MICROWAVE POWER TRANSMISSION RATIO: ITS USE IN ESTIMATING ELECTRON DENSITY

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CONTENTS

SUMMARY

The effect of a cylindrical plasma on transmission of microwave power between broad-beam antennae is calculated in the geometrical optics approximation. The first part extends Wort's (1963) results to an eccentric plasma not filling the whole discharge tube and the second part considers possible enhanced transmission due to a lower density outer plasma. When estimating peak density, the only other parameter which appears to require correcting for is the ratio of discharge to tube radius, which can be estimated by other means.

1. Introduction

In seeking an estimate of electron densities in the Australian National University (A.N.U.) Department of Engineering Physics' toroidal plasma device, hardware problems have so far limited microwave transmission measurements in this laboratory to a single channel. If one thinks of the electron density as expressible as a power series in the (minor) radial distance from some point, it is clear that only one term in the power series can be estimated from a single transmission measurement. The parameter we choose to estimate is the peak electron density. According to the model profile assumed, there are various implied assumptions as to the values of terms not being measured. For example, if a parabolic distribution (Wort, 1963), falling to zero at some distance, r_o, from the centre of the torus, is assumed, then the distribution is expressible as:

$$n_e = n_{e(max.)} \left(1 - \frac{r^2}{r_o^2}\right), \text{ for } r \leq r_o$$

implying that the quadratic term is $(- 1/r_o^2)$ times the constant term and that the coefficients of other powers are zero. Should these implied assumptions not be true, then the estimated term, n_e (max.), is in error; this is essentially the same problem as that of "aliasing" in harmonic analysis (Blackman and Tukey, 1959). In the present context, it would be tedious and not very profitable to assess the individual "aliasing" errors due to higher terms in a power series; instead, the effects of selected parameters on the indicated peak density are estimated.

Part 1 deals with a parabolic density profile for which Wort's (1963) geometrical optics analysis is accepted. However, unlike his plasma, it does not fill the torus and is displaced off centre. The conclusion is, essentially, that the displacement contributes to deflection but not to dispersion and so affects transmission only through the polar patterns of the antennae. However, if the radius of the plasma is as small as half that of the torus, corrections of up to 20% in peak electron density are required.

Measurements interpreted by the method of Part 1 implied loss of a majority of available particles from the main discharge. This raised the question whether the low densities were correct or whether a "halo" of lost particles forming an outer plasma near the walls could raise the signal transmission enough to cause a gross under-estimate of the density of the "core". The conclusion of Part 2 is that this is very unlikely. If the outer plasma is far enough below cut-off to permit transmission towards the centre, its effect on measured central density is negligible.

Plasma resistive attenuation is neglected throughout.

Part 1: Modification of Wort's (1963) Refraction Loss, for Eccentric Plasma Not Filling Discharge Tube

A parabolic distribution of electron density, surrounded by an annulus of negligible density, is assumed.

A. Transverse Plane

Fig. 1a shows a plasma of radius r_o = CA = CB displaced a distance d = OC from the centre of the torus. The torus radius r_1 = PO = OQ.

A ray from a point source, P, meets the plasma boundary at A and is refracted to emerge along BR, as if deflected at D through an angle, ψ. Wort's (1963) angles, ϕ_o, ψ and -2α, are shown and $\tan\delta = \frac{d}{r_1}$.

Ray PA makes an angle, β, with the diameter, PQ, which is also the axis of the transmitter and receiver polar diagrams. In the absence of plasma, the same point R, receiving this ray, would have received the direct ray, PR, at an angle β_o.

Due to the displacement of the plasma, ϕ_o is not neglected compared to the other term in Wort's (1963) equation (8), but small angle approximations are used. Thus, from his page 8 and equation (8),

$$\psi = \frac{\pi}{2} + 2\alpha - 2\phi_o = \frac{2\phi_o K}{1-K}$$

For small β_o, triangle PRD becomes isosceles and

$$\beta_o - \beta = \tfrac{1}{2}\psi = \frac{\phi_o K}{1-K} \qquad (1)$$

From triangle PAC,

$$\frac{\phi_o}{\beta+\delta} \doteq \frac{\sin\phi_o}{\sin(\beta+\delta)} = \frac{PC}{CA} \doteq \frac{r_1}{r_o}$$

