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THE MECHANICS OF BLOCKY MATERIAL

A Thesis submitted for the
Degree of
Doctor of Philosophy
in
The Australian National University
by
Brian Arnold Chappell.
March, 1972.
STATEMENT

The work submitted herein is entirely my own and any assistance or help received is clearly stated in the appropriate parts of the text and in the Acknowledgements.

B.A. Chappell.
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SYNOPSIS

Photoelastic models are constructed to examine the stress distribution in discontinua. To this end photoelastic material namely an epoxy resin (Araldite D) is used and a 22 in. diffusion polariscope constructed.

Initially simple experiments examining slip are performed and these show that by varying the constraints on the surrounding blocks, the stress distribution and deformational response are altered. The effect of rotation appears to be most important in determining what deformational response will result.

Mass reaction of a system of blocks are then examined by measuring both deformations and stress distributions. The deformational measurements show that the assessment of effective moduli is most restrictive as to make the use of continuum concepts in the form of analogue methods suspect.

A numerical method of analysing the photoelastic data is developed and simple tests show that this has great potential in furthering the examination of discontinuum models. In the examination of the blocky mass models the prevalence of tensile stresses and conditions for the development of effective tensile strains are readily observable.

Application of the experimental techniques developed to a field situation at the Mount Isa Mines shows that the experimental methods developed could greatly help in correlating the field situation with the recently developed numerical methods.
CHAPTER 1

MECHANICS OF BLOCKY MATERIAL

1.1 INTRODUCTION

It is generally realised that the continuum concepts of elasticity and plasticity do not adequately predict the stress distributions in the stress space of a loaded body which is discontinuous and where slip between particles occurs. If the stress distribution within the discontinuous body could be assessed at all stages of loading the deformational response and probable modes of failure could be better understood and more confidently predicted. In order to acquire an understanding of the stress distribution in a discontinuum, experiments were performed where both the stresses and deformations were measured.

Therefore, the first and main requirement was to develop experimental techniques which readily allow viable results of the stresses and displacements in a blocky material to be determined. These experimental results are then examined and compared with the analytical results obtained from available theories. From this a numerical method using finite element theory is then shown to offer great potential for determining the load distribution and deformational response.

Some of the queries relevant to the problems associated with the investigation are -

1. what are the causes and types of deformation and how do they affect the stress or load distribution,
2. when and in what situations can the mathematical theories of elasticity or plasticity be applied,

3. what effect do the deformational modes have on the possibility of failure of the discrete elements making up the continuum,

4. what effect does the failure of a discrete element have on the overall stability and how does the stress or load redistribute,

5. what effect does the elasticity of the discrete elements making up the continuum have on the stress or load distribution?

1.2 EXPERIMENTAL METHODS

Photoelasticity is the main experimental method used here but other techniques such as measuring deformations and using numerical methods are extensively employed. Block models made from an epoxy resin, Araldite D, are constructed and examined by measuring both the stresses and deformations after each increment of load is applied to the model. Tests on rock were performed so that collation parameters for a model-prototype problem could be obtained. Rock samples examined varied in size and shape from 1 cm diameter x 2 cm long cylinders to 24 in. x 18 in. x 15 in. rectangles. The prototype in this case were the numbers 2 and 5 ore bodies at Mount Isa Mines Holdings Limited.
1.3 MODES OF DEFORMATION

By experimentally observing the deformation it soon becomes clear that not only does elastic deformation take place but deformation of a different nature for the joints also occurs. If an opening is created by the removal of some of the blocks additional modes of deformation are introduced namely slip between, and rotation of, certain blocks, especially those near and contiguous to the opening. These slips and rotations are found to greatly affect the load distribution and subsequently control the induced stresses within the blocks.

Because of the close correspondence between the mechanical nature of slip and rotation and the modes of deformation involving slip and rotation the latter two modes are often referred to in this work as the mechanisms of slip and rotation. Also if a structural frame representing the rock mass which initially is redundant in that it contains rigid joints, progressively acquires enough pinned jointed hinges to become a mechanism, the acquisition of the final hinge gives what is termed the collapse mechanisms. Other mechanisms used in the literature are those of failure where the inference is that the mode of failure is either by crushing, shear or tension. The latter term of mechanistic failure will be avoided as far as possible in that the type of failure meant will be used and where the precise mode of failure is unknown the word stability will be substituted.

In a discontinuum both load distribution and deformation are most definitely constraint dependent. Consequently the type of constraint and its effect on deformational
response and load distribution should be clearly understood. To this end experiments are performed and reported in Chapter 4.

1.4 LOAD AND STRESS REDISTRIBUTION

The mechanisms of slip and rotation cause the loads to redistribute and the mode in which this is done appears to be -

1. geometry change caused by changing centres of pressure as slip occurs,

2. stress gradients caused by the moments tending to rotate the blocks,

3. joint stiffnesses are either increased or decreased by changing normal loads which in turn attracts or throws off more load.

Many times in this thesis reference is made to the elastic analogue theory as opposed to the elastic theory. The elastic analogue situation is meant to apply where even though the deformational response involves considerable hysteresis the elastic theory is still applicable if only the increasing or decreasing part of the deformational response is considered. This means that the elastic theory and elastic analogue theory are considered distinct because of the nature of the parameters connecting the constitutive relations especially those parameters such as Poisson's ratio.
1.5 EXPERIMENTAL PROGRAMME

Muller and Hofmann (1970) categorize the deformational behaviour of a rock mass in terms of joint system and the theory being used for analysing the design problem. The classification given is -

1. quasi monolith – one body system – mechanics of a continuum

2. jointed rock – imperfect multi-body system – mechanics of a discontinuum

3. cracked rock – perfect multi-body system – mechanics of a discontinuum

4. shattered rock – loose mass – mechanics of mylonite
   statistics and continuum mechanics; soil mechanics.

From the above it becomes clear that the theories of elasticity and plasticity appear to bracket the mechanics of discontinua. It is the author's opinion that the mechanics of discontinua are the study of particulate bodies which as individual elements behave in an elastic or plastic manner, while as a whole they interact as a structure. Therefore these three aspects, namely elasticity, plasticity and structural response must be clearly understood and thoroughly examined. Consequently, in Chapter 2 the concepts of elasticity and plasticity are examined in relation to a discontinuum.
In Chapter 3 the model analysis which forms the basis on which the models were constructed, is examined. The importance of assessing the deformational response of both the model and prototype is stressed because strict similarity is unattainable, while extended similarity is possible. Subsequent to this the construction of a 22 in. diameter diffusion polariscope and loading system is explained and then a description of the materials used and casting techniques developed for the construction of the models is given. Following this an analytical method for reducing the photoelastic data and obtaining the loads imposed on the block is developed. Tests were then performed on single blocks and the boundary loads compared with those obtained from a computer programme which uses the developed numerical analysis. Difficulty is encountered here because the stability of the resultant linear equations from which the boundary loads are evaluated has a small determinant value. This characteristic is even more evident when the values of the isoclinics are included in the equations. Nevertheless this approach has much to offer when investigating discontinuous material.

Simple slip is examined in Chapter 4 and its deformational response in relation to the plastic response discussed. Experiments are performed where it is shown that constraints in the form of fixed and hinged constraining blocks have a marked influence on the resultant deformational response. These constraints in turn affect and control the subsequent stress redistributions. In the latter part of this chapter the blocks are combined and their interaction examined as the system is distorted. It is shown that the removal of a
block or blocks readily induces shear forces in the jointing system and there is a tendency for some blocks to rotate.

On many occasions the elastic analogue method, using the assumption that the material is either isotropic or anisotropic, is applied to the analysis of a discontinuous material. In order to do this apparent or effective moduli are generally determined from samples which may or may not be representative of the blocky mass as a whole. In Chapter 5 this problem of determining effective moduli is examined and it is found that the values determined are so restrictive as to make the elastic analogue approach suspect when determining either the stress distributions or anticipated deformations. A series of experiments on Urquhart Shale samples of different sizes were performed and here again the effective moduli are shown to be very variable and as the samples became larger the moduli values appeared to be controlled more by the defects than by the deformational response of the intact material.

Deformational mechanisms such as slip and rotation redistribute the load. This redistribution of load induces specific stress patterns within the blocks which in turn consistently repeat themselves in the different models examined. In Chapter 6 these stress patterns and their location relative to the opening are examined. An important characteristic of these induced stresses is repeated appearance of tensile stresses and the common situation where effective tensile strains can develop. Another very important factor brought out in this chapter is the fact that here the continuum definitions of stress
and strain do not apply when considering the interaction or load distribution within the blocky mass.

Numerical experiments were performed on blocky models stacked in two different ways and blocks were excavated from these models under various loading conditions and joint properties. These blocky models were also loaded with rigid and semi-rigid foundations and the resulting load distributions compared with the photoelastic results. A description of the numerical model and its implications are given in Chapter 7.

Similar loading patterns and stress distributions on the block boundaries were obtained for both the numerical and physical block models examined. This result is most encouraging because it shows that the load distribution and deformational response is predictable if the blocky mass is considered a structure. Normalisation of the loads in terms of stresses is at the moment not possible in some cases because of the difficulty of incorporating stress gradients into the elasic unit used for defining stress.

In Chapter 8 the geological survey of the Nos. 2 and 5 ore bodies is investigated with particular reference to defects and their orientation and spatial distribution. In the latter part of this chapter the deformational characteristics of continuous and block models depicting the Nos. 2 and 5 ore bodies are investigated.

As will be mentioned many times in this thesis the main criterion being examined is the deformational response and its effect on load distribution within the
blocky material. However, the implications of this do not rest here as the whole mode of subsequent behaviour and analysis relies on understanding the various mechanisms and their progressive behaviour. In Chapter 8 the many factors examined throughout this thesis are brought together and parameters extracted so that the stability of the Nos. 2 and 5 ore bodies are assessed.
Chapter 2.

CONTINUUM AND DISCONTINUUM THEORIES

2.1 PREAMBLE

The continuum theories of elasticity and plasticity have been used in many instances to analyse a discontinuous material such as sand or weathered rock. It has long been recognised, however, that these continuum theories do not satisfactorily analyse materials such as hard broken rock or dense sand. Therefore, in this chapter a brief review of the continuum theories is given and the assumptions on which these theories are based examined.

In many situations an elastic or plastic analogue method is used to analyse the discontinuous material. The basis of this approach is that the deformational response of the discontinuum is the same as that required for the application of either the elastic or plastic theories. This alternative method is fraught with many difficulties and limitations and is examined in some detail in Chapter 5.

It is recognised that in a discontinuous material one of the main modes of deformation is slip. This deformational mode can cause significant geometry change which in turn influences the subsequent deformational response. Besides slip however, there is another important mode of deformation and this is rotation. Rotations can cause significant changes in the stress distribution with the consequent effect on subsequent deformational response.
After considering the problems related to discontinua, the relationship between small yet finite deformations is discussed. This development is important because if any constitutive relationship is to be introduced, the mechanisms of slip and rotation should be clearly recognised and if possible included as they effect the deformational response in markedly different ways. Once this is done it is then possible to examine (Chapter 5) the effects of rotation on the mass deformational response, and the associated problem of stress distribution.

Then accepting the fact that the theories of infinitesimal and anisotropic elasticity plus plasticity are inadequate for analysing a discontinuum the question is posed, how is a discontinuum to be analysed? In an attempt to answer this question work on elastic mechanics is examined and then a structural approach to the problem is adopted. This structural approach is one where loads and deformations rather than stresses and strains are considered. In order to express the resultant loads and deformations in terms of stresses and strains, a representative unit in the discontinuum is required. Basically, if the two dimensional case is considered there are three degrees of freedom at a point, namely the two displacements and rotation. To date the elastic or representative unit considered first one degree of freedom (Trollope, 1968) and then two (Burman, 1971). When considering the third degree of freedom, namely rotation, the representative unit taken in this instance is found to be inadequate for defining the resultant forces and displacements in terms of stresses and strains. This problem is introduced in the latter part of this chapter as it is
important when considering the numerical method of analysing a discontinuum.

2.2 INFINITESIMAL ELASTIC THEORY

The mathematical theory of elasticity requires some restrictive assumptions when defining terms and type of material to which these terms apply (Timoshenko, 1951). These assumptions are -

1. the strain about a point is homogeneous,
2. the strain field is irrotational,
3. the material is elastic.

Simplification of the mathematics is achieved if -

1. the displacements are infinitesimal,
2. a linear stress-strain relationship exists or is assumed,
3. the material properties are isotropic,
4. the material properties are homogeneous.

Once the above factors are satisfied, a solution of the stress-strain state can be obtained if -

1. equilibrium within and at the boundary is satisfied,
2. compatibility in the material is sustained,
3. the moduli connecting the stress-strain relationship are known.
A solution is obtained when the stress or strain state can be determined at any point within the boundary for a specific loading and/or displacement condition. None of the above assumptions allow for rigid body movement in either of the forms, rotation or translation.

The ease or difficulty in attaining a solution of a particular elastic problem depends, in the main, on the configuration of the boundary. If for the two dimensional case, the boundary is simply connected the mathematical solutions are independent of the material constants; whereas if the boundary is multiply connected the material moduli can effect the solution of the mathematical equations. This means that the stress distribution in a model of a hole in a solid piece of material is dependent upon the magnitude of Poisson's ratio (Frocht, 1942). An additional implication when considering multiply connected bodies is that the external contour may be in equilibrium while the internal contours may set up equilibrium conditions independent of the external contour, Figure 2.1, (Clutterbuck, 1958). This equilibrium condition is important when considering progressive failure or slip and the redistribution of loads (Chapter 4). Another important consideration is that if the stress-strain response is non-linear the uniqueness of the solution to a specified boundary condition is lost.

Some of the factors requiring consideration in a blocky material are -

1. the stressed body is generally multiply connected,
MULTIPLY CONNECTED BOUNDARIES

FIG 2.1

Rigid-Plastic (a)  FIG 2.5  Elasto-Plastic (b)
2. the intact material is most likely to be heterogeneous and anisotropic,
3. interaction between contiguous particles is most probable,
4. in situ stresses should be known,
5. large displacements can cause significant geometry changes,
6. rotation of the block material is possible and at the boundary surface most likely,
7. failure of the intact material will possibly occur,
8. non-linearity in the deformational response due to geometry effects is most likely,
9. knowledge of the properties of defects such as joints, faults and weathered zones though difficult to assess is necessary.

It may appear that the difference between a blocky and elastic material inhibits the use of elastic theory for obtaining a solution to the problem of determining the stress distribution. The sum effect of many of the above factors, however, is such that a linear deformational response may result. This would imply that it may be possible to use the elastic analogue approach. In this thesis it is shown that the use of the elastic or plastic analogue theory is so restrictive as to make their use very limited. It is shown that the reason for this is that the values of the moduli are generally stress and material defect dependent.
2.3 INCREMENTAL DEFORMATIONS

When a series of intersecting planes making up a blocky model are loaded, the mass response to incremental load will, in many cases, include slip and this in turn can cause significant geometry changes. The behaviour of these slip mechanisms is found in the main to rely on factors such as the joint characteristics and initial stresses. Consequently in order to correlate results obtained from experimental work, it is necessary to examine incremental deformation theory.

It is clearly evident that when examining block models experimentally, be they surface models, (Chappell 1967), or underground openings, rotation, slip and boundary constraints are basic factors effecting the distribution and redistribution of load. When an initial stress exists, Biot (1965) puts considerable emphasis on the effect changing geometry has on the overall response.

2.4 THEORY OF INCREMENTAL ELASTICITY

In the classical or infinitesimal theory of elasticity a first order differential is used to define the stresses and the independent variables are the co-ordinates of a typical particle in its initial state. With finite deformations a higher second order approximation is used. In this instance the independent variables are taken with reference to the changed co-ordinates of the particle. The question then arises as to whether the stresses and strains should be defined in terms of the original co-ordinates (Lagrangian) or in the distorted co-ordinates (Eulerian).
2.4.1 Definition of incremental strain

A point P is transformed to P' by pure deformation and then a rigid body rotation is imposed to transform P' to P". This sequence of movement refers the pure strains to the rotated axis 1 and 2, Figure 2.2. The pure strains are transformed to the original co-ordinate axes x and y to give the following definitions of strain -

\[ \varepsilon_{11} = \varepsilon_{12} = \frac{1}{2} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{1}{2} \left( \frac{\partial v}{\partial y} - \frac{\partial u}{\partial x} \right) \omega \]

\[ \varepsilon_{11} = \frac{\partial u}{\partial x} + \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \omega + \frac{1}{6} \omega^2 \quad (2.1) \]

\[ \varepsilon_{22} = \frac{\partial v}{\partial y} - \frac{1}{2} \left( \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \omega + \frac{1}{6} \omega^2 \]

where \( \omega = \frac{1}{2} \left( \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \right) \) is a measure of the rotation.

2.4.2 Definition of incremental stress

When defining incremental strain it does not matter whether the material is in a deformed state or not. However, when considering incremental stress, the initial stress state must be considered in the definitions of stress and it becomes important to separate the geometry from the physics of the system.

If the initial stresses are, Figure 2.3

\[ S_{11}, S_{12}, \]

\[ S_{21}, S_{22}, \]

and \( f_{\xi\xi}, f_{\eta\eta} \) and \( f_{\xi\eta} \),
and the point $P(xy)$ is displaced to the point $P'(\xi,\eta)$ the stresses on the block are now 

\[
\sigma_{\xi\xi} = S_{11} + f_{\xi\xi} \\
\sigma_{\eta\eta} = S_{22} + f_{\eta\eta} \\
\sigma_{\xi\eta} = S_{12} + f_{\xi\eta}
\]

(2.2)

If the axes $xy$ are rotated through an angle $\omega$ to $1,2$ the stress $f_{\xi\xi}, f_{\eta\eta}, f_{\xi\eta}$ will have components of stress which are purely of geometric origin. Again if the stresses due to pure deformation are referred to the rotated axes 1 and 2, we have

\[
\sigma_{11} = S_{11} + f_{11} \\
\sigma_{22} = S_{22} + f_{22} \\
\sigma_{12} = S_{12} + f_{12}
\]

(2.3)

The relationship between the stresses $f_{\xi\xi}, f_{\eta\eta}, f_{\xi\eta}$ and $f_{11}, f_{22}, f_{12}$, can be shown to be,

\[
f_{\xi\xi} = f_{11} - 2S_{12} \omega \\
f_{\eta\eta} = f_{22} - 2S_{12} \omega \\
f_{\xi\eta} = f_{12} + (S_{11}-S_{22}) \omega
\]

(2.4)

2.4.3 Stress-strain relationship

In order to express a stress-strain relationship the stresses and strains must be conjugate, that is the same
reference axes must be used to define the stresses and strains. As the strains refer to the axes \( x, y \), before
the incremental deformation is applied so must the stresses refer to these same axes. It can be shown that the
incremental stresses referred to the initial areas, Figure
2.4, are

\[
\begin{align*}
t_{11} &= f_{11} + S_{11} \varepsilon_{22} - S_{12} \varepsilon_{12} \\
t_{12} &= f_{12} + S_{12} \varepsilon_{22} - S_{22} \varepsilon_{12} \\
t_{21} &= f_{21} + S_{12} \varepsilon_{11} - S_{11} \varepsilon_{12} \\
t_{22} &= f_{22} + S_{22} \varepsilon_{11} - S_{12} \varepsilon_{12}
\end{align*}
\] (2.5)

Note that \( t_{12} \neq t_{21} \)

It can be shown that the following constitutive relations are obtained

\[
\begin{align*}
f_{11} &= C_{11}^{11} \text{exx} + (C_{12}^{22} - S_{11}) \text{eyy} + (2C_{11}^{12} + S_{12}) \text{exy} \\
f_{22} &= (C_{11}^{11} - S_{22}) \text{exx} + C_{22}^{22} \text{eyy} + (2C_{12}^{12} + S_{12}) \text{exy} \\
f_{12} &= (C_{12}^{11} - \frac{1}{2}S_{12}) \text{exx} + (C_{22}^{22} - \frac{1}{2}S_{12}) \text{eyy} + \\
&\quad\quad\quad\quad\quad\quad(2C_{12}^{12} + \frac{1}{2}S_{11} + \frac{1}{2}S_{22}) \text{exy}
\end{align*}
\] (2.6)

from which it will be noted that the parameters \( C_{11}^{11} \) and
\( C_{22}^{22} \) are independent of the initial stresses while all the
others are effected by the initial stress state.
To summarise, the main factors from the above analysis are -

1. in second order elasticity where rigid body rotation is significant, apparent strains are non-linear functions of the displacements, equation 2.1.

2. when an initial stress is extant many of the physical parameters are dependent on the initial stress state, equation 2.6.

Another important feature of incremental elasticity is that it does not matter how the initial stress state is attained but it is assumed that with the next increment of distortion an elastic potential exists from which the symmetry of the physical parameters are defined, (Biot, loc cit). Consequently the equilibrium equations are derivable from variational principles using this elastic potential. However it should be noted that if the principle of virtual work is used equilibrium equations determined from virtual work principles are valid if all the components contributing to the virtual work are recognised and included in the analysis.

Even though equilibrium has been established there are still more unknowns than there are equilibrium equations. Also at this stage no material properties have been postulated. Generally however, in order to obtain a solution it is necessary to express the stresses in terms of the strains and this requires the use of the material properties. There are no restrictions on these functional relations between stress and strain as they may be linear or non-linear and express properties such as elasticity, fluid viscosity,
plasticity, viscoelasticity. Herein lies the principle of the elastic analogue approach where if the strain rate is constant or equal to zero an elastic continuum solution, using effective moduli, can represent the stress distribution in the loaded mass.

Finally if a functional relation is obtained, all the distortional modes such as slip and rotation should be included as they in fact contribute to the virtual work.

2.4.4 Functional relation for a blocky material

The response of a blocky material to the imposition of load appears to involve three basic types of transformation, namely -

1. elastic deformation,
2. rotational deformation,
3. translation or slip deformation.

From the above it is evident that when considering an initially stressed continuum, rotation has a marked effect on the magnitude of both stress (equation 2.3) and strain (equation 2.1). Also evident is that non-linearities which are geometrical in origin are introduced. Therefore when defining the functional between stress and strain it is important to distinguish between the physics and geometrics of the relation.

Biot (loc cit) investigated non-linear effects of geometrical origin and established that non-linearity was mainly due to rotation when second order strains were
prevalent. The elastic strains from the intact material in the blocky models examined in this thesis were generally of the order of $10^{-4}$ to $10^{-3}$, whereas the slip strains were at times, far in excess of this (Chapter 5.). Even though equations 2.1 are relevant for a continuum they rely purely on kinematical definition and therefore the same definitions could be postulated for the blocky material examined. Consequently using the relations for second order displacements and small strains the following equations result -

$$\varepsilon_{11} = exx + \frac{1}{2}w^2$$

$$\varepsilon_{22} = eyy + \frac{1}{2}w^2$$

(2.7)

2.5 MATHEMATICAL THEORY OF PLASTICITY

2.5.1 Controlling factors

The nomenclature used here is similar to that employed in the theory of elasticity.

Unknowns of the deformation field are the six stresses and three displacements. Equations of motion constitute the three equations of equilibrium, but the six equations of compatibility which were used in the theory of elasticity are no longer valid. Consequently in order to obtain the requisite number of equations to solve for the redundancies, the use of a yield criterion is invoked.

A judicious choice of this yield criterion can greatly facilitate the solution of the resulting differential equations. The two criteria generally used are -
1. Tresca's criterion which states that at failure or yield the maximum stress is of specific magnitude, namely -

\[ \sigma_1 = k_1 \]

2. Von Mises' criterion which states that the sum of the squares in terms of the deviatoric stress at failure or yield be of specific magnitude,

\[ (\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2 = k^2 \]

Hencky (1924) showed that Von Mises' yield criterion is the state reached by a material when the elastic strain energy of distortion attains a minimum value.

If the material is isotropic the yield criterion should be expressible in terms of the stress field,

\[ f(\sigma_x, \sigma_y, \sigma_z, \tau_{yz}, \tau_{zx}, \tau_{xy}) = 0 \quad (2.8) \]

or

\[ g(\sigma_1, \sigma_2, \sigma_3) = 0 \quad (2.9) \]

where \( f \) and \( g \) are appropriate functions.

The above equations imply that since yielding depends only on the magnitude of the three applied principal stresses and not on their directions, the yield criterion can be expressed as,

\[ h(J_1, J_2, J_3) = 0 \quad (2.10) \]

where \( h \) is some function and \( J_1, J_2, \) and \( J_3 \) are invariants of the stress tensor and are represented as -
\[ J_1 = \sigma_1 + \sigma_2 + \sigma_3, \]
\[ J_2 = - (\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1), \]  \hspace{1cm} (2.11)
\[ J_3 = \sigma_1\sigma_2\sigma_3. \]

The correct choice of a yield criterion depends largely upon observing the stress-strain response for the material, substituting the appropriate failure stresses into the chosen yield criterion, and determining the required yield parameters. Three regimes are generally observed in the stress-strain response, namely -

1. the stress-strain relation for the elastic range,
2. the yield condition if this is discernible, if not, an arbitrary criterion is generally applied, such as the stress at 2\% strain,
3. the stress-strain relation for the plastic range.

Saint Venant, Levy and Von Mises used as their failure criterion the rigid plastic concept, Figure 2.5(a). Experimental work in the first instance by Lad and then confirmed by Taylor and Quinney (1931) showed that for the Von Mises' criterion the yield curve in principal stress space showed marked deviations. Prandtl and Reuss (Prager and Hodge, 1951) obtained better fitting yield curves by utilizing the elastic plastic stress-strain relationship, Figure 2.5(b).

Since there are nine unknowns, six stresses and three displacements, with three equilibrium equations, the result is that six more equations must be defined or six
independent planes of slip must be found to mobilise uniform plastic strain. When plastic deformation occurs, however, it is generally assumed that there is no volume change, that is

$$\dot{\varepsilon}_1 + \dot{\varepsilon}_2 + \dot{\varepsilon}_3 = 0,$$

where $\dot{\varepsilon}_i$ are the principal plastic strain rates. Consequently only five independent slip planes are necessary for the kinematical requirements of plasticity theory.

2.5.2 Residual stresses

Another factor which for a strain hardening material affects the application of the mathematical theory of plasticity, is that the yield stress is a function of the previous plastic distortion, Figure 2.6(a). Consequently if a favourable residual stress exists this can raise the apparent load required to cause further yield, while an unfavourable residual stress can lower the apparent load required to cause yield, (Calladine, 1969). Thus if the deformational response softens the effect of residual stresses could be disastrous. On the other hand, if the mass deformational response is ductile, Figure 2.5, the collapse load is quite unaffected, (Calladine loc cit) by the residual stress.

2.5.3 Boundary and slip constraints

With a blocky rock material constraints such as the type of support and confining pressure greatly affect the deformational response and apparent physical characteristics (Chapter 5). This effect has long been
recognised in soils where the statement that the apparent strength parameters are a function of their environment, is readily accepted. In some cases the Mohr-Coulomb equation, which is used extensively in soil mechanics, is used to describe the strength characteristics of a broken or weathered rock. Rosengren and Jaeger (1968) have shown that for a closely jointed rock, represented in this instance by a heat treated granular calcite the strength parameters $C$ and $\phi$ are greatly affected by interlock and confining pressure. The degree of interlock in the block models examined appears to be dependent upon the stacking geometry, joint orientation and form of constraint (Chapter 6).

2.5.4 Plasticity of soil material

Rankine (1857) using an isotropic, homogeneous, semi-infinite, cohesionless material with a rigid plastic stress-strain response determined the static equilibrium relationship between the principal stresses. The following equations are derived, if Coulomb's criterion for failure, namely -

$$\tau = c + \sigma \tan \phi$$

where $c =$ cohesion,  
$\tau =$ shear stress, 
$\sigma =$ normal stress across plane of failure,  
$\phi =$ angle of friction,

is used in Mohr's general hypothesis of failure, namely -

$$|\tau| = f(\phi)$$
where $f$ is some function, and then

$$\sigma_1 = N_\phi \sigma_3 + 2c\sqrt{N_\phi}, \text{ for the passive case}$$

and

$$\sigma_3 = \frac{\sigma_1}{N_\phi} - \frac{2c}{\sqrt{N_\phi}}, \text{ for the active case}$$

(2.12)

where $N_\phi = \frac{1 - \sin \phi}{1 - \sin \phi}$

This was, in the main, the state of affairs which existed until Drucker, Prager and Greenberg (1952) proved the limit theorems which enabled bounds to be placed on the correct solution. Equations 2.12 are derived using equilibrium, then applying a yield criterion and subsequently determining the principal stress relationships. These expressions are consequently lower bound solutions. Alternatively if any acceptable kinematic mechanism of failure is chosen and the rate of dissipation of internal energy is equated to the rate at which the external forces do work, the resulting relationship between the stresses and material parameters, defining yield, will be greater than the true solution, therefore signifying an upper bound value. One of the main reasons for being able to determine these bounds is brought about by the uncoupling effect of the stresses and strains using a yield criterion (Calladine loc cit). That is at and after yield the stress magnitude is independent of strain. Therefore to apply these limit theorems to a granular material requires that the appropriate deformational response is obtained, Figure 2.5. There are, in most cases, however, basic differences between the ideal response and that of a granular material, namely -
1. the strength of a granular material is not constant but is dependent on the hydrostatic or octohedral stress whereas for a plastic material the strength is independent of hydrostatic stress,

2. a granular material generally dilates whereas a plastic material has zero volume change.

3. the deformational response generally strain hardens or softens.

2.5.5 Coupling up of stresses and strain in a discontinuum

When the problem of stress redistribution is encountered in a blocky material the kinematical mode of failure or collapse becomes most important. This is because in the structural representation of the blocky material collapse hinges or what are often termed virtual plastic hinges (section 2.8) are formed and this leads to eventual collapse. The development of these hinges appear to be largely dependent upon the mechanisms of slip and rotation coupled with the general stress environment.

This means that the loads and deformations are not uncoupled and though the structure may behave plastically it is important to recognise this effect.

2.6 PLASTICITY IN ROCK

Rock can be a hard continuous solid or a relatively soft discontinuous granular material. In between these two extremes there is a range of rocks whose variability
in performance requires the science of rock mechanics.

The mode or mechanism of collapse in a rock mass appears to be primarily a function of the material and defect response to the imposition of load. In turn the rock material may become truly plastic at high confining load or fail in a brittle manner at lower confining loads. Nevertheless the mass response to load which does not involve failure may at times, be quite independent of the individual block response. This latter statement has important implications which are examined in Chapters 6 and 7.

Generally at engineering pressures the intact rock behaves elastically up to brittle failure and if there are few or no constraints the supporting potential of the rock quickly drops off to zero. If, however, constraints are induced or are already there, frictional forces are mobilised which can impart a residual strength, Figure 2.7. Consequently, if the rock mass has a joint system the stress-strain curves may be any one of three types shown in Figure 2.8. The stress-strain response, in this instance, is largely dependent on joint roughness, constraints, and the imposed stress environment (Chapter 4). To summarise, it is proposed that an elastic, elasto-plastic strain hardening or strain softening response can be obtained for certain boundary constraints. It is recognised that what is termed a plastic type response may be caused by slip or friction. There is nevertheless a subtle but strong difference between frictional and plastic behaviour, in that one is a mechanism while the other is flow.
True plastic flow is described as the slip induced in atomic lattices, which in many instances are started at or arrested by a dislocation. Therefore appropriately orientated dislocations can induce or inhibit plastic flow. When flow becomes large enough we have the mesoscopic effect of Luder lines as observed in many metals. This problem of defining slip or gross slip along pre-existent joint sets is the main topic of Chapters 4 and 6, but is mentioned here because it shows the possibility of obtaining gross deformational responses to which either an elastic or plastic analogue type mathematical analysis may possibly be applied. In the work that follows it is shown that the rock mass may respond in either an elastic or plastic mode but the uniqueness or definition of the parameters is lost.

2.7 DISCONTINUUM OR CLASTIC MECHANICS

Many models, such as stress dilatancy developed by Rowe (1962) and Litwiniszyn's (1957) work on stochastic theory, have been developed to predict the mass response of a discontinuum to imposed loads. These approaches require a statistical assessment of the material properties which in some cases is a problem of considerable difficulty. Consequently it is necessary to understand individual mechanisms such as slip, rotation, and unit interactions because the gross effect under different constraints make the interpretation of these separate effects very difficult.

Most theories of failure consider the state of stress imposed on the material when the load sustaining potential
of the material is reduced or completely lost. It is becoming increasingly evident (Reinius, 1955), (Bieniawski, 1967), (Wawersik, 1968), that prior to complete fracture, failure of individual units within the material is occurring, giving rise to the very important question of whether progressive failure is in fact controlling ultimate collapse rather than material strength. This latter type of failure is not only confined to the microscopic and mesoscopic scale of sample testing but possibly to situations of macroscopic scale such as the Vaiont slide, (Muller, 1968).

2.7.1 Structural representation

Reinius (1955) examining the problem of failure in concrete considers that all stress transmission between particles takes place through a three dimensional pin jointed lattice connecting the particles. Two types of connection are considered, Figure 2.9(a) and (b). A double lattice, Figure 2.9(a) exists when the particles are relatively close together and shear forces are transmissible; while a single lattice, Figure 2.9(b) exists when the particles are further apart and only direct tensions or compressions are transmitted. Failure in the lattice is achieved when a specific magnitude of tension in any one of the lattice members is attained. Geometrical parameters, such as lattice lengths, cross-sectional areas and angles of inclination are determined so that a Poisson's ratio of 0.15 is achieved.

Experimental corroboration, though not good, for this structural approach was obtained. Slip deformation is
Types of deformational response

**FIG. 2.8**

(a) Bodies at small distance between bodies, strut connections between particles

(b) Greater distance between bodies

*(after Reinus 1955)*

**FIG. 2.9**

Lateral strain

Longitudinal strain

--- Struts fail in tension

--- Struts also fail in compression

*(after Reinus 1955)*

**FIG. 2.10**
not considered and of consequence the explanation that
the increase in curvature in the observed load-deformation
graph, Figure 2.10, as being solely due to the failure of
the individual particles rather than possible slip is
suspect. Reinius (loc cit) considers the concrete to act
as a structure throughout its loading history. On the
other hand it appears that for intact rock with the load
up to about 70% of its ultimate failure load (Bieniawski,
1967), the deformational response is elastic. However, as
defects increase the deformational response becomes
dependent upon the magnitude of the hydrostatic or insitu
confining pressure and the roughness of the defect faces.
Once slip occurs, however, the stress redistributes
(Section 4.7) and the material can no longer be considered
continuous and is best treated as a structure.

2.7.2 Clastic model

Trollope (1968) from a three dimensional consideration
introduces a two dimensional model where the lines of
action of each force on every circular disc are restricted
to pass through the centroid of the disc, Figure 2.11.
This is the same as saying that the discs act as pin
joints on which six forces or member reactions are imposed.
Consequently the load transmission within the body is
represented as a pin jointed truss, Figure 2.12. In this
pin jointed truss the particle weights \( W \) are applied at
the pinned joints, and the angle \( \theta \) in the framework is
analogous to the distribution angle \( \theta \) defined by Trollope
(1968), Figure 2.11. It is evident that the truss is once
indeterminate, therefore in order to solve the truss the
arching factor \( k \) is invoked. Two limit states are
introduced namely, the no arching case when \( k = 1 \) and the
horizontal member $r = 0$, and the full arching case when $k = 0$ and the diagonal member $q = 0$. This then allows the pinned frame to be solved.

Existence of this arching action is both an experimental and practical fact (Trollope, Speedie and Lee, 1963), (Chappell, 1967). However, definition and behaviour of $k = f(xy)$ within the stressed zone or on the boundary of deformation is most difficult to predict.

Generally it appears that $k$, amongst other things is a function of slip and general stiffness of joints and is associated with the differential boundary deformations. Nevertheless by taking the limit values of $k = 1$ and 0 respectively the pinned framework becomes determinate. A difficulty arises however for the arching case, $k = 0$, Figure 2.13, because the thrusts in the horizontal members become excessively large implying failure is always imminent for this case. If it is realised that $k$ is in the main a function of differential boundary displacements, and that $k$ in fact varies throughout the body, it is not difficult to achieve an equilibrium condition. On the other hand if the frame is symmetrical equilibrium is readily obtained without considering variations of $k$.

Up to this stage the particles have been considered rigid and the limit values of $k = 0$ or 1 are used to make the system statically determinate. This concept derives from the many observations (Rennie, 1959), (Parkin, 1966), (Trollope, 1968) that as arching develops a gap forms between the particles, indicating that the force $q$ tends to zero thus giving the case of full arching $k = 0$ when
Centroidal forces in Systone or representative unit

**FIG. 2.11**

Structural representation

**FIG. 2.12**

Structural frame $q = 0$ with weight acting at each node.

**FIG. 2.13**
q = 0. However if slip or the tendency to slip is induced, shear forces are developed which breaks down the condition that the acting forces pass through the centroid of the disc. The structural frame is no longer pin-jointed and is somewhat redundant.

Trollope and Burman (1970) stress the importance of recognising the path dependence of the failure process especially if this failure is brittle in character. This statement becomes even more relevant when progressive failure has to be considered. In this latter paper full arching $k = 0$ is stated as being the upper limit to the magnitude of stress distribution which when compared with the allowable material stresses, defines the stability potential of the slope. Intuitively this appears to be valid, yet if there is any other mode of stress raiser, for instance material inclusions, openings, defects, slip, rotation, then this intuition becomes suspect.

2.8 SLIP MECHANISMS

In many instances yield in plasticity is attributed to relative slip within the material. However, in this thesis, relative slip does not necessarily depict a plastic deformational response. Also when slip along a system of randomly orientated joints occurs material failure is likely to happen especially if there is a stress increase or decrease in the same area. It is also becoming increasingly evident (Chapter 4) that once slip occurs, say along a block boundary, the analysis using stress and strain is unacceptable and the behaviour is more that of a structure than that of a continuum.
Analysis of the structure is best carried out in terms of loads and displacements, as considered below.

From this the important question is posed, under what circumstances, when slip occurs, can the discrete system still be represented by an appropriate combination of particles giving a representative unit (systone) from which stress can be defined (Trollope, 1968). Analysis in Chapters 6 and 7 show quite clearly that a representative unit can be found in a discrete system where moment transmitted across the joints is not large relative to the normal and shear forces (circular discs, Burman, 1971). However where moments are transmitted across the joint (interlocking system of blocks or discrete particles) an appropriate representative unit has not yet been determined, therefore the load distribution analysis is best performed by a structural approach using forces and displacements rather than stresses or strains.

