consumption. Without this assumption production and consumption decisions would be distinct. In the special case where storage cost and/or transaction cost is zero, the production decision is completely separable from the consumption decision and the optimal extraction path is the same as in Section 2.3.

As shown in Section 2.3, the optimal level of extraction when $U(C) = C$ is at E^* where :

$$\frac{C(E^*)}{E^*} = C'(E^*) \quad (1a)$$

or $\frac{C}{E}$ is maximized at E^* . It can be shown in a similar fashion that when $U(C)$ is strictly concave, the optimal extraction level is at E^{**} where

$$\frac{U(C(E^{**}))}{E^{**}} = U'(C(E^{**})) \cdot C'(E^{**}) \quad (20)$$

It can be seen that $E^{**} < E^*$. Suppose otherwise, i.e. $E^{**} \geq E^*$. Then

$$\frac{U(C(E^*))}{E^*} \leq U'(C(E^*)) \cdot C'(E^*)$$

$$\leq U'(C(E^*)) \cdot \frac{C(E^*)}{E^*}$$

$$\therefore U(C(E^*)) - U(0) \leq U'(C(E^*)) [C(E^*) - 0]$$

which contradicts the strict concavity of $U(C)$ and the assumption that $U(0) \geq 0$.

For the *positive discount* case, E^* and E^{**} can be interpreted as the limiting rates of extraction as the resource stock declines to zero.

Note that *essentially* the result does not change. (11), (19) and (20) are just different expressions for the basic principle that the average "return" per unit extracted be maximized at all points in time if the discount rate is zero. Production is tilt in favour of the present if the discount rate is positive. These intuitive results are in agreement with those of Scott (1967) and Vousden (1973).

(d) Utility with a conservation motive :

In addition to the extraction stream, a resource stock may also contribute to the community's utility *directly*. For example, sand beaches may have recreational values as well as ordinary commercial values. In this case, it may be optimal *not to exhaust* the resource.

Assume that the community's utility function is of the form $U[C(t), R(t)]$ with $U_R > 0$, $U_C > 0$, $U_{CR} > 0$, $U_{CC} < 0$, $U_{RR} < 0$ and $U_{RR}U_{CC} - (U_{RC})^2 > 0$. Type 1 production function is assumed.

Let :

$$L = U[F(K) - \phi(K), R] - \psi F(K) + \lambda_1 \dot{R} + \lambda_2 K \quad (21)$$

The necessary conditions for optimality are :

$$\frac{dL}{dK} = (U_C - \psi - \lambda_1)F' - U_C\phi' + \lambda_2 = 0 \quad (22)$$

$$\dot{\psi} = \delta\psi - U_R \quad (23)$$

$$\dot{R} = -F(K) \quad (24)$$

and also (6), (7) and (8).

From (22), if $\psi \geq 0$, $R > 0$ and $K > 0$,

$$\frac{\partial K}{\partial R} = - U_{CR} [F' - \phi'] / D > 0 \quad (25)$$

where

$$D = U_{CC} [F' - \phi']^2 + (U_C - \psi) F'' - U_C \phi'' < 0 \quad (26)$$

because $F' - \phi' > 0$ and $U_C - \psi > 0$ if $\lambda_1 = \lambda_2 = 0$ and $\psi \geq 0$, by (22).

Similarly,

$$\frac{\partial K}{\partial \psi} = F' / D < 0 \quad (27)$$

Let

$$N(\psi, R) = \delta \psi - U_R \quad (28)$$

$$M(\psi, R) = - F[K(\psi, R)] \quad (29)$$

The slope of the curve $\dot{\psi} = 0$ is given by :

$$\left. \frac{d\psi}{dK} \right|_{\psi=0} = - (\partial N / \partial R) / (\partial N / \partial \psi) \Big|_{N(\psi, R)=0}$$

$$\left. \frac{\partial N}{\partial R} \right|_{N(\psi, R)=0} = - U_{RR} - U_{RC} (F' - \phi') \partial K / \partial R$$

which is positive, using (25), (26) and the strict concavity of $U(C, R)$.

$$\left. \frac{\partial N}{\partial \psi} \right|_{N(\psi, R)=0} = \delta - U_{RC} (F' - \phi') \partial K / \partial \psi > 0$$

Hence, the curve $\dot{\psi} = 0$ is negatively sloped.

Figure 8

Figure 9

The curve $\dot{R} = \text{constant}$ has slope :

$$\left. \frac{d\psi}{dR} \right|_{\dot{R}=\text{constant}} = - \left. (\partial M / \partial R) / (\partial M / \partial \psi) \right|_{\dot{R}=\text{constant}}$$

$$\left. \frac{\partial M}{\partial R} \right|_{\dot{R}=\text{constant}} = - F' \partial K / \partial R < 0$$

$$\left. \frac{\partial M}{\partial \psi} \right|_{\dot{R}=\text{constant}} = - F' \partial K / \partial \psi > 0$$

Hence, the curve $\dot{R} = \text{constant}$ is positively sloped.

The curve $\dot{\psi} = 0$ intersects the ψ -axis at ψ_1 , where ψ_1 is determined by :

$$\psi_1 = U_R(F[K(\psi_1, 0)] - \phi[K(\psi_1, 0)], 0) / \delta = U_R(0, 0) / \delta$$

using (23). The curve $\dot{R} = 0$ intersects the ψ -axis at ψ_2 , where

$$\psi_2 = U_C(0, 0) \cdot [F'(0) - \phi'(0)] / F'(0)$$

using (22) and (24).

If $\psi_2 < \psi_1$, there exists an interior equilibrium : Extraction declines to zero monotonically and the resource is not exhausted. This is illustrated in Figure 9, where α is the optimal path. If $\psi_2 \geq \psi_1$, the optimal path is that which converges to $(\psi_2, 0)$. Among paths satisfying (22), (23) and (24), the optimal path is that which takes the longest time to exhaust the resource. This is path α of Figure 8. Appendix IV proves our assertions.

We note that apart from the possibility that it may not be optimal to exhaust the resource, there are no substantially different results. The following basic results remain :

- i) Marginal product of capital exceeds the marginal cost of borrowing.
- ii) Extraction is monotone decreasing.
- iii) The amount of capital borrowed is monotone decreasing, for along the optimal path,

$$\dot{K} = (\partial K / \partial R) \dot{R} + (\partial K / \partial \psi) \dot{\psi} < 0 \quad (30)$$

2.5 Final Remarks for Section 2.

We have shown that if production depletes the resource stock, it is not optimal to equate the marginal product of capital with the marginal cost of borrowing. In a decentralized framework, assuming firms maximize instantaneous profit (which is likely if the resource is a common property spatially or intertemporally), this implies that fiscal measures are needed. It is shown that an optimal combination of royalty and tax on capital income will ensure the optimal extraction path. The confiscation of rent is not best served by these fiscal correctives, however.

It should be noted that if the resource is exploited by a single firm which maximizes profit over time, its extraction path is socially optimal because there are no externalities, assuming of course that no arbitrary fixed time horizon is imposed on the firm.

As has been pointed out earlier, considerable abstraction has been made to simplify the analysis, the most important one being the absence of capital accumulation (or decumulation). The introduction of a second good and trade would not modify the essential results, however.

3. THE CASE OF A PERFECT WORLD CAPITAL MARKET :

In Section 2 it was assumed that the home country cannot borrow for consumption. Foreign capital was tied to productive uses. The home country either rented foreign capital goods or borrowed from the foreign country in order to purchase foreign capital goods. These two possible courses of action are equivalent : in one case the country pays rental, in the other case, it pays interest. Borrowing for consumption was not allowed.

In this Section, the other extreme situation will be considered. The country has access to a perfect Fisherian capital market. In this case, current consumption need not equal current net output : deficits or surpluses can be financed by the sale or purchase of "bonds" - i.e. by borrowing or lending. The simplest case is that of a small country facing a *given* interest rate. In this case we will show that :

- i) If the resource stock does not enter the community's utility function directly, then production and consumption decisions are separable, and the optimal policies can be characterized simply.
- ii) If there is a conservation motive, however, consumption and production decisions are interdependent in a fundamental way.

3.1 The Model :

a) The production and cost assumptions :

The assumptions of Section 2.2 are retained, except that the cost function $\phi(K)$ is now *linear* : the default-risk argument used in Section 2.2 is not consistent with the existence of a Fisherian capital market.

b) The optimization problem :

The community's welfare is represented by a utility function $U[C(t), R(t)]$. U is strictly concave in C , with $U_C > 0$. If the community does not value the resource for its own sake, $U_R = 0$ identically. If the resource stock contributes to welfare *directly*, then $U_R \neq 0$.

Let δ be the utility discount rate and r the international interest rate. The community's objective is to :

$$\text{Maximize } \int_0^T U[C, R] e^{-\delta t} dt \quad (31)$$

subject to :

$$\int_0^T C(t) e^{-rt} dt \leq \int_0^T [F(K) - r(K - K_d)] e^{-rt} dt - D_0 \quad (32)$$

where K_d is exogenously given, and :

$$\dot{R}(t) = -F[K(t)] \quad (33a)$$

$$R(T) \geq 0 \quad (33b)$$

$$K(t) \geq 0 \quad (33c)$$

$$R(0) = R_0 \quad (33d)$$

$$T \geq 0 \quad (33e)$$

Inequality (32) is the intertemporal budget constraint : the discounted value of the consumption stream must not exceed the discounted value of the net output stream less the initial debt D_0 . ($D_0 < 0$ implies that the country is a net creditor initially).

c) The nature of the intertemporal budget constraint :

The constraint (32) implies that at any time t_1 , the amount of debt is not greater than the current value of capitalized future net income. In order to see this, let us define the net amount borrowed at time t as :

$$B(t) \equiv C(t) - [F(K) - rK_f] \quad (34)$$

where $F(K) - rK_f$ is output net of rental payments.

The stock of debt at time is $D(t)$, where

$$\begin{aligned} D(0) &= D_0 \\ \dot{D}(t) &= B(t) + rD(t) \end{aligned} \quad (35)$$

Thus (32) can be written as :

$$\begin{aligned} D(0) + \int_0^{t_1} e^{-rt} [\dot{D}(t) - \delta D(t)] dt \\ \leq \int_{t_1}^T e^{-rt} [F(K) - rK_f - p(t)c(t)] dt \\ < \int_{t_1}^T e^{-rt} [F(K) - rK_f] dt \end{aligned} \quad (36)$$

Evaluate the first integral by parts and substitute in (36) :

$$e^{-rt} D(t_1) < \int_{t_1}^T e^{-rt} [F(K) - rK_f] dt$$

or

$$D(t_1) < \int_{t_1}^T e^{-r(t-t_1)} [F(K) - rK_f] dt \quad (37)$$

which must hold for all t_1 . This condition corresponds to Arrow and Kurz's condition that "total asset" (capitalized future net incomes minus debt) be positive (Arrow and Kurz (1970), p.157).

3.2 The Optimal Paths :

a) The necessary conditions :

The necessary conditions for problem (31) subject to (32) - (33) are given by Hestenes' theorem 11.1 (1966).

The Hamiltonian is :

$$H = e^{-\delta t} U(C, R) - qF(K) + \lambda e^{-rt} [F(K) - r(K - K_d) - C(t)] + \mu K \quad (38)$$

where, along an optimal path,

$$(i) \quad \lambda \geq 0 \text{ and is a constant} \quad (39)$$

$$(ii) \quad \dot{q} = -\partial H / \partial R = -e^{-\delta t} U_R (= 0 \text{ if } U_R = 0).$$

$$(iii) \quad H \text{ is maximized with respect to } K \text{ and } C :$$

$$e^{-\delta t} U_C = \lambda e^{-rt} \quad (40)$$

implying

$$qF' - \lambda e^{-rt} (F' - r) = 0 \quad (41)$$

$$(iv) \quad \mu \geq 0, \quad \mu K = 0$$

and transversality conditions to take account of constraints (33a) and (33e). To obtain the conditions, we use Hestenes' "control parameter" approach. Let b_1 and b_2 be two control parameters. Define

$$R(T) = X(b_1) \equiv b_1^2 \geq 0$$

$$T = Y(b_2) \equiv b_2^2 \geq 0$$

The transversality conditions are :

$$(v) \quad q(T)dX/db_1 = 0 \Leftrightarrow q(T)R(T) = 0 \quad (42)$$

for $T < \infty$

$$(vi) \quad H(T)dY/db_2 = 0 \Leftrightarrow H(T) = 0 \quad (43)$$

b) The case of no conservation motive :

In this case $U_R = 0$ identically. Differentiating (4) with respect to t yields :

$$\dot{C}(t) = (\delta - r)U_C/U_{CC} \quad (44)$$

Thus consumption is monotone *increasing* if the rate of interest r is greater than the rate of utility discount, and is monotone *decreasing* in the converse case. The sign of $\dot{C}$ is independent of the rate of exploitation or the level of the resource stock. In fact when the resource stock *does not* contribute to utility directly, consumption and production decisions are *separable*. This is just the continuous-time counterpart of the well-known *Fisher's Separation Theorem* usually stated and proved in discrete, finite time models in which present and future prices, properly discounted, define a hyperplane separating production and consumption decisions for economic agents facing given prices. Since we are dealing with a continuous time model, and the time horizon is allowed to be infinite, no such hyperplane can be "visualized". It can be easily seen, however, that the Fisherian theorem also holds in

this case, in the sense that when $U_R \equiv 0$, problem (31) subject to (32) and (33) can be solved in two steps : First, maximize the "net worth" - the right hand side of (32) - subject to (33). Denote the maximum net worth by A_0 . The second step is to maximize the present value of utility, (31), subject to :

$$\dot{A}(t) = -C(t) + rA(t) \quad (45)$$

$$A(0) = A_0, \quad A(t) \geq 0 \quad (46)$$

or, equivalently, subject to :

$$\int_0^T C(t)e^{-rt} dt \leq A_0 \quad (47)$$

In order to see the separability of the problem, suppose that $C^*(t)$, $K^*(t)$, $q^*(t)$ and λ^* be values along an optimal path satisfying (38) - (43). From (iii), $K^*(t)$ must maximize the function :

$$H^0 \equiv \lambda^* e^{-rt} [F(K) - r(K - K_d)] - q^*(t)F(K) \quad (48)$$

Also, from (ii) :

$$\dot{q}^*(t) = 0 \quad (49)$$

But (49) together with (33) and (iv), (v), (vi) are just the necessary conditions for maximizing the right-hand-side of (32), and (48) is the corresponding Hamiltonian.

The second step - maximizing the discounted utility stream subject to (45) and (46) - yields the necessary condition (44). In view of (47) it is easily seen that the problem has no solution if :

$$\dot{C}(t)/C(t) = (\delta-r)U_C/U_{CC}.C > r \quad (50)$$

because if (50) holds, the left-hand-side of (47) will be unbounded. If $\delta > r$, $\dot{C}(t)/C(t)$ is negative, and a solution exists. If $\delta \leq r$, a solution exists if $(r-\delta)/r < -U_{CC}.C/U_C$ and no solution exists if the inequality is reversed. Thus if $U(C) = AC^\alpha$, with $A > 0$, $\alpha \in (0,1)$, $-U_{CC}.C/U_C = 1 - \alpha > 0$ a solution exists if $\alpha > \delta/r$ and no solution exists if $\alpha \leq \delta/r$. If $U(C) = A - C^\beta$, $A > 0$, $\beta < 0$, $-U_{CC}/U_C.C = 1 - \beta > 1$ and a solution exists. Thus, even if the rate of interest is higher than the rate of discount, a solution still exists for the second step, provided that the differential is sufficiently small relative to some measure of the curvature of the utility function.

c) Utility with a conservation motive :

In this case condition (ii) does not reduce to (49). Since along an optimal trajectory $\dot{q}^*(t) = -e^{-\delta t}U_R$, the maximization of "net worth" is not a necessary condition for maximizing welfare. Thus, even though the country has access to a Fisherian capital market, production and consumption decisions are no longer separable. Since the resource enters the utility function directly, this is a source of externality. In the case of a non-separable utility function the externality is all the more obvious, for, from (40) :

$$U_{CC}\dot{C} + U_{CR}\dot{R} = (\delta-r)U_C$$

where $\dot{R} = -F(K)$. Thus consumption, production and borrowing decisions are interrelated in a fundamental way.

APPENDIX IV

PROOF THAT PATH α IS OPTIMAL

Referring to figure 8, α is the path above which all other Pontryagin paths do not exhaust the resource, with ψ rising continuously and below which all other Pontryagin paths exhaust the resource more quickly than path α does. That α is the longest exhaustion path satisfying (22) - (24) follows from property (27). For paths which do not exhaust the resource, extraction falls to zero before T^* , the time taken to exhaust R along path α .

Let path α be denoted by $K^*(t)$, $R^*(t)$, $\psi^*(t)$ and any other path by $K(t)$, $R(t)$, $\psi(t)$. Then comparing α with any other path :

$$\begin{aligned}
 & \int_0^\infty (U^* - U) e^{-\delta t} dt \\
 & \geq \int_0^{T^*} (U^* - U) e^{-\delta t} dt + \int_{T^*}^0 U^*(0,0) [0 - R(T^*)] e^{-\delta t} dt \\
 & = \int_0^{T^*} [U^* - U - U_R^*(R^* - R)] e^{-\delta t} dt \\
 & \quad - \int_0^{T^*} (\dot{\psi}^* - \delta \psi^*) e^{-\delta t} (R^* - R) dt \\
 & \quad - U^*(0,0) R(T^*) / \delta \\
 & = \text{first integral} - (R^* - R) e^{-\delta t} \psi^* \Big|_0^{T^*} \\
 & \quad + \int_0^{T^*} e^{-\delta t} \psi^* (\dot{R}^* - \dot{R}) dt - U^*(0,0) R(T^*) / \delta \\
 & \geq \int_0^{T^*} [u_c^*(C^* - C) - \psi^*(F(K^*) - F(K))] e^{-\delta t} dt
 \end{aligned}$$

(because $\psi^*(T^*) \equiv \psi_2 > U_R(0,0)$).

$$\geq \int_0^{T^*} [U_c^* F^{*'} - \phi^{*'} - \psi^* F^{*'} - \lambda_1] [K^* - K] e^{-\delta t} dt$$

because the production function is of type 1,

≥ 0 by (22).

The case depicted in figure 9 can be proved in the same manner.

2. THE CASE OF AN IMPERFECT WORLD CAPITAL MARKET

As in Chapter IV, we regard as imperfect a capital market which does not allow or severely restricts consumption borrowing. Such an imperfect capital market is assumed in Berdan (1967), Hens (1968) and (1969), Pitchford (1970), Manning (1972) and Wado (1972).

2.1 Summary of Results :

We classify replenishable resources into two categories : those that are "scarce" and those that are "potentially plentiful". The meanings of these words will be made clear later, but broadly they

A version of this Chapter will appear in Pitchford and Turnovsky (1975).

CHAPTER V

INTERNATIONAL BORROWING FOR RESOURCE EXPLOITATION :

REPLENISHABLE RESOURCES.¹

1. INTRODUCTION :

It was shown in Chapter IV that for a country facing an upward sloping supply curve of foreign capital, its marginal cost of borrowing should be kept below the marginal product of capital used in the extraction of its non-replenishable resource. This is because the resource commands an imputed price reflecting the fact that more extraction now means that less is available for the future. Does the same rule apply for replenishable resources? If artificial renewal is possible, what is the relationship between the marginal product of capital in the renewing activity and the marginal cost of borrowing? This Chapter will provide some answers to these questions. The case of a perfect world capital market will also be considered briefly.

2. THE CASE OF AN IMPERFECT WORLD CAPITAL MARKET

As in Chapter IV, we regard as imperfect a capital market which does not allow or severely restricts consumption borrowing. Such an imperfect capital market is assumed in Bardhan (1967), Hamada (1966) and (1969), Pitchford (1970), Manning (1972) and Bade (1972).

2.1 Summary of Results :

We classify replenishable resources into two categories : those that are "scarce" and those that are "potentially plentiful". The meanings of these words will be made clear later, but broadly they

¹ A version of this Chapter will appear in Pitchford and Turnovsky (1975).

are defined relative to technological conditions and the foreign supply of capital.

For scarce resources, restraint on the exploitation is always required - in the sense that the marginal cost of borrowing should be kept below the marginal product of capital in the extraction activity. Thus in the case of a common property resource, a tax on exploitation and a tax on foreign borrowing have to be determined simultaneously.

For potentially plentiful resources, multiple equilibria may occur. Depending on the initial resource stock, the optimal path may lead to an equilibrium at which restraint is required or to an equilibrium at which the marginal cost of borrowing and the marginal product of capital are equalized. The path to the latter type of equilibrium may or may not require restraint on exploitation.

Along a path with no restraint on exploitation, naturally there is no need for artificial renewal. The renewing activity is used only when restraint is exercised on the exploitation. In this case, the marginal cost of borrowing is equated with the *imputed value* of the marginal product of capital in the renewing activity.

If the society has a strong conservation motive (by this we mean that the marginal utility of the resource stock is always positive) or if a larger resource stock always facilitates exploitation, then restraint is always required. This does not necessarily mean that the resource is conserved indefinitely : if the social rate of discount is sufficiently high it may be optimal to exhaust the resource. Along the optimal exhaustion path the gap between the marginal product of capital and the marginal cost of borrowing is positive and increasing.

Under certain assumptions, an equilibrium exists at which the marginal cost of borrowing *exceeds* the marginal product of capital in the exploitation activity. This equilibrium can be shown to be sub-optimal as well as unstable. There is also a saddle trajectory (leading to an equilibrium) along which the marginal cost of borrowing *exceeds* the marginal product of capital. Again this trajectory is shown to be non-optimal.

