ESTIMATION OF DEMAND FOR TEA AND COFFEE

IN AUSTRALIA

by

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A dissertation submitted in partial fulfilment of the requirements for the Degree of Master of Agricultural Development Economics at the Australian National University

August 1977
DECLARATION

Except where otherwise indicated, this dissertation is my own work.

A. Adl

August 1977
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Alireza Adl
This study attempts to review the basic principles of the consumer demand theory in order to estimate the demand for tea and coffee in Australia. Generally the study of consumer demand theory is important because it can be used in development and production planning.

Some of the commonly used systems of demand equations for empirical studies are explained. The study also attempts to explain a habit formation hypothesis and incorporate the hypothesis in the systems of demand equations. On this basis, three models of demand equations with and without habit formation are used to estimate the demand for tea and coffee in Australia.

It is shown that none of the models satisfy all the general restrictions imposed on demand equations, since each one has some advantages and some disadvantages. However, through the comparison of these models it is shown that a habit formation model is justified. Tea in the Australian consumption pattern is an inferior good while coffee is a normal commodity. On the basis of the alternative functional forms, it seems that the double logarithmic system is a better model for estimating the demand for a single commodity such as tea or coffee.
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CHAPTER 1
INTRODUCTION

The concern of this study is to investigate consumer demand theory and its application to commodities such as tea and coffee. There are two primary reasons for this investigation. The first point is related to development planning and the second to production planning. Many countries are now engaged in construction of development programs and to do this adequately it is clearly necessary to have some idea about the changes in consumption that are likely to occur with rising income levels.

According to well-known empirical phenomena observable in many countries, a part of growth in the value of agricultural products available for consumption per capita is attributable to shifts in consumption from less preferred food items to preferred food items. For example, Japan in the post war period experienced rapid changes in consumers' expendable resources, the socio-cultural determinants of consumers' "tastes" and the institutional arrangements as to the procurement and distribution of food. Although the assumption of constant "tastes" may have approximated reality during the pre-war periods of relatively slow growth, this is not the case in the post-war periods when changes occurred so rapidly. When the economy sustains the average annual growth rate in real terms of some 10 per cent for a decade or more, furthermore, the changes in such factors may be continual rather than once-and-for-all.
The structural transformation of the Japanese economy in the 1950's was reflected in the beginning of the absolute decline in the agricultural labour force and in the emergence of a highly sophisticated industrial complex in Japan. This transformation coincides with the radical changes in food consumption patterns of Japanese people. As the real incomes grew rapidly, the institutional and technological framework of Japanese life in general also changed rapidly. Under these circumstances, a drastic transformation took place in methods of food preparation and the patterns of food consumption.

To my mind the Iranian economy has been undergoing a similar path to the Japanese economy. The rapid rate of growth of the last decade following the land reform and transformation of the agricultural labour force to industry is bringing about a tremendous change in consumption patterns of Iranian people. This has been especially true during the last few years as a general increase in the standard of living was achieved by an increase in oil revenues.

I believe this study will enable me to learn the basic concepts of consumer demand theory and its application to the Iranian economy at a proper time and with available data.

Secondly, from a production point of view, there is also a sound justification for this case study. In later chapters changes in consumption patterns due to changes in tastes and habit formation will be investigated. The theory will then be applied to the demand for tea and coffee in Australia. These two commodities, which are mostly produced in developing countries, are traditional commodities in the sense that people in developed countries such as Australia are
used to drinking them. From the producers' point of view it is important to know the short and long run price and income elasticities of a commodity especially when a factor such as habit formation determines the consumption.

In the history of demand analysis two threads, related but separate, can be discussed. These are, first, the work of economists interested in the discovery of general laws governing the operation of individual commodity markets, particularly agricultural markets; and, second, the work of those interested in the laws or behavioural regularities governing what has come to be called consumer preference.

Throughout the eighteenth and nineteenth centuries the empirical approach had made little or no progress in the measurement of demand curves despite its early and promising beginning. In a large part, this was due to the fact that the techniques of regression analysis were not developed by statisticians until late in the nineteenth century. Significant progress was, however, made in the investigation of the influence of income on consumption patterns. In particular, an outstanding contribution was made by Engel who, in 1857, formulated what turned out to be enduring empirical laws governing the relation between income and particular categories of expenditure.

In the late nineteenth century the fusion of the theoretical and empirical approaches in the writings of Marshall was perhaps the catalyst which encouraged agricultural economists to apply the newly discovered technique of correlation to the analysis of single markets. In the present context Marshall's great contribution was the clarification and elaboration of the concept of the elasticity of demand which offered a precise framework within which numerical measurements of market characteristics could be effected.
It was no accident that agricultural commodities were the first to be studied and indeed have provided econometricians with some of their most convincing successes. For partial equilibrium analysis based on fitting single equations requires, ideally, a homogeneous commodity with a simple quantity dimension, stable consumers' preferences, and relatively large fluctuations or trends in supply which are independent of the current market price; and these conditions are most nearly met by many agricultural staples.

By 1939 most of the strengths and weaknesses of what is called classical demand analysis had been probed and most of the techniques still in use had been discovered. It is possible to characterize this classical approach as consisting of the application of variations in least squares single-equation fitting, to both time series and cross-section data, of market models based as far as possible on the theoretical results of Slutsky, Allen and Hicks. Much of this work, together with a great deal of empirical analysis, was drawn together by Shultz in 1938. However, because of the Second World War it was not until the 1950's that fully systematic treatments of this approach were published. The books by Wold and Stone can be regarded as a consolidation of the theoretical and empirical work on static demand models in the first half of this century.

Although there have been a number of important theoretical advances, on the empirical side, certain topics stand out clearly as areas for further investigation. In particular, three might be mentioned: the extension of the analysis to a wide range of commodities, the treatment of the special problems associated with durable goods and application
of the more sophisticated computational and econometric techniques which have since become available.

While the question as to which the classical approach addressed itself was of the type "what is the income or price elasticity of good X?", more recent investigations have posed and begun to answer some more fundamental questions; for example, how should demand functions be specified? What is the best way of allowing for changes in prices? What other important factors should be considered in addition to income and price? In particular, attention has focussed on the theory of demand and its relevance to applied demand analysis. In this context demand theory is regarded not as part of general equilibrium analysis or of welfare theory, but as a tool of empirical investigation.

The problem with which demand analysts are fundamentally concerned is to find out the demand function(s) and know how the demand for a commodity will alter as certain specified exogenous variables change. This information is usually required for a specified moment in time and for some aggregation of individuals, either for all consumers or for some sub-group. If one decides to work in per capita terms in order to remove changes of scale in the population, the problem is to discover how the allocation of the average budget over different commodities will respond to outside changes. In particular one is interested in the effects of changes in real income per head, the structure of relative prices and the distribution of income, and one should like at the same time to have a means of allowing for the introduction of new commodities and changes in tastes and habit formation. All this is of
considerable importance; the increase in the number of large econometric models and the general increase in interest in models for planning and policy formulation offers a wide area for the positive application of any results which are achieved. Consumers' expenditure is the largest item in the gross domestic product of most economies and thus the usefulness of disaggregated planning or prediction is likely to depend on its correct allocation. The changing structure of industry over time depends crucially on the evolution of the elements of consumers' expenditure in response to increasing income while knowledge of price responses is an important element in the formulation of fiscal policy or any other type of economic control.

For some practical purposes it may be sufficient to estimate separately a set of single equation models, one for each category of consumers' expenditure. For example, each equation might express the quantity purchased of each good per head of population as a function of average per capita income, the price of the good relative to some overall price index, time as a catch-all for changes in the distribution of income, the introduction of new products and steady changes in tastes. A functional form must be chosen for the demand equations; it is very convenient to think in terms of elasticities which are dimensionless, and so one might choose a double logarithmic function which gives the elasticities directly as coefficients of the estimating equation. If good i is supposed to be a close substitute or complement for some other good then one may include the price of that good in the demand function; one then estimates another parameter, which is interpreted as a cross price elasticity. This is essentially the method of analysis used by Stone and others in the early fifties and it is
referred to as a "pragmatic" approach. It is pragmatic in the sense that it includes those variables in which one is directly interested, ignoring or summarising others.

However, there are a number of difficulties with such a treatment. For example, one assumption in the double logarithmic model is that the elasticities are the same at all values of the exogenous variables. Although this is convenient, one should not expect it to be true over any but the shortest range, and when working with time series, for econometric purposes, one should like his time span to be as long as possible. Typically, nations become richer over time and one might expect goods which are usually luxuries when the inhabitants of the country are poor to become more and more necessities as real incomes increase. But there is an even more basic problem: if all income elasticities were really constant, those goods with elasticities greater than unity would, as real income increased, come to dominate the budget and eventually would lead to the sum of expenditures on each of the categories being greater than the total expenditure allocated, an obvious absurdity, i.e. it violates the budget restraint of consumer demand theory. Even if the model fits the data well when estimated, one knows that if it is used to project forward it will eventually lead to silly results. Obviously one needs a model with changing elasticities and one needs some theory of how one might expect them to change.

But even if one chooses an alternative form of the demand equations which surmounts these difficulties, one is still faced with problems of another kind. For example, the aggregation problem should be under consideration. The characteristics of individual behaviour
are unlikely to be reproduced in the aggregate. However, it could be argued that one "representative" consumer might represent the whole population. One might then write the demand function so that the aggregation difficulties are met and such that the elasticities for each good change in a sensible way. Even now, there are strong restrictions on the type of behaviour allowed. For example, if income and all prices were to change by the same proportion, real income and relative prices would not change and the quantities bought would remain the same. Although this absence of money illusion is an attractive property for demand functions to possess, it may not be true. Consumers may suffer from a money illusion and it could be argued that it is part of the task of demand analysis to discover whether or not it exists rather than to use a model which precludes it (as a starting point).

The choice of the demand model itself has important implications; strong a priori notions are built into the analysis by the choice of model and these will interact with the data to yield results which will be affected by the model chosen. At the same time such strong preconceptions are inevitable; some functional form must serve as a basis for estimation, and even then when it has been chosen it will, in most circumstances, be possible to estimate only a few parameters for each commodity. This constraint, which is due to the lack of independent variations between prices and income in most time series, rules out the possibility of overcoming some of the specification problems by estimating an equation involving all the prices simultaneously. Faced with all these conditions, and with the necessity of justifying the demand function chosen, it is perhaps natural that investigators have
turned to the theory of demand as a tool for deriving the necessary constraints and for organising their *a priori* assumptions. A model based on preference theory usually offers a practicable alternative to the pragmatic approach and it is this alternative which has been most extensively explored in recent years.

The structure of this study will then be as follows. In the second chapter I will review the consumer demand theory and introduce some of the commonly used systems of demand equations for empirical research. In Chapter 3 I will discuss the hypothesis of habit formation and incorporate this hypothesis into some of the systems of demand equations as an example. In Chapter 4 the demand for tea and coffee in Australia will be estimated. The models to be used for this estimation are based on some of the systems of demand equations of the earlier chapters. This means estimates will be made for the demand for tea and coffee using functions with, and without, habit formation. Finally, Chapter 5 brings together the conclusions of the study.
CHAPTER 2

A BRIEF SURVEY OF DEMAND THEORY

2.1 Introduction

The theory of demand has been surveyed by many economists and there are several good references such as Brown and Deaton (1972), Green (1971) and Phillips (1974). However, in this chapter I will review the stochastic demand functions for a single commodity, say tea or coffee, which are my interest. To begin with, properties of utility functions are discussed.

Although the economists of the nineteenth century explained consumer behaviour on the assumption of cardinal utility functions, it was only a few decades ago that the consumer was assumed to be capable of only ranking commodity combinations consistently in order of preference: Slutsky (1919); Hicks and Allen (1933). This means that the consumer's utility function is not unique. If a particular function describes (approximately) the consumer's preferences, so does any other function which is a monotonic transformation of the chosen function.

It is assumed that the utility function which represents these preferences is strictly quasi-concave, that is, sets such as \( \{X; X \mathcal{R} Y\} \) are strictly convex where \( X \) and \( Y \) are commodity bundles, and \( \mathcal{R} \) is a weak preference relation. This ensures that the equilibrium is a maximum and is unique.

The basic principle of the theory of consumer behaviour is that the consumer maximizes utility subject to his budget constraints;
he has a limited income. The necessary conditions for this maximization will be explained.

The consumer's ordinary demand functions for commodities will be derived from his first order conditions for utility maximization. In general, the quantities demanded of a commodity are a function of all consumer commodity prices and consumer's income. Ordinary demand functions are single valued and homogeneous of degree zero in prices and income. That is, we do initially assume that there is no money illusion. The consumer's compensated demand functions for commodities are constructed by changing his income following a price change in order to leave him at his initial utility level. The compensated demand functions state quantities demanded as a function of all prices and the chosen level of utility. They are also single valued and homogeneous of degree zero in prices.

The consumer's reaction to price and income changes in terms of substitution and income effects will be analyzed. Substitutes and complements will be defined in terms of the sign of the substitution effect for one commodity when the price of the other changes.

The chapter will finish by explaining other characteristics of utility functions such as additivity and separability and, finally, some of the systems of demand equations which are frequently used in empirical studies will be mentioned.

2.2 Consumer

Microeconomics is the branch of economics that deals with the behaviour of individual decision-making units. One of the most important of these units in any economy is the consumer.
The postulate of rationality is the customary point of departure in the theory of the consumer's behaviour. The consumer is assumed to choose among the alternatives available to him that which is preferred to all others within the budget. This implies that he is aware of the alternatives facing him and is capable of evaluating them. All information pertaining to the satisfaction that the consumer derives from various quantities of commodities is contained in the utility function.

2.3 The Nature of the Utility Function

Consider the case that an individual consumer purchases a set of commodities. His ordinal utility function is

\[ U = f(x_1, x_2, ..., x_n) \]  

(2.1)

where \((x_1, x_2, ..., x_n)\) is the vector of quantities consumed of the commodities \(1, ..., n\). It is assumed that (2.1) is a continuous function and has continuous first and second order partial derivatives.

By postulating a utility function, one is actually creating a tool useful for correct description of observed consumer behaviour in the market and a reasonably good forecast of future behaviour. As Phlips (1974) argues, "In the limit, one may say that the utility function exists because we postulate it. Its maximization is the logical consequence of our axioms. It is the economist who maximizes utility to find the 'optimal' quantities corresponding to the quantities that the consumer effectively purchases in the market. The optimization technique is thus simply a procedure that is utilized because it works,
i.e. because it leads to the operational hypotheses which turn out to be valid. Its justification lies in the conclusions that can be derived from it".

