On the detection of spectral distortions in the CMB: recombination to reionization

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To Mummy, Daddy, and Vidya, for everything...
I hereby declare that the work in this thesis is that of the candidate alone, except where indicated below or in the text of the thesis. The work was undertaken between March 2014 and April 2017 at the Australian National University (ANU), Canberra. It has not been submitted in whole or in part for any other degree at this or any other university.

This thesis has been submitted as a Thesis by Compilation in accordance with the relevant ANU policies. Each of the three main chapters is therefore a completely self-contained article, which has been published in, or submitted to, a peer-reviewed journal. The thesis has been excellent preparation for post-doctoral research, as the candidate has experienced the full scientific process from planning and running simulations through to statistical analysis and producing peer-reviewed publications.

The candidate has written each paper in its entirety, incorporating suggestions and feedback from the co-authors and the referees.

Mayuri Sathyanarayana Rao
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The \textit{LCDM} model of cosmology predicts inevitable, weak distortions in the spectrum of the Cosmic Microwave Background (CMB) from that of a blackbody. However, no such deviations have been measured to date. This thesis focuses on CMB spectral distortions arising from the cosmological epochs of recombination and cosmic dawn & reionization. A detection and measurement of these CMB spectral distortions will enable a better understanding of the thermal and ionization history of the Universe and help us probe redshifts that have never been directly observed thus far.

I present a feasibility study for a ground-based detection of extremely weak, ripple-like additive features in the CMB spectrum created by photons emitted during cosmological recombination ($900 \lesssim z \lesssim 7000$). I identify an octave band in the frequency range 2–6 GHz to be optimal for a detection of this CMB spectral distortion. This band maximizes signal-to-noise ratio and has sufficient spectral structure in the signal to aid foreground separation. I introduce the Maximally Smooth (MS) function, an algorithm to distinguish smooth foregrounds from the ripple like signal. Using synthetic spectra, I demonstrate the efficacy of using MS functions over polynomials to separate foregrounds from the cosmological recombination signal. Using Bayesian tests I estimate that using an array of 128 cryogenically cooled, ideal radio-telescopes, spectral ripples from the recombination epoch can be detected with 90\% confidence in 255 observing days. Thus, it is in principle possible to detect these cosmological recombination signals in realistic observing times.

Among others, astronomical foregrounds pose challenges to the detection of CMB spectral distortions. It is thus necessary to have a realistic expectation of the Galactic and extragalactic foreground spectra towards any given direction in the sky. I present GMOSS: Global Model for the Radio Sky Spectrum, a physically motivated model of the radio sky over 22 MHz–23 GHz. GMOSS describes foreground spectra towards all sky directions over $5^\circ$ pixels using processes including synchrotron emission with possible spectral break, emission from composite source populations, free-free emission and thermal absorption.

Using GMOSS I investigate the spectral complexity expected in foregrounds and the
effect of the same on the detection of the global redshifted 21-cm signal from cosmic dawn & reionization ($6 \lesssim z \lesssim 150$). I find that over large beamwidths foregrounds are spectrally smooth and describable using MS functions for various samplings of sky-coverage. However, it is more computationally challenging to describe foreground spectra towards the galactic plane, which is best avoided by experiments seeking to detect CMB spectral distortions from cosmic dawn & reionization (collectively EoR). Once again, I demonstrate the advantage of using MS functions over polynomials to separate foregrounds from the global EoR signal in mock-sky spectra. I find that using MS functions to separate foregrounds, the global signal from EoR can be detected using an ideal instrument with 95% confidence in 10 minutes observing time.

I conclude the thesis with a brief discussion on design criteria for radio-telescopes seeking to detect distortions in the CMB spectrum arising from the epochs of recombination through reionization.
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\[ H \quad \text{Hubble constant} \]
\[ H_0 \quad \text{Hubble constant at redshift } z = 0 \]
\[ \Omega_b \quad \text{Baryon density parameter} \]
\[ \Omega_b h^2 \quad \text{Physical baryon density parameter; } h \text{ is the reduced Hubble constant} \]
\[ \Omega_m \quad \text{Matter density parameter} \]
\[ c \quad \text{Velocity of light in free space} \approx 3 \times 10^8 \text{ m s}^{-1} \]
\[ G \quad \text{Gravitational constant} \approx 6.674 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \]
\[ h \quad \text{Planck constant} \approx 6.626 \times 10^{-34} \text{ m}^2 \text{ kg s}^{-1} \]
\[ k_B \quad \text{Boltzmann constant} \approx 1.380 \times 10^{-23} \text{ m}^2 \text{ kg s}^{-2} \text{ K}^{-1} \]
\[ m_e \quad \text{Electron rest mass} \approx 9.109 \times 10^{-31} \text{ kg} \]
\[ m_p \quad \text{Proton rest mass} \approx 1.672 \times 10^{-27} \text{ kg} \]
1.1 Cosmic Microwave Background

Tune your television to any channel it doesn’t receive and about 1 percent of the dancing static you see is accounted for by this ancient remnant of the Big Bang. The next time you complain that there is nothing on, remember that you can always watch the birth of the universe.

– Bill Bryson, A Short History of Nearly Everything

The study of the origin and evolution of our Universe has evolved through the centuries from the realm of mythology to science. It is now within the folds of scientific study, subject to the scientific method of experimental validation of hypothesis and bears the name ‘Cosmology’. This transition to becoming a rightful field in mainstream science has been enabled by observations of pioneers who not only questioned the popular belief of their time but posed questions in a way that enabled testing by further observations. These observations in turn opened up their share of surprises leading to more questions and this cycle of learning continues today. A cornerstone of such observations, and a classic example of theory and observations working together seamlessly, is the study of the Cosmic Microwave Background (CMB) radiation. It has been a rich source of information that we tease out to deduce the origins of the Universe, which continues to surprise us in its simultaneous simplicity and complexity.

The origin of the Cosmic Microwave Background radiation is best understood by extrapolating the expanding Universe back in time, when it must have been a much smaller and consequently a much hotter place than it is today. In such extreme conditions of the early Universe, matter would be present in the form of its constituent elementary particles. These particles, specifically baryons, are bathed by photons such that for every baryon there are $\sim 1.6 \times 10^9$ photons. There is constant energy exchange between photons and baryons and they are said to be tightly coupled. On the other hand, as the Universe expands
and cools to temperatures below the binding energies of more complex particles such as nuclei and atoms, these particles begin to form. As a large fraction of baryonic matter transitions from primordial plasma to neutral atoms and the scattering of photons off baryons is reduced, the Universe becomes transparent to the previously opaque background photon bath or radiation. The proverbial ‘surface’ over which photons scatter off matter one last time to begin their journey towards the present epoch at redshift $z \approx 0$ is called the surface of last scattering (at $z \approx 1100$) and has a finite width $\Delta z = 80$ (Jones & Wyse, 1985). The cosmic background radiation, after cosmological redshift, has a blackbody spectrum that peaks in the microwave band of the electromagnetic spectrum at 160 GHz and is aptly named the Cosmic Microwave Background radiation. The images of the CMB are in fact images of the surface of last scattering, because the Universe beyond this ‘surface’ is optically thick to CMB photons.

There exist several observable properties of the CMB that have provided a wealth of information about the cosmology of our Universe. One such is its anisotropy or angular fluctuation. While to the zeroth order the CMB is remarkably uniform across the entire sky with a blackbody temperature of $\sim 3$ K, there exist tiny fluctuations in the CMB temperature from one direction of the sky to another. The power spectrum of these anisotropies represents the power present in different angular scales. The near perfect match between the measured CMB power spectrum and that predicted by theory in the Lambda Cold Dark Matter (LCDM) cosmology framework has been key to the acceptance of the LCDM model. In over half a century since the serendipitous discovery of the CMB (Penzias & Wilson, 1965), the measurement and study of CMB anisotropy has been the focus of many major CMB experiments such as the Wilkinson Microwave Anisotropy Probe (WMAP) (Page, 2000) and Planck (Planck Collaboration et al., 2011). The improvement in the anisotropy measurements, as evident in the CMB maps made by COBE, WMAP and Planck as shown in Figure 1.1, come in the form of increased resolution providing the ability to probe smaller angular scales, better foreground subtraction with the use of more frequency channels, and powerful analysis methods.

There exists another aspect of the CMB that has been critical to the success of the LCDM model, namely its spectrum. The best measurement of the CMB spectrum to date comes from the FIRAS instrument aboard the NASA satellite COBE (see Figure 1.2). Limited only by the precision of the instrument calibration, FIRAS measured the CMB spectrum to match that of a blackbody of temperature $2.728 \pm 0.004$ K to within 95% confidence (Fixsen et al., 1996). This remarkably blackbody spectrum of the CMB provides an independent confirmation of the big-bang class of cosmological models that predict a thermal cosmic background radiation. As opposed to anisotropy measurements where there have
1.2 Thesis focus and outline

This thesis discusses the feasibility of a ground based detection of distortions in the CMB spectrum from the epoch of recombination. This includes a method that can be applied to describe foregrounds in such a detection experiment. The thesis also focuses on foreground modelling and subtraction to detect the global CMB spectral distortion from the epoch of reionization (EoR). The thesis is organised as a collection of three papers in chronological order of publication, followed by a chapter with conclusions and a discussion on future work as applied to an experimental detection. A brief outline of these chapters is below.

---

1 Image credit: COBE Project, DMR, NASA
2 Image credit: NASA/WMAP Science Team
3 Image credit: European Space Agency, Planck Collaboration
Chapter 2 Thermal history of the Universe. This chapter presents an overview of the CMB spectrum and why it is so closely thermal. This is followed by a brief discussion on a few broad, inevitable spectral distortions of the CMB, namely $\mu$- and $\gamma$-type distortions. A more thorough discussion on the epochs of recombination and reionization, and the fine-scale CMB spectral distortions that arise from them, is then presented. These fine-scale spectral distortions are the focus of this thesis. The last sections of Chapter 2 make a case for detecting these fine-scale spectral distortions and the challenges therein.

Chapter 3 On the detection of spectral ripples from the recombination epoch. This chapter discusses the feasibility of a ground based detection of the ripple like spectral distortion in the CMB from the recombination epoch. The feasibility study includes identifying a frequency range conducive for detecting the signal that maximizes signal-to-noise ratio and favours foreground separation. The chapter also introduces Maximally Smooth functions that can be used to separate smooth components in a measurement set, such as foreground emission, from the inherently non-smooth ripple like cosmological signal. Estimates of observing time required for signal detection with an ideal instrument for varying levels of confidence using Bayes factors and Markov Chain Monte Carlo (MCMC) simulations are presented. Choice of observing sites conducive to detection considering typical Radio Frequency Interference (RFI) environments is discussed. Finally, the chapter introduces
Chapter 4 GMOSS: Global Model for the radio Sky Spectrum. This chapter presents a physically motivated model for the low-frequency radio sky. This can be used to simulate foregrounds of cosmological signals such as those arising from the EoR. In particular, for the case of the global signal from the EoR where the Galactic synchrotron emission is the dominant foreground, GMOSS can serve to provide a realistic expectation of foregrounds in measurement sets. The input to the model are six all-sky maps over 22 MHz–23 GHz that are all treated to have the same spatial resolution and formats. GMOSS represents spectra over individual pixels of 5° resolution using parameters that describe radiative processes including synchrotron emission, free-free emission and absorption at low frequencies from thermal gas. GMOSS adopts one of two underlying synchrotron emission mechanisms based on the change in spectral index towards high frequencies from that at low frequencies. For the case of spectral steepening the emission is modelled as a break in the injected electron energy distribution and for spectral flattening the spectrum is modelled as composite emission from steep and flat spectrum sources. GMOSS is in agreement to within 17% of the input data sets over most regions of the sky, which is in keeping with typical errors in the sky maps. The estimated parameters are found to be in agreement with independent observations as reported in the literature, giving reason to believe that GMOSS is a good representation of the radio sky and hence can serve as a useful foreground model.

Chapter 5 Modeling the Radio Foreground for detection of CMB spectral distortions from Cosmic Dawn and Epoch of Reionization. With a powerful tool in the form of Maximally Smooth functions, it is useful to investigate whether or not foregrounds are indeed spectrally smooth as has been usually assumed in the literature. The inefficacy of assuming polynomial interpolation based sky models is demonstrated. It is demonstrated that the arbitrary order of such interpolating polynomials can result in unphysical spectral shapes and misleading assumptions of foreground complexity. Foregrounds generated using the physically motivated GMOSS sky model, are spectrally more complex than have been assumed in the literature. Assuming an ideal instrument and using two different frequency-independent beams, the smoothness of mock foreground spectra that are generated adopting the GMOSS sky-model are investigated for different observing sites and sky coverage. The integration time required to detect the EoR signal with 95% confidence assuming an ideal instrument using MS functions for foreground separation is investigated. The deterioration in signal to noise ratio and hence increase
in observing time required to reach the same confidence level if one were to use polynomials of different orders instead of MS functions is also demonstrated. This gives impetus to use MS functions as a foreground separation tool for detecting the global EoR signal. A brief discussion on methods to verify that foregrounds in data are truly smooth and that non-smooth features in residuals are indeed cosmological is also presented.

Chapter 6 Concluding remarks. This chapter presents a brief discussion on the choice of sky models to simulate foregrounds for signals from the epochs of recombination and reionization. Next, it summarizes the work presented in Chapters 3, 4 and 5. Also presented is a discussion on future work with a focus on design criteria for experiments dedicated to detect spectral distortions in the CMB from the epochs of recombination to reionization.
THERMAL HISTORY OF THE UNIVERSE

At first there was only darkness wrapped in darkness. All this was only unillumined water. That One which came to be, enclosed in nothing, arose at last, born of the power of heat.

— Verse 2, Nasadiya Suktam

ΛCDM cosmology predicts inevitable deviations of the CMB spectrum from that of a blackbody. Before venturing into these spectral distortions it is important to understand why at all the CMB spectrum is so closely thermal. The thermal history of the CMB is best described by Figure 2.1 that gives the timeline of the Universe. Placing ourselves at the center of the concentric circles at \( z = 0 \) and looking out into the deep Universe we effectively look back in time. By the cosmological principle we do not occupy a special place in the Universe and hence this concentric view holds good for any observer.

A blackbody is completely defined by its temperature \( T \) and its intensity is given by the Planck law:

\[
B_{\nu}(T) = \frac{2h}{c^2} \frac{\nu^3}{e^{h\nu/kT} - 1} \text{ ergs s}^{-1} \text{ cm}^{-2} \text{ Hz}^{-1} \text{ sr}^{-1},
\]

where \( h, c, k_B \) follow standard notation and \( \nu \) is the frequency. Peculiar to the blackbody is its energy density \( \rho_\gamma \), number density \( N_\gamma \) and photon distribution \( n_\nu \). The photon energy density is obtained by integrating the specific energy density \( U_\nu \) over all frequencies and directions.

\[
\rho_\gamma = \int U_\nu d\nu d\Omega.
\]
Fig. 2.1 The thermal history of the Universe viewed as concentric circles from $z = 0$. The concentric circle at $z \sim 2 \times 10^6$ marks the blackbody photosphere. Energy injection at $z \gtrsim 2 \times 10^6$ results in a shift in the blackbody temperature of the background spectrum. $2 \times 10^6 \lesssim z \lesssim 2 \times 10^5$ is the $\mu$ type distortion zone. $z \lesssim 1.5 \times 10^4$ is the $y$ type zone and the intervening region generates $i$ type distortions. The epoch of recombination spans $8000 \lesssim z \lesssim 900$, with hydrogen recombination peaking at $z \sim 1350$. Redshift $z \sim 1100$ marks the surface of last scattering. Following the dark ages after recombination, the first sources appear and eventually reionize the Universe in the epoch of reionization over $6 \lesssim z \lesssim 40$. Image courtesy: Rishi Khatri (private communication).
Since specific energy density is related to specific intensity as \( U = \frac{B_n}{c} \), for a Planckian distribution of photons we have

\[
\rho_\gamma = \frac{2h}{c^3} \int \frac{v^3}{e^{\frac{hv}{k_B T}} - 1} dv d\Omega
\]

\[
= \frac{8\pi h (k_B T)^4}{c^3} \int \frac{x^3 dx}{e^x - 1}
\]

\[
= \frac{8\pi^5 (k_B T)^4}{15c^3 h^3}
\]

\[
= a_R T^4
\]

\[
= 0.26 \text{ eV} \left( \frac{T}{2.728 \text{ K}} \right)^4,
\]

where \( x = \frac{h v}{k_B T} \) and \( a_R \) is the radiation constant, which is related to the Stefan-Boltzmann constant \( \sigma \) by \( a_R = \frac{4\sigma}{c} \), \( \sigma = \frac{2\pi^5 (k_B T)^4}{15c^3 h^3} \). This gives the photon number density \( N_\gamma \) as

\[
N_\gamma = \int \frac{U_v}{hv} dv d\Omega,
\]

\[
= \frac{2}{c^3} \int \frac{v^2 dx}{e^x - 1}
\]

\[
= \frac{8\pi h (k_B T)^3}{c^3} \left( \frac{k_B T}{h} \right) \int \frac{x^2 dx}{e^x - 1}
\]

\[
\approx 410 \text{ cm}^{-3} \left( \frac{T}{2.728 \text{ K}} \right)^3.
\]

The photon distribution or photon occupation number for a blackbody \( n_v \) is

\[
n_v = \frac{1}{e^x - 1}.
\]

(2.3)

Next we may ask the question: What would happen if we change the temperature from \( T \) to \( T' = T + \delta T \) by injecting some small amount of energy, \( \Delta E \), into the blackbody photon field. Various processes such as electron-positron annihilation and dissipation of small-scale primordial density fluctuations, among others, result in such energy injection. If there is no change in the volume and photon number density, this will simply change the energy density, which is related to the change in temperature by

\[
\Delta E = \frac{\Delta \rho_\gamma}{\rho_\gamma} \sim \left( \frac{\Delta T}{T} \right)^4.
\]

(2.4)
However, merely changing the temperature does not ensure that the photon field has a blackbody spectrum. For that, both the photon number density and distribution of photons also have to change such that the final spectrum is a blackbody with the new temperature. The change in photon number density is related to the change in the energy density by

$$\frac{\Delta N_\gamma}{N_\gamma} = \frac{3}{4} \frac{\Delta \rho_\gamma}{\rho_\gamma}. \quad (2.5)$$

With $x' = h\nu/k_B T'$ and $x = h\nu/k_B T$, where the change in temperature is from $T$ to $T'$, the photon occupation number changes as

$$\Delta n_\nu = \frac{1}{e^{x'} - 1} - \frac{1}{e^x - 1}. \quad (2.6)$$

Restoring a Planckian spectrum after energy injection requires processes that can create or remove photons from the photon bath and change the photon number density as given by Equation 2.5. Double Compton (DC) scattering and Bremsstrahlung are two such radiative processes that participate in changing the photon number density. Another process that determines the shape of the CMB spectrum is Compton scattering. Compton scattering redistributes the energies of photons, conserving photon number. Due to different rates, these processes are effective over different redshifts as shown in Figure 2.2.

In the early Universe at redshifts $z \gtrsim 2 \times 10^6$, DC and Bremsstrahlung processes are effective in thermalizing the CMB via photon-plasma interactions. Signatures of energy injection at these very early times are wiped out and the resultant spectrum is Planckian. This marks the blackbody photosphere, where any energy injection results in a change in blackbody temperature, but the spectrum remains blackbody nonetheless. DC and Bremsstrahlung become ineffective for $z \lesssim 10^6$ and Compton scattering is the dominant process in this regime. When Compton scattering is efficient, as is expected over these redshifts, the spectrum is said to be fully Comptonized and the resulting spectrum has a Bose-Einstein (BE) form with non-zero chemical potential $\mu$. Any energy injected into the background radiation appears as a $\mu$-type distortion of the CMB spectrum (see Figure 2.3). To provide some context, a blackbody spectrum is a special case of the BE spectrum with $\mu = 0$. A detection of a $\mu$-type distortion can place constraints on energy injection over redshifts $2 \times 10^6 \lesssim z \lesssim 2 \times 10^5$, which is called the Bose-Einstein or $\mu$-type zone. COBE has placed upper limits on the $\mu$-type distortion to be $9 \times 10^{-5}$.

Over the redshift range $z \lesssim 1.5 \times 10^4$, where Compton Scattering is no longer efficient, the spectrum does not relax to a BE form and energy injection results in a $\gamma$-type distortion of the CMB spectrum. $\gamma$-type distortions can also arise in the late Universe, post
Fig. 2.2 Rates of radiative processes namely Bremsstrahlung, Compton and DC scattering in the early Universe. The black double-dashed line gives the rate at which electron/baryon plasma achieves thermal equilibrium ($T_e$) and the black solid line shows the Hubble rate. When all three processes are effective for redshifts $z \gtrsim 2 \times 10^6$ the CMB spectrum is blackbody (BB). Spectral distortions in the CMB arise as each of these processes gradually become ineffective in restoring a Planckian form. Compton scattering creates a $\mu$ or Bose-Einstein (BE) type distortion over redshifts $z \gtrsim 10^5$, $y$-type distortion for $z \lesssim 1.5 \times 10^4$ and intermediate or $i$-type distortions in between. Figure reproduced from Khatri & Sunyaev (2012b).
reionization, by the Sunyaev-Zeldovich scattering of CMB photons off hot thermal gas in galaxy groups & clusters and is expected to be the dominant source of total \( y \) distortion. The \( y \)-type distortion is collectively parameterised by the Compton \( y \)-parameter given by

\[
y = \int_0^z \frac{k_B T_e(z')}{m_e c^2} \sigma_T n_e(z') \frac{dt}{dz'} dz' \approx \int_0^z \frac{k_B T_e}{m_e c^2} \sigma_T n_e \frac{dz}{dz'} dz,
\]

which denotes the typical fractional energy gain per scattering times the mean number of scatterings, where \( \sigma_T \) is the Thomson cross-section, \( n_e \) is the electron number density, and \( T_e \) is the electron temperature. While the early \( y \)-type distortions from the pre-galactic era are expected to affect the CMB spectrum at a level of \( 8 \times 10^{-10} \) (Chluba & Sunyaev, 2004), the inevitable late \( y \)-type distortion from galaxy groups and clusters is expected to be at a mean level of \( 1.6 \times 10^{-6} \) (Hill et al., 2015). COBE has placed an upper limit on the \( y \)-parameter to be \( 1.5 \times 10^{-5} \) with 95% confidence (Fixsen et al., 1996). Energy injection over \( 1.5 \times 10^4 \lesssim z \lesssim 2 \times 10^5 \) is predicted to appear as an intermediate or \( i \)-type distortion (Khatri & Sunyaev, 2012a) and this distortion is highly sensitive to the redshift of energy injection. As the name suggests, \( i \)-type distortions are intermediate to the pure \( \mu \)- and pure \( y \)-type distortions and cannot be mimicked by a mixture of the two (Khatri & Sunyaev, 2012a).

All the CMB spectral distortions discussed above, appear as broad distortions of the CMB spectrum and are discussed in greater detail in Chluba & Sunyaev (2012) and references therein. Efforts to detect these broad features will be among the prime science goals of proposed space based CMB missions such as PIXIE (Kogut et al., 2011b) and CoRE (The
There also exist some inevitable ‘fine scale’ spectral features in the CMB spectrum that arise from the epochs of recombination and reionization, as described below.

As the Universe expanded and cooled, electrons combined with helium nuclei and protons at, respectively, redshifts $z \sim 6000$ and $z \sim 1500$ to form the first helium and hydrogen atoms. This is the epoch of recombination. During this epoch the Universe transformed from being completely ionized to almost neutral. As described in Section 2.1, the physics of recombination is governed primarily by $2s \rightarrow 1s$ two-photon decay of hydrogen and escape of Lyman-$\alpha$ photons in an expanding Universe. Recombination is accompanied by the emission of photons that must result in an additive distortion of the underlying background radiation spectrum. As the processes that thermalise the CMB are no longer effective during the epoch of recombination, these additive fine scale distortions of the CMB are expected to survive to date.

Following recombination is a period that is referred to as the Dark Ages, a name that is perhaps equally literal and symbolic as very little is known about this period. The weak interaction of background radiation photons with residual electrons and ions gives us little information about this period. Further, the Dark Ages are marked by the absence of sources of electromagnetic radiation. However, we certainly do see such objects in the local Universe, from our very own Sun to large scale structures such as clusters of Galaxies. There must have been a beginning when these structures formed from the relatively uniform primordial plasma. Furthermore, the Universe we live in today is predominantly ionized, implying that the neutral gas at the end of the epoch of recombination was somehow reionized. This period when the very first sources of electromagnetic radiation formed and the Universe transitioned from being mostly neutral to ionized is collectively referred to as the Cosmic Dawn and the Epoch of Reionization (simply referred to as Epoch of Reionization or EoR henceforth, unless explicitly specified).

The focus of this thesis are fine-scale distortions of the CMB spectrum that arise from the epochs of recombination and reionization. These epochs and the predicted signals arising therein are described in further detail below.

### 2.1 Epoch of recombination

*We, all of us, are what happens when a primordial mixture of hydrogen and helium evolves for so long that it begins to ask where it came from. — Jill Tarter*
The period in the early Universe, over which electrons combined with helium nuclei and protons in the primordial plasma, to form the first helium and hydrogen atoms, is called the epoch of recombination. The recombination of doubly ionized helium, He\textsc{\textit{III}} → He\textsc{\textit{II}}, is believed to have spanned $5000 \lesssim z \lesssim 8000$, followed by singly ionized helium, He\textsc{\textit{II}} → He\textsc{\textit{I}}, over $1600 \lesssim z \lesssim 3000$ and finally hydrogen recombination, H\textsc{\textit{II}} → H\textsc{\textit{I}}, over $900 \lesssim z \lesssim 1600$. Recombination proceeded over several quasi-stationary stages, with the emission of recombination photons that are predicted to appear as additive distortions to the CMB spectrum. The amplitude of these additive distortions is expected to be extremely low as a direct consequence of the small baryon to photon number density ratio $n_{b\gamma} = \frac{n_b}{n_{\gamma}} = 10^{-9}$. The theory of recombination has evolved from the early days that assumed a Saha ionization model to modern day codes that model recombination via electrons trickling down through several atomic substates and account for feedback processes. A brief review of the different models of recombination follows.

### 2.1.1 Saha recombination

The Saha ionization equation

$$\frac{n_{r+1}n_e}{n_r} = \frac{g_{r+1}g_e}{g_r} \frac{(2\pi m_e k_B T)^{3/2}}{\hbar^3} e^{-\frac{\mathcal{X}}{k_B T}},$$  \hspace{1cm} (2.7)

gives the distribution of atomic species over different ionization states, $n_{r+1}$, $n_e$, and $n_r$, in thermal equilibrium, where $g_r$ is the statistical weight of the $r$th ionization state with ionization energy $\mathcal{X}$.

Specifically for the case of hydrogen, under the assumption of thermal equilibrium, we can estimate the ratio of the number density of hydrogen atoms in the ionized state to those in the ground state H(1s). For hydrogen, $r = 1$, $g_r = 4$, the degeneracy of the proton $g_{r+1} = g_p = 2$, which is same as the electron degeneracy $g_e = 2$. The number density of hydrogen atoms in the ground state H(1s), protons, and electrons can be expressed in terms of the ionization fraction $x_e = \frac{n_e}{n_{\text{Htot}}}$ and the total number density of hydrogen atoms $n_{\text{Htot}}$. The resulting Saha equation is

$$n_{\text{Htot}} \frac{x_e^2}{1-x_e} = \left( \frac{m_e T}{2\pi} \right)^{3/2} e^{\frac{-\mathcal{X}_0}{k_B T}},$$  \hspace{1cm} (2.8)

The ionization energy of hydrogen $\mathcal{X}_0$ is 13.6 eV, which corresponds to $15.8 \times 10^4$ Kelvin. With hydrogen comprising 76% of total baryons from Big Bang nucleosynthesis, as per the ΛCDM model, the total number density of hydrogen $n_{\text{Htot}}$ can be estimated as $n_{\text{Htot}} = 0.76 \times n_b = 4.2 \times 10^5 \Omega_b h^2 T_4^3 \text{ cm}^{-3}$, where $T_4$ is the temperature of the photons in $10^4$ Kelvin units and the baryon density parameter $\Omega_b$ is related to the baryon number
2.1 Epoch of recombination

density \( n_b \) as

\[
    n_b = \frac{\Omega_b \rho_c}{m_p} = \frac{3H_0^2 \Omega_b}{m_p 8 \pi G}.
\]

(2.9)

Using Equation 2.9 for \( n_{\text{tot}} \) in Equation 2.8 and rearranging the terms we get

\[
    \frac{x_e^2}{1 - x_e} = \frac{m_p 8 \pi G}{0.76 \times 3 H_0^2 \Omega_b} \frac{5.26 \times 10^4 e^{-\frac{z_0}{0.76 H_0 h^2 T_4}}}{T_4^{3/2}}
    = \frac{5.26 \times 10^{15} e^{-\frac{15.8}{T_4}}}{\Omega_b h^2 T_4^{3/2}},
\]

(2.10)

where \( H_0 = 100 \, h = 3.24 \times 10^{-18} \, h \, \text{km s}^{-1} \, \text{km}^{-1} \). In the above, \( H_0 \) is the Hubble constant and \( h \) is the reduced Hubble constant.

Using \( \Omega_b h^2 = 0.022 \) (Planck Collaboration et al., 2016b) in Equation 2.10 we find that at \( T_4 = 0.374 \) or \( T = 3740 \, \text{K}, x_e = 0.5 \). This corresponds to a redshift of \( z = 1370 \). Thus, by around redshift of \( z \sim 1370 \), half the hydrogen atoms in the Universe are recombined. For ionization fractions of \( x_e = 0.1 \) and \( x_e = 0.01 \), the corresponding redshifts turn out to be \( z = 1250 \) and \( z = 1140 \) respectively. According to the Saha model, recombination proceeds quickly and there is an almost sudden drop in the ionization fraction implying the Universe quickly transitioned from being fully ionized to neutral. However, this model also assumes thermal equilibrium conditions and does not consider all possible mechanisms that affect reaction kinetics. To first order this is addressed by the Peebles model of recombination that assumes a three-level hydrogen atom, as discussed below.

### 2.1.2 Peebles recombination

It was realized in the late 1960s that if recombination proceeded through the Saha model, Lyman-\( \alpha \) (Ly-\( \alpha \)) photons from recombination must have freely escaped to allow the completion of recombination and there must exist a redshifted Ly-\( \alpha \) photon background. This called for a more thorough treatment of the reaction kinetics of the recombination epoch (Zeldovich et al., 1968; Peebles, 1968). A historical account of how this realization came about is in Sunyaev & Chluba (2009a).

Discussed below is the Peebles model of recombination that considers hydrogen as a three-level atom. The three levels being, namely ground state hydrogen \( \text{H}(1s) \), excited state \( \text{H}(2s) \) or \( \text{H}(2p) \) and ionized hydrogen \( (p^+ \text{ and } e^-) \). Recombination can occur by different routes.

We first consider the case where free electrons \((e^-) \) and protons \((p^+) \) recombine directly to hydrogen in the ground state. This is accompanied by the emission of a photon with energy
that is greater than or equal to 13.6 eV. The photoionization cross section $\sigma_{\text{PI}}$ of hydrogen for photons of such energies is $6 \times 10^{-18}$ cm$^{-2}$. The mean free path of recombination photons is given by

$$L_{\text{mfp}} = \frac{1}{\sigma_{\text{PI}} n_{1s}} = \frac{1}{\sigma_{\text{PI}} n_{\text{H}x_1}} = 4 \times 10^{14} x_1^{-1} \text{cm}, \quad (2.11)$$

where $x_1$ is the neutral fraction and the total hydrogen number density at redshifts of recombination ($z \sim 1300$) is 400 cm$^{-3}$. This corresponds to a re-absorption time scale of $10^4 x_1^{-1}$ seconds. For neutral fraction exceeding $10^{-9}$, photoionization rate is faster than Hubble expansion. That is, a direct recombination of a free electron to ground state will immediately photo-ionize a neighbouring neutral hydrogen atom H$(1s)$, thereby rendering the processes ineffective.

Now we consider the case of recombination to excited states, which is referred to as Case-B recombination as is the convention in interstellar medium studies.

$$p^+ + e^- \leftrightarrow H(nl, n \geq 2) + \gamma.$$  

Recombination rate is determined by the total recombination coefficient $\alpha_{\text{tot}}$, which is the average of recombination coefficients of all excited states, that is, for all principal quantum states $2 \leq n \leq \infty$ and angular quantum sub-states $0 \leq l \leq (n - 1)$, averaged over the thermal velocity distribution of electrons:

$$\alpha_{\text{tot}} = \sum_{n=2}^{\infty} \sum_{l=0}^{n-1} \langle \sigma[p^+ e^- \rightarrow H(nl) + \gamma]v \rangle.$$  

Once recombined to an excited state, the electron quickly trickles down to the first excited state, H$(n = 2)$. This can be thought of as recombination to H$(n = 2)$ following the Saha model. The effective rate of recombination to excited states is equivalently the rate at which H$(n = 2)$ is populated. This is given by the difference between the rates at which recombination occurs to various excited states and photoionization of H$(n = 2)$, the latter being determined by photoionization coefficient $\beta$

$$\dot{x}_2 = \alpha_{\text{tot}} n_{\text{Htot}} x_2^2 - \beta x_2. \quad (2.13)$$

From Saha equilibrium

$$\frac{n_e n_p}{n_2} = \frac{x_2^2 n_{\text{Htot}}}{x_2} = \frac{g_e g_p}{g_2} \left( \frac{m_e T}{2 \pi} \right)^{\frac{3}{2}} e^{\frac{\alpha_{\text{tot}} x_2^2}{k_B T}} \Rightarrow x_2 = 4 x_2^2 n_{\text{Htot}} \left( \frac{2 \pi}{m_e T} \right)^{\frac{3}{2}} e^{\frac{\alpha_{\text{tot}} x_2^2}{k_B T}},$$
2.1 Epoch of recombination

where the \( n = 2 \) state has a statistical weight of \( g_2 = 16 \) and ionization energy \( \chi = \frac{Z_0}{4} \).