2.2 The Production and Cost Assumptions :

Let $R(t)$ denote the stock of resource at time t . Without commercial exploitation and artificial renewal, the resource stock grows or declines according to its law of natural growth :

$$\dot{R}(t) = G[R(t)] \quad (1)$$

It is assumed that $G(R)$ is twice differentiable and strictly concave, with $G'(0) > 0$ and :

$$G(0) = G(\bar{R}) = 0 \quad (2)$$

for some $\bar{R} > 0$. (2) and the strict concavity of $G(R)$ implies there is a unique R_M at which $G'(R_M) = 0$. The function $G(R)$ is illustrated in figure 1. A special case of $G(R)$ is the quadratic function used in Plourde (1970) and (1971), Crutchfield and Zellner (1962), Schaefer (1967) Beverton and Holt (1957) and Lotka (1956). Brown (1974) considers a special case where $G(R) = Rf(R)$, with $f' < 0$, $f'' \leq 0$.

There are two production activities : exploitation and artificial renewal. Labour will not appear as an argument in the two production functions, for simplicity. A possible interpretation

is that labour is immobile. Alternatively, if labour is used in the same fixed proportion with capital in both sub-firms, then there is no need to introduce labour explicitly.

The amount of resource exploited is then a function F of capital used $K_1(t)$ and of the resource stock R . We assume $F(K_1, R)$ is concave, with $F(0, R) = 0$, $F_{K_1} > 0$, $F_{K_1 K_1} \leq 0$, $F_{R R} \leq 0$, and $F_{K_1 R} = F_{R K_1} \geq 0$. $F_{K_1} > 0$ implies that for any given amount of capital used, a smaller output is obtained if the resource stock is smaller. The following terminology will be adopted: If $F_{K_1 R} = 0$ for all R , we say that there is no depletion effect on production. When $F_{K_1 R} > 0$ for $R \geq \bar{R}$ for some $\bar{R}$, the depletion effect is locally absent.

The resource stock can be artificially increased by using capital K_2 in the resource renewal activity, $Q(K_2)$. We assume Q is concave, with $Q(0) = 0$, $Q' > 0$, $Q'' \leq 0$. The possibility that $Q(K_2) > 0$ for all K_2 is allowed. $Q(K_2)$ is bounded above by some G .

Figure 1

The boundedness of $Q(K_2)$ implies that $G(R) < \infty$ for large R , since $G(R) < \infty$ for $R > \bar{R}$.

The cost of capital is the same as in Chapter IV (Section 2.2):

$$C(K) = r_K K_d + r(K_p) K_p$$

where $K_p = K_d + K = K_1 + K_2$.

is that labour is immobile. Alternatively, if labour is used in the same fixed proportion with capital in both activities, then there is no need to introduce labour explicitly.

The amount of resource exploited is then a function F of capital used $K_1(t)$ and of the resource mass $R(t)$. We assume $F(K_1, R)$ is concave, with $F(0, R) = 0$, $F_1 > 0$, $F_2 \geq 0$, $F_{11} \leq 0$, $F_{22} \leq 0$, and $F_{12} = F_{21} \geq 0$. $F_2 > 0$ implies that for any given amount of capital used, a smaller output is obtained if the resource stock is smaller. The following terminology will be adopted: If $F_2 = 0$ for all R , we say that there is *no depletion effect* on production. When $F_2 = 0$ for $R \geq R_1$ for some R_1 , the depletion effect is locally absent.

The resource stock can be artificially increased by using capital K_2 in the resource renewal activity, $Q(K_2)$. We assume Q is concave, with $Q(0) = 0$, $Q' \geq 0$, $Q'' \leq 0$. The possibility that $Q(K_2) = 0$ for all K_2 is allowed. $Q(K_2)$ is bounded above by some $\bar{Q}$.

From the above information we write :

$$\dot{R}(t) = -F(K_1, R) + Q(K_2) + G(R)$$

The boundedness of $Q(K_2)$ implies that $\dot{R}(t) < 0$ for large R , since $G(R) < 0$ for $R > \bar{R}$.

The cost of capital is the same as in Chapter IV (Section 2.2) :

$$\phi(K) = r_o K_d + r(K_f) K_f$$

where $K_f + K_d = K = K_1 + K_2$.

As in Chapter IV, it is assumed that capital is perfectly mobile and no capital accumulation takes place.

2.3 The Necessary Conditions :

The social objective is to maximize the discounted stream of net output in terms of the resource good. That is to find $K_i(t)$ and T (possibly infinite) to :

$$\text{Max}_{K_i(t)} \int_0^T [F(K_1, R) - \phi(K_1 + K_2)] \exp(-\delta t) dt.$$

subject to :

$$K_i \geq 0$$

$$R(0) = R_0 > 0$$

$$R(t) \geq 0$$

$$\dot{R}(t) = -F[K_1(t), R(t)] + Q[K_2(t)] + G[R(t)]$$

$$\dot{R} \geq 0 \text{ for } R = 0$$

Following Arrow and Kurz (1970), proposition 5, we form the Lagrangean :

$$\begin{aligned} L = & F(K_1, R) - \phi(K_1 + K_2) + \psi[Q(K_2) - F(K_1, R) + G(R)] \\ & + \lambda_1 K_1 + \lambda_2 K_2 + \lambda_3 \dot{R} \end{aligned} \quad (3)$$

The necessary conditions for an optimal path are :

$$\frac{\partial L}{\partial K_1} = (1 - \psi - \lambda_3) F_1 - \phi' + \lambda_1 = 0 \quad (4)$$

$$\frac{\partial L}{\partial K_2} = (\psi + \lambda_3) Q' - \phi' + \lambda_2 = 0 \quad (5)$$

$$\dot{\psi} = \psi(\delta - G' + F_2) + \lambda_3(-G' + F_2) - F_2 \quad (6)$$

$$\dot{R} = -F(K_1, R) + Q(K_2) + G(R) \quad (7)$$

$$\lambda_3 \geq 0, \lambda_3 \dot{R} = \lambda_3 \dot{R} = 0, \dot{R} \geq 0 \text{ when } R = 0 \quad (8)$$

$$\lambda_2 \geq 0, \lambda_2 K_2 = 0, K_2 \geq 0 \quad (9)$$

$$\lambda_1 \geq 0, \lambda_1 K_1 = 0, K_1 \geq 0 \quad (10)$$

and for T finite, $e^{-\delta T} \psi(T) \geq 0$, $e^{-\delta T} \psi(T) R(T) = 0$.

Note that the Lagrangean (3) is *not concave* in R nor in K_2 when $\psi(t) < 0$. Thus the above necessary conditions are *not sufficient*. In fact we shall show that there is a special case of our model in which there exists a path satisfying all the above necessary conditions which is not optimal.

When $\psi(t) > 1$, the Lagrangean (3) is *not concave* in R nor in K_1 . A close inspection of (4), however, shows that when $\psi(t) \geq 1$, λ_1 must be positive to ensure $\partial L / \partial K_1 = 0$. Intuitively, when the shadow price $\psi(t)$ of the *unharvested* resource exceeds the price of the *harvested* resource good ($= 1$), no exploitation should take place. Similarly, when $\psi(t) \leq 0$, (5) implies that $K_2(t) = 0$.

From the above observations, for given $R(t)$ and $\psi(t)$, $K_1(t)$ is unique if $F(K_1, R)$ is strictly concave in K_1 or if $\phi(K_1 + K_2)$ is strictly convex; $K_2(t)$ is unique if $Q(K_2)$ is identically zero, or if $Q(K_2)$ is strictly concave, or $\phi(K_1 + K_2)$ strictly convex.

$K_1(t)$ and $K_2(t)$ can be positive simultaneously only if $\psi(t) \in (0, 1)$. When $R(t)$ and both $K_1(t)$ and $K_2(t)$ are positive, we can differentiate the system (4) - (5) totally :

$$[(1-\psi)F_{11} - \phi'']dK_1 - \phi''dK_2 = F_1d\psi - (1-\psi)F_{12}dR \quad (11a)$$

$$- \phi''dK_1 + (\psi Q'' - \phi'')dK_2 = -Q'd\psi \quad (11b)$$

The matrix J of the coefficients on the left-hand side of system (11) has negative first order principal minors and positive determinant since $0 < \psi < 1$. Thus by Theorem 4 of Gale and Nikaido (1965) the solution is unique.

Let Δ be the determinant of matrix J . One can solve for the partial derivatives of the "derived demand" equations (4) and (5) (again assuming $\lambda_i = 0$)

$$\frac{\partial K_1}{\partial \psi} = \frac{F_1(\psi Q'' - \phi'') - Q'\phi''}{\Delta} < 0 \quad (12a)$$

$$\frac{\partial K_2}{\partial \psi} = \frac{-[(1-\psi)F_{11} - \phi'']Q' + \phi''F_1}{\Delta} > 0 \quad (12b)$$

$$\frac{\partial K_1}{\partial R} = \frac{(1-\psi)F_{12}(\phi'' - Q''\psi)}{\Delta} \geq 0 \quad (12c)$$

(with strict inequality if and only if $F_{12} > 0$)

$$\frac{\partial K_2}{\partial R} = \frac{-\phi''(1-\psi)F_{12}}{\Delta} \leq 0 \quad (12d)$$

(with strict inequality if and only if $\phi''F_{12} > 0$)

(12a) and (12b) have a simple interpretation: as the shadow price of the resource rises, less capital should be devoted to the exploitation and more should be used for the renewal of the resource. (12c) and (12d) say if $F_{12} > 0$, more capital should be used in the exploitation as the resource stock gets larger, in order to maintain

the equality between the (adjusted) marginal product of capital with its marginal cost; while the amount of capital used in the renewal need to be reduced only if the increase in K_1 affects the marginal cost of borrowing.

When $K_1(t) = 0$ and $K_2(t) > 0$, $\partial K_2 / \partial \psi$ and $\partial K_2 / \partial R$ can be solved using (5) alone :

$$\partial K_2 / \partial \psi = - Q' / (\psi Q'' - \phi'') \quad (12a')$$

which is positive since $K_2(t) > 0$ only if $\psi(t) > 0$.

$$\partial K_2 / \partial R = 0 \quad (12d')$$

Similarly, when $K_2(t) = 0$ and $K_1(t) > 0$,

$$\partial K_1 / \partial \psi = F_1 / [(1-\psi)F_{11} - \phi''] < 0 \quad (12b')$$

$$\partial K_1 / \partial R = - (1-\psi)F_{12} / [(1-\psi)F_{11} - \phi''] \geq 0 \quad (12c')$$

2.4 A Simple Case : No Depletion Effect :

If $F_2 = 0$ identically, we say that there is no depletion effect. The analysis in section 2.3 for the general case can be used, bearing in mind that $F_{12} = F_{21} = F_{22} = 0$ in this case. We will use $F_1(K_1)$ and $F'(K_1)$ interchangeably. From (12), $K_1(t)$ and $K_2(t)$ are functions of $\psi(t)$ alone.

When $R(t) > 0$, we have, from (6) :

$$\dot{\psi}(t) = \psi(t) [\delta - G'(R)] \quad (13)$$

Define

$$N(\psi, R) = \psi[\delta - G'(R)] \quad (14)$$

$$M(\psi, R) = -F[K_1(\psi), R] + Q[K_2(\psi)] + G(R) \quad (15)$$

The locus of points in the (ψ, R) space such that $\dot{\psi}(t) = 0$ is thus the horizontal (R) axis and the vertical line crossing R_δ , where R_δ is the solution of

$$G'(R) = \delta$$

We assume, for the time being, that $G'(0) > \delta$ so that R_δ exists and is positive.

From (13), $\dot{\psi} > 0$ in the set

$$\{(\psi, R) \mid 0 < R < R_\delta \text{ and } \psi < 0\}$$

and the set :

$$\{(\psi, R) \mid R > R_\delta \text{ and } \psi > 0\}$$

and $\dot{\psi} < 0$ in the set :

$$\{(\psi, R) \mid 0 < R < R_\delta \text{ and } \psi > 0\}$$

and the set :

$$\{(\psi, R) \mid R > R_\delta \text{ and } \psi < 0\}$$

The $\dot{R} = 0$ curve has slope

$$\left. \frac{d\psi}{dR} \right|_{R=0} = - (\partial M / \partial R) / (\partial M / \partial \psi) \Big|_{M(\psi, R)=0}$$

where

$$\partial M / \partial R = G'(R) \tag{16a}$$

$$\partial M / \partial \psi = Q' \partial K_2 / \partial \psi - F_1 \partial K_1 / \partial \psi > 0 \tag{16b}$$

Thus $\dot{R} > 0$ above the $\dot{R} = 0$ curve and $\dot{R} < 0$ below the $\dot{R} = 0$ curve. The $\dot{R} = 0$ curve is negatively sloped for $R < R_\delta$ and positively sloped for $R > R_\delta$ due to (13). Note that $\dot{R} = 0$ implies $P(K_1) > 0$ for $0 < R < R_\delta$. Hence

a) Scarce resource:

It is immediate from the definition that if the resource is scarce then $\dot{R} < 0$ when $\psi \geq 0$. This is because when $\psi \geq 0$ no removal takes place ($\dot{R} = 0$) and $\dot{R} < 0$ (hence $P(K_1) > 0$) and $\dot{R} < 0$ (hence $P(K_1) > 0$). These conclusions are derived using (4) and (5).

Figure 2

Thus $\dot{R} > 0$ above the $\dot{R} = 0$ curve and $\dot{R} < 0$ below the $\dot{R} = 0$ curve. The $\dot{R} = 0$ curve is negatively sloped for $R < R_M$ and positively sloped for $R > R_M$, due to (15). Note that $\dot{R} = 0$ implies $F(K_1) > 0$ for $0 < R < \bar{R}$. Hence :

$$\lim_{R \rightarrow 0} \psi \Big|_{\dot{R}=0} = \bar{\psi} \leq 1$$

where $F[K_1(\bar{\psi})] - Q[K_2(\bar{\psi})] = 0$ defines $\bar{\psi}$ implicitly. If $Q = 0$ identically or if $Q'(0) \leq \phi'(0)$ then $K_2 = 0$ for $\psi \leq 1$. In this case $\bar{\psi} = F_1(0)/[F_1(0) - \phi'(0)] < 1$.

The $\dot{R} = 0$ curve may or may not intersect the horizontal axis, depending on the relationship between the natural growth rate $G(R)$ on the one hand, and the production and cost functions on the other hand. Let us define the instantaneous profit maximizing level of output by $F(\bar{K}_1)$ where $\bar{K}_1$ is the solution of $F_1(K_1) = \phi'(K_1)$, K_2 being held at zero. When $F(\bar{K}_1) > G(R_M)$ we say that the resource is "scarce", in the sense that the unrestrained exploitation will eventually exhaust the resource whatever the initial stock is. When the resource is not scarce in this sense, we say the resource is "potentially plentiful". This distinction has important fiscal implications. We shall analyse the two cases separately.

a) Scarce resource:

It is immediate from our definition that if the resource is scarce then $\dot{R} < 0$ when $\psi \leq 0$. This is because when $\psi \leq 0$ no renewal takes place ($K_2=0$) and $K_1 \geq \bar{K}_1$ (hence $F(K_1) - G(R) \geq F(\bar{K}_1) - G(R)$). These conclusions are derived using (4) and (5).

Figure 2 is the phase diagram for this case. The system (6) and (7) has only one stationary point : (ψ_δ, R_δ) . The local behaviour of the system

$$\dot{\psi} = N(\psi, R) \quad (17a)$$

$$\dot{R} = M(\psi, R) \quad (17b)$$

in the neighbourhood of the equilibrium point (ψ_δ, R_δ) can be characterized using linear approximation :

$$\begin{pmatrix} \dot{\psi} \\ \dot{R} \end{pmatrix} = S \begin{pmatrix} \psi(t) - \psi_\delta \\ R(t) - R_\delta \end{pmatrix} \quad (18)$$

where S is the matrix of coefficients :

$$S = \begin{pmatrix} \partial N / \partial \psi & \partial N / \partial R \\ \partial M / \partial \psi & \partial M / \partial R \end{pmatrix} \quad (19)$$

where the partial derivatives are evaluated at the equilibrium point (at (ψ_δ, R_δ) in this case).

The eigenvalues of the matrix S are :

$$\mu_1 = [(\partial N / \partial \psi + \partial M / \partial R) + D^{1/2}] / 2 \quad (20a)$$

$$\mu_2 = [(\partial N / \partial \psi + \partial M / \partial R) - D^{1/2}] / 2 \quad (20b)$$

where

$$D = (\partial N / \partial \psi + \partial M / \partial R)^2 - 4[(\partial N / \partial \psi)(\partial M / \partial R) - (\partial N / \partial R)(\partial M / \partial \psi)] \quad (21)$$

(all the expressions are evaluated at ψ_δ, R_δ).

Using (13) - (16) and the fact that $\psi_\delta > 0$:

$$D = \delta^2 + 4(-\psi_\delta G'')(\partial M/\partial \psi) > 0 \quad (22a)$$

$$\mu_1 = (\delta + D^{1/2})/2 > 0 \quad (22b)$$

$$\mu_2 = (\delta - D^{1/2})/2 < 0 \quad (22c)$$

The eigenvalues are real numbers with opposite signs. Thus the equilibrium has the saddle point property. Given any initial resource stock R_0 , the controller selects ψ_0 such that (R_0, ψ_0) is on the stable branch leading to (ψ_δ, R_δ) . This path is the optimal path, as will be proved in Appendix V.A.

Along the stable branch $\psi > 0$, implying that the marginal product of capital used in the exploitation of the resource should be kept above its marginal cost $((1-\psi)F_1 = \phi')$. As ψ has the interpretation of the marginal social value of the resource, one may think of $(1-\psi)F_1$ as the marginal product of capital K_1 adjusted for the resource cost.

Without further information on the various marginal products one cannot determine whether capital is ever used for the renewal of the resource. If $Q'(0) = \infty$, K_2 will be positive along the optimal path. Note that at the equilibrium $\psi_\delta < 1$, hence $Q' > \phi'$. This is because while Q' is the marginal physical product in terms of the *unharvested* resource good, ϕ' is marginal cost in terms of the *harvested* resource good.

It will be noted that the equilibrium R_δ is a monotonic decreasing function of the social rate of discount (this is not true in general when the depletion effect is introduced, as will be seen later). It is clear that for a sufficiently high rate of discount, there exists no interior equilibrium.

Finally note that the optimal stationary resource stock, R_δ , is independent of small changes in technology and cost (provided that the resource remains scarce). It is a function of the rate of discount and of the natural growth rate alone.

b) Potentially plentiful resource :

The resource is "potentially plentiful" if $G(R_M) \geq F(\bar{K}_1)$. Except for the borderline case where $G(R_M) = F(\bar{K}_1)$, there exist $R_1 < R_M$ and $R_2 > R_M$ such that $G(R_1) = G(R_2) = F(\bar{K}_1)$. When $\psi = 0$ and $R > 0$, the optimal choice of K_1 and K_2 are $K_1 = \bar{K}_1$ and $K_2 = 0$, by (4) and (5). Hence at $(0, R_1)$ and $(0, R_2)$, $\dot{R} = \dot{\psi} = 0$.

The relationship between $F(\bar{K}_1)$ and $G(R_\delta)$ is of interest : if $F(\bar{K}_1) > G(R_\delta)$, then there exists $\psi_\delta > 0$ such that (ψ_δ, R_δ) is an optimal stationary point, in addition to $(0, R_1)$ and $(0, R_2)$. If $F(\bar{K}_1) < G(R_\delta)$, then R_δ is not an optimal stationary resource stock. This is because in order to keep the resource stock stationary at R_δ , exploitation must be subsidized, and such a policy is obviously non-optimal. The two cases

$$i) \quad F(\bar{K}_1) > G(R_\delta), \quad \text{and}$$

$$ii) \quad F(\bar{K}_1) < G(R_\delta)$$

will be considered in some details.

$$\text{Case (i) :} \quad F(\bar{K}_1) > G(R_\delta).$$

Figure 3 is the phase diagram for this case. Since $G(R_1) = F(\bar{K}_1)$ and $R_1 < R_M$ by definition, $G(R_1) > G(R_\delta)$ and $G'(R_1) < G'(R_\delta) = \delta$. There exist three equilibrium : (ψ_δ, R_δ) , $(0, R_1)$ and $(0, R_2)$. The equilibrium $(0, R_1)$ is unstable and the other two

Figure 3

Figure 4

are saddle points. This can be established by an analysis similar to that used in the preceding sub-section.

At (ψ_δ, R_δ) , the eigenvalues are given by (22). They are real numbers with opposite signs.

At $(0, R_2)$, the eigenvalues are :

$$\mu_1 = (\delta + D^{\frac{1}{2}})/2$$

$$\mu_2 = (\delta - D^{\frac{1}{2}})/2$$

$$D = \delta^2 - 4[\delta - G'(R_2)] \cdot G'(R_2) > \delta^2$$

(because $G'(R_2) < 0$).

Therefore μ_1 and μ_2 are real and opposite in sign.

At $(0, R_1)$ the eigenvalues are :

$$\mu_1 = (\delta + D^{\frac{1}{2}})/2$$

$$\mu_2 = (\delta - D^{\frac{1}{2}})/2$$

$$D = \delta^2 - 4[\delta - G'(R_1)] \cdot G'(R_1) < \delta^2$$

(because $G'(R_1) > 0$ and is less than δ).

If $D < 0$, then μ_1 and μ_2 are complex numbers with positive real parts. If $D \geq 0$, μ_1 and μ_2 are both positive real numbers.

In either case, the equilibrium $(0, R_1)$ is unstable.³

The optimal path for given any R_0 can now be analysed. If the initial resource stock is at least as large as R_1 , no social intervention is required : instantaneous profit maximization (in the

³ Figure 3 suggests that $D \geq 0$.

sense $F_1(K_1) = \phi'(K_1)$, and $K_2 = 0$) is optimal. If $R_0 = R_1$, the system stays there. If $R_0 > R_1$, however, the system moves to R_2 along the line $\psi = 0$ as a result of the natural growth of the resource. If $R_0 < R_1$, the system should be controlled to move to R_δ . Along the optimal path, some output restriction is required. The equilibrium at R_1 gives a larger net output than the equilibrium at R_δ but the cost (in terms of consumption foregone) of reaching R_1 from $R_0 < R_1$ is too high. The equilibrium at R_δ corresponds to the well-known "modified golden rule" for optimal capital accumulation.

