Suspension thermal noise and Opto-mechanics in gram-scale flexures

Thanh T-H. Nguyen

A thesis submitted for the degree of Doctor of Philosophy at The Australian National University

11 July 2017
Declaration

This thesis is an account of research undertaken between March 2009 and January 2017 at the Centre for Gravitational Physics, Department of Quantum Science, Research School of Physics and Engineering, The Australian National University, Canberra, Australia.

Except where acknowledged in the customary manner, the material presented in this thesis is, to the best of my knowledge, original and has not been submitted to a degree at any other university.

Thanh Nguyen
January, 2017
Acknowledgments

This has been a long journey and I would not have been able to walk to the end without the help and support of so many people all these years. To you all, thank you.

First and foremost, I would like to thank Professor David McClelland. You have been my mentor since the first day I came to the ANU. Your passion for physics and absolute belief in the detection of Gravitational waves, many years before the actual detection, inspired me to study this field. Your vast knowledge and steady guidance have steered me in the right direction throughout my PhD.

To Dr Bram Slagmolen, I thank you for long hours of discussion and brainstorming sessions. You explained any physics problems to me patiently, always had a ready suggestion to the stream of technical challenges in the lab and let me bounce my ideas off you. For these I’ll always be grateful.

To Professor Daniel Shaddock, any suggestions you made have always been invaluable. You have taught me to approach problems from different angles, and given me the opportunity to learn many useful skills and lessons. Thank you.

In many ways, I would not have been able to finish this journey without Dr Paul Altin. I’m grateful to you for being a dedicated lab partner, an inspirational post doctoral fellow and a wonderful friend. Thank you for all your help, care and kindness.

The Gravitational Physics group at the ANU is truly a wonderful place to work, learn and share knowledge. No matter what projects you are working on, people will lend a helping hand and be willing to participate in a spontaneous brainstorming session. I have been lucky to be on the receiving end of many of those sessions. To Dr Robert Ward, I am constantly amazed with your vast knowledge. ‘Let’s ask Rob’ is the familiar phrase in the group, possibly up there with ‘lunch’ and ‘beer’. To Dr Roland Fledderman, I thank you for your immense patience in answering all my physics questions, no matter how seemingly trivial they were. I also want thank you and Kerstin for your friendship and for sharing your New Year celebrations with me.

To Dr Nick Robins: you deserve my special acknowledgement for your advice and your willingness to listen to me whenever I needed.

I am eternally grateful to many people who read my whole thesis back to front, in particular to Dr Terry McRae for many late nights of thesis correction, Dr Paul Altin for your ‘is tomorrow soon enough’ in reply to my ‘as soon as possible’ and Dr Bram Slagmolen and Professor David McClelland for reading my thesis during your holiday. I also would like to thank Georgia Mansell, Minjet Yap, Andrew Wade, Rose Ahlefeldt, Emily Hathaway and Richard Barry who have taken time to go over chapters.

To the ‘graviteers’: You have made my time at the ANU fun and enjoyable.
Georgia Mansell, Samuel Francis, Shasi Raj, Minjet Yap and Andrew Wade deserve special mentions as they kept me sane during the long thesis writing. I am filled with gratitude for their constant encouragement, friendship, and multiple coffee deliveries. You will always be close to my heart. Other people I would like to acknowledge from our group, with no particular order, include: Tarquin Ralph, Jarrod Dong, Chathura Bandutunga, David McMannus, Lyle Roberts, Paul Sibly, Tim Lam, Jong Chow, John Miller, Silvie Ngo, Danielle Wuchenich, Adam Mullavey, Conor Mow-Lowry, James Dickson and many others.

Over the past year, I have been working at the Australian Bureau of Statistics (ABS) while writing this thesis part-time. I found myself lucky to meet like minded people to work with, but also gain so many friends. To my ABS director Dr Sarah Hinde: thank you for making it as easy as possible for me during my writing time. To Shamali Weeraratne: your giving nature has been a tremendous support. To Robert Smith, Leanne Roberts, Bradley White, Kimberly Grima and Cheng Chen, you have made my time at ABS so much fun. And I thank all of you for constantly reminding me that I had a thesis to write while I was working there.

I would like to thank Emily Hathaway for her unwavering support, for many walks up Mt Ainslie and for providing many meals in the last stage of writing. To Esther and Jason Duffy-Smith, Sasa, Virginia and Tony Maybury: thanks for giving me your warm friendship and sharing with me a world outside of physics. To my housemates Alex and Robin McAndrews: while I was busy in the lab taking measurements or late night thesis writing at uni, you picked up lots of chores and took care of the household. You have both become surrogate siblings to my sister which enabled me to spend more time focussed on my PhD work. Thank you.

To my ‘Canberran’ family, Hien Le and Anh Nguyen: there are truly not enough words to fully express my gratitude. I’m grateful for you not only sharing with me your friendship, but also your family. Now I know where I can go for authentic Vietnamese food.

To my Tassie parents: I will always remember your kind decision on welcoming me into your home, and since then, your care and love have never ceased.

Lastly, to my grandfather, my parents and my sister, thank you for unconditionally supporting me in all my decisions and everything I do. All of my work would be so much more difficult without you.
Abstract

Recent direct detections of gravitational waves by the Advanced Laser Interferometry Gravitational wave Observatory (LIGO) have opened a new field of astronomy. Upgrades to improve the sensitivity of detectors around the world promise a future of rich astronomical observations, which will both expand our knowledge of the universe and complement discoveries from electromagnetic astronomy.

The detectable displacement signals caused by gravitational waves are extremely small and easily masked by noise. Suspension thermal noise is one of them. It couples into the detectors through the isolation systems that suspend and isolate the test mass mirrors, placing a fundamental limit on the displacement sensitivity of interferometric gravitational wave detectors. One way to mitigate this noise source is by careful selection and thorough characterisation of the materials used in these suspension and isolation systems.

We used a Fabry-Perot cavity with one mirror mounted on a gram-scale flexure to study suspension thermal noise and opto-mechanical response of the system. The behaviour of this macroscopic system could then be used to evaluate the responses of a kilogram-scale opto-mechanical system, such as gravitational wave detectors.

Thermal noise is governed by the Fluctuation-Dissipation Theorem, which links mechanical loss to the displacement caused by the thermal energy in each mode. In this thesis, a simple model using this theorem was developed to predict the thermal-noise-induced displacement of the flexures.

We present the frequency distribution of thermal noise for flexures made of aluminium, niobium and silicon. Silicon in particular is a promising material for suspension systems in future gravitational wave detectors. These measurements are in the audio-frequency band between 10 Hz and 10 kHz and span up to an order of magnitude above and below the fundamental flexure resonances. Our analysis indicates that, for aluminium and niobium, structural noise dominates the displacement fluctuation spectra at low frequencies, whereas thermoelastic noise dominates at higher frequencies. The silicon flexure, as a result of careful design, shows a displacement spectrum dominated by structural damping both below and above the fundamental resonance. Results from a second niobium flexure provide evidence for qualitative changes in the displacement spectrum caused by surface damage in addition to a reduction of the mechanical quality factor. The measurement results show good agreement when compared to the simple model.

Lastly, we show experimental results of a statically and dynamically stable opto-mechanical cavity. The system is driven by a single optical field without external feedback control. The cavity exhibits stiffening due to radiation-pressure force, as well as an optically induced damping which cannot be due to radiation pressure. The optical damping is measured to be four orders of magnitude larger than the mechanical damping of the flexure. The cavity is shown to self-lock under the
combined influence of these effects.
6.3.3 Silicon flexure ..................................................... 100
6.4 Chapter summary .................................................. 103

7 A stable single-carrier optical spring 105
7.1 Flexure cavity without feedback control ......................... 105
7.2 Bolometric force model ............................................. 109
7.3 Bolometric feedback model ........................................ 111
7.4 Chapter summary .................................................. 114

8 Conclusions and future work 115
8.1 Summary of thermal noise experiments .......................... 115
  8.1.1 Further work .................................................. 116
8.2 Opto-mechanics ................................................... 117
  8.2.1 Further work .................................................. 117

Bibliography 119
List of Figures

1.1 The quadrupole GW acts on rings of colour-coordinated masses in two basic polarised patterns. .................................................. 2
1.2 (a) Basic configuration of a Michelson interferometer (b) enhanced configuration for better signal to noise ratio readout. ITM: input test mass mirror, PRM: power-recycling mirror, SRM: signal-recycling mirror 3
1.3 LIGO noise budget in its most sensitive frequency band [30, 31]. . . . 5
2.1 An external force is applied onto a mass spring system and displaces the mass by a distance x. ......................................................... 13
2.2 Amplitude spectral density of structural and viscous damping thermal noise of an oscillator of 0.5 g at the resonant frequency of 100 Hz and Q of 2000. These two loss types depends on frequency differently: structural damping is constant across all frequency band, where as viscous damping is a frequency dependent loss. ........................................ 15
2.3 A Maxwell unit of spring and dash pot. ........................................ 17
2.4 Non-linear and non-constant response of silicon thermal expansion coefficient shows temperature dependency. This property is one of the reasons making the material a promising candidate for future gravitational wave detectors [74]. ........................................ 18
2.5 A cartoon of the flexure angular displacement under an applied force. 19
2.6 From left to right: niobium flexures of 300 Hz and 85 Hz and aluminium flexure of 271 Hz. The flexure sizes are comparable to a 10 cents coin. 22
2.7 Silicon flexure design and schematics. ........................................ 22
2.8 Flexure on a circular jig during mirror gluing process. Cotton was padded to help soft from without loosening to keep the flexure in place during the lathe machining process to cut the flexing membrane to micron thickness. ......................................................... 23
2.9 Top row: pictures were taken during test trial on a chipped flexure to determine the required torque before reaching breaking point. Below: schematics and dimensions of the clamp and mount of the silicon flexure. ......................................................... 24
2.10 A close-up picture of the silicon flexure mirror with a Thorlabs mount in the background served as size comparison. Gathered dusts that were conspicuous under torch light but invisible to the naked eyes reinforce the need for clean optics to avoid light scattering which was a real issue to constantly combat in the experiment (see chapters 4 and 5). . 26
2.11 Schematics of the ringdown experiment from top view (left) and side view (right). QPD: Quad photodiode measuring horizontal and vertical beam positions due to flexure displacements. 

2.12 Ringdown measurement for silicon flexure at a pressure of \(2.5 \times 10^{-4}\) mbar. 

2.13 Thermoelastic damping for different flexure thickness of aluminium and silicon materials for this particular designs. Top: silicon flexure, bottom: aluminium flexure. 

2.14 Thermal noise models for aluminium flexure. 

2.15 Thermal noise models for silicon flexure. 

2.16 Estimated equivalent displacements induced by structural and thermoelastic thermal noise of mirror substrates, structural and thermoelastic thermal noise of mirror coatings, and thermal noise of the PZT and mirror combination. 

3.1 A linear Fabry-Perot cavity of length \(L_{\text{cav}}\). The transmittivity, reflectivity and loss coefficients of the front and rear mirrors are \((t_1, r_1, a_1)\) and \((t_2, r_2, a_2)\) respectively. 

3.2 Amplitude and phase of transmitting, reflecting and circulating fields in three cavity conditions: overcoupled \((R_1 = 0.995, R_2 = 0.998)\), impedance matched \((R_1 = R_2 = 0.998)\) and undercoupled \((R_1 = 0.998, R_2 = 0.995)\). 

3.3 A: a diagram of an opto-mechanical cavity with a rear movable mirror. B: Silicon flexure cavity as an example of this system. 

3.4 Radiation pressure force and optical spring amplitude as a function of detuning relative to cavity linewidth. The maximum and minimum turning points of the optical spring are at \(\delta \gamma = \pm \sqrt{3}/\sqrt{3}\). \(P = 100\ \text{mW}, R_1 = R_2 = 0.9959, F = 765\) 

3.5 Theoretical frequency shifts as the mechanical spring is softened/stiffened due to radiation pressure induced spring in quasi-static regime. 

4.1 Simple block diagrams for a standard control system. 

4.2 Bode plot example of an open loop magnitude and phase corresponding to a stable feedback system. The plot illustrates the unity gain frequency and phase margin. The open loop gain has a stable phase margin of \(-90^\circ\), equivalent to a \(1/f\) slope through the UGF in magnitude. At lower frequency, the slope is \(1/f^2\), increasing the amount of suppression. The steeper slope can be described as constructed from 2 poles. A zero can then be added to create the stable \(1/f\) slope at UFGF.
4.3 Normalised Pound-Drever-Hall error signals ($\beta = 1.08$ and $\Omega = 75 \text{ MHz}$) for overcoupled ($R_1 = 0.9993$, $R_2 = 0.9998$), impedance matched ($R_1 = R_2 = 0.9995$) and undercoupled ($R_1 = 0.9998$, $R_2 = 0.9993$) cavities. Reflected powers of the undercoupled and overcoupled cavities are equal and overlaid in the top figure. The error signal size depends on the amount of reflected phase extracted from the cavity. .......................................................... 52

4.4 Pound-Drever-Hall error signals of different modulation depths for the same modulation frequency, input power and cavity conditions. The error signal slope is steeper than the fraction of error signal size over cavity FWHM. .......................................................... 54

4.5 Basic feedback block diagram with typical locations that noise is injected in the loop and affect the measurement output. .......................... 57

4.6 Laser frequency noise equivalent flexural displacement for a 1064 nm Nd:YAG laser. A theoretical prediction for the thermal noise induced flexural displacement for silicon flexure is also included to show the dominance of laser frequency noise in the frequency of measurement. . 60

4.7 Phasor diagram illustrating the smaller scattered light vector, added to the vector presented the main beam. ............................... 61

4.8 Time series output in the presence of parasitic interference. a) and b) are scattered light output as being frequency modulated at different frequencies. c) is scattered light observed when frequency and amplitude modulated. .................................................. 62

4.9 Shelf-like features of parasitic interference (red) in the Fourier domain. The scattered noise is frequency shifted up to high frequency band by $\Omega_M = 2 \text{ kHz}$ and $M = 2.405$. .................................................. 64

4.10 Measured anthropogenic activities in CGP laboratory. ............... 64

4.11 Simplified schematic of analogue and digital implementation for the electronic stage. a) Analogue version. b) Digital version. HV=high voltage; ADC = Analogue to Digital Converter; DAC = Digital to Analogue Converter .................................................. 65

5.1 A schematic diagram shows two main experimental stages: a laser stabilisation (blue tile) and a test cavity lock acquisition (yellow tile). This setup was used mainly for the three flexures presented here. PBS: polarised beam splitter. EOM: electro-opto modulator. g/t: glan-taylor. TPD: photodiode used to measure transmitted power. RPD: photodiode used to measure reflected power. ......................... 68
5.2 Photos of the optical layout in the laboratory. RF-PD1: photodiode measuring reference cavity reflected power. RF-PD2: photodiode measuring test cavity reflected power. PD1: photodiode monitoring input power. PD2: photodiode measuring test cavity transmitted power. PD3 and PD4: photodiodes monitoring laser output powers before electro-optic modulator. The white dashed circle indicates the location of the reference cavity vacuum chamber. The laser is stationed on the bottom right corner with beam running parallel to the edge of the table toward the top where the 2 m tall chamber for the TC are off the picture. The red dashed line traces out where the beam travelled.

5.3 Left: Suspension system. Top right: in-vacuum optical breadboard. Bottom right: Single stage suspension flexure cavity.

5.4 Schematic of experimental layout used to measure thermal noise induced displacements of silicon flexure.

5.5 Electronic setup for modulation and demodulation stages of the experiment.

5.6 Normalised theoretical transmitted powers as a function of frequency for RC (top, blue) and TC (bottom). The cavity lengths are $L_{RC} = 200 \text{ mm}$ (blue), $L_{TC} = 10 \text{ mm}$ (red), and $L_{TC} = 12.7 \text{ mm}$ (green) respectively. On the left the blue, red and green peaks are aligned to illustrate the desired locking point. It takes $\sim 20$ blue peaks to reach the next red or green ones. FSR: Free Spectral Range. FWHM: Full-Width at Half Maximum.

5.7 Low bandwidth open loop gain measured from aluminium flexure cavity feedback loop.

5.8 Measurement of silicon flexure as a PDH readout using SR785.

5.9 Measurement of Niobium flexure before and after servo correction.

5.10 The transmitted power (top) and error signal (bottom) of silicon flexure were obtained when the cavity was swept linearly across the resonance. The measurements (blue) were fitted (red). The error signal slope gives the conversion factor of voltage-to-length equivalence.

5.11 A picture of the cavity length measurement.

5.12 The graph shows the effect of low UGF laser frequency noise on the test cavity measured output. The laser stabilisation servo was moved from optimal operating point to demonstrate the effect of laser frequency noise in the cavity length measurements.

5.13 Spurious interferometer noise displacements measured at different times of day.

5.14 Detection noise for analogue and digital locking schemes measured at the PDH error signal output port.

5.15 Experimental schematic using AOM as the actuator.
5.16 Flexure displacements measured for different locking schemes were in agreements. For PZT and EOM as actuators, the measurements obtained at the error signal ports. For AOM as actuator, the displayed trace was the feedback signal to AOM, and the error signal shows suppressed noise within the bandwidth. 85

5.17 Experimental schematic using EOM as the actuator. 87

6.1 Calculated frequency-dependent loss $\phi_\omega$ for the fundamental oscillation mode of the aluminium flexure, showing the contributions of structural $\phi_{struc}$ and thermoelastic $\phi_{TE}$ damping. The fundamental flexure resonant frequency at 271 Hz is indicated by the vertical dashed line, and the total loss determined from the measured quality factor is marked by the horizontal line. 92

6.2 Measured displacement noise for the aluminium flexure. Readout and gain-limited laser frequency noise are also shown as indicated in the legends. The spectra are clear of other noise sources on either side of the fundamental resonant frequency. 93

6.3 Comparison of thermal noise measurement and model predictions for the aluminium flexure. The traces show the PDH error signal readout (red), the sum of structural and thermoelastic noise as predicted by the model detailed in section § 2.3 (blue), and the quadrature sum of the thermal noise model and other experimental noise contributions (green). The lower plots show the ratio between the measured PDH error signal and the predicted total noise, which in both cases is close to unity between 50 Hz and 5 kHz. 94

6.4 Calculated frequency-dependent loss $\phi_\omega$ for the fundamental oscillation mode of the high-Q niobium flexure, showing the contributions of structural $\phi_{struc}$ and thermoelastic $\phi_{TE}$ damping. The fundamental flexure resonant frequency is indicated by the vertical dashed line, and the total loss determined from the measured quality factor is marked by the horizontal line. 95

6.5 Comparison of thermal noise measurement and model predictions for the high-Q niobium flexure. The traces show the PDH error signal readout (red), the sum of structural and thermoelastic noise (blue), and the quadrature sum of the thermal noise model and other experimental noise contributions (green). The lower plots show the ratio between the measured PDH error signal and the predicted total noise, showing good agreement between 20 Hz and 1 kHz. 96

6.6 Displacement measurement of low-Q niobium flexure shows a viscous response below and above the flexure resonance. 97

6.7 Calculated losses due to surface damage of the low-Q niobium flexure. Top: the frequency-dependent loss $\phi_\omega$ for the fundamental oscillation model showing contributions of structural $\phi_{struc}$ and thermoelastic $\phi_{TE}$ damping without an additional source of viscous damping. Bottom: shows the estimated total loss $\phi_\omega$ when including an extra viscous loss. 98
6.8 Thermal noise induced displacements of surface damaged Niobium flexures showed an increase in viscous damping, potentially due to surface damage. 99

6.9 Microscope image shows a defect in the low-Q niobium flexure. A thin black line in the red circle appeared to run across an entire thickness of the membrane. 99

6.10 Calculated frequency-dependent loss $\phi_\omega$ for the fundamental oscillation mode of the silicon flexure, showing the contributions of structural $\phi_{struc}$ and thermoelastic $\phi_{TE}$ damping. The fundamental flexure resonant frequency is indicated by the vertical dashed line, and the total loss determined from the measured quality factor is marked by the horizontal line. 100

6.11 This graph shows a wide frequency band of silicon displacement (red) together with other potential noise sources such as: the gain limited frequency noise equivalent to flexure displacement in green, the off-resonant noise in magenta and the electronic noise in blue. 101

6.12 The measurements and predicted models are put on the same graph. Apart from the predicted thermal noise of the silicon flexure, the predicted thermal noise of the PZT are also calculated and included in the final model. The inset below shows the ratio difference between the measured trace and the theoretical model trace. 102

7.1 Schematic of the experimental setup in the absence of active feedback. PBS: polarising beam splitter, EOM: electro-optic modulator, PZT: piezo-electric transducer, RPD: photodiode measuring reflected power, TPD: photodiode measuring transmitted power, g/t: Glan-Taylor polariser. 105

7.2 Cavity self-locking with light induced forces. A: Optical spring effects observed on PDH error signals and transmitted signals as the cavity was swept slowly across resonance. B: Self-locking of the cavity which was detuned at $\delta_c = 0.5$. The effect is much stronger with larger input powers. C: Damping of oscillation after a sudden displacement of the mechanical oscillator while the cavity was self-locked. 106

7.3 Detuning dependence of optical spring constant and damping obtained from ringdown measurements as the cavity was manually and slowly swept. In the bottom figure, the dashed line was plotted using the bolometric force model (see section 7.2), the solid line was obtained using the bolometric feedback model (see section 7.3). 107

7.4 Power dependence of the measured maximum optical spring constant and damping rate. The solid lines are the predictions of the model presented in section 7.3, while the dashed lines are the expected behaviour in the presence of radiation pressure alone. 108
7.5 Block diagrams represents the cavity dynamics in the presence of a) radiation pressure and b) radiation pressure and bolometric. $\chi_m$ is the mechanical susceptibility. C is the cavity response transfer function. $R = 2/c$ where $c$ is the speed of light. . . . . . . . . . . . . . . . . . . 111

7.6 Illustration of the bolometric feedback rotating the optical spring constant in the complex plane, resulting in the nonlinear power dependence of the damping rate. . . . . . . . . . . . . . . . . . . 112
## List of Tables

<table>
<thead>
<tr>
<th>Table</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>Quality factor at the fundamental resonance frequencies of aluminium,</td>
<td>28</td>
</tr>
<tr>
<td></td>
<td>niobium and silicon flexures. Designed flexure thicknesses were confirmed</td>
<td></td>
</tr>
<tr>
<td></td>
<td>experimentally through the measured flexure resonant frequencies.</td>
<td></td>
</tr>
<tr>
<td>2.2</td>
<td>A list of parameters used in the theoretical models. [100, 101]</td>
<td>30</td>
</tr>
<tr>
<td>5.1</td>
<td>Flexure cavity parameters</td>
<td>79</td>
</tr>
<tr>
<td>6.1</td>
<td>Parameters of aluminium, niobium and silicon flexures used in thermal</td>
<td>90</td>
</tr>
<tr>
<td></td>
<td>noise experiments.</td>
<td></td>
</tr>
<tr>
<td>6.2</td>
<td>Estimated uncertainties for parameters used to construct the theoretical</td>
<td>91</td>
</tr>
<tr>
<td></td>
<td>thermal noise models.</td>
<td></td>
</tr>
<tr>
<td>6.3</td>
<td>Main losses of aluminium, niobium and silicon flexures.</td>
<td>101</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

“Space-time tells matter how to move, matter tells space-time how to curve.”

– John Archibald Wheeler

On the 14th of September 2015 the first direct detection of gravitational waves (GW) was made by the Laser Interferometry Gravitational-wave Observatory (LIGO) [1, 2]. The astronomical event happened at a distance of $1.3 \times 10^9$ light years away, from a pair of black holes whose masses were 36 and 29 solar masses respectively, spiralling inward and colliding to form a single black hole of 62 solar mass. The energy of equivalent to 3 solar mass was emitted as gravitational waves propagating through space-time and detected on Earth [3]. A few months later, on the 26th of December 2015, a similar event was detected [4]. Both detections occurred with the detectors above their ultimate design sensitivity, promising even higher event rates [5] in the future. With these detected events, the era of gravitational wave astronomy begun.

1.1 The nature of gravitational waves

Over a century ago, in his theory of general relativity Einstein introduced a concept of space-time as a continuum and flexible medium that curves in the presence of heavy masses [6]. He then postulated that when violent astrophysical events happen, such as binary black hole coalescence, the subsequent space-time distortions cause oscillations of space-time. These oscillating waves are called gravitational waves [7, 8].

The lowest mode of the gravitational-wave oscillation for any source is quadrupole dictated by conservation of mass-energy and linear and angular momentum. Figure 1.1 describes the effects of a passing gravitational wave on two rings of free-falling test particles. Over a full period, the rings are simultaneously elongated and compressed on the orthogonal directions, following two basic polarised patterns labelled as $h_+$ and $h_\times$ in figure 1.1.

The gravitational waves interact weakly with matter as they propagate at the speed of light through space-time, carrying the signature of distant astrophysical events. Some of the strongest candidates for gravitational waves are [9–11]:

1
**Inspirals/coalescing binary mergers** - Binary system of two compact objects spiral inwardly toward each other and eventually merge. As their orbital distance decays, they lose angular energy and momentum via gravitational waves. A resulting observed signal would show an increase in both frequency and strength and finally a characteristic ‘chirp’ signal before they asymptote to a violent coalescence. The PSRB1912 + 16 [12–14] and GW150914 [1] are examples of gravitational waves emitted from binary coalescence.

**Spinning massive objects** - The massive spinning objects emit a periodic gravitational waves through loss of angular momentum and energy caused by rotations about their asymmetrical axes. The signal strength depends on the degree of asymmetry in the system. Most candidates, such as non-symmetric pulsars and neutron stars, have predicted signal strength below the detectable sensitivity of the current gravitational wave detectors [15].

**Stochastic background** - Similar to cosmic microwave background radiation, a stochastic background of gravitational waves is a remnant expected from the early formation of the universe. The expected signals provide insight into between $10^{-36}$ and $10^{-32}$ seconds after the Big Bang, a further 300000 years earlier than predicted by the cosmic microwave background [16].

**Burst** - These are non-periodic gravitational wave signals and currently their models are less well developed. They are predicted to originate from non-periodic merging and the collapse of very highly energetic astrophysical systems. The current primary candidate are supernovae [17, 18]. Gravitational wave detections of these sources will give a valuable information on part of the universe that is currently not yet well understood.

