ESSAYS IN THE THEORY

OF

PUBLIC GOODS

AND

INCOME REDISTRIBUTION

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A collection of papers submitted for the Degree of Doctor of Philosophy of the Australian National University by submission of published work.

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In compliance with the requirements for the Degree of Doctor of Philosophy of the Australian National University, I affirm that with the exception of the jointly written papers, Paper Four (and its appendix), appearing as Chapters 2 and 3 of Part II, the work which follows is entirely my own.

I affirm that the papers -

"Pareto Optimal Redistribution Reconsidered"

and

"Hochman & Rodgers on Brennan & Walsh: Reply"

appearing in this document as Paper Four and its appendix, represent genuinely joint research work.

[Signature] Walsh.
PREFACE

The following set of published papers is a selection from a number written over the period 1969-1974 during which I was a staff candidate for the degree of Doctor of Philosophy at the Australian National University.

The selection has been made with the intention of producing as coherent a document as possible; but by the nature of the exercise there is less homogeneity than is typically the case in a thesis of more standard character. The collection resembles in style (if not quality) collections of Essays, like Scitovsky's "Papers on Welfare and Growth" or Johnson's "International Trade and Economic Growth"1 rather than a normal book-length treatment of a single theme.

Nevertheless, the papers are all pretty much in the mainstream of public finance theory, and there are one or two recurring themes throughout the collection. In particular, all except the last two papers are related to the modern theory of public goods; all except the first paper are directly concerned with aspects of income distribution; and most of the papers are concerned, in one way or another, with the question of how the notion of public goods is, or can be, related to distributional issues. In this sense, the title is a fairly accurate indication of the subject matter of the collection.

The papers fall naturally into three groups.

Part I is concerned with public goods theory (the Lindahl paper particularly), and with the distributional implications of public goods provision, on both the product side (paper 2), and the consumption side (paper 3 and its appendix), of individual incomes.

Part II consists of papers relating to various aspects of the theory of so-called "Pareto-desirable redistribution". The question at issue here is that of how far the strict Paretian welfare framework might take us in justifying redistribution of income through the budget. To what extent and in what ways does the redistribution of income yield benefits to donor-taxpayers as well as recipients? Can public transfer programmes be looked on as mutually agreeable coercive arrangements, much like public provision of defence, or law and order? In other words, can redistribution be considered to be a public good, and if so, in what sense? The papers included in this section seek to indicate how it might be, by examining alternative donor motives for redistribution. Paper 4 and its appendix examine a model of donor philanthropy; paper 5 considers the obverse case of donor malice and envy. Paper 6 examines two further motives for transfer, while paper 7 attempts to assess the significance of the Pareto-desirable-redistribution possibility, and indicate some analytical peculiarities of the models.

Part III consists of two papers relating to a problem that arises with a rather more traditional perception of redistributational objectives. This is the problem of how to formulate the notion of horizontal equity...
so as to make it usable in a context where the conceptually ideal arrangement is precluded. Typically, horizontal equity is taken to involve "equal treatment of economic equals", with very little explicit attention to what precisely "more equal treatment" or "less equal treatment" might mean. Since the achievement of horizontal equity appears generally to stop short of the ideal of absolutely identical treatment, it seems desirable to redefine the objective in terms of minimizing the degree of horizontal inequity. The papers seek to do this in two rather different ways - the first by specifying alternative measures of horizontal inequity, the second by drawing on certain analogies with the theory of second-best in welfare economics. The policy implications are drawn out for both approaches.

In one or two cases, either where my views on some aspects of a paper have changed slightly or where I have wanted to comment briefly on subsequent literature, I have added to papers by way of short appendices. For the most part, however, I have resisted the temptation to supplement the papers in this way, and except for the few appendices, and the errata listed at the end of the document, have had to stand by the material as published. I have, however, begun each part with a brief introduction which attempts both to relate the papers to one another, and to indicate the general subject matter of the section. In no case does the introduction run beyond a few pages.

Like most authors, I owe a considerable but rather diffuse debt to a large number of people - particularly to my colleagues in the Faculty of Economics at the Australian National University. I would especially like to thank Cliff Walsh for his stimulating comments on early drafts of most of the papers, and for general assistance in converting those drafts
into a readable form. I should also express my gratitude to Clem Tisdell and Russell Mathews, who in turn acted as Supervisors for the "thesis" during the period of my candidature. By nature of my preoccupation with independent research, both were denied the more standard supervisory tasks; they were however always ready with encouragement and assistance when needed, and in the latter stages Russell Mathews executed the necessary formalities.

Unquestionably though much the most significant debt I owe is to John Head, whose influence on my interests and orientation has been profound. I should like to place on record my deepest gratitude to him both for extensive guidance in my years as a student, and for continued encouragement and stimulation since.

Geoffrey Brennan
September 29, 1975.
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PART I

PUBLIC GOODS AND THEIR DISTRIBUTIONAL IMPLICATIONS
INTRODUCTION

Since the mid-1950's, public finance (at least in its Anglo-American variant) has undergone a complete transformation through the development of modern public expenditure theory.

Prior to this time, public finance had been concerned almost entirely with tax matters. The treatment afforded the expenditure side of the budget had in most cases been little more than perfunctory and often enough, when public expenditures did appear, it was in their role as an alternative means of moderating the aggregate level of economic activity.

The publication of Samuelson's three influential articles on public goods theory in the fifties, and the consequent eruption of interest in the literature throughout the ensuing twenty years has changed all that. Public expenditure theory is now part of the standard analytical apparatus of the public finance theorist, and even the most elementary of modern textbooks in the field offer extensive discussion of the central ingredients of public goods theory.

These central ingredients are simply related. The theory takes as its point of departure the familiar theorem of welfare economics which states that the freely operating competitive market succeeds in achieving an optimal allocation of resources to alternative uses, in the sense that all the potential gains from trade are soaked up. In the face of this theorem, public expenditure theory seeks to establish a case for public intervention in economic processes in two steps. Firstly, it shows that,
for a particular category of goods - "public" goods, defined as those which must be consumed equally and totally by all the market fails to achieve optimal resource allocation. Secondly, the theory seeks to demonstrate that, where the market fails, the political mechanism can be expected to perform rather better ("better" again defined solely in terms of Pareto optimality).

Certain aspects of this two-step procedure have claimed more attention than others. Thus, Samuelson's earlier articles (particularly the second) were concerned mainly with the derivation of the new "optimum conditions" for public goods (the now familiar $\sum MRS = MRT$ condition), rather than with a demonstration that the market could not achieve them. And indeed, the matter of the likely market outcome with public goods, the precise degree of market failure and the determination of what it is about public goods which is primarily responsible for market failure, are still somewhat open questions.

Equally, the crucial issues of the performance of the political mechanism and the demonstration that it can "succeed" where the market fails are matters which are not by any means settled, and which have still not claimed the attention they deserve. All the same, the fundamental questions are now at least widely understood, and by any standards, the profession has in the last twenty years gone a remarkable way towards answering them.

1 or alternatively, those which must be made available to all for equal and total consumption.
At all points, the historical antecedents of modern public expenditure theory are never far from the surface. Unquestionably among the more significant of these is Erik Lindahl's "theory of the budget" which appeared first in German in 1919.\(^2\) Lindahl's theory seems to be an endless source of fascination to public goods theorists and in spite of the passage of time it continues to be read and commented on extensively (even by Lindahl himself\(^3\)). Nor is such interest surprising: Lindahl purports to show that the political mechanism - if amended in a way that is by no means infeasible - can be expected to perform fairly well in allocating resources to alternative uses, a conclusion which is somewhat at variance with more modern discussions of the same question.

The first paper in this section is a reappraisal of Lindahl's model with the aid of the techniques of game-theory. Such a reappraisal seemed to be necessary because the operation of Lindahl's model, with its characteristic (and entirely circular) definition of "equal power", has never been regarded as entirely satisfactory and the techniques of game-theory seemed to hold out some promise of a better treatment. The focus of the paper is, then, very much on the operation of the model and rather less on Lindahl's discussion of its significance, or on how the model ought to be interpreted. John Head's *Finanzarchiv* paper on the Lindahl theory provides an excellent treatment of these other aspects.

\(^2\) It has subsequently been translated into English and appears in Musgrave & Peacock's *Classics in the Theory of Public Finance*.

Since the appearance of my paper, Buchanan’s treatment of the Lindahl model in his book on public goods theory has been brought to my attention. The appendix to this first paper is an appraisal of Buchanan’s version of the operation of the Lindahl model.

The second and third papers in this section are concerned with the distributional effects of public goods supply. Although the rationale for government provision of public goods is based on efficiency considerations, distributional implications of public goods provision have by no means been ignored in the literature. At the same time, the matters considered in paper two (i.e. the factor price effects of public goods provision) have not received very much attention - even though, as I argue in the body of the paper, distributional effects on the sources side of incomes may be quite important. The specific objectives of this paper are twofold: firstly, to develop a public goods analogue of the general equilibrium 2 x 2 model for private goods, so that the optimal level of public goods supply and the distribution which is associated with it can be shown as jointly determined, on the basis of the specified tax institutions and the given distribution of factor ownership; and secondly, to indicate some of the implications of factor price effects for actual fiscal outcomes in a stylized median-voter model of political processes.

In paper three, the more traditional aspect of the public goods/income distribution relationship (i.e. the effects on the consumption side) is examined in the context of a specific question concerning the appropriate

allocation of the benefits from public expenditure when making calculations of income redistribution through the budget. This paper was originally conceived as a comment on an earlier paper by Aaron & McGuire, which appeared in *Econometrica* in 1970. The Aaron-McGuire discussion sought to show that the appropriate attribution of benefits is on the basis of a $\text{MRS}^i \cdot G$ valuation, where $\text{MRS}^i$ is the $i^{th}$ household's marginal valuation of the public good in terms of the private good, and $G$ is the quantity of public goods provided. My comments are based on the conviction that this is not an appropriate valuation procedure, and that the most satisfactory way of attributing public goods benefits is on an equal per-head basis. I attempt to argue this position in paper three. As an appendix to this paper, I have included my rejoinder to Aaron & McGuire's reply.
2. PAPER ONE
Game-Theoretic Aspects of Lindahl's Budget Theory
from Rivista di diritto finanziario e scienza delle finanze
June 1970
GEOFF BRENNAN

GAME-THEORETIC ASPECTS OF LINDHAL'S BUDGET THEORY
GEOFF BRENNAN

GAME-THEORETIC ASPECTS OF LINDHAL'S BUDGET THEORY
GAME-THEORETIC ASPECTS OF LINDAHL'S BUDGET THEORY (*)

It is now almost exactly fifty years since Lindahl's famous theory of budget determination made its entrance into the continental literature (1). Despite the passage of years, it remains one of the most important contributions to the theory of benefit taxation, and as such represents a pillar of much of modern public expenditure theory.

Historical interest aside, Lindahl's theory is significant in that it advances the possibility that the political mechanism, if amended in a not infeasible way, can achieve a Pareto optimum in determining the level of public expenditure and the allocation of its cost. More modern treatment of the political mechanism (2) has been fairly sceptical of its performance as an allocator of resources between public and private sectors of the economy. Indeed, it has become seriously questionable whether the mere demonstration of market failure presents any case for interference by the state at all; political failure analogous to failure in the market can be anticipated, and any case for government intervention in economic processes must, then, rest on a quantitative evaluation and comparison of failure under each type of provision. Thus, Lindahl's theory arguing as it does that the political mechanism can be expected to perform quite well -- even to the point of achieving a Pareto optimum under

(*) I am grateful to John Head and Clem Tisdell for comments on an earlier draft of this paper.
certain reasonable conditions — stands in direct conflict with the larger part of the modern «welfare politics» literature. For this reason, the Lindahl treatment is still very much of interest to the public finance theorist.

The object of this paper is to present a re-examination of the Lindahl model in the light of the techniques of game theory. The predominant concern is with the operation of the model, rather than the interpretation of conclusions (3); and in this connection the questions which seem to be most relevant are these: does the Lindahl construction necessarily give rise to a Pareto optimum? and if so, in this necessarily (or even likely to be) the one isolated by Lindahl as the «fiscal optimum»?

Before directing attention to these questions, however, it seems appropriate to offer a warning about the results which game theory presents.

When game theory first became fashionable, it became very fashionable indeed. Like most fashions, it soon passed out of favour, but there was a time, I think, when it was considered (mostly by those who knew little about it) to hold out promise for solving all the multitude of problems in economics involving conflict of interest. In general, this hope has proved ill-founded. For the class of games which holds most interest for the economist, non-zero-sum games, no general agreement has been possible on the question of what particular outcome represents the unique solution. Thus, we cannot expect that game theory will isolate from the Lindahl situation that outcome (ie. that combination of public expenditure and allocation of cost) which will prevail if the game is played according to Lindahl rules. What can be done, however, is to draw parallels between the Lindahl model and familiar situations from the theory of games in the hope that a certain amount of light will be shed on the sorts of outcomes which might be expected.

1. The Lindahl Model.

On the grounds of a just initial distribution of property, Lindahl argues that a «just» taxation system is characterized by an absence

of coercion, in the sense that no individual is forced to contribute more to the cost of any unit of public expenditure than he receives from that unit in utility terms. Alternatively stated, public expenditure is to be pushed to that point where the marginal unit of tax paid equals the money value of the utility obtained from the marginal unit of public services for each taxpayer. Of course, an infinity of such systems exist, each associated with a different pattern of discrimination between individuals over infra-marginal units of public services. But Lindahl specified that there be no discrimination over units purchased: the cost share for marginal units must be the same as that for infra-marginal units.

The «just» taxation system thus defined, Lindahl proceeds to set up a highly stylized model of budget determination in the political arena in which the achievement of «tax justice» is assured. By contrasting the model with actual political processes, he is able to recommend a number of changes in institutional arrangements, which will increase the likelihood of attaining the fiscal optimum.

The crucial ingredient of the model is the assumption that the electorate partitions itself into two distinct homogeneous groups, which can be designated I and II; group I is assumed to be rich and group II poor. This assumption permits Lindahl to treat the problem of budget determination in a way which is essentially analogous to a two-person game, and thereby allow him to abstract from the multitude of problems associated with large numbers. The assumption may not, of course, be a completely unrealistic one, but it is highly significant. Indeed, it seems that it is precisely this assumption which permits the political mechanism to perform so successfully in the Lindahl context (4).

Government expenditure is presumed to be such that it must be made equally available to all. This is, as we shall see, sufficient to preclude the achievement of a Pareto optimum in the freely operating market.

The Lindahl model is given its familiar depiction in fig. 1. The level of government expenditure, G, is measured on the vertical
axis; on the horizontal axis the respective cost shares, $h$ and $1-h$, of groups I and II are shown. The line $AA'$ is the locus of points depicting the amount of $G$ to which I will contribute as the cost share, $h$, varies from zero to one. Its shape demonstrates that, as $h$ increases from zero, the amount of $G$ which I is prepared to counterbalance falls from infinity when $h$ is zero to $OA'$ when $h$ is one. Likewise, $BB'$ shows the maximum amount of $G$ to which II will contribute for each cost share, $1-h$, between zero and one. Thus $AA'$ and $BB'$ represent the monetary expression of the marginal utility of public expenditure to I and II respectively, and the point of intersection, $P$, of the two curves is recognized as a Pareto optimum: this is Lindahl’s «fiscal optimum».

![Diagram of marginal utility of public expenditure for groups I and II](image)

**Fig. 1.**

It is assumed that each group takes the other's preparedness to contribute at each cost share as given, so that the largest amount of public expenditure which can prevail is determined by the minimum of $G_I$ and $G_{II}$, where $G_I$ is the most $i$ is prepared to
i.e. for any \( h \), \( G = \min (G_{I}^{h}, G_{II}^{1-h}) \)

This condition constrains equilibrium to lie on the segment \( A'PB' \). In fact, however, when faced with the « supply » curve \( BB' \), I will select the point on it, say \( Q \), which if finds most desirable, and likewise II will select some point on \( AA' \) most desirable to it (since II is poor, this will, Lindahl suggests, be at \( A' \)), so that the set of potential equilibria is reduced to the set of points on the segment \( QPA' \). Which point actually prevails depends on the relative strengths of the two groups in defending their own interests: the implication is that if I is, say, stronger than II then the ultimate solution will lie on \( QPA' \) somewhere near \( Q \).

Clearly, in order to ensure the attainment of the fiscal optimum, further strictures on the operation of the model are required. These are embodied in the assumption of « equal power », defined as the ability to exchange to the point of saturation. « It is not difficult to see what equilibrium position corresponds to a situation in which both parties have equally safeguarded the economic rights to which they are entitled under the existing property order... Equilibrium would be established at the intersection point of the two curves, where both parties can exchange up to saturation, and where therefore the money value of the net gain which both parties together derive from public activity is maximized » (5). Lindahl's fiscal optimum is achieved.

The Johansen treatment (6) of the Lindahl model probably presents a clarification to most of us, more familiar as we are with traditional indifference curve analysis. Certainly, it does serve to accentuate a number of very important aspects of the Lindahl construction.

National income, \( Y \), is given and divided justly between groups I and II, so that \( Y_{I} \) is income for I, \( Y_{II} \) is income for II and \( Y_{I} + Y_{II} = Y \). Private consumption for each group is designated by \( X_{I} \); public consumption which I and II enjoy equally according to taste, is \( G \). The marginal cost of \( G \) it taken to be constant. As \( h \) varies from one to zero, I takes successively greater quantities of \( G \) according to the locus \( A'A \), pictured in fig. 2. By mapping \( h \) against \( G \), we obtain fig. 2 in a new aspect (fig. 3); \( A'A \) in fig. 3

(6) JOHANSEN, op. cit.
represents the locus of equilibrium points for I as \( h \) varies and corresponds exactly to \( AA' \) in the standard Lindahl diagram (fig. 1).

\[ BB' \] can be constructed in a similar manner, and with \( 1-h \) as the cost share for group II, we can superimpose the construction on fig. 3, rotate through ninety degrees, and obtain the traditional
Lindahl diagram (fig. 4). We note from fig. 4 a number of interesting points. In the first place, it is quite clear that the point $P$ isolated by Lindahl as the fiscal optimum is a Pareto optimum. In the second place, and equally important, it is apparent that $P$ is only one of a whole set of achievable Pareto optimum points $WW'$ (the locus of points at which indifference curves for I and II are tangential) consistent with the constraint that no-one be made worse off. The status of points $Q$ and $A'$ for I and II is made clear. Faced with the curve $BB'$, the highest indifference curve I can attain is at point $Q$; likewise, the best II can do if he takes $AA'$ as given is to settle for the point $A'$, where I is paying everything.

It has often been observed that Lindahl's description of convergence to the fiscal optimum $P$ is basically unsatisfactory. It is not at all obvious why either group should consider the other's demand curve as given. More plausible, surely, than the envisaged analogue to competition with free agreement and the ability to exchange up to saturation, is a mechanism of the traditional game-theory type where each group attempts to extract from the situation...
the highest possible return to itself; where threat strategies are invoked; where bluffs and counter-bluffs are matched; and where the ultimate «solution» — in the sense of some uniquely determined point — may not even exist, let alone be Pareto optimal.

Consequently, some attention to game theoretic aspects of Lindahl's theory of the budget seems called for. It is such attention which this article seeks to provide.

II. Lindahl As An Unco-operative Game.

The theory of games is a mathematical edifice designed for the study of situations involving conflict of interest (7). The problem that faces each player is that of maximizing his own «pay-off» (utility) when the outcome is not solely dependent on his own actions — it depends crucially on the behaviour of his opponents as well. This general problem is exactly analogous to the situation confronting each group in Lindahl's theory of budget determination — each has to maximize the utility return to itself when the actual budget is determined by neither group alone. Where the interests of players are exactly opposed (one man's gain is the other's loss) as is the case in bridge and chess, the game is called a «zero-sum» game, because for any outcome, the pay-offs to the players sum to zero.

In most social and economic situations, however, players' interests are not exactly opposed, and there is some scope for mutual benefit. Appropriate selection of strategies makes possible the achievement of outcomes which benefit both (all) players. These games are called «non-zero-sum» games, and of course, the Lindahlian budget game is one such.

Within the general category of non-zero-sum games, a further distinction is relevant: that between «co-operative» and «unco-operative» games. A game is called «unco-operative» if all pre-play communication is prohibited. This means that players cannot make agreements between them as to the strategies to be used; nor can they reveal their courses of action before the game begins.

(7) Probably the most readable as well as the most systematic treatment of game theory usable by the not-especially-mathematically-inclined economist is R. D. Luce & H. Raiffa, Games and Decisions, New York 1957. My own debt to that text will be obvious to anyone familiar with it.
commences. It is as if players are forced to act independently. Analogously, games are «co-operative» if such pre-play communication is possible. If we define political action as characterized by the existence of pre-play communication then the Lindahl model treated as an unco-operative game represents an analogue to the provision of public goods in a market situation, as distinct from the political context actually envisaged by Lindahl.

One of the characteristic assumptions of game theory is that the utility functions ordering the outcomes for each player are known by both the player and his opponents alike. This represent a severe restriction on the applicability of game theory to economic situations, and especially to public goods theory, where many of the problems which arise owe much of their perversity precisely to lack of information about utility functions. This assumption is relaxed in section four of this paper, but in the meantime we continue to operate within the traditional game theory framework.

The examination of Lindahl's model an unco-operative context is useful for a number of reasons. Firstly, it is clear that any player will refuse to enter any agreement which gives him a smaller pay-off than he could obtain independently. Thus, what a player can obtain by independent action establishes a lower limit to the pay-off he will accept under any agreement. Secondly, the extent to which each player can penalize his opponent by refusing to enter an agreement (that is, the differences between pay-offs under co-operative and unco-operative play) may be relevant in determining threat power, and hence the likely outcome of the game. Thirdly, in so far as the unco-operative game depicts the market situation, and the co-operative game the political, an analysis of the former may be useful in determining precisely what the problems associated with market provision of «public goods» are — and may serve to throw some light on the questions of whether and in what ways provision through the political mechanism is likely to solve these problems.

It is clear from the Lindahl diagram (figs. 1 and 2) that it is in the interests of each player (8) (each group) to consume some public services even where he has to bear the cost alone. For group 1 this amount is O'A', and for II, OB'. This simply means that the

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(8) Henceforth, I have used the words «player», «group» and «individual» interchangeably. In this context, there is no analytic distinction to be drawn, so the usage should not result in any confusion.
marginal evaluation curve of public goods for I cuts the marginal cost curve at the point where $G$ equals $O'A'$, and the marginal evaluation curve for II intersects the marginal cost curve where $G$ equals $OB'$ (fig. 5) (9). Thus we are not faced with the situation common in the public goods literature where no one unit can itself afford to purchase any of the jointly consumed good.

Let us suppose initially that the groups must announce their strategies simultaneously; this precludes any player from waiting for the other to act, and then adjusting his behaviour in the light of his opponent's actions.

It is also assumed that each player considers only two strategies: he can produce nothing — this strategy $x$; or he can produce at

(9) I have for simplicity assumed that marginal cost is constant; that the marginal evaluation curves are linear; and that the income elasticity of public goods is zero. These assumptions do not seem to impose any undue restriction on the analysis.
the level which yields him maximum utility given that his 'opponent' provides nothing — this is strategy $y$. There are, of course, an infinity of strategies — or at least as many strategies as there are units of public goods up to the point where $ME_i$ equals $MC$, and indeed a number of feasible ones beyond, but it will suffice to start our treatment with this simplification.

The utility accruing to each player from the various strategy combinations can be calculated from fig. 5 by examining the areas under the marginal evaluation curves. For example, when II uses strategy $y$ and I strategy $x$, II's pay-off in utility terms is the consumer surplus $BLU$ (the area between $ME_{II}$ and $MC$). The pay-off to I on the other hand is given by the area $AMB'O$, since by definition I can enjoy the benefits of the $OB'$ units of public goods $G$ supplied by II without contributing anything towards the cost. The strategy combination $(y_I, y_{II})$ results in a total production of $OZ$ of public goods, where $OZ$ is the sum of $OA'$ and $OB'$.

The pay-off in geometric terms are as follows (see fig. 5):

\[
\begin{align*}
&I's \text{ action} \quad II's \text{ action} \\
&x_I (0,0) \quad (AMB'O, BLU) \\
&y_I (ANU, BSA') \quad (AVZO - UNA'O, BWZO - ULB'O)
\end{align*}
\]

The first mentioned area in each bracketed pair is the pay-off to group I, the second is the pay-off to group II. The equilibrium level of public goods (the Pareto optimum) is at point $E$, and the cost shares at that point are $h \left[= \frac{P_I}{MC}\right]$ and $(1 - h) \left[= \frac{P_{II}}{MC}\right]$.

The essential features of the game can perhaps be more readily appreciated if the areas concerned are measured, and a numerical value ascribed to them. The game becomes:

\[
\begin{align*}
&I's \text{ action} \quad II's \text{ action} \\
&x_I (0,0) \quad (10,1) \\
&y_I (2,12) \quad (8,9)
\end{align*}
\]

The most striking characteristic of the game is that there is no equilibrium point — each player's best strategy depends crucially on what his opponent does. If I plays $x$, II's 'best' reply is $y$ in the
sense that playing \( y \) gives II a larger pay-off than playing \( x \). If I plays \( y \), however, II's «best» reply is \( x \). Consequently, II has to decide what I is going to do before he can select his strategy. Clearly, an exactly analogous problem faces I. Thus, it is impossible to say definitely which of the outcomes is most likely. All that we can do is to examine each of the four possible outcomes, and attempt to determine how probable each is.

\[ (x_I, x_{II}) \]

It is sometimes asserted that the problem of market provision of public goods is analogous to the 'prisoners' depicted in game B.

\[
\begin{array}{c|cc}
   & x_{II} & y_{II} \\
\hline
x_I & (4,4) & (10,3) \\
y_I & (3,10) & (8,8) \\
\end{array}
\]

The characteristic feature of this game is, that since \( x \) dominates \( y \) for both groups irrespective of opponents' strategies, rational self-interested players are led away from the outcome which has the largest total pay-off (i.e. \( (y_I, y_{II}) \)) to the «anti-social» outcome \( (x_I, x_{II}) \). Clearly, however, the analogy is imperfect, for the game A does not automatically lead to an outcome in which no public goods are produced. There are, nevertheless, strategic features in the Lindahl game, \((A)\), which make for a certain resemblance to the 'prisoners dilemma' situation.

Budget determination is normally an annual event, so we can suppose the game to be subject to re-intagation at regular intervals. Now, the best outcome that either group can hope for is that in which the other provides all the public goods at his own expense — that is, \( (x_I, y_{II}) \) is best for I and \( (y_I, x_{II}) \) is best for II. Both players known that if they consistently play \( y \), their opponent will play \( x \). Thus, I knows II has no incentive to play \( y \) unless he, himself, plays \( x \). Moreover, the longer I plays \( x \), the greater the pressure on II to play \( y \), since while ever II plays \( x \) he is foregoing utility which he might otherwise obtain. But, the same reasoning causes II to play \( x \). Thus, both players have a strong incentive to play strategy \( x \), because this is most likely to lead to the outcome most favoured by them; but the outcome \( (x_I, x_{II}) \) is desired by neither. The game has become a bargaining situation with \( x \) as the threat strategy and \( (x_I, x_{II}) \) as the conflict point.
In practice, the market situation does not impose the condition that both players must announce their strategies simultaneously. But each player knows that once he has produced public goods up to his own private equilibrium, his opponent will produce nothing at all. Thus, both players have an incentive to let their opponent make the first move; a sort of "I'll-fumble-you-pay" situation arises, and no public goods are produced (10). The situation is the same as prevails with simultaneous announcement of strategies and reiterations of play.

It is interesting to note that it is quite possible that no public goods will be produced even when both parties could perfectly well afford to pay for some units of the public goods by themselves. Thus, the suggestion that complete market failure, in the sense that nothing is produced, arises essentially because no one individual can himself afford to purchase any of the public good (each MU curve lies below the MG curve throughout the entire range) is ill-founded.

\((y_I, x_{II})\):

Since \((x_I, x_{II})\) has only the status of a conflict point in a bargaining game, we would not expect it to be maintained for ever: sooner or later, one of the groups will give in. It seems most plausible to argue that the group which does eventually give in will be that which stands to lose most by not doing so.

Clearly, I's gain when II provides alone is not as great as II's gain when I provides; moreover, I loses more by staying at \((0,0)\) than II does. Thus, the pressure on I to use strategy \(y\) is greater than the corresponding pressure on II; group I is most likely to concede, and \((y_I, x_{II})\) seems the most likely conclusion (11).

\((x_I, y_{II})\):

\((x_I, y_{II})\) will issue out of the bargaining situation only if there is some disguised asymmetry in the budget game. Thus, if group I is in some sense a 'better' bargainer than group II, and sufficiently

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(10) Thus, for example, two neighbouring communities may each fail to provide adequate public transport between the two places not because no one community can afford it, but simply because each expects the other to do it.

(11) This sort of situation is what Olson (op. cit.) refers to as the exploitation of the large by the small. Certainly, this seems to be a common sort of market solution. The presence of external economies in the market seems to arise precisely because such a solution has been attained.
better to outweigh its lower threat potential, then \((x_1, y_{11})\) and not \((y_1, x_{11})\) will be the most likely solution. But this is to incorporate into the analysis aspects which are essentially arbitrary, and difficult to subject to systematic treatment. On the basis of the information provided by the game itself, \((x_1, y_{11})\) is not at all likely.

\[(y_1, y_{11}):\]

As has already been pointed out, the Lindahl game, \(A\), is typically subject to re-iteration; such re-iteration provides scope for implicit collusion (12). Clearly, the bargaining game is costly in the sense that consistent use of \((x_1, x_{11})\) involves both players in foregoing utility they might otherwise have. If players anticipate that \((x_1, x_{11})\) will be sustained for a long period, they may well be anxious to avoid the bargaining game completely. Now, both players may suspect that if they use strategy \(x\) so will their opponent; each may well be content to play \(y\) whilerer the other does, because he knows that failure to do so will only result, eventually in no public goods being provided at all.

This result is in some sense analogous to implicit collusion in oligopoly markets. Even where there is no explicit agreement, no one firm will be prepared to cut prices for fear of unleashing the chaos of a price war. Thus, \((y_1, y_{11})\) may well prevail as the solution, and may be maintained indefinitely. Consequently, provision of public goods in the freely operating market may result in over-expansion of public goods production for strategic reasons. This conclusion is interesting in that it is contrary to the general presumption derived from the literature: that activities which create external economics will generally be undersupplied in the freely operating market (13).

Although in fig. 5 \(OZ\) is greater than \(OE\), so that the outcome \((y_1, y_{11})\) does result in over-expansion of public goods production, this is not necessarily the case, and may indeed be fairly rare. It is not too difficult to show that the condition for such over-expansion relates to the individuals' elasticities of demand for the public good.

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Over-expansion will occur if

\[ \frac{E_{D_I} + E_{D_{II}}}{E_{D_I} E_{D_{II}}} > \frac{1}{OA' \cdot OB'} \]

where \( E_{D_I} \) and \( E_{D_{II}} \) are the individuals' elasticities of demand and \( OA' \) and \( OB' \) the levels of production at which \( ME_I \) and \( ME_{II} \) respectively cut the \( MC \) (fig. 5). Clearly, this condition is most likely to be met when the relevant elasticities are small.

If implicit collusion is possible under re-iteration of the game, then rather more complicated strategy combinations than \( (y_I, y_{II}) \) are feasible. The players may for example gradually reduce their individual production of public goods until a Pareto optimum amount is being provided (amount OE in fig. 5). There are, of course, a number of cost shares compatible with this arrangement, ranging from that in which I derives all the benefits from the movement, to that in which II does. There seems to be no good reason why the cost share corresponding to Lindahl's fiscal optimum (\( P \) in fig. 1) should be the one which actually prevails.

Of the potential «equilibrium» positions, \( (y_I, x_{II}) \) is probably the most likely. But it is unlikely to be achieved quickly, issuing as it does out of a bargaining game in which the conflict point \( (x_I, x_{II}) \) may be sustained for a large number of plays. Thus, although not identical to the prisoners' dilemma, there are distinct similarities between that situation and the problem of market provisions of public goods. And, like the prisoners' dilemma situation, the Lindahl budget game played in an unco-operative context may give rise to implicit collusion over re-iterations, possibly resulting in over-expansion of public goods production. It is even possible that a Pareto optimal level of public goods will eventually be supplied—although there seems no reason why this optimum should be the one isolated by Lindahl as representing 'tax justice'.

In the light of this analysis, it is possible to distinguish two sources of market failure in the provision of public goods. In the first place, complete market failure (where nothing is produced) may result from strategic considerations—both players provide nothing in the hope that their opponents will provide something. Market failure may still prevail when bargaining game comes to an end, because only one of the two groups is contributing to the cost of the public good; and production will occur at the point where the
marginal evaluation of public goods for that group equals the marginal cost of public goods production. In the second place, a substantial cause of the failure to achieve a Pareto optimum seems to arise out of the informational limitations of unco-operative games. Because pre-play communication is impossible, there is no scope under normal market behaviour for exposing that part of the demand curve which is relevant for achieving a Pareto optimum, since at the optimum point, each player is necessarily in the range where his individual marginal evaluation curve lies below the marginal cost curve. As we have seen, where re-iteration makes collusion possible, there is some likelihood that a Pareto optimum may be achieved.

III. Lindahl As A Co-operative Game.

The Lindahl model is intended to depict the determination of the budget in a political context rather than in the freely operating market. Thus, the theory of co-operative games with the characteristic assumptions of scope for pre-play communication and for the making of binding agreements enforceable under the rules of the game, holds out considerable promise for throwing light on Lindahl’s theory. Unfortunately, there is no unified theory of two-person co-operative games; what I have done here is simply to consider what seem to be the most important contributions in the field, and draw analogies with the Lindahl model wherever possible. Before doing this however, it is interesting — and I think, profitable — to examine the precise status of the «solution» concept.

The solutions offered for co-operative games can be considered in three ways:

(a) in a purely descriptive way: under this interpretation, the equilibrating process purports to describe what actually happens, and the solution is supposed to be that outcome which actually prevail.

(b) In a semi-descriptive way: the solution is supposed to demonstrate what would happen if players act according to certain postulates — for example, if players act «rationally» in some well-defined sense. Harsanyi’s treatment of the Zeuthen-Nash solution (14) is an example of this.

(c) in a normative way: here, the solution is that which «should» prevail, or which is «best» or «fairest» in some defined sense. All arbitration schemes are examples of schemes which offer normative solutions.

It is not always clear in which way various solutions are meant to be interpreted. Moreover, the distinction between categories is not always clear in principle, let alone in practice. In this context, our interest is only in solutions which are descriptive; we are not concerned with the question of whether Lindahl's fiscal optimum is a «good» solution, or with what are the characteristics of a «fair» budget. We are concerned simply with the question of what solution the Lindahl budget game is most likely to generate.

The Von Neumann-Morgenstern Solution.

The Von Neumann-Morgenstern analysis of two-person cooperative games does not, strictly speaking, isolate a solution at all. What Von Neumann and Morgenstern do is to isolate a set N, which they argue will contain the solution. N is defined as the set of all undominated pay-offs \((u, v)\) which dominate the maximum pay-off \((u^*, v^*)\).

Their argument that the solution must lie in N rests on the thesis that no player will agree consistently to a pay-off which is less than he could achieve with the consent of his opponent. Thus, in fig. 6, they contend that players will not continually accept pay-off \((u^o, v^o)\) when they could equally well have \((u^1, v^1)\). The negotiation set N consists of all undominated points in fig. 6 to the north-east of \((u^*, v^*)\) — that is, the line segment abc; clearly, this is the set of Pareto optimum points. Thus, in the Lindahl model, the Von Neumann-Morgenstern analysis leads us to the locus of Pareto optimum points W'W (in fig. 4), and assures us that the solution lies somewhere along that line. Precisely which point is most likely, they do not feel disposed to say: all they suggest is that this depends on such things as relative bargaining strengths which are impossible to incorporate systematically into the analysis.

It is interesting to note that this isolation of the Pareto optimal frontier is precisely the result that flows from the application of Fellner's «qualified joint maximization» principle, and is basically the conclusion which Head arrives at in his treatment of the Lindahl model (15).

(15) Head, op. cit.

Thus, the general conclusion of game theory seems to be that some Pareto optimal point will be achieved. But the question arises as to whether that solution will be attained instantaneously, or after a protracted period. Moreover, it still remains to be answered whether Lindahl's fiscal optimum is a likely solution, and if not whether any other points on WW′ can be distinguished as possible contenders.

The Nash Solution.

The Nash bargaining model is concerned with a situation in which two individuals are interested in trading with each other, but where no set pricing arrangements exist. Associated with each trade (that is, each reapportioning of the joint stock of goods) there is a utility pair \((u, v)\) where \(u\) is the utility return to I and \(v\) the utility return to II. The set of all such utility pairs is convex, closed and bounded, and is depicted as \(R\). The «status quo» point \((u^*, v^*)\) is the pay-off prevailing when no trade occurs. Trade can occur only if both players agree — that is, trade will occur only if there exists some point \((u_1, v_1)\) in \(R\) such that \(u_1 > u^*\) and \(v_1 > v^*\). The existence of such points is assumed. Thus, a bargaining game is depicted by a set \(R\) of possible outcomes \((u, v)\) and a status quo point \((u^*, v^*)\).

In Lindahl's model, the protagonists do not exchange goods, but offers to contribute to various levels of government expenditure. The status quo point can be taken as the point of no public expenditure, which in line with the treatment in part II is assumed to have pay-off \((0, 0)\). This point together with the set of pay-offs, \(W\), for all offers which satisfy the constraint that cost shares must sum to unity, constitutes a complete representation of the Nash bargaining model.

The Nash solution is the point \((u_1, v_1)\) such that the product \((u_1 - u^*) (v_1 - v^*)\) is maximized, where \((u^*, v^*)\) is the status quo point. Thus, if we consider a transformation of the axes so that the status quo point from \((u^*, v^*)\) to \((0, 0)\), the Nash solution isolates the point \((u_0, v_0)\) where the product \(u_0v_0\) exceeds the product \(uv\) for any other \((u, v)\) in \(R\). Clearly, the Nash solution is a Pareto optimum, since if there were some point \((u_2, v_2)\) such that \(u_1 > u_2\) and \(v_1 > v_2\) (that is, it is possible from \([u_1, v_1]\) to make everyone better off) then \(u_1 \cdot v_1\) would be greater than \(u_2 \cdot v_2\) — and it would be the Nash solution. In the Lindahl context, we are able to select the level of
government expenditure and the cost share which give rise to that pay-off \((u, v)\) where the product \(uv\) is maximal. Thus, in fig. 5 where the income elasticity of public goods is zero, and hence the Pareto optimal level of public expenditure is unique, we can determine the cost share corresponding to the Nash solution in the following way:

Let us denote the optimal level of government expenditure by \(g\), and let the area under i's marginal evaluation curve at \(g\) be \(V\). Then the net utility to \(I\) is

\[
U_I = U_I - (g \times MC) = u - g
\]

and to \(II\) is

\[
U_{II} = U_{II} - (g \times MC 	imes (1 - h)) = v - g
\]

With \(g, MC\) and \(U_I\) given, we simply select that \(h\) which maximizes \(U_o\).

The rationale for the Nash solution arises from Zeuthen's analysis of the bargaining process. Suppose \(I\) is holding out for a trade with pay-off \((u_i, v_i)\), and \(II\) is holding out for a trade with pay-off \((u_j, v_j)\), where both points are Pareto optimal. If these claims prove to be incompatible, the situation is one of "conflict", and the outcome is the "conflict point" \((\bar{u}, \bar{v})\). The expression

\[
q_I = \frac{u - \bar{u}}{u - \bar{u}}
\]

represents the highest probability of conflict which \(I\) is prepared to face in order to have his offer accepted. Likewise

\[
q_{II} = \frac{v - \bar{v}}{v - \bar{v}}
\]

represents the highest probability of conflict which \(II\) is prepared to face in order to have his offer accepted. The reason for this is that if the probability of conflict is higher than this specified limit then the expected utility of resisting the opponent's offer is less than the expected utility of accepting it. Clearly, this will never be the case where the pay-off under conflict exceeds the pay-off for that player in his opponent's offer; in this case \(q > 1\).

It is assumed that players know one another's utility functions, so that these subjective probabilities or "risk-of-conflict" limits are known by both players. The argument is that the player who is less prepared to face conflict, will "concede": that is, if \(q_i > q_j\), player \(j\) concedes. A concession involves making a new offer, for which the
associated $q$ is lower; this means a claims for a new pay-off $(u, v)$ in which the utility return to oneself is reduced (and hence that to one's opponent is increased). Eventually, this process will lead to a situation where the claims of the players are compatible — the outcome associated with this situation is the solution of the bargaining game, and this solution is the Nash solution. To see this, we observe that for $I$ to make a concession we must have $q_I < q_{II}$.

\[
\frac{u^I - u^{II}}{u^I} < \frac{v^{II} - v^I}{v^{II}} \quad \text{since (0, 0) is (0, 0)}
\]

\[
u^I v^{II} < u^{II} v^I
\]

\[
u^I v^I < u^{II} v^{II}.
\]

Hence, for $I$ to concede to a new offer where $q_I > q_{II}$ implies finding a new offer $(U^I, V^I)$ such that

\[
u^I v^I > u^{II} v^{II}
\]

Thus, the bargaining process must lead to agreement at the point $(u_0, v_0)$ where $u_0, v_0$ is maximized.

All that is required to ensure that this process is an adequate description of the bargaining game is the assumption that each player is rational in the sense that he does not expect his opponent to make a concession in a situation in which he, himself, would not concede (16). This seems eminently plausible, and the analysis does, I think, provide a convincing rationalization of the Nash solution as the solution of two-person co-operative games.

There is no apparent reason why the Nash solution should turn out to be at Lindahl's fiscal optimum, $P$ — in general, we would expect the bargaining process to generate some other point on the line segment $WW'$ (in fig. 4), say $N$. Although this is not our preponderant concern, it is interesting to pose the question whether the Nash solution, which is apparently likely to prevail, may not be «better» in some sense than Lindahl's optimum. In fact, it turns out that the Nash solution can make some claim to 'ethical excellence' in that it is the only solution which has the following four properties:

(i) the solution is independent of the utility units or origin selected;

(16) See Harsanyi (1966), op. cit. and (1965) op. cit.
(ii) the solution is undominated (i.e. Pareto optimal);
(iii) the solution is independent of irrelevant alternatives — if new trades are added, then either the solution is unaffected, or it becomes one of the new trades;
(iv) the solution is symmetric, in the sense that no player gets treatment which would not be bestowed on his opponents were they in the same situation — this means that the solution is independent of labels.

In so far as these properties are quite fair, the moral authority of the Nash solution is unimpeachable.

\[ \text{FIG. 6.} \]

**Threat Strategies:**

Since the Nash solution is defined as that solution \((u, v)\) which maximizes the product \((u - u^*) (v - v^*)\), it is clear that the status quo point is crucial in determining the ultimate solution. The aim of «threat» strategies is to shift the status quo or «conflict point» in such a way as to lead to an ultimate solution more greatly in one's own favour. Clearly, the more costly a player can make conflict for his opponent, the greater the likelihood that his opponent will have to make the next concession. Usually, however, a player can only make conflict more costly to his opponent if he simultaneously makes
it more costly to himself. The planning of optimal threat strategies therefore consists of making the best compromise between increasing the cost of conflict to one's opponent, and preventing it from becoming too costly to oneself. Although examples of the use of threat strategies as a preliminary to bargaining exist in the externalities literature, it is not easy to find non-trivial examples applicable to the Lindahl case. Nevertheless, it is easy enough to see how such threat strategies work. If two neighbouring houses are bargaining over the cost share in erecting a fence between the two properties, and if one house can increase the cost of having no fence to the other (by, say, buying a large Alsatian dog and permitting it to run freely over both properties) without increasing the cost to itself, then it stands to gain by forcing the other to accept a greater proportion of the cost. Thus, if the status quo point moves from \((0, 0)\) to \((-5, 0)\), say, then the Nash solution shifts from \(u_0, v_0\) to a new equilibrium \(u_1, v_1\), such that \(u_1 < u_0\) and \(v_1 > v_0\).

Thus, there is some scope for «anti-social» behaviour on the part of one, or both, parties; and this may be «anti-social» not only in the sense that it serves to reduce the utility of one's opponents, but also the utility accruing to oneself.

**Pre-determined Relative Advantage.**

Raiffa has advanced a scheme for solving games where interpersonal comparisons of utility are meaningful. Consider a game in which security levels are zero for both players; then for any point \((u_1, v_1)\), a forty-five degree line (see fig. 7) through \((u_1, v_1)\) represents a contour of constant relative advantage (that advantage being \(u_1 - v_1\)). Suppose there exists a strategy \(z_1\) by which I can ensure himself a pay-off on or to the right of a particular relative advantage contour, \(cc\) (in fig. 7), and a strategy \(S^H\) by which II can ensure himself a pay-off on or above \(cc\). Then, if I advances a demand for a pay-off at \(M\) (to the right of \(L\)) on the Pareto optimum frontier, II can by using \(S^H\) restrict I to the line \(cc\) at \(L\), thereby inflicting a greater loss on I than on himself. It is therefore reasonable to advance the line \(cc\) as a solution in the sense that, for any solution which implies a relative advantage other than \(c\), at least one player will exercise his effective veto. Thus, there is an incentive for both players to achieve the best possible point on \(cc\), namely the Pareto optimum \(D\) in fig. 7, where \(cc\) cuts \(RS\),
The outcome can be conceived as the solution to two successive games: the first, a zero-sum game to determine relative advantage; and second, a fully co-operative game on the basis of the pre-determined relative advantage. In the latter, the interests of players exactly coincide.

\begin{figure}
\centering
\includegraphics[width=0.5\textwidth]{fig7.png}
\caption{Fig. 7.}
\end{figure}

This solution is, of course, a Pareto optimum, and the procedure seems to have much to recommend it. One of the chief problems with other bargaining procedures is that on each occasion a Pareto optimum may well take a long time to achieve. If a system could be devised whereby the relative advantage (i.e. zerosum) aspects of the conflict could be reconciled once and for all, at say a sort of constitutional level, then a Pareto optimal budget could be ensured each year — there is every incentive for players to co-operate fully to achieve one. The problem here is, however, the difficult of specifying relative advantage in utility terms. Clearly, specifying a particular cost share is not enough (unless it happens to be that corresponding to Lindahl’s fiscal optimum \( P \)).