It is not surprising that when the initial level of resource is sufficiently large that instantaneous profit maximization ($F_1 = \phi'(K_1)$) will not exhaust the resource, there is no need for social intervention to restrict output. This result does not hold, however, if the production is a function of the resource stock so that a smaller resource stock means less output is obtained from given amounts of inputs. This "depletion effect" introduces an element of externality into the problem. It will be shown in Section 2.5 that some tax to restrict output is *always* required if the depletion effect is present for *all* level of resource.

Case (ii) : $F(\bar{K}_1) < G(R_\delta)$.

In this case $R_1 < R_\delta < R_2$. There are three equilibria : $(0, R_1)$, (ψ_δ, R_δ) , and $(0, R_2)$, as depicted in figure 4. The equilibria $(0, R_1)$ and $(0, R_2)$ have the saddle point property while (ψ_δ, R_δ) is completely unstable.

At $(0, R_1)$ the eigenvalues are :

$$\mu_1 = (\delta + D^{\frac{1}{2}})/2$$

$$\mu_2 = (\delta - D^{\frac{1}{2}})/2$$

where $D = \delta^2 - 4[\delta - G'(R_1)] \cdot G'(R_1) > \delta^2$ because $G'(R_1) > G'(R_\delta) = \delta > 0$. Therefore, μ_1 and μ_2 are real and have opposite signs. This result contrasts with case (i).

At $(0, R_2)$, the eigenvalues are the same as in case (i) : they are real and have opposite signs.

At (ψ_δ, R_δ) , the eigenvalues are :

$$\mu_1 = (\delta + D^{\frac{1}{2}})/2$$

$$\mu_2 = (\delta - D^{\frac{1}{2}})/2$$

where $D = \delta^2 + 4(-\psi_\delta G'')(\partial M/\partial \psi) < \delta^2$ because $\psi_\delta < 0$, and $\partial M/\partial \psi > 0$ by (16b). If $D < 0$, then μ_1 and μ_2 are complex numbers with positive real parts. If $D \geq 0$, μ_1 and μ_2 are both positive real numbers. In either case, the equilibrium (ψ_δ, R_δ) is unstable. Figure 4 seems to suggest that the eigenvalues are complex.

The optimal paths for the case $F(\bar{K}_1) < G(R_\delta)$ can now be characterized :

For $R_0 \leq R_1$, the stable branch of $(0, R_1)$ is the optimal trajectory. For $R_0 > R_1$, however, the optimal trajectory is the stable branch of $(0, R_2)$. For $R_1 < R_0 < R_\delta$ there is a path leading to R_1 (the stable branch of R_1 , with negative shadow price) but this path is *non-optimal*. The correct path is the path leading to $(0, R_2)$ along $\psi = 0$. This result is intuitively plausible : along this path the exploitation is *not subsidized nor taxed*, while along the stable branch of $(0, R_1)$ where $\psi < 0$, the

exploitation is encouraged to exceed the instantaneous profit maximizing level and such a path is certainly non-optimal. This serves as an example to show that a stable branch leading to a saddle point equilibrium need not be an optimal path and that necessary conditions alone can mislead.

Another interesting point is that while (ψ_δ, R_δ) is an equilibrium in figure 4, this equilibrium is not optimal even if the initial resource stock is R_δ . We can show that for all $R_0 > R_1$ the optimal trajectory is the stable branch of (ψ_2, R_2) where $\psi_2 = 0$. That is, the path along which $\psi = 0$ is optimal. Appendix V.A proves the assertions made in this and the preceding paragraphs.

2.5 Depletion Effects :

We now consider the case where the resource enters into the production function directly. For definiteness we assume that $F_1 > 0$, $F_2 > 0$, $F_{12} = F_{21} > 0$, $F_{11} < 0$, $F_{22} < 0$, $F(K_1, 0) - F(0, R) = 0$, $F_1(0, R) = \infty$ and $F_2(K_1, 0) = \infty$.

The phase diagram for this case is rather ambiguous. Consider the locus of $\dot{\psi} = 0$. For $R > 0$, $\dot{\psi} = 0$ if and only if $\psi = F_2 / (\delta - G' + F_2)$. Therefore for $R = R_\delta$, $\dot{\psi} = 0$ when $\psi = 1$. For $R < R_\delta$, $\dot{\psi} = 0$ implies either $\psi < 0$ or $\psi > 1$, and $\dot{\psi} < 0$ for $0 > \psi > 1$. For $R > R_\delta$, $\dot{\psi} = 0$ implies $0 < \psi < 1$.

The slope of the $\dot{\psi} = 0$ curve is given by :

$$\frac{d\psi}{dR} = - \frac{\frac{\partial \dot{\psi}}{\partial R}}{\frac{\partial \dot{\psi}}{\partial \psi}} = - \frac{-G''\psi - (1-\psi)(F_{22} + F_{12}\frac{\partial K_1}{\partial R})}{\delta - G' + F_2 - (1-\psi)F_{12}\frac{\partial K_1}{\partial \psi}}$$

which is negative for $R > R_\delta$, because the concavity of $F(K_1, R)$ and (12c) imply that

$$F_{22} + F_{12} \partial K_1 / \partial R < 0.$$

As R tends to infinity, F_2 tends to zero. Hence the curve $\dot{\psi} = 0$ is asymptotic to the R -axis.

For $R > R_\delta$, $\dot{\psi} > 0$ above the $\dot{\psi} = 0$ curve and $\dot{\psi} < 0$ below that curve ($\partial \dot{\psi} / \partial \psi > 0$ for $R > R_\delta$).

The slope of the $\dot{R} = 0$ curve is given by :

$$\frac{d\psi}{dR} = - \frac{\partial \dot{R} / \partial R}{\partial \dot{R} / \partial \psi} = - \frac{G' - F_2 - F_1 \partial K_1 / \partial R}{- F_1 \partial K_1 / \partial \psi + Q' \partial K_2 / \partial \psi}$$

which is positive if $G' < F_2 + F_1 \partial K_1 / \partial R$, which is true for $R > R_M$. For $R < R_M$, the slope of the $\dot{R} = 0$ curve is ambiguous. We know however that the $\dot{R} = 0$ curve is below the line $\psi = 1$ for $R < R_M$, since $F(K_1, R) > 0$ only if $\psi \leq 1$.

From these considerations there must be at least one equilibrium for $R > R_\delta$. There may be more than one equilibria in this range but at least one of them must be a saddle point. In order to see this, note that an equilibrium has the saddle point property if $(\partial \dot{\psi} / \partial \psi)(\partial \dot{R} / \partial R) < (\partial \dot{\psi} / \partial R)(\partial \dot{R} / \partial \psi)$ which is equivalent to

$$(\partial \dot{\psi} / \partial R) / (\partial \dot{\psi} / \partial \psi) > (\partial \dot{R} / \partial R) / (\partial \dot{R} / \partial \psi)$$

because $\partial \dot{\psi} / \partial \psi > 0$ and $\partial \dot{R} / \partial \psi > 0$ for $R > R_\delta$ and $0 < \psi < 1$.

The condition is again equivalent to the requirement that the $\dot{R} = 0$ curve be positive, or negative but less steep than the $\dot{\psi} = 0$ curve (which is negatively sloped for $R > R_\delta$). Figure 5 illustrates a possible situation.

Figure 5

Figure 6

Along the stable branches leading to the saddle point equilibria, the shadow price ψ is positive, implying that the marginal product of capital, F_1 , must exceed its marginal cost. Thus, if $F_2 > 0$ for all R , output restriction is always required, even when the resource is plentiful - as at R_3 and R_4 . Appendix V.A proves that stable branches along which $\psi \geq 0$ are optimal paths.

Using (12a) - (12d) it is easily seen that an equilibrium with a larger R (and hence smaller ψ) yields a larger net output. It is not always optimal to move to an equilibrium associated with a higher net output because the adjustment cost may be too high. Consider figure 5. For $R_5 > R_0 > R_2$ the society is willing to move to R_5 : the adjustment cost is small compared with the (discounted) benefits of higher future consumption. If $R_0 = R_2$, however, the adjustment cost (to move to R_5) becomes so high that the society prefers to stay at R_2 . If $R_0 < R_2$ the cost of building up the resource to R_2 , in terms of "current" consumption foregone, is too high (and so is the cost of maintaining R_0), and the society finds it optimal to sacrifice some welfare of remote future generations in exchange for higher present consumption levels.

2.6 Optimal Exhaustion of a Replenishable Resource :

So far we have assumed that $G'(0) > \delta$. If $G'(0) < \delta$, then :

- i) for the case of a scarce resource, with no depletion effect, there exists no interior equilibrium.
- ii) for other cases, interior equilibria may exist but for a sufficiently small initial resource stock, it is optimal to exhaust the resource.

For simplicity we consider the case where there is no possibility of resource renewal ($Q = 0$ identically) or the cost of renewal is "prohibitive" (i.e. for $\psi \leq 1$, $Q'(K_2) < \phi'(K)$ for all positive K and K_1). Again we assume that $F_1(0, R) = \infty$, $F_2(K_1, 0) = \infty$ and $F(0, R) = F(K_1, 0) = 0$. These assumptions imply that $\dot{R} = 0$ when $R = 0$ and that the $\dot{R} = 0$ curve tends to $\psi = 1$ as R tends to zero.

Figure 6 depicts a situation where there is no equilibrium with $R > 0$. The point $(R = 0, \psi = 1)$ is an optimal terminal point. The path leading to it is the optimal trajectory. It is the longest exhaustion path satisfying the necessary conditions (4) - (10). Paths above it end up with an unused resource stock. Paths below it exhaust the resource more quickly because $\partial K_1 / \partial \psi < 0$. These assertions are easily proved using the arguments made in Appendix IV of the preceding chapter.

2.7 Fiscal Implications :

The necessary condition (4) says that along the optimal path, if ψ is positive, the fiscal implications of this model is the same as those of the previous chapter. In particular, formula (17) of chapter IV applies to this case. The tax rule therefore differs from the static tax formula for an optimal borrowing policy.

There is an important difference between the replenishable resource and the non-replenishable resource cases : In chapter IV, the shadow price ψ is always positive along an optimal path, implying the need for restraint, while in this chapter, it is possible that $\psi = 0$ along an optimal path - in which case the dynamic considerations become irrelevant, and the static tax formula on foreign capital applies without qualifications.

3. THE CASE OF A PERFECT WORLD CAPITAL MARKET :

When consumption borrowing is allowed *and* the foreign supply of capital is perfectly elastic, the analysis in Section 3 of the preceding chapter applies without any essential changes. The only difference is that the first step - maximizing the "net worth" - may not require the restraint on exploitation, due to the abundance of the resource in relation to costs and technology. It is important to note that if foreign capital becomes cheaper, a resource which was potentially plentiful may become scarce, and some restraint on exploitation is then required.

APPENDIX VA

Proof that (i) for any initial resource stock R_0 any trajectory leading to an equilibrium is an optimal path, provided $R > 0$, $\psi \geq 0$ along the path and at the equilibrium, and (ii) given R_0 , the optimal path is unique. This proof differs from the standard proof because the Hamiltonian is *not* concave in K_1 , K_2 and R , as noted in 2.3.

Let the asterisk (*) denote the values taken by variables along the path to be proved as optimal, we compare this path with any other path. $T = \infty$, in the following expressions.

$$\begin{aligned}
 & \int_0^T (F^* - \phi^* - F + \phi) e^{-\delta t} dt \\
 &= \int_0^T e^{-\delta t} \{ F^* - \phi^* - F + \phi + [\psi^* (F_2^* - G^{*'} + \delta) - F_2^* \dot{\psi}^*] (R^* - R) \} dt \\
 &= \int_0^T e^{-\delta t} [F^* - \phi^* - F + \phi - G^{*'} \cdot (R^* - R) + (1 - \psi^*) F_2^* \cdot (R^* - R)] dt \\
 &\quad - \int_0^T (e^{-\delta t} \dot{\psi}^* - \delta \psi^* e^{-\delta t}) (R^* - R) dt \\
 &= \text{first integral} - (R^* - R) e^{-\delta t} \psi^* \Big|_0^T + \int_0^T e^{-\delta t} \psi^* (\dot{R}^* - \dot{R}) dt
 \end{aligned}$$

(by integrating by parts)

$$\begin{aligned}
& \geq \int_0^T e^{-\delta t} [(1-\psi^*)(F^*-F) + \psi^*(Q^*-Q) - (\phi^*-\phi) - (1-\psi^*)F^*_2(R^*-R)] dt \\
& \quad + \int_0^T e^{-\delta t} \psi^*[G^*-G-G^*(R^*-R)] dt \\
& > \int_0^T e^{-\delta t} [(1-\psi^*)(F^*_1)(K^*_1-K_1) + \psi^*Q^{*'}(K^*_2-K_2) - \phi^{*'}(K^*-K)] dt \\
& \geq \int_0^T e^{-\delta t} \{ (K^*_1-K_1) [(1-\psi^*)F^*_1-\phi^{*'}+\lambda^*_1] + (K^*_2-K_2) (\psi^*Q^{*'}-\phi^{*'}+\lambda^*_2) \} dt \\
& = 0
\end{aligned}$$

Thus referring to figure 4 the stable branch of the saddle point $(0, R)$ along which $\psi < 0$ is inferior to the *unstable* branch of the saddle point $(0, R_1)$ - which is the stable branch of the saddle point $(0, R_2)$. The equilibrium point (ψ_δ, R_δ) in figure 4 is non-optimal.

APPENDIX VB

The effect of a small change in the discount rate on the optimal stationary resource stock when the depletion effect is present ($F_2 > 0$).

At an equilibrium (with $R > 0$)

$$\psi(\delta - G' + F_2) - F_2 = 0$$

$$- F(K_1, R) + Q(K_2) + G(R) = 0$$

Differentiate the above system with respect to δ :

$$a \partial \psi / \partial \delta + b \partial R / \partial \delta = -\psi$$

$$c \partial \psi / \partial \delta + d \partial R / \partial \delta = 0$$

where

$$a = \delta - G' + F_2 - (1-\psi)F_{21}K_1/\partial\psi > 0 \text{ for } \delta > G \text{ and } \psi < 1$$

$$b = -\psi G''(1-\psi)(F_{22} + F_{21}K_1/\partial R) > 0 \text{ for } 0 < \psi < 1$$

$$c = -F_1 \partial K_1 / \partial \psi + Q' \partial K_2 / \partial \psi > 0$$

$$d = -F_1 \partial K_1 / \partial R - F_2 + G' \geq 0$$

Hence; for $\psi > 0$, assuming $ad - bc \neq 0$

$$\frac{\partial N}{\partial \delta} = \frac{c\psi}{ad - bc} \geq 0 \text{ according to } ad - bc \geq 0$$

At a saddle point equilibrium $ad - bc < 0$, while $ad - bc > 0$ at an unstable equilibrium.

CHAPTER VI

RESOURCE EXTRACTION UNDER THE UNCERTAINTY ABOUT POSSIBLE NATIONALIZATION.¹

1. INTRODUCTION :

Recent concerns about possible over-exploitation of resources arise from various defects of the market economy. Petroleum reservoirs, for example, may be over-exploited due to their "common property" characteristic. Short-term leases create what may be termed "dynamic common properties". These were considered in Chapter IV. A phenomenon which bears similar characteristics is the exploitation under anticipated nationalization - this is particularly relevant for less developed countries. Resource exploiting firms, mostly foreign-owned companies, have reasons to fear possible nationalization and are likely to formulate their plan, taking this into account. This chapter will look at some aspects of this problem.

Section 2 presents a model in which the probability of being nationalized is a function of time only, and shows how this model can be re-interpreted to allow for (i) no nationalization, (ii) nationalization at a known date, and (iii) no nationalization, but with a rising trend of effective profit tax rate. Solutions for this model is contained in Section 3. Section 4 presents a model in which the probability of nationalization depends on accumulated profit.

¹ A version of this chapter will appear in the Journal of Economic Theory.

2. MODEL ONE

For ease of comparison with earlier models of resource extraction (see, for example, Vousden (1973) and Long (1974a), we use a continuous time model. Let R be a homogeneous resource stock. Extraction is denoted by $Q(t)$, so that :

$$\dot{R} = - Q(t)$$

Profit is a function of extraction alone, and is denoted by $Y(Q)$. A more general formulation would allow profit to be dependent on the cumulative amount already extracted. The effects of cumulative output on the extraction pattern would be very similar to effects of "conservation motive" discussed in Chapter IV, where it was shown that if joint concavity is assumed and $\delta > 0$, the monotonicity of extraction obtained in the simpler model is preserved.

It is reasonable to assume that, from the point of view of the firm, once the resource is nationalized, it remains owned by the "State" for ever. There are three basic sources of uncertainty : (i) whether the firm will be nationalized, (ii) the time at which nationalization occurs if the firm is ever nationalized, and (iii) the amount of compensation at the time of nationalization.

At time 0 the firm possesses subjective (joint) probability that it will be nationalized eventually, but will remain privately owned *at least* until time t . This is denoted by $P(N,t)$. Similarly, the probability that the firm will never be nationalized (and will remain privately owned at least until time t) is $P(\bar{N},t)$. Obviously :

$$P(N,t) \geq P(N,t') \quad \text{for } t < t' \quad (1)$$

$$P(\bar{N},t) = P(\bar{N},t') \quad \text{for all } t, t' \geq 0 \quad (2)$$

The probability that the firm will survive at least until time t is :

$$\Lambda(t) \equiv P(N,t) + P(\bar{N},t) \quad (3)$$

where $\Lambda(0) = 1$.

If $P(N,t)$ is differentiable with respect to t , we can write :

$$dP(N,t)/dt \equiv -\phi(t) \quad (4)$$

and

$$P(N,t) \equiv \int_t^{\infty} \phi(t') dt' \quad (5)$$

$\phi(t)$ is the probability density that the nationalization will occur at time t . In fact the integral in (5) can be interpreted as a Lebesgue - Stieltjes integral and the differentiability of $P(N,t)$ is not required.

If the firm is nationalized at time t , the compensation it receives is a random variable $C[R(t), \theta]$. The *expected* compensation is :

$$EC[R(t), \theta] \equiv V(R(t)). \quad (6)$$

The firm's objective is to maximize the expected value of the sum of discounted returns, by planning a terminal time T and extraction rates $Q(t)$:

$$\text{Maximize}_{T, Q(t)} \int_0^T \{ \Lambda(t) Y[Q(t)] + \phi(t) V[R(t)] \} e^{-\delta t} dt \quad (7)$$

subject to :

$$R(0) = R_0 \quad (\text{given}) \quad (8)$$

$$\dot{R}(t) = -Q(t) \quad (9)$$

$$Q(t) \geq 0 \quad (10)$$

$$R(T) \geq 0 \quad (11)$$

$$T \geq 0 \quad (12)$$

Note that if $Y(Q) \geq 0$ for all Q then (7) is equivalent to an infinite (fixed) time problem. But if $Y(Q) < 0$ for some interval $[0, \bar{Q}]$ (reflecting, for example, some fixed charge as long as the firm remains in business), then it is rational for the firm to choose a terminal time. Constraints (11) and (12) imply that while the end point $R(T)$ and the terminal time T are not fixed, they are not completely free. These constraints will be taken care of by the "control parameters" approach.

The optimal plan $\{T, Q(t)\}$ must be interpreted as a plan which would be followed as long as the firm remains privately owned. Once nationalized, the remaining portion of the plan becomes inoperative.

We observe that the entrepreneur is free to replan and that he is likely to revise his expectations as events are observed. As Arrow (1971) puts it "when beliefs are represented by probabilities, then the observation of an event causes the agent to act in accordance with the conditional probabilities given that event, rather than with the probabilities held before the observation" (p.46).

It might seem at first that our formulation neglects the revision of expectations. It can be shown, however, that expectation

revision is allowed for in our model. (See Appendix VI.A) In particular, if at any time t_1 having observed that no nationalization has occurred, the entrepreneur replans, using revised expectations (conditional probabilities given past events) the new plan is identical to the remainder of the original plan, given the assumption that the Strotz type of inconsistency² due to a variable discount rate does not arise. This is because the revised probabilities at time t_1 , for $t > t_1$, differ from the original probabilities only by a multiplicative constant, $\Lambda(t_1)$, so that :

$$P_{t_1}(N, t) = P(N, t) / \Lambda(t_1)$$

$$P_{t_1}(\bar{N}, t) = P(\bar{N}, t) / \Lambda(t_1)$$

$$\phi_{t_1}(t) = \phi(t) / \Lambda(t_1)$$

$$\Lambda_{t_1}(t) = \Lambda(t) / \Lambda(t_1)$$

The reader should be reminded that the only event that is permitted to be observed is that nationalization has not yet occurred.

A great simplifying assumption is that the implicit utility function is linear. This enables us to take the expected value of the compensation function $C[R(t), \theta]$ and to separate $Y(Q)$ and $C[R(t), \theta]$. If it is known with certainty that no compensation is paid, then the introduction of a non-linear utility function does not complicate the analysis a great deal, nor does it change the basic results (see Chapter IV for a discussion of this point).

² See Strotz (1956)

It is obvious that if the expected compensation is identical to the current present value of the resource then the threat of nationalization has no effect on the firm's behaviour. Formally, let

$$V[R(t_1)] = \text{Maximum} \int_{t_1}^T Y[Q(t)]e^{-\delta t} dt$$

subject to (2.9) - (2.12) and given $R(t_1)$. For any known date of nationalization t_1 , the firm seeks to

$$\text{Maximize} \int_0^{t_1} Y[Q(t)]e^{-\delta t} dt + V[R(t_1)]$$

which is equivalent to :

$$\text{Maximize} \int_0^T Y[Q(t)]e^{-\delta t} dt$$

This is true for all possible t_1 , and hence the uncertainty does not matter.

