DESIGN CALCULATIONS FOR 1988 SERVOCREEP MACHINE

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NOTE BOOK

FEINT
MADE IN AUSTRALIA
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Dynamical radius is proportional to square root of mass,
e.g., take the solar mass.

\[ R = \sqrt{\frac{m}{4\pi\rho}} \]

If the planet's mass is \( m \) and its density is \( \rho \),
then its dynamical radius is given by:

\[ R = \sqrt{\frac{m}{4\pi\rho}} \]

\[ R = \frac{M}{4\pi\rho} \]

\[ R = \frac{L}{4\pi} \]

\[ R = \frac{L}{2\pi} \]
See DJ Shamsa et al. J Am Chem Soc 1994 112 2669-2680 for calculation & measurement. Values of $k$ can be greater than $10^{-12}$ m$^{-1}$.

Use of Kleingeld's reln to correct for pressure.

At $\phi = 0.54$, $k = 1.0 \times 10^{-12}$, calculated for mineral bundle (11.0 fibres).

Crystal values 4.5 x calculated values.
Furnace Insulation - Permeability considerations.

The permeability of the furnace insulation material (Zinc-ASTH and alumina paper) is not known so must be estimated.

For the equivalent channel model, the problem is to estimate the hydraulic radius $R$. For a simple square packing of parallel fibres with spacing $a$ and fibre diameter $d$, we have

$$1 - \phi = \frac{\pi d^2}{4} \frac{1}{\lambda^2}$$

$$\lambda = \left( \frac{\pi d^2}{4(1-\phi)} \right)^{1/2}$$

Zinc-zirconia alumina fibres are being 3 mm diameter, 3.4 density, and zirconia fibres as 3-6 mm; alumina-silica not given.

ASTH insulation is 0.32 g/cm$^3$ density, so if the fibres are ~3.2 g/cm$^3$ density (higher 5%), then $\phi \approx 0.9$.

ASTH 2 paper is 0.14 g/cm$^3$ density but is compacted from 1.23 mm thickness to 0.5 mm thickness, so its density would be 0.34 g/cm$^3$ in application, again $\phi = 0.9$.

Thus for ASTH:

$$\lambda \approx \left( \frac{\pi (3 \times 10^{-6})^2}{4 \times 0.1} \right)^{1/2} = 8.4 \times 10^{-6} \text{ m}$$

ASTH 2:

$$\lambda \approx \left( \frac{\pi (3 \times 10^{-6})^2}{4 \times 0.1} \right)^{1/2} = 8.4 \times 10^{-6} \text{ m}$$

If the fibre dia were 6.10 $^{-6}$, $\lambda = 16.8 \times 10^{-6}$ m.

The hydraulic radius is $\frac{\lambda^2}{d}$, so taking the cell above:

$$R = \frac{\pi d + 4(\lambda - d)}{\pi d + 4(\lambda - d)} = 2.28 \times 10^{-6} \text{ cm for 3 mm fibres}$$

Then the permeability (Patrakian 1983) is given by

$$k = \frac{R^2 \phi}{2}$$

where $T = \frac{1}{\lambda}$ in the tortuosity

$\sim 2.5$ to $4$ (Gagen and Barrie 1983, structure ray 3

for a fully microporous 1899 structure

$$k \approx \frac{R^2 \phi}{2}$$

$$\approx 7.39 \times 10^{-13} \approx 0.8 \times 10^{-12} \text{ m}$$

for both ASTH & ASTH 2, 3 mm fibres

$1.2 \times 10^{-13}$

Take $10^{-13}$
For pulling inside of core & down outside of furnace,

\[ \Delta T \approx \left( \frac{300 - 1000}{1321 - 768} \right) \times 9.8 \times 0.15 = 812 \text{ Pa} \]

\[ \text{Take 800 Pa} \]
Temperature regimes:
Take the case of furnace temperature of 1500 K.
Boundary temperature are estimated as at left.
If we consider a convective cell of mean length 100 mm, the average temp in the upper arm will be of order 1800 K at a guess, 9 in the lower arm of order 500 K. Therefore the driving pressure is

\[ \Delta P = (P_{\text{top}} - P_{\text{bot}}) \times 9.8 \times 0.1 = 318 \text{ Pa}. \]
Take 300 Pa

For internal/external convection (up inside, down outside) we take a mean temperature inside of 700 K (could be 800) and an effective length of 150 mm,

\[ \Delta P = \frac{(P_{\text{top}} - P_{\text{bot}})}{1373} \times 9.8 \times 0.15 = 547 \text{ Pa}. \]
Take 600 Pa

Heat Capacities & Contents:

<table>
<thead>
<tr>
<th>T (K)</th>
<th>(C_p) (J/mol K)</th>
<th>(P) (Pa)</th>
<th>(\Delta H) (J/mol)</th>
<th>Heat Content (J)</th>
</tr>
</thead>
<tbody>
<tr>
<td>300</td>
<td>30.16</td>
<td>33868</td>
<td>1.021 x 10^6</td>
<td>306.10</td>
</tr>
<tr>
<td>500</td>
<td>26.40</td>
<td>28029</td>
<td>0.740 x 10^6</td>
<td>370.10</td>
</tr>
<tr>
<td>700</td>
<td>24.78</td>
<td>23948</td>
<td>0.593 x 10^6</td>
<td>415.10</td>
</tr>
<tr>
<td>800</td>
<td>24.25</td>
<td>22332</td>
<td>0.542 x 10^6</td>
<td>433.10</td>
</tr>
<tr>
<td>1000</td>
<td>23.43</td>
<td>19678</td>
<td>0.462 x 10^6</td>
<td>462.10</td>
</tr>
<tr>
<td>1500</td>
<td>22</td>
<td>15500</td>
<td>0.34 x 10^6</td>
<td>510.10</td>
</tr>
<tr>
<td>2000</td>
<td>21.23</td>
<td>11500</td>
<td>0.23 x 10^6</td>
<td>206.10</td>
</tr>
<tr>
<td>400</td>
<td>27.83</td>
<td>30056</td>
<td>0.853 x 10^6</td>
<td>341.10</td>
</tr>
<tr>
<td>1000</td>
<td>22.84</td>
<td>17649</td>
<td>0.403 x 10^6</td>
<td>483.10</td>
</tr>
</tbody>
</table>
\[ y = 91.6 \times 10^{-6} \text{ at } 673K \text{ 300 MPa} \]

86.7

873
Insulation connection (60 od 27 id) = $2.25 \times 10^{-6} \text{ m}^2$

so half area $A = 1127.10^{-6} \text{ m}^2$ for the connection.

Internal convection. According to

\[ J = \frac{A}{k} \frac{\Delta P}{\Delta x} \]

\[ = \frac{1127.10^{-6} \times 10^{-12}}{90.10^{-6} \times 0.2} \]

\[ = 18.8 \times 10^{-9} \text{ m}^3/\text{s} \]

which will transfer

\[ 18.8 \times 10^{-9} \times 92.10 \]

\[ = 1.7 \text{ W} \]

\[ \sim 2 \text{ W}. \]

There may be smaller cells.

For example, if we take a cell as at left, area $A = 196.10^{-6}$

\[ T_m = 1200 \]

\[ T_i = 400 \]

\[ \Delta P = (p_{1200} - p_{400}) 9.8 \times 0.05 = 249 \text{ Pa} \]

\[ = 196 \times 688 \text{ Pa} \]

\[ J = \frac{700.10^{-6} \times 10^{-12}}{90.10^{-6} \times 0.12} \]

\[ = 16.2 \times 10^{-9} \text{ m}^3/\text{s} \]

which will transfer

\[ 16.2 \times 10^{-9} \times 142.10 \]

\[ = 2.3 \text{ W} \rightarrow 10.2 \text{ W} \]

and there are about 6 of these cells $\rightarrow 61 \text{ W}$. 

So it is hard to see that convection within the fibre insulation will cause more than a few watts of heat loss (assuming $k = 10^{-12} \text{ m}^2$).
Circum = 0.323 m
Area = 0.800323

Could be 20x higher if k=10" instead of 10.12

W0.02 = WES = 140W

Lg R = 70
q = 19 - 15.5
Flow resistance

To calculate the external/external convection, we first calculate the flow resistance of the individual parts, using

\[ J = \frac{A \Delta P}{\eta \Delta x} = \frac{\Delta P}{R_f}, \quad R_f = \frac{\gamma \Delta x}{A_k}. \]

For the main insulation,

\[ R_f = \frac{90 \times 10^{-6} \cdot 0.13}{2260 \times 10^{-6} \cdot 10^{-12}} = 5.18 \times 10^9 \text{ Pas m}^{-3} \]

For the ends

\[ R_f = \frac{100 \times 10^{-6} \cdot 0.03}{323 \times 10^{-6} \cdot 10^{-12}} = 9.29 \times 10^9 \text{ Pas m}^{-3} \]

Total \( R_f = \frac{14.5}{15} \times 10^9 \text{ Pas m}^{-3} \)

so \( J = \frac{\Delta P}{R_f} = \frac{600}{14.5 \times 10^9} = \frac{41.4}{10^{-9}} \text{ m}^3 \text{s}^{-1} \)

and heat transfer = \( \frac{41.4}{10^{-9}} \times 109.10 = 4.5 \times 10^6 \text{ W} \)

ie \( \sim 5 \text{ W convective up the insulation & down the outside.} \)

Then the total convective losses are only estimated to be around 7 W for 1500 K, ie to contribute less than 0.03 W/K to the heat consumption. This could be a bit of an underestimate if the effective temperature differences are underestimated, but not by a lot.

Flow resistance up the inner core insulating sleeve,

\[ R_f = \frac{90 \times 10^{-6} \cdot 0.07}{95 \times 10^{-6} \cdot 10^{-12}} = 66.4 \times 10^9 \text{ Pas m}^{-3} \]

so \( J = \frac{600}{66.13} = 12.0 \times 10^{-9} \text{ m}^3 \text{s}^{-1} \)

and heat transfer = \( 12.0 \times 10^{-9} \times 200 \times 0.6 = 2.4 \text{ W}. \)
Effect of cracks

The Rice-Trezona formula (Peterson 1983) for slots gives

\[ J = A \cdot \frac{1}{3} \cdot \frac{R^2 \Delta P}{\Delta x} \]

where \( R = \frac{c}{2} \) = crack opening

\[ J = A \cdot \frac{c^2 \Delta P}{12 \pi \Delta x} \]

\( i.e. \ R_f = \frac{12 \pi \Delta x}{A c^2} \)

For a circumferential crack on diameter \( d \), \( A = \pi d c \)

\[ \frac{1}{2} \Delta x R_f = \frac{12 \pi \Delta x}{A c^2} \]

where \( \Delta x \) is the wall thickness cracked.

At \( d = 25 \text{mm} \), \( \gamma = 90 \times 10^{-6} \text{ in have} \)

\[ R_f = \frac{0.0138 \Delta x}{c^3} \]

\( \text{m}^{-3} \)

For \( \Delta x = 1.5 \text{mm} \)

\[ R_f = \frac{0.0138 \times 0.0015}{c^3} \]

\[ = \frac{20.63 \times 10^{-6}}{c^3} \]

\[ \frac{R_f}{5 \times 10^9} \text{ when } c = \left( \frac{20.63 \times 10^{-6}}{5 \times 10^9} \right)^{\frac{1}{3}} \]

\( = 16 \times 10^{-10} \text{m}^{-3} \text{ - very small opening} \)

\( (35 \text{mm for } R_f = 0.5 \times 10^9) \)

\( \text{if } R_f = 10^{-11} \)

When \( \Delta x = 6 \text{mm} \), as for overlap of core segments, and

\( c = \frac{30 \text{mm}}{2} \)

\[ R_f = \frac{3.87 \times 10^9}{c^3} \text{ Pa m}^{-3} \text{ - already getting to be a bit low relative to the other resistances - better to avoid.} \]

When \( \Delta x = 1.5 \text{mm} \) and \( c = 0.1 \text{mm} \), \( R_f = 0.021 \times 10^9 \text{ Pa m}^{-3} \)

so already the resistance to flow is negligible compared with the rest of the path.
In case of internal/external convection, the crack short-circuits the bottom closure, for which $R_f \approx 4.5 \times 10^9$.
So total $R_f$ is reduced from $4.5 \times 10^9$ to $10 \times 10^9$ Pa m$^{-3}$
and increasing $J$ to $60 \times 10^{-9}$ m$^2$ s$^{-1}$
so heat flux to $60 \times 10^{-9} \times 10^9 \times 10^6$
which is extra 1.5 W.
If $h = 10^{-11}$ instead of $10^{-12}$ m$^2$, this extra would be 15 W.

If $R_f$ is overestimated by $10^{-11}$, then $c = 42 \times 10^{-6}$ (increased by $10^3$)

Still have to have mean clearances < 40mm for improvement.
Maybe better to stick to a thru 2 path scheme & maybe put some regenerate in the gaps before closing.
Consider this conventional cell.

Take effective length $L = 50\text{mm}$.

$$\Delta P = \frac{(P_{500} - P_{1200})}{1093} \cdot 0.05$$

$$= \frac{198}{68}$$

say $200\text{Pa}$.

$$R_f \approx 3 \cdot 10^9$$ for main insulation

$$+ 4 \cdot 10^9$$ for the end

$$\approx 7 \cdot 10^9$$

$$\therefore J = \frac{200}{7 \cdot 10^9} \times 140 \cdot 10^{-6} = 4\text{ W.}$$

Still not very dramatic relative to the heat being generated at the lower winding, usually at least $200 - 300\text{W}$

Alternative to end: Suppose we try a dead fit, with not more than $C$ clearance at the gaps, length $323\text{mm}$

$$R_f = \frac{12\pi \Delta L}{A C^2} = \frac{12\pi \Delta X}{0.323 \cdot C^3}$$

$$= \frac{12 \cdot 100 \cdot 10^{-6}}{0.323 \cdot C^3} \cdot 0.020$$

$$= \frac{74.3 \cdot 10^{-6}}{C^3}$$

To get $R_f = 10 \cdot 10^9$ (to be better than new)

$$C = \left(\frac{74.3 \cdot 10^{-6}}{10 \cdot 10^{-9}}\right)^{\frac{1}{3}} = 19.5 \cdot 10^{-6}$$

ie the mean clearance has to be not more than about $20\mu\text{m}$.

Difference in thermal expansion is about $\frac{1}{4}(\frac{27.5 \cdot 10^{-6}}{20} - \frac{60.8 \cdot 10^{-6}}{1000-100}) = 0.452\text{ mm per slot}$.

ie we have to put a taper of $0.226$ along each slot face in order to get good matching assuming linear temp gradient which it is not. So it would be very difficult to achieve $20\mu\text{m}$ matching over a whole temp range. Better to stay with $A42$ paper in $1\text{mm}$ slots.
Going back to p81, it is a little hard to see how k could be quite as high as $2 \times 10^{-11}$, but maybe $10^{-11}$ is feasible if the quoted fibre size is a bit on the low side.
Reconsideration of convective losses

From p 64, the total convective losses are estimated to be around 213 W for 1500 K (\(\Delta T = 1200\) K). This is based on \(k = 10^{-12}\) m² K/W, which may be too low.

From p 61, we estimate the following conductive heat losses:

- Radial: 213 W with \(k = 0.3\) for \(\Delta T = 900\) K
- Axial (top): 50 W with mostly girdle, etc.
- Axial (bottom) \(\approx \frac{45}{308}\), or around 0.15 W/K

This would leave \(-0.16\) W/K for convective losses. But the

\[ \frac{308}{45} = 6.8 \]

W/K estimated above giving \#4 is

much below this, so may \#3 has to be increased by a factor

\[ \frac{0.16}{0.06} = 2.7 \times 10^{-12} \]

If we then assume \(k = 2 \times 10^{-12}\) m² K/W for the fiber insulation (both felt and compressed felt), then the convective transfer in internal cells will be of order 2.3 x 20 = 46 W, to be added to the 213 W for radial transfer by conduction, increasing the effective \(k\) by 2.3 to 0.36

The internal/external convection becomes 100 W and the convection up the inner insulating sleeve 48 W.

The effect of cracking the core will reduce total by from

\[ (14.5/20) \times 10^9 = 0.725 \times 10^9 \]

to

\[ 0.493 \times 10^9 \]

Therefore increasing the 100 W internal/external convection loss to 146 W. If the effects of this is to take this heat from the bottom winding, the dramatic effect of cracking the bottom end of the core can be understood.
Why do furnace cores crack?

In the most recent furnace from Minneapolis, the crack config. is as shown at the left. This location has been fairly consistent for all the furnaces, regardless of what kind of alumina was used (Bendigo, Duramic). Consulting Boley & Weiner "Theory of Thermal Stresses" gives the principle that there will be no thermal stresses if the temp. distribution is "a linear function of the rectangular cartesian space coordinates". I understand from this that a linear axial temp. gradient along the core will not give rise to thermal stresses. But a radial gradient will. The situation above probably has a gradient as shown, and the crack may well be a response to the oblique gradient, giving rise to tensile stresses on the inside diameter, normal to which the crack runs. Purely radial I think will tend to give longitudinal cracks on the inside (although there will be an axial tension as well in plane strain).

