Nonlinear dynamics of exciton-polariton Bose-Einstein condensate

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Declaration

This thesis is an account of research undertaken between September 2012 and June 2016 at the Nonlinear Physics Centre, Research School of Physics and Engineering of The Australian National University, Canberra, Australia. Except where acknowledged in the customary manner, this research presents my original results obtained under the supervision of Dr. Elena Ostrovskaya, and has not been submitted in whole or part for a degree in any university.

Guangyao Li
June, 2016
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List of publications


Abstract

Exciton-polariton Bose-Einstein condensates (BECs) are newly emerged quantum systems that are capable of showing macroscopic quantum phenomena with intrinsic open-dissipative nature. The spatial distribution of the polariton density, without any external potential, can be controlled by the geometric shape of the pumping laser, enabling the investigation of polariton dynamics with topologically non-trivial configurations. Meanwhile, exciton-polaritons have spin degrees of freedom inherited from excitons and photons, making it a candidate for the realization of quantum logic gates.

In this thesis, we will investigate theoretically the nonlinear dynamics of exciton-polariton BECs involving both polaritons’ spatial degrees of freedom and spin degrees of freedom, and interactions between them. This thesis is organised as follows: In Chapter 1, we will present an overall review of exciton-polariton systems and important properties of polariton BECs and then introduce the dynamical equations with various interactions that will serve as the main theoretical tool for subsequent chapters. Several polariton pumping and trapping techniques appearing in later chapters will also be introduced. In Chapter 2, we will investigate the superfluidity properties of a single-component polariton condensate under an incoherent annular pumping configuration. By studying the stability properties of polariton persistent currents, we find that the persistent currents can exhibit dynamical instability and energetic-like instability according to different parameter region. A stability phase diagram will be given and its relation with the Landau’s criterion will be discussed. In Chapter 3, we will investigate the spin dynamics of a two-component polariton condensate under a homogeneous pumping configuration. Owing to the Josephson coupling, there exist multiple steady state solutions that allow of controlled spin state switching. A desynchronized region where there exists no stable steady solution is found. In the desynchronized region, a desynchronized state beating periodically over time can exist, which will serve as a building block of spin waves presented in the next chapter. In Chapter 4, by combining results from the previous two chapters we will investigate generally the nonlinear dynamics of polariton condensates under an annular pumping configuration. The spin-orbit interaction provided by the Josephson coupling supports azimuthon states that have simultaneous modulations in both amplitude and phase. The azimuthon states, when viewed in a different polarization basis, form rotating spin waves that can be referred to as the optical ferris wheel. In Chapter 5, results from previous chapters will be extended to micocavities that support the anisotropic TE-TM splitting interaction. Rotating singularities (small-scale vortices) are found as a result. Their properties and experimental observation techniques will be discussed.

Chapter 2-5 provide a theoretical framework for the nonlinear dynamics of polariton condensates. They rely mostly on optical trapping techniques and are ready to be tested in experiments. In Chapter 6, polaritons trapped by an engineered
periodic mesa potential will be discussed. We will investigate the band structure of polaritons under the influence of the periodic potential together with discussions on the phase-modulated interference pattern which corresponds to the polariton Talbot patterns observed in experiments.

New major results presented in this thesis can be summarised as follows:

- Nonlinear dynamics and stability properties for persistent currents supported by the annular incoherent pump are investigated.
- The existence of the desynchronized region for spinor polariton condensates has been proved.
- Azimuthon states are predicted as possible collective excitations of polariton condensates.
- A fully incoherent vortex generation method based on the TE-TM splitting is proposed.
- Observations of Talbot patterns for polariton systems are given for the first time, together with comprehensive theoretical explanations.
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Chapter 1

Introduction

1.1 Semiconductor Microcavity

1.1.1 Excitons in semiconductors

In semiconductors and many other systems, there exist quasiparticles called excitons that are bound states of an electron and a hole attracted to each other by the electrostatic Coulomb interaction. When a photon is absorbed by a semiconductor, it excites an electron from the valence band to the conduction band, and at the same time the electron leaves a positively charged hole in the valence band. Through the Coulomb interaction, an excited electron and a hole form a bound state similar to that of a hydrogen atom [6]. Such bound state is a type of quasiparticle called exciton. In most semiconductors, the Coulomb interaction is strongly screened by electrons in the valence band, leading to weakly bound excitons that are known as Wannier-Mott excitons, whose Bohr radius is about 10-100 Å and therefore extends over tens of atomic sites in the crystal [7,8].

By denoting the creation operator of an electron in the conduction band with momentum $k$ as $\hat{c}^\dagger_k$ and the creation operator of a hole as $\hat{b}^\dagger_k$, the creation operator of an exciton can be found as [6]

$$e^{\dagger}_{K,n} = \sum_{k,k'} \delta_{K,k+k'} \phi_n \left( \frac{m_e k - m_h k'}{m_e + m_h} \right) \hat{c}^{\dagger}_k \hat{b}^{\dagger}_{k'},$$  

(1.1)

where $K$ is the center-of-mass momentum of the exciton, $\phi_n$ is the $n$th relative motion wave function between the electron and the hole, $m_e$ and $m_h$ are their mass respectively, and the symbol $\delta$ should be regarded as the Dirac delta function for the continuous case and the Kronecker delta function for the discrete case. While the open boundary conditions render the in-plane wave vector $k$ continuous, the periodic boundary conditions make $k$ discrete [9]. When the exciton interparticle spacing is much larger than its Bohr radius $n_{exc} \ll a_B^{-3}$, the exciton operators $\hat{e}$ and $\hat{e}^\dagger$ satisfy the Bose commutation relations

$$[\hat{e}_{K',n'}, \hat{e}_{K,n}] = 0, \quad [\hat{e}^{\dagger}_{K',n'}, \hat{e}^{\dagger}_{K,n}] = 0, \quad [\hat{e}_{K',n'}, \hat{e}^{\dagger}_{K,n}] = \delta_{K} \delta_{n,n'},$$  

(1.2)

1.1.2 Excitons trapped in quantum wells

A semiconductor quantum well (QW) is a thin layer of semiconductor with a thickness comparable to the length of the exciton Bohr radius. If the confinement di-
rection is denoted as the $z$ direction, the quantum well can be approximated as a square well potential along $z$ and is trapless in all perpendicular directions so that when the confinement is strong enough only the lowest-energy quantized level of the center-of-mass motion will be relevant [8]. In this case, QW excitons behave as two-dimensional quasiparticles and momentum conservation in an optical transition needs to be satisfied only in the QW plane but not in the $z$ direction. They can couple to light with the same in-plane wave number $k_{xy}$ and arbitrary transverse wave number $k_z$. Owing to the confinement, a QW exciton has a smaller Bohr radius about half the size of that of a bulk exciton [10], with a four times larger binding energy [11]. Therefore, it is more robust compared to a bulk exciton and the probability that an absorbed photon will excite an exciton is also increased with respect to the unconfined case. The robustness of excitons in quantum wells and their increased optical activity inspire embedding quantum wells in microcavities to enhance the light-matter coupling [11].

1.1.3 Photons trapped in microcavities

A microcavity is an optical resonator whose size is close to the wavelength $\lambda$ of the injected laser. In small scale, the trapping of light can be effectively done by using a distributed Bragg reflector (DBR). The DBR is made of layers of alternating high and low refraction index materials, with each layer having an optical thickness of $\lambda/4$. Therefore, light reflected from each interface destructively interfere, making it a high-reflectance mirror. A microcavity forms when two DBRs are put facing each other with a separation of several micrometers.

The trapping ability of a microcavity is represented by the quality factor (Q-factor) defined as

$$Q = \frac{\omega_c}{\delta \omega_c},$$

(1.3)

where $\omega_c$ is the resonant cavity frequency and $\delta \omega_c$ is the linewidth of the cavity mode. The Q-factor, which can exceed $10^5$ [12], is a measure of the rate at which optical energy decays from within the cavity (from absorption, scattering or leakage through the imperfect mirrors) where $Q^{-1}$ is the fraction of energy lost in a single round-trip around the cavity. Equivalently, the exponentially decaying photon number has a lifetime given by $\tau = Q/\omega_c$. A typical structure of a semiconductor microcavity consisting of a $\lambda/2$ cavity layer sandwiched between two distributed DBRs is shown in Fig. 1.1.

Since a planar cavity is by definition invariant under in-plane ($x$-$y$ plane) translations, the in-plane wave vector $\mathbf{k}$ is a good quantum number for the free photon dynamics, which can be described by a Hamiltonian of the form [13]

$$\hat{H}_p = \sum_{\mathbf{k}} E_p(\mathbf{k}) \hat{a}^\dagger_{\mathbf{k}} \hat{a}_{\mathbf{k}}$$

(1.4)

where $\mathbf{k} = (k_x, k_y)$ is the in-plane momentum, $\hat{a}^\dagger_{\mathbf{k}}$ and $\hat{a}_{\mathbf{k}}$ are the creation and annihilation operator for photons respectively, and $E_p(\mathbf{k})$ is the excitation energy. The polarization degree of freedom of photons are ignored temporarily. $\hat{a}_{\mathbf{k}}$ and $\hat{a}^\dagger_{\mathbf{k}}$
satisfy the standard Bose commutation rule

\[ [\hat{a}_k, \hat{a}_{k'}^\dagger] = 0 \quad \text{and} \quad [\hat{a}_k, \hat{a}_{k'}] = \delta_{k,k'}. \quad (1.5) \]

Field operators of photons can be obtained by making Fourier transformations of \( \hat{a}_k \) and \( \hat{a}_k^\dagger \). Interactions between photons that are uncoupled to excitons in microcavities are weak and are usually ignored.

### 1.2 Exciton-polaritons

As mentioned in Sec. 1.1.1, excitons are formed by the combination of electrons and holes, i.e. by two fermions having projections of angular momentum on the structure-growth axis equal to \( J_{e,z} = \pm 1/2 \) for an electron and \( J_{h,z} = \pm 1/2 \) (light-hole) or \( \pm 3/2 \) (heavy-hole) for a hole. Hence, the total angular momentum (spin) of an exciton in the ground state has projection values on the structure axis as: \( \pm 1 \) or \( \pm 2 \). However, optical excitations of the state with \( J_z = \pm 2 \) are prohibited by the selection rules, therefore these “dark” excitons are not coupled to the photonic modes in microcavities [14].

When the \( J_{\text{total}} = 1 \) excitons interact strongly with photons confined in the microcavity and the relaxation time given by decay and decoherence of photons is long enough, exciton-polaritons become new elementary excitations for the semiconductor microcavity system. The Hamiltonian for polaritons can be written as [6]

\[
\hat{H}_{ep} = \hat{H}_e + \hat{H}_p + \hat{H}_I
= \sum_k E_e(k)\hat{e}_k^\dagger \hat{e}_k + \sum_k E_p(k)\hat{a}_k^\dagger \hat{a}_k + \sum_k g_0(\hat{a}_k^\dagger \hat{e}_k + \hat{a}_k \hat{e}_k^\dagger), \quad (1.6)
\]

where \( \hat{e}_k^\dagger \) and \( \hat{a}_k^\dagger \) are creation operators for excitons and photons defined in Sec. 1.1.1 and 1.1.3 respectively with \( E_{p,e}(k) \) the corresponding excitation energy; \( g_0 \) is their interaction strength. Owing to the linearity of \( \hat{H}_{ep} \), it can be diagonalized by the
transformed operators

\[ \hat{P}_k = X_k \hat{e}_k + C_k \hat{a}_k, \]
\[ \hat{Q}_k = -C_k \hat{e}_k + X_k \hat{a}_k, \]  

(1.7)

where $\hat{P}_k$ and $\hat{Q}_k$ are creation operators for lower (LP) and upper (UP) polaritons respectively. The complex coefficients $X$ and $C$ are the Hopfield coefficients, which satisfy $|X|^2 + |C|^2 = 1$. The diagonalized expression of $\hat{H}_{ep}$ is

\[ \hat{H}_{ep} = \sum_k E_{LP}(k) \hat{P}_k^\dagger \hat{P}_k + \sum_k E_{UP}(k) \hat{Q}_k^\dagger \hat{Q}_k, \]  

(1.8)

where $k$ represents the module of $k$. Some examples of polariton dispersions $E_{LP}(k)$ and $E_{UP}(k)$ with different Hopfield coefficients are shown in Fig. 1.2.

**Figure 1.2:** Polariton dispersions with different Hopfield coefficients corresponding to different detunings between the cavity photon and the exciton resonance. Left column: horizontal dashed lines represent exciton dispersion in quantum wells; parabolic dashed curves represent photon dispersion in microcavities; blue curves represent the upper polariton dispersion; red curves represent the lower polariton dipersion. (Figure taken from [6]).

Polaritons as quasiparticles will scatter with phonons and, as a result, will lower
their kinetic energy, which can be described by the semiclassical Boltzmann equation [6]. Meanwhile, polaritons will scatter with polaritons, which will introduce nonlinear interaction into the system. This nonlinear interaction, as we will see in the following section, plays a crucial role in the polariton condensate dynamics.

1.3 Polariton condensate

Polaritons as bosons will exhibit Bose-Einstein condensation (BEC) which is a macroscopic occupation of the lowest energy state resulting from the Bose-Einstein statistics. The polariton dispersion curves given by Sec. 1.2 indicate that the $k = 0$ state of lower polaritons is an ideal candidate for achieving polariton condensation, which has been demonstrated by several pioneering experiments on the spontaneously established coherence for polariton systems [15–17]. Compared with atomic BECs, polaritons have an effective mass of about $10^{-4} \sim 10^{-6} m_e$, where $m_e$ is the electron mass, much smaller than the mass of the elementary constituent of an atomic BEC which is about $10^3 m_e$ [6]. Hence, the critical density for reaching a polariton BEC is eight orders of magnitude lower than that for reaching an atomic BEC at the same temperature; or equivalently, the critical temperature for reaching a polariton BEC is eight orders of magnitude higher than that for reaching an atomic BEC (ranging from $1 nK \sim 1 \mu K$) at the same particle density [6]. Thus, the critical temperature for a polariton BEC can range between $1 K$ to $300 K$, up to the room temperature.

Unlike an atomic BEC, the Bose-Einstein statistics for polariton condensates have not been conclusively demonstrated due to their very short lifetime, which is about several to tens of picoseconds, and their intrinsic nonequilibrium nature. The dynamical and open-dissipative nature of a polariton condensate provides the direct experimental access to the properties of the condensate. Because of the photon leakage, the energy and the in-plane momentum of the polariton condensate can be easily analysed by optical measurements, enabling a direct study of this open-dissipative quantum system. It is one of the main purposes of the current thesis to bridge the gap between the conservative theory and the non-conservative experimental phenomena by extending the knowledge of conservative BEC systems to polariton systems.

1.3.1 Incoherent pumping

The incoherent pumping scheme was used to demonstrate the spontaneous polariton condensation for the first time, which is similar to the spontaneous condensation observed in ultracold gases [16]. It would be the major pumping method considered throughout this thesis. The whole incoherent pumping process is summarized in Fig. 1.3. By setting the energy of the pumping laser well above the dispersion curve of the lower-polariton branch, high energy free carriers are generated. These high energy particles cool down by scattering with the excitations of semiconductor crystal lattice (phonons) and, during this process, their phase correlation is washed out completely, forming a large number of uncorrelated high energy lower polaritons. The polaritons continue to cool down through scattering with phonons until they reach the so-call bottleneck region on the dispersion curve where the cooling process
1.3.2 Coherent pumping

Another polariton pumping method is the coherent pumping scheme. A common realization utilises the parametric down conversion to inject polaritons into the targeted state \((k = 0)\) so that the polariton condensate inherits the phase information from the pumping laser. As shown in Fig. 1.4, the incident angle of the pumping laser is tuned to the inflection point of the lower-polariton branch, which is referred to as the magic angle. The wave number of the resulting signal \((k_S)\) and idler \((k_I)\) fulfill

\[
  k_S = 0 \quad \text{and} \quad 2k_P = k_S + k_I, \tag{1.9}
\]

where \(k_P\) is the wave number of the pumping laser. Under coherent pumping, the phase of the polariton condensate connects directly with the phase of the pumping laser, and therefore, through the phase-velocity relation (the local velocity of a BEC within the mean-field approximation is proportional to the gradient of the phase of its wave function, see Sec. 1.3.3), a direct control of the condensate velocity can be achieved. Specifically, the condensate can acquire rotations from a Laguerre-
1.3 Polariton condensate

Figure 1.4: Coherent pumping scheme. The horizontal dashed line represents the exciton dispersion. The parabolic dashed curve represents the photon dispersion. The upper solid curve represents the upper polariton dispersion. The lower solid curve represents the lower polariton dispersion. A pump laser with an incident angle fulfilling the phase-matched condition generates a pair of polaritons on the lower polariton branch with \( k_S = 0 \) and \( k_I = 2 k_P \), where \( k_P \) is the wave vector of the incident beam (figure taken from [19]).

A Gaussian beam that carries non-zero orbital angular momentum (see Sec. 1.6.2). The possibility of coherent injection of angular momentum to the polariton condensate can be used as a way to control its initial phase, which will be discussed in more detail in Chap. 2 and Chap. 4.

Before we introduce the mean-field dynamic equation for a polariton condensate, it is instructive to revisit the mean-field description of a conservative BEC system, which would be a foundation for our understanding of the open quantum system.

1.3.3 Mean-field descriptions of weakly interacting BEC systems

Assuming that polaritons on the lower dispersion branch near \( \mathbf{k} = 0 \) can be treated as a conservative non-interacting Bose gas described by single particle Hamiltonian \( \hat{H}^{(1)} \) and the condition of thermal equilibrium is fulfilled, the total Hamiltonian can be written as

\[
\hat{H} = \sum_j \hat{H}_j^{(1)},
\]

where each \( \hat{H}_j^{(1)} \) satisfies \( \hat{H}_j^{(1)} \psi_i(\mathbf{r}) = E_i \psi_i(\mathbf{r}) \), with \( \psi_i \) and \( E_i \) its eigenfunction and eigenvalue respectively. The average occupation number \( \bar{n}_i \) for the energy level \( E_i \) is given by [20]

\[
\bar{n}_i = \frac{1}{\exp[\beta(E_i - \mu)] - 1}.
\]
Eq. (1.11) places a constraint $\mu < E_0$ for the chemical potential, otherwise $\bar{n}_i$ would become negative. When $\mu \rightarrow E_0$ the occupation number of the ground state

$$N_0 = \frac{1}{\exp[\beta(E_0 - \mu)] - 1}$$

becomes increasingly large. The Bose-Einstein condensation takes place once $N_0$ becomes macroscopic (approaching the total particle number $N = \sum_i \bar{n}_i$ [20]).

In experiments, however, particles trapped within a certain volume do interact with each other and their interaction will change the properties of the Bose gas significantly. Usually, the range of particle interaction is much smaller than their average distance $d = n^{-1/3}$ where $n$ is the particle density so that the three-body interaction can be safely neglected. This allows us to consider only the two-body interaction.

The polariton operators defined in Sec. 1.2 are given by the diagonalization of the exciton-photon Hamiltonian via the linear combination of electron and hole operators, treating excitons as structureless quasiparticles. Conversely, when the exciton-photon coupling becomes so strong that it is comparable to the binding energy of excitons, a more rigorous approach should be adopted in treating the electron, hole, and photon on equal footing [6, 21]. In all following sections, we assume that polaritons are away from this regime so that they can always be treated as structureless quasiparticles. Consequently, the polariton two-body interaction is dominated by the $s$-wave scattering which is represented by the scattering length $a$ [6].

In writing down the many body Hamiltonian of a Bose system in terms of the field operators $\hat{\Psi}(\mathbf{r})$, defined by the Fourier transform of a given annihilation operator $\hat{\Psi}_k$ which is the annihilation operator $\hat{P}_k$ around $\mathbf{k} = 0$ [see Eq. (1.8)], the abrupt change for a realistic interaction potential $V$ in short-range makes it difficult to obtain solutions for the Bose system. In order to simplify the many-body formalism for conditions of most experiments, it is appropriate to replace the realistic potential $V$ by an effective potential $V_{eff}$ which is slowly-varying in space, provided that $V$ and $V_{eff}$ share the same $s$-wave scattering length $a$. Since the physical properties depend uniquely on the value of the scattering length, this procedure will provide the correct modelling of a BEC system as far as the macroscopic properties of the system are concerned.

Following the procedure, if $|a| \ll d$, the Hamiltonian of the system can be written as

$$\hat{H} = \int \left( \frac{\hbar^2}{2m} \nabla \hat{\Psi}^\dagger \nabla \hat{\Psi} \right) d\mathbf{r} + \int \hat{\Psi}^\dagger V_{ext}(\mathbf{r}) \hat{\Psi} d\mathbf{r} + \frac{1}{2} \int \hat{\Psi}^\dagger \hat{\Psi}^\dagger V(\mathbf{r'} - \mathbf{r}) \hat{\Psi} \hat{\Psi} d\mathbf{r'} d\mathbf{r}, \quad (1.13)$$

where $\hat{\Psi}(\mathbf{r})$ is the field operator, $\hat{\Psi}' = \hat{\Psi}(\mathbf{r}')$, $V_{ext}(\mathbf{r})$ is the external potential provided by the specific structure of the microcavity which will be discuss in Sec. 1.6, and $V(\mathbf{r})$ is the polariton-polariton two-body interaction potential. In the Heisenberg
picture, $\hat{\Psi}$ satisfies
\[i\hbar \frac{\partial}{\partial t} \hat{\Psi}(r, t) = [\hat{\Psi}(r, t), \hat{H}] = \left[ -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(r, t) + \int \hat{\Psi}^{\dagger}(r', t)V(r' - r)\hat{\Psi}(r', t)dr' \right] \hat{\Psi}(r, t),\]
\[(1.14)\]
where the commutation relations for the field operator were used
\[[\hat{\Psi}(r, t), \hat{\Psi}^{\dagger}(r', t)] = \delta(r - r'), \quad [\hat{\Psi}(r, t), \hat{\Psi}(r', t)] = 0, \quad [\hat{\Psi}^{\dagger}(r, t), \hat{\Psi}^{\dagger}(r', t)] = 0.\]
\[(1.15)\]
With the formation of a BEC, the ground state of the given system is macroscopically occupied so that the whole BEC system can be approximated by a single wave function. This point can be seen in that $\langle \hat{\Psi}^{\dagger}\hat{\Psi} \rangle \sim N$ and $\langle \hat{\Psi}\hat{\Psi}^{\dagger} \rangle \sim (N + 1)$, where both quantities are at the macroscopic value $N$ and are much larger than their commutator [Eq. (1.15)] which is at the scale of 1. Therefore, it is reasonable to replace the field operator $\hat{\Psi}$ by a complex field $\psi(r, t)$, the so called mean-field approximation [20].

With the two-body interaction potential $V(r)$ being replaced by the effective potential $V_{\text{eff}}(r)$, by putting $\psi(r, t)$ into Eq. (1.14), we have
\[i\hbar \frac{\partial}{\partial t}\psi(r, t) = \left( -\frac{\hbar^2 \nabla^2}{2m} + V_{\text{ext}}(r, t) + g|\psi(r, t)|^2 \right)\psi(r, t),\]
\[(1.16)\]
where $g = \int V_{\text{eff}}(r)dr = 4\pi\hbar^2a/m$ is the interaction coupling constant dominated by the $s$-wave scattering and $a$ is the $s$-wave scattering length.