The necessary conditions for (7) are given by Hestenes (1966) theorem 11.1, where constraints such as (2.11) and (2.12) are taken care of by the "control parameters" approach. Let b_1 and b_2 be two control parameters. Write

$$R(T) \equiv X(b_1) \equiv b_1^2 \geq 0 \quad (13)$$

$$T \equiv Z(b_2) \equiv b_2^2 \geq 0 \quad (14)$$

The necessary conditions are :

- (i) There exists a Hamiltonian function :

$$H(t, R, Q, p, \mu) = e^{-\delta t} [\lambda(t)Y(Q) + \phi(t)V(R)] + p(-Q) + \mu Q \quad (15)$$

(ii) Q maximizes H for given R, t and p :

$$\frac{\partial H}{\partial Q} = e^{-\delta t} [\Lambda(t)Y'] - p + \mu = 0 \quad (16)$$

(iii) $p(t)$ is continuous, and piece-wise differentiable, with

$$\dot{p} = - \frac{\partial H}{\partial R} = - e^{-\delta t} \phi(t)V' \quad (17)$$

$$(iv) \quad \dot{H} = \frac{\partial H}{\partial t} = - \delta e^{-\delta t} [\Lambda(t)Y(Q) + \phi(t)V(R)] + e^{-\delta t} [\Lambda'Y(Q) + \phi'V(R)] \quad (18)$$

(v) the transversality conditions :³

$$p(T) \frac{dX}{db_1} = 0 \Leftrightarrow p(T)R(T) = 0 \quad \text{for } T < \infty \quad (19)$$

$$H(T) \frac{dZ}{db_2} = 0 \Leftrightarrow H(T) = 0 \quad (20)$$

In addition, μ , is a non-negative, piece-wise continuous function of time, with possible discontinuities when $Q = 0$;

$$\mu \geq 0, \quad \mu Q = 0 \quad (21)$$

For ease of interpretation, we define :

$$\psi(t) \equiv p(t)e^{\delta t} \quad (22)$$

$$\gamma(t) \equiv \mu(t)e^{\delta t} \quad (23)$$

so that,

$$\dot{\psi}(t) = \delta \psi(t) - \phi(t)V' \quad (24)$$

$$\Lambda(t)Y' - \psi + \gamma = 0 \quad (25)$$

³ Condition (19) is not necessary for an infinite time problem or if the terminal date T turns out to be infinite. See Arrow and Kurz (1970). Condition (20) remains valid even if T is infinity.

The effects of uncertainty on the extraction plan are best revealed by studying the certainty cases first. Consider two certainty cases :

- (i) *No nationalization* : the entrepreneur is convinced that the firm will never be nationalized;
- (ii) *Certainty about the date of nationalization*, $\bar{T}$, say.

For case (i), if we define $\Lambda(t) \equiv 1$ and $\phi(t) \equiv 0$ identically, then (7) - (25) remain valid. For case (ii) the objective function is simplified as :

$$\text{Max}_{Q(t), T} \int_0^T Y[Q(t)] e^{-\delta t} dt + e^{-\delta \bar{T}} V[R(T)] \quad (26)$$

where

$$T \leq \bar{T}. \quad (27)$$

All the necessary conditions remain valid except that (19) and (20) are replaced by

$$H(T) = 0 \quad \text{if } T < \bar{T} \quad (28)$$

$$R(T) [p(T) - e^{-\delta \bar{T}} V(R(T))] = 0 \quad (29)$$

and provided that $\Lambda(t) = 1$, $\phi(t) = 0$ for all $t \leq \bar{T}$.

Finally, note that if we put $V(R) \equiv 0$, the problem (7) becomes formally *identical* to a problem with no nationalization, but with an effective profit tax which increases over time. Thus, let $\pi(t)$ be the profit tax rate at time t , define

$$\Lambda(t) \equiv 1 - \pi(t).$$

Since no restriction is imposed on $\Lambda(t)$ apart from $\Lambda'(t) \leq 0$, all the results of the problem with uncertainty about nationalization with no compensation apply to the problem of a mining firm facing an increasing profit tax rate over time.

3. THE EXTRACTION PATTERNS IN MODEL ONE :

3.1 Assumptions and Definitions : If $Y(Q)$ has a maximum, we assume that the maximum is unique, at Q_m , say, with $Y'(Q) > 0$ for $Q < Q_m$, $Y'(Q) < 0$ for $Q > Q_m$. For $Q > 0$, define the *average profit function* $Y(Q)/Q$. Two types of average profit function will be considered :

(i) Type one average profit function, which is generated by a concave $Y(Q)$ with $Y(0) \geq 0$. Obviously, this type of average profit function is decreasing in Q .

(ii) Type two average profit function, which is increasing for $Q \in (0, Q_A)$ and decreasing for $Q > Q_A$. Hence

$$Y(Q_A)/Q_A - Y'(Q_A) = 0. \quad (30)$$

Finally, $Y(Q) > 0$ for at least some interval and $Y''(Q) < 0$ for $Q > Q_A$.

3.2 Solution for the Case of No Nationalization ($\Lambda(t) \equiv 0$). The results for this case are by now well-known (See Chapter III) and are presented (in brief) only for comparison with results for other cases.

If $\delta = 0$, the following proposition can be easily proved.

Proposition 3.1 : If the average profit function has a maximum at

$Q = Q_A$, say, then the optimal extraction path when $\delta = 0$ is

$Q(t) = Q_A$ for $t \leq R_0/Q_A$ (i.e. until the resource is *exhausted*).

If the average profit function has no maximum (e.g. type one average profit function) then no optimal path exists if $\delta = 0$.

A special case of the first part of Proposition 3.1 was stated by Scott (1967), where it is asserted that a competitive mining firm with a U-shaped average cost curve should minimize instantaneous average cost if it wishes to maximize its intertemporal profit. The second part of the proposition is also applicable to a discrete time version of the problem at hand, and was proved in Gale (1967) : A cake-eater with a non-negative, concave utility function, wishing to maximize his non-discounted sum of utilities will find it impossible to do so. Vousden (1973) obtained basically the same results for a special case of his more general model with conservation. Proposition 3.1 can also be proved diagrammatically. See Long (1974b) and Appendix III.A.

Proposition 3.2 : If $\delta > 0$, an optimal extraction path exists for both types of average profit function. For type one average profit function, the resource is exhausted in finite time if $Y'(0) < \infty$ and in infinite time if $Y'(0) = \infty$. For type two average profit function, the resource is exhausted in finite time. As the resource stock $R(t)$ tends to zero, $Q(t)$ tends to Q_A for type two average profit function and to zero for type one. In both cases, $Q(t)$ is a *decreasing* function of time.

Proposition 3.2 is intuitively obvious, given proposition 3.1, since a positive discount rate tends to tilt the extraction toward the present.

3.3 Solution for the Second Certainty Case : Nationalization at $\bar{T}$:

If $V'(R) \equiv 0$, the solution is simple. Let T^* be the time at which extraction would cease if there were no possibility of nationalization (T^* comes out as part of the solution of the problem solved in the preceding sub-section). If it takes infinite time to exhaust the resource, take T^* as infinity. The following proposition can be proved :

Proposition 3.3 : If $\bar{T} \geq T^*$, the optimal path is the same as that of the problem considered in the preceding sub-section. If $\bar{T} < T^*$ and $Y(Q)$ has no maximum, then (i) the resource will be exhausted at time $\bar{T}$; (ii) $Q(t)$ is monotonic non-increasing (decreasing if $\delta > 0$); (iii) $Q(t) > Q_A$ for type two profit function. If $\bar{T} < T^*$ and $Y(Q)$ has a maximum at Q_m then

- If $\bar{T} > R_0/Q_m$, (i) - (iii) holds and $Q(t) < Q_m$
- If $\bar{T} < R_0/Q_m$, the resource will not be exhausted and $Q(t) = Q_m$ for $t \leq \bar{T}$.

For a proof of these results, see Long (1974a) and Chapter IV, Section 2.4(b).

If $V'(R) \geq 0$, but not identically zero, various cases arise.

These are :

- C(i) Extraction ceases at $\bar{T}$, leaving $R(\bar{T}) > 0$
- C(ii) Extraction ceases before $\bar{T}$, leaving $R(\bar{T}) > 0$
- C(iii) Extraction ceases at $\bar{T}$, leaving $R(\bar{T}) = 0$
- C(iv) Extraction ceases before $\bar{T}$, leaving $R(\bar{T}) = 0$

Conditions under which each of these cases can or cannot arise can be found, using the necessary conditions, in particular :

$$\psi(t) \geq Y'[Q(t)] (= \text{for } Q > 0) \quad (31)$$

$$\psi(\bar{T}) = V'[R(\bar{T})] \quad \text{for } R(\bar{T}) > 0 \quad (32)$$

$$\dot{\psi} = \delta\psi \quad (33)$$

Define

$$Q_B \equiv Q_A \quad (\text{for type two profit function}) \quad (34)$$

$$Q_B \equiv 0 \quad (\text{for type one profit function}) \quad (35)$$

The following propositions can be proved :

Proposition 3.4 : Suppose $V(R)$ is concave and $V'(0) \leq Y'(Q_B)$.

Let T^* be defined as in proposition 3.3. Then :

- 1 - C(i) is non-optimal if $\bar{T} \geq T^*$ (since any $R_1 \leq R_0$ to be eventually exhausted, the optimal exhaustion time is $\leq T^*$).
- 2 - C(ii) is non-optimal (since if extraction ceases at $t_1 < \bar{T}$, $\psi(t_1) = Y'(Q_B) \geq V'[R(\bar{T})] = \psi(\bar{T})$. But $\psi(\bar{T}) > \psi(t_1)$ by (33)).
- 3 - C(iii) is non-optimal if $\bar{T} > T^*$. C(iv) is non-optimal if $\bar{T} < T^*$ (see proposition 3.3).

Proposition 3.5 : If $V(R)$ is concave and $V'(0) > Y'(Q_B)$ then (1) and (3) of proposition 3.4 hold, and in addition :

- 1 - Neither C(iii) nor C(iv) is optimal if $\delta = 0$.
- 2 - If $\delta = 0$ and $V'(R) > Y'(Q_B)$ for all $R \leq R_0$, it is optimal that $R(\bar{T}) = R_0$. (This is the case of over-compensation).

The proofs are straightforward and details are left to

Appendix VIB.

3.4 Patterns of Extraction Under Uncertainty : It has been often argued that the effects of uncertainty are in many cases similar to that of raising the discount rate. In this sub-section we shall be concerned with some cases which tend to support this view.

Proposition 3.6 : Assume that $V(R) \equiv 0$, $Y'(0) < \infty$ and $\Lambda'(t) < 0$ for $t \leq T^*$ (T^* is defined in Proposition 3.3). Let subscript u denote variables corresponding to the uncertainty case and subscript c for variables of the first certainty case (i.e. no nationalization). Then :

- (i) The optimal plan under uncertainty exhausts the resource more quickly, i.e. $T_u^* < T_c^* \equiv T^* < \infty$.
- (ii) $Q_u(0) > Q_c(0)$ (i.e. the plan under uncertainty starts at a higher extraction rate).
- (iii) $Q_u(T_u^*) = Q_c(T_c^*) = Q_B$ as defined by (34) and (35).
- (iv) there exists some $t_1 < T_u^*$ such that

$$Q_u(t) > Q_c(t) \quad \text{for } t < t_1$$

$$Q_u(t) < Q_c(t) \quad \text{for } t > t_1$$

Proof : Note that $Y'(0) < \infty$ implies $T_c^* < \infty$ (proposition 3.2). Also from proposition 3.2, as t tends to T_c^* , $Q_c(t)$ tends to Q_B which is defined by (34) and (35). From (2.24) and the continuity of ψ

$$\psi_c(T_c^*) = Y'(Q_B). \quad (36)$$

In a similar manner, it is easily shown that :

$$\psi_u(T_u^*) = Y'(Q_B)\Lambda(T_u^*). \quad (37)$$

Hence (iii) is proved, by noting that $Y'(Q_c(t)) = \psi_c(t)$ and $Y'(Q_u(t)) = \psi_u(t)/\Lambda(t)$, for all t until the resource is exhausted.

From (36), (37) and (24) :

$$\psi_c(0)e^{\delta T_c^*} = \psi_u(0)e^{\delta T_u^*}/\Lambda(T_u^*) \quad (38)$$

To prove (ii) we only need to show :

$$\psi_c(0) > \psi_u(0)/\Lambda(0) \equiv \psi_u(0). \quad (39)$$

Suppose that the converse of (39) is true, then $T_c^* > T_u^*$ (since $\Lambda(T_u^*) < 1$), and also $\psi_c(t) < \psi_u(t) \leq \psi_u(t)/\Lambda(t)$ (since $\dot{\psi}_i = \delta\psi_i$). The last two inequalities imply $Q_c(t) > Q_u(t)$. Hence we obtain the contradictory result that

$$R_0 = \int_0^{T_u^*} Q_u(t)dt < \int_0^{T_c^*} Q_c(t)dt = R_0.$$

This contradiction establishes (39) and hence (ii).

Consider Figure 1. The curves $Q_c(t)$ and $Q_u(t)$ are continuous and non-increasing in the intervals $[0, T_c^*]$ and $[0, T_u^*]$ respectively. Since $Q_c(0) < Q_u(0)$, $Q_c(T_c^*) = Q_u(T_u^*) = Q_B$, and the areas under the curves are equal ($= R_0$), the two curves *must* intersect at least once in the interval $[0, \min(T_c^*, T_u^*)]$. If $T_c^* \leq T_u^*$, $Q_c(t)$ must exceed $Q_u(t)$ for some time interval and the two curves intersect at least twice. If we can prove that the two curves intersect once only in the interval $[0, \min(T_c^*, T_u^*)]$ then we have proved (i) and (iv). Suppose there are two intersections at t_1 and t_2 . Then, by (25)

$$\psi_c(t_i) = \psi_u(t_i)/\Lambda(t_i) \quad (i = 1, 2) \quad (40)$$

Figure 1

Let $t_2 > t_1$,

$$\begin{aligned} \psi_u(t_2)/\Lambda(t_2) &> \psi_u(t_2)/\Lambda(t_1) = \psi_u(t_1)e^{\delta(t_2-t_1)}/\Lambda(t_1) = \psi_c(t_1)e^{\delta(t_2-t_1)} \\ &= \psi_c(t_2) \end{aligned} \quad (41)$$

which contradicts (40) for $i = 2$.

Q.E.D.

Proposition 3.7 : Under uncertainty, $\delta = 0$ does *not* imply the non-existence of an optimal path when the average profit function is of type one. (Compare this with proposition 3.1).

This proposition is obvious since $\Lambda(t)$ plays a role similar to that of an exponential discount function. In particular, if $\Lambda(t)$ is exponential, $\Lambda(t) = e^{-\rho t}$, say, then it is identical to the usual discount factor.

The role of the compensation function $V(R)$ under uncertainty is also interesting. While in subsections 3.b and 3.c, $Q(t)$ is non-increasing, this is no longer obvious under uncertainty. From (24) and (25)

$$\dot{Q}(t) = \frac{\phi(t)[Y' - V'] + \delta\Lambda(t)Y'}{\Lambda(t)Y''} < 0 \quad (42)$$

It would seem that without further restrictions the sign of $\dot{Q}(t)$ along an optimal path is ambiguous.

It was stated in Proposition 3.2 that for $Y(0) \geq 0$, $Y''(Q) < 0$ the resource will be exhausted in finite time if and only if $Y'(0) < \infty$. This is no longer true, since it is possible that extraction is positive for all time t (the resource is asymptotically exhausted).

This possibility arises when, for example $Q_B = 0$, $Y'(Q_B) = V'(0)$,
 $\delta = 0$, $\phi(t) = \rho e^{-\rho t}$.

4. MODEL TWO : PROBABILITY OF NATIONALIZATION DEPENDS ON ACCUMULATED PROFIT

4.1 The Model and the Necessary Conditions :

Define

$$X(t) \equiv \int_0^t Y(Q(\tau)) d\tau \quad (43)$$

$X(t)$ is the "sum" of (non-discounted) past profits. The rate of discount does not appear in (43) mainly for simplicity.

The probability that the firm remains privately owned at the level of accumulated profit X is defined as $\Lambda(X)$, with $\Lambda(0) = 1$ and

$$\Lambda'(X) \leq 0 \quad (44)$$

Assuming no compensation, the objective is :

$$\text{Max}_{Q(t), T} \int_0^T e^{-\delta t} \Lambda[X(t)] Y[Q(t)] dt \quad (45)$$

subject to :

$$\dot{X}(t) = Y[Q(t)] \quad (46)$$

$$\dot{R}(t) = -Q(t) \quad (47)$$

$$Q(t) \geq 0 \quad (48)$$

$$R(T) \geq 0 \quad (49)$$

$$T \geq 0 \quad (50)$$

The Hamiltonian is

$$H = e^{-\delta t} \{ \Lambda[X(t)] Y[Q(t)] \} + p_1 Y[Q(t)] - p_2 Q(t) + \mu Q(t) \quad (51)$$

Along an optimal path,

Q maximizes H for given X, t, p :

$$\frac{\partial H}{\partial Q} = e^{-\delta t} \Lambda(X) Y' + p_1 Y' - p_2 + \mu = 0 \quad (52)$$

$$(\text{where } \mu Q = 0, \mu \geq 0) \quad (53)$$

$$\dot{p}_1 = - \frac{\partial H}{\partial X} = - e^{-\delta t} \Lambda'(X) Y(Q) \quad (54)$$

$$\dot{p}_2 = - \frac{\partial H}{\partial R} = 0 \quad (55)$$

$$\dot{H} = \frac{\partial H}{\partial t} = - \delta e^{-\delta t} [\Lambda(X) Y(Q)] \quad (56)$$

The transversality conditions are :

$$H(T) = 0 \quad (57)$$

$$p_1(T) R(T) = 0, p_1(T) \geq 0 \quad \text{for } T < \infty \quad (58)$$

$$p_2(T) = 0 \quad \text{for } T < \infty \quad (59)$$

For ease of interpretation, we define

$$\psi_1(t) \equiv p_1(t) e^{\delta t} \quad (60)$$

$$\psi_2(t) \equiv p_2(t) e^{\delta t} \quad (61)$$

$$\gamma(t) \equiv \mu(t) e^{\delta t} \quad (62)$$

so that

$$\dot{\psi}_1 = \delta \psi_1(t) - \Lambda'(X) Y(Q) \quad (62)$$

$$\dot{\psi}_2 = \delta \psi_2(t) \quad (63)$$

$$[\Lambda[X(t)] + \psi_1(t)] Y' - \psi_2 + \gamma = 0 \quad (64)$$

Again, we may consider two certainty cases (i) No nationalization

(ii) Nationalization at a certain level of accumulated profit. Case

(ii) is however too artificial to be worth considering. Case (i) has been solved in Section 3.2. We are therefore left with the case where $\Lambda(X)$ is not a "degenerate" function. In particular we assume that $\Lambda(X)$ possesses continuous first derivative.

4.2 The Solution :

Consider first the *zero-discount* case. The following proposition can be proved :

Proposition 4.1 : Let $\delta = 0$. Assume that $\Lambda(X) > 0$ for all relevant X . If the program ends in finite time, the extraction plan is identical to that described in proposition (3.1) for type two average profit function and no optimal plan exists for type one average profit function.

This proposition might seem surprising at first because it implies that uncertainty does not affect the extraction plan (provided the assumptions stated are met). This result contrasts sharply with those obtained in model one, where the probability of nationalization is a function of time (see in particular propositions 3.6 and 3.7). A closer look would convince the reader of the plausibility of proposition 4.1. Since $\delta = 0$ and the probability of nationalization is a function of *accumulated profit alone*, time enters the problem in an *inessential* way.

Proof of proposition 4.1 :

From (57), (60) and (61)

$$\{\Lambda[X(T)] + \psi_1(T)\}Y[Q(T)] - \psi_2(T)Q(T) = 0 \quad (65)$$

For $Q > 0$, from (64)

$$\psi_2(t) = \{\Lambda[X(t)] + \psi_1(t)\}Y'[Q(t)] \quad (66)$$

From (65), (66) and the continuity of $H(t)$

$$\lim_{t \rightarrow T} \{\Lambda[X(t)] + \psi_1(t)\}Y'[Q(t)] - Y'[Q(t)]Q(t) = 0 \quad (67)$$

But, from (59) and the assumption that $\Lambda[X] > 0$

$$\lim_{t \rightarrow T} \Lambda[X(t)] + \psi_1(t) > 0 \quad (68)$$

Hence

$$\lim_{t \rightarrow T} Q(t) = 0 \text{ for type one average profit function} \quad (69)$$

$$= Q_A \text{ for type two average profit function} \quad (70)$$

Now, $\Lambda[X(t)] + \psi_1(t) = \text{constant for all } t \leq T$ (due to (46), (62), (63) and $\delta = 0$).

From (62), (63) and (64)

$$[\Lambda(X) + \psi_1]Y'' \cdot \dot{Q} = \delta\psi_2 - Y'[\Lambda'(X)\dot{X} + \dot{\psi}_1] = 0 \text{ since } \delta = 0 \quad (71)$$

Thus $Q(t) = Q_A$ for $t < T$ for type two average profit function.

For type one average profit function, (69) and (71) imply

$R(T) > 0$, while (66) implies $\psi_2(T) > 0$. Thus the condition (58) is violated. Hence no finite time optimal program exists for type one average profit function.

Q.E.D.

If $\delta > 0$ and $T < \infty$, (65) - (70) remaind valid. Hence :

Proposition 4.2 : If an optimal program with $\delta > 0$ ends in finite time, the limiting rate of extraction as $t \rightarrow T$ is the same as under certainty (no nationalization).

