Macroeconomic Adjustment to External Shocks: Essays on the Behaviour of Individuals and Markets

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A thesis submitted for the degree of Doctor of Philosophy of the Australian National University
With the exception of Chapter Five, I confirm that all of the work presented in this thesis is my own original work unless specifically acknowledged as otherwise. Chapter Five is a piece of work done jointly with Mr. Jeremy Smith, Department of Statistics, The Faculties, A.N.U. Jeremy and I contributed equally to Chapter Five.

David W. R. Gruen
ACKNOWLEDGEMENTS AND DEDICATION

In February, 1980, I submitted a PhD thesis to the University of Cambridge, entitled: 'A statistical mechanical study of the adsorption (sic) of non-polar molecules into lipid bilayer membranes.' It was accepted and I became an accredited biophysicist.

After a couple of 'post-docs' in biophysics, I got restless and began to search for greener fields. I was repeatedly impressed that my father led a more interesting and stimulating life than any of the older scientists I came across. Where they had been worn down and narrowed by decades of academe, he had not. I began to suspect that, while personalities had a lot to do with it, so did the different disciplines. To be any good at science, you have to be extremely focussed and very narrow – it probably helps to take only a limited interest in the outside world. By contrast, the outside world is the central concern of economics.

On 18 March, 1984, I crossed my Rubicon and began reading Macroeconomics by Dornbusch and Fischer. Thus far, I haven't looked back*. So I would like to express my gratitude to my father, both for initial inspiration and for all his subsequent encouragement. He has read each part of the thesis as it was written, and has always made helpful and insightful comments. I would like to dedicate this thesis to him.

In my first economics degree, a Graduate Diploma at ANU, I took a total of seven courses – including three (Graduate Diploma macroeconomics, Masters microeconomics and Masters macroeconomics) taught by Dr. George Fane. When I decided to do a PhD, George agreed to be my supervisor. He has played the pivotal role in teaching me the dominant

* apart from a few forays to the library to examine the Science Citation Index and see if anyone was quoting my old work!
paradigm in economics. Crucially, he has regularly forced me – often against my own worse judgement – to take seriously the general equilibrium aspects of my analysis.

It is an advantage for a student if his supervisor’s world view differs from his own. Provided good humour prevails, the tension is creative. When objections are raised, the student can work on his analysis until it provides a response satisfactory to both. Good humour has prevailed, and I think the intellectual tension between George and me has been creative. I have always found George’s comments and reactions to my work well-founded and stimulating. I am extremely grateful to him for his kindness, for his help and for his intellectual rigour.

I am grateful to my advisor, Professor John Pitchford. He has been generous with his time and always encouraging, especially when my work overlapped with his own (as in Chapter Two).

I thank my mother for accepting with equanimity the prospect of two economists in the family – and for her tireless support in so many ways. My brother always thought my shift to economics was ‘a good thing’ and I am grateful for his expression of that sentiment.

Finally, I would like to thank Jenny Wilkinson, who has put up with having most of this thesis explained to her – often in excruciating detail. I am grateful to her for her useful suggestions, for her companionship and for putting up with me when my mind was elsewhere.
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## Chapter One

### Introduction

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## Chapter Three

### What people know and what economists think they know: a shred of evidence on Ricardian equivalence

1. Introduction and summary
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### Ignorance and Ricardian equivalence

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This thesis consists of three distinct sections. The first two sections present theoretical models of the response of private agents to an unanticipated terms of trade shock and to changes in government net saving, respectively. The third section examines the joint behaviour of interest rates and the exchange rate. The link between the sections is that they were all motivated by the response of the Australian macroeconomy to the 1985/86 adverse terms of trade shock, and by the government’s reaction to that shock.

The first section of the thesis (Chapter Two) analyses the impact on the current account of a permanent terms of trade deterioration in a model in which an immortal intertemporally optimizing household is assumed to exhibit an explicit sensitivity to falling consumption levels. When the household is sufficiently sensitive, it is optimal for its consumption to be continuous over time and then the Harberger-Laursen-Metzler effect holds, i.e., following the trade shock, the current account deteriorates. It is argued that this model is more realistic than earlier models which predict a current account improvement in response to a permanent adverse trade shock.

The second section of the thesis consists of two sub-sections which are both concerned with the Ricardian equivalence hypothesis. The first sub-section (Chapter Three) reports the results of two surveys. Over six hundred economics students were asked to estimate the level of outstanding Australian Federal government debt, and eleven academic economists were asked to predict the proportion of students with ‘a rough idea of the amount of this debt’ (in a sense defined precisely). Student knowledge is very meagre and the average academic overestimates it
The relevance of these results for the Ricardian equivalence hypothesis is discussed.

The second sub-section (Chapter Four) develops a model of an immortal, intertemporally optimizing consumer who is ignorant of the link between bonds and future taxes. The consumer is exposed either to the changing level of Federal government debt in Australia or in the U.S. over the twenty five years, 1963 – 1987. "Best" estimates of the cost of ignorance are about $A2 per annum for an Australian consumer or about $7 per annum for a U.S. consumer – i.e., less than 0.1% of annual income in both instances. When uncertainty about future income and the existence of progressive taxes are included in the model, these estimates are substantially reduced – even from this 0.1% level. It may therefore be optimal for consumers to ignore the link between bonds and future taxes.

The final section of the thesis (Chapter Five) examines the large short-term real interest differential between Australia and the US since late 1984. The chapter provides a detailed examination of an arbitrage condition for a representative US investor. There is some evidence for a risk premium until late 1985. The chapter also examines the possibilities that either the foreign exchange market is inefficient or that the market has continually and rationally expected real depreciation of the $A, but that such depreciation has not yet occurred.
CHAPTER ONE

INTRODUCTION

This thesis contains three sections. In the first two sections, two distinct theoretical models and some survey results are presented, while the third section is an applied piece of work. The sections are linked to the extent that they all evolved from my attempts to grapple with issues raised by the response of the Australian macroeconomy to the 1985/86 adverse terms of trade shock, and by the government's reaction to that shock.

Over the twenty five years 1959/60 to 1983/84, Australia's current account deficit averaged 2.7% of GDP. By contrast, over the five years 1984/85 to 1988/89, it averaged 5.2% of GDP. While there are grounds for arguing that this development should not concern policy makers (Pitchford, 1989), this big increase in the current account deficit – and the associated growth in Australia's net external liabilities – has dominated Australian macroeconomic debate over the last five years.¹

After having been roughly constant for about seven years, Australia's terms of trade fell by about 14% from the March quarter 1985 to the March quarter 1987. This was widely regarded as an important contributing factor both in the subsequent big increase in the Australian current account deficit and in the large fall of the Australian nominal and real exchange rate over roughly the same time.²

¹ It has made the late 1980s in Australia a fascinating time and place to begin a serious interest in macroeconomics.
² The Australian trade-weighted exchange rate index (TWI) fell 36% from the end of December, 1984 to the end of September, 1986. Adjusted for the relative inflation between Australia and its trading partners, this translated into a 29% real depreciation over this time.
On innumerable occasions over the last five years, members of the Federal government have expressed concern about the big increase in Australia's current account deficits. At least partly because of this concern, the government embarked on a programme to increase substantially the net savings of the Australian public sector. This programme has been a remarkable success. As measured by the net public sector borrowing requirement, the public sector moved from a deficit of 6.7% of GDP in 1983/84 to a surplus of 1.0% of GDP in 1988/89.

Each of the above aspects of the behaviour of the Australian macroeconomy provides the motivation for a section of this thesis. The first two sections of the thesis concern, respectively, the response of private agents to an unanticipated terms of trade shock and to changes in government net saving, while the third and final section examines the joint behaviour of interest rates and the exchange rate.

Before delving into the details of the thesis, I draw the reader's attention to a recurring theme. Much neo-classical economics involves maximization of standard objective functions subject to constraints. In this thesis, there are some attempts to go beyond the confines of these standard optimization problems. In most cases, psychology provides the motivation for the proposed modification. The importance of psychology even in markets with high stakes and negligible transaction costs (e.g., securities markets) has recently been highlighted by LeRoy (1989):

'The recent literature on cognitive psychology ... provides a promising avenue for future research. Cognitive psychologists have documented systematic biases in the way people use information and make decisions. Some of these biases are easy to connect, at least informally, with securities market behavior. For example, agents allow their decisions to be distorted by the
presence of points of reference that should be irrelevant ("anchoring"). Further, they systematically overweight current information and underweight background information relative to what Bayes' theorem implies. To be sure, most of the evidence for these biases comes from experiments and questionnaires. Economists have in the past confidently assumed that these biases would disappear in settings where the stakes are high, as in real-world securities markets. However, this line is beginning to wear thin, particularly in the light of economists' continuing inability to explain asset prices using models that assume away cognitive biases.' (p. 1616).

In the remainder of this chapter, a detailed summary of the three sections of the thesis is provided. Attention is drawn to the psychology of individuals and the importance of cognitive biases when they are relevant.

In the first section of the thesis (Chapter Two) a model is developed in which an immortal intertemporally optimizing household is subject to an unanticipated adverse permanent terms of trade shock. Most simple models of intertemporally optimal behaviour predict that such a shock will lead to an immediate *improvement* in the current account (e.g., Obstfeld, 1982; Svensson and Razin, 1983; Fane, 1987). The reason for this paradoxical result is straightforward. The representative household, which makes up the whole domestic economy, has a target level of wealth. The household eventually returns to this target level of wealth after any shock – which ensures that the model exhibits long-run stability. By assumption, the household is forward-looking and cares only about smoothing future consumption. A permanent adverse terms of trade shock reduces wealth below the target. The optimal response is for the household to immediately cut consumption to accumulate foreign assets and eventually re-establish the target level of wealth. In other words, in
adjustment to the new long-run, the domestic economy runs a current account surplus.

In the model of Chapter Two, I retain an intertemporal utility maximizing framework but I introduce an innovation to the standard model which overturns this paradoxical result. All the above models make the standard assumption that instantaneous utility depends on consumption only via the levels of consumption of available goods. But, when the dynamic response of the current account to an adverse shock is of central interest, this traditional assumption is a serious oversimplification. In a long-run stationary state, the representative household has established levels of consumption of all available goods which maximize lifetime utility and balance the current account. If such a household is subject to an unexpected permanent terms of trade shock, these prevailing levels of consumption cannot be sustained indefinitely. But, consumers have an aversion to rapidly falling consumption levels (see below). If this aversion is strong enough, in response to the trade shock, it will be optimal for the household to reduce its consumption gradually. When it does so, for some time following the trade shock, the current account will deteriorate.

Chapter Two invokes Duesenberry's (1949) psychological reasons for consumers asymmetrical response to improving as opposed to worsening future income prospects. Duesenberry explains (p. 25–32) why consumption will quickly rise in response to improving future income prospects. His explanation is in terms of culture, a demonstration effect and status. Thus, 'the basic source of the drive toward higher consumption is to be found in the character of our culture. ... The consumption pattern of the moment is conceived of not as part of a way of life, but only as a temporary adjustment to circumstances. We expect to take the first available chance to change the pattern'. The demonstration
effect is described in these terms: ‘People believe that the consumption of high quality goods for any purpose is desirable and important. If they habitually use one set of goods, they can be made dissatisfied with them by a demonstration of the superiority of others. ... consumption expenditures can be forced up by contact with superior goods’. Finally, the importance of status in influencing consumption is highlighted: ‘Once a group of high income people are recognised as a group of superior status, their consumption standard itself becomes one of the criteria for judging success. ... Once this has occurred, it becomes difficult for anyone to attain a high status position unless he can maintain a high consumption standard’.

That discussion may be compared with Duesenberry’s attitude to falling income prospects, e.g., ‘when high-income families suffer a loss in income ... they continue to live in the same kind of neighborhoods and maintain their contacts with others of the same socio-economic status ... they can absorb a considerable reduction in income by reducing saving without cutting consumption too deeply. Moreover, there is no reason why they should not continue in this position for several years’ (p. 87 – italics added). It is these aspects of the social interaction and psychology of consumers that provide a justification for introducing an aversion to falling consumption directly into the representative consumer’s utility function in Chapter Two.

The second section of the thesis, comprising Chapters Three and Four, is concerned with the response of individuals to changes in the net saving of their government. When a government spends more than it taxes, it makes up the difference by printing money and/or issuing bonds. Other things equal – crucially, government spending and asset holdings through time – an extra bond issued today requires an increase in tax sometime in the future. At least in simple models, the present discounted
value of the future tax increase is exactly equal to the value of the extra bond today.

In what has become known as the Ricardian equivalence hypothesis – noted by David Ricardo and revived by Robert Barro in 1974 – it is suggested that consumers take account of the future tax consequences of their governments’ outstanding bonds. ‘Ricardian’ consumers make their own optimal intertemporal consumption choices – independent of the changing debt position of their government. If the private sector is exclusively inhabited by Ricardians, bond-financed changes in tax have no impact on aggregate demand as they lead to no change in present or future consumption. In an open economy, tax-bond switches have no impact on the current account.

Ricardian equivalence holds exactly when capital markets are perfect, taxes are non-distortionary (lump-sum), and each generation of fully-informed rational consumers is linked to the next by operational intergenerational transfers, so that consumption decisions can be modelled as being made by an immortal representative consumer. Almost all criticisms of Ricardian equivalence amount to a rejection of one or more of these assumptions as being either theoretically or empirically unwarranted.

In Chapter Three, survey evidence on two questions of relevance to the Ricardian equivalence hypothesis is presented. First, do consumers have sufficient knowledge of the level of outstanding government debt to make Ricardian equivalence a plausible model of their behaviour? Secondly, what are academic economists’ perceptions of the community’s knowledge of government debt? Rather than addressing the first question directly, I provide evidence about a group one might expect to have more knowledge than the whole community – undergraduate economics
students. In providing evidence about the second question, I compare the level of student knowledge with academic economists' perceptions of that knowledge.

Student knowledge of the level of Australian Federal government debt is found to be very limited indeed, with less than one in eight students able to provide an estimate of the right order of magnitude and willing to claim their estimate as other than a complete guess. The implication is that Ricardian equivalence seems an implausible model of behaviour – at least for this group of citizens. The contrast between the students' knowledge and academic economists' perceptions of that knowledge is striking – with the average academic overestimating student knowledge fivefold. At least in this case, the academic economists are wildly optimistic about the general level of knowledge of potentially important economic statistics.

To explain these survey results, the work of the cognitive psychologists, Tversky and Kahneman (1973,1974) is invoked. Tversky and Kahneman (1974) identify three 'heuristic principles' which people use to make judgements and decisions in uncertain situations. These heuristics 'reduce the complex tasks of assessing probabilities and predicting values to simpler judgmental operations. In general, [they] are quite useful, but sometimes they lead to severe and systematic errors' (p. 1124).

According to one of these principles, which Tversky and Kahneman call availability, people rely more than they should on readily accessible information, i.e., information which is easily retrievable from memory. This observation is useful when examining the results of the surveys in Chapter Three. For academic economists, the level of outstanding Federal government debt is probably not a number easily retrievable from memory. But using information which is easily retrievable – the value of
the last few Federal budget deficits\(^3\), or perhaps interest payments on outstanding debt – most could quickly come up with a fairly good estimate and be confident that this estimate was within a factor of ten of the true answer. If they use the availability of this information to them in estimating the proportion of students who could do likewise, we should not be so surprised that the academics provide substantial overestimates of student knowledge.

Two conclusions are suggested if the availability heuristic is widely used to make judgements in uncertain situations. First, if in the wider community, the level of outstanding government debt (and presumably, the link between government debt and future taxes) is not easily retrievable from memory, it will not be used to make consumption and saving decisions. As a consequence, tax-bond switches will have systematic effects on the macroeconomy. Second, the people most susceptible to believing in Ricardian equivalence should be those for whom the link between government debt and future taxes is a most obvious and natural one, i.e., professional economists.

A common justification for the use of the rational expectations hypothesis in macroeconomics is that, although people are not always rational, ‘we have no reason to believe that [their] irrationalities cause *systematic and predictable* deviations from rational behavior that a macroeconomist can model’ (Sargent and Wallace, 1976). Chapter Four develops a model in which an individual’s ignorance of the link between government bonds and future taxes leads to exactly such ‘systematic and predictable deviations from rational behavior’, but at no more than a tiny gross cost to her. The implication is that, if the expected cost of informing oneself is

\(^3\) The surveys were conducted before the Federal government began running budget surpluses.
larger than the expected cost of remaining ignorant, then there is an expected net gain from remaining ignorant and it is optimal to do so.\textsuperscript{4}

The individual in Chapter Four is ignorant of the link between bonds and future taxes – but in all other ways she is intertemporally optimizing. At all times, she sets consumption on the basis of her perceived wealth. Thus, for example, when the per capita level of government debt rises, she sets consumption above the level she would choose were she a Ricardian – because she does not understand the future taxes consequences of the increased government debt. \textit{Ex post}, for any given level of initial wealth, her chosen intertemporal allocation of consumption is sub-optimal. Calculating the extent of this sub-optimality enables us to estimate the gross cost of ignorance.

Our representative individual is exposed to the changing level of Australian government debt over the twenty five years, 1963 – 1987. The calculation is then repeated for an individual exposed to US government debt over the same period. “Best” estimates of the gross cost of ignorance are about $A2 per annum for an Australian consumer or about $7 per annum for a U.S. consumer (i.e., less than 0.1% of annual income in both instances). When uncertainty about future income and the existence of progressive taxes are included in the model, these estimates are substantially reduced – even from this 0.1% level.

Clearly, if a government uses bond-financing sufficiently aggressively, it will impose big costs on consumers who are ignorant of its future tax implications. However, as we shall see in Chapter Four, even the fiscal stance of the U.S. Federal government in the 1980’s – judged by many to be

\textsuperscript{4} Although there is no attempt to do so here, it would be possible to build a model in which remaining ignorant maximized expected utility rather than expected net gain.
irresponsibly expansionary – seems to have imposed only very small gross
costs on dynastic families of consumers ignorant of its future tax
consequences.

Note that Chapter Four provides only a partial analysis – no attempt is
made to estimate the expected cost of informing oneself sufficiently to
make an approximately Ricardian consumption choice. But the
difficulties which professional economists rightly perceive in accurately
estimating the changing net worth of the government bring into focus how
severe are the informational requirements of making even an
approximately Ricardian consumption choice in a modern mixed
economy. It seems fanciful to imagine many consumers forming these
estimates for themselves – especially in the light of the results in
Chapter Three. Of course, the estimates could be made by a firm of
economists and sold to consumers. The fact that such a service is not in
widespread use in either Australia or the U.S. strengthens the case
argued in Chapter Four. Provided governments do not change their bond­
financing behaviour substantially, the costs of collecting, analysing (and
disseminating) the information required to make an approximately
Ricardian consumption choice probably outweigh the losses associated
with being ignorant of the link between bonds and future taxes. If so, the
ignorance is rational and bond-financed fiscal policy has significant
systematic effects on aggregate demand and the current account.

Chapter Five is the third and final section of the thesis. It is concerned
with the behaviour of interest rates and the $A exchange rate since the
currency was floated in December 1983. Over the nineteen quarters from
the December quarter 1984 to the June quarter 1989, the *ex post* real
interest differential between Australian and US three month Treasury
bills was positive for sixteen quarters, and averaged 2.4% p.a. Given the
absence of any obvious barriers to the movement of capital into or out of
Australia, this result implies one of three possibilities. The first two possibilities are that there has been a large risk premium on $A assets, or that the foreign exchange market has been inefficient. The third possibility is that the market has continually and rationally expected real depreciation of the $A, but that this depreciation has not occurred – at least not yet. Chapter Five examines each of these possibilities in detail.

There is some survey-based evidence for a risk premium until late 1985. Since then, the survey results are consistent with a null hypothesis of zero risk premium. Chapter Five also presents two \textit{a priori} calculations of the average magnitude of the risk premium necessary to induce a US consumer-investor to hold a small part of her wealth in Australian nominal assets. These theoretical models generate estimates of the average risk premium that are so small as to be negligible (compared, for example, with the average real interest differential between Australia and the US quoted above).

Since a time-varying risk premium provides an inadequate explanation for the real interest differential after late 1985, evidence is provided in favour of the latter two explanations. Chapter Five reviews the international evidence suggesting foreign exchange market inefficiency. One part of this evidence is survey data that suggests that foreign exchange market participants' expectations of the future spot rate are not rational (in the economists' sense of the word).

It is perhaps not surprising that the forward exchange rate is a statistically-significant explanator of foreign exchange market participants' expectations of the future spot rate (Froot and Frankel, 1989)

\footnote{See Section 5.2 for an objection to the use of survey data to measure exchange rate expectations, and for a response to that objection.}

\footnote{The forward exchange rate is the rate at which foreign exchange can be traded now for settlement at a specified date in the future.}
and Chapter Five). In the words of Froot and Frankel: ‘Expectations seem to move very strongly with the forward rate.’ However, there is overwhelming statistical evidence (Froot and Frankel, 1989 and Chapter Five) that the forward rate is a biased predictor of the realised value of the future spot exchange rate. To the extent that surveys of market expectations give reliable information, they imply that expectations are not rational.

A second of the heuristic principles identified by Tversky and Kahneman (1974), called anchoring, may explain why these expectations do not seem to be rational (in the economist’s sense). Anchoring may be described in these terms: ‘In many [uncertain] situations, people make estimates by starting from an initial value that is adjusted to yield the final answer. The initial value, or starting point, may be suggested by the formulation of the problem, or it may be the result of a partial computation. In either case, adjustments are typically insufficient. That is, different starting points yield different estimates, which are biased toward the initial values. We call this phenomenon anchoring. ... [It] is common to naive and to sophisticated subjects, and it is not eliminated by introducing proper scoring rules, which provide incentives for external calibration.’ (p. 1128 and 1129, Tversky and Kahneman, 1974). As this quote suggests, it may be that the forward rate provides an anchor for market participants’ expectations.

Finally, Chapter Five turns to the possibility that the market has continually and rationally expected real depreciation of the $A – but that this depreciation has not (yet) occurred, i.e., that the $A has suffered from a ‘peso problem’.  

7 In regions renowned for earthquakes, many people invest in earthquake insurance. Consider econometricians studying this behaviour during a time in which there have been no earthquakes. They may falsely conclude
There are two observations which make a peso problem for the $A a possibility. Firstly, analysis in Chapter Five demonstrates that unlike all the other currencies examined, over one week and four weeks the $A is subject to infrequent, unpredictable and large depreciations. So, the market may have such events built into their expectations. Secondly, there is an obvious candidate for the cause of a peso problem – that in the longer run the real economy must adjust to stabilize the ratio of Australia’s net external liabilities to GDP – with a lower real and nominal exchange rate during the adjustment process. Chapter Five examines what appear to have been the causes of the ten largest weekly depreciations of the $A from January 1986 to April 1989. Seven appear interpretable in terms of events in the Australian economy. Of these, five seem related to the need for the real economy to adjust to stabilize the ratio of Australia’s net external liabilities to GDP. The $A may suffer from a peso problem because of a market perception that a lower real exchange rate is needed as part of the adjustment to external balance.

Ultimately, Chapter Five remains agnostic about which of the latter two explanations (foreign exchange market inefficiency or a peso problem) is more plausible. To use those immortal words: that is an appropriate topic for future research.

that the behaviour is irrational, because, over their sample, there has been no return from the investment. A similar difficulty exists in the foreign exchange market. In Mexico in the 1970’s, the peso was permanently at a forward discount compared to the $US despite a fixed exchange rate between Mexico and the US which had been in place for years (Krasker, 1980). The market continually expected a devaluation of the peso, and while it did not occur a ‘peso problem’ was said to exist. This term has now become standard to describe this small-sample problem (Hodrick, 1987).
CHAPTER TWO

AGGREGATE SPENDING AND THE TERMS OF TRADE:
THERE IS PROBABLY A HARBERGER-LAURSEN-METZLER EFFECT

2.1 INTRODUCTION AND SUMMARY

In two renowned articles published forty years ago, Harberger (1950) and Laursen and Metzler (1950) argued that as a result of a terms of trade deterioration, a country suffers a fall in its “real income” which, in turn, reduces private savings out of any given level of income (both measured in terms of exportables). Ignoring changes in investment and the government budget deficit, this fall in the flow of savings implies a deterioration of equal magnitude in the country’s current account balance. So, if income (measured in terms of exportables) remains constant, this Harberger-Laursen-Metzler (HLM) effect implies that a deterioration in the terms of trade immediately leads to a deterioration in the current account balance.

Clearly, the HLM effect involves intertemporal choices between consumption and saving. Recognising this, Obstfeld (1982) and Svensson and Razin (1983) re-examined the HLM effect using models which explicitly include intertemporal utility maximizing behavior on the part of a representative household. Obstfeld’s continuous-time model considers a household which maximizes an integral of discounted instantaneous utilities out into the infinite future. Svensson and Razin, as well as providing a detailed analysis of a two period model, also examine an infinite-period model. For this latter model, the representative household exhibits identically homothetically weakly separable preferences, domestic output and the world interest rate, $r$, are the same in every
period and prices are always expected to remain constant for ever. In the model's steady-state, the current account is always balanced.

Svensson and Razin demonstrate that both the stability of this steady-state and the response of the current account to an unexpected permanent trade shock depend on the household's subjective rate of time preference or discount rate, $\delta$. Assuming $\delta$ is a function of per-period utility, $U$, whenever $\delta(U) < r$, the household holds down present consumption (and hence present per-period utility) to accumulate foreign assets which, in turn, allows a higher level of per-period utility in the future. Conversely, when $\delta(U) > r$, the appropriate response is to run down net wealth which therefore reduces per-period utility in the future. Thus, we may identify three types of points on the function $\delta(U)$. A per-period utility $U_1$, where $\delta(U_1) = r$ and $\delta'(U_1) > 0$, represents a locally stable long-run equilibrium. Alternately, a per-period utility $U_2$, where $\delta(U_2) = r$ and $\delta'(U_2) < 0$, represents a locally unstable long-run equilibrium. Finally, if $\delta(U) = r$ and $\delta'(U) = 0$ for $U_3 \leq U \leq U_4$, the household can settle into a neutral long-run equilibrium with any per-period utility between $U_3$ and $U_4$.

Provided the economy stays in the neighborhood of the steady-state, starting from $U_1(U_2)$, an unexpected permanent trade shock leads immediately to a current account surplus (deficit). By contrast, for points between $U_3$ and $U_4$, the response to a permanent trade shock is that consumption immediately jumps to its new long-run value and the current account remains balanced.

Svensson and Razin point out that there are no \textit{a priori} grounds for assuming either a falling or a rising rate of time preference as utility increases. Fisher (1930) assumed the former preferences, while Uzawa (1968), followed by Findlay (1978) and Obstfeld (1982) invoked the latter preferences. Nevertheless, it is a central conclusion of both Obstfeld and
Svensson and Razin that when $\delta(U)$ has a form which leads to either a stable or a neutral stationary state for the infinite-horizon model, an unexpected permanent terms of trade deterioration does not lead to an immediate deterioration in the current account, in disagreement with the HLM effect.¹

This chapter retains an intertemporal utility maximizing framework while challenging this conclusion. Obstfeld, Svensson and Razin, as well as others who have examined the issue [Persson and Svensson (1985), Bean (1986) and Matsuyama (1987)], make the almost universal assumption that instantaneous utility depends on consumption only via the levels of consumption of available goods. But, when the dynamic response of the current account to an adverse shock is of central interest, this traditional assumption is a serious over-simplification. In a long-run stationary state, a representative household has established levels of consumption of all available goods which maximize lifetime utility and balance the current account. If such a household is subject to an unexpected permanent terms of trade shock, these prevailing levels of consumption cannot be sustained indefinitely. But, as has been stressed by several authors (see section 2.4), consumers have an aversion to rapidly falling consumption levels. If this aversion is strong enough, in response to the trade shock, it will be optimal for the household to reduce its consumption gradually. When it does so, for some time following the trade shock, the current account will deteriorate and the Harberger-Laursen-Metzler effect will be observed.

The household's aversion to rapidly changing consumption is introduced by assuming that its instantaneous utility depends not only on the levels of

¹ Note that, when the stationary state is stable, the short-run improvement of the current account follows as a direct consequence of the necessity to satisfy the long-run equilibrium condition, $\delta(U^{LR}) = r$. 

consumption of the available goods, but also on their rates of change with time. Crucially, when consumption is falling over time, it is assumed that the faster it is falling, the lower is the derived instantaneous utility. Within the class of models which consider agents with an infinite horizon, there have been attempts to move away from the assumption that preferences are additively separable over time – the work of Iwai (1972) being an early example which Svensson and Razin (1983) build upon. But, as far as I am aware, the preferences invoked here have not been suggested before. Nevertheless, they are a natural extension of the standard form to encompass consumers' aversion to falling consumption levels. I do not present axiomatic foundations for these new preferences. Rather, in section 2.4, I present what seem the compelling arguments of several authors who have suggested that consumers have an aversion to rapidly falling consumption. As we shall see, conclusions derived from the present model are at variance with the standard conclusions from earlier intertemporal models.

The household maximizes an integral of discounted instantaneous utilities over an infinite horizon by optimally allocating income each moment between consumption and the accumulation of real claims on foreigners. I examine two forms for the subjective rate of time preference, $\delta$, which lead to a stationary state for this infinite-horizon model: $\delta = r$, and the Uzawa (1968) form. A description of the important results in the chapter and its layout follows.

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2 In discrete time models, Hayashi (1985) and Eichenbaum et. al. (1988) assume that consumption goods yield a flow of utility in the period in which they are purchased as well as in subsequent periods. Hayashi's rationale for these preferences is that he is examining the durability of a range of consumption goods. By contrast, Eichenbaum et. al. (1988) explicitly study non-durables and services. As they stress, their assumption of the non-time-separability of preferences is important for the debate about real business cycles. The motivation for, and the consequences of, these preferences are clearly different than for the present model.
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the debate about real business cycles. The motivation for, and the
consequences of, these preferences are clearly different than for the
present model.
Section 2.2 describes the model to be analysed and the optimization problem faced by the representative household. Necessary conditions are derived which must be satisfied by any optimal consumption paths. I demonstrate in Appendix A that, when $\delta = r$ and the instantaneous utility function is additively separable, unique globally optimal paths for consumption exist. When the household is sufficiently sensitive to changing consumption (in a sense defined precisely), these consumption paths are continuous over time, and in immediate response to an unexpected adverse trade shock the current account deteriorates, i.e., the HLM effect holds. Of course, in the long-run following the shock, the current account is again balanced. This dynamic response contrasts with the behavior of the economy when the household has no explicit sensitivity to changing levels of consumption, when, following such a shock, consumption instantly jumps to new long-run values and the current account remains balanced.

Unfortunately, when $\delta$ takes the Uzawa form, it is possible to demonstrate neither uniqueness nor global optimality for consumption paths which satisfy the necessary conditions for optimality. Nevertheless, in Appendix B, I derive a constraint on the household's utility function under which a given discontinuous fall in consumption is a sub-optimal response to a given terms of trade shock. In Section 2.3, simulations are presented of a model economy's response to terms of trade shocks. Consumption paths and the response of the current account are displayed both for a specific case of the Uzawa form and for $\delta = r$. Section 2.4 discusses the chapter's relation to previous literature.
2.2 THE MODEL

Following Obstfeld (1982), the economy consists of a single representative household maximizing utility over its infinite lifetime. Utility is derived from consumption of two goods, an imported 'foreign good' and a 'home good' produced domestically at a fixed rate. Forgoing consumption, the household can save by accumulating an internationally traded bond, assumed to be indexed to the foreign good. The economy is small, in that it can influence neither the terms of trade between home and foreign goods nor the world interest rate. Finally, all prices are expected to remain fixed forever – any price changes take the household by surprise.