![Figure 1.1: The quadrupole GW acts on rings of colour-coordinated masses in two basic polarised patterns.](image-url)
1.2 Ground-based gravitational wave detectors

The Michelson laser interferometric configuration has features that can optimise the detection of gravitational waves on Earth. These features include maximal sensitivity to the quadrupole GW radiation, broadband detection and common mode rejection of laser noise[19]. In a simplified optical configuration shown in figure 1.2 (a), the laser beam is split by a beamsplitter to propagate along two perpendicular paths that are designed to be equal in length. The beams bounce off ‘test mass’ mirrors and destructively interfere at the anti-symmetric port. A passing gravitational wave will act on the space-time of each interferometer arm illustrated as the dashed circles, causing different changes to the arm lengths. This leads to a difference in accrued phase at the beamsplitter, resulting in non-destructive interference and detectable optical signals at the photodetectors. The gravitational waves can now be extracted from the optical readout and presented as strain, which is a measure of the relative arm length fluctuations ($\Delta l$) to the physical Michelson arm length ($L$):

$$h = \frac{2 \Delta l}{L}$$

Figure 1.2: (a) Basic configuration of a Michelson interferometer (b) enhanced configuration for better signal to noise ratio readout. ITM: input test mass mirror, PRM: power-recycling mirror, SRM: signal-recycling mirror

Gravitational waves detected on Earth, even when generated by large astrophysical events, are minuscule. For example, the GW150914 binary black hole coalescence signal which was considered ‘bright’ for its large amplitude was detected at strain of $10^{-21}$. Therefore, to enhance the detector sensitivity, ground-based detectors are more complex with enhancements such as dual-recycling mirrors and optical cavities in Michelson arms, as shown in figure 1.2 (b)[20]. Currently, the LIGO gravitational
wave detectors are going through major upgrades to further increase the detector sensitivity. The international array of gravitational-wave detectors include:

- **Advanced LIGO** [21, 22]- a US led scientific collaboration. The LIGO network comprises three detectors, two interferometers located at Hanford and Livingston (USA) and the third one will be in India [23]. Each detector is a 4km arm interferometer whose sensitivity frequency band ranges from 10 Hz to 7kHz. The expected peak strain sensitivity will be at $3 \times 10^{-24}/\sqrt{\text{Hz}}$ after the upgrade is completed.

- **Advanced VIRGO** [24, 25]- operated by European Gravitational Observatory (EGO), where France and Italy are the main contributors. The advanced Virgo interferometer has 3km arm length and be most sensitive in the frequency band of 10 Hz - 10 kHz. The upgrade will increase the detection sensitivity with expected lowest strain at $3.5 \times 10^{-24}/\sqrt{\text{Hz}}$.

- **GEO-HF** [26, 27]- a German-British GEO colloboration, located near Hannover in Germany. The GEO-HF has folded interferometer arms to enhance the detector sensitivity instead of employing resonating optical cavities as Advanced LIGO and VIRGO. The interferometer arm length is 1200 m. The expected detection band will be in the range of 50 Hz - 6 kHz with expected peak sensitivity at $200 \times 10^{-24}/\sqrt{\text{Hz}}$ in strain.

- **KAGRA** [28, 29]- located underground at Kamioka, Gifu Prefecture, Japan. This will be the first underground gravitational wave detector with part of its interferometer operated under cryogenic condition. The 3 km arm length interferometer will be expected to peak sensitivity of $3 \times 10^{-24}/\sqrt{\text{Hz}}$ in strain in the frequency band of 10 Hz - 5 kHz.

### 1.3 Noise contributions to the detectors

For the target strain sensitivity of order of $10^{-24}/\sqrt{\text{Hz}}$, the ground-based detectors face many noise challenges in both classical and quatum regimes. The predicted noise contribution to Advanced LIGO is shown in figure 1.3 with brief descriptions introduced in this section.

#### 1.3.1 Thermal noise

Thermal noise is a displacement noise resulting from the statistically thermal fluctuation within the test masses and their suspensions. Mitigation of thermal noise is essential to the reduction of the overall displacement noise at the measured output. It is also one of the most challenging aspects of the detector design as materials used to construct the detectors, from test masses, optics to suspension, require low mechanical losses.

There are generally two approaches to lower this limit. The first approach is to investigate and characterise extensively the physical properties of a range of potential
§1.3 Noise contributions to the detectors

materials for the lowest mechanical losses. The second approach is to reduce the environmental temperature by going cryogenic. It is obvious that the most optimal choice for future detectors would be to combine both of the above. Hence the current frontier research in the thermal noise field is materials that are high quality in cryogenic environment such as silicon [32] and sapphire [33, 34].

The primary focus of this thesis is to apply the first approach to silicon material through measuring and analysing displacement spectral density of a silicon flexure.

1.3.2 Quantum noise

Quantum noise arises in the interferometer due to the fluctuating nature of the electromagnetic field that is used to interrogate the differential changes in the detector arm lengths due to passing gravitational waves. This is expected to be the main limiting factor to the second generation of detectors across audio frequency band [35]. The quantum noise can be split into two components: the shot noise and radiation pressure noise as a result of uncertainties in phase and amplitude quadratures of the laser beam.

![LIGO noise budget in its most sensitive frequency band][30, 31].
Introduction

Shot noise

Shot noise manifests through the detectors due to the randomly statistical arrival of photons to the end mirrors. This leads to fluctuations in accrued phase in the two arms, resulting in a measurement that mimics phase signals induced by gravitational waves at the asymmetrical readout port. Shot noise scales inversely proportionally to the square root of optical power, hence it can be lowered by increasing the input power [36].

Radiation pressure noise

Radiation pressure noise introduces to the interferometer as amplitude noise caused by photons’ momentum transferring the mirror displacements. This can imitate the act of gravitational waves modifying the space-time between mirrors, leading to false measurements. The noise source can be reduced by decreasing optical power in the detectors as the noise amplitude is proportional to the square root of optical power [36].

Standard quantum limit

At low power, the shot noise dominates whereas the radiation pressure noise dominate at high power. There exists an optical power at which the contribution of both of these noise sources produce the minimum quantum noise. This minimum is called standard quantum limit or SQL [36].

1.3.3 Seismic noise

Seismic noise can couple into the detector as mirror displacement which can be mistaken as GW signals. Thus, all the freely falling mirrors used in the GW instruments are suspended using isolation systems [37]. At Advanced LIGO, complex multi-stages of pendula and recoiled masses combined with active and passive feedback controls create a soft spring that achieves a displacement spectral density of the order of $10^{-8} \ 1 / \sqrt{\text{Hz}}$, gradually decreasing with increasing in frequency [38].

1.3.4 Gravity gradient noise

Gravity gradient noise, also known as Newtonian noise, is gravitational field fluctuations of the local environment causing an uncorrelated displacement noise on the mirrors. The main sources include anthropogenic activities (e.g: moving cars, trains, people) or fluctuating densities of mass distribution about the detectors (e.g: atmospheric pressure, surface waves) [39]. While understanding of the noise source is still in progress [40], this noise poses a limit to the sensitivity of the ground-based detectors at low frequency.
1.3.5 Other noise sources

Other noise sources that potentially limit the sensitivity of the detectors are: residual gas, feedback control noise and photothermal noise [41, 42]. They are expected to be relatively small and do not expect to limit the sensitivity of the current generation of detectors [35].

1.4 Motivation and thesis structure

With the current target sensitivity, suspension thermal noise is not yet a limiting noise source but it is expected to be in the future generation of detectors. In a Roadmap for the next 20 years, 2015-2035, in developing technologies for a much more sensitive gravitational wave detectors, mitigation of suspension thermal noise by using low loss material such as silicon has been proposed due to its zero-point thermal expansion at 120 K [43]. This is the main motivation driving the research force behind the work presented in this thesis.

Aside from the GW field, controlling thermal noise has become key to many other fields in physics such as designing laser cavities with higher frequency stability [44] and cavity quantum electrodynamics (QED) experiments [45]. This noise has also been the limit of classical noise before reaching quantum state limit in microscopic opto-mechanical experiments.

The goal of this thesis was to measure the thermal force induced displacement spectral density of a silicon mechanical oscillator in the audio frequency band. The work included two themes. First, the apparatus at the ANU was thoroughly characterised by displacement measurements of materials of decreasing mechanical loss: aluminium and niobium. The process helped understanding of the original layout leading to modification for a better sensitivity. Secondly, a simple thermal noise model based on the Fluctuation-Dissipation Theorem was used to predict and explain the obtained measurements.

Another part of the work presented here shows progress in exploring the macroscopic opto-mechanical properties in our system. This is part of a much larger efforts in understanding and eliminating the thermal effects to bring large scale mechanical oscillators to reach standard quantum limit [46].

The thesis structure is as follows:

• Chapter 2: begins with an overview of key developments in the thermal noise research field over the last century. Significant contributions within different areas of this field are integrated in the relevant sections throughout the chapter. Background of the Fluctuation Dissipation Theorem as the most commonly used theory and different loss types are provided to characterise the thermal force effects on a mechanical oscillator. We then introduce aluminium, niobium and silicon flexures and their properties, with a little more focus on the later one as it is a potential material for future detectors. Together with relevant sources of loss, we predict their displacement noise responses in the LIGO sensitive frequency band. At this point it is necessary to discuss other excess
noise produced by the presence of other components in the experiment and could influence the displacement models.

• Chapter 3: summarises key characteristics of a Fabry-Perot cavity. These steps are to prepare the readers for an interferometric sensing technique that conceptualises the experimental design of the thermal noise experiments presented in section 4.2. This chapter also explores the optical spring effect resulted from the restoring radiation pressure force derived from the circulating cavity field in the opto-mechanical cavity. Lastly, the bolometric force induced spring and damping in the cavity is presented as a model that could be used to describe experimental observations shown in chapter 7.

• Chapter 4: describes the interferometric length sensing technique as an optimal tool to detect continuous displacements of mechanical oscillators induced by thermal noise. A brief description of a basic feedback control is provided, together with discussion of unwanted experimental noise sources and their potential effects at the readout. This chapter ends with a short overview of digital and analogue lock acquisition locking systems.

• Chapter 5: details the experimental setup and challenges in noise hunting and system control to reach the desired sensitivity for thermal noise measurements. We dedicated this chapter to address preventive measures and reduction methods for the excess experimental noise sources mentioned in chapter 4. Furthermore calibration methods and measurement uncertainties are also included to clarify the measurements of interest.

• Chapter 6: presents thermal noise induced displacement results from aluminium, niobium and silicon flexures, in order of high to low loss. These results required an increase in sensitivity of the instrument used for measurements. Here the work presented in all previous chapters consolidate the characterisation of measured outputs.

• Chapter 7: presents the experimental observation of a stable single carrier opto-mechanical cavity. Two models were investigated and compared with experimental data.

• Chapter 8: is a summary of the work presented in the thesis and future direction.

1.5 Statement of contribution

I led the experimental characterisation of thermal noise induced measurements of aluminium and silicon oscillators, the modelling of theoretical mechanical loss contribution to displacements of aluminium, niobium and silicon oscillators under the supervision of D. McClelland, B. Slagmolen, P. Altin and D. Shaddock.
The work on the bolometric effect on an opto-mechanical system has been jointly investigated by P. Altin and I under supervision of D. McClelland, B. Slagmolen and R. Ward.

1.6 Publications


Introduction
Chapter 2

Thermal noise in a cantilever

“Nothing happens until something moves”
– Albert Einstein

This chapter contains three main sections. Section 2.1 summarises a thermal noise theory of a mechanical oscillator which was used to model experimental results in this thesis. A brief history of thermal noise discovery and categorising system for different loss types are also included. Section 2.2 describes cantilevers of different materials, niobium, aluminium and silicon, and their theoretical displacement spectra induced by thermal fluctuation under the influence of various losses. This section also contains some ringdown tests and results to further characterise quality factors and resonant frequencies of those cantilevers. The last section provides theoretical models for the excess noise produced by other components in a thermal noise experiment that could potentially be dominant at the measured outputs.

2.1 Thermal noise theory

The study of thermal noise started in the late 18th century with observations made by a Dutch scientist Jan Ingenhousz (1739 – 1799) on random motion of particles suspended in fluid, and later by Brown [48] and Einstein [49]. Nyquist reported similar characteristics in electrical circuits and further relating mechanical dissipation of a resistor $R$ to the voltage variance $V^2$ as [50]:

$$< V^2 > = 4k_B T R \Delta f$$  \hspace{1cm} (2.1)

Here $k_B$ is the Boltzmann constant, $T$ is the absolute temperature and $\Delta f$ is the bandwidth. Equation 2.1 showed a fundamental link between dissipation and fluctuation in a circuit system. Other experiments using solid materials such as reeds and metals also reported the same phenomena. The mechanisms for various types of measured dissipations were theorised using combinations of complex spring and dashpot. [51–54]. In 1951 Callen et al. successfully generalised the irreversible process using time-dependent thermodynamics leading to an important result: the Fluctuation-Dissipation Theorem [55, 56]. This framework enabled analytical calculations of many dissipative induced fluctuations of macroscopic objects once the source of losses had been identified.
There are several ways of categorising different research areas in the study of thermal noise of a mechanical oscillator. One way is to split the field into either mechanical thermal noise or optical thermal noise [57]. The mechanical thermal noise describes fluctuations due to dissipation, or damping loss, within the materials made up the oscillators. Some damping loss examples are thermally driven resulting in random micro-movements of atoms called Brownian noise and dissipative loss caused by stress and strain localised in the mechanical object. In an interferometer like LIGO the above dissipative types applied for both suspension thermal noise, and mirror substrate and coating thermal noise. Sections 2.1.4 and 2.4 provides more detailed descriptions and conditions of those loses and theories of their mechanisms.

Optical thermal noise refers to the effect of temperature fluctuation induced changes in physical properties of mirror materials in sensitive optical experiments such as thermal expansion of mirror coatings [58, 59] or refractive index of those coatings [60]. As a result they could lead to random phase changes and thus inaccurate experimental outputs. The effect of this noise type is further discussed in chapter 7 where we propose the highly dissipated loss resulting in a stable blue-detuned opto-mechanical system.

2.1.1 Fluctuation - Dissipation Theorem

The Fluctuation Dissipation theorem (FDT) is often explained using a system of external force on a mechanical oscillator. An external force $F_{\text{ext}}$ acts on an oscillator that makes the object displace a distance $x$ as illustrated in figure 2.1. Given that this system is linear and in thermodynamic equilibrium, a transfer function of the oscillator can be expressed in the frequency domain ($\omega = 2\pi f$) in terms of the resulted displacement and its applied force as

$$H(\omega) = \frac{x(\omega)}{F_{\text{ext}}(\omega)} \quad (2.2)$$

The mechanical impedance $Z(\omega)$ and mechanical admittance $Y(\omega)$ of the oscillator are given by [9]:

$$Z(\omega) = \frac{F_{\text{ext}}(\omega)}{i\omega x(\omega)} \quad (2.3)$$

$$= \frac{1}{i\omega H(\omega)} \quad (2.4)$$

$$Y(\omega) = \frac{1}{Z(\omega)} \quad (2.5)$$

where $Z(\omega)$ is the sum of the proportionality coefficients and $Y(\omega)$ is the inverse of the mechanical impedance.

The power spectrum of the fluctuating force is linearly proportional to the real
part of the mechanical impedance of the oscillator [61]

\[ F_{\text{therm}}^2(\omega) = 4k_B T \Re[Z(\omega)] \]  

(2.6)

where \( \Re[Z(\omega)] \) is also referred to as a mechanical resistance of the oscillator and represents the dissipative part of the impedance. The power displacement spectrum of the mechanical oscillator can now be written directly as:

\[ x_{\text{therm}}^2(\omega) = \frac{4k_B T}{\omega^2} \Re[Y(\omega)] \]  

(2.7)

where \( \Re[Y(\omega)] \) is called the conductance of the oscillator. Substituting equations 2.4 and 2.5 into the equation above the displacement spectrum can be further expressed in term of the transfer function of the mechanical oscillator:

\[ x_{\text{therm}}^2(\omega) = -\frac{4k_B T}{\omega} \Im[H(\omega)] \]  

(2.8)

Both the mechanical conductance and the imaginary part of the mechanical transfer function represent the dissipation part of the system. These vary depending on the source of losses and their mechanisms. The theorem shows how the mechanical oscillator couples to its surrounding thermal bath quantitatively. It also provides a link between the dissipative losses and the resulting fluctuations of the dissipated oscillator.

### 2.1.2 The FDT in a linear mechanical system

In a linear and thermodynamic equilibrium system, equation 2.8 can be further formalised by finding the transfer functions of the mechanical oscillator for each

\[ \text{Figure 2.1: An external force is applied onto a mass spring system and displaces the mass by a distance } x. \]
Thermal noise in a cantilever

damping case. For a single mode oscillator, the equation of motion is given by:

\[ F_{\text{ext}} = m\ddot{x} + kx \]  \hspace{1cm} (2.9)

where \( m \) is the mass of the oscillator. The thermally driven loss is often included as
the complex component of the spring, \( k = k_{\text{mech}} + i\phi \) where \( k_{\text{mech}} \) is the mechanical
spring constant; \( \phi \) is the imaginary part of the spring constant, and referred to as
the dissipative loss angles. Using the expression \( k_{\text{mech}} = m\omega_0^2 \) the transfer function
of the oscillator at the fundamental resonance now becomes:

\[ H(\omega) = \frac{1}{m\omega_0^2 - m\omega^2 + i\phi(\omega)} \]  \hspace{1cm} (2.10)

Expanding this over all resonances of the oscillator, the general equation of flexure
displacements expressed as a spectral distribution of dissipative losses is given by:

\[ \hat{x}_{\text{total}}^2 = \sum_{i=1}^{n} \frac{4k_B T\phi_i \omega_i^2}{m\omega \left[ (\omega^2 - \omega_i^2)^2 + (\phi_i \omega_i^2)^2 \right]} \]  \hspace{1cm} (2.11)

With

\[ \phi_i(\omega) = \sum_{k=1}^{\infty} \phi_k(\omega) \]  \hspace{1cm} (2.12)

In which \( \sum_{k=1}^{\infty} \phi_k \) is the sum of all mechanical losses of the oscillator at \( i^{th} \) mode. Equation 2.11 will be used to predict and discuss the thermal noise induced displacement
spectra of mechanical oscillators in this thesis.

2.1.3 Categorising loss based on their frequency dependence

The dissipative loss can be divided into two main groups whose frequency response
can either be independent or dependent. The frequency dependent characteristics
have distinct features that can be observed in the off-resonance displacement spectra
induced by each group.

The frequency independent loss is often referred to as structural damping with
\( \phi_k = \frac{1}{Q} \). Most bulk loss and surface loss reported in literature are representatives
of this loss type. The flexure displacement at different regions of the fundamental
resonance (\( i = 0 \)) is approximated to:
\[ x_{th, struc} = \begin{cases} \sqrt{\frac{4k_B T}{m\omega_0^2 Q}} & \text{for } \omega \ll \omega_0 \\ \sqrt{\frac{4k_B T Q}{m\omega_0^3}} & \text{for } \omega = \omega_0 \\ \sqrt{\frac{4k_B T \omega_0^5}{m\omega^3 Q}} & \text{for } \omega \gg \omega_0 \end{cases} \]

where \( \omega_0 \) refers to the frequency (in radians) of the fundamental resonant mode.

The second type of loss is referred to as viscous damping and has a frequency dependent response. Thermoelastic damping is an example of this loss type and was first studied in details by Zener [52, 62] in his investigation on anelastic responses in materials. Other sources of this loss type are gas damping, loss caused by clampings, glues and eddy current. With \( \phi_\omega = \omega/(\omega_0 Q) \) as a general form for velocity-dependent mechanical loss angle, different regions above and below oscillator resonances have different frequency responses.

Figure 2.2: Amplitude spectral density of structural and viscous damping thermal noise of an oscillator of 0.5 g at the resonant frequency of 100 Hz and Q of 2000. These two loss types depend on frequency differently: structural damping is constant across all frequency band, while as viscous damping is a frequency dependent loss.
Thermal noise in a cantilever

\[ x_{th,vis} = \begin{cases} \sqrt{\frac{4k_BT}{m\omega_0^2Q}} & \text{for } \omega \ll \omega_0 \\ \sqrt{\frac{4k_BTQ}{m\omega_0^3}} & \text{for } \omega = \omega_0 \\ \sqrt{\frac{4k_BT\omega_0}{m\omega^2Q}} & \text{for } \omega \gg \omega_0 \end{cases} \]

On resonance (\( \omega = \omega_0 \)) the flexure displacements are independent of frequency for both structural and viscous damping types. In both cases, as Qs get higher, the off-resonant displacements decrease with the square root of Q. Hence, high Q systems with their resonances outside the frequency band of interest are often desired. However, with all the energy concentrated to these resonances, it can lead to increasing displacement amplitude resulting in parametric instability [9].

A typical oscillator displacement spectrum caused by the two loss types is graphically shown in figure 2.2. The red trace (a) is plotted assuming the oscillator experiences a frequency independent damping while the blue trace (b) is with frequency dependent damping. It is important to notice the different slopes of these two types of loss angles. Below the flexure resonant frequency, the spectrum has an \( f^0 \) dependent for frequency dependent loss and \( f^{-1/2} \) for frequency independent loss. The slope of the frequency independent thermal noise spectrum above resonance is \( f^{-5/2} \), which is steeper than the \( f^{-2} \) slope of the mechanical susceptibility. This is of particular interest when attempting to construct an opto-mechanical experiment that is sensitive to radiation pressure noise. For the case of frequency independent damping, having a resonator with a very low resonance frequency means the thermal noise rolls-off faster than radiation pressure noise, which follows the mechanical transfer function of the oscillator. This experiment with the appropriate design parameters can be dominated by radiation pressure noise at frequency above \( f_0 \). However, if the thermal noise exhibits a velocity-damped spectrum, this kind of measurement design is no longer possible.

2.1.4 Sources of loss

The loss mechanism in the system can be categorised into external and internal loss. For the purposes of this thesis internal loss is restricted to any type of loss related to the mechanical oscillator such as structural loss, thermoelastic loss and surface loss. Other losses are considered external, for examples: gas damping, eddy current damping and glue and clamping loss [63, 64]. Some will be discussed later in this chapter.

Structural loss

Structural loss or bulk loss is referred to the intrinsic loss in the micro structure of each material. Saulson [65] described this loss using Maxwell unit as illustrated in figure 2.3. Lossy aluminium could be represented by a soft spring and dashpot while a
series of springs and dashpots could be used to mimic the response of stiffer materials like the rigidity of silicon [54]. The constant loss angles over wide frequency ranges were reported for different materials in [61, 66, 67]. The increase in this intrinsic loss in material is thought to be due to dislocation within the atomic lattice or point defect migrations [68, 69]. In most experiments, this loss is the lowest dissipative loss and thus is often designed to be dominant. This would result in a thermal noise induced displacement of the system that displays a fast roll-off above the resonance. Once this condition is met the loss becomes the fundamental limit to the system sensitivity and can only surpass by cooling and/or replacing the materials used with a lower mechanical loss [70, 71].

Thermoelastic loss

Thermoelastic dissipative loss is a case of anelastic relaxation process in solid materials and was first suggested by Zener [52]. Thermoelastic loss occurs when there is stretch and strain points causing a local micro temperature gradient between these points. The thermal gradient leads to heat flow in an attempt to re-establish an equilibrium state. Consequently, thermoelastic loss depends strongly on the oscillator physical geometry, more specifically the path along which the energy flows, and the temperature dependent properties of the bulk material such as Young modulus [72] and thermal expansion coefficient [69]. Mathematically for a beam-like mechanical oscillator this frequency dependent loss is described with a characteristic strength $\Delta$, and characteristic time $\tau$ [73]:

\[
\phi_{\text{te}}(f) = \frac{\Delta}{1 + (\omega \tau)^2} \tag{2.13}
\]

\[
\Delta = \frac{\alpha^2 E_y T}{\rho C_v} \tag{2.14}
\]

\[
\tau = \frac{\rho C_v t^2}{\kappa \pi^2} \tag{2.15}
\]

here $\alpha$ is the linear thermal expansion coefficient, $E_y$ is the Young modulus, $T$ is the temperature, $\rho$ is the density, $C_v$ is the specific heat, $\kappa$ is the thermal conductivity.
of flexure material and $t$ is the thickness of the membrane. The characteristic time defines the optimal time for heat transfer across the flexure thickness, and an equivalent to a maximum loss in the frequency response.

Thermoelastic loss introduces the upper limit to many well designed micro-mechanical systems [75]. Among those parameters, the thermoelastic loss, the flexure geometry and material thermal expansion could potentially be used to minimise the loss within the frequency band of interest. For example, the suspension wires of the LIGO mirrors were chosen after series of experiments on different potential shapes for as low loss as possible [76]. The profile of the fibers are designed to minimise the thermoelastic damping and they are thick near the bending point and thin along the length [77]. As the characteristic time is proportional to the square of the membrane thickness, this dimension is key to lower the dissipation by shifting the thermoelastic damping out of the frequency range of interest. Another option is employing the strong dependency of the linear thermal expansion coefficient on temperature of the bath. An example of this is shown in figure 2.4 featuring the expansion coefficient of silicon between 6 K and 300 K [74, 78]. The zero-crossings at 14 K and 125 K means the thermoelastic loss would vanish regardless of the oscillator shapes, making silicon a promising candidate for future detectors [77].

![Figure 2.4: Non-linear and non-constant response of silicon thermal expansion coefficient shows temperature dependency. This property is one of the reasons making the material a promising candidate for future gravitational wave detectors [74].](image-url)
Surface loss

Surface loss can occur due to the difference in the Young’s modulus between the outer layer and the bulk of a mechanical oscillator. An oxidised layer on the outer surface of an oscillator is an example of the undesirable cause of this loss due to unintentional exposure to the environment. Improper treatments that lead to rough finishing surfaces and/or chemical residues tend to be the culprits [64]. Finally cracks and lattice defects are also contributing to increase the surface loss of the oscillator [79]. In some cases consequences from cracks could cause a tremendous increase in the thermal fluctuation [47, 80].

Often surface loss is significantly different to the intrinsic bulk loss of the materials [81]. The analytical form of this loss can be expressed as follows:

\[ \phi_s = \alpha_s \frac{S}{V} \]  
\[ \alpha_s = \phi_{bulk} \cdot d_s \]

Parameter \( \mu \) determines by the oscillator mode shape. For a pure bending mode and the thickness to width ratio of the mechanical oscillator is much less than 1, \( \mu \) calculated by [82] has a value of 3. The product of \( \phi_{bulk} \cdot d_s \) depends on the bulk loss of the material underneath and the thickness \( d_s \) of the lossy layer deposited on the surface. As \( \phi_s \) depends on the surface area to volume ratio, thin oscillators such as micro cantilevers are likely dominated by surface loss [83]. For a silicon cantilever of 130 \( \mu \)m thickness this product was reported as small as 0.5 per meter [84].

2.2 Cantilevers

![Diagram of cantilever](image)

Figure 2.5: A cartoon of the flexure angular displacement under an applied force.