From the principle of detailed balance \( \dot{x}_2 = 0 \) and thus

\[
\beta = \frac{\alpha_{\text{tot}}}{4} \left( \frac{m_e T}{2\pi} \right)^{\frac{3}{2}} e^{\frac{x}{q_0^2 T}}. \tag{2.14}
\]

This gives the rate of recombination of electrons to excited states in terms of the change in \( x_2 \):

\[
\dot{x}_2 = \alpha_{\text{tot}} \left( n_{\text{Htot}} x_c^2 - \frac{1}{4} \left( \frac{m_e T}{2\pi} \right)^{\frac{3}{2}} e^{\frac{x}{q_0^2 T}} x_2 \right) \tag{2.15}
\]

Recombination to the ground state can proceed via one of two channels. The first being the Ly-\( \alpha \) transition \( (2p \to 1s) \) and the second being the \( 2s \to 1s \) two photon emission. For the former case, the reaction rate for the Ly-\( \alpha \) transition is given by:

\[
\dot{x}_2 = -\frac{3}{4\pi} A_{2p} x_2 = -\frac{H_0 \omega_{\text{Ly} \alpha}^3}{4\pi^2 n_{\text{Htot}} x_1} x_2. \tag{2.16}
\]

In the above, the depopulation of the \( 2p \) state via Ly-\( \alpha \) emission, with photon angular frequency \( \omega_{\text{Ly} \alpha} \), is governed by a combination of spontaneous transition rate \( A_{2p} x_2 \) and photon escape probability. The latter is given by the inverse of the Sobolev optical depth \( \tau = \frac{3\pi^2 A_{2p} n_{\text{Htot}} x_1}{H_0 \omega_{\text{Ly} \alpha}^3} \). Accounting for detailed balance and allowing for the thermal excitation of the ground state atom to \( n = 2 \) by photons that redshift into Ly-\( \alpha \) we obtain:

\[
\dot{x}_2 = -\Lambda_\alpha \left( x_2 - 4x_1 e^{-\omega_{\text{Ly} \alpha}/T} \right), \tag{2.17}
\]

where \( \Lambda_\alpha = -\frac{H_0 \omega_{\text{Ly} \alpha}^3}{4\pi^2 n_{\text{Htot}} x_1} \). Similarly, for the two photon channel with a decay rate \( \Lambda_{2s} = 8.2\text{s}^{-1} \) we have

\[
\dot{x}_2 = -\Lambda_{2s} \left( \frac{x_2}{4} - x_1 e^{-\omega_{\text{Ly} \alpha}/T} \right). \tag{2.18}
\]

The net rate of production of hydrogen atoms in excited states is given by a combination of all the processes discussed above and is

\[
\dot{x}_2 = \alpha_{\text{tot}} n_{\text{Htot}} x_c^2 - \beta x_2 - \left( \Lambda_{2s} + \frac{H_0 \omega_{\text{Ly} \alpha}^3}{\pi^2 n_{\text{Htot}} x_1} \right) \left( \frac{x_2}{4} - x_1 e^{-\omega_{\text{Ly} \alpha}/T} \right). \tag{2.19}
\]

If the rate of population and depopulation of the \( n = 2 \) are the same, which is expected as the state is short lived, \( \dot{x}_2 = 0 \). This gives

\[
x_2 = 4x_1 \frac{\alpha_{\text{tot}} n_{\text{Htot}} x_c^2 + (\Lambda_\alpha + \Lambda_{2s}) e^{-\omega_{\text{Ly} \alpha}/T}}{\Lambda_\alpha + \Lambda_{2s} + 4\beta}. \tag{2.20}
\]
The change in ionization fraction can be quantified in terms of the rate of loss of free electrons $\dot{x}_e = \dot{x}_2$, which on simplifying gives the Peebles equation for recombination.

$$\dot{x}_e = -\alpha_{\text{tot}} \frac{\Lambda_2 + \Lambda_\alpha}{\Lambda_2 + \Lambda_\alpha + 4\beta} \left( n_{\text{Htot}} x_e^2 + \left( \frac{m_e T}{2\pi} \right)^{\frac{3}{2}} e^{\frac{e_{mT}}{kT}} \right).$$  \hspace{1cm} (2.21)

This implies that $x_e = 0.5$ by $z = 1210$, $x_e = 0.1$ by $z = 980$ and $x_e = 0.01$ by $z = 820$. Thus, the Peebles model indicates that recombination is delayed compared to the Saha model as the Universe is optically thick in Ly-$\alpha$. As the Universe expands and Ly-$\alpha$ is redshifted, recombination proceeds via the Ly-$\alpha$ channel. It is the $2s \rightarrow 1s$ two photon channel that mediates 57% of hydrogen recombination up to redshifts of $z \approx 1400$ (Chluba & Sunyaev, 2006a). Importantly, according to this model there is residual ionization of $\sim 2 \times 10^{-4}$ (Peebles, 1993) toward the end of recombination.

There have been several improvements to the Peebles model that change the predictions at the few percent level, which is significant in the era of precision cosmology. These improvements are described in brief below.

### 2.1.3 Improvements to the Peebles model

The three-level atom model assumes that all higher states of the hydrogen atom are in thermal equilibrium with the CMB field and thus follow the Saha solution. Lack of computing power in the early days limited recombination calculations to this simplified case. In the years since, improvements to this toy-model of recombination have come mainly from a better understanding of atomic physics, increased computing power and refinements in cosmological parameters from experiment and theory. In particular, the recombination calculation codes today are much more refined due to a thorough treatment of the following aspects:

- Inclusion of various higher-level quantum states with detailed treatment of angular momentum substates
- Attention to non-equilibrium processes including forbidden transitions, 2-photon transitions and collisions
- Radiative feedback mechanisms between various states of the same species as well as between different species (hydrogen and helium)

Various recombination codes have addressed the cosmological recombination problem, some noteworthy ones being RECFAST (Seager et al., 1999), RECFAST++ \footnote{http://www.cita.utoronto.ca/jchluba/Science_Jens/Recombination/Recfast++.html}, REC-
Fig. 2.4 Ionization fraction as a function of redshift as predicted by the Saha recombination model (solid orange line) and that predicted by the recombination code RECFAST (dashed green line). The solid horizontal blue line indicates an ionization fraction of 0.5. Note that since RECFAST includes helium recombination, the ionization fraction that is defined as $x_e = \frac{n_e}{n_{\text{Htot}}}$ exceeds 1 at high redshifts.


A comparison of the ionization fraction as a function of redshift as predicted by the Saha model and that given by the recombination calculation code RECFAST (Seager et al., 1999) is given in Figure 2.4. An overview of three of the recombination codes listed above, namely RECFAST, RECFAST++ and COSMOREC, reproduced from Chluba (2013) is presented in Table 2.1. The table presents various aspects over which recombination codes have improved over the years and will potentially improve further in the coming years, giving a more precise picture of cosmological recombination.

2.1.4 Helium recombination

Most purposes helium recombination can be assumed to proceed via Saha recombination. Improvements on the Saha recombination model for helium have come from increasingly refined numerical recombination codes and from improved tables of the helium atomic structure. A qualitative discussion of helium recombination is presented here (Rubiño-Martín et al., 2008).

1. Although helium comprises only 8% of primordial baryonic matter, it contributes 16% of the additive photons to the recombination spectrum.
<table>
<thead>
<tr>
<th>Code</th>
<th>RECFAST</th>
<th>RECFAST++</th>
<th>COSMOREC</th>
</tr>
</thead>
<tbody>
<tr>
<td>Language</td>
<td>Fortran 77/90 &amp; C</td>
<td>C++</td>
<td>C++</td>
</tr>
<tr>
<td>Requirements</td>
<td>–</td>
<td>–</td>
<td>GNU Scientific lab</td>
</tr>
<tr>
<td>Solves for</td>
<td>$X_p, X_{HeI}, T_e$</td>
<td>$X_p, X_{HeI}, T_e$</td>
<td>$X_{ls}, X_{ns}, X_{np}, X_{nd}, T_e$</td>
</tr>
<tr>
<td>ODE Solver</td>
<td>explicit</td>
<td>implicit (Gears method)</td>
<td>implicit (Gears method)</td>
</tr>
<tr>
<td>PDE Solver</td>
<td>–</td>
<td>–</td>
<td>semi-implicit (Crank-nicolson)</td>
</tr>
<tr>
<td>Approach</td>
<td>derivative fudge correction function</td>
<td>physics</td>
<td></td>
</tr>
<tr>
<td>Simplicity</td>
<td>simple</td>
<td>simpler</td>
<td>pretty big code</td>
</tr>
<tr>
<td>Flexibility</td>
<td>limited</td>
<td>better but limited$^2$</td>
<td>very flexible</td>
</tr>
<tr>
<td>Validity</td>
<td>close to standard cosmology</td>
<td>close to standard cosmology</td>
<td>wide range of cosmologies</td>
</tr>
<tr>
<td>Tools</td>
<td>–</td>
<td>ODE solver</td>
<td>HI &amp; He Atom, Solvers, Quadrature routines</td>
</tr>
<tr>
<td>Extras</td>
<td>–</td>
<td>DM annihilation</td>
<td>DM annihilation, high $v$ distortion</td>
</tr>
<tr>
<td>Run time</td>
<td>0.01 s</td>
<td>0.08 s</td>
<td>1.2 – 2 s</td>
</tr>
</tbody>
</table>

Table 2.1 Overview of three recombination codes RECFAST, RECFAST++ and COSMOREC, reproduced from Chluba (2013).

$^2$Flexibility of RECFAST++ is limited compared to CosmoRec, which has provisions for extensions to account for the effect of dark matter annihilation and energy injection by decaying particles.
2. Helium recombination proceeds over two stages, namely He\textsubscript{III} $\rightarrow$ He\textsubscript{II} over $5000 \lesssim z \lesssim 8000$ and He\textsubscript{II} $\rightarrow$ He\textsubscript{I} over $1600 \lesssim z \lesssim 3500$ (Dubrovich & Stolyarov, 1997). The redshifts of helium recombination are higher than those for hydrogen because of higher binding energies, particularly of $^4$He.

3. Although helium recombination does not produce significant observable effects in CMB anisotropy, it does result in distinct and significant features in the CMB spectrum.

4. Due to the abundance of free electrons ‘belonging’ to hydrogen, the He\textsubscript{III}$\rightarrow$He\textsubscript{II} recombination proceeds rapidly, resulting in narrow spectral features.

5. The redshifted recombination lines from He\textsubscript{III}$\rightarrow$He\textsubscript{II} closely coincide in frequency with redshifted hydrogen recombination lines, as the difference in redshifts of the recombination epochs is similar to the difference in the energy of the transitions.

6. Due to the presence of more complex fine structure levels for low quantum shells, He\textsubscript{II}$\rightarrow$He\textsubscript{I} recombination will result in lines different from hydrogenic atoms. Some of these fine-structure lines can be especially bright and notably some are predicted to appear in absorption. Such unique spectral features of cosmological helium recombination could potentially allow for a direct measurement of pre-stellar helium abundance.

7. There also exist several feedback effects between helium and hydrogen recombination. One such is the acceleration of helium recombination once even a small fraction of neutral hydrogen is built up, thus accelerating the He\textsubscript{II} $\rightarrow$ He\textsubscript{I} process. This speed up in recombination implies that the associated photons are released over a short redshift span and result in narrow features in the He recombination spectrum.

### 2.1.5 Spectral distortions from the epoch of recombination

The process of recombination is accompanied by the emission of photons. Particularly because recombination is not instantaneous and proceeds over several quasi stationary stages these photons appear as broad additive distortions to the background radiation spectrum (Rubiño-Martín et al., 2006; Chluba et al., 2007; Chluba & Sunyaev, 2006a). A treatment of the process of recombination of hydrogen and helium atoms via free-bound and bound-bound transitions in a bath of photons in an expanding Universe gives a template of the expected spectral distortion.

While additive distortions from the recombination epoch are predicted to be strongest in the Wien tail of the CMB, the strong cosmic infrared background poses a challenge to
Thermal history of the Universe
detection in this regime. It was pointed out in Dubrovich (1975) that recombination lines from higher quantum states should also be observable as distortions to the CMB spectrum at long wavelengths. Albeit weaker, the detection of these lines could potentially be aided by lower foreground contamination.

The most recent prediction of the total ‘cosmological recombination spectrum’ comes from COSMOSpec (Chluba & Ali-Haïmoud, 2016) as shown in Figure 2.5. An investigation into the detection of these ‘ripple’ like features from the epoch of recombination is the focus of Chapter 3.

![Cosmological Recombination Spectrum](https://via.placeholder.com/150)

**Fig. 2.5** The cosmological recombination spectrum as predicted by COSMOSpec. The strength of the additive distortion is given in brightness units as a function of frequency. The strongest distortions come from Lyman and Balmer transitions of the hydrogen atom and are seen at the highest frequencies. There are distinct features from helium recombination that are present in the template, particularly at high frequencies. A detection and measurement of these spectral distortions would be a direct confirmation of the thermal and ionization history of the Universe, provide an experimental measure of pre-stellar helium abundance and provide independent constraints on certain cosmological parameters.

### 2.2 Epoch of reionization

*The most precious light is the one that visits you in your darkest hour!*
The end of the epoch of recombination is marked by the beginning of the Dark Ages, when no sources of electromagnetic radiation were present and the only ‘light’ was the Cosmic Background Radiation. However, the local Universe is clearly populated by stars and galaxies in a bath of diffuse gas. The period over which the first sources of electromagnetic radiation formed is referred to as the Cosmic Dawn (CD). This was followed by an epoch of cosmic heating predominantly by X-ray sources. The period over which radiation from the first sources reionized the then mostly neutral gas in the Universe is referred to as the Epoch of Reionization. For brevity, the entire period from the beginning of CD to the end of the epoch of reionization will be collectively referred to as the Epoch of Reionization (EoR), unless explicitly specified. EoR is typically predicted to span over the redshift range $6 \lesssim z \lesssim 150$.

EoR is among the most poorly understood periods in cosmology. There exist different probes that have been proposed to study EoR including the Ly-$\alpha$ forest and the 21-cm line of neutral hydrogen. The Ly-$\alpha$ forest refers to the absorption spectra of Lyman continuum from the earliest sources (such as high redshift quasars) by intervening neutral hydrogen. The depth and width of the absorption features are indicative of the density and velocity distribution (thus temperature) of the intervening gas, which can be used to reconstruct the process of reionization. In fact one of the first indications of reionization came from quasar absorption studies. The presence of absorption lines in the Ly-$\alpha$ spectrum of quasar SDSS J1030+0524 at $z = 6.28$, but full transmission of quasar spectrum from J1148+5251 at $z = 6.42$, provided evidence of a rapid temporal and spatially fluctuating radiation field towards the end of reionization (Furlanetto et al., 2006). The complete absorption of continuum and lack of good spectral measurements towards high redshift sources places an upper limit on the redshifts probed by this method.

The 21-cm line from neutral hydrogen provides a complementary and powerful probe to study EoR. This signal from neutral hydrogen that is yet to be ionized can be used to study the process by which the Universe transitioned from being mostly neutral to mostly ionized. The spin-flip transition of the electron between two hyperfine levels of the ground state (1s) hydrogen atom results in the emission of a photon of energy $\sim 5.87433 \mu$eV. This corresponds to a frequency of $\sim 1420$ MHz or equivalently a wavelength of 21-cm. This 21-cm line is a powerful probe of neutral hydrogen and has been used to measure galactic rotation curves and as a probe of the intergalactic medium, of which hydrogen is a significant constituent. The 21-cm line had been proposed to study early structure formation from the early days when Zel’dovich proposed the Zel’dovich pancake model. In this early model, structure formation was a top-down process. Neutral gas infall into
dark matter halos formed large structures that fragmented into smaller structures. This has been replaced by a bottom-up model where smaller structures collapse first, then merge to form larger structures. However, the importance of 21-cm line as probe of early structure formation has lived on.

The 21-cm signal from EoR is expected to be redshifted into the metre-wavelength regime. For most standard models this is typically over the frequency range of $10 \lesssim \nu \lesssim 200$ MHz, corresponding to the redshift range $150 \lesssim z \lesssim 6$. The signal from this epoch can appear as spatio-temporal fluctuations in volume elements (spectral cubes) of the sky and there are experiments underway for a statistical detection of the power spectrum of reionization, such as MWA: Murchison Widefield Array (Bowman et al., 2013); LOFAR : Low Frequency Array (van Haarlem et al., 2013) and PAPER : Precision Array for Probing the Epoch of Re-ionization (Parsons et al., 2010). Upcoming telescopes, such as HERA: Hydrogen Epoch of Reionization Array (DeBoer, 2016) and SKA: Square Kilometer Array (Pritchard et al., 2015), will aim to image the first sources and ionization bubbles from the EoR.

The 21-cm signal is also predicted to be observable as a monopole or global distortion to the CMB spectrum (Shaver et al., 1999). It is this global redshifted 21-cm signal from EoR that is of interest to the work presented in this thesis. There are several experiments aiming to detect the global signal from reionization including SARAS: Shaped Antenna measurement of the background RAdio Spectrum (Patra et al., 2013); EDGES: Experiment to Detect the Global EoR Signature (Bowman & Rogers, 2010); BIGHORNS: Broadband Instrument for Global HydrOgen ReioNisation Signal (Sokolowski et al., 2015); SCI-HI (Voytek et al., 2014) and LEDA: Large Aperture Experiment to Detect the Dark Ages (Greenhill et al., 2012). In addition, a space based mission DARE—Dark Ages Radio Explorer—has been proposed (Burns et al., 2012) to operate on the dark side of the moon to mitigate challenges presented by terrestrial RFI and potential limitations introduced by the ionosphere. The mechanism by which Cosmic Dawn, cosmic heating and reionization introduces global CMB spectral distortions is discussed below.

In the Rayleigh-Jeans approximation, the observed brightness temperature $T^\text{obs}_b$ of a source of radiation when seen through an intervening medium with excitation temperature $T_{ex}$ is given by the radiative transfer equation.

$$T^\text{obs}_b(\nu) = T_{ex}(1 - e^{-\tau\nu}) + T_R(\nu)e^{-\tau\nu},$$

(2.22)

where $T_R$ is the brightness temperature of the source and is extinguished by the optical depth given by $\tau$ as a function of frequency $\nu$. For the specific case of reionization, the CMB constitutes the source that is screened by the excitation temperature of the medium,
2.2 Epoch of reionization

given by the spin temperature of HI. Simply put, the spin temperature represents the ratio of the number of hydrogen atoms in the triplet state (state 1) to those in the single state (state 0)

\[
\frac{n_1}{n_0} = \frac{g_1}{g_0} e^{\frac{-T_s}{T_{\bar{c}}}}.
\] (2.23)

With \(\frac{g_1}{g_0} = 3\) and \(T_s = 0.068\) K, the ratio of populations \(\frac{n_1}{n_0}\) is determined solely by the spin temperature \(T_s\). The interaction of spin temperature with background temperature results in the observable differential brightness temperature

\[
\delta T_{b}^{\text{obs}}(v) = \frac{T_s - T_R}{1+z} (1 - e^{-\tau_v}) \approx \frac{T_s - T_R}{1+z} \tau_v.
\] (2.24)

The optical depth of a cloud of hydrogen is given by the integral of the atomic absorption cross-section along the line of sight \(ds\) as

\[
\tau_v = \int ds (1 - e^{-E_{10}/k_BT_s}) \sigma_0 \phi(v) n_0 \approx \sigma_0 \left( \frac{h \nu}{k_B T_s} \right) n_0 \phi(v).
\] (2.25)

Here, \(E_{10} = h \nu\) is the energy of the hyperfine transition, \(\sigma_0\) is the 21-cm cross section, \(\phi(v)\) is the line profile and \(n_0 = n_H^4\). We now use cosmological quantities to represent \(\phi(v)\) and \(n_H\). The column density of the neutral fraction of hydrogen along the line of sight, \(\ell\), is given by \(n_H = \ell x_1 n_{\text{Hot}}\). This also defines the velocity broadening of the line profile by Hubble expansion \(\phi(v) \sim (c/\ell H(z)) v\). Arriving at an exact solution for optical depth with cosmological parameters (Zaroubi, 2013) and substituting in Equation 2.24 (following standard notation) gives

\[
\delta T_{b}^{\text{obs}}(v) \approx 27 x_{HI} \left( \frac{\Omega_b h^2}{0.022} \right) \left( \frac{0.15}{\Omega_m h^2} \right)^{\frac{1}{2}} \frac{T_s - T_R}{T_s} \left[ \frac{\partial_v v_r}{(1+z)H(z)} \right] \text{mK},
\] (2.26)

where, \(h\) is the reduced Hubble constant and \(\partial_v v_r\) is the derivative of velocities along the line of sight. It is this differential brightness between the absolute 21-cm brightness temperature (spin temperature) and background temperature that is predicted to be observable as a global distortion of the cosmic microwave background spectrum. The positions and amplitudes of the turning points of the signal encodes the astrophysics and cosmology of reionization and hence serves to constrain the same. A thorough treatment of the Epoch of Reionization is in Furlanetto et al. (2006).

A comprehensive description of the physical processes that determine the global signal from dark ages through the end of reionization according to the vanilla model, as shown in Figure 2.6, is in Pritchard & Loeb (2012). A brief description of the same follows.
The shape and strength of the global signal from reionization is determined by the coupling of the spin temperature $T_s$ to the gas kinetic temperature $T_K$ and the background radiation temperature $T_R = T_R$. This coupling is mediated by different physical processes over different redshift regimes.

Following the end of recombination, the very low fraction of residual ionized baryons continue to weakly interact with background photons. Compton scattering couples the $T_K$ to $T_R$ and the high density of the gas collisionally couples $T_s$ to $T_R$. Thus, the mean differential brightness is zero and there is no observable signal from redshifts $200 \lesssim z \lesssim 1000$.

At redshifts $z \approx 200$, baryons and the background radiation start cooling at different rates. While radiation temperature scales as $(1+z)$, non-relativistic baryon temperature scales as $(1+z)^2$. The independent thermal evolution of baryons and photons marks their decoupling. As $T_s \approx T_K$ due to collisional coupling, $T_s$ is ‘dragged down’ with the quickly cooling baryons to temperatures below the background radiation over redshifts $40 \lesssim z \lesssim 200$. At redshifts $z \lesssim 40$, as the Universe continues to expand, collisional coupling is no longer effective due to low gas density. $T_s$ decouples from $T_K$ and is radiatively coupled to $T_R$. Since $T_R > T_K$, $T_s$ is ‘pulled’ back up creating a trough like absorption feature. This first absorption feature is well predicted and guaranteed by $\Lambda$CDM cosmology. The
remaining features of the global signal are model dependent.

The first sources begin to turn on at redshift $z \approx z_*$. The first sources emit Ly-$\alpha$ and X-ray photons. Ly-$\alpha$ readily couples $T_s$ to $T_K$ via the Wouthuysen-Field effect (Wouthuysen, 1952a; Field, 1958). This effect describes the coupling of level populations of the spin-flip transition of hydrogen atoms to kinetic temperature via scattering of Ly-$\alpha$ photons. This can result in a change in the ratio of atoms populating the spin aligned and anti-aligned states and thus $T_s$. This coupling sets $T_s \approx T_K < T_\gamma$. This continues till $z \approx z_\alpha$, when Ly-$\alpha$ coupling saturates when gas everywhere is strongly coupled. As Ly-$\alpha$ coupling saturates, further variation in Ly-$\alpha$ flux does not affect the observable signal. At this stage, heating begins to raise $T_K$. This creates the second absorption dip in the global EoR signal from redshifts $z_\alpha \lesssim z \lesssim z_*$. So long as $T_K < T_\gamma$, the differential brightness temperature is in absorption. As $T_K \gtrsim T_\gamma$, the signal may be seen in emission. By redshift $z_h$ the gas is heated everywhere. Thus over redshifts $z_h \lesssim z \lesssim z_\alpha$ the signal transitions from being observable in absorption to emission.

The signal continues to be observable in emission over redshifts $z_T \lesssim z \lesssim z_h$, till the emission saturates at $z \approx z_T$ when $T_s \approx T_K >> T_\gamma$ and variations in gas kinetic temperature become unimportant. Since, $T_s \approx T_K$, $T_s$ can be dropped from Equation 2.26 and the observable signal depends only on $x_{\text{HI}}$.

By this point there is significant overlap in the ionizing bubbles that surround the first sources and Universe is reionized at $z_{\text{EoR}}$.

At $z \lesssim z_{\text{EoR}}$, any 21-cm signal arises from small pockets of residual neutral hydrogen and the observable signal is close to zero.

Thus, the shape and strength of the global redshifted 21-cm signal critically depends on the astrophysics and cosmology of cosmic dawn, cosmic heating and reionization.

### 2.3 Motivation

*It is the unknown that excites the ardor of scholars, who, in the known alone, would shrivel up with boredom.*

—Wallace Stevens

The spectral distortions of the CMB are clearly determined by the astrophysics and cosmology of the Universe. Listed below are some exciting reasons that motivate a detection
of these distortions, particularly those arising from the epochs of recombination and reionization.

2.3.1 Motivation: Epoch of Recombination

1. Observing the spectral distortions from the epochs of helium and hydrogen recombination would, in principle, provide an additional way to determine some of the key parameters of the Universe. For instance, the dependence of the predicted recombination spectrum on $\Omega_b h^2$ and $T_0$ is shown in Figure 2.7a. It is to be noted that most of the recombination of helium and hydrogen and thus the spectral signatures that arise from these processes occur over redshifts before the formation of the CMB anisotropies near the peak of the Thomson visibility function: spectral lines from transitions associated with the hydrogen atoms arise from a redshift range of 1300–1400. Note that the Thomson visibility function $V = e^{-\tau_T} \times \frac{d\tau_T}{d\epsilon}$ defines the properties of the surface of last scattering in terms of $\tau_T$, the optical depth of the Universe due to Thomson scattering. The peak of the visibility function at $z \approx 1100$ gives the nominal redshift of the surface of last scattering. Hence detections of the recombination lines with increasing accuracy are a way of constraining the thermal evolution of the universe at redshifts beyond the last scattering surface. This also provides a novel method to measure the pre-stellar abundance of helium. An illustration of the dependence of the predicted final recombination spectrum on contribution from HeIII $\rightarrow$ HeII and HeII $\rightarrow$ HeI is given in Figure 2.7b.

2. It permits us to confront our detailed understanding of the recombination process with direct observational evidence.

3. It allows us to gain a better understanding of the thermal history of our Universe on the basis of the different redshifts of hydrogen and helium recombination. For instance, it is possible to compute the recombination spectrum assuming that the ambient radiation field is a distorted blackbody, where the distortion is a $y$-distortion. This $y$-distortion arises from the blue-shifting of photons by Comptonization in the early universe if other processes such as the double-Compton and bremsstrahlung are inefficient in thermalizing the distorted blackbody spectrum. This results in an overall decrease in photons from the Rayleigh-Jeans region and enhancement of the Wien tail. This distortion is described by the parameter $y \propto \int_0^z \frac{kh\epsilon}{m_e} \sigma_T n_e \frac{d\epsilon}{d\epsilon} d\epsilon$. COBE-FIRAS observations have placed the upper limit on any full-sky Comptonization by limiting

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3 Adapted from Sathyanarayana Rao et al. (2015).
2.3 Motivation

Fig. 2.7 Dependence of predicted recombination spectrum on (a) cosmological parameters $\Omega_b h^2$ and $T_0$ and (b) primordial helium recombination. Figures reproduced from Sunyaev & Chluba (2009a).
the average y-distortion to \( y < 2.5 \times 10^{-5} \) (Gawiser & Silk, 2000; Hu, 1995). Figure 2.8a shows the dependence on the y-parameter of the contribution from HI and HeII to the total recombination spectrum. Also shown in Figure 2.8b is the total HI and HeII recombination spectra for different redshifts of energy injection. Thus, the cosmological recombination radiation may allow us to distinguish between Compton y-distortions that were created by energy release before or after recombination.

4. With progress in atomic physics, newly available detailed atomic transition tables of hydrogen and helium are available. This has enabled recombination codes to include angular momentum sub-states and examine the role played by them in cosmological recombination. The current understanding is that the presence of even a small fraction of neutral hydrogen at \( z \lesssim 2400 \) hastens HeII \( \rightarrow \) HeI recombination. Additionally, the time-dependence of emission processes and asymmetry in the emission and absorption profiles of Ly-\( \alpha \) in the case of hydrogen are non-negligible. These are a few aspects that could potentially be verified from observations; therefore, a detection and future detailed study of primordial recombination lines enables a study of the ionization history of the Universe.

2.3.2 Motivation: Epoch of Reionization

There are several unknowns pertaining to the epoch of reionization as demonstrated by the toy-diagram in Figure 2.9. A detection and measurement of the 21-cm signal arising from this epoch can shed light on several aspects that are listed below.

- **When?** The timing of the formation of the first sources and the eventual reionization are poorly constrained. The optical depth to reionization from CMB measurements is a tracer of the redshift of reionization, \( z_{\text{EoR}} \). Depending on the model of reionization this has varied between 11 \( \lesssim z_{\text{EoR}} \lesssim 30 \) from the initial results of the WMAP survey (Kogut et al., 2003) to 7.8 \( \lesssim z_{\text{EoR}} \lesssim 8.8 \) from Planck intermediate results (Planck Collaboration et al., 2016c). The span of the reionization process \( \delta z_{\text{EoR}} \) is also poorly known. The current constraints on \( \delta z_{\text{EoR}} \) place it anywhere between 0.06 (Bowman & Rogers, 2010) and 2.8 (Planck Collaboration et al., 2016c). The large uncertainty in \( z_{\text{EoR}} \) and \( \delta z_{\text{EoR}} \) can be resolved by a precise measurement of the global redshifted 21-cm signal that encodes in its shape and strength the astrophysics and cosmology of reionization.

- **What?** It is not known what the first sources were. Standard models of reionization operating within \( \Lambda \)CDM predict radiative cooling of baryons in dark matter halos by the first stars and galaxies (Naoz et al., 2006; Fialkov et al., 2012). It is likely that the first stars were Population III stars. These stars with extremely low metallicities are expected to have short lifetimes of a few million years and have not been directly
Fig. 2.8 The recombination spectrum as a probe of the thermal history of the Universe. (a) The contribution from HI and HeII recombination towards the total recombination spectrum for varying values of initial $y$-parameter, with energy injection at $z = 4 \times 10^4$. The thin red lines are the overall negative parts of the signal. (b) The total HI+HeII recombination spectra for varying redshifts of energy injection. Figures reproduced from Sunyaev & Chluba (2009a).
Fig. 2.9 While observations of the CMB provide constraints of baryon evolution at high redshifts $z \sim 1000$ and the Ly-$\alpha$ forest is useful at relatively low redshifts of $z \lesssim 6$, the Dark Ages, Comic Dawn, and reionization are poorly understood. The red solid line traces baryon temperature evolution through redshift. The 21-cm signal can provide a window into this poorly known and highly interesting period of the formation of the first sources and the eventual reionization of the Universe. Image credit: Girish Kulkarni (private communication).

observed to date! Indication of such sources from reionization measurements would strengthen our understanding of star formation in extremely metal-poor environments and consequently of stellar physics. While population III stars could have heralded the cosmic dawn, it is likely that population II stars might have contributed to the bulk of reionization. Other than a variety of signatures predicted by varying astrophysical parameters for a class of models of reionization, there exist several other competing classes of models. For instance, it has been suggested that reionization could be driven entirely by X-ray emission from quasars with little to no contribution from star-forming galaxies (Madau & Haardt, 2015). There also exist models (Gibilisco, 1996) suggesting that reionization could have been a consequence of photons from evaporating primordial black holes. With potential sources of first sources and reionization being far and varied, an experimental measurement of the 21-cm signal tracing baryon thermal history from the dark ages through the end of reionization will answer several questions such as: What were the first sources? What was the star formation efficiency? What was the minimum halo mass for star formation? What was the spectral energy distribution of the sources of reionization?

- **How?** The predominant source of reionization is likely to be the UV emission from young hot stars, with ionizing bubbles surrounding individual stars gradually
2.4 Challenges to detection of fine-scale distortions of the CMB

Problems worthy of attack prove their worth by fighting back.
— Piet Hein

Clearly, the spectral distortions in the CMB from the recombination epoch and EoR provide exciting prospects to constrain cosmology and provide important clues to the origin of the first atoms, stars and galaxies in the Universe. However, both the signals listed above are extremely weak distortions of the CMB spectrum. Current receiver technology does not necessarily limit achieving the desired sensitivities. However, a detection of these distortions has remained elusive to date. Some challenges facing a ground-based detection of these spectral distortions are listed below.

- **Foreground modelling** Any radio telescope that is designed for a detection of the global signals from cosmological recombination and reionization will also receive contaminant emission from Galactic & extragalactic sources, the atmosphere and the CMB itself. Together, these form a strong foreground that is several orders of magnitude brighter than the signals of interest. For the case of the global signal from the epoch of reionization, the vanilla model predicts a peak signal strength of $\sim 100$ mK. Over the frequency range of interest $40 \text{ MHz} \lesssim \nu \lesssim 200 \text{ MHz}$ where Galactic synchrotron emission dominates, foregrounds can vary in strength between $10^5$ K to $10^2$ K as a function of frequency and sky direction. For the case of the spectral distortions from the epoch of recombination, typical signal strengths in the decimetre frequency range are $\sim 10$ nK, where the brightest foreground component is the $\sim 3$ K strong CMB signal. Thus the dynamic ranges involved are $10^5 - 10^3$ and $\sim 10^8$ for the reionization and recombination signals respectively. Also, these foregrounds have spectral shapes that have unknown functional forms. Although synchrotron sources may individually be modelled as a power law, a combination of emission from several such sources in the antenna beam and along the line of sight, may no longer be describable as a power law. Variation in spectral indices, emission
and absorption processes by thermal gas also determine the shape of the total sky spectrum. It is from within such a total spectrum that the signals of interest need to be mined. A better understanding by thorough modelling of foregrounds enables devising strategies of foreground separation in measurement sets. Such foreground models are thus required.