The utility function is defined with reference to consumption during a specific period of time. The level of satisfaction that the consumer derives from a particular commodity bundle (or set) depends upon the length of the period during which he consumes it. There is no unique time period for which the utility function should be defined. However, there are restrictions upon the possible length of the periods. An intermediate period is most satisfactory for the static theory of consumer behaviour. This is because if the period is too short the consumer cannot derive utility from variety in his consumption and diversification among the commodities he consumes. On the other hand, tastes (the shape of the function) may change if it is defined for too long a period. Hence, a period such as, say, from one quarter to a year is usually satisfactory.

2.4 Existence of the Utility Function

It is not obvious that real-valued functions that can serve as utility functions exist for all consumers. Consumer preferences must satisfy certain conditions in order to be represented by a utility function. One set of conditions for the existence of a utility function is as follows:

If we consider \( A_1 \) and \( A_2 \) as any pair of commodity combinations, then:
Axiom 1 (completeness):

For all $A_1$ and $A_2$ in $S$ (set of all commodity combinations), either $A_1 R A_2$ or $A_2 R A_1$ or both ($R$ is regarded as "at least as good as").

Axiom 2 (transitivity):

For all $A_1$, $A_2$, $A_3$ in $S$ if $A_1 R A_2$ and $A_2 R A_3$ ($A_1 R A_2 R A_3$), then $A_1 R A_3$.

Axiom 3 (rational choice):

If $A_1$ is chosen from a set of alternatives $S$, then for all $A_2$ in $S$, $A_1 R A_2$.

2.5 The Rate of Commodity Substitution

Consider the simple case where

$$U = f(X) \quad X = (x_1, x_2, \ldots, x_n)$$

(2.2)

The total differential of the utility function is

$$dU = \sum_{i} f_i dx_i$$

(2.3)

varying only quantities of two commodities $i, j$

$$dU = f_i dx_i + f_j dx_j$$

(2.4)

where $f_i$ and $f_j$ are partial derivatives of $U$ with respect to $x_i$ and $x_j$. The total change in utility (compared with an initial situation) caused
by variation in $x_i$ and $x_j$ is approximately the change in $x_i$ multiplied by the change in utility resulting from a unit change of $x_i$ plus the change in $x_j$ multiplied by the change in utility resulting from a unit change in $x_j$. Let the consumer move along his indifference curves by giving up some $x_i$ in exchange for $x_j$. If his consumption of $x_i$ decreases by $dx_i$ (therefore $dx_i < 0$), the resulting loss of utility is approximately $f_i dx_i$. The gain of utility caused by acquiring some $x_j$ is approximately $f_j dx_j$ for similar reasons. Taking arbitrarily small increments, the sum of these two terms must equal zero, since the total change in utility along an indifference curve is zero by definition. Setting $dU = 0$,

$$\frac{dx_i}{dx_j} = -\frac{f_j}{f_i}$$

(2.5)

The slope of an indifference curve, $dx_i/dx_j$ is the rate at which a consumer would be willing to substitute $x_i$ for $x_j$ or $x_j$ for $x_i$, in order to maintain a given level of utility. The negative of the slope, $-dx_i/dx_j$ is the marginal rate of substitution of $x_i$ and $x_j$ or $x_j$ for $x_i$, and it equals the ratio of the partial derivatives of the utility function.

2.6 The Maximization of Utility

The consumer's objective is to maximize his utility from consumption. In pursuing this objective, however, he is constrained by his available resources and the market prices of commodities. For simplicity, I assume that an individual's sole resource, during the period under consideration, is his given disposable income and he
desires to purchase a combination of $x_1, x_2, \ldots, x_n$ from which he derives the highest level of satisfaction. The consumer's budget constraint can be written as

$$\sum_{i=1}^{n} P_i X_i = M \quad (2.6)$$

where $M$ is his given income and $P_1, \ldots, P_n$ are prices of $X_1, \ldots, X_n$ respectively. In order to maximize the utility function subject to a budget constraint, the consumer must find that combination of commodities that satisfies (2.6) and also maximizes the utility function (2.1). To maximize the utility function the Lagrange Multiplier Method is used.

$$L = U(X_1, X_2, \ldots, X_n) + \lambda (M - \sum_{i=1}^{n} P_i X_i) \quad (2.7)$$

where $\lambda$ is the Lagrange multiplier. In economic terms this is the marginal utility of money, $\lambda = \frac{\partial U}{\partial M}$.

It is necessary for a maximum of $U$ subject to the budget constraint that

$$\frac{\partial L}{\partial X_1} = U_1 - \lambda P_1 = 0$$
$$\frac{\partial L}{\partial X_2} = U_2 - \lambda P_2 = 0$$
$$\vdots$$
$$\vdots$$
$$\vdots$$
$$\frac{\partial L}{\partial X_n} = U_n - \lambda P_n = 0$$

$$\sum_{i=1}^{n} P_i X_i = M \quad (2.8)$$
It follows from first \( n \) equations that for all \( i \) and \( j \)

\[
\frac{U_i}{U_j} = \frac{\lambda P_i}{\lambda P_j} = \frac{P_i}{P_j} \quad (2.9)
\]

That is, marginal utilities are proportional to prices. The second order condition for a constrained maximum is that the principal minors of the relevant bordered Hessian,

\[
U = \begin{vmatrix}
0 & U_1 & U_n \\
U_1 & U_{11} & U_{1n} \\
U_n & U_{1n} & U_{nn}
\end{vmatrix}
\]

of orders \( 3, 4, 5, \ldots \) are alternatively positive and negative

\[
\begin{vmatrix}
0 & U_1 & U_2 \\
U_1 & U_{11} & U_{12} \\
U_2 & U_{21} & U_{22}
\end{vmatrix} > 0,
\begin{vmatrix}
0 & U_1 & U_2 & U_3 \\
U_1 & U_{11} & U_{12} & U_{13} \\
U_2 & U_{21} & U_{22} & U_{23} \\
U_3 & U_{31} & U_{32} & U_{33}
\end{vmatrix} < 0, \text{ etc.}
\]

(2.10)

These are satisfied if \( U \) is strictly quasi-concave.

**2.7 Ordinary Demand Functions**

A consumer's ordinary demand function gives the quantity of a commodity that he will buy as a function of commodity prices and his income. The first order condition for maximization (2.8) consists of \((n + 1)\) equations and \((n + 1)\) unknowns. The demand functions are obtained by solving this system for the unknowns. Let such a solution be as follows
\[ X_i = X_i(p_1, \ldots, p_n, M) \quad (i = 1, \ldots, n) \]

\[ \lambda = \lambda(p_1, \ldots, p_n, M) \]

The demand functions are affected by a particular choice of utility function. Any system of demand functions must have the following properties:

\[ \sum_{i=1}^{n} P_i X_i = M \quad \text{(P.I)} \]

\[ X_i = X_i(kp_1, \ldots, kp_n, kM) \quad \text{(P.II)} \]

The first property of the demand system is one of the equilibrium conditions and the second property follows from the fact that if one multiplies \( p_1, \ldots, p_n \) and \( M \) by \( k \) in (2.8) the equilibrium conditions are not altered (homogeneity of degree zero in income and prices).

The homogeneity property has the advantage that it enables one to reduce the number of variables by dividing all prices and income by the price of one of the commodities. For example, in Chapter 4 when using the double logarithmic system all prices and income in the demand equations will be deflated by the consumer price index, which represents the average price of all goods other than tea and coffee. This reduces the explanatory variables to those of the relative price of tea, coffee and sugar and also real income. Alternatively, as in the linear expenditure system and homogeneous indirect translog functions, which will be explained in later sections, one may deflate the explanatory variables in each demand equation by their respective own commodity prices.
2.8 Compensated Demand Functions

Assume one provides a lump-sum adjustment to the consumer's income to achieve his initial utility level after a price change. The consumer's compensated demand functions give the quantities of the commodities that he will buy as functions of commodity prices under these conditions. They are obtained by minimizing the consumer's expenditures subject to the constraint that his utility is at the fixed level $u^0$.

Slutsky (1915) was the first to show that the reaction of the quantity demanded of a good to a change in its price (or to a change in the price of any other good) can be decomposed into an income effect and a substitution effect. The first effect designates the variation in the quantity demanded due to the fact that a price change implies a change in the real income of the consumer.

The substitution effect is that part of the variation in quantity demanded that is due to the fact that if the price of one good changes, its relative price also changes, with the result that less will be consumed of the good whose relative price increases (and more of the goods which are substitutes for it), if one ignores the income effect.

Both effects are the result of one and the same price change. Their sum is equal to the observed variation of quantity demanded. The decomposition can be obtained by the following formula which is known as the Slutsky equation:

$$\frac{\delta x_i}{\delta P_i} = (\frac{dx_i}{dp_i})_{M}, \quad x_i (\frac{\delta x_i}{\delta M}) = K_{ii} - x_i (\frac{\delta x_i}{\delta M}) \quad (2.12)$$
where \( \frac{dX_i}{dP_i} \) is the response of \( X_i \) to a compensated price change (substitution effect) and \( -X_i \frac{\partial X_i}{\partial M} \) is an income effect. In the above case the commodity own price has changed. In the case where it is the price of another good \( (P_j) \) that varies, the Slutsky equation is as follows

\[
\frac{\partial X_i}{\partial P_j} = K_{ij} - X_j \frac{\partial X_i}{\partial M}
\] (2.13)

Generally, in the absence of a particular specification of the utility function, one can say nothing about the sign of the income effect. If \( \frac{\partial X_i}{\partial M} \) is positive, the income effect is negative; if \( \frac{\partial X_i}{\partial M} \) is negative, the income effect is positive. If one assumes that the consumer is given such compensation as to keep his utility level unchanged, two important general restrictions on the substitution effect can then be easily worked out. Here I just explain these restrictions and forgo the proof.  

These general restrictions are that the own (or direct) substitution effect is negative and the matrix of substitution effect \( K \),

\[
K = \begin{bmatrix}
K_{11}, & \ldots, & K_{1n} \\
\vdots & \ddots & \vdots \\
K_{n1}, & \ldots, & K_{nn}
\end{bmatrix}
\]

is symmetric, i.e.

\[
K_{ij} = K_{ji} \quad \text{if} \quad i \neq j \quad \text{(2.14)}
\]

or

\[
\frac{\partial X_i}{\partial P_j} + X_j \frac{\partial X_i}{\partial M} = \frac{\partial X_j}{\partial P_i} + X_i \frac{\partial X_j}{\partial M}
\]

1 For mathematical proof refer to Philips (1974), pp. 42-44.
2.9 Prices and Income Elasticities of Demand

The own price elasticity of demand for a commodity, say commodity $i$, is defined as the proportionate rate of change of $X_i$ divided by the proportionate rate of change of its own price with $P_j$ and $M$ constant.

$$\epsilon_{ii} = \frac{\partial \log X_i}{\partial \log P_i} = \frac{P_i}{X_i} \frac{\partial X_i}{\partial P_i}$$  \hspace{1cm} (2.16)$$

The consumer's expenditure on $X_i$ is $P_i X_i$ and

$$\frac{\partial (P_iX_i)}{\partial P_i} = X_i + P_i \frac{\partial X_i}{\partial P_i} = X_i (1 + P_i \frac{\partial X_i}{\partial P_i}) = X_i (1 + \epsilon_{ii})$$  \hspace{1cm} (2.17)$$

The consumer's expenditure on $X_i$ will increase with $P_i$ if $\epsilon_{ii} > -1$, remain unchanged if $\epsilon_{ii} = -1$ and decrease if $\epsilon_{ii} < -1$.

A cross-price elasticity of demand for the ordinary demand function relates the proportionate change in one quantity to the proportionate change in the price of another commodity $j$. For example,

$$\epsilon_{ji} = \frac{\partial \log X_j}{\partial \log P_i} = \frac{P_i}{X_j} \frac{\partial X_j}{\partial P_i} \hspace{1cm} i \neq j$$  \hspace{1cm} (2.18)$$

Cross-price elasticities may be either positive or negative. In general elasticities are a function of $(P_1, \ldots, P_n, M)$.

An income elasticity of demand for an ordinary demand function is defined as the proportionate change in the purchase of a commodity relative to the proportionate change in income with prices constant.

$$\eta_i = \frac{\partial \log X_i}{\partial \log M} = \frac{M}{X_i} \frac{\partial X_i}{\partial M}$$  \hspace{1cm} (2.19)$$
where $h_i$ denotes the income elasticity of demand for $X_i$. Income elasticities can be positive, negative, or zero but are normally assumed to be positive.

### 2.10 Indirect Utility Functions

The utility functions discussed so far are direct, i.e. they have $X_i$ ($i=1, ..., n$) as arguments. One should know that constrained maximization leads to a system of demand equations of the type:

$$X^0_i = \phi_i(P_1, ..., P_n, M) \quad (2.20)$$

When one replaces $X_i$ by the optimal $X^0_i$ in the direct utility function, one obtains an alternative description of a given preference ordering, called the indirect utility function, which can be written as

$$U^* = f[(\phi_1(P_1, ..., P_n, M), \phi_2(P_1, ..., P_n, M), ..., \phi_n(P_1, ..., P_n, M))]$$

$$= f^*(P_1, ..., P_n, M) \quad (2.21)$$

for ($i=1, ..., n$). The indirect utility function has prices and income as arguments. One should notice that since the demand functions are homogeneous of degree zero (in income and prices) the indirect utility function is also homogeneous of degree zero: as a proportional change in all prices and income does not affect $X^0_i$, it cannot affect $U^*$ either.