Figure of merit against thermal stresses is \( \sigma = \frac{E \cdot \varepsilon}{2(1-\nu)} \) where \( \sigma \) is the fracture strength, \( E \) if we take \( E = 10^3 \) in Coors list, we get

<table>
<thead>
<tr>
<th>Material</th>
<th>Fracture Strength</th>
</tr>
</thead>
<tbody>
<tr>
<td>99.5 Al2O3</td>
<td>0.12</td>
</tr>
<tr>
<td>99.9 Al2O3</td>
<td>0.17</td>
</tr>
<tr>
<td>Mullite</td>
<td>0.21</td>
</tr>
<tr>
<td>Al2O3</td>
<td>0.20</td>
</tr>
<tr>
<td>TZP</td>
<td>0.44</td>
</tr>
<tr>
<td>HPSiN</td>
<td>0.97</td>
</tr>
</tbody>
</table>

MaxTemp: 1750 1900 1700 1750 2400 1200

So maybe the 99.9 Al2O3 will be a substantial improvement. Beyond that, TZP looks good, a possibly Mullite - operation of dimensional stability in both cases.

9/6/93

Experience at Minneapolis is that the Coors 99.9 alumina cracks more readily than the previous 99.8 grades from Bendigo or Duramic. Presumably, it is more brittle in spite of the higher figure of merit indicated above. Cracking occurred by 1100°C.
<table>
<thead>
<tr>
<th>M&amp;Os</th>
<th>$\alpha$ (K$^{-1}$)</th>
<th>$\beta$ (Wm$^{-1}$K$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$8 \times 10^{-6}$</td>
<td>30 at RT</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>at 800°C</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>at 1200°C</td>
</tr>
</tbody>
</table>

**Mullite**

<table>
<thead>
<tr>
<th></th>
<th>9</th>
<th>92.0</th>
<th>$\sim$180°C decreasing at higher $T$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ti</td>
<td>6</td>
<td>130</td>
<td></td>
</tr>
<tr>
<td>Mo</td>
<td>11</td>
<td>60</td>
<td></td>
</tr>
<tr>
<td>Fe</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
on the way up, rather than upon running at 1250-1300°C. It still seems to me that the problem has to do with the radial temperature gradient component. This will give rise to a circumferential tension on the ID & compression on the OD associated with which will be an axial component of the same sign but smaller magnitude in plane strain due to the Poisson effect. Why should the cracking be mainly circumferential rather than longitudinal? Perhaps because of stress raisers associated with the grinding marks. The stress raising effect of the groove on the OD is mainly on the compressive stress side & so should be less effective. So this story does not hang together very well.

Another possibility is that the temperature does not vary as markedly along the OD as along the ID so that the temperature gradient radially is highest at the bottom end of the winding.

The most obvious solution is to go to a duplex core tube. If the wall thickness is halved, the thermal stress should be halved, so the outer tube should be less likely to crack. Also the inner tube will be plain, & so also less likely to crack, if any cracks with a bit of luck, will not be opposite cracked in the outer tube. The inner tube could be alumina or possibly, mullite.

Alternatively, we increase the bore of the ceramic core tube only slightly, say 21 → 22, and insert a 0.5 mm wall thickness metal tube made of iron, Mo, or Ti. The question then arises as to the role of the thermal conductivity of the tube especially at hollow end. The thermal gradient is roughly 1200/60 = 20 K/mm. So for 220°C 21/ID tube, the heat flux would be $q = \frac{\pi}{4} (0.022^2 - 0.021^2) \cdot \frac{1200}{0.06} = 0.6754 \cdot \frac{\text{W}}{\text{cm}^2}$

- $13.5$ W for Ti
- $88$ W for Mo
- $40$ W for Fe
This comparison is about 180 W for the total axial flow for max alumina/1300°C case (p 62) or about 60 W for the min. alumina/1300°C case. So the use of Fe would tend to increase the heat flow rather seriously, whereas Ti would be very suitable. But the situation will not be as bad as this calculation indicates because there is still some gaseous barrier below the Fe, and the increased temp on the ID wall may mitigate some of the radial temp gradient at the bottom of the bottom winding, discussed above. We could also thin down the wall a bit at the bottom, possibly, by opening up the ID a bit.

1/7/93

Following the post-mortem on 002 after its thermal shock in Minneapolis in April (switched on again at full power after cooling somewhat in a power black-out) it is clear that transient heating causes severe cracking. How fast should we allow heating? The time constant for heat penetrating the core is given by 

$$t = \frac{x^2}{D}$$

where $$D = \frac{k}{\rho C}$$

At high temperature, $$k$$ may be around $$5 \text{ Wm}^{-1} \text{K}^{-1}$$ for alumina, $$\rho = 4000 \text{ Kg m}^{-3}, C = 900 \text{ J Kg}^{-1} \text{K}^{-1}$$

$$t = \frac{(0.003)^2}{4000} \frac{4000}{900} = 6.5 \text{s}$$

Therefore, if we wish not to exceed the order of 1K gradient across the alumina, the heating rate should not exceed 1K per 6.5s, i.e. 9 K per minute.

A heating rate of 30 K per minute will thus give rise to an extra temperature gradient of the order of 3K across the core. This ought to be tolerable.
Possible core design modifications

On the presumption that the cracking is due to radial thermal stresses, the most obvious solution is to use two thinner concentric tubes. The outer one will carry the windings & even if it cracks (less likely now due to being thinner) the cracks may not penetrate the inner tube - if it cracks, with a bit of luck the cracks will not be opposite cracks in the outer tube (unless initiated by hot gas through the outer cracks). We may be able to get away with Coors 995 alumina also, which is a cheaper product than 999. 995

<table>
<thead>
<tr>
<th>Density:</th>
<th>3.89</th>
<th>3.96</th>
<th>3.99</th>
</tr>
</thead>
<tbody>
<tr>
<td>Porosity:</td>
<td>0.025</td>
<td>0.0075</td>
<td>0.0025</td>
</tr>
<tr>
<td>Linear shrinkage</td>
<td>0.0084</td>
<td>0.005</td>
<td>0.0025</td>
</tr>
<tr>
<td>To 100% density</td>
<td>0.175 mm</td>
<td>0.05 mm</td>
<td>0.2 mm</td>
</tr>
</tbody>
</table>

ie 21.00 → 20.82  ie 21.00 → 20.75
Since the support area under the load cell is greater than the diameter of the piston, no provision is made for net tensile force in the piston. The attachment threads for the LLC will never be called on to bear a portion of applied tensile load, so are only used for initial assembly. An M52 thread then will leave enough "meat" on OD 35 for this purpose. 0.35 can then be taken as the effective OD of the load cell, with splines projecting beyond this.

Mean diam of splines = 58, circumf = 182.212
Width of spline (15°) = 7.592 mm

If we allow ±0.072, max width = 7.664 or 0.14° = 8.5° above 15°
If we want all splines to contact simultaneously within ±0.020 mm, this comp to ±0.1° would give ±0.025 mm, probably OK. Then we need ±0.06 on the spline width to accommodate this.

Need a clearance of at least 0.025 mm, i.e. ±0.03. So if the spline itself is 15° below 15°, this allows 0.03' clearance + 0.06 on spline groove + 0.03' on the spacing, i.e. 14° 45' ± 0.06' should be just OK - can't aim at anything tighter.

\[
p = (\sigma_n - \sigma_0) \frac{d}{d_0 - d} \\
\sigma_n = \frac{\sigma_0}{d_0 - d} \cdot p
\]
Redesign of Internal Load Cell - Axial Load & Torque

(cont'd from p79, previous book)

Splines for ILC body.

Following on from p75, the effective PCD of the splines of 3 depth is $\phi 58$. Therefore, the radial force on the splines from 1000 Nm torque will be

$$\frac{1000}{0.029} = 34,500 \text{ N}$$

To support this with spline length $L = 30$ and integrated width $W$ with shear stress 180 MPa needs

$$\frac{34,500}{0.030} = 100 \times 10^6 \text{ N}$$

ie your splines of 3mm width x 30mm length would be OK.

The bearing stress on the spline faces will be

$$\sigma_n = \frac{34,500}{0.05 \times 0.030} \text{ MPa}$$

For $N = 4$, $\sigma_n = 96 \text{ MPa}$

$N = 8$, $\sigma_n = 48 \text{ MPa}$

$N = 12$, $\sigma_n = 32 \text{ MPa}$. — finally selected.

Support area on top of piston — pre-amplifier space.

If we choose ILC load cell body hardness of $R_c 45 \pm 2$

420 HB $\pm 20$, then y. stress is about $1300 \pm 100 \text{ MPa}$, ie about 1200 MPa minimum.

If we allow up to 1000 MPa for a confining pressure of up to 600 MPa, then the support area has to be $\frac{500 \times \text{ pressurized area}}{0.60 \text{ pressurized area}}$. If the latter is $\phi 40$, then maximum cutout centrally is $\phi 25$.

If we had a cutout of $\phi 20$, bearing stress = 933 MPa at $p = 700 \text{ HB}$

$\phi 23$ 1045

$\phi 25$ 1150
For 12.0 od, 10 id, stress min. = $3.3 \times 600$ MPa = 1960 MPa, so only about half of the bearing load could be suggested.

Even with $\phi$ 10 rod, this would be OK.

If rod = $\phi$ 10, pd. = 9, axial strength = $\pi \times 0.009 \times 0.004 \times 450 \times 10^6 = 45$ kN

a 25% improvement.

Should be pitch dia 0.0072 giving 36 kN, 3.6 t.
The question then arises of whether the bending/shear stresses in the unsupported zone are safe.

If we make the connection B equal to 8 mm, then the received region A will be supported on an annular area 12 mm wide, 18 mm high by itself, but will give substantial support to the 12 mm region in addition to the shear strength of the annular surface below the 12 mm. If we neglected this support and considered only a region 5 mm deep x 12 mm to support the pressure load on A, then shear stress

\[
\tau = \frac{P \cdot \frac{12}{12.5}}{\pi} = \frac{P \cdot \frac{12}{20}}{\pi} = 0.6P = 360 \text{ MPa}
\]

for \( P = 600 \text{ MPa} \)

This should be strong enough by itself, and so adequate given the extra support.

If we have a 3 mm deep attachment at B, we can probably rely on 4 mm thread engagement, so axial strength

\[
\sigma = \pi \times 0.002 \times 0.004 \times 400 \times 10^6 \text{ N} \text{ taking } E = 400 \text{ GPa}
\]

for 60T rod steel

This should be adequate for holding the two piston segments together, since even under pressure the friction does not exceed this amount, & tensile force only arises under no-pressure conditions. So any 128 connection should be OK.

Taking the whole unsupported area C, shear stress is

\[
\tau = \frac{P}{(4 \times 0.025 \times 0.01)}
\]
\[ <30^\circ \] gives 6 lugs. Torsion strength \( \frac{\pi \cdot 0.042 \cdot 0.005 \cdot 0.021}{2} \)

\[ \gamma = \frac{2000}{\pi \cdot 0.042 \cdot 0.005 \cdot 0.021} \quad \text{for 1000 Nm} \]

\[ = 144 \, \text{MPa} \]

OK - selected.

Tolerances about same as for bottom end (see p 92).

8 splines of 15° wide: Total width \( = 8 \times 15 = 120^\circ \)

\[ \text{Torque strength} = \frac{\pi \cdot 0.042 \cdot 0.005 \cdot 0.021}{3} \]

\[ = 460 \, \text{Nm for} \; \tau = 100 \, \text{MPa} \]

\[ = 924 \, \text{Nm for} \; \tau = 200 \, \text{MPa} \]

\[ = 1016 \, \text{Nm for} \; \tau = 220 \, \text{MPa} \] OK.

Have to have 4-fold symmetry, i.e. 4 x 8 splines. 8 will not fit very well with the cut-outs but 4 is OK. Bearing stress is probably tolerable for 8407 at 40 Rc.
\[
T = \frac{p \cdot \pi \cdot d}{\pi \cdot d \cdot 0.017} \quad \text{for } d = 17 \text{ mm depth}
\]
\[
= \frac{p \cdot d}{0.068} \quad = 176 \text{ MPa for } d = 20 \text{ mm, } p = 600 \text{ MPa}
\]
\[
203 \quad d = 23
\]
\[
220 \quad d = 25
\]
all well within the shear strength ~600 MPa of the steel.

Splines at Top End

(a) Between cap and body:

If we have 8 splines, then each is 360/8 = 45° wide and 5 mm high.

Then shearing strength
\[
= \frac{\pi \cdot 0.04 \cdot 0.005 \cdot T}{8}
\]

\[
= 690 \text{ Nm for } T = 100 \text{ MPa}
\]
\[
= 1040 \text{ Nm for } T = 150 \text{ MPa}
\]

This would be sufficient.

(b) Between cap and specimen extension.

Whatever the number of splines, half the circumference supports the stress.

The torque strength
\[
= \frac{\pi \cdot 0.015 \cdot 0.015 \cdot 0.0075}{2}
\]
\[
= 265 \text{ Nm for } T = 100 \text{ MPa}
\]
\[
= 1060 \text{ Nm for } T = 200 \text{ MPa}
\]

Rather highly stressed but may get away with it, including by extending the length a bit.

Bearing stress
\[
= \frac{2963 \text{ MPa}}{N} = 740 \text{ MPa for } N = 4 \text{ Maybe OK}
\]
\[
= 494 \text{ MPa for } N = 6 \text{ Min. }, \text{ie } 30 \text{ splines}
\]
\[
= 370 \text{ MPa for } N = 8
\]

\[\text{Contact ID 133}\]
Actuator Load Cell/Strings Relations

It seems favorable to change from the Instron load cell (height 96) to a Showa 57 (shear beam type) (height 34) for the following reasons:

1. The Instron requires attachment screws to be inserted on the an outer PCD at each end, thus requiring extra length, possibly in the top end of the actuator to allow removal. (There is also a question of access to these)

2. The Showa can be attached at the outer PCD by screws fed through from the top; the string can be attached by means of a central nut.

3. With total actuator travel now 50 instead of 100 for the Instron, we need to find another 50 daylight for removing the string etc., especially if we go back to having a nut on the main piston instead of screwing the string on to it (a cumbersome operation). Therefore, with the Showa load cell, the string of ~62 mm can go into a spacer below the string, to be removed before removing the string.

The space requirements are the following, based on some assumed dimensions:

\[ A = 25 \text{, say; not less than } 20 \]
\[ B = 15 \text{, say; for nut (1922)} \]
\[ C = 6 \text{ thread} \]
\[ +5 \text{ gap for connections} \]
\[ +10 \text{ lock attachment} \]
\[ +11 \text{ lock projection} \]
\[ A + B + C = 72 \rightarrow 67 \text{ max. if } C = 20 \]

So string has to drop 72 to clear. Only 20 is available as extra travel on actuator (total span = 50) if the string is in the bottom position. So we need another 52 daylight. This can be made up in part by a spacer, + the rest by moving the piston up from bottom position, say 40 + 13 = 53 by spacer if we use 50 spacers.
All this does not allow for a pore-pressure tube sticking out below the dome connection on end of piston.
One can get another 34 mm by removing the load cell & some lesser amount (maybe not a lot) by juggling the strings at an angle.
Length of pore fluid tube sticking out has to be enough to go through the dome plug & still attach HP fittings, ~ 15 - 20 mm

Total amount ~ 45 - 50.

Com'd p101
Thermal Stress in Blown Furnace Cores

At 1300°C the power consumption in the bottom winding is about 500 W (in furnace 007 at Minneapolis, for y 1.5/9.93, when the efficiency was around 0.57 W/C, a bit high). From p. 61, if we take $X$ for the insulation of 0.4, the overall heat loss would be about 375 W, or 190 at $X = 0.2$. So somewhere between 100 to 200 W must be conducted inwards & downwards. The downward loss in the aluminizing core itself should be about 75 W, and the conduction down the AlD3/ZnO piston assembly about 25 W, i.e. 100 W in all, leaving about 0 to 100 W for convection upwards in the furnace. One would expect some heat to be convected upwards, so the heat flow through the core must be greater than 105 W & it could be as much as $125 \text{ W}$. Let us consider 100 W.

Then from $y = \frac{2\pi R^2 \Delta T l}{\ln \frac{R}{r}}$, we have

$$\Delta T = \frac{100 \ln \frac{26}{21}}{2\pi 5 \cdot 0.015} = 45 \text{ K}$$

if we take $X = 5$ for hot aluminizing (it may even be a bit less, but this is probably OK) and assume 15 mm as effective length (allowing for the wires being cooler & less resistive at the lower end).

From Timoshenko & Goodier, at inner surface

$$\sigma = \frac{\Delta \theta}{2} = \frac{\Delta T \Delta \theta}{2}$$

$$= -0.71 \times \sigma \Delta \theta$$

This is a tensile stress

For $\Delta T = 45$, $\alpha = 10^{10}$, $E = \sim 350 \text{ GPa}$,

$$\sigma = 5 \times 96 \text{ MPa}$$

So there could be quite substantial tensile stresses in the core. The tensile strength may be well below 100 MPa at 1300°C.
so internal stress is quite greatly the reason for the cracking of the furnace tubes in the Minneapolis machine at 1250°C. The cracks would then be expected to be initiated at the inner surface and so not be much affected by the grooves on the outside (these have been taken into account above in using an effective OD of 26, so they have a beneficial effect here).

