ON THE THEORY OF INFLATION IN OPEN ECONOMIES

by

Geoffrey Harold Kingston

A dissertation submitted for the degree of Doctor of Philosophy at the Australian National University

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Except where otherwise indicated this dissertation is my own work.
I would like first of all to express my gratitude to Professor S. J. Turnovsky for his invaluable supervision of this dissertation.

I would also like to thank participants in seminars at the Australian National University and the University of Western Ontario for their comments on part of the material in the form of papers. Finally, I wish to thank Marg Gower for typing the manuscript with skill and dispatch.
The theoretical literature provides straightforward answers to questions of the general form: what are the long-run effects on inflation at home and abroad of a sustained increase in monetary growth at home? It is less informative, however, about questions of the kind: what are the long-run effects on monetary growth and inflation at home and abroad of a sustained increase in the gap between public purchases of home output and net explicit taxes on home incomes?

Building on the work of Cagan (1956), Christ (1968), Turnovsky (1977) and others, this dissertation addresses the latter type of question. Various other short-run and long-run effects of expansionary public policy are also considered. Finally, stability, and (less frequently) existence and uniqueness properties are investigated.

For these purposes, a static Keynes-Phillips model is embedded in a dynamic model of the creation of public debt, the formation of CPI and exchange-rate expectations, and movements in the terms of trade; the main analytical novelty being full accounting for the effects of inflation, interest and growth on the budget constraints of the public and private sectors.

Note, however, that the foregoing static and dynamic elements are invoked selectively, depending on considerations of simplicity, the particular problem at hand, and the relevant time horizon (cf. Henderson (1977)). The notion of a hierarchy of successively longer time spans is the central organizing principle of the analysis. Four different time horizons are considered.
In the shortest run, investigated in Chapters III and IV, real intensive output is predetermined. Results in this setting include a variable-inflation analogue of the Dornbusch (1976c) result on exchange-rate overshoot.

Over the next horizon, output is demand-determined and inflation expectations are predetermined. One exercise suggested by this framework is a synthesis of repercussion-multiplier and Phillips-Curve notions; see Chapter IV.

In the longer-run analyses, output is fixed at capacity and expectations are realized. An "inter-run" variant assumes further that national public sectors are able to hold down nominal rates of interest, and/or that external accounts are in a state of "quasi equilibrium" (cf. Mundell (1968)). It is then shown, for example, that the effect on a small country's disposable income of an increase in public spending on the home good is given by the inverse of the standard Marshall-Lerner expression; see Chapters II and III.

In the longest run, nominal rates of interest fully reflect the Fisher effect, and external accounts are in full equilibrium. It is shown that over such a time span, the implementation of a constant monetary growth rule would have to be accompanied by an "accommodating" fiscal policy; see Chapter IV.

On the other hand, suppose that monetary growth is endogenous in every country, and reconsider the problem posed at the outset. Then:

(1) the domestic and foreign inflationary effects of an increased budget deficit at home are independent of relative economic size;
(2) either in a "reserve-currency" country, or in any country under
flexible rates, the disturbance in question will induce a more than proportional increase in domestic monetary growth; (3) rates of monetary growth abroad will rise pari passu or remain unchanged according as whether the domestic economy is a reserve-currency country or under flexible rates; (4) higher budget deficits in "peripheral" countries will not affect monetary growth anywhere.

These and other fiscal analogues of more-familiar propositions concerning the international transmission of monetary disturbances are established in Chapter IV.
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CHAPTER I

INFLATION AND PUBLIC POLICY IN A CLOSED ECONOMY

1. INTRODUCTION: TWO POPULAR EXPLANATIONS OF INFLATION

Students of inflation have now reached a measure of consensus. Most would agree that the Phillips curve--augmented by variables for anticipated domestic and foreign inflation, and possibly reinterpreted as an aggregate supply curve--is the best available proximate explanation of short-run national inflation rates. Most would accept that this curve is approximately vertical in the long run. And most would also accept various long-run equilibrium relationships, such as the purchasing power parity theory (at least in its 'relative' version), the proposition that expectations of inflation and other variables are realized (at least in an expected-value sense), and the proposition that rates of inflation will equal nominal monetary growth less real output growth (at least in the absence of productivity growth). Finally, most analysts would probably agree that the main cause of postwar inflation has been some kind of sustained public-sector disturbance.

But the last-mentioned area of agreement--which, for reasons which will soon become apparent, has been deliberately left vague--is also intimately related to a major area of dispute: to understand the behaviour of the public sector, should we view the fiscal authorities as the 'initiating' agency, with the monetary authorities playing an "accommodating" or "passive" role--or is the opposite perspective more correct?

Parkin (1976) forcefully states the essentials of the former point of view:
"...vote-seeking politicians believe (rightly or wrongly) that by increasing government expenditure on social programs, subsidies and the like, and by holding down interest rates they can improve their electoral chances. The result is an excessive rate of money creation to pay for the programs."¹

And in the long run, of course, inflation will rise more or less pari passu. Parkin made the foregoing observations on postwar British experience. No doubt there are close substitutes in the economic commentary on most other industrial democracies. On this view, the objective of the monetary authorities is the passive one of attempting to maintain 'cheap money' in the face of increased public-sector deficits. The time horizon under consideration may permit a squeezing of the real rate of return on government debt.

More formally, the short-run (economic) aspects of Parkin's scenario are easily handled by the standard Keynes-Phillips apparatus, whereas the pioneering investigations into the associated long-run aspects are Christ (1968), (1969). These papers also consider the secular effects of higher public spending and/or lower net explicit taxes in conditions of endogenously-supplied money. However, they restrict attention to the case of a closed economy characterized by stationary secular nominal income and by net explicit taxes which are at least partly raised by a proportional impost on factor incomes.

From these considerations, and also by abstracting from interest payments, Christ deduced a simple result. Let \( G \) = nominal public spending, \( T \) = nominal net lump-sum taxes, \( D = G - T \) = public sector's 'legislated' deficit, \( Y \) = nominal gross factor income, and \( u \) = rate of tax on factor incomes. Denote the pegged states and steady states of variables by a bar and an asterisk respectively. Then \( \frac{\Delta Y}{\Delta D} = 0 \), which yields \( \frac{dY^*}{dD} = 1/u \).
Or, as Christ (1968) puts it, "long-run static equilibrium requires a balanced budget. It is this feature of the analysis which yields a long-run government purchases [and transfers] multiplier of $1/u$ when tax rates are fixed" (p. 66, loc. cit.).

Christ's 1968 paper postulated a fixed price level, so that the above result was interpreted as applying to real output. More pertinent to our subsequent analysis is his 1969 paper, which postulates fixed real output; then the price level becomes the critical endogenous variable. His overall 'budgetary' approach has stimulated a prodigious amount of research. One consequence of this has been the discovery of many alternative specifications of 'accommodating' or 'passive' central-bank reactions. We shall find it convenient for the subsequent open-economy analysis, however, to focus upon the above-mentioned case of pegged nominal rates of return on interest-bearing public debt.

In contrast to this budgetary approach there is the classical monetary perspective, whereby nominal monetary growth is exogenous in at least one country, and the real rate of interest is exogenous everywhere—see, e.g., Whitman (1975), Swoboda (1977). This entails endogenous rates of government spending and/or net explicit taxes. For example, it is often explicitly or implicitly assumed that increases in the nominal money supply are rebated to the private sector in a lump-sum fashion. That assumption is explicit in Friedman (1969) and the money-in-growth models.

In the special case of a closed economy characterized by no productivity growth (and the latter is also typically assumed by the budgetary analyses), the basic comparative-statics proposition of the
monetary approach is even more simple and well-known than its budgetary counterpart. Let $\mu = \text{rate of nominal monetary growth}$, $p = \text{actual rate of inflation}$, and $n = \text{exogenously-given rate of population and real output growth}$. Then $\ddot{\mu} - p^* - n = \text{0}$, which yields $dp^*/d\mu = 1$.

Or, since long-run static equilibrium requires stationary real per capita balances, a 1 percent increase in nominal monetary growth will raise inflation by 1 percent.

In summary, we can distinguish two popular explanations of the link between public-sector disturbances and accelerated inflation. Each can be summarized by a simple comparative-statics proposition. The question of which explanation is more correct can only be settled empirically.

Nevertheless, joint consideration of these explanations draws attention to other interesting and unsettled questions. Noting that the monetary approach explicitly deals with sustained inflation (and inflation taxes), is there a simple variable-inflation analogue of the basic comparative-statics proposition of the budgetary approach? And noting the explicit, indeed critical role of the public sector constraint in the budgetary approach, how does that constraint fit into the monetary approach? What are the principal effects of our two public-sector disturbances apart from higher inflation? Finally, much is known about stability in the case of endogenous inflation anticipations under fixed real interest and nominal monetary growth rates. But this is not so in the case of endogenous inflation anticipations and endogenous nominal monetary growth under a fixed nominal interest rate.
This chapter addresses these questions in the context of a simple model of a closed economy. That model also underlies an open-economy analysis in later chapters. Its main analytical novelty is full accounting for the effects of inflation, interest and growth on the budget constraints of the public and private sectors. Section 2 exposit the model. Section 3 considers the short run. Section 4 considers the long run. Section 5 examines the stability of the alternative policy regimes. Section 6 contains a summary.
2. THE MODEL

The exposition proceeds as follows. First the main assumptions are listed and discussed. Next the instantaneous relationships are laid out. Then those relationships are embedded in a dynamic model of asset accumulation (or decumulation), and the formation of inflationary expectations. Finally, the model is closed by imposing one or the other of the budgetary and monetary regimes, and initial conditions are introduced.

Assumptions

1. Net explicit taxes are lump-sum imposts and transfers.

2. The public sector does not rebate its revenues from debt creation to the private sector in proportion to initial holdings of public debt.

3. The only assets held by the private sector are those created by the public sector, viz., base money, and a variable-interest short-term 'bond' (like a savings-account deposit).

4. Labour is the sole factor of production, is in infinitely elastic supply at each point in time, and grows at a constant rate. There are constant returns to scale, so that physical capacity output grows at the same rate as the labour stock. The gap between actual and capacity real per capita output is proportional to the gap between actual and anticipated inflation.

5. There is perfect myopic foresight of all variables except possibly interest and inflation rates (whose anticipations are modelled in different ways). Public- and private-sector
consumption plans are fulfilled at each point in time.

6. Private aggregate demand for goods and services (i.e., private absorption) is an increasing function of anticipated private disposable income and actual wealth, and a decreasing function of the anticipated real interest rate. The private sector perceives its holdings of interest-bearing public debt to be part of its wealth (at least in a proximate sense; see below). Private demand for base money is an increasing function of output and a decreasing function of the nominal interest rate.

7. All variables adjust 'smoothly' in the sense of being differentiable with respect to the time except possibly when shift parameters are altered (see below).

8. In the absence of explicit statements to the contrary, we focus on "mild" inflations, by which we mean small perturbations about initial states wherein either monetary growth is zero (i.e., conforms to the full-liquidity rule), or public spending equals net explicit taxes. It turns out that these alternative initial states are equivalent in the steady state, providing real rates of interest initially conform to the golden rule.

These assumptions warrant further comment. Assumption 1 admits various easily-manageable extensions. For example, and in marked contrast to Christ (1968, 1969), it would make little difference to the present analysis if explicit taxes were partly raised by a proportional impost on gross factor incomes. Assumption 2 intentionally introduces
a critical non-neutrality element; see e.g., Mundell (1971, Ch. 2.)

It is also a standard assumption concerning public-sector behaviour. Assumption 3 can be modified by postulating that the bond is a perpetuity rather than one which is traded at fixed par values; the only difference is that the stability analysis becomes more complicated. For further discussion of this point see, e.g., Tobin and Buiter (1976). Assumption 4 reveals that we are abstracting from productivity growth. This, too, is a fairly standard assumption.

Considering Assumptions 3 and 4 together, it is apparent that we are abstracting from physical capital and the financial claims associated with it. This is for simplicity, and is especially useful in the subsequent open-economy chapters. Assumption 5 represents a compromise, in the context of a consistent model, between simplicity and the inclusion of output and employment transients; for further discussion of these issues, see Turnovsky (1977, especially Ch. 3).

With regard to Assumption 6, the absence of durable goods in the model implies that the anticipated real interest rate in private aggregate demand should be interpreted as reflecting substitution between spending and savings rather than physical capital accumulation. For evidence that such effects take place, see, e.g., Weber (1970). On the question of whether government bonds are part of private net worth, note that our formulation of the private sector's flow budget constraint will be shown to imply that the private sector does correctly take into account its "real" tax liabilities, if only in a long-run average sense. For further discussion of this
approach, see, e.g., Dornbusch (1977) (especially n. 2). Finally, one can incorporate the anticipated inflation rate and the real per capita wealth stock into the private expenditure and money demand functions respectively, without fundamentally changing the main results.

Assumption 7, which is fairly standard in the literature, is amplified and discussed below. Finally, we mostly resort to Assumption 8 when analyzing transients. In this context it is used to eliminate certain income effects which would complicate the analysis without fundamentally changing the main results, given conventional assumptions concerning gross substitutability. On the full liquidity rule, see Friedman (1969). The golden rule is used here in a way similar to that of Mathieson (1976).
Instantaneous Relationships

Equations (1.1) set out standard market-clearing assumptions and a definition of the 'legislated deficit':

\[ Q = Z + G, \quad q = z + \bar{g} \] \hspace{1cm} (1.1a), (1.1a)\textsuperscript{t}
\[ C = L, \quad c = \ell \] \hspace{1cm} (1.1b), (1.1b)\textsuperscript{t}
\[ D = G - T, \quad \delta \equiv g - t \] \hspace{1cm} (1.1c), (1.1c)\textsuperscript{t}

where
\[ q \equiv Q/PN = \text{real per capita supply of output}, \]
\[ z \equiv Z/PN = \text{real per capita private demand for output}, \]
\[ g \equiv G/PN = \text{real per capital public demand for output, always taken to be pegged}, \]
\[ c \equiv C/PN = \text{real per capita supply of base money (or, monetized public debt),} \]
\[ \ell \equiv L/PN = \text{real per capita private demand for base money,} \]
\[ \delta \equiv D/PN = \text{real per capita legislated deficit,} \]
\[ t \equiv T/PN = \text{real per capita net explicit taxes,} \]
\[ P, N = \text{price of output and level of the labour force respectively.} \]

Equations (1.2) describe aggregate private demand for goods and money, and the aggregate supply of goods:
\[ z = \bar{z} + (1-s)y - \nu \rho + \omega \omega \] \hspace{1cm} (1.2a)\textsuperscript{t}
\[ \ell = kq^\Phi \exp[-\alpha(\rho + \pi)] \] \hspace{1cm} (1.2b)\textsuperscript{t}
\[ p = \gamma(q-q_F) + \pi \] \hspace{1cm} (1.2c)\textsuperscript{t}

where
\[ \bar{z} = \text{autonomous private expenditure, a constant}, \]
\[ s = \text{marginal propensity to save, a constant}, \]
\[ y = \text{anticipated real per capita private disposable income (see below)}, \]
\( l = \) responsiveness of anticipated savings with respect to the interest rate, a constant,
\( \rho = r - \pi = \) anticipated real rate of interest,
\( \omega = \) responsiveness of spending with respect to wealth, a constant,
\( w = \frac{W}{PN} = \) real per capita private aggregate holdings of public debt,
\( k = \) inverse of the output ('income') velocity of circulation in the initial state (see below), a constant,
\( \phi = \) output ('income') elasticity of money demand, a constant,
\( \alpha(\rho + \pi) = \) interest elasticity of money demand, where \( \alpha \) is constant,
\( \pi = \) anticipated rate of inflation,
\( \rho = \frac{P}{P} = \) actual rate of inflation,
\( \gamma = \) responsiveness of actual inflation with respect to excess demand, a constant,
\( q_F = \) real per capita 'full employment' output, a constant.

All the constants are positive except possibly \( z \), whose precise value is not important here. On the empirical considerations underlying our semilogarithmic specification of money demand, see Section 5 of Chapter 3.

Finally, the respective nominal and real per capita counterparts of our definition of real per capita private disposable income are

\[ Y \equiv Q - T + r(W-C) , \quad y \equiv q - t + \rho(w-c) - \pi c - mw (1.3) , (1.3) \]

where \( Y = \) nominal private disposable income; and \( y = \) anticipated real per capita private income = gross factor earnings \( q \), less net explicit taxes \( t \), plus anticipated earnings from holdings of interest-bearing
public debt \( \rho(w-c) \), less the anticipated inflation tax on monetized public debt \( \pi_c \), less the growth tax on aggregate public debt \( nw \) (\( n = N/N \)), all in real per capita terms.

This formulation reflects our (institutional) Assumption 2: to provide for the anticipated inflation and 'growth' depreciation of its real per capita public debt holdings, the private sector is continually obliged to set aside \( \pi_c \) and \( nw \) respectively of its income in real per capita terms. And as a consequence of Assumption 5—in particular, the possible imperfect myopic foresight of inflation—we distinguish between anticipated and actual real per capita private disposable income, with the former being relevant to private-sector decisions, and the latter being given by \( q-t + (r-p)(w-c) - pc - nw \). The difference between these variables is therefore \( (\pi-p)w \); this variable equals unanticipated private savings, and plays a role in the model analogous to that of unanticipated inventory investment in the traditional model (see Kingston and Turnovsky (1978) for further discussion of this point).

**Dynamic Relationships**

The model contains two dynamic relationships unless the policy regime under consideration involves pegging the rate of nominal monetary growth. In that case we require a third differential equation; see the next subsection.

Consider first the public-sector constraint in nominal and real per capita terms respectively:

\[
\dot{W} = D + r(W-C) \quad \text{and} \quad \dot{w} = \delta + (r-p)(w-c) - pc - nw
\]

Equation (1.4a) states that in nominal terms, the public sector finances its legislated deficit and interest obligations by issuing or
retiring debt. Equation (1.4a)' reveals that in real per capita
terms, the actual inflation tax on monetized debt, \( pc \), and the
growth tax on aggregate public debt, \( nw \), raise revenue for the pub-
lic sector.

Consider next the formation of inflation expectations:

\[
\dot{\pi} = \beta (p - \pi) \tag{1.4b}'
\]

where \( \beta \) = speed of expectations adjustment, a constant, \( \beta > 0 \).

Despite the recent literature on rational expectations (see, e.g.,
Schiller (1978)), this simple adaptive specification can still be
justified. In particular, there are the theoretical arguments of
Friedman (1975), based on information costs; and the empirical studies
by Turnovsky (1970), Pesando (1975), Feige and Pearce (1977) and
Carlson (1977), which provide evidence for autoregressively-generated
inflation expectations. On the other hand, it can be argued that
these considerations are not strictly relevant here, since our model
does not explicitly handle information costs or the general problem of
uncertainty. Thus, for example, there is no stochastic variation of
public policies in our model, and it might be argued that such consider-
ations provide the main justification of autoregressive specifications
of inflation expectations. Accordingly, we shall also pay attention
to the limiting case of perfect myopic foresight of the inflation
rate—in symbols, \( \beta \to \infty \). Note, too, that our treatment of exchange rate
expectations is altogether different; see Chapter III.

As a final point concerning these dynamic relationships, we must
strengthen Assumption 7. In particular, assume \( W, P \) and \( \pi \) adjust
'sluggishly' in the sense of being everywhere continuous with respect
to time (providing \( \beta < \infty \)). Since Assumption 4 implies that \( N \) is also
everywhere continuous with respect to time, it follows that \( w(= W/PN) \) has the same property. Clearly this assumption is necessary if \((1.4a)\) and \((1.4b)\) are to be well defined under parameter shifts. It is certainly not economically justifiable under all conceivable parameter shifts. For example, we would expect a sales tax increase to induce a step increase in \( P \), at least in the first instance. But it seems not unreasonable for the parameter shifts under consideration here, namely, small changes in \( g \), \( t \), and \( \mu \). We now turn to a more detailed discussion of those parameters, in which context we also introduce a final dynamic relationship.

Public Policy

The 'budgetary' and 'monetary' regimes were discussed in Section 1. In symbols the former is given by

\[
t = \check{t} \quad \rho + \pi = \check{r}.
\]

Equations \((1.1)\) to \((1.5)\) together determine the endogenous variables \( q, c, p, \) and \( y \); the predetermined variables \( w, \pi, \) and \( \rho \); and the pegged variables \( g, t, \) and \( r \). Note that \((1.1c)\) and \((1.5a)\) together imply that the legislated deficit \( \delta \) is effectively pegged.

The 'monetary' regime is given by

\[
\check{c} = c(\check{\mu} - p - n), \quad \check{\rho} = \check{\rho}.
\]

Equation \((1.6a)\) utilizes the identity \( \check{c} = c(\check{\mu} - p - n) \), which is well defined only if \( c \) adjusts sluggishly. But that assumption would appear reasonable in the present case. Strictly speaking, \((1.6b)\) should be interpreted as embodying the policy assumption of indexed interest-bearing public debt rather than (say) the market assumption of arbitrage between bonds and claims to durable goods. Equations \((1.1)\) to \((1.4)\), together with \((1.6)\), determine the endogenous
variables q, p, y, and t; the predetermined variables w, δ, and c; and the pegged variables g, μ, and ρ. Now the legislated deficit δ is endogenous. For a summary of the model under each policy regime, see Section 5.

Finally, let us transpose Assumption 8 into symbols:

\[ \mu_o = 0, \delta_o = 0, \rho_o - n = 0, \lambda_o = k \quad (1.7a)', (1.7b)', (1.7c)', (1.7d)' \]

where the subscript o denotes initial values. Under (1.7c)', and in the steady state, the initial state described by (1.7a)' is equivalent to that described by (1.7b)'. Hence one of (1.7a)' and (1.7b)' is redundant. Similarly, we could replace (1.7a)' or (1.7b)' by one or the other of

\[ \pi_o + n = 0, p_o + n = 0 \]

and shall do so as is convenient. Turning to (1.7d)', that condition merely represents a particular choice of units. As a consequence of the other initial conditions and (1.2b)', it is equivalent to

\[ q_o = 1. \]

3. THE SHORT RUN

This section considers some impact effects of expansionary policy under the two alternative regimes. We assume the initial state is the steady state together with conditions (1.7)'.

The Budgetary Regime

Equations (1.1a)', (1.2a)', (1.3)', (1.5)' and (1.7)' together yield the familiar result

\[ \frac{dq}{dg} = \frac{1}{s} \quad (1.8a)' \]

In other words, a unit increase in real per capita public spending will raise real per capita output by the inverse of the marginal propensity to save. Under a pegged interest rate and the initial conditions
specifying a 'mild' inflation, the money-market parameters are irrelevant. The only thing that matters is the elementary multiplier process.

Similarly, we deduce

\[
\frac{dp}{dg} = \gamma, \quad \frac{dc}{dg} = \frac{\partial k}{s}, \quad \frac{dy}{dg} = \frac{1}{s}.
\]

An increase in real per capita public spending will also raise actual inflation, the real per capita supply of high-powered money, and anticipated real per capita private disposable income. The latter variable rises by the same amount as output. Another familiar result (not shown) is that a unitary public spending increase is more expansionary than a unit tax reduction, so that a tax-financed increase in public spending will have a net expansionary effect. In summary, the main point is that the goods market clears instantaneously via a rise in output supported by an accommodating increase in the money supply.

The Monetary Regime

First, a preliminary. Since the real per capita base money stock is predetermined under a pegged rate of nominal monetary growth, and noting that the instantaneous relationships are independent of the rate of nominal monetary growth, it follows that the impact effects of a finite increase in nominal monetary growth on the levels of the instantaneous variables are all zero. Instead, the disturbance in question will affect the rate of change of the instantaneous variables, operating through the rate of change of the real per capita money supply (see (1.6a)).

For example, consider real per capita output. Insert (1.2b) into (1.1b), take logs, differentiate with respect to time, and recall
(1.6)', which yields
\[ \frac{d(\dot{q}/q)}{d\dot{\mu}} = \frac{1}{\phi}. \quad (1.9a)' \]
A 1 percent increase in the rate of nominal monetary growth will raise the percentage rate of growth of real per capita output growth by the inverse of the output elasticity of money demand. Under a pegged real interest rate the goods-market parameters are irrelevant. All that matters is the elementary 'income-velocity' process.

This result, (1.2c)', (1.1a)', (1.2a)', (1.3)' and (1.7)' together yield
\[ \frac{\dot{p}}{\phi} = \frac{\dot{c}}{\phi}, \quad \frac{\dot{c}}{d\dot{\mu}} = \frac{-s}{(1-s)\phi}, \quad \frac{\dot{y}}{d\dot{\mu}} = \frac{1}{(1-s)\phi}. \quad (1.9b)', (1.9c)', (1.9d)' \]
An increase in nominal monetary growth will raise the absolute rate of change of the actual inflation rate, real per capita net explicit transfers, and anticipated real per capita private disposable income. The absolute rate of change of the latter variable rises by more than the absolute rate of change of output, as a consequence of the induced decrease (increase) in the absolute rate of change in net explicit taxes (transfers). These results are 'dual' to those obtained for the effects of expansionary policy under the budgetary regime: the proximate determinant of output response is the money market rather than that the goods market, and the accommodating variable is the absolute rate of change of net explicit taxes rather than the money stock.