Substituting in (1):

$$\beta_o - \beta = \frac{K}{1-K} \cdot \frac{r_1}{r_o} \cdot (\beta+\delta).$$

Putting the ratio $r_o/r_1 \equiv g$,

$$\beta = \frac{g(1-K)}{K+g(1-K)} \cdot \beta_o - \frac{K}{K+g(1-K)} \cdot \delta \qquad (2)$$

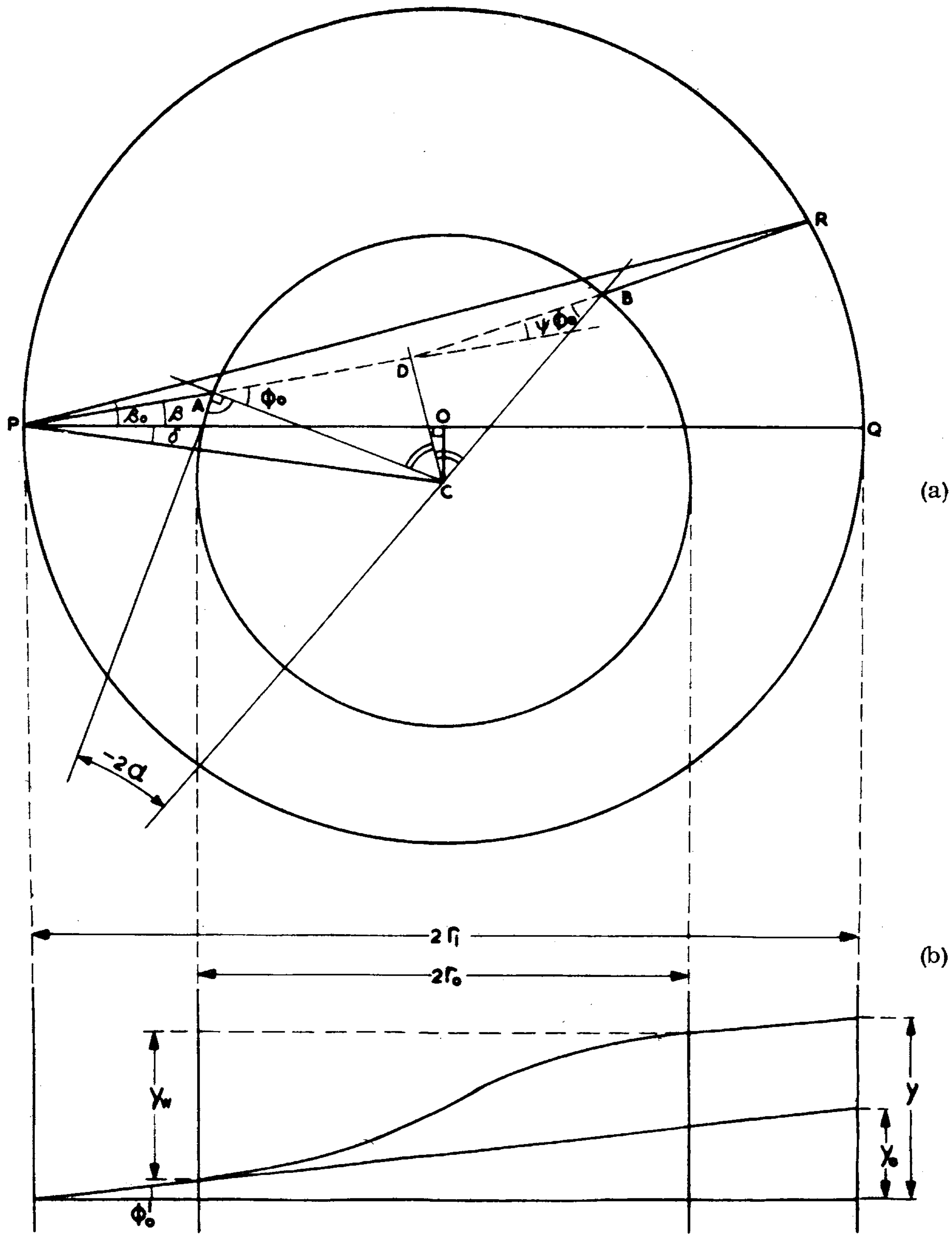

Figure 1 Geometry of ray path in discharge tube with constricted plasma.
(a) Transverse section.
(b) Longitudinal section.