2.8.1 Structural collapse mechanisms

It is clearly evident that the theories of elasticity and plasticity become inadmissible when slip and subsequent stress redistributions occur. Therefore, how can a material responding in such a way, where the continuum theories are inadmissible be analysed? In the following chapters it will be shown that the deformational responses of the blocky material are in nearly all instances dependent upon mechanisms such as slip and rotation. This is most definitely a pre-failure examination of the deformational response and stress distribution. Nevertheless, subsequent material behaviour and therefore ultimate failure is very much dependent on the development of stress.
With the above factors in mind collapse mechanisms in the structural sense due to the formation of virtual hinges from the mechanisms of slip, rotation and material failure are examined. Constraints of course generally restrict the development of hinges which control the impending collapse mechanisms. It should be stressed that these mechanistic solutions do not have the generality or uniqueness as do solutions obtained from elastic or plastic theories.

2.8.2 Virtual hinge formation

It is clearly evident that when creating an opening in a blocky material arching, and beaming of the induced loads occur (Chapter 6.) As this arching action develops it is necessary as a prerequisite for relative deflections to take place. These deflections are caused mainly by slips and rotations and when they take place the formation of virtual hinges are likely to occur. Insight into these phenomena is obtained by examining the Vousoir arch, flying buttress, and plate bande.

2.8.3 Vousoir arch

Initially in the design of a Vousoir arch, concern appeared to be based on the properties of the jointing planes and wedge action of the blocks. Design criteria revolved around the assumption that the line of thrust was normal to the faces of contact between the vousoir blocks. This required the depths of the vousoirs towards the springing to be increased. A French engineer (Couplet) performed some model experiments and showed that the model arches failed by rotation of some of the vousoirs about their edges. From this the assumption was made that the vousoirs were infinitely rough.
DEVELOPMENT OF VIRTUAL HINGES

(a)

FIG 2.14

APPLIED LOAD

DEFORMATION

DEFORMATIONAL RESPONSE

(b)

INCLINED PLATE-BANDE
or FLYING BUTTRESS

FIG 2.15

PASSIVE THRUST LINE

(a)

FIG 2.16

ACTIVE THRUST LINE

(b)
New theories of behaviour were developed in 1800 due to the experimentation of the French engineers Rondelet and Perronnet. They determined that the arches failed by the opening of the joints at the crown, haunches and abutments, Figure 2.14. The doctrine evolved that the line of pressure or thrust must be within the middle third of voussoir contact faces. Determination of this line of thrust has been the subject of much speculation and discussion until Winkler and Castigliano submitted their minimum energy elastic solution to the analysis. The assumptions used were that if the abutments were rigid, and the line of thrust was everywhere within the middle third the arch would behave as a continuous elastic rib. No tension was allowed to develop at any point within the arch. Much doubt has been expressed in the use of Castigliano's method because of the uncertainty as to whether the masonry arch exhibited a linear response between load and displacement.

An exhaustive series of experiments were then carried out in 1890, by the Austrian Society of Engineers to determine the validity of assumptions then used in the design. It was found that until cracks started to develop the measured displacements were approximately proportional to the applied loads; also, the development of these cracks did not cause much alteration to the arch's shape. From this it was accepted that elastic theory was applicable to the design of such structures. However, it was also assumed that the arches behaved as solid ribs and that the thrust could be calculated by Castigliano's method of minimum strain energy. As noted by Pippard et al (1936), this latter proposition does not follow because of the mere fact of a linear
relationship between displacement and loading. This last point is best elucidated by describing the results of one of Pippard's experiments.

Initially a solid rib of mild steel with pinned ends and the same dimensions as the voussoir arch was tested and a relationship between the horizontal thrust and unit load applied at any point in the span of the arch determined. A voussoir arch, also with pinned ends, was now tested and found that, up to certain limiting loads, the behaviour was almost precisely the same as the solid mild steel rib. The limiting load was found to be that load which produced tensile stress in the joint adjacent to the loading point. As the load was steadily increased other areas of tension were developed until eventually the structure collapsed. The development of these tensile regions in the joint plus the consequent cracking of the voussoirs was of the utmost importance in the development of the arching action. The load on an arch was increased from $W_0$ through to $W_4$, Figure 2.14, where $W_1$, $W_2$, $W_3$ and $W_4$ were limiting loads defining the development of cracks, which in turn were defined as virtual hinges (Baker, 1969). It is observed that at the development of each hinge the redundancy of the structure is progressively reduced. A stage is reached where, in this case the fourth hinge, too many hinges are developed and the structure becomes a mechanism.

Pippard and Ashby (1938) also performed many tests using different material strengths (limestone, granite, and concrete), and lime or cement as a jointing material for the voussoirs making up the arch. They determined, amongst other things, that -
1. the middle third criterion using the elastic theory is much too pessimistic,

2. jointing material which allowed tensile stresses to develop increased the range of loading for which the arch behaved as a solid elastic arch rib,

3. generally the strength of the material of which the voussoirs were made did not effect the stability of the arch,

4. if the compressive stress developed in the voussoirs exceeded the material strength, premature failure of the structure occurred,

5. the design of the arch be performed from stability criteria rather than from elastic theory which requires very restrictive assumptions.

Instability of the voussoir arch examined here is regarded as developed when the fourth virtual hinge is formed and the system becomes a mechanism, Figure 2.14. Just before the development of the collapse mechanism, the system is statically determinate and with the formation of the final mechanistic (virtual) hinge the one remaining unknown factor, namely the load causing instability, can be evaluated. Coulomb (1773) well knew this aspect of stability in a masonry structure when he stated that the line of thrust must be contained within the stonework. Heyman (1966) also reports that Coulomb noted that the line of thrust cannot be supposed to pass exactly through corner edge points like A, B, C and D, Figure 2.14, as this would lead to infinite stresses.
Consequently if the nominal axial thrust is reduced to 1/10 of the crushing strength this allows the line of thrust to approach the corner to within 5 per cent of the depth of the section. This latter point will be considered in greater detail when aspects of induced tensile stress controlling strength are examined with reference to stress redistributions in Chapters 6 and 7.

To summarize, the most important aspects of the above discussion are –

1. just before instability, the masonry structure is always statically determinate,

2. instability is caused by transforming the statically determinate structure into a collapse mechanism,

3. instability is reached by the progressive development of virtual hinges,

4. stability of the arch is not necessarily a function of the material's compressive strength,

5. induced tensions, in fact, could be a prime factor in determining the arch's stability,

2.8.4 Flying buttress

An inclined beam, Figure 2.15, is a very useful device for transmitting a horizontal thrust from a higher elevation to a lower one or vice versa. Now if this inclined beam is made up of blocks with little or no bonding material, a situation very similar to the hanging walls of the number 2 and 5 ore bodies at Mount
Isa mines is obtained. Of fundamental importance, however, is that all the basic principles examined in the voussoir arch apply to the flying buttress. However in this instance the deformation modes pertaining to the abutments develop two important lines of thrust, termed the passive and active states, Figure 2.16.

In the first instance the inclined beam is supported between the surfaces A and B, Figure 2.15. If these supports tend to spread, the forces shown in Figure 2.16(a) are developed and the line of thrust, termed the passive case (Heyman loc cit) is mobilised. If on the other hand, the supports are pushed closer together, force H increases and the line of thrust, termed the active state, is produced.

These thrust lines define in effect the point of application and directions of the imposed thrust forces. Therefore, it should be noted, that there is more likelihood of sliding along the joints at the head of the inclined beam when it is in the passive state, Figure 2.16(a). This condition is important when considering the interaction of openings where the relative movements of abutments may cause either a spreading or closure.

2.8.5 Plate band or blocky beam

Trollope (1966) showed vividly in a series of block model tests that by excavating 5/8 ins. smooth plastic cubes a beaming action, which tended to retain the suspended intradosal material, was associated with the arching action. It appears that the formation of this beaming action depends to a large extent, Chapter 6, on the
deformation occurring in the hanging wall and magnitude of thrust imposed. In a blocky beam, Figure 2.17 it is impossible to determine the line of thrust if the blocky material and supporting abutments are assumed rigid. However, from experiments performed the abutments do move and loads redistribute. Consequently the hanging wall tends to sag and in many instances the load on the central span becomes independent of the incumbent overburden.

It therefore appears that it would be best to treat the material making up the beam as elasto-plastic. McDowell et al (1956) considers an elasto-plastic block material making up a masonry wall where tensile forces between blocks cannot develop. After considering the forces and movement initiated by the rotation of two blocks between rigid supports, experimental evidence is presented, which shows remarkable agreement with the derived theory in spite of the simplifying assumptions. It is clear that the thrusts induced are caused by compressive strains produced from the tendency of the blocks to rotate. Also evident is that these induced thrusts can develop effective tensile strains. Questions then arise, what strength parameters control the ultimate behaviour of the beam, are they the moduli of rupture or compressive and/or tensile strength? Here again it is possible that slip and/or rotation mechanisms control these strength parameters (for example effective tension) rather than the parameters generally used or thought to be valid.

In summary it is found that if rigid or semi rigid abutments restrain the blocky beam, its load carrying capacity is four to seven times greater than the load
carrying capacity of a simply supported beam which derives its strength from the flexural modulus of rupture. Nevertheless it should be recognised that if the span to depth ratio is less than 12, elastic flexure is negligible, while if greater, the flexural criterion of strength is important.

In both these papers there appears to be a hesitancy in defining what strength moduli should be used in the ultimate load computations, namely tensile, compressive or flexural. Fortunately the tensile and flexural strengths are of the same order of magnitude and when these are used agreement between theory and experiment is relatively good.

Up to this stage the necessary horizontal stabilising forces have been induced by the rigid supports and block rotations. However, insitu conditions in the underground mining situation have an extant horizontal thrust which influences the intradosal material in much the same manner described above. Trollope (1966) invokes two movements which cause the failure of the blocky beams namely -

(a) individual blocks slide out in shear along vertical joints,
(b) because of bending additional lateral thrusts develop plastic hinges (crushing) leading to eventual collapse.

The most restrictive aspect of the above work, however, is that as the deflection of the beam becomes equal to its depth, collapse is imminent. If we consider that after each stage of progressive slip, a quasi-state of equilibrium exists and if the stage such as Figure 2.17(a)
Funicular Polygon
(b)

Magnitude of Horizontal Force

Mechanistic Beam
(a)

FIG. 2.17
is reached, the only way in which the beam would remain intact, that is without stress redistribution, is for the force system and funicular polygon, Figure 2.17(b) to close. Note that the pole is restricted to the location shown by the magnitude of the horizontal thrust. This funicular polygon shows clearly the obliquity of the resultant forces on the vertical joints. It should be noted that the maximum obliquity occurs at the joints adjoining the abutments and that the shear forces acting on the vertical joints require a redistribution of normal forces on these joints in order to keep the blocks in a state of equilibrium.

2.9 SUMMARY

The examination of deformational response and stress distribution in a blocky material shows that many factors affect these two basic phenomena. It has been indicated that for a blocky material the uncoupling of stress and strain is not a realistic assumption just as it is very questionable to assume that there is a constitutive relation between stress and strain and that an elastic analogue solution is valid.

It is implied that by examining the mechanisms of slip and rotation insight into the stress distribution across a single plane could be obtained and the deformational response anticipated. This statement is corroborated in Chapter 4 where the simple mechanisms of slip and rotation are examined.

When there is a mass of blocks and there are multiple slips and rotations the deformational response and stress
distribution take on a few orders of complexity. Nevertheless by considering the blocky mass as a structure it is possible to analyse the system. It now becomes necessary to try and relate the loads and deformations. That is what are the necessary parameters required in order to analyse the blocky mass?

It is also observed that a blocky structure which initially has infinite redundancy reduces this by the formation of virtual hinges as the load increases. Just before collapse the number of virtual hinges formed are sufficient to make the system statically determinate and the formation of one more hinge creates the collapse mechanism. It should be emphasised that the formation of these virtual hinges is very much load path and constraint dependent. Suggested modes of hinge formation are -

1. crushing failure of material,
2. slip between joints,
3. splitting or tensile failure of material,
4. shear failure of material,
5. rotation of block.

Also apparent is that the formation of these virtual hinges cause the stress to redistribute because of changing geometry. This implies that progressive failure and stress redistribution are interdependent. This should not be confused with progressive failure due to the strain softening characteristics of the virgin material, Figure 2.8. At this stage it is anticipated that these two phenomena are different. In many situations, however, these characteristics are superimposed and mask one another.
Chapter 3.

MODEL ANALYSIS, CONSTRUCTION AND EXPERIMENTAL TECHNIQUE

3.1 PREAMBLE

The approach used when examining the problems associated with the deformational response and stress distribution in a blocky material is the photoelastic method.

In this chapter the model analysis which formed the basis for constructing the blocky models is presented. Then a brief description of the apparatuses constructed, namely the polariscope, loading frame and photoelastic material, is given.

It is at this juncture that an important method of reducing the photoelastic data is developed. This method was used for reducing most of the photoelastic data examined in Chapter 6. Some results obtained from carefully conducted experiments on single blocks are compared with results obtained from the computer programme. These results are discussed and their accuracy assessed.

3.2 MODEL ANALYSIS

Relationships which govern the principles of similitude between a model and its prototype may be determined by one of two ways, namely

1. using the established laws of structural mechanics,

2. using the methods of dimensional analysis.
As established laws of structural mechanics had not yet been realised in this work the latter approach was used for determining the relationships between the relevant parameters.

As the main function in this thesis is to examine deformational response and stress distribution after slip has occurred, a quasi-static state of stress is examined. Elastic criteria are first examined and then subsequently inelastic deformations are considered. For inelastic response slip and rotational mechanisms are considered. This is necessary because generally a design is based on the response obtained from the intact material while the joint characteristics such as strength and orientation are incorporated as strength criteria (Jaeger and Cook, 1969). This, as stated previously, can give a completely erroneous picture to both the mass response and stability characteristics of an engineering structure in blocky material.

3.2.1 Elastic model

In the related structures of model and prototype the stresses and displacements will depend on three major features; the geometry of the structure, the physical properties of the respective materials, and the nature of the applied loads. Using Buckingham's first and second theorems and considering the variables given in Table 1 with the fundamental dimensional units of Mass (M), Length (L) and Time (T) we obtain the relations -
\[
\frac{\sigma_p}{E_p} = f\left( \frac{x_p}{L_p}, \nu_p, \frac{F_p}{E_p L_p^2}, \epsilon_p \right) \quad \text{for the prototype}
\]
\[
\frac{\sigma_m}{E_m} = f\left( \frac{x_m}{L_m}, \nu_m, \frac{F_m}{E_m L_m^2}, \epsilon_m \right) \quad \text{for the model}
\]

where \( f \) is some function.

If equality of all the dimensionless factors is achieved strict similarity exists.

<table>
<thead>
<tr>
<th>Variable</th>
<th>( \sigma )</th>
<th>( E )</th>
<th>( F )</th>
<th>( \nu )</th>
<th>( L )</th>
<th>( x )</th>
<th>( \epsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dimension</td>
<td>( ML^{-1}T )</td>
<td>( ML^{-1}T^{-2} )</td>
<td>( ML^{-1}T^{-2} )</td>
<td>( - )</td>
<td>( L )</td>
<td>( L )</td>
<td>( - )</td>
</tr>
</tbody>
</table>

**TABLE 1.**

If we consider the two dimensional case and if the boundaries of the model and prototype are simply connected the stress distribution for both plane stress and strain would, if the body forces are constant, be controlled by the equation

\[
\nabla^2 (\sigma_x + \sigma_y) = 0
\]

It is at once evident that the stress distribution is independent of the material properties. This however, is not the case if the boundaries of the prototype and model are multiply connected or if the stress distribution is three dimensional, because the stress distribution is then dependent on the value of Poisson's ratio (Frocht, 1942). Consequently it was considered important to have similitude between Poisson's ratio for the intact rock.
and model material, which to all intents and purposes was achieved,

\[
\begin{align*}
\nu \text{ Shale} &= 0.25 \\
\nu \text{ Araldite D} &= 0.25
\end{align*}
\]

It should be noted that for three dimensional stress problems the distribution of stress is even more strongly dependent on Poisson's ratio than it is for the two dimensional multiply connected case. Nevertheless, as an initial study the two dimensional case is examined. Unfortunately in three dimensional photoelasticity, using stress freezing techniques, the model materials presently available have a high Poisson's ratio, namely 0.4 to 0.5 and this, it is felt, has considerable bearing on the mode, development, and subsequent behaviour of mechanisms as examined here.

For strict geometric similarity, therefore, we require

\[
\frac{\varepsilon_{\text{model}}}{\varepsilon_{\text{prototype}}} = \frac{x_{\text{prototype}}}{x_{\text{model}}} = \frac{L_{\text{prototype}}}{L_{\text{model}}} = \lambda \quad 3.3
\]

and for force similarity

\[
\frac{F_p}{E_p L_p^2} = \frac{F_m}{E_m L_m^2}
\]

giving

\[
\frac{F_p}{F_m} = \frac{E_p L_p^2}{E_m L_m^2} = \mu \quad 3.4
\]
also \( v_p = v_m \)

Equations 3.3 and 3.4 give

\[
\frac{E_p}{E_m} = \frac{\sigma_p}{\sigma_m} = \frac{\nu}{\lambda} = \eta
\]

3.2.2 **Inelastic behaviour**

If the deformational response of the prototype is such that inelastic deformations are important, the requirement for similitude is that deformational response at corresponding points in the prototype and model be proportional (Rocha, 1965), Figure 3.1. That is the load deformation graph for the model should be attainable from the response of the prototype by multiplying the ordinates and abcissae by \( \frac{1}{\alpha} \) and \( \frac{1}{\beta} \) respectively, where \( \alpha \) and \( \beta \) are parameters defined in the aforementioned figure. Because of this important requirement much time and effort was expended, Chapter 5, in determining the deformational response of the models.

Using the notation of Figure 3.1, the following relations are obtained -

\[
\delta_m = \frac{\delta_p}{\lambda \beta}
\]

where \( \delta_m \) and \( \delta_p \) are displacements in the model and prototype, and

\[
\sigma_m = \frac{\sigma_p}{\alpha}
\]
**FIG 3.1**

Similarity between deformational responses.

**FIG 3.3**

Line diagram of diffusion polariscope.
where \( \sigma_m \) and \( \sigma_p \) are stresses in the model and prototype.

Force similarity gives

\[
\frac{F_p}{F_m} = \mu ,
\]

and \( \frac{E_p}{E_m} = \eta , \)

therefore \( \delta_m = \eta \frac{F_m}{F_p} \frac{L_m}{L_p} \delta_p \) \hspace{1cm} 3.6

\[ \delta_m = \frac{\eta}{\mu} \frac{1}{\lambda} \delta_p \] \hspace{1cm} 3.7

Because \( \varepsilon_m \) is no longer equal to \( \varepsilon_p \) strict similarity is not achieved. However, by combining some of the non-dimensional groups a single relationship is obtained, giving an "extended non-dimensional value" which is often referred to as extended similarity (Monch, 1964)

3.2.3 Material and defect size

It is important to realise that the mass response and consequently the mechanistic behaviour of the blocky material is, in fact, dependent on the deformational characteristic of the intact material and its strength, especially its tensile value, Chapter 6. Therefore, when considering the intact material the elastic moduli for prototype (\( E_p \)) and model (\( E_m \)) forms the relation

\[
\frac{E_p}{E_m} = \mu = \eta ,
\]
because for the chosen samples determining the moduli relation

\[ L_{\text{model sample}} = L_{\text{prototype sample}} \]

The displacement correlation is given by

\[ \delta_m = \frac{\delta_p}{\lambda} \]

When modelling a rock mass, defects in the rock should be fully assessed. This means that the results of a geological survey with a description of the discontinuities in the area of interest should be available (Chapter 8). The discontinuities of relevance here are mainly faults and joints with a description of their orientation and spatial distributions (Rosengren, 1968, Best, 1970). Strength of the intact material and the slip characteristics along the discontinuities are another two important factors effecting the deformational response of underground openings. Simulation of a natural joint is, to say the least, most difficult. However if an over-riding simplification is made by assuming that the coefficient of friction is 0.5 for the natural joint, which in fact is not far from the truth when interlocking and dilation are not prevalent, then it is possible to achieve slip similarity because the coefficient of friction for the fabricated Araldite D joints is also 0.5

Another important feature is that no matter how accurately the blocks of the model are fabricated and no matter how tightly they are placed or fitted there remain between the contact surfaces, gaps greater than those existing for the prototype. Consequently the size
of the blocks should not be too small and the
frequency of the discontinuities when the model is
assembled should be considerably smaller than that of
the prototype (Fumagalli, 1968) so as to avoid too
great a reduction in the overall modulus of deformation.
Size effect of the blocks on the deformational response
is examined in greater detail in Chapter 5, where it is
determined that for blocks 3/8 ins. square through to
1 ins. square the deformational response between similar
points is much the same in magnitude.

3.2.4 Load application

Loading and excavating specific blocks in the models
may be conducted in one of two ways, namely -

1. the opening is first created and the load subsequently
   applied,

2. the insitu loading is applied before excavating the
   opening.

It is shown (Fumagalli, 1968) that the body forces may
be applied to the surface of the modelled rock mass where
if the density scale \( \delta \) is controlled by circumstances then

\[
\mu = \lambda \delta ,
\]

3.9

where \( \mu \) defines the load scale. This latter technique has
its limitations of which the main one is the relative size
of blocky material to size of opening.
Up to this stage the change in geometry due to phenomena such as slip, rotation, and buckling have not been considered. The first two criteria could possibly be defined in terms of deformation moduli, Chapter 5, while buckling appears to be more difficult to examine and only after insitu measurements have been recorded will it then be possible to correlate data between model and prototype.

3.3 PHOTOELASTIC EQUIPMENT

Since the time Brewster (1816) photoelasticity was, at first, slow to gain acceptance in either design or research. The reason for this was due mainly to the low fringe order of material available for constructing models. Coker and Filon (1931) introduced plastics and refined experimental procedures, such as measuring isoclinics more accurately, and determining the transverse strains. Workers, however, were still loath to use photoelastic techniques to any great extent because the effects of creep were not well understood. Except for the work of Frocht (1941, 1942) no general use of photoelasticity was made until the understanding and production of photoelastic material improved. Detrimental characteristics of available photoelastic material at this stage were creep, edge effect, and stability which is the introduction of supposedly uncontrollable fringe patterns to the model while being manufactured. However, with the use of epoxy resins and improved plastics, materials which overcome the aforementioned deficiencies have been produced, Durelli and Riley (1965), Chappell (1967).
Not only have there been improvements in the production of materials but there have also been substantial improvements in the production of polarisopes and ancillary apparatuses such as comparators, Moire fringe machines, gratings and compensators.

The main experimental procedure used here was the photoelastic method, and to this end a 22 ins. diameter diffusion type polarscope was constructed. Also built were a loading frame, supporting apparatus for the polaroid and quarter wave plates, and a bench from which photographs of basic data such as isoclinics and isochromatics were recorded, Figure 3.2. Material, namely Araldite D (CBY 951) was moulded in the laboratory and subsequently cut to make up the required photoelastic models.

3.3.1 Polarscope.

Two types of polarscope are in general use at the present time, namely the lens and diffusion type. For the model work examined here a diffusion polarscope 22 ins. diameter was built. The diffusion polarscope has been gaining popularity because of the ease with which the model can be aligned to give a clear and undistorted boundary. With a lens polarscope, however, careful alignment of the relatively small area being examined with the optical axis of the compound lenses is essential otherwise optical aberrations arise. Consequently only small areas of the model can be examined at any one particular instance. When studying the mechanisms and interactions of a block system, it is necessary to observe as large an area of the model as possible, (Chappell, 1967).
3.3.2 **Light source**

A line diagram of the diffused light polariscope constructed is shown in Figure 3.3. Three different types of light sources were used so that varying degrees of clarity could be obtained of certain fringe orders, such as the zero fringe. The characteristics of the light sources used are:

1. Green fluorescent lamps, made up to a bank of 6 tubes gives an almost white light and is practically lacking in those wavelengths below 4800 Å. The peak intensity occurs at about 5270 Å with a very sharp secondary peak due to the mercury green line at 5461 Å. Used visually, this source is sufficiently close to white light to permit the easy identification of zero fringe orders which appear as blue grey while the remainder of the field is either a bright blue or yellow green.

2. Mercury vapour lamps give two peaks of light, but by using suitable green filters, Wratten 77, the yellow band in the mercury spectrum is isolated, and a nearly monochromatic source is obtained. Unfiltered mercury light allows the zero order fringe (distinctly black) to be readily differentiated from higher order fringe values (yellowish black).

3. Sodium lamps give a very good monochromatic light source but the intensity of the resulting images are markedly reduced. When observing the isochromatics difficulties arise because there is no difference in shade between the various fringe orders.
Light source & straining frame

FIG 3.4

Camera installation.

FIG 3.2
With sodium lights photographs cannot conveniently be taken with orthochromatic film because the insensitivity of the film to the range of red light. Consequently a pan type film was used, namely Kodalith - Ortho Royal.

These light sources were mounted in containers, Figure 3.4 which had fittings that allowed the light sources to be easily changed. The inside of the containers were painted white in order to diffuse the light. A homogeneous light source field (22 ins. diameter) was obtained by using a flashed opal glass to cover the source. Opal glass gave the additional diffusion necessary to produce a uniform light field.

3.3.3 Polarizing and quarter-wave plates

Most modern polariscopes use sheet polarizers almost without exception. Dolan and Murray (1959) showed the importance of having quarter-wave plates which match each other (i.e. properly set) rather than that they correspond precisely to one quarter of the wave length of light being used. Jessop and Harris (1949) shows analytically the effects of the aforementioned statement when using the Tardy-Sernamont method of measuring fractional relative rotations. Consequently much time was spent in making sure that the pair of quarter-wave plates matched up, that is light transmission was eliminated when the plates were crossed.

3.4 PHOTOGRAPHY

The recording of isochromatics and isoclinics was done mostly by photography; consequently certain techniques
were developed to give the best results.

Green fluorescent light was used in most cases, however, the green light from the mercury vapour lamp was also used. A Linhoff plate camera was set up as shown in Figure 3.2, and the image was focussed onto a ground plate glass screen where it could be examined before photographing.

The photographic material used was a Kodalith Ortho Royal P.200 which gave a high resolution of fine lines and high contrast. The negatives were fully developed to obtain a high contrast between the relevant lines and background. Exposure times varied from 2 to 12 seconds dependent on the use of filters, model material, image lenses and aperture.

3.5 PHOTOELASTIC MATERIAL MANUFACTURE AND PROPERTIES

Stress fields examined in this work are of relatively low magnitude when compared with those developed when studying stress concentrations. Consequently a material with a high order of merit (Young's Modulus/Fringe value) and sensitivity (Ultimate Load/strain) is essential if any viable results from these relatively low order stress fields are to be made.

A thorough examination of photoelastic materials was made, Chappell (1967), and casting techniques developed so that the material and model construction would meet the requirements stated above. Hattersley (1964) performed extensive work developing photoelastic material and from the results of these investigations it was decided to use Epoxy Resin, namely Araldite D cast at room temperature.
The mixing proportions are given in Appendix B.

3.5.1 Material aspects of casting

The term casting used here relates to the mix of Araldite 'D' with an amine type hardener (Ciba hardener 951).

To obtain a stress free casting the mould should be so constructed as to satisfy the following requirements.

3.5.2 Exotherm control

The mould is made of material which controls the rate of heat dissipation away from the casting. It was found that medium hard steel plates 3/8 in. thick and 24 in. square enabled the heat to be controlled for sheet castings up to 1 1/2 in. thick, Figure 3.5. These plates were carefully treated to attain the required smoothness and flatness, and were then subsequently chromium plated.

3.5.3 Expansion and contraction

As the casting polymerises it expands slightly and subsequently contracts. Therefore, the mould must be constructed so as to allow for this movement, otherwise the casting will be stressed and hence photoelastically useless. Rubber tubes were placed over the separating steel spacers, Figure 3.5(b). This slight movement does not affect the overall accuracy of the casting thickness, which is accurate, to within 0.001 in. in overall flatness. This last requirement is also largely dependent on the accuracy to which the moulding plates have been ground and plated.
3.5.4 Surface effects

The excellent adhesive properties of the epoxy resin requires that careful attention be paid to the surface preparation of the mould plates. This attention necessitates polished mould surfaces and the use of a satisfactory release agent, such as releasil 4, Chappell (1967), Appendix B.

3.5.5 Size of castings

In this work photoelastic sheets varying in thickness from \( \frac{1}{4} \)" to 1" and approximately 22 in. wide and 20 in. deep were used. Consequently only sheet castings were required and prepared as described in Appendix B.

3.5.6 Model construction

Chappell (1967) determined that a reasonable photoelastic model would, for 1 in. square blocks, be obtained if the tolerance of \( \pm 0.001 \) in. between adjacent blocks in the columns and rows could be achieved. In the sheet castings the thicknesses could quite easily be controlled to the tolerance of \( \pm 0.0005 \) in. These tolerances were readily attained by cutting the sheet castings with a large diameter (24 in.) diamond rock saw which possessed some rigidity in the cutting plate. By fabricating templates the strips cut from the araldite sheets were accurately controlled. Before cutting however the araldite sheets were stuck to a perspex backing in order to more easily restrict any unwanted movements in the sheet being cut. By doing this all the blocks from the cut sheet retained their relative position and gave a model which
Photoelastic test pieces

**FIG. 3.6**

(a) mould for sheet casting

(b) gasket for sheet casting

**FIG. 3.5**
quite easily met the required tolerances.

3.6 CALIBRATION AND TESTING

3.6.1 Calibration of birefringent material

Once the machine was constructed and functional, a series of simple calibration tests on the birefringent material were performed. Circular test pieces were used when determining the fringe values, Figure 3.6.

\[
\sigma_x = \frac{2P}{\pi t D} \left[ \frac{D^2 - 4x^2}{D^2 + 4x^2} \right] \sigma_1
\]

\[
\sigma_y = \frac{2P}{\pi t D} \left[ \frac{4D^4}{(D^2 + 4x^2)^2} - 1 \right] = \sigma_2
\]

Therefore,

\[
\frac{\sigma_1 - \sigma_2}{2} = \frac{4P}{\pi t D} \cdot \frac{1 - 4 \left( \frac{x}{D} \right)^2}{\left[ 1 + 4 \left( \frac{x}{D} \right)^2 \right]^2} = \frac{4P}{\pi t D} e \left( \frac{x}{D} \right)
\]

but \( \tau_{\text{max}} = nF \)

where \( n = \) number or order of fringe observed,

\( F = \) fringe value measured in psi/fringe.

Thus, \( nF = n \frac{f}{t} = \frac{\sigma_1 - \sigma_2}{2} \)

where \( f = \) fringe value/unit thickness

\( t = \) thickness of disk.
The function \( g \left( \frac{x}{D} \right) \) is usually evaluated at the centre of the disk, where \( g \left( \frac{x}{D} \right) = 1 \).

Consequently, \( \frac{nf}{t} = \frac{4P}{\pi tD} \)

and \( f \) can then be determined.

In many cases it is important to know the deformability potential of the material with which the models are built. This is an important criterion to consider when large deformations change the distribution of stresses in the model. Just as it is possible to derive a stress fringe value so it is also possible to obtain a strain fringe value by the substitution of appropriate constants (Durelli and Riley, 1965)

\[
f_e = \frac{f_0 (1+v)}{E}
\]

where \( f_e \) is the strain fringe value,

\( v \) is Poisson's ratio,

and \( E \) is Young's Modulus.

A quantity which seems to be gaining acceptance for the purpose of assessing deformability is the figure of merit, \( Q \), and is defined as the ratio of the modulus of elasticity to the stress fringe value. The figure of merit for the material used here is given in Table 2, where it is compared with other available materials.
<table>
<thead>
<tr>
<th>SPECIMEN NO</th>
<th>v</th>
<th>E  (p.s.i.)</th>
<th>f*  (p.s.i./fr/in)</th>
<th>( \frac{E}{f} ) (fr/in)</th>
<th>MATERIAL</th>
<th>E  (p.s.i.)</th>
<th>f*  (p.s.i./fr/in)</th>
<th>Q  (fr/in)</th>
<th>CRITICAL TEMPERATURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.42</td>
<td>483,000</td>
<td>74.2</td>
<td>6,520</td>
<td>Araidite 'D'</td>
<td>4,250</td>
<td>2.74</td>
<td>1,895</td>
<td>220°F</td>
</tr>
<tr>
<td>2</td>
<td>0.38</td>
<td>465,000</td>
<td>62.3</td>
<td>7,470</td>
<td>815 + DTA</td>
<td>3,450</td>
<td>1.97</td>
<td>1,755</td>
<td>230°F</td>
</tr>
<tr>
<td>3</td>
<td>0.30</td>
<td>587,000</td>
<td>130.5</td>
<td>4,500</td>
<td>828 + HPA</td>
<td>3,200</td>
<td>2.73</td>
<td>1,172</td>
<td>285°F</td>
</tr>
<tr>
<td>4</td>
<td>0.44</td>
<td>595,000</td>
<td>73.2</td>
<td>8,130</td>
<td>828 + HPA</td>
<td>4,250</td>
<td>2.73</td>
<td>1,585</td>
<td>320°F</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>+ PA</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>0.31</td>
<td>572,000</td>
<td>70.7</td>
<td>8,080</td>
<td>828 + PA</td>
<td>2,950</td>
<td>2.33</td>
<td>1,265</td>
<td>235°F</td>
</tr>
<tr>
<td>6</td>
<td>0.37</td>
<td>535,000</td>
<td>59.8</td>
<td>8,930</td>
<td>'B' + PA</td>
<td>2,500</td>
<td>1.67</td>
<td>1,500</td>
<td>290°F</td>
</tr>
<tr>
<td>7</td>
<td>-</td>
<td>-</td>
<td>378</td>
<td>-</td>
<td>812 + HPA</td>
<td>3,400</td>
<td>8.53</td>
<td>399</td>
<td>170°F</td>
</tr>
</tbody>
</table>

* Direct stress fringe value

**TABLE 2 - SUMMARY OF CALIBRATION TEST RESULTS**
3.7 SINGLE LOADED BLOCK

It could be stated that the two main factors examined in this thesis are the block and its interaction with other blocks.

Consequently the main mode of reducing the photoelastic data was by analysing each block as a separate entity. This procedure is an important feature of this thesis and is fully discussed below. However, in order to test the validity of the theoretical approach, a number of blocks of various sizes and aspect ratios (width to depth) were loaded with known boundary forces and the resulting isoclinics and isochromatics photographed. Numerical values of the isochromatics and isoclinics were substituted, as data, into the computer programme from which the boundary loads were compiled. These output values of the calculated boundary loads could then be compared with the actual imposed loads. From these comparisons the reliability of the experimental procedure could be assessed.

Blocks with aspect ratios of 1, 1⅓, 2 and 2⅔, Figure 3.7 were tested with normal loads imposed on the boundary. Positioning of these loads was varied so that the extremes of stress distribution could be achieved in the blocks. In particular, development of tensile stress was the main point of interest, and the main features of the results are examined in Chapters 6 and 7, where the individual blocks in the mass are considered both experimentally and numerically.
FIG 3.7a
ISOCHROMATIC PATTERNS

ASPECT RATIO $\frac{1}{2}$

ASPECT RATIO 2

ASPECT RATIO $2\frac{1}{2}$
FIG. 3.7b

ISOCLINIC PATTERNS
3.7.1 Photoelastic method

Of the many methods available for reducing photoelastic data in the form of isochromatics (principal stress differences) and isoclinics (principal stress directions) the most commonly used method is that of shear differences. Using various methods such as Moiré fringes, electrical analogue and digital computers, the attainment of sufficient and adequate data for a complete analysis is relatively easy. Yet, the problem in this thesis is such that the stress distributions on the boundaries of individual blocks making up the model are required, and to obtain this the above methods of photoelastic data evaluation are cumbersome and prone to errors. This is because the methods used are based on an additive process in that the evaluation is started at a boundary of known loading and then using photoelastic data and incremental distances, stresses at points within the boundary are progressively evaluated. What the method implies is that any error, be it in the data or accuracy of calculation, will diffuse through the entire process of data reduction because of error propogation (Hartree, 1958). Even the use of computers does not solve this difficulty because it is basically a problem of data interpretation and dimensional evaluation which introduce the expanding errors.

In an attempt to overcome the aforementioned limitations a numerical method reducing the photoelastic date of each individual block is developed. That is any block within the model can be chosen and the relevant photoelastic data extracted. Because of the difficulty in determining accurate values of the isoclinics the analysis was
organised so that it could be made independent of this parameter if required. That is only the values of the isochromatics need be known to determine the boundary loading on the block being analysed.

A computer programme which can handle either square or rectangular blocks of various sizes was compiled and then used to examine some photoelastic work performed on single blocks. Then some photoelastic data obtained from blocks within a blocky mass was evaluated using both the numerical method developed here and the shear differences method developed by Frocht (1942).

3.7.2 Development of controlling equations

In a semi infinite two dimensional half plane which is isotropic and homogeneous a load \( p(s) \) is placed at a point \( (s) \), Figure 3.8(a). Now if the boundary of a virtual block is drawn on this half plane, the stresses induced by the load at the virtual boundary can be evaluated from Flamant's (1892) equation, namely

\[
\sigma_{\text{rad}} = \frac{2}{\pi} \frac{p(s) \cos \phi(s,r)}{R(s,r)}
\]

where \( \sigma_{\text{rad}} \) = the induced radial stress at a point \( r \) on the virtual boundary,

\( R(s,r) \) = distance between the unit load at \( (s) \) and point \( (r) \), Figure 3.8(a)

\( \phi(s,r) \) = angles measured positive anticlockwise from the line of action of the load at a point \( (s) \) to the radius \( R(s,r) \).
If we consider a unit normal load placed at point \((s)\) on the boundary of the virtual block, which has been divided into sixteen segments as shown in Figure 3.9 the normal \((N(s,r))\) and tangential \((T(s,r))\) loads induced at any other point \((r)\) on the boundary will be given by,

\[
N(s,r) = \frac{2 \cos \phi (s,r)}{\pi R(s,r)} \cos^2 \alpha(s,r) \Delta t
\]

\[
T(s,r) = \frac{2 \cos \phi (s,r)}{\pi R(s,r)} \cos \alpha(s,r) \sin \alpha(s,r) \Delta t
\]

where \(\alpha(s,r)\) = angle which radius \(R(s,r)\) makes with the normal to the boundary at point \((r)\).