My point of departure from Obstfeld (1982) is the assumption here that instantaneous utility depends not only on levels of consumption but also on their rates of change with time. Thus, instantaneous utility is given by

\[ U(t) = U(c_f(t), c_h(t), \dot{c}_f(t), \dot{c}_h(t)). \]

Including a sensitivity to changing consumption levels in the instantaneous utility function means that preferences are no longer weakly separable across time. This leads to a richer dynamic adjustment behavior following a shock.

U is assumed continuous and twice differentiable in all arguments. With respect to its first and second arguments (the levels of consumption of foreign and home goods), U is assumed to be strictly increasing and strictly concave. With respect to rates of change of consumption, the household is assumed to exhibit preferences of either 'type one' or 'type two'. Both preference types imply that, for given levels of consumption at any instant, the household's instantaneous utility is highest if these consumption levels are not falling over time. If consumption levels are falling, the faster they are falling, the lower is the derived instantaneous utility.
When the household exhibits type one preferences, for given levels of consumption at any instant, it is also adversely affected when consumption is rising, the more so the faster it is rising. Note, however, that the household’s sensitivity to rising and to falling consumption levels need not be symmetrical. Type two preferences imply that for given levels of consumption, the household’s instantaneous utility is completely unaffected when consumption is rising over time. Introducing the notation $U_i$ to denote the partial derivative of $U$ with respect to its $i^{th}$ argument, for both preference types,

\[ U_3 = 0 \quad \text{when} \quad \dot{c}_f(t) = 0 \quad \text{and} \]
\[ U_4 = 0 \quad \text{when} \quad \dot{c}_h(t) = 0. \quad (1) \]

When the household’s preferences are of type one, $U$ is assumed to be strictly concave in all arguments. When preferences are of type two, $U$ is assumed strictly concave in all arguments when both $\dot{c}_f(t)$ and $\dot{c}_h(t)$ are negative, but not otherwise, since for type two preferences,

\[ U_3 = 0 \quad \text{when} \quad \dot{c}_f(t) \geq 0 \quad \text{and} \]
\[ U_4 = 0 \quad \text{when} \quad \dot{c}_h(t) \geq 0. \quad (2) \]

Clearly, the household’s attitude to falling levels of consumption dictates its short-run response to a permanent adverse terms of trade shock.

Like Obstfeld (1982), to avoid non-interior solutions, it is assumed that

\[ \lim_{c_f(t) \to 0} U_1 = \lim_{c_h(t) \to 0} U_2 = \infty. \quad (3) \]

The representative family is bound by a flow constraint linking the difference between its income and its expenditure to its accumulation of the internationally traded bond. Let $b(t)$ denote bond holdings at time $t$, $p$ the price of the foreign good in terms of the domestic good, $y$ the family’s (fixed) rate of production of the home good and $a(t)$ the family’s total assets or net wealth at time $t$ denominated in foreign goods. Then,

\[ a(t) = b(t) + \frac{y}{rp} \quad (4) \]
and
\[ \dot{\delta}(t) = \frac{y}{p} - c_f(t) - c_h(t) / p + rb(t), \]  
(5)
and hence
\[ \dot{a}(t) = ra(t) - c_f(t) - c_h(t) / p. \]  
(6)

Positive (negative) values of \( \dot{a}(t) \) imply that at time \( t \) the economy is running a current account surplus (deficit), exactly matched, of course, by a capital account deficit (surplus). A constraint must be imposed on \( a(t) \) to rule out the possibility that the household consumes arbitrarily large amounts by meeting its ballooning interest payments with ever increased borrowing. Following Arrow and Kurz (1970), the family's net wealth must always be non-negative, i.e., \( a(t) \geq 0 \), for all \( t \).

We analyse an unanticipated permanent terms of trade shock at \( t = 0 \). Given an initial wealth, \( a(0^+) \), determined from (4) by the household's fixed rate of production of the home good and initial holdings of the foreign bond, \( b(0) \), the household's aim then, is to choose time paths for \( c_f \) and \( c_h \) that

\[
\text{maximize } \int_0^\infty U\left[ c_f(t), c_h(t), \dot{c}_f(t), \dot{c}_h(t) \right] \exp\left[-T(t)\right] \, dt \]  
(7)
subject to

(i) \( T(t) = \int_0^t \delta \left( U\left[ c_f(s), c_h(s), \dot{c}_f(s), \dot{c}_h(s) \right] \right) \, ds \)

(ii) \( \dot{a}(t) = ra(t) - c_f(t) - c_h(t) / p \)

(iii) \( a(t), c_f(t), c_h(t) \geq 0. \)

I consider two forms for \( \delta(U[s]) \): the Uzawa (1968) form where \( \delta(U) \) satisfies

\[ \delta(U), \delta'(U), \delta''(U), \delta(U) - U \delta'(U) > 0, \]  
(8)
and the special case, \( \delta(U[s]) = r \), where \( r \) is the world interest rate. Any consumption paths, \( c_f(t) \) and \( c_h(t) \), which are differentiable with respect to

\(^3\) For any variable \( X \), define \( X(\tau^-) \) as the limit of \( X \) as \( t \to \tau \) from the left, and \( X(\tau^+) \) as its limit as \( t \to \tau \) from the right. Thus, \( 0^- \) and \( 0^+ \) refer to immediately before and immediately after the trade shock respectively. For ease of notation, consumption levels immediately before the trade shock are sometimes denoted \( c_i(0^-), i=f,h \) rather than \( c_i(0^-), i=f,h \).
time at all but a finite number of points and which contain (at most) a
finite number of finite discontinuities can be accommodated within the
structure defined here. For such paths, lifetime utility (7) is either
defined uniquely or infinitely negative. As will become clear, including a
sensitivity to falling consumption levels in the instantaneous utility
function does not preclude discontinuous falls in consumption. Indeed,
under some circumstances, a discontinuous fall in consumption still
represents the optimal response to an adverse trade shock (see Figure 5).

To solve the constrained maximization problem (7), note that provided
\( a(0^+) > 0 \), (3) implies that constraint (iii) is never binding along an optimal
path, and can be ignored when deriving necessary conditions for
optimality. \( a(t) = 0 \) implies that interest payments on outstanding debt are
just being met by current income and hence that from time \( t \) onwards,

\[ \frac{dC_f(t)}{dt} = 0 \]

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just being met by current income and hence that from time \( t \) onwards,
$c_f = c_h = 0$. Following Uzawa (1968) it is also helpful to change variables in (7) from $t$ to $T$. Finally, introducing the notation

$$v_f(t) = \dot{c}_f(t) \quad \text{and} \quad v_h(t) = \dot{c}_h(t),$$

the householder's problem is now to choose time paths for $v_f$ and $v_h$ which

$$\text{maximize} \quad \int_0^\infty \frac{U(c_f, c_h, v_f, v_h)}{\delta U(c_f, c_h, v_f, v_h)} e^{-T} \, dT$$

subject to

(i) $\frac{da}{dT} = \frac{ra - c_f - c_h / p}{\delta}$

(ii) $\frac{dc_f}{dT} = \frac{v_f}{\delta}$

(iii) $\frac{dc_h}{dT} = \frac{v_h}{\delta}$.

The ultimate solution of the optimization problem, time paths for $c_f$ and $c_h$, are uniquely determined from given initial levels of consumption $c_f(0)$ and $c_h(0)$ and time paths for $v_f$ and $v_h$. Equation (10) is in a form which corresponds exactly to proposition 7 on p48 of Arrow and Kurz (1970). $v_f$ and $v_h$ are the instruments used by the household to maximize utility while $c_f$, $c_h$ and $a$ are the state variables whose values are uniquely determined at all times by their initial values and by the differential equations (i) - (iii). It is necessary to introduce three auxiliary functions of time $\lambda_f$, $\lambda_h$, and $\lambda_a$ (respectively, the shadow prices measured in utility terms of consumption of the foreign and home goods and of savings) and to form the current-value Hamiltonian

$$H(x, y, \lambda) = \frac{U(x, y) + \lambda_f v_f + \lambda_h v_h + \lambda_a (ra - c_f - c_h / p)}{\delta U(x, y)}$$

where I have introduced the three vectors $x$, $y$ and $\lambda$: $x = [c_f \ c_h \ a]$, $y = [v_f \ v_h]$ and $\lambda = [\lambda_f \ \lambda_h \ \lambda_a]$. For $v_f$ and $v_h$ to be the solutions to (10), they must maximize the current-value Hamiltonian (11) at all times.
The first order conditions $\partial H/\partial v_f = 0$ and $\partial H/\partial v_h = 0$ lead to implicit equations for $\lambda_f$ and $\lambda_h$:

$$\lambda_f = U_3 (H \delta' - 1)$$

and

$$\lambda_h = U_4 (H \delta' - 1).$$

When (12) is satisfied, we have found an extremum of $H$ but only if the appropriate second order conditions are satisfied can we be confident that $H$ has been maximized. I return to this issue later.

The three auxiliary functions must evolve thus:

\begin{align*}
\dot{\lambda}_f &= \delta \lambda_f + (H \delta' - 1) U_1 + \lambda_a, \\
\dot{\lambda}_h &= \delta \lambda_h + (H \delta' - 1) U_2 + \lambda_a / p \\
\dot{\lambda}_a &= \lambda_a (\delta (U) - r),
\end{align*}

where I have transformed the problem back to real time, and $\dot{\lambda}_i = d\lambda_i / dt$. Differential equations (13), together with the flow constraint on net wealth, $\dot{a} = ra - c_f - c_h / p$, must be satisfied by any optimal path. Past history and the details of the trade shock determine $a(0^+)$, but to solve (13), I also require initial values for the three auxiliary variables.

Examination of (12) and (13) allow us to determine the long-run (LR) equilibrium of the system, when consumption has ceased to change and so $v_f^{LR} = v_h^{LR} = 0$. A priori restrictions on $U$ expressed by equation (1) imply that $U_3^{LR} = U_4^{LR} = 0$ and hence, from (12), that $\lambda_f^{LR} = \lambda_h^{LR} = 0$. Examination of (13) reveals that

$$\lambda_a^{LR} = [1 - (H \delta')]^{LR} U_1^{LR} = [1 - (H \delta')]^{LR} p U_2^{LR}.$$ Provided that $(H \delta')^{LR} \neq 1$, $U_1 (c_f^{LR}, c_h^{LR}, 0, 0) / U_2 (c_f^{LR}, c_h^{LR}, 0, 0) = p$, i.e., the familiar result that the ratio of the marginal utilities is the relative price. Again, provided that $(H \delta')^{LR} \neq 1$, the last equation in (13) implies that $\delta [U (c_f^{LR}, c_h^{LR}, 0, 0)] = r$. Thus, in the long run, when the subjective rate of time preference has the Uzawa form, instantaneous utility always returns to the same level. In this case, the long run
stationary equilibrium to which the system evolves is identical to the long run equilibrium to which Obstfeld's system evolves when his utility function $U_{\text{OBST}}$ satisfies

$$U_{\text{OBST}}[c_f(t), c_h(t)] = U[c_f(t), c_h(t), 0, 0].$$  \hspace{1em} (14)

When $\delta = r$, Appendix A demonstrates the existence of unique, globally optimal paths for $c_h$ and $c_f$ following a permanent adverse trade shock. Provided the household is sufficiently sensitive to falling consumption (precisely, if $J_h$ and $J_f$, defined in Proposition Four, are unbounded), it is a general result that these optimal paths are continuous and hence the current account always deteriorates in immediate response to an unexpected permanent adverse trade shock, i.e., the HLM effect occurs.\footnote{In fact, the HLM effect occurs more generally than this. With the preferences introduced here, whatever the household's sensitivity to falling consumption, the HLM effect occurs after a permanent adverse trade shock if the household is of type two, or provided $c_h^{LR} \leq c_h(0)$ [$c_f^{LR} \leq c_f(0)$ is always satisfied]. In this last case ($c_h^{LR} \leq c_h(0)$, $c_f^{LR} \leq c_f(0)$), consumption must fall monotonically during the adjustment process (see Proposition Two, Appendix A) and hence the current account remains in deficit until the new long-run. The only way the HLM effect can fail to hold is when (i) $c_h^{LR} > c_h(0)$, (ii) the optimal response involves $c_f$ falling discontinuously at $t = 0$, and implausibly, (iii) adjustment to rising consumption is slower than to falling consumption. Then, it is possible for the initial discontinuous fall in $c_f$ to be sufficient for the current account to improve in immediate response to the adverse trade shock.}

By contrast, when the Uzawa form is assumed for $\delta$, we can only be sure that the HLM effect occurs when we can demonstrate that the optimal consumption paths do not contain any discontinuous falls. This is assured only when such falls have an associated infinite utility cost (see Appendix B).

When the Uzawa form is assumed for $\delta$, to satisfy the long-run equilibrium condition, $\delta (U^{LR}) = r$, the household must be a net saver over the adjustment path. Hence, when the HLM effect occurs, the current account must cycle from deficit to surplus before becoming balanced in
the new long-run. Note that the short-run dynamic response of the current account is now determined by the sensitivity of the household to falling consumption, rather than the necessity to satisfy a long-run equilibrium condition.

In the new long-run, the net foreign asset holdings of the representative household depend on the form assumed for $\delta$. When $\delta$ takes the Uzawa form, for $U^{LR}$ to be equal to its pre-shock value, net foreign asset holdings are larger than before the shock and independent of the household's attitude to changing consumption. By contrast, when $\delta = r$, the long-run net asset position depends on sensitivity to changing consumption levels. A greater sensitivity implies lower net holdings of foreign assets, and hence, to balance the current account, a higher level of net exports in the new long-run.

2.3 SIMULATIONS

To illustrate the dynamic behavior of consumption paths and the current account following a terms of trade deterioration, numerical examples are provided in this section. To begin, I compare behavior when the representative household has instantaneous utility

$$U(t) = \ln [c_f(t) \cdot c_h(t)], \quad (15)$$

with behavior when instantaneous utility takes the form

$$U(t) = \ln [c_f(t) \cdot c_h(t)] - [v_f(t)]^2 / 2 - [v_h(t)]^2 / 2. \quad (16a)$$

7 The current account must also cycle from deficit into surplus and back to balance when $\delta = r$, $c_h^{LR} > c_h(0)$, consumption is continuous and adjustment to rising consumption is slower than to falling consumption. Persson and Svensson (1985) find that cycles in the current account occur in response to anticipated terms of trade shocks (either permanent or temporary) or an unanticipated temporary shock. Their results occur because terms of trade changes induce fluctuations in investment-goods real interest rates which feed into investment and hence into the wealth of wage-earning consumers.
The comparison is undertaken for two forms of the rate of time preference,

Case One: \( \delta[U(t)] = r \exp[U(t)] \) and

Case Two: \( \delta[U(t)] = r \).

The system is initially assumed to be in equilibrium with these parameter values:

\[
\begin{align*}
    p &= 0.81 \\
    y &= 1.8 \\
    r &= 0.1 \\
    c_f &= 10/9 \\
    c_h &= 9/10.
\end{align*}
\]

In this equilibrium, the family has no holdings of foreign bonds but its production of home goods is just sufficient to allow it to consume 0.9 units of the home good per annum, and to import and consume 10/9 units of the foreign good. Instantaneous utility (of either form) is zero, and the current account is balanced. This situation corresponds to complete specialization in production assumed by Laursen and Metzler (1950). I examine the dynamic response of the economy to a permanent terms of trade deterioration at time \( t = 0 \) when the price of the foreign good in terms of the domestic good jumps up from \( p = 0.81 \) to \( p = 1 \). Net wealth after the terms of trade deterioration, is therefore \( a(0^+) = b + y/\rho + p = 18 \).

**Case One: \( \delta(t) = r \exp[U(t)] \)**

Assuming that instantaneous utility takes the form (15) reduces the problem to a special case of the one Obstfeld solved. Before the trade shock, \( c_f = 10/9, \ c_h = 9/10 \) and the current account is balanced. After the trade shock, the curves marked 'Obstfeld' in Figures 1 and 2 display the globally optimal consumption paths and current account balance. When (15) is assumed, the household's preferences are weakly separable across time and its income elasticity of demand for both goods is one. Thus, when \( p \) rises to one, \( c_f = c_h \) at all levels of expenditure and hence is displayed as a single line in Figure 1. Figure 2 demonstrates an example of Obstfeld's general point, that the household, faced by a loss of net
wealth, immediately runs a current account surplus in order to establish, in the new long-run equilibrium, levels of consumption consistent with the instantaneous utility enjoyed before the trade shock. For the parameter values chosen, in the new long-run, $c_f^{LR} = c_h^{LR} = 1$, and hence $U^{LR} = 0$, and $b^{LR} = 2$.

Figure 1

![Figure 1: Home good, Foreign good, Obstfeld consumption over time.]

Figure 2

![Figure 2: Current account balance comparison between Obstfeld and Present study.]

The functional forms (15) and (16a) satisfy equation (14) with (15) playing the role of $U_{OBST}$. Thus, the long-run equilibria are the same whether the household's instantaneous utility takes the form (15) or (16a). Unfortunately, when (16a) is assumed, it is not straightforward to derive numerical solutions to the differential equations (13).\(^8\) The first nine years of the numerical solution are displayed in Figures 1 and 2 by the curves marked 'Home good', 'Foreign good' and 'Present study'. Since when (16a) is assumed, the household's preferences are not weakly separable across time, the adjustment paths for consumption of the two goods are different (in contrast to the Obstfeld case).

Unfortunately, it is possible to demonstrate neither uniqueness nor global optimality for the paths derived when (16a) is assumed. To demonstrate global optimality, it is sufficient that proposition 8 on page 49 of Arrow and Kurz (1970) be satisfied. At all times, the current-value Hamiltonian (11)

\(^8\) A brief description of the difficulties follows. To begin, from the analysis following (13), I may derive these long-run parameter values:

$$\begin{align*}
v_f^{LR} &= v_h^{LR} = \lambda_f^{LR} = \lambda_h^{LR} = 0, \\
\lambda_a^{LR} &= c_f^{LR} = c_h^{LR} = 1, \quad a^{LR} = 20.
\end{align*}$$

Any solution of (13) requires specification of initial values of $\lambda_f$, $\lambda_h$ and $\lambda_a$ which lead to these long-run values. At all times, it is possible to establish that $v_h = v_f \cdot \lambda_h / \lambda_f$, and using this fact, the first of the two equations in (12) reduces to a cubic equation in $v_f$. Equations (13) are numerically solved forward in time, and at each time, this cubic equation is solved and, following Pontryagin's maximization principle, that value of $v_f$ which maximizes the current-value Hamiltonian (11) is chosen. The differential equations (13) exhibit considerable instability and I found it necessary to derive asymptotic solutions to them as the long-run is approached. In this asymptotic regime, $c_f(t) = c_h(t) = 1 - \alpha \exp(-\beta [t - \theta])$, where $\alpha$ is a small positive constant, $\theta$ is the time at which the asymptotic solution is to be invoked and $\beta$ is derived from the asymptotic versions of (13). These equations reduce to a quartic equation in $\beta$ with two positive (and hence feasible) roots: $\beta_1 = 0.10138$ (i.e. $\beta_1 = r$) and $\beta_2 = 0.94100$. Careful numerical solution of (13) [including specifying $\lambda_f(0)$ and $\lambda_h(0)$ to eleven significant figures and $\lambda_a(0)$ to six significant figures!] led, after $\theta = 15.5$ years, to values of $\lambda_f$, $\lambda_h$, $\lambda_a$, $v_f$, $v_h$, $c_f$, $c_h$ and $a$ which agreed closely with the asymptotic results using $\alpha = 0.05$ and $\beta = \beta_1$. 
evaluated at the optimal values of $v_f$ and $v_h$ must be a concave function of $x = [c_f \ c_h \ a]$. This is not so even at the new long-run equilibrium where the Hessian $[\partial^2 H(x, y, \lambda) / \partial x_i \partial x_j]$, $i, j = 1, 3$ has two negative eigenvalues and a positive eigenvalue and hence $H$ is not concave. It is therefore not possible to prove that the paths derived assuming (16a) are optimal. Nevertheless, for this case, Proposition One from Appendix A implies that any path involving a discontinuous fall in consumption has an associated lifetime utility infinitely lower that the lifetime utility derived from the paths shown in Figure 1. For instantaneous utility (16a), the globally optimal solution must involve running a current account deficit for some time following a terms of trade deterioration.

Case Two: $\delta(t) = r$

Assuming that instantaneous utility does not depend on rates of change of consumption [i.e., that $U(t)$ takes the form (15)], reduces our problem to one which Obstfeld (1982) deals with in his footnote 16. In immediate response to the trade shock, consumption of both goods jumps to their new

Figure 3
long-run values, $c_t^{LR} = c_h^{LR} = 0.9$ units per annum. The current account remains balanced. By contrast, when utility takes the form (16a), Figures 3 and 4 show the economy's globally optimal dynamic response. The continuity of consumption of both goods on the optimal path is assured because the household is sufficiently sensitive to falling consumption (precisely because for utility (16a), $J_h$ and $J_f$ from Proposition Four of Appendix A are unbounded). Thus, the current account must deteriorate in immediate response to the adverse trade shock. The optimal consumption paths represent a trade-off between a more slowly falling consumption (which enhances instantaneous utility but leads the family to further indebtedness and a lower long-term level of consumption) and a more rapidly falling consumption (which reduces instantaneous utility but allows a higher long-term level of consumption). For the functional form and parameter values chosen, $c_t^{LR} = c_h^{LR} = 0.89$ units per annum and hence $b^{LR} = -0.2$, i.e. in the new long-run, the household is in debt to the rest of the world.
To conclude this section, we examine another form for instantaneous utility which demonstrates an alternative type of adjustment to a trade shock. Assume that \( \delta(t) = r \), and that \( U(t) \) takes the form,

\[
U(t) = \ln \left[ c_f(t) \cdot c_h(t) \right] - \left| v_f(t) - \tan^{-1} v_f(t) \right| - \left| v_h(t) - \tan^{-1} v_h(t) \right|.
\]

In this form, \( U(t) \) is strictly concave in all arguments, but in contrast to (16a), for this form, \( K_i \) and \( J_i, i=f,h \) (defined in Propositions One and Four from Appendix A) are all bounded. As a result, discontinuous falls in consumption may occur as the optimal response to a terms of trade shock.

For the simulation displayed in Figure 5, before the adverse trade shock, \( c_h = 0.5, c_f = 1.5 \) and net wealth is \( a = 30 \), and so, assuming \( r = 0.1 \), the current account is balanced. At \( t = 0 \), an unanticipated permanent adverse trade shock occurs with \( p \) increasing from 1/3 to 1 and net wealth falling to \( a(0^+) = 5.18 \). Figure 5 represents the optimal consumption response. Home good consumption, \( c_h \), is continuous, but foreign good consumption, \( c_f \), falls discontinuously to \( c_f(0^+) = 1.0 \) before falling continuously and smoothly to \( c_f^{LR} = c_h^{LR} = 0.25 \). Even though \( c_f \) falls discontinuously, the response of the current account is qualitatively similar to Figure 4, and so is not shown.

**Figure 5**

![Figure 5](image-url)
2.4 DISCUSSION

The traditional life cycle-permanent income hypothesis is widely regarded as the appropriate framework for analysing how consumption is to be divided between the present and the future. But there has also been a long tradition recognising consumers' aversion to rapidly changing (crucially, falling) consumption levels - for example, Duesenberry (1949), Brown (1952), Houthakker and Taylor (1966) and Pollak (1970). To quote Pollak, there are three reasons for this aversion:

(i) "The consumer may have contractually fixed commitments which prevent him from [quickly] adjusting some portion of his consumption (for example, housing) in response to changes in prices or incomes." When these commitments lapse, this portion of his consumption can adjust.

(ii) "The consumer may be ignorant of consumption possibilities or of his own tastes outside the range of his past consumption experience." Following an adverse terms of trade shock, for some of the goods in the present consumption bundle, it will be necessary to discover what cheaper, lower-quality substitutes exist, and where to buy them, while also learning how to make do with less of others. These are time-consuming adjustment processes.

(iii) "Finally, goods may be 'habit-forming' so that an individual's current preferences depend on his past consumption patterns." The consumer's utility will depend on the rate at which he changes his consumption behavior.

This chapter attempts to integrate these two traditions by conducting the analysis in an intertemporal utility maximizing framework (thereby placing it in the permanent income tradition) while also including explicitly in the consumer's instantaneous utility function an aversion to changing (crucially, falling) levels of consumption. If the representative
household's sensitivity to falling consumption is stronger than its sensitivity to rising consumption, its response to changes in expected future income will be asymmetrical and irreversible (consumption will rise more quickly in response to improved future income prospects than it will fall in response to worsening prospects). These responses correspond closely to the behavior suggested by Duesenberry on the basis of the social interaction and psychology of consumers. Eight pages (p. 25 – 32) are devoted by Duesenberry to detailing why consumption will quickly rise in response to improving future income prospects (see Chapter One of this thesis). That discussion may be compared to his attitude to falling income prospects, e.g., “when high-income families suffer a loss in income ... they continue to live in the same kind of neighborhoods and maintain their contacts with others of the same socio-economic status ... they can absorb a considerable reduction in income by reducing saving without cutting consumption too deeply. Moreover, there is no reason why they should not continue in this position for several years” (p. 87 – italics added).

There have been several extensions of the analysis of Obstfeld (1982) and Svensson and Razin (1983). Both Persson and Svensson (1985) and Bean (1986) examine overlapping generations models with optimal intertemporal consumption choices made by wage earners who live for two periods. Persson and Svensson include optimal investment behavior on the part of firms, while Bean allows a labor supply response by the wage earners. For an unanticipated permanent trade shock, Persson and Svensson demonstrate that the response of saving measured in home goods depends on the rate of time preference in exactly the same way as for the infinite-period model of Svensson and Razin, and under some simplifying assumptions, Bean finds an analogous result for the real trade balance in a two-period model. Matsuyama (1987) analyses an
economy populated by individuals with finite, but stochastic, lives and includes optimal capital accumulation with adjustment costs. In response to an unanticipated permanent loss of wealth (modelled as an increase in foreign economic aid), consumption drops discontinuously at first, then smoothly to the new long-run, and the current account deteriorates. This result, while very similar to those derived here, occurs for a quite different reason. Matsuyama's result is driven by his assumption that his representative consumer's effective rate of time preference, $\delta$ (in his notation, $\theta + p$), always satisfies $\delta > r$, and hence the marginal propensity to consume out of permanent income is less than one. Consumption falls, but not by as much as the loss of wealth; so the current account deteriorates.

While this chapter concentrates on the consequences of an unexpected permanent adverse trade shock, some extensions are straightforward. Provided consumers are sufficiently sensitive to falling consumption (precisely, when $\delta = r$, if $J_f$ and $J_h$ [from Proposition Four of Appendix A] are unbounded, or when $\delta$ takes the Uzawa (1968) form, if $K_f$ and $K_h$ [from Proposition One of Appendix A] are unbounded), optimal consumption paths cannot contain discontinuous falls in consumption. This fact can be used to derive the qualitative response of the economy to any shocks which have an adverse effect on the perceived wealth of the household.

Some policy implications flow from the logic of this chapter. At least for some time following a terms of trade deterioration widely perceived as permanent, it is reasonable to expect the current account to deteriorate. Provided consumers have access to available information, this deterioration should not necessarily be viewed as an alarming development which requires remedial action (for example, contractionary policies designed to reduce the level of imports). Rather, it may be a
consequence of rational private intertemporal choices made by people with a strong aversion to rapidly falling consumption levels. If so, the current account will improve over time without intervention by the authorities.

APPENDIX A

Here I demonstrate that when $\delta = r$, following a permanent adverse terms of trade shock, there is a unique globally optimal consumption response. I impose the assumption that $U(c_f, c_h, v_f, v_h)$ is an additively separable function of its four arguments. Consumption levels immediately before the trade shock are $c_f(0)$ and $c_h(0)$ and household wealth immediately after the shock is $a(0^+)$. I present eight propositions which characterise the unique globally optimal solution.

Proposition One: Define $K_i$ by

$$K_i = \lim_{v_i \to -\infty} \partial U/\partial v_i, \quad i = f, h.$$ 

If $K_f$ and $K_h$ are bounded, the utility derived from a discontinuous fall at $t = 0$ of magnitude $\Delta c_h$ in $c_h$ and $\Delta c_f$ in $c_f$ (with $\Delta c_h$ and $\Delta c_f$ positive) is

$$V = -(\Delta c_h \cdot K_h + \Delta c_f \cdot K_f). \quad (A.1)$$

If $K_f$ ($K_h$) is unbounded, a finite discontinuous fall in consumption of the foreign (home) good has an associated infinite utility cost.

Proof: Over the time interval $0 \leq t \leq \tau$, define $\zeta(t) = [c_f(t) \ c_h(t)]$ by

$$\zeta(t) = \zeta(0) - \Delta \zeta \cdot t / \tau, \text{ with } \Delta \zeta = [\Delta c_f \ \Delta c_h].$$

Introducing the variable $z = t/\tau$, the utility derived over this time interval, $U_\tau$, is given by

$$U_\tau = \int_0^1 \frac{U(\zeta(z), -\Delta \zeta / \tau) \cdot \exp(-r \tau z)}{1/\tau} \, dz.$$

If the discontinuity in consumption occurs at time $t = \tau$, the derived utility is $V \cdot \exp(-T(\tau))$, where $V$ is given by (A.1).
In the limit as $\tau \to 0$, both the numerator and the denominator of (A.2) are unbounded. Applying L'Hôpital's rule gives the required result.

**Remark:** When $\delta$ takes the Uzawa form, a straightforward extension of the above demonstrates that (A.1) remains true.

When $\delta = r$, from (11), $\partial^2 H / \partial v_i \partial v_j = 1/r \cdot \partial^2 U / \partial v_i \partial v_j$, with $i, j = h, f$, and hence the assumed concavity of $U$ in $v_f$ and $v_h$ guarantees the concavity of $H$ in $v_f$ and $v_h$. Thus, when the appropriate first order conditions are satisfied, $v_f$ and $v_h$ maximize the current-value Hamiltonian $H$. Since $\delta = r$, $\delta' = 0$ and the first order conditions take the simple form:

\[
\lambda_f(t) = -U_3(\hat{c}_f(t))
\]
\[
\lambda_h(t) = -U_4(\hat{c}_h(t)).
\]  

(A.3)

From (13), $\lambda_a = 0$ and hence $\lambda_a(t) = \lambda_a^{LR}$. Thus,

\[
\dot{\lambda}_f(t) = r\lambda_f(t) + U_1(c_f^{LR}) - U_1(c_f(t)),
\]
\[
\dot{\lambda}_h(t) = r\lambda_h(t) + U_2(c_h^{LR}) - U_2(c_h(t)),
\]

(A.4)

where $c^{LR}_i$ is the long-run value of $c_i(t)$, $i = f, h$. The restrictions imposed on the utility function at the beginning of this appendix imply that, apart from the fixed terms $U_1(c_f^{LR})$ and $U_2(c_h^{LR})$, the two equations (A.4) are independent and hence can be solved separately. I initially concentrate on the solution of the equations,

\[
\dot{\lambda}_h(t) = r\lambda_h(t) + G(c_h(t), c_h^{LR}),
\]
\[
\lambda_h(t) = -U_4(\hat{c}_h(t)),
\]

(A.5)

\[\text{where } G(c_h(t), c_h^{LR}) = U_2(c_h^{LR}) - U_2(c_h(t)) \text{ for different possible values of } c_h^{LR}. \text{ For any given value of } c_h^{LR}, G(c_h(t), c_h^{LR}) \text{ is a monotonically increasing function of } c_h(t) \text{ with } G(c_h^{LR}, c_h^{LR}) = 0. \text{ I assume that } a(0^+) > 0 (\text{hence } c_h^{LR} > 0) \text{ and that } c_h(0) > c_h^{LR}. \]

10 If $c_h(0) < c_h^{LR}$, there are two possible cases. If household preferences are of type one, the problem is almost identical to that considered in the text, with positive values of $\hat{c}$ replacing negative values. Alternatively, if household preferences are of type two (when the household's instantaneous utility does not depend on $\hat{c}$ for $\hat{c} > 0$), then, $\lambda_h(t) = 0$, and
**Proposition Two:** In the long run, \( \lambda_h(t) \to 0 \) and \( \lambda_h(t) \to 0 \). During adjustment to the long run, \( c_h(t) > c_h^{LR} \), \( \lambda_h(t) < 0 \) and \( \lambda_h(t) > 0 \).