The rate of change of Q is given by :

$$[\Lambda(X) + \psi_1]Y''\dot{Q} = \delta\psi_2 - Y'[\Lambda'(X)\dot{X} + \dot{\psi}_1]$$

Hence

$$\dot{Q}[\Lambda(X) + \psi_1]/\Lambda(X) = \delta Y'/Y'' \quad (72)$$

For the certainty case (no nationalization)

$$\dot{Q} = \delta Y'/Y'' \quad (73)$$

For finite programs, $\psi_1(t) \leq 0$ due to (59) and (62), provided that $Y(Q) \geq 0$. Note in particular that Y'/Y'' is a constant if $Y(Q) = 1 - e^{-aQ}$, $a > 0$. In this sense, one may conclude that for the *positive discount* case, the resource could be extracted more quickly under uncertainty, although it seems that this *need not be true* in general.

5. CONCLUSION :

The introduction of uncertainty shows that in general it is inadequate to allow for it by raising the rate of discount. To what extent this procedure gives a good approximation depends among other things, on the nature of the uncertainty. There is no simple rule for predicting the behaviour of firms under uncertainty. To the extent that nationalization is a private risk with no social counterpart, the extraction patterns under this kind of uncertainty tends to distort the allocation of resources.

APPENDIX VIA

The Consistency of the Original Plan

Suppose that at time t_1 , having observed that no nationalization has occurred, the firm revised its subjective probabilities. The revised probabilities are conditional probabilities, given the observed event. Let $P(N, t | t_1)$ denote the (conditional) probability that the firm will be nationalized eventually but will remain privately owned at least until time $t > t_1$, given that no nationalization has occurred prior to t_1 . Let $P(N, t, t_1)$ denote the corresponding *joint* probabilities (again, $t > t_1$). The (marginal) probability that the firm remains privately owned at least until t_1 is $\Lambda(t_1)$. These three probability measures are related by :

$$P(N, t | t_1) = P(N, t, t_1) / \Lambda(t_1) \quad t > t_1 .$$

But "no nationalization before t " implies "no nationalization before t_1 " for $t > t_1$. Hence :

$$P_{t_1}(N, t) \equiv P(N, t | t_1) = P(N, t) / \Lambda(t_1) \quad t > t_1 .$$

The revised measures $P_{t_1}(\bar{N}, t)$, $\phi_{t_1}(t)$ and $\Lambda_{t_1}(t)$ are obtained similarly. The revised probabilities differ from the original ones only by a multiplicative constant and hence the new plan is identical to the remaining portion of the original plan. It should be noted that, in this model, the only relevant event that is permitted to be observed is whether or not nationalization has occurred.

APPENDIX VIB

PROOFS OF PROPOSITIONS 3.4 AND 3.5
OF CHAPTER VI

1. PROOF OF PROPOSITION 3.4 :

1) Since nationalization is certain at time $\bar{T}$, revenue from compensation is $V[R(\bar{T})]$. Given any $R(\bar{T})$, the profit from the extraction of $R_0 - R(\bar{T})$ is maximized by solving the problem

$$\text{Max} \int_0^T Y[Q(t)] e^{-\delta t} dt$$

subject to

$$S(0) = S_0 \equiv R_0 - R(\bar{T})$$

$$\dot{S}(t) = -Q(t)$$

$$S(T) \geq 0, \quad T \leq \bar{T}$$

The solution of this problem is depicted in the phase diagram below (Figure 1) where β is the optimal path, if T is unconstrained.

If $S_0 = R_0$ and T is unconstrained, then the exhaustion time is $T = T^*$, by definition of T^* . Consider the case $\bar{T} \geq T^*$. The constraint $T \leq \bar{T}$ becomes irrelevant for $S_0 \leq R_0$, and path β is optimal. The exhaustion time for $S_0 < R_0$ is $T < T^* \leq \bar{T}$. Hence $C(i)$ is non-optimal if $\bar{T} \geq T^*$.

2) If extraction ceases at $t_1 < \bar{T}$, then $Q(t_1) = Q_B$. Hence $\psi(t_1) = Y'(Q_B) \geq V'(0) \geq V'[R(\bar{T})] = \psi(\bar{T})$. This contradicts (33).

3) The results follow from proposition 3.3, chapter VI.

Figure 1

2. PROOF OF PROPOSITION 3.4 :

1) If $\delta = 0$, it is always profitable to leave some resource on the ground and wait for the compensation, because $V'(0) > Y'(Q_B)$. More precisely, we can show that by leaving some amount R_1 of resource on the ground, total profit (including compensation) will be increased.

Since $V'(0) > Y'(Q_B)$, there exists some $R_1 > 0$ such that $V'(R_1) > Y'(Q_B)$. Now since $\delta = 0$, the extraction rate is some constant $\bar{Q}$. For any chosen R_1 define

$$I = R_1/\bar{Q}$$

We shall show that the profit foregone by not extracting $\bar{Q}$ over a time interval I is exceeded by the compensation $V(R_1)$.

$$Y(\bar{Q}) \cdot I = I \cdot \bar{Q} \cdot Y(\bar{Q})/\bar{Q}$$

$$\leq I \cdot \bar{Q} \cdot Y'(Q_B)$$

$$< R_1 \cdot V'(R_1)$$

$$< V(R_1) - V(0) = V(R_1).$$

2) The proof of 2) follows from the arguments above.

CHAPTER VII

FOREIGN INVESTMENT AND RESOURCE EXPLOITATION : OPTIMAL REGULATION UNDER ADAPTIVE EXPECTATION.

1. INTRODUCTION :

In Chapters IV and V we studied some models of foreign borrowing for resource exploitation under conditions of certainty. The assumption of certainty was partially relaxed in Chapter VI, where it was assumed that foreign firms have subjective beliefs about possible nationalization. In this Chapter, we will explore the implications of situations where both the foreign country and the home country face uncertainty.

Since the exploitation of natural resource typically introduces a time dimension in an economic model, it seems natural to allow the learning about the uncertainty as events are observed. This problem is not trivial and considerable abstraction has to be made to simplify the analysis. We shall study a two-period model of foreign investment in which both foreign investors and the home government possess imperfect information about the profitability of domestic investment opportunities. Foreign investors are competitive and the home government wishes to maximize intertemporal net social gain. Two types of uncertainty (and hence learning) will be considered : Type one uncertainty is characterized by complete learning after one single experience. This corresponds closely to the type of uncertainty about product quality considered by Nelson (1970). Type two uncertainty, by definition, cannot be removed after any number of experiences although the uncertainty is reduced after each trial. This

is the type of uncertainty studied by Prescott (1972), Crawford (1973) and Kihlstrom (1974).

At the formal level, the first type of uncertainty is similar to that considered by Tisdell (1970) and Turnovsky (1973). Note two differences, however : First, Tisdell and Turnovsky assume that information is a *free good* and will be available in due course while in our model information is obtained only if economic agents choose to experience. Second, in our model *both* sides learn. It will be shown that the home government's optimal decision, taking into account that both sides will learn and will act optimally in the light of new information, will in general be different from a myopic decision or a decision not taking account of learning. In particular, the difference is more marked when an intertemporal resource constraint is imposed.

For type two uncertainty, the Bayesian revision rule is used. The optimal decision depends crucially on the nature of the "noise" which prevents complete learning.

2. THE UNKNOWN RATE OF RETURN AND LEARNING :

Country A, the home country, possesses an investment opportunity with unknown profitability. Assume that the return function is :

$$X_t = xK_t + \epsilon_t G(K_t) \quad (1)$$

with $G(K) \neq 0$ for $K > 0$. x is an unknown constant while ϵ_t is a random disturbance, $E(\epsilon) = 0$. Call x the average rate of return. If ϵ_t is *observable*, learning is complete after one observation, for :

$$x = [X_t - \epsilon_t G(K_t)]/K_t \quad (2)$$

We call this "type one uncertainty". If ϵ_t is *not observable*, e.g. if it is a result of a large number of unknown factors affecting profits, an appeal to the Central Limit Theorem would suggest that ϵ is *normal*, with zero mean and variance σ^2 . For this "type two uncertainty", Bayesian learning is appropriate. Let $q(\epsilon)$ be the probability density function (p.d.f.) of ϵ and $h_1(x)$ be the (subjective) p.d.f. for x . For $K_t > 0$, define the observed rate of return π_t :

$$\pi_t = X_t/K_t \quad (3)$$

The revised p.d.f. for x is given by Bayes' rule¹

$$h_2(x) \equiv h_2(x|\pi_1) = h_1(x)g(\pi_1|x, K_1) \cdot N \quad (4)$$

Where

$$g(\pi_1|x, K_1) = q[(\pi_1 - x)K_1/G(K_1)] \cdot |K_1/G(K_1)|. \quad (5)$$

and N is a normalizing constant, that is,

$$\int_{-\infty}^{\infty} h_{t+1}(x|\pi_t) dx = N \int_{-\infty}^{\infty} h_t(x)g(\pi_t|x, K_t) dx = 1$$

The revised p.d.f. $h_{t+1}(x|\pi_t)$ will be used for the decision making in period $t + 1$. In the words of Arrow (1971) "When beliefs are represented by probabilities, then the observation of an event causes the agent to act in accordance with the conditional probabilities given that event, rather than with the probabilities held before the observation". (p.46)

¹ See Raiffa and Schlaifer (1961), p.29

3. THE ECONOMIC MODEL :

For simplicity, assume that the home country's time horizon comprises only two periods, 1 and 2. The home country does not possess capital suitable for the productive process described by equation (2). Assume further that all (foreign) capital K_t in (2) is equity capital so that the return to the home country is $\theta_t X_t$ where θ_t is the profit tax rate. The expression $\theta_t X_t$ is meaningful if X_t is always non-negative or the home country undertakes to pay foreign investors an amount $\theta_t X_t$ should they make losses ($X_t < 0$). For most of the analysis we will be able to assume that $X_t \geq 0$.

Foreign capitalists own a fixed amount of capital $\bar{K}$, to be allocated between two uses : K is the amount invested in A and $K_B \equiv \bar{K} - K$ is used in country B. The *sure* rate of return in country B is $r_t(K_{Bt})$, with $r'_t < 0$, $r''_t > 0$ and $r(0) = \infty$. $r_1(\cdot)$ is allowed to be *different* from $r_2(\cdot)$, reflecting, e.g., *technological change*.

Foreign capitalists are competitive and we assume that any information about x available to any foreign capitalist is also available to all foreign capitalists. They are assumed to behave as if individually they could not influence the magnitude of K_t . We also assume *risk neutrality* to make the analysis manageable. These assumptions, together with the perfect mobility of capital, ensure that while foreign capitalists may learn, they just maximize the current period expected return.

At the beginning of period t , foreign capitalists have a common expected value for x . This is denoted by m_t^* . Given any

tax rate θ_t , the equilibrium K_t is given by the equalization of expected rates of return :

- i) $(1-\theta_t)m_t^* = r_t(\bar{K}-K_t)$ if $(1-\theta_t)m_t^* > r_t(\bar{K})$;
- ii) $K_t = 0$ if $(1-\theta_t)m_t^* \leq r_t(\bar{K})$.

Define the net rate of return s_t by :

$$s_t = (1-\theta_t)m_t^* \quad (6)$$

The foreign supply of capital can then be written as :

$$K_t = K_t(s_t) \quad (7)$$

with

$$K_t(s_t) = 0 \quad \text{if } s_t \leq r_t(\bar{K})$$

$$K_t(s_t) = \bar{K} - r_t^{-1}(s_t) \quad \text{if } s_t > r_t(\bar{K})$$

where r_t^{-1} is the inverse of the function $r_t(K_{Bt})$. Clearly for $s_t > r_t(\bar{K})$, $K_t' > 0$, $K_t'' < 0$.

Figure 1 illustrates the equilibrium allocation of capital and Figure 2 depicts the function $K_t(s_t)$.

It is assumed that the home government knows foreign capitalists decision rules.

Under type 1 uncertainty, the government will know the value of x at the end of period 1, provided that $K_1 > 0$.

Under type 2 uncertainty we will consider two mutually exclusive cases :

Figure 1

Figure 2

A(i) The government does not revise its subjective beliefs.

That is $E_1(x) = E_2(x)$.

A(ii) The government revises its subjective beliefs according to Bayes' rule.

Under type 1 uncertainty, all foreign capitalists will know the true value of x at the end of period 1, provided of course that $K_1 > 0$.

Under type 2 uncertainty, two mutually exclusive cases will be considered :

B(i) All foreign capitalists do not revise their expectations after observing Π_1 . That is $E_2^*(x) = E_1^*(x)$.

B(ii) Foreign capitalists have identical prior beliefs $h_1^*(x)$ and will revise their beliefs according to Bayes' rule to obtain $h_2^*(x|\Pi_1)$. The asterisk is used to denote foreign expectations.

The home government is risk neutral. It wishes to maximize the *expected value* of "discounted net social gain" of the two periods *taken together*. The "net social gain" of each period is defined as follows :

$$G_t = \theta_t K_t(s_t) \cdot x - D[K_t(s_t)]$$

where $D(K)$ is the "social cost of foreign capital", with :

$$D' > 0, D'' \leq 0, D(0) = 0.$$

The role of the function $D(K)$ is two-fold : First, it represents external diseconomies (whether physical or psychic) of

foreign capital. Second, it ensures that G_t is not proportional to x (the proportionality renders the problem trivial in some cases).

In a framework with learning and expectation revision the precise mathematical expression for the "discounted expected value of net social gain of the two periods taken together" is not trivial. It will be different for different types of uncertainty and different behavioural modes, and therefore must be specified according to cases. In all cases, we will use the "backward induction" method of solution.

4. OPTIMAL REGULATION UNDER TYPE ONE UNCERTAINTY WITHOUT A RESOURCE CONSTRAINT :

In this case it is natural that *both* sides learn after observing X_1 . By the principle of optimality, an optimal strategy for the two period problem is necessarily optimal for the second period. We therefore consider the second period problem first.

4.1 The Second Period Problem and Solution :

An interesting characteristic of the second period problem is a basic *discontinuity* at $K_1 = 0$. If $K_1 = 0$, nothing will be learned in the first period so that the state of information at the beginning of the second period is the same as that at the beginning of the first period. If $K_1 > 0$, x will become known at the beginning of the second period.

Case (a) : No learning in period one ($K_1=0$)

At the beginning of period 2, $m_2^* = m_1^* \equiv E_1^*(x)$ and $m_2 = m_1 = E_1(x)$ (where the asterisk denotes the foreign expectations). The foreign supply function is, by (6) and (7) :

$$K_2 = K_2[(1-\theta_2)m_2] \quad (8)$$

The second period optimization problem for the home government is :

$$\text{Max}_{\theta_2} E(G_2) = \theta_2 \cdot K_2[(1-\theta_2)m_2^*]m_2 - D[K_2[(1-\theta_2)m_2^*]] \quad (9)$$

If m_2^* or m_2 are non-positive, (9) is a trivial problem :
Its solution is characterized by $K_2 = 0$.

For the non-trivial case ($m_2 > 0$ and $m_2^* > 0$), (9) is equivalent to :

$$\text{Max}_{s_2} E(G_2) = (1-s_2/m_2^*)K_2(s_2) \cdot m_2 - D[K_2(s_2)] \quad (10)$$

where $s_2 \equiv (1-\theta_2)m_2^*$ by (6).

We can impose the condition that $s_2 \geq r_2(\bar{K})$, by the nature of the supply function (7). Since $s_2 = r_2(\bar{K})$ implies $K_2 = 0$ and $E(G_2) = 0$, it is clear that $\text{Max } E(G_2) \geq 0$. This implies that $s_2 > m_2^*$ is non-optimal, for $s_2 > m_2^*$ implies $E(G_2) < 0$. Therefore s_2 is effectively bounded above as well as below. The continuity of $E(G_2)$ ensures that a maximum exists since the constraint set is practically compact.

The first order condition for a maximum is :

$$\frac{dE(G_2)}{ds_2} = (m_2 - s_2 m_2 / m_2^* - D') K_2' - K_2 m_2 / m_2^* \leq 0 \quad (11)$$

with < 0 only if $s_2 = r_2(\bar{K})$ and hence $K_2 = 0$.

Define

$$C \equiv \frac{d^2 E(G_2)}{ds_2^2} = (m_2 - s_2 m_2 / m_2^* - D') K_2'' - 2K_2' m_2 / m_2^* - D'' \cdot (K_2')^2 \quad (12)$$

From (11) an interior extremum is characterized by $(m_2 - s_2 m_2 / m_2^* - D') > 0$. Thus $C < 0$ at an interior extremum. Thus we have proved :

Lemma 4.1 : i) either an interior extremum does not exist, and hence $\text{Max } E(G_2) = 0$, at $s_2 = r_2(\bar{K})$ and hence $K_2 = 0$.

ii) or there is only one interior extremum and it is the global maximum ($s_2 > r_2(\bar{K})$ and hence $K_2 > 0$).

These two cases are mutually exclusive and exhaustive. The graphs of $E(S_2)$ for these two cases are depicted in figures 3(i) and 3(ii). In drawing these graphs, use is also made of the properties that $E(G_2) = 0$ for $s_2 \leq r(\bar{K})$ and $E(G_2) < 0$ for $s_2 > m_2^*$.

Referring to figure 3(ii), a necessary and sufficient condition for case (ii) to hold is :

$$\lim_{s_2 \rightarrow r_2(\bar{K})^+} \frac{dE(G_2)}{ds_2} > 0 \quad (13)$$

which is equivalent to :

Lemma 4.2 : An interior maximum exists if and only if :

$$m_2 - r_2(\bar{K}) m_2 / m_2^* > D'(0) \quad (14)$$

$$\text{i.e. } m_2 [1 - r_2(\bar{K}) / m_2^*] > D'(0) \quad (15)$$

Condition (15) simply says that the marginal tax revenue exceeds the marginal social cost of foreign capital (at $K_2 = 0$).

Figure 3(i)

Figure 3(ii)

It is useful to write s_2 as a function α of m_2^* and m_2 . To characterize α , differentiate (11) with respect to m_2^* and m_2 respectively (assuming, of course, that (11) holds with equality) :

$$\frac{\partial s_2}{\partial m_2} = \frac{m_2(s_2 K_2' + K_2)/(m_2^*)^2}{-C} \quad (= \frac{\partial \alpha}{\partial m_2^*}) > 0 \quad (16)$$

$$\frac{\partial s_2}{\partial m_2} = \frac{(1-s_2/m_2^*)K_2' - K_2/m_2^*}{-C} \quad (= \frac{\partial \alpha}{\partial m_2}) > 0 \quad (17)$$

When (11) holds with equality, $C < 0$, hence (16) is positive for the non-trivial case ($m_2 > 0$ and $m_2^* > 0$); (17) is also positive because (11) implies that the numerator of (17) is positive. These results are intuitively reasonable : More favourable expectations imply a higher value for the optimal K_2 . Thus $\alpha(m_2^*, m_2)$ is monotonic increasing when (15) holds (and is $= 0$ otherwise).

Case (b) : Learning occurred in period one ($K_1 > 0$)

In this case, x becomes known at the beginning of period 2, so that $m_2^* = m_2 = x$. All the results in case (a) ($K_1=0$) apply here, with only some minor changes : m_2^* and m_2 are to be replaced by x from equations (8) to (15). Equations (16) and (17) need modifications : since $m_2^* = m_2 = x$, we now write $s_2 = \phi(x)$. Condition (15) for an interior maximum now becomes :

$$x - r_2(\bar{K}) > D'(0) \quad (18)$$

$$\text{Hence } s_2 = \phi(x) = r_2(\bar{K}) \quad \text{if } x \leq D'(0) + r_2(\bar{K}) \quad (19)$$

If $x > D'(0) + r_2(\bar{K})$, $s_2 > r_2(\bar{K})$. The sign of $\phi'(x)$ is determined by differentiating (11) after substituting x for m_2^* and m_2 :

$$\phi'(x) = \frac{ds_2}{dx} = \frac{K'}{-C} > 0 \quad (20)$$

which is again intuitively plausible. The function $\phi(x)$ is illustrated in figure 4. Note that $s_2 = \phi(x) < x$ for all $x > r_2(\bar{K}) + D'(0)$ (to see this, substitute x for m_2 and m_2^* in (11)).

4.2 The First Period Problem and Solution :

We are now in the position to solve the first period problem. The government of country A has to decide on the optimal amount of capital inflow in the first period, bearing in mind that this will have an effect on the second period maximum social gain : If no capital inflow is allowed in period one, the *maximum* social gain in period two is

$$\begin{aligned} V_0 &\equiv \text{Maximum}_{s_2} E(G_2 | K_1=0) \\ &= [1 - \alpha(m_2^*, m_2)/m_2^*] K_2(\alpha)m_2 - D[K_2(\alpha)] \end{aligned} \quad (21)$$

using (10). We used α for $\alpha(m_2^*, m_2)$ for notational simplicity.

On the other hand, if *some* capital inflow is allowed in period one, the *maximum* social gain in period two is, for given x ,

$$\begin{aligned} V &= \text{Maximum}_{s_2} E(G_2 | K_1 > 0) \\ &= [1 - \phi(x)/x] K_2(\phi)x - D[K_2(\phi)] \end{aligned} \quad (22)$$

Figure 4

Since x is unknown at the beginning of period one, expectation must be taken, using the government's subjective beliefs about x . Clearly $E(V) \geq 0$, since $V \geq 0$ for all x .

Let $0 < \delta \leq 1$ be the social discount factor. If $K_1 = 0$ the maximum social gain for the two periods is

$$W_0 = 0 + \delta V_0 \quad (23)$$

If $K_1 > 0$, the expected social gain for the two periods is :

$$W(s_1) = E[G_1(s_1)] + \delta E(V) \quad (24)$$

where

$$E[G_1(s_1)] = [1 - s_1/m_1^*]K_1(s_1)m_1 - D[K_1(s_1)] \quad (25)$$

The following proposition can be proved :

Proposition 4.1 :

(i) If the functions $r_1(.) = r_2(.)$, it will never be optimal to forgo first period investment in order to avoid foreign learning.