As was seen in the previous section, the market fails to provide an optimal quantity of public goods for two distinct sets of reasons:
the first, strategic; and the second, informational. Political provision, defined as provision in a context in which cooperative action is allowed, appears to provide a solution to the latter difficulty. But, the strategic problems still exist.

Most game theory writers seem to be of the opinion that such strategic problems are not sufficient to preclude the eventual attainment of a Pareto optimum. The optimum may, however, take a long time to achieve, particularly if there is scope for bargaining and the use of threat strategies. One possible means of circumventing this lengthy process is to divide the game into two parts: the first a zero-sum game at the «constitutional» level to determine relative advantage; the second, a fully co-operative game to achieve the Pareto optimum associated with that relative advantage. This is possible, however, only when interpersonal comparisons of utility can be meaningfully made.

As to the Pareto optimum point that is eventually achieved, there seems no good reason why it should happen to coincide with Lindahl's fiscal optimum. A fairly persuasive case can be made for the Nash solution, and if some unique outcome is to be specified, the Nash solution does seem to be the most likely contender.

IV. Bargaining with partial ignorance (17).

The assumption that players have full knowledge of opponents' preferences imposes a severe limitation on the applicability of game theory analysis to social and economic questions. Suppose, then, that we make the alternative assumption: that players do not know (and know that they do not know) their opponents' utility functions. Immediately, this has two implications for any conflict model. In the first place, it becomes necessary for players to reveal their preferences to one another as part of the game. In the second place, it is possible for players to use mis-statement of preferences as a strategic device. Of course, if the revelation of preferences has no strategic significance, then the business of informing opponents as to the true nature of one's utility function becomes a mere formality.

(17) Attempts to relax the assumption of perfect knowledge systematically are sparse in the game theory literature. JOHN HARSANYI, «Bargaining in Ignorance of the Opponents Utility Function», Journal of Conflict Resolution, 1962 is probably the most significant, and this section borrows heavily from that paper.
In fact, however, each player does have a strong incentive to understate his true preferences. If I can make II believe that the cost of conflict to himself is small (that is, that $u^*$ is large where $(u^*, v^*)$ is the status quo point) then he can extract from the situation a higher final pay-off than he would obtain if true preferences were known. To exemplify this, let us suppose that in the Lindahl diagram, I pretends to have less of an interest in public goods than he really has. For any cost share, $k$, he pretends to want only some fraction, $kG$, of the amount of public expenditure $G$ indicated by his true preferences, where $0 < k < 1$. This involves a vertical shift in his offer curve in a downward direction (from $AA'$ to $FF'$ in fig. 8) and if II reveals his offer curve accurately, then the fiscal optimum will move strongly in I's favour. If $FF'$ in fig. 8 is I's revealed offer curve and $AA'$ his real offer curve, dissembling of preferences makes it possible for I to shift the «fiscal optimum» from $P$ to $Q$, involving a move to a higher indifference curve for I and to a lower indifference curve for II. The disguising of preferences shifts the apparent status quo point in I's favour, and hence the Nash «solution» moves to some point just below $WW'$ further to the left.
Hence, both players have an incentive to indicate a weaker preference for public goods than they actually have. This tends to result in a situation in which the size of the public sector is consistently smaller than would be optimal. In other words, relaxation of the assumption that all utility functions are known, and admittance of mis-statement of preferences as a strategic device casts serious doubts on the likelihood of the Lindahl budget model achieving a Pareto optimum.

But apart from observing that there is an incentive to disguise preferences, can we, say, anything systematic about what sort of solution is likely to be achieved?

It is clear that in any game the offers which any one player makes depend largely on what he expects his opponent will accept. Now, the best solution which any player can "rationally" expect to achieve is the Nash solution, $A^*$; then, the offer most favourable to a player which his opponent will accept is his expectation of the Nash solution. Thus, for I this is $A_I = e_I A^*$. (i.e. I's expectation of $A^*$)

Likewise, $A_{II} = e_{II} A^*$. But, I knows that II's offer depends on what II thinks he will accept, so he can form a more sophisticated estimate of the ultimate solution:

$$A_I(2) = e_I e_{II} A^*,$$
and
$$A_{II}(2) = e_{II} e_I A^*,$$
and even greater sophistication is possible — for I knows that II's offer depends on what II thinks that I thinks II will accept; and so on. In general, there is a sequence $A_I(n)$ for each player, and the achievement of an ultimate solution depends on:

(a) whether $A_I(n)$ and $A_{II}(n)$ are convergent
and (b) whether they converge to points which are not incompatible.

We can distinguish two possible mechanisms by which convergence is likely to be achieved.

In the first place, the bargaining process which precedes the game may supply a means whereby players are able to reveal their preferences and simultaneously determine those of their opponents. Instead of the bargaining game consisting of an exchange (or exchanges) of ultimatums, it takes the form of a set of tentative offers designed to test the opponent's attitude. This description does seem to accord well with reality, but the success of the information-
gleaning process depends crucially on the absence of excessive bluffing. Knowing that his opponent has a strong incentive to cloak his true preferences, each player will be wary about attaching too much importance to information obtained during the exchanging of offers.

In practice, bargaining costs players something in time and effort, and there will be some point at which bargaining becomes too costly to be continued. This cost serves to set some limit on the amount of bluffing undertaken. We could suppose that for each player, there is a number \( D_i(k) \) such that if the difference between the two offers \( B_I(k) \) and \( B_{II}(k) \) at the \( k \)th trial exceeds \( D_i(k) \), then that player will break off the bargaining procedure, because of the cost of bargaining exceeds the expected gain. Hence, in the Lindahl model, if the budget offers at the \( k \)th trial are more than a certain distance apart in terms of utility to group \( i \), then that group will cease bargaining and the game will reduce to the unco-operative model examined in section II above. The number, \( D_i(k) \) is the «tolerance limit» for the \( i \)th player at the \( k \)th trial, and it will normally be the case that \( D_i(k) \) will decrease as \( k \) increases. And as both players know this, there will be a tendency to ensure that offers become more compatible over time, for fear that the opponent will cease to cooperate. Nevertheless, «tolerance limits» will not normally be known by opponents, and failure to estimate these accurately will result in a collapse in communications, and degeneration into the unco-operative game situation (18).

When utility functions are unknown, then, the possibility of preference revelation during the bargaining process provides scope for achieving a solution not markedly different from the Nash solution (which would prevail if true preference were known). The existence of bluffing, however, tends to cast doubt on the use of bargaining as an informational device, and it is probably easy to overrate this as a mechanism for ensuring a solution.

An alternative mechanism applies if each player enters the game with more or less consistent expectations of his opponent’s utility function. A utility function which is by custom ascribed to a particular group is called a «stereotype utility function», \( U^o \). If I ascribes such a utility function \( V^o \) to group II, it will not normally

(18) This is perhaps exemplified more readily by reference to international political situations than to economic ones — but the same principle seems to apply.
be in II’s interests to act as if his utility function were otherwise, even if it is. For if $V^o$ overstates his preferences, II will probably not be able to change I’s conceptions, since attempts to reveal his true preference will merely be interpreted by I as bluffing. And if $V^o$ underestimates II’s preference, he will have no incentive at all for revealing his true utility function. If the conflict point is $C$, then, the solution which prevails, on Zeuthen-Harsanyi grounds, is the outcome $A$ where

$$(U^o (A) - V^o (C)) (V^o (A) - V^o (C))$$

is maximized; ($U^o$ and $V^o$ are the stereotype utility functions for I and II respectively). This is, of course, the Nash solution, given those particular utility functions. The extent to which this solution approximates the real Nash solution depends on the accuracy of the stereotype functions—i.e., on how closely they approximate reality. But there is no reason to assume that the solution thus achieved will be Pareto optimal; there may well be a strong incentive for both parties to achieve some new point, but problems of suspected bluffing may preclude any such move.

In this connection, one of the most interesting features of Lindahl’s discussion is his attempt to reconcile his own idea of tax justice with the concept of taxation according to ability to pay (19). Recognizing the «administrative» constraints applicable to his model, he argues that taxation according to ability may well present a practical solution to the problem of determining an optimal level of public expenditure and an optimal allocation of its cost. Only in the special case where the money value of marginal utility from public goods is exactly proportional to income will proportional taxation lead to a Pareto optimum, but it is probably not unreasonable to ascribe to groups «stereotype» utility functions which equate the demand for public goods with income levels. In this way, Lindahl has accepted an approach analogous to the «stereotype» utility function approach. This appears to be necessary if his theory of the budget is to be made administratively feasible.

V. Summary and conclusions.

This article began with the express aim of bringing to bear the techniques of game theory on the operation of Lindahl’s famous model

(19) LINDAHL (1928) (op. cit.) in MUSGRAVE & PEACOCK (op. cit.), p. 228-230.
of budget determination. At the outset I raised two basic questions, which I hoped to be able to answer in the course of the discussion:

- does Lindahl's model necessarily give rise to a Pareto optimum?
- is this optimum likely to be the one isolated by Lindahl as representing tax justice?

In answer to the first question, it is probably fair to say that the greater body of game theory is substantially in agreement with Lindahl — a Pareto optimum can (and indeed is fairly likely to) be achieved in the provision of public goods through the political mechanism (i.e. when the budget is determined under the conditions of co-operative games). But the optimum may well be achieved only after a lengthy and socially costly prelude of bargaining, throughout which no public goods are provided.

Where joint action is prohibited by the rules of the game, the situation is rather closer to that prevailing when public goods are supplied by units acting independently in the freely operating market. Here, a Pareto optimum is distinctly less likely, due not so much to strategic problems, but largely to the purely informational constraint imposed by refusing to allow players to co-operate. Where joint action becomes possible over re-iterations of the game, a Pareto optimum becomes more likely.

In the same way, if the characteristic game-theory assumption that all players have full knowledge of opponents' preferences is relaxed, the hopes for the political mechanism as an allocator of resources quickly fade. All that can be hoped for in this context is some point reasonably close to the utility frontier — and this seems to be a fact which Lindahl himself implicitly recognized.

In those cases where a Pareto optimum is achieved, there seems no good reason why it should fall precisely at Lindahl's fiscal optimum. A rather more convincing case can be made for the Nash solution. Nor should it be supposed that failure to achieve «tax justice» in the Lindahl sense is a bad thing: the Nash solution turns out to have some moral authority in its own right, and it is by no means clear why, if at all, Lindahl's solution is to be preferred.

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3. APPENDIX A

BUCHANAN ON LINDAHL

One of the prime weaknesses of Lindahl's formulation of his model seems to be his failure to specify adequately the process by which his 'fiscal optimum' is achieved. While his analytical apparatus is generally taken as being more or less successful in determining a set of points from which, variously interpreted, either bargaining will begin, or the ultimate solution will come, the method by which the fiscal optimum is finally isolated seems completely inadequate. In the face of indeterminacy within the given range (QPA' in fig 1 in the paper), Lindahl introduces his assumption of 'equal power': assuming 'equal power' obtains, only one outcome is possible - to wit, Lindahl's fiscal optimum.

Unfortunately, the definition of 'equal power' is entirely circular: 'equal power' is defined in terms of its ability to isolate the desired outcome. Thus, the 'fiscal optimum' is that outcome which prevails under 'equal power' - and 'equal power' prevails when the 'fiscal optimum' is attained. What the model lacks is some institutional analogue to the equal power assumption. To complete the model, we need to specify some adjustment mechanism which might be said to characterize the notion of equal power, and which will therefore ensure that the fiscal optimum will be reached.
If, instead of the circular 'equal power' assumption, operationally meaningful adjustment processes are applied to the Lindahl model, the isolation of Lindahl's fiscal optimum is revealed as essentially arbitrary - Lindahl's 'optimum' is no more likely than any other point on the utility possibilities frontier, even if the achievement of a Pareto optimum turns out to be certain. Apparently, there is no adjustment process yet known, which represents an analytic counterpart to the 'equal power' assumption.

This gap in the Lindahl formulation has allegedly been remedied by Buchanan in his recent book-length treatment of the public goods concept. By an ingenious welding of Wicksellian marginalism and Lindahl's constant cost-share constraint, Buchanan purports to provide an adjustment process which will ensure that Lindahl's fiscal optimum will be achieved.

The purpose of this short note is to show that Buchanan's claim is unjustified; that his process is no more likely to achieve the Lindahl optimum than it is any other of a number of points within the relevant range; and that in this sense, Buchanan's is no different from other procedures already considered in the literature.

The relevant part of Buchanan's argument can be quoted in full: "Suppose that ... an initial proposal is made to spend an amount $G_1$ on the public good. Tax-sharing schemes are presented along with this spending proposal. In this context, any tax-sharing scheme falling
between $Y_1$ and $Y$ may be approved by both parties, ignoring purely strategic behaviour. For an amount of spending $G_1$, $A$ would, if necessary, finance the whole cost. Similarly, $B$ would if necessary pay a major share as indicated in the scheme at $Y_1$. Agreement becomes possible, on some tax-sharing arrangement and on the spending proposal, anywhere between these limits. Having adopted this initial spending proposal, suppose that a further proposal is made in the second round to expand the level of output incrementally. Agreement remains possible, with many alternative sharing schemes on such increments, but the multiplicity of possible arrangements diminishes rapidly as $G_0$ is approached. At the margin, at $G_0$, only one sharing scheme can command the approval of both parties, that shown by $Y_0$. For all proposals to expand spending beyond $G_0$, no sharing scheme will command the approval of both parties.

The position shown at $P$ is, therefore, the uniquely determinate Wicksellian solution to the problem of public-goods allocation and tax-sharing, given the restrictions of our model. These restrictions include constant marginal tax-shares over quantity."^4

A simple counter-example is sufficient to show that Buchanan's claim is unfounded. Since it is, allegedly, immaterial what cost-share prevails at $G_1$, let us assume without loss of generality that the cost-share is $Y_1$, and initial 'equilibrium' is at $Q$. At $Q$, two things are apparent: firstly, that there exists a set of points bordered by $a_4$ and $b_1$, all of which are Pareto superior to $Q$, and some of which (i.e. those along TS) are actually Pareto optimal; secondly, that Lindahl's fiscal optimum $P$ does not belong to this set. Starting from $Q$, any budget implying a position in the $(h,G)$ plane below $a_4$ serves to make $A$ worse off, and
hence be vetoed by him. Since $a_4$ lies everywhere above $QPA'$, this latter set includes all the points along $QPA'$.

This is not necessarily to suggest that Buchanan's adjustment process (assuming with Buchanan - heroically - that there is no "purely strategic behaviour") will not lead us to some Pareto optimum. At $G_2$, any cost share along $LM$ will give gains to both parties, and likewise, once some point on $LM$ is specified, marginal increases in $G$ will again receive unanimous support providing that the cost-share chosen lies between the relevant indifference curves. Ultimate equilibrium could be established at some point $P_1$ on $TS$. Certainly, beyond this point, no unanimously approved increase in government spending is possible, whatever the cost-share chosen.

What Buchanan apparently overlooks is that there is nothing inherent in his adjustment process which will constrain the successive equilibria $(G_i, Y_i)$ to lie in the area bordered by $QPA'$. If there were, of course, movement along $QPA'$ would always be vetoed by one or other party, because such movements invariably make one of the parties worse off: the $AA'$ and $BB'$ lines of Lindahl's diagram are demand curves - not equilibrium adjustment paths. Buchanan's adjustment process can, in fact, generate any point along $WW'$ between $T$ and $W$, depending on the initial cost share. It will yield up Lindahl's fiscal optimum, $P$, only in the rather special case where the successive budgets always lie in the area bordered by $a_3$ and $b_3$. Such a case is clearly not impossible, but neither is it guaranteed by the Buchanan scheme.

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Such as quantity adjustment, price adjustment, 'cobweb' mechanisms, price leadership and so on. See J.G. Head "Lindahl's Theory of the Budget" Finanzarchiv 1963-64 for an analysis of such adjustment process, as well as a more general discussion of the Lindahl theory.


in our fig. 1

Buchanan op cit p. 135. The transcription is faithful except for symbols.

The basic ingredients in this appendix were written when I was a Visiting Research Fellow at Dalhousie University, Halifax, in early 1971.
4. PAPER TWO
Public Goods and Factor Prices
from
Public Finance/Finances Publiques No 1 1975
I. INTRODUCTION

Unquestionably one of the most significant developments in public finance theory in recent times has been the construction of the modern theory of public expenditure.

Right from the very outset in the development of this theory, the importance of distributional considerations in determining optimal public goods supply has been emphasized. In some ways this is surprising. Bearing in mind that the central contribution of the theory is to point up the essential inadequacy of the market mechanism in supplying public goods, and that this result holds quite independently of the desirability (or otherwise) of the prevailing income distribution, one might have thought that concern over distribution would merely serve to cloud more relevant issues. Yet both Samuelson [13] and Musgrave [11] in their early contributions make much of distributional considerations – Musgrave, because the indeterminacy of the Pareto criterion in specifying a unique optimum seemed to him to represent a distinct source of market failure due to public goods; and Samuelson, because, for him, the conditions for a welfare optimum had to be specified in terms of a welfare framework in which distribution entered as an explicitly “welfare-significant” parameter.

It seems now to be generally recognized¹ that Musgrave’s concern over the distributional indeterminacy of the Pareto criterion in the public goods context is rather misplaced – that is indeterminacy is an inherent characteristic of the Pareto criterion, and applies identically in a private goods world. Nevertheless, one might concede that once market failure is established and provision of public goods through the political mechanism is shown to be necessary, public goods will be associated with a dimension of distributional choice not normally present with private goods. This is so because the market characteristically generates pricing that involves no discrimination over infra-marginal units of the goods supplied: hence for goods provided in the market, there is a one-to-one relationship between the pattern...
of factor ownership and the ultimate distribution of utilities. No such one-to-one relationship appears in the public sector: any particular initial pattern of factor ownership can translate into any of a number of ultimate "distributions of utility," depending on the pattern of infra-marginal "pricing" adopted for public goods. Thus, if market failure is associated with public goods, and if political provision is likely to be involved whenever market failure appears, then it might be argued that public goods imply a degree of distributional indeterminacy not normally associated with private goods.

Samuelson’s concern over distributional effects is at once more understandable and more strange. An explicit treatment of distributional objectives — even if merely formal — seems desirable. Yet if the market and, similarly, the political mechanism fail because economic agents sent the wrong signals, and because decision-makers (even appropriately motivated ones) lack the information to lead the economy to a Pareto optimum, then the design of the appropriate set of lump-sum taxes and subsidies to "... swing the outcome to the ethical observer’s optimum" is likewise impossible. Indeed, the informational requirements associated with distributional objectives are even greater than those associated with efficiency — for clearly one needs to know not only individuals’ marginal valuations of the public good, but also the infra-marginal ones, and these for all private, as well as all public, goods.

Without exception, distributional interest in the public goods literature has focused on the "carving-up" of the utility gains which arise from the expansion in consumption possibilities occurring when public goods are introduced. This paper, by contrast, is concerned with distributional effects of public goods production on the production side — those arising through changes in factor prices.

In spite of the lack of attention paid to these factor price effects, they are not necessarily completely unimportant: it may even be that they dominate distributional effects on the consumption side (as they seem to in tariff policy). In any case, it seems desirable, in the interests of completeness at the very least, to accord them some attention. Such is the general objective of this paper. The more specific aims are threefold: firstly to present a diagrammatic exposition of a model of public goods supply in which factor prices, individual incomes, and the optimal supply of public goods are simultaneously determined — the public goods analogue to "general equilibrium" private goods models; secondly, to present in the light of this a positive model of public goods supply in which factor price effects enter, and to trace out the implications of factor price changes for the operation of this model; thirdly, to indicate how the analysis of tax incidence might alter when taxes are considered in a context in which public expenditure levels (and factor prices thereby) are allowed to adapt to tax changes. These aspects are handled in Sections II, III, and IV in sequence. Section V contains a brief conclusion.
II. THE MODEL

1. The basic ingredients of the model are:
(a) two individuals, I and II;
(b) two commodities, X and G (X is private, so that total production $X$ is equal to the sum of individual consumption i.e., $X = X_I + X_{II}$; G is public, and hence equally and totally consumed by all i.e., $G = G_{I} = G_{II} = G$);
(c) two homogeneous factors, $K$ and $L$ (capital and labour respectively) which are in fixed supply.

It is assumed that both $X$ and $G$ are subject to constant returns to scale in production, and that both factors for both products are purchased in a perfectly competitive market situation.

2. The objective of the model is to depict the way in which the optimal level of public goods supply and the distribution of income are jointly determined, in a setting more or less analogous to the two-sector general equilibrium private-goods model. Needless to say, the basic diagrams of the present section follow fairly closely those of the private-goods model (as explicated, say, by Johnson [6]), though there is quite enough difference to make the present exercise interesting.

What needs perhaps to be emphasized at the outset is that what is presented here cannot be regarded as a general equilibrium model in quite the same way that the private-goods model can, because there is no well-defined set of institutional arrangements (either market or political) which will ensure that the optimal level of public goods is also an equilibrium level. For the purpose of this first section, it will simply be assumed that the optimal level of public goods supply is somehow achieved. This assumption is relaxed in Section II.

3. It is clear that, in this model, the distribution of income will both determine, and be determined by, the optimal level of public goods supply. On the one hand, the distribution of income determines the weighting given to different individuals' preferences in determining fiscal outcomes; and given that individual preferences differ, changes in the distribution of income will in general change the optimal level of public goods supply. On the other hand, each level of public goods supply is also associated with a specific set of factor prices that ensures equilibrium in all factor markets — and this set of factor prices will determine the distribution of income between individual factor owners. When the distribution of income determined by the set of factor prices associated with a given level of public goods supply is the same distribution of income which generates that level of public goods supply as optimal, then that output level (and the associated income distribution) is a general "equilibrium" level, in the special sense of this model.

4. The model develops naturally in two stages. The first step involves deriving, for each point on the production possibilities curve, a distribution of income between...
the two individuals, based on the set of factor prices associated with the production point. From this, a distribution of income curve is constructed and set in the production possibilities plane. This enables us to depict, for any "initial" production point, the distribution of income between individuals; and then derive as the second step in the model, an optimal level of public goods consumption on the basis of the individuals' preferences. This requires us to specify the pricing rule for infra-marginal units of the public good. Where for the given pricing rule the optimal consumption point coincides with the "initial" production point, a total optimum in the general equilibrium sense prevails.

A. Derivation of the Income-Distribution Locus
5. Consider the Edgeworth-Bowley box in Fig. 1. Capital and labour supplies are indicated on the vertical and horizontal axes respectively, and isoquants for $X$ are depicted with $O_X$ as origin while isoquants for $G$ are depicted with $O_G$ as origin. The efficiency locus, $O_XQ_OG$, is determined by the points of tangency between isoquant maps, and indicates that $X$ is relatively capital-intensive and $G$ labour-intensive.

6. Let $Q$ be some point on this efficiency locus. At this point, we can determine: output, relative product prices, and relative factor prices. Factor prices are, of course, given by the slope of the isoquants at $Q$, i.e., the slope of $FF'$. Since constant returns to scale obtain, the outputs of $X$ and $G$ can be determined by the distance of
I

the relevant isoquants from the respective origins. Thus, on the straight line $OG0X$, $OGS$ indicates the quantity of $G$ and $OXR$ the quantity of $X$. Projecting these distances in the vertical and horizontal dimensions respectively and considering $O$ as origin, the quantity of $X$ produced is given as $OX_p$, and of $G$, $OG_p$. Thus, $P$ is a point on the production possibilities curve viewed with $O$ as origin. By taking points on the efficiency locus in the neighbourhood of $Q$, we can obtain a set of points on the production possibilities curve around $P$. By taking the slope of the curve around $P$, we can define national income expressed in terms of private goods, which in this case is $OP'$ along the horizontal axis.

7. This level of national income can now be divided among factors, all of which are to be paid in terms of the private good, $X$. Some of this $X$ will then be re-collected in the form of taxes to pay for the production of $G$. The breakup of $OP'$ between capital and labour can be determined as follows: (i) draw a line from $O_X$ parallel to $FF'$ cutting $OGO'$ (produced) at $M$. Then $O'M$ gives the value (in terms of capital) of $O'O_x$. Labour for factor price $FF'$ so that the ratio $O'M/O'OG$ gives the value of labour to the value of capital at $P$; (ii) draw $MN$ parallel to $O'O_X$ cutting $OOX$ (produced) at $N$. Then the ratio $O_XN/OX$ likewise depicts labour’s and capital’s relative shares; (iii) connect $N$ to $P'$, and draw a line from $O_X$ parallel to $NP'$ cutting $OP'$ at $A$. Then $OA$ indicates the share of national income accruing to capital, and $AP'$ the share accruing to labour.

8. The transformation curve associated with Fig. 1 can be depicted in a more traditional way in Fig. 2 as $RQ$. For any point $P$ and $RQ$, the level of national income, measured in terms of $X$, is $OT$. The distribution of $OT$ between factors is
determined as in Fig. 1, and can be depicted by $YZ$ in Fig. 2 as follows: for each $P$ on $RQ$ let capital's share derived from Fig. 1 be the vertical distance of $YZ$ from the horizontal axis, at that $P$. This defines $YZ$ as the factor distribution of income line, and the line is used as follows. For each $P$ on $RQ$, place the vertical distance of $YZ$ along the vertical axis — this gives $OA$ as capital's share of national income and $AT$ as labour's share.

9. Interest here, however, focuses on the personal distribution of income — i.e., the income shares accruing to individuals $I$ and $II$. Thus, we define a vector $(a, b)$ such that $a$ is $I$'s share of the total capital stock, and $b$ is $I$'s share of the labour stock. Then for any point, $P$, on $RQ$, we derive $OA$ and $AT$ from $YZ$; and then calculate

$$a \cdot OA + b \cdot AT \quad (a, b \leq 1)$$

and

$$(1 - a)OA + (1 - b)AT$$

which are $I$'s and $II$'s share in total income, respectively. On the basis of particular shares $(a, b)$, we can thus transform $YZ$ in Fig. 2, into $UV$ in Fig. 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{B. Derivation of the “Total Optimum”}
\end{figure}

10. We now postulate for $I$ and $II$ preferences between $X$ and $G$ which exhibit standard properties of convexity. The preference maps are depicted in Fig. 3 and $i$'s in the upper quadrant, and $II$'s in the lower. In deriving the optimal supply of public goods, based on these preferences and the pattern of factor ownership, it is necessary to specify some pricing rule for the public good. The most obvious one to start
with is the Lindahl pricing rule — under Lindahl pricing the price for \( G \) is constant over all units of \( G \) and set for each individual at his marginal valuation of \( G \).

On this basis we can define, for any efficient production combination \( P \) on \( RQ \), an associated "consumption optimum" as follows: (i) the point \( P \) determines the marginal rate of transformation (the slope of TS), the level of national income measured in terms of \( X \) (OT), and the personal distribution of that income (OA to I and AT to II, with OB drawn to be equal to AT); (ii) given Lindahl pricing, the locus of potential consumption points for I is the price-consumption curve DE based on level of income OA, and the analogous locus for II is the price-consumption curve FH based on the level of income, OB; (iii) the vertical sum of these price-consumption curves, JK, cuts ST at some point C; at C, the sum of I's and II's marginal rates...
of substitution between \( G \) and \( X \) must equal the marginal rate of transformation at \( P \).

In general, \( C \) will not coincide with \( P \). Thus \( C \) will not be feasible; and \( P \) will not be desired since the total marginal valuation placed on levels of \( G \) at \( P \) will not be equal to marginal cost\(^5\) (in the case shown in Fig. 3, total marginal valuation of \( G \) at \( P \) is less than marginal cost at \( P \), both \( I \) and \( II \) would prefer a reduction in the supply of \( G \)).

12. By taking successive points on \( RQ \) in turn, we can trace out a locus of consumption optima, \( LM \) in Fig. 4(c). Beginning at \( R \) (in Fig. 4a), the distribution of income is given as \( OU \) for \( I \) and \( UR \) (equals \( OB \)) for \( II \). The related price consumption curves are \( DE \) for \( I \) and \( FH \) for \( II \), which sum to give \( JK \). This determines \( L \) where \( JK \) and \( RS \) intersect. Since \( L \) lies to the right of \( R \) some increase in the supply of \( G \) is called for. As we move around \( RQ \), the level of national income (in terms of \( X \)) increases; the marginal rate of transformation of \( X \) for \( G \) (the marginal cost of \( G \) in terms of \( X \)) rises and the distribution of income changes according to \( UV \). At \( Q \), the consumption optimum \( M \) is derived (in Fig. 4b) in a similar way – the distribution of income is given as \( OV' \) for \( I \) and \( TV' \) (equals \( OB' \)) and the associated price-consumption curves are summed to give \( U'K' \), cutting \( TQ \) at \( M \). Since \( M \) lies to the left of \( Q \), some reduction in the supply of \( G \) is called for. Somewhere between the extremes of \( R \) and \( Q \) (\( L \) and \( M \)) the consumption optimum and the production point which gives rise to it will coincide. At this point, \( N \), “general equilibrium” optimality will have been obtained. Elsewhere, \( LM \) will lie entirely outside \( RQ \) since \( RQ \) is convex, all non-coincident consumption optima are infeasible. The locus of consumption optima \( LM \) may touch \( RQ \) at a number of points. There seems to be no reason to think that the number of such points must be odd, or that any of them must be unstable [cf. 6, Johnson].

13. Maintaining the assumption of Lindahl pricing, it is clear that any change in the pattern of factor ownership \((a,b)\) will involve a new distribution of income line, \( UV \), differing from the original one both in slope and position. For any such new income line, a different locus of consumption optima will be established and a new “general equilibrium” optimum will prevail.

14. Let us now alter the pricing rule, while retaining the original \((a,b)\). To take a limiting case, let \( II \) obtain all the consumer surplus from public goods production. For any \( P \) on \( RQ \), there will again be a distribution of national income determined from \( UV \), giving (in Fig. 5), a level of income \( OA \) to \( I \) and \( OB \) to \( II \), where \( OA + OB = OT \). Under the new pricing rule, however, \( I \) is constrained to lie on the indifference curve from \( A \), labelled \( j_i \) in Fig. 5. Then \( II \)’s “consumption possibilities” curve based on \( P \) is \( BW \), determined by vertically subtracting \( j_I \) from \( TS \). This gives a consumption optimum at \( D \) where \( BW \) touches \( j_{II} \) – which yields \( C \) on \( TS \). Again a locus of
potential consumption optima can be derived, one for each point on $RQ$ and one (or more) of which will coincide with the corresponding production point $P$. This “general equilibrium” optimum will be that associated with the new pricing rule, whereby $II$ obtains all the consumer surplus from public goods, and with the pattern of factor ownership $(a, b)$.

![Figure 5](image)

15. It seems that the “consumption equilibrium” given in Fig. 5 can also be interpreted as a Lindahl optimum associated with some other distribution of factors between $I$ and $II$. There exists at least one $(a, b)$ giving rise to an income distribution under which $I$ receives $OA'$ and $II$ receives $OB'$ in Fig. 5 when production is at $P$; and this $(a, b)$ will necessarily give a Lindahl consumption optimum at $C$ when production is at $P$. Thus we can regard any pattern of infra-marginal pricing of the public good as equivalent to a Lindahl solution plus some “lump-sum” redistribution of income. In this sense, a change in the pricing rule is equivalent to a change in the pattern of factor ownership.

16. The central objective of this section has been to derive a diagrammatic model of public goods provision analogous to two-sector “general equilibrium” models for a private goods world, in the sense that prices of factors, and hence the distribution of income, are determined jointly with levels of output rather than independently of them. For any given pattern of factor ownership, and any specified pricing rule for
the public good, the associated "general equilibrium" optimum can be derived. This optimum not only demonstrates the normal optimal conditions for a public goods world, but also is consistent with the distribution of personal income to which the particular level of public goods output gives rise.

III. A POSITIVE MODEL

17. The model set out in the foregoing section involves, of course, a dimension of unreality not present in the completely private goods analogue. As the literature has been at great pains to emphasize, individuals' preferences between private and public goods are characteristically unknown. In order to make the model anything other than a mere technical exercise, it is necessary to postulate some political institution, or decision-making rule, which will provide a link between any specified set of individual preferences and a particular allocation of resources to public use. That is, we require some "positive" model of public goods provision in order to convert the model into a "general equilibrium model" in a more standard sense. The following is offered in the hope that it is of some interest and possible relevance.

18. Suppose that cost-sharing arrangements are fixed ex ante - according to ability-to-pay considerations - and are immutable (or are so at least in respect of public goods choices). Suppose further that the number of voters is large. Each voter is provided with the marginal cost schedule for G and is asked to specify a desired level of public goods production on the basis of the pre-determined cost-sharing arrangements; the actual level of G provided is then set at the median of the individual levels thus registered. Some of the characteristics of this procedure are worth noting.

19. Firstly, each voter-taxpayer has an incentive to reveal his preferences accurately. Because cost-sharing arrangements are independent of revealed preferences, no one stands to gain by deliberate preference misstatement: on the contrary, if I misstates his preference he runs the risk of moving the median outcome away from the output of G he desires. Consider for example a community of (2n - 1) voters ranked according to the quantity of G each desires - the median voter in this case is the nth. Now, if the (n + 1)th voter reveals his preferences accurately (or overstates) he will have no influence on the outcome; but if he understates, and the level of G he opts for is less than that of the nth voter, then the actual level of G taken will be reduced and he will be made worse off. An analogous problem faces the (n - 1)th voter if he overstates. There remains the problem that since the likelihood of any one voter's choice being decisive is very small, voters will rationally be underinformed about the benefits of public spending. Nor of course does accurate revelation of preferences under this procedure make Lindahl pricing immediately feasible, because
any attempt to use information about consumer preference thus obtained to alter cost-sharing arrangements would immediately restore the incentive for preference misstatement.

20. Secondly, the median level of $G$ can in most cases be presumed to approximate the mean. Under a variety of rather special circumstances, the mean of the individual specified levels of $G$ is equal to the optimal level of $G^6$ (i.e., that where $\Sigma MRS$ equals $MRT$), and the welfare loss associated with a procedure that took the mean of the individual $G$'s, would be zero. More generally, such a result will not hold, but in the absence of any information about the slopes of individual demand curves, specifying the mean seems likely to minimize expected welfare loss. The weakness of the mean in this context is that it is susceptible to strategic manipulation, in that individuals could influence the outcome by overstating or understating preferences. An individual would overstate his preferences if he expected himself to have a demand for $G$ in excess of the mean, and understate them in the opposite case. The median is not susceptible to this problem, and must therefore be adopted in preference. But in so far as the median is not too far from the mean in most cases, the welfare loss via such a procedure might be minimal. In this sense the procedure suggested seems likely to generate a supply of $G$ which is not so very far from the truly optimal level, were that knowable.

21. The median level of $G$ is stable in another sense. Were one to opt for, say, the $(n + k)^{th}$ level in a community of $(2n - 1)$ individuals, the smaller number of high-demand individuals may find it more easy to bribe the $(n + k)^{th}$ voters to increase the level of $G$ he nominates. If the median is chosen, the number of potential bribers on either side is the same — it seems no more likely that “high demand” individuals would be able to bribe the median voter to increase $G$ than it is that the necessarily identical number of “low demand” individuals would be able to bribe him to reduce $G$ (unless such a bribe were in fact Pareto desirable). Thus, in the unlikely event that bargaining is feasible in the large numbers case envisaged here, selection of the median voter's choice as decisive substantially diminishes the possibility of non-Pareto-desirable collusion between the median voter and others.

22. It is perhaps worth emphasizing that there is a relationship between the procedure suggested and majority rule: as is well known, if preferences are single-peaked, majority rule will ensure the selection of the median level of $G$ (or at least parties will set their policy platforms based on their expectations of the demands of the median voter). The procedure suggested simply effects explicitly what majority rule, with single-peaked preferences, would approximate anyway. What is here suggested, however, is that a crucial ingredient in ensuring single-peakedness is the requirement that cost-shares be fixed ex ante. In this sense, the recommendation is for a functional separation of equity and efficiency objectives — the “equity branch of
the public household’s is to concern itself with the business of defining equitable cost-sharing arrangements; the “efficiency branch” with determining, on the basis of those cost-sharing arrangements, the optimal level of public goods supply.

23. Adoption of this procedure is sufficient to ensure that the level of public goods supply is related, in a specific way, to the preferences of individual voter-consumers. It is possible that such a link could be established in other ways, but in models of the political mechanism where substantial departure from optimality in annual budget decisions is likely, it cannot be presumed either that individual preferences...
PUBLIC GOODS AND FACTOR PRICES

will be accurately revealed or that they will influence political outcomes in a predictable way. The political decision-making process suggested here, however, is sufficient to permit an exploration of the influence that the pattern of factor ownership is likely to have on both the median voter’s revealed preference and the (related) supply of public goods.

24. Suppose that the fixed cost-sharing arrangement (determined on equity grounds) involves proportional income taxation. Then, any point $P$ on the transformation curve, $RQ$ in Fig. 6, defines both the share of national income going to individual $I$ ($OA$, obtained from $UV$) and a proportional tax rate ($HT/OT$). Since the proportional income tax must always leave the ratio of $I$’s income to $II$’s income unaffected while ever the gross income levels are unchanged, the distribution of the amount of private good available at $P$ must be in the proportions $OA$ to $I$ and $AT$ to $II$. By joining $A$ to $S$ and noting where $PL$ and $AS$ intersect at $N$, the consumption combination for $I$ ($LN$ of $X$ and $OL$ of $G$) can be determined. By taking other points, say $P'$, on $RQ$, a different gross income of $OA'$, and a different net income of $ON'$, for $I$ can be determined. The locus of potential consumption points for $I$, $NN'$, can then be set out on the basis of the given cost-sharing arrangement implied by proportional income taxation, and the given pattern of factor ownership. Individual $I$ will clearly opt for the point $E$ where $NN'$ meets $I$’s highest attainable indifference curve $ij$. It is clear that the position of $NN'$, and hence of $E$, depends crucially on the pattern of factor ownership. If there is a redistribution of factors the position of $UV$ will change, and even if $E$ lies on the new locus of potential consumption points for $I$ (which it will do if $UV$ and $UV'$ intersect at the level of $G$ corresponding to $E$), the slope of that locus will have changed and $E$ will no longer be the point chosen. If for example the new distribution of factors involves $I$ owning less capital and more labour, the distribution of income curve will adopt a position like that of $UV'$: $NN'$ will become steeper, and $I$’s desired position will not be at $E$, but at some other point in his indifference map involving less $G$ and more $X$. The basic observation is that in isolating his most desired quantity of the public good, $G$, the voter takes into account not only the perceived benefits from public goods as such, but also the effects of changing public goods production on his share in total income.

25. This latter influence is essentially absent in an all private goods world: the individual’s contribution to total consumption (and hence production) is relatively so small that it has a negligible effect on total product shares; hence factor proportions and factor prices do not change. With public goods, however, the effects on factor prices will be significant because each individual votes on the total level of public goods supply. For the median voter (whose vote turns out to be decisive in our model) his pattern of factor ownership as well as his indifference pattern as such, will clearly influence his choice of $G$. One immediate implication of this is that
it may be yet more difficult to obtain information about marginal valuations of public goods than has traditionally been thought. Even if a political mechanism can be found whereby preferences for public goods are accurately revealed, these revealed preferences will be overlaid by Pareto-irrelevant factor price effects. And if, as is often argued, the benefits of public goods are only dimly perceived by the electorate, such factor price effects may well predominate.

26. At a rather more general level, we might note that a common feature of most parliamentary democracies is that they are based on “parties”. These parties can be looked on as coalitions of individuals with common interests. Clearly the coalitions which emerge could be based on preferences for public goods per se; but given the large number of public goods, and lack of information about preferences, and given the lack of interest concerning, and knowledge about, public goods benefits rationally possessed by the electorate, it seems at least as likely that dominant coalitions will be organised according to factor groups. If a little casual empiricism can be permitted, we might suppose, for example, that state-provided services are, on the whole, relatively labour intensive — with the exception of defence, which is capital intensive. Private goods on the whole might be supposed to be less labour intensive than public goods and less capital intensive than defence. If so, then the coalitions which we would expect to form might well be: (a) between owners of labour — in favour of an expansion of the public sector generally, although probably against high defence spending; (b) between owners of capital — in favour of a reduction in the size of the public sector (and with a corresponding interest in the expansion of the private sector) although probably strongly in favour of defence spending.

This is, of course, not so very far removed from what is observed in practice in many western countries.

IV. TAX INCIDENCE IMPLICATIONS

27. The foregoing analysis has implications for the analysis of tax incidence which it may be of interest to indicate. Standard incidence analysis proceeds on the basis of equi-revenue comparisons of various taxes (systematized in the “differential incidence framework” suggested by Wicksell and endorsed by Musgrave). It is clear, however, that the demand for public goods itself may depend on the way in which revenue is raised. If such demand is in fact reflected in fiscal outcomes (via the preferences of the median voter, as in the model indicated above, or otherwise) the change in public goods supply, with its attendant redistributive effects arising through factor price changes, emerges as an important influence in the determination of tax incidence.

28. In order to specify these influences, it is necessary to indicate both the effect of
changes in public goods supply on the distribution of income — part of which has been the primary focus of the analysis attempted above — and also the way in which tax changes might alter public goods supply. This last has already been the focus of an entire book by Buchanan [2], but it may be useful to indicate briefly some of the main ways in which the means of raising revenue is likely to influence the demand for public goods. We can, then, isolate the following:

A. Fiscal Illusion

The observed utility cost per tax dollar collected differs according to revenue source because of differential fiscal illusion. Some taxes are readily apparent to the payer and he is therefore highly conscious of paying them; others are less obvious, and indeed where the incidence is unknown or unclear, the taxpayer may have the illusion that he is shifting the tax when it is not shifted, or that the incidence is that of legislative intent when the tax is in fact shifted onto himself. By and large, one might predict that taxpayers are more conscious of taxes on the sources side than those on the uses side of real income, perhaps because the latter tend to be of more equivocal incidence. The company income tax may provide perhaps the best example of fiscal illusion: all groups apparently suspect it to be borne predominantly by someone else. Thus, one might well predict that a particular policy is more likely to receive electoral support if financed by an increase in company income tax rates than if financed by an increase in personal rates. More generally, a substitution involving the replacement of one tax by another which raises revenue less overtly involves a reduction in the apparent utility cost of public goods and hence one would predict an expansion in the size of the public sector. In the incidence context, such an expansion may be of no inherent interest, but since it will affect factor prices it is crucial in determining the ultimate incidence of the tax change.

B. Differential Tax Treatment of Public and Private Goods

Some taxes (such as a sales tax on private goods only) influence the demand for public goods directly via relative prices. This is so because they fall only on the private sector. Thus, a sales tax on private goods and services alone drives a wedge between product prices and factor returns in that sector alone, in which sense it can be analyzed (in the style of Harberger [5], Mieszkowski [9], etc.) as an excise tax levied in the private sector only. Likewise, the corporate income tax (even if completely general in the private sector) might be viewed as a tax on capital in the private sector alone. In a model of the sort suggested in section II, only the income tax seems general and even here there must be some doubt; the rate of return on invested capital in the public sector will be lower than that required in the private sector if, as seems arguable, the "double taxation" of savings is strictly a private
sector phenomenon. An excise tax on a private good will likewise have an incidence influenced according to the complement/substitute relationship between the taxed good and public goods.

C. Differential Income Effects

Any redistribution of income will, in general, be reflected in a change in aggregate demand for any particular good — public goods included, because income elasticities of demand will generally differ between individuals. In the public goods case, such effects are difficult to predict even if, as imagined here, preferences are more or less accurately revealed. Nevertheless, it is clear that since here median preferences are crucial, any reallocation of tax burden away from the majority to the minority at the margin will yield an increase in demand for public goods, since the cost share moves in the median voter's favour. Conversely, a redistribution at the margin from the majority in favour of the minority tends to reduce public goods supply. Such redistributions will be exaggerated or mitigated or simply complicated by attendant changes in factor prices.

D. Distortion Induced

According to Johnson & Pauly (1969), the welfare cost of the tax system is appropriately a cost to be set against public goods production, and on this basis a fully rational, perfectly informed median voter will, ceteris paribus, vote in a larger public sector the smaller the welfare cost induced by the taxes used. Although such effects may not in aggregate prove particularly significant, it is interesting to note that if public goods are labour intensive, labour would have a stronger interest in attaining efficiency in the tax structure than capital would; for the less distorting taxes are (ceteris paribus), the higher the level of public goods supply and hence the greater the factor price change (in labour's favour).

29. All of the above-mentioned influences, then, contribute to the determination of tax incidence in the appropriate "general equilibrium" model, in which the production of public goods enters as a determinant of factor prices. Those influences are not however normally taken into account in the incidence literature. What is really being claimed here is that, while considerable progress has been made in the application of general equilibrium techniques to incidence questions (by Harberger [5], Mieszkowski [9, 10], McLure [7], etc.), those treatments remain partial in one important sense — namely, the level of public expenditure is characteristically taken as fixed. Once it is accepted, however, that taxes represent the "pricing" arrangements for public expenditures, it does seem implausible to argue that changes in those arrangements will always leave the level of public expenditure unchanged. And if some relationship between the level of public expenditure and certain aspects of revenue-raising
can be specified, the way is then open for explicit study of the effects of tax changes on the distribution of income which incorporate the factor price effects wrought by changes in public goods supply.

V. SUMMARY

The objectives of this analysis have been threefold:

Firstly, to present a diagrammatic representation of a model of public goods supply in which factor prices, individual incomes and the optimal supply of public goods are simultaneously determined.

Secondly, to develop a model of public goods supply in which there is a systematic and predictable relationship between individuals’ preferences for public goods and fiscal outcomes, and to trace out in such a model the implications of factor price effects for those fiscal outcomes and for the operation of political processes more generally.