Eq. (1.16) is referred to as the Gross-Pitaevskii (GP) equation in literature and it is the most basic dynamical equation for the mean-field description of a BEC system. It contains a cubic nonlinear interaction term so that its solutions do not fulfill the superposition principle. Such cubic nonlinear interaction is well known in the field of nonlinear optics, and thus Eq. (1.16) is also called the nonlinear Schrödinger equation [22].

The GP equation can also be obtained by applying the variational principle to $\psi(r, t)$ for a time-independent external potential [23]
\[i\hbar \frac{\partial}{\partial t} \psi(r, t) = \frac{\delta E[\psi]}{\delta \psi^*(r, t)},\]
\[(1.17)\]
where ‘*’ represents complex conjugate and
\[E[\psi] = \int \left( \frac{\hbar^2}{2m} |\nabla \psi|^2 + V_{\text{ext}}(r)|\psi|^2 + \frac{g}{2}|\psi|^4 \right)dr \]
\[(1.18)\]
is the energy functional of the system.

A stationary solution where the condensate wave function evolves sinusoidally in time can be found as
\[\psi(r, t) = \sqrt{\rho(r)}e^{-i\mu t/\hbar},\]
\[(1.19)\]
where \( \rho(\mathbf{r}) \) is the particle density and \( \mu \) is the chemical potential. Solution (1.19) has been rigorously proven as the ground state of a BEC system with repulsive \( s \)-wave scattering interaction \[24\].

A general solution of Eq. (1.16) reads

\[
\psi(\mathbf{r}, t) = \sqrt{\rho(\mathbf{r})} e^{i\phi(\mathbf{r}, t)}. \tag{1.20}
\]

By substituting it into the GP equation, we can obtain the continuity equation for particle number conservation

\[
\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0, \tag{1.21}
\]

where \( \rho = n \) is the particle density and \( \mathbf{j} \) is the current density given by

\[
\mathbf{j}(\mathbf{r}, t) = n \frac{\hbar}{m} \nabla \phi(\mathbf{r}, t), \tag{1.22}
\]

and the velocity of the condensate flow is

\[
\mathbf{v} = \frac{\hbar}{m} \nabla \phi(\mathbf{r}, t) \Rightarrow \nabla \times \mathbf{v} = 0. \tag{1.23}
\]

The phase \( \phi \) plays the role of a velocity potential, and the second equation shows that \( \mathbf{v} \) is irrotational, which is connected with superfluidity in BEC systems and will be discussed in Sec. 1.4.

### 1.3.4 Open-dissipative Gross-Pitaevskii equation

The mean-field description derived in Sec. 1.3.3 can also be applied to polariton condensates, provided that suitable phenomenological descriptions of the pumping and decay mechanism have been included. Detailed derivations of dynamic equations for a polariton condensate can be found in \[25–28\].

Under the incoherent pumping conditions, a polariton reservoir should be phenomenologically included as a particle source which is also phase incoherent with the pumping laser. The uncondensed nature of the reservoir allows us to model it as having only the particle density \( n_R(\mathbf{r}, t) \) and without any phase information. The stimulated scattering process from the reservoir into the condensate can be regarded as an increasing function of the reservoir density \( R = R[n_R(\mathbf{r}, t)] \). In practice, the stimulated scattering rate can be chosen as proportional to \( n_R(\mathbf{r}) \) for simplicity. Being uncondensed lower polaritons, the reservoir manifests the polariton repulsive interaction (\( s \)-wave scattering) by increasing the condensate energy if the condensate and the reservoir spatially overlap with each other. The amount of the increased condensate energy is referred to as energy blue-shift in experiments. This effect plays an important role in the process of optically-induced trapping and spin dynamics of the polariton system, which will be discussed in detail in Chap. 3. Finally, polaritons have a finite lifetime between 1 ps \[18\] to 100 ps \[29\] and a thermalization time of about 1 ps \( \sim 10 \) ps \[6\]. (The typical thermalization time and lifetime of an atomic BEC is 1 ms and 1 s respectively \[6\].) Therefore, in principle thermal equilibrium cannot be reached for polaritons having a lifetime shorter than 10 ps. Although many experiments \[16,17\] at present are on the boundary of the
regime where equilibrium can be discussed, we can safely assume that equilibrium or quasi-equilibrium has been reached for polariton condensates discussed in those experiments [18].

When the macroscopic occupation of the polariton condensed state happens, the corresponding mean-field open-dissipative Gross-Pitaevskii equation can be written as [25]

\[ i\hbar \frac{\partial \psi(r,t)}{\partial t} = \left\{ -\frac{\hbar^2}{2m_p} \nabla^2 + g_c |\psi(r,t)|^2 + g_R n_R(r,t) + \frac{i\hbar}{2} \left[ R n_R(r,t) - \gamma_c \right] \right\} \psi(r,t), \]

\[ \frac{\partial n_R(r,t)}{\partial t} = P(r,t) - (\gamma_R + R|\psi|^2) n_R, \]

where \( m_p \) is the effective mass of the lower polariton, which can be obtained in experiments by calculating \( \frac{d^2 E}{d k^2} \big|_{k=0} \) from the lower polariton dispersion curve; \( g_c \) represents the polariton-polariton s-wave scattering within the condensate; \( g_R \) represent the energy blue-shift caused by the reservoir; \( \gamma_c \) is the polariton decay rate given by \( \gamma_c = 1/\tau_c \), where \( \tau_c \) is the mean polariton lifetime; for the reservoir equation, \( P(r,t) \) is the pumping rate and \( \gamma_R \) is the reservoir decay rate.

As will be discussed in Chap. 2, the inclusion of the pumping and decay mechanisms will change properties of a condensate significantly compared with the conservative case. Before entering the detailed discussion, it would be beneficial for us to revisit some important properties of the conservative GP equation.

### 1.3.5 Elementary excitations of conservative condensates

An important class of elementary excitations of the ground state of the GP equation is small-amplitude oscillations. They play an important role in the superfluidity properties for a BEC system. Consider a small amplitude perturbation against the steady state \( \Psi_0(r,t) = \Psi_0(r) e^{-i\mu t/\hbar} \)

\[ \Psi(r,t) = [\Psi_0(r) + \delta \psi(r,t)] e^{-i\mu t/\hbar}, \]

where \( \delta \psi(r,t) \) is the perturbation given by

\[ \delta \psi(r,t) = u(r)e^{-i\omega t} + v^*(r)e^{i\omega t}, \]

\( u \) and \( v \) are the amplitudes of the oscillations (assumed to be small), and \( \omega \) is the frequency of the oscillation.

Inserting Eq. (1.25) and Eq. (1.26) into the GP equation and keeping only the linear terms, we have the equations for \( u \) and \( v \)

\[ \begin{bmatrix} \hat{H}_0 - \mu + 2gn & g\Psi_0^2 \\ -g\Psi_0^2 & -\left( \hat{H}_0 - \mu + 2gn \right) \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \hbar \omega \begin{bmatrix} u \\ v \end{bmatrix}, \]

where \( \hat{H}_0 = -(\hbar^2/2m)\nabla^2 + V_{ext}(r) \) and \( n \) is the particle density. For the ground
state of a uniform gas: $V_{ext} = 0$, $\mu = gn$, and $\Psi_0 = \sqrt{n}$,
\[ u(r) = u e^{ikr} \quad \text{and} \quad v(r) = v e^{ikr}. \] (1.28)

Equation (1.27) then reduces to
\[ \begin{bmatrix} \hbar^2 k^2 / 2m + gn \\ -gn \end{bmatrix} \begin{bmatrix} u \\ v \end{bmatrix} = \hbar \omega \begin{bmatrix} u \\ v \end{bmatrix}. \] (1.29)

Solutions of Eq. (1.29) are given by
\[ (\hbar \omega)^2 = \left( \frac{\hbar^2 k^2}{2m} \right)^2 + \frac{\hbar^2 k^2}{m} gn \] (1.30)

or equivalently,
\[ \omega(k) = \frac{1}{\hbar} \sqrt{\left( \frac{\hbar^2 k^2}{2m} \right)^2 + \frac{\hbar^2 k^2}{m} gn}. \] (1.31)

Eq. (1.31) is the Bogoliubov dispersion law and the perturbation approach is called the Bogoliubov-de Gennes (BdG) analysis [20]. For small $k$, $\omega(k) = k \sqrt{gn/2m}$ depends linearly on $k$, which can be seen in Fig. 1.5.

The energy change caused by $\delta \psi$ is
\[ E = E_0 + E^{(2)}, \] (1.32)
where $E^{(2)}$ is the second order perturbation. The first order perturbation $E^{(1)}$ vanishes by the definition of the steady state $\Psi_0$.

Inserting $\delta \psi$ into the energy expression (1.18), we have
\[ E^{(2)} = \sum_i \int \hbar \omega_i (|u_i|^2 - |v_i|^2) \, dr, \] (1.33)
where the summation goes over all solution branches for $\omega_i$. The quantity $\int \hbar \omega_i (|u_i|^2 - |v_i|^2) \, dr$ must be positive for each mode in order to give a positive $E^{(2)}$ to ensure the stability of the system [20].

The occurrence of negative $E^{(2)}$ is a direct signature of the energetic instability caused by the perturbation. The energetic instability can only destabilize the system in the presence of dissipative terms which drives it towards a lower energy state. By comparison, the dynamical instability means that Eq. (1.30) admits a complex frequency $\omega$ which has a positive imaginary part. The exponential growth of the perturbation will destroy a spatially homogeneous condensate, resulting in a fragmented one. These two kinds of instability correspond to the stability criterion given by the energy functional (1.18) and that of the wave function (1.25) respectively, and they are related to each other. For a conservative BEC, it has been shown that when the system is energetically stable it is also dynamically stable; while the dynamical instability can be revealed by a direct integration of the GP equation, the energetic instability cannot [30]. We will discuss the relationship between elementary excitation spectra and superfluidity properties of a BEC system.
in Sec. 1.4.

1.4 Superfluidity, vorticity and persistent currents

1.4.1 Landau’s criterion for superfluidity

Superfluidity is a common phenomenon seen in different systems including the liquid $^4$He and ultracold atomic gases, which can be exemplified by fluids flowing without viscosity, infinite thermal conductivity, and superconductivity [31]. To understand the origin of superfluidity, we recall Landau’s argument for the critical flow velocity for elementary excitations. Consider a superfluid system in the laboratory frame $K$ having energy $E$ and momentum $P$. When we transform to the frame $K'$ which is co-moving with the superfluid with velocity $V$, the energy and momentum will be Galilean transformed as

$$ E' = E - P \cdot V + \frac{1}{2} MV^2, \quad P' = P - MV, \quad (1.34) $$

where $M$ is the total mass of the fluid.

We first consider the elementary excitation in a co-moving frame $K'$ where the fluid is at rest. If a single excitation with momentum $p$ appears in the fluid then the total energy will increase by $E_0 + \epsilon(p)$, where $E_0$ and $\epsilon(p)$ are the ground state energy and the excitation energy respectively. Now we transform back to the laboratory frame $K$ where the fluid is moving with velocity $v$, its energy $E'$ and momentum $P'$ are

$$ E' = E_0 + \epsilon(p) + p \cdot v + \frac{1}{2} Mv^2, \quad P' = p + Mv. \quad (1.35) $$

Eq. (1.35) shows that $\epsilon(p) + p \cdot v$ is the energy change in the laboratory frame caused by the excitation. The spontaneous creation of excitations can only take place if the excited energy is negative

$$ \epsilon(p) + p \cdot v < 0, \quad (1.36) $$

which is only possible if $v > \epsilon(p)/p$. If the velocity of the fluid is smaller than a threshold velocity

$$ v_c = \min_p \frac{\epsilon(p)}{p}, \quad (1.37) $$

where the minimum is calculated over all values of $p$, then the elementary excitation in the fluid is not energetically possible and the fluid will be flowing without friction. This Landau’s criterion for superfluidity can also be written as

$$ v < v_c, \quad (1.38) $$

where $v_c$ is called the critical velocity.

1.4.2 Critical velocity and Bogoliubov dispersion

From Sec. 1.3.5 we know the dispersion of elementary excitations [Bogoliubov dispersion (1.31)] for the ground state of a uniform BEC. To test if the uniform ground state can exhibit superfluidity, we apply the Landau criterion to the dispersion (1.31)
with \( p = \hbar k \) and \( \epsilon = \hbar \omega \),

\[
\epsilon(p) = \sqrt{\left(\frac{p^2}{2m}\right)^2 + \frac{P^2}{m^2} gn}. \tag{1.39}
\]

We have

\[
v_c = \lim_{p \to 0} \frac{\epsilon(p)}{p} = \sqrt{\frac{gn}{m}}. \tag{1.40}
\]

The quantity \( c = \sqrt{gn/m} \) is the sound velocity, see Fig. 1.5. Therefore, in a weakly interacting uniform BEC system, if the flow speed is smaller than the sound velocity, the fluid will flow without any dissipation, forming, a superfluid.

![Figure 1.5: Bogoliubov dispersion \((p = \hbar k \text{ and } m = gn = 1)\).](image)

Another crucial relation between BEC systems and superfluidity is the connection between the superfluid velocity and the gradient of the phase of the wavefunction. If we apply Landau’s argument to the GP equation (1.16), under the Galilean transformation, one can check that the transformed wave function

\[
\Psi'(r, t) = \Psi(r - v t, t) \exp \left[ \frac{i}{\hbar} \left( m v \cdot r - \frac{1}{2} m v^2 t \right) \right] \tag{1.41}
\]

fulfills the transformed GP equation, where \( \Psi(r - v t, t) \) is the original solution before the Galilean transformation and \( v \) is a constant speed vector. If in the co-moving frame, where the condensate is at rest, a wave function is \( \Psi_0 = \sqrt{n_0} e^{-i\mu t/\hbar} \), then in the laboratory frame, where the fluid is flowing with velocity \( v \), the transformed wave function is \( \Psi = \sqrt{n_0} e^{iS} \), with

\[
S(r, t) = \frac{1}{\hbar} \left[ m v \cdot r - \left( \frac{1}{2} m v^2 + \mu \right) t \right] \tag{1.42}
\]
By applying the velocity relation (1.23), i.e.

$$v_s = \frac{\hbar}{m} \nabla S,$$

(1.43)

one can see that the superfluid motion of a condensate is irrotational.

1.4.3 Irrotationality

Consider a gas confined in a macroscopic cylindrical vessel having radius $R$ and length $L$. One class of important solutions of the GP equation corresponding to rotations around the axis of the cylinder (vortex) is given by

$$\Psi(r) = \Phi(r)e^{is\varphi},$$

(1.44)

where $r$ and $\varphi$ are the cylindrical coordinates. This wave function is an eigenstate of the angular momentum

$$\hat{l}_z = \frac{\hbar}{i} \frac{\partial}{\partial \varphi}$$

(1.45)

with eigenvalue $l_z = s \hbar$, so that the vortex carries a total angular momentum $L_z = N s \hbar$, where $N$ is the total particle number. The tangential velocity around the $z$ axis given by the wave function (1.44) is [20]

$$v_s = \frac{\hbar s}{m r},$$

(1.46)

which decreases with the increase of $r$. It is completely different from the rigid rotation $v = \mathbf{\Omega} \times \mathbf{r}$, where the tangential speed increase with $r$. The circulation of the velocity field (1.46) over a close contour around the $z$ axis is given by

$$\oint v_s \cdot d\mathbf{l} = 2\pi s \frac{\hbar}{m},$$

(1.47)

and it turns out to be quantized in the units of $\hbar/m$.

If an irrotational BEC flow is having an initially imposed rotation whose velocity fulfils the Landau’s criterion of superfluidity, i.e. $v < v_c$, the superfluid will flow indefinitely and form a persistent current.

Persistent currents are most prominent in superconductors, where charges can flow without any resistance. This phenomenon has been explained by the BCS theory [31] that in a superconductor there exists an attractive interaction between electrons that is mediated by the vibrations of the ion lattice, leading to the formation of electron pairs. Below a critical temperature, pairs of electrons (as composite bosons) condense into a BEC which extends through the superconductor. To scatter an electron pair from the condensate it requires a threshold energy higher than the thermal energy available to the system below the critical temperature [see Fig. (1.5)], and thus the superconducting electrons can flow without being scattered, that is, without any resistance. For a charge-natural BEC system, the same principle applies, resulting in charge-natural persistent currents.
1.5 Polariton condensates with spin degrees of freedom

1.5.1 Linear polarization splitting

As discussed in Sec. 1.2, polaritons have spin degrees of freedom inherited from interactions between excitons and photons. Polaritons have spin $J_{\text{total}} = 1$ but only have two available projection values along the crystallographic direction of the solid state lattice, forming a two-state quantum system that corresponds to two different polarization of the photoluminescence emitted from the microcavity. Since this two-state quantum system follows the mathematical description of a spin-$1/2$ particle, the spin degrees of freedom of polaritons does not follow strictly the mathematical framework of a $J_{\text{total}} = 1$ spinor and therefore it is referred to as pseudospin (hereafter we shall omit pseudo while speaking about the polariton spin).

In the circular polarization basis, within the scope of the mean-field theory, the polariton condensate can be represented by a two-component wave function

$$\Psi(r, t) = \bigg[\begin{array}{c}
\psi_+(r, t) \\
\psi_-(r, t)
\end{array} \bigg],$$  \hspace{1cm} (1.48)

where ± corresponds to the right-handed and the left-handed circularly polarized component, respectively.

The dynamical equation for a spinor polariton condensate should consist of same-spin interactions appearing in Eq. (1.24) for each polarization component, as well as cross-spin interactions between two polarization components. The cross-spin s-wave scattering depends only on the density of the other component. Another type of cross-spin interaction is the linear coupling, which is given by the sample-specific linear polarization splitting observed in experiments [32–34] arising from stress at the semiconductor interfaces [17]. The linear coupling will induce tunneling effects between $\psi_+$ and $\psi_-$ similar to the Josephson current observed in superconductor physics [31]. Therefore, the linear coupling between two condensates is also called the Josephson coupling in literature [32,35].

By denoting the linear coupling as $J$, the GP equation for a spinor polariton condensate reads ($\sigma = \pm$)

$$i\hbar \frac{\partial \psi_\sigma}{\partial t} = \bigg\{ -\frac{\hbar^2}{2m} \nabla^2 + u_a |\psi_\sigma|^2 + u_b |\psi_-|^2 + g R n_\sigma + \frac{i\hbar}{2} [R n_\sigma(r) - \gamma_c] \bigg\} \psi_\sigma + J \psi_- \sigma,$$

$$\frac{\partial n_\sigma}{\partial t} = P_\sigma(r, t) - (\gamma_R + R |\psi_\sigma|^2) n_\sigma.$$  \hspace{1cm} (1.49)

Here $m$ is the effective mass of the lower polariton; $\gamma_c$ is the loss rate of polaritons which is the inverse of the polariton life time; $R$ characterises the stimulated scattering rate from the reservoir into the condensate; the interaction constants $u_a$ and $u_b$ characterise the scattering between the polaritons of the same and different polarization states, respectively. It is well established that $|u_b| < |u_a|$ [36], because the dominant interaction process involves electron-electron or hole-hole exchange, which would lead to scattering to dark exciton states for cross-spin polaritons [37]; $g_R$ characterises interactions between the reservoir and the condensate. With weak cross-spin s-wave scattering strength, the cross-spin condensate-reservoir interaction


has been neglected [26]. For the reservoir equation, $\gamma_R$ is the decay rate of the reservoir polaritons and $P_\sigma(r,t)$ is the pumping rate. The time-scale of the short-lifetime reservoir dynamics is assumed to be comparable to that of the condensate, so that the model of static reservoir [26, 28] is no longer applicable.

By making use of the coherent pumping technique (see Sec. 1.3.2), the spin state of a polariton condensate can be optically controlled by tuning the excitation power, frequency and polarization of the pumping laser [34]. Such resonant spin injection technique can be readily incorporated into Eq. (1.49) as a fixed source term appearing on the right-hand side of the equation [34].

1.5.2 TE-TM splitting

Another type of cross-spin interaction is TE-TM splitting. In microcavities, there exists energy splitting between waveguide modes whose electric field is perpendicular to the propagation direction (TE mode) and modes whose magnetic field is perpendicular to the propagation direction (TM mode). Depending on the detuning, the TE-TM splitting for polaritons can reach 1 meV, which is an order of magnitude of the bare-exciton longitudinal-transverse (LT) splitting [38] given by the energy splitting between exciton states with dipole moments parallel and perpendicular to the in-plane wave vector [39]. In this thesis, we do not study the TE-TM splitting and the LT splitting in detail but describe their effects phenomenologically by an effective magnetic field whose direction changes with the azimuthal angle [40], which is shown in Fig. 1.6. Depending on the sign of the TE-TM splitting, the effective magnetic field rotates clockwise for the upper branch polariton and counterclockwise for the lower branch polariton. The difference in signs comes from different exciton oscillator strengths causing different Rabi splittings in TE and TM polarizations [41]. The effective magnetic field can induce precession of the polariton pseudospin, resulting in the optical spin-Hall effect observed in a polariton system pumped by a circularly polarized beam [42].

![Figure 1.6: Effective magnetic field caused by TE-TM splitting (figure taken from [40]).](image)

In describing the effect of the effective magnetic field, it is convenient to introduce
the pseudospin vector which is equivalent to the Stokes vector in optics [43–45]

\[ S = S_x e_x + S_y e_y + S_z e_z \]  

(1.50)

where \( e_{x,y,z} \) is the unit vector of a Cartesian coordinate and

\[ S_x = \text{Re}(\psi^* \psi_-), \quad S_y = \text{Im}(\psi^* \psi_-), \quad S_z = \frac{1}{2}(|\psi_+|^2 - |\psi_-|^2). \]  

(1.51)

By changing its pointing direction, the pseudospin vector can map the polarization of polariton states onto the surface of a unit sphere, the so called Poincaré sphere (Fig. 1.7) where the positive and negative intersection points on the x-axis represent the horizontal and the vertical linearly polarized states respectively; the upper and lower poles correspond to the right-handed and the left-handed circularly polarized states; finally, a general point on the sphere corresponds to an elliptically polarized state.