Furthermore, there is a duality between $f(X_1, ..., X_n)$ and $f^*(P_1, ..., P_n, M)$. Maximization of $f$ with respect to the $X$'s with given prices and income, leads to the same demand equations as
minimization of \( f^* \) with respect to prices and income, with given quantities. To do this one should apply a formula known as Roy's identity (Roy, 1942). At equilibrium one must have \( dU^* = 0 \) and \( \sum X_i dP_i = dM \), or,

\[
\frac{\partial f^*}{\partial P_1} dP_1 + \frac{\partial f^*}{\partial P_2} dP_2 + \ldots + \frac{\partial f^*}{\partial P_n} dP_n = - \frac{\partial f^*}{\partial M} dM
\]  

(2.22)

and

\[
X_1^0 dP_1 + X_2^* dP_2 + \ldots + X_n^0 dP_n = dM
\]

which implies

\[
\frac{\partial f^*/\partial P_1}{X_1^0} = \ldots = \frac{\partial f^*/\partial P_n}{X_n^0} = - \frac{f^*}{\partial M}
\]

(2.23)

or

\[
X_i^0 = - \frac{\partial f^*/\partial P_i}{\partial f^*/\partial M}
\]

(2.24)

Once \( f^* \) has been specified, it suffices to apply this identity to obtain the demand functions.

The main reason to explain indirect utility functions here is that they may be used in place of a direct utility function to generate demand functions. Sometimes it is possible to use indirect utility functions when one does not know the direct dual function (c.f. Manser (1976)).
2.11 Additivity and Separability of Utility Functions

In this section two more restrictions on demand functions are considered. It is possible to break up the utility function into more or less independent "sub" utility functions each relating to some group of goods, perhaps because such goods serve some particular need. This procedure can be carried on to generate as many restrictions as may be desired; in the limit, if one imposes the assumption that preferences are additive so that the marginal utility of each good is independent of the quantities consumed of all the other goods - and this is only plausible for broad categories of goods - then it is possible to derive the magnitudes of all the substitution responses from the income responses and one price response only. These assumptions about the structure of preferences can also be used to provide a solution to the problem of how to combine goods into groups.

If one writes an unspecified utility function as

\[ U = f(X_1, X_2, \ldots, X_n) \]  \hfill (2.25)

to introduce the strong assumption that the utility provided by the consumption of one good is not influenced by the consumption of any other good, the direct utility function is written as

\[ U = f_1(X_1) + f_2(X_2) + \ldots + f_n(X_n) \]  \hfill (2.26)

This function is additive. In view of the ordinal nature of \( U \), a preference ordering, represented by a utility function such as (2.26), is defined as additive if there exists a differentiable function \( F, F' > 0 \),
and n functions $f_i(X_i)$, such that

$$F[f(X_1, \ldots, X_n)] = \sum f_i(X_i) \quad (i=1, \ldots, n) \quad (2.27)$$

The equation (2.27) implies the independence of the marginal utility of good $i$ from the consumption of any other good.

$$\frac{\partial^2 u}{\partial x_i \partial x_j} = 0$$

The less stringent concept of separability has arisen from the work of Leontief (1947) and Sono (1960). One wants to know under what conditions the arguments of the utility function may be aggregated. What one would like to do is to be able to partition the consumption set into subsets which would include commodities that are closer substitutes or complements to each other than to members of other subsets (for example, in such a separation we can partition tea and coffee from soft drinks and alcoholic beverages). Instead of writing, say,

$$U = f(x_1, x_2, x_3, x_4, x_5)$$

one would like to group the variables in the function to make it expressible as, say,

$$U = F(A, B)$$

where $A = f_a(x_1, x_2)$ and $B = f_b(x_3, x_4, x_5)$. The term "expressible" indicates an important condition: one wants the values of $U$ to be the same with or without grouping, i.e. whether $U$ is expressed as a function of all elementary variables or as a function of the groups.
It is a necessary and sufficient condition that for a function to be separable, the marginal rate of substitution between any two variables belonging to the same group be independent of the value of any variable in any other group (Goldman and Ozawa, 1969).

It is necessary here to explain the relevance of the above arguments to this study. Later sections will explain some different demand systems with different functional forms. Some of these functional forms are based on the assumption of additivity and separability. What is being done here is to take the necessary steps for selecting a utility function and derive the required demand functions to estimate consumption of tea and coffee. It is obvious that, depending on the case and for different commodities, one should adjust the assumptions and select a utility function which satisfies both the requirements of consumer theory and also answers the problems of real life.

2.12 Substitution, Complementarity and Independence

The demand for tea and coffee is to be estimated and one knows that in real life people often drink both of them or substitute them for each other; hence it is necessary to explain the theoretical relationship of commodities which might be substitutes or complements for each other.

The hypothesis of weak separability seems to give a more realistic description of the structure of preferences and to provide the appropriate framework to discuss complementarity and substitutability.

From the traditional point of view, two commodities are complements if the increased consumption of \( j \) increases the marginal
utility of $i$ (and vice versa). That is, two commodities are comple-
ments when

$$\frac{\partial^2 U}{\partial x_i \partial x_j} > 0$$

In a similar way, a negative second order cross-partial derivative
defines substitutability.

From an ordinal point of view, the demand equations have the
advantage of being invariant under any monotonic increasing transformation.
As a consequence, the derivatives - and in particular the substitution
effects - of the demand equations are invariant. The sign of the cross-
substitution effect is not only invariant but also undetermined as it
may be positive, negative or zero. This immediately suggests that one
might use the sign of the cross-substitution effects $K_{ij}$ and say that $i$
and $j$ are substitutes whenever $K_{ij}$ is positive. Indeed, a compensated
increase in the price of $j$ leads to an increase in the demand for $i$.
Hicks (1963) has suggested the following definitions:

$K_{ij} > 0$ indicates substitutability

$K_{ij} < 0$ indicates complementarity

$K_{ij} = 0$ indicates independence

These definitions, however, have certain drawbacks. First of
all, they are biased in favour of substitutability. The use of the
cross-substitution effect implies that all goods can be substitutes but
not complements.
Furthermore, in the particular case of a directly additive utility function, the substitution effect reduces to the "general" substitution effect.

\[
K_{ij} = -\frac{\lambda}{\partial \lambda / \partial m} \frac{\partial x_i}{\partial m} \frac{\partial x_i}{\partial m}
\]

On the assumption that marginal utilities are decreasing, all income derivatives are positive, while \( \lambda \) is positive and \( \partial \lambda / \partial M \) is negative, so that \( K_{ij} \) is positive. In the case of independent marginal utilities, all goods are substitutes according to the Hicksian definitions.

The other way to show complementarity and substitutability is as follows:

\[
\frac{\partial x_i}{\partial p_j} > 0 \quad \text{indicates gross substitutability}
\]
\[
\frac{\partial x_i}{\partial p_j} < 0 \quad \text{indicates gross complementarity}
\]
\[
\frac{\partial x_i}{\partial p_j} = 0 \quad \text{indicates gross independence}
\]

In this analysis I hypothesize that coffee and tea are substitutes for each other and other beverages cannot be a good substitute for tea and coffee. I will also test the relationship of sugar with these two commodities to see whether it is a complementary good.

2.13 Proposed System of Demand Functions

In this section some of the commonly used systems of demand equations will be explained. There are, of course, many demand functions used in empirical work, but only those few which are more relevant to estimation of a single commodity are discussed here.
A - The System of Double Logarithmic Functions

A demand function often used in empirical research is of the following form:

\[ X_i = \alpha_i M \prod_{j=1}^{n} \frac{\beta_{ij}}{P_j} \quad (i, j = 1, \ldots, n) \]  

(2.28)

In a logarithmic form (2.28) becomes

\[ \log X_i = \alpha_i + \alpha_i \log M + \sum_{j=1}^{n} \beta_{ij} \log P_j \]  

(2.29)

Where \( X_i \) is the quantity of good \( i \), \( M \) is disposable income, \( P_j \) is the price of all other goods which enter the equation as substitutes or complements.

This system does not satisfy all the conditions imposed on demand equations. More fundamentally, it does not satisfy the Engel aggregation condition, i.e. it is illegitimate except as an approximation to some other legitimate functions.

B - The Indirect Addilog System

One example of a system of demand equations is the following system of demand equations suggested by Houthakker (1960):

\[ X_i = \frac{a_i (M/P_i)^{b_i} i+1}{\sum_{j=1}^{n} a_j (M/P_j)^{b_j}} \quad (i, j = 1, \ldots, n) \]  

(2.30)

* The indirect utility function corresponding to the addilog system is

\[ U = \sum_{i=1}^{n} \frac{(a_i/b_i) (M/P_i)^{b_i}}{n} \]
It can be shown that the system satisfies the property of \( \sum P_i X_i = M \) and the property of homogeneity. To check the Slutsky equation one should differentiate (2.30) with respect to the price of \( P_j \), and obtain the equation,

\[ \frac{\partial x_i}{\partial p_j} = x_i x_j b_j / M \]  

(2.31)

If one differentiates (2.30) with respect to \( M \) and then multiplies it by \( x_j \), one obtains the equation,

\[ x_j \frac{\partial x_i}{\partial M} = \frac{1}{M} \left[ x_j x_i + b_i x_j x_j - x_i x_j \right] \frac{n}{\sum a_i (M/P_i) b_i} \]

(2.32)

By doing the same operation for \( x_j \), one obtains the equation,

\[ \frac{\partial x_j}{\partial p_i} = x_i x_j \frac{b_i}{M} \]

(2.33)

From the above equation it is easy to see that,

\[ \frac{\partial x_i}{\partial p_i} + x_j \frac{\partial x_i}{\partial M} = \frac{\partial x_j}{\partial p_i} + x_i \frac{\partial x_j}{\partial M} \]

The following equation gives the formula for income elasticity

\[ \sigma_i = \frac{M}{X_i} \frac{\partial X_i}{\partial M} = 1 + b_i - \sum b_i a_i \]

(2.34)
Where

\[ \alpha_i = \frac{a_i(M/P_i)^{b_i}}{\sum a_i(M/P_i)^{b_i}} \]  \hspace{1cm} (2.35)
\[ M = \sum_{i=1}^{n} P_i X_i \]

yields the set of demand relations

\[ X_i = c_i + \frac{b_i}{P_i} (M - \sum_{j=1}^{n} P_j c_j) \quad (i, j = 1, \ldots, n) \quad (2.37) \]

Which when written as

\[ P_i X_i = P_i c_i + \frac{b_i}{P_i} (M - \sum_{j=1}^{n} P_j c_j) \quad (i, j = 1, \ldots, n) \quad (2.38) \]

is called the linear expenditure system. This system in the literature is also known as the Stone-Geary Model. This equation says that the expenditure on the \( i \)'th commodity is equal to a certain amount of consumption \( c_i \) (precommitted consumption) valued at current prices plus a certain proportion \( b_i \) of total expenditure less total committed expenditure. The consumer first uses up a certain amount of total expenditure in acquiring the consumption vector \( c = (c_1, \ldots, c_n) \) at current prices, and then distributes the remainder over the set of available commodities in certain fixed proportions given by the elements of \( b = (b_1, \ldots, b_n) \).

The term "precommitted consumption" is used instead of "subsistence consumption". Subsistence consumption may vary across the sample observations, depending on the values of other explanatory variables such as price and income. Precommitted consumption depends on a number of variables which are independent of the economic variables.

To satisfy the budget constraint, \( \sum b_i \) must be equal to one. This constraint is imposed in estimating the vector \( b = (b_1, \ldots, b_n) \).
It is easy to see that the homogeneity property is satisfied if one writes (2.38) as

\[ x_i = c_i + b_i \left( \frac{M}{P_i} - \sum_{j} \frac{P_j}{P_i} c_j \right) \quad (i, j = 1, \ldots, n) \]  

(2.39)

The Slutsky equation is also met by differentiating \( x_i \) with respect to \( P_j \) and \( x_j \) with respect to \( P_i \).

\[ \frac{\partial x_i}{\partial P_j} = -b_i c_j / P_i, \quad \frac{\partial x_j}{\partial P_i} = -b_j c_i / P_j \n
\]

\[ \frac{\partial x_i}{\partial M} x_j = \frac{b_j}{P_j} \left( c_j + b_j \frac{M}{P_j} - b_j \sum \frac{P_j}{P_i} c_i \right) \quad (i, j = 1, \ldots, n) \]

(2.40)

\[ \frac{\partial x_j}{\partial M} x_i = \frac{b_i}{P_i} \left( c_i + b_i \frac{M}{P_i} - b_i \sum \frac{P_i}{P_i} c_i \right) \]

(2.41)

From the above equations it is easy to see that

\[ \frac{\partial x_i}{\partial P_j} + x_j \frac{\partial x_i}{\partial M} = \frac{\partial x_j}{\partial P_i} + x_i \frac{\partial x_j}{\partial M} \]

Finally, one notes that

\[ \sigma_i = \frac{\partial x_i}{\partial M} M / x_i = b_i M / P_i x_i \]

(2.41)

There is no reason why \( \sigma_i \) has to be one for all \( i \). Thus, the linear expenditure system, like the indirect addilog system, satisfies all properties listed earlier.

It has been shown that all goods in the Stone-Geary linear expenditure system must be substituted in the Hicks-Allen sense (this
follows directly because it is essentially a Cobb-Douglas function with a change of origin); there can be no inferior goods and the own-price elasticities cannot exceed unity so that demand for each good must be inelastic with respect to own-price.

However, the linear expenditure model has been extensively applied by many economists for different countries. Stone (1954) and his colleagues continued to use the system with British data, Paelinck (1964) for Belgium, Leoni (1967) for Italy, Parks (1969) for Sweden, Pollack and Wales (1969) for the United States, Yoshihara (1969) for Japan, Goldberger and Gamaletos (1970) for thirteen O.E.C.D. nations, Van Brockhoven (1971) for Belgium and some others.

D - Transcendental Logarithmic Utility Functions

Christensen, Jorgenson and Lau (1975) in their paper have developed tests of demand theory that do not employ additivity or homotheticity as part of the maintained hypothesis. For this reason they have represented the utility functions that are quadratic in the logarithms of the quantities consumed. These utility functions allow expenditure shares to vary with the level of total expenditure and permit a greater variety of substitution patterns among commodities than functions based on constant and equal elasticities of substitution among all pairs of commodities.

They have also exploited the duality between prices and quantities in the theory of demand. A complete model of consumer demand implies the existence of an indirect utility function, defined
on total expenditure and the prices of all commodities. The indirect utility function is homogeneous of degree zero and can be expressed as a function of the ratios of prices of all commodities to total expenditure.

In this study the direct utility function is represented as the direct transcendental logarithmic utility function, or, more simply, the direct translog utility function. The utility function is a transcendental function of the logarithms of quantities consumed. Similarly, the indirect utility function representation is referred to as the indirect transcendental logarithmic utility function, or, more simply, the indirect translog utility function.

