Figure 6.5: Representation of a profile from the Brindabella 20 m DEM using positive wavelets. The original profile is shown dashed and the positive wavelet representation is the solid line. (a) Representation using the first three components; (b) the next three components; (c) the first six, (a and b combined); and (d) the first twenty components.
Figure 6.6: The correlation surfaces derived from the initial profile (a) and from the profile after the first three features are removed (b). The labelled maxima and minima correspond to the labelling of the features in Figure 6.5. Note that the removal of the three dominant features allows lesser correlation peaks to emerge. Scale step 1 is at the coarsest scale and scale step 23 is at the finest scale of the analysis.
no visible difference between the original data and the wavelet representation using all 75 components at the scale of Figure 6.5.

Once the profile has been decomposed into positive wavelet components, the parameters of those components can be analysed to obtain information about the structure of the surface at different scales.

Figure 6.7(a) shows the height (absolute value of amplitude) and scale of all 75 wavelets, with the first six components labelled to correspond with Figure 6.5. Positive and negative components are shown separately and the line at the lower left of the plot represents a correlation \( T \) of 1.8 which is the correlation of the 75th component. Since feature detection proceeds in order of decreasing correlation and the decomposition was terminated at this correlation value, features to the left of this curve have been ignored in this representation. According to Equation (6.5) the height (amplitude) \( m \) of a wavelet component is proportional to the correlation \( T \) divided by the square root of scale \( L \), and the shape \( (m/L) \) is proportional to \( T \) divided by \( L^{3/2} \). These relationships are used to define the constant-correlation lines on Figure 6.7.

At scales between about 200 and 1000 m the positive components (hills) closely follow a straight line on the log-log plot, indicating a power-law relationship with an exponent of 1.5. The negative components are much more scattered but follow a similar trend. This indicates that the hills tend to become more peaked as scale increases within this range. This power law relationship cannot be compared to the spectral power laws of the previous chapter without considerable manipulation so it will not be pursued here, but the comparison will be explored in detail using two-dimensional features in Section 6.5.6.

Figure 6.7(b) shows component shape for all 75 wavelet components, where shape is defined as height divided by scale (length). A large value represents a very peaked feature, while a small value indicates a relatively flat feature. The positive and negative components (hills and valleys) are again plotted separately and the line at lower left indicates correlation at which the representation is truncated. Some interesting features are apparent:

- For scales less than about 600 m, the largest shape values are associated with negative components (valleys), while for scales greater than 600 m the largest shape values are associated with positive components (hills). Presumably this reflects the incised nature of the landscape, with the sharpest valleys at smaller scales than the most peaked hills. The significance of the 600 m threshold (if any) has not been determined.

- The maximum shape values of about 0.12 correspond to maximum slopes of 41% and are associated with features at scales ranging from 100 to 1000 m. This result contrasts markedly with the usual finding based on regular grid representations (Zhang and Montgomery, 1994; Hutchinson, 1996) that the steepest
Figure 6.7: (a) The height (amplitude) and scale of the 75 wavelet components used to represent the Brindabella 20 m DEM profile. (b) Shape (height divided by scale) of the 75 wavelet components. The dotted line at lower left indicates the correlation value at which the decomposition was halted.
slopes are associated with the finest scales. The maximum slope associated with the broadest scale features (scale \( \approx 8000 \) m) is 14\% and with the finest scale features (scale \( \approx 40 \) m) is 21\%.

This exercise has demonstrated that the decomposition appears useful and that the extension to surfaces is warranted. Further analysis of profiles is not considered because profiles do not capture the organisation of the surface topography. In two dimensions the performance of the decomposition should improve as it takes advantage of the two-dimensional organisation of the surface.

6.4 Positive Wavelet Analysis of Surfaces

The positive wavelet analysis procedure readily generalises to two dimensions using the feature described at the beginning of the chapter. The positive wavelet correlation detection integral now involves five parameters:

\[
T(x, y, L, w, \theta) = L^{-1}w^{-1/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} g(u, v)F(u - x, v - y, L, w, \theta)du dv
\]

(6.7)

which is the two-dimensional wavelet transform of the surface \( g(u, v) \) using the wavelet \( F() \) of Equation (6.3). The feature is located at \( x \) and \( y \), \( L \) is its length, \( Lw \) its width and \( \theta \) its orientation. The scale of a feature is \( Lw^{1/2} \) which is proportional to the square root of its area \( L^2w\pi/4 \). The normalising constant for the two-dimensional wavelet is \( C = 28/\pi \).

