Tariff Determination in General Equilibrium:
A Bargain-theoretic Approach to Policy Modelling

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Declaration

Except where otherwise indicated, this thesis is my own original work.

- Hom Moorti Pant
December 24, 1992
To my parents
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ABSTRACT

Government policies with redistributive implications have been a source of many social and political conflicts. Until recently, positive economics has lacked a consistent analytical framework that could explain how such policies are formed and how they respond to exogenous shocks. This study makes a contribution towards this direction by offering a consistent theoretical framework for short-run policy modelling. Assuming that tariff rates are the only available instruments of a government's redistributive policy this study addresses the following two specific questions: (i) how the tariff rates are determined; (ii) and how do they respond to exogenous shocks?

To answer these questions, a general equilibrium model of a political economy is developed by combining a model of the political sphere with a Ricardo-Viner type model of the economic sphere. Policies are determined by strategic interactions between government and the conflicting interest groups in the political sphere which, in turn, determine the welfare of the interest groups in the economic sphere and political support for the government in the political sphere. The conflicting interest groups may spend resources in predatory political activities or may choose to cooperate. A general equilibrium of the political economy is obtained when both the political and the economic spheres are simultaneously in equilibrium. Under fairly general conditions, it is shown that at least one equilibrium exists in a political economy whether it exhibits cooperative behaviour in the political sphere or not.

This study has employed the analytical framework of cooperative-bargaining theory in obtaining a general equilibrium model of the political economy. This approach is taken because a noncooperative equilibrium is not necessarily Pareto efficient. Several interesting results follow from the comparative static properties of the model. In particular, it is shown that: (i) the import-competing sector receives increased protection if the relative price of the home importable falls in the world market; (ii) the protection afforded to a particular sector declines if the domestic endowment of factors moves in favour of that sector; and (iii) so long as the distribution of relative bargaining power remains unaffected by the shocks, the response of the tariff rate to the shocks will be independent of the distribution the bargaining power. These analytical results are very similar to ones that follow from the maximization of a conservative social welfare function. The implications are that: (i) a government's redistributive policy could be modelled as an equilibrium outcome of a bargaining process between the organized interest groups holding conflicting interests on the level of the redistributive policy; and (ii) the bargaining process may be viewed as the mechanism of generating a conservative social welfare function. The self-interest, and public-interest approaches in policy modelling can thus be reconciled.
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CHAPTER 1
INTRODUCTION

1.1 Motivation

Government policies that have redistributive implications have been a source of many social and political conflicts. A striking example is the European farm subsidy program in which the level of conflict has crossed national frontiers. Tariff and non-tariff barriers to trade that exist all over the world are other examples. This study is concerned with the political economy of the formation of a government's redistributive policies.

Until recently positive economics has lacked the sophistication and clarity to explain how policy decisions are made, particularly when redistributive issues are involved. This is so because alternative (purely) redistributive policies cannot be ranked by the Pareto rule, since each one of them is Pareto efficient. The result is that in any serious economic modelling exercise either a rule of social choice, such as maximization of some kind of a social welfare function, has to be specified so that the alternate redistributive policies can be ranked by it or the government policies have to be treated as exogenously given. Since there are serious logical problems in specifying a social choice rule, positive economic models generally regard government policies as exogenously given.1 The question of policy determination therefore has largely remained outside the scope of positive economic models.2 Nevertheless, given a set of government policies and the other exogenous variables, sophisticated models of positive economics, such as computable general equilibrium (CGE) models, can be employed to explain how the optimal allocative decisions are made in an economy. These models can also be used to explain how the Pareto efficient allocation of resources and the distribution of income will change as consequences of a change in government policies.3

However, if the policies are endogenously determined as the recent literature suggests, then as far as the effects of changes in non-policy exogenous variables on endogenous variables are concerned the predictions of a policy-exogenous CGE model may be incorrect. Such policy-exogenous models do not allow for the second round

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1 For a simple exposition of the logical problems with normative social choice rules see particularly chapters 9 and 10 in Mueller (1979).
effects that would take place after the policies respond to the exogenous shocks. Hence, the full effects of, say a world price change, on sectoral employment, output and income distribution would be different from that predicted by a policy-exogenous CGE model. A closer understanding of the true effects of different shocks on the relevant endogenous variables may be obtained from a general equilibrium model that can determine the policy responses of a government endogenously.

How a government’s policies are determined is therefore a vexing question. The present study attempts to answer this question in part by focusing on redistributive policies and assuming that trade taxes/subsidies are the only available redistributive policies. More precisely, this study addresses the following two specific questions. How are the tariff rates in a price taking economy determined? How would the tariff rates change if the politico-economic environment changed?

The choice of the tariff policy as the subject matter of this study is motivated by the following reasons. First, tariff policy is in itself one of the important forms of government policy; second, it has distributive implications; third, its existence is difficult to explain on social welfare grounds under complete and perfect market conditions because, as is well-known it is not the first-best policy; and finally, tariff policy has been a favourite subject matter of previous studies on political economy, and there consequently exists a basis of previous work upon which further extensions can be made.

Clearly, this study is not the first that has raised questions concerned with the formation of redistributive policies in general, and tariff policy in particular. There exists a large body of literature on rent seeking, public choice, and political economy that has addressed these questions in one way or another. This class of literature argues that policy changes can alter the welfare levels of private agents, these agents therefore have incentives to behave strategically in order to influence the policy choices of the government. The government, on the other hand, survives on political support and therefore supplies policies strategically in order to maximize its political support. This

Buchanan, Tollison and Tullock (1980), Bhagwati (1982b), Collander (1984), Rowley, Tollison and Tullock (1988) provide a substantial collection of mainly analytical works that are concerned with rent-seeking and the political economy of endogenous tariff determination.

It is useful to note the difference among the three schools - rent-seeking, public choice and the political economy. The rent-seeking literature is primarily concerned with the welfare cost of rent-seeking when the level of rent generating policy is already given. Nevertheless this literature also describes the strategic behaviour of the rent-seekers in seeking rents created by distortionary policies. The public choice school is concerned with the application of economic tools in explaining the nonmarket decision making or simply the application of economics to political science (Mueller, 1979). This school lacks the foundation of general equilibrium in the economic sphere. The political economy school combines the methodologies of the public choice school with that of positive economics to obtain a political economy framework in which both the political and economic markets are considered and the policies are determined endogenously together with the other economic variables.
A class of literature culminates in the contention that government policies are determined in the political market, where the policy variables play the role of balancing the opposing political forces that are guided by the economic interests of the self-interested agents.

These studies have recognized that the political and the economic spheres are interconnected (for example, Gardner, 1983). Political activities of economic agents or some institutional mechanism that translates economic interests into political preferences provide the link between the two spheres. The agents in such a political economy face a simultaneous choice problem in both spheres such that the choice made in one sphere has an explicit effect on the other sphere. It is therefore natural to expect that rational actors would behave strategically in the political sphere to enhance their positions in the economic sphere and vice versa. But this means that the policies and, consequently, the allocation of resources in the economic sphere depends on the joint action of all players. Therefore, an individual’s welfare no longer depends only on his or her own actions but also on the actions of other individuals as well. For this reason previous studies almost invariably have adopted a game-theoretic framework to study the policy formation process and the individual choice problem in the context of a political economy.

Moreover, once the political sphere is modelled in conjunction with the economic sphere and the strategic behaviours of the individuals are allowed in the model of a political economy the price taking assumption of the Arrow-Debreu model is violated. Agents can choose their lobbying levels (strategies) in the political sphere to affect the prices they face in the economic sphere. Hence, the question of existence of an equilibrium in such a model of a political economy becomes a nontrivial issue.

Previous authors who studied the policy formation process in the framework of a political economy either did not address the question of the existence of equilibrium at all (Yeldan and Roe, 1991, Magee, Brock and Young, 1989) or had no success (Findlay and Wellisz, 1982, 1983, 1984; Wellisz and Willson, 1986; Hall and Nelson, 1992) or have been able to show it only for an exchange economy (Coggins, 1989; Coggins, et al., 1991). Therefore, an attempt to answer the questions posed above in the framework of a political economy begs another more basic question - the question of the existence of an equilibrium. This question is important for models of a political economy for two reasons. First, an analysis based on the equilibrium behaviour of a political economy makes sense only if an equilibrium exists. Second, a prior knowledge of the conditions under which an equilibrium exists is useful in constructing a computable general equilibrium models of a political economy.
Hence, the research questions of this study are as follows. Does an equilibrium exist in a productive political economy? Under what conditions can one be assured of an equilibrium? Is it necessary that a political economy in equilibrium should deviate from free trade? How would the tariff rates in such a political economy respond to exogenous shocks?

In addition to this, there is one more assumption concerning the nature of the political process which needs to be clarified. The existing literature on political economy has modelled the political process of conflict resolution concerning the government's (tariff) policies as a noncooperative game. Such a model does not allow the players to communicate and negotiate with each other and agree on a strategy combination that is mutually beneficial. Therefore an outcome of the noncooperative tariff game may not necessarily be Pareto efficient, since a cooperative outcome may Pareto dominate it. The assumption that the players are rational goal maximizers and the assumption that their political behaviour can be characterized by a noncooperative game thus appear to be potentially inconsistent. Therefore, there is no reason to stay with the assumptions of the noncooperative game theory.

In contrast to the existing studies, this study assumes that the political process provides enough opportunities to negotiate a cooperative strategy through bargaining if the players find it rational to do so. Moreover, if the players could find a cooperative solution it would be the one agreed to by the conflicting parties, and the government would maximize its political support by enforcing it. Therefore, even in situations when a cooperative solution is not self-enforcing, the presence of a politically motivated government guarantees its enforcement. Hence, this study views tariff determination as a bargaining problem. This assumption constitutes the major departure of the present study from the existing literature on endogenous tariffs formation. This study, nevertheless, analyses noncooperative behaviour in the political sphere because the outcome from this process describes what will happen to the players if they fail to reach an agreement during the bargaining process.

Therefore, this study also attempts to answer the following additional questions. How is a bargaining problem defined in the tariff game? What would induce the players to reach an agreement during the bargaining process? How can the bargaining equilibrium be characterized? What would be the bargained tariff rates? How would the bargained tariff rates respond to exogenous shocks? A consistent analytical framework that can provide answers to the above questions will extend the existing frontier of the endogenous tariff literature.
1.2 Objectives of the Study

The broad purpose of this study is to develop a bargain-theoretic framework to model the policy formation process in a political economy.

More specifically this study attempts to

(i) address systematically the analytical issues involved in integrating the political and economic spheres into a single model of a political economy and examine whether a Nash equilibrium exists in such a model;

(ii) describe the bargaining problem in the tariff-setting game, analyze the bargaining process that induces a solution to the problem, and characterize the outcome of bargaining;

(iii) obtain a theoretically consistent general equilibrium model in which the tariff policy of a government is endogenously determined by the bargaining process; and

(iv) study the comparative static behaviour of the bargained tariff rates.

1.3 The Modelling Approach and the Point of Departure

As in previous studies, the working of the economic sphere has been described in this study by a Ricardo-Viner type two-sector general equilibrium model in which the policies of the government are initially regarded as exogenous. The economy is assumed to be small and open in relation to the world market. Moreover, by invoking Lerner's symmetry theorem (Lerner, 1936) only one tariff rate is considered instead of two. This makes the rationalized tariff rate the only instrument of income redistribution and sharpens the focus of the study. The model has been employed to predict the economic consequences of a given policy change and derive the implications for the interests of the individuals. These results, in turn, have been employed to determine individual preferences over alternate policies.

One reason why previous studies failed to obtain an existence result in a political economy is that they did not specify the structure of the general equilibrium model of the economic sphere of the political economy. They worked with general forms of production functions of which the properties of the higher order derivatives were not known. Learning from these results, this study specifies the forms of the production functions. In particular, it is assumed that the production functions can be represented by constant returns to scale CES functions. The choice of this functional form is a pragmatic compromise between simplicity and generality.
Following the contention of the endogenous tariff literature, a government’s choice of the tariff rate has been viewed as an outcome of the policy game played in the political sphere by the government which wants to stay in power, and the owners of the specific factors, who behave strategically to maximize their real rental income determined in the economic sphere of the political economy. The political process has been viewed as an institutional environment in which the conflicts of interests with respect to the tariff policy are resolved.

With this stylization of a political economy this study first investigates the existence of an equilibrium in a political economy under the assumption that the political environment is noncooperative. The noncooperative model of the political economy describes the consequences for the respective groups of predatory lobbying behaviour on their parts, and exposes the rationale for cooperative behaviour. In particular, it is observed that a distinct possibility of saving resources employed in predatory lobbying exists if the players cooperate and agree on (i) any tariff rate, and (ii) not to spend on lobbying the government.

The bargain-theoretic approach has been adopted to study the tariff determination process under a cooperative political environment. The bargaining problem in the tariff game has been solved using the generalized Nash bargaining process in which the players may be endowed with asymmetric bargaining powers. The condition characterizing the bargaining equilibrium has been combined with the model of the economic sphere to derive a policy-endogenous general equilibrium model, the solution of which yields the level of the politico-economic equilibrium tariff rate that is determined endogenously.

A comparative static (counterfactual experiments) approach has been adopted to derive the endogenous response of the bargained tariff rate as the exogenous variables change. Unlike previous studies, which studied the endogenous response of the tariff rate with respect to changes in the world relative price only, this study also derives the response of the tariff rate as the domestic factor endowments change exogenously. The bargaining powers of the players have been held constant, however.

Two central conclusions emerge out of this study. First, though the level of the tariff rate depends on the relative bargaining powers of the players and the relative fear of ruin (Aumann and Kurz, 1977a, 1977b) held by the players, the changes in it do not depend on the distribution of the bargaining power between the players. The extent and direction of change in the bargained tariff rate depend on the movement of the relative fear of ruin held by the players, which is completely explained by the condition of the economic sphere. Second, in general, the bargained tariff rate changes to compensate,
at least partly, for the relative loss of the loser (compared to that of the other player) that arise from changes in the exogenous environment.

The results obtained in this study have some interesting implications for policy modelling in general equilibrium. First, it is found that the comparative static behaviour of the bargained tariff rate is consistent with the predictions that follow from the maximization of a conservative social welfare function (Corden, 1974). The implication of this similarity is that, if bargaining is accepted as the underlying process that generates the (positive) conservative social welfare function, then the problem of identification of the social welfare function vanishes and the implementation of a social welfare function will not be inconsistent with the self-interested behaviour of the government as well. The difference between the political economy approach and the welfare function maximizing approach can be eliminated. Second, the results of this study indicate that a bargain-theoretic framework can be adopted to model the policy formation process in a political economy and this is consistent with the self-interested behaviour of all agents.

Choice of the specific-factor model differentiates the present study from Magee, Brock and Young (1989), and imposition of a particular structure on the forms of production functions differentiates it from the other studies of political economy (for example, Findlay and Wellisz, 1982). This study also differs from the endogenous tariff literature in that it allows cooperative behaviour in the political process of tariff determination, and the general equilibrium of the political economy is defined whenever a solution to the bargaining problem is obtained.

1.4 Scope and Limitation of the Study

Though this study has attempted to be rigorous in deriving the analytical results, it, nevertheless, suffers from a few limitations some of which are unavoidable if the model is to be kept tractable and simple. Further discussion on the limitations and suggestions for future research are provided in chapter 9. Here the reader’s attention is drawn in particular to the following limitations.

First, this study assumes that trade taxes are the only redistributive policy instruments available to a government. This is, of course, an over-simplification. The government has numerous instruments of intervention, none of which is equivalent to the other in every respect in many real world circumstances. The choice of instruments itself can have another political economy story (see Lloyd and Falvey, 1986, and Falvey and Lloyd, 1991). In order to keep the analysis simple and to understand the process of policy determination this particular assumption has to be made.
Second, this study does not consider how the tariff revenue is distributed. It simply assumes that the tariff revenue is transferred to the consumer. But, the response of the tariff rate with respect to the exogenous shocks could be sensitive to the way the tariff revenue is distributed (Long and Vousden, 1991). Our purpose in not modelling the distribution of tariff revenues explicitly is to make the rationalized tariff rate the only instrument of income redistribution.

Third, this study ignores the other government policies that are directed to macroeconomic stabilization and growth. Rausser (1982), Rausser and Foster (1990) have argued that a model that does not consider both ‘pie-expanding’ and ‘pie-sharing’ policies together is likely to yield incorrect answers to the questions of endogenous policy responses. In this sense, the model described here is incomplete. It describes only an aspect of a much larger policy game actually played in a political economy. A direct reflection of this limitation can be observed in determining the minimum expectation of the players in chapter 7.

Within the limitation of these assumptions, the approach adopted in this study is sufficiently general to provide a theoretical basis for larger models with endogenous policies. Such models for different countries can be combined to analyze issues related to regional and global cooperation.

1.5 Overview of the Study

This study is divided into nine chapters. At the beginning of each chapter the purpose and a summary of the main results of the chapter is provided. A brief overview of the study is as follows.

Chapter 2 provides a selective review of endogenous tariff literature. It summarizes the main arguments of previous work holding public-interest and/or self-interest views of the government, and draws some implications for the modelling strategy of this study.

Chapters 3 and 4 describe the economic sphere of a simple open political economy. The structure of a Ricardo-Viner type 2-sector general equilibrium model of the economic sphere is described in chapter 3, assuming that the policies of the government (tariff rates) are given exogenously. The comparative static properties of the endogenous variables are obtained, and on the basis of these results some general properties of the solution functions are deduced. These results are employed in chapter 4 to derive the properties of the second order derivatives of the real rental functions, the rent transformation frontier and its comparative static properties. The rent transformation frontier summarizes the general equilibrium effects of tariff changes on the sectoral rental incomes.
Chapter 5 describes the political process of tariff determination as a noncooperative game and studies the existence of a Nash equilibrium of the political economy. This chapter establishes that if the government behaves as a Stackelberg leader to maximize its political support and offers a pricing function to the conflicting interest groups, and the interest groups behave as Nash followers in lobbying the government in order to maximize their real rental income, then there exists at least one non trivial Nash equilibrium in the political economy. Another interesting aspect of this result is that it shows that the two strands of the existing political economy approach to endogenous tariff theory, which have either considered support maximizing behaviour of the government without considering the reactions of the lobbyists, or considered the lobbying equilibrium for a given pricing function without showing how such a function was obtained, are mutually compatible. They imply the same policy-equilibrium if the political economy admits a unique Nash equilibrium. The formal demonstration of the results derived in chapter 5, though conjectured previously by various authors, are new.

In chapter 6, this study proceeds further and allows the interest groups to search for a cooperative solution if it is individually rational to do so, and from this chapter onwards the political process of tariff determination has been viewed as a bargaining process between the two conflicting interest groups. Assuming that the disagreement payoffs are known, a priori, the necessary and sufficient condition for a unique generalized Nash solution to the bargaining problem in the tariff game has been obtained in this chapter.

Moreover, in chapter 6, the generalization of a Nash solution in the presence of asymmetric bargaining power is discussed in considerable detail. A new characterization of the generalized Nash solution is obtained in terms of the players' generalized fear of ruin. It shows that the equality of players' generalized fear of ruin is an alternate necessary and sufficient condition for the generalized Nash solution to a bargaining problem. This new characterization is important from the point of view of both the endogenous tariff theory and the Nash bargaining theory. Its importance arises because this result provides an intuitive explanation of the Nash bargaining process, and has been employed in providing intuitive explanations of the results obtained in the subsequent chapters.

Chapter 7 combines the results derived in previous chapters to obtain the final structure of the policy endogenous general equilibrium model of the political economy they describe. It also derives the comparative static behaviour of the endogenous tariff rate. The tariff rate that emerges from this model of the political economy guarantees an equilibrium in the economic sphere as well as solving the bargaining problem. This completes the construction of a policy-endogenous general equilibrium model in which the tariff rate (or the redistributive policy) is determined endogenously.
Furthermore, this chapter discusses the problems that arise in identifying the disagreement payoffs and invokes the reference point solution concept to overcome them. It is further argued that the payoffs at the point of players' minimum expectation in the tariff game can serve as a reference point in bargaining. With the concept of 'disagreement' thus made operational, the general equilibrium model of the political economy with cooperative behaviour is subjected to comparative static (counterfactual) experiments in order to see how the tariff rate would change as the exogenous variables of the model change. Throughout these experiments it is assumed that the relative bargaining powers of the players remain unaffected by the shocks. Intuitive explanations for the comparative static results are also provided.

Chapter 8 implements the policy-endogenous general equilibrium model numerically using hypothetical data sets chosen carefully to cover some extreme cases. The procedure adopted in obtaining the minimum expectation payoffs, and calibration of the model are also discussed in detail. The simulation results show that, in general, the directions of the responses of the bargained tariff rate with respect to exogenous shocks are insensitive to the location of the initial equilibrium and the point of minimum expectation. The magnitudes of responses are, of course, observed to be sensitive to these variations. Some hypotheses that follow from these comparative exercises and which appear potentially robust are stated formally. These hypotheses are then checked against the results of previous studies.

Furthermore, the policy-endogenous general equilibrium model has been simulated using one particular data set to predict the consequence of a very large growth in the stock of the specific factor in the import-competing sector. The results showed that the direction of trade in commodities reverses after the shock. The commodity that was imported before the shock will be exported and the commodity that was exported will be imported after the shock, and the commodity that was being taxed before the shock would be subsidized after the shock. This result is used to explain why developing countries tax agriculture and the developed countries subsidize it. Finally, in chapter 9 this study is concluded, and the limitations of the study are discussed.
CHAPTER 2

A SELECTIVE REVIEW OF ENDOGENOUS TARIFF LITERATURE

Introduction

This chapter provides a selective review of the literature to outline the evolution of approaches towards modelling the endogenous determination of the tariff rate. More elaborate reviews of the theoretical as well as empirical works in this area can be found in Baldwin (1982, 1984); Magee (1984); Hughes (1986); Nelson (1988); Magee, Brock and Young (1989); Hillman (1989). A summarized version can also be found in Vousden (1990, chapter 8).

Most of the previous studies have been prompted by the perplexing observation that trade taxes exist in the real world despite the general conclusion that they reduce general "welfare". Income redistribution as the sole cause of trade taxes has been dismissed by economists on the ground that, so long as the government is a "social welfare" maximizer, such taxes do not constitute the first best solutions to the distribution problem (see for example, Bhagwati, 1971).

In trying to explain why trade taxes exist, economists have come up with several possible explanations that trace the evolution of endogenous tariff theory. The analytics of these studies, though, have been geared to predict the response of the tariff rate in the face of increased import competition - that is, when there is a terms of trade gain for whatever reasons. However, one may possibly be able to utilize their models to examine the responses of the tariff rates, when the economy is shocked by any exogenous change.

Previous studies differ considerably in several aspects such as modelling strategies, time horizon, and degrees of refinements. Different reviewers may group them differently depending on the purpose of their review. For example, in a critical survey of the literature Nelson (1988) has classified most of the previous theoretical works into two groups according to their emphasis on the demand side or on the supply side of the political market. This approach treats policies arising solely out of self-interested behaviour of government. Nelson's classification is suitable if one intends to cover only those works that assume self-interested behaviour on the part of government, but it does not provide enough grounds to evaluate those works that emphasize the social welfare maximizing role of government or the works that have attempted to combine the political market approach with the welfare maximizing behaviour of government in their own perspective.
The fundamental differences among previous studies lie in their assumptions regarding the objective of the government, and there are no clear reasons to accept one or other of these assumptions. Therefore, we will first classify previous studies according to their assumptions on government motivation in taking policy decisions: whether the government is assumed to pursue some kind of "public interest" or self-interest, or a combination of both.

The studies based on the assumption that government is motivated by self-interested behaviour constitute the core of the political economy approach to endogenous tariff theory. These studies have tried to model the tariff setting process by augmenting the political market with the usual economic market. While these studies differ in assigning a more prominent role either to the demand side or to the supply side of the political market by being explicit about the underlying market fundamentals, they agree on the point that the equilibrium tariff rate should clear the political market. So, these studies are further classified into three groups according to their coverage in explicit modelling of the political market. Thus, ultimately we will classify the endogenous tariff literature in five different groups (see figure 2.5).

Our purpose in this review is to examine the current state of endogenous tariff theories and their conclusions to obtain some guidelines to make the tariff determination process endogenous in the context of a computable general equilibrium model. In the following sections we will review the endogenous tariff literature with this objective in mind. While doing so, an attempt will be made to summarize the major assumptions and their role in driving the conclusions of each study that are of direct concern to the endogenous tariff literature.

This chapter is divided into five sections. The first section reviews the literature that adopts a public-interest view. The second section reviews the literature that adopts a self-interest view. Section three reviews the literature that adopts a hybrid approach to tariff determination. In particular, this literature maintains that the government responds to political pressure as well as to welfare norms. Section four reviews some works that have addressed the issues of the choice of a specific protective instrument. Finally, section five summarizes the implications to the modelling strategy of the present study.
Figure 2.1

The Family Tree of Endogenous Tariff Literature

Public-interest approach

Self-interest approach (political market)

Demand determined

Supply determined

Market approach

Conservative social welfare maximization: insurance theories

Lobbying contests: Nash-Cournot solution

Bureaucratic interest

Support maximizing politicians

Support maximizing politicians and contesting lobbies

A Combination of welfare maximization and response to private lobbying

A combination of both approaches
2.1 The Public Interest Approach

In this section Corden's concept of the conservative social welfare function (Corden, 1974) will be reviewed first, which will be followed by a review of the insurance theories of tariff determination (Hillman, 1977; Cassing, 1980; Baldwin, 1982; Eaton and Grossman, 1985; and Cassing, Hillman and Long, 1986; Staiger and Tabellini, 1987). All of these works maintain that the government is a social welfare maximizer. One may also include Findlay and Wellisz (1983 and 1984) in the class of "public interest" literature, as they assume that the "prince" is driven to justify his rule by maximizing the output of the composite public good financed by the tariff revenue.

2.1.1 The Conservative Social Welfare Function

Corden (1974) introduced the idea of conservative social welfare function (CSWF), describing it as being particularly helpful for understanding actual trade policies of many countries:

Put in its simplest form it includes the following income distribution target: any significant absolute reductions in real incomes of any significant section of the community should be avoided. This is not quite the same as setting up the existing income distribution as the best, but comes close to it, and so can indeed be described as 'conservative'. In terms of welfare weights, increases in income are given relatively low weights and decreases very high weights (p. 107).

According to Corden, a CSWF expresses the following four ideas.

1. **(Fairness).** Unless there are good reasons or it is unavoidable, it is unfair to allow anyone's real income to be reduced significantly.

2. **(Social Insurance).** Insofar as people are risk averters, everyone's real income is increased when it is known that a government will generally intervene to prevent sudden or large and unexpected income losses. The CSWF is part of the social insurance system.

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1 Yunker (1989) has provided some 'empirical' support to the social welfare maximizing approach to policy determination. Using a general equilibrium model of the US Yunker evaluated different forms of social welfare functions over a range of income tax rates to see whether the actual income tax rate in the US confirms to some welfare maximizing notion or not. He found that the income tax rates that maximize Benthamite (sum of utilities) or Nash (product of utilities) social welfare function are very close to the actual average income tax rate in the US. So, he concluded that the "empirical result is consistent with society (or its government) unconsciously maximizing a social welfare function - suggesting that the concept of social welfare maximization may indeed possess positive as well as normative content (p.111)."
3. **(Social Peace).** Social peace requires that no significant group's income shall fall if that of others is rising. The reason is that social peace might be regarded as a social good in itself or as a basis for political stability and hence perhaps economic development, and even if social peace does not depend on the maintenance of the incomes of the major classes in the community, the survival of the government may.

4. **(Compensation).** If a policy is directed to a certain target, such as protection of an industry or improving the balance of payments, most governments want to minimize the adverse by-product effects on sectional incomes so as not to be involved in political battles incidental to their main purpose.

The concept of the CSWF, as expressed above, is rather informal. It is also not clear whether the four ideas are sufficient to identify a unique social welfare function or not. It is, however, clear that the CSWF represents a positive concept of social welfare.

It is easy to argue that so far as redistributive policies are concerned idea 1 (fairness) implies idea 3 (social peace), and idea 2 (social insurance) implies idea 4 (compensation). Because, if a government intends to avoid a fall in income of any community in general, then it will also avoid a fall in income of one community when that of others is rising. Similarly, if a government's policies are meant to provide social insurance, then it will compensate the loss in income of any community arising out of the adverse by-product effects of its own policies. Hence, in essence we can regard CSWF expressing two ideas or principles: fairness, and social insurance.

However, fairness is a subjective concept. Corden views that any significant absolute reduction in real income of any significant section of the community is unfair. Unless a criterion for significance is explicitly specified, the idea nevertheless remains vague. Moreover, it has not been explained why a government should be concerned with the fairness of its policies at all. In other words, one can always enquire into the motivation that induces a government to remain fair. Moreover, the idea of fairness can be viewed as embodied into the idea of social insurance, because the losers will be compensated unless everyone is a loser. Probably because of this the idea of social insurance has been the one most frequently referred to in the literature.

**A first Best argument for Tariffs**

Corden has made the best use of his CSWF in producing a first best argument for tariffs. He writes

...Yet one can base a first best argument for tariffs on the CWSF. Essentially it depends on the costs or difficulties of obtaining information.
Suppose import prices of particular products fall owing to foreign suppliers becoming more competitive for one reason or another. This will redistribute incomes against producers of import competing goods in favour of consumers or using industries. It may be difficult or even impossible to bring about a redistribution back to the original situation through taxes and subsidies. Quite apart from the institutional difficulties, and collection and disbursement costs, there is the crucial information problem: precisely who gained and lost, and by how much? This is particularly serious if the effects are sudden.

There is only one way of reversing or avoiding the income distribution effects precisely, and that is to impose a tariff which will keep the prices facing domestic consumers and producers exactly where they were before import prices fell. (p. 109-10).

Corden’s first best argument for tariff is somewhat unusual in trade theory. This prompted many others to examine the insurance aspect of the commercial policy more formally.

### 2.1.2 The Insurance Theories

Eaton and Grossman (1985) explicitly focused on the role of commercial policy in providing insurance when the insurance market is incomplete. They started with the assumption that there is some uncertainty regarding the international terms of trade. They further assumed that in an equilibrium prior to the terms of trade shock, all individuals are identical in their taste and factor endowments, and each individual owns two factors which earn at competitive market rates. Furthermore, they have also assumed that each individual must engage his capital, which is sector specific in the short-run, entirely in one activity. Finally they assumed that commercial policy is the only policy available to a social welfare (Benthamite) maximizing government to allocate the risk of terms of trade change.

With these assumptions Eaton and Grossman have shown that tariff intervention favouring the import competing sector can indeed raise social welfare. Moreover, tariffs may dominate production subsidies or taxes as a means of providing such insurance. Thus, Eaton and Grossman’s result not only explains the existence of government intervention on trade but also specifies the exact form of the commercial policy to be adopted.

Cassing (1980), and Cassing, Hillman and Long (1986) have taken a slightly different approach in that these two studies do not explicitly assume that the government intends to maximize a social welfare function of any sort. In the context of a specific factor model of an economy with 2 sectors, where each individual owns one factor only, Cassing, Hillman and Long (1986) have neatly shown, without assuming a social welfare function, that if the insurance market is incomplete, then an ex-ante
commitment to a stable domestic price is individually preferred by all agents provided that agents are sufficiently risk averse.

Thus, Cassing, Hillman and Long's study also suggests that if the insurance market is incomplete, then some form of market intervention, for example tariff intervention, can arise through a general consensus of all agents. One can, however, argue that if everybody prefers commercial policies designed to stabilize domestic prices to an uncertain free trade price, then a price stabilizing policy outcome is definitely welfare improving and therefore Cassing, Hillman and Long's result is consistent with the welfare maximizing hypothesis.

It is important, however, to note one difference between Cassing, Hillman and Long's approach and that of other insurance theorists who assume a benevolent government with a social welfare function. A welfare maximizing government may, at times, choose a policy that is not preferred by some individuals in the society. The only requirement for a policy to be selected by a social welfare maximizing government is that it should raise the aggregate welfare of the society irrespective of the levels of individual welfare. But, in Cassing, Hillman and Long's result a tariff arises as a result of Pareto dominance - that everybody's welfare rises with a government's commitment on terms of trade contingent tariff policy. Thus, in this sense, Cassing, Hillman and Long's result is more powerful than that of other studies concerned with the insurance problem.

All of the above mentioned insurance theories have justified some level of terms-of-trade-contingent tariff protection before the terms of trade changes. This means that people would find their expected welfare increased if the government simply commits to a protective policy in the event that terms of trade change. However, once a government makes some form of commitment it may affect ex-ante allocation of resources and all incentive structures may change accordingly. With this new possibility, how would the government, the producers and the factor owners behave after the shock is actually observed? Once the shock is observed and the uncertainty is resolved, will the social welfare maximizing government behave as expected or committed? Would the same policies still remain optimal? This leads us to the problem of the time-consistency of the optimal policies.

To answer these questions Staiger and Tabellini (1987) took the case of a fall in the price of the home importable and posed the problem in the following way. Suppose that the timing of the decisions after the shock is observed is that, either: (a) first workers relocate and then a tariff is imposed; or (b) the labour relocation and the tariff decisions are made simultaneously.
Staiger and Tabellini have further assumed that (i) the capital incomes and the tariff revenue are distributed to the workers in proportion to their wage incomes, (ii) all workers have identical homothetic utility function, (iii) all workers are endowed with equal amount of labour (iv) there are positive costs of movement for labour, in particular labour will lose a part of its productivity if it has to relocate across the sectors.

With these assumptions they have proved the following three propositions:

1. The optimal tariff policy is either free trade or the imposition of a sufficiently high tariff rate that prevents any sectoral relocation of labour from taking place (p. 831).

2. When free trade is the optimal policy, it is not time-consistent. The time-consistent policy involves a socially excessive level of protection (p. 834).

3. In the time-consistent equilibrium, the production subsidy (rate) is strictly higher than the tariff (rate). If the social gains from redistribution are small enough, the tariff welfare dominates the subsidy (p. 836).

These results are interesting. Therefore, it is worthwhile to see whether they stand when some of the assumptions are relaxed. In particular, we are interested in the validity of the results when labour can move costlessly.

Staiger and Tabellini have argued that if the cost associated with the reallocation of labour is zero, that is if there is no loss in labour productivity when labour moves from one industry to the other, then the time-consistent tariff rate is zero (see their footnote 13). It, therefore, follows that if assumption (iv) is violated then free trade results as the time-consistent policy.

The result that free trade is time-consistent, however, depends critically on the assumption that workers receive capital income and tariff revenue in proportion to their wage income. This assumption implies that initially national income is equally distributed among workers and will remain so after the shock if labour can move costlessly. But, if capital incomes and tariff revenue are not distributed equally before and after the shock because either the initial distribution rule implies an inequality or the rule itself changes with the shock, then the equilibrium incomes of workers after the price change will not necessarily remain equalized. In this case, a social welfare maximizing government may still obtain a non-zero value for its optimal and time-consistent intervention instrument and free trade may not be obtained as a time-consistent policy. Hence, even if labour movement is costless, Staiger and Tabellini’s results may remain valid provided that capital income is not distributed in proportion to wage income.
Hence, what Staiger and Tabellini's work suggest is that if movement of labour is costless, and income is equally distributed, then the optimal and the time-consistent policy is free trade. But, either if there are costs in the inter-sectoral movement of labour, and income is equally distributed among the workers, or if there are no costs in the inter-sectoral movement of labour but the capital income is not equally distributed among the workers, then the only optimal and time-consistent policy response to a terms of trade shock is an offsetting change in the tariff rate so that labour does not move across the industries as the terms of trade change.

Thus the insurance school of endogenous tariff, which views commercial policy as a means of sharing risks associated with terms of trade changes, has formally shown that if the insurance market is incomplete, then free trade is sub-optimal and/or time-inconsistent. An optimal and time-consistent policy involves a compensating change in the tariff rate. In particular, when movement of labour is costless and the distribution of capital income is unequal, time-consistency of the optimal policy requires that the tariff rate should change to offset any change in the terms of trade. Surprisingly enough this conclusion, though derived from maximizing a different (Benthamite) social welfare function comes close to what Corden had deduced on the basis of his conservative social welfare function.

The predictions of the public interest approach to the endogenous tariff theory regarding the endogenous behaviour of the tariffs can be summarized as follows.

1. (insurance). Given that all people are risk averters, everyone will be better off if the risks associated with the international terms of trade changes are insured by the government's tariff policy. So, tariffs will change to offset the effects on the domestic relative prices of any change in the international terms of trade. This conclusion is shared by both Corden (1974) and various other authors subscribing to the insurance school.2

2. (fairness). In general, if any change occurs in the domestic economy that causes the income of one sector to grow, say due to technological progress or to capital accumulation, causing the income of the other sector to decline, then the theory that maximizes CSWF predicts that the policy (tariff rate) will change to protect the losing sector.

---

2 See Vousden (1990: 73-4) for arguments against the use of tariffs as a form of social insurance. He has argued that this form of insurance is not free from the problems of moral hazard and adverse selection, and therefore it will impose another cost on to the society. This raises a question that whether the welfare maximizing government will really choose to provide this kind of insurance at all. For rigorous demonstrations of the result see also Dixit (1987a, 1987b, 1989a, 1989b).
There are two major problems with this approach. First, the social welfare approach lacks explanation on how a CSWF or any other social welfare function is derived (existence) and how one can identify the correct form of the CSWF (uniqueness). Second, even if there exists a unique social welfare function one has to show that the government will actually maximize it while choosing policies before this approach can command some positive value. To claim that a government is a social welfare maximizer we require one more assumption that the government is a benevolent agent. Its preference is to see others (citizens) happier.

Once the government is assumed as a benevolent agent we run into a logical problem. Since, in general, a country is governed by a group of elected or selected representatives, who, when not in government, are assumed to be self-interested, then how can one expect the same people to become benevolent once they are elected or selected to run the government?

Both of these problems do not arise if, instead of assuming that there exists a social welfare function, which is maximized by the benevolent government, one assumes that the government will choose policies so as to maximize its chance of remaining in power or maximize side payments or bribes. This shift in the fundamental assumption takes us to the self-interest theory of tariff formation.

2.2 The Self-interest Approach: Political Economy of Tariff Determination

The authors who assume a self-interested behaviour on the part of the government view policies as simply another commodity which happens to be traded in the political market. As Peltzman (1976: 212) puts it

The essential commodity being transacted in the political market is a transfer of wealth, with constituents on the demand side and their political representatives on the supply side. Viewed in this way, the market here, as elsewhere, will distribute more of the good to those whose effective demand is the highest.

We will review the literature that views policies as the commodities of the political market in three different groups depending on whether they view the quantity of the policies as determined by the supplier or the demanders, or by the market as a whole.

2.2.1 The Supply Side Literature:
Stigler-Peltzman Model and Its Refinements

Formal politico-economic approach to tariff determination begins with Peltzman's (1976) formalization of Stigler's (1971) theory of economic regulation. The theory is based on the assumption that the government, who supplies the regulation, is a
support maximizer. More votes are always desirable because this implies a greater security of tenure, more logrolling possibilities and so on.

The group which benefits from the regulation pays the government with "votes" and "dollars" whereas the losers will reduce their support or increase opposition to the government. Not all gainers and losers are fully informed and so there is some scope for manipulating votes by campaigning, lobbying and so on. Therefore, the "dollar" paid by the beneficiaries is productive to the government.

The Stigler-Peltzman model was initially developed to solve the problem of the regulator confronting a choice of the numerical size of the beneficiary group and was cast in a partial equilibrium setting. The generality of this approach makes it possible to represent its essential features in of a general equilibrium context and to address other redistributive policy issues as well. For example, it is possible to consider the transfers of gains (rents) in general equilibrium from one sector to the other by means of a regulation - the rationalized tariff rate.

Assuming that the owners of the specific factors in the two sector are the two active interest groups we can write the support function of the government as

$$M = M(\Pi_1(P_1), \Pi_2(P_2))$$

where, the support $M$ is increasing and concave in $\Pi_i$ - the real rental income of sector $i=1, 2$; and $P_i$ is the relative price of (the import competing) good $i$ such that real rental incomes $\Pi_1$ and $\Pi_2$ increase and decrease with $P_i$, respectively.

The maximization of the support function by choosing a tariff rate neatly summarizes the choice problem faced by the government. The government may receive an increasing support from sector 1 and a decreasing support from sector 2 by increasing the tariff rate, since $P_i$ increases with the tariff rate.

The first order necessary condition for a maximum support can be written as

$$M_1 \frac{d\Pi_1}{dP_1} + M_2 \frac{d\Pi_2}{dP_2} = 0,$$

and the second order sufficient condition can be written as

$$\frac{d^2M}{dP_1^2} = \left( M_{11} \frac{d\Pi_1}{dP_1} + M_{12} \frac{d\Pi_2}{dP_1} \right) \frac{d\Pi_1}{dP_1} + M_1 \frac{d^2\Pi_1}{dP_1^2} +$$

$$\left( M_{21} \frac{d\Pi_1}{dP_1} + M_{22} \frac{d\Pi_2}{dP_1} \right) \frac{d\Pi_2}{dP_1} + M_2 \frac{d^2\Pi_2}{dP_1^2} < 0.$$
where the subscripts in $M$ denote partial derivatives of the support function.

Assume that the political support function and the real rental functions are sufficiently well behaved so as to admit a unique support maximizing solution. The unique value of $P_1$ that satisfies the above condition will solve the support maximizing problem of the government for a given world price, $P_1^*$. The relation between the world and the domestic relative price then determines the politically equilibrium level of the tariff rate.

Since the economic equilibrium depends on the values of the exogenous variables, the equilibrium rents will respond to changes in the exogenous variables as well. However, for each configuration of the exogenous variables, the equilibrium level of the policy variable has to satisfy the condition of political equilibrium - support maximization.

Differentiating the first order condition for equilibrium totally we get

$$
\frac{d^2 M}{dP_1^2} \cdot \frac{dP_1}{dP^*_1} = 0.
$$

Since $\frac{d^2 M}{dP_1^2} < 0$ by the second order condition, therefore it follows that

$$
\frac{dP_1}{dP^*_1} = 0.
$$

This result suggests that a support maximizing government will always adjust tariff rates so as to offset the effect of the terms of trade changes on the domestic relative price.

Thus, the prediction of the Stigler-Peltzman model, which is based on the self-interested behaviour of government, is the same as that of insurance theory or that of Corden (1974). The reason is simple. Essentially, one can view the political support function as the welfare function being maximized by the government choosing a redistributive policy, which benefits one group and harms the other. The two functions may differ in parameters that are irrelevant in determining the optimal choice of the government.\(^3\)

Hillman (1982), however, contested this result. He argued that political support from each interest group depends not on the levels of rents but on the extent to which they have gained over their free trade levels. His main argument in this regard is that

\[^3\] See also Baldwin (1987) for a similar result.
agents are responsive in their political support only to gains and losses that are due to
the authorities acting to cause the domestic price to deviate from the world price via
tariff intervention. Political support is not affected by changes in variables that are not
due to the authorities' actions. Hence, Hillman (1982) respecified the political support
function so that the government solves the following problem
\[
\max_{\bar{P}} M = M(\Pi_1(P_1) - \Pi_1(P^*_1), \Pi_2(P_1) - \Pi_2(P^*_1)).
\]

By following the steps as in the Stigler-Peltzman model it can be shown that, for
given world relative price \(P^*_1\), the first and second order conditions for the maximum of
the support function are exactly the same as those in the Stigler-Peltzman model. To see
the equilibrium response of the domestic relative price with respect to a small change in
the international relative price we differentiate the first order condition totally and
obtain
\[
d^2M \cdot dP^* = M_{11} \cdot \frac{d\Pi_1^*}{dP_1^*} \cdot \frac{d\Pi_1}{dP_1} + M_{12} \cdot \frac{d\Pi_1^*}{dP_2^*} \cdot \frac{d\Pi_1}{dP_1} + M_{21} \cdot \frac{d\Pi_1^*}{dP_1^*} \cdot \frac{d\Pi_2}{dP_1} + M_{22} \cdot \frac{d\Pi_2^*}{dP_2^*} \cdot \frac{d\Pi_2}{dP_1}.
\]

Given the above properties of the real rental functions and the concavity of the political
support function we can deduce, by employing the second order condition, that\(^4\)
\[
\frac{dP_1}{dP^*} \geq 0
\]
provided \(M_{12} \geq 0\) and \(M_{21} \geq 0\). However, sign ambiguity results if there is envy effect -
that is if \(M_{12} < 0\) and \(M_{21} < 0\).

Hence, in the absence of envy effects, if the price of home importable in the
world market falls, then its price in the home market will also necessarily fall. Thus,
Hillman came to the conclusion that 'a declining industry will continue to decline.'
Despite the existence of protectionist motives the decline of a declining sector can not
be arrested by a politically motivated (support maximizing) government.

If a support maximizing government does not provide a complete protection
against import competition then the rate of return in the declining industry will fall
below that which can be obtained in the other sector. When sufficient time is allowed
for adjustment a part of the capital stock will exit from the declining sector. This will
further lower the employment level in the declining industry. By assuming that the
capacity to provide political support to the government also depends on the size of its
labour employment, Cassing and Hillman (1986) have shown further that the protection

\(^4\) See also Long and Vousden (1991) for this result.
to the declining industry will also continue to decline and such industries will eventually reach a stage of catastrophic collapse.

Hillman's (1982) argument that led to the hypothesis that individuals base their political supports on the divergence of rents from the free trade levels was questioned by Long and Vousden (1991). In particular, they raised two pertinent issues. First, if individuals or groups only praise or blame the government for changes directly attributable to the government's actions, then any changes in the tariff revenue and its distribution associated with a fall in the world price should be included in the political calculus. Second, it may be more reasonable to suppose that people care about changes in welfare relative to the situation before the fall in the world price of the home importable good. Moreover, because Hillman's conclusion was based on a partial equilibrium model its validity in the context of general equilibrium was not clear.

Long and Vousden (1991) studied the policy choice of a support maximizing government in the general equilibrium of a 2-sector specific factor model of a small open economy. In their model the government is assumed to maximize the aggregate of the supports from the three different factor groups given by the following function:

$$M(P, P^*) = \sum a_iV^i(P, Y_i)$$

where each $a_i$ is a positive constant, which they assumed to have been determined by previous lobbying contests, $V^i$ is the indirect utility function, and $Y_i$ is the total income of the factor group $i$, which includes respective factor income and its share of the tariff revenue, which is distributed parameterically among the three factor groups.

Under reasonable assumptions they have shown that if all tariff revenue is given to either the mobile factor or the specific factor in the protected sector, a fall in the world price will lead to a fall in the corresponding domestic price. On the other hand, if the revenue is given entirely to the specific factor in the unprotected sector, or if each factor receives the recycled tariff revenue in the same proportion as it receives factor income, then the domestic price may actually rise or fall as the world price falls depending on the relative risk aversion of the three factor groups. However, they have concluded that in the absence of any good reason why the factor groups differ in their relative risk aversion 'the model would appear to offer good support for the proposition that a declining industry will continue to decline.' (p. 100)

In summary, the supply side literature of the political market is still inconclusive in tracking the endogenous behaviour of the tariff rate. It seems that the authors have agreed on the point that the domestic and world relative prices will not move in the
opposite direction. It is not clear whether the domestic relative price will move at the same rate as in the world market or not.

This ambiguity results simply because different authors assumed different reference points to which the interest groups would compare their current outcome to determine whether they need to adjust their support to the government or not. The Stigler-Peltzman model uses the origin, Hillman preferred the free trade point and Long and Vousden employed the recent past as the reference point. One possible solution to this problem could be to model explicitly the demand side of the political market, or the behaviours of the interest groups and obtain the reference point empirically.

In terms of modelling difficulties, the assumption that the government is a support maximizer and that the government is a social welfare maximizer are not very different. There are difficulties in defining an appropriate political support function as well as there are problems in defining a social welfare function. However, the support maximizing assumption has the advantage that it is internally consistent. It maintains that all agents are self-interested maximizers.

2.2.2 The Demand Side Literature: Nash Equilibrium in a Lobbying Economy

Findlay and Wellisz (1982) were the first to study the question of political equilibrium in conjunction with a general economic equilibrium of a political economy. Although Findlay and Wellisz were mainly interested in demonstrating Tullock's (1967) thesis that the welfare cost of tariffs determined by lobbying contests is much higher than the conventional dead weight loss, their model of the political process of tariff formation as a noncooperative game between interest groups became more important than their welfare result.

Findlay and Wellisz also used a 2-sector, specific-factor model to describe the general equilibrium of a small open economy producing two goods - food and manufactures - with land and capital as the respective specific factors and labour as the mobile factor. Under free trade the country is assumed to have comparative advantage in the production of manufactures. In this context Findlay and Wellisz conjectured, the landed interest would try to introduce a tariff on food at a prohibitive level if they could get away with it, whereas the manufacturing interest would try to preserve free trade. Depending upon the relative strengths and commitments of the two sides it is plausible to think that some tariff between zero and the prohibitive level will emerge. The social value of the resources used up by both sides in this struggle would constitute a welfare cost over and above the familiar deadweight loss associated with whatever tariff level emerges from the political process. (p. 225)
In their model, they represented the supply side of the political market in reduced form by a tariff (supply) function. Specifically, they assumed that a tariff level is determined as a stable function of the resources committed to the political process by each of the two interest groups such that the tariff function is increasing and concave in the resources used in lobbying by the landed interests and decreasing and convex in the resources used in lobbying by the capitalists. They did not specify what sort of government behaviour is implied by this supply function, however.

Given this supply side, the demand side of the political market is described by a noncooperative game between the two interest groups. The landed interest is assumed to maximize its rent, measured in units of food, over and above the free trade level by choosing its lobbying levels, and the capitalists were assumed to maximize their rents, measured in units of manufactures, over and above the autarkic level by choosing their levels of lobbying. Findlay and Wellisz then attempted to solve the game for the lobbying levels at its Nash equilibrium. Once this could be obtained, the tariff function would then yield the equilibrium tariff rate.

However, Findlay and Wellisz were unable to show the existence of a Nash equilibrium in the lobbying game. They simply assumed a unique, stable and interior Nash equilibrium to make the point that the welfare costs of the endogenous tariff determined through the political process exceeds the conventional deadweight loss of a tariff rate. For example, they write

Unfortunately, however, each of these crosspartials is the sum of a long succession of individual terms of conflicting or indeterminate signs. We therefore simply assume that, whatever their slope, the reaction functions have a unique and “stable” intersection defining a Cournot-Nash equilibrium in the “political” sphere.... (p. 229).

In two other papers, Findlay and Wellisz (1983 and 1984) illustrated endogenous determination of the tariff rate in two different regimes - the democratic pluralistic case and the bureaucratic authoritarian case. In the former case they used the same ‘tariff formation function’ as in their (1982) paper to describe the political process. Their description of the game remained relatively informal. Consequently, they had to make appropriate assumptions on the nature of the reaction curves to obtain an equilibrium and thus their implications.

In the bureaucratic case, however, Findlay and Wellisz have outlined the endogenous determination of the tariff rate by assuming that ‘the prince’ is driven to justify his rule by maximizing the 'output' of his regime, considered as a composite public good that requires real resources for its production. The resources have to be acquired from the private sector by means of taxation (Findlay and Wellisz, 1983: 476). This model is more or less equivalent to assuming that the government is a maximizer
of the tariff revenue which is paid in units of labour. It does not allow interest groups to influence the decision taken by the "prince" and therefore, comes more closer to the "public interest" theory of tariff determination.

Similar attempts to analyze the political process of tariff determination as a noncooperative game between the specific factor owners were also made by Wellisz and Wilson (1986) and Hall and Nelson (1992). Wellisz and Wilson have been more explicit in specifying the structure of the 'tariff function'. However, both of these studies failed to establish the existence of a Nash equilibrium formally. For example Wellisz and Wilson (1986: 370) write

Although it is easy to construct examples where this equilibrium exists, a general existence proof does not appear possible because each group's utility-maximizing lobbying effort need not always be a continuous function of the other group's lobbying effort. On the other hand there may exist more than one equilibrium.

Similarly, Hall and Nelson (1992: 72) write

... a long succession of terms involving second derivatives whose signs are not derivable without further assumptions on the model. Whether the reaction functions are positively or negatively sloped we will simply assume that there is a unique, stable solution between the three groups.

The problem of the existence of a Nash equilibrium in a lobbying game between different interest groups was further studied by Coggins, et al., (1991), in the context of a small open exchange economy. Their model had two persons each endowed with a single tradeable commodity. The government's role was to announce the domestic price of the two goods and to take responsibility for clearing the markets by trading in the world market at internationally given prices. While setting domestic prices, the government, in their model, responded to lobbying expenditures of the two agents, which created an incentive to each of the agents to spend resources in lobbying for higher relative price for their commodity.

The supply side of the political market in their model is also summarized by a 'pricing function' announced by the government, which is assumed to be common knowledge. Their pricing function is similar to the tariff formation function of previous studies in that it satisfied similar assumptions - differentiable in lobby expenditures, yields positive but diminishing returns to lobbying expenditures, and sets some form of bounds on the tariff rate or the domestic relative prices - and it transformed the lobbying expenditures of the individuals with conflicting interest into a relative price of the commodities.

With one additional assumption, which they call own good bias, that each person consumes more of the good with which he is endowed than of the other good,
Coggins, et al. (1991), have shown the existence of a non-trivial Nash equilibrium in a lobbying game in an exchange economy. The assumption of own good bias made the indirect utility functions quasiconcave in respective lobby expenditures, which in turn allowed Coggins, et al. to apply Debreu's existence theorem. But this assumption also imposed a restriction on the nature of the utility functions of the players. The utility function with own good bias implies that the indifference curves will have either horizontal segments to the right of the 45 degree line if the individual is endowed with the y-commodity or vertical segments to the left of the 45 degree line if the individual is endowed with the x-commodity.\(^5\) Therefore, as they have indicated, the existence result does not hold generally for any pair of arbitrary utility functions. Nevertheless, their study has made a definite contribution to the literature.

In summary, the authors who have tried to explain the endogenous determination of the tariff rate in the context of conflict resolution have had difficulties in simply proving the existence of a Nash equilibrium in the lobbying game. Moreover, these studies have abstracted from the political market by assuming that there exists a 'tariff formation function' or a 'pricing function' that satisfies certain properties. They have neither shown nor explained what sort of government behaviour is consistent with these assumptions.\(^6\) This approach has remained limited to analysis of the demand side of the political market. To incorporate this approach into a CGE model, at this stage, does not seem straightforward.

2.2.3 The Political Market

Mayer's (1984) approach to the study of endogenous tariff formation is slightly different. He replaced the tariff formation function of Findlay and Wellisz (1982, 1983) by the majority voting rule. Mayer assumed that the interest groups do not lobby for favourable tariff policy, rather they vote for it. In his framework the conflict of interest is resolved by majority voting.

Mayer's model, in contrast to other models, allows a person to own more than one factor of production. By assuming that factor ownership patterns differ among people he demonstrated, both under Heckscher-Ohlin and the specific factor model, that each factor owner has an optimal tariff rate whose value is uniquely related to the

\(^5\) A question of noncommensurability of units of different commodities may arise, which Coggins et al. claim to have solved by their price normalization rule. That is, the units of commodities were chosen so that the prices of the two commodities add up to unity.

\(^6\) It is, however, claimed in Coggins, et al. (1991), that the pricing function 'could be considered a component of a more general model in which the government chooses the pricing function as a Stackelberg leader, with individuals in the economy reacting to the pricing function in an associated lobbying game. Any number of postulates regarding the government behaviour could be consistent with this approach.' (p. 535)
individual's factor ownership. However, in the specific factor model it was necessary to assume that each person possesses at most one type of specific factor in addition to one unit of the mobile factor.

In the special case of majority voting with no voting costs, he showed that it is the median factor owner's optimal tariff rate that will be chosen by the majority. Any shock that changes the position of the median voter would alter the equilibrium tariff rate. By introducing a positive cost of voting, which he assumed to be the same for every voter, Mayer showed that a small minority of big potential gainers had a far greater chance of gaining protection. The large number of losers will find it rational not to vote because the cost of voting exceeds the loss due to increased tariff, thus confirming the hypothesis rigorously of the interest group theories without introducing the lobbying and self-interested behaviour of the government. In fact, in Mayer's model, the government does not play any active role at all. The decisions are made by the voters themselves.

Magee, Brock and Young (1989) - henceforth MBY - have cited previous studies that provide powerful arguments, particularly relevant when redistributive issues are involved, against the majority voting rule. The major argument against the majority rule is the absence of single peaked preferences. Mayer was aware of this limitation, since he acknowledged this problem somewhat indirectly in his assumption that voting takes place on a single issue - that is, in each case only one industry will be trying to gain tariff protection. When all industries try to get protection there may or may not exist any equilibrium under the majority voting rule. This has been regarded as the major conceptual flaw in Mayer's approach (Magee, Brock and Young, 1989: 73)

In a major study MBY, on the other hand, used a probabilistic voting model, which was developed previously by Brock and Magee (1978) to analyze the industry structure of protection. In their model of the political market with two lobbies and two political parties each party aimed to maximize its chance of being elected by offering either a tariff or a subsidy rate, and the lobbies were contributing to the parties for favourable policies to maximize the income of their members. Voters were rationally ignorant. Their choice, however, could be affected positively by the resources spent by the parties in voters' education and negatively by the distortionary effects of the policies.

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7 Let us consider \( n \) proposals, and arrange them in any arbitrary order. If there exists any one voter whose preference over these \( n \) proposals first declines and then rises, then his preference over these \( n \) alternatives is not single peaked. Majority voting rule can produce an equilibrium outcome if voters' preferences are all single peaked. Note that voters' preferences on a single issue are always single peaked. For details see Mueller (1979: 40-44) and references therein.

Each party can obtain more resources to spend on voters' education from the self-interested lobbies of the two factors, who prefer either tariff protection or export subsidy, by deviating more from free trade, but this will also increase the deadweight loss associated with their policy position and antagonise the general voters.

A hierarchy in terms of the possession of relevant political and economic information has been assumed. Political parties, who have all the information regarding the reactions of their lobbies and the voters, were at the top level. Therefore the political parties were assumed to employ Stackelberg strategies against their respective lobbies and the general voters, whereas the parties were assumed to behave as Nash players against each other. The respective lobbies were assumed to have less information than their parties, but better informed than the general voters. So, the lobbies were assumed to follow the parties, but employ Stackelberg strategies against the general voters.

Thus, in MBY's model of the political market each party included its respective lobby's and voters' reactions in its calculus, and each of the lobbies included voters' reactions in their calculus. The parties maximize their probability of election by taking policy positions, lobbies maximize their income by choosing their lobbying contribution, and the general voters, though can be influenced by political campaigns of the lobbies and the parties, generally dislike deviations from free trade. MBY have thus defined a very interesting noncooperative game involving two political parties, two lobbies and the rationally ignorant voters. Their study did not address the question of the existence of an equilibrium in this game. They assumed it.

MBY developed their endogenous tariff theory in general equilibrium (Part II) using a 2 sector Heckscher-Ohlin type general equilibrium model, which assumes perfect factor mobility. Obviously, in such models the conflict of interest arises between labour and capital, not between the industries. Therefore, their politico-economic model yields a long-run behaviour of the tariff rate that is determined by the political behaviour of the agents holding a long-run view on the economic sphere. Such a model is less suited in explaining the short-run behaviour (or immediate response) of the tariff rates.

Moreover, MBY have studied the endogenous behaviour of the tariff rate under either the Leontief or the Cobb-Douglas production technologies only, and the probabilistic voting model was represented by a logit election function. To a certain extent, their conclusions are shaped by these structures as well. Nevertheless, they have obtained many interesting results some of which are as follows.

1. With a Leontief production function, they obtained a 'compensation effect' in policy changes with respect to exogenous shocks. In particular, the following two
results are very interesting in the sense that these results are similar to the predictions of the CSWF or that of the insurance theories.

(i) In general, an increase in a country's terms of trade causes the equilibrium level of protection to rise and the export subsidy to fall. Terms of trade are the ratio of the price of a country's exports over that of its imports. (p. 157)

(ii) Endogenous politics is progressive with respect to exogenous changes in prices and technology. When a factor's income falls, ... the injured factor lobbies harder, and the political system provides policies that generate a partial offset to the initial decline in income. (p. 18).

2. In a simulation of their model with Leontief production functions, MBY observed that multiple equilibria were pervasive. The equilibria contained cases of either 'Prisoner's Dilemma' equilibrium, where each factor was worse off compared with free trade or 'Dominant Player' equilibrium, where one factor gained and the other lost compared with the free trade outcome. The actual outcome was dependant on the factor endowment ratio of the country. Extreme ratio implied a Dominant Player solution and intermediate ratio implied a Prisoner's Dilemma. For example, higher capital-labour ratio implies, in their model, that capital wins and labour loses and vice versa (p. 171).

3. It is then natural to ask why the lobby groups do not cooperate if they get trapped into Prisoner's Dilemma outcomes? MBY's simulation showed that the cooperation that can make both lobbies better off was not feasible because of enforcement problems. Note that their model does not contain a government that benefits from enforcing cooperative agreements. The political parties, who are the potential rulers of the country, have an incentive to make the game more noncooperative.

When Cobb-Douglas production and utility functions were used in defining the general equilibrium structure, MBY obtained a change in some of the results.

First, MBY found that domestic politics is independent of international price. That is, even though the terms of trade change had distributive effects, it had no effect on the expenditures on lobbying, probabilities of election and the policies offered by the parties in equilibrium. This is quite different from the previous result.

Second, an increase in the endowment of a factor had the effect of generating policies favouring the factor itself. The main reason is simple: an increase in the endowment of a factor, say capital, will increase the resource base of the capital lobby which then spends more on lobbying for the pro-capital party; this, in turn, increases the success rate of the pro-capital party as well as improves the policy position of the pro-capital party. Both considerations increase the relative price of the capital intensive
good, increase the return to capital, and lower wages in their expectational equilibrium. This is what they call increasing return on politics. This result is maintained irrespective to a shift in production technologies.

Third, and more surprising, they observed that if the factors were moderately risk averse (with relative coefficient of risk aversion equal to unity) and the factor intensities (measured by the ratio of physical units of the factors) in the two sectors move together, then an economic black hole could result. That is, all of the economy's factor endowment could be exhausted in predatory lobbying while the equilibrium tariff rate may actually fall (p. 223).

The most important aspect of MBY's approach is that it describes both the supply as well as the demand side of the political market, which has been appended to a general equilibrium model of the economic sphere of the political economy. This is apparently the only study which has analysed the endogenous determination of the tariff rate covering both political and economic markets in detail and in which self-interested behaviour on the part of all agents in the model is maintained.

MBY's model of the political market contains a special assumption that deserves scrutiny. It is the assumption that the labour lobby uses labour only in lobbying for the pro-labour party, whereas the capital lobby uses capital only in lobbying for the pro-capital party. Some of MBY's interesting results are the direct consequences of this assumption. For example, consider the compensation effect or the progressivity of the endogenous politics as referred in point 1 above. MBY explain these results as follows. When capital is harmed by a rise in the world price of the labour-intensive good it becomes cheaper for the capital lobby to be involved in political activity whereas it will be relatively more expensive for the labour lobby to spend more labour in lobbying. (see Magee, Brock and Young, 1989: 150).

However, they have not explained why the labour lobby, rather than allowing the capital lobby to seek protection through the political process, cannot utilize cheaper capital in lobbying for the pro-labour party and maintain political power when the price of the labour-intensive good, and hence the wage rate, increases. Similar arguments can be made when the relative price of the capital-intensive good increases in the world market and the wage rate falls in the domestic market. Why do the capital lobby and the pro-capital party not utilise cheaper labour in political campaigning to secure a higher subsidy rate or at least block the labour lobby and the pro-labour party in seeking a higher rate of protection on the labour-intensive good? The question whether their result stands with the factors allowed as substitutes in the political activities remains unanswered.
MBY have also assumed that the political market clears faster than economic markets, as if the parties contest an election every now and then, and therefore the economy may alternate between a tariff regime and a subsidy regime. While making decisions, the agents take the expected prices, not the actual prices, in the economic sphere of the model. The expected prices are determined by the equilibrium probabilities of electoral success and the prices that would result in the event of the success of each of the political parties. Thus, the expectational equilibrium, as they call it, is an important property of their model.

However, to study endogenous tariff formation in the short-run Magee, Brock and Young's model is not directly useful for two reasons: it is based on the Heckscher-Ohlin model, and it does not have a government in place.

To study the behaviour of the policy variables in the short-run the specific-factor model is more suitable to describe the general equilibrium of the economy than the Heckscher-Ohlin model. Similarly, it would be more relevant to consider the political process under one party rule, since elections are actually held at an infrequent interval of, say five years, and one party will be ruling in between any two elections. The incumbent party (or the government), however, would be in constant threat from the opposition and so it will always keep an eye on its re-election prospects. It would be more realistic to assume that the ruling party (or the government) prefers to raise its level of political support.