If we go to 3 mm wall thickness, 340 OD 21 ID, $\Delta T = 28 K$ and $\sigma = 60 MPa$, a substantial improvement. This is the thickness of the inner sleeve in the proposed double-wall arrangement. For the 260 OD 21 ID, $\Delta T = 21 K$ and $\sigma = 48 MPa$, even better.

The reason for the cracking at the bottom end of the bottom winding is not clear, but may be related to some non-linear temperature gradient actually superimposed on the radial one (a linear axial gradient would not give thermal stress).
Piston Extension & Extension Arrangement

We should aim to have the same strength in extension as the actuator is capable of, viz. 100 kN. Thus the reduced section of an extension piston needs to support 100 kN.

If we use ASSAB 8407 at around 48 Rc, the tensile yield stress is around 1350 MPa. Suppose we aim at 1200 MPa stress, ignoring stress concentrations. Then area needed is

\[ \frac{100,000}{1200} = 83 \text{ mm}^2 \]

Or \( d = 10.3 \text{ mm} \). \( d = 10 \text{ mm} \) would lead to 1280 MPa.

If we use a 90° segment bayonet arrangement, the question is how large to make the bore in order to allow up to a max of lateral motion of a peg of φ10 passing through.

\[ \frac{xR}{R} = \frac{R}{R^2} \]

\[ x_{10} = \sqrt{5 - \frac{(8)^2}{2}} = \sqrt{\frac{16 - 25}{2}} = \sqrt{\frac{25 - 8}{2}} \]

\[ x_{R} - x_{10} = \frac{1}{2} \left( \frac{R}{R^2} - \sqrt{\frac{25 - R}{R^2}} \right) \]

Of we want \( x_{R} - x_{10} = 1 \)

\[ \frac{R - x_{10}}{R} = \frac{1}{2} = \frac{R}{R^2} - \sqrt{\frac{25 - R}{R^2}} \]

or \[ \sqrt{2R} = 2\sqrt{25 - R^2} = 1 \]

Which leads to \( R = 5.534 \), i.e. \( D = 10.68 \text{ mm} \).

So we should make the bore 10.6 mm.

Bearing area is half of \( 20D \times 10.6 \text{ mm} = 113 \text{ mm}^2 \) so bearing stress is 885 MPa max at 100 kN, or somewhat more if the assembly is not in perfectly correct azimuth orientation & the segment is reduced in width of only a small amount & the same bearing stress applies in compression.
This could be $d = 19$ so for bearing
\[ \frac{\pi d^2}{4} - \frac{\pi 19^2}{4} = 113 \quad d = 22.5 \]

$M30 \times 3$ (coarse) has min. $d = 23.75$
$M25 \times 2$ (fine) $l = 22.8_{35}$
$M28 \times 2$ (fine) $l = 25.8_{35}$

Length of thread needed
\[ \pi \cdot 25.7 \cdot l = 200(19k, mm) = 100,000 \]
\[ l = \frac{100,000}{\pi \cdot 25.7 \cdot 200} = 6 \text{ mm} \]

So 10 mm length should be OK, $\sim 8$ mm actual thread.

Then thickness of insert can be 10 if bayonet need is 12 we have 2 mm of manoeuvre room.

$M25 \times 2$ thread will be more comfortable in the TLC.
The bearing area for the retaining block, of \( \phi d \), needs also to be about 113 mm\(^2\) if it is of the same hardness.

\[
\frac{\pi d^2}{4} - \frac{\pi (21)^2}{4} = 113
\]

or

\[
d = 24.2
\]

An M30 thread has minor diameter 26.2, so this would be OK. It would also suit a \( \phi 26 \) free block.

The depth \( x \) has to support the tensile ring in shear at \( \phi 20 \)

so

\[
\frac{4}{\pi} x = 100 \text{ kN} \text{ at } 500 \text{ MPa shear stress}
\]

\[
x = \frac{10 \times 10^5}{\pi \times 10 \times 500} = 6.4
\]

so \( x = 10 \) would be plenty; many 12 would give a better bending stiffness if there is room.

The depth of the bayonet head is similarly determined by

\[
\frac{1}{2} \cdot \pi \cdot 10 \cdot 10^5 = \frac{10^5}{500} \quad y = 12.7
\]

At 600 MPa, \( y = 10.6 \)

so probably \( y \) should also be made 12 mm.

We actually need the main drain of the hole to be about \( \phi 18 \) as at left in order to give proper lateral support to the compression rig.

The head of the bayonet is made \( \phi 18 \), its bearing area is \( \frac{1}{2} \cdot \pi \left( 18^2 - 10.6^2 \right) = 83 \text{ mm}^2 \) and so bearing stress at 100 kN is 1200 MPa still tolerable at the limit.

If inner \( \phi = 10.6 + 0.6 \) of bayonet = 17.4, area of bearing = 74.8 mm\(^2\) and stress at 100 kN = 1337 MPa while tensile stress in \( \phi 10.0 \) neck = 1273 MPa. So limit is probably the tensile stress in the neck. Assab 8407 HRC 45 minimum.
Axial Load Cell Details

The elastic element is now 520D 421D, giving a cross-sectional area without cut-outs, of 738 mm$^2$, or a stress of 135 MPa, strain of 0.00068. With four 90° cut-outs, the stress is 203 MPa and strain 0.0010. OK.

The anvil is now supported on segments on top of the elastic element, which are 49.5D 421D. With four 60° segments, the bearing area is $\frac{2}{3} \times 518 = 346 \text{ mm}^2$.

At 100 kN, the bearing stress is 289 MPa, which is 9% (in case of torsion load cell, this may be reduced to half area, giving 579 MPa bearing stress — high but tolerable). The shear stress in the neck A is $\frac{105}{\pi \times 0.042 \times 0.006} = 189 \text{ MPa}$, 8% (doubled it for torsion is still tolerable).

With four 45° segments, bearing area = $\frac{1}{2} 	imes 518 = 259 \text{ mm}^2$ at 100 kN, bearing stress = 385 MPa (torsion load cell would have $\frac{2}{3}$ of this area, giving 579 MPa bearing stress). The shear stress in neck A would be $\frac{105}{\pi \times 0.042 \times 0.006} = 253 \text{ MPa}$ (for torsion 379 MPa, giving fairly high, still marginally tolerable).

For the fluid feed-through 9 port section, shear stress on $\phi 12$ x 11 long is $\frac{48}{\pi \times 0.012 \times 0.0051} = 191 \text{ MPa}$.

Shear stress on $\phi 7.35 \times 5$ long is $\frac{0.0735 \times 700}{4 \times 0.005} = 257 \text{ MPa}$.
Modifications to bottom closure plug

1. Eliminate cross-holes directing gas flow up inside; instead, use flats on the O.D.

The cross-section of the input bore in the pressure vessel is about 2 mm². So if we have four flats, each need only have a cross-section of 1 mm² as so.

\[
\text{Area} = \frac{d^2}{8} \left( \alpha - \sin \alpha \right)
\]

\[
\frac{\sin \alpha}{\alpha} = \frac{2}{d} = 2 \frac{\sqrt{1-\left(\frac{h}{d}\right)^2}}{\left(\frac{h}{d}\right)^2}
\]

\[
\cos \frac{\alpha}{2} = \frac{d-h}{d} = 1 - \frac{h}{d}
\]

So area = \[
\frac{d^2}{8} \left( 2 \arcsin \left( \frac{-2h}{d} \right) - 2 \cdot 2 \frac{\sqrt{1-\left(\frac{h}{d}\right)^2}}{\left(\frac{h}{d}\right)^2} \right)
\]

\[
= \frac{d^2}{4} \left( \arcsin \left( \frac{-2h}{d} \right) - 2 \left(1-\frac{h}{d}\right) \frac{\sqrt{1-\left(\frac{h}{d}\right)^2}}{\left(\frac{h}{d}\right)^2} \right)
\]

With \(d = 65\), \(h = 0.5\), we get area = 3.8 mm²

\(c = 11.4\) mm

So a flat of 15 mm width is plenty.

2. Extend the length of the guide-sleeve to equal that of former sleeve + cap. Now close by an inner 5 mm M62x2 screwed cap, flush.

3. Eliminate the circlip & square off space for the new load cell arm.

Bearing areas for pistons:

1. Piston (2) with two flats, 27AF, 30 OD, 10 ID

\[
\frac{h}{d} = \frac{15}{30} = 0.05, \quad \frac{2h}{d} = 0.1
\]

so area flat = \[
\frac{30^2}{4} \left( \arcsin 0.9 - \frac{18 \cdot 0.05 \cdot 0.95}{\sqrt{1-(0.95)^2}} \right)
\]

= 13.2 mm² per flat, 26.4 mm² total

on 628.3 mm², leaving 602 mm² bearing area.

At 700 MPa pressure, this leads to \(\frac{700 \cdot 602}{10^5} = 822 \text{ MPa}\), plus \(105 \text{ KN force}\), \(\frac{105 \cdot 602}{988} = 166\)

still supportable on steel pistons.
2. Piston / string interface.

From case 1, we have a loss of $6 \times 13.2 = 79.2 \text{ mm}^2$ from area $30 \, \text{BD} \, 22 \, \text{ID} = 32.67 \text{ mm}^2$, leaving 24.7 mm$^2$, less a little bit due to 15° chamfer down to 28.5 mm at corners, thus we have $\sim 240 \text{ mm}^2$ under load $10^5 \text{ kN} \rightarrow 417 \text{ MPa}$.

This should be OK on strings of 718 similar steel at 800 MPa yield stress.

Nut: Given 32 A/F, leaving area is 424 mm$^2$, so bearing stress is 236 MPa - OK.
Shear stress in threads $= \frac{10^5}{\pi \times 20.38 \times 18} = 86 \text{ MPa}$
OK for 707 steel ($\gamma_{\text{str}} = 585 \text{ MPa}$).
Fibre 40 od 27 id 100 l 0.068 l; total gas vol 0.208
Pressure Vessel Volume
Length between mirror face of plug seal = 374
Φ = 65  so total volume = 1.2410 x 10⁻³ m³ (=1)

Volume of top plug inside seal: 65 Φ x 10 l → 0.0332 l
Similar at bottom
1.05 net volume between plugs = 1.175 l.

To calculate the free volume:
Add up the gas spaces, from top down:

Around top piston above furnace 30 od 21 id 10 l
  + Macor insulator 59 od 47 id 6 l
Core volume outside specimen assembly 27 od 21 id 130 l
Vol at bottom of furnace 61 od ~25 id 4 l
Assume 1LC at top of its travel
Volume around top of specimen assembly 19 od 10 id 18 l
Volume in bayonet slots ½ x 18 od 10 id 10 l
Vol around 9d of 1LC 58 od 52 id x ~30 l
Vol around capler plates 42 od 37 id x 12 l
  + 17 od 12 id x 12 l
  + 42 od 12 id x (6+5)
Gaps around components (clearances)
~ 4 x 0.050 x π x 370 l

Vol swept out by 1LC 52 od 30 id 30 l

Gaps around components (clearances)
~ 4 x 0.050 x π x 370 l

= 0.2810 l x 0.9 porosity
Total gas vol 0.3931
Argon cylinder (C14): 48 litres water, 8.6 cm³ argon at 16.917 kPa, 18 °C.
Density = 7.48 mol dm⁻³ (288K)

\[ \text{mol} = \frac{14.9 \text{ kg}}{0.03995 \text{ kg mol}^{-1}} \]

As at 20.1 kPa, \( \approx 17.7 \text{ kg} \)

Energy at 20 kPa, 298 K, enthalpy = 106.24 J mol⁻¹
energy = 2355.2 J mol⁻¹
At 0.1 kPa, energy at 106.24 enthalpy = -1431 + 725.8
= -705.6 J mol⁻¹

An adiabatic expansion from 20 kPa at 298 K, 7.60 kJ gives energy change of 2355.2 - (-705.6) = 3060.8 J mol⁻¹
A bottle contains 374 mol at 20 kPa, so energy available = 1145 kJ
(At 16.9 kPa, the figure is 968 kJ, ie \( \approx 1000 kJ \))
Energy calculations for Potassium enquiring
from Stewart & Jacobson tables (JPCRJRD). The energy of 500 MeV at 308 K is 7.78 J/mol K. So we can follow the expansion adiabatically by following constant entropy down to atmospheric pressure. If here 76 fall in between liquid & gas 9 so one assumes linear proportion of liquid & gas in calculating the enthalpy. The energy release is equal to the enthalpy change, which is 12437 - (-3053) J mol
= 15492 J mol⁻¹ or 397 kJ kg⁻¹.

Density of gas at 500 MeV 308 K is 38.5 mol dm⁻³ = 1.502 kg l⁻¹
so total argon content in pressure vessel is 0.590 kg.

Thus the total energy release is 0.590 x 397 = 234 kJ.

pv = RT would lead to an argon content of 5800 x 0.04065 x 0.039 = 7.93 kg dm⁻³ at 308 K or 8.06 kg at 298 K. For this amount of gas (3.17 kg in pressure vessel at 298K)
E = RT ln (P/P₀) gives 1700 kJ 7 times higher than for adiabatic expansion of real gas.

In case of PSZ furnace insulation, with 40 of 27 id x 110 mm fibres (= 0.068 l), total gas vol = 0.21 l,
or 0.315 kg x 397 kJ kg⁻¹ (above) = 125 kJ
\[ \sigma_r = \frac{2 \sigma_0}{\sqrt{3}} \left[ \ln \frac{A}{\delta} - \frac{D^2 - \delta^2}{2D^2} \right] \]

In plastic zone at inner, \( \delta \):

\[ \sigma_r = \frac{2 \sigma_0}{\sqrt{3}} \left[ \ln \frac{A}{\delta} - \frac{D^2 - \delta^2}{2D^2} \right] \]

\[ \sigma_t = \frac{2 \sigma_0}{\sqrt{3}} \left[ \frac{\delta^2}{D^2} - \frac{\delta^2}{A^2} \right] \]

\[ \sigma = \frac{2 \sigma_0}{\sqrt{3}} \left( \ln \frac{\delta}{A} + \frac{D^2 - \delta^2}{2D^2} \right) \]

(\text{tension positive})

\( \delta \) = dia of plastic front
\( D \) = o.d.
\( d \) = i.d.

\[ P = \frac{2 \sigma_0}{\sqrt{3}} \left( \ln \frac{\delta}{A} + \frac{D^2 - \delta^2}{2D^2} \right) \]

same as - \( \sigma_t \) at inner bore.
Premium Vessel Yielding Calculations with outer sleeve

We now take account of the extra support of the outer (safety/cooling) sleeve. Its actual OD is ~273 but it has semicircular grooves ~6 deep so we shall assume an effective OD of 265. The material used so far is Assab-AI-106 bar type 2L (AlSi1518) for which the minimum yield stress is given as 335 MPa for 16-30mm thickness. Let us assume 330 MPa. The ID of the outer sleeve is 230. To ensure that it remains in its elastic range, its internal pressure (equal to \( \sigma_r \) in the composite vessel) needs to be

\[
\sigma_r < \frac{330}{\sqrt{3}} \left[ \frac{2 \ln \frac{d}{8} - 1}{D} \right]
\]

with \( d = d_i \)

\[
\sigma_r < \frac{330}{\sqrt{3}} \left[ \frac{1}{\frac{265}{230}} - 1 \right] = -47 \text{ MPa}
\]

ie 47 MPa pressure.

The inner vessel yields out to diameter \( S \) when the pressure is

\[
p = \frac{\sigma_r}{\sqrt{3}} \left( 2 \ln \frac{S}{d} + 1 - \frac{S^2}{D^2} \right)
\]

But we also have radial stress component at diameter \( S \) given by

\[
\sigma_r = \frac{6p}{\sqrt{3}} \left( 2 \ln \frac{S}{d} - 1 + \frac{S^2}{D^2} \right)
\]

in the plastic zone.

Limiting \( \sigma_r \) to -47 MPa means

\[
47 = \frac{1100}{\sqrt{3}} \left( 2 \ln \frac{230}{265} - 1 + \frac{265^2}{230^2} \right)
\]
If we take into account the initial internal stress in the outer cylinder due to shrinking it on (see p 108), the T should be reduced to about 107, and hence S to 107 mm, which is reached when $p = 1164$ kPa.

To get 1500 MPa for 100% autofrettage, we need $q = 1596$, say 1600 MPa.

To get 1500 MPa, say 1500; 1489, say 1500; 1383, say 1400;
-47 = \frac{1100}{\sqrt{3}} \left( \frac{(5)}{265} \right)^2 \left( \frac{(8)}{230} \right)^4 \quad \text{from } \frac{8.40}{0.87} \text{ for elastic zone}

\text{or } s = 126

This is reached when

\[ p = \frac{1100}{\sqrt{3}} \left( 2 \ln \frac{126}{65} + 1 - \left( \frac{126}{265} \right)^2 \right) \]

= 1332 \text{ MPa}

ie at 1332 MPa, the yield point has extended out to 126 mm from the outer sleeve is just beginning to yield.

To get to 1500 MPa, another calculation shows that s will have reached \(150\) \(\mu\) with \(\sigma_y = -67\) MPa, so the yield strength of the outer sleeve to resist yielding would have to be \(\frac{67}{47} \times 330 = 470\) MPa.