The mechanical oscillator used in the projects was a vertically orientated cantilever, a term that is used interchangeably with ‘flexure’ in this thesis. The angular displacement \( \delta \theta \) relates linearly to an applied torque \( \tau \) through an angular spring
constant $k_a$. Physical geometry and material elasticity of the flexure defines the above constant governing the flexure responses.

$$k_a = \frac{E_y I_a}{L}$$  \hfill (2.18)

The angular spring constant scales as the inverse to the length (L) of the flexing membrane. The Young modulus $E_y$ (in unit of Pascal) indicates the degree of elasticity response of the flexure under stress and strain. Stiffer materials have higher Young moduli, leading to a higher angular spring and smaller resultant angular displacement. With similar sizes of the flexures used in the project this parameter determined the angular spring and frequency of the flexure. The second moment of inertia $I_a$ incorporates the cross-section area to the effective angular spring constant for different resonating modes. For the bending mode of a beam with rectangular cross-section area of thickness $t$ and width $W$, the second moment of area is

$$I_a = \frac{W t^3}{12}$$  \hfill (2.19)

The resonant frequency of the flexure depends on the squared ratio of the angular spring and flexure moment of inertia $I$. As shown in equation 2.20 it is independent of distance from the axis of rotation to the position of the applied force. The linear translation of the rotation of axis for the resonant modes with small angle approximation give a resonant frequency and an effective mass for that mode.

$$\omega = \sqrt{\frac{k_a}{I}} = \sqrt{\frac{k_l h^2}{I}}$$

$$m_{eff} = \frac{k_l}{\omega^2} = \frac{1}{h^2}$$  \hfill (2.21)

Advantages of employing a cantilever are many. Some are listed below [9, 85]:

- The flexure properties have been well investigated. It is wear free. Under no severe distortion, their structural integrity and axis of rotation will remain constant throughout their lifetime.

- The flexure shape is simple enough to enable monolithic construction to reduce extra clamping or glue loss, thus minimise extra stress or damping to the flexure.

- The flexure resonant modes create smooth and continuous bend/twist along the length of the membrane. The overall thermal noise induced displacement can be calculated analytically from the sum of individual modal thermal fluctuation.

- The flexure has insignificant optical dilution\(^1\). The measured Qs were not increased by this factor [63].

\(^1\)Optical dilution is the diluted factor produced by the ratio of the restored energy due to gravity or tension field to dissipated energy due to the stress and strain (see section 3.3 of [63]).
The range of angular displacements of a cantilever depends on the highest stress allowable at the surface and elasticity characteristics of the material making up that cantilever. Beyond this point, the flexures may display some hysteresis response or worse breaking. In this thesis, forces applied on all the tested flexures are well away from causing these issues. For reference, their estimate breaking points are shown in the next paragraph.

For this type of flexure the load bearing capability is limited by maximum tensile stress response \cite{85}. The critical force is driven by the elasticity of the material, the cross section of the bending area under bending motion and the length over which the load is distributed. Examples of this force is a torque created by the gravitational field pulling on the mass on top of the flexing membrane, or powerful radiation pressure force pushing on the flexure. As the top mass increases, the torque will put more stress along the flexure bending length and eventually snap. The critical force just before this happens can be calculated for a specific flexure length, moment of area and material Young’s modulus \cite{86}:

\[
F_{\text{critical}} = \frac{\pi^2 E_y I_a}{(K L)^2}
\] (2.22)

K is the column effective length factor whose value depends on the boundary conditions of the beam. For a one end fixed beam K takes a value of 2. With a flexing membrane of 6.35 mm width, 110 \(\mu\)m thickness and 1 mm length, equation 2.22 approximates a range of critical forces between 123.4 N and 282.2 N with the low and high Young modulus materials, aluminium and silicon, setting the lower and upper bounds respectively. Niobium material falls in the middle of the range. The critical force can then provide an approximation for the maximum allowable load on the top of the membrane. Assuming the flexure deflection is caused by the gravitational field the estimated maximum loads are 12.6 to 28.8 kilograms with respect to the above metallic materials. With similar assumption applied to the radiation pressure force, gigaWatts of power would be required before the flexure reaches its snapping point, and hence not a concern in our experiment.

### 2.2.1 Aluminium and niobium flexures

Figure 2.6 shows pictures of niobium flexures and aluminium flexure whose flexing membranes were 200 \(\mu\)m, 65 \(\mu\)m and 120 \(\mu\)m resulting in resonant frequencies of 300 Hz, 85 Hz and 271 Hz respectively \cite{87, 88}. The flexure bases were cut along with the rest of the flexure shape to form a single uniform piece as shown in figure 2.6. This moved the mounting point of the flexures further away from the flexing membrane, and help minimise the clamp loss induced flexure fluctuation.

Observations for different loss angles for both aluminium and niobium were reported in \cite{62}. Duffy in \cite{89} reported experimental data for bulk losses of various Aluminium alloys at temperatures ranging from 50 mK to 300 K. The lowest values achieved with procured chemical and heat treatments were \(10^{-7}\) (equivalent to 1,000,000 for pure structural loss). In similar preparation steps Ju reported a bulk loss of Niobium to be around \(10^{-8}\) \cite{90}.
2.2.2 Silicon flexure

Future LIGO generations are predicted to be limited by coating thermal noise and suspension thermal noise [43]. However moving toward cryogenics in the current setup is potentially problematic [91]. For example, fused silica, which is currently used as suspension wires in LIGO, is known to have a wide peak loss due to an increase in thermoelastic loss [92–94] and poor thermal conductivity as temperature cools down to 40 K [95, 96]. Recently attention has shifted to silicon as a material for the LIGO test masses’ suspension wires and suspension blades due to its response of the thermal expansion coefficient as shown in figure 2.4. As the temperature drops the silicon thermal expansion also drops rapidly and crossing zero at 18 K and 125 K. Moreover the surface loss of silicon also increases as the surface to volume ratio decreases. For a 1 µm thick cantilever the silicon surface loss was reported to be 20 times smaller than fused silica one [84, 97].

**Flexure design**

A picture of a silicon flexure and its schematics are shown in figure 2.7. The brittle

![Figure 2.6](image-url)  
*Figure 2.6: From left to right: niobium flexures of 300 Hz and 85 Hz and aluminium flexure of 271 Hz. The flexure sizes are comparable to a 10 cents coin.*

![Figure 2.7](image-url)  
*Figure 2.7: Silicon flexure design and schematics.*
property of silicon eliminates the possibility to explore the option of manufacturing the silicon flexure monolithically as in similar case to aluminium and niobium flexures. Silicon wafers were laser cut into a T-shape to form the overall structure of the flexure. The flexing membrane was cut using a nano-lathe at ANU. The flexure dimensions were restricted by two design requirements:

- **Physical constrains**: the flexure fundamental resonant frequency was devised to be within the current LIGO sensitivity band. Experimental results from aluminium and niobium flexures (see chapter 6) also predicted a dominating and competing presence of structural and thermoelastic damping. A measurement of off-resonant structural damping needs the flexure resonance as far from the thermoelastic relaxation peak as possible to reduce the thermoelastic effect, while still dominated by structural damping within our experimental band of $10 \text{ Hz} - 1 \text{ kHz}$. This put a limit to the dimension of the flexing membrane, especially its thickness on which the thermoelastic damping depends solely.

- **Technical constrains**: silicon brittleness and the oxidisation effect that could lead to increase in surface loss [98] limited how simple and thin the flexing membrane could be cut. This thickness also was a crucial parameter in the resulted flexure resonance which could be adjusted by the top mass. Furthermore the beam height, and therefore mirror position, was constrained by the preceding height from the PZT mirror to the base slab in order to form an optical cavity.

Under cryogenic conditions, silicon material was reported to show surface loss trend below 50 K. Nawrodt [84] stated that the silicon flexure would follow thermoelastic loss down to 50 K.

**Forming a flexure mirror**

![Figure 2.8: Flexure on a circular jig during mirror gluing process. Cotton was padded to help soften without loosening to keep the flexure in place during the lathe machining process to cut the flexing membrane to micron thickness.](image-url)
In a thermal noise experiment any amount of contaminants could be detrimental to desired outputs as they create an oxidisation layer at the outside of the flexure. As a result it increases the total damping in the form of surface loss. To minimise

Figure 2.9: Top row: pictures were taken during test trial on a chipped flexure to determine the required torque before reaching breaking point. Below: schematics and dimensions of the clamp and mount of the silicon flexure.
this scenario we proceeded to clean the flexure thoroughly prior to placing it under vacuum. We tried and discarded the sonic bath method as it led to breaking the flexure at the T-junction. In the end we resolved to ‘bath/soaking’ for 10 minutes in each step:

1. liquinox to get rid of the hydrocarbons;

2. acetone to get rid of the soap in the liquinox;

3. methanol to get rid of the residual acetone.

Once the cleaning process was done we proceeded to glue a 1/4” mirror to form a flexure mirror. The flexure was mounted to a circular jig that was also used with the lathe to cut the flexing membrane as shown in figure 2.8. A small amount equivalent to a needle head of LIGO approved Masterbond EP30-2 [99] was sufficient to glue the mirror in place. The glue mixture was degased while still in its liquid form to eliminate bubbles that may burst under vacuum. As the width of the flexure was designed to be the same as the mirror diameter the edge of the flexure helped guide the position of the mirror. This was to keep the glue strictly at the edge of the mirror so to minimise the contamination of the optical coating on the mirror surface.

**Mount**

Changes in the silicon flexure shape required a new mount to clamp it in place to form a flexure mirror. Pictures and schematics of the mount from different view angles are shown in figure 2.9. The mount is comprised of an aluminium base and a pair of steel rectangular clamps. The aluminium base was designed to enable fine tuning of tilt motion by small increase and decrease of a 3 mm space gap using two M5 screws at the back of the mount. After a test trial of clamping a spare silicon flexure we settled on a torque wrench setting of 6 Nm. The final silicon flexure mirror is presented in figure 2.10.

### 2.2.3 Ringdown measurements

Quality factor Q is a unitless parameter that captures total dissipative losses at resonant frequencies and obtained through a series of ringdown measurements. This value is quintessential in predicting thermal driven fluctuations of a mechanical oscillator. We used the laser beam that was reflected off the flexure mirror to monitor the flexural displacement as shown in the top view of figure 2.11. The flexure was mounted on a metal base inside a vacuum chamber whose pressure ranged from atmospheric to $10^{-6}$ mbar. The reflected beam was detected by a quadrature photodetector to give independent readings of the horizontal and vertical displacements of the flexure. We started a series of experiments in this setup with both horizontal and vertical readouts on the photodetector calibrated to zero offset readings. We repeated the experimental steps as we dropped the pressure in $10^{-1}$ mbar increments to find a point where gas damping effect no longer dominated the total damping.
We excited the flexures by tapping the vacuum chamber casing and observed the ringdown on the oscilloscope. The flexure and mount was placed on top of a heavy base which in turn was bolted onto the vacuum optical breadboard with sorbothane.

Figure 2.10: A close-up picture of the silicon flexure mirror with a Thorlabs mount in the background served as size comparison. Gathered dusts that were conspicuous under torch light but invisible to the naked eyes reinforce the need for clean optics to avoid light scattering which was a real issue to constantly combat in the experiment (see chapters 4 and 5).

Figure 2.11: Schematics of the ringdown experiment from top view (left) and side view (right). QPD: Quad photodiode measuring horizontal and vertical beam positions due to flexure displacements.
sandwiched in-between. The right image of figure 2.11 gives a simplified schematics of the experimental setup from side view. Sorbothane were used after realising that even gentle tapping was still too strong and would overexcite the flexure. The ringdown signal showed a mixed responses of many different resonances and therefore prevented the accuracy of quality factor. Sorbothane hence helped damp part of the external excitation and the strong vibration connecting the vacuum pump and the chamber. Furthermore the optical breadboard was lifted up from the chamber using three long screws to reduce dissipative coupling to the table. The ringdown measurement was recorded vertically and then fitted to calculate flexure quality factor.

Both methods were tested on Niobium flexure to confirm their validity against the Q value previously measured [87]. The envelope of the recorded ringdown was then fitted through with decay time $\tau$ as a single fitting variable using the following equations:

\begin{align*}
x_0 &= Ae^{-t/\tau} \\
Q &= \pi \tau f_0
\end{align*}

where $x_0$ is the recorded flexure displacement, $f_0$ is the flexure resonant frequency and $\tau$ is the decay time as the amplitude A reduced by $1/e$.

![Figure 2.12: Ringdown measurement for silicon flexure at a pressure of $2.5 \times 10^{-4}$ mbar.](image)

A typical ringdown measurement and its fitted result are shown in figure 2.12. Table 2.1 displayed the measured quality factors obtained for fundamental resonances of aluminium, niobium and silicon flexures. As the estimation for Q in a ringdown experiment relied heavily on accurately measuring the time it took for a reduction of
1/e on flexure displacement amplitude, external factors could influence and alter this time leading to underestimating the actual values. Those factors were gas damping, damping from mount, clamp and the vacuum case and vibration of connecting tubes between the pump and the tank. The effect of gas damping in the outcomes were reduced by repeating the ringdown measurement steps at a series of decreasing pressures: atmospheric, $10^{-2}$ mbar, $10^{-3}$ mbar, $10^{-4}$ mbar. The values from each pressure showed an increase in Q value hence a decreasing effect of gas damping. Below $10^{-4}$ mbar there was no noticeable changes in the flexure quality factor. Other factors were reduced through adjusting the experimental setup appropriately as stated above. The vibration through connecting vacuum tube was reduced by cutting the pump off and disconnecting the tube completely. The starting pressure was at $10^{-6}$ mbar and slowly increased after the pump was off. As a result measurements were achieved at pressure as low as $5 \times 10^{-4}$ to avoid residual vibration.

Table 2.1: Quality factor at the fundamental resonance frequencies of aluminium, niobium and silicon flexures. Designed flexure thicknesses were confirmed experimentally through the measured flexure resonant frequencies.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Aluminium</th>
<th>Niobium</th>
<th>Silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Flexure thickness</td>
<td>120 µm</td>
<td>72 µm</td>
<td>100 µm</td>
</tr>
<tr>
<td>Resonant frequency</td>
<td>271</td>
<td>85</td>
<td>167</td>
</tr>
<tr>
<td>[Hz]</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Q</td>
<td>$2100 \pm 100$</td>
<td>$44000 \pm 2000$</td>
<td>$56000 \pm 2800$</td>
</tr>
<tr>
<td>$\phi(f_0)$</td>
<td>$4.76 \times 10^{-4}$</td>
<td>$2.27 \times 10^{-5}$</td>
<td>$1.79 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

2.3 Theoretical thermal noise induced displacements of the cantilevers

Theoretical thermal noise induced displacements for the flexures made of aluminium, niobium and silicon are presented here. The two dominating loss types that are considered for this cantilever shape are structural and thermoelastic damping. Table 2.2 lists parameters used to calculate the thermoelastic damping for different material flexures. As the thermal transfer path increases due to an increase in membrane thickness, the maximum relaxation frequencies shift to a lower frequency region. This means a thicker membrane would result in a higher possibility of thermo-elastical dominance in the regions between $50$ Hz−$1$ kHz.

Figure 2.13 shows the thermoelastic loss angles as a function of frequency for different flexure membrane thickness for aluminium (bottom) and silicon (top). The loss amplitudes are a factor of 10 higher for aluminium than silicon. In fact aluminium material is generally lossier than both niobium and silicon materials due to a much higher thermal expansion coefficient. For the same thickness the characteristic time for aluminium is higher leading to a lower peak frequency than for silicon.
Theoretical thermal noise induced displacements of the cantilevers

thickness of 120 µm the aluminium thermoelastic frequency is predicted to peak at 5.6 kHz while with a similar thickness of 100 µm the silicon relaxation peak is at 15 kHz.

Figure 2.14 and 2.15 shows theoretical thermal noise induced flexure displacement for aluminium and silicon respectively. The models were constructed using the Q values obtained from ringdown measurements assuming structural damping, and theoretically calculated values for thermoelastic loss, respectively. With a resonant frequency of 271 Hz and the measured Q of 2 200 the thermal noise displacements could potentially be dominated by both types of losses around the flexure resonance. Below the resonance the flexure is predicted to follow structural loss induced fluctuation. However above the resonance the dominant response is expected to be thermoelastic with an increase in noise fluctuation about the relaxation peak above 1 kHz.

Niobium flexure displacement spectra induced by thermal noise had similar mixed responses with structural loss dominated below resonance and thermoelastic loss dominated above. Due to the material naturally having much lower loss, the theoretical spectra were an order of magnitude lower than aluminium ones.

The silicon flexure was designed to be dominated by structural loss. Figure 2.15 predicts the thermal noise induced silicon flexure displacement where the structural damping dominates the thermoelastic damping. Above flexure resonance, the thermoelastic damping displacement spectrum drops slower than the thermoelastic one by $f^{-1/2}$ and becomes dominant at high frequency. In the cross-over region, both loss types affects equally to the flexure displacements.

![Graph showing thermoelastic damping for different flexure thickness of aluminium and silicon materials for this particular designs. Top: silicon flexure, bottom: aluminium flexure.](image-url)
Table 2.2: A list of parameters used in the theoretical models. [100, 101]

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Aluminium</th>
<th>Niobium</th>
<th>Silicon</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mirror diameter</td>
<td>$6.35 \times 10^{-3}$</td>
<td>$6.35 \times 10^{-3}$</td>
<td>$6.35 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>Mirror thickness</td>
<td>$2 \times 10^{-3}$</td>
<td>$1 \times 10^{-3}$</td>
<td>$2 \times 10^{-3}$</td>
<td>m</td>
</tr>
<tr>
<td>Young’s modulus ($E_y$)</td>
<td>71</td>
<td>105</td>
<td>162.4</td>
<td>GPa</td>
</tr>
<tr>
<td>Linear thermal expansion coefficient ($\alpha$)</td>
<td>$23 \times 10^{-6}$</td>
<td>$7.3 \times 10^{-6}$</td>
<td>$2.54 \times 10^{-6}$</td>
<td>K$^{-1}$</td>
</tr>
<tr>
<td>Specific heat ($C_v$)</td>
<td>904</td>
<td>265</td>
<td>711</td>
<td>J(kgK)$^{-1}$</td>
</tr>
<tr>
<td>Thermal conductivity ($\kappa$)</td>
<td>138</td>
<td>54</td>
<td>149</td>
<td>W(mK)$^{-1}$</td>
</tr>
<tr>
<td>Density ($\rho$)</td>
<td>2820</td>
<td>8578</td>
<td>2330</td>
<td>kgm$^{-3}$</td>
</tr>
<tr>
<td>Thermoelastic resonance ($f_{te}$)</td>
<td>$5.6 \times 10^3$</td>
<td>$7.2 \times 10^3$</td>
<td>$15 \times 10^3$</td>
<td>Hz</td>
</tr>
</tbody>
</table>

These predicted theoretical traces will be compared against the experimental measurements with no free parameters (see chapter 6), it is therefore utterly important that they are carefully calculated. The theoretical models rely on the accuracy of two parameters: flexure effective masses and thermal dissipative loss. As shown in equation 2.21, the uncertainties of the effective mass depend on the flexure geometrical

![Figure 2.14: Thermal noise models for aluminium flexure.](image-url)
dimensions and the beam height. The uncertainties of the thermal dissipative loss, on the other hand, relies on correctly identifying the types of loss that dominate the system. The loss amplitude can then be either calculated and/or obtained through measurements.

2.4 External noise induced displacement

As the thermal noise displacements of the cantilevers are exceedingly small, the presence of other noise could effectively mask the signals of interest. The contributions are from ambient environment, such as gas damping, and thermal noise induced fluctuations of experimental components, like mirror substrates and coatings, piezo-transducer (PZT), clamps and glue. This section provides some estimation of those losses and ways to minimise them. Figure 2.16 shows the predicted thermal noise resulting in flexure displacement caused by those losses.

2.4.1 Gas damping

Gas damping belongs to the frequency dependent loss category and occurs due to collision of air molecules with the flexure. The extent of the gas damping effect on the flexure depends on factors such as air composition, their density and mean free path and environmental temperature (T). The flexure surface area also plays a significant role as this is where the interactions happen. Theoretically the gas

![Graph](image-url)  
Figure 2.15: Thermal noise models for silicon flexure.
damping loss can be predicted through the following equation [102]:

$$\phi_{\text{gas}}(\omega) \approx \frac{AP}{m\omega} \sqrt{\frac{M}{RT}}$$  (2.25)

where $A$ is the surface area, $P$ is the pressure, $R$ is the gas constant, $M$ is the molar mass of air molecules and $T$ is the environment temperature. As mentioned in ringdown measurements higher pressure environments generally resulted in lower measured quality factors of the flexures. As we lowered the pressure below $10^{-4}$ mbar, the measured Q values remained unchanged, confirming that the flexure was no longer dominated by gas damping. At the pressure of $10^{-6}$ mbar the gas damping loss was below the flexure displacements of interest shown in figure 2.16.

2.4.2 Mirror

Thermal noise from internal friction in the mirror test masses is capable of limiting the interferometer sensitivity and was first identified in LIGO by Gillespie and Raab in 1995 [103]. Mirrors in optics experiments consists of a transparent substrate, often made of fused silica, and layers of coatings to control the transmittivity of light through the substrate. The final mirror thermal noise induced flexure displacements includes separate calculations for the substrate and coatings and then add them in quadrature.

![Figure 2.16](image-url)

**Figure 2.16:** Estimated equivalent displacements induced by structural and thermoelastic thermal noise of mirror substrates, structural and thermoelastic thermal noise of mirror coatings, and thermal noise of the PZT and mirror combination.


**Mirror substrate**

The adiabatic condition for the coating and mirror substrate is when the measured frequency band is above the frequency $\omega_c$, where $\omega_c = \frac{\kappa}{\rho r_0^2}$, and the thermal length in the substrate is larger than the coating thickness and smaller than the beam radius ($r_0$). In the corner frequency expression, $\kappa$ is the conductivity of mirror, $\rho$ is the mirror density and $C$ is the heat capacity. Under this condition Levin applied the Fluctuation Dissipation Theorem to create a more direct method to calculate an overall thermal spectral displacement for the mirror substrate based on substrate loss ($\phi_{\text{sub}}$), substrate Young modulus ($E_{\text{sub}}$), substrate linear expansion ($\alpha_{\text{sub}}$), substrate density and beam radius [104].

$$x_{\text{sub, struc}}^2 = \frac{4 k_B T (1 - \sigma_{\text{sub}}^2)}{\sqrt{2\pi E_{\text{sub}} \omega r_0}} \phi_{\text{sub}}(\omega)$$ (2.26)

$$x_{\text{sub, TE}}^2 = \frac{8 k_B T^2 \alpha_{\text{sub}}^2 (1 - \sigma_{\text{sub}}^2)}{\sqrt{2\pi E_{\text{sub}}^2 \rho^2 r_0^4 \omega}}$$ (2.27)

In the above equation $\sigma_{\text{sub}}$ is a Poisson ratio$^2$ of the substrate and has a value of 0.17.

**Mirror coatings**

Each mirror coating is a stack of multi-alternative layers of thin films that often have different reflective indices and thickness. The two materials often used for dielectric mirrors are fused silica and tantalum oxide. Conventionally each layer of coating is designed to have an optical thickness of a quarter optical wavelength ($\lambda$), so $dT_a = \frac{\lambda}{4nT_a}$ and $dSF = \frac{\lambda}{4nSF}$ where $nT_a$ and $nSF$ are the reflective indices for tantalum oxide and fused silica respectively. Due to the combination of different materials, the analytical expression of coating thermal noise is more complex. As light reflects off the mirror surface, the thermal fluctuation in these layers lead to random phase changes. The thermal expansion ($\alpha_{\text{coat}}$) of the coating depends on those of the dielectric materials and can be calculated as [58]:

$$\alpha_{\text{coat}} = \frac{(\alpha_{T_a} \frac{dT_a}{dT_a + dSF}) E_yT_a (1 - 2 \sigma_{SF})}{E_ySF (1 - 2 \sigma_{T_a})} + \frac{\alpha_{SF} \frac{dSF}{dT_a + dSF}) E_ySF (1 - 2 \sigma_{SF})}{E_ySF (1 - 2 \sigma_{SF})}$$ (2.28)

where $E_ySF$ and $E_yT_a$ are the Young moduli for fused silica and tantalum oxide respectively. The Poisson ratio for fused silica ($\sigma_{SF}$) and tantalum oxide ($\sigma_{T_a}$) are 0.17 and 0.23. The thermal noise of the mirror coating due to thermoelastic damping

$^2$Poisson ratio is the ratio of transverse contraction strain to longitudinal extension strain in the direction of an elastic load, often indicating a degree of tensile deformation [105]
Thermal noise in a cantilever

is expressed as follows:

\[
x_{\text{coat}} = \frac{4 \sqrt{2} \left(1 + \alpha_{SF}\right)^2 \left(\alpha_{\text{coat}} - \alpha_{\text{sub}}\right)^2 d_{\text{total}}^2 k_B T^2}{\pi r_0^3 \sqrt{\kappa \rho_{SF} C} \sqrt{\omega}}
\]

(2.29)

With 19 pairs of coatings the total coating thickness is \(d_{\text{total}} = 20 (d_{Ta} + d_{SF})\).

The theoretical thermal noise of mirrors and coatings are shown in figure 2.16. For the purpose of this thesis these thermal noise fluctuations are not yet the limiting factors. However these predicted traces have large margin of errors due to large uncertainties in the value of thermal expansion of tantalum oxide [106]. For gravitational wave detectors these thermal noise dominates the noise floor and thus lowering the instrument sensitivity. A tremendous amount of research have been directed toward reducing these noise sources through characterising the noise source, searching for low loss materials, improving the mirror substrate and coating quality and implementing various coating techniques [58, 107–109].

### 2.4.3 Piezo-transducer

Piezoelectric transducer (PZT) is made of ceramic material that can expand and contract when voltage is applied. In an optics system PZT combined with a mirror acts as an actuator in a feedback control loop to correct for undesired fluctuations (see chapters 4 and 5). Thermal fluctuation of this component also contribute to the total measured signal at the output. Assuming that structural loss is the dominating loss in a PZT \((\phi_{PZT} = \text{const.})\), the spectral density can be analytically calculated as follows:

\[
x_{PZT} = \frac{4 k_B T \phi_{PZT} \omega_{PZT}^2}{m_{PZT} \omega \left[\left(\omega^2 - \omega_{PZT}^2\right)^2 + (\phi_{PZT} \omega_0^2)^2\right]}
\]

(2.30)

where \(m_{PZT}\) and \(\omega_{PZT}\) are the effective mass and angular frequency of the PZT. Figure 2.16 shows a theoretical thermal noise trace of a PZT that has an estimated mass of 5 g, a measured frequency of 30 kHz and a Q of 40 which was obtained experimentally from a measurement of the mechanical transfer function.