4. THE LONG RUN

The long run is defined by the state of the system when stationarity conditions are imposed on the dynamic equations. Now there are no predetermined variables. First we consider some long-run
properties of the model which are valid for any policy regime. Next we consider expansionary policy under the budgetary regime and the monetary regime respectively.

**Long-Run Equilibrium Relationships**

In the first place, from the stationarity condition to (1.4b)', we obtain

$$\pi^* = p^* .$$  \hspace{1cm} (1.10a)'

Inflation expectations are realized in the long run.

Second, (1.2c)' and (1.10a)' together yield

$$q^* = q_F .$$  \hspace{1cm} (1.10b)'

Output is fixed at 'capacity'. With labour the sole factor of production, this is a version of the 'natural-rate hypothesis'.

Third, from (1.10a)' and the requirement that long-run real per capita money balances be stationary, it follows that

$$\mu^* = \pi^* + n ,$$  \hspace{1cm} (1.10c)'

a proposition which requires no further comment.

Fourth, from (1.10a)' and the stationarity condition to (1.4a)', we obtain

$$\delta^* + \rho^*(w^*-c^*) - \pi^*c^* - mw^* = 0 .$$  \hspace{1cm} (1.10d)'

If public purchases and interest obligations are not covered by net explicit taxes, then revenues from the inflation-cum-growth tax must make up the difference. From this equation, too, follows the equivalence of (1.7a)' and (1.7b)', given (1.7c)', (1.10a)' and (1.10c)'.

Fifth, from (1.10a)', (1.10b)', (1.10d)', (1.10c)', and (1.3)', we deduce

$$y^* = q_F - \bar{g} .$$  \hspace{1cm} (1.10e)'

This is one of the most interesting implications of proper accounting
for the effects of inflation, interest and growth rates in budget con-
straints: in the long run, private disposable income is given by
gross factor income less public spending—not by gross factor in-
come less net explicit taxes. Indeed, steady-state private disposable
income is completely independent of net explicit taxes.

Finally, from (1.10e)' and (1.1a)' we deduce
\[ y^* = z^*. \tag{1.10f}' \]
Income equals spending in the steady state. This is a familiar impli-
cation of simple 'hoarding' models.

As a direct consequence of the foregoing results we have
\[ \frac{dy^*}{dg} = \frac{dz^*}{dg} = -1. \tag{1.11}' \]
In other words, an increase in real per capita public spending will
eventually crowd out an equal amount of real per capita private spending
and income. In addition to holding under either policy regime, these
results do not require Assumptions 6 and 7. On the other hand, we do re-
quire the assumption of a 'mild' inflation—more on this in Chapter IV.

The Budgetary Regime

What are the long-run inflationary effects of expansionary
policy under the budgetary regime? Apply the policy assumptions (1.5)'
and the initial conditions (1.7)' to the budget-balance condition
(1.10d)', to obtain
\[ \frac{d\pi^*}{d\delta} = \frac{1}{w^*}. \tag{1.12a}' \]
In other words, a unit increase in the real per capita legislated
deficit will raise the steady-state rate of inflation by the inverse of
the real per capita stock of aggregate public debt.
This is evidently a variable-inflation 'dual' of Christ's early result (see Section 1). Specifically, the income-tax rate, \( u \), has been supplanted by the relevant inflation-tax base, \( w \). That base is the stock of aggregate public debt—not the real stock of monetized public debt, which is the focus of the traditional theory of the inflation tax. The point is simply that the inflation-tax base available to the public sector is broader than \( c \), insofar as the nominal rate of return on interest-bearing debt can be held down.

Note, too, that the same result holds if we relax the relevant condition concerning the initial rate of inflation, namely \( (1.7a)' \), and introduce the money-market conditions \( (1.1b)' \) and \( (1.2b)' \). For under a pegged nominal interest rate, fixed real per capita output, and no 'wealth' demand for money, we have

\[
\frac{dc^*}{dg} = \frac{dc^*}{dt} = 0 ,
\]

\( (1.12b)' \)

a result which can be used instead of \( (1.7a)' \) to derive \( (1.12a)' \).

Is aggregate real per capita public debt invariant with respect to increased public spending? Insert \( (1.2a)' \) into \( (1.1a)' \), and then recall \( (1.10b)' \) and \( (1.10e)' \), to obtain

\[
w^* = \frac{s(q_F - \tilde{g}) + \theta \rho^* - \tilde{z}}{\omega} \]

\( (1.13)' \)

whence, from \( (1.5b)' \) and \( (1.12a)' \),

\[
\frac{dw^*}{dg} = -\left( s + \frac{\rho}{\omega^*} \right) \frac{w}{w^*} .
\]

\( (1.12c)' \)

That is, an increase in real per capita public spending will drive down real per capita public debt. This is because the associated reduction in real per capita disposable income and the real interest rate both imply reduced 'target' real per capita wealth holdings, where that
target is implicit in our specification of the private expenditure function (1.2a)'. Another implication of (1.13)', together with (1.5)' and (1.12a)', is that a reduction in real per capita net explicit taxes will also drive down real per capita public debt, but by a lesser amount since real income is invariant with respect to net explicit taxes.

The Monetary Regime

What we termed the basic comparative-statics result of the monetary approach, namely \( \frac{d\pi^*}{d\bar{\mu}} = 1 \), follows directly from (1.10a)' and (1.10c)'. This proposition requires no further comment.

More noteworthy is the monetary counterpart of (1.12a)': apply (1.6)' and (1.7)' to (1.11d)', to obtain

\[
\frac{d\delta^*}{d\bar{\mu}} = c^* .
\]

(1.14a)'

A unit increase in the rate of nominal monetary growth will raise the steady-state legislated deficit by the real per capita stock of monetized public debt. This proposition can be related to an implication of Christ's budgetary approach (see Section 1). Specifically, if the initial tax rate is zero, then Christ's analysis yields \( \frac{dD^*}{d\bar{\mu}} = Y^* \).

Equation (1.14a)' is 'dual' to that proposition in the sense that the inflation-tax rate, \( \bar{\mu} \), supplants the income-tax rate, \( u \); and the relevant inflation-tax base, \( c \), takes the place of the income-tax base, \( Y \).

Under a pegged real interest rate, the relevant inflation-tax base is the stock of monetized public debt, as in the conventional theory of inflation taxes.

As with our discussion of (1.12a)', let us relax the initial condition (1.7a)' and introduce the money-market conditions (1.1b)' and (1.2b)'. Under a pegged real interest rate and fixed real per capita output we have
\[ \frac{dc^*}{d\mu} = -\alpha c^* \]  

(1.14b)

which is just the standard result on the 'flight from real balances' in the presence of alternative assets whose real rate of return is invariant with respect to increased inflation.

We are now in a position to generalize (1.14a):

\[ \frac{d\delta^*}{d\mu} = c^*(1 - \alpha r^*) \]  

(1.14c)

where the additional term (as compared with (1.14a)) reflects the reduction in the inflation-tax base induced by higher inflation. The maximum legislated deficit is obtained when

\[ \alpha r^* = 1 , \]

that is, when the interest elasticity of demand for real balances is unity. This is a famous result in the theory of inflation and other taxes (see, e.g., Friedman (1971)). That the unit elasticity is a maximum rather than a minimum may be seen by observing that

\[ \frac{d^2\delta^*}{d\mu^2} = -\alpha c^* < 0 \]

at the turning point under consideration. (More on this in Chapters III and IV.)

Finally, it is clear from (1.13)' that the real per capita stock of aggregate public debt is invariant with respect to monetary expansion when the interest rate and per capita public spending are pegged in real terms. In summary, the only real effects are that net explicit taxes fall (rise) one-to-one with the rise (fall) in inflation-tax revenues, and bond holdings rise one-to-one with the fall in money holdings, in real per capita terms. By contrast, increased public spending under the budgetary regime was shown to have more pronounced real effects, as one would expect.
5. STABILITY

This section considers the local stability of the system under the alternative policy regimes. By way of preliminary we reduce the dimensionality of the system. Insert (1.3)' into (1.2a)', and the resulting expression into (1.1a)'. Similarly, insert (1.2b)' into (1.1b)'; (1.1c)' and (1.2c)' into (1.4a)' and (1.2c)' into (1.4b)'.

This gives

\[ q = \ddot{z} + (1-s)[q-t + \rho(w-c) - nc - nw] - \tau \rho + \omega w - g \]  
(1.15a)

\[ c = k q^\phi \exp[-\alpha(\rho+\pi)] \]  
(1.15b)

\[ \dot{w} = \ddot{g} - t + \rho(w-c) - nc - [n + \gamma(q-q_F)]w \]  
(1.15c)

\[ \pi = \beta \gamma(q-q_F) \]  
(1.15d)

To close the system, impose

\[ t = \ddot{t} \quad \text{and} \quad \rho + \pi = \ddot{r} \]  
(1.15e)

or

\[ \dot{c} = c[\ddot{\mu} - \gamma(q-q_F) - \pi - n] \quad \text{and} \quad \rho = \ddot{\rho} \]  
according as whether the budgetary or the monetary regime is the object of interest. Finally, we invoke the initial conditions (1.7)' when we go to the perturbed counterpart of the above system.

The Budgetary Regime

Under the budgetary regime the system's characteristic polynomial is

\[ K(\lambda; \ddot{t}, \ddot{r}) = \begin{vmatrix} -\lambda & 0 & \beta \gamma \\ -w^* & -\lambda & -\gamma w^* \\ (1-s)w^* - \ell & -w & s \end{vmatrix} \]

Setting \( K(\lambda; \ddot{t}, \ddot{r}) \) equal to zero gives

\[ \lambda^2 s + \lambda \gamma \{w^*[w + \beta(1-s)] - \beta \ell\} + \beta \gamma w^* w^* = 0. \]
The necessary and sufficient condition for stability is that the eigenvalues have negative real parts, i.e., that the coefficients of \( \lambda^i \), \( i=0,1,2 \), each be positive. Given the parameter restrictions introduced in the main text, the only ambiguous term is 

\[
\gamma[w^*w + \beta(1-s)] - \beta u.
\]

Assuming the marginal propensity to consume is positive (\( s < 1 \)), the only destabilizing influence is the responsiveness of private demand to the anticipated real interest rate. Specifically, an increase in the anticipated rate of inflation will lower the real interest rate, which will raise private demand, which raises actual inflation and the rate of change of anticipated inflation. This scenario is the basis of a well-known objection to pegged nominal interest rates.

Offsetting this possible train of events--and dominating it, if the system is stable--are two influences. First, an increase in the expected rate of inflation will reduce anticipated real per capita private disposable income, which reduces private demand, actual inflation, and the absolute rate of change of anticipated inflation. Second, if output rises above capacity, the resulting increase in actual inflation will slow down the rate of change of real per capita public debt, which tends to restore output to capacity via the wealth effect in aggregate demand.

In summary, the inflation tax is necessary for stability, and a positive responsive of private expenditure to a reduced anticipated real interest rate is necessary for instability.

In the case of perfect myopic foresight of the inflation rate \( (\beta - w) \), the system reduces to a first-order differential equation with eigenvalue \( \lambda = -w/[(1-s)w^* - \mu] \). Again, stability is ambiguous,
and the economic considerations underlying its presence or absence are broadly similar to those discussed above. Thus, the possibility of stability in a model with "passive money" (and without price-level jumps and terminal conditions on asset stocks) is not as far fetched as has been supposed (see, e.g., Sargent and Wallace (1973)). The important point is that the inflation tax is a critical stabilizing influence. Its role is analogous to that of the explicit tax on factor incomes in Christ's (1968) stability analysis.

Finally, in the case of fixed inflation expectations (\(\beta = 0\)), the system again reduces to a first-order differential equation, with eigenvalue \(\lambda = -\omega w^*/s\). In this case, therefore, the system is unambiguously stable.

The Monetary Regime

Under the monetary regime the system's characteristic polynomial is

\[
K(\lambda; \mu, \rho) = \begin{vmatrix}
-\lambda & 0 & 0 & \beta_Y & 0 \\
-k & -\lambda & 0 & -\gamma k & 0 \\
-k & 0 & -\lambda & 0 & -1 \\
(1-s)k & 0 & -\omega & s & (1-s) \\
\alpha k & 1 & 0 & -\phi k & 0
\end{vmatrix}.
\]

Setting \(K(\lambda; \mu, \rho)\) equal to zero gives

\[
[\lambda(1-s) + \omega][\lambda^2 \phi + \lambda Y(1-\alpha \beta) + \beta_Y] = 0.
\]

Given the parameter restrictions introduced in the main text, sufficient conditions for stability are

\[s < 1 \text{ and } \alpha \beta < 1.\]

The first of these conditions merely states that the marginal propensity to consume is positive. More interesting is the second condition, which states that product of the responsiveness of money demand to its
opportunity cost, and the adjustment speed of inflationary expectations, is less than unity. This is precisely the criterion obtained by Cagan (1956) in his classic study of hyperinflations. Cagan found that this condition was satisfied for each of the hyperinflations he studied.

In the case of perfect myopic foresight of the inflation rate ($\beta \to \infty$), the system yields a positive eigenvalue $1/\alpha$, so that the system is unstable. This instability, which can also be deduced directly from the condition $\alpha \beta < 1$, is more in line with the findings of Sargent and Wallace (1973). On the other hand, it is worth pointing out that if the real interest rate and real per capita net explicit taxes are endogenous and pegged respectively, rather than the other way round, then pegging monetary growth will stabilize the system under perfect myopic foresight of the inflation rate. The proof of this proposition is straightforward, albeit tedious, and is not shown.

The case of fixed inflation expectations ($\beta = 0$) is of special interest now, since it encompasses the case $\pi = \mu + n$, that is, inflation expectations are based on the fact that the public sector is known to peg monetary growth. This notion has attracted considerable attention lately. It is easily shown that the eigenvalues are then given by $-\omega/(1-s)$ and $-\gamma/\phi$, so that the system is stable. On the other hand, the system is unstable in the aforementioned case of pegged explicit taxes and an endogenous real interest rate. This result is in complete agreement with a result highlighted by Christ (1978). Equations (1.15) easily lend themselves to the task of generalizing it to the case of a positive yet finite expectations adjustment speed ($0 < \beta < \infty$).
We have discussed two policy regimes. One, the 'budgetary' regime, is characterized by pegged rates of nominal interest and real per capita public spending and net explicit taxes. The other, 'monetary' regime is characterized by pegged rates of nominal monetary growth, real interest, and real per capita public spending. It was argued that in either case, expansionary policy under the regime in question can be identified with a popular view on inflation, one asserting 'active' deficits and 'passive' money, and the other vice versa.

Within this framework we performed various exercises in short-run and long-run comparative statics. The short run was characterized by predetermined levels of aggregate public debt, the price level, and the anticipated inflation rate. (For the monetary regime we also took the stock of monetized public debt to be predetermined.) The short-run results yield a conventional income-multiplier or income-velocity explanation of the transmission mechanism, according as whether one adopts the budgetary or monetary characterization of expansionary policy.

The long-run results, on the other hand, contain some novelties, as do those on stability. Recalling that the difference between public spending and net explicit taxes was termed the 'legislated' deficit, our main results can be summarized as follows:
1. Under either policy regime, steady-state private disposable income is given by the difference between output and public spending. It follows that disposable income will decline one-to-one with an increase in public spending, but is invariant with respect to any change in net explicit taxes.

2. Under the budgetary regime, a unit increase in the real per capita legislated deficit will raise the rates of inflation and nominal monetary growth by the inverse of the real per capita stock of aggregate public debt.\(^5\)

3. Under the monetary regime, an increase in nominal monetary growth will raise the legislated deficit by the real per capita stock of monetized public debt, less the product of that variable and the interest elasticity of money demand.\(^6\)

4. Under the budgetary regime, the system may be stable even under perfect myopic foresight of the inflation rate. The inflation tax is necessary for stability, and a positive response of private expenditures to a reduced anticipated real interest rate is necessary for instability.

Finally, under the monetary regime too, the stability findings are noteworthy in that the critical stability criterion is precisely the same as that obtained by Cagan (1956), even though we worked with a third-order system involving the public-sector constraint, unemployment transients, and other complications, whereas Cagan's well-known model is first-order. The limiting case of perfect myopic foresight of the inflation rate yields a second-order system with the familiar saddlepoint instability property.
CHAPTER II

INFLATION AND PUBLIC POLICY IN A SMALL ECONOMY UNDER FIXED EXCHANGE RATES

1. INTRODUCTION

For a decade now, there has been intensive study of the causes and consequences of inflation in a small economy under fixed exchange rates, and the relationship of that issue to domestic policy. Broadly speaking, one can distinguish three stages in the evolution of the theoretical literature.

The first stage was extension of the Keynes-Phillips analysis of a closed economy to the case of a small open economy; see, e.g., Takayama (1969), Helliwell (1969), and Turnovsky and Kaspura (1974). These models tend to be short run, and focus (usually) upon the price level rather than the inflation rate. The effects of domestic fiscal and monetary policies received somewhat more attention than the question of imported inflation, although obviously those issues are closely related.

The second stage was the monetary approach, which in the present context should be taken to refer to an explanation of the balance of payments as well as inflation. The major contributions deal with the multi-country case and often flexible rates as well; see, e.g. Mundell (1971), Johnson (1973), Dornbusch (1973), and Girton and Roper (1977). But there are some noteworthy discussions which specifically proceed from the standpoint of a small open economy under fixed rates; see, e.g. Mundell (1971, Ch. 9), Shinkai (1973), and Dornbusch (1974).
One difference between the monetary and Keynes-Phillips approaches is that the former emphasizes the long-run equilibrium relationships which were outlined at the outset of Chapter I. For example, an elementary consequence of purchasing power parity applied to fixed parities is that a 1 percent increase in the foreign inflation rate will ultimately raise the domestic inflation and monetary growth rates by 1 percent, whereas domestic policy disturbances will have no effect on those rates. Another difference is greater emphasis on stock-adjustment phenomena, especially those relating to desired real balances. Finally (and perhaps this is the most striking difference), the monetary models abstract from the income-multiplier process, that being central to the conventional explanation of 'soft' aggregate demand and its unemployment ramifications.

The third stage has been to combine Keynes-Phillips and monetary ingredients, and others as well. The basic simplifying assumption underlying this approach is to treat variables such as stocks of financial wealth, inflation expectations and relative prices as predetermined or 'state' variables in the short run.

Comparatively early and straightforward examples include Parkin (1974) and Laidler (1975). The interest in additional complications is a little more recent. For example, Kierzkowski (1976) rectifies shortcomings of early work on the so-called 'Scandinavian' case of sheltered and exposed sectors with differing rates of secular productivity growth, Boyer (1977) and Branson (1977) emphasize portfolio effects, Kingston and Turnovsky (1978) pay particular attention to the appropriate specification of flow constraints, and Parkin (1978) goes to the case of rational expectations under either fixed or flexible rates.
These recent models can be summarized as exhibiting Keynes-Phillips properties in the short run, and monetary properties in the long run; although some monetary aspects, particularly in regard to the balance of payments, are apparent in the short run as well.

By opening up the model of the preceding chapter to international trade in goods and interest-bearing public debt, this chapter develops another 'third generation' model of a small economy under fixed rates. Kingston and Turnovsky (1978) is the basic source. Thus, expansionary domestic policy is a major object of interest, as is the proper accounting for inflation, interest and growth in the budget constraints of the public and private sectors. In addition, we consider the effects of an increase in the domestic-currency price of imports (induced either by an increase in the price of foreign currency or by an increase in the foreign-currency price of imports), and an increase in the rate of inflation of the foreign-currency price of imports. Here it is worth pointing out that the focus will be on the traditional definition of a 'small' country, namely, a country that produces a single, endogenously-priced good, which is partly exported and partly consumed domestically; and imports a single consumption good whose foreign-currency price is exogenous.

The remainder of this chapter proceeds as follows. Section 2 sets out the model. Section 3 considers the short run. Section 4 considers the long run. Section 5 examines the stability of the model. Section 6 offers a summary.
2. THE MODEL

Assumptions

Adapting the Chapter I model to the task at hand requires the following modifications of the assumptions set out there. First, the interactions between the home economy and the rest of the world must be specified. Second, some care is required to ensure that the assumed domestic policy regime is consistent with the assumed international environment. Third, for a multi-commodity economy it is natural to define a CPI. Fourth, there is the question of appropriate simplifying initial conditions and units in an open-economy framework.

With regard to the first point, this chapter assumes full international integration of the domestic capital market, partial integration of the goods market, and zero integration of the markets for domestic money stocks and labour. Additionally, at home and abroad the labour force grows at the same exogenously-given rate, and secular productivity growth is zero.

This environment rules out the 'monetary regime' of Chapter I. For it is well known that in a small country under fixed rates and zero productivity growth, the public sector must allow the rate of nominal domestic credit expansion to be consistent with endogeneity and long-run stationarity of the country's real intensive monetary base; see, e.g., Swoboda (1973). Hence the rate of growth of nominal domestic credit must be permitted to be endogenous. On the other hand, recognizing this constraint, and focusing on the case of perfect capital mobility, it is equally well known that the behaviour of domestic credit is unimportant in the sense that over any time horizon the only consequence of a given increase in domestic credit is to drive
out an equal quantity of international reserves. Accordingly, for consistency, simplicity, and without important loss of generality, we assume the domestic public sector pegs the real per capita domestic source component. In nominal terms, therefore, domestic credit expands at a rate equal to domestic price growth plus world labour-force growth. It would make no significant difference if, for instance, the domestic public sector were to peg the proportional reserve backing of the domestic monetary base—as assumed, for example, in Johnson (1973) and Chapter IV below.

Another constraint arises from perfect capital mobility per se. Suppose the home public sector were to issue interest bearing debt at a rate permanently in excess of the sum of the secular inflation rate and growth rate of the world economy. Then in the absence of unequal productivity growth at home and abroad, it is inconceivable that the rest of the world would continue to view the small country's securities as perfect substitutes for its own. Nor is it conceivable that the domestic public sector would issue debt at a rate permanently below the secular global inflation-cum-growth rate; in that event the domestic public sector would accumulate claims on other sectors, in real per capita terms, without limit. In short, the real per capita aggregate debt of the domestic public sector must eventually be stationary in real per capita terms; not just the real per capita monetized debt of the domestic public sector.

It follows that the home fiscal authority must eventually pick rates of spending and net explicit taxation which together ensure stationarity of its real per capita public debt. If rates of nominal monetary growth and real interest in the rest of the world conform to the full-liquidity and golden rules respectively, then it turns out that
the legislated deficit of the home public sector must equal zero. If, on the other hand, nominal monetary growth in the rest of the world is in excess of the full-liquidity rate (with the real interest rate continuing to conform to the full-liquidity rule) then the domestic public sector must spend more than its net explicit tax receipts.

As an empirical matter, however, one would expect this constraint to apply only in the very long run, so that quasi-equilibria of the kind discussed by Mundell (1968), Swoboda (1972), and Turnovsky (1977, Ch. 11) are worthy of attention. Accordingly, we shall consider three kinds of equilibria: the short run; a steady state quasi equilibrium wherein there is a current-account deficit (surplus), financed by a matching capital-account surplus (deficit) and created by a sustained domestic budget deficit (surplus); and long-run full equilibrium.

The remaining aspects of our specification of public policy are as follows. To peg the price of the domestic currency, the domestic public sector is assumed to maintain a stockpile of the interest-bearing liabilities of the rest of the world. This assumption is chosen in preference to the traditional one of noninterest-bearing international reserves partly because of evidence that most reserves is the postwar period have been held in the form of interest-bearing public debt (see, e.g., Whitman (1974)), and partly because this assumption turns out to be easier to work with than the traditional one, providing we assume also that the domestic-currency equivalent of the interest earned by international reserves is rebated to the domestic private sector by distributionally neutral means. But this last-mentioned assumption turns out not to make a great deal of difference until the analysis is extended to two or more countries; see Chapter IV.
The domestic public sector is assumed also to peg its real per capita public spending and net explicit taxes, as in the case of the 'budgetary regime' discussed in Chapter 1. Public spending falls entirely on the home-produced good, explicit taxes are levied entirely on home private incomes, and the public debt is denominated wholly in terms of domestic currency. Each of these assumptions admits various straightforward extensions.