Likewise, for a unit shear load placed at point \((s)\) the normal \((N^*(s,r))\) and tangential \((T^*(s,r))\) loads caused at the point \((r)\) on the boundary will be,

\[
N^*(s,r) = \frac{2 \cos \phi_1 (s,r)}{\pi R(s,r)} \cos^2 \alpha(s,r) \Delta t
\]

\[
T^*(s,r) = \frac{2 \cos \phi_1 (s,r)}{\pi R(s,r)} \sin \alpha(s,r) \cos \alpha(s,r) \Delta t
\]

where \(\phi_1 (s,r)\) = angle measured as positive clockwise from the line of action of the unit shear load at point \((s)\) to the radius \(R(s,r)\), Figure 3.9.

When the half plane is loaded with normal \((p(s))\) and shear \((q(s))\) forces at the point \((s)\), the induced loads at the point \((r)\) on the virtual boundary are,
\[ F_n(r) = \{p(s) \cdot N(s,r) + q(s) \cdot N^*(s,r)\} \Delta t \]
\[ F_t(r) = \{p(s) \cdot T(s,r) + q(s) \cdot T^*(s,r)\} \Delta t \]

where \( F_n(r) = \) normal load induced at point (r), Figure 3.8(b).
\( F_t(r) = \) the tangential load induced at point (r), Figure 3.8(b).

If the virtual block is now removed from the half plane and each side of the block is considered as being at some stage part of the surface of a half plane, the induced stresses at any point (r) on the block due to the imposed loads \( p(s) \) and \( q(s) \) are,

\[ \beta_n(r) = \sum_{s=1}^{16} (p(s) \cdot N(s,r) + q(s) \cdot N^*(s,r)) \Delta t \]
\[ \beta_t(r) = \sum_{s=1}^{16} (p(s) \cdot T(s,r) + q(s) \cdot T^*(s,r)) \Delta t \]

In order to relieve the block from these induced stresses on the boundary, opposite and equal forces are applied to the boundary, namely \(-\beta_n(s)\) and \(-\beta_t(s)\). As the block is in equilibrium under a system of boundary loads \( L_n(r) \) (normal) and \( L_t(r) \) (tangential), the system of loads at point (r) on the boundary are also in equilibrium

\[ \therefore \quad L_n(r) = p(r) - \beta_n(r) \]
\[ L_t(r) = q(r) - \beta_t(r) \]
These expressions are Fredholm equations of the second kind, (Massonnet, 1965), where \( p(r) \) and \( q(r) \) are termed fictitious loads. The Fredholm equation is an integral equation of the type

\[
L(r) = p(r) - \lambda \phi p(s) K(s,r) \, dt \tag{3.16}
\]

where \( p(r) \), \( p(s) \), and \( L(r) \) are scalar or vector quantities distributed on the boundary contour, Figure 3.10 and \( K(s,r) \) is the kernel related to the points \( r \) and \( s \) on the boundary and with the requirement that it must be symmetrical.

Massonnet shows how equation 3.16 can be transformed into the incremental form,

\[
L(r) = p(r) - \lambda \Sigma p(s) K(s,r) \Delta t \]

\[
= p(r) - \beta(p(s)) \tag{3.17}
\]

where

\[
\beta(p(s)) = \lambda \Sigma p(s) K(s,r) \Delta t
\]

Equation 3.17 can be solved for the fictitious loads \( p(r) \) by successive approximations. Initially the given forces \( L(r) \) are taken as the first approximation, \( p_0(s) = L(r) \) and

\[
p_1(r) = L(r) + \beta(p_0(s)) \tag{3.18}
\]

\[
p_2(r) = L(r) + \beta(p_1(s))
\]
This procedure, proposed by Miche (1926) causes an oscillation of \( p_1(r), p_2(r), \ldots, p_n(r) \), around the mean value which in effect is the hydrostatic load value. In general a given force system \( L(r) \) contains the partial components of hydrostatic and deviatoric load. Fortunately by slightly modifying the iteration procedure the above oscillations are progressively reduced. A convergence parameter \( \alpha \) which is a fixed number lying between 0 and 1, is used to correct the distribution of vectors at every stage of the computation. So that-

\[
p_0(r) = \alpha L(r),
\]

\[
p_1(r) = \alpha L(r) + (1-\alpha)p_0 - \alpha \beta(L_0(s)),
\]

\[
p_{n+1}(r) = \alpha L(r) + (1-\alpha)p_n - \alpha \beta(p_n(s))
\]

3.7.3 **Numerical Development**

The stress vector, \( \sigma_{rad} \), on an internal element at point \( n \), Figure 3.11, is

\[
\sigma_{rad} = \frac{2}{\pi} \sum_{s=1}^{16} \left[ L_n(s) \frac{\cos \phi(s,n)}{R(s,n)} + L_t(s) \frac{\cos \phi_1(s,n)}{R(s,n)} \right]
\]

3.20

where \( \sigma_{rad} = \) radial stress at point \( n \),

and all the symbols are as defined above except \( n \) is now an
internal point and not the boundary point \( r \). By resolving
the radial stress into the components \( \sigma_x \), \( \sigma_y \) and \( \tau_{xy} \) in
the \( x \) and \( y \) directions, Figure 3.11, and noting that
\[
\cos \phi_1(s,n) = \sin \phi(s,n),
\]
we obtain,

\[
\sigma_x(n) = \frac{2}{\pi} \sum_{s=1}^{16} \frac{L_n(s) \cos^4 \phi(s,n)}{y(s,n)} - \frac{L_t(s) \sin \phi(s,n) \cos^3 \phi(s,n)}{y(s,n)}
\]

\[
\sigma_y(n) = \frac{2}{\pi} \sum_{s=1}^{16} \frac{L^2_n(s) \sin \phi(s,n) \cos^3 \phi(s,n)}{x(s,n)}
\]

\[
- \frac{L_t(s) \sin^4 \phi(s,n)}{x(s,n)}
\]

\[
\tau_{xy}(n) = \frac{2}{\pi} \sum_{s=1}^{16} \frac{L^2_n(s) \cos^3 \phi(s,n) \sin \phi(s,n)}{y(s,n)}
\]

\[
- \frac{L_t(s) \sin^3 \phi(s,n) \cos \phi(s,n)}{x(s,n)}
\]

3.21

where \( y(s,n) \) and \( x(s,n) \) are the distances from the points
\( s \) to \( n \) in the \( y \) and \( x \) directions respectively, Figure 3.11.

It is easy to show that

\[
(\sigma_1-\sigma_2)_n = \left[ (\sigma_x-\sigma_y)^2 + 4\tau_{xy}^2 \right]^{\frac{1}{2}}
\]

3.22

and \( \tan 2 \theta_n = \frac{2\tau_{xy}}{(\sigma_1-\sigma_2)_n} \)

When equations 3.21 are substituted into equations 3.22
the following equations are obtained, namely
\[(σ_1-σ_2)_n = \frac{2}{π} \sum_{s=1}^{16} \left[ L_n(s) \left\{ \frac{\cos^4 φ(s,n)}{y(s,n)} + \frac{\cos φ(s,n) \sin^3 φ(s,n)}{x(s,n)} \right\} + L_t(s) \left\{ \frac{\sin^4 φ(s,n)}{x(s,n)} + \frac{\sin φ(s,n) \cos^3 φ(s,n)}{y(s,n)} \right\} \right] \]

\[(σ_1-σ_2)_n \sin 2θ_n = \frac{4}{π} \sum_{s=1}^{16} \left[ L_n(s) \cos^3 φ(s,n) \sin φ(s,n) \frac{y(s,n)}{y(s,n)} + L_t(s) \sin^3 φ(s,n) \cos φ(s,n) \frac{x(s,n)}{x(s,n)} \right] \]

3.23

For further development equations 3.23 are put into the condensed form

\[(σ_1-σ_2)_n = \sum_{s=1}^{16} \{L_n(s) P(s,n) + L_t(s) P^*(s,n)\} \]

\[(σ_1-σ_2)_n \sin 2θ_n = \sum_{s=1}^{16} \{L_n(s) Q(s,n) + L_t(s) Q^*(s,n)\} \]

3.24

where \[P(s,n) = \frac{2}{π} \left( \frac{\cos^4 φ(s,n)}{y(s,n)} + \frac{\cos φ(s,n) \sin^3 φ(s,n)}{x(s,n)} \right)\]

\[P^*(s,n) = \frac{2}{π} \left( \frac{\sin^4 φ(s,n)}{x(s,n)} + \frac{\sin φ(s,n) \cos^3 φ(s,n)}{y(s,n)} \right)\]
\[ Q(s,n) = \frac{4}{\pi} \left( \frac{\cos^3 \phi(s,n) \sin \phi(s,n)}{y(s,n)} \right) \]

\[ Q^*(s,n) = \frac{4}{\pi} \left( \frac{\sin^3 \phi(s,n) \cos \phi(s,n)}{x(s,n)} \right) \]

Equations 3.24 are not yet in a form which would allow a solution from experimental results because the coefficients \( P(s,n) \), \( P^*(s,n) \), \( Q(s,n) \) and \( Q^*(s,n) \), which are derived from an equation related to a semi-infinite medium, do not allow for the effects of contiguous boundaries.

3.7.4 Determination of the fictitious stresses from known internal stresses.

If we consider the point \( s \) on the boundary it has been shown that the relationship between the actual loads and fictitious loads are,

\[ L_n(s) = p(s) - \beta_n(s) \]

\[ L_t(s) = q(s) - \beta_t(s) \]

\[  \text{(3.25)} \]

If equation 3.15 are substituted into equations 3.25 the following relations are derived,
\[
(\sigma_1-\sigma_2)_n = \sum_{s=1}^{16} \left[ p(s)P(s,n) - \sum_{r=1}^{16} \{ p(r)(N(r,s)P(s,n) + T(r,s)P^*(s,n)) \} + q(s)P^*(s,n) - \sum_{r=1}^{16} \{ q(r)(T^*(r,s)P^*(s,n) + N^*(r,s)P(s,n)) \} \right] (a)
\]

\[
(\sigma_1-\sigma_2)_n \sin 2\theta_n = \sum_{s=1}^{16} \left[ p(s)Q(s,n) - \sum_{r=1}^{16} \{ p(r)(N(r,s)Q(s,n) + T(r,s)Q^*(s,n)) \} + q(s)Q^*(s,n) - \sum_{r=1}^{16} \{ q(r)(T^*(r,s)Q^*(s,n) + N^*(r,s)Q(s,n)) \} \right] (b)
\]

3.26

Sixteen points \(n\) are considered within the block being examined and at these points the isochromatics and isoclinics, namely \((\sigma_1-\sigma_2)_n\) and \(\theta_n\) respectively, are measured. This gives thirty two known parameters from which thirty two unknowns can be determined, namely the fictitious forces \(p(s)\) and \(q(s)\). Equations 3.26(a) and (b) make up a combination of sixteen equations each. The value of \(\theta_n\) in equation 3.26(b) is the value of the isoclinic where the zero isoclinic is taken parallel to the x axis.
Once the values of $p(s)$ and $q(s)$ are determined, the forces acting on the $r^{th}$ element of the boundary are evaluated using equations 3.15 in the form

$$L_n(r) = p(r) - \sum_{s=1}^{16} \{ p(s)N(s,r) + q(s)N^*(s,r) \} \Delta t \}$$

$$L_t(r) = q(r) - \sum_{s=1}^{16} \{ p(s)T(s,r) + q(s)T^*(s,r) \} \Delta t \}$$

3.7.5 Computer Analysis

The form of equations 3.26 do not lend themselves easily to a solution of the fictitious loads $p(r)$ and $q(r)$ by the normally adopted method of successive approximations, section 3.7.2. Therefore all the coefficients of the fictitious forces are gathered together and the resultant linear equations solved by matrix inversion, or successive approximations.

Initially the co-ordinates for the external points, $s$ and $r$, on the boundary and internal points, $s$ and $n$, within the boundary are generated from the block size. Sixteen points on and within the boundary are considered. This means that 32 boundary forces are evaluated from the 32 experimental readings within the boundary.

The coefficients $N(r,s)$, $N^*(r,s)$, $T(r,s)$, $T^*(r,s)$, $P(s,n)$, $P^*(s,n)$, $Q(s,n)$ and $Q^*(s,n)$ are then compiled by the computer and stored. After this the coefficients of $p(s)$ and $q(s)$ in equation 3.26(a) are calculated and stored in arrays FNRS ($R,N$) and FTRS ($R,N$). The same is
done for the coefficients of \( p(s) \) and \( q(s) \) in equation 3.26(b) and the results stored in arrays FINRS \((R,N)\) and FITRS \((R,N)\). These coefficients are then arranged in a single array \( AMAT \,(R,M) \) and the experimental values of the isochromatics \( \sigma_1 \sim \sigma_2 \) and isoclinics \( \theta_n \) are used to generate the column array, \( PLOAD(N) \). The fictitious loads \( p(r) \) and \( q(r) \) are then evaluated by a matrix inversion method using a subroutine called MTINV(AMAT,NL, PLOAD,K,DETER,ID). From the values of the fictitious forces \( p(s) \) and \( q(s) \) the boundary loads are evaluated and stored in the arrays \( SNORM(S) \) and \( QTAN(S) \).

A listing and brief description of the programme are given in Appendix A.

3.8 SINGLE BLOCK INVESTIGATION

In order to test the validity of the computer programme a series of photoelastic experiments were performed on single blocks which were subjected to carefully imposed loads. In the first instance, applied or induced shear loads on the boundary were prevented as far as possible. The photoelastic data within the blocks were extracted from photographs of the resultant isochromatic and isoclinic patterns.

Difficulty was experienced with the computer programme when the isoclinic data was included whereas with only the isochromatic data from these initial single block tests, some good results were obtained.

Figure 3.12 shows clearly the difference between the applied loads, their contact stress distribution and the
**FIG. 3.12 a**

*Square Block Subjected to Different Loading Conditions*
Actual stress distribution on contact plane

Computer calc. stress dist.

Aspect ratio 1 1/2

Equilibrium check from shear diff. method.
Horiz. forces 8% out.
Vert. forces 5% out.

Equilibrium check from computer analysis
Horiz. forces 11% out
Vert. forces 14% out.

Fig. 3.12c.
computed load values with their average stress distribution. It will be noted that the percentage error is below 14%.

When applied shear was allowed to develop at the boundary and isoclinic data was fed into the computer programme the percentage errors were greater than 30% and this was considered as quite unacceptable. The problem here could be the uncertainty generally experienced in obtaining the isoclinic data or the lack of stress gradients in some of the isochromatic readings in these latter single block experiments. Further work on these factors is needed before a complete understanding of the interaction between the experimental data and computer programme is obtained. It does appear however that where the normal forces are large compared to the shear forces the results obtained are acceptable.

This method of reducing photoelastic data is used in Chapter 6 where specific blocks in a blocky mass are examined in detail.

3.9 EXPERIMENTAL PROGRAMME ON THE BLOCKY MODELS

In practice blocks of varying shape and size are generally encountered, for example, hexagons and rectangles. In this work, however, the rectangle was the shape of block investigated as this is the shape of block more prevalent in the mining situation (Rosengren, 1968) examined in Chapter 8.

As the two types of discontinuity, namely planar and volumetric, (Walsh, 1965) are of prime importance in this work, the testing programmes were organised so
that the response of these two types of discontinuity could be examined. In this section a brief description of these model tests are given, whereas in the Appendix D a more detailed description is presented.

As slip and volume changes occur at nearly all stages of loading and unloading, the loading sequence is of prime importance as the stress distribution is load path dependent. Consequently the bi-axial loading sequence was varied so that the effects of hydrostatic and distortional (stress difference) loads on the deformational response were examined.

3.9.1 Loading system

The overall size of the models varied from 5 in. square to 18 in. square. This therefore required flexibility in being able to apply the varying loads to the boundaries of the models. Consequently, the loading frame, Figure 3.14(a) was made up of channels and angles which were adjustable in that any of the models could be easily accommodated and the biaxial load applied through a series of proving rings or oil pumps, Figure 3.14(b). The displacements were measured by dial gauges set up independently of the model in that no part of the dial gauge had contact with the model or supporting load frame; consequently the relative displacement between specific points in the model could be determined, Figure 3.14(a).

The main object of the loading system was to obtain a uniform pressure on the boundaries of the model and be able to vary the magnitude of vertical load to horizontal
DIAL GAUGE INSTALLATION

FIG 3.14(a)

LOADING FRAME WITH PRES. GAUGES.

FIG 3.14(b)
load. Various loading ratios were applied and the results are considered in Chapters 5 and 7. Loading increments and sequences could be varied to suit the loading history required on the model. For instance a hydrostatic biaxial load could be applied throughout or the horizontal load could be increased and then the vertical load.

Blocks within the model were removed to give various configurations of opening and for each model opening the displacements were measured and recorded for every incremental load increase or decrease. The loading cycles were repeated for most of the tests performed.

At specific stages of the loading and unloading cycle photographs were taken of the resultant stress patterns. A more detailed description of the loading sequences and deformational readings are given in Appendices D and E.
CHAPTER 4

FRICITION AND SLIP MECHANICS

4.1 PREAMBLE

It was Leonardo de Vinci who first recognised that when a body slipped relative to another body, while remaining in contact over the slip surface, a resisting force was initiated and that the magnitude of this force was a function of the normal force acting across the plane of slip. This observation became lost with the progress of time only to be rediscovered by Amonton who examined this frictional force in much greater detail. Amonton showed that -

\[ S = \mu N \]

where \( S \) = shear force,
\( N \) = normal force,
\( \mu \) = coefficient of proportionality or coefficient of friction.

If \( A \) = Gross area upon which sliding is taking place,

then \( \frac{S}{A} = \frac{\mu N}{A} \)

\[ \tau = \mu \sigma \]

where \( \tau \) = shear stress,
and \( \sigma \) = normal stress.
Amonton also determined that the magnitude of $\mu$, for the experiments conducted, was

1. independent of the normal stress,
2. independent of the area of contact,
3. dependent on the type of material in contact.

Bowden and Tabor (1950) examined by experiment and in great detail the phenomenon of friction. The basic tenor of Bowden and Tabor's results was that friction was produced by the welding of points or zones of contact and that when slip occurred these zones were broken only to reform in order to retain a quasi-equilibrium state. This approach to friction is known as the adhesion theory and serves in nearly every instance as the starting point for friction studies.

Because of the importance of slip and hence friction to the deformational response of a blocky material the basic phenomenon of slip along a single plane is examined in this chapter in some detail. The main object here is to determine the relationship between deformational response and stress redistribution as slip occurs.

Accumulative incremental displacements denoting finite magnitudes of strain and the related stresses are discussed as this is the method by which the loading sequences are applied to the models and in most cases, this is the way induced loads are evolved. It is also recognised that if the physical characteristics are load path dependent, a powerful and realistic technique for examining these variable characteristics is by this incremental method.
Subsequent to this incremental study some simple mechanisms are examined in anticipation of the more involved mass block model studies in Chapters 6 and 7.

Another factor examined here is simple slip and the effects of imposed or induced constraint on the resultant deformational response. These constraints will often control the rotational effect imposed on the interacting blocks and hence affect the resultant deformational response and stress redistribution.

In this thesis, the magnitude of elastic strains are generally of the order of 0.1%, whereas when slip occurs the strains are generally about 1.0%. Continued slip can give strains and displacements many orders of magnitude greater than those just stated (up to 5.0% and greater) hence it becomes necessary to consider changes in geometry and the redistribution of stresses due to this effect.

4.2 EXAMINATION OF SLIP

Archard (1958) considered a number of models all of which were assumed to be composed of spherical granules which deformed elastically when subjected to stress. In nearly all cases the number and areas of contact were found to be a function of the normal load. Archard showed that as the surfaces increased in complexity, Figure 4.1 the true contact area became more nearly proportional to the load. The true area of contact Ac was found to vary between KN$^{\frac{1}{2}}$ and KN, where K is a constant of proportionality equal to $\frac{1}{qu}$, where qu is the yield stress in compression, and N is the normal force.
FIG 4.1
Surface roughness (after Archer)

FIG 4.2
stress relation with predicted angle of fracture
If $A_c$ is the actual area of contact between the sliding surfaces,

\[ \alpha \text{ factor varying from } \frac{2}{3} \text{ (elastic deformation) to } 1 \text{ (plastic deformation)}, \]

$S$ is the shearing strength of the material,

$A$ is the total area over which sliding occurs,

$F_{\text{max}}$ is the shear force necessary to cause slip,

then by definition

\[
\frac{F_{\text{max}}}{N} = \mu = SKN^{\alpha-1}
\]

and because $0 < \alpha < 1$, and $0 < 1-\alpha < 1$,

and if $SK = C_1$ (a constant)

then

\[
\mu = \frac{C_1}{N(1-\alpha)}
\]

If the value of $\alpha$ is considered to remain constant, then from equation 4.2 it is evident that $\mu$ decreases as the normal load $N$ across the plane of slip increases. If on the other hand, $\alpha$ increases as $N$ increases the value of $\mu$ will still decrease but at a lesser rate with respect to $N$ and in the limit when $\alpha = 1$, $\mu$ will remain constant.
If \[ \frac{F_{\text{max}}}{N} = \mu , \]

and if \[ F_{\text{max}} = S\text{Ac} = S \frac{N^\alpha }{qu} , \]

then \[ \frac{F_{\text{max}}}{N} = \frac{S}{qu} N^{\alpha -1} \]

and when \( \alpha = 1 \), \( \mu \) is a maximum \( \mu_1 \),

and \[ \frac{S}{qu} = \mu_1 \] 4.3

Consequently from this

\[ \mu = \frac{\mu_1}{N^{1-\alpha}} , \text{ where } \frac{1}{3} \geq 1-\alpha \geq 0 \]

and when \( \alpha = 1 \)

\[ \mu = \mu_1 , \]

but generally \( \mu < \mu_1 \).

Also if \[ F_{\text{max}} = S\text{Ac} \]

and \[ N^\alpha = \text{Ac}^\alpha \sigma^\alpha \]

then \[ F_{\text{max}} = S\text{KAc}^\alpha \sigma^\alpha \]

we obtain the average shear stress by dividing with the total area \( A \) then

\[ \tau = c\sigma^\alpha \] 4.4

where \( c = \frac{S\text{KAc}^\alpha}{A} \) = constant.
If $\alpha$ is considered as constant but less than 1 for a particular joint, friction is a function of the normal load. But when the load reaches such a magnitude, overall plastic yield occurs at the contacts and $\alpha = 1$ and $\mu$ becomes constant. However, in all other instances $\mu < \mu_1$.

4.2.1 Experimental examination of slip

The above work depicts various factors which affect the shear stress causing slip between two relatively smooth surfaces where the maximum protuberances do not exceed 70 micron inches. In order to evaluate the effect of these various factors experiments are generally performed and empirical relationships derived. It is useful to examine these experimental approaches and compare the results with the above derivations.

Hobbs (1966) using experimental curve fitting techniques, obtained values of $C$ and $\alpha$ for certain materials, by assuming the validity of Mohr's failure hypothesis. That is, Mohr's envelope defines the strength and that the angles between the radii normal to the envelope drawn from the centre of the Mohr circle and the normal stress axis, Figure 4.2(a), represents twice the predicted angles of fracture, namely $2\theta$. Also if $\phi$ is the tangent angle of the experimental envelope at the point of interest Figure 4.2(a), then

$$2\theta = \phi + 90$$

and

$$\frac{d\tau}{d\sigma} = \tan \phi = -\cot 2\theta = c \sigma^{\alpha-1}$$  \hspace{1cm} 4.5

By considering equilibrium of a triaxial test piece, Figure 4.2(b), the following equations are obtained,
\[ \sigma_1 = \sigma + \tau \cot \theta \quad (a) \]
\[ \sigma_2 = \sigma - \tau \tan \theta \quad (b) \]

If 4.6(b) is subtracted from 4.6(a)

then \[ \sigma_1 - \sigma_2 = \tau (\cot \theta + \tan \theta) \]

and substituting for \( \tau \) from 4.4

\[ \sigma_1 = c(\cot \theta + \tan \theta) \sigma^\alpha + \sigma_2 \quad 4.7 \]

By experimental curve fitting methods Hobbs (loc cit) obtains the relation

\[ \sigma_1 = c' \sigma^\alpha' + \sigma_2 \quad 4.8 \]

where \( c' \) is a parameter and \( \alpha' \) has the same connotations as \( \alpha \) in the above work. Here it is seen that the parameter \( c' \) is, besides the factors defined for \( c \) in equation 4.4, a function of \( \theta \) hence \( \phi \). For higher values of the normal stress \( \theta \) tends to become constant as well as \( \alpha \). Therefore both the factors \( c \) and \( c' \) become constant. For lower values of normal stress however these factors are not constants.

It is generally accepted that as the deformation of contacts become predominantly plastic, especially at high normal pressures, the value of \( \alpha \) approaches 1 and the linear relation (equation 4.1) is acceptable. However, when the deformation of the contacts is a mixture of brittle failure, elastic distortion and plastic yield, as would be anticipated when the confining or normal load across
the plane of slip is relatively small (say 500 p.s.i.), a simple relationship between $\sigma_1$ and $\sigma_2$ no longer exists nor between $\tau$ and $\sigma$ as in equation 4.1.

4.3 ROLLING

It should be emphasised at this point that the adhesion theory implies that friction is independent of surface roughness. Joint surfaces generally encountered can be considered as rough, (Lambe, 1969) but when minerals such as chlorite or gypsum form a veneer on the joints ($\phi < 10^\circ$) they can be considered as smooth. Lambe (loc cit) discusses the antilubricant effect of water and the diminishing of its effects as the roughness becomes larger, Figure 4.3. The Araldite block joints examined in this Chapter possessed an average roughness of 1 to $3 \times 10^{-3}$ in.

When the material tends to break off at the contacts rolling action becomes part of the process of slip. Lambe (loc cit), states that the points of contact are formed by adhesion as previously described and these junctions are broken by tension not shear due, in the main, to elastic rebound as the normal forces tend to zero. Lambe and Whitman (1969) determined that this rolling friction is generally quite small $\mu_{rol} \ll 0.1$.

4.4 SLIP ASSOCIATED WITH PLASTICITY THEORY

It can be shown that for mobilised slip on a plane, which is at an angle $\theta$ to the maximum principal stress direction, the limit condition gives the relationship,

$$\sigma_1 = \sigma_2 \frac{\tan(\theta+\phi)}{\tan \theta} + \frac{2c \cos \phi}{\cos(\theta+\phi) \sin \theta}$$

4.9
Friction of quartz (after Bromwell (1966) and Dickey (1966))

Relation between coefficient of friction & small roughness value

Interaction between blocks while deforming
and if the joint is considered cohesionless $C = 0$,

$$\frac{\sigma_2}{\sigma_1} = \cot (\theta + \phi) \tan \theta = K$$  \hspace{1cm} 4.10

This equation is the same as the one given by Bray (1967).

By considering the equilibrium of the normal stress on the discontinuity and equating this to zero, that is $\sigma = 0$, Figure 4.2(b), the condition for tensile failure is,

$$\frac{\sigma_2}{\sigma_1} = -\tan^2 \theta = K'$$  \hspace{1cm} 4.11

This indicates that tensile failure will exist if $\sigma_2$ is negative, which occurs when $\theta = 0$, $K = 0$ or when $\theta > 0$, $K > 0$. If $\sigma_1$ becomes negative, tensile failure will occur because the above relations are only valid for the assumption that $\sigma_2$ is negative and $\sigma_1$ is positive.

Slip along the joint will happen when $\frac{\sigma_2}{\sigma_1} < K$. However if $\theta$ is too small platen interference will take place and if $\theta$ is too large $K$ becomes negative implying the development of a tensile condition. Consequently the type of failure in this instance is the fracture of the material surrounding the joint.

Another type of possible failure is that of the intact rock; using the Coulomb–Navier criterion of failure it can easily be shown,

$$\frac{\sigma_2}{\sigma_1} = (1 + \frac{H_2}{\sigma_1}) \tan^2 (45 - \frac{\phi}{2}) - H_1 = K''$$  \hspace{1cm} 4.12
where $\phi_i$ and $c_i$ are the angles of friction and cohesion for the intact rock, and

$$H_i = c_i \cot \phi_i$$

4.4.1 Volume change associated with slip

If a group of discontinuities are considered, Figure 4.4(a), the distorted block configuration, Figure 4.4(b), is load path dependent because for block slips to occur in both directions at the same time, the shear stress system required to bring this about is inadmissible. This type of movement especially if it is load path dependent tends to rotate the individual blocks as can be readily observed by distorting a mass system of blocks, Figure 4.5. It should be noted that this rotative effect tends to add another factor into the cause of dilation, namely volume change due to the change of geometry controlled by imposed or induced constraints.

Whether dilation due to slip or rotation is going to take place appears to be dependent on the imposed or induced stress environment and the constraints inhibiting dilation. A very important characteristic observed in all the experiments was that in all areas where conditions were such as to allow dilation the stresses within the blocks reduced, while in areas where volume contraction occurred the stresses increased within the blocks.

4.5 GEOMETRY CHANGES

It is generally accepted that in a rock mass deformational response is dependent mainly on the distribution
and strength of the discontinuities. In turn the strength of these discontinuities is dependent on factors such as:

1. orientation of the joints in relation to the insitu and induced stress field,

2. nature of the joint whether rough or smooth,

3. nature and state, especially in relation to water, of the infill material,

4. history of the joint, whether slickensided or not,

5. magnitude of insitu stress field,

6. method of extracting rock,

7. volume of rock extracted.

Many of these factors have been examined in some detail by Jaeger and Cook (1969), Horn and Deere (1962), Donath (1964), Hoek (1964) and Rosengren (1968). A factor, however, which has not been examined or reported in detail is the effect of geometry change. Rosengren (loc cit) examined in some detail changes in geometry occurring in a triaxial test piece of jointed rock, where the resultant stresses acting on the joint were assumed to pass through the centre of area of the joint and that the stress distributions, though increasing in intensity, remained uniform over the area on which they acted. This investigation into the effects of geometry change in the triaxial tests was necessary in order to control and understand factors
FIG. 4.5

Tendency for blocks to rotate when deformed
There are two parameters, namely $\phi$ (coefficient of friction) and $E$ (effective modulus), which are used extensively in soil mechanics, and the theory of elasticity. These parameters have been defined in terms of one another by Walsh (1965), and used to analyse experimental data (Bray, 1967). Amonton's equation for a single discontinuity is often extended to multiple discontinuities where the Mohr Coulomb type equation $\tau = c + \sigma_n \tan \phi$ is used to define the strength. Brown (1968), using plaster blocks, has determined the relationship $\tau = c + \sigma^5 \tan \phi$ as being applicable to blocky material.

The above work was performed on material containing systems of multiple joints. Therefore the observed results quite often do not bring out the basic mechanisms causing the observed resultant effects such as deformational response or stress distribution. The approach here is to try and examine these basic mechanisms and then determine their effect on the mass response in a blocky material. Consequently slip along straight joints is examined as a precursor to the more complex systems examined in Chapters 5, 6 and 7.

4.5.2 Slip along a planar discontinuity

Two models of flat serrated notches were constructed as shown in Figure 4.11. The roughness on the faces of the notches was measured with a profilometer and found to vary between 20 to 70 micron mm. Normal loads were applied through parallel side blocks from two proving rings acting through bars which were in turn connected together by means of ball bearings giving a uniform load distribution. As slip occurred the imposed load from the proving rings
was kept constant and the movement of the serrated block was measured with dial gauges.

A normal load was first applied at the required magnitude and the shear load subsequently increased. For each increment of shear load the displacements were recorded. Also when it was obvious that slip had just taken place, the magnitude of the shear load before and after slip was also recorded. Photographs of the isochromatics were taken after each incremental increase in the shear load or when slip had occurred.

A series of photographs indicating the sequence and redistribution of load on the serrated notches are given in Figure 4.12 and 4.13.

4.5.3 Stress redistribution

Price (1958), when considering the dynamics of stress related to a normal fault, assumed that the horizontal stress at right angles to the strike will increase in magnitude and that correspondingly for a thrust fault, the horizontal stress will diminish when slip occurs. Anderson (1951) states that at the central portion of the fault, the shear stresses causing fault movement are relieved while the stress intensity increases at right angles to the fault plane. Therefore at a perpendicular distance of 0.4 times the length of the fault from the plane of the fault the original level of stress is retained, or exceeded. Hence at this distance it is possible that a second main fault may develop. Anderson (loc cit) adds that the stress intensity increases as the ends of the fault plane are approached. These statements recognise
the importance of assessing the stress redistribution before any subsequent behaviour can be determined. This is in fact the approach adopted here.

Interaction between the joint sets has a marked effect on the deformability of the material as a whole, and this interaction or interlock phenomenon is markedly affected by rotation and translation which in turn markedly affect the stress redistribution. With all the models described in this section, the object is to follow the redistribution of the stresses acting on simple joints as slip takes place.

4.6 THREE BLOCK TEST WITH DIFFERING BOUNDARY CONSTRAINTS

4.6.1 Numerical model

It is well known, (Terzaghi, 1943), that the mobilisation of shear strength requires a specific displacement and the ratio of shear force to normal force must reach the maximum angle of obliquity $\phi$, termed the friction angle. Now if the model, Figure 4.6, is loaded from the one side aa, the stress distribution in the middle block will cause maximum strain at this end and zero strain at the free end bb. Also it is assumed that the strain varies linearly from a maximum at the loaded end to zero at the free end then the mobilised shear force which is proportional to this strain, also varies linearly from a value $\mu N$ at the loaded side aa to zero at the end bb. At this stage, if $\mu = 0.5$, torques or moments of magnitude

$$T = 0.3 \text{ NLd}$$

4.13
where \[ N = \text{normal stress}, \]
\[ L = \text{length of block}, \]
\[ d = \text{depth of block}, \]

are imposed on the blocks constraining the middle one. Slip of the middle block takes place once slip at the free end bb occurs, and the magnitude of shear stress at this instance is constant, namely \( \tau = \mu \sigma \), where \( \sigma \) is the imposed or induced normal stress. When slip along the entire joint is imminent the torques acting on the top and bottom blocks are,

\[ T = 0.5 \, NLd \]  

4.6.2 Experimental results

Three square blocks, arranged as shown in Figure 4.7 were tested with dial gauges appropriately placed to measure displacements and the applied normal load set to a specific value. An increasing shear force eventually caused total slip of the middle block. In the tests, the shear force invariably dropped once slip occurred thus arresting further slip. Therefore, in order to induce further slip the load had to be increased to a value equal to or greater than its previous magnitude. During these loading sequences the stress patterns were photographically recorded.

Two types of end constraints were imposed on the constraining blocks, namely fixed or hinged, Figure 4.7(a) and (b). This produces two different kinds of deformational response, Figure 4.8 (a) and (b).
4.6.3 Fixed or rigid end constraints

In this experiment the load was applied through two steel platens with 1/8 in. neoprene sheet interposed between the platens and model. As slip occurred the applied shear load reduced in magnitude because more hydraulic fluid in the loading cylinder was required to sustain the load level. Because the stiffness of the loading system is greater than that of the model, the load displacement response is stable (Jaeger and Cook, 1969). In order to induce further slip, therefore, the shear load has to be increased. It was found in this instance, that the increased load was much the same as the load which had just produced slip, Figure 4.8 (a).

Differences between the ideal situation and the experiment just described are clearly shown in Figure 4.8 (a). The ideal load displacement graph should, once slip is initiated, continue with decreasing shear force because of the reducing area.

The important fact to note is that as slip occurs stress redistributions take place which in turn can effect subsequent deformational responses especially those of a mechanistic nature. For instance the rotative tendency or some initial defect on the slip planes could cause the trailing edge at aa, Figure 4.9., to bite into the contact planes. Consequently local yield at these points is possible. This implies that further slip is inhibited unless an additional load is applied to overcome this ploughing effect brought about by this local yield.
FIG 4.6
Three blocks with constant area of contact

FIG 4.7
End constraints on three block experiments
4.6.4 Free or hinged end constraints

Here again the load is applied through two steel platens, but in this instance ball bearings are inserted between the loading arms and steel platens, Figure 4.7(b). This allows the end loading blocks to rotate more freely. The resulting load-displacement graph and isochromatic stress patterns are shown in Figure 4.8 (b) and 4.9 respectively.

4.6.5 Comparative deformational responses

Two important characteristics from the above experiments become evident,

1. the load-displacement response for the fixed end is nearer to an elasto-plastic type response while that of the free end constraints gives an elasto-plastic strain hardening type response,

2. for both experiments there is a marked redistribution of stress subsequent to each increment of slip.

With pinned end constraints, once slip is initiated an imbalance of the induced torque T on the constraining blocks is mobilized with the consequent redistribution of normal stresses. In this case, however, the rotation of the top and bottom blocks occur more readily, and this mechanism appears to allow more stress redistribution to take place when compared with the fixed end constraining blocks. The net result is that each succeeding slip requires a higher shear force to restart the slip mechanism, Figure 4.8 (b), also there is the tendency for the top and bottom blocks to plough into the sliding
**FIG 4.8**

Experimental results of three block tests (FIG 4.7)
block, consequently requiring the shear load to be increased still further.

With continuing slip, a stage is reached where, even with the hinged constraints (ball bearing in this case), no further change in geometry, besides reduction of area, is likely and the now fixed constraint reaction controls further slip and one could anticipate a levelling off, Figure 4.8 (b) of the force displacement graph. This reaction is a possible explanation for the many instances where strain hardening of a joint is encountered and where the deformational response levels out after some considerable slip, (Rosengren, 1968), Figure 4.10. Spherical ends which are lubricated with Molybdenum Disulphide and tested in a triaxial machine could readily fit the above description.

The above examination suggests that not only can the material response have a deformation-hardening characteristic, but a deformation-hardening effect can also be obtained from the constraints imposed on the surrounding blocks.