### 2.4.4 Clamping and glues

Clamping and glues loss are the another two potential contributors to the overall spectral density of the flexure displacement. Previous work has shown that using LIGO approved glue (Masterbond EP30-2) combined with carefully applied to the needed area were sufficient to avoid making this type of loss become dominant for experimental setup shown in this thesis [87]. Clamping loss was also reduced by monolithically manufacturing the flexures (in niobium and aluminium flexures) and using a harder material such as steel to silicon and metal to metal surface contact as shown in mount design in section 2.2.2 [98].
2.5 Chapter summary

The Fluctuation Dissipation theory was presented as the main theory to model and describe the flexure displacements induced by thermal noise. The flexure resonant frequencies were of 85 Hz, 167 Hz and 270 Hz with masses in the milligrams scale. The Q for these inverted cantilevers ranged from the lowest of 2000 for aluminium and the highest of 60000 for silicon, with no optical dilution. The estimated displacements due to other external noise sources produced by thermal noise of other optics components and gas damping showed that except for PZT thermal noise and gas damping these are not yet the limiting factors in the experiments. Both PZT thermal noise and gas damping induced fluctuations were predicted to be the largest noise contributors among the external noise and should be taken into account when modelling the overall noise.
Thermal noise in a cantilever
In chapter 2 we introduced the properties of the aluminium, niobium and silicon flexures and predicted their frequency responses to thermal noise in the $10 \text{ Hz} - 10 \text{ kHz}$ frequency band. It was clear that the minuscule displacements of the flexures would impose great challenges for observations. In practice, we improved this by combining the flexure with a curved mirror to form a Fabry-Perot cavity, referred to as a flexure cavity in this thesis, to effectively enhance the signals of interest. Section 3.1 provides a theoretical overview of a typical Fabry-Perot cavity and its basic characteristics. A detailed treatment of the optical resonator can be found in many undergraduate physics textbooks such as [110–112].

The cavity rear mirror was mounted on the flexures which formed soft mechanical springs. The mechanical restoring force was similar in strength to light-induced forces such as radiation pressure. These effects thus played a significant role in the dynamics of the cavity. Section 3.2 provides background theory on the flexure displacements under the influence of the combined mechanical and radiation-pressure-induced optical springs. The last section of this chapter explores a theory of bolometric optical spring and damping effects in an opto-mechanical cavity that could potentially explain some of the experimental results presented in chapter 7.

![Figure 3.1: A linear Fabry-Perot cavity of length $L_{cav}$. The transmittivity, reflectivity and loss coefficients of the front and rear mirrors are $(t_1, r_1, a_1)$ and $(t_2, r_2, a_2)$ respectively.](image-url)
3.1 Fabry-Perot cavity

Figure 3.1 shows a Fabry-Perot cavity with two mirrors separated by a distance $L_{\text{cav}}$. The front and rear mirrors have amplitude transmittivities, reflectivities and losses of $(t_1, r_1, a_1)$ and $(t_2, r_2, a_2)$ respectively. The mirrors impose boundary conditions to solve the Helmholtz wave equation for a self-consistent standing wave within the cavity [110, 112]. Conservation of energy implies that $t_1^2 + r_1^2 + a_1^2 = 1$ is true. Note that the mirror power transmittivity, reflectivity and losses are defined as $T_1 = t_2^2$, $R_1 = r_2^2$, and $A_1 = a_2^2 = A_1$. Consider the electromagnetic field ($E_{\text{in}}$) of a laser at frequency $f$ incident on the cavity:

$$E_{\text{in}} = E_0 e^{i\omega t}$$  \hspace{1cm} (3.1)

$$\omega = 2\pi f$$  \hspace{1cm} (3.2)

where $t$ is time of propagation and $E_0$ is the amplitude of the input field. When the round trip phase of the laser field traversing the cavity is equal to multiples of $2\pi$, or the cavity length matches an integer number of half laser wavelengths, the cavity is said to be on resonance. The cavity is called blue-detuned, when the laser frequency is larger than the cavity resonance, and the reverse is referred to as a red-detuned cavity.

Cavity steady state responses

Assuming a lossless cavity ($a = 0$, hence $t_1^2 + r_1^2 = 1$), the self-consistent steady state equations are as follows [111]:

$$E_{\text{refl}} = r_1 E_{\text{in}} + i t_1 r_2 E_{\text{circ}} e^{i2\phi}$$  \hspace{1cm} (3.3)

$$E_{\text{trans}} = i t_2 E_{\text{circ}} e^{i\phi}$$  \hspace{1cm} (3.4)

$$E_{\text{circ}} = i t_1 E_{\text{in}} + r_1 r_2 E_{\text{circ}} e^{i2\phi}$$  \hspace{1cm} (3.5)

$$\phi = -\frac{2\pi L_{\text{cav}}}{\lambda} = -\frac{2\pi L_{\text{cav}} f}{c}$$  \hspace{1cm} (3.6)

where $E_{\text{refl}}$ is the reflected field, $E_{\text{trans}}$ is the transmitted field, $E_{\text{circ}}$ is the circulating field, $\phi$ is the phase gained by the field after travelling half of a round trip of the cavity, and $c$ is the speed of light.

The circulating field ($E_{\text{circ}}$) inside the cavity is built up proportionally to the number of round trips light bounces between the two mirrors. The reflected field ($E_{\text{refl}}$) comprises the promptly reflected field from the front mirror, and the leakage field from the circulating field after each round trip inside the cavity due to the front mirror transmittivity. The promptly reflected field is $\pi$ out of phase with the leakage field as it reflects off the front mirror and indicated by the imaginary term in equation 3.3. The transmitted electric field ($E_{\text{trans}}$) is the leakage of the circulating field due to the transmittivity of the rear mirror.

Rearranging the above equations gives a set of transfer functions as follow:
A linear Fabry-Perot cavity is referred to as ‘impedance-matched’ when the promptly reflected field is equal to the leaked circulating field, resulting in a complete cancellation of the reflected field. An ‘overcoupled’ cavity occurs when the leakage field is larger than the promptly reflected field while the reverse situation is referred
to as an ‘undercoupled’.

Figure 3.2 shows the cavity amplitude and phase responses for three impedance conditions: overcoupled as blue traces, impedance matched as red traces and undercoupled as green traces. At resonance, an impedance matched cavity has zero reflected amplitude and maximum transmission amplitude. Hence, the transmitted power ($P_{\text{trans}} = E_{\text{trans}} E_{\text{trans}}^*$) is equal to the input power making the cavity completely transparent. Both the undercoupled and overcoupled cavities couple light into the cavity equally for the same mirror conditions. They exhibit similar characteristics in the transmitted cavity field amplitudes and phases, the reflected amplitude and the circulating phase, illustrated with the blue traces completely overlaid by the green ones. The circulating fields only vary in amplitude, with more photons coupled into an overcoupled cavity. However, the reflected phase differs significantly between the two impedance conditions. The undercoupled cavity shows a dispersion-like response through resonance with the maximum phase increasing as the cavity impedance gets closer to be matched. The overcoupled cavity, on the other hand, experiences a $2\pi$ phase change through resonance.

Cavity parameters

To effectively describe any Fabry-Perot cavity, a couple of essential fundamental parameters are needed: the free spectral range (FSR), full width at half maximum (FWHM), cavity finesse ($\mathcal{F}$) and cavity quality factor $Q_0$. The free spectral range, or the longitudinal mode spacing, describes a separation between two adjacent resonant modes in units of frequency ($\text{FSR}_\ell$) or length ($\text{FSR}_m$).

\[
\text{FSR}_\ell = \frac{c}{2L_{\text{cav}}} \quad \text{[Hz]} \tag{3.10}
\]

\[
\text{FSR}_m = \frac{\lambda}{2} \quad \text{[m]} \tag{3.11}
\]

Similar to the FSR, the FWHM also has two different units and is defined as the width of the resonant peak at half of its maximum amplitude. The cavity finesse is a dimensionless parameter and can be calculated from the ratio between the free spectral range and the FWHM. For a high finesse cavity, the finesse can be approximated as:

\[
\mathcal{F} = \frac{\text{FSR}}{\text{FWHM}} \approx \frac{\pi \sqrt{r_1 r_2}}{1 - r_1 r_2} \tag{3.12}
\]

The cavity finesse depends solely on the reflectivities of the cavity mirrors, which in turn determines the coupling of light into the cavity. The time taken for light to resonate in the cavity before dissipating to the outside environment as loss gives the stored energy within the cavity. The ratio of the cavity stored energy over the energy loss per cycle is defined as the optical quality factor $Q_0$ of the cavity, similar to the quality factor of a mechanical oscillator. The storage time during which light
§3.2 Optical spring in a flexure cavity

is contained in the cavity before leaking can be mathematically described as:

\[
\tau_{\text{cav}} = \frac{L_{\text{cav}} F}{c \frac{\pi}{3}}
\]  

(3.13)

3.2 Optical spring in a flexure cavity

In the work presented here, the Fabry-Perot cavity was constructed with one of the cavity mirrors mounted on a flexure, forming an opto-mechanical system as shown in figure 3.3. As the cavity is detuned away from resonance, this causes perturbations in the intra-cavity field, which in turn changes the radiation pressure force acted on the flexure. This is the optical spring effects that result in stiffening (blue-detuned) and softening (red detuned) of the mechanical spring of the oscillator. This optical spring effect on large scale oscillator was addressed for free falling suspended mirrors in gravitational wave interferometry[113, 114].

![Figure 3.3: A: a diagram of an opto-mechanical cavity with a rear movable mirror. B: Silicon flexure cavity as an example of this system.](image)

When the cavity photon lifetime is comparable or longer than the mechanical oscillation period, light can be used to extract energy out of the oscillator resulting in cooling or dynamical back-action[115–117], or to drive the oscillator leading to parametric amplification[118]. The opto-mechanical interactions, when under control, can be used to manipulate and access different energy states of the mechanical oscillator, and ultimately reach the standard quantum limit[119, 120]. A more thorough review that provides both classical and quantum approach to this field can be found at[121].

As the flexure cavity presented here is quite short, 10 mm in length, and has a finesse of only 1340; the cavity storage time of 14 ns is negligible compared to other timescales. This section thus only presents a derivation of the adiabatic optical spring where the cavity response is considered instantaneous and much faster than an oscillating period of the mechanical oscillator.
3.2.1 Derivation of optical spring

Photons have momentum. When photons interact with an oscillator, they exert a force, called the radiation pressure force, on the mechanical object. This force is proportional to the beam power and can be expressed as follows:

$$ F_{rp} = \frac{2 P_0}{c} $$

(3.14)

where $P_0$ is the optical power incident on the mirror and the factor of 2 comes from conservation of momentum as light bounces off the mirror.

In our opto-mechanical system, the radiation pressure force on the rear mirror is proportional to the circulating field in the Fabry-Perot cavity (c.f. eq 3.8):

$$ F_{rp} = \frac{2}{c} P_{circ} $$

$$ = \frac{2}{c} \frac{t_1^2}{1 - 2r_1r_2 \cos\left(\frac{4\pi\Delta x}{\lambda}\right) + r_1^2r_2^2} P_{in} $$

(3.15)

where $P_{circ} = E^*E$ is the circulating power, $P_{in}$ is the optical input power and $\Delta x$ is the detuning from resonance in units of length. We set up our opto-mechanical system such that $\Delta x$ is a parameter of mirror displacement from the cavity resonance. Using the 2nd-order Taylor expansion of cosine for small $\Delta x$ leads to

$$ P_{circ} = \frac{t_1^2 P_{in}}{1 - 2r_1r_2\left(1 - \frac{8\pi^2(\Delta x)^2}{\lambda^2}\right) + r_1^2r_2^2} $$

(3.16)

The circulating power can now be described as a function of detuning in units of the cavity linewidth ($\gamma$):

$$ P_{circ} = \frac{\alpha}{1 + (\frac{\Delta x}{\gamma})^2} P_{in} $$

(3.17)

where $\alpha = t_1^2(1 - r_1r_2)^{-2}$ and $\gamma = \frac{\lambda}{4\pi}$ is the cavity linewidth (half-width at half-maximum). Using equation 3.12, the radiation pressure force as a function of detuning can be written as:

$$ F_{rp} = \frac{2}{c} \frac{\alpha}{1 + (\frac{\Delta x}{\gamma})^2} P_{in} $$

(3.18)

Differentiating the radiation pressure force with respect to the detuning $\Delta x$, the linear rate of change is:

$$ \frac{dF_{rp}}{d(\Delta x)} = -\frac{16\alpha F P_{in}}{\lambda} \frac{\delta_{\gamma}}{(1 + \delta_2^2)^2} $$

$$ = -K_{os} $$

(3.19)

(3.20)

where $\delta_{\gamma} \equiv \frac{\Delta x}{\gamma}$. Equation 3.20 is analogous to the spring constant in a mechanical system obeying Hooke’s law. $K_{os}$ is called a radiation-pressure induced optical spring constant and takes positives value for the convention in this thesis. The ‘-’
sign indicates the restoring force in the presence of the optical spring which occurs when blue-detuned from cavity resonance. On the other hand, when the cavity is red-detuned, $k_{os}$ is negative and gives rise to an anti-spring. This interaction of light and mechanical oscillator results in optical bistability in the system [122].

Figure 3.4 shows the radiation pressure force and the respective optical spring constant as a function of cavity detuning in units of linewidth. The maximum spring constant occurs inside the full width half maximum and is at around $\delta_\gamma = \pm 1/\sqrt{3}$.

![Graph showing radiation pressure force and optical spring amplitude as a function of detuning relative to cavity linewidth. The maximum and minimum turning points of the optical spring are at $\delta_\gamma = \pm 1/\sqrt{3}$.](image)

Figure 3.4: Radiation pressure force and optical spring amplitude as a function of detuning relative to cavity linewidth. The maximum and minimum turning points of the optical spring are at $\delta_\gamma = \pm 1/\sqrt{3}$. $P = 100 \text{ mW}$, $R_1 = R_2 = 0.9959$, $\mathcal{F} = 765$

In a system where the optical response time is comparable to or slower than the period of mechanical oscillator, the finite response time of the cavity lags the motion of the oscillator in the velocity quadrature. The radiation pressure force within the cavity, which now relates to the velocity of the oscillator motion, induces optical damping and anti-damping. The blue-detuned side now has an optical spring and anti-damping whereas the red-detuned side has an optical anti-spring and damping.
This single-carrier system dominated by radiation pressure force is therefore either statically or dynamically unstable [123].

Several methods have been explored to stabilise optical springs. A method to overcome the instability is to introduce dual-carrier fields with one detuning for strong optical restoring force and weak anti-damping, and the other providing sufficiently strong optical damping and weak anti-spring. The resulted opto-mechanical cavity is now dominated by a restoring force and optical damping, hence becomes stable [124]. The use of active feedback has been reported to stabilise the opto-mechanical systems by modifying the spring response. The feedback phase introduced as an additional damping could potentially lead to a stable detuned cavity of a combined spring and damping [125]. Lastly, cavity birefringence was exploited to create double carrier fields from a single pump field. It subsequently stabilised the optical spring effects and created an effective optical trap without any external feedback [126].

Figure 3.5: Theoretical frequency shifts as the mechanical spring is softened/stiffened due to radiation pressure induced spring in quasi-static regime.
### 3.2.2 Flexure displacement in opto-mechanical cavity

In a regime where the mechanical frequency is well below the cavity decay rate, the optical damping becomes insignificant. The transfer function of the flexure displacement subject to force $F(\omega)$ in the Fourier domain becomes:

$$
\frac{x(\omega)}{F(\omega)} = \frac{1}{(k_m + k_{os} - m\omega^2) + im\omega\Gamma_m}
$$

(3.21)

where $\Gamma_m$ as the mechanical damping of the flexure and the subscript ‘m’ denotes the mechanical properties of the oscillator. The total spring $k_{tot}$ of the flexure is altered by the sum of both mechanical and optical springs: $k_{tot} = k_0 + k_{os}$. A red-detuned cavity softens the flexure spring, thus shifting the mechanical resonance to a lower frequency. Conversely, for a blue-detuned cavity, the total spring becomes stiffer leading to a higher resonant frequency. The power dependence of the optical spring effect is illustrated in figure 3.5. It shows that the maximum frequency shifts occurs at $\delta_\gamma = 1/\sqrt{3}$ and increases monotonically with input power.

### 3.3 Bolometric effect in an opto-mechanical cavity

In addition to the radiation pressure force, photons can also be absorbed by the mirror on the mechanical oscillator. This can lead to thermo-elastic distortion causing displacements of the oscillator. While classical and quantum models have been well developed for the radiation pressure effect in an opto-mechanical cavity, the theoretical and experimental investigation of this photo-thermal effect, also referred to as the bolometric effect, have been less extensive. In many reported cases, the bolometric force was several order of magnitude greater and induced an opposite response from the system than the radiation pressure force. The consequence was not only a shift of the resonance but also cooling of the mechanical oscillator modes [127–130]. This section presents a theory [131, 132] used to describe the bolometric effect in the classical regime and the induced bolometric spring constant and damping in an opto-mechanical cavity.

#### 3.3.1 Bolometric force in an opto-mechanical cavity

The bolometric effect is generally treated as a force which has a linear response $h(t)$ to the absorbed light [131]:

$$
F_{bol}(t) = \beta \frac{2R}{c} (h * P_{abs})(t)
$$

(3.22)

where $R$ is the reflectivity of the mirror. A common choice for $h(t)$ is

$$
h(t) = (1/\tau_{th}) \Theta(t) e^{-t/\tau_{th}}
$$

(3.23)
where $\Theta(t)$ is the Heaviside function, reflecting thermal relaxation on a timescale of $\tau_{th}$ [131]. Usually, the absorbed power is linearly proportional to the circulating power inside the cavity such that $P_{abs} = AP_{circ}$, where $A$ is the absorption coefficient. The phenomenological parameter $\beta$ is introduced to quantify the relative strength of the radiation pressure and bolometric forces, such that $\beta = (F_{bol}/P_{abs})/(F_{rp}/P_{circ})$.

### 3.3.2 Bolometric force induced spring and damping

Assuming the mechanical oscillator displacement is much smaller than the cavity length ($|x(t)| \ll L_{cav}$) and that the opto-mechanical cavity satisfies the adiabatic condition where the cavity storage time is much smaller than thermal response and slow changes within the cavity, the averaged absorbed power at $x$ over time leads to first order expansion as follows:

$$P_{abs}(x) = P_{abs}(0) + x \frac{dP_{abs}}{dx} \bigg|_{x=0}$$

(3.24)

As the cavity is optically detuned ($\Delta = \omega_{laser} - \omega_{cavity} > 0$), if the cavity storage time is much shorter than the thermal response, the absorption as the cavity is detuned can be described as

$$P_{abs}(t) = P_{abs}(x(t)) + \delta P_{abs}(t)$$

(3.25)

The second term $\delta P_{abs}(t)$ represents the absorption fluctuation around the detuning point. Substituting equations 3.24 and 3.25 into equation 3.22, the bolometric force in term of absorption power in the Fourier domain is:

$$F_{bol}(\omega) = \beta A \frac{2R}{c} h(\omega) \left[ \delta P_{circ}(\omega) + \left( \frac{dP_{circ}}{dx} \right) x(\omega) + P_{circ}(0) \right]$$

(3.26)

where $h(\omega)$ is the Fourier transform of the low pass filter function $h(t)$ as follows:

$$h(\omega) = \frac{1}{1 + i\omega\tau_{th}}$$

The equation of motion of a mechanical oscillator in a cavity dominated by the bolometric force can be described as:

$$m\ddot{x}(t) + m\Gamma_{m}\dot{x}(t) + k_{m}x(t) = F_{bol}(t)$$

(3.27)

Under Fourier transformation, the equation becomes:

$$-m\omega^{2}x(\omega) + im\Gamma_{m}\omega x(\omega) + k_{m}x(\omega) = F_{bol}(\omega)$$

(3.28)

Substituting equation 3.26 into equation 3.28 and rearranging the real and imaginary parts of the equation, the effective damping and spring becomes
\[
\omega_{\text{eff}}^2 = \omega_0^2 + \frac{R\beta A}{m} \frac{1}{1 + \omega^2 \tau_{\text{th}}^2} \frac{2}{c} \frac{dP_{\text{circ}}}{dx} \\
= \omega_0^2 - \frac{R\beta A}{m} \frac{1}{1 + \omega^2 \tau_{\text{th}}^2} \frac{16\alpha FP_m}{\lambda} \frac{\delta_\gamma}{(1 + \delta_\gamma^2)^2} \\
= \frac{k_m}{m} + \frac{k_{\text{bol}}}{m} \\
\Gamma_{\text{eff}} = \Gamma_m + \frac{R\beta A}{m} \frac{\tau_{\text{th}}}{1 + \omega^2 \tau_{\text{th}}^2} \frac{2}{c} \frac{dP_{\text{circ}}}{dx} \\
= \Gamma_m + \frac{R\beta A}{m} \frac{\tau_{\text{th}}}{1 + \omega^2 \tau_{\text{th}}^2} \frac{16\alpha FP_m}{\lambda} \frac{\delta_\gamma}{(1 + \delta_\gamma^2)^2} \\
= \Gamma_m + \Gamma_{\text{bol}}
\]

We can see that the bolometric effect, when treated as a force, gives rise to a spring constant and damping much like the radiation pressure force.

### 3.4 Chapter summary

This chapter has described the theory of a Fabry-Perot cavity which will, in later chapters, play an important role in assisting the experimental readout through coupling flexure movements into intra-cavity phase. Also here we introduced two different potential effects, radiation pressure and bolometric, which arise from photon momentum and absorption in the flexure cavity. The experimental investigation and results into those are presented in chapter 7.
Opto-mechanical cavity
Chapter 3 introduced the Fabry-Perot cavity as a device to enhance the measurement of minuscule flexure displacements by embedding this information in the cavity phase. This cavity phase is used as part of a feedback control system to compensate for flexure displacements and keep the cavity on resonance. The error signal due to the displacement from the control point in this thesis is also used to read out the cavity length changes. Section 4.1 briefly outlines the theory of a basic feedback control of a system. Section 4.2 describes the Pound Drever Hall technique that can be used to both extract the phase information and generate an error signal for the feedback control in the experiments. Section 4.3 describes several potential noise contributions which can easily mask the small displacement amplitudes in high precision metrology. Lastly, section 4.4 gives a brief summary of analogue and digital control systems.

4.1 Basic feedback control system

Feedback control plays a significant role in keeping our experiments operating at a desired point against unwanted perturbations. This section gives relevant background to understand the noise influence on the measured output (section 4.3) and experimental analysis presented in chapters 5-7. A more comprehensive review on the topic can be found in [133–135].

Figure 4.1 shows a simplified circuit for a basic negative feedback scheme. Block diagrams are commonly used as an effective tool to describe a control system. It consists of four elementary components: a plant, a sensor, a servo and an actuator for a typical feedback system. Each of these blocks represents a frequency response transfer function of the element. The plant is a general name for any physical system that requires control. The sensor detects the rate of change from the plant and equates them to an equivalent voltage. The difference between the detected signals and a reference is the error signal (ε). It is then input to a servo that amplifies and shapes the signal to generate appropriate responses to counteract the fluctuations. The actuator takes the servo output and carries out the correction to push the system back toward the desired operating point. The ultimate goal of this negative feedback loop is to drive ε to zero.

When implementing a feedback control in practice, knowledge of the transfer function of each individual components made up the loop is essential. Two transfer function concepts that are helpful in predicting the loop performance and diagnosing
issues are: the in-loop, or suppression, transfer function and the open loop gain. The
suppression transfer function shows the feedback loop performance by comparing the
system response output \( X_{\text{out}} \) caused by an input \( X_{\text{in}} \) to that input signal at the
frequency of interest. \( X_{\text{in}} \) contains a series of sinusoidal signals of small amplitudes
and various frequencies, and experiences the complete feedback loop effect just as
the error signal would. The mathematical form relating \( X_{\text{in}} \) to \( X_{\text{out}} \) is as follows:

\[
X_{\text{out}} = X_{\text{in}} + \epsilon = X_{\text{in}} + (-SPAG(\omega))X_{\text{out}}
\]

\[
\frac{X_{\text{out}}}{X_{\text{in}}} = \frac{1}{1 + SPAG(\omega)}
\]

The product \( SPAG(\omega) \) of all the transfer functions from the individual compo-
nents is the open loop gain of the feedback loop. It has complex values that can
be broken down in amplitude and phase delay, providing an insight to the system
stability.

- When \( SPAG(\omega) \leq 1 \) the system goes through infinite gain or positive feedback.
  In either cases, the result is unstable and/or undesirable.

- When \( SPAG(\omega) > 1 \) the system behaves as a negative feedback control. This
  means the open loop phase delay can not exceed \(-180^\circ\).

Figure 4.2 is an example of a Bode plot, a graphic tool that displays magnitude and
phase responses of the open loop transfer function. The unity gain frequency (UGF)
is defined as the frequency where the gain magnitude crosses 0 dB. The open loop
phase at the unity gain frequency is an important indicator of how stable the system
is. Generally this phase is designed to be at least 30° - 45° from \(-180^\circ\), and the
difference between those two points is called a phase margin. Too close to \(-180^\circ\)
can lead to system ringing or overshooting. In the experiment, we usually tried to
keep the phase margin around \(-90^\circ\) for a stable feedback. This is equivalent to a

\[ \sum \text{Plant } P(\omega) \text{ [plant unit /V]} \]
\[ \text{Actuator } A(\omega) \text{ [V/plant unit]} \]
\[ \text{Sensor } S(\omega) \text{ [V/plant unit]} \]
\[ \text{Reference} \]
\[ \text{Servo } G(\omega) \text{ [V/V]} \]
\[ \epsilon \]
\[ X_{\text{in}} \]
\[ X_{\text{out}} \]

Figure 4.1: Simple block diagrams for a standard control system
1/f response at UGF. Below this point, the slope of the open loop gain is stiffened to effectively suppress a large amount of noise sources that plague the low frequency region.

Figure 4.2: Bode plot example of an open loop magnitude and phase corresponding to a stable feedback system. The plot illustrates the unity gain frequency and phase margin. The open loop gain has a stable phase margin of $-90^\circ$, equivalent to a 1/f slope through the UGF in magnitude. At lower frequency, the slope is 1/f$^2$, increasing the amount of suppression. The steeper slope can be described as constructed from 2 poles. A zero can then be added to create the stable 1/f slope at UFGF.

4.2 Cavity length sensing technique

The Pound Drever Hall (PDH) technique is a sensitive feedback control method to improve laser frequency stability that uses a more stable Fabry-Perot cavity as a frequency reference. Alternatively, we can also use the PDH method to measure or to read out cavity length fluctuations of a test cavity with high precision by comparing the test cavity to a stabilised laser [136, 137]. Moreover, this technique can produce output signals that are shot noise limited and first order immune to laser intensity noise. As a result, the PDH technique has been widely implemented in many experiments ranging from gravitational wave detection, metrology, spectroscopy, laser stabilisation to opto-mechanical sensing [22, 138–140]. Section 4.2.1 describes conceptually the PDH technique to stabilise the laser frequency to a reference cavity and then section 4.2.2 describes the use of this stabilised laser to measure flexure
displacement of a test cavity. Further detailed explanation of the technique can be found in [136, 137].