The ray reaching the centre of the receiving aperture (at Q) has,

$$\beta_o = 0$$

and $$\beta = -\beta_c \equiv \frac{-K}{K + g\,(1 - K)} \cdot \delta \qquad (3)$$

An aperture near Q, having an "acceptance angle" $\Delta\beta_o$ in the absence of plasma, will, with plasma present, accept rays centred at $(-\beta_c)$ within a range,

$$\Delta\beta = \frac{g\,(1 - K)}{K + g\,(1 - K)} \cdot \Delta\beta_o \qquad (4)$$

There are two causes of signal loss due to (2):-

(a) Aerial polar diagrams: The response will be lower around $(\beta = -\beta_c)$ than for the direct ray. Transmission in the A.N.U. torus is between two circular probe tubes of $\frac{1}{4}$ inch bore (say 1.9λ) and $\beta_c < \delta$ for $K < 1$. For a displacement $d = 1$ cm and $r_o = 10$ cm, it is unlikely that β_c will exceed 5^o, for which the H-plane polar diagrams (Chu, 1940) show about 2% loss per aerial, or 4% total. This is well below experimental error and is neglected.

(b) "Diverging Lens" effect: Refraction in the transverse plane gives a reduction of signal in the ratio (from (4)):

$$\Gamma_{(trans)} \equiv \Delta\beta / \Delta\beta_o = \frac{g\,(1 - K)}{K + g\,(1 - K)} \qquad (5)$$

This agrees with Wort's figure when $g = 1$. Other diagnostic results for A.N.U. torus suggest $g = \frac{1}{2}$ giving a factor,

$$\frac{1 - K}{1 + K}$$

i.e. about half Wort's figure near $K = 1$

B Longitudinal Refraction

It is assumed that the ray samples the same density distribution over the same length as the diametral ray, because both the difference between the maximum density (at plasma centre) and the maximum seen by the ray, and the difference between path length and diameter, are small for small deviations.

In Fig. 1b, a ray with inclination to the transverse plane, ϕ_o^1 (Wort's notation) would, in the absence of plasma, deviate a distance,

$$y_o = 2\, r_1\, \phi_o^1$$

With plasma present, the deviation is, in vacuum:

$$2\,(r_1 - r_o)\,\phi_o^1$$

and, in plasma, Wort's deflection:

$$y_w = 2\,r_o\,\phi_o^1\,K^{-\frac{1}{2}}\,\tanh^{-1}(K^{\frac{1}{2}})$$

So, with plasma, the total deflection is:

$$y = 2\,(r_1 - r_o)\,\phi_o^1 + 2\,r_o\,\phi_o^1\,K^{-\frac{1}{2}}\,\tanh^{-1}(K^{\frac{1}{2}})$$

The nett power ratio due to longitudinal deviation is,

$$\Gamma_{(long)} \equiv y_o/y = \left[(1-g) + g\,K^{-\frac{1}{2}}\,\tanh^{-1}(K^{\frac{1}{2}})\right]^{-1} \qquad (6)$$

where, again, $g = 1$ gives Wort's result.

Note that this factor increases as g decreases, also, when Wort's factor is, say 0.5, the factor for $g = \frac{1}{2}$ is 0.66.

C. Combined Effect

The nett effect of refraction in a cylinder with parabolic density distribution is obtained by combining (5) and (6), for the same g (= r_o/r_1):

$$\Gamma_g = \Gamma_{(trans)} \cdot \Gamma_{(long)}$$

$$= \frac{g\,(1-K)}{K + g\,(1-K)} \cdot \left[(1-g) + g\,K^{-\frac{1}{2}}\,\tanh^{-1}(K^{\frac{1}{2}})\right]^{-1} \qquad (7)$$

In particular,

$$\Gamma_{\frac{1}{2}} = \left(\frac{1-K}{1+K}\right) \cdot \frac{2}{(1-g) + g\,K^{-\frac{1}{2}}\,\tanh^{-1}(K^{\frac{1}{2}})}$$

instead of Wort's,

$$\Gamma_1 = (1-K)\,K^{\frac{1}{2}}/\tanh^{-1}(K^{\frac{1}{2}})$$

The last two expressions are plotted for comparison in Fig. 2.