Thirdly, to indicate how the analysis of tax incidence might change when taxes are considered in a model in which public expenditure levels (and factor prices thereby) are allowed to adapt to tax changes.

NOTES

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1 Even by Musgrave himself; see for example Musgrave [12].

2 McLure’s “Expenditure Incidence” paper [8] looks at them in a slightly different way from that developed here.

3 Those not familiar with that treatment may find my discussion rather brief, though not I hope incomprehensible.

4 At C, \( \alpha \beta + \gamma \alpha = \alpha C \). Hence \( (OA - \alpha \beta ) + (OB - \alpha \gamma ) = (OT - \alpha C) \). Dividing both sides by \( OC \), the quantity of \( G \), we have \( \text{MRS}_I + \text{MRS}_II = MRT \), as required.

5 At \( P \), the corresponding point on \( JK \) is \( C' \). At \( C' \) it remains true that \( (OA - \alpha C') + (OB - \alpha \gamma ) = (OT - \alpha C') \) but \( OC' \) exceeds \( OR \), so \( (OT - \alpha C') < (OT - \alpha P) \) so \( (OA - \alpha C') + (OB - \alpha \gamma ) < (OT - \alpha P) \). Dividing both sides by \( OC' \) we have \( \text{MRS}_I + \text{MRS}_II < MRT \), as required.

6 For example where all the individual demand curves for \( G \) are linear and have the same slope: If \( a_i - b_i G \) is i’s demand curve then \( \Sigma a_i - b_i G = MC \) defines the optimal level of \( G \). The individual levels nominated will be those for which \( a_i - b_i G = \alpha_i MC \) where \( \alpha_i \) is the cost-share for \( i \). The mean of these is \( \frac{1}{n} \sum G_i = \frac{1}{n} \sum \frac{a_i - \alpha_i MC}{b_i} \). If the demand curves have the same slope, \( b_i \)

- for all \( i \), the mean is \( \frac{1}{n} \sum G_i = \frac{1}{n} \sum \frac{MC}{b} = \frac{1}{n} \), since \( \sum a_i = 1 \). This is also the optimal level of \( G, G' \).
The reasoning follows closely the line taken in Brennan and McGuire [1]. Such as those of Downs [4], and Buchanan & Tullock [3]. Although to the extent that public goods exhibit some inherently redistributive characteristics on the consumption side, effects of tax changes on the size of the public sector take on a distributional significance independent of those aspects raised in this paper. See Brennan "On the Distributional Implications of Public Goods", Econometrics (forthcoming) and comments by Aaron and McGuire for some relevant dispute.

REFERENCES

Summary: Public Goods and Factor Prices. — Modern public expenditure theory has certainly not ignored the distributional implications of public goods supply, but has focused generally on effects on the consumption side of individual incomes. This paper aims to explore the effects of public goods supply on the distribution of income via changes in factor prices (i.e. on the production or sources side). It does so in two steps. Firstly, it presents a diagrammatic exposition of a model of public goods supply in which factor prices, individual incomes and the optimal level of public goods supply are simultaneously determined — a sort of public-goods analogue to the familiar 2 x 2 general equilibrium private-goods model. This is not a general equilibrium model in the standard sense however, because, as is well known, there is no known market or political institution which translates the optimal level of public goods supply into an equilibrium level. The second phase of the paper involves, therefore, examining a political arrangement — conceivably the best feasible one — under which fiscal outcomes are specifically related to individual preferences; it
then proceeds to examine the implications of factor price changes due to public goods in the context of that political arrangement. Finally, there is a brief discussion of the implications of such factor price changes for the study of tax incidence, given that the choice of tax arrangements will typically influence the level of public goods supply.

Résumené: Biens publics et prix des facteurs. — La théorie moderne des dépenses publiques n’a certes pas ignoré les effets de distribution de l’offre de biens publics, mais a généralement mis l’accent sur les effets relatifs à la consommation des revenus individuels. Cette étude a pour but d’analyser les effets de l’offre de biens publics sur la distribution des revenus à travers les modifications de prix des facteurs (c’est-à-dire sur le côté production ou sources). Elle le fait en deux étapes. Tout d’abord, elle présente sous forme de diagramme un modèle de l’offre de biens publics dans lequel les prix de facteurs, les revenus individuels et le niveau optimal de l’offre de biens publics sont simultanément déterminés — un modèle de biens publics analogue au modèle bien connu de biens privés avec équilibre général par couple de biens. Ce n’est certes pas un modèle d’équilibre général au sens habituel du terme, car, comme on le sait bien, on ne connaît pas de marché ou d’institution politique capable de traduire le niveau optimal de l’offre de biens publics en un niveau d’équilibre. La seconde étape de cette étude en vient cependant à examiner une formule politique — la meilleure qu’il soit possible de concevoir — selon laquelle les versements fiscaux sont spécifiquement fondés sur les préférences individuelles, puis les implications des modifications des prix de facteurs dus aux biens publics dans le cadre de cette formule politique. Pour terminer, on discute brièvement des conséquences des changements dans les prix des facteurs en matière de répercussion fiscale, étant donné que le choix des dispositions fiscales influencera fortement le niveau de l’offre de biens publics.

5. PAPER THREE

ON THE DISTRIBUTIONAL IMPLICATIONS OF PUBLIC GOODS

Geoffrey Brennan*

1. One of the things that public finance practitioners attempt to do, from time to time, is to calculate the total effect of budgetary operations on the distribution of income.

The problems associated with this sort of undertaking are well-known, and generally recognized to be considerable. By no means the least of them revolves around the question of how best to allocate the benefits from public expenditure among the various consumer/taxpayers. Traditionally, the way this problem has been handled seems to have involved the setting out of a number of alternative allocations (equal sum per family, benefits proportional to income and so on) in the hope that the end result might turn out to be not too sensitive to this dimension of the total calculation— a hope which, almost invariably, proves unfounded.

2. At least at the conceptual level, however, the appropriate technique of allocating public goods benefits seems now to be established. In a recent *Econometrica* article, Henry Aaron and Martin McGuire (1970) purport to have provided "... the logically correct method" (1970), p 907; my italics) of determining the distributional impact of public goods supply in the calculation of income redistribution through the budget. Based on "... recent advances in the theory of public goods" (1970, p 907), the authors advance the proposition that "... the income value to each household of the public good equals the product of the marginal rate of substitution for that household between the public good and income, and the amount of the public good." (1970, p 909). Equivalently, the assertion is that 'Lindahl-type' pricing, under which each individual pays a marginal (and average) tax-price for the public good equal to the marginal benefit received, involves

* I am grateful to John Head and Martin McGuire for comments on earlier drafts.
zero redistribution in income-equivalent terms: under such a tax system "... each household pays out in taxes an amount equal to the value of the good received (just as in private markets)". (1970 p 909).

Accordingly, the redistributational impact of the budget can be calculated as a set of fiscal residuals, $F_i$, such that

$$F_i = G \cdot \text{MRS}_{i}^{i} - T_i$$

where $G$ is the level of public goods supply

$T_i$ is the tax paid by the $i$th household

and $\text{MRS}_{i}^{i}$ is the $i$th household's marginal rate of substitution between the public good and income.

Two features of this valuation procedure merit immediate comment.

In the first place, one striking characteristic of the technique is its extremely demanding informational requirements. The MRS'S appearing in the fiscal residuals (equation (1)) are, as Aaron & McGuire remind us, crucial in determining the extent of redistribution through the budget - yet are, given our present state of knowledge, unknown and perhaps even unknowable. Thus, the Aaron-McGuire paper may be seen as suggesting that, in the presence of public goods, the distribution of total income may actually be unspecifiable.

In the second place, it is clear that although not explicitly recognized by the authors, the Aaron-McGuire analysis involves a direct attack on traditional conceptions of the notion of "horizontal equity". If marginal evaluation pricing of the public good is genuinely neutral distributionally, then equal treatment of economic equals requires that individuals with identical private incomes pay not identical taxes (as has normally been thought) but appropriately different taxes regulated according to their tastes between public and private goods. Thus, given that revenues are to finance expenditure on public goods, the Haig-Simons approach to taxation emerges as utterly inappropriate on equity grounds alone: equal tax burdens for individuals with
identical private incomes would, if Aaron & McGuire are right, involve a redistribution of income-equivalents within that group from those with high demand for public goods to those with low demand.

Thus, the Aaron-McGuire technique has implications not only for assessing the distributional impact of the budget but also for a whole range of much broader questions associated with horizontal equity and the definition of the appropriate tax structure for equity purposes.

4. It is of course true - as Aaron & McGuire acknowledge - that their valuation procedure does not measure the full benefit to each household from public goods production, since it ignores the consumer surplus generated over infra-marginal units of the public good. Neglect of consumer surplus is, however, characteristic of "income-equivalent" measures, and it is such neglect that necessitates discussion of the distribution of income, as distinct from, say, the distribution of utilities. By implication, what Aaron and McGuire have sought to do through their valuation procedure is to make assumptions about consumer surplus which are analogous to those adopted in the private goods case. Using marginal valuations of the public goods to calculate income-equivalent benefits apparently seems to them to serve precisely this function.

The basic objective of this paper is to show that this is an illusion - that the use of marginal valuations to calculate income-equivalent benefits in the public goods case is inappropriate. More specifically I aim to demonstrate three things:

(i) that the Aaron-McGuire valuation procedure obscures certain inherently redistributive features of voluntary exchange arrangements in public goods supply;

(ii) that the most appropriate income-equivalent measure of public goods benefits is, ironically, probably the simplest and ostensibly most naive of those in current use - namely, the "equal-share-per-consumption-unit" rule;
and (iii) that the Haig-Simons approach to horizontal equity questions is, after all, the most appropriate even when public goods are explicitly included in the analysis.

SECTION I:

The first step in the line of reasoning involved here is to show that there is something inherent in the nature of public goods which is of redistributio

tal significance in the sense that application of marginal valuation pricing yields a substantively different distribution of utility gains (consumer surplus) in the public goods case than it does in an equivalent private goods case. In other words, MRS. q value involves a fundamentally different thing in distributional terms than it is quantities consumed by different individuals, rather than prices faced them, that are identical.

In order to demonstrate this, we take the simplest possible situation, in which there are just two individuals, A and B, and for which several of the simplifying assumptions that Aaron & McGuire themselves use are adopted - viz:

(a) that each public good is a pure public good (i.e. it is made equally and totally available to both A and B);

(b) that all marginal cost schedules are constant.

Suppose that, initially, there is just one private good, X; and suppose X distributed between A and B in such a way that $X_A = X_B$, i.e. A and B have the e income.

Now we introduce a second private good, Y. In fig 1, A's and B's preferences between X and Y are depicted, and are manifestly different. Both achieve higher difference curves through the introduction of Y - A's new equilibrium is at B, B's at F, and on B. But I am taking it as read that, in spite of the difference
Figure 1.
tastes for Y, A and B are recognized as still having equal incomes: the levels of X equivalent in income terms to $E_A$ and $E_B$ are the same, and equal to $X_A = X_B$.

Hence, we note that, given individuals' preferences between X and Y, equal incomes are associated with levels of utility for A and B given by $E_A$ on a and $E_B$ on b. respectively.

Now suppose that Y is not a private good, but is in fact a public good providing an exactly identical service. We let the price of this public good, G, be twice $P_Y$ so that there is rough compensation in cost terms for the extra utility possibilities necessarily involved in the equal and total consumption opportunity characteristic of public goods supply. Since the public good provides the same service as Y, the indifference maps remain the same: production possibilities, however, change and are now given by the line $TG$, where

$$OT = OX_A + OX_B \text{ (in fig 1).}$$

We note that the locus of possible "Lindahl solutions" for A based on $X_A$'s price-consumption curve for changing prices of G with $X_A$ fixed. Call this curve $Q'O$. The analogous curve for B is $P'P$. The Lindahl solution in our two-person world is, then, given at point R where the vertical sum of $P'P$ and $Q'O$ cuts $TG$. At this point, we have the familiar optimality condition

$$MRS_{XY}^A + MRS_{XY}^B = MRT.$$  

We note further that R must involve a level of Y between $E_A$ and $E_B$. For we note that, along the line segment QE, A's MRS exceeds that at $E_A$ (which equals the slope of $X_A$) and that B's MRS along PE exceeds that at $E_B$ (also equal to the slope of $X_A$). Hence to the left of $E_A$, we know that, since $E_A$ is to the left of $E_B$,

$$MRS_{XY}^A + MRS_{XY}^B > S(X_A, Y_A) + S(X_A, Y_A) = S(TG)$$

where S denotes the slope of the bracketed curve, defined to be positive.
Thus, the Pareto optimum based on $X_A$ and $X_B$ must lie to the right of $E_A$. By analogous reasoning, it must also lie to the left of $E_B$.

Finally, we observe that at $R$ and given Lindahl pricing, $A$ is better off than at $E_A$ and $B$ worse off than $E_B$. Yet $E_A$ and $E_B$ represent the positions achieved by $A$ and $B$ under a situation which, manifestly, left the distribution of income unchanged. Since preferences are unaffected by the move from the private to the public goods case, and marginal valuation pricing is used throughout, the sole difference between the two cases must be that in the private goods case the price of $Y$ is constrained to be identical to $A$ and $B$, and in the public goods case, quantity of $Y$ consumed by $A$ and $B$ is constrained to be the same.

Hence, the appropriate conclusion is this: public goods production in association with Lindahl pricing involves an inherently different distribution of the gains from trade than would prevail with an equivalent private good, priced equally to all consumers. Since the introduction of a private good at the same price to all leaves the income distribution (in income-equivalent terms) unchanged, this observation is sufficient to cast some doubt on the distributional neutrality of Lindahl pricing in the public goods context.

4. The same point can be made rather more simply (though perhaps less rigorously) by appeal to fig. 2. The lines $D_A^D_A$ and $D_B^D_B$ represent $A$'s and $B$'s demand curves for the public good, $G$. We know that a private good, $Y$, providing the same service as $G$ at half the per-unit cost, would (if priced identically at $p_Y = \frac{1}{2}p_G$ to both individuals) leave the income distribution in income equivalent terms unchanged. Under this arrangement, $A$ would consume $q_E$ of $G$, and $B$ would consume $q_K$. Under the Lindahl arrangement for $G$, however, both $A$ and $B$ would consume $q_G$, with $A$ paying a price $ON$, and $B$ a price $OC$.

Using areas under demand curves to estimate (roughly) the extent of the difference between the Lindahl arrangement and the income-equivalent private one,
figure 2
note that:

A gains area FEED by the Lindahl arrangement.
B loses area HCK.

Public goods production with strict benefit taxation is then much more favourable to A and much less favourable to B than the genuinely "income-equivalent" arrangement.

We should note that although the model has been set up in terms of individuals with identical incomes this is by no means necessary for the analysis. Fig 2 could equally well apply to a situation where B's demand for G exceeds A's demand because of income differences. Likewise, the analysis surrounding fig 1 easily be altered to allow \( x_A \) and \( x_B \) to differ. The Lindahl equilibrium would not necessarily lie between the individual equilibria for an equivalent private good, and the high demand individual would necessarily be made worse off and the low demand individual necessarily better off, than each was in the equivalent private goods case.

**HIOE II:**

Given that Lindahl pricing of the public good implies a different distribution of the "gains from trade" than would prevail under perfect market division of an equivalent private good, there are two questions that immediately arise:

firstly, can this difference be conveniently and satisfactorily depicted in "income-equivalent" terms?

secondly, should it be?

In my view, the answers to these questions are not independent, in that if the difference can be depicted in "income-equivalent" terms in a way that is obedient normal conventions, then it should be.
In part, this view springs from my concurrence with what I take to be a widely accepted opinion that the authority and significance of the income distribution as a policy relevant parameter arise because income is the best available index of the level of an individual's economic satisfactions (or utility). With the standard formulation of utility functions indicating a relationship between utility and quantities of goods consumed, the authority of income-equivalents in all-private goods world presumably derives from the fact that, given the same prices faced by all, an individual who has twice the income of another has the option to buy exactly twice as much of each good as that other. The fact that individuals' actual consumption patterns may be quite different indicates that all have been able, by reallocating expenditure from any arbitrarily given basket of goods, to secure extra consumer surplus - but this extra consumer surplus is customarily ignored in calculating income-equivalents', the implicit assumption being that it accrues to individuals more or less in proportion to their income levels. Applying the given set of prices (marginal valuations) to translate any expenditure package into any other, it is clear that an individual consumes (in income equivalents) exactly in proportion to his income.

We should note that with public goods present, it is not completely obvious that the analogous exercise yields similar results. Given that individuals consume some of the public good, it no longer seems quite so clear that the individual with twice the quantity of private goods is necessarily twice as well off: consumption of the public good is necessarily the same for all. And the higher the proportion of individual consumption patterns that public goods represent, the more similar those consumption patterns would seem to be.

In addition, however, my view that the utility redistribution implicit in endrial pricing should be taken into account in calculating the distribution of income equivalents springs from an antagonism to a proposition that seems embodied
in the Aaron-McGuire paper - namely, that there is something uniquely and
inherently authoritative about the use of marginal evaluations (or NRS's) in
calculating individuals' incomes. It seems to me that not only is this not so,
but also that this is a point already widely recognized in traditional incidence
literature. The following example indicates.

Suppose that a tax were imposed on commodity X, and an equal revenue
subsidy imposed on commodity Y. Changes in factor prices aside, traditional
incidence measures would clearly recognize a redistribution of income from consumers
of X to consumers of Y - and this is so in spite of the fact that prices for each
commodity are equal to marginal evaluations in the post-tax as well as in the pre-tax
situation. The same would apply if a tax were imposed on certain consumers of X
and paid to other consumers of X - each would adjust his consumption pattern to the
new set of prices, and for each, marginal evaluation pricing would again obtain. Yet
surely no-one would wish to claim that such an arrangement involves zero redistribution
of income. All the same, this is, it seems to me, precisely what Aaron and McGuire
are claiming in asserting the distributional neutrality of Lindahl pricing.

For consider some private good, Y, which is made available to consumers not
at an equal price to all but rather in a way more or less analogous to that in which
the public good is made available. Specifically, we impose the constraint that
consumption of Y must be the same for all consumers, i.e.

\[
\frac{1}{n} Y = Y_i = Y_j \text{ for all } i \text{ and } j
\]

where Y is total production of Y,

\[
Y_i \text{ is } i's \text{ consumption,}
\]

and n is the number of consumers in the community.

The optimum condition of production under such a constraint is

\[
\text{EMRS}_{YX}^i = n \cdot \text{NRT}_{YX}
\]  

(2)
Suppose that there is a public good $G$ providing an identical service to $Y$ such that

$$MC_G = n.MC_Y$$

The optimum condition for $Y$ in (2) can then be re-written as

$$\frac{MRS^i_{XY}}{i} = \frac{MR_T}{GT}$$

(3)

This is clearly the well-known optimality condition for a public good. Under differential pricing of $Y$ such that $p^i = MRS^i_{XY}$ for all $i$, the system of Lindahl pricing for $G$ is perfectly simulated: the distribution of consumer surpluses will be exactly the same.

The distributional significance of public goods production with Lindahl pricing is then identical with the distributional significance of this artificial constraint on the consumption of $Y$. Manifestly, since in the absence of this constraint each would pay the uniform price of $\frac{1}{n}MC_y (=MC_y)$, the individual is enjoying a per-unit subsidy of

$$\left(\frac{1}{n}MC_G - MRS^i_{CX}\right)$$

(4)

This is of course negative (a tax) if $MRS^i_{CX}$ exceeds $\frac{1}{n}MC_G$.

Thus, the implicit "income-equivalent" redistribution involved in providing public goods with Lindahl pricing is given as a set of subsidies (or taxes)

$$v^i = \frac{1}{n}MC_G - G.MRS^i_{CX}$$

(5)

We can generally, we can simply replace the second term on the right hand side of (5) by $t^i$ to allow for taxing schemes other than the Lindahl one. We can then obtain, a measure of budgetary incidence, a set of fiscal residuals

$$t^i = \frac{1}{n}EC_G - T^i$$

(6)

where $EC_G$ is the level of expenditure on public goods. Clearly, what this measure gives us is the allocation of public goods benefits on the basis of equal shares per consumption unit!
One further set of comments is perhaps desirable to avoid possible confusion.

Clearly, one implication of (6) is that equal lump-sum taxes for all would be distributionally neutral. Equally clearly such a claim is amenable to the same criticism which I have levelled at the Aaron-McGuire measure, namely that the distribution of consumer surpluses differs under such an arrangement from the private goods analogue. Such a criticism is justified, but not decisive. In fact, equation (6) represents an approximation, because of a problem in the choice of quantity weights. In calculating the value of the subsidy to A and its cost to B, I have been obedient to what I understand to be standard convention in income-equivalent analysis — namely, to calculate the value according to the (implicit) revenue transfer, where revenue is given as the subsidy (or tax) rate times the post-operation quantity taken. This involves ignoring the loss in consumer surplus of LEH for A and HLK for B in fig 2 — but this is customarily the case in income-equivalent studies. Technically, it would probably be more desirable to weight the subsidy (or tax) rate by an average of pre- and post-operation quantities (in fig 2, \( \frac{q_A + q_C}{2} \) for A, and \( \frac{q_K + q_C}{2} \) for B), but this would greatly complicate the measure of distributional impact given in equation (6), and would in any case be alien to the normal conventions of incidence discussions. On this basis, we can I think allow (6) to stand.

Finally, it needs to be emphasized that the analysis in no way depends on the actual existence of a private good Y providing an identical service to \( G \) at \( \frac{1}{n} \)th of the cost. This is merely a conceptual device introduced to indicate what the truly income-equivalent case would be, and to show the nature and extent of the difference between this and the case of Lindahl pricing for a public good.
SECTION III:

Some implications of this analysis are perhaps worth spelling out.

In the first place, we have already noted the extreme informational requirements of the Aaron-McGuire procedure. Given predominant ignorance of the SIS in equation (1), one might reasonably despair of ever being able to say much about the true distribution of total income. One of the happier implications of his analysis is that such despair is unnecessary. The measure suggested here is the incidental virtue, quite apart from its conceptual superiority, of being computationally trivial.

We have also noted the implicit attack on standard conceptions of horizontal equity, which is embodied in Aaron & McGuire's analysis. Given the stupidity of equation (6) however, it is clear that the Haig-Simons approach to taxation, under which individuals on identical private incomes pay identical taxes, is indeed treat those individuals equally; the truly distributionally neutral arrangement involves, precisely, the same cost share in financing public goods for - including those on the same incomes.3 To use Lindahl pricing would discriminate between individuals according to the intensity of their preferences for public goods. Terms of fig 1 equal shares would place A on A" in the public goods case, a B', and they would share more or less equally in the allocative cost of the consumption constraint which the public goods concept embodies. If due amount of this is taken in determining the price of G, so that \( p_G < 2p_Y \) by an appropriate amount, the allocative cost is removed, and any redistribution from the distributionally neutral (a, b) situation is minimized.

In the third place, to the extent that the distributional neutrality of Lindahl pricing appears as a crucial ingredient in the possibility of an equity-
iciency separation envisaged by the authors in their earlier theoretical manner (69), that separation emerges as unworkable. Of course, in so far as the McGuire-Aaron scheme purports to be a genuine separation applying to the general case, it has been discredited already. But interpreting their objectives as somewhat less ambitious, McGuire and Aaron could be understood as offering the possibility of an utility-efficiency separation in a context where the distribution is both specified in income-equivalent terms and determined in the light of the income-equivalent assumption possibilities (rather than the full utility possibilities including public goods). Even this extremely modest separation, however, is not possible on terms which McGuire and Aaron set out: the introduction of public goods with Lindahl pricing requires the distribution branch to know the optimal level of public goods supply before the "initial optimal" distribution of income can be determined.

In an environment where strict benefit taxation (or something even moderately close to it) applies, the choice between public and private technology appears to be of distributional significance. Requiring only that public goods demand be positively related to income, we can note that the poor will generally opt for a larger public sector (and the rich a smaller one) quite independently of any ongoing concern with profits on privately owned capital. Thus, in fig. 3, note that E is the initial point in utility space and G'G' the utility possibility frontier with the public good; then let G be the ultimate equilibrium on the basis of Lindahl pricing. On the basis of the foregoing analysis, we know that any private market arrangement will involve a distribution of trade gains more favourable and less favourable to A than in the public goods case given Lindahl pricing. Assuming for the sake of argument that what is distributionally neutral in income-equivalent terms is also distributionally neutral in utility terms, we can conclude that the private market arrangement give a utility distribution along the ray OE.
Then, clearly, B will prefer a less efficient private technology providing it gives rise to a utility possibilities frontier outside $F'T$ since the distribution of utility gains in the private goods case lies along the ray $E$. Likewise, A will prefer the public technology unless the private one is sufficiently more efficient to generate a utility possibilities frontier which lies outside $H'H'$. Thus, under most plausible pricing schemes, the poor will prefer the internalization of externalities which the rich would prefer to leave for the market to sort out. In this sense, the public good/private good distinction may be an important one in understanding "socialist" inclinations in those whose redistribution desires are strongest.

It should be emphasized, finally, that this paper is not to be construed any sort of case for the use of equal allocation of public expenditure between households when measures of the total utility gains (or consumer surpluses) for both private and public goods production are possible. It is simply that in most cases formational constraints require the calculation of redistribution through the target in the more modest terms of income equivalents. In these cases, the equal-allocation-per-household rule emerges as, at once, the simplest and most analytically satisfactory means of allocating public goods benefits.

Dalhousie University, Halifax

Australian National University, Canberra
Henry Aaron & Martin McGuire:

"Public Goods and Income Distribution" Econometrics Vol. 35 No. 6 November 1970 pp. 997-999

James M. Buchanan:

"The Economics of Socialism" Public Choice Vol. 8 Spring 1970 pp. 29-44

John G. Head:


Martin McGuire and Henry Aaron:

Such 'compensation' is rough in that utility possibilities are actually higher in the private goods case - so that for strict allocative equivalence $P_Y$ must slightly exceed $\frac{1}{2}NC_P$. (See Buchanan 1970) But the approximation is sufficiently close for the purposes of the argument at this stage. I discuss this problem in greater detail in footnote 2 below.

This artificial constraint also has some allocative significance - the equal consumption constraint, characteristic of the public goods arrangement, involves a net allocative loss (as Buchanan (1970) has emphasized). But this is in no sense crucial to my argument. Even if A's improvement is less in utility terms than B's loss (i.e. area $FBN$ exceeds area $FBN$ in fig 2) the basic distributional point stands: A is still made better off! If a strictly allocatively equivalent arrangement is considered, the price of the private good $Y$ needs to exceed $\frac{1}{2}NC_P$ somewhat. In such a case, B's loss is smaller and A's gain larger than the analysis here indicates, but this appears to have no real distributional significance. The break-up of the 'gains from trade' will still differ systematically from that prevailing in a case which I take to be universally regarded as distributionally neutral.

This is not to suggest that all should be taxed equally. Presumably we would not wish to treat economic unequals in a distributionally neutral fashion.

see head (1970).
Henry Aaron & Martin McGuire's reply [3] to my criticisms [1] of their earlier paper [2] is extremely difficult to respond to - not so much because I find their argument unanswerable, but rather because I find it difficult to discern precisely what their arguments are. The authors have not attempted to offer much in the way of further justification for their use of marginal rates of substitution in distributing public goods benefits - they have instead concentrated on trying to show that my criticisms are based "on a misperception of the issues." It seems to me that, on the contrary, the misperceptions are mainly theirs, and that their reply has merely served to obscure the central considerations.

In the first place, I cannot agree that what I have done is to present "... a persuasive case for taking account of the distributional consequences of new goods." Like Aaron & McGuire, I prefer for the purposes of this exercise not to redefine the distribution of income continuously when new goods are introduced.

Unlike Aaron & McGuire, however, I believe it to be absolutely crucial that we take into account the distributional effects of price changes. Indeed, I find it revealing that Aaron & McGuire seem to be prepared to ignore these, since much that is at stake in selecting alternative valuation
procedures for public goods turns precisely on the distributional effects of alternative pricing arrangements. Are Aaron & McGuire asserting that, by definition, all points along a price-consumption curve are "income-equivalent"? If so, our differences are indeed semantic; but I cannot see anything to be said for such a definition, and it is certainly not the one used on the tax side of budgetary operations. We do (and it is desirable that we should) take account of price changes in tax incidence analysis, and there is surely an overwhelming case for symmetry in dealing with the expenditure side.

Nor can I agree that our differences of opinion are to be explained in terms of either my incorrect construction of an income index, or my implicit cardinality assumptions. It seems to me to be misleading to suggest that these things are in any way central to the discussion.

In order to focus on these things which are, in my view, central, let us consider the Aaron-McGuire fig. 2. The question at issue is whether the positions indicated by [2] and C are income-equivalent. In order to show that, contrary to Aaron & McGuire's assertion, they are not, I introduced as a benchmark, situation [0]. I argued that, providing we are prepared to ignore the distributional implications of introducing the new good X (as Aaron & McGuire agree that we should), position [0] is recognized as income-equivalent to C: that is, the introduction of a private good, providing the same services as public good X and priced equally to both individuals,
would leave the distribution of income unchanged.\(^1\) Since position [2] is manifestly not income-equivalent to [0], it cannot be income-equivalent to C, and hence the asserted distributional neutrality of Lindahl pricing of \(X\) (position [2]) has to be rejected. Further, I argue that [0] is income-equivalent to [1], and that therefore evaluation of public goods at equal prices to all consumers is appropriate.

Aaron & McGuire's criticisms of this line of reasoning can now be interpreted alternately as follows:

(i) since position [0] offends the equal-consumption characteristic of public goods, it is infeasible and therefore irrelevant;

(ii) the position [1] is not income-equivalent to [0] because there is a welfare loss for both individuals in any move from [0] to [1].

The first of these criticisms can be disposed of fairly easily. As I attempted to make clear in my original piece, position [0] is only a conceptual device - a benchmark - used to indicate what situation would

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\(^1\) If this assumption is deemed to be objectionable, the distributional effects of introducing the new good can be incorporated. Firstly specify [0]. Then specify for A and B the average quantity of X consumed as a proportion of income and thereby determine a new point on each of A's and B's private consumption possibilities curves. This new point [3] would then surely be accepted as distributionally neutral in the sense that a new private good consumed by A and B in proportion to their incomes and priced equally to both would leave the distribution of income unchanged. The difference, somehow measured, between this point [3] and [0] for each individual would represent the redistribution involved in introducing the particular good, X. Then the difference in the "income" of A and B between [0] and [2] remains a measure of the distributional impact of Lindahl pricing. Of course, there are problems in assessing the redistribution from [3] to [0]; whether it involves a redistribution in favour of A or B seems to depend on which good (Y or 'private' X) is taken as numeraire. But these are quite different problems from the ones crucial in the balanced budget incidence context relevant here.
be income-equivalent to C, if it were feasible. Once the distributional equivalence (in income terms) of [0] and C is accepted, the problem becomes that of finding that position which is feasible (i.e. that satisfies the equal-consumption constraint) which is closest in distributional terms to [0]. As I have argued, it does seem to me that position [1] best fits that bill.

The second criticism is somewhat more complicated. For a start, I am taking it as given that Aaron & McGuire are agreed that the move from [0] to [2] involves a redistribution in favour of A and away from B. This emerges whether one uses a Laspeyre or Paasche index and whether one calculates the distributional effects of price changes with given quantity weights, or quantity changes with given price weights. Aaron & McGuire seem to prefer the latter procedure (i.e. to evaluate the move from [0] to [2] by weighting quantity changes by fixed prices). In doing so (whatever prices are used) they must find that the income change in moving from [0] to [2] is the same as the income change in moving from [0] to [1], for both A and B. But this would seem to imply that [2] and [1] are income-equivalent. In other words, Aaron & McGuire may be looked on as asserting that their valuation procedure and mine are the same. This is clearly quite silly, although logical enough if price changes are not taken to be distributionally relevant. If on the other hand, we evaluate the move from [0] to [2] in terms of a price change with fixed quantity weights (which quantities of X ought logically be those at [1] and [2] since they are the only feasible ones) then positions (0) and (1) emerge as income equivalent.
For example, in comparing the move from [0] to [2] with that from [1] to [2], a Paasche (post-operation weights) index would indicate the same income change (for either A or B); the income equivalence of [0] and [1] then follows, and the equal-value-per-consumption-unit rule is vindicated.

Of course, all that is being done here is to attempt to match the analysis normally used on the tax side. In evaluating moves from [0] to [2] in the tax context, positions like [1] are always regarded as income-equivalent to [0]; the welfare loss involved in the move from [0] to [1] is characteristically treated as an allocative (as distinct from a distributional) consideration, while the move from [1] to [2] is regarded as the incidence of the (tax-induced) price change.

Interestingly enough, Aaron & McGuire have not offered the one partial defence of their procedure which seems to make sense. This is that, while MRS valuations are not appropriate for the balanced budget incidence framework to which the original discussion and my comments have been addressed, they are appropriate in a differential expenditure incidence context.\(^2\) There, we wish to determine the distributional effects of replacing one expenditure package with another, while the tax structure (and total revenue) remain unchanged. In such a context, it does seem exactly appropriate to use the change in each individual's MRS multiplied by the quantity of public good provided (defined say as dollars of expenditure) as a measure of the income redistribution. For example, in fig 1, let \(D_A\) be A's demand curve, and \(D_B\) be B's, for an initial expenditure package of q dollars. If \(D_A'\) and \(D_B'\) are

\(^2\) I am indebted to John Head for this important insight.
A's and B's demand curves for an alternative expenditure package, also of q dollars, then we can say that, with fixed taxes, A receives an income increase of q \((MRS'_A - MRS_A)\) and B experiences an income reduction of q \((MRS'_B - MRS_B)\) when the new expenditure package is introduced. In the differential expenditure incidence context, then, marginal-rates-of-substitution values of public expenditure are quite proper: to use equal-value-per-consumption-unit throughout would fail to capture any redistribution at all.

Of course, in the balanced-budget incidence context, which is the one relevant for assessing the distributinal impact of the entire budget, the equal-value rule remains in my view appropriate. Why then the apparent inconsistency between valuation procedures in this and the differential expenditure incidence context? The answer is, I think, that whereas in the balanced-budget incidence context the distributional effects of introducing new goods are not fundamental, they do become crucial in examining the distributional consequences of replacing one set of public goods with another. If this is so, it does seem rather ironical that the one context in which the Aaron-McGuire valuation procedure is valid, is that involving precisely those distributional effects whose relevance Aaron & McGuire so determinedly deny.
   *Econometrica*

   *Econometrica* Nov. 1970 pp 907-920

PART II

PARETO-DESIRABLE REDISTRIBUTION

The effects of government activities on the distribution of income have always been a primary concern for public finance scholars and for social scientists more generally. The policy framework adopted by governments typically contains implicit value judgments about what distributional changes are desirable, as decisions over the level of public services, or over the level of income transfers, have a direct impact on income distribution. However, these decisions are often made in the context of broader economic and social concerns, and the trade-offs involved are complex and often contentious.

In almost all cases, however, the value judgments on the appropriateness of redistribution are based on principles of efficiency and equity. Efficiency generally involves the maximization of social welfare, where social welfare is defined in terms of aggregate consumption or utility. Equity, on the other hand, involves the fair distribution of resources, with a focus on reducing inequalities in income or wealth.

In practice, the possibility that the Pareto criterion, which states that more is better than less, may actually apply, has been recognized for some time. However, until recently, it has not been a possibility that many have seen as particularly relevant. A small group of persistent devotees (most notably Buchanan) has even gone so far as to argue that equality is a social norm, and that it is not necessarily desirable for redistribution to increase equality.

In fact, the possibility that the Pareto criterion, which states that more is better than less, may actually apply, has been recognized for some time. However, until recently, it has not been a possibility that many have seen as particularly relevant. A small group of persistent devotees (most notably Buchanan) has even gone so far as to argue that equality is a social norm, and that it is not necessarily desirable for redistribution to increase equality.
The effects of government activity on the distribution of income has always been a primary concern for public finance theorists and for applied economists more generally. The policy frameworks adopted by them typically contain explicit value judgements about what distributional changes are desirable, or at the very least permit scope for such judgements to be made. In almost all cases, however, the value judgements on distributional matters are quite distinct from those involved in assessing the efficiency implications of policies, and in some senses may be inconsistent with them. Thus, the possibility that transfer programmes designed to redistribute income from rich to poor may be justifiable on precisely the same grounds as public provision of defence, or law and order seems to have been rejected. Yet, such rejection is at odds with the observed tendency of at least some middle and upper income groups (including presumably public finance theorists) to vote upon themselves redistributive programmes designed explicitly to redistribute income away from themselves to others. It is inconsistent with any widespread electoral enthusiasm for anti-poverty programmes.

In fact, the possibility that the Pareto criterion, strictly applied, may actually require redistribution has been recognised for some time; but until recently it has not been a possibility that anyone has taken very seriously outside a small group of persistent devotees (most notably Buchanan). And even where the possibility was acknowledged, it has until recently received little in the way of detailed analytical scrutiny.
In the face of this, Harold Hochman and James Rodgers published a paper in the *American Economic Review* in September 1969 which has turned out to be of widespread interest, and has initiated extensive discussion of the Pareto-desirable redistribution possibility.

The first of the papers (paper four) in this section was originally conceived as a comment on the Hochman-Rodgers article. It focuses specifically on some analytical weaknesses in their exposition, and offers an alternative treatment of the n-person case which appears to be more satisfactory. In the process, however, some major conceptual problems in interpreting the significance of the Paretoan redistribution possibility emerge, and these are pointed up again in more detail in the "perspective" paper (paper seven). A subsequent rejoinder to Hochman & Rodgers' reply appears as an appendix to paper four.

The primary focus of the original Hochman & Rodgers' discussion, and indeed of the major swell of papers which have followed it, has been on the case of donor philanthropy: the possibility of income transfers yielding benefits to all taxpayer-donors arises because of the assumption of universal altruism over some range. But other donor motives are conceivable, and depending on one's tastes probably no less plausible. Papers five and six explore three such alternatives.

Paper five is, perhaps, not to be taken too seriously; but it is interesting to note that a case for universally beneficial transfers can arise in a world where all exhibit malice and envy towards their fellows, as well as in one where all is sweetness and light. The assumption that is important in this connexion is that of increasing marginal disutility of
other's income; this assumption seems to be a perfectly logical way of capturing the spirit of the 'envy' characteristic.

In paper six, donors are assumed to have motives for redistribution based, firstly, on the recognition that there is a risk that they too may at some stage fall on hard times (what I term the "insurance" motive), and secondly, on the desire to avoid the risk of political unrest and/or social violence. The transfer patterns which seem likely to flow from these motives are traced out, and some general discussion of the likely significance of these in justifying public transfer programmes is offered.

Paper seven is a more broad-ranging paper, designed both to assess the significance of Pareto-desirable redistribution in terms of its offering a viable alternative to more traditional formulations of distributional objectives, and also to indicate some of the analytic peculiarities that arise with the models of redistribution we are dealing with.

Because of an understandable (if not exactly laudable) tendency to overestimate the importance of one's own work, there is some overlap between the second part of paper seven (from section IV on) and the earlier papers.
2. PAPER FOUR
Pareto-Optimal Redistribution Reconsidered
from
Public Finance Quarterly April 1973
Pareto-Optimal Redistribution Reconsidered

• GEOFREY BRENNAN
  CLIFF WALSH

Volume 1 Number 2, April 1973
Two important models of "Pareto-optimal" redistribution (one proposed by Harold Hochman and James Rodgers, the other by George von Furstenberg and Dennis Mueller), while starting from similar formulations of the problem, generate substantially different patterns of interpersonal transfers—one involving a floor-level approach (i.e., only the poorest receive transfers), the other involving transfers from each individual to all others poorer than himself. On further investigation, it emerges that this difference can be explained in terms of the erroneous use by Hochman and Rodgers of their transfer-elasticity concept. Appropriately treated, both models generate transfers oriented toward raising the income floor, and, accepting the division of the gains from trade adopted in these models, this result, under most plausible sets of assumptions, seems to apply to the Paretoian approach in general. One of the objectives of our analysis is, however, to expose the extreme sensitivity of the ultimate transfer result to such gains-from-trade assumptions.

AUTHOR'S NOTE: At the time when an earlier draft was prepared, the first-mentioned author was a Visiting Research Fellow at Dalhousie University. With the usual proviso concerning responsibility for remaining errors, we wish to acknowledge a debt to Robert Gregory and to John Head for comments on earlier drafts.

PUBLIC FINANCE QUARTERLY / April 1973, Vol. 1 No. 2
In recent literature, considerable interest has been evinced in the question of Pareto-optimal redistribution. To a substantial extent, this interest has received its stimulus from two recent papers. One, the earlier, is by Harold Hochman and James Rodgers (1969) (hereafter H & R); the other is by George von Furstenberg and Dennis Mueller (1971) (hereafter F & M). In both papers, authors present analytical models in which income transfers beneficial to donors and recipients alike are exhibited and explained in terms of utility interdependencies arising from assumed altruistic attitudes on the part of the rich towards the poor.

In spite of the similarity of subject matter and approach, one conspicuous difference emerges. The H & R model generates interpersonal transfers between each individual and all those below him on the income scale, so that the final transfer pattern will in general involve individuals above the income floor being net recipients. By contrast, in the F & M model, each donor gives only to the poorest, so that only those at the floor level of income can receive transfers.

At first sight, there seems no reason why such divergence should excite surprise—after all, different models often yield different results. But when the generality of the H & R model is appreciated, and when it is noted that the H & R model cannot, under any meaningful values of its parameters, generate F & M transfer patterns, the difference becomes rather more puzzling. And since this is a matter of considerable distributional significance, it seems important to isolate the reasons for it.

A number of possibilities spring immediately to mind. F & M's detailed specification of the form of donors' utility functions may, for example, have some special (and unusual) features which give rise to correspondingly special forms of transfer—that is, it may simply be that F & M's is a peculiar, if interesting, case. At the same time, the H & R analysis is apparently susceptible to some criticism on purely analytic grounds. Meyer and Shipley (1970), for example, have recently pointed up some contradictions in the diagrammatic formulation which H & R use. Perhaps, then, H & R's transfer
patterns are, in fact, not logically consistent with the assumptions they make.

The objective of this discussion is to sort out these various matters. Basically, we shall be concerned with two questions:

(a) to what extent is the H & R treatment amenable to analytic criticism, and how devastating is it;

(b) to what extent is the F & M transfer pattern capable of generalization, and what restrictions on donors' (or recipients') preferences are necessary to ensure the floor-level-of-income approach?

As it turns out, these apparently distinct questions are related. For it becomes apparent on examination that the H & R analysis is indeed faulty, and that, when appropriately formulated, it does give precisely the variety of transfer pattern which F & M isolate.

The argument designed to justify this claim is organized as follows. The first section contains a summary restatement of H & R's two-person analysis with the aid of a simple diagrammatic exposition. This both serves as a convenient point of departure and provides an opportunity to indicate the unimportance of the Meyer-Shipley criticism. In the second section, we critically examine H & R's use of the transfer-elasticity concept by which they extend their analysis to the n-person case in the light of the apparent conflict between H & R's results and those derived by F & M. We attempt to indicate there the inadequacy of this extension technique, and the correspondingly suspect nature of the results derived with its use. In order to avoid the extension problem, we introduce in the third section an alternative analysis of the n-person case, which, although perhaps more difficult to handle than H & R's, is at least analytically explicit and moderately rigorous, and which preserves the crucial generality of the H & R model. Here the focus shifts to the F & M treatment, because the alternative analysis serves to indicate that, under extremely general assumptions (ones which might, in fact, easily be inferred from H & R's discussion), F & M-type floor-level transfer patterns are assured.
In the fourth section, we direct attention to certain important dimensions of the Pareto-optimal redistribution possibility substantially ignored by both contributions; and the last section draws together the appropriate conclusions.

THE TWO-PERSON CASE

(1) Following H & R's analysis, we begin by considering a world consisting of two individuals, M and J, with incomes $Y_M$ and $Y_J$ respectively, and utility functions:

$$U_M = U_M(Y_M, Y_J)$$  \[1\]

$$U_J = U_J(Y_J, Y_M)$$  \[2\]

Attention is restricted to transfers which satisfy the Pareto criterion (i.e., harm neither M nor J) and the utility functions are assumed to embody the following restrictions:

(a) $\frac{\partial U_i}{\partial Y_i} > 0$ \quad \text{i = J, M;}

(b) $MRS_{Y_J Y_M}^i \geq 0$ \quad \text{(i, J, M) for all $Y_J, Y_M$}

which establishes the potential for redistribution

(c) $MRS_{Y_M Y_J}^i < 1$ \quad \text{for $Y_M > Y_J$}

and $MRS_{Y_J Y_M}^M < 1$ \quad \text{for $Y_J > Y_M$}

so that no transfers can be made from poor to rich, and the poor obtain unambiguous benefit from any redistribution in their favor. Further, this ensures that mutually beneficial transfers do not reverse the initial distributional ordering; and

(d) individuals have identical utility functions.

These assumptions are given geometric substance in Figure 1. M’s income is measured on the vertical axis, and J’s on the
horizontal. The budget line for both is AB and is determined by the total (or national) income \((OA = OB = Y_M + Y_J)\), having a slope of \(-1\) since the objective cost of a unit transfer for either individual is a unit reduction in own income. Indifference curves labelled \(I_M\) refer to M's preferences between his own and J's income: they are convex to the origin, and attain a slope of \(-1\) on or above the 45° line OZ. The indifference curves labelled \(I_J\) refer to J's preferences and have similar properties to M's.\(^3\)

The demonstration of the possibility of Pareto-optimal transfers follows fairly simply. Suppose the initial distribution
of income lies somewhere on the line segment AZ—say, at point C. Clearly, M can achieve higher levels of indifference by making transfers to J (M might move from $I_M$ to $I_M$, for example). Now, J is made unambiguously better off by any such transfers which are beneficial to M, so that the extent of mutually beneficial transfers is restricted by M’s preferences alone. If, in keeping with H & R, we allow transfers to be solely a matter of M’s volition, the optimal transfer would then be $Y_F - Y_J$, involving a redistribution to point F. At this point, $MRS_M^J = 1$. Both parties are made unequivocally better off by the move from C to F and the existence of Pareto-optimal redistributions is established.