**Proof:** In the long-run, \( c_h(t) \to c_h^{LR} \) and \( \dot{c}_h(t) \to 0 \). Using (1), examination of (A.5) and (A.6) demonstrate the truth of the first statement. I now eliminate the possibility that at some time \( \tau, \tau \geq 0 \), \( c_h \) falls discontinuously to \( c_h(\tau^+) \), with \( c_h(\tau^+) < c_h^{LR} \). Thus, for a discontinuous fall to occur at \( t = \tau \), \( \lambda_h(\tau^+) = -K_h \). Then, \( \lambda_h(\tau^+) \), as defined in (A.5), is unambiguously negative. But this cannot occur on the optimal path since, for \( t > \tau \), \( \lambda_h(t) \) cannot fall below \( -K_h \). Now assume that at some time \( \tau, c_h(\tau) = c_h^{LR} \), but \( \lambda_h(\tau) \neq 0 \). Then (A.5) and (A.6) imply that \( \lambda_h(t) \) diverges from 0 for \( t > \tau \). Hence, during adjustment to the long run, \( c_h(t) > c_h^{LR} \). If at some time \( \tau \), \( \lambda_h(\tau) \geq 0 \) (with \( c_h(\tau) > c_h^{LR} \)), then (A.5) implies that \( \lambda_h(t) \) increases without bound for \( t > \tau \). Finally, if at some time \( \tau \), \( \dot{\lambda}_h(\tau) \leq 0 \) (with \( \lambda_h(\tau) < 0 \) and \( c_h(\tau) > c_h^{LR} \)), then (A.5) implies that \( \lambda_h(t) \) falls without bound for \( t > \tau \).

**Remark:** \( \lambda_h(t) < 0 \) implies \( \dot{c}_h(t) < 0 \) and so during adjustment to the long-run, \( c_h \) falls monotonically to \( c_h^{LR} \). Over its range of possible values \( \lambda_h \), as defined by (A.6), is a one-to-one non-decreasing function of \( \dot{c}_h \), \( \lambda_h = E(\dot{c}_h) \), and hence can be inverted,

\[
\dot{c}_h = E^{-1}(\lambda_h) = F(\lambda_h). \tag{A.7}
\]

As \( \lambda_h \) increases from \( -K_h \) to 0, \( F(\lambda_h) \) increases monotonically from \( -\infty \) to 0.

**Proposition Three:** For given \( c_h(0) \) and \( c_h^{LR} \), there is a unique finite value \( \lambda_h(0^+) \) which leads to a stable long-run with \( \lambda_h^{LR} = 0 \).

**Proof:** Assume a solution \( \lambda_h^*(t) \) has been found with \( \lambda_h^{*LR} = 0 \). Integration of (A.5) gives

hence, \( \dot{\lambda}_h(t) \equiv 0 \). Thus, immediately the household is aware of a new higher price for the imported good, consumption jumps up from \( c_h(0) \) to \( c_h^{LR} \), where it remains for as long as the information available to the household does not change.
\[ \lambda_h^*(0^+) = -\int_0^\infty e^{-rt} G(c_h(t), c_h^{LR}) \, dt , \]

and since \( G \) is bounded, so is \( \lambda_h^*(0^+) \). I now demonstrate uniqueness. Consider two alternative solutions of (A.5), \( \lambda_1h(t) \) and \( \lambda_2h(t) \), with \( \lambda_1h(0^+) < \lambda_h^*(0^+) < \lambda_2h(0^+) \). Since, \( \lambda_1h(t) < \lambda_h^*(t) \) (A.8) when \( t = 0^+ \), (A.5) implies that,

\[ \lambda_1h(t) < \lambda_h^*(t) \]

(A.9) when \( t = 0^+ \). As a result, a short time, \( \delta t \), later, both (A.8) and

\[ c_1h(t) < c_h^*(t) \]

(A.10) are satisfied with \( t = \delta t \). Hence,

\[ G(c_1h(t), c_h^{LR}) < G(c_h^*(t), c_h^{LR}) \]

(A.11) with \( t = \delta t \) is also satisfied. As time proceeds, (A.8) and (A.11) guarantee the truth of (A.9) which, in turn, ensures that \( \lambda_{1h}^{LR} < 0 \). With respect to \( \lambda_2h(t) \), the argument proceeds as above with \( \lambda_2h \) and \( c_2h \) replacing \( \lambda_{1h} \) and \( c_{1h} \) and with all the inequalities reversed. The conclusion is that \( \lambda_{2h}^{LR} > 0 \). Hence, \( \lambda_h^*(t) \) is the unique solution of (A.5) with \( \lambda_h^{LR} = 0 \). This demonstration suggests an algorithm for finding the unique value, \( \lambda_h^*(0^+) \). Begin with a \( \lambda_h(0^+) \) value which is too high (\( \lambda_h(0^+) = 0 \) is sufficient), and reduce \( \lambda_h(0^+) \) in steps. At each step, solve (A.5) and (A.6) forward in time until either \( c_h(t) \leq c_h^{LR} \), or \( \lambda_h(t) \leq 0 \). If the former occurs first, \( \lambda_h^*(0^+) > \lambda_h(0^+) \); while if the latter occurs first, \( \lambda_h^*(0^+) < \lambda_h(0^+) \). Iteration enables \( \lambda_h^*(0^+) \) to be determined with desired accuracy. That a solution, \( \lambda_h^*(t) \) with \( \lambda_h^{LR} = 0 \) exists is assured by the continuity and differentiability of \( U_2 \).

**Proposition Four:** Defining \( F(\lambda_h) \) by (A.7) and \( K_h \) by (A.1), let \( J_h \) be defined by

---

11 If \( K_h \) [from (A.1)] is finite, it is possible that \( \lambda_h^*(0^+) = -K_h \), in which case, \( \lambda_{1h}(t) \), as defined, cannot exist since \( \lambda_{1h}(0^+) \) cannot be less than \( \lambda_h^*(0^+) \).
\[ \lambda_h = 0 \]
\[ J_h = \int F(\lambda_h) \, d\lambda_h. \] (A.12)
\[ \lambda_h = -K_h \]

If \( J_h \) is unbounded, for all trade shocks, \( c_h(0^+) \) is finite on the optimal path. The analogous statement linking \( J_f \) and \( c_f(0^+) \) is also true.

**Proof:** Assume the contrary: that \( J_h \) is unbounded (infinitely negative) and that \( c_h(0^+) \) is also infinitely negative. Then, \( \lambda_h(0^+) = -K_h \). For \( \tau > 0 \), we have\(^{12}\)

\[
\begin{align*}
&c_h(0) - c_h(\tau) = - \int_{t=0}^{t=\tau} \dot{c}_h(t) \, dt = - \int \frac{\dot{c}_h(t)}{\dot{\lambda}_h} \, d\lambda_h > \frac{-1}{\dot{\lambda}_h^{\max}} \int F(\lambda_h) \, d\lambda_h, \quad (A.13)
&\lambda_h = \dot{\lambda}_h(\tau)
&\lambda_h(\tau)
&\lambda_h = -K_h
\end{align*}
\]

where \( \dot{\lambda}_h^{\max} \) is the largest value taken by \( \dot{\lambda}_h(t) \) in the time interval \( 0 \leq t \leq \tau \). Both \( \dot{\lambda}_h^{\max} \) and \( c_h(0) - c_h(\tau) \) are finite. The definition of \( F(\lambda_h) \) ensures that the integral of \( F(\lambda_h) \) from \( \lambda_h(\tau) \) to 0 is bounded. Hence, since \( J_h \) is infinitely negative, the last integral in (A.13) is also infinitely negative, and the inequality reads \( c_h(0) - c_h(\tau) > \infty \). But this cannot be, so the proposition is true.

**Remark:** The definition of \( \lambda_h \), (A.3), implies that \( J_h \) is a measure of the household's sensitivity to falling consumption (of the home good). If this sensitivity is small, \( J_h \) is bounded (and so is \( K_h \)). For sufficiently severe terms of trade shocks, \( \lambda_h(0^+) = -K_h \), in which case the optimal path involves a finite discontinuous fall in consumption at \( t=0 \). Since the adjustment process is characterised by \( \dot{\lambda}_h(t) > 0 \), a discontinuous fall in consumption can only occur on the optimal path at \( t = 0 \). By contrast, if

\(^{12}\) The first integral in (A.13) could be evaluated by introducing a limiting process similar to the one introduced for the proof of Proposition One. For our purposes here, this is not necessary. It is sufficient to transform this integral over time into an integral over \( \lambda_h \) and recognise that the lower limit of this integral is \( \lambda_h = -K_h \).
the household's sensitivity to falling consumption is sufficiently large, \( J_h \) is unbounded and consumption on the optimal path is continuous.

**Proposition Five:** If \( c_{1h}(t) \) and \( c_{2h}(t) \) [and their associated co-state variables \( \lambda_{1h}(t) \) and \( \lambda_{2h}(t) \)] are the unique solutions of (A.5) and (A.6) with

\[
c_{1h}(0) = c_{2h}(0) \quad \text{and} \quad c_{1h}^{LR} < c_{2h}^{LR}, \quad (A.14)
\]
then, for all \( t > 0 \),

\[
c_{1h}(t) < c_{2h}(t). \quad (A.15)
\]

**Proof:** The proof is by contradiction. There are three conceivable ways in which (A.14) is true while (A.15) is false: (i) \( c_{2h}(t) \) involves a discontinuous fall at \( t = 0 \), such that \( c_{1h}(0^+) \geq c_{2h}(0^+) \), (ii) both \( c_{1h}(t) \) and \( c_{2h}(t) \) are continuous and \( \dot{c}_{1h}(0^+) \geq \dot{c}_{2h}(0^+) \), or (iii) that (A.15) is violated for the first time at time \( \tau > 0 \), when \( c_{1h}(t) \) and \( c_{2h}(t) \) touch or cross. Cases (i) and (ii) may be dealt with together. In case (i), because \( \lambda_{2h}(0^+) = -K_h \), and in case (ii), inverting (A.7),

\[
\lambda_{1h}(t) \geq \lambda_{2h}(t) \quad (A.16)
\]
when \( t = 0^+ \). Further,

\[
G(\dot{c}_{1h}(t), c_{1h}^{LR}) > G(\dot{c}_{2h}(t), c_{2h}^{LR})
\]
when \( t = 0^+ \), and hence, from (A.5),

\[
\dot{\lambda}_{1h}(t) > \dot{\lambda}_{2h}(t) \quad (A.17)
\]
when \( t = 0^+ \). For \( t > 0 \), (A.17) continues to be true and hence, so does (A.16). So using (A.7), for all \( t > 0 \), \( \dot{c}_{1h}(t) \geq \dot{c}_{2h}(t) \) and hence, again for all \( t > 0 \), \( c_{1h}(t) \geq c_{2h}(t) \), contrary to (A.14). In case (iii), \( c_{1h}(\tau) = c_{2h}(\tau) \) and \( \dot{c}_{1h}(\tau) \geq \dot{c}_{2h}(\tau) \). The inverse of (A.7) implies (A.16) with \( t = \tau \), and the argument proceeds as above with time \( \tau \) replacing time 0.

**Remark:** The next two propositions refer to consumption paths with either \( c_{h}(0) > c_{h}^{LR} \) or with \( c_{h}(0) \leq c_{h}^{LR} \).

**Proposition Six:** Let \( c_{h}(t; c_{h}(0), c_{h}^{LR}) \) be the optimal consumption path with \( c_{h}(0^-; c_{h}(0), c_{h}^{LR}) = c_{h}(0) \), and \( c_{h}(\infty; c_{h}(0), c_{h}^{LR}) = c_{h}^{LR} \). Regardless of whether the household is of type one or of type two,
is a monotonically increasing function of its second argument, $c_{h}^{LR}$, with $I(c_{h}(0), c_{h}(0)) = c_{h}(0)/r$.

**Proof:** Follows immediately from Proposition Five and its appropriate modification for the case $c_{h}(0) \leq c_{h}^{LR}$.

**Proposition Seven:** $a(0^{+}) = I(c_{f}(0), c_{f}^{LR}) + I(c_{h}(0), c_{h}^{LR})/p$ (A.18)

**Proof:** Follows immediately from integration of the flow constraint (6).

**Remark:** $c_{f}(0)$ and $c_{h}(0)$ are determined by past history and $c_{f}^{LR}$ is a monotonically increasing function of $c_{h}^{LR}$, determined by the equilibrium condition $U_{1}^{LR} = p U_{2}^{LR}$. The right hand side of (A.18) is, therefore, a monotonically increasing function of $c_{h}^{LR}$, and hence $c_{h}^{LR}$ is uniquely determined by the initial net wealth of the household, $a(0^{+})$. Once the appropriate value of $c_{h}^{LR}$ has been determined, $c_{f}^{LR}$ is determined and the unique solutions to the differential equations (A.4) and the unique time paths $c_{h}(t)$ and $c_{f}(t)$ may be derived.

**Proposition Eight:** The unique time paths $c_{h}(t)$ and $c_{f}(t)$ maximize lifetime utility subject to the flow constraint on net wealth.

**Proof:** This result is assured if proposition 8 on p49 of Arrow and Kurz (1970) is satisfied. It is sufficient to show that, at all times, the current-value Hamiltonian (11) is a concave function of the state variables $c_{f}$, $c_{h}$ and $a$, and that the following six limits are satisfied:

$$\lim_{t \to \infty} e^{-rt} \lambda_{i} \geq 0 \quad \text{and} \quad \lim_{t \to \infty} e^{-rt} \lambda_{i} x_{i} = 0 \quad \text{(A.19)}$$

with $i=1,3$, $x = [c_{f}, c_{h}, a]$, and $\lambda = [\lambda_{f}, \lambda_{h}, \lambda_{a}]$. The long-run properties of the stable solutions of (A.4) ensure that the six limits (A.19) are met. The current-value Hamiltonian is a concave function of $x$ because $U$ is strictly concave with respect to $c_{f}$ and $c_{h}$ and

$$\frac{\partial^{2} H(x, v, \lambda)}{\partial x_{3} \partial x_{i}} = 0 \text{ for } i=1,3.$$
I \left( c_h(0), c_h^{LR} \right) = \int_0^\infty c \left( t; c_h(0), c_h^{LR} \right) e^{-rt} \, dt,

is a monotonically increasing function of its second argument, \( c_h^{LR} \), with
\[ I \left( c_h(0), c_h(0) \right) = c_h(0)/r. \]

**Proof:** Follows immediately from Proposition Five and its appropriate modification for the case \( c_h(0) \leq c_h^{LR} \).

**Proposition Seven:** \( a(0+) = I \left( c_f(0), c_f^{LR} \right) + I \left( c_h(0), c_h^{LR} \right) / p \) (A.18)

**Proof:** Follows immediately from integration of the flow constraint (6).

**Remark:** \( c_f(0) \) and \( c_h(0) \) are determined by past history and \( c_f^{LR} \) is a monotonically increasing function of \( c_h^{LR} \), determined by the equilibrium condition \( U_1^{LR} = p U_2^{LR} \). The right hand side of (A.18) is, therefore, a monotonically increasing function of \( c_h^{LR} \), and hence \( c_h^{LR} \) is uniquely determined by the initial net wealth of the household, \( a(0+) \). Once the appropriate value of \( c_h^{LR} \) has been determined, \( c_f^{LR} \) is determined and the unique solutions to the differential equations (A.4) and the unique time paths \( c_h(t) \) and \( c_f(t) \) may be derived.

**Proposition Eight:** The unique time paths \( c_h(t) \) and \( c_f(t) \) maximize lifetime utility subject to the flow constraint on net wealth.

**Proof:** This result is assured if proposition 8 on p49 of Arrow and Kurz (1970) is satisfied. It is sufficient to show that, at all times, the current-value Hamiltonian (11) is a concave function of the state variables \( c_f, c_h \) and \( a \), and that the following six limits are satisfied:
\[
\lim_{t \to \infty} e^{-rt} \lambda_i \geq 0 \quad \text{and} \quad \lim_{t \to \infty} e^{-rt} \lambda_i x_i = 0 \quad \text{(A.19)}
\]

with \( i=1,3 \), \( \mathbf{x} = [ c_f \ c_h \ a ] \), and \( \lambda = [ \lambda_f \ \lambda_h \ \lambda_a ] \). The long-run properties of the stable solutions of (A.4) ensure that the six limits (A.19) are met. The current-value Hamiltonian is a concave function of \( \mathbf{x} \) because \( U \) is strictly concave with respect to \( c_f \) and \( c_h \) and
\[
\frac{\partial^2 H (\mathbf{x}, \mathbf{y}, \lambda)}{\partial x_3 \partial x_i} = 0 \quad \text{for } i=1,3.
\]
APPENDIX B

For this appendix, I assume \( \delta \) takes the Uzawa form and impose a somewhat weaker assumption than for Appendix A: that the dependence of \( U(c_f, c_h, v_f, v_h) \) on \( v_f \) and \( v_h \) is an additively separable function of \( v_f \) and \( v_h \) (and hence \( U_3 \) and \( U_4 \) are, respectively, functions of \( v_f \) alone and \( v_h \) alone). Assume that in response to a finite terms of trade deterioration, a consumption path \( \tilde{x}(t) = [\tilde{c}_f(t) \quad \tilde{c}_h(t)] \) has been found which satisfies the necessary conditions for optimality (equations (12) and (13)), with \( a(0^+) > 0 \) (if not, see the chapter's fifth footnote), and that \( \tilde{x}(t) \) involves a discontinuous fall in consumption at \( t = 0^+ \):

\[
\lim_{t \to 0^-} \tilde{c}(t) = c_0 \quad \text{and} \quad \lim_{t \to 0^+} \tilde{c}(t) = c_0 - \Delta c
\]

where \( \Delta c = [\Delta c_f \quad \Delta c_h] \) and both \( \Delta c_f \) and \( \Delta c_h \) are non-negative with at most one being zero.

I now derive a sufficient constraint on the instantaneous utility function under which the above consumption path cannot be optimal. To do this involves deriving alternative feasible paths for \( c_f \) and \( c_h \) which are continuous over time and which yield a higher lifetime utility. This alternative path, \( \bar{x}(t) = [c_f(t) \quad c_h(t)] \) satisfies

\[
\bar{x}(t) = x(0) - \Delta c \cdot t / \tau
\]

and hence \( \bar{x}(\tau) = \bar{x}(0) \) and \( \bar{v}(t) = [v_f(t) \quad v_h(t)] = -\Delta c / \tau \)

for \( t \) in the range \( 0 < t < \tau \). When consumption follows this alternative path, wealth is changing over times \( 0 < t < \tau \). At \( t = \tau \), define net wealth to be \( a(\tau) \).

Let \( \bar{w} \) be lifetime utility when consumption follows the path \( \bar{x}(t) \) and \( w \) be lifetime utility when consumption is \( x(t) \). Further, let \( V(\bar{x}^*, a^*) \) be the lifetime utility derived from a consumption path which begins at \( t=0 \).
with $\xi^*$ and $a^*$ as initial values of the state variables, and for which, at all times, the auxiliary variables $\lambda_i$, $i = 1,3$ satisfy equations (12) and (13). Then,

$$\hat{w} = I_0 + \mathcal{V}[\xi^1, a(0^+)]$$  \hspace{1cm} (B.3)

and

$$w = I_\tau + \exp[-T(\tau)].\mathcal{V}[\xi^1, a(\tau)]$$  \hspace{1cm} (B.4)

where

$$I_\tau = \int_0^\tau U(\xi(t), y(t)).\exp[-T(t)] \, dt,$$

and

$$T(t) = \int_0^t \delta[U(\xi(s), y(s))] \, ds.$$  \hspace{1cm} (B.4)

Assuming that $K_f = K_h = K$ [where $K_f$ and $K_h$ are defined in (A.1)],

$$I_0 = \lim_{\tau \to 0} I_\tau = -K(\Delta c_f + \Delta c_h).$$  \hspace{1cm} (B.5)

I assume that $\tau$ is small enough that only terms linear in $\tau$ need be considered. Solving (7) to lowest power in $\tau$ leads to

$$a(0^+)-a(\tau) = \tau[(\text{cad})^0 - dc/2]$$  \hspace{1cm} (B.6)

where $dc = \Delta c_f + \Delta c_h / p$, and $p$ is the price of the foreign good in terms of the domestic good at $t = 0^+$. $(\text{cad})^0 = c_f(0) + c_h(0)/p - r a(0^*)$ is the current account deficit immediately after the terms of trade deterioration but before any change in consumption has occurred. Thus,

$$\mathcal{V}[\xi^1, a(0^+)] - \mathcal{V}[\xi^1, a(\tau)] = [a(0^+)-a(\tau)].\partial \mathcal{V}[\xi^1, a(0^*)]/\partial a(0^*)$$

$$= \tau.[(\text{cad})^0 - dc/2].\lambda_{a^1}$$

\hspace{1cm} 13 Note that this definition of $\mathcal{V}(\xi^*, a^*)$ differs from that of Arrow and Kurz (1970) in two ways. Firstly, $\mathcal{V}(\xi^*, a^*)$ is always defined as beginning at $t=0$ in contrast to the variable $\mathcal{V}(\xi^*, a^*, t)$ of Arrow and Kurz. Secondly, in contrast to the Arrow and Kurz treatment, there is no claim here that $\mathcal{V}(\xi^*, a^*)$ is the globally maximal lifetime utility starting at $t=0$ with $\xi^*$ and $a^*$ as initial values of the state variables. All that is claimed is that $y(t)$ is chosen at all times in such a way that $H(x, y, \bar{\lambda})$ is at a local extremum, and that the auxiliary variables $\lambda_i$, $i = 1,3$ satisfy equations (12) and (13). This is sufficient to guarantee a result that we require: that $\partial \mathcal{V}[\xi^1, a(0^*)]/\partial a(0^*) = \lambda_{a^1}$, where $\lambda_{a^1}$ is the value of the auxiliary variable $\lambda_a$ at $x = [\xi^1, a(0^+)]$ and $t=0^+$ (see Arrow and Kurz, p33-37 and p47-48).
where $\lambda_a$ is the value of the auxiliary variable $\lambda_a$ at $x = [g^1, a(0^+)]$ and $t=0^+$. Again, expanding to the linear term in $\tau$:

$$\exp[-T(\tau)] = 1 - T(\tau) = 1 - \int_0^\tau \delta[U(t)] \, dt = 1 - \tau \bar{\delta}(\tau) \quad (B.8)$$

where $\bar{\delta}(\tau)$ is the average value of $\delta[U(t)]$ over the time interval $0 < t < \tau$.

Combining (B.3) - (B.8) gives a lower bound for $w - \tilde{W}$:

$$w - \tilde{W} \geq I_\tau + [1 - \tau \bar{\delta}(\tau)] \cdot [V[g^1, a(0^+)] - \tau K_1] - I_0 - V[g^1, a(0^+)]. \quad (B.9)$$

If $U_3$ and $U_4$ are unbounded as $\gamma \to -\infty$, then $\tilde{w}$ is infinitely lower than $w$ [see Proposition One in Appendix A]. Clearly, for such a utility function, all feasible consumption paths involving a discontinuous fall in consumption have an associated lifetime utility infinitely lower than the lifetime utility of all feasible continuous consumption paths with $c_r(t), c_h(t) > 0$ for all $t$.

Alternatively, if $U_3$ and $U_4$ are bounded as $\gamma \to -\infty$, a discontinuous fall in consumption involves a finite drop in lifetime utility. Then, when $\tau$ is small enough that ignoring all terms higher than first power in $\tau$ is justified,

$$K > \frac{\tau \{ \lambda_a^1 [( cad )^0 - \frac{1}{2} ( \Delta c_f + \Delta c_h / p )] + \bar{\delta}(\tau) V[g^1,a(0^+)] - I_\tau}{\Delta c_f + \Delta c_h} \quad (B.10)$$

is sufficient to ensure that $w > \tilde{w}$. For given $\tau$ and $\gamma(t)$, the right hand side of (B.10) contains terms which can all be evaluated. Hence, (B.10) imposes a condition on the limiting behavior of $U(g, \gamma)$ as $\gamma \to -\infty$ which, when satisfied, implies that the discontinuous fall in consumption
at $t = 0$ from $g(0)$ to $g^1$ is a sub-optimal response to the terms of trade deterioration.
CHAPTER THREE

WHAT PEOPLE KNOW AND WHAT ECONOMISTS THINK THEY KNOW: A SHRED OF EVIDENCE ON RICARDIAN EQUIVALENCE

3.1 INTRODUCTION AND SUMMARY

This chapter provides survey evidence on two questions of relevance to the Ricardian equivalence hypothesis. These are: (i) do consumers have sufficient knowledge of the level of outstanding government debt to make Ricardian equivalence a plausible model of their behaviour? and (ii) what are academic economists' perceptions of the community's knowledge of government debt? Rather than addressing the first question directly, I provide evidence about a group one might expect to have more knowledge than the whole community – undergraduate economics students. In providing evidence about the second question, I compare the level of knowledge of the students with academic economists' perceptions of that knowledge.

Student knowledge of the level of Australian Federal government debt is found to be very limited indeed, with less than one in eight students able to provide an estimate of the right order of magnitude and willing to claim their estimate as other than a complete guess. The implication is that Ricardian equivalence seems an implausible model of behaviour – at least for this group of citizens. The contrast between the students' level of knowledge and academic economists' perceptions of that knowledge is striking – with the average academic overestimating student knowledge fivefold. At least in this case, the academic economists are wildly optimistic about the general level of knowledge of potentially important economic statistics.
3.2 THE SURVEYS

On Wednesday, 9\textsuperscript{th} September 1987, I handed out a questionnaire to students attending lectures in each of the core subjects of the B.Ec. degree at A.N.U: Economics I, II and III. The questionnaire consisted of six questions and was returned to me within 5-10 minutes. A copy of the questionnaire complete with a partial summary of the results forms Table 1. From a total enrollment of 1026 students in the three years (918 of which sat final exams), 685 completed questionnaires were collected of which 53 were excluded from analysis for these reasons:

48 gave estimates of per capita government debt greater than or equal to $1,000,000. Of these, 27 estimated per capita government debt at more than $10,000,000. It is unclear whether these respondents were providing an answer, however unrealistic, or whether their estimate was of total government debt; in which case it should be divided by the population of Australia (16.2 million).

4 responses were nonsense: I don’t regard $\exp[i\pi]$ as an answer.

1 respondent wrote on his/her questionnaire: “I help put it together.”

All results are derived from the remaining 632 questionnaires. Figures 1 to 3 display the range of estimates of per capita government debt from three cohorts of students. In each case, the median estimate is $1,000 and \(n\) denotes the sample size. Figures relevant to calculating the per capita value of the Australian Federal government's outstanding bonds are given in the Appendix.
TABLE 1

This questionnaire is to help me with my research. I'll be grateful to you if you fill it in. (Numbers in parenthesis are the percentage of the sample who gave each answer.)

<table>
<thead>
<tr>
<th>Age: under 18</th>
<th>(1.1)</th>
<th>As well as studying, do you also do paid work?</th>
</tr>
</thead>
<tbody>
<tr>
<td>18-19</td>
<td>(44.9)</td>
<td>YES</td>
</tr>
<tr>
<td>20-21</td>
<td>(31.3)</td>
<td>NO (47.0)</td>
</tr>
<tr>
<td>22-25</td>
<td>(11.9)</td>
<td>If you do work, roughly how many hours per week do you work?</td>
</tr>
<tr>
<td>26-30</td>
<td>(4.3)</td>
<td>0-4</td>
</tr>
<tr>
<td>over 30</td>
<td>(6.3)</td>
<td>5-8</td>
</tr>
<tr>
<td>(no answer)</td>
<td>0.2</td>
<td>9-15</td>
</tr>
<tr>
<td></td>
<td></td>
<td>16-30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>over 30</td>
</tr>
</tbody>
</table>

If the government spends more than it taxes, it makes up the difference by printing money and/or issuing bonds. Each bond represents a loan from the holder of the bond to the government. The government must eventually repay the loan. What do you think is the value of the Federal government's outstanding bonds?

the population of Australia

Please don't be confused; I am not asking about Australia's foreign debt.

Answer: $ (median answer $1,000)

How accurate do you think your answer is?

A complete guess (66.5)
Within a factor of 10 (8.4)
Within a factor of 2 (6.3)
Within 20% (6.6)
Within 10% (2.4)
(did not estimate how accurate their answer was 2.1)
(did not give an estimate of per capita debt 7.7)

How long do you think you will be living in Australia? Answer 'For ever' if that seems appropriate!

(No answer 14.6)
(Less than 10 years 18.8)
(At least 10 years, or 'For ever' 66.6)
TABLE 1

This questionnaire is to help me with my research. I'll be grateful to you if you fill it in. (Numbers in parenthesis are the percentage of the sample who gave each answer.)

Age: under 18 (1.1)  As well as studying, do you also do paid work?
18-19  (44.9)  YES  (53.0)  NO  (47.0)
20-21  (31.3)  If you do work, roughly how many hours
22-25  (11.9)  per week do you work?
26-30  (4.3)  0-4  (17.6)
over 30  (6.3)  5-8  (26.2)
(no answer 0.2)  9-15  (22.1)
                      16-30  (10.8)
                      over 30  (23.2)

If the government spends more than it taxes, it makes up the difference by printing money and/or issuing bonds. Each bond represents a loan from the holder of the bond to the government. The government must eventually repay the loan. What do you think is the

value of the Federal government’s outstanding bonds

the population of Australia

Please don’t be confused; I am not asking about Australia’s foreign debt.

Answer: $  (median answer $1,000 )

How accurate do you think your answer is?

A complete guess  (66.5)
Within a factor of 10  (8.4)
Within a factor of 2  (6.3)
Within 20%  (6.6)
Within 10%  (2.4)
(did not estimate how accurate their answer was  2.1)
(did not give an estimate of per capita debt  7.7)

How long do you think you will be living in Australia? Answer ‘For ever’ if that seems appropriate!

(No answer 14.6)
(Less than 10 years 18.8)
(At least 10 years, or ‘For ever’ 66.6)
Figure 1
Results from all students

![Bar chart showing proportion of sample (n = 632) for different estimates of per capita debt in $'000. Categories include: No estimate, 0 to 0.1, 0.1 to 0.5, 0.5 to 1.5, 1.5 to 3.5, 3.5 to 8, and more than 8.]

Figure 2
Of those who expect to be living in Australia for at least ten years

![Bar chart showing proportion of sample (n = 421) for different estimates of per capita debt in $'000. Categories include: No estimate, 0 to 0.1, 0.1 to 0.5, 0.5 to 1.5, 1.5 to 3.5, 3.5 to 8, and more than 8.]

Legend:
- Complete guess
- Total
Figure 3

Expect to be living in Australia for \( \geq 10 \) yrs. and are now working > 30 hrs. per week

<table>
<thead>
<tr>
<th>Estimate ($'000 per capita debt)</th>
<th>Complete guess</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>No estimate</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0 to 0.1</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.1 to 0.5</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>0.5 to 1.5</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>1.5 to 3.5</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>3.5 to 8</td>
<td>0.1</td>
<td>0.2</td>
</tr>
<tr>
<td>more than 8</td>
<td>0.1</td>
<td>0.2</td>
</tr>
</tbody>
</table>

Proportion of sample (n = 64)
A questionnaire inquiring about academic economists' perceptions of student knowledge was completed by eleven tenured members of the Economics Department, The Faculties, A.N.U. (one professor, four readers, three senior lecturers and three lecturers). Table 2 compares the level of student knowledge about the amount of outstanding government debt with the academic economists' perceptions of that knowledge.