• **Foreground subtraction** Detecting cosmological signals requires methods of separating the foreground component from signals of interest in the measurement set. Traditional methods use polynomials to model the foreground spectrum (Pritchard & Loeb, 2010b; Bowman & Rogers, 2010). As described above, the precise functional form of foreground spectra are poorly constrained. Thus the order of such a polynomial to completely describe the foreground spectrum is *a priori* unknown. While using polynomials of low order can result in significant residual foreground contribution, high orders risk subsuming the cosmological signal, thus deteriorating signal-to-noise ratio. A method by which foregrounds can be removed without degrading the strength and quality of the cosmological signals is required.

• **Instrument systematics** While Gaussian-type system noise can be reduced by integrating in time and increasing bandwidth, systematic errors in radiometers pose a non-trivial challenge. Particularly, artefacts that mimic signal shapes can lead to erroneous deductions of false-positive and false-negative detections. For instance, observing with antennas that have frequency dependent beams can result in a coupling of spatial structure into spectral structure (Mozdzen et al., 2016). Also, any shapes in the antenna return loss, which serves as a coupling function to the receiver, can result in a spurious modulation of sky power and result in confusing spectral features. Furthermore, antenna beam spillover can result in pick-up of ground radiation, which may be a significant contribution to the system temperature. The antenna design must minimize such beam spillover. Standing waves arising from reflections in cables and connectors that mimic signals of interest can also deteriorate signal detection confidence. Such features also necessitate adopting high-order polynomials to model instrument contributions in the data (Sokolowski et al., 2015). Custom system design, with a focus on precise calibration, can minimize the effects of such instrument related contaminants. Understanding systematics enables data modelling for an unbiased detection experiment.

• **Radio Frequency Interference** Terrestrial Radio Frequency Interference (RFI) poses major challenges (Bower, 2005) to detecting these weak global signals. Firstly, the presence of a strong contaminant can result in several channels of the radio telescope becoming unusable to extract science data. Although RFI is typically present as a narrow feature that is distinct from the rather wide signals of interest,
presence of RFI in several channels can lead to limited usable bandwidth. Additionally, presence of broadband interference, such as from sparking transmission lines or from spillover from the central channel of an especially strong RFI, can necessitate dropping several channels. A second challenge is presented by saturation of receiver electronics in the presence of high power RFI that can result in non-linear system behaviour and potentially irreversible damage to electronics. The frequency band of $40 \, \text{MHz} \lesssim \nu \lesssim 200 \, \text{MHz}$, where the global EoR signal is expected to be present, is severely affected by RFI due to the presence of the Frequency Modulation (FM) radio band between $87.5 \, \text{MHz} \lesssim \nu \lesssim 108 \, \text{MHz}$. While the FM band presents a known challenge, underlying low-level RFI that is detected above signal strength after long integration presents a non-trivial challenge in site selection and post-observing data analysis. For the case of the decimetre-wavelength cosmological recombination signal, a major source of RFI is in the form of satellite down link channels and ground-based telecommunication links, and requires appropriate site-selection and RFI mitigation strategies.

- **Poorly constrained EoR signal** The physics of recombination is relatively well understood. Although there have been subtle changes in signal templates predicted by improved recombination codes, the underlying signal shape has stood the test of time. Detection strategies and data analysis methods can rely on these templates to validate the presence or absence of the signal, and any deviations of measurement from prediction motivates a second look at recombination theory. However, the epoch of reionization is poorly constrained and there exist several models of reionization that predict vastly different signal templates. The nature of the first sources, their luminosity function, heating mechanisms and redshifts at which different processes dominate are variable parameters, to name a few. Although it is this uncertainty that makes EoR studies exciting, it presents challenges in signal recovery and physical interpretation. Inferring the physical parameters of reionization encoded in the signal is a non-trivial challenge (Fialkov & Loeb, 2016; Cohen et al., 2016). Figure 2.10 presents 181 potential expectations for the global EoR signal for a single class of models within the $\Lambda$CDM cosmology. Invoking ‘exotic’ physics such as dark matter annihilation leads to an even larger parameter space and thus different predictions for the signal. It is thus extremely challenging to devise methods to ‘measure’ the EoR signal. An approach is to detect the presence or absence of signals predicted by different models with varying levels of confidence. The challenge is akin to looking for a proverbial needle in a haystack, without knowing what the needle looks like!
Fig. 2.10 Prediction for the global redshifted 21-cm signal as a function of redshift for 181 combinations of astrophysical parameters within the standard model of cosmology. The colour scale indicates the ratio of Ly-α intensity and X-ray heating rate at the deepest trough. Image reproduced from Cohen et al. (2016) (refer original image for a more detailed description).
CHAPTER 3

ON THE DETECTION OF THE CMB SPECTRAL DISTORTIONS FROM THE EPOCH OF RECOMBINATION

This chapter has previously been published\(^1\) as "On the detection of the CMB spectral distortions from the Epoch of Recombination", Sathyanarayana Rao et al., 2015, ApJ, 810, 3. Some additional comments that are not present in the paper are included as footnotes in the chapter.

3.1 Abstract

Photons emitted during cosmological hydrogen \((500 \lesssim z \lesssim 1600)\) and helium recombination \((1600 \lesssim z \lesssim 3500\) for HeII → HeI, \(5000 \lesssim z \lesssim 8000\) for HeIII → HeII) are predicted to appear as broad, weak spectral distortions of the cosmic microwave background. We present a feasibility study for a ground-based experimental detection of these recombination lines, which would provide an observational constraint on the thermal ionization history of the Universe, uniquely probing astrophysical cosmology beyond the last scattering surface. We find that an octave band in the 2–6 GHz window is optimal for such an experiment, both maximizing signal-to-noise ratio and including sufficient signal spectral structure. At these frequencies the predicted signal appears as an additive quasi-sinusoidal component with amplitude about 8 nK that is embedded in a sky spectrum some nine orders of magnitude brighter. We discuss an algorithm to detect these tiny spectral fluctuations in the sky spectrum by foreground modeling. We introduce a maximally smooth function capable of describing the foreground spectrum and distinguishing the signal of interest. We conclude that detection is in principle feasible in realistic observing times provided that radio frequency interference and instrument bandpass calibration are controlled in this band at the required level; using Bayesian tests and mock data, we show that 90%\(^1\)

\(^1\)Note: Since the publication of this paper, there have been improvements to the predicted cosmological recombination signal. The improvements and the effect of the same on the results presented in this work are in Appendix B.
confidence detection is possible with an array of 128 radiometers observing for 255 days of effective integration time. We propose APSERa—Array of Precision Spectrometers for the Epoch of Recombination—a dedicated radio telescope to detect these recombination lines.

3.2 Introduction

The cosmological recombination epoch marks an extended period over which electrons recombine with protons and helium nuclei as cosmological expansion and cooling cause the Universe to transition from a fully ionized primordial plasma to a gas of almost completely neutral hydrogen and helium atoms. In the cosmological concordance model, the cosmological recombination epoch spans redshifts $500 \lesssim z \lesssim 1600$ for hydrogen and for helium recombination the corresponding redshifts are $1600 \lesssim z \lesssim 3500$ for HeII $\rightarrow$ HeI and $5000 \lesssim z \lesssim 8000$ for HeIII $\rightarrow$ HeII. At these redshifts, electrons and photons are tightly coupled through energy exchange via Compton scattering, and the thermodynamic temperatures of electrons and photons are almost exactly equal (at $T \approx 3815\{(1 + z)/1400\}$ K).

During hydrogen recombination, once even a small neutral fraction builds up, photons emitted in any free-bound transition to the ground state would almost immediately be reabsorbed by a nearby neutral hydrogen atom, thereby effectively compensating for the electron captured (Peebles, 1968; Zeldovich et al., 1968; Chluba et al., 2007). Hydrogen recombination therefore relies on free-bound transitions to excited states followed by a trickle down to the ground state, with the subsequent emission of numerous recombination photons. During this process, atoms are frequently photoionized by the huge number of cosmic microwave background (CMB) photons and after multiple dissociations and recaptures the atoms finally reach the ground state, emitting photons in the Lyman-$\alpha$ resonance or the $2s$–$1s$ two-photon continuum (Peebles, 1968; Zeldovich et al., 1968) and, at low frequencies, in transitions among excited states (Dubrovich, 1975; Dubrovich & Stolyarov, 1997, 1995, 1997).

Recombination is expected to be stalled as the ambient radiation temperature in the Lyman-$\alpha$ transition wavelength rises and is balanced in a quasi-static equilibrium with populations in the $1s$ and $2p$ states. Recombination thus depends on the depletion rate of $n = 2$ atoms and removal of the excess brightness in the ambient Lyman-$\alpha$ line by Hubble expansion and two-photon decay from the $2s$ state. These processes occur at a rate that is $\approx 10^7 - 10^8$ times slower than the spontaneous transition probability of the Lyman-$\alpha$ and play a vital role in controlling the dynamics of recombination. About 57% of all hydrogen atoms become neutral through the $2s$–$1s$ two-photon channel (Chluba & Sunyaev, 2006a), which has a vacuum decay rate of only $A_{2s1s} \approx 8.22 \text{s}^{-1}$ (Göppert-Mayer,
3.2 Introduction

1931; Breit & Teller, 1940; Spitzer & Greenstein, 1951; Goldman, 1989; Labzowsky et al., 2005). Consequently, hydrogen recombination is expected to be substantially delayed compared to what might be expected assuming equilibrium Saha recombination at the average densities and temperatures typical for our Universe.

As the number of photons per baryon is roughly \(1.6 \times 10^9\), radiative processes including stimulated recombination, induced emission, and absorption of photons dominate the populating of atomic levels rather than collisions (e.g., Chluba & Sunyaev, 2007; Chluba et al., 2010). As the Universe gradually expands and recombination proceeds by uncompensated bound–bound and free–bound transitions, the level populations of atomic species slowly fall out of equilibrium with the radiation field, which causes departures of the CMB spectrum from an ideal Planck form. The spectral lines corresponding to these transitions are predicted to appear as redshifted additive deviations to the CMB spectrum, with most of the hydrogen lines originating from redshift \(z \approx 1300 – 1400\) (Chluba & Sunyaev, 2006a; Rubiño-Martín et al., 2006); the observable intensity of these spectral lines is furthermore expected to be uniform on the sky and unpolarized. The lines are also substantially broadened owing to the extended recombination time (and electron scattering in the case of HeII recombination; Rubiño-Martín et al. (2008)). At low frequencies, adjacent lines overlap substantially, and hence the cosmological recombination radiation is expected to manifest itself as spectral ‘ripples’ riding on a smooth continuum, which are together an additive component of the extragalactic background light extending over radio, microwaves and near IR wavelengths.

Improvements in our understanding of the recombination history within the framework provided by the concordance cosmology, the role of \(2s–1s\) two-photon decay and other factors governing the depletion of the \(n = 2\) level population, the importance of the contribution of helium to the recombination process (Kholupenko et al., 2007; Wong & Scott, 2007; Switzer & Hirata, 2008a,b), progress in related atomic physics, and, last but not the least, substantial improvements in computing power have all contributed to much improved and more realistic modeling of the epoch in question (see Fendt et al., 2009; Rubiño-Martín et al., 2010, for an overview of recombination physics). Detailed calculations as described in Sunyaev & Chluba (2009b) and references therein provide a fairly precise estimate of the spectral ripples expected as a result of the recombination of hydrogen and helium.

In Fig. 3.1, we show the predictions from 0.1 GHz up to 3000 GHz. At high frequencies we can distinguish features caused by the Lyman-\(\alpha\) line, Balmer-continuum and Balmer-\(\alpha\) line, and Paschen-\(\alpha\) and Brackett-\(\alpha\) lines (Chluba & Sunyaev, 2006a; Rubiño-Martín et al.,
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Helium contributes locally at a typical level\(^2\) of \(\approx 10\% - 30\%\) (Rubiño-Martín et al., 2008; Sunyaev & Chluba, 2009b). Owing to the substantial line broadening resulting from the extended period of recombination, at low frequencies the recombination radiation appears as (i) a smooth continuum to the radio, microwave, and near IR backgrounds, plus (ii) more easily distinguishable spectral ripples that have a quasi-periodic frequency dependence. It is the detection of these ripply features that we focus on in this work because of their distinctive signature that distinguishes them from any other known spectral features in the CMB.

Fig. 3.1 Additive spectral structure expected in the intensity of the uniform extragalactic background light owing to cosmological recombination of hydrogen and helium (Chluba & Sunyaev, 2006a; Rubiño-Martín et al., 2006; Chluba et al., 2007; Zeldovich et al., 1968). The wide frequency range is successively covered in four panels. The additive spectrum is in units of mJy sr\(^{-1}\) in the top panels and Jy sr\(^{-1}\) in the bottom panels.

3.2.1 The importance of an experimental detection of the recombination radiation

Direct observations of the predicted ripples – the unique fingerprint of the recombination era – would be a confirmation of the recombination theory, validating our understanding of the associated atomic physics, as well as the physical processes in these early times, which in itself is a major motivating factor for experimentally detecting them. A detailed

\(^2\)Enhancements of features caused by additional feedback effects (Chluba & Sunyaev, 2010) were neglected for now.
discussion on the dependence of the spectral signatures from the epoch of recombination on various contributing factors that affect their strength, shape, and position is presented in Chluba & Sunyaev (2008a) and Sunyaev & Chluba (2009b). We list here some of the major motivations for an experimental detection.

1. **Determining key cosmological parameters.** Observing the spectral distortions from the epochs of hydrogen and helium recombination would, in principle, provide an additional way to determine some of the key parameters of the Universe. For instance, the dependence of the predicted recombination spectrum on $\Omega_b h^2$ and $T_0$ is shown in Fig. 3 and 4 of Chluba & Sunyaev (2008a). It may be noted here that most of the recombination of hydrogen and helium and thus the spectral signatures that arise from these processes occur over redshifts before the formation of the CMB anisotropies that occurred near the peak of the Thomson visibility function: spectral lines from transitions associated with the hydrogen atoms arise from a redshift range of 1300–1400. Hence, detections of the recombination lines with increasing accuracy provide a way of constraining the thermal evolution of the universe beyond the last scattering surface (see below). This also provides a novel method to measure the *prestellar abundance* of helium and to break parameter degeneracies by combining with CMB anisotropy measurements. An illustration of the dependence of the predicted final recombination spectrum on contributions from HeIII $\rightarrow$ HeII and HeII $\rightarrow$ HeI is given in Fig. 11 of Sunyaev & Chluba (2009b).

2. **Probing energy release in the pre-recombination era.** Detection of cosmological recombination lines allows us to gain a better understanding of the thermal history of our Universe on the basis of the different redshifts of hydrogen and helium recombination. For instance, it is possible to compute the recombination spectrum assuming that the ambient radiation field is a distorted blackbody, where the distortion is of $y$-type (Chluba & Sunyaev, 2009). The $y$-distortion (Zeldovich & Sunyaev, 1969) could arise from the blueshifting of photons by Comptonization in the early universe, owing to energy release at $z \lesssim 50000$, when the redistribution of photons in energy by Compton scattering becomes inefficient (Burgiana et al., 1991; Hu & Silk, 1993; Chluba & Sunyaev, 2012). This distortion is described by the Compton $y$-parameter, $y = \int \frac{k(T_e-T_B)}{m_e c^2} \sigma N_e c \, dt$. COBE/FIRAS observations place an upper limit of $y < 1.5 \times 10^{-5}$ (95% c.l.; Fixsen et al., 1996; Fixsen, 2009). The contributions from hydrogen and helium to the total recombination spectrum depends both on the value of the $y$ parameter and when the distortion was created (Chluba & Sunyaev, 2009). Thus, the cosmological recombination radiation may allow for distinguishing between Compton $y$-distortions that were caused by energy release before or after the epoch of recombination.
3. Testing recombination physics. Today’s most advanced recombination codes (Ali-Haïmoud & Hirata, 2011; Chluba & Thomas, 2011) include many subtle atomic physics and radiative transfer effects (e.g., Dubrovich & Grachev, 2005; Chluba & Sunyaev, 2006b; Kholupenko & Ivanchik, 2006; Kholupenko et al., 2007; Switzer & Hirata, 2008b; Chluba & Sunyaev, 2008b; Hirata, 2008; Grin & Hirata, 2010). These calculations ensure that the science return from Planck and upcoming CMB experiments is not compromised by inaccuracies in the recombination model (Rubíño-Martín et al., 2010; Shaw & Chluba, 2011). However, the detailed dynamics of the recombination process is also reflected in the shape and position of the recombination features. Any departures of the recombination spectrum from the theoretical predictions will indicate the presence of some nonstandard process (e.g., Chluba & Sunyaev, 2009; Chluba, 2010). Thus, by observing the recombination radiation, we can directly confront our understanding of recombination physics with experimental evidence.

Detecting nK amplitude fluctuations in the extragalactic background brightness, which are indeed a tiny perturbation to the orders-of-magnitude larger sky brightness temperature that arises from the CMB, extragalactic sources, and Galactic emission, is a very challenging problem. However, we opine that receiver technology and the ability to produce detector arrays have progressed to the point where it is meaningful to look at the practical issues. In this first paper, we study the feasibility of such a detection by modeling the sky spectrum as recorded by an ideal instrument and examine whether it is at all possible to recover the weak signal embedded in the substantially brighter Galactic and extragalactic foregrounds. We discuss methods for fitting to data for the recovery or detection of the faint recombination line spectrum when observed embedded in the substantially brighter cosmic radio background. We also compute optimal observing frequencies for the detection and associated signal-to-noise ratio for detection with realistic receivers and a purpose-built array of spectral radiometers with due consideration to contribution from atmospheric emission further guided by radio frequency interference (RFI) over the band. Some of the challenges are very similar to those for the detection of the global 21cm signal (see Patra et al., 2013); however, the recombination ripples benefit from their unique frequency dependence, which is hard to mimic by other sources and instrumental effects.

3.3 The signal and noise level for a detection of the recombination radiation

In this section, we discuss the frequency dependence of the spectral signal and that of the background brightness and additive instrument noise to arrive at estimates for signal-to-noise ratio versus frequency for detection of the line structure. The successive peaks of the
3.3 The signal and noise level for a detection of the recombination radiation

recombination spectral features correspond to the spectral lines arising from bound-bound \( n \rightarrow (n - 1) \) electron transitions between adjacent principal quantum states of hydrogen, visible at frequency \( v \simeq 4.7 \, \text{GHz} \, [n/10]^{-3} \) (for emission redshift \( z \simeq 1400 \)). If we subtract a low-order baseline component from the recombination spectrum in the 1.5–7.0 GHz band, the resulting spectral structure is as shown in Fig. 3.2. As a method of removing the foreground and receiver response in an observation with an ideal instrument, if a smooth baseline were to be subtracted from a recorded sky spectrum, Fig. 3.2 is what would be expected as a residual. We now address the problem of identifying the most suitable

![Ripply recombination line signal](image)

**Fig. 3.2** Ripply recombination line signal expected to be detected in the 1.5–7.0 GHz band.

frequency range for the experimental detection of recombination lines from the epoch of recombination by first considering the relative amplitude of the spectral ripples arising from recombination to the noise in the detection. The total system noise is the additive sum of the spatially varying sky background, consisting of contributions from discrete and diffuse Galactic and extragalactic sources in the sky, the CMB and the cosmic infrared background, as well as effects of atmospheric emission, plus receiver noise that is at lowest quantum noise associated with zero-point fluctuations. This sets a fundamental limit on the achievable detection sensitivity. Later in this section we also consider detection using receivers that have more realistic noise temperatures consistent with current technology. We also discuss constraints arising from the requirement that within the observing band we need to cover at least a distinctive segment of the quasi-periodic cosmological signal, which might provide a unique template specific to cosmological recombination that would be difficult to mimic by other astrophysical sources, atmospheric emission, RFI, or instrumental effects. As a representation of the telescope system noise contribution from additive sky radiation, we construct a model spectrum of the background sky brightness
Fig. 3.3 Expected intensity of the recombination signal is shown in black. Our model for the intensity equivalent to the minimal total system noise is shown as a red dotted line, where the receiver noise is assumed to have ideal quantum-noise-limited performance. Also shown as a blue dashed line is the intensity corresponding to system noise when observing with a state-of-the-art cryogenically cooled receiver that has a noise temperature of 1 K and in green is the sky intensity when observing with an uncooled receiver assuming a noise temperature of 14 K. All intensities are in units of Jy sr$^{-1}$ (surface brightness units) with the x-axis in log($v$/GHz).
all the way from 50 MHz up to 4 THz, extending to just beyond the redshifted Lyman-\(\alpha\) line that is essentially the highest frequency at which we may expect spectral features arising from cosmological recombination. We adopt the model presented in Subrahmanyan & Cowsik (2013) for the Galactic and extragalactic contributions to sky brightness and derive sky temperatures toward the Galactic pole at 150, 408, and 1420 MHz. A fit of a power-law form to these brightness estimates yields a temperature spectral index of \(-2.5\) and a normalization corresponding to sky brightness of 438 K at 100 MHz; this model spectrum toward the Galactic pole is adopted to represent the contribution to system noise from Galactic emission plus extragalactic discrete sources.

We also include a component that is an estimate of the far- and mid-infrared background; this model is derived from data in Table 47 of Leinert et al. (1998). For the receiver noise, we compute the system temperature for two cases: (i) assuming a state-of-the-art cryogenically cooled receiver with noise temperature of 1 K above quantum noise (Schleeh et al., 2012) and (ii) that for an uncooled receiver with a more realistic noise temperature of 14 K plus quantum noise (Belostotski & Haslett, 2007; Witvers et al., 2010). We use the am atmospheric model (Paine, 2004) with typical conditions at the Chajnantor\(^3\) plateau site to derive the atmospheric opacity versus frequency. The system temperature is finally translated to above atmosphere using this opacity so that it may be combined with the signal intensity derived above for calculating the signal-to-noise ratio. Our adopted model spectrum for the above-atmosphere system temperature is shown in Fig. 3.3 along with the spectrum of the expected recombination signal.

As a criterion for the minimum bandwidth required at any observing frequency, we choose a spectral window that includes at least two adjacent broad recombination spectral lines that arise from \(n \rightarrow (n-1) (\alpha)\) transitions. This minimum bandwidth would correspond to the spacing between peaks of \(\alpha\) transitions from \(n\) and \(n+2\) shells. Such a spectral window would be expected to include a sufficiently high order of the variation in the signal so as to give it a distinctive signature. Fig. 3.4 shows the ratio of this minimum bandwidth to the nominal center frequency versus the center frequency. The detection of recombination epoch spectral lines clearly requires a larger fractional bandwidth at higher frequencies. A ratio less than unity implies that an octave bandwidth at the observing frequency would contain more than two adjacent recombination lines, satisfying our criterion. The ratio falls below unity for observing frequencies below about 18 GHz, which implies that detecting the cosmological line spectrum would require bandwidths exceeding an octave if the center frequency were to exceed 18 GHz. Receivers with bandwidths wider than an octave are susceptible to self-generated RFI from harmonics of system clocks and local oscillator

\(^3\)https://www.cfa.harvard.edu/~spaine/am/cookbook/unix/other_examples/Chajnantor.amc
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Fig. 3.4 Ratio of spacing between peaks of $\alpha$ transitions from $n$ and $n+2$ quantum shells of hydrogen to the nominal frequency vs. nominal observing frequency in log(Hz) units.

frequencies; therefore, although sensitivity for a detection would undoubtedly improve with wider observing bands, for the ultrasensitive detection experiment being considered herein it would be unwise to attempt detection at frequencies above this value.\textsuperscript{4}

As an estimate of the detectable signal from hydrogen recombination, we require a measure of the peak-to-peak spectral variations versus observing frequency. We first fit a spline through the points in the recombination line intensity template that are at frequencies corresponding to the peaks of the $n \rightarrow (n-1)$ hydrogen transitions. We use a redshift that is a mean value, which we determine by comparing the locations of the peaks in the predicted recombination line spectrum with the corresponding locations given by the Rydberg formula. Subtracting the spline fit gives a residual recombination line spectrum that represents the detectable signal in an observation from which a mean spectral baseline has been subtracted. In Fig. 3.5 we plot the strength of the expected recombination line signal defined as half the peak-to-peak amplitude of the baseline-subtracted ripples expected in the observed band, versus observing frequency. With frequency, the expected ripple amplitude increases by orders of magnitude across the frequency range we consider here. It may be noted that although in terms of spectral intensity this ripple signal amplitude increases with frequency, as shown above the number of spectral ripples within any octave

\footnote{Harmonics from clocks should in principle be very narrow, hence easy to identify and remove from the (spectral) data, encouraging a multi-octave system approach. However, strong harmonics can 'leak’ from a single spectral channel and result in broad spectral features, much like strong RFI. Although this can be mitigated by the use of windowing functions, a single-octave system approach is ‘more clean’ for such a high dynamic range experiment.}
bandwidth decreases with increasing frequency. Thus, a frequency regime that maximizes both aspects is sought. (See Section 3.3.2)

![Graph](image.png)

Fig. 3.5 Variation in the logarithm of amplitude of the ripples in log(Jy sr\(^{-1}\)) units representing the signal from recombination vs. frequency in log(GHz) units.

### 3.3.1 Signal-to-noise ratio: A Matched Filter Approach to Signal Detection

The detection of spectral lines from cosmological recombination involves first subtracting a baseline from the observed spectrum to remove the relatively smoother foregrounds and CMB, after which the residual spectral segment may be examined for the expected spectral ripple using a matched filter. We evaluate here the signal-to-noise ratio for a detection method where an estimate of the amplitude of any spectral ripple that matches a theoretical template is derived from the measurement. The expected signal, following baseline subtraction, is similar to a sinusoidal ripple that has a period that varies systematically across the spectral segment. The noise or uncertainty in the estimate of the amplitude of the ripple is derived from the measurement errors in the measured intensities in the spectral channels, and from the propagation of errors from these measurements to the derived estimate of the amplitude of the signal present in the data. The measurement errors in channel data depend on the system noise, which originates from the sky brightness and receiver noise, and on the channel bandwidth and integration time.

\[^5\]The units in the plot and the caption should be read as Log\((W m^{-2} Hz^{-1} sr^{-1})\)
We consider here the ideal scenario where the measurement data—the bandpass-calibrated spectrum of the cosmic radio background from which a baseline has been subtracted—contain a spectral ripple that is the same as the expected theoretical template without any other residual contamination. In this case where the signal amplitude is estimated using a matched filter derived from the template, we obtain a signal-to-noise ratio that is limited purely by the noise present in the spectral channels. This case study admittedly represents an optimistic estimate of the signal-to-noise ratio attainable.

The residual spectrum after baseline subtraction is assumed to have $N$ frequency channels, each of bandwidth $\delta b$. The sampled residual sinusoidal ripple, which is also the template of the expected ripply recombination lines following baseline subtraction, is given by $a_k$ and has an amplitude of $A_0$. From this template of the expected signal we define a weighting function $w_k$, which is defined such that the weighted sum of $a_k$ is $A_0$. The matched filter is thus the weighted summation of the observed spectrum, and the matched filtering yields an estimate of the amplitude $S$ of the ripple from recombination:

$$S = \sum_{k=0}^{N} a_k w_k$$  \hspace{1cm} (3.1)

The template representing the expectation and the corresponding weighting function may be scaled appropriately to search the data for signatures of the ripple from recombination assuming a range of effective redshifts for the recombination lines. The weighting function is hence similar to the template in that it has the same ripple form but with an amplitude $w_0$. If the signal matches the template, the requirement that $S = \sum_{k=0}^{N} a_k w_k$ ought to equal $A_0$ leads to the condition that $w_0 = 2/N$.

Assuming that there are no gain fluctuations, the uncertainty in the channel data in the residual spectrum $a_k$ is

$$\delta a_k = \frac{T}{\sqrt{\delta b \cdot t}}$$ \hspace{1cm} (3.2)

where $T$ is the system temperature and $t$ is the total integration time. Propagating this error in the channel data to the error $\delta S$ in the estimate of the amplitude of the ripple from

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$^6$ An ideal antenna has been assumed with no beam spillover to the ground and consequently no pick-up of ground radiation.
3.3 The signal and noise level for a detection of the recombination radiation

recombination,

\[
\delta S = \frac{T}{\sqrt{\delta b \cdot t}} \sqrt{\sum_{k=0}^{N-1} (\delta a_k)^2 |w_k|^2}
\]

\[
= T w_0 \sqrt{\frac{N}{2\delta b \cdot t}}.
\]

The last step above follows from the result that the average value of the square of a sinusoidal wave is half the amplitude. Substituting the earlier result that \( w_0 = 2/N \) yields

\[
\delta S = T \sqrt{\frac{2}{N \delta b \cdot t}}. \tag{3.3}
\]

Neglecting calibration uncertainties, the signal-to-noise ratio in the measurement is given by

\[
\text{SNR} = \frac{S}{\delta S} = \frac{A_0}{T} \sqrt{\frac{N \delta b \cdot t}{2}} = \frac{A_0}{T} \sqrt{\frac{B \cdot t}{2}}, \tag{3.4}
\]

where \( B = N \delta b \) is the total bandwidth of the observed spectrum. For this estimate it is assumed that noise in the different channels is uncorrelated.

### 3.3.2 Choice of Frequency for detecting ripples from cosmological recombination

In Fig. 3.6 we show the estimated signal-to-noise ratio versus observing frequency to guide the choice of observing frequency. In creating this plot the signal is assumed to be half the peak-to-peak magnitude of the ripple, which is the amplitude shown in Fig. 3.5. The noise is assumed to be the system temperature (from Fig. 3.3) scaled by a factor \( \sqrt{2/B} \), where \( B \) is the octave bandwidth about a nominal central frequency. The integration time is assumed to be 1 s. The figure shows the signal-to-noise ratio for the case where the system noise corresponds to quantum-noise-limited receivers (in green); we also show in red the signal-to-noise ratio for a cryogenically cooled receiver with 1 K noise temperature and in blue this ratio for the case of uncooled 14 K receivers. All three traces are assuming an ideal case where the spectral ripple in the measurement data matches the predicted template exactly.

Detection with maximum signal-to-noise ratio suggests observing in the 1–6 GHz band. RFI might potentially make detecting the cosmological recombination signal impossible.
Fig. 3.6 Signal-to-noise ratio vs. nominal observing frequency (in log(GHz) scale) for the detection of spectral-line signatures from recombination epoch with an octave bandwidth. The green line shows the ratio assuming ideal quantum-noise-limited receivers; the blue line shows the ratio for the more realistic case of receivers with 14 K receiver noise. The red line shows this ratio for a receiver that is cryogenically cooled to 1 K. The observing time is assumed to be 1 s.

However, ground-based radio observations extending beyond protected frequencies have enabled technologies to mitigate RFI by implementing specialized signal processing techniques in both hardware and software (see, eg., Fisher, 2001a,b). Terrestrial RFI propagates over distances by direct path, ionosphere reflection and waveguide modes in the ionosphere, tropospheric scatter, ground waves, terrain diffraction and diffraction over the curved surface of the Earth, low-altitude surface ducts, and meteor scattering. In the 2-6 GHz band only direct paths and diffraction over the Earth surface are of consequence, and these are severely attenuated beyond the horizon of transmitters. Hence, moving to a remote site on Earth avoids terrestrial RFI. RFI from low-Earth-orbit satellites, which would be the primary source of RFI in the suggested band is ephemeral and may be recognized in the recorded data and excised. More important is the RFI from geostationary satellites, which may be steady, and therefore we propose to perform the detection experiment close to the poles.\textsuperscript{7}

If we avoid the lower end of this band where Galactic HI and terrestrial and satellite-downlink-related RFI are substantial, the 2–6 GHz band is suggested. Observing in octave\textsuperscript{7}

\textsuperscript{7} To overcome challenges posed by terrestrial RFI a space borne experiment (Smirnov et al., 2012) or a moon-based site can be considered. As there is no atmospheric contribution to system temperature, such an experiment would also have higher sensitivity than a ground based experiment. However, deploying and operating an array of radio-telescopes in space presents technical challenges in addition to cost disadvantages.
3.4 A modeling of observations of the cosmological recombination spectrum

bandwidths within this range also satisfies the criterion that at least two cycles of ripples of the recombination line spectrum ought to be contained in the observed spectral segment (see Fig. 3.2).

If RFI indeed proves to be a limiting factor for a ground-based detection of the weak recombination line signal in the 2-6 GHz band, the next optimum band might be the 6-12 GHz band. At higher frequencies, and indeed in this alternate band as well, ground-based detection would be increasingly difficult because of substantial atmospheric emission, which considerably degrades the achievable sensitivity. A detection below 2 GHz is ruled out by the relatively stronger RFI at lower frequencies and the Galactic 21-cm emission from neutral hydrogen in the ISM.

3.4 A modeling of observations of the cosmological recombination spectrum

The recombination lines from cosmological recombination are detected by a receiver along with foregrounds, which are averaged by the telescope beam over its response pattern on the sky. The averaging over a multitude of sources with different emission spectra, over the sky and along line of sight, results in a detected spectrum that would have an unknown form: even if the emissivity of the gas is a power-law form at every location, the spectral index does vary across the sky and along line of sight and, therefore, the averaging by the telescope beam would result in an observed spectrum that deviates from a single power law. A similar effect, related to the superposition of blackbodies inside the beam of an experiment, causes an inevitable $y$-type distortion of the CMB spectrum (Chluba & Sunyaev, 2004); the superposition of blackbodies simply is not a blackbody anymore (Zeldovich et al., 1972; Chluba & Sunyaev, 2004; Stebbins, 2007).

It is necessary to detect the presence of the recombination line spectrum in the observed spectrum although the line spectrum has amplitude that is almost nine orders of magnitude smaller and the additive foreground is a complex spectrum of unknown form! By simulating the sky spectrum as would be observed by an ideal system, this section addresses the question of whether such a detection is at all possible.

We choose an octave band from 3 to 6 GHz as the observing band for the simulations presented here. This band also meets our criterion of having at least two recombination spectral lines within the band. In this section we describe a code we have developed, which simulates an ideal receiver system observing the sky over the identified frequency range and generates the temperature spectrum of the sky as would be produced by such an
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instrument. A calibration method is included in the pipeline.