Under these assumptions, the agents know the actual prices determined by the policy responses of the government. The equilibrium in this type of setting can then be based on ex-post outcomes, rather than on expectations.

A slightly different approach to policy modelling is taken by Yeldan and Roe (1991). They were mainly interested in the implementation of export subsidy rates in the presence of rent seeking activities holding tariffs and tariff-like instruments exogenously constant. In doing so, they have embedded the political sphere of a political economy with its economic sphere described by a conventional CGE model.

Their CGE model contains four economic sectors - agriculture, industry, commercial service, and the public service - of which the public service is nontraded; and eight households - four worker households, one civil-servant household, and three private-sector capitalists (or the producer) households. Worker households receive sectoral wage bill and civil-servant household receives residual profit from public service operation and the 'bribes'. The three producer households get the sectoral rents. They have further assumed that the rent seeking activities are carried out only by the producer households by the payments of bribes out of their rental income.
Given this political and economic structure, they have modelled the supply side of the political market as follows. They assume that the public authority (civil-servant household) responds to rent-seeking activities of the private producers by setting the sectoral subsidy rates in an attempt to maximize the following objective function

$$\Omega = \sum_k I_k V_k, \quad k \in \{\text{producer households}\},$$

subject to the CGE model;

where $V_k$ is the indirect utility level of the producer household $k$, and the influence weights $I_k$ are endogenously determined by the rent-seeking process

$$I_k = e^{-\frac{\beta_k}{R_k}},$$

where, $R_k$ are the monetized costs of the rent-seeking activity or 'bribes' paid by the producer household $k$, and $\beta_k$ are calibration parameters.

The demand side of the political market is described by the utility maximizing behaviour of the producer households. The producer households spend a part of their income in ‘bribing’ the civil-servant household (the authority) to increase their influence weights in the authority’s objective function, which in turn returns with a favourable policy that increases their disposable income, and utilities. The workers’ households were assumed to be nonstrategic. The politico-economic system will be in equilibrium at a policy level when no one wants to adjust his rent-seeking activity or bribes.

The model was implemented using Turkish data of 1981. Simulation results with different closure rules, fixed versus flexible exchange rate, with or without government foreign borrowings, etc. provide an innovative application of the model. The results show that different agents gain differently with different closure rules.

The results, however, may imply that the closure rules, for example fixed or flexible exchange rates, limits to government borrowing, etc. themselves could be the targets of rent-seeking activities, a problem which has not been addressed in the study.

In terms of policy modelling in general equilibrium this study has appended an objective function of the government which is simply a weighted average of the welfare of the rent-seekers, with weights responding positively to the rent-seeking expenditures of the rent-seeking agents. It does not state clearly whether the government is a welfare function maximizer but with weights being affected by political activities of the agents, or is simply a self-interested agent using the Stackelberg strategy vis a vis the rent seekers.
Yeldan and Roe's study has not addressed the existence issue either, and has not explained how the calibration parameters, $\beta$, contained in the influence functions are obtained. These parameters, nevertheless, play a critical role in translating the 'bribes' into the influence weights in the 'objective function' of the government.

2.2.4 Summary of the Political Economy Approach

The endogenous tariff literature that adopts the political economy approach to the question of tariff determination is distinguished by its explicit assumption of self-interested behaviour on the part of government. The literature was classified into three groups according to their emphasis on the dimension of the political market.

The supply side literature assumed support maximizing behaviour of the government and argued that the government would choose a tariff rate which maximizes its political support. But, it did not analyse explicitly how the individual interest groups would react to the policy choice of the government. Moreover, they differ considerably on the appropriate specification of the political support function itself.

The demand side literature has made use of the deduction that there are incentives to the rational agents to behave strategically in influencing the policy decision of the government. For a given policy-supply function (tariff function), this literature has attempted to explain the existence of the tariff 'distortion' as an equilibrium outcome of the policy game played by various interest groups. This class of literature has progressed up to the proof of the existence of a Nash equilibrium in a lobbying game in an exchange economy. It has not been explicit in linking the tariff function to particular behaviour of government. Therefore, the supply side of the political market has remained more or less unexplained.

The literature that has considered both the demand and the supply side of the political market is much richer in several aspects than the studies focusing on either the demand side or the supply side. It shows that government policies are the outcome of a complex interaction among interest groups, political parties and general voters in the political market. However, its economic sphere is described by an expectational equilibrium of a long-run model. Therefore, it is not suitable to study the short-run behaviour of the actors in the political market and the short-run outcomes of the political process. Moreover, the simulation results show that the results are highly sensitive to the choice of the production functions. The conditions under which the existence of an equilibrium can be assured are not known.

The following points emerge as the most common ingredient of the political economy approach to endogenous tariff theory. First, all of the existing studies appear
to agree on the point that given an institutional process of policy supply, different interest groups lobby for favourable tariff policies. Second, the supplier of the (tariff) policy uses it as an instrument for maximizing its own self-interest - of remaining in power or being (re)elected.

2.3 The Hybrid Approach:
A Combination of both Public-interest and Self-interest

There is a third approach to the endogenous tariff theory. It assumes that the actions of the government are determined jointly by its willingness to grant (or perhaps its inability to resist the granting of) tariffs in the face of political pressure and by its desire to maximize social welfare (Feenstra and Bhagwati, 1982: 245).

Feenstra and Bhagwati further explain (p. 246)

Our underlying assumption that one part of the government responds to the protectionist pressures while another tries to maximize welfare subject to this response suggests, as some conference participants wittily remarked, a "left-brain, right-brain" or an "ego versus id" type of approach to the political economy at hand. It does reflect, however, the classic division and confrontation between the (protrade) executive and the (lobbying dominated) legislature in countries such as the United States.

Feenstra and Bhagwati employed a 2 x 2 Heckscher-Ohlin-Samuelson model to describe the general equilibrium of the economy, which is implicitly assumed to be in free trade equilibrium initially. Furthermore, following Findlay and Wellisz (1982) Feenstra and Bhagwati have assumed that the self-interested 'part' of the government offers a tariff function to each factor group, which is increasing and concave in quantities of labour and capital employed by the factor group in lobbying. Given a group-specific tariff function, Feenstra and Bhagwati assume that a factor group intending to lobby the government for tariff protection faces the following "reasonable form for the lobbying cost function"

\[ C(t,w,r) = \frac{t\phi(w,r)}{\max\{0,(p_0^* - p^*(1 + t))\}} \]

where, \( t \) is the tariff rate, \( w \) and \( r \) are the wage rate and the rental rate respectively, \( p_0^* \) and \( p^* \) are the international relative prices of the import competing good respectively before and after the shock. The function \( \phi(w,r) \) is assumed to be increasing and quasiconcave.

By construction, this cost function has some special properties. First, it implies that only one factor will lobby at a time. This is so because, the expression on the denominator of the lobbying cost function implies that a factor will never engage in
lobbying whenever its income is increasing through favourable terms of trade change, because the cost of lobbying remains infinitely large. The factor whose income has fallen, because of an adverse terms of trade change in the international market, will find it rational to engage in lobbying. Second, the injured factor will never lobby for a fully offsetting tariff protection when its income decline is caused by a change in the international terms of trade, because the cost of lobbying will tend to infinity, while the benefits of lobbying will remain finite for every reason. This cost function holds the key to the results obtained by Feenstra and Bhagwati.

With these assumptions Feenstra and Bhagwati obtained some interesting conclusions. For example, take the case of an increased import competition in the context of a specific factor model. The specific factor (or its owner) in the import competing sector will lose as the price of the import competing good falls. To protect its interest it will lobby the government for increased protection.

The government, by assumption, has to respond to the lobbying (political) activity of the injured factor, and may raise the tariff rate on imports. But, this will create a gap between the world prices and the domestic prices, and for obvious reasons, domestic welfare will fall. The government, by assumption, is also a social welfare maximizer. Therefore, it would prefer not to raise tariff rates if other means are available that can provide protection to the injured sector, and would be interested in keeping the domestic prices as close to the world prices as possible.

Under these circumstances the government will necessarily raise the tariff rate on imports if other nondistortionary (lump-sum) taxes are not available or are not feasible. Nevertheless, the government can utilize the tariff revenue strategically so that the injured sector reduces its lobbying activity and the difference between the domestic price and world price is minimized.

In fact, Feenstra and Bhagwati obtain an efficient tariff rate at which the combined effect of the tariff rate and the tariff revenue when transferred fully to the injured factor is just sufficient to make the injured factor indifferent between the reduced level of lobbying that is just sufficient to induce the efficient tariff rate and its optimal lobbying in the absence of such an income transfer. The government's act of bribing the injured factor with the efficient tariff revenue is welfare improving for two reasons. First, the deviation of the domestic price from the world price is reduced with this transfer scheme, and second, the resources absorbed in lobbying for efficient tariff is lower than it would absorb otherwise.

\[9\] Strictly speaking, the basic conflict runs along the factor lines in Feenstra and Bhagwati's model, however, the essential ideas of their model can be captured by a specific factor model as well.
This result accomplishes several things. First, it explains why a tariff exists in the first place. Second, it identifies, in principle, the efficient tariff rate. Third, by virtue of their assumed lobbying cost function, it follows that a declining industry will continue to decline. Fourth, it maintains that the government is basically a benevolent agent.

However, as a guide to policy modelling, this approach is still far from being useful for several reasons. First, it is not clear why the government is supposed to respond to the lobbying effort of the injured factor. Second, the nature and the form of the objective function of the government have not been specified. Third, the lobbying cost-of-tariff function for the injured factor has an ad hoc character. It has not been explained why the cost rises tremendously as the tariff rate approaches to offset the terms of trade change. Fourth, as Baldwin argued in his comments on Feenstra and Bhagwati's paper, it is not clear why the gaining factor does not spend resources in counter lobbying to maintain its gains when the terms of trade change and the injured factor lobbies for an increased tariff protection. Fifth, it is not clear at all whether the hypothesized behaviour of the government has a normative or a positive content. In summary, the approach suggested by Feenstra and Bhagwati, though interesting, does not seem very useful for modelling the process of tariff formation.

2.4 The Form of Protection

The studies reviewed so far have assumed that tariff policy is the only instrument available to the government to pursue its objective, whatever it may be. With this assumption these studies have established that the government will change the tariff rate especially when international terms of trade change. Strictly speaking, these studies have shown that it would be in the interest of the government, for political or welfare reason, to insulate the domestic economy from foreign price shocks. They have not determined whether commercial policy interventions will be in the form of a tariff rate, quota, subsidies or something else.

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10 The reason why the government in their model responds to lobbying by the injured factor is not clear. In the text Feenstra and Bhagwati use a 'tariff function' of Findlay and Wellisz (1982) and say that 'this lobbying function should be interpreted as derived from given political behaviour and institutions, such as the desire of politicians to maximize their probability of reelection' (p. 247), and later on in footnote 10 they write, 'the government's desire to maximize social welfare is consistent with its willingness to grant tariff protection, in that the latter can represent its reaction to distributive equity whereas the former corresponds to allocative efficiency' (p. 257). If the government is prepared to grant protection on equity grounds, then to invoke a 'lobbying function' and a reference to Brock and Magee (1978) is not justified. The social welfare function used by the government should be sufficient enough to generate the optimal tariff rate. On the other hand, if the government is assumed to maximize political support while granting tariff protection to the injured sector, then to claim that it represents the government's reaction to distributive equity is not justified. Thus, the position of Feenstra and Bhagwati is not easily understandable.
There are situations in which the protective instruments are not equivalent. For example, Warr and Parmenter (1986) have shown that in a situation with labour market disequilibrium, protection awarded through government procurement policies may dominate tariffs in terms of maintaining employment elsewhere in the economy, in terms of an increase in employment in the protected sector, or in terms of maintaining the total volume of trade.

How would a self-interested government choose between the tariff and the competitively auctioned quota as a protective instrument, since a relative-price protection to a ‘declining’ industry can be provided in either way? This problem was studied by Cassing and Hillman (1985) in a partial equilibrium setting, but assuming that the industry concerned has a monopoly power in the domestic market.

Cassing and Hillman have shown that the superiority of either instrument depends on the value attached by the government on the tariff revenue. If the government does not value the tariff revenue, and is concerned only with its political support, then tariffs dominate quotas, whereas if the government also values tariff revenue, then the tariff no longer unambiguously dominates the quota as an optimal instrument.

The irrelevance of the welfare comparison of various instruments, when the level of ‘distortion’ is endogenously determined through the political process, was noted by various authors (for example, Rodrik, 1986). Skirting this limitation, however, Rodrik concluded that tariffs could be welfare superior to production subsidies if the protected sector contains a sufficiently large number of firms. His main argument is that under the tariff regime the free rider problem will lead to ‘under-demand’ of protection than relative to the subsidy regime and therefore the level of distortion under the tariff regime will be lower. A Similar conclusion can also be found in Hall and Nelson (1992).

Lloyd and Falvey (1986) studied the choice of policy instrument when there is uncertainty in the international terms of trade using a political economy approach. They have shown that, for a given distribution of the international terms of trade, the distribution of the domestic relative price depends upon the nature of the protective instrument employed to provide protection. Given that all interest groups are risk averse the preference orderings of the protective instruments will differ across the interest groups and the choice of a particular instrument therefore determined by the domestic process of conflict resolution. In particular, they argue that such a choice would be determined by the relative political power of the interest groups.
Lloyd and Falvey's study indicates that any of the numerous protective instruments could be observed in place as an outcome of the political process depending on which of the conditions that guarantee the equivalence of the protective instruments is violated. In our study, we will ignore this issue by assuming that the conditions of equivalence between the protective instruments are satisfied.

2.5 Implications to the Modelling Strategy

We have reviewed the class of endogenous tariff literature which assumes that government is motivated by public-interest and the class of endogenous tariff literature which assumes that government is motivated by its own self-interest. As far as the behaviour of the tariff rate is concerned the predictions of both approaches are similar. Both approaches predict a compensating nature of tariff changes. However, they imply different strategies in modelling the endogenous process of tariff determination.

If one follows the public-interest approach in modelling government behaviour, then the implication of this choice to our modelling problem is that a social welfare function has to be specified somehow and the condition(s) that will be satisfied at the maximum of the social welfare function has to be included in the system of equations describing the general equilibrium of the economic sphere. A solution of this augmented system will yield the welfare maximizing tariff rate in general equilibrium.

However, there are conceptual problems in adopting this approach which need to be solved before such a model can be made operational. In particular, the existence and uniqueness of a social welfare function is a very real problem. Moreover, to guarantee that the social welfare function is maximized through policy choices, it is necessary to assume that the policy makers or the politicians are benevolent and they do not pursue their own self-interest while making policy decisions at the cost of the society. Similar problems arise with the hybrid approach as well. Such problems, however, do not arise if we view the government or the politicians as a class of self-interested agents. In this study, therefore, we will assume that the government is guided by its own self-interest - the interest of remaining in power.

This assumption leads the present study into the class of endogenous tariff literature that adopts a political economy approach to tariff formation. On the basis of the experiences of the previous studies we may draw the following guidelines to model the endogenous process of tariff determination.

\[^{11}\text{For a critical view on welfare economics as such see Sen (1979).}\]
The first issue is related to the stylization of political economy. Several ways of stylizing the political and economic spheres exist in the literature. However, this study selects a particular stylization of these two spheres based on the following consideration.

Most of the previous studies have agreed on the conclusion that domestic tariffs respond to international terms of trade shocks. Since international market condition may change frequently for different exogenous reasons, it is natural to expect that domestic tariff rates are being reviewed continuously. Therefore, this study explicitly assumes that the marginal changes (adjustments) in domestic tariff rates are essentially short-run phenomenon.

Note, however, that this assumption does not necessarily imply a compensating nature of tariff changes. Tariffs may change in either direction in response to a shock. It simply implies that tariff changes do not involve long-run commitments on the part of the policy maker, and because of time inconsistency, the private sector cannot base its optimal decisions involving long-run commitments, including the capacity adjustments in the production sector, on the levels of the existing tariff rates.

This means that the adjustments in the economic markets triggered by tariff changes will be driven by short-run economic interests, and the economic consequences of tariff changes to the private sector are best described by a specific-factor model rather than a model that assumes perfect factor mobility in the economic sphere of the political economy. Because of this reason, in contrast to MBY, this study will employ a specific-factor model to stylize the workings of the economic sphere of the political economy.

On the demand side of the political market this study will allow the players to spend resources in order to obtain favourable tariff policies. In this respect, as in previous studies, it will be assumed that only the owners of the specific factors behave strategically, and the rest behave nonstrategically.12

Whether the country is in a pluralistic or in a dictatorial regime, it will be assumed that the government wants to maximize political support while making policy decisions. The relevant description of the political market will have a 'certain', not an 'expected' government of two or more parties, in place. In contrast to MBY, who studied the politico-economic system based on an expectational equilibrium, this study will be based on the actual (certain) outcomes of the politico-economic process. Thus,

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the present study differs from MBY in its stylization of the politico-economic environment, and provides an exposition of endogenous determination of tariffs in the short-run. Hence, this study may be regarded as a short-run compliment of MBY's long-run model.

Given this politico-economic environment, this study will attempt to attain its objectives in the following steps.

1. Extend Coggins, et al.'s existence result to a productive economy for a given pricing function that satisfies the properties stated in previous studies.

2. Show that the properties satisfied by the pricing function of the government are consistent with the support maximizing behaviour of the government.

These two steps will show that the studies that have focussed on the demand side of the political market and studies that have focussed on the supply side are compatible. Moreover, it will also show that the political economy approach can consistently explain the existence of an 'active' commercial policy, since at least one nontrivial Nash equilibrium exists in the politico-economic system. These results will be obtained in chapter 5.

The noncooperative approach of the existing literature in modelling the demand side of the political market implicitly assumes that the firms in each industry group cooperate with each other, whereas the industry coalitions do not. Adherence to this assumption in modelling the political process may lead to inefficient policy outcomes. For example, it is well known that in a noncooperative game, unless some arbitrary restrictions are imposed, the existence of multiple and sub-optimal equilibria cannot be ruled out. Such has been the experience of MBY who observed a pervasiveness of multiple equilibria, which also displayed the presence of Prisoner’s Dilemma equilibrium, in simulations of models of political economies with endogenous tariff policy.

There is no compelling reason to assume that the political market is characterized by noncooperation only. One can view the political market as an institution which also facilitates communication and negotiation among the interest groups and the political activities as a bargaining process through which the conflicting interests reach an agreement that is individually rational. \[\text{13} \]

\[\text{13} \text{ For applications of cooperative game-theoretic approach in addressing the question of policy determination see Zusman (1976), Zusman and Amiad (1977), Beghin (1990), and Beghin and Karp (1991). Zusman's approach provides a game-theoretic basis for a policy preference function of the government, which is the sum of the government's own objective function and the interest group's objective functions weighted by their respective marginal strength of power over the government. He had} \]
sphere, however, does not rule out the possibility of noncooperation. The players may play the game noncooperatively if they choose to do so.

However, in MBY cooperation did not arise as a viable solution in a tariff-setting game, which they attributed to the presence of multiple equilibria, and the absence of an enforcement mechanism. In the presence of multiple equilibria a double cross in a cooperative solution could lead at least one player to an even worse noncooperative outcome. This result is possible in their model because they had taken a very long-run view in modelling the political economy in which there were two political parties contesting to govern the country. As a result, they had no government in place to enforce a cooperative solution. The problem of enforcement does not arise if one takes a short-run view in modelling the political economy because one can always observe a government in the short-run, which is interested in maximizing political support to remain in power. Enforcement of the cooperative agreement eliminates opposition that would otherwise arise from a policy choice and thus a government will have incentives to enforce cooperative agreements. More importantly, the tariff rates thus obtained at a cooperative solution will be Pareto efficient.

Therefore, in the third step we will extend the scope of the tariff-setting game by permitting cooperation in the strategy sets of the players. The policy making process will be defined in the framework of a Nash bargaining problem in which players will explicitly bargain over the appropriate level of the tariff rates, and also agree on not to be engaged in predatory lobbying activities. We will obtain the condition that will be satisfied by a Nash bargaining solution, and combine it with the conditions of general equilibrium in the economic market. A full model thus obtained will allow us to study the comparative static behaviour of the bargained tariff rate with respect to the exogenous variables of the model. Chapter 6 to chapter 8 will be devoted to this

also suggested a programming technique to estimate the marginal strengths of the interest groups. Zusman's approach was subsequently applied to explain the price policies, and estimate the marginal social powers of different producers and consumers of dairy products under the Israeli Diary Program. Beghin (1990), and Beghin and Karp (1991) also adopted a bargain-theoretic approach in explaining the food pricing policy in Senegal. Their approach differs from that of Zusman in that they have attempted to estimate the game econometrically. However, their specification of the estimating equations is rather ad hoc.

All of the above studies have described the game in general terms in which the government, just like any other player, has interest in policies that are in conflict with the interest of the other private players. Since they have not described a theory of government it is not possible to identify what the objective of the government is. However, in the application of their model to the pricing problem of a particular regulated commodity or a commodity group they have assumed that the government is a revenue maximizer, which could be a reasonable assumption because the game being studied is a small part of the overall policy game in which the government is involved. An application of their approach in a CGE framework does not seem straightforward. However, these studies may provide alternative methods of implementing a bargain-theoretic approach to study the problems of policy formation that could be a matter for future research.
exercise, which will, in totality, represent the major departure of this study from the existing literature.

However, it is observed in the review of the existing literature that the predictions of the political economy approach are surprisingly close to that of the approach that maintains 'public interest' hypothesis in characterizing the government behaviour. Hence, the conclusion of our study can not be expected to be very different from that of these studies. In the event of agreement, the bargain-theoretic approach of the political economy may provide a political economy foundation for the CSWF. The two contending schools can thus be reconciled.
CHAPTER 3
A GENERAL EQUILIBRIUM MODEL OF THE ECONOMIC SPHERE
OF A SMALL OPEN POLITICAL ECONOMY

Introduction

The purpose of this chapter is to describe the workings of the economic sphere of a small open political economy treating the equilibrium outcome of the political sphere, that is the policies of the government, as exogenously given. This chapter is set out in three steps. First, the structure of a general equilibrium model of the economic sphere of a political economy is described in which the government policies, factor endowments, production technologies, and tastes are treated as exogenous. Then, the model is linearized in terms of percentage changes of the model variables around an arbitrary equilibrium point, and solved to obtain the comparative static responses of the endogenous variables. Finally, the properties of the shares and elasticity parameters involved in determining the comparative static responses are employed to deduce the general properties of the equilibrium solution functions that hold at all equilibrium points of the model.

Because government policies are considered exogenous, the model described in this chapter belongs to the class of policy-exogenous general equilibrium models. Moreover, the structure of the general equilibrium model of the economic sphere follows the specific-factor model, which is a sub-class of policy-exogenous models. Therefore, the results of this chapter are fairly well-known in the literature (see for example Jones, 1971; Mussa, 1974; Neary, 1978). This chapter, nevertheless, derives them because these results form the building blocks for the subsequent chapters.

The choice of a specific-factor model is motivated by the following reasons. First, the problem of tariff determination, the specific modelling issue of this study, has already been studied extensively by Magee, Brock and Young (1989) in the context of a Heckscher-Ohlin-Samuelson type general equilibrium model of the economic sphere. Second, tariffs and other redistributive policy changes can be viewed as short-run phenomenon in the sense that they can, in principle, be adjusted quite frequently by the government, whereas the optimal responses of the producers to the tariff and other redistributive policy changes would be constrained by the specificity of some of the productive factors.

Moreover, unless there are reasons, such as credible commitments by the government, to believe that the government’s redistributive policies will not change
over a long period small variations in such policies are unlikely to affect the allocations, within the private sector, of factors that are relatively immobile in the short-run.

This chapter is divided into four sections. The first section outlines the main assumptions of the model. The second section develops the structure of the general equilibrium system. The comparative static results and the properties of the solution functions are studied in the third section. A short summary of the chapter is provided in the fourth section.

3.1 The Economy

In order to simplify and make the model tractable the following assumptions are made. The economy has two single-product sectors, each one with many identical firms. The firms in each sector employ capital and labour according to constant returns to scale CES technology to produce a homogenous good. The existence of intermediate inputs is ignored. All firms are price takers in all commodity, factor and foreign exchange markets. All firms in both production sectors are profit maximizers. Under these assumptions, the aggregation of all firms producing a homogenous commodity into one single sector is valid. It is simply a scalar multiple of a single firm.

In the short run, capital is (firm) sector-specific, whereas (homogenous) labour is perfectly mobile. The endowment of all factors is exogenously given.

Both goods are internationally traded. The economy is small and open relative to the world market of the two goods. Transportation costs and other margins on trade are ignored. As a result, the relative price of the commodities in the domestic market are exogenously determined by the international relative price and government interventions, which can be expressed in equivalent nominal tariff rates.

To simplify further it is also assumed that all factor owners have identical homothetic preferences, and each one is a price taker in goods, factor and foreign exchange markets. The implication of this assumption is that the personal distribution of income plays no role in determining the aggregate demand for commodities, and aggregation of preferences across individuals is valid. More specifically, it is assumed that the economy has a single national (or a representative) consumer with a preference structure that can be represented by a Cobb-Douglas utility function defined on the two commodities, and all final demand is treated as consumption of the national consumer. The consumer receives all factor incomes and the tariff revenue. The consumer is a utility maximizer.

Furthermore, it is assumed that all prices are fully flexible, and all markets clear instantly. All factors are internationally immobile. With these assumptions we proceed
onto derive the general equilibrium structure of the economic sphere of a small open political economy.

3.2 The Structure of the Policy-exogenous General Equilibrium Model

This section specifies the basic structure of the policy-exogenous general equilibrium model (PXGEM). First, under the above assumptions, we analyze a producer's behaviour facing a quantity constraint on the stock of capital. Given the wage rate and the output price, we derive an expression for the virtual rental rate of capital such that the existing stock of capital is optimal to produce a given level of output. We then derive the virtual cost of production at each output level by paying the fixed capital its virtual rental rate and labour its market wage rate. It is then shown that a unique profit maximizing output level can be determined by requiring that average virtual cost be equal to the given output price.

The 'zero-profit' condition and the rental function are then solved to obtain the equilibrium output level and the virtual rental rate in terms of the wage rate and the commodity price. This solves the producer's problem of determining the profit maximizing output level and yields the sector specific rental rate for each sector, (and for each firm) per unit of the specific capital at given prices of commodities and the wage rate.

Moreover, the conditional labour demand functions are expressed in terms of the wage rate and the output prices, since these determine the optimal output levels.

A simple labour market has been specified in which the supply of labour is fixed. Sectoral labour demand functions are aggregated to obtain the market demand for labour, at given commodity prices. The wage rate adjusts to clear the labour market. Thus, the labour market yields the equilibrium wage rate as a function of commodity prices, quantity of labour supplied, and other technological parameters.

Consumer demands are generated by maximizing a Cobb-Douglas utility function.\(^1\) Goods market, and the foreign exchange market clearing conditions simultaneously determine the quantities of net trade on the two goods and the exchange rate.

Thus, the equilibrium conditions of the goods, factors, and the foreign exchange markets are completely specified. This yields a system of equations in fourteen endogenous variables. But, one of the market clearing condition is redundant by

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\(^1\) Cobb-Douglas utility function has been chosen to reduce parametric information, since the demand side is of not much importance to the main purpose of the study.
Walras’ law, we delete the market clearing condition for the foreign exchange, and end up with thirteen equations in fourteen variables. The system is closed by choosing exogenously the nominal exchange rate equal to unity.

The closed system, however, satisfies the homogenous properties - that the real quantities are homogenous of degree zero, and the nominal quantities are homogenous of degree one in commodity prices. This property has been used, as in any other such Walrasian models of small open economies, to normalize the commodity prices by choosing one commodity (exportable) as the numeraire. Furthermore, trade taxes have been rationalized into a single import tariff rate using the Lerner symmetry theorem. This makes the domestic and the foreign price of the numeraire commodity equal to unity, and reduces the number of endogenous variables and the number of equations in the system by one. Thus, we finally obtain a system of twelve equations in twelve endogenous variables that describes the general equilibrium structure of the ‘real’ economic sphere of a small open economy. The following sections derive these relations formally.

3.2.1 The Production Sectors

For each sector $j=1,2$, the production function can be described by the CES function:

\[
Y_j = \left( \alpha_j L_j^{-\rho_j} + \beta_j K_j^{-\rho_j} \right)^{-1/\rho_j};
\]

where, $Y_j$ is the quantity of output, $L_j$ is the units of labour, $K_j$ is the unit of capital\(^2\) employed; $\alpha_j$ and $\beta_j$ are distribution parameters, which are strictly positive and add up to unity; $\sigma_j = 1 / (1 + \rho_j)$ defines the constant elasticity of factor substitution. The parameter $\rho_j$ can take any value such that $\sigma_j$ remains non-negative. It is straightforward to verify that the production function represented by equation (3.1) implies constant returns to scale.

Let $W$ be the market wage rate of labour and let $R$ denote the rental rate of capital, then $C_j(Y_j) = (RK_j + WL_j)$ is the cost of producing output $Y_j$. Given $Y_j$, the problem of a price taking producer is to choose the levels of $L_j$ and $K_j$ so that the cost of producing $Y_j$ is minimized. Formally, suppressing the subscripts\(^3\), the problem may be stated as:

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\(^2\) One may view as capital encompassing all factors that are sector-specific in the short run, and as labour encompassing all factors that are perfectly mobile.

\(^3\) So long as the problems remains symmetric to both the sectors the subscripts denoting the sectors will be suppressed.
subject to \[ Y = \left( \alpha L^{-p} + \beta K^{-p} \right)^{-1/p}. \]

The solution to this minimization problem yields conditional factor demand functions. The conditional demand function for capital stock can be obtained as

\[ K = \beta^{1/(1+p)} R^{-1/(1+p)} \left[ \alpha^{1/(1+p)} W^{\rho/(1+p)} + \beta^{1/(1+p)} R^{\rho/(1+p)} \right]^{1/p} Y; \]

and the conditional demand function for labour can be obtained as

\[ L = \alpha^{1/(1+p)} W^{-1/(1+p)} \left[ \alpha^{1/(1+p)} W^{\rho/(1+p)} + \beta^{1/(1+p)} R^{\rho/(1+p)} \right]^{1/p} Y. \]

Upon substitution, the minimum cost function is given by

\[ C(Y) = \left[ \alpha^{1/(1+p)} W^{\rho/(1+p)} + \beta^{1/(1+p)} R^{\rho/(1+p)} \right]^{(1+p)/p} Y. \]

The equations (3.3) - (3.5) represent a straightforward solution to the cost minimization problem which assumes perfect mobility of factors, both capital and labour, across the sectors in response to higher rewards. This assumption is more reasonable for the longer run than the short run.

However, it is well understood that in the short-run production sectors are unable to adjust their existing stock of some factors to their desired level. One can effectively introduce this constraint imposed by the time horizon into the model by taking some of the factors as sector specific - that their opportunity cost is zero. The specific-factor models could be more relevant in analysing producer behaviour whenever the production sectors are making decisions regarding the short run issues, such as rent-seeking. Therefore, the modelling strategy in this study closely follows the specific factor model (Mussa, 1974; Neary, 1978; and references therein).4 In what follows, the term 'capital' represents all factors that are sector specific in the short-run, and the term 'labour' represents the mobile factors.

Thus, with the assumption that capital stock is given for each sector, equation (3.3) can be solved for the sector-specific rental rate that would make the existing stock of sector specific capital optimal for given wage rate and the level of activity. Solving equation (3.3) we get:

\[ R = \left[ \left( \frac{K}{Y} \right)^p - \beta \right]^{-(1+p)/p} \alpha^{1/p} \beta W. \]

---

4 For a long run view on this issue see Magee, Brock and Young (1989).
The rental rate $R$ obtained from equation (3.6) is the maximum rental rate that a firm would be willing to pay if it had to hire the stock of the specific factor from the rental market to produce an output level $Y$ under the CES production function and a given wage rate $W$. In other words, at the rental rate $R$ given by the equation (3.6) and the wage rate $W$, the producer will find the existing stock of capital optimal to produce $Y$. In line with the literature (for example, Neary and Roberts, 1980) that address the issue of valuation when the choice is quantity constrained, we distinguish this rental rate from the market clearing long run rental rate by stating the following definition:

**Definition 3.1 (Virtual Rental Rate).** For a given level of output and price of the mobile factor, an endogenously determined rental rate that makes the given stock of the specific factor optimal is defined as virtual rental rate of the specific factor.

Substituting the virtual rental rate, $R$, from equation (3.6) into the equation (3.4) and solving for $L$, the labour demand, we get:

$$L = \alpha^\frac{1}{\rho} \left[ K^\rho / (K^\rho - \beta Y^\rho) \right]^{1/\rho} Y. \quad (3.7)$$

This function essentially represents short run conditional demand for labour. The equation (3.7) shows that given the technological parameters, the short run conditional demand for labour depends on the level of output and the stock of fixed factor but not directly on any price including the wage rate. The reason is very simple - if output is to be increased in the short run the only way is to employ more labour in proportion to the level of output. The constant factor of proportion (input-output coefficient) is determined by the stock of the specific factor and the elasticity of substitution. Therefore, this conditional demand for labour can alternately be derived directly from the production function under the assumption that the capital stock is fixed.

Substitution of the virtual rental function from equation (3.6) into the equation (3.5) yields

$$C(Y) = \alpha^\frac{1}{\rho} \left[ K^\rho / (K^\rho - \beta Y^\rho) \right]^{(1+\rho)/\rho} W Y. \quad (3.8)$$

Given a wage rate $W$ and the capital stock $K$, equation (3.8) describes the short run total minimum cost of producing the output level $Y$. In order to distinguish this cost function from the usual cost functions we make the following definitions:

**Definition 3.2 (Virtual Cost).** The minimum cost of producing a given level of output when the mobile factor is paid its market rate and the specific factor is paid its virtual rental rate is defined as the virtual cost of production; the corresponding cost function is defined as the virtual cost function.
**Definition 3.3** (variable Cost). Given a stock of the specific factor, and price of the mobile factor(s), the minimum cost of employing the mobile factor so that a given level of output can be produced is defined as the variable cost of the given level of output. The function that yields the variable cost of production for each level of output is defined as the variable cost function.

In the long run when all factors are variable and command a market price virtual cost becomes equal to the variable cost. However in the short run, when there is at least one sector specific or fixed factor of production, the virtual cost and the variable cost diverge. Virtual cost exceeds variable cost. The cost $C(Y)$, defined in equation (3.8), is an example of virtual cost of producing the output level $Y$. Wage cost only is an example of the variable cost when labour is the only mobile factor of production.

**3.2.2 Determination of Profit Maximizing Output:**

**Zero Pure Profit Condition**

A simple manipulation of equation (3.8), using the production function, yields

$$C(Y)/Y = W / [\alpha(Y / L)^{1+p}].$$  

(3.9)

It can be easily seen that $W / [\alpha(Y / L)^{1+p}]$, the expression on the right hand side of the equation (3.9), represents the marginal cost (wage rate divided by marginal product of labour) of producing $Y$, when labour is the only variable factor of production. This is the derivative of variable cost function. Whereas the expression on the left hand side, $C(Y)/Y$, is the average virtual cost of producing $Y$. *This means that the average virtual cost of production (when capital is paid its virtual price) is exactly equal to the usual marginal cost of production for each value of $Y$.*

Since profit maximization under competitive condition requires that output level be chosen so that marginal cost equals price$^5$, equation (3.9) implies that under similar conditions profit maximization requires that output level be chosen so that average virtual cost be equal to the price. This means that when we are dealing with virtual cost functions profit maximization requires zero profit condition to hold. So long as the production function implies a falling marginal product the average virtual cost curve is

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$^5$ The problem of a profit maximizing sector when the stock of specific factor is fixed at $K$, can be written as $\max L \Pi = \max L \left( P \left( \alpha L^{-p} + \beta K^{-p} \right)^{1/p} - WL \right)$. The condition for profit maximization under competitive conditions then becomes $W = P \alpha(Y / L)^{(1+p)}$ that is, in equilibrium, the mobile factor should be paid the value of its marginal product. Or alternately, the condition may be stated as $P = W / [\alpha(Y / L)^{(1+p)}]$ which means that, in equilibrium, the marginal cost should be equal to the price of output.
upward sloping for given price of mobile factors, and 'zero pure profit' also becomes sufficient condition of profit maximization.

\[ \frac{C(Y)}{Y} \]

\[ P = AR = MR \]

\[ Y^* \text{ Output} \]

Average virtual cost, the usual marginal cost and the zero profit condition

Figure 3.1

For given output price, \( P \), and wage rate \( W \), Figure 3.1 shows the determination of the profit maximizing output level. The horizontal axis measures output level and the vertical axis measures virtual cost and price. The average virtual cost curve is drawn sloping upwards which is implied by the derivative of the average virtual cost function. We know from the equation (3.6) that as output level rises the virtual price of capital also rises. It is precisely so because the marginal productivity of the specific factor, capital, increases and the marginal productivity of labour falls with more employment of labour.

Equation (3.6) can also be rewritten as

\[
R = P \beta \left( \frac{Y}{K} \right)^{1+\rho} \left( \frac{W}{P\alpha(Y/L)^{1+\rho}} \right).
\]

This equation represents nothing more than what can be seen from the first order conditions of cost minimization: that the price paid to each factor be proportional to their corresponding marginal product. However, the way it is expressed in equation (3.6) helps understand the nature of the virtual cost function.

The right hand side of equation (3.6) contains two parts: the first is the value of marginal product of capital and the second is the ratio of wage to the value of marginal product of labour. It shows that if labour is paid less than the value of its marginal product, then the virtual price of a given stock of capital will also be proportionately
less than the value of its marginal product. This means that if producers want to produce less than the profit maximizing level of output at a given wage rate, capital stock and output price, they can hire labour at the market wage rate which will be less than the value of the marginal product. Hence the virtual price of capital will also be less than the value of its marginal product. Therefore, for all output less than that which maximizes profit, it follows that, the virtual cost of production will be less than the value of output and the average virtual cost will be less than the output price.

This explains why in figure 3.1 for all \( Y < Y^* \) we have \( C(Y) / Y \) less than the output price \( P \). Similarly to the right of \( Y^* \) labour has to be paid more than its value of marginal product and so the virtual price of capital will also be more than its value of marginal product. Linear homogeneity of the production function implies that the value of output will be short of the virtual cost of production and hence the curve depicting \( C(Y) / Y \) lies above the revenue line. At point \( E \) average virtual cost equals price, so the value of output is just sufficient to pay for the factors. This means that at \( E \) 'zero pure profit condition' holds and the given stock of capital obtains its maximum feasible virtual rental rate.

It is also clear that the output level at which the zero profit condition holds is unique so long as the production function admits a unique profit maximizing output for any configuration of factor and product price. The implication of this discussion is that imposition of a 'zero pure profit' condition with the virtual cost function is sufficient to identify the profit maximizing output level. The average virtual cost function can serve as a short run output supply function.

Therefore, using equation (3.8) we can write the zero pure profit condition as

\[
P = \alpha^{1/p} \left[ K^p / (K^p - \beta Y^p)^{1+p}/p \right] W.
\]

Solving equation (3.10) for \( Y \) one can obtain the output supply function as

\[
Y = K\beta^{-1/p} \left[ 1 - \alpha^{1/(1+p)} (W / P)^{p(1+p)}/p \right]^{1/p}.
\]

It can be seen from equation (3.11) that the supply of output increases with output price and falls with the wage rate. An increase in the stock of specific factor raises output level at unchanged prices.

---

6 To see clearly that the above assertion holds, let us take any linearly homogenous production function \( Y = f(L,K) \) and hold \( K \) at certain level. Then by Euler's theorem we have
\[
PY = Pf_L L + Pf_K K,
\]
and the virtual cost is given by \( C(Y) = WL + RK \). For output levels less than the profit maximizing level we have \( W < Pf_L \) and \( R < Pf_K \); which implies \( C(Y) < PY \).
For each wage and product price combination, the value of the output given by the equation (3.11) maximizes the quasi-rent. Substitution of the value of Y from (3.11) into equations (3.6) and (3.7) yields the equilibrium rental rate and the equilibrium labour demand functions respectively for given wage rate and the product price.

The equilibrium rental function can, therefore, be written as

(3.12) \[ R = \beta^{-1/p} \left( \frac{P^{\rho/(1+p)}}{W^{\rho/(1+p)}} \right)^{(1+p)/\rho}. \]

Thus, equation (3.12) yields an equilibrium virtual rental rate of the sector-specific capital for given output price and the price of the mobile factor. It shows that in equilibrium the virtual rental rate rises with output price, P, and falls with the price of the mobile factor, W. Making use of the equation (3.11) it can also be seen from the equation (3.12) that the equilibrium rental rate is equal to the value of marginal product of the sector specific factor - capital.

The equilibrium labour demand function can be obtained as

(3.13) \[ L = K\alpha^{1/(1+p)} \beta^{-1/p} \left( \frac{P}{W} \right)^{\rho/(1+p)} - \alpha^{1/(1+p)} \right]^{1/p}. \]

The equation (3.13) shows that equilibrium demand for labour increases with output price and falls with wage rate. Furthermore, at given prices, an increase in the stock of the specific-factor raises the equilibrium demand for the mobile factor.

Thus, the equilibrium behaviour of the production sectors in terms of output supply, labour demand and the payment to their specific factors can be obtained from the equations (3.11)-(3.13) by appending appropriate sectoral subscripts on the variables and the parameters.

3.2.3 The Labour Market

A simple form of labour (mobile factor) market is assumed. The supply of labour is assumed to be exogenously given. For a given output price, production sectors determine their profit maximizing level of employment at each wage rate, which determine the aggregate demand for labour. The aggregate demand for labour has to be consistent with the supply of labour. A flexible nominal wage rate clears the labour market and allocates labour to the production sectors.

Appending appropriate subscripts to denote the sectors in equation (3.13), sectoral demand for labour can be written as

(3.14) \[ L_j = K_j\alpha_j^{\nu_{0+j}/p} \beta_j^{1/p} \left( \frac{P_j}{W} \right)^{\rho_j/(1+p)} - \alpha_j^{1/(1+p)} \right]^{1/p}, \quad j = 1, 2. \]
Let $L$ be the total stock of labour in the economy, then equilibrium in the labour market requires that

$$ (3.15) \quad L = \sum_{j=1}^{2} L_j; \quad j = 1, 2. $$

The equilibrium wage rate is the value of $W$ that solves equations (3.14) and (3.15). Although an exact analytical expression for $W$ is not possible it is clear from equation (3.14) that the equilibrium wage rate is homogenous of degree one in output prices. Because, if all commodity prices and the wage rate double, then equilibrium labour demands remain unchanged and the labour market equilibrium is undisturbed.

### 3.2.4 Demand for Goods

All final demanders are merged to form a single national consumer. The consumer includes all factor owning households, government as an institutional consumer, and all investment activities. This assumption is equivalent to assuming that all of the final consumers have identical and homothetic preferences, and there is no net investment taking place in the economy. This national consumer receives all payments to the primary factors, and tariff revenue. In this framework, lending, borrowing and tax payments within the group cancel out and the aggregate budget constraint holds. The purpose of this assumption is to abstract from the distributional issues.

The preference ordering of this national consumer over the two goods is assumed to be represented by a Cobb-Douglas utility function. The objective of the consumer is to maximize utility, which depends on $(C_1, C_2)$: the quantities of the two goods consumed. In particular, the problem of the consumer can be written as:

$$ (3.16) \quad \max_{C_1, C_2} U(C_1, C_2) = C_1^{\delta_1} C_2^{\delta_2}; \quad \delta_1 + \delta_2 = 1, $$

subject to

$$ (3.17) \quad P_1 C_1 + P_2 C_2 = I $$

where, the income, $I$, of the consumer is given by

$$ (3.18) \quad I = P_1 Y_1 + P_2 Y_2 + Z; $$

and $Z$ is the total tariff revenue.

---

7 For a model that distinguishes between households that own different factors see Long and Vousden (1991).
Solution to this maximization problem leads to the following demand functions:

\[(3.19) \quad C_i = \delta I / P_i; \quad i = 1, 2.\]

The demand functions expressed in equation (3.19) imply that the income elasticities of consumer demand are unity, own price elasticities are -1, and cross price elasticities are zero for both goods. Furthermore, it also implies a constant share of each good in the consumer's budget. These restrictions are the consequences of assuming a Cobb-Douglas utility function. A more realistic representation could be obtained by specifying the utility function accordingly. However, for the purpose of this study the Cobb-Douglas utility function is quite sufficient.8

### 3.2.5 Price Determination

The country under study is assumed to satisfy the small country assumption. This implies that the country is a price taker in the international market. It can affect its domestic prices through various policies, but not the international prices. Let \( \Phi \) denote the nominal exchange rate, \( P_i^* \) denote the international price of good \( i \), and \( T_i \) denote the ad valorem trade tax (positive entry for tax on imports and negative entry for tax on exports) rate on good \( i \), then we can write the domestic price of good \( i \) as

\[(3.20) \quad P_i = \Phi P_i^*(1 + T_i); \quad i = 1, 2.\]

In writing the equation (3.19) it is implicitly assumed that no other taxes and/or controls are used to affect the domestic price of commodity \( i \), and there are no transportation or other margin costs. Of course, these assumptions are for simplification.

### 3.2.6 Equilibrium in Goods and the Foreign Exchange Markets

Let \( M_i(P) \) denote the volume of net import (export, if negative) of commodity \( i \) at domestic price \( P = (P_1, P_2) \), then the domestic market clearing conditions for commodities can be written as

\[(3.21) \quad C_i(P) = Y_i(P) + M_i(P).\]

The tariff revenue collection is given by

\[(3.22) \quad Z = \sum_{i=1}^{2} \Phi P_i^* T_i M_i(P).\]

---

8 A rigorous justification of Cobb-Douglas utility function, however, can be found in Willig (1976)
The value of export at world price represents the supply of, and the value of imports at world price represents the demand for foreign exchange in the domestic economy. Equilibrium in the foreign exchange market requires these two quantities be equal. Therefore we have,

\[(3.23) \quad P_1^*M_1(P) + P_2^*M_2(P) = 0.\]

This equation implies that for all domestic prices the value of domestic exports has to be equal to the value of imports at world price. In other words, trade account at the world price should remain balanced in equilibrium. The implicit assumption here is that there are no capital flows in and out of the country. Of course, this is another simplification.

3.2.7 The Basic Model of the Economic Sphere: PXGEM

Now we can complete the description of the real sector of the economy by collecting equations (3.11) - (3.13), (3.15), (3.19) - (3.21) and (3.23). These equations reflect the equilibrium behaviour on the part of the respective agents, and are rewritten in Table 3.1 to give an overall picture of the economy.

The model consists of fourteen equations in fourteen variables: two domestic output supplies, two consumer demands, two net import quantities, two domestic commodity prices, two sectoral labour demands, two sectoral rental rates, one wage rate and one exchange rate. The system contains seven exogenous variables: two world prices, two tax rates, two quantities of sector specific factors, and one national endowment of mobile factor - labour.

The story told by this system of equations is the following. For given domestic prices, the three equations of the labour market determine the three variables: wage rate, and sectoral allocation of labour. The sectoral output levels can be determined from the two sectoral supply functions, since the market clearing wage rate and the commodity prices are already determined. Similarly, sectoral rental rates are determined from the two rental functions.
Table 3.1
The basic structure of the general equilibrium model of the economic sphere (PXGEM)

The Goods Market

*The supply functions of domestic production sectors*

\[ Y_j = K_j \beta_j^{1/\rho_j} \left[ 1 - \alpha_j^{1/(1+\rho_j)} \left( W / P_j \right)^{\rho_j/(1+\rho_j)} \right]^{1/\rho_j}; \quad j = 1, 2 \]

*Consumer demand functions*

\[ C_j = (\delta_j / P_j) \sum_{i=1}^{2} \left( P_i Y_i + \Phi T_i P_i^* M_i \right) \quad j = 1, 2 \]

*Goods market equilibrium*

\[ C_i = Y_i + M_i, \quad i = 1, 2. \]

The Labour Market

*Sectoral labour demand*

\[ L_j = K_j \alpha_j^{\rho_j/(1+\rho_j)} \beta_j^{1/\rho_j} \left[ \left( P_j / W \right)^{\rho_j/(1+\rho_j)} - \alpha_j^{1/(1+\rho_j)} \right]^{1/\rho_j}; \quad j = 1, 2. \]

*Labour market equilibrium*

\[ L = \sum_{j=1}^{2} L_j. \]

Domestic Price Determination

\[ P_i = \Phi P_i^* (1 + T_i); \quad i = 1, 2. \]

Rental Rates

\[ R_i = \beta_i^{1/\rho_i} \left( P_i^{\rho_i/(1+\rho_i)} - \alpha_i^{1/(1+\rho_i)} W^{\rho_i/(1+\rho_i)} \right)^{(1+\rho_i)/\rho_i}; \quad j = 1, 2 \]

The Foreign Exchange Market

*Equilibrium in the Foreign exchange market*

\[ P_i^* M_i + P_2^* M_2 = 0. \]
Once commodity prices are given, the production side of the economy is completely determined without any reference to the structure and the level of consumer demand. This independence of the supply side from the demand side of the economy occurs because the model describes a small open economy that does not produce any non-tradeable commodity. This feature of a small open economy simplifies the modelling problem considerably, and will be utilized extensively in the subsequent sections and chapters.

The five variables - two consumer demands, two net import quantities and consumer's income (that is, the tariff revenue part of the consumer's income), are simultaneously determined by the five equations, two consumer demand functions and three market clearing conditions, at given product prices. The domestic prices of the two commodities will be determined by the pricing equations (3.20), if the nominal exchange rate is known.

Multiplying both sides of equation (3.20), the market clearing conditions for commodities, by \( P_t \), and using equations (3.17), (3.20), and (3.22) we can obtain equation (3.23). This means that equilibrium in the commodity markets (equation 3.21) implies equilibrium in the foreign exchange market (that is, a balanced trade at world prices). This is, in other words, Walras' Law. Therefore, one of the three market clearing conditions is redundant. We can delete any one of them without losing any information contained in the system. We have chosen to delete the trade balance constraint.

However, once equation (3.23') is dropped out, the system contains only thirteen independent equations in fourteen variables. We are left with two pricing equations (3.20'), and three price variables: exchange rate and two commodity prices. Therefore, one price (or any one nominal quantity) has to be exogenously determined (that is, one more price relation has to be added into the system). This is precisely the point where the money market of the macro economic system becomes quite useful for a model of the present type. In the absence of such a system, we choose the nominal exchange rate to be determined exogenously.

A common feature of the Walrasian general equilibrium models of a small open economy is that the equilibrium real quantities (the demands and the supplies) are homogenous of degree zero in commodity and factor prices, and the nominal quantities (factor prices, and incomes) are homogenous of degree one in commodity prices. Therefore, equilibrium levels of real quantities - outputs, consumption, net trade, and employment - will be unaffected by proportionate change in all commodity prices. This property can be employed to eliminate one of the price variable from the system.
First, we note that the commodity prices are homogenous of degree one in the nominal exchange rate. Hence, a normalization of the nominal exchange rate does not affect the real quantities. So, we set the nominal exchange rate exogenously equal to unity. This normalization has the following consequences. First, all nominal quantities are expressed in terms of the foreign currency. Second, the commodity prices become completely exogenous - determined by foreign prices and the tariff rates, which are exogenous. The government’s policies of affecting the domestic economy will operate through the tariff rates.

Now, we have thirteen equations in thirteen endogenous variables. The system is closed by a (fixed) exogenous exchange rate regime.9

3.2.8 Price Normalization Rule10

It can further be verified from equations (3.12'), (3.14') and (3.15') that even after the exogenous determination of the nominal exchange rate the nominal quantities - the wage rate and the sectoral rental rates are homogenous of degree one in commodity prices, and therefore, it follows from equations (3.11'), (3.14'), and (3.19') that the real quantities - sectoral output supplies, sectoral demands for labour and the consumer's demands - are all homogenous of degree zero in commodity prices.11 These real quantities are unaffected by equiproportionate change, for whatever the reasons, in both commodity prices. Only the nominal quantities will change, which are of no significance.

To eliminate these uninteresting cases of pure nominal changes, commodity prices are normalized and the tariff rates are rationalized in the following way.

---

9 For a discussion on closure rules in a CGE model see Robinson (1989). Robinson has concluded that 'a macro model is needed to determine any two, but no more than two, of the following variables: the domestic aggregate price level, the balance of trade, and the nominal exchange rate.' (p. 921). In our case, we have both tradeable goods, and the trade balance holds by Walras law, the exogenous determination of the nominal exchange rate is sufficient to close the model.


10 A good discussion of price normalization rule and its implication can be found in chapter 6 of Dervis, et al. (1982).

11 Note that the tariff revenue is homogenous of degree one in commodity prices when the nominal exchange rate is exogenously determined, and therefore the consumer’s income is homogenous of degree one in commodity prices. The Marshallian demand is homogenous of degree zero in commodity prices and income.
Table 3.2
A Model of the Real Sector

The Goods Market:

The supply functions of domestic production sectors:

\[ (3.11') \quad Y_j = K_j \beta_j^{1/(1+\rho)} \left[ 1 - \alpha_j^{1/(1+\rho)} (W/P_j)^{\rho/(1+\rho)} \right]^{1/(1+\rho)}; \quad j = 1,2. \]

Consumer demand functions:

\[ (3.19') \quad C_j = \left( \delta_j / P_j \right) \left( \sum_{i=1}^{2} P_i Y_i + T_i^R P^*_i M_i \right) \quad j = 1,2. \]

Goods market equilibrium:

\[ (3.21') \quad C_i(P) = Y_i(P) + M_i(P), \quad i = 1,2. \]

The Labour Market:

Sectoral labour demand:

\[ (3.14') \quad L_j = K_j \alpha_j \beta_j^{-1/(1+\rho)} \left[ \left( P_j / W \right)^{\rho_j/(1+\rho_j)} - \alpha_j^{1/(1+\rho_j)} \right]^{1/(1+\rho_j)}; \quad j = 1,2. \]

Labour market equilibrium:

\[ (3.15') \quad L = \sum_{j=1}^{2} L_j. \]

Domestic price determination:

\[ (3.26') \quad P_i = P_i^* (1 + T_i^R). \]

Rental Rates:

\[ (3.12') \quad R_j = \beta_j^{-1/(1+\rho)} \left( P_j^{\rho_j/(1+\rho_j)} - \alpha_j^{1/(1+\rho_j)} W^{\rho_j/(1+\rho_j)} \right)^{(1+\rho_j)}; \quad j = 1,2. \]

Price Normalization Rule

\[ (3.25) \quad P_2 = 1 \]

List of Endogenous Variables:

- \( Y_j \) : 2 Sectoral outputs
- \( C_j \) : 2 Domestic demands
- \( M_i \) : 2 Net import quantities
- \( L_j \) : 2 Sectoral employment of labour
- \( R_j \) : 2 Sectoral rental rates in units of commodity 2
- \( W \) : 1 Wage rate in units of commodity 2
- \( P_i \) : 1 Price of commodity 1 in units of commodity 2

Total number of endogenous variables: 12.
Total number of equations 12.
First, we invoke the Lemer symmetry theorem (Lemer, 1936) and replace export tax by its equivalent import tax. Suppose that the economy exports good 2, and imports good 1, and let

\[(3.24) \quad T_1^R = [(T_1 - T_2) / (1 + T_2)].\]

Then, the Lemer symmetry theorem implies that in terms of revenue and relative price effects the imposition of a single tariff rate \(T_1^R\) on the imports of good 1 and no tax on exports of good 2 is equivalent to the joint imposition of export tax rate \(T_2\) on good 2, and an import tariff rate \(T_1\) on good 1. We will call \(T_1^R\) the rationalized tariff rate.

Second, with the nominal exchange rate set to unity and a rationalized tariff rate imposed on the imports of good 1, the domestic price of good 2 becomes equal to its international price. Now, we choose good 2 as the numeraire, and express all nominal quantities in units of good 2. As a result we get

\[(3.25) \quad P_2 = P_2^* = 1, \text{ and} \]

\[(3.26) \quad P_1 = P_1^*(1 + T_1^R).\]

Clearly, equations (3.25) and (3.26) replace the original pricing equation (3.20') in the system of equations described in Table 3.1.

Note that, \(T_1^R\) is invariant of pure inflationary (relative price neutral) change in both export and import tax rates.\(^{12}\) Therefore, for given world relative price, \(T_1^R\) is in one-to-one correspondence with the domestic relative price \(P_1\). The sign of \(T_1^R\) indicates whether the economy is subsidizing or taxing foreign trade. In particular, \(T_1^R > 0\) implies a net tax and \(T_1^R < 0\) implies a net subsidy on foreign trade.\(^{13}\)

### 3.2.9 A Model of the Real Economic Sphere

Now, we obtain a general equilibrium model of the real sectors of the economic sphere by applying the normalization rules of the previous section. This concludes the construction of a simple policy-exogenous general equilibrium model of the economic sphere of a small open economy. By deleting the trade balance constraint, writing unity for the nominal exchange rate, and replacing the pricing equation (3.19') by equations (3.25) and (3.26) the system of equation describing the general equilibrium can be written as in Table 3.3.

\(^{12}\) For any positive number \(X\), the relative price neutral tariff rates, \(T_i^*\), are given by

\[(2.27) \quad T_i^* = XT_i + \lambda - 1, \text{ for } i = 1, 2.\]

\(^{13}\) For more discussions on the Lemer symmetry theorem, see chapter 2 in Vousden (1990).
The model of the real sector organized into Table 3.2 contains twelve equations in twelve endogenous variables (excluding the price normalization rule), which are also listed in the table. The exogenous variables are domestic factor endowments $K_1, K_2, L$, the rationalized tariff rate $T$, and the international relative price of commodity 1, $P^*$. This model of the real economic sphere is an example of a policy exogenous general equilibrium model (PXGEM), because the tariff rate is being treated as an exogenous variable. The model is of the standard text-book type the existence of a solution to this model is not a problem (see, for example, Shoven, 1974).

### 3.2.10 Factor and Employment Shares

Let $S_{Lj}$ and $S_{Kj}$ denote respectively the share of labour and the share of the sector specific factor in the value of output (which equals total cost) of sector $j$. Then, by making use of the equations (3.11') and (3.14'), and keeping in mind that $P_2 = 1$ we can write:

\[
S_{Lj} = \frac{WL_j}{PY_j} = \alpha^{1/(1+\rho)}\left(\frac{P_j}{W}\right)^{-\rho/(1+\rho)} \quad j = 1,2.
\]

Using equations (3.11') and (3.12') the share of the specific factor in sector $j$ can be obtained as:

\[
S_{Kj} = \frac{KR_j}{PY_j} = 1 - \alpha^{1/(1+\rho)}\left(\frac{P_j}{W}\right)^{-\rho/(1+\rho)} \quad j = 1,2.
\]

Thus the equations (3.28) and (3.29) show that, in equilibrium, the distributive shares add to one.

Let us further define

\[
\lambda_j = \frac{L_j}{L}; \quad j = 1,2;
\]

\[
\sigma_j = \frac{1}{1+\rho_j}; \quad j = 1,2;
\]

and,

\[
\tau = \frac{1}{1+T^*}
\]

then, $\lambda_j$ represents the employment share of the sector $j$ in total employment of labour, $\sigma_j$ is the elasticity of factor substitution, which is a constant; and $\tau$ represents the rationalized tariff coefficient.
3.3 Comparative Static Results and the Properties of the Solution Functions

For use in subsequent chapters, we will derive the comparative static responses of the endogenous variables under three different tariff regimes: free trade, intermediate, and autarkic. The comparative static responses under a free trade regime are included in Appendix -2A, and that under autarkic regime are included in Appendix - 3B. This section derives the comparative static responses of the endogenous variables under an intermediate tariff regime. These responses are analyzed to derive some general properties of the solution functions. The equilibrium under the intermediate tariff regime has also been called the observed equilibrium, because we assume that the observed ‘general equilibrium data set’ contains a positive rationalized tariff rate. The issues related to the construction of a general equilibrium data set are not addressed in this study. Such a data set is assumed to be given.

The endogenous variables, and the share parameters that are specific to a particular equilibrium point are distinguished by superscripts. Superscript ‘o’ refers to the ‘observed’ equilibrium, superscript ‘*’ refers to the free trade equilibrium, and superscript ‘a’ refers to the autarkic equilibrium.

Let \( E = (Y_1, Y_2, C_1, C_2, L_1, L_2, R_1, R_2, W, P_x) \) be the vector of the solution values of the endogenous variables of the general equilibrium model of the real sector. Let \( E^o \), \( E^* \) and \( E^a \) be the general equilibrium solution vectors of the endogenous variables in the three different tariff regimes but with the same factor endowment and the world relative price. Then the solution vector in observed equilibrium can be written as:

\[
E^o = E^o(K_1, K_2, L, T^*_1, P^*_1);
\]

and the solution vector in free trade equilibrium can be written as

\[
E^* = E^*(K_1, K_2, L, P^*_1);
\]

and the solution vector in autarky can be written as

\[
E^a = E^a(K_1, K_2, L);
\]

where, the \( E \)'s on the right hand side of the above equations represent the appropriate vectors of the solution functions.

In following sections the model of the real sector will be linearized around the observed equilibrium, and then the comparative static results will follow.
3.3.1 Linearization of the Model around the Observed Equilibrium

The model described in the previous section (Table 3.2) contains non linear functions making its analytical solution impossible. Comparative statics, however, can be performed with ease, if the equations are linearized around an equilibrium point. In this subsection, the model will be linearized in terms of percentage changes in the variables around the observed equilibrium point. This technique was initially introduced by Johansen (1960) as a method of solving multi-sectoral models.

The variables of all equations in Table 3.2 are in levels, and they have been written in upper case letters. Let the percentage change of the variables be denoted by corresponding lower case letters with one exception. The exception is that the variable \( t' \) represents change in percentage point of variable \( T'_r \) rather than percentage change in \( T'_r \). This will allow \( T'_r \) to take any value, positive, negative or zero. Once it is understood that \( t \) represents a change in the percentage point of the rationalized tariff rate the superscript becomes redundant, and as it applies to only one sector the subscript is also unnecessary. So, in what follows the superscript and subscript on \( t \) are suppressed.

As mentioned before, consumer demand plays no role in determining output, employment, and prices in the economy. Therefore, changes in consumer demand are of no consequence for changes in the variables of interest - the variables related to the supply side. Consumer demand only affects the quantity of net imports. linearization of consumer demand equations, and goods market equilibrium conditions have been deferred until the comparative statics are performed around the autarky equilibrium where the consumer demands matter. However, it greatly simplifies the algebra.

With these definitions and preliminaries, let us assume that the vector \( E^o \) describes the observed equilibrium of the economy for a given set of values of the exogenous variables. Let us perturb this equilibrium by changing the exogenous variables infinitesimally. By taking logarithmic total differentials around \( E^o \) most of the model equations can be linearized by expressing the variables in their percentage change form. Making use of the defining equations (3.28)-(3.32), the linearized expressions for the model equations can be obtained as follows:

Output supply functions:

Taking logarithmic total differential of equation (3.11') and using equations (3.28), (3.29) and (3.31), which define the cost shares and the elasticities of factor substitution, and evaluating the shares around the 'observed' equilibrium we get
In equation (3.36) \( y^*_j \) is the percentage change in the supply of output in sector \( j \); \( w^* \) is the percentage change in the normalized wage rate; \( p^*_j \) is the percentage change in the normalized price of commodity \( j \); \( k_j \) is the percentage change in the stock of sector specific factor (physical capital) in sector \( j \).

Equation (3.36) shows that the sectoral supply of the equilibrium output increases with the increase in the stock of the sector specific capital, and the equilibrium relative price of the product, and decreases with increase in the equilibrium wage rate. The magnitudes of the effects depend on the degree of 'ease' in factor substitution.

The labour market:

By linearizing the labour demand functions given by equation (3.14') and the labour market equilibrium condition (3.15') around the 'observed' equilibrium and simplifying the resulting expressions by making use of the equations (3.28) - (3.31) we can obtain

**Labour demand functions:**

\[
(3.37) \quad l^*_j = k_j + \sigma_j \left( \frac{S^*_j}{S^*_Kj} \right) (p^*_j - w^*); \quad j = 1, 2; \quad \text{and}
\]

**The labour market equilibrium condition:**

\[
(3.38) \quad l = \sum_{j=1}^{2} \lambda^*_j l^*_j.
\]

where, \( l \) is the percentage change in the total endowment of labour in the economy, and \( l_j \)'s are percentage change in the sectoral employment of labour.