4.6.6 Three blocks with a constant area of contact

Another experimental configuration which has been used for studying slip is shown in Figure 4.6, (Jaeger and Cook, loc cit). Here it will be noticed that the area of contact and the overall geometry remains the same at all stages of slip. Nevertheless, as shear is applied to the planes of slip, a torque is imposed on the side blocks supporting the central one.
Development of digging in effect shown by developing isochromatics as shear load $S$ is increased (unrestrained).
If the material between the contact planes is strong enough to prevent local yield and there is no change in geometry, the load-deformation graph would be as shown in Figure 4.8 (a). If however, the material between the contact planes yielded due to the slight rotation of the supporting blocks, the subsequent applied shear forces would have to overcome the normal shear resistance plus the force required to plough up the material caused by the digging in of the trailing edges aa, Figure 4.9.

4.6.7 Three blocks with a limited but constant area of contact.

A line diagram depicting the geometry of the model examined here is shown in Figure 4.11. Stress redistribution occurs on a minor scale on the contact planes, Figure 4.12, but on a much larger scale within the middle block, Figure 4.12 and 4.13.

Sequential stress redistributions are depicted clearly by the series of photographs of isochromatics in Figure 4.12 and Appendix F. Here again the stress redistributes towards the trailing edge of the slip plane which, in this case, is the trailing contact plane aa, Figure 4.11.

While the shear load was increased and slip induced, the displacements of the centre block were measured with a dial gauge and after each noticeable slip (distinct movement), photographs of the isochromatic patterns were taken.

In order to reduce the effect of changing geometry, the loading system was given a bias to throw load onto
FIG 4.10 (after Rosengren)

Various types of deformational response obtained from slip on a discontinuity in rock sample.
$N = 2000 \text{ lbs.}$

$S = 0$

$S = 340 \text{ lbs.}$

$S = 500 \text{ lbs.}$

**FIG. 4.12**

unconstrained
\( N = 2000 \text{ lbs.} \)
\( S = 0 \)

\( S = 340 \text{ lbs.} \)

\( S = 500 \text{ lbs.} \)

**FIG. 4.13**

constrained
the front part of the model. This, however, did not appear to reduce the subsequent stress redistributions to the trailing edge, aa.

4.7 SLIP IN A STACKED BLOCK SYSTEM

4.7.1 Interaction of an enclosed block

Figure 4.14 depicts a block surrounded by four blocks of equal size. If slip of blocks (A) and (B) occurs with no redistribution of stress the interface between the central blocks (C) and (D) would have to develop an opposing shear force which would in turn provide the opposing torque to satisfy equilibrium. Also if the blocks are the same size as indicated in Figure 4.14, slip of these side blocks would ensue. However, this latter slip is prevented due to interference from the upper and lower block (A) and (B), causing stress concentrations at the points of contact between the blocks (B) (D) and (A) (C) respectively. (Trollope, 1969).

If the blocks in Figure 4.14 are rectangular and the confining stresses are equal, namely

$$ \sigma_a = \sigma_b $$

$$ \frac{N_a}{a} = \frac{N_b}{b} $$

maximum obliquity $\phi_a = \phi_b$ is developed on all edges of the block and incipient slip is prevalent. Consequently the same arguments as apply to the square block are applicable in this instance.
FIG. 4.15
2 blocks X & Y removed

FIG. 4.14 (a) block interaction

FIG. 4.14 (b) block A removed
This edge interaction or interlock is one of the basic phenomena observed in the photoelastic models of the block systems examined in Chapter 6.

The slip mechanisms described above do not consider any stress redistributions which in most cases occur when slip takes place. These stress redistributions have been found in the main to be dependent upon the allowable or induced rotations of the blocks caused by altering the boundary conditions or changing the induced loading. It appears therefore that slip and rotation are two basic factors to be considered when examining the response of a blocky medium to an imposed load, Chapters 5, 6 and 7.

4.7.2 Removal of a single block from a staggered stacked system

A system of blocks, stacked as shown in Figure 4.14(b) are loaded in such manner that no shear stresses occur at the joints, that is the principal stresses are vertical and horizontal. Now if the central block (A) is removed the stress resultant acting between the abutment blocks (D) and (E) now become skew, Figure 4.14b which causes a shear force to be induced at the interface between the abutment blocks and the top and bottom surrounding blocks. If the shear stress exceeds the strength of the joint, slip will occur. By considering equilibrium of the block and joint it becomes evident that as maximum obliquity $\phi_j$ is reached on the joint as described above, Figure 4.14b, so will maximum obliquity $\phi_j$ be attained on the joints surrounding the individual blocks (B) (C) (F) and (G). This means that these blocks are also at a stage of incipient slip.
The subsequent stress redistribution due to the mechanism described above will be considered in more detail in Chapters 6 and 7. Nevertheless, it should be noted that stress concentrations will occur at a, a', a'', and a''', due to the stiffening at 'a' (load increase) and the loosening at 'b' (load decrease). Also if the centre of gravity of the block lay over the opening rather than over the abutment, the rotational effects would be greatly augmented by gravitational forces. In the case of the top blocks, namely (B) and (C), this augmentation would be to increase the rotational effect, while for the bottom blocks (F) and (G) the rotational effect would be reduced.

The abutment blocks (D) and (E), though not rotating are subjected to greatly increased loads at the face of the abutments giving rise to induced tensile strains. This aspect of abutment stability leading to possible spalling will again be considered in greater detail in Chapters 6 and 7.

Many elaborate schemes of stacking; loading and then removing blocks are examined but the factor to be brought out at this juncture is that once slip is instigated the normal application of continuum theory becomes inadmissible and mechanisms due to slip and rotation then determine how the blocky material is going to respond to load.

4.7.3 Removal of two blocks from a stacked system

If the two blocks (X) and (Y) are removed, Figure 4.15, the top abutment blocks (A) and (C) are subjected to greater disturbing moments when compared to the removal of one
block, Section 4.7.2, because of the greater imbalance of induced forces. These rotative effects cause the compressive loads to increase at the corners a,b and reduce at e,d as shown in Figure 4.15. Consequently the stress distribution across the edge between blocks (B) and (C) will be non-uniform and depict a flexural type of stress distribution experienced in a no tension blocky beam.

If the horizontal thrust and therefore the horizontal compressive stress is of sufficient magnitude so as to inhibit the development of tensile conditions at the corner such as e, Figure 4.15, the downward displacement of block (B) is reduced. Because block (B) tends to move downwards more than blocks (C) and (A), the blocks (I) and (J) also experience disturbing and consequent stress concentrations. These stress concentrations, however, would not be as great as those experienced by blocks (A) and (C) because of the reduced relative deflections and therefore lesser rotative effects.

It is clear therefore that the degree of rotation for specific blocks is dependent on the deflections occurring, which in this case is at the midspan of the opening. This rotative effect which produces the mechanistic arching action is inter-related with the mechanistic beaming action. Figure 4.16 shows that as the span of the opening increases while the horizontal thrust remains the same, the rotative effect causing increased stress concentrations is clearly evident.
HOR. Ld. = 315 lbs. × 2
VER. Ld. = 630 lbs. × 2

FIG 4.16 a
Aspect ratio 3
Blocks 3" x 2"
HOR. Ld. = 2 \times 376 \text{ lbs.}
VER. Ld. = 2 \times 756 \text{ lbs.}

Fig 4.16 b
Aspect ratio 4.5
Blocks 3'' x 2''
4.8 SUMMARY

Friction on a simple plane does not appear at all complex in that a simple relation, equation 4.1, connects the shear stress to normal stress. Yet by examining the relation in more detail it becomes evident that this is not the case and in fact the process of frictional slip is quite complex. Experimental values of the coefficient of friction are therefore used when assessing the strength parameters of a jointed material. These experimental parameters are very much constraint dependent.

It has been shown that even if the slip geometry is kept under control, Section 4.6.6 and 4.6.7, the restraining blocks experience an induced moment. These moments cause the restraining blocks to rotate. If this rotation is prevented a rigid restraint is said to apply while if rotation occurs a pinned restraint is acting. The deformational response from these two constraints is very different. In terms of stress redistribution it is observed that for the pinned case this is greater. Consequently the deformational response is definitely related to the stress distribution acting across the planes of slip and the constraints imposed on the blocks making up these slip planes.

A consequent effect of this rotation is the tendency of the restraining blocks making up the slip planes to plough into the material. This effect will again cause the deformational response to strain harden.

In the preloaded staggered block system the removal of one block causes an imbalance in the force system
and introduces shear forces which in turn cause certain
blocks to rotate. This rotation causes the load to
redistribute in what appears to be two basic forms,
namely mechanistic beaming or arching action.

Another important fact is that what was initially
a uniform stress distribution is now non-uniform from
block to block. Therefore to analyse the load distribution
from block to block a structural approach would be
required. Once this is done it would then be appropriate
to consider the stress distribution within the block.
Chapter 5.

DEFORMATIONAL RESPONSE

5.1 PREAMBLE

When a blocky or discontinuous material is encountered in engineering, it is often stated (Muller, 1963) that if the individual units of the sample are small relative to the size of the sample \( \frac{1}{200} \), the measured experimental parameters are representative of the mass as a whole. This approach is found to be valid for the ratio of unit to sample size less than \( \frac{1}{2} \). For higher values of this ratio however the experiments have to be carried out very carefully. Consequently the often quoted ratio of unit size to sample size equal \( \frac{1}{10} \) may appear acceptable. These statements however are not always valid and it is shown here that the measured experimental parameters are very restrictive in that they are relevant only for the configuration of blocks and joints plus the loading sequences to which the models or samples are subjected.

Also in this chapter the compressibility and stiffness characteristics of material containing cracks and pore defects are examined and compared with theoretical bound values. When slip along a crack or movement around a cavity is excessive elastic theory is no longer satisfactory for calculating the stress distribution within the material, nor is the plastic theory suitable for taking over where elastic theory loses relevance. It appears possible that at some stage of deformation when elastic theory or elastic analogue techniques are no longer applicable the material starts to behave as a structure.
What is meant here is that even though the mass deformational response may be elastic or plastic in character the determination of the relevant parameters is so restrictive as to make the analogue approach inapplicable and a structural approach would best serve as a method for analysing the load distribution.

5.2 UPPER AND LOWER BOUND VALUES FOR VOLUME COMPRESSIBILITY

The mass response of a blocky material to an imposed load is an accumulation of effects derived from the response of individual blocks and jointing system. It is common to measure this mass response in the form of experimental parameters such as linear or volume compressibility defined as,

\[
\text{Linear Compressibility } = \beta_L = \frac{\Delta L}{L \Delta \bar{P}} \quad (a)
\]

\[
\text{Volume Compressibility } = \beta_V = \frac{\Delta V}{V \Delta \bar{P}} \quad (b)
\]

where $\Delta \bar{P}$ = incremental change in stress,

$\Delta L$ = corresponding change in length,

$\Delta V$ = corresponding change in volume,

$L$ = original total length,

$V$ = original total volume.

The inverse of $\beta_L$ is often called the effective Young's modulus. Now the question is posed, if the intact moduli of the individual blocks are known, is it possible to obtain the mass modulus of the blocky system? That is,
\[ \beta_{\text{eff}} = \beta + \frac{\Delta n}{\Delta F} \]

where the porosity \( \Delta n \) is positive when its magnitude decreases. Equation 5.2 is independent of the shape or total volume of the pores and requires that the framework of the material containing the pores be elastic.

Pores in a material can generally be defined as belonging to one of two groups, namely volume pores or line cracks. Walsh (1965), using idealised shapes for the first group of pores, obtained theoretical relations from equation 5.2. Brace (1965) experimentally determined values for the volume compressibility, \( \beta_{\text{veff}} \), and compared these results with two summation processes which give the upper and lower bounds for the correct magnitude of intact \( \beta \).

In the solution of an elastic problem both the requirements of equilibrium and compatibility should be satisfied. However, in many problems, in order to acquire a solution, the body is discretized and in so doing, either equilibrium or compatibility is violated.

If a defect material containing pores is split up into regions each containing one defect, and compatibility between the regions is satisfied, Voigt (1928) showed that,

\[ \frac{1}{\beta_v} = \frac{V_a}{\beta_a} + \frac{V_b}{\beta_b} + \frac{V_c}{\beta_c} + \ldots \]

where \( V_a, V_b, \) etc. are the volume percentages of the different materials in the rock, and \( \beta_a, \beta_b, \) etc. are the corresponding compressibilities.
This was shown by Hill (1952) to be a lower bound solution for the compressibility which in turn is the upper bound value for stiffness because of the inverse relationship between the two. Moreover if equilibrium is satisfied between the regions Reuss (1929) showed that,

$$\beta r = V_a\beta a + V_b\beta b + V_c\beta c + \ldots$$  \hspace{1cm} \text{5.4}$$

where $V_a$, $V_b$, etc. and $\beta a$, $\beta b$ are as in 5.3 above.

Equation 5.4 is an upper bound solution for the mass compressibility. Brace (1965) experimentally determined values of compressibility $\beta_{\text{eff}}$ and compared these with values of $\beta v$ and $\beta r$ compiled from component compressibilities $\beta a$, $\beta b$, etc. which in turn were evaluated by sonic methods. In every instance at the lower pressures, less than 2 kilo bars, $\beta_{\text{eff}}$ was greater than $\beta v$ and $\beta r$, whereas at the intermediate pressures, greater than 2 kilo bars but less than 9 kilo bars, $\beta_{\text{eff}}$ was less than $\beta v$ and $\beta r$. At higher pressures greater than 9 kilo bars, $\beta_{\text{eff}}$ was again greater than $\beta v$ and $\beta r$. When the material had relatively large porosities, 0.003 and 0.011, $\beta_{\text{eff}}$ was greater than both $\beta v$ and $\beta r$ and when the material had relatively low porosities $\beta_{\text{eff}}$ lay inbetween $\beta v$ and $\beta r$. The suggestion was made from the above work that material with the higher porosities, and even some with low porosities, the spherical cavities increased the effective compressibility with respect to the intrinsic compressibility, therefore explaining the higher values of measured compressibility. This, however, does not explain the greater values of $\beta_{\text{eff}}$ obtained at the higher pressures. Consequently it is realised that the spherical cavities could greatly effect the validity of the elasticity assumption on which the upper and lower bound values are based.
5.3 ANISOTROPY

The cause of anisotropy in a rock mass may be due to the rock material itself, where inherent factors such as layering or schistosity occur, or it may be due to defect orientations from jointing or faulting. If the defects are randomly distributed the rock mass is generally considered isotropic. However, randomness of defects in nature is the exception rather than the rule, (Lafeber, 1963, Rosengren, 1968). Defects inherent in the material give what is termed intrinsic anisotropy while defects due to jointing or faulting give what is termed induced anisotropy. It is obvious that there is no clear demarcation between the above definitions because inherent anisotropy at high pressures may in fact be induced. At the lower pressures encountered in engineering however the definitions stated above are generally accepted.

Also by considering the superimposition of hard and soft adhering layers Biot (1965) obtained the equivalent modulus of a material mass as,

\[ N = N_1 \alpha_1 + N_2 \alpha_2 \]  \hspace{1cm} (a)

\[ Q = \frac{1}{\frac{\alpha_1}{Q_1} + \frac{\alpha_2}{Q_2}} \]  \hspace{1cm} (b) \hspace{1cm} 5.5

where \( \alpha_1 \) and \( \alpha_2 \) are the percentage thicknesses of the material and \( N \) represents the lineal modulus of extension or compression, and \( Q \) the shear modulus.

The suffixes 1, 2 relate to the hard and soft material and \( \alpha_1, \alpha_2 \) are fractional thicknesses of the hard and soft
material relative to the total thickness being considered.

It should be noted that equations 5.5(a) and (b) have a form similar to equations 5.4 and 5.3 respectively. Equation 5.5(a) is a lower bound solution for a value of the mass lineal modulus $N$, and equation 5.5(b) is an upper bound solution for a value of the mass shear modulus.

In a discontinuous material such as soil or jointed rock where voids between individual particles or blocks occur, continuous parameters such as Young's modulus, Poisson's ratio, Lame's constants, become most difficult to define and equations 5.3, 5.4 and 5.5 lose their relevance. Yet if the discrete particles or blocks are sufficient in number to represent a relatively large mass, the aforementioned equations may again be definable. This latter consequence is essentially the approach used in soil mechanics. There are, however, many situations in soil mechanics where this quasi-continuum approach of analysis is very doubtful, for example, dense sand or over consolidated clay. This dilemma of whether to use elastic or plastic analysis appears to evolve around factors such as a no-tension material and/or relative slip with the corresponding constraints such as prestress and geometric changes. Here therefore, where the above factors are important, the use of elastic anisotropy even from an analogue point of view is inadmissible in that bound solutions do not apply and as will be shown, the experimentally determined parameters are so restrictive as to make their practical use very limited.
5.4 MATERIAL TESTING ON MOUNT ISA URQUHART SHALE

Experimental work was performed to investigate both the induced and intrinsic anisotropic characteristics of Urquhart shale. This work though not complete in depth was found necessary to perform and report in order to distinguish those characteristics affecting deformational response.

5.4.1 Intrinsic anisotropy of the Urquhart shale

Brace (1965) measured the linear compressibility of a gneiss parallel (||) and perpendicular (⊥) to the foliation. From these tests, Figure 5.1, the effects of cracks and crack closure can be summarised. It is apparent that the compressibility is a function of the confining pressure. Compressibilities parallel and perpendicular to the foliation change in relative magnitude. The reason for this characteristic was due to the relative ease with which the cracks closed at the grain boundaries for the lower pressures and at higher pressures $\beta_\perp$ is greater than $\beta_{||}$ because the mica, 10% with a strong preferred orientation, is more compliant normal to the (001) plane of anisotropy.

5.4.2 Unconfined compression tests

From two large blocks of Urquhart Shale, 24 in. x 18 in. x 15 in. 20 NMLC cores were drilled in different directions. One direction was perpendicular to the bedding planes of the shale while two directions were in the plane of the bedding, but at right angles to each other, and another direction was at 45° to the bedding planes. The cores
**FIG. 5.1**

Linear compressibility

![Graph showing linear compressibility vs. pressure.](image)

Pressure, bars (after Brace, 1964)

Linear compressibility of Torrington gneiss

**FIG. 5.5**

Deformational response for Urquhart shale at high confining pressure (35,000 p.s.i.)

![Graph showing load vs. displacement.](image)

1% strain (1.547 Kb) (L 6.25 Kb)

2% strain (1.917 Kb) (L 10.5 Kb)

DISPLACEMENT

LOAD
A is reading from left extensometer.
B is " " right 

FIG 52  Scale lins = 0.008 ins.
were cut to 5 in. lengths and then loaded in a 200T Amsler testing machine.

Two Huggenberger extensometers were located each side of the sample. As the vertical load was applied the deformational response indicated by the extensometers was in many cases uneven, that is, the sample was unevenly strained. It was observed during these experiments that the one extensometer would indicate movement while the other one did not. Yet as the deformation progressed the sequence of movement changed from the one extensometer to the other, Figure 5.2. A plausible explanation for this is that the intact sample contains defects which allow non uniform and progressive slip to take place thus causing local non uniform stress distributions to occur. Nevertheless over large displacements these uneven local slips or deformations give the appearance of being uniform.

In order to substantiate the above observations much more elaborate testing, especially the recording of continuous displacement over varying gauge lengths, would have to be performed. Nevertheless, the main idea of these tests was to determine the intact Young's Modulus and the degree of anisotropy. It becomes evident however, that the so called elastic moduli are ill defined even in an engineering sense.

Of the two blocks from which the samples were taken block 2 had the overall appearance of more jointing, this however, was not evident in the core samples extracted. Nevertheless these particular samples did appear to possess more defects as is evident when comparing the relative magnitude of moduli obtained from the samples of the two
blocks, Table 3. From these results no valid comment on the anisotropic deformational response can be made except that for the unconfined tests the Urquhart Shale does not exhibit any significant intact or inherent anisotropy.

The ultimate failure load for each sample is also given in Table 3. It should be noted that there is no consistent correlation between the loading direction, relative to the bedding planes, and the ultimate strength. Moreover the relationship between the magnitude of the load at fracture and Young's Modulus is not consistent. Here again, not enough testing information is available to show whether the above statements are conclusive, but the trend is obvious. For the purpose of this thesis, therefore, the intact Urquhart Shale, at the relatively low pressures encountered, is considered isotropic and if any anisotropy is observed this is induced from macro defects.

5.4.3 Tests on large Urquhart Shale samples

As a prelude to an attempt to measure the insitu stresses with a Leeman pressure cell a series of calibration tests were performed in the laboratory. The large blocks of rock 24 in. x 18 in. x 15 in. wide, Figure 5.3, from which the samples in Section 5.4.2 were cored, were installed in a large stressing frame (Hoskins, 1967) where flat jacks were used to apply the load. The strain gauge cell was set up in accordance with the instructions pertaining to the cell except in this instance the author used a mix of adhesive glue made up from Araldite D, silica powder, and amine hardener CBY951.

Three block samples, namely 1 block of Trachyte and 2
blocks of Urquhart Shale obtained from Mount Isa Mines were cut to the aforementioned dimensions and when required individual blocks were carefully packed into the straining frame. The strain gauges were then installed and a series of loading cycles from flat jacks were applied to the blocks. As many as 10 to 15 cycles were imposed before the strain gauge hole was overcored and the insitu stresses determined from strain relief of the core.

Experimental strain gauge readings were then fed into a computer programme which evaluated the experimental stresses. Values of the computed stress versus the applied stress are given in Figure 5.4(a), (b) and (c). These graphs are typical of the values obtained from the experiments.

The computer programme calculating the stresses utilizes infinitesimal elastic theory (Leeman, 1968). A Young's modulus of $8 \times 10^3$ psi for the shale was taken from uniaxial test results above and used for the stress compilations. The results clearly show that this assumption is greater for shale block 2 than it is for shale block 1. Just before the blocks were installed in the loading frame they were carefully examined visually and it was clear that though shale block 2 was in one piece it possessed many more defects, such as hard and soft inclusions plus many observable cracks.

It is observed, that even though an incorrect Young's modulus was used for compiling the stresses in the trachyte block it nevertheless behaved anisotropically, Figure 5.4(a). The shale blocks also behaved anisotropically in that they appeared to be dependent on the defect
TRACHYTE BLOCK

Calculating stresses from experimental readings

Applied stress Scale 1" = 200 p.s.i

1 to 1 correlation between calculated and applied stress

FIG 5.3
SHALE BLOCK 1 (sound or intact material)

1<sup>st</sup> LOAD CYCLE

$\text{Scale lines = 200 p.s.i}$

$\text{1 to 1 correlation between calculated and applied stress}$
SHALE BLOCK 1 (sound or intact material)
2nd LOAD CYCLE

Calculated stresses from experimental readings

Scale lines = 200 psi

1:1 correlation between calculated and applied stress.

FIG. 5.4b
SHALE BLOCK 2 (many observable defects)

1st LOAD CYCLE

Calculated stresses from experimental readings
Scale 1 ins. = 400 p.s.i.

1 to 1 correlation between calculated and applied stress

FIG 5.4c
Scale 1 ins. = 200 p.s.i.
<table>
<thead>
<tr>
<th>Block 1 Sample No.</th>
<th>Left Ext. E x 10^5</th>
<th>Right Ext. E x 10^5</th>
<th>Utl. Stress</th>
<th>Block 2 Sample No.</th>
<th>Left Ext. E x 10^5</th>
<th>Right Ext. E x 10^5</th>
<th>Utl. Stress</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>9.1</td>
<td></td>
<td>13,500</td>
<td>2A1</td>
<td>4.5</td>
<td></td>
<td>11,000</td>
</tr>
<tr>
<td>A2</td>
<td>7.08</td>
<td></td>
<td>24,100</td>
<td>2A2</td>
<td>6.5</td>
<td></td>
<td>14,300</td>
</tr>
<tr>
<td>A3</td>
<td>8.05</td>
<td></td>
<td>19,900</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A4</td>
<td>10.6</td>
<td></td>
<td>13,700</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>B1</td>
<td>15.0</td>
<td></td>
<td>30,900</td>
<td>2B1</td>
<td>8.3</td>
<td></td>
<td>24,400</td>
</tr>
<tr>
<td>B2</td>
<td>5.85</td>
<td></td>
<td>16,300</td>
<td>2B2</td>
<td>4.9</td>
<td></td>
<td>12,300</td>
</tr>
<tr>
<td>C1</td>
<td>11.85</td>
<td></td>
<td>19,200</td>
<td>2C1</td>
<td>8.4</td>
<td></td>
<td>12,300</td>
</tr>
<tr>
<td>C2</td>
<td>7.9</td>
<td></td>
<td>14,500</td>
<td>2C2</td>
<td>7.3</td>
<td></td>
<td>19,300</td>
</tr>
<tr>
<td>C3</td>
<td>10.8</td>
<td></td>
<td>18,100</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>C4</td>
<td>8.57</td>
<td></td>
<td>14,200</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D1 45°</td>
<td>7.62</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D2 45°</td>
<td>11.54</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D3 45°</td>
<td>12.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D4 45°</td>
<td>8.1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Vertical load perpendicular to the bedding plane = ⊥
Vertical load parallel to the bedding plane = ||

**TABLE 3.**
characteristics. It should be stressed that this is only an observation and that much more work on this is required before any definite conclusions could be reached.

To summarise it is suggested that small intact samples do not depict the deformational response of a sample which is even moderately larger, for example ten times as large.

5.4.4 Tests at medium confining pressures

Rosengren (1968) performed extensive triaxial tests on Urquhart Shale using confining stresses between 500 p.s.i. to 5,000 p.s.i. In this work it was shown that from the samples tested there were three major modes of failure, namely -

1. failure through a weakness plane,

2. part shear failure in the weakness plane and part tension failure through the intact material,

3. failure through the intact material; unrelated to the weakness plane.

The weakness planes appeared to be veins or bedding joints. Some defect planes of the Urquhart Shale, however, did not constitute planes of weakness as they were either hard dolomite or quartz. As far as the strength of the intact material is concerned, the orientation of these bedding joints and veins introduced an orientated dependent anisotropy. In zones where chlorite veins occurred, this feature is the prime factor causing anisotropic response.
Unfortunately in the work described above no load displacement graphs for the intact material were presented, (this was not required in the context of the work) and Young's modulus was unattainable. From Hoskins (1967), who worked on similar shale material, the values of Young's modulus were taken as,

$$E_{||} = 10.5 \times 10^5 \text{ p.s.i.}$$

$$E_{\perp} = 8.8 \times 10^5 \text{ p.s.i.}$$

It should be noted that anisotropic strength does not imply an anisotropic deformational response. Strength and deformational response though related are such that where mechanisms of slip and rotation are encountered strength is more a function of the deformational mechanisms rather than material strength.

5.4.5 Tests at high confining stresses

Eight samples, 1 cm. in diameter and 2 cm. long, were cut from Urquhart Shale, parallel and perpendicular to the bedding planes. The samples were jacketed with R.R.J tubing and confined in oil at either 25,000 p.s.i. or 35,000 p.s.i. Automatic recording of the load and displacement was performed with a high pressure machine (approximately 100 Kg per sq.cm.) built by Patterson (1963).

Young's modulus was obtained from the results using a secant modulus at 1% and 2% strain giving the results in Table 4.
<table>
<thead>
<tr>
<th>Confining Pressure</th>
<th>25,000 p.s.i.</th>
<th>35,000 p.s.i.</th>
</tr>
</thead>
<tbody>
<tr>
<td>E 1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E 2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E 1%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E 2%</td>
<td></td>
<td></td>
</tr>
<tr>
<td>E 1% ↓</td>
<td>6.3 x 10^6</td>
<td>14.4 x 10^6</td>
</tr>
<tr>
<td>E 2% ↓</td>
<td>6.8 x 10^6</td>
<td>9.1 x 10^6</td>
</tr>
<tr>
<td>E 1% ↓</td>
<td>8.8 x 10^6</td>
<td>11.5 x 10^6</td>
</tr>
<tr>
<td>E 2% ↓</td>
<td>6.6 x 10^6</td>
<td>9.7 x 10^6</td>
</tr>
</tbody>
</table>

**TABLE 4.**

E 1% || is Young's modulus parallel to the bedding plane at 1% strain and E 1% ↓ is Young's modulus perpendicular to the bedding plane at 1% strain. It is evident that at the higher strain, namely 2%, Young's modulus is of less magnitude than the modulus at the lower strain of 1%. This is in accordance with the ductile response depicted by the graph, Figure 5.5. Also at the lower confining stress of 25,000 p.s.i. Young's modulus is of greater magnitude parallel to the bedding plane when compared to the modulus perpendicular to the bedding plane. While at the higher confining stress Young's modulus parallel and perpendicular to the bedding plane is much the same in magnitude. This behaviour is what one would expect when considering the rock sample as a defect material, (Brace, 1965).

To summarise the above work the intact Urquhart Shale is a defect material where the definition of Young's modulus at low confining pressure becomes mechanistically dependent and therefore is only an apparent measure of
deformational response. That is displacement in response to load becomes slip dependent. Of consequence the magnitudes of moduli become difficult to define and a material which may inductively be considered anisotropic may not in fact be so, for example shale. This was depicted by the results of the unconfined compression tests and work performed on the large block samples.

5.5 MODULI FROM BLOCK MODEL TESTS

5.5.1 Preamble

When constructing photoelastic block models in general and the number 2 and 5 ore bodies at Mount Isa mines in particular many questions are posed and those relevant to this thesis are -

1. what effect does the size of the blocks relative to the openings have on the deformational response?

2. what effect does the shape and number of openings have on the deformational response?

3. what effect does the orientation of joints and faults have on the deformational response?

4. what effect does the load path have on the deformational response?

In an attempt to obtain some answers to the above questions and because in the prototype the basic control measurement is one of deformation, it was decided that where possible displacement of salient blocks would be
measured. These block displacements were measured by dial gauges and installed as described in Section 3.6. Consequently the relative displacement between the chosen blocks are easily measured and the effective moduli determined. This information coupled with the photoelastic stress pattern, allowed a clear perspective of the deformational response to be attained.

Individual plots of the block displacements versus loading are given in Appendix E, plus plots of the relative displacement versus loading. Necessary plots for the development of the discussion in this chapter are used when required.

5.5.2 Effect of block size on deformational response

From the geological survey work produced by Rosengren (loc cit) the Urquhart shale in the region of interest has a continuous jointing system along the bedding planes and a discontinuous set of transverse micro faults, Chapter 8. The result is that the Urquhart shale is best represented by a staggered stacking system of the blocks as indicated in Figure 5.6. This was the main configuration used in the experiments though other outlays were also examined where it was anticipated that a better understanding of the mechanisms would result.

In most cases the shape of the blocks were square; however, rectangular blocks with different aspect ratios of length to depth were also examined.

For each block size investigated the first test was one where all the blocks were intact and the loading
cycle applied by equally incrementing the horizontal and vertical loads and then at a predetermined value the horizontal load was kept constant while the vertical load was incremented to a value two or three times the magnitude of the horizontal load. In most cases the load was cycled so that the hysteresis and unloading characteristics were obtained. Subsequent to the test just described, several blocks which varied in number and gave different geometrical opening patterns, were removed and subjected to the same sequence of loading as above.

In the following presentation the deformational results are discussed using the given concepts of moduli definition even though these are subsequently found to be inadequate for a blocky material. By combining the two features of displacement, for example slip and stress redistribution due to this slip, a better appreciation of the deformational response in a blocky mass is obtained, and insight into the behaviour of a discontinuum acquired.

5.5.3 Theoretical effective moduli

If E is defined here as the uniaxial Young's modulus and the material is considered as homogeneous and isotropic, different values of an effective modulus $E_{eff}$ would be required when the imposed loads were not uniaxial.

The basic stress-strain relationships are -

\[
\varepsilon_1 = \frac{1}{E} \left\{ \sigma_1 - \nu (\sigma_2 + \sigma_3) \right\}
\]

\[
\varepsilon_2 = \frac{1}{E} \left\{ \sigma_2 - \nu (\sigma_1 + \sigma_3) \right\}
\]

\[
\varepsilon_3 = \frac{1}{E} \left\{ \sigma_3 - \nu (\sigma_1 + \sigma_2) \right\}
\]
Blocky model as opposed to continuous model used in Fig. 5.7

**Fig. 5.6**

\[
\frac{E_{\text{eff}}}{E} = \frac{1 - \nu}{(1+\nu)(1-2\nu)}
\]

**Fig. 5.7**

Ratio of Effective modulus to Young's modulus.
If constraints are imposed so that $\varepsilon_2$ and $\varepsilon_3$ are zero we have the relationship

$$\sigma_2 = \sigma_3 = \frac{\nu}{1-\nu} \sigma_1 \quad 5.7$$

and the effective modulus in the $\sigma_1$ direction becomes, Figure 5.7 -

$$E_{\text{eff}} = \frac{(1-\nu)}{(1+\nu)(1-2\nu)} E \quad 5.8$$

It is at once apparent that the material now has an induced anisotropy. Consequently it becomes important to define the effective modulus of the intact araldite in terms of the same load constraints imposed on the block models in order to compare the mass block moduli with the intact material moduli. Also if $\varepsilon_2 = 0$ and $\varepsilon_1 \neq 0$, $\varepsilon_3 \neq 0$, $\sigma_3 = 0$ which is another extreme condition of constraint we obtain, Figure 5.7 -

$$E_{\text{eff}} = \frac{E}{1 - \nu^2} \quad 5.9$$

If $\sigma_2$ and $\sigma_3$ are zero, which is the other extreme constraint condition, we have

$$E_{\text{eff}} = E \quad 5.10$$

For $0 < \nu < .5$, $E_{\text{eff}}$ is greater than the uniaxial modulus $E$.

Considering the important practical situation of plane strain, that is if $\varepsilon_2 = 0$, we obtain, Figure 5.7 -
Here it is evident that the effective modulus is dependent on the stress ratio. Consequently, the value of $E_{eff}^3$ may reverse sign. That is $E_{eff}^3$ may become negative suggesting the initiation of a tensile stress in the $\sigma_3$ direction. This result has been borne out on many occasions in engineering and geological situations where $\sigma_3$ has been set to zero and the consequent development of tensile failures from an effective tensile strain occurs.

5.5.4 Practical effective moduli

When the material being deformed is no longer intact but contains joints or open and closed cracks, it appears that the definition of modulus is suspect. However, if the deformation of these defects occurs within a relatively short time and thereafter remains constant, the situation is constant secondary creep $\frac{dc}{dt} = 0$, and the elastic analogue method for calculating stress distributions may appear valid, (Hult, 1966). It should be emphasised, however, that in a defect material hysteresis and residual deformations occur and this makes the total deformation very much load path dependent. Also the magnitude of slip is generally of such magnitude as to make the deformations finite and the consideration of geometry change is essential. Nevertheless for a monotonically increasing or decreasing load deformation
curve it may still be possible to define a practically useful analogue value for Young's modulus.

It is evident by the manner in which the blocks are stacked that the blocky mass has an induced anisotropy in the y and x directions, Figure 5.6. The constitutive stress-strain relations used are:

\[
\varepsilon_y = \frac{\sigma_y}{E_y^{\text{vert}}} - \frac{\nu_{xy}\sigma_x}{E_y^{\text{hor}}} \text{ in the y direction,}
\]

\[
\varepsilon_x = \frac{\sigma_x}{E_x^{\text{hor}}} - \frac{\nu_{yx}\sigma_y}{E_y^{\text{vert}}} \text{ in the x direction,}
\]

\[E_y^{\text{vert}} \text{ and } E_x^{\text{hor}} \text{ are the effective moduli in the direction in which the loads are applied and } \nu_{xy} \text{ and } \nu_{yx} \text{ are strain ratios defined below.}
\]

In order to determine the unknowns \(E_y^{\text{vert}}, E_x^{\text{hor}}, \nu_{xy}\) and \(\nu_{yx}\), specific sequences of loading were applied. Initially the vertical and horizontal loads were equally incremented while the relative deformations were measured with dial gauges, Figure 3.14a. After a definite value of all round stress had been attained, the vertical load was incrementally increased to a value of about three or four times the magnitude of horizontal load.

Consequently under the hydrostatic load increments we have \(\sigma_y = \sigma_x\):

\[
E_{y}^{\text{hyd}} = \frac{\sigma_y(1-\nu_{xy})}{\varepsilon_y}
\]

\[
E_{x}^{\text{hyd}} = \frac{\sigma_y(1-\nu_{yx})}{\varepsilon_x}
\]
where $E_y^{hyd}$ and $E_x^{hyd}$ are the effective moduli when the applied stresses in the vertical and horizontal directions are equal, and $v_{yx}$ and $v_{xy}$ are parameters much like an effective Poisson's type ratio. Then using an analogue type approach $v_{yx}$ and $v_{xy}$ are defined as

$$v_{yx} = -\frac{\varepsilon_{x}}{\varepsilon_{y}^{vert}}$$

and

$$v_{xy} = -\frac{\varepsilon_{y}}{\varepsilon_{x}^{hor}}$$

where $\varepsilon_{y}^{vert}$ and $\varepsilon_{x}^{vert}$ are the vertical and horizontal strains caused by increasing the vertical load only, and $\varepsilon_{y}^{hor}$ and $\varepsilon_{x}^{hor}$ are the vertical and horizontal strains caused by increasing the horizontal load only. Consequently it is easy to show

$$\frac{1}{E_y^{hyd}} = \frac{1}{E_{y}^{vert}} - \frac{v_{xy}}{E_{x}^{hor}}$$

and

$$\frac{1}{E_x^{hyd}} = \frac{1}{E_{x}^{hor}} - \frac{v_{yx}}{E_{y}^{vert}}$$

Values for these experimental effective mass moduli and Poisson's type ratio are plotted against the block size, Figures 5.8 and 5.10. Because some blocks were square while others rectangular it was found more expedient to plot the appropriate values of $E_{eff}$ against the inverse ratio of peripheral contact length to area. This
expediency has some justification in that the defects inherent in the models are in the main dependent upon the peripheral length and size of block.

5.6 EXPERIMENTAL DETERMINATION OF BLOCKY MASS RESPONSE

5.6.1 Intact modulus of model material

The material from which the block models were made was loaded under the same conditions as were the block models and good agreement between the intact Young's modulus and anticipated effective Young's modulus was obtained. The uniaxial Young's modulus was 400,000 p.s.i. and the Poisson's ratio 0.25 and this gives an anticipated Young's modulus under the loading condition imposed as 480,000 p.s.i. whereas the effective experimental value was 450,000 p.s.i. (6.2% error).