The same iterative procedure is used for decomposing the surface: detect the largest magnitude feature by scanning the parameter space and refining the detected feature using local maximisation of the correlation value, then subtract the feature and detect the next one.

In the two-dimensional case it was found that the parameters of features detected from the surface are significantly distorted by the effect of other overlapping features, so the parameters of overlapping features must be jointly optimised. Optimisation of a compound feature by correlation is not possible because it does not have a well-defined scale, so non-linear least-squares optimisation was used (Marquardt, 1963; Press et al., 1989) with the sum of squared differences (SSD) between data and modelled elevations as the objective function to be minimised. The shape-matching properties of the correlation detection algorithm are unfortunately lost using this approach.

6.4.1 Implementation

The implementation of this process required considerable experimentation and modification before satisfactory performance and results were achieved. Firstly, scanning
the entire five-dimensional correlation surface $T(x, y, L, w, \theta)$ for every new feature is computationally very expensive. A simpler algorithm was developed which searches over a small subset of the space that is likely to contain the maximum correlation value. This search uses the following assumptions:

- a circular feature will match the surface well enough compared to a correctly oriented and shaped elliptical feature, for the purposes of selecting candidate features.
- the first feature to be detected will have a length $L$ comparable to the size of the region being analysed; subsequent features will have a scale similar to the previous feature.

Using these assumptions the length, width and orientation are assumed to be known in advance and detection of the next feature is accomplished by a scan across the surface. The search space is thereby reduced from five dimensions to two. The distortion in parameter values introduced by the assumptions is corrected by maximising the correlation after the feature is detected. A good starting point for the optimisation is found by briefly searching along the scale, width and orientation axes. After further experimentation, the initial search was further reduced to a search for the location of maximum amplitude in the residual surface, rather than a maximum correlation. This further simplification is based on the knowledge that the highest amplitudes are almost always associated with the largest scale features remaining in the surface, and large amplitudes and large scales both result in high correlation values.

**Joint Optimisation - Selection of Features**

When jointly optimising overlapping features, the selection of features to optimise requires some care. In the initial implementation, a list was built of features that overlapped the newly added feature, and any further feature that overlapped a feature already in the list was added. This resulted in very large sets of features being selected for joint optimisation resulting in a very slow decomposition. After some experimentation, the selection was restricted to only those features that overlapped the newly introduced feature. Using this method the number of features jointly optimised ranged from two to seven for the data sets used here.

**Optimisation - Stopping Criterion**

Another important choice is at what point to cease optimising a given set of features. The principle adopted here was that optimisation should cease at the point where adding a new feature would produce substantially greater improvement in SSD than the optimisation step. An estimate of the reduction in SSD produced by a new
6.4. **POSITIVE WAVELET ANALYSIS OF SURFACES**

feature is therefore required. An assumption has already been made concerning the scale of the next feature to be detected, and the amplitude of the feature can be approximated using the largest amplitude of the residual surface. If a feature with scale \( L \) and amplitude \( m \) fitted perfectly it would reduce the SSD by:

\[
\Delta SSD = \int_x \int_y F^2 = \frac{L^2 m^2 \pi w}{28}
\]

Based on results using DEM surfaces, it was estimated that detected features typically reduced the SSD by 1/4 this maximum value. The improvement in SSD from each Marquardt optimisation step is compared with the estimated \( \Delta SSD \) for a new feature, and the joint optimisation is terminated when the improvement in SSD is less than one tenth of that expected from the next new feature.

A corresponding procedure was applied to new features: if a newly detected feature does not improve the SSD by at least ten times the amount achieved by the previous optimisation step, it is rejected. The optimisation is then continued for one or more steps subject to the termination condition of the previous paragraph.