(ii) If $r_1(.) = r_2(.)$ and the myopically optimal K_1 is positive, then the non-myopic optimal choice of K_1 coincides with the myopic choice.

(iii) Suppose that the myopic $K_1 = 0$. It is possible that there exists some $K_1 > 0$ such that despite the current loss caused by $K_1 > 0$, the discounted sum of expected social gains with $K_1 > 0$ exceeds the sum W_0 obtained by myopic fiscal policy in each period.

(iv) If $r_1(.) \neq r_2(.)$, reflecting, for example, *technological change* in country B, it is possible that the optimal policy consists of zero foreign investment in period one, even if the myopically optimal K_1 is positive.

Proof :

The government has to perform two computations :

(i) to find s_1 which maximizes $W(s_1)$, subject to the constraint that $K_1 > 0$, or :

$$s_1 > r_1(\bar{K}) \quad (26)$$

(a maximum may not exist)

(ii) to compare W_0 with $\max.W(s_1)$ subject to (26).

We note that $W(s_1)$ may not have a maximum due to the fact that the constraint set $(s_1 > r_1(\bar{K}))$ is not closed. There is a simple way to find a necessary and sufficient condition for the existence of $\max.W$ and to determine $\sup.W$.

Consider the new constraint :

$$s_1 \geq r_1(K) \quad (27)$$

W is a continuous function of s_1 over this constraint set, *provided that* we temporarily take V (in (22)) as independent of s_1 being positive. W has a maximum over (27). If this maximum is an interior one $(s_1 > r_1(\bar{K}))$, then of course $\max.W$ subject to (27) is the same as $\max.W$ subject to (26). If the maximum of W subject to (27) occurs at the corner $(s_1 = r_1(\bar{K}))$, then W has no maximum over (26) but it has a supremum which is equal to $\max.W$ over (27).

Now to maximize W over (27) is the same as to maximize $E(G_1)$ over (27) since V is taken to be independent of s_1 . A necessary and sufficient condition for $E(G_1)$ to have an interior maximum is similar to (15) :

$$m_1[1 - r_1(\bar{K})/m_1^*] > D'(0) \quad (28)$$

since the problem is formally identical to (10). Note that $r_1(\bar{K})$ is allowed to be different from $r_2(\bar{K})$. When $E(G_1)$ does not have an interior maximum, then $\max E(G_1) = 0$, as $s_1 = r_1(\bar{K})$.

Now if $r_1(.) = r_2(.)$, maximum $E[G_1(s_1)] = V_0$. Thus $W_0 \leq W(s_1)$. This establishes (i).

If $E[G_1(s_1)]$ has a maximum at some $s_1 > r_1(\bar{K})$, the myopically optimal K_1 is positive. If, in addition, $r_1(.) = r_2(.)$, then maximum $W(s_1) \geq W_0$. This proves (ii).

If $E(V) > V_0$, we obtain (iii).

Suppose $\max E[G_1(s_1)]$ occurs at some $s_1 > r_1(\bar{K})$, then the myopically optimal K_1 is positive. But if $V_0 > E(V)$ and such that $W_0 > \max W(s_1)$, then the truly optimal K_1 is zero.

Q.E.D.

Remarks on proposition 4.1 : Proposition 4.1 illustrates how the learning possibility may affect decisions. If x were known with certainty by both sides, myopic behaviour would be optimal. With learning, it may pay to incur first period loss in order to learn, as stated in (iii); or it may be optimal to forgo first period gain in order to avoid foreign learning as stated in (iv). Myopic fiscal policy is optimal, however, if $r_1(.) = r_2(.)$ so that the second

period opportunity set without learning is identical to the first period opportunity set and if the myopic K_1 is positive. Any changes in the probability distributions which does not affect the latter condition will not change this myopic property of the optimal policy - provided that $r_1(.) = r_2(.)$. These conclusions are substantially modified if an intertemporal resource constraint is introduced.

5. OPTIMAL REGULATION UNDER TYPE ONE UNCERTAINTY WITH A RESOURCE CONSTRAINT :

When a resource constraint is introduced, the results stated in proposition 4.1 are substantially modified. This is because of the irreversibility of resource exploitation which makes risk-neutral decision makers behave in an apparently conservative manner.

For simplicity, a fixed coefficient technology is assumed. More precisely, the return function (1) is modified as :

$$X_t = x \min (K_t, R_t) + \epsilon_t G[\min (K_t, R_t)] \quad (29)$$

$$R_1 + R_2 \leq \bar{R} \quad (30)$$

R_t is the amount of resource extracted in period t , and $\bar{R}$ is the total resource stock. All the other assumptions are retained.

5.1 The Second Period Problem and Solution :

The solution of the second period problem depends on whether learning took place in the first period.

Case (a) : No learning in period one ($K_1 = 0$).

(10) is to be maximized, subject to the additional constraint

$$K_2(s_2) \leq R_2 = \bar{R} \quad (31)$$

All the conclusions for case (a) of section 4.1 remain valid, with some minor modifications : The first order condition (11) is ≥ 0 if constraint (31) holds with equality, and the term "interior maximum" in Lemma 4.2 must be replaced by "maximum at some $K_1 > 0$ ". The function $s_2 = \alpha(m_2^*, m_2)$ must be modified by adding the restriction

$$s_2 \leq \bar{s}_2$$

where

$$K_2(\bar{s}_2) = \bar{R}$$

Case (b) : Learning took place in period one ($K_1 > 0$).

The second period problem is the same as before, with an additional constraint :

$$K_2(s_2) \leq \bar{R} - K_1 \quad (32)$$

or, equivalently

$$s_2 \leq K_2^{-1}(\bar{R} - K_1) \equiv Z(K_1) \quad (33)$$

K_2^{-1} is the inverse of the function $K_2(s_2)$; this inverse exists for positive $\bar{R} - K_1$. $Z(K_1)$ is monotonic decreasing and tends to $r_2(\bar{K})$ as K_1 tends to $\bar{R}$.

The optimal s_2 can be written as a function of $x, \lambda(x)$ say. This function is *identical* to $\phi(x)$ for all x such that $\phi(x) \leq Z(K_1)$. There is an $\bar{x}(K_1)$ such that $\phi(\bar{x}) = Z(K_1)$. For $x \geq \bar{x}(K_1)$, $\lambda(x) = \phi(\bar{x})$. This is because any increase in s_2 beyond $Z(K_1)$ does not increase output since the resource constraint becomes binding. At this (right-hand) corner maximum :

$$\left. \frac{dE(G_2)}{ds_2} \right|_{s_2=Z(K_1)} = x[1 - s_2/x]K_2' - D'K_2' - K_2 \geq 0 \quad (34)$$

The function $\lambda(x)$ is illustrated in figure 5. Since

$\lambda(x) = \phi[\bar{x}(K_1)]$, we may also write $\lambda(x; K_1)$, bearing in mind that $\lambda(x)$ depends on K_1 only for $x > \bar{x}(K_1)$.

5.2 The First Period Problem and Solution :

Obviously when the resource constraint (30) is irrelevant (e.g. $\bar{R} > 2\bar{K}$) all the results of section (4) hold. These results do not hold in general however. The following proposition will substantiate our assertions :

Proposition 5.1 : Assume that the myopic investment in period 1 is $K_1^{**} > 0$, and the non-myopic investment is K_1^0 . K_1^{**} and K_1^0 are associated with s_1^{**} and s_1^0 . Then $K_1^{**} \geq K_1^0$, with strict inequality if $h(x) > 0$ for some interval belonging to the set $\{x | x > \bar{x}(K_1^0)\}$.

Proof :

At the beginning of period 1, x is unknown. For any choice of $K_1 > 0$, the second period expected net social gain is :

$$\begin{aligned} E[G_2 | s_2 = \lambda(x)] &= \int_0^{\bar{x}(K_1)} \{ [1 - \phi(x)/x] K_2(\phi) x - D[K_2(\phi)] \} h(x) dx \\ &+ \int_{\bar{x}(K_1)}^{\infty} \{ [1 - Z(K_1)/x] K_2[Z(K_1)] x - D[K_2(Z(K_1))] \} h(x) dx \end{aligned} \quad (35)$$

where $h(x)$ is the government subjective p.d.f. of x .

For $K_1^0 > 0$, s_1^0 is characterized by :

Figure 5

Figure 6

$$\left(\frac{dE(G_1)}{ds_1} + \frac{\delta dE[G_2 | s_2 = \lambda(x)]}{ds_1} \right)_{s_1=s_1^0} \geq 0 \quad (36)$$

with $\delta > 0$ only if $s_1^0 = K_1^{-1}(\bar{R})$

After simplifying (36) becomes :

$$E[(x - s_1^0 x / m_1^* - D')K_1' - K_1 x / m_1^*] + \delta \int_{\bar{x}[K_1(s_1^0)]}^{\infty} \{x[1 - Z(K_1)]/x\} K_2' - D'K_2' - K_2\} Z' K_1' h(x) dx \geq 0 \quad (37)$$

The expression $\{....\}$ is evaluated at $s_2 = Z(K_1)$ and hence is positive by (34). Since $Z' < 0$, the integrand is negative. Hence

$$\left. \frac{dE(G_1)}{ds_1} \right|_{s_1^0} > 0 = \left. \frac{dE(G_1)}{ds_1} \right|_{s_1^{**}} \quad (38)$$

(38) and the uniqueness of s_1^{**} imply $s_1^{**} > s_1^0$.

Q.E.D.

Figure 6 illustrates the result of proposition 5.1.

Proposition 5.1 is not just a stochastic version of a result obtained in chapter IV, for it will enable us to compare the optimal s_1^0 when there are learning possibilities with the optimal $\bar{s}_1^0$ when no learning takes place (i.e. when x is known to take its expected value) and to see how s_1^0 changes as uncertainty increases. We shall make use of the following definition and lemma which are due to Rothschild and Stiglitz (1971).

Definition : a random variable z is more uncertain (risky) than y if and only if $z = y + u$ where u is a random variable with $E(u|y) = 0$.

Lemma 5.1 : Suppose that s^0 maximizes a function $E[U(x,s)]$ where x is a random variable. Define

$$Q(x, s^0) = \left. \frac{dU}{ds} \right|_{s^0} \quad (39)$$

If $Q(x, s^0)$ is concave in x , then (i) $s^0 < \bar{s}^0$, where $\bar{s}^0$ maximizes $U[E(x), s]$, and (ii) further increase in uncertainty leads to further reduction in s^0 . s^0 is the optimal choice under uncertainty and $\bar{s}^0$ is the optimal choice under certainty.

Proposition 5.2 : Given the assumptions made in the statement of proposition 5.1 then (i) the optimal investment in the first period is less than the amount which would have been optimal had no learning taken place, and (ii) K_1^0 reduces with further increase in uncertainty.

Proof :

Using (39) and (36), we have, for $x < \bar{x}[K_1(s_1^0)]$

$$Q(x, s_1^0) = (x - s_1^0 x / m_1^* - D') K_1' - K_1 x / m_1^*$$

and for $x \geq \bar{x}[K_1(s_1^0)]$,

$$Q(x, s_1^0) = [x - s_1^0 x / m_1^* - D'] K_1' + \delta [x - Z(K_1)] K_2' Z' K_1' - \delta [D' K_2' + K_2] Z' K_1' - K_1 x / m_1^*$$

For $x < \bar{x}[K_1(s_1^0)]$, $\partial Q / \partial x = (1 - s_1^0 / m_1^*) K_1' - K_1 / m_1^*$

= a say, and for $x > \bar{x}[K_1(s_1^0)]$ $\partial Q / \partial x = a + \delta K_2' Z' K_1' < a$, hence

$Q(x, s_1^0)$ is concave in x . Figure 7a and 7b illustrate two possible shapes of $Q(x, s_1^0)$. An appeal to Lemma 5.1 completes the proof.

Q.E.D.

TYPE TWO UNCERTAINTY: ONLY THE GOVERNMENT EXPECTATIONS

Type two uncertainty is perhaps the most common in the literature on capital markets.

As in the case of type one uncertainty, the government's expectations are assumed to be correct. However, the private sector's expectations are assumed to be incorrect. This is represented by the fact that the private sector's expectations are not equal to the government's expectations. In this case, the private sector's expectations are assumed to be a constant, while the government's expectations are assumed to be a function of the state of the world.

Figure 7a

where $K_1(s_1^0) = [1 - \beta] \int_0^{\infty} \frac{1}{1 + \beta x} dx$. The value of $K_1(s_1^0)$ is determined by the government's expectations of the state of the world. In this case, the private sector's expectations are assumed to be a constant, while the government's expectations are assumed to be a function of the state of the world.

Figure 7b

6. TYPE TWO UNCERTAINTY : ONLY THE GOVERNMENT REVISES EXPECTATIONS :

Type two uncertainty is perhaps more realistic in that learning is not completed in one period. In this section, it will be assumed that only the government learns. If $K_1 > 0$, the home government's revised p.d.f. is $h_2(x)$, with mean m_2 . $h_2(x)$ depends on π_1 and on the magnitude of K_1 , unless $G(K_1)/K_1$ is a constant (see (4) and (5)).

Since foreign capitalists do not learn we would expect that the "value of information" from the home government's point of view, is positive. This can be established formally as follows :

If $K_1 > 0$, $h_2(x)$, and hence m_2 , can be derived using (4). The second period expected social gain is therefore :

$$E_2(G_2) = [1 - s_2/m_2^*]K_2(s_2)m_2(\pi_1, K_1) - D[K_2(s_2)] \quad (40)$$

where E_t signifies that $h_t(x)$ is used. The optimal s_2 which maximizes (4) is an increasing function of m_2 for all m_2 satisfying (14). We write $s_2 = \rho(m_2)$. At the beginning of period 1, the maximum second period expected social gain, viewed from period 1, is :

$$E_{\pi_1|K_1} [\max_{s_2} E_2(G_2)] = E_{\pi_1|K_1} [E_2(G_2|s_2 = \rho(m)))] \quad (41)$$

where the expectation is taken using the "predictive" density function :

$$p(\pi_1|K_1) = \int_{-\infty}^{\infty} g(\pi_1|x, K_1)h_1(x)dx \quad (42)$$

Define the "expected value of information" (E.V.I.) as :

$$E.V.I. \equiv E_{\pi_1|K_1} [\max_{s_2} E_2(G_2)] - D_1[G_2|s_2 = \alpha(m_2^*, m_1)] \quad (43)$$

The second term on the right-hand side of (43) is the maximum second period social gain, had there been no learning (see (10)). We now can prove :

Proposition 6.1 : $E.V.I. \geq 0$.

Proof :

$$\begin{aligned} E_{\pi_1|K_1} [\max_{s_2} E_2(G_2)] &\geq E_{\pi_1|K_1} [E_2(G_2|s_2 = \alpha(m_2^*, m_1))] \\ &= [1 - \alpha(m_2^*, m_1)/m_2^*] K_2(\alpha) E_{\pi_1|K_1} [m_2(\pi_1, K_1)] - D[K_2(\alpha)] \\ &= E_1[G_2|s_2 = \alpha(m_2^*, m_1)] \quad \text{since } E_{\pi_1|K_1} [m_2(\pi_1, K_1)] = m_1 \end{aligned}$$

(see Raiffa and Schlaifer (1961), Section 5.4, Theorem 1).

Q.E.D.

We obtain, as corollaries of proposition 6.1, the following results :

Proposition 6.2 : (i) If the myopically optimal K_1 is positive and only the home government learns, it will never be optimal to forgo first period investment.

(ii) Suppose that the myopically optimal K_1 is zero. It is possible that the truly optimal K_1 is positive since the first period loss may be more than compensated by the gain due to learning.

(iii) Suppose that the myopically optimal K_1 is positive. If $G(K)/K = \text{constant}$, the myopically optimal K_1 is also truly optimal, since any change in $K_1 (> 0)$ does not improve the value of learning.

If $G(K)/K \neq \text{constant}$ the expected value of information would in general depend on the magnitude of K_1 . In order to obtain further results in this direction, we add the assumption that $h_1(x)$ is normal with mean m_1 and variance V_1 . With this normality assumption, and the normality of $q(\epsilon)$, (41) can be transformed into an expression involving the distribution of m_2 by standard transformation procedures :

$$E_{\pi_1|K_1}[E_2(G_2|s_2 = \rho(m_2))] = \int_{-\infty}^{\infty} \{ [1 - \rho(m_2)/m_2^*] K_2(\rho) m_2 - D[K_2(\rho)] \} f(m_2|K_1) dm_2$$

$$\int_{-\infty}^{\infty} L(m_2) f(m_2|K_1) dm_2 \quad (44)$$

where $f(m_2|K_1)$ is the p.d.f. of m_2 . It is normal, with mean $E(m_2) = m_1$ and variance :

$$V(m_2) = \frac{V_1^2}{V_1 + \sigma^2 [G(K_1)]^2 / K_1^2} \quad (45)$$

where σ^2 is the variance of $q(\epsilon)$.

We are now in the position to prove the following result :

Proposition 6.3 : Assume that in the absence of learning, the optimal K_2 is zero, that is, we assume $[1 - r_2(\bar{K})/m_2^*]m_1 < D'(0)$. Under the normality assumption for $h_1(x)$ and $q(\epsilon)$, the expected value of information is an increasing function of V_1 and a decreasing function of σ^2 , for given K_1 .

(iii) Suppose that the myopically optimal K_1 is positive. If $G(K)/K = \text{constant}$, the myopically optimal K_1 is also truly optimal, since any change in $K_1 (> 0)$ does not improve the value of learning.

If $G(K)/K \neq \text{constant}$ the expected value of information would in general depend on the magnitude of K_1 . In order to obtain further results in this direction, we add the assumption that $h_1(x)$ is normal with mean m_1 and variance V_1 . With this normality assumption, and the normality of $q(\epsilon)$, (41) can be transformed into an expression involving the distribution of m_2 by standard transformation procedures :

$$E_{\pi_1|K_1}[E_2(G_2|s_2 = \rho(m_2))] = \int_{-\infty}^{\infty} \{ [1 - \rho(m_2)/m_2^*] K_2(\rho) m_2 - D[K_2(\rho)] \} f(m_2|K_1) dm_2$$

$$\int_{-\infty}^{\infty} L(m_2) f(m_2|K_1) dm_2 \quad (44)$$

where $f(m_2|K_1)$ is the p.d.f. of m_2 . It is normal, with mean $E(m_2) = m_1$ and variance :

$$V(m_2) = \frac{V_1^2}{V_1 + \sigma^2 [G(K_1)]^2 / K_1^2} \quad (45)$$

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Proposition 6.3 : Assume that in the absence of learning, the optimal K_2 is zero, that is, we assume $[1 - r_2(\bar{K})/m_2^*]m_1 < D'(0)$. Under the normality assumption for $h_1(x)$ and $q(\epsilon)$, the expected value of information is an increasing function of V_1 and a decreasing function of σ^2 , for given K_1 .

Proof :

We only need to show that (44) is an increasing function of $V(m_2)$. Consider any two normal p.d.f. $f_a(m_2|K_1)$ and $f_b(m_2|K_1)$ with identical mean m_1 and variances $V_a(m_2) > V_b(m_2)$. We wish to show that :

$$\int_{-\infty}^{\infty} L(m_2) [f_a(m_2|K_1) - f_b(m_2|K_1)] dm_2 > 0 \quad (46)$$

For $m_2 < m_1$ (which is $< D'(0)[1-r_e(\bar{K})/m_1^*]$ by assumption), $L(m_2) = 0$. Hence we only need to show

$$\int_{m_1}^{\infty} L(m_2) [f_a(m_2|K_1) - f_b(m_2|K_1)] dm_2 > 0 \quad (47)$$

There exists a unique $m_0 > m_1$ such that $f_a(m_2|K_1) < f_b(m_2|K_1)$ for $m_2 \in [m_1, m_0)$ and $f_a(m_2|K_1) > f_b(m_2|K_1)$ for $m_2 > m_0$ (To prove this, equate the two density functions and take their logarithms). This fact and $L'(m_2) > 0$ imply :

$$\begin{aligned} & \int_{m_1}^{\infty} L(m_2) [f_a(m_2|K_1) - f_b(m_2|K_1)] dm_2 > \\ & \int_{m_1}^{m_0} L(m_0) [f_a(m_2|K_1) - f_b(m_2|K_1)] dm_2 = \int_{m_0}^{\infty} L(m_0) [f_a(m_2|K_1) - f_b(m_2|K_1)] dm_2 \\ & = L(m_0) \left[\int_{m_1}^{\infty} f_a(m_2|K_1) dm_2 - \int_{m_1}^{\infty} f_b(m_2|K_1) dm_2 \right] = 0 \end{aligned}$$

Q.E.D.

The relationship between the myopically optimal investment in period one, K_1^{**} , and the truly optimal K_1^0 is partially characterized by the following proposition.

Proposition 6.4 : Under the assumptions made in the statement of proposition 5.3, (i) $K_1^{**} \leq K_1^0$ if $G(K)$ is a non-negative, concave function or if $G(K)$ is a non-positive, convex function; (ii) $K_1^{**} \geq K_1^0$ if $G(K)$ is a non-negative, convex function or if $G(K)$ is a non-positive, concave function. Strict inequalities hold for strict convexity or concavity of $G(K)$.