I Direct Translog Utility Function

The direct utility function $U$ can be represented in the form:

$$\ln U = \ln U (X_1, X_2, \ldots, X_n)$$

(2.42)

where $X_i$ is the quantity consumed of the $i$'th commodity. The consumer maximizes utility subject to the budget constraint

$$\sum P_i X_i = M$$

(2.43)

where $P_i$ is the price of the $i$'th commodity and $M$ is the value of total expenditure.

From first order conditions one obtains

$$\frac{\partial \ln U}{\partial \ln X_i} = \left(\frac{P_i X_i}{M}\right) \sum \frac{\partial \ln U}{\partial \ln X_j}$$

(i = 1, 2, ..., n)

(2.44)
To preserve symmetry with the treatment of the indirect utility function given below, one can approximate the negative of the logarithm of the direct utility function by a function quadratic in the logarithms of the quantities consumed:

\[-\ln U = \alpha_0 + \sum_i \alpha_i \ln x_i + \frac{1}{2} \sum_{i,j} \beta_{ij} \ln x_i \ln x_j \quad (i, j = 1, 2, \ldots, n) \]

(2.45)

using this form for the utility function one obtains

\[\alpha_i + \sum_j \beta_{ij} \ln x_j = (P_i x_i / M) \sum_j (\alpha_i + \sum_k \beta_{ki} \ln x_j) \quad (i, j = 1, 2, \ldots, n) \]

(2.46)

To simplify notation one can write

\[\alpha = \sum_k \beta_k \]

(2.47)

\[\beta = \sum_k \beta_{ki} \]

(2.48)

so that

\[\frac{P_i x_i}{M} = \frac{\alpha_i + \sum_j \beta_{ij} \ln x_j}{\alpha + \sum_j \beta \ln x_j} \quad (i = 1, 2, \ldots, n) \]

(2.49)

and

\[x_i = \frac{M}{P_i} \frac{\alpha_i + \sum_j \beta_{ij} \ln x_j}{\alpha + \sum_j \beta \ln x_j} \quad (i = 1, 2, \ldots, n) \]

(2.50)

The budget constraint implies that \(\sum P_i x_i / M = 1\). Also \(\alpha = -1\) since the budget share equations are homogeneous of degree zero in parameters.
II Indirect Translog Utility Function

The indirect utility function $V$ can be represented in the form:

$$\ln V = \ln V (P_1/M, P_2/M, \ldots, P_n/M) \quad (2.51)$$

One can determine the budget share for the $i$'th commodity from the Roy's identity

$$P_i X_i/M = \frac{\partial \ln V}{\partial \ln P_i} / \frac{\partial \ln V}{\partial \ln M} \quad (i = 1, \ldots, n) \quad (2.52)$$

Preserving the same functional form with the direct utility function, one can take the logarithm of the indirect utility function by a function quadratic in the logarithms of the ratios of prices to the value of total expenditure

$$\ln V = \alpha_0 + \sum_i \alpha_i \ln (P_i/M) + \sum_{i,j} \beta_{ij} \ln (P_i/M) \ln (P_j/M) \quad (2.53)$$

Using this form for the utility function one obtains

$$\frac{\partial \ln V}{\partial \ln P_i} = \alpha_i + \sum_j \beta_{ij} \ln (P_j/M) \quad (2.54)$$

$$\frac{\partial \ln V}{\partial \ln M} = \sum_j (\alpha_j + \sum_i \beta_{ij} \ln (P_j/M)) \quad (i,j=1, \ldots,n) \quad (2.55)$$

As before

$$\alpha = \sum_j \alpha_j \quad (2.56)$$

$$\beta = \sum_i \beta_{ij} \quad (j = 1, \ldots, n) \quad (2.57)$$
Therefore, using Roy's Identity,

\[ \frac{P_i X_i}{M} = \frac{\alpha_i + \sum_j \beta_{ij} \ln(P_j/M)}{\alpha + \sum_j \beta \ln(P_j/M)} \]  \hspace{1cm} (2.58)

\[ X_i = \frac{M}{P_i} \frac{\alpha_i + \sum_j \beta_{ij} \ln(P_j/M)}{\alpha + \sum_j \beta \ln(P_j/M)} \]  \hspace{1cm} (2.59)

Where \( \alpha = -1 \) since the budget share equations are homogeneous of degree zero in parameters.

The above function is additive if \( \beta_{ij} = 0 \) (\( i \neq j; i, j = 1, \ldots, n \)). It will be homogeneous if \( \sum_j \beta_{ij} = 0 \) (\( i = 1, \ldots, n \)). If both of these restrictions are imposed simultaneously it will reduce to the linear logarithmic utility function.

Manser (1976) has added new parameters into the indirect utility function (2.53) which can be interpreted as "committed quantities" or "subsistence level of consumption". Suppose the indirect utility function is

\[ -\ln V = \sum_i \ln(P_i/M^S) + \frac{1}{2} \sum_{ij} \beta_{ij} \ln(P_i/M^S) \ln(P_j/M^S) \]  \hspace{1cm} (2.60)

\[ (i, j = 1, \ldots, n) \]

where \( M^S = M - \sum_j P_j Y_j \) is "supernumerary" expenditure. This function is homogeneous of degree zero in money prices and total expenditure as required for an indirect utility function. If the restrictions \( \sum_j \beta_{ij} = 0 \) are imposed then (2.60) implies linear Engel curves.
Substituting $M_i$ into equation (2.60) one has

$$- \ln V = \sum_i \alpha_i \ln \left( \frac{P_i}{M-\Sigma P_j Y_j} \right) + \sum_{ij} \beta_{ij} \ln \left( \frac{P_j}{M-\Sigma P_j Y_j} \right) \ln \left( \frac{P_j}{M-\Sigma P_j Y_j} \right)$$

(2.61)

or

$$- \ln V = \sum_i \alpha_i \ln P_i - \sum_i \alpha_i \ln (M-\Sigma P_j Y_j) + \sum_{ij} \beta_{ij} \ln \left( \frac{P_j}{M-\Sigma P_j Y_j} \right) \ln \left( \frac{P_j}{M-\Sigma P_j Y_j} \right)$$

(2.62)

Differentiating (2.62) logarithmically one gets

$$\frac{\partial \ln V}{\partial \ln P_i} = \alpha_i + \sum_j \beta_{ij} \ln \left( \frac{P_j}{M-\Sigma P_j Y_j} \right)$$

(2.63)

$$\frac{\partial \ln V}{\partial \ln M} = \sum_{k,j} \beta_{kj} \ln \left( \frac{P_j}{M-\Sigma P_j Y_j} \right)$$

(2.64)

since

$$\frac{\partial \ln (M-\Sigma P_j Y_j)}{\partial \ln M} = \frac{\partial \ln (M-\Sigma P_j Y_j)}{\partial (M-\Sigma P_j Y_j)} \cdot \frac{\partial (M-\Sigma P_j Y_j)}{\partial M} \cdot \frac{\partial M}{\partial \ln M} = \frac{M}{M-\Sigma P_j Y_j}$$

(2.65)

and

$$\frac{\partial \ln (M-\Sigma P_j Y_j)}{\partial \ln P_i} = \frac{\partial \ln (M-\Sigma P_j Y_j)}{\partial (M-\Sigma P_j Y_j)} \cdot \frac{\partial (M-\Sigma P_j Y_j)}{\partial P_i} \cdot \frac{\partial P_i}{\partial \ln P_i} = \frac{-\gamma_{ij}}{M-\Sigma P_j Y_j}$$

(2.66)

Now

$$\frac{P_i X_i / M}{\partial \ln P_i} = \frac{\partial \ln V}{\partial \ln P_i} / \frac{\partial \ln V}{\partial \ln M}$$
If $V$ is homogeneous in variables, viz. $\Sigma \beta_{kj} = 0$, and if one imposes the normalization $\Sigma \alpha_k = -1$, one has

$$\frac{P_iX_i}{M} = \gamma_i \left( \frac{-1}{M} \right) + \frac{M - \Sigma P_j Y_j}{M} [\alpha_i + \Sigma \beta_{ij} \ln P_j] \quad (i = 1, ..., n) \quad (2.67)$$

$$\frac{P_iX_i}{M} = \frac{P_i Y_i}{M} + \frac{\alpha_i}{M} \left( M - \Sigma P_j Y_j \right) + \frac{\Sigma \beta_{ij} \ln P_j (M - \Sigma P_j Y_j)}{M} \quad (2.68)$$

$$X_i = \gamma_i \left( \frac{-1}{P_j} \right) + \frac{\alpha_i}{P_j} \left( M - \Sigma P_j Y_j \right) + \frac{\beta_{ij} \ln P_j (M - \Sigma P_j Y_j)}{P_j} \quad (i = 1, ..., n) \quad (2.69)$$

$$X_i = \gamma_i + \frac{P_j}{P_j} + \frac{\alpha_i}{P_j} \left( M - \Sigma P_j Y_j \right) + \frac{\beta_{ij} \ln P_j (M - \Sigma P_j Y_j)}{P_j} + \frac{\gamma_j}{P_j} \quad (2.70)$$

Where again the normalization $\Sigma \alpha_i = -1$ is imposed. As was mentioned, this function has linear Engel curves and one should recognize that it does not reduce to the unrestricted Indirect Translog function.

If the restrictions $\beta_{ij} = 0$, $i \neq j$; $i, j = 1, ..., n$ are imposed (2.69) reduces to the linear expenditure system. That is, the linear expenditure system is a special case of the homogeneous indirect trans-log system.
CHAPTER 3
HABIT FORMATION MODELS

The theory expanded in Chapter 2 leads to a system of demand equations describing the equilibrium values which \( X_i \) will take in any price and income situation. In this analysis it was assumed that habits and tastes are constant. The theory is static in that it assumes an instantaneous adjustment to the new equilibrium values when prices or income change.

It should be obvious that a static approach does not provide a realistic description of how consumers behave in real life. In fact, consumers very often react with some delay to price and income changes, with the implication that the adjustment towards a new equilibrium situation is spread over several time periods. In each time period the adjustment is then partial. In fact the consumer is always adapting himself to a new equilibrium value since prices and income change during time.

There may be two sources of explanation for the origin of these lags. On the one hand, habit formation seems to be a predominant characteristic of consumer behaviour. After a change in price for a good which he developed buying habits, the consumer may appear to buy quantities which are different from the equilibrium values indicated by his static demand equation.

On the other hand, the durability of some consumption goods is the other main source of lags. It is not reasonable to expect a
consumer to change his car to a new brand immediately he gets higher salary. If he had recently bought his car, he might prefer to wait for a while.

Since I am studying the demand for tea and coffee which are non-durable goods, I will concentrate on the first assumption which is habit formation.

Although it is possible to postulate the theory to many other commodities such as alcoholic beverages or even different sorts of meat such as lamb or beef for which consumers may develop a habit, I have selected tea and coffee since they are traditional beverages. They are very good examples for this study because many people are used to drinking these beverages and it is obvious that there is a habit already formed for the consumption of these goods. In addition it is much easier, at least in the present situation in Australia, to get the necessary data for tea and coffee than any other commodity subject to habit formation.

There are several reasons why tastes may change. Philips (1974) has suggested that taste changes may be of two sorts: they either result from better outside information due to external influences on a consumer, or they are of the "built-in" type, being related to past decisions. The first case may be the result of social contacts: a demonstration effect or impression formed by advertising.

The influence of past decisions on current tastes is perhaps more striking. Habit formation clearly falls into this class. Smoking habits are a positive function of past consumption: the more you smoke,
the more you want to smoke. The phenomenon is "auto-regressive" and obeys a "built-in" mechanism.

Demand functions with habit parameter(s) are one sub-set of a family of demand functions derived from a general hypothesis that tastes are not constant. In general these demand functions are derived from utility functions for individual $h = 1, \ldots, H$.

$$U(X) = U(X_{h1}, \ldots, X_{hn}, \gamma_1, \ldots, \gamma_k) = U(X, \gamma)$$

(3.1)

where $\gamma = (\gamma_1, \ldots, \gamma_k)$ is a set of shift parameters. These shift parameters may include such diverse variables as time which adds a time trend to demand functions (Stone, 1966) or consumption of commodities by other individuals ("inter-dependence preference", Pollak, 1976), or prices and real income or habit formation variables or family size (Williams, 1976 and Benus, Kmenta and Shapiro, 1976).

The habit formation hypothesis was first used in demand analysis by Stone (1920, 1954) and Farrell (1952) but the essential idea can be traced back much further. A first hint on how to introduce habits in demand theory was given by Marshall (1920) in his discussion of the limitations of the use of static assumptions. In this discussion Marshall introduces these ideas: (a) adaptation to a change in price is gradual: there is a partial adjustment; (b) the movement along a demand curve is irreversible, when habits have developed in the meantime; (c) the effect of habits is positive.

Some thirty years later Farrell in his paper on "Irreversible Demand Functions" tried to specify and estimate demand functions for
particular commodities subject to habit formation, such as tobacco, beer and spritis. He has argued that the most general form of irreversible demand function would make the individual's demand a function of all his past price-income-consumption position. A simpler form

$$X_{it} = f(M_{t}, P_{it'}, X_{it-1}, M_{t-1}, P_{it-1})$$

(3.2)

would make current demand depend only on current price and income and on the individual's price-income-consumption position in the previous period.

Farrell's assumption is unsatisfactory because it deals only with a single equation which is not derived explicitly from a utility function. It does contain the interesting idea of asymmetry with respect to positive and negative changes but this is probably not worth pursuing in the first instance.

As was explained in Chapter 2, my maximizing any utility function subject to a budget constraint and solving the first order conditions one can get a set of demand functions. These functions should, of course, satisfy the regularity conditions to be theoretically plausible. In general one can specify a utility function and shift one or more or even all of its parameters. The derived demand functions will be the same except that the corresponding parameters are shifted.

The main work on habit formation is that of Pollak, especially Pollak (1970) in which he distinguishes between long-run and short-run functions and alternative habit formation hypotheses.
Pollak's essential idea is that present utility is a function of past consumption or other habit variables, usually past consumption of the individual concerned. The simplest assumption is that the quantity demanded of each good is proportional to consumption of that good in the previous period. A more general assumption is that the quantity demanded of each good is a linear function of consumption of that good in the previous period. In general one can write

\[ U_{it} = U_{it}(X_{hi}, \ldots, X_{hn}, X_{hit-1}) \]

\[ = U(X_{hi}, X_{hit-1}) \]  \hspace{1cm} (3.3)

where \( U_{it} \) is utility function for some individuals (h) for good i in period t.

