Essays on Money, Credit Constraints and Asset Prices

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Declaration

This thesis contains no material that has been accepted for the award of any other degree or diploma in any university. To the best of the author’s knowledge and belief it contains no material previously published or written by another person, except where due reference is made in the text.

Signature ..................

Date .....................
“Monetary theory . . . has esthetic unity born of variety; an apparent simplicity that conceals a sophisticated reality; a surface view that dissolves in ever deeper perspectives.”

Milton Friedman, Preface to *The Optimal Quantity of Money*

“Those of us who were deeply concerned about the danger to freedom and prosperity from the growth of government, from the triumph of welfare-state and Keynesian ideas, were a small beleaguered minority regarded as eccentrics by the great majority of our fellow intellectuals.”

Milton Friedman, Preface to *Capitalism and Freedom* (1982)
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1

Introduction

This thesis consists of three chapters which were written independently. Each chapter answers different questions. But they share a single target: improving the ability of flexible price models in explaining volatile asset price movements. Standard real business cycle models have difficulties in matching asset price volatilities observed in the data. In the literature, usually this problem is solved by introducing the sticky price assumption. This thesis attempts to produce higher asset price volatilities with flexible price models by introducing frictions in the money and credit markets.

This thesis has two original contributions. The second chapter, “segmented money market, credit constraint and asset prices”, is the first in the literature to integrate a segmented market with a credit constraint into a dynamic stochastic general equilibrium model within a flexible price framework. It provides a competing model to explain high asset price volatilities against the popular sticky price models.

The third chapter, “macroeconomic effects of leverage cycles”, is the first in the literature to endogenize the loan-to-value ratio of a Kiyotaki-Moore style credit constraint in a dynamic stochastic general equilibrium model. An endogenous loan-to-value ratio not only produces more volatile asset price movements, it also explains the pro-cyclical

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1See Rouwenhorst (1995) for a more detailed discussion on this feature.

2The Kiyotaki-Moore style credit constraint is an ad hoc assumption on the ability of borrowers to finance for their expenditures. It states that borrowers can at most borrow a fraction of the present value of collateral assets. For example, assume that borrowers use their housing asset $H_{t+1}$ as collateral, their borrowing amount $B_{t+1}$ should satisfy $B_{t+1} \leq \theta E_t \left[ \frac{P_{h_{t+1}} H_{t+1}}{R_{t+1}} \right]$, where $P_{h_{t+1}}$ and $R_{t+1}$ are housing price and discounting interest rate at time $t + 1$ respectively. $\theta$ is the fixed loan-to-value ratio.
Flexible price models have two difficulties in producing significant responses of asset prices to monetary shocks. The combination of segmented market with credit constraint in the second chapter solves the two difficulties simultaneously. The first difficulty is that, in flexible price models (like standard cash-in-advance models), monetary shocks tend to have negative effects on production due to the inflation tax. As a result, asset prices fluctuations are mild in response to monetary shocks. A mechanism needs to be introduced so that production increases in response to positive monetary shocks without resorting to the sticky price assumption. The second difficulty is that, even a mechanism as described above had been discovered, considering the fact that the money injection quantity is negligible compared to the national wealth, it is not convincing that such minor disturbances are capable to produce significant real effects in a stable macroeconomic environment, in which monetary shocks are small.

The idea of uneven money injections, as proposed by Friedman (1968) and the Austrian school economists like Hayek (1969), can solve the first problem. The segmented market models, like Christiano & Eichenbaum (1995), have rigorously included this idea into mainstream macroeconomic models. Due to market segmentations, the agents in the economy receive heterogeneous amount of money injections. Relative prices of commodities preferred by those who receive more money injections will be pushed up due to higher demand, which incurs more production of these commodities. If, and it is a quite realistic assumption, producers (entrepreneurs) receive more money injections, since they prefer to buy capital goods, the prices of capital goods (asset prices) will be pushed up and the accumulation of capital will be increased following the same logic. With more capital available, production in this economy increases. On the contrary, in standard cash in advance models which assume helicopter drop of new issued money, agents receive the same amount of money injections. As a result, monetary policy can only affect the general price level, which reduces production through inflation tax.

However, as suggested by Edmond & Weill (2008), segmented market models are unlikely to produce quantitatively significant effects within reasonable market segmentations. Therefore, it is not surprising that segmented market models turn to the sticky

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3In the literature, the loan-to-value ratio is treated as a constant or an exogenous shock process. But in the real world, loan-to-value ratios are endogenous (determined by banks) and pro-cyclical. See the introduction part in chapter 3 for detail.
price assumption for help. For example, the influential paper by Christiano, Eichenbaum and Evans (2005) integrates sticky price with segmented market in a unified model.

The second difficulty is actually about an amplifying mechanism, which propagates, amplifies and prolongs the initial real effects caused by monetary shocks. And since Kiyotaki and Moore (1997), it is well understood that credit constraints are capable to amplify the effects of initial shocks. The amplifying effects incurred by credit constraints are referred to as financial accelerator in the literature.

Combining segmented market with credit constraint can solve the two difficulties simultaneously. Uneven money injections caused by market segmentations lead to higher asset prices if producers receive more money. The raised asset prices relax the credit constraints, which triggers the financial accelerator mechanism. The financial accelerator then propagates, amplifies and prolongs the initial real effects caused by monetary shocks. This combination provides a competing model in explaining the volatile asset price behaviors within a flexible price framework against the popular sticky price models.

Sticky price models are popular, especially among policy makers. However, the sticky price assumption does not have a solid micro foundation (Williamson (2010)). There are other convincing explanations for the observed sticky price phenomena. In addition, in developing countries, general prices (including wages) are usually much more flexible than in developed countries. Therefore, it is necessary to develop competing flexible price models against the popular sticky price models in explaining volatile asset price movements. There are at least three economic meanings. First, flexible price models have very different policy implications. Since flexible price models are well micro-founded, theoretically, their policy suggestions should be preferred if they can offer competing explanations of volatile asset price movements. Second, in practice, if flexible price models are also capable to explain volatile asset price movements, policy makers should ponder which policy to take rather than just follow the suggestions of sticky price models. Finally, in developing countries, flexible price models which

\[ \text{For observed evidences of sticky prices, see Taylor (1999) and Klenow Malin (2010) for comprehensive surveys. Alchian (1969) and Barzel (1997) convincingly argue that transaction costs could lead to equilibrium sticky prices (the sticky price assumption assumes market disequilibrium.). Alchian and Allen (1974) further suggests that transaction prices are much more flexible than menu prices.} \]
1. INTRODUCTION

are capable to explain volatile asset price movements should be considered first when examining the effects of monetary policy.

In chapter 3, the endogenized loan-to-value ratio of a Kiyotaki-Moore credit constraint provides another mechanism which improves the ability of flexible price models in explaining volatile asset price movements.

In chapter 3, entrepreneurs are assumed to face undiversifiable idiosyncratic shocks in addition to aggregate total factor productivity (TFP) shocks. If the realized idiosyncratic shock is too low, some firms (borrowers) are unable to pay back the money loaned by banks and become bankrupt. Banks have to auction the collateral capital to get some money back. A bankruptcy cost is assumed so that banks can only receive a fraction of the market value of the collateral asset. When there is a positive productivity shock, the default probability decreases, it is profitable to lend more therefore the loan-to-value ratio is increased. On the other hand, more lending puts more asset at risk, which tends to increase the default probability. The reason is that more available funds for entrepreneurs lead to more investment. More investment reduces the return to capital (law of diminishing marginal return), which increases the default chances. The conflicting effects of increased loan-to-value ratio imply there is an optimal loan-to-value ratio such that new investment opportunity is fully exploited and the expected return of banks is maximized.

The endogenous loan-to-value ratio amplifies the financial accelerator effects of a credit constraint. With a Kiyotaki-Moore style credit constraint, a positive TFP shock increases the demand for asset, which pushes up the asset price. The increased market value of collateral asset relaxes the credit constraint, inducing more borrowing as well as more demand for asset, a reinforcing cycle begins and is referred to as financial accelerator. With an endogenous loan-to-value ratio, in response to a positive TFP shock, the optimal loan-to-value ratio is increased as well, which relaxes the credit constraint further. The increased loan-to-value ratio serves as an extra push within each reinforcing cycle. That is, the optimization behavior of banks amplifies the financial accelerator effects.

Chapter 4 extends the model developed in chapter 3 to include the real estates, the most often used collateral asset in the real world. The extended model aims to explain the effects of an endogenous loan-to-value ratio on housing price behaviors. It shows
that the endogenous loan-to-value ratio model is able to produce much larger housing price volatilities than an exogenous loan-to-value ratio model.
1. INTRODUCTION
2

Segmented Money Market, Credit Constraint and Asset Prices

2.1 Introduction

The Great Moderation has challenged the quantitative flexible price models seriously with one economic phenomenon as summarized by the IMF (2000, p77) that “prolonged built-ups and sharp collapses in asset markets have taken place amidst a decline in consumer price inflation and a more stable macroeconomic environment in most of the industrialized world”. Standard quantitative monetary flexible price models (like standard cash-in-advance models) have difficulties in producing large asset price movements with small monetary disturbances.

The first difficulty is that, in standard cash-in-advance models, monetary shocks tend to have negative effects on production and asset prices due to the inflation tax. A mechanism needs to be found so that monetary shocks can have positive effects on production without resorting to the sticky price assumption. The second difficulty is that, even a mechanism had been discovered, considering the reality that money injection quantities are negligible compared to the national wealth, it is not convincing that

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1I have benefitted greatly from the suggestions and comments of Timothy Kam, Pedro Gomis-Porqueras, Timo Henckel, Junsang Lee, Vipin Arora, Chris Edmond and Craig Burnside. I also received very helpful comments in seminar presentations at The Australian National University and the 2011 PhD conference at Queensland University.
such minor disturbances can produce significant real effects in a stable macroeconomic environment.

The idea of uneven money injections proposed by Friedman (1968) and the Austrian school economists like Hayek (1969) can solve the first problem. The segmented market models, like Christiano and Eichenbaum (1995), include this idea in rigorous mainstream macro models. However, as suggested by Edmond and Weill (2008) in their survey paper, segmented market models are unlikely to produce quantitatively significant effects within reasonable market segmentations. Therefore, it is not surprising that segmented market models turn to the sticky price assumption for help. For example, the influential paper by Christiano, Eichenbaum and Evans (2005) includes the segmented market into a new Keynesian dynamic stochastic general equilibrium model.

The second difficulty is actually about an amplifying mechanism, which propagates, prolongs and amplifies the real effects caused by monetary shocks. Ever since Fisher (1933), the amplification mechanism caused by financial market frictions is well understood by economists. However, it is Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) who first include this mechanism into rigorous macroeconomic models. Financial market frictions have been introduced into the sticky price models in no time. The financial accelerator in Bernanke and Gertler (1989) is introduced into a sticky price model by Bernanke, Gertler and Gilchrist (1999). The Kiyotaki-Moore style credit constraint is introduced into a sticky price model by Iacoviello (2005) too. Their simulations show that with the help of sticky prices, volatile asset price behaviors can be largely explained.

There are attempts within the framework of flexible price models too. Cordoba and Ripoll (2004) introduce a Kiyotaki-Moore style credit constraint into a standard cash-in-advance model (essentially the third benchmark model in this chapter). Gust and Lopez-Salido (2011) introduce endogenous segmented markets into a standard cash-in-advance model (essentially the second benchmark model in this chapter). Andolfatto and Williamson (2015) introduce endogenous segmented markets into a third-generation money search model. However, all these attempts are insufficient to solve the two difficulties described above simultaneously.

This chapter integrates segmented market (money is injected unevenly due to market segmentations) with credit constraint into an otherwise standard cash-in-advance
model to solve the two difficulties simultaneously. The model developed in this chapter is based on Christiano and Eichenbaum (1995) and Iacoviello (2005). The mechanism of the model developed in this chapter can be summarized in the diagram below:

\[ M^\uparrow \Rightarrow i^\uparrow \Rightarrow I^\uparrow \Rightarrow q^\uparrow \Rightarrow B^\uparrow \Rightarrow I^\uparrow \Rightarrow q^\uparrow \cdots \]

A positive monetary policy increases the supply of loanable fund, therefore lowers the interest rate. Due to money market segmentations, only entrepreneurs have access to money markets. With lower financing cost, entrepreneurs invest more on capital accumulations. The higher demand of capital pushes up the price of capital, which also serves as collateral for loans. Higher asset price relaxes the credit constraint, and leads to even more borrowing and investment. The financial accelerator starts to function and amplifies the initial effects of monetary shocks. It is therefore possible to produce large asset price movements within a flexible price model.

The simulations show that the combination of segmented market and credit constraint is quite successful. It produces much larger asset price movements in response to small monetary shocks than a standard cash-in-advance model or a model with only credit constraint or segmented market.

The rest of this chapter proceeds as follow: the second section gives a literature review on uneven money injection and models with credit market frictions. Section three presents the main model. Section four reports the simulation results. The last section summarizes this chapter.

### 2.2 Literature Review

In this section, I give a highly selective literature review on uneven money injection and credit constraint. The idea of uneven money injection dates back to as early as Hume (1752) and is first developed as a rigorous segmented market model by Lucas (1990). For a comprehensive survey on segmented market models, see Edmond & Weill (2008). The idea that credit market frictions lead to self-reinforcing cycles dates back to Fisher (1933) and is first developed as rigorous macroeconomic models by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997). For a comprehensive survey on models of financial market frictions, see Brunnermeier, Eisenbach and Sannikov (2012).
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

2.2.1 Uneven Money Injection

In standard real business cycle models with money, money injection is assumed to be of helicopter drop and therefore even. Even money injections only lead to the increase of general price levels and have no real effects. On the other hand, uneven money injections disturb the relative prices and therefore have real effects. Friedman (1994, Chapter 2) gives a vivid example on how uneven money injections cause relative prices to change. In the gold rush era in Melbourne, the prices of luxurious goods increased dramatically as gold miners, the receivers of injected money (gold), preferred an extravagant life.

The relative price between consumption good and capital good could be altered by uneven money injections too. In a segmented money market, if only entrepreneurs have the access to money injections, and since entrepreneurs prefer to buy capital good, money injections will cause the relative price of capital good compared to consumption commodity to increase. The increased price will induce more production of capital goods. This increases the future capital stock. With more capital available, production will be increased. A modified quantity equation of money, \( MV = P_1C + P_2I \), is helpful to explain the mechanism described above. When entrepreneurs receive more of the new issued money than consumers, the nominal price of capital good \( P_2 \) increases faster than the nominal price of consumption good \( P_1 \). As a result, capital good becomes more expensive relative. A higher price of capital leads to more production of capital good. Money injections therefore have real effects.

It is well recognized that Hume (1752) first stated the idea of the quantity theory of money in his influential essay Of Money. His argument that the increase of money supply eventually (that is, in the long-run) only leads to the increase of nominal prices is well verified. However, in that essay, Hume also elucidates two channels through which money injections have real effects. The first channel is unexpected money injections, and this channel is well explored by Lucas (1996). The second channel is uneven money injections. In his essay Of Money, Hume argues that “when any quantity of money is imported into a nation, it is not at first dispersed into many hands but is confined to the coffers of a few persons, who immediately seek to employ it to advantage. [p38]” Then he discusses the effects of uneven money injections in a simplified economy in which manufacturers first receive new-injected money. With more money, manufacturers employ more workers and the production is increased for a while.
2.2 Literature Review

The second channel had been generally ignored in the literature for a long time. It is the Austrian school (for example, Hayek (1969)) that first gives a central role to uneven money injections in the business cycle theory. As surveyed by Garrison (2005), the Austrian school argues that uneven money injections increase the supply of loanable fund, which lowers the interest rate. Since the financing cost is reduced, investment increases.

Friedman (1968) re-states the idea of uneven money injections under the background of modern monetary system. Friedman argues that money injections are conducted in money markets, and only those investors with access to money markets can receive new issued money. Money injections have two opposite effects: liquidity effects and Fisher effects. Liquidity effects tend to reduce the nominal interest rate as more loanable fund is available with money injections. Fisher effects tend to increase nominal interest rate as inflation is expected to increase. Due to market segmentations, money cannot be dispersed to everyone in the economy in the short run, therefore, liquidity effects dominate Fisher effects, and the nominal as well as real interest rate is decreased. But in the long run, Fisher effects dominate, and only the nominal interest rate is increased. In a sense, the argument of Friedman gives a micro-foundation for the uneven money injection theory of the Austrian school.

Lucas (1990) is the first to develop a rigorous model of liquidity effects. In his model, only firms receive money injections. Firms face a cash in advance (CIA) constraint on paying their wage bills. Money injections relax their CIA constraints, leading to higher labor hiring and production. The novelty of the Lucas model is the large family trick which solves the tractability problem of heterogenous money holdings. It is assumed that at the end of each period, entrepreneurs and households merge as a large family, then money holdings become homogenous to each agent. Christiano and Eichenbaum (1995) develop a delicate model based on Lucas (1990) to handle the tractability problem more easily without explicitly using the large family trick. The main model of this paper is based on Christiano and Eichenbaum (1995) to introduce uneven money injections.

\[^2\text{There is no standard version of Austrian business cycle model. Here is just one of them, and hopefully the most accepted one. It could be interpreted in another perspective: if only the investors had the access to new money injections, money injections transfer wealth to investors from savors. Then the richer investors will choose to invest more.}\]
Another approach to model the liquidity effects is to employ the “inventory-theoretic” analysis, which can be seen as a modern version of the Bamboul and Tobin’s money inventory models. This approach is also called as “endogenous segmented market models” in the literature. For a comprehensive survey, see Edmond and Weill (2008).

### 2.2.2 Credit Constraint

The idea that credit market frictions can lead to self-reinforcing cycles (financial accelerator) can be traced back to Fisher (1933). In modern macroeconomic literature, there are mainly two approaches to model financial market frictions. The first one is costly external finance (costly state verification) approach. This approach is first developed by Bernanke and Gertler (BG, 1989). The second approach is limited enforcement. With this approach, a credit constraint which requires assets pledged as collateral guarantees the enforcement of a credit contract. Kiyotaki and Moore (KM, 1997) first develop a rigorous credit constraint model.

The original BG model is a real over-lapping generation model. Charstrom and Fuerst (1997) introduce the costly external finance mechanism into a computable dynamic general equilibrium model. Then Bernanke, Gertler and Gilchrist (BGG, 1999) extend the model of Charstrom and Fuerst (1997) to include money and sticky prices. The BGG model have become the canonical model of the first approach, and there are a lot of extensions based on the BGG model in the literature. See Brunnermeier, Eisenbach and Sannikov (2012) for a comprehensive survey.

Like the BG model, the KM model is a simple real model without money. Iacoviello (2005) introduces a Kiyotaki-Moore style credit constraint into a sticky price dynamic stochastic general equilibrium model. The main model in this chapter is based on Iacoviello (2005) to model the credit constraint. Following important developments include Liu, Wang and Zha (2009, 2013), Kiyotaki, Michaelides and Nikolov (2011) e.t.c.. See the introduction section of chapter 3 for more literature on credit constraint.