**Distance Penalty**

The final constraint on the optimisation of features was to prevent the features from retreating too far over the edge of the data set. Some shapes are well represented by features partially over the edge of the data, but in some circumstances the optimisation of features results in only a tiny proportion of the feature remaining in the data set. This is undesirable because the size of such features does not correspond to the part of the surface they are modelling. This problem was prevented by adding a penalty for features whose centre was outside the boundaries of the DEM.

To be commensurate with SSD, any penalty should be proportional to the square of the amplitude of the feature \( (m^2) \). The penalty should also increase in proportion to the area of the feature outside the data set, or in this case to the square of the distance from the centre of the feature to the edge of the DEM \( (d^2) \), giving a penalty of the form \( m^2d^2 \). However, since the optimisation algorithm uses second derivatives and a continuous second derivative is desirable to maintain stability, a higher power of distance is desirable. Since amplitude and distance have the same units it was deemed reasonable to use \( d^4 \) as the distance penalty rather than \( m^2d^2 \). This penalty performed well. The primary deficiency of this penalty is that it does not prevent very elongated features from extending far outside the DEM while remaining centred within it; this might be prevented by a more sophisticated area-based penalty.
Figure 6.8: The decomposition of the Brindabella DEM. Top: The original and reconstructed DEM. Bottom: the first five features as originally detected, and after joint optimisation with subsequently detected features.
Figure 6.9: The decomposition of the Livingstone DEM. Top: The original and reconstructed DEM. Bottom: the first five features as originally detected, and after joint optimisation with subsequently detected features.
Figure 6.10: The decomposition of a section of the Rinker DEM. Top: The original and reconstructed DEM. Bottom: the first five features as originally detected, and after joint optimisation with subsequently detected features. White indicates elevations higher than the maximum in the DEM, and black elevations lower than the minimum.
6.5. **SURFACE RESULTS**

![Cumulative frequency distribution](image)

Figure 6.11: Cumulative frequency distribution of the number of features as a function of scale in the three surfaces.

### 6.5 Surface Results

#### 6.5.1 Number of Features Detected

The entire Brindabella and Livingstone DEMs and a 5 km × 5 km section of the Rinker DEM were decomposed: the entire Rinker DEM was not processed because it took too long. In each case the decomposition was stopped when the root-mean-squared (RMS) difference (square root of SSD divided by the number of points) was reduced to 1 m. The Brindabella DEM required 946 features to meet this target accuracy, the Livingstone DEM 515 features and the section of the Rinker DEM only 120. The differences in number of features is primarily due to the differences in relief, although the lower complexity of the Rinker surface probably contributed to the low number of features required. Figures 6.8, 6.9 and 6.10 show the original and decomposed surfaces.

Figure 6.11 shows the cumulative frequency distributions of the number of features as a function of scale in the three surfaces. The separation between the Rinker curve on one hand and the Brindabella and Livingstone curves on the other is an indication of the substantial difference between the glacial and fluvial landscapes. However, some of this difference is due to the lower number of features in the Rinker representation: if the decomposition was allowed to proceed further the majority of new features detected would be at fine scales thus shifting the cumulative frequency distribution.
to the left.

6.5.2 Alteration of Features by Joint Optimisation

The initial stages of feature detection and the subsequent alteration of the detected features by finer-scale features is shown in the lower half of Figures 6.8, 6.9 and 6.10. On the left is the surface reconstructed using the first five features as initially detected, while on the right is shown the same five features after joint optimisation with all the subsequently detected features. In all the DEMs it is clear that some relatively fine scale features are detected early in the decomposition. This is because the maximum amplitude of the residual surface is used to locate the next feature, and in some cases this corresponds to a fairly small feature.

The modification of the first five features by joint optimisation with subsequent, finer features usually results in an increase in the amplitude of the broad-scale feature. This is because the feature as initially detected must match all the variation of the surface within its area. The subsequently detected finer scale features are typically of the opposite sign to the broader scale features. The joint optimisation of the overlapping and opposing features allows the broader scale feature to "hand over" the representation of some of the surface shape to the finer features. However this is not always the case: in the Livingstone DEM, the ridge feature projecting from the northwest corner is reduced in amplitude due to the subsequent addition of many finer-scale positive features on top of it.