5.6.2 Mass block moduli with all blocks in place

Mass moduli with all the blocks in place were first measured, however, because of unavoidable defects introduced while manufacturing, Chapter 3, some erratic values were anticipated. Nevertheless an examination of Table 5 and Figures 5.8 to 5.10 show that consistency in the manufacture of the models was achieved.

If the block mass was isotropic its deformational response, under hydrostatic load would give $E_h^x$ and $E_h^y$ as equal. However, Figures 5.8 and 5.9 show that this is not the case, except for the $\frac{3}{8}$ square blocks, and the models are as expected anisotropic. This anisotropy appears to be a function of the stacking
APPARENT MODULUS VS BLOCK SIZE

BLOCK SIZE defined in terms of the ratio of AREA to CONTACT PERIMETER.

LOADING SEQUENCE
Initially a hydrostatic type is applied then the vertical load is subsequently increased.

Displacements measured at the relative points A, B, C, & D.

\[ E_Y (\text{from hydrostatic type loading}) \]

\[ E_Y (\text{from vertical loading}) \]

Block sizes indicated on ordinates.

\[ r = \frac{\text{Area}}{\text{CONTACT PERIMETER}} \text{ for a single block} \]

FIG. 5.8  Scale 1" = 0.1
APPARENT MODULUS $\varepsilon^m$ vs BLOCK SIZE

(refer FIG 5.8)

APPARENT MODULUS $\varepsilon^m$
for hydroseductive type loading

$\varepsilon^m = 4 \times 10^3$ psi.

Scale 1" = 40 x 10^3 psi.

Block size indicated on ordinates.

AREA

CONTACT PERIMETER.

scale 1" = 0.1

FIG 5.9
POISSON TYPE RATIO

$Y = \frac{\text{AREA}}{\text{CONTACT PERIMETER}}$  

scale 1" = 0.01

FIG. 5.10
geometry and block size. Compare experiments $\beta_1$ and $K_1$, Appendix E. In an attempt to follow the variation of the respective effective moduli as defined above, values of the apparent Poisson's ratio in the $x$ and $y$ directions were compiled, as explained in Section 5.5.5 and plotted, Figure 5.10.

For blocks of size smaller than 1 in. the values of the vertical effective Young's moduli are constant, Figure 5.8. However, as the block sizes increase the various moduli also increase in magnitude towards the limiting values of the parent material. Also in every instance, for the staggered stacking of blocks, the horizontal effective modulus $E_x$ is greater in magnitude than the vertical modulus $E_y$. This however, may not apply for blocks smaller than $\frac{3}{4}$ in. square because for this sized block the directional values of moduli approach one another in value. That is the material tends to become isotropic.

It is of interest to note that for all the values of moduli obtained at specific ratios of gauge length to block size (generally 5) the effective moduli are relatively constant. This appears to corroborate the observations of some workers,(Muller loc cit) who have stated that if the sample size is big enough, when compared with the individual unit size such as a single block or grain, the mass response of the sample under a specific load environment is the same or representative of the total mass.

When openings are introduced into the block mass additional deformations, such as slip and rotations are introduced.
The above definitions of mass moduli now become very restrictive indeed. This is because for each particular direction the mass modulus is effected by these additional deformations. This means that the moduli are now mechanistically dependent, and for model-prototype correlation the mechanisms would have to be similar. By examining the blocky models a measure of these effects caused by these additional deformations, is attained.

5.7 BLOCK MODELS WITH OPENINGS

Once it was established that the models with all the blocks intact behaved in a consistent manner, certain blocks were removed to give voids of specific shape. A similar loading sequence as imposed on the previous models was repeated in the following experiments.

Basically the factors examined were -

1. size effects,
2. stress raiser effects due to the voids,
3. slip mechanisms induced by the voids created,
4. the effect of joint orientation on mass response to load,
5. the nature and cause of rotations, that is what is their relation to slip?

The size of blocks tested were -

$\frac{3}{4}$ in. square with jointing vertical and horizontal,
$\frac{3}{4}$ in. square with jointing vertical and horizontal,
1 in. square with jointing vertical and horizontal, mixed block sizes with jointing vertical and horizontal, 2" x 1" rectangular with jointing vertical and horizontal.

In each experiment the displacements of specific blocks (namely four) were measured so that the relative displacements in the vertical and horizontal directions were determined. On most occasions the loading cycles were repeated at least twice and in some instances four times. As the load was reduced, the displacements were recorded at the same load levels used when the loads were increasing. Plots of the load versus displacement for each block displacement measured are given in the Appendix E.

When examining the load displacement graphs it is clear that bodily movements (slip) of the blocks occur and that when considering incremental loads, the slope of the load displacement graph is most difficult to define. However, by considering relatively large load differences (at least 5 load increments) an averaging process, Figure 5.11, is applied to give an effective modulus. These moduli results are summarised in Table 5.

It becomes fairly obvious when examining both the stress and deformation patterns that once some blocks have been removed the effective moduli are position and orientation dependent. That is the magnitude of the modulus obtained depends upon the proximity and position of the block relative to the void created. Consequently, it was decided to concentrate on measuring the effective moduli in the directions of the jointing systems and
in some instances, for inclined jointing, in the direction of the applied loads.

5.7.1 Square blocks

It was anticipated that as certain blocks were extracted, the effective moduli would accordingly decrease in magnitude. However, it is observed from the graphs of moduli versus the number of blocks extracted, Figure 5.12, that in some cases the moduli decrease while in others they increase. Some of the main factors appearing to cause these variations are -

1. relative size of the block system to opening,
2. induced anisotropy,
3. obliquity angle of thrust forces across joint sets,
4. shape, pattern and number of blocks extracted,
5. relative magnitude of the principal stresses,
6. location of block relative to opening from which deformations are recorded,
7. stiffness and characteristics of joint.

In turn the aforementioned factors are those factors which would determine whether joint slip is or is not going to occur. Slip along a single joint has been examined in Chapter 4, where it was determined that, amongst other things, potential slip along a joint is dependent upon the normal stress across the joint which in turn is a function of slip and the applied constraints. Now by creating an opening stress redistributions occur, so that in some regions contiguous to the opening the
FIG. 5.11

Average slope over a number of increments
FIG 5.12
Variation of apparent moduli as lins. sq. blocks extracted.
FIG 5.13.
FIG. 5.14 a

variation of apparent moduli as 

\( \frac{3}{8} \) blocks are extracted
FIG. 5.14 b

variation of apparent moduli as
\( \frac{3}{8} \) blocks are extracted
FIG 5.15
Variation of apparent moduli
as \( \frac{3}{8} \) ins. sq. blocks extracted.
ABUTMENT LOADS

FIG. 5.16
Variation of apparent moduli as \( \frac{3}{4} \) ins. sq. blocks extracted.
FIG. 5.18

Variation of apparent moduli as $\frac{3}{4}$" blocks are extracted.
FIG 5.19
Variation of apparent moduli as 2"x1" rectangles are extracted
LOAD vs. Pt. DEFLECTION

1" x 2" rectangular blocks (μ)</expr>

Fig. 5.20a

LENGTH AB. LENGTH CD. LOADING SEQUENCE
Equal loading increments in the X & Y directions (630 PSI) thereafter

Area = 8.53 sq. ins

Load

Scale 1" = 200 lb
LOAD vs Pt. DISPLACEMENT
1"x2" rectangular blocks with 4 blocks removed (24g)

LENGTH AB = 10"
LENGTH CD = 4"
LOADING SEQUENCE
Equal loading in the X & Y directions up to 630 lbs then vertical loading thereafter.

FIG 5.20b.

DISPLACEMENT scale 1" = 0.02"
FIG. 5.21

Flexure of hanging wall beam
LOAD vs. Pt. DISPLACEMENT

1" x 2" rectangular blocks with 3 blocks removed (μ₈)

LENGTH AB = 8"
LENGTH CD = 2"
LENLOADING SEQUENCE
LOADING INC IN THE X & Y DIRECTIONS UP TO 630 lb. THEN VERTICAL LOADING THEREAFTER.

FIG. 5.22

DISPLACEMENT

scale 1" = 0.01"

LOAD

scale 1" = 200 lbs.
stress intensity will either increase or decrease. Indeed it is generally observed that in regions where the stress increases the modulus or stiffness increases and where the stress decreases the modulus or stiffness also decreases.

5.7.2 1 in. square blocks

In experiments δ4, δ5, δ6 and δ7 blocks were successively removed from the hanging wall. The blocks removed were such as to form a dome of increasing height. In each case after extracting a number of blocks, a loading cycle was applied and deformations of the hanging and foot walls and adjacent vertical supports measured. From these measurements the effective moduli were compiled and plotted, Figures 5.11 and 5.12. An examination of the latter graph indicates that the relative vertical stiffness first reduces as the blocks are extracted and when the height of the dome is increased (10 blocks extracted) the relative vertical stiffness increases. Also the relative horizontal stiffness under hydrostatic loading at first increased to an indefinite value as the first six blocks were extracted and thereafter reduced as the blocks in the hanging wall were removed. The relative horizontal stiffness of these two support pillars under vertical loading also appears to increase slightly as the number of blocks removed increases.

As stated previously, 5.7.1, the effective moduli decrease in magnitude where the stresses decrease and increase in magnitude where the stresses increase. This statement is corroborated by comparing the photoelastic patterns in Figure 5.16 with the deformational moduli in Figure 5.12.
5.7.3 \( \frac{3}{8} \) in. square blocks

It was shown in Section 5.6.2 and Figure 5.8 that for the models tested, those less than 1 in. square behaved mechanistically in much the same manner. Therefore the moduli obtained for the \( \frac{3}{8} \) and \( \frac{3}{4} \) in. blocks could be interpreted as being relevant for the 1 in. blocks with the same geometrical patterns and locations. In these experiments, however, the hanging wall displacements were taken from a block which was initially some distance from the opening, Figure 5.14.

In experiments \( K_2 \) and \( K_3 \) the moduli were measured from blocks located at similar positions and the results plotted as shown in Figures 5.14 and 5.15. The same geometrical configuration and loading conditions existed for the corresponding experiments \( K_2 \) with \( \delta_4 \) and \( K_3 \) with \( \delta_7 \).

In experiment \( K_2 \), however, instead of \( E_{\text{vert}} \) reducing the same amount as \( E_{\text{hyd}} \), the modulus \( E_{\text{vert}} \) was little effected one way or the other by the removal of the blocks. The relative position of the blocks from which displacements were measured are not the same in experiments \( K_2 \) and \( \delta_4 \). In experiment \( K_2 \) the block from which the displacements were measured is some distance away from the opening and therefore away from the zone of disturbance. Consequently the drop in effective modulus would not be as great as that expected from experiment \( \delta_4 \). When comparing experiment \( K_3 \) and \( \delta_7 \) it is seen that the moduli follow the same trends except for the smaller blocks of experiment \( K_3 \), the moduli values were lower. The behaviour of the blocks, hence the various moduli under the hydrostatic
load and then the vertical load, are again strongly dependent on the change of stress environment. When six blocks had been removed in experiment $K_2$ the horizontal modulus $E_{x}^{hyd}$ reduced considerably, while for increasing vertical load the arching action in the hanging wall increased the stress surrounding the abutment blocks, Figure 5.16. As more blocks were removed (18) the stress patterns are again changed with the consequent change in the measured effective moduli tending to follow the increase or decrease of stress environment. In this latter situation, namely 18 blocks removed, the vertical load modulus $E_{y}^{vert}$ is substantially reduced while $E_{x}^{hyd}$ tends to increase slightly.

5.7.4 $\frac{3}{4}$ in. square blocks

Experiments $\Theta_3$, $\Theta_4$, $\Theta_5$ and $\Theta_6$, Figures 5.17 and 5.18 are compared here with the $\frac{3}{8}$ in. and 1 in. square blocks examined above. The blocks from which displacements were measured are depicted in Figure 5.18. The aspect ratio of length to depth of opening is 3 for experiment $\Theta_4$ as compared to 6 for experiments $K_2$ and $\delta_4$. Consequently the arching effect is less extensive for experiment $\Theta_4$ and $E_{y}^{vert}$ increases in magnitude as the hanging wall blocks are removed and the arching of the induced loads is developed instead of the beaming action. Yet as the two foot wall blocks are removed $E_{y}^{vert}$ is reduced in sympathy with the proportionate reduction in stresses around the abutment blocks.
\[ E_I = 480,000 \text{ p.s.i. Young's Modulus for Araldite material.} \]

\[ E_{y}^{\text{hyd}} = \text{effective modulus perpendicular to continuous joint set under hydrostatic loading conditions.} \]

\[ E_{y}^{\text{vert}} = \text{effective modulus perpendicular to continuous joint set under vertical loading in the same direction.} \]

\[ E_{x}^{\text{hyd}} = \text{effective modulus parallel to continuous joint set under hydrostatic loading conditions.} \]

\[ E_{x}^{\text{vert}} = \text{effective modulus parallel to continuous joint set under vertical loading in the same direction.} \]

<table>
<thead>
<tr>
<th>Exp. No.</th>
<th>Block size</th>
<th>No. Blocks removed</th>
<th>Continuous joint set orientation</th>
<th>( E_{y}^{\text{hyd}} ) k.p.i^2</th>
<th>( E_{y}^{\text{vert}} ) k.p.i^2</th>
<th>( E_{x}^{\text{hyd}} ) k.p.i^2</th>
<th>( E_{x}^{\text{vert}} ) k.p.i^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>K1</td>
<td>( \frac{3}{8} ) sq</td>
<td>Intact</td>
<td>Horizontal</td>
<td>36.8</td>
<td>24.5</td>
<td>43.3</td>
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<tr>
<td>K2</td>
<td>( \frac{3}{8} ) sq</td>
<td>6</td>
<td>&quot;</td>
<td>13.9</td>
<td>22.9</td>
<td>17.4</td>
<td>-62.2</td>
</tr>
<tr>
<td>K3</td>
<td>( \frac{3}{8} ) sq</td>
<td>18</td>
<td>&quot;</td>
<td>17.3</td>
<td>12.9</td>
<td>18.0</td>
<td>-36.5</td>
</tr>
<tr>
<td>( \bar{\lambda}_6 )</td>
<td>( \frac{3}{8} ) sq</td>
<td>5</td>
<td>&quot;</td>
<td>3.5</td>
<td>5.2</td>
<td>35.0</td>
<td>-69.8</td>
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<tr>
<td>( \beta_1 )</td>
<td>( \frac{3}{8} ) sq</td>
<td>Intact</td>
<td>&quot;</td>
<td>21.4</td>
<td></td>
<td></td>
<td>-189.5</td>
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<tr>
<td>( \beta_5 )</td>
<td>( \frac{3}{8} ) sq</td>
<td></td>
<td>&quot;</td>
<td></td>
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<tr>
<td>( \bar{\lambda}_4 )</td>
<td>( \frac{3}{8} ) sq</td>
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<td>&quot;</td>
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<td>7.0</td>
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<td>10.5</td>
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<td>( \frac{3}{4} ) sq</td>
<td>Intact</td>
<td>Horizontal</td>
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</tr>
<tr>
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<td>Intact</td>
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<tr>
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<td>&quot;</td>
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<td>( \delta_7 )</td>
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<td>Continuous joint set orientation k.p.i²</td>
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<td>μ⁵</td>
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<td>μ⁶</td>
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<td>&quot;</td>
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<td></td>
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<td>30° to Horiz.</td>
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$E_{CD}$ = effective modulus in the direction of the vertically applied load under hydrostatic load application.

$E_{CD}$ = effective modulus in the direction of the vertically applied load under vertical load application.

$E_{AB}$ = effective modulus in the perpendicular direction of the vertically applied load under hydrostatic load application.

$E_{AB}$ = effective modulus in the perpendicular direction of the vertically applied load under vertical load application.

<table>
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<th>No. Blocks Removed</th>
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**TABLE 5.**
5.7.5 2 in. x 1 in. rectangular blocks

Experiments $\mu_8$ and $\mu_9$, Figures 5.19 and 5.20(a) and (b), indicate that as blocks are extracted the modulus drops in magnitude as expected, yet as in experiment $\theta_4$ ($\frac{3}{4}$ square blocks) $E_{\text{vert}}$ did not reduce as did $E_{\text{hyd}}$. An explanation for this characteristic confirmed from the developing stress patterns is that as the hydrostatic load is increased the hanging wall acts as a horizontal strut which flexes in towards the opening, Figure 5.21. This feature has an important bearing when assessing the stability of the hanging wall.

When comparing the moduli from experiments $\mu_8$, Figure 5.22 and $\mu_9$ it is noted that even though the aspect ratio for the opening in $\mu_9$ is greater than that for $\mu_8$ the moduli for $\mu_9$ especially the vertical ones are greater than those for $\mu_8$. Here it is noted that the difference stems from the different relative block positions from which the displacements were recorded.

5.8 SUMMARY

It may appear plausible in the first instance to use an elastic or plastic analogue approach to determine the stress distribution in a discontinuum. If this is accepted then the problem is to obtain the relevant parameters required for the analysis. This is generally done by acquiring a representative sample and extrapolating the experimental result to the prototype. It has been shown that under carefully controlled conditions if the representative sample size to block unit size is three to five, the parameters derived from the sample could represent
those required for the prototype. This however is about as far as the analogy can go unless the experimental parameters are explicitly determined. That is the situation in the prototype must be carefully modelled in all respects and examined with the same loading history. But now the experimental parameters are so variable over the area of interest as to make the analogue approach well nigh useless.

Tests performed on actual rock shows that a representative size for a sample is most difficult to determine. What may appear to be a simple experimental parameter for a small sample, for instance the uniaxial compression test, becomes more complex when a larger sample is examined. This appears in the main to be due to defects which allow a greater amount of slip or rotation to take place.

The results presented and examined here are an abstract of the experimental work performed, but nevertheless show that the experimental moduli though reproducible for different block sizes are very restrictive in that they are, besides other factors, stress and load path dependent. The other factors are characteristics such as joint orientation and strength, compliability and strength of intact material, and boundary conditions.

From this the question may be posed; what approach could be used to analyse the discontinuum? In the next two chapters namely, 6 and 7, this question is tackled from the point where Chapter 4 closed.
CHAPTER 6.

PHOTOELASTIC MODEL STUDIES

6.1 PREAMBLE

Investigation up to this stage has examined the basic mechanisms of slip and rotation which are likely to take place when a discontinuum is deformed. As these mechanisms operate there is a redistribution of stress. It has also been shown that the use of elastic or plastic analogue methods have the problem of defining and determining appropriate experimental parameters or moduli. Consequently in this chapter the load distribution in a blocky mass is examined and the load patterns imposed on certain blocks investigated. That is the basic mechanisms of slip and rotation have a large influence on the overall load distribution which in turn determines the stress distribution within the individual blocks.

From the work in this chapter it is again shown that from the stress distribution point of view the continuum theories do not apply. This last statement is also relevant to many of the concepts used in clastic mechanics. That is this statement is true where continuum concepts are used to normalise the loads and deformations. This latter point is investigated in greater detail towards the end of this chapter.

A query emanating from this chapter could read; if the analogue continuum theories do not apply to the analysis of a discontinuum, what methods should then be
used to analyse the problem and under what conditions are these methods valid? Many models are examined as described in Chapter 3, and the photoelastic patterns examined here are basically ones which depict, in the author’s opinion, the salient features of load and stress response in a blocky material.

6.2 PHOTOELASTIC BLOCK MODELS

In the first instance the stress distribution around a single opening in a continuum is compared with the results obtained from similar openings in a blocky material subjected to incremental load. The size of the openings have different aspect ratios of length to depth. Subsequent to completing the examination for a single planar opening, blocks in the hanging wall were removed and the loads reapplied. Initially the blocks were extracted before the loads were applied and then subsequently the blocks were extracted after the loads were applied. The orientation of jointing patterns and block stacking were varied as described in Chapter 3. Finally, the creation of a number of openings in the blocky material imposes certain constraints on deformation. This is due to the interaction of the openings which cause stress redistributions and which in turn affect the deformational response.

Besides the work performed on underground openings, the situation of an imposed load from either a semi flexible or rigid foundation on a blocky material making up the supporting subgrade, is also examined. These results are then compared with the results obtained from
a numerical analyses in Chapter 7.

6.3 ELASTIC STRESS DISTRIBUTION AROUND OPENINGS IN A CONTINUUM

By considering holomorphic functions in the zone outside the opening and using conformal transformations Savin (1951) used a specific number of terms in the infinite stress function series,

\[
\phi_0(\delta) = \sum_{n=1}^{\infty} \alpha_n \delta^n \\
\psi(\delta) = \sum_{n=0}^{\infty} \beta_n \delta^n
\]

and from this the stress components \( \sigma_\delta, \sigma_\theta \) and \( \tau_{\delta\theta} \) were obtained, Figure 6.1. From these \( \sigma_{\text{max}}, \sigma_{\text{min}} \) and \( \tau_{\text{max}} \) are compiled for openings with the aspect ratios of 1, 3.2 and 5. Graphs of these stresses and principal stress differences, \( \tau_{\text{max}} \), are given in Figures, 6.2, 6.3 and 6.4. It is a well known fact that for an elastic material the maximum stress \( \sigma_\theta \) occurs on the free boundary of the opening. Therefore in the blocky models examined, the maximum value of \( \sigma_\theta \) was sought and its location. That is, did maximum \( \sigma_\theta \) occur on the boundary or within the blocky mass? This is an important question especially in relation to the interaction of different model openings.

6.4 EFFECTS OF BLOCK SIZE AND ACCURACY ON MODEL RESPONSE

In Section 3.5 a description of the techniques used when constructing the models was given. These techniques
Corner radius = \( \frac{3a}{50} \)

Corner radius = \( \frac{a}{40} \)

Side ratio = \( \frac{\text{Length}}{\text{depth}} \) = 1

\( K \) = concentration factor

FIG. 6.1 a

Side ratio = \( \frac{\text{Length}}{\text{depth}} \) = 5

Load ratio = 2

\( K \) = stress concentration factor

FIG. 6.1 b
Aspect ratio = 3.2
Load ratio = 2

$K$ = stress concentration factor.

Figure 6.1c

aspect ratio = 1
FIG. 6.3
Aspect ratio = 3.2
FIG 6.4
Aspect ratio = 6
(a) relatively good block fit

(b) relatively bad block fit

FIG. 6.5
have been outlined previously (Chappell, 1967). Nevertheless because of their relevance to the work which follows in this and the following chapters and also because additional work and information has become available, the accuracy requirements necessary for the construction of the models are examined here.

If the effects of transferring the applied loads onto the model have been fully recongised, (Chappell, 1967) the following factors were found to control the initial stress state, namely -

1. joint roughness and planarity,
2. squareness or orthogonality of the jointing systems,
3. equality in size between contiguous blocks,
4. stacking geometry,
5. ratio of the magnitude of applied loads,
6. direction of jointing systems relative to applied loads,
7. loading sequence,
8. loading history.

These factors were found in the main to effect the initial stress conditions, in the same way as reported by Ergun (1970). Now if this initial state is disturbed by creating -

1. a slope,
2. an underground opening,
3. or imposing a surface load such as a dam,
mechanisms of deformation are initiated which can greatly control the subsequent load distribution or redistribution. Additional factors of prime importance which control these mechanistic deformations are now introduced, namely, the constraints imposed or caused by the induced loads, and the change of boundary conditions.

If the jointing system is such as to cause no increase in pore space (planar joints) the load distribution under hydrostatic loading should give a uniform load picture, namely, a picture of one tone. If a uniform load state is not achieved the resultant isochromatic picture gives a measure of the lack of fit caused from joint roughness, lack of planarity and squareness, Figure 6.5(a) and (b). These last mentioned factors tend to constitute the joint characteristics which, as shown in Chapter 7, when combined with the intact material and mechanistic response give the mass response of load versus deformation.

Important additional factors require consideration if the mass response of the blocky material is to be understood. These additional factors are -

1. the rotational effect on individual blocks,
2. the change of geometry caused by slip and rigid body movement,
3. the effect of material elasticity on the load distribution and possible redistribution.

As will be noted when examining the aforementioned photographs, the initial defects mentioned above occur in the models, but if they are not relatively large they
do not significantly control the existing stress patterns and therefore do not control subsequent mechanisms resulting from any block extractions. This aspect of model construction was examined by the author (Chappell, 1967).

6.5 SINGLE PLANAR OPENINGS

When a single opening in an elastic continuum is subjected to a uniaxial compressive load, two main features appear, namely -

1. the development of tensile stresses,
2. the development of large compressive stresses.

The compressive stresses are local in nature and if failure ensues the radius of curvature at the corners is increased while the maximum compressive stresses are correspondingly reduced. Figures 6.6 to 6.8 are the isochromatics for single planar openings in blocks of different sizes with varying aspect ratios. The loading sequence was generally such that the horizontal and vertical loads were increased by equal increments to a certain value and then the vertical load subsequently increased to a magnitude 2 or 3 times the horizontal load.

In order to compare the results of the blocky and elastic cases the tangential boundary stresses for the blocky models shown in Figure 6.8 are given in Figure 6.9 for the cases where the load ratios are 1 and 2 respectively.
HOR. Ld. = 376 lbs.
VER. Ld. = 756 lbs.

aspect ratio = 4.5
FIG. 6.6

aspect ratio = 10.0
FIG. 6.7
Hydrostatic type load

vertical load increased

FIG. 6.8
aspect ratio = 8
It is appropriate at this stage to note some of the results of these experiments as they form the basis for comparison.

6.5.1 Arching and beaming action

It is a well known fact that in an elastic continuum where openings of rectangular shape as described above are created the stress intensity in the hanging wall and foot walls will alter to such an extent that the stress changes from compression to tension. This occurs if the horizontal stress is not of sufficient magnitude to inhibit this. Also for a continuum the stress intensity at the abutments increase and there is a doming action of the stress environment above and below the opening, (Denkhaus, 1958). In the blocky models this doming or arching action is also most prevalent, but here the mechanistic actions of slip and rotations are evident, Figure 6.10.

An important factor in the mobilisation of this doming action in a blocky material is the horizontal thrust. If the magnitude of the original horizontal thrust is such as to reduce or minimise slip, the change in stress intensity above and below the opening tends to that of a continuum, and the mechanistic arching action is reduced.

This beaming action was discussed in Chapter 2 where it was shown that in some instances the required horizontal thrust was initiated by rotation of individual blocks or movement of the abutments, while in others the horizontal thrust was extant before the creation of the excavation. These last two actions can give two very different
Pre-loaded 2" x 1" inclined blocks depicting continuous beam type action
HOR. $L_d = 380$ lbs.
VERT. $L_d = 380$ lbs.

FIG. 6.10b
aspect ratio = 3
POST LOADED MODEL
resultant stress patterns in the blocky beam, Figure 6.11. The results of the above photoelastic examination strongly suggest that the main mechanisms in the development of arching and beaming are slip and rotation of specific blocks.

6.6 DEVELOPMENT OF TENSILE STRESSES OR STRAINS

6.6.1 Tensile stresses or strains due to mechanistic action.

In an elastic continuum tensile stresses can readily develop when creating an opening. However in the blocky models examined, tension within the joints is impossible. This latter criterion however does not preclude the development of tension within the individual blocks. In fact the mechanisms of rotation and slip create situations where tensile stresses or strains appear more readily. The existence of these tensile stresses has very important implications to the overall stability of the structure because as stated in Section 2.8, tensile stresses or strains could control strength rather than compressive stresses and strains.

It appears possible therefore that the development of tensile stresses or strains in the underground situation could be due, in the main, to mechanisms such as slip and rotation. Tensile stresses could also develop however from the imposed boundary loads rather than any direct mechanistic interaction of the block or its neighbours.
FIG. 6.11 a

1½" sq. block model depicting a continuous type beaming action
FIG. 5.11 b

1½" sq. block model depicting a mechanistic type beaming action
6.6.2 Experimental evidence of tensile conditions

Four specific cases are examined here where tensile stresses or strains become evident -

1. in the block abutments of the opening, Figure 6.12,
2. in the blocks directly above and below the abutments, Figure 6.13,
3. in the blocks where beaming action occurs, Figure 6.14,
4. in the blocks where arching action occurs, Figure 6.15.

In the case of (1) and (3) the effective tensile strain concept (Brown and Trollope, 1967) is evident and the case of slabbing becomes prevalent. Slabbing is evident in many practical situations, e.g. Mount Isa, Figure 6.16, where abutment or pillar failure has occurred. It can clearly be seen, Figure 6.13, that the mechanistic action of the blocks can readily induce tensile stresses over and above that anticipated from the theory of elasticity. These excess tensile loads are greatly accentuated by the interaction between neighbouring blocks as slip occurs. In the case of (2) and (4) tensile stresses are induced, Figure 6.13 by a punching or digging in of contiguous blocks as their tendency to rotate or slip is inhibited and this phenomenon is referred to here as interaction. It was shown in Section 4.7 that when slip occurs a torsion on the block is induced which in turn tends to cause a rotation of the block. If this rotation is inhibited by a constraint, a punching effect is initiated which can in turn cause a point type load with the resultant inducement of tensile stresses, Figure 6.17.
Typical loads applied or imposed on individual blocks where either a tensile stress or effective tensile strain develops.
FIG. 6.16
Mount Isa, slabbing

FIG. 6.17
(after Goodman)
Induced tension.
6.6.3 **Initial lack of fit in the block models**

Blocks which were too large, Figure 6.18, appeared to cause movements of the surrounding blocks which imposed larger stresses on the oversize block, but tended to have a more local effect than experienced when this block was removed causing larger displacements with the consequent load redistributions. The displacements or movements of the surrounding blocks when removing the oversize block were much greater than occurred when this oversize block was left in place. From a model point of view these oversize or undersize blocks represent areas of stiffer or softer material when compared to the surrounding material.

With the oversize block in the model, the mechanisms of slip and rotation were much the same as those caused when the block was extracted except in this instance the effect was more local. However, the magnitude of load increase was much the same. This suggests that the extent or area affected by the deformational mechanisms is largely dependent upon the magnitude of deflection or movement allowed at the boundary.

6.6.4 **Sloping single planar openings**

Blocks of various sizes and different joint orientations, namely 30°, 45° and 60° to the horizontal, were examined for the same loading sequences as were the horizontal planar openings. For the same loading values, the inclination of the joints to the applied loads induced greater shear forces with a consequent greater mobilisation of the slip mechanism. The basic geometry of the
FIG. 6.18 a

1½" sq. block model with oversize block included
FIG. 6.18 b

1½" sq. block model with oversize block extracted
mechanisms tended to remain the same except lesser movements were experienced in the area opposite to the sides from where the loads were being applied, Figure 6.19. Consequently, there was less load reduction in this area and though the mechanisms mobilised were not as accentuated as in the other areas the load intensities in general were greater. This phenomenon appears to lend strength to the observation that the joint stiffnesses are very much dependent on the stress environment and that in the overall pattern where hydrostatic stress increases the joint stiffness increases and vice versa.

Another way of stating this is that in areas where volume, due to geometry change, decreases the loads tend to increase and in areas where dilatancy occurs the loads tend to decrease.

6.7 COMPARISON OF THEORETICAL RESULTS WITH BLOCKY MODEL EXPERIMENTS

An opening of aspect ratio equal to 3 was obtained in a 3 in. x 2 in. blocky model and the vertical and horizontal loads were increased by equal increments of load until the stress was 84 p.s.i., Figure 6.20. After this the vertical load was increased incrementally until the vertical stress was 168 p.s.i.. The tangential stress around the periphery of the blocky opening is shown in Figure 6.21 in terms of a concentration factor relative to the applied vertical stress.

From Figure 6.1(c) it is noted that the maximum tangential stress for the continuum subjected to the same loading ratio as were the models is not located at the
FIG. 6.19
Load distribution in inclined blocks
FIG. 6.20

3" x 2" block model from which tangential stresses around opening are evaluated (ref. 6.21)
FIG 6.21

Stress concentration around opening for 3"x2" block model.
corners. For the blocky models, however, the maximum concentration factor for the tangential stresses occurred at the corners.

In most cases the compressive concentration factors were not as large as they were for the continuum. This was not however always the case because for the blocky model subjected to equal horizontal and vertical loads the compressive concentration factor in the abutment was 9 while for the continuum it was 6.1. For the continuum this concentration factor is very much dependent on the radius of curvature at the corners of the opening, in this case $\text{rad} = \frac{4}{11}$ the breadth. Consequently the concentration factor of 6.1 could quite easily exceed the value of 9 by decreasing the radius of curvature at the corners.

6.7.1 Experimental and photoelastic computer results

Figures 6.24 and 6.28 show the model and blocks examined by using the computer programme developed in Chapter 3. As will be noted from the photographs specific blocks in these two models appear to have similar stress patterns. For instance the loads on the abutment blocks are similar, blocks surrounding the abutment blocks though certainly not the same size have similar stress patterns, also blocks in the hanging and foot walls have similar stress patterns. Blocks making up the domed arch also possess similar induced stress patterns. With these observations in mind specific blocks with the greatest stress gradients, were chosen and their imposed boundary loads evaluated, Figure 6.24.
EQUILIBRIUM SATISFIED TO WITHIN 15% FOR NUMERICAL METHOD.

**FIG 6.22** (NORMAL LOADS ONLY.)

COMPUTER & SHEAR DIFF. VALUES FROM BLOCK IN FIG 6.24

**FIG. 6.23**
NORMAL LOADS APPLIED TO BLOCK B FIG 6.24
FIG. 6.25a

LOADS IMPOSED ON BLOCKS (FIG. 6.24)
Considering the normal loads acting on blocks A and B it is noted that if a torque is imposed on the block as in Section 4.7, the loads are redistributed on the sides of the blocks so that equilibrium is restored.

Abutment block E has the imposed loads as shown in Figure 6.25(a). Here it is noted that the open face of the abutment is subjected to a high effective tensile strain which could lead to the much experienced slabbing effect.

Blocks C and D were taken from the hanging wall, Figure 25(b), and though the loads on these blocks have been reduced there is still some indication of a slight beaming action. That is the stress distribution on the sides of the blocks indicates that the hanging and foot wall blocks are acting as a beam. Because the blocks are inclined relative to the applied loads, it should be noted that as the vertical load is applied part of this imposed load goes into increasing the thrust on the blocks making up the hanging and foot walls of the opening. Therefore the hanging and foot walls behave more like a continuous beam which develops no tension between the joints rather than a mechanistic beam, Section 6.5.1, which develops open cracks because of the inability of the joints to sustain tensile forces. Figure 6.25(b) shows the development of this beaming action.

6.7.2 Punching effect due to block interaction

If a block slips or rotates relative to contiguous blocks interference from the surrounding blocks occurs. This interference is a punching effect caused by a corner
which makes contact with an enclosing flat surface. Figure 6.27(b) shows an abutment block where the surrounding blocks not only impose a load distribution causing an effective tensile strain, but also creates a punching effect in the upper and lower blocks A₁ and A₂ which in turn develops tensile forces, Figure 6.13.

A characteristic generally observed here is that as the blocks were made more slender, that is the ratio of length to depth increases, this punching effect becomes more prevalent, Figure 6.26a.

6.7.3 Block model loaded and blocks subsequently extracted.

Figure 6.28 shows the same model as 6.24 subjected to the same magnitude of load. In Figure 6.28, however, the load was first applied and the blocks shown subsequently removed. The loads induced on certain blocks are shown in Figure 6.29. The loads imposed on these blocks are readily compared with those of 6.25.

Salient features observed from these experiments are -

1. the abutment blocks for the preloaded case are subjected to stress concentrations larger than they are for the post loaded blocks,

2. the hanging and foot wall blocks, namely, B, contiguous to the abutment blocks, again for the post loaded case, do not experience the same stress concentrations as do the preloaded blocks,
Punching effect (b)

FIG. 6.26
FIG 6.27
NORMAL LOAD INDUCED ON 3"x2" RECTANGULAR BLOCKS FIG 6.26
Relative block positions
FIG 6.29

Induced boundary loads on 1\(\frac{1}{2}\)" sq. blocks
preloaded and forcibly extracted (FIG 6.28)
3. The hanging and foot wall blocks for the post loaded case are subjected to greater flexural forces than are the same blocks from the preloaded case.

From the above features it is concluded that for the preloaded case of block extraction the blocks tend to rotate more when compared with the blocks of the post loaded case. Consequently the joints for the post loaded case are stiffer than those for the preloaded case. This factor of joint stiffness is again examined in Chapter 7 where the load distribution in a blocky material is examined with a numerical or mathematical model. Nevertheless it is quite clearly shown that the resultant load distribution is a function of joint stiffness which in turn is a function of the load environment and sequence.

When the blocks tend to rotate a mechanistic arching action is induced, Figure 6.28, and in this instance the punching effect as discussed in Section 6.7.2 is greatly accentuated.

6.8 EXCAVATION OF THE HANGING WALL WITHIN THE ARCHING DOME

It was found that if little or no beaming action existed, the blocks in the hanging wall could be removed with little or no effect on the load distribution in the surrounding blocks. On the other hand if beaming action either continuous or mechanistic existed a much greater load was thrown into the abutment blocks if the hanging wall blocks were removed. The blocks forming the arch tend to rotate and consequently induce effective tensile forces within these arch supporting blocks. As the key stone area of the arch is approached, mechanistic beaming action above
the arch becomes very prominent, Figure 6.29.

It was also found that when loading the block models, possessing a single opening, the maximum displacement occurs in those blocks making up the face of the hanging wall, and the displacements reduce as the key stone area or summit of the arch is approached.

6.9 INTERACTION OF MULTIPLE OPENINGS

At Mount Isa mines, Chapter 8, the number 2 and 5 ore bodies consist of three openings relatively close to each other. It was found after constructing a continuum model that the material between the lower number 2 and 5 excavations, Figure 6.30, behaved in a manner very similar to a strut. The lower ore body number 2 dilated and number 5 contracted when a hydrostatic type load was applied. Also stress concentrations, besides those at relatively sharp corners, occurred in the material between the upper number 5 and the lower number 2 ore bodies. Tensile areas of stress also occurred in the continuum models, Figure 6.30.

Blocky models were examined with the object of determining the interaction between openings and assessing -

1. the strut action of the blocky material between the openings,

2. the effect the blocky material has on the magnitude of stress concentrations or reductions,
FIG. 6.30
Tangential stresses

FIG. 6.31
Punching effect between openings
3. in what areas the stress concentrations or reductions are likely to occur,

4. if an adjacent opening is being excavated, does the span of the abutments in the existent opening increase or decrease?

