QUANTITATIVE INTERPRETATION
OF
AEROMAGNETIC DATA

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PREFACE

Work reported in this thesis was commenced in the Department of Geophysics at the Australian National University in December, 1953 and involved several aspects. An alternating current susceptibility bridge of the Bruckshaw and Robertson type was designed, built, aligned and used for the measurement of the magnetic susceptibility of rock samples. Field work consisted of several assignments, of which one was for ground geophysical measurements in the Katherine-Darwin region of Australia. Work on a theoretical framework for the interpretation of aeromagnetic data was carried out in conjunction with the above work.

Work reported in this thesis is the work of the author with the exception of the mathematical analysis of the Downward Continuation problem.

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CHAPTER I
INTRODUCTION

The present work is intended as a contribution to the setting up of a framework for the interpretation of total intensity aeromagnetic data. Although instrumentation for this purpose is in good order, little has been published related to the interpretation of data. Quantitative interpretation of magnetic data is only one aspect of the subject. A full appreciation of magnetic interpretation requires a knowledge of rock magnetism or the magnetic properties of rock, an aspect which will only be touched on briefly in this work. The main stress is to be on the use of quantitative methods for the deduction of the shape and, where possible, the average magnetic polarization of the causative body or for the separation of magnetic effects of various types. Good interpretation also requires considerable geological background, and the integration of the geological and geophysical approaches.

In aeromagnetic surveying, the so-called total magnetic intensity anomaly is measured. This is the component of the anomalous field in the direction of the earth's total field vector. If the anomalous field is not too large, the direction of the total field vector may be taken as the normal direction in that area. The total intensity anomaly $\Delta T$ is then a harmonic function and amenable to the same methods of treatment as the other magnetic field components and the gravitational field.

The reason why the total magnetic intensity is measured is that errors in this component due to misorientation are small when compared with errors encountered in the vertical or horizontal components. The instrumentation in use is also simpler than would be necessary for the measurement of these components. Two types of airborne magnetometer are in current use, the Varian proton precession and the "flux-gate".
The Gulf magnetometer is the most commonly used instrument of the "flux-gate" type. The detector element consists of two coils connected in series opposition and physically parallel, containing ferromagnetic cores. These are fed by an alternating current which drives the ferromagnetic core past saturation on each half-cycle. If the cores are biased by a small magnetic field, they reach saturation at slightly different times and differential output voltage "spikes" occur. These are fed to a servo unit that is used to adjust and maintain a direct current through a secondary coil surrounding both sensing elements. The field from this secondary coil exactly balances the field being measured. A measuring circuit is used to determine the nulling current and, hence, the value of the field, which is recorded by a Leeds and Northrup continuously recording potentiometer. The variation of the total magnetic field component of the earth is measured, and this is accomplished by keeping the detector element suitably aligned by means of an orientor system. The two orientor elements are similar to the detector but are mounted at right angles to each other and to the detector. The whole unit is mounted by means of a gimbal system. The orientor elements operate a servo system which orients the unit so that the orientors sense zero field and, hence, the detector responds to the total intensity of the earth's magnetic field.

The Varian airborne magnetometer utilizes the proton precession principle. This is based on the fact that the proton magnetic moments in a sample of water or hydrocarbon will precess about the magnetic field direction at a frequency proportional to the total field intensity. The instrument is simple in principle. In order to detect a signal from the proton precession, the individual protons in the sample must be brought into net alignment by the application of an external field of 100 gauss approximately perpendicular to the earth's field. When this polarizing field is removed, the protons precess about the earth's field for a short time until phase coherence is lost. During this period, the voltage induced by the precessing protons is picked up by the same coil that is used for polarizing. The signal passes through an automatic tuner and pre-amplifier and into a frequency counter.
the type of magnetic bodies encountered in practice. For example, sills and dykes are common geologically. While their magnetic polarization varies spatially within the same body, they are often sufficiently uniform that
The types of field variation encountered in practice are illustrated in Figures 1-1 and 1-2. Figure 1-1 shows an aeromagnetic survey conducted in a shallow-source area, in other words an area where the magnetic anomalies are due to bodies close to the surface of the earth. This is typical of surveys carried out in mineral prospecting. Figure 1-2 shows an aeromagnetic survey conducted in a sedimentary area where the magnetic anomalies originate in a basement of crystalline rocks at considerable depth. In the second case, the anomalies are much less localized than in the first, and the interpretational approach is accordingly different.

Magnetic anomalies are caused by a contrast in magnetic polarization between one rock unit and others. Most rocks have a positive magnetic susceptibility $K$, which gives the rock a polarization in the earth's magnetic field of $i_I = KT$ in the direction of the earth's field. In addition, rocks possess a remanent magnetization $i_R$ which is independent of the earth's field and may be in any direction. $i_R$ may vary from a fraction of $i_I$ to many times $i_I$ in magnitude and has a direction that is often nearly the same as the earth's field but may vary radically from this direction. The total polarization $i_T$ is the vector sum of the induced and remanent components. If $i_R$ is small, this is close in direction to the earth's field. However, if it is larger than $i_I$, the total polarization may be in any direction.

The ambiguity of the gravity interpretation problem is well known. In addition, in magnetic interpretation the possibility of a skewed magnetic polarization brings in further uncertainty. If the depth to the source can be determined from geological information, one source of ambiguity is removed. Also, in some cases it is reasonable to assume magnetic polarization in the direction of the earth's field. Cases of skewed and negative polarization are sometimes discernible.

In the present work, various theoretical models are set up to approximate the type of magnetic bodies encountered in practice. For example, sills and dykes are common geologically. While their magnetic polarization varies spatially within the same body, they are often sufficiently uniform that
their parameters can be deduced using the Dyke model discussed in Chapter IV. The elementary approximations are useful when the dimensions of the causative body are small compared with the depth below the plane of observations. Since many ore bodies and petroleum structures have a limited extent in depth, the dipolar models are useful for detecting them against other effects.

Typical magnetic anomaly profiles in northern and southern latitudes are illustrated in Figure 1-3. These show the anomaly due to a magnetic point pole at a magnetic inclination of 60° and illustrate the correspondence between hemispheres. The parameters calculated for most of the theoretical models discussed in Chapters II to V are also illustrated in this figure. By measuring the same parameters on the actual anomaly profile and comparing with the set of theoretical models, a selection of model can be made. An important aspect of the present work is that where possible the methods are set up to cope with an arbitrary direction of polarization, rather than the usual assumption of magnetic polarization in the direction of the earth's field. Unless otherwise indicated, the derivation of formulas has been made from basic principles.

Another class of problem investigated in this thesis is that of the treatment of data using the Downward Continuation and Derivative methods (Chapters VI and VII). These are useful for improving the resolution of individual anomalies from other effects, such as the detection of weak local anomalies superimposed on a more regional variation. In these methods, values at grid points are obtained by interpolation from the contour map, so that there is a tendency to lose information. Consequently, they are applied most successfully in the early stages of interpretation. More accurate quantitative interpretation is possible with models used in conjunction with the original field intensity data. The present thesis shows the essential similarity between the Downward Continuation and Derivative methods. It is shown that different formulas can be obtained by the variation of a "smoothing parameter" in the original integral for the derived function. Thus, allowance can be made for anomalies of different "wave-length" or areal extent, and for the density and accuracy of the data. The question of choice of formula has given rise to much discussion in the geophysical literature.
FIGURE 1-3: Total intensity anomaly due to point source in (a) northern and (b) southern magnetic hemispheres, showing profile in units of depth at magnetic inclination 45°. The anomalous field component $\Delta T$ is expressed in units of $m_o/T^2$. 
The correlation of aeromagnetic data with ground magnetic data is discussed briefly. The determination of the magnetic susceptibility of rock specimens is illustrated by some measurements made with a susceptibility bridge built by the author. The main problem in the correlation of field intensity measurements with measurements of the magnetic properties of rock samples is the wide variation in magnetic properties of individual rock units, often over short distances.

The problem of the resolution of aeromagnetic data is approached using simple applications of some of the models. This is of considerable practical importance.
CHAPTER II

THE ELEMENTARY APPROXIMATIONS I

We will develop in this chapter four elementary approximations for the total magnetic intensity case - the point pole, point dipole, line of poles and line of dipoles. The line pole and line dipole models are two-dimensional. Type profiles are given for the point sources. For all four type sources, the peak displacement is calculated, and also factors for depth determinations using the two distances from anomaly maximum to half-maximum intensity on the anomaly profile. The more general treatment in Chapter III utilizes in addition the inflection points. Anomalies rarely duplicate the theoretical model curves exactly, but these methods provide useful first approximations.

The applicability of the methods depends on the magnetic inclination, direction of magnetic polarization and dimensions of the source. When the width of the body is small compared to its depth and it is elongated in the direction of polarization, the pole approximations yield the depth to the top. Thus, narrow vertical dykes would yield to a line of poles approximation at high geomagnetic latitudes. For bodies of limited extent in depth, the dipole approximations yield the depth to the center, the point dipole for bodies approaching a sphere in shape, the line of dipoles a horizontal cylinder. Sources that are wide compared with their depth will give broad profiles and depth determinations made from them give values that are too large. This is also the case with complex sources consisting of several closely spaced anomalous bodies whose effects merge to give a single anomaly. The estimated depth will be a maximum value, useful at least for distinguishing shallow from deep-seated sources.

Interpretation of anomalies characterized by nearly circular contours may be approached using a pole or dipole method, while those with contours elongated in one direction require a line of poles or line of dipoles. In Chapter III, methods will be discussed for distinguishing between polar and dipolar effects. Finally, it must be emphasized that the methods serve as a guide to geological reasoning and more elaborate theoretical methods.
Point Pole Approximation

Theory for the total intensity anomaly due to a point pole in the northern hemisphere has been presented (Henderson and Zietz, 1948) but is modified in the present work. In the northern magnetic hemisphere, a narrow body greatly extended in depth whose long axis is close to the direction of polarization, if polarized normally, may be represented by a negative magnetic pole at its upper end.

Using an orthogonal Cartesian co-ordinate system, z-axis vertically downward, pole \(-m_0\) at depth \(\gamma\), the anomalous magnetic potential \(\Delta V\) is given by

\[
\Delta V = -\frac{m_0}{\left(\frac{x^2 + y^2 + (z-\gamma)^2}{2}\right)^{1/2}}
\]  

(2-1)

The x-axis makes an angle \(\beta\) with magnetic north and the y-axis is in the northerly half-plane. \(I\) is the inclination of the total field \(T\), t the unit vector in the direction of \(T\). The total intensity anomaly \(\Delta T\) is given by

\[
\Delta T = -\left[\frac{\partial (\Delta V)}{\partial x} \frac{dx}{dt} + \frac{\partial (\Delta V)}{\partial y} \frac{dy}{dt} + \frac{\partial (\Delta V)}{\partial z} \frac{dz}{dt}\right]
\]

hence, in the xy-plane, the total intensity anomaly \(\Delta T(x,y,0)\) is

\[
\Delta T(x,y,0) = -m_0 \frac{x \cos I \cos \beta + y \cos I \sin \beta - \sin I}{\left(\frac{2}{x^2 + y^2 + \gamma^2}\right)^{3/2}}
\]  

(2-2)

A profile along the y-axis is given by

\[
\Delta T(y) = m_0 \sin I \frac{\gamma - ay}{\left(y^2 + \gamma^2\right)^{3/2}}
\]  

(2-3)

where \(a = \cot I \sin \beta\).

The peak value \(\Delta T_{\text{max}}\) occurs when \(\beta = 90^\circ\) at the point \(y = -\gamma_1\)

\[
\alpha_1 = \left[9 + 8 \cot^2 I\right]^{1/2} - 3 \quad /4 \cot I,
\]  

(2-5)

and its value is

\[
\Delta T_{\text{max}} = m_0 \sin I \left(1 + \alpha_1 \cot I\right) /\gamma^2 \left(\alpha_1^2 + 1\right)^{3/2}
\]  

(2-6)
FIGURE 2-1: Total intensity depth factors $k_1(a)$ and $k'_1(a)$, half-maximum distance ratio $\pi_1, \pi'_1$ for point source.
FIGURE 2-2: Total intensity anomaly from a point source at various magnetic inclinations (northern hemisphere). Profiles in units of depth along magnetic meridian. The anomalous field component $\Delta T$ is in units of $m_o/\gamma^2$. 
The minimum value occurs at

\[ y = \gamma' \mathcal{M}_1, \]

where

\[ \mathcal{M}_1 = \left[ 3 + (9 + 8 \cot^2 \theta) \right]^{1/2} \frac{1}{4 \cot \theta} \]

and its value is

\[ \Delta T_{\min} = m_0 \sin \theta (1 - \mathcal{M}_1 \cot \theta) / \gamma' \left( \mathcal{M}_1^2 + 1 \right)^{3/2}. \] (2-8)

Also,

\[ \Delta T(y) = 0, \text{ when } y = \gamma' \tan \theta. \] (2-9)

We wish to calculate factors which, when multiplied by the distance from maximum to half-maximum intensity along a meridional profile, yield the depth to the point pole. We transform our origin to the peak of the anomaly, taking the y-axis along the magnetic meridian, obtaining

\[ \Delta T(y') = m_0 \sin \theta \frac{(1 + \alpha_1 \cot \theta) - y' \cot \theta}{\left[ \gamma'^2 - 2\alpha_1 \gamma' y' + \gamma'^2 (1 + \alpha_2) \right]^{3/2}}. \] (2-10)

Substituting \( y' = \gamma / k \) in equation 10 and equating with \( \frac{1}{2} \Delta T_{\text{max}} \) from equation (6), we obtain

\[ \frac{1 + \alpha_1 \cot \theta}{2(\alpha_1^2 + 1)^{3/2} k^2} \left[ 1 - 2\alpha_1 k + k^2 (1 + \alpha_2) \right]^{3/2} = \frac{k (1 + \alpha_1 \cot \theta) - \cot \theta}{\left[ \gamma'^2 - 2\alpha_1 \gamma' y' + \gamma'^2 (1 + \alpha_2) \right]^{3/2}}. \] (2-11)

This may be solved numerically for two real roots, a positive \( k_1 \) (a) and a negative \( -k_1' \) (a). They are to be used with \( \gamma \), the distance from the anomaly maximum to half-maximum in a northerly direction, and \( \gamma' \) in a southerly (in the northern hemisphere). These directions are reversed in the southern hemisphere, cf. Figure 1.

The depth \( \gamma = k_1 \gamma = k_1' \gamma' \). The ratio of half-maximum distances \( \gamma : \gamma' \) on the actual anomaly should approximate the theoretical ratio \( \gamma_1 : \gamma_1' = k_1 : k_1' \) if the point pole is an adequate representation of the physical situation. The peak displacement \( \alpha_1 \), the factors \( k_1 \), \( k_1' \) and the ratio \( \gamma_1 : \gamma_1' \) are given in Figure 2-1. A family of \( \Delta T \) profiles corresponding to various values of \( \theta \) is presented in Figure 2-2. Similar notation is used in the subsequent three cases, subscripts 2, 3, 4 corresponding to line of poles, dipole and line of dipoles respectively.
Since a contour aeromagnetic map is normally made up in practice, it was thought sufficient to determine the k-factors for the case $\beta = 90^0$ corresponding to profiles taken through the anomaly maximum, along the magnetic meridian. In a given practical case, the distances $\eta_1$ and $\eta_1'$ are measured on a meridian profile. These may be multiplied by the factors $k_1(a)$ and $k_1'(a)$ respectively to yield two depth determinations. If the maximum is difficult to locate, we may measure the distance $\eta''$ between points of half-maximum intensity and determine $\gamma = k_1 '' \eta''$ where

$$k_1''(a) = \frac{k_1(a) k'_1(a)}{k_1(a) + k_1'(a)} \quad (2-12)$$

A transverse ($\beta = 180^0$) profile through the maximum may serve as a check. This is symmetrical. The width between points of half-maximum intensity $\eta_T''$ per unit depth is given by

$$\eta_T'' = 1.533 (1 + \alpha_1^2)^{1/2}, \quad (2-13)$$

and, therefore, the depth factor $k_T''$ is given by

$$k_T'' = 0.652 (1 + \alpha_1^2)^{-1/2} \quad (2-14)$$

The work of Henderson and Zietz (1948) should be referred to. These authors find the maximum anomaly and one half-maximum point on lines in any direction passing directly above the point pole. Of these lines, only that in the magnetic meridian passes through the absolute anomaly maximum, giving the results presented above. For all other directions, the absolute maximum lies to the south of the line (in the northern hemisphere).

In the southern magnetic hemisphere, our type body would be represented by a positive magnetic pole. The magnetic potential is now

$$\Delta V = \frac{m_0}{\left[ x^2 + y^2 + (z - \gamma)^2 \right]^{1/2}} \quad (2-15)$$

and for a profile along the meridian (y-axis),

$$\Delta T(y) = m_0 \sin I \frac{\gamma' + a y}{(y^2 + \gamma'^2)^{3/2}} \quad (2-16)$$
This expression is merely a mirror image of that for the northern hemisphere. As a result, we may use our northern hemisphere results in the southern hemisphere, provided that we take the distances from maximum to half-maximum intensity $\eta$ in a southerly direction and $\eta'$ in a northerly.

Line of Poles

The theory for this two-dimensional approximation is given by Henderson and Zietz (1948) and is reproduced in part for the sake of clarity. A line of poles parallel to and at a distance $\gamma$ below the x-axis extends infinitely in both directions. The magnetic potential is given by

$$\Delta V_2 = m_1 \ln \left[ \frac{2}{y^2 + (\gamma - z)^2} \right],$$

where $-m_1$ is the pole strength per unit length.

The total magnetic intensity profile along the y-axis is given by

$$\Delta T_2 (y) = \frac{2m_1 \sin I (\gamma - a \gamma)}{y^2 + \gamma^2},$$

(2-18)

The depth factors $k_2$ and $k_2'$ are determined in the same manner as for the point pole. Henderson and Zietz gave the factor $k_2$, where

$$k_2 (a) = \frac{a}{2b} \left[ 1 + \left( \frac{3b-1}{b-1} \right) \frac{1}{2} \right],$$

and

$$b = (a^2 + 1)^{\frac{1}{2}},$$

as a function of inclination $I$ of the total field and angle $\beta$ between the strike of the body and magnetic north. Their values are shown in Figure 2-3, a reproduction of their Figure 4. The author (Smellie, 1956) has calculated the other depth factor $k_2'$, given by

$$-k_2' (a) = \frac{a}{2b} \left[ 1 - \left( \frac{3b-1}{b-1} \right) \frac{1}{2} \right].$$

Values of $k_2' (a)$ as a function of $I$ and $\beta$ are presented in Figure 2-4.
FIGURE 2-3: Curves of constant total intensity depth factor $k_2(a)$ for line pole source.
FIGURE 2-2: Total intensity anomaly from a point source at various magnetic inclinations (northern hemisphere). Profiles in units of depth along magnetic meridian. The anomalous field component $\Delta T$ is in units of $m_0/\tau^2$. 
The depth factors $k_2$ and $k_2'$ are for depth determinations using profiles perpendicular to the strike of the anomaly. The half-maximum distance $\gamma$ must be taken northerly in the northern hemisphere and southerly in the southern and is used with the factor $k_2$ for depth determinations, viz. $\gamma = k_2 \gamma$. The half-maximum distance $\gamma'$ must be taken southerly in the northern hemisphere and northerly in the southern.