We use all-sky maps at 408 MHz (Haslam et al., 1982), 1420 MHz (Reich, 1982; Reich & Reich, 1986; Górski et al., 2005) and 23 GHz (WMAP science data product\(^8\)) as input maps to estimate the combined Galactic and extragalactic brightness contribution to the sky spectrum between 3 and 6 GHz. The sky brightness in the Rayleigh-Jeans limit, at every sky pixel and over frequency channels spaced 10 MHz apart, is derived from these three input temperature maps using a linear interpolation in log–log space. In their final form the three maps are smoothed to a resolution of 1°, are in Galactic coordinates ordered in the nested R8 HEALPix scheme and do not contain the CMB monopole. Preparing the WMAP 23 GHz all-sky map in a form suitable for our sky model involves an additional step of adding the uniform background to the differential measurement map. For this we assume a plane-parallel slab model for the galaxy and use the method adopted in Kogut et al. (2011a), using all-sky images of the absolute brightness of the sky at 150, 408, and 1420 MHz to estimate this uniform component at 23 GHz. The CMB monopole, dipole, and WMAP ILC images, were subtracted from all these images and the pixels with substantial contamination from discrete sources were blanked using the same blanking image that was used in the WMAP analysis. Pixels in each of the images were binned in cosecant Galactic latitude ($\text{cosec}(\theta)$) over the range 1.0–4.0. The runs of median pixel intensity versus mean cosecant($\theta$) were examined in log-log coordinates. A straight line fit to this plot yields an estimate of the mean extragalactic background at each frequency. It may be noted here that we have restricted this analysis to the southern Galactic hemisphere for consistency with the WMAP analysis. For the case of the 23 GHz image, the fit yields an intercept of 0 K with an accuracy of 1 part in $10^9$ as expected for an image has been constructed to have its intercept arbitrarily set to zero. The intercepts estimated for the lower-frequency images were extrapolated using a polynomial fit to get the ‘missing’ uniform background in the 23 GHz image. The resulting value of 493 $\mu$K was added as a constant to the 23 GHz image.

The three maps representing the Galactic and extragalactic radio emission (minus CMB) at 408 MHz, 1420 MHz, and 23 GHz were interpolated at every image pixel separately to estimate sky temperatures at frequencies between 3 and 6 GHz. We interpolate using a first order polynomial in log brightness temperature versus log frequency space, considering the spectrum at every pixel to be a smooth power law in linear space. The total sky brightness toward any sky pixel and frequency is estimated as the sum of the Galactic and extragalactic sky brightness derived from this polynomial interpolation, plus a uniform CMB brightness computed using the Planck formula, plus a uniform component corresponding to the weak

\(^8\)WMAP Science Team
cosmological recombination line spectrum, as shown in Fig. 3.1. Detecting the weak and broad signal requires that the instrument response be stable and flat or at the very least smooth over the entire band. This necessitates precise bandpass calibration. The first component in the signal path is the antenna. In the present simulation we have assumed the pattern and impedance of the antenna to be independent of frequency in the frequency range of our interest, resulting in a uniform response to the sky brightness throughout the band. This ensures that the antenna introduces no spurious frequency structure. We assume the frequency-independent radiation pattern to have a $\cos^2(ZA)$ form, where $ZA$ denotes zenith angle. Careful design can also minimize ohmic losses in the antenna. The antenna bore sight (the optical axis of the antenna) is assumed to be static on the ground at the observing site and directed toward zenith at all times. We neglect polarization in the sky intensity, as well as the telescope response. Further down the receiver chain, complex features can be introduced in measured spectra by additive noise sources with inherent shape, which further couple via multiple reflections due to mismatches in the signal path. These injected spectral features might mimic the cosmological signal, potentially limiting its detection. What is required is a self-calibratable system in which the multiplicative bandpass gain and additive noise contributions are continuously measured in situ so that time-varying bandpass and additive couplings are tracked. A potential architecture is the correlation spectrometer scheme similar to the one described in Patra et al. (2013) in the context of experimentally detecting the redshifted global 21 cm signal from the epoch of reionization (EoR). In brief, the signal from the antenna is split in two, introducing an
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Fig. 3.8 A time-frequency plot showing the synthetic sky spectra over Gauribidanur (latitude: +13°01) as observed over 24 hr starting at LST 19:05. One can see the galaxy rise and set as the sky drifts over the instrument with each spectrum recorded 1 minute apart.

additional phase in one. These identical (except for phase) signals are provided as inputs to a correlation spectrometer. This would enable detection of only the correlated signals including the sky as seen by the antenna. The uncorrelated components, such as noise introduced by the system in the receiver electronics following the point of splitting, would be canceled out. Injecting switched noise from calibrated noise sources at various points along the signal path allows for an online measurement of internal signal reflections. There exist practical challenges in achieving the desired calibration accuracy of one part in $10^9$. However, a better understanding of these challenges would themselves guide the receiver design and calibration schemes adopting the latest advances in technology as they become available.

The spectral response of the observing system at any instant is the weighted sum of spectra toward every pixel in the sky where the weighting is by the beam power pattern toward that sky direction. As stated above, the spectrum toward any sky pixel is not expected to be a simple power law because it is an average along the line of sight of regions that might have different spectral indices. The weighted average spectrum is an average over the sky and along the line of sight, and hence even if each emitting region is of

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9 The immunity of a correlation spectrometer’s output to the noise added after the point of splitting does not extend to the noise estimates of the correlator’s output. The two paths each add uncorrelated noise, which will degrade the overall sensitivity. However, the noise added here may be made an insignificant part of the total system temperature by having sufficient gain ahead of the splitter.
power-law form, the average observed spectrum may have a complex form that is unknown.

Our simulation code has the geographic location of the observing site and local sidereal time (LST) as free parameters. Our code can be easily modified to change the antenna beam pattern as well. For the purposes of testing the algorithms that follow, we simulate the sky spectrum over the Gauribidanur Observatory in southern India at latitude $+13^\circ01$, longitude $77^\circ58$ and at an elevation of $200$ m.\footnote{The mock spectra include atmospheric contribution adopting the correct elevation above sea-level for each observing site.} We choose such a near-equatorial location to maximize variation in the sky spectrum over $24$ hr and hence enable tests of robustness of fitting algorithms with different instances of the sky spectrum.

The generation of spectra takes into account effects of atmospheric refraction and precession; astronomical aberration is negligible even in its severest form for the problem considered here and is dropped from our calculations. Calibration is assumed to be done by recording spectra with a hot ($373.0$ K) and separately a cold ($273.0$ K) load on the antenna and dividing spectra recorded on the sky by the difference between spectra recorded with the hot and cold loads. Bandpass calibration of sufficiently high precision is assumed. Our code generates mock calibrated spectra over time whose mean temperature varies as the sky and Galactic plane drift across the telescope beam. A sample spectrum of the sky is shown in Fig. 3.7.

Fig. 3.8 shows a waterfall plot to illustrate the variation over $24$ hr in the recorded spectra observed by this ideal instrument as the sky drifts across the antenna. The synthetic sky spectra, whose amplitude is of the order of a few K, contains the recombination line spectrum as a small additive component and is representative of mock observations made with a correlation spectral radiometer that does not respond to receiver noise. The challenge is to distinguish the weak signal corresponding to the epoch of cosmological recombination, which is buried in the observed spectrum as a broadband quasi-periodic sinusoid with peak-to-peak amplitude of order $\sim 10$ nK. Not only is the signal a tiny fraction of the total sky spectrum, the recombination line spectrum is an additive component of a foreground spectrum whose functional form is complex and unknown. In the next section we discuss methods to detect the cosmological recombination spectrum and challenges therein.
3.5 The detection of signatures of cosmological recombination

In the previous section we have described the generation of a synthetic spectrum of the radio sky between 3 and 6 GHz, as would be observed by an ideal instrument. The signal from cosmological recombination is expected to appear as quasi-periodic ripples, some 9 orders of magnitude smaller than the Galactic and extragalactic foreground spectrum in which it is additively concealed. The discernment of the recombination signal in such a total spectrum, even under ideal conditions, is pivotal in answering the question of whether an experimental detection is indeed possible. Below we discuss the challenges involved in and describe a possible method for the detection of the cosmological recombination signal in synthetic sky spectra.

The brightness of the sky as measured by a total power radiometer is the sum of the brightness contributions from all the discrete and diffuse Galactic and extragalactic sources that lie within its beam, along the line of sight and across the sky, including the CMB and other cosmological emissions. The precision with which the foreground needs to be modeled so that a subtraction of the model might reveal the cosmological recombination spectrum is extreme. Although the discrete sources lying in the beam might have well-measured spectra and the diffuse sky radiation might have all-sky maps at multiple frequencies, the functional form of the final cumulative spectrum of the foreground is not known \textit{a priori} to the required accuracy. The foreground has necessarily to be fit to the measurement, which implies that the functional form used for the fit to the foreground needs to be of a form that accurately fits to the foreground without also fitting to the recombination spectral features. Only then may a fitting of a foreground model to the measurement set and its subsequent subtraction reveal the recombination line features in the residual.

The thermal and nonthermal processes (e.g., synchrotron and free-free emission) that contribute to the foregrounds have spectra that are smooth because they arise from impulsive emissions in time domain. The electron energy spectra in astrophysical objects and diffuse matter are also believed to be smooth over decades in energy space. Hence, the cumulative spectra in thermal and nonthermal emissions are expected to be featureless over octave bandwidths and therefore ought to be distinguishable in principle from the cosmological recombination radiation.

The problem of recovering broad and weak spectral deviations that are buried in a total sky spectrum that is several order of magnitudes brighter is not unique to cosmological
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Fig. 3.9 Residual obtained on subtracting a sample sky spectrum by an eight order polynomial fitting function in log-frequency and log-temperature space. While one would expect to see signatures of the spectral lines arising from cosmological recombination in the residual at the nK level, the residual is dominated only by thermal noise, with the lines themselves having been absorbed by the fitting function.

hydrogen recombination lines. All-sky (also referred to as global) spectral deviations predicted to arise from the EoR also face a similar challenge in that broad spectral features of about 10–100 mK brightness are buried in foregrounds of several hundred K, the foreground being \( \sim 5 \) orders of magnitude brighter (see Pritchard & Loeb, 2010b). Thus the importance of a method to effectively fit a smooth functional form to the combined Galactic and extragalactic foreground over 1-2 octaves of bandwidth, in which the signal of interest is itself not lost in the process, cannot be overstated. In the context of detecting global EoR signal in the all-sky radio background spectrum, a polynomial functional form of high order has been adopted in the literature as the analytic function to fit to the foreground (see Bowman & Rogers, 2010; Burns et al., 2012; Bernardi et al., 2015). However, broad spectral lines of interest that are present in observed spectra as additive components may also be absorbed in the polynomial fit, particularly if the polynomial is of high order, such that the residual would have little trace of the cosmological signal. As an illustration, on fitting a sample spectrum generated by our simulation with an eighth-order polynomial in log frequency versus log temperature space without any constraints on the
nature of the polynomial, and subtracting the fit spectrum from the original spectrum, we obtain a residual as shown in Fig. 3.9. While one might hope that the polynomial fit to the foreground would leave the recombination line signal untouched, comparison with Fig. 3.2 shows clearly that this is not the case. The ripples from cosmological recombination, the very signal that we are looking to detect, have been eliminated by the fitting process.

What is needed is a careful choice of the functional form that would leave the cosmological signals of interest almost wholly in the residue. Also needed is a method of successive approximation that would model the foreground to the required accuracy so that the faint cosmological signal might dominate the residue.

3.5.1 A complete monotone approach to foreground modeling

The CMB component has a well-defined functional form. The fitting function $f_{fg}(v)$ for the foreground may be defined to be an analytic function over the frequency range of interest, so that the model does not admit discontinuities and its derivatives are defined. The functional form describing the foreground model is expected to be ‘smooth’ in the sense that it must not be able to fit to additive signals that have the sinusoidal structure expected of the recombination spectrum.

A potentially useful functional form is that of a completely monotonic (CM) function. Mathematically, a function $f(x)$ is said to be a complete monotone if for all values of $x$ in the interval $0 \leq x < \infty$, $(-1)^n \times \frac{d^n f(x)}{dx^n} \geq 0$ for every integer $n \geq 0$. An example of a CM function is $f(x) = 1/(a+bx)^c$, where $a \geq 0$, $b \geq 0$, and $c \geq 0$. This implies that power-law form spectra with negative spectral indices are CM functions. It is also known that if $f(x)$ and $g(x)$ are CM, then $af(x) + bg(x)$ where $a$ and $b$ are nonnegative constants is also CM, which implies that the sum of power-law spectra with negative indices would also be CM. A function $f(x)$ may also be CM over finite range $a < x < b$, where $a \geq 0$ and $b \geq 0$: this would be the case if the condition $(-1)^n \times \frac{d^n f(x)}{dx^n} \geq 0$ for every integer $n \geq 0$ holds over the range $a < x < b$. The mathematical definition suggests that if $f_{fg}(v)$, where $v$ is frequency, is the foreground model over a certain bandwidth, then we may adopt a functional form for $f_{fg}(v)$ that is a complete monotone within the bandwidth of interest.

Successive approximation of an analytic functional form to data may be done using a Taylor approximation. The Taylor series of an analytic function always converges about every point in its domain. We may represent $f_{fg}(v)$ as a polynomial whose coefficients are constrained so that $f_{fg}(v)$ is CM and expand the function as a Taylor polynomial by successively estimating the coefficients of the polynomial, stopping when the degree of the polynomial is such that it is a sufficiently good fit to the data (i.e. the observed sky
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spectrum) and the residual is dominated by the embedded cosmological recombination lines. It may be noted here that increasing further the order of the polynomial, whose coefficients are constrained such that the polynomial is a complete monotone, will only yield vanishingly smaller coefficients for the higher-order terms. Since the constraint on the polynomial forces it to remain smooth, once the recombination lines dominate the residual, the residual is not minimized further by the introduction of higher-order terms. Thus, we converge to a smooth CM approximation to the foreground that leaves the recombination line spectral structure as a residual to the fit, which no longer changes significantly with increasing the order of the CM polynomial.

$f_{fg}(v)$ may be modeled as a CM polynomial in brightness temperature versus frequency space. More useful is a modeling in temperature versus log frequency space, where the number of terms in the Taylor approximation would be reduced. The Taylor expansion may be conveniently computed using the lowest frequency in the range of interest as the reference value.

A CM polynomial of arbitrary order $n$, whose constant term $a_0$ is left unconstrained so as to allow arbitrary vertical translations, may be written in the form

$$f(x) = a_0 + \sum_{i=1}^{n} (-1)^i (x-x_0)^i \{ \sum_{j=0}^{n-i} a_{i+j} C_j^i (x_m-x_0)^j \}, \quad (3.5)$$

where $C_k^n$ denotes the binomial coefficient $n! / \{ k!(n-k)! \}$. An example illustrating the algorithm to construct such a CM function and thereby arriving at the functional form presented in Equation 3.5 is described in Appendix A.

A disadvantage of adopting a CM functional form is that the data are fitted in brightness temperature versus log frequency space, where the foreground is expected to be CM. A functional form that is smooth in log temperature versus log frequency space is preferred, since a lower-order polynomial would be sufficient to describe the foreground. Lower numbers of parameters make for more robust fitting with less likelihood that optimization algorithms get trapped in local minima.

### 3.5.2 Modeling the foreground as a Maximally Smooth function

We now consider polynomial functional forms to describe the smooth foreground in log temperature versus log frequency space. The first approximation in this parameterization is a straight line representing the mean spectral index of the sky region. Curvature in the mean spectrum may be represented, to lowest order, by adopting a parabolic form for the spectrum in this log space. This form has a constant second-order derivative.
If we improve on the modeling of the spectral curvature by adding a cubic term to the polynomial, we may constrain the model to be smooth and without inflections by requiring that the second derivative has no zero crossings in the domain. In general, we may model the foreground by an $n^{th}$-order polynomial by imposing that all derivatives of order 2 and higher have no zero crossings within the domain of interest. This is implemented by computing the functions

$$\frac{d^mf(x)}{dx^m} = \{m!/0!\}p_m + \{(m+1)!/1!\}p_{m+1}(x-x_0) + \{(m+2)!/2!\}p_{m+2}(x-x_0)^2 + \{(m+3)!/3!\}p_{m+3}(x-x_0)^3 + \ldots + \{(n-m)!/(n-m)!\}p_n(x-x_0)^{n-m} \quad (3.6)$$

or

$$\frac{d^mf(x)}{dx^m} = \sum_{i=0}^{n-m} \{(m+i)!/i!\}p_{m+i}(x-x_0)^i \quad (3.7)$$

for all $m$ in the range 2, 3, 4, ..., $(n-1)$ and constraining the polynomial coefficients $p_j$ so that there are no zero crossings within the domain for any of these functions. We call the polynomial functions that satisfy these constraints Maximally Smooth functions. They will not have ripples embedded.

We model the foreground in log temperature versus log frequency space using a Taylor series expansion about the lowest frequency $\log_{10}(n_0)$ in our band. The polynomial is written in terms of powers of $\log_{10}(v/v_0)$. If $y(x)$ is the polynomial describing the foreground in log space, then $10^{y(x)}$ is added to a term that describes the CMB spectrum and another term that models the recombination line spectrum to get a model for the total spectrum.

We model the recombination line spectrum component in the mock data using a single scaling parameter: the recombination line component is considered to be this scale factor times a template of the spectral ripple that is nominally expected to be present in the observation. First, the scale factor is in itself a quantitative measure of the recombination template present in the total spectrum. A scale factor close to unity indicates that the predicted template is present in the spectrum, whereas a small value indicates the absence of such a spectral signature. The distribution allowed for this scaling parameter by the goodness of fit yields the confidence in the detection of the predicted recombination template. Second, modeling the recombination lines using a scaled template allows the residuals of the optimization to approach measurement noise if the theoretical framework that led to the template is correct.

The analytic fitting function in its final form, which includes a Maximally Smooth form
for the foreground modeling, is given by

\[ T(v) = \left( \frac{\hbar v}{k} \right) / \left( e^{\frac{\hbar v}{k T_0}} - 1 \right) + p_1 T_{\text{rec}}(v) + 10 \sum_{i=0}^{n} \left( \log_{10} \left( \frac{v}{v_0} \right) \right)^i p_{1+n}, \]  

(3.8)

where \( p_0 \) corresponds to the CMB temperature, \( p_1 \) is the scale factor that multiplies the recombination line template \( T_{\text{rec}}(v) \) and \( p_2 \) through \( p_{2+n} \) are the coefficients of the terms in the \( n^{\text{th}} \)-order Maximally Smooth polynomial that models the foreground. Fig. 3.2 shows the predicted recombination line signature that is expected to be present in the synthetic spectrum as an additive component; as stated earlier, this template has been derived from the predicted recombination line spectrum by subtracting a baseline of low order and is what would be expected as a residual if a smooth baseline were to be subtracted from an observation as a method of removing the foreground. Segments of this template are what form the recombination line template \( T_{\text{rec}}(v) \).

We use the downhill simplex (Nelder & Mead, 1965) optimization algorithm to iteratively fit this model to the synthetic spectrum adopting a successive approximation strategy. We start by fitting to a function that has four coefficients, one for the CMB temperature, one for the amplitude of the recombination line spectrum, and two that describe a first-order model for the foreground. We successively include more terms one by one, giving an initial guess of zero for each new coefficient.

Adopting the method discussed in Section 3, we have generated synthetic spectra over three octave bands — 2.0–4.0, 2.5–5.0 and 3.0–6.0 GHz — spaced uniformly over 24 hr in LST to test the robustness of this modeling algorithm. In this test, the noise in the spectrum was kept small so that the residual spectrum might reveal the recombination line structure obviously, if recovered successfully. As expected, the final fit parameters vary for different realizations of the sky spectrum corresponding to different observation times. Nevertheless, the algorithm did indeed converge for all LSTs and yielded a best-fit scaling factor close to unity at all times. An eighth-order Maximally Smooth polynomial was used to model the foreground. The residuals were consistent with the measurement noise that had been added to the synthetic spectra, and the foregrounds were indeed successfully modeled as eighth-order smooth polynomials, without also fitting to the embedded recombination line spectrum. In Fig. 3.10 we show a sample fit to a mock observation in the 3–6 GHz band. The model-fit CMB plus Maximally Smooth foreground is shown along with a second trace that is constructed by computing the residuals to this fit and adding the residuals to the model with a large rescaling. The plot shows that the model indeed does not fit to the recombination line ripple, which is present in the synthetic spectrum, and does leave the ripple as a residual. The model fitting algorithm based on the Maximally Smooth polynomials we have constructed is capable of separation of the foreground from
Fig. 3.10 Foreground model fit to a synthetic spectrum that represents a mock observation is shown in blue. The residual, scaled by a fraction of the inverse of the chi-square of fit and added back to the foreground model, is shown in red.

embedded ripple, which is nine orders of magnitude smaller. The residual alone is shown in Fig. 3.11, which can be compared with Fig. 3.9 where an eighth-order polynomial was used without any constraints to fit to the synthetic spectrum. We further test whether the foreground spectrum remains smooth to the $\sim 10$ nK level in the presence of Gigahertz Peaked Spectrum (GPS) sources (see O’Dea, 1998; Randall et al., 2011), whose spectral turnover in the gigahertz frequency range might introduce features in the sky spectrum, potentially limiting the ability of the Maximally Smooth function to fit to the foreground with the required accuracy. We introduce by hand a GPS source of variable peak flux density and with a turnover at the band center. Even in the presence of a GPS source with a peak flux density of 100 Jy we find that the final spectrum remains smooth to an extent that the residual on fitting the spectrum with a Maximally Smooth function of high order (order 7) is below the expected recombination signal strength of $\sim 10$ nK. With two GPS sources of varying peak flux densities over reasonable values (up to 10 Jy) and turning over at different frequencies in the 3–6 GHz band, we still find that the spectrum is sufficiently smooth such that the Maximally Smooth function continues to fit to the level of the cosmological recombination signal. This can be attributed to the broad turnover that is practically smooth over an octave bandwidth.

In Fig. 3.12 we show the residuals to the fit for a smooth foreground plus CMB to synthetic spectra for mock observations in different frequency ranges and distributed over LST. The algorithm does recover the recombination ripple in all cases, giving confidence in the
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Fig. 3.11 Residual on subtracting the foreground plus CMB model-fit from the synthetic spectrum.\textsuperscript{11}

Fig. 3.12 Residuals in the 2.0–4.0, 2.5–5.0 and 3–6 GHz bands following fitting and subtracting Maximally Smooth foreground models to synthetic sky spectra. The mock observations are over a 24 hr period, with spectra spaced 2 hr apart, and represent observations made by an ideal instrument as discussed in the text. Shown as a red dotted line is the recombination line template from the theoretical predictions; the overlaid colored dashed lines represent the recovered residuals from different realizations of synthetic sky spectra.

These residuals represent a detection of the cosmological recombination lines. The recovery demonstrates that the smooth functional form proposed here does not also fit to the nanoscale embedded ripple representing the recombination line signature, despite the functional form being allowed to be of arbitrarily high order. The recovery demonstrates

\textsuperscript{11}Note: The signal in red in Fig. 3.10 is the residual scaled by a fraction of the inverse of the chi-square of fit and and added back to the foreground model. This results in the orders of magnitude difference in the y-axis scales of Fig. 3.11 and Fig. 3.10.
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that it is indeed possible to model fit a smooth functional form (e.g., the Maximally Smooth functional) to observations and model the CM foreground—of unknown functional form—to an accuracy better than 10 nK!

3.6 Confidence in Detection

In the previous section, we presented an algorithm by which we recover cosmological recombination lines of amplitude $\sim 10$ nK that are a tiny additive part of a sky spectrum that is nine orders of magnitude brighter. In this section, we first examine the probability distribution for the detected amplitude of the spectral ripple. We then examine the confidence in detection of the expected ripple and the confidence with which we may reject false positives, using a Bayes factor (BF) approach. These lead to inference of the observing time needed for detections with different confidence levels.

3.6.1 Detection based on the fit amplitude of the recombination spectral ripple

An approach to determining whether or not a given sky spectrum contains cosmological hydrogen recombination lines is to jointly fit to the spectrum a model composed of a Planck spectral component for the CMB, a Maximally Smooth polynomial accounting for radiation from point and extended sources in the foreground, and the recombination line template scaled by a factor, which is a variable for the fitting. The functional form for such a model is given by Equation 3.8.

We expect that the fit value of $p_1$, which represents the normalized amplitude of the recombination line ripple, would take on a value of unity if the theory is valid and a value of zero if no lines exist. We choose 0.5 as the threshold between a null detection and a positive detection, with values smaller than 0.5 assumed to indicate a null detection and greater than 0.5 deemed to be indicating a positive detection.

To determine the confidence in detection using such an approach, as well as the probability of a false positive, we synthesize two types of mock observations, one in which the theoretically expected cosmological recombination lines are added and another without. Forty spectra of each type were generated with independent thermal noise, whose level is given by Equation 3.4 for different signal-to-noise ratios. Below we adopt the notation that data and associated terms for spectra with recombination lines present in them would be referred to as data set ‘a’ and those without recombination lines as ‘b.’ We fit each of the synthetic sky spectra separately with the mathematical model given by Equation 3.8 using the successive approximation approach, optimizing all the parameters using the Nelder & Mead (1965) algorithm to minimize the chi-squared difference between the model
and mock data, each time increasing the degree of the Maximally Smooth polynomial in Equation 3.8 by unity until the root mean square of the residual saturates.

To sample the distribution in scaling factor \( p_1 \) and thus derive the confidence for detection using a threshold of 0.5 for the scaling factor, we adopt a Markov Chain Monte Carlo (MCMC) analysis. Specifically, we adopt the EMCEE package (Foreman-Mackey et al., 2013), which is a Python-scripting-language implementation of an affine invariant ensemble sampler as proposed by Goodman & Weare (2010). The EMCEE package wins over traditional implementation of MCMC samplers in its high computational efficiency in generating statistically independent samples from the posterior probability distribution function (PDF). Another major advantage of EMCEE is that it requires tuning of only a couple of parameters by hand in an \( N \)-dimensional space as opposed to \( N^2 \) values in other traditional methods. While data-driven algorithms do exist to arrive at optimum guesses for initial parameters, for conventional methods this comes at the computational cost of lengthy burn-in chains.

Chi-squared minimization of the model \( T(v) \) leads to optimum values for the parameters \( p_0 \) representing the CMB temperature, \( p_1 \) representing the scaling factor, and the polynomial coefficients \( p_2, p_3, p_4, p_5 \) and \( p_6 \) (for a fourth-order Maximally Smooth polynomial). We then invoke the EMCEE sampler with 30 walkers, each of which generates a Markov sampling chain for every parameter. The different chains are initialized at random locations close to the optimum location in the \( N = 7 \) dimensional parameter space. The EMCEE ensemble sampler operates on the log posterior probability distribution function, which we define to be

\[
\ln(\text{Probability}) = \frac{1}{2} \sum \left( \frac{(T(v) - y_{a|b})^2}{\sigma^2} + \ln(2\pi\sigma^2) \right),
\]

(3.9)

where \( y_{a|b} \) is the synthetic sky spectrum with or without recombination lines.

We first run a short burn-in of 1000 steps, discard these values, and run a second burn-in of 6000 steps. We ensure that the length of the second burn-in is at least 10 times the correlation length, and then we discard these as well and run a final useful 500 steps. With 30 walkers exploring 500 steps each, we generate a distribution of 15,000 samples per parameter per synthetic sky spectrum. Marginalizing over the nuisance parameters, which in our case are all parameters with the exception of the scaling factor, we have for 40 sky spectra a distribution of 600,000 scaling factors in all. On running the EMCEE ensemble sampler on data sets ‘a’ and ‘b’ as described above, we have two distributions of scaling factors with 600,000 samples each.
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We generate multiple pairs of data sets ‘a’ and ‘b’ with different noise integration times; as a guide we have adopted times corresponding to different signal-to-noise ratios as defined by Equation 3.4. For each integration time, the fraction of samples of the scaling factor from the Markov chain corresponding to data set ‘a’ that have values exceeding the threshold of 0.5 gives an estimate of the probability of detection. Similarly, the fraction of samples corresponding to data set ‘b’ below 0.5 is an estimate of the probability of rejecting a false positive. These are depicted in Fig. 3.13. Detection of the recombination ripple

![Graph](https://via.placeholder.com/150)

Fig. 3.13 Confidence with which scaling factor (SF) above threshold 0.5 signifies a detection of cosmological recombination lines (shown as a blue solid line with scale on the left of the plot), and percentage likelihood of false positives appearing as a detection for this threshold (shown as a red dotted line with scale on the right side of the plot). The confidence values and percentage likelihoods of false positives are shown vs. integration time assuming cryogenically cooled state-of-the-art 1 K receivers and the co-addition of spectra from an array of 128 independent total-power spectrometers.

with 68% confidence using uncooled receivers requires observing with about $210 \times 10^8$ antenna s, or equivalently about 240000 antenna days. An array of 128 singly polarized\textsuperscript{12} total-power spectrometer elements with uncooled receivers would require about $\sim 5$ yr for detection with this confidence and based on SF threshold. However, using cryogenically cooled state-of-the-art receivers, the observing time for detection with 68% confidence is $\sim 10 \times 10^8$ antenna s or, equivalently 92 observing days with an array of 128 spectral radiometers deploying such cooled receivers. A 90% confidence detection requires observing with 430 days with such an array, and 95% confidence detection requires increasing the observing time for such an array to about 625 days.

\textsuperscript{12}The use of a dual-polarized systems will theoretically improve signal-to-noise ratio and thus reduce the integration time by a factor of $\sqrt{2}$. 
3.6.2 Detection based on BF

Consider some data $D$ and a choice between model $M_1$ characterised by a set of parameters $\theta_1$ and $M_2$ characterized by a set of parameters $\theta_2$, to describe the data. The Bayes factor is given by the ratio of the probability of the data $D$ given the model $M_1$ to the probability of $D$ given $M_2$:

$$BF = \frac{P(D|M_1)}{P(D|M_2)} = \frac{\int P(\theta_1|M_1)P(D|\theta_1,M_1)d\theta_1}{\int P(\theta_2|M_2)P(D|\theta_2,M_2)d\theta_2}. \tag{3.10}$$

An alternative expression of the BF is in terms of posterior and prior odds. Rewriting the probabilities associated with the two models as odds, we obtain

$$\text{Posterior odds} = BF \times \text{Prior odds},$$

or that:

$$\frac{P(M_1|D)}{P(M_2|D)} = BF \times \frac{P(M_1)}{P(M_2)}, \tag{3.11}$$

which leads to:

$$BF = \frac{P(M_1|D)}{P(M_2|D)} \times \frac{P(M_2)}{P(M_1)}.$$

When the prior odds are 1:1 the BF reduces to the likelihood ratio.

We once again adopt the notation used in Section 3.6.1 so that symbols representing data and associated terms for mock observations with the recombination lines present in them would be given subscript ‘a’ and those without recombination lines ‘b.’ Terms representing the null hypothesis would have subscript ‘0’ and alternative hypothesis ‘2.’ In our problem of estimating the confidence in the detection of recombination line ripples in observed spectra, we consider two hypotheses. The first hypothesis is that the spectrum does not contain recombination lines of any detectable amplitude, i.e., within errors the mock spectrum can be adequately modeled by the total of contributions from the CMB and a Maximally Smooth function representing the cumulative spectra from diffuse and point sources in the foreground. We refer to this as the null hypothesis $H_0$. The second hypothesis, which we refer to as the alternative hypothesis $H_2$, is that the observed sky spectrum includes the cosmological hydrogen recombination lines as an additive component along with contributions from the CMB and foregrounds. Hypothesis $H_2$ is that the spectrum in question does contain the recombination lines exactly as given by the theoretical expectations. The BF for a comparison between these alternate hypotheses is given by

$$BF = \frac{P(D|H_2)}{P(D|H_0)} \times \frac{P(H_2)}{P(H_0)}, \tag{3.12}$$

where $P(D|H_2)$ and $P(D|H_0)$ are the respective likelihoods of the alternative and null hypotheses and $P(H_2)$ and $P(H_0)$ are the prior probabilities of the alternative and null hypotheses. Since in this comparison we have no a priori preference or bias towards
On the detection of the CMB spectral distortions from the Epoch of Recombination

the presence or absence of the recombination ripple in data, we may assume equal prior probabilities, and hence the BF reduces to the likelihood ratio given by

\[ BF = \frac{P(D|H_2)}{P(D|H_0)}. \]  

(3.13)

We now proceed to compute the likelihood functions for the above two hypotheses. We generate data set ‘a’, a set of 100 mock observations with independent thermal noise and with each containing the recombination lines as an additive component. We then fit each of these spectra independently with two different models. The first model is the null hypothesis \( H_0 \) with a blackbody function to model the CMB component and a Maximally Smooth polynomial to model the foreground component of the sky spectrum. \( H_0 \) expects that these two components completely describe the spectrum and that the residual on subtracting this model fit from the data must be Gaussian random with zero mean and standard deviation given by the measurement noise. We compute the likelihood of obtaining the data given \( H_0 \) by

\[ P(D_a|H_0) = \prod_{i=1}^{N} e^{-\frac{(y_{\text{res0}}[i])^2}{2\sigma_0^2}} \sqrt{2\pi\sigma_0^2}, \]  

(3.14)

where \( N \) is the number of independent points across the spectrum and \( y_{\text{res0}}[i] \) is the residual spectrum following subtraction of the model corresponding to the null hypothesis.

The variance of the measurement noise, \( \sigma_0^2 \), is estimated from the data themselves. This variance is assumed to be half the Allan variance (AV) of the residual, which is given by

\[ AV = \sum_{i=1}^{N-1} \frac{(y_{\text{res}}[i+1] - y_{\text{res}}[i])^2}{i}, \]  

(3.15)

where \( y_{\text{res}} \) is the residual on subtracting the model from the data. Since in our cases the signal-to-noise ratio is small in the channel data, this approach makes the estimate for measurement error robust and independent of errors in the model fit and any low-order residuals that are not represented in the model. The second model corresponds to the alternative hypothesis \( H_2 \). This model contains the blackbody CMB term, the Maximally Smooth polynomial representing the foreground, and, in addition, a template of the recombination lines that are expected to be present in the spectra for this hypothesis. As before, we proceed by fitting the data with the model, subtracting the best-fit model from the data, and computing the likelihood from the residual. We again make the reasonable assumption that if the data are completely represented by the model, then the residual should be Gaussian random noise corresponding to measurement noise. The likelihood
3.6 Confidence in Detection

Fig. 3.14 BFs computed for data sets ‘a’ and ‘b’ that are mock observations generated with and without the recombination line component, respectively. $\log_{10}(BF)$ is plotted vs. observing time assuming cryogenic cooled state-of-the-art 1 K receiver noise temperatures and a 128-element array of precision total-power spectrometers. Filled circles represent the median values of the BFs computed separately for the two data sets. The horizontally striped region shaded in red represents the range in BFs obtained for the 100 mock observations in data set ‘a,’ and the vertically striped region shaded in green represents the range of BFs obtained for data set ‘b.’