4.2.1 PDH Laser stabilisation

Generally, the PDH technique extracts information from the cavity reflected fields for the relative position of the laser fields and the cavity resonance. An electro-optic modulator (EOM) is used to impose a pair of phase modulated sidebands on the laser (carrier) field at radio frequencies $\pm \Omega_m$ located beyond the cavity linewidth. Adding a phase modulation of frequency $\Omega_m$ changes the input optical field to:

$$E(t) = E_0 e^{i(\omega t + \beta \sin \Omega_m t)}$$

Figure 4.3: Normalised Pound-Drever-Hall error signals ($\beta = 1.08$ and $\Omega = 75$ MHz) for overcoupled ($R_1 = 0.9993$, $R_2 = 0.9998$), impedance matched ($R_1 = R_2 = 0.9995$) and undercoupled ($R_1 = 0.9998$, $R_2 = 0.9993$) cavities. Reflected powers of the undercoupled and overcoupled cavities are equal and overlaid in the top figure. The error signal size depends on the amount of reflected phase extracted from the cavity.
where $\beta$ is the ratio of the frequency deviation to the modulation frequency, frequently called the modulation depth expressed in radians. For the small modulation depths ($\beta \ll 1$) the above equation can be approximated by

$$E(t) = E_0 e^{i\omega t} \left(1 + \frac{\beta}{2} e^{i\Omega_m t} - \frac{\beta}{2} \beta e^{-i\Omega_m t}\right)$$ (4.3)

We can see the upper frequency sideband will have a phase rotation in advance of the carrier whereas the lower frequency sideband will be retarded relative to the carrier.

The mathematical expression for the phase-modulated beam when the modulation index is not small is shown below:

$$E(t) = E_0 e^{i\omega t} (J_0(\beta) + J_1(\beta) e^{i\Omega_m t} + J_1(\beta) e^{-i\Omega_m t})$$ (4.4)

where $(J_0, J_1)$ are Bessel functions of first and second order. These parameters determine the distribution of optical power in the carrier and the sidebands. Using $P = EE^*$, the carrier and sideband optical powers are:

$$P_{\text{carrier}} \approx P_{\text{in}}$$ (4.5)

$$P_{\text{sideband}} \approx \frac{\beta}{2} P_{\text{in}}$$ (4.6)

Around resonance, the carrier interacts with the cavity and the reflected carrier field experiences a phase shift as shown in the cavity field equation 3.6. As the sidebands are located well outside the cavity linewidth, the sideband pair remains unchanged and are promptly reflected and act as a reference. The reflected electric field is therefore modified by the cavity and is now:

$$E_{\text{refl}}(t) = E_0 e^{i\omega t} (J_0(\beta) R(\omega) + J_1(\beta) R(\omega + \Omega_m) e^{i\Omega_m t} + J_1(\beta) R(\omega - \Omega_m) e^{-i\Omega_m t})$$ (4.7)

where $R(\omega) = E_{\text{refl}}/E_{\text{in}}$ as in section 3.1. The photodetector measures the power in the reflected beam as:

$$P_{\text{refl}} = |E_{\text{refl}}|^2$$ (4.8)

This signal is then mixed down with the modulation frequency and low-pass filtered to remove the high order frequencies. Using the small angle approximation, a PDH error signal containing the phase information is subsequently produced. The final mathematical form of the PDH error signal is as follows:

$$\varepsilon = -2 \sqrt{P_{\text{carrier}} P_{\text{sideband}}} \Im(\mathcal{R}(\omega)\mathcal{R}^*(\omega + \Omega_m) - \mathcal{R}^*(\omega)\mathcal{R}(\omega - \Omega_m))$$ (4.9)

The equation shows that the error signal depends on the carrier and sideband powers ($P_{\text{carrier}}, P_{\text{sideband}}$). It is also determined by the cavity impedance conditions indicated in the imaginary part of the products of carrier and sideband reflectivities.

Figure 4.3 illustrates reflected amplitudes, phases, and their respective error signals when scanning the laser frequency relative to the cavity. The figure presents three cavity impedance scenarios: impedance matched (red), under-coupled (green) and over-coupled (blue). The derived error signals are directionally sensitive, and

§4.2 Cavity length sensing technique
therefore help determine which sides of cavity resonance the laser field is on and how much feedback is needed to return to resonance.

As explained in section 3.1 given the same input power, cavity length and finesse, the under-coupled and over-coupled cavities produce similar reflected amplitude (blue trace is underneath the green trace). However the under-coupled cavity gives less cavity phase information due to less light being coupled into the cavity, and therefore the smallest error signal.

![Graph](image)

**Figure 4.4:** *Pound-Drever-Hall error signals of different modulation depths for the same modulation frequency, input power and cavity conditions. The error signal slope is steeper than the fraction of error signal size over cavity FWHM.*

When the cavity is impedance matched, on resonance, the carrier is completely transmitted, therefore the measured shot noise limit at the photodetector is determined by the square root of the sideband powers. For other cavity conditions, some of the carrier field is reflected when the cavity is on resonance, resulting in an increase in DC power on photodetector. Given the same modulation depth, the impedance matched case results in the lowest shot noise limit. For this reason, even though the error signal size of an over-coupled cavity is bigger, it has lower signal to noise ratio than that of the impedance matched one.

Since the ratio of sideband to carrier power relies on the modulation depth, the influence on error signal sizes can be modelled as the effects of different modulation depths on the error signal, as illustrated in figure 4.4. The blue trace with a higher modulation depth than the green one has a steeper slope. This plot also shows that the slope calculated from the ratio of the FWHM over the error signal size does not exactly coincide with the actual error signal slope. Therefore using FWHM and error signal size to find length measurement per volt requires a correction factor as shown
in equation 4.15 and section 5.3.

Given the same input powers and cavity conditions, $\beta$ of 1.08 is the optimal modulation depth [136] to create the steepest slope, hence resolving the highest length information per measured voltage. This modulation depth results in $P_{\text{sideband}}/P_{\text{carrier}} = 0.42$. This value was used to indicate that the experiment operated at the optimal PDH error signal condition (see chapter 5). Furthermore, from equations 4.6 and 4.9, the error signal size is

$$\varepsilon \propto \sqrt{\frac{\beta}{2} P_{\text{in}}}$$

(4.10)

This means both the error signal size and the shot noise limit are proportional to the square root of the modulation depth $\beta$. As a result an increase in the modulation depth can improve the signal to noise ratio, leading to a more sensitive cavity length readout.

Due to the statistical nature of photons, regardless of existing frequency or cavity fluctuation and an optimal error signal, there is a limit to which a cavity length fluctuation can be resolved. Starting with the shot noise limit of the reflected fields, the limit is presented in term of the equivalent cavity length noise. This is referred to as the Schalow-Townes limit [141]:

$$S_L = \sqrt{\frac{hc}{8F\sqrt{P_{\text{carrier}}}}} \sqrt{\frac{\lambda}{\Delta}}$$

(4.11)

A cavity of finesse 1000 and $P_{\text{carrier}} = 10 \text{mW}$ and a laser wavelength of 1064 nm will have a length noise of $1.81 \times 10^{-19} \text{m}/\sqrt{\text{Hz}}$.

### 4.2.2 PDH error signal readout of cavity length fluctuations

For a lossless impedance matched cavity, a change in cavity length due to a perturbation shifts the carrier field off resonance. Subsequently, the cavity phase for a small shift away from cavity resonance can be expressed in terms of multiple of round-trips ($2\pi$) and therefore the cavity length fluctuation can be expressed as a fraction of laser wavelength:

$$\phi = n2\pi + 4\pi \frac{\delta L}{\lambda}$$

(4.12)

The reflectivities of the sidebands due to being promptly reflected take values of 1. Using the small angle approximation and reflection coefficient (equation 3.9), the carrier reflectivity is approximately:

$$\mathcal{R}(\delta L) \simeq \frac{r}{1 - r^2} (4\pi \frac{\delta L}{\lambda})$$

(4.13)
Equation 4.9 can now be simplified to:

$$\varepsilon \approx -16 \sqrt{P_{\text{carrier}} P_{\text{sideband}}} \frac{F}{\lambda} \delta L$$

$$= -8 \sqrt{P_{\text{carrier}} P_{\text{sideband}}} \frac{\delta L}{FWHM_m} \tag{4.14}$$

Equation 4.14 shows a linear relationship between the measured error signal and a fluctuation in cavity length. Hence for the thermal noise experiment where we need to monitor the flexure-displacement caused change in cavity length, the error signal can be used as the experimental readout. Notice that equation 4.9 calculate the instantaneous error signal for a specific fluctuation. The constant $2 \sqrt{P_{\text{carrier}} P_{\text{sideband}}}$ is the zero to peak amplitude of the error signal. The peak-to-peak size of the error signal ($\varepsilon_{pp}$) is twice the constant, and the error signal slope can be calculated as follows:

$$\text{slope of } \varepsilon = -2 \frac{\varepsilon_{pp}}{FWHM_m} \tag{4.15}$$

This equation is an important conversion factor that will be used to find the equivalent length fluctuation of the cavity from the measurement output (section 5.3).

### 4.3 Potential noise sources

We have demonstrated that we can use the PDH technique to obtain cavity length information. In our thermal noise experiment, we wish to limit TC length changes to measurements of flexure displacements of flexure displacement due to thermal noise. However the cavity length fluctuations due to flexure displacements are minuscule, as shown in chapter 2. This section provides a brief description of noise sources that could mask the signal of interest. As most of the listed noise sources here have considerable contributions to the measured spectral readout of the PDH error signal, suppression of these noise sources is required in order to reach the experimental goals which will be addressed in chapter 5.

#### 4.3.1 Servo effects due to different noise injections

In section 4.1 a basic block diagram of a fundamental control feedback loop was presented. Here the control loop is modified to include the effects of unwanted noise injected at various points in this close loop [142, 143]. These noise sources are typical in any feedback control system and can be grouped into 4 types: reference noise ($\eta_1$), servo noise ($\eta_2$), disturbances to the plant which, in addition to the flexure displacement, causes the Fabry-Perot cavity length fluctuations ($\eta_3$) and sensor noise ($\eta_4$). Their effects in a closed loop response at the PDH error signal readout can be expressed as follows:

$$\varepsilon = \frac{\eta_1}{1 + L} - \frac{\eta_2}{1 + L} - \frac{\eta_3 SP}{1 + L} - \frac{\eta_4 S}{1 + L} \tag{4.16}$$
where \( L = SPAG(\omega) \) is the open loop gain. Below the unity gain frequency where \( L \gg 1 \), the error signal read out is dominated by the servo noise \( \eta_2 \) while all other noise are suppressed by the total loop gain. Length disturbances \( \eta_3 \) and sensor noise \( \eta_4 \) measured at \( \varepsilon \) are amplified by the sensor gain, leading to a reduction in the amount of suppression for these noise sources. Above the unity gain frequency where \( L \ll 1 \), servo noise can be approximated to 0 while none of the other noise sources are suppressed. In this region the system is free-running outside the control loop dynamic range. Both sensor noise and disturbances are also fully amplified by the sensor gain.

Reference noise is referred to as perturbations that can alter the desired operating point of a system. Since the PDH error signal is the readout of the cavity length fluctuations, noise at this point will essentially lead to inaccurate readouts and be subsequently over-compensated by the feedback loop, hence moving the operating point away from the original design. In a flexure cavity, reference noise can introduce a DC-offset to the PDH error signals making them asymmetrical. This mimics a detuning from the cavity resonance. In a high power regime where the optical spring is comparable to the mechanical spring of the flexure, a shift of the mechanical resonant frequency can be observed. In the worst case scenario the lock can become unstable as the feedback loop excites the flexure displacements beyond the linear region of the PDH error signal slope. The ratio of peak-to-peak size to DC-offset indicates the effect of the reference noise which can be minimised by either increasing error signal size, if it is not correlated to the DC-offset, or actively adding an offset to correct for this.

Servo noise refers to noise from electronic components of the servo board (for analogue lock acquisition), or the analogue to digital boards and digital to analogue interface boards (for digital lock acquisition). As many actuators such as PZT’s often require hundreds of volts per micron length correction, the servo board has large gain to generate feedback signals at appropriate voltage amplitudes to effectively...
correct for plant fluctuations. However both signal and noise are equally amplified at the output of the servo.

In our experiment, sensing relies on the use of photodetectors, thus sensor noise is generally a combination of both photodetector electronic noise and photodetector shot noise (see section 4.3.3). Sensor noise is designed to be shot noise limited with electronic noise being a few factors smaller than shot noise in amplitude [144]. In this thesis the sensor electronic noise refers to a broader noise source obtained at the mixer output while the photodetector is blocked. This sensor noise now includes photodetector noise and electronic noise of the demodulation stage of PDH error signal generating process.

Lastly, disturbances ($\eta_3$) are noise sources whose responses contribute to PDH error signal and are indistinguishable to the contribution of cavity length fluctuations due to flexural displacements. Hence these noise sources can imitate and effectively mask the thermal-noise-induced flexural displacements that we want to measure. Moreover both these disturbances and the signals will be affected the same way by the open loop gain $L(\omega)$ and sensor gain $S(\omega)$ under lock acquisition. As a result regardless of the loop dynamic and methods of readout these disturbances need to be below the signal of interest.

This subsection has given an overall description of noise effects at different essential points along a feedback control loop. Since measuring flexural displacements induced by thermal noise requires a highly sensitive setup, many noise types that enter the system affect the measured outcomes. These are described below and their measurements are shown in section 5.4.

### 4.3.2 Electronic noise

Since electronic components make up the entire control section the effect and contribution from the electronic noise to the measured output needs to be considered carefully. This noise is related to both sensor noise and servo noise, and depending on the servo dynamic schemes the noise impacts on the overall noise budget differ. Since the error signal point is at the experimental readout, it is easier to address this noise in two parts. The first case we consider is sensor noise, defined above. The second case is all other electronic noise downstream and is referred to as servo or compensator noise. A well known cause of this noise is Nyquist-Johnson noise which is Brownian thermal noise across resistors as described in equation 2.1. For a 10 kΩ resistor at laboratory temperature of 293 K and 1 Hz bandwidth, the thermal noise spectrum averages to $12 \text{nV/}\sqrt{\text{Hz}}$.

### 4.3.3 Photodetector noise

Photodiodes convert photons to electrical current fed through a transimpedance amplifier which can be observed on oscilloscope and spectrum analyser as voltage. The relationship between optical powers onto the diode and measured photovoltage is generally linear within two defined limits. The upper limit to the linear response is the saturation limit occurring at high power of which the detected photovoltage
reaches a plateau. More light upon the detectors does not produce more voltage. The lower limit is the noise floor limit. The power at this point yields a signal to noise ratio of 1 masking the signal of interest. The noise equivalent power (NEP) can then be calculated as follows [145]:

\[
NEP = \sqrt{\eta e \frac{P_{PD}}{\hbar f \sqrt{B}}} \left[ \frac{W}{\sqrt{Hz}} \right]
\]

where \(\eta\) is the quantum efficiency of the diode; \(e\) is the charge of an electron; \(P_{PD}\) is the power upon the photodetector; \(\hbar f\) is the photon energy at the frequency of \(f\) and \(\hbar\) is the reduced Planck constant; and \(B\) is the photodetector bandwidth.

Generally, noise in photodetectors is a sum of dark current noise, electronic noise and shot noise. Dark current occurs due to the statistical nature of photocarriers within photodiodes resulting in a small voltage that can be detected even when the photodetector is completely blocked off light. Among all three noise sources this noise is the smallest and designed not to be the dominating cause at the experimental output. Electronic noise is the Nyquist-Johnson noise mentioned above. Shot noise happens due to the random arrival of photons onto the photodiodes which generates an equivalent random voltage at the output.

In the experiments described in this thesis, photodetector noise contributes part of the overall sensor noise. When the feedback control is acquired, regardless of the servo dynamics, these noise sources are treated similarly to the signal derived from the flexural cavity length fluctuations by the loop suppression. Therefore it is necessary to carefully design for this noise source be lower than the signal of interest.

### 4.3.4 Laser intensity noise

Intensity noise is a source of laser noise produced by the intensity fluctuations of laser power and can be suppressed by activating a ‘noise eater’, a built-in feedback control, located in the laser controller. Generally the intensity noise is not of concern because PDH error signal is immune to this noise source to the first order. However, since photons have momentum, the intensity noise also induces radiation pressure noise causing flexure displacements. In the experiments this effect was measured and found that this classical radiation pressure noise was 2 orders of magnitude lower than the predicted thermal-noise-induced flexural displacements.

### 4.3.5 Laser frequency noise

Frequency noise is the second source of laser noise caused by the fluctuations of optical phase at the laser output. Measuring this phase noise is challenging because the phase evolution requires a reference for comparison. In a flexure cavity experiment the laser frequency fluctuation mimics the phase shift between carriers and sidebands caused by cavity length fluctuations. As a result, those two fluctuations become indistinguishable when measured at the spectral density of PDH error signal. For a Nd:YAG 1064 nm laser, the manufacturer typically quotes the frequency noise to
Noise and cavity length control

Figure 4.6: Laser frequency noise equivalent flexural displacement for a 1064 nm Nd:YAG laser. A theoretical prediction for the thermal noise induced flexural displacement for silicon flexure is also included to show the dominance of laser frequency noise in the frequency of measurement.

be $100 \text{Hz}/\sqrt{\text{Hz}}$ at 100 Hz with a signature 1/f slope response. Figure 4.6 shows the equivalent flexure displacement induced by this noise source against the predicted that by thermal noise fluctuations. It is clear that laser stabilisation is necessary to enable suppression of laser frequency noise below the signal of interest. The stabilisation stage is shown in chapter 5 as the second of the two main feedback control systems in the thermal noise experiments. The gain limited laser frequency results are presented in section 5.4.1.

4.3.6 Scattered loss - parasitic interference

Effect of scattered light in our experiment

Scattered loss occurs when light from unwanted reflections leaves the beam path and is lost from the system. This could cause a large loss in the power intensity. Causes of scattering include dust, air currents, and optical surface imperfections. This loss can be minimised by reducing air flow and maintaining high levels of cleanliness such as wearing gloves and suits. If the scattered light completely diverges from the main path, careful placement of beam dumps will be sufficient to block out random noise distribution to the outputs. However if the scattered light converges back onto the main beam this will create interference where these two fields overlap and appear as unwanted noise at the outputs, known as parasitic interference. One of the main sources causing convergence of the scattered light is reflection off mirror mounts and
other optics components. The combined light source can be expressed as below:

\[ E_{\text{tot}} = E_0 + E_{\text{sc}} = E e^{i\omega_0 t} + \zeta(t) E e^{i[\omega_0 t + \frac{4\pi}{\lambda} \sin(\omega_{sc} t)]} \]

(4.18)

where \( \zeta(t) \) is a product of scatter light amplitude and the degree of coupling between the main beam and the scatter light. When it is time dependent, the scattered light acts as amplitude modulation. The sinusoidal part in the phase component, \( \sin(\omega_{sc} t) \), is a time dependent scattered frequency fluctuation equivalent to phase modulation of the main beam. These modulations due to oscillations or vibrations of the reflective surfaces is the same as cavity length change using PDH error signal readout. When detected on a photodiode the combined intensity produces a constant term and a cross term between the carrier and scattered light.

\[ I_{\text{photodiode}} \approx \text{const.} + \zeta I_0[e^{i\frac{4\pi}{\lambda} \sin(\omega_{sc} t)} + e^{-i\frac{4\pi}{\lambda} \sin(\omega_{sc} t)}] \]

(4.19)

The scattered light amplitude and phase are thus directly proportional to the intensity fluctuation at the measured output. Figure 4.8 shows a modelled time series of scattered light produced by pure frequency modulation and its combination with amplitude modulation. The top two traces are effects of pure phase modulation with the top trace being modulated at faster frequency than the bottom one. The bottom trace simulates the effect of scattered light interference with both phase and amplitude being modulated different frequencies.

Here the scattered light is treated simply as from a single source of constant motion. In reality an observed scattered light is a superposition of many different scattering paths produced by an aggregate of multiple sources. A more generalised treatment of scattered sources can be found in [146]. Depending on the experimental apparatus scattered light can travel back and forth the same paths many times leading to a distinct shelf-like features as shown in figure 4.9. This is called the fringe wrapping or parasitic interferometer effects. Both figures 4.8 and 4.9 often use as visual observation tools in diagnosing the parasitic interferometric effect. Since direct sources of scatter light are often vibration of optics components the scatter light spectrum tends to present in the audio frequency band. In the case where this
unwanted noise source becomes significantly noticeable at the output it is necessary to actively diagnose and isolate the sources of scatter to minimise this effect.

There are several ways to both passively and actively mitigate the scattering sources. Efficient use of beam dumps and tilting of optics when setting up an experiment ensure that the scattered light lost from the main beam do not recombine with the main beam. Other common passive methods to deflect the scattering out of the beam path are reducing air flow and keeping optics cleaned to reduce debris. In the case where scattering is already present in the experiment, identifying the scattered sources would be the first step to reduce this effect. Irises and neutral density filters are used to either block out the scattered light or reduce the scattered light amplitude. Their single or combined use depend on the position of scattered sources in experimental layouts since the neutral density filters would also lead to intensity loss of the main beam.

**Frequency shifting of scattered light**

Most of the above passive methods for reducing parasitic interference require a way to identify the actual noise sources leading to fringe wrapping. Additional optical elements for further isolation may also be needed. Alternatively, we can use an active method to diagnose and upshift the scattered light out of the frequency band of interest. This opto-mechanical frequency shifting technique purposely introduces a phase dithering at a selective modulation depth and frequency above the detection band. A PZT is used to dither the path length between main sources of scattered light with a modulation frequency $\Omega_m$ and modulation depth $M$. This frequency


shift can be written as:

$$E_0 + e^{iM \cos(\Omega_m t)} \ast \zeta(t) E_{sc} = E e^{i\omega_0 t} + e^{iM \cos(\Omega_m t)} \ast \zeta(t) E e^{i[\omega_0 t + \phi_{sc}(t)]}$$ (4.20)

here the amplitude and phase of the scattering field is respectively given by $\zeta(t)$ and $\phi_{sc}(t) = \frac{4\pi}{\lambda} \sin(\omega_{sc} t)$ where $\omega_{sc}$ is the scattered frequency. The path modulation can be written as a sum of Fourier frequencies:

$$e^{iM \cos(\Omega_m t)} = \sum_{l=-\infty}^{+\infty} i^l J_l (M) e^{il\Omega_m t}$$ (4.21)

The intensity detected on a photodiode is therefore:

$$I_{dither} \approx \text{const.} + \zeta I_0 \left[ \sum_{l=-\infty}^{+\infty} i^l J_l (M) e^{il\Omega_m t} \ast e^{i\phi_{sc}(t)} + cc \right]$$ (4.22)

where $cc$ is the complex conjugate.

This technique mixes the frequency fields of scatter noise with the modulated field into a higher frequency band. Selecting an appropriate modulation depth results in a zero net field of the above mixed term in the detection band as it effectively up-shifts the noise to higher frequency band. The distribution of scatter noise depends on waveforms of the dithering frequency and their corresponding weights [146]. Figure 4.9 shows the length equivalent displacement due to scattered noise which was frequency shifted to 2 kHz when the scattered beam path was modulated at 2 kHz with a modulation depth of $M = 2.405$. Other experiments have demonstrated similar applications of this technique using square wave or sinusoidal waves [147].

### 4.3.7 Environmental noise

Environmental noise refers to vibration and anthropogenic noise and considered as disturbances. These seismic ground motion is the dominant source, due to seismic background and personnel movements in the laboratory. In the laboratory of the Centre for Gravitational Physics at the Australian National University the average amplitude spectral density of this environmental noise is around $10^{-9} m/\sqrt{Hz}$ in frequency range of $1 - 100$ Hz as shown in figure 4.10. At $100$ Hz, this is at least 5 order of magnitude higher for aluminium flexure, which was the lossiest materials out of all three used in the experiments. The noise spectral densities were recorded using three STS-2 seismometers while activities were carried out as normal in the laboratory. Since the flexures freely move and their Qs are around 2000 – 60 000, these disturbances can directly excite the flexures and hence mask the minuscule displacements induced by the thermal noise force. Decoupling this noise source from the flexure is required and was achieved in the experiment by a 2.5 m tall suspension isolation system made up of three horizontal and two vertical isolating stages.
Figure 4.9: Shelf-like features of parasitic interference (red) in the Fourier domain. The scattered noise is frequency shifted up to high frequency band by $\Omega_M = 2$ kHz and $M = 2.405$.

Figure 4.10: Measured anthropogenic activities in CGP laboratory.
4.4 Digital and analogue control systems

Here we describe a simplified architecture of the analogue and digital lock acquisition systems that were implemented in this work. A more comprehensive guide to digital control system can be found in Smith [148].

Figure 4.11 presents block diagrams of these control schemes. In comparison, they generally contain similar equivalent processing stages as analogue systems. For the digital system, the equivalent analogue servo board is replaced by a series of components to convert the signals from voltages to computational bits (shown in teal). The main reason behind the transition from analogue to digital was the flexibility of the digital scheme provided in designing and implementing filters for specific experimental needs.

In a digital scheme, signals are prepared through a whitening board and an anti-aliasing chassis prior to going through an analogue to digital conversion board (ADC). An appropriately designed whitening board provide buffers for measured signals of various amplitudes to meet the voltage requirement in order to maximise the dynamic range of the ADC output. An anti-aliasing board removes all frequencies above Nyquist frequency. After the ADC, the signals are recovered by digital filters reversing the shape of the whitening filters. The Control Digital System (CDS) [149, 150] is at the heart of all signal processing and control for our digital feedback scheme. It has built-in applications for multiple functions that otherwise require a combination of laboratory instruments such as a signal generator, an oscilloscope and a signal analyser that can measure frequency spectra and transfer functions. After the CDS, the conversion process from digital to analogue is a mirror of the input. This step includes a digital-to-analogue converter (DAC), an anti-imaging chassis and a de-whitening board.
4.5 Chapter summary

We have shown that the PDH technique provides a sensitive tool for control and sensing of the cavity length fluctuation caused by flexure displacements at the level of those expected from thermal force. The basic control system, together with the influence of different noise sources along the feedback loop on the output signals, are presented. The majority of these sources have potential to dominate the measurements and mask the signal of interest. It is therefore necessary to have preventive methods and suppression techniques in place to mitigate them in order to achieve the sensitivity requirements to observe flexure displacements. These will be discussed in chapter 5, along with the main experimental configuration.
Chapter 5

Thermal noise measurements

In this chapter we present how we built a sensitive apparatus to obtain thermal noise measurements and how we tackled and overcame noise obstacles at the ANU laboratory. The strength of this simple apparatus was the capability to interrogate directly and continuously the broadband flexure displacements. This chapter is structured as follows: section 5.1 describes the experimental configurations to measure flexural displacements; section 5.2 details the lock acquisition and correction for measurements of different locking bandwidths; section 5.3 shows signal conversion from measured output to the equivalent flexural displacements; section 5.4 deals with experimental noise sources listed in chapter 4; and lastly section 5.5 presents two extended locking schemes designed to nullify external mechanical coupling effects on the measured flexure displacements.