**TABLE 2**

COMPARISON OF ECONOMICS STUDENTS' KNOWLEDGE, AND ACADEMIC ECONOMISTS' PERCEPTIONS OF THAT KNOWLEDGE.

Percentage of Economics I, II and III students who (i) estimated $1,250 < per capita Federal government debt < $5,000 and (ii) claimed their answer was not a complete guess compared with academic economists' estimates of the percentage of students who would satisfy both criteria.

<table>
<thead>
<tr>
<th>STUDENT RESULTS</th>
<th>ACADEMIC ECONOMISTS' ESTIMATES</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(11 responses)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>% (sample size)</th>
<th>mean %</th>
<th>standard deviation</th>
<th>range</th>
</tr>
</thead>
<tbody>
<tr>
<td>All students:</td>
<td>4.4 (632)</td>
<td>23.9</td>
<td>16.8</td>
<td>5 to 60</td>
</tr>
<tr>
<td>Of those who expect to be living in Australia for at least 10 years:</td>
<td>4.5 (421)</td>
<td>24.6</td>
<td>16.7</td>
<td>5 to 60</td>
</tr>
<tr>
<td>Of those who expect to be living in Australia for at least 10 years and are over 25 years old:</td>
<td>7.6 (53)</td>
<td>28.9</td>
<td>17.4</td>
<td>5 to 60</td>
</tr>
<tr>
<td>Third year students:</td>
<td>4.9 (102)</td>
<td>31.1</td>
<td>20.0</td>
<td>5 to 70</td>
</tr>
</tbody>
</table>
3.3 DISCUSSION

To make an approximately Ricardian consumption choice in a modern mixed economy, it is necessary to have a reasonably good estimate of net real per capita government debt. To derive such an estimate requires properly accounting for government investments, contingent liabilities and assets (both financial and tangible), as well as estimating liabilities from government superannuation schemes and other social insurance programs (Boskin, 1982, 1987, Eisner and Pieper, 1984). To answer the student questionnaire requires a much more modest level of knowledge.¹

Nevertheless, the striking thing to emerge from the students' answers is how little they know about the level of government debt in Australia – although they do display Socratic wisdom. Less than a quarter (23.7%) are willing to provide any estimate of per capita debt and claim it as other than a complete guess. Requiring their estimates to be of the same order of magnitude as actual per capita debt reduces this proportion substantially. Thus, less than one in eight (12.2%) of the students give an estimate between $500 and $10,000 (not including the endpoints) and are confident that their estimate is not a complete guess.

¹ One question remains: to behave in a Ricardian way, is it necessary to have knowledge of the level of government debt, or is it sufficient to monitor changes to that level? Certainly, to determine the optimal amount to change consumption from one year to the next, it is sufficient to monitor only the change in government debt. But, to determine the optimal (Ricardian) level of consumption at any time, does require knowledge of the total net level of government debt. Even if the government never repays the debt in full, the present discounted value of the interest payments on outstanding debt amounts to the same thing. One could argue that even without having a good idea of what they will be, consumers act 'as if' they took account of future tax liabilities. This seems possible when governments use debt aggressively to finance a large part of their expenditure. It is less convincing when government debt/GDP is at low levels (by historical standards), and when it changes only slowly from year to year (see Chapter Four).
The logic of Ricardian equivalence suggests that those people who think they will be living in Australia for at least ten years should have more than average knowledge of the amount of outstanding government debt – since they expect to be in Australia over an extended period during which the consequences of that debt (increased taxes and/or reduced government spending) are realised. The survey results do not support this logic. Only marginal differences exist between the estimates of all students and of those who expect to be living in Australia for at least ten years (see Figures 1 and 2). For instance, 12.4% of those who expect to be living in Australia for at least ten years (as opposed to 12.2% for all students) estimate per capita government debt between $500 and $10,000 and are confident that their estimate is at least within a factor of ten. The corresponding numbers for estimates in the range $1,250 to $5,000 are 4.5% for those who expect to be living in Australia for at least ten years and 4.4% for all students (from Table 2).

It is worth raising a possible objection to these survey results. Since most undergraduate students are not self-supporting and do not pay income tax, they may have less idea than the wider community of the level of government debt and hence of future taxes. In response to this objection, we can examine the level of knowledge of those students who expect to be living in Australia for at least ten years and who, as well as studying, are working more than thirty hours per week (see Figure 3). This group are (presumably!) paying income tax. They are probably mainly Federal government public servants. As such, many of them come across budget information rather more regularly than the wider community. Nevertheless, even from this cohort, only 17% estimate per capita

---

2 There are about 46,500 public servants in Canberra out of a total population of a quarter of a million.
government debt between $500 and $10,000 and are confident that their estimate is not a complete guess. It is difficult to avoid the inference that knowledge of the level of government debt in the wider community is very meagre indeed.

Even though the sample of academics is very small, their perceptions of student knowledge differ so much from actual student knowledge that meaningful statistical inferences can be drawn. If each academic estimate was drawn from a distribution with a median equal to the actual proportion of students who had 'a rough idea of the level of government debt' (defined precisely in Table 2), then academic underestimates and overestimates would be equally likely. But, of the forty four estimates provided by the academics (four from each academic), two are underestimates of student knowledge and forty two are overestimates. If the estimates are all assumed independent, the probability of an outcome as uneven, or more uneven, than this is $5.6 \times 10^{-11}$. But the suggestion that each academic gives four independent estimates seems far-fetched, so we should apply the same logic to each of the four student cohorts individually. For the first, second and fourth cohorts from Table 2, all the academic economist's estimates are overestimates, while for the third cohort, nine of the eleven academic estimates are overestimates. If the estimates come from a distribution with the correct median, the probabilities of such uneven, or more uneven outcomes are, respectively, $(1/2)^{11} = 0.0005$ and $67 \times (1/2)^{11} = 0.033$. At conventional levels of confidence, the hypothesis that the academic economists estimates are derived from a distribution with a median equal to the actual level of student knowledge is rejected. The best point estimate we have is that the 'average' academic overestimates the proportion of students who have 'a rough idea of the amount of government debt' fivefold: 23.9% instead of 4.4% (from Table 2).
The work of cognitive and social psychologists suggests a reason for the academics' substantial overestimation of student knowledge – beyond wishful thinking that, given the education they've had, their students ought to know better. Tversky and Kahneman (1974) identify three 'heuristic principles' which people use to make judgements and decisions in uncertain situations. According to one of these heuristics, the availability heuristic, people rely more than they should on readily accessible information, i.e., information which is easily retrievable from memory. Tversky and Kahneman (1973, 1974) provide a large number of examples of the availability heuristic at work, and show how it can lead to substantial, and importantly, systematic misjudgements.

For academic economists, the level of outstanding Federal government debt is probably not a number easily retrievable from memory. But using information which is easily retrievable (the value of the last few Federal budget deficits, or perhaps interest payments on outstanding debt), most could quickly come up with a fairly good estimate and be confident that this estimate was within a factor of ten of the true answer. If they use the availability of this information to them in estimating the proportion of students who could do likewise, we should not be so surprised that the academics provide substantial overestimates.

Two conclusions are suggested if we accept that the availability heuristic is widely used to make judgements in uncertain situations. First, if in the wider community, the level of outstanding government debt (and presumably, the link between government debt and future taxes) is not easily retrievable from memory, it is not used in making consumption and

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3 Chapter One provides an introduction to these heuristic principles.
4 The surveys were conducted before the Federal government began running budget surpluses.
saving decisions. Bond financed tax cuts are expansionary. Second, the people most susceptible to believing in Ricardian equivalence should be those for whom the link between government debt and future taxes is a most obvious and natural one, i.e., professional economists.

APPENDIX

These are relevant figures for per capita Australian Federal government debt on 30th June, 1987:

<table>
<thead>
<tr>
<th>Description</th>
<th>Amount</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total securities on issue</td>
<td>$4,192 (1)</td>
</tr>
<tr>
<td>Securities held by:</td>
<td></td>
</tr>
<tr>
<td>the Reserve Bank of Australia</td>
<td>$450 (2)</td>
</tr>
<tr>
<td>Major Commonwealth Trust Funds</td>
<td>$46 (3)</td>
</tr>
<tr>
<td>Public Authorities</td>
<td>$83 (4)</td>
</tr>
<tr>
<td>Gold and foreign exchange holdings of the Reserve Bank on 1st July, 1987</td>
<td>$1,078 (5)</td>
</tr>
</tbody>
</table>

The per capita value of the Federal government's outstanding bonds may be interpreted as (1) − (2) − (3) − (4) = $3,613 or perhaps as (1) − (2) − (3) − (4) − (5) = $2,535. For the purposes of this chapter, it matters little

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5 The numbers are derived from Table 1 and Table 4 of Government Securities on Issue at 30 June 1987. Budget Related Paper No. 1. Quoting from this document: "The securities listed are for the most part Commonwealth securities issued for and on behalf of the Commonwealth, State and Northern Territory Governments although a few State securities, mostly issued prior to the Financial Agreement [May, 1986], are still outstanding. With a few exceptions (for example, Income Equalisation Deposits and some bank loans raised overseas by the Commonwealth) forms of indebtedness not evidenced by the issue of securities are not included in this document. Debt of semi-government and local authorities of the Commonwealth, States and Northern Territory is also excluded, as is the debt of other bodies guaranteed by Governments."


7 In its efforts to influence the short-run value of the $A, the Reserve Bank exchanges Australian government securities for gold and foreign bonds. If these holdings are not subtracted from government's
which number is regarded as more realistic. The Australian Federal
government's fiscal stance in the financial year 1987/88 ensures that
these numbers changed little from the end of June to the 9th September
when the students were quizzed.

outstanding bonds, Reserve Bank foreign exchange operations change the
calculated debt position of the government (often by large amounts in a few
weeks) when, in fact, there have been changes of equal magnitude on both
sides of the Reserve Bank's balance sheet.
"I do not approve of anything which tampers with natural ignorance. Ignorance is like a delicate exotic fruit; touch it and the bloom is gone."

The Importance of Being Earnest, Oscar Wilde.

CHAPTER FOUR

IGNORANCE AND RICARDIAN EQUVALENCE
or Keynesians of the world unite, You have nothing to lose but your bonds

4.1 INTRODUCTION AND SUMMARY

The results presented in Chapter Three suggest that at least the subset of the community represented by undergraduate economics students has almost no idea of the level of outstanding government debt. This chapter systematically examines the cost to a representative intertemporally optimizing consumer of being ignorant of the link between outstanding government debt and future taxes. As well as the survey evidence from Chapter Three, there are two other reasons for assuming ignorance of this link rather than of other determinants of perceived wealth. Firstly, it requires some sophistication to understand the link between bonds and future taxes. Secondly, when the link is understood, to make even an approximately Ricardian choice between consumption and saving, one must analyse a substantial amount of information not readily available to consumers. The difficulties of making an approximately Ricardian choice in a modern developed economy are examined in the Discussion section of this chapter.

The layout of the chapter is as follows. Section 4.2 presents a two-period model in which the behaviour of a Ricardian is compared to the behaviour of someone ignorant of the link between bonds and future taxes (for obvious reasons called a Keynesian). The government introduces a bond-financed tax-cut in the first period of the model financed by increased
lump-sum taxes in the second period. Expressions are derived for the Keynesian’s expansion of first-period consumption and for the wealth cost associated with this sub-optimal intertemporal choice. In section 4.3, a many-period model is used to derive the consumption response of a Keynesian to the changing debt position of her government. Estimates are derived of the wealth cost to this Keynesian of ignorance of the future tax implications of: (i) Australian Federal government debt, 1963 – 1987; (ii) U.S. Federal government debt, 1963 – 1987; and (iii) net Federal, State and local government debt in the U.S., 1957 – 1976. Section 4.4 presents a two-period model which incorporates explicit income uncertainty and distortionary taxes. The final section presents a discussion of several disparate issues raised by the chapter, and a summary of the chapter’s main findings.

The two-period model of section 4.2 bears a close resemblance to the “near-rational” models of Akerlof and Yellen (1985a,b) even though the motivation for the models is rather different. Akerlof and Yellen examine a range of inertial responses by near-rational agents who fail to adjust their behaviour in response to a shock.1 Here, it is assumed that a representative (Keynesian) consumer lacks the information or understanding relevant to making an optimal (Ricardian) intertemporal choice between consumption and saving. In the model of section 4.3, even the Ricardian cannot make fully optimal consumption choices because she does not have complete knowledge about her future income. Because of this, the Akerlof and Yellen analysis is extended to analyse the welfare costs of deviations from an equilibrium which is itself sub-optimal.

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1 They also examine the financial incentive for an individual member of a production cartel to cheat on her colleagues.
The key point of the chapter is that, judged with reference to the actual behaviour of the Australian and U.S. governments over the past twenty-five years, the cost of ignorance for the average citizen is so small that it is probably rational for her to ignore the link between bonds and future taxes when making consumption decisions. If so, bond-financed tax changes do alter the level of aggregate demand because it is not worth an average consumer's time and trouble to keep track of her government's changing level of real per capita indebtedness.

4.2 TWO-PERIOD MODEL

In a two-period model with no uncertainty, define $c_1$ and $c_2$ as the levels of consumption in the two periods of the model, $r$ as the (fixed) real after-tax interest rate between the two periods and $\delta$ as the rate of time preference for a representative consumer. Assuming an additively separable utility function with identical one-period sub-utilities, this consumer seeks to

$$\text{maximize } U = u(c_1) + u(c_2)/(1 + \delta)$$

subject to

$$c_1 + c_2/(1 + r) = W,$$

where $W$ is the present discounted value of assets and after-tax earnings. Provided $u(c)$ is continuous and differentiable, the optimal levels of consumption $c_1^R$ and $c_2^R$ satisfy

$$(1 + r).u'(c_2^R) = (1 + \delta).u'(c_1^R).$$

Assume that, in the first period of the model, the government introduces a bond financed tax-cut of per capita value $B$, fully funded by extra lump-sum taxes in the second period of value $B.(1 + r)$. Issuing this bond has no impact on a Ricardian's consumption as she perceives that her wealth is unchanged and so she consumes at point $R$ in Figure 1.

By contrast, a Keynesian consumer is ignorant of the future tax implications of period-one's outstanding bonds and perceives the increase
FIGURE LEGEND: In period 1, assuming her wealth is $W + B$, the Keynesian expects to consume at $K^E$. In period 2, having discovered her wealth is only $W$, she must consume at $K$. Understanding the link between bonds and future taxes, the Ricardian consumes at $R$. 
in first period after-tax earnings as an increase in wealth to \( W + B \). With the same preferences as our Ricardian, she expects to consume at point \( K^E \) in Figure 1, setting her first period consumption at \( c_1^K = c_1^R + \Delta c_1 \) where (from Appendix A)

\[
\frac{\Delta c_1}{c_1^R} = \left( \frac{(1 + r)^2 \cdot \frac{u''(c_2^R)}{c_2^R}}{(1 + \delta) \cdot \frac{u''(c_1^R)}{c_1^R} + (1 + r)^2 \cdot \frac{u''(c_2^R)}{c_2^R}} \right) \cdot \left( \frac{B}{c_1^R} \right).
\]

(4)

In the second period, unforeseen extra taxes \( B \cdot (1 + r) \) are levied on her and she must reduce consumption to \( c_2^K = c_2^R - \Delta c_1 \cdot (1 + r) \); below the optimal value \( c_2^R \). She is forced to consume at point \( K \) in Figure 1. The cost of her ignorance may be measured by the wealth loss, \( \Delta W \), which would drive the Ricardian to the utility level of the Keynesian at \( K \). Figure 1 shows graphically how \( \Delta W \) can be derived. The cost of ignorance is approximately (see Appendix A)

\[
\frac{\Delta W}{W} = \left[ \frac{-c_2^R \cdot \frac{u''(c_2^R)}{u'(c_2^R)}}{2 c_1^R (1 + \delta + [1 + r]^2) \cdot (1 + r + c_2^R/c_1^R)} \right] \cdot \left[ \frac{B}{c_2^R} \right]^2.
\]

(5)

The first term in equation (5) is closely related to the reciprocal of the elasticity of substitution between consumption in the two periods of the model.\(^2\) The numerical value of the second term is sensitively dependent on the assumed values for \( \delta \) and \( r \), and the assumed form of the utility function.\(^3\)

---

\(^2\) If either \( c_1^R = c_2^R \) or if the one-period sub-utility function takes the isoelastic form, \( u(c) = c^{1-\beta} / (1 - \beta) \), then \( 1/\beta = -u'(c) / c \cdot u''(c) \) is the elasticity of substitution between consumption in the two periods.

\(^3\) For example, if \( \delta = r = 0 \), it takes the value 1/8. Alternatively, assuming that each period of the model represents 35 years, that optimal consumption grows 2% per annum, that the real interest rate is 3% p.a. and that the single period utility function displays a constant elasticity of substitution between consumption in the two periods of the model of 1/2, then it takes the value 1.46.
A crucial point emerges from a comparison of equations (4) and (5). For an ignorant consumer, the fractional increase in her consumption when bonds are issued is proportional to $B/c$, while the fractional wealth cost to her is proportional to $(B/c)^2$. Provided the level of bonds is a modest fraction of the level of consumption, the introduction of the bonds will provide a noticeable increase in aggregate demand, but negligible loss of welfare to the individuals who have increased their consumption. The aim of the next section is to replace the vague “noticeable increase” and “negligible loss” with quantitative estimates.

4.3 MANY-PERIOD MODEL

In this section, a model is developed of the consumption response of a Ricardian and a Keynesian to a continually changing level of government debt over many years. The two consumers are now assumed immortal, and once again are identical in all respects except that the Ricardian understands the link between bonds and future taxes while the Keynesian does not. Before delving into the details of the model, an overview of it is presented. The two consumers are expected utility maximizers (equation 6), and are subject to an expected wealth constraint (equation 7). Each period, they update their expected wealth on the basis of new information and determine their optimal consumption for that period (equation 11). By putting the government’s budget constraint in per capita terms (equation 12), I derive an expression (equation 15) for the contribution which government bond financing makes to the change in an individual’s disposable income from year $t-1$ to year $t$. This contribution is denoted $\Delta y_t^B$. The different consumption behaviour of our two individuals is determined solely by their different attitude to $\Delta y_t^B$: for the Ricardian, $\Delta y_t^B$ is ignored when estimating her expected wealth; while for the Keynesian, $\Delta y_t^B$ directly influences the updating of her expected
wealth. I examine three alternative behavioural assumptions for the impact of $\Delta y^B_t$ on the Keynesian's expectations, including a preferred assumption, in which her expected wealth is gradually updated (equation 18). An expression is derived for the contribution at time $t$ to the wealth cost of the Keynesian's ignorance (equation 25) and a simplification is introduced which enables this wealth cost to be evaluated. Equation 31 defines the wealth cost of Keynesian ignorance. The model has the advantage that estimates of the wealth cost of ignorance can be derived using data on government debt over any length of time. The government debt level at the end of the simulation need not return to its value at the beginning of the simulation, nor is it necessary to make any (necessarily arbitrary) assumptions about the time path of debt after the simulation.

The model is now described in detail. At time $t$, each consumer maximizes lifetime utility of the form

$$E_t U^i = \sum_{j=0}^{\infty} \frac{E_t u(c_{t+j}^i)}{(1+\delta)^j}$$

subject to the constraint

$$E_t W^i_t = \sum_{j=0}^{\infty} \frac{E_t c_{t+j}^i}{(1+r)^j}$$

where $E_t$ is the expectation conditional on information available at $t$ and $i = R$ or $K$ depending on the consumer's identity. The after-tax real interest rate, $r$, and the consumers' subjective rate of time preference, $\delta$, are assumed constant over time. I assume the iso-elastic utility function

$$u(c) = \frac{c^{1-\beta}}{1-\beta}, \quad \beta > 0$$

$1/\beta$ is the elasticity of substitution between consumption in different periods of the model. Following Hall (1978) but using new notation, define $\omega^i_t$ as

$$\omega^i_t = (1-\theta) \cdot [E_t W^i_t - E_{t-1} W^i_t],$$

(9)
where $\theta = (1 + g)/(1 + r)$, $i = R$ or $K$ and $g$ is defined by

$$(1 + g)^\beta = (1 + r)/(1 + \delta). \quad (10)$$

Provided the interest rate is close to the rate of time preference and the stochastic disturbance ($\omega_t^i$) is relatively small, certainty equivalence is justified and it is a close approximation that

$$c_t^i = (1 + g)c_{t-1}^i + \omega_t^i \quad (11)$$

where $i = R$ or $K$. Equation (9) implies that $E_{t-1}\omega_t^i = 0$, while equation (11) implies that $g$ is the growth rate of optimal consumption. So, $\omega_t^i$ is the optimal amount to change consumption at time $t$ due to the change in expected wealth between time $t - 1$ and time $t$. The equations are defined only for $\theta < 1$, and hence for $g < r$.

The government's budget constraint in year $t$ can be written

$$G_t + i_{t-1}B_{t-1}/x_{t-1} - T_t = (B_t - B_{t-1}/x_{t-1}) + (M_t - M_{t-1}/x_{t-1}) \quad (12)$$

where $G$, $T$, $B$ and $M$ are, respectively, real per capita values of government consumption expenditure, taxes, outstanding bonds and the stock of money, $x_{t-1} = (1 + \pi_{t-1}) \cdot (1 + n_{t-1})$ and $i_{t-1}$, $\pi_{t-1}$ and $n_{t-1}$ are, respectively, the after-tax nominal interest rate, the inflation rate and the population growth rate between years $t - 1$ and $t$. The after-tax real interest rate $r$, defined by $1 + r = (1 + i_t)/(1 + \pi_t)$ is assumed constant through time. In year $t$, if our consumer earns an income (from both human and non-human wealth) of $y_t$, then her disposable income $y_t^d$ satisfies

$$y_t^d = y_t - T_t = y_t - G_t + (M_t - M_{t-1})/x_{t-1} + y_t^B, \quad (13)$$

with $y_t^B$ defined by $y_t^B = B_t - (1 + r) \cdot B_{t-1}/(1 + \pi_{t-1})$. \quad (14)

Hence, $\Delta y_t^B$, defined by $\Delta y_t^B = y_t^B - y_{t-1}^B$, \quad (15)

is the contribution which the government's bond financing makes to the change in the consumer's disposable income from year $t - 1$ to year $t$. As explained in the overview, $\Delta y_t^B$ makes no contribution to the Ricardian's
where \( \theta = (1 + g)/(1 + r) \), \( i = R \) or \( K \) and \( g \) is defined by
\[
(1 + g)^\beta = (1 + r)/(1 + \delta). \tag{10}
\]
Provided the interest rate is close to the rate of time preference and the stochastic disturbance (\( \omega_t^i \)) is relatively small, certainty equivalence is justified and it is a close approximation that
\[
c_t^i = (1 + g) c_{t-1}^i + \omega_t^i \tag{11}
\]
where \( i = R \) or \( K \). Equation (9) implies that \( E_{t-1} \omega_t^i = 0 \), while equation (11) implies that \( g \) is the growth rate of optimal consumption. So, \( \omega_t^i \) is the optimal amount to change consumption at time \( t \) due to the change in expected wealth between time \( t - 1 \) and time \( t \). The equations are defined only for \( \theta < 1 \), and hence for \( g < r \).

The government's budget constraint in year \( t \) can be written
\[
G_t + i_{t-1} B_{t-1}/x_{t-1} - T_t = (B_t - B_{t-1}/x_{t-1}) + (M_t - M_{t-1}/x_{t-1}) \tag{12}
\]
where \( G, T, B \) and \( M \) are, respectively, real per capita values of government consumption expenditure, taxes, outstanding bonds and the stock of money, \( x_{t-1} = (1 + \pi_{t-1}) \cdot (1 + n_{t-1}) \) and \( i_{t-1}, \pi_{t-1} \) and \( n_{t-1} \) are, respectively, the after-tax nominal interest rate, the inflation rate and the population growth rate between years \( t - 1 \) and \( t \). The after-tax real interest rate \( r \), defined by \( 1 + r = (1 + i_t)/(1 + \pi_t) \) is assumed constant through time. In year \( t \), if our consumer earns an income (from both human and non-human wealth) of \( y_t \), then her disposable income \( y_{t}^d \) satisfies
\[
y_{t}^d = y_t - T_t = y_t - G_t + (M_t - M_{t-1})/x_{t-1} + y_t^B, \tag{13}
\]
with \( y_t^B \) defined by
\[
y_t^B = B_t - (1 + r) B_{t-1}/(1 + \pi_{t-1}). \tag{14}
\]
Hence, \( \Delta y_t^B \), defined by \( \Delta y_t^B = y_t^B - y_{t-1}^B \),
\[
\tag{15}
\]
is the contribution which the government's bond financing makes to the change in the consumer's disposable income from year \( t - 1 \) to year \( t \). As explained in the overview, \( \Delta y_t^B \) makes no contribution to the Ricardian's
expected wealth. By contrast, because of her ignorance, the Keynesian interprets unanticipated bond-financed changes to disposable income ($\Delta y_t^B$) as changing her expected wealth. To model this link requires an assumption of our Keynesian's perceptions of the permanence of these changes to her disposable income. I examine three alternate behavioural assumptions. Define $W_t^B$ as the contribution which government bond financing makes to our Keynesian's expected wealth. Then, in year $t$, she assumes that bond-financed changes to her disposable income from the previous year

(i) are once-off, and hence her expected wealth has changed by

$$E_t W_t^B - E_{t-1} W_t^B = \Delta y_t^B,$$

or

(ii) are permanent, and hence her expected wealth has changed by

$$E_t W_t^B - E_{t-1} W_t^B = \Delta y_t^B \left/ (1 - \theta) \right.,$$

or

(iii) lead to a gradual updating her expected wealth. Expected wealth is then assumed to have changed by the permanent capitalised value of a weighted sum of past bond-financed changes in disposable income

$$E_t W_t^B - E_{t-1} W_t^B = \frac{\lambda}{1 - \theta} \sum_{j=0}^{m(t)} (1 - \lambda)^j \Delta y_{t-j}^B,$$  

with $\lambda$ assumed to be 1/3 on the basis of Friedman (1957). The value of $m(t)$ is explained in the subsection: Data and results.

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4 Equations (17) and (18) assume that if the expected growth rate of real per capita income is $g$, the Keynesian also expects the unanticipated change in disposable income to grow at a rate $g$. In other words, she expects the proportionate change in disposable income to remain fixed over time.

5 The impact effect on consumption of a $1 bond-financed change in disposable income is $(1-\theta)$, $1 or 33¢ for behavioural assumptions (16), (17) or (18) respectively. Holding government consumption constant, Boskin (1987) finds that a $1 bond-financed change in disposable income leads to a 30 to 40 cent change in consumption, suggesting that equation (18) may be the most realistic of the three behavioural assumptions. However, as Deaton and Muellbauer (1980) point out, it is
As stressed in the overview, the only difference between our two consumers is summarized by their different attitude to $\Delta y_t^B$. Apart from this difference, any personal shock to their expected wealth or any intuition about the future time path of the economy (e.g., of government spending or growth) is assumed to be shared by them both. So, the relationship between the updating of Keynesian expected wealth and the updating of Ricardian expected wealth is simply

$$E_t W_t^K - E_{t-1} W_t^K = [E_t W_t^R - E_{t-1} W_t^R] + [E_t W_t^B - E_{t-1} W_t^B].$$

(19)

Define $\eta_t^i$ by

$$\eta_t^i = \sum_{j=0}^{t-1} (1 + g)^j \cdot \omega_{t-j}^i,$$

(20)

for $i = R, K$ or $B$, with $\omega_{t-j}^i$ defined by equation (9). Then, repeated application of equation (11) backward in time leads to

$$c_t^i = (1 + g)^t \cdot c_0 + \eta_t^i,$$

(21)

for $i = R$ or $K$ where $c_0$ is optimal consumption in period 0, based on available information at that time. I assume $c_0$ to be the same for the two consumers.

To evaluate the welfare cost of Keynesian ignorance, remember that even the Ricardian cannot make fully optimal consumption choices because she does not have perfect knowledge about her future income. The fully optimal consumption at time $t$, $c_t^{opt}$, would be the consumption chosen by an individual facing no uncertainty, and having full knowledge of her actual wealth in the first year of her infinite life, $W_1$. Then,
\[ c_t^{\text{opt}} = (1 + g)^{t-1} \cdot (1 - \theta) \cdot W_1 \]
\[ = (1 + g)^{t-1} \cdot c_t^{\text{opt}} \]  
(22)

Deviations of Ricardian and Keynesian consumption from this optimum optimorum may be defined, 
\[ \varepsilon_t^i = c_t^i - c_t^{\text{opt}}, \quad i = R, K, \]  
(23)
and it follows from equations (9), (19), (20), (21) and (22) that 
\[ \varepsilon_t^K = \varepsilon_t^R + \eta_t^B. \]  
(24)

Provided that the \( \varepsilon_t^i \)'s are small, the welfare cost of the sub-optimal choice \( c_t^i \) is proportional to \((\varepsilon_t^i)^2\). This proposition was demonstrated for a two-period model in section 4.2, and is easily generalized. Thus, the contribution at time \( t \) to the difference between the welfare cost to the Keynesian and the cost to the Ricardian is proportional to 
\[ (\varepsilon_t^K)^2 - (\varepsilon_t^R)^2 = (\eta_t^B)^2 + 2 \varepsilon_t^R \eta_t^B. \]  
(25)

Consideration of a range of possible shocks suggests that the term \( 2 \varepsilon_t^R \eta_t^B \) is of indeterminate sign, and should be small.\(^6\) We assume that, on

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\(^6\) \( \eta_t^B \) is a weighted sum of past and present bond-induced changes to the Keynesian's expected wealth, while \( \varepsilon_t^R \) is the amount by which the Ricardian's consumption at time \( t \) deviates from the value it would take had the Ricardian had full knowledge of all her future income from year 1. Consider an \( n \) year period dominated by an unanticipated event which occurs between times \( t_1 \) and \( t_2 \), with \( 0 < t_1 < t_2 < n \). The event is either (i) a bond-financed lump-sum tax cut at \( t_1 \) paid for by increased lump-sum taxes at \( t_2 \), or (ii) a bond-financed increase in government consumption expenditure at \( t_1 \) again paid for by increased taxes at \( t_2 \), or (iii) a money-financed increase in government consumption expenditure between \( t_1 \) and \( t_2 \) (with the time profile of the real value of bonds kept the same as it would have been without the increase in government spending). Assume the full details of the event are revealed at time \( t_1 \). The first and third events do not lead to any correlation between \( \varepsilon_t^R \) and \( \eta_t^B \) because the former event makes no contribution to \( \varepsilon_t^R \) while the latter makes no contribution to \( \eta_t^B \). The second event leads to a negative contribution to \( \varepsilon_t^R \eta_t^B \) for \( t_1 < t < t_2 \) (because \( \eta_t^B \) is positive while the unanticipated increase in government spending implies a negative contribution to \( \varepsilon_t^R \) for \( t > t_1 \)), and an ambiguous contribution to \( \varepsilon_t^R \eta_t^B \) for \( t > t_2 \) (as \( t \) increases, \( \eta_t^B \) falls and may become negative).
average, this term is sufficiently small that it may be ignored and hence equation (25) reduces to

\[(e_t^K)^2 - (e_t^R)^2 = (\eta_t^B)^2.\]  

(25a)

This is a crucial simplification. It allows us to estimate the welfare costs associated with Keynesian ignorance. We assume Keynesian consumption follows

\[c_t^K = c_{t^{\text{opt}}} + \eta_t^B,\]  

(26)

rather than following equation (21) with \(i = K\). The contribution at time \(t\) to the welfare cost of consumption following \(c_t^K\) from equation (26) rather than \(c_{t^{\text{opt}}}\), is proportional to \((\eta_t^B)^2\). From equation (25a), this is precisely the contribution we require to estimate the cost of being a Keynesian (with consumption following equation (21) with \(i = K\)) rather than a Ricardian (with consumption following equation (21) with \(i = R\)). The simplification eliminates the need to model explicitly all other (personal) deviations of consumption from the optimum.

