UNEMPLOYMENT, INFLATION AND SOME RELEVANT POLICY ISSUES

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STATEMENT

Chapter 4 is derived from two earlier papers which are cited in the bibliography: Stemp (1981) and Stemp and Turnovsky (1982). In the second paper I was responsible for the conclusion that because of the monetary policy rule chosen there are two classes of steady state: one associated with a zero rate of inflation and one associated with a non-zero rate of inflation. I also showed that each class of steady-state is associated with different stability properties. Professor Turnovsky showed that within each class of steady-state the equilibrium may not be unique because of non-linearities derived from the return on bonds (= rb) and the inflation tax on real wealth (= πA). He also provided the example of Section 4.7. For presentation in the thesis parts of the second paper have been revised. In particular, I rewrote Section 4.7 to conform with the approach of Chapter 3.

Chapters 5 and 7 are derived from Stemp and Turnovsky (1983). This paper was written after I had proved the results of Chapter 6 and in response to a suggestion by Professor Turnovsky that Chapter 6 should be rewritten to specifically include jumps in the price level. The proof that the loss function with quadratic adjustment costs is time inconsistent was provided by Professor Turnovsky while the result that there is a loss function (with absolute valued adjustment costs) that can be associated with time consistency was proved by myself.

Apart from these instances, and unless specifically stated otherwise, all of the work reported herein is my own.

Peter Stemp.

Peter J. Stemp
November 1983

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Finally this thesis is dedicated to my mother and to the memory of my father.

ABSTRACT

This thesis is an attempt to consider in some detail a few of the important theoretical and policy issues associated with unemployment and inflation, using a fairly standard framework which many economists would accept as plausible.

Part 1 is concerned with "The Role of Expectations and Deficit Financing." Here we introduce the rational expectations approach to economic modeling and also show how the short-run responses of endogenous variables can be calculated. We do this in conjunction with an examination of the stability properties associated with alternative forms of deficit financing. In Parts 2 and 3 of the thesis we use the techniques developed in Part 1 to examine some policy issues more closely.

Frequently, a government policy maker may want to lower both inflation and unemployment in a manner which is as painless as possible. One solution to the policy maker's problem is presented in Part 2, entitled "Optimal Stabilization Policies Under Perfect Foresight." Here we formalize the policy maker's objective and discuss what policy instruments can be used to achieve that objective. We also spend some time discussing whether it is desirable for prices to jump and if this is the case what is the magnitude of the appropriate jump. Time consistency of the optimal solution is also considered.

"Some Relevant Policy Issues" are discussed in Part 3. When unemployment becomes high, two issues that are frequently raised concern "easy" methods for lowering unemployment and ways of relieving the hardship of the unemployed. One method of lowering unemployment is by lowering the length of the standard working week and hence lowering the aggregate

supply of labor. A way of making life more acceptable to those who are unemployed is by paying higher unemployment benefits to those without work. Both these "solutions" are examined in Part 3.

An important part of this thesis is the way in which the issues mentioned above have been analysed within a consistent framework of models. This framework provides a common thread which, along with the common theme of the issues discussed, unifies the thesis into a coherent whole.

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NOTATION CONVENTIONS

Throughout this thesis, unless otherwise stated, a tilde, " ~ ", denotes the steady state equilibrium value of a variable, and a bar, " — ", denotes an exogenously fixed variable.

Frequently when variables are functions of time, the arguments will be omitted. Also time derivatives will be denoted by dots about the variable concerned. Thus $\dot{x} = \frac{dx}{dt}$.

Let $f = f(x_1, x_2, \ldots, x_n)$ denote a function of n variables. If the jth partial derivative is positive then the following notations will be used interchangeably:

$$f(x_1, x_2, \ldots, \underset{+}{x_j}, \ldots, x_n)$$

$$f_j > 0$$

$$f_{x_j} > 0$$

$$\frac{\partial f}{\partial x_j} > 0$$

A similar convention (with the signs reversed) will apply when the jth partial derivative is negative.

Also we shall let $f_{jk} = \frac{\partial^2 f}{\partial x_j \partial x_k}$.

The notation $g = g(t_0, y_1, y_2, \ldots, y_n)$ where the y_j's are all functions of time will denote that each of the y_j's are evaluated at time, t_0.

CHAPTER 1

INTRODUCTION AND OVERVIEW

From the massive hyperinflations in Central and Eastern Europe after World War I to the world-wide bouts of inflation that followed the oil-price shocks in the 1970's, from the misery that resulted during the Great Depression in the 1930's to the high levels of unemployment that have accompanied the recent world-wide recession, unemployment and inflation have been and probably always will be two of the major concerns of macroeconomics.

This thesis is an attempt to consider in some detail a few of the important theoretical and policy issues associated with unemployment and inflation using a fairly standard framework which many economists would accept as plausible.

A central part of any modern approach to macroeconomic theory must be the mechanism by which expectations, and in particular inflationary expectations, are formed. Accordingly, a significant part of this study deals with the mechanism of expectations formation.[1] Throughout the thesis there is a gradual development in the approach to inflationary expectations.

Chapter 2 discusses the consequences for stability of the economy under the assumption that inflationary expectations are generated by one of the simplest expectations schemes, adaptive expectations. In simple models of the economy adaptive expectations are frequently associated with an economy that converges to equilibrium. We show that when the presence of unemployment is allowed for, stability (in the sense of convergence to an equilibrium) is no longer guaranteed under adaptive expectations, even in the context of a fairly standard model of the economy.

The adaptive expectations mechanism is essentially a mechanistic backward looking extrapolative rule. Thus it gives total emphasis to past values, a rather undesirable property when one is primarily concerned with what will happen in the future. Under adaptive expectations, systematic forecasting errors for many periods in succession are possible. In other words, it is assumed that the agents in the economy make predictable errors yet take no action to revise their rule for forming expectations. For these reasons, adaptive expectations have often been criticized as being ad-hoc and not derivable from any formal optimizing behaviour.

As an attempt to overcome these difficulties associated with adaptive expectations, an alternative expectations scheme, rational expectations, has commanded considerable attention in the past decade. This has been due to several factors. Firstly, it is based on a rather plausible optimizing principle: individuals should not make systematic mistakes in forecasting the future. Secondly, it takes account not just of the past but also of the general equilibrium or system-wide effects. In other words, it is assumed that economic agents make use of all the information that is available within the system. Thirdly, rational expectations is always associated with convergence to long-run equilibrium. The rational expectations hypothesis assumes that optimizing individuals do not make systematic forecasting errors in expectations formation, since if they do there will be an incentive to revise the expectations mechanism so as to eliminate the source of the systematic error. One way that economists can implement this hypothesis in their modeling is to assume that individuals "know" the systematic part of the model and use this to form expectations. These expectations are rational in the sense that when these expectations are fed back into the model the time-path of the economy will imply that there are no systematic forecasting errors which could have been discovered by individuals using information available at the time when expectations were formed. In a model which is characterized by the absence of even random forecasting errors (i.e., a deterministic model), rational expectations reduces to perfect foresight. If systematic influences are much more important than random influences, the assumption of perfect foresight captures much of the spirit of the rational expectations approach with as little complexity as possible.

Another expectations scheme which has many of the properties of perfect foresight and rational expectations is perfect myopic foresight, which occurs in deterministic models if current instantaneous expectations are forecast accurately.

Throughout this thesis, from Chapter 3 onwards we assume that expectations satisfy either perfect myopic foresight or perfect foresight. In dynamic macroeconomic models these assumptions are frequently accompanied by the property of saddlepoint instability. This property is illustrated in the Phase diagram of Figure 1.1. Unless the endogenous variables (x and y in the diagram) happen to lie on the stable arm given by AOA' the economy will diverge from the equilibrium point O. The real world is generally perceived as being relatively stable. Accordingly, at first the saddlepoint property was seen as an undesirable property for rational

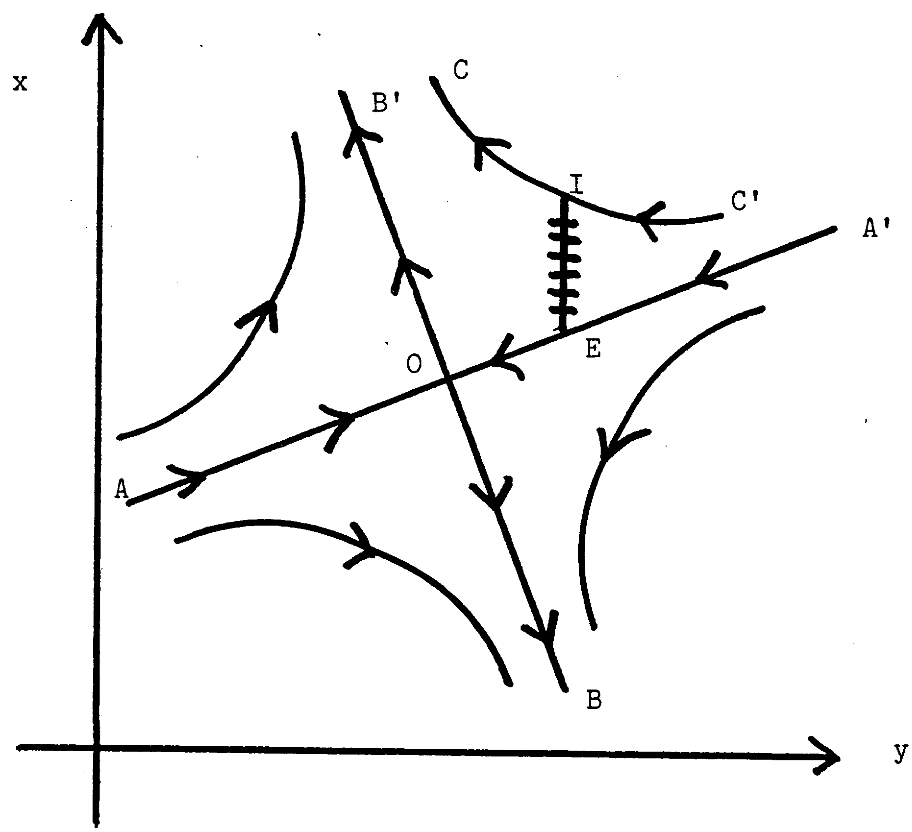


Figure 1.1

EXAMPLE OF SADDLE-POINT INSTABILITY

expectations models, because it was generally associated with instability. However, more recently, rational expectations theorists have argued that if expectations are derived from an optimizing framework then the economy will always converge to its long-run equilibrium. In this rational expectations approach to economic modeling, if the endogenous variables do not lie on the stable arm then decisions by economic agents will ensure that at least one of these variables (i.e., x or y) will "jump", i.e., move instantaneously to a new value, so as to keep the economy on a stable path.

This can be illustrated in Figure 1.1 by the following example: If the economy is at the initial point I then without a jump in the endogenous variables the economy will follow the unstable arm CIC' until it eventually diverges. If a stabilizing jump in the endogenous variable, x, is permitted, then the economy will jump initially to the equilibrium point E and then follow the stable arm AOEA' to the steady-state equilibrium O.

Because of this rational expectations approach to economic modeling and provided sufficient endogenous variables are allowed to jump, rational expectations will lead the economy to always converge to a stable equilibrium. Also, as will be observed, rational expectations are essentially forward looking rather than simply forming forecasts of the future from what has happened in the past.

Although perfect foresight (rational expectations) has many desirable properties, this does not mean that it is without weakness. Obviously perfect foresight cannot be meant to be a literal description of the uncertainties associated with the real world in which we live. How far are we going to relax this assumption? How much information do agents in the economy acquire?

Usually, individuals' information about the world is acquired from linear econometric models. Yet, as demonstrated in Chapter 4, because of non-linearities, equilibria may not be unique and different equilibria might require different short-run responses so that the economy will converge to equilibrium. This raises the question of whether agents in the economy can acquire enough information to react in the manner suggested by the rational expectations approach.

The question of whether jumps in endogenous variables are appropriate responses for the economy is also raised in Part 2 (Chapters 5-7) where it is shown that if policy variables are not fixed but chosen optimally it is possible for the economy to converge to its steady-state equilibrium in an optimal manner without the requirement that endogenous variables jump. Thus it appears no longer clear when and by what formal mechanism endogenous variables will adjust under perfect foresight.

Many other questions have been raised about the appropriateness of the rational expectations assumption. Overall, however, it remains the best expectations scheme that economists have so far been able to develop. Accordingly, the expectations approach that is overwhelmingly emphasized in this thesis is the rational expectations approach. Thus, except in Chapter 2, whenever necessary to attain stability, the rational expectations approach to economic modeling is adopted. In particular, in Part 3 (Chapters 8-10), where we deal with some practical examples, stability is always attained by a short-run jump in the price level.

A central issue in macroeconomics has always been the question of what policy objectives are desirable for fighting unemployment and inflation.

In particular, it has long been held that there is a short-run trade-off between inflation and unemployment. In economic theory, this is embodied in the Phillips curve relationship. Thus when inflationary expectations are static (i.e., they do not change over time), the Phillips curve relationship shows that if government policy is able to reduce inflation in the short-run then unemployment will typically increase. In the past five years, the Fraser Government in Australia and the Thatcher Government in Britain both adopted policies which emphasized "fighting inflation first." The fact that these policies have typically been accompanied by increased unemployment could be interpreted as empirical support for the Phillips Curve short-run trade-off.

The new Hawke Labor Government in Australia "sought and received a very clear mandate from the Australian people . . . . to deal with inflation and unemployment simultaneously."[2] This heralds a new approach to unemployment and inflation in Australia and gives rise to the question of how a government which is concerned with eliminating both these problems simultaneously can best achieve its objectives.

If we can quantify the government's objectives this question lends itself to a formal solution. Thus, if government policy can be used to control inflation, we might formalize this problem in a static framework as follows: Choose inflation, p, so as to

minimize $$L = ap^2 + U^2 \tag{1}$$

subject to

$$p = -\alpha(U - \overline{U}) + \pi \tag{2a}$$

$$\pi = \overline{\pi} \tag{2b}$$

where

p = actual rate of inflation

U = rate of unemployment

$\overline{U}$ = natural rate of unemployment

π = expected rate of inflation

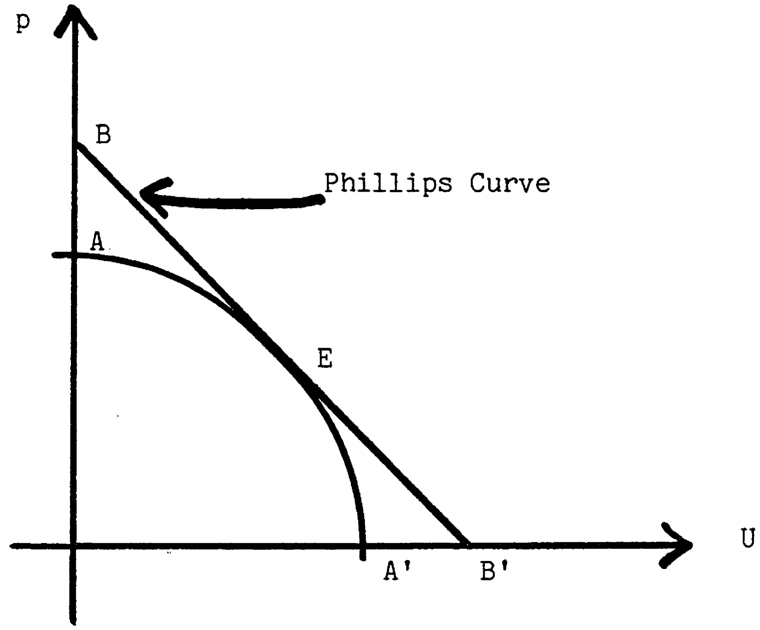
and a, α, $\overline{\pi}$, are fixed constants.

In the simple model above equation (1) is a quadratic loss function; thus as unemployment and inflation diverge from their desirable levels (of zero unemployment and zero inflation) then the loss function increases. Equation (2a) represents a simple Phillips curve with the rate of price inflation increasing with excess demand in the labor market and with the expected rate of inflation. Finally equation (2b) tells us that expectations are static.

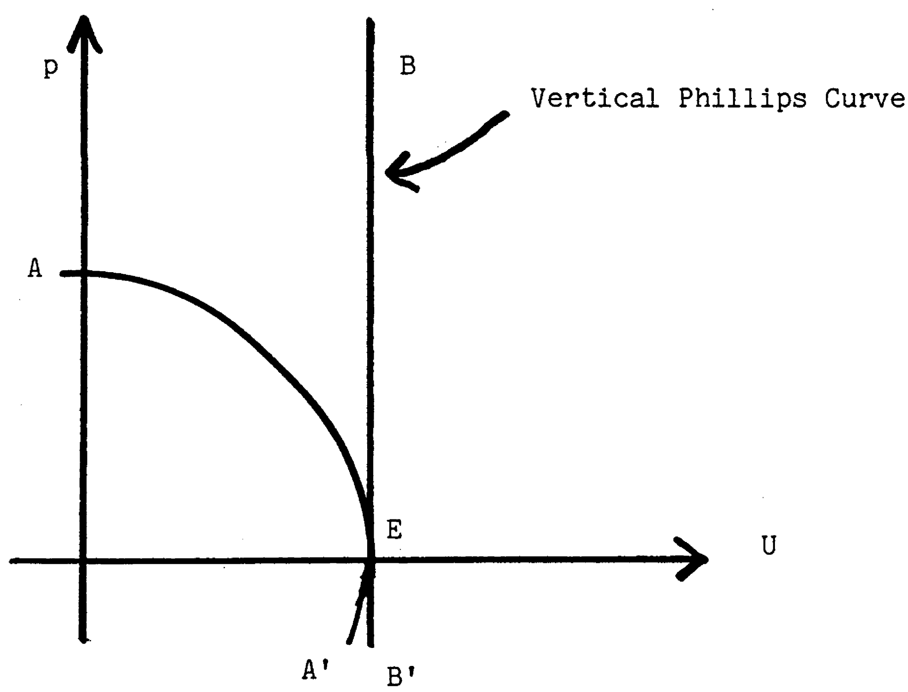
The solution to this problem is represented in Figure 1.2A. Thus the loss function (equation 1) represents a family of concave curves like that given by AEA', the Phillips curve (equations 2a, 2b) is given by BEB' and the optimal solution is given by the intersection of the Phillips curve with the loss function that is closest to the origin. This optimal solution is given in the diagram by E and is associated with a non-zero rate for both inflation and unemployment.[3]

Typically we might expect therefore that if the Australian government is going to fight both inflation and unemployment simultaneously, then they could have to accept possibly higher levels for both variables than they desire.

The approach we have outlined has assumed inflationary expectations are static. If expectations satisfy perfect myopic foresight (rational expectations), then equation (2b) is replaced by



1.2A: STATIC EXPECTATIONS



1.2B: PERFECT MYOPIC FORESIGHT

FIGURE 1.2: OPTIMAL UNEMPLOYMENT/INFLATION TRADE-OFF

$$\pi = p \qquad (2b')$$

and as a result there is no longer any trade-off between unemployment and inflation. Unemployment will always be at its natural rate and inflation will be governed by expectations.

In this case the government cannot control unemployment at all and the policy of fighting both unemployment and inflation simultaneously, has the same policy consequences as "fighting inflation first," i.e., the government must try to drive inflation to its desirable level. Thus it can be seen that the government's interpretation of its policy goals depends, to some extent, on how it perceives expectations behaviour is generated in the real world.

More formally, faced with the objective of fighting both unemployment and inflation simultaneously the government now faces a problem which is given by equations (1, 2a, 2b'). The solution to this problem is given in Figure 1.2B. The Phillips curve, given by BEB', is now vertical and the government's optimal policy decision involves driving inflation instantaneously to zero while unemployment remains at its natural rate.[4]

The preceding discussion indicates that, even in a static framework, the mechanism by which expectations are formed will affect the optimal policy followed by the government in fighting inflation and unemployment simultaneously. In Part 2, we formalize the policy maker's problem in a continuous framework with perfect foresight. The appropriate policy instruments are discussed. We also spend some time discussing whether it is desirable for prices, and hence for unemployment to jump and, if this is the case, what is the magnitude of the appropriate jump.

While the previous Australian Government was preoccupied with fighting inflation, the unemployment rate in Australia had continued to climb. In the last twelve months of the Fraser government, from March 1982 to March 1983, the unemployment rate increased from a seasonally adjusted 6.4% to 10.1%.[5] This was not necessarily a consequence of the government's economic policies but rather may have reflected a worldwide phenomenon that does not have a single cause.

Thus there was a sharp worldwide rise in unemployment following the first oil price shock in 1973. In such countries as Canada, Australia, and the United States, the 1970's had also seen a rising rate of labor force participation which had not been accompanied by a corresponding increase in labor demand thus increasing unemployment trends in these countries over the period.[6] Unemployment had also increased because of the economic consequences of the recent worldwide recession.

In Part 3, we discuss some policy issues which are particularly relevant in times of high unemployment. Two issues that are frequently raised, when unemployment becomes high, concern "easy" methods for lowering unemployment and ways of relieving the economic hardship of the unemployed.

One method of lowering short-run unemployment involves lowering the supply of labor. This can be achieved either by lowering the participation rate (e.g., early retirement, longer schooling) or by reducing the length of the standard working week. In Chapter 8, we examine the economic consequences of the second of these alternatives.

Frequently, efforts to lower unemployment by decreasing the length of the standard working week may be accompanied by workers' attempts to maintain the nominal weekly wage. This will increase the hourly cost of labor. As a result employers may insist on changed work rules in order to achieve productivity gains before they will allow the reduction in the standard working week. In Chapter 8, the consequences of a reduction in the length of the standard working week are examined when, firstly, there is no attempt to maintain the nominal weekly wage (i.e., the hourly nominal wage-rate remains constant) and, secondly, there is a successful attempt to maintain the nominal weekly wage (i.e., the hourly nominal wage-rate increases). The economic consequences of associated productivity gains are also considered.

A way of making life more acceptable to those who are unemployed is by paying higher unemployment benefits to those without work. It has been suggested that such a policy will actually increase long-run unemployment by raising the reservation wages of the unemployed. Hence raising the unemployment benefit will prolong job search and raise the natural rate of unemployment. Nevertheless, this option becomes more politically acceptable in times of high unemployment when it becomes more obvious to a large proportion of the population that those who are without work can do little to change their situation.

In times of high unemployment, increasing the unemployment benefit can increase the fiscal deficit by considerably more than it would in times when unemployment is lower. The full effect on the deficit will depend upon what percentage of the increased unemployment benefit is financed out of taxes and what percentage is not.

It can also be argued that the extent to which individuals in the economy foresee that higher unemployment benefits will have to be financed out of household taxes will influence the household's consumption decision. Thus if individuals foresee that they or their descendants will have to pay back the amount received in unemployment benefits in the form of taxes, then increased unemployment benefits will have little or no effect on consumption. This results since individuals will save more now to compensate for the taxes they or their descendants will have to pay in the future. On the other hand, if individuals believe that they will never have to pay higher taxes as a result of higher unemployment benefits, then an increase in unemployment benefits will lead to an increase in planned consumption.

The consequences for the household sector of whether or not an increase in unemployment benefits is financed out of taxes are examined in Chapter 9, while Chapter 10 examines the full macroeconomic impact of a change in unemployment benefits. This question is examined in a general equilibrium framework which takes account of the effects on unemployment, inflation, output and investment, both initially and in the longer term. The economic consequences of a change in retirement benefits are also examined.

An important part of this thesis is the way in which the issues mentioned above have all been analysed within a consistent framework of models. Specifically the models are rather closely related in that they have the following features in common. Firstly, all the models are deterministic models. Secondly, all models are expressed in continuous-time formulations. This emphasis in continuous-time models is total --

even micro-foundations of the models, when they are derived, are developed within a continuous-time structure. Thirdly, the models are all closed economy models which incorporate inflation, inflationary expectations and (except for Chapter 4) unemployment. Finally the models are dynamic models, thus allowing us to examine the short-run behavior, long-run behavior and intermediate stability properties of the economy.

The present study deals with but a few of the important issues associated with unemployment and inflation. It is believed, however, that the analysis could be extended, without too much difficulty beyond the questions discussed. In particular, similar techniques could be used to examine further related problems. The concluding chapter returns to this theme and provides a number of indications as to what directions future research might take.

PART 1

"THE ROLE OF EXPECTATIONS AND DEFICIT FINANCING"

CHAPTER 2

THE ROLE OF EXPECTATIONS AND THE EFFECTS OF POLICY CHANGES: CONTINUOUS ADJUSTMENT FROM A GIVEN INITIAL POINT

2.1 Introduction

In a recent paper, Turnovsky (1979) considered the dynamic behavior of a simple closed economy as well as the short-run and long-run responses under three alternative forms of passive monetary policy. Two expectations hypotheses -- the adaptive and perfect myopic foresight -- were also considered and their implications related at some length. In order to examine the dynamic properties of the economy, Turnovsky employed a model which consisted of an IS curve, an LM curve and a Phillips curve with the dynamics arising from the accumulation of wealth and the evolution of inflationary expectations. He found that while the forms of passive monetary policy share many common features, they do differ in many important respects. The three policies also illustrated how the effects of expansionary fiscal policies depend critically upon the form of passive monetary policy which is simultaneously in operation, as well as how expectations are formed.

In this chapter, the Turnovsky model is extended to allow for a labor market and the possibility of unemployment in the short-run. In the model developed here unemployment occurs in an essentially Keynesian manner because of the sluggish evolution of the real wage. As a result the short-run responses and the dynamic properties of this model under adaptive expectations differ significantly from the Turnovsky model. This result occurs because of the added complexity of wage dynamics, which are essentially destabilizing under adaptive expectations.

Specifically, we consider the case when all variables evolve continuously from a given initial condition and contrast the different dynamic properties of the model under different forms of expectations and under alternative forms of monetary policy. This analysis excludes the more forward looking analysis of the rational expectations literature (see Sargent and Wallace (1973), Gray and Turnovsky (1979)) in which the initial conditions are treated as endogenous in order to stabilize the model. This rational expectations approach to economy dynamics will be treated in subsequent chapters.

Two mechanisms for the formation of expectations will be considered. The two mechanisms which will be employed here are "adaptive expectations" and "perfect myopic foresight." Expectations derived from an adaptive hypothesis were first introduced by Cagan (1956) and Nerlove (1958). Such expectations are arbitrary in the sense that they have a systematic forecasting error associated with them. This has led to a preference in the more recent literature for expectations derived from the assumption of perfect myopic foresight, where the expected instantaneous rate of change of inflation equals its actual instantaneous rate of change. Interest in perfect myopic foresight also stems from the fact that it is, in a sense which will be made clear subsequently, a limiting case of adaptive expectations, and it may also be considered a deterministic equivalent of rational expectations.

It is a familiar conclusion from the literature that the mechanism for the formation of expectations will influence short-run effects of a policy change as well as the stability properties of the economy (see Tobin and Buiter (1976), Foley and Sidrauski (1971)). By contrasting the different dynamic properties of these expectations mechanisms, we are able to highlight the importance of expectations in the analysis of dynamic models such as those examined here.

Of special importance for the dynamics of the model is the mechanism by which deficit financing and inflationary expectations affect the time-path of the economy. Dynamics arise from the government budget constraint because the government expenditure on goods and services, less taxes, plus the interest owing on outstanding government debt, must be financed either by issuing more bonds or by issuing additional money. It has been shown that different choices of policy for financing the deficit give different results for the time-path and long-run equilibrium of the model (see Ott and Ott (1965), Christ (1968)). In this chapter we shall consider three alternative monetary policies -- these are the same policies as were discussed by Turnovsky (1979) and hence our introduction can be brief:

The three policies are:

(i) A Fixed Real Stock of Money Policy: i.e.,

$$m = \frac{M}{P} = \bar{m}$$

This is often referred to as an "accommodating" monetary policy -- it will be assumed that the government is able to immediately adjust the nominal money supply to compensate for any changes in the price level.

(ii) A Constant Rate of Monetary Growth Policy: i.e.,

$$\dot{M} = \mu M \quad \text{or} \quad \dot{m} = (\mu - p)m$$

This approach yields an equilibrium in which either the real money supply is zero or the rate of inflation is equal to the rate of monetary growth. Throughout this chapter we shall assume that the economy is in the neighborhood of the second of these equilibria.

(iii) A Fixed Real Stock of Bonds Policy: i.e.,

$$b = \frac{B}{P} = \bar{b}$$

This is a corresponding "accommodating" bond policy. Here we shall assume that the government is able to immediately adjust the supply of bonds to accommodate a jump in the price level.

2.2 A Dynamic Macroeconomic Model

The analysis in Chapters 2 and 3 will be based on the following macroeconomic model which is derived under the assumption that labor supply is greater than labor demand, i.e., $N^S > N^D$.

$$Y = D(Y^D, r-\pi, A) + G \tag{1a}$$

$$0 < D_1 < 1, \quad D_2 < 0, \quad D_3 > 0$$

$$Y^D = Y - T + rb - \pi A \tag{1b}$$

$$A = m + b \tag{1c}$$

$$m = L(Y, r, A) \tag{1d}$$

$$L_1 > 0, \quad L_2 < 0, \quad 0 \leqq L_3 \leqq 1$$

$$N^D = N^D(z), \quad N_1^D < 0 \tag{1e}$$

$$N^S = N^S(z), \quad N_1^S > 0 \tag{1f}$$

$$Y = f(N^D) = Y(z), \quad Y_1 < 0 \tag{1g}$$

$$U = \frac{N^S - N^D}{N^S} = U(z), \quad U_1 > 0 \tag{1h}$$

$$w = -\alpha U + \pi, \quad \alpha > 0 \tag{1i}$$

$$\dot{z} = (w - p) \tag{2a}$$

$$\dot{A} = G - T + rb - pA \tag{2b}$$

$$\begin{cases} m = \bar{m} & (3a) \\ \dot{m} = (\mu - p)m & (3b) \\ b = \bar{b} & (3c) \end{cases}$$

$$\begin{cases} \dot{\pi} = \gamma(p-\pi),\ \gamma > 0 & (4a) \\ \pi = p & (4b) \end{cases}$$

where Y = real output

Y^D = real private disposable income

D = real private expenditure

G = real government expenditure

T = real taxes

r = nominal rate of interest

π = expected rate of inflation

$r - \pi$ = real rate of interest

P = price level

$p = \frac{\dot{P}}{P}$ = actual rate of inflation

M = nominal money supply

$m = \frac{M}{P}$ = real money supply

L = real demand for money balances

W = nominal wage-rate

$z = \log\left(\frac{W}{P}\right)$ = log of real wage-rate

$w = \frac{\dot{W}}{W}$ = rate of nominal wage growth

B = nominal supply of bonds

$b = \frac{B}{P}$ = real supply of bonds

A = real private wealth

N^D = aggregate labor demand

N^S = aggregate labor supply

f = production function

U = rate of unemployment (assumed positive)

Equation (1a) is the product market equilibrium condition, in which real private demand increases with real private disposable income and real private wealth and decreases with the real rate of interest. Equation (1b) defines real private disposable income to be real factor income plus returns on government bonds, less expected capital losses on financial wealth (the expected inflation tax on real private wealth) and exogenous real taxes. Real private wealth is defined in equation (1c). Equilibrium in the money market is described by equation (1d) where the demand for real money balances depends upon the real level of income, the nominal rate of interest and real private wealth.

The supply of, and demand for, labor are given by equation (1e) and (1f). Both are dependent on the real wage (with the appropriate sign). Equations (1g) and (1h) describe the production function and the level of unemployment. The production function is written under the assumption that $N^D < N^S$.

Equation (1i) is a simple Phillips curve with the rate of wage growth increasing with excess demand in the labor market and with the expected rate of inflation. It is a standard version of the "expectations hypothesis" with the unitary coefficient on expectations reflecting the "accelerationist" view.

The dynamics of the system are described by equations (2), (3) and (4). Equation (2a) is the definition of the evolution of real wages while equation (2b) defines the evolution of real private wealth. This demonstrates how the time-path of nominal government debt P(G-T) and the rate of return on government bonds (rB) determine the evolution of nominal private wealth ($\dot{M} + \dot{B}$).[1]

The three alternative forms of monetary policy are given by equations (3a-3c). These are, respectively, a fixed real stock of money policy, a constant rate of monetary growth policy and a fixed real stock of bonds policy.

Two alternative policies for the formation of inflationary expectations are also given in equations (4a), (4b). These are, respectively, adaptive expectations and perfect myopic foresight.

The short-run model is given by equations (1a-1i). Following a shock to an exogenous variable (e.g., Government expenditure, G), variables which will change in the short-run include: Y, Y^D, A, r, N^D, N^S, U, w. However, with only these endogenous variables the model is underdetermined since the short-run model is described by 9 equations and we have listed only 8 endogenous variables.

Under perfect myopic foresight (PMF), it is natural to choose the expected rate of inflation, π, as the additional endogenous variable which closes the short-run model.

Under adaptive expectations, we shall assume that the price-level, P, is able to jump to clear the market. This will result in corresponding jumps in z, A, m, and b and lead to jumps in Y and U. If price expectations are of an adaptive form, the jump in the price level may also affect inflationary expectations. This jump in the price level may be considered as a once-for-all jump; then it would be valid to interpret the jump in the price level as having no short-run effect on inflationary expectations, i.e., $d\pi = 0$.[2] This situation will be referred to throughout this chapter as instantaneously static adaptive price expectations (ISAPE). On the other hand, if the jump in the price level is interpreted as the limiting case of a particularly high inflation of exceedingly brief duration then it would be valid to interpret the jump in the price level as affecting expectations, the relationship being given by $d\pi = \gamma\frac{dP}{P}$.[3] This will be referred to as instantaneous jump adaptive price expectations (IJAPE).

When there is a jump in the price level, P, it will be assumed that under the fixed real stock of money policy, i.e., $m = \bar{m}$ (the fixed real stock of bonds policy, i.e., $b = \bar{b}$) that the level of nominal money supply (nominal bond supply) immediately jumps to accommodate the change in the price level, so that $m = \bar{m}$ ($b = \bar{b}$) even in the short-run. This can be justified intuitively if we consider the short-run to be sufficiently long to allow the government to maintain its policies or if we consider that the government knows the new price level instantaneously. On the other hand for the fixed rate of monetary growth policy, any jump in the price level will be reflected in a short-run jump in the real money supply.

After the immediate short-run, the price level evolves continuously, as do the other endogenous variables and the dynamic variables given by z, m, b, π.

It will be noted that the model is essentially a short-run model which abstracts from the dynamics of physical capital accumulation. This has been done primarily to maintain the tractability of the analsis.

2.3 Fixed Real Stock of Money Policy

Let us consider the first form of monetary policy, specified by $m = \bar{m}$. In this case the short-run system under adaptive expectations can be reduced to a set of equations of the form:

$$[(1-D_1)Y_1 - D_A A]\frac{dP}{P} + D_r dr + D_\pi d\pi = D_1 r d\bar{m} - dG \tag{5a}$$

$$[L_1 Y_1 + L_3 A]\frac{dP}{P} - L_2 dr = - d\bar{m} \tag{5b}$$

$$\begin{cases} d\pi = \gamma\frac{dP}{P} \quad \text{(IJAPE)} & (6a) \\ d\pi = 0 \quad \text{(ISAPE)} & (6b) \end{cases}$$

It will be observed that the effect on aggregate demand, D, of a change in the interest rate is given by $D_r = D_1(A-\bar{m}) + D_2$. This effect is made up of a direct effect (substitution effect) and an income effect. Throughout this chapter we shall assume that the direct effect dominates. Hence:

$$D_r = D_1(A-\bar{m}) + D_2 < 0 \tag{7a}$$

Similarly, we shall assume that

$$D_\pi = - D_1 A - D_2 > 0 \tag{7b}$$

and

$$D_A = D_1(r-\pi) + D_3 > 0 \tag{7c}$$

The instantaneous effects on P, r and π of a change in fiscal and monetary policy under adaptive expectations are given in Tables 2.1A, 2.1B. It will be observed that while the short-run effects under the ISAPE assumption can, more easily, be signed unambiguously, the sign of the effects under the IJAPE assumption depend on the speed of price expectations adjustment, γ. This is exhibited most clearly by Figures 2.1A, 2.1B, 2.1C which show examples of how possible short-run comparative static effects are influenced by the size of γ. It will be observed that the polar cases, $\gamma \to 0$ and $\gamma \to \infty$, correspond respectively to the ISAPE assumption and the PMF assumption. The short-run comparative

TABLE 2.1: $m = \bar{m}$ POLICY

Table 2.1A

IJAPE ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN

On	G	$\bar{m}$
P	$-L_2\frac{P}{J}$	$\frac{1}{J}[D_1 r L_2 - D_r]P$
r	$\frac{1}{J}[-L_1Y_1-L_3A]$	$\frac{1}{J}[D_1 r(L_1Y_1+L_3A) - (-(1-D_1)Y_1+D_AA) + \gamma D_\pi]$
π	$-\gamma\frac{L_2}{J}$	$\frac{Y}{J}[D_1L_2r-D_r]$

where $J = L_2[(1-D_1)Y_1-D_AA] + D_r[L_1Y_1+L_3A] + \gamma D_\pi L_2$

Table 2.1B

ISAPE ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN

On	G	$\bar{m}$
P	$-L_2\frac{P}{J}$	$\frac{1}{J}[D_1 r L_2 - D_r]P$
r	$\frac{-L_1Y_1-L_3A}{J}$	$\frac{1}{J}[D_1 r(L_1Y_1+L_3A) - (-(1-D_1)Y_1 + D_AA)]$
π	0	0

where $J = L_2[(1-D_1)Y_1-D_AA] + D_r[L_1Y_1+L_3A]$

Table 2.1C

PMF ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN

On	G	$\bar{m}$
P	0	0
r	0	$\frac{1}{L_2} < 0$
π	$-\frac{1}{D_\pi} < 0$	$\frac{D_1 L_2 r - D_r}{D_\pi L_2}$

Table 2.1D

INTERMEDIATE-RUN EFFECTS OF CHANGES IN

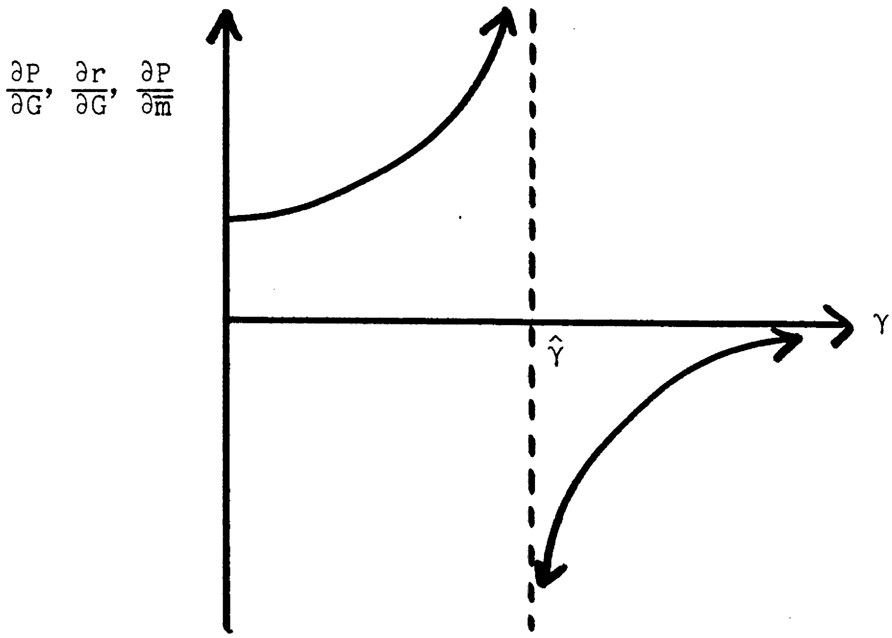
On	A	z
r	$-\frac{L_3}{L_2} > 0$	$-\frac{L_1 Y_1}{L_2} < 0$
π	$(\frac{\partial \pi}{\partial A})_{IR} = (\frac{\partial \dot{\pi}}{\partial \dot{A}})_{IR} = \frac{D_r L_3 - D_A L_2}{D_\pi L_2}$	$(\frac{\partial \pi}{\partial z})_{IR} = (\frac{\partial \dot{\pi}}{\partial \dot{z}})_{IR} = \frac{D_r L_1 Y_1 + L_2 Y_1 (1-D_1)}{D_\pi L_2} < 0$

Table 2.1E

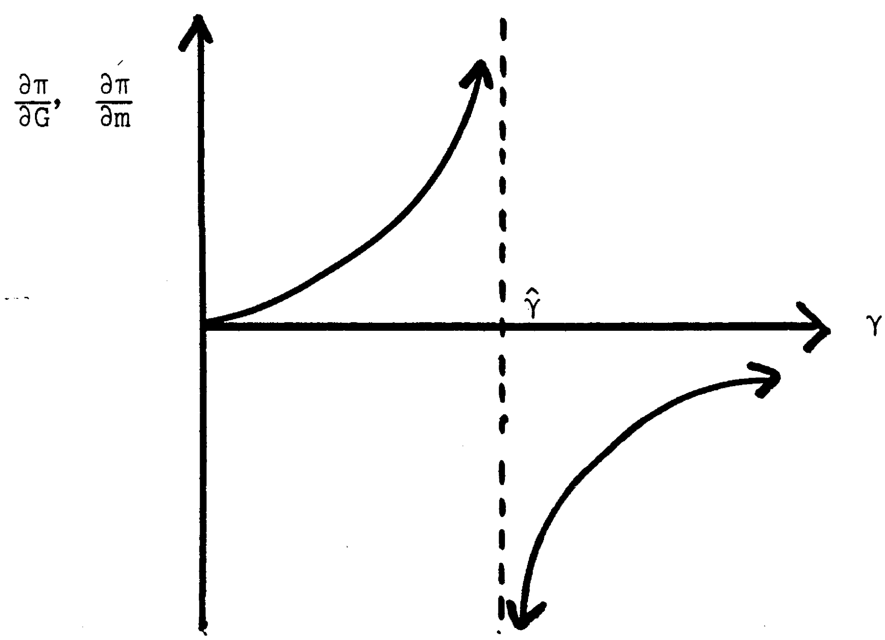
LONG-RUN EFFECTS OF A CHANGE IN

on	G	$\bar{m}$
r	$\frac{1}{J}[L_3D_2 - AL_3(1-D_1)]$	$\frac{1}{J}[D_2(r-\pi) - D_2L_3r - D_3A]$
π	$\frac{1}{J}[L_3(D_2-b(1-D_1)) + L_2((1-D_1)(r-\pi) - D_3)]$	$\frac{1}{J}[D_2(r-\pi) - D_2L_3r + D_3(L_2r-b)]$
A	$\frac{1}{J}[L_2A(1-D_1) - L_2D_2]$	$\frac{1}{J}D_2\bar{m}(1+e)$

where $J = (r-\pi)L_2D_2 + \bar{m}L_3D_2 - L_2D_3A$, $\quad e = \frac{L_2r}{m}$



2.1A

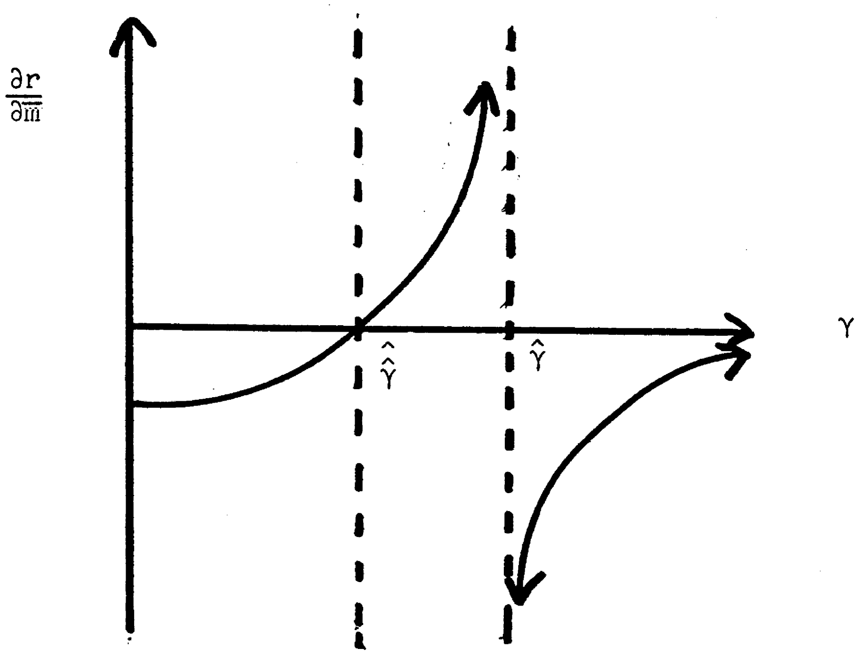


2.1B

Figure 2.1: POSSIBLE GRAPHS OF SHORT-RUN EFFECTS UNDER IJAPE ASSUMPTION

All graphs assume $D_1 r L_2 - D_r > 0$, $L_3 = 0$.

Hence, $$\hat{Y} = \frac{L_2[(1-D_1)Y_1 - D_A A] + D_r L_1 Y_1}{-D_\pi L_2}$$



2.1C

FIGURE 2.1: POSSIBLE GRAPHS OF SHORT-RUN EFFECTS UNDER IJAPE ASSUMPTION (continued)

All graphs assume $D_1 r L_2 - D_r > 0$, $L_3 = 0$. Hence

$$\hat{Y} = \frac{L_2[(1-D_1)Y_1 - D_A A] + D_r L_1 Y_1}{-D_\pi L_2}$$

$$\hat{\hat{Y}} = \frac{L_2[(1-D_1)Y_1 - D_A A] + L_2 D_1 r L_1 Y_1}{-D_\pi L_2}$$

and $\hat{\hat{Y}} < \hat{Y}$

statics under perfect myopic foresight can also be calculated by solving the following set of equations for changes in π and r:

$$D_r dr + D_\pi d\pi = D_1 r d\bar{m} - dG \tag{8a}$$

$$-L_2 dr = -d\bar{m} \tag{8b}$$

Of most interest from the point of view of this thesis is the effect of changes in fiscal and monetary policy on unemployment and the price level or inflation. Under the ISAPE assumption (if γ is close to zero and $D_r L_3$ relatively small) an increase in government expenditure will result in a short-run increase in the price level, leading to a corresponding decline in the real wage and hence to lowered unemployment. The effects on the price level of an increase in the supply of money are indeterminate. Under PMF, monetary and fiscal policy have no short-run effect on the price level or unemployment although an increase in government expenditure will lower inflation and an increase in the supply of money will have an ambiguous effect on inflation.