The ratio of half-maximum distances is given by

$$\frac{\gamma_2}{\gamma_2'} = \frac{\left[(3b-1)^{1/2} - (b-1)^{1/2}\right]}{\left[(3b-1)^{1/2} + (b-1)^{1/2}\right]}$$  \hspace{1cm} (2-20)

and is plotted in Figure 2-5. This enables one to compare the measured ratio with a theoretical value and thus see the degree of approximation obtained.

The shift of the anomaly peak from the position directly above the source is in the negative $y$-direction and of amount

$$\alpha_2 = \frac{(b-1)}{a},$$  \hspace{1cm} (2-21)

and the maximum intensity value

$$\Delta T_{\text{max}} = \frac{2m_1 \sin I}{3} f_2\left(-\alpha_2\right),$$  \hspace{1cm} (2-22)

where $f_2\left(-\alpha_2\right) = \frac{1 + a\alpha_2}{1 + \alpha_2^2}$.

and $m_1$ is the pole strength per unit length.

**Point Dipole Approximation**

We assume a dipole moment $p_o$ in the direction of the earth's field at a depth $\gamma$ below the plane of observations (Figure 2-6). The anomalous magnetic potential in the northern hemisphere for a meridional profile passing directly over the source is then

$$\Delta V_3 = p_o \frac{y \cos I - \gamma \sin I}{(y^2 + \gamma^2)^{3/2}}.$$  \hspace{1cm} (2-23)

Expressed in units of depth, the total intensity anomaly is

$$\Delta T_3 = \frac{p_o}{\gamma^3} f_3(\alpha),$$  \hspace{1cm} (2-24)

where $f_3(\alpha) = \frac{\left[(3 \sin^2 I - 1) - 6 \sin I \cos I \alpha + (3 \cos^2 I - 1)\alpha^2\right]}{(1 + \alpha^2)^{5/2}}$.  \hspace{1cm} (2-25)
FIGURE 2-5: Curves of constant ratio of half-maximum distances $\gamma_2: \gamma_2'$ for line pole source
From this expression, \( f_3(\alpha) \) has been calculated for representative values of \( I \) and the results plotted in Figure 2-7. The mirror images in \( y \) of these curves are suitable for use in the southern hemisphere.

To determine the shift of anomaly maximum from the position directly above the source, we set

\[
\frac{\partial}{\partial \alpha} (\Delta T) = 0, \text{ i.e.,}
\]

\[
(3 \cos^2 I - 1)\alpha^3 - 8 \sin I \cos I \alpha^2 + (7 \sin^2 I - 3)\alpha + 2 \sin I \cos I = 0. \quad (2-26)
\]

The negative real root of this equation yields the required value \( -\alpha_3 \) at the anomaly maximum. This has been calculated for a number of values of \( I \) and the results given in Figure 2-8.

The half-maximum depth factors \( k_3(a) \) and \( k_3'(a) \) were obtained for various values of \( I \) by calculating the maximum value of \( f_3(-\alpha_3) \). Half of this value is then substituted in the expression for \( f_3(\alpha) \) and the resulting equation solved for two real roots \( \beta_3 \) and \( \beta_3' (\beta_3 > \beta_3') \). Then

\[
k_3(a) = (\beta_3 + \alpha_3)^{-1}, \quad k_3'(a) = -\alpha_3 + \beta_3')^{-1},
\]

\[
\eta_3; \eta_3' = (\beta_3 + \alpha_3) / (\alpha_3 + \beta_3').
\]

**Line of Dipoles**

Consider an infinite horizontal line of magnetic dipoles at depth \( \gamma \) whose dipole moment is \( p_1 \) per unit length in the direction of the earth's field and which strikes at an angle \( \beta \) with magnetic north. We have for the magnetic potential in the northern hemisphere

\[
\Delta V_4 = 2p_1 \left[ y \cos I \sin \beta + (z - \gamma) \sin I \right] / \left[ y^2 + (\gamma - z)^2 \right], \quad (2-28)
\]

where the \( y \)-direction is normal to strike and positive in the northern half-plane. The total intensity anomaly for a profile along the \( y \)-axis is given by

\[
\Delta T_4 = \frac{2p_1 \cos^2 I \sin^2 \beta \gamma^2}{\gamma^2} \frac{(\alpha^2 - 1)(1 - q^2) - 4\alpha q}{(1 + \alpha^2)^2}, \quad (2-29)
\]

where \( q = \tan I \csc \beta = a^{-1} \). \( (2-30) \)
FIGURE 2-6: Geometry of the dipole approximation.
FIGURE 2-7: Total intensity anomaly from a dipole source at various magnetic inclinations (northern hemisphere). Profiles in units of depth along magnetic meridian. $\Delta T$ is in units of $p_0 / r^3$. 
FIGURE 2-8: Total intensity depth factors $k_3(a)$ and $k'_3(a)$, half maximum distance ratio $\eta_3; \eta'_3$ and peak displacement $\alpha_3$ for dipole source.
The method of calculation of the half-maximum depth factors is the same as for the point dipole, the parameter used being \( q \). Families of curves of constant \( q \) are used to show the factors in the same way as for the line of poles. The results for \( \alpha_4', k_4'(a) \) and \( \eta_4', \eta_4' \) are shown in Figures 2-9, 2-10, 2-11 and 2-12 respectively.

The total magnetic intensity anomaly may also be expressed in the form

\[
\Delta T_4 = \frac{p}{\gamma^2} f_4(\alpha),
\]

where \( f_4(\alpha) = 2 \frac{(\cos^2 I \sin^2 \beta - \sin^2 I) (\alpha^2 - 1) - [4 \sin I \cos I \sin \beta \alpha]}{1 + \alpha^2}. \]  

Estimation of Magnetic Polarization

(a) Point pole:

The pole strength of the source, \( m_0 \), is found from equation (2-6) relating the maximum intensity \( \Delta T_{\text{max}} \) with the magnetic pole strength \( m_0 \), viz,

\[
\Delta T_{\text{max}} = \frac{m_0 \sin I}{\gamma^2} \frac{1 + \alpha_1 \cot I}{(1 + \alpha_1^2)^{3/2}} = \frac{m_0 \sin I}{\gamma^2} f_1(-\alpha_1),
\]

where \( \alpha_1 \) = peak displacement.

An estimate of the minimum magnetic polarization may be made by assuming that the polarization contrast occurs within an area of \( \gamma^2 \), otherwise the anomaly would depart noticeably from that due to a point pole, i.e.

\[
i_{\text{min}} = \frac{\Delta T_{\text{max}}}{f_1(-\alpha_1) \sin I},
\]

with \( i \) in c.g.s. units.

(b) Line of Poles:

From equation (2-22), the pole strength per unit length is given by:

\[
m_1 = \frac{\Delta T_{\text{max}} \gamma}{2f_2(-\alpha_2) \sin I},
\]

where \( f_2(\alpha) = \frac{1 - \alpha \gamma}{1 + \alpha^2} \).
FIGURE 2-9: Curves of constant shift of anomaly maximum in units of depth, line dipole source.
FIGURE 2-10: Curves of constant total intensity depth factor $k_4(a)$ for line dipole source.
FIGURE 2-11: Curves of constant total intensity depth factor $k_d'(\alpha)$ for line dipole source.
FIGURE 2-12: Curves of constant ratio of half-maximum distances $\eta_4: \eta_{4'}$ for line dipole source.
Assuming the maximum width of the body $f, \Delta T$ in gammas,

$$i_{\text{min}} = \frac{\Delta T_{\text{max}}}{2 f^2_{2}(-\alpha_{2}) \sin I}. \quad (2-34)$$

If the width $w$ is known from geological information,

$$i = \frac{f}{w} \frac{\Delta T_{\text{max}}}{2 f^2_{2}(-\alpha_{2}) \sin I}. \quad (2-35)$$

(c) Point dipole:

From equation (2-24), the dipole moment is given by:

$$p_{0} = \frac{f^{3} \Delta T_{\text{max}}}{f_{3}(-\alpha_{3})} \quad (2-36)$$

where $f_{3}(-\alpha_{3})$ is plotted in Figure 2-13 as a function of $I$. The magnetic material can be considered as localized within a radius equal to the depth $R$ of the center sub-surface, in which case

$$i_{\text{min}} = \left(\frac{f}{R}\right)^{3} \frac{3 \Delta T_{\text{max}}}{4 \Pi f_{3}(-\alpha_{3})}. \quad (2-37)$$

(d) Line of dipoles:

The dipole moment per unit length is given by:

$$p_{1} = \frac{f^{2} \Delta T_{\text{max}}}{f_{4}(-\alpha_{4})} \quad (2-38)$$

There will be a limiting radius to the body, for example the depth of the center sub-surface, denoted by $R$, so that the minimum polarization is given by

$$i_{\text{min}} = \left(\frac{f}{R}\right)^{2} \frac{\Delta T_{\text{max}}}{\Pi f_{4}(-\alpha_{4})}. \quad (2-39)$$

The intensity function $f_{4}(-\alpha_{4})$ is shown in Figure 2-14 as a function of $I$ and $\beta$. 
FIGURE 2-13: Total intensity anomaly maximum function $f_g(-\varphi)$ for dipole source as a function of magnetic inclination $I$. 
FIGURE 2-14: Total intensity anomaly maximum function $f_4(\alpha_4)$ for line dipole source as a function of strike $\beta$ and magnetic inclination $I$. 
In the previous chapter, four elementary approximations useful in the interpretation of aeromagnetic data were discussed. Four new models will be introduced in this chapter. It will be shown that the two-dimensional models, the narrow dyke and horizontal cylinder, yield equations of a type similar to the line of poles and line of dipoles. The vertical cylinder and polarized sphere, however, yield equations that simplify only in special cases. In addition to calculations of the anomaly maximum and half-maximum points, it is shown that calculation of the inflection points serves as a further criterion for distinguishing between type of source.

In all cases, the direction of polarization is arbitrary. This is an extension of work in Chapter II. The derivation of the formulae for the total intensity anomaly is laborious and follows the standard methods of potential theory. The case of the narrow vertical cylinder is discussed in Appendix A as an example of the procedure used.

The cases of the polarized sphere and the horizontal cylinder have been investigated independently by Sutton and Mumme (1957). These authors discussed the relationship between the general cases of arbitrary direction of polarization and the earlier work by the author (Smellie, 1956). Their conclusions were similar to those of the present chapter.

Narrow Vertical Cylinder

We assume a narrow vertical cylinder of cross-sectional area $A$ with its top at $z = \bar{z}$, extending to infinity along the positive $z$-axis. The co-ordinate system is right-handed, with the $xy$-plane horizontal as usual, and the $y$-axis along the magnetic meridian. The magnetic polarization is downward at an angle $\varphi$ from the horizontal, with its azimuth making an angle $\epsilon$ with the $x$-axis measured in the direction of the $y$-axis.

The derivation of this case is given in Appendix A. The magnetic potential $\Delta V$ is given by

\[
\Delta V = -i A \frac{x \cos \varphi \cos \epsilon + y \cos \varphi \sin \epsilon}{T} \frac{\sin \varphi}{\left[ x^2 + y^2 + (z - \bar{z})^2 \right]^{1/2}}
\]
For a profile along the y-axis, one obtains for the total magnetic intensity anomaly the expression

\[ \Delta T = i_T A \left\{ \frac{1}{y^2 + (y-z)^2} \left[ 1 - \frac{y}{(y^2 + y^2)^{3/2}} \right] \cos \Psi \sin \varepsilon \cos \Gamma \right. \nonumber \\
- \left. \left[ 1 - \frac{y}{(y^2 + y^2)^{1/2}} \right] \cos \Psi \sin \varepsilon \cos \Gamma \right\} \]  

This expression simplifies to a point pole type of equation only in the following special cases:

(a) Polarization vertical \((\Psi = 90^\circ)\),

(b) Horizontal component of polarization magnetic east-west \((\varepsilon = 0)\),

(c) High latitudes \((\Gamma = 90^\circ)\).

In these cases, the total intensity anomaly is of the form

\[ \Delta T = \frac{m}{2} \phi \frac{1 - A_1 \alpha}{(1 + \alpha^2)^{3/2}} \]  

where \(\phi = \sin \Gamma\), \(m = i_T A \sin \Psi\),

\[ A = \cot \Gamma \sin \varepsilon + \cot \alpha = \cot \Gamma_A \]  

where \(\Gamma_A\) is defined as the apparent magnetic inclination.

Work on the point pole model described in Chapter II included calculation of the half-maximum depth factors, anomaly asymmetries and the shift of the anomaly maximum as a function of \(\Gamma\). The above analysis shows that these are valid for various values of \(\Gamma_A\), provided that the anomaly is of a point pole type. However, it is sometimes difficult to distinguish between anomalies due to a point pole and a point dipole. Therefore, the inflection points were calculated for a number of cases and a ratio \(R_1\) calculated, where \(R_1\) is the ratio between \(w_{\text{INFL}}\), the width between inner inflection points, and \(w_{1/2}\), the width between half-maximum points. The inner inflection points are defined as the two flanking the anomaly maximum.
By differentiating equation (3-3) twice with respect to $\alpha$ and setting the result equal to zero, we obtain expressions for the determination of the inflection points. These are:

$$
2A_1 \alpha^3 - 4\alpha^2 - 3A_1 \alpha + 1 = 0, \quad A_1 \text{ finite}
$$

$$
2\alpha^3 - 3\alpha = 0, \quad A_1 \text{ infinite.}\tag{3-4}
$$

The inner inflection points are tabulated in Table 3-I, along with other parameters already utilized in Chapter II. The ratio $R_1 = \frac{w_{\text{INFL}}}{w_{1/2}}$ is also included. In Figure 3-1, the parameters $R_1$ and $\eta_1 : \eta'_1$ are shown as a function of $A_1$. Comparison with Figure 3-3 will reveal that the combination of these parameters for the point pole case is consistently different from that for the point dipole case.

**TABLE 3-I**

**VERTICAL CYLINDER (POINT POLE) APPROXIMATION**

<table>
<thead>
<tr>
<th>$i_A$</th>
<th>$A_1$</th>
<th>$\alpha_1$</th>
<th>$\alpha_{\text{INFL}}$</th>
<th>$w_{\text{INFL}}$</th>
<th>$w_{1/2}$</th>
<th>$R_1$</th>
<th>$\eta_1 : \eta'_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0</td>
<td>0</td>
<td>-0.500</td>
<td>0.500</td>
<td>1.000</td>
<td>1.533</td>
<td>0.652</td>
</tr>
<tr>
<td>75°</td>
<td>0.268</td>
<td>0.090</td>
<td>-0.584</td>
<td>0.419</td>
<td>1.003</td>
<td>1.533</td>
<td>0.655</td>
</tr>
<tr>
<td>60°</td>
<td>0.577</td>
<td>0.182</td>
<td>-0.673</td>
<td>0.339</td>
<td>1.012</td>
<td>1.534</td>
<td>0.661</td>
</tr>
<tr>
<td>45°</td>
<td>1.000</td>
<td>0.278</td>
<td>-0.774</td>
<td>0.257</td>
<td>1.030</td>
<td>1.541</td>
<td>0.668</td>
</tr>
<tr>
<td>30°</td>
<td>1.732</td>
<td>0.396</td>
<td>-0.891</td>
<td>0.173</td>
<td>1.064</td>
<td>1.561</td>
<td>0.682</td>
</tr>
<tr>
<td>15°</td>
<td>3.732</td>
<td>0.535</td>
<td>-1.036</td>
<td>0.087</td>
<td>1.123</td>
<td>1.608</td>
<td>0.700</td>
</tr>
<tr>
<td>0°</td>
<td>$\infty$</td>
<td>0</td>
<td>-1.225</td>
<td>0</td>
<td>1.225</td>
<td>1.689</td>
<td>0.726</td>
</tr>
</tbody>
</table>

In Figure 3-1, the parameters $R_1$ and $\eta_1 : \eta'_1$ are plotted as a function of $A_1$.

**Narrow Dyke**

The derivation of the total magnetic intensity anomaly due to an infinite inclined dyke is given in Appendix B. The relevant terms are defined there and in Chapter IV. The result is

$$
\Delta T = 2 \sin \psi \left\{ \left[ i_D \sin I - i_N \cos I \sin \beta \right] \phi_{AB} - \left[ i_N \sin I + i_D \cos I \sin \beta \right] \ln \frac{r_A}{r_B} \right\}.
$$

(3-5)
FIGURE 3-1: Vertical cylinder (pole approximation): Asymmetry $\eta_1; \eta_1'$ and ratio

$$R_1 = \frac{w_{INFL}}{w_{1/2}}$$ as a function of $A_1$. 

\[ A_1 \]
When the dyke becomes narrow compared with its depth below the plane of observations, we have the limiting cases

\[
\begin{align*}
\psi_{AB} & \rightarrow \frac{w}{\gamma} \frac{1}{1 + \alpha^2} , \\
\ln \frac{r_A}{r_B} & \rightarrow \frac{w}{\gamma} \frac{\alpha}{1 + \alpha^2} .
\end{align*}
\]

Thus the expression for the total intensity anomaly becomes

\[
\Delta T = \frac{2 w \sin \psi}{\gamma} \left[ \left( i_D \sin I - i_N \cos I \sin \beta \right) \frac{1}{1 + \alpha^2} \\
- \left( i_N \sin I + i_D \cos I \sin \beta \right) \frac{\alpha}{1 + \alpha^2} \right] .
\]  

(3-6)

The pole strength per unit length of the top of the dyke is denoted by \( m_1 \), where

\[ m_1 = w i_D \sin \psi , \]

and the equation can be expressed in the form

\[
\Delta T = \frac{2m_1}{\gamma} \frac{1 - A^2 \alpha}{\psi_2 \frac{1}{1 + \alpha^2}} ,
\]  

(3-7)

where \( \psi_2 = \sin I - \frac{i_N}{i_D} \cos I \sin \beta \),

and \( A_2 = \frac{i_N + a i_D}{i_D - a i_N} \).