An expression is now derived for the wealth cost of Keynesian ignorance for a simulation running over the years \(t = 1, \ldots, n\). In the first year of the simulation our Keynesian is endowed with wealth, \(W_1 = c_{1^{\text{opt}}} / (1 - \theta)\), and her actual wealth at the beginning of year \(n + 1\) after the simulation has ended is

\[W_{n+1}^K = \frac{c_{1^{\text{opt}}} (1 + r)^n}{1 - \theta} - \sum_{t=1}^{n} c_t^K (1 + r)^{n+1-t}.\]  

(27)

This wealth is optimally spent in an endless flow of consumption growing at an annual rate of \(g\) starting with consumption \(c_{n+1}^K\) in year \(n + 1\), with

\[c_{n+1}^K = W_{n+1}^K \cdot (1 - \theta).\]  

(28)

Given this pattern of consumption, \(c_t^K, t > 0\), the lifetime utility which our Keynesian derives \textit{ex post}, \(U^K\), is therefore
To generate this level of lifetime utility from an optimal consumption stream would require initial wealth, \( W_{\text{opt}} \), where

\[
U^K = \frac{1}{1-\beta} \left\{ \sum_{t=0}^{n-1} \left( \frac{c_{t+1}^K}{(1+\delta)^t} \right)^{1-\beta} + \left( \frac{c_{n+1}^K}{(1+\delta)^n} \right)^{1-\beta} \right\} .
\]  

(29)

Both the Keynesian consumption profile, \( c_t^K, t = 1, \ldots, \infty \), and the optimal consumption profile, \( c_t^{\text{opt}} = c_1^{\text{opt}} (1+g)^{t-1}, t = 1, \ldots, \infty \), satisfy their respective (identical) intertemporal budget constraints, each with wealth at the beginning of the first year of the simulation, \( W_1 \). But the Keynesian solution is sub-optimal and so the equivalent wealth required, \( W_{\text{opt}} \), satisfies the inequality, \( W_{\text{opt}} < W_1 \). The wealth cost of the Keynesian’s ignorance may be defined by the fraction \( (W_1 - W_{\text{opt}}) / W_1 \). However, this wealth cost has been sustained over \( n \) years. If the damage to wealth was sustained at this rate for ever, the wealth cost is

\[
\frac{\Delta W}{W} = \frac{W_1 - W_{\text{opt}}}{W_1 \cdot (1-\theta^n)} ,
\]  

(31)

defined so that \( \Delta W \) is positive. If lifetime wealth is derived from an endless stream of income growing at a constant rate of \( g \) per annum and if the cost of ignorance is spread over the duration of the simulation, equation (31) gives the proportion of income lost in each year of the simulation.

If \( r \leq g \), the wealth required to sustain a stream of consumption growing at an annual rate of \( g \) for ever is unbounded, and the above analysis does not apply. Results derived with \( r \leq g \) assume the representative
consumers have a life of 200 years\(^7\) (beginning with the \(n\) years of the simulation) during which to consume their initial endowment of wealth. By analogy with the above, their initial wealth is set just sufficient to allow \(c_{t}^{\text{opt}}\) units of consumption in year \(t\), \(t = 1, \ldots, 200\). The analysis proceeds analogously to equations (26) to (31), but taking into account the consumer's finite life, during which she must spend all her initial wealth.

*Data and results*

The analysis is undertaken in units of per capita national income in the first year of the simulation with \(c_{1}^{\text{opt}}\) set equal to 0.63.\(^8\) Figure 2 shows several government debt/national income series for Australia and the U.S. These data, along with a national income deflator and population growth data (from sources detailed in Appendix B), are used to provide the input for equation (14). Real per capita national income and the optimal consumption of our two representative individuals are assumed to grow at the same constant rate of \(g\) per annum.\(^9\) In all simulations, for specified values of \(r\), \(\beta\) and \(g\), \(\delta\) is determined by equation (10). Figure 3 shows the wealth cost, \(\Delta W/W\), of being ignorant of the link between U.S. Federal government bonds and future taxes over the period 1963 – 1987 for real interest rates in the range \(-2\%\) p. a. \(\leq r \leq 5\%\) p. a. assuming the growth rate \(g = 1.7\%\) p. a. Results are derived assuming bond-financed changes in

\(^7\) A 200 year life is chosen arbitrarily. The wealth costs are fairly similar (within 25\%) for a 50 year life. For \(r > g\), the difference between the wealth costs derived from the 200 year life model and the infinite life model is less than 10\%.

\(^8\) Over the 25 years 1963 – 1987 for both Australia and the U.S., personal consumption expenditures average 63\% of national income.

\(^9\) It would be possible to allow a different rate of growth for the economy and for optimal consumption. Over the period 1963 – 1987 in the U.S., real per capita national income (private consumption) has grown at 1.7\% p. a. (1.9\% p. a.). Corresponding figures for Australia are 2.2\% p.a. (1.9\% p.a.). Of course, these are *ex post* growth rates, while we require *ex ante* growth rates for both income and optimal consumption.
Figure 2

Government debt series for Australia and U. S. A.
Figure 3


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**FIGURE LEGEND:** Real per capita g.n.p. and optimal consumption are assumed to grow at 1.7 % p. a.
disposable income are immediately perceived as permanent (using equation (17)), as well as gradual updating of perceived wealth (using equation (18)). For each behavioural assumption, results are derived assuming both $\beta = 2$ and $\beta = 15$ in equation (8). Figure 3 does not show results assuming that bond-financed changes in disposable income are always perceived as temporary (using equation (16)). When this assumption is used, the average (root mean square) deviation of Keynesian consumption from optimal consumption is tiny ($< 0.3\%$) while $\Delta W / W$ is even more minute ($< 8 \times 10^{-6}$ when $\beta = 2$).

Figure 4 shows Keynesian consumption, $c_t^K*$, from equation (26), assuming that perceived wealth is gradually updated according to equation (18). The results are derived assuming $r = 3\%$ p. a. but are almost indistinguishable from results using any interest rate in the range $-2\% \leq r \leq 5\%$ p. a. Also shown is actual personal consumption expenditures over this period. Clearly, the Keynesian simulation tracks actual consumption quite well, though it is beyond the scope of this thesis to attempt a detailed econometric analysis.

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10 When equation (18) is used, all the appropriate data in Figure 2 are used to form estimates of $\Delta y^B$. For example, when using U.S. Federal debt, values of $\Delta y^B$ can be derived from 1949 onwards. So, in the 1963–1987 simulations, the minimum number of terms in the summation in equation (18) is fifteen, which occurs for 1963.

11 Optimal consumption (equation (22)) satisfies: $c / g.n.p. = 0.63$. Keynesian consumption, as defined by equation (26), deviates from optimal consumption only as a result of bond-induced changes to Keynesian expected wealth. The aggregate effect of all other (personal) deviations of consumption from the optimum is not modelled explicitly and therefore is not shown.

12 For the period 1963–1987, assuming $r = 3\%$, average (root mean square) deviations from $c / g.n.p. = 0.63$ are: $0.03\%$ when bond–financed tax changes are assumed once-off, $2.5\%$ when expected wealth is assumed to be gradually updated, and $3.7\%$ when bond–financed tax changes are assumed immediately permanent, compared with $2.3\%$ for the actual personal consumption data. If ignorance of the link between bonds and future taxes is an important source of the deviations of aggregate consumption from its optimum, the best estimates of the personal cost of this ignorance should come from simulations which give deviations from
Figure 4

Comparison of Keynesian consumption derived from the model and actual personal consumption in U. S., 1963 - 1987

FIGURE LEGEND: Keynesian consumption is derived assuming that $r = 3\%$ p. a., $g = 1.7\%$ p. a. and that perceived wealth is gradually updated as defined by equation (18) in the text. Actual consumption is total personal consumption on goods and services from the Dept. of Commerce, Survey of Current Business.
In this section, in the restricted framework of a two-period model, I provide estimates of the wealth cost of ignorance in the presence of explicitly modelled future income uncertainty and distortionary taxes. The model is an extended version of a model presented by Barsky et. al. (1986) and it does not assume certainty equivalence. The two periods of the model may be regarded as consecutive periods in an individual life (of length, perhaps, 10 to 35 years), or as consecutive generations. Barsky et. al. detail the available evidence on the extent of income uncertainty both through an individual's life, and between one generation and the next. Of particular interest are the findings of Hall and Mishkin (1982), Jencks (1972) and Olneck (1977).

Hall and Mishkin (1982) hypothesize that household income consists of three components: a deterministic component which models life-cycle and demographic changes, a stochastic component accounting for innovations to perceived lifetime income (modelled as a random walk), and a stationary stochastic component to model transitory influences. They deduce from their data that the annual innovation to perceived lifetime income has a standard deviation of $1,220 or roughly 10% of median household income at the time (1972). The standard error of a forecast of income 10 years into the future is, therefore, $1,220 or roughly 32% of 1972 median household income, while an estimate 35 years into the future has a standard error of roughly 60% of 1972 median household income.

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optimal consumption of a similar magnitude to the actual consumption data, i.e., those that assume gradual updating of expected wealth.
4.4 TWO-PERIOD MODEL INCORPORATING DISTORTIONARY TAXES

In this section, in the restricted framework of a two-period model, I provide estimates of the wealth cost of ignorance in the presence of explicitly modelled future income uncertainty and distortionary taxes. The model is an extended version of a model presented by Barsky et. al. (1986) and it does not assume certainty equivalence. The two periods of the model may be regarded as consecutive periods in an individual life (of length, perhaps, 10 to 35 years), or as consecutive generations. Barsky et. al. detail the available evidence on the extent of income uncertainty both through an individual's life, and between one generation and the next. Of particular interest are the findings of Hall and Mishkin (1982), Jencks (1972) and Olneck (1977).

Hall and Mishkin (1982) hypothesize that household income consists of three components: a deterministic component which models life-cycle and demographic changes, a stochastic component accounting for innovations to perceived lifetime income (modelled as a random walk), and a stationary stochastic component to model transitory influences. They deduce from their data that the annual innovation to perceived lifetime income has a standard deviation of $1,220 or roughly 10% of median household income at the time (1972). The standard error of a forecast of income 10 years into the future is, therefore, $1,220 or roughly 32% of 1972 median household income, while an estimate 35 years into the future has a standard error of roughly 60% of 1972 median household income.

optimal consumption of a similar magnitude to the actual consumption data, i.e., those that assume gradual updating of expected wealth.
Alternatively, if the model is interpreted as applying to consecutive generations, we require an estimate of the uncertainty of an individual's forecast of the income of her children. According to Olneck (1977): "the average difference between brothers on earnings is 87% as large as the difference between random individuals" (p. 137). Jencks (1972) provides data on the variation of annual earnings for full-time year round workers in the U.S. in 1968. The coefficients of variation of earnings (i.e., the standard deviation divided by the mean) for males, females and all workers are, respectively, 0.65, 0.58 and 0.72 (p. 213).\(^{13}\)

The model assumes that the only source of wealth is labour income. In the first period, the representative consumer has a after-tax income \(y_1\), where

\[
y_1 = \frac{y}{1+g}\ 	ext{with certainty.} \quad (32)
\]

Second period after-tax income, \(y_2\), is risky and there do not exist markets in which individuals can reduce this risk.\(^{14}\) Second period income has the distribution

\[
y_2 = \begin{cases} 
(1-x)y & \text{with probability } p, \\
y & \text{with probability } 1 - 2p, \\
(1+x)y & \text{with probability } p.
\end{cases} \quad (33)
\]

Thus, second period income has a coefficient of variation, \(\sigma\), equal to \(x(2p)^{1/2}\). The expected value of second-period income is \(y\), equal to

\(^{13}\) As Barsky et. al. point out, these values overestimate uncertainty by including some transitory variations in income, as well as sampling people at different stages of their life-cycle. But they underestimate uncertainty by not including people with disabilities or those suffering prolonged unemployment.

\(^{14}\) The model relies on the absence of these contingent claims markets although their absence is not explained or modelled – an omission which some find disturbing [see e.g., Bernheim (1987)]. Such contingent claims markets do not appear to exist, perhaps because of the associated adverse selection and moral hazard problems.
(1 + g). y₁ and so g is the growth rate of expected income between the model's two periods. In period one, the government issues a bond financed tax-cut of value B per consumer, and hence first period after-tax income becomes

\[ y₁ = \frac{y}{1 + g} + B \]

with certainty. (32a)

This tax-cut must be paid for in period two and I examine two possible forms for the tax increase. Either it is a flat tax levied at the rate (1 + r). B / y so that tax revenue is

\[
\begin{align*}
(1 + r). (1 - x). B & \quad \text{if } y₂ = (1 - x)y, \\
(1 + r). B & \quad \text{if } y₂ = y,
\end{align*}
\]

and

\[
\begin{align*}
(1 + r). (1 + x). B & \quad \text{if } y₂ = (1 + x)y,
\end{align*}
\]

or it is a progressive income tax which takes the value

\[
\begin{align*}
0 & \quad \text{if } y₂ = (1 - x)y, \\
(1 + r). B & \quad \text{if } y₂ = y,
\end{align*}
\]

and

\[
2(1 + r). B \quad \text{if } y₂ = (1 + x)y.
\]

Note that, provided there are a large number of consumers with an independent identical distribution of second period incomes given by equation (33), the second period tax is just sufficient to pay both the principal and the interest on period one's outstanding bonds. 15 The consumer's optimization problem is now, to

\[
\begin{align*}
\text{maximize} & \quad E(U) = u(c₁) + E[u(c₂)] \big/ (1 + δ) \\
\text{subject to} & \quad c₁ + c₂ / (1 + r) = y₁ + y₂ / (1 + r).
\end{align*}
\]

(1a) (2a)

15 Hall and Mishkin (1982) [who analysed income and consumption data for about two thousand households from the University of Michigan's Panel Study of Income Dynamics in the early 1970's] report that the "overwhelming bulk of the movements in income that give rise to our inference from the data are unrelated to the behavior of the national economy; most are probably highly personal" (p. 480). This suggests that the assumption of independent second period incomes is probably a good one.
When determining $c_1$, the relevant values of $y_1$ and $y_2$ are the perceived values. I again assume that the one-period sub-utility takes the form

$$u(c) = \frac{1 - \beta}{1 - \beta} c, \quad \beta > 0.$$ \hspace{1cm} (8)

In the context of this uncertainty model, $\beta$ is the consumer's relative risk aversion. As before, the Keynesian consumer is ignorant of the future tax implications of period one's bonds, and interprets the bond financed tax-cut as a certain increase in wealth of value $B$. She chooses consumption in the first period, $c_1$, by maximizing expected utility [equation (1a)] subject to her perceived budget constraint [with first and second period incomes given, respectively, by equations (32a) and (33) with $y = y^K$].16 Her actual expected utility, $E(U^K)$, may then be derived using her chosen value of $c_1$ and her actual intertemporal budget constraint which determines $c_2$ for each possible outcome of $y_2$.

Define $y^R$ so that a Ricardian derives expected utility $E(U^K)$ when her incomes are given by equations (32a) and (33) with $y = y^R$. Of course, by her nature, she has a full understanding of the future tax implications of period one's bonds as well as an awareness of the tax regime (flat-rate or progressive) operating in the second period.17 As before, the sub-optimal nature of the Keynesian's consumption decisions ensures that $y^R < y^K$. The wealth cost of ignorance of the link between bonds and future taxes is

$$\Delta W/W = (y^K - y^R)/y^K,$$ \hspace{1cm} (35)

16 Because the model does not assume certainty equivalence, all optimization problems in this section are solved numerically (in double precision arithmetic using a FORTRAN program). In each case, in the neighbourhood of the optimum, 1,000 values for $c_1$ are examined, which differ, one from the next, by $\Delta c = 4 \times 10^{-6} y$. The chosen value of $c_1$ is the one which maximizes expected utility.

17 $y^R$ is derived by a process of iteration.
defined so that $\Delta W$ is positive. This definition ensures that the wealth cost is spread proportionately in the two periods of the model.

A representative set of results is derived assuming these values for parameters: $r = \delta = g = 0$, $\beta = 2$, $B = 0.05, y_1$.\textsuperscript{18} Based on the evidence described earlier, the calculations assume that the coefficient of variation of second period income, $\sigma$, takes the value $\sigma = 0.3$ (appropriate if each period has a length of 10 years) or $\sigma = 0.6$ (more appropriate if each period is 35 years or if the model represents consecutive generations). For given $\sigma$, $x$ may vary in the range $\sigma < x < 1$.\textsuperscript{19} Once $x$ and $\sigma$ are given, the value of $p$ is uniquely determined.

The results in Figure 5 show a substantial reduction in the cost of ignorance when uncertainty is introduced into the model. The reason for this reduction is as follows. First period consumption is determined partly by the necessity for precautionary savings in case of a "bad" outcome in the second period.\textsuperscript{20} When a bond is issued in the first period, the second period tax regime acts as insurance against such a bad outcome, with this insurance effect being more pronounced the more progressive is the tax regime. As Barsky et. al. explain, the Ricardian response to a certain increase in first period income coupled with a second period progressive tax, is to consume part of the certain increase in period one.

\textsuperscript{18} In the absence of income uncertainty, a bond of this magnitude leads to a wealth cost of ignorance, $\Delta W/W$, comparable to those derived from the 1963–1987 simulations assuming gradual updating of expected wealth and $\beta = 2$. Using an alternative set of parameter values ($r = 3\%$ p. a., $\delta = 0$ and $g = 2.3\%$p. a.) leads to very similar results.

\textsuperscript{19} If $x < \sigma$, $p > 1/2$ and hence, $1 - 2p < 0$ while if $x > 1$, one of the second period incomes is negative.

\textsuperscript{20} An example reinforces the point. With $r = \delta = 0$, $x = 1$, $\beta = 2$ and no bonds, optimal first period consumption falls from $c_1 = $100 to $c_1 = $79 to $c_1 = $68 as the coefficient of variation of second period income rises from $\sigma = 0$ (i.e. no uncertainty) to $\sigma = 0.3$ to $\sigma = 0.6$. 
Figure 5

Wealth costs in two-period model with income uncertainty.

**FIGURE LEGEND:** Parameter values are: \( r = \delta = g = 0 \).
The bond issued in the first period has a value of 5% of that period's income. The dotted line shows the wealth cost with no second-period income uncertainty.
Ignorance of the increased future taxes leads to only a small further increase in first period consumption. The Keynesian response differs only marginally from the optimal Ricardian response, and so the associated wealth cost of being ignorant of the future taxes is very small.

4.5 DISCUSSION

1. Allowing the possibility that the private sector contains a fair proportion of Keynesians, the assumption made in this chapter of a constant real interest rate (independent of the changing debt position of the government) is implausible, especially for a large economy like the U.S. Clearly, changing expected future real interest rates have an impact on both Ricardian and Keynesian consumption. For the Keynesian, this follows because it is not necessary to understand the link between bonds and future taxes to act on estimates of future interest rates provided by economists (or one's bank manager). But, within the range examined here, the wealth cost of Keynesian ignorance is almost independent of the fixed real interest rate assumed (see Figure 3). Thus, over a period of varying interest rates, equally well anticipated by our two consumers, the cost of Keynesian ignorance should be close to the estimates provided here.

2. The numbers used for government debt in section 4.3 represent a somewhat naive attempt to estimate net real per capita Federal government debt. As explained, for example, by Boskin (1982, 1987), a more accurate estimate requires properly accounting for government assets, investments and contingent liabilities, as well as estimating

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21 The total increase in first-period Keynesian consumption on introduction of the bond is between 2.5% and 5%, depending on parameter values. For $x = 0.3$ (and $\sigma = 0.3$), the increase is somewhat less than 3%. For larger values of $x$, the first-period Keynesian consumption expansion is larger, reaching nearly 5% for $x = 1$ (with $\sigma = 0.3$ or 0.6). The type of tax regime operating in the second period is irrelevant as these increased taxes are ignored by our Keynesian.
liabilities from social insurance programs. Importantly, including these components can have a dramatic effect on estimates of the government's net debt position. Nevertheless, the fact that there are components of the government's net worth which are very hard to estimate does not invalidate the calculations reported in section 4.3. The government's net debt position (as estimated here) is a piece of information which improves an individual's capacity to smooth her consumption over time. By an argument identical to the one which led to equation (25), what matters is the correlation between the bond-induced deviation from optimal consumption and the deviation from other sources. The calculations presented here simply make the assumption that, on average, this correlation is zero.

3. It is difficult econometrically to estimate the parameter $\beta$ (introduced in equation (8)), or its reciprocal, the intertemporal elasticity of consumption. The reader should refer to Mankiw (1985), Mankiw, Rotemberg and Summers (1985) and Hall (1988) for details and for the range of estimates in the literature. As a consequence of the width of this range, I derive estimates of the cost of ignorance using two widely different values: $\beta = 2$ and $\beta = 15$. For reasons detailed in Appendix C, I regard the former value as providing the best estimates of the cost of ignorance. However, the wealth costs derived in section 4.3 are nearly proportional to $\beta$, so the results quoted may be used to estimate the wealth cost associated with other values of $\beta$.

22 For example, by my measure, net U.S. Federal nominal government debt increased from $223 billion in 1970 to $594 billion in 1980, while a detailed accounting of the government's liabilities and assets (both financial and tangible) reveals an increase in the U.S. Federal government's net worth from $48.3 billion in 1970 to $279.4 billion in 1980 (from Table 2 in Eisner and Pieper (1984)). But even the Eisner and Pieper calculation includes neither the present value of government personnel retirement provisions, nor the projected Social Security commitments under current law, nor other contingent obligations.
4. In 1987, per capita g.n.p. in the U.S. was roughly $18,300 while in Australia, per capita g.d.p. was roughly $A16,300. So, in 1987 prices, a wealth cost, $\Delta W/W$, of 0.001 is equivalent to a loss of after-tax income of roughly $11.50 per annum for an average U.S. consumer or roughly $A10.30 per annum for an average Australian consumer. For an average U.S. consumer, in the 1963–1987 simulations, if unanticipated income changes are immediately assumed permanent and $\beta = 2$ ($\beta = 15$), the gross cost amounts to about $13$ p.a. ($100$ p.a.) whatever real interest rate is assumed. However, the discussion of Figure 4, Boskin's 1987 results, as well as my priors, all suggest that gradual updating of perceived wealth is a more realistic assumption. Then, the gross cost of ignorance is about $7$ p.a. ($46$ p.a.) whatever real interest rate is assumed. The corresponding figures for an average Australian consumer for a 1963–1987 simulation are about $A14$ p.a. ($A150$ p.a.) if unanticipated bond-financed tax changes are immediately assumed permanent and $\beta = 2$ ($\beta = 15$), or about $A2$ p.a. ($A14$ p.a.) if perceived wealth is gradually updated. Since the tax regime operating in all developed economies is more progressive than simply a flat-rate income tax, the simulations described in section 4.4 suggest that all these wealth costs are probably substantial overestimates.

\[\text{23 At constant prices, these numbers grow at the assumed rate of per capita growth of the economy, } g.\]

\[\text{24 Importantly, it is not necessary to agree with the theoretical underpinnings of equation (18) to accept these numbers. Any model which predicts an average deviation between Keynesian and Ricardian consumption of about 2.5\% (as this gradual updating model does) will produce wealth costs very close to these. Estimates were also derived of the cost of ignorance of net Federal, State and local debt in the U.S. (see Figure 2) over the period 1957–1976. Assuming } g = 1.7\% \text{ p. a., } \beta = 2 \text{ and gradual updating of expected wealth, the annual cost is about $10$ if } -2\% \text{ p. a. } \leq r \leq 2\% \text{ p. a., rising to about $18$ when } r = 5\% \text{ p. a.}\]
5. The difficulties discussed earlier which economists (rightly) perceive in estimating the changing net worth of the government bring into focus how severe are the informational requirements of making even an approximately Ricardian choice between consumption and saving in a developed economy. It seems fanciful to imagine many consumers forming these estimates for themselves, especially as the required information is not widely available. Given these observations and the numbers presented in the previous paragraph, the central point of the chapter follows directly. To make an approximately Ricardian consumption choice is sufficiently difficult, and the gross cost of ignoring the link between bonds and future taxes is so small, that it is probably optimal for an average consumer to ignore this link when making consumption and saving choices.

6. Finally, assume for the sake of the argument that the cost of informing oneself sufficiently to make an approximately Ricardian consumption choice is smaller than the expected gross cost of ignorance. Even in this case, how are the ignorant to learn from the Ricardians about the link between bonds and future taxes? There is no “natural selection” market mechanism (of the kind discussed by Alchian (1950) and Friedman (1953)) which would tend to drive ignorant consumers from the marketplace in the way that inefficient firms tend to lose out to more profitable ones. Nor is there a strong incentive for the ignorant to imitate the Ricardians.

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25 At least in Australia, consumers seem to have much less idea of the level of government indebtedness than economists think they have (see Chapter Three). Of course, estimates of government indebtedness could be made by a firm of economists (or others) and sold to consumers. The results of this chapter suggest that, during the recent past, such a service would have had to have been very cheap to have been of net benefit to consumers. Given that and the ease with which individuals could free-ride on such a service, it is not surprising that such a service is not in widespread use in either Australia or the U.S.
Provided governments do not change their bond-financing behaviour substantially, the net gains to Ricardian behaviour are so small that the ignorant will be hard-pressed to observe that their behaviour has had a detrimental effect on their welfare.

**APPENDIX A**

In the first period, the Keynesian intends to expand consumption by \( \Delta c_i \), \( i=1,2 \), where

\[
B = \Delta c_1 + \Delta c_2 / (1 + r).
\]

At these new intended levels of consumption, a condition analogous to (3) holds

\[
(1 + r) u'(c_i R + \Delta c_i) = (1 + \delta) u'(c_i R + \Delta c_i).
\]

(A.2)

Expanding \( u'(c_i R + \Delta c_i) \), \( i=1,2 \) as Taylor expansions around \( u'(c_i R) \), \( i=1,2 \), ignoring derivatives of higher order than \( u''(c_i R) \), and using (A.1) to eliminate \( \Delta c_2 \) leads to (4) in the text. Expanding \( u(c_i K) \), \( i=1,2 \) as Taylor expansions around \( u(c_i R) \), \( i=1,2 \), and again ignoring derivatives of higher order than \( u''(c_i R) \) leads to an estimate of the loss of utility, \( \Delta U \), suffered by our Keynesian consumer as a result of her ignorance. Since \( \partial U / \partial W = \lambda = u'(c_i R) \) where \( \lambda \) is the Lagrange multiplier for the constraint (2) when maximizing (1), \( \Delta W = \Delta U / (\partial U / \partial W) \). Assuming that we can’t sign derivatives of higher order than \( u''(c) \), set \( u''(c_i R) = u''(c_2 R) \), leading to (5) in the text.

**APPENDIX B**

For Australia, the debt series, population data and a price index for g.d.p. come from Occasional Paper No. 8A from the Reserve Bank of Australia entitled *Australian Economic Statistics*, and from *Government Securities on Issue at 30 June 1987. Budget Related Paper No. 1*. The Federal debt series displayed in Figure 2 is defined by:

(Commonwealth Government Securities on Issue minus those held by Public Authorities and by the Reserve Bank) / G. D. P.
For the U.S., the debt series in the first panel of Figure 2 are derived from series published by the Bureau of Economic Analysis (BEA), U.S. Department of Commerce. I define Federal debt as: (Gross amount of debt outstanding: Held by the public minus Federal Reserve Banks, Assets: U.S. Government Securities)/G.N.P. The first two of these series come from the FINANCE section, sub-sections FEDERAL GOVERNMENT FINANCE and BANKING respectively of the BEA’s Survey of Current Business and correspond to series Y 490 and X 304 in The Statistical History of the United States (1976). Quoting from this book: ‘Gross Federal debt is the broadest generally used measure of the Federal debt. It is composed primarily of the public debt (direct borrowing by the Treasury) but also includes agency debt (such as borrowing by the Tennessee Valley Authority or the Postal Service). ... The Federal Reserve System is an independent, federally-chartered, central banking system. As the System is not included in the Federal budget, debt held by the System is included in “debt held by the public.” ... Various writers have contended that the most meaningful measure of the national debt in economic terms is “debt owed to the public.” ... series Y 490 closely corresponds to this concept’.

To make the model in section 4.3 tractable, certainty equivalence is assumed. In each period of the model, the Keynesian consumer forms a point expectation of her wealth and then solves a deterministic optimization problem to derive her consumption in that period. The elasticity of substitution between consumption in any two periods of this model, $\sigma_c$, takes the value $\sigma_c = 1 / \beta$ [where $\beta$ is the coefficient introduced in equation (8)].

Assuming the iso-elastic utility function [equation (8)] for the two-period model of section 4.2, the wealth cost of ignorance, $\Delta W/W$, is proportional to $\beta$ or equivalently, proportional to $1/\sigma_c$ [from equation (5)]. For the simulations of section 4.3, the wealth cost remains roughly proportional to $1/\sigma_c$. Unfortunately, econometric estimation of $\sigma_c$ is a difficult business. Much work from the early 1980's has been convincingly criticised by Hall (1988). The problem is that all consumption data is an average of consumption over a finite period (a month, a quarter etc.) which leads to serial correlation of the error terms in the estimated regression. When combined with the use of lagged endogenous instrumental variables, this serial correlation leads to inconsistent estimators for $\sigma_c$. For several different sets of data, using an appropriate estimator (the Hayashi–Sims estimator), Hall derives estimates of $\sigma_c$ which all satisfy $\sigma_c \leq 0.1$ and which are all statistically insignificantly different from zero.

Three points should be made about Hall's conclusion that $\sigma_c$ is very small (compared to one) and may even be insignificantly different from zero.

(i) There is more than one way to deal with the econometric problem Hall describes. Mankiw, Rotemberg and Summers (1985) [MRS] derive estimates of $1/\sigma_c$ using the same underlying economic model as Hall (in MRS's notation, they test EC imposing the constraint $\gamma = 0$) but using only
fourth quarter consumption data which substantially reduces the problem of serial correlation of the residuals. They generate six different point estimates of $1/\sigma_c$ (using different instrument lists and data on non-durables or non-durables and services) which fall in the range $0.09 < 1/\sigma_c < 0.52$. In all six cases, 95% confidence intervals for $1/\sigma_c$ exclude values of $\sigma_c$ less than one, in stark contrast to Hall’s results.

(ii) Provided the procedure yields consistent estimators, it is preferable to specify a model in such a way that the model’s explanatory variables can be used directly in the estimation (rather than using instruments for them). By using available data on expected real interest rates, Hall (1988) derives consistent estimators of $\sigma_c$ without using instrumental variables. His estimates, $0.346, 0.271$ and $0.066$ for $\sigma_c$ (corresponding to $\beta$ values of 2.9, 3.7 and 15 respectively) are rather larger than his estimates for $\sigma_c$ using instrumental variables, although they are still statistically insignificantly different from zero.

(iii) As Mankiw (1985) argues, and casual observation supports, much of the interest rate sensitivity of consumption should be observed in the consumption of durables (or, more precisely, in the services derived from owning durables) rather than in the consumption of non-durables. All the estimates of Hall (1988) are derived from data on non-durables, while the estimates of MRS are derived from data on non-durables or non-durables and services. By contrast, Mankiw (1985) derives estimates of $1/\sigma_c$ using data on consumer durables (defined to include mostly motor vehicles and parts, furniture and household equipment, but excluding housing) as well as non-durables. Mankiw’s theoretical structure again allows consistent estimators to be derived without using instrumental variables [it leads to his equation (18)]. For three models estimated this way, 95% confidence

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26 To be precise, MRS state that the estimates derived using only fourth quarter data "are essentially identical to" the estimates I’m quoting.
intervals for $1/\sigma_c$ always exclude values of $\sigma_c$ less than 0.4, again in contrast to Hall’s results.