Proceeding from analysis of the immediate short-run we shall now examine the subsequent dynamics of the system. In order to achieve this we can first simplify the model of equations (1-4) by writing the equations in the following form:

$$Y(z) = D[Y(z) - T + (r-\pi)A - r\bar{m},\ r - \pi,\ A] + G \tag{9a}$$

$$\bar{m} = L[Y(z),\ r,\ A] \tag{9b}$$

$$\gamma\dot{z} + \dot{\pi} = -\alpha\gamma U(z) \tag{9c}$$

$$\gamma\dot{A} + \dot{\pi}A = \gamma[G - T + (r-\pi)A - r\bar{m}] \tag{9d}$$

Examination of equations 9 shows that the dynamical system which describes the economy is of the second order. We can approximate the dynamics of this system in the neighborhood of the steady-state by linearizing the system to obtain the linear approximation system.[4] The characteristic equation of this linear approximation system is of the form

$$H_2\lambda^2 + H_1\lambda + H_0 = 0 \tag{10a}$$

The Routh-Hurwitz conditions provide a sufficient condition for the local stability of the model. These conditions are satisfied if H_0, H_1, and H_2 all have the same sign. Provided the system is well-behaved in the neighborhood of the steady-state such conditions also provide a necessary condition for local stability.[5] Throughout the rest of this chapter, we shall assume that the coefficients of the system are such that the Routh-Hurwitz conditions always provide necessary and sufficient conditions for local stability.

By calculation, we can demonstrate that

$$H_2 = L_2[(1-D_1)Y_1 - D_A A] + D_r(L_1Y_1 + L_3A) + \gamma D_\pi L_2 \tag{10b}$$

If $|L_3D_r|$ is sufficiently small and γ is small, then H_2 is unambiguously positive. On the other hand when $\gamma \to \infty$, $H_2 < 0$. Since when $\gamma \to \infty$, the dynamics in the neighborhood of the steady state approach the dynamics under perfect myopic foresight, thus under PMF, $H_2 < 0$.

Similarly, we can show that

$$H_0 = \gamma\alpha U_1[D_rL_3A + D_\pi(-L_2(r-\pi) + L_3(A-\bar{m})) - D_AL_2A] \tag{10c}$$

In order for the model to be locally stable in the neighborhood of the steady-state, it is necessary that H_0 and H_2 have the same sign. Accordingly, given that $|L_3D_r|$ is sufficiently small, under adaptive expectations it is necessary that $H_0 > 0$, while under perfect myopic foresight it is necessary that $H_0 < 0$. In other words, provided $|L_3D_r|$

is sufficiently small, the model cannot be simultaneously stable under both adaptive expectations and perfect myopic foresight; this is a familiar result from the monetary growth literature and was also shown in the Turnovsky (1979) model. Also interesting is the fact that H_2 and H_0, as well as being the coefficients in the characteristic polynominal are also respectively the Jacobians of the short-run system (under the IJAPE assumption) and the long-run system. Hence if the system is to evolve continuously from a given initial point and if the system is stable, then the comparative statics of the long-run model may be of equal but opposite signs, depending on the mechanism by which expectations are formed and the magnitude of $|L_3D_r|$. In fact, as will be shown below, stability of the model of this chapter under adaptive expectations requires that $|L_3D_r|$ is sufficiently large that in general $H_2 < 0$. Hence the conditions for stability under adaptive expectations will be identical to the conditions for stability under perfect myopic foresight (i.e., $H_0 < 0$, $H_1 < 0$, $H_2 < 0$).

In order to examine more closely whether the economy is likely to be stable under either type of expectations mechanism, we can employ Table 2.1D, derived from equations (9a, 9b), which shows the intermediate run effects on r and π of changes in A and z and on $\dot{\pi}$ of changes in $\dot{A}$, $\dot{z}$. Thus after the initial short-run jump to equilibrium occurs, the change in short-run variables will lead to changes in the dynamic variables, $\dot{A}$, A, $\dot{z}$ and z; these will in turn lead to changes in r, π and $\dot{\pi}$, which will lead to further changes in $\dot{A}$, A, $\dot{z}$ and z. Such a procedure will continue until the system either converges to a new steady-state or diverges.

Assume that initially the economy is at steady-state equilibrium and that there is an increase in Government expenditure. Under adaptive expectations (ISAPE assumption) this could lead to an increase in the nominal interest rate, r, and an increase in the price level, P. The latter will lead to short-run falls in real private wealth, A, and the real wage z. Assuming the effect on A is sufficiently small, the result will be a rise in the budget deficit and thus $\dot{A}$ will increase. Using intermediate-run Table 2.1D, an increase in $\dot{A}$ will lead to effects on r and on π and $\dot{\pi}$. If $(\frac{\partial\pi}{\partial A})_{IR}$ is sufficiently large and negative then an increase in $\dot{A}$ will lead to a fall in π and $\dot{\pi}$ and hence will be destabilizing. Since, abstracting from the dynamics of z,

$$dp = (\frac{\partial\pi}{\partial A})_{IR}dA + \frac{1}{\gamma}(\frac{\partial\dot{\pi}}{\partial\dot{A}})_{IR}d\dot{A} \quad (11a)$$

and since $(\frac{\partial\pi}{\partial A})_{IR} = (\frac{\partial\dot{\pi}}{\partial\dot{A}})_{IR}$ thus when $(\frac{\partial\pi}{\partial A})_{IR}$ is sufficiently positive and γ is very small, a small increase in $\dot{A}$ will lead to a very large increase in the rate of inflation. This will lead to a fall in $\dot{A}$ and hence have a stabilizing influence on A.

Abstracting from the dynamics of wealth accumulation, we can now rewrite equation (9c) in the form

$$\gamma\dot{z} + (\frac{\partial\dot{\pi}}{\partial\dot{z}})_{IR}\dot{z} = -\alpha\gamma U(z) \quad (11b)$$

This system is stable if and only if $|(\frac{\partial\dot{\pi}}{\partial\dot{z}})_{IR}| < \gamma$.

Hence, provided $(\frac{\partial\pi}{\partial A})_{IR}$ is positive, γ is sufficiently small and $|(\frac{\partial\dot{\pi}}{\partial\dot{z}})_{IR}| < \gamma$ the economy will be stable. Such parameter configurations are certainly possible and thus stability is attainable.

We can examine this argument more formally by considering the determinants H_0, H_1 and H_2. The following heuristic conditions for stability have been derived

$$(\frac{\partial\pi}{\partial A})_{IR} > 0, \quad (\frac{\partial\dot{\pi}}{\partial z})_{IR} > -\gamma$$

Hence $$(\frac{\partial\pi}{\partial A})_{IR} A + (\frac{\partial\dot{\pi}}{\partial \dot{z}})_{IR} > -\gamma$$

and this implies $H_2 < 0$.

Similarly,

$$H_1 = -D_rL_1Y_1(r-\pi) - (1-D_1)Y_1L_2(r-\pi) + (\gamma+\alpha U_1)(D_rL_3A-D_AL_2A)$$
$$-D_AL_1Y_1\bar{b} + (1-D_1)Y_1L_3\bar{b} - D_\pi L_2\gamma(r-\pi) + D_\pi L_3b + \alpha U_1D_\pi L_2\gamma$$

and hence with $(\frac{\partial\pi}{\partial A})_{IR}$ sufficiently positive, $H_1 < 0$. Similarly

$$H_0 = \alpha U_1[AD_\pi L_2(\frac{\partial\pi}{\partial A})_{IR} + D_\pi(-L_2(r-\pi) + L_3(A-\bar{m}))]$$

and with $(\frac{\partial\pi}{\partial A})_{IR}$ sufficiently positive $H_0 < 0$. Hence H_2, H_1 and H_0 have the same sign and the necessary conditions for stability have been satisfied.

The stability requirements will be observed to differ from preconditions for stability in the Turnovsky (1979) model. This difference can be explained in terms of the incorporation of the real wage-rate, z, which is potentially destabilizing and also from the different specification of the Phillips curve which means that the dynamic path of inflation is affected differently in the intermediate-run.

Under perfect myopic foresight (PMF) and again starting at steady-state equilibrium, an increase in G will lead to a fall in π in the short-run; this will lead to an increase in $\dot{A}$ which will lead to an increase in r and an ambiguous effect on π. If $(\frac{\partial\pi}{\partial A})_{IR}$ is sufficiently negative, this will lead to a further fall in π and a further increase in $\dot{A}$ which will thus diverge. On the other hand, if $(\frac{\partial\pi}{\partial A})_{IR}$ is sufficiently positive this will mean that π increases, the deficit becomes balanced and $\dot{A}$ tends to decline, thus providing a stabilizing influence.

This relationship can be illustrated more formally by calculating the characteristic roots of the model under PMF. These characteristic roots are given by

$$\lambda_1 = -\alpha U_1 \quad < 0 \tag{12a}$$

$$\lambda_2 = \frac{D_\pi L_2(r-\pi) - D_r A L_3 + D_A L_2 A - D_\pi L_3(A-\bar{m})}{D_\pi L_2} \tag{12b}$$

i.e.,

$$\lambda_2 = -A(\frac{\partial\pi}{\partial A})_{IR} + \frac{D_\pi L_2(r-\pi) - D_\pi L_3(A-\bar{m})}{D_\pi L_2} \tag{12b'}$$

Hence, if $(\frac{\partial\pi}{\partial A})_{IR} < 0$, then $\lambda_2 > 0$ and we have exhibited the commonly observed property of saddlepoint instability. It will also be observed that if $(\frac{\partial\pi}{\partial A})_{IR}$ is sufficiently positive then $\lambda_2 < 0$ and the model is stable.

Provided the model is stable the economy will attain its long-run equilibrium. This equilibrium is described by the following set of equations

$$\bar{Y} - D(\bar{Y}-G,\ r-\pi,\ A) - G = 0 \tag{13a}$$

$$\bar{m} - L(\bar{Y},\ r,\ A) = 0 \tag{13b}$$

$$G - T + r(A-\bar{m}) - \pi A = 0 \tag{13c}$$

$$U(z) = 0 \tag{13d}$$

$$w = p = \pi \tag{13e}$$

where $\bar{Y}$ is the level of output fixed by choice of z so that $U(z) = 0$.

In the long-run, there is no unemployment whatsoever and changes in fiscal and monetary policy will have an ambiguous effect on inflation. Furthermore, as previously observed, if the model is stable under adaptive expectations the long-run comparative statics will be of the same sign as when the model is stable under perfect myopic foresight. Under the assumption that the model converges to its steady-state, the long-run effects are given in Table 2.1E.

2.4 Constant Rate of Nominal Monetary Growth Policy

We shall now consider the second form of monetary policy, where the government allows the nominal money supply to increase at a constant rate, μ. In order to examine the short-run effects of a change in G, we can reduce the short-run system under adaptive expectations to a set of equations of the form:

$$[(1-D_1)Y_1 - D_m m - D_b b]\frac{dP}{P} + D_r dr + D_\pi d\pi = -dG \tag{14a}$$

$$[L_1Y_1 - (1-L_3)m + L_3 b]\frac{dP}{P} - L_2 dr = 0 \tag{14b}$$

$$\begin{cases} d\pi = \gamma\frac{dP}{P} \quad \text{(IJAPE)} & (15a) \\ \\ d\pi = 0 \quad \text{(ISAPE)} & (15b) \end{cases}$$

Once again we assume that, in the effects on aggregate demand, the direct effect dominates over the income effect. Hence,

$$D_r = D_1 b + D_2 < 0 \tag{16a}$$

$$D_\pi = -D_1 A - D_2 > 0 \tag{16b}$$

$$D_m = -D_1\pi + D_3 > 0 \qquad (16c)$$

$$D_b = D_1(r-\pi) + D_3 > 0 \qquad (16d)$$

As revealed in Tables 2.2A, 2.2B, the short-run effects on P, r and π of a change in G are similar to those under the $m = \bar{m}$ policy. Once again, while the short-run effects under the ISAPE assumption can, more easily, be signed unambiguously, the sign of the effects under the IJAPE assumption depend on the speed of price expectations adjustment.

However, the instantaneous impact effects of an increase in the rate of monetary growth, μ, on such variables as the price level, P, the nominal rate of interest, r, and the expected rate of inflation, π, are all zero. This reflects the fact that it takes a finite time for a change in the growth rate of money to affect the level of money supply and hence to affect these variables. Instead an increase in μ affects the rates of change of the endogenous variables, i.e., $\dot{z}$, $\dot{r}$, $\dot{\pi}$, $\dot{m}$, and $\dot{b}$. These short-run effects are derived by rewriting equations (1-4) in the following form:

$$Y(z) = D[Y(z) - T + (r-\pi)b - \pi m,\ r-\pi,\ m+b] + G \qquad (17a)$$

$$m = L[Y(z),\ r,\ m+b] \qquad (17b)$$

$$\gamma\dot{z} + \dot{\pi} = -\alpha\gamma U(z) \qquad (17c)$$

$$\gamma\dot{m} + \dot{\pi}m = \gamma(\mu-\pi)m \qquad (17d)$$

$$\gamma\dot{b} + \dot{\pi}b = \gamma[G - T + (r-\pi)b - \mu m] \qquad (17e)$$

These short-run effects are given in Table 2.2D. As will be observed, under both the IJAPE assumption and the ISAPE assumption the sign and

TABLE 2.2: $\dot{m} = (\mu - p)m$ POLICY

Table 2.2A

IJAPE ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN G ON

P	$-\frac{L_2 P}{J}$
r	$-\frac{1}{J}[L_1 Y_1 - (1-L_3)m + L_3 b]$
π	$-\frac{\gamma L_2}{J}$

where $J = L_2[(1-D_1)Y_1 - D_m m - D_b b] + D_r[L_1 Y_1 - (1-L_3)m + L_3 b] + \gamma D_\pi L_2$

Table 2.2B

ISAPE ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN G ON

P	$-\frac{L_2 P}{J}$
r	$-\frac{1}{J}[L_1 Y_1 - (1-L_3)m + L_3 b]$
π	0

where $J = L_2[(1-D_1)Y_1 - D_m m - D_b b] + D_r[L_1 Y_1 - (1-L_3)m + L_3 b]$

Table 2.2C

PMF ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN G ON

P	0
r	0
π	$-\frac{1}{D_\pi} < 0$

Table 2.2D

ISAPE AND IJAPE ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN μ ON

$\dot{z}$	$-\frac{1}{J}\, m[D_1 r L_2 - D_r]$
$\dot{r}$	$\frac{m}{J}\,[AD_m L_3 - A(1-L_3)D_b - (1-D_1)Y_1 - D_1 r L_1 Y_1 - \gamma D_\pi]$
$\dot{\pi}$	$-\frac{\gamma}{J}\, m[D_1 r L_2 - D_r]$
$\dot{m}$	$\frac{m}{J}\,[-L_2(1-D_1)Y_1 + D_r(-L_1 Y_1) + D_r L_3 A + D_b L_2 A - \gamma D_\pi L_2]$
$\dot{b}$	$\frac{m}{J}\,[L_2(1-D_1)Y_1 + D_r L_1 Y_1 - D_r(1-L_3)A - D_m L_2 A + \gamma D_\pi L_2]$

where $J = -L_2(1-D_1)Y_1 - D_r L_1 Y_1 + m[D_r(1-L_3)+D_m L_2] - b[D_r L_3 - D_b L_2] - \gamma D_\pi L_2$

Table 2.2E

PMF ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN μ ON

$\dot{z}$	0
$\dot{r}$	$\frac{m}{L_2} < 0$
$\dot{\pi}$	$-m\left(\frac{D_r - D_1 r L_2}{D_\pi L_2}\right)$
$\dot{m}$	$m > 0$
$\dot{b}$	$-m < 0$

Table 2.2F

INTERMEDIATE-RUN EFFECTS OF CHANGES IN

on	m	b	z
r	$\frac{(1-L_3)}{L_2} < 0$	$-\frac{L_3}{L_2} > 0$	$-\frac{L_1 Y_1}{L_2} < 0$
π	$\left(\frac{\partial\pi}{\partial m}\right)_{IR} = \left(\frac{\partial\dot{\pi}}{\partial\dot{m}}\right)_{IR} = \frac{D_r(1-L_3) + D_m L_2}{-D_\pi L_2}$ < 0	$\left(\frac{\partial\pi}{\partial b}\right)_{IR} = \left(\frac{\partial\dot{\pi}}{\partial\dot{b}}\right)_{IR}$ $\frac{D_r L_3 - D_b L_2}{D_\pi L_2}$	$\left(\frac{\partial\pi}{\partial z}\right)_{IR} = \left(\frac{\partial\dot{\pi}}{\partial\dot{z}}\right)_{IR} = \frac{D_r L_1 Y_1 + L_2(1-D_1)Y_1}{D_\pi L_2}$ < 0

Table 2.2G

LONG-RUN EFFECTS OF A CHANGE IN

on	G	μ
r	$\frac{1}{J}[-(1-D_1)(r(1-L_3)-\mu) + D_3]$	$\frac{1}{J}[D_2(r-\mu) - D_2L_3r - D_3A]$
π	0	1
A	$\frac{[(b-L_2r)(1-D_1) - D_2]}{J}$	$\frac{D_2m(1+e)}{J}$

where $J = (r-\mu)D_2 + L_2D_3r - rD_2L_3 - bD_3$ and where $e = \frac{L_2r}{m}$

magnitude of the short-run effects on the rates of change of the endogenous variables will depend on the magnitude of γ.

The short-run effects of a change in G and μ under perfect myopic foresight are given in Tables 2.2C and 2.2E. It will be observed that these effects are the limit as $\gamma \to \infty$ under the IJAPE assumption. Alternatively, the comparative static effects can be obtained directly from the model. For changes in G this involves solving the system given by equations (8a, 8b). For changes in μ, the effects can be obtained by noticing that since unemployment, U, and the real wage-rate, z, are fixed under PMF thus $\frac{\partial \dot{z}}{\partial \mu} = 0$. For the variables r and π note that these variables are functions solely of the dynamic variables m, A and z. Thus $r = r(z,m,A)$ and

$$\dot{r} = \frac{\partial r}{\partial z}\dot{z} + \frac{\partial r}{\partial m}\dot{m} + \frac{\partial r}{\partial A}\dot{A} \tag{18a}$$

$$\frac{\partial \dot{r}}{\partial \mu} = \frac{\partial r}{\partial m}\frac{\partial \dot{m}}{\partial \mu} = m\left(\frac{\partial r}{\partial m}\right)_{m=\bar{m}} \tag{18b}$$

where $\left(\frac{\partial r}{\partial m}\right)_{m=\bar{m}}$ is as given in Table 2.1C.

In a similar fashion we can show that under perfect myopic foresight $\frac{\partial \dot{\pi}}{\partial \mu} = m\left(\frac{\partial \pi}{\partial m}\right)_{m=\bar{m}}$. Thus an increase in the rate of monetary growth, μ, will tend to lower the future rates of interest, by lowering $\dot{r}$, will increase the rate of monetary growth, $\dot{m}$, and lower the rate of bond growth, $\dot{b}$, while the effects on the future rates of inflation, i.e., on $\dot{\pi}$, will be ambiguous.

Proceeding from an analysis of the immediate short-run we now examine the subsequent dynamics of the system. The dynamical system given by equations (17a-17e) can be observed to be of the third order.

The economy described by this policy will be observed to have at least two steady-state equilibria; one associated with the long-run rate of inflation equal to the rate of monetary growth, i.e., $\mu=\pi$, and one associated with a long-run zero supply of real money, i.e., $m=0$. Since we shall examine the dynamics by linearizing in the neighborhood of the steady-state, it is important to first determine which steady-state is appropriate. Because the economy would collapse long before the real money supply, m, converged to zero and since therefore the model given by equations (1-4) would not describe the economy in the neighborhood of the steady-state, we shall restrict our analysis to the first of these steady-states, i.e., one characterized by $\mu=\pi$.

Approximating the dynamics by linearizing in the neighborhood of this steady-state, the characteristic equation is of the form

$$J_3\lambda^3 + J_2\lambda^2 + J_1\lambda + J_0 = 0$$

In order for the system to be stable, it is necessary that J_3 and J_0 have the same sign. Calculation from the linearized system reveals that:

$$J_3 = L_2[(1-D_1)Y_1 - D_m m - D_b b] + D_r[(L_1 Y_1 - (1-L_3)m + L_3 b] + \gamma D_\pi L_2 \qquad (19a)$$

Provided that the wealth elasticity of the demand for money, i.e., $\frac{L_3 A}{m}$, is less than 1, then $J_3 > 0$ for small γ (i.e., under adaptive expectations) and $J_3 < 0$ if the speed of expectations adjustment, γ, is sufficiently fast. In particular, therefore, under perfect myopic foresight, $J_3 < 0$.

Similarly we can show that

$$J_0 = -\gamma\alpha U_1 m[D_2(r-\mu) - D_2L_3r + D_3L_2r - bD_3] \qquad (19b)$$

J_3 is the determinant for changes in G of the short-run system while J_0 is the determinant of the long-run system. Thus in order for stability to prevail, if the system is to evolve continuously from a given initial point, the comparative statics of a long-run change in G will have different signs, depending on the speed of the expectations adjustment.

To examine under what conditions the economy is likely to be stable we employ Table 2.2F which shows the intermediate run effects on r and π of changes in m, b and z and on $\dot{\pi}$ of changes in $\dot{m}$, $\dot{b}$ and $\dot{z}$.

Assume that initially the economy is at the steady-state equilibrium characterized by $\mu = \pi$ and that there is an increase in Government expenditure. Similar arguments to those in Section 2.3 of this chapter will reveal that if $(\frac{\partial\pi}{\partial b})_{IR}$ is sufficiently positive, b will be stable provided m is stable. However, because $(\frac{\partial\pi}{\partial m})_{IR} = (\frac{\partial\dot{\pi}}{\partial\dot{m}})_{IR} < 0$, examination of equation (17d) reveals that m is most likely to be unstable. This result is consistent with that of Nguyen and Turnovsky (1979) who showed that for a simpler model but the same monetary policy the economy was stable in only 0.06% of cases.

Similar arguments show that, in general, under perfect myopic foresight this model is also unstable. This result is consistent with that of Turnovsky and Nguyen (1980) who for a simpler model but the same policy showed this policy was unstable in all cases examined.

Some feeling for the results under PMF can also be obtained by solving for the characteristic roots. These are given by

$$\lambda_1 = -\alpha U_1 < 0 \tag{20a}$$

and the two characteristic roots of the equation

$$-D_\pi L_2\lambda^2 + [m(D_r - L_2D_1r) - (L_2D_2(r-\mu) + mD_2L_3 - L_2D_3A)]\lambda$$
$$- m[(r-\mu)D_2 + L_2D_3r - rD_2L_3 - bL_3] \tag{20b}$$

In order for stability to prevail, all coefficients of (20b) must be of the same sign.

Note that if $(\frac{\partial\pi}{\partial b})_{IR}$ is large positive this implies that $D_2L_3 - D_3L_2$ is large and positive. This makes the constant term in (20b) more likely to be negative but the coefficient of λ more likely to be positive. In other words, in order for $\dot{b}$ in equation (17e) to be stable, we would like $(\frac{\partial\pi}{\partial b})_{IR}$ large and positive. However, in order for $\dot{m}$ in equation (17d) to be stable we would like $(\frac{\partial\pi}{\partial b})_{IR}$ to be large and negative to counteract the destabilizing effects of the negatively valued $(\frac{\partial\pi}{\partial m})_{IR}$. Overall, this conflict means that it is quite possible that very few combinations of parameters are consistent with stability.

Although stability is expected to be very rare, Table 2.2G gives the comparative static effects under all expectations mechanisms if stability does occur. In the more general case, when the model is unstable, the variables of a system evolving continuously from a given initial condition will diverge unboundedly or possibly begin to converge to the other steady-state equilibrium which is characterized by $m = 0$.

2.5 Fixed Real Stock of Bonds Policy

The third and final policy we shall consider is one where the real level of bonds is kept constant, i.e., $b = \bar{b}$. Under adaptive expectations, the short-run system can be reduced to a set of equations of the form:

$$[(1-D_1)Y_1 - D_A A]\frac{dP}{P} + D_r dr + D_\pi d\pi = -D_1 r d\bar{b} - dG \qquad (21a)$$

$$[L_1 Y_1 - (1-L_3)A]\frac{dP}{P} - L_2 dr = d\bar{b} \qquad (21b)$$

$$\begin{cases} d\pi = \gamma\frac{dP}{P} \quad \text{(IJAPE)} & (22a) \\ d\pi = 0 \quad \text{(ISAPE)} & (22b) \end{cases}$$

Assuming that, in the effects on aggregate demand, D, the direct effect dominates, we have

$$D_r = D_1\bar{b} + D_2 < 0 \qquad (23a)$$

$$D_\pi = -D_1 A - D_2 > 0 \qquad (23b)$$

$$D_A = -D_1\pi + D_3 > 0 \qquad (23c)$$

The instantaneous effects on P, r and π of a change in fiscal and monetary policy are given in Tables 2.3A, 2.3B. The effects of an increase in G are similar to those under the fixed real stock of money policy, while the effects of an increase in $\bar{b}$ are usually opposite in sign to an increase in $\bar{m}$. Thus under the ISAPE assumption an expansionary fiscal policy will result in a short-run increase in the price level, leading to a corresponding decline in the real wage and hence to lowered unemployment. The effects on the price level of lowering the level of real bonds is very similar to the effect of raising the level of real money. Meanwhile the sign and magnitude of short-run comparative static effects under the IJAPE assumption will depend on the speed of expectations adjustment, γ.

Since the short-run comparative statics under PMF, can be derived as the limiting case as $\gamma \to \infty$ of the comparative statics under the IJAPE

TABLE 2.3: $b = \bar{b}$ POLICY

Table 2.3A

IJAPE ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN

on	G	$\bar{b}$
P	$-\frac{L_2 P}{J}$	$-\frac{1}{J}[D_1 r L_2 - D_r]P$
r	$\frac{1}{J}[-L_1 Y_1 + (1-L_3)A]$	$-\frac{1}{J}[D_1 r(L_1 Y_1 - (1-L_3)A)$ $- (-(1-D_1)Y_1 + D_A A) + \gamma D_\pi]$
π	$-\frac{\gamma L_2}{J}$	$-\frac{\gamma}{J}[D_1 r L_2 - D_r]$

where $J = L_2[(1-D_1)Y_1 - D_A A] + D_r[L_1 Y_1 - (1-L_3)A] + \gamma D_\pi L_2$

Table 2.3B

ISAPE ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN

on	G	$\bar{b}$
P	$-\frac{L_2 P}{J} > 0$	$-\frac{1}{J}[D_1 r L_2 - D_r]P$
r	$\frac{1}{J}[-L_1 Y_1 + (1-L_3)A] > 0$	$-\frac{1}{J}[D_1 r(L_1 Y_1 - (1-L_3)A) -$ $(-(1-D_1)Y_1 + D_A A)] > 0$
π	0	0

where $J = L_2[(1-D_1)Y_1 - D_A A] + D_r[L_1 Y_1 - (1-L_3)A] > 0$

Table 2.3C

PMF ASSUMPTION
SHORT-RUN EFFECTS OF A CHANGE IN

on	G	$\bar{b}$
P	0	0
r	0	$-\frac{1}{L_2} > 0$
π	$-\frac{1}{D_\pi} < 0$	$\frac{D_r - D_1 L_2 r}{D_\pi L_2}$

Table 2.3D

INTERMEDIATE-RUN EFFECTS OF CHANGES IN

on	A	z
r	$\frac{1-L_3}{L_2} < 0$	$-\frac{L_1 Y_1}{L_2} < 0$
π	$(\frac{\partial \pi}{\partial A})_{IR} = (\frac{\partial \dot{\pi}}{\partial \dot{A}})_{IR} =$	$(\frac{\partial \pi}{\partial z})_{IR} = (\frac{\partial \dot{\pi}}{\partial \dot{z}})_{IR} =$
	$\frac{D_r(L_3-1) - D_A L_2}{D_\pi L_2} < 0$	$\frac{D_r L_1 Y_1 + L_2 Y_1 (1-D_1)}{D_\pi L_2} < 0$

Table 2.3E

LONG-RUN EFFECTS OF A CHANGE IN

on	G	$\bar{b}$
r	$\frac{1-L_3}{J}[A(1-D_1) - D_2]$	$\frac{1}{J}[-D_2(r-\pi) + D_2L_3r + D_3A]$
π	$\frac{1}{J}[(1-L_3)((1-D_1)\bar{b}-D_2)$ $- L_2(D_3+\pi(1-D_1))]$	$\frac{1}{J}[-D_2(r-\pi) + D_2L_3r$ $- D_3(L_2r-\bar{b})]$
A	$\frac{L_2}{J}[A(1-D_1) - D_2]$	$-\frac{D_2m(1+e)}{J}$

where $J = -\pi L_2D_2 - mD_2(1-L_3) - L_2D_3A$ and $e = \frac{L_2r}{m}$

assumption, the comparative static effects of an increase in G and a decrease in $\bar{b}$ under the fixed real stock of bonds policy will be the same as the comparative static effects of an increase in G and an increase in $\bar{m}$ under the fixed real stock of money policy. Under PMF, monetary and fiscal policy will have no short-run effect on the price level. However, an increase in government expenditure will lower inflation and a change in $\bar{b}$ will have an indeterminate effect on inflation.

In order to examine the dynamics of the system we can simplify the model to the following:

$$Y(z) = D[Y(z) - T + r\bar{b} - \pi A,\ r-\pi,\ A] + G \tag{24a}$$

$$A - \bar{b} = L[Y(z),\ r,\ A] \tag{24b}$$

$$\gamma\dot{z} + \dot{\pi} = -\alpha\gamma U(z) \tag{24c}$$

$$\gamma\dot{A} + \dot{\pi}A = \gamma[G - T + r\bar{b} - \pi A] \tag{24d}$$

The characteristic equation of the system is of the form

$$K_2\lambda^2 + K_1\lambda + K_0 = 0 \tag{25a}$$

and we are able to calculate the coefficients from the linear approximation system. In particular,

$$K_2 = L_2[(1-D_1)Y_1 - D_A A] + D_r[L_1Y_1 - (1-L_3)A] + \gamma D_\pi L_2 \tag{25b}$$

and

$$K_0 = \gamma\alpha U_1[-D_2L_2\pi - mD_2(1-L_3) - L_2D_3A] \tag{25c}$$

Note that these are respectively, the determinants of the short-run system (under IJAPE assumption) and the long-run system. When γ is small and $D_\pi L_2$ is small, $K_2 > 0$, while for γ large (including PMF), $K_2 < 0$.

Thus unless $|D_\pi L_2|$ is very large, it is impossible for the economy to be stable under both adaptive expectations and perfect myopic foresight. In fact, using the intermediate run effects of Table 2.3D, we can show by similar arguments to that of Section 2.3, that because $(\frac{\partial\pi}{\partial A})_{IR} < 0$ the economy will not be stable under adaptive expectations or perfect myopic foresight unless $(\frac{\partial\pi}{\partial A})_{IR} \to 0$, i.e., unless $|D_\pi L_2|$ is very large. Hence, in general, the same stability conditions will exist under both adaptive expectations and perfect myopic foresight (i.e., $K_2 < 0$, $K_1 < 0$, $K_0 < 0$).

This property can be exhibited more formally under the PMF assumption by noting that the characteristic roots are given by:

$$\lambda_1 = -\alpha U_1 < 0 \tag{26a}$$

$$\lambda_2 = \frac{D_2L_2\pi + mD_2(1-L_3) + L_2D_3A}{D_\pi L_2} \tag{26b}$$

Note that $\lambda_2 < 0$ only if $|D_\pi L_2|$ is sufficiently large. In general, therefore, this fixed real stock of bonds policy will be associated with saddlepoint instability under all mechanisms for the formation of expectations. This result is consistent with the results of Turnovsky and Nguyen (1980) who showed that for a simpler model under perfect myopic foresight a fixed real stock of bonds policy is always unstable for a large range of parameter values. However the results are different from those of Nguyen and Turnovsky (1979) who considered the simpler model under adaptive expectations. They showed that for the model the policy of pegging the real stock of bonds was stable in 96% of the cases considered. The different behavior in the model considered here is a result of the destabilizing influence of the incorporation of unemployment and the real wage under adaptive expectations. In the unlikely event that stability occurs, Table 2.3E gives the long-run comparative static effects. In the more general case where instability occurs and the economy is not on the stable arm, the endogenous variables in the economy will diverge.

2.6 Conclusion

In this chapter we have examined a dynamic macroeconomic model which specifically incorporates unemployment. This unemployment is derived in an essentially Keynesian manner from a sluggishly evolving real wage. The model also incorporates the dynamics of asset accumulation and expectations formation.

In particular, we consider the evolution of the economy under the specific assumption that the economy evolves continuously from a given initial condition. The possibility of initial stabilizing jumps, e.g., under rational expectations, will be considered in Chapter 3.

Unlike a simpler model by Turnovsky (1979) which abstracts from the dynamics of wage evolution, the results for the more complex model of this chapter indicate that the stability properties of the model differ very little according to the mechanism by which expectations are formed. This result occurs primarily because of the incorporation of real wage dynamics; as a result the expected rate of inflation, π, as well as its rate of change, $\dot{\pi}$, and hence the actual rate of inflation, p, are always determined by equilibrium in the goods and money markets, irrespective of the mechanism of expectations formation. Hence we obtain very similar conditions for stability under both adaptive expectations and perfect myopic foresight.

By contrast, the short-run comparative statics depend strongly on how expectations are formed instantaneously. We have considered three different mechanisms for the short-run formation of expectations and show how the sign and magnitude of the short-run comparative statics depend on these mechanisms.

Of the three methods of deficit financing that we have considered, the most stable is the fixed real stock of money policy. The fixed rate of monetary growth policy proves to be highly destabilizing while the fixed real stock of bonds policy is also likely to be unstable, although stability cannot be ruled out completely in any case. Under the perfect myopic foresight assumption, saddlepoint instability is quite common.

Overall, by comparing this analysis with that of Turnovsky (1979), we conclude that the effects of a policy change are very sensitive to the model specification. In particular, the incorporation of the real wage and unemployment tends to destabilize this model of the economy under adaptive expectations, but has little effect on the stability properties under perfect myopic foresight.

CHAPTER 3

SHORT-RUN RESPONSES UNDER PERFECT FORESIGHT WITH ENDOGENOUS INITIAL JUMPS

3.1 Introduction

In this chapter we examine the short-run effect of a policy change in a model which is characterized by perfect foresight in the formation of inflationary expectations. The models employed are perfect foresight versions of the models considered in Chapter 2.

Despite the fact that we are assuming perfect foresight in the formation of expectations we shall assume that the consequences of Lucas' (1976) critique are of no great significance. The Lucas critique states that we cannot use conventionally estimated macroeconometric models for evaluating policy changes, e.g., a change in government expenditure, G, because the parameters of the models will change as a result of policy changes. The Lucas critique has been incorporated into the specification of simple macroeconomic models, see McCafferty and Driskill (1980), but since the models of this chapter are essentially ad-hoc models which have not been derived from specific microfoundations, such an analysis is beyond the scope of this chapter.

Under perfect foresight, it is a common result that an economy will be characterized by one or more unstable roots. One procedure for stabilizing such a system has been suggested by Sargent and Wallace (1973) who allow the system to undergo some appropriate endogenously determined initial jump thereby in effect eliminating the unstable path. This procedure is commonly justified by an appeal to transversality conditions, e.g., as in Brock (1974, 1977) who obtains the results for models in which the underlying utility function is separable in real money balances and consumption and for which appropriate "super-Inada" type conditions hold. The extent to which it holds for more general utility functions is not known. Alternatively, the procedure can also be justified by an appeal to the recent empirical work of Flood and Garber (1980) who test the proposition that the market can launch itself into a price bubble with price being driven by arbitrary, self-fulfilling elements in expectations. They show that, for the German hyperinflation, they are unable to reject the proposition that bubbles were absent during this period. Finally, under some conditions all non-convergent paths eventually may be inconsistent with competition and hence are dynamically inefficient in the Samuelson sense (see Burmeister, Caton, Dobell and Ross (1973), Samuelson (1967), Shell and Stiglitz (1967)).

Such arguments as the above are not without their critics. Burmeister (1980) has shown that the arguments that require the use of transversality conditions rely on the assumption of a non-changing world with an infinite horizon and such an assumption is not valid in the finite and ever-changing world in which we live. Also more recent work by Flood, Garber and Scott (1981) has cast some doubts on the earlier empirical results. Finally in the concluding section of his important survey of the rational expectations literature Shiller (1978) discusses a couple of additional problems for the application of rational expectations models. First, they assume individuals possess more knowledge than they could possibly have considering their limited knowledge about the current economic structure and second, it may be unrealistic to suppose that an economy could converge to a rational expectations equilibrium in a reasonable amount of time.

Having noted the doubts that have been expressed in the Sargent and Wallace (1973) methodology we shall merely note that the justification for choosing a stable solution is far from complete but choose the most plausible assumption, i.e., that the economy does follow a stable path.

Even if we accept that the economy converges, it is still necessary to answer the question as to what is the mechanism for attaining a stable solution. Two of the broadest ways we can categorize the endogenous variables which jump to bring the economy onto a stable path (jump variables) is into those variables which are driven by market forces (e.g., the price level) and those variables which jump because the government chooses to intervene to stabilize the economy (e.g., an open market operation). The choice of appropriate "jump" variables is an unanswered question. In this chapter we shall examine in turn what happens when each type of jump variable is employed.

Calculation of the magnitude for the appropriate jump for an endogenous variable in the simplest two dimensional cases can be made via a graphical approach (see, e.g., Gray and Turnovsky (1979)). However very little has been written about the appropriate jumps in more complex models. In the discrete case when the models are described by difference equations, Blanchard and Kahn (1980) have provided an algorithm for solving for appropriate jumps together with conditions for existence and uniqueness of stable solutions. In the continuous case a corresponding theorem has been proved by Buiter (1982). In this chapter we prove three simple theorems which re-establish some of these results and in particular provide a mechanism for calculating the magnitude of appropriate jumps. We also provide some simple applications of the theorems and show how the results are comparable with those obtained under the more usual graphical approach.

The rest of this chapter will proceed as follows: Section 3.2 introduces the theorems. Section 3.3 considers application of the theorems to a perfect foresight model with one unstable eigenvalue while Section 3.4 considers application of the theorems to a perfect foresight model with both one and two unstable eigenvalues. Finally Section 3.5 makes some concluding comments.

3.2 Three Theorems

Assume a dynamical system is described by the following equations[1]

$$\dot{y} = Ay + Bx + c \tag{1a}$$

$$y(0) = Dz(0) + e \tag{1b}$$

$$\lim_{t\to\infty} \left|y(t)\right| < \infty \tag{1c}$$

where

y = (n x 1) vector of endogenous variables

x = (p x 1) vector of exogenous variables, assumed constant

A = (n x n) non-singular matrix[2]

B = (n x p) matrix

c = (n x 1) vector

z = (m x 1) vector of "jump" variables

D = (n x m) matrix of full-rank

e = (n x 1) vector of constants

and where y and z are both functions of time.

Equation (1a) defines the dynamics of the system. Equation (1b) defines the initial conditions for y -- these initial conditions can be changed in order to (say) stabilize the model by an appropriate choice of the jump variables z. Equation (1c) is a non-explosion condition which requires that the endogenous variables, y, remain bounded.

We shall say that a solution to the model exists if, given any matrix of exogenous variables x, we can choose a z(0) so that equations (1a-1c) are satisfied. As will be demonstrated below, depending on the number of jump variables it is possible that a unique solution to the model exists, no solution exists or an infinite number of solutions exist.

Following a similar argument to that of Blanchard and Kahn (1980), A can be transformed into Jordan Canonical Form as follows (see Halmos (1958))

$$A = H^{-1}JH \tag{2a}$$

where J is a diagonal matrix, the diagonal elements of which are eigenvalues of A. Assume that $\bar{m}$ of these eigenvalues are unstable then J can be further decomposed as

$$J = \begin{bmatrix} \underset{\bar{m}x\bar{m}}{J_1} & 0 \\ 0 & \underset{(n-\bar{m})x(n-\bar{m})}{J_2} \end{bmatrix} \tag{2b}$$

where J is partitioned so that all diagonal elements of J_1 have positive real part and all diagonal elements of J_2 have negative or zero real part.

Then writing $\tilde{y}(x) = -A^{-1}[Bx + c]$ we can rewrite equation (1a) in the form

$$\dot{y} = A[y - \tilde{y}(x)] \tag{3a}$$

$$\dot{y} = H^{-1}JH[y - \tilde{y}(x)] \tag{3b}$$

$$H\dot{y} = JH[y - \tilde{y}(x)] \tag{3c}$$

The solution to this differential equation will be a function of both time, t, and the exogenous variable, x, which we have assumed is fixed for all future time periods. Let us denote the solution to equation (3c) by $\Phi(t,x)$. Hence,

$$H[\Phi(t,x) - \tilde{y}(x)] = Ke^{Jt} \tag{3d}$$

where K is a diagonal matrix of constants and e^{Jt} is an (n x 1)-column matrix of exponential functions with exponents equal to the diagonal elements of J multiplied by t.

Let

$$H = \begin{bmatrix} \underset{\bar{m}xn}{H_1} \\ \\ \underset{(n-\bar{m})xn}{H_2} \end{bmatrix}, \quad K = \begin{bmatrix} \underset{\bar{m}x\bar{m}}{K_1} & 0 \\ \\ 0 & \underset{(n-\bar{m})x(n-\bar{m})}{K_2} \end{bmatrix}$$

In order for equation (1c) to be satisfied it is necessary that K_1 is a matrix with all zero entries, i.e., the initial conditions eliminate the unstable eigenvalues,[3] i.e.,

$$H_1[\Phi(0,x) - \tilde{y}(x)] = \underset{\sim}{0} \tag{4a}$$

where $\underset{\sim}{0}$ denotes an $\bar{m}$ x 1 column matrix, all of whose elements are zero.

Given a value of the exogenous variable x, which is expected to hold at all times in the future, equation (4a) describes the initial condition that $\Phi(t,x)$ must satisfy if equation (1c) is to be satisfied, i.e., if $\lim_{t\to\infty}|\Phi(t,x)| < \infty$.

If there is an unexpected change in the exogenous variable x to x + Δx, and if this new value for the exogenous variable is expected to hold at all time in the future, then the new solution which is given by $\Phi(t,x+\Delta x)$ must satisfy the initial condition

$$H_1[\Phi(0,x+\Delta x) - \tilde{y}(x+\Delta x)] = \underset{\sim}{0} \tag{4b}$$

Combining equations (4a, 4b)

$$H_1[\Phi(0,x+\Delta x) - \Phi(0,x)] = H_1[\tilde{y}(x+\Delta x) - \tilde{y}(x)] \tag{4c}$$

Since from equation (1b) the initial conditions for $\Phi(t,x)$ can only be changed through changes in z we can let $|\Delta x| \to 0$ and, using the chain rule, rewrite equation (4c) in the form:

$$H_1D\left[\frac{\partial z(0)}{\partial x}\right] = H_1\left[\frac{\partial \tilde{y}}{\partial x}\right] \quad (4d)$$

where $\left[\frac{\partial \tilde{y}}{\partial x}\right]$ denotes the long-run effect of a change in the exogenous variable, x, on the steady state value of the endogenous variable $\tilde{y}$, i.e., $\frac{\partial \tilde{y}}{\partial x} = -A^{-1}B$ and $\frac{\partial z(0)}{\partial x}$ denotes the short-run change in the jump variable z required to force the endogenous variable to continue to satisfy the non-explosion condition given by equation (1c). H_1D is an $\bar{m}$xm matrix and we shall assume this matrix is of full rank. We can now state the following theorems, which are all a direct consequence of the solution to equation (4d):

THEOREM 3.1: If $\bar{m} = m$, i.e., if the number of jump variables is equal to the number of unstable eigenvalues, then there exists a unique solution for the jump variables so that equations (1a-1c) will be satisfied. This solution is given by

$$\frac{\partial z(0)}{\partial x} = [H_1D]^{-1}H_1\left[\frac{d\tilde{y}}{dx}\right] = -[H_1D]^{-1}H_1[A^{-1}B] \quad (4e)$$

THEOREM 3.2: If $\bar{m} > m$, i.e., if the number of jump variables is less than the number of unstable eigenvalues, then no solution to equations (1a-1c) exists.

THEOREM 3.3: If $\bar{m} < m$, i.e., if the number of jump variables is greater than the number of unstable eigenvalues then an infinite number of solutions to equations (1a-1c) exist.

3.3 Fixed Real Stock of Money Policy

Under perfect foresight, the fixed real stock of money policy discussed in Chapter 2 is described by the following equations:

$$Y(z) = D[Y(z) - T + (r-\pi)A - r\bar{m},\ r - \pi,\ A] + G \tag{5a}$$

$$\bar{m} = L[Y(z),\ r,\ A] \tag{5b}$$

$$\dot{z} = -\alpha U(z) \tag{5c}$$

$$\dot{A} = G - T + (r-\pi)A - r\bar{m} \tag{5d}$$

These equations can be linearized about the steady-state to give a linear dynamical system of the form:

$$\begin{pmatrix} \dot{A} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} A - \tilde{A} \\ z - \tilde{z} \end{pmatrix}$$

where a tilde denotes the steady-state.

This model is characterized either by total stability or a saddle-point instability. Assuming that the parameters are such that a saddle-point instability results, the coefficients of equation (6a) are given by:

$$a_{11} = (r-\pi) + r_A(A-\bar{m}) - \pi_A A$$

$$= \frac{D_\pi L_2(r-\pi) - D_\pi L_3 b - (D_r L_3 - D_A L_2)A}{D_\pi L_2} > 0 \tag{6b}$$

$$a_{12} = r_z(A-\bar{m}) - \pi_z A$$

$$= \frac{-D_2 L_1 Y_1 \bar{m} - L_2 Y_1 (1-D_1)A}{D_\pi L_2} > 0 \tag{6c}$$

$$a_{21} = 0 \tag{6d}$$

$$a_{22} = -\alpha U_1 < 0 \tag{6e}$$

The eigenvalues of the system are given by $\lambda_1 = a_{11} > 0$, $\lambda_2 = a_{22} < 0$ and the system can be solved for explicit solutions for A and z given by:

$$\begin{bmatrix} A - \tilde{A} \\ \\ z - \tilde{z} \end{bmatrix} = \begin{bmatrix} 1 & a_{12} \\ \\ 0 & a_{22}-a_{11} \end{bmatrix} \begin{bmatrix} k_1 e^{a_{11} t} \\ \\ k_2 e^{a_{22} t} \end{bmatrix} \tag{7a}$$

Hence

$$\frac{1}{(a_{11}-a_{22})} \begin{bmatrix} a_{11}-a_{22} & a_{12} \\ \\ 0 & -1 \end{bmatrix} \begin{bmatrix} A - \tilde{A} \\ \\ z - \tilde{z} \end{bmatrix} = \begin{bmatrix} k_1 e^{a_{11} t} \\ \\ k_2 e^{a_{22} t} \end{bmatrix} \tag{7b}$$

Assuming the economy remains on a stable path, the effects on the steady-state of an exogenous change in government expenditure, G, which is to be maintained at all time in the future is given by

$$\frac{\partial \tilde{A}}{\partial G} = \frac{[L_2 A(1-D_1) - L_2 D_2]}{-D_\pi L_2 a_{11}} < 0 \tag{8a}$$

$$\frac{\partial \tilde{z}}{\partial G} = 0 \tag{8b}$$

If we assume that the available jump variable is given by the price level, P, then we can linearize the initial conditions to give

$$\begin{bmatrix} A \\ \\ z \end{bmatrix} = \begin{bmatrix} -\frac{A_0}{P_0} \\ \\ -\frac{1}{P_0} \end{bmatrix} P + \begin{bmatrix} A_0 \\ \\ z_0 \end{bmatrix} \tag{9}$$

Then, we can use theorem 3.1 and equations (7b), (8a-b), (9) to derive the appropriate initial jump in P, which is given by:

$$\frac{\partial P}{\partial G} = \left(\frac{(a_{11}-a_{12})P_0}{-(a_{11}-a_{22})A_0 - a_{12}} \right) \frac{\partial \tilde{A}}{\partial G} > 0 \tag{10}$$

This result which has been derived algebraically can also be illustrated graphically as follows:

From equation (7b), the economy will be on the stable arm if $k_1=0$, i.e., if $(a_{11}-a_{12})(A-\tilde{A}) = -a_{12}(z-\tilde{z})$. Hence the slope of the stable arm is given by

$$\frac{\partial A}{\partial z} = \frac{-a_{12}}{a_{11}-a_{22}} < 0.$$

In Figure 3.1 this stable arm is initially given by the line X_0X_0'. Following a positive jump in the endogenous variable, G, the steady-state is moved to $\tilde{A}_1$ and the corresponding stable arm is given by X_1X_1'. If the economy starts at the equilibrium given by $\tilde{A}_0$ and if the only available jump variable is the price level, P, then the economy must move from the old equilibrium $\tilde{A}_0$ to the new stable arm X_1X_1' along a ray which has slope given by

$$\frac{\partial A}{\partial z} = \frac{A_0}{P_0}$$

(derived from equation (9)). This ray is shown in Figure 3.1 by Y_1Y_1'.