![Figure 1.7: Poincaré sphere.](image)

Following the heuristic formalism presented in Sec. 1.3.3, in the circular polarization basis, the mean-field dynamic equation for the wave function \( \Psi = [\psi_+, \psi_-]^T \) can be derived by applying the variational method to the Hamiltonian density [20,23]:

\[ \mathcal{H} = \mathcal{H}_c + \mathcal{H}_d + \mathcal{H}_{LT}, \]

\[ \mathcal{H}_c = \Psi^\dagger \hat{p}^2 \Psi + \left( \frac{u_a}{4} - \frac{u_b}{2} \right) (\Psi^\dagger \Psi)^2 + \left( \frac{u_a}{4} + \frac{u_b}{2} \right) (\Psi^\dagger \sigma_z \Psi)^2 + g_R \Psi^\dagger \hat{n} \Psi, \]

\[ \mathcal{H}_d = \frac{i}{2} \Psi^\dagger (\hat{\gamma}_c - \hat{n}) \Psi, \quad \text{where} \quad \hat{n} = \begin{bmatrix} n_+ & 0 \\ 0 & n_- \end{bmatrix} \]

\[ \mathcal{H}_{LT} = \Psi^\dagger (\sigma \cdot \Omega_k) \Psi. \]

(1.52)

Here \( \mathcal{H} \) is the Hamiltonian density for the polariton condensate, \( \mathcal{H}_c \) is the conservative part, \( \mathcal{H}_d \) is the dissipative part, and \( \mathcal{H}_{LT} \) is the TE-TM splitting term; \( \hat{p} \) is the momentum operator; \( u_a \) and \( u_b \) represent the same and cross spin s-wave scattering interaction in the polariton condensate; \( n_\pm \) is the spin-dependent reservoir
1.5 Polariton condensates with spin degrees of freedom

Density [42]; \( \sigma \) is the Pauli vector whose components \( \sigma_{x,y,z} \) are the Pauli matrices; finally \( \Omega_k \) is the effective magnetic field introduced by the TE-TM splitting [40,42]. Equation (1.52) has been written in the units of the characteristic time \( \tau_c = \gamma_c^{-1} \), the characteristic length \( L = \sqrt{\hbar/(mc)} \) where \( m \) is the effective mass of lower polaritons near the ground state for both the TE and TM fields [46,47], and the characteristic energy \( E_u = \hbar \gamma_c [2,48] \), and thus it is nondimensionalized.

In this representation, the direction of the effective magnetic field changes by \( 4\pi \) in the \( k \) plane (the momentum plane) when the azimuthal angle changes by \( 2\pi \) around the origin. The corresponding algebraic representation is given by \( \Omega_k = (\Omega_1, \Omega_2) = \beta (k_x^2 - k_y^2, 2k_xk_y) \), where \( \beta \) is the strength of the TE-TM splitting and \( k^2 = k_x^2 + k_y^2 [40] \). In reducing the complexity of numerical simulations, the \( k^2 \) dependency in the denominator is usually replaced by a constant value in the vicinity of a given \( k \) relative to the polariton flow (this constant can be absorbed into \( \beta [46,47] \), we set it to 1 for simplicity), the so called parabolic approximation [42]. This approximation, however, is valid for condensate flows with sufficiently small in-plane momenta. The Hamiltonian density involving \( \Omega_k \) then reads

\[
H_{LT} = \Psi^\dagger (\sigma \cdot \Omega_k) \Psi = \psi_+^\dagger (\Omega_1 - i\Omega_2) \psi_- + \psi_-^\dagger (\Omega_1 + i\Omega_2) \psi_+, \tag{1.53}
\]

where in the last step the real space representation of the wave vector operator \( \mathbf{k} \to -i\nabla \) is used.

The open-dissipative GP equation can be formally obtained by applying the variational equation \( i\partial_t \Psi = \delta H / \delta \Psi^\dagger [20,49] \), which gives

\[
i\partial_t \psi_\pm = \left\{- \frac{1}{2} \nabla^2 + u_a |\psi_\pm|^2 + u_b |\psi_\pm|^2 + g_R n_\pm \right. \\
\left. + \frac{i}{2} [R n_\pm - \gamma_c] \right\} \psi_\pm + \beta (i\partial_x \pm \partial_y)^2 \psi_\pm, \tag{1.54}
\]

where the linear polarization splitting, i.e. the internal Josephson coupling effect has been neglected temporarily. Note that the variational approach is only a formal operation, since \( H \) contains a non-conservative term which violates the condition for the applicability the classical Hamilton’s principle. Theories aiming to remedy this problem has been proposed recently, see Ref. [50] for detailed discussions. Never-theless, our subsequent results are based on Eq. (1.54) and will not suffer from this ambiguous problem.

1.5.3 Physical interpretation of the linear polarization splitting

In the previous section, we have introduced a general framework for representing the cross-spin interaction as an effective magnetic field within the mean-field approximation. The same derivation can also be applied to the linear coupling \( J \). Its
Hamiltonian density is given by:

$$\mathcal{H}_J = J \Psi^\dagger \sigma_x \Psi,$$

(1.55)

where $\Psi$ is the two-component wave function and $\sigma_x$ is the Pauli matrix. Since $\mathcal{H}_J$ can be interpreted as a constant magnetic field pointing along the $x$ axis as $B_J = Je_x$, then

$$\mathcal{H}_J = \Psi^\dagger \sigma \cdot B_J \Psi = \Psi^\dagger (\sigma_x e_x + \sigma_y e_y + \sigma_z e_z) \cdot J e_x \Psi = J \Psi^\dagger \sigma_x \Psi,$$

(1.56)

where $e_{x,y,z}$ are the unit vectors of the Cartesian coordinate system. As in classical electrodynamics, when a (pseudo)spin is subjected to this constant magnetic field, it may align or undergo precession against it, depending on the value of $J$. The effect of $B_J$ can be summarized in Fig. 1.8.

![Figure 1.8: Effects of $B_J$ (red arrow) on the pseudospin vector (black arrow), assuming the pseudospin vector is initially perpendicular to $B_J$. (a) Large $J$, the spin vector tend to align with $B_J$. (b) Medium $J$, the spin vector precesses around $B_J$. (c) Small $J$, little effect on the spin vector.](image)

### 1.6 Experimental pumping and trapping techniques

In this section, we shall introduce briefly some common spatially inhomogeneous optical pumping configurations for generating and replenishing the polariton condensates. Recent progress on fabricated trapping potentials for polariton condensates will also be discussed.

#### 1.6.1 Gaussian pump

A Gaussian beam at the waist position ($z = 0$) can be described as

$$u(x, y) = A \exp \left\{ - \left[ \frac{(x - x_0)^2}{2\sigma_x^2} + \frac{(y - y_0)^2}{2\sigma_y^2} \right] \right\},$$

(1.57)

where $A$ is the amplitude, $(x_0, y_0)$ is the center of the pump spot, $\sigma_{x,y}$ controls the width of the beam along the $x$ and the $y$ direction respectively. For incoherent pumping, only the intensity of the beam $I = |u|^2$ will enter into the GP equation as
the rate of injection of the reservoir polaritons $P(x, y)$. Figure 1.9 shows examples of the intensity distribution of Gaussian beams with different values of $\sigma_{x,y}$. In experiments, the pump beam might not be perpendicular to the sample, leading to the elongated Gaussian distribution shown in Fig. 1.9(b). The elongated Gaussian beam can also be shaped by using cylindrical lens [51]. If the reservoir and the condensate are spin-polarized (see Chap. 3-5), one should consider separately the pumping beam for the $\pm$ components.

1.6.2 Laguerre-Gaussian pump

The Laguerre-Gaussian (LG) beam is given by higher-order mode solutions of the paraxial Helmholtz equation with circularly symmetry. Its expression is [52]

$$u(r, \phi, z) = \frac{A}{w(z)} \left[ \frac{r}{w(z)} \right]^{\frac{|l|}{2}} \exp \left[ -\frac{r^2}{w(z)^2} \right] L_{l_p}^{[l]} \left[ \frac{2r^2}{w(z)^2} \right] \times \exp \left[ -ik \frac{r^2}{2R(z)} \right] \exp(\text{i}l\phi) \exp(-\text{i}kz) \exp(\text{i}\psi(z)), \tag{1.58}$$

where $r, \phi, z$ are the cylindrical coordinate variables; $A$ is a normalization constant; $\psi(z) = (|l| + 2p) \arctan(z/z_R)$ is the Gouy phase; $L_{l_p}^{[l]}$ are the generalized Laguerre polynomials; $w(z)$ and $R(z)$ are functions defined as

$$w(z) = w_0 \sqrt{1 + \left( \frac{z}{z_R} \right)^2}, \quad z_R = \frac{\pi w_0^2}{\lambda}, \quad R(z) = z \left[ 1 + \left( \frac{zR}{z} \right)^2 \right]. \tag{1.59}$$

An example of the intensity and phase distribution of a LG beam (at $z = 0$) can be seen in Fig.1.10.

The effect of the azimuthal mode number $l$, in addition to affecting the generalized Laguerre polynomial $L_{l_p}^{[l]}$, is mainly contained in the phase factor $\exp(\text{i}l\phi)$. 

---

**Figure 1.9:** Spatial distribution of intensity of a Gaussian pump given by Eq. (1.57). (a) $\sigma_x = \sigma_y = 30$. (b) $\sigma_x = 30$ and $\sigma_y = 20$. In the unit of $\mu$m.
When the beam profile is advanced (or retarded) by $l$, it completes a $2\pi$ phase rotation. This is an example of an optical vortex of topological charge $l$ [53], and can be associated with the orbital angular momentum of light in that mode. The positive integer parameter $p$ controls $p + 1$ intensity dips (nodes) in the radial direction, as shown in Fig. 1.10(c-d).

Under the coherent pumping scheme, the orbital angular momentum carried by the LG beam can be directly transferred to a polariton BEC, as shown by the coherent vortex imprinting method in [54]. Under the incoherent pumping scheme, however, the phase information of the pump will be washed out so that only the intensity of the beam will be of concern.

1.6.3 Optical trapping potentials

As discussed in Sec. 1.3, the repulsive interaction between a polariton condensate and its reservoir creates an effective trapping potential whose height is proportional to the strength of the pump [55,56]. Therefore, one can control the coherent properties of a polariton condensate by the proper arrangement of several isolated pump spots provided that they are in close proximity of each other and are able to create phase-locked condensates [57,58]. For example, Fig. 1.11 demonstrates a vortex generation technique based on the chiral pump spot configuration [1]. The optical trapping
Experimental pumping and trapping techniques

1.6.4 Structural and mechanical trapping potentials

Another trapping technique for polaritons relies on confining their photonic or excitonic components via engineered microstructures. Without any external in-plane trapping potential, the size of the polariton BEC is determined mainly by the size of the pumping laser. In comparison, the size of a trapped polariton BEC can be larger (e.g. the Bloch mode in a periodic potential) or smaller (e.g. trapped state in a microstructured trapping potential) than the size of the pumping spot.

With the presence of an external trapping potential, polaritons can condense spontaneously into non-ground energy states [59, 60]. With the help of exciton-exciton and polariton-polariton scattering, polaritons in the reservoir can be scattered into any one of the excited energy states near the bottom of the dispersion curve [59]. To describe properly the relaxation process toward the ground state, one should incorporate energy relaxation mechanisms such as the phonon-assisted relaxation [61, 62] into the open-dissipative GP equation. However, within the scope of current thesis, we will not delve into the energy relaxation processes. We will only mention briefly their effects in Chap. 6.

A rather versatile method of creating polariton traps is based on modulating the cavity layer thickness and transmission with a thin metal mask deposited on the surface of the microcavity, as shown in Fig. 1.12. This would form a shallow square well potential for a polariton condensate. Micropillars can also be etched in a microcavity to provide deep trapping potential for polaritons. An example of pillars is shown in Fig. 1.13.

A polariton trap with a relatively larger area has been formed by mechanically
**Figure 1.12:** A polariton trap created by metal mask deposition on top of the DBR mirror. Shown is figure taken from [63].

**Figure 1.13:** Example of an engineered micropillar for polariton BEC systems (figure taken from [64]).
stressing a planar microcavity wafer. The mechanical stress introduces a strain field which acts as a harmonic potential on the QW excitons and thus on the polaritons as well. A potential up to about 5 meV deep was created by a stressing tip of about 50 $\mu$m in radius. LPs excited away from the trap center drifted over more than 50 $\mu$m into the trap. Condensation of LPs in the trap was also observed, with a threshold density lower than that required to condense without the trap and with a contracted spatial profile. An example of the stress trap is shown in Fig. 1.14.

A detailed review of the existing trapping techniques and modelings can be found in [65].

1.6.5 Periodic lattice potentials

As discussed in [65], a polariton trapping potential can be engineered by introducing a local elongation of the microcavity so that the mesa region acts as an attractive potential for photons. The confinement depth can be tuned by adjusting the height of the mesa and the light-matter detuning, and the inter-site coupling between neighbouring traps is controlled by the distance between them [65]. Trapping potentials created by the etch-and-overgrowth technique can be arranged in 1D (Fig. 1.15) or 2D [60] periodic arrays, creating a “superlattice” potential for exciton-polaritons. For the 1D lattice, the long-range spatial coherence can lead to the existence of extended Bloch modes even with a small-radius pumping spot. Discussions about the formation of the extended Bloch modes and polariton Talbot patterns generated by interference of the leaked polaritons can be found in Chap. 6. In optics, the Talbot effect is a near-field diffraction effect as a plane wave is incident upon a periodic diffraction grating. Based on the Huygens-Fresnel principle, superposition of periodic point sources generates an interference pattern that reflects the period of the original grating [66]. As polaritons in the linear regime (insignificant self-interaction) obey the linear Schrödinger equation, the superposition principle can also give rise to a non-trivial polariton interference, resulting in the Talbot pattern.
1.7 Motivation

Let us conclude the essential introduction into the exciton-polariton physics by briefly discussing motivation for the research presented in this Thesis. Exciton-polariton condensates, due to their controllable nonlinear interaction strength and tunable effective mass [6], are an ideal system for the investigation of superfluidity in open quantum systems. Although several experiments have been done to demonstrate the superfluidity of a polariton fluid [68,69], the true nature of open-dissipative superfluidity is still under debate [70]. Thus, a systematic theoretical/experimental approach, rather than phenomenological descriptions of isolated phenomena, is required to fully address the problem. In this thesis, we will investigate the superfluidity and nonlinearity-driven dynamics of an open-dissipative polariton BEC system with the specific aim of extending the conservative stability analysis to polariton BECs while incorporating pumping and decay mechanisms together with their spin degrees of freedom. Our theoretical research framework will complement polariton superfluidity experiments by providing testable predictions related to the control of topological excitations and numerical models of experimental results. Our results can be applied to the design of ultra-fast coherent polaritonic devices.

The stability analysis of polariton persistent currents would be the main theme of Chap. 2-4. Having the stability knowledge in mind, in Chap. 5 we will present method for generating persistent currents by making use of the TE-TM splitting effective magnetic field. Finally, in Chap. 6, we will further extend our understanding of a single polariton flow to interference between multiple polariton flows, which will in turn provide a feasible method of extrapolating the phase information of a given coherent source of polaritons.
Chapter 2

Stability of persistent flows in optically induced traps

2.1 Persistent currents in exciton-polariton condensates

As discussed in Sec. 1.4, superfluidity is an ability of a fluid to flow without friction, which has been studied in various systems including the superfluid helium [71–73], superconductivity [31], BECs of dilute atomic gases [20], and, more recently, exciton-polariton BECs in semiconductor microcavities [6,13,18,68].

One of the most important predictions of quantum hydrodynamics is the formation of persistent currents of a superfluid confined in an annular trap with an initially imposed rotation. Apart from the fundamental interest in this problem, ultra-sensitive interferometric devices based on persistent currents have been suggested [74,75]. The ability to use the Laguerre-Gaussian (LG) mode of an optical laser to trap atomic BECs and transfer orbital angular momentum (OAM) from photons to atoms [74,76,77] fuelled intensive studies of persistent currents in atomic condensates. Stability analysis [30,78–81] confirmed that persistent currents in ultracold atomic gases are metastable states with lifetimes limited only by longevity of the BEC [77]. Although these theoretical studies agree with experiments, their scope is limited to conservative systems at thermal equilibrium.

Applicability of the existing theories to the novel quantum fluids formed by exciton-polariton condensates is questionable. As discussed in Sec. 1.3, regardless of the excitation scheme and in contrast to ultracold atomic gases, an exciton-polariton BEC is an intrinsically non-equilibrium system because of the pumping and radiative decay of polaritons. With the rapid growth of interest in persistent flows of open-dissipative condensates [1,82–84], the urgent open question is how the intrinsic gain and loss would affect their stability.

In this chapter, we address this problem by constructing a comprehensive theory of polariton condensates with non-zero OAM supported by an \textit{optically induced annular confinement}. We focus on the incoherent, off-resonant pumping scheme which offers the possibility to engineer a trapping potential landscape by shaping the optical pump beam (see Sec. 1.3.1) [1,55–57,85] and ensures that the condensate’s phase evolution is not driven by the pump. From previous research, it has been argued that dynamical instability is the major mechanism responsible for the breakdown of a superflow of atomic BECs in optical lattices [30,86]. The dynamical instability results from the exponential growth of an initially small-amplitude per-
turbation which will introduce fragmentation to an initially smoothly distributed condensate wave function. As it will be shown in the following discussions, a polariton condensate is also subjected to this decoherent mechanism, which has been observed recently in organic microcavities [87]. We predict that persistent currents of polaritons with sufficiently high angular momenta are always prone to oscillatory dynamical instabilities where the amplitude of the perturbation keep oscillating during the exponential growth. However, in sizeable regions of the parameter space, the quantized circulation can persist almost indefinitely.

2.2 Dynamical equations

2.2.1 The model

The off-resonantly pumped polariton condensate can be described by the mean-field dissipative Gross-Pitaevskii (GP) equation for the macroscopic wave function, $\psi$, coupled to the rate equation for the density of the excitonic reservoir $n_R$, as discussed in Sec. 1.3.4:

$$\frac{i\hbar}{\partial t} \psi = \left[ -\frac{\hbar^2}{2m} \nabla^2 + g_c |\psi|^2 + g_R n_R + \frac{i\hbar}{2} (R n_R - \gamma_c) \right] \psi,$$

$$\frac{\partial n_R}{\partial t} = P - (\gamma_R + R|\psi|^2) n_R,$$  \hspace{1cm} (2.1)

where $P(r, t)$ is the pumping rate, $g_c$ and $g_R$ characterise polariton-polariton and polariton-exciton interactions, respectively. The relaxation rates $\gamma_c$ and $\gamma_R$ quantify the finite lifetime of condensed polaritons and the reservoir, respectively. The stimulated scattering rate, $R$, controls growth of the condensate density.

The mean-field model (2.1) is phenomenological, and cannot properly describe the condensation dynamics because it is not frequency selective [62]. Nevertheless, here we focus on the problem of stability of a formed condensate with certain angular momentum, and therefore the assumption of a macroscopic polariton occupation in a condensed state is appropriate. Furthermore, the model conforms to the assumption that the far-off-resonant, incoherent optical cw excitation prohibits transfer of the pump phase to the condensate due to the accompanying phonon and exciton relaxation processes.

In what follows, we will consider the dimensionless form of Eq. (2.1) obtained by introducing the characteristic time $T = \gamma_c^{-1}$, length $L = \sqrt{\hbar/(M\gamma_c)}$, and energy $E = \hbar \gamma_c$ scales.

Optical trapping techniques [55–57, 82–85] rely on effective trapping potentials for polaritons created due to polariton flows and interaction with the reservoir. In the spirit of this approach, the annular condensate can be supported by the LG pump beam, with the LG mode intensity defining the spatial distribution of the condensed state. The OAM carried by the LG beam will not be transferred to the condensate because it is replenished from the incoherent excitonic reservoir, which “scrambles” the phase. Within the homogeneous approximation, the threshold of pumping rate to build up a non-zero condensate density is $P_{th} = \gamma_R \gamma_c/R$ [25].

In this chapter we will use physical values that are reasonably close to those
§2.2 Dynamical equations

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Figure 2.1: Persistent currents in an optically induced annular trap. (a) Schematics of the radial structure of a LG pump beam; Steady state (b) density and (d) phase of the condensate for \( l = 2 \); (c) Radial profiles of the steady states with \( l = 0, 2, 10 \) supported by the LG\(_{50}\) mode.

measured for AlGaAs microcavity polaritons [88,89]: \( g_c = 6 \times 10^{-3} \text{ meV}\mu\text{m}^2 \), \( g_R = 2g_c \), \( \gamma_c = 0.33 \text{ ps}^{-1} \), and \( R = 0.01 \text{ \mu m}^2 \text{ ps}^{-1} \). The lower polariton effective mass entering our model is related to the mass of free electron as: \( m = 10^{-4}m_e \). The condensate nonlinear interaction coefficient \( g_c \) is taken from the two-dimensional expression for the QW exciton coupling-constant [90]; the value of \( g_R \) is chosen to match the experimentally measured energy blueshift \( \sim 1 \text{ meV} \) [88]; the polariton lifetime can span between \( 2 \sim 3 \text{ ps} \) in low-Q samples and potentially be \( 10 \sim 100 \text{ ps} \) in high-Q samples [6]; the stimulated scattering coefficient \( R \) can be roughly estimated by the threshold pumping power \( P_{th} = \gamma_R \gamma_c / R \) [25].

2.2.2 Steady currents

Equations (2.1) with the radially symmetric pump, \( P(r) \), admit steady states of the form: \( \psi = \Psi(r, \theta) \exp(il\theta) \exp(-i\mu\tau) \), where \( \mu \) is the energy (chemical potential) of the steady state, \( (r, \theta) \) are the polar coordinates, and \( l \) is the phase winding number [91] (topological charge of a vortex) with the ground state corresponding to \( l = 0 \). Such steady states can be found by solving (2.1) numerically, with the initially imposed vorticity \( \psi(0) = Ar^{|l|} \exp(il\theta) \), and some examples for \( l = 0 \) and \( l \neq 0 \) are presented in Fig. 2.1. Remarkably, the radial profiles of the condensate show an extremely weak dependence on \( l \) [Fig. 2.1 (c)]. In experiment, the initial vorticity can be imprinted, e.g. by a pulsed resonant transfer of the orbital angular momentum onto an established \( l = 0 \) state [54].

Within the pump area, quantised superfluid flows are supported purely by the
Stability of persistent flows in optically induced traps

balance of gain and loss, and therefore resemble dissipative vortex solitons in a focusing optical media [92]. At the same time, the steady state maintains inward and outward polariton flows outside the pump area, which creates an effective trapping potential in the radial direction [48].

In numerical simulations, the steady states are characterised by the conserved real part of the full energy functional, $E_0$ [see Eq. (1.18)], and angular momentum, $L_z$:

$$E_0 = \int \left[ \frac{1}{2} |\nabla \psi|^2 + g_R n_R |\psi|^2 + \frac{g_c}{2} |\psi|^4 \right] r dr d\theta,$$

(2.2)

$$L_z = \frac{i}{2} \int \left( \psi^* \frac{\partial \psi}{\partial \theta} - \psi \frac{\partial \psi^*}{\partial \theta} \right) r dr d\theta.$$

(2.3)

For a steady state with azimuthally homogeneous density, the normalised angular momentum is equal to the topological charge of the vortex: $L_z/\int |\psi|^2 r dr d\theta = l$.

When the radius of the LG beam is much larger than the width of the annulus, i.e. $r_0 \gg a$, one can separate the radial and azimuthal dependence of the condensate wave function [78, 81, 93, 94] and derive a one-dimensional model, which was shown to agree with its higher-dimensional counterparts in the conservative case [79]. To this end, we set $\psi(r, \theta) = \Phi_0(r) \psi(\theta, t)$ [95], where $r \in [r_0 - a, r_0 + a]$ and $\Phi_0(r)$ is assumed to take a constant value over the width of the ring, $2a$. Substituting this ansatz into our model, and integrating out the radial dependence, we arrive at the reduced 1D model:

$$i \frac{\partial \psi(\theta, t)}{\partial t} = \left\{ -\frac{1}{2r_0^2} \frac{\partial^2}{\partial \theta^2} + g_c n_c^0 |\psi(\theta, t)|^2 + g_R n_R(\theta, t) 
+ \frac{i}{2} [Rn_R(\theta, t) - \gamma_c] \right\} \psi(\theta, t),$$

(2.4)

$$\frac{\partial n_R(\theta, t)}{\partial t} = P(\theta) - (\gamma_R + Rn_c^0 |\psi(\theta, t)|^2) n_R(\theta, t),$$

where, assuming our normalisation, $\gamma_c = 1$.