To summarize, the best evidence (from a model which includes both durables and non-durables and does not require the use of instrumental variables) suggests that $\sigma_c$ is probably not less than 0.4 (and may be substantially more than 0.4). But we cannot be too confident. This unsatisfactory state of affairs led me to derive estimates of the cost of ignorance of the link between bonds and future taxes using two widely different values for $\sigma_c$: $\sigma_c = 1/2$ ($\beta = 2$) and $\sigma_c = 1/15$ ($\beta = 15$). I regard the former value as providing the “best” estimates of the cost of ignorance.

In contrast to the above, in the uncertainty model of section 4.4, $\beta$ is the (constant) relative risk aversion of the consumer. As Hall (1988) points out, in an uncertain intertemporal environment, a consumer's relative risk aversion is not in general equal to the reciprocal of her intertemporal elasticity of consumption. This (undesirable) coincidence occurs because of the single-parameter functional form chosen here for the one-period utility function. Hall (1988) argues that his evidence on the smallness of the intertemporal substitution of consumption does not imply that a representative consumer has a large relative risk aversion. Accepting this point, the results in section 4.4 are all derived assuming $\beta = 2$. This value is in the middle of the range of values quoted in the literature [see e.g., Barsky et. al. (1986)] although, again, one cannot have too much confidence in the econometric evidence from which it was derived.
CHAPTER FIVE *

A RANDOM WALK AROUND THE $A:
EXPECTATIONS, RISK, INTEREST RATES AND
CONSEQUENCES FOR EXTERNAL IMBALANCE

5.1 INTRODUCTION

Australia is a small open economy operating with essentially no impediments to the movement of capital into or out of the country. As a consequence, Australian interest rates should satisfy international arbitrage conditions. The arbitrage condition for a representative US investor can be expressed either in terms of nominal or expected real interest rates. Thus,

\[ i_A = i_{US} - E(\Delta S / S) + \rho p \]  \hspace{1cm} (1)

or

\[ E r_A = E r_{US} - E(\Delta S^R / S^R) + \rho p^R, \]  \hspace{1cm} (2)

where \( i \) and \( E r \) denote the nominal and expected real interest rates for some asset, \( S \) is the nominal US$/A exchange rate, \( \Delta S \) is the change in \( S \) over the life of the asset, and the superscript \( R \) denotes real. The risk premia, \( \rho p \) and \( \rho p^R \), are the excess returns demanded by a US investor to hold the Australian denominated asset.\(^1\)

This chapter presents a detailed examination of these two equations. The almost exclusive focus is on short-term nominal assets with the horizon of our analysis ranging from one week to three months. The chapter is laid out as follows. Section 5.2 presents evidence on the forward rate and

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\* This chapter was written jointly with Mr. Jeremy Smith.

\(^1\) The term 'risk premium' is often used loosely to mean the excess return demanded by investors to compensate them for the 'risk' of an exchange rate fall. In this chapter, we use the term in its technical sense. For a given expectation of the return on an asset, the risk premium is the excess return required because of the expected distribution of that return which may be summarised by the expected higher moments of the return on the asset. Equations (1) and (2) are approximations because, for example, \((1 + i_A)/(1 + i_{US})\) is only approximately equal to \(1 + i_A - i_{US}\). In the chapter, the exact expressions are used when required. See Appendix A for a discussion of the relevance of tax for the equations.
on survey market expectations as predictors of the future spot exchange rate. In section 5.3, after drawing implications from the survey on the size of the risk premium, two theoretical models are presented of the risk premium necessary to induce a representative US consumer-investor to hold a small proportion of assets in short-term nominal Australian assets. Section 5.4 discusses the behaviour over the past fifteen years of consumer price inflation and short-term nominal and real interest rates for several OECD economies. The fifth section in this chapter demonstrates that since Dec 83, the $A, unlike all the other currencies we examine, has exhibited significant skewness because of many large rapid unpredictable depreciations. The final section discusses the results of the chapter in terms of either a time-varying risk premium, or an inefficient foreign exchange market, or a peso problem (see definition in Chapter One) for the $A. Since late 85, all our evidence is that the risk premium has been much too small to explain the short-term real interest differential between Australia and the US, and so we focus on the latter two explanations. Evidence on the inefficiency of the foreign exchange market can provide a rationalization of the results – but a puzzle remains. As an alternative, we provide evidence that the $A suffers from a peso problem because of a market perception that, in the longer run, the real economy must adjust to stabilize the ratio of Australia's net external liabilities to GDP – with a lower real and nominal exchange rate during the adjustment process.

5.2 EXCHANGE RATE EXPECTATIONS AND THE FORWARD RATE

To begin, we define the notation to be used in the chapter for spot and forward exchange rates. Let $S_t[a/b]$ be the spot price of currency 'a' measured in currency 'b' at time t (so that an increase in $S_t[a/b]$ represents an appreciation of currency b with respect to currency a) and let $F_{t,k}[a/b]$ be the price at time t, k periods forward. Further, let lower
case exchange rate variables denote the natural log of upper case exchange rate variables \((s_t \equiv \ln S_t)\) and \((f_{t,k} \equiv \ln F_{t,k})\).

The forward rate

A standard way to test whether the forward rate is a biased predictor of the future spot rate (see e.g., Hodrick (1987), Goodhart (1988)) is to estimate the equation

\[
\Delta s_{t+k} = \alpha + \beta f_{d_{t,k}} + \eta_{t+k},
\]

where \(\Delta s_{t+k} = s_{t+k} - s_t\), \(f_{d_{t,k}} = f_{t,k} - s_t\) is the current k-period forward discount\(^2\) and \(\eta_{t+k}\) is the error term. If the change at time \(t\) predicted by the (log) forward discount is an unbiased predictor of the actual (log) change in the spot exchange rate, then \(\alpha = 0\) and \(\beta = 1\). By contrast, if \(\beta = 0\), the forward rate (or forward discount) tells us nothing about the future movement of the spot rate. Finally, if \(\beta < 0\), on average over the next \(k\) periods, the deviation from trend of the spot rate is in the opposite direction to the deviation predicted by the forward discount.

Table 1 reports regressions of equation (3) for the $A / $US market over two different time periods for both a four week and a thirteen week horizon. The reason for the two different time periods and technical details concerning the regressions are discussed in Appendix B. Details about the exchange rate dataset (dataset A) are provided in the Data Appendix. In all cases in which data from overlapping time periods are used (Table 1 is the first example), the standard errors (SE) of the estimates are evaluated using the technique of Hansen (1982).

There is by now "considerable evidence for a variety of currencies and sample periods ... that indicates a strong rejection of the proposition that

---

\(^2\) Given our definitions of \(s\) and \(f\), if for example \(s_t\) is the log $US/$A rate and \(f_{d_{t,k}}\) is positive, then in the forward market the $US is trading at a discount compared to the spot market.
the forward rate is an unbiased predictor of the future spot rate" (Hodrick, 1987, p. 54). To give a recent example, Goodhart (1988) estimates equation (3) for nine datasets (using daily, weekly and monthly exchange rate data on four currencies against the $US). Six of his point estimates of \( \beta \) are negative and in five cases his estimate is significantly (more than two standard errors) less than 1. By contrast, none of his estimates are significantly different from 0. That is, he cannot reject the hypothesis that the forward discount has no capacity to explain future movement of the spot rate.

**TABLE 1**

**ESTIMATES OF COEFFICIENTS FOR EQUATION (3)**

<table>
<thead>
<tr>
<th>k (Weeks)</th>
<th>Intercept Coefficient</th>
<th>Intercept SE</th>
<th>Coefficient on Forward discount</th>
<th>Coefficient SE</th>
<th>( R^2 )</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Four*</td>
<td>-0.011</td>
<td>0.009</td>
<td>-1.93</td>
<td>1.59</td>
<td>0.023</td>
<td>0.40</td>
</tr>
<tr>
<td>Four+</td>
<td>-0.007</td>
<td>0.014</td>
<td>-1.67</td>
<td>2.15</td>
<td>0.011</td>
<td>0.47</td>
</tr>
<tr>
<td>Thirteen*</td>
<td>-0.028</td>
<td>0.016</td>
<td>-0.35</td>
<td>1.22</td>
<td>0.003</td>
<td>0.12</td>
</tr>
<tr>
<td>Thirteen+</td>
<td>-0.039</td>
<td>0.025</td>
<td>-1.20</td>
<td>1.72</td>
<td>0.022</td>
<td>0.12</td>
</tr>
</tbody>
</table>

Dataset A: * 6 Jan 84 to 21 Apr 89; + 15 Feb 85 to 21 Apr 89.

For Australia, Thorpe et. al. (1988) estimate equation (3) over one month, three month and six month horizons using a trade-weighted exchange rate measure (formed from bilateral exchange rates with the US, Japan, West Germany and Britain). Their period of estimation ranges from Dec 83 – Sept 87 to Nov 84 –Jan 88, and each of their point estimates of \( \beta \) is negative – but the level of significance is low. Only for a one month horizon can they reject \( H_0: \beta = 1 \) at a 5% level of significance.\(^3\) All the regressions in Table 1 are consistent with this pattern – negative point estimates of \( \beta \) – but again the level of significance is low. Only in the case

\(^3\) For this regression, they also reject \( \beta = 0 \) at a 5% level of significance.
of the first reported regression can the hypothesis $H_0: \beta = 1$ be rejected at the 5% level in favour of the alternative $H_1: \beta < 1$.

Survey data on exchange rate expectations

Between Mar 85 and Sept 87, *The Australian* newspaper published the results of a market survey of expectations of the $US/A$ conducted by Dr. Ben Hunt. For almost the whole sample period sixteen of the major companies involved in the foreign exchange market took part in the survey. Every Friday between 2pm and 5pm each company's chief foreign exchange dealer was asked his/her expectation of the spot value of the $US/A$ at 3pm the following Friday and at 3pm in four weeks time (see Hunt, 1987). Further details of the survey are in the Data Appendix. In this sub-section we analyse the four week expectational data.

Figures 1 and 2 show comparisons of the four week exchange rate expectations with the one month forward rate, the four-weekly inflation differential between the US and Australia ($\pi^u_t - \pi^A_t$),\(^4\) and the behaviour of the spot rate. Define $s^e_{t+k}$ as the (log of the) mean of the market participants' expectations of the spot rate in $k$ weeks,\(^5\) and $\Delta s^e_{t+k}$ by $\Delta s^e_{t+k} = s^e_{t+k} - s_t$. Judged by the root mean square error (RMSE) over

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\(^4\) To derive inflation, we use price indices which had been published when the expectations were formed. Australian quarterly CPI numbers were always published by the end of the month immediately following the end of each quarter. For each Friday in month $i$ we therefore define annual Australian inflation as $\frac{P_{i,4}}{P_{i-16}} - 1$, where $P_i$ is the CPI for the quarter containing month $i$. For these numbers, we adjust for the 'Medicare effect' – which affects the estimates for the first twenty one weeks. Defining $p_i$ as the monthly CPI for the US, annual US inflation is defined as $\frac{p_{i,3}}{p_{i-15}} - 1$. Four weekly inflation is derived from these annual inflation figures.

\(^5\) Much existing literature imagines that there is a single expectation that is homogeneously held by investors. We assume that this expectation is being measured by the mean of the responses. One can take the somewhat more sophisticated view that we measure the 'true investor expectation' with a measurement error (Froot and Frankel, 1989). Our econometric tests remain valid provided this measurement error is random (and hence uncorrelated with information available when the expectation is formed).
the sample period, both the average market participants' four-week forecast \( s_{t+4}^e \) and the forward rate \( f_{t,1\text{ month}} \) are marginally worse forecasts of the spot rate in four weeks than the no-change forecast \( s_{t+4}^e = s_t \) one would use if one thought the exchange rate was a martingale or a random walk without drift. The root mean square errors are respectively, 3.3% and 3.2% compared with a RMSE for the no-change forecast of 3.1%.\(^6\) Table 2 reports estimates of equation (3) for the time period of the expectational data as well as four other equations.

\(^6\) This result tallies with others who find that over a short horizon, market participants' forecasts of the future exchange rate are often worse, but never significantly better than a 'no-change forecast' – Lowe and Trevor (1986), Hunt (1987) and Manzur (1988).
Figure 1
Average four week forecast, forward rate
and change in spot rate for $US/$A
Figure 2

Average four week forecast compared with the forward rate and the inflation differential

% change

Average change predicted by market participants

Change predicted by inflation differential

Change predicted by forward rate

$US/$A exchange rate
### TABLE 2

**ESTIMATES OF COEFFICIENTS FOR EQUATIONS (3 – 7)**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Estimates</th>
<th>Standard errors (SE)</th>
<th>R²</th>
<th>DW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equation (3) ( \Delta s_{t+4} = \alpha + \beta fd_{t,1 \text{ month}} )</td>
<td>-0.044</td>
<td>(0.023)</td>
<td>0.098</td>
<td>0.47</td>
</tr>
<tr>
<td>Equation (4) ( \Delta s_{t+4} = \alpha + \beta fd_{t,1 \text{ month}} + \gamma (\pi^u_{t} - \pi^A_{t}) )</td>
<td>-6.22</td>
<td>(3.24)</td>
<td>0.10</td>
<td>0.098</td>
</tr>
<tr>
<td>Equation (5) ( \Delta s_{t+4}^{e} = \alpha + \beta fd_{t,1 \text{ month}} )</td>
<td>0.002</td>
<td>(0.005)</td>
<td>1.20</td>
<td>0.071</td>
</tr>
<tr>
<td>Equation (6) ( \Delta s_{t+4}^{e} = \alpha + \beta fd_{t,1 \text{ month}} + \gamma (\pi^u_{t} - \pi^A_{t}) )</td>
<td>0.0065</td>
<td>(0.0040)</td>
<td>0.514</td>
<td>2.10</td>
</tr>
<tr>
<td>Equation (7) ( s_{t+4}^{e} - s_{t+4} = \alpha + \beta fd_{t,1 \text{ month}} )</td>
<td>0.047</td>
<td>(0.023)</td>
<td>7.38</td>
<td>0.131</td>
</tr>
</tbody>
</table>

Survey and exchange rate data: Weekly (Fridays), 8 Mar 85 to 18 Sept 87.
Estimation of equation (3) over the period of the exchange rate surveys again shows a negative point estimate of $\beta$ (Table 2). The hypothesis $H_0: \beta = 1$ is rejected at the 5% level in favour of the alternative $H_1: \beta < 1$. Equation (5) shows that, on its own, the forward rate has significant explanatory power for the average exchange rate change expected by market participants. On the basis of estimates of this equation for their (fairly extensive) survey expectations data, Froot and Frankel (1989) conclude that "expectations seem to move very strongly with the forward rate", although they don't examine alternative explanators for expectations. The evidence from equation (6) is that our survey expectations seem to move strongly with the inflation differential rather than the forward rate. Importantly, this is in stark contrast with the actual exchange rate which shows no tendency to move in the direction predicted by either the forward discount or the inflation differential either over the sample period (equation 4) or during the whole period of the $A$ float (not shown). Equations (3) and (5) can be combined to give equation (7) which demonstrates that the average expectational error at time $t$ exhibits statistically significant correlation with information available to the market at time $t$ (the one month forward discount at $t$). This is evidence that either market expectations are not rational, or that over this period, the $A$/US market suffered from a 'peso problem'. These possibilities are examined in the discussion section.

Over the sample, the average annual inflation differential between Australia and the US was 5.4% p.a., the average annual rate of depreciation implied by the one month forward rate was 8.2% p.a., while market participants expected depreciation at an average rate of 7.8% p.a. On average, the actual rate appreciated at 2.7% p.a. Corresponding figures for the period Nov 85 – Sept 87 were 6.5% p.a. for the inflation differential, 8.5% p.a. for the one month forward rate, 10.5% p.a. for the
average market participant compared to an average rate of appreciation of the actual rate of 2.8% p.a.

Thorpe et al. (1988) report an extensive survey of market participants' exchange rate expectations at a one month horizon. The surveys were conducted weekly from Nov 84 to Mar 88 and used to estimate expectations of a trade-weighted $A exchange rate (TWA, to distinguish it from the Reserve Bank's TWI). Their study shows interesting similarities with our survey results. They fit equation (7) for their expectations data and conclude, as we do, that the average expectational error at time t is correlated with the forward discount at t. From Nov 84 to Dec 85, their market participants expected nominal appreciation of their TWA at an average rate of 4.7% p.a. This seems to correspond quite well with our expectations results for most of 1985 (see Figure 2). From Jan 86 to the end of their sample in Mar 88, their participants expected nominal depreciation of their TWA at an average rate of 8.6% p.a. – again somewhat comparable to our results.

An objection to the use of survey data to draw inferences about market expectations is this: "Consider the incentive problem of a trader who possesses private information that he has used to construct a portfolio of positions based on the deviations of his expectations from the current forward rates. When [someone] calls him for his expectations, will he reveal his information, or will he lie and quote something like the forward rate? Does he even know that the two are different?" (Hodrick, 1987 p. 135)

Two points are worth making in response to this objection. Firstly, Goodhart (1988) provides evidence that foreign exchange traders – and the banks they work for – very seldom take open positions over a period as long as a month, which suggests that if traders have private information they very rarely think it sufficiently reliable to use it in the marketplace. Secondly, it is easy to understand the nature of private information in, for
example, the stockmarket. In that market, there are obvious examples of potentially private information likely to affect a company's future profitability (e.g., the discovery of a valuable mineral ore deposit) and hence its stock price. But what is the nature of private information which, on average, helps to predict the value of the $US/$A exchange rate in four weeks time? If such information exists, it has been kept secret from macroeconomists for some time. Our best models of exchange rate determination over a four-week (or substantially longer) horizon do not perform significantly better than a random walk forecast (Meese and Rogoff, 1983). This second point sounds flippant, but is meant in all seriousness.

5.3 THE RISK PREMIUM

To begin, we briefly use some of the results from the previous section to provide measures of the nominal risk premium. Figure 3 shows a decomposition suggested by Froot and Frankel (1989) of the average market participant's expectational error, $s_{t+4} - s_{t+4}^e$, into the nominal risk premium, $rp = s_{t+4}^e - f_{t,1\text{month}}$, and the forward rate error, $fre = s_{t+4} - f_{t,1\text{month}}$,\(^7\) $s_{t+4} - s_{t+4}^e = fre - rp$.

For the estimate of the risk premium in Figure 3 to be accurate, it is necessary that the 'true investor expectation' is measured with no error—a requirement that seems rather stringent (see footnote 5). We can alternatively use equation (5) in Table 2 to test the null hypothesis that the risk premium is zero (equivalent to $\Delta s_{t+4}^e = f_{d_{t,1\text{month}}}$; hence the null

\(^7\) It would make minimal difference to construct a four-week forward rate.
hypothesis is $\alpha = 0, \beta = 1$. Applying this test to our whole sample survey at a 5% level of significance, we accept this null of zero risk premium.⁸

Figure 3

Nominal risk premium of the average market participant and the forward rate error

% per annum

---

Thorpe et. al. (1988) estimate equation (5) for market expectations of their trade-weighted $A$. They find their point estimate $\beta = 3.52$ is very significantly different from $\beta = 1$, and hence they reject the null of zero risk premium. This result is dominated by their survey expectations from Nov 84 to late 85. For expectations formed from late 85 to the end of their survey in Mar 88, they accept the null of zero risk premium. Analysis of the sub-sample of our survey data from Mar to Oct 85 also reveals a statistically significant risk premium, which is not present in the sub-sample from Nov 85 to the end of the sample in Sept 87.
In the rest of this section we provide a priori estimates of the risk premium necessary to induce a US consumer-investor to hold a proportion of wealth in short-term Australian denominated interest-bearing bills. We present two simple single-period models in which a US consumer-investor maximizes end-of-period expected utility. This utility is derived from consumption of a basket of goods. The period of the model is four weeks.

In the first model, four weeks is assumed sufficiently short that the price index for the basket of goods at the end of this time, $P_{t+1}^{us}$, is known at the beginning of the period (i.e., it is not a source of uncertainty). The investor has an initial real wealth, $W_t$, and chooses to hold a proportion of this wealth, $x$, as an Australian bill with nominal return $i^A$ with the remainder $(1-x)$ held as a US bill with nominal return $i^{us}$. Real end-of-period wealth, $W_{t+1}$, is therefore

$$W_{t+1} = \frac{W_t}{P_{t+1}^{us}/P_t^{us}} \left[ (1-x)(1+i^{us}) + x(1+i^A) (S_{t+1}/S_t) \right]$$

where, $S$ is the spot exchange rate, $S = \$US/\$A$. Defining $\delta$ by

$$1 + \delta = (S_{t+1}/S_t)(1+i^A)/(1+i^{us}),$$

end-of-period wealth is given by

$$W_{t+1} = W_t (1+i^{us}) \cdot [1+x\delta] / (P_{t+1}^{us}/P_t^{us}).$$

The investor is assumed to have a constant relative risk aversion, $\beta$, and hence seeks to maximize expected utility of the form

$$EU = E [ c^{1-\beta}/(1-\beta) ],$$

where consumption, $c$, satisfies $c = W_{t+1}$. By construction, the only source of uncertainty arises from movements in the $\$US/\$A$ exchange rate which lead to $\delta$ being a random variable. As we find evidence that changes in the $\$US/\$A$ exchange rate exhibit both skewness and leptokurtosis (see section 5.5), we consider the first four moments of the distribution of $\delta$. Thus, expanding $f(\delta) = [1 + x\delta]^{1-\beta}$ as a Taylor
expansion around \( f(0) \) and including terms up to \( \delta^4 \), leads to an expression for expected utility, \( \text{EU}(x) \), as a function of the share of initial wealth held in Australia, \( x \). Expected utility maximization implies that \( \frac{d\text{EU}(x)}{dx} = 0 \), and imposing this condition gives

\[
\text{EU}(x) = \beta x \delta^2 - \frac{\beta (\beta + 1)}{2} x^2 \delta^3 + \frac{\beta (\beta + 1)(\beta + 2)}{6} x^3 \delta^4. \tag{9}
\]

This equation gives the nominal risk premium, \( E \delta \) – equivalent to \( \text{rp} \) in equation (1) – which the Australian bill must pay to induce the investor to hold a proportion \( x \) of wealth in Australia, as a function of the higher moments of the distribution of \( \delta \). The coefficient of relative risk aversion, \( \beta \), is thought to be about two (see, for example, Friend and Blume (1975) or Newberry and Stiglitz (1981)). Quantitative estimates of the risk premium are derived from Dataset A using the period Jan 84 to Apr 89 and assuming \( \beta = 2 \). As mentioned above, the period of the model is four weeks. Sample estimates of the unconditional four-week moments of \( \delta \) are: \( E\delta^2 = 1.27 \times 10^{-3} \), \( E\delta^3 = -1.99 \times 10^{-5} \), and \( E\delta^4 = 5.69 \times 10^{-6} \). These numbers are used in equation (9) to calculate the nominal risk premium as a

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9 Despite suggestions to the contrary (Juttner and Luedcke, 1988), covered interest parity seems a very good approximation for the Australian–US foreign exchange market after the float of the $A (Frankel and Froot, 1987; Smith, 1989a). We therefore use the approximation: \((1 + i_t^A)/(1 + i_t^{US}) = (S_t/F_t,1_{month})^{12/13}\). The exponent 12/13 is used to convert from one month forward to four weeks forward. \( \delta \) is therefore defined by: \( 1 + \delta = (S_{t+4}/S_t)(S_t/F_t,1_{month})^{12/13} \), where the subscript \( t \) is defined in weeks. Of course, almost all the variation in \( \delta \) arises from the term \( (S_{t+4}/S_t) \).

10 As Pagan (1988) stresses, the risk premium at any time is determined by the moments of \( \delta \) conditional on information available at that time. The unconditional moments give an estimate of the average conditional moments over some extended period of time – a few weeks or a few months (see Frankel, 1988). Appendix C addresses this issue, but it does not change the conclusions presented here.
function of the fraction of the portfolio held in Australia and the results are displayed in Figure 4. 11

An estimate of the proportion of Australian assets held in an international asset portfolio can be derived from a survey reported in *The Economist* March 25, 1989. Nine international investment banks were asked for their best mix of investments (equities, bonds and 'cash') over the next 12 months. Their suggestions for the geographical location of their equity holdings (but not their bond or cash holdings) were reported. Of six who suggested putting part of their portfolio in Australia, the range of recommended Australian portfolio shares was $0.1\% \leq x \leq 2.5\%$. When examining Figure 4, we may use these numbers as a guide to estimating portfolio shares for Australian bills. 12 For $x \leq 2.5\%$, the risk premium required to induce such a holding is less than $0.09\%$ p.a., and the increase in risk premium required to induce an investor to raise Australian asset holdings by 10% is less than $0.01\%$p.a.

---

11 To stress the point: we do not evaluate the actual excess return on Australian bills over the sample period. Rather, we calculate the excess nominal return that a utility maximizing US investor requires because of the volatility – measured in $\$US$ – of the nominal return on her Australian asset. It is this excess return that constitutes the risk premium. We tested the sensitivity of the results to changes in some of the assumptions. Deriving the moments of $\delta$ using Dataset B, or alternatively, superimposing a constant depreciation of the $\$A$ of $10\%$ p.a. (equivalent to $0.736\%$ every four weeks) on the actual exchange rate data both lead to estimates of the risk premium almost identical to those shown in the Figure.

12 As will be discussed shortly, the representative consumer is probably not free to put all her financial resources into an international asset portfolio, and hence these numbers should be overestimates.
For the estimated values of the second, third and fourth moments of $\delta$, and for values of $x$ relevant to Australia, the effects of skewness and leptokurtosis on the risk premium are negligible – only the $E\delta^2$ term on the rhs of equation (9) contributes to the risk premium. Truncating the analysis beyond this term is equivalent to applying mean-variance analysis. Such analysis can be conducted without some of the restrictive assumptions used above, and it forms the basis of our second simple model. Thus, we now (i) assume that the price index for the basket of goods at the end of the period, $P_{us,t+1}$, is unknown at time $t$, and (ii) allow our investor to choose nominal assets in Australia and four other
countries (Canada, Japan, U.K. and West Germany) as well as the US. Let \( \rho^j \) be the real return on the nominal asset from country \( j \), defined by

\[
1 + \rho^j \equiv \frac{1 + i^j_{US}}{P_{t+1}^{US}/P_t^{US}} \cdot (S_{t+1}^{j/US}/S_t^{j/US})
\]

where \( S^{j/US} \) is the price of currency \( j \) in US dollars (equal to one when \( j = US \)). Define \( \rho \) as the column vector of the five non-US real returns, \( \rho^j, j \neq US \), and \( z \) as the column vector of real returns relative to the real return on the US asset:

\[
z \equiv \rho - \iota \rho^{US}
\]

where \( \iota \) is a column vector of five ones.\(^{13}\) Define \( x \) as the column vector of the five non-US portfolio shares – and thus the share allocated to the US asset is \( (1 - x' \iota) \). Finally, define \( \Omega \) as the variance-covariance matrix of the real excess return of the non-US assets

\[
\Omega \equiv E (z - Ez)(z - Ez)'
\]

where the expectation is formed at the beginning of the period under consideration. If the investor maximizes a function of the expected value and variance of her end-of-period wealth, then Frankel and Engel (1984) show that

\[
E z = \beta \text{cov}(z, \rho^{US}) + \beta \Omega x.
\]

We now use equation (10) to derive quantitative estimates of \( E z^A \), the expected excess real return on the Australian asset over the US asset, or equivalently, the real risk premium, \( r_{pR} \) from equation (2), which a US investor requires to hold a fraction \( x^A \) of her portfolio in Australian bills. The first term on the rhs of equation (10) is zero if it is assumed that the price index \( P_{US,t+1}^{US} \) is known at time \( t \). Even if the price index is unknown, this term should be very small, because inflation rates are so much less

\(^{13}\) \( z^A \), the Australian element of \( z \) \((z^A \equiv \rho^A - \rho^{US})\), satisfies \( z^A = \delta(1+i^{US})P_{US,t}/P_{US,t+1}^{US} \), where \( \delta \) is defined by equation (8).
countries (Canada, Japan, U.K. and West Germany) as well as the US. Let $\rho^j$ be the real return on the nominal asset from country $j$, defined by

$$1 + \rho^j = \frac{1+i^j}{P_{t+1}^j/P_t^j} \cdot \left( S_t+1[j/\$US]/S_t[j/\$US] \right)$$

where $S[j/\$US]$ is the price of currency $j$ in US dollars (equal to one when $j = \text{US}$). Define $\rho$ as the column vector of the five non-US real returns, $\rho^j$, $j \neq \text{US}$, and $z$ as the column vector of real returns relative to the real return on the US asset:

$$z = \rho - 1 \rho^\text{us}$$

where 1 is a column vector of five ones. Define $x$ as the column vector of the five non-US portfolio shares — and thus the share allocated to the US asset is $(1 - x)^\prime 1$. Finally, define $\Omega$ as the variance-covariance matrix of the real excess return of the non-US assets

$$\Omega = E(z - Ez)(z - Ez)^\prime$$

where the expectation is formed at the beginning of the period under consideration. If the investor maximizes a function of the expected value and variance of her end-of-period wealth, then Frankel and Engel (1984) show that

$$Ez = \beta \text{cov}(z, \rho^\text{us}) + \beta \Omega x. \quad (10)$$

We now use equation (10) to derive quantitative estimates of $Ez^A$, the expected excess real return on the Australian asset over the US asset, or equivalently, the real risk premium, $rp^R$ from equation (2), which a US investor requires to hold a fraction $x^A$ of her portfolio in Australian bills. The first term on the rhs of equation (10) is zero if it is assumed that the price index $P_{t+1}^\text{us}$ is known at time $t$. Even if the price index is unknown, this term should be very small, because inflation rates are so much less

---

13 $z^A$, the Australian element of $z$ ($z^A = \rho^A - \rho^\text{us}$), satisfies $z^A = \delta.(1+i^\text{us}).P_t^\text{us}/P_{t+1}^\text{us}$, where $\delta$ is defined by equation (8).
volatile than exchange rates. Sample estimates of the unconditional covariances and variance needed to calculate $E z^A$ are provided in Table 3.\textsuperscript{14} Appendix D describes how the estimates are derived.

**TABLE 3**

**ESTIMATES OF VARIANCE AND COVARIANCES OVER FOUR WEEKS**

<table>
<thead>
<tr>
<th></th>
<th>Estimate 1</th>
<th>Estimate 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\text{var} (z^A)$</td>
<td>$11.9 \times 10^{-4}$</td>
<td>$3.23 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(2.55 \times 10^{-4})$</td>
<td>$(1.60 \times 10^{-4})$</td>
</tr>
<tr>
<td>$\text{cov} (z^A, p^\text{us})$</td>
<td>$0.249 \times 10^{-4}$</td>
<td>$2.76 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(0.183 \times 10^{-4})$</td>
<td>$(1.76 \times 10^{-4})$</td>
</tr>
<tr>
<td>$\text{cov} (z^A, z^Y)$</td>
<td>$2.58 \times 10^{-4}$</td>
<td>$-0.90 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(1.58 \times 10^{-4})$</td>
<td>$(0.37 \times 10^{-4})$</td>
</tr>
<tr>
<td>$\text{cov} (z^A, z^C)$</td>
<td>$2.58 \times 10^{-4}$</td>
<td>$-0.90 \times 10^{-4}$</td>
</tr>
<tr>
<td></td>
<td>$(1.58 \times 10^{-4})$</td>
<td>$(0.37 \times 10^{-4})$</td>
</tr>
</tbody>
</table>

To estimate $E z^A$, it is also necessary to have estimates of the portfolio share held outside the US. Remember that wealth, as defined here, is consumed at the end of the model. For the sake of realism, this wealth should include labour income, since this makes up a large proportion of average consumers' resources. Hence, for a representative consumer, we should expect that only a small proportion of wealth is available to be held as a portfolio of foreign assets.\textsuperscript{15} As an illustrative calculation, assume that the total portfolio share held in countries other than the US or Australia is 0.10, and that this share is divided in proportion to GNP (so that $x^Y = 0.053$, $x^\text{DM} = 0.024$, $x^E = 0.015$, $x^C = 0.008$).\textsuperscript{16} Substituting these

\textsuperscript{14} Again, it is the conditional moments which determine investor behaviour and the unconditional moments give an estimate of the average conditional moments over some extended period of time (a few weeks or a few months). See Appendix C.