It can be observed from Figure 3.1 that following an increase in G an appropriate jump will be one which initially reduces A and z, i.e., the appropriate jump will be a rise in P. This result is the same as that suggested by equation (10). Once the stable arm has been reached the economy will then evolve along X_1X_1' until the new steady-state, $\tilde{A}_1$, is reached.

Since the steady state effects of a change in $\bar{m}$ are given by

$$\frac{\partial \tilde{A}}{\partial \bar{m}} = \frac{D_2\bar{m}(1+e)}{-D_\pi L_2 a_{11}} < 0 \qquad (11a)$$

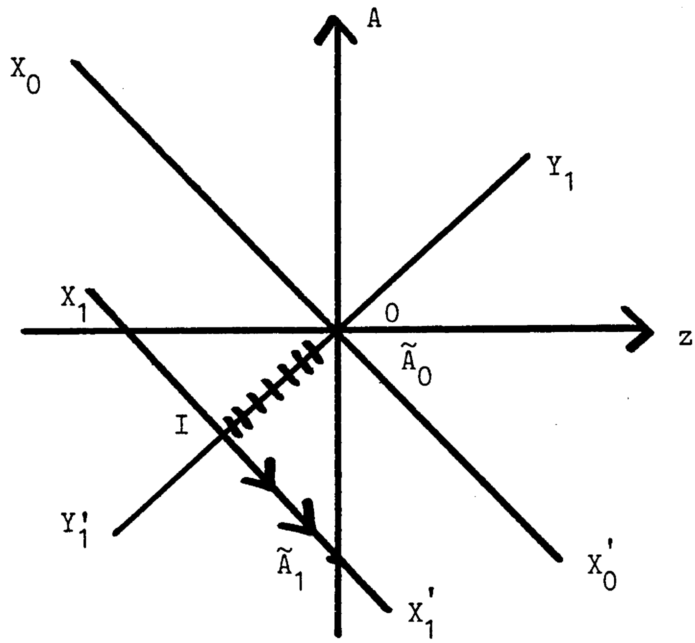


Figure 3.1

PERFECT FORESIGHT: SADDLEPOINT INSTABILITY WITH PRICE JUMPS; FIXED REAL STOCK OF MONEY POLICY

$$\frac{\partial P}{\partial G} > 0$$

$$\frac{\partial \tilde{z}}{\partial \bar{m}} = 0 \tag{11b}$$

where the sign of $\frac{\partial \tilde{A}}{\partial \bar{m}}$ assumes that $e = \frac{L_2 r}{m} > -1$.

Hence the appropriate short-run response of the price level, P, would be qualitatively the same under an increase in $\bar{m}$ as under an increase in G, although the magnitudes may differ.

3.4 Fixed Rate of Monetary Growth Policy

We can describe the fixed rate of monetary growth policy under perfect foresight (introduced in Chapter 2) by the following equations:

$$Y(z) = D[Y(z) - T + (r-\pi)b - \pi m,\ r - \pi,\ m + b] + G \tag{12a}$$

$$m = L[Y(z),\ r,\ m + b] \tag{12b}$$

$$\dot{z} = -\alpha U(z) \tag{12c}$$

$$\dot{m} = (\mu-\pi)m \tag{12d}$$

$$\dot{b} = G - T + (r-\pi)b - \mu m \tag{12e}$$

Once again, the equations can be linearized about the steady-state to give a linear dynamical system of the form:

$$\begin{pmatrix} \dot{m} \\ \dot{b} \\ \dot{z} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} m - \tilde{m} \\ b - \tilde{b} \\ z - \tilde{z} \end{pmatrix} \tag{13a}$$

where

$$a_{11} = -\pi_m m = \frac{[D_r(1-L_3) + D_m L_2]m}{D_\pi L_2} > 0 \tag{13b}$$

$$a_{12} = -\pi_b m = \frac{[D_b L_2 - D_r L_3]m}{D_\pi L_2} \gtrless 0 \tag{13c}$$

$$a_{13} = -\pi_z m$$

$$= \frac{-[D_r L_1 Y_1 + L_2(1-D_1)Y_1]m}{D_\pi L_2} > 0 \qquad (13d)$$

$$a_{21} = (r_m - \pi_m)b - \mu$$

$$= \frac{[-D_1 m(1-L_3) + D_m L_2] - \mu D_\pi L_2}{D_\pi L_2} \gtrless 0 \qquad (13e)$$

$$a_{22} = (r_b - \pi_b)b + (r-\pi)$$

$$= \frac{[D_1 m L_3 + D_b L_2]b + D_\pi L_2(r-\pi)}{D_\pi L_2} \gtrless 0 \qquad (13f)$$

$$a_{23} = (r_z - \pi_z)b$$

$$= \frac{[D_1 m L_1 Y_1 - L_2(1-D_1)Y_1]b}{D_\pi L_2} > 0 \qquad (13g)$$

$$a_{31} = 0 \qquad (13h)$$

$$a_{32} = 0 \qquad (13i)$$

$$a_{33} = -\alpha U_1 < 0 \qquad (13j)$$

The eigenvalues of the system satisfy the following equalities:

$$\lambda_1 + \lambda_2 = a_{11} + a_{22} \qquad (14a)$$

$$\lambda_1\lambda_2 = a_{11}a_{22} - a_{12}a_{21} \qquad (14b)$$

$$\lambda_3 = -\alpha U_1 < 0 \qquad (14c)$$

We shall consider two cases. Case 1 will be characterized by $a_{12} > 0$, $a_{21} > 0$, $a_{22} < 0$ (this can be obtained if μ is small, $\bar{m}$ large and L_3 positive but close to zero). In this case one of the eigenvalues is positive, i.e., $\lambda_1 > 0$, while the rest are negative.

Case 2 will be characterized by $a_{12} > 0$, $a_{21} < 0$, $a_{22} > 0$ (which can be obtained if μ is large positive and there are no wealth effects in the demand for money, i.e., $L_3 = 0$). In this case two of the eigenvalues have positive real parts. Either both these eigenvalues are real, i.e., $\lambda_1 > 0$, $\lambda_2 > 0$ or the eigenvalues are complex conjugates, i.e., $\lambda_1 = \eta + i\mu$, $\lambda_2 = \eta - i\mu$, where $\eta > 0$.

In both cases the solution to the dynamical system is given by:

$$\begin{pmatrix} m - \tilde{m} \\ b - \tilde{b} \\ z - \tilde{z} \end{pmatrix} = \begin{pmatrix} a_{12} & a_{12} & q_{13} \\ \lambda_1 - a_{11} & \lambda_2 - a_{11} & q_{23} \\ 0 & 0 & q_{33} \end{pmatrix} \begin{pmatrix} k_1 e^{\lambda_1 t} \\ k_2 e^{\lambda_2 t} \\ k_3 e^{\lambda_3 t} \end{pmatrix} \tag{15a}$$

where

$$q_{13} = a_{13}(a_{33}-a_{22}) + a_{23}a_{12} \tag{15b}$$

$$q_{23} = a_{13}a_{21} + a_{23}(a_{33}-a_{11}) \tag{15c}$$

$$q_{33} = -a_{21}a_{12} + (a_{33}-a_{11})(a_{33}-a_{22}) \tag{15d}$$

and this dynamical system can be rewritten in the form:[4]

$$\frac{1}{a_{12}(\lambda_2-\lambda_1)} \begin{pmatrix} \lambda_2 - a_{11} & -a_{12} & h_{13} \\ -(\lambda_1 - a_{11}) & a_{12} & h_{23} \\ 0 & 0 & h_{33} \end{pmatrix} \begin{pmatrix} m - \tilde{m} \\ b - \tilde{b} \\ z - \tilde{z} \end{pmatrix}$$

$$= \begin{pmatrix} k_1 e^{\lambda_1 t} \\ k_2 e^{\lambda_2 t} \\ k_3 e^{\lambda_3 t} \end{pmatrix} \tag{16a}$$

where

$$h_{13} = \frac{a_{12}q_{23} - (\lambda_2 - a_{11})q_{13}}{q_{33}} \tag{16b}$$

$$h_{23} = \frac{(\lambda_1 - a_{11})q_{13} - a_{12}q_{23}}{q_{33}}. \tag{16c}$$

$$h_{33} = \frac{a_{12}(\lambda_2 - \lambda_1)}{q_{33}} \tag{16d}$$

The effects on the steady-state of an exogenous change in Government expenditure, G, which is to be maintained at all time in the future is given by

$$\frac{\partial \tilde{m}}{\partial G} = \frac{[-D_2L_3 + D_3L_2 + (1-D_1)(-L_2(r-\mu)+L_3b)]\tilde{m}}{D_\pi L_2(a_{11}a_{22} - a_{12}a_{21})} \tag{17a}$$

$$\frac{\partial \tilde{b}}{\partial G} = \frac{[-D_2(1-L_3) - D_3L_2 - (1-D_1)(\mu L_2 - b(1-L_3))]\tilde{m}}{D_\pi L_2(a_{11}a_{22} - a_{12}a_{21})} \tag{17b}$$

$$\frac{\partial \tilde{z}}{\partial G} = 0 \tag{17c}$$

The numerator of Equation (17b) is always positive while the numerator of Equation (17a) is positive provided D_3 is close to zero (which we shall assume throughout the rest of this chapter). Hence the steady-state effects, $\frac{\partial \tilde{m}}{\partial G}$ and $\frac{\partial \tilde{b}}{\partial G}$ have the opposite sign to $(a_{11}a_{22} - a_{12}a_{21})$, i.e., in case 1 which is the case of a saddlepoint instability, $\frac{\partial \tilde{m}}{\partial G} > 0$, $\frac{\partial \tilde{b}}{\partial G} > 0$. While in case 2, the case of two unstable eigenvalues, $\frac{\partial \tilde{m}}{\partial G} < 0$, $\frac{\partial \tilde{b}}{\partial G} < 0$.

In order to stabilize the model we shall allow either a jump in the price level, P, which will be assumed to be achieved by market forces which act in a way as to stabilize the economy, or an open market operation, which assumes that the government deliberately acts to keep the economy on a stable path. In the case of one unstable eigenvalue only one of these forces will need to come into play in order to stabilize the economy. If both forces come into effect, Theorem 3.3 shows that there are an infinite number of combinations of jumps which will keep the economy stable.

In the case of two unstable eigenvalues it will be necessary for both forces to be at work in order to stabilize the economy. If only one variable jumps initially (or if no variables jump initially), Theorem 3.2 shows that it will be impossible to stabilize the economy.

The initial conditions corresponding to a jump in P and an open market operation (constrained jumps in m and b) are given by Equations (18a) and (18b) respectively:

$$\begin{pmatrix} m \\ b \\ z \end{pmatrix} = \begin{pmatrix} -\frac{M_0}{P_0^2} \\ -\frac{B_0}{P_0^2} \\ -\frac{1}{P_0} \end{pmatrix} P + \begin{pmatrix} m_0 \\ b_0 \\ z_0 \end{pmatrix} \qquad (18a)$$

$$\begin{pmatrix} m \\ b \\ z \end{pmatrix} = \begin{pmatrix} +1 \\ -1 \\ 0 \end{pmatrix} (m-m_0) + \begin{pmatrix} m_0 \\ b_0 \\ z_0 \end{pmatrix} \qquad (18b)$$

Given Equations (12-18) we are now in a position to analyze more closely the appropriate jumps needed to stabilize the economy.

3.4.1 Case 1: One Unstable Eigenvalue

If there is one unstable eigenvalue and if the appropriate stabilizing policy is an open market operation then the appropriate initial jump in m is given by

$$\frac{\partial m}{\partial G} = \frac{\left[(\lambda_2 - a_{11})\frac{\partial \tilde{m}}{\partial G} - a_{12}\frac{\partial \tilde{b}}{\partial G}\right]}{(\lambda_2 - a_{11} + a_{12})} \gtrless 0 \tag{19}$$

Hence $\frac{\partial m}{\partial G} \gtrless 0$ according as $\lambda_2 - a_{11} + a_{12} \lessgtr 0$. This result is illustrated graphically in Figures 3.2 and 3.3. In these figures, the slope of the stable arm is given by $\frac{\partial m}{\partial b} = \frac{a_{12}}{\lambda_2 - a_{11}} < 0$. Initially this stable arm is given by X_0X_0'. Following an increase in Government expenditure, the equilibrium moves to E and the new stable arm is given by X_1X_1'. If the only mechanism for stabilizing the economy is an open market operation and if the economy is initially at the equilibrium given by 0, then the economy must move to the new stable arm along the ray Y_1Y_1' which has a slope of -1.

Figure 3.2 illustrates the case when the slope of X_1X_1' is greater than the slope of Y_1Y_1', i.e., $\frac{a_{12}}{\lambda_2 - a_{11}} > -1$. It will be observed that the appropriate response is an increase in m with a compensating decrease in b. Since $\frac{a_{12}}{\lambda_2 - a_{11}} > -1$ implies $a_{12} + \lambda_2 - a_{11} < 0$, this result is equivalent to that suggested by Equation (19). Once the economy has reached its short-run equilibrium (denoted by I) it will evolve slowly along the stable arm until it reaches its new equilibrium.

Figure 3.3 illustrates the case when $\frac{a_{12}}{\lambda_2 - a_{11}} > -1$. This results in a short-run fall in m, which is also equivalent to the impact effect suggested by Equation 19.

It is also interesting to note that in Figure 3.2, the initial jump in m overshoots the steady state equilibrium value of $\tilde{m}$. This is in a sense a closed economy analogue of the Dornbusch (1976) overshooting of

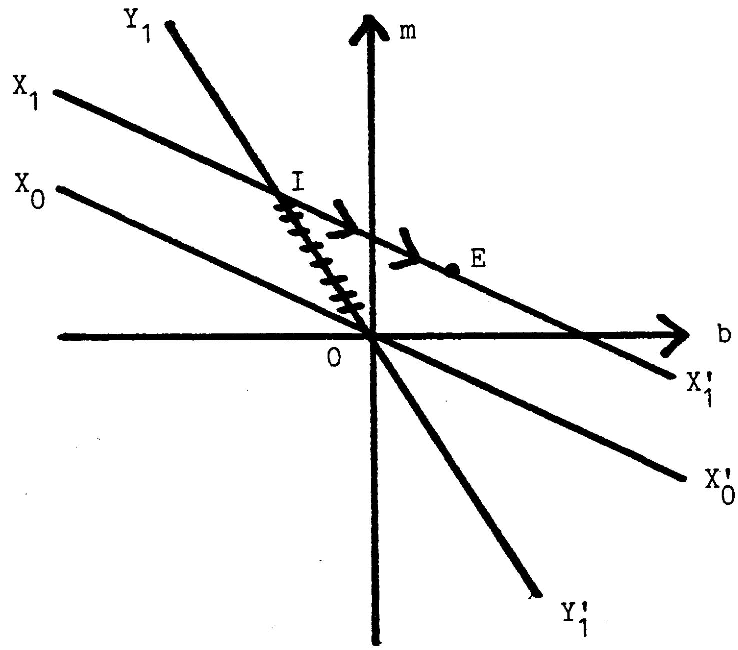


Figure 3.2

PERFECT FORESIGHT: SADDLEPOINT INSTABILITY WITH OPEN MARKET OPERATION; FIXED RATE OF MONETARY GROWTH POLICY

$$\frac{\partial m}{\partial G} > 0$$

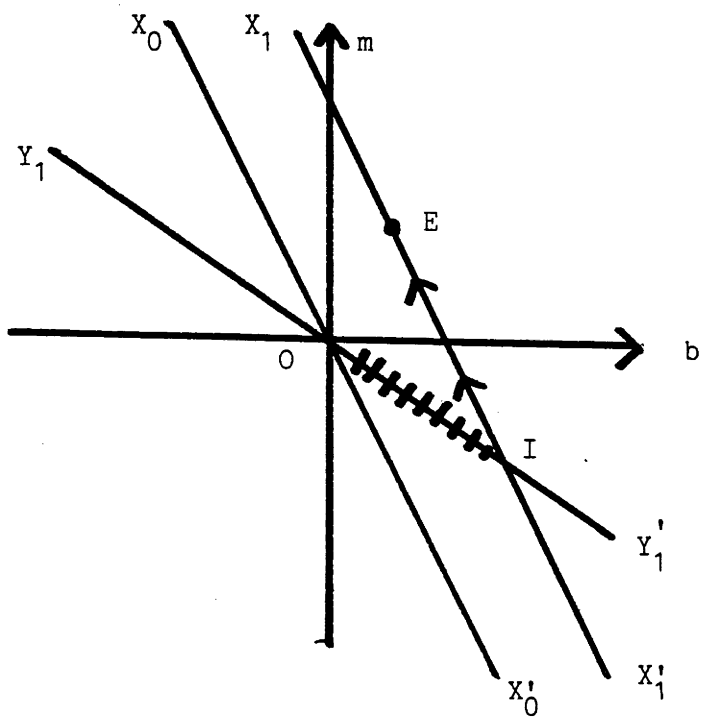


Figure 3.3

PERFECT FORESIGHT: SADDLEPOINT INSTABILITY WITH OPEN MARKET OPERATION; FIXED RATE OF MONETARY GROWTH POLICY

$$\frac{\partial m}{\partial G} < 0$$

the exchange rate in an economy characterized by rational expectations.

If the appropriate stabilizing policy is a jump in the price level, rather than an open market operation, then the short-run jump is given by:

$$\frac{\partial P}{\partial G} = \frac{-\left[(\lambda_2 - a_{11})\frac{\partial \tilde{m}}{\partial G} - a_{12}\frac{\partial \tilde{b}}{\partial G}\right]}{\left[(\lambda_2 - a_{11})\frac{M_0}{P_0^2} - a_{12}\frac{B_0}{P_0^2} + h_{13}\frac{1}{P_0}\right]} \gtrless 0 \qquad (20)$$

Hence $\frac{\partial P}{\partial G} \gtrless 0$ according as

$$(\lambda_2 - a_{11})\frac{M_0}{P_0^2} - a_{12}\frac{B_0}{P_0^2} + h_{13}\frac{1}{P_0} \gtrless 0$$

The magnitude of the short-run effect can thus be calculated provided the parameter values of the model are known. However, the results cannot be exhibited graphically because this would require a three-dimensional diagram. Hence, Theorem 3.1 provides a mechanism for calculating the short-run effects which is more general than the graphical approach.

3.4.2 Case 2: Two Unstable Eigenvalues

If there are two unstable eigenvalues, in order to stabilize the economy we require two variables to jump, e.g., the price level and an open market operation. Application of Theorem 3.1 gives as the appropriate jumps:

$$\begin{bmatrix} \frac{\partial P}{\partial G} \\ \frac{\partial m}{\partial G} \end{bmatrix} = \left\{ \begin{bmatrix} \lambda_2 - a_{11} & -a_{12} & h_{13} \\ -(\lambda_2 - a_{11}) & a_{12} & h_{23} \end{bmatrix} \begin{bmatrix} -\frac{M_0}{P_0^2} & 1 \\ -\frac{B_0}{P_0^2} & -1 \\ -\frac{1}{P_0} & 0 \end{bmatrix} \right\}^{-1} \begin{bmatrix} (\lambda_2 - a_{11})\frac{\partial \tilde{m}}{\partial G} - a_{12}\frac{\partial \tilde{b}}{\partial G} \\ -(\lambda_1 - a_{11})\frac{\partial \tilde{m}}{\partial G} + a_{12}\frac{\partial \tilde{b}}{\partial G} \end{bmatrix} \qquad (21)$$

Equation (21) applies whether the eigenvalues are real-valued or complex-valued. (If $\lambda_1=\overline{\lambda}_2$, then $h_{13}=-\overline{h}_{23}$, and the appropriate jumps, $\frac{\partial P}{\partial G}$ and $\frac{\partial m}{\partial G}$, are still real-valued). In either case, if we know the parameter values of the model, we can calculate the required jumps.

We can also simplify the model by abstracting from the dynamics of the real wage; i.e., by removing Equation (12c) from the model and assuming Y is fixed at its full employment level given by $\overline{Y}$. Then the linearized version of the model is of the form:

$$\begin{bmatrix} \dot{m} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} m - \tilde{m} \\ b - \tilde{b} \end{bmatrix} \tag{22}$$

where the a_{ij}'s are as given in Equations (13b, 13c, 13e, 13f). Then if there are two jump variables, the number of jump variables will be equal to the number of unstable eigenvalues and to the number of dynamic variables. Then Theorem 3.1 shows that the approprate jumps can be given by

$$\begin{bmatrix} \frac{\partial P}{\partial G} \\ \frac{\partial m}{\partial G} \end{bmatrix} = [HD]^{-1}H \begin{bmatrix} \frac{\partial \tilde{m}}{\partial G} \\ \frac{\partial \tilde{b}}{\partial G} \end{bmatrix} \tag{23a}$$

But H, D are both non-singular 2x2 matrices. Hence,

$$\begin{bmatrix} \frac{\partial P}{\partial G} \\ \frac{\partial m}{\partial G} \end{bmatrix} = D^{-1} \begin{bmatrix} \frac{\partial \tilde{m}}{\partial G} \\ \frac{\partial \tilde{b}}{\partial G} \end{bmatrix} \tag{23b}$$

and so

$$\begin{bmatrix} \frac{\partial m(0)}{\partial G} \\ \frac{\partial b(0)}{\partial G} \end{bmatrix} = \begin{bmatrix} \frac{\partial \tilde{m}}{\partial G} \\ \frac{\partial \tilde{b}}{\partial G} \end{bmatrix} \tag{23c}$$

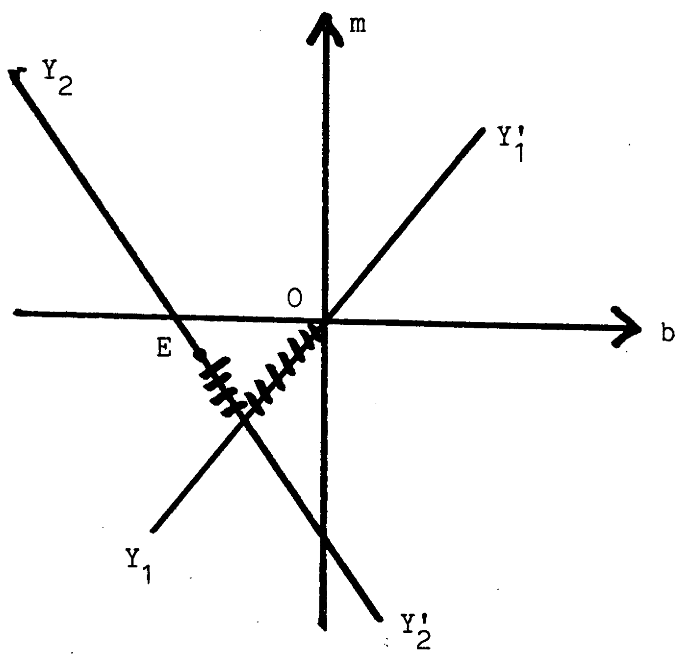


Figure 3.4

PERFECT FORESIGHT: TWO UNSTABLE EIGENVALUES, TWO JUMP VARIABLES AND TWO DYNAMIC VARIABLES -- THE CASE OF A JUMP FROM OLD EQUILIBRIUM TO NEW EQUILIBRIUM WITH NO INTERMEDIATE DYNAMIC PATH i.e., if an exogenous change in G results in a movement of the steady-state to a new equilibrium, the appropriate stabilizing jumps will be ones which move the initial values from the old equilibrium to the new equilibrium.

This result is revealed graphically in Figure 3.4. Because there are no stable eigenvalues there will be no stable arm and the only way the stable equilibrium can be reached is by movement along the ray associated with a jump in P (denoted by Y_1Y_1') which has a slope of $\frac{M_0}{B_0} > 0$ followed by a movement along the ray associated with an open market operation (denoted by Y_2Y_2') which has a slope of -1. Any jumps which do not bring the economy to the new equilibrium denoted by E will be unstable and not appropriate. Also as can be seen from the diagram, with movement along one ray (equivalent to only one jump variable being available), the new equilibrium cannot, in general, be reached and the resulting trajectory for the economy will be unstable (thus confirming the conclusions of Theorem 3.2).

3.5 Conclusion

In this chapter we have examined the results of a policy change when the economy stays on a convergent path. By comparison with Chapter 2 it will be observed that the short-run effects when the initial conditions for the endogenous variables are fixed can differ drastically from the short-run effects when the economy is forward looking and seeks to keep the economy on a stable path. Also the dynamic properties of the economy will also differ greatly.

By deriving a simple theorem along the lines of the more complex results of Blanchard and Kahn (1980) and Buiter (1982) we have established the familiar results that depending on the number of jump variables and the number of unstable eigenvalues, it is possible to have no solution which stabilizes the economy, a unique solution which stabilizes the economy or an infinite number of solutions which will stabilize the economy. Theorem 3.1 has also been used to calculate the magnitude of appropriate jumps in the endogenous variables in order to stabilize the economy. These calculated jumps have been shown to be equivalent to jumps obtained by a graphical approach.

CHAPTER 4

THE NON-UNIQUENESS OF EQUILIBRIA, STABILITY PROPERTIES AND SHORT-RUN RESPONSES UNDER PERFECT FORESIGHT

4.1 Introduction

In Chapters 2 and 3 of this thesis we have examined in some detail the stability properties of the economy when inflationary expectations satisfy perfect foresight and assuming various policies for financing the deficit. The stability of similar monetary models under perfect foresight (rational expectations) has been widely discussed in the recent macroeconomic literature. One of the familiar conclusions to emerge is that the dynamics of such models will typically be associated with unstable characteristic roots. In the simplest cases this may involve just one unstable root; see e.g., Sargent and Wallace (1973). But in other cases, this instability may be accompanied by sluggish behavior elsewhere in the system, leading to "saddlepoint" type instability; see e.g., Sidrauski (1967), Burmeister, Caton, Dobell, and Ross (1973), Burmeister, Flood, and Turnovsky (1981), Dornbusch and Fisher (1980), etc. As Chapter 3 demonstrates it is possible to find parameter values and models which give examples of a wide range of stability properties.

In any case, the introduction of an unexpected shock requires the system to undergo an instantaneous jump, in order for it to remain bounded following the disturbance. In the Sargent and Wallace example, the system must jump instantaneously from the initial to the new steady state. In the example of a saddlepoint instability, the system must jump onto the stable arm, which it then follows continuously into the new steady state. Examples of appropriate stabilizing behaviour have also been discussed in Chapter 3. The rationale for this behavior is typically the absence of "speculative bubbles," or more formally, an appeal to transversality conditions, applicable in associated optimizing models and which usually impose such boundedness; see Brock (1974, 1977).[1]

Many existing models make particularly simple assumptions regarding the financing of the government budget deficit, assuming, often only implicitly, that it is entirely bond-financed. The present chapter uses a simple dynamic inflationary model, which is a simplified version of the model discussed in Chapters 2 and 3. The model considered in this chapter was developed in a previous paper by Turnovsky (1979) and is employed here to consider two, closely related, issues. First, we show how the stability properties of the model depend critically upon the interaction between: (i) the sign and magnitude of the government deficit; and (ii) the method of deficit financing. Secondly, we discuss how the presence of interest payments and inflation taxes in the government budget constraint and elsewhere renders the model intrinsically nonlinear, giving rise to problems of nonexistence and nonuniqueness of equilibria.[2] Moreover, the stability properties associated with a particular combination of government deficit and mode of deficit financing are closely aligned with the choice of steady state. Thus the saddlepoint instability typically associated with perfect foresight is shown to depend critically upon these two aspects, as well as upon other parameter values of the model. In general, the dynamic behavior of the model can encompass: (a) total instability (all eigenvalues having positive real parts); (b) saddlepoint instability; (c) total stability (all eigenvalues having negative real parts), depending upon the choice of steady state, the size of the deficit, and its chosen mode of finance.

While problems of nonuniqueness of equilibria arising from nonlinearities are not new to macroeconomics, they are not generally considered in the rational expectations literature. This is because this literature has typically dealt with (stochastic) systems which are linear in the endogenous variables and treat the coefficients as given parameters.[3] Under these assumptions, these problems of nonexistence and nonuniqueness cannot arise. However, recent work on rational expectations models has begun endogenizing the coefficients by deriving them from some underlying microeconomic optimization. In this case, the model becomes inherently nonlinear in the endogenous variables, giving rise to the kinds of problems encountered in this chapter; see e.g., McCafferty and Driskill (1980).

In what one might call traditional economic dynamics, the problem of nonuniqueness of equilibria is frequently (but not always) resolved by appealing to Samuelson's Correspondence Principle. In effect, this proposition asserts that only the stable equilibria are economically relevant. However, this approach is inapplicable in the rational expectations context, where frequently <u>all</u> equilibria are associated with unstable roots. The essence of the rational expectations solution methodology is to eliminate the effects of these unstable roots by allowing for some initial jump in the system. This raises certain methodological issues. Thus some alternative to the correspondence principle, one based on some welfare criteria, is required to decide which of the multiple equilibria may be relevant.

The remainder of the chapter proceeds as follows. The basic model is outlined in Section 4.2. Section 4.3 discusses the problems of non-existence and nonuniqueness of steady-state equilibria, resulting from the nonlinearities in the model, and in doing so, pays particular attention to the question of the economic feasibility of the equilibrium. The following section considers the dynamics and stability of the system in the respective neighborhoods of the alternative steady states. Section 4.5 comments briefly on the choice between alternative steady states, while Section 4.6 indicates the alternative responses of the system to exogenous shocks, these responses depending upon the particular stability property of the associated equilibrium. Section 4.7 considers a specific example, while the main conclusions of our analysis are reviewed in the final section.

4.2 Specification of the Model

$$Y = D(Y^D, r-\pi, A) + G \qquad 0 < D_1 < 1,\ D_2 < 0,\ D_3 > 0 \tag{1a}$$

$$Y^D = Y - T + rb - \pi A \tag{1b}$$

$$A = m + b \tag{1c}$$

$$m = L(Y, r, A) \qquad L_1 > 0,\ L_2 < 0,\ 0 \leqq L_3 \leqq 1 \tag{1d}$$

$$p = \alpha(Y - \bar{Y}) + \pi \qquad \alpha > 0 \tag{1e}$$

$$\pi = p \tag{1f}$$

$$\dot{m} = \vartheta(G - T + rb) - pm \qquad 0 \leqq \vartheta \leqq 1 \tag{2a}$$

$$\dot{b} = (1-\vartheta)(G - T + rb) - pb \tag{2b}$$

where Y = real output (income),

$\bar{Y}$ = real capacity output,

Y^D = real private disposable income

D = real private expenditure,

G = real government expenditure,

T = real taxes, assumed to be exogenous,

r = nominal rate of interest,

π = expected rate of inflation,

A = real private wealth,

m = M/P = real money supply,

M = nominal money supply,

P = price level

b = B/P = real stock of government bonds,

B = nominal stock of government bonds,

p = $\dot{P}/P$ = rate of inflation, and

L = real demand for money balances.

The equations (1a-1f) are very similar to the equations of the model considered in Chapters 2 and 3, except that model also included unemployment and the dynamics of wage adjustment. Our treatment can therefore be very brief. Equation (1a) is the IS curve, with private expenditure demand depending upon real disposable income, the real rate of interest, and wealth. Disposable income is defined in (1b) and wealth in (1c). The LM curve is described by Equation (1d), with the demand for money being postulated to depend upon real income, the rate of interest and wealth. Equation (1e) is a simple expectations augmented Phillips curve, embodying the 'natural rate' hypothesis, while (1f) asserts that inflationary expectations satisfy perfect myopic foresight, i.e., the instantaneous expected rate of inflation equals the instantaneous actual rate of inflation. The dynamics of the system are described by equations (2). Equations (2a) and (2b) are derived by assuming $\dot{M}/P$ and

$\dot{B}/P$ are fixed proportions of the government budget which comprises government expenditure plus the interest payments on the outstanding government debt less the revenue to government from taxation. Two key policy parameters are introduced into the model. These include the current government deficit, G-T, and the method of deficit financing, described by ϑ. An increase in ϑ implies increased money financing and decreased bond financing. Conversely, a decrease in ϑ implies decreased money financing and increased bond financing.

Combining (1e) and (1f), it is seen that output is pegged at its full employment level, $\bar{Y}$, and the model simplifies to

$$\bar{Y} = D(\bar{Y} - T + (r-\pi)b - \pi m,\ r-\pi,\ m+b) + G \tag{3a}$$

$$m = L(\bar{Y},\ r,\ m+b) \tag{3b}$$

$$\dot{m} = \vartheta(G - T + rb) - \pi m \tag{4a}$$

$$\dot{b} = (1-\vartheta)(G - T + rb) - \pi b \tag{4b}$$

where r, π, m and b are endogenous variables and G, T, and ϑ are exogenous variables. It is seen that an increase in $r(\pi)$ will have both a positive (negative) income effect and a negative (positive) substitution effect on aggregate demand, D. Assuming that the substitution effect dominates in each case we impose the restrictions

$$D_r \equiv D_1 b + D_2 < 0 \tag{5a}$$

$$D_\pi \equiv -D_1 A - D_2 > 0 \tag{5b}$$

Likewise an increase in wealth will have an income and wealth effect on aggregate demand, D. The sign of the income effect will depend upon whether the additional wealth occurs in the form of bonds or money. In both cases we assume that the positive wealth effect dominates, enabling us to impose the restrictions

$$D_b \equiv D_3 + (r-\pi)D_1 > 0 \quad (5c)$$

$$D_m \equiv D_3 - \pi D_1 > 0 \quad (5d)$$

The model we have chosen is essentially a short-run model which abstracts from the dynamics of physical capital accumulation. This assumption has been adopted to make the analysis of the model more tractable.

4.3 Steady-State Equilibrium

We begin by considering the steady-state equilibrium for the economy. This is obtained by setting $\dot{m} = \dot{b} = 0$ in (4a), (4b), yielding the following equations

$$\bar{Y} = D(\bar{Y} - G, \tilde{r} - \tilde{\pi}, \tilde{m} + \tilde{b}) + G \quad (6a)$$

$$\tilde{m} = L(\bar{Y}, \tilde{r}, \tilde{m} + \tilde{b}) \quad (6b)$$

$$\vartheta(G - T + \widetilde{rb}) = \widetilde{\pi m} \quad (6c)$$

$$(1-\vartheta)(G - T + \widetilde{rb}) = \widetilde{\pi b} \quad (6d)$$

where the tilde denotes the fact that the variables are evaluated at their steady-state values.

For each choice of G-T and ϑ there are two classes of steady states. This possibility arises because of the nonlinearities stemming from the inflation taxes on money and bonds (πm, πb) together with the interest payments on government bonds (rb). One of the classes of steady states which may arise is described by equations (6a)-(6d) which in general is characterized by a non-zero steady state rate of inflation (i.e., $\tilde{\pi} \neq 0$) and is dependent upon both government policy parameters G-T and

ϑ. We shall refer to this as the <u>inflationary</u> steady state.

Alternatively, (6a)-(6d) will also be met if

$$\bar{Y} = D(\bar{Y} - G, \tilde{r}, \tilde{m} + \tilde{b}) + G \tag{6a'}$$

$$\tilde{m} = L(\bar{Y}, \tilde{r}, \tilde{m} + \tilde{b}) \tag{6b'}$$

$$G - T + \tilde{r}\tilde{b} = 0 \tag{6c'}$$

$$\tilde{\pi} = 0 \tag{6d'}$$

In this case the steady state involves government budget balance at zero inflation and shall be referred to as the <u>stable price</u> steady state. Note that while it depends upon the fiscal policy parameter G-T, it is <u>independent of</u> the financial policy parameter ϑ.

In examining the two classes of steady states, it will be shown that each class is associated with the possibility of zero, one, or multiple, steady-state equilibria. Throughout this chapter we shall restrict ourselves to equilibria which are economically feasible. These are defined to be equilibria in which the steady state solutions for the money supply, bond supply, and nominal interest rate are all non-negative (i.e., $\tilde{m} \geq 0$, $\tilde{b} \geq 0$, $\tilde{r} \geq 0$); the steady-state output $\bar{Y}$ is non-negative by assumption.[4]

We shall now examine the two alternative classes of equilibria in turn.

4.3A. <u>Inflationary Steady State</u>

Since we are assuming that $\tilde{\pi} \neq 0$, equations (6c) and (6d) may be combined to give

$$\bar{Y} = D(\bar{Y}-G, \tilde{r}-\tilde{\pi}, \tilde{m}+\tilde{b}) + G \tag{7a}$$

$$\tilde{m} = L(\bar{Y}, \tilde{r}, \tilde{m}+\tilde{b}) \tag{7b}$$

$$\tilde{m} = \vartheta(\tilde{m}+\tilde{b}) \tag{7c}$$

$$\vartheta(G-T) + \tilde{r}(1-\vartheta)\tilde{m} = \tilde{\pi}\tilde{m} \tag{7d}$$

The possibility of there being zero, one, or more than one economically meaningful steady states arises from the fact that the steady-state government budget constraint is a quadratic function. The problems can be illustrated most clearly by assuming that D(.) and L(.) are linear homogeneous functions. In this case we may solve (7a)-(7c) for $\tilde{r}$ and $\tilde{\pi}$ in the form

$$\tilde{r} = \frac{-D_2\vartheta L_1\bar{Y} + D_2(\vartheta-L_3)\tilde{m}}{\vartheta L_2 D_2} \tag{8a}$$

$$\tilde{\pi} = \frac{-D_2\vartheta L_1\bar{Y} - L_2\vartheta(Y-G)(1-D_1) + [L_2D_3-D_2(L_3-\vartheta)]\tilde{m}}{\vartheta L_2 D_2} \tag{8b}$$

and substituting these expressions into (7d), we derive the following quadratic equation in $\tilde{m}$

$$[D_2\vartheta(L_3-\vartheta) - L_2D_3]\tilde{m}^2 + [D_2\vartheta^2L_1\bar{Y} + L_2\vartheta(\bar{Y}-G)(1-D_1)]\tilde{m}$$
$$+ L_2D_2\vartheta^2(G-T) = 0 \tag{9}$$

A necessary condition for an economically meaningful solution for the system to exist, is that the solution for $\tilde{m}$ be real, and that at least one root be non-negative. For the existence of real roots, the discriminant of (9) must be positive; i.e.

$$\vartheta^2\{[D_2\vartheta L_1\bar{Y} + L_2(\bar{Y}-G)(1-D_1)]^2$$
$$- 4L_2D_2(G-T)[D_2\vartheta(L_3-\vartheta) - L_2D_3]\} > 0 \tag{10}$$

Inequality (10) may, or may not, be met. Conditions conducive to it being satisfied include: (i) a high income elasticity in the demand for money, reflected by a large L_1, when ϑ is non-zero; (ii) a budget surplus if the fraction of money financing, ϑ, is small (but not zero), or large (near unity); and (iii) a budget deficit for appropriate values of ϑ lying in between these two extremes.[5] Conversely, if these conditions are reversed, the nonexistence of a solution for $\tilde{m}$ increases in likelihood.

From (7c) it is seen that an economically meaningful solution for $\tilde{b}$ obtains if and only if such a solution for $\tilde{m}$ exists. Finally, whether the implied solution for $\tilde{r}$ is economically feasible is determined by substituting the solution for $\tilde{m}$ obtained from (9) into (8a) above.

Looking at extreme cases, if $\vartheta = 0$ (i.e., the deficit is bond financed), equation (9) reduces to $\tilde{m} = 0$. That is, if the nominal money supply is held fixed, then with a non-zero rate of inflation, the real money supply must eventually be reduced to zero. The interest rate $\tilde{r}$ is unambiguously positive. At the other extreme, if the deficit is entirely money-financed (ϑ=1), the coefficient of $\tilde{m}^2$ in (9) is positive. Assuming that (10) is met, so that a solution exists, there will be two solutions for $\tilde{m}$ if the government runs a current deficit (G-T > 0). These may be associated with 0, 1, or 2 positive values for $\tilde{r}$. However, if the government runs a current surplus (G-T < 0), one root of (9) will be negative and there will be only one economically feasible solution for $\tilde{m}$. This will be associated with a positive value for $\tilde{r}$ (if D_3 is large enough) or a negative value for $\tilde{r}$ (if D_3 is close to zero).

The critical aspect in the determination of the economically feasible solution is the root structure for the solutions of $\tilde{m}$, denoted by $\tilde{m}_1$ and $\tilde{m}_2$ say. More generally, these depend upon both (i) the size of the current government deficit G-T, and (ii) the mode of deficit financing ϑ. Assuming that the roots are real we have the following:

$$G - T > 0 \qquad \vartheta + \frac{L_2 D_3}{D_2 \vartheta} > L_3 \qquad \tilde{m}_1 > 0, \tilde{m}_2 > 0$$

$$G - T < 0 \qquad \vartheta + \frac{L_2 D_3}{D_2 \vartheta} > L_3 \qquad \tilde{m}_1 > 0, \tilde{m}_2 < 0$$

$$G - T > 0 \qquad \vartheta + \frac{L_2 D_3}{D_2 \vartheta} < L_3 \qquad \tilde{m}_1 < 0, \tilde{m}_2 > 0$$

$$G - T < 0 \qquad \vartheta + \frac{L_2 D_3}{D_2 \vartheta} < L_3 \qquad \tilde{m}_1 < 0, \tilde{m}_2 < 0$$

Finally, we should note that if the linear homogeneous functions D(.) and L(.) are replaced by affine functions, i.e., linear functions plus a positive or negative constant, then the sign and magnitudes of the steady state equilibria may change drastically, as may the number of economically feasible solutions for given values of G - T and ϑ. Basically, the constants, D_0 and L_0 say, of the affine transformation may shift the equilibria in either the positive or negative direction, depending upon their respective signs. If the functions D(.) and L(.) are nonlinear, the problems of nonuniqueness and nonexistence are obviously compounded further in complexity.

4.3B. Stable-Price Steady State

This is described by (6a')-(6d') and it is immediately evident from the third equation of this set that this will not have an economically feasible solution if G - T > 0, since then either $\tilde{r}$ or $\tilde{b}$ will be negative. Again, assuming that D(.) and L(.) are linear and homogenous, we may solve (6a') and (6b') for $\tilde{r}$ and $\tilde{b}$. Substituting the resulting expressions (not reported) into (6c') yields the following quadratic equation in $\tilde{m}$

$$-D_3[D_2(1-L_3) + L_2D_3]\tilde{m}^2 + \{D_3L_1\bar{Y}[D_2(2-L_3) + L_2D_3] + (\bar{Y}-G)(1-D_1)[L_3D_2(1-L_3) + D_3L_2(1+L_3)]\}\tilde{m} + (D_2L_3-L_2D_3)^2(G-T) - [(1-D_1)(\bar{Y}-G)L_3 + L_1\bar{Y}D_3][D_2L_1\bar{Y} + L_2(1-D_1)(\bar{Y}-G)] = 0$$

If G - T < 0, but is not too large in magnitude, then there will be two economically feasible solutions for $\tilde{m}$. One of these is associated with positive values of $\tilde{r}$ and $\tilde{b}$ while the other is associated with negative solutions for both $\tilde{r}$ and $\tilde{b}$. There is therefore only one totally feasible solution. As the size of the surplus G - T increases in magnitude, the coefficient of the constant term becomes negative, implying that there is only one feasible solution for $\tilde{m}$. This solution is associated with positive or negative values for both $\tilde{r}$ and $\tilde{b}$, depending upon whether $D_2L_3 - L_2D_3 \gtrless 0$.

If $D_3 = 0$ and G - T < 0 but not too large, only one economically feasible solution for $\tilde{m}$ will exist. However, this will be associated with negative values for both $\tilde{r}$ and $\tilde{b}$, and therefore is not feasible overall.

Taking Cases 4.3A and 4.3B together, we draw the following conclusions regarding the steady state equilibria. If the government is running a deficit, then the only feasible long-run equilibrium is given in 4.3A, in which there may be anything between 0 and 2 feasible solutions, depending upon parameter values. If the government is running either a balanced budget or a small surplus, there is one feasible stable price equilibrium, together with either 0 or 1 inflationary equilibria, depending upon parameter values. As the size of the surplus increases, the possibility arises of there being no stable price equilibrium. Indeed, it now becomes possible for no feasible solution for either class of steady state to exist. This occurs, for example, if $G - T < 0$, $D_3 = 0$, while $\vartheta < L_3$.

4.4 Dynamics and Stability

We shall now examine the dynamics of the model in the neighborhood of each steady state. The general system given by equations (3) and (4) can be linearized about the steady state as follows

$$\begin{bmatrix} \dot{m} \\ \dot{b} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} m-\tilde{m} \\ b-\tilde{b} \end{bmatrix} \tag{12}$$

where the a_{ij}'s are functions of the parameters of the model, evaluated at the steady state as demonstrated in the Appendix, and where $\tilde{m}$ and $\tilde{b}$ denote the relevant steady state values. We shall assume that by the appropriate introduction of additive constants D_0 and L_0, these are economically meaningful. Since our concern in this section is with the transitional dynamics about the steady state, this assumption is unimportant for present purposes.