Here should be explained the definitions of short-run and long-run utility functions. In a dynamic context, the short-run is defined as a framework of analysis in which the variables under study have no time to adjust fully so as to attain a steady state. The long-run is then, by definition, such that the variables may reach a position of rest at an equilibrium point. The "steady state" or long-run equilibrium will be characterized by the fact that \( X_{it} = X_{it-1} \) for all t or \( X_{it} - X_{it-1} = 0 \).

This study is interested in short-run demand functions derived from a utility function with changing tastes although one can obtain long-run demand functions from the short-run demand functions (see Pollak 1970, 1976).
In deriving a set of dynamic demand functions, therefore, three choices have to be made:

(i) choice of utility function;

(ii) choice of shift parameters in this function;

(iii) choice of habit formation hypothesis.

Concerning (i), a wide range of utility functions has now been treated in conjunction with habit formation hypotheses including Stone-Geary utility functions (Pollak and Wales 1969, Pollak 1970), two-stage C.E.S. functions (Brown and Heien 1972), one-stage C.E.S. function (Pollak 1970), quadratic utility functions (Houthakker and Taylor 1970), direct addilog utility function (Basman 1968, Sato 1972), indirect translog utility functions (Manser 1976), and Cobb-Douglas functions (Peston 1967).

Concerning (ii), the usual method is to introduce the concept of a committed bundle of commodities and to assume that these parameters shift as some function of past consumption (Pollak and Wales 1969), (Brown and Heien 1972), (Pollak 1970), Manser 1976) and many others. But this is not essential. In general, the habit may shift any of the parameters as Pollak (1970) and Manser (1976) have mentioned. Peston (1967) and Manser (1976) both develop functions without precommitted consumption. However, one may be able to develop an a priori reason for supposing that there are precommitted bundles which shift (Pollak 1970).

Concerning (iii), most of the economists mentioned above have used linear or proportional hypotheses: e.g., Brown and Heien (1972); Pollak and Wales (1969), but these are a special case of multi-period

The simplest assumption of habit formation is that the necessary quantity of each good is proportional to consumption of that good in the previous period, that is:

$$b_{it} = \beta \cdot x_{it-1} \quad 0 \leq \beta < 1$$ (3.4)

where $b_{it}$ is the value of $b$ (necessary collection of goods) in period $t$, $x_{it}$ is the value of $x$ in period $t$ and $\beta_i$ is a "habit formation coefficient". A more general assumption is that the necessary quantity of each good is a linear function of consumption of that good in the previous period. That is

$$b_{it} = b^* + \beta_i \cdot x_{it-1} \quad 0 \leq \beta_i < 1$$ (3.5)

where $b^*_i$ is a constant.

Pollak (1970) has interpreted $b^*_i$ as a "physiologically necessary" component of $b_{it}$ and $\beta_i \cdot x_{it-1}$ as the "psychologically necessary" component.

The "habit formation" assumption of (3.4) and (3.5) implies that consumption in the previous period influences current preference and demand, but that consumption in the more distant past does not. One can generalize the assumption by allowing the necessary quantity of each good to depend on a geometrically weighted average of all past consumption of that good. However, for simplicity in estimation, I will
only use assumption (3.5) in this analysis. One could also argue that since habit to drink tea or coffee is quite different from the addiction to alcohol or smoking the period of one year is a reasonable time for the introduction of past consumption.

In the analysis of demand for any choice of the utility function, shift parameters and habit formation hypothesis there is still the problem of aggregation. The habit formation hypothesis concerns one individual, individual h, but the demand function estimated is a market demand function for all consumers. Here there are two choices. The hypothesis can be specified in terms of each individual and aggregate explicitly, viz.:

\[ X_{it} = \sum_{h} X_{hit} = \sum_{h} X_{it}(P, M, \gamma) \]  

where \( X_{hit} \) is the quantity demanded of good i for individual h in time t, \( P \) is the price in time t, \( M \) is the income of individual h in time t, \( \gamma \) is the shift parameter for individual h and \( X_{it} \) is total demand for good i in time t. Alternatively, the hypothesis can be stated in terms of an aggregate or social utility function for all consumers, viz.:

\[ U = U(X, \gamma) \]

\[ X = (X_1, \ldots, X_h) = (\sum h X_{hi}, \ldots, \sum h X_{hn}) \]

\[ \gamma = \gamma(X_{t-1}, \ldots, X_{t-s}) \]  

therefore

\[ X_{it} = X_{it}(P, M, \gamma) \]
Considering the above explanations and with the aim to estimate a single commodity demand function for tea and coffee, one can apply habit formation hypothesis to a few well known utility functions.

There are many systems of demand equations which have already been tested empirically by many economists. The choice of the following models is just to demonstrate that the habit formation hypothesis is applicable to any theoretical demand system.

I Stone-Geary Utility Functions

Beginning with an additive utility function of the form

\[
U(X_t) = \sum_{i=1}^{n} a_i \log(X_{it} - b_{it})
\]

\[a_i > 0 \quad \sum_{i=1}^{n} a_i = 1 \quad (X_{it} - b_{it}) > 0\] (3.8)

Maximizing (3.8) subject to the budget constraint

\[
\sum_{i=1}^{n} p_{it} X_{it} = M_t\] (3.9)

As shown in Chapter 2, this gives the linear expenditure system

\[
p_{it} X_{it} = p_{it} b_{it} - a_i \sum_{k=1}^{n} p_{kt} b_{kt} + a_i M_t\] (3.10)

\[(i=1, \ldots, n)\]

or the system of commodity demand functions

\[
X_{it} = b_{it} - \sum_{k=1}^{n} (a_i b_{kt})(P_{kt}/P_{it}) + a_i (M_t/P_{it})\] (3.11)

\[(i=1, \ldots, n)\]
As was explained in Chapter 2, it is usually assumed that $b_{it}$ is precommitted consumption and the consumer after satisfying his committed expenditure will spend his "supernumerary" income $(M_t - \sum P_k b_{kt})$ proportionally among the other goods. Shifting the parameters of the utility function and introducing habit formation one can have three different demand functions:

(a) Incorporating habit formation (3.5) into the model by allowing only the $b_i$'s to depend linearly on immediate past consumption. Substituting equation (3.5) into (3.10) gives

$$P_{it} X_{it} = P_{it} (b^* + \beta_i X_{it-1}) - a_i \sum_{k=1}^{n} P_{kt} (b^*_k + \beta_k X_{kt-1}) + a_i M_t$$

(i=1, ..., n) (3.12)

simplifying (3.12) and dividing through by relative prices gives the demand functions

$$X_{it} = b^* + \beta_i X_{it-1} - a_i \sum_{k=1}^{n} (a_i b^*_k)(P_{it}/P_{kt}) - \sum_{k=1}^{n} (a_i \beta_k)(P_{it}/P_{kt}) X_{kt-1} + a_i (M_t/P_{it})$$

(i=1, ..., n) (3.13)

(b) Incorporating the habit formation into the model by allowing only $a_i$'s to shift.

This means the consumer spend his supernumerary income subject to habit formation. The assumption is that the marginal propensities to
consume out of supernumerary income is a linear function of past consumption.

\[ a_{it} = a^*_i + d_i X_{iit-1} \quad 0 \leq d_i < 1 \] (3.14)

where, again, \( d_i \) is the habit coefficient and \( a^*_i \) is a constant.

Substituting (3.14) into equation (3.10) gives

\[
P_{it}X_{it} = P_{it}b_{it} - (a^*_i + d_i X_{iit-1}) \sum_{k=1}^{n} P_{kt}b_{kt} + (a^*_i + d_i X_{iit-1}) M_{it} \]

\[ (i=1, \ldots, n) \] (3.15)

or

\[
X_{it} = b_{it} - \sum_{k=1}^{n} (a^*_i b_{kt}) (P_{kt}/P_{it}) - \sum_{k=1}^{n} (d_i X_{iit-1} b_{kt}) (P_{kt}/P_{it}) + a^*_i (M_{it}/P_{it}) + d_i X_{iit-1} (M_{it}/P_{it}) \]

\[ (i=1, \ldots, n) \] (3.16)

(c) Incorporating habit formation into the model by allowing both \( a_i \)'s and \( b_i \)'s shift.

In this case the demand systems which are the result of substituting (3.5) and (3.14) into (3.10) are of the following form:

\[
P_{it}X_{it} = P_{it} (b^*_i + \beta_i X_{iit-1}) - (a^*_i + d_i X_{iit-1}) \sum_{k=1}^{n} P_{kt} (b^*_k + \beta_k X_{kkt-1}) + M_{it} (a^*_i + d_i X_{iit-1}) \]

\[ (i=1, \ldots, n) \] (3.17)

Simplifying (3.17) and dividing through by \( P_{it} \) gives
\[
X_{it} = b^* + \beta_i X_{it-1} - \sum_{k=1}^{n} (a^* b^*) (P_{kt}/P_{it}) - \sum_{k=1}^{n} (a^* \beta_i X_{kt-l}) (P_{kt}/P_{it}) - \\
(b^* d_i X_{it-l}) (P_{kt}/P_{it}) - \sum_{k=1}^{n} (d_i X_{it-l} \beta_i X_{kt-l}) (P_{kt}/P_{it}) + \\
a^*(M/P_{it}) + d_i X_{it-l} (M/P_{it})
\]

\[\text{(3.18)}\]

(ii) The System of Double Logarithmic Functions

As was explained in Chapter 2, the demand functions of this group may be expressed as

\[
\log X_{it} = \alpha_{it} + \gamma_i \log M_t + \sum_{j=1}^{n} \beta_{ij} \log P_{jt} (i=1, \ldots, n)
\]

\[\text{(3.19)}\]

Enforcing the homogeneity constraint, one can write (3.19) in the following form

\[
\log X_{it} = \alpha_{it} + \gamma_i \log (M_t/\pi) + \sum_{j=1}^{n} \beta_{ij} \log (P_{jt}/\pi)
\]

\[\text{(3.20)}\]

where \(\pi\) is some price index of all prices (including that of good \(i\)).

Now it can be assumed that \(\alpha_{it}\) is a linear function of past consumption

\[
\alpha_{it} = \alpha^* + c_i X_{it-1} \quad 0 \leq c_i < 1
\]

\[\text{(3.21)}\]

Substituting (3.21) into (3.20) gives

\[
\log X_{it} = \alpha^* + c_i X_{it-1} + \gamma_i \log (M_t/\pi) + \sum_{j=1}^{n} \beta_{ij} \log (P_{jt}/\pi)
\]

\[\text{(3.22)}\]
III Indirect Translog Functions

In Chapter 2 it was explained that the demand functions of this group are of the following forms:

A - Without Precommitted Levels of Consumption

\[ X_{it} = \frac{M_{it}}{P_{it}} \cdot \frac{\alpha_{it} + \sum_{j} \beta_{ij} \ln \left( \frac{P_{jt}}{M_t} \right)}{\alpha + \sum_{j} \beta_{ij} \ln \left( \frac{P_{jt}}{M_t} \right)} \]  

(3.23)

where \( \alpha = \sum_{j} \alpha_j = -1 \) and \( \beta = \sum_{i=1}^{n} \beta_{ij} \)  

\( j = 1, \ldots, n \)

B - With Precommitted Levels of Consumption

\[ X_{it} = \gamma_{it} + \alpha_i \left( \frac{M_t}{P_{it}} \right) - \sum_{j \neq i} \alpha_j \gamma_j \left( \frac{P_{jt}}{P_{it}} \right) + \sum_{j} \beta_{ij} \ln \left( \frac{P_{jt}}{P_{it}} \right) - \sum_{j} \beta_{ij} \ln \left( \frac{\gamma_j \cdot P_{jt}}{P_{it}} \right) \]

(3.24)

\( i = 1, \ldots, n \)

where \( \sum \alpha_i = -1 \)

For (3.23), one can specify that \( \alpha_i \) depends linearly on consumption in the immediately preceding period.

\[ \alpha_{it} = \alpha^*_i + d_i X_{it-1} \]  

(3.25)

Substituting (3.25) into (3.23) gives

\[ X_{it} = \frac{M_t}{P_{it}} \cdot \frac{\alpha^*_i + d_i X_{it-1} + \sum_{j} \beta_{ij} \ln \left( \frac{P_{jt}}{M_t} \right)}{\alpha + \sum_{j} \beta_{ij} \ln \left( \frac{P_{jt}}{M_t} \right)} \]

(3.26)

\( i = 1, \ldots, n \)
For (3.24) one can specify that the committed quantity of good $i$ depends linearly on consumption of good $i$ in the immediately preceding period.

$$Y_{it} = \gamma_i^* + a_i x_{i, t-1} \quad \text{where } 0 \leq a < 1 \tag{3.27}$$

Substituting (3.27) into (3.24) gives

$$x_{i, t} = \gamma_i^* + a_i x_{i, t-1} + \sum_{j \neq l} a_j \gamma_j (p_{j, t}/p_{i, t}) + \sum_{j} \beta_{ij} \ln p_{j, t} \left( \frac{M_{i, t}}{P_{i, t}} \right) - \sum_{j} \beta_{ij} \ln p_{j, t} \left( \frac{\sum_j \gamma_j p_{j, t}}{p_{i, t}} \right)$$

$$\quad (i=1, \ldots, n) \tag{3.28}$$

In Chapter 4, equations will be estimated for only four commodities ($n = 4$). These commodities are tea, coffee, sugar and a composite which is all other goods except tea, coffee and sugar. To estimate demand for tea and coffee in such equations one has to impose the earlier assumption of separability. This means that I am assuming that the marginal rate of substitution between tea and coffee is independent of the consumption of the composite good which is an aggregate of all other goods.

Regarding habit formation, for simplicity it will be assumed that consumption of each good depends on its own past consumption and not on that of other goods.
4.1 Introduction

The objective in this chapter is to estimate equations (3.11), (3.13), (3.20), (3.22), (3.24) and (3.28), which were explained in the last chapter, for two commodities - tea and coffee. The primary reason that the analysis is restricted to only two equations is the time limit imposed on the completion of this study. There is no theoretical reason why it is not possible to estimate all equations with different shift parameters. There are some estimation problems when the estimation is restricted to the few equations discussed in Chapter 3. These problems relate to the availability of proper estimation techniques and a proper package for computing work.