\textsuperscript{15} Consumers may not be able to make large borrowings against future labour income and/or may have substantial financial commitments (e.g., borrowing for a house) which limit their capacity to invest in foreign assets.

\textsuperscript{16} Based on 87 GNP using average exchange rates in Dec 87.
numbers and the point estimates in Table 3 into equation (10), and again assuming $\beta = 2$ gives

$$\text{E} z^A = [0.128 + 3.09 x^A] \text{ } \% \text{ per annum.}$$

Thus, for example, to induce our investor to hold $x^A = 0.01$ of wealth in Australia (as well as 0.10 in other countries in the above proportions), a real return in Australia 0.16% p.a. higher than the real return in the US is required. The reader is invited to choose alternative portfolio shares, and evaluate the corresponding risk premia.

Finally, we estimate the increase in risk premium needed to induce our investor to raise the proportion of wealth held in Australia by $\Delta x^A$. It is no longer necessary to make specific assumptions about the portfolio shares held in other countries. Provided the increase in Australian holdings comes at the expense of assets held in the US, equation (10) implies

$$\Delta \text{E} z^A = 3.09 \Delta x^A \text{ } \% \text{ per annum.}^{17}$$

To induce the representative consumer to put an extra 0.01 (that is, 1%) of wealth in Australia, an extra average return in Australia of 0.031% p.a. is required. At the risk of appearing uncontroversial, this number seems rather small to us.

5.4 INFLATION AND INTEREST RATES

Figures 5 and 6 display, respectively, short-term nominal interest rates and 12 month ended CPI inflation rates for seven OECD countries over the period 1975 – 1989.\textsuperscript{18} The expected short-term real interest rate differential between each country $j$ and the US, $r^j_{\text{diff}}$, is given by\textsuperscript{19}

\begin{equation}
\end{equation}

\textsuperscript{17} With the minor exception of Canada, if the rise in Australian holdings is at the expense of other foreign holdings, the increase in risk premium is even smaller than equation (11) because of the positive values of $\text{cov}(z^A, z^j)$, $j \neq C$.

\textsuperscript{18} The interest rates are 3 month Treasury bill rates for Australia, US and Canada; the 6 month Treasury bill rate for Italy, and 3 month Eurocurrency rates for Japan, West Germany and United Kingdom. For Australia, the CPI numbers are quarterly, while for all other countries,
where $\pi_j$ is expected inflation in country $j$. Figure 7 shows this differential using CPI inflation over the previous 12 months for $\pi_j$.\(^{20}\)

Over the 59 months Nov 84 –Sept 89, this measure of the short-term real interest differential between Australia and the US was positive for all but four months, and averaged +2.6% p.a. An alternative measure of expected real interest rates can be derived using the realized inflation rate during the ex post 3 month period as a proxy for expected inflation. By this measure, over the nineteen quarters from Dec Q 84 to Jun Q 89 the short-term real interest differential between Australia and the US was positive for sixteen quarters, and averaged +2.4% p.a.\(^{21}\)

---

19 The relationship is approximate because $E (1 + \pi)^{-1} = (1 + E\pi)^{-1}$.
20 To compare like with like, we use the 3 month US Eurocurrency interest rate for $i_{US}$ when evaluating the real interest differential for Japan, West Germany and United Kingdom.
21 Estimated in the same way over the same period, the real interest differential on 10 year government bonds between Australia and the US averaged – 0.6% p.a. Since we have some evidence for a risk premium in 84-85, but not since then, it is interesting to evaluate the ex post real interest rate differential between Australia and the US over the fourteen quarters Mar Q 86 to Jun Q 89. It averaged 2.8% p.a. for 3 month Treasury bills and 0.2% p.a. for 10 year bonds. McKibbin and Morling (1989) examine alternative measures of the real interest rate differential for 90 day bank bills between Australia and US. From Dec Q 84 to Mar Q 89, using the quarterly change in the GDP (GNP) deflator for Australia (US) to estimate expected inflation, gives an average real interest rate differential of 2.3% p.a. Changing to a forecasting equation to estimate expected quarterly Australian GDP inflation (CPI inflation) leads to an average real interest differential over the period of 1.7% p.a. (2.5% p.a.). Finally, using a forecasting equation to estimate quarterly US GNP inflation and the forecasting equation to estimate expected quarterly Australian GDP inflation (CPI inflation) leads to an average real interest differential over the period of 1.9% p.a. (2.7% p.a.) – Steve Morling, personal communication.
Frankel and MacArthur (1988) used this approach to measure the 3 month real interest differential between 24 countries and the US over the period Sept 82–Oct 86. Excluding the four closed economies in their sample (Bahrain, Greece, Mexico and South Africa), four of the remaining twenty countries (Austria, Denmark, Hong Kong and Switzerland) have an average real interest differential more than 2% p.a. below the US, while none have a real interest differential more than 2% p.a. above the US. As Frankel and MacArthur point out, this asymmetry occurs because of the high real interest rates in the US over this period. But the US continues to experience fairly high real interest rates. Hence, this comparison with Frankel and MacArthur emphasises how unusually high Australian short term real interest rates have been since late 84.

A different perspective is provided by Table 4 which shows the excess nominal return a US investor would have achieved by investing (and continually rolling over the investment) in Australian 3 month Treasury bills rather than US bills. Of course, the results include the exchange rate change which occurs between the purchase and the sale of the bills. Over thirty years, from Jan 60 to Dec 89, the average excess return from investing in Australian short-term bills has been negligible (0.4% p.a.). But the Australian real interest premium since late 84 would have returned a US investor a nominal excess return of 6.3% p.a. on an

---

22 The ex post real US interest rate on 3 month Treasury bills averaged 5.4% p.a. from Sept 82 to Oct 86, 3.3% p.a. from Oct 84 to Jun 89, compared to an average of 2.0% p.a. from Jan 75 to Jun 89.

23 We are grateful to George Fane for suggesting this Table. It is constructed using end year exchange rates and annual average yields on 3 month Treasury notes from Tables S.7 and S.11 in Norton and Aylmer (1988) and from more recent RBA Bulletins.
investment from Jan 85 to Dec 89 – because over this time the $A experienced real appreciation against the $US at an average rate of about 3% p.a. Even from Jan 84 – which includes virtually the whole period of the $A float – the excess nominal return has averaged 3.9% p.a. as over
this time the $A appreciated against the $US at an average real rate of about 1% p.a.
Figure 7

Real Interest Rate Differential (cf U.S.)

%  

- Australia  

- West Germany  

- Japan  

%  

- United Kingdom  

- Canada  

- Italy  

Jan-75 Jan-77 Jan-79 Jan-81 Jan-83 Jan-85 Jan-87 Jan-89
# TABLE 4

Nominal excess return from investing in 3 month Australian Treasury bills rather than 3 month U.S. Treasury bills (expressed as % p.a.)

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
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<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1964</td>
<td>0.6</td>
<td>-0.5</td>
<td>3.4</td>
<td>-2.2</td>
<td>-4.6</td>
<td>-7.1</td>
<td>-11.7</td>
<td>6.3</td>
<td>15.9</td>
<td>24.5</td>
<td>0.1</td>
</tr>
<tr>
<td>1969</td>
<td>0.0</td>
<td>1.4</td>
<td>0.6</td>
<td>-3.4</td>
<td>-5.8</td>
<td>-9.4</td>
<td>2.8</td>
<td>11.0</td>
<td>20.1</td>
<td>11.6</td>
<td></td>
</tr>
<tr>
<td>1974</td>
<td>1.1</td>
<td>0.2</td>
<td>0.6</td>
<td>-4.2</td>
<td>-4.2</td>
<td>-4.5</td>
<td>3.1</td>
<td>11.0</td>
<td>15.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1979</td>
<td>0.3</td>
<td>0.2</td>
<td>0.6</td>
<td>-3.4</td>
<td>-1.9</td>
<td>-4.5</td>
<td>11.0</td>
<td>15.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1980</td>
<td>-0.7</td>
<td>-1.0</td>
<td>-1.2</td>
<td>-2.0</td>
<td>-1.9</td>
<td>0.3</td>
<td>2.8</td>
<td>7.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1984</td>
<td>-1.2</td>
<td>-1.6</td>
<td>-1.4</td>
<td>-2.0</td>
<td>0.8</td>
<td>4.7</td>
<td>7.9</td>
<td>6.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1985</td>
<td>-0.9</td>
<td>-1.2</td>
<td>-0.5</td>
<td>-0.3</td>
<td>0.3</td>
<td>2.8</td>
<td>7.9</td>
<td>6.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1986</td>
<td>-0.3</td>
<td>-0.5</td>
<td>0.7</td>
<td>-0.3</td>
<td>0.7</td>
<td>4.7</td>
<td>7.9</td>
<td>6.3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1987</td>
<td>0.4</td>
<td>0.4</td>
<td>0.6</td>
<td>-0.3</td>
<td>0.7</td>
<td>3.9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1988</td>
<td>0.4</td>
<td>0.4</td>
<td>0.6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1989</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
This section provides evidence about the skewness of the $A against a range of currencies and against the trade-weighted index (TWI). There have been many studies examining the skewness of exchange rate returns over very short horizons (a day or less – see for example, Bewley et.al., (1987)). By contrast, our analysis examines skewness over a week and four weeks because we are interested in an horizon relevant to those investing in Australian assets for substantially longer than a day. In all cases we examine the behaviour of the random variable $D_i [a/b]$, defined as

$$ D_i [a/b] = 100 \times ( s_{t+i} [a/b] - s_t [a/b] ). $$

(12)

There are two reasons for studying $D_i [a/b]$. Firstly, defining $\Delta_i S_t [a/b] = S_{t+i} [a/b] - S_t [a/b]$, it follows that

$$ D_i [a/b] = 100 \times \ln \left( \frac{1 + \Delta_i S_t [a/b]}{S_t [a/b]} \right) = 100 \times \frac{\Delta_i S_t [a/b]}{S_t [a/b]}, $$

provided $\Delta_i S_t / S_t$ is small compared to 1. Hence, $D_i [a/b]$ is approximately the percentage return from converting currency a into currency b for $i$ periods. Secondly, by definition, $D_i [b/a] = -D_i [a/b]$ and so $-D_i [a/b]$ is approximately the percentage return from converting currency b into currency a for $i$ periods.

**TABLE 5: THIRD CENTRAL MOMENT OF $D_i [a/$A], i = ONE WEEK**

<table>
<thead>
<tr>
<th>Currency a</th>
<th>$US</th>
<th>¥</th>
<th>£</th>
<th>DM</th>
<th>$C</th>
<th>TWI</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Period</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Jan 79 – Dec 83</td>
<td>-3.9</td>
<td>-4.5</td>
<td>-3.3</td>
<td>-5.9</td>
<td>-3.9</td>
<td>-3.3</td>
</tr>
<tr>
<td>Jan 84 – Apr 89</td>
<td>-6.1**</td>
<td>-9.8**</td>
<td>-8.8*</td>
<td>-8.7**</td>
<td>-5.3**</td>
<td>-6.2**</td>
</tr>
<tr>
<td>Jan 86 – Apr 89</td>
<td>-5.6*</td>
<td>-12**</td>
<td>-7.6</td>
<td>-12*</td>
<td>na</td>
<td>-6.4*</td>
</tr>
</tbody>
</table>

* Periods 79 – 83 and 84 – 89: From dataset A. Period 86 – 89: From dataset B.

* Different from zero at 10% level of significance

** Different from zero at 5% level of significance
TABLE 6

THIRD CENTRAL MOMENT OF $D_i \ [a / $A]\$
Dataset A: weekly (Friday) data, Jan 84 – Apr 89.

<table>
<thead>
<tr>
<th>Period</th>
<th>$US</th>
<th>£</th>
<th>DM</th>
<th>TWI</th>
</tr>
</thead>
<tbody>
<tr>
<td>One week</td>
<td>-6.11**</td>
<td>-8.83*</td>
<td>-8.74**</td>
<td>-6.22**</td>
</tr>
<tr>
<td></td>
<td>(2.49)</td>
<td>(4.92)</td>
<td>(4.39)</td>
<td>(2.87)</td>
</tr>
<tr>
<td>Two weeks</td>
<td>-10.85*</td>
<td>-16.99</td>
<td>-16.93</td>
<td>-13.75**</td>
</tr>
<tr>
<td></td>
<td>(6.52)</td>
<td>(12.02)</td>
<td>(11.92)</td>
<td>(6.23)</td>
</tr>
<tr>
<td>Four weeks</td>
<td>-40.10</td>
<td>-88.74</td>
<td>-69.89</td>
<td>-39.05*</td>
</tr>
<tr>
<td></td>
<td>(31.70)</td>
<td>(69.64)</td>
<td>(48.16)</td>
<td>(22.22)</td>
</tr>
</tbody>
</table>

THIRD CENTRAL MOMENT OF $D_i \ [a / $A]\$
Dataset B: daily data, Jan 86 – Apr 89.

<table>
<thead>
<tr>
<th>Period</th>
<th>$US</th>
<th>£</th>
<th>DM</th>
<th>TWI</th>
</tr>
</thead>
<tbody>
<tr>
<td>One week</td>
<td>-5.57*</td>
<td>-7.63</td>
<td>-11.67*</td>
<td>-6.42*</td>
</tr>
<tr>
<td></td>
<td>(2.96)</td>
<td>(4.94)</td>
<td>(6.29)</td>
<td>(3.42)</td>
</tr>
<tr>
<td>Two weeks</td>
<td>-15.25*</td>
<td>-25.67</td>
<td>-38.51*</td>
<td>-19.19*</td>
</tr>
<tr>
<td></td>
<td>(8.67)</td>
<td>(18.46)</td>
<td>(22.52)</td>
<td>(10.59)</td>
</tr>
<tr>
<td>Month (22 working days)</td>
<td>-40.61</td>
<td>-51.69</td>
<td>-114.17</td>
<td>-54.97</td>
</tr>
<tr>
<td></td>
<td>(32.52)</td>
<td>(60.64)</td>
<td>(87.27)</td>
<td>(38.64)</td>
</tr>
</tbody>
</table>

Standard errors in brackets
*
Significantly different from zero at the 10% level
**
Significantly different from zero at the 5% level
### Table 7

**CROSS COUNTRY COMPARISON OF TCM OF D<sub>i</sub> [a / b], i = ONE WEEK**

<table>
<thead>
<tr>
<th>Currency b</th>
<th>$A</th>
<th>$US</th>
<th>¥</th>
<th>£</th>
<th>DM</th>
<th>$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A</td>
<td>•</td>
<td>-6.1**</td>
<td>-9.8**</td>
<td>-8.8*</td>
<td>-8.7**</td>
<td>-5.3**</td>
</tr>
<tr>
<td>$US</td>
<td>6.1**</td>
<td>•</td>
<td>-1.9</td>
<td>-2.0</td>
<td>-1.4</td>
<td>0.0</td>
</tr>
<tr>
<td>¥</td>
<td>9.8**</td>
<td>1.9</td>
<td>•</td>
<td>0.7</td>
<td>0.0</td>
<td>1.6</td>
</tr>
<tr>
<td>£</td>
<td>8.8*</td>
<td>2.0</td>
<td>-0.7</td>
<td>•</td>
<td>-0.3</td>
<td>1.4</td>
</tr>
<tr>
<td>DM</td>
<td>8.7**</td>
<td>1.4</td>
<td>0.0</td>
<td>0.3</td>
<td>•</td>
<td>1.0</td>
</tr>
<tr>
<td>$C</td>
<td>5.3**</td>
<td>0.0</td>
<td>-1.6</td>
<td>-1.4</td>
<td>-1.0</td>
<td>•</td>
</tr>
</tbody>
</table>

**CROSS COUNTRY COMPARISON OF TCM OF D<sub>i</sub> [a / b], i = FOUR WEEKS**

<table>
<thead>
<tr>
<th>Currency b</th>
<th>$A</th>
<th>$US</th>
<th>¥</th>
<th>£</th>
<th>DM</th>
<th>$C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A</td>
<td>•</td>
<td>-40</td>
<td>-79*</td>
<td>-89</td>
<td>-70+</td>
<td>-22</td>
</tr>
<tr>
<td>$US</td>
<td>40</td>
<td>•</td>
<td>-23+</td>
<td>-27</td>
<td>-10</td>
<td>0.9</td>
</tr>
<tr>
<td>¥</td>
<td>79*</td>
<td>23+</td>
<td>•</td>
<td>-1.2</td>
<td>-0.3</td>
<td>20</td>
</tr>
<tr>
<td>£</td>
<td>89</td>
<td>27</td>
<td>1.2</td>
<td>•</td>
<td>-3.4</td>
<td>18</td>
</tr>
<tr>
<td>DM</td>
<td>70+</td>
<td>10</td>
<td>0.3</td>
<td>3.4</td>
<td>•</td>
<td>6.8</td>
</tr>
<tr>
<td>$C</td>
<td>22</td>
<td>-0.9</td>
<td>-20</td>
<td>-18</td>
<td>-6.8</td>
<td>•</td>
</tr>
</tbody>
</table>

Dataset A: weekly (Friday) data, Jan 84 – Apr 89.

+ Significantly different from zero at 20% level
  (only used for four-weekly results)

* Significantly different from zero at the 10% level

** Significantly different from zero at the 5% level
TABLE 8

CROSS COUNTRY COMPARISON OF TCM OF $D_i [a / b]$, 
i = ONE WEEK (FIVE WORKING DAYS)

<table>
<thead>
<tr>
<th>Currency b</th>
<th>Currency a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
</tr>
<tr>
<td>$A$</td>
<td>•</td>
</tr>
<tr>
<td>$US$</td>
<td>5.6*</td>
</tr>
<tr>
<td>¥</td>
<td>12**</td>
</tr>
<tr>
<td>£</td>
<td>7.6</td>
</tr>
<tr>
<td>DM</td>
<td>12*</td>
</tr>
</tbody>
</table>

Dataset B: daily data, Jan 86 – Apr 89.
+ Significantly different from zero at 20% level
* Significantly different from zero at the 10% level
** Significantly different from zero at the 5% level

CROSS COUNTRY COMPARISON OF TCM OF $D_i [a / b]$, 
i = ONE MONTH (TWENTY TWO WORKING DAYS)

<table>
<thead>
<tr>
<th>Currency b</th>
<th>Currency a</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$A$</td>
</tr>
<tr>
<td>$A$</td>
<td>•</td>
</tr>
<tr>
<td>$US$</td>
<td>41</td>
</tr>
<tr>
<td>¥</td>
<td>120+</td>
</tr>
<tr>
<td>£</td>
<td>52</td>
</tr>
<tr>
<td>DM</td>
<td>114+</td>
</tr>
</tbody>
</table>
TABLE 9

NON-PARAMETRIC TEST OF SKEWNESS OF \( D_i \) \([a / b]\),
\( i = \text{ONE WEEK (FIVE WORKING DAYS)}\)

<table>
<thead>
<tr>
<th>Currency b</th>
<th>( $A )</th>
<th>( $US )</th>
<th>( \¥ )</th>
<th>( \£ )</th>
<th>( \text{DM} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( $A )</td>
<td>•</td>
<td>((-)) 5**</td>
<td>((-)) 3**</td>
<td>((-)) 2*</td>
<td>((-)) 3**</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2*</td>
<td>1*</td>
</tr>
<tr>
<td>( $US )</td>
<td>(+) 5**</td>
<td>•</td>
<td>((-)) 1*</td>
<td>0</td>
<td>((-)) 1*</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>(+) 1*</td>
<td></td>
</tr>
<tr>
<td>( \¥ )</td>
<td>(+) 3**</td>
<td>(+) 1*</td>
<td>•</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>2*</td>
<td></td>
</tr>
<tr>
<td>( \£ )</td>
<td>(+) 2*</td>
<td>0</td>
<td>0</td>
<td>•</td>
<td>0</td>
</tr>
<tr>
<td>( \text{DM} )</td>
<td>(+) 3**</td>
<td>(+) 1*</td>
<td>0</td>
<td>0</td>
<td>•</td>
</tr>
</tbody>
</table>

Dataset B: daily data, Jan 86 – Apr 89.

Results of one-sided non-parametric test described in Appendix E. The sign (+) [(-)] indicates that significant positive [negative] skewness is found. The number indicates the number of working days of the week which exhibit significant skewness. The stars indicate the level of significance: * significant at 5% level; ** significant at 1% level.

In Tables 5 – 8, we examine the sample values of the third central moment \((tcm)\) of \( D_i \) \([a/b]\), i.e., \( E \{ D_i \} - E D_i \) \([a/b]\)^3, for a range of currencies and time periods. Table 5 demonstrates that for weekly changes, the \( \$A \) may have been negatively skewed against most currencies since 1979, but that this skewness has become much larger
and statistically significant since the $A was floated in Dec 83. Since Jan 86, this skewness has remained large (and may have become larger – see the comparison of two week changes in Table 6).

Tables 7 and 8 show that the $A is much more skewed than other currencies. This is clear both from the size of the point estimates of the third central moment, and from their level of significance. These conclusions are supported by the results of a non-parametric test shown in Table 9. This test, which is described in Appendix E, has the advantage that under the null hypothesis, it is not necessary to assume that $D_i [a/b] is normally distributed. The test is also approximately valid when the observations of $D_i [a/b] come from different distributions.

Figures 8 and 9 show the distributions of $D_i [TWI/$A], $D_i [¥/$A] and $D_i [¥/$US] for $i = one week, and for $i = four weeks since Jan 86. In Figure 9, $D_i [¥/$US] is chosen because it is one of the most skewed distributions which does not involve the $A (see Table 8). In both Figures, the significant proportion of big depreciations of the $A is clearly not matched by an equal proportion of big appreciations.

---

24 In common with others (e.g., Bewley et. al. 1987), we find evidence of leptokurtosis (which should be a disease, but in fact means a distribution with a larger fourth central moment than the normal distribution with the same variance). We find leptokurtosis in weekly log changes of the $A against most currencies since the float but not before it.

25 The distributions are derived by a non-parametric technique kindly suggested to us by Adrian Pagan. Given observations $x_i, i = 1,...,N, from an unknown density function, f(x), the density function $f(x*) at any point $x* is estimated by

$$f(x*) = \frac{1}{\sqrt{2\pi} \ N h} \exp \left( -\frac{1}{2} \left[ \frac{x_i - x^*}{h} \right]^2 \right)$$

where $h = \sigma_x, N^{-1/5} and \sigma_x is the standard error of the observations $x_i, i = 1,...,N.
Figure 8

Distribution of weekly and four weekly returns on the $A against the TWI

In both cases, the normal distribution has the same mean and variance as the actual distribution. Results are derived from dataset B: daily rates Jan 86 – Apr 89.
Figure 9

Distribution of weekly and four weekly returns: ¥/$A and ¥/$US

The ¥/$US distributions have been shifted by +0.08% and +0.38% respectively, so that their means coincide with the means of the ¥/$A distributions. Results are derived from dataset B: daily rates Jan 86 – Apr 89.
TABLE 10

Analysis of the ten largest weekly falls (Friday to Friday) of $A against the TWI, Jan 86 to Apr 89 using Dataset B.

<table>
<thead>
<tr>
<th>Week</th>
<th>%</th>
<th>Reason – judged by reports during the week in the <em>Australian Financial Review.</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>31 Jan - 7 Feb 86</td>
<td>3.2</td>
<td>No obvious event.</td>
</tr>
<tr>
<td>30 May - 6 June 86</td>
<td>4.6</td>
<td>No obvious event. &quot;US investors are beginning to get nervous about the magnitude of the economic problems facing the Australian Government.&quot; Lead article, 5 June. This article is almost exclusively about adjustment to the external trade imbalance.</td>
</tr>
<tr>
<td>20 - 27 June 86</td>
<td>3.6</td>
<td>On 24 June, <em>The Wall Street Journal</em> editorial page asks: &quot;Will Australia become the next Banana Republic?&quot;</td>
</tr>
<tr>
<td>27 June - 4 July 86</td>
<td>5.0</td>
<td>Rumours that Mr. Keating had resigned as Treasurer. Removal of exemptions for withholding tax. Push by unions for wider superannuation coverage. Waterside Workers nationwide strike (resolved on 3 July).</td>
</tr>
<tr>
<td>18 - 25 July 86</td>
<td>4.5</td>
<td>Unexpectedly large June quarter CPI figure announced (up 1.7%).</td>
</tr>
<tr>
<td>15 - 22 Aug 86</td>
<td>3.1</td>
<td>Unexpectedly large July current account deficit announced ($1.35bn).</td>
</tr>
<tr>
<td>9 - 16 Jan 87</td>
<td>4.2</td>
<td>No obvious event. &quot;Sudden change of sentiment in foreign exchange market.&quot; Front page, 14 Jan. Perhaps related to the EMS realignment occurring at the time.</td>
</tr>
<tr>
<td>23 - 30 Oct 87</td>
<td>6.7</td>
<td>Delayed reaction to stock market crash (on 19, 20 Oct). &quot;The world stockmarket crash has ...[left] Australia exposed because of its high foreign debt burden and dependence on commodity exports.&quot; (30 Oct)</td>
</tr>
<tr>
<td>24 June - 1 July 88</td>
<td>2.9</td>
<td>Global strength of $US</td>
</tr>
<tr>
<td>10 - 17 Feb 89</td>
<td>6.8</td>
<td>Unexpectedly large Jan current account deficit announced ($1.54bn) &quot;The dollar is now diminishing our competitiveness. When demand conditions moderate I expect, and indeed hope, that the dollar will fall. And certainly, the day that starts we will not be standing in the way of stopping it.&quot; Mr. Keating, 16 Feb.</td>
</tr>
</tbody>
</table>

The data consists of 170 weekly changes. The median change is 0.19%. Of the ten largest deviations from this median (values of $y_j$ from Appendix E), all ten are falls. They form the basis for this Table.
In this section, we have focussed our attention particularly on the period since Jan 86. Since then, most of the exogenous "news" from Australia's terms of trade has been favourable. The terms of trade had already fallen from 100.6 in Mar Q 85 to 91.9 in Dec Q 85. In 86, the fall continued at a slower rate to a low of 87.0 in Mar Q 87, then climbed rapidly to 113.4 by Mar Q 89 (from RBA Bulletin, Table K.3, Dec 88 and Jun 89). So, it is hard to sustain the argument that since Jan 86 Australia's external sector has suffered a series of unfavourable exogenous shocks which have required a succession of falls in the $A.\text{26}

Table 10 \text{27} provides evidence on the possible causes of the ten largest weekly falls of the $A$ from Jan 86 to Apr 89.\text{28} The large fall which followed the 1987 stockmarket crash suggests that the exchange rate is (sometimes) very sensitive to external shocks. Most of the other events which were identified seem related to the unexpectedly slow progress being made by the real economy to stabilize the ratio of net external

\text{26} Of course, one could claim that the outcome was unfavourable compared to what was expected. But for the outcome to have appeared unfavourable, the expectations must have been for a massive improvement in the terms of trade. We are not aware of any such optimistic expectations.

\text{27} We use an Australian financial newspaper for Table 10 rather than a foreign one because Wong (1988) finds that most of the movements in the $A/$US exchange rate occur while the foreign exchange markets are open in Australia.

\text{28} As Frank Milne pointed out to us, the Table suffers from an important weakness. Rather than examining all events of a particular kind (e.g. all current account announcements) to test whether, on average, unexpected outcomes have an impact on the exchange rate, we search for the 'causes' of the biggest falls, \textit{ex post}. We did use Dataset B to examine the percentage change in the $A$ against the TWI from the day before monthly current account announcements to four working days after \((s_t+4 - s_{t-1})\) where \(t\) is the day of the release). We find a correlation coefficient between \((s_t+4 - s_{t-1})\) and the nominal $A$ value of the announced current account deficit, of -0.39. Surprisingly, we find an average value of \((s_t+4 - s_{t-1})\) of 0.27% over the 39 announcements in the sample, compared to a mean weekly change for the whole sample of 0.006%. So, on average, during a week which included a current account announcement, the $A$ appreciated substantially more than during an arbitrarily chosen week.
liabilities to GDP. This comment applies most obviously to the second and third events as well as to the two current account announcements. We return to this theme in the final section of the chapter.

5.6 DISCUSSION

Despite continuous repetition of the claim that the efficient markets hypothesis implies that exchange rates should move as random walks with no drift, the claim is false. In fact, the joint hypothesis that (i) market-participants are risk-neutral, (ii) transactions costs are small enough to be ignored and (iii) the market is efficient (or equivalently, market-participants form and use rational expectations) implies that the exchange rate must undergo a random walk around the forward rate. As we have seen, there is now very strong international evidence — supported by our analysis of the $US/$A market — that exchange rates do not do this. There are three possible interpretations of this evidence. Firstly, there could be a time-varying risk premium which investors demand to hold, say, $A nominal assets. Secondly, the market could be inefficient. Thirdly, there could be a 'peso problem'. We examine each of these interpretations in turn. For convenience, we repeat the arbitrage conditions introduced at the beginning of the chapter which should be satisfied by Australian interest rates:

\[ i^A = i^{US} - E(\Delta S / S) + \rho_p \]  

or  

\[ E r^A = E r^{US} - E(\Delta S^R / S^R) + \rho_R. \]  

29 The net liabilities/GDP ratio will increase (potentially without bound) if net exports are in deficit (as they are in Australia) and if the effective nominal interest rate on the liabilities exceeds the nominal growth rate of GDP (which, at least for $A-denominated debt, it does). At present, the real economy is on an unsustainable external liabilities/GDP path.

30 This should be true for large enough transactions. Goodhart and Taylor (1987) provide detailed estimates of transactions costs in the London and Chicago futures markets.

31 To be precise, the three conditions imply that the forward rate must be an unbiased predictor of the future spot rate or, equivalently, that \( s_{t+k} - f_t \) must be a martingale.
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\[
\begin{align*}
    i^A & = i^{US} - E\left(\frac{\Delta S}{S}\right) + rp \\
    \text{or} \quad E r^A & = E r^{US} - E\left(\frac{\Delta S^R}{S^R}\right) + rp^R.
\end{align*}
\]

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\[ i_A = i_{US} - E(\Delta S/S) + rp \]

or

\[ E r_A = E r_{US} - E(\Delta S^R/S^R) + rp^R. \]  

(1)

(2)

29 The net liabilities/GDP ratio will increase (potentially without bound) if net exports are in deficit (as they are in Australia) and if the effective nominal interest rate on the liabilities exceeds the nominal growth rate of GDP (which, at least for $A-denominated debt, it does). At present, the real economy is on an unsustainable external liabilities/GDP path.