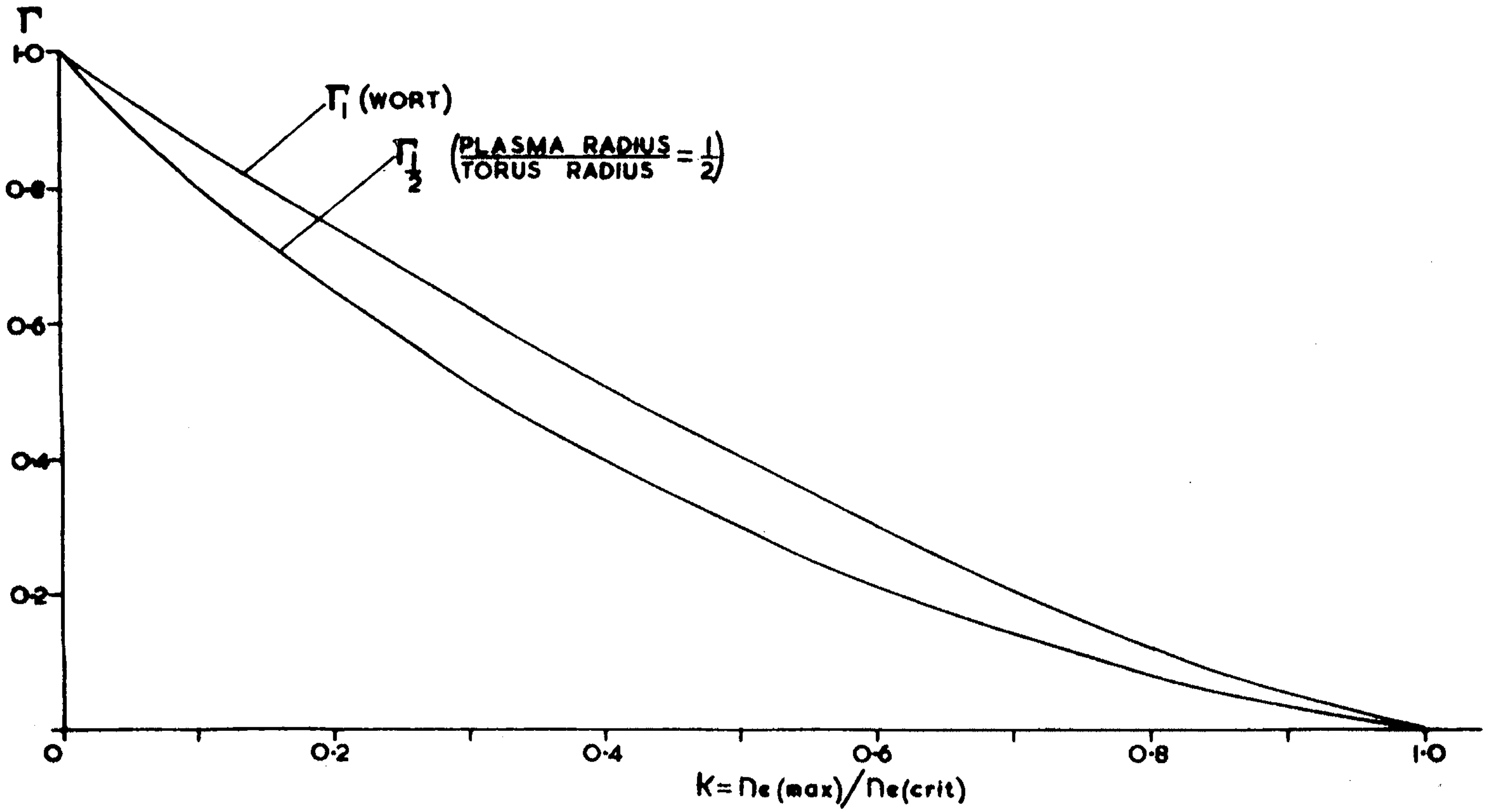

Figure 2 Effect of plasma constriction on microwave transmission.