Equation (3.37) shows that the change in the sectoral equilibrium demand for labour (employment) is governed by change in the quantity of the sector specific factor, change in the product price, change in the equilibrium wage rate, and the 'ease' of factor substitution. Increased product price or increased stock of the sector-specific factors would increase sectoral labour demand, and an increased wage rate would depress the sectoral demand for labour. The equation also shows that the real wage elasticity of sectoral labour demand is equal to \( \sigma_j / S^*_Kj \).
Equation (3.38) means that any change in the national endowment of labour has to be accommodated by changes in the equilibrium sectoral employments of labour. Other things remaining the same, this is possible only if the wage rate, measured in units of commodity 2, falls.

**Sectoral rental rates:**

Similarly, the equilibrium rental function can be linearized by logarithmic differentiation. Simplifying the expression by using equations (3.28) and (3.29) and the fact that the factor shares add up to unity we get

\[
(3.39) \quad r_j^* = \frac{1}{S_k^j} (p_j^* - S_k^j w^*); \quad j = 1, 2
\]

where, \( r_j^* \) is the percentage change in the equilibrium virtual rental rate in sector \( j \). The equation (3.39) shows that the equilibrium virtual rental rate in each sector rises with relative price of own output and falls with the wage rate.

**Price equations:**

Similarly, linearizing the price equations (3.25) and (3.26) we get

\[
(3.40) \quad p_1^* = p_1^* + \tau^* t, \quad \text{and}
\]

\[
(3.41) \quad p_2^* = 0
\]

where \( \tau^* \) is defined in equation (3.32), and \( t \), as indicated above, is 100 \( x dT^* \) the change in the percentage point of the rationalized tariff rate.

Equation (3.40) shows that the normalized price of import competing good, good 1, in the domestic market moves with international price of good 1. So long as the economy is not adopting a net subsidy (subsidy payments exceeding tariff collection) policy in international trade, the rationalized tariff rate, \( T_1^r \), is always positive. It can be seen from the defining equation (3.32) that \( \tau^* < 1 \). So, at an unchanged international relative price, the rise in the domestic relative price with respect to an increase in the rationalized tariff rate is less than proportionate. However, the elasticity of the domestic relative price of import competing good with respect to the rate of protection is unity.\(^{14}\) Equation (3.41) simply shows that the price of good 2, the numeraire, does not change.

---

\(^{14}\) The rate of protection is given by \( (1 + T_1^r) \), and the percentage change in the rate of protection is given by \( \tau^* t \).
### Table 3.3

**Linearized version of the PXGEM**

(Around the Observed Equilibrium of the Economy)

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3.36)</td>
<td>[ y_j^o = k_j + \sigma_j \left( \frac{S^o_j}{S^o_k} \right) (p_j^o - w^o) ]; ( j = 1, 2 ).</td>
</tr>
<tr>
<td>(3.37)</td>
<td>[ l_j^o = k_j + \frac{\sigma_j}{S^o_k} (p_j^o - w^o); ] ( j = 1, 2 ).</td>
</tr>
<tr>
<td>(3.38)</td>
<td>[ l = \sum_{j=1}^{2} \lambda_j^o l_j^o. ]</td>
</tr>
<tr>
<td>(3.39)</td>
<td>[ r_j^o = \frac{1}{S_k^o} (p_j^o - S_L^o w^o); ] ( j = 1, 2 ).</td>
</tr>
<tr>
<td>(3.40)</td>
<td>[ p_1^o = p_1^* + \tau^o t, ] ( \text{and} )</td>
</tr>
<tr>
<td>(3.41)</td>
<td>[ p_2^o = 0. ]</td>
</tr>
</tbody>
</table>
The system of the linearized equations is brought together in Table 3.3. The exclusion of the demand sides of the goods markets has left the linearized system with only eight variables in eight equations. This system will be used in the following section to derive the comparative static results around the 'observed' equilibrium.

3.3.2 Comparative Statics around the Observed Equilibrium

Wage effects:

Substituting the equilibrium labour demand functions from the equation (3.37) into the market clearing equation (3.38) and solving for the wage rate, $w$, we get:

$$w^o = A^{-1}[\lambda_1^o k_1 + \lambda_2^o k_2 + (\lambda_1^o \sigma_1 / S_{x_1}^o) p_i^o - l],$$

where,

$$A = \frac{\lambda_1^o \sigma_1 / S_{x_1}^o + \lambda_2^o \sigma_2 / S_{x_2}^o}{S_{x_1}^o} > 0$$

is the magnitude of the real wage elasticity of the aggregate labour demand.

Equation (3.42) fully describes the behaviour of the equilibrium wage rate with respect to changes in the exogenous variables in the model. Since $p_i$ is completely and independently explained by exogenous variables - foreign price and tariff rate, it can be regarded as a given datum. By setting all exogenous variables equal to zero except one at a time we can obtain the elasticity of wage rate with respect to the exogenous variables.

In particular, by setting changes in all endowment variables equal to zero, the elasticity of wage rate with respect to the relative price of commodity 1 can be obtained as

$$\eta^o = \frac{w^o}{p_i^o} = A^{-1}(\lambda_1^o \sigma_1 / S_{x_1}^o).$$

Both the numerator and the denominator of the right hand side of the expression (3.43) are positive, because all the share parameters and the elasticities of factor substitution are positive. Since, the numerator of the term is less than the denominator, it follows that $0 < \eta < 1$. That is, as the relative price of good 1 increases, the wage rate will increase but by less than in proportion.

Similarly, it can be inferred from equation (3.42) that the equilibrium wage rate will increase with an increase in the stock of the sector specific factor in either sector but will fall with an increase in the stock of labour in the economy.
Employment effects:

Substituting back the response of the equilibrium wage rate from equation (3.42) into equation (3.37) we can obtain the changes in the sectoral employment of labour as

\[ I_1^e = A^{-1} \left[ \frac{\lambda_2^* \sigma_2}{S_{K2}} \left( \frac{\sigma_1}{S_{K1}} p_1^e + k_1 \right) + \frac{\sigma_1}{S_{K1}} \left( I - \lambda_2^* k_2 \right) \right] \]

and,

\[ I_2^e = A^{-1} \left[ \frac{\lambda_2^* \sigma_2}{S_{K1}} k_2 - \frac{\sigma_2}{S_{K2}} \left( \lambda_1^* k_1 + \frac{\lambda_2^* \sigma_1}{S_{K1}} p_1^e - I \right) \right] \]

From these two equations (3.44) and (3.45) we can obtain the extent and direction of changes in sectoral employment with respect to given changes in the exogenous variables. For example, the equilibrium employment of labour in sector 1 expands if (1) the relative price of commodity 1 increases, (2) the stock of capital in sector 1 increases, (3) the stock of labour in the economy increases, and (4) the stock of capital in sector 2 decreases.

Similarly, in equilibrium, the employment of labour in sector 2 increases if (1) the capital stock in sector 2 expands, (2) the stock of labour in the economy expands, (3) the stock of capital in sector 1 decreases, and (4) the relative price of commodity 1 falls (that is the relative price of good 2 rises). The rate of change of sectoral employment with respect to the exogenous variables can be obtained as in the case of the wage rate.

Some properties of the labour demand functions follow from equation (3.44). These properties are generally valid irrespective of the location of the point of equilibrium. Since the equilibrium level of employment of labour in sector 1, ceteris paribus, is a strictly increasing function of the relative price of good 1 (equation 3.44), it follows form labour market equilibrium condition that if the tariff rate (that is, the relative price of good 1) is increased in steps, then \( \lambda_1 \), the employment share of sector 1, will increase at successive equilibrium points. Hence, we write the following proposition for future reference.

**Proposition 3.1** Ceteris paribus, the employment share of sector 1, \( \lambda_1(P_1) \), is a strictly increasing function of the relative price \( P_1 \). That is, \( d\lambda_1 / dP_1 > 0 \). In other words, increased protection of the import competing sector increases the share of the import competing sector in total employment of labour in the economy.
In a non-specialization case $0 < \lambda_i < 1$, and $\lambda_i = 1$ when the economy specializes in the production of good 1. It can be seen from the equation (3.43) that the relative price elasticity of the wage rate is an increasing function of the employment share of sector 1. Hence using proposition 3.1 we can write the following proposition regarding the property of the price elasticity of the normalized wage rate:

**Proposition 3.2**  The relative-price elasticity of the wage rate (measured in units of commodity 2) is an increasing function of the relative price of good 1. That is, $d\eta / dP_1 > 0$.

The propositions 3.1 and 3.2 capture the second order effects of a relative price change. In particular, proposition 3.2 means that the relative price elasticity of the wage rate depends upon the size of the price change. The larger the price change the larger the elasticity of the wage rate. However, the relative-price elasticity of the wage rate will never exceed unity. Hence, we can state the following proposition:

**Proposition 3.3**  Protection of the import competing sector always lowers the real wage faced by the import competing sector, and raises the real wage faced by the exporting sector.

**Effects on the sectoral rental rates:**

The endogenous variable, the wage rate, can be eliminated from the rental equation (3.39) by using the equation (3.42). This yields expressions for the percentage changes in the sectoral rental rates in terms of the percentage changes in the exogenous variables. After simplification, the sectoral rental rates can be written as

$$ (3.46) \quad r_1^e = \frac{1}{AS_{k1}^e} \left[ \left( \frac{\lambda_1^e \sigma_1}{S_{k2}^e} + \lambda_1^e \sigma_1 \right) p_1^e - S_{li}^e \left( \lambda_1^e k_1 + \lambda_2^e k_2 - l \right) \right] $$

and,

$$ (3.47) \quad r_2^e = -\frac{S_{li}^e}{AS_{k2}^e} \left( \frac{\lambda_1^e \sigma_1}{S_{k1}^e} p_1^e + \lambda_1^e k_1 + \lambda_2^e k_2 - l \right) $$

The equations (3.46) and (3.47) show that the rental rates in both sectors increase as the supply of labour in the economy increases; and the rental rates in both sectors fall if the stock of capital increases in either sector. In other words, an increase in the stock of capital in sector 1 not only lowers the rental rate in sector 1 but also lowers the rental rate in sector 2 as well. It is because, an exogenous increase in the stock of capital in sector 1 increases the demand for labour, which bids the wage rate up
causing the rental rate in sector 2 to fall. Similar reasoning holds if the stock of capital in sector 2 increases exogenously.

The elasticities of the sectoral rental rates with respective to any exogenous variable can be computed from the equations (3.46) and (3.47). In particular, by setting all endowment changes to zero the elasticities of the rental rates with respect to the relative price of good 1 can be obtained as

\[
\frac{\rho^*}{P^1} = A^{-1} \left( \frac{\lambda^*_1 \sigma_1}{S_{K1}^0} + \frac{\lambda^*_2 \sigma_2}{S_{K1}^0 S_{K2}^0} \right) > 1, \text{ and}
\]

\[
\frac{\rho^*_2}{P^1} = -\frac{S_{K2}^0}{S_{K1}^0 A} \left( \frac{\lambda^*_2 \sigma_1}{S_{K1}^0} \right) < 0.
\]

Equation (3.48) shows that the elasticity of the rental rate in sector 1 with respect to the relative price of commodity 1 is greater than unity. Mussa (1974) has provided the following interpretation of this result. As the price of commodity 1 rises, the value of the share of initial output of sector 1 that went to fixed capital also rises proportionately. Since the wage rate does not rise in proportion to the output price, some surplus arises from part of the initial output, which went to the payment of wages, but that will now accrue to the capital. Moreover, there will be an increase in output, which means a part of the incremental output will again go to the given stock of capital. Hence the rental rate in each sector will rise more than proportionately as the relative price of its output rises.

Equation (3.49) shows that the elasticity of the rental rate in sector 2 with respective to the relative price of good 1 is negative. The magnitude of the elasticity depends on the elasticity of the wage with respect to the relative price of good 1, and the labour intensity in sector 2. The absolute value of the elasticity will be higher, the higher the distributive share of labour in sector 2, and the higher the relative price elasticity of the wage rate.

From these results we can state the following proposition regarding the property of the rental functions:

**Proposition 3.4** The rental rate in sector 1, \( R_1 \), is a strictly increasing, and the rental rate in sector 2, \( R_2 \), is a strictly decreasing function of the relative price of commodity 1, \( P_1 \). That is \( dR_1 / dP_1 > 0 \), and \( dR_2 / dP_1 < 0 \).
Output effects:

Using the wage equation (3.42) the output supply functions given by the equation (3.36) can be rewritten as

\[
(3.50) \quad y_1^* = A^{-1}\left[\left(\lambda_1^0\sigma_{12} + \lambda_1^0\sigma_1\right)k_1 + \frac{\sigma_1 s_{11}^*}{s_{x1}^0} \left(\lambda_2^0\sigma_2 + p_1^\circ - \lambda_2^0k_2 + l\right)\right],
\]

and

\[
(3.51) \quad y_2^* = A^{-1}\left[\left(\lambda_2^0\sigma_{12} + \lambda_2^0\sigma_2\right)k_2 - \frac{\sigma_2 s_{12}^*}{s_{x2}^0} \left(\lambda_1^0k_1 + \frac{\lambda_1^0\sigma_1}{s_{x1}^0} - p_1^\circ - l\right)\right].
\]

From these two equations it is clear that an increase in the relative price of good 1 results in an increase in the output of sector 1 and a fall in the output of sector 2; and an increase in the stock of labour in the economy leads to an increase in the output of both sectors. These effects are quite intuitive. What is interesting is that an increase in the stock of capital in sector 1 leads to an increase in the output of sector 1 and a fall in the output of sector 2. Similarly, an increase in the stock of capital in sector 2 leads to an increase in the output of sector 2 and a fall in the output of sector 1. In other words, irrespective of the capital intensity of the sectors, an increase in the capital stock in one sector leads to a fall in the output of the other sector. This effect is quite different from the Rybczynski Theorem, which predicts that increase in the stock of capital will lead to an increase in the output of capital intensive good and a fall in the output of labour intensive good.

The reason is quite simple. An increase in the stock of capital in sector 1 (the sector specific factor) raises the productivity of labour (the mobile factor) in sector 1 and therefore, at unchanged prices, the demand for labour goes up. Consequently, wage rate will go up. The result is, as can be seen from the equation (3.37), that the equilibrium employment for labour in sector 2 falls, and hence the output of sector 2 falls.

Since the rent in sector 1 can be increased through increased protection, and if the resulting surplus is ploughed back to increase the stock of capital in sector 1, then over time the output of sector 1 will exceed its domestic demand and will be exported at the world price, and the output of sector 2 will be insufficient and will have to be imported to meet domestic demand. This means that commodity 2 will turn into an import competing good.

Thus, under the assumption that the new rent generated through increased protection will be reinvested within the sector, the comparative static results in a general equilibrium framework provide some support to the classical 'infant industry' argument of a protective trade policy.
3.4 Summary

A simple general equilibrium model of a small open economy has been described in this chapter. The model assumes that the tariff rate that creates a wedge between domestic and foreign relative price is exogenously determined.

Comparative static results have shown that the rental income of each sector is a strictly increasing function of the relative price of own commodity and a strictly decreasing function of the relative price of other's commodity. This remains valid whether the rental income is measured in units of own commodity or in units of the numeraire commodity.

This result clearly demonstrates that changes in tariff rates are capable of redistributing rents from one sector to the other. The mechanism behind this transfer is the difference in the real wage faced by the two sectors, which induces a reallocation of labour between the two sectors, and hence the outputs and the rents.

Therefore it can be inferred that, for given endowment of factors, technologies of production and the international prices, the choice of a particular tariff rate by the government determines a particular combination of sectoral rental incomes in the general equilibrium of the economy. In the next chapter, an attempt will be made to obtain an analytical relationship between the sectoral rental incomes that summarize the mechanism and redistributive effects of the tariff changes in general equilibrium.
Appendix-3A: Comparative Statics around the Free Trade Equilibrium

The free trade environment can be mimicked by setting the tariff rate to zero in the model of the real sector listed in Table 3.3. This means nothing more than that the domestic relative price will be equal to the international relative price. Given the parameters of the model the free trade equilibrium levels of the endogenous variables can be obtained by replacing \( P_i \) by \( P_i^* \) and solving the model for endogenous variables.

Since, at the observed level of other exogenous variables, \( E^* \) denotes the vector of the equilibrium levels of the endogenous variables in the free trade environment, a linearized version of the model around \( E^* \) can be obtained by following the same procedure as was done in linearizing the model around the observed equilibrium. The linearized system of equations is listed in Table 3A.1.

| Table 3A.1 |
| Linearized version of the PXGEM |
| (Around the free trade equilibrium) |
| The supply functions: |
| (3A.1) \[ y_j^* = k_j + \sigma_j \left( \frac{S^*_{xj}}{S^*_{Kj}} \right) (p_j^* - w^*); \]  \( j = 1, 2 \). |
| Labour demands functions: |
| (3A.2) \[ I_j^* = k_j + \sigma_j \left( \frac{S^*_{xj}}{S^*_{Kj}} \right) (p_j^* - w^*); \]  \( j = 1, 2 \). |
| Labour market equilibrium condition: |
| (3A.3) \[ l = \sum_{j=1}^{2} \lambda_j^* I_j^* \]. |
| The rental rates: |
| (3A.4) \[ r_j^* = \frac{1}{S^*_{Kj}} (p_j^* - S^*_{xj} w^*); \]  \( j = 1, 2 \). |
| The price normalization rule |
| (3A.5) \[ p_2^* = 0 \]. |

The set of equations listed in Table 3A.1 contain three different changes compared to the equations listed in Table 3.3 of the text. First, the domestic relative price has been replaced by the international relative price. Second, all shares have been evaluated at the free trade equilibrium point. Third, the endogenous variables now represent a percentage change over their free trade equilibrium levels. However, the
levels of the exogenous variables are assumed to be the same in all the three states hence they represent a percentage change over their observed levels.

The comparative static results will be similar to that described by equations (3.42) - (3.51) except that the shares correspond to free trade equilibrium and the percentage change in the domestic relative price $p_1^*$ has been replaced by the percentage change in the international price ratio $p_i^*$. 

In particular, the response of the rental rates around free trade equilibrium will be given by

\begin{equation}
\begin{aligned}
    r_1^* &= \frac{1}{S_{K1}^* \left( \frac{\lambda_i^* \sigma_1}{S_{K1}^*} + \frac{\lambda_i^* \sigma_2}{S_{K2}^*} \right)} \left[ \left( \frac{\lambda_i^* \sigma_2}{S_{K2}^*} + \lambda_i^* \sigma_1 \right) p_i^* - S_{L1}^* \left( \lambda_i^* k_1 + \lambda_i^* k_2 - l \right) \right]; \\
    r_2^* &= -\frac{S_{L2}^* \left( \frac{\lambda_i^* \sigma_1}{S_{K1}^*} + \frac{\lambda_i^* \sigma_2}{S_{K2}^*} \right)}{S_{K2}^*} \left( \lambda_i^* k_1 + \lambda_i^* k_2 - l \right). 
\end{aligned}
\end{equation}

The natures of these rental response functions are similar to those of the rental response functions (3.46) and (3.47). The difference is in the magnitudes of their elasticities. These equations have been derived for future reference.

**Appendix-3B: Comparative Statics around the Autarkic Equilibrium**

The other extreme of free trade is the autarky, in which the economy does not trade with the rest of the world. An equilibrium under autarky requires that the domestic demand and supply of each commodity be equal. Thus, an economy under autarky can be described by the presence of domestic market clearing conditions.

Since there are no imports, collection of tariff revenue will be zero. The consumer's income will be equal to total value added in the economy. Since, the consumer satisfies the budget constraint, the market for one commodity clears implies that the market of the other commodity clears (Walras' Law). Hence, by choosing commodity 2 as the numeraire, as before, and ignoring the market clearing condition for commodity 2, the autarkic representation of the economy can be obtained as listed in Table 3B.1 simply by modifying the model of a trading economy listed in the Table 3.3.
The Goods Market:

**The supply functions of domestic production sectors:**

\[(3.11') \quad Y_j = K_j \beta_j^{-1/\rho_j} \left[1 - \alpha_j^{1/(1+\rho_j)} \left(\frac{W}{P_j}\right)^{\rho_j/(1+\rho_j)}\right]^{1/\rho_j}; \quad j = 1, 2\]

**Consumer demand functions:**

\[(3B.1) \quad C_j = \left(\delta_j / P_j\right) \sum_{i=1}^{2} P_i Y_i; \quad j = 1, 2.\]

**Market clearing conditions for commodities:**

\[(3.29') \quad C_i = Y_i\]

Labour Market:

**Sectoral labour demand:**

\[(3.14') \quad L_j = K_j \alpha_j^{-1/\rho_j} \beta_j^{-1/\rho_j} \left[(P_j / W)^{\rho_j/(1+\rho_j)} - \alpha_j^{1/(1+\rho_j)}\right]^{1/\rho_j}; \quad j = 1, 2.\]

**Labour market equilibrium:**

\[(3.15') \quad L = \sum_{j=1}^{2} L_j; \quad j = 1, 2.\]

Rental Rates:

\[(3.12') \quad R_j = \beta_j^{-1/\rho_j} \left(P_j^{\rho_j/(1+\rho_j)} - \alpha_j^{1/(1+\rho_j)} W^{\rho_j/(1+\rho_j)}\right)\left(1+\rho_j\right)^{1/\rho_j}; \quad j = 1, 2\]

**Price Normalization Rule:**

\[(3.30) \quad P_2 = 1\]

**List of Endogenous Variables:**

- \(Y_j\) \quad j = 1, 2. : 2 Sectoral outputs
- \(C_j\) \quad j = 1, 2. : 2 Domestic demands
- \(L_j\) \quad j = 1, 2. : 2 Sectoral employment of labour
- \(R_j\) \quad j = 1, 2. : 2 Sectoral rental rates in units of commodity 2
- \(W\) : 1 Wage rate in units of commodity 2
- \(P_1\) : 1 Price of commodity 1 in units of commodity 2

Total number of endogenous variables: 10.

Total number of equations 10.
The dimension of the model in autarky is reduced to ten equations in ten variables. The autarkic relative price is completely endogenous, there is no foreign price effect. Trade taxes will be adjusted to maintain the autarkic equilibrium. Therefore, there will be no trade, and tariff revenue collections will be zero.

Assume that \( E^a \) represents the vector of the level of endogenous variables at autarky equilibrium. Then the model can be linearized around \( E^a \). The linearized version of the model is listed in Table 3B.3.

### Table 3B.2
Linearized version of the CGE model in autarky

<table>
<thead>
<tr>
<th>The supply functions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3B.2) ( y_j^a = k_j + \sigma \left( \frac{S_{lj}^a}{S_{kj}^a} \right) (p_j^a - w^a) ); ( j = 1, 2 ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Commodity demand functions</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3B.3) ( c_j^a = -p_j^a + \sum_{j=1}^{2} H_i^a (p_i^a + y_i^a) ); ( j = 1, 2 )</td>
</tr>
</tbody>
</table>

where, 
\( H_i^a = \frac{p_i^a y_i^a}{\sum_{j=1}^{2} p_j^a y_j^a} \).

<table>
<thead>
<tr>
<th>Labour demands functions:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3B.4) ( l_j^a = k_j + \sigma \left( \frac{S_{kj}^a}{S_{kj}^a} \right) (p_j^a - w^a) ); ( j = 1, 2 ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Labour market equilibrium condition:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3B.5) ( l = \sum_{j=1}^{2} \lambda_j^a l_j^a ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>The rental rates:</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3B.6) ( r_j^a = \frac{1}{S_{kj}^a} (p_j^a - S_{kj}^a w^a) ); ( j = 1, 2 ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Market clearing condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3B.7) ( c_i = y_i ).</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Price Normalization Rule</th>
</tr>
</thead>
<tbody>
<tr>
<td>(3B.8) ( p_2^a = 0 ).</td>
</tr>
</tbody>
</table>

One of the important features of the set of equations presented in Table 3B.2 is the presence of demand functions, which were absent in other models. The reason is that the demand functions may play an important role in determining relative
commodity price in autarky whereas in the observed state of the economy the relative price is determined by foreign price and tariff rate.

Substituting the demand function for commodity 1 from the equation (3B.3) into the market clearing condition (3B.7) and solving for the relative price, noting that \( p_2^e = 0 \) and that \( \sum_i H_i^e = 1 \), yields

\[
(3B.9) \quad p_1^e = y_2^e - y_1^e.
\]

The equation (3B.9) has a strong\(^{15}\) rule of price change. It shows that the relative price of the commodity, whose supply grows at a relatively faster rate falls. The rate of fall is equal to the differential in the growth rate of outputs in the two sectors.

For future reference we call it the \textit{autarky rule of price}.

Substituting the output supply function from the equation (3B.2) into the equation (3B.9) and solving for price we get

\[
(3B.10) \quad p_1^e = \mu_0 (k_2 - k_1 + \mu_1 w^e),
\]

where,

\[
(3B.11) \quad \mu_0 \equiv 1/\left(1 + \sigma_1 S_{K1}^e / S_{K1}^e\right) > 0 \quad \text{and} \quad \mu_1 \equiv \frac{\sigma_2 S_{K1}^e}{S_{K1}^e} - \frac{\sigma_2 S_{K2}^e}{S_{K2}^e}.
\]

Solving the labour market equilibrium condition (3B.5) for the wage response using the demand functions given by the equation (3B.4) yields

\[
(3B.13) \quad w^e = \psi_0 (\lambda_1^e k_1 + \lambda_2^e k_2 - l) + \psi_1 p_1^e,
\]

where,

\[
(3B.14) \quad \psi_0 = 1/[\lambda_1^e \sigma_1 / S_{K1}^e + \lambda_2^e \sigma_2 / S_{K2}^e] > 0 \quad \text{and} \quad \psi_1 = \psi_0 \lambda_1^e / S_{K1}^e > 0.
\]

\(^{15}\) The above assertion is based on the underlying assumptions that the economy does not trade, and that the preferences of the consumer can be represented by a Cobb-Douglas utility function. It is shown by Willig (1976) that if the price elasticities of demand are locally constant, then the only form of utility function that is consistent with neo-classical utility maximization is the Cobb-Douglas. In a two good case it can be easily seen that the direction of price change (not the magnitude) implied by the equation (2B.9) is valid with any utility function that yields demand functions with income elasticity equal to unity. Finally, if the economy trades with the rest of the world, then the autarky model could be applied to the world economy. The market clearing condition that will be applicable at a global level will imply a similar conclusion at global level. But the conclusion will not necessarily hold at a national level.
The equations (3B.10) and (3B.13) can be solved for the two variables $p^*_i$ and $w^*$. The solution yields

$$w^* = \frac{1}{1 - \psi_j \mu_0 \mu_1} \left[ (\psi_0 \lambda^*_i - \psi_j \mu_0) k_1 + (\psi_j \mu_0 + \lambda^*_2 \psi_0) k_2 - \psi_0 \right]$$

and

$$p^*_i = \frac{\mu_0}{1 - \psi_j \mu_0 \mu_1} \left[ (1 + \mu_1 \lambda^*_2 \psi_0) k_2 - (1 - \mu_1 \lambda^*_1 \psi_0) k_1 - \mu_1 \psi_0 \right].$$

The effects of changes in endowment variables on the equilibrium wage rate and the relative price of commodity 1 are given by the equations (3B.16) and (3B.17). Substitution of the values of $p^*_i$ and $w^*$ into the equations (3B.2), (3B.4) and (3B.6) yield the effects on sectoral outputs, employment and the rental rates respectively. The elasticities of the endogenous variables with respect to the change in endowment variables can be evaluated in turn by changing one endowment variable at a time. In particular, the response of the rental rate in sector 2 can be obtained from the equation (3B.6) as

$$r^*_2 = -\frac{S_{k2}}{S_{x2}(1 - \psi_j \mu_0 \mu_1)} \left[ (\psi_0 \lambda^*_2 - \psi_j \mu_0) k_1 + (\psi_j \mu_0 + \lambda^*_2 \psi_0) k_2 - \psi_0 \right].$$

Since the price of commodity 2 is held fixed (it is the numeraire), it is obvious that the channel through which a change in exogenous variable may affect the rent in sector 2 is the wage rate. If the shock leads to an increase in the wage rate the rental rate will fall, and if it leads to a decrease in the wage rate the rental rate in sector 2 will rise.

The mechanism through which the effects of an exogenous shock are transmitted to the wage rate is as follows. Ceteris paribus an increase in the stock of capital in sector 2 produces effects on the wage rate in two rounds. In the first round, the productivity and hence the demand for labour is increased in sector 2. Therefore, the aggregate demand for labour increases and the wage rate will go up to clear the labour market. As a result, the employment of labour and the level of output in sector 2 rise whereas the employment of labour and the output in sector 1 fall. It will produce the second round effect - the relative price effect. The positive relative growth of output in sector 2 will raise the relative price of good 1 (equation 3B.9) which, in turn, raises the demand for labour of sector 1 and the wage rate will rise further.

Similar arguments can be provided to explain the effects of an increase in the stock of capital in sector 1. The first round effect on the wage rate of an increase in the stock of capital in sector 1 is quite similar - it will lead to an increase in the wage rate. However, the relative price effect is different. As the stock of capital in sector 1 increases, the employment of labour and output in sector 1 increase and the
employment of labour and output in sector 2 fall. The relative decline in the output of sector 2 lowers the price of commodity 1 at constant price of commodity 2. The real wage rate faced by sector 1 increases and the labour demand schedule shifts back producing a fall in the wage rate.

The net effect of a change in $K_1$ on the wage rate, therefore, depends on the relative size of the productivity effect and the relative price effect. It can be shown that the productivity effect dominates the relative price effect if the elasticity of factor substitution in sector 1 is less than unity$^{16}$ That is, the wage rate rises if the capital stock in sector 1 cannot easily substitute labour.

Thus it is clear from the equation (3B.18) that the rental rate in sector 2 will increase in autarky if (1) the supply of labour in the economy increases, or (2) stock of capital in sector 2 declines, or (3) stock of capital in sector 1 increases with $\sigma_i > 1$ or stock of capital in sector 1 decreases with $\sigma_i < 1$. If $\sigma_i = 1$ then change in the stock of capital in sector 1 will have no impact on the rental rate of sector 2.

$^{16}$ Using the defining equations (2B.11), (2B.14) and (2B.15) it can be shown that

$$\psi_0 \lambda_1 - \psi_1 \mu_0 = \frac{\lambda_1^e}{\lambda_1^e} \frac{S_{K1}^e (1 - \sigma_1)}{[\lambda_1^e \sigma_1 / S_{K1}^e + \lambda_2^e \sigma_2 / S_{K2}^e] (S_{K1}^e + \sigma_1 S_{L1})}.$$ 

Therefore, the sign of the coefficient of $k_1$ in the equation (3.74) depends on the size of $\sigma_1$. 
CHAPTER 4

THE RENT TRANSFORMATION FRONTIER

Introduction

It was shown in the previous chapter that, as far as the owners of the sector specific factors are concerned, any change in the rationalized tariff rate means a reduction in the rental income of one sector and a gain in the rental income of the other. An increase in the rationalized tariff rate increases the rent to the specific factor in the import competing sector, and decreases the rent to the specific factor in the exporting sector. In other words, a change in the tariff rate, ceteris paribus, effects a transfer of rents between the owners of the sector specific factors. The purpose of this chapter is to derive a rent transformation frontier that describes the combinations of Pareto efficient distributions of rental incomes, and to study its nature and comparative static properties. This chapter together with chapter 3 completes the description of the economic sphere of a small open political economy.

A study of the nature and comparative static properties of the rent transformation frontier is important for two reasons. First, it shows the condition under which the rent possibility set is compact and convex, which is a sufficient condition for a unique bargaining solution. This, in turn, provides a preview of the applicability of the bargain-theoretic approach in endogenizing the tariff formation process. Second, it also describes the way the rent possibility set responds to the changes in the exogenous variables. A clear understanding of this feature of the rent possibility set is necessary in understanding the comparative static behaviour of the bargaining solution that will be derived in the later chapters.

This chapter is divided into five sections. The first section analyzes the second order properties of the rental functions and derives the rent transformation function. The second section discusses the shape of the frontier. The third section compares the rent transformation frontier with the product transformation frontier. The fourth section derives the comparative static properties of the frontier. The chapter is concluded in section five.

4.1 Derivation of the Rent Transformation Function

Given the world relative price, domestic factor endowments, technologies and tastes, a general economic equilibrium can be defined for each level of tariff rate. A vector of levels of endogenous variables that solves the system of equations (model) listed in Table 3.2 constitutes the economic equilibrium. Since, the rental incomes to the
sector specific factors are among the endogenous variables of the model, a particular
combination of the rental incomes is associated with each tariff rate. Using the fact that
the domestic relative price is in one-to-one correspondence with the tariff rate a one-to
one correspondence can be defined between the domestic relative price and the
combination of sectoral rental rates.

Let us define, for each sector $i$

\begin{equation}
\tilde{R}_i \equiv \frac{R_i}{P_i}
\end{equation}

where $R_1$ and $R_2$ measure the equilibrium rental rates in units of the numeraire, the
commodity 2. It is clear from the defining equation (4.1) that $\tilde{R}_i$ simply measures the
(virtual) rental rate of the specific factor in each sector in units of the respective
commodity - that is commodity $i$. We will call $\tilde{R}_i$ the real rental rate of the specific
factor.

We know from equation (3.48) that the real rental rate in each sector are
increasing functions of the relative price of their own commodity. When commodity 2
is chosen as the numeraire and the domestic price of commodity 1, $P_i$, is expressed in
terms of commodity 2, then $P_1$ becomes the only price variable in the model that is
affected by tariff (policy) changes. Moreover, the result (3.48) implies that $\tilde{R}_i$ is an
increasing function of $P_1$ whereas $R_2$ is a decreasing function of $P_1$. The second order
properties of the rental functions can be assessed in the following way.

We can obtain from equations (3.6), (3.11) and (3.12) that

\begin{equation}
\tilde{R}_i = \beta_i \left( \frac{Y_i}{K_i} \right)^{1+\rho_i}
\end{equation}

Equation (4.2) simply restates that the real rental rate in each sector is equal to the
marginal physical product of the specific factor. Differentiating both sides of equation
(4.2) totally with respect to $P_i$ yields

\begin{equation}
\frac{d\tilde{R}_i}{dP_i} = \beta_i K_i^{-1(1+\rho_i)(1+\rho_i)} Y_i^{\rho_i} \frac{dY_i}{dP_i}
\end{equation}

Equation (4.3) confirms the first order property of the rental function mentioned
above. Differentiating both sides of equation (4.3) again with respective to $P_i$ we get

\begin{equation}
\frac{d^2\tilde{R}_i}{dP_i^2} = \beta_i K_i^{-1(1+\rho_i)} (1+\rho_i) Y_i^{\rho_i} \left[ \rho_i Y_i^{-1} \left( \frac{dY_i}{dP_i} \right)^2 + \frac{d^2Y_i}{dP_i^2} \right]
\end{equation}

We know that for each sector $i$,
\[ \beta_i x_i^{(1 + \rho_i)} (1 + \rho_i) y_i^{\rho_i} > 0, \text{ and} \]
\[ y_i^{-1} \left( \frac{d y_i}{d p_i} \right)^2 > 0. \]

Therefore, for each sector \( i \), the sufficient conditions for \( (4.5) \) are that

(i) \( -1 < \rho_i \leq 0; \) and
(ii) \( \frac{d^2 y_i}{d p_i^2} < 0 \)

Now we can interpret the second order property of the rental function of each sector by taking the commodity of the other sector as the numeraire in turn. In other words, while evaluating the second order property of the rental function (and also the output supply function) of sector 1, we will consider commodity 2 as the numeraire and express the price of commodity 1 in units of commodity 2, and while evaluating the second order property of the rental function (and the output supply function) of sector 2, we will consider commodity 1 as the numeraire.

The first of the above two conditions requires that in both sectors the long run elasticities of factor substitution be finite and greater than or equal to unity. The second condition requires that, in general equilibrium, the supply of output of each sector grow at a decreasing rate as its relative price (in terms of units of the other commodity) increases. Whether condition (ii) is met generally or not when condition (i) is satisfied, is a pertinent question. Instead of evaluating the second order property of the supply functions we make the following assumptions on the production technologies (and, hence on output supply functions).

**Assumption 4.1** In what follows we will assume that the above conditions (i) and (ii) hold. In other words, we assume that the elasticities of factor substitution in both sectors are at least unity; and that in each sector the output supply is a concave function of its relative price.\(^1\)

\(^1\) The statement that the output supply of each sector is a concave function of its relative price is, in fact, a conjecture rather than an assumption. The reason is that given the production functions, and the factor endowments it should be possible to obtain the exact natures of the sectoral output supply functions. For example, if the production functions are Cobb-Douglas in both sectors, then the condition
It follows from condition (4.5) that if Assumption 4.1 holds, then the real rental function of each sector is increasing and concave in the relative price of own commodity. Since the relative price of commodity 1 is the inverse of the relative price commodity 2 and vice versa, the following proposition follows:

**Proposition 4.1** The equilibrium real rental rate in each sector is an increasing and a concave function of the relative price of own output, and decreasing function of the relative price of the output of the other sector.

Now, clearly if the domestic relative price of commodity 1 is $P_1$, then the combination of the equilibrium rental rates of the two sectors is given by the pair $(\bar{R}_1(P_1), \bar{R}_2(P_1))$. Various combinations of equilibrium rental rates can thus be obtained by varying the domestic relative price of commodity 1 to obtain a rent transformation frontier analogous to Gardner's (1983, 1987) surplus transformation curve.\(^2\)

**Definition 4.1** The locus of the combinations of equilibrium rental incomes (or rates) in units of own output corresponding to each tariff rate (or domestic relative price) is defined as the Rent Transformation Frontier (RTF). A function that describes the locus is the Rent Transformation Function.

(ii) directly follows from the sectoral output supply functions given by equation (3.36), the elasticity of the equilibrium wage rate given by equation (3.43), and Proposition 3.2. This result can be seen as follows.

Holding capital stocks fixed, the equilibrium sectoral output supply functions given by equation (3.36) can be expressed as

$$\frac{dY_i}{dP_i} = \frac{Y_i}{P_i} \left( \frac{S_{Li}}{S_{Ki}} \right) (1 - \eta(P_i))$$

Differentiating this equation with respect to $P_i$ we get

$$\frac{d^2Y_i}{dP_i^2} = \frac{S_{Li}}{S_{Ki}P_i^2} \left[ Y_i \theta(\theta - 1) + P_i Y_i \frac{d\theta}{dP_i} \right]$$

where $\theta(P_i) = (1 - \eta(P_i))$.

Since $0 < \theta(P_i) < 1$ by equation (3.43), and $\theta(P_i)$ is a decreasing function of $P_i$ by Proposition 3.2, it follows that

$$\frac{d^2Y_i}{dP_i^2} < 0,$$

if production functions are Cobb-Douglas.

Whether this result holds for any arbitrary CES production function with elasticity of factor substitution finitely greater than unity or not is still a question. When the elasticities of substitution are greater than unity, then the distributive shares will not remain constant as the relative price changes. For example, if the relative price of commodity 1 increases, then the distributive share of the mobile factor in sector 1 increases and the distributive share of mobile the factor in sector 2 decreases. Evaluation of the second order derivatives of the real rental functions when the elasticities of factor substitution are greater than unity thus became quite involved and is left for future studies.

\(^2\) See also Bullock (1992) for an application of Gardner's concept of surplus transformation curve.
Given the relative price of commodity 1, the equilibrium real rental rates for the two sectors, \( \tilde{R}_1(P_1), \tilde{R}_2(P_1) \), can be obtained from equation (3.12') as functions of the wage rate and the relative price of commodity 1. Since both sectors face the same wage rate, the rental function for sector 2 can be used to eliminate wage rate from the rental function for sector 1. As a result we can obtain the Rental Transformation Function as

\[
(4.6) \quad \tilde{R}_1 = P_1^{-1} \beta_1 \frac{\sigma_1}{1-\sigma_1} \left[ P_1^{1-\sigma_1} - \alpha_1 \alpha_2 \left( 1 - \beta_2^{\sigma_2} R_2^{1-\sigma_3} \right)^{1-\sigma_3} \right]^{1-\sigma_1}.
\]

Equation (4.6) expresses the rental rate to the specific factor in sector 1 as a function of the rental rate to the specific factor in sector 2. It is a condition that will be satisfied by the rental rates at all equilibrium points and defines a locus in \( \tilde{R}_1 \times R_2 \) plane which coincides with the sectoral output plane.\(^3\)

The slope of the rental transformation function can be obtained from the properties of the rental functions. Since,

\[
\frac{d\tilde{R}_1}{dR_2} = \frac{d\tilde{R}_1}{dP_1} \frac{dP_1}{dR_2} = \left( \frac{R_1 / P_1}{R_2} \right) \left( \frac{r_1 / p_1 - 1}{r_2 / p_1} \right)
\]

and from equation (3.48) we have \( r_1 / p_1 > 1 \), and from equation (3.49) we have \( r_2 / p_1 < 0 \), it follows that

Three points may be noted here. First, the correct expression that describes the transformation of the real rental incomes as the relative price change can be obtained as follows.

Let \( \Pi_i = \tilde{R}_i K_i \). Then \( \Pi_i \) measures the real rental income in sector \( i \). By following the same procedures we can obtain the rent transformation frontier as

\[
(4.6') \quad \frac{\Pi_1}{K_1} = P_1^{-1} \beta_1 \frac{\sigma_1}{1-\sigma_1} \left[ P_1^{1-\sigma_1} - \alpha_1 \alpha_2 \left( 1 - \beta_2^{\sigma_2} \frac{\Pi_2}{K_2} \right)^{1-\sigma_3} \right]^{1-\sigma_1}.
\]

It is clear that equation (4.6') can be obtained from equation (4.6) simply by dividing and multiplying the rental rate terms in (4.6) by the respective values of the sectoral capital stocks. The properties of the two forms of expression are essentially the same. The frontier described by equation (4.6) will be referred as the rental transformation frontier, and that by equation (4.6') will be referred as the rent transformation frontier.

Second, so long as commodity 2 is used as the numeraire the nominal rental rate equals the real rental rate in sector 2, and so the nominal rental income equals the real rental income in sector 2. Therefore, we will continue to use \( R_2 \) to denote the real rental rate \( \tilde{R}_2 \) in sector 2.
Thus, the rental transformation frontier slopes downward to the right. This result holds for any arbitrary equilibrium of the economic sphere, and does not depend on the values of the share parameters at any specific equilibrium point. Hence, a proposition follows:

**Proposition 4.2**  At every equilibrium of the economic sphere, a tariff change benefits the owner of the specific factor in one sector and hurts the owner of the specific factor in the other sector. Therefore, the owners of sector specific factors will have a conflict of interest with respect to tariff changes.\(^4\)

### 4.2 The Shape of the Rent Transformation Frontier

Substituting the expressions for \( r_i / p_i \) and \( r_j / p_j \) from the equations (3.48) and (3.49) into the equation (4.7) and simplifying the expression making use of equation (3.43), which gives the price elasticity of the wage rate, yields

\[
\frac{d\tilde{R}_1}{dR_2} = \frac{\tilde{R}_1}{R_2} \left( \frac{\lambda_2 \sigma_2 S_{L1}}{\lambda_1 \sigma_1 S_{L2}} \right).
\]

Differentiating both sides of equation (4.8) with respect to \( R_2 \) yields, on further simplification,

\[
\frac{d^2\tilde{R}_1}{dR_2^2} = \frac{\sigma_2 \lambda_2 S_{L1}}{\sigma_1 \lambda_1 S_{L2}} \left( \frac{\tilde{R}_1}{R_2} \right) \times
\]

\[
\left[ \left( \frac{R_2 d\tilde{R}_1}{R_1 dR_2} - 1 \right) + R_2 \left( \frac{d\lambda_2}{\lambda_2 dR_2} - \frac{d\lambda_1}{\lambda_1 dR_2} \right) + \left( \frac{dS_{L1}}{S_{L1} dR_2} - \frac{dS_{L2}}{S_{L2} dR_2} \right) \right].
\]

Since \( \lambda_i = L_i / L \), and since the motive variable for change in this general equilibrium system is the domestic relative price, then

\[^4\text{This may not be true if we are analysing endogenous tariff changes \textit{ex ante} in which case the owners of the specific factors may agree in the direction of tariff changes. For a demonstration of this and other interesting results see Cassing, Hillman, and Long (1986). Furthermore, this apparent conflict between the owners of the sector specific factors is a consequence of the adoption of the specific factor model. If we take a long run view and adopt Heckscher-Ohlin-Samuelson model that allows capital to move across the sectors, then the basic conflict will be between labour and capital as predicted by the Stolper-Samuelson theorem (see for example, Brock, Magee and Young, 1989). In this case it would be more useful to obtain the factor income transformation frontier and evaluate its properties than to stay with the RTF defined here.}\]
(4.10) \[ \frac{dS_{l_2}}{dR_2} = \frac{dS_{l_1}}{dR_2} \frac{dP_j}{dP_i}; \quad j = 1,2; \] and
\[ \frac{dL_j}{dR_2} = \frac{dL_j}{dR_2} \frac{dP_i}{dP_j}; \quad i = 1,2. \]

Using equation (4.8), equation (4.9) can be rewritten as
\[ d^2 \tilde{R}_1 = \left( \frac{\sigma_2 \lambda_2 S_{l_2}}{\sigma_1 \lambda_1 S_{l_1}} \right) \left( \frac{\tilde{R}_1}{R_2^2} \right) \times \left[ \left( -\frac{\sigma_2 \lambda_2 S_{l_1}}{\sigma_1 \lambda_1 S_{l_2}} - 1 \right) + \frac{(l_2 / p_1 - l_1 / p_1) + (s_{l_1} / p_1 - s_{l_2} / p_1)}{r_2 / p_1} \right]. \]

Making use of the equations (3.28), (3.44) and (3.45) around any arbitrary equilibrium equation (4.12), after a simplification, can be rewritten as
\[ d^2 \tilde{R}_1 = \left( \frac{\sigma_2 \lambda_2 S_{l_2}}{\sigma_1 \lambda_1 S_{l_1}} \right) \left( \frac{\tilde{R}_1}{R_2^2} \right) \times \left[ -(\sigma_2 \lambda_2 S_{l_1} + \sigma_1 \lambda_1 S_{l_2}) + \sigma_1 \sigma_2 (1 - \sigma_1)(\lambda_2 \sigma_2 - \sigma_2 \lambda_2 S_{l_1}) + (1 - \sigma_2)(\lambda_1 \sigma_1 - \sigma_1 \lambda_1 S_{l_2}) \right]. \]

On a further simplification using the adding up property of the distributive and employment shares the equation (4.13) can be rewritten as
\[ d^2 \tilde{R}_1 = \left( \frac{\sigma_2 \lambda_2 S_{l_1}}{\sigma_1 \lambda_1 S_{l_2}} \right) \left( \frac{\tilde{R}_1}{R_2^2} \right) \times \left[ \sigma_2 \lambda_2 S_{l_1} (\sigma_1 - 1) + \sigma_1 \lambda_1 S_{l_2} (\sigma_2 - 1) + \sigma_1 \lambda_1 S_{x_2} + \sigma_2 \lambda_2 S_{x_1} \right]. \]

Thus, it is clear from the right hand side of the equation (4.14) that the condition \( \sigma_1 \geq 1 \) and \( \sigma_2 \geq 1 \) is sufficient for \( \frac{d^2 \tilde{R}_1}{dR_2^2} < 0 \). These conditions hold under assumption 4.1, and thus we have the following proposition:

**Proposition 4.2**  
If the rental rates are measured in units of own output; and the long run elasticities of factor substitution are at least unity in both the sectors, then the rental transformation function is concave and negatively sloped. That is, the rental transformation frontier slopes downward to the right and is concave to the origin in the rental plane, and the set of feasible combinations of rental rates (and rental income) is convex.
4.3 The Product and Rent Transformation Frontiers

On the basis of these results, the nature of the rent transformation frontier can be illustrated geometrically. Figure 4.1 shows the rental and product transformation frontiers for given world price, endowment of sector specific factors, and the economy wide supply of the mobile factor.

![Product and Rent Transformation Frontiers](image)

The rent and product transformation frontiers

Figure 4.1

In this four quadrant figure, quadrants II and IV show the production functions of sector 1 and sector 2 respectively. The total endowment of labour in the economy is $OL (= OL')$. The x-axis in quadrant I measures the output of commodity 2, and the y-axis measures the output of commodity 1. The curve AB represents the usual product transformation frontier - it shows the combination of maximum attainable output of one sector given the output level of the other sector. For example, if all labour is employed in sector 2, $OB$ units of commodity 2 will be produced. Alternately, if all labour is employed in sector 1, $OA$ units of commodity 1 will be produced, while the output of commodity 2 will be zero.\(^5\) The curve AB shows the transformation possibilities of commodity 2 into commodity 1 and vice versa.

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\(^5\) Note that if the production functions are not characterized by unitary elasticity of factor substitution, then the output of sector 2 will not be zero even if all of the mobile factor is employed in sector 1. Similarly, the output of sector 1 will not be zero even if all of the mobile factor is employed in sector 2. Each sector can produce a minimum quantity of its output by employing the sector specific...
The product transformations in equilibrium, under the maintained hypothesis that production sectors are profit maximizers, can always be induced by exogenous changes in domestic relative price of commodities. The mechanism behind this transformation is that changes in the relative price of the commodities alter the equilibrium wage rate, which will induce a reallocation of the mobile factor - labour, between the sectors. Thus each point on the curve AB is a point of production equilibrium that corresponds to a particular domestic relative price of commodities.

Since the mobile factor (labour) is paid its marginal product, AC represents the wage bill of sector 1 and OC represents the rent to the specific factor in sector 1 when all labour is employed by sector 1. Rent to the specific factor in sector 1 is zero (see quadrant II). Similarly, when all labour is employed by sector 2, DB represents the wage bill and OD represents the rent to the specific factor in sector 2, while the rent to the specific factor in sector 2 is zero (see quadrant IV). The curve CD traces out the combination of the real rents to the two sector-specific factors through reallocation of labour between the two sectors induced by exogenous change in the domestic relative price of commodities.

The product transformation frontier and domestic price ratios are well known tools of economists that help to locate the equilibrium product mix. It is natural to enquire about the location of equilibrium rents for given commodity prices.

We know from equations (3.11) and (3.12) that the specific factors, in equilibrium, are paid their corresponding value of marginal product. Therefore, using equation (3.6) and noting that commodity 2 is the numeraire, we can write

\[
\frac{R_1}{R_2} = \frac{\beta_1 (Y_1 / K_1)^{1+\rho_1}}{\beta_2 (Y_2 / K_2)^{1+\rho_2}}.
\]

Hence, by expressing the rents in units of own output the equation (4.15) can be rewritten as

\[
\frac{K_1 \tilde{R}_1}{K_2 R_2} = \frac{\beta_1 K_1^{\rho_1}}{\beta_2 K_2^{\rho_2}} \left( \frac{Y_1^{1+\rho_1}}{Y_2^{1+\rho_2}} \right).
\]

Given the parameters of the production functions and the stock of the specific factors, equation (4.16) provides the translation from output levels to the ratio of

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factor only. In this case both the product and the rent transformation frontiers will have some linear segments on both ends. However, the above properties of the frontiers will remain unchanged. In what follows we will ignore this possibility until chapter 7, since at this level of aggregation corner solutions are rare possibilities.
sectoral rental incomes. In general, it can be seen from equation (4.16) that the relation between rental mix and output mix is non-linear.

In particular, if the production function in both the sectors are Cobb-Douglas, then both $\rho_1$ and $\rho_2$ tend to zero and the equation (4.16) reduces to

\begin{equation}
\frac{K_1\tilde{R}_1}{K_2\tilde{R}_2} = \frac{\beta_1}{\beta_2} \left( \frac{Y_1}{Y_2} \right).
\end{equation}

Now it is easy to see that the rental mix (ratio of rental incomes) is proportional to the output mix (ratio of sectoral outputs). This relationship can be illustrated graphically.

As in Figure 4.1, AB and CD represent the product transformation frontier represents the rent transformation frontier respectively. Given that the relative price of commodity 1 is $P_1$, the optimal output mix is determined by the point E where the absolute value of the slope of the product transformation frontier is $1/P_1$. Sectoral outputs of the two sectors are respectively $Y_1$ and $Y_2$. The slope of the ray OE shows the equilibrium output mix.

If the shares of capital in the two sectors are equal (that is, if they have identical Cobb-Douglas production functions), then it follows from equation (4.17) that the rental
mix will also be given by the slope of the same ray OE. The rental equilibrium will be at the point F on the rental transformation frontier.\textsuperscript{6} If the share of capital in sector 1 is greater than the share of capital in sector 2, (that is, if $\beta_1 > \beta_2$) the equilibrium rental mix will be given by the slope of the ray, such as OG, which lies above the ray OE. If $\beta_1 < \beta_2$ then the rental mix will be given by the slope of the ray such as OH which lies below the ray OE. But the relation between the two rays that show the output mix and the rental mix is stable in the sense that the slopes of the two rays always remain in constant proportion. In general, there is no reason to assume that the production technologies of the two sectors are identical and Cobb-Douglas, and therefore, the value of the rent transformation frontier in locating distributive equilibrium geometrically is quite limited. Nevertheless, with sufficient structure imposed, it can be a useful illustrative device.

4.4 Comparative Static Properties of the RTF

This section identifies the variables that bring shifts and movements along the rent transformation frontier. It then describes the nature of the effects of changes in the exogenous variables on the RTF.

(a) Changes in endowment

Changes in the endowments of the factors in the economy bring about changes in the capacity of the economy to produce. In general, these changes are responsible for the shifts in the RTF. For example, an increase in the stock of capital in sector 1 will increase the level of output that it can produce in specialization through the resulting increase in the productivity of labour. If the consequent increase in the marginal product of labour is not very high, so as not to raise wage rates prohibitively high, then the rental income of sector 1 will also increase. However, the rental income of sector 2 under specialization will be unaffected by change in the stock of capital in sector 1. Therefore, the RTF will shift in favour of sector 1. A similar argument applies to the case with an increase in the stock of capital in sector 2. It will induce a shift in the RTF in favour of sector 2.

\textsuperscript{6} In fact, using the definition of cost share of labour from the equation (3.28), employment share from (3.30), and noting that rental mix is given by equation (4.17), it can be seen from equation (4.8), by normalizing the sectoral stock of capital to unity, that the slope of the rental transformation function under Cobb-Douglas technologies is given by

$$\frac{dR_1}{dR_2} = \frac{\beta_1}{\beta_2} \frac{1}{P_1}.$$  

Therefore, if the share of capital in the two sectors are equal then in equilibrium the slope of the rental transformation frontier will be equal to the slope of the product transformation frontier. That is, the product transformation frontier is a proportionate blow out of the rent transformation frontier.
Similarly, an increase in the stock of labour in the economy, ceteris paribus, lowers the wage rate which, in turn, contributes to an increase in the rental income of either sector. The extent of shift in the rental income under specialization, as will be indicated below, depends on the distributive shares of labour and the elasticities of factor substitution in the two sectors. The intuitive mechanism can be tracked from equation (3.42) which yields the response of the wage rate as labour supply changes, and from equation (3.39), which yields the response of rental rate as wage rate changes.

Analytically, the effects of the endowment changes in the sectoral rental rate can be observed from the equations (3.46) and (3.47). The extreme points of the RTF can be observed by creating situations of specialization in the employment of the mobile factor, which can be ensured by setting $\lambda_1 = 1$ and $\lambda_2 = 1$ respectively.

When $\lambda_1 = 1$, the effect of an endowment change in the rental rate of sector 1, at constant commodity prices, can be obtained from equation (3.46) and is rewritten as

$$r_1 = -\frac{S_{L_1}}{\sigma_1} (k_1 - l).$$

Therefore, ceteris paribus, for $k_1 > 0$, the percentage change in the maximal rental income of sector 1 can be written as $k_1 + r_1 = (1 - S_{L_1} / \sigma_1) k_1$ that is positive for $\sigma_1 \geq 1$ and; for $l > 0$, we have $r_1 = (S_{L_1} / \sigma_1) l$ which is always positive. That is, the rental income of sector 1 will increase under specialization as the national endowment of labour and/or the stock of capital in sector 1 increases, and the elasticities of factor substitution are not very small.

Similarly, by setting $\lambda_2 = 1$ we can obtain the effect of endowment changes on the rental income of sector 2. For $k_2 > 0$, ceteris paribus, we can obtain from the equation (3.47) that $k_2 + r_2 = (1 - S_{L_2} / \sigma_2) k_2$ which is positive for $\sigma_2 \geq 1$, and for $l > 0$, we get $r_2 = (S_{L_2} / \sigma_2) l$ which is always positive. These results show that the maximal rental income of sector 2 will increase with an increase in the endowment of labour and/or with an increase in the stock of capital in sector 2, provided the elasticity of factor substitution is not very small.

Figure 4.3 shows typical patterns of shifts on the RTF as endowments change. Panel (a) shows the effect of an increase in the stock of capital in sector 1, panel (b) shows the effect of an increase in the stock of capital in sector 2, and panel (c) shows the effect of an increase in the endowment of the mobile factor on the RTF. The axes measure the rental incomes in units of own output. Numbers on the axis represent the respective sectors or the owners of the specific factors.
(b) *Changes in price variables:*

Any change in the domestic relative price, by definition of RTF, causes a movement along the RTF. Changes in the domestic relative price can be caused by a change in the tariff rate and or by a change in the international relative price. Therefore, changes in the world price and the tariff rate cause a movement along the frontier.

To illustrate, let $P_1^*$ be the initial world price ratio and $T_1$ be the tariff rate. The figure illustrates that the imposition of tariff rate raises the rental income of sector 1 and
lowers the rental income of sector 2. Suppose the world relative price of commodity 1 falls, and the world relative price becomes $\bar{P}_1^*$. Then, at an unchanged tariff rate the domestic relative price of commodity 1 also falls and consequently, the combination of equilibrium rental incomes of the sectors is given by the point $R(\bar{P}_1^*(1 + T_i))$. If the tariff rate is increased to compensate for the fall in world price then the rental income will again be represented by the point marked $R(\bar{P}_1^*(1 + T_i))$ but with a different value for the tariff rate variable, $T_i$.

Thus, if the source of the shock is the tariff rate or the international relative price, then the effect will be a movement along the RTF. If the source of the shock is a change in one of the factor endowments, then the effect will be a shift on the RTF.

4.5 Summary

Basing on a simple general equilibrium model of a small open economy described in the previous chapter a rent transformation function has been derived in this chapter. The corresponding rent transformation frontier traces the equilibrium combinations of the sectoral rental incomes as the tariff rate changes. Along a frontier an increase in the rental income of one sector necessarily reduces the rental income of the other sector. The mechanism behind this transfer is the induced difference in the real wages faced by the two sectors, which further induces a reallocation of labour between the two sectors and hence the outputs and the rents.

It has been shown that if the long run elasticity of factor substitution is at least unity in both sectors and rents are measured in units of own output, then the rental transformation frontier is concave to the origin and lies inside the product transformation frontier. This shows that the sectoral rental functions, that yield rental income at each relative price, are bounded, and the real-rent possibility set is convex.

Any change in the endowment of factors will cause a shift on the frontier and a change in the tariff rate or the international price ratio will cause a movement along the frontier. Since, the tariff rates are such powerful instruments of income redistribution an obvious question that follows is how the tariff rates are determined. An attempt to answer this question using a political economy approach will be made in the next chapter.
CHAPTER 5

EXISTENCE OF A NASH EQUILIBRIUM IN THE TARIFF GAME

Introduction

Coggins, et al. (1991) showed the existence of a noncooperative Nash equilibrium in a general equilibrium framework of a lobbying 2-person exchange economy. Their proof involved a restriction on individual preferences, namely own good bias as they have called it. Other assumptions in their model are mainly directed to the nature of the pricing function, which are similar to that of previous authors (for example, Findlay and Wellisz, 1982).

Coggins, et al.'s model had two persons each endowed with a single commodity. Both commodities were internationally traded. The government's role was to announce the domestic price of the two goods and take the responsibility of clearing the markets by trading in the world market at internationally given prices. In setting domestic prices the government responded to the lobbying expenditures of the two agents and satisfied its budget constraint (government feasibility). Such a responsive behaviour on the part of the government created an incentive to each of the agents to spend resources in lobbying for higher relative price for their commodity. In all previous studies with noncooperative lobbying behaviour, a stable pricing (or tariff) function that possesses certain properties is assumed. It has not been shown anywhere, however, what kind of government behaviour would produce such a pricing function.

The purpose of this chapter is to extend the existence result obtained in Coggins, et al. (1991), on to a productive economy and also demonstrate that the properties of the pricing function commonly assumed in the literature is consistent with Peltzman-type political support maximizing behaviour of the government. This result demonstrates that the authors who studied the behaviour of the tariff rate from the supply side of the political market by simply maximizing a political support function, and the authors who studied the tariff formation process from the demand side of the political market by assuming a tariff (or pricing) function are mutually consistent. The conclusions of the two approaches would be the same if the lobbying game admits a unique Nash equilibrium.

This chapter is divided into six sections. The first section provides a general description of the economy. Its purpose is to recall what are relevant from the previous chapters and to provide the motivation for the construction of a tariff setting game. The second section specifies the properties of the government's pricing function as in previous studies.
Given a pricing function of the government, the problem of the lobbies (or the interest groups) has been specified in section three. Here, we differ from Coggins, et al. in (the form of) one assumption. Coggins, et al. have assumed that the players are utility maximizers, whereas we followed Findlay and Wellisz (1982) and assumed that the players, the owners of sector specific factors, are payoff - real rental income less lobbying expenditure - maximizers.

In section four, the tariff game is specified in strategic form. The existence theorem and related Lemmas are proved in section five. In addition to the existence results, this section also shows that in all Nash equilibria of the tariff game each player spends a nonzero amount of its real rental income in predatory lobbying. Moreover, it is also shown that under a reasonable assumption, the conjecture of Findlay and Wellisz (1982) that a positive tariff rate will result in the Nash equilibrium of the game is correct. Section six concludes the chapter. Finally, Appendix-5A demonstrates that the properties of the pricing function assumed in section 2 are, in fact, consistent with the support maximizing behaviour of the government. As far as the author is aware, all of these results are new additions to the literature.

5.1 The Economy

As described in Chapter 3, the economy produces 2 tradeable goods, good 1 and good 2; good 1 is import competing, and good 2 is exportable under free trade. The two goods are produced by two different sectors using intersectorally mobile and homogenous labour, and sector specific capital. Factor endowments are given. The economy is assumed to be small and open, so changes in the economy are assumed to be incapable of influencing international relative prices.

We have also assumed that all consuming units have identical and homothetic utility functions. This assumption is not of much significance for our study, which is mainly concerned with the supply side behaviour. The supply side of the economy is independent of the demand side by virtue of small and open economy assumption. This assumption greatly simplifies the algebra. It is further assumed that the owners of the sector specific factors receive quasi-rent - the residual income after wage payments, labour gets its wage at the market clearing rate, production sectors are competitive, and while making production and consumption decisions the agents take all prices as given.

Clearly, under this structure, movements of the domestic relative price caused by changes in the government's policy variable (tariff rate) will have different impacts on the real incomes received by the owners of different factors. This has been extensively discussed in chapters 3 and 4. It is natural for the factor owners to pursue all possible ways of influencing the government's decision in their favour.
As in previous studies, we further assume that the government and the owners of the specific factors are the only agents who behave strategically. The owners of the specific factors want to maximize the real rental income net of lobbying contribution, and the government wants to maximize political support by offering a lobby-sensitive tariff (or pricing) function. The owners of the mobile factor, however, are assumed to be nonstrategic due to the chronic free rider problem and rational ignorance.

Intuitively, the nature of the lobbying game to be studied in this chapter can be described as follows. Given a free trade situation and a responsive behaviour of the government to the lobby efforts, there is an incentive to the import competing sector to spend some resources in lobbying the government for an imposition of a tariff on the imports of good 1. This will raise domestic prices of the import competing good and hence the rents to the specific factors in the import competing sector. But, a tariff on the imports of good 1 also means a lower relative price of the export good and a higher price of the mobile factor, which, in turn, means a lower rent to the specific factor in the exporting sector. Thus, the owners of the specific factor in the exporting sector will also have some incentive to lobby the government not to raise the price of import competing good, that is not to impose tariffs on imports.

Conversely, if the economy is in a high tariff regime, the exporting sector has an incentive to spend resources in lobbying the government to lower the tariff rates, which will raise the relative price of the exportable, and consequently raise the rents to the specific factor in the exporting sector. This means that there will be an incentive to the import competing sector to spend resources on counter-lobbying to maintain the tariff level and the level of its rents.

Since the tariff policy (or the pricing policy) of the government responds to the lobbying efforts of the two sectors, the extent of increase in the rent in each sector depends also on how the other sector behaves. As one sector increases its lobbying effort if the other sector also increases its effort in counter-lobbying, then it is possible that the equilibrium tariff rate may not change at all and hence, the relative price and the rents may remain unchanged.