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31 To be precise, the three conditions imply that the forward rate must be an unbiased predictor of the future spot rate or, equivalently, that \( s_t + k - f_{t+k} \) must be a martingale.
A time-varying risk premium

Both our evidence, and the more extensive survey evidence of Thorpe et. al. (1988) suggest that there was a statistically significant gap between the forward discount and market expectations of depreciation – and hence a risk premium – from late 84 to late 85. Since then, the survey data leads us both to accept the null hypothesis of zero risk premium. Section 5.3 presents two a priori calculations of the average magnitude of the risk premium necessary to induce a US consumer-investor to hold a small part of her wealth in Australian nominal assets. Within the chosen framework the calculations are as realistic as we could make them, but in some respects they are pretty naive.\footnote{32}{Investors are assumed to be homogeneous with a constant relative risk-aversion and to do their intertemporal optimization one period at a time.} Be that as it may, these theoretically-based estimates of the average risk premium are so small as to be negligible (compared, for example, with the average real interest differentials between Australia and the US derived in section 5.4).\footnote{33}{The average risk premium is also negligible compared to the average after-tax real interest differential between Australia and the US (see Appendix A).} These calculations are consistent with the fairly well established inability of current models of time-varying risk premia to account for interest rate differences between countries with essentially no impediments to the movement of capital (e.g., Hodrick, 1987; Cumby, 1988).

An inefficient foreign exchange market

Consider the Dornbusch (1976) model of a small open economy with perfect capital mobility, rational market participants, sticky goods prices and no risk premia.\footnote{34}{This seminal model forms the basis of most modern open-economy macro-models including, in the Australian context, the Murphy (1988) model and the MSG2 (McKibbin and Elliott, 1989 and McKibbin and Sachs, 1989) model.} In this model, an unanticipated increase in...
domestic nominal and real interest rates (i.e., a tightening of monetary policy in the small economy) leads to a market anticipation that in the long-run, domestic inflation will be lower than previously expected. As a direct consequence, in the long-run, the domestic nominal exchange rate will be higher than previously expected, while the long-run real exchange rate will be unchanged. Immediately the tightening is recognised by the market, the domestic exchange rate jumps up – overshooting its long-run nominal appreciation. This jump is necessary so that, during adjustment to the long-run, the exchange rate depreciates (in both real and nominal terms) at exactly the rate necessary to equate the return on domestic and world short-term nominal assets. If there are repeated shocks, they cause repeated jumps in the nominal exchange rate, but during periods in which there is no relevant new information, the return on short-term domestic nominal assets is the same as it is on foreign ones.

Unfortunately, the world does not seem to work like this model. Apparently unanticipated changes in domestic interest rates do not lead to jumps in the exchange rate (Goodhart, 1988). Tight domestic monetary policy does not lead to an adjustment path for the exchange rate like that described in the previous paragraph.35 It is an oft repeated claim – and it seems to be true – that the domestic real (and nominal) exchange rate of an open economy tends to be held up during periods when the domestic

35 In discussing the fact that the long-term real interest differential between the US and its trading partners increased by about 5 percentage points from 1980 to mid-1984, and the real appreciation of the $US from 1980 to 1985, Dornbusch and Frankel (1987) comment: [the overshooting model implies that] "the entire increase in ... the value of the dollar should have occurred in one (or two or three) big jumps, for example when it was discovered that monetary policy was going to be tighter than previously expected, or fiscal policy looser. Yet the appreciation in fact took place month-by-month, over four years (with investor expectations, as reflected in the forward discount, interest differential or survey data, all forecasting a depreciation)."
real interest rate is higher than the rest of the world.\textsuperscript{36} What is rarely remarked is that this claim strongly suggests that the foreign exchange market is inefficient. This follows because, provided risk premia are small (which, at least since late 85, all our evidence suggests they are), the expectation that the exchange rate will be held up combined with high domestic real interest rates should lead to a flood of foreign demands for domestic interest-bearing assets – setting off the adjustment process described in the previous paragraph. In the real world, instead of a flood, there is a trickle, which, on average, slowly appreciates the domestic currency.\textsuperscript{37} \textbf{As a consequence, for an extended period (many months) there is the opportunity for substantial gain (with what seems an associated small risk) but this opportunity is seized by comparatively few.}

These observations can be rationalized by the evidence of Froot and Frankel (1989) that survey expectations follow the forward rate. Because of covered interest parity, when domestic nominal interest rates are higher than world nominal interest rates, the forward rate predicts depreciation of the domestic currency (at a rate which, if it were realised, would equalise the yield from domestic and foreign assets). For market participants who use the forward rate as an ‘anchor’ for their expectations of the future spot rate,\textsuperscript{38} the gain from higher domestic

\textsuperscript{36} As an example of this it is clear from Table 4 that there has been a substantial return from holding short-term $A nominal assets during the extended period of high real interest rates on these assets since late 84 (or late 85).

\textsuperscript{37} Given covered interest parity, regressions of equation (3) support this statement. Both the regressions in Table 1, as well as most of those quoted by Goodhart (1988), Obstfeld (1988), Thorpe et. al. (1988) and many others, find negative estimates of $\beta$ – although they are usually insignificantly different from zero. These regressions suggest that a higher domestic nominal interest rate leads, on average over the next month or three months, to a higher (or perhaps unchanged) domestic nominal exchange rate than would otherwise have been the case. See also Meese and Rogoff (1988).

\textsuperscript{38} See Chapter One for a discussion of ‘anchoring’. Expectations formed by such a process are not rational in an economist’s sense.
nominal interest rates is completely offset by an expected depreciation. This may explain why there is no massive foreign demand for domestic nominal assets when domestic interest rates are high. **But if that is so, what remains is the substantial puzzle that the actual behaviour of exchange rates does not alter market expectations.**

*A peso problem* 39

A peso problem is often invoked in defence of the hypothesis of rational market expectations, but our comments in this sub-section also apply if the market’s expectations are not rational.

We have some evidence that the $A does suffer from a peso problem. Firstly, our survey expectations data from Mar 85 (Nov 85) to Sept 87 imply that, over this period, market participants expected a real depreciation of the $A at an average rate of 2.4% p.a. (4% p.a.) while the actual rate appreciated. Secondly, since late 84 (late 85), Australian ex post short-term real interest rates have been at an average premium of 2.4% p.a. (2.8% p.a.) compared to the US.

By themselves, these observations do not constitute overwhelming evidence that the $A suffers from a peso problem. They could be explained by the argument presented at the end of the last sub-section. 40

What makes a peso problem for Australia a distinct possibility are two further observations. Firstly, our skewness analysis demonstrates that, unlike all the other currencies we examined, over one week and four weeks the $A is subject to infrequent, unpredictable and large

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39 See Chapter One for a definition of this term.
40 With reference to that argument, the average magnitude of depreciation predicted by the Australian market participants is close to that predicted by the forward rate (see Figure 2 and associated discussion). Note however that we find that the inflation differential dominates the forward rate as an explanator of market expectations (equation 6).
depreciations. So it seems reasonable that the market should have such events built into their expectations. Secondly, there is an obvious candidate for the cause of a peso problem – that in the longer run the real economy must adjust to put Australia on a sustainable net external liabilities/GDP path – with a lower real (and nominal) exchange rate during the adjustment process. Examining what appear to have been the causes of the ten largest weekly depreciations of the $A since Jan 86 (Table 10), seven appear interpretable in terms of events in the Australian economy (all but the first, seventh and ninth falls in the Table). Of these seven, five (the second, third, sixth, eighth and tenth) appear related to the need for the real economy to adjust to put us on a sustainable net external liabilities/GDP path. This evidence supports the view that if the $A suffers from a peso problem, its cause is a market perception of the need for a lower real exchange rate as part of the adjustment required to stabilize the ratio of Australia’s net external liabilities to GDP.41

We briefly deal with two questions suggested by this analysis. Firstly: if there is a widespread and long-held expectation that the real value of the $A will fall, why doesn’t it? The answer is probably that high short-term real interest rates have held it above the level it would otherwise have been.42

41 An alternative candidate for the cause of a peso problem is a market perception that there is a non-negligible chance that Australian inflation will dramatically accelerate in the future and depreciate the nominal exchange rate. A belief in the possibility of either accelerated monetary expansion or the collapse of the Prices and Incomes Accord could be the source of this expectation. With the benefit of hindsight, although Australian inflation over the past four years has been substantially higher than comparable countries (see Figure 6) it has shown few signs of acceleration. If inflationary expectations were the source of a peso problem, agents should have been continuously surprised that the event(s) they were anticipating did not eventuate, and presumably have revised their expectations. Only one of the events in Table 10 seems related to inflation (the CPI announcement).
42 Nevertheless, we remain impressed by the conclusions of Meese and Rogoff (1983, 1988) that macroeconomic fundamentals (and interest rate
Secondly: if a peso problem exists, what will solve it? From the end of May to the end of July 86, the $A fell 15% against the $US and 19% against the TWI. The survey of market participants (Figure 2) for the ten Fridays immediately following this depreciation show that, on average, they expected a depreciation of the $A of 1.03% over the next four weeks; equivalent to depreciation at an annual rate of 12.6%. This number is again substantially higher than the depreciation predicted by the inflation differential, 7.2% p.a., but closer to the annualized depreciation predicted by the 1 month forward rate over these ten weeks (10.8% p.a.).

This suggests that even a reasonably large depreciation may not be enough to influence expectations sufficiently to eliminate a peso problem. In principle, there should be a depreciation large enough to turn expectations around – but it is hard to know how large is sufficient. An alternative possibility is that the peso problem will not be eliminated until the real economy is clearly seen to be moving onto a sustainable path for net external liabilities / GDP.

Consequences for external imbalance

Finally, we comment on the relevance of these observations for the current debate on Australia’s external imbalance. For an extended period since late 85, market participants have expected the real value of the $A to fall against the $US at roughly 4% p.a. This may explain why a substantial real interest premium on short-term $A denominated assets differentials in particular) have little significant capacity to explain movements in the nominal or real exchange rate, even over periods as long as a year.

43 Derived using annual inflation rates which had been published when the expectations were formed – as for Figure 2.

44 Interestingly, market participants’ reaction to the previous rapid fall in the $A had been quite different. From the beginning of Feb to 8 Mar 85, the $A fell 16% against the $US and 13% against the TWI. Over the next ten Fridays, they predict an average appreciation of the $A at an annual rate of 8.8%. Perhaps the subsequent behaviour of the $A (and the terms of trade and current account deficit) changed their attitude.
can persist without setting off an adjustment of the Dornbusch (1976) type. If the market perceives significant real depreciation is necessary to put Australia onto a sustainable external liabilities/GDP path, these consequences follow. Firstly, while the monetary authorities keep short-term interest rates high, the Australian economy pays a real interest premium on the substantial proportion of short-term $A-denominated external debt. Secondly, when the monetary authorities reduce Australian short-term nominal interest rates, at some unpredictable time there may be a big fall in demand for Australian nominal assets and a large rapid depreciation. Presumably after a sufficiently large depreciation the market will cease to expect further real depreciation — but the depreciation required to change market expectations may be very large indeed. The hope is that such an abrupt exchange rate adjustment does not have serious adverse consequences for the wider economy.
DATA APPENDIX

Exchange rate data

**Dataset A: Weekly (Friday) exchange rates, 5 Jan 79 to 21 Apr 89.**

The spot rate, one month forward rate, and three month forward rate for the $A/$US market and the Australian trade-weighted index (TWI) are from I. P. Sharp Associates 'Australian Financial Markets Data Base'. The first three of these series are, in turn, from the Commonwealth Bank of Australia, and are the average of buy and sell rates at the close of trade on each Friday. The TWI is from the Reserve Bank of Australia (RBA) at 4 p.m. The spot rates $US/Y, $US/£, $US/DM, $US/$C are from I. P. Sharp Associates 'Currency Exchange Rates Data Base', and are the noon buying price in $US in New York. These data are from the Federal Reserve System, N.Y. Bank.

**Dataset B: Daily exchange rates, 1 Jan 86 to 11 Apr 89.**

Rates are the daily representative rates from the RBA for $US/$A, Y/$A, £/$A, DM/$A, and the TWI.

For both datasets, cross-rates are derived by dividing the appropriate rates (e.g., for dataset B, Y/£ is derived as \( \frac{Y/A}{£/A} \)).

**Survey data on exchange rate expectations**

"An attempt was made to contact the same individual each week, however if the usual respondent was unavailable then [foreign exchange] expectations would be elicited from an alternative forecaster. This method of survey guaranteed a high response rate." (Hunt, 1987). The sixteen companies in the survey were: A.N.Z. Bank, B.N.P., The Australian Bank, Barclays, B.A., B.T., Citicorp, Commonwealth Bank, Elders, Lloyds, Macquarie Bank, National Australia Bank, Rural and Industry Bank, Schroders, State Bank of N.S.W. and Westpac. Each Friday, we have the lowest, the highest, and the arithmetic mean
expectation of the sixteen participants. The expectations data runs from Friday 8 Mar 85 to Friday 18 Sept 87, with five missing weeks: leaving 128 weeks of data. The regressions and figures in this section require spot and forward rates for the $US/$A. We use the Friday close spot rate in the wholesale market, and the Friday one month forward average of buy and sell rates (quoted in the *Australian Financial Review* on the following Monday).

The data for section 5.4 comes from these sources: 3 month Treasury bill interest rates and 10 year bond rates for the US and Australia as well as Australian CPI, net external debt and terms of trade are from RBA Bulletins (various issues). Other short-term interest rates and foreign CPI are from IMF International Financial Statistics (IFS). To evaluate ex post 3 month real interest rates, we used the average yield on Australian 3 month Treasury bills for the last tender in each quarter (from RBA Bulletin), and the quoted yield on 3 month US Treasury bills on the last trading day of the quarter (from the New York Times) along with realised CPI inflation over the next 3 months.

**APPENDIX A**

In this appendix, we deal briefly with the important issue of tax. The arbitrage conditions given by equations (1) and (2) are valid for a representative US investor provided the tax treatment is the same for the income earnt on US nominal assets as it is for income earnt on Australian nominal assets. In general, the income earnt on Australian nominal assets consists of both interest and capital gain/loss. The tax treatment of capital gain is the same as nominal income, but a maximum of $US 3,000 per annum capital loss can be written off against other income for US tax-payers. For large investors, this may be a significant

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45 The information in this appendix was supplied by the Internal Revenue Service Section of the US Consulate in Sydney on 15 March, 1990.
distortion. However, in a diversified portfolio which includes nominal assets from several countries, a capital loss on holdings in one currency can be offset against capital gain on holdings in other currencies. This should reduce the distortion, though not eliminate it.

APPENDIX B

Here, technical details concerning the estimation of equation (3) are discussed. The results in Table 1 with \( k = \) four weeks use the one month forward rate to generate the variable \( f_{d,t,k} \), while the results with \( k = \) thirteen weeks use the three month forward rate. The fact that the forward rates are defined for a slightly different time length than the change in the spot rate makes minimal difference for our purposes. For example, for the first regression in Table 1, it amounts to ignoring the difference between \( s_{t+28} \) and \( s_{t+30} \) (with \( t \) measured in days). Viewed from time \( t \), \( s_{t+30} - s_{t+28} \) is very closely modelled as a random variable with zero mean, and so may be included in the error term in equation (3). This timing issue is, however, critical for alternative tests of the efficiency of the foreign exchange market (see Tease, 1988).

Recursive least squares regression of equation (3) with \( k = \) four weeks using data beginning on 6 Jan 84 shows that the point estimate of the coefficient \( \beta \) is strongly positive (as large as 17) and unstable up to the beginning of 1985, after which it becomes negative and fairly stable\(^{46}\) to the end of the sample (21 Apr 89). Therefore in Table 1, we report regressions starting in Jan 84 and in Feb 85. The latter date is chosen to correspond as closely as possible to Tease (1988).

Using data on the exchange rates of five countries against the US and assuming \( H_0: \alpha = 0, \beta = 1 \), Cumby and Obstfeld (1984) strongly reject the

\(^{46}\) Despite this fair degree of stability, estimates of equation (3) for sub-periods can produce rather different results – see equation (3) in Table 2.
assumption of conditional homoscedasticity for the errors in equation (3) for four of the exchange rates. Applying their test to our problems leads to test statistics of $2.02$ and $2.93$ for $k =$ four weeks and $1.73$ and $0.897$ for $k =$ thirteen weeks. The test statistic is asymptotically distributed $\chi^2(2)$ which has a critical value of $5.99$ at the 5% level. Thus, with this test, we cannot reject the null hypothesis of conditional homoscedasticity in all cases.

APPENDIX C

This appendix examines the statistical properties of the exchange rates in Datasets A and B, including the conditional variance, covariance and skewness of the exchange rates. We provide a summary of our findings – further details are given in Smith (1989b).

For all the exchange rates in the two datasets we establish the following results. Using Perron and Phillips (1987) tests, we accept the null hypothesis that the log of the spot exchange rate at a one week interval (Dataset A) and one day interval (Dataset B) has no significant time trend but requires, at least, one unit difference to be stationary. Using Augmented Dickey-Fuller (1979) tests, we reject the hypothesis that the log exchange rate requires a second unit difference to be stationary. Thus, over the sample periods, these tests do not reject the hypothesis that each exchange rate follows a random walk with no drift.

The conditional variance of log changes in the spot exchange rate is successively modelled as autoregressive conditional heteroscedasticity [ARCH] (Engle, 1982), generalised ARCH [GARCH] (Bollerslev, 1986) and exponential GARCH [EGARCH] (Nelson, 1988). An ARCH(5) model is fitted to both datasets, and evidence for ARCH is found – although the explanatory power of the models is low. A GARCH(1,1) model is then fitted to both datasets, and the likelihood function shows this model to be
far superior to the ARCH model. These two models impose a symmetrical distribution for the estimated conditional variance, while the EGARCH model does not. Given the significant skewness reported in section 5.5, we expected to find evidence of EGARCH. In almost all cases, the parameter point estimates in our EGARCH model imply that there is greater volatility in the immediate aftermath of a fall in the $A than in the immediate aftermath of a rise. Unfortunately, the standard errors of the estimates are so large that this asymmetry is not statistically significant.

Rather than quote all the parameter estimates for each of the models, Figure 10 shows non-parametric estimates of the variance, skewness and two covariances for four-weekly changes in s[$US/$A], conditional on the change in s[$US/$A] over the previous week. We established that conditioning on the change in s[$US/$A] over the previous week provided more variability in the estimates than conditioning on the change in s[$US/$A] over the previous four weeks. Define $s_t - s_{t-1} = g(t)$, and imagine a variable $G(t+4,t)$ defined in terms of exchange rates at times $t$ and $t+4$ (e.g., $G(t+4,t) = [s_{t+4} - s_t]^2$ or $G(t+4,t) = [s_{t+4} - s_t]^3$). With a sample $s_t$, $t = 1, \ldots, N$, the non-parametric estimator of $G(t+4,t)$ conditional on $g(t) = g^*$ is

\[
G(t+4, \tau \mid g(\tau) = g^*) = \sum_{t=2}^{N-4} G(t+4,t) \cdot \omega_{t \cdot g^*}
\]

where

\[
\omega_{t \cdot g^*} = \frac{\exp \left( -\frac{1}{2} \left[ \frac{g(t) - g^*}{h} \right]^2 \right)}{\sum_{t=2}^{N-4} \exp \left( -\frac{1}{2} \left[ \frac{g(t) - g^*}{h} \right]^2 \right)},
\]

and $h = \sigma_g (N - 5)^{-1/5}$ and $\sigma_g$ is the standard error of the observations $g(t)$, $t = 2, \ldots, N - 4$. This non-parametric estimator is very similar to one
suggested by Pagan and Schwert (1989). Figure 10 shows conditional estimates of $(s_{t+4} [\text{US/A}] - s_t [\text{US/A}])^2$, $(s_{t+4} [\text{US/A}] - s_t [\text{US/A}])^3$, $(s_{t+4} [\text{US/A}] - s_t [\text{US/A}])$, $(s_{t+4} [\text{US/Y}] - s_t [\text{US/Y}])$ and $(s_{t+4} [\text{US/A}] - s_t [\text{US/A}])$, $(s_{t+4} [\text{US/DM}] - s_t [\text{US/DM}])$ based on Dataset A from Jan, 84 to Apr, 89. The third and fourth of these estimators correspond to covariances provided the expected change in the exchange rate over four weeks is zero, which is a good approximation. The unconditional sample estimates of the four variables above are, respectively, $12.63 \times 10^{-4}$, $-47.91 \times 10^{-4}$, $2.30 \times 10^{-4}$, and $2.46 \times 10^{-4}$.

Within the sample, Figure 10 shows that the behaviour of the exchange rate over the next four weeks depends to a considerable extent on its movement in the previous week. Based on this figure, the risk premium required by a US investor after a fall of the $A$ of between 4% and 5% in the previous week should be substantial (using equation (10) and assuming a portfolio share in Australia of 0.02 as well as 0.10 in other foreign countries in the proportions used in the text, the risk premium is 0.5% p.a.). But, both this analysis and the analysis summarized above based on parametric approaches (ARCH, GARCH and EGARCH) suggest that the conditional moments of the change in the exchange rate over the next four weeks only depend on the exchange rate over the previous few weeks (probably no more than three weeks). As a consequence, these results do not undermine the conclusions reached in section 5.3 of the text. On average over extended periods (several weeks or

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47 To deal with the problem of outliers in the $g(\tau)$ data, Pagan and Schwert (1989) suggest a slightly different estimate. We deal with this problem in a different way: after ordering the $g(\tau)$ sample from the most negative to the most positive, we only estimate $G(\tau+4, \tau)$ for $g^*$ values between the fifth smallest value of $g(\tau)$ and the fifth largest value of $g(\tau)$.

48 However, the results of Pagan and Schwert (1989) suggest that, out of sample, the predictive capacity implied by the Figure is probably substantially overstated.
Figure 10
Conditional second and third moments and covariances of four-week changes in $s[\$US/\$A]$ using Dataset A, Jan 84 to Apr 89

\[ [s(t+4) - s(t)]^3 \]

\[ [s(t+4) - s(t)]^2 \]

\[ \text{Cov} (\$US/\$A, \$US/DM) \]

\[ \text{Cov} (\$US/\$A, \$US/Y) \]
several months), the risk premium which a utility maximizing US consumer-investor should demand for holding short-term nominal Australian assets seems to be negligible – compared, for example, to the average short-term real interest differential between Australia and the US from late 84.

**APPENDIX D**

This appendix describes how the estimates in Table 3 were derived. We measure time in weeks and use exchange rate Dataset A from 6 Jan 84 to 21 Apr 89. We do not have interest-rate data for each country, so we use the approximation \((1 + i^j) = 1\), for all \(j\). Then,

\[
\sigma^j = p^j - p^\text{us} = \frac{P^\text{us}}{P_t + 4} \frac{\Delta S^j_t}{S_t} - (1 + i^\text{us}) \approx \frac{P^\text{us}}{P_t + 4} \frac{\Delta S^j_t}{S_t},
\]

where \(\Delta S^j_t = S_{t+4} [j / \text{US}] - S_t [j / \text{US}]\). Since the period of analysis is four weeks, the approximation, \((1 + i^j) = 1\), introduces an average error of the order of (or less than) 1%. Further, as noted previously, almost all the variation in \(\sigma^j\) arises from exchange rate variation.

Price data comes from monthly US CPI data from OECD Main Economic Indicators (various issues). All Fridays in any given month are assigned the price index for that month. To evaluate \(\text{cov}(\sigma^A, p^\text{us})\), we require \(E_t \left[ \frac{p^\text{us}}{P_t + 4} \right]\). We assume a simple form of adaptive expectations:

\[E_t \left[ \frac{p^\text{us}}{P_t + 4} \right] = \frac{P^\text{us}_{t-16}}{P^\text{us}_{t-12}}.\]

The sixteen week lag is used to ensure that only published price indices are used in forming the expectation. A more sophisticated expectation formation assumption\(^{49}\) should reduce \(E_t \left[ \frac{p^\text{us}}{P_t + 4} \right] = E_t \left[ \frac{p^\text{us}}{P_t + 4} \right] \) and hence reduce our estimate of

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\(^{49}\) Given the dramatic change in the income velocity of money in the 1980s (see, for example, Friedman, 1988 – especially Figure 2), *ex ante* it might have been quite difficult to have had more accurate inflation expectations than the backward looking ones used here.
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\[
\frac{z^j}{P_t} = \rho - \rho^u_s \approx \frac{P^u_s}{P^u_s} \left[ (1 + i^j) \left(1 + \frac{\Delta S_t^j}{S_t} \right) - (1 + i^u_s) \right] = \frac{P^u_s}{P^u_s} \frac{\Delta S_t^j}{S_t},
\]

where \(\Delta S_t^j = S_{t+4}^j / S_t / \text{US} - S_t^j / \text{US}^j\). Since the period of analysis is four weeks, the approximation, \((1 + i^j) = 1\), introduces an average error of the order of (or less than) 1%. Further, as noted previously, almost all the variation in \(z^j\) arises from exchange rate variation.

Price data comes from monthly US CPI data from OECD Main Economic Indicators (various issues). All Fridays in any given month are assigned the price index for that month. To evaluate \(\text{cov} (z^A, \rho^u_s)\), we require \(E_t [P_{us_t} / P^u_{us+t+4}]\). We assume a simple form of adaptive expectations: \(E_t [P_{us_t} / P^u_{us+t+4}] = P_{us_{t-16}} / P_{us_{t-12}}\). The sixteen week lag is used to ensure that only published price indices are used in forming the expectation. A more sophisticated expectation formation assumption\(^49\) should reduce \(P_{us_t} / P^u_{us+t+4} - E_t [P_{us_t} / P^u_{us+t+4}]\), and hence reduce our estimate of

\(^49\) Given the dramatic change in the income velocity of money in the 1980s (see, for example, Friedman, 1988 – especially Figure 2), *ex ante* it might have been quite difficult to have had more accurate inflation expectations than the backward looking ones used here.
cov ( z^A , p^{us} ). It suits our purposes if our estimate is an overestimate. To evaluate the other covariances and the variance, we assume

$$E_t [ (P_{ust} / P_{ust+4} ) (\Delta S_i / S_i) ] = 0,$$

(D.1)

because, over four weeks, exchange rates changes for the currencies we consider are well approximated as an unpredictable random variable with zero mean. We have established that using the one month forward discount in equation (D.1) as the expected depreciation of the $A against the $US makes only a small change to our estimate of var( z^A ) – it increases the estimate from $11.9 \times 10^{-4}$ to $12.5 \times 10^{-4}$.

**APPENDIX E**

We describe here a non-parametric statistical test for the skewness of the distribution $D_i [a/b]$. Assume we have a sample with an odd number $(2n + 1)$ of independent\(^{50}\) observations from $D_i [a/b]$.\(^{51}\) Order the sample from the most negative to the most positive and define $d_j$ as the $j$\(^{th}\) observation (so $d_{j-1} < d_j < d_{j+1}$ for $1 < j < 2n + 1$). $d_{n+1}$ is the median of the sample. Define $y_j = d_j - d_{n+1}$, $j = 1, \ldots, 2n+1$, and form the random variables $Y_k , k = 1, \ldots, 2n$, defined by

$Y_k = -1$ when $y_j$ is the $k$\(^{th}\) largest of the $y_j$'s in absolute value and $y_j$ is negative;

$Y_k = 1$ when $y_j$ is the $k$\(^{th}\) largest of the $y_j$'s in absolute value and $y_j$ is non-negative. Finally, define the random walk $Z_k$, by

\(^{50}\)The assumption of independence makes the analysis exact. We examine the removal of this assumption at the end of this appendix.

\(^{51}\)If we have an even number $(2n + 2)$ of independent observations, we define $d_j$ as described, but now define $y_j = d_j - (d_{n+1} + d_{n+2} ) / 2$, $j = 1, \ldots, 2n+2$. The random variables $Y_k , k = 1, \ldots, 2n$, are defined as described and equation (E.1) is again the basis of our non-parametric test for the skewness of $D_i [a/b]$. 
Z_0 = 0, \quad Z_k = \sum_{j=1}^{k} Y_j, \quad k = 1, \ldots, 2n.

Provided that d_j \neq d_{n+1} for all j \neq n + 1,^52 there are exactly n '−1' values and n '+1' values taken by the random variables Y_k, k = 1, \ldots, 2n, and hence the random walk, Z_k, walks from Z_0 = 0 to Z_{2n} = 0. Crucially, under the null hypothesis that the distribution D_i[a/b] is symmetric, all distributions of the n '−1' values and n '+1' values among the random variables Y_k, k = 1, \ldots, 2n, are equally likely and all walks Z_k from Z_0 = 0 to Z_{2n} = 0 are also equally likely. By contrast, if D_i[a/b] is negatively (positively) skewed, Z_k, k = 1, \ldots, 2n will be more likely to walk to large negative (positive) values before returning to zero when k = 2n. No specific assumption about the distribution of D_i[a/b] is necessary – the null hypothesis is simply that D_i[a/b] is symmetric.

Define the random variable W_M as the number of the random variables Y_k, k = 1, \ldots, M, which take the value '−1'. Under H_0, Pr(W_M = w), is

\[
Pr( W_M = w ) = \binom{M}{w} \cdot \binom{2n-M}{n-w} / \binom{2n}{n} \quad \text{(E.1)}
\]

Our one-sided test for negative [positive] skewness involves evaluating the probability, Pr(W_M ≥ w) [ Pr(W_M ≤ w)]. For the results in Table 9, n = 84, and M = 10 was chosen. Evaluation of (E.1) gives: Pr(W_{10} = 0) = Pr(W_{10} = 10) = 0.0007, Pr(W_{10} ≤ 1) = Pr(W_{10} ≥ 9) = 0.0090, Pr(W_{10} ≤ 2) = Pr(W_{10} ≥ 8) = 0.0494. Thus, sample values of W_{10} of 9 or 10 (0 or 1) imply rejection of H_0 at the 1% level of significance against the alternative of

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^52 With the exception of the TWI data (which is quoted to three figures), all our exchange rate data is quoted to (at least) four significant figures, so it is unlikely that any two values of d_j would be the same.
negative (positive) skewness, while a value of 8 (2) implies rejection at the 5% level. Sample values $3 \leq W_{10} \leq 7$ are insignificant.\footnote{An alternative test based on the distribution of the maximum (or minimum) value taken by the walk, $Z_k$, $k = 1, \ldots, 2n$, was also examined but found to have little power.}

As discussed in Appendix C, in general the distribution of $D_i [a/b]$ at time $t$ ($D^t_i [a/b]$) depends on observations of $s_{\tau+i} - s_{\tau}, \tau < t$. Clearly, this invalidates our assumption of the independence of the observations, and our test of skewness must be modified. The null hypothesis is now that each of the $D^t_i [a/b]$ distributions is symmetric with a common mean, $\mu$. Under this null, the distribution of $W_{10}$ depends on how different are the distributions $D^t_i [a/b], t = 1, \ldots, 2n+1$. At one extreme is the case already examined when all the distributions are identical, and each $Y_k, k = 1, \ldots, 2n$ has an equal chance of coming from any of the $D^t_i [a/b], t = 1, \ldots, 2n+1$. At another extreme, assume that under the null there are only two distinct (symmetrical) distributions: for ten particular times, $t(j), j = 1, \ldots, 10$, the distributions $D^{t(j)}_i [a/b] = D^+$, and at all other times, $\tau, \tau \neq t(j), j = 1, \ldots, 10, D^t_i [a/b] = D^*$. $D^*$ is assumed to have all its probability weight "near" $\mu$, while $D^+$ is assumed to have all its probability weight in two tails "far from" $\mu$, so that $D^*$ and $D^+$ have no overlap. In this contrived case, we can be sure that for $j = 1, \ldots, 10, Y_j$ must come from $D^+$ and hence from the ten particular times, $t(j), j = 1, \ldots, 10$. Then under the null hypothesis, $\Pr(W_{10} = w)$ is simply

$$\Pr(W_{10} = w) = \binom{10}{w} / 2^{10}.$$  \hspace{1cm} (E.2)

Equation (E.2) gives: $\Pr(W_{10} = 0) = \Pr(W_{10} = 10) = 0.00098, \Pr(W_{10} \leq 1) = \Pr(W_{10} \geq 9) = 0.011, \Pr(W_{10} \leq 2) = \Pr(W_{10} \geq 8) = 0.055$. Thus, even in this extreme case, the critical values of $W_{10}$ are only changed slightly.
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