The "100% autofrettage pressure", the max. to avoid reverse yielding on pressure release, is

\[ P < \frac{D^2 - d^2}{D^2} \sigma_y \quad \text{on Tresca criterion} \]

\[ = \frac{265^2 - 65^2}{265^2} \times 1100 = 1034 \text{ MPa} \]

To get to 1200 MPa for 100% autofrettage, we need \(\sigma_y = 1100 \times \frac{1200}{1034} = 1275 \text{ MPa} \)

or Brinell hardness \(\sim 410 \text{ (HRC = 4.4)}\)

Should one consider using managing steel, eg. Vascmax 300

\[ \sigma_y \sim 1400 \text{ MPa} \text{ (initial yield at the bore at } \sim 750 \text{ MPa).} \]
Stress Effect of Shrink-fitting Over Sleeve

Using Jaeger & Cook 1976 p.123, if we have an interference of \( \Delta u = 24 \)
where \( \Delta u \) is the displacement at the contact surface of inner & outer cyl.
rep. and if inner cylinder is ID = d, OD = D & outer cyl. SD rep.

\[
\Delta u = \frac{p s^3}{4 (\lambda + 2)} \left[ \frac{p d^2 s}{9 (s^2 - d^2)} + \frac{s^3}{4 (\lambda + 2) (s^2 - d^2)^3} \right] + \frac{p s d^2 s}{9 (s^2 - d^2)^2} + \frac{-p s d^2 s}{9 (s^2 - d^2)^3}
\]

If we put \( d = 65, S = 230, D = 265 \), we have

\[
\Delta u = \frac{\alpha 230}{4 (\lambda + 2)} (-1.9666) + \frac{\alpha 230}{4} (-3.9666)
\]

\[
\frac{\Delta u}{\alpha 230} = -1.9666 . \frac{230}{4 (\lambda + 2)} = -3.9666 . \frac{230}{\alpha 230} = 5.15 \times 10^{-6} \text{ MPa}
\]

\[
\alpha = 0.3, \quad \frac{E}{(1 + 2)} = 210 \text{ GPa}, \quad \frac{E}{(1 + 2)} = 80,770 \text{ MPa}
\]

\[
\frac{E}{(2 + 2)} = \frac{E}{(2 + 2)(1 - 2)} = 807, 692 \text{ MPa}
\]

\[
\frac{\Delta u}{\alpha 230} = \frac{230}{\alpha 230} \cdot 5.15 \times 10^{-6} \text{ MPa}
\]

ie \( \frac{\Delta u}{\alpha 230} = 0.4.4 \text{ MPa} \)

For an interference of 0.080mm (No 4 machine) we thus have \( p = 6.75 \text{ MPa} \) ie nearly 7 MPa.

This gives a hoop tension on the bore of the outer cylinder of

\[
\frac{D^2 + S^2}{D^4 - S^4} = 7.107 \times 6.75 = 48.0 \text{ MPa}
\]
Preliminary consideration for 14.9 GPa pressure vessel.

The best approach is probably that of Davidson & Kendall (1970 Pushed book p86) of using a duplex design with partial autofrettage of the inner cylinder and shrinking on the outer cylinder. We can initially consider staying with the same diameters as for the deformation rig's 700 MPa vessel. A preliminary calculation suggests that the use of 145 OD inner vessel of maraging steel, maybe 200 grade or, if we don't get a fully elastic cycling with this, the 250 grade, and we can probably use ASSAB 718 (600 MPa yield stress) on the outer vessel, saving heat treatment & restoring some additional safety.

The yield pressure of the outer vessel of 230 OD 145 ID, ie $K = \frac{2D}{D} = 1.586 \quad K^2 = 2.516$

in $\sigma_y = \frac{K^2 - 1}{2K^2} \frac{800}{5.032} = 1.516 \quad 800 = 241 \text{ MPa}$

So the inner vessel cannot be allowed to exert more than 241 MPa pressure on the outer shell. The application of this pressure produces a hoop compression in the inner cylinder of

$\sigma_0 = \frac{2 \times 145^2}{145^2 - 65^2} \frac{800}{241} = 603 \text{ MPa}$

which will in effect be an initial stress reducing the hoop tension at max. pressure by 600 MPa. Without this, the hoop tension at 1.4 GPa will be $1400 \times \frac{230^2 - 65^2}{230^2 - 65^2} = 1643 \text{ MPa}$, so the initial stress can support 600 of this, leaving 1643 MPa to be supported by the inner vessel itself. But the plastic yield pressure of the composite vessel is $\frac{200^2 - 65^2}{2 \times 200^2} = 632 \text{ MPa}$, so there needs to be another additional internal stress from autofrettage. This may be necessary to go to 250 maraging steel if not enough can be obtained in this way.
Possible Solid Insulation in Furnace

There still seems to be a problem with the development of gaps between the fibre insulation & the furnace core, at temperature — even at 1200 K (925°C). The AST insulation travels out over the high temp part of the core & presumably cannot be pushed back by the outer compliant layers either because of a hoop support effect from too small radial slots (these have been opened up in Minneapolis 926 furnace but not yet tested) or, perhaps more probably, from endwire support from the lower temperature parts of the AST. There are two possible approaches to overcoming this problem:

1) Make the insulation in three sections, with one of these being over the hot zone & separated from the two end zones by a layer of alumina paper which hopefully would all independent spring loading of the central section core to the hot zone of the core

2) Make the insulation immediately over the core out of a “solid” insulation such as porous alumina which will not develop the crumbling effect (it would have to be taken up in the inner alumina paper so maybe we would still have the same problem)

3) A combination of these two approaches.

In order to specify a solid insulation sleeve (still to be split into 120° sectors) we need to calculate its OD.

Take the case of 1700 K furnace temperature, a solid sleeve of $\kappa = 2 \text{ W m}^{-1} \text{ K}^{-1}$ & outer AST of 0.4 (361)

To find the optimum diameter of changeover, we need to solve
$K_i$: Porous alumina $\sim 2$, or 3.3
PSZ
mullite
Macoor
ASH
Stainless steel

reported at 273K, 16.5 - 373K, 24.5 - 973K.

For $x_2 = 35$

1. If we need $\frac{k_2}{k_1} > 0.8$, $k_2 > 0.8k_1$, $x_1 < 0.25k_1$

2. If $k_2 = 1.7$ (Macoor), we need $x_1 < 2.1$

3. If $k_2 = 2.4$ (PSZ), $k_1 < 0.8$

Thus, process 1 and 2 is OK with PSZ or Macoor only.

Thus, all is OK with ASH only.
\[
\theta = 2\pi \Delta T_1 e^{x/x_1} = 2\pi \Delta T_2 e^{x/x_2} \quad \text{with} \quad \Delta T = \Delta T_1 + \Delta T_2
\]

\[
\ln \frac{x_2}{x_1} = \frac{\Delta T - \Delta T_1}{\Delta T_1} \cdot \frac{x_2}{x_1}
\]

If we take \( \Delta T = 1400 \), \( \Delta T_1 = 400 \), then \(\ln \frac{55}{27} = 0.5 \cdot \ln \frac{x_2}{x_1} \cdot \frac{x_2}{x_1}
\]

Fix \( \frac{x_2}{x_1} = 0.4 = 5 \) we have \( x_2 = 28.6 \) \( x_2 = 25.7 \)

\( x_1 = 1.76 \) \( x_1 = 31.2 \)

\( x_1 = 2 \) \( x_1 = 30.7 \)

\( x_1 = 2.57 \) \( x_1 = 30 \)

So to drop the temp at \( x_2 \) to 1300 K from 1700 K we need the outer insulation to be not more than \( Y_2 = 0.39 \) times the thermal conductivity of the inner.

For \( \frac{x_2}{x_1} = 0.4 = 0.2 \), we have \( \ln \frac{55}{27} = 0.25 \) and \( \ln \frac{x_2}{x_1} = 1.42 \)

\( x_1 = 38.2 \)

\( x_1 = 35 \)

Thus, with a 35 OD sleeve, to drop the temp on the OD from 1700 K to 1300 K, the insulation of the inner sleeve needs to be at least \( 1 + 0.8 = 1.25 \) times that of the outer sleeve. If the outer were PS2, the inner needs \( x_1 \approx 1.9 \). If outer is HVAC, inner \( X \approx 1.4 \). If outer is ASHT at \( x_2 = 0.4 \), inner \( X \approx 0.32 \).

A possible configuration that would be...
The X of the 40-layer is turned nearest important near all part of.

\[ F = \frac{g_2 (g_2 + g_3) + g_3}{0.4 (g_2 + g_3)} \]

\[ g = 0.4 \]

\[ F = \frac{0.4 (g_2 + g_3)}{0.4} \]

\[ F = g_2 + g_3 \]

\[ g_2 = 0.1 \]

\[ g_3 = 0.3 \]

\[ F = 0.4 + 0.3 \]

\[ F = 0.7 \]
effective so long as the outer ceramic tolerated the temperature gradients would be:

The outer layer could be ASHT or the inner SALI. Alternatively, from the point of view of thermal conductivity & temp limits, the outer could be PSZ or Macor if the inner were porous alumina (e.g. Degussa AL25 if that will keep its dimensions). However, the Macor would certainly crack & the PSZ may do so. Maybe the outer could be porous alumina too. It could also be mullite. Or:

Over the 60 mm central section we would expect a radial heat loss of 
\[ q = \frac{2\pi \cdot 0.06}{\ln \frac{35}{27} + \ln \frac{50}{35} / 2.1 + 1.7} \]
\[ = 0.86 \text{ W K}^{-1} \]
\[ = 1.43 \text{ W K}^{-1} \]
\[ = 0.39 \text{ (0.31 for A40)} \]
\[ = 0.52 \text{ (0.01)} \]
\[ = 0.65 \text{ (0.071)} \]
\[ = 0.19 \text{ (0.3 say)} \]
\[ = 0.25 \text{ (0.3 say)} \]
\[ = 0.32 \text{ (0.3 say)} \]

Use of a φ40 x 90 long SALI in above any smaller would halve the gas vol in the pressure vessel.
If \( x_1 = x_2 \), \( \frac{\Delta T}{\Delta t} \) at \( \phi 40 = 0.49 \), i.e. \( \Delta T \) = 686 \( \frac{\text{mm}}{\text{AT}} = 1400 \text{K} \)

\( 1700 \text{K} \rightarrow 987 \text{K} \left( 714 \text{°C} \right) \).
Temperature drop over first layer:

\[
\frac{y}{2\pi l} = \frac{\Delta T}{\ln \frac{r_2}{r_1} + \ln \frac{r_3}{r_2}} = \frac{\Delta T_1}{\ln \frac{r_2}{r_1}} = \frac{\Delta T_2}{\ln \frac{r_3}{r_2}}
\]

\[
= \frac{\Delta T K_1 K_2}{K_2 \ln \frac{r_2}{r_1} + K_1 \ln \frac{r_3}{r_2}} = \frac{\Delta T_1 K_1}{\ln \frac{r_2}{r_1}} = \frac{\Delta T_2 K_2}{\ln \frac{r_3}{r_2}}
\]

\[
\Rightarrow \frac{\Delta T_1}{\Delta T} = \frac{K_2 \ln \frac{r_2}{r_1}}{K_2 \ln \frac{r_2}{r_1} + K_1 \ln \frac{r_3}{r_2}} = \frac{1}{1 + \frac{K_1 \ln \frac{r_3}{r_2}}{K_2 \ln \frac{r_2}{r_1}}}
\]

For \( r_1 = 27 \), \( r_2 = 40 \), \( r_3 = 60 \) & \( x_1 = 0.04 \) (SAL1) \( K_L = 2.2 \) (PSZ)

we have \( \frac{\Delta T_1}{\Delta T} = 0.842 \) \( \frac{\Delta T_2}{\Delta T} = 0.1579 \)

\( \Delta T = 1400 \text{ K} \), \( \Delta T_1 = 1179 \text{ K} \) \( \Delta T_2 = 221 \text{ K} \)

If we apply this to the aluminia paper & SAL1 combination, if assume their conductivities are the same, \( \frac{\Delta T_1}{\Delta T} = \frac{1}{1 + \frac{\ln \frac{0.0518}{0.27}}{\ln \frac{27}{27}}} = 0.074 \)

so temperature of the 1179 K temp drop about 85 K, or the order of 100 K occurs across the aluminia paper layer.
Heat loss due to convection through a clearance

Suppose we had a clearance of $C$ on the radiuses around a core tube that extended from the hot zone to the top of the furnace, a distance of 100 mm, and that there is a flow of gas driven by the temperature inside that outside, i.e.,

$$\frac{\Delta P}{L} = \left(\frac{\rho_{in} - \rho_{out}}{\rho_{in}}\right) \times 9.8 \text{ at 310 KPa}$$

$$= \left(\frac{1373 - (\text{gas temp})}{\rho_{in}}\right) \times 9.8$$

$$= 7500 \text{ Pa m}^{-1}$$

The Poiseuille formula then gives a fluid flow of (p85)

$$J = \frac{\pi d C^2 \Delta P}{12 L}$$

For $d = 0.027$, $C = 90 \times 10^{-6}$, we have

$$J = \frac{\pi \times 0.027 \times C^2}{12 \times 90 \times 10^{-6}}$$

and allowing $220 \times 10^6$ $m^{-3}$ heat transfer (p82) we have a heat flux of

$$Q = \frac{\pi \times 0.027 \times 7500 \times 220 \times 10^6 \times C^3}{12 \times 90 \times 10^{-6}}$$

$$= 1.296 \times 10^{14} \text{ W m}^{-2}$$

For $C = 10 \mu m$,

<table>
<thead>
<tr>
<th>$C$ (mm)</th>
<th>$Q$ (W)</th>
<th>$1600 \text{ K}$</th>
<th>$0.0001$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.1</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>20</td>
<td>1</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>30</td>
<td>3.5</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>50</td>
<td>16</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>75</td>
<td>55</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>120</td>
<td>130</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>150</td>
<td>437</td>
<td>0.006</td>
<td>0.01</td>
</tr>
<tr>
<td>200</td>
<td>1037</td>
<td>0.006</td>
<td>0.01</td>
</tr>
</tbody>
</table>

This indicates that we don't get serious convective losses up such a clearance until it is of the order of 50-100 $\mu$m (100-200 $\mu$m on the diameter), which seems to vary with experience.
Equivalence to gap with alumina paper.

Poiseuille flow in gap: \( J_1 = \frac{\pi d c_1^3 \Delta P}{8} \)

Permeation through paper in gap: \( J_2 = \frac{\pi d c_2 \frac{k}{2}}{L} \)

so \( J_1 = J_2 \) when \( \frac{c_1^3}{12} = c_2 k \) or \( c_2 = \frac{c_1^3}{12k} \).

<table>
<thead>
<tr>
<th>h/\mu m</th>
<th>( k = 10^{-11} \text{ m}^2 )</th>
<th>( k = 10^{-12} \text{ m}^2 )</th>
<th>( k = 10^{-13} \text{ m}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>8</td>
<td>83</td>
<td>830</td>
</tr>
<tr>
<td>20</td>
<td>67</td>
<td>670</td>
<td>6,700</td>
</tr>
<tr>
<td>30</td>
<td>225</td>
<td>2,250</td>
<td>22,500</td>
</tr>
<tr>
<td>50</td>
<td>1040</td>
<td>10,400</td>
<td>104,000</td>
</tr>
<tr>
<td>75</td>
<td>3,500</td>
<td>35,000</td>
<td>350,000</td>
</tr>
<tr>
<td>100</td>
<td>8300</td>
<td>83,000</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>28,000</td>
<td>281,000</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>67,000</td>
<td>670,000</td>
<td></td>
</tr>
</tbody>
</table>

Thus if alumina paper of \( k = 10^{-12} \) fills a gap of \( 0.67 \mu m \), it will have the same flow as a free gap of 20 \( \mu m \), and the area of the paper is several times greater, this free gap would be only slightly larger. If the \( k = 10^{-11} \), a gap of 0.5 \( \mu m \) is equivalent to a gap of \( 0.04 \mu m \). Thus a free gap of 20-40 \( \mu m \) is equivalent to a paper layer of order of 0.5 \( \mu m \), i.e. one compressed layer of A4/A2 paper. But from the figures on the previous page, such a gap would give rise to less than 0.01 W/K heat loss, which still seems to be contrary to experience.
If we try a dead fit, without alumina paper, we could have the situation:

\[ \Delta = \Delta r - \frac{\Delta r}{2} = \frac{1}{2} \Delta r. \]

In case of differential thermal expansion with temp diff \( \Delta T \),

\[ \Delta r = y \cdot \alpha \cdot \Delta T \]

= 27.8, 10, 200

= 0.043, 16, 43 \mu m.

So a relatively small radial temp. gradient will produce a good mating if initially the inner diameter is a few tens of pm less than the outer one. We can probably get away with dead fits between core & insulation outside the windings zone, with a single layer of alumina padding between the segments, if a cut of 0.75 mm is feasible, otherwise a double layer in a cut of 1.5 mm, with a double layer on the outside for elastic loading.
Upper limit to wall temperature based on radial flow of heat:

\[ \Delta T = \frac{Q}{T} \ln \frac{H}{d} \]

At 0.6 W/K, power at 650 K = 800 W over length of 100 mm.