(2) This diagram differs from H & R’s own in that they attempt a transformation of Figure 1 (which is obedient to their algebraic formulation) into the transfer/differential space (mapping $(Y_M^0 - Y_M^J)$ against $(Y_J^0 - Y_J^F)$). For such a transformation to be possible, the indifference map must be such that outward shifts of AB in Figure 1 generate on “income-consumption” path through F parallel to OZ, and the Meyer-Shipley criticism is based on the problems associated with such a constraint in the light of H & R’s analysis.

Clearly, though, by adhering to the graphical depiction set out in Figure 1 and ignoring the transfer/differential space, any pattern of “income effects” (and, hence, IC lines in H & R’s terms) in M’s preference map can be depicted and analyzed. Thus, the H & R treatment can be resurrected fairly painlessly, in which sense the Meyer-Shipley criticism emerges as, at best, not very serious.

THE N-PERSON CASE

(1) From the viewpoint of policy prescription, however, interest centers around the extension of the two-person analysis to the n-person case and, adding the assumption of no strategic behavior, H & R effect this extension through the development of the transfer-elasticity concept.
M's transfer-elasticity ($E_M$) is defined as the elasticity of his transfer demand ($T$) with respect to a change in the initial income differential. Taking his differential with $J$ to illustrate:

$$E_M = \frac{dT}{d(Y_M - Y_J)} \cdot \frac{(Y_M - Y_J)}{T} \tag{3}$$

For different assumptions about the value of $E_M$ and the shape of the size distribution of income, H & R derive allegedly Pareto-optimal patterns of redistribution in three steps. First, for every individual, the initial differential between his income and that of all those poorer than himself is calculated; second, the assumed value of $E_M$ is used to estimate, for each calculated initial differential, the desired transfer; and, finally, the transfers received and given by any individual are subtracted to obtain his fiscal residual, yielding the patterns of net transfers exhibited in H & R's Tables 3 through 5 (Hochman and Rodgers, 1969: 550-552).

(2) It is at this stage profitable to contrast the H & R model described so far with that adopted by F & M. In that latter model, individuals are partitioned into $n$ groups—1, 2, . . . , $n$—and for each individual $i$ a utility function is postulated which, with $a > b > 0$, takes the form

$$U_i = Y_i^{a-b} (Y_1, Y_2, . . . , Y_{i-1})^b \tag{4}$$

Clearly, the incomes of all those poorer than $i$ enter positively into $i$'s utility—and it is equally clear that this is simply one of a class of utility functions all of which satisfy the H & R restrictions set out in a through d above.

The derivation of transfer patterns based on a floor level of income follows immediately. We note that, differentiating equation 4 with respect to $Y_k$, we have:

$$\frac{\partial U_i}{\partial Y_k} = b \frac{U_i}{Y_k} \text{ for } k < i \tag{5}$$
which is clearly maximal when $Y_k$ is minimal. In other words, gains most from transfers to the poorest throughout the range, and rational behavior dictates giving to that group and it alone.

We should note, though, that while the F & M utility formulation is obedient to restrictions used in H & R's analysis, there is no finite value of H & R's "transfer elasticity" which can generate floor-level transfer patterns. The H & R treatment of the n-person case seems constrained to ensure positive net transfers for some groups above the income floor. There is clearly some need to reconcile this apparent inconsistency.

Our view is that this inconsistency arises out of H & R's use of the transfer-elasticity concept to extend their analysis to the n-person case: this approach, with its simple bilateral comparisons, involves a partial framework in a context where only a general framework with simultaneous multilateral comparisons is adequate. The basic problem is that the H & R extension technique necessarily involves the assumption that M's desired transfer to any one individual is independent of his transfers to others. In fact, however, in making a transfer to one individual, M necessarily alters the differential between his own income and the incomes of all those others to whom he might also wish to make transfers: this will, in general, influence the size of the transfers he wishes to make to those others. This observation involves much more than a criticism of the description which H & R offer of the transfer process—it contains the elements of a serious challenge to the qualitative structure of their so-called Pareto-optimal transfer patterns. In particular, one possibility which clearly might arise is that the process of making transfers to one individual may completely exhaust the initial desire to give to others—in which case, the poorest individual might well turn out to be the sole recipient of any transfers made.

(3) In order to elucidate further, let us focus on the three-person case. This is sufficient to depict the qualitative nature of the n-person case and is used for its relative simplicity. There are three individuals M, J, and K, with initial incomes $Y_M^0$, $Y_J^0$, and $Y_K^0$ respectively such that
We assume that J desires to make no transfers, and that at the margin (though not necessarily in the initial situation) K likewise desires to make no transfers. Then we can focus on M's preferences in determining optimal redistribution. Clearly, from M's viewpoint, the optimal distribution of national income \((Y_M^0, Y_K^0, Y_J^0)\) is described by the point at which his indifference bowl in \((Y_M, Y_K, Y_J)\) space is tangential to the income possibilities surface. If this tangency occurs at some point other than \((Y_M^0, Y_K^0, Y_J^0)\), there may be transfers from M which make everyone better off.

This situation can, under the assumption of separability, be depicted in two-dimensional terms by taking in turn different perspectives of the three-dimensional situation, and setting them together as in Figure 2. The initial income vector \((Y_M^0, Y_K^0, Y_J^0)\) determines the positions of the "income" lines AA', BB', CC', and the initial distribution points L, N, and P on those lines; and the indifference curves shown in each quadrant are different perspectives of M's total indifference surfaces. Thus, the southeast quadrant depicts M's preferences between income for own use and J's income; while the northwest quadrant depicts his preferences between income for own use and K's income. The northeast quadrant depicts M's preferences between \(Y_J\) and \(Y_K\).

As can be seen, M desires initially to give to both J and K—for in the southeast quadrant he desires to achieve point G from P involving a transfer to J of PX; and in the northwest quadrant, he desires to achieve F from N, involving a transfer to K of NY. But one cannot conclude from this initial situation that M will, in fact, give to both, because he cannot give to J without affecting his position vis-à-vis K.

As M gives to J, effecting a movement along CC' from left to right, \(Y_J\) is clearly increasing, and, hence, in the northeast quadrant A'A moves out from O, and L moves to the right, tracing out a horizontal path LL'. At the same time, \(Y_M\) is clearly decreasing, and hence in the northwest quadrant B'B is moving in toward O, and N traces out a horizontal path NN' toward the horizontal axis OY_K.
As M gives to J, P moves toward G (making M better off), L moves outward (making M better off), and N moves inward (making M worse off). Given normal indifference surfaces, it is clear that as N moves inward toward N', F will in general move downward toward F' and at some point between N and N', F may move below NN'.
The crucial point is, of course, that the mere act of giving to J can exhaust M's desire to give to K. Hence, a simple examination of initial differentials and M's concomitant transfer desires in bilateral terms (as depicted in Figure 1, and the northwest and southeast quadrants of Figure 2) cannot determine the optimal transfer pattern, or the appropriate set of fiscal residuals.

It should perhaps be noted in passing that M's giving to J can also exhaust K's desire to give to J, even if K desires to make transfers to J in the initial situation. No demonstration of this obvious result seems necessary, and, in any case, the basic point is quite familiar from existing literature: M's transfers to J may generate strictly inframarginal external benefits for K. It may be worth emphasizing that the situation outlined above seems to depend on considerations quite different analytically from the marginal-inframarginal externality distinction.

(4) Although the diagrammatic presentation (Figure 2) is convenient for showing up the weaknesses of the H & R approach, it is a rather clumsy tool for showing the nature and position of the total equilibrium, and hence for deriving the optimal transfer pattern. The reasons for this are twofold: in the first place, the precise relationship between the indifference maps in the various quadrants is rather difficult to specify, and in the second, the calculation of the total optimum seems to require an assessment of distances between successive indifference curves in the different dimensions. Such distances are not amenable to rigorous analysis.

Thus, any attempt to calculate the correct pattern of transfers and fiscal residuals demands the use of a different technique. Accordingly, we have indicated in what follows an alternative (algebraic) formulation of the problem, by which the total "optimum" can be derived and examined. As before, we focus on the three-person case, since this seems sufficient to indicate the nature of the n-person situation.
AN ALTERNATIVE MODEL

(1) There are three individuals, M, K, and J, with initial incomes \( Y_M, Y_K, \) and \( Y_J \) such that
\[
Y_M^0 > Y_K^0 > Y_J^0
\]
and with utility function
\[
U_M = U_M(Y_M, Y_K, Y_J) \quad [7]
\]
\[
U_K = U_K(Y_K, Y_J, Y_M) \quad [8]
\]
\[
U_J = U_J(Y_J, Y_K, Y_M) \quad [9]
\]
in which
\[
\frac{\partial U_J}{\partial Y_J} > \frac{\partial U_J}{\partial Y_K} > \frac{\partial U_J}{\partial Y_M} > 0 \quad \text{and} \quad \frac{\partial U_K}{\partial Y_K} > \frac{\partial U_K}{\partial Y_M} > 0. \quad [10]
\]
This allows the possibility that K may desire to give to J, although J does not wish to give to either M or K. In fact, however, we suppose initially that K has no Pareto-relevant demand for transfers to J—the multiple-donor case has already been elegantly handled by F & M, and in any case our prime concern is with the multiple recipient case. Accordingly, we focus on M's preferences and, as in the two-person case, allow M to extract his maximum benefit from transfers by explicitly precluding "strategic behavior" on the part of any individual.

M, in attempting to achieve his utility maximum, aims for a distribution of income such that
\[
\frac{\partial U_M}{\partial Y_M} = \frac{\partial U_M}{\partial Y_K} = \frac{\partial U_M}{\partial Y_J} \quad [11]
\]
So much is standard analysis. What is analytically peculiar in this case is that M may not be able to achieve this equalization. The reason for this is that he is constrained in his ability to reallocate funds between \( Y_J \) and \( Y_K \), and, in this sense, M's decision calculus is essentially different from that applying in the case where he is faced with a problem of allocating income between three goods. Here, M is obliged to consume minimum
quantities of \( Y_J \) and \( Y_K \)--and one (or both) of these may already be too large for M's utility maximum. He is thus faced with an interesting and unusual problem of "second-best" optimization: equation 11 describes M's objective only in an unconstrained state.

(2) Equation 11 does, however, provide a convenient point of departure. To translate it into a specific pattern of redistribution, further restrictions on M's utility function must be imposed. Accordingly, we assume:

(a) that \( Y_K \) and \( Y_J \) enter separably into M's utility function, so that M's demand for transfers to J is not directly affected by \( Y_K \) (or vice versa);
(b) that M's preferences over J and K exhibit the characteristics of "horizontal equity."

The first assumption is made for analytic convenience, and requires little justification. The second is clearly of a different order, but it seems an appealing one to us for a number of reasons:

(1) given more traditional discussions of the "equity" concept, it strikes us as being a moderately interesting assumption;
(2) it has often been asserted that horizontal equity represents a noncontentious dimension in redistributitional questions, in which case it seems reasonable to assume that M's preferences will embody this notion; \(^6\)
(3) only if horizontal equity applies is it possible to treat individuals and groups of individuals with the same incomes interchangeably. Since both H & R and F & M do this in applying their models to U.S. data, the assumption is implied in both discussions. (Indeed, it is difficult to see how any empirical results at all could flow from such models without horizontal equity obtaining.) In this sense, we are simply following out the implications of the earlier papers;
(4) if horizontal equity does not obtain, M must have a systematic preference for one or other of the individuals unrelated to their
Although not apparently exceptional, the horizontal equity assumption turns out to be quite powerful, and its implications far-reaching. Its significance can be seen in two ways. Diagrammatically, horizontal equity can be indicated by the properties of M’s indifference map in $Y_K/Y_J$ space (the northeast quadrant of Figure 2). The indifference curves must, in the first place, be symmetric about a $45^\circ$ ray through the origin, since any other form would indicate that M’s preferences were not independent of labels (that is, that M had a systematic preference for one individual). Second, they must be convex throughout the entire range, to ensure that when $Y_K$ and $Y_J$ are identical, it is utility-maximizing for M to treat them identically. Figure 2 is in fact drawn to exhibit the relevant characteristics. Clearly, from Figure 2,

$$\frac{\partial U_M}{\partial Y_K} < \frac{\partial U_M}{\partial Y_J} \quad \text{for} \quad Y_J < Y_K$$

and

$$\frac{\partial U_M}{\partial Y_K} > \frac{\partial U_M}{\partial Y_J} \quad \text{for} \quad Y_J > Y_K$$

(i.e., the indifference curve has a slope steeper than $A'A$ above the $45^\circ$ ray through 0, and is less steep above that ray). It follows that

$$\frac{\partial U_M}{\partial Y_K} = \frac{\partial U_M}{\partial Y_J} \quad \text{if and only if} \quad Y_M = Y_K$$

Condition 11 translates, thereby, into the requirement that

$$Y_M > Y_K = Y_J$$

The same point can be derived directly by noting that assumptions a and b above ensure that
\[ \frac{\partial U_M}{\partial Y_i} = f(Y_M, Y_j) \quad \text{for } i = J, K \]

where \( f \) is the same function for both \( J \) and \( K \). Further, we know from Figure 1 (and common sense) that

\[ \frac{\partial^2 U_M}{\partial Y_i^2} < 0 \]

so that \( f(Y_M, Y_i) \) is greater for whichever \( Y_i \) is lower, and can only take identical values for \( J \) and \( K \) when \( Y_J \) and \( Y_K \) are equal. Equation 13 follows directly.

The implication of the horizontal equity assumption in this context is, then, that \( M \)'s desired income distribution is one in which the incomes of all those poorer than himself are equalized, at some level below his own. Clearly, this is a case in which \( F \& M \)'s floor-level-of-income transfer pattern would prevail.

(3) We cannot, however, immediately conclude that horizontal equity will imply a floor-level approach more generally, because, as we have emphasized, equation 13 (following from 11) is only a depiction of \( M \)'s desired state in unconstrained circumstances, not of the ultimate optimum. And this is so for two reasons:

1. even though the initiative for redistribution is assumed to come from \( M \), optimality may require \( K \)'s sharing in the cost of transfers;
2. \( Y^0_K \) (and \( Y^0_I \) for that matter) may initially exceed the level \( M \) desires, and since \( M \) cannot force anyone to reduce his income, \( M \)'s ideal distribution may remain unattainable.

(4) The remainder of this section is designed to show that these complications do not, in fact, influence the floor-level implications of the horizontal equity assumption. The results follow fairly immediately.

In the multiple donor case, given Lindahl pricing and identical donor preferences (though not, of course, identical
incomes) we know that no one is made worse off by transfers and, hence, that all cost-shares lie between zero and one. Even if K receives "transfers" from M in this case, they cannot serve to increase \( Y_K \) — they are simply bribes to induce K to give to J.

In the case of multiple-potential recipients, the prospect that \( Y_K \) may exceed M's desired level for \( Y_K \) (as determined in 13 somewhat complicates M's decision calculus in that transfers to J result in a situation where

\[
\frac{\partial U_M}{\partial Y_M} = \frac{\partial U_M}{\partial Y_K}
\]

while

\[
\frac{\partial U_M}{\partial Y_M} < \frac{\partial U_M}{\partial Y_J}
\]

still obtains. In this case, further transfers from M to J benefit M in that the differential \( (Y_M - Y_J) \) is reduced, but reduce M's utility in that the differential \( (Y_M - Y_K) \) is reduced. M will continue to make transfers to J until the benefit equals the loss, at which point in general

\[
Y_M > Y_K > Y_J.
\]

But nowhere in this process will M make transfers to K. Hence, the floor level of income approach still obtains.

(5) So much, then, for the three-person case. When the number of individuals is increased, the pattern of fiscal residuals can become rather more complicated, but one important generalization can be made—no one can receive positive net transfers as a result of the redistributional process while his income exceeds somebody else's. The set of n individuals can then be partitioned into three groups:

(a) the relatively rich who make transfers, and have negative fiscal residuals;
(b) some middle-income individuals who neither give nor receive, and, hence, have zero fiscal residuals (this set may be empty); and
(c) the poor who receive transfers and have positive fiscal residuals and who have identical posttransfer incomes.

The pattern which emerges is one in which redistributional policy is directed toward raising the floor level of income, with the cost of the program allocated (according to marginal evaluations of transfer) among those whose incomes lie above that floor level.

This is, of course, precisely the transfer pattern isolated by F & M—and the conclusion to be drawn from this analysis is that the F & M results are capable of extensive generalization. All that seems to be required is the assumption of “horizontal equity” in donors’ preferences. And what needs to be emphasized is the extreme weakness of this assumption; any model of redistribution within the Paretian framework which seeks to explain transfers in terms of income levels alone must imply horizontal equity and hence the floor-level-of-income approach.

Since the H & R model fits this class, the analysis contained therein emerges as demonstrably unsatisfactory; the transfer pattern they derive, in which groups with differing posttransfer incomes can have positive fiscal residuals, is inconsistent with the assumptions they make. Their errors arise, we believe, because their transfer elasticity concept, as defined, is inherently inadequate to extend the two-person to the n-person case.

SOME FURTHER PROBLEMS

One further point should be made concerning the assumptions of our model. We use (as both H & R and F & M do, the latter implicitly) the assumption of “no strategic behavior” in order to delineate a unique set of transfers from the large number of Pareto-optimal ones.

Thus, in Figure 1, we have focused on the point F, because if we derive M’s marginal evaluation curve for transfers, by taking
his marginal rate of substitution between $Y_M$ and $Y_J$ for various points along the consumption possibilities line AB, then F represents the unique point for which that marginal evaluation equals the (constant) marginal cost of transfer. However, it is clearly possible to mulct M's consumer-surplus and pay its money value to J without interfering with the marginal conditions, thus affecting a move to some point on the line segment FS. It is obvious from Figure 1 that any posttransfer distribution on the segment FS involves a Pareto-optimal redistribution from C. The point F corresponds, in fact, to a situation of minimum redistribution, with M extracting his maximum gains from trade, while S corresponds to a situation of maximum redistribution, with J obtaining all the gains from trade. In terms of the Pareto framework itself, F has no more authority than any other point in the Pareto-optimal range – S would do just as well.

Likewise, in the three-person case, where both K and M potentially desire to give to J (and transfers to J are hence potentially a traditional public good), we have assumed that K and M pay for transfers according to their marginal evaluations. (Thus, if K has a zero marginal evaluation, he contributes nothing, even if he had desired to give to J initially.) As in the two-person case, donors retain their maximum consumer surplus from the transfer process. It is this assumption that permits us to concentrate on M's preferences in the three-person analysis.

Relaxation of this assumption clearly results in considerable changes in the nature of the outcome. If all possible utility gains were, for example, to accrue to K, rather different transfer patterns will result. All of these patterns would, however, be Pareto-optimal, providing no one was made worse off than he was in the initial situation.

The function of the “no strategic behavior” assumption in the analysis is to determine the distribution of the total utility gains which issue from the transfer process. The assumption is made essentially for analytical convenience. It has no particular justification in positive terms, because there seems to be no
good reason why any government (even one wedded to the notion of a Pareto optimum) should select F (in Figure 1) out of the infinite number of Pareto-optimal redistributions. Nor in terms of the Paretian framework does this assumption about the distribution of utility gains have any normative justification. Yet, and this needs to be emphasized, the final distributional result is extremely sensitive to changes in the distribution of utility gains. If, for example, the indifference maps in Figure 1 are symmetric about the income-consumption line, S involves precisely twice as much transfer (i.e., twice as high a floor level of income) as F does. Similarly, in the three-person case, K might receive almost as much transfer as J does if M's consumer-surplus is paid entirely to K.

In other words, both the H & R and F & M models are involved in making arbitrary and distributionally crucial assumptions which remain implicit throughout the analysis. For this reason, we do not claim for the Pareto-optimal redistribution possibility, as, for example, H & R do, that it provides the basis for a synthesis of the allocative and redistributive functions of government. Indeed, it is precisely because the Paretian framework is agnostic over a possibly large range of distributional issues (those concerned with the division of utility gains) that we believe such a synthesis to be impossible! In no sense is the present analysis to be viewed as an alternative to the more traditional treatments of distributional issues: "external norms" are still required.9

Rather, the significance of what we (and, by implication, H & R) have done lies in the provision of an analysis which considers the question of an "efficient output" of a rather unusual produce—namely, income transfers. The justification for the endeavor lies in the dual facts that income transfers constitute a commodity of particular interest, and that the analytics of the case are rather unusual—not at all in the hope that the tricky normative decisions involved in distributional policy may thereby be avoided.
CONCLUSION

Recent papers by H & R and F & M have succeeded in demonstrating that income redistribution can be required by the Pareto criterion. The conspicuous differences in transfer patterns derived, however, would suggest that not much can be said about the actual redistributions likely to issue from the Paretian approach. Such a conclusion is indeed justified, but for reasons entirely other than those contributing to the differences in results derived by H & R and F & M. For while the ultimate distributional result is extremely sensitive to differences in assumptions about the division of gains from “trade,” F & M and H & R are involved in making similar assumptions in that dimension. Rather, F & M’s results diverge from H & R’s, because the latter use an inadequate technique to extend their two-person model to the n-person case. Their transfer-elasticity concept is a “partial” tool, holding constant things which, by the nature of the exercise, must change. When appropriately corrected, the H & R analysis yields patterns of transfer essentially identical to F & M’s—namely, transfers directed at raising the income floor. Indeed, our analysis serves to suggest that any model which seeks to explain redistribution within the Paretian framework on the basis of donor altruism, and which relates transfers given and received to income levels only, must (under standard assumptions about the division of gains from trade) result in a transfer pattern which exhibits the floor-level-of-income approach.

Of course, transfer patterns like H & R’s could almost certainly be produced under some (possibly macabre) set of assumptions concerning the distribution of utility gains from the redistributive process. But this must be small comfort to H & R, for it is impossible to rescue their transfer result without exposing the extreme distributional significance of such assumptions. Once this point is noted, it becomes apparent that the Pareto criterion alone cannot choose between the extensive variety of Pareto-optimal redistributions corresponding to any initial (nonoptimal) distribution of income. For this reason, the Pareto-optimal redistribution possibility in no way obviates the
need for appeal to externally determined distributional judgments. It is difficult to see, therefore, how H & R’s attractive (if surprising) claim to have provided a synthesis of the distributional and allocative objectives of government can be sustained. How can the goal of vertical equity be contained within the Paretian concept of efficiency when the latter framework is consonant with redistributonal patterns which vary so conspicuously in nature and magnitude?

NOTES

1. Other comments, bearing on different aspects of the paper, have been offered by Musgrave (1970) and Goldfarb (1970). Replies to all comments can be found in Hochman and Rodgers (1970).

2. This list of properties contains all the assumptions and restrictions imposed by Hochman and Rodgers on their two-person analysis. A detailed explanation can be obtained from Hochman and Rodgers (1969: 543-545), while our Figure I illustrates their significance.

3. Specifically, J’s indifference curves are mirror images of M’s with OZ the axis of symmetry (following from restriction d, above). It is stressed that this assumption of identical utility functions is stronger than necessary, but it is the one made by H & R, and it does help to simplify the n-person case.

4. They are, of course, related by virtue of their mutual connection with the total indifference bowls.

5. If both were, no transfers would result.

6. Musgrave (1970: 160) refers to horizontal equity as “perhaps the most widely accepted principle of equity in taxation.”

7. Symmetry would, for example, be consistent with indifference curves which have a local minimum along the 45° ray from 0.

8. It is not possible, in the case considered here, for M to obtain all the gains from trade, because the “trade” involves increasing YJ. From assumption b in section I, J values the increase in his own income more than he regrets the reduction in YM. This is not to say that traditional treatments of redistribution are perfect by any means, but the point needs to be emphasized that the goal of economic efficiency cannot of itself generate a unique point on the utility-possibilities frontier (or, in general, the production-possibilities surface) without distributional specifications.

9. It is perhaps worth pointing out that a similar problem arises in treatment of “equity” as a merit good (see Head, 1966) for, if we assign a shadow price to equity, incorporating an allowance for “irrationality” and “imperfect knowledge,” we are still left with the question of how to divide up the appropriately “corrected” utility gains.
REFERENCES


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3. APPENDIX TO PAPER FOUR

Hochman & Rodgers on Brennan & Walsh: Reply from

Public Finance Quarterly July 1974
Harold Hochman and James Rodgers’ response to our reconsideration of their discussion of Pareto redistribution indicates that their model and ours are quite distinct. However, the differences emerge from a number of assumptions that had not been explicitly raised in their earlier discussions of the model; and while these assumptions are not implausible, it is not clear that systematic analysis of them would yield the sort of redistributive patterns that their formal models have produced. Moreover, their response totally misinterprets our comments on the transfer-elasticity concept (which we argue obscures aspects of the transfer process which have an important bearing on the final redistributive pattern), and on the normative significance of the notion of Pareto desirable redistribution (which we believe to be less than H&R have suggested).

HOCHMAN AND RODGERS ON
BRENNAN AND WALSH:
REPLY

GEOFFREY BRENNAN
CLIFF WALSH
Australian National University

Hochman and Rodgers’ fairly lengthy—but nonetheless extremely entertaining—reply to our reconsideration of Pareto-optimal redistribution has certainly served to clarify a number of the issues arising out of their original analysis.1 It has also served (properly, no doubt) to administer a timely rebuke for our excessive zeal in matters analytic. Whether it has really come to grips with the basic points we sought to make is rather less clear; and consequently some sort of rejoinder does seem necessary.

AUTHORS’ NOTE: We wish to acknowledge financial support from our wives, and assorted friends too numerous to mention.
For convenience, we collect our observations into two sections: the first dealing with problems associated with the actual models of redistribution; and the second with the more general question of how the Pareto desirable redistribution possibility is to be interpreted.

MODELS

H&R's central response to our reconsideration is fairly clear. After charging us with being rather too concerned with niceties of technique, and thereby threatening to divert attention in the Paretian redistribution discussion away from its philosophical and sociological frontiers (whatever they may be), H&R assert that our analysis “fails” anyway, because we have misinterpreted their model.

It does now appear that H&R’s model and ours are quite distinct. Whether we can be held responsible for our “misinterpretation” is, however, debatable, because much that is involved in the H&R construction has emerged only as a result of their further discussion of their model.

What we had not fully appreciated—although it seems clear from H&R’s most recent discussion—is that individual preferences are actually defined in terms of specific transfer elasticity values which will generate the sets of ultimate transfer patterns H&R “derive.” In this sense, any attempt to indicate a logical inconsistency in their derivation of results is necessarily doomed to failure, because the relation between their results and the underlying donor preferences is tautological!

H&R’s attempts to justify their “model” now, predictably, take the form of observations which they believe will lend credence to their final transfer patterns. Thus, they produce a number of arguments—some of them quite persuasive—which suggest that transfers may be required to maintain the initial income ordering strictly. These observations, H&R suggest, are actually assumptions implicitly embodied in their analysis. They are:
(a) donors may have an inherent preference for the maintenance of some income differential between all individuals who have different pretransfer incomes;

(b) the rich may have an aversion to transfers which discourage effort;

(c) "empathy" between donor and recipient may be strongest for those closest in social and cultural—and hence income—terms.

Furthermore, they observe that real world transfers tend to be "distribution-condensing" rather than "distribution-truncating" (as, for example in Gillespie's results which H&R cite in their original paper), and this, they believe, suggests further support for their model. A number of points might be made briefly about this attempted self-justification. Firstly, it is surely misleading to adduce anything about donor preferences for redistribution by observing the total distributional impact of the budget, even if one has complete faith in a Gillespie-like procedure. It is one thing to argue that actual redistribution may reflect something of the distributional demands of would-be voluntary donors, and something else entirely to suggest that it reflects nothing but! Moreover, the distributional impact of the total budget (rather than just the directly redistributational part) would seem to involve considerations other than purely distributional motives (e.g., the distributional implications of public goods provision).

Secondly, while the arguments that H&R bring to bear in defence of their model are not at all implausible, it is far from obvious that a systematic analysis of them would yield up H&R-type results. For example, the empathy motive might well suggest transfers inversely related to initial differentials; donor aversion to discouragement of effort might well involve minimum wage legislation, or subsidies for leisure substitutes; and so on. Of course, whether these particular suggestions turn out to be right or wrong is beside the point: the central message is that there is a need to handle the various possibilities in an analytically explicit and systematic manner.
In our own modest way, this is precisely what we sought to do for the H&R model as we originally understood it. In defence of our apparent preoccupation with such a task, it is perhaps worth pointing out that—H&R’s suggestions notwithstanding—they did not “initiate discussion” of the Pareto-optimal redistribution possibility. That much had already been done—by Buchanan (1960), Graaf (1957), Head (1966), Olsen (1969), Vickrey (1962), and a number of others. What was new was H&R’s attempt to provide a systematic analysis of the possibility, and it is surely in this that the significance of the original H&R piece is to be interpreted. Given such a historical context, mere analytical inadequacies—if only in that too many assumptions were left implicit—do seem to be of some importance.

So we return to square one. And even taking account of H&R’s new revelations about their model, we still find ourselves in some difficulty. The key to the technical discussion in their original contribution is the transfer-elasticity concept; and unfortunately the true nature of that concept remains obscure. Indeed, H&R take exception to our attempt to unlock its secrets, arguing that their use of the transfer (or income) elasticity is no different from that employed in the conventional consumer choice model. This assertion seems to us to be misleading in two respects:

(a) H&R ignore one of the crucial implications of our discussion of their elasticity concept—namely, that there is an important difference between choice over ordinary goods and services, and choice over income transfers. If the consumer has an initial basket of goods consisting of, say, bread, butter, and jam, he can obtain more bread by giving up some butter, or some jam, or some of both. But when we are dealing with an individual who is purchasing “income differentials,” it is impossible for him to purchase a decrease in one differential without simultaneously purchasing a change in all other differentials. For this reason, redistributions cannot be treated as being analytically equivalent to consumer choices in other contexts.
(b) In the more familiar, conventional, consumer choice models we accept the use of price or income elasticity measures because (rightly or wrongly) we believe that we understand what they imply about the underlying preferences of individuals, and, hence, their behaviour patterns. And our confident use of elasticity measures, we suggest, stems from the fact that the basic analysis of consumer (and producer) choice has been subject to rigorous analysis.

In the case of the transfer-elasticity concept, however, the required knowledge of the more basic elements of rational donor choice is absent, and the use of the concept has served to obscure, rather than to illuminate, these important questions. Nor is it really sufficient for H&R to argue that their analysis is illustrative rather than definitive. Given the assumptions they have made about the donors’ preferences (including those revealed in their recent reply), we do not need the transfer elasticity to tell us that transfer activities are likely to be “distribution-condensing”; and their employment of the concept to generate hypothetical transfer patterns serves to create an altogether spurious impression of analytical precision and empirical relevance.

Our own rather lengthy sequential analysis of the transfer process was not (as H&R imply) intended as a description of the consumer allocating his income, or as an attempt to overturn the standard paradigm of consumer choice. Rather, it should be seen as an attempt to map out what is, after all, relatively unfamiliar terrain.

**INTERPRETATION**

Turning to the welfare, or normative, significance of the Pareto-optimal redistribution possibility, H&R again accuse us of misinterpreting their arguments. In particular, they make three assertions:

(a) we incorrectly deny the essential implication of Paretian redistributions—namely, that opportunities for gains from trade narrow the
range of efficient income-distributions and in this sense produce a partial collapse of vertical equity into efficiency;

(b) our concern over the allocation of consumer's surplus—and hence the patterns and magnitude of redistributions—is misplaced because voluntary transfer processes, within the prevailing structure of property rights, allow the rich total authority over the disposition of their incomes;

(c) the (positive) assumption of recipient passivity can, in any case, be rationalised on grounds of risk-aversion on the part of potential recipients, and appears justified by casual observations of actual redistribution patterns.

Let us consider these more or less in turn.

We would not in any sense want to deny the possibility that Pareto-optimal redistributions might narrow the range of efficient income distributions. However, this does not seem to us to imply necessarily a collapse (even a "partial" one) of equity into efficiency. The concurrence of the patterns of redistribution generated in H&R's model of philanthropic giving with standard notions of equity arises essentially because the assumptions they adopt guarantee that this will be the case. Even so, as H&R themselves suggest, it is conceivable that, because of the strong feeling of empathy between cultural and social peers, much of the redistribution that occurs may have little effect on the equity of the income structure because it flows among those close to one another on the income scale. Moreover, if we step outside the altruism model to examine other sources of mutually beneficial income transfers, the required redistributions of income may increase, not reduce, disparities in income levels. It is one thing to argue that the traditional separation of efficiency and distributional questions is broken down, but quite another to suggest that equity and efficiency are thereby necessarily partially reconciled. Perhaps H&R would agree—in which case our apparent difference on this is simply a semantic confusion.

Our differences over the second and third of H&R's assertions are more basic, and H&R's position on these more
difficult to comprehend. It seems likely, we would admit, that if the voluntary market processes were not subject to free-rider problems, Pareto-optimal redistributions would proceed to the point where the rich obtain the maximum gains from trade. But H&R appear to be arguing that this is the only outcome consistent with voluntary donations by the rich. And this is manifestly not true. We can retain the consistency of income transfers with individual preferences while permitting redistribution to be effected through the political mechanism in either of two ways: by postulating an institutional government equipped with the necessary information, and motivated in a Paretian way; or by assuming a decision-making rule of unanimity for policy acceptability. In either of these cases our arguments about the allocation of consumer's surplus (and hence about the possibility of redistributions going much further than would occur in a perfectly operating market) retain their force. Surely H&R cannot have (unconsciously) moved from the convenient expository device of assuming the absence of market failure into a completely unwarranted presumption that the market outcome is the only one consistent with individual preferences. Yet what else can they be saying?

This is, we insist, a point of considerable importance in interpreting the significance of the Pareto desirable redistribution possibility; and we would like to take this opportunity to try to spell out in detail just why we believe that to be so. In the process, we shall attempt to say something, generally, more about how the whole matter of Paretian redistribution might be interpreted.

At the most general level, it seems clear that the normative significance of the Pareto-optimal redistribution possibility depends very much on the sort of Paretian one is. If, in the first place, one interprets the Pareto criterion as being sufficient, but not necessary, for policy desirability, then the possibility of Paretian redistributions has some operational significance, but is not ultimately crucial. For members of this school of thought, the Pareto criterion is regarded as a partial specification of the social welfare function, and distributional matters are looked on
as being finally resolved by a more detailed specification of the social welfare function—beyond its Paretian characteristics. The Pareto desirable redistribution possibility then emerges as a means of putting some content into an otherwise empty treatment of distributional questions without the need for recourse to anything more than standard tools of analysis. The economist qua economist can now contribute authoritatively to discussion of distributional issues: even though he may not know very much about the extent of the redistribution permissible on Paretian grounds, he may be able to specify something about the appropriate nature and form of such redistributions.

However, for those who have interpreted the Pareto criterion as both necessary and sufficient for policy desirability—a view which seems to be taken by Buchanan, for example—Paretian redistributions attain even greater significance. For this school of thought, the real normative thrust of the Pareto desirable redistribution possibility resides in the fact that it suggests that the Paretian framework is much less conservative distributionally than we have traditionally thought—the status quo orientation is substantially removed. If, though, the strict Paretian framework is to be sold to a wider market among policy makers on the basis of the Paretian redistribution possibility, it is presumably crucial to know just how far such redistributions can take us in the direction of a more equitable distribution in the standard sense. But this turns out to be a difficult question to answer in at least two senses. Firstly, market failure will generally arise in the redistribution process, and it then becomes difficult to give the redistribution possibility any significant operational content since we face the standard preference revelation problems associated with public goods. Secondly, application of the Pareto criterion to redistribution questions in general serves only to specify a range of distributional outcomes—a range which may itself be very extensive. In this sense, the strict adherents to the Pareto criterion are saddled with a rather embarrassing agnosticism over a potentially enormous range of distributional outcomes.
One could, of course, take a somewhat different view of the Pareto redistribution possibility—namely, that it is seen as providing a justification for what is currently done by way of redistribution. In this sense, the Pareto criterion does appear less conservative than we have previously thought, but in this context it emerges as conservative in another sense: it can justify what we have, but would insist on no departure from current practice—in either direction!

Finally, one might take a more positive view of the notion of Pareto redistributions, regarding the notion as embodying some explanatory power. It is conceivable that “Pareto redistribution” explains what we observe—but on the whole it seems most unlikely. A much simpler and much more persuasive explanation of redistribution occurring through the political mechanism would focus on the fact that the distributions of market and political power differ. In any case, the Pareto redistribution possibility is consistent with such a wide range of ultimate distributional outcomes that its true predictive power is severely limited.

On the other hand, focusing on the existence of mutual gains from redistribution does emphasise the fact that redistribution is, up to a point a non-zero-sum activity4—a fact which may help us to explain the form that transfers take, if not their magnitude. Thus, for example, if redistribution involved a pure conflict (i.e., zero-sum) situation one might expect all redistribution to take place as cash transfers: yet much that in practice does pass for redistribution is effected through free (or highly subsidised) provision of goods and services, even though price-exclusion appears economically feasible. Is such redistribution-in-kind perhaps to be explained by donor preferences, and, if so, is it not, thereby, essentially Paretoian? No doubt other explanations exist, but this one seems persuasive.

All-in-all, though, it appears that while the phenomenon of Pareto-optimal redistribution is not uninteresting, its claims to significance must necessarily be modest—and certainly more modest than H&R would have us believe. Nonetheless, what the
Paretian redistribution concept has emphasised is that standard tools of analysis, and a weak set of value judgments, may allow the economist qua economist a more substantial role in the discussion of distributional questions than has hitherto been accepted by a profession still struggling to break loose from the shackles imposed by Robbins’ critique of welfare theory.

NOTES

1. See Hochman and Rodgers (1969, 1970) for their original analysis. Our reconsideration is to be found in Brennan and Walsh (1973).

2. While H&R’s use of the terms “distribution-condensing” and “distribution-truncating” carry a degree of descriptive accuracy, they may be misleading. Measured in terms of the Gini coefficients, a distribution-condensing pattern of redistribution does not necessarily imply a greater degree of (post-transfer) equalisation than one which is distribution-truncating.

3. See, for example, Brennan (1973) for a discussion of some aspects of Paretian redistribution which depend on motives other than altruistic attitudes of the rich toward the poor.

4. Schelling (1967) has emphasised the non-zero-sum aspects of military conflict, and argues that interpreting war as a zero-sum activity is dangerously misleading. The same may be said of redistribution.

REFERENCES


4. PAPER FIVE

Pareto-Desirable Redistribution:
The Case of Malice and Envy
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PARETO DESIRABLE REDISTRIBUTION:
THE CASE OF MALICE AND ENVY *

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1. Much attention has been directed in recent literature to the possibility of using familiar allocative welfare norms to handle policy questions relating to income distribution.¹ Most of the contributions in this area have been concerned, either implicitly or specifically, with models in which the rich derive benefits of an essentially altruistic kind from transfers to the poor. In such an environment, the possibility of Pareto desirable redistribution follows more or less automatically: universally beneficial transfers arise because all are universally beneficent. The environment is, however, constrained by the crucial prior assumption that the rich are philanthropically motivated.

Such an assumption is, of course, analytically legitimate, and may indeed be of some relevance: individuals’ conceptions of man are no doubt matters of taste and personal experience. But it is surely just as reasonable to take precisely the opposite view—that, far from exhibiting generosity, kindness and concern for others, man is characterized by attitudes of envy and malice towards his fellows. And such a view would, it seems to me, have the added virtue of being more in keeping with the spirit of this dismal science, and with the nature of homo economicus as traditionally conceived.

With the analytical environment thus changed, the question arises as to whether the possibility of Pareto desirable redistribution evaporates.

* Gratitude is due to John Head and an unknown referee for comments on earlier drafts.

¹ See, for example, Hochman and Rodgers (1969, 1970, 1972); Olsen (1967); Pauly (1970); and Von Furstenberg and Mueller (1971).
Might the Pareto criterion require redistribution even in this ‘fallen’ world?

It is to this question that the present paper is addressed. Specifically, I aim to demonstrate the validity of three propositions:

(i) that actions undertaken for reasons of malice and envy can be Pareto relevant;
(ii) that Pareto relevant malice and envy can generate redistribution;
(iii) that such Pareto desirable redistribution will in general be from rich to poor under the most reasonable assumptions.

The basic justification for the exercise lies in the demonstration that the possibility of Pareto optimal redistribution is not dependent on (possibly implausible) assumptions concerning the sweet and lovely nature of man. The Pareto test can, then, (at least conceptually) be used to justify income transfers in cases other than those of universal philanthropy.

In what follows, we consider propositions (i), (ii) and (iii) above, more or less in turn.

2. The assertion that motives of malice and envy may be Pareto relevant is (or perhaps ought to be) mildly offensive to intuition. After all, the basic objective of actions undertaken for reasons of malice and envy is to harm someone — and one might have thought that such actions would necessarily be precluded within the Pareto framework, a characteristic requirement of that framework being that no one should ever be made worse off. Indeed, in an environment where all are malicious towards and envious of one another, a fundamental role of the Paretoian government would seem to be precisely to protect each individual from the envy and malice of his fellows.

All this seems perfectly valid. At the same time, if all are motivated by malice and envy, and if malicious and envious acts are precluded under the initially prevailing “constitution”, all may gain by a relaxation of rules, which permits each to inflict suffering on others over some range.

It is clear, for example, that when two individuals agree to engage in physical combat, they are involved in a voluntary exchange of sorts (in this case a trade of blows), and arguing from a standard presumption in welfare theory, it seems reasonable to suppose that both are made better off. Individuals may, of course, fight for a number of reasons — because some prize hangs on the outcome, or because they miscalculate
the probability of success, or whatever. But they may also fight simply because they hate one another, in which case the gains from "trade" arise because the utility of inflicting suffering on one's opponent exceeds, for each, the disutility of suffering endured by oneself. This case can I think be reasonably described as depicting motives of "malice and envy". If battle were precluded in such a case, all might well gain from a change in rules which permitted physical combat over some range, and within the Pareto framework at least, such a rule change would be required for optimality. In this (quite standard) sense, malice and envy can be Pareto relevant.

3. Even so, the prospect of generating universally beneficial transfers in such circumstances seems remote. One might have thought that, in general terms, the presence of malice and envy would serve to render all conflicts of interest more intense, thus making the utility possibilities schedule even steeper than implied by the normal constraints of limited resources, and thereby the possibility of upward-sloping segments (apparently characteristic of the Pareto desirable redistribution possibility) correspondingly less likely.

This is true — but not necessarily decisive. If malice and envy operate in such a way as to make some conflicts more intense than others, then the possibility of interpersonal transfers which provide universal benefits remains. And we should note here that an inherent feature of the envy concept is that conflicts of interest are likely to be more intense when at least one of the parties is rich — individuals are characteristically more envious of the rich than of the poor.

4. Accordingly, we consider the following model.

There are three individuals, A, B and C, with incomes $Y_A$, $Y_B$, $Y_C$ respectively. Initially, these take values $Y_A^0$, $Y_B^0$ and $Y_C^0$, such that $Y_A^0$ and $Y_B^0$ are approximately equal and greatly in excess of $Y_C^0$.

Each is endowed with the same utility function,

$$U_i = u(Y_i, Y_j, Y_k) \quad \text{for all } i, j, k$$

which is assumed to exhibit the following properties:

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2 See Polinsky (1971) and Graaff (1957).
G. Brennan, Malice and envy

(a) positive marginal utility of own income, i.e.

$$\frac{\partial U_i}{\partial Y_i} > 0,$$

(b) negative marginal utility of others’ income (to indicate malice), i.e.

$$\frac{\partial U_i}{\partial Y_j}, \frac{\partial U_i}{\partial Y_k} < 0,$$

(c) increasing marginal disutility of others’ income (to indicate envy), i.e.

$$\frac{\partial^2 U_i}{\partial Y_j^2} < 0,$$

(d) diminishing marginal utility of own income, i.e.

$$\frac{\partial^2 U_i}{\partial Y_i^2} < 0,$$

(e) malice and envy are superior in consumption, i.e.

$$\frac{\partial^2 U_i}{\partial Y_j \partial Y_i} < 0.$$

We further assume that over some range the malice and envy implied in these properties is Pareto relevant, so that individuals are prepared to suffer in order to inflict suffering on others. In this model, all such suffering takes the form of monetary losses.

5. Focussing initially on A’s and B’s preferences over \( Y_A \) and \( Y_B \), we can depict these assumptions geometrically in fig. 1. Accordingly, we observe that:

(a) all indifference curves have positive slope;
(b) indifference curves further from the origin in a horizontal (vertical) direction indicate higher levels of utility for A (B);
(c) A’s indifference curves become less steep, and B’s become more steep as \( Y_A \) and \( Y_B \) increase;
(d) for A (B), successive indifference curves along a constant own-income line become less (more) steep as other-income increases, indicating a preparedness to give up more to inflict harm of a given money value on the other as other’s income rises;
(e) for A (B), successive indifference curves along a constant other-income line become less (more) steep as own income increases, indicating a preparedness to give up more to inflict harm of a given money value on the other as own income rises.

In order to ensure Pareto relevant malice in this simple case, we need to assume that over some range each is prepared to endure more than a dollar’s loss to inflict a dollar’s worth of suffering on the other. This implies that for some non-infinite income combination, the indifference maps achieve slopes of unity — and hence slopes of more than unity for B (and less than unity for A) for income combinations beyond that point. It follows that there exists a locus ZZ’ of points of tangency between indifference curves, and that this locus will generally take a position as depicted in fig. 1 — downward-sloping from left to right. ³

If the initial income combination \((Y_A^x, Y_B^x)\) lies above and to the right of ZZ, then not only will A and B desire to inflict harm on one

³ It seems reasonable that the curve ZZ should be convex, as depicted but this is not necessary for the model.
another, but it will be Pareto optimal to allow them to do so. In the area outside $Z'Z$, both are prepared to accept losses in order to be able to inflict losses on the other: both can be made better off by the destruction of each other’s income, up to some point on $Z'Z$. Thus, from the initial point $X$, the shift to some point on $ST$ (of $Z'Z$), say $E$, moves both to higher indifference curves, and the slope of $EX$ determines the proportions in which $Y_A$ and $Y_B$ are reduced. Since $C$ is envious of/malicious towards $A$ and $B$, the reduction in $Y_A$ and $Y_B$ serves in itself to make $C$ better off, so that no one in the community has his position worsened by this “trade”.