Within the range of interest, ($g \leq 1$) it can be shown that Γ_g increases monotonically with g. Thus curves for ($\frac{1}{2} < g < 1$) lie between those plotted in Fig. 2. For $g = \frac{1}{2}$, i.e. a plasma of half the discharge tube diameter, the largest correction is for a transmission of about 0.35, where the modified formula gives a calculated electron density at the centre 20% less than Wort's figure.

3. Part 2: Possible Focussing Due to a Region of Lower Electron Density and an Outer Plasma Layer

In some experiments, signal transmission has been less than expected on the basis of refraction by a quiescent plasma and attempts have been made to account for this in terms of plasma turbulence (Wort, 1966). In this laboratory, the observed signal is sometimes greater than expected on the basis of full ionisation of the available atoms. One looks for "converging lens" effects.

The following simplified analysis shows the effect of a region of zero electron density between an inner plasma core, whose density is in question, and an outer region of plasma.

In Fig. 3, it is assumed that the inner plasma has a uniform density, n_1, and refractive index, μ_1, out to radius r_o. There is an outer plasma of electron density n_2, refractive index μ_2, inner radius r_1 and outer radius r_2. The discharge tube radius is r_3. All boundaries have a common centre, O.

For transmitting and receiving points P and P', we have, with no plasma, a direct ray at an angle $\beta_o = \angle OPP'$ to the diameter. In the presence of plasma the ray path is PABCDEFGP', whose initial segment PA makes an angle,

$\beta = \angle OPA$ with the diameter.

It is readily shown (as in Wort, 1963, equation 4), that the angles ϕ' and ϕ between the ray and the inward and outward radial directions are the same at corresponding points (such as C and E) so that the whole ray path is symmetrical about OD, the perpendicular bisector of CE.

The ratio: $\dfrac{\text{transmitted signal with plasma}}{\text{transmitted signal without plasma}}$

is: $$\Gamma = \beta / \beta_o \qquad (8)$$

We use small angle approximations, i.e.

(1) Use an angle in radians in place of its sine both in Snell's law and in the sine rule for triangles.