Proof :

Define s_1^{**} and s_1^0 by $K_1^{**} = K_1(s_1^{**})$, $K_1^0 = K_1(s_1^0)$. s_1^0 maximizes $E(S_1) + \delta E.V.I.$ Hence :

$$\left(\frac{dE_1(G_1)}{ds_1} = \delta \frac{dE.V.I.}{ds_1} \right)_{s_1=s_1^0} = 0 \quad (48)$$

But

$$\frac{dE.V.I.}{ds_1} = \frac{dE.V.I.}{dV(m_2)} \cdot \frac{dV(m_2)}{dK_1} \cdot \frac{dK_1}{ds_1} \quad (49)$$

From the proof of proposition 6.3, $dE.V.I./dV(m_2) > 0$. Therefore sign $dE.V.I./ds_1 = \text{sign } dV(m_2)/K_1 = - \text{sign } G(K)[G' - G(K)/K]$ by (45). Since by assumption $G(K) = 0$ if and only if $K = 0$,

$G(K) = G(K) - G(0) \geq (K-0)G'$ depending on $G'' \leq 0$. Hence

$dV(m_2)/dK_1 \geq 0$ if $G(K)$ is concave and non-negative and non-positive or convex and non-negative. Thus $dE_1(G_1)/ds_1$, evaluated at s_1^0 , is non-positive in the first case, implying $s_1^0 \leq s_1^{**}$, and non-negative in the second case, implying $s_1^0 \geq s_1^{**}$ (the same argument was used in (38), proposition 5.1).

Q.E.D.

Proposition 6.4 has an intuitive interpretation: when the "noise" $|G(K)|/K$ declines with K , it pays to increase K_1 beyond its myopic level K_1^{**} . It is interesting to note that, unlike the

type one uncertainty case, the production function plays an important role in the determination of the optimal level of investment in period one.

7. TYPE TWO UNCERTAINTY : OTHER CASES :

Suppose that the foreign capitalists are Bayesians while the home government is not. It does not revise expectations but it knows that foreign capitalists are Bayesians. In this case, unlike section V, it is not possible to show that the expected value of information, from the point of view of the home government, is non-negative. This is to be expected since information tends to benefit foreigners. The home government, however, can benefit from foreign learning under the condition stated below :

Proposition 7.1 : Suppose that in the second period it will be optimal not to have any foreign investment if foreign capitalists do not revise their expectation upwards; that is $E_1[G_2 | s_2 = \alpha(m_1^*, m_1)] = 0$. Then the expected value of information is non-negative.

Proof : Obvious, since $E_{\pi_1 | K_1} [\max_{s_2} E_1(G_2)] \geq 0$.

Q.E.D.

Though trivial, proposition 7.1 has an interesting implication : Suppose the myopic K_1 and K_2 are zero. It may be optimal however to encourage foreign capital in period one, since in the second period, the home country would benefit from any upward revision by foreign capitalists and would not lose anything on a result of a downward revision.

Further analysis can be pursued using the normality assumption for x . Results similar to propositions 6.3 and 6.4 can be obtained. When both sides of the market are Bayesian, the analysis becomes complicated, unless both sides have the same subjective beliefs, in which case we can obtain results similar to propositions 6.3, 6.4 and 7.1.

8. CONCLUSION :

We have shown that in general the truly optimal policy must take into account the affects of present decisions on learning possibilities as well as the effects of future learning on the future optimization problem. When information is not a free good, the truly optimal policy implies an optimal information policy. Our foreign investment model provides a concrete example as to how informational considerations affect policies toward foreign investment : It may be optimal to incur losses in the first period in order to learn or to avoid foreign learning, depending on various situations. Of particular interest is the case with an intertemporal exhaustible resource constraint where it is shown that less resource is committed in the first period than would be the case if there were no learning possibilities.

Under type one uncertainty, the information policy consists of an all-or-nothing choice. For type two uncertainty, however, except for the case $G'(K) = \text{constant}$, the "amount" or rather "quality" of information can be controlled, and the truly optimal level of investment generally departs from the myopic level.

The implications of learning for economic optimization are rich. Although our emphasis is on the normative aspects of the problem, our analysis is not without positive significance. The same concepts and methods can be used to study market behaviour.

CHAPTER VIII

ATTITUDES TO RISK AND THE OPTIMAL ALLOCATION OF NATURAL RESOURCES BETWEEN THE DOMESTIC AND FOREIGN SECTORS

1. INTRODUCTION :

In Chapter VII we assumed risk neutrality in order to simplify our analysis of the problem of optimal regulation under adaptive expectation. It was also assumed that only foreign capitalists exploit domestic resources. These simplifying assumptions enabled us to focus on the problems associated with expectation revision and learning. This Chapter will relax these assumptions. Our emphasis will be on attitudes to risk and their effects on the optimal allocation of natural resources between the domestic and foreign sectors. In particular, we shall attempt to provide some answers to questions such as : Suppose that the investment in natural resources is a risky activity, how should a country allocate its natural resources between the domestic and foreign sectors? Should it allocate more natural resources to domestic investors if the country becomes wealthier? How does the optimal allocation depend on certain technological parameters?

2. THE BASIC MODEL AND ASSUMPTIONS :

The home country possesses a stock $\bar{R}$ of non-depletable¹ resource (say land), and a stock of capital $\bar{K}_d$. Let R_f be the amount of domestic resource to be exploited by foreign capital and R_d by domestic capital, we have

¹ Perhaps all resources are depletable. But if the deterioration due to use is negligible, then non-depletability is a convenient assumption.

$$R_f + R_d \leq \bar{R} \quad (1)$$

One unit of resource requires b_d units of domestic capital, so that the amount of domestic capital used in the resource exploiting activity is :

$$K_d = R_d/b_d \leq \bar{K}_d \quad (2)$$

Similarly, the amount of foreign capital used is

$$K_f = R_f/b_f \geq 0 \quad (3)$$

It is assumed that foreign capital is not used in any other sector in the home country. The amount of domestic capital not used in the resource sector, $(\bar{K}_d - K_d)$ is invested elsewhere, earning a return w per unit of capital. Each unit of resource (combined with the appropriate amount of capital) yields a return x which is a *random variable*.

Thus, the total return to domestic capital is

$$w(\bar{K}_d - K_d) + K_d x/b_d \quad (4)$$

and foreign capital earns

$$K_f(1-\theta)x/b_f \quad (5)$$

where θ is the tax rate on foreign earnings.

Given foreign capitalists' (subjective) probability distributions of x (which need not be identical), each foreign capitalist varies his investment with the tax rate θ :

$$K_f^i = K_f^i(\theta) \quad (6)$$

If all individual supply functions are monotonic decreasing, the total foreign supply of capital is obtained by summing up individual supply functions. Hence we may write

$$K_f = K_f(\theta) \quad (7)$$

with $K'_f(\theta) < 0$.

The home country wishes to maximize its expected utility (by choosing K_d and θ).

$$\text{Max. } EU[w(\bar{K}_d - K_d) + xK_d/b_d + \theta xK_f(\theta)/b_f] \quad (8)$$

subject to

$$K_d \geq 0 \quad (9)$$

and (1), (2) and (3).

3. THE FOREIGN SUPPLY OF CAPITAL :

In section 2 it was assumed that the supply of foreign capital is monotonic decreasing in θ . This section will provide some justification for this assumption. Two alternative justifications are :

i) Foreign capitalists are risk neutral and production in the foreign country is subject to diminishing returns. This is the approach adopted in Chapter VII where we derived a monotonic increasing function

$$K_f = K_f(s)$$

where $s = (1-\theta)E(x)/b_f$ ($b_f = 1$ in Chapter VII). Hence K_f is monotonic decreasing in θ .

ii) Alternatively, we may assume that foreign capitalists are "risk averse" and their "absolute risk aversion" is a non-increasing function of wealth. In addition, the return to capital in the foreign country is a constant, $\bar{w}$. This will be explained more precisely below.

Each individual foreign capitalist owns an amount $\bar{K}_f^i$ of capital. He wishes to allocate $K_f^i \leq \bar{K}_f^i$ to the resource exploiting activity in the home country and the remainder, $(\bar{K}_f^i - K_f^i)$, is used in his own country. The allocation must maximize his expected utility :

$$\text{Maximize } EU^i [K_f^i(1-\theta)x/b_f + (\bar{K}_f^i - K_f^i)\bar{w}]$$

Note that the investor takes θ as given. Assuming an interior solution, the first order condition is²

$$EU' \cdot [(1-\theta)x/b - \bar{w}] = 0 \quad (8)$$

and the second order condition is

$$D \equiv EU'' \cdot [(1-\theta)x/b - \bar{w}]^2 < 0 \quad (9)$$

If the investor is risk averse (i.e. $U'' < 0$) then $D < 0$.

We wish to examine the effect of an increase in the tax rate θ on the supply of risky capital K_f^i . Differentiating (8), we obtain

$$\frac{dK_f^i}{d\theta} = - \frac{1}{D} \{ EU' \cdot (-x/b) - EU'' \cdot [(1-\theta)x/b - \bar{w}] K_f^i x/b \} \quad (10)$$

The first term in {...} is negative but without further assumption the sign of the second term is ambiguous. Following Arrow (1971) and

² Superscripts and subscripts are dropped when there is no confusion.

Pratt (1964), we define the absolute risk aversion function :

$$R_A(Y) = - U''(Y)/U'(Y) \quad (11)$$

Absolute risk aversion is said to be decreasing, constant or increasing depending on whether $R'_A(Y)$ is negative, zero, or positive. Using a technique developed by Sandmo (1971) we will prove the following lemma :

Lemma 3.1 : Assume that x is a non-negative random variable.

Then

$$EU'' \cdot [(1-\theta)x/b - \bar{w}] K_f^i x/b$$

is positive, zero or negative depending on whether absolute risk aversion is decreasing, constant or increasing.

Proof : Note that in (10), K_f^i is fixed at its optimal value. For given K_f^i , the income Y is a function of the random variable x .

$$Y = K_f^i(1-\theta)x/b + (\bar{K}_f^i - K_f^i)\bar{w} \quad (12)$$

Let x^0 be the value of x such that

$$(1-\theta)x^0/b - \bar{w} = 0 \quad (13)$$

The associated income level is $Y(x^0)$. For $x > x^0$, $Y(x) > Y(x^0)$ by (12). Decreasing absolute risk aversion implies :

$$\frac{-U''[Y(x)]}{U'[Y(x)]} < \frac{-U''[Y(x^0)]}{U'[Y(x^0)]} \equiv T \quad (14)$$

for $x > x^0$. Also, for $x > x_0$,

$$[(1-\theta)x/b - \bar{w}]K_f^i x/b > 0 \quad (15)$$

because $x > 0$. Hence for $x > x^0$

$$\begin{aligned} T.U'[Y(x)] \cdot [(1-\theta)x/b - \bar{w}]K_f^i x/b \\ > -U''[Y(x)] \cdot [(1-\theta)x/b - \bar{w}]K_f^i x/b \end{aligned} \quad (16)$$

For $x < x^0$, both inequalities (14) and (15) are reversed. Hence (16) holds for all x . Taking expected values of both sides and noting (8), we conclude that the right-hand side of (16) is negative. Thus Lemma 3.1 is proved for the case of decreasing absolute risk aversion. The proofs for other cases are similar.

We can now state proposition 3.1 :

Proposition 3.1³ : Suppose $x \geq 0$ and foreign investors are risk averse. Then

$$\frac{dK_f}{d\theta} < 0$$

if absolute risk aversion is constant or decreasing.

The proof follows from (9), (10) and Lemma 3.1.

The related questions of interest are

- (i) whether more capital is invested in the risky activity as wealth $(\bar{K}_f^i)$ increases; and
- (ii) whether the *proportion* of wealth invested in the risky activity increases with wealth.

³ Stiglitz (1969) stated essentially the same result (his Proposition 3.a) but did not offer a direct proof.

Stiglitz (1969) showed that, for a *competitive* investor,

- (a) as wealth increases, more is invested in the risky activity if and only if *absolute risk aversion* is decreasing
- (b) as wealth increases, the proportion of one's wealth in the riskless activity decreases if and only if *relative risk aversion* is decreasing, where relative risk aversion is defined by

$$R_R(Y) = - YU''(Y)/U'(Y)$$

Result (b) does not hold in general if there are more than one risky activity. Cass and Stiglitz (1972) provided a set of sufficient conditions for (b) to hold when there are $n > 2$ assets, one of which is riskless. These conditions are rather restrictive. In fact some weakening of these conditions is possible.⁴

4. THE OPTIMAL ALLOCATION OF NATURAL RESOURCES BETWEEN THE DOMESTIC AND FOREIGN SECTORS :

Since $K_f(\theta)$ is monotonic decreasing, it can be inverted

$$\theta = \theta(K_f) \quad (17)$$

(8) can then be written as

$$EU[w\bar{K}_d + K_d(x/b_d - w) + \theta(K_f)K_fx/b_f] \quad (18)$$

In order to sharpen the analysis we shall make the following assumptions

⁴ See Appendix VIII.

i) The home country has sufficient capital to exploit all its natural resource (although it may not choose to do so, especially in view of the availability of foreign capital). That is, we assume

$$\bar{R}/b_d < \bar{K}_d \quad (19)$$

ii) Were foreign capital not available, the home country would find it optimal to exploit all its natural resource. To ensure this, we assume the dominance of the resource exploiting activity

$$x/b_d \geq \underline{x}/b_d \geq w \quad (20)$$

Condition (20) implies that investment in the resource industry is "more profitable". (If the domestic economy were decentralized, this would require a rent payable for the use of the resource so as to eliminate the excess demand for the rights to exploit the resource.)

It is important to note that conditions (19) and (20) *do not* necessarily imply that the optimizing home country will use its capital to exploit all of its natural resource. This might seem "paradoxical" at first, in view of (20). But upon reflection, it is quite simple: Instead of exploiting all the resource by the use of domestic capital, the home country may withdraw b_d units of domestic capital from the resource sector and use these in the non-resource sector. The loss to domestic capital is $(x - wb_d)$ but the gain (in the form of tax revenue from foreign capital replacing the withdrawn domestic capital) is $\hat{K}_f \theta(\hat{K}_f) x / b_f$ where $\hat{K}_f = b_f$ (the amount of foreign capital required for each unit of resource). The optimal allocation must strike some balance between the gain and the loss. In the case of a *risk-neutral* social utility function, the balance is at the point where expected marginal gain equals expected marginal loss (as will be proved).

It should be obvious, from (19) and (20), that all resource will be exploited. This result will be stated as Lemma 4.1.

Lemma 4.1 :

$$\bar{R} = R_d + R_f = K_d b_d + K_f b_f \quad (21)$$

Proof : Form the Lagrangean :

$$\begin{aligned} L = & EU[w\bar{K}_d + K_d(x/b_d - w) + \theta(K_f)K_f x/b_f] \\ & + \lambda_1(\bar{K}_d - K_d) + \lambda_2 K_d + \lambda_3 K_f \\ & + \lambda_4(\bar{R} - K_f b_f - K_d b_d) \end{aligned}$$

The first order conditions are :

$$\partial L / \partial K_d = EU' \cdot [x/b_d - w] - \lambda_1 + \lambda_2 - \lambda_4 b_d = 0 \quad (22)$$

$$\begin{aligned} \partial L / \partial K_f = & EU' \cdot [\theta(K_f)x/b_f - \theta'(K_f)K_f x/b_f] \\ & + \lambda_3 - \lambda_4 b_f = 0 \end{aligned} \quad (23)$$

$\lambda_i \geq 0$ (= 0 if the associated constraint holds with equality).

(20) and (22) imply that $\lambda_1 + \lambda_4 b_d > 0$. We wish to show that $\lambda_4 > 0$. Suppose $\lambda_4 = 0$ while $\lambda_1 > 0$. This implies that $K_d = \bar{K}_d$, which, in view of (19), implies that $\bar{R} - K_f b_f - K_d b_d < 0$, which is a contradiction. Hence $\lambda_4 > 0$, i.e. Lemma 4.1 is proved.

In view of Lemma 4.1, we can re-write (18) as

$$EU[w\bar{K}_d + (\bar{R}/b_d - K_f/b_d)(x/b_d - w) + K_f \theta(K_f)x/b_f] \quad (24)$$

Assuming an interior maximum, the first order condition is, from (24),

$$EU' \cdot [-(b_f/b_d)(x/b_d - w) + \theta(K_f)x/b_f + K_f\theta'(K_f)x/b_f] = 0 \quad (25)$$

$\theta(K_f)x/b_f + K_f\theta'(K_f)x/b_f$ is the marginal gain (MG) from employing one extra unit of foreign capital in the resource sector.

$(b_f/b_d)(x/b_d - w)$ is the marginal loss (ML) caused by shifting b_f/b_d units of domestic capital to a less productive use. Note that both the marginal gain and marginal loss are probabilistic. Equation (25) says that the marginal gain, weighted by probabilities and by marginal utility, equals the marginal loss (similarly weighted).

The second order condition for a maximum is :

$$D = EU'' \cdot [MG - ML]^2 + EU' \cdot [(2\theta' + \theta'' \cdot K_f)x/b_f] < 0 \quad (26)$$

(26) is satisfied if the total gain function $\theta(K_f)K_fx/b_f$ is concave in K_f and if the utility function exhibits risk aversion ($U'' < 0$) or risk neutrality ($U'' = 0$). Mild risk preference may also be consistent with (26). It is assumed that $D < 0$ at an extremum.

We may now compare the optimal amount of foreign capital (and hence the optimal quantity of resource allocated to foreigners) under uncertainty with the optimal amount under "certainty", as the situation where the random variable x takes its expected value (from the home country's point of view). The last qualification is of some importance : it is possible that x is certain from the point of view of the home country while it is regarded as stochastic by foreigners.

Under certainty, the optimal amount of foreign capital used is given by :

$$-(b_d/b_f)(Ex/b_d - w) + [\theta(K_f^C) + K_f^C\theta'(K_f^C)]Ex/b_f = 0 \quad (27)$$

where the subscript c stands for certainty.

Under uncertainty, the first order condition is (25). Let the associated optimal K_f be denoted by K_f^u (u standing for uncertainty). The following proposition will be proved :

Proposition 4.2 : K_f^u is greater than, equal to or less than K_f^c depending on whether the utility function $U(Y)$ of the home country exhibits risk aversion, risk neutrality, or risk preference ($U'' < 0$, $U'' = 0$, or $U'' > 0$).

Proof : If $U'' = 0$, U' is a constant, hence (25) and (27) are equivalent expressions. Thus $K_f^c = K_f^u$.

If $U'' < 0$, then for any Y_1, Y_2 with $Y_2 > Y_1$, we must have $U'(Y_1) > U'(Y_2)$. Fix K_f at K_f^c , Y is then a function of x .⁵ For $x > Ex$,

$$Y(K_f^c, x) > Y(K_f^c, Ex) \quad (28)$$

and hence

$$U'[Y(K_f^c, x)] < U'[Y(K_f^c, Ex)] \quad (29)$$

Also, for $x > Ex$,

$$[\theta(K_f^c) + K_f^c \theta'] x / b_f > (b_d / b_f) (x / b_d - w) \quad (30)$$

in view of (27). Hence, for $x > Ex$,

⁵ This method of proof is adapted from Sandmo (1971).

$$\begin{aligned} & \{ [\theta(K_f^C) + K_f^{C\theta'}]x/b_f - (b_d/b_f)(x/b_d - w) \} U' [Y(K_f^C, x)] \\ & < \{ [\theta(K_f^C) + K_f^{C\theta'}]x/b_f - (b_d/b_f)(x/b_d - w) \} U' [Y(K_f^C, Ex)] \end{aligned} \quad (31)$$

It is easily seen that for $x < Ex$, both inequalities (29) and (30) are reversed, so that (31) in fact holds for *all* x .

Taking expected values for both sides of (31), and noting (27), it follows that

$$E\{ [\theta(K_f^C) + K_f^{C\theta'}]x/b_f - (b_d/b_f)(x/b_d - w) \} U' [Y(K_f^C, x)] < 0 \quad (32)$$

Hence K_f^u , which satisfies (25), must be greater than K_f^C , due to the inequality (32) and the second order condition $D < 0$ at K_f^u .

The case of risk preference can be proved in exactly the same manner, provided that the risk preference is sufficiently mild so that the second order condition is not violated.

Q.E.D.

5. EFFECTS OF CHANGES IN WEALTH AND IN TECHNOLOGICAL PARAMETERS :

5.1 Changes in The Capital Stock :

Given the same $\bar{R}$, a country with a larger capital stock $\bar{K}_d$ may allocate more or less of its natural resources to the foreign investors, depending on the nature of its absolute risk aversion function. This will be made precise by the following proposition.

Proposition 5.1 : If absolute risk aversion is decreasing, then an increase in $\bar{K}_d$ will increase K_f^u . The converse applies if absolute risk aversion is increasing.

Proof : Differentiate (25) with respect to $\bar{K}_d$, taking K_f^u as a function of $\bar{K}_d$.

$$\begin{aligned} dK_f^u/d\bar{K}_d = & \frac{-1}{D} EU'' \cdot w \cdot [- (b_d/b_f) (x/b_d - w) \\ & + \theta(K_f^u) x/b_f + K_f^u \theta' x/b_f] \end{aligned} \quad (33)$$

where $D < 0$ is given by (26). Using the method of proof in Lemma 3.1, we obtain

$$dK_f^u/d\bar{K}_d \leq 0$$

if and only if

$$R'_A(Y) \geq 0.$$

5.2 Changes in the Resource Stock :

The effect on K_f^u of an increase in $\bar{R}$ for given $\bar{K}_d$ is very similar to the effect of an increase in $\bar{K}_d$ for given $\bar{R}$:

Proposition 5.2 : An increase in $\bar{R}$ will increase, leave constant, or decrease K_f^u depending on whether absolute risk aversion is decreasing, constant, or increasing.