6. In a market context with large number of $A$'s and $B$'s such Pareto desirable “trades” would not be forthcoming. Since the most desirable outcome for each individual would occur when all others indulged in mutual income destruction, a problem of “free riding” exactly analogous to the public goods case arises. Each individual has an incentive to act as if his malice were much less intense than it actually is. Government intervention is thus necessary if the Pareto desirable income destruction for $A$ and $B$ is to occur.

Government intervention will of course also be necessary, in general, to contain the infliction of losses within the range of Pareto relevance. Both individuals may prefer to have malicious acts outlawed entirely if the only possible alternative is a complete free-for-all. Thus in fig. 1, if the “three-for-all” results in a position in the $(Y_A, Y_B)$ plane below and to the left of $V$, both will prefer $X$. Consequently, even if there are universal gains from indulging malice and envy over some range, some government intervention to prevent persecution will in general be required whenever “anti-social” motives of this sort are present.

We note that, as in the altruism case, the relevance is also less stringent in the large numbers case. A reduction in the income of any one individual yields benefits to all, which must be (vertically) summed to obtain total benefit; hence Pareto relevance does not require that each is prepared to pay a dollar to inflict a dollar’s suffering on another—only that each is prepared to pay at least $100/n$ cents where $n$ is the relevant group size. Alternatively put, the total monetary loss inflicted in a programme of mutual income destruction involving all of the equal-income rich, is directly proportional to the numbers involved: if

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4 See Goldfarb (1970).
a dollar’s reduction in own-income purchases a dollar’s reduction in all others’, then it presumably provides a greater utility increment the larger the number of individuals participating.

Thus when numbers are large, it is in the first place more likely that malice will be Pareto relevant; and in the second, more likely that this Pareto relevant malice will have to be vented through the political mechanism if it is to be vented at all. The Pareto framework would, of course, insist that it should be.

7. Use of the political mechanism to effect the desired income destruction for A and B serves to open options which are not really available in the market. The reason for this is that market organization (if perfect) would imply a specific distribution of the gains from “trade” which is different from the one which might be expected in the political context in this instance. The difference in the break-up of gains arises because C is necessarily a party to all (unanimous) political decisions, whereas he is not a party to market trades between A and B (beyond the constraint that he is not actually made worse off).

In particular, it is now possible for A and B to inflict damage on one another not by destroying income, but by redistributing it to C. To take first a simple case let us suppose that

\[ \frac{\partial U_A}{\partial Y_C} = \frac{\partial U_B}{\partial Y_C} = 0 \quad \text{when} \quad 2Y_C < Y_A = Y_B. \]

Then while \( Y_C \) remains less than half of \( Y_A \), A feels no envy or malice towards C at all. Assuming this condition to hold, A and B are indifferent between destruction of income and redistribution to C. Since C himself prefers the latter, then the Pareto criterion isolates this as the appropriate policy: redistribution from A and B to C is necessary to achieve Pareto optimality.

8. In the more general case,

\[ \frac{\partial U_A}{\partial Y_C} - \frac{\partial U_B}{\partial Y_C} < 0 \quad \text{for all} \quad Y_C. \]

Thus, A and B are malicious towards (envious of) C, in spite of C’s poverty. In this case, A and B prefer income destruction to redistribu-
tion in C’s favour. However, given market failure, redistribution from A and B to C makes everyone better off: because by the assumption of increasing marginal disutility of other’s income, A (B) is made better off by any redistribution from B (A) to C. Hence, while redistribution is not actually necessary for Pareto optimality, it does serve to make everyone better off. And unless the maximum possible gains from trade are explicitly to be ascribed to the richest groups alone some redistribution to C will be involved in attaining an efficient outcome.

In this general case, we can obtain the optimum conditions by maximizing \( U_A \), subject to the constraint that B be made no worse off and with income destruction explicitly precluded (i.e. \( Y_A + Y_B + Y_C = K \) throughout). This latter restriction is sufficient to ensure that C is not made worse off, for C must necessarily receive any transfers and, given positive marginal utility of own income, these do make C better off. Thus, for Pareto optimality, we require:

\[
\left[ \frac{\partial U_A}{\partial Y_B} - \frac{\partial U_A}{\partial Y_C} \right] \cdot \left[ \frac{\partial U_B}{\partial Y_A} - \frac{\partial U_B}{\partial Y_C} \right] = 0
\]

\[
= \left[ \frac{\partial U_A}{\partial Y_A} - \frac{\partial U_A}{\partial Y_C} \right] \cdot \left[ \frac{\partial U_B}{\partial Y_B} - \frac{\partial U_B}{\partial Y_C} \right]. \tag{7}
\]

Starting from a situation in which \( Y_A \) and \( Y_B \) are large and \( Y_C \) small, we know that:

(a) \( \frac{\partial U_A}{\partial Y_B} \), \( \frac{\partial U_B}{\partial Y_A} \) are large and negative;
(b) \( \frac{\partial U_A}{\partial Y_A} \), \( \frac{\partial U_B}{\partial Y_B} \) are small and positive;
(c) \( \frac{\partial U_A}{\partial Y_C} \), \( \frac{\partial U_B}{\partial Y_C} \) are small and negative;

Thus, the left-hand side of (7) tends to be large, the right-hand side to be small, establishing scope for Pareto optimal transfers. 5 As income

5 The case where \( \frac{\partial U_A}{\partial Y_C} \), \( \frac{\partial U_B}{\partial Y_C} = 0 \) is clearly a special case; the optimal conditions become

\[
\frac{\partial U_A}{\partial Y_B} = \frac{\partial U_A}{\partial Y_A} \cdot \frac{\partial U_B}{\partial Y_B}
\]

or \( \text{MRS}^A_{Y_A Y_B} = \text{MRS}^B_{Y_A Y_B} \) (as depicted in fig. 1). We note that since \( \frac{\partial U_A}{\partial Y_C} \), \( \frac{\partial U_B}{\partial Y_C} \) are negative, they affect (7) by reducing the left-hand side and increasing the right-hand side, for any initial set of values, as well as hastening the move to equilibrium via changes in \( \frac{\partial U_A}{\partial Y_C} \), \( \frac{\partial U_B}{\partial Y_C} \); thus, as expected, redistribution is less substantial in the more general case.
is transferred from A and B to C, three influences operate to establish the equality in (7). As \( Y_A \) and \( Y_B \) decline \( \partial U_A / \partial Y_A \) and \( \partial U_B / \partial Y_B \) rise, and the absolute values of \( \partial U_A / \partial Y_C \), \( \partial U_B / \partial Y_A \) fall. And as \( Y_C \) rises, the absolute values of \( \partial U_A / \partial Y_C \), \( \partial U_B / \partial Y_C \) rise likewise.\(^6\) Thus, \( (7) \) is established at some new point with \( Y_C \) larger, and \( Y_A \) and \( Y_B \) smaller, than initially.

9. The model thus given is sufficient to indicate that malice/envy may establish a case for redistribution within the Pareto framework. It also serves to suggest that such redistribution, as in the altruism cases now familiar from the literature, will be from rich to poor. The crucial ingredient in generating this result seems to be the assumption of increasing marginal disutility of other individual’s income — an assumption which emerges naturally as an implication of the envy concept.

Apart from any inherent interest this analysis may claim, its significance lies precisely in the fact that it goes some way towards freeing the possibility of Pareto optimal redistribution from the need to assume universal philanthropy. The limitations of this traditional framework are, in fact, more severe than they may seem. As emphasized by Von Furstenberg and Mueller,\(^7\) given the ‘practical’ requirements of horizontal equity and smoothness of rate structure, Pareto optimal redistribution can only be effected if all individuals with identical incomes

\(^6\) Because of the assumed structure of this model, \( Y_C \) rises by two dollars for every dollar reduction in \( Y_A \) and \( Y_B \) and the third influence tends to predominate in the optimizing process. If we assume (to accord with reality) that the income distribution is skewed towards lower incomes, the number of C’s becomes very large relative to the number of A’s and B’s so that \( Y \) rises very slowly in relation to reductions in \( Y_A \) and \( Y_B \). Because of this, we can reasonably put \( \frac{\partial U_A}{\partial Y_C} \) and \( \frac{\partial U_B}{\partial Y_C} \) close to zero in (10), and the special case if \( \frac{\partial U_A}{\partial Y_C} = 0 \) examined earlier becomes relevant. This is so, of course, not so much because A and B feel no malice towards C, but rather because the changes in \( Y_C \) are of quite small magnitude, relative to the changes in \( Y_A \) and \( Y_B \).

This point established an important qualitative difference between the malice/envy case and the case of pure philanthropy. In the latter, realistic assumption about the skewness of the income distribution suggests that the cost of achieving the desired level of consumption for the poor may be very high even when all middle and higher income groups are contributing. Setting the ratio of recipients to suppliers at practically relevant magnitudes eats substantially into the plausibility of the model. By contrast, in this model the more recipients per supplier and hence the less significant per individual the gifts are, the more substantial in toto redistribution is likely to be.

\(^7\) See Von Furstenberg and Mueller (1971) p. 636 for a more detailed discussion.
have identical tastes. Thus, it seems that if there exist some rich individuals who are not philanthropically inclined any operational redistributive package will make these individuals worse off and hence not satisfy the Pareto test. Appropriate compensation may of course be worked out for such individuals via vote trading (or otherwise), but the possibility of genuine redistribution over the entire range is seriously impaired by this possibility. Hence, if a persuasive case for the Pareto desirability of government redistributive programmes is to be mustered, it seems necessary to assume that all the rich are philanthropically motivated.

What the foregoing analysis indicates is that such an assumption is not in fact required. Individuals motivated by malice and envy may also be prepared to contribute to redistributive programmes, not because they value increased consumption by the poor, but because they value reduced consumption by the rich. Redistribution from rich to poor retains its ‘public goods’ properties — all the rich derive positive benefits from the transfer programme over some range: it is simply that the nature of benefits conferred varies conspicuously according to motivation. Thus the problem of non-uniform tastes may well be less intractable here than with other public goods.

All in all, it does appear as if malice and envy may not be wholly unmitigated evils — which is perhaps reassuring, since they do seem to exist in some abundance.

References


8 They may value redistribution for other reasons to — the raising of the floor level of income may provide an insurance facility for all, in the face of periodic cataclysm; or transfers may, via income effects, increase the poor’s consumption of goods (health, housing, fire protection, education etc.) which generate external benefits for the rich; or redistribution may simply be a bribe to the poor designed to reduce the probability of revolution or crime. See Head (1966) and Brennan (1973) for a brief discussion of such possibilities.

9 Explicitly contrary to the Von Furstenberg — Mueller assertion (see footnote 7).
PARETO DESIRABLE REDISTRIBUTION:  
THE NON-ALTRUISTIC DIMENSION  

Geoffrey Brennan *

The question of the nature and extent of redistribution required by pure efficiency considerations has become something of a feature of recent welfare literature.¹

Very largely, the contributions involved have concerned themselves with the possibility of Pareto desirable redistribution arising out of altruistic attitudes taken by the rich towards the poor, and with the precise patterns of redistribution associated with specific assumptions about the relevant utility functions.

Although it has always been clearly recognized that economic methodology admits altruism as a possibility in explaining individual and social behaviour, economists have nevertheless traditionally been reluctant to rely too much on "heroic" conceptions of man in developing their theories, and the view of human motivations characteristically taken in economic theory has tended to be a fairly cynical one. Accordingly, it is of some interest to raise the question as to whether redistribution can be justified within the Pareitian framework by appeal to considerations other than those depending on the assumption of a philanthropically-inclined rich.

The purpose of this paper is to explore two such considerations. As a point of departure, we observe that:

... even in relatively stable societies, incomes of individuals of the same age, and even of similar educational background and other socio-economic characteristics, show considerable dispersion. In societies which have been ravaged by great wars, revolutions, depressions, hyper-inflations and other political and social upheavals, such uncertainty may clearly be intense. At least to a substantial degree, such income-uncertainty is likely to be privately unavoidable and uninsurable; and in a society in which most individuals are risk-avers, this uncertainty may be reflected in redistributive government policies, and midly egalitarian conceptions of the proper state of distribution [2, p. 10].

The central assertion is that, quite apart from any philanthropic considerations, an individual may be prepared to contribute towards redistributive programmes; firstly, because in the face of certain privately uninsurable contingencies, raising the floor level of income precludes or substantially reduces the possibility of complete destitution; and, secondly, because increased

*¹I am grateful to John Head and Tom McGuire for stimulating and profitable discussion.

consumption by the poor may actually reduce the possibility of such contingencies occurring—for example, raising the income levels of the poor may reduce the likelihood of crime, social unrest and/or revolution. The former motive we refer to as the “insurance motive,” and the latter as the “self-protection motive.” When either motive is operative, income transfers from rich to poor will give benefits to both groups—all will be made better off and, hence, efficiency considerations will insist that the transfers be undertaken.

Clearly, the existence of these motives does not in any way preclude giving for philanthropic reasons as well. Rather, redistribution can be expected to serve all functions simultaneously. This implies, in turn, that the aggregate transfer pattern will reflect attributes which derive from each of the motives stated, given that all are operative at the margin. In this sense, the present analysis should be regarded more as an extension of, and complement to, existing Paretian treatments of income redistribution based on the assumption of altruism, than as a contrary or competing view.

In analysing the insurance and self-protection motives for income transfers, three specific questions are of interest:

first, how likely is it that the conditions under which each motive is Pareto relevant will be met? or, more generally, just how plausible are the various models?

second, what is the likely pattern of transfers which results? (that is, who are the likely recipients, and how will the cost be shared among taxpayers?)

third, is the redistribution (if any) which flows from the model necessary for Pareto optimality, or simply the result of one means of achieving the utility possibilities frontier, given a specific distribution of the gains from trade? Clearly, if the latter, then any redistribution which occurs flows from the assumption determining the division of the gains from trade and not from the Pareto criterion in itself. While such assumptions about the division of trade gains may be legitimate (and even desirable); they are in no sense essential to the Paretian framework, and indeed represent something of an intrusion into it, the only distributional assumption inherent in the Paretian system being that which insists that no one be made worse off. Clearly, then, where the Pareto criterion is consistent with zero redistribution, efficiency considerations in themselves do not logically require net income transfers, and any redistribution that occurs in such cases cannot appropriately be categorized as “Pareto desirable.”

With these questions in mind, we shall examine the insurance and self-protection motives in turn—in section I and II respectively—and in section III draw the appropriate conclusions.
I. THE INSURANCE MOTIVE

It is well-known (following Buchanan and Tullock) that at the constitutional level,² where individuals have characteristically zero knowledge of their future income positions, risk aversion will lead individuals to desire an allocation of property rights which generates identical incomes (or perhaps more strictly, to a selection of fiscal institutions which will ensure proximate income equality). One of the crucial ingredients in generating the desire for income equality is, of course, the assumption of completely imperfect knowledge concerning future income position. In the context of “in-period” choices, the assumption of zero information is thoroughly implausible, so that one should not expect the desire for equality to carry over entirely from the constitutional level. Nevertheless, one might imagine that given some uncertainty about future income, something of a desire for limited income disparities may be reflected by those who are risk averse over the relevant range. In this sense, the model to be explored here can be viewed as an attempt to generalise the Buchanan-Tullock result to the context of in-period (i.e. non-constitutional) choice. It represents an extension of the Buchanan-Tullock model in another sense, as well. Here, we assume that all individuals are both risk averse and risk loving over relevant ranges, so as to permit the desire for redistribution to be consistent with an observed propensity to gamble (as well as to insure).

As a point of departure, we assume the existence of certain disastrous contingencies which are privately unavoidable, privately uninsurable, and which if they occur will reduce the income of the affected person (or persons) to some near-zero level. The occurrence and/or incidence of these “cataclysms” is subject to uncertainty, and individuals are risk averse over the relevant range. This being so, all are prepared to pay something to insure against potential cataclysms; and we note that raising the floor level of income in society to some non-zero level, \( y_0 \), provides some (though not full) such insurance. Focusing on such a policy, we are concerned to determine, firstly, the likely pattern of tax shares if strict benefit taxation is applied (i.e. the distribution of benefits at the margin) and secondly, the likely magnitude of such benefits. Thus, we begin by asking the question: what sort of transfer pattern would be most likely to issue from the insurance motive, given that the motive is Pareto relevant? Later we consider whether the assumption of Pareto relevance is in fact plausible. In this sense, we are beginning by conducting an analysis for the insurance motive analogous to that undertaken by Von Furstenberg and Mueller [10] for the altruism case, with the explicit intention of comparing the Von Furstenberg-Mueller transfer patterns with those arising with the insurance motive under what seem to be the most plausible assumptions.

²See Buchanan & Tullock [1, ch. 13]. Buchanan & Tullock do consider the disincentive effects of income redistribution, which of course mitigate the desire for equality somewhat.
A. The Transfer Pattern

Since we are assuming Lindahl pricing arrangements, the transfer pattern for any given level of transfer, is determined by the distribution of marginal benefits from the transfer operation. In order to obtain this distribution of benefits, the prevailing utility function must be specified. Here, for analytical convenience, we assume that all individuals have utility functions of the same form and in the interest of completeness, the form taken is one which is consistent with both insurance and gambling. The simplest such form is that depicted in figure 1, where
The characteristic features of this depiction are that:

a) $U(y^i)$ is everywhere positive;

b) $U_{y^i}$ is everywhere positive;

c) $U_{y^i} < 0$ if $y^i < y_{A^i}$

$> 0$ if $y^i > y_{A^i}$

Let the probability of cataclysm for the $i$th individual be $p_i$, and let post-cataclysm income be $y_{O^i}$. Then expected utility is given by

$$E(U^i) = p_i U(y_{O^i}) + (1-p_i) U(y_{A^i})$$

We assume initially that cataclysm is no respecter of income differentials, so that $p_i$ is the same for all $i$.

The policy under consideration involves raising $y_{O^i}$, and distributing the cost according to marginal benefit (i.e. "Lindahl pricing"). The marginal benefit for $i$ is denoted $G^i$ where

$$G^i = p_i U_{y^i} (y_{O^i})$$

$y_{o^i}$ being the new floor level of income, and $U_{y^i}$ the partial derivative of $U$ with respect to $y^i$.

In evaluating (2), we focus on condition c) above, and taking the simplest possible case, we let $U_{y^i} = 2a(y^i - y_{A^i})$

then $U_{y^i} = a(y^i)^2 - 2ay_{A^i} + f(y_{A^i})$, by integration

Now, $U_{y^i} > 0$ from b) above,

and to ensure that this is everywhere true, we impose certain restrictions on the constant in integration, $f(y_{A^i})$, viz:

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3See Savage & Friedman [8] for a detailed treatment of this formulation. Other treatments of behaviour under uncertainty are of course possible (see for example Luce & Raiffa [5 ch. 13]). Preference for the formulation used does not reflect any conviction of superiority—simply convenience. The discussion here is, of course, meant to be indicative rather than definitive.
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\[ f(y_{A_i}) > (y_{A_i})^2, \text{ since this gives} \]

\[ U_{y_i} = a(y^i - y_{A_i})^2 + c \]  

(4)

which is everywhere positive.

Then, \[ U_{y_i} > a(y^i - y_{A_i})^2 \]

so that marginal benefit is a quadratic function of the difference between \( y_{A_i} \) and \( y_0 \); i.e., \[ G > ap(y_0 - y_{A_i})^2 \]

(5)

To exemplify the translation of this result into a set of Lindahl cost-shares, and thus into a set of tax rates, we can take the data provided by Von Furstenberg and Mueller [10] for the U.S. Thus, the income distribution is broken up into deciles, and a cost-share associated with the median income-earned in each class. We can thereby derive table 1 which is an amalgamation and extension of Von Furstenberg and Mueller’s table 1 and 2. Manifestly, whereas Von Furstenberg and Mueller derive, for a plausible formulation of the utility functions in the altruism context, a set of cost shares giving a proportional income tax structure, in this case the tax structure is quite progressive. The transfer pattern is hence even more in the interests of vertical equity (as normally conceived) than that which the philanthropy model tends to generate.

So far we have assumed that the individual cannot insure privately against the cataclysms in question. As an alternative, we could allow the individual to purchase some (albeit inadequate) private insurance. In that case \( p^1 \) would vary

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\( ^4 \) On the basis of utility functions of the form

\[ U_i = a \log y_i + b \sum \log \frac{y_j}{y_i} \]

Von Furstenberg & Mueller calculate the Lindahl cost-share for each income group, given a policy of raising the floor level of income. Thus, the income levels of those poorer than oneself enter positively into one’s utility function. Costs are calculated on the assumption that raising the floor level of income to $3,000 costs only $1,432 (the difference between $3,000 and $1,568), the assumption being that individuals will continue to work at the pre-transfer level. Different (and perhaps more practically relevant) assumptions are possible, but we adhere to the Von Furstenberg-Mueller convention for purposes of comparison.
<table>
<thead>
<tr>
<th>Decile</th>
<th>Family Income Range</th>
<th>Median Income</th>
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<th>Y_n = $5000</th>
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<td></td>
<td></td>
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<td>Insurance</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Cost Share</td>
<td>Tax Rate</td>
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<td>Altruism</td>
<td>Insurance</td>
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<tr>
<td></td>
<td></td>
<td></td>
<td>Cost Share</td>
<td>Tax Rate</td>
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<td>-</td>
<td>-</td>
</tr>
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<td>4230</td>
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<td>.0015</td>
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<td>6866 - 8427</td>
<td>7652</td>
<td>.0717</td>
<td>.0215</td>
</tr>
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<td>.1666</td>
<td>.2174</td>
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<tr>
<td>10</td>
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<td>23578</td>
<td>.2209</td>
<td>.4208</td>
</tr>
</tbody>
</table>

(all figures are in terms of U.S. dollars 1970)
### Table II

**Benefit Taxation Patterns - Insurance (Some Private Facilities)**

<table>
<thead>
<tr>
<th>Decile</th>
<th>Family Income Range</th>
<th>Median Cost Share</th>
<th>Tax Rate</th>
<th>Cost Share</th>
<th>Tax Rate</th>
<th>Cost Share</th>
<th>Tax Rate</th>
<th>Cost Share</th>
<th>Tax Rate</th>
<th>Cost Share</th>
<th>Tax Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 - 3100</td>
<td>1568</td>
<td>-</td>
<td>-</td>
<td>-</td>
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<td>-</td>
<td>-</td>
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<tr>
<td>2</td>
<td>3101 - 5187</td>
<td>4230</td>
<td>.0058</td>
<td>.021%</td>
<td>.0194</td>
<td>.066%</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>5188 - 6865</td>
<td>6052</td>
<td>.0248</td>
<td>.059%</td>
<td>.0584</td>
<td>1.38%</td>
<td>.0050</td>
<td>.035%</td>
<td>.0017</td>
<td>.081%</td>
<td>-</td>
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<tr>
<td>4</td>
<td>6866 - 8427</td>
<td>7652</td>
<td>.0459</td>
<td>.086%</td>
<td>.0849</td>
<td>1.59%</td>
<td>.0224</td>
<td>1.23%</td>
<td>.0464</td>
<td>2.54%</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>8428 - 10,000</td>
<td>9205</td>
<td>.0679</td>
<td>1.06%</td>
<td>.1043</td>
<td>1.63%</td>
<td>.0467</td>
<td>2.13%</td>
<td>.0806</td>
<td>3.68%</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>10,001 - 11,695</td>
<td>10,825</td>
<td>.0918</td>
<td>1.21%</td>
<td>.1199</td>
<td>1.58%</td>
<td>.0763</td>
<td>2.96%</td>
<td>.1118</td>
<td>4.34%</td>
<td>-</td>
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<tr>
<td>7</td>
<td>11,696 - 13,650</td>
<td>12,635</td>
<td>.1192</td>
<td>1.35%</td>
<td>.1335</td>
<td>1.51%</td>
<td>.1121</td>
<td>3.73%</td>
<td>.1409</td>
<td>4.69%</td>
<td>-</td>
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<tr>
<td>8</td>
<td>13,651 - 16,142</td>
<td>14,802</td>
<td>.1528</td>
<td>1.48%</td>
<td>.1460</td>
<td>1.41%</td>
<td>.1579</td>
<td>4.48%</td>
<td>.1693</td>
<td>4.81%</td>
<td>-</td>
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<tr>
<td>9</td>
<td>16,143 - 20,000</td>
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<td>.1998</td>
<td>1.61%</td>
<td>.1597</td>
<td>1.28%</td>
<td>.2237</td>
<td>5.26%</td>
<td>.1996</td>
<td>4.71%</td>
<td>-</td>
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<tr>
<td>10</td>
<td>20,001 - 23,578</td>
<td>23,578</td>
<td>.2919</td>
<td>1.77%</td>
<td>.1749</td>
<td>1.06%</td>
<td>.3559</td>
<td>6.34%</td>
<td>.2397</td>
<td>4.27%</td>
<td>-</td>
</tr>
</tbody>
</table>
between individuals, presumably according to the income level, and we might variously assume that

\[ p_i^1 = \frac{b_i}{y_i} \]

\[ p_i^2 = \frac{b_i}{(y_i^o)^2} \]

where \( b \) is some constant.

Then, we would have alternativelly

\[ G > \frac{ab_i}{y_i^o} \left[ y_i^o - y_i \right]^2 \] (6)

\[ G > \frac{ab_i}{(y_i^o)^2} \left[ y_i^o - y_i \right]^2 \] (7)

Application of (6) and (7) to the cases already considered would yield table II. The former translates into uniformly progressive taxation (though not, of course, as progressive as in the application of (5) above—cf table I); even the latter, while regressive over the latter part of the range, is not obviously less equitable (in the traditional sense) than the proportional tax rates characteristic of the Von Furstenberg-Mueller model.

2. The Plausibility of the Model:

What we have done so far is to examine the pattern of transfers which might arise from the insurance motive given cost-sharing arrangements based on marginal benefits, and given minimal assumptions about individuals' attitudes towards risk, and about the nature of the contingencies involved. We have said nothing about the likely magnitude of such redistribution.

By attributing specific values to parameters, no doubt the optimal level of \( y_0 \) could be calculated. In the interests of continued generality, however, this step is avoided. Instead, we examine, in more abstract terms, the sorts of conditions necessary to ensure that the insurance motive will be Pareto relevant over a range not insignificant in distributional terms. Consequently we need to explore the circumstances under which individuals will prefer a public transfer programme, to, in the first instance, insuring through private market facilities, and in the second, simply enduring the risk of cataclysm.
We begin by examining the cost of providing insurance by raising the floor level of income. Suppose that floor level were set at $Y_0$; and consider the cost of raising $Y_0$ by one dollar. Let the proportion of individuals earning no more than $(Y_0 + 1)$ dollars be $\alpha_0$, and suppose initially that the transfer process induces no substitution effects between effort and leisure (i.e. that $\alpha_0$ is itself not dependent on the means of insurance). Then the total cost of raising $Y_0$ by one dollar is $\alpha_0N$, where $N$ is the population size. This cost will, under the benefit principle, be distributed among the $(1-\alpha_0)N$ “donors” according to their distance in income terms from $Y_0$ (as outlined in the foregoing analysis)—but speaking loosely, the cost to the “average donor” will be $\frac{\alpha_0}{1-\alpha_0}$ dollars for each dollar by which $Y_0$ is raised.

If $Y_0$ were set, for example, at the top of the lowest decile of income earners, this cost to the average donor would be about eleven cents per dollar increase.

Clearly, then, insurance via income transfer is pretty costly, and when disincentive effects are allowed for even this is a gross understatement. Moreover, as $Y_0$ rises, the cost of the operation per donor will generally rise at a faster rate, since not only is the number of recipients rising, but also the number of donors is falling—and probably, over the relevant range, $\alpha_0$ rises faster than $Y_0$.

The high cost of insuring via redistribution immediately poses two questions. Firstly, is it really conceivable that private market facilities may not be able to do as well? Secondly, is the risk of catastrophe sufficiently great to justify an expenditure of this magnitude?

In the first place, it is clear that there are, indeed, many contingencies which are pretty well completely uninsurable privately (nuclear war, domestic revolutions or hyper-inflation perhaps exemplify). What is not clear is that a public redistribution programme is likely to provide more effective insurance. Financial hedging against nuclear holocaust, internal upheaval or hyper-inflation may well be impossible through market channels; but governments themselves seem no less susceptible to war and revolution than individuals are, and in situations of hyperinflation, income transfers are inherently inadequate in providing the sort of protection required. There are of course other less catastrophic events such as fire, flood and earthquake which are customarily more localized in their impact and for which the redistributive policy may work, but for disasters of this kind, private insurance facilities usually exist as well. It is, in fact, far from obvious how public transfer programmes could improve on private arrangements under any circumstances.

The superiority of government in the insurance field rests on the existence of economies of scale in information collection and risk-pooling which can be mopped up by the government, but not by private firms—this presumably flows directly
from the government's larger size. Given the international structure of many of the larger insurance firms, such superiority is highly questionable. Moreover, insurance via public redistribution imposes additional costs on insurers which are of considerable significance. Firstly, transfer programs of the "floor-income" type insist that "sum assured" be the same for all individuals; given differing incomes and/or tastes, the imposition of this constraint implies a cost for those whose preferences exhibit risk-aversion most strongly (most conspicuously, the rich). Secondly, payments are made independently of the occurrence of cataclysm: many recipients are victims not so much of disaster as of simple poverty. Thus, unless cataclysm is sufficiently common to render the greater proportion of poor individuals in the former class, it seems unlikely that private facilities could not provide insurance to donors more effectively than transfer policy does. Moreover, even if market provision were hopelessly inadequate, unless the probability of cataclysm is high, many would rationally opt to sustain the risk; and this brings us to our second question.

At first sight, it seems inconceivable, even for highly unstable societies, that $p$ could approach the sort of value required to justify redistribution of any significance. If the "optimal" level of $Y_p$ were, for example, that associated with the top of the first decile of the income distribution, $p$ would surely have to take a value of at least one in twenty to justify the transfer policy on insurance grounds alone.\(^5\) Cataclysms would, in fact, seem to be somewhat less common.

Yet is such a value so implausible? Intuitive inclinations to think so arise because one tends to focus on annual probabilities as being relevant. If however the probability of cataclysm is considered in life-time terms, substantial values of $p$ may well be reasonable. For, if $x$ is the probability that the $i$th individual will be involved in a cataclysm in any year, the probability of avoiding cataclysm over an (expected) life-span of $n$ years is

\[
(1 - x)^n
\]

Thus, the life-time probability of cataclysm is given by

\[
\bar{p} = 1 - (1 - x)^n \tag{8}
\]

which for normal values of $n$, and not inconceivable values of $x$, can have reasonably significant values. For example, for $n = 70$ and $x = 0.3\%$ we have $\bar{p} = 0.2$.

We should note though that with parameters expressed in lifetime terms the cost of cataclysm tends to be correspondingly less significant, in which sense the attempted

\(^5\) Given risk aversion over the range, the minimum Pareto relevant value of $p$ will be less than one in nine.
resurrection is far from complete. Thus, the appropriate conclusion would still seem to be that most individuals in the “donor” class would, for any plausible value of p, opt to endure the risk of cataclysm in preference to a transfer programme of any magnitude.

Hence, even if private insurance facilities are grossly inadequate, it appears unlikely that the insurance motive will justify much in the way of redistribution within the Pareitian welfare framework. The Pareto relevance of the insurance motive in redistributational policy might, however, be established by appeal to two slightly different considerations.

In the first place, although the apparent cost of insuring via transfer is large, raising the floor level of income may serve to further objectives for the donor in addition to insurance. If, for example, the rich are also altruistically motivated, then raising the floor level of income provides both an insurance facility and satisfies certain philanthropic ends. Thus, while the insurance motive may not in itself go far towards justifying redistribution on a substantial scale within the Pareitian framework, it may nevertheless be an important ingredient in the total picture. This fact provides perhaps some justification for examining the pattern of transfers associated with the insurance motive, without overmuch concern with the extent of redistribution which this motive generates in isolation.

Where present together, the insurance and philanthropic aspects of income redistribution may be related in a more technical sense. If, as suggested by Vickrey [9, p. 42] individuals tend to be philanthropically inclined (“empathetic” in his terminology) towards those nearer to themselves on the income scale, the orientation which altruism takes in an uncertain world may be precisely towards reducing the risk of destitution for one’s “friends.” In such a case, the insurance concern applies not only to oneself, but also to others near one on the income scale. This has the effect of magnifying the p of equation (1), and makes Pareto relevant redistribution on the basis of the insurance motive more likely. It also suggests that fairly progressive redistribution may issue out of the Vickrey case, in spite of the fact that altruistic interest, there, only extends to those in a similar income position.

In the second place, the process of insurance may itself serve to reduce the probability of the cataclysm’s occurring. This is the case for example with unemployment insurance: payment offsets the reduction in income (and hence in spending) which would otherwise have occurred in the downward multiplier.

\[6\] If, as we have tended to argue, it is the high cost of insurance via redistribution rather than the inadequacy of demand for insurance per se which makes the model implausible, then, since simultaneous altruism acts to reduce the price, it may well have a very substantial, discontinuous, effect on demand for redistribution (i.e. there is some presumption that insurance demand for redistribution is highly elastic).
process, thus reducing the severity of the departure from full employment. In such cases, the change in the probability of cataclysm exhibits elements of "publicness"—it is enjoyed equally and totally by all, so that irrespective of whether the insurance is provided by private firms or not, some public intervention may be called for. Since individuals do not enjoy the full benefits of the insurance they themselves undertake in such cases, market failure can be expected.

It would seem in such cases that the "first-best" solution would be for the government to impose the appropriately calculated set of insurance subsidies, or to provide insurance itself at somewhat less than its market cost. This is, however, only one possibility another might be for the government to conduct the appropriate transfer programme oriented towards raising the income floor. In the unemployment case, for example, the guaranteed income level would work in a way analogous to the dole— with an identical built-in stabilising effect. If the market failure is at all severe, all may be made better off by a transfer programme of the optimal magnitude. In this case, however, the transfer programme is not necessary to achieve optimality: an efficient outcome could, presumably, also be achieved by subsidizing private insurance.

In some instances, however, the reduction in the probability of cataclysm is explicitly associated with the redistributive process, which means that the transfer programme may be necessary for efficiency independently of the distribution of the gains from trade. One of the more striking examples of this latter situation, perhaps, arises where it is fear of social unrest or crime which generates the need for insurance. Consideration of such cases brings us immediately to the "self-protection" motive, and it is to this that we now turn.

II. THE SELF-PROTECTION MOTIVE.

Throughout this discussion, we shall focus on the case in which it is fear of popular revolution which motivates redistribution. A similar analysis could no doubt be applied to the case of crime reduction, or even reduction in the probability of war (by appropriate international transfers) and many of the conclusions may well carry over to these situations. Here, however, we concern ourselves with the revolution case as being one which has traditionally been of some interest in the redistributive context.

The assertion that large income disparities tend to generate political upheaval is, after all, scarcely an unfamiliar one in equity literature. Indeed, one of the

standard points of reference in justifying income redistribution within traditional normative framework, has been the "potential political explosiveness" of inequitable income structures. The immediate implication is that removal of such inequities will serve to reduce the likelihood of revolutionary activity.

Whether or not such assertions are really justified is unclear. Standard literature has not, to my knowledge, established a persuasive case for income redistribution on such grounds, and in any case (as Tullock [9] has recently emphasized) much of the accepted writing on revolutions is not very useful in understanding the revolutionary process itself because it focuses on the "public goods" aspects and not on the strictly appropriable benefits and costs relevant for the analysis of individual behaviour. Moreover, even if it can be shown that redistribution from rich to poor works in the desired direction, the existing property rights structure may well be more efficiently defended by policies of a different sort.

The present analysis has, then, three basic objectives:

(i) to indicate that income redistribution is a legitimate anti-revolutionary strategy;
(ii) to demonstrate some presumption towards its superiority over other possible strategies;
(iii) to suggest the nature of the appropriate transfer pattern in this sort of context.

The discussion centres around a simple model of revolutionary behaviour similar to that outlined by Tullock in a recent issue of this journal (9). Unlike Tullock, however, the prime concern here is with a situation in which revolution has not yet become active: policy is oriented towards preventing revolution ever beginning, rather than stopping it from being successful, once begun. Thus, we consider a community which has for some time been stably partitioned into two groups:

a) group A, which is a small, possibly hierarchically structured, elite. It holds most of the formal political power and most of the community's wealth;

b) group B, which contains the major proportion of the population. Income levels in this group are not far above subsistence, and members are virtually impotent in formal politics.

In fact, any income distribution will do, but such a simple one makes for ease of analysis. All individuals are rational and non-altruistic: thus, in considering the initiation of revolutionary activity each considers only the potential costs and benefits accruing to himself. The object of any revolution undertaken, it follows, is
to shift the existing property-right structure in the revolutionary's favour. Any improvement in efficiency of resource use or the equity of the income distribution in a Lorenz curve sense is purely incidental to revolutionary objectives: these consist simply of increasing own income at someone else's expense. Thus, revolution can be viewed as being directed towards changing the identity or hierarchy of the elite, rather than the size.

In this context anyone may become a revolutionary—even the king himself if he can thereby shift property rights even more in his own favour. At the same time, all individuals have an interest in preventing revolutions from occurring—and this for two reasons:

First, the business of revolution is explicitly taken to involve a potential cost to all in terms of the risk of property destruction, injury and/or loss of life. Thus, no one derives positive utility from the revolutionary process as such, and all will be prepared to pay something to avoid the costs associated with revolution. We can assume that this cost is the same for all, independent of income position or participation.

Second, since non-participants cannot really expect to be made better off by the revolution, and must concede the possibility that they will be made worse off, most will be prepared to pay something to avoid that possibility. If the revolution succeeds, loss is of course more certain and more costly for current members of the elite. For the poor, the chance that they will be made better off by a change in regime is more significant, but there is also the possibility that they will be made worse off, and hence, risk-averse members of B at least will generally prefer the certainty of the status-quo situation. As we shall show, risk-lovers will not generally be non-participants. Clearly, then, if there exists a set of bribes to potential revolutionaries which serves to prevent revolution being undertaken, all may gain by the payment of such. What we need to determine are the characteristics of those most likely to initiate revolution—and the policies most likely to be efficient in discouraging them.

Consider, as a point of departure, the potential revolutionary's decision calculus. The $i^{th}$ individual will initiate revolutionary activity if his expected gain exceeds his expected cost, i.e. if

Thus we have no "entertainment value" term as Tullock has, although this is not a point of significance since Tullock himself is skeptical of its importance (cf. Tullock [7, p. 92]).

This may not be too unreasonable if the rich are better able to afford protection. Costs proportional to income may be just as good an approximation, but the choice is of no significance.
\[ E(U_{REV}^i) \geq E(U_{NON}^i) \]  

(9)

where \( E(U_{REV}^i) \) is i's expected utility if he starts the revolution

and \( E(U_{NON}^i) \) is his expected utility if he does not start the revolution.

Since the initial situation is one of stability, we can take it that i ignores the possibility that anyone else will start the revolution. Thus,

\[ U_{NON}^i = U(y^i) \]

(10)

where \( y^i \) is i's initial income.

In the simplest case,

\[ E(U_{REV}^i) = p_i U(y^i_s) + (1-p_i) U(y^i_o) - p_H H \]

(11)

where \( p_i \) is the probability for i that the revolution will be successful

\( y^i_s \) is the payoff if the revolution is successful

\( y^i_o \) is the payoff if the revolution is unsuccessful

\( U^i \) is i's utility function

\( p_H \) is the probability of injury or property loss

and \( H \) is the cost of injury or property loss.

The product \( p_H H \) is, as we have assumed, identical for all individuals.

Now, let us suppose that \( y^i_s \) and \( y^i_o \) are the same for all i—that is, all expect the same possible rewards and punishments—and further that the utility functions \( U^i \) are identical in respect of the valuations placed on \( y^i_s \) and \( y^i_o \). Then, we can rewrite (11) as

\[ E(U_{REV}^i) = p_i U^i + (1-p_i) U^i - p_H H \]

(11.a)

Then, individual i will initiate revolutionary activity if

\[ p_i U^i + (1-p_i) U^i - p_H H > U^i(y^i) \]

(12)

\[ \text{This is identical to Tullock's equation (4), p. 90, in all essential respects.} \]
This formulation immediately suggests a classification of policies designed to prevent revolution from occurring. We can thus distinguish between:

(i) "appeasement policies"—those which are designed to increase \( y^1 \), and hence the right-hand side of (12), the object being to increase the opportunity cost of revolutionary activity;
(ii) "repression policies"—those which are designed to reduce \( p_s \) (and increase \( 1 - p_s \)) correspondingly;
(iii) "threat policies"—those which operate by making the prospect of losing so terrible, that no one will run the risk (i.e., reduce \( y^1 \) and hence \( U_0 \) to lower levels).

Although we can distinguish these various effects easily enough in principle via their effects in equation (12), certain policies may well have multiple effects so that the distinction in practice may not always be apparent. Thus, effective threats might require current action, so that for example the imprisonment and torture of suspected political activists may achieve both repression and threat objectives simultaneously. Likewise, policies may work in opposite directions: repression by means of political censorship and legislation against private meetings may worsen individuals' valuations of the status quo, and thus encourage revolution via a reduction in \( U(y^1) \). More particularly, since the probability of revolutionary success, \( p_s \), is almost certainly dependent on the revolutionary's command over resources, increases in \( y^1 \) designed to increase the opportunity cost of revolution also work in the opposite direction via income effects on \( p_s \). In the light of this observation it is not even clear that appeasement policies will work in the desired direction, let alone be more efficient than other policy types.

Bearing in mind that appeasement and repression policies will normally work in opposite directions, the choice between the two largely hinges on whether the income effects on \( p_s \) outweigh, or are outweighed by, the increase in the opportunity cost of revolution associated with an increase in \( U(y^1) \). Let us suppose that the probability of success is a linear function of income, i.e.,

\[
p^*_s = a y^1
\]

(13)

There seems no particular reason why this should not represent a fair approximation. Then we need to show, in order to establish the legitimacy of appeasement policies, that an increase in \( y^1 \) will in general have a more than proportional increase in utility (i.e., will raise the opportunity cost of revolution by more than that proportion).

The analysis presented here is essentially the embodiment of three simple propositions:
1. that potential revolutionaries are risk-lovers;
2. that the poor are more likely to revolt than the rich;
3. that the appropriate transfer policy in this context is one based on the notion of equality of opportunity rather than that of income equalization.

The first of these propositions follows more or less automatically from the formulation of revolutionary calculus depicted in (12). Focusing on the left-hand side of the inequality, it is clear that risk-averse individuals will find the "gamble" between $U_A$ and $U_B$ unattractive relative to risk-lovers, and further that they will be more repelled by the possibility of accident, loss of life, damage to property. Thus, by appeal to figure 2, we can contrast the restrictions on $p_3$ relevant for initiating
revolution for risk-averse and risk-loving individuals, given common values of $y^1$, $U_x$, $U_o$ and $H$. The curve, $C_1$, applies for the risk-lover, and $C_2$ for risk-aveter. With $C_1$, revolution will be undertaken for values of $p_s$ such that

$$p_s(1) > \frac{y_o x}{y_o y_s} \quad (14)$$

With $C_2$, revolution will be initiated only if $p_s$ takes a value such that

$$p_s(2) > \frac{y_o y}{y_o y_s} \quad (15)$$

Manifestly, $p_s(1) < p_s(2)$: the conditions applying to the probability of success are much more stringent for risk-avers, so it follows that potential revolutionaries are most likely to be risk-loving.

This should not be taken as implying that such individuals necessarily derive utility from taking risks—it may simply mean that the marginal utility of income is increasing over the range (that is, that the revolution-initiator places a high valuation on the post-revolutionary outcome as compared with smaller income increases in the early range). Nor does it imply that risk-avers will never join revolutions—it is clearly conceivable that (14) may hold. But there does seem a very strong presumption that the revolution will not be initiated by such—the “hard-core” revolutionaries (those who start things going) are risk-lovers.

Manifestly, with $C_1$ operative, a proportionate increase in $y^1$ implies a more than proportionate increase in $U(2) - U$ is convex from above. It follows immediately then that increases in $y^1$ have a greater effect via the opportunity cost mechanism in discouraging revolution than in encouraging it through income effects on $p_s$ (given that (13) applies). Thus, in figure 3, if $y^1$ rises to $y^1_2$ such that $(y^1_2 - y_o)$ is twice $(y^1 - y_o)$, then we note that:

$$p_s(y^1) = \frac{y_o x}{y_o y_s}$$

11The effect of $p_s$ is simply to reduce $y^x$ and $y^o$ (expectations of gain and loss), if $y^x$ and $y^o$ are understood as not of $p_s$, then that cost can be ignored in the calculations. Thus (14) and (15) and the subsequent analysis abstracts from differential attitudes to the $p_s$ term, although these differences work in the same direction basically as those considered.
If ABB' applies,

$$p_s(y_2') = \frac{y_0 x'}{y_0' s} = \frac{2y_0 x_1}{y_0' s}$$

But ABD applies, so

$$p_s(y_2') = \frac{y_0 x_2}{y_0' s} > \frac{y_0 x'}{y_0' s} \quad (= 2 \frac{y_0 x_1}{y_0' s})$$
Since failure in the revolution may well mean torture and death, we can take it that $y_0$ is negligibly different from zero—in which case, the difference between $y_1$ and $y_0$ can be taken as representing absolute income. A proportionate increase in income, then, implies more than proportionately increased stringency in the conditions for revolutionary success.

This gives us our second proposition: there is clearly a presumption that the poor are more likely to revolt than the rich. Further, it appears that appeasement policies will in general work in the desired direction.