The equations to be estimated in this chapter relate to three different models: the linear expenditure system, the double logarithmic system, and the homogeneous indirect translog system. In each model the two equations will be estimated with habit formation and without habit formation. As was explained in Chapter 3, the form of habit formation selected to be tested in this study is the linear hypothesis with the lagged value of the dependent variable. So in the following paragraphs habit formation is referred to as models having lagged own consumption as the habit formation variable.

All three models are of the following functional form:
\[ X_1 = f_1 (P_1, P_2, P_3, P_4, M) \]  
\[ X_2 = f_2 (P_1, P_2, P_3, P_4, M) \]

where \( X_1 \) and \( X_2 \) are the quantity of tea and coffee demanded, \( P_1, P_2 \) and \( P_3 \) are the prices of tea, coffee and sugar respectively and \( M \) is personal disposable income. Since it was not possible to get an average price of all the other commodities in consumer expenditure, the consumer price index \( \pi \) was selected to approximate the price of all other commodities \( P_4 \) which might affect the consumption of tea and coffee. There are, however, two points to be mentioned: first, that the price of sugar and the price of all other commodities \( \pi \) were not statistically significant in any of the estimated equations. So all the statistical results in this chapter report only the price of tea and coffee and personal disposable income as explanatory variables. It should also be noted that in the double logarithmic system, since all prices and income are deflated by the consumer price index, \( \pi \) is automatically omitted from the equations due to the logarithmic characteristics of the function.

The second point is that \( X_1 \) and \( X_2 \) in this analysis are regarded as the dependent variables. This choice is due to theoretical requisites and statistical arguments: (i) supply in the short run is considered to be elastic, and (ii) the range of variations in the dependent variables should be as large as possible. The first argument requires further discussion.

To analyse the price/consumption relationship it is necessary to examine the "identification problem", that is to decide which
variables to treat as independent. The justification that prices or quantities, or neither of these, are predetermined must be examined. The choice of either price or consumption as predetermined requires an explicit and restrictive assumption to be made concerning the short run elasticity of supply of the commodities consumed.

To assume supply is highly price elastic leads to the acceptance of prices as predetermined and to the estimation of demand elasticities. An inelastic supply assumption leads to a model specification where price is some function of the level of demand and the estimated model coefficients are not elasticities but price flexibilities. Inability to postulate either assumption necessitates the use of multi-equational models where both prices and quantities are regarded as current endogenous variables.

Since I am estimating the demand for tea and coffee in Australia where neither are produced in quantity, it can be assumed that prices are determined exogenously by the international market. On the other hand, Australian consumption of tea and coffee is, comparatively, so small that changes in the present consumption pattern cannot change the prices. Overall, it seems reasonable to assume that prices are exogeneous. Consumers are thus faced with given prices and vary their purchases accordingly. Thus the proposed models of Chapter 3 approximate the ordinary demand functions.

4.2 Stochastic Specification and Empirical Results

The Data

The data available for analysis of the consumption of tea and coffee in Australia is far from adequate. The available data is
based on yearly per capita consumption of tea and coffee for the whole country.

In such a situation the analysis is limited to estimation of aggregate demand functions. It would have been far better, of course, to have had time series and cross-section data for different states or at least for the capital cities of states. Were such data available it would be possible to analyse the changes in consumption for the different income levels represented by the states. Ideally, consumption data by income classes is required. It would then be possible to investigate habit formation for the different income groups and different sub-classes of the population. For example, one may be interested in analysing habit formation with regard to different ethnic groups with different cultures and naturally different tastes and consumption patterns.

Unfortunately the household expenditure survey 1974-75 (ABS, Bulletin 2), which is the first major survey of this kind to be undertaken by the Australian Bureau of Statistics, aggregates all non-alcoholic beverages into one category so that the results are useless for our purposes. It might, however, be possible to get consumption for major cities by personal investigation of the records of the major tea and coffee packing companies. But this was not possible in the limited time available for this study. As will be seen, the results of this study suggest that the collection of such data might be most worthwhile and should be encouraged.

My confession then is that the data available for use in this study is far from satisfactory. It is composed of twenty-six yearly
observations for Australia starting from 1950. These observations are for per capita consumption of tea and coffee; retail prices of tea, coffee and sugar; and personal disposable income (Table A.1).

The effect of population growth on demand is allowed by using per capita figures. The consumption figures are extracted from Australian Yearbooks and publications of the Bureau of Statistics (1). A yearly weighted average for retail prices of tea, coffee and sugar is calculated from the quarterly publication of ABS (2). Some of the original figures for the retail price of coffee are not published but were kindly provided by the Australian Bureau of Statistics (ABS). Personal disposable income is extracted from the Australian National Accounts (3).

I Double Logarithmic System

The corresponding equations in this model are of the following form:

(i) Without lagged consumption

\[
\log X_1 = \alpha_1 + \beta_{11} \log \frac{P_1}{\pi} + \beta_{12} \log \frac{P_2}{\pi} + \gamma_1 \log \frac{M}{\pi} + U_1 \quad (4.3)
\]

\[
\log X_2 = \alpha_2 + \beta_{21} \log \frac{P_1}{\pi} + \beta_{22} \log \frac{P_2}{\pi} + \gamma_2 \log \frac{M}{\pi} + U_2 \quad (4.4)
\]

(1) ABS: "Apparent Consumption of Tea and Coffee".

(2) ABS: "Average Retail Prices of Selected Food Items. Six State Capital Cities and Canberra".

(3) ABS: "Australian National Income Expenditure".
(ii) With lagged consumption

\[ \log X_{1t} = \log \alpha_1^* + C_1 \log X_{1t-1} + \beta_{11} \log \frac{P_{1t}}{\pi} + \beta_{12} \log \frac{P_{2t}}{\pi} + \gamma_1 \log \frac{M_t}{\pi} + U_{1t} \]

\[ \log X_{2t} = \log \alpha_2^* + C_2 \log X_{2t-1} + \beta_{21} \log \frac{P_{1t}}{\pi} + \beta_{22} \log \frac{P_{2t}}{\pi} + \gamma_2 \log \frac{M_t}{\pi} + U_{2t} \]

where in these equations \( \alpha \)'s are constants, \( \beta \)'s, \( \gamma \)'s and \( C \)'s are parameters to be estimated and \( U \)'s are disturbance terms. To estimate these parameters equations (4.3 - (4.6) which are "just identified" can be fitted separately by using ordinary least squares estimators (O.L.S.).

The classical method of least squares regression has been shown to give the best linear unbiased estimates (BLUE) of the coefficients when certain well known conditions are fulfilled. These conditions are the assumptions about the distribution of the disturbance term \( U \) which are as follows:

1. \( E(U_{it}) = 0 \)
2. \( E(U_{it}^2) = \sigma_i^2 \)
3. \( E(U_{it} U_{jt}) = 0 \quad i \neq j \)

That is, the expected value of each \( U_{it} \) is zero, its variance is constant over time, and the \( U \)'s are independent across goods and time periods.
Tables 4.1 and 4.2 summarize the results of regression on individual equations (4.3) - (4.6). The negative own price elasticity for both tea and coffee indicates that with any price increase the consumption of these commodities will decrease. However, from the magnitude of these elasticities it is noticed that consumption of tea is own-price inelastic while consumption of coffee is own-price elastic. The sign of cross price elasticities indicate that tea and coffee are substitutes as expected. From the t statistics mentioned in Table 4.1, it is evident that all parameters are significant at 95 per cent level of confidence.

The sign of income elasticity for tea is negative. This means that at higher income levels tea is considered to be an inferior commodity. If this result is true, it is possible to accept the argument that the ever increasing per capita consumption of coffee in the last few years is an indication of a new habit formation in drinking coffee. It has been suggested that for some Australians the consumption of coffee has been regarded as being fashionable, especially in higher income groups.

In Table 4.2 it is noticed that except for lagged consumption (the habit coefficient) other parameters have lost their significance. This is, of course, a common result when one introduces a lagged variable in demand equations and in fact this loss of significance is the price one pays for introducing lagged variables into the equation.

The value of the coefficient of determination ($R^2$) is quite high in both cases and in fact improves by about two per cent in the model with the habit parameter. The $R^2$ provides a standard measure of
TABLE 4.1
ELASTICITIES OF DEMAND FOR TEA AND COFFEE
O.L.S. ESTIMATES FOR EQUATIONS (4.3) AND (4.4)
WITHOUT LAGGED CONSUMPTION
(ESTIMATED SEPARATELY)

<table>
<thead>
<tr>
<th>Per Capita Consumption</th>
<th>Tea Price ($P_1$)</th>
<th>Coffee Price ($P_2$)</th>
<th>Income ($M$)</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tea ($X_1$)</td>
<td>-0.130</td>
<td>0.244</td>
<td>-0.452</td>
<td>1.872</td>
</tr>
<tr>
<td></td>
<td>(-2.64)</td>
<td>(3.642)</td>
<td>(-3.915)</td>
<td></td>
</tr>
<tr>
<td>$R^2 = 0.937$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee ($X_2$)</td>
<td>1.265</td>
<td>-1.129</td>
<td>1.421</td>
<td>-4.255</td>
</tr>
<tr>
<td></td>
<td>(10.373)</td>
<td>(-6.767)</td>
<td>(4.953)</td>
<td></td>
</tr>
</tbody>
</table>

t statistics are shown in parentheses.
<table>
<thead>
<tr>
<th>Per Capita Consumption</th>
<th>Tea Price ($P_1$)</th>
<th>Coffee Price ($P_2$)</th>
<th>Income ($M$)</th>
<th>Lagged Consumption (Habit Coefficient)</th>
<th>Constant</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tea ($X_1$)</td>
<td>-0.009</td>
<td>-0.153</td>
<td>0.672</td>
<td></td>
<td>0.619</td>
</tr>
<tr>
<td></td>
<td>(-0.184)</td>
<td>(0.099)</td>
<td>(-1.364)</td>
<td>(4.108)</td>
<td></td>
</tr>
<tr>
<td>$R^2 = 0.964$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D = 2.200$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Coffee ($X_2$)</td>
<td>0.354</td>
<td>-0.213</td>
<td>0.345</td>
<td>0.748</td>
<td>-0.973</td>
</tr>
<tr>
<td></td>
<td>(1.625)</td>
<td>(-0.947)</td>
<td>(1.189)</td>
<td>(5.038)</td>
<td></td>
</tr>
<tr>
<td>$R^2 = 0.982$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$D = 2.270$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

t statistics are shown in parentheses.
how well a specific relation fits the sample data by explaining the ratio of variations in the dependent variable which is explained by independent variables. This measure can be used as a criterion for comparison of (1) the goodness of fit between any two alternative (but not necessarily mutually exclusive) sets of independent variables and, (2) the goodness of fit of two functional forms given the set of independent variables.

The last statistic that should be explained is the Durbin-Watson statistic. This statistic is used to determine the degree of autocorrelation (time series correlation) in the disturbance term. In Table 4.1, D is very low for equation (4.3) indicating the presence of autocorrelation in the data. The value of D is equation (4.4) is significant at 95 per cent level and shows that there is no autocorrelation in the disturbance term of this equation.

In the usual linear regression models the successive disturbances may be correlated for various reasons, thereby violating the assumption of spherical disturbances which is very important in the application of the ordinary least squares method. Autocorrelation of the first and higher orders in the disturbances can arise because of (1) the faulty functional form assumed for the model; (2) the omission of variables in the analysis of a relation either as a result of the lack of data or of mere ignorance; (3) the tendency for observation errors in economic variables to persist; (4) the estimation of missing data by either averaging or extrapolating; and (5) the lagged effect of temporary shocks distributed over a number of time periods.
Although there are techniques such as the Cochrane-Orcutt transformation for correction of autocorrelation there is no necessity for such a correction here. This is because this problem is automatically corrected in the equations with lagged consumption which are in fact a new specification of the model. The explanation of this phenomenon is very easy. One could say that, in the context of the habit models which depend on lagged consumption, it is a plausible assumption that there is no autocorrelation. One would expect autocorrelation of the U's if a higher level of consumption of the i'th good yesterday is associated with a higher level of consumption of the i'th good today. But in the habit models which depend on lagged consumption, this relationship has already been taken into account. In these models a higher level of \( U_{it-1} \) implies a higher level of \( X_{it-1} \), which in turn implies a higher level of \( \alpha_{it} \) and \( X_{it} \).

In the equations estimated so far, an investigation of the correlation matrix shows a very high collinearity (sometimes more than 0.9) among some variables such as income and price and income and consumption. According to some econometric theory multicollinearity is a data problem. In general, the existence of multicollinearity results in large sample variances of the coefficient estimators, the uncertain specification of the model with respect to the inclusion of variables, and also the resulting difficulty in the interpretation of the estimated coefficients.

As David S. Huang says:

Keep in mind that the principles of least squares are not invalidated by the existence of multicollinearity; it is not least squares that is at fault. The fact
is that the data will simply not allow any
method to distinguish between the effects of
collinear variables on the dependent variable.

One solution of multicollinearity is to drop a number of
independent variables that are highly collinear with the others and
to re-estimate the relation with the remaining variables.

Any departure from the use of variables other than the ones
correctly specified would bring about biases in the O.L.S. estimator so,
if high collinearity exists among the data for the correctly included
variables, the chances are that the researcher will either drop some
variables or will impose linear restrictions on the parameters of the
true model for re-estimation. Estimates obtained in this way are biased,
since dropping variables (zero restriction) or imposing linear restric­
tions, constitutes a departure from what is presumed to be a correct
original model.

In economic research one often does not know if one has
correctly included the relevant variables in a relation, or if the
correct functional form has been chosen. Here, the specification in
the sense of exclusion or inclusion of variables is an open Pandora's
Box if the included and the excluded variables suffer from multi­
collinearity in their data separately or jointly. For any set of
included variables, high multicollinearity plays havoc with the accurate
determination of the coefficient estimates; hence, the validity of the
specific set of included variables can be questioned. Any change in
the list of included and excluded variables raises the question of
possible bias in the re-estimated coefficients. The unbiasedness pro­
\perty of the O.L.S. procedure is undermined by poor specification.
Multicollinearity in the data cuts across the estimation, the specification, and the interpretation of empirical results. Often, in economic research, one is searching for a better and better specification of a relation, but at each alteration of specification there are problems of estimation and interpretation when data that are multicollinear are used.