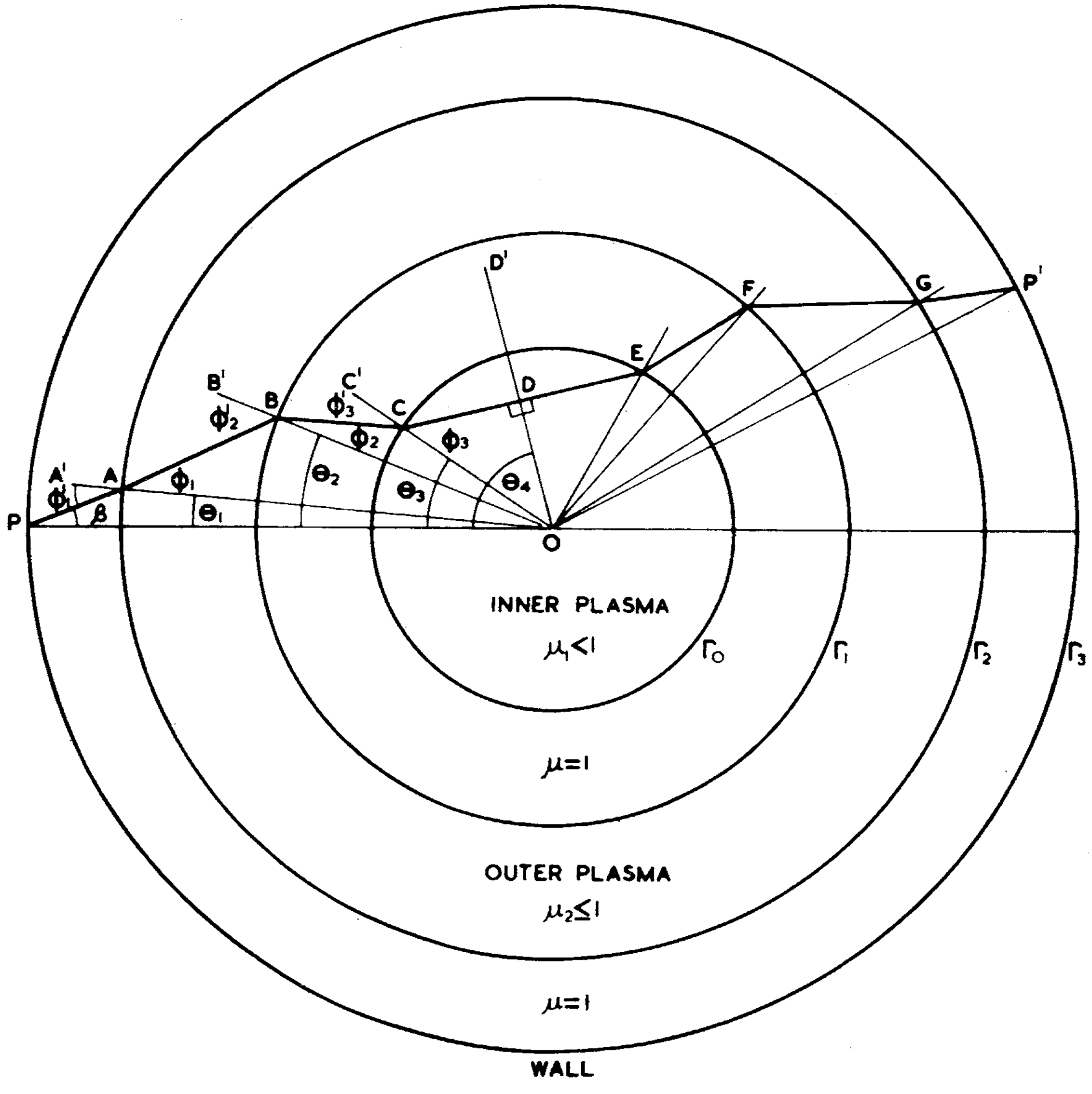

Figure 3 Geometry of ray path in two-layer plasma.

(2) PA $\doteq r_3 - r_2$, AB $\doteq r_2 - r_1$, etc.

In addition, only a typical ray, lying in the transverse plane, is traced.

The ray may be traced starting from P:

$$\phi'_1 = \beta + \theta_1 = \frac{r_3}{r_2}\beta \qquad\qquad \theta_1 = \frac{r_3 - r_2}{r_2}\cdot\beta$$

$$\phi_1 = \frac{1}{\mu_2}\phi'_1 = \frac{1}{\mu_2}\cdot\frac{r_3}{r_2}\cdot\beta \qquad\qquad \theta_2 - \theta_1 = \frac{r_2 - r_1}{r_1}\cdot\phi_1$$

$$\phi'_2 = \frac{r_2}{r_1}\cdot\phi_1 = \frac{r_3}{r_1}\cdot\frac{\beta}{\mu_2} \qquad\qquad = \frac{r_2 - r_1}{r_1}\cdot\frac{r_3}{r_2}\cdot\frac{\beta}{\mu_2}$$

$$\phi_2 = \mu_2\phi'_2 = \frac{r_3}{r_1}\cdot\beta \qquad\qquad \theta_3 - \theta_2 = \frac{r_1 - r_o}{r_o}\cdot\phi_2$$

$$\phi'_3 = \frac{r_1}{r_o}\phi_2 = \frac{r_3}{r_o}\cdot\beta \qquad\qquad = \frac{r_1 - r_o}{r_o}\cdot\frac{r_3}{r_1}\beta$$

$$\phi_3 = \frac{1}{\mu_1}\phi'_3 = \frac{r_3}{r_o}\cdot\frac{\beta}{\mu_1} \tag{9}$$

From the symmetry argument above, the direct ray PP' is perpendicular to OD. Thus,

$$\frac{\pi}{2} - \beta_o = \theta_4 = \theta_3 + \frac{\pi}{2} - \phi_3$$

$$\text{or} \quad \beta_o = \phi_3 - \theta_3 \tag{10}$$

From the right-hand sequence of equations,

$$\theta_3/\beta = \frac{r_3 - r_2}{r_2} + \frac{r_2 - r_1}{r_1}\cdot\frac{r_3}{r_2}\cdot\frac{1}{\mu_2} + \frac{r_1 - r_o}{r_o}\cdot\frac{r_3}{r_1} \tag{11}$$