Bearing in mind that appeasement and repression policies operate in conflict over some range, the conclusion here tends to suggest that repression policies may often be dangerous in the pre-revolutionary context. Since the relevant group (i.e. of risk-averses) is moderately sensitive to changes in the opportunity cost of revolution, policies which serve to reduce the desirability of the status quo situation are somewhat suspect. Of course, such policies do not necessarily operate via income effects alone—but in some cases they seem to, and where they do, they seem unlikely to be of much effect in discouraging potential revolutionaries from becoming active.

We should note in passing that since $C_1$ is relatively flat in the neighbourhood of $y_0$, threat policies are less influential here than they would be, say, with the risk-averse. At the same time, threat policies do work in the desired direction, and we would generally expect the punishments for treason (even in stable societies) to be severe. In so far as these threat policies influence the status quo situation, however, the same problem applies as with repression.

One conclusion which seems to flow from all this is that redistribution from rich to poor will lead to an aggregate reduction in the likelihood of revolution. Such a conclusion seems at best doubtful. What we have shown via the second proposition is that an increase in income has a larger effect on the revolutionary inclinations of the poor than of the rich: but we have ignored any budgetary constraints. Once transfer policy is considered, budgetary constraints become immediately apparent: any increase in $y_1$ for members of B implies a corresponding reduction in $y_1$ for members of A. Given the assumed distributional structure, the reduction in income for members of A vastly exceeds the increase for members of B, and the net effect on the likelihood of revolution remains unclear.

What does emerge is that the distinction between poor and rich crucial in standard equity literature is insignificant in this context relative to the distinction between risk-lovers and risk-averse. Raising the floor level of income which seemed appropriate in the insurance case is unattractive here. Clearly more efficient is a redistribution from the risk-averse to the risk lovers (if possible organized in such a
way that the income effect on the probability of success is minimized) so that the opportunity cost of revolution is increased for the relevant group alone. With $C_1$ operative, we note that the expected utility of a one in $\frac{1}{n}$ chance of obtaining an income increase of $\Delta y$ vastly exceeds the utility of a certain income increase of $\frac{\Delta y}{n}$. Thus, a greater increase in the opportunity cost of revolution to the relevant group is achieved by offering them a small chance of a large income increment than a revenue-equivalent equal payment to all. \(^\text{12}\) Clearly, the reverse preference holds for the risk-averse. In this way, a policy which collects an equal sum (or an equal proportion of income) from all and returns it to all in the form of a small chance to win a large prize in a well-ordered lottery serves precisely the function of redistributing from risk-averse to risk-lovers—from the population in general to potential revolutionaries in particular.

In the light of this observation, it is interesting to note the importance placed on the concept of "equality of opportunity" in much recent equity literature. What this notion is taken as meaning, in most contexts, is that every individual, irrespective of income, should be given an equal chance to win a "prize" in some well-ordered income lottery, where prizes are often of large size and few in number. The emphasis is explicitly not on the equalization of incomes per se, but rather on the equalization of chances in a lottery in which prizes may be very disparate. Generally, since such equality of opportunity extends only over part of the income determination process, the notion translates in practice into the admonition that under-privileged groups should be given "some chance" to ascend out of the slough of poverty.

Consider, for example, the selection of individuals who are to receive higher education, the cost of which is to be borne entirely (or predominantly) by the public sector (i.e., by individuals other than those receiving the education). In this context, the notion of equality of opportunity usually implies that such selection should be made solely on the basis of "academic ability" (somehow measured), and should explicitly not be related to family income (the implicit assumption being that academic ability and family income are not positively correlated). Indeed, many have recommended full public support of higher education precisely so that such a selection procedure might be implemented.

Given that full government support prevails, and that academic ability and prevailing family income are unrelated, selection according to academic ability is equivalent in distributional (though not necessarily in academic) terms to selecting an individual with probability $\alpha$ to receive a large prize—viz. the discounted value of

\(^{12}\text{The income effect on the probability of success may well be less significant too, given that the income increase only accrues to one individual—inaactivity is purchased by the prospect of gain rather than the gain itself.} \)
the net benefit stream which accrues to the individual from higher education. The parameter $\alpha$ is the proportion of the population actually selected.

It is intuitively clear that if this prize is large enough, and $\alpha$ small enough, the total tax-expenditure policy may easily serve to increase the variance of the income distribution: all (including the poor in some measure) will contribute to cost, whereas the prizes are as likely to go to the rich as the poor, and in any case whoever gets a prize will be rich ex post. Since in many countries, the prize is large and $\alpha$ is small, educational policies\textsuperscript{13} instituted in the interests of equality of opportunity may well work contrary to the interests of equity in the traditional sense. Given that such policies tend to increase income dispersion, the phenomenon which needs to be explained is why demand for such policies is so persistent. The foregoing analysis supplies a possible answer: the risk-averse believe in equality of opportunity because it purchases protection from revolution; potential revolutionaries believe in it because they are risk-lovers. In spite of possible offence to standard equity measures, all may gain over a considerable range from transfer policies of this type.\textsuperscript{14}

III. SUMMARY AND CONCLUSIONS

The objective of this paper has been to explore two considerations which might generate Pareto desirable redistribution without requiring the prior assumption of a philanthropically inclined rich. These two considerations related alternately to two observations: firstly, that raising the floor level of income provides, in the face of disastrous and privately uninsurable contingencies, a form of insurance facility; secondly, that redistribution from rich to poor may be, over some range, an efficient way for the rich to enforce the existing property right structure.

On the basis of this analysis it seems unlikely that the "insurance motive" will often be Pareto relevant \textit{in isolation}, in which sense the insurance motive goes little

\textsuperscript{13}The education example may have special appeal in this context, since the intelligent may be precisely the most dangerous revolutionaries once activated. In so far as educational opportunity buys off the intelligent as well as the risk-loving, it must represent an attractive policy to those concerned to ensure the perpetuation of the existing property right arrangements.

\textsuperscript{14}All this may be substantially different in a situation where revolution is under way. Those who seek to enforce the existing property-right structure may well operate by attempting to minimize participation say by polarizing the hard-core revolutionaries--and this would imply policy oriented explicitly towards maintaining the passivity of the risk-averse. If redistribution were used in such a context, transfer patterns of a more traditional, floor-raising type would seem desirable. However, in this situation threat policies appear more promising, since the risk-averse are highly sensitive to changes in $Y_x$ (see $C_2$ in fig. 2).
way in itself towards freeing the possibility of Pareto desirable transfers from the universal philanthropy assumption. On the other hand, insurance considerations may be an important ingredient in determining the aggregate transfer pattern when other motives are operative. Where for this or other reasons the insurance motive is policy relevant, the distribution of marginal benefits tends to be progressive under a variety of assumptions concerning the relation between income and probability of disaster, and voluntary exchange pricing will generate a correspondingly progressive tax system. The outcome under the insurance motive is thus qualitatively similar to (although possibly more progressive than) that issuing from the philanthropy case.

By contrast, the self-protection motive, concerned with pre-empting the possibility of revolutionary activity by the poor, tends to generate a transfer scheme oriented towards "equality of opportunity" rather than income equalization. The reason for this is that those who initiate revolution are most likely to be risk-lovers, and hence a greater increase in the opportunity cost of revolution to these potential revolutionaries can be achieved by the offer of a small chance of a large income increase than by raising the floor level of income uniformly. Again unlike the insurance motive, there seems some likelihood that the self-protection motive may be relevant in isolation. Given that repression policies tend to worsen the status quo, and that risk-lovers are relatively more sensitive to changes in the status quo situation than to changes in the probability of success, appeasement policies of the appropriate type seem preferable. In this sense, there are good reasons for believing that self-protection motives may lead to Pareto desirable redistribution over a considerable range. It should be emphasised however that such redistribution is not necessarily equitable in the traditional sense.

Apart from the specific analysis of these two, hopefully interesting, cases and the distance covered in freeing the possibility of Pareto desirable redistribution from the need to assume universal philanthropy on the part of the rich, this discussion perhaps performs an additional function. It emphasises that redistribution may mean many things to many men—and hence, that in the Paretian framework at least, the pattern of transfers should reveal attributes which reflect the multiple ends those transfers serve.

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"Pareto-Optimal Redistribution": A Perspective

by

Geoffrey Brennan *

1. Introduction

1. It has generally been agreed that a severe limitation of the Paretian welfare framework in providing a basis for a prescriptive theory of public policy lies in its fundamental conservatism in questions of income redistribution. Because the central notion in the Paretian framework is that of the Pareto criterion, and because it is an essential feature of that criterion that no one be made worse off, deliberately redistributive policies involving the transfer of purchasing power from one group to another appear to be quite explicitly precluded within the Paretian welfare system: such policies make some individuals better off, but only at the expense of others.

This is not to say of course that all distributional judgment is inadmissible within the Paretian framework: some scope for distributional judgment remains in terms of the decision as to how the utility gains from Pareto desirable policies should be divided between individuals1. Such judgments would, however, be quite distinct from – and in a sense rather alien to – the Paretian framework itself. Moreover, they have not traditionally been thought to operate (at least under normal circumstances) in any way which would justify redistributions of the magnitude required to make a genuine assault on poverty, or to remove any except perhaps the most completely "repulsive" of inequities. Thus, the Paretian welfare framework apparently serves only to render the existing – ethically arbitrary – distribution of income sacrosanct.

Now, not all self-styled Paretians take such a strict view of the criterion's distributional constraint. Under some interpretations2, the Pareto criterion...
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is seen as providing a sufficient but not necessary condition for policy desirability. In the hands of this school, the Paretian framework can perhaps best be viewed as the partial specification of a social welfare function, known only up to its Paretian characteristics; the precise distributional attributes remain undetermined. The spirit of such an interpretation is almost indistinguishable from the hypothetical compensation variant of the New Welfare school—and, in fact, the appropriate formalization of the Paretian framework thus interpreted would seem to be provided by "social welfare function" exponents of the Bergson/Samuelson type, whose characteristic treatment of distributional questions is completely empty in any except a purely formal sense.

At the same time, other interpreters of the Paretian framework (in particular, those of the Buchanan school) have adhered to the distributional constraint much more strictly. For these writers, the Pareto criterion becomes both a sufficient and a necessary condition for policy desirability—the restriction that no one be made worse off is not so much an analytic convenience for the temporary waiving of distributional issues as an ethical principle to be pursued for its inherent virtue.

Fig. 1

Thus, if in fig. 1 a government measure permits an outward shift of the utility possibilities frontier from $\gamma_1$ to $\gamma_2$ and the initial situation is depicted by S on $\gamma_1$, any movement in a north-easterly direction from S is Pareto-desirable, and all points on the closed interval, TV, are equally acceptable. Points on $\gamma_0$ outside TV such as K and L are, variously interpreted, either quite unacceptable, or simply non-evaluable within the terms specified by the Paretian framework. Since the Pareto criterion is agnostic between points on TV, additional welfare judgments might be applied to isolate the distributionally most desirable of such points. These judgments would represent, however, something of an intrusion into the framework; the only distribu-


and under the Baumol interpretation of the Pareto criterion are logically precluded, since the social welfare function is inaccessible to the economist.
tional judgment intrinsic to the Paretian system is that which ensures, at least, the preservation of the initial utility levels for each individual.

2. A considerable amount of effort in the welfare literature has been directed towards freeing the Paretian framework from its distributional straitjacket – and indeed, much of the last two decades of discussion in welfare theory can be interpreted in precisely this way. Thus, for example, the hypothetical compensationist variant of the new welfare tradition by requiring, for a policy to be desirable, only that compensation be possible (not necessarily paid) sought thereby to extend the range of distributional agnosticism over all points on the dominant utility possibilities frontier. (In fig. 1, all points on the segment yO would be superior to S or any other point on AF.) By such means, it was hoped to permit a clear separation of "efficiency" considerations from the apparently more contentious distributional ones. Distributional judgments could be superimposed by those moral supermen qualified to make them without prejudicing the desirability of efficiency-oriented policies implemented within the hypothetical compensationist framework.

In an alternative direction, Buchanan and Tullock have suggested that the Paretian framework will admit some redistribution, because in the context of constitutional choice, some redistribution is the price which all are prepared to pay in order to avoid the large decision-making costs associated with decision-making rules which approach unanimity (i.e. which ensure that no one is made worse off). Hence, if Pareto-desirable policies are redefined to be those implemented under a decision-making rule which is itself unanimously accepted, then certain redistribution will in general be involved in policies undertaken within the Paretian framework.

By and large, however, the point of reference in such discussions has been to widen the Pareto criterion in such a way as to render it consonant with redistribution. By contrast, the prime concern in this paper is with circumstances under which the strictly applied Paretian welfare framework actually requires redistribution, and with the characteristics of the redistribution thus required.

Over the past few years, a considerable literature has grown up around this question. Beginning more or less with Buchanan [6] in his review of Musgrave's "Theory of Public Finance", interest can be traced through Head [10], Johnson & Ireland [16] and Olson [20] to Hochman & Rodgers' influential A.E.R. paper in 1969 [13], since when widespread concern over the possibility of Paretian redistribution has become something of a feature of recent welfare literature.


Most writers on the subject have proceeded via the observation that the distributional constraint inherent in the Pareto framework is expressed in terms of utility rather than income. Once the link between own income and own utility is broken, the possibility of mutually beneficial income transfers becomes apparent: "income transfers" become a commodity much like any other, and the Pareto criterion can be applied to determine the efficient output. If that efficient output is in fact non-zero, then the Pareto welfare framework will insist that the appropriate redistribution be made.

In a slightly more general way, we can look on all such situations as being characterized by a utility-possibilities frontier which is upward-sloping over some range (for example, as depicted in fig. 2). The upward-sloping segment indicates that over the relevant range the interests of all parties are coincident in that in order to achieve a utility increase for some individual, it is necessary to increase the utility level of another. Thus, in figure 2, F lies on the utility possibilities frontier, but is dominated by (i.e. less preferred to) all points within the feasible area FES. In this diagram, each individual achieves his utility maximum at a point where the other's is non-zero – in contrast to the normal formulation where the utility-possibilities frontier is downward-sloping throughout the entire range. Manifestly, the distribution of utilities consistent with the Pareto framework is delimited to the range determined by the position of points G and E: once all Pareto-desirable policies have been implemented, the ultimate utility distribution must lie on CE.

II. The Significance of Pareto-Optimal Redistribution

1. The significance one attaches to the possibility of Pareto-desirable redistribution depends largely on the sort of Pareto one is. If, as in the Buchanan interpretation, the framework's distributional constraint is to be interpreted strictly, the existence of Pareto-optimal transfers implies that...
the Paretian framework, even narrowly applied to "in-period" decisions (i.e. without recourse to the constitutional level of decision-making), may nevertheless be much less conservative distributionally than has traditionally been thought. If, on the other hand, one is a Paretian of the Baumol type (or more generally a "social welfare function" exponent of the Samuelsonian variety), the prime interest in Pareto-optimal redistribution derives from the possibility of putting some teeth into an otherwise completely empty, formalistic treatment of distribution questions, without the need for recourse to anything other than standard tools of analysis.

There are, then, two questions of interest. Firstly, to what extent does the possibility of Pareto-desirable redistribution relieve the Paretian framework of its status quo orientation in distributional matters? Secondly, what is the precise nature of the transfer pattern that issues, under various assumptions, from a purely Paretian formulation of redistributive "objectives"? We shall deal with each of these questions in turn. For the remainder of this section and most of section III, the primary point of reference is the former aspect; while towards the end of section III and throughout section IV, the main concern is the pattern of redistribution which one might expect in a Paretian world.

The extent to which Pareto-optimal redistribution succeeds in freeing the Paretian framework from its apparent distributional conservatism depends on two features of the situation. In the first place, and most obviously, it depends on the extent of the range over which the upward-sloping portion of the utility-possibilities curve applies. If the curve is sausage-shaped, as in fig. 3 (a), the range of possible utility distributions which are acceptable within the Paretian framework is strictly delimited — utility distributions outside the range set by the slopes of OE and OG are inconsistent with the Paretian framework, in the sense that distributions of utility outside that range cannot exist once all Pareto-desirable policies have been implemented. In this case, major redistributational adjustments are an integral part of Pareto-desirable policy. If on the other hand, the utility-possibilities curve is predominately of traditional shape (as in fig. 3 (b)), traditional concern over the distributional conservatism of the Paretian framework applies: quite glaring inequities may still remain after all Pareto-desirable policies have been put into effect.

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Fig. 3a

Fig. 3b

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In the absence of market failure, an ex post test of the extent of Pareto-optimal redistribution would be provided by examining the amount of redistribution occurring voluntarily through private channels. In that case, the position of E in fig. 2 could be induced from the distribution of utilities ultimately prevailing in the market. Since it has normally been assumed that the free market distribution of income, even after voluntary philanthropy, will be violently inequitable, the demonstration of market failure (in the Paretian sense) emerges as a crucial ingredient in demonstrating the distributional congeniality of the Pareto criterion.

Market failure is important in another sense. The ultimate utility distribution clearly depends on the way in which the utility gains in moving away from dominated points on the frontier are divided between the various parties. From F (in fig. 2), all points on the line segment ES are Pareto-desirable, but manifestly S is rather less "conservative" distributionally than E. The division of utility gains (i.e. the choice of points from ES) depends essentially on the mechanism through which the move is effected. The market mechanism, on the one hand, given normal assumptions of large numbers, perfect information and so on, specifies that division of utility gains associated with absence of discrimination over infra-marginal units of goods exchanged. There is, however, no institutional counterpart to this uniform pricing rule in the political context. Thus, it seems reasonable to expect that the division of utility gains will be different from that prevailing under perfect market adjustment. Since the normal market outcome turns out to be that associated with minimum redistribution consistent with the Pareto criterion, the political mechanism is therefore less conservative distributionally than the market.

2. These arguments may perhaps be most easily demonstrated by appeal to the following example: Let us assume that upper income groups derive altruistic satisfaction from increased consumption by the poor, and let us suppose that over some range short of complete income equalization, the rich derive greater utility from the poor's consumption than from the equivalent amount of own consumption. In the simplest model, consider a two-person community in which I is rich and II is poor. Then

\[
U_I = \mu_I (Y_I, Y_{II}), \\
U_{II} = \mu_{II} (Y_{II})
\]  

where \(U_i\) is i's utility level, and \(Y_I\) is income for own use. Let \(Y = Y_I + Y_{II}\) be national income, and let the preferences of I and II be those as depicted in fig. 4. If the initial distribution of income is given by \(C (Y_{II}, Y_{II}^*)\), then it is clear that transfers from I to II will, over some range, shift both individuals to higher indifference curves (for example, to \((Y_1^*, Y_{II}^*)\) at E).

1 Although in the simultaneous presence of public redistribution there can be no ex post test of this, the proposition may not be impossible to test (for example, by examining the sensitivity of private philanthropic giving to changes in the degree of progression in the impact of the total budget).

2 imposing a conceptual one, namely the Lindahl-rule.

3 Although there is a conceptual one, namely the Lindahl-rule.
In the basis of the given level of national income and the preference maps as given, it is possible to translate the income-possibilities plane (fig. 4) into a utility-possibilities curve by taking the various “indifference” combinations along AB. This yields a curve of the sort depicted in fig. 5 with the characteristic upward-sloping portion over the range A'E'.

From an initial distribution of income at C, I will give to II until he (I) reaches his utility maximum at E. This represents the point at which I's marginal evaluation curve for transfers (derived by taking the slope of successive indifference curves along AB) cuts the marginal cost of transfers curve (which is horizontal at unity, since the objective cost to I of a dollar's transfer to II is simply a dollar). At E, I's demand for transfers is exhausted.

In the situation as depicted, there seems no reason for doubting that the relevant transfers can be affected through a voluntary market mechanism, and hence no reason for supposing that the ultimate outcome might be any point other than E. It should be clear that E does represent a Pareto-optimum—there are no further gains from trade beyond this point. In some ways this is a surprising result. Although the recipient benefits at the margin from transfers made to him by the donor, it is nevertheless possible to achieve Pareto optimality while ignoring recipient preferences. This situation seems to contrast with what appear to be similar interdependencies arising in the case of public goods (or externalities more generally) where the preferences of all members of society who benefit from an activity are relevant in determining the optimal level of that activity. Thus, if I buys some defence from
which II benefits at the margin, optimality requires that II’s preferences also be taken into account in determining the amount of defence provided. This is apparently not the case with “transfers”: although the utility function formulation in (1) above has the interdependence relationship normally taken to be characteristic of externality situations, the expected Paretian conditions do not apply. This is a point we shall have cause to return to.

If, instead, I is a group of equal-income/preference-identical individuals, then redistribution to II becomes a “public good” for each member of I, in the sense that each would enjoy the benefits of redistribution to II irrespective of whether he himself contributed to its cost. Thus, as in other cases, each member of I has an incentive not to contribute to any voluntary transfer programme, and redistribution would tend to be “underexpanded” in the freely operating market.

Suppose that, in response to market failure, redistribution is effected through the political mechanism. In order to maintain the Paretian normative framework, we might alternately postulate an institutional government (say a “benevolent” dictator) assumed to be equipped with all the necessary information, and motivated in a Paretian way, or a decision-making rule

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which requires unanimity for policy acceptability. Given the former environment, the government (dictator) will be constrained to select some point on the line segment ES in fig. 4 (corresponding to E'S' in fig. 5). Manifestly, S corresponds to the point of maximum redistribution consistent with the Paretian constraint that neither party is to be made worse off than he was initially (at E); at S, all the gains from redistribution accrue to I. The market equilibrium, E, corresponds to the point of minimum redistribution – here, the donor (I) achieves maximum gains from redistribution. There seems to be no particular reason why the assumed dictator will select E from the infinity of points along ES – some other point will almost certainly prevail.

Or, if we take instead an unanimity decision-rule, two basic differences between the political and market mechanisms are relevant. Firstly, the transfer process in the market is incremental: as the movement along CE is effected, I moves to successively higher indifference curves and the Paretian constraint becomes operative for a continuously changing "initial" point. By contrast, in the political arena, parties vote on a total tax-expenditure operation, selecting that which they like most: the process is not essentially incremental, and the choice facing I is between a set of all-or-nothing proposals. Secondly, in the market process (as we have seen) II's preferences do not count beyond the consideration that he be made no worse off – the "optimal" outcome is determined by the point of tangency between I's indifference
curve and the budget line. In the political context, II’s preferences do count because policies must receive II’s explicit approval before they can be implemented.

In either case, the expectation is that the market and political mechanisms will differ with respect to the division of the gains from trade. Since the market division of these gains is, in this two-person case, that associated with minimum redistribution, a political mechanism of this Paretian sort will almost certainly carry redistribution further than the market would.

Given this result, three interesting corollaries follow. Firstly, under publicly organized redistribution, I will always complain that redistribution has been carried too far, even though he is made better off by the total transfer operation. The reason for this is that at points beyond E, I’s valuation of a dollar’s transfer is less than a dollar. Of course, this in itself implies nothing about the efficiency (or otherwise) of the redistribution, since it ignores II’s preferences for transfers from I to II.

Secondly, under a Paretian government, I will always attempt to conduct as much redistribution as possible privately through the market, since this ensures him a higher proportion of the utility gains. If in fig. 6, the slope of \( FF' \) indicates the division of utility gains under political provision (\( FF' \) being less steep than CE), then political provision from C yields the point P. Thus, if I can conduct redistribution through the market to point D, say, he has every incentive to do so, since he can ensure thereby that Q and not P will be the final outcome. Thirdly, and relatedly, the ultimate utility distribution will be more equitable the more inequitable, ceteris paribus, the initial distribution.

4. This simple model has served to indicate that the distributional conservatism of the Paretian framework depends on two considerations: a) on the extent of the upward-sloping portion of the utility-possibilities frontier (i.e. where E lies on AB in fig. 4); b) on the extent to which redistribution is implemented publicly rather than through private channels.

Three general questions immediately present themselves. Firstly, what situations can generate upward-sloping portions in the utility-possibilities curve, and how prevalent are these? Secondly, to what extent do these situations establish a case for public rather than private redistributive programmes? Thirdly, what are the expected characteristics of the redistributive patterns these various situations generate?

We examine these questions more or less in turn.

III. Paretian Redistribution and Public Goods

1. It is possible to distinguish two types of situation where the utility-possibilities curve is upward-sloping over some range: a) where income transfers in the narrow sense give utility gains to donors (and by assumption recipients) and are Pareto-relevant over part of the range; b) where there are positive utility interdependencies of a more general sort.
In fact, we shall concern ourselves with the latter circumstances first, and this both because it is analytically illuminating in relation to the more specific cases, and also because it enables us to focus later on those aspects of the income transfers case which cannot be conveniently handled in general terms.

The basic result that we are concerned to show is that the phenomenon of upward-sloping segments of the utility-possibilities frontier arises whenever there exist Pareto-relevant positive utility interdependencies. We show this by reference to the paradigmatic case — by examining the change in utility possibilities which is wrought when public goods are introduced into an erstwhile private goods world.

In fig. 7, TT' depicts production possibilities of two goods, X and Y, and the preference maps subscripted I and II indicate the utility functions of two individuals I and II. These remain invariant throughout the analysis.

Suppose initially that X and Y are private goods, in the standard sense that the more of X or Y that I consumes, the less is available for II. The utility maximum for I in the two-person world occurs at E_I where I consumes X_I of X and Y_I of Y. Since II is consuming zero quantities of both goods at I's utility maximum, II enjoys zero utility at that point. At II's utility maximum at E_{II}, II is likewise consuming all the available quantities of both
goods ($X_H$ of $X$ and $Y_H$ of $Y$). In both cases then, one party's utility maximum is associated with zero utility for the other: the utility-possibilities frontier takes traditional shape (as in fig. 1, for example).

Suppose now that, without any change in production possibilities or preference functions, we allow $X$ to be a public good¹, while $Y$ remains private². The maximum utility level for I is again $E_I$, consuming $X_I$ of $X$ and $Y_I$ of $Y$. At $E_I$, since $X$ is public, II consumes $OX_II$ of $X$. Hence, II enjoys utility given by the indifference level $ii^r$. In order to reduce II's utility level to zero, I must consume at $T$ (involving zero public goods consumption). Manifestly, $ii^T$ represents a lower level of utility than $ii^r$. In the same way, I's utility maximum at $E_{II}$ implies consumption of $OX_{II}$ of $X$ for I as well as for II, although II consumes all the private good available at that point. To ensure a zero level of utility for I, II must accept indifference level $ii^n$ at $T$, which involves a utility loss to both parties³. The corresponding utility possibilities schedule is thus as depicted in fig. 8a): it is characterized by upward-sloping portions in the early parts of the range (i.e. close to the axes). Thus, apart from its more important and more celebrated allocative connotations, the existence of public goods does seem to imply something of distributional significance, in the sense that it delimits the range of utility distributions which are acceptable under the criterion of economic efficiency (in this case, to $E_{II}$ in fig. 8a)⁴).

Having made this point, it is as well to be clear what it does not imply. Many writers have focussed on externalities arising from altruistic motivations as being of a distributional significance in their own right. Thus, for example, if I wishes II to consume education for altruistic reasons (as distinct from any benefits of a more self-oriented kind, such as reductions in social unrest, or the benefits of an "educated" community), then II's education will enter as an argument in I's utility function. Hence, II's education is a non-private

¹ A public good is defined as one which is made equally available for consumption to all, so that consumption by each individual is equal to total production: $X_I = X_H = X$.

² This situation does not purport to have any economic realism – the technique is for analytic purposes only, the objective being to indicate how the utility possibilities frontier alters shape, not position.

³ It may seem strange that a Pareto optimum can be defined at $E_I$ say, when III's marginal evaluation of public goods (the slope of $ii^T$ at $X_I$) is non-zero. The interpretation to be given to III's 'marginal evaluation' in this case is however, not the amount of $Y$ which III would be prepared to give up to consume the marginal unit of $X$ at $X_I$ (since III has no $Y$) but rather the amount of $Y$, III would need to receive in order to compensate him for a marginal reduction in $X$ production.

⁴ Note that in this context, II will (in the political mechanism) vote for as much public good as possible (i.e. production at $T'$ in fig. 7) since this makes him best off. The second-best utility possibilities frontier associated with increased public goods output beyond $E_I$ yields the dotted line $E_I'T''$ in fig. 8. This may establish $T''$ as a quasi-threat point in the political "game", and II may be able to ensure himself a higher utility level than $ii^T$ (at least $ii^{T''}$) giving the ultimate solution somewhere on the segment $F_{II}T''$. In this context too, then, the political mechanism seems likely to be less conservative distributionally than the market outcome; although here there is no necessity for state intervention unless I is redefined as a group of individuals identical with respect to income and preference patterns.
good, while II's consumption of beer is not. In this case, the problem has normally been formulated as a standard externality problem and the point made that this will normally imply the payment of a subsidy to II designed to increase II's consumption of education. The distributive implications of such subsidies are clear. From an analytic point of view, however, the origin of the externality in altruistic motivations is of no significance whatever—the problem would be the same if II's education were valued by I for purely self-oriented reasons. Yet, if this formulation of the situation as a standard

![Diagram](image)

externality problem is a valid one, no particular distributional significance necessarily attaches. For it would be possible to organize a Pareto-desirable policy which ensured an optimal outcome and yet left II no better off (as is possible in all externality situations)—and the Pareto-criterion would find this outcome perfectly acceptable. Of course, given some pricing rule which serves to divide up the gains from "transfers-in-kind" in a way which gives II something, II's utility will in fact be increased. But this point is identical with that considered in relation to fig. 1. The Pareto-criterion does not actually require redistribution in a logical sense, any more than it does when two individuals trade private goods, and are both (potentially) made better off.

This can imply either of two things: that writers have been confused about the distributional implications of externality situations; or that this particular formulation of the problems (as a standard externality situation) is an inadequate specification of the underlying notion. An alternative, and ostensibly more satisfactory, formulation would be one in which both II's
education and II's income enter I's utility function as arguments. If such were the appropriate formulation, however, only the entrance of II's income into I's utility function is of distributive significance; the education preference can then be handled as a standard externality.

Thus, we do not deny that in some instances donors may wish to give specific goods rather than income to the poor. What is denied is that this can be formulated satisfactorily simply as a standard externality problem. The internalization of externalities no more implies anything of distributational significance than does private trading. The only significance of externalities in the distributional context is simply the much more general point that has been outlined above with respect to public goods - namely, that a "corner solution" can obtain where the full appropriation of II's utility gain from public goods production is impossible because II has inadequate private goods initially to make such appropriation feasible.

Seen in this light, one would not want to make too much of the general point. The existence of publicly provided defence, health, law and order and so on does imply that those with zero disposable money income do enjoy the benefits of protection, reduced probability of disease and so forth - but this may not be much comfort in the absence of food, shelter, and adequate clothing. In other words, the mere existence of public goods does not go anything like far enough in ensuring the sort of utility redistributions normally regarded as necessary for the satisfaction of equity standards.

2. In an all public goods world, however, the upward-sloping segments of the utility-possibilities frontier are much more significant. Without any change in production possibilities or consumers' preferences, let us assume that fig. 7 depicts a world in which all goods are public - i.e. both X and Y are totally and equally consumed by I and II. Then, the corresponding utility possibilities surface is that depicted in fig. 8b). For at I's utility maximum, $E_I$, both I and II consume the output combination $(X_I, Y_I)$ - yielding a utility combination of $u_I$ for I and $u_{II}$ for II. At II's utility maximum, $E_{II}$, both consume the output combination $(X_{II}, Y_{II})$ and I achieves indifference level $i_I$ for I. To reduce the utility of either party to zero, the levels of X and Y produced must be zero, so that $U_I$ and $U_{II}$ go to zero simultaneously. The set of Pareto-optimal points in output space is given by the line segment $E_I E_{II}$ on T'T. The overall effect is to delimit the range of Pareto-optimal utility distributions quite severely: nothing less than $i_I'$ for I or $i_{II}'$ for II is acceptable within the Pareto framework.

It may be tempting to regard this point as one of some analytic interest, but zero practical relevance. After all, it has been generally thought that the all-public goods analysis is of use only in examining committee decisions and that no normal economic analogue exists. It is clear, however, that the essential characteristics of an all-public-goods world apply precisely in the environment of prime interest here: namely, where the incomes of all individuals enter into the utility functions of each. For in this case, each individual's

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income is a public good (at least in some senses) and no purely private goods enter the model.

This implies that situations of utility interdependence generating Pareto-desirable income redistributions need to be handled in the analytical context set by a world in which there are no private goods. And this is not a trivial observation, because such a world is rather different from the more familiar private goods world. It is appropriate to set out some of its peculiarities here.

In the first place, and perhaps most significant, since no private goods exist, there is no numeraire - no good in terms of which "side payments" can be made. One characteristic property of trade in normal situations is that it is "market-clearing", by which we mean that it reconciles the apparently conflicting desires of differing parties. After the institutional framework for trade has been specified, each party operates in the interests of the other - trading implies the transformation of the individual's interests so that they are coincident with others. Adam Smith's butcher works not because he wants to, but because others want him to; by paying the butcher for his services, these others ensure that he "voluntarily" operates in their interests. In the absence of a numeraire, however, reconciliation of conflict by "side payments" is impossible.

This means, in turn, that welfare criteria like those employed in the hypothetical compensation variant of the New Welfare School are strictly
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speaking non-operational. "Compensation" even of a hypothetical kind is in a basic sense inconceivable. To put the same point in a slightly different way, the optimum condition of exchange (the analogue to the $\sum MRS = MRT$ rule in a private-public context) necessarily embodies an explicit distributional parameter: we cannot separate out (even conceptually) distributional from allocative considerations.

The relevant optimum conditions for an all-public-goods world are, in fact, the following:

a) marginal rates of technical substitution between factors must be equal in all industries (as in the private goods world) to ensure production on the production possibilities frontier;  
b) the marginal rate of transformation should lie between the marginal rates of substitution of the various parties at the individual maxima along the production possibilities frontier. The point in the range $E_1 E_2$ in fig. 7 for which we finally settle is dependent solely on distributional criteria.

3. We have already looked briefly (in section II) at an example of Pareto-desirable redistribution arising from altruism on the part of the rich. Recognizing this situation as one in which all goods are public, we can apply the optimum conditions analysis for an all-public-goods world to explain some of the apparent peculiarities of that example:

1 This delimits the range of production points $TT'$ in fig. 8b.

2 Let $G, H$ be public goods - produced according to $f(G, H) = 0$.

Let $A, B$ be two individuals, such that $\mu_A = U^A (G, H)$  
$\mu_B = U^B (G, H)$

To obtain Pareto optimal conditions, let $\mu_B = \mu_B$ and maximize $\mu_A$. Then, we maximize

$$U^A (G, H) + \lambda [U^B (G, H) - \mu_A] - qf (G, H) = 0$$  
$$U^A + \lambda U^B - qf_B = 0$$  
$$U^A + \lambda U^B - qf_B = 0$$

from (2) $q = \frac{U^A + \lambda U^B}{f_B}$

Thus, if $\mu_A > \mu_B$, then, $U^A = \frac{f_B}{f_A}$ is required for optimality.

(i.e. optimal output locus is between individual utility maxima on the transformation curve.)
we should not expect that optimality in transfers will be given where 
\[ 2\text{MRS}^I = \text{MRT} \]

since this is an optimality condition for an entirely different situation;

(ii) likewise, however, the standard private world 
\[ \text{MRS}^I = \text{MRT} \]

conditions, which was recognized as sufficient for optimality in that case, is exposed as only one of a number of possibilities – and, moreover, a limiting case. More generally, we require

\[ \text{MRS}^I < \text{MRT} < \text{MRS}^II \text{ when } \text{MRS}^II > \text{MRS}^I \]

(or \[ \text{MRS}^I > \text{MRT} > \text{MRS}^II \text{ when } \text{MRS}^I > \text{MRS}^II \]).

Thus, for example, if we derive the donor's marginal evaluation curve for transfers (as in fig. 9) we require that this marginal evaluation be less than or equal to marginal cost. Since, however, we cannot under the Pareto-criterion make the donor worse off than at zero transfer, we are constrained to lie in the range \( Y_{l^b} \leq Y_{l^d} \), where \( Y_{l^b} \) is calculated such that triangles \( BEV \) and \( CEX \) have equal area. Put another way, we can without contravening the Pareto-criterion appropriate \( l \)'s consumer surplus from the transfer process and pay it to \( Ii \): since it must be "paid" in terms of further transfers, however, this gives the equilibrium output of transfers at a point on \( l \)'s marginal evaluation curve below the marginal cost curve. \( l \)'s consumer surplus is paid to \( Ii \) in terms of the good itself – but this does not imply any allocative distortion;

(iii) we might expect in a standard externality case that if \( Ii \) were motivated by envy towards \( I \) so that the poor actually derive marginal disutility from the high incomes of the rich, then a case for redistribution would be
immediately established: Pareto-optimality would require appropriate transfers to internalize the externality. Manifestly, no bribe is possible. Individual II cannot pay I to reduce I’s income, since the very act of payment increases \( Y_I \) and reduces \( Y_{II} \). Thus, not all sorts of utility interdependence involving income will necessarily generate redistribution. This is of course clear if we direct attention to the relevant utility-possibilities frontier: mutual antagonism between rich and poor in no way immediately generates the upward sloping segment characteristic of the altruism case — rather the opposite, since if higher \( U_I \) reduces \( U_{II} \) (and vice versa) over and above the reduction in the consumption possibilities implied by production constraints, the welfare frontier becomes yet steeper\(^1\).

The general point to be made is that extrapolation from a public-private (or purely private) goods world yields results which are unlikely to apply to the income distribution case. One cannot simply trust to educated intuition to indicate optimal outcome, because this is not familiar terrain: results which appear to be efficient may easily turn out not to be, and conversely.

4. Let us summarize the main thrust of this section:

Having recognized the importance of upward-sloping segments of the utility-possibilities frontier in freeing the Pareto framework from its ostensible distributional conservatism, we have been concerned to isolate those situations where this phenomenon occurs.

Starting at the most general level, one such instance arises when public goods are present. In this context, however, it is necessary to distinguish sharply between positive externalities generally which are of no inherent distributional significance, since all the gains can accrue to either party, and “corner solutions” in cost-sharing which do have some inherent distributional implications (albeit minor ones). The importance of this latter phenomenon is, however, much greater in an all-public-goods world, and this sort of world seems to provide precisely the generalised analytical depiction of the situation prevailing when each individual’s income enters positively into each other’s utility function.

This observation carries with it three implications:

(i) the Pareto conditions themselves involve explicitly distributional parameters — no unique “efficient” outcome can be defined without further distributional judgements;

(ii) in the absence of a numeraire, the “hypothetical compensationist” interpretation of the Pareto framework requires redefinition, since no “compensation” of a hypothetical nature is conceptually feasible;

(iii) generalization of standard “efficiency” results based on situations where private goods exist are apt to be misleading.

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\(^1\) This is not to say that Pareto optimal redistribution is impossible in the malice-envy environment, but it depends on a rather more complicated body of reasoning than straight-forward extrapolation from the externalities case.
"Pareto-Optimal Redistribution": A Perspective

Of these, only the first is in any way important – yet it in itself is quite devastating enough. For even if the various models of Pareto-desirable redistribution turn out to be plausible, the Pareto-criterion cannot specify anything more than a range of distributional outcomes – and the extent of indeterminacy may well be as substantial as the amount of redistribution required (i.e. in fig. 4, distances CE and ES may be identical).

Thus, those circumstances in which the Pareto framework succeeds in establishing a case for substantial redistribution from rich to poor are precisely those in which little of any specific nature can be said about the ultimately desirable distribution.

This should not be construed as implying that the economist who constrains himself to say only those things which fall within the ambit of efficiency considerations can say nothing of any interest about desirable income redistributions – for certain income distributions (hopefully the most inequitable ones) can, on the basis of the Pareto-test, be completely excluded. But one cannot induce from this that there is no further need for external distributional norms once the possibility of Pareto-desirable redistribution is accepted. If the central object of the Pareto-optimal redistribution literature is to "sell" the Pareto framework to those who in the past abjured it for reasons of its distributional conservatism, it has failed. For one cannot lay to rest the presumption of conservatism without simultaneously raising the equally appalling spectre of policy indifference over a potentially enormous range of distributional outcomes.

Far from establishing a case for sole reliance on the Pareto-criterion to solve both allocative and distributional objectives, it seems to me that an appropriate message from the literature is that, where substantial Pareto-desirable redistributions seem warranted, those who wish to make redistributive prescriptions can do so, confident that the conflict between distributional and allocative objectives is probably much less severe than has traditionally been thought. Even if the ultimate distribution does not serve to make everyone better off, at least the allocative cost of the inefficient initial income distribution has been removed and this gain can be set against the allocative cost (via inevitable substitution effects between effort and leisure, consumption and saving etc.) of the redistributive process itself. In the context of the familiar Musgrave welfare framework, the equity and efficiency goals may over a large range be seen as working essentially in the same direction.

For this reason, it is still very much of interest to investigate the nature and extent of the redistribution required by efficiency norms. Hopefully, we should at least be able to indicate what distributions the Pareto test excludes.

Moreover, the case where incomes of all individuals are public is of some inherent analytical interest, since it is one of the few cases where the all-public-goods analysis can be applied.

Thus, the various cases of mutually beneficial income transfers still need to be explored, and it is to such exploration that we now turn.
IV. Pareto-Desirable Income Transfers. Some Specific Cases

It is not the objective of this section to provide a complete analysis of all the possible instances where transfers which are beneficial to both donor and recipient may arise. Rather, we hope to provide a bird's eye view of the main results and focus attention on certain salient features of the various models. Bearing in mind our point of reference in interpreting the literature, a number of specific questions assume major importance:

(i) in which cases is the redistribution necessary for Pareto-optimality rather than one means of achieving it associated with a special division of the gains from trade? That is, in which cases do we observe an upward-sloping utility-possibilities curve, and in which cases not?
(ii) what conditions need to be imposed on the models to ensure that the redistribution is Pareto-relevant, and how plausible are these?
(iii) under what circumstances is market failure associated with the redistributive desires of individuals, so that public transfers are necessary?
(iv) what are the essential characteristics of the transfer pattern which results from the various motives, and what is the likely distribution of costs among tax-payers?

The standard method of categorizing Pareto-optimal redistribution is according to donor's motive, and it is convenient to adopt that method here. Accordingly, we distinguish between:

(i) the philanthropic motive;
(ii) the insurance motive;
(iii) the protection-from-revolution motive;
(iv) the motives of malice and envy;
(v) the desire to optimize individual income-streams over time in the face of capital market imperfections.

The list does not necessarily purport to be exhaustive.

We should note that these motives are in no sense mutually exclusive — all may be operative simultaneously. Thus, "income-transfers" viewed as a commodity in donor's consumption exhibit Marshallian joint supply in the sense that one dollar of transfer may provide a number of distinct services. The ultimate transfer pattern and the overall plausibility of the possibility of Pareto-desirable transfers depends then on all the motives taken together — for the optimal output quality and quantity depends on the "sum" of demands associated with the individual motives. Thus, if $D_1$ is the philanthropic demand for redistribution in fig. 10, $D_2$ the insurance demand, $D_3$ the protection demand and so on, optimal transfer output is given at $q_0$, where $D_1D_2$ (the vertical sum of the $D_3D_4$) cuts the marginal cost curve, $MC$. Manifestly, not all motives need be marginally relevant; but in general we would expect that more than one motive would be.

The result is that a motive which seems unlikely to be Pareto-relevant when reviewed in isolation may well turn out to be an important ingredient in
the total redistributive package. Thus, while we shall, in the analysis to follow, consider each motive separately, it is important to bear in mind that they do not operate independently.

![Diagram of Redistribution](image)

**Fig. 10**

**A. The Philanthropic Motive**

1. It seems clear, from both casual observation and introspection, that philanthropic motives do exist and are economically relevant: people do give things away, and for apparently philanthropic reasons. Yet the actual motivation for such acts is not always obvious. Even without being unduly cynical, it seems possible to suggest that much which passes for philanthropy may well, in fact, involve an informal *quid pro quo* - for example, intra-family gifts, Christmas presents, bonuses given as a "gift" by employer to employee might all fit this category. Of course, it may be very difficult in practice to distinguish such cases (even by introspection) or indeed even to define precisely what is meant by philanthropic or altruistic behaviour. Fortunately, many of the fine distinctions involved are of prime interest only to the moral philosopher – but it does seem necessary here to isolate three possible cases. The characteristic used for classification is the item which appears as an argument in the donor's utility function; thus, we distinguish the following possibilities:

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(i) where it is the act of giving itself which generates utility for the donor;
(ii) where it is the increase in the recipient’s well-being which gives the donor
satisfaction;
(iii) where the gift disguises some implicit transaction, in which case it is the
item received in exchange which enters the utility function and not the
gift as such.

Such distinctions seem necessary for two reasons. Firstly, the cases differ
conspicuously in their distributional significance — in particular, case (iii) in
no way results in a genuine transfer of income from “donor” to “recipient”,
no more in fact than is involved in the purchase of a loaf of bread or the serv-
ices of a hairdresser. In this sense, case (iii) differs markedly from the other
two possibilities. Secondly, the various cases differ with respect to the degree
and nature of market failure involved. For example, case (iii) involves no
market failure (or at least none that is readily apparent). The second case, by
contrast, does, as we have already seen, generally involve considerable market
failure, especially where the number of potential donors is large. Since the
utility the donor derives is independent of whether he himself gives, each
prefers that the recipient’s well-being be increased through the contributions
of donors other than himself; no one donor will give voluntarily, although all
would gain by the appropriate, mutually coercive, arrangement.