\( k = 29 \) for H/3, 0.351 for mild steel. Take an average of 0.35

Then \( \Delta T = \frac{800}{0.1} \ln \frac{210}{65} \approx 52 K \)

The max temp of wall is \( \approx 62^\circ C \) if all heat radial. But a lot goes out axially so the wall temp should not rise more than \( \approx 20-30^\circ C \), i.e. 70-30-40^\circ C
Volume Change Measurement with Intensifier

State of intensifier = 302 mm
No clearance needed over floor, is about 340 allowing 12 for the corner & 25 freeboard below it. Present ground clearance is about 220, so it would be necessary to raise the intensifier by 120 mm.

Thus would be about 530 mm of space for the LVDT, allowing a ±125 DC-LVDT length 512
or a ±75

If we choose the ±125 range = 250 travel, then
Displacement resolution at 12-bit is 0.061 mm, corresponding to a volume resolution (Φ41 piston) of 80.6 mm³. This could be improved to 12.1 mm³ with the ±75 range & 14-bit resolution.

The other limiting factor is the resolution of pressure detection. In a 250 MPa with a meter resolution of 1 in 20,000 (2.14bit), minimum pressure fluctuation detectable is 0.025 MPa. The compressibility of argon at room temperature is 300 MPa is about 0.84 GPa⁻¹ or 0.00084 1/MPa⁻¹. The volume of argon is ~200,000 mm³ of solid insulation in furnace & 400,000 with all fiber plus ~200,000 for half the intensifier volume, so we can reckon on something like 600,000 mm³

\[ \text{so } \Delta V = \text{400,000} \times 0.035 \times 0.00084 \]

\[ = 12 \text{ mm}³ \]

That is, the resolution for the LVDT is about the same as for the pressure control sensitivity, and is limited to around 12 mm³ under the most favorable circumstances—probably can't reckon in better than twice this in practice. 57% of a Φ20 x 20 (long spec is 79 mm³) so this is 1/15th of such a volume.
Another limitation comes from temperature control on the gas. In the best case of solid insulation except over the windings, the volume at temperature is about

\[
(\phi 21 - \phi 15.5) \times 150 \text{ length} \quad \text{inside core} = 30,000 \text{ mm}^3
\]

\[
(\phi 40 - \phi 27) \times 110 \text{ length} \quad \text{outside core} = 75,000 \text{ mm}^3
\]

At 300 MW & 1000 K, \( \frac{1}{V} \frac{dV}{dT} = 19.698 \left( \frac{20.293 - 19.134}{100} \right) \)

\[
= 585 \times 10^{-6} \text{ K}^{-1}
\]

so for 105,000 mm\(^3\), \( \frac{\Delta V}{\Delta T} = 61.5 \text{ mm}^3 \text{ K}^{-1} \)

so to achieve 12 mm\(^3\) sensitivity the temperature control has to be within 0.2 K i.e. \( \pm 0.1 \text{ K} \), which is not very realistic.

Thus with volume measurement by intensifier monitoring it would be difficult to do better than 50-100 mm\(^3\) sensitivity compared with 1570 mm\(^3\) specimen volume [at high temperature & pressure], without going into apparatus distortion corrections (these should cancel out at count pressure).

**Comparison with volumometer in por fluid system**

If we had a close-fitting sleeve over the specimen assembly, with bellows at the bottom sealed at both ends, connected to the por fluid volumometer & kept at the same pressure as the conjoining pressure, the volume in the assembly might be \((\phi 15 - \phi 10) \times 25 \text{ length} = 2500 \text{ mm}^3\) plus the volume in the volumometer, say 4 stroke = 125 \( \times \phi 7 = 450 \)

say 3000 mm\(^3\) altogether instead of the 400,000 in the intensifier case. The pressure fluctuation limit on the \( \Delta V \) detection is thus reduced by a factor of 300 \( \approx 1/100 \) or \( \approx 0.1 \text{ mm}^3 \). The limit is set by the displacement resolution, say 1/20,000.
50 mm for Turn 0, again 0.1 mm

The temperature fluctuation limit is $\frac{3000}{10^5} \times 61.5 = 1.8 \text{ mm}^3 \text{ K}^{-1}$

as one may be able to get 1-2 mm$^3$ resolution or about 0.1% of the maximum volume.

Could have an arrangement as at left, connected to a columnometer (may need an additional one to the pore fluid system). The main jacket is sealed with O-rings on the ID.

Could form an outer jacket by expanding an inner tube or then putting in corrugations test by running a tool into a split die, e.g., in milling machine by revolving the table.

The seals at the end could possibly be made by mesh taper fits, with some gasket goo, & having the tube up against shoulder for compression exists.

Seal mesh goo

Seal mesh in oil

Seal mesh good because no pressure differential
Electrical Connections for Physical Properties Measurement

There are three types of measurement that might be of interest:
- Electrical conductivity
- Thermal conductivity
- Elastic wave speed

Electrical conductivity: At its simplest, one active lead and one return. But in a more sophisticated measure (wire method), at least three active leads would be needed, maybe two from one end and one from the other.

Thermal conductivity: Need at least three thermocouples or divided bar method, i.e., 6 leads.

Elastic wave speed: One transducer needs one active lead, P&S speeds can be got from one passdown of the right cut. However, for S polarization, studies probably two leads would be needed, for 2 passdowns.

In general, the measurements need to be made inside the specimen jacket. The exception to this could be in the case of elastic measurements involving pulse transmission with a receiver at the opposite end to the sender. Then we may need a lead through the bottom end. It could involve a transducer set in the bottom closure to the jacket with a banana type plug entering a socket attached to the feed-through.

For measurements inside the jacket, leads could be taken down a hollow piston and then out through grooves in split ceramic pistons.

Another requirement for both electrical & thermal conductivity will be an insulating sleeve around the specimen, with connection for conduction through it & the non-jacket.
For PSZ (Ts) $\alpha = 4.9 \times 10^{-6}$ at 25 - 400°C

Stavax $\alpha = 1.4 \times 10^{-6}$ 25 - 400°C

$\Rightarrow$ = PSZ

$\equiv$ = steel
INTERNAL LOAD CELL PRESSURE COMPENSATION

We assume that the dielectric is purely argon so as to avoid complications due to Mylar films. The plates are supported on a combination of steel (5THFN) and PSZ.

From Kaye & Labey, $K$ for steel is 165 (footnote) = 166 (55) + 169 (175). We take a value of $K = 166$ for the steel in GPa.

From Niles data, $K = 206$ (GPa), $v = 0.31$, $K = \frac{E}{3(1-v)} = 180$ GPa \( \text{m/PSZ} \).

The coefficients of thermal expansion are very close the same & so temp. compensation should be OK whatever the proportions of steel & PSZ.

Consider the situation at the left when pressure is applied (above the 18 gap). The steel shortening should be exactly compensated, and refer all displacements to the base.

Displacement in a length $L$ is given by

$$ \delta = \frac{PL}{3K} \quad \text{where} \quad P = \text{pressure} \quad K = \text{bulk modulus} $$

So

$$ \frac{\delta}{P} = \frac{L}{498} \quad \text{for steel} \quad \frac{L}{540} \quad \text{for PSZ} \text{ in \ N/\m}^2 $$

Then at A, for downward displacement $\delta_A$,

$$ \frac{\delta_A}{P} = \frac{18 - \frac{5}{498}}{498} - \frac{1.5}{540} = \frac{13 - 1.5}{498} \frac{540}{540} \quad \text{mm/\m}^2 $$

At B,

$$ \frac{\delta_B}{P} = \frac{11.2 - \frac{x}{498}}{540} \quad \text{mm/\m}^2 $$

Then the increase in the gap AB =

$$ \frac{\delta_B - \delta_A}{P} = \frac{-1.8 + x}{498} + \frac{2 + 1.5}{540} = \frac{DAAB}{P} $$

At C,

$$ \frac{\delta_C}{P} = \frac{6.2 - \frac{x}{540}}{498} $$

At D,

$$ \frac{\delta_D}{P} = \frac{18 - \frac{7.6}{498} - \frac{4.5}{540}}{498} \quad \frac{10.4 - 4.5}{540} $$

So increase in gap CD =

$$ \frac{\delta_D - \delta_C}{P} = \frac{4.2 + x}{498} - \frac{4.5 + x}{540} = \frac{DACD}{P} $$

Since increasing $x$ goes in the wrong direction for compensating pressure effects on the zero, there is no point in keeping $x$ variable, so we put $x = 1.5$, leading to
490D 421D - Section 500 mm², axial displacement 64 mm (1.48x) 15 mm (1.74x)

480D 421D = 62.4

Yield 500D 421D, cross-section = 578 mm² instead of 738 for 5242T.
Incorporates stresses below by 1.28, giving axial displacement = 55 mm.

Actual length = 54 m, of which 23 mm has cut-outs reducing its cross-section to 2/3, & the remaining 31 is full. So axial displacement
at 100 kN = 0.000661 \times \left(31 + \frac{3}{2} \times 2.3\right)
= 0.0228 mm = 22.8 mm 43.3 mm

ie. of order of 20 mm.

540D 431D = 671.5 mm², 100 kN \to 148.9 kPa, strain = 0.000726 on solid
0.001090 on cut-out part.

In torsion, with mean dia = 49 say, shear force = 20,400 N, a shear
stress on 671.5 mm² = 30.4 MPa, strain = 0.000380 on solid
0.000570 on cut-out part.

Shear displace = \left(0.000380 \times \left(31 + \frac{3}{2} \times 2.3\right)\right)
= 0.0249 \approx 25 mm

\tau = \frac{16}{\pi} \frac{TD}{D^4 - d^4}
= \frac{16}{\pi} \frac{1000 \times 0.054}{(0.054)^4 - (0.043)^4} = 54.1 MPa \approx 0.000576 \text{ strain on solid}
0.000041 on cut-out part.

Circumferential displace = \left(0.000676 \times \left(31 + \frac{3}{2} \times 2.3\right)\right)
= 44.3 mm.
Increase in upper gap AB: \[
\frac{\Delta AB}{P} = -\frac{3.3}{498} + \frac{3.0}{540}
\]
\[
= -0.001071 \text{ mm/MPa}
\]
\[\text{an increase}\]

Increase in lower gap CD: \[
\frac{\Delta CD}{P} = +\frac{0.000335}{\text{mm/MPa}}
\]
\[\text{an increase}\]

We can write \[
\frac{\Delta AB}{P} = -0.000368 - 0.000703
\]
and \[
\frac{\Delta CD}{P} = -0.000368 + 0.000703
\]

So relative to a balance point, there is an imbalance of \[0.000703 \text{ mm/MPa}\] in the direction corresponding to compressive loading.

The cross-section of the elastic element is \[52.02 \times 42.12 = 788.3 \text{ mm}^2\]
so 100 kN gives a stress of \[100 \times 1000 / 788.3 = 130.5 \text{ MPa}\] or a strain of \[130.5 / 205,000 = 0.000641\] or a displacement over the 25 mm length of \[0.0165 \text{ mm} = 16.5 \mu \text{m}\] (may be of greater due end only).

Thus the above predicted zero shift of \[0.000703 = 0.7 \mu \text{m/MPa}\]
or \[0.013 \mu \text{m for 300 MPa}\] is equivalent to \[0.00048\]
\[0.00013 \text{ of full scale} = 1.3 \text{ kN in 100 kN scale}.\] This is not very serious. It could be reduced to 0.9 by reducing the PSZ insulator to 1 mm instead of 1.5 mm.

\[= 0.32 \text{ kN for 1 mm gap at 100 MPa} \]
\[\text{ie 0.1 kN per 100 MPa}\]

Effect of Mylar film

If the capacitance gaps are not equal, there will be different proportions of Mylar film & argon in each so a pressure effect on the load zero could arise.

The relative permittivity of Mylar is 3.2 Ceradale service center data and that of argon (assumed proportional to density) & interpolating between normal pressure and liquid argon (1.53 at 82K at rel. density 1.66, 82K) is \[0.1 \text{ MPa: 1000, 104 MPa, 105}\]
\[30 MPa, 1.15, 100 MPa, 1.30 \& 300 MPa, 1.41.\]}
Rel. displacement of plates with pressure

\[ \Delta = \frac{1}{3} \frac{P}{K_{\text{steel, element}}} - \frac{1}{3} \frac{P}{K_{\text{steel, plate}}} - \frac{1}{3} \frac{P}{K_{\text{PSZ}}} \]

If we count 4 x PSZ spacers of thickness \( x \), we have

\[ \Delta = \frac{1}{3} \frac{72/1.60}{16600} \left( 4.5x \right) - \frac{1}{3} \frac{P}{180000} \]

For \( x = 0.81 \) mm and \( P = 300 \) MPa,

\[ \Delta = \frac{21}{1660} \cdot \frac{4.5}{1800} = 0.0002 \text{ mm} = 0.21 \mu \text{m} \]

If PSZ digit for 100 kN = 43.3 \( \mu \)m (previous page),

Then \( \Delta \) comes to \( \frac{0.21 \times 100}{43.3} = 0.49 \) kN from 300 MPa

or \( 0.00016 \) kN per 100 MPa.

Actual shift (9/1/95) is 0.5 \( \mu \)N for 300 MPa, it about 25 \% that predicted.
\[ \frac{\Delta u}{V} = \frac{0.3 - 0.52}{0.52 + 0.32} - \frac{0.3 + 0.3 + A}{0.60 + 1.44} \]

\[ = \frac{1 + 1.44 A}{0.52} - \frac{1 + 1.44 A}{0.60} \]

\[ = \frac{2 + 5.26 A}{2 + 5.67 A} - \frac{2 + 9.57 A + 10.89 A^2}{2 + 21.86 A + 29.82 A^2} \]

\[ = - \frac{0.41 A + 1.37 A^2}{4 + 21.86 A + 29.82 A^2} \approx - \frac{0.10 A}{1 + 5.47 A} \]

\[ \approx - 0.10 A + 0.55 A^2 \]

Thus for \( A = 0.01 \) mm, \( \frac{\Delta u}{V} = 0.0010 \)

Full scale \( \Delta u = \frac{E}{2d} = \frac{0.0165}{0.6} = 0.0275 \), so \( \frac{\Delta u}{V} = 0.001 \) corresponds to 0.036 of full scale, i.e. to \( \sim 3.6 \text{ kV} \), i.e. a 10\% of...
For $\Delta = 0.03$ mm, a 10% gap difference, $\frac{\Delta V}{V} = 0.003$ compared with full-scale load $\frac{\Delta V}{V} = 0.0275$, i.e. a 0.11% full-scale zero shift, or 11 kN -- quite significant.

For 1 kN shift $\Delta = 0.00275 = 0.01$, i.e. $\frac{\Delta V}{V} = 0.002275 = 0.1\%$

so $\Delta = 0.003$ mm on 0.3 mm, i.e. the gaps have to be exactly equal to within 1% when a 0.1 mm Mylar film is inserted in a 0.3 mm gap.

More generally, from p.124, $\frac{\Delta V}{V} = \frac{1}{1 + \frac{\varepsilon_2}{\varepsilon_1}}$

If we put $d_1 = d$, $d_2 = d + \Delta$

$\varepsilon_1 = (0.3 - 0.1) \varepsilon + 3.2 \times 0.1 = 0.32 + 0.2\varepsilon$

$\varepsilon_2 = (0.3 + \Delta - 0.1) \varepsilon + 0.31 = 0.32 + (0.2 + \Delta) \varepsilon$

Pressure effect in $\frac{(V)}{V}$ atm

$\Delta V = \frac{\left(0.32 + 0.2\varepsilon\right)(1 + \Delta)}{0.32 + (0.2 + \Delta)\varepsilon} - \frac{0.52}{0.52 + \Delta}

= \frac{0.32 + 0.2\varepsilon}{0.32 + (0.2 + \Delta)\varepsilon} \left(1 + \frac{\Delta}{d}\right)

\left(0.52 + \Delta\right) + \frac{0.32 + 0.2\varepsilon}{0.32 + (0.2 + \Delta)\varepsilon}

With $d = 0.3$, $\Delta = 0.03$

$\frac{\Delta V}{V} = \frac{0.32 + 0.2\varepsilon}{0.32 + 0.23\varepsilon} - \frac{0.9455 - \varepsilon}{0.9455 - \varepsilon}

= \frac{0.9091 + 0.9455 + \varepsilon}{0.9455 - \varepsilon} + 1.04 f(\varepsilon)

\frac{0.9455 - \varepsilon}{0.9455 - \varepsilon} + f(\varepsilon)

\frac{0.9091 + 0.9455 + \varepsilon}{1.8546 + 2.04 f(\varepsilon)}

\frac{0.9342}{1.415}

\frac{0.9342}{500/112}

\frac{0.9030}{100 kN load (p.124), it is equivalent to 10.9 kN offset due to imbalance stress with the presence of the Mylar film.
\[
\text{mol/dm}^3 \times 10^3 \times 0.039 = \text{kg m}^{-3}
\]
Experience with half-bridge 2 capacitance load cell

The following circuit has been recently used:

After many problems a reading was obtained that gave reasonable correspondence between internal & external load cells. However, the load cell zero shifts upwards (compression direction) as the pressure increases, about 12 kN for 300 MPa pressure. The increase is non-linear, rather like that of the argon permittivity, but it shows some hysteresis which is hard to account for. The main effect is probably due to a phase shift between A & B leading to a dip in the initial annulment of the voltages at these points. Maybe the raising from the change in capacitances in the load cell due to increasing gas density. Maybe the hysteresis is somehow an additional effect due to changed capacitance to earth in a yield-through or something like that, although such an effect would be expected to be small.