Function $P(D_a|H_2)$ is given by

$$P(D_a|H_2) = \prod_{i=1}^{N} e^{-\frac{y_{res2[i]}^2}{2\sigma_2^2}} \sqrt{2\pi\sigma_2^2}, \quad (3.16)$$

where $y_{res2[i]}$ is the residual on subtracting the alternative hypothesis model from the data, and variance $\sigma_2^2$ is half the AV of this residual. Since the AV gives the measurement noise independent of the model used to fit to the data, we expect $\sigma_0^2$ and $\sigma_2^2$ to be the same.

Having computed the likelihoods for both models, we arrive at the BF by computing the likelihood ratio:

$$BF_a = \frac{P(D_a|H_2)}{P(D_a|H_0)}. \quad (3.17)$$

We repeat this exercise with a separate data set ‘b,’ which is a set of 100 spectra corresponding to mock observations with the same measurement noise variance as in data set ‘a,’ but with no recombination lines in the synthetic spectra. We once again fit these spectra with the two models as before. We compute the likelihood functions for the null and
Fig. 3.15 Confidence with which BF above unity might signify a detection and simultaneously reject the detection to be a false positive, vs. integration time. We assume cryogenically cooled state-of-the-art 1 K receiver noise temperatures and a 128-element array of precision total-power spectrometers.

alternative hypotheses given by $P(D_b|H_0)$ and $P(D_b|H_2)$ respectively, under the reasonable assumption that when a model that completely represents the data, except for random noise, is subtracted from the data, the residual must be the measurement noise. Thus, the BF for the 100 mock observations that are generated assuming that the recombination line component is absent is given by

$$BF_b = \frac{P(D_b|H_2)}{P(D_b|H_0)}.$$  \hspace{1cm} (3.18)

Fig. 3.14 shows a plot of $\log_{10}(BF_a)$ and $\log_{10}(BF_b)$ versus integration time. With increasing integration time the measurement noise reduces and the two BFs diverge, as expected.

For any data set, the BF indicates the relative preference of the data for the two hypotheses. The median $BF_a$ rises above unity with increasing integration time, increasingly preferring the hypothesis $H_2$ over $H_0$. In contrast, the median $BF_b$ drops below unity with increasing integration time, increasingly preferring the hypothesis that the recombination lines are absent in data set ‘b.’ We may set a threshold at unity and examine the confidence with which this threshold might discriminate between observations that contain a recombination ripple and those in which this additive component is absent. At any integration time, the distribution of BFs indicates the probability that $BF_a$ is above unity, which gives the confidence in a detection. For the same integration time, the distribution of $BF_b$ yields the
probability of obtaining a value above unity, which gives the likelihood of a false positive. Using the multiple data sets ‘a’ and ‘b,’ we have computed the run of the confidence in detection (and simultaneous rejection of false positives) versus integration time: this is depicted in Fig. 3.15.

If we adopt the Bayes factor as a detection statistic, detection of the recombination ripple with 68% confidence using uncooled receivers requires observing with about $52 \times 10^8$ antenna secs, or equivalently about 60,000 antenna days. An array of 128 singly-polarized total-power spectrometer elements with uncooled receivers would require about 1.3 yr for detection with this confidence. Using cryogenically cooled state-of-the-art receivers, the observing time for such detection with 68% confidence is $2.56 \times 10^8$ antenna s or equivalently, 23 observing days with an array of 128 spectral radiometers deploying such cooled receivers. A 90% confidence detection requires observing with 255 days with such an array, and a 95% confidence detection requires about 440 days.

Fig. 3.16 shows the confidence in the detection of cosmological recombination lines in synthetic sky spectra as a function of integration time, estimated using both the BF method and using the fit to the amplitude of the recombination ripple (Section 3.6.1). The BF approach to detection is simplest in that it only asks whether in an observed spectrum the lines as predicted by theory are present or totally absent. The method that derives an amplitude for the ripple, as estimated by a best fit to a scaling factor for a template of the expected ripple, attempts to ask a more informative question in that it attempts to evaluate the amplitude of the recombination line ripple. Unsurprisingly, the BF method has greater confidence in its answer for any integration time.

3.7 Summary

We demonstrated that it is, in principle\textsuperscript{13}, feasible with present-day technology to experimentally detect the cosmological hydrogen and helium recombination lines at low frequencies, although the lines are embedded in a foreground that is about nine orders of magnitude brighter with \textit{a priori} unknown precise spectral shape\textsuperscript{14}. The recombination radiation has a smooth component that is difficult to distinguish from the smooth foregrounds; however, the recombination radiation also has a unique ripply component that may be distinguished from the foregrounds and instrumental effect. We estimate the amplitude of the ripply signal, the instrument noise arising from receivers and foregrounds, and estimate the signal-to-noise ratio in detections using ground-based spectrometers.

\textsuperscript{13}\textit{i.e.}, with an ideal system
\textsuperscript{14}The foreground has an \textit{a priori} unknown precise spectral shape.
Fig. 3.16 Comparison of the confidence in detecting cosmological recombination lines in sky spectra using BFs and that given by a comparison of the best-fit ripple amplitude to a threshold. The confidence values are shown vs. integration time assuming cryogenically cooled state-of-the-art 1 K receiver noise temperatures and a 128-element array of precision total-power spectrometers.

Detection may ideally be attempted using an octave band in the 2–6 GHz window; an octave band would have a spectral segment of the recombination ripple with sufficient structural complexity so as to be distinguishable from the relatively smoother foreground. We have developed an algorithm to detect the recombination line ripple by foreground modeling that models the foreground as a *Maximally Smooth* polynomial, which we define enforcing no zero crossings in derivatives of order $n \geq 2$. A similar approach may be applicable to modeling and detecting the global 21-cm signal at lower frequencies.

We then evaluate the confidence with which these cosmological recombination lines may be isolated using the aforementioned algorithm and estimate the integration times required for detection with varying degrees of confidence. In its simplest form, a detection is testing an observed spectral segment addressing the question of whether or not the theoretically motivated spectral ripple is present in the sky spectrum or absent. To answer this question with 90% confidence requires an integration time of 32,640 antenna days with a total-power spectral radiometer with cryogenically cooled state-of-the-art receivers; however, with today’s best uncooled receivers the integration time for such a detection increases to as much as 660,000 antenna days. This makes a compelling case for the use of cryogenically cooled receivers. Moreover, since the antenna element for such all-sky global signals does not gain from antenna directivity, very small antenna elements of centimeter dimensions may be deployed, perhaps as a compact cluster of antenna elements housed in
3.8 Future Investigations

Ripple-like spectral signatures arising from the epochs of hydrogen and helium recombination are predicted to appear as additive distortions to the sky spectrum at a level $\sim 9$ orders of magnitude weaker than the total brightness of the radio sky. We have demonstrated that it is in principle possible to detect these signals. Owing to their small amplitude, any small departure in the spectral radiometer from the ideal behavior assumed herein could critically affect the detection likelihood. Studies of the effects of potential instrumental non-idealities and confusion arising from additive astrophysical contaminants, which have not been considered here, will form the next step in our future work. The key investigations are listed below.

1. The spectral radiometer radio telescope used to detect the weak cosmological recombination lines must be capable of achieving the required spurious-free dynamic range so as to recover the embedded cosmological signal in the measurement. This implies that the receiver configuration and calibration techniques must be specifically designed to minimize systematics and that any residual systematics are either well characterized or emendable to accurate modeling so as to enable unambiguous detection of the weak signal. A future work is the design of a suitable receiver configuration, observing strategies, gain stability, and calibration methods to achieve the required accuracy in the measurement.

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15 www.rri.res.in/DISTORTION
16 http://www.rri.res.in/
2. In our simulations, we have generated mock sky spectra as observed by an ideal frequency-independent antenna beam. The spectral response of such an antenna is *maximally smooth* if all sources in the beam have spectra that are complete monotones. However, designing and fabricating an antenna that is frequency independent over an octave bandwidth is nontrivial. Frequency dependence in the beam pattern and its sidelobes may result in a response that is no longer smooth and may confuse the detection of cosmological recombination. A study of this mode coupling of sky structure into spectral structure in a spectral radiometer, which depends on the type of frequency dependence in the beam pattern, would lead to design tolerances on the antenna element for this detection experiment.

3. The all-sky ‘ripply’ signal arising from the epochs of hydrogen and helium recombination is inherently unpolarized, and this spectral feature may be detected with an antenna that responds to any single polarization mode, either linear or circular. However, the foreground synchrotron emission from extragalactic sources as well as Galactic emission, is linearly polarized, and Faraday rotation during the line-of-sight propagation results in the received polarization position angle varying with observing frequency. This causes linearly polarized sources to appear with a ‘ripply’ spectral structure in the response of linearly polarized antennas. One possible way of eliminating beam asymmetries and the effect of beam rotation across the sky is to rotate the array itself. A study of this mode coupling of polarized sky emission to spectral structure is a future study that will lead to design tolerances on the polarization properties of the antenna element and on the polarization calibration. The potential of using the unpolarized nature of the cosmological signal to discriminate it from polarized foregrounds is another aspect to be explored.

4. The spectral template of the additive ripple-like feature from cosmological hydrogen and helium recombination has a fairly accurate theoretical prediction. The near quasi-sinusoidal nature of this signal acts as a fingerprint providing a means to distinguish it from other weak cosmological signals. Two such cosmological signals are the $\mu$- and $y$-distortions of the CMB (Zeldovich & Sunyaev, 1969; Sunyaev & Zeldovich, 1970). Even within the standard cosmological model, these distortions are created at amplitudes that are typically larger than the cosmological recombination radiation (e.g., Chluba & Sunyaev, 2012; Sunyaev & Khatri, 2013; Chluba, 2013; P. Andre et al., 2014). A large average Compton $y$-distortion, at the level of $y \approx 10^{-7} - 10^{-6}$, is expected from reionization and structure formation (Sunyaev & Zeldovich, 1972; Hu et al., 1994; Cen & Ostriker, 1999; Oh et al., 2003) and unresolved clusters and filaments (Miniati et al., 2000; Refregier et al., 2000; Zhang et al., 2004). At low

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17A circularly polarized antenna is truly circularly polarized only along the boresight of the antenna. A dual polarization antenna with full polarization information is more useful for foreground separation.
frequencies, this type of distortion only leads to a weak frequency-dependent signal, 
\[ T(v) \approx -2y(1 - x^2/12) \] with \( x \approx 0.018 \, [\text{GHz}] \), making it less problematic. For a \( \mu \)-distortion, the low-frequency spectrum shows much richer structure, especially when varying time dependence of the energy release mechanism (Fig. 14 of Chluba & Sunyaev, 2012). Each of these introduces spectral features in the CMB, with the \( \mu \)-distortion peaking at about 1 GHz. In an experiment optimized to detect ripples originating from the epochs of hydrogen and helium recombination, it would be interesting to study whether \( \mu \)- and \( y \)-distortions of the CMB are a possible source of confusion or would themselves be detected as a positive by-product. Similarly, the anomalous microwave emission from spinning dust (see Draine & Lazarian, 1998; Ali-Haïmoud et al., 2009; Planck Collaboration et al., 2014) will add another spectral dependence to the low-frequency regime, which at this point we have not investigated.

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Chapter 4

GMOSS: Global Model for the Radio Sky Spectrum

This chapter has previously been published as "GMOSS: All-sky model of spectral radio brightness based on physical components and associated radiative processes", Sathyanarayana Rao et al., 2017, AJ, 153, 26. Some additional comments that are not present in the paper are included as footnotes in the chapter.

4.1 Abstract

We present the Global Model for the Radio Sky Spectrum (GMOSS), a novel, physically motivated model of the low-frequency radio sky from 22 MHz to 23 GHz. GMOSS invokes different physical components and associated radiative processes to describe the sky spectrum over 3072 pixels of $5^\circ$ resolution. The spectra are allowed to be convex, concave or of more complex form with contributions from synchrotron emission, thermal emission, and free-free absorption included. Physical parameters that describe the model are optimized to best fit four all-sky maps at 150 MHz, 408 MHz, 1420 MHz, and 23 GHz and two maps at 22 and 45 MHz generated using the Global Sky Model of de Oliveira-Costa et al. (2008). The fractional deviation of the model from data has a median value of 6% and is less than 17% for 99% of the pixels. Though aimed at the modeling of foregrounds for the global signal arising from the redshifted 21-cm line of hydrogen during the Cosmic Dawn and the Epoch of Reionization (EoR), over redshifts $150 < z < 6$, GMOSS is well suited for any application that requires simulating spectra of the low-frequency radio sky as would be observed by the beam of any instrument. The complexity in spectral structure that naturally arises from the underlying physics of the model provides a useful expectation for departures from smoothness in EoR foreground spectra and hence may guide the development of algorithms for EoR signal detection. This aspect is further explored in a subsequent paper.
4.2 Introduction

Interest in the low-frequency radio sky goes back to the very early days of radio astronomy. Early measurements at 20.5 MHz by Karl Jansky showed that the Galaxy itself is a strong emitter of radiation at low frequencies. The source of such Galactic emission was poorly understood and was attributed to a wide variety of phenomena, right from radio stars to dust grains. It is now understood that the predominant radiative mechanism contributing to this Galactic emission at long wavelengths is synchrotron radiation from relativistic electrons spiraling around Galactic magnetic field lines. With many experiments currently underway attempting to detect redshifted 21-cm spectral signatures arising from the Cosmic Dawn and the Epoch of Reionization (EoR), there is renewed interest in understanding the precise spectral shapes of the low-frequency radio sky, which forms a strong foreground to the weak cosmological signal.

The radio sky is composed of signals from our Galaxy, extragalactic radio sources, and the cosmic microwave background (CMB), with added spectral distortions related to the cosmic thermal history of baryons, structure formation, energy release in the early universe, and interactions between propagating radiation and gas. These include distortions from Cosmic Dawn and the Epoch of Reionization. The emergence of the first sources in the Cosmic Dawn and the transition of baryonic matter in the Universe from being almost completely neutral to its present mostly ionized form during EoR is an interesting and poorly constrained period in cosmology. During these times the cosmological evolution of the spin temperature of hydrogen and reionization as a result of first light from the first collapsed objects is expected to leave an imprint as redshifted 21 cm emission and absorption (see Madau et al., 1997; Shaver et al., 1999). These 21-cm spectral distortions are a probe of the thermal history of the gas and also of the sources and timing of reionization (Glover et al., 2014) in the redshift range of 6 to about 150. For a comprehensive review of the subject, see Furlanetto et al. (2006). There are global, all-sky isotropic spectral features as well as angular variations in spectral structure, embedded as tiny additive components in the radio background at frequencies $\lesssim 200$ MHz. Radio emission from Galactic and extragalactic sources forms strong foregrounds to the cosmological signal and are orders of magnitude brighter.

It is necessary to have a realistic expectation for the radio foreground that would be observed by EoR detection experiments (see Bowman et al., 2008; Patra et al., 2013; Pober et al., 2014, 2015; Voytek et al., 2014; Ali et al., 2015; Bernardi et al., 2015). Although the sky spectrum as measured by individual experiments will be instrument-specific, a generic model representative of the spectral distribution of intensities in the low-frequency radio sky and a method to simulate the expected contribution of the same to spectra observed in
4.3 Motivation

EoR detection telescopes will be a powerful tool in formulating data analysis methods.

Here we present a physically motivated model of the low-frequency radio sky: the Global Model for the Radio Sky Spectrum (GMOSS). GMOSS is a generic model and can be used to generate spectra of the low-frequency radio sky for other applications as well. One such application is to simulate the expected foregrounds for other spectral distortions of the CMB, such as those arising from the Epoch of Recombination (Sunyaev & Chluba, 2009b). As noted in Sathyanarayana Rao et al. (2015), the optimal frequency for a ground-based detection of cosmological recombination lines is an octave band in the range of 2–6 GHz. Over these frequencies the recombination signal is expected to have a quasi-periodic sinusoidal shape, whereas the foregrounds are expected to be smooth. However, since the cosmological recombination signal is at least eight orders of magnitude weaker than the foreground, a thorough treatment of the expectation of the spectral shapes inherent in the foreground can be provided by GMOSS.

The motivation for GMOSS in comparison to existing sky models is given in Section 5.3. GMOSS itself is described in Section 5.4, and a discussion on the distribution of parameters and the goodness of fit is presented in Section 4.5.

4.3 Motivation

Precise measurement of foreground spectra is a goal of experiments aiming to detect redshifted 21-cm signatures from the Cosmic Dawn and EoR (hereafter referred to in totality as the EoR). Disentangling spectral structure specific to the EoR signal from those of the foreground (Harker, 2015) and instrument (Switzer & Liu, 2014) is a challenge. In generic models (such as Figure 1 of Pritchard & Loeb, 2010a), the redshifted 21-cm signal (in the 10–200 MHz window) is expected to show multiple turning points arising from various physical processes, such as X-ray and UV heating, ionization of the gas, and Wouthuysen-Field coupling of spin to kinetic temperature via Lyman-α to name a few (for a description of the physics that determine the turning points in the EoR signal, see Pritchard & Loeb, 2010b). On the other hand, it is assumed that over the same frequency range, foregrounds are smooth (see Petrovic & Oh, 2011) and that ‘smoothness’ is captured by low-order polynomials in log brightness temperature versus log frequency space (we hereinafter refer to this domain as log-log space). It has been proposed to exploit this supposed smoothness of foregrounds to distinguish them from the global EoR signal, which in the generic form is expected to show multiple turning points between 10 and 200 MHz. However, it is uncertain if this assumption of an inherently smooth foreground is indeed correct. For instance, a mechanism that might result in inflections of the observed
spectrum is the combined emission from steep and flat spectrum sources along with radiation from sources that have a break in the electron energy distribution. Furthermore, flattening of this combined spectrum due to absorption at low frequencies by a thermal interstellar medium along the line of sight and at high frequencies by free-free emission can introduce additional shapes in the spectrum. Though the assumption is that averaging of multiple spectra of various shapes across the sky and along the line of sight must result in an observed spectrum that is devoid of sharp spectral features, the underlying spectral energy distribution (SED) that results in the radiation would ultimately determine the inherent smoothness of the observed spectrum. An interpolating function that is guided by physical processes provides a non-trivial treatment of the shape of the foreground spectrum.

Waelkens et al. (2009) present a tool to simulate maps of the total and polarized synchrotron emission of the radio sky, including effects of Faraday rotation. Other popular sky models, such as the Global Sky Model (GSM; de Oliveira-Costa et al. (2008) with recent improvements made by Zheng et al. (2016)) use data-driven methods to generate snapshots of the sky at frequencies between those where large-area maps are available. This is useful in creating all-sky maps at discrete frequencies and may also be used to generate spectra in any sky direction by computing the sky brightness over a contiguous range of frequencies. It is estimated that for sub-GHz frequencies predicted maps from the GSM are at worst in error by \( \lesssim 10\% \) depending on the region of the sky. To date, in the literature, mock spectra have been generated by first computing the beam-weighted temperatures from maps at discrete frequencies and then interpolating with either power laws or low-order polynomials to simulate EoR foregrounds. For instance, Pritchard & Loeb (2010b) generate a mock spectrum by allowing for the GSM-generated sky to drift over the zenith of an ideal frequency-independent \( \cos^2 \) antenna beam for 24 hr. They find that their mock spectra can be fit to mK precision with a cubic polynomial in log-log space. Independently, Bernardi et al. (2015) find that to fit the spectrum of synchrotron emission from mono-energetic cosmic ray electrons to within 100 mK maximum errors, a polynomial of the sixth order is required in log-log space. For the case of synchrotron emission from an evolved population of cosmic ray electrons diffusing through the Galactic halo, the order of the polynomial required reduces to four. Harker et al. (2016) produce mock sky spectra that contain foregrounds generated using polynomial interpolation and fit them with polynomials of similar order to remove the foregrounds, eliminating any ambiguity arising from using polynomials of a different order. While this helps in focusing on statistical inference methods for the parameters of the cosmological signal itself, it remains to be examined what the precise shape of the foreground spectrum might be and to which extent it might confuse detection of the 21-cm signal from EoR.

What is required is a model of the foreground which encapsulates the underlying physics
that gives rise to the foreground spectrum. Whether or not the resultant spectrum is smooth would guide the strategies employed to detect the EoR signal that is embedded in a measurement set. If polynomials are used to model the foreground spectrum, the order of the polynomial required to fit the foreground with the precision required for detection of the EoR signal should be guided by the spectral complexity determined by the underlying physical radiative processes.

In GMOSS we present a physical sky model that includes the effects of plausible radiative processes arising from different components of the radio sky. Additionally, the physical parameters used to describe GMOSS are guided by all-sky maps at different frequencies, thus enabling a check on the goodness of the model.

4.4 GMOSS : A spatially resolved spectral model for the radio sky

Previous efforts to determine the complexity of EoR foregrounds have usually simulated sky spectra by interpolating with polynomials between brightness temperatures that were first computed for large instrument beam widths (see Harker, 2015). However, even if the spectra of individual sources that lie in the beam are of power-law form, the summation of many such spectra with a distribution in spectral indices across the beam and along the line of sight would result in an observed spectrum that would no longer be a power law but something spectrally more complex. Thus, there exists a need to generate a generic sky-model at higher resolutions than typical single antenna beams, which may then be convolved with a large antenna beam to generate mock spectra that are qualitatively more representative of the cumulative emission. GMOSS provides such a sky model wherein plausible physical processes are used to estimate the sky spectrum toward each direction. Furthermore, the parameters describing the spectral shape toward each sky pixel and hence describing the sky model are constrained by existing all-sky maps.

We use 4 available all-sky maps at 150 MHz (Landecker & Wielebinski, 1970), 408 MHz (Haslam et al., 1982), 1420 MHz (Reich, 1982; Reich & Reich, 1986; Górski et al., 2005), and 23 GHz (WMAP science data product \(^1\)) to generate GMOSS. Additionally, we also use all-sky maps at 22 and 45 MHz generated using the GSM (de Oliveira-Costa et al., 2008). These last two all-sky maps are expected to closely match the raw data maps that were inputs to the GSM at the corresponding frequencies. The CMB monopole temperature, when present, was subtracted from the maps. All the maps were reduced to

\(^1\)WMAP Science Team.
a common resolution of $5^\circ$ and represented with the ‘R4’ nested HEALPix\(^2\) scheme in galactic coordinates. Additionally, the temperature scale of the 150 MHz all-sky map of Landecker & Wielebinski (1970) was corrected by subtracting an offset of 21.4 K and scaling the pixel intensities by a factor 1.05 to improve the accuracy of the representation (see Patra et al., 2015). An offset of 493 $\mu$K was added to the 23 GHz map to include an estimate of the uniform component missing in the differential map, as described in Sathyanarayana Rao et al. (2015). Using the resultant images, with identical beams and pixelation, we generate a physical sky model for the Galactic and extragalactic emission (excluding the CMB) in the 22 MHz–23 GHz band, as described below.

### 4.4.1 GMOSS: Physics

Herein we describe GMOSS. The six maps used as inputs to generate GMOSS are shown in Figure 4.1. The dominant mechanism of emission at low radio frequencies is synchrotron radiation. The total spectrum emitted by an ensemble of electrons $N(\gamma)$, with a distribution $N(\gamma)d\gamma \propto \gamma^{-p}d\gamma$ and energies ranging between $\gamma_{\text{min}}$ and $\gamma_{\text{max}}$, is given by

$$P_{\text{tot}}(\nu) = C \int_{\gamma_{\text{min}}}^{\gamma_{\text{max}}} P(\nu) \gamma^{-p}d\gamma. \quad (4.1)$$

Here $\gamma$ is the Lorentz factor of the electrons, indicative of electron energy, and $P(\nu)$ is the emission spectrum from individual electrons. $p$ is the index of electron energy distribution $N(\gamma)$. We define the temperature spectral index ‘$\alpha$’ such that the brightness temperature $T(\nu) \propto \nu^{-\alpha}$, where $\alpha$ is related to the electron distribution index by $\alpha = \frac{p+3}{2}$. Further, $P(\nu) \propto F(x)$ with

$$F(x) = x \int_{x}^{\infty} K_{\frac{5}{2}}(\xi)d\xi, \quad (4.2)$$

where $x = \frac{\nu}{\nu_c}$ and $K_{\frac{5}{2}}$ is the Bessel function of the second kind. $\nu_c$ is the critical frequency given by

$$\nu_c = \frac{3\gamma^2 q B \sin\alpha}{4\pi mc}, \quad (4.3)$$

where $B$ is the magnetic field, $\alpha$ the pitch angle between the particle (electron) velocity and magnetic field, $c$ the velocity of light in free space, $q$ the charge of the particle (electron), and $m$ its mass. We refer the reader to Rybicki & Lightman (1986) for a detailed treatment of the synchrotron radiation process.

\(^2\)http://healpix.sourceforge.net
Fig. 4.1 Maps used as input to GMOSS. Maps at (a) 22 MHz and (b) 45 MHz are generated from GSM (de Oliveira-Costa et al., 2008). Map (c) is at 150 MHz (Landecker & Wielebinski, 1970) with corrections applied from Patra et al. (2015). Map (d) 408 MHz (Haslam et al., 1982), (e) 1420 MHz (Reich, 1982; Reich & Reich, 1986; Górski et al., 2005) and (f) 23 GHz (WMAP science data product\(^1\)) with a uniform component added as described in Sathyanarayana Rao et al. (2015). All maps are in units of Kelvin at a common resolution of 5° and in galactic coordinates, represented in with nested ‘R4’ scheme of HEALPix.
Spectral shapes may be described as convex, concave, or more complex. Toy models of each of the shapes, exaggerated for purposes of representation, are shown in Figure 4.2. In GMOSS, convex spectra are modeled as synchrotron emission arising from an ensemble of electrons having a break in their energy distribution, with steeper energy indices at higher frequencies. Concave spectra are modeled as composites of steep and flat spectrum components wherein the steep spectral component dominates at lower frequencies and the flat spectral component at higher frequencies. Spectra with more complex shapes are modeled as convex or concave with significant additional thermal absorption at low frequencies and/or optically thin free-free emission at high frequencies. The parameters that optimally describe the physical model are $C_1$, $\alpha_1$, $\delta_\alpha$, $\nu_{br}$, $C_2$, $T_e$, $I_x$ and $\nu_t$. We
4.4 GMOSS : A spatially resolved spectral model for the radio sky

denote by $\alpha_1$ and $\alpha_2$ the low- and high-frequency spectral indices, respectively, for the synchrotron component of the model; parameter $\delta_\alpha$ is defined by the relation $\alpha_2 = \alpha_1 + \delta_\alpha$. The remaining parameters are described below, where we separately consider the modeling of pixels with synchrotron components that are convex and concave.

1. Case of pixels where $\alpha_2 > \alpha_1$: We refer to such spectra as convex. We model such spectra as primarily arising from synchrotron emission from electrons with power-law energy distributions with a break, which consequently causes the emission spectrum to be a broken power law and hence of convex form. We also allow for thermal absorption at low frequencies and optically thin thermal emission toward higher frequencies to account respectively for any low-frequency flattening and for any high-frequency excess. The functional form of the model describing the sky brightness temperature $T(v)$ is given by

$$T(v) = C_1 \left( v^{-2} \sum_{\gamma_{\text{break}}}^{\gamma_{\text{break}}} F(x) \times \gamma^{-(2\alpha_1-3)} \, d\gamma \right) + I_x v^{-2.1} + T_e \left( 1 - e^{-\left(\frac{\nu_t}{\nu}\right)^{2.1}} \right).$$

(4.4)

The two spectral indices $\alpha_1$ and $\alpha_2 = \alpha_1 + \delta_\alpha$ ($\delta_\alpha > 0$) are appropriately converted to electron distribution indices to derive the emission spectra. The break frequency $\nu_{\text{br}}$ is related to the Lorentz factor $\gamma_{\text{break}}$ at which the energy spectrum has a break in its power-law form; the two are related by Equation (4.3). An optically thin thermal component with brightness parameterized by $I_x$ is added to the synchrotron emission to account for any excess at high frequencies. The total emission is assumed to be absorbed at a turnover frequency $\nu_t$ by a separate thermal foreground medium; this medium is assumed to have an electron temperature $T_e$ and a constant emission measure and would also add its own emission to the sky brightness. This is captured by the terms $e^{-\left(\nu_{\text{br}}/\nu\right)^{2.1}}$ and $T_e \left( 1 - e^{-\left(\nu_t/\nu\right)^{2.1}} \right)$ respectively. Finally, the normalization parameter $C_1$ provides scaling to match the observational data in temperature units.

2. Case of pixels where $\alpha_2 < \alpha_1$: The spectra of these pixels are concave and are modeled as a composite of flat and steep spectrum synchrotron emission components with individual brightness temperatures of spectral indices $\alpha_1$ and $\alpha_2 = \alpha_1 + \delta_\alpha$ ($\delta_\alpha < 0$). Concave spectra could, in principle, be modeled as arising from concave energy distributions; however, that would be unphysical and hence not meaningful. Additionally, it is also computationally easier to model concave spectra as composed
of steep and flat spectrum components than as a concave electron energy distribution. The functional form adopted here is given by

$$T(v) = C_1 \left( v^{-\alpha_1} + \frac{C_2}{C_1} v^{-\alpha_2} + I_x v^{-2.1} \right) e^{-\left( \frac{v}{v_{\text{br}}} \right)^{2.1}} + T_e \left( 1 - e^{-\left( \frac{v}{v_{\text{br}}} \right)^{2.1}} \right).$$  \hspace{1cm} (4.5)

We drop the parameter \( v_{\text{br}} \) and instead have a second normalization parameter \( C_2/C_1 \), which denotes the ratio of contributions from flat and steep spectrum components. The other parameters are the same as in the case of convex spectra. In both cases there are seven parameters to be fit for.

### 4.4.2 GMOSS - Methods

We employ the downhill simplex algorithm (Nelder & Mead, 1965) to optimize the seven free physical parameters of the convex and concave spectral models in GMOSS by minimizing a goodness of fit \( \chi^2 \) toward every pixel. \( \chi^2 \) is computed to be

$$\chi^2 = \frac{1}{N} \sum_{i=1}^{N} \left( \frac{T_{\text{data}}(v_i) - T_{\text{model}}(v_i)}{T_{\text{data}}(v_i)} \right)^2,$$ \hspace{1cm} (4.6)

where \( T_{\text{data}} \) denotes the image data at frequency \( v_i \), \( T_{\text{model}} \) denotes the model prediction at the frequency corresponding to the image data, and \( N \) is the number of data points available at the sky pixel at which the fit is being done. This constrains the model to fit the measurements with minimal fractional errors, which is appropriate if all the radio images have the same fractional errors. Despite each sky pixel being modeled with a larger number of parameters (7) than data points (6), the model spectrum for any pixel does not exactly fit all the data points for that pixel. The deviations from the data are caused by the defined nature of the model in terms of physically motivated constraints and emission processes that allow only a specific family of curves. The high dimensionality of the modeling problem requires considerable sophistication in the algorithm to obtain a good fit that converges to the global minimum in the multi-dimensional parameter space. Markov chain Monte Carlo (MCMC) is more computationally expensive than the downhill simplex algorithm adopted herein. Future upgrades of GMOSS will adopt an implementation using MCMC as more all-sky radio images become available. The model can accommodate more input maps, including maps with partial sky coverage, and can also weight images in a way that is appropriate to their individual fractional errors.

---

3It would be desirable to weight the fits by some parameter relating to the reliability of the input maps, such as error bars of the input maps. However realistic error bars are not readily available, hence fits are done presuming all the input images have the same fractional errors. It is also difficult to rigorously demonstrate that overfitting to the GMOSS model has not occurred due to the same reason.
4.4 GMOSS: A spatially resolved spectral model for the radio sky

Initial guesses for parameters $\alpha_1$ and $\alpha_2$ (and hence for $\delta_\alpha$) are obtained by computing 2-point temperature spectral indices between respectively the data at 45 and 150 MHz and between the data at 408 and 1420 MHz, respectively. These determine the model employed to be of either the convex or the concave form. Initial guesses for normalization parameters $C_1$ and $C_2$ are evaluated at 1420 MHz. Initial guesses for parameters $T_e$ and $v_t$ are set to the nominal expectations of 8000 K and 1 MHz, respectively. $I_x$ is computed from the difference between the brightness temperature in the data at 23 GHz and that corresponding to the model evaluated after the optically thin high-frequency emission is omitted; however, if the difference is negative, then the initial guess for this parameter is set to be vanishingly small. Further, to aid the optimization toward realistic solutions, certain parameters are constrained to be within physically acceptable limits. The temperature spectral indices are constrained to lie between 2.0 and 3.0 (Bennett et al., 1992) and the physical temperature $T_e$ for the thermal foreground component that provides the low-frequency thermal absorption is not allowed to exceed 10,000 K (Haffner et al., 2009).

The synchrotron spectrum requires integration over spectra arising from individual electrons, where a single electron spectrum itself requires integration of the modified Bessel function of the second kind. Choice of the numerical integration technique is critical to accurately and efficiently implementing GMOSS. While an adaptive integration method aids in hastening computing time, care must be taken so that the numerical approximations in the algorithm do not introduce unphysical discontinuities or excessive error in the spectrum owing to computational noise. We use a combination of adaptive integration methods to optimize speed of computation and numerical accuracy. Adaptive Gauss-Kronrod quadrature is used for all integrals while estimating the parameters and the open Rhomberg adaptive method is used for integrals that generate the output spectra in specific bands where an accurate representation is desired. For example, for generating foreground spectra that may serve for evaluating algorithms for detecting EoR signatures, the open Rhomberg adaptive method is used for integrals that generate the spectrum in a contiguous set of frequencies in the 40–200 MHz band, which is of interest to EoR science. The method of integration allows for a trade-off between speed and accuracy as desired.