This is an equation of the line of poles type. Inflection points may be calculated as before, as a function of \( A_2 \). The determining equations are

\[
A_2 \alpha^3 - 3\alpha^2 - 3A_2 \alpha + 1 = 0 , \quad A_2 \text{ finite},
\]  

(3-8)

\[
\alpha^3 - 3\alpha = 0 , \quad A_2 \text{ infinite}.
\]

The results are presented in Table 3-II. The parameters \( R_2 \) and \( \gamma_2, \gamma_2' \) are shown graphically in Figure 3-2.
FIGURE 3-2: Narrow dyke (line pole): Asymmetry $\gamma_2 : \gamma_2'$ and ratio $R_2 = \frac{w_{\text{INFL}}}{w_{1/2}^2}$ as a function of $A_2$. 
### Table 3-II

**Narrow Dyke (Line of Poles)**

<table>
<thead>
<tr>
<th>( I )</th>
<th>( A_2 )</th>
<th>( \alpha_{INFL} )</th>
<th>( \omega_{INFL} )</th>
<th>( \omega_{1/2} )</th>
<th>( \eta_{2}/\eta_{1} )</th>
<th>( R_2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0</td>
<td>-0.577</td>
<td>0.577</td>
<td>1.154</td>
<td>2.000</td>
<td>1.000</td>
</tr>
<tr>
<td>75°</td>
<td>0.268</td>
<td>-0.700</td>
<td>0.466</td>
<td>1.166</td>
<td>2.034</td>
<td>0.770</td>
</tr>
<tr>
<td>60°</td>
<td>0.577</td>
<td>-0.832</td>
<td>0.364</td>
<td>1.196</td>
<td>2.142</td>
<td>0.600</td>
</tr>
<tr>
<td>45°</td>
<td>1.000</td>
<td>-1.000</td>
<td>0.268</td>
<td>1.268</td>
<td>2.336</td>
<td>0.474</td>
</tr>
<tr>
<td>30°</td>
<td>1.732</td>
<td>-1.192</td>
<td>0.176</td>
<td>1.368</td>
<td>2.578</td>
<td>0.382</td>
</tr>
<tr>
<td>15°</td>
<td>3.732</td>
<td>-1.428</td>
<td>0.088</td>
<td>1.516</td>
<td>2.951</td>
<td>0.316</td>
</tr>
<tr>
<td>0°</td>
<td>∞</td>
<td>-1.732</td>
<td>0</td>
<td>1.732</td>
<td>3.464</td>
<td>0.268</td>
</tr>
</tbody>
</table>

**Polarized Sphere**

If we assume a uniformly polarized sphere, with components of the dipole moment \( p_x, p_y, p_z \), located at depth \( f \) below the origin of co-ordinates along the \( z \)-axis, the \( x \)-axis making an angle \( \beta \) with the magnetic meridian, the potential at point \((x_0, y, z)\) is given in the northern hemisphere by

\[
\Delta V_3 = \frac{p_x + p_y + p_z (f - z)}{[x^2 + y^2 + (f - z)^2]^{3/2}},
\]

and the general equation for the total magnetic intensity anomaly is

\[
\Delta T_3 = -\frac{p_x p_x' + p_y p_y' + p_z p_z'}{[x^2 + y^2 + (f - z)^2]^{5/2}},
\]

where

\[
p_x = \left[(y^2 + (f - z)^2 - 2x^2)\right] \cos I \cos \beta - 3xy \cos I \sin \beta + 3x (f - z) \sin I,
\]

\[
p_y = \left[(f - z)^2 - 2y^2 + x^2\right] \cos I \sin \beta - 3xy \cos I \sin \beta + 3y (f - z) \sin I,
\]

\[
p_z = 3x(f - z) \cos I \cos \beta + 3y(f - z) \cos I \sin \beta + \left[y^2 - 2(f - z)^2 + x^2\right] \sin I.
\]

If we assume dipole moments \( p_x', p_y', p_z' \), referred to a stationary system with \( y \)-axis along the magnetic meridian and take a \( y \)-profile \((x = 0)\) in the plane \( z = 0 \),

\[
p_z = p_z'; \quad p_y = p_y' \sin \beta - p_x' \cos \beta;
\]

\[
p_x = p_x' \sin \beta + p_y' \cos \beta.
\]
With the profile expressed in units of the depth, i.e., \( y = \gamma \alpha \), and 

\[ p_x = \gamma_1 p_0, \quad p_y = \gamma_2 p_0, \quad p_z = \gamma_3 p_0, \]

where \( \gamma_1, \gamma_2, \gamma_3 \) are the direction cosines of the dipole moment, the total intensity anomaly becomes

\[ \Delta T_3(\alpha) = \frac{p_0}{\gamma_3^3} \left( \frac{1 + B \alpha + C \alpha^2}{(1 + \alpha^2)^{5/2}} \right), \]

where

\[ \phi_3 = 2 \gamma_3 \sin I - \gamma_2 \cos I, \]

\[ B = \frac{3(\gamma_1 \cos \beta - \gamma_2 \sin \beta \sin I - 3 \gamma_3 \sin \beta \cos I)}{2 \gamma_3 \sin I - \gamma_2 \cos I}, \]

\[ C = \frac{\gamma_2(2 \sin \beta - 1) \cos I - 3 \gamma_1 \cos I \sin \beta \cos I - \gamma_3 \sin I}{2 \gamma_3 \sin I - \gamma_2 \cos I}. \]

This general case is obviously cumbersome. One fact that becomes obvious, however, when dealing with the dipole approximation, is that if we deal with the positive portion of the anomaly curve where it is larger than the negative portion, calculated values of the anomaly half-width, width between inflection points and R-values vary by only a few per cent. Thus, in spite of the complexity of the dipole solution, it does not seem worthwhile to calculate a large number of cases, particularly for geophysical applications where in non-ideal circumstances the dipole moment is the leading term but is perturbed by higher multipole moments.

It appears that, for cases when the anomaly maximum is larger than the minimum, the anomaly half-width is equal to the depth and the ratio \( R_3 = \frac{w_{\text{INFL}}}{w_{1/2}} \) is constant within a few per cent. This simplification does not apply to the negative portion of the anomaly curve.

The case of polarization in the direction of the earth's field yields a simpler formula, with

\[ \beta = 90^\circ, \quad \gamma_1 = 0, \quad \gamma_2 = \cos I, \quad \gamma_3 = \sin I, \]

\[ \phi_3 = 3 \sin^2 I - 1, \]

\[ B = \frac{-6 \sin I \cos I}{3 \sin^2 I - 1}, \]

\[ C = \frac{3 \cos^2 I - 1}{3 \sin^2 I - 1}. \]

(3-13)
Inflection points may be calculated by solution of the equation

\[(2C-5) - 15 B \alpha + (30 - 21C) \alpha^2 + 20 B \alpha^3 + 12 C \alpha^4 = 0.\]

Results for some cases are shown in Table 3-III. At \(I = 30^\circ\), the anomaly minimum is becoming stronger than the maximum, and the parameters \(w_{1/2}', w_{\text{INFL}}\) and \(R_3\) are beginning to deviate from the near-constant values of the cases \(I = 90^\circ, 60^\circ\) and \(45^\circ\).

**TABLE 3-III**

<table>
<thead>
<tr>
<th>I</th>
<th>B</th>
<th>C</th>
<th>3</th>
<th>(w_{1/2})</th>
<th>(w_{\text{INFL}})</th>
<th>(\gamma)</th>
<th>R</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0</td>
<td>-0.500</td>
<td>0</td>
<td>1.000</td>
<td>0.776</td>
<td>1.000</td>
<td>0.776</td>
</tr>
<tr>
<td>60°</td>
<td>-2.078</td>
<td>-0.200</td>
<td>0.273</td>
<td>1.020</td>
<td>0.799</td>
<td>0.705</td>
<td>0.783</td>
</tr>
<tr>
<td>45°</td>
<td>-6.000</td>
<td>1.000</td>
<td>0.431</td>
<td>1.051</td>
<td>0.849</td>
<td>0.594</td>
<td>0.808</td>
</tr>
<tr>
<td>30°</td>
<td>10.390</td>
<td>-5.000</td>
<td>0.622</td>
<td>1.121</td>
<td>0.927</td>
<td>0.499</td>
<td>0.825</td>
</tr>
</tbody>
</table>

The ratios \(R_3\) and \(\gamma_3\) are plotted in Figure 3-3. \(R_3\) is only shown for the range from \(90^\circ\) to \(30^\circ\).

**Polarized Horizontal Cylinder**

If we assume an infinite horizontal cylinder with a component of polarization normal to the cylinder axis inclined at \(\phi\) with the horizontal, the magnetic potential is

\[
\Delta V = 2p_1 \frac{y \cos \phi + (z-J) \sin \phi}{y^2 + (J-z)^2}.
\]

(3-14)

The axial component of polarization makes no contribution. The total magnetic intensity anomaly is given by

\[
\Delta T(\alpha) = 2p_1 \frac{(\cos \phi \cos I \sin \beta - \sin \phi \sin I)(\alpha^2 - 1) - 2(\sin \phi \cos I \sin \beta + \cos \phi \sin I)\alpha}{J^2 (1 + \alpha^2)^2}.
\]

(3-15)
FIGURE 3-3: Polarized sphere (dipole): Asymmetry $\eta_3 \eta_3'$ and ratio $R_3 = \frac{W_{\text{INFL}}}{W_{1/2}}$ as a function of magnetic inclination I for normal polarization.
This may also be expressed in the form

$$\Delta T_4(\alpha) = \frac{2p_1}{f} \frac{4 \phi}{f} f(4)$$

(3-16)

with $$f_4(\alpha) = \frac{\alpha^2 - 1 - A_4^2}{(1 + \alpha^2)^2}$$,

$$\phi = \cos \phi \cos I \sin \beta - \sin \phi \sin I,$$

$$A_4 = \frac{2 (\sin \phi \cos I \sin \beta + \cos \phi \sin I)}{\cos \phi \cos I \sin \beta - \sin \phi \sin I}.$$

The simpler case of polarization in the direction of the earth's field, discussed previously, is obtained from the above by making the conversions

$$p_1 \cos \phi \rightarrow p_1' \cos I \sin \beta,$$

$$p_1 \sin \phi \rightarrow p_1' \sin I,$$

with the result

$$\Delta T_4 = \frac{2p_1}{f} \frac{4 \phi}{f} \frac{(\cos^2 I \sin^2 \beta - \sin^2 I)(\alpha^2 - 1) - 4 \sin I \cos I \sin \beta}{(1 + \alpha^2)^2} \alpha,$$

$$\phi = \cos^2 I \sin^2 \beta - \sin^2 I,$$

$$A_4 = \frac{4 \cos I \sin I \sin \beta}{\cos^2 I \sin^2 \beta - \sin^2 I} \phi \neq 0.$$

The parallelism between the general and special cases is apparent, and it may be noted that, if the anomaly is of a line-dipolar type, the deduction of $$A_4$$ can lead to an estimate of depth.

In Table 3-IV, the results of calculations of the inflection points are presented for a range of $$I_4$$ from 90° to 45°, in which range $$A_4$$ varies from 0 to 0°. As in the case of other two-dimensional models, there is a relationship between the negative anomaly profile at low apparent inclinations and the positive anomaly profile at high apparent inclinations.

It may be noted that, for east-west strikes and normal polarization,

$$f_4(\alpha) = \frac{(\cos^2 I - \sin^2 I)(\alpha^2 - 1) - 4 \alpha \sin I \cos I}{(1 + \alpha^2)^2},$$

so that with a reversal of abscissae, the positive anomaly at inclination $$I > 45°$$ is the same as the negative anomaly at inclination (90° - I).
The inflection points were calculated by solution of the equations

\[ 1 - 2A_4^3 \alpha - 6 \alpha^2 + 2A_4^3 \alpha + \alpha = 0, \quad A_4 \neq \infty, \]
\[ \alpha^3 - \alpha = 0, \quad A_4 = \infty. \]

### Table 3-IV

<table>
<thead>
<tr>
<th>( I/A )</th>
<th>( A_4 )</th>
<th>( w_{\text{INFL}} )</th>
<th>( w_{1/2} )</th>
<th>( R/A )</th>
<th>( \eta_4/\eta_4' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°</td>
<td>0</td>
<td>0.828</td>
<td>0.972</td>
<td>0.851</td>
<td>1.000</td>
</tr>
<tr>
<td>75°</td>
<td>1.155</td>
<td>0.847</td>
<td>0.997</td>
<td>0.850</td>
<td>0.784</td>
</tr>
<tr>
<td>60°</td>
<td>3.464</td>
<td>0.898</td>
<td>1.074</td>
<td>0.836</td>
<td>0.620</td>
</tr>
<tr>
<td>45°</td>
<td>( \infty )</td>
<td>1.000</td>
<td>1.214</td>
<td>0.824</td>
<td>0.501</td>
</tr>
</tbody>
</table>

In Figure 3-4, the factors \( R/A \) and \( \eta_4/\eta_4' \) are presented graphically as a function of \( A_4 \).

### Field Applications

**Spruce Brook Anomaly:** This anomaly is shown in Figure 3-5. Its maximum intensity level is 4280 gammas, compared with an average value of 3840 gammas on surrounding neutral ground, giving an anomalous intensity of 440 gammas.

The distances from maximum to half-maximum intensity are 850 feet in an easterly and 1100 feet in a westerly direction, the distance between inflection points 1300 feet (along the flight line), hence \( R = 0.67 \), \( \eta_4/\eta_4' = 0.77 \). These values satisfy the criteria for a vertical cylinder type of anomaly in high latitudes with \( A_1 = 0.5 \). The apparent inclination for this anomaly is

\[ I/A = \cot^{-1} 0.5 = 63.5°. \]

Referring to Figure 2-2, we find the depth \( \gamma = k_1(a) \eta_E = k_1(a) \eta_W = 1270 \text{ feet}, \) where \( \eta_E \) and \( \eta_W \) are the easterly and westerly half-maximum distances respectively. The minimum polarization is

\[ i_{\text{min}} = 4.9 \times 10^{-3}, \]
FIGURE 3-4: Polarized horizontal cylinder (line dipole): Asymmetry $\eta_4: \eta'_4$ and ratio $R_4 = \frac{W}{W_{1/2}}$ as a function of $A_4$. 
FIGURE 3-5: Portion of Geological Survey of Canada Geophysics Paper 275, Serpentine, Newfoundland, showing the "Spruce Brook" anomaly.
FIGURE 3-6 Total magnetic intensity map showing the Marmora anomaly, from Geological Survey of Canada aeromagnetic sheet 31C/5, Campbellford. Scale one inch equals one half mile. Flight lines shown.
Marmora Anomaly (Figure 3-6): As observed earlier (Smellie, 1956), the point pole approximation is not suitable on this anomaly. If we assume a line of poles striking at $\beta = 45^\circ$ as indicated by the trend of the contours, we have (solid profile) $\eta = 580$ ft., $\eta' = 700$ ft., $\eta:\eta' = 0.82$, which is the theoretical value for a line of poles at $I = 75^\circ$, $\beta = 45^\circ$. The inflection point width of about 750 ft., provides further confirmation of this. From Figure 2-4 and Figure 4 of the paper by Henderson and Zietz (1948), we obtain two depth estimates, $J_2 = k_2(a) \eta = 630$ ft., and $J_2' = k_2'(a) \eta' = 630$ ft. The magnetic ore mass causing this anomaly is 2400 ft. long, 500 ft. wide at its maximum width, with a northwest-southeast strike and a dip southwest of 70° to 80°. It is capped by 120 ft. of limestone, which added to the flight elevation yields a depth to the top of about 620 ft. Using eq (2-34), the minimum polarization is $33 \times 10^{-3}$ c.g.s. units.
CHAPTER IV
THE DYKE MODEL

The dyke model is the most useful for the interpretation of magnetic data since many geological bodies are sill-like or dyke-like in form. They are often sufficiently two-dimensional and uniformly magnetized so that the dyke model can be used for an estimate of their dimensions and average magnetic polarization.

The Fundamental Formulas

A section normal to the infinite inclined dyke is shown in Figure 4-1. \( \phi \) is the angle subtended by the top of the dyke, \( r_A \) is the distance from the point of observation to the side A of the top of the dyke, \( r_B \) is the distance from the point of observation to the side B of the top of the dyke, \( i_D \) is the component of magnetic polarization parallel to the side of the dyke and positive downward, perpendicular to the strike, \( i_N \) is the component of magnetic polarization perpendicular to the sides of the dyke and positive northward, \( \psi \) is the dip of the dyke.

The derivation of the total magnetic intensity anomaly is carried out in Appendix B. It is accomplished by representing an elemental strip of the surface of the dyke by a line of poles and integrating over the entire surface. The result is

\[
\Delta T = 2 \sin \psi \left\{ i_D \sin \mathbf{I} - i_N \cos \mathbf{I} \sin \phi \right\} \frac{r_A}{r_B} \ln \frac{r_A}{r_B} \]  

(4-1)

It is of interest to note the vertical intensity \( \Delta Z \) and horizontal intensity \( \Delta H \) anomalies, obtainable from the above by setting \( \mathbf{I} \) equal to 90° and 0° respectively. These are

\[
\Delta Z = 2 \sin \psi \left( i_D \phi_{AB} - i_N \ln \frac{r_A}{r_B} \right), \quad \text{and} \]  

(4-2)

\[
\Delta H = -2 \sin \psi \sin \phi \left( i_N \phi_{AB} + i_D \ln \frac{r_A}{r_B} \right). \]  

(4-3)

These expressions may be compared with some formulas current in the literature. For example, eq (4-3) may be expressed in terms of the vertical polarization \( i_z \) and meridional horizontal polarization \( i_H \) by means of the conversions

\[
i_N = i_z \cos \psi + i_H \sin \psi \sin \phi, \quad \text{and} \]  

(4-4)
FIGURE 4-1: Geometry of the dyke model.
\[ i_D = i_T \sin \psi - i_H \cos \psi \sin \phi. \]

The result is

\[ \Delta H = -2 \sin \psi \sin \phi \left\{ \left[ i_z \cos \psi + i_H \sin \psi \sin \phi \right] \varphi_{AB} + \left[ i_z \sin \psi - i_H \cos \psi \sin \phi \right] \ln \frac{r_A}{r_B} \right\}. \]

This may be compared with eq (11) of Egyed (1948) by making a suitable change of notation. The above expression, eq (4-5), differs from eq (11) of Egyed (1948) by a factor \( \sin \phi \), indicating that this author has calculated the horizontal magnetic field component normal to the strike of the dyke rather than along the magnetic meridian. This error appears to be widespread in the literature.

The general equation (4-1) for the total intensity anomaly is of the form

\[ \Delta T = \phi_D \left[ \phi_{AB} - A_D \ln \frac{r_A}{r_B} \right], \]

where

\[ \phi_D = 2 \sin I \sin \psi \left[ i_D - a \cdot i_N \right], \]

\[ A_D = \frac{i_N + a \cdot i_D}{i_D - a \cdot i_N}, \]

and it may be recalled that \( a = \cot I \sin \phi \).

For the special case of magnetic polarization \( i_T \) in the direction of the earth's field,

\[ i_D = i_T (\sin \psi \sin I - \cos \psi \cos I \sin \phi), \]

\[ i_N = i_T (\cos \psi \sin I + \sin \psi \cos I \sin \phi). \]

The total magnetic intensity anomaly is now given by

\[ \frac{\Delta T}{2 i_T \sin \psi} = \left[ \sin \psi \left( \sin^2 I - \cos^2 I \sin^2 \phi \right) - 2 \cos \psi \sin I \cos I \sin \phi \right] \varphi_{AB} \]

\[ - \left[ \cos \psi \left( \sin^2 I - \cos^2 I \sin^2 \phi \right) + 2 \sin \psi \sin I \cos I \sin \phi \right] \ln \frac{r_A}{r_B}. \]

In the general notation of eq (4-6),

\[ \phi_D = 2 i_T \sin \psi \left[ \sin \psi \left( \sin^2 I - \cos^2 I \sin^2 \phi \right) - 2 \cos \psi \sin I \cos I \sin \phi \right], \]

\[ A_D = 2 \frac{i_T \sin \psi \sin I \cos I \sin \phi + \cos \psi \left( \sin^2 I - \cos^2 I \sin^2 \phi \right)}{\sin \psi \left( \sin^2 I - \cos^2 I \sin^2 \phi \right) - 2 \cos \psi \sin I \cos I \sin \phi}. \]

In this case, \( i_T \) and \( \psi \) are determinable. However, normal polarization cannot generally be assumed.
Type Curves

For the case of normal polarization, type curves showing the effects of width, dip and magnetic inclination are shown in Figures 4-2, 4-3 and 4-4 respectively. Tables 4-I and 4-II give the functions $\phi_{AB}$ and $\ln \frac{r_A}{r_B}$ for the calculation of other cases, $\phi_{AB}$ being an even function and $\ln \frac{r_A}{r_B}$ being an odd function. The origin for these tables is a point directly above the center of the top of the dyke. Curve fitting can be very laborious, and in the next section a direct method will be developed.

Referring to Figure 4-4, one can see the variation in function with magnetic inclination for normal polarization, from a pure $\phi$-function at $I = 90^\circ$ to a $\ln$-function at $I = 45^\circ$ and back to a negative $\phi$-function at $I = 0^\circ$, with mixed functions at intermediate cases. In southern magnetic latitudes, the functions are mirror images of those for the corresponding northern latitudes. In the method to be described in the next section, it is necessary to use the positive portion of the anomaly profile in high latitudes, the negative portion in low latitudes and to keep in mind the reversal between hemispheres.