For a steady state, which has homogeneous density in the azimuthal direction, gain balances loss: $Rn_R^0 = \gamma_c$, were $n_R^0$ is the steady state reservoir density. The chemical potential of the stationary condensate with the azimuthal wave function $\psi(\theta, t) = \psi_0^e = \exp(il\theta) \exp(-i\mu t)$ is given by $\mu = p^2/(2r_0^2) + g_c n_c^0 + g_R n_R^0$, where $n_c^0 = \Phi_0^2 = \gamma_R(\overline{P} - 1)/R$ is the condensate density, and $\overline{P} = P/P_{th}$.

2.3 Bogoliubov-de-Gennes stability analysis

The stability of the steady states with non-zero angular momentum can be analysed following the standard Bogoliubov-de Gennes (BdG) approach (see Sec. 1.3.5) [20], by calculating the spectrum of the elementary excitations of the condensate and the reservoir in our reduced one-dimensional model: $\psi(\theta, t) = \psi_0^e(1 + \delta \psi)$, and $n_R(t) = n_R^0 + \delta n_R$. The excitations of the steady state and its reservoir are introduced in the form [25, 62]:

$$\delta \psi = u_0 e^{ik\theta - i\omega t} + v_0^* e^{-ik\theta + i\omega t},$$

$$\delta n = u_0 e^{ik\theta - i\omega t} + v_0^* e^{-ik\theta + i\omega t}.$$
§2.3 Bogoliubov-de-Gennes stability analysis

Figure 2.2: Dispersion curves \( \text{Re}[\omega(k)] = \Omega(k) \) and \( \text{Im}[\omega(k)] = \Gamma(k) \) for fixed \( \bar{P} = 2.5 \) and \( l = 60 \), where \( k \) is the azimuthal wave vector defined in Eq. (2.5). Red and blue curves correspond to condensate modes. Green curve corresponds to the reservoir mode. (a) \( P_0 < \gamma_c/\gamma_R \), (b) \( P_0 = \gamma_c/\gamma_R \), (c) \( P_0 > \gamma_c/\gamma_R \). Insets show corresponding dispersion for \( l = 0 \).

Inserting \( \psi(\theta) \) and \( n_R \) into Eq. (2.4), and keeping only linear terms, we obtain the BdG equations:

\[
\mathcal{L}_m(k) \mathcal{U} = \omega \mathcal{U}, \quad \mathcal{U} = (u_0, v_0, w_0)^T,
\]

\[
\mathcal{L}_m(k) = \begin{bmatrix}
(k_+ + g_c n_c^0) & g_c n_c^0 & (g_R + i \frac{1}{2} R) \\
-g_c n_c^0 & (k_+ - g_c n_c^0) & -(g_R - i \frac{1}{2} R) \\
-i \gamma_c n_c^0 & -i \gamma_c n_c^0 & -i(\gamma_R + Rn_c^0)
\end{bmatrix}.
\]

Here \( k_\pm = \pm (1/2)(\bar{k}^2 \pm 2\bar{k}\bar{l}) \) and \( \{\bar{k}, \bar{l}\} = \{k/r_0, l/r_0\} \).

The spectrum of elementary excitations for \( l = 0 \) is well known [13,25,28,96,97]. For \( l \neq 0 \), the dispersion relation given by the BdG equations is:

\[
\omega^3 - \omega^2(2\bar{k}\bar{l} - i\gamma_R) - \omega(\omega_B^2 - \bar{k}^2\bar{l}^2 + R\bar{\gamma}_c - 2i\bar{\gamma}_R\bar{k}\bar{l}) - f(\bar{k}, \bar{l}) = 0,
\]

where \( f(\bar{k}, \bar{l}) = \bar{\gamma}_c(i g_R \bar{k}^2 + R\bar{k}\bar{l}) + i\bar{\gamma}_R(\omega_B^2 - \bar{k}^2\bar{l}^2) \), \( \omega_B^2 = \bar{k}^4/4 + g_c \Phi_0^2 \bar{k}^2 \) is the standard Bogoliubov dispersion, and we introduced the shorthand notations: \( \bar{\gamma}_c = \gamma_c \Phi_0^2 = P_{th}(\bar{P} - 1) \) and \( \bar{\gamma}_R = \gamma_R + R\Phi_0^2 = \gamma_R \bar{P} \).
At $k = 0$, the real part of the excitation frequency $\omega(0) = \Omega(0) + i\Gamma(0)$ is found as $\Omega^2(0) = R\tilde{\gamma}_c - \tilde{\gamma}_R^2/4$. Consequently, it turns to zero for a critical pumping power $P_0 \equiv \bar{P}^2/[4(\bar{P} - 1)] = \gamma_c/\gamma_R$, and the spectrum near $k = 0$ resembles the Bogoliubov dispersion (see Sec. 1.3.5). For $P_0 < \gamma_c/\gamma_R$, the Goldstone mode ($\omega = 0$) at $k = 0$ is separated from the non-zero mode by a gap of the size $\Omega^2(0) > 0$ [98]. For $P_0 > \gamma_c/\gamma_R$, $\Omega^2(0) < 0$ and the excitations exhibit a diffusive behaviour near $k = 0$ [25, 96, 99]. The gapped and diffusive characters of the excitation spectra can be linked, respectively, to the underdamped and overdamped oscillations of the reservoir discussed in [98]. Fig. 2.2 shows typical dispersion curves for the $l \neq 0$ in the gapped (a) and diffusive (c) regimes and the marginal case $P_0 = \gamma_c/\gamma_R$ [Fig. 2.2(b)]. In what follows, without loss of generality, we assume the condition $l > 0$.

![Figure 2.3](image)

**Figure 2.3**: Critical values of $k_1^*$ and $k_2^*$ defining the modulational instability domains (shaded) in the regimes (a) I and (b) II corresponding to Fig. 2.2 (a) and (c), respectively. (c) Phase diagram of $l_{ds}/r_0$. Dashed line, $\bar{P} = (g_R/g_c)(\gamma_c/\gamma_R)$, separates the MI regimes I and II. Solid line is given by $P_0 = \gamma_c/\gamma_R$ (see text); (d) Stability domains in the $l$ vs $r_0$ plane. Dots correspond to the 2D numerical simulations, the boundary (dashed), $l_{ds}$, of the dynamically stable (shaded) region is given by the 1D theory.

When the imaginary part of the excitation frequency becomes positive, $\Gamma(k) > 0$, for an interval $k_2^* < k < k_1^*$, the corresponding steady state experiences modulational (dynamical) instability (MI). Although the rotational symmetry of the flow is preserved [100], its instability stems from the open-dissipative nature of the polariton system [25]. As seen from Fig. 2.2, for $l \neq 0$ the corresponding real part of the excitation frequency is always non-zero, $\Omega(k) \neq 0$, which indicates the oscillatory nature...
of the instability. The polariton current exhibits MI only above certain critical OAM which is defined as \( l_{ds} \). It is given by \( \Gamma(k) \) crossing into the positive half-plane, at which point \( k_1^* = k_2^* \neq 0 \). Two regimes of instability can be identified:

**Regime I** corresponds to \( l_{ds} = 0 \) and is defined by the condition \( \bar{P} < (g_R/g_c)(\gamma_c/\gamma_R) \) [97]. In this regime, the ground state \( l = 0 \) is modulationally unstable, and \( k_2^* = 0 \), as shown in the inset on the right panel of Fig. 2.2(a) and in Fig. 2.3(a). The real part of the corresponding excitation frequency is zero, \( \Omega(k) = 0 \), so that perturbations of the \( l = 0 \) state grow exponentially and lead to fragmentation of the azimuthally homogeneous steady state. In this parameter range, due to the saturable nature of gain in this system, the effective nonlinearity becomes attractive for low condensate densities [97]. As seen from Fig. 2.3(c) (below dashed line), this regime mostly overlaps with the gapped domain of the excitation spectra (below the solid line). Physically, this behaviour appears to be most relevant near the condensation threshold, \( \bar{P} \sim 1 \) due to the long lifetimes of the reservoir compared to condensate polaritons \( \gamma_R/\gamma_c < 1 \).

**Regime II** corresponds to \( l_{ds} > 0 \) and \( \bar{P} > (g_R/g_c)(\gamma_c/\gamma_R) \) [Fig. 2.3(c), above dashed line]. In this regime the ground state \( l = 0 \) is dynamically stable, and the 1D theory predicts dynamical stability of the flow against azimuthal density modulations up to reasonably high values of \( l_{ds} \) [Fig. 2.3(b,c)].

### 2.4 Comparison to simulation results

Numerical simulations of the full 2D model (2.1) with a weak, incoherent perturbation applied to the steady current, show remarkable agreement with the predictions of the 1D stability theory. Indeed, in the regime II, for \( l > l_{ds} \), the initial stage of the instability development manifests in oscillating and rapidly growing density perturbations [Fig. 2.4(d)], whereby the condensate fragments [Fig. 2.4(b)]. Fluctuations around the steady state grow without the formation of surface modes [101,102], confirming the validity of our 1D approximation. During the long-term, nonlinear stage of instability development, the azimuthal flow “heals” [Fig. 2.4(c)], and the system attains a new, dynamically stable steady state [Fig. 2.4(a)]. Fig. 2.4 shows a typical scenario of the oscillatory instability development causing the system to enter a steady state with a reduced OAM and energy.

In contrast, in the regime I, where \( l_{ds} = 0 \), once the dynamical instability of the persistent current is triggered, the steady flow never recovers [Fig. 2.5]. The rate of instability-triggered decay depends on the maximum instability growth rate, \( \max(\Gamma) \), which accounts for the broad transition region from dynamically unstable to stable regime depicted in Fig. 2.3(d).

In the annular geometry, steady state transitions _can be observed only in the MI domain, \( l > l_{ds} \)._ Indeed, in this parameter domain, the modulationally unstable current undergoes fragmentation and experiences phase slips, thus enabling the condensate to reduce its OAM (see Fig. 2.4). Outside the MI domain, the exponential decay of excitations _suppresses_ the development of instability, and the vortex flow remains dynamically stable. Indeed, for \( l \ll l_{ds} \) [red dots in Fig. 2.3(d)], we do not observe decay of the persistent flow in our full 2D numerical simulations of the model.
**Figure 2.4:** Evolution of a dynamically (modulationally) unstable state with $l > l_{ds}$ in the regime II. (a) Energy and (normalised) angular momentum evolution during steady state switching triggered by the oscillatory instability; (b)-(c) density and (e)-(f) phase distribution at the (b,e) intermediate and (c,f) final stages of evolution. (d) Peak density evolution at an arbitrary point. Dashed line in (a,d) indicates introduction of a weak pulsed perturbation.

**Figure 2.5:** Evolution of a dynamically (modulationally) unstable state with $l > 0$ in the regime I. (a) Energy (2.2) and (normalised) angular momentum (2.3) evolution during decay of the $l = 7$ current triggered by oscillatory instability; (b)-(c) density and (e)-(f) phase distribution at the (b,e) intermediate and (c,f) final stages of evolution. (d) Peak density evolution at an arbitrary point. Dashed line in (a,d) indicates introduction of a weak pulsed perturbation.
The above reasoning is consistent with the previous numerical study of a single charge polariton vortex in a *wide annulus* geometry [32]. The loss of the spatial coherence of the condensate shown to be associated with the decay of the circulation [32], is a typical signature of the MI triggered by fluctuations [103].

### 2.5 Conclusions

We have analysed the dynamical (modulational) instability of the persistent currents in dissipative polariton condensates confined by all-optical annular traps and predicted that a current with an orbital angular momentum within the modulationally stable domain will persist. Above critical values of the OAM, the flows suffer from oscillatory instabilities, which leads to either dynamical switching to a new metastable steady state with a lower OAM, or to the destruction of the superfluid flow. However, the scope of these results, including the transition between steady states, should be restricted to dynamical instability phenomena (rather than being energetic instability phenomena), because it has been proven that the conservative GP equation cannot reveal energetic instability in numerics without the dissipation or relaxation term [30]. For our phenomenological polariton model, whether such limitation remains or not is unclear. Therefore, we stick to the dynamical instability interpretation. Our results, together with recent experimental development of the engineered spin-orbit-coupling structure to support OAM in polariton system [64]; and the generation of vortex arrays by using the LG beam [104], pave the way for the future utilization of persistent current in polaritonic devices.

Owing to the complexity of the open-dissipative nature of polariton systems, there exist several different phenomenological models that feature complex pumping and decay mechanisms [28]. It is therefore instructive to discuss the relevance of our results to specific features of our model.

Firstly, the influence of the reservoir on the condensate dynamics gives rise to the appearance of *regime I*, in which even the ground state with zero orbital angular momentum is modulationally unstable. As argued in Ref. [25], and supported by later studies [97,103,105], when the reservoir mode participates fully in the dynamics, the so called *hole-burning effect* will take place and lead to the fragmentation of the condensate. This can be seen in Fig. 2.2(a) where the instability is indeed given by the reservoir mode (green curves). When the $\gamma_R/\gamma_c$ ratio or the $\tilde{P}$ value is increased, however, the reservoir has little influence on the condensate dynamics. It can be seen in Fig. 2.2(c) that the reservoir mode is more (dynamically) stable than the condensate ones. In this regime, the reservoir can be regarded as indifferent to small perturbations and can be treated as static. This regime roughly corresponds to the area above both the solid line and the dashed line in the phase diagram Fig. 2.3(c). Many models [26–28,106] adopted the static reservoir assumption. The stability analysis in the framework of these models corresponds to our *regime II* or to the boundary area (Fig. 2.3(c) solid curve) where the Bogoliubov-like dispersion of elementary excitation is obtained [28].

Secondly, the stability of the persistent current is dramatically influenced by the open-dissipative nature of the system. The transition to a lower OAM steady state through phonon emission, which is typical for conservative condensates [107], may
be suppressed by the presence of the energy influx in a pumped system. When the transition between steady states occurs in the polariton system (Fig. 2.4), it is driven by the dynamically unstable modes that enable condensate fragmentation. This result can also be obtained by using models that account for energy relaxation, such as that in [106], since the dispersion curves for elementary excitations are similar to those found in our model. The observation of persistent currents in polariton systems is possible if they are dynamically stable.

Lastly, a direct observation of the existence of dynamical instability is recently confirmed by an experiment on organic microcavities [87]. Although in their setup the polariton condensate is pumped by pulsed excitations, the result seems to confirm the relevance of Wouters-Carusotto model [Eq. (2.1)] and the existence of the modulational (dynamical) instability phenomenon depicted in Fig. 2.5.

The possibility to create a polariton condensate in an optically induced annular trap has already been explored experimentally [82,84]. Provided that the coherent imprinting of the orbital angular momentum [54] can be realised for these systems, experimental tests of our predictions are feasible.
Chapter 3

Homogeneous spin dynamics

3.1 Spinor polariton condensate

As discussed in Sec. 1.5, the two-component spinor exciton-polariton condensates provide a rich resource for exploring the effects of internal (spin) degrees of freedom on the dynamics of non-equilibrium superfluids. A remarkable range of spin-related phenomena has been explored in systems with a coherent resonant optical pump, which offers the possibility to directly inject the spin state of exciton-polaritons and explore the ultrafast spin-switching dynamics [33, 34, 108, 109]. On the contrary, the non-resonant incoherent excitation responsible for the spontaneous formation of coherent exciton-polariton condensates is only beginning to be experimentally explored in the context of spin-dependent effects. The reason for this disparity is a long-held belief that the incoherent reservoir (uncondensed hot exciton-polaritons), inevitably created by the incoherent excitations far above the energy of the condensed exciton-polariton quasiparticles, lacks spin-selectivity, i.e. owing to the fast spin-relaxation processes within the reservoir, the pump polarization cannot affect the spin of the condensed polaritons. Thus, many theoretical models of an incoherently excited exciton-polariton system with spin degrees of freedom treat the reservoir as a non-polarized scalar density state [26,33].

Relatively recent experimental studies of the optical spin-Hall effect [42] employed microcavities with short exciton lifetimes and succeeded in creating a spin-polarized reservoir of high-energy “excitonic” polaritons. Such a reservoir can be controllably replenished by a pump laser of a selected polarization, thus opening up the way to create and manipulate spinor condensates of exciton-polaritons in the regime of incoherent far off-resonant optical excitation. The experiments confirming spin-polarized nature of the reservoir call for a general analysis of spin dynamics under off-resonant excitation conditions, which will assist in understanding stationary spin structures and spin waves [110–112], as well as in the design of optically-controlled transistors and switching devices based on the dynamical selection of polariton spin states [113,114].

In this chapter, we consider the exciton-polariton condensate in the regime of cw far off-resonant incoherent excitation, in the absence of external magnetic field. We analyse the non-stationary dynamics of the condensate pseudospin (polarization vector) and the relaxation dynamics towards possible steady states for both spatially homogeneous and inhomogeneous cw pump. We show that the spin-polarized incoherent reservoir allows efficient switching between different spin states of the
Homogeneous spin dynamics

polariton condensate, both in the spatially homogeneous and some inhomogeneous pumping regimes. Furthermore, we analyse perturbations of steady states by a spatially inhomogeneous optical pulse and demonstrate that heavily damped spin waves excited by the perturbations can lead to the formation of stable non-trivial spin textures. It should be noted that our general analysis, with a suitable modification of physical parameters, can be applied to a quasispin model [115] of weakly coupled, spatially separated polariton condensates, to determine the steady state populations and relative phases, as well as possible dynamical regimes (see, e.g. Ref. [116]).

3.2 Dynamical models

3.2.1 Two-component Gross-Pitaevskii equations

As established in Sec. 1.5.1, the dynamical equations for the condensate wave functions corresponding to the left $\psi_-$ and right $\psi_+$ handed circular polarization states are written as ($\sigma = \pm$) [42]:

$$i\hbar \frac{\partial \psi_\sigma}{\partial t} = \left(-\frac{\hbar^2}{2m} \nabla^2 + u_a |\psi_\sigma|^2 + u_b |\psi_-\sigma|^2 + g_R n_\sigma + i\hbar \frac{\gamma_c}{2} \left[ R n_\sigma (r) - \gamma_c \right] \right) \psi_\sigma + J \psi_-\sigma.$$

(3.1)

The equations for polariton wave functions are coupled to the equations for the “hot”, exciton-like reservoir created by the incoherent cw pump. To begin with, we will consider the simplest model of spin-dependent reservoir, which was introduced in [42] and discuss the possible effects of cross-spin stimulated scattering later:

$$\frac{\partial n_\sigma}{\partial t} = P_\sigma(r, t) - \left( \gamma_R + R |\psi_\sigma|^2 \right) n_\sigma,$$

(3.2)

where $\gamma_R$ is the decay rate of the reservoir polaritons and $P_\sigma(r, t)$ is the pumping rate. The time-scale of the short-lifetime reservoir dynamics is assumed to be comparable to that of the condensate, so that the model of static reservoir [26, 28] is no longer applicable. A rapidly decaying reservoir is a relevant assumption when the relaxation within the reservoir is rapid enough so that it is able to adiabatically follow the evolution of the condensate [25]. As mentioned in Sec. 1.5.1, the resonant spin injection technique can be readily incorporated into Eq. (3.1). By introducing the characteristic units, Eq. (3.1) can be written in the dimensionless form as discussed in Sec. 2.2.2.

In the numerical calculations, the following physical values of the parameters are chosen as experimentally accessible [26,88,89]: $m_{LP} = 10^{-4} m_e$ (where $m_e$ is the free electron mass), $u_a = 6 \times 10^{-3}$ meV $\mu m^2$, $u_b = -0.1 u_a$, $g_R = 2 u_a$, $\gamma_c = 0.33 \text{ ps}^{-1}$, $\gamma_R = 1.5 \gamma_c$, $R = 0.01 \mu m^2 \text{ ps}^{-1}$, and $J \approx 0.1 \sim 0.2$ meV. So, the dimensionless quantities are: $u_a = 7.7 \times 10^{-3}$, $u_b = -7.7 \times 10^{-4}$, $g_R = 1.5 \times 10^{-2}$, $\gamma_R = 1.5$, $R = 8.4 \times 10^{-3}$, and $J \approx 0.1 \sim 1$. This choice of parameters corresponds to an assumption of a very short lifetime of reservoir excitons, however the results do not depend on a particular value of $\gamma_R$. 
3.2.2 Homogeneous steady states

We start with the analysis of the spin-mixing dynamics for a spatially homogeneous condensate. In experiments, the quasi-homogeneous distribution of condensate density can be achieved, e.g. by a flat-top “super-Gaussian” excitation [117]. In this case, it is convenient to separate the amplitude and phase dynamics of the order parameter by using the transformation:

$$\psi_\sigma(t) = \sqrt{\rho_\sigma(t)} \exp[i\phi_\sigma(t)]$$

where $\rho_\sigma(t)$ is the density of the spinor component. The relative phase difference is defined as $\theta = \phi_- - \phi_+$. Substituting these spatially homogeneous states into Eq. (3.1), we obtain the dynamical equations for the condensate density $\rho_\sigma$, the relative phase $\theta$, and the reservoir density $n_\sigma$:

$$\dot{\rho}_\sigma = (Rn_\sigma - \gamma_c)\rho_\sigma + \sigma 2J\sqrt{\rho_\sigma\rho_{-\sigma}} \sin \theta$$

$$\dot{\theta} = \left( U - \frac{J\cos \theta}{\sqrt{\rho_+\rho_-}} \right) (\rho_+ - \rho_-) + g_R \delta n_R$$

$$\dot{n}_\sigma = P_\sigma - (\gamma_R + R\rho_\sigma)n_\sigma,$$

(3.3)

where $U = u_a - u_b$ and $\delta n_R = n_+ - n_-$. The dynamics described by Eq. (3.3) is rather universal, and would apply to the description of any two-mode polariton system with linear (Josephson-type) coupling such as condensates in a double-well potential [118] or a two-level system [119], provided that their spatial variation can be ignored.

The amplitude and phase of each component in the condensate determine the polarization of the coherent photoluminescence emitted from the cavity. We will use the pseudospin representation introduced in Sec. 1.5.2 to describe pseudospin states whose total condensate density $n_c = \rho_+ + \rho_-$ defines the length of the Stokes vector:

$$S_0^2 = S_x^2 + S_y^2 + S_z^2 = n_c^2/4.$$  

Note that both the pseudospin and the total condensate density, $S_0 = n_c/2$, are functions of time, therefore it makes sense to consider the evolution of this vector in the normalized pseudospin space, $s_i(t) = S_i(t)/S_0(t)$, i.e. on the Poincaré sphere of unit radius (cf. Fig. 1.7). Although we will use the normalized Poincaré sphere representation throughout the text, one should keep it in mind that $S_0$ can vary with time.