Third, turning to the question of a CPI, several authors have postulated a multiplicatively-weighted average of the price of the home good and the domestic-currency price of the foreign good; see, e.g., Boyer (1977), Kingston and Turnovsky (1978). Thus we introduce

\[ P_d = P^{1-m}(P_w \bar{E})^m \]

where \( w \) used as a subscript denotes foreign variables, and

- \( P_d \) = domestic-currency cost of living in the home country,
- \( P \) = domestic-currency price of home output,
- \( P_w \) = foreign-currency price of foreign output, exogenous to the small domestic economy,
- \( \bar{E} \) = price of foreign currency, taken to be pegged,
- \( m \) = share of the foreign good in domestic private spending when the terms of trade are unity, \( m \) constant, \( 0 < m < 1 \).

Whilst this form is typically chosen for convenience, it does have some theoretical merit. It is the 'true' cost of living index if the domestic residents' utility function, defined with domestic goods and imports as its arguments, is Cobb-Douglas; see, e.g., Samuelson and Swamy (1974).
This and subsequent chapters utilize more frequently its equivalent versions

\[ P_d / P = \sigma^m, \quad P_d = (1-m)p + m(p_w + e) \]

where \( \sigma \equiv EP_w / P = \) inverse of the commodity terms of trade faced by the home country,

\[ P_d \equiv \frac{\dot{P}_d}{P_d} = \text{domestic-currency rate of inflation of the home cost of living}, \]

\[ p \equiv \frac{\dot{P}}{P} = \text{domestic-currency rate of inflation of the price of domestic output}, \]

\[ p_w \equiv \frac{\dot{P}_w}{P_w} = \text{foreign-currency rate of inflation of the price of foreign output}, \]

\[ e \equiv \frac{\dot{E}}{E} = \text{rate of inflation of the price of foreign currency, or, the exchange depreciation rate, with } e = 0 \text{ under fixed rates.} \]

Finally, with regard to initial conditions and convenient units, we shall continue to invoke Assumption 8 and equations (1.7)' as is convenient. Those conditions turn out to imply that trade is balanced in the initial steady state. In addition, suppose that domestic portfolios are 'covered' in the initial state, by which we mean that a proportion \( m \) of domestic private wealth is allocated to bonds denominated in foreign currencies. In consequence, domestic wealth in terms of overall purchasing power is invariant at each instant with respect to a devaluation (see Section 3). If devaluations and revaluations of the domestic currency are viewed by the domestic private sector as equiprobable events (and that is the natural starting point of devaluation analysis), then this portfolio allocation will be the optimal
hedging strategy, providing investors are risk averse. See Girton and Henderson (1977), and Boyer (1977) for analyses of 'uncovered' initial portfolio allocations. And on the question of appropriate units, we set the price of foreign exchange and the terms of trade equal to unity in the initial state.

The Model

Exposition of the model follows the general format of the Chapter I counterpart of this section. First, market-clearing assumptions, and a definition of the legislated deficit are set out (cf. equations (1.1)):

\[ Q = Z + G + X, \quad q = z + \bar{g} + x \]  
\[ C + F = L, \quad \bar{c} + \bar{f} = \bar{\ell} \]  
\[ D = G - T, \quad \delta = \bar{g} - \bar{t} \]

where \( q = Q/PN = \) per capita supply of the domestic good, measured in terms of itself,

\( z = Z/PN = \) per capita supply of aggregate domestic private demand, measured in terms of the domestic good,

\( x = X/PN = \) per capita domestic trade balance, measured in terms of the domestic good,

\( c = C/PN = \) per capita domestic source component of the domestic monetary base, measured in terms of the domestic good, and taken to be pegged,

\( f = F/PN = \) per capita foreign source component of the domestic monetary base, measured in terms of the domestic good;

and \( g, t, \ell \) and \( \delta \) require no further discussion.
Equation (2.1b)' embodies our assumption that the 'real' per capita domestic source component of the domestic monetary base is pegged. By defining 'real' here to mean purchasing power over the home good, it is implied that the domestic public sector does not monetize the capital gains on its reserves that ensue from a devaluation. See Mundell (1971, Ch. 9) for a detailed discussion both of this case, and of alternative budgetary processes whereby the public sector views such capital gains as a source of revenue available for widening its legislated deficit.

Consider now the equations explaining private demand for goods and money, the aggregate supply of goods, and the trade balance:

\[ z = \tilde{z} + (1-s)y - \ell(r_w - \pi) + \omega \]  
\[ \ell = kq^\phi \exp(-ar_w) \]  
\[ p = \gamma(q - q_F) + \pi \]  
\[ x = \xi_wh^\eta - mz \theta^{1-\eta} \]

where \( y \) = per capita anticipated private disposable income, measured in terms of the domestic cost of living (see (2.3)' below, \( r - \pi \) = anticipated real interest rate, measured in terms of the domestic cost of living, \( r = r_w \) = domestic nominal interest rate, tied to the exogenously-given world rate \( r_w \), \( \pi \) = anticipated rate of inflation of the domestic cost of living,
\[ w \equiv \frac{W}{P_dN} = \text{per capita wealth of the domestic private sector, measured in terms of the domestic cost of living,} \]

foreign demand for real per capita domestic output in the initial state, a constant,

\[ \eta_w = \text{foreign Slutsky-compensated price elasticity of demand for domestic output, a constant,} \]

\[ \eta_f = \text{domestic Slutsky-compensated price elasticity of demand for foreign output, a constant.} \]

The choice of units for money demand warrants further comment. It might appear more reasonable to write the left-hand side of (2.2b) as \( L/P_{dN} \) rather than \( L/PN \). But if we reduce the right-hand side to the elementary 'Cambridge' case \( \phi = 1 \) (a unitary 'income' elasticity of money-demand) and \( \alpha = 0 \) (no opportunity cost of holding real balances), then our specification reduces to the familiar \( L = kQ \), whereas the above-mentioned alternative gives rise to the non-standard formulation \( L = k\sigma^mQ \). Also, the empirical study by Grassman (1976) suggests that \( P \) rather than \( P_d \) is the appropriate choice of deflator.

All the constants in (1.2) are positive. Our constant-elasticity specifications of export and import demands are for convenience. The export function is written solely in terms of relative prices, reflecting the fact that all the other determinants of export demand are exogenous to the small domestic economy and can therefore be embodied in the constant term \( \xi_w \).

Consider next the equations for nominal and real per capita private disposable income:
Equation (2.3) parallels its closed-economy counterpart (1.3). To see the role of our assumption that interest payments on international reserves are rebated to the private sector, observe that domestic private holdings of the interest-bearing debt issued by the domestic or foreign public sectors is given by \( W - L \) (not \( W - C \)), and note that (2.3) can be written as

\[
Y = Q - T + r_w (W - L) + r_f W
\]

The components of (2.3)' are measured in terms of the domestic cost of living (rather than the domestic good), so that the model incorporates both the income and wealth effects of a change in the terms of trade (see, e.g., Laursen and Metzler (1950) and Dornbusch (1973) respectively). Evidently we have opted for an index-number solution to the problem of incorporating these effects, as proposed some time ago in the case of the income effect (see Machlup (1956)).

We now turn to the dynamic equations of the model. These describe the accumulation of assets, the formation of inflation expectations and the evolution of the terms of trade.

The equation for the accumulation of assets by the domestic private sector may be derived from two other equations. One is the domestic public-sector constraint and the other is the domestic balance of payments. The former has nominal and real per capita forms

\[
\dot{V} = D + r_w (V - C), \quad \dot{\bar{V}} = \bar{\delta} + (r_w - p)(v - \bar{C}) - p\bar{C} - \bar{n}v
\]

where \( v = V/PN = \) per capita aggregate public debt issued by the domestic public sector, measured in units of the domestic good, so that \( v - C \) is the domestic public sector's real per capita interest-bearing debt. These equations parallel their closed-economy counterparts.
counterparts (1.4a) and (1.4a)'. To see the role of our assumption that interest payments on international reserves are rebated to the private sector, simply add \( \frac{r_F}{w} \) to both sides of (2.4a).

The equation for the domestic balance of payments has nominal and real per capita forms

\[
\dot{F} = X + r_w (W - V) + (V - C) - (W - L), \quad \dot{f} = x + (r_w - p)(w - v) + (v - c) - (w - \ell) - n(w - v).
\]

Equation (2.4b) says that the balance of payments is equal to the trade balance \( X \), plus interest earnings from the net indebtedness of the rest of the world to the domestic country, \( r_w (W - V) \), plus the capital account balance, \( (V - C) - (W - L) \), where each of these items is expressed in terms of the domestic currency. Equation (2.4b)' reveals that in real intensive terms, net domestic earnings from the net indebtedness of the rest of the world are reduced by the inflation-cum-growth tax on that indebtedness, i.e. \( (p+n)(w-v) \).

To explain the accumulation of assets by the domestic private sector, use (2.1b) to eliminate \( L \) and \( F \) from (2.4b), yielding

\[
X + r_w (W - V) + (V - W) = 0,
\]

and then use (2.4a) to eliminate \( V \), which leads to the nominal and real per capita forms of the wealth accumulation equation:

\[
\dot{W} = D + X + r_w (W - C), \quad (2.5a)
\]

\[
\dot{w} = \frac{\dot{W}}{\sigma m} + \left[ r_w - (1-m)p + mp_w \right] \left( w - \frac{w}{\sigma m} \right) - \left[ (1-m) p + mp_w \right] \frac{c}{\sigma m} - nw \quad (2.5a)'
\]

Equation (2.5a) says that the rate of accumulation of aggregate public debt by the domestic private sector is equal to the domestic legislated deficit \( D \), plus the trade surplus \( X \), plus interest earned from domestic private holdings of interest-bearing public debt together with
domestic public holdings of reserves, \( r_w(W-C) = r_w(W-L) + r_wF \) where each of these items is expressed in terms of the domestic currency. Equation (2.5a)' reveals that in real intensive terms, where 'real' here means overall domestic purchasing power, inflation and growth taxes must be taken into account. It also incorporates tedious but straightforward conversions, involving \( \sigma \), to ensure items are commensurate.

The formation of inflationary expectations is explained by
\[
\hat{\pi} = \beta[(1-m)p + m\pi_W - \pi], \tag{2.5b}'
\]
which is just the analogue of (1.4b)' in a two-commodity world.

The evolution of the terms of trade is described by
\[
\hat{\tau} = \sigma(p_w - p) \tag{2.5c}'
\]
which follows from the identity \( \hat{\tau} = \sigma(p_w + e - p) \) together with our assumption \( e = 0 \), where the identity in question holds only for points in time at which \( P, P_w \) and \( E \) are differentiable with respect to time. With respect to our assumptions concerning 'sluggish adjustment', we assume that \( W, P, P_w, N \) and \( \pi \) have that property. Thus two impacts of a devaluation are first, the level of the domestic-currency price of imports rises by the full amount of the devaluation, and second, the domestic-currency price of the home good does not change.

This concludes the exposition of the dynamic relationships. The model as a whole, therefore, contains the instantaneous relationships (2.1)' to (2.3)', which determine \( q, z, x, \ell, f, p, \) and \( y \); the dynamic relationships (2.5)', which proximately explain the predetermined variables \( w, \pi, \) and \( \sigma \); the policy variables, \( \bar{g}, \bar{t}, \bar{c}, \) and \( \bar{E} \); and various exogenous variables and parameters including \( P_w, p_w \) and \( r_w \).
The real per capita aggregate public debt of the domestic public sector, $v$, is determined recursively by the above equations together with (2.4a)', and in the absence of jumps in $k$ or $c$, real per capita reserves $f$ are proximately explained by (2.4b)'. For a summary of the model, see Section 5.

The exposition of the model as a whole is finalised by reproducing our closed-economy initial conditions (1.7)' and introducing new initial conditions. Thus:

$$\mu_o = 0, \delta_o = 0, \rho_o-n = 0, k = k \quad (2.6a)', (2.6b)', (2.6c)'; (2.6d)'$$

where $\mu = C/C = rate of growth of nominal domestic credit. Under (2.6c)', and in the steady state, (2.6a)' is equivalent to (2.6b)', and/or any one of

$$\pi_o + n = 0, p_o + n = 0, p + n = 0, (L/L)_o = 0, (F/F)_o = 0, x_o = 0,$$

where the $x_o = 0$ equivalence holds only in full equilibrium. Further aspects of these equivalences are discussed in Section 4. As in Chapter I, $q_o = 1$. Finally, units are chosen so that

$$\sigma_o = 1, \bar{E}_o = 1. \quad (2.6e)', (2.6f)$$

Our next topic is the comparative statics of the short run.

3. THE SHORT RUN

Fiscal Expansion

Consider first the impacts of increased spending by the domestic public sector. Equations (2.1a)', (2.2a)', (2.2d)' and (2.6)' together yield

$$\frac{dg}{dg} = \frac{1}{1-h}, \quad (2.7a)'$$
where \( h = (1-s)(1-m) \) = marginal propensity of the domestic private sector to spend its income on the home good.

Equation (1.7a)' is merely a way of expressing the standard foreign trade multiplier. Stability considerations suggest \( h < 1 \) (see Section 5). Assume that inequality.

This result leads to the impacts

\[
\begin{align*}
\frac{dp}{dg} &= \frac{\gamma}{1-h} , \quad \frac{dp_d}{dg} = \frac{\gamma(1-m)}{1-h} \\
\frac{df}{dg} &= \frac{dk}{dg} = \frac{\phi k}{1-h} \\
\frac{dz}{dg} &= \frac{1}{1-h} , \quad \frac{dx}{dg} = \frac{-\gamma h}{1-h} \quad -1 \\
\frac{dy}{dg} (= \frac{d\gamma}{dg}) &= \frac{1}{1-h} 
\end{align*}
\]

These results, too, are straightforward. The increase in domestic output will raise the rate of inflation of the price of the domestic good. The rate of inflation of the overall cost of living will rise also, though not by as much. The increased transactions demand for money, coupled with the pegged domestic source component, will induce a stock-shift improvement in the economy's external reserves. The increase in aggregate domestic spending, on the other hand, and (in particular) increased imports, leads to a deterioration in the domestic trade balance. Finally, anticipated real per capita private disposable income will rise by the same amount as output.
Devaluation

Consider next the effects of a devaluation which is unanticipated and expected never to be repeated once it occurs. It is convenient to begin with the impact on domestic wealth and the cost of living. Since \( w = W/P_dN \) we have
\[
\frac{dw}{dE} = \frac{P_dN(dW/dE) - WN(dP_d/dE)}{(P_d N)^2} = mw - W = 0.
\]

With 'covered' domestic portfolios, a 1 percent devaluation will raise nominal domestic wealth by \( m \) percent. On the other hand, it will also raise the domestic-currency price of imports and thence the domestic cost of living by \( m \) percent, so that the overall purchasing power of domestic wealth is unchanged. This result is contrary to its analogue in simple monetary models (see, e.g., Mundell (1971), Swoboda (1977)), which assume that domestic currency is the only asset in domestic portfolios, so that in the short run a devaluation will reduce the overall purchasing power of domestic wealth, and can be likened to a capital levy by the domestic public sector on the domestic private sector (Mundell (1971, Ch. 9)).

Turning to the effect of the disturbance in question on domestic output, first recall (2.1a)', (2.2a)', (2.2d)', (2.6)', and the result \( dw/dE = 0 \); then note that \( y = z \) and \( mz = \varepsilon_w \) in the neighbourhood of the initial steady state. This yields
\[
\frac{dq}{dE} = \frac{\varepsilon_w[\eta + \eta_w - (1+h)]}{1 - h}
\]
Assuming \( h < 1 \), our criterion for a devaluation to raise output in the short run is that the sum of the Slutsky-compensated price elasticities of import and export demand, \( \eta + \eta_w \), exceed unity plus the marginal propensity of domestic residents to consume the home good, \( 1 + h \). This is a more stringent condition than one often found in the literature, namely, \( \eta + \eta_w - 1 > 0 \) (see, e.g., Johnson (1978)). The reason for the difference is that the literature often abstracts from the income effect of a currency depreciation (with Laursen and Metzler (1950) being the classic exception to that popular omission).

Using this result we deduce the impacts

\[
\frac{dp}{dE} = \frac{\gamma \xi_w (\eta + \eta_w - (1+h))}{1-h}, \quad \frac{dp_d}{dE} = \frac{\gamma (1-m) \xi_w (\eta + \eta_w - (1+h))}{1-h}
\]

\[
\frac{df}{dE} = \frac{dk \xi_w (\eta + \eta_w - (1+h))}{1-h}
\]

\[
\frac{dz}{dE} = \frac{(1-s) \xi_w (\eta + \eta_w - 2)}{1-h}, \quad \frac{dx}{dE} = \frac{\xi_w [s (\eta + \eta_w - 1) + (1-s-h)]}{1-h}
\]

\[
\frac{dy}{dE} = \frac{\xi_w (\eta + \eta_w - 2)}{1-h}
\]

The rates of inflation of the price of the domestic good and the overall cost of living, and the level of reserves in domestic currency, will all increase or decrease according as whether domestic output rises or falls; the explanation is essentially the same as that given in connection with the public spending multipliers. Note that the criterion for a devaluation to improve reserves measured in terms of
foreign currency is more stringent than $\eta + \eta_w > 1 + h$, since
$$d(PNf/E)/dE = PN(df/dE - f) \leq PN(df/dE).$$

The criterion for a devaluation to improve per capita domestic private spending measured in terms of the domestic good, $z$, and anticipated per capita private disposable income measured in terms of the overall cost of living, $y$, are also more stringent than the output criterion. This is a consequence of the income (or 'terms-of-trade') effect discussed above. Finally, the ordinary Marshall-Lerner criterion, i.e., $\eta + \eta_w - 1 > 0$, is sufficient to ensure that a devaluation will improve the trade balance, given a positive marginal propensity to spend on imports out of disposable income, i.e., $1-h-s > 0$.

Foreign Inflation

Consider finally the effects of an increase in the foreign-currency price of imports $P_w$, and of an increase in the rate of inflation of the foreign-currency price of imports, $p_w$.

Once again it is convenient to begin with the impact on domestic wealth and the cost of living. We have
$$\frac{dw}{dp_w} = \frac{P_d N(dW/dP_w)}{(P_d N)^2} - WN(dp_d/dP_w) = - m_w.$$  

In contrast to our treatment of the price of foreign exchange, we have set up the model in a way that does not let the private sector hedge against increases in the foreign-currency price of imports.
Hence an increase in the foreign-currency price of imports will reduce the overall purchasing power of domestic wealth, as does an increase in \( P_d \) or \( E \) in simple monetary models.

Turning to the effect of the disturbance in question on domestic output, proceed as for the derivation of (2.8a)', to obtain

\[
\frac{dq}{dP_w} = \frac{dq}{dE} \left( \frac{(h+s)\mu \eta \theta}{1-h} \right).
\]

(2.9)'

The wealth effect of an increase in the foreign-currency price of imports means that a devaluation which raises the domestic-currency price of imports by the same amount is more expansionary (or less contractionary) than the disturbance presently under consideration.

But the following illustrative calculation suggests that the difference is unlikely to be substantial. Consider (2.2a)'. If aggregate domestic spending is approximately homogeneous of degree one in the neighbourhood of the golden-rule rate of interest, then \( z = \ln \). Moreover, we shall see that \( y \approx z \) in the neighbourhood of the initial full steady state. These considerations imply that private spending and wealth stand in the relation \( sz \approx \mu \eta \). Together with (2.8a)' and the familiar approximation \( x \approx 0 \), that relationship yields

\[
\frac{dq}{dP_w} \approx \frac{\mu \eta \theta}{\mu \eta \theta} \left[ 1 + h + s(h+s) \right] \frac{1}{1-h}.
\]

If \( h \approx .7 \) and \( s \approx .1 \) then \( s(h+s) \approx .08 \), a number which is unlikely to be critical in determining the magnitude of \( dq/dP_w \) either on its own or vis-à-vis the magnitude of \( dq/dE \).
The upshot of this illustrative calculation is that in the short run, the elasticity and income effects are likely to dominate the wealth effect of the disturbance in question. This line of reasoning, which carries over to the other impact multipliers (in particular $df/dP_w$ and $dx/dP_w$), is contrary to the thrust of much recent analysis; see, e.g., Johnson (1978).

Other impacts of an increase in the foreign-currency price of imports may be derived and analyzed in an entirely similar fashion. There is nothing of substance to be added to our derivations and discussions of (2.8b)' and (2.9a)'.

We conclude this section by considering the impacts of an increase in $p_w$. Suppose first that there is no accompanying increase in the foreign nominal interest rate $r_w$. Then the Chapter I discussion of the impacts of an increased rate of nominal monetary growth is relevant. Specifically, the levels of the instantaneous variables are independent of $p_w$ (with the impact $dp_d/dp_w = m$ being a trivial exception) but their rates of change are not.

In the present case, the relevant transmission mechanisms are threefold, involving the rates of change of real per capita wealth, inflation expectations, and the terms of trade. Equations (2.5)' and (2.6)' together yield
\[
\frac{dw}{dp} = -mw, \quad \frac{dT}{dp} = m\beta, \quad \frac{d\sigma}{dp} = \sigma.
\]

Observing that \( q = q(w, \pi, \sigma) \) and proceeding as for (2.8a)' and (2.9a)', we deduce

\[
\frac{dq}{dp} = \frac{dq}{dp} = \frac{m(1-m)\beta}{1-h}
\]

so that the interpretation of (2.10)', parallels that of (2.9)', except that now there is an additional positive term due to the negative impact of an increase in \( p_w \) on the absolute rate of change of the anticipated real interest rate, \( (r_w - \pi) \). Analogous reasoning carries over to the other \( p_w \) impacts.

Suppose now that an increase in \( p_w \) is accompanied by an increase in the foreign nominal interest rate \( r_w \). Then it is clear from the foregoing that all that matters for the levels of the instantaneous variables (except for \( P_d \)) is the increase in \( r_w \). Indeed, the disturbance in question and its short-run effects are completely analogous to those of an interest-rate hike in a closed economy under the budgetary regime (see Chapter I). Thus, for example, there are substitution and income effects in the product market; and since the former are conventionally viewed as dominant, there is a presumption that domestic output and the rate of inflation of its price will both fall.

We conclude this section by drawing attention to a pitfall of our analysis of the domestic effects of increases in the level and
rate of change of the foreign-currency price of imports. If one views inflation as being some kind of sustained public-sector disturbance, then the analysis of 'imported inflation' should proceed from a specific model of the behavior of the foreign public sector, and should recognize that its domestic impacts will operate through the entire vector of foreign variables impinging on the domestic economy, namely \((P_w', P_w, r_w, \xi_w)\). In short, a multi-country analysis is required.

4. THE LONG RUN

First this section considers some long-run equilibrium relationships which are valid under either quasi or full long-run equilibrium. Then it distinguishes between those alternative characterizations of the long run, and gives an account of their specific properties.

Long-Run Equilibrium Relationships

The definition of \(p_d\) and \(\rho\), and equations (2.1)' to (2.5)', together yield the steady-state relationships

\[
\begin{align*}
 p^* &= p_c^* = \eta^* = p_w^* \quad (2.11a)' \\
 \rho^* &= r^* - \tau^* = r_w - p_w^* = \rho_w^* \quad (2.11b)' \\
 q^* &= q_F^* \quad (2.11c)' \\
 (L/L)^* &= \mu^* = (F/F)^* = (W/W)^* = p_w^* + n \quad (2.11d)' \\
 z^* &= q_F^* - \bar{g} - x^* \quad (2.11e)' \\
 y^* &= z^*(r^*)^{-m} \quad (2.11f)'
\end{align*}
\]

Equations (2.11a)' and (2.11b)' state that anticipated and actual rates of domestic inflation and real interest are tied to their exogenously-given world counterparts. Equation (2.11c)' says that domestic real per capita output is tied to its 'natural rate'.
Equations (2.11d)' say that the rates of growth of all nominal domestic financial quantities (with the "quasi exception" of V) are tied to the inflation-cum-growth rate of the world economy. Finally, (2.11e)' and (2.11f)' are just the open-economy counterparts of (1.11e)' and (1.11f)'.

As a direct consequence of these relationships we observe that the long-run effects of fiscal policy on domestic output, inflation and interest rates are zero, as are those of a devaluation or an increase in the foreign price level. A 1 percent increase in the foreign inflation rate will raise the rates of growth of domestic nominal variables (with the possible exception of V) by 1 percent. These relationships, together with (2.1b)' and (2.2b)', show that the real per capita domestic money supply and reserve stock, measured in terms of the domestic good, are also invariant with respect to the above disturbances, unless an increase in the foreign inflation rate is accompanied by an increase in the foreign nominal interest rate, in which event the effects parallel those of increased nominal monetary growth in a closed economy (see Chapter I). Finally, any increase in domestic real per capita public spending will reduce the sum of real per capita foreign and private domestic spending on the home good, $z^* + x^*$, by the same amount.