Direct Method of Interpretation

A working method is desirable for the direct interpretation of dyke parameters without recourse to curve-fitting. This is accomplished by calculating certain factors theoretically and using these in conjunction with the same factors on the anomaly profile.

The total intensity function is of the form

$$\Delta T = \phi_D \left[ \phi_{AB} + A_D \ln \frac{r_B}{r_A} \right].$$

(4-12)

Two cases may be set up for convenience.

Case 1: $\phi_D \neq 0$,

$$\Delta F_1 = \frac{\Delta T}{\phi_D} = \phi_{AB} + A_D \ln \frac{r_B}{r_A}.$$

(4-13)

Case 2: $\phi_D = 0$,

$$\Delta F_2 = 2 \ln \frac{r_B}{r_A}.$$

(4-14)
### TABLE 4-I

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### TABLE 4-II

| $\ln \frac{r_A}{r_B}$ for the Dyke Model |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0  | 0.50 | 1.0 | 1.5 | 2.0 | 2.5 | 3.0 | 3.5 | 4.0 | 4.5 | 5.0 | 5.5 | 6.0 | 6.5 | 7.0 | 8.0 |
| 1  | 0  | 0.347 | 0.478 | 0.465 | 0.401 | 0.347 | 0.265 | 0.212 | 0.163 |   |   |   |   |   |   |
| 2  | 0  | 0.478 | 0.805 | 0.879 | 0.805 | 0.703 | 0.612 | 0.478 | 0.375 | 0.327 |   |   |   |   |   |   |
| 5  | 0  | 0.347 | 1.070 | 1.639 | 1.459 | 1.152 | 0.936 | 0.787 |   |   |   |   |   |   |   |   |
| 10 | 0  | 0.389 | 0.805 | 1.281 | 1.557 | 1.857 | 2.090 | 2.308 | 2.191 | 2.056 | 1.857 | 1.683 | 1.417 |   |   |   |
FIGURE 4-2: Vertical Dyke at $I = 90^\circ$, showing effect of varying width to depth ratio $w$, profile in units of depth to top, total intensity anomaly normalized to unity, vertical polarization.
FIGURE 4-3: Total intensity anomaly $\Delta T$ in units of $2l_T$, profile in units of depth, for Dyke at $I = 60^\circ$, E-W strike, $w = 10$, showing effect of inclination. $\alpha = 60^\circ, 90^\circ, 120^\circ$, normal polarization.
FIGURE 4-4: Dyke of width ratio $w = 5$, vertical, strike E-W, showing effect of varying magnetic inclination $I$. Profile in units of depth to the top. Total intensity anomaly $\Delta T$ in units of $2i_T$, normal polarization.
In order to set up a direct method for the interpretation of magnetic anomalies due to dyke-like bodies, certain values were calculated for \( w = 1, 2, 5 \) and \( 10 \), with \( A_0 = 0, 0.5, 1, 2 \) (Case 1) and for Case 2. Values for the narrow dyke \( (w = 0) \) are given in Chapter III. These were the maximum value of the function and the abscissae of the maximum, minimum, half-maximum intensity and inflection points. The origin for \( x \) is taken directly above the north edge of the top of the dyke.

Case 1: 
\[
\Delta F_1 = \tan^{-1} (x + w) - \tan^{-1} x + \frac{AD}{2} \left[ \ln \left(1 + x^2\right) - \ln \left(1 + (x + w)^2\right)\right]
\]

To determine the extrema of the function, a single differentiation is performed and the result set equal to zero. For the maximum,

\[
x_m = \frac{-\left(A_0 - 2\right) - \sqrt{A_0 + 2 + 4A_0(A_0 + w)}}{2A_0}, \quad A_D \neq 0,
\]

\[
x_m = -\frac{w}{2}, \quad A_D = 0.
\]

The maximum value of the function is given by

\[
\Delta F_{\text{max}} = \tan^{-1} \frac{w}{1 + x_m(x_m + w)} + \frac{A_0}{2} \ln \frac{1 + x_m^2}{1 + (x_m + w)^2}, \quad A_0 \neq 0,
\]

\[
\Delta F_{\text{max}} = 2 \tan^{-1} \frac{w}{2}, \quad A_0 = 0.
\]

Half this value is substituted into the original equation for determination of the abscissae for half-maximum intensity. The points of inflection are obtained by performing a second differentiation and equating to zero, yielding the equation:

\[
a_1 x + b_1 x^2 + c_1 x^3 + d_1 x^4 + e_1 x^5 = 0,
\]

where

\[
a_1 = -2w + A_0(3w^2 + w^4),
\]

\[
b_1 = 4w^2 + 2w^4 + A_0(6w + 4w^3),
\]

\[
c_1 = 4w + 8w^3 + A_0(6w^2 - w^4),
\]

\[
d_1 = 12w^2 + A_0(4w - 4w^3),
\]

\[
e_1 = 6w - 5A_0w,
\]

\[
f_1 = -2A_0w.
\]
Case 2: \[ \Delta F_2 = \ln \left[ 1 + x^2 \right] - \ln \left[ 1 + (x+w)^2 \right]. \]

In this case,
\[ x_m = \frac{1}{2} \left\lbrack -w - \sqrt{w^2 + 4} \right\rbrack. \]

Setting \( \frac{\Delta F_{\text{max}}}{2} = b, a = e \), the half-maximum points are given by
\[ x_1 = \frac{1}{a-1} \left\lbrack -aw + \sqrt{aw^2 + 2a - a^2 - 1} \right\rbrack. \]

The polynomial equation for the determination of the inflection points has the following coefficients:

\[ a_2 = 3w + w^3, \]
\[ b_2 = 6 + 4w^2, \]
\[ c_2 = 6w - w^3, \]
\[ d_2 = 4 - 4w^2, \]
\[ e_2 = -5w, \]
\[ f_2 = -2. \]

Computations for this problem were carried out on a Datation 205 computer. The results are shown in Table 4-III.

One pair of criteria for distinguishing between cases is the ratio \( \frac{R_D}{\eta_D} \) between inflection point width and half-intensity width and the ratio \( \frac{\eta_D}{\eta_D} \) of distances from anomaly maximum to half-maximum intensity. These are plotted in Figure 4-7 for various cases.

From the relation between \( w_{1/2} \) and \( w \) (Figure 4-5), and the measured values of \( w_{1/2} \) on the anomaly profile, the depth may be determined. From the A-value and Figure 4-6, estimates of magnetic polarization may be made in some cases.

**Examples**

(a) **Thubun River Anomaly** (Figure 4-9)

This anomaly, in the Great Slave Lake area of the Northwest Territories, is not strictly two-dimensional, but a profile across it is compared in
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FIGURE 4-5: Anomaly half-width as a function of dyke width w.
FIGURE 4-6: Anomaly maximum as a function of dyke width $w$ for various $\Lambda$ values.

$(\Delta F_1)_{\text{max}}$

$(\Delta F_2)_{\text{max}}$
FIGURE 4-7: Ratio $R_D$ between inflection point width and half-intensity width, ratio $\gamma_D : \gamma_D$ of distances from anomaly maximum to half-maximum intensities as a function of $A$. 
FIGURE 4-10: "Thubun River" anomaly, showing profile AB from Figure 4-8 and theoretical vertical intensity anomaly due to an infinite inclined dyke dipping southeastward at 45°, depth 1000 feet, $w_r = 2$, assuming normal polarization of $i_T = 3.3 \times 10^{-3}$ c.g.s.
Figure 4-10 with a theoretical anomaly due to a normally polarized dyke with
$w = 2$, $\psi = 45^0$, polarization $3.3 \times 10^{-3}$ c.g.s., top at the surface (sub-flight 1000 feet).

(b) **Boolcoomata Anomaly** (Figure 4-10)

This anomaly, in the Boolcoomata area of South Australia, reaches a peak intensity of 6200 gammas above a general level of 3000 gammas. The southerly half-maximum profile distance $\eta = 1400$ feet, the northerly $\eta' = 2400$ feet, the distance between inner inflection points is 2500 feet, hence $R = 0.66$, $\eta : \eta' = 0.58$. These values are close to those for $A_p = 1$, $w_r = 1.5$ (Figure 4-7). The corresponding anomaly half-width is 2.75 depth units, hence the depth to the top of the anomalous dyke-like body is $3800/2.75 = 1380$ feet sub-flight.
FIGURE 4-11: Portion of Bureau of Mineral Resources, Geology and Geophysics aeromagnetic sheet Booloomata, South Australia, showing the "Booloomata" anomaly.
CHAPTER V

FAULT, PRISM AND CONTACT MODELS

Fault Model

Magnetic bodies that are extensive both laterally and in depth may be displaced by faulting. Often, the fault can be considered two-dimensional, and the surface of the body can be shown in section normal to the strike of the fault as in Figure 5-1, where the discontinuity due to the fault is AB.

The total intensity anomaly due to such a discontinuity is derived in Appendix C. An elemental strip of the surface is represented by a line of magnetic poles and this is integrated over the entire surface. \( r_A \) is the distance from the point of observation to the side A of the discontinuity, \( r_B \) is the distance from the point of observation to the side B of the discontinuity, \( \alpha_{AB} \) is the angle subtended by the discontinuity, \( \delta \) is the angle of inclination of the fault discontinuity.

In terms of components of magnetic polarization \( i_H \) horizontal and normal to the strike, and \( i_z \) vertical, the total magnetic intensity is given by

\[
\Delta T = 2 \sin \delta \left\{ \left[ i_H (\sin \delta \sin I - \cos \delta \cos I \sin \phi) + i_z (\sin \delta \cos I \sin \phi + \cos \delta \sin I) \right] \ln \frac{r_A}{r_B} + \left[ i_H (\sin \delta \cos I \sin \phi + \cos \delta \sin I) + i_z (\cos \delta \cos I \sin \phi - \sin \delta \sin I) \right] \alpha_{AB} \right\} \]  

(5-1)

As in the case of the Dyke model, the corresponding horizontal intensity anomaly differs by a factor \( \sin \phi \) from the expression in the current literature (Heiland, 1940, eq 8-61e). This is again due to the fact that earlier authors have calculated the anomalous field component perpendicular to the strike of the body rather than the horizontal component along the magnetic meridian.

For the special case of polarization \( i_T \) in the direction of the earth's field,

\[
\frac{\Delta T}{2 i_T \sin \delta} = \left\{ 2 \sin \delta \sin I \cos I \sin \phi + \cos \delta \left( \sin^2 I - \cos^2 I \sin^2 \phi \right) \right\} \ln \frac{r_A}{r_B} + \left[ 2 \cos \delta \sin I \cos I \sin \phi - \sin \delta \left( \sin^2 I - \cos^2 I \sin^2 \phi \right) \right] \alpha_{AB}. \]  

(5-2)
FIGURE 5-1: Geometry for the Fault model.
The total intensity anomaly equations are of the form

\[ \Delta T = \varphi_F \left[ \ln \frac{r_A}{r_B} + A_F \varphi_{AB} \right], \] (5-3)

where

\[ \varphi_F = 2 \, i \, T \sin \delta \left[ 2 \sin \delta \sin I \cos I \sin \beta + \cos \delta \left( \sin^2 I - \cos^2 I \sin^2 \beta \right) \right], \]

and

\[ A_F = \frac{2 \, \cos \delta \sin I \cos I \sin \beta - \sin \delta \left( \sin^2 I - \cos^2 I \sin^2 \beta \right)}{2 \sin \delta \sin I \cos I \sin \beta + \cos \delta \left( \sin^2 I - \cos^2 I \sin^2 \beta \right)}. \]

A direct method for the interpretation of fault-type anomalies has been developed recently by the author and will be published separately. In the present work, some type curves will be presented for the case of normal polarization. These may be calculated using tables 5-I and 5-II.

The origin is directly above the top of the fault discontinuity, and all dimensions are in units of the depth to the top of the discontinuity. In these units, the vertical dip slip is denoted by \( d \), the horizontal dip slip by \( g \). Thus,

\[ r_A = \left[ (1+d)^2 + (x+g)^2 \right]^{1/2}, \]

\[ r_B = \left[ 1 + x^2 \right]^{1/2}, \]

\[ \varphi_{AB} = \tan^{-1} \frac{x+g}{1+d} - \tan^{-1} x. \] (5-4)

Common to all models are the terms \( \ln r_B \) and \( \tan^{-1} x \). These are given in the first row of tables 5-I and 5-II respectively. To obtain the terms \( \ln r_A \) and \( \tan^{-1} (x+g)/(1+d) \) from the tables, one need only choose the correct "d" row and shift the values by "g" units before subtraction from the corresponding terms \( \ln r_A \) and \( \tan^{-1} x \).

Figure 5-2 shows the total magnetic intensity anomaly at \( I = 90^\circ \) and vertical polarization, for a dip slip of 3 and horizontal dip slips of 2, 5 and 10.

Figure 5-3 shows the total magnetic intensity anomaly due to a vertical fault of dip slip 3 and normal polarization at various magnetic inclinations.
TABLE 5-I

\[ \ln \left[ x^2 + (1+d)^2 \right]^{1/2} \] for the fault problem

<table>
<thead>
<tr>
<th>d/x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.347</td>
<td>0.805</td>
<td>1.152</td>
<td>1.417</td>
<td>1.629</td>
<td>1.806</td>
<td>1.956</td>
<td>2.087</td>
<td>2.203</td>
<td>2.308</td>
<td>2.402</td>
<td>2.489</td>
<td>2.568</td>
<td>2.642</td>
<td>2.710</td>
</tr>
<tr>
<td>0.5</td>
<td>0.000</td>
<td>0.406</td>
<td>0.590</td>
<td>0.917</td>
<td>1.210</td>
<td>1.452</td>
<td>1.653</td>
<td>1.823</td>
<td>1.969</td>
<td>2.097</td>
<td>2.211</td>
<td>2.313</td>
<td>2.407</td>
<td>2.492</td>
<td>2.571</td>
<td>2.644</td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.693</td>
<td>0.805</td>
<td>1.040</td>
<td>1.282</td>
<td>1.498</td>
<td>1.683</td>
<td>1.845</td>
<td>1.985</td>
<td>2.110</td>
<td>2.222</td>
<td>2.322</td>
<td>2.414</td>
<td>2.498</td>
<td>2.572</td>
<td>2.649</td>
</tr>
<tr>
<td>2</td>
<td>1.098</td>
<td>1.151</td>
<td>1.282</td>
<td>1.445</td>
<td>1.610</td>
<td>1.763</td>
<td>1.903</td>
<td>2.030</td>
<td>2.145</td>
<td>2.250</td>
<td>2.345</td>
<td>2.433</td>
<td>2.515</td>
<td>2.591</td>
<td>2.661</td>
<td>2.727</td>
</tr>
<tr>
<td>3</td>
<td>1.386</td>
<td>1.417</td>
<td>1.498</td>
<td>1.601</td>
<td>1.733</td>
<td>1.856</td>
<td>1.975</td>
<td>2.087</td>
<td>2.191</td>
<td>2.287</td>
<td>2.376</td>
<td>2.460</td>
<td>2.537</td>
<td>2.610</td>
<td>2.678</td>
<td>2.742</td>
</tr>
</tbody>
</table>

TABLE 5-II

\[ \tan^{-1} \frac{x}{1 + d} \] for the fault problem

<table>
<thead>
<tr>
<th>d/x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
<td>0.785</td>
<td>1.107</td>
<td>1.249</td>
<td>1.326</td>
<td>1.373</td>
<td>1.406</td>
<td>1.429</td>
<td>1.446</td>
<td>1.460</td>
<td>1.471</td>
<td>1.480</td>
<td>1.488</td>
<td>1.494</td>
<td>1.499</td>
<td>1.504</td>
</tr>
<tr>
<td>0.5</td>
<td>0.000</td>
<td>0.588</td>
<td>0.927</td>
<td>1.107</td>
<td>1.279</td>
<td>1.326</td>
<td>1.360</td>
<td>1.386</td>
<td>1.406</td>
<td>1.422</td>
<td>1.435</td>
<td>1.446</td>
<td>1.456</td>
<td>1.464</td>
<td>1.471</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>0.000</td>
<td>0.464</td>
<td>0.785</td>
<td>0.983</td>
<td>1.107</td>
<td>1.190</td>
<td>1.249</td>
<td>1.292</td>
<td>1.326</td>
<td>1.352</td>
<td>1.373</td>
<td>1.391</td>
<td>1.406</td>
<td>1.419</td>
<td>1.429</td>
<td>1.438</td>
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<tr>
<td>2</td>
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<td>0.322</td>
<td>0.588</td>
<td>0.785</td>
<td>0.927</td>
<td>1.030</td>
<td>1.107</td>
<td>1.166</td>
<td>1.190</td>
<td>1.249</td>
<td>1.279</td>
<td>1.305</td>
<td>1.326</td>
<td>1.344</td>
<td>1.360</td>
<td>1.374</td>
</tr>
<tr>
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<td>0.245</td>
<td>0.464</td>
<td>0.644</td>
<td>0.785</td>
<td>0.896</td>
<td>0.983</td>
<td>1.052</td>
<td>1.107</td>
<td>1.153</td>
<td>1.190</td>
<td>1.222</td>
<td>1.249</td>
<td>1.272</td>
<td>1.292</td>
<td>1.310</td>
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<tr>
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<td>0.000</td>
<td>0.099</td>
<td>0.197</td>
<td>0.291</td>
<td>0.380</td>
<td>0.463</td>
<td>0.540</td>
<td>0.611</td>
<td>0.673</td>
<td>0.733</td>
<td>0.785</td>
<td>0.833</td>
<td>0.876</td>
<td>0.914</td>
<td>0.951</td>
<td>0.983</td>
</tr>
<tr>
<td>19</td>
<td>0.000</td>
<td>0.050</td>
<td>0.099</td>
<td>0.148</td>
<td>0.197</td>
<td>0.245</td>
<td>0.291</td>
<td>0.337</td>
<td>1.380</td>
<td>0.440</td>
<td>0.463</td>
<td>0.503</td>
<td>0.540</td>
<td>0.576</td>
<td>0.611</td>
<td>0.624</td>
</tr>
</tbody>
</table>
FIGURE 5-2: Inclined fault or contact at $\gamma = 90^\circ$, vertical dip slip $d = 3$, horizontal dip slip $g = 2, 5, 10$. 

TOTAL INTENSITY ANOMALY (UNITS OF $2 \times 10^6$)
FIGURE 5-3: Normally polarized vertical fault, vertical slip $d_F = 3$, at various magnetic inclinations.
FIGURE 5-4: Terrain effect of trough in magnetic material.
Fault Model Application - Terrain Effects

An ever-present problem in rugged topography is the terrain effect. An idealized representation of this is shown in Figure 5-4, which portrays the effect of a trough in magnetic material of normal polarization at high latitudes. From our earlier work on the Fault model, the anomaly is given by

\[
\frac{\Delta T}{1_T} = \frac{\Delta Z}{1_Z} = \ln \left( \frac{(1+d)^2 + x^2}{1 + (x+d)^2} \right)^{1/2} - \ln \left( 1 + (x-d)^2 \right)^{1/2} + \tan^{-1}(x-d) - \tan^{-1}(x+d).
\]

(5-5)

For comparison, the peak negative vertical intensity anomaly due to a dyke-like polarization contrast 1000 feet wide, contrast \(-0.5 \times 10^{-3}\) in the vertical direction, follows. These values represent a situation of common interest in structural interpretation and it will be noted that the fall-off in terrain effect is faster than for the dyke-like body. For a given application an optimum terrain clearance can be selected to minimize terrain effects and still show desired effects.