5.1 Experimental setup

The first step in characterising and understanding the thermal noise of the cantilevers was to be able to detect displacements of the flexure at the level of expected thermal noise. The analysis of thermal noise presented in chapter 2 has predicted the flexure displacements to be on the order of $10^{-15} \text{ m/}\sqrt{\text{Hz}}$ at 100 Hz. This small amplitude required a highly sensitive apparatus to detect the signal of interest and prevent it from being masked by other noise sources in the laboratory environment. It was necessary therefore to keep the experimental design as simple as possible and ensure the combined noise from the experimental components and the environment was not limiting the measurements. The challenge in suppressing noise sources throughout this experiment is discussed later in this chapter.

Our experiment involved detecting the length change in an opto-mechanical Fabry-Perot test cavity due to movement of one mirror mounted to the flexure. The test cavity transduces length change due to flexure displacements into a change in cavity phase which can then be detected in the cavity reflected power (section 3.1). The Pound-Drever-Hall locking scheme, as detailed in section 4.2, uses this phase change to continuously track the cavity fluctuation around a resonant locking point through its inherent sensitivity to small cavity length change. The PDH control system was sensitive to phase noise but insensitive to laser intensity noise and variation in input power. It also provided a means to calibrate the measured signal to thermal-noise-induced flexure displacements.
Figure 5.1 shows a schematic of the experimental layout consisting of 3 main stages: an injection stage (white), a laser stabilisation stage (blue) and a readout stage of the thermal noise induced flexure displacement (yellow).

In the injection stage, the main laser beam, shown in red in figure 5.1, was used to interrogate the flexure cavity, named test cavity (TC). A Faraday isolator was used to reduce optical feedback to the laser. It also helped maintain a pure vertically polarised field entering the electro-optic modulator (EOM) to minimise amplitude modulation in the output. We used a resonant EOM to impose radio frequency (RF) side-bands of high modulation depth at 75.8 MHz on the laser beam for the PDH locking scheme. The optical layout in the laboratory is shown in figure 5.2.
§5.1 Experimental setup

The laser stabilisation is laid out in the cyan tile with a simplified electronic stage for the feedback control. The purpose of this stage is to stabilise our main laser source by locking it to a more thermally stable low-expansion reference cavity (RC). The RC was formed from two Zerodur substrate mirrors optically contacted 20 cm apart with hollow cylindrical spacers of the same material. This resulted in a high mechanical Q cavity with a finesse of 6000. To further ensure stability of the cavity and low coupling to the environment, the RC was suspended on a single stage blade spring system under $10^{-5}$ mbar pressure. The feedback control and the effect of gain-limited frequency noise on the TC are discussed in section 5.4.1.

The last stage is the main part of the experiment where the thermal-noise-
Thermal noise measurements induced displacement of niobium, aluminium and silicon flexures were obtained. These flexures were formed as part of flexure cavities whose responses were monitored at the reflected photodiode (RPD) and the transmitted photodiode (TPD). The cavity length fluctuation was measured using the PDH error signal and read out at the RPD. We used this arrangement to measure the displacement spectra of 85 Hz niobium and 302 Hz niobium flexures, a 271 Hz aluminium flexure and a 167 Hz silicon flexure.

**Test cavity with niobium or aluminium flexure**

The laser beam to the TC travelled through a couple of polarised beam splitters (PBS) and waveplates for power and polarisation control as shown in figure 5.1. Once inside the vacuum chamber, the beam bounced off two steering mirrors before coupling into the TC. On transmission, we used two other steering mirrors and a lens to guide the transmitted beam to the optical bench outside. The TCs of the aluminium and 85 Hz niobium flexures were formed by a front mirror mounted on a piezo electric transducer (PZT) and a 1/4” mirror glued on to the flexure. They were separated by 12 mm and mounted on a 2 kg steel slab. The 302 Hz niobium flexure cavity was built similarly except that the mirror mounted on the flexure was 7 mm in diameter. The radius of curvature of the front mirror was 50 cm. The test cavity ensemble was suspended on the optical breadboard which in turn was hung at the bottom of a 5-stage isolation structure, shown in figure 5.3. Under the single stage suspension,

![Image](image-url)

Figure 5.3: *Left:* Suspension system. *Top right:* in-vacuum optical breadboard. *Bottom right:* Single stage suspension flexure cavity.
the pendulum motion of the TC was passively damped by a pair of small Eddy current dampers.

The assembly process and the beam alignment and mode matching for the test cavity required great care and were often time consuming because of the narrow cavity length and small diameter mirrors. Moreover, since the TC was suspended, it took time for the motion of the isolation systems to decay in between each adjustment to the optical layout on the breadboard inside the vacuum chambers. Once the chamber was closed, alignment had to be performed from the outside with a long lever arm, creating another challenge in coupling light into the cavity.

The test cavity was locked to the stabilised laser using the PDH locking technique. With the cavity full-width at half-maximum of 20 MHz, the side-bands were located well outside of the linewidth of the cavity and acted as a reference to detect the relative phase shift of the carrier after travelling through the cavity. We used the PDH error signal as the main experimental output and as a test point to monitor the length fluctuation for locking the TC to the laser using the PZT as the actuator.

![Experimental layout used to measure thermal noise induced displacements of silicon flexure.](image)

**Test cavity with silicon flexure**

Measuring the silicon flexure displacement induced by thermal noise required reducing the noise floor of the system described above because silicon has lower loss which leads to smaller displacement (section 2.3). We made several changes as shown in figure 5.4 to reach the required sensitivity level. Mostly, the alterations were aimed at further simplifying the experimental setup to reduce cavity alignment challenges.
Thermal noise measurements

and improve the signal-to-noise ratio. The changes are listed below with a noise analysis described in section 5.4:

- We used a second Faraday isolator to allow the TC to be interrogated with a linearly polarised beam. It also heavily filtered out scattering fringes.

- The combination of half and quarter waveplates finely tuned the input and output field polarisation, showing a 20% improvement in the reflected field detected. This helped correct for undesired small angle rotation of the laser field polarisation as the beam bounced off the steering mirrors.

- The PDH modulation depth was increased from $\beta = 0.75$ to $\beta = 1.08$ to improve the signal to noise ratio.

- The front mirror was replaced with a less reflectivity mirror to reduce the cavity undercoupling and increase the reflected signal.

- We simplified the optical layout on the breadboard inside the vacuum chamber. This reduced cyclic error (discussed in section 5.4.2) and further limited the small rotation of polarisations from bouncing off steering mirrors. With the beam input now being directed into the cavity from outside, the challenge of cavity alignment was found to be greatly reduced.

- To further assist the alignment, we replaced the 1" diameter steering mirrors on the periscope and inside the vacuum chamber with 2" ones.

- Finally, rebalancing of the suspension system was required due to a complete overhaul of the mass distribution on the optical breadboard. This step was time consuming, nevertheless proved to be significant toward achieving a working isolation system and making the re-alignment easier once the chamber was closed.

5.2 Experimental control

In this section, we introduce the control and sensing aspect of the experiment including the electronic configuration for modulation and demodulation stages, and signal acquisition and loop correction.

RF electronics

The RF modulation and demodulation stage of the experiment is shown in figure 5.5. We sourced the 75.8 MHz RF signals for the modulation and demodulation from a common signal generator. The signal was split and independently conditioned for both the EOM and the local oscillator (LO). This ensured flexibility to achieve the optimal modulation depth for the PDH error signal and produce the appropriate LO amplitudes for the mixers. The RF field of the reflected beam from both TC and RC were combined with their respective LO signals using level 17 and 23 mixers. For
each servo, the low frequency signal was isolated with a low pass filter to produce PDH error signals. Prior to the thermal noise measurement of silicon, we had used an active electronic mixer board for the laser stabilisation loop, where the relative phase of the LO and reflected RF signal could be adjusted. However we observed some electronic back reflection from the board and later replaced this with a passive mixer (Mini-circuit level 23). The dynamic range was improved by an order of magnitude and we were able to suppress laser frequency noise by the same factor. To further ensure no electronic back reflection within the demodulation path, a combination of high pass filters and 50 Ω terminators were also installed at the mixer outputs of both TC and RC.

**Acquisition scheme**

In the experiment we stabilised the laser to the RC by feeding back to the PZT and temperature control of the laser. The TC was in turn locked to the laser frequency by changing the test cavity length via the high voltage PZT. Due to the large TC free spectral range of $11 - 15 \text{ GHz}$, the TC-PZT did not have sufficient dynamic range to reliably capture the resonance. By tuning the laser temperature, we could bring the laser closer to resonance in both the RC and TC to within a few volts accessible by

---

Figure 5.5: Electronic setup for modulation and demodulation stages of the experiment
Thermal noise measurements

the TC-PZT. This is demonstrated in figure 5.6. The top plot illustrates multiple resonances of the RC (blue) while the bottom plot is a single free spectral range of a 10 mm long TC (red) and a 12.7 mm long TC (green). It took approximately 15 to 20 RC resonant peaks to travel a single TC free spectral range. We also found that options for temperature range that satisfied the above condition were limited due to mode hops. One way to reduce this challenge was to increase the macroscopic TC length. However, for the same RC length, a smaller TC resulted in a smaller suppressed-frequency-noise equivalent TC length fluctuation (see section 5.4.1). Hence, the cavity length of 10 − 12 mm provided a good balance for this experiment.

The most straight-forward strategy was to set the DC inputs to TC PZT, laser temperature and laser PZT to 0 volts before adjusting the laser temperature to maintain sufficient dynamic range. Once the range was established, the laser thermal offset was used to fine tune the relative position of the laser to both RC and TC resonances.

The TC feedback control comprised a servo board of flat gain, a high voltage (HV) amplifier board and a 1 Hz low pass filter (LPF) defining the $1/f$ slope for the overall feedback loop. The output of the LPF was fed into the TC PZT. Figure 5.7

![Reference cavity](image1)

![Test cavity](image2)

Figure 5.6: Normalised theoretical transmitted powers as a function of frequency for RC (top, blue) and TC (bottom). The cavity lengths are $L_{RC} = 200$ mm (blue), $L_{TC} = 10$ mm (red), and $L_{TC} = 12.7$ mm (green) respectively. On the left the blue, red and greed peaks are aligned to illustrate the desired locking point. It takes $\sim 20$ blue peaks to reach the next red or green ones. FSR: Free Spectral Range. FWHM: Full-Width at Half Maximum.
§5.2 Experimental control shows an open loop transfer function of the TC feedback, obtained from a swept sine measurement. Below 1 Hz, an integrator provided higher gain to compensate the lower frequency drift and was responsible for the phase drop-off. In most cases, the unity gain frequency (UGF) of the TC lock was about $10 - 50$ Hz. At low UGF, as long as the laser was simultaneously pre-stabilised to correct for the slow temperature drift, the lock was relatively stable with a phase margin of $-90$ degrees and could stay acquired for hours.

![Magnitude and Phase Plot](image)

Figure 5.7: Low bandwidth open loop gain measured from aluminium flexure cavity feedback loop.

High-UGF lock acquisition was facilitated by using a high voltage amplifier before the last low pass filter in the electronic path to minimise amplifying high frequency noise. A unity gain frequency of 1 kHz (above the fundamental resonance) was achieved only for the niobium flexure but even so the stability was unreliable [87]. Through monitoring with the oscilloscope, we discovered that the main cause for instability was largely due to the build up of flexure excitation over time, leading to a loss of lock when flexure displacements became larger than the cavity FWHM. Investigation into the cause showed mechanical coupling between the PZT and the flexure through the TC base. We tested this in the open loop condition by applying a sine wave to the TC PZT at the flexure fundamental frequency while observing the flexure ring up at the transmitted outputs. The time taken to amplify the resonant peaks and cause observable oscillations at the flexure resonance was much smaller for the high Q silicon flexure than the low Q aluminium one.
PDH error signal loop corrections

As mentioned in section 4.2, we recorded the PDH error signal spectra of the test cavity as our experimental outputs to probe for flexure displacements. For high UGF, the readouts were multiplied by the inverse of the loop suppression when converting these spectra to flexure displacement equivalent. For low UGF, the PDH error signal was not affected significantly above the unity gain frequency, so the correction for in-loop suppression was not necessary. However, in practice, for consistent treatment of all measured spectra and to ensure no residual loop effects, the correction was applied to all measurements.

Figures 5.8 and 5.9 provide examples of the raw recorded outputs and the loop corrected outputs for two scenarios of low and high UGF locking of silicon and niobium flexure cavities respectively. The first graph shows residual loop suppression around the unity gain frequency of 40 Hz as indicated by the red circle. In the second graph, the suppression of the feedback loop affected the entire spectrum up to 1 kHz.

5.3 Cavity length calibration measurements

After loop compensation, the traces were converted to equivalent cavity length fluctuations. This section steps through this process in detail and presents various efforts on obtaining as accurate values as possible.

The conversion to equivalent cavity length displacement ($\delta x$) is based on the slope
§5.3  Cavity length calibration measurements

Figure 5.9: Measurement of Niobium flexure before and after servo correction.

Figure 5.10: The transmitted power (top) and error signal (bottom) of silicon flexure were obtained when the cavity was swept linearly across the resonance. The measurements (blue) were fitted (red). The error signal slope gives the conversion factor of voltage-to-length equivalence.
Thermal noise measurements

of the error signal: (c.f. equation 4.15):

\[ \delta x = \varepsilon \times \frac{\text{FWHM}_m}{2} \times \varepsilon_{pp} \]  

(5.1)

where \( \varepsilon \) is the measured PDH error signal and \( \varepsilon_{pp} \) is the peak-to-peak voltage of the error signal size. The units now become:

\[ \left[ \frac{m}{\sqrt{\text{Hz}}} \right] = \left[ \frac{V}{\sqrt{\text{Hz}}} \right] \times \left[ \frac{m}{V} \right] \]

The FWHM\(_m\) and the \( \varepsilon_{pp} \) were obtained by fitting to the cavity transmission and error signal. Figure 5.10 shows a typical fitting result. The plot was a measurement from a linear sweep of either the cavity length or the laser frequency at low input power to prevent the nonlinear distortion of the cavity resonance due to the optical spring effect. The sweep amplitudes and frequencies were chosen such that the time series data of the transmitted response included the sidebands, and the shapes appeared Lorentzian. Separated from the carrier at a fixed modulation frequency of 75.8 MHz, the sideband peaks were used as a frequency reference to convert the horizontal time axis to the equivalent frequency. This process was also used to calculate the Hz/volt response of the TC-PZT.

To perform the fitting routine, we had to determine both the TC length and the mirror transmittivities using a calibrated photodiode. The cavity length was measured by placing a ruler parallel to the cavity base as shown in figure 5.11. Using equation 3.10 the frequency separation between two consecutive cavity resonances was subsequently calculated. The mirror transmittivities were obtained by taking the ratios of beam powers before and after the mirrors. The precision of the method relied on the detector sensitivity to measure a weak beam passing through the highly reflective mirrors, and a scaling factor depending on the quantum efficiency, the electronic amplification and impedance of the photodetectors. In our laboratory this factor was found by recording volt outputs corresponding to a series of optical

Figure 5.11: A picture of the cavity length measurement.
Table 5.1: Flexure cavity parameters

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Aluminium</th>
<th>Nobium</th>
<th>Silicon</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius of curvature (PZT mirror)</td>
<td>10 cm</td>
<td>10 cm</td>
<td>20 cm</td>
</tr>
<tr>
<td>Flexure mirror diameter</td>
<td>6.35 mm</td>
<td>6.35 mm</td>
<td>6.35 mm</td>
</tr>
<tr>
<td>Cavity length $L_{cav}$</td>
<td>12.7 mm</td>
<td>12.3 mm</td>
<td>10 mm</td>
</tr>
<tr>
<td>Free spectral range $FSR$</td>
<td>11.8 GHz</td>
<td>12.2 GHz</td>
<td>15 GHz</td>
</tr>
<tr>
<td>Finesse $F$</td>
<td>600</td>
<td>775</td>
<td>1364</td>
</tr>
<tr>
<td>Linewidth $FWHM_{Hz}$</td>
<td>19.7 MHz</td>
<td>15.7 MHz</td>
<td>11 MHz</td>
</tr>
<tr>
<td>Linewidth $FWHM_m$</td>
<td>89 nm</td>
<td>69 nm</td>
<td>39 nm</td>
</tr>
</tbody>
</table>

powers on the photodetector. The slope of a linear fit through the data was taken as the scaling factor. Furthermore this measurement also helped define the linear region before saturation of the photodetector.

For a given input power and modulation depth, the above parameters were used to fit a Lorentzian to the measured transmitted power and results in the expected loss and mirror reflectivities of the test cavity. The cavity finesse $F$ was calculated using equation 3.12 and the $FWHM_{Hz}$ was the ratio of $FSR_{Hz}$ and $F$. The length equivalent of the full-width at half-maximum $FWHM_m$ was calculated as follows:

$$FWHM_m = \frac{\lambda}{2} \frac{FWHM_{Hz}}{FSR_{Hz}}$$ (5.2)

A theoretical error signal was produced using the cavity parameters from the fitting routine and a scaling factor corresponding to the reflected photodetectors and any extra gain in the demodulation stage, and compared with the measured to validate the fitted results. The good agreement between the measured and theoretical error signals gave us confidence to use the results obtained from the fitting routine in the conversion process. Table 5.1 lists the parameters of aluminium, niobium and silicon flexure cavities. For the silicon flexure cavity the ratio of length fluctuations per volt measured was $2.9 \times 10^{-11}$ m/V.

Digital system conversion factors

Conversion factors from digital units of counts to flexure displacement required not only the error signal slope of meters per volts but also a unique gain factor from the input to the whitening board to the measured point within the CDS. We calibrated this figure by injecting signals of known amplitudes and reading out values at the error signal test point in the digital feedback loop. For the channel that measured the error signal, the conversion was measured at 1852 counts/V.

5.4 Experimental noise budget

This section presents our experimental approach to reduce the contributions of the potential noise sources to the thermal noise measurements (described in section 4.3). With careful design and effective suppression, we show that the TC length fluctuations

§5.4 Experimental noise budget
Thermal noise measurements

were not dominated by any of these noise sources, and therefore dominated by thermal noise.

### 5.4.1 Suppressed frequency noise equivalent to TC cavity length displacement

The PDH error signal readout measured only relative change of the laser frequency and test cavity resonance. Noise in the laser frequency is indistinguishable from a cavity length change caused by flexure displacement. Without active laser stabilisation, this noise source would have dominated the measured spectrum as shown in figure 4.6.

Stabilisation to a more thermally stable RC was achieved by using a PDH locking scheme to actively feed back to both the laser PZT and temperature control [87]. The open loop gain of this feedback control consists of three regions: below 1 Hz, the loop gain had a $1/f^2$ response to correct for any slow thermal drift through controlling the temperature of the laser crystal. Between 1 Hz and 60 kHz, the RC loop followed roughly a $1/f$ response which was used to correct for laser frequency fluctuations by actuating the laser PZT. The UGF was limited by the reference cavity pole above 60 kHz, and hence defined the maximum suppression achieved for the stabilisation loop.

The gain-limited laser frequency noise spectrum was obtained from the PDH error signal of the laser stabilisation loop. To show the limitation of the laser frequency

![Graph](image)

**Figure 5.12:** The graph shows the effect of low UGF laser frequency noise on the test cavity measured output. The laser stabilisation servo was moved from optimal operating point to demonstrate the effect of laser frequency noise in the cavity length measurements.
noise to the TC error signal output, the gain limited laser frequency noise was converted to the equivalent flexure displacement based on the following relationship:

\[
\frac{\delta L_{TC}}{L_{TC}} = \frac{\delta f_{laser}}{f_{laser}} = \frac{\delta L_{RC}}{L_{RC}}
\]  

(5.3)

where \( \delta L_{TC} \) and \( \delta L_{RC} \) are cavity length fluctuations of the test cavity and reference cavity respectively; \( L_{TC} \) and \( L_{RC} \) are the test cavity and reference cavity lengths; and \( f_{laser} \) and \( \delta f_{laser} \) are the laser frequency and its frequency fluctuations. Assuming a perfect lock, the equation 5.3 relates a fractional change in the laser frequency to the equivalent cavity length fluctuation. Hence,

\[
\delta L_{TC} = \delta L_{RC} \frac{L_{TC}}{L_{RC}}
\]  

(5.4)

The extent of equivalent flexure displacement induced by suppressed frequency noise depends on the length ratio of the test cavity and reference cavity. For example, the silicon flexure cavity of 10 mm and RC of 200 mm provided a factor of 20 reduction, on top of loop suppression, in frequency noise influence to the test cavity. Figure 5.12 shows the effect of laser frequency noise on the measurement readout. The blue trace is the PDH error signal equivalent to silicon flexure displacement while the dashed magenta trace is the gain limited laser frequency noise equivalent to flexure displacement. The overall gain of the stabilising loop was reduced from its optimal operating point, the magenta trace was measured at the error signal monitor port and subsequently converted to the flexure displacement following equations 5.1 and 5.4. Above 4 kHz the blue trace follows the magenta one, but below 4 kHz, in the frequency band of interest, frequency noise does not dominate.

### 5.4.2 Effects of parasitic interferometry

Parasitic interferometry, or cyclic error, can occur when light scattered from optics in the beam path interferes with the main beam. If the optics moves, this results in a time-dependent phase shift leading to an undesired noise contribution. The theory and the preventive and diagnostic methods of this noise is presented in section 4.3.6. In practice, we applied a slight tilt of all transmissive optic components to minimise re-entering of scattered light into the main optical path.

There was a significant increase in cyclic errors when the thermal noise experiment for the silicon flexure was assembled. Figure 5.13 shows the distinctive shelf-like features of parasitic interferometry at different frequencies at various time between 6 pm and 9 am. Suppressing this noise required us to identify the source by either tapping on optics while monitoring the PDH readout; or using the frequency shift technique (section 4.3.6). This process was time-consuming but essential to enable observation of flexure displacements induced by thermal noise.

Investigation into the cause pointed to the natural 1 Hz swinging motion of the TC slab on the breadboard indicating that the structure was not effectively damped. Since the measured noise traces showed a more amplified cyclic errors than previously observed, it demonstrated a greater coupling of environmental noise to the TC slab,
hence suggesting that the performance of the isolation system was degrading. We reduced this noise source by about an order of magnitude at 100 Hz as shown in the (dashed) red trace of figure 5.13 through the following steps:

- Simplifying the optical configuration on the optical breadboard inside the vacuum chamber (section 5.1);
- Re-optimising the suspension (section 5.4.4);
- Adjusting the positions of the magnets bolted on the breadboard to the aluminium plates on the slab to increase the passive eddy current damping.

5.4.3 Measured detection noise

The term “detection noise” was used to describe all noise sources introduced in the electronic path between the photodetector and the PDH error signal output. This included sensor noise and electronic noise. The analysis in section 4.3 showed that these noise sources experienced similar amplification and filtering by the electronic components of the demodulation stage to the signal of interest. The detection noise in the PDH error signal places a bound below which signals of interest were no longer distinguishable from noise. In the laboratory, we diagnose the noise sources through measurements of individual components from the PD downstream to the monitor port, and an overall noise profile when the PD was blocked from the incoming optical field.

The components of the digital locking system are shown in figure 4.11. Separate measurements of these led to the conclusion that the major contributor to the noise was...

Figure 5.13: Spurious interferometer noise displacements measured at different times of day.
Experimental noise budget

Detection noise was the whitening board. At the time, the board had a zero at 1.3 Hz, and a pole at 78 Hz, and a flat gain of 40 dB above 78 Hz, making it perform more like a broadband amplifier than a true whitening board. Since the filter shape did not tailor for a specific signal shape and the output amplitude was clamped at 15 V, the presence of a large resonant peak in the input signal would degrade the signal-to-noise ratio of the measured amplitudes in the off-resonant regions, and lead to them being dominated by the noise floor of the whitening board at the output. To accurately measure this noise, we injected a sinusoidal signal at the flexure resonant frequency and amplitude. We then monitored the readout using the same built-in signal analyser program as in measuring the PDH error signal spectra in the CDS. The blue trace in figure 5.14 shows a typical measurement of this process.

In the analogue system, electronic noise of all passive components contributed to the overall detection noise. The noise spectra were detected with a SR785 spectrum analyser which has a $10 \text{ nV}/\sqrt{\text{Hz}}$ noise floor, shown as the yellow trace in figure 5.14. The red trace was measured when the test cavity was off-resonant, representing the sum of both scattering noise and detection noise. It was also a readout of any disturbances mimicking cavity length fluctuation which were demodulated at the output. The difference between the off-resonant noise and detection noise indicated the accrued phase noise in the optical path. Above 1 kHz, detection noise formed the minimum observable limit.

In both the digital and analogue locking schemes, detection noise was close to but not above the signal of interest in the 10 Hz to a few kHz frequency band. Since these noise sources were uncorrelated with the error signals, they contributed quadratically

![Figure 5.14: Detection noise for analogue and digital locking schemes measured at the PDH error signal output port.](image-url)
5.4.4 Environmental noise

Seismic noise was a dominating noise source as shown in figure 4.10. Over the last two decades, the vibration isolation system at ANU has been through two generation of construction and upgrade to successfully lower this noise source below the required sensitivity in the measurement frequency band \([48, 87]\). There were no major changes to these systems for either the RC or the TC during this work. However, to perform at the optimal desired level, maintenance and re-alignment were required before assembling a new test cavity on the optical board and before closing the vacuum chambers.

5.5 Locking schemes using AOM and EOM

As mentioned in section 5.2, a considerable impediment to the high UGF locking of the TC to the laser was the mechanical coupling of the TC-PZT to the high Q
§5.5 Locking schemes using AOM and EOM

flexure through the steel base. A high bandwidth lock required a larger feedback signal to the TC-PZT above the flexure resonance. This excited the flexure leading to a continuous increase in displacements which eventually went beyond the PDH linear region and the cavity dropped lock. In this section, we present two high bandwidth locking schemes using actuators located externally to the TC. The purposes were: 1) to achieve a stable high bandwidth lock; 2) To determine whether mechanical coupling introduced noise in off-resonant regions.