**Long-Run Quasi Equilibrium**

Proceeding as for equations (2.11)',

\[
-x^* = \bar{\delta} - (p_w + n)\bar{c} \quad (2.12a)'
\]

\[
\bar{v}^* = \bar{\delta} - (p_w + n)\bar{c} \quad (2.12b)'
\]
Equation (2.12a)' says that as a consequence of the cessation of domestic wealth accumulation in real per capita terms, and if the world interest rate conforms to the golden rule, then the quasi-equilibrium domestic trade deficit will equal the domestic 'legislated' budget deficit minus the inflation-cum-growth tax on the real per capita monetized debt of the domestic public sector.

This proposition stands in marked contrast to the 'New School' theory of external balance, which asserts \( x^* + \delta^* = 0 \) (see, e.g., Godley and Cripps (1973)). That theory proceeds from two premises. First, as an accounting identity, what we have termed the 'legislated' deficit will equal the trade deficit plus the gap between private savings and investment. Second, the equilibrium gap between private savings and investment is zero.

The incorrect reasoning in this argument is failure to recognize that the relevant tax variable is net explicit taxes plus the inflation-cum-growth tax, which means that one or the other of the above premises must be modified. The procedure adopted in this dissertation, of course, has been to accept the nominal counterpart of the foregoing accounting identity (and also to set investment equal to zero), and then go on to show that the relevant tax variable in the real per capita counterpart of this and related identities includes the inflation-cum-growth impost.

As a final comment on this issue, it is worth pointing out that the trade deficit of most industrialized countries nowadays is seldom more than the 'legislated' deficit, as would be predicted by our approach.

Equations (2.12a)' and (2.12b)' together embody the familiar
proposition that an equilibrium trade deficit is financed by a matching capital account surplus (observe that $v^* = (v^* - c) = (v^* - w^*)$), and can be attributed to the domestic public sector's foreign borrowing.

What are the comparative-static implications of a long-run quasi equilibrium? In the first place, equations (2.12)' and (2.1c)'

together imply

\[
\begin{align*}
\frac{dx^*}{dg} = \frac{dv^*}{dg} &= 1 \\
\frac{dx^*}{dt} = \frac{dv^*}{dt} &= -1
\end{align*}
\]

That is, any increase in spending by the domestic public sector on the home good will raise the home trade deficit and rate of external borrowing, by the same amount. The reverse is true for any increase in the net explicit taxes raised by the domestic public sector from the home private sector. These are standard propositions in the theory of small open economies under perfect capital mobility; see, e.g., Mundell (1968) and Turnovsky (1977, Ch. 11).

Second, we have seen that any increase in spending by the domestic public sector on the home good will also lower the quantity of domestic output consumed by other sectors, $z^* + x^*$, by the same amount. Analogous reasoning establishes that $z^* + x^*$ is invariant with respect to any change in domestic net explicit taxes. These propositions, (2.11f)', and (2.13a)' together imply

\[
\begin{align*}
\frac{dz^*}{dg} = \frac{d[y^*(v^*)^m]}{dg} &= 0 \\
\frac{dz^*}{dt} = \frac{d[y^*(v^*)^m]}{dt} &= -1
\end{align*}
\]

That is, domestic private spending and income, measured in terms of
the home good, are invariant with respect to any increase in domestic public spending on the home good, but will decline one-to-one with any increase in the net explicit taxes imposed on the domestic private sector. This is precisely the reverse of one of our key closed-economy propositions (see Chapter I).

Third, using (2.2d)', (2.6e)' and (2.13b)', we can establish

\[
\begin{align*}
\frac{d\sigma^*}{dg} &= -\frac{1}{\xi(\eta + \eta_w - 1)}, & \frac{dy^*}{dg} &= \frac{1}{\eta + \eta_w - 1} \\
\frac{d\sigma^*}{dt} &= \frac{1 - m}{\xi(\eta + \eta_w - 1)}, & \frac{dy^*}{dt} &= -1 - \frac{1 - m}{\eta + \eta_w - 1}
\end{align*}
\]

(2.13c)'

Each of these expressions depends on the standard Marshall-Lerner criterion, \( \eta + \eta_w - 1 \). In the standard barter model (see, e.g., Mundell (1968, Part I), and our model too (see Section 5), we require \( \eta + \eta_w - 1 > 0 \) for stability. Accordingly, assume that this inequality is satisfied.

It follows that an increase in public spending will improve the home terms of trade, and raise domestic disposable income. The point underlying the former result is simple: if the system in general and the goods market in particular are stable, then increased demand for the domestic good will raise its relative price. The latter result is equally straightforward; we saw above that domestic private income and spending in terms of the home good is invariant with respect to a change in \( g \), but we have seen just now that an increase in \( g \) will enable its producers to trade on better terms.

An increase in net explicit taxes will worsen the home terms of trade, and reduce domestic disposable income. To interpret these results, note that the share of the domestic good in foreign aggregate private spending, \( m_w \) say, is necessarily zero—as a consequence
of our small country assumption. Then the expression for $\frac{d\nu^*}{d\tilde{t}}$ is seen to be precisely that yielded by the barter theory of transfers. The explanation is that an increase in the explicit tax on the domestic private sector will induce a one-to-one diversion of income (in terms of the home good) from the home private sector to the rest of the world, and the resulting excess demand for the domestic good (recall $m_m^w + m_w < 1$) requires a reduction in its relative price. Moreover, once we evaluate domestic disposable income in terms of its overall purchasing power, we find that the tax increase imposes a 'secondary burden' on the home private sector (the 'transferor').

Analogous considerations apply to the other comparative-static properties of a quasi equilibrium. First, domestic private wealth in terms of its overall domestic purchasing power, $w^*$, may be seen from (2.2a)' and (2.11c)' to respond in the same way as domestic private income, $y^*$. Second, an increase in foreign inflation $p_w$ will have effects on the variables just examined which are qualitatively the same as those of an increase in $t$—providing the foreign real interest rate remains constant (i.e., if $dr_w/dp_w = 1$), and providing the domestic monetary base is partly domestic in source (i.e., if $c > 0$). On the other hand, these variables are invariant with respect to a change in $p_w$, accompanied by no change in $r_w$, in the event that the domestic monetary base is wholly foreign in source (i.e., if $c = 0$, so that $\ell = f$). This is a straightforward consequence of our assumptions that international reserves earn market rates of interest and that interest earnings from reserves are rebated to the domestic private sector.

Finally, an increase in $p_w$ with $dr_w/dp_w > 0$ will induce
the domestic private sector to substitute money for bonds. The analogy with increased monetary growth in a closed economy is not quite exact, because here the accommodating variable is the rate of foreign borrowing rather than domestic net explicit taxes.

**Long-Run Full Equilibrium**

The preceding subsection has revealed several reasons why the public sector of a small open economy will have an incentive to finance part of its legislated deficit by borrowing on the world's capital market. But we should also consider the time horizon over which a small country's borrowing power is limited.

Analytically, a tractable way of incorporating this long-run constraint is to assume domestic explicit taxes vary as is necessary to ensure the cessation of aggregate debt creation in real per capita terms. Accordingly, replace equations (2.12)' by

\[ g^* - t^* - (p_w + n)c = 0 \]  \hspace{1cm} (2.14a)

whence

\[ x^* = 0. \]  \hspace{1cm} (2.14b)

That is, in long-run full equilibrium the public sector's budget and the nation's trade must balance. Section 5 below suggests one possible adjustment path for \( t \).

Instead of (2.13a)' we now have

\[ \frac{dx^*}{dg} = \frac{d\phi^*}{dg} = \frac{d\delta^*}{dg} = 0 \]  \hspace{1cm} (2.15a)

Any increase in public spending must be wholly financed by explicit taxes. And instead of (2.13b)' we have

\[ \frac{dz^*}{dg} = \frac{d[y^*(\sigma^*)^m]}{dg} = -1, \]  \hspace{1cm} (2.15b)

as in a closed economy.
Similarly, the full-equilibrium counterparts of (2.13c)' are

\[
\frac{d\sigma^*}{dg} = - \frac{m}{\xi (\eta + \eta_w - 1)} , \quad \frac{dy^*}{dg} = - 1 + \frac{m}{\eta + \eta_w - 1} .
\] (2.15c)'

Once again there are exact analogues of propositions in standard barter theory (see, e.g., Mundell (1968, Part I)). The expression for \(d\sigma^*/dg\) is identical to its barter counterpart for the effect of a productivity decrease on the terms of trade. The relevant story is also the same: in a stable market, a reduction in the amount of the home good available to the domestic private sector and the rest of the world will raise its relative price, provided that the other good is not a perfect substitute \((\eta, \eta_w < \infty)\), and that there is trade at the margin \((m > 0)\). And the expression for \(dy^*/dg\) is identical to its barter counterpart for the effect of a productivity decrease on real income: in a stable market, a reduction in the amount of the home good available to the domestic private sector and the rest of the world will lower domestic private factor incomes in terms of the home good, but will also improve the terms on which the good is traded, so that there is an ambiguous movement of domestic private income in terms of its overall purchasing power.

Turning to the other comparative-static properties of a full equilibrium, our remarks in the preceding subsection on the response of domestic private wealth continue to apply. The effects of an increase in the foreign rate of inflation, accompanied by a matching increase in the foreign nominal interest rate and in the presence of a positive domestic source component exactly parallel those of an increase in monetary growth in a closed economy. On the other hand, if the domestic monetary base is wholly foreign in source, then the
only domestic real effect of an increase in \( p_w \) (with \( \frac{dp_w}{dp_w} = 1 \))
is a one-to-one substitution by domestic residents of money for bonds. All other domestic real variables are invariant with respect to this particular disturbance.

5. STABILITY

This section considers the local stability of the system. As in the Chapter I counterpart of this section (cf. equations (1.15)'), we reduce the dimensionality of the system. Insert (2.3)' into (2.2a)', and (2.2d)' into (2.1a)'; and recall (2.2c)', (2.5a)', (2.5b)', and (2.5c)' . This gives

\[
z = z + (1-s)[(q-\hat{q})\sigma^m + (r_{-1})(w-c\sigma^m) - \eta c\sigma^m - mw] - t(r_w - \eta) + \omega w
\]

\[
q = z + \hat{q} + \xi_w \sigma^m - mz\sigma^{1-\eta}
\]

\[
p = \gamma (q-q_p) + \pi
\]

\[
\dot{w} = \bar{\gamma}^m + \xi_w \sigma^m - m\gamma \sigma^{1-\eta}
\]

\[
+ [r_w (1-m)p - mp_w] (w-\bar{\sigma}^m) - [(1-m)p+mp_w] \bar{\sigma}^m - mw
\]

\[
\dot{\eta} = \beta [(1-m)p + mp_w - \pi]
\]

\[
\gamma = \sigma (p_w - p)
\]

Equations (1.15)' describe a quasi equilibrium. The case of a full equilibrium (see Section 4) is dealt with below.

Upon invoking the initial conditions (2.6)' we obtain the characteristic polynomial
Setting $K(\lambda)$ equal to zero gives
\[
\lambda^3 (1-h) + \lambda^2 \left[ m [\beta (1-h) + w] + (1-m)^2 y [w^* + \beta ((1-s)w^* - \eta)] + \xi (l-w^*) \right] + \lambda \left[ \beta y [w^2 + y (1-m)^2 w^*] + \xi (l-h) + (1+\beta) (\eta + w^* - 1 - h) \right] + \beta y \xi (l-w^*) = 0.
\]

The necessary and sufficient condition for stability is that the eigenvalues have real parts; i.e., that the coefficients of $\lambda^i$, $i = 0, 1, 2, 3$ each be positive; and that the product of the coefficients of $\lambda^3$ and $\lambda^0$ exceed the product of the coefficients of $\lambda^2$ and $\lambda^1$.

Focusing on the sign condition for stability, we see that there is little to add to the analysis of the budgetary regime in Chapter I, except for two considerations. First, it is no longer necessary for stability that the relevant inflation-tax base, $w^*$, be positive. On the other hand, it is necessary that the Marshall-Lerner condition be satisfied, i.e., $\eta + w^* - 1 > 0$.

In the case of perfect myopic foresight of the CPI inflation rate ($\beta \rightarrow \infty$), the system reduces to a second-order differential equation. Given the Marshall-Lerner condition and the parameter
restrictions in the text, the necessary condition for instability turns out to be a positive response of private expenditure to a reduced real interest rate \((\ell > 0)\), so that a vertical domestic IS curve \((\ell = 0)\) is sufficient for stability.

In the case of fixed inflation expectations \((\beta = 0)\), the system again reduces to a second-order equation. Assuming \(h < 1\), a sufficient condition for stability is that the strong version of the Marshall-Lerner condition discussed in Section 3 be satisfied, i.e.,

\[
\eta + \eta_w - (1 + h) > 0.
\]

To put this another way, for instability we require the home good to be a Giffen good in domestic and foreign budgets.

Finally, what can be said about the stability of the real per capita stock of interest-bearing public debt issued by the domestic public sector, i.e., \(v - c\)? It is clear from (2.4a)' and conditions (2.6)' that \((v - c)^* = 0\), i.e., in the initial-steady state the variable in question is stationary. Recalling also (2.12b)', we conclude that \((v - c)\) is only neutrally stable if both real per capita domestic credit, \(c\), and net explicit taxes, \(t\), are pegged.

Towards the end of Section 4 we canvassed the possibility that net explicit taxes are endogenous, adjusting as is necessary to ensure the stationarity of \(v - c\). A point-in-time counterpart of that policy was not spelt out. One possibility is

\[
t = \dot{g} + (r_w - p_w)(v - c) - p_w c - nv,
\]

which meets our requirement that the domestic public sector eventually adjusts its net explicit taxes as is necessary to ensure that it does not issue any new debt in real per capita terms. Under the initial conditions (2.6)' this yields
\[ \hat{t} = 0 \]

where the hat notation denotes the perturbed state of a variable.

Hence the previous discussion of the stability of \( \sigma, \pi, w, z, q, \) and \( p \) under \( t = \hat{t} \) continues to apply. With regard to \( v \), \( (2.4a)' \), \( (2.5c)' \) and \( (2.1)' \) together imply

\[ \hat{v} = v \ln(\hat{v}) \]

so that \( v \)'s trajectory is monotonically related to that of the inverse of the terms of trade. To put this another way, the domestic public sector issues (retires) debt according as whether its terms of trade are deteriorating (improving).

In summary, there exists at least one policy rule which will drive the entire system to full equilibrium if and only if the system is stable in variables other than \( v \) (or \( v-c \)) under the quasi-equilibrium regime. A single stability analysis is sufficient.

6. SUMMARY

This chapter has extended our closed-economy model to the case of a small economy, albeit with some monopoly power in the market for its exportable, under fixed exchange rates. Recognizing that our assumption of perfect capital mobility implies the rest of the world effectively pegs the domestic nominal interest rate, most of this chapter has focused on a policy regime analogous to Chapter I's budgetary regime. On the other hand, recognizing that there is an 'ultra-long' time horizon over which the domestic public sector is also obliged to relinquish control over its legislated deficit, this chapter has also considered a policy regime more akin to Chapter I's monetary regime. The former regime was described as a quasi
equilibrium (see Mundell (1968), Swoboda (1972) and Turnovsky (1977)), and the latter was described as a full equilibrium, in which case legislated deficits can no longer be financed by foreign borrowing.

The model can be viewed as a synthesis of Keynes-Phillips and international-monetary elements, and our short-run results reflect that. Here we wish to draw attention to two points. First, we have gone to some lengths to distinguish between private disposable income measured in terms of its purchasing power over the home good, and private disposable income measured in terms of its overall purchasing power. Whereas this distinction is central to the standard barter theory of international trade, it is increasingly neglected in short-run analyses—at the same time the wealth effects of devaluation have been receiving more attention. Second, compensating simplifications were gained from using the import share parameter to weight the domestic CPI, and by confining attention to a highly specific initial state.

In the long run, domestic rates of inflation are tied to the world rate. This is an elementary consequence of the small-country assumption, and purchasing power parity under fixed parities. It follows that domestic rates of inflation are invariant with respect to expansionary domestic policies.

On the other hand, expansionary domestic policies will affect domestic private disposable income, in the long run as well as the short run. Besides being of interest in their own right, these effects yield some insight into the reasons why national public sectors may choose to implement expansionary policies. And if many small countries implement such policies, inflation rates may be
affected; see Chapter IV. Accordingly, it is worth highlighting the following results concerning the effects of expansionary domestic policies on domestic steady-state private disposable income:

1. Under either quasi or full equilibrium, domestic steady-state private disposable income in terms of the home good is given by capacity output, plus the trade deficit in terms of the home good, minus the domestic public sector's purchases of the home good. It follows that expansionary domestic policy will raise private income in terms of overall purchasing power only if it improves the terms of trade or worsens the trade deficit.

2. Under a quasi equilibrium, the public sector's spending multiplier with respect to private income in terms of overall purchasing power is given by the inverse of the Marshall-Lerner criterion, that expression being the same here as its counterpart in the standard barter model. Assuming stability, in which event the Marshall-Lerner condition is satisfied (along with various closed-economy stability criteria), we conclude that increased public spending financed by foreign borrowing will raise private income in terms of overall purchasing power.

3. Under a quasi equilibrium, the income effect of an income tax (financed by foreign lending) is precisely the same as that predicted by the barter model for the effect of a transfer of the home good by the domestic country to the rest of the world. Hence domestic private income in terms of the home good will rise one-to-one with a reduction in explicit taxes. And domestic private income in terms of overall purchasing power
must rise by more than that, since a tax cut financed by foreign borrowing will also improve the terms of trade.

4. Under a full equilibrium, the income effect of an increase in public spending on the home good is precisely the same as that predicted by the barter model for the effect of a productivity decline at home ("immiserizing growth" and all that). Hence domestic private income in terms of the home good will decline one-to-one with an increase in domestic public spending. But the movement in domestic private income in terms of overall purchasing power is not necessarily negative, for even a tax-financed increase in public spending will improve the terms of trade, albeit to a lesser extent than a spending increase financed by borrowing abroad.
1. INTRODUCTION

We have seen that the theoretical literature on inflation and public policy in a small economy under fixed exchange rates splits into three parts: the Keynes-Phillips approach, the monetary approach, and an eclectic approach based on synthesis of the other two. This classification could also serve for the case of a small economy under flexible rates. However it is more convenient to organize these introductory comments by focusing on the question of price anticipations—especially the anticipated price of foreign currency.

The traditional Keynes-Phillips model is based on the assumption of static price expectations, meaning that the domestic-currency prices of foreign currency, domestic output and foreign output are all expected to be constant. This is true, for example, of the Fleming-Mundell model; see, Fleming (1962), Mundell (1968, Ch. 17). Yet in the case of exchange-rate anticipations alone, intuition and evidence both suggest that in the absence of a credible commitment by national public sectors to fixed parities, the forward rate will be highly sensitive to current economic disturbances; for evidence, see, e.g., Black (1973).

The difficulties with static exchange-rate expectations are brought to the fore when we consider this assumption in conjunction with perfect capital mobility. With predetermined commodity prices,
domestic monetary expansion is able to shift the exchange rate only by shifting domestic output in the first instance. But monetary disturbances probably affect the spot rate much more promptly than that.

Several recent papers have resolved this problem by postulating that exchange-rate expectations are regressive. This general hypothesis has two variants at present. One postulates that the anticipated rate of exchange depreciation is negatively related to the gap between the exogenously-given 'normal' level of both the forward and spot rate, and the current spot rate; see, e.g., Argy and Porter (1972), Dornbusch (1976a,b), and Girton and Henderson (1976a), (1977). With conventional specifications of money demand (either with or without a 'wealth' demand for money), and regardless of whether or not output rates are taken to be predetermined, money supply shocks will influence the spot rate.

The other variant is due to Dornbusch (1976c). The distinguishing feature of his expectational scheme is that the 'normal' rate is equated to its steady-state solution value, where that solution is implicit in a simple general-equilibrium model whose steady state has the standard neutrality properties. Again, monetary shocks influence the spot rate regardless of whether or not output is predetermined. Indeed, the model necessarily exhibits the much-discussed 'over-shooting' phenomenon: a 1 percent expansion of the nominal domestic money supply that was previously unforeseen but is expected to persist will depreciate the spot rate by more than 1 percent in the short run (and by precisely 1 percent in the long run).
Another interesting proposition yielded by the Dornbusch model concerns the insulating properties of flexible rates. A 1 percent increase in the foreign-currency price of imports will appreciate the spot rate by precisely 1 percent in both the short run and the long, so that domestic real variables are fully insulated against this particular disturbance.

From the standpoint of explaining a sustained inflation, however, either version of the regressive hypothesis is subject to a critical limitation. Each proceeds in terms of levels of the nominal domestic money supply and the prices of foreign currency and output. Accordingly, Turnovsky and Kingston (1977) proceeded in terms of the percentage rates of change of prices, exchange rates, and money supplies, rather than in terms of the levels of those variables. That paper dealt with the expectations issue by assuming perfect myopic foresight of all variables, including the exchange depreciation rate.

This hypothesis has several desirable features. First, it is simple. Second, it is the continuous deterministic analogue of rational expectations, so that participants in the foreign exchange market are assumed not to make systematic forecasting errors. This is an advantage because the econometric evidence currently available on foreign exchange markets favours the hypothesis of efficiency in foreign exchange markets (see, e.g., Magee (1976)). Third, it permits the condition for stock equilibrium in the money market to play a proximate role in exchange-rate determination, which is also in line with current theory and tests (see, e.g., Magee (1976) and Girton and Roper (1977)).
On the other hand, several difficulties stem from the specification of anticipations in the Turnovsky-Kingston model. First, some of the results are counter-intuitive. For example, an increase in the rate of domestic monetary growth does not immediately raise the exchange depreciation rate; instead, it will temporarily exert pressure in the reverse direction. (In the long run, of course, a 1 percent increase in the domestic monetary growth rate will raise the domestic exchange depreciation rate by 1 percent.) This short-run result is a direct consequence of the fact that, given interest rate parity and perfect myopic foresight, the rate of exchange depreciation is directly analogous to the domestic nominal interest rate and therefore responds negatively to a monetary expansion. Second, the stability of the model is far from assured. This is not surprising, since the assumption of perfect myopic foresight typically introduces saddlepoint instabilities into financial markets and thence into complete systems in which they are embedded; Chapter I above discusses this problem in the context of a closed economy, and Kouri (1976) and Gray and Turnovsky (1978) pursue the topic in the context of a small open economy under flexible rates. Third, in Chapter I it was noted that the assumption of rational inflation expectations has never been very convincingly supported by empirical evidence. In particular, postwar United States data currently seem to favour the traditional autoregressive explanation of inflation expectations.

In the light of these considerations, and noting that the Dornbusch (1976c) variant of the regressive hypothesis can, in a sense, be reconciled with the perfect foresight hypothesis (more on
the remainder of this chapter considers a small economy under flexible exchange rates, adaptive inflation expectations, and a modification of the Dornbusch hypothesis which permits secular differences in national monetary growth and inflation rates along with the possibility of sustained revenue gains to national public sectors from expansionary policy. Section 2 sets out the model. Although this facilitates comparison with the preceding chapters, the complete model turns out to have a degree of complexity which does not repay analysis of its transients. Accordingly, Section 3 considers the short-run and inter-run responses of domestic inflation and exchange-depreciation rates, to increased domestic monetary growth and foreign price growth, in the special case of fixed real per capita domestic output.

Sections 4 and 5 deal with the long run, in which case anticipations are relevant only insofar as they affect the model's stability. Section 4 considers the long-run response of the domestic real economy to expansionary domestic policies. The emphasis there is on analogies to the barter model in the context of both quasi and full equilibria (see Chapter II). Section 5 considers the question of the optimal rate of exchange depreciation, that question having originally been raised by Mathieson (1976). We shall find that an analogue to the standard theory of the closed-economy inflation tax (see Chapter I) readily presents itself.

2. THE MODEL

Assumptions

The Chapter II model requires augmentation in two respects. First, if the domestic public sector no longer pegs the exchange rate, what alternative monetary norm should we postulate? Second (and this
is related to the first point) now we must spell out the above-mentioned modification of the Dornbusch expectational hypothesis to conditions of sustained inflation and monetary growth.

With regard to the first point, it is natural to assume that the domestic public sector pegs either the domestic nominal interest rate or the rate of growth of the domestic nominal monetary base—if only to enable useful comparisons with the preceding chapters. In the context of our simple model this rules out a regime of dirty floating—see, e.g., Girton and Roper (1977) for an analysis which encompasses that case. Additionally, the case of a pegged domestic nominal interest rate in conjunction with perfect capital mobility and perfect myopic foresight of the exchange depreciation rate essentially takes us back to the case of a small economy under fixed rates. In symbols: 

\[ r = \tilde{r} \quad \text{and} \quad r = r_w + e, \]  

whence 

\[ e = \tilde{r} - r_w = \tilde{e}. \]  

Finally, we have not been able to discover any appealing specifications of regressive exchange-rate expectations in conjunction with a pegged nominal interest rate. This leaves the case of a pegged rate of nominal monetary growth which, accordingly, is assumed everywhere in this chapter, except Section 4—for reasons which will become apparent therein.