The stationary states of the system (3.3) define fixed points on the Poincaré sphere, and the stable ones correspond to phase-locked stationary solutions of Eq. (3.1) with a well-defined polarization state of the polaritons. Desynchronized states with oscillating densities and relative phase correspond to limit cycles of the dynamical system Eq. (3.3). From the early studies of the bosonic Josephson junction in an ultracold atomic system [120], it is known that its dynamics can be mapped to that of a superconductor Josephson junction driven by a constant bias current. For the system of Eqs. (3.3), in the limit of a stationary reservoir, this analogy is valid in a narrow parameter regime identified in [26, 32], namely when (a) scattering into the polariton condensate dominates the relaxation dynamics of the reservoir $R\rho_\pm \gg \gamma_R$, (b) the polariton interaction energy is greater than the tunnelling energy $Un_c \gg J$, and (c) populations of the two spin components are similar (quasi-linear polarization) $n_c \gg S_z$. Under these conditions, following the procedure outlined in [26], one can derive the approximate self-consistent equation
for the relative phase:

$$\ddot{\theta} + \gamma_c \dot{\theta} - 2U J n_c^0 \sin \theta = (U + g_R) \delta P,$$

(3.4)

here $\delta P = P_+ - P_-$, $n_c^0 = P/\gamma_c$, and $P = P_+ + P_-$. Analysis of the phase space of this dynamical system [121] shows that, for the equal pumping of the two spin components, i.e., linearly polarized pump, $\delta P = 0$, only phase-locked steady states exist. For an elliptically polarized pump when $|I_c| = (|\delta P|/P \gamma_c(U + g_R)/(2UJ)) > 1$, the dynamics is dominated by limit cycles. For $|I_c| < 1$ the system admits steady states determined by the condition $\sin \theta = I_c$ and corresponding to the distinct steady states of the homogeneous system for each value of the pump rate.

Although equation (3.4) is obtained under a very restrictive assumption of a nearly linear polarization of the polaritonic state, it can describe some experimental results such as in Ref. [118]. Notably, when the pump rate is large, conditions (a) and (b) can be automatically fulfilled once condition (c) is satisfied (detailed discussion can be found in Ref. [26]). However, in general, Eq. (3.4) does not apply, especially when the condensate densities are small and the interaction energy is comparable to Josephson coupling. Therefore, in what follows, we will consider the variety of the steady states and dynamical behaviour governed by Eq. (3.3) under various pumping conditions, which fall outside the regime captured by the Josephson junction model (3.4).

3.3 Linearly polarized pumping

Most commonly, the optical pump producing a polariton condensate in non-spin-resolved experiments is linearly polarized. In our model, this means that the pump is balanced: $P_+ = P_-$. Under this condition, from the first of Eq. (3.3), it follows that the synchronized state can be reached when $Rn_\sigma = \gamma_c$ and the internal Josephson current $I_J = 2J \sqrt{\rho_+ \rho_-} \sin \theta$ becomes zero. Consequently, the phases are locked to $\theta = 0$ or $\theta = \pi$ (mod $2\pi$). The two fixed points on the Poincaré sphere corresponding to these phase locked linearly polarized states are $s_1 = (1,0,0)$ (corresponding to $\theta = 0$), and $s_2 = (-1,0,0)$ (corresponding to $\theta = \pi$). The density of the steady state grows linearly with the pump rate: $\rho_\sigma = (P_{th}/\gamma_c) (P_\sigma/P_{th} - 1)$, see Fig. 3.1 (a-c), where the normalized pump rate is defined as $\bar{P}_\sigma = P_\sigma/P_{th}$ and for the linearly polarized pump particularly $\bar{P}_+ = \bar{P}_- = \bar{P}$.

Following the standard Lyapunov stability analysis [121], we deduce that, for our choice of the sign of $J$, the $\pi$ out-of-phase state $s_2$ (the so called anti-bonding state) is stable and the in-phase state $s_1$ is unstable. Direct numerical integration of Eq. (3.3) with different initial conditions also confirm that $s_2$ is stable Fig. 3.2(a-b), where the interaction constant $g_R$ takes the same value as that in Fig. 3.1. We note that this selection of the anti-bonding steady state in a weakly (linearly) coupled two-state system has also been recently confirmed in [116,122]. This effect persists even for the inhomogeneous (Gaussian) pumping [116], as shown in Fig. 3.1(d) and discussed in Sec. 3.6 below.

The other steady states arise when $\rho_\sigma > (P_{th}/\gamma_c) (P_\sigma/P_{th} - 1)$, where the sign of the inequality is the opposite for the other polarization component, see Fig. 3.1 (a-c) dashed branch 1 and 3. They are sustained by a non-zero internal Josephson...
3.3 Linearly polarized pumping

Figure 3.1: Linearly polarized pumping. (a)-(c) Stationary solutions of Eq. (3.3) for linearly polarized pump. Branches 1 and 3 are elliptically polarized states; branches 2 and 4 are bonding ($s_1$) and anti-bonding ($s_2$) linearly polarized states, respectively. Only the solid branch is stable (see text). (d) Dynamical evolution of a spatially inhomogeneous state governed by Eq. (3.1) represented by integrated Stokes parameters (see Sec. 3.6). The pump is linearly polarized ($P_0 = 5$, $P_{-0} / P_0 = 0.5$) and has a Gaussian profile with the widths $a_x = a_y = 5$. Red dot: initial state of evolution; green rectangle: final state of evolution corresponding to the marked point in (a-c). Inset: evolution of the energy functional (3.5). Here $J = 0.5$ and other parameters are given in Sec. 3.2.1.
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Figure 3.2: Linearly polarized pumping fixed points. Trajectories of the pseudospin vector obtained from numerical solutions of Eq. (3.3) for linearly polarized, spatially homogeneous pump, $\mathbf{P}_+ = \mathbf{P}_- = 1.2$ and $J = 0.5$; the interaction strength $g_R$ between the condensate and the reservoir takes the same value as in Fig. 3.1 for (a)(b); for (c) and (d) an artificially enlarged or reduced $g_R$ is used respectively. Red dots: initial states of evolution; Green rectangles: final states of evolution. Initial conditions are arbitrary. Parameters are: (a) $g_R = 1.5 \times 10^{-2}$, $\theta(0) = 0.5$; (b) $g_R = 1.5 \times 10^{-2}$, $\theta(0) = 2$; (c) Artificially enlarged $g_R = 1.5 \times 10^{-1}$; (d) Artificially reduced $g_R = 1.5 \times 10^{-3}$. Other parameters are given in Sec. 3.2.1.

current which will grow with the increasing pump rate. Lyapunov analysis reveals that they are unstable.

Introduced by the interaction between the condensate and the reservoir, the blue-shift term which is proportional to $g_R$ plays an important role in the spin dynamics. It can modify the properties of steady states when its energy is compatible with the Josephson tunneling energy. Figure 3.2(c-d) illustrates how different values of $g_R$ can affect the evolution of Stokes parameters in phase space. Here the initial conditions are chosen arbitrarily, and the trajectories are obtained by numerical integration of Eqs. (3.3). Indeed, both the positions of fixed points and their stability can change, e.g. when $g_R = 1.5 \times 10^{-3}$, both $s_2$ and $s_1$ become stable, see Fig. 3.2(d); higher value of $g_R$ will lead to the system in an elliptically polarized state, see Fig. 3.2(c). The elliptically polarized steady states formed under linearly polarized pumping conditions corresponds to the so called self-trapped states with strongly imbalanced population of the polarization components. It is a consequence of nonlinear interactions [120,123] and, has been studied in the context of a double-well polariton system both experimentally [124] and theoretically [122].
3.4 Elliptically polarized pumping

When the imbalanced polarization-selective pumping is introduced, the dependence of the density of the orthogonal spin components on the pump rate displays a hysteresis-like profile. However, it does not correspond to the typical bistability behaviour, since most of the blue curve in Fig. 3.3 corresponds to oscillatory unstable fixed points. Fig. 3.3 (a-c) shows a typical fixed point distribution against small pumping imbalance, where all unstable solution branches with at least one vanishing $\rho_\sigma$ are not shown for clarity. Stable fixed points exist in semi-circular polarized pumping region (blue solid), which will be discussed in the next subsection, and the semi-linearly polarized pumping region (red solid), where at the point $\bar{P}_- = \bar{P}_+$ the fixed point coincides with the single anti-bonding stable state $\rho_+ = \rho_-$ shown in Fig. 3.1.

One should note that the above discussion is based on the Lyapunov stability analysis of the steady states, which describes the linear stability to a long-wavelength (spatially homogeneous) perturbation with the wave vector $k = 0$. The steady state may become unstable to a spatially modulated perturbation with $k \neq 0$, see, e.g. discussions about the modulational instability in 2D [26] and 1D [125] cases discussed in Chap. 2.

As a verification of the existence of the stable branches, we performed a pulsed excitation of a steady state in Fig. 3.3 (a-c) where the red dots and green rectangles represent stable steady states before and during the pulse. They fit the inhomogeneous simulation result, Fig. 3.3 (d), qualitatively well (see Sec. 3.6 for details).

3.5 Circularly polarized pumping

The steady state shown in Fig. 3.3 (a-c) at $\bar{P}_- = 0$ corresponds to the situation when only one spin component is pumped, which is physically akin to the strongly imbalanced dissipative double-well Josephson junction [122] (the opposite case, $\bar{P}_+ = 0$, is physically identical). In this limiting case, the fixed point solutions of Eq. (3.3) are shown in Fig. 3.4, where the steady state for $\bar{P}_+ = 2.4$ is the same as that represented by the dark blue solid curve in Fig. 3.3 (a-c) at $\bar{P}_- = 0$.

Again, Fig. 3.4 (d) shows us the inhomogeneous simulation result verifying the position of fixed points predicted by the homogeneous model.

Naturally, the condensate supported by a spin-polarized reservoir created by the incoherent circularly polarized pump retains its nearly circular polarization, with the orthogonal polarization component only weakly populated by the Josephson coupling [Fig. 3.4(b)]. This dynamical behaviour of the mean-field model is consistent with the conclusion that the polariton population transfer between the two spin components is due to the linear polarization splitting [41]. In principle, spin-relaxation mechanisms for the condensed polaritons and reservoir excitons involve indirect scattering processes and momentum relaxation due to phonon scattering, and therefore should be treated within the quantum kinetics formalism [41,44]. Within the framework of our mean-field model, we can account for spin-relaxation processes by introducing phenomenological cross-spin reservoir depletion terms [26] into the model equations (3.1): $Rn_\sigma \rightarrow Ra_n + R_b n_{-\sigma}$ and $R|\psi_\sigma|^2 \rightarrow Ra|\psi_\sigma|^2 + R_b|\psi_{-\sigma}|^2$. 
Figure 3.3: Elliptically polarized pumping. (a)-(c) Selected stationary solutions of Eq. (3.3) with an elliptically polarized pump ($\tilde{P}_+ = 2.4$). Only the solid branches are stable. Red dot (green rectangle): the steady states before (during) a pulsed perturbation is added. (d) Dynamics of spatially inhomogeneous states under pulsed perturbation represented by the integrated Stokes parameters (see Sec. 3.6). Steady states are supported by a Gaussian cw pump with $\tilde{P}_0 = 4$, $P_0^- / P_0 = 0.4$, $a_x = a_y = 5$ and perturbed by a pulse with $\tilde{P}_0 = 0.12$, $P_0^- / P_0 = 1$, $a_{px} = a_{py} = 5$. Red dot: initial state of evolution; green rectangle: final state of evolution. Inset: evolution of the energy functional (3.5). Here $J = 0.5$ and other parameters are given in Sec. 3.2.1.

As long as the natural assumption $R_b \ll R_a$ holds (and in fact, up to $R_b \approx 0.4R_a$ for our parameter values), the number and the type of the steady states under the various pumping conditions considered above remain the same. In particular, the circularly polarized pump will inevitably produce a steady state with nearly circular polarization. However, for larger values of the cross-spin scattering coefficient $R_b$, the dynamics is dominated by the desynchronised states with oscillating polarization.

3.6 Switching of spin states

In the spatially inhomogeneous situation, it is well known that the in-plane magnetic fields, either being externally applied or caused by the TM-TE splitting [41], can lead to nontrivial spatial structures for both density and spin distributions [26, 40, 42]. The linear spin coupling can also be regarded as an effective magnetic field pointing along the $x$ axis, which can lead to the appearance of spin patterns. However, here we analyse steady states of the polariton spinor systems sustained by a cw
Figure 3.4: Circularly polarized pumping. (a)-(c) Stationary, spatially homogeneous solutions of Eq. (3.3) with \( \dot{P}_- = 0 \). Only the solid branch is stable (see text). (d) Dynamical evolution of a dynamically inhomogeneous state governed by Eq. (3.1) represented by integrated Stokes parameters (see Sec. 3.6). The pump is circularly polarized \( (\dot{P}_0 = 3, P_{0-} = 0) \), and has a Gaussian profile with the widths \( a_x = a_y = 5 \). Red dot: initial state of evolution; green rectangle: final state of evolution corresponding to the stable steady state indicated in (a-c). Inset: evolution of the energy functional (3.5). Here \( J = 0.5 \) and other parameters are given in Sec. 3.2.1.
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pump with a Gaussian shape and show that the spatially averaged evolution of the spin components qualitatively follows the prediction of the homogeneous model discussed in Sec. 3.2.2. The results obtained within the homogeneous approximation can therefore be used as a guide for spin-switching manipulation with a realistic, spatially inhomogeneous pump.

To determine the evolution of pseudospin under spatially inhomogeneous excitation, we solve directly the open-dissipative GP equations (3.1) with a radially symmetric Gaussian pump by using the split-step method [126]. The pump rates for each of the polarization components are defined as $P^\sigma(x, y) = P^\sigma_0 e^{-(x^2/a_x^2 + y^2/a_y^2)}$, where $a_{x,y}$ are the widths of the common Gaussian pump profile along the $x$ and the $y$ directions, and the total peak rate is: $P_0 = P^+_0 + P^-_0$. As before, the peak value of the pump rate is conveniently normalized by the homogeneous threshold value $\bar{P}_0 = P^0_0/P_{th}$, and $\delta P$ denotes $P^+_0 - P^-_0$.

By adjusting the pump rate and the degree of its circular polarization $\delta P/P_0$, the system can be driven to a synchronized steady state. In numerical simulations, the condensate mean-field energy functional, $E = E_+ + E_- + E_+$, serving as the measure of kinetic and interaction energy of the condensate [127], can be used to detect whether the steady state is reached:

$$E_\sigma = \int d^2r \left[ \frac{1}{2} |\nabla \psi_\sigma|^2 + \frac{u_a}{2} |\psi_\sigma|^4 + g R n_\sigma |\psi_\sigma|^2 + \frac{u_b}{2} |\psi_\sigma|^2 |\psi_{-\sigma}|^2 + J \text{Re}(\psi_\sigma \psi_{-\sigma}^*) \right],$$

(3.5)

where the integration is over the area where the condensate density is non-negligible.

To map the evolution of the inhomogeneous condensate to that of the Stokes parameters, we trace the evolution of the spatially integrated quantities as $s^\text{int}_i(t) = \int S_i(x, y) d^2r/S^\text{int}_0$, where $S^\text{int}_0$ is given by the spatially integrated length of the Stokes vector. The integration area is the same as that used to determine the steady state energy (3.5). In the following, unless it is indicated explicitly; we shall refer to the normalized $s^\text{int}_i(t)$ simply as integrated Stokes parameters. This normalization procedure, although eliminating information on the spatial distribution of the pseudospin, allows us to recover the spatially averaged polarization state. This averaged spin dynamics could be observable experimentally even without performing the spatially resolved polarimetry of the cavity photoluminescence.

Polarization dynamics for the linearly and circularly polarized pump are shown in Fig. 3.1(d) and Fig. 3.4(d), respectively. As we can see, the spatially averaged dynamics in the case of inhomogeneous pumping fits the homogeneous predictions well.

For a typical steady state under elliptically polarized pumping, the value of the cross section densities and spatial distribution of the pseudospin is shown in Fig. 3.5(a). Its build up process is similar to that of the linearly-polarized pumping case, see Fig. 3.5(b-c). The linear coupling $J$ causes switching of the dominant density between the two components as polaritons spread out from the pump source, which leads to the appearance of radially symmetric domains of polarization density $s_z$. The larger $J$ leads to the denser spin pattern and loss of correspondence between the averaged spin dynamics in inhomogeneous and homogeneous cases. If
§3.6 Switching of spin states

Figure 3.5: Synchronized and desynchronized states. (a)-(c) Evolution towards a synchronized steady state with $\bar{P}_0 = 4, P_0^-/P_0 = 0.4, a_x = a_y = 5$, and $J = 0.5$: (a) density profiles (along $y = 0$) and spatial distribution of $s_z$ for the steady state, (b) evolution of integrated Stokes parameters and energy functional (3.5) starting from an arbitrary small initial condition, (c) evolution of integrated Stokes parameters on the Poincaré sphere from the initial (red dot) to the final (green rectangle) steady state. (d) Evolution of integrated Stokes parameters towards a desynchronized state with $\bar{P}_0 = 8, P_0^-/P_0 = 0.4, a_x = a_y = 5$, and $J = 0.5$. Initial state is marked by the red dot the final state is a closed orbit. Inset: evolution of the mean-field energy functional (3.5) with the integration performed over the area where $n_c(x, y) > 10^{-3}$. Other parameters are given in Sec. 3.2.1.
Figure 3.6: Steady states. Steady state integrated quantities against the ratio $P_0/P_0$ for $\bar{P}_0 = 4$, $a_x = a_y = 5$, $J = 0.5$. Within the gap $|\delta P/P_0| < 0.16$, one needs to increase $\bar{P}_0$ in order to reach steady states. Other parameters are given in Sec. 3.2.1.

$J$ is sufficiently small, which is the case in the planar semiconductor microcavities, the polarization state is almost homogeneous across most of the integration area. Importantly and similarly to the case of the miscible atomic condensates [128], the two spin components always coexist in space, so that spatial spin domains do not form.

The degree of control of the averaged polarization state of the condensate attainable by an incoherent, far off-resonant spin-polarized excitation according to our model is presented in Fig. 3.6 and could be tested in an experiment, which would validate our assumptions. Due to our particular choice of the spatial pump profile, in Fig. 3.6, within the range $|\delta P/P_0| < 0.16$ one needs to increase $\bar{P}_0$ in order to reach steady states. With a larger pump rate, however, the degree of control of polarization decreases because the system will easily fall into a desynchronized state even for a small value of imbalance.

For example, for an elliptically polarized pump, within the region marked with “D” in Fig. 3.3, none of the stationary state branches are stable, which does not rule out the existence of the closed orbits (limit cycles) on the Poincaré sphere corresponding to desynchronized states. Evolution of the Stokes vector in a desynchronized state is similar to that shown in Fig. 3.5 (d), where the angle of the trajectory precession can vary depending on the system parameters. This regime is not captured by Eq. (3.4) because the latter can only describe limit cycles in the vicinity of the $s_z = 0$ plane. Indeed, it can be revealed by inhomogeneous simulations of the full model equations that, in such a state, the spatial distribution of the spin pattern keeps breathing. The corresponding Stokes parameters circle around the Poincaré sphere [129], reducing the degree of (time-averaged) polarization of its luminescence. A similar pinning and depinning effect has been observed experimentally [33]. Here this effect is associated with moving in and out of the phase-locked synchronized regime of the condensate dynamics caused by intrinsic interactions between the two spin components, so that it can arise even without taking into account the structural disorder in the sample.

The coexistence of multiple stable branches described in Sec. 3.2.2 indicates that an inhomogeneous pump might be used to switch the system between different steady states. Indeed, we find that the efficient switching between different spin and
density states can be achieved by applying a radially symmetric incoherent *pulsed excitation* of the form: 

\[
P^\sigma(r,t) = \frac{1}{4} P_0^\sigma \left\{ 1 + \tanh[\tau_p(t - t_0)] \right\} \left\{ 1 + \tanh[\tau_p(t_1 - t)] \right\} e^{-\left(\frac{x^2}{a_{px}^2} + \frac{y^2}{a_{py}^2}\right)},
\]

where \(a_{px,py}\) determines the pulse width, \(t_0\) and \(t_1\) (\(t_1 > t_0\)) are the pulse switch on and switch off time respectively, and \(\tau_p\) is a coefficient controlling adiabaticity of the excitation (large \(\tau_p\) corresponds to a non-adiabatic pulse). We assume that the combined pump intensity is given by an interference of the in-phase cw and pulsed laser beams.

**Figure 3.7:** Switching of spin states. Spin switching dynamics for the case of a Gaussian pump. (a-c) Energy of the initial and final state after (or during) the switching pulse. Dash line: pulse switched on. Dash-dot line: pulse switched off. The system can be controllably switched from one stable fixed point to another stable fixed point or a stable limit cycle. (d-f) Integrated total density and integrated Stokes parameters during the switching process for switching between the steady states shown in (c). Parameters of the cw pump and pulsed perturbation in (c) are the same as in Fig. 3.3 (d), parameters of the pulsed perturbation in (a) are: \(\bar{P}_0 = 0.4, P_0^- / P_0^+ = 0.65\); and in (b) \(\bar{P}_0 = 0.9, P_0^- / P_0^+ = 0.65\). Other parameters are given in Sec. 3.2.1.

A radially symmetric non-adiabatic pulse, \(a_{px} = a_{py}\), causes an abrupt change in the condensate energy and a strong outward density flows combined with the internal Josephson currents leading to spin mixing dynamics [Fig. 3.7(d-f)]. As a result, after an initial transient dynamics, the system can enter a different steady state [Fig. 3.7(a,c)], or a phase desynchronized state [Fig. 3.7(b)]. If the original steady state is stable, the perturbed system restores its initial density and polarization state after the pulse is switched off. We note that the transient dynamics shown in Fig. 3.7 represents time-dependence of a spatially averaged Stokes vector, and therefore does not necessarily represent a spin wave excited by the pulse [112].

Regardless of the fact that with strong spatial variation the homogeneous results would not be generally applicable, one can still compare the spatially-averaged inhomogeneous results of Fig. 3.7(d-f) with the stationary solutions of Eq. (3.3), as long as the former begins and ends with steady states. Figure 3.3 shows two of the
relevant fixed point solution branches that are qualitatively comparable to the simulation data. The initial (red dot) and final (green square) steady states correspond to the flipping of $s_x$ shown in Fig. 3.7(d) before and after the pulse was added. These results confirm that the spin switching dynamics that may be observed in experiments under spatially inhomogeneous excitation conditions can be qualitatively inferred from the structure and stability of the steady states of the corresponding homogeneous model.

### 3.7 Excitation of nontrivial spin textures

While the density and spin wave excitation in the regime of a radially-symmetric cw pump and pulse leads to the efficient switching between different spin and density states, the spatial dynamics can be captured by the homogeneous or spatial averaging approximation, as long as the radial symmetry of the spatial density distributions for both spin components is preserved. However, the situation changes dramatically when the radial symmetry of the system is broken. This can be achieved by combining a radially symmetric normal incidence pump with elongated excitation, e.g. at a steep angle to the sample surface, or vice versa. Fig. 3.8 demonstrates the consequence of pulsed perturbation of a steady state established with an elongated ($a_x = 16$, $a_y = 5$) pump. The pulse is a strong, off-resonant, tightly focused Gaussian beam with $a_{px} = a_{py} = 4$ (a-d) or $a_{px} = a_{py} = 3$ (e-h) and peak power comparable to that of the cw pump. It was set to having $\pi/2$ phase difference with the pump and thus no interference pattern appeared. Remarkably, the symmetry of the density flows caused by the pulse is now broken, leading to different flow speeds along the two symmetry axes. As a result, larger area pulse produces a phase fold associated with a pronounced dip in the condensate density Fig. 3.8 (a-c). An even tighter pulse will result in the two phase folds terminating on vortices, leading to formation of a stable and stationary configuration of two vortex-antivortex pairs.