**NEGATIVE ANOMALY DUE TO TROUGH 500 FEET DEEP IN MATERIAL OF MAGNETIC POLARIZATION \(10^{-3}\) C.G.S. UNITS**

<table>
<thead>
<tr>
<th>Terrain Clearance</th>
<th>(\Delta T_{\text{min}})</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 feet</td>
<td>-242</td>
</tr>
<tr>
<td>500 feet</td>
<td>-38</td>
</tr>
<tr>
<td>1000 feet</td>
<td>-34</td>
</tr>
</tbody>
</table>

**Anomaly Due to Vertical Dyke**

---

1000 Feet Wide, \(i_z = -0.5 \times 10^{-3}\)

- 275

- 160

- 90

**Vertical Prism Model**

The excellent memoir by Vacquier et al (1951) treats the case of the vertical prism, and this model is used for the calculation of a number of cases at various geomagnetic latitudes. Models of various sizes are used, and both total intensity and "curvature" are calculated in each case. The models find their chief application in the case of sedimentary basins, where magnetic polarization contrasts are used for determining the depth to the top of the basement.
In mineral surveys, one is usually dealing with magnetic bodies that outcrop or come very close to the surface. Generally speaking, there is insufficient flight-line coverage to permit calculation of curvatures on a grid-spacing equal to the depth to the top of the source, so that only the total intensity map and models may be used.

The first approximation to the depth to the top of a prism-like body is made using the distances of maximum gradient measured along the flanks of the anomaly. This is a rather crude approach, particularly because the distance of maximum gradient is difficult to determine. Having estimated the depth, the dimensions of the body are approximated by the inflection points on the anomaly and expressed in units of the depth. A model may then be selected, and by comparing percentage changes in intensity on the model and on the map, a number of estimates can be made of the depth and averaged. If the depth estimates are too inconsistent, a different model must be selected.

Contact Problem

The question of a long regional slope of a magnetic member such as one would encounter at the edge of a basin area is approached by using the fault formula and letting \( d \) become very large. The functions \( \ln r_A \) and \( \tan^{-1} \left( \frac{x+g}{1+d} \right) \) are then slowly varying, so that near the upper edge of the slope the anomaly is of the form

\[
\Delta T = -\phi_F \left[ \ln r_B + A_F \tan^{-1} x \right].
\]  

(5-6)

A well-known special case of this is the anomalous variation in high latitudes due to a vertically polarized vertical contact extending to great depth,

\[
\Delta Z = -\phi_F A_F \tan^{-1} x.
\]  

(5-7)
CHAPTER VI

THE DOWNWARD CONTINUATION METHOD

The Downward Continuation method is used to calculate the field below the plane of observations corresponding with a measured field distribution, thus obtaining greater resolution since the field mapping is closer to the source of the anomalous field. In practice, our observations can give no information about changes in anomalous physical properties within horizontal distances small compared with the depth of the anomaly source. Thus, improvement in resolution is limited. However, some knowledge of the depth of the anomalous body is obtained by the fact that below the depth of burial Downward Continuation calculations should begin to give a solution that oscillates rapidly due to the lack of convergence of the mathematical solution. Peters (1949) in his direct approach to the interpretation of vertical intensity magnetic data describes a method of Downward Continuation based on Taylor's Expansion using the field and its vertical derivatives on the surface. Bullard and Cooper (1948) in discussing Downward Continuation of gravity utilize two methods. One is an analytic method based on a Fourier-Bessel integral and incorporating an exponential smoothing function. The degree of smoothing used discriminates against anomalies more local than the depth of Downward Continuation. The other uses a finite difference approximation to Laplace's equation and eliminates anomalies more local than the grid used for the continuation process, which is the same as the depth of Downward Continuation. The aim of the present work is to show how the Downward Continuation method may be used in a flexible manner to cope with anomalies of varying dimensions. In this way, anomalies of the particular type one is interested in may be emphasized. Also, a variable continuation step may be made while still retaining anomalies of a given "wave-length". A limitation is that errors may build up to the point where they are larger than the anomalies sought.
The Fundamental Problem

Aeromagnetic surveys fall into two broad classes, those whose object is to show anomalies due to magnetic bodies at a shallow depth below the level of observations (mineral exploration) and those flown over sedimentary basins where the object is information on the underlying basement complex where magnetic bodies may be at depths of thousands of feet.

In the shallow source problem, we are often dealing with localized anomalies, some relatively unmerged with neighbouring ones, and their corresponding magnetic effects are expressible as the summation of the effects due to multipole moments of various orders, viz. poles, dipoles, quadrupoles.

In the deep source problem anomalies are merged together so that large-scale effects give the strongest anomalies and those due to structural uplifts in the magnetic basement are small perturbations on these. The field may be expressed in various ways, one of which is the Fourier-Bessel integral.

The Downward Continuation Formulation

The problem of the Downward Continuation of gravity data has been treated by Bullard and Cooper (1948) who included in their analytical approach both a two-dimensional and a three-dimensional formulation. This treatment of the gravity case is taken over for the treatment of the total magnetic intensity case.

(a) Two dimensions. It is assumed that a simple harmonic potential field variation

\[ A \cos px \]

corresponds with a field variation at depth \( z \) of

\[ Ae^{pz} \cos px. \]

If now the field at depth \( z \) is assumed to consist of a number of components and the surface field is smoothed exponentially, viz.

\[ \tilde{g}_0 (\xi) = \sqrt{\frac{\beta}{\pi}} \int_{-\infty}^{\infty} g_o(x) e^{-\beta(x \cdot \xi)^2} dx, \]

(6-3)
then the field at depth $z$ turns out to be

$$
\bar{g}_z(x) = \int_{-\infty}^{\infty} g_0(x_1) \lambda'(x_1 - x) \, dx_1,
$$

where $\lambda'(x_1 - x) = \frac{1}{\pi} \int_{0}^{\infty} e^{-\frac{p^2}{2\beta}} \cos p(x_1 - x) \, dp$.

(6-5)

$\lambda'(x_1 - x)$ is a weighting function depending on $\beta$. The smoothing parameter determines the fineness of the field variation produced. With small $\beta$ (heavy smoothing), the Fourier expansion is attenuated at a certain wave length limit so that only the longer wave-length components are extrapolated downward and a relatively smoothed field results. If, however, a large $\beta$ (light smoothing) is used, short wave length components are included. Since these are magnified according to $e^{p\beta}$, the weighting function becomes sharply peaked and of large amplitude. As a result, there is a danger that errors will be magnified disproportionately. The critical aspect of the application is to select smoothing that is light enough to pass desirable components but not so light that extraneous effects are included.

If the continuation step is used as the unit of length and we set $z \sqrt{\beta} \rightarrow \sqrt{\beta}$, the weighting function becomes

$$
\lambda(x_1 - x) = \frac{1}{\pi} \int_{0}^{\infty} e^{p} e^{-\frac{p^2}{4\beta}} \cos p(x_1 - x) \, dp.
$$

This function is plotted for three values of $\beta$ in Figure 6-1.

(b) Three dimensions. In this case, components are assumed of the form $e^{p\beta} J_0(\beta r) \cos n \phi$.

$$
e^{p\beta} J_0(\beta r) \cos n \phi.
$$

(6-7)

The surface field is smoothed according to

$$
\bar{g}_z(\phi) = \frac{2\pi}{\beta^2} \int_{0}^{\pi/2} \int_{0}^{2\pi} \bar{g}_z(\rho, \phi_2) \, e^{-\frac{\beta^2}{4}} d\rho d\phi_2,
$$

and the final result is

$$
\bar{g}_z(\phi) = \int_{0}^{\infty} g_0(r_1) \Lambda^s(r_1) \, dr_1,
$$

where

$$
\Lambda^s(r_1) = r_1 \int_{0}^{\infty} e^{p\beta} e^{-\frac{p^2}{4\beta}} J_0(\beta r_1) p \, dp.
$$

(6-10)
FIGURE 6-1: Two-dimensional downward continuation weighting functions showing effect of smoothing parameter.

Horizontal scale $2^{1/2} \text{ in} = 1 \text{ unit}$
In non-dimensional form, i.e., \( z \sqrt{\gamma} \rightarrow \sqrt{\gamma} \),

\[
\sum (r_1) = r_1 \int_0^\infty e^{\frac{p}{\sqrt{\gamma}}} \int_0^1 J_0 (pr_1) p \, dp.
\]  

(6-11)

Bullard and Cooper (1948) calculated this function for \( \gamma = 1 \) and the extension to other values of the smoothing parameter will be treated subsequently.

**Variation of Smoothing Parameter (Two Dimensions)**

The effects on the Downward Continuation weighting function of the variation of smoothing parameter may be seen by considering the two-dimensional case. In Figure 6-1, the \( \delta = 1 \) and \( \delta = 4 \) cases are taken from the paper by Bullard and Cooper (1948), while the \( \delta = \frac{1}{4} \) case has been calculated using a function given by Born (1933, p.486). The very heavy smoothing of the \( \delta = \frac{1}{4} \) case means that adjacent values are weighting the centre point heavily and this has an averaging effect which will smooth out many desired effects. For \( \delta = 4 \), the weighting function is sharply peaked and can, therefore, operate on field components whose wave lengths are relatively short. Another point to keep in mind, of course, is the density of the data. Sufficient data must be available within the rapidly-oscillating portion of the weighting function to permit derivation of a relatively true picture of the continued field.

Grid approximation formulas for two-dimensional Downward Continuation may be made up, such as the following for a step of one unit.

\[
\begin{align*}
\delta &= 4: & T_{1.0} &= 28.4 \, T(o) - 16.2 \left[ T(0.4) + T(-0.4) \right] + 2.5 \left[ T(0.8) + T(-0.8) \right] \\
\delta &= 1: & T_{1.0} &= 1.08 \, T(o) - 0.63 \left[ T(0.4) + T(-0.4) \right] - 0.07 \left[ T(0.8) + T(-0.8) \right] \\
\delta &= \frac{1}{4}: & T_{1.0} &= 0.26 \left[ T(1.2) + T(-1.2) \right] - 0.14 \left[ T(1.6) + T(-1.6) \right] - 0.05 \left[ T(2.0) + T(-2.0) \right] \\
\delta &= 0.26 \left[ T(2.2 \rightarrow \infty) + T(-2.2 \rightarrow -\infty) \right].
\end{align*}
\]  

(6-12)

Points spaced along the profiles at intervals of 0.4 times the step are used. In the case of \( \delta = 4 \), this interval represents a practical limit of coarseness, since all the weight of the initial positive portion of the curve is applied to the center point.
These formulas were tested on two theoretical models, the vertical fields due to a vertically polarized cylinder and a narrow vertically polarized dyke, at 1 unit depth, taking a downward step of 0.625 units, the initial fields normalized to unity. Only the anomaly maximum is calculated.

(a) Vertically Polarized Cylinder  
(b) Narrow Vertically Polarized Dyke

\[
\begin{align*}
\text{Theor. } T & = 7.11 \\
T(4) & = 3.90 \\
T(1) & = 1.78 \\
\text{Theor. } T & = 2.67 \\
T(4) & = 5.00 \\
T(1) & = 1.61
\end{align*}
\]

It will be recalled that the half-width of the anomaly in case (b) is about twice that in case (a). Application of the \( \phi = 4 \) formula appears to be attempting to continue wave-lengths that are too short and the combination of five points used for the calculation can be fitted by a Fourier expansion that includes short wave-length terms.

**Variation of Smoothing Parameter (Three Dimensions)**

Calculations of the three-dimensional Downward Continuation weighting function have been carried out for the cases of smoothing parameter \( \tau = 0.5, 1.5, 2.0, 3.0 \) and \( 4.0 \). Added to the case \( \tau = 1 \), these permit a more flexible application of the method. By using lighter smoothing, it will be possible to continue shorter wave-length components with a given set of data. Also, by utilizing a varying smoothing parameter, it should be possible to vary the continuation step and retain the same wave-length cut-off. Otherwise, with a fixed smoothing parameter and variable step, the wave-length continued increases with the size of the step.

The three-dimensional Downward Continuation function was evaluated by a method of Numerical Quadrature, using a Datatron 205 high-speed digital computer.

The function may be expressed in the form

\[
\Lambda (r_1) = r_1 e^{\tau} \int_0^\infty e^{(p-2r_1^2)} J_0 (pr_1) \, p \, dp.
\]  

(6-13)
Goodwin (1949) has discussed the approximation
\[
\int_{-\infty}^{\infty} e^{-x^2} f(x) dx = h \sum_{n=-\infty}^{\infty} e^{-n^2h^2} f(nh) + \epsilon(h).
\]  
(6-14)
With a simple change of variable and the function defined to be zero in 
\(-\infty < x < \infty\), one obtains an applicable formula.

The Bessel function is approximated by Chebyshev expansions in the 
range \((0, 4)\) and \((4, \infty)\).

"h" provided a simple and flexible control over the error term, and was varied by as much as a factor of 3 depending on the values of \(r_1\) and \(\gamma\).

"n_{max}" was derived as a function of \(\gamma\) in order that the required accuracy of 0.1% could be obtained.

Computations were carried out on a Datatron 205 computer. The Downward Continuation weighting functions are shown in Figure 6-2 in graphical form. I am indebted to Mr. F. Howley of Adalia Computations Limited for carrying out the mathematical analysis and computer programming.

**Grid Approximation**

The alternatives in applying a weighting function of the \(\wedge(r_1)\) type are to use it in template form or as a grid approximation. With a template, work is laborious and time-consuming. While not quite as accurate, a grid approximation method is realistic when dealing with aeromagnetic data, since the grid spacing can be made equal to the flight-line spacing. In the work to be described, values were interpolated to the corners of a square grid. For the calculations at a given point, the center point and five rings were utilized as shown in Figure 6-3.

The proper weighting of the various circles is not easy, since the circle radii are not uniformly spaced. One approach was used by Peters (1949) and involved weighting each circle with half the value of the integral between it and each adjacent circle. While this is accurate in some cases, there are situations when it is not. For example, if the
FIGURE 6-2: Three-dimensional downward continuation weighting functions for smoothing parameter $\tau = 0.5, 1.0, 1.5, 2.0, 3.0, 4.0$. 
FIGURE 6-3: Grid for downward continuation.
weighting function changes sign, its integral between one circle and the next may be close to zero. On the other hand, by taking an integration interval straddling the circle, a more suitable value may be obtained. It is impossible with the grid arrangement as shown to select integration limits which exactly straddle each circle. In the system which follows, taking the grid spacing as unity, the centre point was weighted with the integral from 0 to 0.6 and the first ring with the integral from 0.6 to 1.2. This weights each point with an equal area of the region of integration. The selection of the remaining intervals is somewhat arbitrary, since it is impossible to weight each point with an equal area of the integral and at the same time have these limits fall evenly between circles. The following arrangement has been provisionally adopted.

<table>
<thead>
<tr>
<th>RING NUMBER</th>
<th>C \cdot P</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radius</td>
<td>0</td>
<td>1.000</td>
<td>1.414</td>
<td>2.236</td>
<td>2.914</td>
<td>4.121</td>
</tr>
<tr>
<td>Integration Range</td>
<td>0 0.6</td>
<td>1.2</td>
<td>1.8</td>
<td>2.5</td>
<td>3.5</td>
<td></td>
</tr>
</tbody>
</table>

The weighting function \( \Lambda(r_1) \) is expressed in units of the Downward Continuation step for each \( \gamma \) value. This means that as the step is varied, the integration ranges must be varied in an opposite manner. For example, for a step of twice the grid spacing, the integration ranges become

\[
0 \quad 0.3 \quad 0.6 \quad 0.9 \quad 1.25 \quad 1.75
\]

With a fixed \( \gamma \) value, since the smoothing parameter is geared to the step taken, a step of two grid units will have a low wave-length cut-off of twice that with a step of one grid unit, and the process will extrapolate downward longer wave-length components. Thus, it is best to vary the smoothing parameter with the step taken. Grid approximation formulas using a smoothing parameter varying with the step will be used in the next few sections.
Test With Polar Sources

The Downward Continuation method was tested on two sets of artificial data. These were the vertical field due to a polar source and to two polar sources separated by a distance equal to the depth. This type of anomaly is obtained, for example, in the vertical gravity field component due to a dense sphere and in the vertical magnetic field due to a single magnetic pole (vertically polarized vertical plug). Theoretical field values were calculated on a square grid of unit mesh. The sources were at a depth of two grid units and in the case of the double source, separated by two grid units. The formulas used for the test are given in Table 6-1, together with the depths of continuation and the smoothing parameters used. Results are shown diagrammatically in Figures 6-4 to 6-9. It can be observed that formulas 1, 2 and 3, which represent a short wave-length formulation are much more effective in reproducing the field than the long wave-length formulations represented by formulas 4, 5, 6 and 7, 8, 9. Results of the application of formula 6 do not fit on scale and are not shown.

Table 6-1
GRID APPROXIMATION FORMULAS FOR TEST OF DOWNWARD CONTINUATION METHOD

<table>
<thead>
<tr>
<th>SMOOTHING FORMULA</th>
<th>STEP PARAMETER</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.5 1</td>
<td>+4.68</td>
<td>-2.38</td>
<td>-0.60</td>
<td>-0.70</td>
<td>-0.50</td>
<td>-0.50</td>
</tr>
<tr>
<td>2</td>
<td>3.0 2</td>
<td>+17.75</td>
<td>-10.0</td>
<td>-8.5</td>
<td>+0.75</td>
<td>+0.5</td>
<td>+0.5</td>
</tr>
<tr>
<td>3</td>
<td>4.0 2.67</td>
<td>+43.1</td>
<td>-23.6</td>
<td>-29.1</td>
<td>+12.5</td>
<td>-1.0</td>
<td>-0.9</td>
</tr>
<tr>
<td>4</td>
<td>1.0 1</td>
<td>+2.30</td>
<td>+0.51</td>
<td>-1.10</td>
<td>-0.30</td>
<td>-0.10</td>
<td>-0.30</td>
</tr>
<tr>
<td>5</td>
<td>2.0 2</td>
<td>+4.85</td>
<td>+1.98</td>
<td>-4.12</td>
<td>-2.11</td>
<td>+0.20</td>
<td>+0.20</td>
</tr>
<tr>
<td>6</td>
<td>4.0 4</td>
<td>+25.1</td>
<td>+20.9</td>
<td>-19.4</td>
<td>-29.0</td>
<td>+9.2</td>
<td>+3.1</td>
</tr>
<tr>
<td>7</td>
<td>0.5 1</td>
<td>+0.66</td>
<td>+1.00</td>
<td>+0.21</td>
<td>-0.31</td>
<td>-0.28</td>
<td>-0.28</td>
</tr>
<tr>
<td>8</td>
<td>1.0 2</td>
<td>+0.80</td>
<td>+1.50</td>
<td>+0.65</td>
<td>-0.24</td>
<td>-0.94</td>
<td>-0.77</td>
</tr>
<tr>
<td>9</td>
<td>1.5 3</td>
<td>+1.00</td>
<td>+2.18</td>
<td>+1.47</td>
<td>-0.19</td>
<td>-2.21</td>
<td>-1.25</td>
</tr>
</tbody>
</table>

\[ T = AT(0) + B \bar{T}(1) + C \bar{T}(2) + DT(3) + DT(4) + \bar{T}(5) \]
FIGURE 6-4: Single polar source - downward continuation to half depth of source.
FIGURE 6-5: Single polar source - downward continuation to depth of source.
FIGURE 6-8: Single polar source - downward continuation to below source.
FIGURE 6-7: Double polar source - downward continuation to half depth of source.
FIGURE 6-8: Double polar source - downward continuation to depth of source.
FIGURE 6-9: Double polar source - downward continuation to below source.
Test With New Brunswick Two-Level Data

The application to a two-level aeromagnetic survey carried out by the Geological Survey of Canada in New Brunswick is shown in Figures 6-10, 6-11 and 6-12. The high-level data was taken at 4000 feet above sea level on east-west flight lines at an average line spacing of one-half mile, and is shown in Figure 6-10. The Downward Continuation shown in Figure 6-11, to 1000 feet above terrain, was carried out using variable steps depending on the true height of the low-level survey above sea level. A grid spacing of 2800 feet and the $\gamma = 1$ formulas as follows were used.