Turning to the question of exchange-rate anticipations, the Dornbusch hypothesis is given by 

\[ \varepsilon = \theta (\ln E^* - \ln E), \]  

where \( \varepsilon \) is the anticipated rate of inflation of the price of foreign exchange, and \( \theta \) is an adjustment coefficient, taken to be a positive constant. Our approach is first to differentiate the above with respect to time, yielding 

\[ \dot{\varepsilon} = \theta (e^* - e); \]  

and then to note that \( e^* \) has an explicit representation in terms of exogenous variables and parameters in the
present case, since the policy assumption $\mu = \tilde{\mu}$, and the long-run equilibrium relationship $e^* = \mu^* - n - p_w$ (see below), together imply $e^* = \mu - n - p_w$.

It follows that $\dot{e} = \theta(\mu - n - p_w - e)$, or, the absolute rate of change of the anticipated rate of exchange depreciation is proportional to the gap between the exogenously-given rate of domestic monetary expansion (less domestic real output growth) and the rate of inflation of the domestic-currency price of imports.

The Model

Consider first the static relationships

$$ q = z + g + x $$

$$ z = \tilde{z} + (1-s)y - \ell(r_w + \varepsilon - \pi) + uw $$

$$ x = \xi_w \sigma_w - m z \sigma - \gamma $$

$$ y = (q-\gamma) \sigma - m + (r_w + \varepsilon - \pi)(w - c \sigma - m) - \pi \sigma - m - nw $$

$$ c = k q \phi \exp[- \alpha(r_w + e)] $$

$$ p = \gamma(q - q_F) + \pi $$

$$ p_d = (1-m)p + m(p_w + e) $$

In the light of the preceding chapters, these relationships require no further explanation, although it might be worth noting that (3.1e) incorporates the assumption that at some stage in the past the domestic public sector has dissipated its foreign assets—whence $f = 0$, in addition to $f = 0$, so that $\ell = c$.

Consider next the dynamic relationships

$$ \dot{w} = (\tilde{\sigma} + x) \sigma - m + (r_w + \varepsilon - p_d)(w - c \sigma - m) - p_d c \sigma - m - (\varepsilon - e) m w - nw $$

$$ \dot{\pi} = \beta(p_d - \pi) $$

$$ \dot{\sigma} = \sigma(p_w + e - p) $$
\[ \dot{c} = c(\bar{\mu} - n - p) \]  
\[ \dot{v} = \bar{\delta} + (r_w + \varepsilon - p)(v - c) - pc - nv \]  
\[ \dot{\varepsilon} = \theta(\bar{\mu} - n - p_w - e) . \]  

(3.2d) 
(3.2e) 
(3.2f)

Again, previous explanations obviate the need for detailed discussion of these relationships, although note that the capital-loss item \((\varepsilon - e)\) in (3.2a) is a consequence of our assumption of initially 'covered' domestic portfolios (see Chapter II) and the fact that the ex post domestic return on the interest-bearing debt of the domestic public sector in terms of overall domestic purchasing power is \(r - p_d\) \((= r_w + \varepsilon - p_d)\), whereas the ex post domestic return on the interest-bearing debt of the foreign public sector is \(r_w + e - p_d\).

Consider finally the conditions

\[
\begin{align*}
\mu_o &= \delta_o = 0 \\
\pi_o + n &= p_o + n = p_w + n = 0 \\
\rho_o - n &= \rho_w - n = 0 \\
\ell_o &= k \\
\sigma_o &= E_o = 1
\end{align*}
\]

(3.3)

where \(\rho_o = r_w + \varepsilon - \pi_o\). The interpretations of these conditions are straightforward and can be found in Chapters I and II.

In summary, there are thirteen equations, (3.1)' and (3.2)'; thirteen endogenous variables, \(q, z, x, y, \varepsilon, \pi, w, \sigma, c, p, p_d, e\) and \(v\); and thirteen nominal counterparts of those variables which in principle can be determined at any time (whether or not the system is perturbed) by reference to the aforementioned equations in conjunction with the conditions (3.3)'.
convenient to proceed directly to the model's long-run equilibrium relationships.

**Long-Run Equilibrium Relationships**

The following steady-state equations are straightforward consequences of equations (3.1)' and (3.2)'

\[ \begin{align*}
    p^* &= p_c^* = \pi^* = p_w + e^* \\
    \rho^* &= r^* + e^* - \pi^* = r_w - p_w = \rho_w \\
    q^* &= q_F \\
    \mu^* &= (\omega^*)^* = p_w + e^* + n \\
    z^* &= q_F - \bar{g} - x^* \\
    y^* &= z^*(\sigma^*)^{-m} \\
    e^* &= e^* \\
\end{align*} \]

(3.4a)', (3.4b)', (3.4c)', (3.4d)', (3.4e)', (3.4f)', (3.4g)'

In view of the discussion in the Chapter II counterpart of this section, equations (3.4) warrant just two comments. First, domestic steady-state rates of commodity price inflation are tied down by the gap between domestic nominal monetary growth and world real output growth—not by the rate of inflation of the foreign-currency price of imports. Second, the rate of inflation of the price of foreign exchange (whose anticipated and actual values coincide in the steady state) is given by domestic nominal monetary growth less the sum of world nominal price growth (in terms of foreign currency) and world real output growth, so that the domestic nominal interest rate will exceed the world rate by the amount just stated.
3. FIXED OUTPUT

If output is predetermined at capacity \((q = q_F)\), then the model can be reduced to two recursive submodels. The first describes the dynamics of the rates of inflation of the domestic-currency prices of domestic output, living costs, and foreign exchange, together with their respective expectations. The second describes the dynamics of interest-bearing and noninterest-bearing public debt, and the evolution of the terms of trade. Attention is focused on the former, given by:

\[ \begin{align*}
\ddot{\mu} &= p + n - \alpha \epsilon \\
p &= \pi \\
\dot{\epsilon} &= \theta(\mu - n - p_w - \epsilon) \\
\dot{\pi} &= \beta[(1-m)p + m(p_w + \epsilon) - \pi] .
\end{align*} \]

Equation (3.5a)' is obtained by differentiating (3.1e)' with respect to time and using (3.2d)' to eliminate \(\dot{c}\). Equation (3.5b)' is (3.1f)' in the special case \(q = q_F\). Equation (3.5c)' merely reproduces (3.2f)', and (3.5d)' is obtained by substituting for \(p_d\) in (3.2b)'

This subsystem can be viewed as an 'inflation' analogue of the Dornbusch (1976c) model. As well as requiring \(q = q_F\), its simple structure is also a consequence of our assumption of no 'wealth' demand for money. If this latter assumption is dropped, then the system can no longer be dealt with in a simple recursive fashion.

By eliminating \(p\) and \(\epsilon\) the submodel can be reduced to one in \(e\) and \(\pi\) only:
Providing inflation expectations adjust sluggishly, (3.6a)' can be viewed as a reduced-form equation. Accordingly, we can read off the impacts of various disturbances on the rate of exchange depreciation, and apply the results to the rate of inflation of domestic living costs, $p_d$.

**Comparative Statics**

Consider first the impacts of increased domestic monetary growth

\[
\frac{de}{d\mu} = 1 + \frac{1}{\alpha \Theta} \quad (3.7a)'
\]

\[
\frac{dp}{d\mu} = m(1 + \frac{1}{\alpha \Theta}) \quad (3.7b)'
\]

The result (3.7a)' shows that a 1 percent increase in domestic monetary growth, that was previously unforeseen but is expected to persist once it occurs, will raise the actual rate of exchange depreciation by more than 1 percent in the short run. Allowing for the fact that working in terms of rates of change rather than levels, the analogy with Dornbusch (1976c) readily presents itself. (In our notation the relevant Dornbusch result is $d(ln E)/d(ln C) = 1 + 1/\alpha \Theta$.) A 1 percent monetary expansion exerts downward pressure on the domestic interest rate, and generates expectations of an increased rate of exchange depreciation in the long run. To compensate for the imminent reduction in the domestic interest rate, the instantaneous increase
in the exchange depreciation rate must exceed the long-run increase; otherwise interest rate parity cannot hold on the path to long-run equilibrium.

A high interest elasticity of demand for real balances and fast adjustment of exchange-depreciation expectations are both dampening influences. Thus, the disturbance in question will raise the rate of inflation of the CPI by more or less than 1 percent according as whether

\[ \omega \geq \frac{m}{1-m} , \]

where the expression on the right-hand side is evidently an index of the openness of the domestic economy to trade in goods and services.

Consider next the foreign inflation impacts. Differentiation of (3.6a)' and \( p_d \) with respect to \( p_w \) yields

\[ \frac{de}{dp_w} = -1 \quad \text{(3.8a)'} \]

\[ \frac{dp_d}{dp_w} = 0 \quad \text{(3.8b)'} \]

A 1 percent increase in foreign price growth will lower the short-run exchange depreciation rate by exactly 1 percent. In consequence, the domestic CPI inflation rate will remain the same. Note that these results obtain regardless of whether or not there is a matching increase in the foreign nominal interest rate (i.e., \( \frac{dr}{dp_w} = 1 \)). This is due to the recursive nature of the present model. Thus, the steady state of the more general model set out in Section 2 has this insulation property only if the foreign real interest rate remains unchanged (i.e., \( \frac{dr}{dp_w} = 1 \)). Moreover, its point-in-time state
has this property only if there is also a matching movement in the anticipated exchange depreciation rate (i.e., \( \frac{de}{dp_w} = \frac{de}{dp_w} \)).

Whether or not the present model and its Section 2 counterpart satisfy this condition is a moot point which is examined in more detail below.

**Dynamics**

The remainder of this section is concerned with the behaviour over time of anticipated CPI inflation, and the actual rates of inflation of the price of domestic output and foreign exchange. First we focus on the time paths associated with constant rates of domestic monetary growth and foreign price growth; then we reconsider the effects of an increase in domestic monetary growth.

Beginning with the anticipated rate of CPI inflation, (3.6a)' into (3.6b)' yields

\[
\dot{\pi} = \beta m(1 + \frac{1}{\lambda\theta})(\bar{\mu} - n - \pi).
\]

(3.9a)'

Evidently the present model is stable. The general adjustment speed of inflation expectations to the equilibrium \( \bar{\mu} - n = \pi^* \) is given by the product of the partial adjustment speed \( \beta \) and the impact multiplier \( \frac{dp_d}{d\bar{\mu}} \). And with \( p = \pi \) (recall (3.5b)'), identical considerations apply to the actual rate of inflation of the domestic-currency price of foreign output.

Consider now the anticipated and actual rates of exchange depreciation. In the former case, use (3.6a)' to eliminate \( e \) from (3.5c)'; and in the latter case differentiate (3.6a)' with respect to time, permitting only \( \pi \) to vary, and then use (3.9a)'}
eliminate $\pi$. This yields
\[ \dot{e} = -\frac{1}{\alpha}(\dot{\mu} - n - \pi) \quad (3.9b) \]
\[ \dot{e} = -\frac{\beta}{\alpha}(1 + \frac{1}{\alpha}) (\dot{\mu} - n - \pi). \quad (3.9c) \]

A necessary condition for perfect foresight expectations is that the anticipated exchange depreciation rate converges to equilibrium at the same rate as its ex post counterpart. The above pair of equations imply that this condition is satisfied if and only if
\[ \frac{\dot{e}^2}{\beta m} - \theta - \frac{1}{\alpha} = 0 , \]
that is
\[ \theta = \beta m(1 \pm \sqrt{\frac{1}{4} + \frac{1}{\alpha \beta m}}) . \]
Assuming the Cagan condition is satisfied (i.e., $\alpha \beta < 1$), then the two solutions for $\theta$ will be opposite in sign, so that there is only one positive solution consistent with perfect foresight anticipations:
\[ \theta = \beta m(1 + \sqrt{\frac{1}{4} + \frac{1}{\alpha \beta m}}) . \]

There is a diagram which illustrates most of the points made in this section—and also draws attention to a difficulty with the foregoing notion of "consistent" expectations. Figure 1's horizontal axis shows time. Its vertical axis shows the percentage rates of change of the domestic nominal monetary base, the anticipated domestic CPI, the actual domestic-currency price of home output, the spot price of foreign currency, and the actual domestic CPI. Assume that prior
to time $\tau_0$, the system is in steady-state equilibrium, with $n = 0$ for simplicity, and that the initial conditions (3.3)' are satisfied. At time $\tau_0$ the domestic monetary growth rate jumps to 1 percent, and thereafter remains at that rate.

**FIGURE 1**

Transitory Effects of Increased Domestic Monetary Growth in a Small Economy Under Flexible Exchange Rates
Under the assumption that inflation anticipates adjust sluggishly, the anticipated domestic CPI inflation rate, $\pi$, does not jump at time $\tau_0$. Figure 1 depicts the monotonic convergence of inflation expectations to the new, higher rate of domestic monetary growth. And with output fixed at capacity, the trajectory of the rate of inflation of the actual domestic-currency price of home output, $p$, is the same.

The initial jump in the actual exchange depreciation rate, $e$, is given by (3.7a)', and its subsequent time path may be deduced from (3.9c)'. Again there is monotonic convergence to the new rate of domestic monetary growth.

The initial jump in the actual domestic CPI inflation rate, $p_d$, is given by (3.7b)'. Figure 1 illustrates the case of undershooting ($dp_d/d\mu < 1$), although we cannot rule out the possibility of overshooting ($dp_d/d\mu > 1$). The subsequent time path of $p_d$ is evidently a weighted average of the paths of $p$ and $e$ (see above), and is easily deduced from that consideration.

Finally, conspicuous by its absence from the diagram is the trajectory of the anticipated exchange depreciation rate, $e$. The reason for this omission is as follows. To determine the behaviour of $e$ subsequent to time $\tau_0$ is straightforward; its general path may be deduced from (3.9b)', and its path in the special case of "consistent expectations" (see Dornbusch (1976c) and above) may be deduced from (3.9c)'. On the other hand, focusing on the case of consistent expectations, we also require a jump in $e$, of magnitude $1 + 1/\alpha e$, at time $\tau_0$. But the model lacks the formal analytical
contrivance necessary to ensure this outcome. Indeed, the money market condition (3.5a)', and the methodology we have used hitherto, together suggest that the anticipated exchange depreciation rate adjusts sluggishly—in which event its trajectory subsequent to \( \tau_0 \) would be below the horizontal axis of Figure 1. That adjustment path is clearly implausible.

The Dornbusch (1976c) model would appear to give rise to essentially the same problem; in that model it is the level of the forward rate (rather than its percentage rate of change) which should jump at the instant a monetary expansion takes place. One way out of this difficulty is to postulate rational exchange-rate expectations together with a terminal condition on real money balances—à la Sargent and Wallace (1973). This is done by Gray and Turnovsky (1978), who show that the initial jump is again \( 1 + 1/\varphi \).

4. COMPARISON WITH THE BARTER MODEL

The most novel aspect of our analysis of the fixed-rate case is undoubtedly that concerning analogies to the barter theory of international trade. This section investigates whether the parallels pointed out there carry over to the flexible-rates case. As in Chapter II, we distinguish between quasi and full long-run equilibrium, and consider those cases in turn.

**Long-Run Quasi Equilibrium**

Noting equations (3.3)', and assuming the exogenously-given foreign real interest rate takes its golden-rule value (cf. equations (2.12)'), equations (3.2a)' and (3.2e)' imply
These equations parallel their fixed-rates counterparts (2.12)', and our detailed comments on those equations apply here as well. But the analogy is not exact for two reasons. First, the relevant inflation-cum-growth tax rate is \( \tilde{\mu} = p_w + n + e^* \), not \( p_w + n \). Second, the relevant inflation-cum-growth tax base is \( c^* \), not \( \tilde{c} \).

Since

\[
c^* = k q_F \phi \exp(- \alpha \tilde{\mu})
\]

when the foreign interest rate conforms to the golden rule, it follows that the domestic public sector also indirectly controls the relevant inflation-tax base in the present case, although there is a well-known upper bound on the amount of revenue that can be raised; see next section.

The comparative statics of increased public spending and net explicit taxes are precisely the same as in the fixed-rate case (cf. equations (2.13)').

\[
-x^* = \tilde{\delta} - \tilde{\mu} c^*
\]

\[
\dot{v}^* = \tilde{\delta} - \tilde{\mu} c^*
\]
\[
\begin{align*}
- \frac{dx^*}{dg} &= \frac{d\tilde{y}^*}{dg} = 1 \\
- \frac{dx^*}{dt} &= \frac{d\tilde{v}^*}{dt} = -1 \\
\frac{dz^*}{dg} &= \frac{d[y^*(\sigma^*)^m]}{dg} = 0 \\
\frac{dz^*}{dt} &= \frac{d[y^*(\sigma^*)^m]}{dt} = -1 \\
\frac{d\sigma^*}{dg} &= -\frac{1}{\xi(\eta + \eta_w - 1)} , \quad \frac{dy^*}{dg} = \frac{1}{\eta + \eta_w - 1} \\
\frac{d\sigma^*}{dt} &= \frac{1 - m}{\xi(\eta + \eta_w - 1)} , \quad \frac{dy^*}{dt} = -1 - \frac{1 - m}{\eta + \eta_w - 1}
\end{align*}
\]

The interpretation of these results is the same as given in connection with equations (2.13).

An increase in domestic monetary growth \( \bar{\mu} \) will raise or lower the receipts of the domestic public sector from the inflation-cum-growth tax on its noninterest-bearing debt according as whether the monetary growth at the margin is below or above its revenue-maximising rate; see below. Accordingly, the effects on the terms of trade, and domestic private income in terms of its overall purchasing power, are qualitatively the same as an increase or decrease in net explicit taxes, depending on the margin just mentioned.

This story is not quite the same as that given in connection with an increase in \( p_w \) under fixed rates; see Chapter II. The difference can be viewed as a consequence of our specification of domestic public sector policy in the fixed-rate case. If, for example, the
assumptions were that reserves are invested in noninterest-bearing assets, and that the domestic source component varies so as to ensure that reserves comprise a constant fraction of the domestic monetary base, then essentially the same explanation would suffice for either exchange-rate regime.

**Long-Run Full Equilibrium**

We analyzed long-run full equilibrium in the fixed-rates counterpart of this section by postulating that real per capita public spending is pegged and that net explicit taxes adjust endogenously as required to ensure the cessation of net foreign borrowing. This subsection takes another look at this case. But in the light of our analysis of the 'monetary' regime in a closed economy, and especially the preceding subsection, it is not difficult to foresee that our flexible-rates analysis of this case will contain few surprises.

On the other hand, there is an alternative policy regime which ensures long-run full equilibrium, and also says something new about the 'budgetary' view of the causes and consequences of inflation (see Chapter I). Specifically, we introduce the case whereby real per capita public spending and net explicit taxes are pegged, and nominal monetary growth adjusts endogenously as is necessary to ensure the cessation of net foreign borrowing.

In the case of endogenous net explicit taxes and exogenous monetary growth there is, instead of equations (3.10)',


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In the case of endogenous net explicit taxes and exogenous monetary growth there is, instead of equations (3.10)'},
\[ g - t^* - \mu kq_F^*(\alpha \mu) = 0 \]  

whence

\[ x^* = 0 \]  

That is, in long-run full equilibrium the public sector's budget and the nation's trade must balance—cf. equations (2.14). 

Instead of (3.11), we now have:

\[
\begin{align*}
\frac{dx^*}{dg} &= \frac{dy^*}{dg} = \frac{d\delta^*}{dg} = 0 \\
\frac{dz^*}{dg} &= \frac{d[y^*(\sigma^*)^m]}{dg} = -1 \\
\frac{d\sigma^*}{dg} &= - \frac{m}{\varepsilon (\eta + \eta_w - 1)}, \quad \frac{dy^*}{dg} = -1 + \frac{m}{\eta + \eta_w - 1}
\end{align*}
\]

These equations are the same as their fixed-rates counterparts (2.15). Accordingly, they require no further comment. 

In the case of exogenous net explicit taxes and endogenous monetary growth, on the other hand,

\[ g - t^* - \mu^* kq_F^*(\alpha \mu^*) = 0 \]  

and allowing \( \mu^* \) to take any positive value,

\[ \frac{d\mu^*}{d\delta} = \frac{de^*}{d\delta} = \frac{d\pi^*}{d\delta} = \frac{1}{c^*(1 - \alpha \mu^*)} \]  

To interpret (3.15), recall two of the key Chapter I results. Specifically, in the case of the budgetary regime, \( d\mu^*/d\delta = d\pi^*/d\delta = 1/w^* \); and in the case of the monetary regime, \( d\delta^*/d\mu^* = c^*(1 - \alpha \mu^*) \). Equations (3.15) can be viewed as a hybrid of these two results: monetary growth is endogenous, as in the budgetary approach; and the
real interest rate is tied to its exogenously-given golden-rule rate, as in the monetary approach.

The upshot is that an increase in the legislated deficit is inflationary or deflationary according as whether monetary growth, at the margin, is below or above its revenue-maximizing rate. In symbols, $\frac{d\pi^*}{d\delta} \geq 0$ according as whether $\mu^* \leq 1/\alpha$. In the former case, and providing the domestic private sector is holding some interest-bearing public debt, a unit increase in the legislated deficit is always more inflationary than when the nominal interest rate is pegged, because the required increase in inflation-tax revenues must be generated from a narrower base. To put this another way, in the present case the domestic public sector is unable to improve the real terms on which it trades its interest-bearing public debt. It must accept the exogenously-given global real rate of interest.

Even in a closed economy there are a variety of assets (apart from money) which compete with interest-bearing public debt, and thereby limiting the public sector's scope for squeezing the real rate of return on its interest-bearing debt. Chapter I's analysis of the budgetary approach abstracted from such effects. The present analysis therefore provides a salutary reminder of the long-run limits on the inflation-tax base available to the public sector. On the other hand, the present analysis disregards important existence and stability issues. These are taken up in Chapter IV.

As a final observation on the policy regime in question, note that insofar as an increase in the legislated deficit is effected
by higher public spending, equations (3.13) continue to hold; whereas an increase in the legislated deficit which is effected wholly by reduced net explicit taxes will have no effect on the terms of trade and disposable income.

5. IS THERE AN OPTIMAL CRAWL?

This section applies some elements of the analysis in this and preceding chapters, and also some standard evidence on the demand for money, to a claim by Mathieson (1976) concerning the optimal secular rate of exchange depreciation, or "crawl". Mathieson argues that the crawl which induces a unitary interest elasticity of money demand will maximize domestic private consumption. We shall argue that the unit-elasticity crawl will minimize domestic private consumption, and that that crawl in question represents a straightforward application of the standard theory of the inflation tax in a closed economy (see Chapter I) to the case of a small open economy under flexible rates. We also draw attention to other aspects of the unit-elasticity crawl, but the two foregoing points are the main ones.

Mathieson's model diverges from the steady-state counterpart of our Section 2 model in four noteworthy respects. First, it assumes perfect goods-market integration, so that the terms of trade are fixed. Second, it abstracts from real-interest and wealth effects in the spending-saving choice, although this turns out to be unimportant in the present context. Third, the growth tax on domestic private holdings of aggregate public debt, \( mw \), appear in the wealth accumulation equation rather than in the definition of private disposable income. We prefer the latter alternative for several reasons; for example,
it ensures that long-run full-equilibrium private disposable income
is given by long-run full-equilibrium private spending, which seems
a desirable property in the context of zero productivity growth.
Again, however, this turns out to be unimportant in the present con-
text. Fourth the domestic public sector raises no explicit taxes.

With these considerations in mind, we turn to the key steady-
state relationship in Mathieson's paper (p.189; Mathieson's notation):

\[ \frac{dC}{de} = - \frac{(1-s)n[m + L'y(i^* + e)]}{n - s(i^* - \pi^*)} \]

This relationship is a straightforward implication of the following
notations, definitions, and assumptions:

\begin{align*}
C &= (1-s)y_D = \text{real per capita private spending}, \\
\epsilon &= \text{rate of crawl, taken to be pegged}, \\
s &= \text{marginal propensity to save, a constant}, \\
y_D &= y + i(w-m) - \pi w = \text{private disposable income}, \\
y &= \text{real per capita output, taken to be exogenous}, \\
i^* &= i - e = \text{foreign nominal interest rate, taken to be exogenous}, \\
w &= \int (sy_D - nw) = \text{real per capita private wealth}, \\
\pi &= \pi^* + \epsilon = \text{inflation rate}, \\
\pi^* &= \text{foreign inflation rate, taken to be exogenous}, \\
m &= L(i)y = \text{supply of real per capita money balances}, \\
n &= \text{world rate of population growth, a constant}, \\
L(i)y &= \text{per capita demand for balances, with } L' \equiv \frac{\partial L}{\partial i}.
\end{align*}

All these variables refer to the domestic economy unless specifically
indicated to the contrary.