Although nontrivial phase structures appear only in one of the spin components [see Figs. 3.8 (d), (h)], they are maintained and stabilised by the internal Josephson current between the spin components and do not exist in the absence of the linear coupling ($J = 0$). Therefore, as has been established for spinor systems of ultracold atoms [130, 131], further study of these structures in polariton condensates should be performed in the context of heavily damped spin wave excitation [112]. We also note that no phase singularities appear in the linearly-polarized basis, which sets these structures apart from the half-charge and spin-vortices in polaritonic [132–134] and atomic [135,136] condensates.

### 3.8 Conclusions

In conclusion, we have investigated in detail the formation and polarization structure of the steady states of an exciton-polariton condensate created by far off-resonant spin-polarized pump in a semiconductor microcavity with linear polarization splitting. We model the dynamics of the system under the assumption that the incoherent pump creates a fast-responding and rapidly decaying reservoir, which significantly affects the condensate dynamics. Our analysis includes effectively spatially
homogeneous pumping conditions, as well as spatially inhomogeneous pumping with a Gaussian profile. The polarization dynamics in the latter case can be mapped to the homogeneous dynamics through a spatially averaging procedure. We have demonstrated that the phase-locking conditions for the existence of stationary states naturally leads to the self-trapped states with a nontrivial phase relationship between the condensate components, including $\pi$ out-of-phase (anti-bonding) states. The existence of both phase-locked and spin-beating states can enable efficient control and switching of the polarization states in the system. Experimentally, these findings can be tested with spatially averaged polarization measurements under incoherent spin-selective excitation conditions.

In addition, we have analysed the formation of non-trivial spin-textures under pulsed, spatially inhomogeneous excitation, and demonstrated numerically the reliable creation of stable vortex-antivortex pairs.

Our analysis, with suitable modification of parameters, is widely applicable to a general two-state polariton system with linear coupling, including the weakly linked spatially separated condensates or the multi-mode condensates in shallow potential traps.
Figure 3.8: Excitation of vortex pairs for the case of an elongated elliptically polarized pump with $P_0^- / P_0 = 0.4$ and a tightly focused radially symmetric pulse with $P_0^- / P_0 = 0.65$ and different widths (see text). (a) Density profile of the $\psi_+^+$ component showing a density dip associated with a phase fold (b). (c) Cross-section of density and phase across the fold. (d) Density profile of the $\psi_-^+$ component corresponding to (a); (e) Density profile of the $\psi_+^-$ component showing stationary configuration of vortex pairs connected by a phase fold (f); (g) Cross-section of density and phase across the phase fold. (h) Density profile of the $\psi_-^-$ component corresponding to (e). All density plots are logarithmic. Other parameters are given in Sec. 3.2.1.
Chapter 4

Azimuthons and pattern formation

4.1 Beyond the homogeneous spin dynamics

Exciton-polariton condensates in optically annular confinement, which has been discussed in Chap. 2, is capable of supporting superfluid polariton currents that are of potential use for the proposed interferometry and sensing devices based on microcavity polaritons [18]. The annular polariton flow, its stability and disruption [2, 82, 83], as well as its connection to nontrivial vortices in two-component (spinor) polariton systems [84] have been vigorously investigated both experimentally and theoretically. The straightforward spin state detection method has enabled a multitude of experimental studies of half-solitons, half-vortices, spin vortices, and other non-trivial spin textures spontaneously occuring in exciton-polariton condensates [42, 84, 133, 137–139].

The majority of non-trivial spin dynamics in polariton condensates is associated with effective magnetic field induced by the momentum-dependent TE-TM energy splitting between the polariton modes (see Sec. 1.5.2) [40, 140]. However, spontaneous formation of spin patterns and non-trivial spin dynamics [3, 26] can also be caused by the asymmetry-induced, momentum-independent linear coupling whose effects have been discussed in Chap. 3.

It has been shown previously, both for the conservative GP equation, as well as the nonlinear Schrödinger equation and the Ginzburg-Landau equation, that vortex states are special cases of more general steady states called azimuthons in optics and soliton train (ST) states in atomic BEC systems. Azimuthons are solutions of a cubic nonlinear equation that are having spatial modulations in density and phase simultaneously. These states are first discussed in atomic BEC systems [141, 142] and later in nonlinear optics [95]. (In the following we will not discriminate between the azimuthon and the ST state.) Various types of azimuthons have been studied extensively [95, 143, 144] and have been observed in experiments [145, 146]. Although vortexes [2] and dark solitons [147] have been investigated for polariton systems, no systematic extension of azimuthon states for two-component polariton systems has been established yet.

In this chapter, we examine non-trivial spin states of the dynamical system describing non-equilibrium, incoherently pumped BEC of exciton-polaritons trapped by an annular potential induced by the pump. We show that this pumping configuration supports steady vortex states with azimuthally modulated density (azimuthons) which can be interpreted as Josephson vortices [148–151]. Steady rotation with THz-
range frequency associated with this states results in optical ferris wheels [152] in the cavity photoluminescence. Recently, it has been shown that two polariton condensation centers can exhibit self-induced oscillations that cover a wide range of frequency [153]. Thus, the optical ferris wheels can be viewed as spin oscillations at the THz-range being manifested in the spatial domain. We also describe the stationary pattern formation supported by nonlinear instabilities of the annular polariton flow, and show that the noise naturally present in the system due to, e.g. thermal effects, allows for spontaneous formation of vortex azimuthons. In a two-component system, if the vortex azimuthon states of each component are not fully synchronized, the linear coupling will introduce spatially modulated particle tunneling between two components, forming, Josephson vortices [148]. We will discuss this situation in detail in Sec. 4.4.

4.2 Inhomogeneous spinor polariton condensate

From Sec. 1.5.1, the mean-field dynamics of a two-component (spinor) polariton condensate can be described by the open-dissipative Gross-Pitaevskii equation coupled to the rate equations for spin-polarized reservoir of hot excitonic polaritons created and replenished by a non-resonant optical pump, see Eq. (1.49). We repeat the dynamic equation here in order to be self-contained

\[ i\partial_t \psi_\sigma = \left\{ \frac{-1}{2} \nabla^2 + u_a |\psi_\sigma|^2 + u_b |\psi_{-\sigma}|^2 + g_R n_\sigma + \frac{i}{2} [R n_\sigma - \gamma_c] \right\} \psi_\sigma + J \psi_{-\sigma}, \]

\[ \partial_t n_\sigma = P_\sigma(r) - (\gamma_R + R |\psi_\sigma|^2)n_\sigma, \] (4.1)

The anisotropic TE-TM splitting effect is assumed to be weak and thus is not taken into account [112]. We also assume that the cross-spin stimulated scattering is negligible comparing with the same-spin counterpart [26]. As shown in [3], weak cross-spin stimulated scattering will not significantly affect the spin dynamics. In experiments, the polariton condensate can either separate from the reservoir in space or not, depending on the strength of the repulsive interaction between the reservoir and the condensate. For the former case, a detailed study of its spin dynamics can be found in [114]; here we will only discuss the latter case where the condensate spreads over the whole pumping area.

Equation (4.1) is written in the dimensionless form by using the characteristic scales of time \( \tau_c = 3 \) ps, length \( L = \sqrt{\hbar/(m\gamma_c)} = 2 \) \( \mu \)m, and energy \( E_u = h\gamma_c = 0.66 \) meV. We assume parabolic dispersion approximation near the polariton ground state, where \( m \) is the effective mass of the lower polaritons. All unspecified parameters in Eq. (4.1) take the default numerical values listed at the end of this section. For these parameters, the unit of time, \( t = 1 \), used in dynamical simulations throughout this work, corresponds to 3 ps. All quantities in Figures 4.1-4.7 are plotted in dimensionless units.

Although generally the energy functional corresponding to Eq. (4.1) takes complex values, when the pumping and decay reach equilibrium there exist dynamically stable steady states whose energy functionals are strictly real [2]. This suggests that
the condensate dynamics can be approximately characterized by the real part of the energy functional [127], which remains unchanged when a steady state or a vortex azimuthon is reached, where for the former the pumping and dissipation compensate each other at every moment, whereas for the latter the same equilibrium is reached only on average.

The spatial distribution of the condensate is controlled by the pump rate $P_{\sigma}(r)$ which is proportional to the spatial intensity distribution of the pump beam. In this section, we use a LG beam to form an annular pumping configuration. The pumping power of the beam is normalized by the threshold power for polariton condensation $\bar{P}$, where $P_{\text{th}} = \gamma R \gamma_c / R$ is the pumping threshold given by the homogeneous pump approximation [25], and $P_{\text{max}}$ is the peak intensity of the LG beam. For a spinor system, the intensity of the LG beam is split into each component as $\bar{P} = \bar{P}_+ + \bar{P}_-$. We denote the polarization bias of the pump as $PL = \bar{P}_- / \bar{P}$, which represents the fraction of power in the $-$ component, e.g. for a linearly polarized pump $PL = 0$, while for a right-handed circularly polarized pump $PL = 0$.

In the following sections, all simulation results are given by solving Eq. (4.1) via the split-step method [126]. In the numerical calculations, the following values of parameters are chosen as experimentally accessible [88]: $m_{LP} = 10^{-4} m_e$ (where $m_e$ is the free electron mass), $u_a = 6 \times 10^{-3}$ meV $\mu$m$^2$, $u_b = -0.1 u_a$, $g_R = 2 u_a$, $\gamma_c = 0.33$ ps$^{-1}$, $\gamma_R = 1.5 \gamma_c$, $R = 0.01 \mu$m$^2$ ps$^{-1}$, and $J \approx 0.01 \sim 0.5$ meV [154]. So, the dimensionless quantities are: $u_a = 7.7 \times 10^{-3}$, $u_b = -7.7 \times 10^{-4}$, $g_R = 1.5 \times 10^{-2}$, $\gamma_R = 1.5$, $R = 8.4 \times 10^{-3}$, and $J \approx 0.01 \sim 0.5$. This choice of parameters corresponds to an assumption of a very short lifetime of reservoir excitons, however the results do not depend on the particular value of $\gamma_R$.

### 4.3 Vortex states

The full picture of dynamical phenomena described by Eq. (4.1) is given by the interplay between the nonlinear interactions and the linear coupling. We start the discussion by reviewing some of the existing results as limiting cases of this dynamical model and then extend our understanding to the more intricate situations.

If $J = 0$ and the cross-spin nonlinear interaction is vanishingly small, two polarization components become effectively decoupled from each other and Eq. (4.1) reduces to two sets of single-component equations. To obtain a steady state, one can require the equilibrium between pumping and decay $R n_\sigma - \gamma_c = 0$ and steady reservoir density $\partial_t n_\sigma = 0$. Together these conditions lead to $|\psi_\sigma|^2 \propto P_\sigma$, i.e. the condensate density distribution follows the intensity distribution of the pump and is therefore azimuthally homogeneous. Under a LG pump $P_\sigma$, the condensate density distribution has an annular shape that can support vortex states. The existence and stability properties of single-component vortex states have been discussed in detail in Chap. 2. In this chapter we will only consider the regime where vortex states are modulationally (dynamically) stable.

The above conclusion remains valid even in the presence of the cross-spin interaction [26]. Thus, the topological charge of vortex states for each polarization component can be different from each other. When the condensate is pumped by a
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Figure 4.1: Anti-bonding states formed under an LG$_{50}$ mode pumping. (a) Radial profiles of the condensate density and pumping rate along the dashed line in (b). (b) density and (d) phase distribution of the $+$ component. (c) Phase distribution of the $-$ component. Parameters: $\bar{P} = 6$, $PL = 0.5$, and $J = 0.5$.

linearly polarized ($PL = 0.5$) LG beam and one component acquires non-zero angular momentum, e.g. $m_+ = 1$ and $m_- = 0$, where $m$ is the topological charge [2], the system forms the so called half-vortex state [49, 132], which has been observed in experiment [133]. When viewed in the linearly polarized basis, such a state forms a rotating vortex with a $\pi$ phase jump around the azimuthal coordinate and a density dip [134], where the horizontal and vertical polarization components in the linearly polarized basis are defined, respectively, as

$$\psi_H = \frac{1}{\sqrt{2}} (\psi_+ + \psi_-) \quad \text{and} \quad \psi_V = \frac{i}{\sqrt{2}} (\psi_- - \psi_+).$$

If $J \neq 0$ (without loss of generality we assume $J > 0$ throughout this work), there exists energy splitting between two polarization components [47], which will lead to particle exchange between $\psi_+$ and $\psi_-$, the so-called Josephson currents [see Eq. (4.5)], and will introduce spin dynamics into our system [32]. Assuming that the spatial variation of the condensate can be ignored (the homogeneous approximation), the previous section has shown that, under a linearly polarized ($PL = 0.5$) pump, with sufficiently large $J$ the condensate will fall into the anti-bonding state, where the relative phase between the two components maintains a $\pi$ difference [3]. Now we model the pumping configuration with a linearly polarized LG beam whose cross-section profile is shown in Fig. 4.1(a). The condensate falls into the anti-bonding
4.4 Azimuthons and spin waves

Vortex states in the annular trap created by the optical LG pump have azimuthally homogeneous density distributions and azimuthally linear phase distributions over the pumped area. Stable ST states in a single-component polariton system under the incoherent LG pumping scheme are not possible as a result of driven-dissipative nature of the system. As mentioned above, a steady-state condensate and reservoir density distribution should be proportional to the pump rate, i.e. for an annular azimuthally homogeneous pump their density should be azimuthally homogeneous. This argument no longer holds true if the system supports Josephson vortices [148–151] that stem from internal Josephson currents between the two condensate components. In the simplest case, a Josephson vortex will introduce a sine-shape spatial distribution of Josephson current between two components [149]. If the Josephson vortex does not fully compensate the density difference between the two components, one can expect that the density modulation of a vortex state will form cnoidal waves [155] that mimic the conservative soliton state.

The particle density imbalance can be introduced by a spin-biased pump. The homogeneous spin dynamics considered in [3] dictates that, if the polarization of the pump slightly deviates from the linear one ($PL = 0.5$), the condensate will still form a steady state with a fixed relative phase which is close to but not exactly $\pi$. We denote such a state as semi-anti-bonding (SAB) state and the corresponding relative phase is denoted by $\theta_s$. The relative phase between $\psi_+$ and $\psi_-$ is defined by

$$\theta(\mathbf{r}, t) = \phi_-(\mathbf{r}, t) - \phi_+(\mathbf{r}, t),$$

where $\phi_{\pm}$ is the phase of $\psi_{\pm}$. In such a state, the Josephson current maintains the relative phase $\theta_s$ between two components throughout the whole pumped area. In an annular pumping configuration, if both components acquire non-zero angular momentum $m_{\pm}$ (not necessarily the same), the spatial flow of the condensate will lead to spatial variation of the phase in each polarization component. This variation is governed by the relation $\mathbf{v} \sim \nabla \phi$, where $\mathbf{v}$ is the local velocity of the condensate.
flow and $\phi$ is the phase, and will lead to the deviation of the relative phase between two components from $\theta_s$. The competition between the azimuthal flow governed by the nonlinear interactions within each component and the Josephson current given by the linear coupling between the two components results in cnoidal rotating waves that are very similar to that of the ST states found in atomic BEC systems.

**Figure 4.2:** Spatial distribution and time evolution of a soliton train state. (a)-(b) density and (d)-(e) phase distribution for $\psi_+$ (a) (d) and $\psi_-$ (b) (e). White arrows indicate the rotation direction of the density dips. (c) Time evolution of the energy and normalized orbital angular momentum [2]. Vertical dotted line: perturbation added time. (f) Azimuthal distribution of density and phase along the dashed line in (a). Parameters: $P = 25$, $P_L = 0.4$, $m_+ = -2$, $m_- = 1$, and $J = 0.17$.

Fig. 4.2 shows a snapshot of the ST state. In this case, the $+$ component was pumped strongly than the $-$ component, which is demonstrated by the pseudocolor representation of the polariton density. Although each component acquired different angular momentum, $m_+ = -2$ and $m_- = 1$, their densities rotated in the same direction as indicated by the white arrows. Density dips can be seen in their density distribution for both components. The number of density dips is given by the phase winding difference between the two components, and in the current case specifically $j = |m_+ - m_-| = 3$. The ST state is spatially inhomogeneous and the dimensionality reduction method used in [2] is no longer applicable. Nevertheless, the condensate can be qualitatively represented by the area pumped most strongly by the LG beam, as indicated by the white dashed ring in Fig. 4.2(a). Fig. 4.2(f) shows the density and phase distribution around the ring for both components. The Josephson current cannot fully compensate the density difference around the ring, and the azimuthal density dip distributions in the two components are complementary and are associated with the azimuthally nonlinear distribution of phase. Fig. 4.2(c) further demonstrates that the ST state is stable to a weak broadband perturbation defined in [2].
To verify cnoidal wave rotations, Fig. 4.3(a) shows time series of the condensate density recorded along the dashed ring, which demonstrates the periodic rotation around the center of the condensate, with the period at about $T_P \sim 10$ ps, at the frequency of terahertz. Fig. 4.3(b) shows instantaneous density and phase distribution for $\psi_+$ along the dashed ring, as well as plots fitted by using the expression for cnoidal waves derived in [156,157],

$$|\psi_+(\varphi)|^2 \sim \text{cn}^2(\tilde{\varphi}, k) \quad \text{and} \quad \phi_+(\varphi) \sim \Pi(\xi, \tilde{\varphi}, k),$$

(4.4)

where $\text{cn}$ and $\Pi$ are Jacobi elliptic function and incomplete elliptic integral of the third kind respectively, $\tilde{\varphi} = j K(k) (\varphi - \varphi_0) / \pi$ is the reduced azimuthal coordinate with $j$ the number of density dips, $\varphi_0$ a constant phase shift, and $K(k)$ the complete elliptic integral of the first kind, where $k \in [0,1]$, and $\xi$ are fitting parameters. In contrast to [157], instead of linear dependency, the densities of the other component are related by the elliptic relation: $(|\psi_+(\varphi)|^2)^2/a + (|\psi_-(\varphi)|^2)^2/b = 1$, where the numerical coefficients $a$ and $b$ define the axes of the ellipse (translated to the origin) shown in Fig. 4.3(c).

The phase winding number difference between two polarization components gives rise to circulating internal Josephson currents that form the Josephson vortex [148–150]. The expression of the internal Josephson current for polariton systems is given by [32]

$$I_J(r, t) = |\psi_+||\psi_-| \sin(\theta),$$

(4.5)

where $\theta$ is the relative phase defined in Eq. (4.3), and the positive value of $I_J$ indicates particle flows from the $-$ component toward the $+$ component and vice versa. Fig. 4.4(a) and (b) show the corresponding $\theta$ and $I_J$ of the ST state in Fig. 4.2. Both of them are time-dependent and rotating at the same speed as the cnodial wave. In contrast to the optical vortex azimuthon, the topological charge [158] of $I_J$ has the same value as the number of the density dips in the azimuthal density distribution, specifically, three in our case. Here we emphasize that, unlike azimuthons supported purely by nonlinear interactions [95, 143, 144], the ST states we are considering are supported by the Josephson vortex given by the Josephson (linear) coupling.

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Figure 4.4: Properties of the ST state shown in Fig. 4.2. (a) and (b) Spatial distributions of the relative phase $\theta$ and the Josephson current $I_J$. (c) and (d) Density distributions in the linearly polarized basis for the horizontal $H$ and the vertical $V$ component. Arrows indicate rotation direction.

recently in Ref. [112] for polariton systems. The spatial propagation of spin waves manifests itself in the linearly polarized basis [159]. As shown in Fig. 4.4(c) and (d), in the linearly polarized basis both $H$ and $V$ components of the ST state exhibit periodic density modulation with high contrast. The modulated densities, which rotate at the same speed as the density dips in Fig. 4.3(a), represent the optical ferris wheels [152] in the cavity photoluminescence. They might be applicable in the design of polariton spin switch [160] for ultra-fast polaritonic devices.

4.5 Emergence of fluctuating patterns

It is well-known that both self-interference effects and nonlinear instabilities in driven-dissipative systems can lead to formation of stationary and fluctuating patterns [161, 162]. In polariton systems, pattern formations has been observed in experiments, e.g. the sunflower state [163] and the self-ordered state [164]. Recently, it has been proposed that modulational instability can result in the appearance of phase defects in polariton systems [110]. As it will be shown below, similarly, instabilities introduced by the Josephson vortex can lead to pattern formation in density and phase for a polariton condensate.

With the increase of the linear coupling $J$, particle densities carried by the internal Josephson current become comparable to the azimuthal flow within each polarization component, so that they can break the azimuthal flow by perturbations in the form of long thin stripes with sharp phase gradients. Fig. 4.5 shows snapshots of
non-stationary striped states. While both components keep their overall azimuthal flows, their density distributions are cut by density stripes that either originate from the centre of the condensate (open stripe) or form a closed loop (closed stripe). Similar to the Faraday waves observed in atomic BEC systems [165–167], the stripes are oriented perpendicularly across the condensate ring.

Figure 4.5: Fluctuating states. (a)-(c) Density distribution of the less pumped component $|\psi_-|^2$ for different $J$. (d)-(f) The corresponding relative phase distribution. The white box indicates the magnification area shown in Fig. 4.6(a). Other parameters are the same as in Fig. 4.2.

The locations of those density stripes are determined by that of the Josephson vortices, which is linked to the relative phase $\theta$. Fig. 4.5(d)-(f) show the corresponding spatial distribution of $\theta(r)$. As we can see, the stripes are distributed randomly on top of a uniform phase background, which corresponds to a fixed relative phase $\theta_s$, i.e. that of the SAB state. In the transverse direction of every stripe, the relative phase has a $2\pi$ phase change which consists of two $\pi$ phase changes in both polarization components separately. Within a given polarization component, if its particle flow is represented as a path with directions, then, when the path crosses the boundary of a stripe, there will be a $\pi$ phase change for the flow. Specifically, Fig. 4.6(a) shows a magnified plot for the relative phase in the area highlighted by a white box in Fig. 4.5 and a schematic plot of a path crossing the closed stripe. There are $\pi$ phase changes with opposite sign for both components when the path passes the marked points $A$ and $B$. The overall effect is that the path has zero phase gain after crossing the boundary of a closed stripe twice. Therefore, the phase winding number for each polarization component will not change by passing through a closed stripe. In contrast, open stripes that originate from the middle of the condensate toward the outside of the pumping region such as Fig. 4.5(f), can change the phase winding number.