Thus, in the end what a sector will obtain depends not only on how it behaves but also on how the other sector(s) behaves as well. While taking decisions on whether to lobby or not to lobby the government, each sector has to consider the possible reaction from the other sector(s). Therefore, the lobbying contest between the two sectors can be viewed as a 2-person game with lobby expenditures as their strategies. The objective of the game is to obtain a favourable tariff policy which, in turn, means a favourable domestic price ratio.
Since for each price ratio there is a unique equilibrium in the economic sphere of the political economy (which has been described by a CGE model), we will obtain a full equilibrium of the political economy if there is also an equilibrium in the tariff game, which represents its political side. Therefore, the immediate question is whether there exists any equilibrium in such a game? This question will be answered formally in the following sections.

5.2 The Pricing Function

Let the strategic response of the government to the lobbying efforts of the interest groups be summarized by a pricing function

\[ P_i = P_i(\eta_i, \eta_{-i}); \quad i = 1, 2. \]

where, \( \eta_i \) and \( \eta_{-i} \) are non-negative real numbers that represent the lobby expenditures\(^1\) of sector \( i \) and sector(s) \(-i\) (all other sectors but not \( i \)) respectively;\(^2\) and \( P_i \) is the normalized relative price of good \( i \) in terms of the other good.\(^3\) It is assumed that the pricing function of the government is common knowledge.

As in previous studies (in particular, Findlay and Wellisz, 1982; Wellisz and Wilson, 1986; and Coggins, et al., 1991), we assume that the pricing function satisfies the following properties:

(A1) The function \( P_i = P_i(\eta_i, \eta_{-i}) \) is continuous and differentiable in \( \eta_i \) and \( \eta_{-i} \).

(A2) \( P_i(0,0) = P_i^* \).

Assumption (A2) guarantees that if no one chooses to lobby then the world price prevails. This assumption presupposes that the government has no reason of its own to deviate from the world price, and if we observe a deviation it is because at least one agent in the economy wants the domestic price to be different from the world price.

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1 Lobbying activities may take different forms - for example, mobilizing public support to the government or organization of a street demonstration against the government, or an outright bribe. This activity may also involve the use of factors. However, in this study the lobbying effort of each sector has been measured in equivalent units of output spent or lost in lobbying activities.

2 Note that \(-i\) is the complementary set of \( i \) with respect to the player set, which is defined in section 5.4. However, in a 2-player game \( i = 1 \), if and only if \(-i = 2 \) and vice versa.

3 Here, we do not fix the numeraire, but assume that for each sector the commodity produced by the other sector is the numeraire. Therefore, the relative prices of the two commodities satisfy \( P_{-i} = P_i^{-1} \). In other words, the relative price of good 1 is the inverse of the relative price of good 2.
(A3) **(Productive Lobbying)** $P_i = P_i(\eta_i, \eta_{-i})$ is strictly increasing and concave in $\eta_i$ and strictly decreasing and convex in $\eta_{-i}$.

Satisfaction of assumption (A3) guarantees that for a given lobbying effort of player 2, if player 1 increases its lobbying effort, then the domestic price of good 1 will increase relative to the price of good 2 (monotonicity). But successive increases in lobbying efforts will bring increasingly smaller increase in the relative price of good 1 (concavity).

(A4) **(Bounded Pricing)** There exist two finite and positive real numbers $P_i'$ and $P_i''$ such that for all $\eta_i$ and $\eta_{-i}$, $P_i(\eta_i, \eta_{-i}) \in [P_i', P_i'']$.

Assumption (A4) means that the government will choose tariffs in such a way that, for each $i$, the induced domestic relative price of good $i$ will be within the bounds regardless of the level of lobbying expenditure of the private agents. The bounds include world relative prices in the feasible set by assumption (A2).

The rationale behind such limiting prices is that if the domestic relative price is too far from off the world price and/or autarky price the overall dead weight loss may be sufficiently high to invite a successful political opposition.4

Let $X = \{P_i : \mathbb{R}_+^2 \to [P_i', P_i''] \mid P_i(\eta_i, \eta_{-i}) \text{ satisfies (A1) through (A4)} \}$. Then elements of $X$ are admissible pricing functions.

**An example of the pricing function**

We can construct an example of such pricing functions to illustrate the main features. Since it is easier to construct examples of pricing functions that map lobbying expenditures into the unit interval (simplex), for the sake of this example, let us assume that prices of the two commodities satisfy $P_1 + P_2 = 1$. Then, the price ratio can be identified from the two normalized prices. For nonzero lobby expenditures let us specify a pricing function as follows:

$P_i(\eta_i, \eta_{-i}) = P_i' + (P_i'' - P_i') \left( \frac{\eta_i^\alpha}{\eta_i^\alpha + \eta_{-i}^\beta} \right)$

where $0 < \alpha < 1$ and $0 < \beta < 1$ are constants. If we take normalized free trade and autarky prices as the bounds, then equation (5.2) can display a family of price - lobby expenditure relationship satisfying the assumptions (A1) through (A4).5 The parameters

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4 For deadweight loss consideration see Becker (1983).
5 Note that this particular pricing function is not defined at the origin of the lobbying space. When no one lobbies domestic normalized prices are defined by assumption (A2). As mentioned before this
\( \alpha \), and \( \beta \) reflect the asymmetric costs to the two players of obtaining a given change in the relative price.

It is clear that, in a 2-sector case, when sector 2 spends infinitely large amount of resources in lobbying for a given lobby expenditure of sector 1 then \( P_1(\eta_1, \eta_2) = P_1' \),

assumption implies that the government has no interest in maintaining a domestic price ratio that is different from the world price ratio, and any distorted price can only be maintained by a constant flow of lobbying activities by the players.

An alternate assumption could be that the government is not committed to maintain any price ratio. It is simply interested in maximizing its political support. A more general pricing function could be specified as follows:

\[
(4.2') \quad P_i(\eta_i, \eta_{-i}) = P_i^* + (P_i^{**} - P_i^*)e^{-\beta_i(\eta_i - \eta_i^*)} - (P_i^* - P_i')e^{-\beta_i(\eta_i - \eta_i^*)}
\]

where \( 0 < \beta_i, \beta_{-i} < 1 \). Clearly, this pricing function satisfies assumptions (A1), and (A4). Instead of assumption (A2) it will now satisfy

\[
(A2') \quad P_i(\eta_i, \eta_{-i}) = P_i^* e^{[P_i', P_i^*]}, \text{where } (\eta_i^*, \eta_{-i}^*) \text{ is some minimum amount of lobbying expenditures set by the government.}
\]

Instead of (A3), the productive lobbying assumption, which states that the productivity of lobbying increases at decreasing rate, it will now allow to have increasing returns at very low levels of lobbying expenditure. However, the pricing function will remain increasing and quasiconcave in the lobbying expenditure of player \( i \), and decreasing and quasiconvex in the lobbying expenditure of player \(-i\). This assumption is certainly weaker than (A3).

Furthermore, given expenditure of player \(-i\), the productivity of lobbying expenditure of player \( i \) will increase at a decreasing rate for all \( \eta_i \geq \eta_i^* + 1 \). Similarly (A4) is modified as follows:

\[
(A4') \quad \text{If } P_i^* \neq P_i', \text{ then we require } P_i^* e^{[P_i', P_i^*]} \text{. In other words, the bounds should include world price.}
\]

With this pricing function (5.2') offered by the government one can identify various motives of the government. For example,

(i) if the government has chosen \( \beta_i = \beta_{-i} = 0 \), then it means that the government does not care about lobbying expenditures of the private players. For all levels of lobbying expenditures we will have \( P_i(\eta_i, \eta_{-i}) = P_i^* \). Which further means that when setting domestic prices the government is interested in something else, for example in maximizing a genuine social welfare function;

(ii) if the government has set \( P_i^* = P_i^* \), and \( \eta_i^* = \eta_{-i}^* = 0 \), then this implies that the government as such has no interest in deviating from free trade if no private players lobbies for a different domestic price;

(iii) if (i) and (ii) hold, then it follows that the government is committed to free trade;

(iv) if \( P_i^* = P_i^{**} \), then this means that the government is determined to protect the interest of player \( i \), but is unable to ignore the political power of player \(-i\). Similarly, if \( P_i^* = P_i' \), then this means that the government is willing to serve the interest of player \(-i\), by setting the lower bound price, but it is unable to ignore the political power of player \( i \). In either case it will be a game between one of the players and the government. The other player does not have to spend on lobbying the government, the player is already in a favourable position;

(v) if the government is a genuine welfare maximizer but the self-interested players are also so powerful that it can not ignore their lobbying activity, then the pricing function offered by the government may contain elements that reflect the government's mixed preferences. Such a case was studied by Feenstra and Bhagwati (1982). If this is the case, then \( P_i^* \) would be the welfare maximizing price with nonzero values for both \( \beta_i \) and \( \beta_{-i} \), and so on.

Moreover, the pricing function \( P_i(\eta_i, \eta_{-i}) \) given by equation (5.2') is well defined, continuous, differentiable, and bounded for all nonnegative values of lobbying expenditures. It is increasing and quasiconcave in the lobbying expenditure of player \( i \), decreasing and quasiconvex in that of player \(-i\).

Another example of the pricing function can be found in Coggins (1989). To maintain comparability with previous works we will continue assuming that the pricing function satisfies (A1) through (A4). Support maximizing behaviour of the government will be considered in the Appendix.
that is the relative price of good 1 takes its lower bound, and when sector 1 spends such a large amount of resource in lobbying for a given lobby expenditure of sector 2 then \( P_1(\eta_1, \eta_2) = P''_1 \), that is the distorted price of good 1 will take its upper bound.

The pricing function (5.2) is very sensitive to small lobby expenditures around the origin in the lobby space. For example, if player 2 does not lobby at all, then player 1 can obtain its upper bound price just by spending a cent in lobbying. Will the government announce such a pricing function? Herein lies the strategic behaviour of the government. Because, when no one is lobbying, it costs less to obtain a large favour from the government every player will be lured to spend some resources in lobbying. When one player starts lobbying with a very small sum and is able to obtain a good deal in price changes then it becomes almost a necessity to the other player to spend resources in counter lobbying. The dynamics of competitive lobbying will be set into motion. The government then can rely on the pricing function to induce competitive lobbying involving larger sums. So, a government may offer such a very sensitive pricing function if it wants to induce more competitive lobbying from the interest groups.

Assumptions (A1)-(A3) may be found in Coggins, et al. (1991). In addition to these three assumptions they have one more assumption, which they call as Bounded Lobbying. This means that, given the lobbying expenditure(s) of the other agent(s), there exists a finite maximum lobbying expenditure for each agent that exhausts all of his resources. This implies that the agent is incapable of increasing lobby expenditure beyond that bound. Evaluation of the bound for agent \( i \) when others are not lobbying at all gives the greatest upper bound for agent \( i \)'s lobby expenditure. The price that is obtained with this greatest upper bound lobby expenditure yields the ceiling for the price that agent \( i \) can ever obtain. Thus Bounded Lobbying implies Bounded Pricing. As will be argued below, while describing the nature (compactness) of the strategy set of players, that bounded pricing assumption implies bounded lobbying if the agents are payoff maximizers.

Instead of taking this indirect route the bounded lobbying assumption has been replaced by its equivalent form - the bounded pricing assumption which does not impose any extra restriction, for it has been maintained that the agents are payoff maximizers. As will be seen below, this shift in the assumption has been very helpful in proving the existence of an equilibrium in the lobbying economy.
5.3 The Problem of the Lobbies

Let us denote the stock of sector-specific capital in sector \( i \) by \( K_i \) and the equilibrium real rental rate\(^6\) (measured in units of own output) at given price \( P_i(\eta_i, \eta_{-i}) \) be \( R_i(P_i(\eta_i, \eta_{-i})) \). The payoff to player \( i \), when players are behaving strategically, at any strategy combination \((\eta_i, \eta_{-i})\) can be written as

\[
\Pi_i(\eta_i, \eta_{-i}) = K_i R_i(P_i(\eta_i, \eta_{-i})) - \eta_i .
\]

Given the pricing rule of the government and lobby expenditure \( \eta_{-i} \), of the other player(s), the problem of sector \( i \) is to choose a level of lobby expenditure \( \eta_i \) such that the problem

\[
\max_{\eta_i} \Pi_i(\eta_i, \eta_{-i}) = \max_{\eta_i} [(K_i R_i(P_i(\eta_i, \eta_{-i})) - \eta_i); \quad i = 1, 2
\]

is solved.

It is already known that \( R_i \) is bounded, and strictly increasing in \( P_i \). In addition, we have also assumed that \( R_i \) is concave in \( P_i \) (Assumption 3.1).\(^7\) Since \( P_i \) is increasing in lobby expenditure of sector \( i \) and decreasing in lobby expenditure of the other sector, sector \(-i\), each sector can raise its rental income by increasing expenditure on lobbying. Concavity of the pricing function implies that each player needs to spend an increasing amount on lobbying to obtain constant increases in the relative price of their commodity.

Under the assumption that each player shows Nash behaviour with respect to the other player, the optimization problem of each player, represented by (5.3), is to determine the level of lobby expenditure that maximizes their respective payoff (the total rent net of lobby expenditure). The constraints to these problems are implicit in the nature of the equilibrium rental functions and in the nature of the pricing function.

5.4 The Game in Strategic Form

With these preliminaries we can proceed to describe the tariff game in a strategic form. For each agent \( i \), and for given \( \eta_{-i} \) of other player(s), let us define

\(^6\) For simplicity, we will change the notation for the real rental rates. In chapter 2, \( R \) was used to denote the nominal rental rate, and \( \bar{R} \) was defined in chapter 3 to denote the real rental rate measured in units of own output. In this chapter we will be discussing in terms of real rents and to avoid confusion \( \bar{R} \) has been replaced by \( R \) with appropriate subscript to denote the player or the sector, which have been used interchangeably.

\(^7\) Note that, in chapter 3 we have shown that this assumption is true for Cobb-Douglas production functions, but whether it is generally true is not known. Here, we have assumed it to be true generally.
(5.5) \[ \hat{\eta}_i(\eta_{-i}) = K_i R_i(P_i(\hat{\eta}_i, \eta_{-i})) \], and
(5.6) \[ \bar{\eta}_i = \max_{\eta_{-i}} \hat{\eta}_i(\eta_{-i}) \]

then, \( \hat{\eta}_i \) represents the maximum lobbying expenditure that agent \( i \) can incur if it is a payoff maximizer, when player \(-i\) is spending \( \eta_{-i} \); and \( \bar{\eta}_i \) represents the level of lobbying expenditure which will never be exceeded by the payoff maximizing agent \( i \), whatever be the lobbying expenditure of the other player. Thus, \( H_i = [0, \bar{\eta}_i] \) defines the strategy set of player \( i \).

From the defining equations (5.5) and (5.6) it is clear that \( \hat{\eta}_i \leq \bar{\eta}_i \) for each player \( i \). This implies that for any given \( \eta_{-i} \) the choice set of player \( i \) is limited to a subset of the strategy set \( H_i \). Let \( \varphi_i \) be a correspondence defined on the strategy space such that \( \varphi_i(\eta_{-i}) = [0, \hat{\eta}_i] \), then \( \varphi_i(\eta_{-i}) \subseteq H_i \) and \( \varphi_i(\eta_{-i}) \) represents player \( i \)'s constraint correspondence - a mapping - that determines the choice set of player \( i \) given the strategy choice of other players.

Let \( I = \{1, 2\} \) be the player set, then \( H = \times_{i \in I} H_i \) is the set of ordered pairs that describes all possible strategy combinations of the two players, and \( \Pi_i \) is the associated payoff function for each player \( i \).

**Definition 5.1.** Any \( \Gamma = \{(H_i, \Pi_i, \varphi_i)_{i \in I}\} \) is a collection of tariff games in strategic form.

**Definition 5.2.** A pair \( (\eta_i^*, \eta_{-i}^*) \) is a noncooperative Nash equilibrium of the tariff game if \( \eta_i^* \) solves

\[ \max_{\eta_i \in \varphi_i(\eta_{-i})} \Pi_i(\eta_i, \eta_{-i}^*); \quad i \in I \]

### 5.5 The Existence Results

The sufficient conditions for the existence of an equilibrium of a game in \( \Gamma \) are listed in the following theorem.

**Theorem 5.1** (Debreu, 1982: 702-3). If, for every \( i \in I \), the set \( H_i \) is a non-empty, compact, convex subset of a Euclidean Space, \( \Pi_i \) is a continuous real-valued function on \( H = \times_{i \in I} H_i \) that is quasiconcave in its \( i \)th variable, and \( \varphi_i \) is a continuous, convex-valued correspondence from \( H \) to \( H_i \), then the social system \( (H_i, \Pi_i, \varphi_i)_{i \in I} \) has an equilibrium.

The immediate problem is to check whether all of the conditions mentioned in the theorem are satisfied in the tariff game. We will check these conditions for the arbitrary player \( i \). If the conditions of the theorem are satisfied for player \( i \), then it will
be valid for all players, and the theorem will be applicable to our problem. We will accomplish that by a series of Lemmas.

**Lemma 5.1** (Coggins, et al., 1991). Let \( X \subset \mathbb{R}, \ Y \subset \mathbb{R}, \) and let \( g: X \to Y \) be differentiable, then \( g \) is quasiconcave if and only if for every pair of elements \( x, x' \) of \( X \) \[ g'(x) < 0 \text{ and } x' > x \] together imply \( g'(x') < 0. \)

**Lemma 5.2.** If the pricing function satisfies (A1) through (A4), and \( R_i(P_i) \) is differentiable, strictly increasing and concave in \( P_i \), then for given \( \eta_{-i} \) the payoff function

\[
\Pi_i(\eta_i, \eta_{-i}) = K_i R_i(P_i(\eta_i, \eta_{-i}))) - \eta_i
\]

is quasiconcave in \( \eta_i; \ \eta_i \in \mathbb{R}_+ \).

**Proof:** First note that \( \Pi_i(0, \eta_{-i}) > 0 \) for the price has a lower bound, and the rental rate is positive at this price. Even though the rental rate increases with \( P_i \), and \( P_i \) increases with the lobby expenditure, the payoff \( \Pi_i(\eta_i, \eta_{-i}) \) eventually becomes negative as \( \eta_i \) increases. To see this, since the pricing function satisfies (A4), \( P_i \) has its upper bound \( P_i^* \). Now, let \( \eta_i' = K_i R_i(P_i^*) \), then for all \( \eta_i > \eta_i' \), it follows from equation (5.8) that we must have \( \Pi_i(\eta_i, \eta_{-i}) < 0. \)

That the payoff becomes negative from positive also suggests that there must be a \( \eta_i'' \in [0, \eta_i'] \) such that \( \partial \Pi_i / \partial \eta_i < 0 \) for \( \eta_i = \eta_i'' \).

Differentiation of equation (5.8) partially with respect to \( \eta_i \) yields

\[
\partial \Pi_i / \partial \eta_i = K_i (dR_i / dP_i)(\partial P_i / \partial \eta_i) - 1.
\]

Since \( \partial \Pi_i / \partial \eta_i < 0 \) for \( \eta_i = \eta_i'' \), and \( R_i(P_i) \) is strictly increasing in \( P_i \) implies

\[
(\partial P_i / \partial \eta_i)|_{\eta_i=\eta_i''} < 1 / [K_i(dR_i / dP_i)].
\]

Assumption (A3), the concavity of \( P_i \) implies that the slope of \( P_i \) falls as \( \eta_i \) increases. Therefore, for all \( \eta_i > \eta_i'' \) we must have

\[
(\partial P_i / \partial \eta_i) < (\partial P_i / \partial \eta_i)|_{\eta_i=\eta_i''} < 1 / [K_i(dR_i / dP_i)].
\]

which follows from concavity of \( R_i \) in \( P_i \), and monotonicity of \( P_i \) in \( \eta_i \). Therefore, that for all \( \eta_i > \eta_i'' \), we have \( \partial \Pi_i / \partial \eta_i < 0. \) Hence by Lemma 5.1 the payoff function \( \Pi_i(\eta_i, \eta_{-i}) \) is quasiconcave in \( \eta_i. \) Q. E. D.

The argument used in the proof of Lemma 5.2 asserts that if the payoff function becomes negative for some level of lobby expenditure then it remains negative for all higher levels of lobby expenditures. This effectively precludes nonconvexities in the
choice set and the constraint correspondence remains convex valued which is shown in the following Lemma.

**Lemma 5.3.** If the pricing function satisfies assumptions (A1) through (A4) and the rental function $R_i$ is strictly increasing, concave and differentiable in $P_i$, then the strategy set of player $i$, $H_i$, is nonempty, compact and convex subset of $\mathbb{R}$, the real line, and further, the constraint correspondence $\varphi_i(\eta_{-i})$ is convex-valued and continuous.

Proof. By definition the strategy set of player $i$ is $H_i = [0, \vec{\eta}_i]$. The player can always choose not to lobby, implying that $0 \in H_i$, therefore, $H_i \neq \emptyset$. The set is obviously closed for it also includes boundary points. Compactness requires that the set be bounded.

Since $P_i$ is bounded from above by $P_i^*$ [Assumption (A4)], and $R_i$ is increasing in $P_i$, implies that $R_i(P_i^*) < +\infty$. It follows from equations (5.5), (5.6) and $0 < K_i < +\infty$ that for all non-negative values of $\eta_{-i}$ we must have $\vec{\eta}_i \leq K_i R_i(P_i^*)$. Therefore, $H_i$ is compact. Convexity of $H_i$ is automatically satisfied because it is a closed interval of a real line.

To show that the choice set is convex or the constraint correspondence is convex valued, let us consider the definition of $\varphi_i(\eta_{-i})$. Since $\varphi_i(\eta_{-i})$ is a collection of strategies (lobby levels) available to player $i$ when player $-i$ has chosen to lobby $\eta_{-i}$ amount of resources, $\varphi_i(\eta_{-i})$ can also be defined by

$$\tag{5.11} \varphi_i(\eta_{-i}) \equiv \{ \eta_i \in H_i | \Pi_i(\eta_i, \eta_{-i}) \geq 0 \}$$

But this means $\varphi_i(\eta_{-i})$ is a better set for the payoff function, which is quasiconcave by Lemma 5.2 implies that $\varphi_i(\eta_{-i})$ is a convex set.

Lastly, it remains to be shown that the constraint correspondence $\varphi_i(\eta_{-i})$ is continuous. Continuity of $\varphi_i(\eta_{-i})$ has been shown in Coggins, et al. (1991), which is restated here for the sake of completeness.

It suffices to show that $\varphi_i(\eta_{-i})$ is upper and lower hemicontinuous. Since $P_i$ is continuous in $H_i$, and $R_i$ is continuous in $P_i$, it follows from equation (5.5) that $\hat{\eta}_i(\eta_{-i})$.

---

8 Note that the nature of the rental functions depend on the economic system and the rental rates are defined for all prices, whereas the bounds of the pricing function are in the domain of the government. For any $P_i > P_i^*$ it follows from the monotonicity of the real rent that $R_i(P_i) > R_i(P_i^*)$.

9 A correspondence $\varphi$ from a subset $S$ of a Euclidean space to a subset $T$ of a Euclidean space is upper hemicontinuous (u. h. c.) at a point $x^0$ of $S$ if there is a neighbourhood of $x^0$ in which $\varphi$ is bounded, and for every sequence $x^q$ in $S$ converging to $x^0$ in $S$, and every sequence $y^q$ in $T$ converging to $y^0$ in $T$ such that for every $q$, one has $y^q \in \varphi(x^q)$, then $y^0 \in \varphi(x^0)$. Upper hemicontinuity of $\varphi$ on $S$ is defined as upper hemicontinuity at every point of $S$. 
is continuous in \([0, \bar{\eta}_i]\). Thus the graph of \(\varphi_i(\eta_{-i})\) is closed. As \(\varphi_i(\eta_{-i})\) is also compact valued by the above argument, it is upper hemicontinuous. To show lower hemicontinuity, consider a sequence \(\{\eta_{-i}^n\}\) converging to \(\eta_{-i}^*\), and take any arbitrary \(\eta_i^* \in \varphi(\eta_{-i}^*)\). If \(\eta_i^n < \hat{\eta}_i(\eta_{-i}^*)\), then for \(N\) large we may set \(\eta_i^n = \eta_i^*\) for \(n > N\). Then clearly \(\eta_i^n \to \eta_i^*\), and the conditions for lower hemicontinuity are satisfied. If \(\eta_i^n = \hat{\eta}_i(\eta_{-i}^*)\), then let \(\eta_i^n = \hat{\eta}_i(\eta_{-i}^*)\). As \(\hat{\eta}_i(\eta_{-i})\) is continuous, the conditions for lower hemicontinuity are again satisfied. It is concluded that \(\varphi_i(\eta_{-i})\) is lower hemicontinuous. Thus, it is continuous. Q. E. D.

**Theorem 5.2. (Existence of an equilibrium in a tariff game).** Given a Tariff Game in \(\Gamma = (H_i, \Pi_i, \varphi_i)_{i \in I}\) as defined above, if the rental function \(R_i\) is real-valued, strictly increasing, concave and differentiable in \(P_i\), and the pricing function \(P_i(\eta_i, \eta_{-i})\) satisfies assumptions (A1) to (A4), then there exists at least one noncooperative Nash equilibrium.

Proof: Under the assumptions of the Theorem 5.2, Lemma 5.2 and Lemma 5.3 hold. Since, by assumption, the pricing function is continuous in lobby expenditures \(\eta_i\) and \(\eta_{-i}\). By the assumption of the theorem the rental rate \(R_i\) is continuous and real valued in \(P_i\). This implies that the payoff function of any arbitrary player \(i\) defined by the equation (5.3) is continuous and real valued in \(H\) and quasiconcave in its \(i\)th variable. Hence, the conditions of Theorem 5.1 are satisfied, and the theorem is proved. Q. E. D.

Theorem 5.2 has guaranteed that at least a Nash equilibrium will exist in an economy where tariff policy responds to the lobby efforts of the interest groups. Since \(0 \in H\), no lobbying is always a feasible strategy to both the players, it is a candidate for an equilibrium. It is natural to ask, at this point, whether \((\eta_i, \eta_{-i}) = (0, 0)\) can be a Nash equilibrium of the game. The following Lemma answers this question.

**Lemma 5.4.** In addition to the conditions of Theorem 5.2, if the pricing function is such that the partial elasticities of price with respect to the lobbying expenditures are locally constant, then \((\eta_i, \eta_{-i}) = (0, 0)\) cannot be a Nash equilibrium.

---

Similarly, A correspondence \(\varphi\) from a subset \(S\) of a Euclidean space to a subset \(T\) of a Euclidean space is lower hemicontinuous (l. h. c.) at a point \(x^0\) of \(S\) if, for every sequence \((x^q)\) in \(S\) converging to \(x^0\) in \(S\), and every \(y^0\) converging to \(\varphi(x^0)\), there is a sequence \((y^q)\) in \(T\) converging to \(y^0\) such that for every \(q\), one has \(y^q \in \varphi(x^q)\). Lower hemicontinuity on \(S\) is defined as lower hemicontinuity at every point of \(S\).

Continuity of a Correspondence at a point or on a set is defined as the conjunction of upper and lower hemicontinuity. (Debreu, 1982: 698-701).
Proof: Let us consider the behaviour of player $i$ when player -$i$ chooses not to spend on lobbying, that is $\eta_{-i} = 0$. The payoff to player $i$ is given by
\[
\Pi_i(\eta_i,0) = R_i(P_i(\eta_i,0)) - \eta_i, \quad \eta_i \in \varphi_i(0).
\]
The first order condition for the maximum requires that player $i$ choose lobby expenditure satisfying
\[
(\partial R_i / \partial P_i)(\partial P_i / \partial \eta_i) \leq 1, \text{ and } \eta_i(\partial \Pi_i / \partial \eta_i) = 0.
\]
The conditions will yield a solution of $\eta_i = 0$ if and only if $(\partial R_i / \partial P_i)(\partial P_i / \partial \eta_i) < 1$ for all $\eta_i \in \varphi_i(0)$. But
\[
\left(\frac{\partial R_i}{\partial P_i}\right)\left(\frac{\partial P_i}{\partial \eta_i}\right) = \left(\frac{P_i}{R_i}\frac{\partial R_i}{\partial P_i}\right)\left(\frac{\eta_i}{P_i}\frac{\partial P_i}{\partial \eta_i}\right)\left(\frac{R_i}{\eta_i}\right).
\]
Since, under the conditions of the Lemma, the first two terms on the right (elasticities) are always positive, and locally constant around the origin of the lobbying space; and the real rental income is positive for all prices therefore, in the neighbour of the origin, we have
\[
\lim_{{\eta_i \to 0}} \left(\frac{\partial R_i}{\partial P_i}\right)\left(\frac{\partial P_i}{\partial \eta_i}\right) = \lim_{{\eta_i \to 0}} \left(\frac{P_i}{R_i}\frac{\partial R_i}{\partial P_i}\right)\left(\frac{\eta_i}{P_i}\frac{\partial P_i}{\partial \eta_i}\right)\left(\frac{R_i}{\eta_i}\right) = +\infty.
\]
That is, $(\partial R_i / \partial P_i)(\partial P_i / \partial \eta_i) > 1$ in the neighbourhood of $\eta_i = 0$, and the first order necessary condition for a maximum is not satisfied. Therefore, $\eta_i = 0$ cannot be a best reply of player $i$ to the strategy $\eta_{-i} = 0$ of player -$i$. Since choice of $i$ is arbitrary, we find that $(\eta_i, \eta_{-i}) = (0,0)$ cannot be a Nash equilibrium (see Definition 5.2) in the tariff game.

Q.E.D.

Thus, under the assumed behaviour of the pricing function and the behaviour of the rental functions, Lemma 5.4 guaranteed that in any Nash equilibrium we will observe that the players will be spending positive amounts on lobbying. It is tempting to conclude that all prices corresponding to Nash equilibrium are necessarily different from free trade prices. This is not always the case, however. In fact, as Coggins, et al. (1991) also have shown, if the lobby expenditure of one player is matched appropriately by another player in equilibrium then the free trade price may result. The arguments are as follows:

\[10\] If there are increasing returns to scale, then at very low price the rental income may be negative even if the slope of the rental function with respect to price is positive. In that case, not spending on lobbying might be the best reply for the player and hence Lemma 5.4 may not hold. However, this study is based on the assumption of constant returns to scale in both sectors.
Let $\eta_i$ and $\eta_{-i}$ be any feasible level of lobby expenditures of the two players. Then by the assumption of productive lobbying (A3) we have

$$P_i(\eta_i, 0) > P_i(0, 0) = P^*_i > P_i(0, \eta_{-i}).$$

By the mean value theorem, there exist feasible expenditures $\bar{\eta}_i \in (0, \eta_i)$ and $\bar{\eta}_{-i} \in (0, \eta_{-i})$ such that $P_i(\bar{\eta}_i, \bar{\eta}_{-i}) = P^*_i$.

Thus, the result that non-zero lobby levels may generate a free trade price ratio is still possible provided the government has no interest of its own in deviating from free trade. It is not yet known whether such combination of lobby expenditures can constitute an equilibrium of the game. What has been shown so far is insufficient to rule out the possibility that at least one Nash equilibrium outcome of the game will yield the free trade price ratio.

**Definition 5.3.** Given a description of a small open Walrasian economy a domestic relative price, induced by a tariff policy, is self-financing if the total collection of tariff revenue net of subsidies is non-negative.

It is evident that not all relative prices are self-financing. For example, if the government wishes to make the domestic relative price of exportable higher than that in the world market [that is, free trade price] then it would require an import subsidy and/or export subsidy. In total, the policy would be one of net trade subsidy.

Similarly, if the government wished to make the domestic relative price of the import competing good higher than the relative price that would prevail in autarky, then the export of an otherwise import competing good has to be subsidized as would the import of an otherwise exportable good. In both cases the collection of tariff revenue would be negative.

It is possible, however, to finance such a tariff policy either by resorting to external borrowing or by imposing some form of domestic tax. Financing a trade subsidy through the imposition of a domestic tax means a reduction in the domestic living standard to a level below that could be attained otherwise. A government would require one or more complementary policies that can generate surplus revenues in order to sustain a tariff policy belonging to this class. If so, then the effects of such complimentary policies on the payoffs of different agents need to be analysed together with the impact of tariff rates.
This means that tariff policy cannot be studied in isolation. Inclusion of tariff policies that imply a net subsidy immediately leads outside the scope of this study.\textsuperscript{11} This study assumes that the rationalized tariff rate is the only instrument available to a government for income redistribution.

In most real world situations, the collection of tariff revenue net of subsidy is nonnegative. It is, therefore, safe to assume that the government, as far as tariff policy is concerned, never adopts a net subsidy policy.\textsuperscript{12} This means that the government will choose tariff/subsidy rates in such a way that the domestic price ratio will be within the bounds set by free trade and autarky price ratios. Formally, we state the following property of the pricing function, which is implicit in Findlay and Wellisz (1982):

\[ (A5) \quad \text{Self-financing}. \quad P^T_i(\eta_i, \eta_{-i}) \in [P^*, P^*_i] \text{ for all } \eta_i \text{ and } \eta_{-i}, \text{ where } P^*_i \text{ is the relative price of good } i \text{ in autarky.} \]

Assumption (A5) does not exclude the possibility of simultaneous existence of export subsidy and import tax in the announcement of government policies. This is perfectly consistent with assumption (A5). What it rules out are the combinations of those subsidy and tax rates that lead to higher subsidy payments than the tariff revenue collections. In other words, this assumption requires that the rationalized tariff rate be nonnegative.

Note that assumption (A5) by itself is not sufficient to preclude the possibility of free trade. Players may choose not to lobby at all or what they spend may counteract each other so that free trade prices are maintained. Now we will show that Findlay and Wellisz's (1982) conjecture is correct.

\textbf{Corollary 5.1.} Suppose the conditions of Theorem 5.2 are satisfied with the pricing function satisfying (A1) - (A3) and (A5). If \((\tilde{\eta}_i, \tilde{\eta}_{-i})\) is a Nash equilibrium of the Tariff Game, then \(P_i(\tilde{\eta}_i, \tilde{\eta}_{-i}) \neq P^*_i\).

\textsuperscript{11} In Coggins, et al (1991), the government directly sets the prices of commodities in response to private lobby expenditures. They have overcome the problem of complimentary financing policy by stating a government feasibility condition. The condition required that the sum of lobby income of the government and trade balance be non-negative. That is, all prices that can not induce lobby expenditures sufficient to finance the consequent trade deficit are considered unfeasible. This assumption is reasonable in their two person exchange economy but may not be so in a productive economy with many agents some of whom are non strategic. Government, for example, may use other taxes to get a transfer from nonstrategic agents.

\textsuperscript{12} It is not easy to judge the validity of this assumption on the basis of published figures. It involves an evaluation of effects of nontariff barriers as well and it is not always possible to estimate figures for the payments of subsidies in different forms, hence the statement at best is a conjecture. However, as far as developing countries are concerned published figures show that tariff revenue has remained one of the major source of government revenue. Motivated by this fact Feehan (1992) has derived efficiency rules for public input provision when the tariff is the sole source of revenue. For economies, whose government depends on tariff revenues, this assumption should be directly relevant.
Proof: Suppose not, and assume that \( P_i(\tilde{\eta}_i, \tilde{\eta}_{-i}) = P_i^* \). Since \( P_i^* \) is the lower bound of \( P_i \) we must have \( P_i(0, \eta_{-i}) \geq P_i^* \) for all \( \eta_{-i} \in H_{-i} \). That is, \( P_i \) will not fall below \( P_i^* \) whatever be the lobby expenditure of the other player(s). So,

\[
\Pi_i(0, \tilde{\eta}_{-i}) \geq K_i R_i(P_i^*)
\]

\[
> K_i R_i(P_i^*) - \tilde{\eta}_i
\]

\[
= K_i R_i(P_i(\tilde{\eta}_i, \tilde{\eta}_{-i})) - \tilde{\eta}_i
\]

Hence \( \eta_i = 0 \) yields a higher payoff than \( \eta_i = \tilde{\eta}_i \) for \( \eta_{-i} = \tilde{\eta}_{-i} \). This implies that \( \tilde{\eta}_i \) is not a best response to \( \tilde{\eta}_{-i} \), which further implies that \( (\tilde{\eta}_i, \tilde{\eta}_{-i}) \neq (0,0) \) is not a Nash equilibrium of the game. A contradiction to the hypothesis. Q. E. D.

Thus the corollary shows that a Nash equilibrium of a Tariff Game will lead to a domestic price ratio that will always be different from the free trade price ratio. This completes the answer to the question of existence of an equilibrium in a productive economy that allows agents to behave strategically at least with respect to the issues related to the choice of tariff policies.

So far nothing has been said about the uniqueness of equilibrium of the game. The sufficient conditions for the uniqueness of equilibrium can be found in Friedman (1986). It suffices to note that the condition requires, together with other conditions, that the payoff function of each player be concave in their respective strategy set. Lemma 5.2 has shown it to be quasiconcave. Unless further restrictions are imposed on the rental function it cannot be guaranteed that the condition will be satisfied generally.

5.6 Conclusion

This chapter has extended the existence result obtained by Coggins, et al. for a productive economy that allows agents to behave strategically. It has shown that if the government is responsive to the lobby pressures of the interest groups, then there exists at least one non-trivial Nash equilibrium in a tariff game. Moreover, if the government is constrained to choose a tariff policy that is self-financing then all of the Nash equilibria will imply a positive rationalized tariff rate. That is, the domestic price ratio in a Nash equilibrium will be different from the free trade price ratio.

In order to show the existence of an equilibrium in a lobbying exchange economy Coggins, et al. had to impose one restriction, which they called 'own good bias', on preferences. The proof outlined in this chapter did not require that assumption for a productive economy. This result occurred partly because in the Coggins, et al.'s model the endowment of good for each player was fixed but in our case sectoral output
levels, and hence the rental incomes are variable, and partly because Coggins, et al. measured player's payoffs in indirect utilities whereas this study used real net rental income as the payoffs to the players. It was thus possible to obtain much stronger results compared to previous studies with less severe assumptions. In fact, we obtained the existence results with the set of assumptions that were in Findlay and Wellisz (1982).

One of the limitations of the noncooperative Nash solution is that if there are multiple equilibria (see Magee, Brock and Young, 1989 for such cases) it cannot specify the mechanism through which one of the equilibria will be selected. It is also frequently the case that the Nash process may lead to a suboptimal solution of the game. To resolve this problem some mechanism of refining the Nash equilibria, such as subgame perfectness, are required.

One of the fundamental assumptions of the noncooperative game is that the players make decisions in isolation. If there is a distinct possibility of increasing their payoff through communication, negotiation and adoption of a joint maximizing strategy then the assumption that rational players in a competitive environment always play a noncooperative game does not seem plausible. In the tariff game, for example, each player may spend resources in competitive lobbying just to protect himself against the predatory lobbying of the other. It is possible that they both might have been receiving lower payoffs than they would receive if they had agreed not to compete against each other.

In the next chapter we change the rules of the game, and allow the players to discuss, negotiate, and make binding agreements that are payoff (welfare) improving. This takes us to the study of the game in a cooperative form. However, as Kreps (1990: 505) has remarked 'this is not the cooperation borne of altruism or fondness of one's fellow player. This is cooperation arising from a self-interested calculation of the benefits and losses that may accrue from "polite" behaviour'.

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13 We noted in the review that Magee, Brock and Young (1989) have found several Prisoner's Dilemma equilibria in their simulations of the noncooperative game with Leontief production functions.
Appendix-5A: Derivation of the Properties of the Pricing Function of a Support Maximizing Government in a Lobbying Economy

The purpose of this Appendix is to demonstrate that the properties of the pricing function assumed in section 2 of the text are consistent with the support maximizing behaviour of the government. In other words, we will show that the properties of the pricing function are sufficient for the maximum of the government's political support function.

We assume that in the political market the government exchanges policies for political support. In particular, this means that, besides policies that benefit everyone, if a policy change benefits one person or group and harms the other, and the beneficiary is able and prepared to offer increased support to the government that exceeds the loss in support from the losing person or group, then both the government and the winning player will be better off from this deal. Policy will thus be adjusted accordingly. This assumption denies any benevolence on the part of the government. It implies that policy changes are solely guided by the self-interest of the government driven by its desire to remain in power and are thus politically motivated.14

We further assume that with respect to each policy change involving a distributional issue the population can be divided into three groups according to their interest on the proposed policy change: the group that anticipates direct benefits and supports it; the group that anticipates a loss and therefore opposes it; and the third group that is uncertain about the effects of policy change on its welfare, and therefore is in a state of confusion as to whether or not to support the proposed policy change.

To be more specific, let us consider the case of an increase in the tariff rate on the imports of good 1. Clearly, the owners of the specific factor in sector 1 benefit from this change and therefore they form the first group - people who support the tariff increase. The owners of the specific factor in sector 2 lose from this change and therefore they form the second group - people who oppose the tariff change. The owners of the mobile factor, labour, may lose or gain from this change depending on their consumption pattern (tastes) known only to them, and the sizes of their income and commodity price change. If the workers are well informed, then they would, of

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14 This assumption on the motivation of the government follows the tradition of the public choice literature. "The central feature of that tradition is", as Brennan (1992: 13) describes, "the searching out of invisible hand institutions, arrangements that achieve benign results without undue reliance on benevolent motivations. To test out whether such institutions are present or not, one must assume non-benevolent motivations: one does not test whether a plate is oven-proof by placing it in the cupboard."
course, either individually support or oppose the change, but there are information costs in being well informed.

We further assume that ownership of the specific factors is concentrated among a few and their gains or losses are relatively large compared with the information cost. Therefore they are fully aware of the likely consequences to them of the tariff changes. The owners of the mobile factor, workers in particular, are numerous. For anyone in this group the cost of information exceeds the benefit. Therefore, they remain rationally ignorant (Downs, 1957; Magee, Brock and Young, 1989). However, they make use of the ‘free’ information - voters' education - provided by the two ‘interest groups’ or the players. Each player can obtain more workers' support for the government by spending more resources on ‘voters' education’. Hence the two interests will be contending in the political market in mobilizing workers' support to the government that will implement policies beneficial to them.

As Peltzman (1976: 213) has aptly summarized -

It is not enough for the successful group to recognize its interests; it must organize to translate this interest into support for the politician who will implement it. This means not only mobilizing its own vote, but contributing resources to the support of the appropriate political party or policy: to finance campaigns, to persuade the voters to support or at least not oppose the policy or candidate, perhaps occasionally to bribe those in office.

With this stylization of the political process, the problem of the two players and the government being resolved in the political market can be described as follows.

Given the responsiveness of the government's pricing policy to increased mobilization of political support, each player \(i\) chooses its lobbying expenditure, \(\eta_i\), to maximize

\[
\Pi_i = R_i(P_i(\eta)) - \eta_i; \quad i = 1,2
\]

where, the variables \(\Pi_i\) and \(R_i\), as defined in the text, are the payoff and real rental income of player \(i\) respectively, measured in units of own output, and \(P_i\) is the normalized price of good \(i\). Note that once \(P_1\) is determined, \(P_2\) will be determined since it is the inverse of \(P_1\). Therefore, we will focus on the government's choice of \(P_1\) only and express the functions accordingly.

The government, on the other hand, is fully aware of the self-interested behaviour of the players. It understands that each player will spend resources in mobilizing political support for it only if the price policy it supplies is going to yield them a higher payoff than otherwise. The government also understands that if a policy change hurts a player, then the player will divert his or her lobbying effort to oppose the
policy changes and possibly finance the political campaign of the opposition. Therefore, the government, taking into account the reactions of the players, chooses $P_1$ to maximize the Peltzman-type political support function

\[(5A.2) \quad S = S(\Pi_1(P_1(\eta)), \Pi_2(P_1(\eta)))\]

where, the support function $S$ is strictly increasing and concave in its arguments.\(^{15}\)

The act of maximizing support by the government and the act of maximizing payoffs by the private players interlock the three in a political process in which each of the private players holds an opportunity to trade fruitfully with the government by mobilizing political support for a favourable price (or trade) policy and vice versa. For example, player 1 can mobilize increased support for the government by spending more on 'voters' education', if the government is prepared to provide enough protection to it by raising the tariff rate (or lowering the export subsidy). Similarly, player 2 can also mobilize additional political support for the government if it returns with lower protection to the import competing sector or with increased export subsidy. The government can provide favourable tariff protection to any of the players if he or she is prepared to spend resources sufficient to mobilize more political support than it would lose by implementing the policy.

The problems of the government and of the two players can be viewed as a Stackelberg game in which the government is the leader, and the two private players are Nash followers. The government, assumed to conjecture the reactions of the Nash players correctly, and takes into account all the reactions of the private players (subject to the general equilibrium of the economy) while maximizing its support function. It then obtains a solution function $P_1(\eta_1, \eta_2)$ to offer to the private players. This function will extract maximum support for it from the society whenever the players are in a Nash equilibrium. Our purpose in this Appendix is to uncover the properties of this pricing function.

Before we go onto analyse the nature of the pricing function in detail, let us clarify the scope of the appendix first. As a Stackelberg leader it may be possible for the government to choose a pricing function maximizing (5A.2) such that, given this pricing function, each player finds no-lobbying as his best strategy, then the price that maximizes (5A.2) with $\eta_1 = \eta_2 = 0$ also solves (5A.1) and (5A.2) simultaneously.

\(^{15}\) This is an indirect political support function that is based on the assumption that, besides players' own voting decisions, the political support/opposition of the 'ignorant' voter group depends on the lobbying efforts of the players which, in turn, are determined by the respective payoffs of the players (or the price function offered by the government) follows from the first-order condition (5A.3). See also Peltzman (1976).
Alternately, a government may induce no lobbying on the part of private players if it offers a constant pricing function, say \( P_i(\eta_1, \eta_2) = \hat{P}_i \) for all \( \eta_1 \) and \( \eta_2 \), where \( \hat{P}_i \) could be a price that emerges at some Nash equilibrium and also maximizes its political support. In the later case, no lobbying certainly increases the payoffs of both players from what they would obtain at the corresponding Nash equilibrium with positive lobbying. In both cases no player is dissatisfied, and no player will spend in lobbying against the government or financing the political campaign of the opposition. Such a pricing mechanism would also raise the level of political support for the government. If such possibilities exist, then such a price corresponds to the (self-enforcing) cooperative outcome, which we shall study in the next chapter. However, if any one of the players aspires for a higher or lower price than \( \hat{P}_i \), that is he is not satisfied with the 'fixed-price' offered by the government, then he may still spend resources in opposing the government, and supporting the opposition. The player can behave strategically to 'educate' the voters that will penalize the incumbent in the next election. To mitigate this opposition, the government has to offer a pricing function that induces a positive lobbying effort from the beneficiary. It is this situation that we analyze in this appendix.

The first order condition of the maximum to the Nash player \( i \) is:

\[
\frac{d \Pi_i}{d \eta_i} - 1 = 0.
\]

The condition (5A.3) implies that each player will spend on lobbying as long as a unit of output spent on lobbying yields one extra unit of output in rental income. This condition can be rewritten as

\[
\frac{d R_i}{d P_i} = \frac{1}{\partial P_i / \partial \eta_i}.
\]

The necessary condition for the maximization of the government's support function can be written as

\[
\frac{d S}{d P_i} = \sum_i \frac{\partial S}{\partial \Pi_i} \left[ \frac{d R_i}{d P_i} - \frac{\partial \eta_i}{\partial P_i} \right] = 0.
\]

This condition states that the government will choose the level of relative price so that at the margin the gain in support is exactly balanced by the loss in support that arises due to the pricing policy. It follows that (5A.5) is satisfied whenever (5A.4) is satisfied and that the 'lobbying derivatives' (Baldwin, 1987) exist such that

\[
\partial \eta_i / \partial P_i = \frac{1}{\partial P_i / \partial \eta_i} \quad \text{for each } i.
\]
The condition (5A.6) will follow automatically if the pricing function satisfies assumption (A1) of the text - that it is continuous in \( \eta_1 \) and \( \eta_2 \), and that \( \partial P / \partial \eta_i \neq 0 \) for each \( i \).

We know that \( dR_1 / dP_1 > 0 \) and \( dR_2 / dP_1 < 0 \). Satisfaction of the first order condition of support maximization follows from the satisfaction of the first order condition (5A.3) if the government announces a pricing function satisfying (5A.6), and

\[
(5A.7) \quad \frac{\partial P_1}{\partial \eta_1} > 0 \quad \text{and} \quad \frac{\partial P_1}{\partial \eta_2} < 0.
\]

at all nonzero values of \( \eta_1 \) and \( \eta_2 \). This means that the pricing function should be such that the relative price of commodity 1 increases with increased lobbying effort of player 1 and decreases with increased lobbying effort of player 2.

**Second Order Conditions:**

For each \( i \), differentiating the first order condition (5A.3) with respect to \( \eta_i \) we get

\[
(5A.8) \quad \frac{d^2 \Pi_i}{d\eta_i^2} = \frac{d^2 R_i}{dP_i^2} \left( \frac{\partial P_i}{\partial \eta_i} \right)^2 + \frac{dR_i}{dP_i} \frac{\partial^2 P_i}{\partial \eta_i^2}.
\]

We know that \( (\partial P_i / \partial \eta_i)^2 > 0 \), \( dR_i / dP_i > 0 \), and we get the following conditions:

(i) If \( d^2 R_i / dP_i^2 \leq 0 \), then the sufficient (but not necessary) conditions for \( d^2 \Pi_i / d\eta_i^2 < 0 \) are that

\[
\partial^2 P_i / \partial \eta_i^2 < 0 \quad \text{and} \quad \partial^2 P_i / \partial \eta_2^2 > 0.
\]

(ii) If \( d^2 R_i / dP_i^2 > 0 \), then the conditions that \( \partial^2 P_i / \partial \eta_i^2 < 0 \) and \( \partial^2 P_i / \partial \eta_2^2 > 0 \) are necessary but not sufficient for \( d^2 \Pi_i / d\eta_i^2 < 0 \). A sufficient condition would require sufficiently large magnitude of \( \partial^2 P_i / \partial \eta_i^2 \).

This means that if the government supplies a pricing function such that the conditions

\[
(5A.9) \quad \partial^2 P_i / \partial \eta_i^2 < 0, \quad \text{and} \quad \partial^2 P_i / \partial \eta_2^2 > 0
\]

hold with sufficiently large magnitudes of the second order derivatives of the pricing function at the point where the first order conditions are satisfied, then the second order
conditions of the maximization of the payoff functions of the two players are always satisfied. The pricing function, however, will meet this requirement, by Lemma 5.2, if it is bounded, assumption (A4) in the text, and satisfies the condition (5A.6) and (5A.9). But the conditions (5A.6) and (5A.9) are the same as assumption (A3) of the text.

We now check whether the second order condition holds for the government's support maximization whenever it is satisfied for the private players. Differentiating equation (5A.5) with respect to $P_i$ yields

\[
\frac{d^2 S}{dP_i^2} = \sum_i \left[ \left( \frac{dR_i}{dP_i} - \frac{d\eta_i}{dP_i} \right) \frac{\partial^2 S}{\partial P_i \partial \Pi_i} + \frac{\partial S}{\partial \Pi_i} \left( \frac{d^2 R_i}{dP_i^2} - \frac{d^2 \eta_i}{dP_i^2} \right) \right]
\]

When the first order conditions are satisfied for the Nash players we have $dR_i / dP_i = d\eta_i / dP_i$, therefore, it follows that

\[
\frac{d^2 S}{dP_i^2} < 0 \text{ if and only if } \sum_i \left[ \frac{\partial S}{\partial \Pi_i} \left( \frac{d^2 R_i}{dP_i^2} - \frac{d^2 \eta_i}{dP_i^2} \right) \right] < 0
\]

Since $\frac{\partial S}{\partial \Pi_i} > 0$, a sufficient (but not necessary) condition for

\[
\sum_i \left[ \frac{\partial S}{\partial \Pi_i} \left( \frac{d^2 R_i}{dP_i^2} - \frac{d^2 \eta_i}{dP_i^2} \right) \right] < 0 \quad \text{is that}
\]

\[
\frac{d^2 R_i}{dP_i^2} - \frac{d^2 \eta_i}{dP_i^2} < 0
\]

for each player $i$.

Now it will be shown that the condition (5A.12) is satisfied whenever $d^2 \Pi_i / d\eta_i^2 < 0$, and the first order condition (5A.5) is satisfied. In other words, the second order condition for the maximization of the government's support function is satisfied whenever the payoff functions of the two players are simultaneously maximized.

Clearly, for $d^2 R_i / dP_i^2 \leq 0$ the condition (5A.12) is immediately satisfied, since by differentiating equation (5A.6) we can see that

\[
\frac{\partial^2 \eta_i}{\partial P_i^2} = -\left( \frac{\partial P_i}{\partial \eta_i} \right)^3 \frac{\partial^2 R_i}{\partial \eta_i^2}
\]

which is positive by condition (5A.9).

---

16 This condition is noted in Willisz and Wilson (1986: footnote 2) as a requirement for the existence of a Nash equilibrium.
In general, it follows from equations (5A.8) and (5A.4) that

\[(5A.14) \quad \frac{d^2 R_i}{d P_i} < \left( \frac{\partial P_i}{\partial \eta_i} \right)^3 \frac{\partial^2 P_i}{\partial \eta_i^2}.\]

Thus, equations (5A.13) and (5A.14) together imply that

\[\frac{d^2 R_i}{d P_i} < \frac{\partial^2 \eta_i}{\partial P_i^2}\]

for each player \(i\), and therefore, the condition (5A.12) holds.

Hence, if the government supplies a pricing function \(P_i(\eta_1, \eta_2)\) such that (i) for all feasible \((\eta_1, \eta_2) \neq (0,0)\), it is continuous, and differentiable - i.e., (A1) is satisfied; (ii) it is strictly increasing and concave in the lobbying expenditure, \(\eta_1\), of player 1 and decreasing and convex in the lobbying expenditure, \(\eta_2\), of player 2 - i.e., (A3) is satisfied; and (iii) it is bounded - i.e., (A4) is satisfied, then the support function of the government is maximized whenever the payoffs of the Nash players are maximized simultaneously. If there are more than one pricing functions that meet the above conditions but yield different levels of total political support, then the government would offer that pricing function which yields the maximum political support for it.

It has been shown in the text that at least one noncooperative Nash equilibrium exists in the lobbying game between the private players if the pricing function offered by the government satisfies assumptions (A1)-(A4). Note that maximization of the government's support does not preclude the possibility of multiple equilibria in the associated lobbying game. The players may be using their resources in different combinations to mobilize support for the government at different Nash equilibria so that the total support induced by the pricing function is constant. Therefore, the relative price obtained at the Nash equilibrium of the lobbying game subject to a given pricing function on the one hand, and the relative price obtained from maximizing the political support function on the other hand are identical if and only if the Nash equilibrium in the associated lobbying game is unique.

Since, the assumption of self-finance (A5) is a special case of the bounded pricing assumption (A4), the government will be maximizing its support within the self-financing constraint if it offers a pricing function that satisfies (A1)-(A3), and (A5). Therefore, a government offering a pricing function that satisfies assumptions (A1)-(A4) or (A1)-(A3) and (A5) will henceforth be called a 'popular' or 'support maximizing' government.
CHAPTER 6
BARGAINING IN THE TARIFF GAME

Introduction

In the previous chapter, it was assumed that the government offers a pricing function that responds to the lobbying efforts of the private players. Each player, therefore, had an opportunity to obtain a favourable price by spending resources in lobbying the government. Players were assumed to maximize rental payoffs - their sectoral rental income net of lobbying expenditure. Nash behaviour was assumed on the part of each player while choosing their optimal lobbying expenditures. In other words, given the pricing function of the government, it was assumed that each player takes the lobbying expenditure of the other player as given, and chooses his own lobbying expenditure to maximize his rental payoff. It was shown that at least one noncooperative Nash equilibrium exists in a lobbying economy of this type.

However, the assumptions of a noncooperative game also imply that players do not communicate and cooperate with each other in adopting joint strategies even if it may yield strictly higher payoffs to both the players. Recognizing the restrictiveness of this approach we are changing the rules of the game in this chapter. Here, we will allow players to communicate, negotiate and enter into a binding agreement if it is individually rational to do so.

Each player is now free to choose between noncooperation and cooperation with the opponent. This possibility has expanded the strategy set of each player. Not spending on lobbying the government and demanding a particular tariff rate with the opponent are new additions to the noncooperative strategy sets. Now, a player may choose to lobby the government, may unilaterally decide not to lobby the government, or may demand a binding agreement from the opponent on a particular tariff rate for not spending on lobbying the government for tariff changes.

These changes in the rules of the game transform the tariff game into a bargaining problem - where players bargain over the tariff rate (or domestic relative price). Player 1, the import competing sector, prefers a higher tariff rate and player 2, the exporting sector, prefers a lower tariff rate (prefers an export subsidy!). The main purpose of this chapter is to obtain the tariff rate that solves the bargaining problem in the tariff game.

We attain the objective in a sequence of steps. Nash's original solution to bargaining problems in which players hold unequal bargaining powers yields payoff
distributions that depend on the way players are ordered in the vector space. We show this well-known result, having specified the process of tariff determination as a standard fixed-threat Nash bargaining problem. We then provide a more precise definition of the bargaining power of a player, and argue that the distribution of bargaining power has to be included in the mathematical description of the game. We call it a generalized bargaining game, which is defined by the bargaining set, the disagreement payoffs, and the distribution of the bargaining powers of the players. We then show that the so-called asymmetric Nash solution to a bargaining problem is in fact symmetric. We provide this result as a corollary to Roth's theorem that characterizes the Nash solution to an arbitrary bargaining game with asymmetric bargaining powers. Since the solution is symmetric, we have called it the generalized Nash solution rather than calling the 'asymmetric Nash solution'.

We ask, then, why players reach an agreement in a bargaining game. Binmore, Rubinstein, and Wolinsky (1986) have argued that players reach agreement because (a) they have a high time-preference rate (so that they value current above future gain), this induces an agreement and/or (b) they fear a third party may intervene; the opportunity of gain would then be entirely lost and a disagreement would result. This increases their temptation to conclude the deal. They have further shown that the difference in players' fear of disagreement, and the difference in players' time-preference rates can be captured by the difference in the player's bargaining powers.

Aumann and Kurz (1977a) have employed the concept of 'fear of ruin' resulting from possible disagreement. At the Nash equilibrium, they show, players hold identical fear of ruin.

We have argued that Binmore, Rubinstein, and Wolinsky's 'fear of disagreement' and that addressed by Aumann and Kurz are different, and that each one can provide a separate motivation for the players to reach agreement. We have further argued that Aumann and Kurz's concept of 'fear of ruin' can not be captured by differences in players' bargaining powers, because it is not a constant number. It changes during the bargaining process as the players attain different levels of gains. Since, Aumann, and Kurz's concept of 'fear of ruin' was defined for bargaining games with equal bargaining powers, we have attempted to generalize this concept for an arbitrary distribution of the bargaining power among the players so that all sources of the 'fear of ruin' could be addressed simultaneously.

We have proved that the equality of the generalized fear of ruin constitutes a separate characterization of the generalized Nash solution to an arbitrary Nash bargaining problem, including the tariff game. This result is new, and more importantly, it holds the key to the results of our subsequent chapters. We have also shown that if the
fear functions are well-behaved, then the generalized Nash solution to the tariff game is stable.

Our result is different from that obtained by Svejnar (1986), who showed that the generalized Nash solution also implies equality of generalized fear of ruin, and that equality of generalized fear of ruin together with usual axioms implies the generalized Nash solution. We have shown that equality of generalized fear of ruin, when each player holds a strictly positive fear of ruin, yields the generalized Nash solution without any reference to the other axioms. The advantage of this result is that we can now obtain the generalized Nash bargaining solution in a different way - using players' fear of ruin and the distribution of bargaining power. This result seems to be useful in simple and intuitive demonstration of the bargaining process and the generalized Nash bargaining solution.

Finally, with a summary of the bargaining problem, we have stated the necessary and sufficient condition for the Nash solution to the bargaining problem of the tariff game. We have also attempted to identify the fear of ruin with the concept of endogenous bargaining power of the players.

This chapter is organised into five sections. The first section describes the setting of the game in the form of a standard bargaining problem. The second section describes the Nash solution to the bargaining problem. It is argued that the generalized Nash solution to a bargaining problem that allows unequal bargaining power is not asymmetric as is commonly believed. In the third section, the bargaining process is described in terms the concept of fear of ruin, where we argue that the fear of disagreement referred to by Aumann and Kurz and the fear of disagreement referred by Binmore, Rubinstein and Wolinsky are different, and a generalization of the concept of fear of ruin has been proposed. In the fourth section, the main result that the equality of generalized fear of ruin constitutes a separate characterization of the generalized Nash bargaining solution, is proved. Finally, in the fifth section the basic contention of the chapter has been summarized.

The contribution this chapter makes to the thesis is that it examines the tariff game in a bargain-theoretic framework thoroughly. It obtains the necessary and sufficient condition that characterizes the generalized Nash solution to the bargaining problem in the tariff game. The next chapter will take up this condition and embed it into the policy-exogenous general equilibrium model (described in chapter 3) to obtain a tariff-endogenous general equilibrium model of the economy.
6.1 Rules of the Game and the Bargaining Problem

We continue to assume that the government is a support maximizer. So, if the two players - owners of the specific factors in the two sectors - agree on some issue, the best policy for the government is to implement the agreement. This will guarantee the maximum support to the government. If they disagree, then the game will be played as described in the previous chapter: the government will choose a policy that maximizes its political support subject to the reaction functions of the two coalitions (players). The government behaves as a Stackelberg leader vis-a-vis the private players - it announces its policy function. The two players behave as Nash players against each other taking the government's policy function as given.

Thus, in the tariff game, the new rules of the game would mean the following to the players: If the two players agree on a particular tariff rate, then the government will announce and implement that rate. Otherwise, (if they fail to agree on any rate) the government's pricing function will be in effect and the two players will play a non-cooperative game. The tariff rate that arises at the Nash equilibrium of the game will be implemented. Therefore, if the two parties cooperate with each other - that is, they agree on a tariff rate, and agree not to lobby the government, then they receive the resulting rental income as their payoffs. If they cannot cooperate, then they receive the

---

1 For analytical similarity between the choices of a support maximizing government and a welfare maximizing government see Baldwin (1987).
resulting rents less the lobbying expenditures as their payoffs. This situation is illustrated in Figure 6.1.

For simplicity, in figure 6.1, identical Cobb-Douglas production technologies are assumed. The curve CD represents the rent transformation frontier (RTF) for given endowment of factors, and international prices (see chapter 4). Suppose that the parties could not agree on any tariff rate, and chose to play a noncooperative game. Suppose further that a unique noncooperative Nash equilibrium is obtained with lobbying expenditures $\tilde{\eta}_1$ and $\tilde{\eta}_2$ yielding a domestic price ratio $\hat{\tilde{P}}$. Then the sectoral payoffs, which are rental incomes net of lobbying expenditures, are given by $\tilde{\Pi}_1$ and $\tilde{\Pi}_2$ for sector 1 and sector 2 respectively.

Recall that sectoral rental incomes are measured in units of own output. Let the point E (in figure 6.1) denote the payoff combination and the point R denote the combination of sectoral rental incomes at the noncooperative Nash equilibrium. Then E defines the disagreement payoffs - the payoff to each player if they can not reach an agreement.

However, if the two players cooperate, then they need not spend resources in competitive lobbying, and they can stay on the rent transformation frontier CD. For example, if the players had accepted the price $\hat{\tilde{P}}$ and agreed not to participate in competitive lobbying, both would have received payoffs corresponding to the point R, which represents a combination of higher payoffs than that corresponding to the point E.

Thus, given that E would be the outcome of disagreement, the shaded area EFG represents the set of payoff combinations that the two players can improve upon by choosing cooperation rather than noncooperation. The arc FG represents the set of feasible and Pareto efficient payoffs. Any point on it is strictly better than point E, and any movement along it hurts one player or the other.

As indicated in the previous chapter, a noncooperative Nash equilibrium may not be unique. In the case where there are multiple equilibria it is not possible to identify the disagreement payoffs of the game with a particular noncooperative Nash equilibrium of the game. Since all the Nash equilibria are characterized by non-zero lobbying by all players (Lemma 5.4), the resulting equilibrium payoffs in each of the noncooperative Nash equilibria are not on the curve CD, but lie inside it. Therefore, the above argument is equally applicable irrespective of which of the noncooperative Nash equilibria is attained.

In general, all payoff combinations represented by points in the area OCD can be regarded as the set of feasible outcomes and that the set of noncooperative Nash
equilibria, which is a subset of feasible outcomes, can serve as potential disagreement payoffs. The sets of points in the areas like EFG are always non-empty and contain points that, in terms of payoffs, strictly dominate points like E. Therefore, there is always an incentive to the players to get involved in a bargaining process and search for a mutually agreeable tariff rate.