The bottleneck in computing time is significantly reduced by using analytic approximations for the integral of the Bessel function. A first-order analytical approximation for the integral is given in Rybicki & Lightman (1986). This is further simplified using the mathematical software tool ‘Mathematica’ (Wolfram Research, Inc., 2016) to reduce the number of integrations by an order of magnitude. For values of $x \geq 3$ the Equation (4.2)
can be approximated by

\[
F(x) = \left( 13 \sqrt{\pi} \left\{ 2429625 + 2x\left( -1922325 + 5418382x + 83221732x^2 \right) \right\} \right)
- 1196306216 e^x \pi x^{7/2} \text{Erfc}\left(\sqrt{x}\right) \frac{1}{967458816 \sqrt{2} x^{5/2}} e^{-x}.
\]

(4.7)

Here Erfc is the complementary error function. For \( x < 3 \), Equation (4.2) is approximated by

\[
F(x) = -\frac{\pi x}{\sqrt{3}} + \frac{9 x^{11/3} \Gamma\left( -\frac{2}{3} \right)}{160 2^{2/3}} - \frac{x^{1/3} \left( 16 + 3x^2 \right) \Gamma\left( -\frac{4}{3} \right)}{24 2^{1/3}}.
\]

(4.8)

Here, \( \Gamma(x) \) is the Gamma function. Both approximations deviate from the exact treatment by less than 0.1%.

### 4.5 GMOSS parameter values and goodness of fit

The physical sky model fits the data at all 3072 pixels with a median \( \chi^2 \) of 0.0034, corresponding to a mean fractional departure of 6%. Ninety-nine percent of the pixels have \( \chi^2 \) less than 0.03, corresponding to a mean fractional error of 17%, and a histogram of \( \chi^2 \) is shown in Figure 4.3. Also shown in Figure 4.3 is an all-sky map of \( \chi^2 \) on a Mollweide projection of the sky in Galactic coordinates. Surprisingly, the errors are relatively smaller where the Galaxy dominates, both in the plane and in the region of the north polar spur. In these regions the model corresponding to a convex spectrum was selected by the fitting algorithm.

Shown in Figure 4.4 are sample spectra derived from GMOSS (blue solid lines) toward three pixels that are representative of convex (pixel 36), concave (pixel 2060), and more complex (pixel 1130) spectral shapes. The measurement data from the six maps are also shown in the three panels using filled red circles. Figure 4.4 also shows in a panel the locations of these pixels as an image in Galactic coordinates. The chosen pixels lie toward different sky regions, where the spectral shapes are dominated by emission from different components. As demonstrated by the fit solution to sample pixel 1130, not only is the emission from the Galactic center region brighter, as expected, than emission away from the plane, but the spectral structure is also more complex, necessitating modeling with a
Fig. 4.3 (a) Histogram of goodness of fit $\chi^2$ computed for all 3072 pixels. The median of the distribution is 0.0034, mean is 0.0054 and maximum value is 0.2280. (b) The distribution of the $\chi^2$ values computed over the 3072 sky pixels is shown in Mollweide projection, Galactic coordinates with $5^\circ$ resolution.
broken power-law synchrotron emission plus significant low-frequency turnover due to thermal absorption and excess free-free emission at the high-frequency end. This is to be expected from the variety of components and processes that are unique to the Galactic plane, particularly the Galactic center, including HII regions and supernova remnants, to name a few. Pixel 36 is in the vicinity of the central bulge, and the fit solution is that of a convex spectrum in which the emission is modeled in GMOSS as a broken power-law synchrotron spectrum, without significant thermal effects needed, which are mainly present toward the Galactic plane. The spectrum at pixel 2060 is in the vicinity of the Galactic pole and has a concave form. The spectrum toward this pixel is modeled by GMOSS as composite emission from flat and steep spectrum components. Two sample all-sky maps

![Graphs](image1.png)

Fig. 4.4 Data points (filled red circles) towards representative pixels with overlaid GMOSS generated spectra (solid blue lines). Pixel positions selected for display contain (a) convex shape at pixel 36, (b) concave shape at pixel 2060 and (c) complex shape at pixel 1130. The plots are in log-temperature versus log-frequency scale. Panel (d) shows the positions of these pixels on a Mollweide projection of the sky in Galactic coordinates, where the solid green line traces the ecliptic.

at 50 and 200 MHz generated using GMOSS are shown in Figure 4.5. The Mollweide projection maps are in Galactic coordinates with 5° resolution. With this resolution, the coarse Galactic features clearly arise in the maps, and the mean temperatures are higher in the lower-frequency map as expected. These maps have a median deviation from their
nearest input maps—namely the ones at 45 MHz and 150 MHz—by 25% and 50%\(^4\) respectively. The panels in Figure 4.6 show the distribution of optimized parameters\(^5\)

![Figure 4.5](image)

Fig. 4.5 All-sky maps derived from GMOSS at (a) 50 MHz, (b) 200 MHz. The maps are in units of Kelvin at a resolution of 5° and in Galactic coordinates.

across the 3072 sky pixels on Mollweide projections of the sky in Galactic coordinates. Panels showing the break frequency and secondary normalization parameters in convex and concave spectra, respectively, are mutually exclusive in modeling the sky spectrum in GMOSS. In each case, the pixels that have spectra of the other form are masked and are in gray. The median values of some of the optimized parameters are given in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Median value</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\alpha_1)</td>
<td>2.50</td>
</tr>
<tr>
<td>(\alpha_2)</td>
<td>2.58</td>
</tr>
<tr>
<td>(v_{br})</td>
<td>0.36 GHz</td>
</tr>
<tr>
<td>(I_x)</td>
<td>(8.39 \times 10^{-10})</td>
</tr>
<tr>
<td>(T_e)</td>
<td>2060 K</td>
</tr>
<tr>
<td>(v_t)</td>
<td>0.3 MHz</td>
</tr>
</tbody>
</table>

Table 4.1 Median values of optimized parameters over all 3072 pixels that describe the sky models.

Figure 4.6(a) shows the relative distributions of pixels that have convex and concave spectra and thus entail two different models for the relativistic electrons. It is interesting to note that these pixels employing different models are distributed toward distinct regions of the sky. The Galactic plane and the north polar spur are distinct and require convex spectra employing a break in the power-law synchrotron emission, with the spectral fit toward pixel 36 in Figure 4.4 serving as a representative example. In contrast, the radio spectra in

\(^4\)This seemingly large deviation is expected simply from the frequency deviation between the maps being compared.

\(^5\)The distributions include variance owing to errors in the input images.
Fig. 4.6 The sky distribution of the optimized values of parameters. All maps are at a resolution of $5^\circ$ and in galactic coordinates. (a) Distribution of pixels with convex (black) and concave (white) spectra. The Galactic plane and north polar spur are clearly of distinct type suggesting different physics in the electron populations compared to the extragalactic sky (b) $\alpha_1$ (c) $\delta_\alpha$ given as positive when $\alpha_2 > \alpha_1$ and negative when $\alpha_2 < \alpha_1$ such that $\alpha_2 = \alpha_1 + \delta_\alpha$ (d) Temperature $T_e$ (e) break frequency $\nu_{br}$ for convex spectra in GHz (f) the additional normalization for the flat spectrum sources, $C_2$ for concave spectra (g) the frequency of thermal absorption turnover, $\nu_t$ and (h) the parameter representing optically thin free-free emission, $I_x$. 
regions off the Galactic plane and the spur are concave and hence required a modeling as a composite of flat and steep spectrum components; the spectral fit toward pixel 2060 in Figure 4.4 is a representative example of this type.

The above difference is likely a reflection of the difference in origin of relativistic electron populations. In the Galaxy, electrons are believed to be created in shock accelerations associated with supernovae, and these then diffuse and migrate off the plane, acquiring a break in the electron energy distribution because of aging and loss mechanisms (Lisenfeld & Völk, 2000). On the other hand, the extragalactic radio emission is dominated at low frequencies by powerful radio galaxies in which the acceleration is in hot spots at the ends of relativistic jets and perhaps in-situ re-acceleration in cocoons. At high frequencies, the dominant emission is from flat spectrum cores of active galactic nuclei (see, for example, Miley, 1980). This consistency between GMOSS results and expectations lends confidence to the modeling presented here.

Figure 4.6(b) gives the distribution of the temperature spectral index $\alpha_1$ of the low-frequency synchrotron emission across the sky. The parameter $\delta_\alpha$ that represents the change in spectral index toward high frequencies is shown in Figure 4.6(c). Unsurprisingly, pixels that have positive values of $\delta_\alpha$ are those that have convex spectra, represented in black in Figure 4.6(a) and those with negative values correspond to pixels that have spectra of the concave shape, given as white pixels in Figure 4.6(a).

The temperature of the thermal medium that models the absorption at low frequencies is shown in Figure 4.6(d), and the frequency of the thermal absorption turnover is in Figure 4.6(g). Pixels with the highest temperatures and the highest turnover frequencies lie mostly along the Galactic plane and toward the north polar spur. Pixels in blue in Figure 4.6(g) have very low values, indicating that the extragalactic sky does not require any significant thermal absorption in the physical modeling. The median thermal absorption turnover frequency, $\nu_t$, in the GMOSS modeling is 0.3 MHz, which is consistent with the estimates in literature (Novaco & Brown, 1978; Cane, 1979) that this value is in the ballpark of 1 MHz. The median value of the GMOSS parameter $T_e$ for the ionized medium (WIM) that models this turnover is 2060 K, which is in the range of electron temperatures estimated for the interstellar medium: from observations of radio recombination lines (RRLs) at 328, 75, and 34.5 MHz Kantharia & Anantharamaiah (2001) estimate the extended low-density warm ionized medium (ELDWIM) responsible for carbon recombination lines to have $T_e$ as low as 30–300 K, and based on 1.4 GHz recombination lines, Heiles et al. (1996) estimate the ELDWIM to have $T_e$ of 7000 K.

Figures 4.6(e) and 4.6(f) represent two mutually exclusive parameters of the physical
models in GMOSS—namely, the break frequency $\nu_{br}$ for pixels that have convex spectra, and a secondary normalization $C_2$ for the flat-spectrum emission component in the case of pixels that have concave spectra. The median value of $\nu_{br}$ is 360 MHz (see Table 4.1). This is consistent with the pure diffusion model of Galactic cosmic ray electron propagation in Strong et al. (2011), for a break in the electron injection spectrum of 4.0 GeV, if the magnetic field is 1.4 $\mu$G.

Lastly, Figure 4.6(h) shows the parameter $I_x$, which represents optically thin free-free emission that dominates at high frequencies. This parameter clearly arises from a component that lies along the north polar spur and a region of the Galactic plane with a relatively smaller scale-height. The pixels with spectra showing spectral steepening as well as significant optically thin free-free emission would most certainly have a spectral shape that is complex, due to the convex spectrum at low frequencies followed by an upturn at high frequencies, where the excess emission dominates; the spectrum toward pixel 1130, as shown in Figure 4.4, is an example of this type. Using GMOSS-generated model spectra at each pixel, we have computed the 2-point spectral indices for the pair of frequencies 50 and 150 MHz, and separately for the pair 400 and 1200 MHz. Histograms of these 2-point spectral indices are shown in Figure 4.7. The computed spectral indices have a mean value of 2.49 at the lower set of frequencies, where the spectral indices are distributed over the range 2.27–2.76. Within the errors, this is the same as the median value of 2.50 for the $a_1$ model parameter (listed in Table 4.1), which represents the spectral index of the synchrotron component below any break. This low-frequency spectral index is also consistent with the observations of Rogers & Bowman (2008). At relatively higher frequencies, the 2-point spectral index between 400 and 1200 MHz is 2.56, and in this band the range is over 2.24–2.85. The median value of the spectral index parameter $\alpha_2$ above any break is 2.58 and is in the same ballpark as the median 2-point spectral index. The high-frequency spectral index is again in agreement with measurements of the absolute radio sky at cm wavelengths (see, for example, Kogut et al., 2011a).

4.6 Summary

We have presented a physically motivated sky model of the low-frequency radio sky from 22 MHz to 23 GHz. Spectra over this frequency range are presented over 3072 pixels covering the sky in R4 HEALPix pixels; the resolution is 5°. GMOSS models the spectrum at each pixel primarily as optically thin synchrotron emission, adopting either electron distribution of a broken power-law form or a composite of steep and flat spectrum components. Additionally, optically thin thermal emission is included to correct for any deficit at the high frequency end, and thermal absorption is added as a foreground screen to account
Fig. 4.7 Histogram of 2-point spectral indices derived from GMOSS at all 3072 pixels between (a) 50 MHz and 150 MHz (b) 400 and 1200 MHz.
for any low-frequency flattening. The 7-parameter model is fit to six all-sky images at 0.022, 0.045, 0.150, 0.408, 1.420, and 23 GHz to derive the GMOSS model spectra. The fractional differences between the GMOSS model spectra and the input data have a median value of 6%. This is in keeping with the systematic calibration errors in the input maps, which range from 1% to 20%. Furthermore, derived physical parameters of the model are in reasonable agreement with expected values, providing confidence in the physical model.

With a resolution of 5°, which is much finer than that of typical antenna beams used in experiments attempting to detect the global signature of EoR, the model can be convolved with appropriate antenna beams in simulations of these experiments to generate mock sky spectra. GMOSS provides an expectation of the EoR foreground to help one arrive at appropriate component separation strategies without any inherent assumptions of smoothness in the foreground spectrum. It may be noted here that GMOSS is intended to represent the radio continuum emission, and the physical processes included are those relevant to the radio continuum in the MHz-to-GHz frequency range. This is useful for modeling the foreground contamination of wideband cosmological spectral signals or distortions of the CMB spectrum in these wavelengths. RRLs from the Galactic thermal component would be expected to add sharp spectral features and would hence be distinguishable from wideband spectral signals such as those expected from the recombination and reionization epochs and the Dark Ages and Cosmic Dawn in between. Hence, RRLs are not included in GMOSS.

Also, it may be noted that the all-sky radio maps used as inputs to GMOSS – and indeed also all-sky maps that may be available in the foreseeable future – do not have the accuracy anywhere near the level of the cosmological signals mentioned above. Including more images at intermediate frequencies in the coming years, as new images become available, will most certainly improve GMOSS. If the fits yield residuals outside the error bars of the images, then more spectral complexity would necessarily have to be added to the model. However, allowing for multiple breaks (which are indeed observed in sources where multiple cooling processes are presumed to occur) and/or multiple spectral components would most likely satisfy improved data, without requiring additional radiative processes. It is not impossible that such inclusions might result in increased complexity of the synthetic spectra and hence prove to be of greater confusion to the detection of cosmological signals.

A study of the implications of GMOSS predicted sky spectra for EoR signal detection and signal extraction strategies are presented in a subsequent paper (Sathyanarayana Rao et
al., in preparation\textsuperscript{6}). This subsequent paper presents among other things a comparison of GMOSS with existing polynomial models for foregrounds as well as a discussion on regions of the sky best suited for EoR signal detection experiments.

However, since GMOSS is not tailored to EoR science but is based entirely on plausible physics that produce the radio sky spectrum, it may be used for any problem that requires simulating the radio sky spectrum. Simulation studies of the foreground contamination in cm-wavelength detection experiments of signals arising from the epoch of recombination are one such area where GMOSS can be applied. GMOSS is being made publicly available at \url{www.rri.res.in/DISTORTION/} and will be updated as other maps and processes are included. The code used to generate the model is flexible and can include more maps, including those with partial sky coverage.

### 4.7 Acknowledgements

We acknowledge the use of the Legacy Archive for Microwave Background Data Analysis (LAMBDA), part of the High Energy Astrophysics Science Archive Center (HEASARC). HEASARC/LAMBDA is a service of the Astrophysics Science Division at the NASA Goddard Space Flight Center. Some of the results in this paper have been derived using the HEALPix (\cite{Gorski2005}) package. MSR would like to thank K. S. Dwarakanath for their useful discussions on Galactic radio emission. JC is supported by the Royal Society as a Royal Society University Research Fellow at the University of Manchester, U.K.

\textsuperscript{6}Accepted for publication in ApJ, 2017

\textsuperscript{7}Also available at https://github.com/RRIDISTORTION/GMOSS
5.1 Abstract

Cosmic baryon evolution during the Cosmic Dawn and Reionization results in redshifted 21-cm spectral distortions in the cosmic microwave background (CMB). These encode information about the nature and timing of first sources over redshifts 30–6 and appear at meter wavelengths as a tiny CMB distortion along with the Galactic and extragalactic radio sky, which is orders of magnitude brighter. Therefore, detection requires precise methods to model foregrounds. We present a method of foreground fitting using maximally smooth (MS) functions. We demonstrate the usefulness of MS functions over traditionally used polynomials to separate foregrounds from the Epoch of Reionization (EoR) signal. We also examine the level of spectral complexity in plausible foregrounds using GMOSS, a physically motivated model of the radio sky, and find that they are indeed smooth and can be modeled by MS functions to levels sufficient to discern the vanilla model of the EoR signal. We show that MS functions are loss resistant and robustly preserve EoR signal strength and turning points in the residuals. Finally, we demonstrate that in using a well-calibrated spectral radiometer and modeling foregrounds with MS functions, the global EoR signal can be detected with a Bayesian approach with 90% confidence in 10 minutes’ integration.
5.2 Introduction

The epoch of reionization (EoR) represents one of the important transitions in the physical state of the universe, when the almost neutral baryonic matter from the dark ages transitioned to its mostly ionized form. Another important transition occurred earlier at even higher redshifts, when during the epoch of recombination the primordial plasma recombined and the universe became neutral, with atomic hydrogen, helium, and a small fraction of light elements. While the physics of the epoch of recombination is well understood from the theoretical point of view (Ali-Haïmoud & Hirata, 2011; Chluba & Thomas, 2011; Glover et al., 2014) and constrained by observations of the cosmic microwave background (CMB) (Calabrese et al., 2013; Farhang et al., 2013; Planck Collaboration et al., 2016a), the EoR is relatively poorly understood. Among other aspects, the thermal evolution of baryons and nature of the first sources, the exact redshift and duration of reionization, and the dominant mechanisms that affect reionization are poorly constrained (Planck Collaboration et al., 2016b). The evolution of the strength of the redshifted 21-cm line of hydrogen against the radiation from CMB is expected to trace the thermal history of the gas across the EoR. Thus, the redshifted 21-cm signal is predicted to appear as a distortion of the CMB spectrum, encoding the physics of the EoR in the signal structure. A comprehensive review of EoR physics is presented in Furlanetto et al. (2006).

There are a multitude of scenarios that predict different global redshifted 21-cm signatures (see Pritchard & Loeb, 2010b). The form of the generic signature from the EoR is shown in Fig. 5.1. The high-redshift absorption feature ‘A’ at frequencies below about 30 MHz arises from collisional coupling of the spin temperature to the relatively low gas kinetic temperature. At relatively higher frequencies and below redshifts of about 40, a second absorption dip ‘C’ arises from Wouthuysen-Field (Wouthuysen, 1952b; Field, 1958) driving of spin to the kinetic temperature by Ly-α from the first collapsed objects. The subsequent X-ray and UV heating of the gas kinetic temperature, and consequently the spin temperature as well, by energetic radiation from the first stars and galaxies transforms the appearance of the gas from absorption to emission (at ‘D’); the ionizing radiation then progressively results in the disappearance of the baryons in redshifted 21 cm and the gas is fully ionized and vanishes from this diagnostic by redshift 6 (at ‘E’). The critical spectral features representing events in the thermal history, which appear as turning points and successive absorption and emission features, are in the frequency range 10–200 MHz corresponding to events at redshifts of about 150 to 6. Identifying the true reionization signal amongst the multitude of plausible forms currently allowed by observations to date is a way to constrain the parameter space of the sources of reionization and their spatial-temporal distribution. At meter and decameter wavelengths, where the global EoR signal resides, foregrounds due to Galactic and extragalactic radio sources are relatively
5.3 Motivation

There are several ongoing and proposed experiments to detect the global EoR signal. Nevertheless, the methodology for the modeling, subtraction of foregrounds, and extraction of the much smaller EoR signal from spectral data, or for joint modeling of foregrounds and EoR signals, continues to be an open problem.

Most global EoR signal detection experiments employ a spectral radiometer connected to an elemental antenna. This provides as its response a measurement set that contains the signature arising from baryon thermal evolution and reionization of hydrogen gas...
along with the orders-of-magnitude larger foreground component. This radio foreground component appears as an average of sky spectra over the beam pattern of the telescope antenna. Even though the foreground might be composed of emitting volume elements that individually have spectra of power-law form, the variation in the spectral indices of these sources along the line of sight and across the sky within the antenna beam results in an observed spectrum in which the foreground component has an unknown form and cannot be modeled as a simple power law anymore. As discussed below, experiments have employed high-order polynomials to model these foregrounds and also instrument systematics.

The EDGES experiment (Bowman & Rogers, 2010) observed the sky spectrum in the frequency range 100–200 MHz and excluded rapid reionization with timescales less than that corresponding to $\Delta z < 0.06$. Polynomials of different orders in different frequency windows were used to jointly fit and model the foreground and instrumental systematics. The BIGHORNS experiment (Sokolowski et al., 2015) used a ninth-order polynomial to model the receiver noise and the Global Sky Model (GSM) of de Oliveira-Costa et al. (2008), together with the simulated radiation pattern of their antenna, to estimate the foreground contribution in their data. The SCI-HI 21-cm all-sky experiment (Voytek et al., 2014) also used the interpolated GSM to calibrate the foreground in their data. Experiments to date have indeed had to use high-order polynomials to fit out both foregrounds and uncalibrated bandpass; however, in this work, we focus entirely on complexity inherent in foregrounds and assume a smooth bandpass of the observing instrument.

The order of the polynomial deemed necessary to fit foreground components in spectral data to $\sim$ mK level, lower than the expected signal, has varied in literature depending on the assumed sky model. Pritchard & Loeb (2010b) argued that averaging sky spectra over large angular scales by wide-angle beams of telescopes designed for the global EoR detection would result in smooth frequency dependence for the foregrounds in spectral data and a third-order polynomial may suffice to model the foreground component and reduce their residuals to well below the expected EoR signal. They note that these residuals are dominated by the numerical limitations of available sky models and also recognize that a simple polynomial approach to modeling the foreground—with order sufficient to describe the foreground and systematics in spectral data—could substantially diminish the desired EoR signal in the residual. More recent results suggest the need to adopt polynomials of higher order ($> 4$) to model foregrounds to mK levels to be able to discern the global EoR signal (see Bernardi et al., 2015). While polynomials of lower order might not model the foreground components with sufficient precision, polynomials of higher orders risk over fitting the foregrounds and partially removing the EoR signal. Adoption of sky models with increasing complexity while generating synthetic spectra and mock observations in general leads to concluding that a higher order in the polynomial is required to fit to
the foreground components in the mock observations so as to be able to discern the EoR signal. This uncertainty in inferences of the degree of the polynomial required to model the foreground in measurements with sufficient precision arises both due to differences in the assumed spectral structure in the sky brightness and due to instrumental effects. Since both the global EoR signal as well as the model for the low-frequency sky (\( \lesssim 200 \) MHz) are uncertain, appropriate choice of the functional form adopted to model the foreground in spectral data is critical to interpreting any global EoR signal detection experiment, failing which results in unknown biases in interpretation of data.

To summarize, the aforementioned approaches come with two problems. First, modeling with polynomials of arbitrary order, inclusive of GSM-based models, which also employ mathematical interpolation between measurements from publicly available all-sky radio surveys, can be unrealistic in their spectral shapes. Second, estimates of the percentage loss in EoR signal on subtracting a polynomial baseline and the level of foreground contamination that may remain in the residual, suggest that such a simple polynomial or GSM approach to modeling the foreground component in spectral data is far from ideal (Pritchard & Loeb, 2010b; Voytek et al., 2014; Harker, 2015).

This paper represents an improvement on previous work in two ways. First, we use a new physically motivated model of the radio sky, GMOSS (Sathyanarayana Rao et al., 2017), which models the radio spectrum over the entire sky with parameters that describe continuum radiative processes. By beam-weighting such a physically motivated sky model, we generate synthetic spectra representing mock observations that have physical and plausible spectral forms.

Second, we suggest a new functional form for describing the foreground component, namely maximally smooth (MS) functions. We demonstrate that this is an improvement over polynomial or other interpolations between measured sky brightness at discrete and widely spaced frequencies, which would result in spectra with unphysical shapes and arbitrary spectral complexity. This functional form for the foreground was recently applied to simulations of the detection of primordial recombination-line ripples from redshift \( z \approx 1000 \) (Sathyanarayana Rao et al., 2015), where it was demonstrated that this approach is indeed powerful. We demonstrate that for the more realistic physical GMOSS model of the foreground, the proposed modeling of foregrounds in measurement sets using MS functions is robust and superior to the more simplistic polynomial-form representation. We find that foregrounds in mock spectra generated using GMOSS, though spectrally more complex than previously assumed, are indeed smooth and describable by MS functions.

Most importantly, MS functions are loss resistant in the sense that when they are used to
fit out foregrounds, they do not substantially attenuate the embedded EoR signal and also preserve the turning points in the EoR signature, unlike when high-order polynomials are used for foreground fitting. We also examine detection likelihoods when mock observations are jointly modeled using theoretical EoR templates and MS functions to describe the foreground.

5.4 Toward a spectral model for the radio sky

Previous efforts to simulate data analysis methods for global EoR signal detection experiments have assumed simple sky models that are, in log-temperature versus log-frequency space (hereinafter referred to as log($T$) versus log($n$) space), either power laws or low-order polynomials (see, for example, Pritchard & Loeb, 2010b; Harker, 2015). The method adopted to separate the signal from foregrounds in mock observations is to fit the simulated spectra with polynomials of low order, possibly identical to those used to generate the input model sky. Clearly, since the EoR signal contains higher order inflections than those used to describe the sky spectrum and model the foreground in data, this would leave behind a substantial part of the EoR signal in the residual. Such a simplistic input model for the sky with subsequent modeling of the foreground components in mock observations using polynomials of matching order belies the challenge when dealing with real measurements where the true functional form of the sky spectrum and that of the foreground component in data are unknown.

In this section, we first use a set of all-sky maps to generate synthetic sky spectra, without any EoR component added. We then combine spectra from individual pixels over wide beams to generate mock observations. We estimate the degree of polynomial required to model the foreground in these mock data to mK precision in each case. Combining spectra from individual pixels can result in a final beam-weighted mock spectrum with different spectral complexity. Spectra with greater spectral complexity, in log($T$) versus log($n$) space, require a polynomial of higher degree to fit to the desired level.

We investigate the consequences of adopting three separate sky models on the deduced order of the polynomial required to fit the final beam-weighted foregrounds to mK level. The three different models we have investigated are (i) power laws or linear functions in log($T$) versus log($v$) space, (ii) polynomials in log($T$) versus log($v$) space and (iii) the Global MOdel for the radio Sky Spectrum (GMOSS; Sathyanarayana Rao et al. 2017).

Inputs to the sky models. A set of six all-sky maps are used to generate the sky models. The maps used are at 150 MHz (Landecker & Wielebinski, 1970), 408 MHz (Haslam et al.,
5.4 Toward a spectral model for the radio sky

1982), 1420 MHz (Reich, 1982; Reich & Reich, 1986; Górski et al., 2005) and 23 GHz (WMAP science data product). The treatment of various maps to bring them to their final usable form are presented in Sathyanarayana Rao et al. (2017). Maps at 22 and 45 MHz are generated from the GSM (see de Oliveira-Costa et al., 2008). These are at frequencies at which input maps are available for GSM, hence they are close to the original maps used by the GSM. In their final form, all input maps have a common resolution of 5° and are represented in galactic coordinates in nested ‘R4’ scheme of HEALPix.

For each of the three sky models adopted here, spectra are derived via fits that are done separately at each sky pixel. The spectral fits are then averaged over wider beams to generate the final mock spectrum as might be observed with a telescope that has a wide field of view.

Mock spectra and calibration. Our code uses the aforementioned sky images to generate synthetic sky spectra at any location on Earth and at any local time, including appropriate corrections for effects including precession and atmospheric refraction. In this section we assume observing with an antenna having a frequency-independent cosine beam with half-power beam width (HPBW) of 78°. The antenna is assumed to be pointed toward zenith and observes the sky over Gauribidanur Observatory in southern India, which is at latitude 13°01 N and longitude 77°58 E. Data are assumed to be recorded by a correlation spectrometer with in-built calibration for instrument bandpass with 1 MHz frequency resolution. We focus on the frequency range 40–200 MHz, where signatures of the Cosmic Dawn and of Reionization are predicted to be present; these signatures are substantially more uncertain today than those from the Dark Ages, which lie below about 40 MHz.

Assumptions. We assume a Dicke-switching calibration scheme wherein differencing the spectra recorded separately with hot and cold loads (373.0 K and 273.0 K, respectively) on the antenna serves to calibrate the bandpass and absolute scale. Admittedly, such schemes are limited in their accuracy owing to internal reflections within the signal path, which would be different for the sky signal and the injected calibration signal. Here we focus on the spectral complexity of foregrounds and ignore calibration errors, instrumental systematics, and mode coupling of spatial structures in the sky to the measured spectrum in frequency (see Mozdzen et al., 2016).

1 WMAP Science Team.
5.4.1 Case of power-law Form for Sky Spectra

Galactic synchrotron emission, which dominates the low-frequency radio sky (in our context $\nu \lesssim 1$ GHz), may in its simplest form be described as a power law:

$$T(\nu, n) = T_{V_0, n} \times \left(\frac{\nu}{V_0}\right)^{\alpha(n)},$$  \hspace{1cm} (5.1)

which in log($T$) versus log($\nu$) space has a linear form:

$$\log(T(\nu, n)) = \log(T_{V_0, n}) + \alpha(n) \times \log\left(\frac{\nu}{V_0}\right).$$  \hspace{1cm} (5.2)

Here $T(\nu, n)$ is the brightness temperature at frequency $\nu$ toward pixel $n$ and $\alpha(n)$ is the temperature spectral index of the power-law emission in that sky pixel. $T_{V_0, n}$ is the temperature at frequency $\nu = V_0$ toward pixel $n$.

We realize an all-sky model by computing at each sky pixel such a power-law spectrum by a straight-line fit to the data points given by the six maps toward that pixel. Fits done in log($T$) versus log($\nu$) space, as given by Equation (5.2), weight the data to have uniform fractional errors. Mock spectra, without any EoR signal added, were generated at times spaced 1 hr apart over the full LST range 0–24 hr by averaging the pixel spectra with a weighting defined by the telescope beam at each sidereal time. We show in Fig. (5.2) the form of the residuals on fitting with polynomials of a few orders for representative spectra, which correspond to LSTs when the beam is on and off the Galactic plane. The amplitudes of residuals on fitting these spectra with a polynomial of order two are a few mK, with residuals somewhat larger for mock observations toward the Galactic plane. Fig. (5.2) thus indicates that if we assume a power-law spectral form at every pixel that lies in a telescope beam, the beam-averaged foreground spectra might be fit with a quadratic polynomial with an accuracy that allows discerning the generic EoR signal. However, it is not possible to fit a substantial number of sky pixels with such a simplistic power-law spectrum, and in sky regions having intrinsically curved spectra, the errors in the fit exceed measurement errors; therefore, the model considered above is implausible.

5.4.2 Case of a polynomial sky model

As mentioned above, the six data points in the input sky maps do not lie on a straight line at every pixel in log($T$) versus log($\nu$) space. Clearly, the emission from the foreground, even in single pixels, is not precisely describable as a single power law. We allow for curvature in the sky spectrum by fitting the six data points at each pixel with a fifth-order polynomial.
Fig. 5.2 Residuals obtained on fitting to mock observations of the foreground sky in log($T$)-log($n$) space with polynomials with orders between 1 and 3. The sky spectra were generated from a model sky that assumes a simple power-law form radio spectrum at each sky pixel. The panel on the left are residuals obtained for a mock observation that is away from the Galactic plane and the one on the right corresponds to a mock spectrum that includes the Galactic plane.

in log($T$) versus log($v$) space:

$$
\log(T(v,n)) = \log(T_{0,n}) + \alpha(n) \times \log\left(\frac{V}{V_0}\right) + \beta_2(n) \times \log\left(\frac{V}{V_0}\right)^2 + \beta_3(n) \times \log\left(\frac{V}{V_0}\right)^3 + \beta_4(n) \times \log\left(\frac{V}{V_0}\right)^4 + \beta_5(n) \times \log\left(\frac{V}{V_0}\right)^5,
$$

where the $\beta_i(n)$ coefficients provide for higher order curvature in the spectrum.

The optimized best-fit coefficients to such a quintic form at every pixel provides an input sky model. We generate beam-convolved sky spectra for different LSTs over 0–24 hr and fit these with polynomials of varying order. The residuals so obtained for two such realizations of the sky for LSTs corresponding to off and on the Galactic plane are in the left and right panels of Fig. (5.3), respectively. Clearly, if we adopt a quintic polynomial model for sky spectra and seek to model the foreground components in measurements to an accuracy so as to limit residuals to mK levels, polynomials of order four are required off the Galactic plane and orders about 5–6 are required for telescope pointings toward the Galactic plane.

The above exercise demonstrates that if a higher level of complexity is allowed for the spectral structure in the model for the foreground, a higher order polynomial is necessary to fit the mock observations that simulate beam-averaged spectra. The spectral structure inherent in the sky model in simulations thus critically guides strategies devel-
Fig. 5.3 Residuals on fitting mock spectra, generated from a model that assumes a quintic form for the spectrum at each pixel, with polynomials of increasing orders. The panel on the left are residuals obtained for an observation that is away from the Galactic plane and the one on the right is for a mock spectrum that is toward the Galactic plane.