From (8), (9), (10) and (11):

$$\frac{1}{\Gamma} = \frac{\beta_0}{\beta} = \frac{r_3}{r_o \mu_1} - (\frac{r_3}{r_2} - 1) - (\frac{r_2}{r_1} - 1) \cdot \frac{r_3}{r_2} \cdot \frac{1}{\mu_2} - (\frac{r_1}{r_o} - 1) \frac{r_3}{r_1}$$

We are mainly interested in the case of an outer plasma near the torus wall

$$(r_3 - r_2 \ll r_2)$$

Then, letting $r_3 = r_2$ = torus radius:

$$\frac{1}{\Gamma} = \frac{r_2}{r_o \mu_1} - (\frac{r_2}{r_1} - 1) \frac{1}{\mu_2} - (\frac{r_1}{r_o} - 1) \frac{r_2}{r_1} \qquad (12)$$

Absence of the outer plasma can be represented either by letting $\mu_2 = 1$ or by letting $r_1 = r_2$; in either case we have a transmission ratio Γ_o given by,

$$\frac{1}{\Gamma_o} = \frac{r_2}{r_o \mu_1} + (1 - \frac{r_2}{r_o}) \qquad (13)$$

The influence of the outer layer is exhibited by:

$$\frac{1}{\Gamma_o} - \frac{1}{\Gamma} = (\frac{r_2}{r_1} - 1)(\frac{1}{\mu_2} - 1)$$

$$= L, \text{ say} \qquad (14)$$

one factor of which vanishes when $r_1 = r_2$, the other when $\mu_2 = 1$, consistent with the remarks above. Notice that (14) does not contain the parameters (r_o, μ_1) of the <u>inner</u> plasma. It is then likely that (14) applies for any profile of the plasma core, including the parabolic one, i.e. we assume (14) holds even when (13) is replaced by other relations, such as those in Part 1.

If an outer plasma layer is present both the above factors are positive so that $\Gamma > \Gamma_o$ and, if one ignores this outer layer, one underestimates the density of the plasma core.

The correction term from equation (14) is given in Table I for selected ratios of n_2/n_c and r_1/r_2,

TABLE I

Outer Sheath		$L = \left[(1/\mu_2) - 1 \right] \left[(r_2/r_1) - 1 \right]$		
n_2/n_c	$(1/\mu_2) - 1$	$r_1/r_2 = 0.75$	0.8	0.9
0.4	0.39	0.130	0.098	0.043
0.5	0.41	0.138	0.102	0.046
0.6	0.58	0.194	0.145	0.064
0.7	0.83	0.276	0.208	0.092
0.8	1.24	0.413	0.310	0.138
0.9	2.16	0.721	0.450	0.240
0.95	3 48	1.160	0.870	0.387
0.98	6.09	2.36	1.77	0.788

For example, suppose the measured transmission ratio is,

$$\Gamma = 0.15$$

(for which Fig. 2 would give a central density 0.7 of the cut-off density).

If there were an outer layer with,

$$n_2/n_c = 0.9, \quad r_1/r_2 = 0.75,$$

then equation (14) and Table I give,

$$\frac{1}{\Gamma_o} = 6.667 + 0.721$$

$$= 7.39$$

$$\Gamma_o = 0.136$$

(for which Fig. 1 gives n_e (max.)$/n_c$ = 0.72).

The central electron density is thus about 3% higher than would be calculated by neglecting the outer layer.

For a given outer plasma, the correction becomes insignificant as the central density approaches cut-off, as long as the density of the outer is below 90% of the cut-off value.

4. Conclusion

Despite the fact that a single channel microwave power transmission produces one number and can be used to calculate only one parameter of an electron density distribution, it has been shown that, if one estimates peak electron density on the assumption of some simple model profiles, this parameter is not very much perturbed by plausible variation of the (higher order) terms one is not estimating.

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