There are three reasons to believe that all agreements reached in a tariff game will be enforceable. First, playing a noncooperative Nash equilibrium strategy is always a credible threat that can be issued by any player against the other player, and that works as a deterrent against possible deviant behaviour of either player. Therefore, when the players agree not to spend on lobbying the government and agree on a tariff rate, they will most likely find it not rational to deviate from it. Second, as Subik (1982) argued, constitutional arrangements and the presence of government as the enforcing agency makes players almost incapable of deviating from the agreement. Moreover, if the government is a support maximizer, as we have assumed, then it will have an incentive to implement such cooperative agreements. Third, the tariff game is a periodic game. It is played repeatedly. Cooperative outcomes of repeated games are, in general, self-enforcing (Fudenberg and Maskin, 1986; Friedman, 1986).

Definition 6.1 (Bargaining Set). For given \( \Pi^d \equiv (\Pi_1^d, \Pi_2^d) \) let

\[
\mathcal{R} = \left\{ (\Pi_1^d, \Pi_2^d) | \Pi_1 \leq K_1 P_1^{-\alpha_1(1-\alpha_1)} \times \left[ P_1^{-\alpha_1} - \alpha_1 \alpha_2^{-\alpha_2(1-\alpha_2)} (1 - \beta_2^\alpha (\Pi_2 / K_2)^{-\alpha_2(1-\alpha_2)}) \frac{1}{1/(1-\alpha_2)} \right] \Pi_1 \geq \Pi_1^d \right\}.
\]

Then, \( \mathcal{R} \) is a collection of all payoff combinations that are individually rational, and over which the players may bargain. \( \mathcal{R} \) is defined as the bargaining set.

Note that the Pareto efficient boundary of the bargaining set is the segment of the rent transformation frontier (see equation 4.6') that has individually rational points. It has already been shown that if the elasticities of factor substitution in both the sectors are at least unity then the boundary of the bargaining set is concave to the origin (Proposition 4.2, chapter 4). Therefore, the points in OCD form a convex set. Since the maximum output that a sector can produce in the event of specialization is finite (because of finite endowment of mobile factor(s) and concave production function), and since also the rental income is always less than the level of output, the set \( \mathcal{R} \) is also bounded. The set includes its boundary points, hence it is closed. The set \( \mathcal{R} \) is also a subset of a 2-dimensional Euclidean space, therefore it is compact and convex.

If the elasticities of factor substitution are sufficiently low to undermine the convexity of the bargaining set defined over certain outcomes, then as in the standard
case (for example Nash, 1950) we can invoke the expected payoff approach that will guarantee the convexity of the corresponding bargaining set. Any concave function defined over $\mathcal{R}$ will have its maximum in it.

If the disagreement payoff $\Pi^d$ is assumed to be predetermined and fixed, then the underlying bargaining problem of the tariff game satisfies the conditions of the existence theorem proved by Nash (1953). In the case that the disagreement payoff is not known a priori, then the bargaining problem in a tariff game satisfies the conditions of the existence theorem proved in Harsanyi (1963).

Thus, whether the disagreement payoffs are assumed to be predetermined or not (variable), the tariff game in cooperative form can be viewed as a standard bargaining problem. However, in this chapter we will assume that the disagreement payoff is predetermined and therefore, the bargaining problem is defined by the pair $(\mathcal{R}, \Pi^d)$. Let $\mathcal{B}$ be the class of such 2-person bargaining games for which the underlying noncooperative game is given by $T$ (see Chapter 5).

6.2 Nash Solution to the Bargaining Problem in a Tariff Game

Several solution concepts have been proposed in the literature. The Nash solution is one of them. A short review of these solution concepts can be found in Datt (1989). He has also convincingly argued that Nash solution has a degree of generality not shared by other solution concepts. Several other studies that attempted to solve standard bargaining problems by introducing various frictions into it have concluded that Nash solution is robust (see, Binmore, Rubinstein and Wolinsky, 1986; van Damme, 1986; Chun, 1988b; Chun and Thomson, 1990; Carlson, 1991).

---

2 It is very important to note that this study assumes that players are interested in maximization of the sectoral rents. How they use the rental income - whether they spend all in the consumption of the two goods, or save and invest - is not analysed. It is, in turn, assumed that the owners of the specific factors would behave nonstrategically as consumers.

This stylization implies that the two players bargain in terms of rental payoffs, or in terms of relative price or in terms of tariff rate but not in terms of utilities as the standard Nash bargaining process is formulated. In this sense the bargaining game that is studied here is an adapted version of the standard Nash bargaining game. Binmore (1987c) has studied such an adapted game for an exchange economy, and he preferred to call it a bartering game. In his bartering game players communicate not in terms of utilities but in terms of real quantities of commodities that are to be traded. We follow Binmore in this respect because it is more sensible to assume that players have perfect information on each other's rental prospects at different prices than to assume that they know each other's rental prospects and the utilities of the rental incomes at various prices as well. Furthermore, the assumption that production sectors strive for maximization of profit is fairly standard, and is quite common in studies of labour market that use bargain theoretic approach (for example, Datt, 1989).

3 The problem of identifying the disagreement payoffs will be dealt in the next chapter.
Nash (1950, 1953), who studied 2-person bargaining games, defined a solution of a bargaining game in $B$ as a function $\mu: B \rightarrow \mathbb{R}^2$ such that for any $(\mathcal{R}, \Pi^d) \in B$ we have $\mu(\mathcal{R}, \Pi^d) \in \mathcal{R}$. In other words, a solution of the game is a rule for assigning a feasible payoff to each game. Roth (1979) has argued that this rule can also be interpreted as an arbitration procedure.

Nash (1953), in his seminal paper, used both strategic and axiomatic approaches to analyse the bargaining problem. In the strategic approach, he constructed a negotiation model, so-called "demand game", in which each player (in a 2-person game) demands a particular utility payoff. If the demands of both the players are jointly compatible, then each one gets what is demanded; otherwise each receives the predetermined disagreement payoff. Nash has shown that rational bargainers will agree on the payoff division that maximizes the Nash product over the feasible set.

In the axiomatic approach, Nash (1953) listed a set of eight general properties that any reasonable solution to the bargaining problem should possess. A deduction of logical solutions that satisfy the "desirable" properties led Nash to the same solution that he obtained from the negotiation model - that the solution should maximize the Nash product! Nash, therefore, wrote,

It is rather significant that this quite different approach yields the same solution. This indicates that the solution is appropriate for a wider variety of situations than those which satisfy the assumptions we made in the approach via the model (p. 136).

Subsequent authors have improved upon Nash's work and have neatly summarized his works. For example, Binmore (1987a) has listed "somewhat freely adapted versions of Nash's axioms" as follows:

**Axiom 1** (feasibility and strong individual rationality).

$$\Pi^d < \mu(\mathcal{R}, \Pi^d) \in \mathcal{R}.$$  

That is, a solution to the bargaining problem should be feasible, and that it must be strongly individually rational. The second characteristic requires that each player should receive more in bargaining "equilibrium" than that can be obtained in disagreement.

---

4 In fact, Harsanyi (1963) has shown quite generally that there exists an optimal threat strategy combination such that if the cooperation breaks down each of the player will find it best to play against the other player(s)
Axiom 2  (invariance).

For any increasing affine\(^5\) transformation \(\alpha: \mathbb{R}^2 \to \mathbb{R}^2\),

\[ \mu(\alpha \mathcal{R}, \alpha \Pi^d) = \alpha \mu(\mathcal{R}, \Pi^d). \]

In other words, if the units of measurement and the origins of payoffs of the players were changed\(^6\) by any affine transformation, then the outcome of the new bargaining game will be equal to the conformable transformation of the original bargaining solution. This requires that the "real" solution should not depend on the chosen units of measurement. It should always be recoverable by inverse transformation on the solution. This axiom is also called Independence of Equivalent Utility (Payoffs) Representation.

Axiom 3  (independence of irrelevant alternatives).

\[ \mu(\mathcal{R}, \Pi^d) \in \varphi \subseteq \mathcal{R} \Rightarrow \mu(\varphi, \Pi^d) = \mu(\mathcal{R}, \Pi^d). \]

This axiom states that if \(\varphi\) is a subset of the original bargaining set \(\mathcal{R}\) and contains the original solution point \(\mu(\mathcal{R}, \Pi^d)\), then \(\mu(\mathcal{R}, \Pi^d)\) should also be the solution to the bargaining game \((\varphi, \Pi^d)\). The intuition is that since the set \(\varphi\) was available when the bargaining set was \(\mathcal{R}\) and the players nevertheless chose \(\mu(\mathcal{R}, \Pi^d)\) should mean that they will choose \(\mu(\mathcal{R}, \Pi^d)\) when it is available in the game \((\varphi, \Pi^d)\). This is a stipulation that the players be consistent.

Axiom 4  (efficiency).

\[ M > \mu(\mathcal{R}, \Pi^d) \Rightarrow M \notin \mathcal{R}. \]

In other words, the axiom of efficiency requires that the solution picked by the \(\mu\) rule should not be dominated by any other payoff combinations in the bargaining set. This axiom requires that the solution be Pareto optimal. There should be no possibility of increasing the payoff of one player without reducing the payoff of the other player.

---

\(^5\) \(\alpha\) is called an increasing affine transformation if \(\alpha(y) = ay + b\) with \(a > 0\), and any real number \(b\).

\(^6\) Note that the numerical value of rental income can be changed by changing the unit of output measurement, and a change in the origin can be brought by effecting a lump sum income transfer to and from a sector. We know that a lump sum tax, for example, does not affect the output decision of a production sector, whereas it affects the break even point, where a sector obtains a zero rental income. It is implicitly assumed that the lump sum taxes are never greater than the disagreement payoffs. This assumption is necessary to ensure that specialization will not be induced by the imposition of lump sum taxes or by the presence of some fixed costs. This requirement slightly restricts the nature of affine transformation that can be applied when we are dealing with rental income rather than "utilities". So long as the proofs of the theorems are concerned it is of no consequence.
**Axiom 5** (symmetry).

If $\tau : (\Pi_1, \Pi_2) \rightarrow (\Pi_2, \Pi_1)$, then

$$\mu(\tau \mathcal{R}, \tau \Pi^d) = \tau \mu(\mathcal{R}, \Pi^d).$$

This axiom means that whoever is called the first player is immaterial. Each player will get the same payoff whether she is called player 1 or player 2 - measured along the x-axis or measured along the y-axis. This axiom has strong implications and needs some scrutiny.

**Definition 6.2** A bargaining set $\mathcal{R}$ is called symmetric if

$$(\Pi_1, \Pi_2) \in \mathcal{R} \iff (\Pi_2, \Pi_1) \in \mathcal{R}.$$

A bargaining game $(\mathcal{R}, \Pi^d)$ is called a symmetric game if $\mathcal{R}$ is symmetric and $\Pi_1^d = \Pi_2^d$.

If $(\mathcal{R}, \Pi^d)$ is a symmetric game, then axiom 5 requires that $\mu_1(\mathcal{R}, \Pi^d) = \mu_2(\mathcal{R}, \Pi^d)$. The argument is as follows: Since the game is symmetric, nothing will be changed by a permutation of the players. In particular, $(\tau \mathcal{R}, \tau \Pi^d) = (\mathcal{R}, \Pi^d)$, therefore, $\mu(\tau \mathcal{R}, \tau \Pi^d) = \mu(\mathcal{R}, \Pi^d)$. That is, $\mu_1(\tau \mathcal{R}, \tau \Pi^d) = \mu_1(\mathcal{R}, \Pi^d)$. But $\mu_1(\tau \mathcal{R}, \tau \Pi^d) = \tau \mu_1(\mathcal{R}, \Pi^d)$ imply that $\mu_1(\mathcal{R}, \Pi^d) = \mu_2(\mathcal{R}, \Pi^d)$. Both players will gain equally over the disagreement payoffs. Therefore, in a symmetric game with $\Pi^d = 0$ if the bargaining solution rule $\mu$ satisfies the axiom of symmetry then the solution always lies on the ray of 45 degrees.

**Theorem 6.1** (Nash's theorem). There is a unique solution possessing axioms 1-5. It is the function $\mu$ defined by $\mu(\mathcal{R}, \Pi^d) = \overline{\Pi}$ such that $\overline{\Pi} \geq \Pi^d$ and $(\overline{\Pi}_1 - \Pi_1^d)(\overline{\Pi}_2 - \Pi_2^d) > (\Pi_1 - \Pi_1^d)(\Pi_2 - \Pi_2^d)$ for all $\Pi \in \mathcal{R}$and $\Pi \neq \overline{\Pi}$.

**Proof:** See Nash (1950, 1953), and Roth (1979).

Roth (1979) also showed, however, that a solution that satisfies axioms 1-3 also satisfies the axiom 4. Therefore, in the presence of axioms 1-3 and axiom 5, axiom 4 - that the solution is required to be Pareto optimal - is redundant. The beauty of Nash's solution, nevertheless, lies in the result that the bargaining problem can be solved by maximizing the Nash product over the bargaining set.

Nash's bargaining solution is highly restrictive. It is based on the assumption that all "significant differences between the players are those which are included in the mathematical description of the game" (Nash, 1953: p. 137). This implies that when the
game is converted into the symmetric form, all differences between the players disappear. Nash (1953: p. 138) further argued that

With people who are sufficiently intelligent and rational there should not be any question of "bargaining ability", a term which suggests something like skill in duping the other fellow. The usual haggling process is based on imperfect information, the hagglers trying to propagandize each other into misconceptions of the utilities involved. Our assumption of complete information makes such an attempt meaningless.

Precisely because of this interpretation of the bargaining situation, a permutation of the players in a symmetric bargaining game brought no change in the bargaining set as well as the position of level curves of the Nash product. Therefore, identical payoffs were obtained by the players at the Nash equilibrium of a symmetric game. This argument clearly dismisses all possibilities but the bargaining set as that which differentiates the players. In view of recent developments in cooperative game theory, Nash's argument maintains, in particular, that the players have equal bargaining power or ability.

Recent studies on the bargaining problem have not only raised questions concerning the completeness of information about each other's utility functions, but also have shown that there may exist factors influencing the outcome of bargaining that are not accounted by the bargaining set and the disagreement point - the constituents of a mathematical description of a bargaining problem. In particular, they contend, players may well be endowed with uneven bargaining powers or weights.

For example, Kalai (1977a) has shown that if an n-person bargaining game is played by two coalitions of size p and q with p+q=n such that within each coalition players have identical utility functions, then a non-symmetric Nash solution may arise even if the n-person game yields a symmetric Nash solution. In this case the source of apparent 'bargaining power' of the coalition is its membership. This was not envisaged by Nash. Similarly, the other sources of asymmetry are: players having different degrees of risk aversion (Roth, 1979), difference in the time preference rate (Rubinstein, 1982), different probability attached to the risk of breakdown of the negotiation (Binmore, Rubinstein, and Wolinsky, 1986), bargaining skill (Ohyama, 1989) and players possessing imperfect knowledge about each other (Harsanyi and Selton, 1972).

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7 Nash, in his 1950 paper, expressed this assumption explicitly, which he rejected in the 1953 paper.

8 Further references of the works that have independently obtained asymmetric Nash bargaining solutions can be found in Binmore (1987d: p.94).
In the case that players have unequal 'bargaining power', for whatever the reason, Nash's solution payoff to each player depends on the order in which the players are represented. In other words, Nash's solution, as characterized by maximization of the 'Nash product', may not satisfy the axiom of symmetry.

To see this consider a symmetric bargaining game \((\mathcal{X}, \Pi^d)\). Let the ordered pair \((\Pi_1, \Pi_2)\) such that \(\Pi_1 \neq \Pi_2\) be the Nash product maximizing solution of the game. Let \(\tau\) be any player permutation. Since \((\mathcal{X}, \Pi^d)\) is a symmetric game, we must have \((\mathcal{X}, \Pi^d) = (\tau\mathcal{X}, \tau\Pi^d)\). The Nash product maximizing solution vector of the permuted game \((\tau\mathcal{X}, \tau\Pi^d)\) should also be equal to \((\Pi_1, \Pi_2)\). But this means that player 2 will receive \(n_1\) and player 1 will receive \(n_2\). Hence, the solution depends on order in which the players are represented in the game.

Therefore, Nash's solution to the bargaining problem required an extension so that a solution can be obtained even in the presence of unequal bargaining powers.

In theories of bargaining the concept of 'bargaining power' is imprecisely defined (Binmore, Rubinstein, and Wolinsky, 1986: 186). To make the term more precise we state the following definition.

Consider a controlled bargaining situation \((A,0)\), where \(A = \{X \in R^2 | X_i \geq 0, \sum X_i \leq 1\}\). In this game the players have identical prospects in agreement and in disagreement. If their payoffs at the solution of the game differ consistently every time they play the game \((A,0)\), then the result can be attributed to some unaccounted rule of the game that creates a difference between otherwise identical players.

**Definition 6.3** (Bargaining Power). Suppose that, in agreement, player 1 always gets \(\delta\) times what player 2 gets in the controlled bargaining game \((A,0)\), where \(\delta\) is some positive number. Then, the number \(\delta\) measures some kind of power of player 1 over player 2, and therefore, the number \(\delta\) is defined as the relative bargaining power of player 1.

This definition of bargaining power accords well with that of Chamberlain and Kuhn (1965: p.170), who defined 'bargaining power as the ability to secure another's agreement on one's own terms.'

Let \(Z\) be a vector of all variables, such as the time preference rate of the players, the players' belief that the opportunity of gain will evaporate or be snatched by a third party, the difference in players' skill of negotiation or coalitional strength of members, the difference between the degrees of free rider problem within each coalition and other unaccounted factors in the political environment etc., that influence the bargaining
outcome but do not belong to the choice set of the bargainers. Then, we can infer, on the basis of previous studies referred above, that

$$\delta = \delta(Z).$$

The functional dependence of relative bargaining power of player 1 on \( Z \) acknowledges that the relative bargaining power of a player is essentially a dynamic concept. It may change as the exogenous environment changes.

In a given environment \( Z \), a value of \( \delta \) equal to unity implies that the two players are equally powerful, neither is favoured against the other, and \( \delta > 1 \) implies that player 1 possesses more bargaining power than player 2, and vice versa.

We normalize the measure of bargaining power by defining

$$\Theta_1 = \frac{\delta}{1 + \delta}, \text{ and } \Theta_2 = \frac{1}{1 + \delta}.$$

Then, the parameter \( \Theta_i(Z) \) satisfies \( 0 < \Theta_i(Z) < 1 \), and \( \sum_i \Theta_i(Z) = 1 \); and therefore, it can be called the normalized bargaining power of player \( i \).

With this definition of bargaining power, we can proceed on to the extension of Nash's bargaining solution in the presence of unequal bargaining power of the players. The following important theorem in this direction was proved by Roth (1979).\(^9\)

**Theorem 6.2** (Roth, 1979: Theorem 3). For each strictly positive vector \( \Theta \), such that \( \sum_i \Theta_i = 1 \), there is a unique solution \( \Pi \) satisfying the axioms of feasibility, invariance and independence, such that \( \mu(A, 0) = \Theta \) where \( A = \{ X \in \mathbb{R}^2 | X_i \geq 0, \sum X_i \leq 1 \} \). For any bargaining game \( (\mathcal{R}, \Pi^d) \), \( \mu(\mathcal{R}, \Pi^d) = \Pi \in \mathcal{R} \), such that \( \Pi \geq \Pi^d \) and

\[
(\Pi_1 - \Pi_1^d)^{\Theta_1} (\Pi_2 - \Pi_2^d)^{\Theta_2} > (\Pi_1 - \Pi_1^d)^{\Theta_1} (\Pi_2 - \Pi_2^d)^{\Theta_2}
\]

for all \( \Pi \in \mathcal{R} \) such that \( \Pi \geq \Pi^d \) and \( \Pi \neq \Pi^d \).

**Proof:** See Roth (1979: 16-17), Binmore (1987a: 34-37), and Appendix-6B.

The first part of the theorem states that if the players possess different weights given by the vector \( \Theta \), and bargain over a 'pie' of size unity, represented by the symmetric bargaining set \( A \) with disagreement payoff equal to zero, then player \( i \)'s share in the unit pie is just \( \Theta_i \). This explains why \( \Theta_i \) has been defined as the bargaining

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\(^9\) In a 2-person case the theorem was first proved by Kalai (1977a).
power of player $i$ (see also Binmore, 1987a and 1987b). If all players have equal bargaining power, then the theorem implies that each of the players will get an equal share of the pie. In particular, if the set $A$ is defined for a 'pie' of size $n$, that is $A = \{ X \in \mathbb{R}^2 | X_i \geq 0, \sum X_i \leq n \}$, and $\Theta_i$'s are suitably normalized so that $\sum_{i=1}^{n} \Theta_i = n$, then each player will get one "pie". This is the standard Nash bargaining solution for a symmetric game, which is also called the symmetric Nash bargaining solution. Thus, the theorem 6.1 (Nash's theorem) is a special case of Theorem 6.2 in which the players hold equal bargaining power.\footnote{Nash's solution to any arbitrary bargaining problem can be obtained easily using this property of the underlying symmetric bargaining problem.}

The second part of the theorem shows that in any bargaining game with predetermined disagreement payoffs, and a compact and convex bargaining set the solution to the bargaining problem that satisfies axiom 1-3 is the one that maximizes the asymmetric Nash product over the bargaining set. The strict inequality implies that the solution is unique. That is, the payoff distribution that maximizes the asymmetric Nash product and the solution that satisfies the "desirable" properties are one and the same. One implies the other.

A note of clarification is warranted here. It may appear that the solution to the bargaining game $(\mathcal{R}, \Pi^d)$ that satisfies axioms 1-3 does not necessarily satisfy axiom 5 - the axiom of symmetry. Binmore (1987a), for example, has explicitly stated that the asymmetric Nash solution satisfies axiom 5 if and only if the players have equal bargaining powers. But this would mean that the solution depends on the way players are represented if players do not have equal bargaining powers. If a player's payoffs are now measured along the x-axis, then she will receive a different payoff at the solution of the game than she would obtain had her payoffs were measured along the y-axis. This situation, certainly, is not satisfying.

However, once we isolate the axiom of symmetry from the hangover of identical bargaining power of the players and treat them independently it can be easily seen that the "asymmetric" Nash solution is in fact symmetric. We will show this result as a Corollary to Roth's Theorem. First, to highlight the role of bargaining power in the mathematical description of a bargaining game we state the following definition:

**Definition 6.4** (Generalized Bargaining game). A triplet $(\mathcal{R}, \Pi^d, \Theta)$ is defined as a generalized bargaining game, where $\mathcal{R}$ is a compact and convex bargaining set,
\(\Pi^d \in \mathcal{R}\) is a predetermined disagreement payoff vector, and \(\Theta\) is, as defined above, a given vector of (normalized) bargaining power of the players.

Definition 6.4 recognizes the independent status of the information regarding the bargaining power distribution of the players. Therefore, it is included in the mathematical description of the game. Curiously enough, previous writers, who recognized the role of the distribution of bargaining power in determining the solution of a bargaining game, did not include it in the description of the game. For example, in order to specify a solution to any bargaining game \((\mathcal{R}, \Pi^d)\), Roth's theorem (theorem 6.2) requires a priori information on \(\Theta\). This is essentially equivalent to say that the game is defined by a triplet \((\mathcal{R}, \Pi^d, \Theta)\). Clearly, Roth's theorem holds for the bargaining problem \((\mathcal{R}, \Pi^d, \Theta)\) as well as it holds for a bargaining problem \((\mathcal{R}, \Pi^d)\) with given \(\Theta\).

With this definition of a generalized bargaining game, all of the axioms listed above can be restated 11 accordingly by simply changing the description of the bargaining problem. From now on the description \((\mathcal{R}, \Pi^d)\) will be replaced by \((\mathcal{R}, \Pi^d, \Theta)\) to mean a bargaining problem with bargaining set \(\mathcal{R}\), a predetermined disagreement point \(\Pi^d \in \mathcal{R}\) and an exogenously given allocation of bargaining power \(\Theta\).

**Corollary 6.1** For any bargaining game \((\mathcal{R}, \Pi^d, \Theta)\), the solution \(\mu(\mathcal{R}, \Pi^d, \Theta) = \Pi \in \mathcal{R}\) such that

\[
(\Pi_1 - \Pi^d_1)^{\Theta_1} (\Pi_2 - \Pi^d_2)^{\Theta_2} > (\Pi_1 - \Pi^d_1)^{\Theta_1} (\Pi_2 - \Pi^d_2)^{\Theta_2}
\]

for all \(\Pi \in \mathcal{R}\), \(\Pi \neq \Pi^d\) and \(\Pi^d \neq \Pi^d\) satisfies axiom 5, and therefore is symmetric.

**Proof:** Let \(H(\Pi_1, \Pi_2) = 0\) be the Pareto efficient boundary of the bargaining set \(\mathcal{R}\). Let \(\Pi\) maximize \((\Pi_1 - \Pi^d_1)^{\Theta_1} (\Pi_2 - \Pi^d_2)^{\Theta_2}\) subject to \(H(\Pi_1, \Pi_2) = 0\). To visualize the effect of permutation, assume initially that player 1's payoffs are measured along the y-axis and player 2's payoffs are measured along the x-axis. Let \((\Pi_2, \Pi_1)\) represent the initial solution as shown in the panel (a) of the following figure 6.2.

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11 Axiom 2 requires a different treatment. It should be read as follows:

For any increasing affine transformation \(\alpha: R^2 \rightarrow R^2\),

\[\mu(\alpha \mathcal{R}, \alpha \Pi^d, \Theta) = \alpha \mu(\mathcal{R}, \Pi^d, \Theta)\]

This is because bargaining power distribution is independent of unit of measurement of rental income. It has been normalized separately.
Now let us permute the players and measure player 1's payoffs along the x-axis and player 2's payoffs along the y-axis. The parameters $\Theta_1$ and $\Theta_2$ continue to measure the bargaining powers of players 1 and 2 respectively (they have been permuted too). Compared to panel (a) the appearance of the bargaining set, and the curvature of the level curve of the asymmetric Nash product have changed in panel (b). However, the problem is still to maximize $(\Pi_1 - \Pi_1^*)^{\Theta_1} (\Pi_2 - \Pi_2^*)^{\Theta_2}$ subject to $H(\Pi_1, \Pi_2) = 0$.

Therefore, the solution vector is $(\Pi_1, \Pi_2)$ which is the permutation of the initial solution vector $(\Pi_2, \Pi_1)$.

Thus, the unique solution vector that maximizes the "asymmetric" Nash product over the bargaining set is simply permuted by a permutation of the players. The payoff received by each of the players is unaffected by the permutation. Q. E. D.

Now, in the light of Definition 6.4, we restate Definition 6.2 as follows:

**Definition 6.2'** A bargaining set $\mathcal{R}$ is called symmetric if

$$(\Pi_1, \Pi_2) \in \mathcal{R} \iff (\Pi_2, \Pi_1) \in \mathcal{R}.$$ 

A generalized bargaining game $(\mathcal{R}, \Pi^d, \Theta)$ is a symmetric game if and only if $\mathcal{R}$ is symmetric, $\Pi_1^d = \Pi_2^d$ and that $\Theta_1 = \Theta_2$.

It is obvious that if a generalized bargaining game is symmetric, then the players will receive equal payoffs at the solution of the game. The argument is as follows: The game is symmetric implies that for any permutation $\tau$ on players we have
\((R, \pi^d, \Theta) = (\tau R, \tau \pi^d, \tau \Theta)\). Therefore, \(\mu(R, \pi^d, \Theta) = \mu(\tau R, \tau \pi^d, \tau \Theta)\). Since, the solution function \(\mu\) is symmetric, we must have \(\mu(R, \pi^d, \tau) = \tau \mu(R, \pi^d, \Theta)\). That is, \((\mu_1, \mu_2) = \tau(\mu_1, \mu_2) = (\mu_2, \mu_1)\). Therefore, we have \(\mu_1 = \mu_2\).

We know from theorem 6.2 that a solution to the bargaining problem satisfies the axioms 1-3 if and only if it maximizes the "asymmetric" Nash product. The Corollary 6.1 shows that the solution also satisfies the axiom of symmetry. This result holds as long as the bargaining powers are assigned to the players, not to the axes. Since Roth has shown that satisfaction of axiom 1-3 implies satisfaction of the axiom of efficiency, it follows that the solution to any arbitrary bargaining game, for given distribution of bargaining powers, satisfies all the five axioms as satisfied by the original Nash's solution if and only if it maximizes the corresponding asymmetric Nash product over the bargaining set. The only difference is that the original Nash solution applies in the case when all players have equal bargaining power. In the presence of uneven bargaining power it loses its symmetric property. The solution that follows from theorem 6.2, however, applies to any arbitrary distribution of bargaining power provided that bargaining power of each player is strictly positive.12

For this reason, in what follows the so-called 'asymmetric' Nash solution to any bargaining game with arbitrary distribution of bargaining power will be called the generalized Nash solution. The product \((\Pi_1 - \Pi_1^0)^a (\Pi_2 - \Pi_2^0)^b\) will be called the generalized Nash product.

### 6.3 The Bargaining Process

Since the bargaining problem in the tariff game is essentially the same as any abstract bargaining game studied by game theorists, we can obtain insights into the underlying bargaining process from their studies as well. The explanation of the bargaining process most frequently referred to is that of Zuthen (1930). He assumed that in a bargaining process players offer proposals to each other, and postulated that each party will make concessions to his opponent once he finds that his opponent's determination is firmer than his own. Harsanyi (1956) has shown that this process of negotiation is mathematically equivalent to that of Nash's solution.13

In recent times, game theorists have started to study the bargaining process by specifying every move of the players (for example, Rubinstein, 1982). This is often called the sequential strategic approach. Rubinstein showed the existence of a unique

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12 Otherwise the solution will not be unique. See Binmore (1987b).

13 Further details can be found in Harsanyi (1956), Roth (1979: 28-31).
perfect Nash equilibrium in any bargaining game, if players have sufficiently high time-preference rates and/or every player has to bear a fixed cost of bargaining for each period.

Binmore, Rubinstein and, Wolinsky (1986) not only studied bargaining problems using the sequential strategic approach but also studied the relationship between the solutions obtained from this approach and that of the axiomatic approach or static representation of a bargaining problem. Moreover, they examined the two motives behind the bargaining process that may induce the players to agree rather than to insist indefinitely on incompatible demands.

One of the motives studied by Binmore, Rubinstein and, Wolinsky (hereafter BRW) was the player's "impatience to enjoy the fruits of an agreement", which is concerned with the relative time preference of the players, and the other motive was the player's relative fear of disagreement.

If the players do not have a high enough time-preference rate, then they may keep on insisting on incompatible demands and no agreements may be reached. Making use of Rubinstein's seminal study (Rubinstein, 1982) Binmore (1987b) showed that the relative difference in the time preference rate of the players can be a source of unequal bargaining power of the players in the static representation of the game. Player with relatively higher time preference rate will have lower bargaining power. This was also obtained by BRW.

In their comprehensive study of the strategic models of bargaining BRW also studied a game in which players faced an exogenous risk of breakdown of negotiations. They also found the existence of a unique perfect equilibrium in this game. They showed that if the players differ in their beliefs concerning the likelihood of a breakdown of the negotiation, then the unique perfect equilibrium of the game approaches to the 'asymmetric' Nash bargaining solution to the static version of the game. The correspondence is that if a player's estimate of the probability of breakdown is higher relative to his opponent, his bargaining power will be correspondingly lowered.

These results have two important implications. First, if players differ in their time-preference rate and/or in the probability of exogenous breakdown of negotiation, then the solution to the bargaining problem is given by the generalized Nash bargaining solution, whereas if the players do not differ in their beliefs (hold equal probability of breakdown, and have identical time preference rates), then the original Nash solution is applicable. Second, if each player assigns a constant probability to the breakdown of the negotiation, then the fear of breakdown consistent with this probability is captured by
the asymmetric bargaining powers of the players. The player who is relatively more fearful will have lower bargaining power.

6.3.1 Fear of Ruin: Another Fear of Disagreement

To probe the fear of disagreement further, let us assume that the players hold identical beliefs about the external environments, have identical time preferences, etc., such that the players end with having equal bargaining powers in BRW's sense. Now let us consider a situation as described in figure 6.3.

Assume that RF is the rent transformation frontier (the Pareto efficient boundary of the bargaining set in an arbitrary bargaining game) and $\Pi^*$ is a distribution of rents at any mutually agreeable\textsuperscript{14} relative price $P^*$. Without loss of generality, let us take the case of player 1. Suppose further that for a small change of $\Delta P_1$ in $P_1$ the resulting distribution of rent in market equilibrium is given by the point $\Pi^* + \Delta \Pi$. This means that player 1 gains by $\Delta \Pi_1$ and player 2 looses by $\Delta \Pi_2$ ($\Delta \Pi_2 < 0$) if the price increase actually takes place. Therefore, player 1 may insist on such a price increase and player 2 is likely to oppose (or reject) it. Would player 1 insist on a price increase?

If player 1 insists on an increase in $P_1$, player 2 has two options: accept it, or quit the negotiation table and play a noncooperative game. Under this circumstance,

\textsuperscript{14} To each player we can always take an offer of the other party as mutually agreeable price. Because, it will be agreed upon if the player in question accepts it.
insisting on such a price increase implies a gamble on the part of player 1. If it is accepted by player 2, player 1 will get a gain of $\Delta \Pi_1^e$; if it is rejected by player 2, and player 2 quits the table (player 1 is ruined), player 1 will lose the entire gain of $(\Pi_1^e - \Pi_1^4)$, and end up with the disagreement payoff of $\Pi_1^d$.

Suppose that at $\Pi_1^e$, player 1 believes that if he insists on a small increase in price, then the probability that player 2 quits the negotiation and the disagreement results is $q_1(\Pi_1^e)$. Let $\overline{q}_i(\Pi_1^e, \Delta \Pi_i)$ be such that player 1 is indifferent between insisting on a price increase for a contingent gain of $\Delta \Pi_i$ with probability $(1 - q_i)$ and accepting $\Pi_1^e$ with certainty. That is, let

$$(6.3) \quad \Pi_1^e = (1 - \overline{q}_i)(\Pi_1^e + \Delta \Pi_i) + \overline{q}_i \Pi_1^4.$$ 

Then, $\overline{q}_i$ provides the threshold probability at $\Pi_1^e$ such that $\overline{q}_i > q_i$ implies that player 1 will insist on a price increase; $\overline{q}_i \leq q_i$ implies that player 1 will prefer the certain outcome $\Pi_1^e$ and will not insist on a higher price for his commodities. This means that $\overline{q}_i$ measures the 'boldness' of player 1 in an environment defined by $(\Pi_1^e, \Pi_1^4)$.

Certainly, player 1, as can be seen from equation (6.3), will not risk the same amount of gain $(\Pi_1^e - \Pi_1^4)$ for smaller amounts of contingent gains with identical probability distribution. To induce him to risk $(\Pi_1^e - \Pi_1^4)$ for a gain that is smaller than $\Delta \Pi_i$ and remain indifferent ex ante, the probability of 'ruin' - that player 2 quits and disagreement results - has to be smaller than that corresponds to $\Delta \Pi_i$. Therefore, for given $(\Pi_1^e - \Pi_1^4)$, the threshold probability $\overline{q}_i(\Pi_1^e, \Delta \Pi_i)$ declines as $\Delta \Pi_i$ gets smaller and smaller, and needs to be standardized to make a measure of player 1's boldness. Symmetric arguments can be made for player 2. Hence the following definition due to Aumann and Kurz (1977a).

**Definition 6.5 (fear of ruin).** Let

$$q_i^*(P_1^e) = \lim_{\Delta_1 \to 0} \frac{\overline{q}_i(\Pi_1^e, \Delta \Pi_i)}{|\Delta P_1|}.$$ 

Then, $q_i^*(P_1^e)$ is a measure of boldness of player $i$ at $\Pi_1^e$ or at price $P_1^e$ and,

$f_i = (q_i^*(P_1^e))^{-1}$ is defined as a measure of player $i$'s fear of ruin.15

The measure of boldness thus defined is 'the maximum probability of ruin per dollar of additional gains which player $i$ is prepared to tolerate, for very small potential gains' (Roth, 1979: p. 50).

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15 See also Roth (1979) for similar arguments and definitions in terms of utilities.
Moreover, it can be inferred directly from equation (6.3) that for a given contingent gain and his disagreement payoff, the maximum probability of ruin that player \( i \) is ready to accept declines as his payoff, \( \Pi_i \), increases. Hence \( q_i(\Pi_i^o; \Delta \Pi_i) \) is a decreasing function of \( \Pi_i \), and so, the fear of ruin is an increasing function of \( \Pi_i \).

Solving equation (6.3) for \( q_i(\Pi_i^o; \Delta \Pi_i) \) we get

\[
q_i(\Pi_i^o; \Delta \Pi_i) = \frac{(\Pi_i^o + \Delta \Pi_i) - \Pi_i^e}{(\Pi_i^o + \Delta \Pi_i) - \Pi_i^d}.
\]

Now, using equation (6.1) in equation (6.5) and substituting the resulting expression for \( q_i(\Pi_i^o; \Delta \Pi_i) \) into the equation (6.4) we get

\[
q_i^*(P_i^o) = \lim_{\Delta \to 0} \left[ \frac{[(\Pi_i^o + \Delta \Pi_i) - \Pi_i^e] / \Delta P^1}{(\Pi_i^o + \Delta \Pi_i) - \Pi_i^d} \right].
\]

That is, player \( i \)'s boldness at \( \Pi_i^o \) is given by

\[
q_i^*(P_i^o) = \frac{d\Pi_i}{dP^1}_{\Pi_i^o}.
\]

Therefore, player \( i \)'s fear of ruin\(^{16}\), which is the inverse of the measure of player \( i \)'s boldness, is given by

\[
f_i(P_i^o) = \frac{\Pi_i^o - \Pi_i^d}{d\Pi_i / dP^1}_{\Pi_i^o}.
\]

Thus, equation (6.8) defines the adapted version of Aumann and Kurz's concept of the fear of ruin. In general, at any relative price \( P_1 \) of commodity 1 such that \( \Pi \in \mathbb{R} \), we can write the fear functions, corrected for sign effects in case of player 2, as follows.

\[
f_1(P_1) = \frac{\Pi_1(P_1) - \Pi_1^d}{d\Pi_1(P_1) / dP_1} \quad \text{and} \quad \text{(6.9a)}
\]

\[
f_2(P_1) = -\frac{\Pi_2(P_1) - \Pi_2^d}{d\Pi_2(P_1) / dP_1}. \quad \text{(6.9b)}
\]

\(^{16}\) In Aumann and Kurz (1977a) and Roth (1979) an equation similar to the equation (6.8) is directly used to define the fear of ruin that is later on shown to be related with the probabilistic measure of boldness. The order has been reversed in the present study because the concept of fear of ruin or boldness is best expressed in the belief system of the player. In any case the implication to the overall analysis is unchanged.
The derivative $d\Pi_i / dP_i$ measures the marginal ability of player $i$ to obtain rents through price changes. It shows the change in the total rents obtained in sector $i$ when the relative price of commodity 1 changes by one unit. Therefore, the magnitude of $d\Pi_i / dP_i$ is determined completely by the economic system.

**Definition 6.6** The fear functions $f_1(P_1)$ and $f_2(P_1)$ are said to be well behaved if for all $P_i$ that are relevant to bargaining (that is, such that $\Pi(P_i) \in \mathcal{R}$) if $f_1(P_1) > 0$ and $f_2'(P_1) < 0$.

Note that the fear functions are well behaved under Assumption 3.1 - that the real rental functions are concave in the relative price of own commodity. The conditions which are sufficient in general for $f_1'(P_1) > 0$ and $f_2'(P_1) < 0$, are derived in Appendix -5A.

The derivative properties of the fear functions indicate that player 1's fear of ruin increases as $P_1$ increases and player 2's fear of ruin increases as $P_1$ falls (or the relative price of commodity 2 increases).

![Graph](image)

**Figure 6.4**

Aumann and Kurz (1977a, 1977b) discovered an important property of the Nash solution to a bargaining game. They showed that in a bargaining game "the Nash solution calls for that compromise which makes the two players equally fearful of ruin, where ruin is here taken to mean disagreement" (Aumann, 1977a: 1149). This condition, equality of fear of ruin, turns out to be identical to the first order condition of maximizing 'the Nash product over the bargaining set. The intuitive reason behind the equilibrating process is that the player who fears the most will concede. This explanation is similar to that of Zuthen's referred above.
The Nash bargaining equilibrium is illustrated in figure 6.4. $P_x$ is the bargained price where both the players are equally fearful of ruin. For all prices that are less than $P_x$, player 2's fear of ruin exceeds player 1's fear of ruin. Hence player 2 concedes and prices are allowed to rise. For all prices greater than $P_x$, player 1's fear of ruin exceeds the player 2's fear of ruin; therefore player 1 concedes. The price moves in favour of player 2.

It is important to distinguish between the fear of disagreement created by a positive probability of breakdown of the negotiation in BRW and the fear of ruin in Aumann and Kurz. In BRW the implied fear to each player is constant until the agreement is reached, whereas in Aumann and Kurz a player's fear increases as he receives an increasing gain from the bargain. Along the Pareto efficient frontier as the fear of one player increases the fear of the other player necessarily decreases. This is not the case with the concept of fear in BRW.

The types of fear of disagreement in these two studies are intrinsically different. This is because the sources of fear are different. In BRW players fear because they think that if an agreement cannot be reached quickly, then the opportunity might be snatched by a third party, or may vanish by itself. For both players the source of fear, therefore, is the third party or a time factor. In the case of Aumann and Kurz, however, each player fears because the other player may refuse to agree. That is why the measure of the fear of ruin, as defined by Aumann and Kurz, increases with the gain of the player relative to the disagreement payoff. The source of the fear of ruin for each player is the other player (opponent). The fear of ruin exists even in the absence of threat of a third party snatching the opportunity.

The implication of this discussion is that although the difference in players' perception of the risk of breakdown or the fear of disagreement as viewed by BRW is captured by the difference in bargaining power of the players, the fear of ruin as defined by Aumann and Kurz is not. Therefore, the fear of ruin constitutes a separate, independent motive that not only affects the outcome of the game but also induces the players to reach an agreement through constant revisions of their incompatible demands.

6.3.2 Generalized Fear of Ruin

However, the property of the Nash bargaining solution, as discovered by Aumann and Kurz, that in equilibrium the players should be equally fearful of ruin holds, as in the case of the original Nash solution, only with equal bargaining powers. It is because the above deduction is based on the assumption that the players hold equal bargaining powers. An immediate problem is to explore whether or not a similar
condition holds when there are reasons to believe that players hold unequal bargaining powers.

For example, one of the reasons that players have unequal bargaining power is that they have unequal time preference rates. We know that \( p_i < p_2 \) implies \( \Theta_1 > \Theta_2 \), where \( p_i \) is the time preference rate of player \( i \) (Rubinstein 1982; Binmore 1987b: 71). Other things being equal, a player with higher time preference rate will be ready to reduce his demand by more than a player with lower time preference rate. This means that the player with higher time preference rate will have a greater fear of ruin than the player with a lower rate.

Similarly, let us consider the case of unequal fear of disagreement in BRW's sense - the fear that the bargaining opportunity will be snatched by a third party. Now, the fear of ruin held by each player should take account of fears from two sources: (1) the fear that a third party will intervene; and (2) the fear that the opponent will quit the bargaining table and disagreement will result. The measure of boldness as defined by the equation (6.4) and given by the equation (6.7) has to be understood in a broader context than it was defined for. In the case that there are many sources of fear, a measure of boldness (and a measure of fear of ruin) should represent the maximum tolerable probability of occurrence of either of the events - for example, a third party intervenes and seizes the bargaining opportunity, or the opponent disagrees.

Therefore, it is now necessary to differentiate between a player's total boldness and a player's boldness vis a vis her bargaining opponent. We argue (on the basis of player's indifference) that, in any situation, a player has a fixed capacity to risk already attained gain. That measures her total boldness and is given by the equation (6.7). In order that a player insist on a favourable price (tariff) change and assume the risk of breakdown of negotiation the capacity to risk the gain should exceed the sum of risks that arise from all sources. A higher risk from a third party, therefore, reduces the capacity to risk the conflict with the bargaining opponent. That is, the higher a player's BRW type fear of disagreement (relative to the other player) the lower the player's boldness vis a vis his opponent and therefore the higher is the player's Aumann and Kurz type fear of ruin. Other things being equal, if the players have equal capacity to risk the conflict, the player who fears more than his opponent that a third party will snatch the opportunity will also display a greater fear of ruin vis a vis her opponent.

Let us consider again the situation described in figure 6.3 If player 1 demands a price increase it will induce a bargaining process during which player 2 may disagree or a third party may intervene. If any of the events occur the disagreement payoff will result. The higher the demand the more it is likely that either of the events will occur.
As in BRW, let \( a_i(\Delta P_i) \) be defined as follows:

\[
(6.10a) \quad a_i(\Delta P_i) = \begin{cases} 
1 - e^{-\xi_i \Delta P_i} & \Delta P_i > 0; \\
0 & \Delta P_i \leq 0
\end{cases}
\]

\[
(6.10b) \quad a_i(\Delta P_i) = \begin{cases} 
1 - e^{-\xi_i(-\Delta P_i)} & \Delta P_i < 0; \\
0 & \Delta P_i \geq 0
\end{cases}
\]

where, \( a_i^* > 0 \) and is independent of the current price. Then \( a_i(\Delta P_i) \) denotes player \( i \)'s subjective probability that a third party intervention occurs and that players have to receive the disagreement payoffs if she insists on a price change of \( \Delta P_i \).

Similarly we can define Aumann and Kurz type fear of ruin as follows. Let,

\[
(6.11a) \quad b_i(\Delta P_i; P^*) = \begin{cases} 
1 - e^{-\xi_i(\Delta P_i)}; & \Delta P_i > 0; \\
0; & \Delta P_i \leq 0
\end{cases}
\]

\[
(6.11b) \quad b_i(\Delta P_i; P^*) = \begin{cases} 
1 - e^{-\xi_i(-\Delta P_i)}; & \Delta P_i < 0; \\
0; & \Delta P_i \geq 0
\end{cases}
\]

where, \( b_i^* > 0 \) for each \( i \) such that \( b_i^* \) increases and \( b_i^* \) decreases with \( P_i \).

Then, \( b_i(\Delta P_i; P_i^*) \) denotes player 1's subjective probability that player 2 will declare disagreement if she demands a price increase of \( \Delta P_i \) and \( b_i(\Delta P_i; P_i^*) \) denotes player 2's subjective probability that player 1 will declare disagreement if he demands a price fall of \( \Delta P_i \) given that \( P_i^* \) is a mutually acceptable price.

It is clear from these definitions that

\[
(6.12) \quad a_i^* = \lim_{\Delta P_i \to 0} (a_i / \Delta P_i) \text{ and } b_i^*(P_i^*) = \lim_{\Delta P_i \to 0} (b_i / \Delta P_i),
\]

where, we take right-hand limit for player 1 and left-hand limit for player 2. It is clear from equation (6.12) that \( a_i^* \) and \( b_i^* \) yield player \( i \)'s subjective valuation of the

\[\text{It is implicitly assumed that the length of a bargaining period depends on the size of price increase demanded. Insistence on larger price increases increase the likelihood that the opponent will reject it and disagreement results. If the opponent does not disagree, he will certainly take longer time in haggling before he accepts it. But this increases the chance that a third party will intervene and the outcome will be the disagreement.}

A slight abuse of notation may require some clarification. The relevant value of \( \Delta P_i \) is positive for player 1 and negative for player 2. That is, player 1 is interested in price increases and player 2 is interested in decreases of the relative price of commodity 1. By considering only the absolute value of \( \Delta P_i \) we can reduce unnecessary repetition of analogous equations. This is what is done here.
likelihood of the two types of risks per unit of price change when the proposed price changes are small.

Given this environment, we can search for the threshold probability of conflict with the opponent that player \( i \) can accept for small gains. At price \( P^*_i \), at most, player \( i \) will demand a price change of size \( \Delta P^i \) such that the following equality holds:

\[
(6.13) \quad \Pi^i = [a_i(\Delta P^i) + b_i(\Delta P^i) - a_i(\Delta P^i)b_i(\Delta P^i)]\Pi^i + (1 - a_i(\Delta P^i))(1 - b_i(\Delta P^i))(\Pi^i + \Delta \Pi^i).
\]

Note that \([a_i(\Delta P^i) + b_i(\Delta P^i) - a_i(\Delta P^i)b_i(\Delta P^i)]\) measures the probability that either of the events occurs and disagreement results, and \([1 - a_i(\Delta P^i)][1 - b_i(\Delta P^i)]\) measures the probability that agreement results and player \( i \) will obtain the contingent gain of \( \Delta \Pi^i \) when he demands a price change of \( \Delta P^i \).

From equation (6.13) we get

\[
(6.14) \quad [a_i(\Delta P^i) + b_i(\Delta P^i) - a_i(\Delta P^i)b_i(\Delta P^i)] = \bar{q}^i(\Delta P^i),
\]

where

\[
(6.15) \quad \bar{q}^i(\Delta P^i) = \frac{(\Pi^i + \Delta \Pi^i) - \Pi^i}{(\Pi^i + \Delta \Pi^i) - \Pi^i}.
\]

For small price changes, taking limits to both sides of (6.14) and (6.15) yields

\[
(6.16) \quad a_i^* + b_i^* = q_i^*
\]

where \( q_i^* \) is as defined in equation (6.7). Thus, equation (6.16) shows that the maximum probability of conflict with the opponent that player \( i \) is prepared to tolerate is the difference between the measure of her total boldness, \( q_i^* \), and her subjective probability that a third party intervenes \( q_i^* \). That is, given \( a_i^* \), while negotiating for gains, \( b_i^* \) measures player \( i \)'s boldness at \( \Pi^i \) vis a vis her opponent.

**Definition 6.7** (Total Boldness). The maximum probability of conflict with the external world that a player is prepared to accept for a small current gain is a measure of the player's total boldness in a given environment. The inverse of this measure is the player's total fear of ruin.

**Definition 6.8** (Residual Boldness). Given other exogenous risks, the maximum probability of conflict with the opponent that a player is prepared to accept for small gains is defined as a measure of the player's residual boldness. The inverse of this measure is the player's residual fear of ruin.
In the absence of a third party risk, as can be seen from equation (6.16), residual boldness of a player coincides with her total boldness that is equal to the Aumann and Kurz's measure of boldness as defined above.

A difference in players' perception of a third party risk has effects in two directions. On the one hand, it affects players' bargaining power, as shown by BRW; on the other hand, as shown above, it also affects player's boldness vis a vis her opponent. A player assigns a higher probability to a third party intervention than his opponent means that he will have lower bargaining power relative to his opponent. For given total boldness of each player (not necessarily equal), he assigns higher probability to a third party intervention also implies that he will be less bolder vis a vis his opponent.

This argument leads us to the conclusion that either we should consider residual boldness (or residual fear of ruin) in characterizing the bargaining solution or we should allow, in one way or the other, for the difference in players' perception of a third party risk to play a role with a measure of player's total boldness in determining the bargaining outcome or characterizing the bargaining solution.

The case of a third party risk also suggests that if there are other sources of difference in bargaining power between the players besides third party risk, then those factors may also affect a player's fear of ruin vis a vis her opponent in a bargaining process. If a player has relatively more bargaining power than her opponent, then the reason behind the bargaining power differential should also work to dampen the fear of the player vis a vis her opponent. Since there can be several sources of fears or the characteristics that differentiate the players, and Aumann and Kurz's measure of fear of ruin is a total measure, the following generalization is proposed.

Definition 6.9 (Generalized Fear of Ruin). Let $f_i^* = f_i / \Theta_i$, then $f_i^*$ is defined as player $i$'s generalized fear of ruin.

Thus the generalized fear of ruin will be equal to Aumann and Kurz's concept of fear of ruin if and only if players have equal bargaining power, that is, if the players are identical except the bargaining set.

Now, we state the following axiom\textsuperscript{18} initially proposed by Svejnar (1986):

Axiom 6 (Equality of Generalized Fear of Ruin).

A solution to a bargaining game satisfies $f_i^* = f_j^*$ for $i \neq j$.

\textsuperscript{18} In Svejnar (1986) this axiom has been called "equality of fear of disagreement relative to bargaining power".
6.4 Generalized Nash Bargaining Solution in a Tariff Game

and the Generalized Fear of Ruin

In an n-person bargaining problem in which the object of bargaining is a division of a fixed 'pie'; the players endowed with unequal bargaining power have concave utility functions, $U_i$ for each player $i$, defined on the size of their share in the 'pie'; the zero normalized (disagreement payoff transformed into zero by change of origin) bargaining set, denoted by $S$, is compact and convex in utility space; Svejnar proved quite generally that:

**Theorem 6.3** (Svejnar). There is a unique solution which satisfies Axioms 1-3, and Axiom 6. It is the solution that maximizes $\prod_{i=1}^{n} U_i^\Theta$, for all $U \in S$.

**Proof** See Svejnar (1986).

The proof of theorem 6.3 outlined by Svejnar is quite general. So long as the bargaining set is compact and convex, and contains at least one point that strictly dominates the disagreement payoff, then the proof of the equivalence between (a) the bargaining solution satisfying the four axioms, and (b) the solution that maximizes the generalized Nash product over the bargaining set, remains valid.

However, theorem 6.2 (Roth's theorem) shows that the solution to the bargaining problem can be characterized without referring to axiom 6. This implies that the axiom 6 is redundant in the presence of axioms 1-3. This axiom will be more useful if it could characterize the generalized Nash bargaining solution in the absence of at least some of the axioms 1-3. The following theorem shows that this is in fact the case.

Recall that if the international price ratio $P^*_i$, and the domestic factor endowments $K_1, K_2$ and $L$ are exogenously given, and the elasticities of factor substitution in both the sectors are at least unity, then the bargaining problem $(\mathcal{R}, \Pi^d, \Theta)$ is well defined for exogenously given disagreement payoff $\Pi^d$, and bargaining power distribution $\Theta$.

**Theorem 6.4** If the fear functions are well defined - that is, the derivatives of the rental functions with respect to the relative price do not vanish at all prices, then there exists a unique solution $\mu(\mathcal{R}, \Pi^d, \Theta) = \Pi(P) \in \mathcal{R}$ to any bargaining problem $(\mathcal{R}, \Pi^d, \Theta)$ in a tariff game, which can be characterized by any of the following equivalent statements:

(i) $P = \arg \max_{\Pi(P) \in \mathcal{R}} \left[ \Pi_1(P) - \Pi_1^d \right]^{\Theta_1} \left[ \Pi_2(P) - \Pi_2^d \right]^{\Theta_2}$. That is, $\Pi(P)$ maximizes the generalized Nash product over the bargaining set.
(ii) \( \mu(\mathcal{R}, \Pi^d, \Theta) \) satisfies axioms 1-3. That is, the solution to the bargaining problem \( (\mathcal{R}, \Pi^d, \Theta) \) satisfying the axioms 1-3 selects \( \Pi(\bar{P}_1) \) as the outcome of the bargaining.

(iii) \( f_i^*(\bar{P}_1) > 0 \) for each \( i \), and \( f_i^*(\bar{P}_1) = f_j^*(\bar{P}_1) \), \( i \neq j \) such that \( \Pi(\bar{P}_1) \in \mathcal{R} \). That is, at \( \bar{P}_1 \) each player \( i \) has a positive fear of ruin, and that players' generalized fears of ruin are equalized.

**Proof:** (i) holds, if and only if (ii) holds follows from Theorem 6.2. Therefore, it suffices to show that (i) holds if and only if (iii) holds.\(^{19}\)

(a) First we will show that (i) implies (iii).

Suppose (i) holds. This implies that at \( P=\bar{P}_1 \) the level curve of generalized Nash product is tangent to the boundary of the bargaining set. Their slopes are equal.

The slope of any level curve of Nash product is given by

\[
\frac{d\Pi_1}{d\Pi_2} \bigg|_{\text{NP}} = -\frac{\Theta_2}{\Theta_1} \left( \frac{\Pi_1(\bar{P}_1) - \Pi^d_1}{\Pi_2(\bar{P}_1) - \Pi^d_2} \right)
\]

and since RTF is the boundary of the bargaining set, its slope is given by

\[
\frac{d\Pi_1}{d\Pi_2} \bigg|_{\text{RTF}} = \frac{d\Pi_1(\bar{P}_1) / d\bar{P}_1}{d\Pi_2(\bar{P}_1) / d\bar{P}_1}.
\]

Therefore, at the solution point we must have

\[
\frac{d\Pi_1(\bar{P}_1) / d\bar{P}_1}{d\Pi_2(\bar{P}_1) / d\bar{P}_1} = -\frac{\Theta_2}{\Theta_1} \left( \frac{\Pi_1(\bar{P}_1) - \Pi^d_1}{\Pi_2(\bar{P}_1) - \Pi^d_2} \right),
\]

which means that

\[
\frac{1}{\Theta_1} \left( \frac{\Pi_1(\bar{P}_1) - \Pi^d_1}{d\Pi_1(\bar{P}_1) / d\bar{P}_1} \right) = \frac{1}{\Theta_2} \left( \frac{\Pi_2(\bar{P}_1) - \Pi^d_2}{d\Pi_2(\bar{P}_1) / d\bar{P}_1} \right).
\]

Therefore, it follows from equation (6.9) that \( f_i^*(\bar{P}_1) = f_2^*(\bar{P}_1) \). Moreover, \( \Pi_1(\bar{P}_1) > \Pi^d_1 \) implies that \( f_i^*(\bar{P}_1) > 0 \) for each \( i \). Thus, at the solution point of the game each player holds strictly positive fear of ruin and the generalized fears of ruin are equalized across the players.

(b) Conversely, we will show that (i) follows from (iii).

\(^{19}\) See Appendix-6C for (ii) implies (i).
Suppose that (iii) holds. First note that \( f_i^*(\bar{P}_1) > 0 \) implies that \( \Pi(\bar{P}_1) \) lies on the boundary of the bargaining set. If \( \Pi(\bar{P}_1) \) does not lie on the boundary of the bargaining set, then players can increase their payoffs without affecting the relative price simply by cutting down their lobbying expenditure. This means that \( \left| \frac{d\Pi_i(\bar{P}_1)}{d\bar{P}_1} \right| = \infty \) in the neighbourhood of \( \bar{P}_1 \). As a consequence, each player's fear of ruin of at \( \bar{P}_1 \) is zero. This contradicts with the hypothesis that \( f_1(\bar{P}_1) > 0 \). Therefore, \( f_i^*(\bar{P}_1) > 0 \) for each \( i \) imply that \( \Pi(\bar{P}_1) \) lies on the boundary of the bargaining set.

By reversing the above steps it can be seen that \( f_i^*(\bar{P}_1) = f_2^*(\bar{P}_1) \) implies the satisfaction of the condition for tangency of a level curve of generalized Nash product to the boundary of the bargaining set at \( \Pi(\bar{P}_1) \). Concavity of the generalised Nash product and convexity of the bargaining set guarantee that the point of tangency is unique, and that second order condition of maximization of the generalized Nash product is satisfied. Therefore, (iii) holds implies that (i) holds. Hence the theorem.

Q. E. D.

**Corollary 6.2 (Bargained Tariff Rate).** A tariff rate \( \bar{T} \) is a unique outcome of bargaining in a tariff game if \( \bar{T} \) satisfies

\[
\bar{P}_1 = p_1^*(1 + \bar{T}).
\]

**Proof:** Since \( \bar{P}_1 \) is unique by Theorem 6.4, the proof follows from the assumption that tariff is the only wedge between the world and the domestic relative prices. Q. E. D.

**Corollary 6.3** If the fear functions are well behaved, then the bargaining solution obtained under the conditions of Theorem 6.4 is stable.

**Proof:** This corollary will be proved by way of a graphic illustration.

That fear functions are well behaved implies, by definition, that player 1's fear of ruin increases and player 2's fear of ruin decreases as \( P_1 \) increases.

Let RF in figure 6.5 represent the part of the RTF that forms the boundary of the bargaining set, and a level curve of generalized Nash product labelled NN be tangent to the curve RF at the point C. Let \( \bar{P}_1 \) be the domestic price ratio that corresponds to the point C, and \( \Pi(\bar{P}_1) \), be the payoff combination at price \( \bar{P}_1 \). It follows from Theorem 6.4 that at point C the generalized fears of ruin are equalized across the players.

Now, suppose that, for some reason, the equilibrium is disturbed and the economy is at the point B. Given unchanged values of the exogenous variables, the
relative price should rise and the economy should move to the point C if the equilibrium at the point C is a stable one.

At the point B the generalized Nash product is not maximized. Obviously, B is not an equilibrium point. The two players will continue bargaining. Player 1 would like the price to rise and player 2 would not like the price to rise. Since the absolute slope of the RTF is greater than the absolute slope of the Nash product curve at B, we can write

\[
-\frac{d\Pi_1(\bar{P})}{d\Pi_2(\bar{P})} \frac{\theta_2}{\theta_1} > \frac{\theta_2}{\theta_1} \left( \frac{\Pi_1(\bar{P}) - \Pi^*_1}{\Pi_2(\bar{P}) - \Pi^*_2} \right).
\]

But this means that

\[
-\frac{1}{\theta_2} \left( \frac{\Pi_2(\bar{P}) - \Pi^*_2}{d\Pi_2(\bar{P})/dP} \right) \left( \frac{d\Pi_1(\bar{P})/dP}{d\Pi_2(\bar{P})/dP} \right) > \frac{1}{\theta_1} \left( \frac{\Pi_1(\bar{P}) - \Pi^*_1}{d\Pi_1(\bar{P})/dP} \right).
\]

That is, at the point B, by definition, we have \( f^*_2 > f^*_1 \). This means that player 2 will concede and the domestic relative price will rise. The process will continue until we reach the point C, where both players are equally fearful of each other quitting the negotiation.

Similarly, it can be seen that if the economy moves to points like A, then player 1's fear of ruin will exceed player 2's fear of ruin. Player 1 will concede and the price will fall.
6.5 Summary

It is clear from the above results that the Nash bargained solution in a tariff game can be obtained as a solution to the following maximizing problem:

\[(6.23) \quad \max \left[ \Pi_1 - \Pi_1^d \right] \left[ \Pi_2 - \Pi_2^d \right], \]

subject to the RTF,

\[(6.24) \quad \Pi_1 \leq K_1 P_1^{-\rho_1} \beta_1^{1-\rho_1} R_1^{1-\rho_1} - \alpha_1^\sigma \alpha_2^{1-\rho_1} \left( 1 - \beta_2^\sigma \left( \frac{\Pi_2}{K_2} \right)^{1-\rho_1} \right)^{1-\sigma} \]  

and that \[\Pi_i \geq \Pi_i^d,\] for each player \(i = 1, 2\).

The equation (6.24) is obtained from the equation (4.6) by replacing \(R_i\) by \(\Pi_i\). This substitution is allowed by the rule that in a cooperative game players need not spend resources in lobbying the government. The real-rent combinations along the RTF also are available to the players. Therefore, the boundary of the feasible set is the RTF. The inequality in (6.24) indicates that players may choose inefficient outcomes - either by inefficient allocation of resources or by playing noncooperation. The above bargaining problem is illustrated in figure 6.6.

An illustration of a Nash bargaining solution

Figure 6.6
The level curves of the generalized Nash product defined by the function (6.23) are rectangular hyperbolas with asymptotes $\Pi_1^d$ and $\Pi_2^d$. The presence of these asymptotes effectively restricts the solution payoffs to lie above the disagreement payoffs. The effective bargaining set is $\mathcal{R}$, which lies to the north east of the disagreement point, $(\Pi_2^d, \Pi_1^d)$. The tilt of the curve is determined by the magnitude of $\Theta_i$. Higher value of $\Theta_i$ produces more tilt towards the $i$th axis. The magnitude of $\Theta_i$ is an institutionally given datum and represents the 'bargaining power' of player $i$.

The maximand is concave and continuous; the constraint set is compact and convex; therefore there exists a unique maximum of the generalized Nash product in $\mathcal{R}$. The first order condition of maximization problem, which is also sufficient, is that the Nash product (NP) curve be tangent to the Pareto efficient boundary of the feasible set - that is, the northeast boundary of the feasible set described by the equation (6.24). Replacing the sign of inequality by equality in (6.24) we can obtain the expression for the boundary of the feasible set, which is the rental transformation frontier (RTF).