**Acousto-optic modulator as an actuator**

The implementation of an acousto-optic modulator (AOM) into the experiment required some modification to the configuration in the TC layout area without any changes to the laser stabilisation. The basic concept was to frequency shift light going to the test cavity using an AOM and to lock this frequency to the TC by feeding back to the AOM driven frequency. The AOM was added downstream from the pick off wedge for the RC as shown in figure 5.15(a). We employed the AOM double pass technique [151] to prevent beam steering misalignment and to fold the extra beam path back to the original optical setup. The AOM was driven from a function generator to frequency shift the main beam by 200 MHz.

The phasor diagram shown in figure 5.15(b) illustrates the concept behind this

![Figure 5.16](image)

Figure 5.16: Flexure displacements measured for different locking schemes were in agreements. For PZT and EOM as actuators, the measurements obtained at the error signal ports. For AOM as actuator, the displayed trace was the feedback signal to AOM, and the error signal shows suppressed noise within the bandwidth.
lock scheme, with the PDH technique still being used to produce the error signals. The AOM introduces a frequency shift to the carrier and sideband pairs (shown as red arrows) at $\omega_{AOM}$ away from the original (shown as black arrows). The RC is now locked to $\omega$ with $\omega \pm \Omega$ as PDH sidebands while the flexural cavity is driven by $\omega + \omega_{AOM}$. The blue profile represents the test cavity and the red sidebands become its frequency references for PDH error signal readout. When the lock is engaged, drifts between the test cavity resonance and the laser frequency are fed back to the AOM adjusting $\omega_{AOM}$ in order to bring the red carrier back on resonance with the cavity. All these are done while leaving the laser source (black arrows), and the cavity itself, untouched.

In the laboratory, after the laser stabilisation was engaged, we employed a SR560 lock-in amplifier as a servo for the AOM feedback control loop. A DC voltage to the PZT was required to bring the test cavity close to resonance. Approximately 90% of the PDH error signal was put into the SR560, amplified and low pass filtered with a corner frequency of 100 Hz. The SR560 output was fed into the modulation input to the function generator. The signal generator was frequency modulated from the external input at 840 kHz per volt to adjust the $\omega_{AOM}$ frequency. Due to the low dynamic range, this could maintain lock for only a few minutes. To fix this, the remaining 10% of the error signal was used to feed back to the TC PZT with a UGF of 10 Hz. The overall bandwidth of the loop was 2 kHz. This provided evident that high bandwidth locking was not possible using the TC PZT as the actuator due to the mechanical coupling of the PZT to the flexure via the cavity base.

With the laser frequency noise effectively suppressed, there were two ways of reading out the measurement of interest for this configuration. The first was the in-loop PDH error signal which required loop correction as discussed in section 5.2. The second was to use the feedback signal to the AOM as a direct readout of the actual fluctuations of the cavity length in the frequency band of 10 Hz - 2 kHz. The conversion for length equivalence of the AOM actuating signal ($\kappa$) was:

$$\delta x \left[ \frac{m}{\sqrt{(Hz)}} \right] = \kappa \left[ \frac{V_{\text{rms}}}{V} \right] \times 840 \left[ \frac{kHz}{V} \right] \times \frac{L_{TC}}{f_{\text{laser}}} \left[ \frac{m}{Hz} \right]$$

(5.5)

Figure 5.16 presents the length equivalent spectra of the feedback signal to the AOM (green) and the error signal readout without loop correction (dashed black line). These two traces overlap beyond the UGF at about 2 kHz. Below 2 kHz, the error signal was suppressed by the in-loop gain whereas the AOM feedback signal indicated the amount of cavity length fluctuations in the system. Above 2 kHz, since the system was out-of-loop, the AOM feedback signal follows the error signal trace. The flexure displacement measured using low bandwidth feedback to the PZT (red trace) was in agreement with the measurements obtained when locked at high bandwidth using the AOM. It indicated that the mechanical coupling and PZT to the flexure did not affect the off-resonant measurements.
Electro-optic modulator as actuator

Due to the limited dynamic range of the AOM implementation, we used a fibre electro-optic modulator (EOM) as an actuator in an effort to remove the feedback to TC-PZT completely. The EOspace EOM, originally intended to be in the TC path, had a larger modulating bandwidth than the previously used AOM. However as it increased the cyclic error noise by nearly a factor of 10 we decided to add the fibre EOM to the laser stabilisation path, resulting in a modified configuration as shown in figure 5.17.

Figure 5.17(b) illustrates the concept behind this locking scheme. The original EOM imposed the $\Omega = 75.86$ MHz sidebands on the laser field which were used in the PDH locking of the TC to the laser. The second fibre EOM was driven by a Voltage-Controlled Oscillator (VCO)(ZX95-100) to impose a second set of sidebands, shown as black arrow pair. They were at $\omega \pm \omega_{EOM}$ at 150 MHz away from the main carrier. The $\omega - \Omega$ sideband and the $\omega - \omega_{EOM}$ sideband were used in a PDH locking to stabilise the laser frequency to the RC by feeding back to both laser PZT and temperature. The TC feedback loop used the main carrier ($\omega$) and the $\omega + \Omega$ sideband at 75.86 MHz to generate a PDH error signal. Fluctuations between the laser frequency and the TC resonance would provide a feedback signal to change the modulation frequency of the second EOM and shift the laser frequency back on the TC resonance.

In the laboratory we used the same servos as for the low bandwidth PZT locking

![Figure 5.17: Experimental schematic using EOM as the actuator.](image-url)
to perform the laser stabilisation and TC feedback control. The system stayed stably locked without feedback to the TC-PZT, except for a separate DC source required to bring the TC on resonance. We compared the suppressed laser frequency noise before and after installation of the second EOM to ensure that no additional noise sources were introduced to the RC lock loop. Moreover, the trace was continuously monitored while the laser was locked to the TC. This gave us confidence that the laser stabilisation performance was not degraded. In the TC control loop, the output of the TC servo was low pass filtered with a 1 Hz corner frequency before being fed back to the VCO driving the EOM. The lock was stable and the overall unity gain frequency was $80 - 100\text{Hz}$. We recorded the measurements using the SR785 and calibrated them to the equivalent flexure displacement.

Figure 5.16 presents the measured flexure displacements using this EOM locking against the other two schemes. All three traces are quite in agreement. Being able to achieve high bandwidth lock using faster actuators confirmed that the coupling between the PZT and the flexure through the cavity base limited the stability of the TC feedback control. The agreement also validated the measured spectra and showed that the mechanical coupling did not introduce extra noise to the off-resonant regions.

\section*{5.6 Chapter summary}

This chapter has presented the experimental apparatus and length conversion used to obtain the measurement readout spectra for aluminium, niobium and silicon flexure cavities. Through a series of tests and measurements described here we concluded that the influence of potential noise sources listed in chapter 4 were successfully either suppressed or mitigated at the sensitivity level we expected to observe displacements of the flexures induced by thermal fluctuations. The length equivalent spectra of TC error signals are now ready for comparison against the predicted thermal noise models. This investigation is presented in chapter 6.
Chapter 6

Thermal noise results

In this chapter we present direct measurements of the frequency dependence of thermal noise in our aluminium, niobium and silicon flexures. Our measurements cover the audio-frequency band from about 10 Hz to 10 kHz, which is of particular relevance to ground-based interferometric gravitational wave detectors. These measurements span up to an order of magnitude above and below the fundamental flexure resonances. Our results are well explained by the simple model introduced in chapter 2 in which both structural and thermoelastic loss play a role.

The ability of such a model to explain this interplay is important for experiments where the sensitivity is approaching the standard quantum limit. When dominated by structural damping [65], thermal noise above the mechanical resonance rolls off with frequency faster than quantum radiation pressure noise. This allows the standard quantum limit to be observed at sufficiently high frequencies. However, thermal noise when dominated by viscous damping exhibits the same frequency dependence as quantum radiation pressure noise. This dependence makes it necessary to cool such a system (typically to below 1 K) in order to observe the standard quantum limit. Furthermore, measurements on one of the niobium flexures provided evidence that surface damage can affect the frequency dependence of thermal noise in addition to reducing the quality factor. This result can aid the understanding of how ageing effects impact on thermal noise behaviour, an important consideration for GW detector systems.

Our analysis indicates that, for three out of four flexures, structural noise dominates the displacement fluctuation spectra at low frequencies, whereas thermoelastic noise dominates at higher frequencies. Laser frequency noise limited the measurements above 5 – 10 kHz. Previous measurements have reported showing coating and mirror thermal noise as the dominant source of fluctuations in various regions of the displacement spectrum [107, 108, 152, 153]. Our flexure resonances fall in the intermediate region where structural and thermoelastic loss have comparable magnitudes. While structural and viscous damping have been studied before [154–156], to our knowledge no experiments to date have explored this crossover regime.

Section 6.1 gives a brief synopsis of our experimental parameters the details of which can be found in chapters 2 and 5. Section 6.2 highlights results used in this chapter that were derived in chapter 2 when discussing the theoretical model of thermal-noise-induced flexure displacements. Section 6.3 presents the main content of this chapter: displacement spectra of the aluminium, niobium and silicon flexures.
Thermal noise results are discussed and compared against the theoretical thermal noise models.

The majority of work in this chapter has been published in:


### 6.1 Experimental summary

The inverted aluminium, niobium and silicon flexures tested here were described in chapter 2 and we summarise the details in Table 6.1 for convenience. The layout of the experiment is shown in figure 5.1 and 5.4. To measure the extremely small thermal displacements of the flexures with a high signal-to-noise ratio, the test cavity was placed into a vacuum chamber and suspended from a multi-stage isolation system. The test cavity was 10 − 12 mm long and comprised a front mirror glued to a PZT in addition to the back mirror on the flexure. The cavity finesse was 600 and 700 for aluminum and niobium respectively, and 1200 for the silicon flexure experiments. The length fluctuations of the test cavity imprinted a phase shift onto the light which was then amplified by approximately a factor of the cavity finesse. The test cavity was kept on resonance using the PDH locking technique and fluctuations in the cavity length were read out via the error signal. The spectrum was then corrected using the measured closed-loop servo response as described in section 5.2. Excess noise was reduced to below the equivalent thermal noise displacement, using the techniques described in section 5.4.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Al</th>
<th>Nb (high Q)</th>
<th>Nb (low Q)</th>
<th>Si</th>
</tr>
</thead>
<tbody>
<tr>
<td>Width × height (mm × mm)</td>
<td>5 × 1</td>
<td>6.35 × 1</td>
<td>6.35 × 1</td>
<td>6.35 × 3</td>
</tr>
<tr>
<td>Thickness (µm)</td>
<td>120</td>
<td>72</td>
<td>200</td>
<td>100</td>
</tr>
<tr>
<td>Effective mass (g)</td>
<td>0.4</td>
<td>0.7</td>
<td>0.3</td>
<td>0.3</td>
</tr>
<tr>
<td>Resonance f₀ (Hz)</td>
<td>271</td>
<td>85</td>
<td>302</td>
<td>167</td>
</tr>
<tr>
<td>Mirror size</td>
<td>1/4&quot;</td>
<td>1/4&quot;</td>
<td>7 mm</td>
<td>1/4&quot;</td>
</tr>
<tr>
<td>Q</td>
<td>2100 ± 100</td>
<td>44000 ± 2000</td>
<td>1550 ± 200</td>
<td>56000 ± 2800</td>
</tr>
</tbody>
</table>

Table 6.1: Parameters of aluminium, niobium and silicon flexures used in thermal noise experiments.

### 6.2 Theoretical thermal noise model overview

Recall from chapter 2, the dimensionless loss parameter φᵢ of a flexure, shown in equation 2.11, is the linear sum of all losses for the i-th mode. As discussed in that
chapter, the two most prominent loss contributions to the flexures were structural and thermoelastic. Therefore, the total loss becomes:

\[ \phi_{\text{tot},i}(\omega) = \phi_{\text{struc},i}(\omega) + \phi_{\text{TE},i}(\omega) \]  

(6.1)

where \( \phi_{\text{struc}} \) and \( \phi_{\text{TE}} \) are the structural and thermoelastic loss respectively.

In this experiment, we calculate the thermoelastic loss \( \phi_{\text{TE}}(\omega) \) from the flexure geometry and thermal related parameters as shown in equation 2.13. The total loss of the flexure at the fundamental frequency \( \phi(f_0) \) is obtained through fitted ringdown measurements (see section 2.2). The structural loss is then inferred from the calculated thermoelastic loss:

\[ \phi_{\text{struc}}(f_0) = \phi_{\text{tot}}(f_0) - \phi_{\text{TE}}(f_0) \]  

(6.2)

As the structural loss is frequency independent, \( \phi_{\text{struc}}(\omega) \) is constant across all frequency. The overall loss over the measured frequency band for the fundamental mode is found using equation 6.1. We can then predict the thermal noise induced displacements of the flexure as the quadratic sum of thermal-noise-induced displacements at the individual modes as shown in equation 2.11.

Table 6.2 presents estimated uncertainties for the parameters used to construct the theoretical thermal noise models.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Uncertainties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective mass</td>
<td>5%</td>
</tr>
<tr>
<td>Temperature</td>
<td>1%</td>
</tr>
<tr>
<td>Resonance frequency</td>
<td>1 - 2%</td>
</tr>
<tr>
<td>Q from ringdown</td>
<td>5%</td>
</tr>
<tr>
<td>Flexure thickness</td>
<td>5%</td>
</tr>
</tbody>
</table>

Table 6.2: Estimated uncertainties for parameters used to construct the theoretical thermal noise models.

6.3 Thermal-noise-induced flexure displacements

This section contains the main results of the thermal noise experiments. With no free parameters, we compare the measured displacements with the predicted models of thermal noise shown in chapter 2. The flexure displacement measurements will be presented in order of decreasing mechanical loss of materials, i.e: aluminium, then niobium and finally silicon. This is equivalent to an increase in mechanical quality factors which scale inversely to the square of measurable amplitudes of the flexure off-resonant displacements induced by thermal fluctuation. Measurements using the aluminium flexure resulted in the highest displacement thermal noise among the three materials, whereas the silicon flexure displayed the lowest.
6.3.1 Aluminium flexure

Figure 6.1 shows the predicted frequency-dependent loss, made up of structural and thermoelastic components, for the aluminium flexure in the audio frequency band. The fundamental resonant frequency is indicated by vertical dashed line and the total loss calculated from the measured quality factor is shown as a horizontal dashed line. The thermoelastic loss (red) was calculated from equation 2.13 using the physical geometry of this flexure, which resulted in a relaxation peak frequency at 6 kHz. The inferred structural loss was then obtained from the measured Q and thermoelastic loss at the fundamental frequency, as explained in section 6.2. At 271 Hz, the thermoelastic loss was $2.1 \times 10^{-4}$ while the inferred structural loss was $2.7 \times 10^{-4}$. As the structural damping is frequency independent, the loss trace (green) is constant over the frequency band of interest. Both of the losses contribute roughly equally resulting in an overall loss of $4.8 \times 10^{-4}$. The linear sum of these two losses presented in blue shows an overall frequency-dependent trend with structural loss dominating below the flexure resonant frequency while thermoelastic loss dominated significantly above this frequency.

Figure 6.2 presents the measured flexure displacement spectrum (red line). The

![Graph showing frequency-dependent loss and contributions](image-url)
Thermal-noise-induced flexure displacements

The plot also presents gain-limited residual laser frequency noise converted to an equivalent displacement noise (green trace) and the electronic readout noise (blue trace). The flexure displacement was significantly above these noise sources at frequencies up to approximately 1 kHz. For reference we also show the expected thermal noise of the mirror coatings and PZT, estimated using equation 2.29 and 2.30.

The combined thermoelastic and inferred structural loss were substituted directly into equation 2.11 to produce a model for the total thermal noise induced flexural displacement. This is shown in figure 6.3. In the region between 40 Hz to 1 kHz, below the resonant frequency, the flexure displays a $1/f^{-1/2}$ slope, consistent with structural damping, while above this frequency, the displacement displays a viscous damping, falling off as $1/f^{-3/2}$. We also included the theoretical thermal noise induced displacement of the second harmonic of the flexure. The total loss for the second harmonic was calculated based on equation 2.12 with the structural loss kept constant. (Thermoelastic loss is insignificant for the shear mode [98]).

The green trace in figure 6.3 shows the sum of the model and all measured noise sources. The predicted trace agrees well with the measured displacement spectrum between 10 Hz and 10 kHz. The lower panel in figure 6.3 shows the ratio between the theoretical noise prediction and the measurement. Substantial deviation from unity appears below 40 Hz due to the presence of the residual suspension noise and multiple electronic peaks from the whitening board measurement. Visual observation of figure 6.1 shows the crossing point between the two losses occurred above the

![Figure 6.2: Measured displacement noise for the aluminium flexure. Readout and gain-limited laser frequency noise are also shown as indicated in the legends. The spectra are clear of other noise sources on either side of the fundamental resonant frequency.](image)
Thermal noise results

flexure resonance. This is in excellent agreement with the mixed behaviours seen in recorded measurements shown in figure 6.3. Based on the values listed in table 6.2, the overall uncertainty in these results was 23%. We can conclude that the measured PDH error signal is dominated by thermal noise induced flexure displacement in frequency range of 50 Hz and 10 kHz.

6.3.2 Niobium flexure

In this section, the displacement measurements of two niobium flexures are compared to their thermal noise models. Similar to the aluminium flexure above, the models presented are constructed simply from structural and thermoelastic losses. One of the niobium flexures has a higher Q of 44 000 and a fundamental resonance at 85 Hz. The second has a lower Q of 1 550 and at a resonance of 302 Hz. A difference in Q was expected due to the different thickness of the flexure membranes which changes amplitude of thermoelastic loss. However, we found that an additional viscous loss also contributed to the significant decrease in Q of the 302 Hz flexure.

Figure 6.3: Comparison of thermal noise measurement and model predictions for the aluminium flexure. The traces show the PDH error signal readout (red), the sum of structural and thermoelastic noise as predicted by the model detailed in section § 2.3 (blue), and the quadrature sum of the thermal noise model and other experimental noise contributions (green). The lower plots show the ratio between the measured PDH error signal and the predicted total noise, which in both cases is close to unity between 50 Hz and 5 kHz.
The high-Q niobium flexure

Figure 6.4 shows the structural and thermoelastic losses of the high Q niobium flexure. The frequency dependent thermoelastic loss is presented in red showing a relaxation frequency of 6.5 kHz. The constant structural loss is shown as a green trace and the overall frequency dependent loss is shown in blue. At 85 Hz the calculated thermoelastic loss was $8.5 \times 10^{-6}$ resulting in a structural loss of $1.5 \times 10^{-5}$. Around the crossing point of the red and green traces the structural and thermoelastic losses are comparable in amplitudes. As the fundamental resonance resides in this vicinity, this explains the mixed response of the measured niobium flexural displacement shown in figure 6.5.

As with the aluminium flexure, the total loss was used to generate thermal noise models for the high Q niobium flexure. Figure 6.5 shows the predicted thermal-noise-induced flexure displacement in blue while the measured displacement is presented in red. With a total uncertainty of 17%, a visual observation shows a good agreement in the off-resonant regions between these two traces up to 1 kHz. The green trace in the figure is the result of combining the theoretical model and external noise sources that contributed significantly at high frequency. This agreement is further clarified.
Figure 6.5: Comparison of thermal noise measurement and model predictions for the high-Q niobium flexure. The traces show the PDH error signal readout (red), the sum of structural and thermoelastic noise (blue), and the quadrature sum of the thermal noise model and other experimental noise contributions (green). The lower plots show the ratio between the measured PDH error signal and the predicted total noise, showing good agreement between 20 Hz and 1 kHz.

in the bottom plot of figure 6.5 where the ratio between the red and green traces is close to 1.

The low-Q niobium flexure

Figure 6.6 presents the measured displacement noise of the 302 Hz niobium flexure. The measured output shows viscous damping thermal noise features of $1/f^0$ below the resonant peak and $1/f^{-2}$ above the peak. The blue trace is the length-equivalent PDH error signal when the cavity was off resonance. The green trace is the gain-limited frequency noise. The Q of this flexure was 1550 obtained from the ringdown measurement and was confirmed through a transfer function measurement.

Our model indicates the increased membrane thickness of the low Q niobium flexure should shift the thermoelastic peak to lower frequencies. This would lead to an increase in thermoelastic loss around 302 Hz. As shown in figure 6.7 top, the calculated thermoelastic loss at the flexure fundamental resonance of 302 Hz is $2.1 \times 10^{-4}$, which is only a factor of 2 less than the maximum damping amplitude at the thermoelastic peak. When we inferred the structural loss from the measured Q and calculated thermoelastic loss, the result was $4.1 \times 10^{-5}$ shown as the red trace.
§6.3 Thermal-noise-induced flexure displacements

Displacement noise \[ \text{[m/}\sqrt{\text{Hz}}] \]

Frequency [Hz]

Off-resonant noise
Gain limited frequency noise
Measured data

Figure 6.6: Displacement measurement of low-Q niobium flexure shows a viscous response below and above the flexure resonance.

in the top figure 6.7. The total loss presented as the blue trace shows a viscous-dominating response, which provides a reasonable explanation to the measured data in figure 6.6. However, the concern with the model of this flexure is the very high structural loss whose value has not been verifiable in literature. The loss amplitude is also more than a factor of 20 higher than that of the high Q niobium flexure. This raised questions considering they were both manufactured from the same batches at the same time.

As a result we concluded that this flexure has lower Q due to the existence of an additional viscous loss. External loss mechanisms were investigated, including damping from residual background gas collisions, loss from the flexure clamping, and the amount and type of glue used (initially “Vac-Seal”, a vacuum compatible two-part epoxy, and subsequently superglue). None of these tests showed significant changes to the off-resonant thermal noise. We then took a different approach and assumed that the additional viscous loss was internal. The total loss for this flexure would then become:

\[
\phi_{0,Nb} = \phi_{\text{struc}} + \phi_{TE} + \phi_V
\] (6.3)

where \( \phi_V \) is the additional viscous loss.

The bottom graph of figure 6.7 shows the scenario of loss distribution in the presence of an extra viscous loss. The structural loss \( \phi_{\text{struc}} \) was the same as for the high-Q niobium flexure and is shown as trace 2. The thermoelastic loss shown in the bottom figure as trace 3 is the same as the thermoelastic in the top figure. Together with the measured Q of 1550, the inferred additional viscous loss shown as trace 4 is \( 4.1 \times 10^{-4} \) at the resonant frequency. In this scenario, both thermoelastic loss and viscous loss are the dominating factors.

The sum of the losses in figure 6.7 was used to generate the models shown in figure
Figure 6.7: Calculated losses due to surface damage of the low-Q niobium flexure. Top: the frequency-dependent loss $\phi_\omega$ for the fundamental oscillation model showing contributions of structural $\phi_{\text{struc}}$ and thermoelastic $\phi_{\text{TE}}$ damping without an additional source of viscous damping. Bottom: shows the estimated total loss $\phi_\omega$ when including an extra viscous loss.

6.8. The measured noise is shown in red. The green trace is the theoretical model with the extra viscous loss whereas the blue trace is the model with high structural loss and no extra viscous loss. Both traces are the combination of the thermal noise models and the experimental external noise sources such as electronic noise and gain limited frequency noise. Qualitatively, the green trace shows a better agreement to the measured output, especially below the resonant peak, consistent with the theory of an extra viscous loss. The ratios of the measured data and each of the models are shown in the pane below figure 6.8. Comparing these two ratios confirms that the
§6.3 Thermal-noise-induced flexure displacements

Figure 6.8: Thermal noise induced displacements of surface damaged Niobium flexures showed an increase in viscous damping, potentially due to surface damage.

model with an additional viscous model provides a better explanation of the viscous damping response of the flexure displacement.

Figure 6.9: Microscope image shows a defect in the low-Q niobium flexure. A thin black line in the red circle appeared to run across an entire thickness of the membrane.

One possible explanation for the additional viscous loss was the existence of a small defect on the side of the membrane. This was observed under the microscope [87] and could cause ‘rubbing’ leading to increased damping of the flexure. Even though this effect has not been well documented, it was clear such a small defect
Thermal noise results could be quite detrimental to a highly sensitive experiment. Noticeably there was a report from the VIGRO collaboration for evidence of surface damage of a blade spring that was part of the interferometer’s core suspension system [157]. The damage increased the total loss of the suspension performance, changing the overall thermal noise shape of the instrument from structural to a slower fall off of $1/f^{-1/2}$.

6.3.3 Silicon flexure

Among all the materials we tested, silicon showed the lowest structural loss. Therefore to achieve the required sensitivity, many changes in the experimental setup were required as detailed in section 5.1. The experimental redesign and suspension rebalance resulted in the lowest overall noise floor ever achieved for this apparatus.

Similar to the previous flexures, the measured Q at the fundamental resonance was used as an upper bound for the total loss at that frequency. The calculated thermoelastic noise is shown as red trace in figure 6.10 and the inferred structural loss is shown as a green trace. The overall frequency dependent loss is presented in blue.

At the resonant frequency of 167 Hz, the thermoelastic loss is $2.3 \times 10^{-6}$, seven times smaller than the inferred structural loss. The crossing between thermoelastic loss and other frequency independent loss is located at about 1 kHz. Contrary to the previous flexures whose resonant frequencies fell between the two frequency dependency regimes, the silicon loss was designed to have more structural features.

![Figure 6.10: Calculated frequency-dependent loss $\phi_\omega$ for the fundamental oscillation mode of the silicon flexure, showing the contributions of structural $\phi_{struc}$ and thermoelastic $\phi_{TE}$ damping. The fundamental flexure resonant frequency is indicated by the vertical dashed line, and the total loss determined from the measured quality factor is marked by the horizontal line.](image)
in the off-resonant regions around the flexure resonance. The loss at the flexure resonance is $1.8 \times 10^{-5}$ and predominantly structural. A summary of the main losses for different flexures are presented in table 6.3.

Table 6.3: Main losses of aluminium, niobium and silicon flexures.

<table>
<thead>
<tr>
<th>Material</th>
<th>$\theta_{\text{struc}}$</th>
<th>$\theta_{\text{TE}}$</th>
<th>$\theta_{\text{total}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aluminium</td>
<td>$2.7 \times 10^{-4}$</td>
<td>$2.1 \times 10^{-4}$</td>
<td>$4.8 \times 10^{-4}$</td>
</tr>
<tr>
<td>Niobium (high Q)</td>
<td>$1.5 \times 10^{-5}$</td>
<td>$8.5 \times 10^{-6}$</td>
<td>$2.3 \times 10^{-5}$</td>
</tr>
<tr>
<td>Silicon</td>
<td>$1.6 \times 10^{-5}$</td>
<td>$2.3 \times 10^{-6}$</td>
<td>$1.8 \times 10^{-5}$</td>
</tr>
</tbody>
</table>

Our experimental measurements are presented in figure 6.11. The red trace presents the measured PDH error signal while the cavity was locked to the laser. The green trace is the gain limited frequency noise equivalent flexure displacement. The blue trace is the electronic noise measured at the output port with the photo-detector blocked. The electronic noise includes the photo-detector dark current, the electronic read out noise and the minimum noise floor of the detecting instrument SR785. The combination of the two latter traces place the minimum noise floor below the

![Figure 6.11](image-url)

Figure 6.11: This graph shows a wide frequency band of silicon displacement (red) together with other potential noise sources such as: the gain limited frequency noise equivalent to flexure displacement in green, the off-resonant noise in magenta and the electronic noise in blue.

level at which the silicon flexure displacements were measured. Included in this figure is an off-resonant noise trace in magenta. This trace was obtained at the output port while the cavity was tuned far away from its resonance and the laser was stabilised to the reference cavity. The trace captures all other excess noise sources, such as spurious interferometric noise, that could potentially mask the true
Thermal noise results

flexural displacement signals. It shows the sensing noise floor to be below the signal of interest and eventually dominated by electronic noise at approximately 1 kHz.