Hence we can approximate the dynamics of m and b in the neighborhood of the steady state by the following dynamic equations

$$\begin{bmatrix} m-\tilde{m} \\ b-\tilde{b} \end{bmatrix} = \begin{bmatrix} a_{12} & a_{12} \\ \lambda_1-a_{11} & \lambda_2-a_{11} \end{bmatrix} \begin{bmatrix} k_1e^{\lambda_1 t} \\ k_2e^{\lambda_2 t} \end{bmatrix} \tag{13}$$

where k_1 and k_2 are constants and λ_1 and λ_2 are the eigenvalues associated with the steady state.

It is clear from (13) that the stability properties depend upon the sign configurations of the eigenvalues. If the real parts of both roots are negative, the model is locally stable; if one root is positive and the other negative, the model will exhibit "saddlepoint" type instability; if the real parts of both eigenvalues are positive, the model is totally unstable in the neighborhood of the steady state.

We shall now examine λ_1, λ_2 for the two classes of steady state equilibria in turn.

4.4A. Inflationary Steady State

As demonstrated in the Appendix, the eigenvalues for this equilibrium are given by

$$\lambda_1 = -\tilde{\pi} \tag{14a}$$

$$\lambda_2 = \Big\{-D_2\tilde{A}\vartheta^2 + D_2\tilde{A}L_3\vartheta + \frac{D_2L_2(T-G)}{\tilde{A}} - D_3L_2\tilde{A}\Big\}/(D_1\tilde{A} + D_2)L_2 \tag{14b}$$

Using (7c) and (8b), these expressions may be written as

$$\lambda_1 = \frac{D_2\vartheta L_1\bar{Y} + L_2\vartheta(\bar{Y}-G)(1-D_1) - [L_2D_3 - D_2(L_3-\vartheta)]\tilde{m}}{\vartheta L_2D_2} \tag{14a'}$$

$$\lambda_2 = \frac{-[\vartheta(\vartheta-L_3)D_2 + D_3L_2]\tilde{m}^2 - D_2L_2(G-T)\vartheta^2}{\tilde{m}\vartheta(D_1\tilde{A} + D_2)L_2} \tag{14b'}$$

where $\tilde{m}$ is the feasible solution to equation (9).

By considering equations (14a'), (14b') in conjunction with (9), it is seen that a variety of sign patterns for the eigenvalues, and therefore a range of dynamic behavior is possible. To illustrate these possibilities it is convenient to focus on the special case $D_3 = 0$, when there is no wealth effect in aggregate expenditure demand. Using this case, we can characterize examples of the three possible types of eigenvalue configurations in terms of the two policy parameters as follows.[6]

(i) Total Stability: When $G - T > 0$ and $\vartheta < L_3 < 1$, (14a') and (14b') imply $\lambda_1 < 0$, $\lambda_2 < 0$.

(ii) Saddlepoint Instability: For $G - T > 0$ and $0 < L_3 < \vartheta = 1$ (i.e., the case of all money financing) it can be shown that for the larger steady-state value of $\tilde{m}$, given by (9), we have $\lambda_1 < 0$, $\lambda_2 > 0$.

(iii) Total Instability: For $G - T < 0$ and $0 < L_3 < \vartheta$, (14b') implies $\lambda_2 > 0$. If further $\vartheta = 1$ (i.e., all money financing), then in the neighborhood of the feasible equilibrium $\tilde{m}$, $\lambda_1 > 0$, yielding an example of total instability.

4.4B. Stable Price Steady State

As demonstrated in the Appendix, the eigenvalues for this case have the following properties

$$\lambda_1 + \lambda_2 = [(1-\vartheta)(\tilde{r}+r_b\tilde{b}) + \vartheta r_m\tilde{b}] - \tilde{m}\pi_m - \tilde{b}\pi_b \tag{15a}$$

$$\lambda_1\lambda_2 = (\vartheta\tilde{A}-\tilde{m})[(\tilde{r}+r_b\tilde{b})\pi_m - r_m\tilde{b}\pi_b] \tag{15b}$$

Thus there are two cases to consider. First, when $\tilde{m} > \vartheta\tilde{A}$, $\lambda_1\lambda_2 > 0$ and $\lambda_1 + \lambda_2 > 0$. Hence in this case the real parts of λ_1 and λ_2 must be positive; i.e., the model will possess two unstable roots in the neighborhood of the steady state. Secondly, when $\tilde{m} < \vartheta\tilde{A}$, $\lambda_1\lambda_2 < 0$ and hence the model possesses one positive and one negative characteristic root; there is therefore saddlepoint instability.

It will be recalled that the equilibrium $\tilde{m}$ (and hence $\tilde{A}$), which may or may not be unique, depending upon G - T, is independent of the debt financing parameter ϑ. Thus whether a particular steady state is associated with two unstable roots or one unstable root depends simply upon whether $\vartheta \gtrless \tilde{m}/\tilde{A}$. Saddlepoint instability will tend to be associated with predominantly money-financed deficits, total instability will tend to result when the deficit is financed primarily be selling bonds.[7]

4.5 Choice of Appropriate Steady State

In principle it is possible, for given values of the parameters ϑ, G - T, to find several possible steady states as potential equilibria. In the case where functions D(.) and L(.) are linear (discussed in Section 4.3), up to two feasible equilibria may exist if the government is running a small surplus. If D(.) and L(.) are nonlinear, the problems of uniqueness become more acute. Unfortunately, the model lacks any procedure for choosing among them. As we have seen, most of the equilibria are associated with unstable root(s), so that Samuelson's correspondence principle, which would eliminate such equilibria is not helpful. Furthermore, as noted in the introduction, the rational expectations approach eliminates the effects of unstable roots where they exist, by allowing for some appropriate initial jump.

One procedure followed by rational expectations theorists to choose between multiple equilibria in nonlinear models is to introduce a welfare function and to assume that the economy will be driven to that equilibrium which maximizes welfare. However, it still remains unclear what mechanism in the model ensures that the utility maximizing steady state rather than some other steady state equilibrium will be reached.

4.6 Responses to Exogenous Changes

In Section 4.4 we established that it is possible for the economy described in this model to have zero, one, or two unstable characteristic roots, depending upon the choice of fiscal and monetary policy and other parameters of the economy. As a result of this, the requirement that the system be stable can impose very different types of adjustment on the economy following an exogenous disturbance, depending upon the initial steady state and its associated root structure.

4.6A. Locally Stable Equilibrium

In the case of a (locally) stable equilibrium, i.e., an equilibrium which is associated with two stable characteristic roots, the adjustment is what may be referred to as of the traditional type. The state variables in the basic system (m and b) are predetermined. Any exogenous disturbance in the system gives rise to instantaneous jumps in the interest rate r and the inflation rate π, in accordance with equations (3a) and (3b). These generate change in $\dot{m}$ and $\dot{b}$, and thereafter the dynamics of the system ensures that the economy will evolve continuously towards the new steady state.

4.6B. Saddlepoint Instability

In the case of saddlepoint instability, i.e., an equilibrium which is associated with one stable root, λ_1 say, the unstable root λ_2 is eliminated by setting $k_2 = 0$ in (13). In the case when this unstable root is eliminated by a jump in the price level, P, an appropriate jump following an exogenous change in government expenditure, G, say can be derived using Theorem 3.1 of Chapter 3. This jump is given by

$$\frac{\partial P}{\partial G} = \frac{-[(a_{11}-\lambda_1)\frac{\partial \tilde{m}}{\partial G} + a_{12}\frac{\partial \tilde{b}}{\partial G}]P_0}{m_0(a_{11}-\lambda_1) + a_{12}b_0} \tag{16a}$$

Once this initial jump has been made, leading to corresponding jumps in m and b the economy will then evolve towards the steady-state along the stable arm of the saddlepoint, the equation of which is given by

$$m - \tilde{m} = \frac{a_{12}}{\lambda_1 - a_{11}}(b - \tilde{b}) \tag{16b}$$

4.6C. Total Instability

The case where both roots are unstable may pose some difficulties. As shown recently by Blanchard and Kahn (1980), Buiter (1982) and also in Theorems 3.1 to 3.3 of Chapter 3, a dynamic system containing unstable roots can be stabilized only if there are as many variables which are free to undergo independent jumps ("jump" variables) as there are unstable roots in the system. In effect, it is the appropriate initial jumps in these variables which eliminate the instabilities from the adjustment. In the case of the saddlepoint most frequently encountered and just discussed, this is no problem; the required jumps in m and b are achieved by an appropriate jump in the price level P. When there are two unstable roots the choice of a second jump variable is somewhat less obvious; we shall assume that this jump in the price level is accompanied by an open-market operation which is undertaken by the monetary authority in order to stabilize the economy.

When there are two unstable roots and two jump variables, application of Theorem 3.1 of Chapter 3 shows that m and b will jump instantaneously to the steady-state following an exogenous shock, i.e., at all times.

$$m = \tilde{m} \tag{17a}$$

$$b = \tilde{b} \tag{17b}$$

Application of Theorem 3.1 when the appropriate short-run responses are given by a jump in the price level and an open market operation tells us that the appropriate jumps following an exogenous change in say, Government expenditure, G, satisfy the following equation system.

$$\begin{pmatrix} \frac{\partial M}{\partial G} \\ \frac{\partial P}{\partial G} \end{pmatrix} = \frac{P_0}{m_0 + b_0} \begin{pmatrix} b_0 & -m_0 \\ -1 & -1 \end{pmatrix} \begin{pmatrix} \frac{\partial \tilde{m}}{\partial G} \\ \frac{\partial \tilde{b}}{\partial G} \end{pmatrix} \tag{18}$$

4.7 An Example

To illustrate some of the issues we have been discussing, and in particular to show the interdependence between the dynamic adjustment of the system and the steady state, we shall analyze the following specific example. We shall assume that the government is running a small surplus and that it increases its expenditure, but by a sufficiently small amount, to ensure that the budget remains in surplus. We shall assume that the deficit is money financed, so that $0 < L_3 < \vartheta = 1$, and that $D_3 = 0$. Under these conditions, there is a unique feasible inflationary equilibrium, in the neighborhood of which both characteristic roots are positive, so that the system is totally unstable. For this same range of parameter values, we have noted that the stable price equilibrium will be associated with negative values of $\tilde{r}$ and $\tilde{b}$, and hence is infeasible. However, we shall assume that by an appropriate affine transformation, the constants D_0 and L_0 in the demand functions and other parameters are chosen to ensure that feasibility of all variables does obtain. Assuming this to be the case, the stationary price equilibrium is associated with characteristic roots $\lambda_1 > 0$, $\lambda_2 < 0$, so that this equilibrium is a saddlepoint. We shall now discuss the adjustment of the economy from each equilibrium in turn.

In the case of the inflationary equilibrium, setting $\vartheta = 1$, $D_3 = 0$ in (6a)-(6d), the steady state reduces to

$$\bar{Y} = D_0 + D_1(\bar{Y}-G) + D_2(\tilde{r}-\tilde{\pi}) + G \tag{19a}$$

$$\tilde{m} = L_0 + L_1\bar{Y} + L_2\tilde{r} + L_3\tilde{m} \tag{19b}$$

$$G - T = \tilde{\pi}\tilde{m} \tag{19c}$$

$$\tilde{b} = 0 \tag{19d}$$

where the constants L_0 and D_0 are chosen to ensure that $\tilde{r} > 0$. With $G - T < 0$, it follows from (19c) that the associated equilibrium rate of price change must be deflationary, i.e., $\tilde{\pi} < 0$.

The response of the steady state to an increase in G is given by the following expressions

$$\frac{\partial \tilde{r}}{\partial G} = \frac{(1-L_3)}{J}[-D_2 + (1-D_1)\tilde{m}] > 0 \tag{20a}$$

$$\frac{\partial \tilde{m}}{\partial G} = \frac{1}{J}[(1-D_1)L_2\tilde{m} - L_2D_2] < 0 \tag{20b}$$

$$\frac{\partial\tilde{\pi}}{\partial G} = -\frac{1}{J}[D_2(1-L_3) + L_2(1-D_1)\tilde{\pi}] \gtrless 0 \tag{20c}$$

where $J \equiv -\tilde{m}(1-L_3)D_2 - L_2D_2\tilde{\pi} > 0$ in the neighborhood of equilibrium. Thus an increase in G leads to an increase in the steady-state rate of interest, accompanied by a reduction in the real money stock. The rate of inflation may either rise or fall, while the real stock of bonds remains zero.

Following the discussion of Section 4.6B, since both $\lambda_1 > 0$ and $\lambda_2 > 0$, the increase in G must be accompanied by an appropriate open market operation in order to permit the system to remain in steady state. Using equations (18)-(20), the required jumps in M and P are given by

$$\frac{\partial M}{\partial G} = \left(\frac{P_0 b_0}{m_0 + b_0}\right)\frac{\partial\tilde{m}}{\partial G} < 0 \tag{21a}$$

$$\frac{\partial P}{\partial G} = -\left(\frac{P_0}{m_0 + b_0}\right)\frac{\partial\tilde{m}}{\partial G} > 0 \tag{21b}$$

If the government accompanies the increase in its expenditure with an open market sale of bonds which yields the increase in money supply given by equation (21a) and if the price level increases in accord with equation (21b) then the overall result will be that the real stock of bonds will remain constant, while the real stock of money will fall and the economy will remain in equilibrium.

Setting $D_3 = 0$ in (6a')-(6d'), the stable price steady state is described by

$$\bar{Y} = D_0 + D_1(\bar{Y}-G) + D_2\tilde{r} + G \tag{22a}$$

$$\tilde{m} = L_0 + L_1\bar{Y} + L_2\tilde{r} + L_3(\tilde{m} + \tilde{b}) \tag{22b}$$

$$G - T + \tilde{r}\tilde{b} = 0 \tag{22c}$$

$$\tilde{\pi} = 0 \tag{22d}$$

Differentiating these equations with respect to G, the response of the steady state is given by

$$\frac{\partial\tilde{r}}{\partial G} = -\frac{(1-D_1)}{D_2} > 0 \tag{23a}$$

$$\frac{\partial\tilde{m}}{\partial G} = \frac{(1-D_1)[\tilde{r}L_2-L_3\tilde{b}] + L_3D_2}{-D_2(1-L_3)\tilde{r}} < 0 \tag{23b}$$

$$\frac{\partial\tilde{b}}{\partial G} = \frac{D_2 - \tilde{b}(1-D_1)}{-D_2\tilde{r}} < 0 \tag{23c}$$

Thus the steady-state rate of interest will rise, while the real stocks of money and bonds will fall.

This case is characterized by a saddlepoint behaviour of the dynamic adjustment process and the appropriate response in the price level following a change in government expenditure is given by equation (16a). The sign of this adjustment is dependent on the sign of the terms a_{11} and a_{12}, which are given in the Appendix. With $\vartheta = 1$, $D_3 = 0$, these reduce to

$$a_{11} = \frac{1-L_3}{L_2}\left(\frac{\tilde{b}D_r - \tilde{m}D_2}{D_1(\tilde{m}+\tilde{b}) + D_2}\right) \tag{24a}$$

$$a_{12} = \frac{\tilde{r}D_r}{D_1(\tilde{m}+\tilde{b})+D_2} + \frac{L_3}{L_2}\left(\frac{\tilde{m}D_2-\tilde{b}D_r}{D_1(\tilde{m}+\tilde{b})+D_2}\right) \tag{24b}$$

Both of these are indeterminate in sign so that the sign of the jump in the price level can be either positive or negative, depending upon parameter values.

Assume plausibly, for example, that $a_{11} > 0$, $a_{12} > 0$. Then in this case substitution with equation (16a) shows that $\partial P/\partial G > 0$. In other words, following an exogenous increase in government expenditure, G, the price level will rise. This will result in corresponding falls in both m and b and then the economy will follow a stable path given by equation (16b) until equilibrium is reached.

As has been done in Chapter 3 it is also possible to illustrate the jump and subsequent dynamic path using a graphical approach. The equivalence of the two approaches has been suitably emphasized in Chapter 3 and we shall not repeat those arguments here.

The examples given here suffice to show how contrasting behavior in the system can result from exogenous disturbances for identical parameter values, but beginning from different steady states.

4.8 Conclusion

In this chapter we have developed a simple nonlinear monetary macro model under perfect foresight. We have emphasized here how the inherent nonlinearities associated with inflation taxes and interest payments give rise to problems of nonexistence and nonuniqueness of equilibria. For any choice of fiscal and monetary policy parameters, we have identified two sets of steady state equilibria. One of these is associated with a non-zero rate of inflation and depends upon both the monetary and fiscal policy parameters. The other is associated with a stable price level and is independent of the monetary policy parameter. For either set of equilibria, there may be zero, one, or multiple solutions, depending upon other parameter values.

We have also considered the stability properties that arise as a result of the interaction between the sign and size of the current budget deficit and alternative modes of deficit financing. We find that these stability properties depend critically upon the associated steady state. For the class of inflationary steady states we find that the dynamic system may: (i) be totally stable; (ii) have saddlepoint type instability, (iii) be totally unstable, depending upon parameter values and the mode of deficit financing. In the neighborhood of the stable price equilibrium, the dynamics is either: (i) totally unstable, or (ii) a saddlepoint. For either equilibrium, the dynamic response of the system to an exogenous shock depends critically upon the root structure of the associated equilibrium. While jumps commonly found in models of perfect foresight frequently arise in this model, they are not the only response. Thus, despite the fact that none of endogenous variables have been constrained to move sluggishly, it is quite possible for the system to respond in a "smooth" manner, in line with dynamic models of a traditional "sluggish" variety.[8] In fact, depending upon the type of fiscal and monetary policy chosen, all types of dynamic response are possible.

Finally, we have taken a particular set of policy rules and shown how the choice of steady state will influence both the dynamic properties of the system and the system's appropriate response to an exogenous shock. The qualitative difference in the system's responses serves to underline the fact that in a real world which is characterized by nonlinearities, the possession of limited knowledge about linear relationships (as in, for example, a simplified econometric model), may not be sufficient for people in the economy to foresee accurately the appropriate response of the economy to exogenous changes, in the manner predicted by the rational expectations literature.

Throughout the rest of this thesis we shall be emphasizing the rational expectations approach to economic dynamics. However, some of the questions raised in this chapter about the non-uniqueness of short-run responses to an exogenous shock and whether individuals have enough information so as to make such responses attainable will not be treated further. Chapter 4 has been included in this thesis to emphasize that the issues are not as clear as some of the recent literature would have us believe; rather it should be emphasized that, while there is adequate theory to justify the optimality of stabilizing jumps (see Brock (1974, 1977)), it is still necessary for economists to develop a micro-theory as to how individuals have sufficient information available to them so that the stabilizing jumps suggested by the rational expectations literature can be achieved. This issue, although recognized as a need of high priority by the author, will not be treated in this thesis.

APPENDIX TO CHAPTER 4

The dynamics of the linearized model in the neighborhood of the steady state are described by the following equation

$$\begin{pmatrix} \dot{m} \\ \dot{b} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} m-\tilde{m} \\ b-\tilde{b} \end{pmatrix} \quad (A.1)$$

where the a_{ij}'s are evaluated at the steady state. This means that

$$a_{11} \equiv \frac{\partial \dot{m}}{\partial m} = \vartheta r_m \tilde{b} - \tilde{\pi} - \pi_m \tilde{m} \quad (A.2a)$$

$$a_{12} \equiv \frac{\partial \dot{m}}{\partial b} = \vartheta(\tilde{r} + r_b \tilde{b}) - \pi_b \tilde{m} \quad (A.2b)$$

$$a_{21} \equiv \frac{\partial \dot{b}}{\partial m} = (1-\vartheta) r_m \tilde{b} - \pi_m \tilde{b} \quad (A.2c)$$

$$a_{22} \equiv \frac{\partial \dot{b}}{\partial b} = (1-\vartheta)(\tilde{r} + r_b \tilde{b}) - \tilde{\pi} - \pi_b \tilde{b} \quad (A.2d)$$

where the tilde denotes that the corresponding variables are evaluated at the appropriate steady state and where

$$r_m = \frac{1 - L_3}{L_2} < 0 \quad (A.3a)$$

$$r_b = \frac{-L_3}{L_2} > 0 \quad (A.3b)$$

$$\pi_m = \frac{(D_1 \tilde{b} + D_2)(1 - L_3) - (D_1 \tilde{\pi} - D_3) L_2}{(D_1 \tilde{A} + D_2) L_2} \quad (A.3c)$$

$$\pi_b = \frac{-(D_1\tilde{b} + D_2)L_3 + (D_1(\tilde{r}-\tilde{\pi}) + D_3)L_2}{(D_1\tilde{A} + D_2)L_2} \qquad \text{(A.3d)}$$

The characteristic equation of this dynamic system is given by

$$\lambda^2 - (a_{11} + a_{22})\lambda + (a_{11}a_{22} - a_{12}a_{21}) = 0 \qquad \text{(A.4)}$$

Let λ_1, λ_2 denote the eigenvalues or characteristic roots of the system. Then

$$\lambda_1 + \lambda_2 = a_{11} + a_{22}$$

$$= [(1-\vartheta)(\tilde{r} + r_b\tilde{b}) + \vartheta r_m\tilde{b}] - \tilde{m}\pi_m - \tilde{b}\pi_b - 2\tilde{\pi} \qquad \text{(A.5a)}$$

$$\lambda_1\lambda_2 = a_{11}a_{22} - a_{12}a_{21}$$

$$= -\tilde{\pi}[(1-\vartheta)(\tilde{r} + r_b\tilde{b}) + \vartheta r_m\tilde{b} - \tilde{\pi} - \tilde{m}\pi_m - \tilde{b}\pi_b]$$

$$+ (\vartheta\tilde{A} - \tilde{m})[(\tilde{r} + r_b\tilde{b})\pi_m - r_m\tilde{b}\pi_b] \qquad \text{(A.5b)}$$

As shown in the text, for each choice of ϑ there are two possible steady states. In the first of these steady states $\tilde{\pi} \neq 0$, $\tilde{m} = \vartheta\tilde{A}$ and $\tilde{b} = (1-\vartheta)\tilde{A}$. In the second possible steady state, $\tilde{\pi} = 0$ and $\tilde{r}\tilde{b} = T - G$.

Using equations (A.5) it is possible to describe the properties of the characteristic roots in the neighborhood of each steady state.

A. <u>Inflationary Steady State $\tilde{\pi} \neq 0$</u>

In this case,

$$\lambda_1 = -\tilde{\pi} \qquad \text{(A.6a)}$$

$$\lambda_2 = (1-\vartheta)(\tilde{r} + r_b\tilde{b}) + \vartheta r_m\tilde{b} - \tilde{\pi} - \tilde{m}\pi_m - \tilde{b}\pi_b$$

$$= \frac{[-\vartheta^2 D_2\tilde{A} + \vartheta(D_2\tilde{A}L_3 - D_2L_2\tilde{r}) + (D_2L_2(\tilde{r}-\tilde{\pi}) - D_3L_2\tilde{A})]}{(D_1\tilde{A} + D_2)L_2}$$

$$= \frac{-\vartheta^2 D_2\tilde{A} + \vartheta D_2\tilde{A}L_3 + D_2L_2[(T-G)/\tilde{A}] - D_3L_2\tilde{A}}{(D_1\tilde{A} + D_2)L_2} \quad (A.6b)$$

B. Stable-Price Steady State, $\tilde{\pi} = 0$

In this case the two roots satisfy

$$\lambda_1 + \lambda_2 = [(1-\vartheta)(\tilde{r}+r_b\tilde{b}) + \vartheta r_m\tilde{b}] - \tilde{m}\pi_m - \tilde{b}\pi_b \quad (A.7a)$$

$$\lambda_1\lambda_2 = (\vartheta\tilde{A}-\tilde{m})[(\tilde{r}+r_b\tilde{b})\pi_m - r_m\tilde{b}\pi_b] \quad (A.7b)$$

It is readily shown that the term $[(\tilde{r}+r_b\tilde{b})\pi_m - r_m\tilde{b}\pi_b] < 0$, so that $\text{sgn}\,\lambda_1\lambda_2 = -\text{sgn}(\vartheta\tilde{A} - \tilde{m})$. Turning to (A.7a), the following argument shows that $\lambda_1 + \lambda_2 > 0$ whenever $\tilde{m} > \vartheta\tilde{A}$, provided the following additional mild restrictions are added: (i) $\pi_b < 0$, and (ii) either $L_3 = 0$ or $\tilde{b} \geq \tilde{m}$.

First, substituting from (A.3a) and (A.3b)

$$[(1-\vartheta)r_b + \vartheta r_m]b = \left(\frac{\vartheta - L_3}{L_2}\right)b. \quad (A.8a)$$

Thus, (A.7a) may be written as

$$\lambda_1 + \lambda_2 = (1-\vartheta)\tilde{r} + \left(\frac{\vartheta-L_3}{L_2}\right)\tilde{b} - (\tilde{m}\pi_m + \tilde{b}\pi_b) \quad (A.8b)$$

But

$$-m\pi_m \geqq \frac{m(D_1 b + D_2)(L_3 - 1)}{(D_1 A + D_2)L_2} \geqq \frac{m(L_3 - 1)}{L_2} \qquad \text{(A.8c)}$$

Accordingly, if $\pi_b < 0$,

$$\lambda_1 + \lambda_2 \geqq \left(\frac{\vartheta - L_3}{L_2}\right)\tilde{b} - \tilde{m}\pi_m \qquad \text{(A.8d)}$$

$$\geqq \left(\frac{\vartheta - L_3}{L_2}\right)\tilde{b} + \frac{\tilde{m}(L_3 - 1)}{L_2} \qquad \text{(A.8e)}$$

$$= \frac{L_3(\tilde{m} - \tilde{b})}{L_2} + \frac{\vartheta\tilde{b} - \tilde{m}}{L_2} \qquad \text{(A.8f)}$$

$$\geqq \frac{L_3(\tilde{m} - \tilde{b})}{L_2} + \frac{\vartheta\tilde{A} - \tilde{m}}{L_2} \qquad \text{(A.8g)}$$

$$> 0 \text{ if, } \tilde{b} \geqq \tilde{m} \text{ or } L_3 = 0, \text{ and } \tilde{m} > \vartheta\tilde{A} \qquad \text{(A.8h)}$$

Therefore, in general, when $\tilde{m} > \vartheta\tilde{A}$, $\lambda_1 + \lambda_2 > 0$ and $\lambda_1\lambda_2 > 0$ (the real parts of λ_1 and λ_2 are positive) and, when $\tilde{m} < \vartheta\tilde{A}$, $\lambda_1\lambda_2 < 0$ (i.e., λ_1 and λ_2 are real and of opposite sign).

PART 2

"OPTIMAL STABILIZATION POLICIES UNDER PERFECT FORESIGHT"

CHAPTER 5

SPECIFICATION OF AN OPTIMAL STABILIZATION PROBLEM

5.1 Introduction

The problem of choosing between inflation and unemployment rates continues to be a fundamental one in most modern economies. The question of the optimal choice of tradeoff between them has been investigated by a number of authors. The earliest studies, conducted in the mid 1960's were purely static; see, e.g., Lipsey (1965) and Brechling (1968). Subsequently, the analysis was extended to a dynamic context on the assumption that inflationary expectations, known to be a critical aspect of the tradeoff, follow some gradual evolutionary process such as an adaptive scheme; see, e.g., Phelps (1967, 1972), Turnovsky (1981). These authors derive an optimal path in which the inflation rate adjusts gradually toward some steady-state equilibrium, while the unemployment rate converges slowly toward its natural rate. The transitional dynamic adjustment path depends upon the parameters characterizing the economy and the preferences of the policy maker, including in particular, the rate of time discount.

In Chapters 3 and 4 of this thesis we have introduced several related continuous-time macro-models and used them to emphasize the dynamic properties of the economy when inflationary expectations satisfy perfect foresight, which is the deterministic analogue of rational expectations. In this chapter we take a similar model which includes both unemployment and inflation and add a loss function in order to examine the optimal inflation-unemployment trade-off when expectations satisfy perfect foresight.

In the rational expectations approach to economic dynamics, we have already shown how the dynamics of the economy are typically associated with instability and how there will be a jump in one or more endogenous variables (e.g., a jump in the price level or an open market operation) in order to stabilize the economy.

In Chapter 6 we examine whether or not we can choose an appropriate mix of fiscal and monetary policies in order to minimize the chosen loss function. At the same time we require that the optimal policy is consistent with the objective that real money and real bonds converge to a finite value. The latter objective is not postulated as being a formal requirement of the model but can be justified by appealing to the arguments of the rational expectations approach to economic modeling.[1]

When these objectives are simultaneously satisfied it is no longer necessary that the price level jump in order to stabilize the economy. Thus it is consistent with the rational expectations approach to economic dynamics that no jumps in the price level occur. This is the assumption adopted in Chapter 6. However if we accept that the price level can jump to stabilize the economy as the rational expectations literature does, then surely it is equally plausible for the price level to jump in order to minimize the loss function. In Chapter 7 we consider the case when the price level is allowed to jump to minimize the loss function.

Whether or not jumps in the price level are allowed we show in Chapters 6 and 7 that under perfect foresight the optimal strategy is to drive the economy instantaneously to a zero rate of inflation. However the time path of unemployment depends on whether or not initial jumps in the price level occur. In the rest of this chapter we develop a modeling framework which will be used for the analysis of these subsequent chapters.

5.2 A Dynamic Macroeconomic Model

The analysis of Chapters 5 to 7 will be based on the following macroeconomic model which is derived under the assumption that labor supply is greater than labor demand; i.e., $N^S > N^D$.

$$Y = D(Y^D, r-\pi, A) + G \qquad (1a)$$

$$0 < D_1 < 1,\ D_2 < 0,\ D_3 > 0$$

$$Y^D = Y - T + rb - \pi A \qquad (1b)$$

$$A = m + b \qquad (1c)$$

$$m = L(Y, r, A) \qquad L_1 > 0,\ L_2 < 0,\ 0 < L_3 < 1 \qquad (1d)$$

$$N^D = N^D(z) \qquad N^{D'} < 0 \qquad (1e)$$

$$N^S = N^S(z) \qquad N^{S'} > 0 \qquad (1f)$$

$$Y = f(N^D) = Y(z) \qquad Y' < 0 \qquad (1g)$$

$$U = (N^S - N^D)/N^S = U(z) \qquad U' > 0 \qquad (1h)$$

$$w = \alpha(\bar{U} - U) + \pi \qquad (1i)$$

$$\pi = p, \text{ except at points where } P \text{ jumps} \qquad (1j)$$

$$\dot{z} = w - p \qquad (2a)$$

$$\dot{m} = \vartheta(G - T + rb) - pm \qquad (2b)$$

$$\dot{b} = (1 - \vartheta)(G - T + rb) - pb \qquad (2c)$$

where

Y = real output

Y^D = real private disposable income

D = real private expenditure

G = real government expenditure

r = nominal interest rate

π = expected rate of inflation

$r-\pi$ = real interest rate

P = price level

$p = \dot{P}/P$ = actual rate of inflation

M = nominal money supply

$m = M/P$ = real money supply

B = nominal supply of bonds

$b = B/P$ = real supply of bonds

A = real private wealth

W = nominal wage rate

$z = \log(W/P)$ = log of real wage rate

$w = \dot{W}/W$ = rate of nominal wage inflation

N^D = demand for labor

N^S = supply of labor

f = production function

U = rate of unemployment

$\bar{U}$ = natural rate of unemployment

The model essentially follows the framework of the model analysed in Chapters 2 and 3 combined with the monetary policies analysed in Chapter 4.

Equation (1a) is the product market equilibrium condition, in which real private demand increases with real private disposable income and real private wealth and decreases with the real rate of interest. Equation (1b) defines real private disposable income to be real factor income plus returns on government bonds, less expected capital losses on financial wealth (the expected inflation tax on real private wealth) and exogenous real taxes. Real private wealth is defined in equation (1c). Equilibrium in the money market is described by equation (1d) where the demand for real money balances depends upon the real level of income, the nominal rate of interest and real private wealth.

The supply of, and demand for, labor are given by equations (1e) and (1f). Both are dependent upon the real wage (with the appropriate sign). Equations (1g) and (1h) describe the production function and the rate of unemployment as being the difference between the supply of and demand for labor as a percentage of the labor supply. The production function is written on the assumption that $N^D < N^S$.

Equation (1i) is a simple Phillips curve with the rate of wage inflation increasing with excess demand in the labor market and with the expected rate of inflation. It is a standard version of the "expectations hypothesis" with the unitary coefficient on expectations reflecting the "accelerationist" view. Equation (1j) states that inflationary expectations satisfy perfect myopic foresight; i.e., the instantaneous expected rate of inflation equals the instantaneous actual rate of inflation. This equation is assumed to hold everywhere, except at points where there are jumps in the price level, in which case p becomes infinite.

The dynamics of the system are described by equations (2a)-(2c). The first of these equations defines the evolution of real wages, while equations (2b) and (2c) describe government financial policy and the evolution in the stocks of financial assets it generates. Specifically, these equations assert that the government deficit consists of government expenditure plus the interest payments on government debt less tax revenues and that a fraction ϑ of this deficit is financed by money creation, with the balance $(1-\vartheta)$ being financed by issuing bonds.

5.3 Specification of an Optimization Problem

Equations (1) and (2) describe the economy faced by the policy maker. We assume that the policy maker regards a state of zero inflation (stable price level) and a fixed rate of unemployment, δ say, as globally optimal. Given these long-run targets we assume that the policy maker's objective is to choose some combination of government expenditure policy (G), taxation policy (T) and monetary policy (ϑ) so as to minimize the following loss function

$$L \equiv c|U(0) - U_0| + \frac{1}{2}\int_0^{\infty} e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \qquad (3)$$

where U_0 is the previously inherited rate of unemployment, and U(0) is the endogenously determined initial unemployment rate, following the initial jump in the price level, if it occurs (see below).

Note that in general $\bar{U} \neq \delta$. This reflects the fact that the rate of unemployment which the economy tends to approach, i.e., the natural rate level, $\bar{U}$, may not coincide with the level that the policy maker finds socially optimal. The parameter a $(0 < a < \infty)$ reflects the relative importance attached to inflation and unemployment in the intertemporal objective. As a increases, the policy maker is concerned increasingly with inflation; as a decreases the objective is weighted more heavily toward unemployment. The parameter γ $(0 < \gamma < \infty)$, which measures the rate of time preference, reflects the degree of myopia of the policy maker; the larger γ the more myopic he is.

It will be demonstrated later that if an initial jump in the price level occurs then by an appropriate choice of this initial jump in the price level, P, the inital rate of unemployment U(0) can be chosen. This instantaneous initial jump in unemployment from its inherited rate is assumed to impose adjustment costs on the economy and these are reflected in the cost parameter c. Note that these initial costs are assumed to be proportional to the absolute magnitude of the jump.

In order to solve the policy optimization problem it is first convenient to use equations (1) to express the short-run rate of inflation and the short-run interest rate in terms of the endogenous variables U, m, and b, and the policy variables G and T. These solutions are given by[2]

$$p = p(U, m, b, G, T) \tag{4a}$$

$$p_U < 0, \; p_m < 0, \; p_b \gtrless 0, \; p_G < 0, \; p_T > 0$$

$$r = r(U, m, b) \tag{4b}$$

$$r_U < 0, \; r_m < 0, \; r_b > 0$$

Note that (4a) implies that an increase in government expenditure is deflationary, while an increase in exogenous tax receipts is inflationary. The reason for these seemingly perverse effects is seen from (1a) and (1d). In order for product market equilibrium to be maintained, an increase in G must be matched by a reduction in private demand, which with output and wealth fixed instantaneously, is met by an increase in the real interest rate. But since the nominal interest rate is independent of G, this increase takes the form of a reduction in the inflation rate. And likewise for a change in T. However, it should be stressed that these effects are only partial; they do not allow for the jumps in the price level which may occur. If changes in the price level follow policy changes, then they in turn will impact on the rate of inflation. Given that U = U[log(W/P)], m = M/P, and b = B/P, and that the nominal wage rate W, and the nominal asset supplies M and B are all constrained to move continuously, the complete effect of an increase in G on the instantaneous rate of inflation is given by the expression

$$\frac{dp}{dG} = -\left(p_U U' \frac{1}{P} + p_m \frac{M}{P^2} + p_b \frac{B}{P^2}\right) \frac{\partial P}{\partial G} + p_G$$

In addition to the partial effect which is negative, as noted, there are the induced effects operating through the jump in the price level and this has several induced positive effects. First, by lowering the real wage this reduced unemployment, thereby stimulating inflation. Secondly it reduces the real stock of money and bonds and a sufficient condition for the combined net effect of this to be inflationary is that the wealth elasticity of the demand for money be less than unity. In short, if a jump in the price level occurs, the induced effect of increase in G are all inflationary and indeed on balance are likely to dominate the perverse partial deflationary effect.

Differentiating (1h) with respect to t and combining with (1i) and (2a), we obtain

$$\dot{U} = \beta(\bar{U} - U) + \varepsilon(\pi - p) \tag{5}$$

where for convenience we let $\beta = U'\alpha$, $\varepsilon = U'$.

The question arises as to when the jumps in the price level, if they occur, will take place. Intuitively it seems reasonable that this will occur at the points where the policy variables are likely to undergo discrete changes, which is at the beginning of the period of optimization.[3] Invoking this assumption it follows that $\pi = p$, thereafter.

The objective facing the policy maker can now be summarized by the following optimization problem: Choose appropriate policy instruments so as to

$$\text{Min } c|U(0) - U_0| + \frac{1}{2}\int_0^\infty e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \tag{6}$$

subject to[4]

$$p = p(U, m, b, G, T) \tag{7a}$$

$$r = r(U, m, b) \tag{7b}$$

$$\dot{m} = \vartheta(G - T + rb) - pm \tag{7c}$$

$$\dot{b} = (1 - \vartheta)(G - T + rb) - pb \tag{7d}$$

$$\dot{U} = \beta(\bar{U} - U) \tag{7e}$$

where $W(0) = W_0$, $M(0) = M_o$, and $B(0) = B_0$ are all predetermined and P_0 is either endogenous or fixed depending upon whether the price level is or is not allowed to jump in order to minimize the loss function. The initial quantities m(0), b(0), and U(0) are also endogenous or fixed and satisfy the constraint

$$m(0) = M_0/P(0) \tag{8a}$$

$$b(0) = B_0/P(0) \tag{8b}$$

$$U(0) = U[\log(W_0/P(0)] \tag{8c}$$

In Chapter 6 we examine the number and type of policy instruments that will be required in order for the economy to remain stable when there are no jumps in the price level. In examining this question we also look at the associated optimal inflation/unemployment mix for the economy. Chapter 7 then examines optimal stabilization policy when prices are allowed to jump endogenously.

CHAPTER 6

OPTIMAL STABILIZATION POLICY WITH NO JUMPS IN THE PRICE LEVEL

6.1 Introduction

We now consider the situation when the policy maker has up to three policy instruments available to him. These policy instruments involve two fiscal instruments: Government expenditure (G) and taxation (T), as well as a monetary instrument (ϑ). We assume that the policy maker will choose some or all of these policy instruments to satisfy two objectives:

(i) to minimize the given loss function which is a function of both unemployment and inflation; and

(ii) to keep the economy on a path which will eventually converge to a stable long-run equilibrium.

When the price level is not allowed to jump endogenously in order to stabilize the economy the question arises as to how many and what type of policy instruments the policy maker will have to employ to simultaneously sustain both these objectives. This is the question addressed in Chapter 6.

6.2 The Problem Re-Stated

When there are no jumps in the price level, the problem facing the policy maker, first given in equations (6), (7a)-(7e), (8a)-(8c) of Chapter 5 can be summarized as follows:

Choose appropriate policy instruments so as to

$$\text{Min } \frac{1}{2}\int_0^\infty e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \qquad (1)$$

subject to

$$p = p(U, m, b, G, T) \qquad (2a)$$

$$r = r(U, m, b) \qquad (2b)$$

$$\dot{m} = \vartheta(G - T + rb) - pm \qquad (2c)$$

$$\dot{b} = (1 - \vartheta)(G - T + rb) - pb \qquad (2d)$$

$$\dot{U} = \beta(\bar{U} - U) \qquad (2e)$$

where

$$m(0) = m_0, \; b(0) = b_0, \; z(0) = z_0$$

are predetermined and thus so is $U(0) = U(z_0)$.

The policy maker is also required to choose policy instruments so that

$$\lim_{t\to\infty} |m(t)| < \infty \qquad (3a)$$

$$\lim_{t\to\infty} |b(t)| < \infty \qquad (3b)$$

Since $\lim_{t\to\infty} U(t) = \bar{U}$, equations (3a)-(3b) ensure that the economy converges to its steady-state equilibrium.

As stated earlier the policy-maker is free to choose from two fiscal instruments (G and T) and a monetary instrument (ϑ). In examining whether or not an optimal solution to this problem exists, each combination of available instruments is considered in turn.

6.3 Determination of the Optimal Solution

To solve the optimization problem given by equations (1), (2a)-(2e), first write down the Hamiltonian functions as follows:

$$H = \frac{1}{2} e^{-\gamma t}[ap^2 + (U - \delta)^2] + \mu e^{-\gamma t}[\dot{m} + pm - \vartheta(G - T + rb)]$$

$$+ \varphi e^{-\gamma t}[\dot{b} + pb - (1 - \vartheta)(G - T + rb)]$$

$$+ \eta e^{-\gamma t}[\dot{U} - \beta(\bar{U} - U)] \qquad (4)$$

where $\mu e^{-\gamma t}$, $\varphi e^{-\gamma t}$, $\eta e^{-\gamma t}$ are the discounted Lagrange multipliers associated with the dynamic equations (2c)-(2e) respectively. The Euler equations with respect to m, b, U, G, T and ϑ are respectively given by[1]

$$\dot{\mu} = app_m + \mu[-\vartheta r_m b + p_m m + p + \gamma]$$

$$+ \varphi[-(1 - \vartheta)r_m b + p_m b] \qquad (5a)$$

$$\dot{\varphi} = app_b + \mu[-\vartheta(r_b b + r) + p_b m]$$

$$+ \varphi[-(1 - \vartheta)(r_b b + r) + p_b b + p + \gamma] \qquad (5b)$$

$$\dot{\eta} = (U - \delta) + app_U + \mu[-\vartheta r_U b + p_U m]$$

$$+ \varphi[-(1 - \vartheta)r_U b + p_U b] + (\beta + \gamma)\eta \qquad (5c)$$

$$app_G + \mu[-\vartheta + p_G m] + \varphi[-(1 - \vartheta) + p_G b] = 0 \qquad (5d)$$

$$app_T + \mu[\vartheta + p_T m] + \varphi[(1 - \vartheta) + p_T b] = 0 \qquad (5e)$$

$$\mu = \varphi \tag{5f}$$

In addition the dynamics of m, b, and U, are given by equations (2c)-(2e). The optimal solution must also satisfy the following transversality conditions as $t \to \infty$

$$\lim_{t \to \infty}(m\mu e^{-\gamma t}) = 0 \tag{6a}$$

$$\lim_{t \to \infty}(b\varphi e^{-\gamma t}) = 0 \tag{6b}$$

$$\lim_{t \to \infty}(U\eta e^{-\gamma t}) = 0 \tag{6c}$$

Finally, we also require that the solution is stable, thus satisfying equations (3a)-(3b).

We now consider in turn: the case when the policy maker is able to manipulate all three of the instruments (this will be known as the three instrument problem), the case when the policy maker is able to manipulate any two of these instruments (the two instrument problem) and the case when the policy maker is able to manipulate only one of these instruments (the one instrument problem). We examine the nature of the optimal stabilization policy and associated optimal time-paths for unemployment and inflation in each case; we also examine whether the economy converges to equilibrium.

6.4 The Three Instrument Problem

For the three instrument problem, i.e., when Government expenditure (G), taxes (T) and monetary policy (ϑ) are all available instruments, the necessary conditions are given by equations (5a)-(5f) and (2c)-(2e).

In particular, equations (5d), (5e) imply

$$(p_T + p_G)[\mu\vartheta + \varphi(1 - \vartheta)] = 0 \tag{7a}$$

Hence, since $p_T + p_G \neq 0$,

$$\mu\vartheta + \varphi(1-\vartheta) = 0 \tag{7b}$$

Combining equations (7b), (5f)

$$\mu = \varphi = 0 \tag{7c}$$

and then substituting equation (7c) in equation (5d)

$$p = 0 \tag{7d}$$

Hence the optimal path is given by

$$\dot{\eta} = (U-\delta) + (\beta+\gamma)\eta \tag{8a}$$

$$p = 0 \tag{8b}$$

$$\dot{m} = \vartheta(G - T + rb) \tag{8c}$$

$$\dot{b} = (1-\vartheta)(G - T + rb) \tag{8d}$$

$$\dot{U} = \beta(\bar{U} - U) \tag{8e}$$

In fact, with three policy instruments, each of p, m, b can be controlled independently by allocating one policy instrument to each variable. Thus m can be controlled by the appropriate choice of the budget deficit, G - T. Given any budget deficit the time-path of b can be controlled by the choice of ϑ.[2] Then given any time-path for real money, m, and real bonds, b, any time-path for inflation, p, can be derived by an appropriate choice of government expenditure, G. In particular, therefore, the optimal time-path for inflation, given by $p = 0$ can be derived by an appropriate choice of government expenditure, given suitable choices of G - T and ϑ which lead to stable values for real money and real bonds. Such an optimal policy can be achieved with an instantaneous jump in the rate of inflation to zero, while m and b evolve continuously from the initial condition towards their steady state equilibria.

6.5 The Two Instrument Problem

6.5.1 A Monetary and a Fiscal Instrument

When a monetary instrument (i.e., ϑ) and a fiscal instrument (say G) are available to the policy maker in order to attain his policy objectives, then the relevant necessary conditions are given by equations (5a)-(5d), (5f), (2c)-(2e). In particular by using equations (5a), (5b), (5d), (5f) we can demonstrate that for the optimal solution

$$p = \mu = \varphi = 0 \tag{9}$$

Hence the necessary conditions for the optimal path reduce to equations (8a)-(8e). We have already shown in section 6.4 of this chapter that this path can be attained using three policy instruments. In fact it can be attained using only one fiscal instrument and a monetary instrument.