The necessary and sufficient condition for the Nash solution to the bargaining problem is given by

$$\left(\frac{\Pi_1 - \Pi_1^d}{\Pi_2 - \Pi_2^d}\right) = -\frac{\Theta_1}{\Theta_2} \frac{d\Pi_1}{d\Pi_2} \bigg|_{\text{RTF}}.$$

The payoff combination $\Pi = (\Pi_1, \Pi_2)$, that satisfies the condition given by the equation (6.25) is the solution to the bargaining problem. The relative price, and hence the tariff rate, that corresponds to this solution payoff is the outcome of the bargaining in a tariff game. More formally, the rationalized tariff rate $\bar{T}$ that satisfies

$$\bar{T} = \arg\max \left\{ \left[\Pi_1(P_i^*(1 + T)) - \Pi_1^d\right]^{\varphi_1} \left[\Pi_2(P_2^*(1 + T)) - \Pi_2^d\right]^{\varphi_2} \big| \Pi \in \mathcal{R} \right\}$$

is the unique Nash bargained solution to the tariff game.

Theorem 6.2 (Roth's theorem) has shown that the generalized Nash bargained solution is unique. It is also individually rational, independent of scale of measurement, and independent of irrelevant alternatives. Corollary 6.1 has shown that the solution is also symmetric. Roth's result also guarantees that the solution is efficient.

Aumann and Kurz have shown that at the Nash bargaining solution players' fear of ruin are equalized. Theorem 6.4 took it a step forward and has shown that equality of fear of ruin can constitute a separate characterization of the Nash bargaining solution. Aumann and Kurz's result holds when players have equal bargaining power. But the characterization of the generalized Nash bargaining solution in terms of generalized fear of ruin provided by the theorem 6.4 holds for any arbitrary distribution of
bargaining power among the players. It is shown that at the generalized Nash bargaining solution players' generalized fears of ruin are equalized. Clearly, Aumann and Kurz's result is a special case of this result.

Furthermore, it also shown in corollary 6.3 that the generalized Nash bargaining solution is stable when the fear functions are well behaved. The appendix provides a generally sufficient condition for fear functions to be well behaved.

This chapter has differentiated the concept of fear of disagreement as suggested by BRW and that suggested by Aumann and Kurz. It is argued here that they have addressed two different sources of fear. In BRW the source of fear is either the external world or time, whereas in Aumann and Kurz the source of fear is the opponent. The proposed generalization of fear of ruin not only allows these two types of fear concepts but also it can accommodate any other difference between the players that are capable of affecting the bargaining powers of the players.

Now, it can be seen from the equation (6.25) that at the bargained outcome, the gain of player 1 relative to the gain of player 2 over the disagreement payoff depends on two terms: the ratio of the institutionally given bargaining power of the players, $\Theta_1 / \Theta_2$, and the slope of the rental transformation frontier. Begin and Karp (1991) have named the second term as the 'endogenous' bargaining power and the first term as 'exogenous' bargaining power of the players.

The role of these two terms in determining the outcome can best be viewed with the help of the following figure.

![Effects of changes in endogenous and exogenous bargaining powers on the bargaining outcome](Figure 6.7)
The two panels in the above figure reflect the effect of the difference in the two types of bargaining powers of the players. In panel (a) two generalized Nash product curves that correspond to different exogenous bargaining power distribution, holding the endogenous bargaining power constant, are drawn. The curves were drawn on the assumption that $\Theta_1 < \Theta_1$. The result is that under unchanged economic circumstances, increased exogenous bargaining power of player 1 has the effect of increasing payoff to player 1.

In panel (b) two rental possibility frontiers are drawn to reflect an exogenous expansion in sector 1. The exogenous bargaining power distribution has been held fixed. The new rent possibility set defined by the transformation frontier C'D contains the rent possibility set defined by the rent transformation frontier CD. Hence in equilibrium, they are able to attain a higher Nash product with the rental possibility curve C'D than with CD. The figure has been so drawn that the gain goes to player 1 only. Thus the relative difference in the gain over the disagreement payoff can be attributed to change in the 'endogenous' bargaining power of the players.

Thus the 'endogenous' bargaining power can be viewed as being determined by the technologies of production, and distribution of the players' market powers. This study assumes that all markets are competitive, therefore, possession of a meaningful market power by any one of the players has been ruled out. However, some advantages due to technological and installed capacity differences are still there.

If the slope of the RTF is defined to measure the distribution of endogenous bargaining powers among players, then a movement along a rental transformation frontier implies a change in the slope of the frontier and therefore, in turn, implies a change in the endogenous bargaining power of the players. However, given a disagreement payoff, the RTF, and a distribution of exogenous bargaining power, one and only one point of the frontier will constitute a solution and other points of the frontier will be irrelevant. The endogenous bargaining power will be determined by the solution of the game, not the other way round. If endogenous bargaining power is to determine the solution of the game then such a movement along a given RTF should not be regarded as change in the endogenous bargaining power of the players. It follows that a frontier defines a particular configuration of 'endogenous' bargaining power of the players. A change in 'endogenous' bargaining power can arise only if the location of the RTF is changed.

In fact, the slopes of a given RTF show the economic limitation of the political system in transferring rents from one sector to the other sector. A shift in the frontier changes the constraint faced by the political system. For example, at each rental income of sector 1, the rent transformation frontier C'D has a greater absolute slope than the
rental transformation frontier CD. This means that player 1 can obtain more rents per unit loss in the rental income of player 2 along the frontier CD than along the frontier CD. Therefore, it can be argued that the frontier CD implies a greater 'endogenous' bargaining power of player 1 than implied by the frontier CD.

It can be seen from equation (6.9) that the slope of the RTF plays a crucial role in determining each player's total fear of ruin. For given disagreement payoffs, at each payoff level of player 1 the steeper the RTF the less fearful is player 1. Therefore, a biased outward shift of the RTF favouring player 1 has the effect of making player 1 bolder (that is, less fearful of ruin), and making player 2 less bold (that is more fearful of ruin). Thus an increase in the 'endogenous' bargaining power of a player can also be viewed as an increase in the total boldness of the player and vice versa. The interpretation of the generalized Nash bargaining solution in terms of generalized fear of ruin is, therefore, consistent with endogenous and exogenous dichotomy of bargaining power of the players.

If the exogenous bargaining power coefficients, $\Theta'$s, are known, then inclusion of the equation (6.25), with appropriate definitional equations, into the policy exogenous general equilibrium model described in chapter 2 would be sufficient to solve for the bargained tariff rate. However, in the absence of reliable information on the exogenous bargaining power of the players we can study the comparative static properties of the equilibrium tariff rate under the assumption that the distribution of the exogenous bargaining powers of the players are not affected by changes in the exogenous variables.

In the next chapter, we will combine the generalized Nash bargaining game with the policy exogenous general equilibrium model of an open economy to obtain tariff endogenous general equilibrium model of the economy. A combination of the condition (6.25) that identifies the generalized Nash bargaining solution with the equilibrium conditions of a policy exogenous general equilibrium model developed in chapter 2 will yield the set of conditions that characterize a full equilibrium of the economy. The conditions will then be used to evaluate the comparative static properties of the bargained tariff rate.

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20 This interpretation of endogenous bargaining power is consistent with Beghin and Karp's explanation endogenous bargaining power. See Beghin and Karp (1991), footnote 4.
Appendix-6A: Derivative Properties of the Fear Functions

Let us consider Aumann and Kurz’s fear of ruin at price $P_1$. Player 1’s fear of ruin is defined by

\[(6A.1) \quad f_1(P_1) = \frac{\Pi_1(P_1) - \Pi_1^d}{d\Pi_1 / dP_1}, \]

and player 2’s fear of ruin is defined by

\[(6A.2) \quad f_2(P_1) = -\frac{\Pi_2(P_1) - \Pi_2^d}{d\Pi_2 / dP_1}. \]

Differentiating equation (6A.1) with respect to the relative price yields

\[(6A.3) \quad \frac{df_1(P_1)}{dP_1} = \frac{1}{(\Pi_1')^2} \left( (\Pi_1')^2 - (\Pi_1 - \Pi_1^d) \frac{d^2\Pi_1}{dP_1^2} \right). \]

Therefore, the sufficient condition for $\frac{df_1(P_1)}{dP_1} > 0$ is that $\frac{d^2\Pi_1}{dP_1^2} < 0$.

Satisfaction of the condition means that player 1’s fear of ruin increases with $P_1$. Given the differentiability of the rental function, the necessary and sufficient condition for $\frac{df_1(P_1)}{dP_1} > 0$ is that the term in the parentheses on the right be positive.

Similarly, differentiating (6A.2) with respect to $P_1$ yields

\[(6A.4) \quad \frac{df_2(P_1)}{dP_1} = -\frac{1}{(\Pi_2')^2} \left( (\Pi_2')^2 - (\Pi_2 - \Pi_2^d) \frac{d^2\Pi_2}{dP_1^2} \right) \]

A sufficient condition for $\frac{df_2(P_1)}{dP_1} < 0$ is that $\frac{d^2\Pi_2}{dP_1^2} < 0$. If the condition is satisfied then an increase in $P_1$ implies a fall in player 2’s fear of ruin. That is, at a higher relative price of commodity 1, player 1 becomes more fearful of ruin and player 2 becomes bolder. At a lower relative price of commodity 1, player 1 becomes bolder and player 2 becomes more fearful of ruin.

If the second order derivative of the payoff function with respect to $P_1$ is nonpositive, we are done. If this is not the case, that is if $\frac{d^2\Pi_1}{dP_1^2} > 0$, then the information is not sufficient to determine the sign of the derivative of the fear of ruin as relative price changes.
The purpose of this Appendix is to obtain conditions under which $\frac{df_1(P_1)}{dP_1} > 0$ and $\frac{df_2(P_1)}{dP_1} < 0$ when the second order derivatives of the payoff functions are positive.

If the underlying production functions are continuously differentiable and the elasticities of factor substitution are finite then the sectoral payoff functions are continuously differentiable in $P$. Therefore, under these standard conditions, we can use Taylor series to approximate the sectoral rental functions. The series can, then, be used to evaluate the slope of fear of ruin functions of the players.

First, consider the case of player 1. Since Taylor series can be constructed around any arbitrary value of $P$ provided the derivatives exist, we can choose $P^*$ such that $\Pi_1(P^*) = \Pi_1^e$. Then, for all $P_1$ such that $P_1 \in R$, we have $P_1 \geq P_1^*$. The second order Taylor series expansion of the payoff function $\Pi_1(P_1)$ around $P_1^*$ can be written as

$$(6A.5) \quad \Pi_1(P_1) = \Pi_1^e + a(P_1 - P_1^*) + b(P_1 - P_1^*)^2,$$

where, $a = \Pi'_1(P_1^*) > 0; \quad b = (1/2)\Pi''_1(P_1^*) > 0$.

and the derivative of the payoff function is given by

$$(6A.6) \quad d\Pi_1 / dP_1 = a + 2b(P_1 - P_1^*)$$

Therefore, the fear function can be written as

$$(6A.7) \quad f_1(P_1) = \frac{a(P_1 - P_1^*) + b(P_1 - P_1^*)^2}{a + 2b(P_1 - P_1^*)}. $$

Differentiating the fear function with respect to $P_1$ yields after simplification

$$(6A.8) \quad \frac{df_1(P_1)}{dP_1} = \frac{a^2 + 2[2b(P_1 - P_1^*)]^2 + 2ab(P_1 - P_1^*)}{[a + 2b(P_1 - P_1^*)]^2} > 0$$

Similarly, we can obtain a Taylor series approximation of the rental income function for player 2 by expanding around $P_1^a$ where $\Pi_2(P_1^a) = \Pi_2^e$. Then for all $P_1$ such that $\Pi(P_1) \in R$ we must have $P_1 \leq P_1^a$.

For player 2, we can write

$$(6A.9) \quad \Pi_2(P_1) = \Pi_2^e + c(P_1 - P_1^a) + d(P_1 - P_1^a)^2$$

where, $c = \Pi'_2(P_1^a) < 0$ and $d = \Pi''_2(P_1^a) / 2$

If $d < 0$, then we are done. So suppose that $d > 0$. 
Following similar steps we can obtain

\[
\frac{df_2(P_1)}{dP_1} = -\frac{c^2 + 2d(P_1 - P_1^*)^2 + 2cd(P_1 - P_1^*)}{[c + 2d(P_1 - P_1^*)]^2}
\]

\[
= -\frac{[c + d(P_1 - P_1^*)^2 + [d(P_1 - P_1^*)]^2}{[c + 2d(P_1 - P_1^*)]^2} > 0;
\]

where,

\[
f_2(P_1) = -\frac{c(P_1 - P_1^*) + d(P_1 - P_1^*)^2}{c + 2d(P_1 - P_1^*)}.
\]

These results remain valid if we use Taylor series of third order, provided that the third order derivative of the rental function remain nonnegative for player 1 and nonpositive for player 2 at the point of expansion. As we increase the order of expansion, the signs of the derivatives of the fear functions become ambiguous.

Therefore, the sufficient conditions for fear functions to be well behaved are that

(i) the elasticities of factor substitution be finite; and

(ii) either of the following two conditions holds:

(a) the second order derivatives of the rental functions with respective to \( P_1 \) are negative everywhere;

(b) all third and higher order derivatives of the rental functions with respect to the relative price vanish at the point of expansion.

The first condition assures that there are no corner solutions. This condition together with the continuous differentiability of the output supply functions with respect to price, then, guarantees the differentiability of the rental functions\(^{21}\) at each price.

The second condition is related to the higher order derivatives of the rental functions. This condition is satisfied if the rental functions, in the relevant range for bargaining, are approximately quadratic (concave or convex) in relative price of either commodity. The presence of nonlinearities makes it difficult to assess whether the condition (ii) is satisfied or not.

However, the rental functions are bounded and strictly increasing in relative price of own commodity, eventually at higher prices, the second order derivatives of the

\[\text{Recall that rental income, measured in units of own output, at each price is equal to output supply less the cost of mobile factor.}\]
rental functions with respect to the relative price of own commodity have to be nonpositive.

Condition (i) is normally satisfied. We do not normally expect factors of production to be perfectly substitutable. Condition (ii) may be violated in some cases. Particularly this condition may be violated if the rental functions are wavy, though strictly increasing in relative price of own commodity. If this is so, then the bargaining solution that equalizes generalized fear may not be stable. Player 1's generalized fear or ruin may fall as the relative price of commodity 1 rises and player 2's generalized fear of ruin may fall as the relative price of commodity 2 rises. Thus, each player may like to put demands that are likely to be incompatible. Hence, we regard condition (ii) as a condition for stability of the bargaining solution. However, in this study, we assume that the conditions (i) and (ii) are satisfied.

Appendix -6B: Proof of Roth's Theorem

We will prove Roth's theorem by way of the following lemma. The proof basically follows Binmore (1987a).

Lemma 6.1 Consider a bargaining problem (A,0) where,

\[ A = \{(x_1, x_2) | x_1 + x_2 \leq 1; x_1 \geq 0, x_2 \geq 0\}. \]

If \( \mu(A,0) = (x_1^*, x_2^*) \) is a solution to the bargaining problem (A,0) that satisfies axioms of feasibility, invariance and independence, then

(i) \( \mu(A,0) = (x_1^*, x_2^*) \) is efficient;

(ii) there exists a \( \tau \in (0,1) \) such that \( x_1^* = \tau \) and \( x_2^* = 1 - \tau \); and

(iii) \( (x_1^*, x_2^*) = \max_{(x_1, x_2) \in A} x_1^* x_2^{(1-\tau)}. \)

Conversely, for any given \( \tau \in (0,1) \), if \( (x_1^*, x_2^*) = \max_{(x_1, x_2) \in A} x_1^* x_2^{(1-\tau)} \), then \( x_1^* = \tau \) and \( x_2^* = 1 - \tau \), and the solution \( x^* \) satisfies axioms of feasibility, invariance, and independence. Moreover, the solution \( x^* \) is unique.

Proof: (i) Let us consider the following figure:
Let the area OBC represent the symmetric bargaining set A, of which the line BC represents the Pareto efficient boundary of the set A. Let $\mu(A,0) = x^*$ be a solution of the bargaining problem that satisfies the three axioms.

Since $x^*$ satisfies the axiom of feasibility and it is individually rational, $x^* > 0$. Suppose that the solution $x^*$, which satisfies the three axioms, does not lie on BC. Let BD be the line passing through the point B and $x^*$. Then the area OBD forms a subset of the area OBC. Let us call it the set A'. The set A' contains both the disagreement point 0, and the solution point $x^*$, and is a subset of the set A. Therefore, by the axiom of independence, $\mu(A',0) = x^*$.

Let $\alpha$ be an affine transformation which maps $0 \rightarrow 0, B \rightarrow B,$ and $D \rightarrow C$. Then by similarity, $\alpha x^*$ lies on BC and so $\alpha x^* \neq x^*$. By invariance, $\mu(\alpha A',0) = \alpha \mu(A',0) = \alpha x^*$. But, since $\alpha A' = A$, therefore, $\mu(\alpha A',0) = \mu(A,0) = x^*$. But this is a contradiction and therefore, $x^*$ lies on BC, and so it is efficient.

(ii) Let E be the point on BC which represents the solution vector $x^*$. Choose $\tau$ so that

$$\tau = \frac{BE}{BC}.$$ 

Since BC is a 45 degree line, it follows that $x_1^* = \tau$ and $x_2^* = 1 - \tau$.

(iii) For $\tau$ given by (ii), a straightforward solution of the following maximization problem

$$\max x_1^* x_2^{(1-\tau)} \text{subject to } x_1 + x_2 = 1$$

is that $x_1^* = \tau$ and $x_2^* = 1 - \tau$. 
To show the converse we will reverse the steps. For given \( \tau \), \( x^* \) solves

\[
\max x_1^\tau x_2^{(1-\tau)} \text{ subject to } x_1 + x_2 = 1
\]

implies that \( x_1^* = \tau \) and \( x_2^* = 1 - \tau \).

Since the Pareto efficient boundary of the bargaining set \( A \) is a subset of the points satisfying \( x_1 + x_2 = 1 \), and \( x^* \) satisfies this equation, it is efficient. Obviously, \( x^* > 0 \) for \( \tau \in (0,1) \), therefore it is individually rational and feasible.

Since the first order condition of the maximization problem remains unaltered by any positive affine transformation of the set \( A \), and the maximand, therefore \( x^* \) satisfies the axiom of invariance.

Since, \( x^* \) is also the maximizer of \( x_1^\tau x_2^{(1-\tau)} \) over any subset of the feasible set \( A \) that also contains \( x^* \) and 0, therefore, it satisfies the axiom of independence.

Moreover, the maximand is concave and the feasible set is compact and convex, the solution \( x^* \) is unique. Q. E. D.

The major point of the lemma is that every solution to a bargaining game \((A,0)\) that satisfies the three axioms induces a distribution parameter \( \tau \in (0,1) \), and every distribution parameter induces a solution to the bargaining problem \((A,0)\) that satisfies the three axioms. For a given \( \tau \in (0,1) \), the induced solution to the bargaining problem is unique, whereas there can be many solutions that satisfy the three axioms and induce many values of \( \tau \) such that \( \tau \in (0,1) \). In fact, all points on the boundary of the feasible set satisfy the three axioms and are potential solutions to the bargaining problem \((A,0)\).

The minimum information, in addition to \((A,0)\), required to obtain a unique solution is the ratio at which the players will receive their payoffs at the solution to the bargaining problem \((A,0)\). If this ratio, say \( x_1 / x_2 = \delta \), is institutionally provided, then those points in the set \( A \) that lie on the ray of slope equal to \( \delta \) become potential solution points. It is useful to note that if other axioms are not satisfied, then the condition that payoffs be divided according to \( \delta \) alone is not sufficient to yield a unique outcome of the bargaining. For each \( \delta > 0 \), we can define a unique \( \tau = \delta / (1 + \delta) \in (0,1) \).

Since all solutions satisfying the three axiom lie on the efficient frontier, the intersection of the ray with the efficient boundary of the set \( A \) constitutes the unique solution to the bargaining problem \((A,0)\) with given \( \delta \). Note that, \( \delta = 1 \) yields the original Nash solution.
Appendix-6C: Proof of Theorem 6.4

Theorem 6.4 For any bargaining problem $(\mathcal{R}, \Pi^d, \Theta)$ in a tariff game, if the fear functions are well defined, the following statements are equivalent:

(i) \( \bar{P}_1 = \arg \max_{\Pi \in \mathcal{R}} [\Pi_1(P_1) - \Pi_1^d][\Pi_2(P_2) - \Pi_2^d]^{\Theta} \). That is, \( \bar{P}_1 \) maximizes the generalized Nash product over the bargaining set.

(ii) \( \mu(\mathcal{R}, \Pi^d, \Theta) = \Pi(\bar{P}) \), that is \( \bar{P}_1 \) is the price at the generalized Nash solution to the bargaining game \( (\mathcal{R}, \Pi^d, \Theta) \) with bargaining power distribution \( \Theta \) that satisfies the axioms 1-3.

Proof: We have to show that (i) holds if and only if (ii) holds. Since the implication of (ii) by (i) is straightforward, we will show that (ii) implies (i).

Let us assume that \( \bar{\Pi} = \Pi(\bar{P}) \) is the payoff combination that maximizes the generalized Nash product over the bargaining set \( \mathcal{R} \). Assume that \( \mu(\mathcal{R}, \Pi^d, \Theta) \) is the solution of the bargaining problem that satisfies axioms 1-3. We have to show that \( \mu(\mathcal{R}, \Pi^d, \Theta) = \Pi(\bar{P}) \).

Let us define a set \( \tilde{\mathcal{R}} \) such that

\[
\tilde{\mathcal{R}} = \left\{ (\bar{\Pi}_1, \bar{\Pi}_2) \mid \bar{\Pi}_i = \Theta_i \left( \frac{\Pi_i - \Pi_i^d}{\Pi_i - \Pi_i^d} \right) ; \Pi \in \mathcal{R} \right\}
\]

It can be seen that \( \tilde{\mathcal{R}} \) is obtained from \( \mathcal{R} \) by a positive affine transformation. The point \( \bar{\Pi} \in \mathcal{R} \) is transformed into \( \Theta \in \tilde{\mathcal{R}} \), and \( \Pi^d \in \mathcal{R} \) is transformed into \( 0 \in \tilde{\mathcal{R}} \). We know that the payoff that maximizes the generalized Nash product is invariant under such transformations, therefore \( \Theta \) solves the problem

\[
\max_{\Pi \in \mathcal{R}} \tilde{\Pi}_1^\Theta \tilde{\Pi}_2^\Theta.
\]

We are done, if we can show that

\[
\mu(\tilde{\mathcal{R}}, 0, \Theta) = \Theta.
\]

Since better-than-\( \Theta \) set and the set \( \tilde{\mathcal{R}} \) are both convex, \( \Theta \) is in both the sets, hence by the separating hyper plane theorem, there is a unique hyper plane passing through \( \Theta \) that is tangent to both the sets at \( \Theta \). AB is that line (hyper plane).
The slope of the generalized Nash product curve passing through $\Theta$ is given by

$$\frac{d\tilde{\Pi}_1}{d\tilde{\Pi}_2} = \frac{\Theta_2}{\Theta_1} \cdot \frac{\tilde{\Pi}_1}{\tilde{\Pi}_2}.$$ 

Therefore, the slope of the tangent AB that passes through the point $(\tilde{\Pi}_1, \tilde{\Pi}_2) = (\Theta_1, \Theta_2)$ is -1.

Now, construct a set $H$ of all points from the area OAB. Then the set $H$ is symmetric. Therefore, by Lemma 6.1 $\mu(H, 0, \Theta) = \Theta$.

But, since $\Theta \in \tilde{\mathcal{K}} \subset H$, and the solution $\mu$ is independent of irrelevant alternatives, therefore, $\mu(\tilde{\mathcal{K}}, 0, \Theta) = \Theta$. Now by applying inverse positive affine transformation, using the invariance property of $\mu$, we can see that

$$\mu(\mathcal{K}, \Pi^d, \Theta) = \Pi(\overline{\mathcal{P}}).$$

Hence the condition (ii) implies the condition (i). Q. E. D.
CHAPTER 7

A GENERAL EQUILIBRIUM MODEL OF A POLITICAL ECONOMY

AND COMPARATIVE STATIC RESULTS

Introduction

In chapter 2, we reviewed a selection of the existing theories of tariff determination to examine the current state-of-the-art in modelling the tariff formation process. It was clear from the review that there seems a general consensus among the economists in that a tariff can consistently be viewed as an equilibrium outcome of the political market. The political process can be described by a noncooperative game in which the policy maker as well as the private interest groups behave strategically to further their own self-interests.

In chapter 3 a stylized 2-sector, 3-factor policy exogenous general equilibrium model was obtained to describe an economic equilibrium. Using the solutions of this model for different tariff rates, a rent transformation frontier was derived in chapter 4. This frontier describes a locus of the equilibrium combinations of rental incomes at different tariff rates. It showed that the owners of specific factors have conflicting interest on government’s tariff policy.

Following previous work, we studied the strategic interaction between the government and private interest groups in Chapter 5. We argued that the government, as a Stackelberg leader, offers a lobbying-sensitive pricing function to the interest groups in order to extract the maximum political support for it. The owners of specific factors (the interest groups) play a noncooperative tariff game by choosing the amount of their predatory lobbying expenditure, affecting the political support for the government, to maximize their rental income. It was shown that for such political economies at least one noncooperative Nash equilibrium exists.

By recognizing the fact that a cooperative solution would dominate noncooperative solutions, we studied the problem of tariff determination as a Nash bargaining problem in chapter 6. This framework views all political activities as integral parts of the bargaining process. There, we derived the necessary and sufficient condition for a generalized Nash solution to the bargaining problem of the underlying tariff game.

The purpose of this chapter is to combine the results obtained in the previous chapters and derive a set of conditions that characterizes the equilibrium in a tariff (policy)-endogenous general equilibrium model (PEGEM) of a small open economy. In
particular, the conditions describing the bargaining equilibrium in the tariff game, which is the condition for an equilibrium in the political market, is combined with the conditions of general economic equilibrium in the policy-exogenous model (PXGEM) developed in chapter 3. A simultaneous satisfaction of these conditions implies a general (politico-economic) equilibrium of the stylized political economy. The political economy in a general equilibrium is then subjected to comparative static experiments to analyze the (comparative static) responses of the bargained tariff rate to exogenous shocks.

The chapter is organised into seven sections. The first section describes the full model by collecting all the equilibrium conditions. The second section discusses the problems concerned with the identification of the disagreement payoffs in a bargaining problem in the tariff game. We argue that the concept of minimum expectation, suggested by Roth (1977), can be employed as a reference point in bargaining to operationalize the concept of disagreement.

In section three we obtain the players' minimum expectation in the tariff game under two different political environments. First, we consider a coercive type of government, which rules by force, and derive players' minimum expectation under this regime. It is shown that the minimum expectation of each player, in this case, is zero. Then, we consider the case with a support maximizing government, which offers a pricing function that satisfies assumptions (A1)-(A3), and (A5). We argue that, with a popular government, the minimum expectation of the exporting sector (player 2) is the payoff at the autarkic equilibrium, and the minimum expectation of the import-competing sector (player 1) is the payoff at the free trade equilibrium. This completes the groundwork to operationalize the PEGEM.

The comparative static responses of the bargained tariff rate are derived in section four. Two separate subsections are devoted to the two types of government behaviour.

In the first subsection, we derive and explain the comparative static results under the assumption that the government is coercive. In particular, it is shown that, ceteris paribus, (i) any change in the international terms of trade will induce a tariff change that will exactly offset the effect of the international terms of trade change on the domestic relative prices, (ii) an exogenous increase in the stock of specific factor in either sector will induce a fall in the rate of protection awarded to the expanding sector and a rise in the rate of protection awarded to the other sector, (iii) the effect of an increase in the supply of the mobile factor on the rate of protection, however, is ambiguous. It depends on the relative labour intensities and relative ease of factor
substitution in the two sectors. Intuitive explanations of these results are provided with geometric illustrations.

In the second subsection, we have attempted to examine the comparative static behaviour of the bargained tariff rate under a support-maximizing government. More specifically, we required that the government choose tariff rates within the range so that the domestic relative price falls between the closed interval defined by the relative prices at the free trade and the autarkic equilibrium. This made the payoffs at the minimum expectation sensitive to changes in all of the exogenous variables. Analytical results under CES production functions thus became quite complex. We assumed Cobb-Douglas production functions to simplify the algebra. Though it was possible to show the validity of Hillman's result that the domestic relative price moves with the world price, we could not determine the direction of the response of the bargained tariff rate with respect to each of the exogenous variables.

Section five summarizes the chapter. The difficulties in obtaining clear-cut analytical results highlight the importance of simulation exercises. Under the specific conditions of the economy in question, it also highlights the importance of a computable PEGEM in understanding the behaviour of the bargained tariff rate. This exercise is deferred until the next chapter.

7.1 The PEGEM: A Tariff-endogenous General Equilibrium Model

We know that the necessary and sufficient condition for the generalized Nash solution to the bargaining problem in the tariff game is given by equation (6.25) as

\[(6.25') \quad \frac{\pi_1 - \pi_1'}{\pi_2 - \pi_2'} = -\frac{\theta_1}{\theta_2} \frac{d\pi_1}{d\Pi_2} \]

Recall that \(\pi_1 = K_1 \tilde{R}_1\), is the rental income of sector 1 in units of commodity 1, where \(\tilde{R}_1 = R_1 / P_1\), and \(\pi_2 = K_2 R_2\) is the rental income of sector 2 in units of commodity 2. Commodity 2 is the numeraire. Unless the sectoral capital stocks change we can normalize capital stocks by setting \(K_1 = K_2 = 1\). With this normalization the rental rate equals the rental income for each sector.

We can obtain from equation (4.8) that the slope of the RTF is given by

\[(7.10) \quad \frac{d\pi_1}{d\Pi_2} \bigg|_{RTF} = \frac{K_1 d\tilde{R}_1}{K_2 dR_2} = -\frac{K_1 \tilde{R}_1}{K_2 R_2} \left( \frac{\lambda_2 \sigma_2 S_{L2}}{\lambda_1} \right) = -\frac{\pi_1}{\Pi_2} \left( \frac{\lambda_2 \sigma_2 S_{L2}}{\lambda_1 \sigma_1 S_{L2}} \right) \]

The right hand side of equation (7.10) can be simplified by noting that \(\lambda_i = L_i / L\) is the share of sector \(i\) in the total employment of labour, and the distributive share of
labour $S_L = W_L / P_L Y_L$ where $P_2 = 1$ by the choice of numeraire. Using these definitions, equation (7.10) can be rewritten as

$$(7.11a) \quad \frac{d\Pi_1}{d\Pi_2}_{RTF} = -\frac{\Pi_1}{\Pi_2} \left( \frac{\sigma_2 Y_2}{\sigma_1 Y_1 P_1} \right).$$

Alternately, we can also write

$$(7.11b) \quad \frac{d\Pi_1}{d\Pi_2}_{RTF} = -\frac{\sigma_2 S_{K1}}{\sigma_1 S_{K2} P_1}$$

where, $S_{K_i} = \Pi_i / Y_i$ is the distributive share of capital in sectoral output in each sector $i$.

Now substituting the expression for the slope of the RTF from equation (7.11a) into equation (6.25') we get

$$(7.12) \quad \frac{\Pi_1 - \Pi_1^d}{\Pi_2 - \Pi_2^d} = \Theta_1 \frac{\Pi_1}{\Pi_2} \left( \frac{\sigma_2 Y_2}{\sigma_1 Y_1 P_1} \right).$$

Alternately, the equation (7.12) can also be written as

$$(7.13) \quad \frac{\Pi_1 - \Pi_1^d}{\Pi_2 - \Pi_2^d} = \Theta_2 \left( \frac{\sigma_2 S_{K1}}{\sigma_1 S_{K2} P_1} \right).$$

This condition can be explained as follows. We know that $\Pi_1$ is a strictly increasing function of $P_1$ and $\Pi_2$ is a strictly decreasing function of $P_1$, the left-hand side of equation (7.13) is a strictly increasing function of $P_1$ for given a value of $\Pi_1^d$. Given that $\sigma_1 \geq 1$ and $\sigma_2 \geq 1$, we also know that $S_{K1}$ is a nonincreasing function of $P_1$, and $S_{K2}$ is a nondecreasing (constant in the Cobb-Douglas case) function of $P_1$. Therefore, for a given distribution of bargaining power, the right-hand side of equation (7.13) is a strictly decreasing function of $P_1$. Thus, a unique domestic relative price satisfies, as discussed in the previous chapter, the necessary and sufficient condition for the solution to the bargaining problem. A non unique situation may arise if $\sigma_1$ is close to zero and $\sigma_2$ is very large.

Now we can combine equation (7.12) with the policy-exogenous general equilibrium model developed in chapter 3. Recalling equations from Table 3.2, the system of equations that describes the full equilibrium of the economy can be written as in the following Table 7.1.
The Political Market (or Sphere):

Condition for bargaining equilibrium:

\[
\frac{\Pi_1 - \Pi_1^d}{\Pi_2 - \Pi_2^d} = \frac{\Theta_1}{\Theta_2} \left( \frac{\sigma_2 Y_2}{\sigma_1 Y_1^* P_1^*} \right).
\]

Definition:

\[
\Pi_i = K_i R_i / P_i, \quad i=1,2.
\]

The Economic Markets (or Sphere)

(a) The Goods Market:

The supply functions of domestic production sectors:

\[
Y_j = K_j \beta_j^{-1/P_j} \left[ 1 - \alpha_j^{1/(1+\rho_j)} \right] \left( W / P_j \right)^{\rho_j/(1+\rho_j)} P_j^{1/(1+\rho_j)}, \quad j=1,2.
\]

Consumer demand functions:

\[
C_j = \left( \delta_j / P_j \right) \left[ \sum_{i=1}^{2} P_i Y_i + T_i^{*} P_i M_i \right], \quad j=1,2.
\]

Equilibrium in the market of good 1.

\[
C_1 = Y_1 + M_1.
\]

(b) The Foreign Exchange Market:

Trade balance constraint:

\[
P_1^* M_1 + P_2^* M_2 = 0.
\]

(c) The Labour Market:

Sectoral labour demand:

\[
L_j = K_j \alpha_j^{1/(1+\rho_j)} \beta_j^{-1/P_j} \left[ \left( P_j / W \right)^{\rho_j/(1+\rho_j)} - \alpha_j^{1/(1+\rho_j)} \right]^{1/(1+\rho_j)}, \quad j=1,2.
\]

Labour market equilibrium:

\[
L = \sum_{j=1}^{2} L_j.
\]

Domestic price determination:

\[
P_1 = P_1^{*} (1 + T_1^{*}).
\]

Virtual rental rates:

\[
R_j = \beta_j^{-1/P_j} \left( P_j^{\rho_j/(1+\rho_j)} - \alpha_j^{1/(1+\rho_j)} W_j^{\rho_j/(1+\rho_j)} \right)^{1/(1+\rho_j)}, \quad j=1,2.
\]

Total number of equations 15.
Table 7.1 (continued)

List of Endogenous Variables:

\[ \Pi_j \quad j = 1, 2 \quad : \quad \text{2 Sectoral rental incomes} \]
\[ Y_j \quad j = 1, 2. \quad : \quad \text{2 Sectoral outputs} \]
\[ C_j \quad j = 1, 2. \quad : \quad \text{2 Domestic demands} \]
\[ M_j \quad j = 1, 2 \quad : \quad \text{2 Net Import quantities} \]
\[ L_j \quad j = 1, 2 \quad : \quad \text{2 Sectoral employment of labour} \]
\[ R_j \quad j = 1, 2 \quad : \quad \text{2 Sectoral rental rates in units of commodity 2} \]
\[ W \quad : \quad \text{1 Wage rate in units of commodity 2} \]
\[ P_1 \quad : \quad \text{1 Price of commodity 1 in units of commodity 2} \]
\[ T^R_1 \quad : \quad \text{1 Rationalized tariff rate} \]

Total number of endogenous variables: 15.

List of exogenous variables:

\[ K_j \quad j = 1, 2. \quad : \quad \text{2 Endowments of sector specific capital stocks} \]
\[ L \quad : \quad \text{1 Endowment of Labour in the economy} \]
\[ P^*_1 \quad : \quad \text{1 International relative price of commodity 1} \]
\[ \Pi_j' \quad j = 1, 2 \quad : \quad \text{2 Disagreement payoffs} \]
\[ P_2 \quad : \quad \text{Price of the numeraire commodity (always unity).} \]

Total number of exogenous variables: 7.

Parameters:

\[ \sigma_j = 1 / (1 + \rho_j) \quad j = 1, 2 \quad : \quad \text{2 Elasticities of factor substitution} \]
\[ \alpha_j, \beta_j \quad j = 1, 2 \quad : \quad \text{4 Distributive parameters of CES production functions} \]
\[ \delta_j \quad j = 1, 2 \quad : \quad \text{2 Budget share parameter of C-D utility function} \]
\[ \Theta_1, \Theta_2 \quad : \quad \text{2 Parameters reflecting the bargaining powers} \]

Total number of parameters: 8.
Thus Table 7.1 describes a system of 15 equations in 15 endogenous variables, including the rationalized tariff rate. For given values of exogenous variables and model parameters the system, in principle, can be solved for the 13 endogenous variables. The solution vector of endogenous variables describes the full equilibrium of the politico-economic system of the stylized economy.

This system demands more information than the conventional policy-exogenous CGE models. In addition to the information required by a conventional CGE model, it requires information on the distribution of the players' bargaining powers and information on disagreement payoffs, $\Pi^d$. With respect to the distribution of bargaining powers, we make the following assumption.

**Assumption 7.1**  *The distribution of bargaining power between the players is exogenously given and it is unaffected by small changes in the values of exogenous variables.*

In chapter 6, it was argued that bargaining powers are affected by factors such as the time-preference rates of the players, their subjective probability that a third party will snatch the bargaining opportunity and so on. In this light, assumption 7.1 does not seem to be too restrictive. However, it paves a clear way towards the comparative static analysis of the PEGEM. We will move to analyse the problem of identifying the disagreement payoffs to the players that constitutes the most critical information required by the PEGEM.

### 7.2 Identification of the Disagreement Payoffs

The disagreement payoff is the payoff that players will receive in the event that they fail to reach an agreement. A natural candidate for this is the payoffs at a noncooperative Nash equilibrium of the tariff game. But, there are two problems, which make the use of payoffs at a noncooperative Nash equilibrium less attractive. First, the possibility of multiple Nash equilibria can not be dismissed a priori, and there seems no clear way of identifying which one of them will be attained in the event of a disagreement. Second, even if there are reasons to believe that a unique Nash equilibrium will be attained, the government's pricing function has to be specified before any Nash equilibrium can be computed. This would further require a good knowledge of the government's political support function.

The solution to the bargaining problem discussed in chapter 6, however, is based on given disagreement payoffs. So long as it is known before the bargaining game is played, any arbitrary pair of payoffs can be a candidate for the disagreement payoffs. All results obtained in chapter 6 remain valid, since no result obtained in chapter 6 is
based on the assumption that the disagreement payoff is a noncooperative Nash solution.

A particularly interesting alternate candidate for the disagreement payoff is the minimum expectation payoff proposed by Roth (1977). Roth's concept of minimum expectation can be defined in the following way.

Let $\mathcal{R}$ be the set of all feasible payoff combinations. For each player $i$, define

$$\Pi_i^{\max} = \max\{\Pi_i \mid (\Pi_i, \Pi_{-i}) \in \mathcal{R}\};$$

and

$$\Pi_i^{\min} = \max\{\Pi_i \mid (\Pi_i, \Pi_{-i}^{\max}) \in \mathcal{R}\}.$$

**Definition 7.1 (minimum expectation).** The payoff combination $\Pi^{\min} = (\Pi_1^{\min}, \Pi_2^{\min})$ represents the minimum expectation of the players in cooperation. The payoff combination $\Pi^{\max} = (\Pi_1^{\max}, \Pi_2^{\max})$, which is also called the ideal point, represents the aspiration levels\(^1\) of the players in cooperation (see figure 7.1).

![Diagram](image)

The point of minimum expectation

Figure 7.1

One attractive feature of the point of minimum expectation is that it represents the payoff to each player when the bargaining opponent has been able to obtain the best feasible outcome for herself, say by forming a coalition with the government or by preemptive lobbying. The payoff combination $\Pi^{\min}$ therefore represents the worst outcome to each player. No rational player will choose a strategy that yields his opponent a payoff less than that corresponds to $\Pi^{\min}$ because to do so would bring no benefit, possibly a reduction, in his own payoff. In other words, for each player, the payoff combination $\Pi^{\min}$ corresponds to the opponent's dictatorial solution.

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\(^1\) See also Friedman (1986) pp. 160-62.
Roth (1977) has shown that the Nash solution to the class of bargaining problems in which the disagreement payoff is given by $\Pi^m$ satisfies all the axioms as satisfied by the original Nash solution. The only difference between the two is that Nash's solution is independent of irrelevant alternatives (axiom of independence) other than the disagreement point, whereas the new solution will be independent of irrelevant alternatives other than the point of minimum expectation.

Thomson (1981) has argued that the disagreement point in a bargaining problem simply serves as a reference point to which players find it natural to compare any proposed compromise. He has suggested several possible candidates for a reference point. However, the list of reference points can be considerably shortened by requiring that the point satisfy some desirable properties. One such property, as suggested by Thomson, is that the point of reference be sensitive to changes in the set of feasible outcomes. In this respect the point of minimum expectation, suggested by Roth, displays another attractive feature that is not possessed by other points like the status quo - the point of minimum expectation could be changed by changes in the boundary of the feasible set.

Furthermore, Thomson also proposed the following two essential\(^2\) properties of a reference point:

(i) That it be invariant with respect to positive affine transformations; and

(ii) That it be invariant with respect to symmetrization of almost symmetric bargaining problems.

Thomson has found (in his Lemma 2) that the point of minimum expectation satisfies the two desirable properties of a reference point\(^3\).

It is important to keep in mind that the solution to a bargaining problem obtained by defining the minimum expectation payoffs as the disagreement payoffs is not necessarily equal to the Nash solution if the disagreement payoffs are already well defined by some other rule of the game (for example, the status quo or zero). Therefore, the solution that is obtained by the rule that disagreement payoffs are the payoffs at the player's minimum expectation yields, at best, a Nash-like solution to a bargaining problem, which satisfies axioms similar to those as satisfied by the Nash solution.

---

\(^2\) These properties are required in establishing the correspondence between a solution that maximizes the Nash product and a solution that satisfies all the axioms that characterizes a bargaining solution.

\(^3\) See Thomson (1981) for details and for other candidates for reference points.
In general, the operational definition of the disagreement payoffs in the theory of bargaining has remained unclear. Identification of the disagreement payoffs has, therefore, been suggested as a matter of modelling judgement (Binmore, Rubinstein, and Wolinsky, 1986). The concept of minimum expectation, nevertheless, is well defined, operational and payoffs at minimum expectation possess some desirable properties of a reference point in bargaining. Thus the payoffs at the point of minimum expectation appear to be natural candidates for the disagreement payoffs.

However, the point of minimum expectation, as pointed out by Thomson, has a serious limitation that it is *not always* continuous in the feasible region. A small change in the feasible region, in some cases, may lead to radical changes in the point of minimum expectation. For example, consider the following figure:

![Figure 7.2](image)

Suppose that the initial set of feasible outcomes is OAEB. Then player 1's minimum expectation is 0, and player 2's minimum expectation is OD. Now if the set of feasible outcomes is expanded to OCEB, (point C very close to point A) then the minimum expectation of each player will be zero. Therefore, with a small increase in the feasible set the point of minimum expectation will move to the origin from the point D. If the payoff at the point of minimum expectation is used as the disagreement payoff, the bargaining solution may also change abruptly for small changes in the set of feasible outcomes. This situation is also not very satisfactory, particularly for the validity of comparative static results. The seriousness of the discontinuity of the point of minimum expectation with respect to the feasible set in the tariff game is, therefore, worth examining.
7.3 Minimum Expectation in the Tariff Game

In this section we derive expressions for the minimum expectation payoffs of the players under two different political environments. First, we consider a special case in which the government rules by force. We allow for the possibility that any of the players may form a coalition with the government and choose a policy that best suits the winning player. The government balances its budget, if necessary, by taxing the losing player and other nonstrategic agents in the economy. We shall call this type of government a coercive government and obtain players' minimum expectations under it.

Next, we consider a support maximizing government, which offers a pricing function satisfying assumption (A1)-(A3) and (A5). We shall call this type of government a popular government, and obtain players' minimum expectation under it.

7.3.1 Minimum Expectation under A Coercive Government: 
A Special Case

Specifically for illustrative purposes, we consider a special case in which the government can completely be captured by one of the players. In other words, we consider a case in which the government will form a coalition with the winning player and use its coercive power to provide maximum benefit to the winner at the cost of the loser. As an extreme case, we can think of a coercive government in terms of Posner's classification.4

If a player fails to form a coalition with the 'coercive' government, then the result is that his rental income can be taxed to sustain the price that benefits the winning player most. This means that, in disagreement, the worst outcome to each player could be that the player has to surrender all of his rental income to finance the government's budget deficit created by the price policy favouring the opponent. This in turn implies that the payoff at the minimum expectation of each player with a coercive government is zero. It is interesting to note that the point of minimum expectation, in this case, has been located without first locating the aspiration level of the players. Therefore, we write

\[ \Pi^{\text{min}} = (0,0). \]

Continuity of the point of minimum expectation in this case is, therefore, not a problem at all since the point of minimum expectation, under coercive behaviour of the

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4 Posner (1974) distinguished three forms of political system: (i) entrepreneurial - which sells favourable legislation to industries that value it most; (ii) coercive - which awards legislation to groups that are able to make credible threats to retaliate with violence if society does not give them favourable treatment; (iii) democratic - legislation is awarded by the vote of elected representatives of the people.
government, does not change as the RTF shifts due to changes in the factor endowments. However, it loses its sensitivity with respect to changes in the boundary of the feasible (or the bargaining) set.

The case with $\Pi^d = \Pi^{\text{min}} = (0,0)$ is interesting for three reasons. First, it greatly simplifies the model and analytical results are possible. It can illustrate the mechanism of endogenous determination of the tariff rate. Second, it corresponds to a potentially dictatorial type of government. The results, therefore, will show the behaviour of bargained tariff rates under a particular political environment where the government can be captured by one of the bargaining party if no agreement is reached during the bargaining process. Third, more interestingly, $\Pi^d = 0$ corresponds to Brock, Magee, and Young's economic black hole and could be considered as the worst possible noncooperative Nash equilibrium outcome in the tariff game.

Therefore, the disagreement payoffs for the bargaining problem will be measured by the payoffs at the minimum expectation of the players. However, depending upon the nature of the government the disagreement payoffs or payoffs at the minimum expectation of the players may be $(0,0)$ or $(\Pi^*_1, \Pi^*_2)$. Both cases will be considered in the comparative statics of the model.

**7.3.2 Minimum Expectation under a Support Maximizing Government**

It follows from definition 7.1 that location of the point of minimum expectation requires a knowledge of the aspiration level (ideal point) of the players. In the tariff game, where players are attempting to maximize the rental incomes and the rental income of player 1 is a strictly increasing function of the tariff rate, and the rental income of player 2 is a strictly decreasing function of the tariff rate, it is natural to think that the best outcome for each player is attained when the employment of labour (mobile factor) is completely specialized in his favour. In other words, player 1 would aspire to a tariff rate that eliminates employment of labour in the exporting sector and player 2 would aspire for a tariff rate that eliminates the employment of labour in the import competing sector.

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5 Magee, Brock and Young have defined an economic black hole, in the context of a long-run model in which both capital and labour are involved in predatory lobbying, as a situation in which all of the economy's factor endowment is exhausted in predatory lobbying. See Magee, Brock and Young (1989: 223). A short-run analogue of their economic black hole can be defined as a situation in which the owners of the specific factors exhaust all of their rental incomes in predatory lobbying. So, if in a noncooperative Nash equilibrium each player exhausts his rents in predatory lobbying and obtains zero payoffs, then the equilibrium tariff thus determined may be defined as the black hole tariff.

6 In such a case one sector will employ all labour and the other sector will be producing outputs employing its sector-specific capital stock only. Production of outputs with a single factor is possible under CES production function. The worst outcome for each sector is not to be able to employ the mobile factor. Therefore, the rental income and the payoff at the point of minimum expectation of each sector
However, a price (or tariff rate) will require a complementary financing policy if it is not self-financing. Rational players will take into account the effects of such policies on their payoffs while determining their aspiration levels. For example, player 1 may not like to push for a domestic relative price that is higher than that at autarky even though it implies a higher rental income, if she has to bear all the subsidy cost. This is equivalent to subsidizing her production out of her own rental income. The payoffs at such prices will be correspondingly less than the rental incomes by the amount of subsidy cost. Nevertheless, player 1 would certainly prefer to have such a higher-than-autarky price if it is financed by taxing someone else in the economy. Symmetric arguments can be made for player 2 - the exporting sector.

In the model studied here, tariff revenue is transferred to the national consumer (see equation (3.19') in Table 7.1), which encompasses the government, owners of the sector-specific factors, and the labourers. The model does not specify the policies and mechanisms for the transfer of the tariff revenue to each individual that will be adopted in maintaining a balanced budget of the government. This means that even though the government's budget constraint is always satisfied (with the aggregate budget constraint) the extent of the gain from the tariff revenue or the cost of the subsidy to each individual remains unknown. Thus the model can not yield the payoffs to the players gross of tariff revenue or net of subsidy costs. As a result, the problem of identifying each player's aspiration level, and hence the point of minimum expectation, remains unresolved. We have considered this limitation as the price to be paid for the simplicity of the present model.

However, even in a model that attempts to address the distributional issues explicitly, numerous ways of financing a tariff induced budget or trade deficit can be conceived. At least a somewhat arbitrary rule of distribution of the tariff revenue has to be spelled out. Therefore, the point of minimum expectation or the aspiration level of the players can be identified only after the mode of balancing the government's budget is specified.

We get around this problem by invoking the properties of the pricing function offered by the government. It has been assumed (see chapter 5) that the pricing function

will be equal to the level of output thus produced. Comparative static results with minimum expectation thus defined are reported in Appendix 7B.

For a clear approach to the problem of distributing the recycled tariff revenue and its impact on the endogenous tariff rate see Long and Vousden (1991). They have shown that if the players do not differ in their risk preferences significantly, then a failure to account for the tariff revenue changes in the players' payoffs will have no qualitative consequence on the final result. It simply makes the modelling aspect more complicated. However, if there are reasons to believe that the players differ in their risk preferences significantly, then it follows from their result that the omission may affect the result qualitatively. We ignore the tariff revenue changes in accounting players' payoffs by assuming that players have almost identical risk preferences.
of the government satisfies assumptions (A5), which states that, in disagreement, the government's pricing function yields a domestic relative price that will always fall in between the free trade and the autarky relative price.

This means that, given the properties of the government's pricing function, the aspiration level of player 1 is the payoff that is obtained when price takes its upper bound - that is the autarkic price, and the aspiration level of player 2 is the payoff that is obtained when the price takes its lower bound - that is the free trade price. This implies that player 1's (import competing sector) minimum expectation is the payoff at the free trade price and player 2's (exporting sector) minimum expectation is the payoff at the autarkic price - at the tariff rate that eliminates all imports (see figure 7.3). Therefore, we write

$$\Pi^\text{min} = (\Pi_1^*, \Pi_2^*)$$

where, $\Pi_1^*$ is the rental income of the import competing sector at the free trade price, and $\Pi_2^*$ is the rental income of the exporting sector at the autarky price.

![Minimum Expectation under a support maximizing government](Figure 7.3)

The following reasons justify $\Pi^\text{min} = (\Pi_1^*, \Pi_2^*)$ as a reasonable reference point in bargaining:

1. Free trade and autarky equilibria are well defined concepts, and exist for all non-zero factor endowment configurations;
(2) For a given factor endowment, payoffs at the free trade price and at the autarky price are unique;

(3) It is claimed (without a rigorous proof) that \( \Pi^\text{min} = (\Pi^*, \Pi^*_s) \) is a continuous function of the feasible set. The argument is: so long as production functions are continuously differentiable in factors, and commodity demand functions are differentiable in commodity prices, all rental functions and the autarky equilibrium price are continuous functions of factor endowments. This implies that the RTF is continuous in factor endowments, and therefore, the payoffs at the free trade price and at the autarky price are continuous in factor endowments. Therefore, \( \Pi^\text{min} = (\Pi^1, \Pi^*_s) \) is continuous in the feasible set, since factor endowment changes are the only sources that bring change in the feasible set;

(4) Clearly, as the RTF shifts with a change in factor endowments, so do the payoffs at these prices. Therefore, \( \Pi^\text{min} = (\Pi^1, \Pi^*_s) \) is sensitive to changes in the boundary of the feasible set.

(5) Moreover, under the assumption of self-finance, \( \Pi^\text{min} = (\Pi^1, \Pi^*_s) \) also satisfies Thomson's two properties of a reference point as referred above.

Therefore, we will use \( \Pi^\text{min} = (\Pi^1, \Pi^*_s) \) as the reference point of the players for the bargaining problem in the tariff game. It is clear from the above discussion that this point of minimum expectation is obtained under the assumed properties of the pricing function. As indicated in chapter 5, these properties of the pricing function are consistent with the support maximizing behaviour of the government.

7.4 Comparative Static Results

7.4.1 Comparative Static Results I: Coercive Government

If the government does not care about its support because it is coercive or dictatorial, then the pricing function offered by the government need not satisfy some of these assumptions - especially, the bounded pricing and self-financing assumptions may not be satisfied. Minimum expectation of the players in this political environment can be quite different from that in a political environment where the government is based on popular politics. For this reason we have constructed the following illustrative special case.

7.4.1.1 Modification of the Model

In this section, we derive the version of the PEGEM that is applicable under a coercive government. We know that under a coercive government the minimum
expectation of the players is the origin. Therefore, substituting $\Pi^d = 0$ into equation (7.12) and solving for $P_1$ we obtain

\[(7.17)\quad P_1 = \left(\frac{\Theta_1 \sigma_2}{\Theta_2 \sigma_1}\right) \frac{Y_2}{Y_1}.
\]

Linearizing equation (7.17) around the 'observed' full equilibrium we obtain

\[(7.18)\quad P^e = (\theta_1 - \theta_2) + (y_2^e - y_1^e).
\]

At unchanged bargaining powers, whatever they may be, equation (7.18) reduces to

\[(7.19)\quad P^e = y_2^e - y_1^e.
\]

Table 7.2

Linearized version of PEGEM: coercive government behaviour

---

**The Political Market:**

\[(7.19')\quad P_1^e = y_2^e - y_1^e.
\]

**Economic Markets:**

**The supply functions:**

\[(3.36')\quad y_j^e = k_j + \sigma_j \left(\frac{S_{ij}^e}{S_{kj}^e}\right) (p_j^e - w^e);\quad j = 1, 2.
\]

**Labour demands functions:**

\[(3.37')\quad l_j^e = k_j + \frac{\sigma_j}{S_{kj}^e} (p_j^e - w^e);\quad j = 1, 2.
\]

**The Labour market equilibrium condition:**

\[(3.38')\quad l = \sum_{j=1}^{2} \lambda_j l_j^e.
\]

**Sectoral rental rates:**

\[(3.39')\quad r_j^e = \frac{1}{S_{kj}^e} (p_j^e - S_{ij}^e w^e);\quad j = 1, 2.
\]

**Price equations:**

\[(3.40')\quad p_1^e = p_1^* + \tau^e t,\quad \text{and}
\]

\[(3.41')\quad p_2^e = 0.
\]

This equation is exactly similar to the autarky rule of price that was obtained in Appendix-2B. It shows that if some exogenous shock led the outputs of the two sectors,
in the new equilibrium, to grow at different rates, then the relative price of the commodity growing at a faster rate will fall.\(^8\)

In order to get a more precise meaning out of the equation (7.19) we combine it with the linearized version of the PXGEM described in Table 3.3. This yields the linearized version of the PEGEM that is listed in Table 7.2. Once again, we have ignored the four equations of the demand side, which are of no consequence to our analysis.

### 7.4.1.2 Responses of the Tariff Rate

This system contains 9 equations in 9 endogenous variables, including the tariff rate. The system can be solved for each of the endogenous variables in terms of changes in the exogenous variables to obtain the elasticity formulae.

In fact, part of the job has already been done in chapter 3. Using equations (3.50), and (3.51) that yield the responses of sectoral output levels we can obtain the expression for the differential of the sectoral output growth as

\[
(7.20) \quad y_2^* - y_1^* = \frac{1}{A^o}\left[\left(\frac{\lambda_1^o \sigma_1}{S_{K1}^o} + \frac{\lambda_2^o \sigma_1 S_{L1}^o}{S_{K1}^o} + \lambda_2^o \sigma_2\right)k_2 - \left(\frac{\lambda_2^o \sigma_2}{S_{K2}^o} + \frac{\lambda_1^o \sigma_2 S_{L2}^o}{S_{K2}^o} + \lambda_1^o \sigma_1\right)k_1\right] \\
- \left(\frac{\sigma_2 S_{L2}^o}{S_{K2}^o} \frac{\lambda_1^o \sigma_1}{S_{K1}^o} + \frac{\sigma_1 S_{L1}^o}{S_{K1}^o} \lambda_2^o \sigma_2\right)p_1 + \left(\frac{\sigma_2 S_{L2}^o}{S_{K2}^o} - \frac{\sigma_1 S_{L1}^o}{S_{K1}^o}\right)\right]
\]

where,

\[
A^o = \lambda_1^o \sigma_1 / S_{K1}^o + \lambda_2^o \sigma_2 / S_{K2}^o > 0.
\]

Solving equation (7.19), and (7.20) for \(p^o\) we get

\[
(7.21) \quad p^o = B^{-1}\left[\left(\frac{\lambda_1^o \sigma_1}{S_{K1}^o} + \frac{\lambda_2^o \sigma_1 S_{L1}^o}{S_{K1}^o} + \lambda_2^o \sigma_2\right)k_2 - \left(\frac{\lambda_2^o \sigma_2}{S_{K2}^o} + \frac{\lambda_1^o \sigma_2 S_{L2}^o}{S_{K2}^o} + \lambda_1^o \sigma_1\right)k_1\right] \\
+ \left(\frac{\sigma_2 S_{L2}^o}{S_{K2}^o} - \frac{\sigma_1 S_{L1}^o}{S_{K1}^o}\right)\right];
\]

where,

\[
B = A^o + \left(\frac{\sigma_2 S_{L2}^o}{S_{K2}^o} \frac{\lambda_1^o \sigma_1}{S_{K1}^o} + \frac{\sigma_1 S_{L1}^o}{S_{K1}^o} \frac{\lambda_2^o \sigma_2}{S_{K2}^o}\right) > 0.
\]

Note that equation (7.19) remains independent of the way tariff is distributed. Therefore, as far as the results of this section are concerned, the distribution of the tariff revenue does not matter.
Recalling that the factor creating a wedge between the domestic relative price and the world relative price is the tariff rate (see equation (3.40') in Table 7.2) it follows from equation (7.21) that

\[
\tau t = -p_1^* + B^{-1} \left[ \frac{\lambda_2^o \sigma_1}{S_{K1}^o} + \frac{\lambda_2^o \sigma_1 S_{L1}^o}{S_{K1}^o} + \lambda_2^o \sigma_2 \right] k_2
\]

Some interesting results follow from equation (7.22). Before we discuss those results, we recall the definitions of the variables and parameters involved in the equations.

First, \( \tau = 1 / (1 + T_1^R) \), where, \( T_1^R = (T_1 - T_2) / (1 + T_2) \) (equations 3.24, and 3.32) is the rationalized tariff rate - a single tariff rate that is equivalent to the joint imposition of an import tax at rate \( T_1 \), and an export subsidy (tax if negative) at rate \( T_2 \), and \( t = 100 \times dT_1^R \) is the change in the percentage point (not the percentage change in the tariff rate) of the rationalized tariff rate.

The term \( (1 + T_1^R) \) can be viewed as the rate of protection offered to the import competing sector, and therefore, the term \( \tau t \), which represents the percentage change in \( (1 + T_1^R) \), can also be viewed as the percentage change in the rate of protection awarded to the import competing sector. If, in the initial full equilibrium, \( T_1^R > 0 \) - that is, the economy was taxing trade - then we obtain \( 0 < \tau < 1 \). Similarly, \( \tau = 1 \) for \( T_1^R = 0 \), and \( \tau > 1 \) for \(-1 < T_1^R < 0 \). If we exclude the possibility of subsidizing imports or taxing exports at rates greater than 100% as practically implausible, then the parameter \( \tau \) is always positive.

Furthermore, the share parameters \( \lambda_1, \lambda_2, S_{K1}, S_{K2}, S_{L1}, \) and \( S_{L2} \) are always positive. The elasticities of factor substitution \( \sigma_1 \) and \( \sigma_2 \) are also positive.

Now by setting any three of the four exogenous variables - \( p_1^*, k_1, k_2, \) and \( l_1 \), in turn equal to zero we can obtain the comparative static results as follows:

\[
\frac{\tau t}{p_1^*} = -1 < 0;
\]

\[
\frac{\tau t}{k_1} = -B^{-1} \left( \frac{\lambda_2^o \sigma_2}{S_{K2}^o} + \frac{\lambda_2^o \sigma_1 S_{L2}^o}{S_{K2}^o} + \lambda_1^o \sigma_1 \right) < 0;
\]

\[
\frac{\tau t}{k_2} = B^{-1} \left( \frac{\lambda_2^o \sigma_1}{S_{K1}^o} + \frac{\lambda_2^o \sigma_1 S_{L1}^o}{S_{K1}^o} + \lambda_2^o \sigma_2 \right) > 0; \text{ and}
\]
These results can be summarized as follows. For small changes, other things remaining the same -

**Result 7.1** Any change in the relative price of the import competing good in the world market is exactly compensated by domestic tariff changes leaving the domestic relative price of the import competing good unchanged.

**Result 7.2** If a sector experiences an exogenous increase in the stock of its specific factor, then the rate of protection awarded to this (growing) sector will decline and the rate of protection awarded to the other sector will rise.

**Result 7.3** An exogenous increase in the supply of the mobile factor (labour) in the economy may lead to a fall or a rise in the rate of protection awarded to a sector depending on the relative ease of factor substitution and factor intensity between the two sectors.

**7.4.1.3 Discussion of the Results**

**Result 7.1** This result implies that if, say, the price of the domestic exportable rises in the world market, ceteris paribus, then either export subsidies will fall or import taxes will increase to such an extent that the increase in the rationalized tariff rate will exactly offset the effect of the world price change. The domestic economy will be fully insulated against terms of trade shocks. No reallocation of resources will take place. Why do we get this result?

Suppose that \( \Pi(P^*_t) \) represent the distribution of rents at free trade, and \( \Pi(P^*_t) \) represent the payoffs at the generalized Nash bargaining equilibrium. Therefore, at the domestic relative price \( P^*_t \) the players' generalized fears of ruin are equal. Suppose further that, other things the same, the price of the home exportable good rises in the world market. Let the new world relative price of the import competing good be \( P^*_t \) such that \( P^*_t < P^*_t \).

At an unchanged tariff rate (policy) the relative price of the import competing good in the domestic market will fall at the same proportional rate as it did in the world market. Let the new domestic relative price be \( P^*_t \). Consequently, PXGEM predicts that the rental income of the import competing sector will fall and that of the exporting sector will rise compared to the initial equilibrium. Thus, the players will slide from the point \( \Pi(P^*_t) \) to the point \( \Pi(P^*_t) \) along the RTF (see figure 7.4). As a result the gain of player 1, relative to the reference point (origin), declines, and that of player 2 increases.
We know that each player's fear of ruin (disagreement) depends directly on the size of the gain relative to the minimum expectation it follows that at $\bar{P}$, player 2 will be more fearful of player 1 declaring disagreement than player 1 fears of player 2 declaring disagreement.\(^9\) To restore equality in players' generalized fear of ruin player 2's payoffs has to be reduced and that of player 1 has to raised by tariff changes. Therefore, in the new sequence of bargaining process prompted by the world price change player 2 will ultimately concede, leading to a higher tariff rate in the new equilibrium.

The new tariff rate will be such that the induced domestic relative price remains unaffected, since the RTF and the level curves of the generalized Nash product are unaffected by changes in the international terms of trade, and therefore, the equilibrium point will also remain unaffected.

**Result 7.2:** The following explanation can be given to the result that an exogenous increase in the stock of the specific factor in the import competing sector would lead to a fall in the rationalized tariff rate and/or a rise in the export subsidy. The remaining results (the effect of an increase in the stock of the specific factor in the exporting sector, and the Result 7.3, that is the effect of an increase in the supply of the mobile factor) can be explained in a similar way.

---

\(^9\) For details see the discussion on point B in the proof of Corollary 6.3.
In order to simplify the diagrammatic exposition, while drawing the following diagram (figure 7.5), we have assumed that production functions are Cobb-Douglas in both sectors.