It may be noted here that there do exist foreground models in the literature with greater sophistication than polynomial fits in log(T) versus log(v) space. Notably the Global Sky Model (GSM) (de Oliveira-Costa et al., 2008; Zheng et al., 2016) uses a data-driven method to arrive at a foreground model using many data sets of large-area sky maps. Some studies in the literature that have examined foreground spectral complexity, as mentioned in Section 5.3, use GSM in one of two ways. One approach is to first generate sky maps at widely spaced frequencies using GSM, construct beam-averaged temperature maps, then adopt polynomial interpolation between the image pixels to derive mock spectra. This is similar to the case of polynomial interpolation described above and potentially results in unphysical spectral shapes. A second approach is to use GSM to derive sky brightness temperature at every pixel over finely sampled frequencies and simulate pixel-by-pixel foreground spectra. In both approaches, GSM uses a cubic-spline interpolation to derive principal components at all frequencies other than those in the input data set. The spline interpolation is a mathematical fit without physical motivation, and there is no physics involved in deriving the principal components of GSM. This may lead to spectra with unphysical spectral shapes.\(^2\)

\(^2\)Data-driven models are complementary to physically motivated models such as GMOSS, each best suited to address different science goals and challenges.
Since there are a limited number of maps (data points) available and the uncertainties in individual maps are orders of magnitude greater than the precision required for detection of the EoR, and with no physical rational to guide the order of the interpolating polynomial, we move toward the physically motivated sky model described below.

5.4.3 GMOSS: Global MOdel for the radio Sky Spectrum

GMOSS (Sathyanarayana Rao et al., 2017) is a physically motivated model of the radio sky spectrum in which seven physical parameters define the radiative processes that generate the spectrum of the brightness at each sky pixel. GMOSS incorporates plausible physics including (i) synchrotron emission that is assumed to arise from a power-law form electron energy spectrum that may have a break as well, (ii) composite emission from flat and steep spectrum synchrotron sources, (iii) thermal absorption at low frequencies, and (iv) optically thin thermal free-free emission at high frequencies. GMOSS provides synthetic spectra representative of the intensity distribution toward HEALPix pixels of 5° over the whole sky. GMOSS parameters are optimized by fits of the spectra to six all-sky maps at frequencies distributed between 22 MHz and 23 GHz, same as for the case of sky models described in Sections 5.4.1 and 5.4.2. Details of GMOSS and the goodness of the model are described in detail in Sathyanarayana Rao et al. (2017). GMOSS describes spectra of individual 5° pixels toward different sky directions using one of two forms, described below, depending on whether the pixel spectra are convex or concave.

For the case when the spectral index toward high frequencies is steeper than that toward low frequencies, GMOSS adopts an underlying model of synchrotron spectral steepening in which the frequency dependence of brightness temperature \( T(v) \) is described by

\[
T(v) = C_1 \left( v^{-2} \int_{\gamma_{\text{min}}}^{\gamma_{\text{break}}} F(x) x^{\gamma_{\text{break}}-3} d\gamma + \int_{\gamma_{\text{break}}}^{\gamma_{\text{max}}} F(x) x^{\gamma_{\text{break}}-3} d\gamma \right) + I_\nu v^{-2.1} e^{-\left(\frac{\nu}{\nu_1}\right)^{2.1}} + T_e \left( 1 - e^{-\left(\frac{\nu}{\nu_1}\right)^{2.1}} \right).
\]  

(5.4)

Here \( \alpha_1 \) and \( \alpha_2 \) denote the spectral indices of electrons over Lorentz-factor distributions in the range \( \gamma_{\text{min}} \) and \( \gamma_{\text{max}} \) with a break at \( \gamma_{\text{break}} \). \( \alpha_1 \) and \( \alpha_2 \) are both constrained to lie in the typically observed range \( 2 \leq \alpha_1, \alpha_2 \leq 3 \) and are related by the change in spectral index \( \delta \alpha = \alpha_2 - \alpha_1 \), with \( \delta \alpha \geq 0 \). \( T_e \) parameterizes the electron temperature and is constrained to be no greater than \( 10^4 \) K. \( \nu_1 \) represents the low-frequency absorption turnover where the medium becomes optically thick. \( I_\nu \) represents the optically thin thermal component that dominates toward higher frequencies. \( C_1 \) represents the normalization. \( v \) and \( F(x) \) represent...
follow standard notation in synchrotron radiation literature for which we refer the reader to Rybicki & Lightman (1986).

For the case of pixels with high-frequency spectral flattening, the model adopted is a composite of emission from steep and flat spectrum sources in which the spectrum is described by

\[ T(v) = C_1 \left( v^{-\alpha_1} + \frac{C_2}{C_1} v^{-\alpha_2} + I_x v^{-2.1} \right) e^{-\left(\frac{v}{v_T}\right)^{2.1}} + T_e \left( 1 - e^{-\left(\frac{v}{v_T}\right)^{2.1}} \right). \] (5.5)

In this model, \( \frac{C_1}{C_2} \) denotes the relative contributions from steep and flat spectrum sources with spectral indices \( \alpha_1 \) and \( \alpha_2 = \alpha_1 + \delta_\alpha \), \( \delta_\alpha \leq 0 \). Other notations and constraints are the same as those for Equation 5.4 above.

There are seven parameters describing the physical processes to fit six data points at each sky pixel, which may seem an over fit. However, the total spectra that incorporate thermal free-free and synchrotron processes simply cannot fit any arbitrary set of six measurement points for two reasons. First, the overall shape of the spectrum is strongly constrained to be of the form corresponding to the sum of free-free and synchrotron emission. Second, physically motivated constraints have been placed on the electron temperature of the thermal foreground medium (\( T_e \)) and the two spectral indices describing the synchrotron emission. Additionally, there exists interdependence between these two spectral indices: the second spectral index is parameterized as an offset from the first, while still being bound by the aforementioned constraints. Thus, the seven parameters that describe the physical models of GMOSS do not over fit the six data points at each pixel. Consistent with this expectation, GMOSS spectra do not exactly pass through all six data points and the residuals to the fit are in the ballpark of the measurement errors in the data.

The synthetic spectra from GMOSS over the 40-200 MHz band and over HEALPix pixels of 5° over the whole sky are averaged over telescope beams to yield mock observations. Residuals on fitting polynomials of varying order to such mock observations are given in Fig. (5.4a) and Fig. (5.4b) for two LSTs where the telescope beam points off and on the Galactic plane, respectively. The complexity in spectral structure due to the underlying physical processes necessitates a polynomial of order seven in \( \log(T) \) versus \( \log(v) \) space to fit the mock observations of the foreground to an accuracy sufficient to be able to limit residuals to a few mK and well below that of the EoR signal. We infer that increasing the complexity of the model that describes the foreground demands, unsurprisingly, higher order polynomials to model the mock observations to sufficient accuracy for detection of the EoR. This trend is shown in Fig. (5.5), which plots the root mean square (RMS) of
Fig. 5.4 Residuals on fitting mock spectra, generated from a model that assumes GMOSS as the sky model, with polynomials of increasing orders. The panel on the left are residuals obtained for a mock observation that is away from the Galactic plane and the one on the right is for a mock spectrum that is toward the Galactic plane.

residuals against the polynomial order; these residuals were obtained on fitting mock observations generated assuming the three different sky models. When the order of the fitting polynomial is increased to a sufficiently large value, all curves will eventually reach an asymptotic RMS level corresponding to the numerical noise inherent in the mock spectra. The solid lines represent residuals corresponding to spectra away from the Galactic plane and the dashed lines are for those looking at the Galactic plane. Although at first glance a low RMS residual might be encouraging, it may be noted that increasing the order of the fitting polynomial would also fit to the EoR signal, thus subsuming the EoR signal into the estimate of the foreground and hence compromising the detection. This is demonstrated by fitting polynomials of varying order to mock observations that also contain the generic vanilla model for the EoR signal. GMOSS is adopted for describing the foreground sky spectrum since it is physically motivated and most realistic. The residuals in this case are shown in Fig. (5.6). Increasing the order of the polynomial changes the form of the residual: the peak amplitude of the residual progressively diminishes and the number of turning points in the residual increases, and thus the residual progressively departs from the form of the generic vanilla model for the EoR signal. To fit and subtract foregrounds or for joint modeling of the foreground with the EoR signal, without compromising the information contained in the EoR signal and preserving the turning points and amplitude of the signal, we propose below that foreground components of spectral radiometer measurements be modeled not with polynomials but with Maximally Smooth (MS) functions.
Modeling the Radio Foreground for detection of CMB spectral distortions from Cosmic Dawn and Epoch of Reionization

Fig. 5.5 rms of residuals on fitting mock observations of foregrounds, which are generated using different sky models, versus the order of the fitting polynomial. Solid lines are for mock observations that are recorded away from the Galactic plane and the dashed lines are for those that are toward the Galactic plane.

Fig. 5.6 Residuals obtained on fitting a mock observation of the sky spectrum, which uses GMOSS as the sky model and also contains the EoR signal, with polynomials of orders 4–7. Overlaid as a solid black line is the generic EoR signal. Increasing the order of the polynomial used, in log($T$) vs. log($n$) space, to model and remove the foreground component of the spectrum results in a residual that progressively departs from the generic EoR signal. Not only does the amplitude of the EoR signal reduce, additional turning points are also introduced.
5.5 Modeling the foreground using MS functions

We hereinafter adopt GMOSS as the sky model due to its physically motivated nature. As described above and in Sathyanarayana Rao et al. (2017), GMOSS describes individual pixels (5° wide) to have varying spectral shapes such as concave, convex, or of more complex form. We now examine whether beam-averaged mock spectra generated using GMOSS are smooth in the sense that they do not have embedded small-amplitude ripples that may confuse a detection of fine-scale cosmological signals from reionization.

We attempt to fit the mock spectra using MS functions (see Sathyanarayana Rao et al., 2015). An MS function \( f(x) \) is a polynomial of degree \( n \)

\[
f(x) = p_0 + p_1(x-x_0) + p_2(x-x_0)^2 + p_3(x-x_0)^3 + p_4(x-x_0)^4 + \ldots + p_n(x-x_0)^n \tag{5.6}
\]

in which the polynomial coefficients \( p_j \) are constrained so that there are no zero crossings within the domain for any derivative of order \( m \geq 2 \). The order-\( m \) derivative of the polynomial is

\[
\frac{d^m f(x)}{dx^m} = \sum_{i=0}^{n-m} \{ (m + i)! / i! \} p_{m+i}(x-x_0)^i \tag{5.7}
\]

and the coefficients \( p_j \) are constrained so that for all \( m \) in the range 2, 3, 4, ..., \( (n-1) \) the above derivative functions are never zero within the domain of interest. Thus the smooth component in a data set is described by an MS function with its coefficients optimized by minimizing the chi-square (\( \chi^2 \)) between data and the MS functional form:

\[
\chi^2 = \frac{\left( \sum \left[ \log_{10}(x/x_0)^i \right] p_i - \text{data} \right)^2}{\text{number of data points}}. \tag{5.8}
\]

Coefficients \( p_i (i \geq 2; \ i \in \mathbb{Z}) \) are disallowed from changing signs within the domain of the spectrum and hence the fitting polynomial is consistent with the condition for smoothness.

Critical to the modeling is the formulation of a robust method for solving for the parameters: the MS function is defined in log-temperature versus log-frequency space. In this space, the function is written as an expansion about \( \log(x/0) \), where \( x_0 \) is a pivot frequency. The coefficients of the expansion are constrained so that there are no zero crossings in the second derivative and higher derivatives. Beginning with a low-order polynomial \( f(x) \), the order \( n \) of the MS function is incremented by unity in each iteration and the parameters returned in the previous step are used as initial guess in the next.

Constraining the polynomial in this manner while fitting to the measured sky spectrum
Fig. 5.7 Residuals on fitting the global EoR signal with varying orders of (a) polynomials and (b) MS functions; fits are made in log($T$)-log($n$) space to the assumed global EoR signal plus CMB with temperature $T_0 = 2.73$ K. Residuals to polynomial fits substantially differ from the EoR signal assumed, displaying reduced amplitude as well as multiple turning points. On the other hand, fitting the EoR signal with an MS function of arbitrary order preserves turning points and after the smooth component of the signal is entirely removed, the residual saturates and does not change any further on increasing the order of the MS function as evidenced by the curves of all residuals lying perfectly one above the other. Shown as a black solid line in both panels is the global EoR signal that is being fit to. A polynomial of order at least 7 is adopted as is necessitated by GMOSS, as described in Section 5.4.3.

in log($T$)-log($n$) space allows the function to fit to the mean spectral index, a constant spectral curvature and higher order curvatures without allowing the polynomial to follow any ripple or multiple turning points in embedded spectral components. As a first step, to demonstrate that there is no loss in the EoR signal on modeling the foreground using an MS function, we examine the residuals on fitting the adopted generic form of the EoR signal with MS functions and compare with residuals on fitting them with polynomials that have no constraints on coefficients. We present in Fig. (5.7) residuals on fitting the global EoR signal, which has multiple turning points over the 40–200 MHz band and is hence not MS, with polynomials and MS functions of varying orders. We add the CMB monopole temperature to the global EoR signal and then fit the resulting spectrum in log($T$)-log($n$) space. Since foregrounds generated using GMOSS as the sky model require at least a seventh order polynomial in log($T$)-log($n$) space to fit to the accuracy needed for global EoR detection, we fit foregrounds with polynomials of orders 7, 10 and 20. As shown in Panel (a) of Fig. (5.7), modeling the EoR signal with polynomials results in turning points being introduced in the residual, which were not present in the EoR signal itself. As a consequence, the shape of the residual is no longer qualitatively similar to the cosmological signal. Further, the amplitude of the residual progressively reduces and if fit with a sufficiently high-order polynomial the signal risks being subsumed in any polynomial model for the foreground and hence being entirely fitted out. As a comparison, residuals on modeling the same spectrum with MS functions of orders 7, 10 and 20 are
shown in Panel (b) of Fig. (5.7). Clearly, the residual obtained on fitting MS functions retains all the turning points of the EoR signal. The residual is similar but not identical to the EoR signal itself and represents a smooth baseline subtracted version of the signal. Additionally, increasing the order of the MS function does not deteriorate the amplitude of the signal in the residual once all the smooth components in the data have been removed. This suggests that if the only non-smooth component in an EoR detection experiment is the signal itself, fitting an MS function of arbitrarily high order should leave behind in the residual a distinctive signature of the EoR signal with all turning points intact and with minimal degradation of signal strength.

As a second step, we examine whether beam-averaged mock sky spectra generated using GMOSS as the sky model are indeed maximally smooth, at least to the precision needed to detect an embedded generic EoR signal. We generate mock sky spectra at different observing sites to include different parts of the sky, and hence investigate the smoothness of spectra over different samplings of sky coverage and spectral complexity. At any observing site, we generate 24 mock sky spectra one hour apart and with a uniform sampling over LST. The observing sites adopted for the simulations were (a) Hanle in North India where the Indian Astronomical Observatory is situated (Latitude = 32.79°N) and (b) the Murchison Radio Observatory (MRO) site in Western Australia (Latitude = 26.68°S). With large antenna beams as are typically employed by most global EoR detection experiments (see Raghunathan et al., 2013), the Galactic plane will come into the beam at some time in the day. Mock sky spectra were generated by simulating observations with (a) an ideal frequency-independent $\cos^2$ beam pattern that has maximum toward zenith (see Figure 5.8a) and (b) adopting the short-monopole type beam pattern of the SARAS\textsuperscript{3} antenna that has null toward zenith, null toward horizon and maximum toward elevation of about 30° (see Figure 5.8b). The choice of observing sites, close to +30° and –30° of latitude, and beams that are contrasting in shapes, was made to have a wide variety in plausible spectra. The final calibrated mock spectra in each case are generated over the 40–200 MHz band and have, in addition to the foreground spectrum, contribution from a Planck form CMB with no spectral distortions.

Though in the frequency range of interest (40–200 MHz) the difference between the Planck form and its Rayleigh-Jeans approximation is about 1 Jy sr\textsuperscript{−1} in specific intensity, we still explicitly fit to the CMB using its Planckian form to avoid any spectral shapes that may arise from approximately modeling the CMB as a constant in brightness temperature.

\textsuperscript{3}see http://www.rri.res.in/DISTORTION/saras.html
Fig. 5.8 Frequency-independent antenna beam patterns adopted in simulations of mock sky spectra. The \( \cos^2 \) beam pattern in panel (a) represents a beam with a peak toward the zenith; this is the radiation pattern of a conical spiral antenna. The SARAS 2 beam in panel (b), on the other hand, has a null toward the zenith and a peak toward an elevation of 30°; this is the radiation pattern of a short monopole. Both beams have a null toward the horizon. The beam shapes have been chosen to be complementary to one another and provide different beam-weighted samplings of the sky.

Thus the functional form used to describe the mock spectrum is

\[
T(v) = \frac{h \nu}{k} \left( \frac{e^{h \nu/kT} - 1}{10^{\sum_{i=1}^{N} \log_{10}(v/p_i)}} \right) + 10^{\sum_{i=1}^{N} \log_{10}(v/p_i)} p_{i+2}.
\]  

(5.9)

Here, \( p_0 \) is the CMB temperature, \( p_1 \) is the frequency about which the expansion is centered, and \( p_2 \) through \( p_{2+N} \) are the coefficients of the terms of the \( N^{th} \)-order MS function that models the foreground. The remaining terms follow standard notation.

The mock spectra representing the foreground and undistorted CMB at different LSTs and for different sites and telescope beams were all uniformly fit with MS functions of order 10. Sample spectra corresponding to LSTs when the antenna temperatures are minimum and maximum, the corresponding sky coverage by the telescope beam for each of these spectra, and the residuals on fitting these spectra with the function given by above Equation 5.9, are shown in Figures 5.9, 5.10, 5.11 and 5.12. The sky-coverage plots give the all-sky map at 150 MHz with a resolution of 5° such that pixels that lie outside the beam are blanked. To examine for just the smoothness of foregrounds, these mock spectra have not had the EoR signal added.

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4The pixels within the beam are weighted by the response of the antenna; this is not shown in the sky images displayed.
5.5 Modeling the foreground using MS functions

(a) \( \cos^2 \) beam Hanle off Galactic plane

(b) \( \cos^2 \) beam Hanle on Galactic plane

(c) Spectrum off Galactic Plane

(d) Spectrum on Galactic Plane

(e) Residual

(f) Residual

Fig. 5.9 Testing the smoothness of foregrounds in mock sky spectra that are simulated using GMOSS as the sky model for the sky over Hanle (Latitude = 32.79°N). The first column shows the pixels that lie in the antenna beam used to generate spectra; these are displayed on an all-sky map at 150 MHz with a resolution of 5°. Pixels that lie outside the beam are blanked. The spectra themselves are shown in the second column. The third column shows the residual on fitting these spectra with the form given by Equation 5.9, which represents our adopted Maximally Smooth model for the foreground. Spectra are simulated for a frequency-independent beam with a cosine\(^2\) pattern. Two spectra at the LSTs in which they have maximum and minimum foreground brightness are shown as examples. Three figures that follow show corresponding plots for the sky at other locations and for different assumed telescope beams.
Fig. 5.10 Same as Fig. 5.9, but with mock sky spectra simulated using GMOSS for the sky over MRO (Latitude = 26.68°S). A frequency-independent beam with cosine^2 pattern. Two spectra at the LSTs in which they have maximum and minimum foreground brightness are shown, as examples.
Fig. 5.11 Same as Fig. 5.9, but with beam adopted to be that of the SARAS 2 short monopole antenna.
Fig. 5.12 Same as Fig. 5.10, but with beam adopted to be that of the SARAS 2 short monopole antenna.
If the residual on fitting these spectra as the sum of a Planckian and MS function is at the level of a few mK, the foregrounds may be considered to be smooth at a level required to discern the EoR signal. This would provide impetus to use the MS function to separate foregrounds from the more complex cosmological signal. However, if the foreground spectra are themselves not smooth, the residuals in this exercise would be large, possibly larger than the EoR signal, which would indicate that there may potentially be spectral structure in the foregrounds that may limit or prohibit detection of the EoR signal, particularly if adopting signal extraction strategies that assume smooth foregrounds. Inferences from the above exercise, whose results are presented in Figures 5.9, 5.10, 5.11, and 5.12, are summarized below:

1. When mock observations of sky spectra are generated using GMOSS and made with wide frequency-independent beams and over the entire LST range, MS functions are capable of accurately modeling the shape of the foreground component to leave residuals less than 5 mK, demonstrating that the foreground spectrum is indeed smooth to such a precision. It follows that if the mock spectrum contains the generic EoR signal, which is inherently not smooth over the 40–200 MHz wide band, the residual on fitting such a spectrum with an MS function would be expected to be dominated by the EoR signal, with a smooth baseline subtracted. The smooth foreground would be largely removed and thus separated from the EoR signal. This expectation, which is based on the above analysis, is demonstrated below in Section 5.6.

2. Spectra that have relatively lower contamination from the Galactic plane and Galactic center are fit by MS functions to greater absolute precision and yield residuals that have RMS of \( \sim 0.01 \) mK, which is substantially lower than the amplitude of the expected EoR signal. However, spectra at LSTs at which there is relatively large contribution from the Galactic plane and Galactic center yield residuals with RMS in the range 2–5 mK when fit with MS functions. As discussed in Sathyanarayana Rao et al. (2017), spectra toward the Galactic plane indeed have more complex spectral shapes.

Figure 5.13 shows the RMS of residuals obtained on fitting MS functions to model foregrounds versus the mean temperature of the corresponding spectrum. This is shown for the four sets of data corresponding to the two sites and two beams described above. The mock spectra were synthesized at separations of one hour and distributed over the entire 24 hr in LST. Clearly there exists a correlation between the mean temperature of the spectrum and the RMS of the residual.

3. In the case of spectral measurements that are made with beams well off the Galactic center, with minimal contribution from the Galactic plane, the mean temperatures
Fig. 5.13 Mean temperature of mock spectra versus LST is shown in panel (a). The RMS of residuals on fitting MS functions to these spectra is shown in panel (b). The RMS residuals for spectra simulating observations with a \( \cos^2 \) beam at Hanle and MRO are shown as filled circles and filled stars, respectively. The RMS of residuals for spectra simulating observations with the beam of the SARAS 2 antenna, at Hanle and MRO, are given by filled squares and triangles, respectively. Clearly, the RMS residuals are larger for spectra that are brighter, which are those having relatively greater contribution from the Galactic plane. A spectrum recorded at MRO by an antenna with \( \cos^2 \) beam would have a maximum temperature at the LST when the Galactic center is at zenith. This also corresponds to the spectrum having the largest RMS residual amongst the various mock spectra considered here.
are low and the corresponding residuals have an RMS lesser than 1 mK. We find that an eighth-order MS function is sufficient to fit the foreground to a level that is lower than most predicted EoR signals. Therefore, provided observations are made at LSTs where Galactic contamination is relatively low, the foreground will be removed to relatively high precision. Toward these sky directions, any global EoR signal in a class of models similar to the vanilla model, with multiple inflections in the band, is expected to be detectable in the residual.

4. On the other hand, for spectra toward regions of the sky that are intrinsically brighter, such as those toward the Galactic center, the RMS of the residual on fitting MS functions is higher. The highest RMS is expected for spectra recorded at MRO with a $\cos^2$ beam, when the Galactic center transits nearly overhead. This is despite fitting with an MS function of order 10 and there is no significant change in the residual on increasing the order of the MS function to even 20. However, the RMS is at worst a few mK, which implies that even toward these directions that have significantly larger brightness, generic models of the EoR signal can nevertheless be usefully distinguished from the foreground. On the other hand, since these regions do have relatively brighter emission with more complex spectral shapes, and the fitting process is computationally challenging, it is best to avoid the Galactic plane ($\pm 10^{\circ}$) and the Galactic center while attempting to detect the weak global EoR cosmological signal.

For various large-area samplings of the sky, we have thus demonstrated that the foreground component in a beam-averaged spectrum is adequately describable by an MS function. Thus for observations with a frequency-independent beam, MS functions can be used to separate foregrounds from the global EoR signal. We next discuss mathematical statistical detection based on MS-form modeling of foregrounds.

### 5.6 On the detection of the global reionization signal

A mock observation of the radio sky as observed by a $\cos^2$ beam is shown in Fig. (5.14). A GMOSS model for the sky has been adopted and CMB with a spectral distortion corresponding to generic global EoR has been added. We fit this mock observation of the sky spectrum with a function given by the sum of an MS function to model the foreground and a Planck spectrum to account for the undistorted CMB, as described by Equation 5.9. We use the downhill simplex (Nelder & Mead, 1965) optimization algorithm to iteratively fit this model to the synthetic spectrum, adopting a successive approximation strategy. The MS polynomial is expressed as a polynomial expansion about a frequency given by the parameter $p_1$. The first iteration fits a function that has four parameters: the CMB temperature $p_0$, the pivot frequency $p_1$ and two coefficients that describe a single power law for the
Modeling the Radio Foreground for detection of CMB spectral distortions from Cosmic Dawn and Epoch of Reionization foreground. We successively include more terms using the optimized parameters from the previous iteration as initial guess and zero for the new coefficient introduced. The residuals on subtracting a model with a Planckian function and MS functions of degree 2, 5, 7, 8, 10, and 15 are also shown in Fig. (5.14). This may be compared to the residuals obtained above on fitting unconstrained polynomials to the same mock observation containing the EoR signal, which was shown in Fig. (5.6). As noted earlier, for the case where the foregrounds were modeled as unconstrained polynomials, the residual amplitude gets progressively lower with increasing polynomial order and there are additional turning points introduced. On the other hand, while using MS functions, the residual remains unchanged once the MS function encounters a shape that cannot be described as smooth and thenceforth increasing the order of the MS function no longer changes the residual. This would continue to be the case on fitting the spectrum with MS functions of arbitrarily higher order. The residual recovered on fitting the mock spectrum with such components that may accurately model the undistorted CMB and foreground appears qualitatively similar to the adopted global EoR signal, which was included in the simulation. We now proceed to quantify the confidence in signal detection and the likelihood of false positives.

We adopt a Bayes Factor (BF) approach to quantify signal detection as described in Sathyanarayana Rao et al. (2015). The BF is the Bayesian equivalent of the Maximum likelihood of detection in frequentist statistics. Given a dataset $D$, BF gives the more probable of two models $M_1$ and $M_2$ with parameters $\theta_1$ and $\theta_2$ respectively. BF is expressed as

$$BF = \frac{P(D|M_1)}{P(D|M_2)} = \frac{\int P(\theta_1|M_1)P(D|\theta_1,M_1)d\theta_1}{\int P(\theta_2|M_2)P(D|\theta_2,M_2)d\theta_2}. \quad (5.10)$$

For the case where both the models are equally likely, the ratio of priors is unity. It is assumed that it is as likely that an EoR model is present in an observation as not, and all plausible EoR models are a priori equally likely. If the outcome of the test is that model $M_1$ is more likely than $M_2$, then the BF is expected to be large (greater than unity) and if $M_2$ describes the data better than $M_1$, then BF is expected to be small (less than unity).

The likelihood for any model $M$ is given by

$$P(D|M) = \prod_{i=1}^{N} \frac{e^{-y_{\text{res},i}^2 / 2\sigma^2}}{\sqrt{2\pi \sigma^2}}, \quad (5.11)$$

where $N$ is the number of independent points across the spectrum and $y_{\text{res},i}$ is the residual spectrum following subtraction of the corresponding model from the data $D$. The variance of the measurement noise, $\sigma^2$, is assumed to be half that estimated by differencing neighboring channels. The resulting expression is similar to the functional
5.6 On the detection of the global reionization signal

Fig. 5.14 (a) Mock spectrum generated on observing the GMOSS sky with a frequency-independent $\cos^2$ beam. The spectrum also contains CMB with a distortion given by the vanilla model of the global EoR signal. (b) Residuals obtained on fitting the spectrum in panel (a) with a Planck function plus MS functions of degree 2, 5, 7, 8, 10, and 15. Once the MS function has fit out the bulk of the foreground and reduced its residual to well below the EoR signal, the residual no longer changes on increasing the order of the MS function. This residual essentially retains a large part of the EoR signal, with all turning points preserved.
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form used to describe the non-Gaussian phase noise in clocks, referred to as Allan Variance and is given by the relation

$$\sigma^2 = \frac{1}{2(N-1)} \sum_{i=1}^{N-1} \left( y_{\text{res}}[i+1] - y_{\text{res}}[i] \right)^2. \quad (5.12)$$

Computing variance via Allen variance provides an estimate that is insensitive to residual signals that may be present following subtraction of the model.

In this work, for the purpose of demonstrating the value of MS function approach to modeling the foreground, we consider only the vanilla model for global EoR and use the BF test to compare between two models $M_1$ and $M_2$ that correspond, respectively, to a function describing foreground plus CMB plus EoR and a second function that models only the foreground and undistorted CMB:

$$M_1: \quad T(n) = \left( \frac{h \nu}{k} \right) / \left( e^{\nu/n_0} - 1 \right) + 10^{\sum_{i=0}^{N} \log_{10}(\nu/p_1)[i]} p_{i+2} + y_i(\nu) \quad (5.13)$$

and

$$M_2: \quad T(n) = \left( \frac{h \nu}{k} \right) / \left( e^{\nu/n_0} - 1 \right) + 10^{\sum_{i=0}^{N} \log_{10}(\nu/p_1)[i]} p_{i+2}. \quad (5.14)$$

Here, $y_i(\nu)$ is the template of the EoR signal at frequency $\nu$ and the remaining terms are the same as in Equation 5.9.

Consider two mock spectra that might separately represent the dataset $D$, one that contains the EoR signal and the other that does not. For the two models $M_1$ and $M_2$ defined above, we would expect the BF for the spectrum that contains the EoR signal to be larger than unity and that for the spectrum that does not contain the signal to be less than unity. However, this may not necessarily be the case in the presence of noise. For large noise and poor signal to noise ratio, it is statistically possible that the particular mock spectrum that does not contain EoR erroneously yields a large value for BF. This would lead to a mistaken conclusion that the EoR signal is indeed present in the data when there is none, thus giving a false positive detection. On the other hand, it is also statistically possible for the spectrum containing the EoR to yield a small BF leading to the erroneous interpretation that the signal is not present in the measurement; a false negative. Both such incorrect inferences are best avoided by observing for sufficient time and attaining adequate sensitivity. An examination of the distribution of BFs for varying amounts of noise, or equivalently the integration time, can guide the interpretation of the BF by assigning appropriate confidence to outcomes of experiments.
We generate two data sets ‘a’ and ‘b’ of 100 mock spectra each. While spectra in dataset ‘a’ contain the vanilla model of the EoR signal added to them, spectra in dataset ‘b’ do not. We also introduce Gaussian random noise in spectra of both datasets. The noise corresponds to that in a correlation spectrometer measuring a difference spectrum between the antenna temperature and reference load; we assume a scheme similar to that adopted in Patra et al. (2015). The variance $\Delta T$ of the added Gaussian noise is given by

$$\Delta T = \sqrt{\frac{(5/4)(T_a + T_{\text{ref}})^2 + (T_a + T_{\text{ref}})(T_{n1} + T_{n2}) + T_{n1}T_{n2}}{2\Delta v\Delta t}}.$$ (5.15)

Here, $T_a$ is the antenna temperature, $T_{\text{ref}}$ is the temperature of a reference load that serves to provide a baseline against which the antenna temperature is measured, $T_{n1}$ and $T_{n2}$ give the effective amplifier noise temperatures in the two paths of the receiver that feed into the correlation spectrometer. We assume values $T_{\text{ref}} = 300 \, \text{K}$, $T_{n1} = T_{n2} = 50 \, \text{K}$. $T_a$ in each channel of bandwidth $\Delta v = 1 \, \text{MHz}$ is given by the noise-free antenna temperature of the mock spectrum in the corresponding channel. Spectra with varying amounts of noise are generated by varying the integration time $\Delta t$.

For different integration times, the simulation thus yields sets of 100 mock spectra each with and without EoR signal added, with different noise realizations. For every spectrum from each of the two data sets we obtained two residuals by fitting models $M1$ and $M2$ given by Equations 5.13 and 5.14 respectively. The corresponding likelihoods are computed using Equation 5.11 and these are then used to compute the BF for the said spectrum using Equation 5.10. We next determine the median and range in which the BF$s$ for each integration time are expected to lie for a given confidence level. For a given integration time, this range for the BF$s$ is wider for a higher confidence level and narrower for a lower confidence level.

If the ranges of BF$s$ for the data sets ‘a’ and ‘b’ for any integration time have significant overlap, then a result that is a BF from the overlap domain would be uncertain and results that are outside this overlap domain would be more conclusive. If the detection strategy is a useful one, the BF ranges would be increasingly disjoint for decreasing noise and increasing integration time, thus giving a high confidence level of detection, high rejection of false positives, and low likelihood of false negatives.

The median and range in BF$s$ for the pair of data sets were computed for three different integration times. In Fig. (5.15a) we show these ranges for detection with 75% confidence and delineate the divergence in the distributions with increasing integration time. The median values of the distributions are marked using filled circles about which the range is presented as a red shaded region for spectra from data set ‘a’ and as green shaded
region for spectra from data set "b." In order to represent regions with 75% confidence in this panel (a), the width is chosen to be the region in which 75% of the BF samples lie. For greater confidence in any detection, we include a correspondingly greater fraction of samples in the delineated regions and the regions would diverge at correspondingly greater integration times. For example, Fig. (5.15b) is for detection with 95% confidence. Since increasing the MS function to arbitrarily high orders does not change the residuals, as demonstrated by Fig. (5.7b), increasing the order of the MS function does not change the results presented in Fig. (5.15). The above approach that uses MS functions to model the foreground and BFs to arrive at an interpretation may be compared with the equivalent BF distribution plots for the case where the foreground component in the models M1 and M2 are described using unconstrained polynomials instead of MS functions. The order of the polynomials is fixed to be 7 as discussed in Section 5.4.3. This is the necessary and sufficient order for polynomials to fit foregrounds that are generated using a GMOSS sky model. The resulting BF ranges for 95% confidence when using polynomials of order 7 and 20 are shown in Fig. (5.16).