The two approaches have different modelling strategies. The costly external finance approach is more micro-founded because in costly external finance models, the financial contract is optimally determined by financial intermediaries, but in the limited enforcement models, credit constraint is just an ad hoc assumption. However, despite all these differences, it should be stressed that the main mechanism of the two approaches are almost the same. They are usually referred to as financial accelerator models.
2.3 The Model

The modelling strategy differences between these two approaches are similar to that of the two modelling strategies in econometrics. The costly external finance approach is like the structural form model, while the credit constraint approach is like the reduced form model. There are no absolute advantages for either approach. Rather, it is the research targets determine which approach should be used. For the purpose of this chapter, I adopt the credit constraint approach.

2.3 The Model

There are three types of agents in the model: households, financial intermediaries and entrepreneurs. Households are endowed with labor, entrepreneurs are endowed with capital and possess the technology to produce final products and capital goods. Financial intermediaries are owned by households and are the only channel through which to inject money.

![Cash flow Chart](image)

**Figure 1. The Cash flow Chart.**

Left, 1A, Cash flow at the beginning of each period; Right, 1B, cash flow at the end of each period.

At the beginning of time $t$, households allocate their money holdings between the cash reserve for consumption goods $C_t$ and short term savings to financial intermediaries $N_t$. Financial intermediaries collect the short time savings from households and accept new money injections $g_tM_t$ (here $M_t$ is the money stock, $g_t$ is the money growth rate) from the central bank. Financial intermediaries then lend the money collected from households and injected by the central bank to entrepreneurs. Entrepreneurs are
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

assumed to face cash-in-advance constraint for wage bills (working capital). They bor-
rows cash from financial intermediaries to pay for their wage bills to households, as
shown in Figure 1A.

With the short term cash loan from financial intermediaries, entrepreneurs employ
labor from households and combine labor with capital to produce the final products.
Then households use the reserved money to buy consumption good from entrepreneurs.
With this income, entrepreneurs pay back the loans with interest to financial interme-
diaries. Then financial intermediaries pay back households their short term saving with
interests. The above processes are shown in figure 1B.

Table 1. The sequence of events in a given period $t$

At the end of period $t$, both households and entrepreneurs make their decisions
for investment. Since it is assumed that only entrepreneurs have access to investment
technology, households would like to lend their leftover of income to entrepreneurs. It is
assumed that due to financial frictions, households make their lending to entrepreneurs
through financial intermediaries. Financial intermediaries collect long-term savings $L_{t+1}$
from households, then lends long-term loan $B_{t+1}$ to entrepreneurs. The long-term
loan is much more risky than the short-term cash loan, for safety reasons (costly en-
forcement), financial intermediaries require entrepreneurs pledge capital as collateral for
long-term loans. With new long-term borrowing from financial intermediaries and prof-
its, entrepreneurs pay back last period long term loan plus interests $R_{t}B_{t}$ to financial
intermediaries and make decisions new investment $I_{t}$. Finally, financial intermediaries
pay back last period long-term saving plus interests to households.
2.3 The Model

The timing of this model is summarized in Table 1.

At any period, a representative bank has two types of liabilities and assets, as shown in figure 2. However, financial intermediaries in this model are very simple and not micro-founded. Rather, they are more like a modelling strategy to introduce segmented markets and credit constraints. In addition, the short term liability and asset of a representative bank is in cash, and its long term liability and asset is in credit (commodity).

<table>
<thead>
<tr>
<th>Liabilities</th>
<th>Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td>Short-term Savings</td>
<td>Short-term Loans</td>
</tr>
<tr>
<td>Long-term Savings</td>
<td>Long-term Loans</td>
</tr>
</tbody>
</table>

Figure 2. The Balance sheet of a representative bank

The details of the model is presented below. Since the choices of households are standard, I will start with the household’s problem.

2.3.1 Households

The population of households are normalized to be of measure 1. Each household owns one unit of labor, and tries to maximize the lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, 1 - H_t)$$

while faces a cash-in-advance constraint and a flow budget constraint:

$$P_tC_t + N_t = M_t$$

$$P_tw_t H_t + R^c_t N_t + L_t R_t = [M_t - P_tC_t - N_t] + L_{t+1} + M_{t+1}$$
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

where $E_0$ is the expectation operator at time zero, $\beta \in (0, 1)$ is the discount factor, $C_t$ is the consumption at $t$. $H_t$ is the labor supply with wage real $w_t$, and $(1 - H_t)$ is the leisure. The nominal price at $t$ is $P_t$. $N_t$ is the short term saving described above, and part of household’s money holding; $L_t$ is the nominal long term savings. $R^n_t$ and $R_t$ are the nominal gross returns paid by financial intermediaries. Note in equation (2.3), $R_t$ and $R^n_t$ are different, $R_t$ is determined in period $t - 1$, while $R^n_t$ is determined at period $t$.

To normalize the nominal variables, denote $m_t = \frac{M_t}{P_{t-1}}$ as the real money balance at period $t$, $n_t = \frac{N_t}{P_t}$ as the real saving, and $\Pi_t = \frac{P_{t+1}}{P_t}$ as the gross inflation rate.

The households choose consumption $C_t$, labor supply $H_t$, short-term saving $N_t$, long-term saving $L_{t+1}$ as well as money holding $M_{t+1}$ to maximize their utility. The first order conditions are reported in Appendix A, where $\lambda_t$ is the Lagrangian multiplier for households’ income.

2.3.2 Entrepreneurs

The population of entrepreneurs are normalized to be of measure 1 too. Entrepreneurs own capital and make the decisions of investment and labor hiring to maximize their lifetime utility over consumption $C^e_t$:

$$E_0 \sum_{t=0}^{\infty} \gamma^t U (C^e_t)$$

(2.4)

where $0 < \gamma < \beta < 1$, i.e., entrepreneurs are less patient than households. This assumption is to make sure that the credit constraint (2.9) binding around the steady state.

Entrepreneurs face five constraints: the technology to produce the capital, the technology to produce the final products, a Kiyotaki-Moore style credit constraint with capital as collateral and a cash-in-advance constraint to finance for working capital, and finally, a flow-budget constraint.

$$K_{t+1} = (1 - \delta) K_t + I_t$$

(2.5)

$$I_t = \Phi \left( \frac{L_t}{K_t} \right) K_t$$

(2.6)
2.3 The Model

\[ Y_t = z_tF(K_t, H_t) \]  \hspace{1cm} (2.7)

\[ P_tw_tH_t = M_t^c \]  \hspace{1cm} (2.8)

\[ b_{t+1} \leq \theta E_t[q_{t+1}K_{t+1}\Pi_{t+1}/R_{t+1}] \]  \hspace{1cm} (2.9)

\[ Y_t + b_{t+1} = C_t^e + q_tI_t + [w_tI_t - b_t^c] + m_t^eR_t^c + R_t b_t \Pi_t \]  \hspace{1cm} (2.10)

where \( K_t \) is the capital stock at time \( t \), \( I_t \) is the investment, \( \delta \) is depreciation rate of capital. \( Y_t \) is the production at \( t \), and \( z_tF(K_t, H_t) \) is the production function, which uses capital and labor as inputs to produce the final products, and faces a productivity shock \( z_t \). \( b_t = \frac{B_t}{P_t-1} \) is the real long-term loan from financial intermediaries, and \( m_t^e = \frac{M_t^e}{P_t} \) is the real short term loan from financial intermediaries. \( \Pi_t \) is the real price of capital good, or the “Tobin’s Q”. \( R_t^c \) is the gross nominal interest rate paid to financial intermediaries, while \( R_t \) is the gross interest rate paid to households.

Equation (2.7) is the standard dynamic equation of capital; equation (2.7) is the production function; equation (2.10) is the flow budget constraint for entrepreneurs.

Equation (2.6) is the adjustment cost for investment. It pins down the price of capital \( q_t \). Equation (2.8) is the cash-in-advance constraint for entrepreneurs, where \( M_t^c \) is the money borrowed from financial intermediaries to finance for working capital (the nominal wage bill). Equation (2.9) is the credit constraint for long-term borrowing. It says that the long term borrowing requires capital pledged as collateral and entrepreneurs can at most borrow a fraction (0 < \( \theta \) < 1) of the present value of the collateral asset.

Since \( M_t^c \) is to pay for the wage bills, It is obvious from the flow budget constraint (equation (2.10)) that the long term borrowing \( b_{t+1} \) is used to finance for capital. The short-term borrowing \( M_t^c \) is paid back at the end of period \( t \), therefore its interest rate \( R_t^c \) is determined at period \( t \). The long-term borrowing \( b_t \) is loaned at time \( t - 1 \) and paid back at period \( t \), therefore its interest rate \( R_t \) is determined at time \( t - 1 \).
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

2.3.3 Financial Intermediaries

As described at the beginning of this section, financial intermediaries have two types of liabilities and assets. For short term cash loan to entrepreneurs, financial intermediaries have two sources of funding: the short term cash savings $N_t$ from households and new money injections $g_t M_t$. For long term commodity (credit) loans to entrepreneurs, financial intermediaries have only one funding source: the long-term savings from $L_{t+1}$ from households. Financial intermediaries are in a complete competitive market therefore make zero profit:

$$R_t^e M_t^e = R_t^n N_t$$  \hspace{1cm} (2.11)

$$R_{t+1} L_{t+1} = R_{t+1} B_{t+1}$$  \hspace{1cm} (2.12)

In equilibrium we have:

$$M_t^e = (N_t + g_t M_t)$$  \hspace{1cm} (2.13)

where $g_t$ is the money growth rate and controlled by the monetary authority. The monetary policy in this model is too simple to be alike how monetary policy is conducted in the real world. However, Christiano & Eichenbaum (1995) show that this simple way is equivalent to conducting monetary policy by trading government bonds. Adding the government bond trading only complicates the computations. Equation (2.13) says that in equilibrium, the money borrowed by entrepreneurs from financial intermediaries should equal to the saving from households $N_t$ plus the new injected money. Monetary shock $g_t$ is assumed to follow an AR(1) process:

$$\ln (g_{t+1}) = \rho_g \ln (g_t) + \varepsilon_{t+1}^g$$  \hspace{1cm} (2.14)

where $0 < \rho_g < 1$ is the persistent parameter, $\varepsilon_{t+1}^g$ is the i.i.d shock with zero mean.

It is obvious from equations (2.11) and (2.13) that an increase of money supply would lead to an decrease of the short term loan interest rate $R_t^e$. Since only entrepreneurs get the access to the money market, the lowered short term loan interest rate only benefits entrepreneurs.
2.3 The Model

2.3.4 Equilibrium

Suppose $U(C_t, H_t) = \ln(C_t) + \xi \ln(1 - H_t)$, $U(C_t^e) = \ln(C_t^e)$, $z_t F(K_t, H_t) = z_t K_t^\alpha H_t^{1-\alpha}$, and $I_t$ subjected to an adjustment cost $\chi^2 (\frac{I_t}{K_t} - \delta)^2 K_t$. $z_t$ is the exogenous productivity shock and follows an AR(1) process:

$$\ln(z_{t+1}) = \rho z \ln(z_t) + \varepsilon_{t+1}$$

where $0 < \rho < 1$ is the persistent parameter, $\varepsilon_{t+1}$ is the i.i.d shock with zero mean.

The we can derive all the first order conditions as reported in Appendix A.

In a competitive equilibrium, the markets for goods, labor, credits all clear. The goods market clearing condition is:

$$Y_t = C_t + C_t^e + q_t I_t$$

The market clearing condition for short term loan market is:

$$M_t^e = (N_t + g_t M_t)$$

The market clearing condition for short term loan market is:

$$L_{t+1} = B_{t+1}$$

A competitive equilibrium then can be defined as sequences of prices $\{W_t, q_t, R_t, R_t^e, R_t^n\}^\infty_{t=0}$ and sequences of allocations $\{Y_t, C_t, C_t^e, I_t, L_t, H_t, N_t, B_t, K_t, M_t, \Pi_t\}^\infty_{t=0}$ such that (i) taking prices as given, the allocations solve the optimizing problems for households and entrepreneurs, and (ii) all markets clear. A full characterization of the equilibrium is presented in Appendix A.

The steady state values are presented in appendix B. Let hatted variables denote percentage changes from the steady state, and those without time subscript denote steady state values. Then the model can be reduced to a linearized system, as reported in Appendix C.
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

2.4 Parameterizations and Simulations

The parameter values are calibrated using the US quarterly data, and reported in table 2. \( \beta \) is calibrated so that the risk-free interest rate is 1%. Consistent with Iacoviello (2005), \( \gamma \) is set as 0.96. \( \xi \) is calibrated so that the steady state value of labor (\( H \)) is \( 1/3 \). \( \alpha \) is calibrated so that it equals to the proportion capital income accounts for in GDP. The depreciation rate \( \delta \) is set as 0.0025, which is traditional in the literature. The marginal adjustment cost \( \chi \) is estimated by Christensen & Dib (2008) as 0.58. The persistent parameters \( \rho_z \) and \( \rho_g \) are set as 0.95. The steady state gross inflation rate \( \Pi \) is set as 1.01, equivalent to 4% annual inflation. These parameter values are conventional in standard real business cycle models. Mendoza (2006) estimates that the loan-to-value ratio \( \theta \) ranges from 0.14-0.4 with an average at around 0.3. Therefore in this chapter and the next two chapters, \( \theta \) is set as 0.3. The simulation results in this chapter is robust for \( 0 < \theta < 1 \).

<table>
<thead>
<tr>
<th>parameter</th>
<th>notation</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>households’ discount factor</td>
<td>( \beta )</td>
<td>0.99</td>
</tr>
<tr>
<td>entrepreneurs’ discount factor</td>
<td>( \gamma )</td>
<td>0.96</td>
</tr>
<tr>
<td>credit constraint</td>
<td>( \theta )</td>
<td>0.3</td>
</tr>
<tr>
<td>Gross Inflation(Steady State)</td>
<td>( \Pi )</td>
<td>1.01</td>
</tr>
<tr>
<td>marginal adjustment cost</td>
<td>( \chi )</td>
<td>0.58</td>
</tr>
<tr>
<td>depreciation rate</td>
<td>( \delta )</td>
<td>0.025</td>
</tr>
<tr>
<td>autocorrelation of shocks</td>
<td>( \rho_z )</td>
<td>0.95</td>
</tr>
<tr>
<td>autocorrelation of shocks</td>
<td>( \rho_g )</td>
<td>0.95</td>
</tr>
</tbody>
</table>

Table 2. Parameter Values

To understand the effects of the combination of segmented market and credit constraint, three benchmark models are developed for comparisons. The first benchmark model is a standard real business cycle model with a cash-in-advance constraint for consumption and an adjustment cost for capital accumulation. The second benchmark model has uneven money injection (segmented market) but no credit constraint. The third benchmark model faces binding credit constraint but money is injected evenly.\(^3\)

\(^3\) Mathematically, the first benchmark model does not have the credit constraint equation (2.9), the zero-profit condition (2.13) for financial intermediaries is simply \( N_t = M_t^r \), and the CIA constraint for households (2.2) is \( C_t + N_t = M_t + T_t \), where \( T_t = g_t M_t \) is the money injection. The second benchmark model does not have the credit constraint equation (2.9). The third benchmark model has the credit constraint equation (2.9), but its zero-profit condition equation and CIA constraint equation for households are the same with benchmark model 1.
2.4 Parameterizations and Simulations

<table>
<thead>
<tr>
<th>Data</th>
<th>MM (% of Data)</th>
<th>BM1 (% of Data)</th>
<th>BM2 (% of Data)</th>
<th>BM3 (% of Data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y, C)</td>
<td>0.95</td>
<td>15</td>
<td>86</td>
<td>12</td>
</tr>
<tr>
<td>(Y, I)</td>
<td>0.96</td>
<td>98</td>
<td>-9</td>
<td>85</td>
</tr>
<tr>
<td>(Y, B)</td>
<td>0.67</td>
<td>99</td>
<td>23</td>
<td>65</td>
</tr>
<tr>
<td>(Y, q)</td>
<td>0.54</td>
<td>87</td>
<td>26</td>
<td>72</td>
</tr>
<tr>
<td>(Y, M)</td>
<td>0.32</td>
<td>112</td>
<td>-18</td>
<td>93</td>
</tr>
<tr>
<td>(Y, Π)</td>
<td>0.16</td>
<td>104</td>
<td>-6</td>
<td>87</td>
</tr>
</tbody>
</table>

Table 3: Contemporaneous correlations with output

BMi stands for the ith benchmark model

MM stands for the main model

Data: HP filtered U.S. Quarterly data, 1980.01-2006.12

Data source: Fred (Federal Reserve of Economic Data)

Note: The data range and source reported here apply to all tables in this thesis; here consumption is the sum of consumptions of households and entrepreneurs by assuming that they have equal shares.

Table 3 presents the correlations between the main variables with output for the four models and the data. In general, the main model fits the data better than the benchmark models, and the first benchmark model fits the data worst. The exception is the correlation between output and consumption. In this case, the first benchmark model outperforms the main model and other 2 benchmark models. It is only slightly positive in the main model, comparing with almost 1 in the data. In the main model, the correlation between output and the consumption of households actually is negative. The reason is that, in the main model, only entrepreneurs receive money injections. As a result, expansionary monetary policy transfers wealth from the household sector to the entrepreneur sector. With less wealth, consumption is reduced accordingly. Negative correlation between output and households’ consumption is a normal result in segmented market models.

Table 4: Percentage standard deviation relative to output

<table>
<thead>
<tr>
<th>Data</th>
<th>MM (% of Data)</th>
<th>BM1 (% of Data)</th>
<th>BM2 (% of Data)</th>
<th>BM3 (% of Data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption, C</td>
<td>0.45</td>
<td>34</td>
<td>18</td>
<td>89</td>
</tr>
<tr>
<td>Investment, I</td>
<td>1.86</td>
<td>87</td>
<td>30</td>
<td>68</td>
</tr>
<tr>
<td>Asset Price, q</td>
<td>6.94</td>
<td>45</td>
<td>6</td>
<td>22</td>
</tr>
</tbody>
</table>

Table 4. Percentage standard deviation relative to output

See chapter 12 of *The ABCs of RBCs* (McCandless (2008)) for more discussions.
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES


Table 5 reports the simulated standard deviations of the main variables. In general, the main model outperforms all three benchmark models, and the first benchmark performs the worst. Most importantly, the asset price volatility in the main model is significantly much higher than in all three benchmark models. This result shows that combining segmented market with credit constraint is capable to produce volatile asset price behaviors without the sticky price assumption.

Figure 3 reports the impulse responses of the main variables to a 1% positive monetary shock. There are a few interesting features. First, the main model has much larger impulse responses than all three benchmark models. Second, the labor first increases in response to expansionary monetary shocks, then decreases. The decrease is caused by the inflation tax. Due to the inflation tax, the relative price of leisure drops, making leisure a more attractive commodity, which decreases the labor supply. This result is standard in cash-in-advance models, and adding segmented market or credit constraint does not change this special feature of cash-in-advance models.
Figure 4. Responses of asset price and inflation to a 1% positive monetary shock

Figure 4 presents the responses of asset prices and inflation rate after a positive monetary shock. It shows that asset price movements are most volatile in response to monetary shocks in the main model than the three benchmark models. Figure 5 shows the effects of expansionary monetary policy on long-term credit ($h_t$) for the four models. The credit expands most dramatically in the main model than the benchmark models. Together with figure 4, it shows that asset prices move with credit expansion. This is one of the typical facts summarized by the IMF annual report (2000).
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

Figure 5. Effects of expansionary monetary policy on long-term credit

2.5 Concluding Remarks

This chapter combines a segmented money market with a credit constraint into a dynamic stochastic general equilibrium to explain the volatile behavior of asset prices within a flexible price model. A segmented money market produces positive responses to output and asset prices after positive monetary shocks. The financial accelerator effects incurred by a credit constraint then amplify the initial effects of expansionary monetary policies. It is therefore possible to produce much larger asset price volatilities within a flexible price framework.