The Josephson vortex embodied in a stripe stems from differences in particle flows within two polarization components. Fig. 4.6(c) shows a pseudo-3D plot for the Josephson current $I_J$ corresponding to Fig. 4.6(a). The x-y plane represents the 2D
spatial distribution of $I_J$, where the darker and lighter color indicate the positive and negative value of $I_J$ respectively. Above the plane the + component is placed, while below the plane there is the − component. The dark solid arrow (above the plane) indicates the particle flow in $\psi_+$ when passing through the boundary of the stripe (straight dashed line); the gray dashed arrow (below the plane) indicates the particle flow in $\psi_-$ at the same position, where both components share approximately the same flow direction as indicated. The difference between the velocities of two flows, which in turn leads to a $2\pi$ phase change in $\theta$, results in the appearance of internal Josephson currents around the boundary of the stripe. Whereas particles tunnel from $\psi_-$ to $\psi_+$ in the darker blue area (red arrow), the direction of the tunneling flow reverses in the lighter color area (green arrows), thus forming the Josephson vortex. Unlike the ST state and many superconducting systems, where the Josephson vortex is given by tunneling between two counter-propagating flows [151], here it is given by tunneling between two co-propagating flows, and both its amplitude and position are time-dependent. With the continuous change of the spatial distribution of particle flows in $\psi_{\pm}$, the position of the closed stripe will change as a result. For the current case, Fig. 4.6(a), the closed stripe will expand or merge with another closed stripe until it disappears.

Clearly, in this system the Josephson vortex has unconventional flow properties. However, we retain the terminology because of the same physical origin, i.e. circular exchange of particle due to the linear coupling.

**Figure 4.6:** Josephson vortex. (a) Magnified relative phase plot. White arrow: schematic illustration of a path having phase changes when crossing the boundary (green dots) of a closed stripe. (c) A schematic diagram showing the flow direction of the internal Josephson current (see text). (b) and (d) density ($|\psi_-|^2$) and relative phase distribution of a steadily rotating striped state. White arrows indicate the rotation direction. Parameters are the same as in Fig. 4.2 except that $m_+ = -2$, $m_- = 2$, and $J = 0.26$. 


With the further increase of the linear coupling $J$, the effect of the SAB state is more obvious. As can be seen in Fig. 4.5, less density dips and relative phase stripes form with larger $J$, and the phase becomes more uniform across the pumped area. In Fig. 4.5(e), the number of closed stripes reduces significantly, while keeping the number of the open stripes the same (equal to the number of density dips in Fig. 4.2). It means that the overall phase winding number for each component can still be different from each other. In Fig. 4.5(f), no closed stripes exist, and the number of the open stripes is not equal to three. In this case, both components share the same phase winding number, and such a state is very close to the SAB state. The remaining open stripes are not static. They appear and collide with each other and disappear periodically.

A different set of parameters supports the existence of steady rotating stripes. Figures 4.6(b) and (d) show such an example. Four stripes rotated anti-clockwise steadily, with the period at about $t = 96$ ps. Also, the arrangements for the $2\pi$ phase change direction in $\theta$ embodied in the stripes differs from that of Fig. 4.5(f), preventing them from merging. As the linear coupling $J$ increases further, the condensate falls into the SAB state independently of the initial winding number in each component, and the relative phase is fixed at $\theta_s$ everywhere.

Distinction should be made between defects in the relative phase of the striped states and the shock line defects in frozen states which are solutions of the complex Ginzburg-Landau equation [168, 169]. The shock line defects are caused by the phase difference between two nearby vortices [168, 169], which are basically single-component phenomena for an open-dissipative system. In fact, shock lines and frozen states have been observed numerically for a single-component polariton condensate under an homogeneous incoherent pump in [110]. And it has been observed in experiment that a spiraling state called the sunflower state [163] exist under a Gaussian-shape incoherent pump, which might correspond to the frozen states after spiraling waves are established [110]. In contrast, the striped states considered here are intrinsically two-component phenomena which highly depend on the strength of the linear coupling $J$ (see Fig. 4.5).

Note that Eq. (4.1) only considers the same-spin condensate-reservoir interaction [42, 170]. If the cross-spin condensate-reservoir interaction were included, assuming that it has the same strength as the same-spin one, the interaction changes the parameter regimes of vortex states, azimuthons, and patterned states.

The competition between the nonlinear interaction and the linear coupling is ubiquitous in many multi-component nonlinear systems. Thus, the pattern formation leading to striped states could be applicable to other dissipative nonlinear systems such as atomic BECs and nonlinear optics.

### 4.6 Spontaneous vortex generation

In the above two sections, the orbital angular momentum of each component was imprinted by an external coherent LG pulse in the initial stage of the condensate evolution towards a steady state. While ensuring the controlled generation of angular momentum, this coherent imprinting is not essential for obtaining non-zero OAM for a polariton condensate. In fact, each component can acquire angular momentum
Azimuthons and pattern formation

independently starting from white noise in the process of mode selection governed by the specific spatial configuration of the incoherent pump. In this section, we will show the process of ST state generation from white noise. And the final state will exhibit asymmetric density distribution due to the random capture of the initially formed vortices.

We start discussing generally the condensate growing process that applies to a wide range of pumping configurations. This process, which is governed by the model equations (4.1), is essentially a single-component phenomenon and can be illustrated by the example of a polariton condensate supported by a fully circularly polarized pump \((PL = 0)\). In this case the pumped polarization component, \(\psi = \psi_+\), dominates the whole dynamical process. Before the pump reaches the threshold power, the particle density \(|\psi|^2\) is zero or takes a negligibly small value. When the pumping threshold power is reached, the \(|\psi|^2 = 0\) state is no longer dynamically stable and the condensate density will grow exponentially [171, 172]. If we assume that the correlation length of \(\psi\) extends to the whole pumped region, then \(\psi\) will grow like \(\psi \sim |\psi_0| e^{i\phi_0} e^{i\omega_1 t} e^{i\omega_2 t}\), where \(\phi_0\) is the initial phase and \(\omega_{1,2}\) are the real and imaginary part of the eigenfrequency of the unstable mode, respectively [2]. This is a typical homogeneous growth scenario where the whole condensate shares the same growth rate (given by \(\omega_2\)), and the final state would inherit the angular momentum completely from \(\phi_0\) (under a radially symmetric pump). If \(\phi_0\) has no OAM, then there is none in the final state.

When noise is present in the initial state or in the driven-dissipative GP equation, the condensate will experience inhomogeneous growth and vortices will appear. A practical white noise can be generated independently at each point from a random variable \(Y = n_s X\), where the random variable \(X \sim N(0,1)\) follows the standard normal distribution [173] and \(n_s\) represents the noise strength. Depending on the noise strength, the noise in the phase of the condensate will reduce the correlation length of the condensate, and when \(n_s\) reaches a critical value, the correlation length becomes zero and thus the growth rate of the condensate will differ from point to point. Localized defects such as vortices can form during this process.

With the continuous growth of the condensate, those initial vortices will be captured or repelled by the condensate depending on the specific pumping configuration. Fig. 4.7 shows snapshots of the inhomogeneous growth of the condensate and the spontaneous formation of vortices. The condensate is seeded by a sufficiently strong white noise. As we can see from Fig. 4.7(a)-(c), vortices grow locally and are randomly captured within the pumped ring. Vortices with opposite charges annihilate and, if they do not cancel out completely, the remaining charge will be inherited by the bulk condensate as in Fig. 4.7(d). Note that in Fig. 4.7 the linear Josephson coupling \(J\) is set to support a ST state. The spontaneous formation of vorticity for each component can be regarded as independent from each other for the current pumping configuration. The resulting state is an imperfect azimuthon vortex state. The same process also applies to spontaneous formation of homogeneous vortex states and striped states.
4.7 Conclusions

In conclusion, by using a dynamical mean-field model to describe a two-component exciton-polariton condensate formed in the incoherent spin-polarized pumping regime, we have demonstrated the existence and dynamical stability of vortex azimuthons and spin patterns in an annular trapping geometry imposed by the pumping geometry. We have investigated the intrinsic connection between these nontrivial spin structures and internal Josephson currents supported by a linear polarization splitting. In experiments, the polarization splitting can be controlled by different methods, e.g. by applying stress [174], electric fields [175], or magnetic fields [45]. It has been reported that the polarization splitting can reach 0.2 meV [154], when normalized by $E_u = 0.66$ meV, it corresponds to $J = 0.3$, high enough to support azimuthons (cf. $J = 0.17$ in Fig. 4.2). Therefore, it is feasible to observe azimuthons with the existing experimental techniques. With a given value of polarization splitting, one can tune other parameters, e.g. the polarization bias of the incoherent pump, to probe different regimes. Our results are generally applicable to other open-dissipative systems in the context of atomic BECs and nonlinear optics.

If there exists TE-TM splitting in the microcavity sample, preliminary results show that azimuthons can still be stable but their density will deform into an elliptical distribution. If the total energy relaxation [176] is included, the decay rate of the condensate will increase. This effect can be compensated by enhancing the external pumping power so that our current results will not be altered. However, further research is required to fully understand the impacts of TE-TM splitting and energy relaxation to spinor polariton systems.
Figure 4.7: Inhomogeneous growth of the condensate and spontaneous formation of a vortex state. (a)-(d) Density distribution of $|\psi_+|^2$. (e)-(h) Phase distribution $\phi_+$ for the corresponding time.
Chapter 5

Incoherent generation of persistent currents in spinor exciton-polariton condensates

5.1 Bypassing the coherent vortex generation technique

In Chap. 4, we have discussed the spatially inhomogeneous azimuthon states for exciton-polariton BECs under an effective annular trap provided by an incoherent pump. In generating the azimuthon states, one can either use the coherent phase imprinting technique to set the initial angular momentum, or rely on the random capture of small-scale vortices (Fig. 4.7). Generally, among various vortex generation techniques, the coherent phase imprinting is easier, since the phase of the condensate can be partially controlled by the external laser. For a spinor polariton BEC, an experimentally demonstrated method is to fix the density and phase for one polarization component by a coherent laser, and then the cross-spin interaction given by the TE-TM splitting (Sec. 1.5.2) will fix the effective potential exerting on the other component, resulting in a charge-two vortex [177]. (Both the strengths of the TE-TM splitting and the linear splitting can be controlled by the design of the microcavity [154,178]).

For a fully incoherently pumped condensate, however, the realization of such an idea is difficult, since the pumping laser can only affect the condensate indirectly through the reservoir and cannot impart any phase information. To circumvent this difficulty, the investigation of possibilities for vortex generation without relying on any coherent control has recently begun [1, 84, 88, 110, 164]. A flexible method which utilizes multiple pumping spots can generate vortex arrays [58,179] or isolated vortices [1] in a controllable manner. Besides that, by making use of an asymmetric potential together with the microcavity that supports long-lifetime polaritons (> 100 ps) it is also possible to generate a charge-one vortex incoherently [84]. However, all the resulting vortices from the mentioned methods are small-scale whose sizes are at the scale of the healing length (a couples of µm), rendering those methods falling short of generating large-radius persistent currents.

In this chapter, we will introduce an incoherent persistent current generation method based on the TE-TM splitting designed to overcome the existing difficulties. In the absence of a frequency-selective pumping mechanism, the development of a condensate depends strongly on its initial conditions [106]. This phenomenon,
combined with an incoherently pumped LG beam, forms a polariton-condensate topology magnifier which can amplify small-amplitude singularities (vortices) into an experimentally detectable persistent current (bulk vortex state), where the seed singularities can be generated by making use of the optical spin-Hall effect \[40\] resulting from the TE-TM splitting discussed in Sec. 1.5.2. Our method can be used to generate persistent currents with oppositely rotating polarization components without any specially designed external potential \[84\] or chiral pumping spot configuration \[1\].

5.2 Dynamical equations

From Sec. 1.5.2, the open-dissipative GP equation in the present of TE-TM splitting can be written in the non-dimensionalyzed form as

\[
i \partial_t \psi_\pm = \left\{-\frac{1}{2} \nabla^2 + u_a |\psi_\pm|^2 + u_b |\psi_\mp|^2 + g_R n_\pm + \frac{i}{2} \left[R n_\pm - \gamma_c\right]\right\} \psi_\pm
+ \beta(i \partial_x \pm \partial_y)^2 \psi_\mp,
\]

\[
\partial_t n_\pm = P_\pm(r) - (\gamma R + R|\psi_\pm|^2) n_\pm,
\]

where the dynamical equation for the reservoir was included phenomenologically to describe the incoherently pumping process \[25,42\]. The internal Josephson tunneling effect is assumed to be small and therefore is neglected \[42,46,49\].

5.3 Optical spin-Hall effect

A polariton condensate pumped incoherently with a linearly-polarized Gaussian beam will end up with a Gaussian-shape linearly-polarized steady state around the pumped area in the absence of the TE-TM splitting (Chap. 3) \[3, 48\]. With the TE-TM splitting, polarization domains will form under the influence of the effective magnetic field (Sec. 1.5.2), this phenomenon is called the optical spin-Hall effect \[40\]. Fig. 5.1 shows the resulting steady state together with phase singularities in the vicinity of the highly populated area. These singularities result from precession of the pseudospin vector according to the continuously changing orientation of the effective magnetic field \[46\] and are indifferent to initial noise. Therefore, they are ideal seeds for the generation of persistent currents.

In the optical spin-Hall effect, the pump is set to be linearly polarized and the singularities are static. With an elliptically polarized pump, however, the singularities can be rotated by the effective potential exerted by the dominant polarization component to the subsidiary one. To see this, we recall that the pseudospin precession is represented by the precession of $S_k$ around the effective magnetic field $\Omega_k$ (Sec. 1.5.2). If both polarization components do not share the same $k$, e.g. $\psi_+ = \psi_+(k_1)$ and $\psi_- = \psi_-(k_2)$, then $k$ would be a function of $k_{1,2}$ denoted as $k = f(k_1, k_2)$. Suppose the polarization imbalance of the pump is so large that one polarization component dominates (e.g. Sec. 3.5), say $\psi_+(k_1)$, the orientation of the effective magnetic field will be fixed at $\Omega_k \approx \Omega_{k_1}$, and $S_k \approx S_{k_1}$ will precess around
5.3 Optical spin-Hall effect

Figure 5.1: Steady states under an incoherent linearly polarized Gaussian pump. (a),(b) Density distributions of the + and − components plotted in logarithmic scale. (c),(d) Phase distributions of the + and − components. Arrows indicate singularities. Parameters: $\bar{P} = 4, P_L = 0.5, \alpha_p = 12,$ and $\beta = 0.02.$

$\Omega_{k_1}$. And this precession will be mainly provided by the evolution of $\psi_-$. In terms of the dynamical equation (5.1), now $\psi_+$ can be treated as static $\partial_t \psi_+ \sim 0$ and the effective magnetic filed acting on $\psi_-$ can be regarded as an time-independent effective external potential

$$V_-(r) = \beta (i \partial_x - \partial_y)^2 \psi_+(r).$$

(5.2)

This effective potential is similar to the spin-orbit-coupling interaction in atomic BEC systems \[180\] except that it is a quadratic function of $k$ instead of a linear one. For the current biased pumping case, the spatial distribution of $\psi_+$ is shown in Fig. 5.2(b,e), which can be approximated by a Gaussian distribution as

$$\psi^0_+ = A e^{-r^2/2a^2} e^{-i2\pi \theta^2}.$$

(5.3)

where $A$ is the amplitude, $a$ is the width, and the trivial harmonic oscillation in time is ignored at the moment. The effective potential generated by $\psi^0_+$ is

$$V^0_- = 2A \beta \left( \frac{r^2}{a^4} e^{-r^2/2a^2} \right) e^{i(28 \theta^2 - \frac{r^2}{2a^2} - \frac{\pi}{2})}.$$

(5.4)

The spatial distribution of amplitude and phase for $V^0_-$ is shown in Fig. 5.2(a,d). Its
amplitude vanishes at the coordinate origin and peaks at $r = a\sqrt{2}$, with $4\pi$ phase change around a given loop that encloses the origin.

As discussed above, $V_0^-$ will introduce singularities to $\psi_-$ after the build-up process. At the start of the condensate build-up, both $\psi_\pm$ have small-amplitude noisy distributions of density and phase. When the biased Gaussian pump is turned on, the averaged density of $\psi_+$, being pumped stronger, will grow faster than that of $\psi_-$. The effective potential $V_-$ soon evolves into $V_0^-$ and induces two vortices in $\psi_-$ whose density is still small. Since the Gaussian-shape pumping configuration without any external trapping potential is unable to support any stable vortex solution [48], both vortices appearing in $\psi_-$ will be expelled from the high density region to the surroundings. The final state, supported by the effective potential $V_-$, consists of two singularities (vortices) that are rotating on the periphery of the high-density area, as shown in Fig. 5.2(c,f). For each of those singularities, the period of rotation around the origin is about $60\,\text{ps}$ according to our chosen parameters. The existence of the rotating singularities stemming from the TE-TM splitting is one of the main results of the current chapter.

The optical spin-Hall effect can also be explained qualitatively by the effective potential. In Fig. 5.2(c), the rotating singularities cause the density distribution of $\psi_-$ to deviate from the two-dimensional Gaussian distribution, resulting in an elongated one. If the biased Gaussian pump is set to be unbiased, i.e. linearly polarized, for a steady state the densities of both components should be at the same value, especially in the center of the pump. The effective magnetic field then generates two singularities in each component and deforms their density distributions along opposite directions. Since $S_k$ and $\Omega_k$ share the same value of $k$, the precession of $S_k$ follows the description of the optical spin-Hall effect, forming polarization
domains. The final steady state is the one shown in Fig. 5.1, with the mutual effective potentials for the two components shown in Fig. 5.3.

5.4 Persistent current generation

To generate persistent currents, in addition to the central Gaussian beam, a subsequent LG beam is turned on adiabatically. The adiabaticity results from the long response time (tens of nanoseconds) of the controlling electronic devices compared with the polariton lifetime (tens of picoseconds), hence we describe the time evolution of its normalized pumping power by a slowly-growing hyperbolic tangent function, see Fig. 5.5(b) solid curve. The width of the LG beam is large enough to overlap with both singularities, and its power is slightly larger than the Gaussian one. The spatial distribution of the overall pumping rate is shown in Fig. 5.5(a).

As it has been discussed in previous section, with a biased Gaussian pump ($PL = 0.4$), rotating singularities would appear in the weakly populated component $\psi_-$. If a linearly polarized LG beam ($PL = 0.5$) is adiabatically added to such a state, a bulk vortex state (or equivalently, persistent currents) with even density distribution along $\theta$ will form in the dominant component $\psi_+$. Fig. 5.4 shows several characteristic snapshots of this process, where the final state is essentially a persistent current supported by the LG beam [2]. The normalized angular momentum for
Incoherent generation of persistent currents in spinor exciton-polariton condensates

each polarization component is defined as

\[ L_{z\pm} = -i \int \psi_\pm^*(x \partial_y - y \partial_x) \psi_\pm \, dr \over \int |\psi_\pm|^2 \, dr, \] (5.5)

where the integration area covers most of the simulation grid. (In simulations, only the real part of Eq. (5.5) will be taken in order to eliminate numerical errors.) As can be seen in the time evolution of \( L_{z\pm} \) shown in Fig. 5.5(b), only the + component acquires angular momentum \( m = -2 \) after the density build-up process.

Figure 5.4: Vortex generation. Left (right) column: density (phase) distribution of \( \psi_+ \). From top to bottom, figures correspond to points 1-4 in Fig. 5.5.

In the process of persistent current generation, the artificial potential serves as a propelling factor for \( \psi_+ \), whereas for \( \psi_- \) it has the effect of screening. Fig. 5.6 shows data from an intermediate state corresponding to the green dot 2 in Fig. 5.5(b), with (a,c) the density and phase distribution of \( \psi_- \); and (b,d) the amplitude and phase
5.5 Conclusions

To summarise, we demonstrate the existence of rotating singularities under an imbalanced Gaussian incoherent pump. These singularities can be used as seeds for the generation of persistent currents. The same procedure can also be applied to the controlled generation of azimuthons, provided that the strengths of TE-TM splitting and the linear coupling support the existence of azimuthons. This vortex generation technique does not rely on any specially designed external potential and would be generally applicable to semiconductor microcavity with suitable TE-TM splitting strength.
Figure 5.6: Effective potentials. (a) and (c) density and phase distribution for $\psi_-$. (b) and (d) amplitude and phase distribution of the effective potential $V_-$. Data corresponds to the green dot 2 in Fig. 5.5(b). White circles with arrow indicate the rotation direction of the singularities. Dashed line circles in (a-b) indicate the maximum value of $V_-$ in real space. Dashed line circles in (c-d) indicate the phase rotation direction of $V_-$. 
Talbot effect for exciton-polaritons

6.1 From single to multiple polariton flows

Chap. 2-5 investigate systematically the stability and generation methods of circular polariton persistent currents, assuming that macroscopic coherence has been established throughout the polariton condensate. This macroscopic coherence can be manifested by the interference pattern of multiple polariton flows. To observe the interference pattern, instead of separating an existing polariton flow and then recombining it, here we adopt a different approach by spatially shaping a polariton condensate into a one-dimensional periodic source array. This arrangement is achieved by engineering an 1D mesa array into the microcavity and by changing the free polariton dispersion curve into a Bloch wave dispersion so that polaritons can spontaneously condense into nonground states near the bottom of the lower polariton branch (Sec. 1.6.4). In the direction perpendicular to the mesa array, polariton flows inherit phase modulations of the Bloch mode inside the mesa array, which will affect the Talbot pattern. Conversely, from the Talbot pattern we can deduce the phase information of a Bloch state inside a given mesa array.

6.2 Background

The Talbot effect is a manifestation of near-field (Fresnel) diffraction of a coherent plane wave incident on a periodic grating, which results in a nontrivial 2D pattern of fringes often referred to as a “Talbot carpet”. According to the Huygens-Fresnel principle, it is interpreted as interference of coherent spherical waves originating from the apertures of the grating. Nearly two centuries after the discovery of the optical Talbot effect [66], it continues to be re-discovered and re-examined in the context of matter and optical waves of various physical nature and spatial scales. Apart from the linear and nonlinear optics [181], the Talbot effect has been observed with atoms [182–184], electrons [185], X-rays [186, 187], single photons [188], and surface plasmon-polaritons (SPPs) [189–191]. The visually stunning effect is not purely of aesthetic value. Talbot interference has a deep connection with number theory and theory of quantum revivals [192, 193], and serves a range of practical purposes. Indeed, the periodic self-imaging of the source resulting from the spatial Talbot effect [194] gives rise to grating-based imaging techniques [186, 187, 195]. Various applications of temporal and spatial Talbot effects in metrology, data transmission, atomic lithography, and optical manipulation have also been suggested [181].
The common prerequisite for the observation of the spatial Talbot effect is a periodic arrangement of sources of coherent spherical waves that can propagate in the direction perpendicular to the direction of the array. In optics, this is naturally achieved by diffraction of incident light on an array of apertures [181]. However, this effect can also be reproduced by other means. For example, in surface plasmon polariton physics Talbot interference of waves from periodically arranged sources rather than diffraction of a plane wave on a periodic grating has been observed [191].

![Figure 6.1: Mesa configuration. Schematics of (a) an elliptical optical pump illuminating a 1D array of mesa traps in a microcavity and (b) polariton flows (arrows) responsible for the Talbot effect in (d). Real space images of the exciton-polariton emission (c) at low excitation power, below the condensation threshold and (d) at high excitation power, above the condensation threshold. The Talbot effect is visible in (d).](image)

Recently, experimental investigations of macroscopic coherent quantum states (bosonic condensates) of microcavity exciton-polaritons [13, 18, 196] in one-dimensional (1D) periodic potentials has received a lot of attention [67, 197–200]. When a microcavity is optically pumped, the exciton-polaritons populate the energy bands of the resulting periodic potentials. Propagation of coherent polariton waves in the plane of a quantum well, away of the periodic array, has never been investigated in detail because it is difficult, if not impossible, to avoid trapping of polaritons in stationary (extended or localised) states of the 1D array [198,199].