**DOWNWARD CONTINUATION FORMULAS - $\gamma = 1$** (6-15)

\[
T_{0.6} = 3.08 T(0) - 1.53 T(1) - 0.21 T(2) - 0.10 T(3) - 0.06 T(4) - 0.16 T(5)
\]

\[
T_{0.8} = 2.74 T(0) - 0.61 T(1) - 0.72 T(2) - 0.13 T(3) - 0.10 T(4) - 0.23 T(5)
\]

\[
T_{1.0} = 2.30 T(0) + 0.51 T(1) - 1.10 T(2) - 0.31 T(3) - 0.10 T(4) - 0.30 T(5)
\]

\[
T_{1.5} = 1.30 T(0) + 1.65 T(1) + 0.08 T(2) - 0.63 T(3) - 0.11 T(4) - 0.29 T(5)
\]

\[
T_{2.0} = 0.80 T(0) + 1.50 T(1) + 0.65 T(2) - 0.24 T(3) - 0.94 T(4) - 0.77 T(5)
\]

The actual low level survey at 1000 feet above terrain on north-south flight lines at an average spacing of one-half mile appears as Figure 6-12. The comparison is favourable in the case of the main anomaly, part of the difference being attributable to the difference in flight line direction and the inadequate line spacing. One would not expect very narrow anomalies to be continued adequately without using a lighter smoothing parameter. This example was carried out using a variable step but fixed smoothing parameter. Since the land surface varied from sea level to some 1200 feet above sea level, the variation in step was not large.
FIGURE 6-10: Aeromagnetic survey at 4000 feet above sea level, New Brunswick, Canada. Flight lines east-west at half mile spacing.
FIGURE 6-12: Aeromagnetic survey at 1000 feet mean terrain clearance, New Brunswick, Canada. Flight lines north-south at half mile spacing.
The Downward Continuation method was tested on an aero-magnetic survey carried out adjacent to the 49th parallel of latitude by the Geological Survey of Canada. The flight line spacing for this was one mile, permitting only the broader magnetic features to be continued downward accurately. Another limitation was the fact that only twelve lines were flown. Since a five-ring formula was used, and a grid spacing of one mile, this left a strip of calculated values only four miles wide. To obtain a set of formulas, the following were derived from the calculated weighting functions:

\[ \gamma = 0.5 : T \]
\[ = 1.66T(0) - 0.10T(1) - 0.25T(2) - 0.15T(3) - 0.10T(4) - 0.06T(5) \]
\[ \gamma = 1.0 : T \]
\[ = 2.30T(0) + 0.51T(1) - 1.10T(2) - 0.30T(3) - 0.10T(4) - 0.30T(5) \]
\[ \gamma = 1.5 : T \]
\[ = 3.22T(0) + 1.33T(1) - 2.21T(2) - 0.60T(3) - 0.50T(4) - 0.24T(5) \]

From these, a working set was obtained by interpolation, with the step as usual taken as a fraction of the grid spacing of one mile.

### TABLE 6-II

**DOWNWARD CONTINUATION GRID APPROXIMATION FORMULAS - 49TH PARALLEL TEST**

\[ T = AT(0) + BT(1) + CT(2) + DT(3) + ET(4) + FT(5) \]

<table>
<thead>
<tr>
<th>FORMULA</th>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>SUB-FLIGHT</th>
<th>GROUND ELEVATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.83</td>
<td>0.11</td>
<td>-0.50</td>
<td>-0.22</td>
<td>-0.14</td>
<td>-0.08</td>
<td>3,000-4,000</td>
<td>5,000-6,000</td>
</tr>
<tr>
<td>2</td>
<td>2.09</td>
<td>0.30</td>
<td>-0.82</td>
<td>-0.30</td>
<td>-0.17</td>
<td>-0.10</td>
<td>4,000-5,000</td>
<td>4,000-5,000</td>
</tr>
<tr>
<td>3</td>
<td>2.36</td>
<td>0.54</td>
<td>-1.18</td>
<td>-0.39</td>
<td>-0.21</td>
<td>-0.12</td>
<td>5,000-6,000</td>
<td>3,000-4,000</td>
</tr>
<tr>
<td>4</td>
<td>2.68</td>
<td>0.82</td>
<td>-1.59</td>
<td>-0.48</td>
<td>-0.28</td>
<td>-0.15</td>
<td>6,000-7,000</td>
<td>2,000-3,000</td>
</tr>
</tbody>
</table>

Each formula was designed to continue downward from 9,000 feet to the midpoint of the range shown on the right hand side of Table 6-II. All four formulas were applied at each point using a Datatron 205 computer and the value selected according to the ground elevation at that point.

A strip of grid-interpolated data 144 miles east-west by 11 miles north-south was treated and a contour map made up of the continued field,
FIGURE 6-13: Portion of total intensity aeromagnetic map at 9,000 feet above sea level. Christina Lake area, British Columbia. Scale one inch equals two miles.
FIGURE 6-14: Downward continuation of total magnetic intensity to ground level, Christina Lake area, British Columbia. Scale one inch equals two miles.
a portion of which is shown in Figure 6-14. Figure 6-13 shows the portion of the original total intensity map in the same area, in the vicinity of Christina Lake, British Columbia. The step here is from 9,000 feet to about 2,000 feet above sea level. As pointed out earlier, only the broad magnetic variation is shown. The downward continuation shows improved resolution over the original data and shows the trends of the basic intrusive masses more clearly. In the case of the southerly lobe of this mass, a westerly extension beneath Christina Lake is indicated.
CHAPTER VII

DERIVATIVE AND RESIDUAL METHODS

The use of the "Second Derivative" method for resolving local anomalies in the potential field methods has been discussed by a number of writers (Elkins, 1951; Rosenbach, 1953; Peters, 1949; Henderson and Zietz, 1949). Recently Nettleton (1954) has reviewed these various formulas and pointed out the similarity between them and the "average value method" for eliminating regional gravity effects described by Griffin (1949). Both Nettleton and Griffin remark on the necessity of careful selection of grid spacing if one is to obtain satisfactory results.

The application of the First Derivative in the gravitational case has been discussed by several writers (Evjen, 1936; Baranov, 1953; Ackerman and Dix, 1955). The First and Second Derivatives are similar in type and may be calculated using a square grid of points. It will be shown in the present work that a smoothing parameter has a similar effect on the derivative formulations to that on the Downward Continuation formulation.

Second Derivative Method

The Second Derivative method has been by far the most popular method for treating potential-field data. In a preliminary study of the method on shallow-source data, the "center-point-and-one-ring" method was used, viz.

\[
\frac{\partial^2 T}{\partial z^2} = \frac{4}{s^2} \left[ T(o) - \bar{T}(s) \right],
\]

where \( s \) is the radius of the ring, \( T(o) \) the value of the field component at the central point and \( \bar{T}(s) \) the average value on the ring of radius \( s \).
This method was tested on an aeromagnetic map kindly provided by Dominion Gulf Company of Toronto, Canada. This map, on a scale of approximately one inch equals 1320 feet, is of the Barlow Lake area, lat. 49° 54'N, long. 74° 40'E, Province of Quebec. Flight elevation averaged 500 feet above terrain. A portion of the map is shown in Figure 7-1. The anomaly blocked off is due to an ultrabasic sill.

Figure (7-2(a) shows the Second Derivative calculated with a grid spacing of 1/4" or 330 feet actual distance on a square grid. This is perhaps pushing the method unrealistically, since the flight lines average 1320 feet apart. However, closures on the Second Derivative map generally coincide with closures on a ground magnetic map of the area (Figure 8-1). Irregularity of the contours in places shows the effects of the fine grid spacing and interpolation between flight lines.

Figure 7-2(b) shows the Second Derivative for a grid spacing of 1" (1320 feet). It is a smoother map than the previous, although some of the detail is lost. Calculations were made at points 1/4" apart on N-S lines drawn 1" apart. The zero intensity contours are wider apart than in the previous example.

Formula 13 of Henderson and Zietz (1949) was applied to the New Brunswick test area (Chapter VI) with s = 4000 feet (Figure 7-3).

In his development of a multipole method for the interpretation of gravitational anomalies, Grant (1952) has discussed the calculation of derivatives of the gravitational potential from measured vertical gravity field values. Since no calculations were carried out for $U_{zzz}$, the second vertical derivative of the vertical gravitational field, these have been carried out. First of all, the basis for the calculations will be sketched briefly. The vertical gravitational field on a plane below the plane of observations is expressed as in the case of the Downward Continuation problem by a Fourier-Bessel integral,
Total intensity aeromagnetic map, Barlow Lake Area, Courtesy Dominion Gulf Company. Scale one inch equals 1320 feet; contour interval 20 gamma. Solid line outlines area of second derivative maps.
FIGURE 7-2: Second vertical derivative of total magnetic intensity, Barlow Lake anomaly, calculated using a center point and ring-of-four, scale one inch equals 1320 feet. (a) Grid spacing $\frac{1}{4}''$, contour interval 100,000 gammas /mile$^2$. (b) Grid spacing 1'', contour interval 25,000 gammas /mile$^2$. 
The solution for the Second Derivative is

\[ g_{zz} = U^0_{zzz} = \frac{1}{\beta^2} \int_0^\infty g(\rho) \Lambda_1^{(x)}(\rho^-) \, d\rho, \]

where \( \Lambda_1^{(x)}(\rho^-) = \gamma^2 e^{-\gamma \rho^2} \text{F}_1(-1; 1; \gamma \rho^2), \)

where \( \text{F}_1 \) denotes the confluent hypergeometric function.

If \( \rho \) is expressed in units of \( s \), the grid spacing \( \beta \) may be replaced by \( \gamma = \beta s^2 \), and the form of the function remains unchanged except for a dimensional factor.

We now have

\[ g_{zz} = \frac{4}{\beta^2} \int_0^\infty g(\rho^1) \Lambda_1^{(x)}(\rho^1) \, d\rho^1, \]

where \( \Lambda_1^{(x)}(\rho^1) = \gamma^2 e^{-\gamma \rho^1^2} \text{F}_1(-1; 1; \gamma \rho^1^2), \) and \( \rho^1 = \rho / s \).

In Table VII-1, the weighting function \( \Lambda_1^{(x)}(\rho^-) \) is tabulated for \( \gamma = 1 \) from tables of the confluent hypergeometric function as a function of \( \rho^1 \), the square root of the argument. To convert to other values of \( \gamma \), the \( \rho^1 \) column is divided by \( \sqrt{\gamma} \) and the \( \Lambda_1^{(x)}(\rho^1) \) multiplied by \( \gamma \).

**TABLE VII-1**

<table>
<thead>
<tr>
<th>( \rho^1 )</th>
<th>( \Lambda_1^{(x)}(\rho^-) )</th>
<th>( \rho^1 )</th>
<th>( \Lambda_1^{(x)}(\rho^1) )</th>
<th>( \rho^1 )</th>
<th>( \Lambda_1^{(x)}(\rho^-) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0.8062</td>
<td>0.147</td>
<td>1.378</td>
<td>0.212</td>
</tr>
<tr>
<td>0.1414</td>
<td>0.136</td>
<td>0.8367</td>
<td>0.125</td>
<td>1.414</td>
<td>0.191</td>
</tr>
<tr>
<td>0.2000</td>
<td>0.184</td>
<td>0.8660</td>
<td>0.102</td>
<td>1.483</td>
<td>0.196</td>
</tr>
<tr>
<td>0.2550</td>
<td>0.226</td>
<td>0.8944</td>
<td>0.080</td>
<td>1.549</td>
<td>0.197</td>
</tr>
<tr>
<td>0.2828</td>
<td>0.240</td>
<td>0.9274</td>
<td>0.059</td>
<td>1.612</td>
<td>0.192</td>
</tr>
<tr>
<td>0.3162</td>
<td>0.257</td>
<td>0.9487</td>
<td>0.039</td>
<td>1.673</td>
<td>0.188</td>
</tr>
<tr>
<td>0.3873</td>
<td>0.283</td>
<td>0.9747</td>
<td>0.019</td>
<td>1.732</td>
<td>0.173</td>
</tr>
<tr>
<td>0.4472</td>
<td>0.293</td>
<td>1.0000</td>
<td>0</td>
<td>1.871</td>
<td>0.141</td>
</tr>
<tr>
<td>0.5000</td>
<td>0.292</td>
<td>1.049</td>
<td>-0.035</td>
<td>2.000</td>
<td>0.110</td>
</tr>
<tr>
<td>0.5479</td>
<td>0.284</td>
<td>1.095</td>
<td>-0.066</td>
<td>2.121</td>
<td>0.084</td>
</tr>
<tr>
<td>0.5916</td>
<td>0.271</td>
<td>1.140</td>
<td>-0.093</td>
<td>2.236</td>
<td>0.060</td>
</tr>
<tr>
<td>0.6325</td>
<td>0.254</td>
<td>1.183</td>
<td>-0.117</td>
<td>2.345</td>
<td>0.043</td>
</tr>
<tr>
<td>0.6708</td>
<td>0.235</td>
<td>1.225</td>
<td>-0.137</td>
<td>2.449</td>
<td>0.031</td>
</tr>
<tr>
<td>0.7071</td>
<td>0.214</td>
<td>1.265</td>
<td>-0.153</td>
<td>2.550</td>
<td>0.022</td>
</tr>
<tr>
<td>0.7416</td>
<td>0.193</td>
<td>1.304</td>
<td>-0.167</td>
<td>2.646</td>
<td>0.014</td>
</tr>
<tr>
<td>0.7746</td>
<td>0.162</td>
<td>1.342</td>
<td>-0.177</td>
<td>2.739</td>
<td>0.011</td>
</tr>
<tr>
<td>0.8040</td>
<td>0.142</td>
<td>1.378</td>
<td>-0.187</td>
<td>2.828</td>
<td>0.006</td>
</tr>
</tbody>
</table>
The function $\Lambda_1(x)(f_1)$ is shown graphically in Figure 7-4. If a grid approximation is made, in the same manner as for the Downward Continuation problem, one obtains for the $\gamma = 1$ case and grid spacing unity,

$$\gamma = 1 : \frac{\partial^2 g}{\partial z^2} = 0.497 T(0) + 0.184 T(1) - 0.426 T(2) - 0.197 T(3)$$
$$- 0.029 T(4) - 0.029 T(5).$$  \hspace{1cm} (7-7)

If a formula is desired for shorter wave-length anomalies, $\gamma$ may be increased. A practical limit is set when the entire positive portion of the weighting function is used for the coefficient of the center-point value, viz.

$$\gamma = 2.78 : \frac{\partial^2 g}{\partial z^2} = 2.03 T(0) - 1.63 T(1) - 0.37 T(2) - 0.03 T(3).$$  \hspace{1cm} (7-8)

It will be observed that in this case, most of the negative weight is on the first ring, and weights on rings beyond the second are negligible. This formula is therefore similar, except for scaling, to the "center-point-and-one-ring" formula used earlier.

**First Derivative**

Grant (1952) has developed the theory for the first vertical derivative of gravity along with certain other derivatives. The first derivative weighting function $\Lambda^{(v)}(f_1)$ is amenable to a grid approximation, and formulas have been derived for the cases $\gamma = 1$ and $\gamma = 4$, as follows:

$$\gamma = 1 : \frac{\partial g}{\partial z} = 0.48 T(0) + 0.35 T(1) - 0.09 T(2) - 0.43 T(3) - 0.07 T(4)$$
$$- 0.34 T(5).$$  \hspace{1cm} (7-9)

$$\gamma = 4 : \frac{\partial g}{\partial z} = 1.86 T(0) - 0.88 T(1) - 0.32 T(2) - 0.30 T(3) - 0.60 T(4)$$
$$- 0.30 T(5).$$  \hspace{1cm} (7-10)

These formulas have been tested on the polar models used for the Downward Continuation test (Figures 7-5 and 7-6), and the increased resolution with light smoothing can be seen. A certain similarity can be seen between first and second derivative formulas for a given smoothing parameter and between these and the Downward Continuation formulas for a step equal to the grid spacing. As would be expected, the second derivative formula is more sharply peaked, emphasizing as it does a curvature rather than a gradient. It should be added that the grid approximation formulas give a
FIGURE 7-4: Second derivative weighting function \( \lambda_1(x)(r_1), \ T = 1. \)
FIGURE 7-5: First derivative test on single polar source, $\tau = 1$ and $\tau = 4$. 
FIGURE 7-6: First derivative test on double polar source, $\tau = 1$ and $4$. 
result that only approaches the true derivative value under favourable conditions, and one must use caution in attempting to deduce multipole moments using them.

Residual Methods

Residual anomaly determination involves subtraction from the anomalous field at a point of an average value determined at several surrounding points, such as the first ring of four or the first and second rings of four each. This process is similar to certain first and second derivative calculations, if we consider applying the entire positive portion of the weighting function to the center point and the entire negative portion to the surrounding points. If we consider the Downward Continuation function for $\gamma = 1$, we see that the integral of the positive portion is 3, the negative portion 2, so that the formula

$$ T(0) + 2 \left[ T(0) - T(1) \right], \quad (7-11) $$

viz., the intensity at the point plus twice the residual anomaly at the point, effects a very crude Downward Continuation approximation to a depth of some two-thirds the first ring radius.
Chapter VIII

Correlation of Aeromagnetic and Ground Data

The problem of calculating a potential field distribution on a plane above the plane on which observations have been taken is solved by the upward continuation process. Methods of achieving this have been described by Henderson and Zietz (1949b) who were interested in total magnetic intensity anomalies and Peters (1949) who included it in his general treatment of the interpretation of vertical magnetic intensity data. The latter used data taken on a square grid, the former interpolated to grid points from a contour aeromagnetic map.

Theoretical Aeromagnetic From Vertical Ground Magnetic Anomaly

Qualitative comparisons of ground magnetic and aeromagnetic anomalies have appeared in the literature, for example in Vacquier et al. (1951). A quantitative calculation of total intensity aeromagnetic from vertical intensity ground data enables one to determine the anomaly intensity to be expected in an airborne survey, the flight-line spacing necessary for adequate detailing of that particular type of anomaly and the response of the airborne instrument. Otherwise, practical applications of this method are limited.