Although there are a few advantages in combining credit constraint with segmented money market models, as shown in the simulation section, there are a few shortcomings. First, asset price volatility is much larger, but not large enough. This suggests that there are other factors that have significant impacts on asset prices but are not examined in this model. Second, due to the existence of large inflation tax, cash-in-advance is not an ideal way to introduce money into general equilibrium models. It is well recognized in the literature that in the short run, expansionary monetary policy has positive effects on production and employment. However, strong inflation tax effects in cash-in-advance
2.6 Appendix 1: Characterization of Equilibrium

models usually lead to more unemployment after expansionary monetary shocks. Other more micro-founded ways to introduce money into macroeconomic models shall be explored.

2.6 Appendix 1: Characterization of Equilibrium

2.6.1 F.O.Cs:

F.O.C with respect to $N_t$:

$$\frac{1}{C_t} = \lambda_t R^n_t$$  \hspace{1cm} (2.19)

Labor supply equation:

$$\lambda_t = \frac{\xi}{1 - H_t w_t}$$  \hspace{1cm} (2.20)

Euler Equation for households:

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_{t+1}}{\Pi_{t+1}}$$  \hspace{1cm} (2.21)

No arbitrage condition:

$$E_t R^n_{t+1} = E_t R_{t+1}$$  \hspace{1cm} (2.22)

Labor demand equation:

$$\left(1 - \alpha\right) \frac{Y_t}{H_t} = R^n_t w_t$$  \hspace{1cm} (2.23)

Euler equation for entrepreneurs:

$$\frac{1}{C^e_t} \left[ q_t - \theta E_t q_{t+1} \frac{\Pi_{t+1}}{R_{t+1}} \right] = \gamma E_t \frac{1}{C^e_{t+1}} \left[ q_{t+1} (1 - \delta - \theta) + \alpha \frac{Y_{t+1}}{K_{t+1}} \right]$$  \hspace{1cm} (2.24)

Tobin’s Q:

$$q_t = 1 + \chi \left( \frac{I_t}{K_t} - \delta \right)$$  \hspace{1cm} (2.25)

2.6.2 Zero-Profit Condition:

$$R^n_t = \frac{n_t}{n_t + g_t M_t} R^n_t$$  \hspace{1cm} (2.26)
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

2.6.3 Market Clearing Conditions:

Production function:

\[ Y_t = z_t K_t^\alpha H_t^{1-\alpha} \]  (2.27)

Consumption goods market clearing condition:

\[ Y_t = C_t + C_t^e + q_t I_t \]  (2.28)

Credit constraint:

\[ b_{t+1} = \theta E_t q_{t+1} K_{t+1} \frac{\Pi_{t+1}}{R_{t+1}} \]  (2.29)

CIA constraint for entrepreneurs:

\[ w_t H_t = n_t + g_t \frac{m_t}{\Pi_t} \]  (2.30)

Flow budget constraint for households:

\[ w_t H_t = b_{t+1} - b_t \frac{R_t}{\Pi_t} + m_{t+1} - R_t \]  (2.31)

Evolution of money:

\[ m_{t+1} = (1 + g_t) \frac{m_t}{\Pi_t} \]  (2.32)

CIA constraint for households:

\[ C_t + n_t = \frac{m_t}{\Pi_t} \]  (2.33)

Evolution of capital:

\[ K_{t+1} = (1 - \delta) K_t + I_t \]  (2.34)

2.6.4 Exogenous Shocks

\[ \ln (z_{t+1}) = \rho_z \ln (z_t) + \varepsilon_{t+1} \]  (2.35)

\[ \ln (g_{t+1}) = g + \rho_g \ln (g_t) + z_{t+1}^g \]  (2.36)
2.7 Appendix 2: Steady State

\[ R = \frac{\Pi}{\beta} \]  
\[ R^n = R \]  
\[ \frac{K}{Y} = \frac{1}{\alpha} \left[ 1 - \frac{\theta \Pi}{\gamma} - 1 + \delta + \theta \right] \]  
\[ \frac{H}{Y} = \left( \frac{K}{Y} \right) \frac{\alpha}{\alpha - 1} \]  
\[ H = \frac{1}{3} \]  
\[ Q = 1 \]  
\[ \Pi = 1 + g \]  
\[ B = \theta K \frac{\Pi}{R} \]  
\[ N = (1 - \alpha) \frac{Y}{R} \]  
\[ M = \Pi \left[ N (1 + R) + \left( \frac{R}{\Pi} - 1 \right) B \right] \]  
\[ C = \frac{M}{\Pi} - N \]  
\[ C^e = Y - C - \delta K \]  
\[ R^e = \frac{N}{N + g \frac{M}{\Pi}} R \]
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES

\[ W = (1 - \alpha) \frac{Y}{H R^c} \]  
\[ \lambda = \frac{1}{CR} \]  
\[ \xi = \lambda (1 - H) w \]

2.8 Appendix 3: Log-Linearized System

\[ \hat{c}_t = -\hat{\lambda}_t - \hat{r}_t^n \]  
\[ \hat{\lambda}_t = \frac{H}{1 - H} \hat{h}_t - \hat{w}_t \]  
\[ \hat{\lambda}_t = E_t \left[ \hat{\lambda}_{t+1} + \hat{r}_{t+1} - \hat{\pi}_{t+1} \right] \]  
\[ E_t \hat{r}_{t+1} = E_t \hat{r}_{t+1} \]  
\[ \hat{y}_t = \hat{r}_t^e + \hat{w}_t + \hat{h}_t \]  
\[ \hat{q}_t - \theta \frac{\Pi}{R} E_t (\hat{q}_{t+1} + \hat{\pi}_{t+1} - \hat{r}_{t+1}) + \left( \theta \frac{\Pi}{R} - 1 \right) \hat{c}_t^e \]  
\[ = \gamma (1 - \delta - \theta) \hat{q}_{t+1} + \gamma \alpha \frac{Y}{K} (\hat{y}_{t+1} - \hat{k}_{t+1}) + \left( \theta \frac{\Pi}{R} - 1 \right) \hat{c}_{t+1}^e \]  
\[ \hat{q}_t = \chi (\hat{i}_t - \hat{k}_t) \]  
\[ \hat{r}_t = \hat{r}_t^e + \frac{R - R^e}{R} (\hat{g}_t + \hat{m}_t - \hat{\pi}_t - \hat{n}_t) \]  
\[ \hat{y}_t = \hat{z}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{h}_t \]  
\[ \hat{y}_t = \frac{C}{Y} \hat{c}_t + \frac{C^e}{Y} \hat{c}_t^e + \frac{I}{Y} \hat{h}_t \]

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2.8 Appendix 3: Log-Linearized System

\[ \hat{b}_{t+1} = E_t \left( \hat{q}_{t+1} + \hat{k}_{t+1} + \hat{\pi}_{t+1} - \hat{r}_{t+1} \right) \] (2.63)

\[ WH \left( \hat{w}_t + \hat{h}_t \right) = N\hat{\eta}_t + g \frac{M}{\Pi} (\hat{g}_t + \hat{m}_t - \hat{\pi}_t) \] (2.64)

\[ WH \left( \hat{w}_t + \hat{h}_t \right) = B \left[ b_{t+1} -\frac{R}{\Pi} \left( \hat{b}_t + \hat{r}_t - \hat{\pi}_t \right) \right] + M\hat{m}_{t+1} - NR (\hat{r}_t^n + n_t) \] (2.65)

\[ \hat{m}_{t+1} = \hat{g}_t + \hat{m}_t - \hat{\pi}_t \] (2.66)

\[ \hat{c}_t + \frac{N}{C} \hat{\eta}_t = \frac{M}{C\Pi} (\hat{m}_t - \hat{\pi}_t) \] (2.67)

\[ k_{t+1} = (1 - \delta) k_t + \delta \hat{\iota}_t \] (2.68)

\[ \hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon_{t+1}^z \] (2.69)

\[ \hat{g}_{t+1} = \rho_g \hat{g}_t + \varepsilon_{t+1}^g \] (2.70)

Where \( \lambda_t \) is the Lagrangian multiplier for household’s income.
2. SEGMENTED MONEY MARKET, CREDIT CONSTRAINT AND ASSET PRICES
3

Macroeconomic Effects of Leverage Cycles

3.1 Introduction

A credit constraint with assets pledged as collateral is able to produce reinforcing cycles which amplify the effects of initial exogenous shocks. This amplifying mechanism is referred to as financial accelerator in the literature. Financial accelerator is recognised as an important amplifying mechanism in macroeconomics. This is due to that if initial exogenous shocks were the main causes of economic fluctuations as predicted by the standard real business cycle (RBC) models, they have to be unreasonably large to fully explain the cyclical behaviours of macroeconomic variables, but a model with credit constraint is possible to produce large and persistent fluctuations even with small initial shocks.

Though the idea of financial accelerator induced by credit constraint is old, it is Kiyotaki and Moore (1997) who first develop a rigorous model. The literature built on the seminal work by Kiyotaki and Moore is burgeoning and belongs to a broader strand of literature of financial frictions. It is worth pointing out that Iacoviello (2005) is the first to introduce a Kiyotaki-Moore style credit into a dynamic stochastic general

\[^1\] I have benefitted greatly from the comments and suggestions of Timothy Kam, Pedro Gomis-Porqueras, Chung Tran, Richard Dennis, Vipin Arora, Timo Henckel and three anonymous thesis examiners.

\[^2\] As noted in Chapter 2, there is another way to induce financial accelerator by using the costly-state-verification approach as in Bernanke and Gertler (1989).

\[^3\] For a comprehensive survey of this literature, see Brunnermeier, Eisenbach and Sannikov (2012).
3. MACROECONOMIC EFFECTS OF LEVERAGE CYCLES

equilibrium (DSGE) model. The model developed in this chapter is based on the Iacoviello model.

However, the quantitative importance of collateral constraints has been challenged. Kocherlakota (2000) suggests that the quantitative effects of collateral constraints might not be significant. Recent quantitative studies present contradictory evidences. Some confirm the conjecture of Kocherlakota (2000), some disprove. Cordoba and Ripoll (2004) find only weak amplifying effects of a credit constraint; Iacoviello and Neri (2010) report negative co-movements between asset prices and investment. On the other hand, Iacoviello (2005), Kiyotaki, Michaelides and Nikolov (2011), Khan and Thomas (2013) among a few others provide evidences supporting strong amplifying effects of collateral constraints. Liu, Wang and Zha (2009, 2013) investigate this problem by identifying which kind of shocks would induce significantly amplifying effects of collateral constraints. They find that shocks which would shift the demand for collateral assets (like patience shocks and financial shocks) would be significantly amplified by collateral constraints, on the other hand, the amplifying effects to total factor productivity (TFP) shocks are limited due to that the prices of collateral assets and the risk free interest rate move in the same direction, as a result, the present values of collateral assets do not change significantly with TFP shocks.

In the literature of collateral constraints, the loan-to-value (LTV) ratio has been assumed to be exogenous. It is either assumed as a constant, examples include Kiyitaki and Moore (1997), Iacoviello (2005), Iacoviello and Neri (2010), Kiyotaki, Michaelides and Nikolov (2011), or assumed to be following an exogenous random process, examples include Liu, Wang and Zha (2009, 2013), Gerali, Neri, Sessa And Signoretti (2010), Khan and Thomas (2013). LTV ratios following an exogenous random process are called financial shocks in the literature.

But in the real world, empirical studies (examples include Levitin and Wachter (2010), Duca, Muellbauer and Murphysee (2011), Fostel and Geanakoplos (2014)) show that the LTV ratio is not only endogenous but also pro-cyclical. The works of Geanakoplos and others intend to provide theoretical explanations for an endogenous LTV ratio. Geanakoplos (1997) proves the existence of an equilibrium with an

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5For a survey of this literature, see Fostel and Geanakoplos (2014).
optimal (therefore endogenous) LTV ratio in a simple binomial model; Geanakoplos (2003) then proves that the equilibrium LTV ratio is pro-cyclical: it is higher when the aggregate state of the economy is high and lower when the opposite happens. The pro-cyclical LTV ratio movements are called leverage cycles in the literature.

This chapter intends to fill the gap by developing a DSGE model with endogenous LTV ratio to study the macroeconomic effects of leverage cycles. The results show that by assuming an exogenous loan-to-value (LTV) ratio of a collateral constraint, either being constant or following an exogenous random process, the amplifying effects of a collateral constraint is undermined, even to TFP shocks.

The model developed in this chapter is a variant of the models of Iacoviello (2005) and Bernanke, Gertler and Gilchrist (1999). A banking sector is introduced and banks are allowed to optimally determine the loan-to-value ratio. The key innovation is that, instead of adopting a regular financial contract, which specifies the loan quantity, interest rate and terms of maturity among other items, banks are assumed to adopt a contract which requires assets pledged as collateral and specifies the loan-to-value ratio among other items. For banking loan contracts, it is costly to monitor the behaviors of borrowers once the loans are distributed. Collateral requirement serve as a tool to reduce adverse selection as well as moral hazard. It is not surprising that collateral loans on average account for more than 35% of the assets of banks in US. If interest rates are regulated by the authorities, like in China, adjusting LTV ratios becomes even more important for commercial banks to control risks.

In this chapter, following Bernanke, Gertler and Gilchrist (1999), firms (entrepreneurs) are assumed to face undiversifiable idiosyncratic shocks in addition to an aggregate productivity shock. If the realized idiosyncratic shock is too low, some firms de-

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6The model developed in this chapter is partially inspired by Stiglitz and Greenwald (2004). They state that (P51) collateral requirements could affect the returns of banks, however, “for simplicity”, they “shall ignore” the collateral requirements along with other determinants. Indeed, the trade-off faced by a bank in this model is quite similar to that of a bank in the seminal work of Stiglitz and Weiss (1981). In Stiglitz and Weiss (1981), the profit of a bank is not necessarily increased by charging higher interest rate as more risk is induced; in this chapter, the profit of a bank is not necessarily increased by setting a higher LTV ratio as more risk is induced.

7See chapter 10 of Mishkin (2011) for a detailed explanation.

8The data here only includes mortgage loans. This ratio would be even higher if loans collateralized with other types of assets are included.

9Since the idiosyncratic shock cannot be diversified, then financial intermediaries can emerge and specialize in dealing with this issue. This is consistent with the sprit of the influential Diamond and
fault. Banks then have to auction the collateralized asset to get some money back. A bankruptcy cost is assumed so that banks can only receive a fraction of the market value of the collateralized asset. When there is a positive total factor productivity shock, the default probability decreases, it is profitable to lend more therefore the loan-to-value is increased. On the other hand, more lending puts more assets at risks, which tends to increase the default probability. The reason is that for firms, more available funds lead to more investment, the increased capital accumulation reduces the return to capital (law of diminishing marginal return), which increases the default chances. The conflicting effects of increasing loan-to-value ratio imply that there is an optimal loan-to-value ratio such that new investment opportunity is fully exploited and the expected return of banks is maximized.

With a Kiyotaki-Moore style credit constraint, a positive shock increases the demand for asset, which pushes up the asset price. The increased market value of collateralized asset relaxes the credit constraint, inducing more borrowing as well as more demand for asset, a reinforcing cycle begins, which is referred to as financial accelerator. The mechanism of a collateral constraint with constant loan-to-value ratio can be summarized in the following diagram:

\[
\begin{align*}
z^\uparrow & \implies q^\uparrow \implies B^\uparrow \implies I^\uparrow \implies q^\uparrow \implies \cdots
\end{align*}
\]

With an endogenous loan-to-value ratio, in response to a positive shock, the optimal loan-to-value ratio is increased as well, which relaxes the credit constraint further. The increased loan-to-value ratio is an extra push within every reinforcing cycle. The mechanism of a collateral constraint with endogenous loan-to-value ratio can be summarized in the following diagram:

\[
\begin{align*}
z^\uparrow & \implies q^\uparrow \implies \theta^\uparrow \implies B^{\uparrow\downarrow} \implies I^{\uparrow\downarrow} \implies q^{\uparrow\downarrow} \implies \theta^{\uparrow\downarrow} \implies \cdots
\end{align*}
\]

Comparing the mechanism diagrams above, it is obvious that the endogenous loan-to-value ratio amplifies the financial accelerator effects of a credit constraint.

The model with endogenous LTV ratio is calibrated with US quarterly data. Simulation results of this model support the arguments above. In addition, compared with a model with constant loan-to-value ratio, the model developed here fits the data better.

in first and second order moments in general. Especially, asset price movement is significantly more volatile in the endogenous loan-to-value model, a fact often explained poorly by standard real business cycle models as noted in the first chapter of this thesis.

The rest of the chapter proceeds as follow. Section 2 discusses the details of the model. Section 3 deals with parameterizations and simulations. Section 4 concludes.

3.2 The Model

The model developed in this section closely follows Iacoviello (2005), which is a DSGE model with exogenous loan-to-value ratio. This aims to compare and show the effects of endogenous loan-to-value ratio. There are three types of agents in the model: households, entrepreneurs and financial intermediaries. Each is normalized to be of measure one. Following Kiyotaki & Moore (1997) and Iacoviello (2005), households are heterogeneous to entrepreneurs in preferences and endowments. With respect to preferences, households are more patient than entrepreneurs. This induces borrowing from households to entrepreneurs. With respect to endowments, households own labor, entrepreneurs own capital. At each period, entrepreneurs hire labor from households, then produce final products combining labor with capital. Financial intermediaries collect savings from households and lend them to entrepreneurs.

In addition to an aggregate total factor productivity shock, entrepreneurs suffer from undiversifiable idiosyncratic shocks as well. This may cause bankruptcy if the realized shock is too low. To ensure the safety of their loans, financial intermediaries require capital pledged as collateral. When entrepreneurs are bankrupt, financial intermediaries auction the collateralized capital to get some money back. However, this action is costly because of bankruptcy cost. The key departure of this model from Kiyotaki & Moore (1997) and Iacoviello (2005) is that, due to the possibility of bankruptcy and the existence of bankruptcy cost, to maximize their expected profit, financial intermediaries optimally choose a loan-to-value ratio, instead of accepting a constant loan-to-value ratio. The decisions of financial intermediaries on loan-to-value ratio have conflicting effects. They need to lend more (therefore increase the loan-to-value ratio) in order to make more profits, but lending more also puts more money at risk. This implies for a representative financial intermediary, there is an optimal loan-to-value ratio to be set at the beginning of each period so that its profit is maximized.
3. MACROECONOMIC EFFECTS OF LEVERAGE CYCLES

3.2.1 Households

A representative household owns one unit of labor. At the beginning of time \( t \), a representative household optimally determines his consumption \( C_t \), labor supply \( L_t \) and saving \( D_{t+1} \) to maximize the discounted expected lifetime utility

\[
\max_{\{C_t, L_t, D_{t+1}\}} \left\{ E_0 \sum_{t=0}^{\infty} \beta^t \left[ \ln (C_t) + \xi \ln (1 - L_t) \right] \right\}
\]

(3.1)

where \( E_0 \) is the expectation operator. The parameter \( \beta \in (0, 1) \) is a subjective discount factor.