It follows directly that

\[ \frac{dC}{de} = 0 \Rightarrow \chi = -1 \]
where \( \chi \equiv (i^* + \bar{e})\frac{L'}{L} \) = elasticity of demand for real balances with respect to the nominal interest rate, and that
\[
\bar{e}_* = -i^* - \frac{L}{L'}
\]
where \( \bar{e}_* \) is the rate of crawl which induces a unitary interest elasticity of money demand.

The crucial issue is that nature of the turning point associated with \( \chi = -1 \). Mathieson argues (p. 129, n.15):

"We can show that the optimal crawl [\( \bar{e}_* \)] maximizes rather than minimizes consumption. Given the solution for \( dc/d\bar{e} \), we will have \( d^2c/d\bar{e}^2 = -(1-s)n(y(2L' + L''(i^* + \bar{e}))/ (n - s(i^* - \pi^*)) \).

If \( n > s(i^* - \pi^*) \) (which is likely to be true in a Golden Rule world where \( i^* - \pi^* = \) the 'real' interest rate \( \approx n \)), then \( d^2c/d\bar{e}^2 < 0 \) if
\[
2L' + L''(i^* + \bar{e}_*) > 0.
\]
Since \( \bar{e}_* = i^* - \frac{L}{L'} \) we can rewrite the condition for \( d^2c/d\bar{e}^2 < 0 \) as
\[
2L' - L''L/L' > 0.
\]

Now if \( \chi = -1 \),
\[
\frac{\partial \chi}{\partial i} = \frac{2L' + L''(i^* + \bar{e}_*)}{L}.
\]

Hence \( 2L' + L''(i^* + \bar{e}_*) > 0 \) only if \( \partial \chi/\partial i > 0 \). But there is strong evidence that the interest elasticity of demand for real balances is a non-increasing function of the nominal interest rate (i.e., \( \partial \chi/\partial i \leq 0 \)).
What is the evidence from episodes of rapid inflation? First, a preliminary. Investigators of rapid inflations have measured the opportunity cost of holding real money balances by the expected inflation rate rather than by the nominal interest rate. But if (following Mathieson and others) one assumes the expected real interest rate is constant, then the response of the interest elasticity of demand for money to an increase in the nominal interest rate will have the same sign as the response of the inflation elasticity of demand for money to an increase in the (expected) inflation rate. To put this another way, assume \( i - \pi \) is constant, recall that \( \chi \equiv (i/m)(\partial m/\partial i) \), and introduce \( \widetilde{\chi} \equiv (\pi/m)(\partial m/\partial \pi) \). Then \( \partial \chi/\partial i \leq 0 \) according as whether \( \partial \chi/\partial \pi \leq 0 \). Indeed, it is well known that there is only a negligible loss of accuracy if one chooses to use \( \chi \) and \( \widetilde{\chi} \) interchangeably in conditions of rapid inflation.

The classic reference on the demand for money during rapid inflations is Cagan (1956). He obtained good results with the semi-logarithmic functional form \( m = \tilde{k} \exp(-\tilde{\alpha} \pi) \), where \( \tilde{k} \) and \( \tilde{\alpha} \) are positive constants. If this specification is correct—and numerous subsequent investigations have supported Cagan’s basic findings—then \( \partial \chi/\partial i < 0 \). And following the pioneering work of Cagan, the semi-logarithmic functional form has become dominant in the literature on the inflation tax.

The demand for money during the hyperinflation in Weimar Germany has been painstakingly re-examined by Frenkel (1977). One aspect of his study is especially relevant, namely, his Box-Cox estimate of \( \lambda \) in the function
\[
\log m = \beta_0 + \beta_1 \left( \frac{\lambda - 1}{\lambda} \right) + u, \ u = \text{error term}.
\]

Equation (7) is especially relevant because it is consistent with \( \tilde{\chi} < 0 \) together with each of the three cases \( \frac{\partial \tilde{\chi}}{\partial \pi} < 0 \). It turns out that \( \lambda \approx 1 \) is the best estimate by most criteria, although the estimate \( \lambda \approx 0 \) has some merit. The case \( \lambda = 1 \) takes us back to the semi-logarithmic specification; see above. The case \( \lambda \rightarrow 0 \), on the other hand, corresponds to the double-logarithmic specification

\[
\log m = \tilde{\gamma} + \tilde{\chi} \pi, \text{ where } \tilde{\gamma} \text{ and } \tilde{\chi} \text{ are constants. If this form is correct, then } \frac{\partial \tilde{\chi}}{\partial \pi} = 0. \text{ Since } \tilde{\chi} < 0 \text{ and } \frac{\partial \tilde{\chi}}{\partial \pi} > 0 \text{ are both true only if } \lambda < 0, \text{ it is pertinent that Frenkel apparently finds no support for the hypothesis } \lambda < 0.
\]

What is the relevant evidence from periods of moderate inflation? The semilogarithmic form has again yielded good results. Thus, for example, Girton and Roper (1977) recently obtained such results with \( m = k \exp(-\alpha i) \), where \( k \) and \( \alpha \) are constants. (This dissertation, of course, has adopted this form in all the chapters thus far.) If this specification is correct, then \( \frac{\partial \chi}{\partial i} < 0 \). On the other hand, the double-logarithmic specification \( \log m = \gamma + \chi i \), where \( \gamma \) and \( \chi \) are constants, has been very popular in studies of the demand for money during the moderate inflations in postwar Western countries. Of course the double-logarithmic specification implies \( \frac{\partial \chi}{\partial i} = 0 \).

From moderate inflations, too, there is a body of evidence which is especially relevant, as in the case of rapid inflations. Here we refer to the large number of tests of the liquidity-trap hypothesis. "If the liquidity-trap hypothesis is true, it must be the case that the \([\text{absolute value of the}]\) interest elasticity of
the demand for money becomes greater as the rate of interest falls..." (Laidler (1977), p. 130). Laidler concludes: 'the evidence on the liquidity trap is not quite clear-cut. On the whole, the evidence goes against the hypothesis...' (Laidler (1977), p. 132).

We infer that the turning point associated with $\chi = -1$ will minimize (not maximize) domestic real per capita private consumption. Given the structure of Mathieson's model this is simply because the turning point in question is associated with maximal inflation-cum-growth payments by the domestic private sector on its holdings of real balances, and no offsetting reduction in the net explicit taxes paid by that sector. That a unit elasticity of money demand will maximize public revenue from inflation has been recognized at least since Cagan (1956). (See also Friedman (1971).)

The question then arises: which sector will derive maximum benefit from the consumption-minimizing crawl? This is not clear in Mathieson (1976); the rate of crawl is assumed to be controlled by the domestic public sector, yet the relevant section of Mathieson's paper also assumes the domestic monetary base is wholly foreign in source. Despite his argument to the contrary (see pp. 189-190), it is difficult to see how the domestic public sector can control the crawl. Furthermore, it is not clear whether equilibrium in question is one that admits or rules out sustained borrowing abroad. In any event, the latter alternative would seem much more reasonable for the kind of long-run welfare (or "illfare") analysis under consideration here.
One way out of these difficulties (and any other would appear to be a close substitute) is to assume that the home monetary base is wholly domestic in source, that real per capita net explicit taxes are fixed, and that real per capita public spending is endogenous, adjusting as necessary to ensure long-run full equilibrium. Assume further that real per capita net explicit taxes are zero, and that the world real interest rate conforms to the golden rule. Note that domestic monetary growth is equal to the sum of foreign price growth (in terms of foreign currency), world output growth and the rate of crawl. Hence the following analogue of equations (3.12a)' and (3.14)'

\[ g - (\pi^* + n + \bar{\epsilon})yL(\pi^* + n + \bar{\epsilon}), \]

where \( g \) denotes steady-state real per capita domestic public spending. At the unit-elasticity crawl \( \bar{\epsilon} = \bar{\epsilon} \), \( g \) attains a maximum. Recalling (3.4e)' in the case of balanced trade, and noting that there is no need to distinguish between alternative definitions of 'real' consumption when domestic and foreign goods are perfect substitutes, we deduce that maximal \( g \) will coincide with minimal domestic private consumption, as required.

As a final question, what is the consumption-maximizing crawl? In the light of the foregoing analysis, now the object of the exercise is to minimize domestic public spending \( g \). The answer is evidently given by the corner solution \( \pi^* + n + \bar{\epsilon} = 0 \), whence \( \bar{\epsilon} = - (\pi^* + n) = - i^* \). This, too, is readily interpreted in terms of standard closed-economy theory: is merely that rate of crawl which ensures that domestic monetary growth conforms to the
full-liquidity rule; for further details, see Friedman (1969).

6. SUMMARY

This chapter has extended the analysis to the case of a small economy under flexible exchange rates.

Under the preferred specification of inflation and exchange-depreciation expectations, the model in the preceding chapters turns out to be too complex for useful short-run and stability analysis. Accordingly, to investigate transients we went to the special case of exogenous real per capita output. The resulting submodel is stable in the sense of ensuring stationary inflation and exchange depreciation rates.

In this fixed-output framework we considered the impacts of a 1 percent increase in the rate of domestic monetary growth, and an increase in the rate of foreign price growth in terms of foreign currency. These disturbances were assumed to be previously unforeseen, but expected to persist once they occur.

The domestic monetary disturbance will only shift the anticipated and actual domestic-currency rates of inflation of the price of home output over time. By contrast, it will induce an immediate increase in the domestic-currency rate of inflation of the price of foreign output in excess of 1 percent (cf. Dornbusch (1976c)). The overall impact on the rate of inflation of domestic living costs is positive, but ambiguous in magnitude.

The foreign price disturbance will not affect the rate of inflation of domestic living costs, or any other domestic variable,
either instantaneously or in the long run. This insulation property holds in more general models if and only if two conditions hold. First, exchange-depreciation expectations must satisfy the requirement of perfect myopic foresight. Second, any increase in the foreign inflation rate must be accompanied by a matching increase in the foreign nominal interest rate (cf. Turnovsky and Kingston (1977)).

Sections 4 and 5 dealt with the long run, wherein exchange-rate expectations are realized as a matter of definition. Real per capita output was again fixed, although now as a consequence of the (definitional) equalization of the actual rate of inflation of the domestic-currency price of domestic output and the anticipated rate of inflation of domestic living costs. As in Chapter II, it was useful to distinguish between quasi and full equilibria.

Again following Chapter II, analogies with the standard barter model were investigated. Not surprisingly, the analogies proposed there were found to carry over to the case of flexible rates.

A more interesting exercise undertaken in that context proceeded from the observation that full equilibrium under flexible rates can also be attained by permitted domestic monetary growth (rather than the legislated deficit) to vary as is required to shut off net foreign borrowing so that the domestic legislated deficit is exogenous. We then considered the effect of an increase in the legislated deficit on the rates of growth of domestic nominal variables. There is an important difference between this policy regime and the budgetary regime of Chapter I: now the domestic public sector is unable to alter the real terms on which it trades
its interest-bearing debt for money, since in the long run those terms are given to the small domestic country by the global capital market.

One implication of this alternative characterization of the budgetary regime is that starting from a position of moderate inflation, each successive incremental widening of the legislated deficit is necessarily more inflationary than its predecessor; and as domestic monetary growth approaches its revenue-maximizing rate, the inflation increments approach infinity. Does this help explain the observations of Cagan (1956) and subsequent investigators concerning the extreme volatility of inflations in the neighbourhood of their revenue-maximizing rates?

As a final exercise in the comparative statics of the long run we examined the optimal crawl proposed by Mathieson (1976). The crawl in question is that which induces a unitary interest elasticity of money demand. Appealing to standard evidence on the demand for money, and drawing attention to parallels with the standard closed-economy theory of the inflation tax, we argued that the proposed crawl will minimize rather than maximize private consumption. This policy was shown to admit a reinterpretation in terms of maximal consumption by the domestic public sector. Attention was drawn to the crawl which induces a zero interest elasticity of money demand. This alternative policy will maximize domestic private consumption. This merely restates a version of the full-liquidity rule: if the world real interest rate conforms to the golden rule, then the domestic nominal money stock should be constant.
CHAPTER IV
INFLATION AND PUBLIC POLICY IN
THE WORLD ECONOMY

1. INTRODUCTION

This chapter considers the role of national economic policies, especially budgetary policies, in the causes and consequences of inflation at the global level. This section contains a critical review of some of the relevant literature. Most of the main ideas in the long-run literature have been introduced in Chapters I and II, and some remaining gaps will be filled in Sections 2 and 3 below, so the focus here is on contributions pertaining to the short run.

The repercussion multiplier literature (see, e.g., Mundell (1968, Ch. 7), Stern (1973)) is a natural starting point. Given the usual short-run recursive link between inflation and excess demand (see below), this literature is relevant to the issues at hand. Assuming the terms of trade are predetermined, the basic implication for short-run inflation theory is simple: if any national public sector undertakes an expansionary fiscal policy, then aggregate demand in every trading country in the world will rise, so that all trading countries' CPI inflation rates will rise.

In the absence of restrictive assumptions on IS-curve and Phillips-curve parameters, however, we cannot say more than that. To see this, consider the following elementary Keynes-Phillips world, consisting of countries A and B, and an equalized, pegged nominal
interest rate $\bar{r}$ (more on this later):

\[ q_a = z_a + \bar{g}_a + x_a \quad , \quad q_b = z_b + \bar{g}_b + x_b \]
\[ z_a = (1-s_a)q_a - \tau_a \bar{r} \quad , \quad z_b = (1-s_b)q_b - \tau_b \bar{r} \]
\[ x_a = (1-s_a)m_a z_a - (1-s_a)z_a \quad , \quad x_b = x_a \]
\[ p_a = \gamma_a (q_a - \bar{q}_a) + \bar{\pi}_a \quad , \quad p_b = \gamma_b (q_b - \bar{q}_b) + \bar{\pi}_b \]
\[ p^*_a = (1-m_a)p_a + m_a p_b \quad , \quad p^*_b = (1-m_b)p_b + m_b p_a \]

where a bar denotes a pegged, fixed or predetermined variable; subscripts denote countries; and A's endogenous variables are

$q_a =$ real per capita output in A, $z_a =$ real per capita aggregate private spending by residents of A, $x_a =$ real per capita trade surplus, $p_a =$ rate of inflation of the price of A's output, $p^*_a =$ rate of inflation of A's CPI. Additionally, $g_a =$ real per capita spending by A's public spending on A's output. Assume $0 < s_a, m_a < 1$ and $\gamma_a, \tau_a > 0$. Analogous considerations apply to B.

Since stocks of factors of production and commodity prices in terms of the producers' currency are taken to be predetermined throughout this section, it is not essential to distinguish between nominal and real per capita magnitudes. We have done so, however, because some of these variables are re-used in Sections 2 and 3 below, in which context the distinction is important.

An obvious question to ask of this model is whether expansionary fiscal policy in A induces a greater increase in A's CPI inflation rate than in B's CPI inflation rate. Straightforward manipulation of the foregoing model yields
\[
\frac{d(p^*-p^*_b)}{d\sigma_a} = \frac{(1-m_a -m_b)}{\Gamma} [\gamma_a (1-h_b) - \gamma_b (1-h_a -s_a)] < 0
\]

where \(0 < h_i = (1-s_i) (1-m_i) < 1, \quad i = a, b,\)

and \(0 < \Gamma = (1-h_a) (1-h_b) - (1-h_a -s_a) (1-h_b -s_b) .\)

Even under the "classical presumption" of transfer theory, namely, \(m_a + m_b < 1\) (and note that our CPI specification facilitates a better-behaved framework than is usually the case; see Chapter II), we are unable to answer the question without introducing unappealing restrictions on IS-curve and Phillips-curve parameters.

Working with output sectors essentially the same as that in the model just examined, analysts have introduced relative-price elements (see, e.g., Laursen and Metzler (1950), Meade (1951), Mundell (1968, Ch. 18)); and monetary sectors (see, e.g., Mundell (1968, Ch. 18), Cooper (1969), Roper (1971), Swoboda and Dornbusch (1973)). Since Mundell's analysis extends in both directions, it serves as a convenient focus for further discussion of some of the problems in the construction of short-run global models.

The model in question contains a pair of national IS curves, a pair of national LM curves, a simple 'gold-standard' model of the international system in the case of fixed rates, and a traditional static expectations model of exchange-rate determination in the case of flexible rates. It assumes perfect international integration of national capital markets, partial integration of national commodity markets, and zero international integration of national markets for money stocks and labour.
Its fixed-rate version effectively captures the main ingredients of the short-run monetary approach to the balance of payments, and does not generate any anomalous propositions as a consequence of that. More debatable (albeit entirely conventional) is its symmetrical "gold-standard" explanation of international payments arrangements: there is a fixed nominal quantity of international reserves (gold?), which is shifted back and forth between the two national economies in response to goods-market and money-market disturbances. This framework is at variance with the postwar facts in three respects.

First, there is ample evidence that the predominant international asset in postwar official portfolios has been the interest-bearing debt issued by the public sectors of a few key-currency countries, especially the United States. For evidence, see, e.g., Whitman (1974), (1975). Second, Whitman also reports that the United States has generally been found to have an offset coefficient of zero. This, too, is inconsistent with the gold-standard model. Third (and we shall postpone tackling this point), the stock of international reserves has obviously not been stationary in nominal terms. These considerations, which clearly must play a crucial role in the explanation of worldwide postwar inflation under fixed rates, have been extensively investigated. Roper (1973), Girton and Henderson (1976a, b), (1977), and Girton and Roper (1977) are the main analytical contributions.

A simple way of modifying the Mundell (1968, Ch. 18) model so as to take these considerations and contributions into account is as follows. Recall the two IS curves set forth earlier, and relax the assumption that the interest rate r is pegged. Append the national monetary sectors
\[ \tilde{c}_a = \lambda_a(q_a, r), \quad \tilde{c}_b + f = \lambda_b(q_b, r) \]

where \( c_a \) = real per capita monetized debt issued by the public sector of the reserve-currency country, \( c_b \) = real per capita monetized debt issued by the public sector of the 'peripheral' country, and \( f \) = real per capita international assets held by the public sector of the 'peripheral' country, assumed to consist of interest-bearing debt of A's public sector. In other words, A takes the role of the "nth" country, with no official demand for foreign assets (easily generalized to admit the case of no "systematic" official demand for foreign assets), so that B supports the parity.

Since

\[ q_a = (1-s_a)q_a - \lambda_a r + g_a + m_b[(1-s_b)q_b - \lambda_b r] - m_a[(1-s_a)q_a - \lambda_a r] \]
\[ q_b = (1-s_b)q_b - \lambda_b r + g_b + m_a[(1-s_a)q_a - \lambda_a r] - m_b[(1-s_b)q_b - \lambda_b r], \]

we have four equations in the four unknowns \( q_a, q_b, r, \) and \( f. \)

Observe that the condition for equilibrium in B's money market is completely redundant for the determination of \( q_a, q_b, \) and \( r. \) It follows that any change in \( c_b \) will merely induce an equal and offsetting movement in B's international reserves, just as in the standard small-economy model under fixed rates and perfect capital mobility. Any change in \( c_a \) on the other hand will affect \( q_a \) and \( q_b \) (and \( r \)) as in a closed economy, thereby shifting inflation rates at home and abroad. Another way of looking at this monetary asymmetry is to assume that the national outputs \( q_a \) and \( q_b \) are pre-determined rather than endogenously determined, as proposed by
standard offset analysis (see, e.g., Kouri and Porter (1974)). In that event, A's offset coefficient would be zero, whereas B's offset coefficient would be minus one. Finally, reverting to the case of endogenous $q_a$ and $q_b$, we can show that there are also national asymmetries with regard to the impacts of fiscal policy. But the model's implications in this regard are that fiscal asymmetries are not as sharp as monetary asymmetries (contra. the long-run fixed-rates analysis of Section 3 below).

Turning to the flexible-rates version of Mundell's model, one objection to the static-expectations approach to exchange-rate determination was noted in Chapter III. There are two other anomalous propositions stemming from that approach. First, fiscal expansion by a small country (under flexible exchange rates and perfect capital mobility) will have no effect on aggregate demand in that country. Second, monetary expansion by a country of arbitrary relative size will lower aggregate demand in the rest of the world.

With respect to either of these propositions, empirical findings by Helliwell and associates for the postwar North American economy suggest that the opposite is true; see, e.g., Helliwell and Maxwell (1974), Helliwell (1974). Here the main ingredient would appear to be a regressive element in the modelling of exchange-rate expectations. As with the case of static exchange-rate expectations, however, Chapter III drew attention to some difficulties with popular regressive expectational hypotheses. More generally, no straightforward method of adequately incorporating exchange-rate transients into the traditional short-run model would appear to have been
devised, as yet. Thus we are not really in a position to offer constructive suggestions for improving the flexible-rates version of Mundell (1968, Ch. 18).

Accordingly, the remainder of this chapter focuses on secular aspects of global inflation. Both fixed and flexible rates are considered. Henceforth assume fixed real per capita national rates of output, and perfect international integration of national commodity markets. On the other hand it is convenient to retain the above-mentioned assumptions of perfect capital mobility and zero international integration of national markets for money stocks and labour. Section 2 investigates quasi equilibria under fixed nominal interest rates. Section 3 investigates full equilibria under fixed real interest rates. Section 4 summarizes this chapter.

2. QUASI EQUILIBRIUM UNDER FIXED NOMINAL INTEREST RATES

This section analyzes inter-run interactions between national inflations, budget deficits, and trade deficits. It provides a multi-country counterpart of the "budgetary" explanation of inflation (see Chapter I). Specifically, the following questions are addressed: In a world of pegged nominal interest rates, what are the global inflationary effects of an increase in the legislated deficit in one or more countries? Under what conditions will higher public-sector deficits "matter", in the sense of affecting private disposable incomes at the national and global levels? And, if private incomes are affected at the national level, what is the role of trade deficits in international income redistributions? Is stability possible in a world of pegged nominal interest rates and endogenous monetary growth rates?

A necessary preliminary is a more precise specification of national public-sector policies.
Public Policy

Each public sector pegs its real per capita spending and net explicit tax receipts. There are no public-sector imports, nor are there any net explicit taxes raised from non-residents. Net explicit taxes are taken to be lump-sum imposts and transfers (proportional taxes on factor incomes would make no difference to the analysis). Each public sector denominates its debt issues in terms of its own noninterest-bearing debt.

The $n^{th}$ public sector always pegs the nominal rate of return on its interest-bearing debt. Additionally, it never intervenes in foreign exchange markets and holds no international reserves. Each of the other public sectors pegs either the price of its noninterest-bearing debt in terms of the $n^{th}$ currency, or the own-currency nominal rate of return on its interest-bearing debt. In the fixed-rates case, international reserves are held, in the form of the interest-bearing debt of the $n^{th}$ public sector; the own-currency equivalent of the interest proceeds are rebated to the local private sector by distributionally neutral means. Domestic credit is expanded as is necessary to ensure a constant international reserve ratio (cf. Johnson (1973)). In the flexible-rates case, no international reserves are held.

Public-Sector Deficits and Inflation

This subsection establishes the following proposition: A unit increase in the global legislated deficit will raise the steady-state rate of inflation in each country by the inverse of the global real per capita stock of aggregate public debt.$^1$
Under the assumptions introduced thus far in this chapter, the proposition in question holds regardless of whether the exchange rate of any public sector is fixed or flexible. Nor need we specify which subset of the \( n \) countries is responsible for the disturbance in question. On the other hand, the proposition holds only in the neighbourhood of "mild" inflations. Also, real rates of interest must initially conform to the golden rule. Previous chapters introduced the single-country counterparts of these initial conditions; their multi-country counterpart will be formalized shortly. Also as in preceding chapters, assume that national labour stocks grow at a common rate, \( \bar{n} \), say.

As a first step towards establishing the proposition in question, let us write the public-sector constraint, \( i=1, \ldots, n \), in nominal and real per capita terms respectively:

\[
\dot{V}_i = D_i + r_i (V_i - C_i), \quad \dot{V}_i = \delta_i + \rho_i (V_i - C_i) - \pi_i c_i - n_i v_i \quad (4.1),(4.1),
\]

where the subscript denotes variables referring to the \( i \)th country; and

\[
v_i = \frac{V_i}{P_i N_i} = \text{real per capita aggregate debt issued by } i \text{'s public sector},
\]

\[
v_i - c_i = \frac{(V_i - C_i)}{P_i N_i} = \text{real per capita interest-bearing debt issued by } i \text{'s public sector},
\]

\[
c_i = \frac{C_i}{P_i N_i} = \text{real per capita monetized debt issued by } i \text{'s public sector},
\]

\[
\delta_i = \frac{D_i}{P_i N_i} = g_i - t_i = \text{real per capita legislated deficit of } i \text{'s public sector},
\]
\[ g_i \equiv \frac{G_i}{P_i N_i} = \text{real per capita spending by i's public sector}, \]
\[ t_i \equiv \frac{T_i}{P_i N_i} = \text{real per capita net explicit taxes raised by i's public sector}, \]
\[ \rho_i \equiv r_i - \pi_i = \text{real rate of return on the interest-bearing debt issued by i's public sector}, \]
\[ \pi_i \equiv \frac{P_i}{P_i} = \text{rate of inflation of the local-currency price of i's output}, \]
\[ n_i \equiv \frac{N_i}{N_i} = \text{rate of growth of i's labour stock, a constant}. \]

Note that our policy assumption concerning the redistribution of interest earnings on reserves, from the public sector to the private sector, is necessary to ensure that the public-sector constraints for the 'peripheral' countries are symmetrical with the public-sector constraint for the \( n \)th country.