The first case is rather more subtle. Clearly, in this case, the donor only
derives utility from those gifts which he himself makes, in which sense giving
is a purely private activity and the free-rider problem conspicuous in case (ii)
does not arise. However, problems can arise on the recipient side of the trans-
actions. At first sight, the relation between recipients appears to be strictly
“zero-sum” (one man’s gain is another’s loss) and hence Pareto-irrelevant. In
fact, however, recipients’ demand can be relevant in two possible directions.
Firstly, many charitable gifts of this type are handled by “charitable
institutions” of one sort or another. In the ideal case, these institutions
would in fact perform some valued function, exemplifying the allocative
advantages of the greater division of labour. But since donors are in this
instance essentially indifferent to the use these gifts are put, resources can be
“wasted” in the production of glossy brochures, the support of a large ad-
ministrative and advertising staff, and otherwise directed to non-transfer
uses. While donors are largely indifferent about this outcome, potential
recipients are not. And it is not too implausible to suggest that recipients may
be able to “bribe” donors and the charitable institutions themselves (for any

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1 This is the case which has received most attention in the literature. For two
important examples see H. Hochman & J. Rodgers: “Pareto Optimal Redistribu-
D.C. Mueller: “The Pareto Optimal Approach to Redistribution: A Fiscal Applica-

2 It is of course possible that charitable institutions provide exactly the services
which donors (perhaps subconsciously) desire. Donors may, for example, like to have
their generosity and the “good works” themselves widely broadcast — both to ad-
vertise their own virtue, and to remind themselves or of it. Brochure production
and other advertising may be an important ingredient in this process.
rent forgone) to redirect such funds to direct redistributive uses and all be better off. With large numbers of potential recipients, however, such a rearrangement will not be likely without collective action (i.e. state intervention).

Secondly, the arbitrary nature of the "open-handed charity" characteristic of those cases where recipient's identity is irrelevant will generally translate into an arbitrary pattern of transfers. If the recipient group is risk-averse, all would prefer an arrangement whereby gifts were shared equally to one in which each faced the uncertain possibility of a proportionately larger sum or nothing. Of course, if market insurance facilities were perfect, optimal pooling of such risks could be achieved within the private market; generally, market facilities will not be satisfactory but even if they were, administrative convenience may suggest a budgetary solution. (This problem may also arise in the context of case (ii) above, particularly if donors' preferences are subject to occasional variation.)

Because the voluntary nature of the giving process is crucial to the donor's utility in the "giving-for-giving's-sake" case, coercive action will almost certainly be inappropriate for correcting any such market failure: redirecting the redistribution through the bureaucracy via the direct tax-transfer process will effectively destroy the source of donor satisfaction. Generally, a more suitable corrective device will lie in tax remission (of the income tax, say) to encourage donors to place their gifts in the desired way—perhaps via the medium of certain "approved" charities.

Likewise, if market failure arises (in either case (i) or case (ii)) because individuals derive utility from the philanthropy of others, independently of any concern for the recipient (i.e. individuals think it good that others should be generous, and are prepared to encourage them to be so), government action must preserve the voluntary nature of the giving process. Again, taxing "donors" and transferring the revenue to recipients is completely inappropriate.

By contrast, with the more substantial market failure characteristic of case (ii), it may be much simpler and administratively more convenient to redistribute directly in this way. Thus, it may be perfectly consistent and indeed thoroughly desirable for the government both to subsidize donations to private charities and simultaneously to conduct large scale redistribution itself through direct budgetary operations.

2. The large numbers which represent an important ingredient in market failure also imply something for the plausibility of conditions under which the philanthropic motive is Pareto-relevant. For as the number of potential donors grows, the contribution which each must make to increase recipients' incomes falls proportionately: whereas in the one donor/one recipient case, mutually beneficial transfers require that a dollar transfer be worth one dollar of own-income to the donor, in the n donor case, it need be worth only \( \frac{1}{n} \) dollars to each. Of course, recipients are also numerous in practice and income distributions are customarily skewed towards lower income groups, so that although the basic point seems acceptable, it is not clear how far it takes us.
For example, Von Furstenberg & Mueller [23] indicate that to achieve a floor level of family income of $5,000 in the U.S. case, a proportional income tax of approximately 4.1% would be required. Since this would involve something of the order of a ten percent increase in total tax receipts, it is clearly far from negligible. And even so, the calculation abstracts completely from any disincentive effects. Thus, it is assumed that those on incomes less than $5,000 will continue to work at the pre-tax income level and will simply have their incomes supplemented to achieve the $5,000 lower limit. Given the infeasibility of such “lump sum” transfers, we might assume that welfare recipients (at least) will choose to modify their effort-leisure choices. If, for example, they simply cease to work at all (by no means implausible) the cost of the operation would be increased by a factor of 2.4: the proportional income tax rate required would rise to 10% (roughly equivalent to a thirty percent increase in tax receipts).

Moreover, it is not obvious that all apparent philanthropy is likely to be directed towards the poor: much giving may have little effect on the equity of the income structure because it involves transfers among peers in the income scale. Church-giving, donations to artistic or academic institutions and even some more narrowly charitable giving may exemplify. It is after all plausible to argue (as Vickrey has) that philanthropy tends to be bestowed on those with whom one feels “empathy”, and that such empathy rarely extends to individuals distant from oneself in geographical, cultural, social — and hence income — terms. Thus, realistic models of redistribution based on donor altruism may well turn out to be only moderately redistributive in the equity sense, even where the extent of interpersonal transfer is considerable. In other words, upward-sloping segments of the utility-possibilities frontier occurring in the middle of the range, however extensive, are of little significance in terms of achieving distributive justice as traditionally interpreted: interpersonal transfers occur but their success in freeing the Paretian framework from its distributional conservatism in the relevant sense is severely limited. And it is far from clear that this is not the sort of redistribution one might expect to prevail in a Paretian world.

By and large, the models actually employed in the literature have tended to assume that altruistic interest on the part of the rich is oriented predominantly towards the poor and have attempted to posit plausible formulations of utility functions which depict this sort of situation. Together with some assumptions (often implicit) about the break-up of the utility gains from the redistributive process and about the values taken by crucial parameters, it has been possible to indicate the sorts of transfer patterns which might be

1 Such charitable giving may, for example, be directed towards those in one’s own income bracket who have temporarily fallen on hard times, and in this sense represent an informal insurance facility. Such insurance, irrespective of the formality of its contractual basis, implies a redistribution among the insured at any point in time from “policy holders” generally to claimants — but over a lifetime need have negligible effect on the equity of the income structure (e.g. the Gini co-efficient).

expected under Paretoian rule. Not surprisingly, these transfer patterns turn out to be reasonably redistributive. Indeed, on the basis of quite weak assumptions (embodiying essentially the notion of horizontal equity) it is possible to show that voluntary exchange pricing among donors will yield maximum progression on the recipient side – that is, transfers are directed towards raising the floor level of income so that all recipients have identical post-transfer incomes. Von Furstenberg & Mueller [23] have indicated, for one form of the utility function, how the cost of such an operation would be shared among donors for likely magnitudes of the income floor and given voluntary exchange pricing. In their case, taxation is roughly proportional.

Such exercises are of course ultimately arbitrary: they can only serve to indicate what might happen, given that the rich do desire to give to the poor for altruistic reasons. They make no attempt to induce such motives from independent observation or reasoning, and it hardly needs be emphasized that their "exhibition" of progressiveness constitutes an input of the models, not an output. It may nevertheless be comforting to know that progressiveness in transfer patterns can be generated on the basis of not too implausible assumptions.

In summary, then, it seems quite persuasive to argue that philanthropic motives may partly determine the taxpayer's attitude towards income redistribution. Yet the magnitude and nature of these motives remain unknown: one is obliged to argue on the basis of intuition, casual empiricism, and introspection. While, on the one hand, the conditions for Pareto-relevance, at least over the early part of the range, seem sufficiently weak for some interpersonal transfers to be necessary for efficiency, it is far from clear that these transfers are likely to be substantial or that they will necessarily be directed to those in direst need.

B. The Insurance Motive

1. It is well known, following Buchanan & Tullock1, that at the constitutional level with characteristically zero information about future income positions, risk-averse individuals have an incentive to opt for a set of fiscal institutions which generate income equality.

Although zero knowledge never prevails at the "in-period" level relevant for budgetary choice, some of the qualitative nature of the constitutional situation may nevertheless prevail if information is less than perfect. It is at least clear that redistribution from rich to poor does provide an insurance facility for all, in that unforeseen fluctuations in income position are inevitably less marked than they would otherwise be. Hence in the face of uncertainty, all would gain by the appropriate redistributive policy if they are risk-averse over the relevant range.

Since the redistributive process does not in general (and possibly cannot) distinguish between those who have temporarily fallen on hard times, and

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those who are inherently poor, the only transfer policy which is “riskless” is a policy of raising the income floor – any other policy involves an expectation of zero protection. This being so, it follows that insurance through the transfer process involves two conspicuous differences from normal market-type insurance facilities. In the first place, the “sum assured” in the transfer case is necessarily the same for all. In the second place, “claims” are made by and paid to individuals other than those who have experienced a sudden change in income-position (to those other than “policy-holders” as it were). Both these differences serve to make market-type insurance facilities more attractive to the insurer, and hence it seems that insurance via the transfer process can only be Pareto-desirable if the corresponding market facilities are disastrously inadequate.

In this context, it has been claimed that there are some contingencies which are privately uninsurable and for which transfer policy might provide appropriate protection. The evidence of privately uninsurable risks, however, in no sense establishes a case for government intervention: the superiority of the transfer solution over market facilities can only be demonstrated if market failure can be shown to be significant. Such market failure may arise from two possible sources:

(i) the failure of market facilities to soak up all the gains from risk-pooling;
(ii) the existence of external effects in the provision of insurance.

The first possibility relates to the observations that the process of risk-pooling itself involves increasing returns to scale: the larger the number of policy holders, the closer the number of claims will approximate the expected value. The possible greater size of the public sector argues in favour of government intervention in this field, but bearing in mind the international scope of some of the larger private insurance firms, such a case is not immediately persuasive. Moreover, beyond a certain point, gains from further risk-pooling decline, and it is not clear that market failure due to this increasing returns phenomenon would be sufficient to justify transfer programmes as an alternative.

In some instances, the process of insurance involves spill-over effects which are enjoyed by non-policy-holders. One conspicuous example of this is unemployment insurance – when any unemployed individual has unemployment insurance, his consumption falls less when he becomes unemployed, the downward multiplier process is less severe and aggregate unemployment less marked. In this sense, some of the benefits of that insurance accrue to those without policies, and a case for government intervention can be made. No such argument seems to apply more generally, however, except perhaps in the anti-revolution case which we consider later.

Thus, transfer policy on the basis of the insurance facility it provides seems unlikely to be justifiable on efficiency grounds in isolation. At the same time, given that other motives for redistribution are operative, insurance aspects may be relevant in determining the “Pareto-optimal” quantity and nature of redistribution.

2. The transfer pattern most likely to issue from the insurance motive is similar in kind to that associated with altruism under similar assumptions about the distribution of utility gains. On the recipient side, the establishment of a floor level income seems appropriate, given that individuals are risk-averse, since other forms of redistribution involve risk of receiving nothing when the donor falls on hard times. On the taxpaying side, Lindahl-pricing under moderately plausible assumptions about the formulation of donors’ utility functions seems to generate reasonably progressive rate structures under a variety of assumptions about the relation between income and probability of destitution. Thus, the same transfer programme may satisfy altruistic and insurance motives simultaneously. This outcome is to be contrasted with that arising under yet other motives, where the transfer programme required is quite different in type.

C. The Self-Protection Motive

1. It is frequently asserted that distributional disparities tend to be politically explosive – the immediate implication being that the removal of these disparities serves to reduce the likelihood of domestic unrest, and ultimately of revolution. Given that the process of revolution implies a cost to all (participant or not) in terms of danger of loss of life, injury and/or damage to property, all may gain by policies which reduce the probability of revolution occurring. Moreover, since revolution implies introducing a large element of uncertainty into the income determination process, risk-averse individuals will be prepared to pay something in order to ensure the certainty of the status quo situation. If a set of bribes can be designed from the community at large to potential revolutionaries in particular, ensuring the maintenance of the initial distribution of property rights, then all will gain. The payment of such bribes is, however, only one possibility in the armory of anti-revolutionary weapons. The initial property-right arrangement may be more efficiently defended by policies designed to reduce the likelihood of revolution being successful (which will of course have a disincentive effect on those contemplating the initiation of revolution) or policies aiming at making the punishments for revolutionary activity so severe that no one will run the risk.

Moreover, while payment of bribes serves to increase the opportunity cost of revolution to recipient groups, it also increases their command over resources and thereby the likelihood of success in any revolution undertaken. It is far from clear, therefore, that often-voiced assertions about the relation between political unrest and distributional justice are justified. In order to make any case for redistribution on self-protection grounds, we need to show, firstly, that the effect of bribes in aggregate is to reduce the likelihood of revolution and secondly, that bribes may well be more efficient in so doing (i.e. less expensive for donors) than other possible policies.


Since repression and threat policies tend to reduce the utility derived from the status quo situation (for all) in the process of making revolution less attractive to potential revolutionaries, much in both problems revolves around whether potential revolutionaries are relatively more sensitive to changes in the status quo (the "opportunity cost" of revolution) than to changes in the probability of revolutionary success. If this were so, then not only would appeasement policies be likely to work in the desired direction but also there would be some presumption against repression and threat policies.

2. As a point of departure, we need to note some of the characteristics of potential revolutionaries. Since revolution involves a gamble for those who initiate it — the chance of a substantial gain if the revolution is successful and the chance of a substantial loss if it is unsuccessful — individual's attitudes to "gambling" are basic in determining their revolutionary inclinations. Such attitudes are in turn determined by the individual's attitudes to risk and income changes — those with increasing marginal utility of income and little distaste for risk find gambling attractive, and hence are more likely to become revolutionaries. Now, the characteristic feature of increasing marginal utility of income is that it generates a convex total utility of income schedule, and the convexity property immediately insures that, if zero income involves zero utility, a proportional increase in income involves a more than proportionate increase in the probability of revolutionary success necessary to induce the potential revolutionary to become active.
Thus, suppose that individuals A will initiate revolution if and only if his expected gain exceeds his expected cost, i.e., if and only if

\[ p_s U_s + (1 - p_s) U_0 > U_1 \]

where \( p_s \) is the probability of revolutionary success

- \( U_s \) is the utility of income \( Y_s \)
- \( Y_s \) is A's income if the revolution is successful
- \( U_0 \) is the utility of income \( Y_0 \)
- \( Y_0 \) is A's income (assumed zero) if the revolution is unsuccessful
- \( U_1 \) is the utility of current income \( Y_1 \).

If, in the more likely case, A is a gambler, so that he exhibits a utility of income schedule as indicated in fig. 11, then clearly a proportionate increase in \( Y_1 \) increases utility \( U_1 \) in more than that proportion, and hence for given values of \( U_s \) and \( U_0 \), we require a more than proportionate increase in \( p_s \) to justify participation in the revolution. Thus, for potential revolutionaries (i.e., gamblers), increases in current income will generally reduce revolutionary tendencies unless (as seems unlikely) the probability of success is also related to income according to some convex function: there seems a fair presumption that bribes to potential revolutionaries will work in the desired direction. Obversely, policies which simultaneously reduce \( Y_1 \) and \( p_s \) (as threat and repression tend to do) are required to have a rather larger effect on \( p_s \) than on \( Y_1 \) in order to work against revolutions.

3. Supposing that we accept the case (only a weakly presumptive one) in favour of redistribution for self-protection purposes, what can we say of the transfer pattern likely to issue from this case? Clearly, what is required is redistribution from the community generally to potential revolutionaries in particular. Equally clearly, income transfers of the standard sort are unlikely to simulate the pattern required — for while there is some presumption that the poor are more likely to revolt than the rich (following the foregoing analysis), income redistribution from rich to poor serves to increase the revolutionary inclinations of richer individuals as much (if not more, given the normal sort of income structure) as it reduces those of the poorer. In aggregate, redistribution from rich to poor is likely to have effects on the probability of revolution which are at best ambiguous.

Instead, we require redistribution from risk-averse to risk-loving individuals — from those who are aggregate insurers over the relevant range, to those who are gamblers. The obvious way of achieving such redistribution is to collect an appropriate sum from everyone (related perhaps to income in the way indicated by the insurance motive) and use the revenue collected to finance a large-scale lottery, with a small number of large prizes. The rationale for this is simply that a much larger increase in the utility associated with the status quo for potential revolutionaries can be achieved by offering a small chance of a large prize than the certainty of an equivalently smaller gain — potential revolutionaries are, after all, gamblers.

This translates into a recommendation for transfer policies oriented towards "equality of opportunity" rather than income equalization. Whereas income equalization is designed to improve the distribution of income in a
Lorenz-curve sense, equality of opportunity is designed to give to all – the poor included – an equal chance in the income lottery. Needless to say, greater "equality of opportunity" may increase disparities in income levels.

To take a standard case, if we consider the selection of individuals to receive higher education provided by the state, the notion of "equality of opportunity" is normally taken to imply that selection should be based on some measure of ability, and explicitly not in any way related to income. If we accept the implicit assumption that ability and income are unrelated, selection according to ability is equivalent in distributional (though perhaps not academic) terms to a random selection of a small number of individuals, each of whom is to receive a large prize (viz. the discounted value of the increased future income streams associated with the receipt of education). The line of reasoning outlined here is that such income redistribution may involve gains to all – not just those who receive the education – in that the policy might well serve to reduce the likelihood of revolution ever being initiated. It is surely not too implausible to argue that societies which exhibit easy upward movement in the social scale are less likely to be afflicted by internal unrest than those where such movement is extremely difficult. If so, public policy to facilitate social mobility – in this case via income transfers – emerges as a legitimate "self-protection" activity and indeed may well be the most efficient and convenient way of defending the existing property-rights structure.

D. Motives of Malice and Envy

The line of reasoning associated with motives of malice and envy depends on three propositions:

(i) that actions undertaken for reasons of malice and envy can be Pareto-relevant;

(ii) that such Pareto-relevant malice and envy can generate redistribution;

(iii) and that such redistribution can under reasonable assumptions involve income transfers from rich to poor.

Since each of these propositions is (or perhaps ought to appear) mildly counter-intuitive, it may be profitable to indicate how and in what sense each applies.

It may seem that, since the objective of actions undertaken for reasons of malice and envy is to make someone else worse off, such actions would necessarily be precluded within the Paretian framework. Indeed, in a context where all are malicious towards, and envious of, all others, an appropriate role of the Paretian government would seem to be to protect each member of the community from the malice and envy of his fellows. In general, all would surely prefer such a situation to one in which no such protection was afforded. At the same time, in a context where all malicious (envious) acts are precluded by law, all individuals may be made better off by a relaxation of rules over some range, permitting each to inflict losses on his fellows at the cost of having to endure some losses himself. This will clearly be the case wherever the utility of causing harm to someone else exceeds for each the disutility of the harm he himself sustains. (This sort of motivation may for example apply
when individuals voluntarily agree to engage in physical combat—and they may agree to fight without any presumption that they would not be in favour of general laws against assault.)

Accepting this possibility, however, does not immediately suggest that universally beneficial transfers might arise as a consequence. On the contrary, since all conflicts of interest in a malice/envy environment are just that much more intense, the possibility of upward-sloping segments of the utility-possibilities frontier seems correspondingly that much more remote. While in aggregate this must be valid, it is not necessarily decisive; for although malice and envy may serve to make all conflicts more intense, some may become more intense than others.

In particular, if in the spirit of the “envy” characteristic, all are more envious of (or malicious towards) the rich than the poor, income transfers of a traditional equity-oriented type may make all better off. If, for example, I and II are rich and III is poor, I and II may gain from transfers from rich to poor, because each derives more utility from the reduction in the other rich individual’s income than he loses by virtue of the reduction in his own consumption possibilities, and the increase in III’s income.

With large numbers of I’s and II’s, such redistribution would have to be conducted publicly. Each would prefer a situation where all others gave to III, since he could then “consume” the reduction in the incomes of all the other rich at no cost to himself—a “free-rider” problem of the type familiar from public goods analysis arises. In this sense, the malice/envy case is rather like the altruism case. Like that case, also, the larger numbers seem to imply rather less stringency in the conditions for Pareto-relevance, since the cost of inflicting harm on any individual is substantially reduced with additional contributors.

Indeed, the malice/envy case adorned by the assumption of increasing marginal disutility of other individual’s income, matches almost exactly with the altruism case. As in that context, the ultimate transfer pattern is indeterminate on the basis of the Pareto-criterion alone, but under a variety of assumptions about the distribution of gains from trade, transfers do go from the rich to the poor—and often, from the richest to the poorest.

E. The Temporal Consumption Motive

It has been argued (originally by Buchanan, and more recently by Polinsky) that a case can be made for progressive taxation, with its attendant redistributive implications, within the Pareto framework on the grounds that it is one way of correcting for asymmetries and inefficiencies in the capital market. The line of reasoning involved runs more or less as follows:


Generally, it is easier to lend in capital markets than it is to borrow for current consumption. In this sense, it is more costly to advance consumption temporally than to postpone it, even in real discounted terms. Lifetime income tends to be intertemporally distributed in such a way that higher incomes are earned in the later years of life—for example, it might grow exponentially up to age sixty, and then decline. By contrast, desired lifetime consumption might be, say, distributed fairly evenly over the relevant time span, or even heavily weighted to earlier years. With a perfect capital market, the divergence between the desired consumption and income time-paths would not matter. Given the asymmetries assumed, however, actual and desired consumption paths diverge by an amount reflecting the deviation of the interest paid on borrowing for consumption purposes from the allocatively efficient rate.

For example, if in fig. 12, CC depicts the desired consumption path, and YY the actual income path, the actual consumption path might be EE.

If we now consider the addition of a tax-expenditure operation which involves progressive taxation, together with the provision of a publicly provided benefit stream which is constant over time, then disposable income takes the shape of the line Y'Y' and effective income (cum public benefit stream) takes the form of the line ZZ. Clearly, ZZ is closer to CC than YY is—and hence, so will the new actual consumption path E'E' be (not shown in fig. 12). The tax-expenditure operation permits a closer approximation to the desired consumption path, and clearly, the more progressive the tax and the
more substantial the tax expenditure operation is, the closer to optimality the time-path of consumption will lie. It is of course possible to conceive of an expenditure operation which is also progressive (e.g. means-tested cash transfers) – and clearly if the total operation were progressive enough and CC rose throughout much of the early range, ZZ may actually involve too much consumption too early. Since there is no assumed inefficiency within the capital market as far as *postponing* consumption is concerned, however, CC can be achieved in this case at zero cost.

What we have implicitly assumed so far is that all individuals have identical lifetime incomes; such an assumption is necessary if redistribution is completely inter-temporal, and not inter-personal in lifetime terms. Under the situation with identical incomes, interpersonal redistribution between individuals of different ages will occur at any point in time, but over each individual’s lifetime, total income remains unchanged. Once this assumption is dropped, interpersonal redistribution will occur in lifetime terms. Clearly, unless the gains from the intertemporal reallocation of lifetime income are substantial, not much in the way of interpersonal redistribution (of lifetime income) can be justified at all. The extent of such gains depends on:

(i) the extent to which desired and actual consumption paths diverge in the absence of government intervention;

(ii) the size of the substitution effect between present and future consumption (which determines the allocative cost of the path divergence);

and (iii) the allocative cost of the taxing operation (i.e. due to the “dis-incentive” effects on effort, risk-taking etc.).

Bearing in mind that the validity of intertemporal preferences is open to some questions (since the “desired” lifetime consumption path tends to be strongly influenced by the point in time at which the preference is calculated) and given the possibility of more direct policy intervention to correct for capital market inefficiencies, not much in the way of genuine redistribution seems to spring from this case. One could, of course, make the now familiar point that where other motives for (interpersonal) redistribution apply, the intertemporal considerations may become relevant. It does not seem likely, however, that they would assume much significance in the total redistributive package.

F. Conclusions

1. It is clear that no one of the motives which we have considered unequivocally generates a significant amount of redistribution when considered by itself. To some extent, though, the number of motives, all potentially operative and all possibly generating some demand on the part of donors for redistribution, encourages one to believe that the extent of Pareto-desirable redistribution may, after all, turn out to be significant. Such a belief would, of course, be rather more tenable if the nature of the redistribution required under each motive was the same. In fact, however, this is not so. In the first place, the self-protection motive requires interpersonal transfers which may well run in precisely the opposite direction over much of the range to those
associated with philanthropic (or other) motives. In other words, redistribution in the interests of "equality of opportunity" may well conflict with distributional justice in the traditional quality of income sense. In the second place, it is by no means obvious that philanthropic concern will invariably be oriented towards the poor, and if it is not, philanthropic giving may be in conflict with the insurance and malice and envy cases, which tend to generate redistribution towards the poorest.

In any case, a final "optimal" level of transfers can only be decided on the basis of a specific assumption about the distribution of the utility gains from the redistributive process. Such assumptions have in fact been made in the foregoing discussion, but it needs to be emphasized both that these assumptions are arbitrary from a Paretian viewpoint and that the final distributional result is extremely sensitive to them. In most cases, the Lindahl-solution with its implied neglect of recipients' preferences seems to indicate the minimum amount of redistribution consistent with the Pareto-criterion, in that the Lindahl-solution implies the lowest efficient floor level of income. But the Lindahl-solution also involves a distribution of costs among donors which is far from minimally progressive. It is indeed conceivable that if the richest appropriate all the consumer surplus accruing to the donor class (so that the poorer donors gain no utility from the transfer process), the distribution of income may, in aggregate, deteriorate in a Lorenz-curve sense: although the floor level of income is raised and the maximum level of income reduced, there are also redistributions from poorer to richer in the middle of the range. Perhaps, from a traditional equity viewpoint, that should not be taken as mattering too much. The crucial point is, however, that efficiency considerations alone are not sufficient to tell us very much about either the extent of Pareto-optimal redistribution or the nature of the transfer pattern which the Pareto-criterion requires.

If this is so, then the approach can hardly be very useful as a tool in specifying the distributional attributes of the social welfare function. At best, it can exclude some distributions of income – hopefully, but not necessarily, the most inequitable ones – from consideration. Nor can it be very useful, from a purely positive viewpoint, in explaining what actually occurs or in predicting responses to changing conditions: since the Paretian framework is consistent with such a wide range of distributional outcomes from any given set of donor preferences, it may be quite difficult to prove that what actually prevails does not reflect these donor preferences in some way. But it must be similarly difficult to prove that it does.

What then is the significance of the exercise? The answer is, I think, that although the Paretian framework cannot isolate a unique distributional result or even constrain the result to lie within a moderate range, it can provide a potential justification for what is currently being done in terms of income redistribution. That is, we can at least indicate that what is actually being done may well be desirable on the basis of pure efficiency considerations. In this sense, the Paretian framework does appear to be rather less conservative distributionally than generally thought. But it emerges in this context as conservative in another (perhaps true) sense – its application implies that no
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departure from current redistributional practices in either direction can really be justified. Whether this will be sufficient to make strict application of the Paretoian welfare framework (à la Buchanan) attractive to a wider professional public, is impossible to predict. It would certainly be rather strange if it did, because it would involve the acceptance not only of current redistributional practices but also of the principle that over a potentially enormous range, one distribution is as good as another.

None of this is to be construed as implying that the Paretoian perspective on distributional questions is not interesting, nor in its own way illuminating. The argument is simply that its claims to significance must necessarily be fairly modest.

References

PART III

SECOND-BEST HORIZONTAL EQUITY

It has long been recognised that, in many respects, in today's imperfect world, the question of the best possible tax system will inevitably be of less importance than the other, more practical, problems of today's world. The horizontal equity goal continues to be chosen as that of equity. The policy problem was that of choosing between perfectly applicable systems and schemes, rather than between alternative tax systems all of which have some degree of horizontal inequality. It is rare to see any consideration of what precisely goes wrong with imperfect horizontal equity goals, and often enough the recommendations take the form of pleading for changes which would bring the prevailing tax system closer to some ideal system, or that one which would be best if only it could be realised. Here is in the case that combining the relevant analysis explicitly recognises complex trade-offs in this area. It is important to consider the practical net change in this area, and in that sense or policy recommendations may be more or less pretentious on the precise formulation of the problem adopted.
INTRODUCTION

"Whoever hopes a faultless tax to see, 
Hopes what ne'er was, or is, or e'er shall be"

McCulloch's version of Pope, as cited in 
J. Stamp The Fundamental Principles of Taxation

It has long been recognized that, in tax matters as in other aspects of economic life, the world is an imperfect place. The quest for the perfectly equitable tax system is widely acknowledged to be a chimerical one, and it seems to be generally accepted that the best feasible tax system will invariably fall short of that which might be imagined if the operative constraints did not exist.

Strangely enough, in the horizontal equity dimension, relatively little has been done to confront this problem in an analytically explicit manner. The horizontal equity goal continues to be cited as that of achieving identical tax treatment for economic equals (variously defined), as if the policy problem were that of choosing between perfectly equitable tax systems and others, rather than between alternative tax systems all of which exhibit some degree of horizontal inequity. It is rare to see any discussion of what precisely more (or less) horizontal equity might be taken to mean, and often enough tax recommendations take the form of piecemeal changes which would bring the prevailing tax system 'closer', in some ill-defined sense, to that tax system which would be best if only it could be achieved. Nor is it the case that conducting the relevant analysis explicitly is a purely formal exercise. There are some important conceptual and practical issues at stake in this, and it does seem as if policy recommendations may be quite sensitive to the precise formulation of the problem adopted.
The two papers included in this section are designed to go some way towards filling this gap in the horizontal equity literature. The first originally grew out of a dissatisfaction with an earlier attempt along similar lines by Shirley Johnson and Thomas Mayer published in the Quarterly Journal of Economics in the early sixties. The approach taken both in this earlier paper and in my "extension" (paper eight) involves specifying a set of measures of the "degree of horizontal inequity" and then drawing out some of the policy implications by examining what tax arrangements are required to minimize the degree of horizontal inequity, thus measured. Of the alternative measures, the "number of inequities", originally propounded by Johnson & Mayer, seems to me to be entirely unsatisfactory - it would imply, for example, that the most equitable tax system feasible might well be one which raised all the revenue required from just one individual out of a group of economic equals. Not surprisingly perhaps, the "money-value of inequities" measures look to be much more promising. A number of different money-value measures are examined, and general policy prescriptions derived on the basis of them.

One regrettable feature of this approach, however, is that it becomes extremely complicated and unwieldy in anything other than a world composed entirely of economic equals. In response to this difficulty, the second paper (paper nine) adopts an alternative approach based on the clear analogy between the horizontal equity problem and the more familiar 'second-best' problem in standard Paretian welfare economics. Using this approach it is possible to take cognizance of vertical equity requirements explicitly, without the analysis getting out of hand. The basic conclusion
is not unexpected, given the similarities between this and the more standard 'second-best' analysis. It is shown that the tax system should not necessarily tax the various components of the tax base in the individually most equitable way: any particular inequity may be desirable because it offsets other inequities elsewhere in the tax system. The best feasible system may well seem a fairly repulsive animal to those bred on the aesthetic niceties of the Haig-Simons approach; but the central conclusion seems to be that it does not matter if the tax system is a bit of a mess, providing it's a mess of compensating 'errors'.
2. PAPER EIGHT

Horizontal Equity -
An Extension of an Extension
from
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HORIZONTAL EQUITY: AN EXTENSION OF AN EXTENSION

by

GEOFFREY BRENNAN

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1. The advocates of Sidgwick's principle of horizontal equity have propounded the ethical excellence of equal tax treatment for economic equals with such vigour that it is now an accepted tenet of tax policy in almost all Western economies. It is, therefore, perhaps a trifle regrettable that the principle is, strictly speaking, non-operational. The basic irrelevance of the normal formulation arises because it asserts the desirability of the abolition of horizontal inequities in a context in which the relevant choice is invariably between tax systems all of which are necessarily horizontally inequitable to some extent. As has been recognized, explicitly by Johnson and Mayer ¹ and implicitly by other writers in tax theory, what is required is some criterion for deciding which of two inequitable tax systems is the less inequitable: that is, Sidgwick's principle must be extended in some way to incorporate a dictum on the degree of horizontal equity that a tax system exhibits. It is toward this end that the present analysis is directed.

To some extent, the question has already been treated by Johnson and Mayer in their "Extension of Sidgwick's Equity Principle", and hence some justification for my discussion seems to be in order. The extended horizontal equity criteria which Johnson and Mayer suggest are phrased in terms of minimizing either the number of inequities or the aggregate money-value of inequities. Much of the Johnson and Mayer treatment and most of their interesting conclusions flow from the "number of inequities" case. As I shall argue, although this case permits much simpler analysis, the simplicity is bought at great cost since most of the persuasive force of the original formulation evaporates.

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rates. Indeed, in many instances, it yields conclusions which are entirely alien to the spirit of Sidgwick’s principle. Consequently, in contrast with the Johnson and Mayer article, my predominant concern is with the “money-value of inequities” case. This turns up a number of additional problems, which Johnson and Mayer either treat superficially or fail to consider at all.

This paper is then conceived as an attempt to criticize — and to extend — the “extension” of Sidgwick’s principle originally propounded by Johnson and Mayer.

2. The “number of inequities” case, for all its inadequacies, is nevertheless uncomplicated analytically, and provides a convenient point of departure for the present analysis.

Let us consider a community, $S$, of individuals, and examine the inequities which arise when the tax which is levied divides $S$ into two mutually exclusive sets: $A$, consisting of those who are treated favourably by the tax; and $A'$, consisting of individuals who are treated unfavourably. We assume that there is horizontal equity within sets, and the task is to determine the number of horizontal inequities in this situation. Clearly, the number of inequities is $|A|\times|A'|$, where $|A|$ represents the number of individuals in $A$. For example, if $A = (a,b)$ (that is, the set consisting of individuals $a$ and $b$) and $A = (a',b',c')$ the number of inequities is $2 \times 3 = 6$, and can be enumerated as

$a < -- > a'; \quad a < -- > b'; \quad a < -- > c'$;
$b < -- > a'; \quad b < -- > b'; \quad b < -- > c'$.

More generally, the tax will divide the population into any number of mutually exclusive sets with respect to tax treatment, horizontal inequities existing between, but not within, sets. If we designate these sets $A, B, ..., K$ then the number of inequities is given by the expression

$I = |A|\times|B| + |A|\times|C| + |B|\times|C| + |C|\times|K| + ...$

Clearly, $I$ will be minimized (actually zero) when $|A| = |S|$ and $|B| = |C| = ... = |K| = 0$. This occurs when all the population is

The precise definition of horizontal equity — or more particularly, of “equal treatment” and “economic equals” — is ignored throughout this discussion. This is of course a very important question, and can scarcely be considered to have been settled. But the problems I am concerned with here arise independently of just how equal treatment and economic equals are defined.
treated with the same degree of favouritism (that is, when no horizontal inequities exist) and this is the case whose virtue Sidgwick extols.

Given, however, that there will, in general, be some horizontal inequity in the tax system, the problem reverts to a selection of that tax system for which $I$ is minimal. It is therefore important to examine those changes which reduce $I$—these are tax changes which under the new criterion we could recommend on the grounds of horizontal equity.

The chief conclusions can be stated in terms of a number of simple propositions, which follow automatically from the definition of $I$ given above. The proofs are omitted in the interests of brevity.

**Proposition I:**

$I$ is maximized when $(A) = (B) = (C) = \ldots = (K)$.

**Corollary:**

$I$ is minimized when the discrepancy between group size is maximized, for a given number of groups. The minimum number of inequities for $m+1$ groups occurs when $m$ of the groups have size $1$, and the other, size $(S) - m$. Then the number of inequities is

$$I = m((S) - m) + m \left(\frac{m-1}{2}\right).$$

**Proposition II:**

A tax change which reduces $A$ by $x$ and increases $B$ by $x$ and leaves other groups unchanged will reduce $I$ if $(B) > ((A) - x)$.

**Proposition III:**

A tax change which amalgamates two groups and leaves the size of other groups unchanged, will reduce $I$.

**Proposition IV:**

If all groups are of equal size, and the number of groups is reduced, with the groups remaining of equal size in the new situation, the number of inequities is reduced.

**Proposition V:**

If the number of inequities is minimized for a given number of groups, reduction of the number of groups reduces the inequities.

These propositions suggest that there are two possible dimensions in reducing the number of inequities in the tax system: we can reduce the number of groups, either by amalgamating any two groups, or by reducing the number of groups while keeping the structure of relative group sizes unchanged. This result seems to argue in favour of simplicity in the tax system—that it is better to have a few clear cut
inequities, rather than a tax system in which there are a lot of individually inequitable taxes which when operating together partially cancel out the inequities induced by each other. Since the latter is the situation which tends to prevail in practice, this seems to be a powerful result.

Alternatively, the degree of inequity can be reduced by making group sizes more unequal, for example by shifting individuals from a smaller group to a larger group.

It is rarely easy to assess the virtue, or otherwise, of an ethical principle when it is stated in abstraction. Often, a principle which appears innocent, or even intuitively appealing, when stated as a bare assertion, translates into situations which almost everyone would regard as quite objectionable.

It is my contention that this is true of the new principle of horizontal equity which Johnson and Mayer propose. While the aim of minimizing the number of inequities seems, intuitively, to be a reasonable extension of the original Sidgwick formulation, it turns up conclusions which are most unappealing, and certainly at variance with the spirit of Sidgwick's principle as usually interpreted. To see this in the most striking way, it is simply necessary to observe that, following from Corollary I, the most equitable tax system in the Johnson and Mayer sense occurs when all the revenue is raised by a lump sum tax on one person. Such an arrangement clearly serves to minimize the number of inequities and hence, according to the Johnson and Mayer criterion, is desirable on horizontal equity grounds. But it is scarcely horizontally equitable in the Sidgwick sense.

The basic point is that a tax system which exhibits a large number of inequities is usually accepted as providing reasonable horizontal equity given that those inequities are all fairly small in money value. Certainly, the Haig-Simons approach to income taxation has proven highly appealing even though there exist various types of imputed income and various costs (such as the disutility of effort) which it is administratively impossible to allow for in the calculation of tax liability. On the other hand, a tax system which has a small number of very large inequities (for example, a complete tax exemption for all whites in Rhodesia *) would be unacceptable to almost everyone (with the possible exception of those favourably treated).

(*) By way of explanation, there were (at 21 December 1968) about 250,000 whites in Rhodesia in a total population of almost five million (i.e. whites represent about 5 % of the total).
HORIZONTAL EQUITY: AN EXTENSION

It seems clear then that the question of the number of inequities is substantially irrelevant. Policy prescriptions which flow from the number-of-inequities analysis are misleading in the extreme: tax changes which such analysis suggests may well work against horizontal equity in the normal sense of that term. Clearly, some criterion is required for deciding which of two inequitable tax systems is the less inequitable; equally clearly the number of inequities, alone, does not provide such a criterion.

3. The obvious alternative to using the number of inequities as a measure of the degree of horizontal inequity in a tax system is to use the aggregate money-value of inequities. This will ensure that inequities which are large in money value will be given a correspondingly heavier weight in the index of horizontal inequity, than they are given in the number-of-inequities case (where of course each inequity gets equal weight irrespective of size).

The money value of inequities can be measured in a number of ways. The measure which Johnson and Mayer suggest is a logical extension of the number-of-inequities case: we simply weight each inequity by its size in dollars. So that if \( t(A_i) \) is the tax paid by each individual in group \( i \), then the money value of inequities in a tax system which divides the total population into mutually exclusive sets \( A_i \), is given by \( M \), where

\[
M = \sum_{i=1}^{n} \sum_{j=1}^{n} \{ A_i \cap A_j \} |t(A_i) - t(A_j)|
\]

where \( |t(A_i) - t(A_j)| \) is defined so as to always be positive.

One argument against this measure may be precisely that it arises as a logical extension of the number-of-inequities case, and hence tends to have embedded in it all the biases which make the pure number case so unsatisfactory. Consequently, it may be felt desirable to ignore all questions of the number of inequities and consider only the size of inequities that exist. One way of doing this is to use the measure \( F \) where

\[
F = \sum_{i=1}^{n} \sum_{j=1}^{n} \frac{|t(A_i) - t(A_j)|}{n - 1} ; > j
\]

or \( F' = \frac{1}{n-1} \sum_{i=1}^{n} |t(A_i) - t'| \)
where \( t \) is the straight average of tax rates \( \frac{\sum t(A_i)}{n} \) and \( n \) is the number of groups.

Under such measures the number of inequities is abstracted from completely — each difference is given the same weight irrespective of the number of people affected. However, it is precisely because \( F \) does not distinguish a situation where the number of large inequities is very small from a situation where the number of large inequities is substantial, that it is unsatisfactory.

An alternative measure which suggests itself is the standard deviation of actual tax payments about the mean. If \( t \) is the average tax paid by each individual, then \( t = \frac{R}{S} \) where \( R \) is total tax revenue, and the standard deviation \( ^* \) of the distribution of tax payments about the mean is given by

\[
M' = \sqrt{\frac{\sum (A_i - t)^2}{S - 1}}
\]

A measure which is equivalent is \( (M')^4 \) — the variance of the distribution. The case for the standard deviation (or variance) can be made on two grounds: firstly, it tends to weight distant observations (i.e. large inequities) more heavily than does the Johnson and Mayer measure, \( M \); and secondly, it is more logical from a statistical point of view because of its convenient mathematical properties.

All these measures may be criticized on the grounds that they depend on the units in which each inequity is measured. If each inequity is valued in cents, the measure will yield different results from the case where inequities are measured in dollar. Thus, there is a strong case for using some scale-free measure such as the "co-efficient of variation", \( V \), where

\[
V = \frac{M'}{t}
\]
rather than \( M' \) as such. \(^6\)

\(^*\) Since \( S \) is constant for any given population, there seems no real need to be divided by \( S - 1 \). This step would of course be necessary in cross-sectional studies.

\(^5\) Melvin and Anne White have used the co-efficient of variation as a measure of horizontal inequality in "Horizontal Inequality in the Federal Income Tax Treatments of Homeowners and Tenants", National Tax Journal, September 1965.

\(^6\) This is probably necessary only for cross-sectional studies, where units vary. Where as is more likely, one is comparing different taxes in the same community.
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If, however, it is true that we are prepared to tolerate small inequities, while wishing to weight more substantial inequities more heavily, then the co-efficient of variation might be considered an inadequate measure of the degree of inequity. Two distributions (such as Fig. 1 and Fig. 2) may have the same standard deviation, but one may have long tails (Fig. 1) and the other (Fig. 2) no tails at all. The tax distribution with long tails shows that there are a small number of tax-paying units who bear a markedly greater burden — and a small number who enjoy a markedly smaller burden — than the main bulk of taxpayers. On the other hand, the distribution with no tails confines the size of any inequity within fairly reasonable limits. It does appear that the latter is preferable.

This seems to suggest that higher moments than the second may be relevant. In particular, we could well choose to aim at minimizing the kurtosis of the distribution — where the kurtosis $k$ is given by

$$k = \frac{u_4}{u_2^2} - 3$$

where $u_4$ is the fourth moment of the distribution and $u_2$ is the variance. This again is a scale-free measure (i.e. independent of the units in which tax liability is measured), and has the precise advantage that it weights large deviations from the mean more heavily than does the standard deviation. Hence, in the comparison of Fig. 1 and 2, Fig. 2 has a considerably lower kurtosis, even though the standard deviations are identical.

Of these measures, my own preference is for the kurtosis, on the grounds that this measure weights large inequities heavily. By and large however, all the measures, except $F$ and $F'$, give roughly the same results for most tax comparisons, and I have used $M$ and $M'$ in no problem is likely to arise in using the standard deviation as such, providing a reasonable convention is followed in the use of units.

![Fig. 1](image1.png)
![Fig. 2](image2.png)
some contexts below because their relative simplicity makes for less complicated mathematical analysis.

4. Irrespective of the measures used, the consideration of the money-value (as distinct from the number) of inequities introduces of itself some complications into the analysis.

In the first place, we are now dealing with an additional set of variables — namely, the effective tax rates faced by each group — and it becomes correspondingly more difficult to make simple ex ante propositions about tax changes which will reduce horizontal inequity. Needless to say, the theorems proved with respect to the number-of-inequities case no longer hold. Moving individuals in such a way as to make a large group larger will no longer necessarily reduce inequity since everything depends crucially on where in the spectrum of tax treatments the group lies. Nor is it true that inequity will be maximized when groups are of equal size. If we consider the inequity involved in a system where there are three groups of equal size differentially treated, a movement of individuals from the central group to one of the extreme groups will serve to increase the degree of horizontal inequity even though group size has been made less even. But apart from this obvious analytic complication, there are two important considerations which merit special attention: the question of equi-revenue comparisons; and the significance of vertical equity considerations.

A. EQUI-REVENUE COMPARISONS

It now becomes strictly necessary to consider only tax changes which leave aggregate revenue unchanged. Failure to do this results in ignoring the crucial influence which the size of inequities wields on the conclusions drawn. To exemplify the need, consider a population of ten taxpayers: one of them pays $1,000 in tax — the other nine pay nothing. A change is instituted whereby five individuals now pay $1,000 and only five manage to escape tax. Using the average deviation measure, we can derive the following results:

<table>
<thead>
<tr>
<th>Size Grp. 1</th>
<th>Tax (1)</th>
<th>Size (2)</th>
<th>Tax (2)</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-change</td>
<td>9</td>
<td>—</td>
<td>1</td>
<td>1,000</td>
</tr>
<tr>
<td>Post-change</td>
<td>5</td>
<td>—</td>
<td>5</td>
<td>1,000</td>
</tr>
</tbody>
</table>
This leads to the conclusion that the change is undesirable. But this is to miss the crucial point that the extra revenue made available permits a reduction in the rate of tax on those actually subject to taxation. Thus, the appropriate post-change situation for consideration is that given below:

| Post-change | 5  | — | 5  | 200 | 5,000 |

Hence, if it is valid to assume that we can use the extra revenue obtained to reduce all rates of tax in proportion, then increasing the coverage of the tax in this example is clearly desirable.

In fact, it can be shown that, under equi-revenue comparisons, it will never be advantageous to allow taxpayers to escape (avoid or evade) tax completely, whatever the size of the tax-free group. If it is possible to increase the coverage of the tax, it will be desirable to do so.

To see this, let \( A \) be the size of the group which evades or avoids, and \( B \) the group which pays tax. Then \( t_a = 0 \) and \( t_B = \frac{R}{\{B\}} \) where \( R \) is revenue.

\[
M = (A)(B) \left( \frac{R}{(B)} - 0 \right) = (A)R
\]

and \( \frac{M}{\partial(A)} = R > 0 \), so that a decrease in \( A \) implies a reduction in \( M \). In other words, any reduction in the degree of evasion (or avoidance) will reduce the amount of horizontal inequity in money value terms, simply because the increased revenue can be used to cut tax rates: this in turn reduces the size of the inequities which still exist.