6.9.1 *Horizontal and vertical joints relative to the applied loads.*

In single openings the loads imposed on the abutment blocks are such that an effective tensile strain is induced and failure by slabbing may occur. Now in the case of multiple openings the blocks between the openings appear to be subjected from both sides to the punching phenomena described in Section 6.7.2, Figure 6.31. This means that greater areas of material are subjected to tensile forces in one direction whereas at right angles to these tensile forces large compressive loads are imposed on the blocks.

This creates a situation where the rock may be in a meta stable state. That is if an opening is made in the area where the induced tensile forces coupled with the associated compressive loads act, then failure of the material around the opening is likely to ensue.

Another important feature of multiple openings is that as an opening is being created in the proximity of an existent opening, the span of the existent opening may either increase or decrease. This could quite easily create an active or passive situation, (Section 2.8.3), for load distributed in the extant hanging wall beam and therefore affect the stability of the opening.
6.9.2 Inclined joints relative to the applied loads

If Figures 6.32 and 6.33 are compared the relative position of the underlying opening will determine whether the overlying abutment block will tend to rotate clockwise, Figure 6.33, or anticlockwise, Figure 6.32. From this it could be noted that the load intensity is thrown off from the face of an abutment by judiciously placing the underlying opening.

Here again in the inclined openings the basic mechanisms are much the same as those encountered for horizontal openings. Except in this instance if the vertical load is increased part of this force goes into producing a thrust force on the blocks and shear effect on both systems of joints, Figure 6.33. This tends to induce more shear forces in the joint systems and if the shear stiffness of the joints is not large, rotation of the blocks more readily occurs.

6.9.3 Strut action

In these experiments, performed on blocks with inclined joints to the loading system, the initial stresses within the blocks had not been properly relieved. Time prevented the equalisation of moisture content conditions within the blocks as this took up to six months to achieve. The initial stresses are shown in Figure 6.34.

Strut action is clearly evident when examining the load distribution between the blocks, Figure 6.36. It appears that as the thickness of the strut is decreased the load intensity in the strut increases. This, however, does not
FIG. 6.34.

FIG. 6.35.
FIG. 6.36
Strut action
affect the mechanistic modes causing the load distributions. That is the mobilised mechanisms of slip and rotation do not in this instance interfere with the adjoining blocky struts. Basically the opening causes the release of transverse confining loads along the continuous joint set, each row of blocks tends to act independently of the others in the cases examined.

6.10 FOUNDATION LOADS IMPOSED ON A BLOCKY SUBGRADE

The load distribution imposed on an isotropic, semi-infinite, homogeneous, elastic subgrade is dependent on the relative stiffness, $K$, (Borowicka, 1939), of the foundation to the subgrade. Two foundation blocks, the semi-flexible foundation with $K = 0.167$ and the rigid foundation with $K = 1.000$, were used to impose loads first on an intact then blocky subgrade material.

6.10.1 Contact stresses

In Figure 6.37 the stresses between the foundation blocks and the intact subgrade are depicted. For the rigid foundation it was found that between the loads of 425 lbs. and 525 lbs. the contact zone between the foundation and subgrade slipped. This slip involved a redistribution of the contact forces as shown. This tended to affect a change from a rigid to semi-flexible stress distribution.

6.10.2 Blocky subgrade

Contact stresses in the blocky material which had no side restraint are shown in Figure 6.38. It should be noted that the stress distribution in the contact blocks
Figure 6.37

Foundation load imposed on continuous subgrade

Area Check

\[ K=10 \quad P=425 \text{ lbs} \quad \frac{\sigma}{P'} \quad \frac{\sigma'}{P'} \quad \text{GRAPH} \]

\[ K=10 \quad P'=525 \text{ lbs} \quad \frac{\sigma}{P} \quad \text{GRAPH} \]
$K = 0.167$

$\Sigma A = 1.005P$

$\Sigma A = 0.995P$

**FIG. 6.38.**
reflects the characteristics of the stress distribution defined by the stiffness of the foundation block, namely whether it is semi-flexible or rigid.

If the stacking geometry of the subgrade is varied as shown in the relevant diagrams, the contact stresses between the foundation block and subgrade blocks do not appear to vary significantly. The stress distribution between the blocks, however, do vary, Figure 6.39(a) and (b). This implies that the stacking geometry of the blocks is a major factor in determining the stress distribution within the blocks, Section 7.8.2. Another important factor is the horizontal forces or constraints applied to the blocks; because, if the side forces were large enough so as to inhibit slip the stress distribution would tend to that experienced by a continuum.

6.11 SUMMARY

Many models were examined by measuring the deformational response and stress distribution as the loads were incrementally increased or decreased. It was found that with many different models investigated and loading paths imposed, the basic mechanisms mobilised did not vary significantly. These basic mechanisms are those of slip and rotation and what has been termed punching. Therefore the load distributions arising from these mechanisms were investigated. To this end, it was found that the same pattern of induced loads on certain blocks appeared repeatedly even if the boundary conditions differed. This would support the statement that the mechanisms control the load distribution and the boundary conditions tend to determine the magnitude of load attained.
Another very important feature in the work of this chapter is that the mechanisms cause a load distribution which in nearly every instance induces a tensile stress or creates a state of effective tensile strain. This development is most prominent near the surface of an opening or contact surface between an applied load and subgrade. That is the mechanism of rotation is most prominent near the surface or boundary where the loading conditions are altered.

Areas of hard or soft (oversize and undersize blocks) inclusions appear to set in motion similar mechanisms causing similar load distributions as discussed above. The magnitude of the loads appearing however, depend largely on the magnitude of applied load and the amount of relative deformation permitted at the boundary.

Generally, it was observed that the maximum compressive concentration factor for the blocky models was less than comparable values obtained from the continuum. However, the induced or redistributed loads impose conditions where tensile stresses or effective tensile strains are readily developed. This again raises the question of whether compressive or tensile forces control the strength and therefore stability of a blocky material. If the maximum compressive loads are reduced and the maximum tensile stresses increased there appears to be strong evidence that tensile stresses in fact control the stability of a blocky material.

A very important question arises from both the physical and numerical experiments namely, what effect does the elasticity or deformability of the individual blocks have
on the stress or load distribution? With this in mind it is very important to note the particular features of this chapter and correlate these with those from Chapter 7.

Another very important characteristic of discontinua is the development of shear forces in the joints. For example if a system of blocks is subjected to a hydrostatic load, shear forces may be induced in the joints which appear to be dependent on the stacking geometry and the relative stiffness of joints to blocks. This aspect is examined in more detail in Chapter 7. It is stressed here, however, because the whole basis of slip is the development of shear forces which in turn tend to cause rotations.
CHAPTER 7.

ANALYTICAL STRESS DISTRIBUTION

7.1 PREAMBLE

Stress distribution in a discontinuous material cannot be determined with any generality or confidence by the methods of elasticity or plasticity. Nevertheless many approaches using these theories are still being attempted. For example -

1. a blocky material has been assumed elastically anisotropic, (Erlikhman, 1971),

2. a sand has been assumed plastic with strain hardening characteristics (Roscoe, et al, 1963) or with a non associated flow rule behaviour (Davis, 1967),

Presently, much effort is being expended using the finite element approach, (Goodman et al, 1968), (Best, 1970). Here elements representing planar joints are incorporated with elements representing the continuum material.

In this chapter a finite element type programme is used to analyse a loaded system of blocks from which varying numbers of specific blocks have been removed. The results of these numerical experiments are compared with those of the photoelastic experiments performed in Chapter 6.
Also examined here are the load distributions in a blocky subgrade when the load is imposed from either a semi-flexible ($K = 0.167$) or rigid ($K = 1.000$) foundation.

Though the elastic theory is inadequate for determining the stress distribution in a discontinuum, the theory is nevertheless most useful as forming the basis for comparison and formulating how fracture around an opening may develop. Consequently it was deemed necessary to examine the main features of stress distribution in a linear elastic continuum with multiple openings evaluated in this instance from another finite element programme.

An important feature of this chapter is that it confirms the observation that a discontinuum cannot be analysed unless it is treated as a structure. That is the use of force in terms of thrust with both shear and moment are essential in determining the load distribution. The computer programme used in these numerical experiments was developed by Burman (1971), and a very brief review, involving factors which are relevant to the work performed herein, is given.

A résumé and description of the input and output data pertaining to the computer programme are given in Appendix C.

7.2 STRESS DISTRIBUTION IN AN ELASTIC CONTINUUM WITH MULTIPLE OPENINGS

The starting point for the analysis of stress around excavations is knowledge of the initial stress state existing before the excavation was made. This initial
stress state depends on factors such as -

1. the superincumbent rock,
2. geological history,
3. formation of discontinuities such as faults, joints, bedding planes and dykes,
4. properties of rock such as homogeneity and isotropy,
5. inherent stresses such as stresses induced when the rock solidifies.

Major simplifying assumptions are generally made in order to obtain tractable expressions from which the pre-stress conditions are estimated. Leeman (1968) has developed techniques for measuring the insitu stress but this is fraught with major difficulties when considering a discontinuum, Section 5.4. Nevertheless by judicious assumptions and careful observations an idea of the pre-stress condition is attainable, Chapter 8.

Another question now arises: is it possible to acquire a clear picture of the stress state for the two dimensional case where in fact a three dimensional one exists? Enough work has been performed (Denkhaus, 1965) to show that, for simple shapes such as the sphere or spheroid, the magnitude of the three dimensional stress state is less than that obtained for the two dimensional one. It should be emphasised, however, that this statement relates to stress distribution and not to failure criteria. Consequently simplification to the two dimensional situation has much to commend it in that it is relatively simple and tends to give a higher magnitude of stress.
If the influence of increasing gravitational pressure along the periphery of the opening is neglected and the opening is considered as being contained in an infinite body subjected to an external pressure equal to the gravitational pressure, the "flat plate" theory is said to apply. Though this statement is really only valid for a continuum, it does nevertheless lend some validity to the flat plate theory approach used for the model experiments. On the other hand it should be recognised that the actuating force in the hanging wall of a blocky material is gravity and as such is a prime factor affecting the stability of the opening. This latter point is considered as ineffective in the initial analysis when using a "flat plate" theory but is most important when considering the final stability of the opening, Chapter 8.

Though failure of the intact material is not the main factor examined in this thesis, it nevertheless is of fundamental importance when considering the redistribution of stress around an excavation. However by examining the possible modes of failure, insight into the stability of underground openings is better appreciated.

Figure 7.1 shows the discretization of the continuum examined and Figure 7.2 the stresses induced near the boundaries of the openings and within the material.

7.3 DESCRIPTION OF THE MECHANISTIC FINITE ELEMENT METHOD

Burman (1971) using a method similar to the one used by Goodman (1968) replaced the elastic blocks with rigid ones and used a joint element which includes both the block and joint response to stress versus strain and moment
FIG. 1. Discretization of continuum for multiple openings.
Boundary concentration factors relative to the vertical load

FIG. 7.2
versus rotation. What this basically does is replace the discontinuum, which here is a system of blocks, with a structural frame, Figure 7.3. The interaction between the blocks and joints is then represented by joint elements placed between the centroids of the blocks.

7.4 DEVELOPMENT OF FRACTURE ZONE SURROUNDING UNDERGROUND EXCAVATION

The creation of an underground opening in an elastic continuum causes the compressive stress to increase and at times induces tensile stress. Consequently it is possible and in fact most likely that failure of the surrounding material takes place. This in turn creates a fracture zone which alters the geometry and loading conditions.

It is generally accepted (Denkhaus, 1958) that failure occurs in regions where either the maximum shear stress exceeds the shear strength or one of the principal stresses becomes tensile in character and exceeds the tensile strength. In the fractured zone the material is discontinuous and as such responds with mechanistic modes of deformation and as such can still support and distribute load.

By assuming the blocks to be rigid the degrees of freedom are reduced to three, namely the two displacements and rotation at the centroid of the block. The displacements at a corner or edge of the block are then related to the deformations at the centroid. As the corners or edge of the block make up the nodes of the joint elements
it then becomes necessary to relate the nodal forces to the nodal displacements by standard finite element techniques (Zienkiewicz, 1967). Transverse isotropy ($E_1, v_1$ in the plane of the joint and $E_2, v_2$, and $G_2$ normal to the joint) was used in the formulation of the constitutive equations. Burman (loc cit) showed that the nodal strain components due to rotation $\theta_c$ about the block centroid is equivalent to that resulting from rotation about the mid point of the joint element. From these rotations the displacements are evaluated and the average local shear and normal stresses are calculated at the centre of the joint element.

7.4.1 Characteristics of non-linear block interaction

In this computer programme there are three deformation modes which contribute most significantly to the non-linear or in fact non-elastic response of the structure to load. These are,

1. the response normal to the joint is limited by the tensile strength,

2. the shear strength of the joint is limited by Coulomb's shear strength criterion,

3. a non-linear stress strain response for the joint may apply.

Analysis of non-linear behaviour can be approached by a constant or variable elastic matrix process as depicted by Zienkiewicz and Valliappan (1969). The computer programme used in this chapter employs a
constant elasticity approach in which the initial stress or strain is the variable. It is found that the initial stress approach operates successfully for conditions where a small increment of stress is accompanied by a relatively large change in strain, for example when residual or incremental shear stress mobilises large strains.

A non-linear relationship is obtained which connects each degree of freedom at the block's centroid with the normal and shear stress at the centroid of each joint element surrounding the rigid block. The resultant data from this programme gives the average stresses in the joint elements which in fact give the forces acting on the boundaries of the rigid blocks at the centroid of the chosen joint elements.

7.5 REPRESENTATION OF FORCES IN TERMS OF STRESSES IN A DISCONTINUUM.

It is generally a requirement in engineering to try and normalise forces and displacements to give stresses and strains. This normalisation of forces in a discontinuum, especially a blocky material, encounters many difficulties. To date the main procedure when dealing with a discontinuum, is to take a representative unit, often called a systone (Trollope, 1968 and Section 2.7), and from this define the stresses. Consequently the average stresses over the systone unit represents the forces acting over the actual areas of contact. This approach does not consider the variation of stress over the face or boundary of the systone. That is stress gradients are neglected. This representation is adequate
when stress gradients are not prevalent within or between systone units. This means that the systone unit represents the behaviour of a discontinuum when only direct forces and shear forces are acting. When moments within and between units are introduced stress gradients have to be considered, Chapter 4, and the systone unit as defined in Section 2.7.2 is found to be inadequate.

Now when moments are considered in a two dimensional continuum another constant termed the characteristic length L is introduced (Mindlin and Tiersten, 1962). If the continuum is now discretized to give an equivalent structural lattice (Banks and Sokolowski, 1968) this characteristic length is now a function of both the lattice geometry and material properties.

The implication here is that where moments and hence stress gradients are prevalent in a discontinuum, the choice of a representative unit has to consider the above characteristic length before the continuum concepts of stress and strain can be used. As this is at the moment unavailable, a structural approach to analysing a discontinuum is used, in that the resultant forces and moments are not normalized to give average stresses.

By considering circular discs Trollope, (1968) defined a systone unit where the normal and shear forces were used to define equilibrium in terms of stresses. If the area of contact, which in this case is between circular discs, Figure 7.4, is relatively small compared to the discs themselves and the magnitude of the transmitted moments has a relatively small affect on the average stress distribution across the joint, the systone unit adequately
represents the force system in terms of stress. For a blocky material, however, the elements are large compared to the size of the blocks and the effects of moments transmitted across the block faces cause significant stress gradients. In order to represent the forces acting on the faces of the block in terms of equivalent stresses the characteristic length \( L \) would have to be known.

### 7.5.1 Influence of couple stresses

Cauchy showed (Love, 1944) that if central forces were assumed to act at the nodal points of the structural crystal lattice in a homogeneously deforming isotropic elastic material then Poisson's ratio should be \( \frac{1}{2} \). For metals, however, the Poisson's ratio is greater than \( \frac{1}{2} \) and this is taken as evidence for the existence of couple stresses imposed on the structural lattice making up the material. It was found that the gradient of couple stresses affects stress analysis rather than just the magnitude of a couple stress.

If \( \chi_{ij} \) is the intensity per unit area of these stress couples, where \( i \) denotes area and \( j \) the vector direction, it can be shown that for equilibrium

\[
\sigma_{ij} - \sigma_{ji} = \sum_{L=1}^{3} \frac{3 \chi_{ij}}{3L}
\]

where \( i, j, \) and \( k \) are all different and \( L \) is a characteristic length. Consequently, when considering the two
dimensional case, and when a couple stress exists, equilibrium is found to give the relation \( \sigma_{xy} \neq \sigma_{yx} \). When this happens it follows that the path dependent movements in a blocky mass, Section 4.7, may quite readily occur. That is, the path dependent movements in a blocky mass as propounded by Bray (1965) are not as restrictive as they may first appear, and in fact where moments across joints are prevalent this mode of deformation takes place.

7.6 NUMERICAL EXPERIMENTS

Between 125 and 132 blocks were used in the experiments with different sequences of loading, joint properties and block stackings. Primarily, the object was to investigate the load distribution for excavations in and foundation loads imposed on a blocky material.

A series of numerical experiments were performed from which the loads on specific blocks could be assessed. From these imposed block loads, stresses induced within the blocks could then be evaluated using the theory of infinitesimal elasticity.

Initially the experiments were performed where certain blocks were removed before the loads were imposed, and then these experiments were repeated with all the blocks intact before the loads were imposed and the same blocks subsequently removed.

7.6.1 Hydrostatic loading

Two stacking systems were used as shown in Figure 7.3. The first diagram, Figure 7.3(a) represents a three joint
FIG. 13  BLOCK ARRANGEMENT

A. LOCATION WITH ELEMENT ARRANGEMENT

THREE JOINT INTERSECTION SYSTEM

STAGGERED STACKING

COLUMN STACKING
FOUR JOINT INTERSECTION SYSTEM
**FIG. 7.4**

FINITE ELEMENTS REPRESENTING CONTACT AREA BETWEEN CIRCULAR DISCS.

4 JOINT INTERSECTION SYSTEM.

3 JOINT INTERSECTION SYSTEM.

3 JOINT INTERSECTION SYSTEM.

**FIG. 7.5**

JOINTS SYSTEMS FROM VARIOUS STACKING SYSTEMS
intersection problem (Lafeber, 1963) and Figure 7.3(b) depicts a four joint one. Another three jointed system is the hexagonal one which does not however intersect at right angles, Figure 7.5. This latter system induces an order of complexity higher than that experienced for the square blocks, and in the opinion of the author is a specialised instance much less common than that encountered with the square blocks.

When the blocks are stacked to give a model with staggered vertical joints and then a hydrostatic load imposed, shear forces in the jointing system are developed, Figure 7.6(a) and (b). These induced shear forces are dependent on the magnitude of shear modulus of the joints and the relative movement between blocks in contiguous rows. Figure 7.6(a) shows the force field caused by the hydrostatic loading when the shear stiffness is relatively high while 7.6(b) depicts the force field when the shear stiffness is relatively low. The terms relatively high and low shear stiffnesses are defined as the ratio of the joint shear modulus to Young's modulus normal to the length of the joint. A value of $\frac{1}{3}$ is considered high while $\frac{1}{10}$ is low.

If the clastic unit, namely systone as described in Section 2.7 is used to define stress the consequent stress system satisfies equilibrium to within 1%.

7.7 BLOCK EXTRACTION

In a stacked block system it was found, that the development of shear force in the jointing system under hydrostatic load was dependent in the main on factors
FIG. 7.6

HYDROSTATIC TYPE APPLIED LOAD AND THE EFFECT OF STACKING GEOMETRY ON LOAD DISTRIBUTION

(a)

(b)

SHEAR MODULUS = 3000 psi.
such as stacking geometry and shear stiffness of the jointing system. Because of the symmetry of the induced shear forces, however, there is no tendency for the blocks to rotate. If some blocks are now removed from the stacked system additional shear forces are induced, Section 4.7. This imposes a rotative tendency on certain blocks and destroys the symmetry of the forces acting on the blocks be they normal or shear forces.

It was found in Chapter 6 that when certain blocks were extracted the mechanisms of slip and rotation were initiated. These mechanisms impose specific load distributions on the blocks which in turn control the stress distributions within the blocks. Though failure is not studied here it is shown that in every instance where rotation of the block occurs either a tensile stress or effective tensile strain is induced. It should go without saying that this characteristic has very important implications in the subsequent behaviour or response of the blocky mass.

To have an analytical basis for calculating the load distribution on the blocks and then determining the stress intensity within the blocks is not only of fundamental importance but also of great practical significance. Consequently some simple block models are numerically evaluated and the results then compared with corresponding experimental values.

7.7.1 Blocks extracted and load subsequently imposed

First 3, 5 and then 6 blocks were removed from the model and after each group of blocks had been extracted the
boundary loads were applied. When the shear moduli in the joints were relatively high the induced shear forces caused the interacting forces to have a large angle of obliquity, Figure 7.7(b). This leads to a system less conducive to slip along the joints, because of the high shear stiffness. When the shear moduli of the joints are reduced, the loads transmitted across the joints have a smaller comparative obliquity, Figure 7.7(a). Nevertheless, this still imposes a rotative effect on certain blocks but with a much reduced shear force. It was found, as expected, that less slip occurred when the shear stiffness of the joints was high.

It was noted that when the joints were stiff in shear, continuous beaming type forces were mobilised and when the joints were of low stiffness, mechanistic beaming type forces were mobilised, Section 6.5.1. When comparing the numerical and experimental results it was found that the load distribution for the numerically soft joints agreed remarkably well with the experimental case where the blocks were loaded and then certain blocks extracted explosively, Figure 7.8. Good correlation between the numerical joints of low stiffness and the physical case was obtained where the blocks were first extracted and the model subsequently loaded. That is a mechanistic type beaming action was initiated, Figure 7.9.

Another important factor observed from these numerical experiments was that as soon as the moment transmitted across the joint became significant equilibrium of the clastic unit, Section 2.7 was no longer satisfied (20% to 30% error). However in areas of the same load field, generally some distance away from the opening where the
FIG. 7.7

Effect of Joint Shear Modulus
On Load Distribution

Joint Shear Modulus = 3000 psi

Joint Shear Modulus = 3000 psi
FIG. 7.8
Joints of high shear stiffness with blocks explosively (hammered) extracted
FIG. 7.9
Post-loaded blocks
moments transmitted across the joints were small, equilibrium of the clastic units was satisfied within 2%.

7.7.2 Blocks extracted or mined out after the loads are applied

In these numerical experiments the loads were first imposed and then certain blocks in groups of 3, 5 and 6 were mined out. The resultant magnitudes and patterns of force, Figures 7.10 to 7.12, were the same as those obtained when the blocks were first extracted and the loads subsequently applied. This would imply that the load distribution is independent on the load path, which is basically what Best (1970) and Burman (1971), among others, determined. The physical experiments showed, Section 6.7.3, that this determination is certainly not correct. It has been determined that this numerical result of load path independence is a result of the numerical iteration process, (Byrne, 1972).

The shear stress response in relation to shear strain and normal stress is shown in Figure 7.13. Essentially the shear stress is, in the main, a linear function of normal stress and the process of solving the finite element equations is by satisfying compatibility. Therefore for each imbalance of strain at a node the strain correction brings the stress back onto another shear versus normal stress curve, Figure 7.13. This results in a linear relationship between shear and normal stress and makes the whole deformational response load path independent, even with the introduction of the supposed non-linearities, Section 7.4.1. It is therefore important and necessary to investigate this numerical problem in greater detail.
FIG. 7.10

Effect of Joint Shear Modulus on Load Distribution

Joint Shear Modulus = 300,000 psi

Joint Shear Modulus = 30,000 psi
Fig. 7.11
Block Extraction for Different Load Ratios

Load Ratio: 1st/10th

Load Ratio: 2nd/10th

Diagram (a)

Diagram (b)
FIG. 7.12

Block Extraction for Different Load Ratios

LOAD RATIO 1 Vert.:1 Hor.

LOAD RATIO 2 Vert.:1 Hor.
For the moment however, if the aforementioned limitation is recognised it is still possible to glean some very useful information from the numerical work and compare this with the experimental work on physical models, Chapter 6.

7.7.3 Individual blocks

Again in the numerical experiments performed it was found that the load distribution for the different geometries of block openings created was very much dependent on the mechanisms taking place and in the numerical examples this was rotation. Nevertheless, even with this one mechanism included in the numerical model it was found that comparable results with those of the photoelastic models were attainable and specific patterns of load distribution repeatedly appeared on certain blocks. Therefore once the overall load distribution had been determined individual blocks were examined as in Chapter 6.

Figure 7.3(a) gives the block arrangement and their numbers of identification. Initially the three blocks 68, 69 and 70 were removed then 56 and 57 and finally block 44. Figures 7.14(a) and (b) show the loads acting on the horizontal surfaces of the blocks as the blocks stated are removed.

7.7.4 Abutment block number 67.

When the block mass is subjected to a hydrostatic type load ratio of 1 there is a greater tendency for the abutment block 67 to rotate, Figure 7.15(a). For a load ratio of 2, Figure 7.15(b), the load and stress gradients across to top and bottom sides of the abutment
FIG. 7.146  NORMAL LOADS

K = CONCENTRATION FACTOR.

38 39 40 41 42 43 44 45

K = 1.0

51 52 53 54 55 56 57

K = 1.0

63 64 65 66 67 68 69 70

FIG. 7.14G (BLOCK DESIGNATION FIG. 7.3)

NOTE
--- A.3.1 3 BLOCKS REMOVED
--- A.5.1 5 BLOCKS REMOVED
--- A.6.1 6 BLOCKS REMOVED

BLOCKS MINED OUT AFTER LOAD HAS BEEN APPLIED
HYDROSTATIC TYPE LOADING.
MOMENTS

38 39 40 41 42 43 44 45

K = 1.0

51 52 53 54 55 56 57

K = 1.0

63 64 65 66 67 68 69 70
**FIG. 7.16**

(a) Underlying block supporting abutment block.

3 Blocks removed after loading
Loading ratio 1vert:1hor.

(b) 5 Blocks removed after loading
Loading ratio 1vert:1hor.

(c) 6 Blocks removed after loading
Loading ratio 1vert:1hor.
overlying block resting on abutment block.

Loading ratio $I_{vert} : I_{hor}$

(a)

3 Blocks removed after loading

Loading ratio $I_{vert} : I_{hor}$

(b)

5 Blocks removed after loading

Loading ratio $I_{vert} : I_{hor}$
6 Blocks removed after loading
Loading ratio Ivert:1hor

3 Blocks removed after loading
Loading ratio Ivert:1hor.

**FIG. 7.18**
Hanging wall block.
block increase but the tendency for the block to rotate is reduced. This latter result increases the effective tensile strain with the consequent potential for slabbing to occur.

7.7.5 **Lower supporting abutment block number 80.**

As the blocks were extracted the loads on the supporting abutment block, number 80, remain much the same except for the horizontal thrust which increases, Figure 7.16(a), (b) and (c).

7.7.6 **Upper supporting abutment block number 55.**

When only three blocks had been removed the load distribution on the upper and lower abutment supporting blocks number 55 and 80, Figure 7.17(a) and 7.16(a), were much the same. However as the hanging wall blocks were extracted, the stress gradients on the top of the abutment block 67 reduced, Figure 7.15(a), and when six blocks had been removed this stress gradient reversed sign. This shows that as the blocks are removed from the wall the stress gradient acting on the top joint of the abutment block appear to decrease. This is contrary to what was determined from the photoelastic models, where in fact the stress gradient increases.

7.7.7 **Hanging wall block number 56.**

Figure 7.16 shows the stress distribution imposed on a hanging wall block. Here it is clearly seen that this is the type of stress distribution arrived at when the blocks react as a blocky beam responding with the
mechanistic mode of rotation.

7.8 LOAD DISTRIBUTION DUE TO SURFACE LOADS

When using the infinitesimal theory of elasticity it is general to consider small displacements as being homogeneous at a point and differentials of degree higher than 1 as being negligible in magnitude. The general deformation is then decomposed into rigid body motion (asymmetric tensor) and pure deformation (symmetric tensor). The asymmetric tensor makes up the motion of translation and rotation. By a suitable choice of axes the asymmetric tensor is eliminated. However if an asymmetric stress condition exists an asymmetric strain condition is brought about and this makes the elimination of rigid rotation by partition of the tensor into a symmetric and a skew symmetric a mathematical fiction. This is another reason why small displacement elasticity becomes inadequate for defining the deformational response of discontinuous media.

It is clear (Lee, 1963 and Chappell, 1967) that under certain conditions of foundation rigidity and Poisson's ratio the contact shear forces have little affect on the distribution of the normal contact forces. These contact shear forces however can change the normal load distribution considerably if slip and rotation of the material making up the supporting subgrade is possible.

7.8.1 Contact load distribution

In Figure 7.19 the contact stresses between a rigid and semi flexible foundation on a blocky subgrade are
depicted. Here it is evident that the elastic interaction between the blocks and foundation is masked by slip and rotation (moment transmission) of individual blocks, and as this slip is inhibited, the stress distribution tends to that of a continuum. The important point to note here is that the elastic interaction of the blocks and foundation give a definite stress distribution on the boundary of the blocks. This in effect can control subsequent mechanisms especially rotation and, perhaps just as important, subsequent failure mechanisms.

Difficulty is encountered when incorporating the elasticity of each block into a general computer programme because of the large storage capacity required. Nevertheless if the load distribution in a blocky material is to be examined with a computer, especially if the failure of individual blocks is to be included, the elastic response of individual blocks should be considered.

7.8.2 Load distribution within a blocky subgrade

With the numerical experiments performed here, two stacking geometries were examined, Figure 7.3(a) and (b), and the shear stiffness in these models were also varied so that the extremes of stiff and soft joints were investigated. Also examined was the effect load distribution had on the blocks when either a flexible or rigid load distribution was applied to the blocky subgrade. The resultant load distributions from these experiments are depicted in Figures 7.20 and 7.21.
CONTACT STRESSES ON BLOCKY SUBGRADE
Stress distribution at contact plane from foundations with varying stiffness.

FIG. 7.19
FIG. 7.20 LOAD DISTRIBUTION IN A BLOCK SUBGRADE

FLEXIBLE FOUNDATION

SEMIRIGID FOUNDATION
FIG. 7.21  LOAD DISTRIBUTION IN A BLOCKY SUBGRADE
7.9 **SUMMARY**

Though the numerical method used in the experiments performed in this chapter show general agreement with the form of results produced from the photoelastic method, the method still has limitations to its general applicability. Only one basic mechanism is included in the numerical model where in fact in the photoelastic models two are operative, namely slip and rotation. Another important factor not included in the numerical models was the elasticity of the individual blocks.

In the choice of elements and their distributions the programme is not sensitive enough to pick up the development of tension. That is, tension is relieved once it is experienced at the centroid of the chosen element and the load re-evaluated with the load relieved in this tension element. Now if the element is relatively long, tension may occur but not at the centroid of the element. Hence the loads are not re-evaluated and the impossible situation develops where the resultant load acting on the element is outside the chosen element, Figure 7.7(b).

Even with these limitations it would appear that a good assessment of the load distribution in a blocky material is obtained with the numerical model used.

There are however some very important limitations to the numerical method brought out in the comparison with the photoelastic experimental results. The stress concentration factors from the numerical examples are very much less than those obtained from the theoretical and model experiments, Figure 7.22. It appears that the main
factor causing these differences is the one of slip. Because in the numerical model slip is as yet not correctly modelled whereas in the photoelastic models it is a reality and its effect is of some significance. The load distributions in individual blocks for the numerical and photoelastic models are again much the same except the stress concentrations on the blocks are reduced for the numerical models.
CHAPTER 8.

NOS. 2 AND 5 ORE BODIES AT MOUNT ISA MINES

8.1 PREAMBLE

The Mount Isa Mines give an excellent field situation where the response of blocky material subjected to varying environmental conditions can be examined. Work in this chapter is in the main a development of the work performed by Hoskins (1967) and Rosengren (1968). Hoskins, among other things, performed stress measurements in the Rio Grande copper ore bodies and Rosengren tested, examined, classified and performed statistical analyses on rock defects in the Black Star area for the specific purpose of assessing the slope stability of a proposed open cut.

Work in this chapter is an attempt to assess the stability of underground openings with specific reference to the numbers 2 and 5 ore bodies of the Black Star area. Stability and stope dilution are inextricably related to the rock mass response as the imposed loads change. It is necessary in this particular problem that the work of Hoskins (loc cit) and Rosengren (loc cit) be considered when evaluating the raw data collected from the field investigation of the Nos. 2 and 5 ore bodies.

Insitu stress measurements used in conjunction with geological structural investigations, (Hegert, 1968), give at the present time, the best assessment of the insitu stresses encountered at Mount Isa Mines. Further work for determining the insitu stresses is being
performed by the Mount Isa Mines Research Department. Calibration of the Leeman cell was carried out by the author at the Australian National University.

Data obtained from the drilling programme are analysed in much the same way as proposed by Rosengren (loc cit). However due to improvements in technique accruing from the use of an NMLC Craelius core orientating device, Figure 8.1, and improved classification of the joints, the different conditions pertaining to the Nos. 2 and 5 ore bodies require reconsideration and examination. Particular emphasis is placed on defining the spatial distribution of defects as complementary to the reporting of joint orientations.

In order to perform a realistic model study of the Nos. 2 and 5 ore bodies, information pertaining to the insitu material properties and stress conditions is necessary. However, in the initial stages of the project none of this information was available from the relevant ore bodies; hence, work performed by Rosengren (loc cit) and Hoskins (loc cit) was used as a basis for constructing and testing the models. As the field information became known towards the end of the time available for the project, it became evident that the joint patterns, because of folding, were in some cases not orientated in the directions originally assumed. However, because of the wide range of models examined, in which the block sizes, joint orientations, and profiles of the blocks extracted were varied, some useful and meaningful interpretations could still be made.
NMLC Craelius core orientating device

**FIG. 8.1**

---

Core logging trays

**FIG. 8.5**
8.2 GENERAL LOCATION AND ENVIRONMENT

Mount Isa is situated in the Precambrian shield region of north-west Queensland at latitude 20° 47' S, longitude 139° 29' E. The Urquhart Shale is a member of the Mount Isa group which is a 15,000 ft. thick conformable sequence of silt stone, shale and bedded carbonates. This in turn forms part of the western limb of a north-south striking and north plunging anticline. The general bedding plane trend exhibits a predominant north-south strike and a dip of 60° - 65°W and persists over a distance of 17 miles north and 20 miles south of the mine (Bennet, 1965). About 2000 ft west of the mine the formations have been truncated by the Mount Isa Fault which strikes north-south and dips steeply west.

The Mount Isa group is made up of the following zones which are given in the sequence of west to east which is the youngest to oldest -

Magazine Shale,
Kennedy Siltstone,
Spear Siltstone,
Urquhart Shale,
Native Bee Siltstone,
Breakaway Shale,
Moondarra Siltstone.

The Urquhart Shale contains the ore bodies which are presently being mined. These ore bodies consist of adjacent but quite separate lead-zinc-silver and copper zones. The lead-zinc-silver ore occurs as fine grained galena and sphalerite together with abundant pyrite and
pyrrhotite in the bedding planes of the shale, and there is general agreement that it is of sedimentary origin. The copper ore occurs as chalcopyrite in large masses of deformed and recrystallised rock known as "silica dolomite", which slightly transgresses the enclosing shales. It has been suggested that the copper ore is also sedimentary in origin, (Bennett, 1965), however this hypothesis is still controversial.

The mine is divided up into four sections which in turn are named after the original leases of the area. The northern part of the mine is called Black Star section, Figure 8.2, and contains the Nos. 2 and 5 ore bodies and the 650 copper ore body. Sub-level open mine stoping has been used in this area because the country rock appears to be fresh and strong. Further to the east of the Black Star area, a series of narrow lead ore bodies are being mined by cut and fill methods and the name applied to this section is the Racecourse lead ore bodies. In the central section, called the Black Rock area, a series of transverse faults occur and this has allowed the carbonates to be leached to depths in excess of 200 ft. Consequently the 500 copper ore body in this area is weak in mechanical strength and porous. It is in this area that both the open cut and sub-level caving methods were used for extracting the ore. However the open cut was forced to stop 30 ft. short of the 520 ft. planned depth, because of the instability of the hanging wall, (Edwards, 1967). Sub-level caving of the 500 ft. ore body has also been terminated to ensure general stability of the Black Rock region. In the southern section, called the Rio Grande, Figure 8.2, the very large 1100 copper ore body terminates against a greenstone
FIG. 8.3
CROSS SECTIONAL PLAN
FIG. 8.3c

Nº 2 & 5 ORE BODIES
SEQUENCE OF EXTRACTION
basement some 3000 ft. below the surface. Open stoping methods are being used for this section because the host rock, silica dolomite, is intact and strong.

8.3 NOS. 2 and 5 ORE BODIES

As stated above the Nos. 2 and 5 ore bodies are located in the Black Star section and are lead-zinc-silver ore bodies in a host rock of shale, Figure 8.3(a). The method to be used for extracting the areas of mineralization is by sub-level caving and the proposed design is given in Figure 8.3(b). Extraction of the ore is to be carried out in the sequence marked on the cross section, namely one through to eight. The anticipated behaviour of the design for ore body No. 2 is that while the sequences one and two are being executed the hanging wall zones marked 5 and 6 will remain intact. However, if they do collapse at this stage the economic recovery will not be affected because these regions are zones of mineralization. When zone 4 is extracted, that is when the pillar is recovered, a critical stage could be attained on the economic recovery of the ore extracted at the scrums. This comes about because as the pillar is being demolished initial or additional collapse of regions 5 and 6 is possible and if this has already occurred the material coming down would cause dilution. So the question arises; is it possible to determine the stability of the hanging wall for the environment encountered and the sequence of extraction proposed? Ore body No. 5 has only two sequences of extraction, which is to be subsequent to the completion of extracting ore body No. 2, but because of the proximity of ore body No. 2 the behaviour of the hanging wall is unknown. It should be noted that silica
dolomite transgresses the shale material and that near to the hanging wall of the No. 2 ore body massive silica dolomite occurs. Also of possible and great significance are the existing stopes A\textsuperscript{W} 80 - 81 and B\textsuperscript{W} 80. This latter aspect will be considered towards the end of this chapter.

In order to give a viable answer to the problem posed above, as much field information as attainable was collected and using the work and experience of previous investigators a programme of experimental and computational work based on theoretical premises was planned. In this chapter the geological data is analysed and the relevant information is extracted so that the design can be made and the behaviour of the anticipated ore stopes predicted.