Suppose that $E_0$, the point of tangency of Nash product $N_0$ to the rent transformation frontier CD, is the initial equilibrium of the tariff game. Suppose further that the capital stock in sector 1 increases exogenously. Then, the RTF will shift upwards to CD. PXGEM predicts that, at an unchanged tariff rate and therefore at an unchanged domestic relative price, the output and rental income of sector 1 will increase and the output and rental income of sector 2 will fall. The point $E_1$ describes the combination of the rental incomes (in economic equilibrium) of the two sectors.

From equation (7.11b) it can be seen that the slope of the RTF depends only on the relative price, since the distributive shares are constant under Cobb-Douglas production functions. If the point $E_1$ on CD and the point $E_0$ on CD correspond to the same relative price, then the slopes of the frontiers at $E_0$ and at $E_1$ respectively should be equal. However, the absolute slope of the Nash product curve at $E_1$ will exceed the slope of the Nash product curve at $E_0$ (by homotheticity\(^{10}\), see the curve labelled $N_1$).

---

\(^{10}\) When the payoffs at the point of minimum expectation are zero for both players, then the generalized Nash product reduces to a Cobb-Douglas type function. So, the generalized Nash product is
Therefore, the curve $N_1$ will not be tangent to the frontier $CD$ at $E_x$. However, their slopes indicate that at $E_x$ player 1’s fear of ruin will exceed player 2’s fear of ruin. Therefore, in the new bargaining process, induced by the shock, player 1 will concede and the tariff rate on imports of commodity 1 will fall, leading to a fall in the relative price of commodity 1 in the domestic market. The new bargained equilibrium will be attained at $E_2$ on $CD$ where both players will be equally fearful of ruin.

The results, though consistent with general intuition in terms of the direction of responses, are very strong. In particular, the result that the domestic economy will be fully insulated against any terms of trade change in the world market can be debated. However, it should be noted that the results are subject to the assumption that the government is expected to be unconstrained with respect to tariff rates, and as a result the point of minimum expectation was the origin. The reference point, thus assumed, was insensitive to changes in the economic environment.

### 7.4.2 Comparative Static Results II: Support Maximizing Government

In this section we will study the comparative static properties of the bargained tariff rate under the assumption that the government is a support maximizer. It has been shown in the appendix to chapter 5 that if the government is a support maximizer, then it will offer a pricing functions that satisfies assumptions (A1) - (A3), and (A5). In particular, assumption (A5) implies that the prices offered by the government always falls between the autarkic and the free trade equilibrium price.

In the following subsections we first modify the model to accommodate the consequence of this assumption on the minimum expectations of the players which is followed by the comparative static result when the world price changes. Details of the other comparative static results are provided in the Appendix- 7A.

#### 7.4.2.1 Modification of the Model

Under the assumptions (A1)- (A3) and (A5), the payoff at the minimum expectation of the import competing sector will be equal to the payoff at the free trade price and the minimum expectation payoff of the exporting sector will be equal to the payoff at the autarkic price. Therefore we write

$$\Pi^d = \Pi^{\min} = (\Pi'^1, \Pi'^2)$$

Clearly homothetic for the same reason as the Cobb-Douglas production function is - that the slope of the level curves at any point depend only on the ratio of the payoffs at that point.
Table 7.3
Linearized version of the PEGEM under Cobb-Douglas production functions and the self-financing assumption

The Political Market:
(7.28') \( D_1(\pi_1^e - \pi_1^e) - D_2(\pi_2^e - \pi_2^e) = y_2^e - y_1^e - p_1^e \).
(7.30') \( \pi_i^e = k_i + r_i^e - p_i^e \).

Economic Markets:
Output supply functions:
(3.36') \( y_j^e = k_j + \left( \frac{S_{ij}^e}{S_{Kj}^e} \right) (p_j^e - w^e); \quad j = 1, 2. \)

Labour demands functions:
(3.37') \( l_j^e = k_j + \frac{1}{S_{Kj}^e} (p_j^e - w^e); \quad j = 1, 2. \)

The labour market equilibrium condition:
(3.38') \( l = \sum_{j=1}^{2} \lambda_j^e l_j^e. \)

Sectoral rental rates:
(3.39') \( r_j^e = \frac{1}{S_{Kj}^e} (p_j^e - S_{ij}^e w^e); \quad j = 1, 2. \)

Price equations:
(3.40') \( p_1^e = p_1^e + \tau^e \tau, \quad \text{and} \)
(3.41') \( p_2^e = 0 \)

To obtain analytical results, which are not possible otherwise, we assume that the production functions in both sectors are Cobb-Douglas (not necessarily identical). This assumption implies that

\[ \sigma_1 = \sigma_2 = 1 \]

and all distributive shares will be constant at all equilibria. In particular, it can be seen from equations (3.28) and (3.29) that \( S_{Li} = \alpha_i \), and \( S_{Ki} = \beta_i \) for each sector \( i \).

With these modifications the first order condition of bargaining equilibrium (7.12) can be written as

\[
\frac{\prod_1 - \Pi_1^*}{\prod_2 - \Pi_2^*} = \frac{\prod_1}{\prod_2} \frac{\Theta_2 Y_2}{\Theta_2 P_1 Y_1}.
\]

Holding bargaining powers constant, equation (7.27) can be linearized and written in terms of percentage changes of the variables as
(7.28) \[ D_1(\pi_1^* - \pi_1^*) - D_2(\pi_2^* - \pi_2^*) = y_2^* - y_1^* - p_1^* \]

where,

(7.29a) \[ D_1 = \frac{\Pi_1^*}{\Pi_1^* - \Pi_1} > 0, \text{ and} \]

(7.29b) \[ D_2 = \frac{\Pi_2^*}{\Pi_2^* - \Pi_2} > 0. \]

Linearizing the equation (7.14), the percentage changes in rental incomes can be expressed in terms of percentage changes in rental rates as follows:

(7.30) \[ \pi_i^* = k_i + r_i^* - p_i^* \]

where it is understood that commodity 2 is the numeraire, and therefore, \( p_2^* = 0 \).

Now the linearized version of the PEGEM can be written as in Table 7.3. Note that the systems of equations in Table 7.3 and 7.2 differ in two respects. First, the elasticities of factor substitution are set to unity in Table 7.3 because we are assuming Cobb-Douglas production functions. Second, the point of minimum expectation in Table 7.3 is not the origin. The system presented in Table 7.3 allows the payoff at the reference point (disagreement payoff) to respond to changes in exogenous variables.

7.4.2.2 World Price Changes and Response of the Tariff Rate

Now we apply a shock to the economy of a small change in the relative price of the import competing good in the international market assuming that the factor endowments remain unchanged.

Since \( k_i = 0 \) for each sector \( i \), it follows from equation (7.30) that

\[ \pi_i^* = r_i^* - p_i^* , \text{ and } \pi_i^* = \pi_i^* - p_i^*. \]

Hence using equations (3.46) and (3A.6) that describe the response of rental functions in PXGEM (chapter 3) we can write

(7.31) \[ \pi_i^* = \frac{p_i^* A_2^* S_{\ell_1}}{A^* S_{K_1} S_{K_2}}, \text{ and } \pi_i^* = \frac{p_i^* A_2^* S_{\ell_1}}{A^* S_{K_1} S_{K_2}}. \]

It has also been shown in chapter 3 that the RTF does not shift with respect to any change in the international terms of trade (or relative price). It simply induces a movement along the RTF. In other words, the shape of the product transformation frontier is unaffected by the terms of trade change in the international market.
Therefore, for given tastes, the autarkic equilibrium is unaffected by changes in the international terms of trade. This implies that

\[ \pi_2^* = 0. \]

From equation (3.47) and equation (7.30) we get

\[ \pi_2^* = r_2^* = -\frac{\lambda_1^s S_{L2} p_1^*}{A^s S_{K1} S_{K2}}. \]  

(7.32)

Similarly, by setting elasticities of factor substitution equal to unity, and holding endowment of factors constant, we obtain from equation (7.20) that

\[ y_2^* - y_1^* = -\frac{1}{A^s S_{K1} S_{K2}} (\lambda_1^c S_{L2} + \lambda_2^c S_{L1}) p_1^*. \]  

(7.33)

Substituting these results that follow from PXGEM into equation (7.28) of linearized PEGEM (Table 7.3) we get

\[ D_1 \left( \frac{\lambda_2^c S_{L1}}{A^s S_{K1} S_{K2}} p_1^* - \frac{\lambda_2^c S_{L1}}{A^s S_{K1} S_{K2}} p_1^* \right) + D_2 \frac{\lambda_2^c S_{L2}}{A^s S_{K1} S_{K2}} p_1^* \]

\[ = -\frac{1}{A^s S_{K1} S_{K2}} (\lambda_1^c S_{L2} + \lambda_2^c S_{L1}) p_1^* - p_1^*. \]

Solving this equation for \( p_1^* \) we obtain

\[ \frac{1}{A^s S_{K1} S_{K2}} (D_1 \lambda_2^c S_{L1} + D_2 \lambda_1^c S_{L2} + 1) p_1^* = D_1 \frac{\lambda_2^c S_{L1}}{A^s S_{K1} S_{K2}} p_1^*. \]  

(7.35)

The coefficients on both sides of equation (7.35) are positive, it follows that

\[ \frac{p_1^*}{p_1} > 0. \]  

(7.36)

Equation (7.36) shows that the relative price of commodity 1 in the domestic market moves with the relative price of commodity 1 in the world market. In particular, it states that if the relative price of the domestic exportable increases in the world market, then the relative price of the exportable in the domestic market will also increase. Equation (7.35) can be solved, for the percentage change in the protection rate, as

\[ \tau = \frac{1}{1 + D_1 \lambda_2^c S_{L1} + D_2 \lambda_1^c S_{L2}} \left[ D_1 A^s S_{L1} \left( \frac{\lambda_2^c}{A^*} - \frac{\lambda_2^c}{A^*} \right) - D_2 \lambda_2^c S_{L2} - 1 \right] p_1^*. \]  

(7.37)
It can be shown that $\lambda_2^* / A^* > \lambda_1^* / A^*$ since $\lambda_2^* > \lambda_1^*$ by proposition 2.1, it can not be said a priori that the tariff rate will fall or rise as the relative price of the import competing good rises in the world market. Thus all the unambiguous results regarding the behaviour of the bargained tariff rate obtained in the previous case disappeared as we allowed the point of minimum expectation to respond to change in the exogenous variables.

The situation becomes still worse as we perform comparative static experiments by changing factor endowments. In these cases even the direction of changes of the domestic relative price appeared ambiguous let alone the response of the bargained tariff rate (see Appendix-7A).

However, it is important to note that equations (7.28) and (7.29) hold for any arbitrary CES production functions provided that the distribution of the bargaining power is unaffected by the shocks. Since, equations (7.28) and (7.29) do not contain the terms representing the players’ relative bargaining powers the solution of the linearized version of PEGEM is independent of the distribution of the bargaining power. This means that the responses of the bargained tariff rate with any arbitrary CES production function are independent of the distribution of bargaining power. The distribution of bargaining power may affect the level of the tariff rate but not the magnitude of response in the tariff rate as the exogenous variables change. This is nonetheless a very remarkable result.

7.5 Summary

In this chapter, we obtained a policy endogenous general equilibrium model (PEGEM) by combining the conditions that characterize the Nash bargaining equilibrium in the political sphere, and the conditions that characterize an equilibrium in the economic sphere of a political economy. By arguing that the payoffs at the point of minimum expectation of the players can be a reasonably good reference point for bargaining in the tariff game, the concept of disagreement was made operational.

The comparative static responses of the bargained tariff rate were studied under two different assumptions on the nature of the government.

First, we considered the case of a coercive government, which had no restraint in choosing the domestic relative price. Consequently, the minimum expectations of both players were zero. The comparative static results, in this case, were conclusive and it was found, in particular, that the bargained tariff rate moved in the opposite direction to exactly offset the change in the world relative price of the domestic import competing good.
When it was assumed that the government is a support maximizer, which maintains some restraint in choosing domestic relative price, the minimum expectations of the player 1 and 2 were given by the free trade and the autarkic equilibrium payoffs respectively. In this case, the minimum expectation payoffs became not only nonzero but also sensitive to changes in the exogenous variables. It was not possible to sign the comparative static responses of the tariff rate as the exogenous variables change. However, as in previous studies, it was found that ceteris paribus the domestic relative price always moves in the same direction as the international relative price moves. A clear-cut answer can be obtained easily by simulating the model numerically.

Therefore, in the next chapter we will simulate the PEGEM with some hypothetical data set that will also show how the model can be implemented.
Appendix-7A: Derivation of Complete Comparative Static Results under a Popular Government and Cobb-Douglas Production Functions

From equation (7.30) we have,

\[
\pi_1^* - \pi_1 = (k_1 + r_1^* - p_1^*) - (k_1 + r_1^* - p_1^*) = (r_1^* - p_1^*) - (r_1^* - p_1^*).
\]

(7A.1)

For a given tariff rate, equation (3.46) can be used to obtain

\[
r_1^* - p_1^* = \frac{\lambda_1^* S_{K1}}{A^o S_{K1} S_{K2}} - \frac{S_{K1}}{A^o S_{K1}} (\lambda_1^* k_1 + \lambda_2^* k_2 - l)
\]

(7A.2)

Symmetrically, we can obtain from equation (3A.6) that

\[
r_1^* - p_1^* = \frac{\lambda_2^* S_{K1}}{A^o S_{K1} S_{K2}} - \frac{S_{K2}}{A^o S_{K1}} (\lambda_2^* k_1 + \lambda_2^* k_2 - l).
\]

(7A.3)

Using equations (7A.2) and (7A.3) into equation (7A.1) we can express \((\pi_1^* - \pi_1^*)\) in terms of percentage change in the relative price, and percentage change in factor quantities.

Similarly, from equation (7.30), noting that commodity 2 is the numeraire in PXGEM, we can also write

\[
(\pi_2^* - \pi_2^*) = (k_2 + r_2^* - p_2^*) - (k_2 + r_2^* - p_2^*) = r_2^* - r_2^*.
\]

(7A.4)

Substituting \(\sigma_1 = \sigma_2 = 1\) in defining equations (3B.11) - (3B.15) and then simplifying the terms in equation (3B.18) yields

\[
r_2^* = S_{L2} (k_2 - l).
\]

(7A.5)

This equation shows that under a Cobb-Douglas production function, the rental income in sector 2 is independent of capital stock in sector 1.\(^{11}\)

Therefore, making use of the result obtained in equation (3.47) together with equation (7A.5) we can write

\(^{11}\) See also the discussion following equation (2B.18) in Appendix 2B. One implication of this result is that the payoff of player 2 in the autarkic equilibrium, which is the minimum expectation of player 2, remains unaffected by changes in the capital stock in sector 1 since, the price of commodity 2 has been held fixed by the choice of the numeraire.
Substituting equation (7A.6) in equation (7A.4) we can express \( (\pi_2^* - \pi_1^*) \) in terms of percentage change in the relative price and percentage change in factor quantities.

Finally, using equation (7.20) we can obtain

\[
(7A.7) \quad y_2^* - y_1^* - p_1^* = \frac{1}{A^o S_{K1}} k_2 - \frac{1}{A^o S_{K2}} k_1 + \left( \frac{S_{K1} - S_{K2}}{A^o S_{K1} S_{K2}} \right) l - \frac{1}{A^o S_{K1} S_{K2}} p_1^*.
\]

Substituting equations (7A.1), (7A.4), and (7A.7) in equation (7.28) of the text we obtain

\[
(7A.8) \quad \left( D_1 \frac{\lambda_2^o S_{L1}}{A^o S_{K1} S_{K2}} + D_2 \frac{\lambda_3^o S_{L2}}{A^o S_{K1} S_{K2}} + \frac{1}{A^o S_{K1} S_{K2}} \right) p_1^* \\
= D_1 \frac{\lambda_2^o S_{L1}}{A^o S_{K1} S_{K2}} p_1^* + \left[ D_1 \frac{S_{L1}}{S_{K1}} \left( \frac{\lambda_2^o}{A^o} - \frac{\lambda_2}{A^o} \right) - \frac{1}{A^o S_{K2}} \right] k_1 \\
+ \left[ D_1 \frac{S_{L1}}{S_{K1}} \left( \frac{\lambda_2^o}{A^o} \right) - D_2 S_{L2} \left( \frac{\lambda_3^o}{A^o S_{K2}} + 1 \right) + \frac{1}{A^o S_{K1}} \right] k_2 \\
+ \left[ D_1 \frac{S_{L1}}{S_{K1}} \left( \frac{1}{A^o} - \frac{1}{A^o} \right) + D_2 S_{L2} \left( \frac{1}{A^o S_{K2}} + 1 \right) + S_{K1} - S_{K2} \right] l.
\]

It is obvious that the coefficients of the variables \( p_1^* \), and \( p_1^* \) are positive, whereas the coefficients of the endowment variables \( k_1 \), \( k_2 \) and \( l \) are ambiguous (we need specific values of the parameters to determine) in their signs. Thus, algebraically, we have the following comparative static results:

(i) \( \frac{p_1^*}{p_1^*} > 0; \)

(ii) \( \frac{p_1^*}{k_1} < 0; \)

(iii) \( \frac{p_1^*}{k_2} < 0; \) and

(iv) \( \frac{p_1^*}{l} < 0. \)
Moreover, as we go to estimate the response of the tariff rate, even the effect of relative price becomes indeterminate. The nice and clear result obtained in the previous case disappears here suggesting a need for a numerical simulation of the model.

Appendix-7B: Minimum Expectation and the Comparative Static Results when Specialization in Labour Employment is Feasible

If specialization in the employment of labour is feasible, then the aspiration level of player 1 is the rental income that can be obtained when all labour in the economy is employed in sector 1. Sector 2 will produce its output employing capital only. Therefore the payoff to the specific factors in sector 2, in this case, will be equal to its output. This represents the minimum expectation of player 2. Similarly, the aspiration level of player 1 is the rental income that it can obtain when all labour in the economy is employed in sector 2. Sector 1 will produce its output employing its capital stock only. Therefore, if this happens, the payoff to the specific factor in sector 1 will be equal to the quantity of output produced in sector 1.

We know that if $L_1 = 0$, then $Y_1 = \beta_1^{-1/\rho_i} K_i$ since the production functions are assumed to be CES in both sectors. Therefore, the minimum expectation of each player $i$ is given by

$$\Pi_i^{\text{min}} = \beta_i^{-1/\rho_i} K_i.$$  

(7B.1)

Now, replacing $\Pi_i^o$ by $\Pi_i^{\text{min}}$ defined by equation (7B.1) and linearizing the PEGEM given in Table 7.1 we can obtain the linearized version of the model as listed in table 7.4. Where, as in equation (7.19), for each player $i$, $D'_i$ is defined as

$$D'_i = \frac{\Pi_i^{\text{min}}}{\Pi_i^o - \Pi_i^{\text{min}}} = \frac{\beta_i^{-1/\rho_i}}{(R_i^o / P_i^o) - \beta_i^{-1/\rho_i}} > 0.$$  

(7B.2)

Comparative Static Results:

Making use of equations (7B.4) and (7B.5) equation (7B.3) can be written as

$$D'_1 (\sigma_1^o - p_1^o) - D'_2 (\sigma_2^o) = y^e_1 - y^e_2 - p^o_i.$$  

(7B.6)

We can obtain from equation (3.46) that

$$r_i^o - p_i^o = \frac{2 \lambda_1^o \sigma_1 S_{i1}}{A_{1} S_{11} S_{21}} p_i^o - \frac{S_{i1}^o}{A_{1} S_{11}^o} \left( \lambda_1^o k_1 + \lambda_2^o k_2 - 1 \right),$$  

(7B.7)

and from equation (3.47) we obtain
Substituting the expressions for \((r^o - p^o)\) from equation (7B.7) and for \(r^o\) from equation (7B.8) into equation (7B.6) we obtain

\[
(7B.9) \quad y^o_2 - y^o_1 - p^o_i = \left( \frac{D_2^o \sigma_2 S^o_{k1}}{A^o S^o_{k1} S^o_{k2}} + \frac{D'_2^o \sigma_1 S^o_{k2}}{A^o S^o_{k1} S^o_{k2}} \right) p^o_i - \left( \frac{D^o S^o_{k1}}{A^o S^o_{k1}} - \frac{D^o S^o_{k2}}{A^o S^o_{k2}} \right) \left( \lambda^o_2 k_1 + \lambda^o_2 k_2 - 1 \right).
\]

Substituting the expression for \((y^o_2 - y^o_1)\) from equation (7.20) into equation (7B.9) and solving for \(p^o_i\) we obtain

\[
(7B.10) \quad p^o_i = \Omega^{-1} \left[ \left( \frac{D^o S^o_{k1}}{A^o S^o_{k1}} - \frac{D^o S^o_{k2}}{A^o S^o_{k2}} \right) \lambda^o_1 - \frac{1}{A^o S^o_{k2}} \left( \lambda^o_2 \sigma_2 + S^o_{k2} \lambda^o_1 \sigma_2 + \lambda^o_1 \sigma_1 S^o_{k2} \right) \right] k_1

+ \Omega^{-1} \left[ \left( \frac{D^o S^o_{k1}}{A^o S^o_{k1}} - \frac{D^o S^o_{k2}}{A^o S^o_{k2}} \right) \lambda^o_2 + \frac{1}{A^o S^o_{k1}} \left( \lambda^o_1 \sigma_1 + S^o_{k1} \lambda^o_2 \sigma_1 + \lambda^o_2 \sigma_2 S^o_{k1} \right) \right] k_2

+ \Omega^{-1} \left[ \frac{1}{A^o} \left( \frac{S^o_{k2}}{S^o_{k2}} \right) - \frac{D^o S^o_{k1}}{A^o S^o_{k1}} - \frac{D^o S^o_{k2}}{A^o S^o_{k2}} \right] l.
\]

where,

\[
(7B.11) \quad \Omega = \left( \frac{D_1^o \sigma_1 S^o_{k1}}{A^o S^o_{k1} S^o_{k2}} + \frac{D'_1^o \sigma_1 S^o_{k2}}{A^o S^o_{k1} S^o_{k2}} \right) + \frac{\sigma_2}{A^o S^o_{k1} S^o_{k2}} \left( \lambda^o S^o_{k2} + \lambda^o S^o_{k1} \right) + 1 > 0.
\]

As in the case of coercive government, it follows from equation (7B.10) that changes in tariff rate will exactly offset any change in terms of trade in the international market. Thus the domestic economy will be insulated from terms of trade shock. This result follows because the variable \(p^o_i\) responds only to domestic endowment changes, but not to changes in the international relative price.

The effects of changes in the endowment variables in the domestic relative price is unclear. The size and direction of effects can only be obtained after the numerical simulation of the model.
Table 7.4
Linearized version of PEGEM:
Specialized labour employment in disagreement

The Political Market

\( (7B.3) \quad D_1'(\pi_1^e - \pi_1^{min}) - D_2'(\pi_2^e - \pi_2^{min}) = y_2^e - y_1^e - p_i^e. \)

\( (7B.4) \quad \pi_i^e = k_i + r_i^e - p_i^e \)

\( (7B.5) \quad \pi_i^{min} = k_i \quad i=1, 2. \)

Economic Markets:

Output supply functions:

\( (3.36') \quad y_j^* = k_j + \sigma_j \left( \frac{S_j^e}{S_{kj}} \right) (p_j^e - w^e); \quad j = 1, 2. \)

Labour demands functions:

\( (3.37') \quad l_j^* = k_j + \sigma_j \frac{S_{kj}}{S_j^e} (p_j^e - w^e); \quad j = 1, 2. \)

The labour market equilibrium condition:

\( (3.38') \quad l = \sum_{j=1}^{2} \lambda_j l_j^*. \)

Sectoral rental rates:

\( (3.39') \quad r_j^* = \frac{1}{S_{kj}^e} (p_j^e - S_{kj}^e w^e); \quad j = 1, 2. \)

Price equations:

\( (3.40') \quad p_1^e = p_1^* + \tau^* t, \quad \text{and} \)

\( (3.41') \quad p_2^e = 0 \)
CHAPTER 8
SOME ILLUSTRATIVE SIMULATIONS OF THE PEGEM

Introduction

The comparative static results presented in the last chapter have indicated that, even in this simple model, if the point of minimum expectation responds to exogenous shocks, then a priori predictions on the behaviour of the bargained tariff rate are not straightforward matters. To be able to do so one has to evaluate complex algebraic expressions involving various shares corresponding to different equilibrium points and the payoffs at the point of minimum expectations. One way to overcome this problem is to solve the PEGEM numerically.

Therefore, the main purpose of this chapter is to implement the computable version of the PEGEM numerically. This implementation serves the following purposes. First, it demonstrates that the PEGEM is operational - that is, PEGEM can be simulated to observe the endogenous behaviour of the policy variables as the economy faces exogenous shocks. Second, it yields some predictions regarding the behaviour of the bargained tariff rate, which can then be compared with the findings and predictions of previous studies.

Though, numerical simulation is useful in obtaining definite answers to the questions of our interest, it is not free from disadvantages. The major disadvantage of this technique is that the responses of the endogenous variables thus obtained are not valid generally. They reflect the behaviour of the endogenous variables under a particular environment, which is determined by the chosen, estimated or the observed values of the parameters and the exogenous variables of the model. If the model is calibrated to the data set of a particular country, then its predictions would be valid only for that country. To retain the flavour of generality we have, therefore, opted to calibrate the model to different sets of hypothetical data and parameter values covering some extreme cases.

In particular, we have considered the equilibrium behaviour of the economy in three different cases. Case (a) depicts the economy as if it were in an environment of almost free trade. This situation is characterized by a very low tariff rate, and a large volume of trade. Case (b) depicts the economy as if it were in an environment of almost autarky. This situation is characterized by a very high tariff rate, and almost no trade with the rest of the world. Case (c) depicts an intermediate case in which the economy is described as if it had a moderate tariff rate and moderate volume of trade with the rest of the world.
Simulation of the model under these three different cases allowed us to examine whether or not the results imply a general pattern in the responses of the bargained tariff rate, irrespective of the nature of the government and the parametric configuration.

In summary, the simulation results displayed a general pattern of the bargained-tariff response with respect to changes in the exogenous variables. Irrespective of the trade regime, the direction of the bargained-tariff response under a popular government followed the same pattern as under a coercive government, as described in the previous chapter. The precise magnitudes of the responses are of course different.

More importantly, it is observed that the behaviour of the bargained tariff rate is consistent with the predictions that follow from the maximization of a conservative social welfare function. This observation has far reaching consequences. First, it shows that the bargain-theoretic approach can be used to obtain a social welfare function that is consistent with self-interested behaviour of the politico-economic agents. Second, if welfare theorists are prepared to accept the assertion that the positive social welfare function is an outcome of a (Nash) bargaining process, then the results presented here may help resolve the existing difference between the schools that believe in the ‘self-interest’ and those that believe in the ‘public-interest’ as a motive force working behind the policy making process.

This chapter is divided into seven sections. The first section describes the simulation model, which is simply the linearized version of the PEGEM. The second section describes the calibration and simulation strategy, linearization errors, and the nature of the basic data sets to be considered to maintain a flavour of generality in the results.

The third section discusses the simulation results in two parts. In the first part, we hold the elasticities of factor substitution at unity, and consider economies at different tariff regimes. Here, we provide estimates of the bargaining powers of the players under the assumption that the observed data set represents the full equilibrium. We provide a detailed discussion of the simulation results by comparing the predictions of the policy-exogenous model with that of the policy-endogenous model. Finally, we explain why the bargained tariff rate responds to the shocks as predicted by the PEGEM. In the second part, we consider the case of an intermediate tariff regime and

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1 For an axiomatic derivation of the Nash social welfare function see Kaneko and Nakamura (1979). For arguments against the use of the Nash social welfare function see (Ng, 1981) and also Yaari (1981). It is, however, noteworthy that all of these studies are limited up to the original version of Nash's solution that does not allow for unequal bargaining powers of the players. Whether the presence of asymmetric bargaining powers can dilute Ng's is a topic for further research.

2 See Martin (1990) for a comparative discussion of both theory and evidence on private and public interest approach in policy formulation.
examine the behaviour of the tariff rate with different production technologies. The testable propositions that follow from these simulation exercises are listed in section four.

In section five, we evaluate the credibility of the hypotheses forwarded by this study against the results of previous studies. It is found that each of the predictions of the PEGEM can find support in previous empirical or theoretical works. In section six we have applied this model to provide an alternate explanation of some of the commonly raised issues such as why developing countries tax agriculture and developed countries subsidize it. Finally, this chapter is concluded in section seven.

8.1 PEGEM: The Simulation Model

The linearized version of the PEGEM in its full form is listed in Table 8.1. This model describes the conditions for equilibrium in the political market as well as in the economic market. The condition for equilibrium in the political market is the condition for a generalized Nash solution to the bargaining problem in the tariff game. The conditions for equilibrium in the economic markets are the conditions that describe the PXGEM. The endogenous economic variables are determined by the economic markets and the policy variable - the tariff rate, is determined by the political market. Full equilibrium is attained when both markets are simultaneously in equilibrium. This occurs when the values of the economic variables determined in the economic market also satisfy the condition for equilibrium in the political market.

The economic markets contain three sub-models of which one is factual and two are counterfactual (see Table 8.1). The first component, which is the factual one, is the basic sub-model that lists the conditions of equilibrium in the observed state of the economic markets. It is simply the linearized version of the PXGEM component of the PEGEM listed in Table 7.1. The set of equations in the basic sub-model differs from the set of equations listed in Table 7.3 or in Table 7.4 (see Appendix 7B) in that it includes the demand side of the economy, namely equations (8.1), (8.3), (8.4), and (8.6), which have been ignored so far. These equations, pertaining to the demand side, can be obtained in the following way.

Linearizing the consumer demand equation (3.19') we can obtain

$$(8.1)\quad c_j^e = -p_j^e + \sum_{i=1}^{2} H_i^e (p_i^e + y_i^e) + H_3^e z^e, \quad j = 12.$$
\( H_i^o = \frac{P_i^o Y_i^o}{\sum_{i=1} P_i^o Y_i^o + Z^o}, \quad i = 1, 2; \) 

\( H_3^o = \frac{Z^o}{\sum_{i=1} P_i^o Y_i^o + Z^o} \) and

\( Z^o \) represents the tariff revenue in the observed equilibrium of the economy. In percentage change form it can be expressed as

\( z^o = p_{1}^* + m_{1}^o + t / T^o \)

where, \( t \) is the change in percentage points of the rationalized tariff rate, \( T^o \), on imports of good 1.

The defining equations (8.2a) and (8.2b) show that the \( H_i^o \) terms are the shares of sectoral value-added and tariff revenue respectively in total income of the national (representative) consumer, which is the sum of the sectoral value-added and the tariff revenue.

To clear the market of good 1 we must have \( C_i^o = Y_i^o + M_i^o \). Expressing this condition in percentage change form we get

\( c_i^o = J_i^o Y_i^o + J_2^o m_i^o \)

where,

\( J_i^o = \frac{P_i^o Y_i^o}{P_1^o C_i^o} \) and

\( J_2^o = \frac{P_2^o M_1^o}{P_1^o C_i^o} \)

are the shares of domestic output and import in the domestic consumption of good 1 respectively.

We have already noted that the trade balance constraint represents the market clearing condition in the foreign exchange market. Expressing the trade balance constraint in percentage change form and noting that good 2 is the numeraire we get

\( m_2^o = p_2^* + m_2^o \).
### The Full Model

**The Political Market:**

(7.28') \( D_1(\pi_1^* - \pi_1^*) - D_2(\pi_2^* - \pi_2^*) = y_2^* - y_1^* - p_1^* \).

(7.30') \( \pi_i^* = k_i + r_i^* - p_i^* \)

**Economic Markets: PXGEM**

**The Basic Sub-model**

(a) **Commodity Markets**

*Output supply functions*

(3.36') \( y_j^* = k_j + \sigma_j \left( \frac{S_{2j}^e}{S_{kj}^e} \right) (p_j^* - w^*) \); \( j = 1, 2 \).

*Consumer demand, and tariff revenue*

(8.1) \( c_j^* = -p_j^* + \sum_{i=1}^{2} H_i^o (p_i^* + y_i^*) + H_j^o z^o \), \( j = 1, 2 \).

(8.3) \( z^o = p_1^* + m_1^* + t / T_1^o \)

*Market clearing equations*

(8.4) \( c_j^* = J_j^o y_j^* + J_j^o m_j^* \)

*(Trade balance constraint)*

(8.6) \( m_2^* = p_2^* + m_2^* \)

(b) **The Labour Market**

*Labour demands functions*

(3.37') \( l_j^* = k_j + \sigma_{kj} \left( \frac{S_{kj}^e}{S_{kj}^e} \right) (p_j^* - w^*) \); \( j = 1, 2 \).

*The labour market equilibrium condition*

(3.38') \( l = \sum_{j=1}^{2} \lambda_j^* l_j^* \).

(c) **Sectoral Rental Rates**

(3.39') \( r_j^* = \frac{1}{S_{kj}^o} (p_j^* - S_j^o w^*) \); \( j = 1, 2 \)

(c) **The Price Equations**

(3.40') \( p_1^* = p_1^* + \tau^o t \), and

(3.41') \( p_2^* = 0 \)
The Free Trade Sub-model

(a) Commodity Markets

Output supply functions

\[(3A.1') \quad y_j^* = k_j + \sigma_j \left( \frac{S_{ij}}{S_{ij}^*} \right) (p_j^* - w^*); \quad j = 1, 2.\]

Consumer demand

\[(8.7) \quad c_j^* = p_j^* + \sum_{i=1}^{2} H_i^* (p_i^* + y_i^*), \quad j = 1, 2.\]

Market clearing conditions

\[(8.8) \quad c_i^* = J_i^* y_i^* + J_2^* m_i^* \]

(Trade balance)

\[(8.9) \quad m_2^* = p_1^* + m_1^* \]

(b) The Labour Market

Labour demands functions

\[(3A.2') \quad l_j^* = k_j + \sigma_j \left( \frac{S_{ij}}{S_{ij}^*} \right) (p_j^* - w^*); \quad j = 1, 2.\]

The labour market equilibrium condition

\[(3A.3') \quad l = \sum_{j=1}^{2} \lambda_j^* l_j^*.\]

(c) The Rental Rates

\[(3A.4') \quad r_j^* = \frac{1}{S_{ij}^*} (p_j^* - S_{ij}^* w^*); \quad j = 1, 2.\]

(d) Price Normalisation Rule

\[(3A.5') \quad p_2^* = 0.\]
Table 8.1 (contd.)

**Autarkic Sub-model:**

(a) **Commodity Markets**

*Output supply functions:*

\[(3B.2) \quad y_j^s = k_j + \sigma_j \left( \frac{S_{ij}^s}{S_{kj}^s} \right) (p_j^s - w^s) ; \quad j = 1, 2.\]

*Commodity demand functions*

\[(3B.3') \quad c_j^s = -p_j^s + \sum_{i=1}^{2} H_i^s (p_i^s + y_i^s) ; \quad j = 1, 2.\]

**Market clearing condition**

\[(3B.7') \quad c_i = y_i.\]

(b) **The Labour Market**

*Labour demands functions*

\[(3B.4') \quad l_j^s = k_j + \sigma_j \left( \frac{S_{ij}^s}{S_{kj}^s} \right) (p_j^s - w^s) ; \quad j = 1, 2.\]

**The labour market equilibrium condition**

\[(3B.5') \quad l = \sum_{j=1}^{2} \lambda_j^s l_j^s.\]

(c) **The Rental Rates**

\[(3B.6') \quad r_j^s = \frac{1}{S_{kj}^s} (p_j^s - S_j^s w^s) ; \quad j = 1, 2.\]

(d) **Price Normalisation Rule**

\[(3B.8') \quad p_2^s = 0.\]
Note that equations (8.4) and (8.6) require that the markets for good 1 and foreign exchange continue to clear after each disturbance to the economy, and the labour market will continue to clear by equation (3.38'). It does not say anything about the market of good 2. It will continue to clear by Walras' Law.

The second component of the full model has been called the free trade sub-model. It describes the equilibrium of the economy under free trade. This set of equations has been obtained from the set listed in Table 3A.1 by adding, as in the main model, equations (8.7) - (8.9) that represent the demand side of the economy. The way these demand side equations have been derived is similar to that in the main model. Note that there are no variables to represent tariff revenue and tariff rate since they do not exist under free trade regime. The purpose of including this component into the full model is to obtain expression for the payoff at the minimum expectation of player 1 (owner of the specific factor in the import competing sector). Thus the free-trade sub-model will yield the changes in the minimum expectation of player 1 as exogenous variables change.

The third component of the full model has been called the autarky sub-model. The equations listed in this group have been copied from Table 3B.2. Note that under autarky the market of each commodity clears domestically. There are no net imports and therefore, whatever the tariff rate, tariff revenue collection is always zero. We have stated the market clearing condition for good 1 and the market of good 2 will clear by Walras' Law. This component of the model yields the behaviour of the endogenous variables if the economy maintains an autarkic regime before and after a given shock. In particular, this sub-model yields the expression for the minimum expectation of player 2 - the exporting sector.

8.2 Some Strategic Considerations

This section contains three subsections. The first subsection outlines the simulation strategy that will be employed in obtaining the counterfactual data sets. The second subsection discusses the linearization errors and consequently examines the accuracy of the counterfactual data sets. In the third subsection we consider the issue concerned with maintaining the flavour of generality of the simulation results while choosing the hypothetical basic data sets.

8.2.1 Simulation Strategy and Calibration of the PEGEM

It can be seen from Table 8.1 that the implementation of the model requires both factual and counterfactual information. Factual information, that includes the elasticities of factor substitution in the two sectors and various quantity and value shares at the
observed state of equilibrium (or 'base year'), is required to calibrate the basic sub-model. Counterfactual information, that includes the quantity and value shares both at the free trade equilibrium and at the autarkic equilibrium, is required to calibrate the free trade sub-model and the autarkic sub-model of the PEGEM. This information is also required to calculate the coefficients $D_1$, and $D_2$ (see equation 7.29). Therefore, the following 3-step strategy has been adopted to generate the counterfactual data sets (see figure 8.1).

In the first step we calibrate the basic sub-model using the base year data set. The quantity and value shares are calculated from the base year data set (for example from the input-output table) under the normalization that the base year domestic prices of both goods are equal to unity. The general nature of the data set required to obtain sufficient information to calibrate the basic sub-model and generic rules to calculate the relevant shares are outlined in the Appendix 8A. We assume that the elasticities of factor substitution are known from extraneous sources and remain constant throughout experiments.

The basic sub-model contains 13 variables in 13 equations. The endogenous variables are the percentage changes of the following variables: 2 sectoral outputs, 2 domestic demands, 2 net import quantities, 2 sectoral employments of labour, 2 rental rates, 1 wage rate, 1 domestic relative price, and 1 tariff revenue. It contains five exogenous variables four of which - 2 stocks of sector specific capital, 1 economy wide supply of labour, 1 relative price of good 1 in the world market - are in percentage change form, and one policy variable - the rationalized tariff rate - is in the form of change in percentage point.

In this model net capital inflow is assumed to be zero. Therefore, the assumptions that goods and foreign exchange markets clear imply that trade remains balanced at the world price. The above division of the variables into endogenous and exogenous variables is sufficient to close the model, and in what follows it is referred to as the natural closure of the basic sub-model. Of course, commodity 2 is the numeraire and therefore the change in its price is set to zero. Once the model has been parameterized (either by calibration or estimation) it can be simulated for any given change in the exogenous variables.
Simulation strategy

1. Estimation of the Disagreement payoffs
2. Calibration of the PEGEM
3. Simulation of the PEGEM for Different Shocks.

Figure 8.1
In step 2 we simulate the basic sub-model with two different closures. In the first simulation, we use the natural closure, and eliminate the observed tariffs completely and evaluate the behaviour of the economy under free trade. For example, if, in the observed equilibrium, the rationalized tariff rate is 25 per cent, then we set the change in percentage points of the rationalized tariff rate, \( \tau = -25 \), holding other exogenous variables constant. Simulation of the basic sub-model yields the effects of this tariff cut on the endogenous variables of the model. The result is then used to update the observed (or base year) data set. The updated data set, then, describes the state of the economy at the free trade equilibrium. In other words, this will depict the state of the economy had it followed the free trade policy instead of following what has been the observed policy. The updated data set is then used to calibrate the free trade sub-model of the PEGEM.

In the second simulation we change the closure of the model. This time we treat the tariff rate as an endogenous variable, and the net import of good 1 as the policy variable. This swapping of the endogenous and exogenous variable does not affect the numbers of the endogenous and the exogenous variables. Holding other exogenous variables constant, we apply an exogenous policy shock of 100 per cent cut in the import of good 1 and simulate the model to obtain its effect on the set of endogenous variables. The result of the simulation yields, among others, a percentage point increase in the rationalized tariff rate that would be consistent with zero imports of good 1. This will also imply a 100 per cent reduction in the export of good 2 since the model maintains a binding trade balance constraint. Thus, we obtain the tariff rate that induces zero trade in both commodities. Updating the base year data set using these results yields the levels of the endogenous variables that describe the economy under autarkic equilibrium. The updated data set is then used to calibrate the autarky sub-model of the PEGEM listed in Table 8.1.

In step 3 we make use of these two counterfactual descriptions of the economy with the observed one to calibrate the full model, which is then used to evaluate the behaviour of the bargained tariff rate as the exogenous variables change.

In the updated (counterfactual) data sets, nominal magnitudes are measured in units of commodity 2, since commodity 2 is the numeraire. The rental payment in sector 2 is the payoff of player 2 under autarky. Therefore, the rental payments of sector 2 under autarky represents the minimum expectation of player 2. To obtain the payoff of player 1 under free trade, which is measured in units of commodity 1, we have to divide the rental payments of sector 1 by the relative price of commodity 1 at free trade equilibrium. The relative price of commodity 1 can be obtained from the updated data set. For example, if, after the elimination of the tariff, the relative price of commodity 1
falls by 10 per cent, then the relative price of commodity 1 at the free trade equilibrium will be 0.90, since it is equal to unity at the observed equilibrium. The parameters D1, and D2 can be estimated by combining the payoffs at the point of minimum expectation with the observed payoffs according to the defining equation (7.29) that completes the calibration of the PEGEM.

8.2.2 Simulation of the Basic Sub-model and Linearization Errors

The system of equations in the basic sub-model is the linearized version of the system of nonlinear equations describing the PXGEM (the economic markets of PEGEM listed in Table 7.1). In other words, the equations listed here (Table 8.1) are the linear approximations to the actual relationships among the model variables. Each curve, describing the actual functional relation, is replaced by its tangent at the point of observed equilibrium. Therefore the solutions obtained from the simulation of the linearized system will show movements along the tangents not along the actual curves thus producing linearization errors.

Linearization errors can be ignored for small changes or shocks. However, if the shocks are large in magnitude producing large changes in the endogenous variables, then the errors due to linearization may be significant.

In both of our simulations, the shocks can be quite large. Therefore, the linear solutions could produce large errors, and the description of the economy under the free trade regime or under the autarkic regime, obtained by following the above procedure, may be inaccurate. Clearly, the nature of the problem warrants the use of a solution procedure that can approximate the exact nonlinear solutions as accurately as desired.

We have used GEMPACK3 version 4.0.2 to simulate the models. This version of the software has the capability to obtain multistep solutions by updating the data base after each step of simulation. For example if we selected a 2-step solution of the model, then it will first divide the shock into two equal parts and performs linear simulation of the model for the first part of the shock. It then updates the base data, recalculates the shares, etc. and recalibrates the model at the new point defined by the updated data set. Then, it simulates the model for the second part of the shock. The basic principle can be understood as a polynomial approximation to a curve. By increasing the number of linear segments (of a given shock) the final solution can be made very close to the actual solution. Moreover, it also provides a solution using the Richardson

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3 The GEMPACK Software System for solving large economic models was developed by the Impact Project, University of Melbourne, Melbourne Australia. Details of user guidelines, syntax and semantic can be found in Pearson and Codsi (1991a, 1991b). In the GEMPACK referencing system the documents are normally identified as GED-30, and GED-31 respectively.
extrapolation\textsuperscript{4} based on the results of 2 or 3 multi-step solutions. The errors due to linear approximations can, therefore, be reduced considerably by increasing the number of steps in the multi-step simulation.

While simulating the basic sub-model, we have used tariff revenue as a criterion to judge the accuracy of the results, since tariff revenue has to be zero at both the free trade equilibrium and the autarkic equilibrium. In each simulation we initially requested the extrapolated solution based on three multi-step - 2-step, 4-step, and 8-step solutions. If the updated data after the simulation did not yield a zero tariff revenue collection, then the numbers of steps were changed to 5-steps, 10-steps, and 20-steps. If the tariff revenue still did not vanish in the updated data set, then the numbers of steps were increased further and so on until the tariff revenue vanished or became approximately zero. The updated data set is, then, checked to ensure that markets clear domestically at the autarkic equilibrium and the trade account balances at the free trade equilibrium. This final check ascertains that the update commands have been correctly specified and executed.

Though our simulation is intended to be illustrative only, we have insisted, in all cases, on extrapolated results.

8.2.3 Preliminary Considerations on Basic Data Sets

Now, we ask what sort of hypothetical data sets would be required to capture the possible extreme responses of the bargained tariff rate? On the basis of the analytical results obtained from the comparative static exercise of the previous chapter, one may suspect that the direction of response of the bargained tariff rate with respect to the exogenous variables can possibly be altered if there are extreme changes in the magnitude of the parameters \(D_1\) and \(D_2\). Particularly, if \(D_1\) is much larger than \(D_2\), then the tariff rate may rise when the price of good 1 rises in the world market, and vice versa. This is equivalent to hypothesizing that the direction of tariff change for a given shock depends on where the economy is on the production (or rent) transformation frontier, since \(D_1\) takes its largest value if the observed equilibrium is very close to the free trade equilibrium and its smallest value if the observed equilibrium is close to the autarkic equilibrium. Similarly, \(D_2\) will take its largest value if the observed equilibrium is very close to the autarkic equilibrium and its smallest value if the observed equilibrium is very close to the free trade equilibrium. Therefore, one may argue that the direction of response of the bargained tariff rate will be different if the

\textsuperscript{4} For details on Richardson extrapolation see Pearson (1991).
observed equilibrium is close to the free trade equilibrium instead of being close to the autarkic equilibrium.

Such an argument may not be valid because it fails to consider the general equilibrium effects of a movement of the observed equilibrium point on other parameters. For example, consider the case with Cobb-Douglas production functions. It can be seen from equation (7A.8) or (7.37) that the effect that a terms of trade loss has on the tariff rate can be positive if $D_1 > D_2$. However, if we consider the case of an observed equilibrium which is close to the free trade equilibrium so that we get $D_1 > D_2$, then we will also obtain the result that each sector's share in total employment of labour will not differ by much from that at the free trade equilibrium. Therefore, the term $(\lambda_2^* / A^* - \lambda_2^0 / A^0)$ in equation (7.37) will decline and tend to zero as the observed equilibrium moves closer towards the free trade equilibrium. Thus, in effect, the decline in the value of the term $(\lambda_2^* / A^* - \lambda_2^0 / A^0)$ may more than offset the increase in the value of $D_1$ preventing the reversal of the direction of the response of the domestic tariff rate with respect to the international terms of trade. Moreover, if the production functions are not characterized by unitary elasticities of factor substitution, then the distributive shares will also change and the exact behaviour of the tariff rate as exogenous variables change remains, nevertheless, an empirical issue.

Thus, it follows that the parameters of concern are the observed tariff rate and the elasticities of factor substitution. The first one positions the observed equilibrium somewhere between free trade and the autarkic equilibrium, and the second one affects the behaviour of the distributive share parameters. Therefore, as an illustration of how this issue can be resolved and also maintain a flavour of some degree of generality we perform numerical simulations in two parts.

In part I we maintain that the production functions are Cobb-Douglas, and simulate the model to observe the behaviour of the tariff rate with respect to the exogenous variables in three different cases: Case (a) depicts the observed equilibrium of the economy as if it was in an environment of *almost free trade*. This situation is characterized by a very low tariff rate, and a large volume of trade. Case (b) depicts the economy as if it was in an environment of *almost autarky*. This situation is characterized by a very high tariff rate, and almost no trade with rest of the world. Case (c) depicts an *intermediate case* in which the economy is described as if it had a moderate tariff rate and a moderate volume of trade with rest of the world. These experiments will allow us to examine the pattern of the endogenous response of the bargained tariff rate under different tariff regimes. The hypothetical data sets used in the simulations are presented in Table 8.2. Such a variation in the tariff regime could have been caused by various reasons, including different distribution of bargaining powers of the players.
In part II, we observe the behaviour of the tariff rate by varying the combination of the elasticities of factor substitution while holding the observed equilibrium at the intermediate tariff regime (case c).

These simulation exercises are not the same as a sensitivity analysis, in its strict sense. They can, nevertheless, be viewed as a weak sensitivity check of the direction of effects.

8.3 Simulations of the PEGEM

8.3.1 Calibration, Simulation, and Discussion of the Results: Part I

The descriptions of the observed equilibrium of three different economies under three different tariff regimes are provided in Table 8.2. The figures in Table 8.2 are values at domestic prices, where the units were chosen so as to make the domestic base-prices of both goods equal to unity. Therefore, the figures can also be regarded as expressions in units of either good, whatever is convenient. From the values of net imports it is apparent that the tariff revenue in cases (a) and (b) is 1 unit of the numeraire, whereas in case (c) it is 5 units of the numeraire. The rationalized tariff rate is 2 per cent in case (a), 100 per cent in case (b) and 25 per cent in case (c), whatever had been the actual rates of tariffs and subsidies.5

In each of the three cases, we followed the three-step simulation strategy, described above, to calibrate the PEGEM. The counterfactual data, which describe the equilibrium under autarky and the equilibrium under free trade, were generated in the first two steps of the simulations of the basic sub-model, and are presented in the top portion of the Tables 8.4-8.6 respectively.

Given the 'observed' equilibrium of the economy, the figures under simulated autarky indicate the state of the economy in equilibrium, if the country adopts autarkic trade policy, which represents the worst outcome to the exporting sector. Similarly, the figures under simulated free trade indicate the state of the equilibrium, if the economy adopts a free trade policy, which represents the worst outcome to the import competing sector. Clearly, these two parts represent the counterfactual information yielding the minimum expectations of the players. To repeat again, these figures were obtained by multi-step simulations of the PXGEM for shocks that eliminate all imports, and tariffs respectively.

These factual and counterfactual data sets describe the bargaining environments fully. In the next section, we will estimate the implied bargaining powers of the players

---

5 See Appendix -8A for methods of calculation.
under the assumption that the ‘observed’ data set also represents the bargaining equilibrium - that is, it corresponds to the full equilibrium of the political economy.

<table>
<thead>
<tr>
<th>Case (a): Almost free trade regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
</tr>
<tr>
<td>Sector 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case (b): Almost autarkic regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
</tr>
<tr>
<td>Sector 2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Case (c): Intermediate regime</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
</tr>
<tr>
<td>Sector 2</td>
</tr>
</tbody>
</table>

8.3.1.1 Estimates of the Bargaining Powers of the Players

Since the ‘observed’ data corresponds to the full equilibrium of the economy, and the minimum expectation of player 1 is obtained at the free trade equilibrium and the minimum expectation of player 2 is obtained at the autarkic equilibrium, the data presented in the first part of each Table 8.4-8.6 should satisfy the necessary and sufficient condition of the generalized Nash solution to the bargaining problem in the tariff game. Rewriting condition (7.12) as

\[
\frac{\Theta_1}{\Theta_2} = \frac{\Pi_2^*}{\Pi_1^*} \left( \frac{\Pi_1^* - \Pi_1^*}{\Pi_2^* - \Pi_2^*} \right) \left( \frac{\sigma_2 Y_2^*}{\sigma_1 P_1^* Y_1^*} \right).
\]

The right hand side of this equation contains known terms, and therefore it can be used to solve for the values of the parameters \(\Theta_1\) and \(\Theta_2\), which measure the bargaining powers of player 1 and player 2 respectively, in the unit interval. The solutions thus obtained yield normalized bargaining powers of the players.

Under Cobb-Douglas production function the elasticities of factor substitution are unity. The values of output (value-added) in both sectors are chosen, for simplicity, to be equal. We can cancel the numerator and the denominator of the last term on the
right hand side of the above equation. The bargaining powers of the players can now be estimated by making use of the data presented on the first part (top portion) of the Tables 8.4-8.6, in each of the three cases as follows.

Thus, it can be seen that the exporting sector holds more bargaining power (approximately 24 times) than the import competing sector in case (a), whereas the import competing sector holds more bargaining power (approximately 28 times) than the exporting sector in case (b). The players seem to hold almost equal bargaining power in case (c). Though the numbers obtained accord well with the notion of bargaining power, the three cases do not necessarily represent the same economy and therefore are not directly comparable.

Table 8.3
The implied bargaining powers of the players in the three cases

<table>
<thead>
<tr>
<th>Θ₁/Θ₂: Bargaining power of player 1 relative to player 2</th>
<th>case(a)</th>
<th>case(b)</th>
<th>case(c)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.04082</td>
<td>28.00</td>
<td>1.07143</td>
<td></td>
</tr>
<tr>
<td>Θ₁: Bargaining power of player 1</td>
<td>0.04</td>
<td>0.97</td>
<td>0.52</td>
</tr>
<tr>
<td>Θ₂: Bargaining power of player 2</td>
<td>0.96</td>
<td>0.03</td>
<td>0.48</td>
</tr>
</tbody>
</table>

Note that these estimates of bargaining powers are not very robust. Granted the validity of the fictitious numbers used in their estimation, the estimates of the bargaining powers are subject to the choice of the production functions and on the way the disagreement payoffs are operationalized. The first issue is clearly empirical, and it is concerned with robust estimation of the production functions. The second issue, however, can be taken up to the conceptual level, and it is concerned with question such as whether the point of minimum expectation is actually the reference point of the players in bargaining or not.

Following Roth (1977) and Thomson (1981), this study argues that the point of minimum expectation can serve as a reasonable reference point to bargaining problems in the tariff game, and the point of the minimum expectation of player 1, who has stakes in the import competing sector only, is the payoff at the free trade equilibrium, and the minimum expectation of player 2, who has stakes in the exporting sector only, is the payoff at the autarkic equilibrium. If the reference point or the disagreement payoff can better be specified in a different way, then the estimates of the bargaining powers can certainly be different from what we have obtained here. For example, if the disagreement payoffs can be shown to be zero, then the bargaining powers implied by
our hypothetical data set under Cobb-Douglas production functions would be equal for each player.

These estimates are, nevertheless, based on a consistent set of assumptions therefore, the approach can provide at least a useful benchmark for future studies.

8.3.1.2 Discussion of the Simulation Results

In the second part of the Tables 8.4 - 8.6 we have presented the comparative static responses of the relevant endogenous variables as predicted by PEGEM and PXGEM as the exogenous variables increase by a one percent. Before we discuss these results, it might be helpful to recall that PXGEM and PEGEM differ in their treatment of the tariff rate. PEGEM determines the tariff rate endogenously, whereas PXGEM treats tariffs as exogenously determined. Therefore, the results obtained from simulating the PXGEM represent the responses of the endogenous variables with respect to the exogenous variables had the tariff rates been held constant at the observed level. The PEGEM allows players to bargain and adopt a new tariff rate, if that is agreeable, as exogenous variables change. Hence, the results obtained by simulating the PEGEM also take into account the endogenous responses of the tariff rate while predicting the responses of other economic variables. Naturally, the predictions of the PXGEM and PEGEM are different.

For example, if the international terms of trade changes by a one percent, then PXGEM has to predict, by definition, that the domestic relative price will also change in the same direction by a one percent, whereas in PEGEM the tariff rate may also change and therefore, the change in the domestic relative price, as predicted by the PEGEM, will not be the same as predicted by the PXGEM. The domestic relative price may change by more or less than a one percent. As a consequence, the other endogenous variables pertaining to the economic market, which are governed by changes in the domestic relative price, will also show a different response under PEGEM compared to the prediction of the PXGEM.

We have presented the predictions of both models for each of the three cases. The results presented under both models are extrapolated results based on 1-step, 2-step, and 4-step simulations, and therefore, are comparable.

The simulation results are discussed in the following three sections. First, we explain only those aspects of the results that are due to the particular structures assumed in the model. These results will not necessarily hold when those particular structures, for example Cobb-Douglas production functions, are replaced by different ones. Then, we compare the responses of the endogenous variables under PXGEM with those of the PEGEM, and explain why the responses of the endogenous variables differ when the
policy variable is also endogenous. Finally, we explain, both intuitively and mechanically, why the policy variable, the bargained tariff rate, changes in the direction as shown by the simulation results.

8.3.1.3 The Consequences of the Assumed Structures

The four columns in the lower part of each of the Tables 8.4 - 8.6 list the response (elasticities) of the endogenous variables in terms of their percentage changes (with two exceptions) over their observed values for a one percent increase in the exogenous variables. The exogenous variables are the relative price of the import competing good in the international market, stock of specific factor in sector 1, stock of specific factor in sector 2, and the economy-wide endowment of labour respectively. The two exceptions are: the variables $t$ and $z$ (not listed in the Table). The variable $t$ measures the change in the percentage point of the rationalized tariff rate and the variable $z$ measures change (instead of percentage change) in tariff revenue collected. It is because the tariff rate and the tariff revenue can sometimes take the value zero (for example at the free trade equilibrium) in which case percentage changes of these variables are not defined.

The following patterns appeared quite distinctly in the simulation results presented under PEGEM in Tables 8.4 - 8.6, which are either simply due to the assumption of Cobb-Douglas production functions or because of the choice of the initial data sets and the small country assumption.

First, in each case and for each shock, we have obtained that the response of sectoral output, $y_i^*$, and the response of the payoff to player $i$, $\pi_i^*$, are identical. This is due to the assumption of Cobb-Douglas production functions, which implies that the factor shares in each sector remain globally constant.

Second, the rental income of player 2 at the autarkic equilibrium has remained constant (that is, its percentage changes are zeroes) on two occasions: when world price changes, and when the stock of capital in sector 1 changes. The first result follows from the independence of the autarkic equilibrium in the domestic market from world price changes. The second result is the consequence of an assumption of Cobb-Douglas production functions (See Appendix-3B) in both sectors.
Table 8.4
Simulation results: Part I
Case (a): Almost free trade

<table>
<thead>
<tr>
<th>Sector 1</th>
<th>Sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>Rent</td>
</tr>
<tr>
<td>Sector 1</td>
<td>70</td>
</tr>
<tr>
<td>Sector 2</td>
<td>40</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector 1</th>
<th>Sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated autarky</td>
<td>Simulated free trade</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector 1</th>
<th>Sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>Rent</td>
</tr>
<tr>
<td>Sector 1</td>
<td>143</td>
</tr>
<tr>
<td>Sector 2</td>
<td>27</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector 1</th>
<th>Sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simulated free trade</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Sector 1</th>
<th>Sector 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage</td>
<td>Rent</td>
</tr>
<tr>
<td>Sector 1</td>
<td>68</td>
</tr>
<tr>
<td>Sector 2</td>
<td>40</td>
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</tbody>
</table>

Simulation results

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
<th>PECEGM: Policy-endogenous General Equilibrium Model</th>
<th>PXGEM: Policy-exogenous General Equilibrium Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_1^*$</td>
<td>$K_1$</td>
<td>$K_2$</td>
</tr>
<tr>
<td>$y_1^*$</td>
<td>0.48</td>
<td>0.44</td>
</tr>
<tr>
<td>$y_2^*$</td>
<td>-0.49</td>
<td>-0.14</td>
</tr>
<tr>
<td>$r_1^*$</td>
<td>1.43</td>
<td>-0.57</td>
</tr>
<tr>
<td>$r_2^*$</td>
<td>-0.49</td>
<td>-0.14</td>
</tr>
<tr>
<td>$w$</td>
<td>0.73</td>
<td>0.22</td>
</tr>
<tr>
<td>$p_1^*$</td>
<td>0.94</td>
<td>-0.02</td>
</tr>
<tr>
<td>$t$ (change in percentage pt.)</td>
<td>-0.06</td>
<td>-0.02</td>
</tr>
<tr>
<td>$\pi_1^*$</td>
<td>0.48</td>
<td>0.44</td>
</tr>
<tr>
<td>$\pi_2^*$</td>
<td>-0.49</td>
<td>-0.14</td>
</tr>
<tr>
<td>$\pi_1^*$</td>
<td>0.52</td>
<td>0.46</td>
</tr>
<tr>
<td>$\pi_2^*$</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

| $y_1^*$              | 0.51                                             | 0.45                                             | -0.31                                           | 0.85                                           |
| $y_2^*$              | -0.52                                            | -0.15                                            | 0.91                                            | 0.24                                           |
| $r_1^*$              | 1.52                                             | -0.54                                            | -0.31                                           | 0.85                                           |
| $r_2^*$              | -0.52                                            | -0.15                                            | -0.09                                           | 0.24                                           |
| $w$                  | 0.78                                             | 0.23                                             | 0.13                                            | -0.36                                          |
| $p_1^*$              | 1.00                                             | 0.00                                             | 0.00                                            | 0.00                                           |
Table 8.5
Simulation results: Part I
Case (b): Almost autarkic

<table>
<thead>
<tr>
<th>The observed data</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>70</td>
<td>30</td>
<td>100</td>
<td>102</td>
<td>2</td>
<td>1.0</td>
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<tr>
<td>Sector 2</td>
<td>40</td>
<td>60</td>
<td>100</td>
<td>99</td>
<td>-1</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulated autarky</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>71</td>
<td>31</td>
<td>102</td>
<td>102</td>
<td>0</td>
<td>1.01</td>
</tr>
<tr>
<td>Sector 2</td>
<td>40</td>
<td>59</td>
<td>99</td>
<td>99</td>
<td>0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulated free trade</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>19</td>
<td>8</td>
<td>27</td>
<td>81</td>
<td>54</td>
<td>0.50</td>
</tr>
<tr>
<td>Sector 2</td>
<td>53</td>
<td>80</td>
<td>133</td>
<td>79</td>
<td>-54</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Simulation Results

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
<th>$P_1^*$</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$L$</th>
</tr>
</thead>
</table>

**PEGEM**: Policy-endogenous General Equilibrium Model

<table>
<thead>
<tr>
<th>$y_1^*$</th>
<th>0.02</th>
<th>0.31</th>
<th>-0.01</th>
<th>0.70</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_2^*$</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0.61</td>
<td>0.39</td>
</tr>
<tr>
<td>$r_1^*$</td>
<td>0.07</td>
<td>-0.97</td>
<td>0.57</td>
<td>0.41</td>
</tr>
<tr>
<td>$r_2^*$</td>
<td>-0.02</td>
<td>-0.01</td>
<td>-0.39</td>
<td>0.39</td>
</tr>
<tr>
<td>$w$</td>
<td>0.04</td>
<td>0.01</td>
<td>0.58</td>
<td>-0.59</td>
</tr>
<tr>
<td>$p_1^*$</td>
<td>0.05</td>
<td>-0.28</td>
<td>0.58</td>
<td>-0.29</td>
</tr>
</tbody>
</table>
| $t$ (change in percentage pt.) | $-1.89$ | $-0.57$ | $1.16$ | $-0.58$

<table>
<thead>
<tr>
<th>$\pi_1^*$</th>
<th>0.02</th>
<th>0.31</th>
<th>-0.01</th>
<th>0.70</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi_2^*$</td>
<td>-0.02</td>
<td>-0.01</td>
<td>0.61</td>
<td>0.39</td>
</tr>
<tr>
<td>$\pi_1^*$</td>
<td>1.37</td>
<td>0.70</td>
<td>-0.82</td>
<td>1.12</td>
</tr>
<tr>
<td>$\pi_2^*$</td>
<td>0.00</td>
<td>0.00</td>
<td>0.59</td>
<td>0.40</td>
</tr>
</tbody>
</table>

**PXGEM**: Policy-exogenous General Equilibrium Model

<table>
<thead>
<tr>
<th>$y_1^*$</th>
<th>0.51</th>
<th>0.45</th>
<th>-0.31</th>
<th>0.85</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_2^*$</td>
<td>-0.52</td>
<td>-0.15</td>
<td>0.91</td>
<td>0.24</td>
</tr>
<tr>
<td>$r_1^*$</td>
<td>1.52</td>
<td>-0.54</td>
<td>-0.31</td>
<td>0.85</td>
</tr>
<tr>
<td>$r_2^*$</td>
<td>-0.52</td>
<td>-0.15</td>
<td>-0.09</td>
<td>0.24</td>
</tr>
<tr>
<td>$w$</td>
<td>0.78</td>
<td>0.23</td>
<td>0.13</td>
<td>-0.36</td>
</tr>
<tr>
<td>$p_1^*$</td>
<td>1.00</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>
Table 8.6
Simulation Results Part I:
Case (c) - Intermediate tariff regime and sector I relatively labour intensive

<table>
<thead>
<tr>
<th>The observed data</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>70</td>
<td>30</td>
<td>100</td>
<td>125</td>
<td>25</td>
<td>1.0</td>
</tr>
<tr>
<td>Sector 2</td>
<td>40</td>
<td>60</td>
<td>100</td>
<td>80</td>
<td>-20</td>
<td>1.0</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th>Simulated autarky</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
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<td>41</td>
<td>137</td>
<td>137</td>
<td>0</td>
<td>1.24</td>
</tr>
<tr>
<td>Sector 2</td>
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<td>53</td>
<td>88</td>
<td>88</td>
<td>0</td>
<td>1.00</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Simulated free trade</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>49</td>
<td>21</td>
<td>70</td>
<td>111</td>
<td>41</td>
<td>0.8</td>
</tr>
<tr>
<td>Sector 2</td>
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<td>67</td>
<td>112</td>
<td>71</td>
<td>-41</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Simulation Results

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
<th>Effects of 1% Increase in</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$P_1^*$</td>
</tr>
</tbody>
</table>

**PEGEM: Policy-endogenous General Equilibrium Model**

<table>
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<td>$w$</td>
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**PXGEM: Policy-exogenous General Equilibrium Model**

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<td>1.52</td>
<td>-0.52</td>
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$y_1^*$, $y_2^*$, $r_1^*$, $r_2^*$, $w$, $p_1^*$, $t$, $\pi_1^*$, $\pi_2^*$, $P_1^*$, $K_1$, $K_2$, $L$
Table 8.7
Simulation Results Part I:
Case (c) - Intermediate tariff regime and sector 1 relatively capital intensive

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<th></th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Cons. Exp.</th>
<th>Net Imp.</th>
<th>Price Level</th>
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**Simulation results**

<table>
<thead>
<tr>
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<tr>
<td><strong>Endogenous Variables</strong></td>
<td></td>
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<tr>
<td><strong>PEGEM: Policy-endogenous General Equilibrium Model</strong></td>
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<tr>
<td><strong>PXGEM: Policy-exogenous General Equilibrium Model</strong></td>
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</table>
Finally, the responses of the endogenous variables with respect to each of the exogenous variables under PXGEM are exactly the same in the three cases. This should be of no surprise because of two reasons: the small country assumption, and the identical supply-side information. The small country assumption links domestic price only with the world price, so the domestic relative price is independent of any change in domestic demand and supply condition. Identical supply-side information means that the two sectors in the three cases employ the same amount of factors, apply the same production technology, produce the same quantities of outputs, and face the same prices. The three cases differ only in the structure of the consumer's demand. Therefore, the responses of the supply side variables have to be the same. This provides a good basis against which the predictions of the PEGEM can be compared.