Thus multicollinearity is easy to detect but difficult to solve. It is well known that most economic variables are correlated with one another, or that they move together. And, since economic data are not generated by experimentally controlled conditions, multicollinearity is a prevalent phenomenon in the observations for independent variables. Thus the problem is not so much in detecting the existence of collinearity but rather to find out the severity of multicollinearity. Usually, as a part of standard computations for regression analysis, one calculates the simple correlation for the pairs of the independent variables and forms the correlation matrix, say, \( R = (r_{ij}) \) where \( r_{ij} \) is the sample correlation between \( X_i \) and \( X_j \), say, \( i=1, 2, ..., k \). If \( r_{ij} \) are large, say, 0.8 or greater, pair wise collinearity is serious. But how high \( r_{ij} \) should be before it is intolerable is a question that can be answered only by the investigator according to individual problems. Klein (1962) suggests a rule of thumb that multicollinearity is "tolerable" if \( r_{ij} < R \), where \( R \) is the square root of the coefficient of multiple determination, \( R^2 \).

If multicollinearity is intolerable in the sense of high simple correlations among any two independent variables or in the sense of not being able to obtain well-determined estimates of the regression
coefficients, what then is the solution? One frequently used procedure is to drop the variable or variables with which the other independent variables are highly collinear - the zero restriction. One of the problems of zero restrictions is that when one drops a variable on the basis of the statistically insignificance of that variable, for instance, through the usual t test, and then re-estimates the equation, the estimates obtained will suffer from pre-testing bias (pre-testing bias arises in an estimator when the estimator no longer has the probability distribution implied by the original model) as well as from the bias of sequential estimators.

As mentioned before, the price of sugar was omitted from the equations because of its statistical insignificance. In re-estimating the equations after dropping the price of sugar it was noticed that the multicollinearity still exists and it is not logical to drop any further variables such as the price of tea or coffee. Thus, according to the rule of thumb mentioned before; since in most cases $r_{ij} < R$, it is assumed that the present multicollinearity is tolerable.

Generally speaking, the results of double logarithmic equations show that the variables chosen to explain the consumption of tea and coffee are quite plausible and the magnitude of the coefficient of determination proves the presence of a very good fit. Also, demand equations for coffee show a marginally better functional form and goodness of fit. Coffee shows to be a normal commodity and elastic to changes in income and price. The results of the equations with lagged consumption, however, do not strongly emphasize the appropriateness of the functional form. If the present functional form is acceptable, one
should say that since only the habit parameter is significant in the equations, the variation in demand for tea and coffee is mostly explained by habit formation. Although the habit coefficient may be a significant parameter, it is hard to deny the significance of income and prices in these demand relationships. This discussion will be postponed until the result of the analysis of other models is introduced.

II Linear Expenditure System

The equations in this model with \( n = 4 \) are:

(i) Without lagged consumption

\[
X_1 = b_1 - (a_1 b_2) \left( \frac{P_2}{P_1} \right) - (a_1 b_3) \left( \frac{P_3}{P_1} \right) - (a_1 b_4) \left( \frac{\pi}{P_1} \right) + a_1 \left( \frac{M}{P_1} \right) \tag{4.7}
\]

\[
X_2 = b_2 - (a_2 b_1) \left( \frac{P_1}{P_2} \right) - (a_2 b_3) \left( \frac{P_3}{P_2} \right) - (a_2 b_4) \left( \frac{\pi}{P_2} \right) + a_2 \left( \frac{M}{P_2} \right) \tag{4.8}
\]

(ii) With lagged consumption

\[
X_{1t} = b_1 + \beta_1 X_{1t-1} - (a_1 b_2) \left( \frac{P_{2t}}{P_{1t}} \right) - (a_1 b_3) \left( \frac{P_{3t}}{P_{1t}} \right) - (a_1 b_4) \left( \frac{\pi_t}{P_{1t}} \right) + a_1 \left( \frac{M_t}{P_{1t}} \right) \tag{4.9}
\]

\[
X_{2t} = b_2 + \beta_2 X_{2t-1} - (a_2 b_1) \left( \frac{P_{1t}}{P_{2t}} \right) - (a_2 b_3) \left( \frac{P_{3t}}{P_{2t}} \right) - (a_2 b_4) \left( \frac{\pi_t}{P_{2t}} \right) + a_2 \left( \frac{M_t}{P_{2t}} \right) \tag{4.10}
\]

where, again, a's, b's and \( \beta \)'s are the parameters to be estimated. To estimate the above model it was noticed that because of the nonlinear restrictions across the equations the estimates of O.L.S. are not consistent.
The technique selected to estimate this model was maximum likelihood estimates with full information.* The program package used for this estimation is RESIMUL which was written by C.R. Wymer of the London School of Economics. The program is based on maximum likelihood estimates and readers may refer to Wymer (1969) or any econometric textbook for further information. However, a brief explanation of the principles of maximum likelihood estimates is in order.

In equation \( Y_i = \hat{a} + \hat{b}X_i \) the interval estimates of \( a \) and \( b \) using the least squares method are based on the assumption that the \( X_i \) values are fixed. In practical applications, however, \( X_i \) may also be random variables.

Let \( f(X_i), i=1, 2, ..., n \) be the probability distribution of \( X_i \) and assume that the conditional probability distribution of the \( Y_i \) given \( X_i \) are normal and independent with expected values \( (a + bX_i) \) and constant variance \( \sigma_u^2 \). Then the joint distribution of \( X_i \) and \( Y_i \) can be expressed as:

\[
L = f(X_1) ... f(X_n) ... f(Y_1 | X_1) ... f(Y_n | X_n) =
\]

\[
f(X_1) ... f(X_n) \frac{1}{2\pi \sigma_u^2 n/2} \exp \left[ -\frac{1}{2\sigma_u^2} \sum (Y_i - a - bX_i)^2 \right] \quad (4.11)
\]

Since the logarithm of a function has the maximum point at the respective position as the original function, we take the logarithm of \( L \), which is:

* I am indebted for the estimation of this model to Julian Morris from the Industries Assistance Commission in Canberra who kindly helped me to use the program and estimate the model.
Log $L = \log f(X_1) + \ldots + f(X_n) - \left( \frac{n}{2} \right) \log 2\pi - \left( \frac{n}{2} \right) \log \sigma^2_u \sum \frac{1}{2\sigma^2_u} (Y_i - a - bX_i)^2$ \hspace{1cm} (4.12)

Partially differentiating $\log L$ with respect to $a$, $b$, and $\sigma^2_u$ gives:

$$\frac{\partial \log L}{\partial a} = \frac{1}{\sigma^2_u} \sum (Y_i - a - bX_i)$$

$$\frac{\partial \log L}{\partial b} = \frac{1}{\sigma^2_u} \sum X_i (Y_i - a - bX_i)$$ \hspace{1cm} (4.13)

$$\frac{\partial \log L}{\partial \sigma^2_u} = -\frac{n}{2\sigma^2_u} + \frac{1}{2\sigma^4_u} \sum (Y_i - a - bX_i)^2$$

Equating them to zero, we obtain the maximum likelihood estimates of $a$ and $b$:

$$\hat{a} = \frac{\sum XY - (\sum X)(\sum Y)}{n \sum X^2 - (\sum X)^2} \quad \text{or} \quad \frac{\sum XY}{\sum X^2}$$

$$\hat{b} = \frac{\sum X Y - (\sum X) (\sum Y)}{n \sum X^2 - (\sum X)^2}$$

The estimated form of equations (4.7 - (4.10) with the above technique is as follows. It should be noted that the sugar price and also the price of "other goods" are omitted from the equations because of statistical insignificance.

(i) Without lagged consumption

$$X_1 = 3.280 + (0.00004) \frac{P_2}{P_1} - (0.00018) \frac{M}{P_1}$$

$$R^2 = 0.96 \quad D = 1.61$$

(38.66) (5.04) (8.01) \hspace{1cm} (4.7)
\[ X_2 = 0.229 - (0.0007) \left( \frac{1}{P_2} \right) + (0.0002) \left( \frac{M}{P_2} \right) \]  \hspace{1cm} (4.8)

\[ R^2 = 0.92 \quad D = 1.04 \]

(ii) With lagged consumption

\[ X_{1t} = 0.589 + 0.806 X_{1t-1} + (0.000002) \left( \frac{P_{2t}}{P_{1t}} \right) - (0.00003) \left( \frac{M_{t}}{P_{1t}} \right) \]  \hspace{1cm} (4.9)

\[ R^2 = 0.96 \quad D = 1.95 \]

\[ X_{2t} = 0.068 + 1.020 X_{2t-1} + (0.00008) \left( \frac{P_{1t}}{P_{2t}} \right) - (0.00001) \left( \frac{M_{t}}{P_{2t}} \right) \]  \hspace{1cm} (4.10)

\[ R^2 = 0.98 \quad D = 1.98 \]

It should be mentioned that the program does not provide \( R^2 \) and Durbin-Watson statistics. \( R^2 \) and D mentioned above are derived from the O.L.S. technique used for the above model. The income and price elasticities of these equations are summarized in Tables 4.3 and 4.4. It is noticed that the magnitudes of price elasticities are very low. This may suggest that no great further increase in consumption per head could be expected as a consequence of economic changes.

The sign of own-price elasticity for coffee in Table 4.3, based on equation (4.8), is positive which is rejected on theoretical grounds. It is interesting to note that there is a considerable difference in results between the equations without lagged consumption
### TABLE 4.3
ELASTICITIES OF DEMAND FOR TEA AND COFFEE
MAXIMUM LIKELIHOOD ESTIMATES FOR THE SUB-SYSTEM OF LINEAR EXPENDITURE SYSTEM - EQUATIONS (4.7) AND (4.8)
(WITHOUT LAGGED CONSUMPTION)

<table>
<thead>
<tr>
<th>Per Capita Consumption</th>
<th>Tea Price ($P_1$)</th>
<th>Coffee Price ($P_2$)</th>
<th>Income ($M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tea ($X_1$)</td>
<td>-0.00002</td>
<td>0.000007</td>
<td>-0.283</td>
</tr>
<tr>
<td>Coffee ($X_2$)</td>
<td>-0.0005</td>
<td>0.0005</td>
<td>0.603</td>
</tr>
</tbody>
</table>

### TABLE 4.4
ELASTICITIES OF DEMAND FOR TEA AND COFFEE
MAXIMUM LIKELIHOOD ESTIMATES FOR THE SUB-SYSTEM OF LINEAR EXPENDITURE SYSTEM - EQUATIONS (4.9) AND (4.10)
(WITH LAGGED CONSUMPTION)

<table>
<thead>
<tr>
<th>Per Capita Consumption</th>
<th>Tea Price ($P_1$)</th>
<th>Coffee Price ($P_2$)</th>
<th>Income ($M$)</th>
<th>Lagged Consumption (Habit Coefficient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tea ($X_1$)</td>
<td>-0.000001</td>
<td>0.000001</td>
<td>-0.047</td>
<td>0.822</td>
</tr>
<tr>
<td>Coffee ($X_2$)</td>
<td>0.000006</td>
<td>-0.000006</td>
<td>-0.030</td>
<td>0.976</td>
</tr>
</tbody>
</table>
and equations with lagged consumption in this model. In equations without the habit formation parameter the signs of the cross price elasticities are not acceptable (Table 4.3) with reference to common-sense that tea and coffee are substitutes. The signs of cross price elasticities for tea and coffee in equations with the habit parameter are positive (Table 4.4) and confirm the idea of substitutability.

The income elasticity for tea in both equations with and without lagged consumption is again negative. In Chapter 2 it was mentioned that in the linear expenditure system there can be no inferior good. This is because the marginal utility of each good in this model is a function of its own quantity only and decreases when the consumption of the good increases, and the consumption of each good increases with an income rise. From this viewpoint one should conclude that this model is not suitable for estimation of a single commodity such as tea and the model should be used for very broad aggregates such as food, clothing and housing. Although statistically not significant, the income elasticity for coffee is also negative in equation (4.10).

The habit parameter in equations (4.9) and (4.10) is strongly significant and it is interesting to note that, except for the price ratios of tea and coffee and the income price ratio for coffee, the other variables have maintained their significance at 95 per cent confidence level. Like the previous model, the habit elasticity of coffee is higher than tea and again confirms the idea that habit formation for coffee is stronger than for tea. However, there is a major difference between the results of this model and the double logarithmic system. In equation (4.10) it is noticeable that the magnitude of the habit coefficient for coffee is more than one which contradicts the assumption of $0 \leq \beta < 1$. 
III Homogeneous Indirect Translog Functions

The corresponding equations in this model are of the following form:

(i) Without lagged consumption

\[
X_1 = \left(1 - \alpha_1 \right) + \alpha_1 \left(\frac{M}{P_1}\right) - \alpha_1 \left(\frac{M}{P_1}\right) \sum_{j=2,3} \gamma_j \left(\frac{M}{P_1}\right) + \left(\frac{M}{P_1}\right) \sum_{j=2,3} \beta_{1j} \ln \left(\frac{P_j}{P_1}\right)
\]

\[
- \left[ \sum_{j=2,3} \gamma_j \left(\frac{P_j}{P_1}\right) \right] \left[ \sum_{j=2,3} \beta_{1j} \ln \left(\frac{P_j}{P_1}\right) \right] - \gamma_1 \sum_{j=2,3} \beta_{1j} \ln \left(\frac{P_j}{P_1}\right) \tag{4.14}
\]

\[
X_2 = \gamma_2 \left(1 - \alpha_2 \right) + \alpha_2 \left(\frac{M}{P_2}\right) - \alpha_2 \left(\frac{M}{P_2}\right) \sum_{j=1,3} \gamma_j \left(\frac{M}{P_2}\right) + \left(\frac{M}{P_2}\right) \sum_{j=1,3} \beta_{2j} \ln \left(\frac{P_j}{P_2}\right)
\]

\[
- \left[ \sum_{j=1,3} \gamma_j \left(\frac{P_j}{P_2}\right) \right] \left[ \sum_{j=1,3} \beta_{2j} \ln \left(\frac{P_j}{P_2}\right) \right] - \gamma_2 \sum_{j=1,3} \beta_{2j} \ln \left(\frac{P_j}{P_2}\right) \tag{4.15}
\]