The maximization behavior of a representative household subjects to a flow budget constraint. Define \( W_t \) as the wage at time \( t \) and \( R_t \) as the gross interest rate for the saving \( D_t \) from last period, the flow budget constraint for the household is:

\[
W_t L_t + R_t D_t = C_t + D_{t+1}
\]

(3.2)

Solving the optimization problem of the household yields first order conditions for labor supply (equation (3.3)) and consumption (equation (3.4)):

\[
\frac{\xi}{1 - L_t} = \frac{1}{C_t} W_t
\]

(3.3)

\[
\frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1}} - R_{t+1} \right]
\]

(3.4)

Equation (3.3) is a standard labor supply equation. Equation (3.4) is the Euler’s equation for households.

3.2.2 Entrepreneurs

A representative entrepreneur \( j \), combines the labor he/she hires and his/her own capital to produce final products. The production function is a Cobb-Douglas function:

\[
Y^j_t = \omega^j_t z_t K_t^\alpha L_t^{(1-\alpha)}
\]

(3.5)

where \( Y^j_t \), \( K_t \) and \( L_t \) are final product, capital stock and labor demand at time \( t \) respectively, and \( z_t \) is the total factor productivity shock which follows an AR(1) process:

\[
\ln (z_{t+1}) = \rho_z \ln (z_t) + \varepsilon_{t+1}^z
\]

(3.6)
where the parameter $\rho_z$ measures the degree of persistence, $\varepsilon_{t+1}$ is an i.i.d random process with zero mean and variance given by $\sigma^2_z$.

The idiosyncratic shock to productivity $\omega^j_t$ has a support domain over $[0, \infty)$ and $E\left(\omega^j_t\right) = 1$. The fixed continuous and once-differentiable distribution function of $\omega^j_t$ is known to all agents in this economy as $F\left(\omega^j_t\right)$. The idiosyncratic shock $\omega^j_t$ is undiversifiable, and can not be observed ex ante but is free to observe once it is realized. To simplify the problem, it is assumed that $\omega^j_t$ is realized after all decisions of entrepreneurs have been made. Because $E\left(\omega^j_t\right) = \int_0^\infty \omega^j_t dF\left(\omega^j_t\right) = 1$, the aggregate behaviors of entrepreneurs are not affected by $\omega^j_t$. More precisely, the representative agent approach still works for entrepreneurs.

A representative entrepreneur has the utility function

$$E_0 \sum_{t=0}^{\infty} \gamma^t \ln \left(C^e_t\right)$$

where $C^e_t$ is the entrepreneur’s consumption, the parameter $\gamma$ is the discount factor. It is assumed that $\gamma < \beta$, which means that entrepreneurs are less patient than households. This assumption implies that entrepreneurs borrow from households (through financial intermediaries) to finance their production expenditure. In addition, it makes sure that the credit constraint (equation (3.10)) is binding around the steady state.

The flow budget constraint for the representative entrepreneur is:

$$Y_t + B_{t+1} = C^e_t + W_t L_t + q_t I_t + B_t R^e_t$$

where $W_t$, $q_t$, $R^e_t$ denote the wage, price for capital (Tobin’s q) and interest rate charged by financial intermediaries respectively, $I_t$ is the investment, $B_{t+1}$ is the borrowing from financial intermediaries. The capital dynamics is described by

$$K_{t+1} = (1 - \delta) K_t + I_t$$

where $\delta$ is the depreciation ratio.

---

\[\text{Footnote: Following Bernanke, Gertler and Gilchrist (1999), to keep the aggregate numbers of entrepreneurs constant, the same amount of new entrepreneurs are born to substitute those bankrupt entrepreneurs, and they are endowed with the same capital stock with other survived entrepreneurs. This assumption simplifies the computation.}\]
3. MACROECONOMIC EFFECTS OF LEVERAGE CYCLES

The borrowing amount \( B_{t+1} \) is limited to a fraction of the market value of the collateral asset (capital):

\[
B_{t+1} \leq \theta_t E_t \left[ q_{t+1} K_{t+1} \frac{1}{R^k_{t+1}} \right]
\]  \hspace{1cm} (3.10)

where \( \theta_t \) is the optimal loan-to-value ratio determined by financial intermediaries at the beginning of each time \( t \). This equation is the Kiyotaki-Moore style credit constraint with an endogenous loan-to-value ratio.

A representative entrepreneur chooses \( C^e_t, L_t, I_t \) and \( K_{t+1} \) to maximize \((3.7)\) subject to \((3.5), (3.8), (3.9) \) and \((3.10)\).

The two first order conditions for labor demand and capital stock are respectively:

\[
W_t = (1 - \alpha) \frac{Y_t}{L_t}
\]  \hspace{1cm} (3.11)

\[
\frac{1}{C^e_t} \left[ q_t - \theta_t E_t q_{t+1} \frac{1}{R^k_{t+1}} \right] = \gamma E_t \frac{1}{C^e_{t+1}} \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta - \theta_t) q_{t+1} \right]
\]  \hspace{1cm} (3.12)

Equation (3.11) is a standard labor demand equation. Equation (3.12) is the Euler’s equation for entrepreneurs.

Finally, by definition, the return to capital \( R^k_{t+1} \) is

\[
R^k_{t+1} = E_t \left[ \frac{\alpha Y_{t+1}}{R^k_{t+1}} + (1 - \delta) q_{t+1} \right] \frac{1}{q_t} \]
\]  \hspace{1cm} (3.13)

which says that the return to capital is the marginal product of capital plus the capital gains.

The above equation can be derived strictly using the approach by Luk and Vines (2011). Assume that entrepreneurs do not conduct the production by themselves. Instead, they rent out their capital to firms by charging \( R^k_{t+1} \) as unit rent. Then the firms’ problem is to maximize their expected profits:

\[
\max_{K_{t+1}, L_{t+1}} \left\{ Y_{t+1} + (1 - \delta) q_{t+1} K_{t+1} - W_t L_t - R^k_{t+1} K_t \right\}
\]

The first order condition with respect to \( K_{t+1} \) yields equation (3.13).
### 3.2 The Model

#### 3.2.3 Capital Producers

Entrepreneurs own the technology to produce capital goods too. They conduct the production of capital goods through capital producers. Consistent with chapter 2 and following Christensen and Dib (2008), capital producers adopt a linear technology to transfer a fraction of consumption goods to capital goods while subject to a quadratic adjustment cost specified as \( \chi \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \).

Capital producers’ problem is to maximize their profit by choosing the quantity of investment \( I_t \):

\[
\max_{I_t} E_t \left[ q_t I_t - I_t - \chi \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right]
\]  

(3.14)

The first order condition with respect to \( I_t \) is

\[
E_t \left[ q_t - 1 - \chi \left( \frac{I_t}{K_t} - \delta \right) \right] = 0
\]  

(3.15)

This equation is a standard Tobin’s Q equation that relates the price of capital to its marginal adjustment cost. Since at the steady state, \( \bar{I} = \delta \bar{K} \), the steady state value of \( q_t \) equals to 1. The technology to produce capital is constant return-to-scale, therefore the profit of capital producers is zero.

#### 3.2.4 Financial Intermediaries

Financial intermediaries are owned by households. A representative Financial intermediary collects savings \( D_{t+1} \) from households, then lends the savings to entrepreneurs. To describe the behavior of financial intermediaries, the bankruptcy (default) probability function of entrepreneurs needs to be defined first. Since at each period, \( \omega_t \) is unique for each entrepreneur, it is convenient to index entrepreneurs by \( \omega_t \), though \( \omega_t \) does not affect the aggregate behaviors of entrepreneurs, as elaborated at the beginning of section 3.2.

At the end of each period \( t + 1 \), both individual and aggregate shocks are realized. Entrepreneurs sell their products and use this liquidity income to pay for the wage bill to workers, and use the left over to pay back the loans plus interests to financial intermediaries. There is a threshold level \( \omega_{t+1} \), such that the left over is just enough
to pay for the loans plus the interests. Define this threshold level $\omega_{t+1}$ as $\bar{\omega}_{t+1}$, and it should satisfy:

$$\bar{\omega}_{t+1} Y_{t+1} - W_{t+1} L_{t+1} - R_{t+1} E_{t+1} B_{t+1} = 0 \quad (3.16)$$

The above equation says that the income for the specific entrepreneur $\bar{\omega}_{t+1}$ is just enough to pay for the wage bill and the due loan plus interests. The distribution function of $\omega_{t+1}$ is fixed and common knowledge. Define

$$\Gamma (\bar{\omega}_{t+1}) = \int_{0}^{\bar{\omega}_{t+1}} \omega_{t+1} dF (\omega_{t+1}) \quad (3.17)$$

as the bankruptcy (default) probability function of entrepreneurs. If $\omega_{t+1} \geq \bar{\omega}_{t+1}$, entrepreneurs survive, financial intermediaries will make new loans to them for the next period. If $\omega_{t+1} < \bar{\omega}_{t+1}$, these entrepreneurs are claimed to be bankrupt. The banks have to auction their asset (capital) to get some money back. Assume the bankruptcy cost is $(1 - \mu)$, i.e., after the auction, banks can only get $\mu$ fraction of the market value of the asset. Then the banks’ problem at the beginning of time $t$ is to set an optimal loan-to-value ratio $\theta_t$ to maximize the expected return

$$\max_{\theta_t} E_t \left\{ \left[ 1 - \Gamma (\bar{\omega}_{t+1}) \right] R_{t+1} E_t \left( q_{t+1} K_{t+1} \frac{1}{R_{t+1}} \right) \right. \right.$$  

$$\left. + \mu \Gamma (\bar{\omega}_{t+1}) (1 - \delta) E_t \left( q_{t+1} K_{t+1} \right) \right\} \quad (3.18)$$

where $\theta_t E_t \left( q_{t+1} K_{t+1} \frac{1}{R_{t+1}} \right) = B_{t+1}$ is the amount loaned to entrepreneurs. $[1 - \Gamma (\bar{\omega}_{t+1})]$ is the fraction of loan that is safe. Therefore $[1 - \Gamma (\bar{\omega}_{t+1})] R_{t+1} E_t \left( q_{t+1} K_{t+1} \frac{1}{R_{t+1}} \right)$ is the return from safe loan. $\Gamma (\bar{\omega}_{t+1})$ is the fraction of entrepreneurs that defaults, $(1 - \delta) E_t \left( q_{t+1} K_{t+1} \right)$ is the market value of their capital stock at time $t+1$, and $\mu$ is the fraction that can be collected back after the auction. So $\mu \Gamma (\bar{\omega}_{t+1}) (1 - \delta) E_t \left( q_{t+1} K_{t+1} \right)$ is the return from bankrupt entrepreneurs.

The optimal behavior of financial intermediaries yielding a first order condition for $\theta_t$

$$\theta_t = \frac{1 - \Gamma (\bar{\omega}_{t+1})}{\partial \Gamma (\bar{\omega}_{t+1})} + \mu (1 - \delta) \quad (3.19)$$

Potential buyers could be survived entrepreneurs, new-born entrepreneurs or capital producers, who then sell them to entrepreneurs. They all buy the auctioned asset at the market value $q_{t+1}$.
Finally, it is supposed that financial intermediaries are in a complete competitive market thus face zero-profit constraint, i.e.,

\[ [1 - \Gamma (\bar{\omega}_{t+1})] R_{t+1}^e B_{t+1} + \mu \Gamma (\bar{\omega}_{t+1}) (1 - \delta) E_t (q_{t+1} K_{t+1}) = R_{t+1} D_{t+1} \] (3.20)

### 3.2.5 Properties of \( \Gamma (\bar{\omega}_{t+1}) \) and \( \theta_t \)

To proceed to the next section, the properties of \( \Gamma (\bar{\omega}_{t+1}) \) and \( \theta_t \) need to be examined first. From equations (3.17) and (3.19), we have:

\[ \Gamma (\bar{\omega}_{t+1}) = \int_{\bar{\omega}_{t+1}}^{\omega_{t+1}} \omega_{t+1} dF (\omega_{t+1}) \] (3.21)

\[ \theta_t = 1 - \Gamma (\bar{\omega}_{t+1}) + \mu (1 - \delta) \] (3.22)

It will be shown later in this section that both the default probability function \( \Gamma (\bar{\omega}_{t+1}) \) and the optimal loan-to-value ratio \( \theta_t \) are implicit functions of \( Y_{t+1} \), \( B_{t+1} \) and \( R_{t+1}^e \). When we do log-linearization of these two implicit functions around the steady state in the next section, only first order derivatives matter. The purpose of this section is to determine the qualitative relationship of \( \Gamma (\bar{\omega}_{t+1}) \) and \( \theta_t \) with respect to \( Y_{t+1} \), \( B_{t+1} \) and \( R_{t+1}^e \). This determines the signs of their first order derivatives.

Notice that the only choice variable for financial intermediaries is \( \theta_t \). An individual financial intermediary is atomic. When it makes decision about \( \theta_t \), it should take all prices as well as variables determined by entrepreneurs and households as given. Specifically, financial intermediaries take \( Y_{t+1} \), \( B_{t+1} \) and \( R_{t+1}^e \) as given. Since financial intermediaries are homogeneous, the representative intermediary’s choice is also the aggregate behavior of the financial intermediary sector.

Similar to Bernanke, Gertler and Gilchrist (1999), the optimal loan-to-value ratio problem can be seen as a partial-equilibrium financial contract problem. In a partial
equilibrium, financial intermediaries take $Y_{t+1}$, $B_{t+1}$ and $R^e_{t+1}$ as given and optimally choose the loan-to-value ratio $\theta_t$ to maximize their expected return. Then $Y_{t+1}$, $B_{t+1}$ and $R^e_{t+1}$ are determined in a general equilibrium.

First derive the threshold value $\bar{\omega}_{t+1}$ from equation (3.16), and use the property of constant return-to-scale production function that $W_{t+1}L_{t+1} = (1 - \alpha)Y_{t+1}$:

$$\bar{\omega}_{t+1} = (1 - \alpha) + \frac{R^e_{t+1}B_{t+1}}{Y_{t+1}}$$ (3.23)

From the above equation, $\bar{\omega}_{t+1}$ is a function of $Y_{t+1}$, $R^e_{t+1}$ and $B_{t+1}$. Take the derivatives of all three variables respectively:

$$\frac{\partial \bar{\omega}_{t+1}}{\partial Y_{t+1}} = -\frac{R^e_{t+1}B_{t+1}}{(Y_{t+1})^2} < 0$$ (3.24)

$$\frac{\partial \bar{\omega}_{t+1}}{\partial R^e_{t+1}} = \frac{B_{t+1}}{Y_{t+1}} > 0$$ (3.25)

$$\frac{\partial \bar{\omega}_{t+1}}{\partial B_{t+1}} = \frac{R^e_{t+1}}{Y_{t+1}} > 0$$ (3.26)

Since $\Gamma(\bar{\omega}_{t+1}) = \int_{0}^{\bar{\omega}} \omega_{t+1} dF(\omega_{t+1})$ increases in $\bar{\omega}_{t+1}$, that is, $\frac{\partial \Gamma(\bar{\omega}_{t+1})}{\partial \bar{\omega}_{t+1}} > 0$, therefore

$$\frac{\partial \Gamma(\bar{\omega}_{t+1})}{\partial Y_{t+1}} = -\frac{\partial \Gamma(\bar{\omega}_{t+1})}{\partial \bar{\omega}_{t+1}} \frac{R^e_{t+1}B_{t+1}}{(Y_{t+1})^2} < 0$$ (3.27)

$$\frac{\partial \Gamma(\bar{\omega}_{t+1})}{\partial B_{t+1}} = \frac{\partial \Gamma(\bar{\omega}_{t+1})}{\partial \bar{\omega}_{t+1}} \frac{R^e_{t+1}}{Y_{t+1}} > 0$$ (3.28)

$$\frac{\partial \Gamma(\bar{\omega}_{t+1})}{\partial R^e_{t+1}} = \frac{\partial \Gamma(\bar{\omega}_{t+1})}{\partial \bar{\omega}_{t+1}} \frac{B_{t+1}}{Y_{t+1}} > 0$$ (3.29)

The economic meanings of above three equations are very clear. The first equation says that financial intermediaries rationally expect the default probability decreases if production $Y_{t+1}$ increases given other variables; the second equation means that the is log-linearized around the steady state. The only thing matters in a linear system is the first order derivatives, whose signs are already determined. I use the same partial equilibrium/general equilibrium trick in this section.
3.2 The Model

increase of borrowing cost $R_{t+1}^e$ leads to more bankruptcy; the third equation states that lending more puts more asset at risk.\footnote{Like in Bernanke, Gertler and Gilchrist (1999), these qualitative relationships are rational expectations of financial intermediaries in a partial equilibrium, and they are true when the general equilibrium is reached because they are “rationally expected.”}

Since $B_{t+1} = \theta_tE_t\left(q_{t+1}K_{t+1}\frac{1}{R_{t+1}}\right)$, so $\frac{\partial B_{t+1}}{\partial \theta_t} = q_{t+1}K_{t+1}\frac{1}{R_t} > 0$. Combining with equation \[(3.28), \text{we have}

$$
\frac{\partial \Gamma (\bar{\omega}_{t+1})}{\partial \theta_t} = \frac{\partial \Gamma (\bar{\omega}_{t+1})}{\partial B_{t+1}} \frac{\partial B_{t+1}}{\partial \theta_t} = \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \omega_t} q_{t+1}K_{t+1} > 0
$$

Equation \[(3.22)\] characterizes the optimal decision rule for $\theta_t$. Applying the properties of $\Gamma (\bar{\omega}_{t+1})$ to equation \[(3.22), \text{we have the properties of } \theta_t:\]

$$
\frac{\partial \theta_t}{\partial Y_{t+1}} = \frac{-\partial \Gamma (\bar{\omega}_{t+1})}{\partial Y_{t+1}} > 0 \quad (3.31)
$$

$$
\frac{\partial \theta_t}{\partial B_{t+1}} = \frac{-\partial \Gamma (\bar{\omega}_{t+1})}{\partial B_{t+1}} < 0 \quad (3.32)
$$

$$
\frac{\partial \theta_t}{\partial R_{t+1}^e} = \frac{-\partial \Gamma (\bar{\omega}_{t+1})}{\partial R_{t+1}^e} < 0 \quad (3.33)
$$

The economic meanings of the above three equations are consistent with intuitions. It says that in response to an increase of a variable which is expected to reduce the bankruptcy probability ($Y_{t+1}$), the optimal choice for financial intermediaries is to increase the loan-to-value ratio $\theta_t$; in response to an increase of variables which are expected to raise the bankruptcy probability ($B_{t+1}, R_{t+1}^e$), the optimal choice for financial intermediaries is to lower the loan-to-value ratio $\theta_t$.\footnote{Like in Bernanke, Gertler and Gilchrist (1999), these qualitative relationships are rational expectations of financial intermediaries in a partial equilibrium, and they are true when the general equilibrium is reached because they are “rationally expected.”}
For convenience, define $\Gamma (\bar{\omega}_{t+1})$ and $\theta_t$ as implicit functions of $Y_{t+1}$, $B_{t+1}$ and $R^e_{t+1}$. That is:

$$\Gamma (\bar{\omega}_{t+1}) = \psi \left( Y_{t+1}^-, B_{t+1}^+, R^e_{t+1}^+ \right)$$ (3.34)

$$\theta_t = \phi \left( Y_{t+1}^+, B_{t+1}^-, R^e_{t+1}^- \right)$$ (3.35)

The signs above the variables stand for the qualitative relationship with $\theta_t$ or $\Gamma (\bar{\omega}_{t+1})$ respectively. Then in the next section, the two implicit functions are log-linearized around the steady state. The only thing matters in a linear system is first order derivatives, of which signs and values have been settled in this section.