Changes in scale, width and orientation also occur as the surface decomposition proceeds. The broad scale valley feature on the eastern edge of the Brindabella DEM both shortens and widens (relative to its length) as finer scale features take over the task of representing the valleys in the southeast corner of the surface. In the Rinker DEM, the feature on the western edge has rotated significantly due mainly to the introduction of a new feature that accounts for the peak near the northeastern end of this feature.

6.5.3 Computational Cost

The decomposition is quite costly in computer time, and is dominated not by the detection of features but by the joint optimisation of features. If the area of the DEM being decomposed is doubled, the area of the largest features can be expected to double and the number of features of a given size will also double. The doubling of area of the largest features almost doubles the time for each joint optimisation step. The increased number of features similarly increases the number of joint optimisations to be performed (since a joint optimisation is performed after each new feature is detected). The increase in range of scales as the DEM area is increased by a factor of
6.5. SURFACE RESULTS

$k$ also results in an increased number of features in each joint optimisation by a factor proportional to $\log k$. This induces a further increase in computational effort by a factor $(\log k)^3$ due to the matrix computations required for the optimisation. This informal analysis suggests the computational cost increases with the number of DEM points $N$ at least by order $N^2(\log N)^3$. By comparison, a direct implementation of the Fourier transform requires order $N^2$ effort and the Fast Fourier Transform (FFT) requires order $N \log N$.

The decomposition of the Brindabella DEM into its 946 features required about 48 hours of processing time on a Sun SparcStation 20; the Livingstone DEM with 515 features required 26 hours and the Rinker DEM section with 120 features 10 hours: an earlier attempt to decompose the entire Rinker DEM (251001 points) had not approached the target accuracy after a week of processing, so was abandoned. Each feature requires six parameters, so the 54126 point Brindabella DEM was reduced to 5688 parameters, a compression of almost 10:1; the 48816 point Livingstone DEM was reduced to 3090 parameters (15:1) and the 63001 point Rinker DEM section to 720 parameters (87:1).

The method in its current form is impractical for surfaces larger than about 100,000 points on workstation style computers. The optimisations would vectorise well so a significant improvement in performance could be expected using supercomputers. More fundamental improvements in the efficiency of the algorithm might also be possible.

6.5.4 Assessing the Quality of the Decomposition

The quality of the resulting decomposition can be assessed in a number of ways. The following assessments will be discussed:

- the number, size and depth of depressions in the original and decomposed surfaces;
- the representation of the drainage network; and
- the representation of particular features in the landscape

Depressions

The absence of depressions and sinks (the lowest point of a depression) is an important property of some landscapes, particularly old fluvial landscapes such as those at the Brindabella and Livingstone sites. In such landscapes the reconstructed surfaces should not introduce depressions where they do not exist in the original DEM, and
6.5. SURFACE RESULTS

Figure 6.12: Depressions (in red) overlaid on contours of the Brindabella (50 m contour interval), Livingstone (20 m interval) and Rinker (10 m interval) surfaces. Depressions from original surface are on left and from reconstructed surface on right. The contours are from the original surface in all cases, and scale is the same (about 1:76 000) for all three surfaces.
the preservation of small depressions is much less important than the prevention of new ones.

In other landscapes, particularly in karst and recently glaciated terrain, depressions and sinks are relatively common. The Rinker site is an example of a glaciated landscape with many depressions. In such landscapes, preservation of the depressions that are in the original DEM is important. The introduction of additional small depressions should be avoided but is less of a problem than in depressionless fluvial landscapes.

Figure 6.12 shows the contours of the original surfaces and, in red, the depressions present in the original and reconstructed surfaces. The three surfaces are shown at the same scale. The Brindabella DEM contains two single-cell depressions 0.20 and 0.28 m deep. The reconstructed surface reproduces these single-cell depressions as 1.12 and 0.29 m deep respectively, and introduces a further 13 depressions ranging from 0.02 to 1.24 m deep. All of the sinks in the reconstructed surface (including the two in the original DEM) occur in the bottoms of valleys. Given their location and small size, the presence of these depressions do not significantly compromise the structure of the surface.