To avoid this effect it may be necessary to return to a full-bridge arrangement. This would also double the output & may lend itself well to the use of the Analog Devices AD578 voltage signal conditioning card, which measures A-B & A+B, i.e. radiometrically, making it insensitive to excitation fluctuation and to phasing problems.
from p78, \( \frac{v}{\sqrt{V}} = \frac{d_1}{d+d_1} \) since \( \cot \theta = \frac{d_1}{d+d_1} \)

so if \( d_1 = d+S \), \( d = d-S \)

\[ \frac{v}{\sqrt{V}} = \frac{1}{\sqrt{d+S+d-S}} = \sqrt{V} \]

\[ \frac{d-S}{\sqrt{d-S+d+S}} = a \]

\[ \sqrt{V} = \frac{d+S-(d-S)}{2d} = \frac{1}{\sqrt{d}} \]

\[ \sqrt{V} = \frac{d+S+d-S}{\sqrt{d+S+d-S}} = \frac{1}{\sqrt{V}} \]

\[ \frac{V_A - V_B}{V_A + V_B} = \frac{s}{d} \]

Full bridge circuit with 5 plates: In this case, the capacitance to earth of the outer plates is much larger than for the central plate, so there is an out of balance inside the pressure vessel when it is pressurized for a balanced capacitance bridge.

Thus this is an unsatisfactory arrangement
$C_1$ and $C_2$ are $\approx 60 \, \mu F$ at 0.1 MHz, $\approx 80 \, \mu F$ at 300 kHz.
$C_5$, $C_6$ are mainly capacitors to earth from outer plates, $C_3$, $C_4$ from leads in stem.
$Z_7$ is impedance to earth mainly through feed line.

$60 \, \mu F = 0.76 \, \Omega \text{ at } 3.5 \, \text{kHz}, 0.53 \, \Omega \text{ at } 5 \, \text{kHz}, 0.35 \, \Omega \text{ at } 7.5 \, \text{kHz}$

So $Z_9$ and $Z_{10}$ could be about $500 \, \Omega$ each, with a small pot in between for initial zeroing. Alternatively, they could be capacitances of about $60 \, \mu F$ for 5 kHz.

$45 \, \mu F$ for 7.5 kHz.
Optimum Circuit for Half-Bridge Load Cell (Capacitance)

Mike Gladwin would like to see \( z_9, z_{10} \) as an upper
centre-tapped differential anti-transformer, which is balanced so that \( E + V \) are at the same voltage at each measurement,
recording the change in balance point of the transformer.
However, we are operating in out-of-balance mode so are sensitive to the value of \( z_7 \), the impedance to earth through the feed through from the centre plate,
in the case of a three-plate load cell with excitation on the outer plates.

If impedance \( z_7 \) is infinite, we have \( V = V_a \left( \frac{z_1 - z_4}{z_1 + z_2} \right) \)
But when \( z_7 \) is finite, we see a voltage
of \( V \times \frac{z_7}{\frac{1}{z_2} + \frac{1}{z_6}} = \frac{1}{\frac{z_7}{\frac{1}{z_2} + \frac{1}{z_6}} + 1} \)

If \( z_7, z_2, z_6 \) are all pure capacitances, we get \( \frac{1}{\frac{z_7}{\frac{1}{z_2} + \frac{1}{z_6}}} \)
Thus it is desirable that the impedance of \( z_7 \) be small
compared with \( z_1 \) and \( z_2 \). \( z_7 \) is made up of the
capacitance of the feed through and the resistance in parallel

Provided \( C_3 \) and \( C_4 \) are equal \( C_5 \) and \( C_6 \) should also be
made equal, so that \( \frac{1}{z_9} \times \frac{1}{z_2} + \frac{1}{z_5} \)

\[
\frac{1}{z_9} + \frac{1}{z_2} + \frac{1}{z_5} = \frac{1}{z_2} = \frac{z_2}{z_1}
\]

Since \( z_2 / z_1 \) should stay constant during raising pressure,
and also \( z_9, z_3, z_0 + z_6 \) should stay constant, then it
is essential that \( z_5 = z_6 \) for pressure independence when
we initially set \( z_1 = z_2 \) (i.e. \( C_1 = C_2 \)).
Temperature Sensitivity of Load Cell Zero

For PSZ (TS grade) \( \alpha = 9.9 \cdot 10^{-6} \text{K}^{-1} \) at 25-400°C

"STAVAX" \( \alpha = 11.4 \cdot 10^{-6} \text{K}^{-1} " \)

Difference is \( 1.5 \cdot 10^{-6} \) (eqn. 11.12)

In original 3-plate version there is 4.2 mm of PSZ insulators that are in parallel with STAVAX.

So differential expansion = \( 4.2 \times 1.5 \cdot 10^{-6} \) mm per K

\[ = 0.0063 \text{ mm K}^{-1} \]

At 100 kN load, displ = \( \frac{43}{100} \text{ mm} \) (eq. 12.3)

So 1 K gives effect of \( \frac{0.0063 \cdot 100}{43} = 0.015 \text{ kN} \)

10 K " " " " 0.15 kN

50 K " " " " 0.75 kN

Increase in temperature will be equivalent to applying tensile load.

We could halve this effect by going to 0.5 mm insulators, \( = \text{total} 2.0 \text{ mm PSZ} \).
Hysteresis in Internal Load Cell

Revealed in tests in Potadham with spring – observed displacement corrected for apparatus distortion on basis of load vs external force with solid block in place of spring.

Max width Δ of loop was 

~ 4 kN for amplitude 60 kN

This may have its origin in friction at the bearing surface S between the loaded supporting block and elastic element E of the load cell.

The load is transferred from the box to the element E mainly in shear but there will presumably be some Poisson expansion associated with the axial loading on loading surface itself, which is 250 D 191D. Spreading the load to 35 D 191D gives an effective cross-sections of 680 mm, or a normal stress of 1.5 MPa, a strain of 7.10^-6 per kN force.

The Poisson expansion strain is then 2.3 x 10^-6 per kN.

On the elastic element, the bearing stress on the 520 D 42D = 738 mm² is 1.36 MPa per kN force, but this is supported on only half the area – however, the unloaded part has to be stretched too so may be we take it averaged over the whole. 1.36 MPa = 6.5 x 10^-6 axial strain / kN or Poisson expansion strain of 2.2 x 10^-6. At mean diameter 47, this means a displacement of 101 nm per kN.

In the box the Poisson strain is to be referred to a smaller diameter, ~ 27, giving a mean displacement of 63 nm.
Thus there will tend to be a mismatch of $40 \mu m$ per kN. If this is spread out to a depth about equal to the width of the bearing area on $E_v$, viz. $5 \mu m$, it corresponds to a shear strain of $8 \times 10^{-6}$ or a shear stress of $0.04 \text{ MPa}$, a shear stress of $470 \text{ N}$ for a normal force of $1 \text{ kN}$. So will be coefficient of friction of $0.4$ for dry steel, this section might be at the verge of slipping (but the friction is about $0.8$, so the sliding should not begin).

However, since the frictional resistance & the build-up of Poisson's expansion mismatch, if any, are both linearly proportional to the axial load, the situation should not change with increasing load. Thus an explanation of the slope change from this source is hard to believe.

There is another possible effect from Poisson's expansion of the slot B, viz., bending on the plate carrier C, which is not under axial load. At the location $L$, the internal diameter of the carrier is $36.990/36.025 \text{ mm}$ and the OD of the boss is $35.991/35.975 \text{ mm}$. So the diametral clearance is between $0.009$ and $0.050 \text{ mm}$. If we take a Poisson's expansion strain of around $23 \times 10^{-6} \text{ per kN}$, estimated above, for the boss, then the decrease in clearance at $36 \text{ kN}$ will be $83 \times 10^{-6} \text{ per kN}$ or $0.003 \text{ mm}$ at $35 \text{ kN}$. If we take into account the stress concentration due to the load being supported on four $45^\circ$ lugs, this quantity might be nearly doubled to around $0.005 \text{ mm}$ or so. But it is still hard to imagine that it will lead to binding of B in C. Another component of strain could come from the bending effect due to the load being applied at small radius of support at B. In fact the carrier C is supported...
at an azimuth 45° from where the head is supported on C. Might the "tending" effect could lead to a 30 binding. Opposite the lugs could have a greater effect on the plates, being applied between the points of support of the cameo C.

If the 0.005 mm expansion estimated above were increased to double by the "tending" effect, then we could be looking at 0.010 mm, maybe just beginning to bind, at ~ 35 kN. At 100 kN, the effect would then become ~ 0.030 mm, very likely binding firmly.

So while the explanation for the hysteresis effect in this way is marginal, it seems not impossible, in view of the complex nature of the stress situation.

However, the magnitude of the effect is very considerable via a 25% increase in effective compliance, can this be accounted for?

At the $36$ at the root of the supporting lugs of B, the area is $\frac{1}{2} \times 30.6 = 15.35 \text{ mm}^2$ giving a shear force of 3.54 MPa or a shear strain of $4.4 \times 10^{-5}$ or a displacement of 12.10 mm per kN axial load, over the 3 mm radial clearance between the supports E & the boss B. From p 123, the total displacement in E is 0.23 mm = 228.10 mm, so it would be easy to increase this by 25% by adding only a portion of the extra 132.10 mm (this would be reduced by the need to deform the cameo C), viz 57.10.

So the explanation of the hysteresis from binding of B in C may be feasible during loading. But why shouldn't the unloading be along the same line?? The story does not work for unloading — we still need a frictional sliding that comes into effect after a certain amount of displacement from either end of the hysteresis loop.

Examination turned out to be frictional binding between insulators & one plate & clearance hole in other plate. Reduced diameter of insulators.
Total Twist in length is
\[ \theta = \frac{32 \ell M}{\pi (D^2 - d^2) G} \]

\[ = \frac{32 \times 0.054 \times 1000}{\pi (0.049^2 - 0.042^2)} \approx 0.10^9 \]

\[ = 0.0026 \text{ radians} \]

\[ = 0.15^\circ \]
Internal Force Cell for Torsion: Plate Arrangement

With any angle at left, the same excitation is used for both axial and torque loads.

With the plates 10D, 14D, and six cut-outs, we have an area of 935 mm². There has to be a further cut-out for the torsion plates of roughly 10 x 10 x 2 = 200 mm², so net capacitance area is of the order of 750 mm², giving a capacitance of

$$\frac{\varepsilon \varepsilon_0 A}{d} = \frac{1.3 \times 8.85 \times 10^{-12} \times 700 \times 10^{-6}}{0.0003} = 26.8 \times 10^{-12}$$

$$= 26.8 \mu F$$

For the vertical plates, we can get about 8 x 40 area x 2, or about 640 mm², nearly as good as for the axial plates.

For minimal coupling of the torsional signal to the axial (the axial load will tend to contaminate the torque signal, but not vice versa), we need to maximize spacing x (above) and minimize the area of the plate carrying the vertical plates. If we can make x = 6 mm, giving 20% of axial signal and the torque plate have only half the area of the others, we get 40% of axial signal as contamination, ie a 2 1/2% correction for axial load to be applied to torque reading — probably have to live with this.
Spline dimensions:

For our summary & review the spline dimensions provided for the torsion internal load cell:

**Bottom plug/load cell body**

OD over splines = 61.000 / 61.074

ID over splines = 55.000 / 55.074

Length = 32

12 splines 14° ± 06', equispaced ± 03'

Spline groove 15° ± 06', equisp. ± 03'

Clearance on circ: = 0.126 ± 0.0102 ± 0.025

But unlikely to be so close (see p92)

Shear stress in root: 

\[
\frac{1000}{0.0275} = \frac{1}{\pi \cdot 0.035 \cdot 0.032} \cdot \frac{2}{12} = 13 \text{ MPa}
\]

Bearing stress on splines: 

\[
\frac{1000}{0.0275} \cdot \frac{0.003 \cdot 0.032}{12} = 32 \text{ MPa}
\]

Load cell body/spec. anvil carrier:

6 lugs, location by lugs themselves.

Root diam of lugs = 37

Thickness = 6

Shear stress in root:

\[
\frac{1000}{0.0185} \cdot \frac{2}{\pi \cdot 0.037 \cdot 0.006} = 155 \text{ MPa}
\]

Shear stress in load cell body:

\[
\frac{1000}{0.0235} \cdot \frac{0.0185}{4 \cdot (0.02^{2} - 0.02^{2})} = 461 \text{ MPa}
\]

Bearing stress:

\[
\frac{1000}{0.0235} \cdot \frac{0.006 \cdot 0.005 \cdot 1}{6} = 236 \text{ MPa}
\]

Slots for lugs:

30° ± 0.06', equispaced ± 0.03'

Clearance on circ: = 0.103 ± 0.082 ± 0.021

Min clearance = 0.021 ± 0.021 = 0

Max = 0.185 ± 0.021 = 0.206
Species and carrier / piston expansion
6 bars, location by OD above g捆ines.
Root of large g捆ines = 0.9, length = 8
Shear stress in root = \( \frac{1000}{0.0075 \times \frac{2}{\pi \times 0.0196}} \) = 44.1 MPa
Bearing stress = \( \frac{1000}{0.01125 \times \frac{1}{0.0035}} \) = 529 MPa

Now we need ample clearance for run-in to the touch point but close tolerance in order to get even loading on the g捆ines.

For spec. \( \phi = 15^\circ \), angle of twist \( \theta \)

\[
x = \frac{r}{\tan \phi}, \quad x = yL
\]

\[
\theta = \frac{x}{y} = \frac{r}{y} = 0.15 \text{ for shear}
\]

shaev \( 0.001 \) of the order of elastic range. No \( \phi \) twist gives us about 62 times elastic for run-in, 2\( \phi \) gives 13X.

2\( \phi \) is 0.6% of a full turn \( \approx 110 \text{ revs of motor at } 20000 : 1 \text{ reduction} \) (2 sec. on motor at full speed)

If one turn of motor is 1 bit, then 2\( \phi \) comes to about 100 bits. Probably OK but we can't do much less than 2\( \phi \) run-in. Can't afford more than 2.5\( \phi \) which would increase root shear stress by 11%.
Pressure Effects in UC Zero – further analysis of Potsdam behavior

(of earlier analysis p 122)

After reducing the diameter of the PSZ insulators, the hysteresis problem (p150) in the Potsdam load cell was essentially eliminated. There may be a ±0.2 kg hysteresis still present, but this can be neglected on a 0.5% accuracy scale.

However, there is still a hysteresis associated with pressure cycling. There is a load zero shift of around 2 kN for increase in pressure, most of which occurs in the first 100 MPa, and is presumably due to some residual mis-fit between the body of the piston and the pressure bringing about full contact in the first 100 MPa. The effect involves an effective distortion of the base of the UC of ~0.8 mm; presumably, the piston contacts first at the 0.20 μm then distorts. Contact is established out to 0.40 μm, thereby bringing the plates closer together. Raising the UC gives a relative displacement of the plates of ~0.8 mm may require several mm of mis-match of piston and UC base.

Time constant:

Each time the pressure is changed, it takes some time for the load zero to settle down. For the same time scale as for the pressure. If we take $x = \frac{\sqrt{DE}}{C}$

$\tau = \frac{x^2}{D} = \frac{x^2}{C} = 1700 \times \frac{460}{25} = 143520 \times \frac{x}{1000}$

So for 50 mm, $\tau = 360$ seconds = 6 minutes for about 1% effect, perhaps 24 minutes for too, about the time-scale observed – i.e. for heat to travel through vessel wall, or along the load cell.
net area = 3 mm²

area = 22.5 mm²

PSZ plate

6D, 2.71D
When the pressure is changed, the temperature changes very appreciably due to adiabatic heating or cooling of the case by amounts of up to 30 K or so as observed on thermocouples. At the same time the load cell zero can change by the order of 4 kN, corresponding to a 1.6 mm rel. def. of the plate. Over the elastic element length of 50 mm, this corresponds to a temperature difference of 0.0016/50 × 10^(-8) = 3 K, which is easily envisioned if there is a temp. change in the gas of 30 K, and the plate carries it well thermally connected than the elastic element.

At the same time there will be a temperature gradient set up between the plate supports and the screws (sketch at left). There is in fact a difference in temp. coeff. between the STANAX supports (α = 11.0 × 10^(-6) and PSZ = 100 × 10^(-6)) and the screw (probably ~4140, α = 127 × 10^(-6)), it effectively δα = 2.10^(-6) in favour of the screw. If we increase the temperature by 30 K, then relative displacement for a total screw length of ~20 mm = 30.10^(-5) × 20 = 0.60 mm or 20 μm, equivalent 63 kN. This is also approaching the zero transient zero shift of 4 kN observed so both the above effects may be contributing to the transient zero shift.

Now consider the effect of a transient temperature gradient between the plate support & screw. Suppose this were of the order of 5 K. Then, for an effective screw length of 20 mm, the extra elastic strain in the screw is 5.10^(-5) × 12 = 0.0006, comes to a stress of 12 MPa in the screw, which would probably be higher than the initial tightening tension. Thus:

- During pumping: excess positive zero shift will occur transiently, with overstretching of the screw.
- During release: excessive negative zero shift transiently, and the screw will be relatively loose, maybe completely.
load
200
press up
Press down
Hysteresis

area
22.5
3

4.4
MPa

333 MPa

Support screw

For 100 N
The screw tightening of the screw on the pumping phase will tend to give a permanent stretching of the supports due to stretching the screw, ie leave a residual positive shift of the head gus.