### 3.2.6 Market Clearing Conditions and Equilibrium

In a competitive equilibrium, the markets for goods, labor, loanable fund all clear. The goods market clearing condition is:

$$Y_t = C_t + C^e_t + q_t I_t$$ (3.36)

The market clearing condition for credit market is:

$$B_{t+1} = D_{t+1}$$ (3.37)

A competitive equilibrium then can be defined as sequences of prices $\{W_t, q_t, R_t, R^e_t\}_{t=0}^\infty$ and sequences of allocations $\{Y_t, C_t, C^e_t, I_t, L_t, \theta_t, \Gamma (\bar{\omega}_t), D_t, B_t, K_t\}_{t=0}^\infty$ such that (i) taking prices as given, the allocations solve the optimizing problems for households, entrepreneurs and financial intermediaries and (ii) all markets clear. A full characterization of the equilibrium is presented in Appendix A.

Appendix B describes the steady state. Let hatted variables denote percentage deviations from the steady state, and those without subscript time index denote steady state values. The model can be reduced to a log-linearized system described in Appendix C.

The log-linearized form of the two implicit functions (3.34) and (3.35) are respectively:

$$\hat{\Gamma} (\bar{\omega}_t) = G_y \hat{y}_t + G_b \hat{b}_t + G_r \hat{r}^e_t$$ (3.38)
\[ \dot{\theta}_t = T_y \dot{y}_{t+1} + T_b \dot{b}_{t+1} + T_r \dot{r}_t \]

where

\[ G_y = \frac{1}{\Gamma(\bar{\omega}_t)} \left. \frac{\partial \psi (Y_t, B_t, R^e_t)}{\partial Y_t} \right|_{Y_t = \dot{Y}, B_t = \dot{B}, R^e_t = \dot{R}} \]

\[ G_b = \frac{1}{\Gamma(\bar{\omega}_t)} \left. \frac{\partial \psi (Y_t, B_t, R^e_t)}{\partial B_t} \right|_{Y_t = \dot{Y}, B_t = \dot{B}, R^e_t = \dot{R}} \]

\[ G_r = \frac{1}{\Gamma(\bar{\omega}_t)} \left. \frac{\partial \psi (Y_t, B_t, R^e_t)}{\partial R^e_t} \right|_{Y_t = \dot{Y}, B_t = \dot{B}, R^e_t = \dot{R}} \]

\[ T_y = \frac{1}{\theta} \left. \frac{\partial \phi (Y_{t+1}, B_{t+1}, R^e_{t+1})}{\partial Y_t} \right|_{Y_{t+1} = \dot{Y}, B_{t+1} = \dot{B}, R^e_{t+1} = \dot{R}} \]

\[ T_b = \frac{1}{\theta} \left. \frac{\partial \phi (Y_{t+1}, B_{t+1}, R^e_{t+1})}{\partial B_t} \right|_{Y_{t+1} = \dot{Y}, B_{t+1} = \dot{B}, R^e_{t+1} = \dot{R}} \]

\[ T_r = \frac{1}{\theta} \left. \frac{\partial \phi (Y_{t+1}, B_{t+1}, R^e_{t+1})}{\partial R^e_t} \right|_{Y_{t+1} = \dot{Y}, B_{t+1} = \dot{B}, R^e_{t+1} = \dot{R}} \]

The properties of \( \Gamma(\bar{\omega}_{t+1}) \) and \( \theta_t \) presented above restricted the signs of these six parameters, that is, \( G_y, T_b \) and \( T_r \) are negative, \( T_y, G_b \) and \( G_r \) are positive. However, the exact values depend on the function form of the distribution function \( F(\bar{\omega}_t) \), especially, depend on the value of \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \), which is the first order derivative of the distribution function of \( \Gamma(\bar{\omega}_t) \) with respect to \( \omega_t \) taking value at the steady state. See appendix D for details on deriving these six parameters.

### 3.3 Model Simulations

In this section, the results of some numerical experiments are presented. To compare, a benchmark model is developed. This benchmark model shuts down the optimal loan-to-value ratio channel while shares the other features of the model described in the last section. Instead of optimally determining a loan-to-value ratio, financial intermediaries in the benchmark model fixate the loan-to-value ratio to its steady state value.  

\[ \text{For} \]

Mathematically, in the benchmark model, equation (3.34) is taken away, and \( \theta_t \) equals to its steady state value \( \theta \) in all equations characterize the competitive equilibrium.
convenience, in the following parts, the model developed in the last section is referred to as the endogenous loan-to-value ratio model (endo. LTV in short), the benchmark model is referred to as the exogenous loan-to-value ratio model (exo. LTV in short).

The simulation results show that a model with endogenous loan-to-value ratio can generate much larger and more persistence impulse response to total factor productivity shocks. The first and second order moments generated by the two models are presented and compared with the US data. In general, the endogenous loan-to-value ratio model fits the data better than the benchmark model. Especially, asset price movements are more volatile in the endogenous loan-to-value model, a fact that is explained poorly by standard real business cycle models.

3.3.1 Model Parameterizations

The parameter values are reported in table 1. The coefficient of Cobb-Douglas production function $\alpha$, the depreciation ratio $\delta$ and the persistence parameter of real shock $\rho_z$ are set as (or around) $1/3$, $0.025$ and $0.95$ respectively. The households’ discounting factor $\beta$ is calibrated such that the quarterly risk free interest rate equals to $1.01\%$. The parameter $\xi$ for leisure $(1 - L_t)$ is calibrated such that the steady state of labor is $1/3$. These parameter values are conventional in standard real business cycle models. Consistent with Kiyotaki and Moore (1997) and Iacoviello (2005), the discounting factor of entrepreneurs $\gamma = 0.96$. The marginal adjust cost $\chi$ is estimated by Christensen and Dib (2008) as $0.58$. Consistent with Bernanke, Gertler and Gilchrist (1999), The steady state bankruptcy rate of $\Gamma(\tilde{\omega}_t)$ is set as $0.05$, which is the long-run average bankruptcy rate in US. The bankruptcy cost $(1 - \mu)$ is calibrated such that the steady state value of the quarterly risky interest rate $R^e$ equals to $1.03$, which is the long-run average risky interest rate in US. Consistent with chapter 2, the steady state value of the loan-to-value ratio $\theta$ is set as $0.3$.

The first order derivative of the distribution function $\Gamma(\omega_t)$ with respect to $\omega_t$ taking value at the steady state, $\left. \frac{\partial \Gamma(\omega_t)}{\partial \omega_t} \right|_{\omega_t = \tilde{\omega}}$, is a key parameter to the model developed above. It largely determines the values of the parameters of equations (3.38) and (3.39), the two key equations in characterizing the model. However, it is also non-standard and specific to the model above. The strategy adopted here is to pick an ad hoc value for $\left. \frac{\partial \Gamma(\tilde{\omega}_t)}{\partial \tilde{\omega}_t} \right|_{\tilde{\omega}_t = \tilde{\omega}}$, such that the parameter values for equations (3.38) and (3.39) are
3.3 Model Simulations

"reasonable" in the sense that they are neither too large nor too small. The value of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \) is set as 0.005 in the first place. As shown in appendix D, the value of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \) together with steady state values of \( \theta, Y, K, B \) and \( R^e \) determine the values of \( G_y, G_b, G_r \) and \( T_y, T_b, T_r \). And the higher \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \), the higher absolute values of \( G_y, G_b \) and \( G_r \). The values of \( G_y, G_b, G_r \) and \( T_y, T_b, T_r \) together with the sensitivity analysis are reported in appendix E. The sensitivity analysis is about the robustness of the simulation results with respect to different values of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \).

It shows that as long as the value of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \) is low, qualitatively the simulation results are robust.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Notation</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>household’s discount factor</td>
<td>( \beta )</td>
<td>0.99</td>
</tr>
<tr>
<td>entrepreneur’s discount factor</td>
<td>( \gamma )</td>
<td>0.96</td>
</tr>
<tr>
<td>depreciation ratio</td>
<td>( \delta )</td>
<td>0.025</td>
</tr>
<tr>
<td>total capital income share</td>
<td>( \alpha )</td>
<td>1/3</td>
</tr>
<tr>
<td>marginal adjustment cost</td>
<td>( \chi )</td>
<td>0.58</td>
</tr>
<tr>
<td>parameter for leisure</td>
<td>( \xi )</td>
<td>0.77</td>
</tr>
<tr>
<td>steady state LTV ratio</td>
<td>( \theta )</td>
<td>0.3</td>
</tr>
<tr>
<td>bankruptcy cost</td>
<td>( 1 - \mu )</td>
<td>0.32</td>
</tr>
<tr>
<td>persistence of shock</td>
<td>( \rho_z )</td>
<td>0.95</td>
</tr>
<tr>
<td>Slope of ( \Gamma (\bar{\omega}) )</td>
<td>( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg</td>
<td>_{\bar{\omega}_t = \bar{\omega}} )</td>
</tr>
</tbody>
</table>

Table 1. Parameter Values

3.3.2 Simulation Results

Table 2 presents the percentage standard deviations of the main variables relative to output for the date and the two models. All two models tend to over-predict the volatilities of variables except for asset price \( q_t \) and working hour \( L_t \). The exogenous loan-to-value ratio model only accounts for 9% of the asset price volatility. The endogenous loan-to-value ratio model accounts for 22%. This is far from enough, but it is a more than 100% improvement.

\[ \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \] is equivalent to set the standard deviation of the distribution function \( F (\omega) \). Since the mean of \( \omega \) equals to one by assumption, then given the value of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} \), we can solve for the standard deviation of \( F (\omega) \) as long as we know the distribution form.

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3. MACROECONOMIC EFFECTS OF LEVERAGE CYCLES

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>Endo. LTV (% of data)</th>
<th>Exo. LTV (% of data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption, C</td>
<td>0.45</td>
<td>169</td>
<td>155</td>
</tr>
<tr>
<td>investment, I</td>
<td>1.86</td>
<td>103</td>
<td>78</td>
</tr>
<tr>
<td>Labor, L</td>
<td>0.53</td>
<td>61</td>
<td>46</td>
</tr>
<tr>
<td>asset price, q</td>
<td>6.94</td>
<td>22</td>
<td>9</td>
</tr>
</tbody>
</table>

Table 2. Percentage standard deviation relative to output

Note: in this section, consumption is defined as the sum of consumptions of both the household sector and the entrepreneur sector.

Table 3 presents the contemporaneous correlations between the main variables for the data and the two models. The exogenous loan-to-value ratio model over-predicts all correlations. The endogenous loan-to-value ratio model also over-predicts correlations between output & investment and output & bank loan, and slightly fits the data better than the exogenous loan-to-value ratio model. In particular, with respect to correlations between output & consumption, the endogenous loan-to-value ratio model accounts for more than 99% of the data, while the exogenous loan-to-value ratio model over-predicts. It is interesting that both models fit the data quite well for the correlation between bank loan & asset price. This verifies the importance of credit constraint (with or without optimal loan-to-value ratio) to explain the typical fact of positive co-movement between credit and asset prices.

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>Endo. LTV (% of data)</th>
<th>Exo. LTV (% of data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y, C)</td>
<td>0.95</td>
<td>99</td>
<td>112</td>
</tr>
<tr>
<td>(Y, I)</td>
<td>0.96</td>
<td>106</td>
<td>114</td>
</tr>
<tr>
<td>(Y, B)</td>
<td>0.67</td>
<td>187</td>
<td>192</td>
</tr>
<tr>
<td>(Y, q)</td>
<td>0.54</td>
<td>83</td>
<td>104</td>
</tr>
</tbody>
</table>

Table 3, Contemporaneous correlations

In terms of first-order autocorrelations, as we can see from Table 4, in general, the two models over-predict the autocorrelations of all main variables. But the endogenous loan-to-value ratio model performs slightly better than the exogenous loan-to-value ratio model. Especially, the endogenous loan-to-value ratio model predicts almost perfectly about the autocorrelation of output, while the benchmark model over-predicts.

The annual report of IMF (2000) summarized this typical fact among others. In the second chapter I compared different models with or without the credit constraint, the results also support this conclusion.
3.3 Model Simulations

To summarize, the credit constraint models tend to over-predict the first and second order moments of variables except for the volatility of asset price and labor. However, the credit constraint model with endogenous loan-to-value ratio fits the data better in general. In particular, an endogenous loan-to-value ratio mechanism significantly improves the explanatory power of flexible price models in accounting for asset price volatilities. However, including this mechanism only partially succeeds in fully explaining the high volatilities of asset prices.

<table>
<thead>
<tr>
<th></th>
<th>data</th>
<th>Endo. LTV (% of data)</th>
<th>Exo. LTV (% of data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>output, Y</td>
<td>0.98</td>
<td>99</td>
<td>105</td>
</tr>
<tr>
<td>consumption, C</td>
<td>0.90</td>
<td>114</td>
<td>120</td>
</tr>
<tr>
<td>investment, I</td>
<td>0.96</td>
<td>112</td>
<td>118</td>
</tr>
<tr>
<td>bank loan, B</td>
<td>0.92</td>
<td>105</td>
<td>116</td>
</tr>
<tr>
<td>asset price, q</td>
<td>0.89</td>
<td>103</td>
<td>107</td>
</tr>
</tbody>
</table>

Table 4. First-order autocorrelations

Figure 1 and 2 are impulse responses of the main variables in response to a 1% positive productivity shock. For both models, the bankruptcy rate $\Gamma(\bar{\omega}_t)$ decreases immediately after the shock. To exploit this opportunity, banks in the endogenous loan-to-value ratio model increase the loan-to-value ratio $\theta_t$. By assumption, there is no change of $\theta_t$ in the exogenous loan-to-value ratio model. Compared to the exogenous loan-to-value ratio model, the increase of $\theta_t$ in the endogenous loan-to-value ratio model leads to a higher production $Y_t$, a much higher capital accumulation $K_t$, a more significant rise in asset price $q_t$, and spurs a much larger bank loans to entrepreneurs by relaxing the credit constraint $(B_{t+1} = \theta_tE\left[K_{t+1}\frac{1}{R_{t+1}}\right])$ further.
There are additional interesting features reflected in these two figures. In figure 2, the bankruptcy rate \( \Gamma (\bar{\omega}_t) \) decreases less in the endogenous loan-to-value ratio model than in the exogenous loan-to-value ratio model. This reflects the fact that in the endogenous loan-to-value ratio model, banks exploit the new investment opportunity and increase their loans to entrepreneurs for more returns. Lending more puts more money at risks and increases the bankruptcy rate, as stated in equation (3.29). Entrepreneurs buy more capital with more bank loans, which reduces the return to capital \( R^k_t \) because of the law of diminishing marginal return to capital, as shown on the left top on figure 2. This provides an evidence that by allowing banks optimally setting a loan-to-value ratio, the economy operates more efficient in the sense that no opportunity is wasted.

Finally, more bank loans in the endogenous loan-to-value ratio model also leads to a higher risk premium \( (R^e_t - R_t) \), as shown in the bottom right on figure 2. This is consistent with the data as well as intuition. In a world of uncertainty, more lending puts more money at risk (a higher bankruptcy chances), which require a higher risk premium to compensate the extra risks.
3.4 Conclusions

This chapter develops a real dynamic stochastic general equilibrium model with credit constraint in which the loan-to-value ratio is endogenized. The loan-to-value ratio is optimally determined by financial intermediaries to maximize their expected returns. The model developed in this chapter is cable to explain the fact of pro-cyclical loan-to-value ratios. In addition, the endogenous loan-to-value ratio mechanism amplifies the effects of financial accelerator and produces much larger asset price volatilities. This improves the ability of flexible price models to explain volatile asset price movements.

As suggested by Liu, Wang and Zha (2009, 2013), credit constraint with fixed loan-to-value ratio is incapable to significantly amplify the effects caused by TFP shocks. This is due to that the risk-free interest rate and the asset prices move into the same direction in response to TFP shocks, as a result, the present value of collateral assets only increases moderately which in turn leads to moderate credit increases. This

Figure 2. impulse response to a 1% positive TFP shock
chapter shows that the amplifying effects of credit constraints are underestimated by assuming a constant loan-to-value ratio. When the loan-to-value ratio is endogenous and pro-cyclical, it functions like an accelerator to the financial accelerator. Compared with a constant loan-to-value ratio model, the endogenous loan-to-value ratio model is able to produce much larger amplifying effects to TFP shocks.

Liu, Wang and Zha (2009, 2013) find that the amplifying effects of credit constraints are significant with respect to demand shocks including the loan-to-value ratio shocks. It is interesting to compare the effects of loan-to-value ratio shocks in an exogenous loan-to-value ratio model to the effects of TFP shocks in an endogenous loan-to-value ratio model. The amplifying effects of loan-to-value ratio shocks would be larger as the amplifying effects of endogenous loan-to-value ratio are partially comprised by the increase of risk-free interest rate. However, the endogenous loan-to-value ratio model developed in this chapter suggests that the loan-to-value ratio is not exogenous, either it being a constant or random shock. With endogenous loan-to-value ratio, credit constraints would produce even larger amplifying effects with respect to other demand shocks, which would be underestimated when the loan-to-value ratio is assumed being random shocks.