8.3.1.4 A Comparison between the Predictions of PXGEM and PEGEM

Consider first the shock to the domestic economy, which was initially in almost free trade equilibrium, of a one per cent increase in the relative price of the home importable in the international market - Table 8.4. PXGEM predicts that domestic relative price of good 1 will increase by a one percent, and consequently the output of good 1 will increase by 0.51 per cent, output of good 2 will fall by 0.52 per cent, the rental rate in sector 1 will increase by 1.52 per cent, the rental rate in sector 2 will fall by 0.52 per cent and the wage rate will rise by 0.78 per cent, all measured in units of commodity 2. These results are consistent with the predictions of standard general equilibrium models.

However, the PEGEM predicts that as the relative price of the home importable good rises in the world market by a one percent the protection awarded to the domestic import competing sector will be reduced through a tariff cut of 0.08 percentage points. So, the new tariff rate will be 1.92 per cent ad valorem on imports of commodity 1. The domestic relative price will increase by 0.92 per cent only. Consequently, the responses of other economic variables are reduced accordingly. The output of commodity 1 increases by 0.48 per cent, the output of commodity 2 falls by 0.49 per cent, the rental rate in sector 1 increases by 1.43 per cent, the rental rate in sector 2 falls by 0.49 per cent, the wage rate rises by 0.73 per cent, the payoff of player 1 increases by 0.48 per cent and the payoff of player 2 falls by 0.49 per cent. For each of the variables, the PEGEM has predicted a smaller change (gain or loss) than the PXGEM. This occurs simply because policy (tariff) change has prevented a foreign price change to be transmitted fully into the domestic economy.
Now let us consider the effects of a one percent increase in each of the endowment variables, ceteris paribus, in the case (c), which assumes that the economy was initially in an intermediate equilibrium (Table 8.6).

When $K_1$ increases by a one percent the bargained tariff rate falls by 0.17 percentage points, the gain in sector 1, obtained through capacity expansion, is partly eroded by a decline in the price of its output. The output in sector 1 grows by 0.38 per cent only, which is less than 0.45 per cent as predicted by the PXGEM. The output of sector 2 falls but by less than that predicted by the PXGEM. Similarly, as $K_2$ expands by a one percent the tariff rate increases by 0.35 percentage points, which increases the protection awarded to the import competing sector. As a result, output in the exporting sector increases and the output in the import competing sector falls but by less than that predicted by the PXGEM. A similar interpretation can be given to other figures.

The effects of an increase in the supply of labour in the economy display the effect of the difference of factor intensity between the two sectors. Output of the sector which is labour intensive expands at a faster rate than the output of the sector which is capital intensive.

The simulation results for case (b) are presented in Table 8.5. Note that in this case the economy is in almost autarkic equilibrium, and the tariff rate at the observed equilibrium is 100 per cent and so, the value of $\tau^o$ is 0.5.

When the models were shocked by a one percent increase in the relative price of good 1 in the international market PXGEM predicted that the output of the import competing sector will increase by 0.51 per cent and the output of the exporting sector will decrease by 0.52 per cent.

The predictions of the PEGEM are quite different. It predicts that the tariff rate will fall by 1.89 percentage points. So, the relative price of commodity 1 in the domestic market will increase by 0.05 per cent only. Consequently, the changes in other variables remained very small. The output of sector 1 increased by 0.02 per cent and that of sector 2 decreased by 0.02 per cent and so on.

Similarly, as $K_1$ increased by a one percent, ceteris paribus, PEGEM predicted a decline in the protection awarded to sector 1. The tariff rate fell by 0.57 percentage points and the domestic relative price fell by 0.28 per cent. This price change partially compensated for the loss in the rental income of player 2 arising out of the expansion of sector 1, which otherwise would bid some of the mobile factor away from sector 2. Consequently, the output and the rental income of sector 2 remained almost unaffected by the shock.
As $K_2$ increased by a one percent, ceteris paribus, the tariff rate increased by 1.16 percentage points raising the protection to sector 1. The relative price of commodity 1 increased by 0.58 per cent. Consequently, output in sector 1 did not decline by as much as was predicted by the PXGEM. In fact, it remained almost unchanged. Output in sector 2 increased by 0.61 per cent. The most surprising result is that the rental rate in sector 1 increased by 0.56 per cent! We would expect it to fall as in case (a). The PXGEM predicts that the rental rates in both sectors fall as the capital stock increases in either sector. This unusual result can be explained as follows.

Since, the relative price of commodity 1 increased by 0.58 per cent and the wage rate also increased by 0.58 per cent in the new equilibrium, the real wage faced by sector 1 remained almost unchanged. Output and employment of labour (not shown, but can be inferred from equation (3.37') in Table 8.1) in sector 1 has remained almost unchanged. Under Cobb-Douglas production functions the distributive shares are constant implying that the share of output of sector 1 that goes to the specific factor has not changed either. The relative price of commodity 1 has gone up by 0.57 per cent implies, therefore, that the rental rate and the rental income of sector 1 (in units of commodity 2) should go up by 0.57 per cent keeping the payoff to player 1 (rental income in units of commodity 1) unchanged. This is precisely what has happened.

8.3.1.5 The Exogenous Shocks and the Bargaining Equilibrium

The fundamental question is that why does the tariff rate change in the first place as the exogenous variables change? Once this factor is understood properly the responses of all other variables can be explained as in a conventional policy exogenous CGE model. In terms of the arguments developed so far, the perturbation, as the exogenous variables change, in the equality of the players' generalized fear of ruin holds the key.

More precisely, we have seen (in chapter 6) that the generalized Nash solution to the bargaining problem in the tariff game is characterized by the condition

$$\frac{f_1}{\Theta_1} = \frac{f_2}{\Theta_2}$$

where,

$$\frac{f_1}{f_2} = \frac{\Pi_1^* - \Pi_1^*}{\Pi_2^* - \Pi_2^*} \left( \frac{-1}{d \Pi_1 / d \Pi_2} \right)$$

is the ratio of the two players' fear of ruin (see Appendix 6A), and $\Theta_i$ is the 'exogenous' bargaining power of player $i$. 
Recall that a player's fear of ruin is simply the inverse of his boldness, which is the maximum probability of conflict that the player is prepared to accept for a small gain in the payoff. It has been shown in chapter 6 that a player's boldness declines as his payoff relative to the minimum expectation payoff increases. In other words, a player's fear of ruin (conflict) will increase if either his payoff has increased with unchanged minimum expectation or his payoff at the minimum expectation has fallen at unchanged current payoff level or a combination of both. Intuitively, this means that the more a player has been able to obtain a net gain relative to his minimum expectation the more fearful he will be of conflict with his bargaining opponent. In a Nash bargaining process the player who fears more relative to his bargaining power will reduce his demand. A Nash equilibrium in the bargaining process is attained when both players' fear of ruin are proportional to their bargaining powers.

Now, if an exogenous shock perturbs the distribution of the payoffs without affecting both the minimum expectation payoffs and the bargaining powers of the players, then the player's fear of ruin, who gains from the shock, will increase and the other player's fear of ruin, who loses will fall. A similar reasoning holds when both the current payoffs (after the shock at the unchanged policy) and the payoffs at the point of minimum expectation are affected by the shock. The player's fear of ruin, who has gained more relative to his new minimum expectation, will also increase by more than that of the other player, who has gained less relative to his new minimum expectation payoff.

Let us consider, for simplicity, Cobb-Douglas production functions again. Then, using equation (7.11), which yields the slope of the RTF, we can write

$$\chi \equiv \frac{f_1}{f_2} = \frac{\Pi_1^e - \Pi_1^*}{\Pi_2^e - \Pi_2^*} \left( \frac{S_{x_2}P_{1}^e}{S_{x_1}} \right).$$

Therefore, the generalized Nash solution is obtained when we have

$$\chi = \frac{\Theta_1}{\Theta_2}.$$  

With an unchanged tariff rate after a shock if $\chi > \Theta_1 / \Theta_2$ in the immediate economic equilibrium, then player 1’s generalized fear of ruin exceeds that of player 2 and the political market will be out of equilibrium. Therefore, in the new bargaining process player 1 will concede and the tariff rate will fall. The direction of tariff change will be reversed if $\chi < \Theta_1 / \Theta_2$.

After each shock the PXGEM- updated value of $\chi$ can be obtained as
\[
\chi^* = \frac{\Pi_1^{\circ}(1 + \frac{k_1}{100} + \frac{\pi_1^c}{100} - \frac{p_1^c}{100}) - \Pi_1^{\circ}(1 + \frac{\pi_1^*}{100})}{\Pi_2^{\circ}(1 + \frac{k_2}{100} + \frac{\pi_2^c}{100}) - \Pi_2^{\circ}(1 + \frac{\pi_2^*}{100})} \left( S_{x2} p_1^{\circ} \left(1 + \frac{p_1^c}{100}\right) \right),
\]

where all endogenous variables are as predicted by PXGEM. Note that the PEGEM predictions of \( \pi_1^* \) and \( \pi_2^* \) are the same as that of PXGEM.

By comparing the updated value of \( \chi \) with the relative bargaining power of player 1, in each of the three cases and for each of the shocks, we can predict the direction of change in the bargained tariff rate. For example, in case (a) we have \( \Theta_1 / \Theta_2 = 0.0408 \), and \( \chi^* \) equals 0.0421, 0.0412, 0.0404, and 0.0409 respectively as the world relative price of good 1, stock of specific factor in sector 1, stock of specific factor in sector 2, and the supply of labour increases by a one percent in turn. That is, at the unchanged tariff rate \( \chi^* \) exceeds \( \Theta_1 / \Theta_2 \) as the world relative price of good 1, the stock of the specific factor in sector 1, or the supply of labour in the economy increases by a one percent, and \( \chi^* \) falls short of \( \Theta_1 / \Theta_2 \) as the stock of specific factor in sector 2 increases by a one percent. This, in turn, means that player 1’s generalized fear of ruin relative to that of player 2 increases. Therefore the bargained tariff rate falls, as the world price of commodity 1, the specific factor in sector 1, or the supply of labour in the economy increase, and player 2’s generalized fear of ruin relative to that of player 1 increases. Therefore the bargained tariff rate will thus rise, as the stock of the specific factor in sector 2 increases.

The exact magnitude of change in the bargained tariff rate is determined by parametric configurations and therefore we need PEGEM to evaluate them. One can verify that the PEGEM updated value of \( \chi \), in each case is equal to \( \Theta_1 / \Theta_2 \) - the relative bargaining power of player 1. This implies that at the PEGEM solution the players’ generalized fears of ruin are equalized. Thus, the simulation results demonstrate the overall consistency and the implementation of the PEGEM developed in the previous chapters at least in cases with Cobb-Douglas production functions.

8.3.2 Recalibration, Simulation and Discussion of the Results: Part II

To see the behaviour of the tariff rate under CES production functions we performed two different sets of simulations: one with \( \sigma_1 = 1.5 \) and \( \sigma_2 = 2 \), and the other with \( \sigma_1 = 2 \) and \( \sigma_2 = 1.5 \). The results are presented in Tables 7.7 and 7.8 respectively. In both simulations the initial characterization of the economy was as in case (c). The

6 Note that the numbers given in the Tables have been rounded several times. Therefore, we can see that the PEGEM results satisfy this condition only approximately.
state of the economy at autarky and at the free trade also differ from that obtained in Table 8.6, since the elasticities of factor substitution are different from unity. Therefore, PXGEM was simulated again with new parameter values to obtain the counterfactual information required to calibrate the PEGEM.

The simulation results presented in Tables 8.8 - 8.9 display similar patterns of behaviour of the endogenous variables as was displayed by the results presented in Tables 8.4 - 8.6. These results can be understood by the same set of arguments. The differences, as expected, are that the responses of the payoffs and the sectorial outputs are no longer identical; the minimum expectation of player 2 now responds to changes in the stock of specific factor in sector 1. This is simply the consequence of assuming that the production functions are not Cobb-Douglas.7

The most important observation is that the bargained tariff rate has fallen with an increase in the world relative price of the home importable, the stock of the specific factor in the import competing sector, or the supply of labour in the economy (it is labour intensive), and has risen with an increase in the stock of the specific factor in the exporting sector. The pattern of the responses of the tariff rate has remained intact even when the factor intensities of the sectors were reversed (see Table 8.7) or when the assumption that the production functions are Cobb-Douglas was dropped and the functions were characterized by CES production functions.

Thus, the pattern of the behaviour of the bargained tariff rate is similar to that analytical results obtained in the case of a 'coercive' government (chapter 7). The difference is that the tariff changes in these cases, unlike the case with a 'coercive' government, do not fully insulate the economy from world price changes. This is because the point of minimum expectation responds to the changes in the exogenous variables, and unlike the case with a coercive government the point of minimum expectation does not coincide with the origin. The responsiveness of the point of minimum expectation to the shocks not only changes the bargaining set, but also alters the curvature of the level curves of the generalized Nash product by shifting their asymptotes.

---

7 We have assumed that the long run elasticities of factor substitution are at least unity. The production functions are not Cobb-Douglas implying that the elasticities are greater than unity. Then, it follows from equation (3B.18) and the subsequent discussion in Appendix -3B that the rental income of player 2 will in fact fall as the stock of the specific factor in sector 1 increases.
Table 8.8
Simulation Results Part II:
Case (c) - Intermediate tariff regime and CES production functions with $\sigma_1=1.5, \sigma_2=2$

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<th></th>
<th>Wage</th>
<th>Rent</th>
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<th>Price Level</th>
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<tr>
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Simulation results

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Table 8.9
Simulation Results Part II:
Case (c) - Intermediate tariff regime and
CES production functions with $\sigma_1=2$, $\sigma_2=1.5$

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<tr>
<th></th>
<th>Wage</th>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Sector 1</td>
<td>70</td>
<td>30</td>
<td>100</td>
<td>125</td>
<td>25</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td>Sector 2</td>
<td>40</td>
<td>60</td>
<td>100</td>
<td>80</td>
<td>-20</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td><strong>Simulated autarky</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector 1</td>
<td>96</td>
<td>38</td>
<td>134</td>
<td>134</td>
<td>0</td>
<td>1.18</td>
<td></td>
</tr>
<tr>
<td>Sector 2</td>
<td>31</td>
<td>54</td>
<td>85</td>
<td>85</td>
<td>0</td>
<td>1.00</td>
<td></td>
</tr>
<tr>
<td><strong>Simulated free trade</strong></td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Sector 1</td>
<td>41</td>
<td>21</td>
<td>62</td>
<td>111</td>
<td>49</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>Sector 2</td>
<td>52</td>
<td>68</td>
<td>120</td>
<td>71</td>
<td>-49</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Simulation results

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
<th>$P_1^*$</th>
<th>$K_1$</th>
<th>$K_2$</th>
<th>$L$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>PEGEM</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$y_1^*$</td>
<td>0.39</td>
<td>0.34</td>
<td>-0.11</td>
<td>0.77</td>
</tr>
<tr>
<td>$y_2^*$</td>
<td>-0.39</td>
<td>-0.04</td>
<td>0.71</td>
<td>0.33</td>
</tr>
<tr>
<td>$r_1^*$</td>
<td>0.67</td>
<td>-0.43</td>
<td>0.20</td>
<td>0.22</td>
</tr>
<tr>
<td>$r_2^*$</td>
<td>-0.26</td>
<td>-0.03</td>
<td>-0.19</td>
<td>0.22</td>
</tr>
<tr>
<td>$w$</td>
<td>0.39</td>
<td>0.04</td>
<td>0.29</td>
<td>-0.33</td>
</tr>
<tr>
<td>$p_1^*$</td>
<td>0.47</td>
<td>-0.10</td>
<td>0.26</td>
<td>-0.16</td>
</tr>
<tr>
<td>$t$ (change in percentage pt.)</td>
<td>-0.65</td>
<td>-0.12</td>
<td>0.33</td>
<td>-0.20</td>
</tr>
<tr>
<td>$\pi_1^*$</td>
<td>0.19</td>
<td>0.67</td>
<td>-0.06</td>
<td>0.39</td>
</tr>
<tr>
<td>$\pi_2^*$</td>
<td>-0.26</td>
<td>-0.03</td>
<td>0.81</td>
<td>0.22</td>
</tr>
<tr>
<td>$\pi_1^*$</td>
<td>0.70</td>
<td>0.79</td>
<td>-0.27</td>
<td>0.48</td>
</tr>
<tr>
<td>$\pi_2^*$</td>
<td>0.00</td>
<td>0.03</td>
<td>0.71</td>
<td>0.26</td>
</tr>
</tbody>
</table>
Intuitively the mechanism behind the endogenous response of the tariff rate can be explained as follows. An increase in the world relative price of good 1 not only increases the rental income of player 1 at the unchanged tariff rate but also raises the payoff of player 1 at the free trade equilibrium, which is the minimum expectation of player 1. The minimum expectation of player 2 remains unaffected, since the autarkic equilibrium is unaffected by changes in the world prices. Since an increase in the payoff at the minimum expectation has the effect of reducing a player's fear of ruin, therefore, for a given increase in the world price of good 1, the increase in player 1's fear of ruin under a support maximizing government will be lower than that under a coercive government, whereas player 2's fear of ruin declines by the same amount in both cases. This means that player 1 will have to forgo less, in terms of a tariff cut, to attain a new bargaining equilibrium under a support maximizing government than under a coercive government, in which case the point of minimum expectation remains at the origin irrespective of the shocks. This, in turn, means that endogenous changes in the tariff rate under a support-maximizing government will not be sufficient to offset the world price changes.

We conclude this section with a remark. Since, the simulation results show that the bargained tariff rate, and hence the domestic relative price, responds to changes in the endowment variables even if the bargaining powers of the players are unchanged. This means that the domestic relative price may lie anywhere between the autarky and the free trade price had the endowment variables been configured (or change) appropriately.

Therefore, it is important to note the fact that a player holding more bargaining power than his opponent is neither necessary nor sufficient for the location of the full equilibrium either closer to the free trade equilibrium or closer to the autarkic equilibrium. The location of the full equilibrium point depends not only on the bargaining powers of the players but also on the configuration of the endowment variables as well.

8.4 Some Testable Hypotheses

On the basis of these results we may put forward the following hypotheses or refutable propositions regarding the behaviour of the bargained tariff rate. Ceteris paribus -

(H1) If the international relative price of the home importable falls (rises) in the world market, then its relative price in the home market also falls (rises), but the bargained tariff rate will rise (fall).
(H2) If the stock of the specific factor in the import competing sector increases (decreases) exogenously, then the bargained tariff rate will fall (rise).

(H3) If the stock of the specific factor in the exporting sector increases (decreases), then the bargained tariff rate will rise (fall).

(H4) If the supply of labour (or the mobile factor) in the economy increases (decreases) exogenously, then the bargained tariff rate will fall (rise) provided that the import-competing sector is more labour (capital) intensive compared to the exporting sector. The tariff rate will remain unaffected by changes in the supply of labour if both sectors are equally labour intensive.

Before we go on to apply the model to explain some issues of practical interest it is necessary to validate the model itself. One approach towards the validation of the model is to check whether the hypotheses that follow from the model are consistent with the observed facts. An elaborate statistical test of these hypotheses is beyond our scope. For the present purpose we will simply compare the predictions of PEGEM with the hypotheses, observations, and findings of previous studies.

8.5 Existing Literature and Credibility of the Hypotheses

The first hypothesis is consistent with Hillman's (1982) hypothesis that a declining industry will continue to decline even if there are politically motivated tariffs to retard the rate of decline of such industries. Long and Vousden (1991), in a more general setting than that considered by Hillman, also obtained the result that, though the result depends on the way tariff revenue is distributed among the factor owners, Hillman's hypothesis remains robust provided that the owner of specific factor in the unprotected sector is not significantly less risk averse than the owner of the specific factor in the protected sector.

The remaining hypotheses can be related to the findings of Magee, Brock and Young (1989), who have deduced that any increase in the endowment of a factor in an economy always leads to an increase in the policy favoured by the factor (p. 209). This means that an increase in the stock of the specific factor in sector 1 should lead to an increase in the tariff rate and an increase in the stock of the specific factor in sector 2 should lead to a fall in the tariff rate. The hypotheses (H2) and (H3), in particular, indicate the contrary. This apparent disagreement between Magee, Brock and Young's result and the predictions of the PEGEM can be reconciled by noting that they have not distinguished between the interests of the owners of the specific factors (capital) in the two sectors because they do not have the specific factors. Theirs is a long-run model. Magee, Brock and Young's result is based on the assumption that capital is capital wherever it is employed, which runs against the implication of their three tests of
Stolper-Samuelson Theorem (pp. 101-10; and p. 293-4), where it is shown that both capital and labour favour protection if they are in the import competing sector and favour free trade if they are in the exporting sector. PEGEM is based on this distinction. Therefore the predictions of the two models differ.

Moreover, observing that tariff rates in the US declined over this century, Brock, Magee, and Young (1989) also attempted to correlate this decline with the movements of factor endowments, and the US terms of trade changes. They found that changes in the labour-capital ratio had a statistically insignificant effect on tariff rates, whereas terms of trade changes had the expected sign and significant effect on the tariff rate. However, when the first difference of the labour-capital ratio was employed to explain tariff changes it was found to be significant.

The predictions of PEGEM imply that changes in the capital stock in different sectors have opposing effects on the tariff rate, and therefore, nothing a priori can be said about the effect of a change in the aggregate stock of capital on the tariff rates. For example, if the capital stocks in both sectors increase in proportion to their effects on the tariff rate, then their effects on the tariff rate may cancel each other, and one may observe that the tariff rate did not change even with an increase in the aggregate stock of the capital. Several other possibilities can be conceived in which the movement of the tariff rate may appear independent of changes in the aggregate endowments of the factors. Therefore, we conjecture that if the stock of capital was disaggregated by import competing and exporting sectors, it is likely that we may observe the type of the relationship consistent with the prediction of the PEGEM.

8.6 An Application of the Model and some Additional Hypotheses

Of the four hypotheses, (H2) and (H3) relate the level of domestic protection to the structure of capital formation in the domestic industries. The hypotheses imply that if an exogenous increase in the stock of the specific factor in the import-competing sector is sufficiently large to more than offset the positive effect on the tariff rate of an increase in the stock of capital in the exporting sector, then the country will lower the protection awarded to the import competing sector.

The process of capital formation may take several forms. Protection by itself can be a source of surplus generation in the import competing sector which, attracted by the higher rents under protection, can be reinvested within the sector thereby further increasing the stock of the specific factor in the import competing sector. Moreover, new investors may come in, or the sector may experience specific factor augmenting technical progress.
Table 8.10
The responses of endogenous variables
to an increase of a 200% in $K_1$
(Details of the multi-step and extrapolated simulation results)

<table>
<thead>
<tr>
<th>Endogenous Variables</th>
<th>100-step Results</th>
<th>200-step Results</th>
<th>400-step Results</th>
<th>Extrapolated Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y_1^*$</td>
<td>146.470</td>
<td>146.360</td>
<td>146.314</td>
<td>146.274</td>
</tr>
<tr>
<td>$y_2^*$</td>
<td>-17.7247</td>
<td>-17.6953</td>
<td>-17.6875</td>
<td>-17.6843</td>
</tr>
<tr>
<td>$c_1^*$</td>
<td>77.6859</td>
<td>77.5900</td>
<td>77.1922</td>
<td>76.5611</td>
</tr>
<tr>
<td>$c_2^*$</td>
<td>27.4964</td>
<td>27.4809</td>
<td>27.2405</td>
<td>26.8450</td>
</tr>
<tr>
<td>$m_1^*$</td>
<td>-198.609</td>
<td>-198.400</td>
<td>-197.393</td>
<td>-195.783</td>
</tr>
<tr>
<td>$m_2^*$</td>
<td>-198.609</td>
<td>-198.400</td>
<td>-197.393</td>
<td>-195.783</td>
</tr>
<tr>
<td>$l_1^*$</td>
<td>55.5755</td>
<td>55.4609</td>
<td>55.4240</td>
<td>55.4007</td>
</tr>
<tr>
<td>$l_2^*$</td>
<td>-27.7752</td>
<td>-27.7241</td>
<td>-27.7088</td>
<td>-27.7002</td>
</tr>
<tr>
<td>$r_1^*$</td>
<td>-41.3285</td>
<td>-41.1904</td>
<td>-41.1105</td>
<td>-41.0235</td>
</tr>
<tr>
<td>$r_2^*$</td>
<td>-17.7247</td>
<td>-17.6953</td>
<td>-17.6875</td>
<td>-17.6843</td>
</tr>
<tr>
<td>$w$</td>
<td>13.8654</td>
<td>13.8508</td>
<td>13.8499</td>
<td>13.8531</td>
</tr>
<tr>
<td>$z$ (change)</td>
<td>-2.95317</td>
<td>-2.91209</td>
<td>-3.39410</td>
<td>-4.21115</td>
</tr>
<tr>
<td>$t$ (change)</td>
<td>-35.4785</td>
<td>-35.3554</td>
<td>-35.2807</td>
<td>-35.1972</td>
</tr>
<tr>
<td>$\pi_1^*$</td>
<td>146.470</td>
<td>146.360</td>
<td>146.314</td>
<td>146.274</td>
</tr>
<tr>
<td>$\pi_2^*$</td>
<td>-17.7247</td>
<td>-17.6953</td>
<td>-17.6875</td>
<td>-17.6843</td>
</tr>
<tr>
<td>$\pi_1^*$</td>
<td>174.082</td>
<td>173.968</td>
<td>173.911</td>
<td>173.853</td>
</tr>
<tr>
<td>$\pi_2^*$</td>
<td>2.19E-07</td>
<td>.000000</td>
<td>.000000</td>
<td>7.30E-08</td>
</tr>
</tbody>
</table>
Table 8.11
Effects of a 200% increase in $K_1$ on the levels of the main variables
(Values are in units of commodity 2)

<table>
<thead>
<tr>
<th>Sector</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp. Exp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>30</td>
<td>70</td>
<td>100</td>
<td>125</td>
<td>25</td>
<td>1.0</td>
</tr>
<tr>
<td>Sector 2</td>
<td>60</td>
<td>40</td>
<td>100</td>
<td>80</td>
<td>-20</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The Economy After the Shock:

<table>
<thead>
<tr>
<th>Sector</th>
<th>Wage</th>
<th>Rent</th>
<th>Value-added</th>
<th>Con. Exp.</th>
<th>Net Imp. Exp.</th>
<th>Price Level</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1</td>
<td>53.08</td>
<td>123.84</td>
<td>176.92</td>
<td>158.55</td>
<td>-18.37</td>
<td>0.72</td>
</tr>
<tr>
<td>Sector 2</td>
<td>49.39</td>
<td>32.93</td>
<td>82.31</td>
<td>101.47</td>
<td>19.16</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Assume that the import competing sector is more capital intensive than the exporting sector. Assume further that the initial equilibrium of the economy is as given in Table 8.7. We may now ask what happens to the bargained tariff rate if sector 1 experiences a sufficiently high growth in its stock of the specific factor - say, by a 200 per cent? We can use PEGEM to answer this question. It also provides a ground to compare two otherwise similar economies that differ only in the stock of capital in sector 1.

We used the factual and counterfactual data from Table 8.7, and simulated the PEGEM to obtain the result shown in Tables 8.10 and 8.11.

Since the size of the shock was very large a convergence problem was encountered. The numbers of steps were increased to 100, 200 and 400. The three multistep solutions and the extrapolated solution of the model are presented in Table 8.10, and the updates of the base data (based on the extrapolated solution) are given in Table 8.11.

The results shown in Table 8.10 display the same feature as was seen in the previous results: the output and employment of labour in sector 1 expand while those in sector 2 contract; rental rates in both sectors fall; wage rate increases; tariff rate falls by 35 percentage points, relative price of commodity 1 falls; and so on. The percentage change in sectoral output is identical with the percentage change in the payoff of the respective player, and the minimum expectation of player 2 has remained unaffected - both consequences of the assumption of Cobb-Douglas production functions.\(^8\)

\(^8\) However, some of the variables are not showing the sign of convergence. Convergence requires that the difference between 400-step and 200-step solution be less than the difference between 200-step and 100-step solution. Tariff revenue, consumer demand, and quantities of net imports have not satisfied this criterion. Out of these three nonconverging variables, the source of the problem is the tariff revenue
The updated data base (Table 8.11) shows that commodity 1 will be exported in the new equilibrium instead of being imported, and the exports of commodity 1 will be taxed at the rate of 10 per cent ad valorem. Initially, the imports of commodity 1 were being taxed at the rate of 25 per cent ad valorem. On the basis of this exercise we draw the following additional hypotheses:

A sufficient increase in the stock of a specific factor in the import competing sector will lead to

(H5) A reversal in the direction of trade: commodity 1 will be exported and commodity 2 will be imported.

During the process of the reversal in the direction of trade, sector 2 contracts and sector 1 expands in terms of output and employment. This result is consistent with the empirical findings of Martin and Warr (1992), who observed that capital accumulation and technical change, biased against agriculture, have been the most important determinants of the decline in agriculture's share of GDP in Thailand. In case of Indonesia, capital accumulation has been the principal cause of decline in agriculture's share of GDP (Warr, 1991).

Martin and Warr have explained this phenomenon by invoking the Rybczynski effect, which implies that an increase in the stock of capital would lead to an increase in the output of capital intensive sector (non-agriculture) and a decline in the output of labour intensive sector (agriculture). The mechanism of this effect involves a movement of both capital and labour away from agriculture to the non-agriculture sector, which is more capital intensive than agriculture, and therefore this is equivalent to an increase in the stock of capital in non-agriculture sector and a decline in the stock of capital in agricultural sector. Both changes, in PEGEM, imply a decline of agriculture and expansion of non-agriculture sector.

(H6) A reversal in the direction of tariff protection: exports of commodity 1 or the imports of commodity 2 will be taxed or the production of commodity 2 will be subsidized.

Initial tariff policy implied a tax on imports of commodity 1 or a tax on the exports of commodity 2 or a subsidy in the production of commodity 1. Thus change variable. It is fluctuating, thereby affecting the consumer's income which in turn has affected the consumer demand and consequently, the quantities of net imports. Since the size of the error seems very small, we decided not to increase the number of steps any further.
PEGEM can possibly provide some additional explanation as to why developing countries tax agriculture and subsidize manufacturing, and developed countries subsidize agriculture and tax manufactures (Anderson and Hayami, 1986; Honma and Hayami, 1986; Krueger, Schiff and Valdes, 1988).

(H7) If a country continues to 'over-invest' in the import-competing sector relative to the exporting sector, then over time it will adopt a policy of unilateral trade liberalization. In other words, trade liberalization will eventually turn out to be the dominant strategy for a highly protective economy.

This hypothesis appears to be consistent with Drysdale and Garnaut's (1992) observation that recent trade liberalization in the Western Pacific countries has been mostly non-discriminatory and unilateral. They argue that the 'observation of the highly beneficial effect of one country's liberalization on its own trade expansion, has led each Western Pacific economy to calculate that, whatever the policies of others, it will benefit more from keeping its own borders open to trade than from protection' (p.5).

(H8) Over a long period, some countries may display tariff cycles.

The base of the rationalized tariff changes as the capital stock in the import competing sector keeps on increasing, and the tariff rate on the imports of previously exported commodity (commodity 2) starts rising. This provides a leverage to sector 2 through protection, and the capital stock in sector 2 may start to accumulate. The tax on exports of good 1 or on imports of good 2 will start to fall as capital accumulation in sector 2 takes momentum, and hence the tariff cycle.

The approach underlying the construction of the PEGEM may help explain the existence of tariff cycles. The current model is too crude to be able to provide a complete description of the tariff cycles. A more detailed explanation can be found in Cassing, McKeown, and Ochs (1986), who, by assuming that players have spatially concentrated asset distribution and markets in such assets are incomplete, have been able to show not only the existence of tariff cycles but also that tariff cycles match the pattern of the business cycles. Some of the empirical evidence cited in their paper are not inconsistent with our hypothesis, however.

(H9) For given technologies of production, tastes of the domestic consumer, and the total supply of the mobile factor (labour), there is a unique configuration of the stocks of sector specific-factors that will yield free trade as the generalized Nash solution to the bargaining problem in the tariff game. In other words, any
imbalances among factor endowments, tastes and technologies will lead to a positive tax on either domestic exports or on imports, whichever way it is viewed.

Though this hypothesis follows from (H2), it is not easy to check its empirical validity because of its nonrefutable character. However, if this hypothesis is correct, then one may expect to find almost every country with a positive tax on trade, since it will be almost impossible to maintain the equilibrium configuration of the factor endowments.

8.7 Summary

In this chapter, a computable version of the PEGEM was developed and some illustrative simulations were performed on hypothetical data sets. The results demonstrate that the bargain-theoretic approach can be used to construct a policy-endogenous general equilibrium model and that the behaviour of the endogenous policies can be predicted as the economic and political environments change.

The numbers that have been used in simulating the model are all hypothetical. Therefore, the magnitudes of the elasticities are not meaningful either. However, the simulation results have shown some degree of consistency in the direction of change of the response of the bargained tariff rate with respect to changes in the exogenous variables. They were similar to the directions that were obtained analytically in the previous chapter for a special case of a ‘coercive’ government. These results prompted a number of hypotheses regarding the behaviour of the tariff rate.

All of the above hypotheses, implied by the PEGEM, very closely paralleled the predictions of the approach that maximizes a conservative social welfare function (Corden, 1974), in which increases in income are given relatively low weights and decreases very high weights (Corden, 1974: p.107). At the heart of this function lies the idea that any significant reduction in real incomes of any significant section of the community should be avoided (Corden, 1974: p.107). A number of reasons such as fairness, social insurance, and avoidance of social and political conflict have been forwarded in defence of this approach.

For example, if the relative price of the import competing good falls in the world market, then the tariff rate increases by (H1) implying that the policy change compensates, at least partly, for the loss in the real income of player 1. Similarly, the response of the tariff rate implied by (H2) - (H4) can also be predicted by conservative social welfare theorists on grounds of fairness, and so on.
The predictions of the PEGEM, however, were not derived by maximizing a conservative social welfare function. They are the outcomes of bargaining between the two players, where the government is viewed as a self-interested agent, which maximizes its political support by implementing the agreement reached by the two players.

Hillman (1982) has suggested a point of difference between the two approaches. He argued that the conservative social welfare maximizing approach implies that policy change ought to be directed at arresting industry decline, whereas (H1) or a politically motivated behaviour of the government implies that a declining industry will continue to decline. However, as we saw in chapter 7 if the government is 'coercive' in nature, then the policy supplied by the government will meet his criteria of maximizing the conservative social welfare function, which is also the prediction of Stigler-Peltzman model (see Hillman, 1982) that policy changes will fully compensate terms of trade changes.

Therefore, it follows that there are problems in differentiating a conservative social welfare-maximizing government, and a self-interested, politically motivated, support-maximizing government. This is true in the case of a 'coercive' government. It is impossible to characterize a government simply by observing the policies chosen by it. Some further method needs to be devised. However, as argued by Posner (1974) and Vousden (1990), the social welfare-maximizing approach lacks explanation on how such functions are formed and translated into legislation, whereas the bargain-theoretic approach, while being able to predict as much, does not suffer from this criticism.

However, if the conservative social welfare function is considered as an outcome of a (Nash) bargaining process, then this study can be used to reconcile the difference between the conservative social welfare maximizing approach and the political support maximizing approach to the policy determination.

In the next chapter, this work will be summarized and the limitations will be discussed.
Appendix-8A: Generic calculation of share parameters

Table 8A.1
The Base year data set

<table>
<thead>
<tr>
<th>Sector</th>
<th>Wage</th>
<th>Rent</th>
<th>Value Added</th>
<th>Consumption Expenditure</th>
<th>Value of Net Import</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>WL1</td>
<td>R1 K1</td>
<td>P1 Y1</td>
<td>P1 C1</td>
<td>P1 M1</td>
</tr>
<tr>
<td>2</td>
<td>WL2</td>
<td>R2 K2</td>
<td>P2 Y2</td>
<td>P2 C2</td>
<td>P2 M2</td>
</tr>
</tbody>
</table>

These data should satisfy the following restriction:

(a) Zero profit condition - that is for each \( i \) we must have

\[ W_{Li} + R_i K_i = P_i Y_i \]

(b) Market clearing conditions - that is for each \( i \) we must have

\[ P_i C_i = P_i Y_i + P_i M_i. \]

It is clear that conditions (a) and (b) together imply that the aggregate budget constraint of the national consumer holds.

From this data set, we can make the following calculations:

1. Tariff revenue equals the sum of the values of net imports at domestic price. That is

\[ Z = P_1 M_1 + P_2 M_2 \]

by the condition that trade balances at world prices.

2. The rationalized tariff rate

\[ T_1 = - Z / P_2 M_2 \]

3. Employment shares:

\[ \lambda_i = \frac{W_{Li}}{W_{L1} + W_{L2}} \]

by the assumption of perfect mobility and homogeneity of labour.

4. Distributive shares:

\[ S_{Li} = WL_i / P_i Y_i \] and
\[ S_{ki} = \frac{R_i K_i}{P_i Y_i} \]

because zero profit condition holds.

(5) Tariff coefficient

\[ \tau = \frac{1}{1 + T_i} . \]

The share parameters H's and J's can be calculated by using the defining equations (8.2) and (8.5) respectively.
CHAPTER 9
SUMMARY, CONCLUSION AND LIMITATIONS

The major concern of this study has been to provide a coherent analytical framework which can explain the process of the formation of a government’s redistributive policies. Tariff policy has been chosen as the specific subject matter of this study because, among other things, it provides both a sharp focus on the issue of redistribution, and a methodological basis developed from existing literature on endogenous tariff. Consequently, the immediate questions addressed by this study have been how the tariff rates in a political economy are determined and how they respond to exogenous shocks.

Thus, this study explained the process of endogenous tariff formation while, at the same time, illustrating the formation of a government’s redistributive policies. In so doing the study adopted a political economy approach to policy formation and assumed that the political process admits cooperative behaviour among the conflicting interest groups. The logical sequence involved in deriving a general equilibrium model of a political economy that determines the tariff rate endogenously can be summarized as follows.

9.1 Summary

The literature concerned with the endogenous determination of the tariffs was reviewed in chapter 2 in order to obtain some guidelines for modelling the process of tariff formation. Basically, two competing approaches emerged, yielding similar predictions. One approach is based on the assumptions that there exists a social welfare function of some kind (conservative or other), and that the government is a social welfare maximizer. Thus, in effect, this approach assumes the maximization of social welfare to be the process that determines a government’s policies. The other approach is based on the assumption that a government is a political support maximizer. This approach puts forward a model of complex strategic interactions between government and the conflicting private interest groups in the political sphere in the pursuit of self-interest as the process of determining government policies. In this model, each of the private interest groups behaves strategically in mobilizing political support for the government so as to enhance their own welfare in the economic sphere; the government behaves strategically in supplying policies affecting the economic sphere, in particular the welfare of the interest groups, so as to maximize political support for itself in the political sphere.
The first approach suffers from two major problems: (i) how to ascertain the existence and uniqueness of a social welfare function; and (ii) the logical problem of assuming that the government is a benevolent agent, which does not suffer from the classic principal-agent problem, and that the other individuals are self-interested agents. The second approach views all agents in the political economy as self-interested and attempts to obtain policies as a device of conflict resolution. This approach does not suffer from the above two problems. Thus, the review clearly indicated that the (tariff) policy formation process could be coherently modelled if one adopts the second approach rather than the first.

Though the political economy approach did provide a consistent analytical framework within which the policy formation process could be modelled, the existing literature subscribing to this view has nevertheless left several fundamental questions unresolved. In particular, the existence of an equilibrium in a productive political economy was an open question. Logically, it is necessary that the existence of an equilibrium in a political economy be established prior to the analysis of its equilibrium behaviour. The question of the existence of an equilibrium in a political economy had therefore to be addressed before the political economy approach could be employed to address the questions of endogenous determination of the tariff rate and its comparative static behaviour. Hence, this study developed a general equilibrium model of a political economy by integrating a model of the political sphere with a model of the underlying economic sphere, and addressed the question of the existence of an equilibrium in such an economy. Subsequently, the questions concerned with the formation of tariff policy were addressed in their logical order.

The modelling work of this study began in chapter 3, where a Ricardo-Viner type simple 2-sector general equilibrium model of the economic sphere of a political economy was described, assuming that the outcome of the political sphere, that is the policies of the government (tariff rates), is exogenously given. The comparative static properties of the endogenous variables in the economic sphere were obtained, and on the basis of those results some general properties of the solution functions of the general equilibrium model of the economic sphere were deduced.

In chapter 4, the results obtained in chapter 3 were employed to obtain some further results that were critical to the later chapters. First, it was shown that if in each sector (i) the output supply function is concave in own relative price and (ii) the elasticity of factor substitution is at least unity, then the real rental functions are increasing and concave in own relative prices. Second, the rent transformation frontier, which describes the Pareto efficient distribution of rental incomes between the two sectors, was derived and its comparative static properties were analysed. The rent
transformation frontier summarized the general equilibrium effects of tariff changes on
the sectoral rental incomes.

The first order property of the rent transformation frontier showed that the two
owners of the specific factors (or simply sectors) have opposing interest in the
government's tariff policy. The second order property of the frontier showed that the
real-rent possibility set is convex provided that the elasticities of factor substitution are
at least unity. It is important, however, to note that these results are based on the
assumption that the production functions in both sectors are characterized by constant
returns to scale and CES. As far as CGE models are concerned this assumption is fairly
common. Chapters 3 and 4 together completed the description of the economic sphere.

Chapter 5 described the political process of tariff determination as a
noncooperative game and studied the existence of a Nash equilibrium in the political
economy in two steps.

In the first step, as in previous studies, it was assumed that the government
offers a lobbying sensitive pricing function, which satisfies some reasonable properties,
to the conflicting interest groups. Each player was assumed to choose his or her own
lobbying effort to maximize his or her real rental income subject to the general
equilibrium of the economic sphere by taking the lobbying effort of the other player as
given. For a given pricing function offered by the government, then, this study proved
the existence of a Nash equilibrium in the tariff game by analysing the Nash behaviour
of the two players. It also considered the case where, in offering a pricing function, the
government is constrained by the budgetary implications of that pricing function (in that
it could not afford a net subsidy on foreign trade while supporting the equilibrium
price). In this case, although free trade was feasible, a Nash equilibrium necessarily
implied a positive tax on trade.

In the second step, it was demonstrated that the assumed properties of the
government's pricing function are sufficient to guarantee the maximum of a Peltzman-
type political support function of the government at each Nash equilibrium of the tariff
game. These results in effect established that there exists an equilibrium in the political
economy, where the government behaves as a Stackelberg leader in maximizing its
political support, and the private interest groups behave as Nash followers in lobbying
the government in order to maximize their real rental income.

Another interesting aspect of this result is that it showed that the two strands of
the existing political economy approach to endogenous tariff theory, which have either
considered support-maximizing behaviour of the government without considering the
reactions of the lobbyists, or considered the lobbying equilibrium for a given pricing
function without showing how such function was obtained, are mutually compatible. They imply the same policy-equilibrium if the political economy admits a unique equilibrium.

This noncooperative Stackelberg-Nash equilibrium describes the frontier of the existing endogenous tariff literature. The formal demonstration of the results derived in chapter 5, though conjectured previously by various authors, are new.

Though the political economy thus described in terms of a noncooperative game was capable of integrating the political sphere with the underlying economic sphere, it was not yet sufficient to capture a complete description of the political process that would lead to an equilibrium in a political economy. In particular, it did not allow for the possibility that the interest groups may communicate and negotiate with each other in order to obtain a cooperative solution that will Pareto dominate the noncooperative outcome in the tariff game. Since a cooperative solution will be the one agreed by the two conflicting parties, the government will maximize its political support by enforcing it. There were no compelling reasons to continue assuming that the political sphere is characterized by noncooperation.

Therefore, in chapter 6, this study proceeded further and allowed the interest groups to search for a cooperative solution if it was individually rational, and from that chapter onwards the political process of tariff determination has been viewed as a bargaining problem between the two conflicting interest groups.

The bargaining problem in the tariff game was formally defined in chapter 6 assuming that the disagreement payoffs are known a priori. The rent transformation frontier, derived in chapter 4, was employed to define the bargaining set. The results obtained in chapters 3 and 4 guarantee that the bargaining set is compact and convex. The bargaining problem thus defined satisfied the conditions required for the existence and uniqueness of a Nash solution to the bargaining problem set out by earlier studies. The sufficient condition for a unique Nash bargaining solution is that the elasticities of factor substitution be at least unity. On the other hand, if the polity does not admit cooperative behaviour, then the sufficient conditions for the existence of a noncooperative Nash equilibrium are (i) the elasticities of factor substitution be at least unity and (ii) the output supply functions in each sector be strictly increasing and concave in own-relative-price. Thus, the question of whether an equilibrium in a political economy exists is fully answered. This study has shown that an equilibrium exists in a political economy regardless of whether the political sphere admits a cooperative behaviour or not. These results allowed us to characterize a general equilibrium in a political economy and study its properties. Therefore, we proceeded to
obtain the necessary and sufficient condition for a unique Nash bargaining solution in
the tariff game in order to address the remaining questions.

The original Nash solution to any arbitrary bargaining game, and issues related
to its generalization in the presence of asymmetric bargaining powers were discussed in
considerable detail which did yield some useful results. First, it was shown that if the
distribution of bargaining power is included in the mathematical description of the
bargaining game, then the Nash solution with asymmetric bargaining power is also
symmetric. Second, it was shown that an alternate necessary and sufficient conditions
for the generalized Nash solution to a bargaining problem are that (i) the players’ fear
of ruin be strictly positive, and (ii) the players’ fear of ruin relative to their bargaining
power be equal. This is a new and interesting characterization of the Nash bargaining
solution. In particular, this result implies that a player who expects to gain clearly in the
economic sphere under a changed circumstance (after a shock) will concede in the
political market. He will offer a concession to the opponent by agreeing on policy
adjustments. It is because he will be more fearful of ruin (disagreement) relative to the
other player who does not expect to gain by as much. This result is important not only
from the point of view of the endogenous tariff theory but also from the point of view
of the Nash bargaining theory. This is because the result provides an intuitive
explanation of the Nash bargaining process. It has been employed to explain the results
obtained in subsequent chapters. Thus, in summary, chapter 6 provided the condition
characterizing the bargaining equilibrium and a mechanism to summarize the
bargaining process, which explains the directions of movement of the bargaining
equilibrium in response to exogenous shocks.

In chapter 7, the condition characterizing the Nash bargaining solution was
combined with the conditions of general equilibrium in the economic sphere, described
in chapter 3, and a policy-endogenous general equilibrium model of the political
economy was obtained. The bargained tariff rate, which depended on the entire politico-
economic environment, was obtained as the solution of the model. This completed the
construction of a policy-endogenous general equilibrium model in which the tariff rate
is the only policy variable of the government.

However, the condition of a bargaining equilibrium contained terms
representing the disagreement payoffs, which were not yet identified. The problem of
identification of the disagreement payoffs was resolved by adopting the reference point
solution concept. It was further argued that the payoffs at the point of players’
minimum expectation in the tariff game could serve as a reference point during the
bargaining process. Minimum expectation payoffs were identified for two different
types of government namely, coercive and popular. The coercive government,
introduced for the sake of reference, was defined to be one that can be captured by the
winning player and is prepared to subsidize trade to benefit the winning player. Players' minimum expectation under such a government was shown to be always zero. A popular government, on the other hand, was characterized by support-maximizing behaviour with self-financing policy constraint. Under such a government, the minimum expectation of the owner of the specific factor in the import competing sector was shown to be the payoff at the free trade relative price and that in the exporting sector it was shown to be the payoff at the autarkic relative price.

With these two different reference points the model of the political economy was subjected to two different sets of comparative static experiments in order to see how the tariff rate would change as the exogenous variables of the model change. Throughout those experiments it was assumed that the relative bargaining powers of the players remained unaffected by the shocks.

The model did admit conclusive analytical solutions under a coercive government. It was shown that the bargained tariff rate with a coercive government provides a perfect insulation to the domestic relative price from the international terms of trade shocks. Moreover, the bargained tariff rate was also found to be responsive to changes in the domestic endowments of the factors of production. In general, it was observed that tariff changes tend to compensate the loser for the relative loss arising out of the exogenous shocks. The results were explained intuitively.

However, the analytical results of the comparative static exercise with a popular government were not entirely conclusive. This was because the point of minimum expectation responded to the shocks; the expressions yielding the response of the bargained tariff rate proved complex enough to make any deductions very difficult. Nevertheless, with Cobb-Douglas production functions it was shown that domestic relative price moves in the same direction as the international relative price.

In chapter 8, the policy-endogenous general equilibrium model was simulated numerically using hypothetical data sets that covered some extreme cases. The procedures adopted in obtaining the minimum expectation payoffs, and the calibration of the model were also discussed in detail. The simulation results showed that, in general, the directions of response of the bargained tariff rate under a popular government were not different from that under a coercive government. The magnitudes of the responses were, of course, different. Some hypotheses that followed from these comparative exercises were stated formally. Those hypotheses were then checked and found to be consistent with the results of previous studies. It was seen that, in general, the bargained tariff rate changes in response to an exogenous shock so as to compensate, at least partly, for the relative loss of the losing player as a result of the
shock. Uneven changes in the players' generalized fear of ruin was shown to be the principal mechanism leading to the comparative static results obtained in this study.

Furthermore, the policy-endogenous general equilibrium model was simulated to predict the consequence of a very large growth in the stock of the specific factor in the import competing sector. The results showed a reversal in the direction of trade. The commodity that was imported before the shock was exported after the shock and vice versa. The commodity that was being taxed before the shock was subsidized after the shock. Several interesting hypotheses followed from the results of this simulation, which are consistent with the predictions of previous studies and the stylized facts. For example, if the characteristic feature that differentiates between developed and developing countries is the faster rate of capital accumulation and technological growth in the non-agricultural sector relative to the agricultural sector, then on the basis of the simulation results one could predict that a developing country exporting agricultural products will be taxing the farmers and a developed country exporting mainly non-agricultural products will be taxing the producers of non-agricultural products. Thus, we may employ the political economy approach to explain why developing countries tax agriculture and developed countries subsidize it. This completes our study of the process of endogenous tariff formation and the comparative static behaviour of the tariff rate.

9.2 Central Conclusions

The central conclusion of this study can be stated as follows.

Whether a political economy admits a cooperative behaviour or not, there exists at least one equilibrium under fairly general conditions.

A government's redistributive policy can consistently be viewed as an equilibrium outcome of a bargaining process between the organized interest groups holding conflicting interests on the level of the redistributive policy. If some exogenous shock disturbs the bargaining equilibrium, then in the new equilibrium the level of redistributive policy of the government will change so as to compensate, at least partly, for the relative loss of the losing player.

The distribution of the relative bargaining power between the interest groups is one of the factors that determines the equilibrium level of the redistributive policy. But, so long as the distribution of relative bargaining powers is unaffected by the shocks, the changes in the redistributive policy in response to the shocks will be independent of the distribution of the bargaining power.
The mechanism underlying the process of policy adjustments is the dynamics of the fear of ruin, which moves in the same direction as the relative gain of a player. In particular, if an exogenous shock (a) causes a decline in the gain perceived by the owner of the specific factor in the import-competing sector relative to his minimum expectation, while (b) that of the owner of the specific factor in the exporting sector rises relative to his minimum expectation, then the fear of ruin held by the owner of the specific factor in the import competing sector falls, while that of the owner of the specific factor in the exporting sector rises. In the new bargaining equilibrium the rationalized tariff rate will rise.

9.3 Implication

This study has two implications to policy modelling. First, it demonstrates that the bargain-theoretic approach can provide a theoretically consistent and numerically implementable analytical framework in modelling the policy formation process in a political economy. Second, this approach is potentially capable in reconciling the two diverging approaches to policy modelling that are based on two different views on the motivation of the government, namely, that of self-interest and that of the public-interest. Such a possibility arises because the comparative static behaviour of the bargained tariff rate turns out to be strikingly similar to the predictions that follow from the maximization of a conservative social welfare function (see chapter 2). So, if bargaining is accepted as the underlying process that generates the (positive) conservative social welfare function (CSWF), then the problem of identification of the social welfare function vanishes. Furthermore, the implementation of a social welfare function will not be inconsistent with the self-interested behaviour of the government. The difference between the political economy approach and the welfare function maximizing approach can be disregarded.

The fact that the predictions of Corden's CSWF and that of the policy-endogenous general equilibrium model (PEGEM) based on the bargain-theoretic approach are indistinguishable prompts us to investigate the internal relationship between the two approaches. CSWF maintains that policy changes should be directed to avoid any significant absolute reductions in real incomes of any significant section of the community. No function has been specified to represent this CSWF, however.

What constitutes a significant level of reduction and what defines a significant section of the community are questions which are yet to be addressed by the CSWF theorists. However, we can now specify one functional form that is consistent with the CSWF by assuming some answers to the above questions.
The first question concerns the segments of population whose welfare is considered important by the state. This is essentially equivalent to the question of number of players in the game. The second question is essentially related to the question of the reference point in the Nash bargaining game, since in a Nash bargaining game the payoff of no player is allowed to fall below the 'disagreement' payoffs, i.e., the level of reference payoffs. One may logically disagree on a particular choice of the reference point, but to solve a Nash bargaining problem one has to assign a value to it in one way or the other.

If the players in the bargaining game are the significant sections of the community, and the reference point used in the bargaining problem is the same as the reference point implied by the CSWF, then it follows that the CSWF is indistinguishable from the generalized Nash product which will be maximized by the choice of the tariff rate subject to the rent transformation function. Therefore, the generalized Nash product may be viewed as a specific form of the conservative social welfare function and the bargaining process in the political sphere as the underlying mechanism of generating it.

9.4 Limitations

Though this study attempted to derive the analytical results rigorously, it, nevertheless, suffers from distinct limitations, some of which were necessary to keep the model tractable and simple. The major limitations can be described as follows.

First, this study assumed that the trade tax is the only redistributive policy instrument available to a government. This was, of course, a simplification. The government has numerous instruments of intervention, none of which is in the real world completely equivalent to any other in every respect (see, Warr and Parmenter, 1986; Hertel, 1989). The choice of instruments itself may require separate politico-economic explanations (see Lloyd and Falvey, 1986, and Falvey and Lloyd, 1991). These may eventually extend the generality of the model presented in this study. In order to keep the analysis simple, well focused on a specific issue and to understand the process of policy determination as clearly as possible an assumption of this sort was necessary.

Second, this study did not consider how the tariff revenue was distributed. It assumed that the tariff revenue is simply a transfer to the consumer. But, the response of the tariff rate with respect to the exogenous shocks could be sensitive to the way tariff revenue is distributed among the factor groups (Long and Vousden, 1991). The purpose in not modelling the distribution of tariff revenue explicitly was to make the rationalized tariff rate the only instrument of income redistribution. Otherwise, tariff
revenue itself would be another instrument in affecting the income distribution. Consistency in the model of a political economy would require that some sort of revenue-seeking activity also be modelled.

Third, this study has ignored other policies of a government that could be directed to macroeconomic stabilization and growth. Rausser (1983) and Rausser and Foster (1990) have argued that a model that does not consider both 'pie-expanding' and 'pie-sharing' policies together is likely to yield incorrect answers to the questions of endogenous policy responses. In this sense, the model described in this study is not exhaustive, as it described only a sub-game of a much larger policy game actually played in a political economy. A direct reflection of this limitation was observed in chapter 7 while determining the minimum expectation of the players.

Fourth, this study also assumed that the mobile factor is fully employed at the wage rate that clears the market. This is another simplification. However, this assumption could be relaxed by specifying the mobile factor market in a more realistic way. For example, one may consider that the wage rate is exogenously determined and model the unemployment/employment level as an endogenous variable. Alternately, a labour supply function that responds to changes in the wage rate can be built into it. These are possible extensions that can be handled easily within the present framework.

Fifth, this study has not modelled a coalition of the owners of the mobile factor as a separate, independent player. In the political sphere, they are considered as rationally ignorant voters. However, one may model enterprise bargaining, and then consider the existence of unemployment and the strategic behaviour of the owners of the mobile factors at the same time. Wallerstein (1987) could provide some guidelines in modelling these two issues related to the labour market.

Sixth, the interest groups in this study were assumed to maximize the real rental income, not the utility that they can obtain from it. Hence the existence result obtained in this study needs to be understood with appropriate qualifications. Moreover, since in the standard Nash bargaining theory players' payoffs are defined in terms of utilities, the solution to the bargaining problem obtained in this study is therefore a Nash-like solution, because the players' payoffs are measured in terms of real rental income. This simply underscores the importance of further research in this area. Binmore (1987c) and Rubinstein, Safra and Thomson (1992) may provide the axiomatic foundation for such studies.

Seventh, in a more realistic model, one may introduce the inter-industry relationship by incorporating the input-output structure into the model of the economic
sphere. The technique for doing so is already well developed in the literature (see, for example, Dixon, et al., 1982).

Eighth, the solution to the bargaining problem has been made operational by taking a somewhat ad hoc approach in specifying the disagreement payoffs. The point of minimum expectation, which has been assumed to summarize the worst scenario in disagreement, has been identified by imposing a particular restriction, namely the self-financing, on the government’s behaviour. This restriction is justified so long as the tariff rate is the only policy instrument available to the government. One may, however, systematically disagree on the imposition of this restriction and on the use of minimum expectation payoffs as the reference point in the bargaining process. A better theory of disagreement, if available, could improve the theoretical structure of the policy-endogenous general equilibrium model.

Ninth, this study also assumed that the distribution of relative bargaining power among the players is exogenously given. This was justified by Binmore, Rubinstein, and Wolinsky’s demonstration that difference among players in their time preference rates, and the fear of disagreement due to exogenous intervention could be captured in the static representation of the game by the asymmetric distribution of the bargaining power. However, there could be other strategic behaviours of the players, not modelled, that affect the opponent’s perception of exogenous risk, and hence the distribution of the relative bargaining power. A systematic analysis of factors that determine a player’s bargaining power would therefore enrich the model presented in this study.

9.5 Suggestions for Further Research

The above limitations of this study suggest some areas of future research in policy modelling. In particular, future research in this area of policy modelling may extend this study in three different directions.

The first direction of research might be towards the application of the general equilibrium model of a political economy described in this study to country specific data sets which could then be pooled together to obtain a regional model of general equilibrium. Such work has already been done with policy-exogenous models (see, for example, Industry Commission, 1991; Whalley, 1985). Use of policy-endogenous general equilibrium models instead in the construction of a regional model would allow for the political constraints faced by the governments of each country, and therefore, would offer a more general analytical framework to study the possibilities of regional cooperation. This framework would provide sufficient ‘behind-the-scene’ information to models of trade wars and negotiation such as studied by Harrison, Rutstrom and Wigle (1989), Markusen and Wigle (1989) and Harrison and Rutstrom (1991).
The second direction of research could be towards the generalization of the present model. As indicated above, the present model suffers from many limitations due to several simplifying assumptions. In particular, the following areas of research seem to be interesting and feasible.

(a) The existence result provided in this study is based on a particular structure of the production functions, namely the CES functions, and real rent maximizing behaviour of the owners of the specific factors. An analytically challenging work is to study the existence of an equilibrium in a political economy with general production functions, and utility maximizing behaviour on the part of the owners of the specific factors. As a first step one may impose some restrictions on the nature of the utility functions that would define a one-to-one correspondence with the real rental income and the utility level it yields.

(b) This study followed the Nash bargaining approach to solve the bargaining problem in the tariff game. One may test the robustness of the results obtained in this study by following other solution concepts provided in cooperative game theory. Moreover, to operationalize the concept of disagreement, one might also examine whether there exist other uniquely defined reference points, which are more appealing in terms of their empirical relevance than the point of minimum expectation and whether a shift in the reference point affects the conclusion of this study qualitatively.

(c) A more challenging area of research in policy modelling would, of course, be to attempt to model the government expenditure on productive activities as well rather than to assume that the (tariff) revenue of a government is transferred to the ‘individuals’ in a lump-sum manner. A model of political economy that also describes government expenditures more realistically would bring the ‘pie-expanding’ policies of the government into the purview of general equilibrium modelling. In such models, government policies will create not only a movement along a given rent transformation frontier but also the possibilities of shifts on the frontier as well. The results obtained from such models would be more realistic than the results obtained from models that allow for the redistributive policies only, such as the predictions of the present study (see, Weingast, et al., 1981; Rausser, 1983; and Rausser and Foster, 1990).

(d) There exist several possibilities to extend the structure of the present model by improving the specification of the general equilibrium structure of the economic sphere. For example, one may consider the following straightforward cases. (i) This study considered the case of two traded goods only. One may introduce a non-traded good into the model and study the endogenous behaviour of the tariff
rate. The model presented in Cassing and Warr (1985) could be employed to
describe the behaviour of the economic sphere of a political economy with three
goods of which one is non-traded. A model of the political economy could be
obtained by superimposing the political sphere onto it. (ii) One may also consider
introducing inter-industry dependency by incorporating the input-output structure
in the description of the economic sphere. A simple modification of the
description of the economic sphere of the present study is sufficient to do this
one may consider a generalization to n-commodity case. However, with arbitrarily
chosen n-commodity aggregates and the corresponding input-output structure one
may not always obtain a well-behaved bargaining set. This will make the model
solution sufficiently complex and one may have to look into mixed strategy space
for the solution.

(e) Finally, the third direction of research could be towards empirical validation of
the political economy approach in policy modelling. This can be done in two
ways. First, one may take the predictions of the analytical models or calibrated
numerical models and test whether the predictions are consistent with history. For
example, the prediction of this study that if capital accumulation and/or
technological progress in the import competing sector dominates that in the
exporting sector, then the protection afforded to the import competing sector
declines is refutable. Second, one may attempt to estimate the game
econometrically and administer the diagnostic tests rigorously. Data limitations to
the second type of tests is likely to be a limiting factor.

Studies in these directions are important for the following reasons.

First, if policy changes are actually governed by the politico-economic structure
of the society, then the mere realization of this fact may save resources of the society at
large from going to the design and implementation of policies that do not confirm to
this reality. The society as a whole can be made better off by policy changes that were
from the beginning directed to the political reality of the society.

Second, if bargaining is a fact of life, then economists can better guide society
by predicting the bargaining outcomes than by proposing changes that will never be
implemented.
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