This is not to say, of course, that where a tax is subject to substantial evasion, it may not be in the interests of horizontal equity to repeal the tax completely and raise revenue in some other way. The point is simply that the size of the group escaping tax is irrelevant in deciding whether to widen the effective tax base, or not.

A certain amount of care must be taken in setting up equal revenue comparisons. It is clearly illegitimate to assume as Johnson and Mayer do that the revenue can be made up by levying a tax on the favoured group alone, since if this were so, it would be possible to remove inequities completely simply by raising rates on the favoured until they
were treated equally with the (originally) unfavoured. Nor is it permissible to assume that the revenue can be raised in some non-equitable way somewhere else in the system, since if this were so, it would be possible to remove the tax in question completely and simply increase the rates of the "non-inequitable" tax — thus removing all horizontal inequity. There is no real problem. Or, if the constraint takes the peculiar form of insisting that the "inequitable" tax must be imposed, then we can simply apply the lowest rates compatible with the constraint, and raise the revenue lost by completely horizontally equitable taxes as before.

The impression given by Johnson and Mayer that by and large the conclusions which are reached in the number of inequities case flow over to the money-value case arises precisely from their inadequate treatment of the equal revenue constraint — or more particularly, from their illegitimate assumptions as to how any lost revenue is to be made up.

In order to make the problem non-trivial, it is necessary to consider a framework in which we have a large number of possible taxes, each individually inequitable, but which when used in association, serve to cancel out some of the inequities induced by each other. In such a context, it is important to realize that the money value of inequities induced by a particular tax depends on the rate at which that tax is imposed. This is so simply because, other things being equal, the higher is the average tax paid, the larger the standard deviation from that average. If we increase all tax rates in proportion, then each inequity will rise in that proportion, and the standard deviation will be correspondingly higher. This is of course a trivial observation, but some interesting conclusions do flow from it. For example if we have two individually inequitable taxes, it will reduce horizontal inequities to impose them simultaneously, even if there is no systematic relation between favoured and unfavoured groups under the two taxes. To see this, suppose we have a community of twenty people and wish to raise $1,000 in tax revenue. We have two possible taxes, each of which exempts half the population. Thus, under tax 1 we have two groups A and B, each of ten people; and under tax 2, there are two groups C and D, each of ten people. If AC (etc.) is the intersection of A and C, levying the taxes together gives:
Levying either tax on its own, however, gives a value of 50,000 to \((M')^2\).

In other words, the degree of horizontal inequity is less when the two taxes are levied together than when either tax is levied alone, given equal revenue.

A mathematical proof of this general proposition is easily given. Suppose \(T_i\) and \(T_j\) are equal revenue taxes with different incidence, but the same degree of horizontal inequity. Then by definition \(\text{var} \ (T_i) = \text{var} \ (T_j)\). We are to compare \(T_i\) and \(T_j\) levied together, with \(T_i\) levied on its own, but at twice the rate. Then we are concerned to compare \(\text{var} \ (T_i + T_j)\) with \(\text{var} \ (2T_i)\).

\[
\text{var} \ (2T_i) = 4 \text{var} \ T_i \\
\text{var} \ (T_i + T_j) = \text{var} \ T_i + \text{var} \ T_j + 2r_{T_iT_j} \sqrt{\text{var} \ T_i \times \text{var} \ T_j} \quad \text{(A)}
\]

by a familiar statistical theorem

\[
= 2 \text{var} \ T_i + 2r_{T_iT_j} \sqrt{\text{var} \ T_i \times \text{var} \ T_j} \quad \text{since} \quad \text{var} \ T_i = \text{var} \ T_j = 2 \text{var} \ T_i (1 + r_{T_iT_j})
\]

where \(r_{T_iT_j}\) is the covariance of \(T_i\) and \(T_j\).

Clearly, if \(r_{T_iT_j} < 1\), \(\text{var} \ (T_i + T_j) < \text{var} \ (2T_i)\).

Now, \(r_{T_iT_j}\) takes values from \(-1\) to \(+1\), depending on how the incidence of the two taxes is related. If \(r_{T_iT_j} = -1\), then individuals paying high tax under \(T_i\) are paying correspondingly low tax under \(T_j\); the inequities introduced by \(T_i\) are exactly offset by those introduced by \(T_j\). If \(r_{T_iT_j} = 1\) the taxes have identical incidence. Since the taxes have different incidence, \(r_{T_iT_j} < 1\); and clearly this is all that is required for the theorem to hold. Equation \(\text{(A)}\) generalises to the case of a distribution made up of \(n\) independent components — so that

\[
\text{var} \ (\sum T_i) = \sum \text{var} \ T_i + \sum \sum 2r_{ij} \sqrt{\text{var} \ T_i \times \text{var} \ T_j}
\]

and the proposition is valid for the case where there are \(n\) taxes.

Moreover, even if one tax is more horizontally inequitable than the other (when equal revenue is collected), it may still be in the interests of horizontal equity to impose them together. Suppose that \(T_i\) and \(T_j\) are two taxes, such that when levied at rates which ensure equal revenue from each, the relationship between variances is
$s' \text{var}(T_i) = \text{var}(T_s), \text{ where } s > 0$

Comparing the use of $T_i$ at twice the rate, with $T_i$ and $T_s$ together we have:

$$\text{var}(2T_i) - \text{var}(T_i + T_s) = 4 \text{var} T_i - [\text{var} T_i + s'\text{var} T_s + 2r_{T_iT_s}\text{var} T_i]$$

$$= \text{var} T_i (3 - s' - 2r_{T_iT_s}) > 0 \text{ if } s' + 2rs - 3 < 0$$

(where $r$ stands for $r_{T_iT_s}$)

Clearly $s' + 2rs - 3 < 0$ as $0 < s < -r + \sqrt{r^2 + 3}$

If $r = -1$, we need have $s < 3$ ($s' < S$)

If $r = 0$, we need have $s < \sqrt{3}$ ($s' < 3$)

If $r = 1$, we need have $s < 1$ ($s' < 1$). In this latter case of course since the taxes are of equal incidence, it will pay to introduce $T_s$ only if it is more equitable than $T_i$.

So that for example, if the incidence of the two taxes is such that $r_{T_iT_s}$ is zero, then it will improve horizontal equity to introduce them together even if one is almost "three times more inequitable" than the other in the defined sense.

The general proposition is, then, that for any two taxes which do not have substantially similar incidence patterns, it will normally be in the interests of horizontal equity to impose them together, rather than one on its own, even if one is, by itself, rather less equitable than the other.\textsuperscript{1}

This conclusion is to be contrasted with that which flows from the number-of-inequities case, where the presumption is that the fewest taxes in operation the more equitable the system will be.

This result is an interesting and in fact a highly applicable one. To exemplify how the result can be applied, suppose we consider the case for levying a capital gains tax. It is sometimes argued that the taxation of capital gains is undesirable on horizontal equity grounds because it taxes fictitious as well as real capital gains, and that imposition of such a tax may well create as many inequities as it removes. If we depict the set of individuals who make real capital gains by $A$, then under an income tax which exempts capital gains the set $A$ is favoured and the set $(S - A)$ is relatively unfavoured. If $B$ is the set of people who make fictitious capital gains, then under an income tax which is augmented by a capital gains tax, we have four sets of people:

\textsuperscript{1} Further improvements in horizontal equity may obtain if tax rates (and hence, the importance of different taxes in revenue terms) are optimally related to $r_{T_iT_s}$ but I have ignored this additional question here.
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A, B real and fictitious capital gainers
A. \((S - B)\) real but not fictitious capital gainers
\((S - A) \cdot B\) fictitious but not real capital gainers
\((S - A) \cdot (S - B)\) no capital gains, fictitious or otherwise.

Arguing from the general proposition, unless there are reasons for thinking that proportionately more of \((S - A)\) than of \(A\) make fictitious capital gains in which case the covariance of the two taxes may be close to unity (and most evidence if anything suggests the contrary), then introduction of a capital gains tax almost certainly increases horizontally equity. And this so, not so much because we have removed inequities between capital gainers and others, since we may well have introduced more inequities than we removed \((if \ B > \ A)\), but because the addition to revenue obtained by including capital gains (both real and fictitious) in the tax base, can be used to cut tax rates, and the size of all inequities can be reduced thereby.

Obversely, if revenue is to be lost in a tax change, an increase in tax rates will be necessary after the change to make up the revenue lost, and this will tend to increase inequities. Thus, some care must be taken in recommending exemptions \((or\ tax\ changes\ which\ reduce\ revenue)\ which\ seem\ desirable\ on\ horizontal\ equity\ grounds,\ since\ the\ necessary\ increase\ in\ tax\ rates\ will\ tend\ to\ increase\ the\ value\ of\ inequities.\ This\ latter\ increase\ has\ to\ be\ offset\ against\ the\ apparent\ reduction\ in\ inequities\ induced\ by\ the\ tax\ change\ itself.

This can be exemplified by considering the case for introducing averaging provisions, in the following circumstance. Suppose we have a community of twenty people on the same lifetime income — there are no averaging provisions, and ten of these people earn income in a lumpy fashion. We call this group \(A\) — the remainder group \(B\). Because of the progressive rate structure, individuals in group \(A\) pay $50 p.a. — those in \(B\) $10 p.a. A system of averaging is introduced, but is imperfect. Group \(B\) is unaffected; group \(A\) is split — five still pay $50 p.a. — this is group \(A_1\); five pay only $20 p.a. — this is \(A_2\).

Then, comparing the two situations:
initially, total revenue is $600; average tax per individual $30.
then, \((M') = 10 (50 - 30) + 10 (30 - 10) = 8,000\)

after imperfect averaging, total revenue falls to $450, so an increase in all tax rates by a proportion of one third is necessary.

Then \(t_{A_1} = 50 \left(\frac{4}{3}\right) = 66 2/3; t_{A_2} = 20 \left(\frac{4}{3}\right) = 26 2/3\)
average tax per individual is now $30
then \((M')^4 = 5(662/3 - 30)^4 + 5(30 - 26 2/3)^4 + 10(30 - 13 1/3)^4 = 9,000 \text{ (approx.)}
\]
Hence, the degree of horizontal inequity has risen — and this because of the increase in tax rates, necessitated by the loss in revenue associated with the tax change.
Finally, it should be pointed out with reference to equal-revenue comparisons, that measures of the degree of horizontal equity such as the variance or kurtosis of the distribution of tax payments, can incorporate equal-revenue considerations fairly simply. Since the mean tax rate \(t\) is given by
\[
t = \frac{R}{\{S\}}
\]
where \(R\) is revenue, equal revenue comparisons are simply those for which \(t\) does not change (since \(\{S\}\) is constant). Thus, we simply examine the effects of various tax changes on the kurtosis (or variance) subject to the constraint that the mean is fixed.

B. VERTICAL EQUITY CONSIDERATIONS

Having obtained a figure for the total money value of inequities, it is still unclear what significance the measure ought to be given. It is far from appealing that we should give an inequity of $1,000 the same weight irrespective of the incomes of those individuals concerned. Surely a horizontal inequity of $1,000 is more significant if it is between individuals earning $1,500 than it would be if the individuals were millionaires. If this is so, we cannot ignore the question of vertical equity entirely. As has been observed by Musgrave and others, the questions of horizontal and vertical equity are merely aspects of the same problem: there is no point in insisting on equal tax treatment for economic equals if it is not intended to treat economic unequals unequally. The framework Johnson and Mayer provide is then only

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The concept of horizontal equity is valid when we have an entire population of economic equals. This hardly is a practically relevant case. Moreover, it has always been quite feasible to design a perfectly horizontally equitable system simply by instituting a system of lump sum taxes of equal amounts on all individuals. That this has seldom been seriously advocated for developed economies indicates the overriding importance of vertical equity considerations. It also points up the danger of considering horizontal equity questions in abstraction — one can easily be led into making policy prescriptions which have simply no practical relevance.

There are a number of ways in which vertical equity considerations can be incorporated into the analysis. One is to seek to maximize horizontal equity subject to some constraint which limits the tolerable deviation from the vertical equality goal in some way. Another possibility is to attempt to incorporate horizontal and vertical equity considerations into the one objective, and simply measure all inequities (both horizontal and vertical) in the same manner. This latter framework permits a certain amount of play-off of one goal against another — we can now sacrifice a little horizontal equity to obtain increased vertical equity (or vice versa). Under the constraint formulation, such play-off is effectively precluded. The degree of vertical equity is given — no vertical equity may be sacrificed, either to achieve increased horizontal equity, or otherwise. Sometimes, of course, increased horizontal equity tends to work in the interests of vertical equity as well. Loopholes in the tax base often tend to favour the rich, in that it is they who have the greatest opportunity — and the greatest incentive — to shift income into avenues which are tax-free. An income tax structure which looks highly progressive at first sight may well turn out to be barely proportional in terms of total income (defined, say, according to net accretions). On the other hand, horizontal and vertical equity considerations are often in direct conflict — for example, progression aids vertical equity but, in the absence of satisfactory averaging provisions, works against horizontal equity by discriminating against those with lumpy income streams. In such situations, the scope for the play-off of objectives seems desirable. Besides, there does appear to be some methodological advantage in combining in the one objective two practically and conceptually related goals.

Suppose that \( p(y) \) is a function which shows how tax liability should change with respect to income. The precise structure of this function is not specified, but it is presumed to involve a progressive rate structure. Let \( T_i \) be the tax that the \( i \)th individual pays. Then the
requirement for a perfectly equitable tax system is that
\[ T_i = kp(y_i) \quad \text{for all } i \]
where k is a constant determined by the desired allocation of resources between public and private sectors (i.e. the revenue demands of the government). Clearly this requirement subsumes horizontal equity requirements since tax units in equal economic positions (e.g. on equal incomes) necessarily pay the same tax.

Thus, any deviation from what we might term the "conceptually optimal" level of taxation represents an inequity, and we can measure the degree of inequity by
\[
\frac{\sum_i \left| T_i - kp(y_i) \right|}{kp(y_i)} = \frac{\sum_i \left| T_i - kp(y_i) \right|}{kp(y_i)}
\]
where \( kp(y_i) \) is the "conceptually optimal" level of tax for that income and \( T_i \) is the actual tax paid by the individual in question. We can, now, define "favourable" or "unfavourable" tax treatment according to whether actual tax paid exceeds or is less than the conceptually appropriate tax, and the degree of "favourability" will depend on the ratio of actual to optimal tax burden. Thus, everyone for whom \( \frac{T_i}{kp(y_i)} \) equals, say, 1.3 can be considered as belonging to the same class \( A_j \) with respect to the degree of favourability enjoyed.

Hence, any class of taxpayers \( A_j \) will contain economic unequals, but all members of \( A_j \) are enjoying the same degree of preferential (or non-preferential) treatment. We can now recalculate the degree of inequity as the variance
\[
Q = \frac{1}{m-1} \sum_i A_j \left( \frac{T_i}{kp(y_i)} - 1 \right)^2
\]
or
\[
Q_i = \frac{1}{m-1} \sum_i A_j \left( \frac{T_i}{p(y_i)} - k \right)^2
\]
where k is a constant determined by revenue considerations, and m is the number of groups \( A_j \). Again, the kurtosis of the distribution, because it weights large deviations more heavily may represent an attractive alternative.

Needless to say, analysis now becomes much more complicated. Equal revenue considerations are harder to handle, and it becomes much more difficult to make general statements about the sorts of tax changes which are desirable. Moreover, we have now left the rela-
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trively safe ground of pure horizontal equity questions, where there is a fair amount of consensus, and made a partial move into the much more contentious field of vertical equity. Although here I have tried to remain pretty well agnostic on the question of the precise nature of function \( p(y) \), any actual application of the criterion will obviously require its complete specification. This may seem a heavy price to pay, in that the general acceptability, which the original Sidgwick formulation enjoys, no longer obtains. Yet it does seem clear that such a price is necessary if the notion of horizontal equity is to be rescued from irrelevance.

C. SUMMARY

The traditional formulation of the horizontal equity principle is irrelevant for policy purposes, because it can only distinguish between perfectly equitable tax systems and others. Since in practice the policy problem is to choose from among a number of possible (inequitable) tax systems that which is the most equitable or least inequitable, a measure of the degree of horizontal equity (or inequity) is necessary.

One possible measure, which has already received some attention in the literature, is the number of inequities. If the horizontal equity notion is extended in this way, the objective becomes the minimization of the number of inequities. This criterion is not, however, a reasonable extension of the original formulation, because it turns up recommendations conspicuously at variance with the spirit of Sidgwick’s rule. In particular, faithful administration of this criterion would result in a tax system when all revenue was raised by levying a lump-sum tax on one individual (or a minimum number of individuals).

A preferable notion is the money-value of inequities, but this can be measured in a number of ways. My own preference is for measures which weight large inequities heavily, since this seems to accord most readily with intuitive ideas about horizontal equity. Of these, the variance and the kurtosis of actual tax burdens about the mean are perhaps the most logical because of their convenient statistical properties. If minimization of the variance is taken as the horizontal equity objective, then the theorems which flow from the number-of-inequities case no longer hold.

The money-value of inequities depends upon the rate of tax applying, and it is therefore necessary to restrict attention in this case
to equal revenue comparisons of taxes. If we do this, we obtain the interesting result that the more general a tax system the more likely it is to be horizontally equitable — not so much because the number of inequities is reduced, but rather because the greater coverage implies a lower rate of tax and hence a smaller "distance" (in money terms) between those favourably and those unfavourably treated.

Except in the trivial case where the population consists entirely of economic equals, it is necessary to incorporate vertical equity criteria into the analysis. Otherwise we are involved with making the implicit assumption that any inequity of a given size is worth equal weight independent of the proportion of income or tax liability that the inequity represents. In any case, vertical equity considerations protrude automatically because no policy recommendation which contravened them violently would prove acceptable. The incorporation of vertical aspects of the equity question does not change the analytic technique in any substantial way. The conceptually optimal tax burden for each individual is specified, and proportionate deviations from this are measured as inequities, irrespective of whether they arise from horizontal or vertical failings. This, of course, complicates the analysis considerably, and forces us into the relatively contentious area of vertical equity. It is, however, impossible to see how such a step can be avoided.

Summary: Horizontal Equity: An Extension of an Extension. — The traditional formulation of the horizontal equity principle is irrelevant for policy purposes, because it can distinguish only between perfectly equitable tax systems and others, whereas the relevant policy choice is invariably between tax systems all of which exhibit some inequities. A measure of the degree of horizontal inequity is required, and this paper seeks to provide one. The number of inequities, suggested as a measure by Johnson & Mayer in their "Extension of Sidgwick's Equity Principle", (Quarterly Journal of Economics, 1962) is examined and found to be unsatisfactory. Only measures which incorporate the money value of inequities are acceptable, and among these preference is shown for those which weight large inequities relatively heavily (e.g. the variance or kurtosis of the distribution of tax burdens). Since the money-value of inequities depends on the tax rate applying, equal-revenue tax comparisons are essential. On this basis, the customary preference for taxes with wide coverage is shown to be justified — not because the number of inequities is reduced (it may be increased), but because the greater coverage implies a lower rate of tax per person, and thus a smaller distance in money terms between different taxpayers. It is also shown that, contrary to Johnson & Mayer's conclusions, it is
never in the interests of horizontal equity to allow any taxpayer to evade tax. An attempt is made to incorporate vertical equity considerations into the analysis. This is unnecessary only if the population consists entirely of economic equals — which is scarcely a likely contingency.

Résumé: Equité horizontale: l'Extension d'une Extension. — La formulation traditionnelle du principe d'équité horizontale est inapplicable à des fins politiques, parce qu'elle peut seulement distinguer entre des systèmes fiscaux parfaitement justes et les autres, alors que le choix politique en question est invariablement à faire entre systèmes fiscaux qui, tous, présentent certaines inéquités. Il faut rechercher une façon de mesurer le degré d'inéquité horizontale. C'est ce que cette étude se propose de faire. Le nombre des inéquités, proposé comme instrument de mesure par Johnson et Mayer dans leur „Extension du principe d'équité de Sidgwick“ („Extension of Sidgwick's Equity Principle", Quarterly Journal of Economics, 1962) est étudié et jugé non satisfaisant. On ne peut prendre en considération que les instruments de mesure que incluent la valeur monétaire des inéquités, et parmi celles-ci, on donne la préférence à ceux qui donnent une pondération relativement lourde aux grandes inéquités (par exemple la variance de la répartition des charges fiscales). Puisque la valeur monétaire des inéquités dépend du taux d'imposition appliqué, il est essentiel de procéder à des comparaisons fiscales à revenu égal. Sur cette base, la préférence coutumi ère pour les impôts à large assiette se révèle comme justifiée — non parce que le nombre des inéquités est réduit (il peut être augmenté), mais parce que la plus grande assiette implique un taux d'imposition plus bas par personne, et ainsi moins de différence en termes monétaires entre les différents contribuables. On montre également que, contrairement aux conclusions de Johnson et Mayer, ce n'est jamais bon pour l'équité horizontale de permettre à un contribuable de se dérober à l'impôt. On essaie d'inclure des considérations d'équité verticale à l'analyse. C'est superflu seulement si la population se compose entièrement d'égaux sur le plan économique — ce qui n'est presque jamais le cas.

Steuern mit weit gefäßer Steuerbemessungsgrundlage gerechtfertigt ist — nicht weil die Anzahl der Ungleichheiten reduziert wird (sie kann sogar erhöht werden), sondern weil die breitere Bemessungsgrundlage einen niedrigeren Steuersatz je Person und somit geringere monetäre Unterschiede zwischen den verschiedenen Steuerzahlern mit sich bringt. Es wird ebenfalls gezeigt, daß es — entgegen der Schlußfolgerung von Johnson und Mayer — keinesfalls im Sinne der horizontalen Gleichheit ist, irgendeinem Steuerzahler Steuerhinterziehungen zu gestatten. Weiter wird versucht, auch den Aspekt der vertikalen Gleichheit in die Analyse einzubeziehen. Dies ist nur dann überflüssig, wenn die Bevölkerung gänzlich aus ökonomisch Gleichgestellten bestünde — was wohl kaum je der Fall sein dürfte.
3. PAPER NINE
Second-Best Aspects of Horizontal Equity Questions from
Public Finance/Finances Publiques No 3 1972
Horizontal equity — the equal treatment of economic equals — is generally agreed to be one of the basic prerequisites of justice in taxation. Together with the related concept of vertical equity (the appropriately unequal treatment of economic unequals) it has traditionally provided the fundamental criterion for assessing the distributional desirability of tax systems and changes in them.

As has been occasionally observed, the practically relevant problem which one faces in horizontal equity questions almost invariably involves making a prescription in a setting where the perfectly equitable solution is unattainable. The non-availability of the first best solution may arise for a number of different reasons. In the first place, the large-scale changes required to achieve complete horizontal equity throughout the entire tax system will often lie outside the scope of the individual practitioner’s mandate. In this case, he is obliged to take as given certain aspects of the tax structure which themselves impose considerable inequity within the system. In the second place, even if the individual has the power to make all the necessary changes, it may nevertheless be administratively impossible (or too costly) to tax certain components of the ideal tax base. For example, while accepting the conceptual desirability of netting out of income the disutility of effort involved in earning it, one might nevertheless despair of ever being able to do this in any way that was not completely arbitrary. In the third place, the area of conflict which necessarily exists between distributional and other objectives of tax policy may render the simultaneous achievement of all goals impossible if the number of fiscal instruments is insufficient. In such a case, the achievement of complete horizontal equity may not be technically impossible, but simply too costly in terms of the required sacrifice in other policy objectives.

Given that the perfectly equitable solution is, in fact, unattainable, the horizontal equity problem is seen to be similar in kind to the problem of the “second best” which arises in the Paretoian welfare framework: the objective is to select the best from among a set of suboptimal positions, the full optimum being precluded by the nature of the problem.

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1 See, for example, Johnson and Mayer [4], White and White [7]; Brennan [1], Shoup [6], pp. 23–27, and Dixon [2].
2 For example, a system of equal lump sum taxes levied on all economic units represent a feasible and administratively simple tax structure which exhibits perfect horizontal equity, but involves a violent contradiction of vertical equity objectives.
3 See Lipsey and Lancaster [5].
Recognition of the essentially second-best nature of horizontal equity questions carries with it certain implications. It suggests, for example, that the technique of analysis common in equity literature which proceeds by specifying the nature of the “first best” tax system, and then assesses actual tax systems according to their apparent similarity to this conceptually ideal one may be thoroughly inappropriate. Perhaps more strikingly, it suggests that it may not always be desirable to treat particular components of the total tax base (or to levy individual taxes) in the individually most equitable way: the inequities existing in the treatment of a specific component may be entirely in the interests of horizontal equity for the tax system as a whole, because they serve to offset inequities extant elsewhere in the system. In a similar way, it would seem that piecemeal policies designed to permit a gradual movement towards the conceptual ideal may in the process of such movement actually increase the degree of inequity within the system. All this can perhaps best be summed up by saying that there is a possibility that, as is the case in Paretian welfare theory, the technical characteristics of the second best optimum may differ very markedly from those of the “first best”.

If this is so, traditional conclusions may well be quite misleading and traditional analytic techniques dangerously inadequate. One of the aims of this analysis is to demonstrate that this is, indeed, the case.

Now, the analogy between the horizontal equity problem and the problem of second best serves to suggest not only the possible dimensions of existing inadequacies in the theory, but also a possible approach to the development of a theory capable of handling horizontal equity questions in a second best world. Most of the existing theoretical analysis relevant for this class of problems has proceeded by attempting to specify measures of the degree of horizontal inequity which a tax system exhibits, and then examining the effects of various tax changes on such measures — the ultimate objective being, of course, the minimization of the degree of horizontal inequity subject to the operative constraints. One of the major weaknesses of this approach is that it seems to be incapable of generating any results of interest except in a world composed entirely of economic equals: the analytics of the general case turn out to be fiendishly complicated. Since an acceptable degree of vertical equity is often one of the most important constraints applying, an important dimension of the second best problem is ignored if the existence of economic unequals is assumed away.

The approach taken here borrows heavily from the style of second best analysis in economic welfare theory. In contrast with other attempts, it does seem capable of incorporating vertical equity considerations to some extent. The central aim of the paper is to derive something in the way of theoretical presumptions as to the nature of the (feasible) most equitable tax system, and the characteristics of tax changes most likely to advance general equity objectives. On the basis of these, it is demonstrated that the characteristics of the second best solution are rather different from those of the ideal, so that the doubts about traditional techniques of analysis seem well justified.

For the purposes of analysis, let us suppose that economic equals are defined as being those who enjoy identical economic satisfactions (or “utility”). Additionally let us suppose

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4 See Brennan [1] for an attempt by the present author at analysis along these lines.
that the best index of utility is given by "total income", \( Y_z \), which consists of a number of components, \( Z_i \).

Then \( Y_z \) is the conceptually best tax base, where

\[
(1) \quad Y_z = Z_1 + Z_2 + \ldots + Z_n
\]

The most equitable arrangement applies when tax liability is calculated according to \( Y_z \) for each individual — that is

\[
(2) \quad T_i = t_i \cdot Y_i = f(Y_i)
\]

where \( T_i \) is the tax paid by individual \( i \), \( Y_i \) is his income, and \( t_i \) is his tax rate.

Manifestly, the rate, \( t_i \), is given as some function of the tax base, so that

\[
(3) \quad t_i = g(Y_i) = \frac{1}{Y_i} \cdot f(Y_i)
\]

with the same function, \( g \), applying to all individuals.

Under such an arrangement, individuals with equal total incomes pay the same tax: economic equals are treated equally. Individuals with higher incomes will pay "appropriately more" tax, the extent of the tax differential for different incomes being determined by the rate structure, as given by the function \( g \) (equation (3)).

Now, there are two basic characteristics of this ideal tax system. Firstly, the same rate of tax applies to all components of the tax base; that is

\[
(4) \quad T_i = t_i \cdot Y_z \quad \text{for all} \quad i
\]

\[
= t_i \cdot Z_1 + t_i \cdot Z_2 + \ldots + t_i \cdot Z_n
\]

Secondly, and relatedly, since \( t_i \) depends on \( Y_z \), the sum of the components, \( Z_j \) it is desirable to tax all components together: there is one tax with \( Y_z \) as base, not \( n \) taxes with \( Z_j \) as their respective bases, since the latter arrangement makes it virtually impossible to determine the appropriate tax rate. Disaggregation of the tax base will not, in the first best world, operate in the interests of horizontal equity.

Let us suppose that for administrative reasons, some component (say, \( Z_n \)) of the conceptually ideal tax base is necessarily exempt from tax. Arguing by analogy from a familiar result in the theory of second best, it seems attractive to suggest that it may no longer be desirable to tax all other components of income at the same rate. This suspicion can easily be substantiated.

5 The particular problem to which this paper is addressed arises quite independently of the precise specification of the ideal tax base and the definition of the individual components. Whatever the concept of "economic equals" is taken to mean exactly, the second best problem is still generally operative. Of course, some definition of the ideal tax base is necessary, before any specific second best recommendations can be made.
Suppose that component of income, $Z_n$, is related to other components via the following relation:

\[ Z_n = z(Z_1, Z_2, \ldots, Z_{n-1}) \]

By Taylor’s expansion, we can derive the following approximation:\(^6\)

\[ Z_n = z(0, \ldots, 0) + Z_1 \frac{\partial Z_n}{\partial Z_1} + Z_2 \frac{\partial Z_n}{\partial Z_2} + \ldots + Z_{n-1} \frac{\partial Z_n}{\partial Z_{n-1}} \]

Hence,

\[ Y_z = Z_1 + \ldots + Z_{n-1} + z(0, \ldots, 0) \]

Consequently, the optimal tax liability for individual $i$ is given by

\[ T_i = t_i Y_i \]

\[ = \{ t_i (1 + \frac{\partial Z_n}{\partial Z_1}) Z_1^i + \ldots + t_i (1 + \frac{\partial Z_n}{\partial Z_{n-1}}) Z_{n-1}^i \} + t_i z(0, \ldots, 0) \]

\[ = s_1^i \cdot Z_1^i + s_2^i \cdot Z_2^i + \ldots + s_{n-1}^i \cdot Z_{n-1}^i + t_i z(0, \ldots, 0) \]

\[ = s_j^i \cdot Z_j^i \]

where $s_j^i = t_i (1 + \frac{\partial Z_n}{\partial Z_j})$.

It follows immediately from (9) that it is no longer in the interests of horizontal equity to levy the same rate of tax on all components of income actually subject to tax. Rates of tax on different components will differ according to the magnitude of $\frac{\partial Z_n}{\partial Z_j}$. This partial derivative, $\frac{\partial Z_n}{\partial Z_j}$, measures the extent to which income components $Z_n$ and $Z_j$ tend to be earned in association. The rate of tax applicable to components of income which tend to be highly positively associated with $Z_n$ will be correspondingly higher than that applicable to other components of income. Conversely, if a certain component is negatively associated with $Z_n$, the tax rate will tend to be lower than general. More strictly, if average “degree of association” is given by $d$, where $d = \frac{1}{n} \Sigma \frac{\partial Z_n}{\partial Z_j}$, then the tax rate applicable to component, $Z_n$,

\[ 6 \text{ This expression (6) is exactly correct if } z(Z_1, \ldots, Z_{n-1}) \text{ is linear.} \]
will be greater than the average if \( \frac{\partial Z_n}{\partial Z_n} > d \).

Since, in general, different rates of tax should apply to different income components, we seem to have also established something of a presumptive case in favour of the disaggregation of the tax base – the first best system in which there is only one tax with the appropriately broad base gives way to a system in which there are many independent taxes each bearing a different rate.

It is important, however, to bear in mind that the feasible best tax rate applying to any particular income component depends not only on \( \frac{\partial Z_n}{\partial Z_n} \) but also on the rate, \( t' \) (see equation (10)). Now, \( t' \) is determined by total income – or, in this case, on the feasible best approximation to it (equation (7)). Since \( Y_z \) must also be known in order to calculate the appropriate tax rate, the case for disaggregation of the tax base is more apparent than real: unless the size of other components is taken into account in taxing any one component, no very close approximation to horizontal equity can be expected. In other words, equation (8) notwithstanding, this approximation of \( Y_z \) remains, in general, the predominant determinant of the total tax burden and, hence, of the rate appropriate to any one component of the base.

In many cases, therefore, the feasible best tax system will involve not so much differential rates of tax on different components of the tax base, but rather respecification of those various components in such a way as to change the effective rate in the appropriate direction. If we view the relation in (8) above in a slightly different way, we have:

\[
-T_j = t' \cdot Y_z
\]

(11) \( + \sum (1 + \frac{\partial Z_n}{\partial Z_j} Z_1 + (1 + \frac{\partial Z_n}{\partial Z_2} Z_2 + ... + (1 + \frac{\partial Z_n}{\partial Z_{n-1}} Z_{n-1} + Z(0, ..., 0)) \]

In this expression, the adjustment required to take account of the impossibility of taxing component, \( Z_n \), occurs not in the rates of tax but in the definitions of components. For example, component \( Z_j \) is redefined in such a way as to give earners of that class of income relatively favourable or unfavourable treatment according to whether \( \frac{\partial Z_n}{\partial Z_j} \) is less than or greater than the average degree of association. In the former case, this might be done by allowing deductions which in the "first best" world are not regarded as equitable, or by narrowing the base in some other way. Alternatively, it may in some cases be easier to achieve by giving all other components rather harsher treatment by refusing to allow deductions which in the conceptually ideal situation are perfectly legitimate. By such means, effective rates of tax on the various components can be changed in accordance with equation (10) without any disaggregation of the tax base.

Such redefinition is not, of course, always easy – the effective rate on a particular component may only approximate the second best one. Thus, the technique of altering the specifications of the base will not in general be as successful in achieving second best equity objectives as is explicit variation of tax rates for different components, so that some equity cost is imposed by doing things in this way. The point is simply that explicit variation implies
disaggregation of the base and hence, itself, involves a substantial equity cost — almost certainly a larger one.

It is worth noting that the case against disaggregation depends on the desirability of a progressive rate structure. In a world composed entirely of economic equals, or one in which a proportional rate structure is regarded as sufficient for vertical equity purposes, there is no equity cost involved in disaggregating the tax base, so that explicit rate variation (as in equation (10)) is quite acceptable. This is so because the tax rate on total income is the same for all individuals:

\[ T = t \cdot Y = t \cdot \sum_{i} \left( \left( t(1+\frac{\partial Z_n}{\partial Z_1}) \right) Z_1^i + \ldots + \left( t(1+\frac{\partial Z_n}{\partial Z_{n-1}}) \right) Z_{n-1}^i + t \cdot z(0,...,0) \right) \]

Manifestly, the rate of tax applicable to any component is independent of \( i \). In this sense at least, focusing on horizontal equity questions in abstraction from the vertical aspects would lead to conclusions markedly different from those which result when a more integrated view of the equity goal is taken.

It should perhaps also be pointed out that the analysis is completely independent of the sign of \( Z_n \) — if \( Z_n \) were negative then equation (7) would apply except that the sign of each partial derivative \( \frac{\partial Z_n}{\partial Z_j} \) would be negative, and \( z(0,...,0) \) would represent a lump sum deduction from the tax base not an addition — that is, equation (7) becomes

\[ Y_z = Z_1 + \ldots + Z_{n-1} - Z_n - t \cdot z(0,...,0) \]

Thus, the analysis applies identically to the case where failure to achieve the conceptually ideal tax base arises through the administrative difficulties of exempting from tax some component which should be netted out of the tax base.

The central conclusions of this theoretical analysis are, then, as follows: a) in the case where administrative constraints preclude the taxation of some component of the conceptually ideal tax base (or the exemption of some non-component) it will not, in general, be desirable from a horizontal equity point of view to levy identical effective rates of tax on remaining components of the tax base; b) because of vertical equity considerations, variations in the effective rate of tax as between components will in general be better achieved through respecification of these components than through disaggregation of the tax base and explicit differentials between components. This implies the conscious introduction of (avoidable) inequities in order to cancel out others (unavoidable ones) prevailing in the system.

These conclusions are, I think, sufficient to demonstrate that, as in Paretian welfare theory, the technical characteristics of the second best solution differ substantially from those of the first best. Thus, the doubts about the validity of traditional conclusions and the relevance of traditional analytic techniques seem to be justified.
II.

It is perhaps profitable, in order to indicate how these results might be applied, to consider a number of simple examples.

In the first place, let us suppose that income is broken up into three components: $Z_1$, labour income; $Z_2$, property income; and $Z_3$, leisure. It is assumed to be administratively impossible to tax leisure, (or the imputed income therefrom), so that a tax system which levies the same rate $t$ on both $Z_1$ and $Z_2$ may be rejected on horizontal equity grounds in favour of a system in which $Z_1$ and $Z_2$ are taxed at different rates. In particular since

$$\frac{\partial Z_3}{\partial Z_1} < 0 \quad \text{and} \quad \frac{\partial Z_3}{\partial Z_2} \approx 0,$$

a system which differentiates appropriately in favour of labour income and against property income, will be more horizontally equitable than a system which taxes labour and property income at the same rate. Hence, the discrimination which exists against property income earners under the income tax with net accretions base (normally referred to as the “double taxation of savings”) may be justified on the grounds that the imputed income from leisure is untaxed. This is not, of course, an unfamiliar conclusion, but it does serve to exemplify the sort of results which apply in practice. A more unusual example is provided by the taxation of capital gains in the context of the following model. Suppose that for some reason, capital gains cannot be taxed. The question then arises — what is the second best solution, given that other components of income — wages, dividends and so on — can all be taxed? To answer this question, we consider a model in which the investor makes a choice between two types of investment — the purchase of “growth” stocks, from which he anticipates a capital gain, and the purchase of “income” stocks, which will yield a dividend. The rational investor will act so as to maximize his net return (i.e., the return net of tax and of risk premium). In equilibrium, the net rate of return for growth and income shares must be the same — otherwise, investors would move from the less profitable to the more profitable field of investment. In this model, assume that the firm cannot alter the nature of its stocks. If the distinction between growth and income shares is primarily a question of the investment policy of the firm, we assume that such policy is largely determined by the nature of the firm’s operations. If the firm could change plough-back policy at will, and thereby change the nature of its stock, the supply of the different asset types would become highly elastic. The assumption here is that the supply of income (and growth) shares is fixed.

Suppose initially there is no tax, and consider the imposition of a tax on dividends (capital gains being exempt). There will be a movement of capital from income shares towards growth shares, since the net rate of return on growth shares is higher. The price of growth shares rises (and the price of income shares falls) until the net rate of return is again the same for all types of asset. Hence, the tax, in the long run, is effectively spread over all assets. The net rate of return on all assets has fallen by the same proportion, and this proportion is less than the rate of tax on dividends. Thus, the redistributional effects included by the tax on dividends are threefold: (i) a capital loss to current holder of income shares, (ii) a capital gain to current holders of growth shares, (iii) a reduction in the net rate of return on all capital

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7 See, for example, Head [3].
consider to: $Z_1$, relatively which ends in $0$, then property income double issue is unify the tax of the question become — a model phase of income — his net if return from the the firm shares is policy is plough different income towards if growth the same sets. The con is less the tax on capital gain all capital investment by an amount which represents a smaller proportion of total capital income than the tax rate on dividends.

The capital gains and losses occur in the period when the imposition of the tax is announced, and represent a once-and-for-all redistribution in favour of growth share owners at that time. Throughout the subsequent life of the tax, the horizontal inequities induced by the failure to tax capital gains are not between receivers of capital gains (untaxed) and other property income recipients, but between property income earners generally (including those who make capital gains) and labour income earners. Purchasers of growth shares have to pay a higher price (and purchasers of income shares a lower price) than they would have paid in the absence of the tax on dividend income. The effective (as distinct from nominal) rates of tax on the differing types of asset are thereby equalized, though at a level lower than the nominal rate on dividend income and hence lower than the effective rate applying to non-property income.

Let us now consider the case for attempting to plug the capital gains loophole in such a context by incorporating capital gains into the tax base. Again, the redistributional effects will be threefold — but in this case we have: a capital gain to current owners of income shares; a capital loss to current owners of growth shares; and a further reduction in the overall rate of return on capital. Clearly, there is no guarantee that those who gain (lose) relatively by the imposition of the capital gains tax will be those who lost (gained) when the tax on dividends was imposed. On the contrary, since those who have the highest marginal tax rates have the greatest incentive to move into tax free areas, we would expect that the response to the tax on dividend income would involve a reallocation of share types — growth shares to the rich, and income shares to the poor. This means, of course, that the lump sum effects of closing the capital gains loophole are likely to be vertically equitable. But given that characteristics other than income (such as attitudes to risk) will in general influence portfolio selection, they may be far from horizontally equitable. More generally, the usual persuasive case put forward for a capital gains tax may be somewhat misplaced, because it operates on the naive incidence assumption that no capitalization and amortization occur either in response to the tax or its absence.

However, all this is an aside. The point relevant for the current analysis is that, in this model, since the net rate of return is equalized for all assets, then capital gains and dividend income tend (in the long run) to be positively associated. Hence, $\frac{\partial Z_3}{\partial Z_2} > 0$ and $\frac{\partial Z_3}{\partial Z_1} = 0$,

where $Z_1$, $Z_2$ and $Z_3$ are labour income, dividend income and capital gains respectively. Thus, if as assumed capital gains cannot for various administrative reasons be incorporated in the income tax base, a reasonable second best alternative may be to tax such property income as can be taxed at a rather higher rate than labour income. Although the immediate capitalization effects may be undesirable, it may not be too difficult to counteract these by a series of lump sum transfers, and the long run effects are likely to be in the interests of horizontal equity since the effective tax rates on labour and capital income are brought closer to equality.

Increasing the effective rate of tax on property income might be achieved in a number of different ways. Firstly, we might replace the existing general income tax (with capital gains exempt) by two taxes — one on property income (other than capital gains) and one on labour income. Both would exhibit progressive rate structures, but the rate on property
income would be uniformly higher than that on labour income. Such a scheme would, however, discriminate against individuals who earned income predominantly in one or other form, and operate favourably towards those whose income source was spread evenly between property and labour effort. As an alternative, therefore, one might retain the general income tax, but alter the definitions of labour and property incomes for tax purposes. Deductions for property incomes (such as that applying for interest paid on borrowed funds where those funds are used to purchase income-bearing assets) which in a conceptually ideal world are thoroughly legitimate, may be disallowed, and/or exemptions quite unwarranted in the first best situation may be instituted for wage and salary incomes. In this way, the effective rate of tax on property income is raised and that on labour income lowered without dis-aggregation of the tax base being involved. Of course, this latter method, in general, involves the introduction of further inequities (such as between those who borrow for investment and those who invest out of personal funds), and the last state of the world may turn out to be worse than the first. Whether or not this is so is essentially an empirical question, which we cannot consider here; but there is, as we have seen, nothing in the way of general theoretical presumption which would necessarily incline us to such a view. On the contrary, the very nature of the second best solution is such that the explicit introduction of inequities will in general be required to lead us towards equity objectives.

REFERENCES


Summary: Second-best Aspects of Horizontal Equity Questions. — The practically relevant problem which one faces in horizontal equity questions almost invariably involves making a policy choice in a setting where the perfectly equitable solution is unattainable. In this sense, the horizontal equity problem is similar in kind to the problem of the "second-best" in standard Paretian welfare theory. Such similarity immediately suggests that the nature of the best feasible tax system may differ very sharply from that of the conceptual ideal — just as second-best optima differ substantially from "first-best" in efficiency matters. It is one of the objectives of the paper to show that this is indeed the case. Beyond this, however, the second-best analogy serves to indicate how criteria may be developed capable of handling horizontal equity questions in the practically relevant setting. By contrast with other attempts at devising such criteria, this approach is capable of incorporating vertical equity considerations into the analysis explicitly. The central conclusion is that it will in general not be desirable to treat particular components of the tax base in the individually most equitable way: the inequities existing in the treatment of a specific component may be entirely in the interests of horizontal equity as a whole, because they offset inequities elsewhere in the system. Likewise, to achieve the most equitable tax system possible, it will normally be necessary explicitly to introduce inequities into the tax system, designed so as to compensate for those which are genuinely unavoidable.
Résumé : A propos du "Second-Best" et de l'équité horizontale. — Le problème pratiquement important que l'on rencontre en matière d'équité horizontale implique presque toujours un choix politique tel qu'il est impossible de parvenir à une solution parfaitement équitable. En ce sens, le problème de l'équité horizontale est du même genre que celui du "deuxième rang" dans la théorie du bien-être par tient standard. Une telle analogie sug gere immédiatement que la nature du meilleur système fiscal praticable peut différer très profondément de l'idéal théorique — tout comme les optimas de deuxième rang diffèrent profondément de ceux de premier rang en matière d'efficacité. L'un des buts de cette étude est de montrer que tel est bien le cas. En outre, cependant, l'analogie avec le deuxième rang sert à montrer comment les critères peuvent être développés, de façon à saisir les questions d'équité horizontale sur le plan de l'importance pratique. Contrairement aux autres tentatives faites pour découvrir de tels critères, cette approche est capable d'intégrer explicitement dans l'analyse des considérations d'équité verticale. La conclusion essentielle est qu'il ne sera généralement pas souhaitable de traiter les divers éléments de la base imposable de la manière individuellement la plus équitable : les inégalités existant dans les traitements d'un de ces éléments spécifiques peuvent être conformes à l'équité horizontale considérée comme un tout, car elles annulent des inégalités existant par ailleurs dans le système. De la même manière, pour obtenir le système fiscal le plus équitable possible, il sera généralement nécessaire d'introduire explicitement des inégalités de traitement fiscal qui serviront à compenser celles qui sont naturellement inévitables.

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Paper One: Lindahl's Theory of the Budget

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Paper Two: Public Goods and Factor Prices

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Paper Five: Pareto-Desirable Redistribution: The Case of Malice and Envy

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Paper Seven: 'Pareto-Optimal Redistribution': A Perspective

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