Both the original and reconstructed Livingstone surface contain a number of shallow depressions, 23 in the original and 25 in the reconstruction. Most of these lie along the channel in the southeastern corner of the DEM. The reconstructed surface preserves most of the depressions in the original DEM, although often of different depths and covering larger areas than the original depressions. The deepest depressions in both surfaces are nearly the same depth (1.68 m and 1.67 m) but this is coincidental as they are in different locations. Ten of the depressions in the reconstructed surface do not occur in the original DEM while four in the original do not appear in the reconstruction; some close but unconnected depressions in the original DEM are represented as a single larger depression in the reconstruction.

The Rinker surface is different to the other two in that it contains relatively broad depressions in the original surface, although still quite shallow (2.9 m maximum depth). This reflects the different type of landscape, formed recently by glacial action with a still poorly organised drainage structure including some lakes. Some of the depressions are perched upon uplands, as in the nearly circular depression near the southwest corner. There are 16 depressions in the original surface containing 20 grid points (8000 m²) or more, as well as about another 30 depressions containing less than 20 points, mostly only one point. The reconstructed surface also contains a number of depressions (about 40) some of which are very large (up to 430 000 m²), and the deepest is 6.1 m. The pattern of depressions is quite different from the original DEM, with less than half the depressions overlapping (by visual comparison). One of the largest depressions in the reconstructed surface occurs where there is none in the original while the largest (but not deepest) depression in the original is represented by
a much smaller and shallower depression in the reconstruction. Nevertheless, most of the sinks in the reconstruction still occur along drainage lines. The hilltop depression of the original DEM has not been represented. A number of single-point depressions in the original DEM that fall along a drainage line have been subsumed within a very elongated depression in the reconstructed surface.

The ability of the decomposition to represent the presence and absence of depressions is clearly dependent on the nature of the landscape, with better representation in incised, high relief areas such as the Brindabella region than in disorganised, low relief landscapes like Rinker. In the Brindabella and Livingstone surfaces the reconstructed surface represents most of the depressions in the original surface and introduces a small number of additional depressions almost exclusively along drainage lines. The performance is less satisfactory in the Rinker surface. It could be argued that continuing the decomposition of the Rinker surface to include the same number of features (and thus presumably the same relative accuracy) as the Brindabella decomposition would provide much better representation of the depressions in the Rinker surface. However the fact that the decomposition is fitting features by elevation (thereby ignoring the existence of sink points), together with the very flat valley bottoms in the Rinker DEM, makes it unlikely that even a very fine decomposition would capture the structure of depressions accurately in this landscape.

Drainage Network

In this feature-based model the drainage network should be represented by negative features. Moreover narrow valleys require narrow negative features to represent them, so the centrelines of the negative features should approximate the drainage network. Centrelines can be defined in various ways but must be sensitive to the degree of elongation of the features: a nearly circular feature should have a very short centreline because it contributes little to the directional character of the surface. The centreline chosen here is the line between the two foci of the elliptical contour where the wavelet amplitude is 20% of its maximum. The 20% level is fairly arbitrary but is a reasonable compromise between 0% (the outline of the wavelet) which overestimates the region of influence of the feature and 50% (for example) which underestimates it. As an assessment of quality, we wish to examine the degree to which these centrelines reproduce the drainage network of the surface.

Figure 6.13 shows the derived centrelines and contours for each surface. Some of the centrelines extend beyond the boundaries of the DEMs but these extensions have not been shown. In the Rinker surface, most of the dominant drainage lines have been represented by elongated negative features: the channel proceeding from the northeast corner has been represented by a single very long feature; the curved valley in the northwest corner has been represented by a set of shorter features; and
Figure 6.13: The centrelines of all negative features for the three surfaces. Contour interval is 50 m for Brindabella, 20 m for Livingstone and 10 m for Rinker.
the channels in the southwestern quadrant have been captured. Not all the negative features are confined to single drainage channels: the line crossing the southeastern corner of the DEM corresponds to a valley at the southern edge but continues along a ridge and through a saddle. Two approximately aligned valleys and the intervening saddle are thus represented by a single feature.