3.5 Appendix A: Characterization of Equilibrium

labor supply equation:
\[ \frac{\xi}{1 - L_t} = W_t \frac{1}{C_t} \]  
(3.46)

Euler equation of households:
\[ \frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1}} R_{t+1} \right] \]  
(3.47)

labor demand equation:
\[ W_t = (1 - \alpha) \frac{Y_t}{L_t} \]  
(3.48)

Euler equation of entrepreneurs:
\[ \frac{1}{C_t} \left[ q_t - \theta_t E_t q_{t+1} \frac{1}{R_{t+1}} \right] = \gamma E_t \frac{1}{C_{t+1}} \left[ \alpha \frac{Y_{t+1}}{K_{t+1}} + (1 - \delta - \theta_t) q_{t+1} \right] \]  
(3.49)

marginal adjustment cost of investment (Tobin’s Q):
\[ q_t = 1 + \chi \left( \frac{I_t}{K_t} - \delta \right) \]  
(3.50)
Zero-Profit Condition:

\[ [1 - \Gamma (\tilde{\omega}_{t+1})] R_{t+1}^e B_{t+1} + \mu \Gamma (\tilde{\omega}_{t+1}) (1 - \delta) E_t (q_{t+1} K_{t+1}) = R_{t+1} D_{t+1} \] (3.51)

optimal loan-to-value ratio:

\[ \theta_t = \phi (Y_{t+1}, B_{t+1}, R_{t+1}^e) \] (3.52)

bankruptcy probability function:

\[ \Gamma (\tilde{\omega}_{t+1}) = \psi (Y_{t+1}, B_{t+1}, R_{t+1}^e) \] (3.53)

production function:

\[ Y_t = z_t K_t^\alpha L_t^{1-\alpha} \] (3.54)

cConsumption market clear condition:

\[ Y_t = C_t + C_t^e + q_t I_t \] (3.55)

credit constraint:

\[ B_{t+1} = \theta_t E_t q_{t+1} K_{t+1} \frac{1}{R_{t+1}} \] (3.56)

credit market clear condition:

\[ B_{t+1} = D_{t+1} \] (3.57)

budget constraint of households:

\[ W_t L_t + R_t D_t = C_t + D_{t+1} \] (3.58)

Evolution of capital:

\[ K_{t+1} = (1 - \delta) K_t + I_t \] (3.59)

Exogenous Shock:

\[ \ln (z_{t+1}) = \rho_z \ln (z_t) + \varepsilon_{t+1}^z \] (3.60)
3. MACROECONOMIC EFFECTS OF LEVERAGE CYCLES

3.6 Appendix B: Steady State

\[ R = \frac{1}{\beta} \]  \hspace{1cm} (3.61)

\[ 1 - \theta \frac{1}{R^e} = \gamma \left[ \frac{Y}{K} + 1 - \delta - \theta \right] \] \hspace{1cm} (3.62)

\[ Y = K^\alpha L^{1-\alpha} \] \hspace{1cm} (3.63)

\[ B = \theta K \frac{1}{R^e} \] \hspace{1cm} (3.64)

\[ B = D \] \hspace{1cm} (3.65)

\[ W = (1 - \alpha) \frac{Y}{L} \] \hspace{1cm} (3.66)

\[ [1 - \Gamma (\bar{\omega})] R^e B + \mu \Gamma (\bar{\omega}) (1 - \delta) K = RD \] \hspace{1cm} (3.67)

\[ \Gamma (\bar{\omega}) = \int_{0}^{\infty} \bar{\omega} dF (\bar{\omega}) \] \hspace{1cm} (3.68)

\[ \bar{\omega} = (1 - \alpha) + \frac{R^e B}{Y} \] \hspace{1cm} (3.69)

\[ \theta = \frac{1 - \Gamma (\bar{\omega})}{\partial \Gamma (\bar{\omega}) / \partial \bar{\omega}} + \mu (1 - \delta) \] \hspace{1cm} (3.70)

\[ WL + DR = C + D \] \hspace{1cm} (3.71)

\[ C^e = Y - C - \delta K \] \hspace{1cm} (3.72)

\[ \lambda = \frac{1}{CR} \] \hspace{1cm} (3.73)

\[ \xi = \lambda (1 - L) w \] \hspace{1cm} (3.74)
3.7 Appendix C: Log-linearized System

\[
\frac{L}{1-L} \hat{L} = -\hat{c}_t + \hat{w}_t
\]  
(3.75)

\[-\hat{c}_t = E_t [-\hat{c}_{t+1} + \hat{r}_{t+1}]\]  
(3.76)

\[
\hat{y}_t = \hat{w}_t + \hat{l}_t
\]  
(3.77)

\[
\left( \theta \frac{1}{R^e} - 1 \right) \hat{c}^e_t + \hat{q}_t - \theta \frac{1}{R^e} \left( \hat{\theta}_t + \hat{\dot{q}}_{t+1} + \hat{r}^e_{t+1} \right) = \left( \theta \frac{1}{R^e} - 1 \right) \hat{c}^e_{t+1} + \gamma \left[ \alpha \frac{Y}{K} \left( \hat{y}_{t+1} - \hat{k}_{t+1} \right) + (1 - \delta - \theta) \hat{q}_{t+1} - \theta \hat{\theta}_t \right]
\]  
(3.78)

\[
\hat{q}_t = \chi \left( \hat{i}_t - \hat{k}_t \right)
\]  
(3.79)

\[
\hat{\Gamma} (\bar{\omega}_t) = G_y \hat{y}_t + G_b \hat{b}_t + G_r \hat{r}^e_t
\]  
(3.80)

\[
\hat{\theta}_t = T_y \hat{y}_{t+1} + T_b \hat{b}_{t+1} + T_r \hat{r}^e_{t+1}
\]  
(3.81)

\[
\hat{y}_t = \hat{z}_t + \alpha \hat{k}_t + (1 - \alpha) \hat{l}_t
\]  
(3.82)

\[
\hat{y}_t = \frac{C}{Y} \hat{c}_t + \frac{C^e}{Y} \hat{c}^e_t + \frac{I}{Y} \hat{l}_t
\]  
(3.83)

\[
\hat{b}_{t+1} = E_t \left( \hat{\theta}_t + \hat{q}_{t+1} + \hat{k}_{t+1} - \hat{r}_{t+1} \right)
\]  
(3.84)

\[
\hat{b}_{t+1} = \hat{\dot{a}}_{t+1}
\]  
(3.85)

\[
WL \left( \hat{w}_t + \hat{l}_t \right) + RD \left( \hat{r}_t + \hat{a}_t \right) = C \hat{c}_t + D \hat{\dot{a}}_{t+1}
\]  
(3.86)

\[
\hat{k}_{t+1} = (1 - \delta) \hat{k}_t + \delta \hat{i}_t
\]  
(3.87)

\[
\hat{z}_{t+1} = \rho_z \hat{z}_t + \varepsilon^z_{t+1}
\]  
(3.88)
3. MACROECONOMIC EFFECTS OF LEVERAGE CYCLES

3.8 Appendix D: log-linearization of $\Gamma(\bar{\omega}_t + 1)$ and $\theta_t$

From section 3.3, we know that, $\Gamma(\bar{\omega}_t + 1)$ and $\theta_t$ are implicit functions of $Y_{t+1}$, $B_{t+1}$, and $R_{\ell+1}^e$. (Equations (3.34) and (3.35) in the text):

$$\Gamma(\bar{\omega}_t) = \psi\left(Y_t, B_t, R_t^e\right) \quad (3.89)$$

$$\theta_t = \phi\left(Y_{t+1}, B_{t+1}, R_{\ell+1}^e\right) \quad (3.90)$$

We take Taylor’s first-order extensions on the implicit functions of $\Gamma(\bar{\omega}_t) = \psi(Y_t, B_t, R_t^e)$ and $\theta_t = \phi(Y_{t+1}, B_{t+1}, R_{\ell+1}^e)$ with respect to $Y_t, B_t$ at the steady state, then use the log-linearization approximation formula, we have:

$$\hat{\Gamma}(\bar{\omega}_t) = G_y \hat{y}_t + G_b \hat{b}_t + G_r \hat{r}_t^e \quad (3.91)$$

$$\hat{\theta}_t = T_y \hat{y}_{t+1} + T_b \hat{b}_{t+1} + T_r \hat{r}_{t+1}^e \quad (3.92)$$

where

$$G_y = \frac{1}{\psi(Y_t, B_t, R_t^e)} \left. \frac{\partial \psi(Y_t, B_t, R_t^e)}{\partial Y_t} \right|_{Y_t=Y, B_t=B, R_t^e=R}$$

$$= \frac{1}{\psi(Y_t, B_t, R_t^e)} \left( - \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \omega_t} \frac{R_t^e B_t}{Y_t^2} \right) \left|_{Y_t=Y, B_t=B, R_t^e=R} \right. \right.$$  

$$= - \frac{1}{\Gamma(\bar{\omega})} \frac{R^e B}{Y^2} \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \omega_t} \bigg|_{\omega_t=\bar{\omega}} \quad (3.93)$$

The second step uses the result from section 3.3: equation (3.27). Similarly, we can get the values for the other five parameters using the properties of $\Gamma(\bar{\omega}_{t+1})$ and $\theta_t$ from the section 3.3:
3.9 Appendix E: Sensitivity Analysis

\( G_r = \frac{1}{\Gamma (\bar{\omega})} \left. \frac{\partial \psi (Y_t, B_t, R_t^e)}{\partial R_t^e} \right|_{Y_t = Y, B_t = B, R_t^e = R} \quad (3.94) \)

\( T_y = \left. \frac{1}{\bar{\theta}} \frac{\partial \phi (Y_{t+1}, B_{t+1}, R_{t+1}^e)}{\partial Y_t} \right|_{Y_{t+1} = Y, B_{t+1} = B, R_{t+1}^e = R} \quad (3.95) \)

\( T_b = \left. \frac{1}{\bar{\theta}} \frac{\partial \phi (Y_{t+1}, B_{t+1}, R_{t+1}^e)}{\partial B_t} \right|_{Y_{t+1} = Y, B_{t+1} = B, R_{t+1}^e = R} \quad (3.96) \)

\( T_r = \left. \frac{1}{\bar{\theta}} \frac{\partial \phi (Y_{t+1}, B_{t+1}, R_{t+1}^e)}{\partial R_t^e} \right|_{Y_{t+1} = Y, B_{t+1} = B, R_{t+1}^e = R} \quad (3.97) \)

And it is clearly from the formulas above that the values of these six parameters depend on the steady state values of \( \Gamma (\bar{\omega}_t+1) \), \( \bar{\theta}_t \), \( Y_t \), \( B_t \), \( K_t \) and \( R_t^e \) (whose values are listed in appendix B) as well as \( \left. \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \right|_{\bar{\omega}_t = \bar{\omega}} \).

3.9 Appendix E: Sensitivity Analysis

This appendix reports the sensitivity analysis for the six parameters in these two equations with respect to different values of \( \left. \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \right|_{\bar{\omega}_t = \bar{\omega}} \):

\( \hat{\Gamma} (\bar{\omega}_t) = G_y \hat{y}_t + G_b \hat{b}_t + G_r \hat{r}_t^e \quad (3.98) \)

\( \hat{\theta}_t = T_y \hat{y}_{t+1} + T_b \hat{b}_{t+1} + T_r \hat{r}_{t+1}^e \quad (3.99) \)
Table 1 reports the values of $G_y$, $G_b$, $G_r$ and $T_y$, $T_b$, $T_r$ for different values of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}}$. It is clear from this table that the higher value of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}}$, the larger the gap between $G_y$ and $G_b$, $G_r$ as well as $T_y$ and $T_b$, $T_r$.

<table>
<thead>
<tr>
<th>$\frac{\partial \Gamma (\bar{\omega})}{\partial \bar{\omega}}$</th>
<th>$G_y$</th>
<th>$G_b$</th>
<th>$G_r$</th>
<th>$T_y$</th>
<th>$T_b$</th>
<th>$T_r$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
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<td>0.0185</td>
<td>0.9495</td>
<td>-0.0189</td>
<td>-0.9709</td>
<td></td>
</tr>
<tr>
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<td>0.1276</td>
<td>2.4940</td>
<td>0.7433</td>
<td>-0.0497</td>
<td>-0.9709</td>
</tr>
<tr>
<td>0.01</td>
<td>-3.8936</td>
<td>0.2931</td>
<td>3.7802</td>
<td>0.8538</td>
<td>-0.0753</td>
<td>-0.9709</td>
</tr>
</tbody>
</table>

Table E1: $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}}$ and the six parameters

Figure E1 reports the impulse response of $\theta_t$ and $Y_t$ for different values of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}}$. The first row is for $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} = 0.001$, the second row is for $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} = 0.01$. The results of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} = 0.005$ are reported in the main text.
3.9 Appendix E: Sensitivity Analysis

Figure E1. impulse response of $Y_T$ and $\theta_t$ for different values of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}}$

Figure E2 presents the impulse response of $\Gamma_t$ for different values of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}}$.

The first row is for $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} = 0.001$, the second row is for $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} = 0.01$.

The results of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}} = 0.005$ are reported in the main text.

Figure E2. impulse response of $\Gamma_t$ for different values of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}}$

The sensitivity analysis shows that, first, the qualitative relationship is robust to the values of $\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t = \bar{\omega}}$ as long as it is quite low.
3. MACROECONOMIC EFFECTS OF LEVERAGE CYCLES
Leverage Cycles and Housing Prices

4.1 Introduction

In chapter 3 a real DSGE model with endogenous loan-to-value (LTV) ratio is developed which requires tangible capital ($K_t$) as the collateral asset. It shows that the endogenous LTV ratio mechanism improves the ability of credit constraint models to explain the volatile movements of the price of capital (Tobin’s Q). However, in the real world, not all capital is suitable for collateral. Land and houses are two most favorable tangible assets for collateral requirement. This fact is partially due to their immobility, partially due to their relatively high liquidity compared with other tangible capital (like machines) and intangible assets (like human capital). It is almost costless to guard land or houses in case the borrowers cheat, and it is relatively easy to auction a land or house in case the borrowers default. It is not surprising that mortgage loans on average account for more than 35% of American banks’ asset for decades since the second world war.

As a result, it is not surprising that land/houses are the collateral assets in most credit constraint models ever since Kiyotaki and Moore (1997), who develop the first rigorous credit constraint model with land as collateral. Some writers aim to examine the extent to which collateral constraints can explain housing price movements. For example, Iacoviello (2005) introduces a Kiyotaki-Moore style credit constraint into a

\footnote{I have benefitted greatly from the comments and suggestions from Timothy Kam, Pedro Gomis-Porqueras, Richard Dennis, Chung Tran and Timo Henckel and three anonymous thesis examiners.}
New Keynesian model and verifies the significant amplifying effects of credit constraint to demand shocks on production as well as housing prices calibrated and estimated with the US data; with land as collateral in their model, Liu, Wang and Zha (2009, 2013) have identified shocks which make the positive co-movements between land prices and investment a driving force of the land-price dynamics on the macro-economy.

In this chapter, the model developed in the last chapter is extended to include a housing sector. This aims to examine the effects of credit constraints with endogenous LTV ratio on housing prices. The endogenous and pro-cyclical movements of LTV ratios are referred to as leverage cycles in the literature (Fostel and Geanakoplos (2014)). As such, this chapter is also about the relationship between leverage cycles and housing prices.

Empirical studies find that fluctuations in housing prices are primarily driven by changes in land prices (Davis and Heathcote (2007)). And since the supply of land is fixed in most cases, it does little harm to the ability of a model in explaining the behavior of housing prices by assuming the housing supply being fixed. Indeed, the research by Iacoviello and Neri (2010) finds that the housing supply dynamics have no significant effects on the housing price dynamics. For these reasons, the housing supply is assumed to be constant in the model of this chapter. And housing price and land price are interchangeable in this chapter.

In the last chapter, it is argued that collateral requirements serve as a method to reduce moral hazards and setting an optimal LTV ratio is an important way to control risks for banks. A financial contract specifying the LTV ratio is not a standard financial contract in the macroeconomic literature, which usually only specifies the loan quantity and interest rate among other items. However, it worth mentioning that a standard mortgage loan contract in the real world usually specifies the down-payment ratio (which equals to \(1 - \text{LTV ratio}\)), interest rates and other items. The optimal financial contract derived in the last chapter is quite consistent with the mortgage loan contract in the real world. Therefore, it is quite suitable to extend the model developed in the last chapter to include houses as collateral. With houses as collateral, the optimal financial contract derived in this chapter is even more consistent with a mortgage loan in the real world.

---

2Cheung (1970) argues that usually a contract is structural and there is an optimal structure for a contract with different transaction costs. The discrepancies between a standard financial contract
The model is calibrated with the US data and the simulation results are consistent with the last chapter: the housing price is much more volatile in response to total factor productivity (TFP) shocks, and in general, the endogenous LTV ratio model fits the data better than the exogenous LTV ratio model.

The rest of this chapter proceeds as follow. Part 2 discusses the details of the model. Part 3 deals with parameterizations and simulations. Part 4 concludes this chapter.

4.2 The Model

The model developed here closely follows the model of chapter 3. There are three types of agents in the model: households, entrepreneurs and financial intermediaries. Each is normalized to be of measure one. Households own labor, entrepreneurs own capital. At each period, entrepreneurs hire labor from households, and produce final products combining labor with capital and houses. Households are more patient than entrepreneurs. This assumption introduces borrowing from households to entrepreneurs. Financial intermediaries collect savings from households and lend them to entrepreneurs. Finally, the house supply is assumed to be constant, as explained in the introduction section.

4.2.1 Households

A representative household owns one unit of labor. At the beginning of time $t$, a representative household optimally determine his/her consumption $C_t$, labor supply $L_t$, saving $D_{t+1}$ and housing stock $H_{t+1}$ to maximize the discounted expected lifetime utility

$$
\max \left\{ E_0 \sum_{t=0}^{\infty} \beta^t [\ln (C_t) + \xi_h \ln (H_{t+1}^c) + \xi_l \ln (1 - L_t)] \right\}
$$

(4.1)

where $E$ is the expectation operator. The parameter $\beta \in (0, 1)$ is a subjective discount factor. The utility of households comes from consumption $C_t$, housing service $H_{t+1}^c$ and leisure $(1 - L_t)$. The maximization behavior of the household subject to a flow budget constraint. Denotes $W_t$ as the wage, $R_t$ as the gross risk-free interest rate in the macroeconomic literature and a standard mortgage loan contract in the real world show that macroeconomists care little about the structure of financial contract. The models developed in this chapter and the last chapter intend to narrow this gap in a sense.
4. LEVERAGE CYCLES AND HOUSING PRICES

for the saving $D_t$ from last period, $P_t^h$ as housing price at time $t$, the flow budget constraint for the household is:

$$W_t L_t + P_t^h H_t^c + R_t D_t = P_t^h H_{t+1}^c + C_t + D_{t+1}$$

(4.2)

Solving the optimization problem of the household yields first order conditions for labor supply (equation (4.3)) and two Euler equations with respect to $D_{t+1}$ (equation (4.4)) and $H_{t+1}^c$ (equation (4.5))

$$\frac{\xi}{1 - L_t} = \frac{1}{C_t} W_t$$

(4.3)

$$\frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1}} R_{t+1} \right]$$

(4.4)

$$\frac{1}{C_t} P_t^h = \beta E_t \left[ \frac{1}{C_{t+1}} P_{t+1}^h + \xi H \frac{1}{H_t^c} \right]$$

(4.5)

Equation (4.3) is the standard labor supply equation. Equation (4.4) is the standard Euler equation. Equation (4.5) can be re-written as:

$$\frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1}} \frac{P_{t+1}^h}{P_t^h} + \xi H \frac{1}{P_t^h H_t^c} \right]$$

(4.6)

Compare equation (4.6) with equation (4.4), according to the no arbitrage condition, we have:

$$\left[ \frac{1}{C_{t+1}} R_{t+1} \right] = \left[ \frac{1}{C_{t+1}} \frac{P_{t+1}^h}{P_t^h} + \xi H \frac{1}{P_t^h H_t^c} \right]$$

(4.7)

From the equation above, we can see that households are willing to tolerate a relatively lower return from housing investment ($\frac{P_{t+1}^h}{P_t^h}$) compared with the return from financial asset $D_{t+1}$, since there is a housing service premium $\xi H \frac{1}{P_t^h H_t^c}$. 
4.2 The Model

4.2.2 Entrepreneurs

A representative entrepreneur $j$, combines hired labor, owned capital and houses to produce final product. The production function is a Cobb-Douglas function:

$$Y_t^j = \omega_t^j z_t \left[ (H_t^t)^\eta K_t^{1-\eta} \right]^{\alpha} L_t^{1-\alpha}$$  \hspace{1cm} (4.8)

where $Y_t^j$, $H_t^t$, $K_t$ and $L_t$ denote final product, owned houses, capital stock and labor demand at time $t$ respectively. The parameters $\alpha \in (0,1)$ and $\eta \in (0,1)$ measure the output elasticities of these three inputs. $z_t$ is the productivity shock which follows an AR(1) process:

$$\ln (z_{t+1}) = \rho_z \ln (z_t) + \varepsilon_{t+1}$$  \hspace{1cm} (4.9)

where the parameter $\rho_z$ measures the degree of persistence, $\varepsilon_{t+1}$ is an i.i.d random process with zero mean and variance given by $\sigma_z^2$.