(ii) With lagged consumption

\[
X_{1t} = \gamma^*_1 + \alpha_1 X_{1t-1} + \alpha_1 \left(\frac{M_{1t}}{P_{1t}}\right) - \alpha_1 \left(\frac{M_{1t}}{P_{1t}}\right) \sum_{j=2,3} \gamma_j \left(\frac{M_{1t}}{P_{1t}}\right) + \left(\frac{M_{1t}}{P_{1t}}\right) \sum_{j=2,3} \beta_{1j} \ln \left(\frac{P_{jt}}{P_{1t}}\right)
\]

\[
- \left[ \sum_{j=2,3} \gamma_j \left(\frac{P_{jt}}{P_{1t}}\right) \right] \left[ \sum_{j=2,3} \beta_{1j} \ln \left(\frac{P_{jt}}{P_{1t}}\right) \right] - \gamma_1 \sum_{j=2,3} \beta_{1j} \ln \left(\frac{P_{jt}}{P_{1t}}\right) \tag{4.16}
\]

\[
X_{2t} = \gamma^*_2 + \alpha_2 X_{2t-1} + \alpha_2 \left(\frac{M_{2t}}{P_{2t}}\right) - \alpha_2 \left(\frac{M_{2t}}{P_{2t}}\right) \sum_{j=1,3} \gamma_j \left(\frac{M_{2t}}{P_{2t}}\right) + \left(\frac{M_{2t}}{P_{2t}}\right) \sum_{j=1,3} \beta_{2j} \ln \left(\frac{P_{jt}}{P_{2t}}\right)
\]

\[
- \left[ \sum_{j=1,3} \gamma_j \left(\frac{P_{jt}}{P_{2t}}\right) \right] \left[ \sum_{j=1,3} \beta_{2j} \ln \left(\frac{P_{jt}}{P_{2t}}\right) \right] - \gamma_2 \sum_{j=1,3} \beta_{2j} \ln \left(\frac{P_{jt}}{P_{2t}}\right) \tag{4.17}
\]
As was mentioned before, sugar is omitted from the above equations. So $p_3$ in the above equations represents $\pi$. Again $\alpha'$s, $\beta'$s, $\gamma'$s and $\delta'$s are the parameters to be estimated.

The method chosen to estimate the above equations is generalized least squares (G.L.S.) as well as O.L.S. The argument to select G.L.S. technique is as follows.*

In an earlier section it was explained that one of the assumptions in O.L.S. about the disturbance term is that $u$ has a mean equal to zero and a constant variance. In this notation it is not specified just what distribution (the shape as well as the size of spread) $u$ has. This is both an advantage and a disadvantage. It is an advantage in the sense that in many situations the distribution of $u$ is not exactly known but yet one wants to proceed with an analysis of the linear dependence relation by the use of a linear regression model (where one assumes only that $u$ has a zero mean and a constant variance), and a disadvantage in the sense that what can be done in the way of statistical inference in a regression model when $u$'s shape of distribution is unknown may be rather limited. Now, if it is assumed that the distribution of $u$ is normal, in addition to the assumption made in O.L.S., a linear regression model sometimes referred to as the classical normal linear regression model is obtained.

Equations (4.14) - (4.17) are estimated both with O.L.S. and G.L.S. but since the results of the two techniques are very close only G.L.S. estimates are reported below. It should also be mentioned that

* Unfortunately it was not possible to use the RESIMUL package for this model.
in the above equations using \( \pi \) as the price of "other goods" all
parameters lost their significance so the composite good was omitted
from the equations. The re-estimated form of the above equations is
as follows:

\[
x_1 = 2.314 - (0.0018) \frac{M}{P_1} - (0.522) \frac{P_2}{P_1} - (0.000006) \frac{M}{P_1} \ln \left( \frac{P_2}{P_1} \right) + (0.218) \frac{P_2}{P_1} \ln \left( \frac{P_2}{P_1} \right)
\]

\( R^2 = 0.964 \) \( D = 1.74 \) \[(4.14)\]

\[
x_2 = 0.0067 + (0.001) \frac{M}{P_2} + (0.495) \frac{P_1}{P_2} + (0.000482) \frac{M}{P_2} \ln \left( \frac{P_1}{P_2} \right) - (0.115) \frac{P_1}{P_2} \ln \left( \frac{P_1}{P_2} \right)
\]

\( R^2 = 0.969 \) \( D = 2.04 \) \[(4.15)\]

(ii) With lagged consumption

\[
x_{1t} = 1.66 + 0.288 x_{1t-1} - (0.00013) \frac{M_t}{P_{1t}} - (0.359) \frac{P_{2t}}{P_{1t}}
\]

\( R^2 = 0.96 \) \( D = 1.999 \) \[(4.16)\]
X_{2t} = 0.051 + 0.746X_{2t-1} + 0.00034\left(\frac{M_t}{P_{2t}}\right) + (0.262)\left(\frac{P_{1t}}{P_{2t}}\right) + \\
(0.761)(5.675) \quad (2.248) \quad (1.131)

\left(0.00018\right)\ln\left(\frac{P_{1t}}{P_{2t}}\right) + (0.0032)\ln\left(\frac{P_{1t}}{P_{2t}}\right)

(2.355) \quad (0.015)

(R^2 = 0.984) \quad D = 2.26 \quad (4.17)

(t statistics are shown in parentheses)

Tables 4.5 and 4.6 summarize the price and income elasticities of the above equations. Most of the parameters which are significant at 95 per cent level in equations without lagged consumption maintain their significance in equations with lagged consumption (contrary to the double logarithmic model). The only exception was the income price ratio for tea which is not significant at 95 per cent confidence level but significant at 90 per cent. Also, the habit parameter for tea is significant only at 90 per cent confidence level. However, some of the elasticity signs are not acceptable on theoretical grounds. For instance, the own-price elasticity of tea in equations with and without lagged consumption is positive. On theoretical grounds this should be rejected. The income elasticity for tea is again negative confirming the results of the double logarithmic model and linear expenditure system. These is also another indication for inferiority of tea. The cross price elasticity for tea in both equations with and without lagged consumption is negative. For example, in equation (4.14) with a one per cent increase in the price of coffee, consumption of tea will decrease by
### TABLE 4.5
ELASTICITIES OF DEMAND FOR TEA AND COFFEE
G.L.S. ESTIMATES FOR THE SUB-SYSTEM OF INDIRECT TRANSLOG FUNCTIONS
EQUATIONS (4.14) AND (4.15) WITHOUT LAGGED CONSUMPTION

<table>
<thead>
<tr>
<th>Per Capita Consumption</th>
<th>Tea Price ($P_1$)</th>
<th>Coffee Price ($P_2$)</th>
<th>Income ($M$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tea ($X_1$)</td>
<td>0.674</td>
<td>-0.266</td>
<td>-0.283</td>
</tr>
<tr>
<td>Coffee ($X_2$)</td>
<td>1.835</td>
<td>-1.889</td>
<td>1.420</td>
</tr>
</tbody>
</table>

### TABLE 4.6
ELASTICITIES OF DEMAND FOR TEA AND COFFEE
G.L.S. ESTIMATES FOR THE SUB-SYSTEM OF INDIRECT TRANSLOG FUNCTIONS
EQUATIONS (4.16) AND (4.17) WITH LAGGED CONSUMPTION

<table>
<thead>
<tr>
<th>Per Capita Consumption</th>
<th>Tea Price ($P_1$)</th>
<th>Coffee Price ($P_2$)</th>
<th>Income ($M$)</th>
<th>Lagged Consumption (Habit Coefficient)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tea ($X_1$)</td>
<td>0.057</td>
<td>-0.183</td>
<td>-0.203</td>
<td>0.294</td>
</tr>
<tr>
<td>Coffee ($X_2$)</td>
<td>0.733</td>
<td>-0.614</td>
<td>0.421</td>
<td>0.714</td>
</tr>
</tbody>
</table>
0.27 per cent. This value is 0.18 per cent in equation (4.15) with lagged consumption. Considering the cross price elasticity for coffee in equations (4.15) and (4.17) which are both positive, one can conclude that while coffee is a strong substitute for tea the reverse situation is not applicable.

The magnitude of the habit parameters for tea and coffee in Table 4.5 indicates that habit formation for coffee is stronger than habit formation for tea. With a one per cent change in habit, consumption of tea will change by 0.29 per cent while with a one per cent change in habit for coffee, its consumption will change by 0.71 per cent.

It should be mentioned that with the same arguments in the linear expenditure system, estimation of an inferior commodity such as tea in this model violates the assumptions of this system of demand equations and is not appropriate. Besides, by using the G.L.S. technique of estimation instead of maximum likelihood estimates it was practically impossible to impose all the restrictions across the above equations. So it is concluded that the above results are just an approximation of real estimates.
In this study the demand for tea and coffee in Australia was estimated by three models: a pragmatic approach using a double logarithmic system; a linear expenditure system; and homogeneous indirect translog functions. In each of these models a habit formation hypothesis based on linear lagged consumption was tested.

Although the results in each model are in one way or another different from each other it is concluded that:

(1) Estimation results are good in some respects such as a high $R^2$ and significant explanatory variable but not in others (e.g. autocorrelation, negative income elasticities in the linear expenditure system and homogeneous indirect translog functions). This is not surprising in a relatively complex system and one in which changes in functional form make significant differences. Clearly it will take a long time to obtain completely satisfactory estimates for commodities subject to habit formation.

(2) The habit formation model is justified. This is particularly concluded because:

(a) In almost all models it improved the goodness of fit;
(b) It removed autocorrelation;
(c) In almost all the models the coefficients of linear habit formation are significant.
(3) In all estimated models the income elasticity for tea is negative suggesting tea is an inferior commodity in the Australian consumption pattern. With an increase in income consumers will substitute coffee for tea. Coffee is a normal commodity with a positive income elasticity.

(4) Comparison of own-price elasticities show that tea is own-price inelastic and coffee is own-price elastic.

(5) Tea and coffee are gross substitutes.

(6) It can be concluded that the preferred equation for estimating the demand for tea and coffee may be the double logarithmic system, despite its aggregation deficiencies. At least, unlike the linear expenditure systems and homogeneous indirect translog functions, it permits inferior goods. This, of course, does not mean that the linear expenditure systems and indirect translog functions are not satisfactory models. In this case study, the demand for a single commodity like tea which came out as an inferior good, is estimated, while the above models are designed to be quite satisfactory for broad categories of goods which are normal.

(7) It seems obvious that more work should be done on models with habit formation. This means that alternative functional forms should be tested with different habit formation hypotheses. The estimation techniques should also improve to enable the researcher to estimate some relatively sophisticated models.
There are however some other points to be discussed here. As was mentioned in Chapter 3, in this study only short-run demand functions are studied. If one is interested in long-run demand functions it is possible to obtain these functions from short-run demand functions. Given the consumption vector of period 0, and given prices and income of period 1, the short-run demand functions yield a consumption vector for period 1. In a "steady state" or "long-run equilibrium" the optimal consumption vector for period 1 will be identical with the consumption vector of period 0. And, if prices and income remain constant over time, the optimal consumption vector in every subsequent period will also be equal to the consumption vector of period 0. The long-run equilibrium consumption vector could be found by solving the short-run demand functions (3.11) under the assumption that $X_{it-1} = X_i$ for all $i$. However, the discussion of the properties of the long-run demand functions with habit formation is postponed to further studies and is not developed in this analysis.

It was impossible to get cross-section data of Australian consumption of tea and coffee. It was also impossible to carry on the study on the basis of States or major capital cities because of the inadequacy of the data. In studies related to habit formation and consumer's taste, the researcher may be interested to study the consumption pattern of a special group or a section of the community for further marketing studies for example. It is obvious that under such conditions the availability of data plays a very important role.
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Shultz, H. The Theory and Measurement of Demand, Chicago University Press.


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__________ "Full Information Maximum Likelihood Estimation with Non-Linear Restrictions", mimeograph.


### Appendix

**Table A.1 Observations**

<table>
<thead>
<tr>
<th>Year</th>
<th>$X_1$</th>
<th>$X_2$</th>
<th>PS</th>
<th>PC</th>
<th>PT</th>
<th>CPI*</th>
<th>M</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>3.20</td>
<td>0.32</td>
<td>167</td>
<td>425</td>
<td>150</td>
<td>63</td>
<td>701.88</td>
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<tr>
<td>51</td>
<td>3.17</td>
<td>0.34</td>
<td>192</td>
<td>432</td>
<td>192</td>
<td>66</td>
<td>717.29</td>
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<tr>
<td>52</td>
<td>2.94</td>
<td>0.34</td>
<td>267</td>
<td>445</td>
<td>197</td>
<td>69</td>
<td>732.70</td>
</tr>
<tr>
<td>53</td>
<td>3.01</td>
<td>0.41</td>
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<td>457</td>
<td>217</td>
<td>72</td>
<td>748.11</td>
</tr>
<tr>
<td>54</td>
<td>2.90</td>
<td>0.50</td>
<td>300</td>
<td>482</td>
<td>261</td>
<td>72</td>
<td>763.52</td>
</tr>
<tr>
<td>55</td>
<td>2.70</td>
<td>0.54</td>
<td>300</td>
<td>494</td>
<td>352</td>
<td>75</td>
<td>805.27</td>
</tr>
<tr>
<td>56</td>
<td>2.74</td>
<td>0.66</td>
<td>321</td>
<td>489</td>
<td>341</td>
<td>80</td>
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<tr>
<td>57</td>
<td>2.77</td>
<td>0.70</td>
<td>334</td>
<td>473</td>
<td>331</td>
<td>79</td>
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<tr>
<td>58</td>
<td>2.68</td>
<td>0.80</td>
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<td>450</td>
<td>329</td>
<td>80</td>
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<td>945.27</td>
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<tr>
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<td>494</td>
<td>423</td>
<td>170</td>
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</tbody>
</table>

$X_1$ = Per capita consumption of tea  
$X_2$ = Per capita consumption of coffee  
PS = Price of sugar in cents (4 lbs pkt)  
PC = Price of coffee in cents (2 oz jar)  
PT = Price of tea in cents ($\frac{1}{2}$ lb pkt)  
CPI = Consumer price index  
M = Personal disposable income in dollars

* 31 June 1952 = 100; 31 June 1966 = 100