The Brindabella and Livingstone plots are more complex. A notable feature of the Brindabella centrelines is the network in the valley at the north of the DEM which shows features aligned with the major valley axis and others aligned with the tributaries to either side. There are several long centrelines including the two running east-west across the lower half of the DEM that, like the line in the Rinker DEM, represent two valleys and an intervening saddle as a single feature. The narrow channel at the southern edge of the Livingstone DEM is represented by a series of short line segments and a longer line capturing the broader scale orientation of the valley. In both of these surfaces most of the lines are oriented along valleys while some are located in peculiar positions that do not relate to the drainage structure. These are presumably capturing more generalised low areas rather than drainage structures.

The collections of centrelines shown here are poor representations of the surfaces' drainage networks, although the dominant valley features have been identified. The lack of connectivity of valley features is an obvious deficiency of this representation. Its tendency to integrate two valleys draining in opposite directions from a saddle into a single feature is not very attractive, except that it does draw attention to alignments that could be related to geological structures.

Landscape Features

From each DEM two prominent features in the landscape were selected, one hill and one valley, and the wavelets that represent these features were extracted from the representation for examination. The purpose of this analysis is to determine how well the wavelet features representing a specific feature capture the shape of the landscape feature and its broader scale context. The sites were selected without a priori knowledge of the wavelet features present at those sites.

The features chosen for study are:

- Brindabella:
  - a small hill on the main range about 1/3 of the way from the southern edge of the DEM; and
  - a drainage confluence in the dominant northern valley

- Livingstone:
Figure 6.14: Representations of two selected topographic features from each site. The elevation colour scale is the same as for the original DEM. White ellipses are positive features and black ellipses are negative features.
6.5. **SURFACE RESULTS**

- the hill to the east of the centre of the DEM that separates the northern and southern catchments; and
- a drainage confluence in the western part of the northern catchment

- **Rinker:**
  - the dominant hill in the southwestern quadrant; and
  - the approximately circular depression in the northwestern quadrant

The wavelet features contributing to these topographic features were extracted from the full feature set using the criterion that the amplitude must be more than 1% of its maximum to be counted as contributing to the shape at the given location. This level was selected to exclude features that overlap the site only marginally and are not contributing significantly to the shape of the surface at the site. Figure 6.14 shows the surfaces reconstructed using only the selected features, and the elliptical outlines of those features (the angular appearance of some ellipses is an artifact of the plotting process).

The Brindabella hill is represented by five features: the large positive feature representing the main range, an east-west negative feature forming valleys and a saddle, two moderately sized negative features forming valleys to the west and northeast and finally a small positive feature representing the hill itself. As a description of the landscape at this location this appears reasonable: a hill located on the top of the main range at the head of two valleys with a saddle and broad valleys to the south.

The Brindabella valley site is represented by eight features, three positive and five negative. The positive features are the main range, a smaller positive feature centred on the main range but projecting across the valley, and a finer feature of similar alignment centred on the valley. The main negative feature represents the broad valley structure while the four smaller negative features represent the finer valleys meeting at the site. Apart from the two positive features oriented across the valley, this is a good description of the location: it is within the influence of the main range but on its lower slopes, and is situated within a broad valley with several narrow valleys meeting at the site. The undesirable positive features owe their existence to the peak at the northwest corner of the DEM (for the larger feature) and a pair of ridges extending down into both sides of the valley (both features).

The Livingstone hill is represented by four positive and four negative features. The broad scale negative features represent the northern and southern catchments and the broad scale positive features represent the generalised outline of the hill and a ridge extending towards the northeast corner of the DEM. Within these broad scale structures there are two fine-scale negative features, representing valleys in the side of the hill, and two positive features, one oriented in the same direction as the broad-scale hill representing the shape of the hilltop itself and an elongated feature representing a
rise to the southeast of the hill and the adjoining ridge. This representation matches
the known structure of the hill very well at the broad scale: a hill situated between
two catchments with a significant ridge extending to the the north. The smaller
features account for recognisable fine scale structures around the hilltop.