In equation (4.8), $\omega_t^j \in [0, \infty)$ is the idiosyncratic shock to productivity with $E \left( \omega_t^j \right) = 1$. The fixed continuous distribution function of $\omega_t^j$ is known to all agents in this economy as $F \left( \omega_t^j \right)$. The idiosyncratic shock $\omega_t^j$ can not be observed ex ante but is free to observe once it is realized. To simplify the problem, it is assumed that $\omega_t^j$ is realized after all decisions of entrepreneurs are done. Because $E \left( \omega_t^j \right) = \int_0^\infty \omega_t^j dF \left( \omega_t^j \right) = 1$, the aggregate behaviors of entrepreneurs are not affected by $\omega_t^j$. More precisely, the representative agent approach still works for entrepreneurs.

A representative entrepreneur has the utility function

$$E \sum_{t=0}^{\infty} \gamma^t \left[ \ln \left( C_t^e \right) \right]$$  \hspace{1cm} (4.10)

where $C_t^e$ denotes the entrepreneur’s consumption, the parameter $\gamma$ is the subjective discount factor. It is assumed that $0 < \gamma < \beta$, which means that entrepreneurs are less patient than households. This assumption implies that entrepreneurs borrow from households (through financial intermediaries) to finance for their expenditure. In addition, it makes sure that the credit constraint (equation (4.13)) is binding around the steady state. This means that for small shocks, the credit constraint is always binding, and therefore can be log-linearized around its steady state.
4. LEVERAGE CYCLES AND HOUSING PRICES

The flow budget constraint for the representative entrepreneur is

\[ Y_t + B_{t+1} + P^h H_t^e = C_t^e + W_t L_t + q_t I_t + P^h H_{t+1}^e + B_t R_t^e \]  

(4.11)

where \( W_t, q_t, R_t^e \) denote the wage, price for capital (Tobin’s q) and interest rate charged by financial intermediaries respectively. \( H_t^e \) is the houses owned by entrepreneurs at time \( t \). \( I_t \) denotes investment. And the capital dynamics is described by

\[ K_{t+1} = (1 - \delta) K_t + I_t \]  

(4.12)

where \( \delta \) is the depreciation ratio.

\( B_{t+1} \) denotes the borrowing from financial intermediaries. The borrowing amount is limited to a fraction of the market value of the collateral assets (capital and houses)

\[ B_{t+1} \leq \theta_t E_t \left[ q_{t+1} K_{t+1} + P^h_{t+1} H_t^e \right] \frac{1}{R_{t+1}} \]  

(4.13)

where \( \theta_t \) is the optimal loan-to-value ratio determined by financial intermediaries at the beginning of each time \( t \). This equation is a Kiyotaki-Moore style credit constraint equation with an endogenous loan-to-value ratio.

A representative entrepreneur chooses \( C_t^e, L_t, I_t H_{t+1}^e \) and \( K_{t+1} \) to maximize (4.10) subject to (4.8), (4.11), (4.12) and (4.13).

The first order conditions for labor demand, capital stock and houses are respectively

\[ W_t = (1 - \alpha) \frac{Y_t}{L_t} \]  

(4.14)

\[ \frac{1}{C_t^e} \left[ q_t - \theta_t E_t q_{t+1} \frac{1}{R_{t+1}} \right] = \gamma E_t \frac{1}{C_{t+1}^e} \left[ q_{t+1} (1 - \delta - \theta_t) + \alpha (1 - \eta) \frac{Y_{t+1}}{K_{t+1}} \right] \]  

(4.15)

\[ \frac{1}{C_t^e} \left[ P^h_t - \theta_t E_t P^h_{t+1} \frac{1}{R_{t+1}} \right] = \gamma E_t \frac{1}{C_{t+1}^e} \left[ P^h_{t+1} (1 - \theta_t) + \alpha \eta \frac{Y_{t+1}}{H_{t+1}} \right] \]  

(4.16)

Equation (4.14) is the standard labor demand equation. Equations (4.15) and (4.16) are the two Euler equations with respect to \( K_t \) and \( H_t^e \). The economic meaning of these two equations will be discussed later in the simulation section.

---

\(^3\) Following Bernanke, Gertler and Gilchrist (1999), to keep the aggregate numbers of entrepreneurs constant, the same amount of new entrepreneurs are born to substitute those bankrupt entrepreneurs, and they are endowed with the same capital stock with other survived entrepreneurs. This assumption simplifies the computation.
4.2 The Model

To ensure that the financial accelerator mechanism is working, we need to make an extra assumption:

\[ \gamma + \left( \frac{1}{R^e} - \gamma \right) \theta > 0 \] (4.17)

The assumption above makes sure that the shadow price of houses is positive at the steady state. This ensures that the financial accelerator is working when faces small productivity shocks.

To ensure that the endogenous LTV ratio mechanism is working, it is assumed that:

\[ \left( \frac{1}{R^e} - \gamma \right) \theta > 0 \] (4.18)

The assumption above makes sure that the shadow price of houses is increasing in the endogenous LTV ratio \( \theta_t \). This guarantees the endogenous LTV ratio mechanism is working around the steady state. The above two assumptions are easy to satisfy for most calibrations. We will discuss the mechanism in detail in the simulation section.

Finally, by definition, the return to capital \( R_{t+1}^k \) is

\[ R_{t+1}^k = E_t \left[ \frac{\alpha \eta Y_{t+1} + (1 - \delta) q_{t+1}}{q_t} \right] \] (4.19)

which says that the return to capital is the marginal product of capital plus the capital gains.

4.2.3 Capital Producers

Entrepreneurs own the technology to produce capital goods as well. They conduct the production of capital goods through capital producers. Following Christensen and Dib (2008), capital producers adopt a linear technology to transfer a fraction of consumption goods to capital goods subject to a quadratic adjustment cost specified as

\[ \frac{\chi}{2} \left( \frac{I_t}{K_t} - \delta \right)^2 K_t. \]

This equation can be derived strictly using the approach by Luk and Vines (2011). Assume that entrepreneurs do not conduct the production by themselves. Instead, they rent out their capital to firms by charging \( R_{t+1}^k \) as unit rent. Then the firms’ problem is to maximize their expected profits:

\[ \max_{K_{t+1}, L_{t+1}} \left\{ Y_{t+1} + (1 - \delta) q_{t+1} K_{t+1} - W_t L_t - R_{t+1}^k K_t \right\} \]

The first order condition with respect to \( K_{t+1} \) yields equation (4.19).
Capital producers’ problem is to maximize their profit by choosing the quantity of investment $I_t$:

$$\max_{I_t} E_t \left[ q_t I_t - I_t - \chi \left( \frac{I_t}{K_t} - \delta \right)^2 K_t \right]$$

(4.20)

The first order condition with respect to $I_t$ is

$$E_t \left[ q_t - 1 - \chi \left( \frac{I_t}{K_t} - \delta \right) \right] = 0$$

(4.21)

This equation is a standard Tobin’s Q equation that relates the price of capital to its marginal adjustment cost. Since in steady state, $\bar{I} = \delta \bar{K}$, the steady state value of $q_t$ is 1. The technology to produce capital is constant return-to-scale, therefore producer’s profit is zero.

### 4.2.4 Financial Intermediaries

Financial intermediaries are owned by households. A representative Financial intermediary collects the savings $D_{t+1}$ from households, then lends the savings to entrepreneurs. To describe the behavior of financial intermediaries, we need to first define the bankruptcy (default) probability function of entrepreneurs.

At the end of each period $t+1$, both individual and aggregate shocks are realized. Entrepreneurs sell their products and use this liquidity income to pay for the wage bill to workers, and use the left over to pay back the loans plus interests to financial intermediaries. There is a threshold level $\omega_{t+1}$ such that the left over is just enough to pay for the loans plus the interests. Define this threshold level $\omega_{t+1}$ as $\bar{\omega}_{t+1}$, it should satisfy:

$$\bar{\omega}_{t+1} Y_{t+1} - W_{t+1} L_{t+1} - R_{t+1}^c B_{t+1} = 0$$

(4.22)

The above equation says that the income for the specific entrepreneur $\bar{\omega}_{t+1}$ is just enough to pay for the wage bill and due loan plus interest.

Since the distribution function of $\omega_{t+1}$ is fixed and common knowledge, then define

$$\Gamma (\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} dF (\omega_{t+1})$$

(4.23)
4.2 The Model

as the bankruptcy (default) probability function of entrepreneurs. If \( \omega_{t+1} \geq \bar{\omega}_{t+1} \), the entrepreneurs survive, financial intermediaries will make new loans to them for next period. If \( \omega_{t+1} < \bar{\omega}_{t+1} \), these entrepreneurs are claimed as bankrupt. The banks have to auction their asset (capital) to get some money back. Assume the bankruptcy cost is \( (1 - \mu) \), i.e., after the auction, banks can only get \( \mu \) fraction of the asset market value. Then the banks’s problem at the beginning of time \( t \) is to set an optimal loan-to-value ratio \( \theta_t \) to maximize the expected return

\[
\max_{\theta_t} E_t \left\{ 1 - \Gamma (\bar{\omega}_{t+1}) \right\} R_{t+1}^e \theta_t E_t \left[ \left( q_{t+1} K_{t+1} + P_{t+1}^h H_{t+1}^e \right) \frac{1}{R_{t+1}^e} \right] \]

where \( \theta_t E_t \left[ \left( q_{t+1} K_{t+1} + P_{t+1}^h H_{t+1}^e \right) \frac{1}{R_{t+1}^e} \right] = B_{t+1} \) is the amount loaned to entrepreneurs. \( 1 - \Gamma (\bar{\omega}_{t+1}) \) is the fraction of loan that is safe. Therefore \( 1 - \Gamma (\bar{\omega}_{t+1}) \) \( R_{t+1}^e B_{t+1} \) is the return from safe loan. \( \Gamma (\bar{\omega}_{t+1}) \) is the fraction of entrepreneurs that defaults, \( E_t \left[ q_{t+1} (1 - \delta) K_{t+1} + P_{t+1}^h H_{t+1}^e \right] \) is the market value of collateralized assets at time \( t + 1 \), and \( \mu \) is the fraction that can be collected back after the auction. So \( \mu \Gamma (\bar{\omega}_{t+1}) E_t \left[ q_{t+1} (1 - \delta) K_{t+1} + P_{t+1}^h H_{t+1}^e \right] \) is the return from bankrupt entrepreneurs.

The optimal behavior of financial intermediaries yielding a first order condition for \( \theta_t \)

\[
\theta_t = \frac{1 - \Gamma (\bar{\omega}_{t+1})}{\partial \Gamma (\bar{\omega}_{t+1}) / \partial \theta_t} + \mu (1 - \delta) \quad (4.25)
\]

Finally, it is supposed that financial intermediaries are in a complete competitive market thus face zero-profit constraint, i.e.,

\[
[1 - \Gamma (\bar{\omega}_{t+1})] R_{t+1}^e B_{t+1} + \mu \Gamma (\bar{\omega}_{t+1}) E_t \left[ (1 - \delta) q_{t+1} K_{t+1} + P_{t+1}^h H_{t+1}^e \right] = R_{t+1} D_{t+1} \quad (4.26)
\]

4.2.5 Properties of \( \Gamma (\bar{\omega}_{t+1}) \) and \( \theta_t \)

We first examine the properties of the default probability function \( \Gamma (\bar{\omega}_{t+1}) \) and the optimal LTV ratio function \( \theta_t \) before conducting the simulations. From equation
4. LEVERAGE CYCLES AND HOUSING PRICES

and apply the property of constant return-to-scale production function that $W_{t+1}L_{t+1} = (1 - \alpha) Y_{t+1}$, we have

$$\bar{\omega}_{t+1} = (1 - \alpha) + \frac{(R^e_{t+1} - 1) B_{t+1}}{Y_{t+1}}$$ (4.27)

From the last subsection, we have the explicit function expressions of $\Gamma (\bar{\omega}_{t+1})$ and $\theta_t$:

$$\Gamma (\bar{\omega}_{t+1}) = \int_0^{\bar{\omega}_{t+1}} \omega_{t+1} dF (\omega_{t+1})$$ (4.28)

$$\theta_t = \frac{1 - \Gamma (\bar{\omega}_{t+1})}{\partial \Gamma (\bar{\omega}_{t+1}) / \partial \theta_t} + \mu (1 - \delta)$$ (4.29)

The above three equations show that the two functions $\Gamma (\bar{\omega}_{t+1})$ and $\theta_t$ are implicit functions of $Y_{t+1}$, $B_{t+1}$ and $R^e_{t+1}$. As in the last chapter, using the partial equilibrium trick, we can determine the qualitative relationships between $\Gamma (\bar{\omega}_{t+1})$ and $\theta_t$ with $Y_{t+1}$, $B_{t+1}$ and $R^e_{t+1}$. Then for convenience, we define

$$\Gamma (\bar{\omega}_{t+1}) = \psi \left( Y_{t+1}^-, B_{t+1}^+, R^e_{t+1}^+ \right)$$ (4.30)

$$\theta_t = \phi \left( Y_{t+1}^+, B_{t+1}^-, R^e_{t+1}^- \right)$$ (4.31)

The signs above the variables stand for the qualitative relationship with $\theta_t$ or $\Gamma (\bar{\omega}_{t+1})$ respectively. Then in the next section, the two implicit functions are log-linearized around the steady state as the only thing matters in a linear system is the first order derivatives.

4.2.6 Market Clearing Conditions and Equilibrium

In a competitive equilibrium, the markets for goods, labor, houses and loanable fund all clear. The goods market clearing condition is:

$$Y_t = C_t + C^e_t + q_t I_t$$ (4.32)

The housing market clearing condition is:

$$\bar{H} = H^e_t + H^e_t$$ (4.33)
where $\bar{H}$ is the fixed supply of houses.

The market clearing condition for loanable fund market is:

$$B_{t+1} = D_{t+1}$$  \hspace{1cm} (4.34)

A competitive equilibrium then can be defined as sequences of prices \(\{W_t, q_t, P^h_t, R_t, R^e_t\}_{t=0}^\infty\) and sequences of allocations \(\{Y_t, C_t, C^e_t, I_t, L_t, \theta_t, \Gamma(\tilde{\omega}_t), D_t, B_t, K_t, H^e_t, H^f_t\}_{t=0}^\infty\) such that (i) taking prices as given, the allocations solve the optimizing problems for households, entrepreneurs and financial intermediaries and (ii) all markets clear. A full characterization of the equilibrium is presented in Appendix A.

Appendix B describes the steady state. Let hatted variables denote percentage deviations from the steady state, and those without subscript time index denote steady state values. The model can be reduced to a log-linearized system described in Appendix C and D.

### 4.3 Model Simulations

In this section, the results of some numerical experiments are presented. To compare, a benchmark model is developed. This benchmark model shut-off the optimal loan-to-value ratio channel. Instead of optimally determining a loan-to-value ratio, financial intermediaries in the benchmark model fix the loan-to-value ratio to its steady state value. For convenience, in the following parts, the model developed in the last section is referred to as the endogenous loan-to-value ratio model (endo. LTV in short), the benchmark model is referred to as the exogenous loan-to-value ratio model (exo. LTV in short). The simulation results show that a model with endogenous loan-to-value ratio can generate much larger and more persistence impulse response to productivity shocks. The first and second order moments generated by the two models are also presented and compared to the US data. In general, the endogenous loan-to-value ratio model fits the data better than the benchmark model. Especially, housing price movements is much more volatile in the endogenous loan-to-value model. This again suggests that the endogenous LTV ratio mechanism is capable to increase the explanation power of flexible price models on volatile asset price behaviors.

\textsuperscript{5}Mathematically, in the benchmark model, equation (4.31) is taken away, and $\theta_t$ equals to its steady state value $\theta$ in all equations characterize the competitive equilibrium.
4. LEVERAGE CYCLES AND HOUSING PRICES

4.3.1 Model Parameterizations

Table 1 presents the parameter values. Consistent with standard real business cycle models, the values of \((1 - \alpha)\), \(\delta\) and \(\rho_z\) are set as (or around) 0.67, 0.025 and 0.95 respectively. Consistent with the estimation of Iacoviello (2005), the values for \(\alpha \eta\) and \(\alpha (1 - \eta)\) are set as 0.3 and 0.03 respectively. The discounting factor of households \(\beta\) is calibrated such that the quarterly risk free interest rate equals to 1.01%. Consistent with Kiyotaki and Moore (1997) and Iacoviello (2005), the discounting factor of entrepreneurs \(\gamma = 0.96\). The marginal adjust cost \(\chi\) is estimated by Christensen and Dib (2008) as 0.58. The parameter \(\xi_l\) for leisure \((1 - L_t)\) is calibrated such that the steady state of labor is 1/3. Consistent with Iacoviello (2005), the value of \(\xi_H\) is calibrated such that the housing-output ratio \((P_h \bar{H}) / Y\) is 6.6. The steady state bankruptcy rate of \(\Gamma (\bar{\omega}_t)\) is set as 0.05, which is the long-run average bankruptcy rate in US. The bankruptcy cost \((1 - \mu)\) is calibrated such that the steady state value of the quarterly risky interest rate \(R_e\) equals to 1.03, which is the long-run average risky interest rate in US. Consistent with chapter 2, the steady state value of the loan-to-value ratio \(\theta\) is set as 0.3.