The loosening of the screw during pressure release will counteract the latter effect & may enable the screw to get another "grip" lower down, so giving a residual shortening of the support - leaving a residual negative shift of the head gus.

To counteract the above effects, 1. The rate of pressure change should be minimized in order to minimize the transient temperature gradients & 2. The screw should be as "tight" as possible initially.

The screw tightening should be such that the screw is near its elastic limit. Suppose this corresponded to a strain of 0.002. Then the thermal strain of $10^{-5} \Delta T$ equals this when $\Delta T = 200$ K.

In practice, the screw may be much less tight. If it were tightened to 100 N, one area 3 mm² gives a stress of 33 MPa.

The change in stress due to a temp diff $\Delta T$ between screw & support $= E \Delta T / 2 \sigma = 1.2 \times 10^{-5}$ per K i.e 12 MPa for $\Delta T = 10$ K or 37 MPa for $\Delta T = 30$ K.

Complete loosening of the screw on release of pressure would require 30 K if the screw were initially tightened to a little over 100 N.

It should be possible to tighten the screws to a point where neither excessive yielding nor pressure nor screw loosening on depressurizing will occur.

In order to relax the screw entirely, we have to put the thermal expansion into the support by an amount equal to the strain in the screw. At 100 N screw load, it
strain is $3.205 \times 10^{-6}$ for complete relaxation.

ie $\Delta T = \frac{100}{3.205 \times 10^{-6}} = 13.6 K$

For 300 N in screw, $\Delta T = 40 K$.

Thus, during pressure release, there could be a slackening of the screw if these $\Delta T$ were exceeded. Conversely, during pumping, the loading of the screw would be doubled for the same $\Delta T$.

The main transient in the load cell zero probably comes from $\Delta T$ between elastic element & plate carrier, but the hysteresis may come from the working of the support screws, with a minor contribution to the transient from this also.

However, the effect still seems rather large. Thus, the hysteresis is of the order of 1 kN or 0.4 mm.

Taking the plate support length $= 20 mm$, then the hysteresis corresponds to a change in stress in the support of $\Delta \sigma = \frac{0.004}{20} = 4.3 \text{ MPa}$ or 97 N loading on the screw. In the screw would need to slacken 100 N residually, after the thermal transient, are levelled out relative to what it was during pressurization, or we need 0.4 mm of movement in the screws position in its hole, corresponding to 0.8 mm of setting on the screw thread & shank, or lateral movement of 0.013 mm, or 0.13 mm, not a lot of contribution. So it is not clear how we can get 0.4 mm of hysteresis movement in the plate support, as well as setting of the screws under thermal transients.

To slacken the screw 97 N means changing its strain by $\frac{97}{3.205 \times 10^{-6}} = 32 \text{ kN}$ in it $= \frac{32}{205,000} \times \frac{1}{3} = 0.003 \text{ mm}$, or 0.13 mm, not a lot of contribution. So it is not clear how we can get 0.4 mm of hysteresis movement in the plate support, as well as setting of the screws under thermal transients.
Should focus attention on the support & the force on it.
The effects occur with sliding on the threads during diurnal contraction or expansion (see crossref out above).

Another way of looking at the problem:

If we raise the temp. by \( \Delta T \), thermal expansion strains is \( \Delta \sigma \), total extra strain in support = \( \Delta \sigma \Delta T - \frac{\Delta F}{23} \cdot \frac{1}{200,000} \) where \( \Delta F \) is extra force from screw

\[ \text{Area} = 23 \text{mm}^2 \]

This must be equal to extra stain in screw = \( \frac{\Delta F}{200,000} \)

\[ \Delta \sigma = \frac{\Delta F}{200,000} \]

\[ \Delta \sigma = \frac{\Delta F}{200,000} \cdot \Delta T \]

\[ \Delta F = \frac{200,000 \cdot \Delta T}{\frac{1}{3} + \frac{\Delta T}{23}} \]

or \( \Delta T = \frac{\Delta F}{6.37} \)

Thus, for \( \Delta T = 36 \text{K} \), \( \Delta F = 191 \text{N} \) \( \Delta T = 16 \text{K} \), \( \Delta F = 102 \text{N} \)

Thus, with a \( \Delta T \) of \( \sim 15-30 \text{K} \) on cooling, the stress in the screw could be transiently fully relaxed and then tighten up again when temp equilibrium is again reached. The relaxation time for this transient will be of order (p136) 0.144 (10) = 14 sec, which may make 18 K of \( \Delta T \) feasible if the heating rate is fairly rapid, since the gas temperature can drop maybe 100 K with fairly rapid depressurization over a minute or so. So it is not unreasonable to expect re-tighten at steps on re-presurizing to a level high than at steps during depressurizing, giving a hysteresis in the direction observed.

However, there is still a problem to explain how full recovery of the zero to occurs back at atmospheric pressure (the longer time scale may be due to less heat transfer through the low density argon, although it is rather long for this, \( \sim 1 \text{hour} \) — is this time scale due to recovery in the screw face, in which case why doesn't it occur at 100 KPa).
Modification to Internal Load Cell Piston Fixing

The difficulties in getting a perfect contact between load cell body & piston, to avoid zero drift with pressure, highlight the desirability of being able to keep the two together. To do so, the screw attachment would have to be made separate. A way to do this is shown at left.

The original design with thread integral with the piston was made with a view to it being possible to support the full 100 kN on the thread in the case of extension tests at atmospheric or low pressure. However, such tests could not be carried out with the present piston config because the ILC piston is connected to the compression piston, only by the pressure loading, coming to the pressure acting over 0.30. However, this loading is always less than that over the 0.40 seal between ILC & load cell piston, & so the load cell is always retained in contact by pressure loading on its piston up to the stage of parting of the two pistons. The threads are therefore never loaded in tension. For tests at zero pressure, a new piston arrangement would be needed, & the ILC would also be redundant, so an entirely new attachment of the specimen to ELC may as well be used. There is therefore no loss of function in instituting the change shown at left. Also in future, a longer rotation length at 0.49 can be provided.

The screw drive in the piston head should still be OK with the 0.44 shoulder to support the nut, & a slot can be provided in the nut for tightening — only has to compress the O-ring.
In Mike Gladwin's circuit, there is a resistor \( R \) to earth in front of the FET at the top of the LCC stem. The value suggested by Mike was 1000 \( 10\Omega \). However, in the reworked Potsdam load cell, there was some erratic noise as zero pressure was approached, which Alan F. thought might be static electricity, unable to leak away fast enough on the large \( R \). So for the MIT cell he put in a 10 \( 10\Omega \) resistor.

We now seem to see a marked pressure dependence of the calibration. Could this be due to the lower value of \( R \)?

The equivalent circuit is shown at left simplified by assuming initial reactance balance. Capacitor \( C \) includes both capacitance to earth in the leads (maybe \( \sim 60 \mu \text{F} \)) & the LCC capacitances themselves (\( \sim 30 \mu \text{F} \)).

We have

\[
i_1 = i_2 + i_3
\]

\[
i = \frac{V - V_5}{Z}, \quad \frac{V_3 + V - i_1}{Z_2}, \quad \frac{V_5}{Z}
\]

\[
\omega = \frac{V_5 e^{\frac{j\pi}{2}}}{Z_1}, \quad \frac{V_5 - V_0 e^{-\frac{j\pi}{2}}}{Z_2}, \quad \frac{V_5}{Z_3}
\]

\[
Z_1 = \frac{e^{-j\frac{\pi}{2}}}{\omega (C + \Delta C)} \quad Z_2 = \frac{e^{-j\frac{\pi}{2}}}{\omega (C - \Delta C)} \quad Z_3 = \frac{1}{R}
\]

Putting these in & reducing leads to

\[
\omega = 2\pi f
\]

\[
4\pi f R C \gg 1, \quad \text{we get}
\]

\[
\frac{V_5}{V} = \frac{2AC}{1 - \frac{1}{2} \left(4\pi f R C\right)^2}
\]

\[
\frac{V_5}{V} \approx \frac{2AC}{\left(1 - \frac{1}{2} \left(4\pi f R C\right)^2\right)} \sim 3 \times 10^{-5} \text{ with } R = 10^{10}, \quad f = 10, \quad C = 100, \quad \frac{4\pi f R C}{125}
\]
DIMENSIONS IN MILLIMETRES
DO NOT SCALE
TOLERANCES ± 0.3 UNLESS STATED

ASSAB 7/8
AS RECEIVED
BLACK OXIDE FINISH

PATerson INSTRUMENTS PTY LTD
HPT TESTING MACHINE
TORSION ACTUATOR
DETAIL 7 - TOP PLATE

ACN 008 644 273
\[ i_1 = i_2 + i_3 \]

\[ Z_1 = \frac{e^{-\frac{i\pi}{2}}}{\omega(C+AC)} \quad Z_2 = \frac{e^{-\frac{i\pi}{2}}}{\omega(C-AC)} \]

\[ (V_o e^{\frac{i\pi}{2}} - V_s)(C+AC) = \left( V_s - V_o e^{\frac{-i\pi}{2}} \right)(C-AC) + \frac{V_s}{\omega R} e^{\frac{i\pi}{2}} \]

\[ (V_o e^{\frac{i\pi}{2}} - V_s - V_s + V_o e^{\frac{-i\pi}{2}})C + (V_o e^{\frac{i\pi}{2}} - V_s + V_s - V_o e^{\frac{-i\pi}{2}})DC = \frac{V_s}{\omega R} e^{\frac{i\pi}{2}} \]

\[ e^{\frac{i\pi}{2}} + e^{-\frac{i\pi}{2}} = 0 \]

\[ e^{\frac{i\pi}{2}} - e^{-\frac{i\pi}{2}} = 2i \]

\[ -2V_s C + 2V_o DC \]

\[ V_s = \frac{2AC V_o - V_s}{2C + \frac{1}{\omega R} e^{-\frac{i\pi}{2}}} = 2V_o \Delta C \]

\[ \frac{V_s}{V_o} = \frac{2AC}{2C + \frac{1}{\omega R} e^{-\frac{i\pi}{2}}} = \frac{AC}{C} \frac{1}{\frac{1}{2} + \frac{1}{2\omega R} e^{-i\pi/2}} \]

\[ \approx \frac{iAC}{C} \left( 1 - \frac{\pi}{2} e^{-i\pi/2} \right) \text{ if } \frac{1}{2\omega R} \gg 1 \]

\[ \left| \frac{V_s}{V_o} \right| \approx \frac{AC}{C} \left( 1 - \frac{i\pi}{2} \right) \left( \frac{1}{4\pi^2 f R C^2} \right)^2 \]

\[ \omega = 2\pi f \]

\[ 3.10 \text{ mW} \quad f = 10^7 \quad R = 10^5 \quad C = 100 \text{ pF} \]
At this level, changing C from 100 to 120 pF due to increase in permittivity of the argon would have a negligible effect. Have to take R down to ~ 0.1 MΩ to see the effect of R to be significant; maybe just detectable at 1 MΩ.
Pressure Dependence of Internal Load Cell - Theoretical

From Rohr & Smith, or from \( E = \frac{9kG}{3k + G} + \frac{dE}{dp} \cdot \frac{dG}{dp} = 15.5 \)

one gets \( \frac{dE}{dp} = 5.1 \)

\( \Rightarrow \quad E = E_0 + 5.1 \frac{p}{\text{Pa}} \)

\( = E_0 (1 + 5.1 \frac{p}{E_0}) \)

For \( p = 100 \text{ MPa} \) and \( E_0 = 210 \text{ GPa} \), we have

\[ E = E_0 (1 + 0.0024) \text{ per } 100 \text{ MPa} \]

or 0.24% per 100 MPa.

In MIT machine, from 100 to 500 MPa, the ILC calibration slope (above 20 kN) increased about 0.8% per 100 MPa, so there seems to be some effect beyond that due to deflop in the elastic element. Maybe some second-order effect from change in permittivity of the gas, although it is hard to see it.
<table>
<thead>
<tr>
<th>$d_{mm}$</th>
<th>$T/MPa$</th>
<th>$\theta$/rad</th>
<th>$T$</th>
<th>$\theta$</th>
</tr>
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<tbody>
<tr>
<td>15</td>
<td>755</td>
<td>0.307</td>
<td>1510</td>
<td>0.372</td>
</tr>
<tr>
<td>16</td>
<td>622</td>
<td>0.237</td>
<td>1243</td>
<td></td>
</tr>
<tr>
<td>17</td>
<td>518</td>
<td>0.186</td>
<td>1037</td>
<td>0.372</td>
</tr>
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<td>18</td>
<td>437</td>
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<td>878</td>
<td>0.296</td>
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<tr>
<td>19</td>
<td>371</td>
<td>0.119</td>
<td>743</td>
<td>0.238</td>
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<td>20</td>
<td>318</td>
<td>0.097</td>
<td>637</td>
<td>0.192</td>
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<td>550</td>
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<td>478</td>
<td>0.133</td>
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<td>23</td>
<td>209</td>
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<td>419</td>
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<td>326</td>
<td>0.080</td>
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<tr>
<td>28</td>
<td>116</td>
<td>0.025</td>
<td>232</td>
<td>0.051</td>
</tr>
<tr>
<td>29</td>
<td>104</td>
<td>0.022</td>
<td>209</td>
<td>0.044</td>
</tr>
<tr>
<td>30</td>
<td>94</td>
<td>0.019</td>
<td>187</td>
<td>0.038</td>
</tr>
</tbody>
</table>
Calibration of Torsion Load Cell

Simpliest to use an elastic rod as at left. If made of 8407, $G = 82\, \text{GPa}$ probably within 1% (could ask Asat).

From Salmen 1.246, we have

\[
\text{angle of twist } \theta = \frac{2\pi L}{Gd} = \frac{32ML}{\pi Gd^4}
\]

and

\[
M = \frac{\pi d^3 \tau}{16}
\]

If we choose $d = 15\, \text{mm}$ and $M = 7550\, \text{Nm}$,

\[
th \theta = \frac{16.5}{7.015^3} = 755 \, \text{MPa}
\]

This is probably OK for 8407 steel at 50/52 Re.

Then the angle of twist is

\[
\theta = \frac{2.755 \times 10^6 \times 0.025}{82.10^9 \times 0.015} = 0.31 \, \text{radians}
\]

which is reasonably measurable.

So one might make one rod of 15 mm diameter and one of 26 mm. From the combination, the twist in the calibration rod at the test apparatus distortion could be separated. Then the $\phi 26$ rod could be used for calibration to 1000 Nm if desired.

Max strain in piston for external load cell

Torque $M = \frac{\pi d^3 \tau}{16}$

So

\[
\frac{\Delta \varepsilon}{G} = \frac{16M}{\pi G d^3} = \frac{16000}{\pi \times 80.10^9 \times 0.03^3}
\]

For $d = 30\, \text{mm}$ and $M = 1000\, \text{Nm}$

\[
\Delta \varepsilon = 0.0024
\]

So this is a suitable value for a torsional load cell based on elastic resistance strain gauge.
Furnace Instability

A problem has arisen with instability in the 852-insulated furnaces. After being at high temperature, there is a rapid change at relatively low temperature if a rapid change in furnace characteristic as the temperature is changed, making control very difficult. In the returned Potsdam furnace 011 and in the ETH furnace 014, this changeover occurred at around 1100 K, making it hard to operate between 1000 K and 1200 K.

(A) Below 1000 K, the power consumption was high and 0.85 kW.
(B) Above 1200 K, the power consumption was lower and 0.65 kW.

In the transition from (A) to (B), the most marked effect was a relative decrease in the amount of power in the bottom winding, with not much change in the power in the top winding, and an input to the centre winding (power here was zero in (A)). There were corresponding changes in the potentiometer settings and in the furnace thermocouple readings.

The ETH furnace 014 was opened, and an extra layer of AH2 paper was inserted on the OD of the 852 (this proved to be too much and led to galling of the 85 sleeve). The main effect here was that the power consumption in regime (B) was not changed much, but in (A) it was increased, so that the transition from (A) to (B) was less marked. Associated with this modification was an increase in bottom furnace temp. and decrease in top furnace temperature (relative values changed over), with corresponding changes in pot. settings and powers i.e. more power had to be put into the bottom winding, especially at the higher temperatures. Evidently the insulation fit onto the core & windings was disturbed in some opening the furnace.
The first question that arises is whether the extra power dissipation is due to a convective stream into the crack in the lower part of the core (in both cases the core cracks on the first run) and somehow out of the top of the furnace, or whether it is excess conductive loss due to increase in temperature on the OD of the SATI layer due to increased convection in it (as a result of separation between the core & the SATI on thermal cycling). Pending a check with an iron sheen to core the crack, the following calculation lends credence to the latter hypothesis (whereas changing the inner alumina paper seal seemed not to affect the losses).

From earlier (p. 84), the heat conducted radially out from the SATI layer through the PSZ outer sleeve is

$$ g = \frac{2\pi l k}{\ln \frac{r_2}{r_1}} \Delta T = \frac{2\pi \cdot 0.1 \cdot 2.2}{\ln \frac{60}{40}} \Delta T = 3.41 \Delta T $$

where $\Delta T$ is the temperature drop across the PSZ.