In both cases the ranges of BFs for data sets with and without the EoR signal diverge at integration times larger than those for the case when using MS functions. Firstly, fitting data with such polynomials degrades the SNR by fitting out a substantial part of the EoR signal itself along with the foreground. The functional form of the foreground spectrum is a priori unknown and using a polynomial of arbitrary order degrades the SNR. This downside is all the more severe when polynomials of high orders are adopted to describe the foreground. Clearly, the integration time required to distinguish between the presence or absence of the generic EoR signal using a BF test is larger when a polynomial of order 20 is used compared to the case when a polynomial of order 7 is used, which is in turn larger than the case when MS functions are used. Second, residuals of such fits with high-order polynomials may well mimic plausible EoR signal shapes, hence leading to degeneracy between the EoR signal and residuals produced on fitting spectra with polynomials. Thus the ability to distinguish between the presence or absence of the EoR signal is compromised by modeling the foregronds as polynomials of high order as opposed to the more robust method of using MS functions to model foregrounds. The integration time beyond which the ranges for the BFs with and without EoR signal cease to overlap is the integration time required for detection of EoR with that confidence. If we assume that we use the BF method to detect the presence of the vanilla model of global EoR, and compare the two hypotheses corresponding to the presence of the vanilla model versus absence of any EoR signature, we may examine the confidence level at which the BF distributions diverge versus integration time. The resulting curve is shown as a solid line in Fig. (5.17). We see that using a BF test with MS functions to model foregrounds, it is possible to detect the presence of the EoR signal with 95% confidence in 10 minutes effective integration
5.6 On the detection of the global reionization signal

Fig. 5.15 Distribution of Bayes factors versus integration time for spectra containing the EoR signal (red shaded region) and spectra without the EoR signal (green shaded region). The median Bayes factors are shown using filled circles. The point at which the two shaded regions diverge represents the integration time required to distinguish between the presence and absence of the signal in a measurement with a certain confidence. The confidence is given by the width of the shaded regions about the median value. For detection with greater confidence, the regions diverge later, i.e., at greater integration times. It may be noted that the models used to describe the spectra for BF estimation employ MS functions of order seven to fit the foreground component.
Fig. 5.16 Distribution of Bayes factors versus integration time for spectra containing the EoR signal and spectra without the EoR signal. The median Bayes factors are shown using filled circles. The point at which the two shaded regions diverge represents the integration time required to distinguish between the presence and absence of the signal in a measurement with 95% confidence. The red and green shaded regions represent Bayes Factor distributions on using a polynomial of order 7 to describe foregrounds. The black and blue regions denote Bayes Factors using a polynomial of order 20 to describe foregrounds. As expected, increasing the order of polynomial results in the two sets of Bayes Factors (with and without the EoR signal) diverging at larger integration times. This is because the signal-to-noise ratio degrades as more of the signal is erroneously subsumed in the foreground as the order of the fitting polynomial is increased.
5.7 Conclusions and Summary

Toward detection of the global redshifted 21-cm signal from reionization, we have done simulations of spectral radiometer observations in the 40–200 MHz band with wide-field antennas. The specific aim has been to study the spectral shapes expected for the foreground, whether these might confuse or limit detection of the expected wide-band cosmological signatures. This work is an improvement over previous studies in that we examined the efficacy of modeling the foreground as MS functions for the case of plausible spectral shapes as given by the physically motivated sky model GMOSS. The new approach to global EoR detection has been compared with previous modeling of foregrounds that used polynomials in log(\(T\)) versus log(\(n\)) space. Mock data corresponding to contrasting beam time using a correlation spectrometer with system parameters as described above. It may be noted here that this result may likely apply to other models that are similar in nature to the vanilla model of the EoR signal, which has multiple turning points in the frequency range of 40–200 MHz, and has a shape that is distinct and distinguishable from smooth foregrounds using MS functions.

The BF test proposed here considers a statistical detection of the signal over the entire bandwidth used, in this case 40-200 MHz. In other words, the BF test represents a cumulative likelihood for detection considering the entire observing band and all spectral channels in unison. This results in an observing time that is much smaller than estimated using the radiometer equation over the resolution bandwidth of the spectrometer used (1 MHz); the relevant bandwidth is the effective bandwidth of the signal and depends on the precise signal shape or distribution over the band. For the corresponding case of modeling of foregrounds using polynomials of order 7 and 20, the confidence in detection of signals and in rejection of false positives are given as a dashed-dotted line and dashed line, respectively in Fig. (5.17). We see that if a BF test is done using a polynomial of order 7, an effective integration time of at least 18 minutes is required to detect the presence of the EoR signal with 95% confidence, which is double the time required when using MS functions. This number increases to 72 minutes when using a polynomial of order 20. Thus, choosing a polynomial of different orders can result in different confidence in detection for the same integration time with the same instrument. The interpretation as signifying either the presence or absence of the EoR signal in a measurement set can critically depend on the order of the polynomials used to describe foregrounds or system parameters, unless supported by other methods to quantify signal detection. Additionally and more importantly, MS functions are robust in that even with increasing order they do not subsume EoR models with inflections; therefore, it is preferable to use MS functions to model foregrounds and distinguish the cosmological 21-cm signal.
Fig. 5.17 The confidence in detection of a signal using the BF test versus integration time. The BF test that employs models using MS function to fit to the foreground component in spectra is shown as a solid green line. For noise as given by Equation (5.6), corresponding to a correlation spectrometer scheme described in Patra et al. (2015), an effective integration time of 0.16 hours or 10 minutes results in a 95% confidence detection of the vanilla model global EoR signal, along with the rejection of false positives with the same confidence. Also shown are confidence in detection using the BF test for models that use polynomials of order 7 (blue dashed-dotted line) and order 20 (dashed red line) to model the foreground component in spectra. As seen from the figure, increasing the order of the polynomial used results in a larger integration time required to reach the same level of confidence; we interpret this as being due to the degradation in the signal-to-noise ratio since more of the global EoR signal is subsumed in the foreground model. For a polynomial of order 7, an integration time of 18 minutes results in a 95% confidence detection and also confidence in rejection of false positives. For the case of an order 20 polynomial, this increases to 72 minutes. In both cases the integration time required is larger than when using MS functions to model the foreground.

shapes, site latitudes, and the full range of observing sidereal times have been considered. In this work we do not consider complications of instrument systematics such as residual errors in bandpass calibration and mode coupling of sky structure to spectral confusion via a frequency dependent beam.

The work leads to the conclusion that wide-band wide-field sky spectra as measured by well-calibrated radiometers attached to frequency-independent antennas are expected to be MS to the precision necessary for detection of vanilla or generic predictions for the global EoR signal. We have demonstrated that modeling foregrounds using Maximally Smooth functions is advantageous for the detection of global Epoch of Reionization signals that have multiple turning points in the observing frequency band. The MS function can
selectively fit to any smooth foreground and CMB with arbitrary precision by allowing for a sufficiently large order for the fitting function, despite the high order such a modeling would leave a residual that contains most of the global EoR signal. Most importantly, the residual following MS function fitting preserves turning points that define the critical epochs in the cosmological evolution of the baryons during First Light or the Cosmic Dawn and reionization.

The MS-function approach to modeling the foreground has been demonstrated to be amenable to a successive approximation type of solution for the model parameters. The MS-function coefficients may be solved for iteratively by increasing the polynomial order successively while progressively approximating the foreground with greater accuracy. There is no loss of signal in the residual if the order of the MS function were increased arbitrarily and hence the approximation may be continued to degrees well beyond what is needed and until the higher order fitting coefficients are found to be negligible. The amplitude of the EoR features is thus largely recovered and, unlike in the case of the hitherto adopted polynomial fitting approach, the amplitude is undiminished with increasing order of the fitting function.

The MS-function modeling approach suggested herein may be viewed as a filter that progressively and in successive approximations removes or filters out the smooth component of the data while leaving behind the non-smooth component, for increasing orders of the MS function. After having filtered out all the smooth components of the signal, further increasing the order of the MS function no longer affects the signal, resulting in a "saturated residual." The demonstrated success in reducing residual to below EoR signal detection levels by fitting foregrounds with MS functions, and the success in accurately modeling the foregrounds for mock observations corresponding to different sky directions, site latitudes, telescope beam types, and observing LSTs demonstrates the robustness of the MS-function approach as opposed to simple polynomial fits.

Our work that uses the physically motivated GMOSS sky model suggests that the foreground is indeed MS to the required precision for detection of global EoR. This also suggests a path forward for further study involving system design where bandpass distortions arising from internal reflections are constrained to be ‘smooth’ and representable by MS functions. Nevertheless, we conclude by briefly discussing the consequence of non-smooth foreground components, which may manifest as spurious residuals following MS modeling of spectral data, particularly in observations made toward the Galactic Center and plane. If the foreground spectrum is itself not smooth at a level above the EoR signal, fitting the sky spectrum with an MS function will fail to fit the non-smooth part of the foreground, and the EoR signal will appear confused in this foreground residual. This consideration leads to the question that if an observation of the sky spectrum is fit with an MS function of a high order, and if the residual is not diminished any further by increasing...
the order of the MS function, then how do we distinguish between residuals that are EoR signals from any non-smooth foreground? A solution to this problem may be found in joint analysis of sky spectra observed separated in LST so that they contain different samplings of the sky, and hence different foreground spectra. Saturated residuals that differ between different regions of the sky is indicative of spectral structure in foreground spectra and a changing residual may be used to “discover foreground components” and hence separate this non-smooth part of the foreground from global EoR (Switzer & Liu, 2014). However, if the saturated residuals toward different regions of the sky after fitting with MS functions of arbitrarily high order are the same in structure and amplitude, it would be highly suggestive that the residual is global in nature, making a case for EoR signal detection. Comparing the residual with smooth baseline subtracted templates, which may be obtained by subtracting MS functions from predictions for global EoR signals in various models, would provide the ability to distinguish between EoR scenarios.

We have presented a BF approach to estimate the integration time necessary to detect with varying levels of confidence the presence of global EoR signal in sky spectra, when MS functions are used to model the foreground. We have also compared with the case where polynomials are used to model the foreground. We have found that a fit to recorded sky spectra using an MS function provides the advantage of signal detection with improved signal-to-noise ratio as compared to the polynomial case. A receiver noise temperature of $\sim 50$ K, as has been assumed in our analysis above, is achievable. However, making a frequency-independent antenna over multi-octave bandwidths is non-trivial. Nevertheless, with such a well designed antenna and spectral radiometer, our analysis indicates that with about 10 minute effective integration the global EoR signal may be detected with 95% confidence using MS functions.

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CONCLUDING REMARKS

In this thesis I have studied aspects of detecting fine scale distortions in the Cosmic Microwave Background spectrum. I have focussed on CMB spectral distortions arising from the epochs of recombination and reionization.

6.1 Choice of sky model

I have synthesised mock sky spectra to investigate various aspects of signal detection such as feasibility, nature of foregrounds and foreground separation. The sky model assumed is critical to synthesising realistic foreground spectra and making any useful deductions from them. Existing models of the radio-sky that are used to simulate foregrounds are based on mathematical interpolation of sky temperatures at widely spaced frequencies and do not have any physical basis. Such models that are physically uninformed will not serve the end purpose of simulating spectra that are realistic representations of the true sky.

Mock-spectra for the cosmological recombination lines, over an octave band in the frequency range of 2–6 GHz, are simulated by beam-averaging pixel spectra assuming a sky model that is a power-law at each pixel. This is in keeping with the continuum emission expected from most radio sources that has a form which is closely power-law at least over an octave in frequency. A power-law sky model also provides the numerical precision necessary over the large dynamic range between sky-temperature and signal strength.

Over frequencies of 40–200 MHz, where the EoR signal is predicted to be observable, emission from Galactic and extragalactic sources forms the dominant foreground component. The cumulative emission from Galactic and extragalactic sources has a functional form that is poorly constrained. Additionally, the predicted signal from cosmic dawn and the epoch of reionization is expected to span several octaves in frequency. It is more likely for foreground spectra to exhibit higher spectral complexity over multi-octave bandwidths than a single octave window as identified for the spectrally rich cosmological recombination signal. Thus GMOSS, which uses physical parameters to describe Galactic and extragalactic radio emission over 22 MHz–23 GHz, serves
as a useful sky model for simulating foreground spectra for the global EoR signal. Additionally, GMOSS spectra are numerical noise limited at the level of \(\sim nK\), which serves well to simulate foregrounds for the EoR signal that is predicted to have strengths of 10–100 mK in standard models. Although it is desirable to adopt GMOSS as the sky model for simulating foreground spectra for the spectral ripples from the recombination epoch, the current numerical precision of GMOSS is insufficient. Future improvements in the numerical computations in GMOSS could reduce these errors to below nK, in which case GMOSS might be used to describe foregrounds for developing detection methods and algorithms for cosmological recombination spectral distortions.

Although the exact functional form of foreground spectra are \textit{a priori} unknown, methods to separate foregrounds from fine-scale CMB spectral distortions can be devised by choosing realistic sky-models in simulations. Simulations in this thesis demonstrate that beam averaged foreground spectra for both the monopole CMB spectral distortions can be described using Maximally Smooth (MS) functions. Since both fine-scale CMB spectral distortions are expected to be non-smooth, MS functions are a promising method for foreground separation.

\subsection{Summary}

A summary of the main chapters of this thesis is presented below.

\subsubsection{Chapter 3}

Photons emitted in bound-bound transitions over the epoch of helium and hydrogen recombination \((900 \lesssim z \lesssim 7000)\) are expected to appear as weak, additive distortions of the CMB spectrum. I have presented a feasibility study for a ground based detection of these spectral signatures of cosmological recombination.

In the study I first identified a frequency range most conducive to such a detection. The two criteria that were used to identify the frequency range were (a) presence of multiple turning points in the cosmological signal in an octave band to aid foreground separation and (b) maximum signal-to-noise ratio. I found the optimal frequency range to be an octave band in the frequency range 2–6 GHz. In this frequency range the sky brightness temperature is typically of the order of \(\sim\)Kelvin whereas the cosmological signal has a peak-to-peak strength of \(\sim\)20 nK.

Adopting a sky model that is a power-law at each pixel, I synthesized mock spectra as would be produced by an ideal telescope that observed the sky. I demonstrated that using high-order polynomials to model foregrounds in these mock spectra results in the spectral ripples from the recombination epoch being subsumed by the fitting procedure. This results in degradation of signal strength and thus discourages the use of polynomials for foreground separation. I have presented a useful alternative for foreground separation using ‘Maximally Smooth’ (MS) functions. MS functions do not have zero crossings in derivatives of the second order and above. This prevents MS functions from describing ripple-like non-smooth features. The spectral signature from the recombination epoch over the identified frequency range is non-smooth by definition. I have
demonstrated that using MS functions to model foregrounds in mock spectra leaves in the residual a distinct smooth-baseline-subtracted template of the cosmological signal with signal strength and turning points preserved. Unlike polynomials, increasing the order of MS functions does not progressively degrade signal strength. Using MS functions, I have demonstrated a method that can be used to separate foregrounds in sky spectra for detecting spectral ripples from the recombination epoch. Using Bayesian tests I have investigated the amount of effective integration time required to detect this signal using an array of 128 ideal radio telescopes with cryogenically cooled receivers. The effective integration time for a 95% confidence detection with a Bayes Factor test is 255 days. For a 95% confidence measurement of the amplitude of the recombination spectral ripple, the effective integration time required is 430 days.

In the feasibility study I have demonstrated that it is in principle possible to detect the weak signal from the epoch of recombination in reasonable time using receiver technology that is either presently or soon to be available.

6.2.2 Chapter 4

I present GMOSS: Global Model for the Radio Sky Spectrum, a physically motivated model for the low frequency radio sky. GMOSS describes sky spectra towards different sky directions over individual pixels of $5^0$ resolution. Individual pixel spectra in GMOSS are derived by fitting seven physical parameters to functional forms describing plausible physics towards these pixels. The parameters are fit to data points derived from all-sky maps at 22, 45, 150, 408, 1420 MHz and 23 GHz. The raw sky maps are treated such that in their final forms they are at a common resolution of $5^0$ and have the CMB monopole and dipole temperatures subtracted. In addition, a scaling and offset correction is applied to the 150 MHz (Landecker & Wielebinski, 1970) map as suggested by (Patra et al., 2015). An offset of 493 $\mu$K is added to the 23 GHz WMAP differential sky map to provide the missing DC component.

GMOSS spectra toward individual pixels are modelled by one of two functional forms based on the observed change in spectral index between low and high radio frequencies. If the spectral index steepens towards high frequencies, the underlying emission is modelled as synchrotron emission with a steepening in electron energy distribution. This creates a spectral shape that can be described as convex. This convex spectrum is parameterised by a low frequency spectral index $\alpha_1 \left( T(\nu) \propto \nu^{-\alpha_1} \right)$, change in spectral index $\delta_\alpha$ ($\delta_\alpha \geq 0$), break in electron energy $\nu_{\text{break}}$ and normalisation $C_1$. For the case of flattening in high-frequency spectral index, the underlying emission mechanism is modelled as a composite of synchrotron emission from spectra with flat and steep spectral indices. Such a spectrum has a concave spectral shape and is parameterized by a low-frequency spectral index, change in spectral index, normalisation and relative contribution of steep and flat spectrum sources. Both models are modified by thermal emission from diffuse gas and absorption by the thermal medium as it becomes optically thick at low frequencies. These processes are described by additional parameters, namely, electron temperature $T_e$, frequency $\nu$, where the optical depth of the medium becomes unity and an excess thermal emission component with brightness $I_x$. 
**Concluding remarks**

GMOSS effectively describes the data points within typical errors (~17%) in the input data maps. The parameters derived from GMOSS are in agreement with independent measurements in literature. The galactic plane and the north polar spur arise as distinct features in parameter maps indicating that the physics towards these sky directions is distinct from other directions that have larger contribution from extragalactic emission, as is expected.

Thus, with GMOSS, I provide a physically motivated model of the low-frequency radio sky between 22 MHz and 23 GHz that is a realistic expectation for sky emission. I proceed to use GMOSS to simulate plausible sky spectra over the 40–200 MHz frequency range where the global 21-cm EoR signal is expected to be present.

### 6.2.3 Chapter 5

I discuss three aspects relating to foregrounds of the global redshifted 21-cm signal from EoR. The first aspect addresses the question of spectral complexity of foregrounds. I first demonstrate that polynomial interpolation of sky temperatures at individual pixels of sky maps at widely spaced frequencies can result in erroneous deductions of foreground spectral complexity. The degree of polynomial that will appear to be necessary to separate foregrounds in mock spectra depends on the order of the interpolating polynomial used in generating the sky model. Thus such unphysical foreground models are best avoided.

I next adopt GMOSS as the sky model to generate beam-averaged mock sky spectra and investigate the order of polynomial required to describe foregrounds with accuracy (mK) well below the expected amplitude of EoR signals. I find that a polynomial of at least order 7 is required to model foregrounds in such spectra. Also, foreground spectra are more complex towards the Galactic plane and require higher-order polynomials to describe those foregrounds to mK levels.

I next examine whether foreground spectra are smooth as defined by Maximally Smooth functions. I simulate several expectations of mock sky spectra using a GMOSS sky model as observed by two frequency independent antenna beams, namely a cosine² beam and the beam of the SARAS 2 antenna. These mock spectra, which are expected to be a near realistic representation of true sky spectra, can be described by MS functions of at least order 8. The required order of the MS function increases towards the Galactic plane. Importantly, I deduce that foreground spectra are indeed smooth and this smoothness can be used as a criterion to separate foregrounds from non-smooth models of global EoR signals.

With a Bayes Factor test I find that when using MS functions of order 8 to separate foregrounds in mock spectra a 95% confidence signal detection requires 10 minutes effective integration. Using polynomials of the the same order deteriorates signal-to-noise ratio by subsuming a part of the EoR signal and thus requires a larger integration time of 18 minutes for a 95% confidence detection.

Increasing the polynomial order further to order 20 degrades the signal strength and introduces more turning points thus increasing the integration time further to 72 minutes for a detection with the same level of confidence.

In summary, in this chapter I have demonstrated the efficacy of using MS functions to separate realistic foregrounds from the global EoR signal.
6.3 Towards an experimental detection of fine scale CMB spectral distortions

There are currently five ongoing experiments aiming to detect the global signal from the epoch of reionization; however, there is only one experiment in the works to detect the cosmological recombination lines APSERa. APSERa—Array of Precision Spectrometers for the Epoch of Recombination—is a venture of the Raman Research Institute, with a dedicated science goal of detecting the spectral ripples from the epoch of recombination. On completion, APSERa will comprise an array of 128 spectral radiometers with cryogenically cooled receivers. APSERa will operate over the frequency range of 2–4 GHz at a radio quiet site possibly close to the poles.

I present here some specifications to guide system design for experiments seeking to detect fine-scale CMB spectral distortions such as those from recombination to reionization.

- **Antenna design** The antenna is the first element in a radio telescope. It acts as a sensor of electromagnetic radiation, coupling sky radiation to receiver electronics. It is of prime importance that the antenna does not severely attenuate or modify the shape of the received signal at this first stage since the sky-signal has not yet been amplified by any electronics. A few essential antenna design criteria are frequency independence and smooth & high reflection efficiency and radiation efficiency. When an antenna is frequency dependent, the antenna beam and thus sky coverage varies as a function of frequency. This results in a ‘leakage’ of spatial structures into the frequency domain potentially introducing spectral shapes that may confuse signal detection. Such a leakage is termed mode-mixing in literature (see, for example, Liu & Tegmark, 2011). It is thus desirable to have an antenna that is frequency independent over the entire band of interest. Antennas that are electrically short can potentially be frequency independent. It is also desirable for the antenna have a smooth return loss or equivalently reflection efficiency. The return loss gives the ratio of the power reflected by the antenna to the input power. The antenna return loss determines the coupling between the sky-power and the receiver. Thus, any spectral shape in the return loss will appear as a multiplicative shape in the sky spectrum. Finally, the fraction of sky power coupled by the antenna to the receiver electronics is measured by the total efficiency which is a combination of the antenna return loss (reflection efficiency) and the radiation efficiency. Losses to the ground and ohmic heating are possible mechanisms that deteriorate radiation efficiency. Since both the cosmological signals of interest are inherently weak, poor antenna efficiency results in increased observing time to reach desired sensitivities.

Some examples of antennas that have been custom designed to detect CMB spectral distortions from the epoch of recombination and reionization are shown in Figure 6.1. A discussion on some ultra-wideband antennas custom designed to detect the monopole CMB spectral distortions is presented in Subrahmanyan et al. (2016). While the antennas described in Subrahmanyan et al. (2016) have some properties that are favourable to detecting CMB spectral distortions, these designs require further improvements to achieve the desired science goal.
Concluding remarks

Fig. 6.1 (a) A fat, profiled dipole antenna operating over 87.5–175 MHz above an absorptive ground plane adopted by SARAS. This antenna was custom designed to be frequency independent over the entire bandwidth of operation, a feature that is essential for detection of weak spectral distortions in the CMB. Figure reproduced from Patra et al. (2013). (b) A frequency independent fat profiled monopole antenna designed for operation over 2–4 GHz. This antenna has a beam that varies less than 10% over the octave bandwidth and has a smooth return loss, favorable to detection of spectral ripples from the epoch of recombination. However the resistive loss in the antenna and dependence on earth properties below the electrical ground plane call for further improvements in antenna design. Figure reproduced from Raghunathan et al. (2015).

Thus antenna design is a critical challenge that requires special attention and possibly several iterations to arrive at a final configuration.

- **Bandpass Calibration** The ripples from cosmological recombination and the global EoR signal are to be measured as distortions in the sky spectrum and the importance of a clean bandpass cannot be overemphasised. A clean bandpass is essential to measure weak features in the sky spectrum and to be able to confidently attribute these to the cosmological recombination or reionization signal. It is of utmost importance to design elements and assemble the same in the receiver such that the calibrated bandpass is flat. Several additive and multiplicative effects can contaminate the bandpass by introducing features that confuse signal detection. Of importance among these are reflections in connectors and cables between discrete components in the receiver. Reflections create standing wave patterns in the bandpass that are by definition non-flat. Such standing waves can be avoided/minimized by using extremely short cable lengths. While good bandpass calibration typically ensures that multiplicative terms are removed, additive terms introduced by interference of multi-path reflections present a non-trivial challenge.
A design that serves to reduce some receiver systematics is the correlation spectrometer scheme shown in Figure 6.2 as adopted in SARAS (Patra et al., 2013). Noise injection at
the antenna using a directional coupler provides bandpass calibration. Splitting the signal into two separate receiver chains that are later correlated serves to minimise receiver noise that are uncorrelated between the two chains. A switched noise injection scheme using a crossover switch provides absolute calibration and allows for cancelling internal common-mode additive noise. System design for detecting the signal from the epoch of recombination can be guided by the above scheme adopted in SARAS. However some design principles would have to be modified because of the weak sky signals in the 2–4 GHz band compared to the extremely bright galactic synchrotron emission in the SARAS band of 87.5–175 MHz.

- **Receiver noise** Over the frequencies of 40–200 MHz observed by EoR signal detection experiments the sky noise dominates receiver noise. However, over the recommended frequency range of 2–6 GHz for detecting the cosmological recombination signal, receiver noise is likely to dominate the relatively ‘cold’ sky. This necessitates reducing overall system noise by appropriate choice of components with low individual noise-temperature contributions. Further, since the sky signal is weak, it is important to amplify the signal at the earliest possible stage to avoid any degradation of signal-to-noise ratio by receiver electronics. This can be achieved by using a low-noise amplifier connected directly to the antenna. Cryogenically cooling the LNA can serve to reduce LNA and thus system noise temperature.

### 6.4 Final remarks

I have discussed the feasibility of detection of spectral distortions in the CMB from cosmological helium and hydrogen recombination. I have also presented a method of separating smooth foregrounds from the intrinsically non-smooth CMB spectral distortion, using Maximally Smooth functions. I estimate that it is in principle possible to detect the CMB spectral distortion from the epoch of recombination with 95% confidence using an array of 128 radio telescopes with cryogenically cooled receivers in 255 days effective integration time.

I extend the application of MS functions to separate foregrounds from the global EoR signal in mock sky spectra. As the exact nature of the EoR signal is *a priori* unknown and because it is expected to span over multiple octaves in frequency where Galactic and extragalactic emission dominate, a realistic model of the radio sky is necessary to make any useful deductions. I present such a physically motivated model of the radio sky in GMOSS: Global Model for the Radio Sky Spectrum. With GMOSS as the sky model and using MS functions for foreground separation, I find that the global 21-cm signal from EoR can be detected with 90% confidence in 10 minutes integration time with a single-element, uncooled, ideal radio telescope.

Designing and building radio telescopes that meet specifications required to detect CMB spectral distortions is non-trivial. However, identifying these specifications helps lay the path ahead.

In this thesis I have provided some methods and tools that will bring us one step closer to detecting fine-scale spectral distortions in the CMB from recombination to reionization.
Concluding remarks

Fig. 6.2 A correlation spectrometer scheme adopted in SARAS (Patra et al., 2013). Noise source injection provides bandpass and absolute calibration. A crossover switch alternately connects the antenna and the reference noise source to different ports of a 180° power splitter/combiner. While the correlation spectrometer naturally reduces response to uncorrelated noise in the two signal chains downstream, the switching allows for cancellation of common-mode noise. The measurement set of this scheme is a complex spectrum where the sky power is expected to appear only in the real part and the spurious systematics from reflections would appear in both the real and imaginary parts. This property can be used to separate systematics from sky power. Figure reproduced from (Patra et al., 2013); refer to the paper for more details.
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APPENDIX A

THE COMPLETELY MONOTONE (CM) FUNCTION

Mathematically, a function \( f(x) \) is said to be a complete monotone (CM) if for all values of \( x \) in the interval \( 0 \leq x < \infty \), \((-1)^{n} \times \frac{d^{n}f(x)}{dx^{n}} \geq 0 \) for every integer \( n \geq 0 \). The method by which a CM function of any said order may be constructed is best explained by an example, such as that for a cubic function as given below. We construct a cubic function \( f(x) \) which is CM in the range defined by \([x_{0},x_{m}]\):

\[
f(x) = p_{0} - p_{1}(x-x_{0}) + p_{2}(x-x_{0})^{2} - p_{3}(x-x_{0})^{3}, \quad (A.1)
\]

where \( p_{i} \) for all \( 0 \leq i \leq 3 \) are the coefficients that are constrained so that \((-1)^{n} \times \frac{d^{n}f(x)}{dx^{n}} \geq 0 \) for every integer \( n \geq 0 \) and for all \( x \) in \([x_{0},x_{m}]\). The third derivative of \( f(x) \) is

\[
\frac{d^{3}f(x)}{dx^{3}} = -6p_{3}. \quad (A.2)
\]

To constrain \( f(x) \) to be CM we require that \((-1)^{3} \times \frac{d^{3}f(x)}{dx^{3}} \geq 0 \), or that \( p_{3} \geq 0 \). Let \( p_{3} = a_{3} \), where \( a_{3} \) is a positive value. The second derivative of \( f(x) \) is

\[
\frac{d^{2}f(x)}{dx^{2}} = 2p_{2} - 6p_{3}(x-x_{0})
\]

\[
\equiv 2p_{2} - 6a_{3}(x-x_{0})
\]

Once again to constrain \( f(x) \) to be CM we require that \((-1)^{2} \times \frac{d^{2}f(x)}{dx^{2}} \geq 0 \), which leads to the condition that \( 2p_{2} - 6a_{3}(x-x_{0}) \geq 0 \). This is satisfied if \( p_{2} \) is selected such that \( 2p_{2} - 6a_{3}(x_{m}-x_{0}) \geq 0 \), which ensures that the criterion is satisfied for all \( x \) in \([x_{0},x_{m}]\). Therefore, we express \( p_{2} \) as

\[
p_{2} = a_{2} + 3a_{3}(x_{\text{max}} - x_{0}), \quad \text{where} \quad a_{2} > 0.
\]

The first derivative of \( f(x) \) is

\[
\frac{df(x)}{dx} = -p_{1} + 2p_{2}(x-x_{0}) - 3p_{3}(x-x_{0})^{2}
\]

\[
\equiv -p_{1} + 2\{a_{2} + 3a_{3}(x_{m}-x_{0})\}(x-x_{0}) - 3a_{3}(x-x_{0})^{2}
\]

To constrain \( f(x) \) to be CM, we once again require that \((-1) \times \frac{df(x)}{dx} \geq 0 \). This requires that we determine \( p_{1} \) such that \(-p_{1} + 2\{a_{2} + 6a_{3}(x_{m}-x_{0})\}(x_{m}-x_{0}) + 3(a_{3})(x_{m}-x_{0})^{2} \geq 0 \), ensuring that the criteria is satisfied for all \( x \) in \([x_{0},x_{m}]\). To guarantee this we express \( p_{1} \) as \( p_{1} = a_{1} + 2a_{2}(x_{m}-x_{0}) + 3a_{3}(x_{m}-x_{0})^{2} \), where \( a_{1} \geq 0 \). Since the constant term \( p_{0} \) represents a vertical translation of
the function and does not affect the shape—the smoothness of the function itself—we allow it to be a free parameter \(a_0\). In summary, we describe the cubic CM function to have the form

\[
f(x) = a_0 - \{a_1 + 2a_2(x_m - x_0) + 3a_3(x_m - x_0)^2\}(x-x_0) + \{a_2 + 3a_3(x_m - x_0)\}(x-x_0)^2 - a_3(x-x_0)^3,
\]

(A.3)

where all of the coefficients \(a_0, a_1, a_2\) and \(a_3\) are non-negative. In this functional form, setting \(a_3 = 0\) reduces the function to CM of order 2 and if we further set \(a_2 = 0\) we have a CM polynomial of order unity. This formulation allows a successive approximation to data by first fitting for a CM polynomial of order unity as a Taylor expansion at the reference point \(x = x_0\) to optimize for coefficients \(a_0\) and \(a_1\), then adding a higher order term with initial guess \(a_2 = 0\) and optimizing all three coefficients, and finally adding a cubic term with initial guess \(a_3 = 0\) and optimizing all four coefficients. All coefficients are constrained to be positive by expressing them in the form \(a_i = 10^{b_i}\), where the \(b_i\)'s are allowed to be real numbers.

Thus a CM polynomial of arbitrary order \(n\) may be written in the form

\[
f(x) = a_0 + \sum_{i=1}^{n} (-1)^i (x-x_0)^i \sum_{j=0}^{n-i} a_{i+j} C_j^{i+j} (x_m - x_0)^j,
\]

(A.4)

where \(C_k^n\) denotes the binomial coefficient \(n!/[k!(n-k)!]\) and the constant term \(a_0\) is left unconstrained so as to allow arbitrary vertical translations.
Since the work presented in Chapter 3 was published the cosmological recombination spectrum template has been refined further. This comes from the latest recombination calculations from Cosmospec presented in Chluba & Ali-Haïmoud (2016). The main improvement comes from interspecies photon feedback. This changes the strength and shape of recombination radiation. This also changes the number of photons emitted by the hydrogen atom from 5 in previous calculations to $\sim 6.1$ considering all three recombination eras namely, HeIII $\rightarrow$ HeII, HeII $\rightarrow$ HeI, and HII $\rightarrow$ HI. The radiative feedback between hydrogen and helium leads to non-trivial features in the GHz band, particularly at 1 GHz where the signal strength is reduced compared to previous estimates. However, in the band of interest over 2–6 GHz, the peak-to-peak amplitude of the recombination signal is increased from $\sim 20$ nK to $\sim 40$ nK, without significant change in spectral shape. This is encouraging in terms of improved signal-to-noise ratio. A comparison of the total recombination spectrum assumed from the latest calculations at the time of publication of the paper presented in Chapter 3 and the current prediction from Cosmospec is presented in Figure B.1. The baseline subtracted cosmological recombination signal from Cosmospec over the 1–7 GHz band is shown in Figure B.2.
Improved template of the cosmological recombination signal

Fig. B.1 The current prediction of the total recombination spectrum from CosmoSpec (Chluba & Ali-Haïmoud, 2016) overlaid with the spectrum assumed from the latest calculations at the time of publication of the paper presented in Chapter 3.

Fig. B.2 A smooth baseline subtracted template of CMB spectral distortion from the recombination epoch over 1–7 GHz as predicted by CosmoSpec (Chluba & Ali-Haïmoud, 2016).