Next, transpose the 'budgetary' policy regime into symbols.

We have
\[ g_i = \dot{g}_i, \quad t_i = \dot{t}_i \quad (4.2)', (4.3)', \]
for each country; and
\[ e_i = 0 \text{ with } f_i = \psi_i(c_i + f_i) \quad (4.4)', (4.5)', \]
or
\[ f_i = 0 \text{ with } \rho_i + \pi_i = \dot{r}_i \quad (4.6)', (4.7)', \]
according as whether country \( i, i \neq n \), is maintaining a fixed or flexible exchange rate. For the \( n \)th country, on the other hand, \( e_n = 0, f_n = 0 \), and \( \rho_n + \pi_n = \dot{r}_n \), irrespective of the exchange-rate regime.
Equations (4.2)' to (4.7)', which have obvious nominal counterparts (4.2) to (4.7), introduce the variables:

$$
e_i = \frac{E_i}{E} = \text{rate of depreciation of the price of } i\text{'s currency in terms of } n\text{'s currency},$$

$$f_i = \frac{F_i}{PNI_i} = \text{real per capita international reserves held by the } i^{th} \text{ public sector},$$

$$\psi_i = \text{proportional international backing of } i\text{'s monetary base, a constant.}$$

Similarly, transpose the initial state into symbols:

$$\pi_i^0 + n_i = \pi_j^0 + n_j = 0$$

$$\delta_i^0 = \delta_j^0 = 0$$

$$\rho_i^0 - n_i = \rho_j^0 - n_j = 0$$

where \(i, j = 1, \ldots, n\), and the superscript (0) denotes initial values. It can be shown that any one of these three pairs of equations is redundant in the initial steady state, given the other two pairs.

Now select units for goods, labour stocks and currency which set 'nuisance' constants equal to unity, and transpose the arbitrage and 'growth parity' assumptions into symbols (the absence of a subscript denotes transnational variables):

$$P = P_n = \frac{P_i}{E_i}, \quad \pi = \pi_n = \pi_i - \epsilon_i$$

$$r = r_n = r_i - \epsilon_i, \quad \rho = \rho_n = \rho_i$$

$$N = N_n = N_i, \quad n = n_n = n_i$$

where \(i = 1, \ldots, n\); the equation numbers on the right-hand side refer to equations (not the identities); and
\[ \pi \equiv \frac{P}{P} = \text{rate of inflation of the price of the numéraire}\]

(i.e., the \( n \text{th} \) country's output),

\[ \rho \equiv r - \pi = \text{real rate of return on the interest-bearing debt} \]

issued by the numéraire country's public sector,

\[ \bar{n} \equiv \frac{N}{N} = \text{rate of growth of the numéraire country's labour stock}. \]

Equations (4.9) to (4.12) enable us to consolidate the national public-sector constraints, thereby aggregating up to the global public-sector constraint in nominal and real per capita terms respectively:\[\text{2}\]

\[ V = \Delta r(V-C) + \sum_{i=1}^{n-1} \epsilon_i \cdot c_i / E_i, ~ \check{V} = \delta + (v-c) - \bar{n} \bar{v} + \sum_{i=1}^{n-1} \epsilon_i \cdot c_i \] (4.12), (4.12),

where

\[ v \equiv \frac{V}{PN} \equiv \sum_{i=1}^{n} v_i = \text{real per capita aggregate debt issued by the world's public sector}, \]

\[ v-c \equiv \frac{(V-C)}{PN} \equiv \sum_{i=1}^{n} (v_i - c_i) = \text{real per capita interest-bearing debt issued by the world's public sector}, \]

\[ c \equiv \frac{C}{PN} \equiv \sum_{i=1}^{n} c_i = \text{real per capita monetized debt issued by the world's public sector}, \]

\[ \delta \equiv \frac{D}{PN} \equiv \sum_{i=1}^{n} \delta_i = g-t = \text{real per capita legislated deficit of the world's public sector}, \]

\[ g \equiv \frac{G}{PN} \equiv \sum_{i=1}^{n} g_i = \text{real per capita spending by the world's public sector}, \]

\[ t \equiv \frac{T}{PN} = \sum_{i=1}^{n} t_i = \text{real per capita net explicit taxes raised by the world's public sector}. \]
Equation (4.12) states that in terms of the numeraire currency, the world's public sector finances its legislated deficit and interest obligations by issuing debt and/or by depreciating the numeraire currency. Equation (4.12)' reveals that in real per capita terms, the inflation tax on monetized debt, \( \pi_c \), and the growth tax on aggregate debt, \( \bar{\nu} \), raise revenue for the world's public sector.

Finally, to establish the proposition in question, consider the steady-state counterpart of (4.12)', to which we apply the policy equations (4.2)', (4.3)', (4.4)', and (4.5)'; the initial conditions (4.8)'; and equations (4.9)', (4.10)' and (4.11)'--the conditions for purchasing power parity, interest rate parity and growth parity respectively. This yields

\[
d\hat{\pi}*/d\hat{\delta} = 1/\nu* ,
\]

as required.

This result warrants further comment. In the first place, it represents a straightforward generalization of one of our main closed-economy results; see Chapter I. (That result, in turn, is a variable-inflation counterpart of one of the main results in Christ (1968), (1969).)

Second, there is symmetry between countries. Specifically, each public sector has equal scope for raising inflation-tax revenues from the world's private sector. By contrast, the best-known multicountry extensions of inflation-tax theory, namely, Mundell (1971), (1972), suggest that the \( n \)th public sector will have much more scope for taxing the rest of the world than the other \( n-1 \) public sectors, at least in the case of fixed exchange rates. The
difference can be explained partly by the fact that Mundell characterizes expansionary public policy in the n\textsuperscript{th} country as a fiscal-accommodated monetary expansion (see Chapter I), which the other n-1 public sectors are unable to emulate because they must permit nominal monetary growth to be endogenous. Another source of difference is that he is concerned with full equilibria rather than quasi equilibria. (We shall find that Mundellian asymmetries in the fixed-rates case re-emerge when we go to the case of full equilibrium; see Section 3.) Finally, he assumes that reserves are held in the form of noninterest-bearing debt of the n\textsuperscript{th} public sector.

Third, there is symmetry between exchange-rate regimes. This is in line with an observation made by, for example, Porter (1976): in a world of perfect capital mobility and fixed nominal interest rates, flexible rates will not provide relatively more insulation of the domestic inflation rate. The underlying reasoning, which was outlined in Chapter III, is very simple: if national nominal interest rates are pegged, then international arbitrage will effectively peg exchange depreciation rates. As might be expected, this is another instance wherein symmetry does not hold under the "ultra-long-run" conditions investigated in Section 3 below.

**Public-Sector Deficits and Private Disposable Income**

This subsection establishes the following propositions: in each country, steady-state private disposable income is given by home output plus the home trade deficit minus home public spending, where each of these variables is in real per capita terms. Hence
global steady-state private disposable income will decline one-to-one with any increase in global public spending, but is invariant with respect to any change in global net explicit taxes.

The first step towards deriving this proposition is to introduce i's private-sector budget constraint in nominal and real per capita terms respectively:

\[
W_i = Y_i - Z_i, \quad \hat{W}_i = y_i - z_i
\]

where \(i = 1, \ldots, n\), and

\[
w_i \equiv W_i / P_i N_i = \text{real per capita aggregate public debt held by i's private sector},
\]

\[
y_i \equiv q_i - \tau_i + \rho_i (w_i - c_i) - n \pi_i - n w_i = \text{i's real per capita private disposable income},
\]

\[
\hat{q}_i \equiv Q_i / P_i N_i = \text{i's real per capita output, taken to be fixed},
\]

\[
z_i \equiv Z_i / P_i N_i = \text{real per capita aggregate spending by i's private sector}.
\]

As in the case of the public-sector constraints (4.1), the policy assumption that interest earnings from international reserves are rebated is important to ensure symmetry across countries and exchange-rate regimes.

Consider next i's national expenditure identity, in nominal and real per capita terms respectively:

\[
Q_i \equiv Z_i + G_i + X_i, \quad q_i \equiv z_i + g_i + x_i
\]

where \(x_i \equiv X_i / P_i N_i = \text{i's real per capita trade surplus}, i = 1, \ldots, n\), and

\[
\Sigma x_i = 0.
\]
These notions require no further comment.

The steady-state counterpart of (4.14)', the identity (4.15)', and the assumption that $q_i$ is fixed, together imply the first part of the proposition under consideration:

$$y^* = q_i - x_i - g_i,$$

where $i = 1, \ldots, n$. And summation over all countries yields the second part:

$$y^* = q - g,$$

where $y = \sum_{i=1}^{n} y_i = \text{real per capita global disposable income},$

$$q = \sum_{i=1}^{n} q_i = \text{real per capita global output},$$

$$g = \sum_{i=1}^{n} g_i = \text{real per capita global public spending}.$$

Equations (4.16)' and (4.17)', which have the elementary "crowding-out" implications noted at the beginning of this subsection, may be seen to rely on the assumption of perfect goods-market integration. Otherwise there would be terms-of-trade complications; see Chapter II. Less obviously, they rely on the assumption of a 'mild' inflation; see Section 3 below.

One corollary of (4.17)' is $\frac{dy^*}{dt} = 0$, which is analogous to what Buchanan and Wagner (1976) have termed the "Ricardian Invariance Theorem"--or, it makes no eventual difference to private disposable income whether a given increase in government outlays is financed by printing money or by selling bonds. Our claim is that it makes no eventual difference to aggregate private disposable income
whether a given increase in government outlays is financed by explicit taxes or by inflation taxes.

**Public-Sector Deficits and Trade Deficits**

This subsection establishes the following proposition: A unit increase in the global real per capita legislated deficit will worsen the real per capita trade balance of each country by the home public sector's share in global increases in legislated deficits, minus the home private sector's share in global aggregate public debt.

The first step towards establishing this proposition is to explain the sources of accumulation (or decumulation) of public debt by each country's private sector. Insert the definition of \( y_i \) into (4.14), and use (4.15) to eliminate \( z_i \). This yields

\[
\dot{\hat{w}}_i = \delta_i + x_i + \rho_i \left( \hat{w}_i - c_i \right) - \pi_i \hat{c}_i - n_i \hat{w}_i, 
\]

(4.18)

\( i = 1, \ldots, n \), in real per capita terms; its nominal counterpart (4.18) (not shown) is equally straightforward to derive.

Next, recall the operations immediately preceding (4.13). This yields the required result:

\[
- \frac{dx_i^*}{d\hat{\delta}} = \frac{\dot{\hat{\delta}}}{\hat{\delta}} - \frac{\dot{\hat{w}}_i}{\hat{w}^*},
\]

where \( i = 1, \ldots, n \); a hat (\(^\hat{}\)) denotes small deviations in a variable; and

\[
w = v = \text{global real per capita public debt.}
\]

We make two comments on this result. First, a standard proposition in the theory of small open economies (discussed at length in Chapter II) is that an increase in public spending or a reduction in net explicit taxes will raise the trade deficit by the
same amount. The above result reduces to that proposition if the \( i^{th} \)
public sector has a unit share in any increases in the world's legis-
lated deficit (i.e., \( \hat{\delta}_j = 0 \) for \( j \neq i \)) and the \( i^{th} \)
private sector has a zero share in global public debt (i.e., \( v_i/w = 0 \)). Both these
assumptions underlie the standard small-country analysis. Our multi-
country approach to the problem draws attention to the restrictive
nature of these assumptions, especially the former. This reveals an-
other pitfall in the "New School's" approach to balance-of-payments
forecasting (see Chapter II), since that approach typically disre-
gards fiscal developments in the rest of the world; see, e.g.,
Godley and Cripps (1973).

Second, define a "co-ordinated" global fiscal expansion as one
which does not disturb the initial balance of trade. (From conditions
(4.8)' and the steady-state counterpart of (4.18)', we see that trade
is everywhere balanced in the initial steady state.) Then the neces-
sary and sufficient condition for such an expansion is that national
public-sector shares in the increased global legislated deficit equal
national private-sector shares in the initial stock of public debt.
Stability

This subsection establishes the following proposition: the policy regime under consideration will, at best, stabilize all variables except the share of each national public sector in the global supply of interest-bearing public debt—that variable is, at best, only neutrally stable.

Thus far we have made minimal use of assumptions on private-sector behaviour. Such assumptions must now be introduced. Postulate the private expenditure functions

$$z_i = z_i + (1-s_i)y_i - \ell_1 + \omega_i w_i,$$

and the money-market equilibrium conditions

$$c_i + f_i = \ell_i(q_i, r_i, w_i),$$

where $0 < s_i < 1$ and $\omega_i > 0$ for $i=1,\ldots,n$; $f_i \geq 0$ for $i=1,\ldots,n-1$; and $f_n = 0$. Equations (4.19)' require no further comment. Equations (4.20)' embody a more general specification of money demand than we have utilized hitherto, because a restrictive formulation turns out to be unnecessary in the present context.
It can be shown that by introducing (4.19)' and (4.20)' we have closed the model; the details are not set out. The complete model is conveniently recursive, especially in deviation form.

Even so, to ascertain the stability of the model in the case of an arbitrary number of countries is a complex task which does not repay the effort involved.

Accordingly, consider the two-country dynamic subsystem:

\[
\dot{\hat{x}}_i = \hat{x}_i - \hat{w}_i, \\
0 = \omega_i \dot{\hat{x}}_i + \hat{x}_i - [(1-s_i)\hat{w}_i - \lambda_i] \hat{n}, \\
0 = \sum \hat{x}_i,
\]

where \(i=1,2\); a hat (\(^\hat{\cdot}\)) denotes a small deviation from the steady-state value of a variable; and the details of how these equations are derived is relegated to a footnote.\(^4\) The characteristic polynomial of this system is easily found to be

\[
\Gamma(\lambda) = \begin{vmatrix}
-\lambda & 0 & 1 & 0 & -\hat{w}_1^* \\
0 & -\lambda & 0 & 1 & -\hat{w}_2^* \\
\omega_1 & 0 & 1 & 0 & -[(1-s_1)\hat{w}_1^* - \lambda_1] \\
0 & \omega_2 & 0 & 1 & -[(1-s_2)\hat{w}_2^* - \lambda_2] \\
0 & 0 & 1 & 1 & 0
\end{vmatrix},
\]

which yields the characteristic equation
\[\lambda^2 \left\{ [(1-s_1)w_1^* - \lambda_1] + [(1-s_2)w_2^* - \lambda_2] \right\} + \lambda \left\{ \omega_1 w_1^* + \omega_2 w_2^* + \omega_1 [(1-s_2)w_2^* - \lambda_2] + \omega_2 [(1-s_1)w_1^* - \lambda_1] \right\} + \omega_1 \omega_2 w^* = 0.\]

As essentially is the case in a closed economy under the "budgetary regime" (see Chapter I), the critical expression is \([(1-s_i)w_i^* - \lambda_i], i=1,2\) "critical" in the sense that a necessary condition for stability is \(\Sigma [(1-s_i)w_i^* - \lambda_i] > 0\), and a sufficient condition for instability is the reverse of that inequality. The relevant economic considerations were discussed at length in Chapter I.

Consider now the following subsystem involving asset creation by national public sectors:

\[\dot{\nu} = -\nu_i^* \hat{n}\]

where \(i=1,2\); and the equations in question follow from (4.1)' and the operations immediately preceding (4.13)'. To investigate the stability of \(\nu_i, i=1,2\), form the following 7 x 7 bordered determinant:

\[\Delta(\lambda) = \begin{vmatrix} \Gamma(\lambda) & 0 & 0 \\ \vdots & \vdots & \vdots \\ 0 & 0 & 0 \\ \end{vmatrix} - \begin{vmatrix} 0 \cdots & -\nu_1^* -\lambda & 0 \\ \vdots & 0 & 0 \\ 0 \cdots & -\nu_2^* & 0 -\lambda \\ \end{vmatrix},\]

which yields the characteristic equation

\[\lambda^2 \Gamma(\lambda) = 0.\]
Clearly there are (at least) two zero eigenvalues, so that the subsystem in question is (at best) neutrally stable.

The remaining step towards establishing stability properties of the complete model is to verify that with the exception of \((v_1 - c_1)\) and \((v_2 - c_2)\), all the endogenous variables in their perturbed states are either zero or linear combinations of \(\hat{w}_i\). The details are quite straightforward, albeit tedious, and are not set out.

3. FULL EQUILIBRIUM UNDER FIXED REAL INTEREST RATES

Thus far our multi-country analysis has proceeded on the assumption that national public sectors are able to hold down the nominal rate of return on their interest-bearing debt—even in the face of a sustained rise in the inflation rate. It might be objected that the model does not capture substitution considerations which militate against the long-run feasibility of this policy. One example is the neglect of the margin of substitution between money and durable goods.

Another questionable aspect of the analysis, from a longer-run standpoint, is the (at best) neutral stability of the stocks of interest-bearing liabilities of national public sectors.

Finally, the analysis of the preceding section is heavily dependent on the assumption that inflation is confined to the neighbourhood of its full-liquidity rate—which, of course, means a negative rate of inflation, providing the rate of population and real output growth is significantly positive, and the real rate of interest is in the neighbourhood of its golden-rule rate. What can we say about the
budgetary aspects of global inflations which proceed more briskly than that?

This section addresses these issues, in a somewhat different setting to that of Section 2. The reason is that simply to impose an alternative policy regime on our Section 2 model would ensue in excessive complication, and would also raise the awkward problem of defining real private disposable income when the deadweight losses arising from suboptimal holdings of real balances are no longer of the second order of smalls. Our approach is to set aside goods-market equilibrium conditions and the question of the appropriate definition of private disposable income, instead bringing to the fore the interactions between public-sector budget constraints and money-market equilibrium conditions.

Put another way, our approach is to take the well-known Cagan-Bailey model of inflation in a closed economy; superimpose some of the extensions of that model proposed by Mundell (1972), Cathcart (1974) and others (the former reference extends the Cagan-Bailey model to the case of two countries); and investigate the consequences of at least one national public sector choosing to peg its real per capita legislated deficit rather than the rate of growth of its nominal monetized debt. In doing so, and in contrast to most of the preceding analysis, we are also able to ascertain existence and uniqueness properties, owing to the simplicity of the model, and to a modified version of the novel and revealing diagram developed in Turnovsky (1978).
Assumptions

Despite these and other differences between the Section 2 model and the one here, we retain the assumptions concerning the extent to which various markets are integrated internationally; fixed national real per capita rates of output (i.e., $q_i = \bar{q}_i$); perfect myopic foresight of the exchange depreciation rate; and a common global growth rate, $n$ say. Also as in Section 2, international reserves earn interest, and are held if and only if the local parity is pegged.

We also bring back assumptions sometimes used in preceding chapters. Specifically, recall the semilogarithmic specification of money demand (i.e., $\ell_1^i = k_1 q_1^i \exp[-\alpha_1 (\rho_1 + \pi_1^i)]$); the assumption that the interest-bearing debt of national public sectors is indexed and yields a golden-rule real rate of return (i.e., $\rho_1^i = n_1^i$); and the adaptive specification of inflation expectations.

On the other hand, there is the new assumption that the real per capita interest-bearing debt of each public sector is pegged—in nominal terms, $(V_i^t - C_i^t)/(V_i^0 - C_i^0) = \pi_i^t + n_i^t$. This serves to exclude quasi equilibria of the kind discussed in the preceding section. Note that the policy regime to be studied in this section would be incompatible with the Section 2 model, since it would over-determine that system.

In the present case, there are not yet enough policy rules to close the system. Nevertheless it is worth pausing to formalize the discussion thus far:

$$\dot{c}_a = \delta_a - (p_a + n)c_a, \quad \dot{c}_b = \delta_b - (p_b + n)c_b \quad (4.25a)', \quad (4.25b)'$$
\[ c_a = k_a \exp[\gamma_a (n + \pi_a)], \quad c_b + f = k_b \exp[\gamma_b (n + \pi_b)] \quad (4.26a'), (4.26b') \]

\[ \dot{\pi}_a = \beta_a (p_a - \pi_a), \quad \dot{\pi}_b = \beta_b (p_b - \pi_b) \quad (4.27a'), (4.27b') \]

\[ \epsilon = p_b - p_a \quad (4.28') \]

Equations (4.25)' merely restate equations (4.1)' in the special case of two countries (A and B once again), and the policy regime described earlier in this section. With regard to (4.26)', note that they reflect \( q_a = q_b = 1 \) (see Chapter I); and that real per capita reserves \( f \) exceed zero if and only if the rate of exchange depreciation \( \epsilon \) is pegged at zero (see below). Concerning (4.27)', and recalling that inflation expectations were characterized by the perfect myopic foresight hypothesis in Section 2, the reversion to an adaptive hypothesis might be puzzling. The explanation is that the model yields two kinds of steady-state equilibria, one kind necessarily characterized by rates of inflation in the hyperinflation range, and the other kind being consistent with inflations less rapid than that. We wish to focus on the latter. Under perfect myopic foresight and sluggish price-level adjustment, the former kind of equilibrium is stable and the latter kind is unstable. But under adaptive expectations, and given the Cagan stability condition, the reverse situation is true. Hence equations (4.27)'). Finally, equation (4.28)' constitutes an elementary purchasing-power-parity explanation of exchange rate movements, as is required when A and B goods are perfect substitutes.
Flexible Exchange Rates

Since it is simpler, we begin with the flexible-rates case. We close the foregoing model by imposing

\[
\begin{align*}
\delta_a &= \delta_a \\
\delta_b &= \delta_b \\
f &= 0
\end{align*}
\]

so that the endogenous variables are \( c_a, c_b, p_a, p_b, \pi_a, \pi_b, \) and \( \varepsilon. \)

Note, too, that we shall refer as is convenient to national nominal rates of monetary growth \( \mu_a \) and \( \mu_b, \) given by

\[
\begin{align*}
\mu_a &= \dot{c}_a /c_a + p_a + n, \quad \mu_b &= \dot{c}_b /c_b + p_b + n,
\end{align*}
\]

and ex ante national nominal rates of interest, given by

\[
\begin{align*}
r_a &= \pi_a + n, \quad r_b &= \pi_b + n.
\end{align*}
\]

It follows that \( \mu_a^*, r_a^* \) and \( \pi_a^* + n \) may be used interchangeably (as can their B counterparts), which considerably simplifies the analysis.

The ensuing overall model of a flexible-rates world is both recursive and decomposable. Thus, for example, each country's internal variables are wholly independent of those in the other country, so that without loss of generality we can confine attention to one country, \( \text{A} \) say.

That country's steady state can be summarized by the pair of equations
\[ \delta_a - \mu^*_a c_a^* = 0 \quad (4.32a)' \]
\[ c_a^* - k_a \exp(-\alpha_a^* \mu_a^*) = 0. \quad (4.33a)' \]

Equations (4.32a)' and (4.33a)' have an interesting geometrical representation; see Figure 2 below.

Figure 2 embodies notions similar to those of Figure 1 of Turnovsky (1978) (though Turnovsky postulates a linear money demand function). Using essentially the same notation as Turnovsky (1978), the GAGA schedule shows the combinations of \(c_a^*\) and \(\mu_a^*\) that are consistent with A's public sector balancing its steady-state budget. It is a rectangular hyperbola. (In the Cagan-Bailey model, by contrast, the appropriate policy schedule is a horizontal line, representing the authorities' choice of a particular rate of nominal monetary growth.)

The LALA schedule shows the combinations of \(c_a^*\) and \(\mu_a^*\) that are consistent with A's private sector balancing its steady-state portfolio. It belongs to the exponential family.