The Livingstone valley site is represented by only two features: the broad-scale
negative feature representing the northern catchment and a narrow feature represent-
ing the valley itself. The lack of additional features representing additional structure
is surprising because this site, like the Brindabella valley site, is located at the con-
fluence of several tributaries and additional features were expected to account for the
adjacent landforms. Presumably the flatness of the valley floor results in adjacent fea-
tures — positive and negative — terminating before they cross the valley centreline,
unlike the Brindabella situation.

The hill in the Rinker DEM is represented by a broad scale positive feature over-
laying the broad scale negative feature forming the southern depression, with a finer
scale positive feature adding to the height of the hill and forming a ridge on its north-
western side. The depression site in the Rinker DEM is represented by a negative
feature overlaid on the end of a narrow positive feature that represents the hill to
the northeast of the depression. A small negative feature within the larger feature
deepens and shapes the depression. Both these representations are reasonable and
consistent with the topographic settings of the features.

At the sites studied, the decomposition provides a good representation of the shape
of the topographic features and their broader-scale setting. In one case (the Brind-
abella valley) the representation included apparently superfluous features related to
the terrain some distance away rather than at the site of interest; in another case
(the Livingstone valley) the representation had fewer features than seemed necessary
to represent the topographic context.

6.5.5 Derived Terrain Parameters

From this new representation constructed from overlapping features at different scales,
novel terrain parameters can be derived that capture properties of the surface that
could not be measured from simpler surface representations such as grids. At ev-
ery point on the surface a number of features of different scales and amplitudes are
summed to produce the elevation at that point. The parameters of the contributing
features can be combined to produce derived parameters describing various aspects
of the shape or structure of the surface at the point. Three such parameters have
been devised:

- terrain complexity — the sum of the wavelet amplitudes (without multiplying
  by the height, so all wavelet amplitudes are in \([0, 1]\));
Figure 6.15: Derived parameters from the Brindabella decomposition.
Figure 6.16: Derived parameters from the Livingstone decomposition.
Figure 6.17: Derived parameters from the Rinker decomposition.
6.5. **SURFACE RESULTS**

- positivity — the sum of feature amplitudes (wavelet amplitude multiplied by height) divided by the sum of the absolute value of feature amplitudes; this ranges from $-1$ when all contributing amplitudes are negative to $+1$ when they are all positive;

- orientation — the average orientation of features, weighted according to degree of elongation.

Figures 6.15, 6.16 and 6.17 show the spatial patterns of the three derived parameters for each of the three study sites.

**Terrain Complexity**

Terrain complexity is highest where many features contribute to the surface elevation. This is not the same as complexity of contour shape, but is meant to indicate the degree to which the surface requires multiple features to represent its shape; these multiple features will generally (but not necessarily) cover a range of scales.

In the Brindabella surface the largest values occur mostly on the upper reaches of drainage lines, and most valleys have moderate to high complexity. Areas of low complexity include the large ridge extending northeast from the centre of the DEM and another ridge extending eastwards from the main range about $2/5$ of the way up from the southern edge of the DEM.

The Livingstone surface has highest complexity along the eastern edge of the hilly area in the southwest of the site; as in the Brindabella surface, the highest values are in the upper reaches of stream lines. However unlike the Brindabella surface the lower valleys have low complexity, apart from the stream on the middle of the southern edge of the DEM, and the moderate to high complexity areas are the ridge lines.

The Rinker surface has broad areas of low complexity, and the highest complexity areas are where drainage lines cut through hills.

These results correspond qualitatively with the spatial variations in drainage density described in Section 1.6.1. At the Brindabella site the drainage density and terrain complexity are both higher in the valley bottoms than in the upper regions of the landscape, while at the Rinker and Livingstone sites the highest complexity is in the dissected uplands and lowest in the valleys. The difference between upland and lowland areas at the Livingstone site is much more pronounced in the terrain complexity results (3–4 for upland and 0–1 for lowland) than in the drainage density values (7.0 km$^{-1}$ for upland 6.3 km$^{-1}$ for lowland). This could be due to the low relief of the valley areas, with small convergent regions in the valleys being detected by the manual drainage density measurements but missed by the feature decomposition.