We can easily choose G so that inflation is driven instantaneously to zero. In order to stabilize the economy we need only choose ϑ so that m and b converge to their steady state. This can be achieved as follows:

Assuming that G converges to $\tilde{G}$ we can show that the steady state values of $\tilde{m}$ and $\tilde{b}$ will satisfy

$$p(\bar{U}, \tilde{m}, \tilde{b}, \tilde{G}) = 0 \tag{10a}$$

$$r(\bar{U}, \tilde{m}, \tilde{b})\tilde{b} = T - \tilde{G} \tag{10b}$$

Then the time-paths for the variables m, b and U can be described by the following equations

$$\begin{pmatrix} \dot{m} \\ \dot{b} \\ \dot{U} \end{pmatrix} = \begin{pmatrix} \vartheta r_m \tilde{b} & \vartheta(r_b\tilde{b}+\tilde{r}) & \vartheta r_U \tilde{b} \\ (1-\vartheta) r_m \tilde{b} & (1-\vartheta)(r_b\tilde{b}+\tilde{r}) & (1-\vartheta) r_U \tilde{b} \\ 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} m-\tilde{m} \\ b-\tilde{b} \\ U-\bar{U} \end{pmatrix} + \begin{pmatrix} \vartheta \\ 1-\vartheta \\ 0 \end{pmatrix} (G-\tilde{G}) \tag{11a}$$

But p=0, hence

$$G - \tilde{G} = -\frac{p_m}{p_G}(m-\tilde{m}) - \frac{p_b}{p_G}(b-\tilde{b}) - \frac{p_U}{p_G}(U-\bar{U}) \tag{11b}$$

Hence, equation (11a) can be rewritten as

$$\begin{pmatrix} \dot{m} \\ \dot{b} \\ \dot{U} \end{pmatrix} = \begin{pmatrix} \vartheta(r_m\tilde{b} - \frac{p_m}{p_G}) & \vartheta(r_b\tilde{b}+\tilde{r} - \frac{p_b}{p_G}) & \vartheta(r_U\tilde{b} - \frac{p_U}{p_G}) \\ (1-\vartheta)(r_m\tilde{b} - \frac{p_m}{p_G}) & (1-\vartheta)(r_b\tilde{b}+\tilde{r} - \frac{p_B}{p_G}) & (1-\vartheta)(r_U\tilde{b} - \frac{p_U}{p_G}) \\ 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} m-\tilde{m} \\ b-\tilde{b} \\ U-\bar{U} \end{pmatrix} \tag{11c}$$

The characteristic roots of this system are given by

$$\lambda_1 = 0 \tag{12a}$$

$$\lambda_2 = \vartheta[r_m\tilde{b} - \frac{p_m}{p_G}] + (1-\vartheta)[r_b\tilde{b} + \tilde{r} - \frac{p_b}{p_G}] \tag{12b}$$

$$\lambda_3 = -\beta \tag{12c}$$

By appropriate choice of ϑ we can force $\lambda_2 < 0$ and thus m and b will converge.

6.5.2 Two Fiscal Instruments

When the policy maker has two fiscal instruments (i.e., Government expenditure, G, and taxes, T) which are available to him to attain his policy objectives the necessary conditions for the policy maker's optimization problem are given by equations (5a)-(5e), (2c)-(2e).

In particular as shown in equation (7b)

$$\mu\vartheta + \varphi(1-\vartheta) = 0 \tag{13a}$$

Substituting (13a) in equations (5a), (5b) and combining the results gives

$$[\vartheta p_m + (1-\vartheta)p_b][\vartheta ap - \varphi((1-\vartheta)m - \vartheta b)] = 0 \tag{13b}$$

Since in general $\vartheta p_m + (1-\vartheta)p_b \neq 0$, thus

$$\vartheta ap = \varphi[(1-\vartheta)m - \vartheta b] \tag{13c}$$

Incorporating equation (13c) we can summarize the necessary conditions as

$$\dot{\eta} = (U - \delta) + (\beta + \gamma)\eta \tag{14a}$$

$$\dot{\varphi} = \varphi(p + \gamma) \tag{14b}$$

$$\vartheta ap = \varphi[(1-\vartheta)m - \vartheta b] \tag{14c}$$

$$\dot{m} = \vartheta(G - T + rb) - pm \tag{14d}$$

$$\dot{b} = (1-\vartheta)(G - T + rb) - pb \tag{14e}$$

$$\dot{U} = \beta(\bar{U} - U) \tag{14f}$$

From equations (14b), (14c) the steady state values of p are given by[3]

$$\tilde{p} = \tilde{\varphi} = 0 \quad \text{or} \quad \tilde{p} = -\gamma \tag{15}$$

where a tilde denotes a steady-state value. However when $\tilde{p} = -\gamma$ substitution in equations (14d), (14e) reveals that $(1-\vartheta)m = \vartheta b$ and hence from (14c), $\tilde{p} = 0$. Hence $\tilde{p} = 0$ describes the only legitimate steady-state. Then substituting equation (14c) in equation (14b) gives that

$$\dot{\varphi} = \frac{\varphi^2[(1-\vartheta)m - \vartheta b]}{\vartheta a} + \varphi\gamma \tag{16a}$$

and linearizing about the steady state (noting that $\tilde{\varphi} = 0$) gives that

$$\dot{\varphi} = \gamma\varphi \tag{16b}$$

i.e.,

$$\varphi(t) = k_1 e^{\gamma t} \tag{16c}$$

Then, in order to ensure that the transversality condition, equation (6b), is satisfied, we must set $k_1 = 0$ and thus $\varphi(t) = 0$ for all t; but then substituting in equation (14c), p = 0 along the total optimal path. As it is unaffected by any of the policy instruments, unemployment will follow the usual path towards $\bar{U}$.

The optimal path for inflation can always be attained by an appropriate choice of one of the fiscal instruments, say taxes, T, e.g., choose

$$T - \tilde{T} = \frac{-p_G}{p_T}(G - \tilde{G}) - \frac{p_U}{p_T}(U - \bar{U}) - \frac{p_m}{p_T}(m - \tilde{m}) - \frac{p_b}{p_T}(b - \tilde{b}) \tag{17a}$$

The following argument demonstrates that government expenditure, G, can then be used to drive m and b to a stable equilibrium.

Substituting equation (17a) into equations (2c)-(2e) and linearizing about the steady state we obtain

$$\begin{pmatrix} \dot{m} \\ \dot{b} \\ \dot{U} \end{pmatrix} =$$

$$\begin{pmatrix} \vartheta(r_m\tilde{b} + \frac{p_m}{p_T}) & \vartheta(r_b\tilde{b}+\tilde{r} + \frac{p_b}{p_T}) & \vartheta(r_U\tilde{b} + \frac{p_U}{p_T}) \\ (1-\vartheta)(r_m\tilde{b} + \frac{p_m}{p_T}) & (1-\vartheta)(r_b\tilde{b}+\tilde{r} + \frac{p_b}{p_T}) & (1-\vartheta)(r_U\tilde{b} + \frac{p_U}{p_T}) \\ 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} m-\tilde{m} \\ b-\tilde{b} \\ U-\bar{U} \end{pmatrix}$$

$$+ \begin{pmatrix} \vartheta \\ 1-\vartheta \\ 0 \end{pmatrix} (1 + \frac{p_G}{p_T})(G - \tilde{G}) \tag{17b}$$

If we choose

$$G - \tilde{G} = -\left(\frac{p_T}{p_T + p_G}\right)\Big\{(1 + r_m\tilde{b} + \frac{p_m}{p_T})(m-\tilde{m})$$

$$+ (r_b\tilde{b} + \tilde{r} + \frac{p_b}{p_T})(b-\tilde{b})\Big\} \tag{17c}$$

then the resulting dynamical system will be stable and thus equations (3a), (3b) will be satisfied.

6.6 The One Instrument Problem

In a similar manner to the arguments of Section 6.4 and 6.5 we can show that when the policy maker can only control one instrument the solution characterized by a zero rate of inflation satisfies the necessary conditions; we shall restrict our analysis here to examining whether such a solution is feasible and whether it is consistent with a stable equilibrium for the economy.

6.6.1 A Monetary Instrument Alone

The requirement that p = 0 can be rewritten in the form

$$p_m(m-\tilde{m}) + p_b(b-\tilde{b}) + p_U(U-\bar{U}) = 0 \tag{18a}$$

Given that U is initially fixed, inflation can thus be driven instantaneously to zero by an appropriate open market operation.[4] Once inflation has been driven to zero in order that inflation remain there we require that $\dot{p} = 0$. Noting that

$$\dot{p} = p_m(\dot{m}) + p_b(\dot{b}) + p_U(\dot{U}) \tag{18b}$$

i.e.,

$$\dot{p} = (\vartheta p_m + (1-\vartheta)p_b)(G - T + rb) + p_U\beta(\bar{U}-U) \tag{18c}$$

we can force $\dot{p} = 0$ by an appropriate choice of monetary policy ϑ, and thus the policy of driving inflation instantaneously to zero, where it remains thereafter, is attainable with just a monetary instrument.

Is such a policy associated with a stable economy? The dynamics of m, b and U are given by

$$\begin{pmatrix} \dot{m} \\ \dot{b} \\ \dot{U} \end{pmatrix} = \begin{pmatrix} \vartheta r_m\tilde{b} & \vartheta(r_b\tilde{b}+\tilde{r}) & \vartheta r_U\tilde{b} \\ (1-\vartheta)r_m\tilde{b} & (1-\vartheta)(r_b\tilde{b}+\tilde{r}) & (1-\vartheta)r_U\tilde{b} \\ 0 & 0 & -\beta \end{pmatrix} \begin{pmatrix} m-\tilde{m} \\ b-\tilde{b} \\ U-\bar{U} \end{pmatrix} \tag{19}$$

The characteristic roots of this dynamical system are given by

$$\lambda_1 = 0 \tag{20a}$$

$$\lambda_2 = \vartheta(r_m\tilde{b}) + (1-\vartheta)(r_b\tilde{b} + \tilde{r}) \tag{20b}$$

$$\lambda_3 = -\beta \tag{20c}$$

In order for the system to be stable we require that $\lambda_2 < 0$, i.e., the asymptotic value of ϑ satisfies $[(1-\vartheta)/\vartheta] < [(-r_m\tilde{b})/(r_b\tilde{b} + \tilde{r})]$. This equality may or may not be satisfied[5] and thus we cannot be sure whether or not the system is stable.

6.6.2 A Fiscal Instrument Alone

We can always choose a fiscal instrument, say G, so that inflation is driven to and kept at zero. As shown in equations (11) and (12) whether or not such a policy is associated with a stable dynamic system will depend upon whether the exogenously fixed monetary policy (i.e., ϑ) is such that

$$\lambda_2 = \vartheta[r_m\tilde{b} - \frac{p_m}{p_G}] + (1-\vartheta)[r_b\tilde{b} + \tilde{r} - \frac{p_b}{p_G}] < 0 \tag{21}$$

This may or may not be the case and thus if only a fiscal instrument is available to the policy maker the economy may well not be stable (i.e., conditions (3a) and (3b) may not be satisfied).

The use of only one fiscal instrument can, however, in some circumstances be unambiguously associated with stability, e.g., consider the case when there are no bonds in the model. (This situation is characterized by $\tilde{b} = 0$, $\vartheta = 1$, whence $\lambda_2 = -(p_m/p_G) < 0$.)

6.7 Summary of Optimal Strategy

In all cases considered, the optimal time-path for unemployment and inflation under perfect foresight is given by:

$$p = 0 \tag{22a}$$

$$U = \bar{U} + (U_0 - \bar{U})e^{-\beta t} \tag{22b}$$

Although the optimal solution has been derived formally, we can always decompose the policy maker's problem into three objectives. Firstly, the policy maker aims to minimize the loss associated with inflation; i.e., to minimize

$$\int_0^\infty p^2 e^{-\gamma t} dt \tag{23}$$

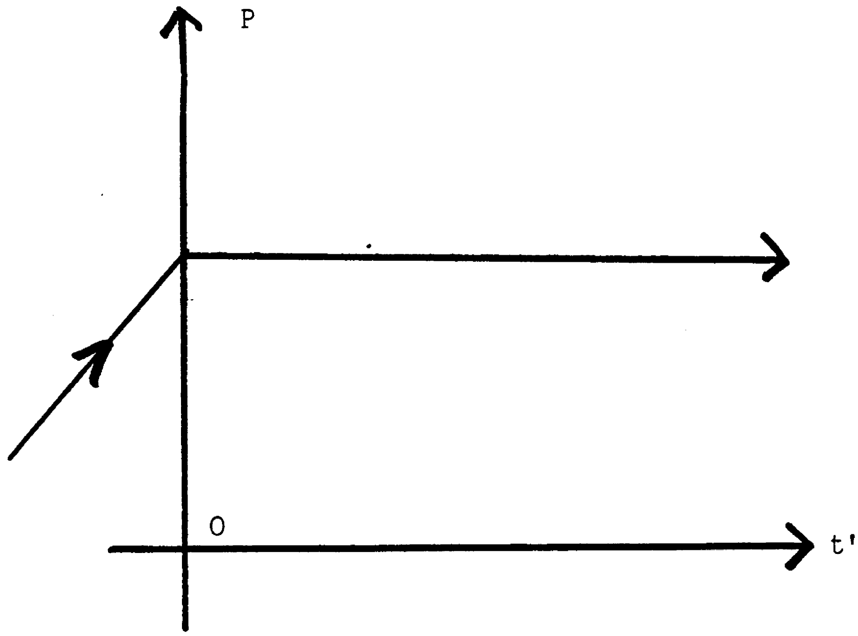
The global minimum solution to this problem is given by equation (22a) and this minimum can always be obtained provided the policy maker has at least one policy instrument available to him.

Secondly, the policy maker aims to minimize the loss associated with unemployment, i.e., to minimize

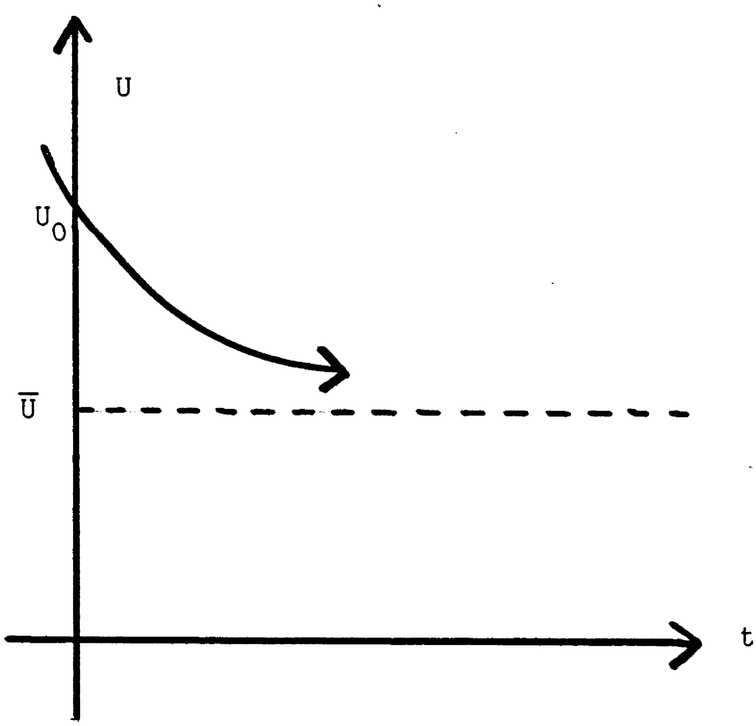
$$\int_0^\infty e^{-\gamma t}(U-\delta)^2 dt \tag{24}$$

Since no policy instruments will influence unemployment and since no jumps of the price level are permitted, the solution to this problem is always given by equation (22b).

Finally, the policy maker aims to ensure that equations (3a) and (3b) are satisfied; i.e., to ensure the stability of the economy. When the policy maker has a second policy instrument available to him, this policy instrument can be devoted totally to this purpose, thus ensuring stability occurs. However when the policy maker has only one policy instrument, this policy instrument will have to ensure that equations



6.1A: TIME-PATH FOR PRICE LEVEL



6.1B: TIME-PATH FOR UNEMPLOYMENT

FIGURE 6.1

(23), (3a), (3b) are satisfied simultaneously. Depending on parameters, these dual objectives may not be able to be satisfied.

Figures 6.1A and 6.1B give examples of optimal time-paths for unemployment and the price level. As discussed, depending on the number of instruments such time-paths may or may not be consistent with a stable economy.

6.8 Conclusion

In Chapter 6, we have shown how, for the given macro-model under perfect foresight when there are no jumps in the price level, the optimum stabilization policy involves driving inflation instantaneously to zero and allowing unemployment to evolve towards its natural rate. We have also examined how the number of available policy instruments influences the stability properties of the economy.

In the chosen model there are three exogenous variables which can potentially be used as policy instruments: government expenditure, taxes and the method of deficit financing. However, a typical policy maker may not be free to manipulate all of these instruments in order to achieve his policy objectives (e.g., the policy maker may be free to change government expenditure and monetary policy but for political reasons may be obliged to keep tax revenue below certain limits). Accordingly we have examined in turn the implications of the availability of all possible combinations of policy instruments. We have shown that the fewer policy instruments that are available to the policy maker the more likely is the optimum policy to be associated with an unstable economy.

In the usual rational expectations approach to economic dynamics, any instability is removed by a jump in appropriate endogenous variables (e.g., the price level). Thus when instability occurs (e.g., when only one policy instrument, say G, is available and monetary policy, ϑ, is such that the economy is unstable) the appropriate solution methodology would require deriving a reduced form for the time-path of P that restricts m and b to stable time-paths. Such a solution would be conditioned on an arbitrary path for the policy instrument (i.e., G). Once this is achieved, the optimal path for the price level can be derived by maximizing over the policy instrument (i.e., over G). The time-path for P would have to take into consideration not only whether P stabilized the economy but also how the time-path for P affected the loss function given by equation (1). This is an extremely difficult problem and has not been treated in this chapter where we assume that no jumps in the price level are permitted.

When there are at least two policy instruments, the optimal stabilization policy will always be associated with stability and thus the price level will not need to jump to ensure that the economy converges to its steady-state equilibrium. Thus there is no need for the price level to jump merely to stabilize the economy and the analysis of this chapter is quite consistent with the rational expectations approach when two or more instruments are available. If the price level does jump, therefore, it will only do so to minimize the loss function. Analysis of this problem becomes more tractable and is the subject of Chapter 7.

CHAPTER 7

OPTIMAL STABILIZATION POLICY WITH ENDOGENOUS JUMPS IN THE PRICE LEVEL

7.1 Introduction

In Chapter 6 we have shown that if the policy maker has a fiscal instrument (say G) and a monetary instrument (say ϑ) available to him and if the price level is constrained to evolve continuously from a given initial point then the optimal policy will be for the economy to jump instantaneously to a zero rate of inflation while unemployment evolves towards its natural rate. This time-path is typical of the time-paths for the economy when the optimal policy mix is such that the economy is stable. In this chapter, we consider what happens when, for this policy mix and expectations satisfying perfect foresight, the price level is also able to jump endogenously to minimize the loss function. We show that while the economy can still jump instantaneously to a zero rate of inflation, there is a tradeoff between an initial once-and-for-all jump in the price level and the subsequent gradual adjustment of unemployment to its natural rate.

An important, and widely discussed, aspect of optimal policy determination under rational expectations, when there are endogenous jumps in state variables, concerns the question of the time consistency of the optimal policy; see, e.g., Kydland and Prescott (1977), Turnovsky and Brock (1980). This question of time inconsistency did not arise with the analysis of Chapter 6, nor does it arise subsequently in this thesis with the microfoundations in Chapters 8 and 9, because in all these cases the economy is required to evolve continuously from a given initial point. Thus there are no endogenous initial jumps in the state variables. However the question does arise in this chapter because of the endogenous jumps in the price level.

For the problem considered here, we show that with the objective function we consider, the solution is indeed time consistent for a wide range of parameter values, although for a relatively modest modification of the cost function, the optimal solution is rendered always time inconsistent.

7.2 The Problem Re-Stated

It is appropriate here to restate the problem which was first given in equations (6), (7a)-(7e), (8a)-(8c) of Chapter 5.

The objective facing the policy maker can be summarized by the following optimization problem:

$$\min_{\vartheta, G} c|U(0) - U_0| + \frac{1}{2}\int_0^\infty e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \qquad (1)$$

subject to

$$p = p(U, m, b, G, T) \qquad (2a)$$

$$r = r(U, m, b) \qquad (2b)$$

$$\dot{m} = \vartheta[G - T + rb] - pm \qquad (2c)$$

$$\dot{b} = (1-\vartheta)(G - T + rb) - pb \qquad (2d)$$

$$\dot{U} = \beta(\bar{U} - U) \qquad (2e)$$

where $W(0) = W_0$, $M(0) = M_0$, and $B(0) = B_0$ are all predetermined and $P(0)$ is endogenous. The initial quantities $m(0)$, $b(0)$, and $U(0)$ are endogenous and satisfy the constraint

$$m(0) = M_0/P(0) \tag{3a}$$

$$b(0) = B_0/P(0) \tag{3b}$$

$$U(0) = U[\log(W_0/P(0)] \tag{3c}$$

It will be observed that the magnitude of the instantaneous initial jump in unemployment from its inherited rate U_0, is assumed to impose adjustment costs on the economy and that these initial costs given by $c|U(0) - U_0|$ in equation (1) are assumed to be proportional to the absolute magnitude of the jump. This form of cost function turns out to be time consistent for a wide range of parameter values. By contrast, if these initial costs are represented by the more familiar quadratic cost function, time inconsistency always obtains.

Since the function describing the loss associated with the initial jump in the price level, i.e., $c|U(0) - U_0|$, is nondifferentiable at $U(0) = U_0$, the optimization problem specified by equations (1), (2a)-(2e), (3a)-(3c) can most easily be solved by decomposing it into the following two problems

Problem 1

$$\text{Find } L_1 = \text{Min } c(U(0) - U_0) + \frac{1}{2}\int_0^\infty e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \tag{4a}$$

$$\text{subject to } U(0) \geqq U_0 \tag{4b}$$

and equations (2a)-(2e), (3a)-(3c)

Problem 2:

$$\text{Find } L_2 = \text{Min } c[U_0 - U(0)] + \frac{1}{2}\int_0^\infty e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \tag{5a}$$

$$\text{subject to } U_0 \geqq U(0) \tag{5b}$$

and equations (2a)-(2e), (3a)-(3c)

The solution to the original problem associated with the loss function given by (1) is then a solution for which $U(0) \geqq U_0$ if $L_1 \leqq L_2$, or a solution for which $U_0 \geqq U(0)$ if $L_2 \leqq L_1$.

7.3 Determination of the Optimal Solution

To solve the optimization problems specified by both Problems 1 and 2 we first write down the Hamiltonian function as follows

$$H \equiv \frac{1}{2} e^{-\gamma t}[ap^2 + (U - \delta)^2] + \mu e^{-\gamma t}[\dot{m} - \vartheta(G - T + rb) + pm]$$
$$+ \lambda e^{-\gamma t}[\dot{b} - (1-\vartheta)(G - T + rb) + pb]$$
$$+ \eta e^{-\gamma t}[\dot{U} - \beta(\bar{U} - U)] \tag{6}$$

where $\mu e^{-\gamma t}$, $\lambda e^{-\gamma t}$, and $\eta e^{-\gamma t}$ are the discounted Lagrange multipliers associated with the dynamic equations (2c)-(2e) respectively. The Euler equations with respect to m, b, U, G and ϑ are respectively given by[1]

$$\dot{\mu} = app_m + \mu[-\vartheta r_m b + p_m m + p + \gamma] + \lambda[-(1-\vartheta) r_m b + p_m b] \tag{7a}$$

$$\dot{\lambda} = app_b + \mu[-\vartheta(r_b b + r) + p_b m] + \lambda[-(1-\vartheta)(r_b b + r)$$
$$+ p_b b + p + \gamma] \tag{7b}$$

$$\dot{\eta} = U - \delta + app_U + \mu[-\vartheta r_U b + p_U m] + \lambda[-(1-\vartheta) r_U b + p_U b]$$
$$+ (\beta + \gamma)\eta \tag{7c}$$

$$app_G + \mu[-\vartheta + p_G m] + \lambda[-(1-\vartheta) + p_G b] = 0 \tag{7d}$$

$$\mu = \lambda \tag{7e}$$

In addition the dynamics of m, b, and U are given by equations (2c)-(2e). Equations (7a)-(7e) can be solved to give the evolution of the variables

μ, λ, η, π, m, b, and U along the policy maker's optimal path.

In addition, the optimal solution must satisfy the following transversality conditions as $t \to \infty$

$$\lim_{t\to\infty} (m\mu e^{-\gamma t}) = 0 \tag{8a}$$

$$\lim_{t\to\infty} (b\lambda e^{-\gamma t}) = 0 \tag{8b}$$

$$\lim_{t\to\infty} (U\eta e^{-\gamma t}) = 0 \tag{8c}$$

Furthermore, the fact that m(0), b(0) and U(0) are endogenously determined in accordance with (3a)-(3c) imposes the constraints

$$b(0) = \varphi_1 m(0), \text{ where } \varphi_1 \equiv B_0/M_0 \tag{9a}$$

$$U(0) = U[\log \varphi_2 m(0)], \text{ where } \varphi_2 \equiv W_0/M_0 \tag{9b}$$

Accordingly, the following transversality conditions must also be satisfied at the initial time 0; see, e.g., Kamien and Schwartz (1971).

For Problem 1

If $U(0) > U_0$, then

$$\mu(0) + \varphi_1\lambda(0) + [U'/m(0)][\eta(0) - c] = 0 \tag{10a}$$

if $U(0) = U_0$, then

$$\mu(0) + \varphi_1\lambda(0) + [U'/m(0)][\eta(0) - c] \leqq 0 \tag{10b}$$

For Problem 2

If $U_0 > U(0)$, then

$$\mu(0) + \varphi_1\lambda(0) + [U'/m(0)][\eta(0) + c] = 0 \tag{11a}$$

if $U_0 = U(0)$, then

$$\mu(0) + \varphi_1\lambda(0) + [U'/m(0)][\eta(0) + c] \geqq 0 \tag{11b}$$

The solution given by the system of equations (7)-(11) can be simplified considerably. First, (7d) and (7e) imply

$$p = \frac{\mu[1 - p_G A]}{a p_G} \tag{12a}$$

Also, combining equations (7a), (7b) and (7e) implies a second relationship between p and μ

$$p = \frac{\mu[r_m b - r_b b - r + (p_b - p_m)A]}{a(p_m - p_b)} \tag{12b}$$

In general, equations (12a) and (12b) cannot both be satisfied simultaneously unless

$$p = \mu = 0, \text{ for all } t \tag{12c}$$

Moreover, combining (12c) with (7e) implies

$$\lambda = 0, \text{ for all } t \tag{12d}$$

Hence, using (12a)-(12d), the optimal path reduces to

$$\dot{\eta} = (\beta + \gamma)\eta + U - \delta \tag{13a}$$

$$\dot{U} = \beta(\bar{U} - U) \tag{13b}$$

$$p = 0 \tag{13c}$$

$$\dot{m} = \vartheta[G - T + rb] \tag{13d}$$

$$\dot{b} = (1-\vartheta)(G - T + rb) \tag{13e}$$

$$\lim_{t\to\infty} (U\eta e^{-\gamma t}) = 0 \tag{13f}$$

Equations (13a)-(13f), together with the transversality conditions (10a,b), (11a,b) determine the optimal path for unemployment and inflation. Using (12c) and (12d), the latter simplify to the following:

For <u>Problem 1</u>:

If $U(0) > U_0$, then $\eta(0) = c$ (14a)

$U(0) = U_0$, then $\eta(0) \leqq c$ (14b)

For <u>Problem 2</u>:

If $U_0 > U(0)$, then $\eta(0) = -c$ (15a)

$U_0 = U(0)$, then $\eta(0) \geqq -c$ (15b)

Equations (13a), (13b), (13f), and the transversality conditions given by equations (14) and (15) can then be used to solve for the optimal path for U as follows. Integrating (13b), yields

$$U(t) = \bar{U} + [U(0) - \bar{U}]e^{-\beta t} \tag{16a}$$

where U(0) is the initial value of U(t), following the jump, which is to be determined. Substituting (16a) into (13a) and integrating again, we find that

$$\eta(t) = -\frac{(\bar{U} - \delta)}{\beta + \gamma} + \frac{[\bar{U} - U(0)]}{2\beta + \gamma} e^{-\beta t} + De^{(\beta + \gamma)t} \tag{16b}$$

where D is an arbitrary constant. In order for the transversality condition (13f) to be satisfied we require D = 0. Then substituting equations (14), (15) into (16b) and comparing the loss functions for Problems 1 and 2, we can show that the optimal solution to the general problem is given by

$$U(0) = \bar{U} + \frac{(2\beta + \gamma)}{\beta + \gamma}[\delta - \bar{U}] - (2\beta + \gamma)c;$$

$$\text{if } U_0 < U(0) \qquad (17a)$$

$$\bar{U} + \frac{2\beta + \gamma}{\beta + \gamma}(\delta - \bar{U}) - (2\beta + \gamma)c \leqq U(0)$$

$$\leqq \bar{U} + \frac{(2\beta + \gamma)}{\beta + \gamma}(\delta - \bar{U}) + (2\beta + \gamma)c; \text{ if } U_0 = U(0) \qquad (17b)$$

$$U(0) = \bar{U} + \frac{(2\beta + \gamma)}{\beta + \gamma}(\delta - \bar{U}) + (2\beta + \gamma)c; \text{ if } U_0 > U(0) \qquad (17c)$$

It will be observed from (17b) that if the inherited unemployment rate, U_0, lies in a specific closed interval, bounded by

$$\bar{U} + \frac{(2\beta + \gamma)}{\beta + \gamma}(\delta - \bar{U}) - (2\beta + \gamma)c \leqq U_0$$

$$\leqq \bar{U} + \frac{(2\beta + \gamma)}{\beta + \gamma}(\delta - \bar{U}) + (2\beta + \gamma)c \qquad (18)$$

then it will be optimal for no initial jump in the unemployment rate, U, to occur. However, if U_0 lies outside this interval, then the unemployment rate will jump instantaneously to the nearest boundary of this closed interval. The boundaries of the closed interval depend upon: the natural rate of unemployment $\bar{U}$; the target rate of unemployment δ; the cost associated with the initial jump in the unemployment rate, c; the coefficient of unemployment in the Phillips curve, β; the rate of time preference of the policy maker, γ. Several special cases are worth noting.

(i) If the cost associated with the initial jump $c \to \infty$, then it will never be optimal to have an initial jump in unemployment, i.e., $U(0) = U_0$.

(ii) If the cost associated with the initial jump, $c \to 0$, then the initial unemployment rate will always jump to a level given by

$$U(0) = \bar{U} + \frac{2\beta + \gamma}{\beta + \gamma}(\delta - \bar{U}) \tag{19}$$

(iii) If the policy maker is perfectly myopic, i.e., $\gamma \to \infty$, then it will never be optimal to have an initial jump in unemployment, provided $c > 0$,

(iv) If the desired target rate of unemployment equals the natural rate, i.e., $\delta = \bar{U}$, then the boundaries of the closed interval, viz $\bar{U} \pm (2\beta + \gamma)c$ are symmetric about the natural rate, $\bar{U}$.

Note that these jumps in the unemployment rate, when they occur, are mirrored by jumps in the price level. Indeed, it is the initial jump in the price level which, given the instantaneous stickiness of the nominal wage, permits the initial jump in the unemployment rate to occur. Differentiating (3c), the relationship between the two jump variables is given by

$$\frac{dP}{P_0} = \frac{U_0 - U(0)}{U'} \tag{20}$$

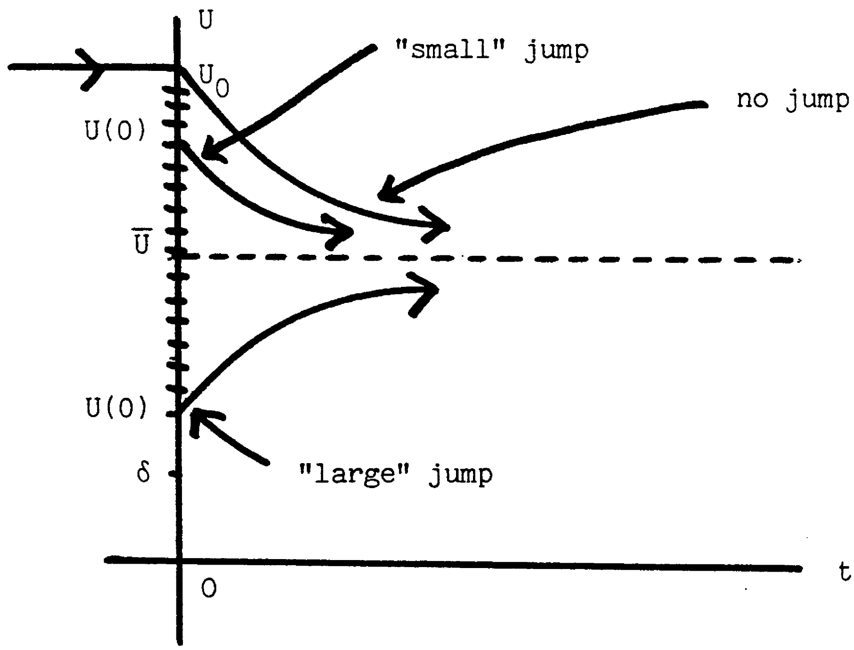
Once the initial value of U(0) has been obtained the implied optimal path for U is derived by substituting for U(0) into the solution given by equation (16a). To characterize the path of unemployment more precisely, we shall assume throughout the rest of this chapter that

$$U_0 > \bar{U} \geq \delta \qquad (21)$$

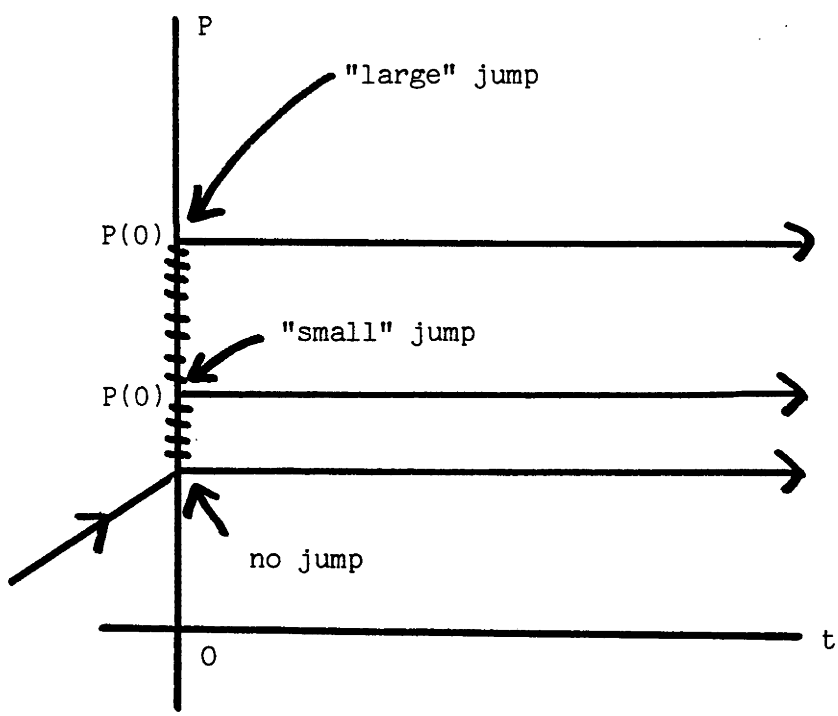
i.e., we assume that the inherited unemployment rate exceeds the natural rate, which in turn is at least as great as the socially desirable target. Using this assumption, Figures 7.1A and 7.1B illustrate the time paths of the unemployment rate and price level in the three cases corresponding to: (i) a "small" initial downward jump in the unemployment rate, such that $U_0 > U(0) > \bar{U} \geq \delta$; (ii) a "large" initial downward jump in unemployment, such that $U_0 > \bar{U} > U(0) > \delta$; and (iii) no initial jump in the unemployment rate, i.e., $U(0) = U_0$.

Starting from inherited values of U_0 and p_0, the optimal policy calls for inflation to be reduced instantaneously to zero and for unemployment to jump to $U(0)$, after which it evolves towards its natural rate $\bar{U}$. In the case where the parameters c, β, γ are such as to render only a small jump as being optimal, the unemployment rate approaches its natural rate from above. In the case where a large initial jump is optimal, the unemployment rate falls initially below the natural rate and subsequently approaches it from below. In the final case of no initial jump, the optimal unemployment rate simply evolves continuously (from above) towards its natural rate. The two figures also illustrate the tradeoff between the initial once-and-for-all jump in the price level and the subsequent intertemporal adjustment in the unemployment rate. The lower the rate of unemployment desired along the optimal path, the larger is the required initial increase in the price level.

Although the optimal solution has been derived formally a more heuristic approach to the problem is also possible. Thus it can be seen that the optimal objectives can be decomposed into several components,



7.1A: TIME PATH FOR UNEMPLOYMENT



7.1B: TIME PATH FOR PRICE LEVEL

FIGURE 7.1

each of which is associated with a different policy instrument.

The first objective is to minimize the loss associated with inflation; i.e., to minimize

$$\int_0^\infty p^2 e^{-\gamma t} dt \tag{22}$$

Noting

$$p = p(U, m, b, G, T) \tag{2a}$$

then given values of U, m, b, and T, it is possible to ensure that p = 0 for all t, by appropriately adjusting government expenditure G.

The second objective is to choose the money-bond mix (ϑ) so that the values of m and b converge to finite steady-state equilibria. As stated earlier, such an objective has not been postulated as being a formal requirement of the model but can be justified by requiring that the dynamics of the economy are consistent with the rational expectations approach to economic modeling, i.e., the economy converges to its steady-state equilibrium.

The third objective is to minimize the loss function associated with the time path for unemployment; i.e., to minimize

$$c|U(0) - U_0| + \frac{1}{2}\int_0^\infty e^{-\gamma t}(U - \delta)^2 dt \tag{23}$$

Since U is independent of fiscal and monetary policy, a global minimum can be achieved by choosing an optimal value for the initial condition U(0). Recalling (16a), the formal solution of this problem gives precisely the same results as those given by equations (17a)-(17c).[2]

7.4 Time Consistency of the Optimal Solution

Given the possibility of jumps through the price level, the question of the time consistency of the solution to the optimal policy problem specified by equations (1), (2a)-(2e) naturally arises. More precisely, is the optimal path chosen at time t=0 consistent with the optimal path that would be chosen at some time in the future, say at t = 1? Clearly it is desirable that this be the case.

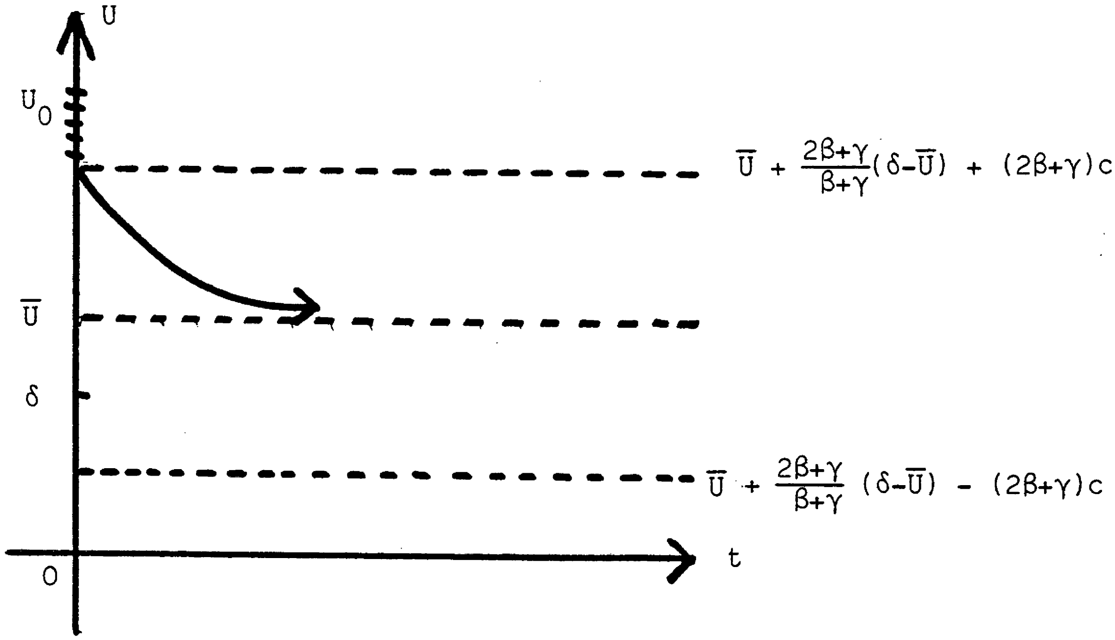
Figure 7.2A gives an example which is time consistent. This time consistency will occur provided $\bar{U}$ lies within the closed interval

$$\bar{U} + \frac{2\beta + \gamma}{\beta + \gamma}(\delta - \bar{U}) - (2\beta + \gamma)c \leqq \bar{U}$$

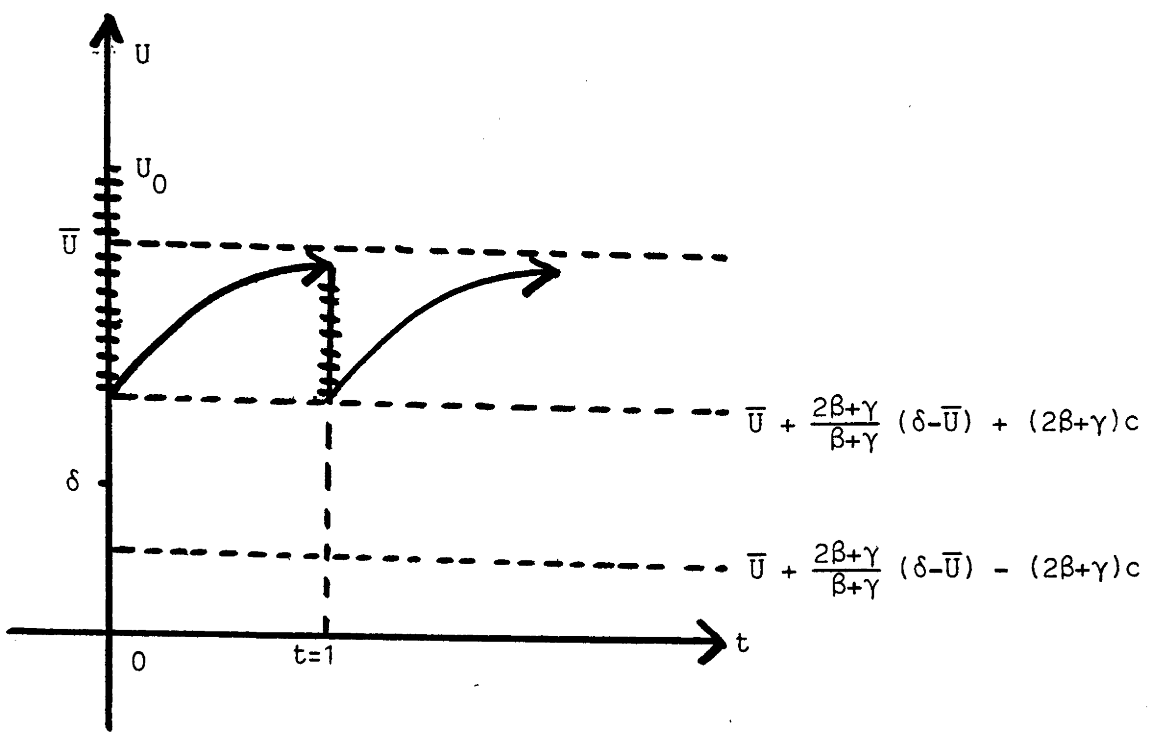
$$\leqq \bar{U} + \frac{2\beta + \gamma}{\beta + \gamma}(\delta - \bar{U}) + (2\beta + \gamma)c \qquad (24)$$

In this example, an initial jump will take the unemployment rate to the boundary of this closed interval. Thereafter, the dynamic path given by equation (16a) will drive U monotonically towards $\bar{U}$, which by (24) lies within the closed interval. Since the dynamic path never leaves the closed interval after the initial jump and since it is not optimal to jump within the closed interval, this solution is time consistent.

Figure 7.2B illustrates what will happen if inequalities (24) do not hold. In this case the solution is time inconsistent. As before, the initial jump takes unemployment to the boundary of the closed interval. However, if (24) does not hold, $\bar{U}$ lies outside the closed interval. Accordingly, the dynamic path given by (16a), which following the initial jump directs U monotonically to $\bar{U}$, takes U out of the closed interval. Figure 7.2B shows what happens if the decision maker recomputes the optimal policy at t = 1. Clearly a jump back to the boundary of the



7.2A: EXAMPLE OF TIME CONSISTENT SOLUTION



7.2B: EXAMPLE OF TIME INCONSISTENT SOLUTION

Figure 7.2

closed interval is optimal and hence the solution is time inconsistent.

Note that if the desired target rate of unemployment equals the natural rate ($\delta = \bar{U}$) then the inequalities (24) are always satisfied and the time path of unemployment is time consistent. Also, if we consider the examples given in Figures 7.1A and 7.1B, then the case of a small jump is represented by Figure 7.2A and is thus time consistent, while that of a large jump is illustrated by Figure 7.2B and is clearly time inconsistent. In general, given values of the natural rate of unemployment, $\bar{U}$, and the parameters β, γ, and c, it is always possible for the policy maker to choose his desired target rate of unemployment δ so as to ensure that the time path for the optimal unemployment rate be time consistent.

It is important to be aware that this property of the time consistency of the optimal path is sensitive to the specification of the loss function attached to the initial jumps. If, for example, this is replaced by the seemingly natural quadratic function, so that the objective function (1) becomes

$$L = \frac{1}{2} c[U(0) - U_0]^2 + \frac{1}{2} \int_0^\infty e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \qquad (1')$$

than it is shown in the Appendix that the optimal value for U(0) is given by

$$U(0) = \frac{cU_0 + \frac{\bar{U}}{2\beta + \gamma} + \frac{\delta - \bar{U}}{\beta + \gamma}}{c + \frac{1}{2\beta + \gamma}} \qquad (25a)$$

so that

$$U(0) - \bar{U} = k_1 \left((U_0 - \bar{U}) + \frac{\delta - U}{c(\beta + \gamma)} \right) \qquad (25b)$$

where

$$k_1 = \frac{c(2\beta + \gamma)}{1 + c(2\beta + \gamma)}$$

However, if following an initial jump from U_0 to $U(0)$, the policy maker immediately recomputes his optimal policy now using $U(0)$ as a given value, the corresponding optimal desired initial condition, $U(0+)$ say, now becomes

$$U(0+) - \bar{U} = k_1 \left(U(0) - \bar{U} + \frac{\delta - \bar{U}}{c(\beta + \gamma)} \right)$$

$$= k_1^2 (U_0 - \bar{U}) + k_1 (k_1 + 1) \left(\frac{\delta - \bar{U}}{c(\beta + \gamma)} \right) \qquad (26)$$

Since $U(0+) - \bar{U} \neq U(0) - \bar{U}$, the recomputed path is not continuous following the initial jump and thus the problem with a loss function specified by (1') does not have a time-consistent solution.

7.5 Concluding Remarks

This chapter has analyzed the optimal intertemporal choice of inflation and unemployment in a dynamic macro model in which inflationary expectations satisfy perfect foresight and in which endogenous jumps in the price level are permitted. We have shown that the inflation rate can be driven to zero, although there is a trade-off between an initial once-and-for-all jump in the price level, and an associated jump in the unemployment rate (if such jumps are indeed optimal), and the subsequent evolution of unemployment towards its natural rate.