In this chapter, we employ an exciton-polariton condensate in a 1D buried mesa array of polariton traps [see the schematics in Fig. 6.1(a,b)] to observe the Talbot interference patterns with coherent hybrid light-matter waves. The Talbot effect for exciton-polaritons is uniquely enabled by the ability of exciton-polaritons to condense into a non-ground extended (Bloch) state of the 1D array [60,67], as well as by the nature of the mesa traps, which are embedded into the microcavity [60] rather than forming free-standing pillars on the substrate [198]. When the Bloch mode is characterized by a periodic distribution of polariton density maxima
located in the barriers between the mesa traps, the barrier regions act as sources of polariton waves, which are free to propagate in the plane of the quantum well [see Fig. 6.1(b)]. The periodic array of such sources creates a Talbot carpet shown in Fig. 6.1(d). Moreover, we demonstrate that this system mimics both amplitude and phase gratings for the light-matter waves.

### 6.3 Experiments

![Figure 6.2: Low power dispersion. (a) Parabolic dispersion of exciton-polaritons and (b) k-space image of the emission in the planar region below the condensation threshold.](image)

The experiment was performed using 1D mesa arrays microstructured in an AlAs/AlGaAs microcavity with GaAs quantum wells, as described in [60]. Mesas of 3.5 µm diameter are separated centre-to-centre by the distance of 5.5 µm, with the effective polariton potential depth of \( \sim 5 \) meV for each mesa. The exciton-polariton condensate is formed spontaneously by pumping the microcavity with a cw laser injecting free carriers well above the polariton energy. The pump beam has a FWHM dimension of \( 2.5 \times 36 \) µm, which illuminates approximately six mesas, as shown in Fig. 6.1(c). Real and reciprocal space imaging of the cavity photoluminescence resulting from the polariton decay is used to analyse the spatial density distribution and dispersion of exciton-polaritons.

In the regime of low excitation powers, the dispersion (energy versus in-plane momentum) of exciton-polaritons created outside and in the mesa array are remarkably different. Outside the mesa array, the parabolic dispersion \( E(k) \) near the in-plane momentum \( k = 0 \) is typical of the lower polariton dispersion branch in a planar microcavity, as seen in Fig. 6.2. For the pump powers below condensation threshold, \( P < P_{th} \) polaritons in the planar region display the typical parabolic dispersion shown in Fig. 6.2(a). Correspondingly, the \( k \)-space image of the low-power
emission in Fig. 6.2(b) displays the characteristic circular shape. The dispersion in Fig. 6.2(a) is used to fit the effective mass of the free polariton, $m_p$, using the relation $E = E_0 + \hbar^2 k^2 / 2m_p$. Here $E_0 = 1.57059$ eV, which yields $m_p = 4.447 \times 10^{-5} m_e$, where $m_e$ is the free electron mass. The wavelength of the coherent exciton-polariton wave above the condensation threshold calculated using this value of the effective mass is consistent with that deduced from the transverse period of the Talbot carpet. In contrast, the emission from polaritons located in mesa traps reveals the band-gap energy structure [201] imposed by the periodicity of the trapping potential in the lateral ($x$) direction [Fig. 6.3(a)], as described in [60, 67]. Both the discrete energy states in the individual mesas and the characteristic band-gap spectrum of extended Bloch states can be seen in Fig. 6.3(a). The lowest band of Bloch states is formed above the excited energy state in the individual mesas, similarly to the spectra of the deep photonic wires in organic microcavities [202, 203]. The gaps between the energy bands become progressively narrower, as can be seen from the spectrum in Fig. 6.3(a).

![Figure 6.3](image)

**Figure 6.3**: Dispersions and $k$ space distributions. (a,b) Dispersion measurement of polariton emission from the mesa traps (a) below and (b) above condensation threshold. Both localised and extended energy states are seen in (a). Condensation in two gap states in the first (G1) and the third (G3) spectral gaps are visible in (b). Dashed yellow lines correspond to $\pm k_B/2$ and mark the first Brillouin zone of the array. The solid lines in (a) show the first four extended Bloch bands calculated from Eq. (6.1). (c,d) Energy filtered reciprocal space image of the condensate emission from the G3 state (c) measured in the experiment, and (d) calculated theoretically from the field distribution $f_T(r)$ (see text).
At the higher pump powers, exciton-polaritons undergo a transition to bosonic condensation. The majority of the condensate populates non-ground steady states marked by G1 and G3 arrows in Fig. 6.3(b). These weakly spatially localised states [204] are located in the narrow first and third spectral gaps, respectively, and are indistinguishable from Bloch modes modulated by a broad Gaussian envelope. In particular, the highly populated steady state G3 forms in the very narrow third energy gap of the linear spectrum in Fig. 6.3(a). Its density and phase distributions are inherited from the stationary Bloch state at the top of the third spectral band. Unlike propagating polariton waves, the steady states trapped in the array are nearly monochromatic.

The reciprocal \((k)\) space image of the emission intensity also undergoes dramatic changes above the exciton-polariton condensation threshold. At the low power, the \(k\)-space image exhibits a ring-shape distribution due to the flow of the untrapped polaritons in all directions. Once the pump power exceeds the condensation threshold, the polaritons condense predominantly in the G3 mode determined by the extended Bloch state of the mesa array. This state is characterised by two peaks at the edges of the third Brillouin zone \(k_x = \pm 2.85 \mu m^{-1}\) visible in Fig. 6.3(b) and the corresponding maxima in the \(k\)-space emission pattern Fig. 6.3(c). Moreover, Fig. 6.3(c) demonstrates discrete distribution of the polariton emission intensity in the \(k\) space which suggests additional periodicity in the transverse \((y)\) direction. This is the direct consequence of the Talbot effect, which is revealed in the spatial distribution of the polariton density.

Lower energy gap state G1 is also clearly visible in the photoluminescence spectrum shown in Fig. 6.3(b). Polaritons in this state are well contained within the mesa traps and do not escape into the planar region. The real-space image of the emission spectrally filtered at the G1 state shown in Fig. 6.4(a) reveals that this state appears as a result of hybridization of higher-order 2D trapped states of individual mesas. These modes show a localized large-density spot in the mesa centre and a ring around the mesa edge. The \(k\)-space emission demonstrates the behaviour which is dramatically different from that of the G3 state. Namely, \(k\)-space emission corresponding to the G1 state is discrete along the \(k_x\) direction, but continuous in the \(k_y\) direction [Fig. 6.4(b)] because of the absence of the Talbot interference.

**Figure 6.4:** G1 state. Energy-filtered (a) real-space and (b) \(k\)-space images of the polariton density in the gap state G1.
In the G1 state, the majority of the polariton density is contained within the individual mesas. In contrast, the real space density distribution of the G3 state along the $y = 0$ line shown in Figs 6.6(a,d) displays larger polariton density in the potential barrier regions between the individual mesas. Polaritons in the barrier regions are free to propagate in the plane of the quantum well, thus generating coherent flow of polaritons in the transverse direction. The real space image of the polariton flow outside the mesa array [Figs 6.6(a)] shows the interference structure consistent with the linear Talbot effect, with the spatial periodicity both in the lateral and transverse directions.

With the growing pumping power above the condensation threshold, the contrast of the Talbot fringes becomes enhanced. The Talbot effect becomes more pronounced as the power is raised substantially above threshold. In addition to the images of the Talbot effect shown in the main text, in Fig. 6.5 we present real space images of the exciton-polariton emission at different pump powers for the same mesa array. The images are not filtered by energy, but the emission is clearly dominated by the G3 state and the resulting Talbot carpet. Comparison of the first revival distance, $L/2$, in different panels shows that the increased above-threshold pump power does not affect the Talbot length. This indicates that the polariton wavelength (blueshift) is not undergoing significant change in the presented range of powers. The lateral period does not change either, which is consistent with the fact that the Talbot period is determined by the period of the grating (i.e. the lateral period of the mesa array).

![Figure 6.5: Power dependence of the Talbot effect. Real-space images of the polariton density at different pump powers below and above the condensation threshold $P_{th}$. Talbot effect is clearly seen above the threshold.](image)

### 6.4 Theory

The full dynamics of exciton-polariton condensation in the one-dimensional mesa array for moderate pump powers above threshold can be reliably reproduced by the two-dimensional mean-field dynamical model taking into account energy relaxation due to quantum and thermal fluctuations in the system [205]. The detailed description of the model, which describes transition between different energy states occupied by the condensate in the mesa traps can be found in [67]. It consists of the open-dissipative Gross-Pitaevskii equation for the condensate wave function incorporating stochastic fluctuations and coupled to the rate equation for the excitonic...
reservoir created by the off-resonant cw pump [67, 205]. Numerical modelling with
the parameters corresponding to our experiment reproduces condensation into a non-
ground steady state with the real space density distribution shown in Fig. 6.6(b). It
reveals the Talbot pattern in qualitative agreement with the experiment. In what
follows, we present a simple, intuitive theory of this effect based on its analogy with
the linear near-field diffraction.

In the low density regime, i.e. for excitation powers below the condensation
threshold, the exciton-polaritons occupy the band-gap ladder of single-particle en-
ergy states in a periodic potential (see, e.g. [60] for details of the potential character-
isation). The energy band structure \( E_n(k) \), where \( n \) is the band index, can
be calculated directly by solving the stationary single-particle Schrödinger equation
for the macroscopic wave function of the polariton condensate \( \psi(x,y) \) in a peri-
odic in-plane potential. Because of the 1D nature of the lateral periodicity, the
dimensionality reduction can be performed, and eigenvalues approximated by the
spectrum of the factorised eigenstates \( \psi(x,y) = \chi(y)\phi_k(x) \exp(-ikx) \), where \( \chi(y) \)
is a transverse mode of an individual mesa trap, and \( \phi_k(x) = \phi_k(x+a) \) are the
extended polariton Bloch states in an effective 1D potential \( V(x) = V(x+a) \) with
the in-plane momentum \( k \equiv k_x \). The 1D Bloch states obey the following equation:

\[
\frac{\hbar^2}{2m_p} \left( k - i\nabla_x \right)^2 + V(x) \right) \phi_{n,k}(x) = E_n(k)\phi_{n,k}(x). \tag{6.1}
\]

Here \( m_p \approx 4.45 \times 10^{-5} m_e \) is the effective polariton mass in the planar region, and
\( m_e \) is the free electron mass. We approximate the 1D periodic potential created by
the mesas by the anharmonic analytical function \( V(x) = V_0 [F(x) - 1] \), where:

\[
F(x) = \frac{(1 + s)^2 [1 + \cos(k_B x)]}{2 [1 + s^2 + 2s \cos(k_B x)]},
\]

and \( k_B = 2\pi/a \) is the size of the Brillouin zone of the periodic potential. The shapes
of the potential for varying degrees of anharmonicity are described in [206].

The band-gap spectrum \( E_n(k_x) \) calculated using Eq. (6.1) for the anharmonicity
parameter \( s = -0.2 \) and the potential depth \( V_0 = 5.2 \) meV, assuming the transverse
ground state of the mesa \( \chi(y) = \chi_0(y) \), demonstrates good agreement with the ex-
perimentally measured spectrum, as seen in Fig. 6.3(a). We note, however, that the
multitude of populated energy bands visible in Fig. 6.3(a) includes extended states
formed by hybridisation of the higher-order two-dimensional modes of the individual
mesa traps. One of such bands, weakly populated by low-density polaritons, is seen
in Fig. 6.3(b) below the state G1.

The lateral density distribution for the polariton Bloch mode giving rise to the
G3 state emission in Fig. 6.3(b) is well reproduced by our model. The calculated
density of the Bloch state \( \phi_B(x) \equiv \phi_{3,k_B/2}(x) \) and the corresponding experimental
intensity of the polariton emission along the \( x \) axis are shown in Fig. 6.6(d). The
highest peaks of \( |\phi_B(x)| \) are located in the local maxima of \( V(x) \) which correspond to
the barrier regions of the mesa array. This Bloch state belongs to the top edge of the
third energy band \( (n = 3) \) and therefore is a “π-state” with a staggered phase; i.e. its
adjacent density peaks have \( \pi \) phase difference. This phase difference is inherited by
the transverse flow of polaritons, which in turn leads to some characteristic features in the Talbot pattern. Conversely, from the intensity of the polariton emission in the Talbot pattern one can reliably infer the phase distribution of the condensate wave function $\phi_B(x)$ in the mesa array.

The Talbot pattern can be reproduced by applying the linear Huygens-Fresnel principle to the polariton flow, thus assuming that polariton waves propagating away from the mesa array are of sufficiently low particle density, so that the nonlinearity can be ignored. As indicated by the lateral period of the Talbot pattern, only the highest peaks of $|\phi_B(x)|^2$ act as sources of the coherent polariton flow. According to the Huygens-Fresnel principle, the condensate in the barrier regions can be represented by an array of point sources of decaying radial waves whose field is given by [205]:

$$ f_m \sim (1/\sqrt{r}) \exp[i(k_p r_m + \theta_m) - \kappa r_m], $$

where $r_m = \sqrt{|r - r_m|^2}$ is the relative distance between a point on the plane, $r$, and the position of the $m$-th source, $r_m$, $k_p = 2\pi/\lambda_p$ is the polariton wave vector, $\kappa = \gamma m_p/(2\hbar k_p)$ is determined by the polariton decay rate $\gamma$, and $\theta_m$ is the initial phase inherited from $\phi_B(x)$. The total field of the Talbot carpet is given by the linear superposition of waves from all sources: $f_T(r) = \sum_m f_m$, which, in our case, are located on the line $y = 0$.

Figure 6.6(c) shows the Talbot pattern reproduced by the linear theory assuming the polariton wavelength $\lambda_p = 2.6 \ \mu$m and the polariton lifetime $\gamma^{-1} \sim 10 \ \text{ps}$. The
real space image is in excellent agreement with the experimental pattern Fig. 6.6(a). The dark lines located between bright lobes result from destructive interference between the adjacent sources due to the relative $\pi$ phase difference. Away from the $x$ axis, the pattern distorts and eventually vanishes because of the finite number of sources and polariton decay. Nevertheless, the pattern persists for distances sufficient to re-image the source twice. Finally, the calculated Fourier transform of the Talbot pattern $f_T(r)$ matches the experimental $k$-space signature very well [cf. Figs. 6.3(c) and 6.3(d)]. The discrepancies between the calculated and experimental images stem from the theoretical assumption of distributed point sources located on $y = 0$, whereas in practice each source has a finite transverse extent.

The Talbot length is defined as the transverse distance at which the phase shift of all plane wave sources is equal to $2\pi N$, where $N$ is an integer. At this distance, the original density distribution at $y = 0$ is revived. Remarkably, in our case the Talbot pattern demonstrates the complete revival at half the Talbot length $L/2$ [see Figs. 6.6(a,c)]. This is because the array of coherent polariton sources created by the Bloch state mimics both amplitude and phase gratings. At the transverse distance $L/2$, the phase of each source acquires a $\pi$ shift, thus reproducing the staggered phase structure and the polariton density (emission intensity) pattern of the origin. The Talbot length determined from the transverse period of the pattern in the experimental real-space image is $L \approx 20 \mu$m. The wavelength of the coherent polariton wave is comparable to the period of the mesa array ($a = 5.5 \mu$m), and can be calculated using the non-paraxial correction to the Rayleigh formula [191, 194]:

$$L = \lambda_p\left[1 - \sqrt{1 - \frac{\lambda_p^2}{a^2}}\right]^{-1},$$

which yields $\lambda_p \approx 2.8 \mu$m. This value agrees well both with that assumed in our theoretical calculations above, and with the de Broglie wavelength of the transversely free polaritons with the energy $E_p = 5.81$ meV [state G3 in Fig. 6.3(b)]:

$$\lambda_{dB} = h/\sqrt{2m_pE_p} = 2.4 \mu$m.

The Talbot effect for exciton-polaritons illustrates the possibility to shape two-dimensional exciton-polariton flows in the plane of a quantum well. This requires loading of exciton-polaritons into a one-dimensional array with a particular density and phase structure at a sufficiently high energy, so that leaking of the polaritons into the planar region is significant. Such an array of sources forms a “flat lens” analogous to those demonstrated for surface plasmon-polaritons (see, e.g. [191,207]) and shapes the in-plane polariton density, as well as the corresponding cavity photoluminescence.

Figure 6.7 demonstrates three different designs for the polaritonic flat lens (right panels) and the resulting interference pattern (left pattern) in real space calculated using the Huygens-Fresnel principle. The lenses are formed by 1D arrays of polariton sources arranged along the $x$ axis, at $y = 0$. Lens Fig. 6.7(a) is created by coherent exciton-polaritons loaded into an equidistant array of traps similar to that described in the main text. However, here both the density and phase of polaritons have a very strong gradient, which may be achieved, e.g. by a focused excitation spot, as suggested in [201]. The resulting interference produces well-collimated flow of polaritons in the transverse direction. Lens Fig. 6.7(b) is produced by an array of microstructured mesas with progressively decreasing distance between individual traps. Polaritons loaded into the mesa array occupy a steady state with $\pi$ phase difference between the neighbouring density peaks, and the additional density mod-
Figure 6.7: Polariton flat-lens designs. Theoretically calculated real-space images (left panels) of the polariton density (a.u.) formed by the flat lenses with different density and phase distributions (right panels) of the polariton sources. The polariton wavelength is \( \lambda_p = 3.6 \mu m \) in (a), and \( \lambda_p = 2.3 \mu m \) in (b,c).

ulation can be produced by using two optical excitation spots rather than one. The resulting pattern is reminiscent of the diffraction-limited focusing effect achieved by optical metasurfaces [208]. Finally, lens Fig. 6.7(c) employs a steady state with a flat phase in the same microstructured array as in (b), which produces a dramatically different “beam splitting” pattern. We emphasise that, while Fig. 6.7 demonstrates the concept of polaritonic flat lenses, precise mechanisms for dynamical loading of exciton-polaritons into the corresponding 1D steady states should be further investigated before these designs can be demonstrated in an experiment. In addition, designs (b) and (c) require fabrication of a tailored microstructured sample.

Finally, we note that the Talbot interference of exciton-polaritons is an ubiquitous effect. However, not every higher-order exciton-polariton mode responsible for leaking into the planar regions and generating Talbot patterns may be captured by our simple 1D theory. The Talbot interference of exciton-polaritons can be produced by a variety of higher-order exciton-polariton modes in the mesa array that leaking into the planar regions. Some of such modes are genuinely two-dimensional. As an
example, Fig. 6.8(a) demonstrates a Talbot pattern generated by an extended Bloch state formed by hybridisation of 2D mesa states in an array of 2 µm mesas with the lateral period of 5 µm. Although spatial density distribution of this Bloch mode cannot be described by our simple 1D theory, the mechanism for producing Talbot fringes is the same: a four-lobe density pattern in the barrier regions [marked in Fig. 6.8(a)] acts as a source of polaritons propagating in the direction transverse to the mesa array. Furthermore, as this high-energy state is only weakly trapped, leakage of polaritons out of the mesas also occurs, which further complicates the pattern. The $k$-space imaging in Fig. 6.8(b), in this case, shows a fine discrete structure due to the sophisticated spatial pattern of the Talbot carpet.

### 6.5 Conclusions

To summarise, we demonstrate the Talbot effect for an exciton-polariton condensate loaded into a one-dimensional array of mesa traps. The Talbot pattern is formed due to condensation of exciton-polaritons into a non-ground energy state which has pronounced density peaks in the barrier regions between mesas. Polaritons at these locations are free to propagate transversely to the mesa array and therefore the barrier regions act as a periodic array of sources of coherent polariton waves. Due to the nontrivial phase of the non-ground state in the array, these sources mimic both amplitude and phase gratings for the light-matter waves, which links our observations to the Lohmann effect [209]. Moreover, the period of the grating is comparable to the wavelength of the coherent light-matter waves, which provides opportunity for exploring non-paraxial effects on the microscale. Numerical calculations exploiting a mean-field nonlinear model of polariton condensation, as well as the linear Huygens-Fresnel diffraction theory, allow us to reproduce the spatial distribution and spectral signatures of the Talbot effect.
Talbot effect for exciton-polaritons
Conclusion and perspective

Motivation for the current thesis is driven by the polariton condensate being a highly controllable quantum system which can exhibit superfluidity even at room temperature [210]. In the development of superfluid-based coherent devices and circuits, such as ultra-fast nonlinear switches [160] and precision rotational and gravitational sensors [211], a concrete understanding of the nature of polariton superfluid flow is indispensable. Our results describe some general dynamical properties of polariton condensates under the incoherent pumping scheme, thereby paving the way for the utilization of their coherent properties in the design of polaritonic devices. With the continuous development of fabrication techniques, we expect that complicated microstructures can be embedded in high quality semiconductor microcavities, providing better confining potentials to control polariton systems.

Starting from our current results, we can envisage the following three directions for further research:

1) Large scale polariton quantum fluid dynamics

The Landau’s criterion of superfluidity applies well to single-component conservative BEC systems, because their total energy and elementary excitation energy are ready to be calculated. Later, by calculating the revised elementary excitation energy the concept of critical velocity can be extended to two-component conservative systems [212]. These results, however, apply only to conservative BEC systems. By studying how a polariton current with spin degrees of freedom responds to a static defect we may address the modifications to the Landau’s criterion in open-dissipative systems. Resolving this problem for a spinor polariton superfluid is of critical importance for the development of polariton superfluid circuit applications. With suitable control over the strength of spin-dependent interactions such as enhancing the TE-TM splitting by an open cavity structure [213], a non-Abelian gauge potential can be introduced to polariton systems [47]. This topic will also study the influence of the non-Abelian gauge potential has on the superfluidity of polaritons.

2) Dynamics of topological excitations in polariton condensate

Topological excitations are elementary excitations of two-dimensional systems embedding topological order, such as vortices, and which are found to have a strong influence on superfluidity in other BEC systems [214]. The quantization and den-
sity distribution of vortices in various BEC systems have been discussed in detail in [20]. The open-dissipative nature of the polariton condensate is expected to modify strongly the properties of quantized vortices, including their core structure, stability and dynamics. In addition, novel structures of “vortex matter” formed with ensembles of vortices are expected [58]. The properties and dynamics of quantized vortices and other topological excitations will be studied, along with the dynamics of the polariton condensate under rotation and defect excitation. Also, it is of importance to understand and characterize the pairwise interactions between vortices and how they are modified by the presence of gain/loss.

3) Polariton flows under the influence of external potential

In the presence of a periodic external potential, band-structures form, and polaritons may be confined to states with novel properties such as the Dirac cone in hexagonal lattices, or the flat band of a Kagome lattice. Proposals [215] exist where the combination of spin-orbit coupling in a flat band geometry may lead to the creation of a new state of matter resembling the fractional quantum Hall effect (FQHE) [216] which would exhibit dissipationless edge-states. It is given by partially filling a flat band with non-zero Chern number resulting from geometrically frustrated systems such as the frustrated Kagome lattice [215]. Further theoretical proposals [217] predict how two isolated energy levels (corresponding to surface currents with different spins) within an engineered band gap might be created in polaritons in a topologically robust manner, resulting in a polariton topological insulator [218].

All of these topics will result in new insights into the fundamental properties of this unique open-dissipative quantum system.
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