8.4 STRUCTURAL INVESTIGATION

In the initial stages of the investigation the results obtained in Rosengren's (loc cit) work was used to assess the anticipated model response of the Nos. 2 and 5 ore bodies, as this work related to an open cut in the upper reaches of the No. 2 ore body relatively close to the material of interest.

However in performing his work, Rosengren assumed with validity, that the bedding planes were consistently orientated. Circumstances at the initial stage of this work, such as limited development of the area and the non availability of circulating air, restricted drilling to the number 11 level. Also when inspecting the X74 hanging wall, it becomes apparent that this region has been subjected to folding. This therefore makes it essential to acquire a more definite method for orientating the core.
<table>
<thead>
<tr>
<th>Hole number</th>
<th>Level</th>
<th>Co-ordinates</th>
<th>Bearing</th>
<th>Dip</th>
<th>R.L. (feet)</th>
<th>Length (feet)</th>
<th>Core Size</th>
</tr>
</thead>
<tbody>
<tr>
<td>RM 16 Y 80 W Dec</td>
<td>11</td>
<td>7948 1546</td>
<td>272 53</td>
<td>-29</td>
<td>9706</td>
<td>127</td>
<td>NMLC</td>
</tr>
<tr>
<td>RM 17 Y80 W Hor</td>
<td>11</td>
<td>7948 1546</td>
<td>272 53</td>
<td>+4</td>
<td>9709</td>
<td>80</td>
<td>NMLC</td>
</tr>
<tr>
<td>RM 18 Y80 E Inc</td>
<td>11</td>
<td>7949 1557</td>
<td>086 51</td>
<td>+29</td>
<td>9710</td>
<td>103</td>
<td>NMLC</td>
</tr>
<tr>
<td>RM 19 Y80 E Hor</td>
<td>11</td>
<td>7949 1557</td>
<td>086 51</td>
<td>+2</td>
<td>9708</td>
<td>60</td>
<td>NMLC</td>
</tr>
<tr>
<td>RM 20 Y80 NW Hor</td>
<td>11</td>
<td>7953 1546</td>
<td>324 39</td>
<td>0</td>
<td>9708</td>
<td>143</td>
<td>NMLC</td>
</tr>
<tr>
<td>RM 21 Y80 SW Hor</td>
<td>11</td>
<td>7937 1546</td>
<td>220 53</td>
<td>0</td>
<td>9708</td>
<td>145</td>
<td>NILC</td>
</tr>
<tr>
<td>RM 22 Z81 NE Hor</td>
<td>11</td>
<td>8076 1478</td>
<td>064 22</td>
<td>-0</td>
<td>30</td>
<td>9708</td>
<td>123</td>
</tr>
<tr>
<td>RM 23 Z82 SE Hor</td>
<td>11</td>
<td>8199 1430</td>
<td>122 49</td>
<td>+0</td>
<td>25</td>
<td>9709</td>
<td>181</td>
</tr>
<tr>
<td>RM 24 Z83 NE Hor</td>
<td>11</td>
<td>8202 1430</td>
<td>059 24</td>
<td>+1</td>
<td>25</td>
<td>9709</td>
<td>222</td>
</tr>
<tr>
<td>RM 25 Z84 SE Hor No. 1</td>
<td>11</td>
<td>8308 1426</td>
<td>136 50</td>
<td>-0</td>
<td>35</td>
<td>9710</td>
<td>153</td>
</tr>
<tr>
<td>RM 26 Z84 SE Hor No. 2</td>
<td>11</td>
<td>8332 1416</td>
<td>107 06</td>
<td>+0</td>
<td>25</td>
<td>9709</td>
<td>204</td>
</tr>
</tbody>
</table>

**Figure 8.4** - Holes Drilled to Investigate the Structural Characteristics of the Numbers 2 and 5 ore bodies, Mount Isa Mines
Unfortunately a suitable method of core orientation was not available until the last hole, namely RM 24, was being drilled. Nevertheless, as will be shown, by comparing and adjusting the stereograms of the various cores a means of orientating the other cores is achieved.

Eleven holes were drilled at the locations denoted in Figure 8.4. As will be noted these holes give a good coverage of the hanging wall of the No. 2 ore body. Structural information of the No. 5 ore body and its hanging wall were not investigated by drilling but the development drives and cross cuts allowed enough investigational work to be executed to come to the conclusion that in this area folding is almost absent and that the material is very much like that which was thoroughly investigated previously.

8.4.1 Structural classification

The material which hosts the two ore bodies of interest here is Urquhart shale while the material close to the hanging walls of these ore bodies is Silica Dolomite, Figure 8.3a, which appears to transgress the shale material. The rock mass is a composite body of rock material and defects. Most structural analyses to date have been concerned with defect orientation, (Muller, 1963), whereas the spatial distribution of the defects appears to have been neglected. Joint spacing and continuity have been clearly recognised as affecting the response of a rock mass to a change in load, (Muller, 1963). However if the drilling hole is long and region large and there is a possible interaction between contiguous excavations, the averaging of joint spacing along the bore
hole length and continuity over the region of interest becomes suspect.

Adequate definition of the spatial distribution at the present time has not been produced. Much of the difficulty appears to stem from the requirement of defining the spatial distribution along the drill hole and then inter-relating this distribution to information obtained from contiguous boreholes. Nevertheless an attempt is made in this structural analysis to incorporate more precisely the distribution of jointing along the length of the bore hole.

8.4.2 Borehole logging

The logging technique described below is that developed and used at Mount Isa Mines. In essence the procedure is to lay the core, which has been pieced together, on angle iron rails, Figure 8.5, and then a reference line is drawn preferably along the true top of the drill core. Defects such as planar breaks are classified, in that, their depths along the core are noted, and their orientations relative to the reference line are measured and recorded. The reference line is obtained by an NMLC Craelius core orientating device, Figure 8.1. Operation of this orientating device is simple and effective in producing the desired accuracy in orientations, namely 2° to 3°. Its operation is to obtain a profile from the movable prongs by means of pressing the device onto the intact core before it has been extracted from the hole, and while this profile is in contact with the core an indentation mark defining the true bottom of the hole is made with a hardened ball bearing, (Bridges and Best, 1971).
It is also important that the orientation of the hole relative to the magnetic or true north with its inclination or declination be known. If the bore hole deviates, which is common in the longer holes and in anisotropic ground, the entire length of the hole may require surveying. Fortunately the bore holes in relation to the present investigation were relatively short.

Because of the prevalence of folding in the hanging wall of the No. 2 ore body the consistency of the bedding plane dip could not be used as a reliable orientating criterion (Rosengren loc cit). Yet it was at times possible to adjust the unorientated holes by using the orientated hole, R.M. 24, as a base and manipulating the data of the contiguous holes so as to obtain correlation between the various suites of defects in the corresponding regions. To do this satisfactorily the reference line used in the unorientated core should be consistent. Because in some holes this was not so, the recorded data had a variable base line and it becomes most difficult to obtain meaningful orientation of the unorientated data. Nevertheless the data of the unorientated hole was plotted out in stacked histograms, Figure 8.6, and used to obtain a qualitative concept of the region drilled. With this information and that of the orientated holes a structural evaluation of the hanging wall of the No. 2 ore body can be made (Bridges and Best, loc cit).

8.4.3 Defect classification

The region of interest here is not traversed by any large transverse faults as are other areas of the mine, (Herget, 1968). However a prominent north-south fault with an associated shear zone is located in the X74 hanging
wall drive. Also evident from the core logging results are a number of relatively small faults and shear zones.

From an engineering point of view the defects are split up into three main groups and then further divided into sub-groups as indicated in detail in Appendix G. The three main groups of discontinuities are caused by bedding breaks (B1, B2, B3, and B4), jointing (J1, J2, J3 and J4), and fractures due to faulting and shear zones (F1, F2 and F3).

Veins in the Urquhart shales are plentiful and appear, in the main, to consist of two types of infill material namely calcite and quartz. However defects described in the previous paragraph bear more relevance to the investigation because the lack of cohesion markedly affects the strength and slip mechanisms. Therefore these cohesionless joints will be examined in greater detail.

8.5 ANALYSIS OF DEFECTS

The technique of stereographic projection analysis described here was developed by Bridges and Best (loc cit) and Rosengren (loc cit), and because of the fundamental importance of this technique and because of the author's use of this method for defect analysis a brief description of the technique is given.

8.5.1 Bedding joint orientations

After the cores have been logged the defects are plotted as stacked histograms in Figure 8.6. Initially the bedding joints are grouped and plotted together and then the
transverse joints. On examining these histograms which are basically the number of defects per foot run, it is generally possible to divide the length of bore into segments in which certain joints may or may not be prominent. Assistance in making this choice of segmentation can be gained by plotting the core bedding angle along the length of the core. In order to produce this plot, however, a consistent reference line is required and this as mentioned previously was not achieved in the majority of holes drilled.

When a reference line is available, the core bedding angles can be plotted so that there is no ambiguity when considering the direction of dip of the bedding relative to the reference line, Figure 8.6. From this a general picture of the folding can be attained. Referring to Figure 8.6, it is noted that from 0 to 20 feet the bedding angle is regular whereas from 20 feet to 60 feet there is a gradual decrease in the core bedding angle to zero followed by an increase from zero with a reverse dip direction; this in fact is a transition of the core from one limb of the fold to the other. The remaining section of this core indicates a consistent bedding angle between 50° and 75°. Nevertheless there are sudden local changes in the core bedding angle which are caused by tight minor folds on the limbs of the main fold.

Subsequent to the segmentation of the core from the stacked histograms, stereographic plots of various joint suites were plotted, Figure 8.7. From 0 to 58 feet, which is one side of the limb of the fold, the bedding plane joints and breaks cluster, Figure 8.7, giving a dip of 40° in the direction 0 20°M.N. From 58 feet to 87 feet the
clustering is more indefinite; the bedding breaks however do show the same clustering on the other limb of the synclinal fold which gives a bedding dip of 70° in the direction 270° M.N.. These bedding joints and breaks cluster to give an excellent fit in a great circle which represents the fold axis, giving a plunge of 36° in a direction 343° M.N..

The five minor folds which have also been plotted on the stereograms, are summarised in Figure 8.7. Poles to these great circles represent the minor fold axes, which, as the stereogram indicates have widely differing orientations. These fold axes, however, lie on the same great circle which represents a plane dipping 72° in the direction 267° M.N. It is also evident that the axis of the main fold also lies in this plane. Therefore, although the fold axes have different orientations, they all lie in the same plane which is also the attitude of bedding in the west dipping limb of the fold. This in turn is also the general attitude of the bedding of the Urquhart Shale throughout the mine area. From this it is suggested that all the folds encountered by the drill core R.M. 24 are part of the same phase of folding.

8.5.2 Transverse joint orientations

Joints were classified and plotted as four distinct types, namely \( J_1, J_2, J_3 \) and \( J_4 \) as defined in Appendix G. \( J_2 \) and \( J_3 \) were combined however because they indicate that the particular joint in the core depicts partial fracture in that the joint is terminated or dies out. \( J_1 \) indicates a true joint while \( J_4 \) depicts a weakness plane with some degree of cohesive strength. Again stereograms of the
Fig. 8.7 Z83 N.E. Hor. II/1 (RM24)

Analysis of Bedding Features

Sterograms showing analysis of structural data from oriented core

- Poles of 81 & 83 joints
- Poles of 83 & 84 joints
- (See appendix for classification of bedding joints & trends)
- Poles of routine bedding measurements
- Poles of bedding measurements from folded axes in core

(After Best)

(a) 0°-30°

(b) 30°-60°

(c) 60°-90°

(d) 90°-120°

(e) Contoured plot of poles to all bedding measurements (252 poles)

(f) Plot of minor folds showing orientation of fold axes

Main fold axis @ 37/32°N
Z83 NE.HOR.11/L (RM.24) - ANALYSIS OF JOINTING

STEREOGRAMS SHOWING ANALYSIS OF STRUCTURAL DATA FROM ORIENTED CORE

Fig. 8.8

- Poles of J1 joints
- Poles of J2 & J3 joints
- Poles of J4 breaks

(See Appendix for joint classification system)

Percentage distribution of joint poles

Fig. 8.8

(a) 0 - 50'
(b) 50' - 90'
(c) 90' - 180'
(d) 180' - 222'

Contoured plot of poles to all "J" measurements, showing joint sets

Summary plot of "J" measurements for entire hole (236 poles)
<table>
<thead>
<tr>
<th>Joint Set</th>
<th>Average spacing in feet</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0' - 58'</td>
<td>58' - 87'</td>
<td>87' - 160'</td>
<td>160' - 222'</td>
<td></td>
</tr>
<tr>
<td>'North dippers'</td>
<td>1.5</td>
<td>1.7</td>
<td>3.1</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>Axial plane</td>
<td>1.5</td>
<td>2.4</td>
<td>4.5</td>
<td>5.5</td>
<td></td>
</tr>
</tbody>
</table>

**Joint Spacing**

Using the structural log of RM 24, the calculated values of the ratio $\frac{\sum J_1 + \sum J_4}{\sum J_2 + \sum J_3}$ are as follows -

<table>
<thead>
<tr>
<th></th>
<th>0' - 48'</th>
<th>58' - 87'</th>
<th>87' - 160'</th>
<th>160' - 222'</th>
<th>0' - 222'</th>
</tr>
</thead>
<tbody>
<tr>
<td>'North dippers'</td>
<td>2.4</td>
<td>2.0</td>
<td>1.5</td>
<td>2.2</td>
<td>2.1</td>
</tr>
<tr>
<td>Axial plane</td>
<td>11.0</td>
<td>10.0</td>
<td>0.9</td>
<td>0.8</td>
<td>3.0</td>
</tr>
</tbody>
</table>

**Continuity of Joints**

**TABLE 6.**
drill hole segments were plotted, Figure 8.8 and then combined to give an overall assessment.

Figure 8.8 shows a summary of the joints which clearly gives two prominent sets of joints. The one set which dips steeply north (85° N 13° M.N.) is generally known as the "north dippers" and the other set, which dips steeply west (82° W 260° M.N.) are not generally apparent in the mine area and therefore have no specific designation. The relationship between these west dipping joints being in the same plane as the fold axis, indicates the probability of these joints being caused by axial cleavage. It is clear that the "north dippers" are most prominently developed in the segment between 160 feet and 222 feet.

8.5.3 Spacing

It becomes evident when examining a core log that for any section of the hole the average joint spacing can be calculated. This requires the assumption that the joints are evenly spaced along the section being analysed. It often happens that the jointing may be local and that over a larger section the figure of average spacing is meaningless. However by plotting the stacked histogram for the various types of joints, local concentrations are observed. From this histogram the average distance between the bedding joints is corrected by multiplying the apparent thickness with the sine of the average core bedding angle. The average spacing of the joints are given in Table 6. . At this stage the relative continuity of the joints has not been considered therefore average spacings are computed by adding all the J breaks in the
sections of the drill hole and dividing by the distance perpendicular to the sets.

8.5.4 Continuity

Continuity of a joint in a mass of rock is, at the present state of the art, most difficult if not impossible to determine accurately. Nevertheless useful parameters giving a measure of continuity can be assessed from the core analysis.

$J_1$ and $J_4$ breaks denote continuous joints while $J_2$ and $J_3$ breaks are joints which are partially continuous. Therefore by taking the ratio of the sum of the $J_1$ and $J_4$ breaks to the total number of breaks in the segment under consideration a measure of the proportion of continuous joints is attained. Consequently the nearer this ratio is to 1 the more continuous the joints of that segment are likely to be, Table 6.

8.6 FIELD CONDITIONS RELEVANT TO MODEL CONSTRUCTION

Cross-sectional profiles and areal extent of the Nos. 2 and 5 ore bodies are shown in Figure 8.3. In these diagrams the relative position of the Silica Dolomite and Urquhart Shale are depicted. The presence of this Silica Dolomite has a major influence on the behaviour of the No 2 ore body.

Rosengren (loc cit) and Herget (loc cit) showed that the Urquhart Shale in the Black Star area has a continuous set of joints parallel to the bedding planes which dip at an angle of 75° west with a north-south strike. A set
of micro joints perpendicular to the bedding planes also occur and are approximately 50% continuous. Additionally, work performed by Hoskins (1967) in the Silica Dolomite of the Rio Grande region which is located south of Black Star region, Figure 8.2, indicated that the maximum principal stress dipped 25° East in a west-to-east direction and the intermediate principal stress dipped 65° West in an east-west direction and finally the minor principal stress was horizontal acting in a north-south direction. The magnitudes of the principal stresses using the door stopper technique are (Hoskins, loc cit) -

\[
\sigma_1 = 3150 \text{ p.s.i.} \\
\sigma_2 = 2250 \text{ p.s.i.} \\
\sigma_3 = 1850 \text{ p.s.i.}
\]

From this work it could be concluded that the region was made up with blocks of shale, approximately 2 ft. square stacked much in the way of a stretcher bond wall, where there is a continuous joint set (bedding plane joints) and a transverse discontinuous joint set. The jointing in the third dimension though not continuous is such that the two dimensional representation would be conservative in assessing the stress or load distributions.

Some of the queries involved in this particular part of the investigation are -

1. what is the stress distribution caused by the excavations?
2. how is the stability of the hanging walls affected by the stress redistributions?

3. what dilution can be expected from the partial caving-in of the hanging walls?

4. what effect will the sequence of extraction have on the overall stability, especially the stability and ease of extraction?

5. would it be preferable to undermine the hanging wall; that is leave a veneer of relatively rich ore in the hanging wall, so that if cave-ins do occur there will be minimal dilution of the ore extracted?

A fundamental question posed in the testing and examination of the models is; what effect does the sequence of excavation and loading have on the stress and deformational patterns? Namely, is it valid to remove specific blocks and subsequently impose the load, or should the load first be applied and the blocks then removed. The latter sequence has certain limitations because the difficulty of instrumentation and the destruction of the model. Nevertheless this sequence was examined though the first sequence, namely excavation then loading, was used for most of the experiments performed. Consequently, by comparison, the difference between the two sequences can be distinguished and the test results appropriately interpreted.
8.7 EXPERIMENTAL PROGRAMME

8.7.1 Continuous model

From the information available excavations to a scale of 1 in. = 70 ft. were modelled using Araldite D as the photoelastic model material. In the first instance the mine models were of continuous material and then they were constructed from blocks of various sizes.

It soon became evident that, in most cases, the deformational responses of the hanging and foot walls were non-linear in the continuous model. Moreover this non-linear response was not of the same magnitude and in some instances moved in a direction contrary to what would be anticipated; that is where one would expect a contraction to occur an expansion would take place.

Many different loading sequences were applied to the continuous model in an effort to acquire as comprehensive a picture of the elastic responses of the proposed structure. The results could then be used as a basis for comparison with the blocky models. Some load displacement curves are depicted in Figure 8.9.

The model instrumentation and installation was much the same as that described for the blocky material, Chapter 3. For the purposes of description and explanation the excavations will be referred to here as 1, 2 and 3, Figure 8.10.
FIG 8.10

NOMENCLATURE

$ s = $ shorter span
$ L = $ Longer span

$ v = $ vert. ld.
$ \alpha = $ orientation

$ H = $ Hor. ld.
$ \beta = $ orientation

TABLE 7.

<table>
<thead>
<tr>
<th>$ \alpha $ orient.</th>
<th>App. Modulus</th>
<th>p.s.i.</th>
<th>$ \beta $ orient.</th>
<th>App. Modulus</th>
<th>p.s.i.</th>
<th>$ % $ diff. of $ E $ for hyd. type C/D</th>
</tr>
</thead>
<tbody>
<tr>
<td>$ E_{L,1}^{H,1} $</td>
<td>15,000</td>
<td></td>
<td>$ E_{L,1}^{H,1} $</td>
<td>14,000</td>
<td></td>
<td>7.0</td>
</tr>
<tr>
<td>$ E_{L,1}^{V,1} $</td>
<td>35,800</td>
<td></td>
<td>$ E_{L,1}^{V,1} $</td>
<td>56,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ E_{S,1}^{H,1} $</td>
<td>24,800</td>
<td></td>
<td>$ E_{S,1}^{H,2} $</td>
<td>24,500</td>
<td></td>
<td>1.2</td>
</tr>
<tr>
<td>$ E_{S,1}^{V,1} $</td>
<td>12,500</td>
<td></td>
<td>$ E_{S,1}^{V,1} $</td>
<td>25,100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ E_{L,1}^{H,2} $</td>
<td>19,700</td>
<td></td>
<td>$ E_{L,1}^{H,2} $</td>
<td>20,500</td>
<td></td>
<td>4.0</td>
</tr>
<tr>
<td>$ E_{L,1}^{V,2} $</td>
<td>20,900</td>
<td></td>
<td>$ E_{L,1}^{V,2} $</td>
<td>-190,000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ E_{S,1}^{H,2} $</td>
<td>-32,900</td>
<td></td>
<td>$ E_{S,1}^{H,2} $</td>
<td>-30,500</td>
<td></td>
<td>7.9</td>
</tr>
<tr>
<td>$ E_{S,1}^{V,2} $</td>
<td>-101,100</td>
<td></td>
<td>$ E_{S,1}^{V,2} $</td>
<td>137,500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ E_{L,2}^{V,3} $</td>
<td>24,600</td>
<td></td>
<td>$ E_{L,2}^{V,3} $</td>
<td>22,400</td>
<td></td>
<td>9.8</td>
</tr>
<tr>
<td>$ E_{L,2}^{H,3} $</td>
<td>13,900</td>
<td></td>
<td>$ E_{L,2}^{H,3} $</td>
<td>-92,200</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$ E_{S,2}^{V,3} $</td>
<td>7,800</td>
<td></td>
<td>$ E_{S,2}^{V,3} $</td>
<td>9,800</td>
<td></td>
<td>25.7</td>
</tr>
<tr>
<td>$ E_{S,2}^{H,3} $</td>
<td>-22,500</td>
<td></td>
<td>$ E_{S,2}^{H,3} $</td>
<td>-9,000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
8.7.2 Experimental procedure

In order to ensure that the experiments were independent of any external influences, such as deformation of the supporting frame or method of securing the model in place, the models were set in either one of two positions, Figure 8.10, namely $\alpha$ with the greater span of opening vertical and $\beta$ with the greater span of opening horizontal. All applied loads were equally incremented to a specified all round load and then the vertical load was incrementally increased to twice the magnitude of the horizontal load. Now when the model was secured in the $\beta$ position the same loading sequence as just described was imposed. Consequently the directional moduli, such as that measured across the short span $E_S^H$ or long span $E_L^H$, for the hydrostatic type loading (equal all round increments) should be equal for the two model orientations $\alpha$ and $\beta$. An examination of the moduli results, Table 7 indicates that relatively good agreement was attained.

Another important factor in the assessment of the experimental results is their repeatability. Therefore throughout the entire experimental programme at least 20% of the experiments performed were repeated. Also for each experimental set up the loading was cycled at least twice and in many cases where hysteresis and deformational hardening were evident as many as four repetitions were applied. With the elastic models examined good repeatability of the load deformation curves and moduli values were obtained.
8.8 DEFORMATIONAL MODULI

By extracting material from the models the deformational resistance of the remaining material is correspondingly reduced. The magnitude of this reduced resistance to deformation appears to depend on -

1. the amount of material extracted,
2. the profile geometry created by the extracted material,
3. the magnitude and direction of imposed loads in relation to the resultant profile of opening.

For the sake of brevity and explanation the following nomenclature will be adopted when discussing the experimental moduli. In the symbol,

\[ E_{S,\alpha}^{H,2} \]

the subscript \( S \) will mean the shorter span of the opening, while the subscript \( \alpha \) will specify the orientation of the opening, Figure 8.10.; the superscript \( H \) will specify the hydrostatic type loading condition and the superscript 2 specifies the opening being considered, namely 1, 2 or 3, Figure 8.10. In the symbol -

\[ E_{L,\beta}^{V,2} \]

the subscript \( L \) represents the longer span of the opening, \( \beta \) denotes the orientation of the openings, Figure 8.10, the superscript \( V \) depicts the incremental application of the vertical load, and the superscript 2 represents the
openings 1, 2 and 3 of the Nos. 2 and 5 ore bodies, Figures 8.3 or 8.10.

8.8.1 Continuous model

It is generally accepted that the insertion of an opening in a continuum will increase the overall compliability, namely the deformational response to applied load. Yet when measuring the deformational characteristics of the Nos. 2 and 5 ore bodies the overall deformability in some directions increased while in others it decreased. Examination of Table 7 which depicts the results of the overall moduli when the longitudinal axes are initially vertical (α) and then horizontal (β), portrays the effect just stated and Table 8 indicates how the impression of isotropy may be attained from measuring deformations under a hydrostatic type loading.

As the deformations are measured closer to the edges of the openings so are the moduli correspondingly reduced in magnitude. Deformation of the hanging and foot walls have distinct and independent movement. Nevertheless, by considering their relative movements (moduli measurements) an idea of whether the opening is contracting or dilating is obtained. A modulus of positive sign defines contraction while that of negative sign indicates dilation.

With a single square opening under hydrostatic loading, the modulus appears to reduce accordingly; yet with an undirectional increase in loading perpendicular to either the long or short axis of the opening, the modulus appears to decrease and increase parallel and perpendicular
to the direction of loading, Table 7. From the profiles of the Nos. 2 and 5 ore bodies, however, the movement within the openings are not consistent in that in some instances the openings contract while on other occasions they dilate depending on the loading sequences applied. Nevertheless, it is clear by comparing the α and β orientations that the deformational response of an opening is dependent upon the size and profile of the neighbouring openings. Moreover, the importance of the relative magnitude and direction of the principal stresses relative to the geometry of the profiles is fundamental to understanding and/or anticipating the relative convergence expected in the excavation.

8.8.2 Blocky models

As expected, the moduli across the openings modelling the Nos. 2 and 5 ore bodies were correspondingly reduced due to the introduction of the discontinuities. Measurements across opening number 1, Table 9, were performed in models of various block sizes and subjected to the same loading conditions as were the continuous models at β orientation. These results for the moduli across the short span for the different block sizes are depicted in Table 9, values of the moduli across the long span were so haphazard as to make any plot appear meaningless; yet values of magnitudes for these moduli are also given in Table 9.

Some useful information becomes evident by examining the graph from Figure 8.11 and comparing this with Figure 8.12. The form of both the graphs are similar in that the moduli for hydrostatic loading are greater than those
for uniaxial loading. Also for the square blocks the magnitude of the moduli are much the same. For the rectangular blocks, however, the moduli appear to increase as the block size increases. This could be due to either one or both of the following reasons:

1. the shape of the blocks could affect the deformational response of the model,

2. the size of the opening relative to the size of the blocks could be such, that this becomes the controlling influence.

Another important factor is that in the blocky material the openings appear to behave more in the way one would expect of a single opening. That is the mechanisms of slip and rotation take over the load bearing and load distribution property of the blocky material. Yet the interaction of the openings now becomes most important in that these mechanisms are greatly accentuated and the development of tensile forces or strains becomes more prevalent.

8.9 MODEL - PROTOTYPE DISPLACEMENT CORRELATION

In Chapter 3 it was shown that strict similitude is very difficult to achieve but by combining some non-dimensional groups a single relationship is obtained giving what is termed extended similarity.

From equation 3.18 we have -
**TABLE 8**

H = HYD. TYPE APP. LO.  
V = VERT. LO. APP.

**α orientation**  

**DISPLACEMENTS MEASURED AT POINTS A, B, C, AND D.**

<table>
<thead>
<tr>
<th>App. Mod.</th>
<th>p.s.i</th>
<th>App. Mod.</th>
<th>p.s.i</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E_{CD}^V$</td>
<td>88,900</td>
<td>$E_{CD}^V$</td>
<td>118,000</td>
</tr>
<tr>
<td>$E_{CD}^N$</td>
<td>279,000</td>
<td>$E_{CD}^N$</td>
<td>750,000</td>
</tr>
<tr>
<td>$E_{AB}^V$</td>
<td>-232,200</td>
<td>$E_{AB}^V$</td>
<td>118,900</td>
</tr>
<tr>
<td>$E_{AB}^N$</td>
<td>260,000</td>
<td>$E_{AB}^N$</td>
<td>800,000</td>
</tr>
</tbody>
</table>
TABLE 9
DISPLACEMENTS MEASURED AT
POINT MARKED A, B, C, AND D

Hyd. = disp. meas. when a hyd. type ld. applied.
Vert. = " " " vert. ld. applied.

<table>
<thead>
<tr>
<th>TEST β₂ (1&quot; x 2&quot; rectangles) ref. App. E</th>
<th>App. Mod.</th>
<th>p.s.i</th>
</tr>
</thead>
<tbody>
<tr>
<td>A.</td>
<td>Eᵥcd</td>
<td>17,150</td>
</tr>
<tr>
<td>B.</td>
<td>Eᵥhyd.</td>
<td>3,750</td>
</tr>
<tr>
<td>D.</td>
<td>Eᵥvert.</td>
<td>-36,900</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST 8₂ (1&quot; x sq blocks)</th>
<th>Eᵥvert.</th>
<th>41,300</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eᵥcd (ld. 650, -9451)</td>
<td>Eᵥvert.</td>
<td>13,180</td>
</tr>
<tr>
<td>Eᵥcd (ld. 315, -630)</td>
<td>Eᵥvert.</td>
<td>-ve ∞</td>
</tr>
<tr>
<td>Eᵥc</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST λ₇ (½ sq. blocks)</th>
<th>Eᵥhyd.</th>
<th>5,670</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eᵥvert.</td>
<td>7,380</td>
<td></td>
</tr>
<tr>
<td>Eᵥhyd.</td>
<td>-34,500</td>
<td></td>
</tr>
<tr>
<td>Eᵥvert.</td>
<td>∞</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TEST λ₇ (½ sq. blocks)</th>
<th>Eᵥvert.</th>
<th>18,650</th>
</tr>
</thead>
<tbody>
<tr>
<td>Eᵥvert.</td>
<td>3,760</td>
<td></td>
</tr>
</tbody>
</table>
DISPLACEMENTS MEASURED AT POINTS A, B, C and D.

Hyd. = Disp. meas. when a hyd. type ld. applied.

Vert = Disp meas. when only a vert. ld. is applied.

FIG 8.11

\[ r = \frac{\text{Area}}{\text{Contact perimeter}} \]

Scale 1" = 0.1

FIG 8.12

\[ r = \frac{\text{Area}}{\text{Contact perimeter}} \]

Scale 1" = 0.1
\[ \delta_m = \frac{E_p}{E_m} \frac{F_m}{F_p} \frac{L_p}{L_m} \delta_p \]

where subscript \( m \) refers to the model and subscript \( p \) to the prototype. Hence the following relation is easily derived, namely -

\[ \delta_m = \frac{E_p}{E_m} \frac{\sigma_m}{\sigma_p} \frac{L_m}{L_p} \delta_p \]  

8.1

Application of this equation to a practical situation is however most difficult to perform. The basic requirement in deriving this equation is that the deformational response for the model and prototype be similar. Now \( E_m \) for the models has been measured with some degree of consistency, however, \( E_p \) for the prototype is basically unknown. In Chapter 5, even with the limited tests performed, it becomes clearly evident that the mass deformational response is a complex phenomenon. Consequently the ratio of \( \frac{E_p}{E_m} \) is found to be difficult to define and use in equation 8.1.

Results from the continuum models of the Nos. 2 and 5 ore bodies combined with an analogue \( E_p \) of \( 7 \times 10^5 \) lbs. per in\(^2\) gives for the elastic prototype the following deflections at the mid-spans of the openings -

<table>
<thead>
<tr>
<th>Opening</th>
<th>Deflection at mid-span in.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.1</td>
</tr>
<tr>
<td>2</td>
<td>-1.8</td>
</tr>
<tr>
<td>3</td>
<td>-1.1</td>
</tr>
</tbody>
</table>

LOAD RATIO = 2 \( \perp \) to long span
The negative sign denotes a dilational effect at the mid-span.

Considering opening 1 it was found that the deflection in the prototype from the 2 in. x 1 in. rectangular blocks would be approximately 17 in. while for the square blocks this deflection would be 24 in.

It should be stressed that these figures rely heavily on the assessment of the ratio of $\frac{E_p}{E_m}$ and this at the moment is most difficult to ascertain, especially the factor $E_p$.

8.10 STRESS DISTRIBUTION

If the average applied stress in the model is 150 p.s.i. and the fringe value of the birefringent material has a fringe value of 50 p.s.i. per fringe for the chosen model thickness then by counting the number of fringes passing through a point the concentration factor of stress can be determined at that point by dividing the number of fringes by 3. If the basic deformation mechanisms have been correctly modelled this concentration factor would also be representative of the prototype situation. This concentration factor, however, would be very much dependent on the load path and resultant stress environment, yet with a predominance of certain jointing systems the stress distributions could be controlled by the specific mechanisms of slip and rotation and the concentration factor determined could then be pertinent.
8.10.1 General testing procedure

In the initial stages of the testing programme it was shown conclusively that the mechanisms which occur or are likely to occur are stress dependent. During the loading and unloading cycles both the stress patterns and deformation at certain points were recorded. The stress patterns obtained were representative of the whole region of interest in the model.

8.10.2 Continuous model

If it is assumed that the model gives a similar stress pattern to that in the prototype, that is, the stress concentration factors are equal, we have -

\[ \sigma_p = \sigma_p \text{ average } K, \]

\[ \sigma_m = \sigma_m \text{ average } K \]

where \( \sigma_p \) is the stress value in the prototype on the boundary of the opening

\( \sigma_p \text{ average} \) is the average stress before creating the excavation

\( K \) is the stress concentration factor obtained from the model study

\( m \) refers to the model.

From the isochromatics cutting the boundary of the excavation it was determined that the tangential stress for an applied stress ratio of 2 to 1, Figure 8.13, would vary from approximately 500 p.s.i. to 14,000 p.s.i.
No. 2 and 5 ore bodies

Multiple openings  
FIG. 8.14 a
FIG. 8.15a.
FIG 8.16 Interaction of openings in a blocky material.

Three openings, 7 blocks removed.

Three openings, 10 blocks removed.
The high stress concentrations occurred in specific and generally small areas. That is the stress concentrations have a local influence. Where the stresses reduce however the areal extent is more pronounced.

8.10.3 Blocky models

In these experiments the results of all the work in this thesis applies. That is the stress distribution is now that of a structure where mechanisms of slip and rotation are important and the effects of geometry change must be considered.

Photographs of some of the experiments performed are shown in Figures 8.14 to 8.16. Here again all the work discussed in the previous chapters applies. Of note, however, is that the stress concentrations and reductions are much more prevalent and concentrated over larger areas. That is regions which were only locally affected by stress redistributions are now much more extensive and susceptible to failure. Tensile stresses, effective tensile strains and increased compressive stresses are readily located in the models.
CHAPTER 9

SUMMARY AND CONCLUSIONS

9.1 INTRODUCTION

Photoelasticity is the main experimental method used in this thesis. However, before being able to use this method, photoelastic material possessing specific properties had to be manufactured, models produced and experimental techniques developed. Some of these requirements were investigated in some previous work performed by the author (1967). Nevertheless further development work was required to meet the specific needs of this thesis and these were -

1. construction of a 22 in. diffusion polariscope,

2. manufacture of models to meet the required optical and deformational properties plus the required geometrical tolerances,

3. method of recording the photoelastic data from relatively large stress fields,

4. numerical method of reducing the photoelastic data.

Parallel with and subsequent to the above development the deformational response and stress distribution of simple and relatively complex physical models were examined. Also parallel with the above work the analytical problem of determining the stress distribution in blocky models was investigated.
9.2 REDUCTION OF EXPERIMENTAL READINGS

In many cases the shear difference method was used to reduce the experimental readings taken in the form of the principal stress difference (isochromatics) and the principal stress directions (isoclinics). This method expands any initial error as the computation progresses from a boundary to areas within the stress field. Consequently this coupled with the finite difference approach of the shear difference method requires that the isochromatics be accurate to within ± 0.1 p.s.i. and the isoclinics to ± 5°. Only in a few specific cases was it possible to obtain the required accuracy when reading the isoclinics. With the above limitations in mind a numerical method was developed, Chapter 3, and programmed for a PDP.10 computer. Here it is noted that though the results could be evaluated independent of the isoclinics the accuracy required of the isochromatics is somewhat increased to a value of something like ± 0.05 p.s.i. These tolerances given here were determined by using both the shear difference and numerical methods and if overall equilibrium was satisfied to within 10% the experimental results were considered satisfactory.

As noted above the numerical reduction of the photoelastic data requires the readings of the isochromatics to be accurate to within ± 0.05 p.s.i.. The stress gradients encountered in the areas or blocks of interest generally allowed the values of the isochromatics to be determined to within ± 0.1 p.s.i.. This means that much of the photoelastic data obtained could not readily be evaluated with the computer programme. Nevertheless with a Babnet-Soleil compensator with an accuracy of ± 0.02 p.s.i. the potential of the numerical method is
FIG. 9.1

NORM  Ld = 315 lbs

1st Slip

2nd Slip

3rd Slip
considerably increased.

The grid of the points from which the photoelastic readings were taken varied in size from 0.2 in. to 2.0 in. and on many occasions where stress concentrations occurred the grid system completely enclosed the isochromatics and the stress concentration was therefore lost in the numerical calculation.

9.2.1 Material production

By examining Table 2 the excellent optical and physical characteristics of the amine cured epoxy resins are at once evident. Of fundamental importance is the greatly reduced primary creep with the consequent reduction of stress redistribution due to this cause.

Edge effects in the epoxy resin are also much reduced and after each casting the epoxy sheet was stored for a period of two to three months. In this relatively short time the moisture gradient across the thickness of the sheet is reduced by moisture diffusion and the edge effects also correspondingly reduced and at times entirely eliminated. To achieve this final condition, the humidity and temperature must be kept reasonably constant.

In order to reduce buckling effects on the two dimensional models, the larger models were constructed from Araldite sheets at least $\frac{3}{4}$ in. thick and in many instances 1 in. thick.

At times residual stresses occurred at the corners of the unloaded blocks giving a fringe order of one. Whereas
the fringe values introduced into the loaded models were at times of the order of 17 to 23. This deficiency was overcome by taking isochromatic readings before and after the models were loaded.

9.2.2 Model construction

It was found that if the dimensions of the blocks were controlled to within ± 0.001 in. the block models gave a reasonably uniform picture of stress distribution for a hydrostatic (equal vertical and horizontal type load. On the other hand if the models were constructed with the dimensional tolerances of ± 0.01 in. these misfits gave a non-uniform stress picture in that local stress concentrations could be observed.