Whether or not there is an initial jump in the price level has been shown to depend upon certain given aspects of the economy, such as the natural rate of unemployment, the slope of the unemployment-wage inflation in the Phillips curve, and the cost imposed on the economy by the initial jump in unemployment. In addition, it depends upon certain parameters that can be chosen by the policy maker, such as the socially desired target rate of unemployment and his rate of time preference. These factors, together with the precise specification of the loss function, affect the time consistency or otherwise of the optimal policy.

The results of this and the previous chapter differ from the optimal policies obtained for models in which expectations evolve sluggishly such as when they are generated adaptively (see e.g., Phelps [1967, 1972] and Turnovsky [1981]). Whether or not endogenous jumps in the price level are permitted has been shown to have little effect on the optimal path for inflation which, under perfect foresight, is always driven instantaneously to zero. Thus there is no real trade-off between inflation and unemployment. An important consequence of this is that preconceptions about the mechanisms by which expectations are formed can radically alter the policy maker's optimal strategy in controlling inflation and unemployment.

APPENDIX TO CHAPTER 7

In this Appendix we briefly discuss the case of initial quadratic costs, so that the objective of the policy maker is to minimize

$$L = \frac{1}{2} c[U(0) - U_0]^2 + \frac{1}{2}\int_0^\infty e^{-\gamma t}[ap^2 + (U - \delta)^2]dt \qquad (A.1)$$

subject to equations (2a)-(2e), (3a)-(3c) of the text.

The optimal path again reduces to equations (13a)-(13f), together with a transversality condition. The following transversality condition must now be satisfied at the initial time 0

$$\mu(0) + \varphi_1\lambda(0) + \frac{U'}{m(0)}[\eta(0) - c(U(0) - U_0)] = 0 \qquad (A.2)$$

and using (12c), (12d) this reduces to

$$\eta(0) = c[U(0) - U_0] \qquad (A.3)$$

The optimal solution for $\eta(t)$ must again satisfy (16b), where we require $D = 0$. Then substituting (A.3) into (16b) we derive

$$U(0) = \frac{cU_0 + \frac{\bar{U}}{2\beta + \gamma} + \frac{\delta - \bar{U}}{\beta + \gamma}}{c + \frac{1}{2\beta + \gamma}} \qquad (A.4)$$

The implied optimal time path for U is obtained by substituting for U(0) from (A.3) into the solution (16a). At the same time, the rate of inflation is set to zero in accordance with equation (13c).

PART 3

"SOME RELEVANT POLICY ISSUES"

CHAPTER 8

THE EFFECTS OF CHANGING THE LENGTH OF THE STANDARD WORKING WEEK

8.1 Introduction

> "All the 16 Member countries (of the Organisation for Economic Co-operation and Development (OECD)) for which fairly precise data are available have experienced decreases in actual average annual hours (worked per person) over the last 30 years. The downward trend has gathered momentum over this period from an average annual decrease of 0.3 per cent in the 1950s to one of 0.8 per cent in the 1960s and 0.9 per cent in the 1970s....
>
> "The length of the working week has also been substantially reduced between three and six hours over the last ten years in nearly half the member countries."
>
> Organisation for Economic Co-operation and Development (1982, pp. 18-19).[1]

The idea of reducing the length of the standard working week has become particularly popular because it is seen by some as a method of work-sharing that may be able to reduce the high levels of unemployment that have persisted in recent years. Popular press reports indicate that this trend is particularly marked in some European countries (especially France, but also Belgium, the Netherlands, and Italy).

This notion that the reduction of the working week will lead to a form of work-sharing assumes that workers will be willing to work fewer hours for the same hourly wage, thus creating new jobs at no cost to employers. Frequently, however, this is not the view of employees who see a reduction in the length of the standard working week, with the nominal weekly wage kept constant, as simply a surreptitious way of raising the nominal hourly wage-rate. When workers insist that their nominal weekly wage-packet remains constant, employers frequently require that any reduction in the working week is accompanied by associated productivity gains. Such gains may be achieved by changes in work rules, shorter lunch and coffee breaks, or the relaxation of union regulations (e.g., union regulations related to the number of workers required to carry out a particular set of tasks may be relaxed). In general, following a reduction in the working week, the extent to which productivity gains are achieved and nominal weekly wage-packets are reduced will be the outcome of collective bargaining and will depend on the political effectiveness of various employer and union groups.

We do not examine the process of collective bargaining here, but given that an agreement to reduce the length of the standard working week has been reached, we provide a methodology for examining the economic consequences of that agreement. The cases when the nominal hourly wage-rate and the nominal weekly wage are kept constant are both considered. Different combinations of work-week reduction and productivity gain are also examined to see what effect they have on such variables as the real wage, unemployment and the process of capital accumulation.

In order to examine the question in a scientific manner, a dynamic macro-model, which incorporates the process of capital accumulation as well as the dynamics of wage fixation, is constructed. In this model, the dynamics of wage fixation are determined by a Phillips curve. The manner of incorporating the dynamics of capital accumulation is related to earlier works (see Blinder and Solow (1973), Infante and Stein (1976), Tobin and Buiter (1976)). In particular we derive a model which is much in the spirit of the work of Turnovsky (1977, 1980) and which incorporates an equity market, and inflation, as well as inflationary expectations which satisfy perfect myopic foresight. The particular economy that we examine is assumed to be highly capital intensive, with significant capital depreciation and to be characterized by the absence of speculative bubbles, as suggested by the empirical work of Flood and Garber (1980).

The rest of this chapter will proceed as follows: Sections 8.2 and 8.3 provide microfoundations for the household sector and the corporate sector. Sections 8.4 and 8.5 incorporate the derived functions in a larger macro-framework. Section 8.6 examines the long-run comparative statics while the dynamics of the economy and the short-run responses required to maintain stability are examined in Sections 8.7 and 8.8. Section 8.9 provides some concluding comments.

8.2 The Household Sector

We assume that households all have similar optimization decisions to that of a basic representative household. We shall assume that members of all households must spend a total of $\overline{N}$ hours in each time period, either working or looking for work. The objective of this representative household is then to choose its planned consumption, demand for money, demand for government bonds and demand for equities so as to maximize the intertemporal utility function

$$\int_0^T e^{-\beta t} U(c, \overline{N}, m)dt + S(A(T)) \tag{1a}$$

$$U_1 > 0,\ U_2 < 0,\ U_3 > 0$$

$$U_{ii} < 0, \text{ for all } i$$

$$U_{ij} = 0, \; i \neq j$$

$$S' > 0, \; S'' < 0$$

subject to

$$c + \dot{m} + Q\dot{E} + \dot{b} = \rho v\overline{N} + dQE + ib - \pi m - T_h \tag{1b}$$

$$A = m + QE + b \tag{1c}$$

$$A(0) = A_0 \tag{1d}$$

where:

c = real private consumption plans by households

$\overline{N}$ = real supply of labor by households (assumed exogenous) expressed in man-hours

m = demand for real money balances

b = demand for real government bonds

E = demand for equities

Q = (expected) price of equities

ρ = (expected) employment rate

A = real private wealth

v = (expected) real hourly wage-rate

T_h = taxes, assume exogenous

D = (expected) real dividends

$d = D/(QE)$ = (expected) dividend yield, taken to be parametrically given to household sector

r = nominal rate of interest on bonds and equities which are assumed perfect substitutes

π = (expected) rate of inflation of price level

$q = \dot{Q}/Q$ = (expected) rate of inflation of equity prices

$i = r - \pi = d + q$ = real rate of return on equities and bonds which are assumed perfect substitutes

β = consumer's rate of time preference.

The utility function is assumed to be concave in its three arguments, c, $\overline{N}$ and m. Following Brock (1974) and Brock and Turnovsky (1981), real money balances have been incorporated into the utility function as a mechanism for capturing the reasons for holding money in a world of certainty.[2] The utility function is assumed additively separable in all arguments purely as a means of simplifying the analysis. Equation (1a) then defines the aggregate utility of the household sector as dependent on aggregate consumption, labor supply and money demand over all future time periods and also dependent on the utility obtained from leaving a bequest.

The budget constraint faced by the household is given by equation (1b) which is expressed in real flow terms. At each point in time the composite household can acquire income from a variety of sources. Individuals in this household have an expected probability, ρ, of being employed and, if employed, they supply labor to firms, at a real hourly wage-rate, v; they receive dividend yields, d, on their holdings of equities and they face an inflation tax ($=\pi m$) on their holdings of real money. They also pay taxes. Their income can be spent on the purchase of real consumption goods or in the acquisition of new holdings of real private wealth. Equation (1c) tells us that wealth can be held either in the form of government bonds, equities or money. Equation (1d) gives the household's initial endowment of real private wealth.

Crucial for the development of the model is the mechanism by which expectations are formed. Throughout this chapter we shall assume that, given the current time period t, the value of all variables prior to and including t are known with certainty. In the derivation of the microfoundations of the household sector we shall then assume that expectations of future values of particular variables are determined by assuming that all future values are the same as the current value. Stated another way: current and past expectations are assumed to satisfy perfect myopic foresight; future expectations satisfy a static expectation hypothesis.

The notion of a static expectations hypothesis needs to be made more explicit. This model will satisfy a static expectations hypothesis in which the future values of the variables which face each individual in the economy are formed by static expectations. These variables are the employment rate (ρ), the real hourly wage-rate (v), the real rate of interest (i) and the rate of inflation (π).

Given a mechanism for the formation of expectations, the household's optimization decision can be described as follows: at any current instant of time, all members of the household know with certainty the value of all variables. Using its expectations about the future, the household determines its optimal consumption and demand for assets for that instant and for all future time. When at the next instant of time the values of the appropriate variables become known, the household compares actual values with the expectations and, if these diverge, then the household's decision variables are revised on the basis of the new value of the exogenous variables and current expectations.

We are now in a position to examine the solution to the household's optimization problem. Firstly, we rewrite equation (1b) as

$$c + \dot{A} = \rho v\overline{N} + iA - (i + \pi)m - T_h \qquad (1b')$$

The necessary conditions for the problem given by (1a), (1b'), (1c), (1d) can be derived as follows:

Let

$$H = e^{-\beta t}U(c,\overline{N},m) + e^{-\beta t}\psi[\rho v\overline{N} + iA - (i+\pi)m - c - T_h]$$

where H is a Hamiltonian function and $e^{-\beta t}\psi$ is the discounted co-state variable.

Then the necessary conditions are given by[3]:

$$U_1 = \psi \tag{2a}$$

$$U_3 = (i+\pi)\psi \tag{2b}$$

$$\dot{\psi} = (\beta-i)\psi \tag{2c}$$

$$e^{-\beta T}\psi(T) = S'(A(T)) \tag{2d}$$

By equations (2c), (2d),

$$\psi(t) = e^{(\beta-i)(t-T)}\psi(T) \tag{3a}$$

$$\psi(t) = e^{\beta t-i(t-T)}S'(A(T)) \tag{3b}$$

Hence from equations (2), (3)

$$c = c(\underset{-}{\psi}) \tag{4a}$$

$$m = m(\underset{-}{\psi},\ \underset{-}{i+\pi}) \tag{4b}$$

$$\psi = \psi(\underset{+}{i},\ \underset{-}{A(T)}) \tag{4c}$$

But using equation (1b') and equation (4a)

$$A(T) = f(t,\ \underset{+}{A},\ \underset{+}{i},\ \underset{+}{\rho v\overline{N}},\ \underset{+}{\psi},\ \underset{+}{-(i+\pi)m},\ \underset{-}{T_h}) \tag{5a}$$

Then using equation (4c),

$$\psi = g(t,\ \underset{-}{A},\ \underset{+}{i},\ \underset{-}{\rho v\overline{N}},\ \underset{-}{-(i+\pi)m},\ \underset{+}{T_h}) \tag{5b}$$

Note that the sign of the partial derivative w.r.t. i depends on a negative wealth effect derived from equation (5a) and a positive substitution effect derived from equation (4c). Throughout this chapter we have assumed that the positive substitution effect dominates.

Substituting equation (5b) into equation (4b) gives

$$m = m(t,\ \underset{+}{A},\ \underset{-}{i},\ \underset{+}{\rho v\bar{N}},\ \underset{-}{i+\pi},\ \underset{-}{T_h}) \tag{5c}$$

Then substituting (5c) into (5b) we obtain

$$\psi = \psi(t,\ \underset{-}{A},\ \underset{+}{i},\ \underset{-}{\rho v\bar{N}},\ \underset{+}{i+\pi},\ \underset{+}{T_h}) \tag{5d}$$

Note that the sign of the partial derivative w.r.t. each variable in equation (5b) is dependent on a direct effect and also on an indirect effect which comes through the variable m. We have followed conventional practice in equation (5d) and assumed that the direct effect dominates in each case. (In other words, we have assumed that the effect through m is minimal.)

Hence using equations (4a), (4b), (5d) we can define the household sector as follows:

$$C = C(\underset{+}{A},\ \underset{-}{i},\ \underset{+}{\rho v\bar{N}},\ \underset{+}{-\pi},\ \underset{-}{T_h}) \tag{6a}$$

$$m^d = L(\underset{+}{A},\ \underset{-}{i},\ \underset{+}{\rho v\bar{N}},\ \underset{+}{-\pi},\ \underset{-}{T_h}) \tag{6b}$$

where C denotes planned consumption, m^d denotes aggregate demand for money and the demand for bonds-cum-equities has been eliminated by Walras' Law, since the household will later be incorporated within an equilibrium model.[4]

8.3 The Corporate Sector

The corporate sector will be represented by a single representative firm which finances all of its investments through the sale of equities to stockholders (from the household sector) and which returns all profits to its stockholders in the form of dividends.[5] The constraints facing the firm can be summarized as follows[6]:

$$Y = \eta(\overline{N})F(K,N^D) = \eta(\overline{N})N^D F(\frac{K}{N^D}) \tag{7a}$$

$$F' > 0,\ F'' < 0,\ \eta' < 0$$

$$\Pi = Y - vN^D - h(I) \qquad h' > 0,\ h'' > 0 \tag{7b}$$

$$\Pi = D + T_f \tag{7c}$$

$$I = \dot{K} + \lambda K \tag{7d}$$

$$I = Q\dot{E} \tag{7e}$$

$$K(0) = K_0,\ E(0) = E_0 \tag{7f}$$

where

Y = real output

K = demand for physical capital by the representative firm

N^D = real demand for labor by the representative firm expressed in man-hours

Π = real gross profit

Q = price of equities

E = quantity of equities, issued by the representative firm

D = dividends to the stockholders

T_f = corporate taxes (assumed exogenous)

$\overline{N}$ = supply of labor by the households to the representative firm expressed in man-hours

v = real hourly wage-rate, taken as exogenous by the representative firm

$\eta(\overline{N})$ = productivity factor for given supply of labor, $\overline{N}$

λ = rate of capital depreciation (assumed constant).

Equation (7a) describes the production function of the representative firm, which is assumed to have the usual neoclassical properties of positive but diminishing marginal productivities and constant returns to scale. In addition it is assumed that if the management lowers the number of hours that workers are required to work it can use this to negotiate productivity gains within the firm. These productivity gains are represented by the function $\eta(\overline{N})$.

Equation (7b) defines profits as depending on real output, less wage-costs and less the costs of adjustment associated with new investment. Following Treadway (1969) and Gould (1968), we have assumed that the adjustment cost function is convex to the origin. Equation (7c) defines the distribution of profits which are paid either as dividends to the stockholders or as corporate taxes to the government.

The firm's investment function is given by equation (7d) which tells us that there is a constant rate of depreciation, λ, and that all new investment is used to either replace depreciated capital or increase the capital stock. Equation (7e) states that the firm finances all of its investment through the sale of equities to stockholders.

The market value of the firm's securities outstanding at time t, is given by

$$V(t) = Q(t)E(t)$$

Because the representative firm has a large number of stockholders with different life-expectancies, we shall assume that the firm tries to maximize the initial present discounted value of these securities. Now,

$$V = QE \tag{8a}$$

$$\Rightarrow \quad \dot{V} = Q\dot{E} + \dot{Q}E \tag{8b}$$

$$\Rightarrow \quad \dot{V} = \dot{K} + \dot{Q}E + \lambda K \tag{8c}$$

$$\Rightarrow \quad \dot{V} + \Pi = \dot{K} + \dot{Q}E + D + T_f + \lambda K \tag{8d}$$

$$\Rightarrow \quad \dot{V} + \Pi = \dot{K} + iQE + T_f + \lambda K \tag{8e}$$

$$\Rightarrow \quad \dot{V} + \gamma = iV \tag{8f}$$

where

$$\gamma = \Pi - \dot{K} - T_f - \lambda K \tag{8g}$$

i.e.,

$$\gamma = \eta(\overline{N})F(K,N^D) - vN^D - h(I) - I - T_f \tag{8h}$$

As for the household sector, it is necessary to define a mechanism for the formation of expectations. We shall again assume that current and past expectations satisfy perfect myopic foresight; future expectations satisfy a static expectations hypothesis. In this instance static expectations means that all future values of the rate of return on equities (i) and the real wage-rate (v) are assumed to be constant.

Then from equation (8f)

$$V(s) = e^{is}\,[X - \int_0^s e^{-it}\gamma(t)dt] \tag{9a}$$

and, assuming $i > 0$, then in order for $V(s)$ to remain finite as $s \to \infty$

we require that

$$X = \int_0^\infty e^{-it}\gamma(t)dt \tag{9b}$$

and hence the value of the firm at any arbitrary time s in the future is given by

$$V(s) = QE(s) = \int_s^\infty e^{i(s-t)}\gamma(t)dt \tag{9c}$$

At any time s, we shall assume that the representative firm's objective is to maximize the real market value of its securities, $V(s) = QE(s)$. Since this is equivalent to maximizing the initial value of its securities $V(0) = QE(0)$, the firm's decision problem is to choose production decisions K, N^D and supply of equities, QE, so as to maximize an objective function of the form

$$V(0) = \int_0^\infty e^{-it}[\eta(\overline{N})F(K,N^D) - vN^D - h(I) - I - T_f]dt \tag{10a}$$

subject to

$$\dot{K} = I - \lambda K \tag{10b}$$

The Hamiltonian for this problem is given by

$$H = e^{-it}\{\eta(\overline{N})F(K,N^D) - vN^D - h(I) - I - T_f\}$$
$$+ e^{-it}\psi\{I - \lambda K\} \tag{11}$$

where $e^{-it}\psi$ is the discounted co-state variable associated with K.

Then the necessary conditions for this problem are given by:

$$\psi = h'(I) + 1 \tag{12a}$$

$$\dot{\psi} = (i + \lambda)\psi - \eta(\overline{N})F'(\frac{K}{N^D}) \tag{12b}$$

$$v = \eta(\overline{N})G(\frac{K}{N^D}) \tag{12c}$$

$$I = \dot{K} + \lambda K \tag{12d}$$

$$\lim_{t\to\infty} e^{-it}\psi K = 0 \tag{12e}$$

where $G(\frac{K}{N^D}) = F(\frac{K}{N^D}) - (\frac{K}{N^D}) F' (\frac{K}{N^D})$

and $G' (\frac{K}{N^D}) = - (\frac{K}{N^D}) F'' (\frac{K}{N^D}) > 0.$

Now from equation (12b)

$$\psi(s) = e^{(i+\lambda)s} \{ X - \int_0^s e^{-(i+\lambda)t}\eta(\overline{N})F'(\frac{K}{N^D})dt\} \tag{13a}$$

Application of the transversaltiy condition given by equation (12e) gives:

$$X = \int_0^\infty e^{-(i+\lambda)t}\eta(\overline{N})F'(\frac{K}{N^D}) \tag{13b}$$

Hence

$$\psi(s) = e^{(i+\lambda)s}\{ \int_s^\infty e^{-(i+\lambda)t}\eta(\overline{N})F'(\frac{K}{N^D}) \, dt \} \tag{13c}$$

Hence

$$\psi = \psi(\underset{-}{i}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}) \tag{13d}$$

Then using equation (12a)

$$I = I(\underset{-}{i}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}) \tag{14a}$$

But from equation (12c)

$$\frac{K}{N^D} = \frac{K}{N^D}(\underset{+}{v}, \underset{-}{\eta(\overline{N})}) \qquad (14b)$$

Hence,

$$N^D = N^D(\underset{+}{K}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}) \qquad (14c)$$

From equation (9c),

$$QE(s) = e^{is}\int_s^\infty e^{-it}[\eta(\overline{N})N^D F(\frac{K}{N^D}) - vN^D - h(I) - I - T_f]dt \qquad (15a)$$

But, from equation (12c),

$$\eta(\overline{N})N^D F(\frac{K}{N^D}) - vN^D = \eta(\overline{N})KF'(\frac{K}{N^D}) \qquad (15b)$$

Hence,

$$QE(s) = e^{is}\int_s^\infty e^{-it}[\eta(\overline{N})KF'(\frac{K}{N^D}) - h(I) - I - T_f]dt \qquad (15c)$$

Thus using equations (14a)-(14c) we can derive the following equation for the supply of equities:

$$QE = QE(\underset{+}{K}, \underset{-}{i}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}, \underset{-}{T_f}) \qquad (16)$$

where the sign of the partial derivatives w.r.t. i, v and $\eta(\overline{N})$ assume that the partial derivatives of investment given in equation (14a) are relatively small.[7]

8.4 A Dynamic Macro-Model

We are now in a position to specify a macro-model which incorporates the household sector and corporate sector that have been derived in Sections 8.2 and 8.3 of this chapter. The model, which assumes that labor demand is less than labor supply, i.e., $N^D < \overline{N}$, is given by the following equations:

$$Y = C + I + G_o \tag{17a}$$

$$C = C(\underset{+}{A}, \underset{-}{i}, \underset{+}{\rho v \overline{N}}, \underset{+}{-\pi}, \underset{-}{T_h}) \tag{17b}$$

$$m = L(\underset{+}{A}, \underset{-}{i}, \underset{+}{\rho v \overline{N}}, \underset{+}{-\pi}, \underset{-}{T_h}) \tag{17c}$$

$$I = I(\underset{-}{i}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}) \tag{17d}$$

$$QE = QE(\underset{+}{K}, \underset{-}{i}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}, \underset{-}{T_f}) \tag{17e}$$

$$A = m + b + QE \tag{17f}$$

$$Y = \eta(\overline{N})N^D F(\frac{K}{N^D}) \tag{17g}$$

$$v = \eta(\overline{N})G(\frac{K}{N^D}) = \eta(\overline{N})\{F(\frac{K}{N^D}) - (\frac{K}{N^D})F'(\frac{K}{N^D})\} \tag{17h}$$

$$U = \frac{\overline{N} - N^D}{\overline{N}} \tag{17i}$$

$$w = -\alpha(U) + \pi \tag{17j}$$

$$\rho = \frac{N^D}{\overline{N}} \tag{18a}$$

$$\pi = p \tag{18b}$$

$$\dot{K} = I - \lambda K \tag{19a}$$

$$\dot{v} = v(w - p) \tag{19b}$$

$$\dot{m} + \dot{b} = G_0 - T + rb - p(m + b) \tag{19c}$$

$$m = \overline{m} \tag{19d}$$

where

Y = real national output

C = aggregate planned consumption

I = aggregate planned investment

G_o = real government expenditure assumed exogenous

A = real private wealth

r = nominal rate of return on bonds-cum-equities

π = expected rate of inflation

$i = r - \pi$ = real rate of return on bonds-cum-equities

ρ = expected employment rate

v = real hourly wage-rate

$\overline{N}$ = supply of labor (expressed in man-hours and assumed exogenous)

T_h = level of taxes on households assumed exogenous

T_f = level of corporate taxes assumed exogenous

$T = T_h + T_f$ = total tax receipts

m = real supply of (and demand for) money

b = real supply of (and demand for) bonds

QE = real supply of (and demand for) equities

K = real level of physical capital

N^D = real demand for labor (expressed in man-hours)

p = actual inflation rate

U = unemployment rate

w = rate of wage growth

Equilibrium in the product market is described by equation (17a) while equations (17b)-(17e) describe planned consumption, the demand for money, planned investment and the supply of equities respectively. Equation (17f) is the definition of real private wealth. The production function is defined in (17g) while equation (17h) tells us that the real wage-rate is equal to the marginal product of labor. Equation (17i) defines the unemployment rate and equation (17j) is a standard version of the Phillips Curve.

Equations (18a)-(18b) define the expected employment rate and the expected rate of inflation both of which are assumed to satisfy perfect myopic foresight.

The dynamics of the model are given by equations (19a)-(19d). The rate of capital accumulation is defined as equal to the rate of investment less capital depreciation. The evolution of the real wage is defined in the standard manner and the supply of money and bonds is constrained by the government budget constraints. The monetary policy, given by equation (19d), is such that the real supply of money is kept constant.

The short-run model is represented by 12 equations (equations 17a-17j, 18a-18b). In the short-run the dynamic variables (K, v, m and b) are fixed and the 12 endogenously determined variables are given by Y, C, I, A, U, N^D, w, QE, i, π, ρ, p. In subsequent parts of this chapter we shall restrict our analyses to examining the short-run and long-run effects of a change in the supply of labor $\overline{N}$. As can be seen from the model, changes in $\overline{N}$ will have an effect in the short-run through changes in the level of unemployment, negotiated changes in the nominal hourly wage-rate and through induced changes in productivity (i.e., $\eta(\overline{N})$). In the long-run there is no unemployment, and changes in $\overline{N}$ will have an effect on the full employment level of output as well as on such variables as the hourly real wage-rate and the long-run capital stock.

8.5 Simplifying the Model

In subsequent sections of this chapter, merely in order to simplify the analysis, we shall assume that the wealth effects in planned consumption and the demand for money are close to zero and can be ignored. The model given by equations (17)-(19) can then be rewritten in the form:

$$Y = C(\underset{-}{i}, \underset{+}{vN^D}, \underset{+}{-\pi}) + I(\underset{-}{i}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}) + G \tag{21a}$$

$$m = L(\underset{-}{i}, \underset{+}{vN^D}, \underset{+}{-\pi}) \tag{21b}$$

$$Y = \eta(\overline{N})N^D F\left(\frac{K}{N^D}\right), \quad F' > 0 \tag{21c}$$

$$v = \eta(\overline{N})G\left(\frac{K}{N^D}\right), \quad G' > 0 \Rightarrow N^D = N^D(\underset{+}{K}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}) \tag{21d}$$

$$\dot{v} = -\alpha v U(\underset{-}{K}, \underset{+}{v}, \underset{+}{\overline{N}}, \underset{-}{\eta(\overline{N})}) \tag{22a}$$

$$\dot{K} = J(\underset{-}{K}, \underset{-}{i}, \underset{-}{v}, \underset{+}{\eta(\overline{N})}) \tag{22b}$$

$$\dot{m} + \dot{b} = G - T + ib - \pi m \tag{22c}$$

$$m = \overline{m} \tag{22d}$$

Throughout this chapter we assume that the economy described by the model is highly capital intensive (i.e., characterized by a high capital/labor ratio, $K/(N^D)$). As a result, G' (= $[K/(N^D)]F''$) will be large. In this case the cost of labor will be a relatively unimportant part of the firm's decision process and hence the demand for labor, N^D, the unemployment rate, U, and the aggregate output, Y, will be relatively insensitive to the real wage-rate.[8] Also investment will only be affected by the real wage-rate to the extent that the interest rate is affected by the real wage, i.e., $I = I(i, \eta(\overline{N}))$.

Henceforth we shall let z (= log v) denote the logarithm of the real wage-rate. Then, assuming that the economy is highly capital intensive, equations (21a)-(21d) can be solved to give short-run solutions for the real rate of interest, i, and the rate of inflation, π, in terms of the dynamic variables z, K, and m and the exogenous variables $\eta(\overline{N})$. In general, because of the complexity of the model, the partial derivatives of i and π are ambiguous. However it is possible to simplify the model given by equations (21a)-(21d) so that plausible effects can be calculated. If we give emphasis to the dominant effects these equations can be reduced to the following IS/LM model

$$\eta(\overline{N})Z(\underset{+}{K}) = D(\underset{+}{z}, \underset{-}{i}, \underset{-}{\pi}) + G \qquad (23a)$$

$$m = L(\underset{-}{i}, \underset{-}{\pi}) \qquad (23b)$$

Then if we further assume that D_π is non-zero but sufficiently small, the short-run solutions for i and π are given by

$$i = i(\underset{-}{K}, \underset{+}{z}, \underset{+}{m}, \underset{-}{\eta(\overline{N})}) \qquad (24a)$$

$$\pi = \pi(\underset{+}{K}, \underset{-}{z}, \underset{-}{m}, \underset{+}{\eta(\overline{N})}) \qquad (24b)$$

These sign values are consistent with results derived under perfect myopic foresight in simpler models (see Table 2.1C of Chapter 2 and equations (4a), (4b) of Chapter 5).

The short-run solutions for i and π have some consequences which might contradict the preconceived notions of some economists. Thus for example, an increase in the real wage is deflationary. The reason for this seemingly perverse effect can be seen from equations (23a)-(23b). In order for product market equilibrium to be maintained, an increase in z must be matched by an increase in i (since D_π is close to zero). Then, in order to maintain equilibrium in the money market, this increase in i must be matched by a corresponding decrease in π.

Intuitively this can be justified by noting that the level of inflation, which is most closely affected by the price level, is directly influenced by the relationship between the price of goods (P) and the cost of labor (W). Thus if the price level increases (falls) by more than the nominal wage then z will fall (rise) and inflation will tend to rise (fall).

Having derived equations for i and π we can now describe a simplified version of the model by the following equations:

$$i = i(\underset{-}{K}, \underset{+}{z}, \underset{+}{m}, \underset{-}{\eta(\overline{N})}) \qquad (24a)$$

$$\pi = \pi(\underset{+}{K}, \underset{-}{z}, \underset{-}{m}, \underset{+}{\eta(\overline{N})}) \qquad (24b)$$

$$\dot{z} = -\alpha U(\underset{-}{K}, \underset{+}{\overline{N}}, \underset{-}{\eta(\overline{N})}) \qquad (24c)$$

$$\dot{K} = J(\underset{-}{K},\ \underset{-}{i},\ \underset{+}{\eta(\overline{N})}) \tag{24d}$$

$$\dot{m} + \dot{b} = G_0 - T + ib - \pi m \tag{24e}$$

$$m = \overline{m} \tag{24f}$$

8.6 Long-Run Comparative Statics

In order to examine the effects of a change in the standard working week, i.e., $\overline{N}$, it is first necessary to examine the long-run comparative statics.

The long-run model derived from equations (24a)-(24f) is given by the following set of equations:

$$U(\underset{-}{K},\ \underset{+}{\overline{N}},\ \underset{-}{\eta(\overline{N})}) = 0 \tag{25a}$$

$$J(\underset{-}{K},\ \underset{-}{i},\ \underset{+}{\eta(\overline{N})}) = 0 \tag{25b}$$

$$G_0 - T + ib - \pi\overline{m} = 0 \tag{25c}$$

$$i = i(\underset{-}{K},\ \underset{+}{z},\ \underset{+}{\overline{m}},\ \underset{-}{\eta(\overline{N})}) \tag{25d}$$

$$\pi = \pi(\underset{+}{K},\ \underset{-}{z},\ \underset{-}{\overline{m}},\ \underset{+}{\eta(\overline{N})}) \tag{25e}$$

In the long-run unemployment is driven to zero and all capital accumulation and wealth accumulation cease. Then rewriting equation (25c) as

$$V(\underset{+}{i},\ \underset{-}{\pi},\ \underset{+}{b},\ \underset{-}{\overline{m}}) = 0 \tag{25c'}$$

we can substitute equations (25d)-(25e) into equations (25a), (25b), (25c') to obtain the following steady-state system[9]:

$$U(\underset{-}{K}, \underset{+}{\overline{N}}, \underset{-}{\eta(\overline{N})}) = 0 \tag{26a}$$

$$J(\underset{-}{K}, \underset{-}{z}, \underset{-}{\overline{m}}, \underset{+}{\eta(\overline{N})}) = 0 \tag{26b}$$

$$V(\underset{-}{K}, \underset{+}{z}, \underset{+}{b}, \underset{?}{\overline{m}}, \underset{-}{\eta(\overline{N})}) = 0 \tag{26c}$$

The long-run comparative static effects of a change in $\overline{N}$ are then given by:

$$\frac{\partial \tilde{K}}{\partial \overline{N}} = \frac{-U_2 - U_3\eta'}{U_1} > 0 \tag{27a}$$

$$\frac{\partial \tilde{z}}{\partial \overline{N}} = \frac{1}{\Delta}\{-U_1J_4\eta'V_3 + (U_2 + U_3\eta')J_1V_3\} < 0 \tag{27b}$$

$$\frac{\partial \tilde{b}}{\partial \overline{N}} = \frac{1}{\Delta}\{U_1(V_2J_4\eta' - J_2V_5\eta') + (U_2 + U_3\eta')(J_2V_1 - J_1V_2)\}$$

$$\gtrless 0 \tag{27c}$$

where $\Delta = U_1J_2V_3 > 0$ and a tilde denotes the steady-state.

In general, therefore, a lowering of the length of the standard working week (i.e., a decrease in $\overline{N}$) will lead to a fall in the steady-state level of capital stock and a rise in the steady-state hourly real wage-rate. If the reduction in the length of the standard working week also leads to gains in productivity (i.e., $\eta' < 0$) then the sign of these effects will be the same although the magnitude will be increased.

Intuitively, the decrease in $\overline{N}$ has made labor more scarce and thus pushed up its price (i.e., pushed up the hourly real wage-rate). Also by lowering the equilibrium demand for labor the decrease in $\overline{N}$ has lowered the equilibrium level of capital stock that is required to support this work-force.

It is interesting to note that while the worker's hourly real wage-rate will definitely increase, we cannot say for certain whether the worker's weekly real wage-packet will increase or decrease. However, in general we can conclude that the weekly real wage would only increase if productivity gains were sufficiently large (i.e., η' sufficiently large negative).

In general the decrease in $\overline{N}$ will push up the real interest rate, lower the long-run level of inflation and will also lead to a decrease in the demand for labor, N^D, and a decrease in real output, Y, unless productivity gains offset the other results of the change in $\overline{N}$, while the effect on the equilibrium level of bonds is ambiguous. However, if we assume that V_1 and V_2 are relatively insignificant compared with U_1, J_1 and J_2 (i.e., the effect of a change in the capital stock or the real wage on the government budget constraint is relatively insignificant) then a decrease in the length of the standard working week will raise the supply of bonds in the long-run (i.e., $\frac{\partial\tilde{b}}{\partial\overline{N}} < 0$). Henceforth we shall assume that this is the case.

8.7 Dynamics

Having examined the long-run effects of a change in the length of the standard working week, we can now analyze the dynamics of the economy. The dynamic system given by equations (24a)-(24f) can be linearized about the steady state to give the following linearized version of the model:

$$\begin{pmatrix} \dot{K} \\ \dot{z} \\ \dot{b} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & 0 & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} K - \tilde{K} \\ z - \tilde{z} \\ b - \tilde{b} \end{pmatrix} \tag{28}$$

where

$$a_{11} = J_K < 0 \tag{29a}$$

$$a_{12} = J_z < 0 \tag{29b}$$

$$a_{21} = -\alpha U_K > 0 \tag{29c}$$

$$a_{31} = i_K b - \pi_K \overline{m} < 0 \tag{29d}$$

$$a_{32} = i_z b - \pi_z \overline{m} > 0 \tag{29e}$$

$$a_{33} = i > 0 \tag{29f}$$

The characteristic roots of this system then satisfy the following constraints:

$$\lambda_1 + \lambda_2 = a_{11} < 0 \tag{30a}$$

$$\lambda_1 \lambda_2 = -a_{12} a_{21} > 0 \tag{30b}$$

$$\lambda_3 = i > 0 \tag{30c}$$

Hence λ_1 and λ_2 have negative real parts and the system has one unstable root given by λ_3. We can now solve equation (28) to give a dynamic system of the form

$$\begin{pmatrix} K - \tilde{K} \\ z - \tilde{z} \\ b - \tilde{b} \end{pmatrix} = \begin{pmatrix} \lambda_1 & \lambda_2 & 0 \\ a_{21} & a_{21} & 0 \\ q_{31} & q_{32} & 1 \end{pmatrix} \begin{pmatrix} k_1 e^{\lambda_1 t} \\ k_2 e^{\lambda_2 t} \\ k_3 e^{\lambda_3 t} \end{pmatrix} \tag{31}$$

where

$$q_{3j} = \frac{a_{31}\lambda_j + a_{32}a_{21}}{\lambda_j - a_{33}}, \quad j = 1,2 \qquad (32)$$

The linearized system given by equation (31) can be rewritten in the form[10]:

$$\frac{1}{a_{21}(\lambda_2-\lambda_1)}\begin{bmatrix} -a_{21} & \lambda_2 & 0 \\ a_{21} & -\lambda_1 & 0 \\ h_{31} & h_{32} & a_{21}(\lambda_2-\lambda_1) \end{bmatrix}\begin{bmatrix} K-\tilde{K} \\ z-\tilde{z} \\ b-\tilde{b} \end{bmatrix} = \begin{bmatrix} k_1e^{\lambda_1 t} \\ k_2e^{\lambda_2 t} \\ k_3e^{\lambda_3 t} \end{bmatrix} \qquad (33)$$

where

$$h_{31} = a_{21}(q_{31} - q_{32}) \qquad (34a)$$

$$h_{32} = \lambda_1 q_{32} - \lambda_2 q_{31} \qquad (34b)$$

Hence

$$h'_{31} = \frac{h_{31}}{a_{21}(\lambda_2-\lambda_1)} = \frac{a_{33}a_{31} + a_{32}a_{21}}{(\lambda_1-a_{33})(\lambda_2-a_{33})} \gtrless 0 \qquad (34a')$$

and

$$h'_{32} = \frac{h_{32}}{a_{21}(\lambda_2-\lambda_1)}$$

$$= \frac{-a_{31}\lambda_1\lambda_2 + a_{33}a_{32}a_{21} - a_{32}a_{21}(\lambda_1 + \lambda_2)}{a_{21}(\lambda_2 - a_{33})(\lambda_1 - a_{33})} > 0 \qquad (34b')$$

For the purposes of exposition we shall henceforth assume that $h'_{31} < 0$.

8.8 Short-Run Comparative Statics

We are now in a position to use equation (33) to examine the short-run responses of the economy to a change in the length of the standard working week. Since we are assuming the absence of speculative bubbles, any short-run response of the economy will be such as to keep the economy on a stable path which converges to the long-run equilibrium described by equations (25a)-(25e).

The most obvious way in which this stable path could be achieved is by a jump in the price level, P, brought about by market forces. Although there are still no formal microfoundations which justify this market response by the price level to stabilize the economy there is some justification for the absence of speculative bubbles in the empirical work of Flood and Garber (1980) and the theoretical work of Brock (1974, 1977)[11] and we shall follow the rational expectations approach to economic modeling by assuming that the price level acts in the short-run so as to stabilize the economy.

In the short-run the capital stock will remain constant. Also in a capital intensive economy labor demand is independent of the price level. Thus following a reduction in the length of the standard working week, unemployment will fall in the short-run and aggregate output will remain constant or may even rise if there are associated productivity gains (i.e., $\eta' < 0$). If the reduction in the length of the working week is not accompanied by any exogenous change in the nominal hourly wage-rate, W, then the short-run response of the price level, P, is given by

$$\frac{\partial P}{\partial \overline{N}} = \frac{-\left(h'_{31}\frac{\partial \tilde{K}}{\partial \overline{N}} + h'_{32}\frac{\partial \tilde{z}}{\partial \overline{N}} + \frac{\partial \tilde{b}}{\partial \overline{N}}\right)P_0}{h'_{32} + b} > 0 \qquad (35)$$

Thus, a decrease in the length of the standard working week, which we have already shown will lead to a decline in the long-run rate of inflation may also lead to a fall in the short-run price level. Accordingly, a decrease in the length of the standard working week may also lead to an increase in the hourly real wage-rate and, unless productivity gains are significant (i.e., unless η' is large negative), may also lead to a decline in the short-run rate of inflation. Whether or not the increase in the hourly real wage-rate is enough to maintain the weekly wage-packet will depend upon the parameters of the economy and, in general, is indeterminate.

Intuitively, the decline in the price level following a reduction in the length of the standard working week can be justified by observing that the rational expectations approach to economic modeling is essentially a forward looking approach. Thus the agents in the economy observe that in the long-run output will be reduced and so will inflation and take account of this in setting their current prices, thus revising current prices downward.

The short-run response to a reduction in the length of the standard working week may not only involve a jump in the price level, P, but may also involve an exogenous response in the nominal hourly wage-rate, W, which could result from political pressure exerted by workers to maintain their nominal weekly wage-packet despite a reduction in the length of the working week. For example, if the working week is reduced from 40 hours to 35 hours then if the nominal weekly wage-packet is to remain the same, this would require an increase of the nominal hourly wage-rate by a factor of 40/35. More generally, if W_0 is the nominal wage before the change in the length of the standard working week, then the resulting change in the nominal hourly wage-rate, which is required to keep the nominal weekly wage constant, can be approximated by

$$\frac{\partial W}{\partial \overline{N}} = - \frac{W_0}{\overline{N}} < 0 \tag{36}$$

When such an exogenous shock to the nominal hourly wage-rate is incorporated into the calculation of the short-run response of the price level, then the short-run multiplier is given by

$$\frac{\partial P}{\partial \overline{N}} = \frac{-\left[h_{31}^{'}\frac{\partial \tilde{K}}{\partial \overline{N}} + h_{32}^{'}\left\{\frac{\partial \tilde{z}}{\partial \overline{N}} + \frac{1}{\overline{N}}\right\} + \frac{\partial \tilde{b}}{\partial \overline{N}}\right] P_0}{h_{32}^{'} + b} \tag{37}$$

Thus by comparing equations (35) and (37) we can observe that an exogenous shock to the nominal hourly wage-rate will increase the price level above what it would have been if no attempt to maintain the nominal weekly wage-packet had occurred. Whether or not the exogenous increase in the nominal wage is inflationary or deflationary will depend upon whether the real wage decreases or increases as a result.

Another important question is whether or not wage-earners can maintain their real weekly wage by trying to maintain their nominal weekly wage.

In order to examine these questions let us assume that the multiplier given by equation (35) is zero, i.e., assume that if there were no exogenous response in the nominal hourly wage-rate, W, to a change in $\overline{N}$, then the real hourly wage-rate, v, would remain constant. Now assume that when working hours are reduced there is an exogenous response in the nominal hourly wage-rate which is designed to maintain the nominal weekly wage-packet. We now ask what will happen to the real wage in the short-run after the full impact effects on prices have worked their way through the economy?

As a result of the exogenous shock to W, equation (36) shows that the impact effects on the real wage, with the price level held constant at $P = P_0$ will be given by

$$\left(\frac{\partial v}{\partial \overline{N}}\right)_{P=P_0} = -\frac{W}{P_0\overline{N}} \qquad (38a)$$

Also, from equation (37), the effect on prices will be given by

$$\frac{\partial P}{\partial \overline{N}} = -\left(\frac{h_{32}'}{h_{32}' + b}\right)\frac{P_0}{\overline{N}} \qquad (38b)$$

Hence, the total effect on the real wage is given by

$$\frac{\partial v}{\partial \overline{N}} = \left(\frac{\partial v}{\partial \overline{N}}\right)_{P=P_0} - \frac{W}{P_0^2}\frac{\partial P}{\partial \overline{N}} \qquad (38c)$$

$$= -\frac{W}{P_0\overline{N}} + \frac{W}{P_0\overline{N}}\left(\frac{h_{32}'}{h_{32}' + b}\right) \qquad (38d)$$

$$= -\left(\frac{b}{h_{32}' + b}\right)\frac{W}{P_0\overline{N}} < 0 \qquad (38e)$$

In other words, the result of a conscious attempt to maintain nominal weekly wages, will be that wage-earners are better off in real terms than they would have been if no such attempt had been made. However, the attempt to maintain nominal weekly wages will not of itself be sufficient to maintain real weekly wages. If workers are to be better off (in real weekly wage terms) than they were prior to the reduction in the length of the standard working week, then this can only come about because prices fall sufficiently after working hours are reduced (i.e., because $\partial P/\partial \overline{N}$ is sufficiently large positive in equation (35)). This may or may not occur depending on parameter values in the economy and is really out of the workers' or even the government's control.

Because the real hourly wage-rate rises as a result of a conscious attempt to maintain the nominal weekly wage we can also conclude that such an attempt will tend to lower inflation. Intuitively this can be explained because relative to nominal wages, prices have fallen and thus inflation will tend to fall as well.

8.9 Conclusion

In this chapter, we have examined the effects of reducing the length of the standard working week in a capital intensive economy which is characterized by a high capital/labor ratio. In this model all expectations satisfy perfect myopic foresight and we assume the absence of speculative bubbles (i.e., we assume that the economy converges to its long-run equilibrium).

We have considered the effect of reducing the standard working week when the workers seek to maintain either their nominal hourly wage-rate or their nominal weekly wage-rate. We have also considered what happens when some of the effects of a reduced working week are offset by increased productivity gains which are achieved prior to the change by negotiations between workers and management.

In the short-run, a reduction in the length of the standard working week will always lower unemployment and, if productivity gains have been achieved, raise aggregate output. Also if the nominal hourly wage-rate has remained constant, prices may fall, inflation may fall and the real hourly wage may rise. If workers have been successful in maintaining the nominal weekly wage-rate then this will increase the short-run real hourly wage-rate even further and will also exert short-run deflationary pressure on the economy.

In the long-run, which is characterized by full employment in the labor market, the level of capital stock will always be lower. While the real hourly wage-rate will be higher than it would be otherwise, (the real interest rate), the real weekly wage-packet, profits and the level of aggregate output may actually be (larger) smaller unless significant productivity gains have been achieved in the negotiations prior to the implementation of reduced working hours.

In general we can conclude that the only definite benefits of reduced working hours would be lower unemployment in the short-run and increased leisure time as well as reduced long-run inflation (unless productivity gains are significant). If the nominal hourly wage-rate has remained constant in the short-run, a reduction in working hours will also be accompanied by probable lower short-run prices, lower inflation and higher real hourly wage-rates. An attempt to maintain the weekly nominal wage-rate would also tend to lower inflation by raising prices less than it raises the nominal wage-rate. However these gains would come at the expense of a lower long-run national output (unless productivity gains are significant). Also there is no guarantee either in the short-term or the long-term that workers would be able to maintain their real weekly wage-packet. As a rule, any gains made by wage-earners as a result of reducing the length of the standard working week would come at the expense of employers, although both sides could gain if the associated productivity gains are sufficiently large.