The development proceeds from the theory of upward continuation. Peters (1949, eq 32) gives for the magnetic potential at a point \((x, y, z)\) above a plane on which the vertical magnetic intensity \(\Delta Z\) is measured at points \((x_1, y_1, 0)\) the following expression:

\[
\Delta V(x, y, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{\Delta Z(x_1, y_1, 0) \, dx_1 \, dy_1}{\left[ (x-x_1)^2 + (y-y_1)^2 + z^2 \right]^{1/2}} \tag{8-1}
\]

The total intensity anomaly in the northern hemisphere is then

\[
\Delta T = \frac{1}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \frac{(x_1-x) \cos I - z \sin I}{\left[ (x-x_1)^2 + (y-y_1)^2 + z^2 \right]^{3/2}} \Delta Z(x_1, y_1, 0) \, dx_1 \, dy_1 \tag{8-2}
\]
Since we may choose the origin to be any point in the xy-plane, we choose it at our point of computation. Also, we are interested in the total intensity anomaly at height $h$ above the ground surface. Substituting $x = y = 0$ and $z = -h$, and changing to polar co-ordinates $x_1 = r \cos \theta$, $y_1 = r \sin \theta$,

$$\Delta T = \frac{1}{2\pi} \int_0^{2\pi} \int_0^r \frac{r \cos \theta \cos I + h \sin I}{(r^2 + h^2)^{3/2}} \Delta Z \, r \, dr \, d \theta. \quad (8-3)$$

In high geomagnetic latitudes, a circular template is generally suitable, divided into compartments bounded by arcs of radii $r_n$, $r_{n-1}$ and radial lines at angles $\theta_n$, $\theta_{n-1}$ with respect to the magnetic meridian. If $\Delta Z$ is the average value of the vertical intensity anomaly within such a compartment, then the contribution to the theoretical total intensity anomaly is given by:

$$\frac{\Delta Z}{2\pi} \int_{\theta_{n-1}}^{\theta_n} \int_{r_{n-1}}^{r_n} \frac{r \cos \theta \cos I + h \sin I}{(r^2 + h^2)^{3/2}} \, r \, dr \, d \theta$$

$$= \frac{\Delta Z}{2\pi} \left\{ \cos I \left[ \log_e \frac{(1 + u_n^2)^{1/2} + u_n}{(1 + u_{n-1}^2)^{1/2} + u_{n-1}} + \frac{u_{n-1}}{(1 + u_n^2)^{1/2}} - \frac{u_n}{(1 + u_{n-1}^2)^{1/2}} \right] \\ + \sin I (\theta_n - \theta_{n-1}) \left[ \frac{1}{(1 + u_{n-1}^2)^{1/2}} - \frac{1}{(1 + u_n^2)^{1/2}} \right] \right\} \quad (8-4)$$

where $u_n = r_n/h$.

**Example - Barlow Lake Area**

A study of the "Second Derivative" method on an aeromagnetic map of the Barlow Lake area has already been discussed. In addition to an aeromagnetic map, a vertical intensity ground magnetic map on a scale of 400 feet to the inch was also provided by Dominion Gulf Company. Converted to a scale of 1320 feet to the inch, this is shown as Figure 8-1. For the calculations, the map was used on its original scale. A template was constructed as shown in Figure 8-2. The radii used corresponded to ground distances of 125, 250, 500, 750 and 1250 feet. Neglect of contributions...
FIGURE 8-1: Vertical intensity ground magnetic map, Barlow Lake area. Scale one inch equals 1320 feet, contour interval 5000 gammas.
FIGURE 9-2: Template for calculation of total intensity aeromagnetic anomaly from vertical intensity ground magnetic map.
FIGURE 3-3: Theoretical total intensity aeromagnetic anomaly, Barlow Lake area. Scale one inch equals 1020 feet, contour interval 1000 gammas.
to the integral from greater distances means that a portion of the field varying slowly with position is neglected. Using the magnetic elements $D = 18^\circ 24^\prime W$, $I = 78^\circ N$, the coefficients of Table 8-I were calculated.

<table>
<thead>
<tr>
<th>COMPARTMENT</th>
<th>COEFF.</th>
<th>COMP.</th>
<th>COEFF.</th>
<th>COMP.</th>
<th>COEFF.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0293</td>
<td>11</td>
<td>0.0204</td>
<td>21</td>
<td>0.0220</td>
</tr>
<tr>
<td>2</td>
<td>0.0199</td>
<td>12</td>
<td>0.0229</td>
<td>22</td>
<td>0.0314</td>
</tr>
<tr>
<td>3</td>
<td>0.0185</td>
<td>13</td>
<td>0.0255</td>
<td>23</td>
<td>0.0288</td>
</tr>
<tr>
<td>4</td>
<td>0.0172</td>
<td>14</td>
<td>0.0234</td>
<td>24</td>
<td>0.0225</td>
</tr>
<tr>
<td>5</td>
<td>0.0185</td>
<td>15</td>
<td>0.0220</td>
<td>25</td>
<td>0.0161</td>
</tr>
<tr>
<td>6</td>
<td>0.0264</td>
<td>16</td>
<td>0.0186</td>
<td>26</td>
<td>0.0135</td>
</tr>
<tr>
<td>7</td>
<td>0.0255</td>
<td>17</td>
<td>0.0153</td>
<td>27</td>
<td>0.0161</td>
</tr>
<tr>
<td>8</td>
<td>0.0229</td>
<td>18</td>
<td>0.0139</td>
<td>28</td>
<td>0.0225</td>
</tr>
<tr>
<td>9</td>
<td>0.0204</td>
<td>19</td>
<td>0.0153</td>
<td>29</td>
<td>0.0288</td>
</tr>
<tr>
<td>10</td>
<td>0.0194</td>
<td>20</td>
<td>0.0186</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The product of coefficient and average $\Delta Z$ value within each compartment is summed over the entire template to yield the theoretical total magnetic intensity anomaly. A map of the results is shown in Figure 8-3 and compares favourably with the measured intensity shown in the block of Figure 7-1. No structure of the theoretical anomaly is not shown in the measured data. The peak intensities are essentially the same and the relative positioning is as good as one could expect with a base map made from an air photo mosaic and a flight-line spacing of one-quarter mile. However, the anomaly is noticeably narrower, partly due to the fact that the effects of neighbouring anomalies were not considered. The aeromagnetic data were taken with an earlier model of the Gulf airborne magnetometer which had a slower response than later models, and this may be an additional reason for the broader measured aeromagnetic anomaly.

A further example will be given in Chapter X.
CHAPTER IX
MEASUREMENT OF MAGNETIC SUSCEPTIBILITY

Magnetic susceptibility and remanent magnetization measurements of field samples are of value in correlating with magnetic polarizations estimated from field intensity measurements. Accordingly, the author undertook the construction of a susceptibility bridge at the Department of Geophysics as a first step. In this chapter, measurements are described that are correlated with field results in Chapter X.

Apparatus

The susceptibility bridge of the type constructed has been described by Bruckshaw and Robertson (1948) and is shown schematically in Figure 9-1. The magnetizing field is provided by an alternating current through the Helmholtz coil at a frequency of 50 cycles per second. The number of turns of the coil $W_2$ is adjusted so that the voltages induced by the primary field in coils $W_1$ and $W_2$ are of equal magnitude. These coils are connected in series opposition so that there is no resultant output voltage due to the primary field. With a rock sample in the position C as shown, the oscillating induced magnetization produced in the sample by the primary field gives rise to a differential output voltage from the double coil $W_1 W_2$. This is compared with the reference voltage from $W_3$ by means of a slide-wire potentiometer and detector G.

Details of the apparatus built by the author at the Australian National University follow. Coil data are as follows:

<table>
<thead>
<tr>
<th>Coil</th>
<th>Inner Radius</th>
<th>Winding Section</th>
<th>No. of Turns</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>13.8 cm</td>
<td>0.4 x 0.8</td>
<td>16</td>
</tr>
<tr>
<td>$W_1$</td>
<td>2.26</td>
<td>1.67 x 2.08</td>
<td>15,400</td>
</tr>
<tr>
<td>$W_2$</td>
<td>4.34</td>
<td>1.67 x 1.02</td>
<td>11,400</td>
</tr>
<tr>
<td>$W_3$</td>
<td>5.36</td>
<td></td>
<td>17</td>
</tr>
</tbody>
</table>

The centers of the primary coils forming the Helmholtz coil were 16.5 cm apart. Specimens cut as cylinders were placed coaxially with the coil arrangement, and with center d cm from the center of the pickup coils.
FIGURE 9-1: Alternating current susceptibility bridge, after Bruckshaw and Robertson.
With this arrangement, and $d = 3.86$,

$$KV = 1.27 \frac{r}{R} ,$$

where $r$ = resistance tap on potentiometer,

$R$ = total resistance of potentiometer,

$K$ = magnetic susceptibility of sample,

$V$ = volume of sample.

The r.m.s. primary field at the center of the Helmholtz coil is 0.685 oersteds per ampere, the peak value 0.88. Bruckshaw and Robertson made measurements at a peak value of 0.5 oersted, but the author is of the opinion that an r.m.s. value of 0.5 oersted would have more significance in correlating with actual induced magnetization in the earth's field.

Detection of bridge balance was carried out using an R-C coupled amplifier and detector system of the type developed by Johnson et al (1949), modified for use at the mains frequency of 50 cycles by changing the values of all coupling condensers. A better amplifier design might have been used, but this arrangement gave satisfactory results down to the limit of sensitivity desired, $10^{-4}$ c.g.s. units of susceptibility. The amplifier is readily convertible to low-frequency use for the measurement of remanent magnetization as originally intended by Johnson et al.

Measurement of Rum Jungle Samples

A suite of samples was kindly provided by Territory Enterprises Proprietary Limited from drill core obtained in hole CD152, a vertical hole at 1000 south on traverse line 9400 west of the Rum Jungle co-ordinate system (see Figure 10-6).

Cylindrical specimens were cut of diameter 21 mm and length 21 mm, the cylinder axis being taken along the axis of the core. The susceptibility is, therefore, given by

$$K = 0.175 \frac{r}{R} \text{ c.g.s. units per c.c.}$$
Two sets of measurements were made, one at a primary field current \( I = 0.66 \) amperes (\( H_{\text{RMS}} = 0.45 \) oersted) and one at \( I = 0.28 \) amperes (\( H_{\text{RMS}} = 0.19 \) oersted). Changes in susceptibility with applied field were small. The list of measured values is given in Table 9-1.

<table>
<thead>
<tr>
<th>SAMPLE NO.</th>
<th>DEPTH</th>
<th>( H_{\text{RMS}} = 0.45 )</th>
<th>( H_{\text{RMS}} = 0.19 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>349.5 ft</td>
<td>( 1.7 \times 10^{-3} )</td>
<td>( 2.5 \times 10^{-3} )</td>
</tr>
<tr>
<td>2</td>
<td>375</td>
<td>4.4</td>
<td>4.1</td>
</tr>
<tr>
<td>3</td>
<td>400</td>
<td>4.1</td>
<td>3.7</td>
</tr>
<tr>
<td>4</td>
<td>424</td>
<td>( &lt; 0.1 )</td>
<td>( &lt; 0.1 )</td>
</tr>
<tr>
<td>5</td>
<td>450</td>
<td>1.3</td>
<td>1.3</td>
</tr>
<tr>
<td>6</td>
<td>473.5</td>
<td></td>
<td>4.8</td>
</tr>
<tr>
<td>7</td>
<td>500</td>
<td>7.5</td>
<td>6.8</td>
</tr>
<tr>
<td>8</td>
<td>523</td>
<td>3.7</td>
<td>3.4</td>
</tr>
<tr>
<td>9</td>
<td>550</td>
<td>3.0</td>
<td>2.5</td>
</tr>
<tr>
<td>10</td>
<td>325.5</td>
<td>6.5</td>
<td>6.2</td>
</tr>
</tbody>
</table>

The results are plotted against depth in Figure 9-1.

Measurements were also carried out on dolerite samples from drill hole No. 5001 at the Great Lake, Tasmania, and have been referred to elsewhere (Joplin and Jaeger, 1957). These results illustrate the wide variation in susceptibility values over a given body. Correlation based on a few measurements is inclined to be misleading.
FIGURE 9-2: Reversible magnetic susceptibility in $10^{-3}$ c.g.s. units, from drill core at various depths in hole CD152, Rum Jungle, N. T. Dashed curve, primary current 0.65 ampere, solid curve 0.28 ampere.
CHAPTER X
APPLICATION TO THE KATHERINE-DARWIN AREA

Extensive airborne magnetic coverage has been made of the Katherine-Darwin region of the Northern Territory of Australia by the Bureau of Mineral Resources, Geology and Geophysics of the Commonwealth of Australia using shoran positional control and an average flight-line spacing of 1,000 feet. During the 1954 field season, the author conducted some ground magnetic investigation in the area, and interpreted much of the airborne data. Magnetic relief in the area is in general low, but there are a few prominent anomalies.

Aeromagnetic Interpretation

A portion of BMR aeromagnetic sheet G71-92A, Rum Jungle District, appears as Figure 10-1. We will discuss some of the anomalies.

(a) Brodribb anomaly.
This is relatively complex and extends from four miles south of Darwin River siding to the Stuart Highway about one mile northwest of Manton Dam. The anomaly follows structural trends as shown by air photographs and geological mapping faithfully. However, the anomaly is rather complex and was approached in several ways. Maximum gradients at selected points indicated, by comparison with the depth indices of Vacquier et al (1951), depths in the range 700 - 1300 feet. The Dyke model indicated, taking a profile as shown across the center of the anomaly, a dyke with $A = 2$ and $w = 3$ corresponding to a depth of 830 feet, with a corresponding magnetic polarization that is rather high for a sediment. However, this profile was not typical, the anomaly being broader at other points along strike. The residual method was tested on the anomaly. Eight points were taken on the periphery of a square of side $2s$ and the average value subtracted from the value at the center point. The results for $s = 1/4$ and $1/2$ mile are shown in Figure 10-2. The loss of detail at $s = 1/2$ mile is apparent. From this, and an examination of some of the minor magnetic closures, one can conclude that the local features of the anomaly originate at depths less than $1/4$ mile. One possible interpretation is an irregularly
LEGEND

PRIMARY CONTOUR
SECONDARY CONTOUR
RADIOACTIVE PLATEAU
TOPOGRAPHIC SYMBOLS
MAIN ROADS
TRAILS
STREAMS

AEROMAGNETIC SURVEY

RUM JUNGLE AREA

AUSTRALIA

SCALE

MILE 0 1 2 MILES

FIGURE 10-1: Total intensity aeromagnetic map Rum Jungle area, N.T., a portion of BMGG aeromagnetic sheet G71-91A, scale one inch equals one mile contour interval 100 gammas.
FIGURE 16-3: Non-anal blank intensity anomaly, Brenda Lake area, using center point with square of eight points. Grid spacing (a) 1/4 mile, (b) 1/2 mile.
polarized sedimentary horizon, but the indicated magnetization is rather high for a sediment. Another possible interpretation is that of a basic intrusive at variable depth below the surface, flanking the north contact of the Rum Jungle granite. However, there is no positive evidence for the latter.

(b) Rum Jungle granite anomaly.

Magnetic contours at the north end of this anomaly follow the mapped edge of the granite approximately. The general change in magnetic level over the northern portion of the granite is about 200 gammas, indicating a possible polarization contrast of

$$i_T = \frac{\Delta T}{2\pi \sin^2 I} = \frac{2.0 \times 10^{-3}}{6.28(0.643)^2} = 0.8 \times 10^{-3}$$

(contact problem, normal polarization), and local relief is higher in places. The southern portion of the granite shows little magnetic relief but the airborne scintillometer indicated a radioactive "plateau". Regional gravity work (unpublished) has indicated that there is also a density contrast, the northern segment being more dense than the southern. Thus the geophysical evidence suggests lateral differentiation of the intrusive, the more acidic phase toward the south and adjacent to the mineralized zones at Rum Jungle.

(c) Hematitic Boulder Conglomerate anomaly.

The sharper portions of this anomaly (above the 2000 gamma level) indicate a thin dyke (line of poles) at depth 500 feet. This is superimposed on what appears to be a more deep-seated feature.

(d) Mt. Fitch – Rum Jungle anomaly.

Prism models indicated depths on individual closures a, b, c, d ranging from 500 to 1000 feet and from the flanks of the overall anomaly, a range 550 to 900 feet. The anomaly is too complex to apply an exact model. A rough estimate of magnetic polarization can be obtained by using a 6 x 6 prismatic model at $I = 45^0$, obtaining for a map intensity of 1100 gammas a polarization contrast of $3.7 \times 10^{-3}$. One qualitative
interpretation of this feature could be a sill with a main anticlinal axis as shown and cross-warping bringing the sill nearer surface at local closures a, b, c, and d. Alternatively, the local closures could be due to non-uniform magnetization.

(e) Brown’s anomaly.
A portion of the original compilation sheet showing this anomaly is shown in Figure 10-3. The flight line direction is unsatisfactory for the use of inflection points, but the strong negative indicates a probable dipolar type source. Although the anomaly profile changes along strike in distances comparable with the "half-width" of the anomaly, half-maximum distances were measured on three representative profiles along the anomaly, indicated on the figure. Anomaly "half-widths" are 1320, 1320 and 990 feet on profiles 1, 2 and 3 respectively, with the asymmetries 0.61, 0.8, 0.7. The interpretation of a line-dipolar type source at depths approximately equal to the profile half-widths would indicate that the depth decreases to the south-west. This was borne out by ground magnetic work to be described later. Considering a maximum intensity of 1900 gammas on profile 2, and the magnetic material localized within a radius of 820 feet (tangential to the surface), the minimum magnetic polarization is \(4.5 \times 10^{-3}\). This agrees with measured magnetic susceptibilities described earlier.

(f) Brock’s Creek anomaly.
In Figure 10-4, a portion of the "Burnside" aeromagnetic sheet is shown. An E-W profile was taken as indicated across the amphibolite sill west of the Brock’s Creek granite. The amphibolite sill here appears to be about 900 feet wide and folded into a tight anticline, according to geological mapping (Sullivan and Iten, 1952). The model used to reproduce this anomaly consisted of two infinite inclined dykes, one dipping west at 60°, the other east at 60°, each of width \(w = 1\) and touching, striking north-south. The comparison between theoretical and actual anomaly profiles is shown in Figure 10-5. If polarization is in the direction of the earth’s field, a value of \(4.6 \times 10^{-3}\) c.g.s. would be required to produce this intensity.
FIGURE 10-8: Total intensity aeromagnetic map showing Brown's anomaly, Rum Jungle, N. T. Figure provided from a compilation sheet by the Bureau of Mineral Resources, Geology and Geophysics of the Commonwealth of Australia. Scale one inch equals one mile. Flight lines shown.
FIGURE 10-4. Total intensity aeromagnetic map, Burnside area, N.T., a portion of BMRGG aeromagnetic sheet G159-2, scale one inch equals one mile, contour interval 100 gammas.
FIGURE 10-5: Comparison of aeromagnetic anomaly profile (dashed curve) with theoretical composite dyke model anomaly (solid curve), Brocks Creek anomaly.
Ground Magnetic Work, Rum Jungle Area

Ground magnetic work was carried out by the author during the 1954 field season along with other work (Langron, 1956), employing both vertical and horizontal intensity measurements.

The locations of the traverses are shown on Figure 10-6, and were for ground investigation of Brown's anomaly. The survey as such will not be described, but only the application of certain interpretational techniques.

Brown's anomaly on line 7800W, using the two-dimensional theory of Bullard and Cooper was continued downward to a depth of 200 feet using the analytical method thence to 400 feet using the finite difference method. The results are shown in Figure 10-7. A resolution of the anomaly into two peaks shows that the source is of the same type as is found in the central part of the anomaly.

Traverse lines 9400 and 10,400 west were traversed using both horizontal and vertical intensity instruments (Figure 10-8). A downward continuation to 200 feet depth was carried out on the vertical intensity data of line 10,400 west using the two-dimensional analytical method and shows improved resolution of the anomaly (Figure 10-7).

The theoretical aeromagnetic anomaly corresponding to a two-dimensional profile as on line 10,400 west was carried out using two-dimensional theory. The method will be described briefly.