Figure 6.12 presents measurements shown in figure 6.11 together with other theoretical models and overall predicted noise for comparison. As the output of this experiment is the test cavity PDH error signal obtained from the stabilised laser, the readout contains all noise sources that could cause length fluctuation within the test cavity. The new traces in this figure is the theoretical thermal noise of the PZT actuator that was used to keep the cavity on resonance. The PZT resonance was determined through a transfer function measurement. This was then used as part of the thermal noise calculation for the PZT presented in this figure. The final predicted noise trace shown in blue was the quadrature sum of the thermal noise of all the flexure resonances and the equivalent displacements induced by the measured dark noise and gain limited frequency noise.

At high frequency, together with electronic noise and gain-limited frequency noise, the off-resonance of other flexure modes have small contribution toward the measured noise because the thermal-noise-induced displacements of the fundamental mode rolls off below those other displacements. The high order modes were fitted to the spectra assuming that the main dominating losses for twist and shear modes were still thermoelastic and structural.

Without any free parameters, the predicted noise trace and the measurement are
in agreement in both the low frequency region below the first higher order mode and the high frequency region until the laser frequency noise started to dominate. In the lower graph of figure 6.12, the agreement is again highlighted by the ratio being close to 1. The off-resonant region around the fundamental peak shows a structural response up to 500 Hz. Beyond the second resonance, the measurement was also influenced by a complicated combination of thermoelastic damping, thermal noise of higher-order mode, environmental noise. An extra noise of unknown origin is observed around the second, third and fourth harmonics. The complexity in the frequency span between 2 kHz and 20 kHz makes it challenging to separate and resolve each peak and hence understand their true contribution to the overall measured trace. The overall propagating uncertainty for the silicon flexure measurement was estimated at 19%.

6.4 Chapter summary

We have presented measurements of off-resonant thermal noise for aluminium, niobium and silicon flexures in the audio-frequency band between 10 Hz and 10 kHz at room temperature. Our experimental results show good agreement with a simple model that is dominated by frequency-independent structural and frequency-dependent thermoelastic damping. The current apparatus has a sensitivity on the order of $10^{-17}$ m/$\sqrt{\text{Hz}}$ and can be used as a thermal noise characterisation tool to measure the mechanical loss in materials.

The low-Q niobium result provides evidence for changes in the displacement spectrum caused by surface damage in addition to a decrease in the mechanical Q. These results are of particular interest for the design of suspension systems for future generation gravitational wave observatories, as well as the understanding of ageing effects in existing detectors.

Silicon is considered a potential low loss material to achieve higher sensitivity for the future gravitational wave detectors due to its zero-coefficient of thermal expansion at cryogenic temperatures. The room temperature displacement spectrum presented here has shown the off-resonant thermal-noise-induced displacement of silicon flexure is small. It also indicates that an appropriate choice of physical parameters should allow the structural damping to dominate thermal noise well above the mechanical resonance, an important requirement in designing suspension system in gravitational wave detectors.

Lastly, systems dominated by thermoelastic loss have the same $1/f^{-2}$ frequency dependence as those dominated by radiation pressure noise. Therefore quantum radiation pressure noise can be masked unless the system is cooled to cryogenic temperatures. On the other hand, structural-loss-dominated systems exhibit a $1/f^{5/2}$ roll off so that radiation pressure noise can dominate at high frequency. Thus, careful tailoring of the flexure geometry to shift the thermoelastic loss outside the region of interest will be an important consideration in designing quantum radiation pressure noise experiments.
Thermal noise results
This section presents the experimental observation of a single-carrier self-locked silicon flexure cavity. The cavity could be tuned to be statically and dynamically stable. We characterise the power and detuning dependences of the observed effect and show that the optically induced damping is four orders of magnitude larger than the mechanical damping in this system. Two different models are presented and compared to the data.

### 7.1 Flexure cavity without feedback control

Figure 7.1 shows the experimental apparatus. The main optical path remained unchanged from the one shown previously in figure 5.4. As this experiment was intended to test the optical spring behaviour under the influence of light forces alone, the feedback stage of the test cavity was not used. The 167 Hz silicon flexure has an effective mass of about 0.3 g and a mechanical spring constant of approximately 10^5 \text{N/m}.
A stable single-carrier optical spring

330 N/m. The flexure cavity was 10 mm long with a finesse of 1340 resulting in a cavity storage time $\tau_{\text{cav}}$ of approximately 14 ns. The cavity response time was much faster than the oscillating period of the flexure, hence the damping due to radiation pressure was negligible.

Figure 7.2: Cavity self-locking with light induced forces. A: Optical spring effects observed on PDH error signals and transmitted signals as the cavity was swept slowly across resonance. B: Self-locking of the cavity which was detuned at $\delta = 0.5$. The effect is much stronger with larger input powers. C: Damping of oscillation after a sudden displacement of the mechanical oscillator while the cavity was self-locked.

Figure 7.2A shows classic optical bistability observed while scanning the cavity resonance via the PZT [122]. We obtained the error signals and the transmitted powers of both the sidebands and the carrier by sweeping the TC slowly via the PZT for three different input powers. When the input power was increased by factors of
5 and then 20, the transmitted responses became increasingly asymmetrical. This demonstrated that the effect was optically-induced. As described in section 3.2, an opto-mechanical cavity dominated by radiation pressure forces will exhibit a restoring force on one side of resonance and an anti-spring restoring force on the other side. When the laser beam was detuned from the cavity resonance to the anti-spring side, the cavity was unstable and wanted to accelerate quickly either away from resonance or over to the stable side. This is demonstrated clearly in the sharp wall of the cavity response at 100 mW (red trace). On the other side of the cavity resonance, the presence of a spring introduced a restoring force leading to a more stable response.

At large input power, for example at 100 mW, the cavity self-locked when detuned near $\delta_\gamma = 1/\sqrt{3}$, as shown experimentally in figure 7.2B. When placed manually at this detuning via the TC-PZT, the time taken for the transmission and error signal to drift away from the detuning point became longer as the input power increased. At 100 mW input, the restoring force from the optical spring was large enough to compensate for cavity and laser frequency drifts resulting in self-locking.
to the detuning point for over an hour. During this time, sudden disturbances were quickly subdued bringing the cavity back to the detuning point (Figure 7.2C). This demonstrated optically-induced damping on the same side as optical stiffening of the mechanical spring.

To experimentally quantify this, the flexure was excited at constant intervals while we slowly scanned through the TC resonance. The subsequent ringdown in both transmission and error signal were recorded and fitted to derive the effective spring constant and damping rate. This step was repeated for different cavity detunings at several optical powers. The results of the measurements are presented in figures 7.3 and 7.4.

Figure 7.3 shows the measured spring constant and damping rate as a function of detuning for several input powers. We found that the measured spring constant and
damping rate could be described roughly by the following detuning dependences:

\[ k_{\text{tot}} \propto \frac{\delta_\gamma}{(1 + \delta_\gamma^2)^2} \]  
\[ \Gamma_{\text{tot}} \propto \frac{\delta_\gamma}{(1 + \delta_\gamma^2)^3} \]

The measured data was fit using equations 7.1 and 7.2 with a single scaling factor \((\zeta)\) as the fitting parameter.

Figure 7.4 shows the maximum optically induced spring constant and damping rate as a function of input power when the cavity was blue-detuned. Each data point was obtained from a curve like those shown in figure 7.3. The spring constant shows a linear dependence on the input power \((k_{\text{tot}} \propto P_{\text{in}}})\) while the damping rate increases quadratically \((\Gamma_{\text{tot}} \propto P_{\text{in}}^2})\).

The maximum measured damping rate was four orders of magnitude larger than the mechanical damping of the flexure \(\Gamma_m = \omega_0/Q = 2\pi \times 3\) mHz. Since the radiation pressure induced damping rate was also small \((\Gamma_{\text{os}} = 2\pi \times 1 - 10\) mHz), these observations cannot be explained by radiation pressure alone. As presented in section 3.3, the bolometric effect can also give rise to a power dependent spring constant and damping rate. In the next section, we discuss whether the bolometric effect could be responsible for the observed damping.

## 7.2 Bolometric force model

Assuming that both radiation pressure and thermal expansion can be treated as forces which add linearly, a simple model can be built to incorporate both effects. From sections 3.2 and 3.3, the spring constant and damping rate of an opto-mechanical system exhibiting both forces become:

\[ \omega_{\text{eff}}^2 = \frac{k_m}{m} + \frac{k_{\text{os}}}{m} + \frac{k_{\text{bol}}}{m} \]  
\[ \approx \omega_0^2 - \left( \frac{1}{m} + \frac{R\beta A}{m} \frac{1}{1 + \omega^2 \tau_{\text{th}}^2} \right) \frac{16\alpha F P_{\text{in}}}{\lambda} \frac{\delta_\gamma}{(1 + \delta_\gamma^2)^2} \]  
\[ \Gamma_{\text{eff}} = \Gamma_m + \Gamma_{\text{os}} + \Gamma_{\text{bol}} \]  
\[ \approx \Gamma_m + \frac{R\beta A}{m} \frac{1}{1 + \omega^2 \tau_{\text{th}}^2} \frac{16\alpha F P_{\text{in}}}{\lambda} \frac{\delta_\gamma}{(1 + \delta_\gamma^2)^2} \]

where \(\Gamma_{\text{os}}\) is negligible, and (c.f. equation 3.20):

\[ \frac{16\alpha F P_{\text{in}}}{\lambda} \frac{\delta_\gamma}{(1 + \delta_\gamma^2)^2} = -\frac{2}{c} \frac{dP_{\text{circ}}}{dx} \]  
\[ \tau_{\text{th}} = \frac{\rho Cr_0^2}{2\kappa} \]
where $\rho$, $C$, $\kappa$ and $r_0$ are the density, specific heat, thermal conductivity and the beam radius respectively. For SiO$_2$, $\rho = 2200$ kg/m$^3$, $C = 746$ J/kg/K, and $\kappa = 1.38$ W/m/K [59] and a spot size of 129 $\mu$m, the thermal time-scale was calculated to be about 10 ms.

The total spring constant is now the sum of mechanical, radiation pressure and bolometric contributions. The second term in equation 7.4 shows the dependence on cavity detuning of the optically induced spring constant. The relationship was in agreement with the best fits to the spring constant data in both figures 7.3 and 7.4 with $\zeta$ of $-0.128$ as the fitting parameter.

Equation 7.6 is obtained by assuming a linear relationship between circulating power and absorbed power, $P_{abs} = AP_{circ}$ as reported in other systems [128, 132]. However, the measured optical induced damping rate shows a different relationship to both input power and detuning than indicated in equation 7.6. While the linear absorption leads to damping as a function of $\delta_z/(1 + \delta_z^2)^3$, the measured damping rate is proportional to $\delta_z/(1 + \delta_z^2)^3$. In addition, the measured damping rate is proportional to the square of $P_{in}$ instead of a linear response.

The observed damping rate could be explained if we assume that the absorption is non-linear such that $P_{abs} = A^2 P_{circ}^2$, where $A^2$ represents a second-order non-linear absorption coefficient. Equation 7.7 then becomes:

$$
\frac{d(P_{circ}^2)}{dx} = 2P_{circ} \frac{dP_{circ}}{dx} \propto \frac{P_{in}^2 \delta_z}{(1 + \delta_z^2)^3} \quad (7.9)
$$

Thus, the damping rate is now proportional to $\delta_z/(1 + \delta_z^2)^3$ and to $P_{in}^2$ as experimentally observed. The fitting parameter $\zeta$ is equivalent to $\beta A^2$ in this non-linear absorption model. The negative value of $\beta A$ indicates that the radiation pressure and bolometric forces act in opposite directions, which makes intuitive sense since the radiation-pressure-induced restoring force tends to expand the cavity whereas the thermal expansion of the mirrors effectively shortens it. Since the optical restoring and damping are on opposite sides of resonance, cavities in the presence of only radiation pressure force are generally statically or dynamically unstable. Here, since bolometric damping occurs on the same side of resonance as the radiation pressure restoring, the model predicts that the cavity would be stable when it was blue-detuned. This agrees with the observed self-locking as described previously in figure 7.2.

It is worth noting that this model relies on the assumption that radiation pressure and bolometric can be treated as independent forces. To fit our observations, it would also require that the $P_{abs} \propto P_{in}^2$.

To test this hypothesis, the absorption of the cavity mirrors as a function of incident power was measured by A. Bell, J. Steinlechner and I. Martin at the University of Glasgow. No evidence of nonlinear absorption was found up to intensities of 200 kW/cm$^2$, which is the maximum intensity achieved in our experiment. This result suggests that the bolometric force model does not capture the physics at work in our system.
7.3 Bolometric feedback model

An alternative model can be constructed by treating bolometric effects as a displacement instead of a force. Absorption of light causes a displacement of the mirror surface which in turn alters the intra-cavity field. Hence the bolometric effect interacts with the radiation pressure spring in a similar manner to a feedback servo [125]. Figure 7.5 presents the block diagrams that describe the cavity dynamics: a) with only radiation pressure present; and b) with both bolometric and radiation pressure effects. Recall that the mechanical susceptibility of a damped harmonic oscillator is

\[ \chi_m = \frac{1}{k_m - m\omega^2 + i \Gamma_m \omega m} \]  

(7.10)

The radiation-pressure induced change in the susceptibility depicted in figure 7.5 a) is as follows:

\[ \frac{X_{out}}{F_{in}} = \frac{1}{1/\chi_m - RC} \]  

(7.11)

\[ = \frac{1}{k_m + k_{os} - m\omega^2 + i \Gamma_m \omega m} \]  

(7.12)

where \(-RC = k_{os}\) is the radiation-pressure induced optical spring constant as shown in equation 7.7 (the optical damping is negligible for our system). Equation 7.12 describes a spring which is stiffened on the blue side of resonance and softened on the red side, in agreement with equation 3.21.

When the bolometric effect is included, the cavity output drives expansion of the mirror which changes the cavity length as illustrated in figure 7.5. The circulating
power inside the cavity responds to a cavity length change due to the bolometric
displacements and the radiation-pressure induced flexure displacements. The transfer
function of the nested feedback loop is:

\[
\frac{X_{out}}{F_{in}} = \frac{1}{1/\chi_m + k_{os} - CB/\chi_m}
\]

(7.13)

\[
= \frac{1}{1 - CB \frac{1}{\chi_m} + k_{os}/(1 - CB)}
\]

(7.14)

where \(B\) is the bolometric response to a change in absorbed power in units of \(m/W\).
Here we define \(CB\) such that thermal expansion of the mirror effectively decreases
the cavity length:

\[
G_b \equiv CB = \frac{dP_{circ}}{dL} \frac{-\xi A}{1 + i\omega \tau_{th}}
\]

(7.15)

where \(\tau_{th}\) is calculated using equation 7.8; \(A\) is the linear absorption coefficient of
the mirror \(P_{circ} = AP_{in}\), measured to be 20 ppm; and \(\xi = \bar{\alpha}/\kappa\), with \(\bar{\alpha} = 2(1 + \sigma)\alpha\)
the effective expansion coefficient \([59, 158]\).

Equation 7.14 shows that the optical spring has been modified by the bolometric
feedback:

\[
k_{mod} = \frac{k_{os}}{1 - G_b}
\]

(7.16)

Similar to a feedback servo, the gain \(G_b\) can rotate \(k_{os}\) in the complex plane, giving
it a non-negligible imaginary part which would be observed as damping. Using a
Taylor expansion for \(G_b \ll 1\), we can separate the real and imaginary parts of the

![Figure 7.6](image-url)
modified optical spring constant $k_{\text{mod}}$:

\[
\Re(k_{\text{mod}}) = k_{\text{os}} (1 + \Re(G_b)) = k_{\text{os}} \left(1 + \frac{k_{\text{os}} \xi A}{R} \frac{1 + \omega^2 \tau_{th}^2}{1 + \omega^2 \tau_{th}^2}\right)
\]  

(7.17)

\[
\Im(k_{\text{mod}}) = k_{\text{os}} \left(\Im(G_b)\right) = -k_{\text{os}}^2 \frac{\xi A \omega \tau_{th}}{R} \frac{1 + \omega^2 \tau_{th}}{1 + \omega^2 \tau_{th}^2}
\]  

(7.18)

$\Re(k_{\text{mod}})$ and $\Im(k_{\text{mod}})$ are the modified optical spring constant and damping rate of the system. The magnitudes of the optical spring constant and the damping rate as a function of detuning are

\[
\Re(k_{\text{mod}}) \propto P_{\text{in}} \frac{\delta_{\gamma}}{(1 + \delta_{\gamma}^2)^2}
\]  

(7.19)

\[
\Im(k_{\text{mod}}) \propto P_{\text{in}}^2 \frac{\delta_{\gamma}^2}{(1 + \delta_{\gamma}^2)^4}
\]  

(7.20)

Here, the optical spring constant shows a linear dependence on the optical input power whereas the damping rate responds quadratically to the power. This is consistent with the measurements depicted in figure 7.4. The bolometric feedback rotates the optical spring constant in the complex plane leading to the nonlinear power dependence of the damping rate as illustrated in figure 7.6.

The expressions for $\Re(k_{\text{mod}})$ (equation 7.17) and $|\Im(k_{\text{mod}})|$ (equation 7.18) can be used to describe the measured spring constant and damping rate as shown in figure 7.3 (solid lines). The change in the spring constant due to bolometric effect is negligible. On the other hand, it is clear that $\delta_{\gamma}^2/(1 + \delta_{\gamma}^2)^4$ shows better agreement with the measured data than the detuning dependence of $\delta_{\gamma}/(1 + \delta_{\gamma}^2)^3$ (dashed line) obtained from the bolometric force model.

Even though the magnitude of the damping rate from the feedback model describes the measured data, it has the opposite sign. Under the assumption that the bolometric effect will shorten the cavity length, the model predicts an anti-damping when the cavity is blue-detuned, making the cavity dynamically unstable. This is inconsistent with the experimental observations presented in figure 7.1. If the absorbed power caused the cavity to lengthen instead, then this model would agree with the experimental data. The physical mechanism behind the experimental observation is still under investigation. One possible explanation is that the high reflective coating of the mirror was incorrectly marked and as a result, this surface now faced outwardly to the cavity. If this were true, then thermal expansion of the mirror would effectively lengthen the cavity, which would explain why we observe damping instead of anti-damping. Unfortunately, at the time of writing, we did not have access to the cavity mirror to measure the reflectivity and the silicon flexure was damaged in preparation for absorption measurements.
7.4 Chapter summary

In this chapter, we presented the experimental observation of a stable single carrier flexure cavity in the absence of any feedback control. The data suggested the presence of an optical effect other than radiation pressure. The measured damping was four orders of magnitude larger than the mechanical damping and scaled with the square of the input optical power. Two models were built to explain the observations, hence gain further understanding of this opto-mechanical system, however neither agrees completely with observation. Work is ongoing to better understand this simple opto-mechanical system.
In this thesis, we used a short Fabry-Perot cavity with one mirror on a flexure to investigate the dynamics of a gram-scale opto-mechanical system. The frequency dependence of thermal-induced displacements was investigated, and the opto-mechanical dynamics were studied. The findings contribute to the understanding of the underlying physics of thermal fluctuations and light-mechanical interactions. This chapter summarises experimental results and observations, and suggestions for future investigations.

8.1 Summary of thermal noise experiments

- The broadband thermal-noise-induced displacements of gram-scale flexures made of aluminium, niobium and silicon were presented. The recorded frequency response falls within the LIGO detection band. The aluminium and 85 Hz niobium flexures show structural-dominated damping below the flexure resonances and thermoelastic-dominated damping in the higher frequency regions. The silicon flexure was designed to push the thermoelastic damping peak to higher frequency, such that structural damping would be the dominant source of fluctuations both below and above the fundamental frequency. Experimental measurements of the flexure displacement spectrum confirmed that it was dominated by structural damping.

- A simple model constructed from the Fluctuation-Dissipation Theorem [9] was used to predict the thermal-noise-induced flexure displacements. The comparison between the experimental results and the model showed good agreements, without any free parameters. The ability of this simple model to accurately predict thermal noise spectra is important for investigations of quantum-radiation-pressure noise and the standard quantum limit.

- A major part of the project was working toward achieving a highly sensitive apparatus to measure the minuscule flexure displacements caused by thermal fluctuations. Several techniques and implementations were applied to diagnose and isolate the sources of noise, and then subsequently mitigate them in the frequency band of interest. Scattered light, for example, was diagnosed using the frequency shift technique and eventually minimised by increasing eddy current damping of the test cavity motion and optimising the suspension system.
Conclusions and future work

The noise floor in the silicon flexure measurements was improved by nearly an order of magnitude to $10^{-17}$ m/$\sqrt{\text{Hz}}$ in the frequency band between 80 Hz and 2 kHz.

- Three different control schemes for stabilising the test cavity were successfully implemented, using a PZT, an AOM and an EOM as the actuators. Low UGF feedback of $10 - 50$ Hz (below the flexure resonance) was achieved by feeding back to the PZT, whereas control using the AOM and EOM attained a stable lock with a UGF of 2 kHz. All three flexure displacement measurements were consistent up to 2 kHz.

8.1.1 Further work

The sensitivity of the current apparatus is already exceptional for a bench-top experiment. The natural next step is to lower the current displacement noise floor further and simultaneously broaden the measurement bandwidth, so that displacement spectra of higher $Q$ oscillators could be measured. The current limiting factors are:

- The reference cavity used to stabilise the laser frequency placed a bandwidth limit on the laser frequency stabilisation due to the cavity pole at 60 kHz. Adding a zero at this frequency to the laser stabilisation servo could nullify the cavity low-pass filter effect, hence increasing the control bandwidth. Eventually this stabilisation loop will be limited by the frequency response of the laser PZT. Increasing the loop bandwidth further may be possible by adding a faster actuator such as an AOM or EOM for frequency feedback.

- Characterisation of the mechanical coupling of the PZT to the flexure via the cavity base should be carried out prior to the redesign of this cavity. Furthermore, the estimated PZT thermal noise is a limiting factor to measure extremely small displacements at room temperature. One option is to build a cavity without an actuator, stabilised by a fast external actuator such as an EOM as demonstrated in section 5.5.

The displacement measurements of the silicon flexure at room temperature show promise as a potential material for future gravitational wave detectors. The next step is to measure this displacement at cryogenic temperatures (120 K) where the thermal expansion of silicon crosses zero, resulting in the lowest thermoelastic damping. At 120 K, the two dominant loss mechanisms are expected to be surface loss and structural loss [84]. Tailoring the design will be important to ensure that the structural loss dominates in the frequency band of interest.

Some modifications of the current experimental apparatus will be required to accommodate for additional cooling components. The final suspension stage used to isolate the ensemble, consisting of the 2 kg base, the input mirrors and the flexure, will need to be enclosed in a radiation shield which is actively cooled to around 123 K using liquid nitrogen. An intermediate cooling gas will be used to bring the ensemble
(and the whole vacuum system) down to 120 K. Once this temperature is reached, the vacuum pumps will be engaged to bring the pressure down to $10^{-6}$ mbar and the radiative shield will maintain the flexure base at the design temperature.

Alternatively, a remotely operated, removable link between the radiative shield and the flexure base could be used to bring the flexure down to 120 K. Once this is reached the removable link could be disengaged from the flexure base and the radiative shield would maintain the flexure base at the operating temperature.

### 8.2 Opto-mechanics

The silicon flexure cavity exhibited both static and dynamic stability when the cavity was blue-detuned. This is in contrast to what is expected for an opto-mechanical cavity dominated by radiation pressure, where both sides of resonance should be unstable. The measured optical spring constant shows a dependence on optical power and detuning as expected. The observed optically induced damping was four orders of magnitude larger than the mechanical damping of the flexure and could not be explain by radiation pressure. Although work is ongoing to identify the mechanism responsible for stabilising the spring, two models of thermal expansion of the mirror coatings were presented.

The first model treated the bolometric effect as a force, based on the work of Restrepo et. al. [131] which has been used to model a gold cantilever [132]. This model could explain the observations if the mirror absorption was nonlinear with incident power.

The second model treated the bolometric effect as a displacement which modified the overall dynamics of the optical spring, similarly to a feedback servo. This model successfully predicts the magnitude of the damping but not the sign, indicating an anti-damping on the blue-side of resonance. This was not consistent with the observations unless the cavity was lengthened as the result of the bolometric effect.

#### 8.2.1 Further work

Investigation into the physical mechanism responsible for the damping will be the first obvious step toward fully understanding the opto-mechanical flexure interactions. This may include re-acquiring the original mirrors to measure the mirror reflectivities; or a rebuild of the silicon flexure cavity to repeat the experiments with a new mirror. Understanding how different optical spring effects scaled with various parameters will allow us to predict whether these effects will need to be addressed in larger scale systems. With the knowledge gained, the feasibility of a quantum radiation pressure noise experiment could be studied.


[23] Proposal of the consortium for indian initiative in gravitational-wave observations (indigo). In *IndIGO Consortium*, 2011. 4


[31] . https://awiki.ligo-wa.caltech.edu/aLIGO/GWINC.


[63] K. Agatsuma. *Study of pendulum thermal noise in gravitational wave detectors*. PhD thesis, Department of Physics, Faculty of Science, University of Tokyo, December 2009. 16, 20


[88] A. Mullavey. Experimental demonstration of radiation pressure effects inside a detuned fabry-perot cavity. Master’s thesis, Physics Department, Australian National University, Canberra, Australia, 2006. 21


[122] B. Sheard. Arm locking for space-based gravitational wave detectors and optomechanical effects in interferometers. PhD thesis, Physics Department, Australian National University, Canberra, Australia, 2005. 43, 106


[146] A. Wade. *Quantum measurements limited in gravitational detectors*. PhD thesis, Physics Department, Australian National University, Canberra, Australia, 2016. 61, 63


[149] R. Bork. CDS real-time data acquisition scheme. LIGO Document Number LIGO–T0999638-v1. 65