CHAPTER 9

A CONTINUOUS-TIME SPECIFICATION OF THE HOUSEHOLD SECTOR INCORPORATING BOTH UNEMPLOYMENT BENEFITS AND PENSIONS

9.1 Introduction

In this chapter we extend the derivation of the household sector which was developed in Section 8.2 of Chapter 8, to allow for unemployment benefits and pensions as well as inter-generation bequests. We also take this opportunity to examine the related issue of how the sale of government bonds to households will affect the household sector functions.

The derived household sector will then be used in Chapter 10 to examine the effects of changing unemployment benefits and pensions in a larger macroeconomic framework which includes the dynamics of asset accumulation, the dynamics of capital accumulation and unemployment. This question has not, as far as we know, been examined in such a framework before or in conjunction with so many other issues.

For the purposes of this chapter the household will be assumed to consume goods, supply labor and demand assets (i.e., both money and the composite good known as bonds-cum-equities). The household sector is thus uniquely defined in an aggregate sense by the four functions which define the demand (and supply) for these quantities. One such specification of the household is given by the following equations:

$$C = C(\underset{+}{Y^D}, \underset{-}{i}, \underset{+}{A}) \tag{1a}$$

$$m^d = L(\underset{+}{Y}, \underset{-}{i}, \underset{+}{-\pi}, \underset{+}{A}) \tag{1b}$$

$$b^d + QE^d = J(\underset{+}{Y}, \underset{+}{i}, \underset{-}{-\pi}, \underset{+}{A}) \tag{1c}$$

$$N^S = N^S(\underset{+}{v}, \underset{+}{i}, \underset{-}{A}) \tag{1d}$$

Equation (1a) is a consumption function such as might typically be derived from the life-cycle hypothesis and defines planned consumption as a function of disposable income, the real rate of interest and the level of real assets. Equations (1b, 1c) define demand for money and demand for bonds-cum-equities in a manner suggested by Tobin (1969) -- each asset is assumed positively related to national income and the level of assets, positively related to its own rate of return and negatively related to the rate of return of possible substitutes. Equation (1d) defines labor supply in a manner suggested by Lucas and Rapping (1969) -- in this formulation labor supply is assumed related to the real wage, the rate of interest and the supply of real assets. In practice, Walras' Law allows us to eliminate one market, usually the bonds-cum-equity market, and so the household sector can be defined by three equations; i.e. equations (1a, 1b, 1d).

In this formulation of the household sector, government bonds would influence the household functions through disposable income, Y^D, and real assets, A. Unemployment benefits and pensions would influence only the consumption function, through changes in Y^D. Unfortunately, equations (1a-1d) are not derived from the same optimizing model and accordingly may not be consistent.

To derive a consistent household sector which includes unemployment benefits and pensions it is necessary to derive all the household functions from the same optimizing model. One simpler prototype model was provided by Barro (1974) based on the Samuelson (1958) consumption-loan model. Using this model, Barro argued that the introduction of government bonds and government financed pensions did not affect the functions of the household sector and therefore did not affect saving or the dynamics of capital accumulation -- mainly because people foresee that in the future they, or their heirs, will have to pay higher taxes, and adjust their bequests so as to just compensate. Feldstein (1974, 1981) argued (and had empirical results to prove) that the introduction of government financed pensions did affect the dynamics of capital accumulation. The degree to which pensions have an affect on the dynamics of capital accumulation is an empirical question which is beyond the scope of this paper. We shall, however, consider the theoretical issues which underlie the different hypotheses and, in an integrated manner, we shall consider how the household sector would have to be formulated in order to be consistent with the different hypotheses. Finally a specification of the household sector will be provided. This specification, it is argued, provides the most plausible functional form for the household functions. Also because the specification is derived from a continuous framework it can be readily integrated into the literature on the dynamics of asset accumulation (see Blinder and Solow (1973), Tobin and Buiter (1976), Turnovsky (1977)).

9.2 An Optimizing Model

For the purposes of this chapter, time is divided into periods which are precisely one unit long. The jth period begins at time j and goes until j + 1. Individuals are assumed to live precisely two periods. In the first period of life, individuals are assumed to be in the Labor Force (and hence either employed or unemployed) while in the second period individuals are assumed to be retired (and hence not in the Labor Force). At the beginning of each period exactly one generation is born. Accordingly at any instant of time there is precisely one young generation whose members are in the Labor Force and one old generation whose members have retired. There is no growth in this model. The generation which begins life at time j will be known as the jth generation. The objective of the jth generation is to choose planned consumption, c_j, supply of labor, l_j, and demand for money, m_j, so as to maximize:

$$V_j = \int_j^{j+2} e^{-\beta t} U(c_j, l_j, m_j)\, dt + \alpha V_{j+1} \tag{2a}$$

where $U_1 > 0,\ U_2 < 0,\ U_3 > 0$

$U_{ii} < 0$, for all i

$U_{ij} = 0,\ i \neq j$

$0 \leqq \alpha < e^{\beta}$

subject to

$$c_j + \dot{m}_j + Q\dot{E}_j + \dot{b}_j = \rho v l_j + (1-\rho)u + dQE_j + ib_j - \pi m_j - \delta T$$

$$\text{for } t \in [j, j+1) \tag{2b}$$

$$c_j + \dot{m}_j + Q\dot{E}_j + \dot{b}_j = s + dQE_j + ib_j - \pi m_j - (1-\delta)T$$

$$\text{for } t \in [j+1, j+2) \tag{2c}$$

$$l_j = 0 \quad \text{for } t \in [j+1, j+2) \tag{2d}$$

$$T = T_0 + \gamma[i(b_j^s + b_{j-1}^s) + (1-\rho)u + s - (\dot{b}_j^s + \dot{b}_{j-1}^s)]$$

$$\text{for } t \in [j, j+1) \tag{2e}$$

$$\Omega_j = b_j^h + QE_j + m_j \qquad \text{for } t \in [j, j+2) \tag{2f}$$

$$A = \Omega_j + \Omega_{j-1} \qquad \text{for } t \in [j, j+1) \tag{2g}$$

$$b_j^h(j) = 0 \tag{2h}$$

$$\Omega_j(j) = 0 \tag{2i}$$

$$\Omega_j(j+1) = A(j+1^-) \tag{2j}$$

$$\Omega_j(j+2^-) \geqq 0 \tag{2k}$$

In this model, T_0, δ and γ are fixed constants. All other variables are functions of time and

c_j = real private consumption plans by jth generation

l_j = labor supply (in hours) by jth generation

b_j^s = amount of government bonds that have been supplied to the jth generation either indirectly, through the bequests of previous generations, or directly, through the actions of government during the jth generation's lifetime

b_j^h = amount of government bonds held by the jth generation

m_j = real demand for money by jth generation

E_j = demand for equities by jth generation

Ω_j = real private wealth held by jth generation

A = aggregate real private wealth

Q = (expected) price of equities

ρ = (expected) employment rate

W = (expected) nominal hourly wage-rate

P = (expected) price level

$v = \frac{W}{P}$ = (expected) real hourly wage rate

$q = \frac{\dot{Q}}{Q}$ = (expected) rate of inflation of equity prices

$\pi = \frac{\dot{P}}{P}$ = (expected) rate of inflation of price level

β = consumer's rate of time preference

D_j = (expected) real dividends by jth generation

$d = \frac{D_j + D_{j-1}}{Q(E_j + E_{j-1})}$ = (expected) dividend yield, taken to be parametrically given to the household sector and defined for $t \in [j, j+1)$

u = (expected) real unemployment benefit paid to members of a generation who are currently unemployed

s = (expected) real pension paid to any members of a living generation who are retired

$i = d + q$ = (expected) real rate of return on equities and bonds which are assumed to be perfect substitutes

T = aggregate real tax receipts

δ = fixed proportion of taxes paid by the generation which is currently working, $0 \leqq \delta \leqq 1$

γ = the extent to which the buying back (and sale) of government bonds, pensions and unemployment benefits are expected to be financed by taxation, $0 \leqq \gamma \leqq 1$

The utility function is assumed to be concave in its three arguments, c_j, l_j, and m_j. As in Section 8.2 of Chapter 8, real money balances have been incorporated into the utility function as a mechanism for capturing the reasons for holding money in a world of certainty and the utility function is assumed additively separable in all arguments purely as a means of simplifying the analysis. Equation (1a) then defines the aggregate utility of the current generation as dependent upon consumption, labor supply and labor demand over its members' lifetime plus a fraction of the utility of the next generation.

The budget constraint faced by the jth generation in its first period of life (i.e., while a member of the Labor Force) is given by equation (2b), which is expressed in real flow terms. At each point in time households can acquire income from a variety of sources. They have an expected probability, ρ, of being employed and, if employed, they supply labor to firms, at a real wage-rate, v, they receive dividend yields, d, on their holdings of equities and they face an inflation tax ($= \pi m$) on their holdings of real money. They also receive unemployment benefits if unemployed and pay taxes. Their income can be spent on the purchase of real consumption goods or in the acquisition of new holdings of real private wealth. Wealth can be held either in the form of government bonds, equities or money.

In the second period of life (i.e., while members of the jth generation are retired) the budget constraint is given by equations (2c, 2d). In this period, labor supply is zero. Income comprises dividends, pensions and returns on government bonds less government taxes and the inflation tax on money.

The precise form of taxes faced by the households is described by equation (2e). The case when $\gamma = 1$ corresponds to the situation when the buying back (and sale) of government bonds, debt on government bonds, pensions and unemployment benefits are expected to be financed fully by taxation. The case when $\gamma = 0$ corresponds to the situation when the tax level is exogenously fixed.

Real private wealth held by the jth generation is defined in equation (2f) and aggregate real private wealth in any time period is the sum of the real private wealth held by the two generations that are still living (equation (2g)). Because of the taxation structure, the amount of government bonds held by the jth generation, b_j^h, in general will differ from the supply of government bonds to the jth generation, b_j^s. Both b_j^h and b_j^s are explicitly defined in the next section. At the beginning of their lifetimes, all generations possess zero net wealth (equation (2h)). In particular, they possess no government bonds (equation (2i)). At the end of its lifetime no generation can be in debt. Thus people must have zero net wealth at the end of their life or have a positive bequest (equation (2k)). All bequests by the jth generation (whether in the form of bonds, money or equities) must be left to the next generation (j+1). This gives rise to the constraint given by equation (2j) which embodies a discrete jump in the jth generation's real private wealth if it has been left a positive bequest. The jth generation's bequest is given by $\Omega_j(j+2^-)$.[1] Note that while the jth generation's net wealth (Ω_j) may have discrete jumps at the end of any period, the aggregate net wealth function (A) whose evolution is defined by equations (2b, 2c), must be continuous. An initial endowment of real private wealth provides the initial condition that will close the model.

In the household's optimization decision the variables c_j, l_j, m_j, and QE_j are determined by the household. All other variables are taken as given by the household.

9.3 Net Holdings of Government Bonds

The optimizing model essentially allows work and retirement in a framework which incorporates unemployment benefits, pensions and government bonds. These are all assumed to be financed either through a budget deficit or through taxes. In particular, we assume that the government determines the rate of supply of government bonds to each generation (i.e., $\dot{b}_j^s$) and that each generation is compelled by legislation to buy these bonds. The variable, b_j^s, denotes the amount of government bonds that have been supplied to the jth generation, either indirectly, through the bequests of previous generations, or directly, through the actions of government during the jth generation's lifetime. Accordingly, for $t \varepsilon [j, j+1)$

$$b_j^s = \int_j^t \dot{b}_j^s \, dt \tag{3a}$$

and

$$b_{j-1}^s = \int_j^t \dot{b}_{j-1}^s \, dt + \sum_{i=-\infty}^{j} \int_{i-1}^{i} (\dot{b}_i^s + \dot{b}_{i-1}^s) dt \tag{3b}$$

Not all the government bonds supplied to the jth generation will be held by the jth generation; some bonds will be used to pay tax liabilities. In order to examine the relationship between the amount of bonds held by the jth generation, b_j^h, and the amount of bonds supplied to the jth generation, b_j^s, it is necessary to examine the budget constraints of the household sector more closely. In order to do this, equations (2b, 2c, 2e) can be combined to give the following equations:

$$c_j + \dot{m}_j + Q\dot{E}_j + [\dot{b}^s_j - \delta\gamma(\dot{b}^s_j + \dot{b}^s_{j-1})]$$

$$= \rho v l_j + dQE_j + (1 - \delta\gamma)(1 - \rho)u - (\delta\gamma)s$$

$$- \pi m_j - \delta T_0 + i[b^s_j - \delta\gamma(b^s_j + b^s_{j-1})]$$

$$\text{for} \quad t \in [j, j+1) \tag{4a}$$

$$c_j + \dot{m}_j + Q\dot{E}_j + [\dot{b}^s_j - \gamma(1 - \delta)(\dot{b}^s_{j+1} + \dot{b}^s_j)]$$

$$= dQE_j + [1 - \gamma(1 - \delta)]s - \gamma(1 - \delta)(1 - \rho)u$$

$$- \pi m_j - (1 - \delta)T_0 + i[b^s_j - \gamma(1 - \delta)(b^s_j + b^s_{j+1})]$$

$$\text{for} \quad t \in [j+1, j+2) \tag{4b}$$

Hence the rate of growth of bonds held by the jth generation, b^h_j, can be given by

$$\dot{b}^h_j = \dot{b}^s_j(1 - \delta\gamma) - \delta\gamma(\dot{b}^s_{j-1}) \tag{5a}$$

$$\dot{b}^h_{j-1} = \dot{b}^s_{j-1}(1 - \gamma(1 - \delta)) - \gamma(1 - \delta)\dot{b}^s_j \tag{5b}$$

$$\text{for} \quad t \in [j, j+1)$$

Hence, the aggregate holding of bonds, at any time, is given by

$$b^h_j + b^h_{j-1} = (1 - \gamma)(b^s_j + b^s_{j-1}) \tag{6a}$$

$$= (1 - \gamma)\left(\sum_{i=-\infty}^{j} \int_{i-1}^{i} (\dot{b}^s_i + \dot{b}^s_{i-1})dt + \int_j^t (\dot{b}^s_j + \dot{b}^s_{j-1})dt\right) \tag{6b}$$

Since

$$b_j^h(j) = 0 \tag{6c}$$

and

$$b_j^h(j+1) = b_j^h(j+1^-) + b_{j-1}^h(j+1^-) \tag{6d}$$

then, the current holding of bonds by the jth generation, i.e., b_j^h, is different from the supply of bonds to the jth generation, i.e., b_j^s, but both variables are uniquely defined by all previous rates of supply of bonds, i.e., $\dot{b}_j^s$.

We can then use equation (6a) to make the following observations about the relationship between the type of taxation (in this case the choice of γ) and the public's holding of government bonds.

(i) When the funding of bond sales is fully paid for by taxes (i.e., when $\gamma = 1$), the aggregate holdings of bonds will always be zero.

(ii) When the tax level is exogenously fixed (i.e., when $\gamma = 0$), the aggregate holdings of bonds is equal to the aggregate amount supplied.

(iii) In intermediate cases (i.e., for $0 < \gamma < 1$), the aggregate holdings of bonds will be positive but less than the amount supplied.

9.4 Simplifying the Model

Having derived equations (5, 6) which define the amount of bonds held by each generation and noting that bonds and equities are assumed to be perfect substitutes, we can rewrite the household's optimization problem given by equations (2) as follows:

$$\max_{c_j, l_j, m_j} V_j = \int_j^{j+2} e^{-\beta t} U(c_j, l_j, m_j) dt + \alpha V_{j+1} \tag{7a}$$

subject to

$$c_j + \dot{\Omega}_j = \rho v l_j + i\Omega_j - (i + \pi)m_j + y_j \qquad \text{(7b)}$$

$$\text{for} \quad t \in [j, j+2)$$

where

$$y_j = \begin{cases} (1-\delta\gamma)(1-\rho)u - \delta\gamma s - \delta T_0, & \text{for } t \in [j, j+1) \qquad \text{(7c)} \\ -\gamma(1-\delta)(1-\rho)u + [1-\gamma(1-\delta)]s - (1-\delta)T_0 & \qquad \text{(7d)} \\ \qquad \text{for } t \in [j+1, j+2) & \end{cases}$$

$$l_j = 0, \text{ for } t \in [j+1, j+2) \qquad \text{(7e)}$$

$$\Omega_j(j) = 0 \qquad \text{(7f)}$$

$$\Omega_j(j+1) = \Omega_j(j+1^-) + \Omega_{j-1}(j+1^-) \qquad \text{(7g)}$$

$$\Omega_j(j+2^-) \geqq 0 \qquad \text{(7h)}$$

9.5 The Role of Expectations

As was the case in Sections 8.2 and 8.3 of Chapter 8, the mechanism by which expectations are formed is crucial for the development of the model. We shall again assume that current and past expectations satisfy perfect myopic foresight; future expectations satisfy a static expectations hypothesis.

This notion of a static expectation hypothesis needs to be made more explicit. More precisely we require that the variables which face each individual in the economy are formed by static expectations. These variables are the employment rate (ρ), the real-wage (v), the rate of return on bonds and equities (i), the rate of inflation (π), the pension (s) and the unemployment benefit (u).

Given a mechanism for the formation of expectations, each generation's optimization decision can be described as follows: at any current instant of time, each generation knows with certainty the value of all the exogenous variables which it takes as given. Using its expectations about the future, the household sector determines its optimal consumption, labor supply and labor demand for that instant and for all future time. When at the next instant of time the values of the appropriate variables become known, the household compares actual values with the expectations and, if these diverge, then the household's decision variables are revised on the basis of the new value of the exogenous variables and current expectations.

9.6 Solution to the Optimizing Model

As demonstrated in the Appendix, provided the rate of return on bonds and equities is greater than the rate of return on money, i.e., $i + \pi > 0$,[2] then a unique solution to the optimizing model exists and conditions for such a solution are given by the following equations:

$$U_1 = \psi_j \tag{8a}$$

$$U_3 = (i + \pi)\psi_j \tag{8b}$$

$$U_2 = -(\rho v)\psi_j \qquad \text{for } t \in [j, j+1) \tag{8c}$$

$$l_j = 0 \qquad \text{for } t \in [j+1, j+2) \tag{8d}$$

$$\dot{\psi}_j = (\beta - i)\psi_j \tag{8e}$$

$$\psi_j(j+1^-) = \psi_j(j+1) = Z_j'[A(j+1)] = V_j'[\Omega_{j-1}(j+1^-)] \tag{8f}$$

$$\psi_j(j+2^-) = \alpha\psi_{j+1}(j+2^-) + \mu_j \tag{8g}$$

for $t \in (j+1,\ j+2)$

where $\mu_j\Omega_j(j+2^-) = 0$

and where

$$Z_j[A(j+1)] = \int_{j+1}^{j+2} e^{-\beta t}U(c_j,\ l_j,\ m_j)dt + V_{j+1}$$

subject to

$$\Omega_j(j+1) = A(j+1)$$

Given A_0 an optimal program is determined as follows: $A(0) = A_0$ determines $\psi_{-1}(0)$ (equation (8f)) which determines c_{-1} and m_{-1} (equations 8a, 8b). These in turn determine the -1th generation's bequest (= $\Omega_{-1}(1^-)$) to generation 0 (using equation (7b)). This bequest determines A(1) (using equation (8f)) and the time path for c_0, m_0, l_0, (using equations 8a, 8b, 8c, 8d, 8g) which determine the 0th generation's bequest (= $\Omega_0(2^-)$) to generation +1, and so on.

Given a fixed initial condition (e.g., $A(0) = A_0$), the following observations can be made about the optimal solution:

(i) As α decreases, the jth generation's bequest (= $\Omega_j(j+2^-)$) decreases. Hence as α decreases, the probability increases that the jth generation leaves no bequest (i.e., $\Omega_j(j+2^-) = 0$).[3]

(ii) In particular, when α=0 there is no bequest (i.e., $\Omega_j(j+2^-) = 0$). This is reasonable since, in this case, the next generation's utility function does not appear in the current generation's utility function and thus there is no motive to leave a bequest.

(iii) Whenever the jth generation's bequest (= $\Omega_j(j+2^-)$) is non-zero,

$$c_j \lesseqgtr c_{j+1}, \quad m_j \lesseqgtr m_{j+1}, \quad \text{and } l_j \gtreqless l_{j+1}$$

according as $\alpha \gtreqless 1$.

In the rest of this chapter two cases will be considered:

(a) the case when the optimal bequest (i.e., $\Omega_j(j+2^-)$) is $\geqq 0$ (i.e., $\mu_j = 0$). This will be referred to as the Operative Bequest Motive Case.

(b) the case when the optimal bequest is < 0 (i.e., $\mu_j > 0$). This case will be referred to as the Non-operative Bequest Motive Case and will be characterized by a bequest which is constrained to equal zero.

9.7 Operative Bequest Motive

When the bequest motive is operative, equation (8g) shows that $\psi_j = \alpha\psi_{j+1}$ for $t \in [j+1, j+2)$. Hence, assuming that α is fixed, we can use equations (8a-8d) to show that for $t \in (j, j+1)$.

$$c_{j-1} = c_{j-1}(\underset{-}{\psi_j}) \tag{9a}$$

$$m_{j-1} = m_{j-1}(i \underset{-}{+} \pi, \ \underset{-}{\psi_j}) \tag{9b}$$

$$l_{j-1} = 0 \tag{9c}$$

$$c_j = c_j(\underset{-}{\psi_j}) \tag{9d}$$

$$m_j = m_j(\underset{-}{i + \pi}, \underset{-}{\psi_j}) \tag{9e}$$

$$l_j = l_j(\underset{+}{\rho v}, \underset{+}{\psi_j}) \tag{9f}$$

Also equation (7b) can be combined for two successive generations to give

$$\dot{A} = \rho v l_j - (c_j+c_{j-1}) + iA - (i+\pi)(m_j+m_{j-1}) + (y_j+y_{j-1}) \tag{10}$$

Hence if we assume that the expectations for all time periods after t satisfy static individual expectations, then[4]

$$A(j+1) = f[t, \underset{+}{A}, \underset{+}{\psi_j}, \underset{+}{i}, \underset{+}{\rho v}, \underset{+}{-(i+\pi)(m_j+m_{j-1})}, \underset{+}{(y_j+y_{j-1})}] \tag{11a}$$

But from equation (8f),

$$\psi_j(j+1) = Z_j'(A(j+1)) \tag{11b}$$

and from equation (8e),

$$\psi_j(j+1) = e^{(\beta-i)(j+1-t)}\psi_j(t) \tag{11c}$$

Hence,

$$\psi_j(t) = e^{(\beta-i)(t-j-1)}\psi_j(j+1) \tag{11d}$$

$$= e^{(\beta-i)(t-j-1)}Z_j'(A(j+1)) \tag{11e}$$

Hence, using the concavity of Z_j (proved in the Appendix)[5]

$$\psi_j(t) = g[t, \underset{-}{A}, \underset{+}{i}, \underset{-}{\rho v}, \underset{-}{-(i+\pi)(m_j+m_{j-1})}, \underset{-}{(y_j+y_{j-1})}] \tag{11f}$$

Then using equations (9b, 9e)

$$m_j(t) = m_j[t, \underset{+}{A}, \underset{-}{i}, \underset{+}{\rho v}, \underset{-}{i+\pi}, \underset{+}{y_j+y_{j-1}}] \qquad (11g)$$

Hence, from equations (11f, 11g)

$$\psi_j(t) = h[t, \underset{-}{A}, \underset{+}{i}, \underset{-}{\rho v}, \underset{-}{-\pi}, \underset{-}{y_j+y_{j-1}}] \qquad (11h)$$

and, from equations (9a-9f);[6]

$$c_j = c_j[t, \underset{+}{A}, \underset{-}{i}, \underset{+}{\rho v}, \underset{+}{-\pi}, \underset{+}{y_j+y_{j-1}}] \qquad (12a)$$

for $t \in [j, j+2)$

$$m_j = m_j[t, \underset{+}{A}, \underset{-}{i}, \underset{+}{\rho v}, \underset{+}{-\pi}, \underset{+}{y_j+y_{j-1}}] \qquad (12b)$$

for $t \in [j, j+2)$

$$l_j = l_j[t, \underset{-}{A}, \underset{+}{i}, \underset{+}{\rho v}, \underset{-}{-\pi}, \underset{-}{y_j+y_{j-1}}] \qquad (12c)$$

for $t \in [j, j+1)$

$$l_j = 0 \qquad \text{for } t \in [j+1, j+2) \qquad (12d)$$

where $y_j + y_{j-1} = (1-\gamma)[(1-\rho)u + s] - T_0$ is the aggregate income derived from unemployment benefits and pensions.

The effect that government bonds have on the household sector will depend upon the way government bonds influence real private wealth (A). As shown in equations (6a-6b) the supply of government bonds will have some effect on the household sector unless they are fully financed out of taxation and not even partially, by means of a budget deficit (i.e., they will have some effects unless $\gamma = 1$). This is essentially the result of Barro's (1974) exposition. As the degree of taxation financing decreases (i.e., as γ decreases), the effect of government bonds on the household sector will increase.

In a similar fashion it is clear that unemployment benefits and pensions will have some effect on the household sector unless $\gamma = 1$. As γ decreases, the effect of unemployment benefits and pensions on the household sector will increase.

When we derive aggregate planned consumption, C, aggregate demand for money, m^d, and aggregate supply of labor, N^S, where for $t \in [j, j+1)$

$$C = c_j + c_{j-1} \tag{13a}$$

$$m^d = m_j + m_{j-1} \tag{13b}$$

$$N^S = l_j \tag{13c}$$

then the aggregate functions will have the same properties as the individual component functions given by equations (12a-12c), i.e., these aggregate household sector functions are given by

$$C = C[\underset{+}{A}, \underset{+}{-\pi}, \underset{-}{i}, \underset{+}{\rho v}, \underset{+}{(1-\gamma)(1-\rho)u}, \underset{+}{(1-\gamma)s}, \underset{-}{T_0}] \tag{13a'}$$

$$m^d = L[\underset{+}{A}, \underset{+}{-\pi}, \underset{-}{i}, \underset{+}{\rho v}, \underset{+}{(1-\gamma)(1-\rho)u}, \underset{+}{(1-\gamma)s}, \underset{-}{T_0}] \tag{13b'}$$

$$N^S = N^S[\underset{-}{A}, \underset{-}{-\pi}, \underset{+}{i}, \underset{+}{\rho v}, \underset{-}{(1-\gamma)(1-\rho)u}, \underset{-}{(1-\gamma)s}, \underset{-}{T_0}] \tag{13c'}$$

where C = aggregate planned consumption, m^d = aggregate demand for money, N^S = aggregate labor supply, and A is aggregate real private wealth including holdings of government bonds.

9.8 Non-Operative Bequest Motive

When the bequest motive for the jth generation is non-operative, then $\Omega_j(j+2^-) = 0$. Also from equations (8a-8d),

$$c_j = c_j(\underset{-}{\psi_j}) \tag{14a}$$

$$m_j = m_j(\underset{-}{i+\pi}, \underset{-}{\psi_j}) \tag{14b}$$

$$l_j = l_j(\underset{+}{\rho v}, \underset{+}{\psi_j}) \qquad \text{for } t \in [j, j+1) \tag{14c}$$

$$l_j = 0 \qquad \text{for } t \in [j+1, j+2) \tag{14d}$$

Then using equations (7b, 14a-14d), and assuming the bequest motive is also non-operative for generation j-1, i.e., $\Omega_{j-1}(j+1^-) = 0$, we can show that

$$\Omega_j(j+2^-) = f(t, \underset{+}{\Omega_j}, \underset{+}{\psi_j}, \underset{+}{i}, \underset{+}{-(i+\pi)m_j}, \underset{+}{y_j}, \underset{+\text{ or }0}{\rho v}) \tag{15}$$

where the partial derivative w.r.t. ρv is positive if $t \in [j, j+1)$ and zero if $t \in [j+1, j+2)$.

Hence, since $\Omega_j(j+2^-) = 0$, from equation (15),

$$\psi_j(t) = \psi_j(t, \underset{-}{\Omega_j}, \underset{-}{i}, \underset{-}{-(i+\pi)m_j}, \underset{-}{y_j}, \underset{-\text{ or }0}{\rho v}) \tag{16}$$

and hence, using equation (14b), we obtain[7]

$$m_j(t) = m_j(t, \underset{+}{\Omega_j}, \underset{+}{-\pi}, \underset{-}{i+\pi}, \underset{+}{y_j}, \underset{+\text{ or }0}{\rho v}) \tag{17a}$$

Hence, we can rewrite equation (16) as

$$\psi_j(t) = \psi_j(t,\ \underset{-}{\Omega_j},\ \underset{-}{-\pi},\ \underset{+}{i+\pi},\ \underset{-}{y_j},\ \underset{-\text{ or }0}{\rho v}) \tag{17b}$$

Then, from equations (14), the optimal solution for the jth generation, is given by[8]

$$c_j = c_j(t,\ \underset{+}{\Omega_j},\ \underset{+}{-\pi},\ \underset{-}{i+\pi},\ \underset{+}{y_j},\ \underset{+\text{ or }0}{\rho v}) \tag{18a}$$

$$\text{for } t \in [j,\ j+2)$$

$$m_j = m_j(t,\ \underset{+}{\Omega_j},\ \underset{+}{-\pi},\ \underset{-}{i+\pi},\ \underset{+}{y_j},\ \underset{+\text{ or }0}{\rho v}) \tag{18b}$$

$$\text{for } t \in [j,\ j+2)$$

$$l_j = l_j(t,\ \underset{-}{\Omega_j},\ \underset{-}{-\pi},\ \underset{+}{i+\pi},\ \underset{-}{y_j},\ \underset{+}{\rho v}) \tag{18c}$$

$$\text{for } t \in [j,\ j+1)$$

$$l_j = 0 \qquad \text{for } t \in [j+1,\ j+2) \tag{18d}$$

Once again we can derive aggregate planned consumption, C, aggregate demand for money, m^d, and aggregate supply of labor, N^s, where, for $t \in [j, j+1)$

$$C = C(\underset{+}{\Omega_j},\ \underset{+}{\Omega_{j-1}},\ \underset{+}{-\pi},\ \underset{-}{i},\ \underset{+}{\rho v},\ \underset{+}{y_j},\ \underset{+}{y_{j-1}}) \tag{19a}$$

$$m^d = L(\underset{+}{\Omega_j},\ \underset{+}{\Omega_{j-1}},\ \underset{+}{-\pi},\ \underset{-}{i},\ \underset{+}{\rho v},\ \underset{+}{y_j},\ \underset{+}{y_{j-1}}) \tag{19b}$$

$$N^s = N^s(\underset{-}{\Omega_j},\ \underset{-}{\Omega_{j-1}},\ \underset{-}{-\pi},\ \underset{+}{i},\ \underset{+}{\rho v},\ \underset{-}{y_j},\ \underset{-}{y_{j-1}}) \tag{19c}$$

In general, the coefficients of Ω_j and Ω_{j-1} will not be equal, nor will the coefficients of y_j and y_{j-1}. As a first approximation however we will assume in what follows that these coefficients are approximately the same. Then planned consumption and demand for money (supply of labor) will be positively (negatively) related to:

$$b_j^h + b_{j-1}^h = (1-\gamma)(b_j^s + b_{j-1}^s) \tag{20a}$$

and

$$y_j + y_{j-1} = (1-\gamma)[(1-\rho)u + s] - T_0 \tag{20b}$$

Hence, when $\gamma \neq 1$ these respective functions will be positively (negatively) related to the supply of government bonds, the unemployment benefit and the pension and negatively (positively) related to the exogenously fixed level of taxes, T_0.

9.9 A General Specification of the Household Sector

In the analyses of the previous sections 9.7 and 9.8 of this chapter, we have demonstrated that, in a world in which there are government bonds, pensions and unemployment benefits, the household sector functions will, in general, depend on these variables. This relationship will occur unless expenditure on these items is completely financed out of taxes (i.e., $\gamma = 1$) and each generation has an operative bequest motive.

Even in this special case there are other reasons why the household sector functions may still depend on the variables:

(i) The real world is made not simply of two generations but of many different individuals who earn different wages, pay different taxes and receive different amounts of bonds, pensions and unemployment benefits as a result of government policies.

(ii) Feldstein (1976) and Kotlikoff (1979) have pointed out that in reality an individual's retirement age is not fixed but rather endogenous.

(iii) Drazen (1978) has shown that the solution will change if investment has been made in a form (e.g., human capital) which cannot readily be converted back into equities.

(iv) There has also been some debate about the implications of incorporating growth in the model (see Feldstein (1975), Barro (1976), Carmichael (1982)).

These issues mean that with almost any uniform tax structure the household sector functions will still depend to some extent on government bonds, pensions and unemployment benefits.

Furthermore, the derivations of the household functions have relied on information about the income and wealth of specific generations. In general, we would expect that the typical policy maker would only have aggregate information available to him. Accordingly, for the purposes of tractability, some approximations will have to be made in defining a general specification of the household sector.

Some general principles can, however, be observed concerning the general structure of the household sector functions. As a rule, these functions depend on income (in the form of expected wages (ρv)), as well as the tax base (= T_0), the rates of return on money (= $-\pi$) and bonds-cum-equities (= i), the current holdings of real private wealth excluding the level of government bonds (= A^x), and also the expected returns from unemployment benefit (= $(1-\rho)u$), pensions (= s) and the supply of government bonds (= b). As a first approximation these last three variables will all be multiplied by $1-\gamma$, i.e., the extent to which expenditure on these items is not financed out of taxation. A general specification of the household sector can then be written as follows

$$C = C[\underset{+}{\rho v}, \underset{-}{T_0}, \underset{-}{i}, \underset{+}{-\pi}, \underset{+}{A^x}, \underset{+}{(1-\gamma)b}, \underset{+}{(1-\gamma)(1-\rho)u}, \underset{+}{(1-\gamma)s}] \tag{21a}$$

$$m^d = L[\underset{+}{\rho v}, \underset{-}{T_0}, \underset{-}{i}, \underset{+}{-\pi}, \underset{+}{A^x}, \underset{+}{(1-\gamma)b}, \underset{+}{(1-\gamma)(1-\rho)u}, \underset{+}{(1-\gamma)s}] \tag{21b}$$

$$N^s = N^s[\underset{+}{\rho v}, \underset{+}{T_0}, \underset{+}{i}, \underset{-}{-\pi}, \underset{-}{A^x}, \underset{-}{(1-\gamma)b}, \underset{-}{(1-\gamma)(1-\rho)u}, \underset{-}{(1-\gamma)s}] \tag{21c}$$

where C denotes aggregate planned consumption, m^d denotes the aggregate real demand for money and N^s denotes the aggregate supply of labor. As a rule, we would expect that the sign of the effect of b, $(1-\rho)u$ and s would be as shown. However, when the bequest motive is non-operative, the arguments of Section 9.8 show that this may not always be the case.

9.10 Concluding Remarks

In this chapter, we have examined the effects of changing unemployment benefits and pensions on the specification of the household sector. We have chosen to do this by extending Barro's (1974) model to a continuous-time framework in which there is unemployment and equities as well as bequests, government bonds, unemployment benefits and pensions. This formal approach rather than an ad-hoc approach has been chosen for two reasons.

First, it makes clear the type of assumptions that underlie the specification. In particular, the assumptions with respect to the formation of expectations which are needed to develop the solution to the model.

Secondly, it also ensures that the derived household sector is internally consistent.

Barro's results, which conclude that government bonds, pensions and unemployment benefits do not affect the household functions, have been shown to apply only under a set of very restrictive assumptions; it has been argued that even when only some of these restrictive assumptions apply, more realistic assumptions will mean that the household functions do respond to changes in social welfare payments and bonds.

The most important conclusion to emerge from this chapter is that, contrary to what is suggested by the ad-hoc specification of the household sector given in equations (1a-1d), if unemployment benefits and pensions occur in the specification of the consumption function then they will also occur in the specification of the demand for money and labor supply. Thus, as will be demonstrated more clearly in the next chapter, a change in unemployment benefits and pensions may well have different effects than a change in Government expenditure, which will only affect aggregate demand.

Appendix to Chapter 9

In this Appendix we provide a more formal derivation of conditions for the solution of the optimizing model. The proof uses Hestenes (1965) theorem as expounded by Long and Vousden (1977).[9] For convenience we shall drop all superscripts and unnecessary subscripts. The problem given by equation (1) can then be divided into two parts:

<u>Problem 1</u>

Given Ω_{-1}^{**}

$$\max_{c,l,m} \int_0^1 e^{-\beta t}U(c,l,m)dt + Z(\Omega_{-1}^{**} + \Omega^{*}) = V(\Omega_{-1}^{**}) \tag{A.1a}$$

subject to

$$\dot{\Omega} = \rho v l - c + i\Omega - (i+\pi)m + y \tag{A.1b}$$

$$\Omega_{-1}^{**} = \Omega_{-1}(1^{-}) \tag{A.1c}$$

$$\Omega^{*} = \Omega(1^{-}) \tag{A.1d}$$

and where Z is as defined in Problem 2.

<u>Problem 2</u>

Given $\Omega_{-1}^{**} + \Omega^{*}$

$$\max_{c,m} \int_1^2 e^{-\beta t}U(c,o,m)dt + \alpha V_{+1}(\Omega^{**}) = Z(\Omega_{-1}^{**} + \Omega^{*}) \tag{A.2a}$$

subject to

$$\dot{\Omega} = i\Omega - c - (i+\pi)m + y \tag{A.2b}$$

$$\Omega_{-1}^{**} = \Omega_{-1}(1^-) \tag{A.2c}$$

$$\Omega^{*} = \Omega(1^-) \tag{A.2d}$$

$$\Omega^{**} = \Omega(2^-) \geqq 0 \tag{A.2e}$$

Solution to Problem 1

The Hamiltonian for the first problem is given by:

$$H = e^{-\beta t}U(c,l,m) + e^{-\beta t}\psi[\rho v l - c + i\Omega - (i+\pi)m + y] \tag{A.3}$$

where $e^{-\beta t}\psi$ is the discounted co-state variable associated with the state variable Ω. Application of the necessary conditions gives:

$$U_1 = \psi \tag{A.4a}$$

$$U_2 = -\psi(\rho v) \tag{A.4b}$$

$$U_3 = (i+\pi)\psi \tag{A.4c}$$

$$\dot{\psi} = (\beta - i)\psi \tag{A.4d}$$

Application of Hestenes theorem gives the transversality condition

$$\psi(1) = Z'(\Omega_{-1}^{**} + \Omega^{*}) \tag{A.4e}$$

Solution to Problem 2

The Hamiltonian for the second problem is given by

$$H = e^{-\beta t}U(c,o,m) + e^{-\beta t}[i\Omega - c - (i+\pi)m + y] \tag{A.5}$$

where $e^{-\beta t}\psi$ is the discounted co-state variable associated with the state variable ψ. Application of the necessary conditions gives:

$$U_1 = \psi \tag{A.6a}$$

$$U_3 = (i+\pi)\psi \tag{A.6b}$$

$$\dot{\psi} = (\beta-i)\psi \tag{A.6c}$$

The transversality conditions are given by

$$\psi(1) = Z'(\Omega_{-1}^{**} + \Omega^{*}) \tag{A.6d}$$

$$\psi(2) = \alpha V'_{+1}(\Omega^{**}) + \mu \tag{A.6e}$$

where $\mu \geqq 0$, $\mu\Omega^{**} = 0$.

Solution of Combined Problem

We shall prove that, for the optimal solution,

$$Z'(\Omega_{-1}^{**} + \Omega^{*}) = V'(\Omega_{-1}^{**})$$

Sketch of Proof: Let $\overline{c}$, $\overline{I}$, $\overline{m}$ be a set of solutions defined for $t \in [0,1)$ and for which $\Omega = \overline{\Omega}$. Let

$$X(\Omega_{-1}^{**}) = X(\overline{c}, \overline{I}, \overline{m}, \overline{\Omega}, \Omega_{-1}^{**})$$

$$= \int_0^1 e^{-\beta t} U(\overline{c},\overline{I},\overline{m})dt + Z(\Omega_{-1}^{**} + \overline{\Omega}).$$

Hence $X'(\Omega_{-1}^{**}) = Z'(\Omega_{-1}^{**} + \overline{\Omega})$.

Let $(\hat{c}, \hat{I}, \hat{m}, \hat{\Omega})$ denote the optimal solution. If $(\overline{c}, \overline{I}, \overline{m}, \overline{\Omega}) = (\hat{c}, \hat{I}, \hat{m}, \hat{\Omega})$ then $X(\Omega_{-1}^{**}) = V(\Omega_{-1}^{**})$, i.e., $X'(\Omega_{-1}^{**}) = V'(\Omega_{-1}^{**})$. Hence $V'(\Omega_{-1}^{**}) = Z'(\hat{\Omega} + \Omega_{-1}^{**})$.

Then, noting that the transversality conditions given by equations (A.6d) and (A.4e) are identical, we can write down the necessary conditions for the solution to this problem as given in Equation (8).

Sufficiency and Uniqueness Theorems

In order to prove the sufficiency of the necessary conditions we must first prove that $V(\Omega_{-1}^{**})$ is concave.

Sketch of Proof

Choose two separate initial conditions Ω_{-1}^{**1} and Ω_{-1}^{**2} with corresponding optimal paths given by $(c^1, l^1, m^1, \Omega^1, \Omega_{-1}^{**1})$ and $(c^2, l^2, m^2, \Omega^2, \Omega_{-1}^{**2})$. Then $\lambda\Omega_{-1}^{**1} + (1-\lambda)\Omega_{-1}^{**2}$ is the initial condition for a feasible path given by $\lambda c^1 + (1-\lambda)c^2$, $\lambda l^1 + (1-\lambda)l^2$, $\lambda m^1 + (1-\lambda)m^2$, $\lambda\Omega^1 + (1-\lambda)\Omega^2$. This follows since such a solution satisfies equations (7a-7h).

Hence $V(\lambda\Omega_{-1}^{**1} + (1-\lambda)\Omega_{-1}^{**2})$

$$\geqq \int_0^2 e^{-\beta t}U(\lambda c^1 + (1-\lambda)c^2, \lambda l^1 + (1-\lambda)l^2, \lambda m^1 + (1-\lambda)m^2)dt + \alpha V_{+1}(\lambda\Omega^{**1} + (1-\lambda)\Omega^{**2})$$

(by definition of optimality)

$$> \lambda \int_0^2 e^{-\beta t}U(c^1, l^1, m^1)dt + (1-\lambda)\int_0^2 e^{-\beta t}U(c^2, l^2, m^2)dt + \alpha V_{+1}(\lambda\Omega^{**1} + (1-\lambda)\Omega^{**2})$$

(by concavity of U)

$$\geqq \lambda V(\Omega_{-1}^{**1}) + (1-\lambda)V(\Omega_{-1}^{**2}).$$

(Noting that $V_{+1}, V_{+2}, \ldots, V_{+n}, \ldots$ have the same properties as V.) Hence by definition V is strictly concave.

Then, assuming V is twice differentiable, $V'' < 0$.

The sufficiency theorem given in Long and Vousden (1977, pp. 25-26, Theorem 6) can be used to show that the program which satisfies the necessary conditions is optimal.

Application of Long and Vousden (1977, p. 30, Theorem 8) then establishes the uniqueness of the optimal solution if it exists.

Existence of Optimal Solution

It is still necessary to prove that the optimal solution is feasible. A solution which satisfies the necessary conditions can be constructed as follows:

First note that the solution to the equation $\dot{\psi} = (\beta - i)\psi$ (equation 8e) exists given any initial condition. Thus an optimal solution exists provided an appropriate initial condition exists which satisfies the transversality condition given by $\psi(1) = Z'(\Omega^* + \Omega_{-1}^{**})$.

To prove that such an initial condition exists, note firstly that $V'(\Omega_{-1}^{**}) = Z'(\Omega^* + \Omega_{-1}^{**})$. Hence $V''(\Omega_{-1}^{**}) = Z''(\Omega^* + \Omega_{-1}^{**})$ and since V is concave, so is Z.

Assume Ω_{-1}^{**} is given and fixed and $\psi(1) < Z'(\Omega^* + \Omega_{-1}^{**})$.

Then $\psi\uparrow \Rightarrow m(0)\downarrow, \Omega(0)\uparrow$, (by equation 1)

$\Rightarrow m\downarrow, \Omega\uparrow$ for all t

$\Rightarrow \Omega^*\uparrow$, (since $i + \pi > 0$)

$\Rightarrow Z'(\Omega^* + \Omega_{-1}^{**})\downarrow$, (by concavity of Z).

Hence by continuity of ψ and Z we can choose $\psi(1)$ so that a solution exists which will satisfy the necessary conditions. A similar proof obtains if $\psi(1) > Z'(\Omega^* + \Omega_{-1}^{**})$.

The optimal solution can thus be obtained by choosing suitable initial values of m and QE. In general since demand for money will be non-zero and since $m(0) + QE(0) = 0$, this will involve the household initially borrowing equities which it will have to repay at the market rate of interest (= i).

CHAPTER 10

THE EFFECTS OF CHANGING UNEMPLOYMENT BENEFITS AND PENSIONS

10.1 Introduction

> "The (Australian) Government has already increased, from May, 1983, the rate of unemployment benefit for single persons aged 18 and over by $4.25 per week to $68.65 per week at a cost of about $100 million in 1983-84. The Government has now decided to increase the rate again in November 1983 by $4.95 per week, or by $2 a week more than needed to allow for inflation, to bring the rate to $73.60 per week. Commencing in May 1984, this rate will be automatically indexed for increases in the Consumer Price Index: at that time a $2 supplement will again be provided. On current reckoning, this will amount to a further increase of about $4.80 per week. The total increase since the Government came to office will then be around $14 per week or more than 21 percent; over the same period the combined married rate is likely to have increased, with existing indexation arrangements, by about 16 percent to around $148.90 per week."
>
> Australian 1983/84 Budget Speech[1]

The increase in unemployment benefits initiated by the recently elected Australian Labor Government was also accompanied by restrictions on eligibility for old-age pensions which will effectively lower the average benefit received from the government by those who have retired. It seems quite likely that the election of any new government in Australia may frequently be accompanied by changes in the benefits accrued to the general population from welfare payments. This will occur because the change in government is frequently accompanied by a change in ideology as to what welfare policy can be considered equitable or socially desirable. Accordingly, changes in unemployment benefits and pensions which are financed by the government can be expected to recur in Australian historical experience. Here we develop a framework for the analysis of the effects of such policy changes and also use this framework to examine the consequences of such changes.