Figure 2 depicts the special case \(0 < \mu_a^* < 1/\alpha_a\) or, equivalently, \(0 < \delta_a < k_a(2.7 \times \alpha_a)^{-1}\); the alternatives are discussed subsequently, as is the choice of \(P_A\) as the relevant equilibrium even though \(L_A\) cuts \(G_A\) at \(Q_A\) as well. It is easily shown that the two schedules have the same slopes at \(R_A\) and \(R_A'\), where those points correspond to \(\mu_a^* = 1/\alpha_a\). The shaded rectangle shows A's steady-state legislated deficit--or, in conventional terminology, its steady-state revenue from money creation. The shaded triangular area shows the deadweight loss arising from the fact that A's public sector is printing money at a rate faster than that prescribed by the full-liquidity rule.
Full-Equilibrium Budget and Portfolio Balance in an Intermediate-Sized Economy under Fixed Real Interest and Flexible Exchange Rates

Figure 2 leads to the following simple observation: under full equilibrium and fixed real interest rates, the system would be over-determined in the event that a public sector were independently to peg both its legislated deficit and the rate at which it creates nominal noninterest-bearing debt. By pegging the legislated deficit (i.e., by anchoring the $G_A$ schedule), a public sector determines its total revenues from money creation. And by exhibiting a "stable" demand for money (cf. the $L_A$ schedule), the private sector takes up the system's remaining degree of freedom.
Despite its rudimentary character, this line of argument has a policy implication which is both important and little understood: advocacy of a constant monetary growth rule should go hand in hand with advocacy of an accommodating fiscal policy. If, for example, nominal monetary growth is reduced from \( x \) percent per annum to zero, and then held at that rate, then fiscal policy should be conducted so as to ensure that the difference between public spending and net explicit taxes (expressed as a fraction of the base money stock) is able to follow suit.\(^6\)

As a final exercise in comparative steady states under flexible rates, consider the implications of successively more inflationary initial states.

At \( \delta_a^0 = 0 \) (i.e., \( \mu_a^0 = 0 \)) there exists a unique equilibrium, at the intersection of \( \text{LL}_A \) and the horizontal axis of Figure 2. The relationship \( \mu_a^* = \pi_a^* + n \) and equation (4.32a)' together imply

\[
\left( \frac{d\pi_a^*}{d\delta_a} \right)_{\mu_a^0 = 0^+} = \frac{1}{c_a^*} \quad (= 1/k_a) .
\]

Alternatively (for future reference):

\[
\left( \frac{d\mu_a^*}{\mu_a^*} \right)_{\delta_a^0 = 0^+} = 1 .
\]

The overall result is comparable to (4.13)'. There are two noteworthy differences, however. First, the relevant inflation-tax base is narrower, i.e., \( c_a^* < \omega^* \). Second, increased deficits in \( B \) have no effect on \( A \)'s inflation rate, and vice-versa. The classical "insulation property" of flexible-rate regimes is reinstated.
At \( 0 < \delta^0_a < k_a(2.7x \alpha_a)^{-1} \) there exist two equilibria, one below the rate of monetary growth associated with maximal revenue from money creation (i.e., \( \mu^*_a = 1/\alpha_a \)), and the other above that rate; see Figure 2. Focusing on the former case, from (4.32a)', (4.33a)' and the relationship \( \mu^*_a = \pi^*_a + n \), we deduce

\[
\left( \frac{d\pi^*_a}{d\delta_a} \right) < 0 < \mu^0_a < 1/\alpha_a
\]

\[
\left( \frac{d\mu^*_a}{d\mu_a} \right) = 1/(1-\alpha^*_a) > 1.
\]

The restoration of budget balance after an increase in the legislated deficit requires a more than proportional increase in the rate of monetary growth, as a consequence of the induced reduction in the relevant tax base. In the terminology of Mundell (1972), there is a secondary "volume" effect in addition to the primary "value" effect.

At \( \delta^0_a = k_a(2.7x \alpha_a)^{-1} \) (i.e., \( \mu^0_a = 1/\alpha_a \)) there exists one equilibrium, given by the tangency of \( G_A \) to \( L_A \) in Figure 2. Points \( R_A \) and \( R'_A \) will coincide. Figure 2 also suggests that the pure waste of inflationary deficit finance, as compared with noninflationary finance, reaches a maximum at this point. In fact this "collection cost" is easily shown to equal roughly 170 percent of the legislated deficit; cf. Bailey (1956). On the other hand, calculations of this type are of questionable relevance inasmuch as they neglect the costs of raising explicit taxes; cf. Phelps (1972). The responses of inflation
and nominal monetary growth at the margin under consideration are given by

\[ (\frac{d\pi^*_a}{d\delta_a})_o = \frac{(d\mu_a^*/\mu_a^*)}{(d\delta_a/\delta_a)}_o = +\infty, \]

as noted in Chapter III.

At \( \delta^0_a > k_a (2.7 \times \alpha_a)^{-1} \), no equilibrium exists. This suggests that actual inflations in excess of the revenue-maximizing rate \( \pi^*_a = 1/\alpha_a \) - \( n \) either are transitory phenomena or are generated by a policy regime different to that considered here. For example, providing \( \alpha_a \beta_a \) were less than unity for some \( \mu_a^* > 1 \), the Cagan-Bailey policy \( \mu_a = \mu_a^* \) would be capable of generating such an inflation; see Cagan (1956) or Chapter I above.

**Fixed Exchange Rates**

Now we adopt the model consisting of equations (4.25)' to (4.28)' to the case of fixed exchange rates. There are two interesting ways of introducing that regime into the model. One is to postulate that B (as well as A) pegs its legislated deficit \( \delta_b \), thereby relinquishing proximate control over the composition, \( f/(c_b + f) \), as well as the level \( c_b + f \), of its real per capita monetary base. The other is to assume that B does wish to exert proximate control over the composition of its monetary base, in which event it must relinquish proximate control over its legislated deficit. We shall deal with these two cases in turn.

Formally, the first variant is described by
\[
\begin{align*}
\delta_a &= \delta_a \\
\delta_b &= \delta_b \\
e &= 0
\end{align*}
\] (4.34)'

so that the endogenous variables are \( c_a, c_b, P_a, P_b, \pi_a, \pi_b, \) and \( \varepsilon \).

Additionally, equations (4.30)' and (4.31)' continue to apply, providing \( \mu_b \) is interpreted as referring to \( \dot{c}_b/C_b \) rather than \( \dot{L}_b/L_b \). (Of course, this distinction need not be made when one is comparing steady states.)

Unlike the flexible rates model, the overall fixed-rate model cannot be split into subsystems describing each national economy in isolation. But it is still conveniently recursive. This is apparent from the following summary of the steady-state relationships:

\[
\begin{align*}
\delta_a - \mu_a c_a^* &= 0 \quad (4.35a)'
\\
c_a^* - k_a \exp(-\alpha_a \mu_a^*) &= 0 \quad (4.36a)'
\\
\delta_b - \mu_a c_b^* &= 0 \quad (4.35b)'
\\
c_b^* + f^* - k_b \exp(-\alpha_b \mu_a^*) &= 0 \quad (4.36b)'
\end{align*}
\]

The nature of the recursivity is quite simple. The world rate of growth of nominal variables is simultaneously determined by A's budget balance schedule (4.35a)' and its portfolio-balance schedule (4.36a)'. B must accept this rate \( \mu_a^* \), which together with B's legislated deficit \( \delta_b \), determines the domestic source component of B's monetary base \( c_b^* \); see (4.35b)'. Finally, these relationships together with B's money-market equilibrium condition (4.36b)', determine the level of B's reserves \( f^* \).
In summary, and in contrast to Section 2, we find a re-emergence of Mundellian asymmetries between key-currency countries and the rest of the world. Indeed, the rest of the world's loss of policy autonomy is more severe than is often thought to be the case, inasmuch as B's public sector cannot peg both its legislated deficit $\delta_b$ and its international reserves as a proportion of B's monetary base $\psi (F/L_b)$, even if B is not a small country.

The next step is to consider the fixed-rates counterpart of Figure 2. The left-hand panel of Figure 3 below requires no further comment. In the right-hand panel, $G_B G_B$ shows the combinations of $c_b^*$ that are consistent with B's public sector balancing its steady-state budget, for exogenously-given values of $\mu_a^*$ and $\delta_b$. The $L_B L_b$ schedule shows the combinations of $f^*$ that are consistent with B's private sector balancing its portfolio, for exogenously-given values of $\mu_a^*$ and $c_b^*$. Other features depicted by Figure 3 should be obvious in the light of the foregoing discussion of Figure 2. Note that as it stands, Figure 3 suffices to represent the case wherein $\psi$ rather than $\delta_b$ is pegged; see below.

In examining the comparative-static properties of (4.35a) - (4.36b), we confine attention to the intermediate case $0 < \delta_a^0 < k_a (2.7 x \alpha_a)^{-1}$ and $0 < \mu_a^0 < \min(1/\alpha_a^0; 1/\alpha_b^0)$. (It follows that $\delta_b$ cannot be pegged at a level less than zero.) The effect of an increase in A's legislated deficit on the world rate of growth of nominal
FIGURE 3

Full-Equilibrium Budget and Portfolio Balance in a Two-Country World Under Fixed Real Interest and Exchange Rates
variables is the same as the effect of an increase in $\delta_a$ on $\pi_a^*$ or $\mu_a^*$ in the case of flexible rates (see above), and therefore requires no further discussion.

Other fixed-rate effects of an increase in $\delta_a^*$ include

$$\frac{dc_b^*}{c_b^*} = \frac{1}{1 - \alpha_a^* \mu_a^*} < -1,$$

$$\text{sgn}[df^*/d\delta_a^*] = \text{sgn}[1 - \psi^* - \alpha_b^* \mu_a^*] > 0.$$

The first result is straightforward. An increase in A's legislated deficit will raise B's inflation-cum-growth tax rate $\mu_b^* (\equiv \mu_a^*)$, which necessitates a reduction in B's inflation-cum-growth tax base $c_b^*$ in order to preserve budget balance in B. The second result, on the other hand, is a little more complicated. An increase in A's legislated deficit will lower the opportunity cost of holding real per capita money balances in B--specifically, $r_b^* (\equiv \pi_a^*)$ will rise. Hence $\kappa_b^*$ will decline. Offsetting this negative effect on B's real per capita reserves is a decline in the domestic source component $c_b^*$ of B's monetary base $\kappa_b^*$. The overall effect is ambiguous, and will depend on the initial relative magnitudes of the steady-state proportion of domestic assets in B's monetary base, $1 - \psi^*$, and the steady-state interest elasticity of money demand by B residents, $\alpha_b^* \mu_a^*$. This ambiguity is readily deduced from Figure 3.

Observe, too, that

$$\frac{dc_b^*}{c_b^*} = \frac{1}{1 - \psi^*} < 0.$$

In other words, an increase in B's legislated deficit will raise the
domestic source component of B's monetary base and lower the foreign source component by the same amount, leaving B's overall monetary base unchanged. In the light of the previous explanations these results require no further comment.

Consider next the case of a pegged rather than endogenous international reserve ratio $\psi$, whence B's legislated deficit $\delta_b$ is endogenous rather than pegged. Formally, replace (4.34)' by

$$\delta_a = \frac{\delta_a}{\psi}$$

$$(4.37)'$$

(Assume $0 < \psi < 1$, for simplicity.)

Similarly, replace the steady-state equations (4.35b)' and (4.36b)' by

$$\delta_b - \mu_a (1 - \psi) k_b \exp(-\alpha_b \mu_a) = 0, \quad (4.38b)'$$

$$f - \psi k_b \exp(-\alpha_b \mu_a) = 0. \quad (4.39b)'$$

Together with (4.35a)' and (4.36a)', these equations yield a recursive system. We proceed directly to the comparison of steady states in the intermediate case $0 < \delta_a < k_a (2.7 x \alpha_a)^{-1}$ and $0 < \mu_a < \min(1/\alpha_a, 1/\alpha_b)$.

Again the effect of an increase in $\delta_a$ on the world rate of growth of nominal variables is the same as the effect of an increase in $\delta_a$ on $\pi_a$ and $\mu_a$ in the case of flexible rates. Other comparative-statical results include:

$$\frac{d\delta_b / \delta_b}{d\delta_a / \delta_a} = \frac{1 - \alpha_b \mu_a}{1 - \alpha_a \mu_a} > 0, \quad \frac{df / f}{d\delta_a / \delta_a} = \frac{-\alpha_b \mu_a}{1 - \alpha_a \mu_a} < 0,$$
An increase in A's legislated deficit will raise the world rate of growth of nominal variables. Hence B's real per capita monetary base will decline. With money-market equilibrium and a pegged reserve ratio in B, this induces a reduction in both the domestic and foreign source components of B's monetary base. In the intermediate range of inflation rates under consideration, B's revenues from money creation will rise, and must be dissipated by an increased legislated deficit in B.

An increase in B's reserve ratio will increase (reduce) the foreign (domestic) source component of B's monetary base. Since the domestic source component of B's monetary base is also B's inflation-cum-growth tax base, and with B's monetary growth determined elsewhere, her revenues from money creation will fall. Hence B's legislated deficit, too, will decline.

**Stability**

The equilibria just investigated are locally stable, given the assumed bounds on parameter values.

Consider first the case of a flexible exchange rate. Focus on the subsystem describing country A, namely, (4.25a)', (4.26a)', and (4.27a)'. (Entirely symmetrical arguments apply to B.) The equations under consideration are easily reduced to the following first-order equation in terms of \( \hat{c}_a \):

\[
\frac{d\delta^*_b/\delta^*_b}{d\psi/\psi} = \frac{\psi}{1-\psi} < 0, \quad \frac{df^*/f^*}{d\psi/\psi} = 1.
\]

\[
\hat{c}_a = \frac{\beta_a}{1 - \alpha_a} \left[ -\alpha_a \frac{\delta^*_a}{\hat{c}_a} + \hat{c}_a \cdot \hat{\lambda}_a \right]. \tag{4.40a}'
\]
Upon linearizing in the neighbourhood of equilibrium we obtain the following eigenvalue, denoted by $-\lambda_a$ for future reference:

$$\lambda = -\lambda_a = -\frac{\beta_a (1 - \alpha b)}{1 - \alpha_a \beta_a^*}.$$  \hfill (4.41a)'

From Figure 2, and given the Cagan condition $\alpha_a \beta_a < 1$, we see that the equilibrium $P_A$ is locally stable, whereas the equilibrium $Q_A$ is locally unstable.

Consider next the case whereby Country B pegs the exchange rate as well as its legislated deficit. Take (4.25b)', (4.27b)' and (4.28), set $e = 0$ and $\delta_b = \tilde{\delta}_b$, and use (4.25a)' and (4.40a)' to eliminate $p_b (= p_a)$ and $\tilde{c}_a$ respectively, to obtain two equations in $c_a$, $c_b$, and $\tau_b$:

$$\dot{c}_b = -\delta_b - \frac{c_b}{c_a} \left[ \delta_a - \frac{\beta_a}{1 - \alpha_b \beta_a} \left[ -\alpha_b \delta_a + c_a \ln \left( \frac{c_a}{c_b} \right) \right] \right],$$  \hfill (4.42)'

$$\dot{\tau}_b = \frac{\beta_b}{c_a} \left[ \delta_a - \frac{\beta_a}{1 - \alpha_b \beta_a} \left[ -\alpha_b \delta_a + c_a \ln \left( \frac{c_a}{c_b} \right) \right] - c_a (\tau_b + n) \right].$$  \hfill (4.43)'

These two equations, together with (4.41a)', define a third-order system in terms of $c_a$, $c_b$, and $\tau_b$. Linearize in the neighbourhood of equilibrium, and recall (4.40a)' to obtain the characteristic polynomial

$$\begin{vmatrix}
-\lambda_a & -\lambda & 0 \\ 
-\lambda_a & -\mu_a & 0 \\ 
\beta_a (\lambda/c_a - \mu_a/c_b^*) & 0 & -\beta_b - \lambda
\end{vmatrix},$$

which yields the simple characteristic equation
The necessary and sufficient conditions for local stability are threefold. First, the reserve-currency country A must be stable in isolation—or, equivalently, it must be stable under floating rates. Second, A must maintain a positive rate of nominal money creation—or, equivalently, a positive legislated deficit. This is because the world rate of inflation tax functions as an automatic stabilizer in restoring budget balance, in close analogy to the factor-income tax in Christ (1968) and the literature building on that paper; see Chapter I. Third, to ensure the eventual convergence of anticipated and actual inflation rates in the rest of the world, the speed of expectations adjustment must be positive in B. From Figure 3, we see that the equilibrium \((P_A, P_B)\) is consistent with these conditions.

Consider finally the case of fixed exchange rates and a pegged reserve ratio in country B. Now it suffices to restrict attention to the dynamic equations \((4.41a)'\) and \((4.43)'\). From the above characteristic polynomial it is easy to deduce this system's eigenvalues, namely, \(-\lambda_a\) and \(-\beta_b\). These require no further discussion.
4. SUMMARY

This chapter has extended the analysis to the case of a multi-country global economy. In contrast to the preceding chapters, attention was focused almost exclusively on the 'budgetary' characterization of public policy. On the other hand, somewhat more attention was devoted to disaggregating time horizons. Thus, Section 1 considered the case of predetermined inflation expectations, and also (briefly) the case of predetermined output. Section 2 went to the longer-run case of perfect myopic foresight and fixed output. At the same time, the time horizon there was postulated to be sufficiently short to permit nominal rate of interest to be pegged, and quasi equilibria to occur, i.e., sustained borrowing and lending between national public sectors. Section 3 envisaged a yet longer time horizon in which rates of interest always conform to the golden rule, and public policies ensure that sustained borrowing and lending between national public sectors does not take place.

As in preceding chapters, the longer-run findings are more novel than those pertaining to the short run, so we restrict attention to Sections 2 and 3. In both sections, and in contrast to most of the analysis in preceding chapters, we went to the limiting case of perfect goods-market integration.

Section 2 investigated steady-state interactions between inflations, budget deficits and trade deficits. In fact this 'steady state' turned out to be at best neutrally stable. Hence our use of the term 'quasi equilibrium' (coined by Mundell (1968)). The following three results from that analysis hold irrespective of whether the
accompanying exchange-rate regime consists of fixed- or freely-floating exchange rates. On the other hand, they do presuppose a mild inflation in the initial state:

1. In each country, steady-state private disposable income is given by home output, plus the home trade deficit, minus home public spending. Hence global steady-state private disposable income will decline one-to-one with any increase in global public spending, but is invariant with respect to any change in global net explicit taxes.

2. A unit increase in the global legislated deficit will raise the steady-state rate of inflation in each country by the inverse of the global stock of aggregate public debt.

3. A unit increase in the global legislated deficit will worsen the trade balance of each country by the home public sector's share in global increases in legislated deficits, minus the home private sector's share in global aggregate public debt.

Section 3 adapted and extended the Cagan-Bailey inflation model to the case of pegged legislated deficits and a two-country global economy. As in Section 2, steady states were the main objects of interest. The term "full equilibrium" was appropriate, since any equilibria between zero and revenue-maximizing rates of inflation turned out to be stable, given the well-known Cagan condition on the product of money-demand and expectational parameters. Note the following implications of that framework:
1. Once inflations and monetary growth are allowed to proceed at any positive rate short of revenue maximization, there are no simple links between public spending, net explicit taxes and steady-state private disposable income, since the deadweight losses associated with inflationary finance are not necessarily of second-order magnitude.

2. Under fixed exchange rates, rates of monetary growth and inflation are equalized internationally. A 1 percent increase in the legislated deficit of the reserve-currency country will induce a more than proportional rise in the steady-state global rates of monetary growth, and global inflation will rise. A 1 percent increase in the legislated deficit of the rest of the world, by contrast, will merely raise the domestic source component of its monetary base by 1 percent, thereby reducing global international reserves; global inflation will not be affected.

3. Under flexible exchange rates, an increase in the legislated deficit of any given country will raise the steady-state rates of monetary growth and inflation in that country only. The extent to which these local rates will rise is entirely analagous to that enunciated in the preceding proposition.
4. Under either fixed or flexible rates, public sectors have fewer degrees of freedom than might be supposed. Specifically, under fixed exchange rates, the rest of the world's public sector cannot peg both its legislated deficit and the proportional reserve backing of its monetary base. And under flexible rates, no national public sector can peg both its legislated deficit and the rate of growth of its monetized debt. It follows that a constant monetary growth rule must be accompanied by an accommodating fiscal policy.
Notes to Chapter I

1. P. 111, loc. cit. Parkin terms this scenario the 'monetary-pull' view. See Brunner (1976) for a similar argument (albeit in the absence of interest-bearing public debt, and in the presence of the two countries rather than one).


3. When an equation has both a nominal and a real per capita form, we shall denote the latter by a prime. Its nominal counterpart is set out only if it is of independent interest.

4. Our derivation uses the propositions \(dq/d\mu = 0\) and \(q_o = 1\). Also, the stated interpretation of (1.9c)' and (1.9d)' presupposes a positive marginal propensity to consume \((s < 1)\), which, in turn, is intimately related to stability; see Section 5.

5, 6. These interpretations, of (1.12a)' and (1.14c)' respectively, are useful from the point of view of comparison with the relevant Christ results; see Section 4 above. In some contexts, however, it is more useful to think in terms of their dimensionless counterparts; see especially Section 4 of Chapter IV. In symbols we have

\[
\frac{d\pi^*/\pi^*}{d\delta/\delta} = \frac{\delta}{\pi^*\omega^*} = \frac{D^*}{\pi^*\omega^*}, \quad \frac{d\delta^*/\delta^*}{d\mu/\mu} = 1 - \alpha\xi^*.
\]

The interpretation of such expressions is given in Chapter IV.
Notes to Chapter II

1 However W is not predetermined in the event of a change in the domestic parity; see Section 3 below for details.

2 To obtain \((L/L)^* = p_w + n\), for example, observe that
\[ \ell^* = k q_F^\phi \exp(-\alpha r_w), \]
whence \(\dot{\ell}^* = 0\).

3 Our subsequent analysis focuses on the case \(p_w = r_w - p_w = n\), i.e., the foreign interest rate conforms to the golden rule. Of course that assumption is implied by (2.6c)' together with (2.11b)'.

Notes to Chapter III

1 The first four sections of this paper are based on Turnovsky Kingston (1977), and especially Kingston and Turnovsky (1978). Section 5 builds on Clarke and Kingston (forthcoming).

2 In addition, neither of these articles actually sets out a price equation. But they have usually been interpreted as having something to say about "inflationary pressure". Moreover, subsequent studies in the Fleming-Mundell framework have explicitly included a "naive" Phillips curve in which the exogenous or predetermined nature of inflation expectations is either implicit, as in Takayama (1969), Helliwell (1969), or explicit, as in Turnovsky and Kaspura (1974).
The first pair of equations follows directly from (3.10)', and the definition $\delta = g - t$. The second pair follows from the first pair, together with (3.4e)' and (3.4f)'. The third pair follows from the other two pairs, together with (3.1c)' and the initial conditions $x_0 = 0$, $\sigma_0 = 1$.

The first row of equalities follows directly from (3.12)'. The second row follows from the first row, together with (3.4e)' and (3.4f)'. The third row follows from the other two rows, together with (3.12b)' and the initial condition $\sigma_0 = 1$.

Apart from our brief mention of the stability implications of $\pi = \bar{\mu}$ (see Chapter I), and our treatment of the short-run effects of devaluation (see Chapter II), hitherto in this dissertation we have assumed that expectations are conditioned on the actual past and present state of the system rather than the anticipated future state of exogenous variables.

Notes to Chapter IV

1 Of course nominal monetary growth in each country will rise pari passu.

2 To derive these equations note especially (4.10) above.

3 See, too, Brunner (1976) for an extension of Christ's model to the fixed-rates two-country case. His model proceeds in terms of price levels (rather than inflation rates), abstracts from capital markets, deals with the case of partial rather than full goods-market integration, and yields a quasi equilibrium in the form of non-stationary reserves rather than non-stationary net foreign borrowing.
Each of these equations employs the operations immediately preceding (4.13'); (4.21)' uses (4.18)'; (4.22)' uses (4.15)', (4.19)', the definition of $y_i$ and the assumption $q_i = q_i$; and (4.23)' uses $\Sigma x_i = 0$.

See Cagan (1956) and Bailey (1956).

This overdeterminacy result may be related to an instability resulted reported--but not interpreted--by Christ (1978), Turnovsky and Kingston (1977), and Marxsen (1978). Specifically, a Keynesian closed economy model, augmented by relationships for the government budget constraint and price adjustment under static inflation expectations (see Christ (1978), and the brief mention of its extension to adaptive expectations, in Chapter I above), such a model under flexible exchange rates and adaptive inflation expectations (see Turnovsky and Kingston (1977)), or a popular money-in-growth model (cf. Marxsen), will all exhibit instability if all of the following are pegged: real intensive government spending; the rate of tax on real intensive factor incomes, regardless of whether that rate is specified in lump-sum or proportional terms; and the monetary growth rate.

It is worth pointing out that the policy assumption utilized to rule out quasi equilibrium, namely, that each national public sector pegs its outstanding real per capita stock of interest-bearing debt, is not as restrictive as might be supposed. For the steady-state equations we worked with in Section 3 would be yielded by any public policy which ensured full equilibrium under the international environment assumed there, the critical assumptions being equalized growth rates, and
zero international mobility of labour and money stocks. Thus, the steady-state results of Section 4 hold under imperfect goods mobility, i.e., relative purchasing power parity; and/or zero capital mobility, i.e., no interest arbitrage internationally. Note, too, that the extension to an arbitrary number of countries is straightforward.
BIBLIOGRAPHY