It was generally found however that when the mechanisms of slip and rotation were introduced the misfits did not appear to control the subsequent load distributions, especially if the magnitudes of the slips and rotations were such as to be greater than the dimensional magnitude of the misfits. More work on this aspect would be required to put the tolerances stated above into dimensionless form. Nevertheless the above observation is very important in that the magnitude of slip or rotation if sufficient, namely greater than the elastic deformation of the intact material or greater than the size of misfit, these mechanisms control the load distribution and consequent stress distribution within the block.
9.3 MECHANISMS OF SLIP AND ROTATION

When simple slip on a planar discontinuity occurs the material or blocks making up the discontinuity tend to rotate. If the rotation is inhibited by so-called rigid constraints the resultant deformational response is that of an elastic plastic material; whereas if the material or blocks are allowed to rotate the resultant deformational response is that of an elastic strain hardening material. The point to be made here is that the resultant deformational response is constraint dependent. When slip occurs the magnitude of displacements are such as to give second order strains and it then becomes necessary to separate the geometry from the physics of deformation. Biot (1965) lucidly shows what effects second order strains have on the kinematic and constitutive relations in that the second order effects are accounted for by introducing a rotation \( \omega \) which is defined in terms of the displacements. Physically this rotation has a marked effect on the stress redistribution in that an induced moment caused by the increasing shear forces and at times changing geometry, cause the normal stresses to redistribute from average values to ones which are now variable because of the induced moment. This alone can cause secondary effects such as digging in at the corners with the consequent effect on the subsequent deformational response. Now if slip occurs there is a geometry change and the resultant forces cause a further alteration to the normal stresses with the consequent effects on further digging in at the corners and deformational response.

The point to be made here is that when slip occurs there
is an associated rotative effect and the subsequent deformational behaviour will depend largely upon whether constraints for the surrounding material or blocks will or will not allow this rotation to occur.

When a single block is removed from a system of stacked blocks which has been subjected to loading the forces initially induced are upset and a redistribution of these forces especially those surrounding the removed block is required. The shear forces around those blocks surrounding the opening increase and tend to cause these blocks to rotate. This rotation or tendency to rotate induces moments which in turn require the normal stresses across the discontinuities to redistribute. Associated with this rotation is the occurrence especially at the corners of the block for digging into occur.

Two of the consequent effects of the above stress redistributions are -

1. if the compressive normal stresses become tensile when the moments are large enough and in turn if the discontinuities cannot support tension there must be a further redistribution of load,

2. if slip occurs a geometric effect is introduced which results in yet another mode of load redistribution.

The consequence of the above factors is that the continuum theories no longer apply. Yet it may appear that there is still the possibility of using analogue methods incorporating the continuum theories to predict the behaviour of a blocky
or discontinuous material. This last mentioned approach is fraught with many dangers and difficulties and some of these are examined by trying to determine appropriate experimental parameters for use in the analogue theories.

9.3.1 Deformational response

When testing shale samples of different sizes under various loading conditions it soon becomes evident that effective Young's modulus is not an invariant for the material and in fact is quite variable depending upon the size of sample, loading conditions, boundary constraints and the points where the displacements are measured which all could in turn be a function of the macro defects in the sample.

The Urquhart Shale which was the rock examined could intuitively be considered anisotropic. Yet the unconfined compression tests on medium sized samples, 2 in. x 5 in. did not show the rock to be anisotropic whereas small samples, 1 cm diameter x 2 cm long behaved in a manner generally consistent with what would be expected from an anisotropic material. Work on the larger samples showed the material being tested responded in an anisotropic manner but this time not in unison with the intuitively anticipated anisotropy (perpendicular and parallel to the bedding planes) but more in unison with the macro defects. Whether this apparent anisotropy in this case is due to the boundary constraints, loading conditions or observable (macro) defects is not at all clear and further work would have to be done to clarify this. Yet the fact is that an elastic analogue parameter is a variable factor very much
dependent on the size of sample and the imposed testing constraints.

If Young's modulus is derived from a simple uniaxial unconfined compression test and then if constraints are imposed on the sample the resultant apparent modulus becomes ill defined for certain conditions of constraint. Therefore if an effective or apparent modulus is going to be used another ratio defining the relationships between the stresses is introduced namely, \( \sigma_3/\sigma_1 = -\nu \) apparent which is only an experimental parameter.

9.3.2 Blocky model response

Repeatability of the effective moduli obtained from the experiments performed on the blocky models with all the blocks intact shows that good consistency in the construction of the models was achieved. As the block size increased above, 1 in. square, so did the magnitude of the effective moduli increase. This increase appeared in the main to be due to the lesser number of discontinuities across which the deformations were measured when the larger blocks were encountered. That is the effective moduli in these particular experiments were a measure of the joint defects in the models.

When an opening is created in the blocky models the effective moduli are drastically altered. If the displacements are measured from the hanging and foot walls of the opening the resultant effective modulus is reduced for a vertical load while if the displacements are taken from the abutments the modulus is increased. This increase appears to be dependent in the main on the horizontal confining loads.
Many characteristics can be observed when examining Chapter 5, but the main point to be stressed is that the effective moduli are not only load path dependent but also very much position dependent. That is the moduli vary in the area surrounding the hole.

Besides recording the displacements as the loads were applied they were also noted as the loads were reduced and then subsequently recorded in graphic form, Appendix E. As expected the moduli reduced as the loads were reduced and at first they were very high and then as the loads approached zero the moduli attained their lowest value of the whole cycle. It becomes obvious here that the deformational responses for the loading and unloading cycles are different and that the load and stress distributions would correspondingly be different. On some occasions, (experiments $\beta_2$, $\gamma_{31}$, $\gamma_{32}$, $\mu_5$), as the loads were reduced the displacements still increased. The reason for this is due to the interaction of the blocks as the loads are decreased. This last mentioned action becomes even more evident when multiple openings are examined because the interaction of the openings greatly affect the directions in which the displacements will most likely occur.

An examination of Tables 5 and 9 will show quite clearly that the moduli for a blocky material with an opening in it is a variable parameter even if only the load increasing part of the deformational response is considered. Consequently the use of the elastic analogue method of analysis becomes most difficult to perform because of -
1. the variable nature of the effective moduli, and
2. the difficulty in determining these moduli,

9.4 STRESS DISTRIBUTION

When the blocky models are loaded the response passes through various phases of deformation and as this happens a redistribution of induced loads occurs. If slip does not take place the model behaves as if it was elastic which, in this instance, is precisely what it is. As slip occurs there is the consequent redistribution of stress due in the main to geometry changes and rotations caused by induced moments. Now if a certain load level is attained without the occurrence of slip the stress distribution would be that of an elastic material. However if slip occurs while this specified load is being attained the stress distribution is now load path dependent. Consequently, in all the experiments performed an incremental load approach was adopted in that the stress patterns (namely isochromatics) were recorded after each increment of load.

An examination of the developing isochromatic pictures, Chapter 6, Figures 9.1 and 9.2, show that for the case of increasing applied loads the mechanisms of slip and rotation control the developing stress patterns. During these experiments two features were consistently observed -

1. as the loads were increased the stress patterns would develop and distribute throughout the stress field in a continuous manner until an equilibrium state was attained,
FIG. 9.2a

2" x 1" blocks.
FIG. 9.2b

3" x 2" blocks.
2. at various load levels slip occurred with the consequent and immediate redistribution of stress.

9.4.1 Single opening

Figures 9.1 and 9.2 show two single openings with different aspect ratios subjected to an incremental loading sequence. The main point to note is that as the load is increased for the opening with an aspect ratio of 3 tension in the hanging wall block (tension isochromatic of fringe value one) is developed; whereas for the opening with the aspect ratio of 8 no tensile fringes are developed at all. Arching of the load in the more slender opening is also clearly evident in that as the vertical load is increased the load is thrown onto the abutments and loads within the arch are in fact reduced.

It is easy to show (Denkhaus, 1958) that arching of the stresses occur even for a continuum but here the action is mechanistic in that slips and rotations occur and this has been shown by measuring the deformations which occur, Chapter 5. Here the blocky mass acts as a structure in that those areas which have been stiffened by an increase of confining load or normal load across the joints take up more load as it is applied than those areas which are of less stiffness because of a reduction in the confining or normal load across the joints. Even though there are differences in the resulting stress patterns for the openings of different sizes and shapes, the basic mechanisms which occur repeat themselves and the differences are one of degree rather than different types of mechanism.
As stated above the blocky mass appears to act as a structure where the basic modes of load transfer between the blocks are thrust shear and moment. When slip occurs the blocky mass appears to carry and transfer its load by what could be termed a mechanistic arching and/or mechanistic beaming action. At times if the confining or normal loads across the joints are of sufficient magnitude to prevent slip the material acts as a continuum.

9.4.2 Models preloaded and blocks subsequently removed

As the blocks were removed the hammering action induced dynamic pressure waves which caused some areas to momentarily relieve the confining load and therefore allow the formation of gaps, Figure 9.3. Figures 9.4 and 9.5 show the development of the stress patterns as the blocks are removed. The basic mechanisms are much the same as for the post loaded case except the mechanistic action appears to be more accentuated especially the mechanistic beaming action, Figure 9.4. Explanation for this lies in what was stated above, namely that the dynamic shock waves momentarily allow the blocks to slip more and therefore the blocks more readily develop the mechanistic arching and beaming actions. This was corroborated by performing experiments where the preloaded blocks were removed with a minimum amount of disturbance, Figure 9.4 and then repeating the experiments where the block had to be hammered out, Figure 9.5.

9.5 BASIC MECHANISMS

Many times in this thesis it has been stated that the mechanisms of slip and rotation cause the loads to
FIG. 9.3
1½" sq. blocks pre-loaded
1/2" sq. blocks pre-loaded & forcibly extracted

FIG. 9.4
$1 \frac{1}{2}''$ sq. blocks

FIG. 9, 5
distribute in such a way that either tensile stresses or effective tensile strains readily develop within the blocks. This characteristic is most important to recognise because the strength and stability of a blocky mass is inextricably tied up with this development.

Though not examined in depth, but an approach offering great insight into the strength and stability of an opening, is to allow these blocks which develop either tensile stresses or strains to fail and then follow the subsequent load redistribution. It is felt that what would be a highly redundant structure to start off with would become statically determinate just before collapse, Section 2.8. This approach would require careful experimentation but the techniques developed here could offer much to utilize this particular approach.

9.6 ANALYTICAL ANALYSIS

The various methods of analysing the deformational response or stress distribution in a discontinuous medium were examined in Chapter 2. Knowledge and appreciation of these various theories is a pre-requisite before examining discontinua and its behaviour under load. To date most of the theories have been concerned in the main with failure criteria with little reference to the mode or manner of achieving this state. As is well known if the intact material has a stable deformational response that is it does not strain soften, this approach may be accepted. There are to the author's mind three very important situations where this approach is very dangerous, namely —
1. the intact material may strain often,
2. progressive failure of the blocky structure may occur,
3. though the material may not strain soften the geometry change may cause the resultant deformational response to soften.

If these characteristics are evident it then becomes very important to know how and where the dangerous stresses, be they compression or tension or dangerous strains, occur.

With this in mind the result of this thesis shows the continuum approach is inapplicable.

The approach where the individual units making up the discontinuum are combined together (systone) to give definition to stress or strain is shown to be suspect where moments across the joints, hence stress gradients, have to be considered.

Nevertheless if the blocky system is considered a structure in which thrust shears and moments are transmissible the system then becomes one in which the load distribution and deformational response is attainable. The finite element solution suits this approach very well indeed as shown in Chapter 7. There are still many problems to be overcome in this numerical approach as is stated in the aforementioned chapter.

Some of the immediate problems confronting the numerical method at the moment are -
1. The elastic response of the blocks should be included in the analysis which at the moment considers the blocks as rigid.

2. The size of the elements making up the joints should be small enough to pick up the development of tension due to moments transmitted across the joints.

3. The mode or way of coming back onto the response curve should be thoroughly examined because at the moment this makes the load distribution independent of the load path which is, by experiment, not the case.

4. The geometry changes which occur in the physical model should be incorporated into the numerical model.

CONCLUSIONS

The experimental techniques developed have shown that photoelasticity is a viable method for examining the stress distribution in a discontinuum. There are nevertheless many factors which still require careful consideration. Primarily an improved method of extracting more accurate photoelastic data is required as this will allow better correlation with numerical and practical situations. It has been shown that this is possible with a slight improvement in the experimental techniques.

In a discontinuum the mechanisms of slip and rotation are most important in controlling the subsequent load distributions and eventually the ultimate collapse. The aforementioned mechanisms readily induce either or both a tensile stress or effective tensile strain within the blocks. This, it is anticipated, will have a basic
effect on the stability of the discontinuum. Another important factor related to the stability of the discontinuum is that of constraints. Here it has been shown that where constraints have been removed the mechanism of rotation readily occurs.

The use of analogue techniques has been shown to be most restrictive in that the determination of effective moduli is environment dependent and their determination requires model techniques rather than testing a so called representative sample. This statement applies where the block size to opening size is relatively large. There is definitely an element of homogeneity in the values of the effective moduli (\(\frac{3}{9}\) sq. block tests) when all the blocks are intact, this however is not necessarily the case when an opening is created. Consequently to investigate this latter point much larger block models are required so that the effect of block size to opening size can be studied in greater detail.
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APPENDIX A.

The basis of this programme is set out in section 3.7. After generating the co-ordinate values for the particular block size, the relevant parameters are compiled. Then the fictitious loads (PLOAD) are evaluated using an iterative procedure or at times by matrix inversion. The fictitious loads are then used to evaluate the boundary loads acting on the block. A listing of one of the programmes using the iterative procedure is enclosed.

For Listing refer Appendage
APPENDIX B

PHOTOELASTIC MATERIALS

Desirable Properties of Photoelastic Material

The desirable features of a photoelastic material are -

1. The material should exhibit linear characteristics with respect to -

   (a) stress-strain properties,
   (b) stress-fringe-order properties,
   (c) strain-fringe-order properties.

When the prototype material exhibits a non-linear stress-strain property (some types of rock) the model material should have a similar characteristic. In the study of rocks this could give scope to many materials which are generally considered unsuitable because of their non-linear deformational response.

2. The material should exhibit a high modulus of elasticity so as to inhibit excessive deflections which could change the boundary conditions. A high ultimate strength is also desirable.

3. the birefringence of the material should be sensitive in that it has a low material fringe value. That is the stress per fringe per unit model thickness.
4. The material characteristics should not change with small changes in temperature.

5. Creep should not be excessive. This however can be overcome if the material has linear viscous-optic characteristics and the calibration of the material is determined with respect to time.

6. Time edge effect should be absent. It has been established that time edge effect is caused by the diffusion of water vapour from the air into the material, or from the material into the air. Hence a material which will allow moisture diffusion at a reasonable rate is desirable.

7. The material should be transparent to the light employed in the polariscope. When a lens polariscope is being used this requires a high grade (glass) finish to the model surface.

8. The material should be homogeneous and possess both mechanical and optic isotropy. Cast materials generally possess these characteristics while extruded plastics could be anisotropic.

9. The material should be free from residual stresses. This is a condition which controls the construction of the moulds used in casting the material.

10. Additional requirements are that the material be easily machinable and not prohibitively expensive.
Epoxy Resins

Epoxy resins belong to a group of thermosetting resins which change irreversibly under the influence of heat from a flexible and soluble material into one which is infusible and insoluble through the formation of a covalently crosslinked bond. The production of this crosslinking, termed curing, is performed by many types of materials, however the two additives (curing agents) considered are the acid anhydrides and amines.

The reactions of the basic monomer with amine additive involves opening the epoxide ring to give a B ‑ hydroxyamino linkage —

\[
\text{Epoxide ring} = -[\text{O} \begin{array}{c} \text{C} \end{array} \begin{array}{c} \text{O} \end{array} \text{0 CH}_2 \text{CH} \text{CH}_2]_x^- \]

plus

\[
\text{Amine} = \text{N H}_2 (\text{C}_2\text{H}_4\text{NH})_2 \text{C}_2 \text{H}_4 \text{N H}_2
\]

gives

\[
\text{Hydroxyamino linkage} = - \text{C H}_2 \text{C H C H}_2 + \text{R N H}_2
\]

Acid anhydrides crosslink by reacting through esterification of the secondary hydroxyl groups on the epoxy resin and on the epoxide ring.

It has been determined that the photoelastic properties of a polymerised material is more or less independent of the basic epoxy used, but that the curing agent is the
significant modifier.

Of these two curing agents the acid anhydrides have been considered superior in the production of a photoelastic material because the exothermic reaction is such as to allow a relatively large casting (hence model) to be obtained. The exotherm of the amine curing agent has restricted its general use except in the production of relatively small models (less than 500 gms) in the form of sheets.

The figure of merit is the ratio of Young's Modulus of Elasticity to material fringe value and is an indirect factor assessing the material's ability to produce a significant optical stress pattern without excessive deformation. Hence generally the higher the figure of merit the more suitable the material is for stress analysis.

It can be seen that generally the amine cured resins have a figure of merit three to four times greater than anhydride cured resins and other materials such as C.R. 39 and Catalin. Therefore it was decided to develop casting techniques using Araldite D and Ciba Hardner 951 (amine type).

**Araldite 'D' Casting Resin**

Araldite 'D' (Listed by Ciba as C Y 230) is a solvent-free, liquid epoxy resin modified with a plasticizer to assist its application as a casting material.

The curing agent, Ciba Hardner 951, is reported in the literature as being a triethylene tetramine. This is a viscous yellow liquid, soluble in water and has a boiling
point of 278°C. The specific gravity is 0.982.

Initially experiments were performed using the proportions given by Hattersley (1964) however as larger castings required greater quantities of resin the proportions of mixing had to be modified so as to reduce the heat generated during polymerization. If this heat is excessive, the casting will bubble and crack and in this state the material is said to be degraded.

The resin and hardner polymerizes at room temperature and if the exotherm is properly controlled the heat produced can assist in the removal of casting stresses and improve the physical properties. It has been observed that if the temperature of the mix is ever raised to approximately 100°C the casting changes from a light yellow colour to amber. This has been established as due to the breakdown of the amine compounds present, giving off dark coloured nitrates.

Four important factors are considered if optically good castings are to be produced, namely -

Mix Proportions

The proportions of curing agent (hardner 951) to the monomer (Araldite 'D') must be carefully controlled.

The proportions used varied from 8 p.p.h. (parts per hundred of resin) to 10 p.p.h.

The less hardner used the less the exotherm produced. Unfortunately the modulus of elasticity also reduces in
magnitude as the hardner proportion is reduced. Eight parts per hundred is considered as the minimum hardner content which allows complete polymerization. The general criterion used was that the greater the mass of Araldite 'D' used the less hardner added.

**Bubble Formation**

Bubble formation in the resin has been found to be due to a number of causes, most of which can be eliminated by careful mixing and pouring methods.

The most troublesome cause of bubble formation is that due to moisture. This moisture has been found to come from three sources -

(a) moisture in the material supplied commercially,
(b) moisture adhering to the container in which the material is poured,
(c) moisture in the atmosphere.

It is observed that while mixing the Araldite, before the hardner is added, and the temperature of the mix is lowered (40°C to 35°C) a cloudiness appears in the mix. These are moisture droplets which do not come out on casting.

Moisture in the supplied material is eliminated by heating the resin in the oven at 90 - 100°C for a period between 15 - 24 hours. The mixing containers are carefully dried and the mixing is performed in an environment of inert gas, namely CO₂ obtained from dry ice.
**Mixing Heat**

The viscosity of Araldite 'D' resin, at room temperature, prevents satisfactory mixing. This is because bubbles are readily formed (mixing and pouring) and the uniform distribution of the curing agent in the period of 4 - 5 minutes is most difficult to achieve. Consequently it has been established that the temperature of 35°C is the minimum at which mixing should be performed. For small castings a higher minimum temperature can be used, namely up to 45°C. This is because of the lower total exotherm produced by the smaller castings.

To prevent different rates of polymerization (hence defects) in the casting it is essential that the heat be evenly distributed throughout the mix before the hardner is added. The following is found to be a good means of achieving this even heat distribution.

The Araldite 'D' is heated to 60 - 70°C and then stirred continuously until the temperature of 35 - 45°C was reached. The reason for this requirement is that the resin is a very poor conductor of heat and makes it difficult to achieve uniformity of the viscosity of the resin. Consequently when the hardner is added, difficulty in distributing the hardner is experienced and uneven polymerization in the mix results in optical defects. These defects can be seen in the form of 'folds' or 'swirl'.

**Impurities**

The resins which are supplied commercially are not free
from contamination. This is generally of no consequence when the resins are applied commercially. The contamination is in the form of dust particles, moisture and higher molecular weight material. For good optical work it is found necessary to filter the material before use.

The Araldite 'D' was heated for 4 - 5 hours at 80 - 100°C and subsequently filtered through glass wool held in a filter funnel. The filtered material was allowed to cool to enable the higher molecular weight material to settle, in the form of a gel, to the bottom of the beaker. The filtered material was then decanted off.

This heavier gel is also a contributing factor to the formation of 'folds' or 'swirl'.

Mould Aspects of Casting

The term casting here relates to the mix of Araldite 'D' and hardner.

To obtain a stress free casting the mould should be constructed in such a way as to satisfy the following requirements.

Exotherm Control

The material used in making the mould must control the rate of heat dissipation away from the casting. This requires a suitable mould material to be selected and the thickness of the mould chosen to fulfil this aforementioned requirement.
This heat dissipation is perhaps the controlling factor as to the maximum size of casting producible. Epoxy resin is itself a bad conductor of heat and therefore a thick casting will prevent the escape of heat in the central regions.

**Expansion and Contraction Control**

As the casting polymerises it expands slightly and subsequently contracts. Therefore the mould must be so constructed as to allow this movement to take place otherwise the casting will be stressed and hence for high grade work, photoelastically useless.

This slight movement is achieved by using suitable gaskets and mould material.

**Surface Effects**

The excellent adhesive properties of the epoxy resins require that careful attention be paid to the surface preparation of the mould plates. This attention necessitates polished mould surfaces and the use of a satisfactory release agent.

A suitable release agent termed Releasil 4 made up in the proportions:-

- Silicon Grease 33% of total weight
- Toluene 67% of total weight.

The commercial product toluol is inadequate to use as a solute as it contains too many impurities.
The use of waxes to polish the mould is not recommended as the electrostatic changes set up attract dust particles to the surface. Brasso, which contains Oleic acid, has been found suitable as a polisher.

Sheet Castings

For sheets up to \( \frac{3}{8} \) in thick a good armour plate glass, \( \frac{1}{4} \) in thick, is used for the mould plates. Ordinary glass was found to contain too many micro cracks which caused the adherence of the casting irrespective of the releasil agent used. Sheets up to \( \frac{1}{2} \) in thick were cast successfully using Chromium plates, \( \frac{1}{4} \) in mild steel, for the mould. This allows a greater dissipation rate of heat than would glass.

The procedure used for all sheet casting was as follows -

1. The clean dry mould plates, being either glass or metal, were polished with Brasso. The polish was allowed to dry before polishing with a soft cloth.

2. Releasil 4 was applied to the polished surface in such a manner as to give an even distribution. The plates were then left in the oven at 40°C for about \( \frac{1}{2} \) hour. This was to remove the striation effect that the releasil would otherwise have on the surface finish.

3. The mould plates were assembled using G clamps and rubber gaskets. The thickness adjusted as required.

4. The filtered resin is pre-heated to 60 - 70°C then allowed to cool to 35 - 40°C (depending upon the sheet
thickness being cast). As the resin is cooling it is stirred with a geared-type stirrer. Care is taken to ensure that the surface of the resin is at no time broken hence precluding the entrapment of air.

5. The curing agent, hardener 951, is added and stirred for a further 4 - 6 minutes. During this time the curing agent must be thoroughly mixed into the resin.

The approximate time taken for polymerization to be initiated, is 15 - 20 minutes from the period when the hardner is added.

6. Pouring into the mould is accomplished by holding the mould at an angle and allowing the mix to run in a continuous flow down the sides of the mould.

The mould is then set vertical and allowed to cure for a period varying from 12 - 24 hours. The vertical position of the mould encourages any entrapped air to rise.
APPENDIX C.

MECHANISTIC FINITE ELEMENT PROGRAMME

In this explanation the nomenclature shown in Figure C.1 is used. Tensile stresses are considered positive and moments which tend to cause closure of the joint in the region of positive x are also considered positive. Strains correspond to the same sign convention just mentioned.

Stresses and strains are computed at the joint centres relative to the local co-ordinates and are the basic parameters used to determine the joint performance.

The joint is made up of four noded rectangular finite element in which the displacement field is based on a Lagrangian interpolation between nodal values. This provides for a linear variation of displacements in both directions with the associated shape functions -

\[ N_i = \lambda_i (1 + x_o) (1 + y_o) \quad i = 1, \ldots, 4 \]

where \( x_o = \frac{x'}{x_1} \quad y_o = \frac{y'}{y_1} \)

From this the relationship between nodal forces and displacements are determined using standard finite element techniques (Zienkiewicz, 1967). Transverse isotropy of the joint is used where the shear modulus is independent of the normal modulus such that \( E_1 v_1 \) is perpendicular to the plane of the joint and \( E_2 v_2 \) and \( G_2 \) in the plane of the joint.
Basically the blocks are assumed rigid and instead of considering the joint as the element the lines connecting the centre of the joint to the centre of the two blocks may be considered as the element possessing the properties of the rectangular joint (Burman, 1971).

**Displacement Transformation**

If an inextensible line, Figure C.2, is given a rotation \( \theta_c \) the resultant displacements are -

\[
\begin{align*}
\{\delta_u\} &= \begin{bmatrix} \cos \theta_c - 1 - \sin \theta_c \\ \sin \theta_c \cos \theta_c - 1 \end{bmatrix} \{x', y', z'\} \\
\{\delta_v\} &= \begin{bmatrix} \sin \theta_c \cos \theta_c - 1 \\ \sin \theta_c \cos \theta_c - 1 \end{bmatrix} \{x', y', z'\}
\end{align*}
\]

Also if the fourth and higher order terms in the power series representation of the trigonometric functions are neglected the result gives -

\[
\begin{align*}
\delta_u &= -\frac{1}{2} x' \theta_c^2 - z' \theta_c (1 - \frac{\theta_c^2}{6}) \\
\delta_v &= x' \theta_c (1 - \frac{\theta_c^2}{6}) - \frac{1}{2} z' \theta_c^2
\end{align*}
\]

and if the rigid body translations are excluded

\[
\begin{align*}
\{u_p\} &= \begin{bmatrix} 1, 0, x' \theta_c^2 \\ -z' + z' \frac{\theta_c^2}{6} - \frac{1}{2} x' \theta_c \frac{\theta_c^2}{6} \\ 0, 1, \frac{\theta_c^2}{6} - \frac{1}{2} z' \theta_c \frac{\theta_c^2}{6} \end{bmatrix} \{u_c\} \\
\{v_p\} &= \begin{bmatrix} 0, 1, \frac{\theta_c^2}{6} - \frac{1}{2} z' \theta_c \frac{\theta_c^2}{6} \end{bmatrix} \{v_c\}
\end{align*}
\]

or

\[
\{f\}_p = | T A | \{\delta_c\}
\]
where \( \{f\}_p \) is the displacement vector of a point on the boundary of a rigid block whose centroid has undergone the rigid displacement given by the vector \( \{\delta_c\} \). The above equations are in global co-ordinates and are transferred to local ones.

If the general boundary point \( p \) is taken as the nodes of the rectangular joint element in turn, a relation between the nodal joint displacements to the global centroid displacements is found in the local co-ordinate system. The local normal strain component due to rotation \( \theta_c \) about the centroid is equivalent to that resulting from a rotation about the mid-point of the longitudinal boundary of the joint, (Burman loc cit).

The above concepts plus those stated in Chapter 7 are incorporated into a finite element programme from which the results discussed in the aforementioned chapter are obtained.
APPENDIX D.

DEFORMATIONAL RESPONSE

From the above experiments the relative deformations of certain blocks in response to the applied load could be determined and the stress distribution at each stage of the response recorded. The graphs of the deformational response of the blocks on which measurements were taken, are given below. Also given are the graphs of relative deformations in the horizontal and vertical directions and the sequence of photoelastic stress patterns obtained for the loading cycle where three, five and seven blocks were removed.

Below is a summary of the testing procedure which sets out the basic techniques adopted in the experiments.

\[
\frac{3}{8} \text{ in square blocks}
\]

Initially, the deformational response of the model with all the blocks in place was determined. The stacking of the blocks was varied, Figures \( \beta_1 \) and \( \beta_5 \) in order to represent two different defect patterns.

In the first case the blocks were stacked in such a way that the joints were continuous in both the horizontal and vertical direction, Figure \( \beta_1 \). The horizontal load was increased in increments of 31.5 lb. to 126 lb. then the vertical load was increased to 252 lb. in increments of 31.5 lb. After this the vertical load was reduced by increments of 31.5 lb.. For each load increment or decrement the displacements of specific blocks, namely two in
the horizontal and two in the vertical plane were read from the dial gauges and recorded.

In the second case the blocks were stacked so that the horizontal joints were continuous and the vertical joints were 50% continuous (John, 1961). The loading sequences and displacements as described in the first case were repeated. In another experiment on the 50% continuous joint model the loading sequence was varied in that both the vertical and horizontal loads were increased by equal increments of 31.5 lb. to 157.5 lb., then subsequent to this the vertical load was increased by the same increments to 315 lb. Subsequent to this again the vertical load was reduced in decrements of 31.5 lb. to 157.5 lb. and then both the vertical and horizontal loads were reduced to zero in equal decrements of 31.5 lb. Throughout each stage of loading and unloading the deformation of specific blocks, Figure $\beta_5$, was recorded.

In the next experiment performed, $K_2$, six blocks were removed, Figure $K_2$, and the dial gauges were located as shown. The same loading sequence as above was executed with the addition however of photographs being taken of the stress patterns from the first loading cycle.

Eighteen blocks were then removed, Figure $K_3$, and with the same dial gauge arrangement as in $K_2$, experiment $K_3$ was performed in a similar manner.

1 in. square blocks

In the first experiment, $\delta_1$, all the blocks were intact and stacked as shown, Figure $\delta_1$. An all round hydrostatic
load was applied to the model in increments of 126 lb. up to 630 lb. Subsequently the vertical load was increased by increments of 126 lb. to 1260 lb. Subsequent to this the vertical load was reduced in decrements of 126 lb. until the hydrostatic load was again 630 lb., then both loads were reduced in decrements of 126 lb.. At each load increment or decrement, the deformational response of specific blocks, is recorded. This cycle was repeated three times.

For experiment δ₂, eight blocks were removed. This pattern has a similar configuration to the Mount Isa Nos. 2 and 5 ore bodies. It should be noted that the dial gauges are located in the same positions as for experiment δ₁. The same loading and recording sequence as experiment δ₁ was repeated; however, in this case photographs were taken of the changing stress patterns.

Experiment δ₃ was the same as experiment δ₂ except, in this instance, the dial gauges were located on different blocks, Figure δ₃, and photographs of the stress patterns were not taken.

Six blocks were removed for experiment δ₄. Here again the loading and recording procedures were the same as for experiment δ₁. Only three photographs were taken in this experiment, namely when the hydrostatic load was 630 lb. and then when the vertical load had been increased to 1260 lb. and finally when the vertical load had been reduced to give again the hydrostatic load of 630 lb.

In experiment δ₅, 10 blocks were removed and the dial gauges were arranged as in Figure δ₇. The same loading,
recording and photographic procedures as experiment 5 were repeated.

For experiments 6 and 7, 14 and 19 blocks were removed respectively to give the opening configurations and dial gauge placements, Figure 5. In these last two experiments, however, no photographs were taken.

It should be noted that in experiments 5, 5, 6 and 7, the displacement of the bottom block in the vertical plane, namely D, could not be measured because the restraining forces on this block were not of sufficient magnitude to retain the block in position. In experiment 7, enough restraining forces had developed at a hydrostatic load of 126 lb.

2 in x 1 in Rectangular Blocks

The first experiment, namely 6, in this series was performed with all the blocks intact and stacked as shown in Figure 6. Both the horizontal and vertical loads were increased by equal increments of 126 lb. to give a hydrostatically increasing load up to the value of 630 lb. From this load on, only the vertical load was increased in increments of 126 lb. to 1260 lb. Displacements of the specific blocks as depicted were recorded for each increment and decrement of load.

Experiment 7 had seven blocks removed, Figure 7, in such a way as to give a cross sectional void profile similar to that of Nos. 2 and 5 ore bodies. Incremental loads as in experiment 6 were applied in the same way and the deformation characteristics recorded as before.
Photographs of the loading cycle were also taken of the developing stress patterns. Precisely the same procedure was carried out for experiment $\mu_8$ except in this case only three blocks were removed as shown in Figure $\mu_8$.

From all these experiments the displacements of the specific blocks at certain loads were graphed. Also depicted were the relative displacements in the vertical and horizontal directions as shown.

3 in x 2 in Rectangular Blocks

In the first experiment, $v_5$, all the blocks were intact and arranged as shown in Figure $v_5$. The dial gauges were located on specific blocks as shown. In the first part of the experiment the vertical and horizontal loads were increased by increments of 126 lb. to an all round load of 252 lb. Thereafter the vertical load alone was increased by increments of 126 lb. to a maximum load of 1260 lb. Once the maximum load was reached this procedure was reversed. At all stages of the loading and unloading sequences the readings from the dial gauges were recorded. The above procedure was repeated for the second part of the experiment except in this case the horizontal and vertical loads were increased to 630 lb. and then the vertical load was increased to 1260 lb. by the same increments as before. Here again a careful record of the deformational characteristic of specific blocks, Figure $v_5$, was compiled.

Experiment $v_6$ was precisely the same as $v_5$ except, in this instance, three blocks were removed and the dial
gauges were located as shown in Figure \( v_6 \).

Loading and recording procedures, as were performed for experiment \( v_5 \) were repeated for experiments \( v_7 \) and \( v_8 \), although in this instance 5 blocks and 2 blocks were removed from models \( v_7 \) and \( v_8 \) respectively. The dial gauges for these models were located as shown.

Graphs of the block displacements versus applied loads and relative displacement in the vertical and horizontal directions versus applied loads are given.

Block models with joints inclined to the direction of the applied loads.

\( \frac{1}{2} \) in Square Blocks

In the first, \( \beta_2 \), of this series of experiments the continuous set of joints were placed at 60\(^\circ\) to the horizontal while the discontinuous set, \( S = 50\% \) (John, 1962) were placed at 150\(^\circ\) to the horizontal, Figure \( \beta_2 \), with all the blocks intact. As will be observed the dial gauges were located so that the relative displacements in the horizontal and vertical directions could be measured. The loading sequence was such that the horizontal load was increased in increments of 31.5 lb. to 157.5 lb. then the vertical load was increased by the same increments to a magnitude of 315 lb.. At each stage of applying an incremental or decremental load, readings of the dial gauges were taken and recorded. This cycle of loadings was repeated twice for this particular experiment.
For experiment $\beta_3$ the continuous set of joints were placed at $30^\circ$ to the horizontal while the discontinuous set was placed at $120^\circ$ to the horizontal. Again the dial gauges were located on blocks which allowed the horizontal and vertical relative displacements to be observed, Figure $\beta_3$. The same loading and recording technique used for the previous experiment, $\beta_2$ was repeated.

1½ in Square Blocks

Stacking of the 1½ in block model, for this series of tests, was arranged as shown in Figure $\gamma_5$. In this instance all the blocks were intact and the continuous joint set arranged at $60^\circ$ to the horizontal while the discontinuous joint set lay at $150^\circ$ to the horizontal. Placement of the dial gauges in this series of experiments, except for $\gamma_{11}$, was parallel and perpendicular to the continuous set of joints, Figure $\gamma_5$.

In the first experiment of this series, namely the load in the vertical and horizontal direction increased by equal increments of 126 lb. until the hydrostatic magnitude of 756 lb. was attained. Once the maximum load was achieved the load was reduced by decrements of 126 lb. Three cycles of loading and unloading were performed in this test. At each stage of applying the incremental loads the dial gauge readings were observed and recorded.

In the next two experiments, namely $\gamma_6$ and $\gamma_7$, three blocks and then five blocks were removed, Figure $\gamma_6$. The main purpose of these two experiments was to observe the effects of pore size on the stiffness within the
region of concern and the deformational response of the blocks making up the regional area containing the pores. Loading and unloading of the model was again hydrostatic and in increments of 126 lb., and the maximum load attained was 882 lb. Again a record of the dial gauge readings at each stage of the loading and unloading was compiled. Experiment $\gamma_{10}$ was similar to experiment $\gamma_{7}$ except in this case the dial gauges perpendicular to the continuous joint set were placed differently, Figure $\gamma_{10}$. Also experiment $\gamma_{8}$ was the same as experiment $\gamma_{6}$ but here the deformational characteristics of the hanging wall blocks were examined more closely.

Another experiment, $\gamma_{11}$, was performed with all the 1½ in square blocks intact but the difference was that the location of the dial gauges were such that the relative displacements in the horizontal and vertical directions could be measured, Figure $\gamma_{11}$.

Photographs of the changing stress patterns in experiments $\gamma_{6}$, $\gamma_{7}$ and $\gamma_{9}$ were recorded.

Extraction of Blocks while model is under imposed load

In the mining situation, material is extracted while in a state of stress. Therefore, a series of experiments were conducted where blocks were removed while the model was in a preloaded state.

The first experiment $\alpha_{3}$ was performed on the 2 in x 1 in model, Figure $\alpha_{1}$, where the continuous joints were at 30° to the horizontal and the 50% discontinuous joints were at 150° to the horizontal. With the load still imposed, the
block was removed by hammering it out. After the block had been extracted photographs of the resulting stress pattern were taken. Further blocks indicated by 1, 2, 3, etc. were removed in the same sequence as the numbers shown and after each block was removed the resultant stress pattern in the model photographed.

Experiments $\alpha_4$ and $\alpha_5$ were performed on 1½ in square blocks, where the continuous joint set was placed at $60^\circ$ to the horizontal and the 50% discontinuous joint set $150^\circ$ to the horizontal. Here again the main objects of the experiments were to observe the changing stress patterns as specific blocks were removed.