At first glance, an increase in welfare payments financed by the government, would be expected to have similar effects to a change in any other form of government expenditure, increasing aggregate demand and, if not financed out of taxation revenue, increasing the budget deficit. However the household sector developed in Chapter 9 shows that there are important differences. Firstly, changes in welfare payments will only have an effect if they are not fully financed out of taxation revenue. Secondly, if changes in welfare payments affect aggregate demand they will also have an influence on the demand for money and the supply of labor.[2]

In this chapter we examine the short-run and long-run effects of changing unemployment benefits and pensions in a model of the economy which includes the intrinsic dynamics of asset accumulation as well as the dynamics of capital accumulation. In the past, the effect of changing welfare payments have not been considered in conjunction with these issues. One approach has been to examine changing welfare payments within a partial equilibrium framework (e.g., Feldstein (1977)). Another approach has been to examine changing welfare payments within the framework of Diamond's (1965) neoclassical growth model. The most extensive model by Hu (1979) is a general equilibrium model capable of analyzing long-run effects. However his model assumes that the budget is always balanced and abstracts from the stability properties of the economy. Here we extend the analysis of Hu by incorporating some of the assumptions of the Barro (1974)/Feldstein (1974, 1981) debate within the framework of the dynamics of asset accumulation (see Blinder and Solow (1973), Tobin and Buiter (1976), Turnovsky (1977)). This is an important extension because the mechanism by which unemployment benefits and pensions are financed not only affects the form of the household sector functions but through its effect on the government surplus/deficit will also influence the dynamics of the economy. The stability properties of the economy will also affect the short-run and long-run responses to a change in benefits and pensions.

10.2 A Dynamic Macro-Model

We base the analyses of this chapter on a model which is similar in many respects to the model of Chapter 8. This model assumes that all investment is equity financed, unemployment is positive (i.e., $N^S > N^D$), and also assumes the absence of government bonds and hence the total money financing of the government deficit. The model also incorporates unemployment benefits and pensions and assumes that a fixed percentage of expenditure on these is financed out of taxation on the household sector and the rest is financed by printing money. This model is given by the following equations:

$$Y = C + I + G_0 \tag{1a}$$

$$C = C(\underset{+}{A}, \underset{-}{i}, \underset{+}{\rho v}, \underset{+}{-\pi}, \underset{+}{(1-\gamma)(1-\rho)u}, \underset{+}{(1-\gamma)s}, \underset{-}{T_0'}) \tag{1b}$$

$$m = L(\underset{+}{A}, \underset{-}{i}, \underset{+}{\rho v}, \underset{+}{-\pi}, \underset{+}{(1-\gamma)(1-\rho)u}, \underset{+}{(1-\gamma)s}, \underset{-}{T_0'}) \tag{1c}$$

$$I = I(\underset{-}{i}, \underset{-}{v}) \tag{1d}$$

$$A = m + QE \tag{1e}$$

$$QE = QE(\underset{+}{K}, \underset{-}{i}, \underset{-}{v}, \underset{-}{T_f}) \tag{1f}$$

$$Y = N^D F(K/N^D),\ F' > 0,\ F'' < 0 \tag{1g}$$

$$v = G(K/N^D) = F(K/N^D) - (K/N^D)F'(K/N^D) \tag{1h}$$

$$N^S = N^S(\underset{-}{A},\ \underset{+}{i},\ \underset{+}{\rho v},\ \underset{-}{-\pi},\ \underset{-}{(1-\gamma)(1-\rho)u},\ \underset{-}{(1-\gamma)s},\ \underset{+}{T_0'}) \tag{1i}$$

$$U = \frac{N^S - N^D}{N^S} \tag{1j}$$

$$w = -\alpha(U-\bar{U}) + \pi \tag{1k}$$

$$\rho = \frac{N^D}{N^S} \tag{2a}$$

$$\pi = p \tag{2b}$$

$$\dot{K} = I - \lambda K \tag{3a}$$

$$\dot{v} = v(w-p) \tag{3b}$$

$$\dot{m} = G_0 - T_0 + (1-\gamma)(1-\rho)u + (1-\gamma)s - pm \tag{3c}$$

where

Y = real national output

C = aggregate planned consumption

I = aggregate planned investment

G_0 = real government expenditure assumed endogenous

A = real private wealth

r = nominal rate of return on equities

π = expected rate of inflation

$i = r-\pi$ = real rate of return on equities

ρ = expected employment rate

u = real unemployment benefit

v = real wage-rate

s = real pension

N^S = supply of labor

N^D = demand for labor

m = real supply of (and demand for) money

QE = real supply of (and demand for) equities

K = real level of physical capital

p = actual inflation rate

U = unemployment rate

$\bar{U}$ = natural unemployment rate

w = rate of wage growth

T_0' = level of taxes on households other than taxes used to finance unemployment benefits and pensions

γ = fixed proportion of expenditure on unemployment benefits and pensions that is financed by taxation on household sector, $0 \leqq \gamma \leqq 1$

T_f = corporate taxes (assumed exogenous)

$T_0 = T_0' + T_f$ = aggregate taxation other than taxes which are used to finance unemployment benefits and pensions

Equilibrium in the product market is described by equation (1a). Equations (1b) and (1c) describe planned consumption and the demand for money under the assumption that aggregate household taxation includes a fixed proportion of expenditure on unemployment benefits and pensions (i.e., equal to $T_0' + \gamma(1-\rho)u + \gamma s$). These functions were derived from an optimizing framework in the preceding chapter. Planned investment is defined in equation (1d), real private wealth is defined in equation (1e) and the supply of equities is defined in equation (1f). The production function is defined in equation (1g) while the demand for and supply of labor can be derived from equations (1h) and (1i). Equation (1j) defines the unemployment rate and equation (1k) is a standard version of the Phillips curve.

Equations (2a)-(2b) define the expected employment rate and the expected rate of inflation, both of which are assumed to satisfy perfect myopic foresight.

The dynamics of the model are given by equations (3a)-(3c). The rate of capital accumulation is defined as equal to the rate of investment less capital depreciation. The evolution of the real wage is defined in the standard manner and the supply of money is constrained by the government budget constraint. In particular equation (3c) is derived by assuming that $\dot{M}/P$ is given by the difference between total government expenditure (= $G_0 + (1-\rho)u + s$) less total taxation receipts (= $T_0 + \gamma(1-\rho)u + \gamma s$).

A possible extension of this model would be to assume that the natural rate of unemployment $\bar{U}$ is a function of unemployment benefits; e.g., we could assume $\bar{U} = \bar{U}(\underset{+}{u})$. In other words an increase in the unemployment benefit will raise the natural rate of unemployment. This specification has some support in the popular press and could also be derived from a search theory approach to unemployment. It turns out that provided $\gamma < 1$ this specification will not change the general conclusions of our approach except that an increase in unemployment benefits will raise the long-run unemployment rate. Accordingly, this richer specification will not be considered in detail here.

We could also consider the case when a fixed proportion of the expenditure on unemployment benefits and pensions is financed out of taxes on the corporate sector. However taxes on the corporate sector do not affect the investment decision, or the process of capital accumulation and their only influence on the corporate sector comes about by lowering the value of equities.[3] In the economy that we shall subsequently be considering, wealth effects in the consumption function and the demand for money function are minimal. Accordingly, partially financing expenditure on unemployment benefits and pensions out of corporate taxes will simply lower the budget deficit. As a result the same general conclusions will apply whether unemployment benefits and pensions are partially financed out of corporate taxes or not and we shall not consider this case further in this chapter.

10.3 Simplifying the Model

In the rest of this chapter, we shall examine the effects of an exogenous change in unemployment benefits and pensions. We shall also assume, merely to simplify the analysis, that the wealth effects in planned consumption and the demand for money are close to zero and can be ignored. The model given by equations (1)-(3) can then be rewritten in the form:

$$Y = C(i,\ \rho v,\ -\pi,\ (1-\gamma)(1-\rho)u,\ (1-\gamma)s) + I(i,v) + G_0 \qquad (4a)$$

$$m = L(i,\ \rho v,\ -\pi,\ (1-\gamma)(1-\rho)u,\ (1-\gamma)s) \qquad (4b)$$

$$Y = N^D F(K/N^D) \qquad (4c)$$

$$v = G(K/N^D) \Rightarrow N^D = N^D(K,v) \qquad (4d)$$

$$U = U(\underset{+}{i},\ \underset{+}{v},\ \underset{-}{-\pi},\ \underset{-}{(1-\gamma)(1-\rho)u},\ \underset{-}{(1-\gamma)s},\ \underset{-}{K}) \qquad (4e)$$

$$1 - \rho = U \qquad (4f)$$

$$\dot{v} = -\alpha v(U-\bar{U}) \tag{4g}$$

$$\dot{K} = J(\underset{-}{K}, \underset{-}{i}, \underset{-}{v}) \tag{4h}$$

$$\dot{m} = G - T_0 + (1-\gamma)(1-\rho)u + (1-\gamma)s - \pi m \tag{4i}$$

The dynamic variables of this model are given by v, K and m while the short-run model given by the six equations (4a)-(4f) has the endogenous variables, Y, i, π, N^D, ρ and U.

As in Chapter 8 we assume that the economy described by the model is highly capital intensive (i.e., characterized by a high capital/ labor ratio). Hence the firm's optimization decision will be relatively independent of the returns to labor. Also the demand for labor, N^D, and aggregate output, Y, will be relatively insensitive to the real wage-rate. Investment, I, will only be affected by the real wage to the extent that the interest rate is affected by the real wage (i.e., I = I(i)). Unlike in Chapter 8, where labor supply $\bar{N}$ is fixed, the labor supply, N^S, derived in this model from the household optimization problem of Chapter 9, will still depend on the real wage-rate; thus the unemployment rate, U, will also still depend on the real wage.

In a similar manner to Section 8.5 of Chapter 8, the short-run model given by equations (4a)-(4f) can be solved for the real rate of interest, i, and the rate of inflation, π, in terms of the variables v, K, m and U, as well as the exogenous variables u and s. The equations are given by:

$$i = i(K, v, m, (1-\gamma)(1-\rho)u, (1-\gamma)s) \tag{5a}$$

$$\pi = \pi(K, v, m, (1-\gamma)(1-\rho)u, (1-\gamma)s) \tag{5b}$$

Throughout this chapter, we assume that the real money supply is the primary determinant of interest rates and inflation. This assumption is consistent with the stance of recent Australian budgets, which have emphasized the use of a monetary instrument to control inflation and also has some foundation in recent U.S. historical experience (or at least the public perception of this recent experience as represented by the reaction of the U.S. stock market to actions by the U.S. Federal Reserve System). It is important to note that we are not assuming that the other forces do not affect inflation and real interest rates; rather what we are saying is that these other forces are secondary compared with the impact of the money supply. If we make this assumption and also assume plausible sign values for the partial derivatives that are consistent with the sign values adopted in earlier chapters, then a simplified version of the dynamic model can be described by the following equations.

$$i = i(\underset{+}{m}) \tag{6a}$$

$$\pi = \pi(\underset{-}{m}) \tag{6b}$$

$$U = U(\underset{-}{K}, \underset{+}{z}, \underset{+}{i}, \underset{-}{-\pi}, \underset{-}{(1-\gamma)Uu}, \underset{-}{(1-\gamma)s}) \tag{6c}$$

$$\dot{z} = -\alpha(U-\bar{U}) \tag{6d}$$

$$\dot{K} = J(\underset{-}{K}, \underset{-}{i}) \tag{6e}$$

$$\dot{m} = G_0 - T_0 + (1-\gamma)Uu + (1-\gamma)s - \pi m \tag{6f}$$

It will be observed that since the nominal money supply, M, is fixed in the short-run, any changes in the real money supply, m, will come through changes in the price level. Thus an increase in the price level will raise inflation and lower real interest rates in the short-run and a fall in the price level will have the reverse effect.

Since the primary objective of this chapter is to examine the effects of a change in u or s, we can observe that such variables will only affect the economy if $\gamma < 1$, i.e., if pensions are not totally financed out of household sector taxes. For the rest of this chapter we shall consider the effects of a change in pensions and unemployment benefits when this is the case, i.e., when $0 \leqq \gamma < 1$.

10.4 Long-Run Comparative Statics

We shall now examine the long-run effects of a change in unemployment benefits, u, and pensions, s. To do this we must first consider the long-run model derived from equations (6a)-(6f) which is given by the following set of equations:

$$U(\underset{-}{K}, \underset{+}{z}, \underset{+}{i}, \underset{-}{-\pi}, \underset{-}{(1-\gamma)\bar{U}u}, \underset{-}{(1-\gamma)s}) = \bar{U} \tag{7a}$$

$$J(\underset{-}{K}, \underset{-}{i}) = 0 \tag{7b}$$

$$G_0 - T_0 + (1-\gamma)\bar{U}u + (1-\gamma)s - \pi m = 0 \tag{7c}$$

$$i = i(\underset{+}{m}) \tag{7d}$$

$$\pi = \pi(\underset{-}{m}) \tag{7e}$$

In the long-run unemployment is driven to its natural rate, $\bar{U}$, and all capital accumulation and wealth accumulation cease. Provided $0 \leqq \gamma < 1$, any change in pensions will affect the steady state. Also, provided $0 \leqq \gamma < 1$ and $\bar{U} > 0$, any change in unemployment benefits will also affect the steady state. Assuming these are the case we can rewrite equation (7c) as

$$\underset{-}{V}(\underset{-}{\pi}, \underset{-}{m}, \underset{+}{(1-\gamma)\bar{U}u}, \underset{+}{(1-\gamma)s}) = 0 \qquad (7c')$$

and we can substitute equations (7d)-(7e) into equations (7a), (7b), (7c') to obtain the steady-state system, which is given by the following equations.[4]

$$U(\underset{-}{K}, \underset{+}{z}, \underset{-}{m}, \underset{-}{(1-\gamma)\bar{U}u}, \underset{-}{(1-\gamma)s}) = \bar{U} \qquad (8a)$$

$$J(\underset{-}{K}, \underset{-}{m}) = 0 \qquad (8b)$$

$$V(\underset{+}{m}, \underset{+}{(1-\gamma)\bar{U}u}, \underset{+}{(1-\gamma)s}) = 0 \qquad (8c)$$

The long-run comparative static effects of a change in s are then given by:

$$\frac{\partial \tilde{K}}{\partial s} = \frac{1}{\Delta}[(1-\gamma)U_z J_m V_s)] > 0 \qquad (9a)$$

$$\frac{\partial \tilde{z}}{\partial s} = \frac{(1-\gamma)}{\Delta}[U_m J_k V_s - U_s J_k V_m - U_k J_m V_s] \gtrless 0 \qquad (9b)$$

$$\frac{\partial \tilde{m}}{\partial s} = \frac{1}{\Delta}[-(1-\gamma)U_z J_k V_s] < 0 \qquad (9c)$$

where $\Delta = U_z J_k V_m < 0$ and the tilde denotes that these are steady-state effects.

Thus, provided $0 \leqq \gamma < 1$, an increase in pensions will raise the long-run capital stock, lower the real money supply and have an ambiguous effect on the real wage-rate. Intuitively these results can be justified as follows: first, the increase in pensions will raise the government deficit thus lowering the real money supply necessary to balance the budget. In turn, the lowering of equilibrium money supply will lower the real interest rate, hence raising the level of investment and hence increasing the long-run capital stock. The ambiguity in the effect on the real wage can be explained as follows: The increase in capital stock will raise the demand for labor thus tending to lower the unemployment rate. On the other hand, the reduced real money supply will raise the supply of labor thus having a tendency to raise the unemployment rate. If the overall effect is to reduce medium term unemployment then labor will be more scarce and hence the real wage will rise. If the overall affect is to increase medium-term unemployment then labor will be more abundant and hence the returns to labor will be reduced (i.e., the real wage will fall).

As a result of the above effects, an increase in pensions will also increase the long-run level of inflation and raise output while regardless of the magnitude of the effects steady state unemployment will remain constant at its natural rate. Also, the larger the percentage of the change in pensions that is financed out of taxation, the smaller will be the magnitude of the steady-state effects.

All the effects that result from a change in unemployment benefits, u, have the same sign as a corresponding change in pensions, s, and can be justified in a similar manner. These long-run effects of a change in u will be greater the larger the natural unemployment rate $\bar{U}$ and will become smaller as $\gamma \to 1$.

10.5 Dynamics

We shall now examine the dynamic properties of the model given by equations (6a)-(6f). This model can be linearized about its steady state to give the following dynamic system.

$$\begin{pmatrix} \dot{z} \\ \dot{K} \\ \dot{m} \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} z-\tilde{z} \\ K-\tilde{K} \\ m-\tilde{m} \end{pmatrix} \tag{10}$$

where, assuming plausible sign values,

$$a_{11} = -\alpha U_z < 0 \tag{11a}$$

$$a_{12} = -\alpha U_K > 0 \tag{11b}$$

$$a_{13} = -\alpha U_m > 0 \tag{11c}$$

$$a_{21} = J_z < 0 \tag{11d}$$

$$a_{22} = J_K < 0 \tag{11e}$$

$$a_{23} = J_m < 0 \tag{11f}$$

$$a_{31} = (1-\gamma)uU_z - \pi_z m > 0 \tag{11g}$$

$$a_{32} = (1-\gamma)uU_K - \pi_K m < 0 \tag{11h}$$

$$a_{33} = (1-\gamma)uU_m - \pi_m m - \pi > 0 \tag{11i}$$

If we follow the assumptions of Section 10.4 of this chapter and assume that the economy described by this dynamic system is capital intensive and that the money supply is the primary determinant of interest rates and inflation then, as a first approximation, we can let $a_{21} = 0$. If in addition we assume that the unemployment benefit, u, is relatively small then we can also let $a_{31} = a_{32} = 0$. We shall make these assumptions throughout the rest of this chapter.

Accordingly the characteristic roots of the dynamic system can be approximated by:

$$\lambda_1 = a_{11} < 0 \tag{12a}$$

$$\lambda_2 = a_{22} < 0 \tag{12b}$$

$$\lambda_3 = a_{33} > 0 \tag{12c}$$

Hence the system has three real characteristic roots, two of which are stable and one which is unstable. We can now solve equation (10) to give a dynamic system of the form:

$$\begin{pmatrix} z-\tilde{z} \\ K-\tilde{K} \\ m-\tilde{m} \end{pmatrix} = \begin{pmatrix} 1 & a_{12}a_{23} & a_{12}a_{23}+a_{13}(a_{33}-a_{22}) \\ 0 & a_{23}(a_{22}-a_{11}) & a_{23}(a_{33}-a_{11}) \\ 0 & 0 & (a_{33}-a_{22})(a_{33}-a_{11}) \end{pmatrix} \begin{pmatrix} k_1 e^{a_{11}t} \\ k_2 e^{a_{22}t} \\ k_3 e^{a_{33}t} \end{pmatrix} \tag{13}$$

Since $a_{33} > 0$ the economy will follow an ever exploding path unless the economy lies on a stable arm which is characterized by $k_3 = 0$. The economy will only be on this path if the real money supply is always at its steady-state, i.e., $m = \tilde{m}$. We shall use this property of the model in the next section to examine the short run response of the economy to an exogenous change in unemployment benefits and pensions.

10.6 Short-Run Comparative Statics

Following the work of Sargent and Wallace (1973), Brock (1974, 1977) and Flood and Garber (1980), we now assume the absence of speculative bubbles and hence that the price level will jump in the short-run in such a way as to keep the economy on a stable path which converges to the long-run equilibrium described by equations (7a)-(7e).

We know that along the stable path $m=\tilde{m}$ at all times. Accordingly, following an exogenous change in the level of pensions, there will be a short-run change in m given by

$$\frac{\partial m}{\partial s} = \frac{\partial \tilde{m}}{\partial s} < 0 \qquad (14a)$$

This can be achieved by a short-run jump in the price level given by

$$\frac{\partial P}{\partial s} = -\frac{P}{m} \cdot \frac{\partial \tilde{m}}{\partial s} > 0 \qquad (14b)$$

in other words, an increase in the pension will lead to a corresponding upward jump in the price level.

As a result of this short-run change in prices, in the short-term inflation will rise and real interest rates as well as the real wage will fall. Meanwhile, the effects on short-run unemployment will be indeterminate. Finally, investment will increase, physical capital stock will remain constant and output, which is primarily dependent on the level of physical capital stock will remain close to constant.

The effects of an increase in unemployment benefits will have the same sign as the effect of an increase in pensions but the magnitude will depend on the parameters in the economy.

10.7 Conclusion

We have now examined the short-run and long-run effects as well as the dynamic path associated with a change in unemployment benefits and pensions. Consistent with the arguments of Barro (1974) and Feldstein (1974, 1981) the effect of a change in welfare payments depends upon the extent to which these welfare payments are financed out of household sector taxes. Thus when all welfare payments are financed out of household sector taxes (i.e., when $\gamma = 1$) a change will have no effect whatsoever. On the other hand, when no welfare payments are financed out of household sector taxes (i.e., when $\gamma = 0$) a change will have its greatest effect.

In the case when $\gamma < 1$, in the short-run we have shown that an increase in pensions will increase the price level thus lowering the real money supply and the real wage. As a result, inflation, which is assumed to be primarily a monetary phenomenon will increase.

Following these changes the economy will evolve towards a long-run equilibrium which is characterized by higher inflation, a higher capital stock, higher output and a natural rate of unemployment.

In this chapter we have restricted our analysis to positive economic questions. As a result we have not examined the welfare effects of a policy change nor have we considered optimum levels for the pension or unemployment benefits. However, since we have shown that the effects of a policy change will differ depending on how such pensions and unemployment benefits are financed thus we can conclude that these questions must be resolved before, or as an integral part of, any normative study.

CHAPTER 11

SOME CONCLUDING REMARKS

In the preceding chapters we have demonstrated some of the techniques that can be employed to examine a few of the important issues related to unemployment and inflation. Each chapter includes its own introduction and conclusion. Thus, in summarizing the results of the thesis here we can be very brief. For broader clarification the reader is referred to the relevant chapter.

Following a general introduction to the thesis in Chapter 1, "The Role of Expectations and Deficit Financing" was examined in Part 1 (Chapters 2-4). In Chapter 2 the relationship between adaptive expectations and perfect myopic foresight was discussed. It was shown that even in models of limited complexity it is quite likely for adaptive expectations to be associated with instability. In Chapter 3 the rational expectations approach to economic modeling, which involves the application of appropriate endogenous stabilizing jumps, was introduced. Also a general procedure was developed for the calculation of appropriate short-run responses following an exogenous shock. This procedure was used throughout the rest of the thesis in the calculation of appropriate short-run responses. Finally, in Chapter 4, the possibility of non-linearities leading to non-unique steady-state equilibria and non-unique stability properties was discussed. Throughout Part 1, examples of different stability properties were provided by considering the consequences of different methods of Deficit Financing.

Part 2 (Chapters 5-7) addressed the issue of "Optimal Stabilization Policies Under Perfect Foresight" in two different ways. Thus, in Chapter

5 the objectives of a policy maker who wants to fight unemployment and inflation simultaneously was formalized. In Chapter 6, we examined the solution to this problem when prices were constrained to move sluggishly, while Chapter 7 examined the solution to this problem when prices were allowed to jump in the short-run. We showed that, in both cases, the optimal policy involved driving inflation to zero instantaneously. However if prices were allowed to jump the optimal path for unemployment was possibly different than if prices evolved sluggishly.

It was also shown that a meaningful solution could not always be obtained if the policy maker had only one policy instrument available to attain his objectives. Also, depending on parameters which can be determined by the policy maker, the optimal jump in the price level may or may not be associated with time inconsistency.

"Some Relevant Policy Issues" were discussed in Part 3 (Chapters 8-10). Here we introduced the most complex models of the thesis. Thus for the first time the dynamics of capital accumulation as well as a crude stock-market were introduced. Also the more unusual properties of the models were derived from optimizing behaviour.

In Chapter 8 we examined the consequences of lowering the length of the standard working week. As expected, the implementation of this policy lowered short-run unemployment. Surprisingly, however, unless accompanied by significant productivity gains, such a policy also lowered long-run inflation and output. As a consequence of the forward looking nature of the rational expectations approach, short-run inflation was then also lowered.

In Chapter 9 and 10 it was shown that, in general, increasing unemployment benefits and pensions will raise short-run inflation and have an indeterminate effect on unemployment. In the long-run, inflation will be higher and unemployment will converge to its natural rate.

Having summarized the general conclusions of the thesis, we now devote the rest of this chapter to an examination of possible extensions of the results.

Even without perfect foresight, if expectations adjust sufficiently quickly in the real world then the economy will typically exhibit the instability properties associated with rational expectations. Thus, in the real world, it is possible to have many of the properties associated with rational expectations even though expectations are not truly rational. Among the unresolved questions from this thesis is whether or not there is actually sufficient information available in a world without perfect foresight for individuals to be able to stabilize the economy in the manner suggested by the rational expectations literature.

Another question concerns the choice of appropriate jump variables which will stabilize the economy. In models with static expectations or adaptive expectations, which have stable dynamic paths, the typical short-run response involves an adjustment to short-run equilibrium and can be achieved by means of the traditional tâtonnement process. In the rational expectations approach, the literature shows that it is optimal for stability to be achieved by jumps in the appropriate endogenous variables. However there is no equivalent to the tâtonnement mechanism which describe what makes the jumps occur. The choice of appropriate jump variable(s) is to some extent an open question and is frequently treated in an ad-hoc manner. Basically it depends upon how quickly relevant dynamic variables react to new information ("news") as it becomes available. In a closed economy model like those treated in this thesis, we frequently choose the price level as the jump variable. In other contexts, other variables might be more appropraite choices for jump variables. For example, in the exchange market literature, the goods market is often assumed to clear slowly in relationship to financial markets. In this case the exchange rate is typically taken to be the jump variable, with the price of output being constrained to move sluggishly. The choice of appropriate jump variables and the mechanism by which the jump is achieved present an avenue for future research.

By taking several alternative forms of deficit financing in Part 1 we have also been able to show that the stability properties of the economy are sensitive to monetary policy and also to the interrelationship between monetary policy and the sign and magnitude of the government deficit. This issue could have been emphasized throughout the thesis. Thus in Part 3 we could have treated the effects of changing the length of the standard working week, unemployment benefits and pensions in conjunction with the means for financing the deficit. Since this would have clouded the central issues, we instead adopted an approach of highlighting one monetary policy and examining the consequences in full detail. The results of Part 1 illustrate that the choice of a different mechanism for financing the deficit could well affect the conclusions of the rest of the thesis.

Part 2 showed that the optimal policy for fighting inflation and unemployment under perfect foresight involves driving inflation instantaneously to zero and thus no short-run trade-off between inflation and unemployment exists. This is in marked contrast to results derived under adaptive expectations where inflation converges along a dynamic path towards its equilibrium. The question arises whether it would ever be possible to obtain such a trade-off under perfect foresight.

Essentially the no trade-off results are a consequence of the specification of the Phillips Curve in Chapter 5. Thus it appears likely that some form of trade-off would occur if we adopted a less conventional formulation of the Phillips Curve, e.g., we could replace equations (1i, 2b, 2c) in Section 5.2 of Chapter 5 by the following equations

$$w = \alpha(\overline{U} - U) + \mu \qquad (1i')$$

$$\dot{m} = (\mu - p)m \quad (\text{i.e., } \dot{M} = \mu M) \qquad (2b')$$

$$\dot{m} + \dot{b} = G - T + rb - p(m+b) \qquad (2c')$$

In this equation system, the rate of monetary growth μ is also taken to represent some underlying core rate of inflation.

Another alternative would be to treat the economy as an open economy. Thus we could replace equations (1a, 1b, 1j) by

$$Y = D(Y^D, r - \delta\pi - (1-\delta)\pi^*, A) + G \qquad (1a')$$

$$Y^D = Y - T + rb - (\delta\pi + (1-\delta)\pi^*)A \qquad (1b')$$

$$\delta\pi + (1-\delta)\pi^* = p, \text{ except at points where P jumps} \qquad (1j')$$

and leave equation (1i) as

$$w = \alpha(\overline{U} - U) + \pi \qquad (1i)$$

where π is expected inflation of domestically produced goods and π^* is expected inflation of foreign produced goods, assumed exogenous.

Thus equations (1a', 1b', 1j') are derived under the assumption that inflation of domestic price levels is a trade-weighted average of inflation of domestic and foreign produced goods, whereas equation (1i) assumes that wage claims are based on the price increases of domestic produced goods.

Either of these alternative specifications could give an optimal short-run trade-off between unemployment and inflation, and provides an opportunity for future research.

The models of Part 3 could be generalized to incorporate different methods of deficit financing as well as alternative methods for financing investment but we have chosen to highlight only the central issues. The interrelationships between several issues would probably best be handled in a perfect foresight equilibrium framework (e.g., as in Brock and Turnovsky (1981)). This would be a suitable extension of Part 3 of the thesis.

The topic of unemployment and inflation is so broad that it encompasses nearly all of macroeconomics. We have examined only a few issues in a closed economy framework. The next step would be to examine some important issue as pertaining to open economies. One obvious issue would be to examine the effects of imported inflation as a result of exogenous shocks (e.g., the Oil Price Shocks). Another issue would be to examine the effect of particular government policies in one country (e.g., tight money policy in U.S.) on other countries. Such work may well involve the use of Game Theory (e.g., as in Canzoneri and Gray (1983)).

In conclusion, it should be emphasized once again that this thesis is not in any way meant to be an exhaustive study of inflation and unemployment. Rather it is meant to be a presentation of a few techniques that can be used to examine the economic implications of government policy decisions. Only a few topics have been examined; many topics are yet to be considered.

NOTES

Notes to Chapter 1

1. A more extensive approach to the importance of the role of expectations is given in Begg (1982).

2. This is a quote from the 1983/84 Australian Budget Speech which was delivered to the Australian Parliament on 23 August 1983 by the Honourable P.J. Keating, M.P., Treasurer of the Commonwealth of Australia.

3. We can also show this result algebraically as follows:

$$L = ap^2 + U^2 = a[\alpha^2(U-\overline{U})^2 - 2\alpha\pi(U-\overline{U}) + \overline{\pi}^2] + U^2$$

$$\frac{\partial L}{\partial U} = a[\alpha^2 2(U-\overline{U}) - 2\alpha\overline{\pi}] + 2U = 0$$

Hence letting a ^ denote optimal solutions

$$\hat{U} = \frac{a\alpha^2\overline{U} + a\alpha\overline{\pi}}{a\alpha^2 + 1} > 0$$

$$\hat{p} = \frac{\alpha\overline{U} + \overline{\pi}}{a\alpha^2 + 1} > 0$$

where the signs assume $\overline{\pi} > 0$, $\overline{U} > 0$.

4. We can show this result algebraically as follows:

$$L = ap^2 + \overline{U}^2$$

$$\frac{\partial L}{\partial p} = 2ap = 0$$

Hence $\hat{p} = 0$.

5. These figures were provided by the Australian Bureau of Statistics.

6. This issue is discussed further in Organisation for Economic Co-operation and Development (1982).

Notes to Chapter 2

1. The government budget constraint is given by $\dot{M} + \dot{B} = P(G-T) + rB$. Using the fact that $\dot{M}/P = \dot{m} + mp$, $\dot{B}/P = \dot{b} + bp$, equation (2b) follows.

2. This assumption was adopted by Foley and Sidrauski (1971). In a more formal sense the zero change in the expected inflation rate can be interpreted as the integral of an infinite jump over a set of measure zero.

3. This assumption has been formulated in this heuristic manner by Boyer (1976). We can formalize the notion more rigorously as follows:

Let us represent the price level, P, in the neighbourhood of the jump by

$$P = P_0 + H(t-\bar{t})dP$$

where $H(t-\bar{t})$ denotes the Heaviside function, i.e.,

$$H(t-\bar{t}) = 0 \text{ for } t < \bar{t}, \; H(t-\bar{t}) = 1 \text{ for } t \geqq \bar{t}$$

Taking the derivative of P at $t = \bar{t}$ we can express

$$\dot{P}/P = \delta(t-\bar{t})[dP/P]$$

where $\delta(t-\bar{t})$ denotes the Dirac delta function $\delta(t-\bar{t}) = 0, \; t \neq \bar{t}$ $\int_0^\infty \delta(t-\bar{t})dt = 1$ and is a generalized function representing an impulse at time t. Since

$$\gamma(\dot{P}/P) = \dot{\pi} + \gamma\pi$$

hence $$\gamma\delta(t-\bar{t})(dP/P) = \dot{\pi} + \gamma\pi$$

and integrating both sides

$$\pi = e^{-\gamma(t-\bar{t})}\gamma H(t-\bar{t})(dP/P)$$

in the neighbourhood of $t = \bar{t}$

Hence, at $t = \bar{t}$, $d\pi = \gamma(dP/P)$

4. The nonlinearities in this model occur because of the return on government bonds, rb, and the inflation tax on wealth, πA, even if all other functional forms are linear.

5. When the coefficients are not well-behaved, Burmeister (1980) has pointed out that it is possible to have a deterministic model for which the resulting time series appears stochastic: it does not converge to a point, does not diverge and is not cyclic; see also Diamond (1976).

Notes to Chapter 3

1. Here we simply require that $\lim_{t\to\infty} |y(t)| < \infty$. More formally we would require that the present discounted value of the y variables converge. Such a requirement could be justified by considering the model as having been derived from underlying optimization models. The assumption (1c) is actually a stronger requirement than the above, and is therefore sufficient to demonstrate the basic methodology of the rational expectations approach.

2. The symbol A will be used to represent real private wealth and an nxn-matrix. Similarly z will have the meaning of a jump variable and the log of the real wage and D will be used to represent aggregate demand and an nxm-matrix. The correct meanings should be clear from the context.

3. This equation and the rest of this section apply for both real-valued and complex-valued eigenvalues.

4. This result is obtained by noting that

$$\begin{pmatrix} A & B \\ 0 & C \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & -A^{-1}BC^{-1} \\ 0 & C^{-1} \end{pmatrix}$$

Notes to Chapter 4

1. This result has been obtained for models in which the underlying utility function is separable in real money balances and consumption and for which appropriate "super Inada" type conditions hold. The extent to which it holds for more general utility functions is not known.

2. Non-linearities and corresponding non-uniqueness of solutions are not restricted to the models considered in this chapter. Such non-linearities also arise in the models considered in Chapters 2 and 3. The issue of non-uniqueness of equilibria was ignored in these earlier chapters by assuming plausible sign values and considering only specific examples.

3. This source of uniqueness is not to be confused with non-uniqueness encountered in linear stochastic rational expectations models, arising from the arbitrariness of the constant of integration. This type of non-uniqueness has been extensively discussed by Taylor (1977) and procedures such as finite variance criteria and minimum variance criteria have been proposed for determining the arbitrary constant in these cases.

4. Turnovsky and Brock (1980) develop a model in which through lump sum taxation an equilibrium stock of bonds $b < 0$ is feasible and indeed optimal. We, however, do not consider such equilibria in this analysis.

5. This statement needs to be made somewhat more precise. The critical expression $D_2\vartheta(L_3-\vartheta) - L_2D_3$ is quadratic in ϑ, and is positive for both $\vartheta = 0$ and $\vartheta = 1$. Provided $D_2L_3^2 < 4L_2D_3$, this quadratic quantity changes sign for ϑ lying in the range $\vartheta_1 < \vartheta < \vartheta_2$, where ϑ_1, and ϑ_2 are the roots to $-D_2\vartheta^2 + D_2L_3\vartheta - L_2D_3 = 0$.

6. These conditions have been determined for the case of linear homogeneous functions D(.) and L(.). They can be suitably modified for the affine case.

7. Scarth (1980) and Turnovsky and Scarth (1982) produce results which are consistent with these. Their results are produced in a stochastic framework with no inflation and considering only the polar cases (i.e., $\vartheta = 0$, $\vartheta = 1$).

8. Burmeister and Turnovsky (1978) discuss the case when perfect foresight, coupled with sluggish adjustment of the appropriate market,

can give rise to perfect foresight (rational expectations) being associated with total stability.

Notes to Chapter 5

1. More formally we would require that the present discounted values of money and bonds converge. This requirement can be justified by considering the model as having been derived from an underlying optimization for the consumer. The assumption that real money and bonds converge to a finite value is actually a stronger requirement than the above, and is therefore sufficient to demonstrate the basic conclusions of the rational expectations approach.

2. In deriving the restrictions on the partial derivatives in (4a) and (4b), we have assumed that

$$D_1 m + D_2 < 0$$
$$D_1 b + D_2 < 0$$

3. This result can be proved formally. Using a modification of the proof of Vind's Theorem (see Arrow and Kurz (1970, p. 51)) it is possible to show that since the Hamiltonian is convex in U, an optimal solution will only jump at the initial time point.

4. Throughout the rest of Chapters 5-7 we shall assume that p and r can be described by affine functions.

Notes to Chapter 6

1. More precisely equation (5f) reduces to $(\mu-\varphi)(G-T+rb) = 0$. Since monetary policy would be ineffective if $G-T+rb = 0$, we have assumed that G and T have been chosen so that $G-T+rb \neq 0$ for all finite values of time, t. In this case equation (5f) reduces to $\mu=\varphi$ as given.

2. The conclusion that m and b can be controlled by G-T and ϑ respectively rests on the assumptions that $\vartheta \neq 0$, $G-T+rb \neq 0$.

3. The steady-state equilibrium $\tilde{p} = -\gamma$ will be recognized by the reader as the Friedman (1969) full liquidity rule.

4. An open market operation can be considered a special case of monetary policy ϑ involving an instantaneous infinite value for ϑ.

This notion can be developed more formally as follows: Suppose that G-T+rb is non-zero and we wish b to jump from b, to $b_1 + db$ and m to jump from m_1 to $m_1 - db$ at time $t = \bar{t}$. We can represent b as:

$$b = b_1 + H(t - \bar{t})db$$

where $H(t - \bar{t})$ denotes the Heaviside function, i.e., $H(t - \bar{t}) = 0$ for $t < \bar{t}$, $H(t - \bar{t}) = 1$ for $t \geqq \bar{t}$.

Taking the derivative of b at $t = \bar{t}$, we can express

$$\dot{b} = \delta(t - \bar{t})db$$

where $\delta(t - \bar{t})$ denotes the Dirac delta function. In general this is defined by

$$\delta(t - \bar{t}) = 0, \; t \neq \bar{t}, \; \int_0^\infty \delta(t - \bar{t}) = 1$$

and is a generalized function representing an impulse at time $\bar{t}$.

Thus if we choose

$$\delta(t - \bar{t})db = \vartheta(G - T + rb) - \pi b$$

where r, b, G, T, π and b are evaluated after the jump in b has occurred, then b will jump from b_1 to $b_1 + db$. In other words we need only choose

$$\vartheta = \frac{\delta(t - \bar{t})db + \pi b}{G - T + rb}$$

Since π, b, r, G and T will all remain finite such a procedure involves an infinite instantaneous jump in ϑ.

5. This can be proved more formally as follows. Assume initially $\dot{U} = 0$ and substitute into equation (18b).

Hence $\dot{m} = \dfrac{-p_b}{p_m} (\dot{b})$

and thus

$$\frac{1-\vartheta}{\vartheta} = \frac{-p_m}{p_b}$$

Since p_b can be given neither sign nor magnitude the inequality which has been derived under the assumption that $\vartheta > 0$, may or may not be satisfied.

Notes to Chapter 7

1. The equation $\partial H/\partial\vartheta = 0$ reduces to $(\mu - \lambda)(G - T + rb) = 0$. Since monetary policy would be ineffective if $G - T + rb = 0$, we have assumed that G and T have been chosen so that $G - T + rb \neq 0$ for all finite values of time, t. In this case (7e) reduces to $\mu = \lambda$, as given.

2. This result can be derived formally as follows. Suppose the objective is to minimize

$$L = c|U(0) - U_0| + \frac{1}{2}\int_0^\infty e^{-\gamma t}(U - \delta)^2 dt$$

Substituting for U(t) from (16a) gives

$$L = c|U(0)-U_0| + \frac{1}{2}\left(\frac{(\bar{U}-\delta)^2}{\gamma} + \frac{2[U(0)-\bar{U}][\bar{U}-\delta]}{\beta + \gamma} + \frac{[U(0)-\bar{U}]^2}{2\beta + \gamma}\right)$$

$$\equiv c|U(0)-U_0| + \varphi[U(0)].$$

We can now divide the general problem into two subproblems.

Problem 1:

Minimize $L_1 = c[U(0) - U_0] + \varphi[U(0)]$

subject to $U(0) \geqq U_0$

and Problem 2:

Minimize $L_2 = c[U_0 - U(0)] + \varphi[U(0)]$

subject to $U_0 \geqq U(0)$

Differentiating the appropriate expressions for L_1 and L_2 with respect to U(0) then yields expressions equivalent to (17a)-(17c).

Notes to Chapter 8

1. The Members of OECD are Australia, Austria, Belgium, Canada, Denmark, Finland, France, the Federal Republic of Germany, Greece, Iceland, Ireland, Italy, Japan, Luxembourg, the Netherlands, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, Turkey, the United Kingdom and the United States.

2. Similarly to Brock and Turnovsky (1981), we do not wish to suggest that putting money into the utility function is good monetary theory. What we are saying is that until fully satisfactory micro-foundations of the demand for money are developed, the procedure we are following is a useful analytical tool.

3. We assume here and throughout the rest of this thesis that all solutions to optimization problems are interior solutions, unless otherwise stated.

4. The use of Walras' Law here is a special one which is not uncommon in macroeconomics. Walras' Law, which can be derived from the various budget constraints (see Turnovsky (1977)) tells us that if the labor market does not clear then another market (i.e., goods, bonds, money or equities) will not clear as well. Here we follow Patinkin (1965, p. 333) and dodge this difficulty by attributing to workers a completely passive behavior pattern accordingly to which they adjust the expected supply of labor to the amount demanded by employers. Hence, by definition, "equilibrium" always exists in the labor market.

5. It is also possible for a firm to finance its investments out of retained earnings (i.e., by not returning all of its profits to its stockholders) or by the sale of private bonds. These cases have not been considered here.

6. Note that F is assume homogeneous of degree one. The following notations will be used interchangeably:

$$N^D F(K/N^D) = N^D F[(K/N^D), 1] = F[K, N^D]$$

7. This assumption can be justified if

$$h(I) = h_2 I^2 + h_1 I$$

where h_2 is sufficiently large.

8. More formally

$$\frac{\partial N^D}{\partial v} = \frac{-N^{D2}}{\eta(\overline{N})G'K} < 0$$

$$\frac{\partial Y}{\partial v} = \eta(\overline{N})\frac{\partial N^D}{\partial v} F\left(\frac{K}{N^D}\right) + N^D F'\left(\frac{K}{N^D}\right)\frac{1}{G}$$

Assuming $G' \rightarrow \infty$, $\frac{\partial N^D}{\partial v} \rightarrow 0$ and $\frac{\partial Y}{\partial v} \rightarrow 0$.

9. The sign of $J_1 (< 0)$ has been derived by assuming that the rate of capital depreciation, λ, is sufficiently large.

10. Equation (33) can be derived by noting that

$$\begin{pmatrix} A & 0 \\ B & C \end{pmatrix}^{-1} = \begin{pmatrix} A^{-1} & 0 \\ -C^{-1}BA^{-1} & C^{-1} \end{pmatrix}$$

11. There is a possible inconsistency here in that, in the derivation of the microfoundations of both the household sector and the corporate sector, we have assumed that expectations are formed by a combination of perfect myopic foresight and static expectations whereas much of the theory behind the absence of speculative bubbles (Brock (1974, 1977)) assumes perfect foresight. However, Flood and Garber (1980) merely tested the existence of speculative bubbles in the real world. Since the basis of our microfoundations is probably reasonably close to decision making in the real world where individuals do not have perfect foresight, we shall rely on the Flood and Garber (1980) analysis to justify the absence of speculative bubbles here.

<u>Notes to Chapter 9</u>

1. Throughout this paper $\Omega_j(j+2^-)$ denotes the limit of Ω_j as t approaches j+2 from below. Note that $\Omega_j(j+2)$ is undefined. Similarly, when the previous generation's bequest motive is operative, $\Omega_j(j+1^-) \neq \Omega_j(j+1)$.

2. If $i + \pi < 0$, i.e., $i < -\pi$, then the rate of return on bonds-cum-equities is less than the rate of return on money and there is no reason to hold bonds-cum-equities in the model.

3. This result is consistent with Drazen (1978) who has shown that crucial in determining if the bequest motive is operative is the rate at which the current generation discounts their descendant's utility. The higher the discount rate, the more likely is the bequest motive not to be operative.

4. Throughout this chapter, when a variable is written as the function of time, t, and several other functions of time, this will denote that each of the functions are evaluated at t.

5. The effect of i on $\psi_j(t)$ is made up of a wealth effect and a substitution effect which have conflicting signs. Throughout the rest of this chapter we shall assume that the substitution effect is dominant.

6. The sign of the partial derivative w.r.t. ρv in equation (11c) depends upon whether a "discouraged worker" or "additional worker" effect dominates. The "discouraged worker" effect measures the number of people who in times of high unemployment abandon the futile search for jobs and drop out of the labor force or supply less labor. A dominant discouraged worker effect thus corresponds to $(\partial l/\partial \rho) > 0$. The "additional worker" effect results from the hypothesis that in married couple families in which the husband is unemployed, the current fall in current income below permanent levels drives the wives to seek work. A dominant "additional worker" effect corresponds to $(\partial l/\partial v) < 0$. Empirical results for the United States (see Barth (1968) and Wachter (1972)) and Australia (see Gregory and Sheehan (1975) and Scherer (1978)) confirm the dominant discouraged worker effect. Although more recent work in the United States by Fleisher and Rhodes (1976) has argued that this result is caused by aggregation problems and that the true net effect is insignificant, we have assumed throughout this chapter that the discouraged worker effect predominates and accordingly that the sign of the partial derivative w.r.t. ρv in the labor supply functions (e.g., equation (11c)) is positive.

7. Again, we assume that the substitution effect is dominant (see footnote 5).

8. Again, we assume that the discouraged worker effect predominates (see footnote 6).

9. The method used in this Appendix is a generalization of the methods used by Pitchford (undated) who took Barro and Grossman's (1976) work/retire model and derived the household sector rigorously using optimal control theory. As this work/retire model involved only consumption, labor and money, with no discounting, no bequests, no unemployment and no equities the analysis of this Appendix involves a considerable extension.

Notes to Chapter 10

1. The 1983/84 Australian Budget Speech was delivered on 23 August 1983 by the Honourable P.J. Keating, M.P., Treasurer of the Commonwealth of Australia.

2. Government expenditure would also have an effect on the demand for money and labor supply if the household sector was derived from an optimizing framework which assumed that Government expenditure influenced individual's preferences (i.e., if Government expenditure was included as an argument in the individual's utility function). While this is a plausible assumption, the standard approach to the household sector does not assume this.

3. In a world of all equity financing before-tax profits are allocated between stockholders and the government as follows: $\Pi = D + T_f$. The statement that corporate taxes do not affect the investment decision or the process of capital accumulation assumes an additive tax function, i.e., $T_f = T_0'' + \gamma(1-\rho)u + \gamma s$. This statement no longer remains true if we assume a multiplicative form for the tax function, i.e., $T_f = \beta\Pi$, where $\beta = \beta\{\underset{+}{(1-\rho)u},\underset{+}{s}\}$.

4. The effect of a change in the real money supply on the government deficit depends on a positive effect through the inflation rate as well as a direct negative effect. Throughout this chapter we assume that the positive effect dominates. The effect of a change in the real money supply on unemployment is also ambiguous. We assume here that the negative effect through inflation dominates.

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