Proof : Differentiate (25) with respect to $\bar{R}$, taking K_f^u as a function of $\bar{R}$:

$$\begin{aligned} dK_f^u/d\bar{R} = & \frac{-1}{(D \cdot b_d)} EU'' \cdot (x/b_d - w) [- (b_d/b_f) (x/b_d - x) \\ & + \theta(K_f^u) x/b_f + K_f^u \theta' x/b_f] \end{aligned} \quad (34)$$

Note that (20) is assumed, so that the method of proof in Lemma 3.1 works in this case, too. Hence

$$dK_f^u/d\bar{R} \leq 0$$

if and only if

$$R'_A(Y) \geq 0$$

5.3 Changes in Technological Parameters :

For convenience, we define

$$\alpha = 1/b_d$$

An increase in α (a decrease in b_d) is a technological improvement since less domestic capital will be required for each unit of resource. An increase in w is also an improvement. Their effects are not identical, however. This is because income is a linear function of w but not a linear function of α , as an inspection of (24) reveals.

Proposition 5.3 : If absolute risk aversion is decreasing or constant, an increase in w will increase K_f^u . The effect of an increase in w is ambiguous if absolute risk aversion is increasing.

The effect of an increase in α on K_f^u is quite ambiguous, except in the risk neutrality case, where K_f^u will decrease. K_f^u also decreases if it is sufficiently small and absolute risk aversion is constant or increasing.

Proof : Differentiating (25) with respect to w yields

$$dK_f^u/dw = \frac{-1}{D} \{ EU' \cdot (\alpha b_f) + EU'' \cdot [\bar{K}_d - K_d] [-\alpha b_f(x\alpha - w) + \theta(K_f)x/b_f + K_f\theta'x/b_f] \} \quad (35)$$

The first term, $EU' \cdot (\alpha b_f)$ is clearly positive, while the second term is positive, zero, or negative depending on whether absolute risk aversion

is decreasing, constant, or increasing (using the arguments in Lemma 3.1). This proves the first part of the proposition.

Differentiating (25) with respect to α yields :

$$\begin{aligned} dK_f^u/d\alpha = & \frac{-1}{D} \{ EU' \cdot [-b_f(x\alpha-w) - b_f x\alpha] \\ & + EU'' \cdot [K_d x\alpha - K_f b_f \alpha(x\alpha-w)] [-ab_f(x\alpha-w) + \theta(K_f)x/b_f + K_f \theta' x/b_f] \} \end{aligned} \quad (36)$$

The first expectation term is negative, while the second term is ambiguous, except when K_f is sufficiently small ($K_d > K_f b_f \alpha$), then it is positive, zero, or negative depending on decreasing, constant, or increasing absolute risk aversion. The second term vanishes in the risk neutrality case.

Q.E.D.

The effect of a change in b_f is ambiguous, since the supply function $\theta(K_f)$ (and hence its derivative) has b_f as a parameter. While it is possible to determine the effect of b_f on $\theta(K_f)$, its effect on $\theta'(K_f)$ is quite ambiguous.

6. CONCLUSION :

We have been able to explore some effects of attitudes to risk on the optimal allocation of natural resources between the domestic and foreign sectors. It is interesting that the knowledge of the "primary" attitudes to risk (risk aversion, risk neutrality, or risk preference) is often not enough. More detailed information about attitudes to risk, such as the derivatives of the absolute and relative risk aversion functions are required. If individuals are concerned

about the "average" level of utility of their community, they will tend to disagree on the optimal amount of foreign investment if their attitudes to risk (as well as evaluations of risk) are different. This may explain *partly* some conflicts of opinions on foreign investment in resources.

Recent paper Cass and Stiglitz (1972) showed that if there are n securities in states of nature, and if the risky securities yield returns in only one state of nature, and all assets are held in positive amounts, then the (wealth) elasticity of demand for the riskless asset is greater than, equal to, or less than unity as relative risk aversion increases, remains constant, or decreases with wealth (Theorem 1", p.349). They also stated: "Under weaker, stronger theorems are not seen available", "... with a slightly more complicated pattern of returns, the wealth elasticity of demand for (the riskless asset) may be greater than or less than unity, regardless of whether there is increasing or decreasing relative risk aversion" (p.341).

We shall show that in fact a stronger theorem is available. That is, there are sufficient conditions which are weaker than those given by Cass and Stiglitz. Before stating our theorem it is useful to recall Cass and Stiglitz's notation:

There are n securities and s states of nature. The j th security yields a return per dollar invested in it of r_{js} in state s . The matrix of returns is a square matrix R :

$$R = (r_{js}) \quad j, s = 1, 2, \dots, n \quad (1)$$

¹ This Appendix will appear in the Review of Economic Studies.

APPENDIX VIII

RISK AVERSION AND WEALTH EFFECTS ON PORTFOLIOS

WITH MANY ASSETS : AN EXTENSION¹

In a recent paper Cass and Stiglitz (1972) showed that if there are as many securities as states of nature, each of the risky securities yields returns in only one state of nature, and all assets are held in positive amounts, then the (wealth) elasticity of demand (for the riskless asset) is greater than, equal to, or less than unity as relative risk aversion increases, remains constant, or decreases with wealth (Theorem 1", p.340). They also stated : "Unfortunately, stronger theorems do not seem available", ... "with a slightly more complicated pattern of returns, the wealth elasticity of demand for (the riskless asset) may be greater than or less than unity, regardless of whether there is increasing or decreasing relative risk aversion" (p.341).

We shall show that in fact a stronger theorem is available. That is, there are sufficient conditions which are weaker than those given by Cass and Stiglitz. Before stating our theorem it is useful to recall Cass and Stiglitz's notation.

There are n securities and n states of nature. The i^{th} security yields a return per dollar invested in it of $\rho_i \theta$ in state θ . The matrix of returns is a square matrix P :

$$P = [\rho_i \theta] \quad i, \theta = 1, 2, \dots, n \quad (1)$$

¹ This Appendix will appear in the *Review of Economic Studies*.

The first security is a safe asset. By assumption $\rho_{1\theta} = \rho_M > 0$ for all θ . Cass and Stiglitz assumed that the pattern of returns for the $(n-1)$ risky assets is :

$$\rho_{i\theta} = 0 \quad \text{for } \theta \neq i \ (i > 1) \quad (2)$$

$$\rho_{i\theta} = \rho_i > 0 \quad \text{for } \theta = i \ (i > 1) \quad (3)$$

Conditions (2) and (3) are rather stringent. We offer the following weaker conditions on the pattern of returns for the risky assets :

- (i) There exists some state of nature (which we shall assume to be the first state of nature, without loss of generality) such that

$$\rho_{i1} = 0 \quad (i > 1) \quad (4)$$

- (ii) Let S be the submatrix obtained by deleting the first row and the first column of P . We require that S is non-singular, (given (i) and the assumption that the first asset is safe (ii) is equivalent to the assumption that P is non-singular).

Obviously our assumptions are weaker than those of Cass and Stiglitz since we do not require that S be a diagonal matrix nor do we assume that the diagonal elements of S be strictly positive.

We shall prove the following theorem, which is a generalization of Cass and Stiglitz's Theorem 1' :

Theorem : If there are as many securities as states of nature, there exists some state of nature in which all the $(n-1)$ risky securities

yield zero returns, the n patterns of returns are linearly independent (i.e. rank $P = n$) and all assets are held in positive amounts² (in the optimal portfolio), then the wealth elasticity of demand for the riskless asset is greater than, equal to, or less than unity as relative risk aversion increases, remains constant, or decreases with wealth.

It might seem at first that any $(n-1) \times (n-1)$ submatrix S could be shown equivalent to the Cass-Stiglitz diagonal submatrix simply by a change of basis (a redefinition of the basic risky assets) provided that S is non-singular and/or S has $n - 1$ independent eigenvectors. This conjecture turns out to be incorrect because of the following constraints :

- (i) the Cass-Stiglitz submatrix must have *positive* diagonal elements, and
- (ii) by definition of a matrix of returns, each row is the pattern of returns *per dollar* invested in the corresponding assets. Hence each of the newly defined patterns of returns must be a linear combination, with weights adding to one, of the original patterns of returns.

Consider the following counter-example :

$$S = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

S is non-singular and has two distinct eigenvalues (and hence two linearly independent eigenvectors), but there are no $t_1, t_2, r_1 > 0, r_2 > 0$ such that :

² This assumption may be weakened. See footnote 5.

$$t_1(1,2) - (1-t_1)(3,4) = (r_1, 0)$$

$$t_2(1,2) - (1-t_2)(3,4) = (0, r_2)$$

Before proving the theorem we need the following results which are hinted at in an earlier paper by Cass and Stiglitz (1970) but no precise statements nor formal proofs were given³.

Proposition 1 : Let $Q = (q_{ij})$ be an $n \times n$ semi-positive, non-singular matrix of returns. Let b_{ij} be the (i,j) entry of Q^{-1} .

If $\sum_{j=1}^n b_{ij} \leq 0$ for some i , then there exists some original security dominated by a mutual fund (i.e. linear combination, with weights adding to one, of the original securities).

As a trivial corollary of proposition 1, we state :

Proposition 2 : Let Q be an $n \times n$ semi-positive, non-singular matrix of returns. If there is no original security which is dominated by some mutual fund, then $Q^{-1}e > 0$, where e is a (conformal) column vector with all its entries equal to unity.

Proof of Proposition 1 : Assume $\sum_{j=1}^n b_{ij} \leq 0$ for some i . Define $\mu = (\mu_1, \mu_2, \dots, \mu_n)$ where

$$\mu_h = b_{ih}, \quad h = 2, 3, \dots, n \quad (5)$$

$$\mu_1 = b_{i1} - \sum_{j=1}^n b_{ij} \quad (6)$$

³ See Appendix I of Cass and Stiglitz (1970), especially the sixth paragraph on page 152.

$$\mu Q = \left(-\sum_{j=1}^n b_{ij}, 0, 0, \dots, 0, 0\right)Q + (b_{i1}, b_{i2}, \dots, b_{in})Q$$

$$\therefore \mu Q \geq (b_{i1}, b_{i2}, \dots, b_{in})Q \quad (7)$$

The right-hand-side of (7) is the i^{th} row of the matrix $Q^{-1}Q = I$. Hence

$$\mu Q \geq \underline{0}' \quad (8)$$

with strict inequality for at least the i^{th} component. ($\underline{0}'$ is a zero row vector in R^n).

Note that $\mu \neq \underline{0}'$ by (8) and $\sum_{i=1}^n \mu_i = 0$ by (5) and (6). Hence $\mu_k < 0$ for some k .

Note that μQ is a linear combination of the rows of Q . Let Q_j be the j^{th} row of Q . From (8)

$$\sum_{j=1}^n \mu_j Q_j \geq \underline{0}' \quad (9)$$

with strict inequality for at least the i^{th} component. Hence, for some $\mu_k < 0$, (9) gives

$$\sum_{j \neq k} (\mu_j / -\mu_k) (Q_j) \geq Q_k + (1 / -\mu_k) \underline{0}' = Q_k \quad (10)$$

The security k is therefore dominated by the mutual fund described by the left-hand-side of (10).

Q.E.D.

The proof of proposition 2 is obvious since the statement $X \Rightarrow Y$ is equivalent to $\text{Non } Y \Rightarrow \text{Non } X$.

We are now ready to prove our theorem.

Proof of the Theorem : The assumption that the optimal portfolio consists of positive amounts of all securities implies that none of the securities is dominated by a mutual fund. That is, no row of P is dominated by a linear combination, with weights adding to unity, of other rows of P . Hence the submatrix S (obtained by deleting the first row and the first column of P) contains no dominated row. Therefore $S^{-1}e > \underline{0}$ by proposition 2. Define an $(n-1) \times (n-1)$ diagonal matrix $\bar{S}$ with positive diagonal elements $\bar{\rho}_2, \bar{\rho}_3, \dots, \bar{\rho}_n$ which are inverses of the elements of the column vector $S^{-1}e$. In symbols :

$$\bar{S}^{-1}e = S^{-1}e > \underline{0} \quad (11)$$

Partition the matrix P into four submatrices :

$$P \equiv \left[\begin{array}{c|cccc} \rho_M & \rho_M & \dots & \rho_M \\ \hline 0 & \rho_{22} & \dots & \rho_{2n} \\ \vdots & & & \\ \vdots & & & \\ 0 & \rho_{n2} & & \rho_{nn} \end{array} \right] \equiv \left[\begin{array}{c|c} \rho_M & J \\ \hline 0 & S \end{array} \right] \quad (12)$$

Define the "equivalent matrix of returns" $\bar{P}$:

$$\bar{P} \equiv \left[\begin{array}{c|cccc} \rho_M & \rho_M & \dots & \rho_M \\ \hline 0 & \bar{\rho}_2 & 0 & \dots & 0 \\ \vdots & 0 & \bar{\rho}_3 & \dots & 0 \\ \vdots & & & & \\ 0 & 0 & \dots & \dots & \bar{\rho}_n \end{array} \right] \equiv \left[\begin{array}{c|c} \rho_M & J \\ \hline 0 & \bar{S} \end{array} \right] \quad (13)$$

Let $a = (a_1, a_2, \dots, a_n)$ where a_i is the proportion of wealth allocated to the i^{th} asset, with $\sum_{i=1}^n a_i = 1$, Partition a into :

$$a \equiv a(a_M | a_R)$$

where $a_R \equiv (a_2, a_3, \dots, a_n)$ and $a_M \equiv a_1$ (14)

For any portfolio allocation a , the resulting pattern of returns is :

$$aP = (a_M^0 | a_M^J + a_R S) \quad (15)$$

We wish to show that for any allocation a , there exists an equivalent allocation $\bar{a} = (\bar{a}_1, \bar{a}_2, \dots, \bar{a}_n)$, $\sum_{i=1}^n \bar{a}_i = 1$ such that :

$$aP = \bar{a}\bar{P} \quad (16)$$

Define :

$$\bar{a} \equiv (a_M | \bar{a}_R) \quad (17)$$

where

$$\bar{a}_R \equiv a_R S \bar{S}^{-1} \quad (18)$$

and a_M is identical to a_M in (14). Obviously by (17), (18) and (11) :

$$\begin{aligned} \sum_{i=1}^n a_i &= a_M + \bar{a}_R e = a_M + a_R S \bar{S}^{-1} e = a_M + a_R e \\ &= \sum_{i=1}^n a_i = 1 \end{aligned} \quad (19)$$

and

$$\bar{a}\bar{P} = (a_M^0 | a_M^J + \bar{a}_R \bar{S}) = aP \quad (20)$$

by virtue of (18) and (15).

By a similar argument, it is easy to show that for any allocation $\bar{a}(\sum_{i=1}^n \bar{a}_i = 1)$ one can find a such that (16) holds.

Note that if a_R is a positive or semi-positive vector then so is $\bar{a}_R$ as defined by (18), since S is a semi-positive, non-singular matrix and $\bar{S}^{-1}$ is a diagonal matrix with positive diagonal elements.

Our matrix $\bar{P}$ and allocation $\bar{a}$ satisfy all the conditions of Cass and Stiglitz's theorem 1". Hence, using their theorem^{4,5},

$$\frac{d\bar{a}_1}{dW_0} \gtrless 0 \quad \text{as} \quad R'[W] \gtrless 0 \quad (21)$$

where

$$R[W] \equiv -u''[W] \cdot W / u'[W].$$

But $\bar{a}_1 \equiv a_1 \equiv a_M$ by (14) and (17). Hence our theorem is proved.

Q.E.D.

⁴ Note the following misprints in Cass and Stiglitz's paper

- i) a_1 on lines 7 and 9 (p.341) should be $\hat{a}_1$
- ii) Equation (16) (p.338) should be :

$$\frac{d \ln a_{\theta^*}}{d \ln W_0} = \frac{1/R_{\theta^*}}{\sum_{\theta} (a_{\theta}/R_{\theta})} - 1$$

(This equation is the relevant part of Lemma 1 for the purpose of theorem 1").

⁵ The assumption that all assets are held in positive amounts is made so as to ensure

- i) that no security is dominated;
- ii) that consumption is positive for all states of nature; and
- iii) that $\min \hat{\rho}_i \hat{a}_i \geq \hat{\rho}_1 \hat{a}_1$ (p.341 of Cass and Stiglitz.)

With $\hat{\rho}_j \hat{a}_j > \hat{\rho}_j \hat{a}_j$ for some j . For (iii) the non-negative holding of all risky assets, with strict positivity for at least one risky asset, is sufficient.

Thus the assumption $a_i > 0$ for all i can be replaced by weaker assumptions (a) no dominated securities, (b) positive consumption and (c) $a_i \geq 0$ for all $i > 1$ with $a_j > 0$ for some $j > 1$.

CHAPTER IX

CONCLUSION

The aim of this thesis has been to study some basic aspects of foreign investment and foreign borrowing for resource exploitation. It is now time to summarize our main conclusions.

We have constructed idealized mathematical models in an attempt to isolate what we consider as basic aspects of the problem under study. Our emphasis has always been on *optimal* planning and regulation, but the consequences of the lack of regulation and planning must be clear, as they have been pointed out more or less explicitly in our analysis.

Chapter IV was primarily concerned with optimal fiscal mixes - mainly of tax on foreign capital and royalty. It was shown that along the optimal extraction path, the marginal product of capital must *exceed* the marginal cost of borrowing. This is because the resource commands a *positive imputed price*. This imputed price is properly taken into account by private producers only if they maximize profits over time without any constraint on the terminal date and also provided that the resource is not a common property. In this case the Kemp-MacDougall tax rule for foreign capital remains optimal, though of course the actual tax rate varies with time as foreign capital is phased out. There is no need for a royalty in this case; in fact royalties would distort production plans. An appropriate means for extracting "rent" is the competitive bidding for mining rights. If firms do not take the long-run view, perhaps because of constraints on their time horizon or because of common property exploitation, then

a royalty is a good corrective. It should reflect the imputed price of the resource, and thus should increase over time if the rate of interest is positive. Without an appropriate royalty, the Kemp-MacDougall tax must be modified to take into account the imputed price of the resource. Thus for any given royalty rate, there is a corresponding optimal tax on foreign capital.

Chapter IV also looked at the idealized case of a perfect international loan market. There we showed that consumption may be monotonic increasing, decreasing, or constant, depending on the relationship between the social rate of time preference, the market rate interest and the curvature of the utility function. The access to an international capital market thus can change significantly the time profile of consumption. We also noted that the production and consumption of natural resources for a small country facing a perfect international capital market may constitute an important exception to the Fisher Separation Theorem if resources also have aesthetic values not reflected in market prices.

While in Chapter IV, resources always command positive imputed prices, Chapter V pointed out the existence of cases in which the imputed price is zero, at least at some stationary states. This is because Chapter V was concerned with replenishable resources. Even if the resource good is always *wanted* without satiation, the cost of exploitation - which partially depends on the supply price of foreign capital - may be so high that the optimal quantity extracted per period does not exceed the maximum sustainable natural replenishment. In this case, there may be no need for any regulation of exploitation, even when the resource is a common property. If foreign capital becomes

cheaper, however, then the resource will command a positive imputed price, and regulation is required for the case of common property exploitation. When the imputed price is zero, the Kemp-MacDougall tax on foreign capital is optimal. When the imputed price is positive but not properly taken into account by firms, then for any given level of royalty, there is a corresponding modified Kemp-MacDougall tax which is optimal.

In both Chapters IV and V, we considered the effect of changes in the discount rate. One reason for doing so arises from a commonly expressed suspicion that foreign capitalists may discount the future at a higher rate than the one which is deemed socially appropriate. This may be the case if foreign capitalists feel the risk of nationalization - a private risk without a social counterpart. Chapter VI looked at this problem more closely, by comparing the privately "optimal" behaviour with the socially optimal exploitation path. There we showed that the uncertainty about possible nationalization has an effect on the exploitation path which is similar to the effect of a higher (and generally variable) rate of discount. Under certain assumptions, it was established that foreign capitalists would plan for higher "initial" extraction rates and exhaust the resource prematurely. To correct for this possible bias, a profit tax rate which varies over time may be appropriate.

Uncertainty of a more intricate nature was introduced in Chapter VII where we studied optimal regulation under the adaptive expectation about the profitability of resource extraction. In order to isolate the effects of the irreversibility of resource depletion, we first considered the case where the resource is ample (infinite) in

supply, and then we compared the results of this case with the results obtained in the case of a scarce resource. In the first case, the presence of learning does not affect decision fundamentally, while in the second case (resource scarcity), the introduction of learning will cause the optimal behaviour of the home government to exhibit properties akin to those obtained under risk aversion, even though we assumed risk neutrality. More precisely, our model told us that the government should proceed more cautiously than under certainty, and should preserve more resource for the future, perhaps because it will benefit from learning and hence will be able to determine the optimal amount of foreign capital in a better light. This conclusion may justify partly the call for more conservation. It is subject to an important qualification : it was obtained under the assumption that the information obtained in any period is independent of the scale of operation in that period. When this assumption was removed, the optimal amount of foreign investment was shown to depend crucially on the relationship between the random variable and the controlled variable (the scale of exploitation). This is a not surprising result for models in which learning is important. Artificial as our analysis may seem, it points to the importance of careful specification of the nature of uncertainty and careful analysis before a decision is taken.

Another aspect of uncertainty is the attitudes to risk taking. This was considered in Chapter VIII, where we looked at the problem of optimal allocation of natural resources between the domestic and foreign sectors. We took the attitudes to risk as given - an integral part of the community's preference structure. We were able to show that the optimal allocation depends crucially on the attitudes to risk -

not only on primary characterizations such as risk aversion or risk preference, but also on secondary properties such as the elasticity of marginal utility (a measure of relative risk aversion) and the derivative of the absolute risk aversion function. This showed, for example, how risk averse individuals with identical wealths may disagree not only on the magnitude but also on the *direction* of the optimal change in foreign investment in natural resources as the community's wealth changes by a given amount. This may partly explain some conflict of opinions between those people who are equally concerned about the welfare of the community.

As we have noted, important simplifications have been made in order to facilitate our analysis. In particular, we have ignored the problem of capital accumulation and international trade in our study of foreign investment in natural resources. These extensions are interesting topics for further research.

Finally, we wish to reiterate that we have ignored the problem of imperfections and externalities of central planning, not because it is unimportant, but rather because it lies outside the scope of our analysis.

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