Referring to Chapter VIII, we set our y-axis normal to the strike of the anomaly, assume \( \Delta V \) as a function of \( y_1 \) only and integrate out the contributions in the x-direction to the integrals for \( \Delta T \), obtaining for the southern hemisphere case

\[
\Delta T = \frac{1}{4\pi} \int_{-\infty}^{\infty} \frac{h \sin I + y_1 \cos I \sin I}{y_1^2 + h} \Delta Z (y_1) \, dy_1.
\]
FIGURE 10-6: Location plan of grid system, Rum Jungle area, N.T.
Figure 10.7

Line 104 W
Theoretical Total Intensity Anomaly
500 ft above Ground Level

Measured Vertical Magnetic Intensity

Calculated Downward Continuation of
Vertical Magnetic Intensity to 200 ft below Surface

Calculated Downward Continuation of
Vertical Magnetic Intensity to 400 ft below Surface

1800 1600 1400 1200 1000 800 600 400 200 0 200 400 600
South
SCALE
GAMMAS
ANALYTIC CONTINUATION OF ANOMALIES
TABLE 10-I

COEFFICIENTS FOR CALCULATION OF TWO-DIMENSIONAL AEROMAGNETIC
FROM VERTICAL GROUND MAGNETIC ANOMALY, SOUTHERN HEMISPHERE,
$\text{I} = 40^\circ$, $\beta = 120^\circ$

<table>
<thead>
<tr>
<th>$y$</th>
<th>North</th>
<th>South</th>
</tr>
</thead>
<tbody>
<tr>
<td>0–100</td>
<td>0.1138</td>
<td>0.1402</td>
</tr>
<tr>
<td>100–200</td>
<td>0.0815</td>
<td>0.1543</td>
</tr>
<tr>
<td>200–300</td>
<td>0.0501</td>
<td>0.1554</td>
</tr>
<tr>
<td>300–400</td>
<td>0.0243</td>
<td>0.1484</td>
</tr>
<tr>
<td>400–500</td>
<td>0.0054</td>
<td>0.1370</td>
</tr>
<tr>
<td>500–750</td>
<td>0.0343</td>
<td>0.2832</td>
</tr>
<tr>
<td>750–1000</td>
<td>-0.0630</td>
<td>0.2232</td>
</tr>
<tr>
<td>1000–1500</td>
<td>-0.1390</td>
<td>0.3214</td>
</tr>
<tr>
<td>1500–$\infty$</td>
<td>-0.5570</td>
<td>0.9710</td>
</tr>
</tbody>
</table>

The resultant short profile (Figure 10-7) shows the merging of the two peaks
and shift of the anomaly maximum northwards. This profile was calculated on
the basis of 2000 gammas intensity over neutral ground and the curve pre-
sented in Figure 10-7 is corrected for this. The calculated intensity is
approximately twice the observed but this is understandable on two counts:

(a) The ground anomaly is abnormally strong on this particular profile, so
that two-dimensional theory yields too high a result.

(b) It is possible that the nearest flight line did not pass directly over
the anomaly peak.
CHAPTER XI

RESOLUTION

An important aspect of the application of the aeromagnetic method is the problem of resolution. This is common to other geophysical methods and has been discussed in a general way by the author elsewhere (Smellie, 1959). There are two principle aspects – the survey should resolve geophysical effects of interest and the flight-line spacing should be sufficiently close that anomalies of interest are not either missed completely or not recognized for their true worth.

Resolution Of Width Effects

Aeromagnetic data are recorded along discrete flight-lines that normally cross the regional geological strike at right angles. Many magnetic formations can be considered two-dimensional from the point of view of geophysical interpretation. A model that is widely applicable is the dyke. Curves such as Figure 4-5 show that when the width of the dyke becomes smaller than the depth sub-flight, the anomaly half-width and other parameters are close to those for the thin dyke and change slowly with changes in width. Therefore, width estimates are correspondingly inaccurate. This is one type of resolution problem.

Another problem can be illustrated using the dyke model. That is the size limit of detectability for dykes of a given magnetic polarization. An example would be the diabase dykes that are so common on the Canadian Precambrian Shield.

We assume a high magnetic latitude and vertical polarization. Equation (2-34) is used, with \( \gamma \) and \( w \) both expressed in feet. If we assume a minimum anomaly of 2 gammas for detectability, a dyke of polarization \( 10^{-3} \) c.g.s. units, and a dyke clearance of 1,000 ft,

\[
w = \frac{\gamma \Delta T_{\text{max}}}{T_r} = \frac{1000}{2 \times 10^{-5}} = 10 \text{ ft.}
\]

To be certain of appearing on an aeromagnetic map with a contour interval of 10 gammas, the width would have to be 50 ft.
Resolution Of Closely-Spaced Sources

The problem of the resolution of the effects due to two closely-spaced sources has been approached theoretically by Elkins and Hammer (1938). They show how the anomalies due to the two sources become merged into a single anomaly as the height of the plane of observation above the bodies is increased. Although the examples quoted are for vertical magnetic intensity and gravity, some of them apply to total magnetic intensity results in high latitudes.

For example, the result for the gravitational field due to two infinitely long horizontal cylinders is the same as the result for the vertical magnetic field due to two vertically polarized narrow dykes, representable by two lines of poles.

The Resolution Limit

\[ L = \frac{2s}{3} = 1.155 \]

where \( 2s \) represents the separation between the poles at depth \( J \). Other formulas are given for various fields and their derivatives and from these we find other elementary cases, shown in Table 3–V.

**TABLE 3–V**

**RESOLUTION LIMITS**

<table>
<thead>
<tr>
<th>Type of Source</th>
<th>( L )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pole</td>
<td>1.0</td>
</tr>
<tr>
<td>Line of Poles</td>
<td>1.155</td>
</tr>
<tr>
<td>Dipole</td>
<td>0.779</td>
</tr>
<tr>
<td>Line of Dipoles</td>
<td>0.828</td>
</tr>
</tbody>
</table>

\( L \) is expressed as separation/depth for two elementary sources.

The above are, of course, for the intensity anomaly, and as we shall see later, improved resolution can be effected by the use of certain treatment methods.
Depth Estimation With Complex Anomalies

A complex anomaly is defined as one caused by a body neither of simple geometric form nor of uniform magnetic properties. A detailed study of the magnetic properties of any magnetic body discloses a rapid variation within small distances. Ground magnetic surveys, in addition, reveal localized anomalies which from their sharpness indicate a very shallow origin, superimposed on more widespread anomalies. While the main aim of this work is to present models for interpreting anomalies due to the average bulk effect of large bodies, it must be added that these local polarization contrasts give rise to anomalies that are superimposed on the main anomaly and give rise to local minor magnetic closures. Depth estimates on these minor closures, assuming a dipolar source, can give depths to the centers of the polarization contrasts producing these minor anomalies, and these values form a useful adjunct to depth estimates based on features on the flanks of the anomaly.

As the depth of the body increases, these minor anomalies become merged until eventually the only discernible anomaly is the overall effect of the entire body. Thus the complexity of an anomaly of given dimensions can be used as a further check on depth estimates using block models. For example, at a depth of 2,000 feet, we know that at high latitudes the anomalies from dipolar sources 1,560 feet apart will be merged to form a single anomaly along a profile directly above them. However, sources separated by a greater distance will be partially resolved.

Application - Flight-Line Spacing

In some applications of the airborne magnetic method, flight-line spacing is critical. The necessary flight-line spacing for detection of certain localized bodies can be suggested by the elementary approximations. For general structure of extensive magnetic horizons, the line-spacing need only be less than the dimensions of significant structural variation. On the application to deep-basement areas, current practice is a line-spacing at least equal to the basement depth.
With localized magnetic plugs, the point pole provides a limiting model when lateral dimensions are small compared with the depth to the top. The half-width in a north-south direction is given by

\[ h^"_l = h^"_l + h'^"_l = \frac{1}{k^"_1} + \frac{1}{k'^"_1} = \frac{1}{k^"}_l \]

per unit depth. The half-width for a transverse profile (E-W magnetic) passing directly through the anomaly maximum is

\[ h^"_T = 1.533 \left(1 + \alpha^2_1 \right)^{1/2} \]

per unit depth. It is desirable to space the flight lines so that at least one of them will record at least half the maximum intensity of the anomaly and in order to do this, the flight line spacing must be equal to the half-width. If it is greater than the half-width \( h^"_n \), then the probability of recording an intensity of at least half the maximum anomaly intensity is given by

\[ P = \frac{h^"_n}{L} \]

where \( \gamma \) is the depth and \( L \) the line-spacing. Thus line-spacings of at least one-eighth mile at terrain clearances of 500 feet are desirable for detecting this type of source.

A similar approach may be made for bodies limited in depth and lateral extent and, hence, representable by a dipole. Anomaly half-widths for this type of source are tabulated in Table 3-III. We have seen that the "half-width" is equal to the depth, and the line-spacing would need to be at least equal to the terrain clearance plus the expected depth sub-surface.
APPENDIX A

Derivation of the Vertical Cylinder Anomaly

We assume a narrow vertical cylinder of cross-sectional area $A$ as shown in Figure 12-1. The co-ordinate system is right-handed, with the xy-plane horizontal as usual, and the y-axis along the magnetic meridian. The cylinder extends from $z = -\infty$ to $z = \infty$. The magnetic polarization is downward at an angle $\psi$ from the horizontal, with its azimuth making an angle $\epsilon$ with the x-axis measured in the direction of the y-axis.

The magnetic potential due to the top of the cylinder of cross-sectional area $A$ is given by

$$ (\Delta V)_1 = -i_T \frac{A \sin \psi}{\sqrt{x^2 + y^2 + (y - z)^2}}^{1/2} $$

The magnetic potential due to the x- and y-components of polarization is obtained by integrating the contribution due to the elemental dipole contained in a length $dz_1$ of the cylinder, viz:

$$ dz_1 \frac{A i_T \cos \psi}{x \cos \epsilon + y \sin \epsilon} \frac{x \cos \epsilon + y \sin \epsilon}{\sqrt{x^2 + y^2 + (z - z_1)^2}}^{3/2} dz_1 $$

Integrating from $z_1 = -\infty$ to $z_1 = \infty$, one obtains

$$ \Delta V_2 = i_T A \cos \psi \frac{x \cos \epsilon + y \sin \epsilon}{(x^2 + y^2)} \left[ 1 - \frac{y - z}{\sqrt{x^2 + y^2 + (y - z)^2}}^{1/2} \right] $$

Hence, the magnetic potential is given by

$$ \Delta V = -i_T A \left\{ \sin \psi \left[ \frac{x \cos \epsilon \cos \psi + y \sin \epsilon \cos \psi}{x^2 + y^2} \right]^{1/2} - \frac{x \cos \epsilon \cos \psi + y \sin \epsilon \cos \psi}{x^2 + y^2} \sqrt{\frac{y - z}{x^2 + y^2 + (y - z)^2}} \right\} $$

$$ \times \left[ 1 - \frac{y - z}{\sqrt{x^2 + y^2 + (y - z)^2}}^{1/2} \right] $$
FIGURE 12-1: The vertical cylinder.
Derivation of the Dyke Anomaly

A section normal to the infinite inclined dyke is shown in Figure 12-2. The anomalous field due to an elemental strip of the surface of the dyke is representable by the line of poles equation. For the top, this is

\[ d(\Delta T) = 2 \sin I \int_z^{z_1} \frac{dx}{\sqrt{z_1^2 + (x-x_1)^2}} \frac{z_1 - a(x-x_1)}{z_1^2 + (x-x_1)^2} \]

with \( z_1 = d \).

For the right hand side,

\[ d(\Delta T) = -2 \sin I \int_z^{z_1} \frac{dz}{\sin \psi} \frac{dz_1}{\sqrt{z_1^2 + (x-x_1)^2}} \frac{z_1 - a(x-x_1)}{z_1^2 + (x-x_1)^2} \]

with \( x_1 = -(z_1 - d) \cot \psi \).

For the left hand side,

\[ d(\Delta T) = 2 \sin I \int_z^{z_1} \frac{dz}{\sin \psi} \frac{z_1 - a(x-x_1)}{z_1^2 + (x-x_1)^2} \]

with \( x_1 = -w - (z - d) \cot \psi \).

The complete expression for the total magnetic intensity anomaly is given by

\[
\frac{\Delta T}{2 \sin I} = \int_z^{z_1} \frac{dx_1}{\sqrt{d^2 + (x-x_1)^2}} \frac{d - a(x-x_1)}{z_1^2 + (x-x_1)^2} = \int_z^{z_1} \frac{dz}{\sin \psi} \frac{z_1 - a(x + (z_1 - d) \cot \psi)}{z_1^2 + (x + (z_1 - d) \cot \psi)^2} \]

\[
\int_z^{z_1} \frac{dx_1}{2 \sin I \frac{d + a(x-x_1)}{d^2 + (x-x_1)^2}} = \frac{1}{2} \ln \frac{x^2 + d^2}{(x+w)^2 + d^2} \]

\[
+ \tan^{-1} \frac{x+w}{d} - \tan^{-1} \frac{x}{d} \]

\[
\int_z^{z_1} \frac{dz}{z_1^2 + (x + (z_1 - d) \cot \psi)^2} = (1 - a \cot \psi) \int_d^{d(\Delta T)} \frac{dz_1}{z_1^2} \csc^2 \psi + z_1 \cot \psi (x - d \cot \psi) + (x - d \cot \psi)^2 \]

\[
- a(x - d \cot \psi) \int_d^{d(\Delta T)} \frac{dz_1}{z_1^2} \csc^2 \psi + z_1 \cot \psi (x - d \cot \psi) + (x - d \cot \psi)^2 \]

\[
= \frac{(1 - a \cot \psi) \sin^2 \psi}{2} - \ln \left( \frac{x^2 + d^2}{(x + d)^2} \right) - (a \sin^2 \psi + \sin \psi \cos \psi) \]

\[
\frac{1}{\tan \frac{x - d \cot \psi}{x - d \cot \psi}} \int_d^{d(\Delta T)} \frac{dz_1}{z_1^2} \csc^2 \psi + z_1 \cot \psi (x - d \cot \psi) + (x - d \cot \psi)^2 \]

\[
\frac{1}{\tan \frac{x - d \cot \psi}{x - d \cot \psi}} \int_d^{d(\Delta T)} \frac{dz_1}{z_1^2} \csc^2 \psi + z_1 \cot \psi (x - d \cot \psi) + (x - d \cot \psi)^2 \]
FIGURE 12-2: The infinite inclined Dyke.
\[
\int \frac{dz}{d} 1 \frac{z_1 - a \left[ x + w + (z_1 - d) \cot \psi \right]}{z_1^2 + \left[ x + w + (z_1 - d) \cot \psi \right]^2} = \frac{(1 - a \cot \psi) \sin^2 \psi}{2}
\]

\[
\times \ln \left[ (x+w)^2 + d^2 \right] - (a \sin^2 \psi + \sin \psi \cos \psi) \tan \frac{-1 \csc \psi \cdot z_1 + \cot \psi \cdot (x+w-d \cot \psi)}{x + w - d \cot \psi}
\]

Combining the second and third integrals, one obtains

\[
\frac{(1-a \cot \psi) \sin^2 \psi}{2} \ln \frac{x^2 + d^2}{(x+w)^2 + d^2} - (a \sin^2 \psi + \sin \psi \cos \psi) \left[ \tan \frac{-1 x+w}{d} - \tan \frac{-1 x}{d} \right]
\]

Thus the complete expression is

\[
\Delta T = \left[ i_2^* \cdot 2 \sin I - i_N^* (2 \cos I \sin \beta \sin \psi + 2 \sin I \cos \psi) \right] \phi_{AB}
\]

\[
+ \left[ i_2^* \cdot 2 \cos I \sin \beta + i_N^* (2 \sin I \sin \psi - 2 \cos I \sin \beta \cos \psi) \right] \ln \frac{r_B}{r_A}
\]
APPENDIX C

Derivation of the Fault Anomaly

A section normal to the infinite fault discontinuity is shown in Figure 12-3. \( i_z \) is the vertical component of magnetic polarization, \( i_N \) is the component of polarization normal to the fault discontinuity. At the point of observation, the anomaly due to an elemental strip may be represented by the line of poles equation. Integrating over the surface of the magnetic body,

\[
\frac{\Delta T}{2 \sin I} = i_z \int_{-g}^{g} dx \frac{(d+1) - a(x-x_1)}{(d+1)^2 + (x-x_1)^2} + i_z \int_{-g}^{g} dx \frac{1 - a(x-x_1)}{1 + (x-x_1)^2}
\]

\[
+ \frac{i_N}{\cos \delta} \int_{-g}^{g} dx \frac{(1-x_1 \tan \delta) - a(x-x_1)}{(1-x \tan \delta)^2 + (x-x_1)^2}
\]

\[
\int_{-g}^{g} dx \frac{(d+1) + a(x-x_1)}{(d+1)^2 + (x-x_1)^2} = \frac{a}{2} \ln \left[ (g+x)^2 + (d+1)^2 \right] - \tan^{-1} \frac{x+g}{d+1}
\]

\[
- \lim_{x_1 \to -\infty} \frac{a}{2} \ln \left[ (x_1-x)^2 + (d+1)^2 \right] + \tan^{-1} \frac{x_1-x}{d+1}
\]

\[
\int_{-g}^{g} dx \frac{1 - a(x-x_1)}{1 + (x-x_1)^2} = \frac{a}{2} \ln (1+x^2) + \tan^{-1} x
\]

\[
+ \lim_{x_1 \to -\infty} \frac{a}{2} \ln \left[ (x_1-x)^2 + 1 \right] + \tan^{-1} (x_1-x)
\]

\[
\int_{-g}^{g} dx \frac{1 - x_1 \tan \delta - a(x-x_1)}{(1-x_1 \tan \delta)^2 + (x-x_1)^2} = (a - \tan \delta) \int_{-g}^{g} dx \frac{x_1}{1 + \tan^2 \delta - 2x_1 (x+\tan \delta)}
\]

\[
= \frac{a - \tan \delta}{2(1+\tan^2 \delta)} \ln \frac{1 + x^2}{(1+d)^2 + (x+g)^2} + \frac{(1+a \tan \delta)(1-x \tan \delta)}{1 + \tan^2 \delta} x
\]

\[
\int_{-g}^{g} dx \frac{1}{(1+x^2) - 2(x+\tan \delta)x_1 + (1+\tan^2 \delta)x_1^2}
\]

\[
= \frac{a - \tan \delta}{1 + \tan^2 \delta} \ln \frac{1 + x^2}{(1+d)^2 + (x+g)^2} + \frac{1 + a \tan \delta}{1 + \tan^2 \delta} \left( \tan^{-1} \frac{x+g}{1+d} - \tan^{-1} x \right)
\]

\[
\Delta T = \left[ i_z (2 \cos I \sin \beta) - i_N (2 \cos I \sin \beta \cos \delta - 2 \sin I \sin \delta) \right] \ln \frac{r_A}{r_B} + \left[ i_N (2 \sin I \cos \delta + 2 \cos I \sin \beta \sin \delta) - i_z \cdot 2 \sin I \right] \phi_{AB}
\]

Eq (5 - 1) is obtained from this expression by the substitution

\[
i_N = i_z \cos \delta + i_H \sin \delta
\]
FIGURE 12-3: The Fault.
References


Rosenbach, O., 1953, A contribution to the computation of the "second derivative" from gravity data: Geophysics, v. 18, p. 894-912.


Smellie, D. W., 1958, Downward continuation and the second derivative: Geophysical Prospecting, v. 6, p. 77 (abstract).