The increased power consumption in regime B is around 0.2 W/K or say 200 W for a specimen temperature of ~1300K. Therefore the change in $\Delta T$ will be 159 K, i.e. from 191 K to 249 K.

Such a change would seem not unreasonable but would have to result from an increased convective flow of gas around the SATI due to less resistance in the upward branch of the circulation.

From p. 84/85, the flow resistances for permeation in fibre and flow through cracks are:

$$ R_f = \frac{2}{\pi \Delta x_1} \text{ and } R_c = \frac{12 \gamma}{\pi \Delta x_1} $$

For a square cell, $\Delta x_1 = \sim 10 \text{ mm}$ and $\Delta x_2 = \sim 2.5 \text{ mm}$ for remaining.

$$ \Delta x_1 = 7.5 \text{ mm} $$

Relative resistances with & without crack are

$$ \left( \frac{2}{\pi \Delta x_1^2} \Delta x_1^3 + \frac{12 \gamma \Delta x_1^4}{\pi \Delta x_1} \right) / \frac{2}{\pi \Delta x_1} $$
\[
\frac{3}{4} + \frac{12 \Delta k}{4 \pi \eta c^3} = \frac{3}{4} + \frac{\Delta k}{dc^3}
\]

This becomes \( \frac{3}{4} \) when \( \frac{\Delta k}{dc^3} = 0 \).

\[
\frac{4 \Delta k}{d} = \frac{4 \cdot \frac{\pi}{4} (40^2 - 34^2) \times 10^{-6}}{0.027}
\]

\[
= 1.39 \times 10^{-13}
\]

or \( c > 51 \times 10^{-6} \) m

\[\text{i.e. } c > 51 \mu\text{m} \]

There are possible uncertainties in some of these quantities, but the result serves to indicate that convection up a crack could greatly increase the heat transfer across the SAI 1 and so give rise to extra heat conduction across the PSZ.

If we wish to account for 0.85 or 0.65 W/K, we have

\[
\frac{3}{4} + \frac{\Delta k}{dc^3} = \frac{4}{3} \quad \text{i.e. } \frac{\Delta k}{dc^3} = \frac{4}{3} - 3 = 0.5833
\]

or \( c^3 = \frac{\Delta k}{0.5839} \)

\[
0.5839 = \frac{\pi}{4} (40^2 - 34^2) \times 10^{-6} \times 2.7 \times 10^{-12}
\]

\[
= 5.98 \times 10^{-14}
\]

\[\text{i.e. } c = 39 \times 10^{-6} \text{ m} = 39 \mu\text{m} \]

or \( c = 14 \mu\text{m} \) if \( \Delta k = 10^{-12} \)
Review of piston & LHC clearances

There have been pick-up problems from time to time between the load cell body (or piston head) and the guide portion of the bottom plug, and between the load cell piston and the $\phi 30$ bore in the bottom plug.

In a letter of 22/3/94 to Kohlstedt, it was recommended that for the actual pairs of the load cell guide of 58.910 (in spec. 58.000/58.030) the head of the load cell piston should be 57.965/57.945, i.e. with clearance of 0.045/0.065. For the piston stem ($\phi 30$), a diameter of $\phi$ 29.950/29.935 was recommended. But the bore of the plug is not recorded, probably $\phi$ 30.910, giving clearance 0.060/0.075.

8/3/95: The load cell guide was knocked out a bit, on account of pick-up. No clear record of final clearance. Clearances are $\phi 30$ load cell piston = 0.062 $\phi 30$ comp. piston = 0.057 $\phi 42$ comp. piston = 0.055.

6/9/94: Opened up load cell guide to 58.040, giving clearance 0.040 to 0.050.

Thus we should probably aim at the following minimum clearances:

<table>
<thead>
<tr>
<th></th>
<th>Over LHC in guide ($\phi 58$)</th>
<th>Over LHC stem ($\phi 30$)</th>
<th>Over Comp piston $\phi 42$</th>
<th>Over Comp piston $\phi 30$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diameter</td>
<td>57.960/57.940</td>
<td>29.945/29.930</td>
<td>42.360/42.344</td>
<td>29.945/29.930</td>
</tr>
<tr>
<td>Clearances</td>
<td>0.040</td>
<td>0.055</td>
<td>0.040</td>
<td>0.055</td>
</tr>
</tbody>
</table>

Thus values allow approx. min. clearance of 0.025 plus 0.030 for Poisson expansion on $\phi 30$ & 0.015 on $\phi 42$. The guide values allow 0.020 plus 0.020 for eccentricity. Tolerances on diameters give extra room on these minimum values.
Temperature drop over outer core

If we put the windings on the inner core, what is the temperature on the outer side?

From p. 113,

\[
\frac{\Delta T}{\Delta x} = \frac{\Delta T_1}{x_1} + \frac{\Delta T_2}{x_2} + \frac{\Delta T_3}{x_3}
\]

\[
\Delta T = \frac{\Delta T_3}{\ln \frac{x_3}{x_2}}
\]

\[
\ln \frac{x_3}{x_2} = \frac{37}{6}
\]

\[
\frac{\Delta T_3}{\Delta T} = \frac{\ln \frac{60}{27} + \ln \frac{40}{27} + \ln \frac{37}{36}}{18.43 + 0.983 + 0.0196}
\]

\[
= 0.0165
\]

So for \(\Delta T = 1000K\), \(\Delta T_3 = 16.5K\). This would double if the \(K\) for SATI were doubled. In any case, the temperature drop across the outer Al_2O_3 core sleeve is probably less than 30K. This is not much less than over the windings themselves, but would be smoothed out somewhat.
Overall gauge thicknesses seem to be less than 0.1 mm, so with glue the thickness will not be much more than 0.1 mm.

If $3, \phi$ thickness is $1/2 \times (0.5$ deep groove$)$

Then $P.15 = 125 P = \frac{65}{2650} \text{MPa}$ at $P = 580 \text{MPa}$. Still OK, just acceptable.
Stain-gauge load cell

New no cut-out. For elastic element OD 50, we need 1D such that we get a strain of ~ 0.001 at 100 kN, ie
\[
\frac{100 \times 10^6}{\pi (0.05^2 - d^2) \times 210 \times 10^9} = 10^{-3}
\]
\[
\frac{1}{4} \frac{0.05^2 - d^2}{0.05^2 - d^2} = \frac{\pi}{4} \frac{210 \times 10^6}{210 \times 10^9} = \frac{2100 \times \pi}{2100 \times \pi} = \frac{1}{4}
\]
\[
d^2 = (0.050)^2 - 210 \times 10^6 \times \frac{1}{2100 \times \pi} \approx 0.0435
\]
\[
d = 0.0435
\]
ie d ≈ 4.4 mm.

ie 3 mm wall thickness.

But we need much greater thickness than this in order to accommodate the stain-gauge cavity with minimal bending in it if the strain gauge is fairly wide. OD 53, ID 47 gives exactly 0.0010 strain at 100 kN.

If we use Micro measurement, UA-125BZ gages, there has a backing 7.4 x 3.3. Therefore a cut-out of 8 x 4 (or a little less) would be suitable, with a depth of about 0.5. Thus the outer 28 cylinder could be 1.25 mm thick & the inner one 1.75 (accommodating the 0.5 depth with 1.25 of "meat" under the cut-out).

The max average shear stress at section AB would be
\[
p \times \frac{2}{1.25} = 1.6p = 800 \text{ MPa at } p = 500 \text{ MPa}
\]
\[
p \times 1.16 = 700 \text{ MPa at } p = 700 \text{ MPa}
\]

if we take the cut-out as being infinitely long. But it is actually 8 long, circumference 24 (w 22 with opening at one end) so average shear stress over whole perimeter = \[
p \times \frac{32}{1.25 \times 22} = 1.16p = 582 \text{ MPa at } p = 500 \text{ MPa}
\]
815 at 700

Actual mean shear stress will be somewhere between these limits, probably ~650 MPa at 500 psi.
\[(25.75 - h)^2 + 4^2 = (25.75)^2\]
\[(h - 25.75) = (25.75)^2 - 16\]
\[h = 25.75 - \sqrt{(25.75)^2 - 16}\]
\[= 0.31\]

As a groove 8 mm around the circumference & 0.7 mm deep at centre will be 0.4 deg at ends.

\[E = 194.68\]

from CALMAX
If we use A516 GR10 steel at \( \sim 55 \) Rc, the 0.2 yield stress is about 1850 MPa. Its shear yield stress is around 1600 MPa. Should be OK.

In NASA p 449 there is a formula for a rectangular plate along 6 under \( C' = 5 \) which gives max stress:
\[
\sigma = K \left( \frac{P}{t^2} \right) \frac{2}{(1 + C^2)}
\]
where \( K \) for mild steel is
0.48 (fixed)
0.72 (free supported)

For \( a = 8 \), \( b = 4 \), \( t = 1.25 \) we have
\[
\sigma = 0.5 \cdot \frac{4}{(1.25)^2} \cdot \frac{2}{1 + 0.15} = \frac{4P}{(1.25)^3} = 2.05P
\]

Thus for \( P = 700 \) MPa, \( \sigma = 1434 \) MPa, well within elastic range of CARMAX.

So it looks as if we could go up to an 8 x 4 cut-out for the strain gauges. There will be some stress concentration & permanent deformation at the edges but probably not enough to prevent dismantling again.

Closure of gap between cylinders.

The change in radius of a cylinder of id 2R, od 2R is
\[
\Delta r = \frac{P r}{E} \left( \frac{1 + \left( \frac{R}{2} \right)^2}{1 - \left( \frac{R}{2} \right)^2} \right) \text{ in plane stress (Note Book 9 p12)}
\]

If we have two cylinders 47/50.5 and 50.5/53 diameters, then the differential change
\[
\Delta r = \frac{P \cdot 50.5}{E} \left( \frac{1 + \left( \frac{47}{50.5} \right)^2}{1 - \left( \frac{47}{50.5} \right)^2} + \frac{1 + \left( \frac{50.5}{53} \right)^2}{1 - \left( \frac{50.5}{53} \right)^2} \right) + 2 \Delta r
\]
\[
= \frac{P \cdot 50.5}{194,000} \left( 13.95 + 14.26 + 0.6 \right) = 0.00375\, \text{mm}
\]

Thus a gap of 0.030 mm would be closed by a pressure of
\[
p = \frac{0.030 \cdot 2}{0.00375} = 8 \, \text{MPa}
\]

Thus bottle pressure should close the gap, and the effective pressure between the cylinders can be taken as the confining pressure minus around 8 MPa.
\[ P = \frac{1.75 \times 200}{13.5 \mu} \]
Slippage between cylinders.

An attempt would be made to have the inner cylinder contact the outer at both ends so that they behave as one cylinder from atmospheric pressure. But such a mating is not going to be perfect. If there is a mismatch of even 1 µm, this will correspond to an axial load of

\[
\frac{0.001 \times 1}{0.001} = 1.4 \text{ kN}
\]

so there is likely to be some non-linearity in the first few kN at zero pressure.

Under pressure the cylinders are pressed together and the friction between them will tend to transfer load between them. Thus at the level of the strain gauge, the loading of one cylinder by friction on the other will be

\[
\frac{135 \mu P}{1.75} = 135 \mu P \frac{1.25}{1.25} = 135 \mu P \text{ minimum}
\]

which has to be equal to the stress associated with the maximum strain in the elastic element of the load cell, i.e. 135 MPa = 200

\[
\frac{1.75 \mu}{1.75} = 200 = 117 \mu P
\]

or \( P = \frac{175 \mu P}{1.75} = 200 \mu P = 26.875 \text{ MPa} \) for \( \mu = 1 \)

and \( 525 \text{ MPa} \) for \( \mu = 0.5 \)

To these pressures one has to add 8 MPa for closing the gap (assuming it originally is 0.030 µm) so the minimum confining pressure to give full elastic behavior to the load cell is around 30-60 MPa.
Effect of Outer Core on Power for Torsion Furnace of p1500

Original torsion furnace had a core #271D, 290D. Suppose that over the windings section (100 mm long) we put an outer core of 291D, 310D. What will be the power increase? Consider the radial heat loss:

\[Q = \frac{2\pi L_0}{\ln \frac{R_0}{R_1}} + \ln \frac{R_1}{R_0}\]

Taking \(L = 100\), \(\Delta T = 1000\) K, we have for single core:

\[\frac{27.0.1.1000}{0.184 + 0.983} = 636\text{ W}\]

(Observe power = 704 W)

and for double core:

\[\frac{27.0.1.1000}{2.2.0.184 + 0.4 + 0.983} = 755\text{ W} \text{ i.e. extra 119 W}\]

Using full 180 mm for hot zone will be an exaggeration. Probably 75 mm should be a better figure since the peak temp. is over 50 mm. So the extra power may be of order of 90 W. So power consumption will go up from about 0.71 W/K to about 0.80 W/K. Still tolerable probably, although current may run a little high.
Electrical connections of SS 14 C

AXIAL:

B', D', alternative to B, D in case these are too pressure sensitive

Bottom of body annular space at head of stem

Black, Red, Pink, White, exciter

1005 - 10/98
area = 2091

net area = 1693

net area = 1693

nominal intensity = 23.5%

ie for 500 MPa, bearing pressure = 618 MPa

This is OK.
Zero shift problem in strain-gauge load cell

Tests at CF2 showed that the torque gage was very little affected by pressure, but the axial load gage was severely affected, being shifted by some 20% or more of FS for 300 MPa, ie 300 MPa shifts the 100 gage by 20 - 30 kN.

This effect is probably due to the bulging inward of the grooves in which the circumferential gage sets, effectively increasing the diameter and stretching the gage. The axial gage is not affected in the same way. Hence we have to move the 'dummy' gage to a location where it experiences the same strain as the axial gage during pressure change. The most likely position is on the bottom face of the insert carrying the gages. Here it will experience almost the same strain as the axial gage due to the pressure. The leads can be brought down a groove on the od of the insert & across this face also.
Position of dummy gauges for axial load. It now seems to me best to put the dummy gauges in the vertical groove going up to the active gauge but below the elastic element section. Here it will feel the effect from applying pressure immediately, and the 0.63 (Poisson) effects when the splined section closes in on the strain gauge section. Thus there will be some zero shift in the instrument (could be minimized by a closer fit) and possibly some imbalance (giving further zero shift) due to strain concentration in the groove; but the latter will only affect the $e_1$ & $e_2$ and hence only the $e_1$ component through a Poisson effect.

Position of Active Gauge

In order to minimize the amount of pressure needed to settle the load cell (lock the two elements together) it is best to position the gauges as low as possible. From the analysis on p.133, the locking up pressure is

$$p = \frac{300 \cdot 0.75}{\mu L}$$

For steel on steel, clean & dry, $\mu = 0.78$ and greasy $\mu = 0.1$. With oxide film, it is given as $\mu = 0.27$ (Parks, p.219), so produced by heating in air at 100-500°C. The surfaces are as ground & so we should have at least $\mu = 0.5$ unless the O-rings exude oils.

The strain gauge should be about twice the wall thickness up from the position A, i.e. about 6 mm up, leaving $h = 30$.

Thus $$p - 8 = \frac{300 \cdot 0.75}{0.5 \cdot 30} = 35$$, i.e. $p = 43$ MPa.

allow for clamping 0.80 gap.

i.e. The load cell should be OK to full scale at $p = 50$ MPa provided the surfaces are dry. 9. It should be OK to 50 kN at half this pressure, i.e. a bit above bottle pressure.
change of post of charging gauge

use of SiF4 in furnace

length of winding
Pressure dependence of ILC - Theoretical
Calibration of torsion ILC
External torsion load cell - strain
Furnace instability
Piston and ILC clearance
Temperature drop over outer core
Strain-gauge load cell
- slippage between cylinders
Effect of outer core on torsion furnace power
81 Furnace insulation — permeability considerations; flow resistances; effect of cracks
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92 Redesign of ILC — axial load & torque; splines
95 Actuator/load cell/steerup relations
97 Thermal stress in aluminising furnace cores
99 Piston extension for extension tests
101 Axial load cell details; mod. to bottom plug; heating examples
104 Pressure vessel volumes — net gas volume
105 Energy calculation for Potsdam enquiry
106 Pressure vessel yielding with outer sleeve accounted for.
108 Stress from shrink-fitting outer sleeve
109 Preliminary concept on 1.45 afl vessel
110 Possible solid insulations in furnace
113 — temp drop over first layer
114 Heat loss due to convection through a clearance
115 — equivalence to gap filled with aluminising paper
116 Fits without aluminising paper
117 Upper limit to wall temperature.
118 Use of intensifier for volume change measurement sensitivity
119 — CF volumetric sensitivity
121 Electrical connections for phys. prop. measurements
122 Internal load cell pressure compensation — see also p. 136
123 — effect of higher fill
126 Experience with half bridge load cell
128 — optimal circuit for " " "
129 Temp. sensitivity of load cell gno.
130 Hysteresis in internal load cell.
133 Internal load cell for torsion — plate arrangements
136 Pressure effects in ILC gno. — further analysis of Potsdam
141 Modification to ILC piston fixing
142 Influence of resistor value on ILC Behaviour