<table>
<thead>
<tr>
<th>parameter</th>
<th>notation</th>
<th>value</th>
</tr>
</thead>
<tbody>
<tr>
<td>household’s discount factor</td>
<td>(\beta)</td>
<td>0.99</td>
</tr>
<tr>
<td>entrepreneur’s discount factor</td>
<td>(\gamma)</td>
<td>0.8</td>
</tr>
<tr>
<td>depreciation ratio</td>
<td>(\delta)</td>
<td>0.025</td>
</tr>
<tr>
<td>total labor income share</td>
<td>((1 - \alpha))</td>
<td>0.67</td>
</tr>
<tr>
<td>total capital income share</td>
<td>(\alpha (1 - \eta))</td>
<td>0.3</td>
</tr>
<tr>
<td>total housing income share</td>
<td>(\alpha \eta)</td>
<td>0.03</td>
</tr>
<tr>
<td>marginal adjustment cost</td>
<td>(\chi)</td>
<td>0.58</td>
</tr>
<tr>
<td>parameter for leisure</td>
<td>(\xi)</td>
<td>0.7698</td>
</tr>
<tr>
<td>parameter for housing service</td>
<td>(\xi_H)</td>
<td>0.1</td>
</tr>
<tr>
<td>steady state LTV ratio</td>
<td>(\theta)</td>
<td>0.94</td>
</tr>
<tr>
<td>bankruptcy cost</td>
<td>(1 - \mu)</td>
<td>0.27</td>
</tr>
<tr>
<td>persistence of shock</td>
<td>(\rho_z)</td>
<td>0.95</td>
</tr>
<tr>
<td>Slope of (\Gamma (\bar{\omega}_t))</td>
<td>(\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t}) \bigg</td>
<td>_{\bar{\omega}_t = \bar{\omega}}</td>
</tr>
</tbody>
</table>

Table 1. Parameter Values

The value of the first order derivative of the distribution function of \(\Gamma (\omega_t)\) with respect to \(\omega_t\) taking value at the steady state \(\frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t}\) \bigg|_{\bar{\omega}_t = \bar{\omega}} is set as 0.0005 in the first
4.3 Model Simulations

place. As shown in appendix D, the value of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \) together with steady state values of \( \theta, Y, K, B \) and \( R^e \) determines the values of \( G_y, G_b, G_r \) and \( T_y, T_b, T_r \), the log-linearized parameters of the two key equations (4.30) and (4.31). And the higher \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \), the higher absolute values of \( G_y, G_b \) and \( G_r \). The value of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \) is set very low so that the values for parameters \( G_y, G_b \) and \( G_r \) are reasonable in the sense that they are neither too large nor too small. The values of \( G_y, G_b, G_r \) and \( T_y, T_b, T_r \) are reported in appendix E.

In appendix E, sensitivity analysis is conducted with respect to different values of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \). It shows that as long as the value of \( \frac{\partial \Gamma (\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \) is quite low, the simulation results are qualitatively robust.

Finally, it is worth mentioning that with the calibrations above, the two assumptions about the shadow price of houses (equations (4.17) and (4.18)) are satisfied.

4.3.2 Simulation Results

Table 2 presents the standard deviations relative to output for the data and the two models. All two models tend to over-predict the volatility of consumption \( C_t \) but under-predict the volatilities of other variables, though in general, the model with endogenous LTV ratio matches the data slightly better. Especially, the endogenous LTV ratio model produces much higher volatilities in asset prices (Tobin’s q \( q_t \) and housing price \( P^h_t \)) than the exogenous LTV ratio model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>data</th>
<th>Endo. LTV (% of data)</th>
<th>Exo. LTV (% of data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>consumption</td>
<td>0.45</td>
<td>123</td>
<td>144</td>
</tr>
<tr>
<td>investment, I</td>
<td>1.86</td>
<td>82</td>
<td>65</td>
</tr>
<tr>
<td>Labor, L</td>
<td>0.53</td>
<td>49</td>
<td>54</td>
</tr>
<tr>
<td>Tobin’s q</td>
<td>6.94</td>
<td>17</td>
<td>9</td>
</tr>
<tr>
<td>housing price, ( P^h )</td>
<td>1.28</td>
<td>84</td>
<td>60</td>
</tr>
</tbody>
</table>

Table 2. Percentage standard deviation relative to output

Note: here consumption = \( C + C^e \). This definition applies to all the tables in this section.

Table 3 presents the contemporaneous correlations between main variables for the data and the two models. The two models over-predicts all correlations, though the endogenous LTV ratio model matches the data slightly better.
4. LEVERAGE CYCLES AND HOUSING PRICES

<table>
<thead>
<tr>
<th>data</th>
<th>Endo. LTV (% of data)</th>
<th>Exo. LTV (% of data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Y, C)</td>
<td>0.95</td>
<td>104</td>
</tr>
<tr>
<td>(Y, I)</td>
<td>0.96</td>
<td>102</td>
</tr>
<tr>
<td>(Y, B)</td>
<td>0.67</td>
<td>185</td>
</tr>
<tr>
<td>(B, q)</td>
<td>0.54</td>
<td>146</td>
</tr>
<tr>
<td>(B, (P^h))</td>
<td>0.58</td>
<td>153</td>
</tr>
</tbody>
</table>

Table 3, Contemporaneous correlation

In terms of first-order autocorrelations, as one can see from Table 4, in general, the two models over-predict the autocorrelations of all main variables. However, the endogenous LTV ratio model performs slightly better than the exogenous LTV ratio model.

<table>
<thead>
<tr>
<th>data</th>
<th>Endo. LTV (% of data)</th>
<th>Exo. LTV (% of data)</th>
</tr>
</thead>
<tbody>
<tr>
<td>output, Y</td>
<td>0.98</td>
<td>102</td>
</tr>
<tr>
<td>consumption, C</td>
<td>0.90</td>
<td>105</td>
</tr>
<tr>
<td>investment, I</td>
<td>0.96</td>
<td>107</td>
</tr>
<tr>
<td>bank loan, B</td>
<td>0.92</td>
<td>103</td>
</tr>
<tr>
<td>Tobin’s q</td>
<td>0.89</td>
<td>103</td>
</tr>
<tr>
<td>housing price, (P^h)</td>
<td>0.83</td>
<td>108</td>
</tr>
</tbody>
</table>

Table 4. First-order autocorrelations

To summarize, the credit constraint models tend to over-predict the first and second order moments of variables except for the volatility of asset prices. However, the credit constraint model with endogenous LTV ratio fits the data slightly better in general. In particular, the endogenous LTV ratio model can produce much more volatile asset prices (\(q_t\) and \(P^h_t\) in this chapter) movements. This is an significant improvement considering that the standard real business cycles are notorious for producing low volatility of asset prices.
Figures 1 - 4 are impulse responses of main variables in response to a 1% positive productivity shock. For both models, the bankruptcy rate $\Gamma (\bar{\omega}_t)$ (top right on figure 2) decreases immediately after the shock. To exploit this opportunity and earn more expected returns, banks in the endogenous LTV ratio model increase the LTV ratio $\theta_t$ (bottom right on figure 1). By assumption, there is no change of $\theta_t$ in the exogenous LTV ratio model. Compared to the exogenous LTV ratio model, the increase of $\theta_t$ in the endogenous LTV ratio model leads to a higher production $Y_t$, a much higher capital accumulation $K_t$, and a higher consumption $C_t$ by relaxing the credit constraint $(B_{t+1} \leq \theta_tE_t[\bar{q}_{t+1}K_{t+1} + P^h_{t+1}H^e_{t+1}] - \frac{1}{R_{t+1}})$ further, which spurs a much larger bank loans to entrepreneurs.
4. LEVERAGE CYCLES AND HOUSING PRICES

There are a few interesting features reflected in these figures. In figure 2, the bankruptcy probability $\Gamma_t$ decreases more in the exogenous LTV ratio model than in the endogenous LTV ratio model. This reflects the fact that in the endogenous LTV ratio model, banks exploit the new investment opportunity and increase their loan to entrepreneurs for more expected returns. This puts more stakes at risks and tends to increase the bankruptcy rate, as stated in equation (??). Entrepreneurs invest more on capital with more bank loans, which reduces the return to capital $R^k_t$ because of diminishing marginal return to capital, as shown on the top left on figure 2. It can be interpreted that by allowing banks optimally setting a loan-to-value ratio, the economy operates more efficient in the sense that no opportunity is wasted. In an economy with endogenous LTV ratio, the capital accumulation as well as production increased much larger than in the exogenous LTV ratio model.

In addition, more bank loans in the endogenous LTV ratio model also leads to a higher risk premium ($R^e_t - R_t$), as shown in the bottom right on figure 2. This is consistent with the data as well as intuition. In a world of uncertainty, more lending
puts more money at risk (a higher bankruptcy chances), which require a higher risk premium to compensate the extra risks.

Figure 3. impulse response to a 1% positive TFP shock

Figure 3 presents the impulse responses of asset prices, Tobin’s Q $q_t$ and housing price $P^h_t$. The endogenous LTV ratio model produces larger responses in asset prices than the exogenous LTV model. With an endogenous LTV ratio, the credit constraint is relaxed further, which leads to higher borrowing ($B_t$) from the banks, as shown on bottom left of figure 2. With more fund available, entrepreneurs can invest more on capital stock and buy more houses to produce. Higher demand pushes up the prices higher.
4. LEVERAGE CYCLES AND HOUSING PRICES

Figure 4 presents the responses of house holdings of households and entrepreneurs. Since for credit constrained entrepreneurs, houses are qualified as collateral, they have extra values for entrepreneurs. When more investment opportunity appears, entrepreneurs are more desired to acquire houses than households. Since the supply of houses is fixed, house holdings of entrepreneurs in both models increase accompanying an equivalent decrease in the house holdings of households, as shown in figure 4. With an endogenous LTV ratio which increases during a boom, the “collateral premium” described above is even larger. This explains why house holdings of entrepreneurs (households) increase (decrease) more in the endogenous LTV ratio model. This mechanism can be explained more clearly with the Euler equation with respect to $H_{t+1}^c$:

$$p_h^t = E_t \left\{ \gamma \frac{C_e^t}{C_{t+1}} \left[ \alpha \eta Y_{t+1} H_{t+1}^c + \mu_t p_h^{t+1} \right] \right\}$$

(4.35)

where $\mu_t = \gamma \frac{C_e^t}{C_{t+1}} (1 - \theta_t) + \theta_t \frac{1}{R_{t+1}}$ is the shadow price of credit constraint. Notice this equation is just another form of equation (4.16). This Euler equation indicates
4.4 Conclusions

that, the marginal cost to acquire one unit of houses $p_t^h$ equals to its marginal product plus the collateral premium: $\mu_t p_t^{h+1}$. Since with our calibrations, the assumptions that $\gamma + (\frac{1}{R} - \gamma) \theta > 0$ and $(\frac{1}{R} - \gamma) > 0$ (equations (4.17) and (4.18)) are both satisfied, therefore the shadow price $\mu_t p_t^{h+1}$ is positive around the steady state, and a larger $\theta_t$ causes a higher collateral premium. As a result, in the endogenous LTV ratio model, entrepreneurs hold more houses than in the exogenous LTV ratio model when they are facing productivity shocks.

4.4 Conclusions

This chapter extends the model of the last chapter to include one more collateral asset: the real estate. The aim is to study the effects of an endogenous loan-to-value (LTV) ratio on the behaviors of housing price in a real dynamic stochastic general equilibrium model with houses as collateral assets.

The simulation results show that compared with a constant LTV ratio model, the endogenous LTV ratio model produces more significant and persistent impulse responses of all main variables to TFP shocks. The first and second moments of the endogenous LTV ratio model fit the data slightly better than the exogenous LTV ratio model. More importantly, asset prices, including both Tobin’s Q and the housing price, are much more volatile in the endogenous LTV ratio model. This shows that the endogenous LTV ratio improves the ability of flexible price models in explaining volatile asset price movements.

The results suggest that the endogenous loan-to-value ratio acts like an accelerator to the financial accelerator incurred by credit constraints and a constant loan-to-value ratio model might underestimates the amplifying effects of credit constraints. Considering the facts that mortgage loans accounts for more than 1/3 of the banking assets and the loan-to-value ratio (1 - down-payment ratio) is pro-cyclical, the endogenous loan-to-value ratio model has the potential to better explain the behaviors of housing prices.
4. LEVERAGE CYCLES AND HOUSING PRICES

4.5 Appendix A: Characterization of Equilibrium

labor supply equation:
\[ \frac{\xi}{1 - L_t} = W_t \frac{1}{C_t} \quad (4.1) \]

Euler equation with respect to \( D_{t+1} \):
\[ \frac{1}{C_t} = \beta E_t \left[ \frac{1}{C_{t+1} - R_{t+1}} \right] \quad (4.2) \]

Euler equation with respect to \( H_{t+1}^c \):
\[ \frac{1}{C_t} p_t^h = \beta E_t \left[ \frac{1}{C_{t+1} + \xi_h} \frac{1}{H_{t+1}^c} \right] \quad (4.3) \]

labor demand equation:
\[ W_t = (1 - \alpha) \frac{Y_t}{L_t} \quad (4.4) \]

Euler equation with respect to \( K_{t+1} \):
\[ \frac{1}{C_t} \left[ q_t - \theta_t E_t q_{t+1} \frac{1}{R_{t+1}^c} \right] = \gamma E_t \frac{1}{C_{t+1}} \left[ q_{t+1} (1 - \delta - \theta_t) + \alpha (1 - \phi) \frac{Y_{t+1}}{K_{t+1}} \right] \quad (4.5) \]

Euler equation with respect to \( H_{t+1}^e \):
\[ \frac{1}{C_t} \left[ p_t^e_h - \theta_t E_t p_{t+1}^h \frac{1}{R_{t+1}^e} \right] = \gamma E_t \frac{1}{C_{t+1}^e} \left[ p_{t+1}^h (1 - \theta_t) + \alpha \phi \frac{Y_{t+1}}{H_{t+1}^e} \right] \quad (4.6) \]

return to capital:
\[ R_t^k = \frac{\alpha Y_t}{K_t} + (1 - \delta) q_t \quad (4.7) \]

Tobin’s Q:
\[ q_t = 1 + \chi \left( \frac{I_t}{K_t} - \delta \right) \quad (4.8) \]

Zero-Profit Condition:
\[ [1 - \Gamma (\bar{\omega}_{t+1})] R_{t+1}^c B_{t+1} + \mu \Gamma (\bar{\omega}_{t+1}) E_t \left[ (1 - \delta) q_{t+1} K_{t+1} + p_{t+1}^h H_{t+1}^c \right] = R_{t+1} D_{t+1} \quad (4.9) \]

optimal loan-to-value ratio:
\[ \theta_t = \phi (Y_{t+1}, B_{t+1}, R_{t+1}^c) \quad (4.10) \]

bankruptcy probability function:
\[ \Gamma (\bar{\omega}_{t+1}) = \psi (Y_{t+1}, B_{t+1}, R_{t+1}^c) \quad (4.11) \]
4.6 Appendix B: Steady State

credit market clear condition:
\[ B_t = D_t \] (4.12)

production function:
\[ Y_t = z_t \left[ (H_t^e)^\phi K_t^{1-\phi} \right]^\alpha L_t^{1-\alpha} \] (4.13)

consumption goods market clear condition:
\[ Y_t = C_t + C_t^e + q_t I_t \] (4.14)

housing market clear condition:
\[ H = H_t^e + H_t^c \] (4.15)

credit constraint:
\[ B_{t+1} = \theta_t E_t \left[ q_{t+1} K_{t+1} + p_{t+1}^h H_{t+1}^c \right] \frac{1}{R_{t+1}} \] (4.16)

budget constraint for households:
\[ W_t L_t + p_t^h H_t^c + R_t D_t = p_t^h H_{t+1}^c + C_t + D_{t+1} \] (4.17)

evolution of capital:
\[ K_{t+1} = (1 - \delta) K_t + I_t \] (4.18)

Exogenous Shock:
\[ \ln (z_{t+1}) = \rho z \ln (z_t) + \varepsilon_{t+1} \] (4.19)

4.6 Appendix B: Steady State

\[ P^h H^c = \beta \left[ P^h H^c + \xi_h C \right] \] (4.1)

\[ 1 - \theta \frac{1}{R^c} = \gamma \left[ 1 - \delta - \theta + \alpha (1 - \phi) \frac{Y}{K} \right] \] (4.2)

\[ P^h \left( 1 - \theta \frac{1}{R^c} \right) = \gamma \left[ P^h (1 - \theta) + \alpha \phi \frac{Y}{H^c} \right] \] (4.3)

\[ R^k = \alpha (1 - \phi) \frac{Y}{K} + 1 - \delta \] (4.4)
4. LEVERAGE CYCLES AND HOUSING PRICES

\[ Y = \left( K^{1-\phi} (H^e)\phi \right)^\alpha L^{1-\alpha} \] (4.5)

\[ B = \theta \left( \left( P^h H^e + K \right) \frac{1}{R^e} \right) \] (4.6)

\[ B = D \] (4.7)

\[ [1 - \Gamma(\bar{\omega})] R^e B + \mu \Gamma(\bar{\omega})(1 - \delta) K = RD \] (4.8)

\[ \Gamma(\bar{\omega}) = \int_0^\infty \bar{\omega} dF(\bar{\omega}) \] (4.9)

\[ \bar{\omega} = (1 - \alpha) + \frac{R^e B}{Y} \] (4.10)

\[ \theta = \frac{1 - \Gamma(\bar{\omega})}{\partial \Gamma(\bar{\omega}) / \partial \bar{\omega}} + \mu (1 - \delta) \] (4.11)

\[ W = (1 - \alpha) \frac{Y}{L} \] (4.12)

\[ [1 - \Gamma(\bar{\omega})] R^e B + \mu \Gamma(\bar{\omega}) \left[ (1 - \delta) K + P^h H^e \right] = RD \] (4.13)

\[ WL + DR = C + D \] (4.14)

4.7 Appendix C: Log-linearized System

\[ \frac{L}{1-L} \dot{l}_t = -\dot{c}_t + \dot{w}_t \] (4.1)

\[ -\dot{c}_t = E_t [-\dot{c}_{t+1} + \dot{r}_{t+1}] \] (4.2)

\[ \frac{p^h}{C} \left( \dot{p}^h_t - \dot{c}_t \right) = \beta E_t \left[ \frac{p^h}{C} \left( \dot{p}^h_{t+1} - \dot{c}_{t+1} \right) - \frac{\xi_h}{H_e} \dot{H}^e_{t+1} \right] \] (4.3)
4.7 Appendix C: Log-linearized System

\[ \dot{y}_t = \dot{w}_t + \dot{l}_t \]  
\[ (\theta \frac{1}{R^e} - 1) \dot{c}_t^e + \dot{q}_t - \theta \frac{1}{R^e} (\dot{\theta}_t + \dot{q}_t) = (4.4) \]

\[ (\theta \frac{1}{R^e} - 1) \dot{c}_t^{e+1} + \gamma \left[ \alpha (1 - \phi) \frac{Y}{K} \left( \dot{y}_{t+1} - \dot{k}_{t+1} \right) + (1 - \delta) \dot{q}_{t+1} - \theta \dot{\theta}_t \right] \]  
\[ = (4.5) \]

\[ \left( \theta \frac{P^h}{R^e} - 1 \right) \dot{c}_t^e + P^h \dot{p}_t^h - \theta \frac{P^h}{R^e} (\dot{\theta}_t + \dot{p}_t^h) = (4.6) \]

\[ R^h \dot{r}_t^k = \alpha \frac{Y}{K} \left( \dot{y}_t - \dot{k}_t \right) + (1 - \delta) \dot{q}_t - R^h \dot{q}_{t-1} \]  
\[ = (4.7) \]

\[ \dot{q}_t = \chi \left( \dot{\lambda}_t - \dot{k}_t \right) \]  
\[ = (4.8) \]

\[ -\Gamma (\hat{\omega}) R^e B \hat{\Gamma} (\hat{\omega}_{t+1}) + \left[ 1 - \Gamma (\hat{\omega}) \right] R^e B \left( \dot{r}_{t+1} + \dot{\theta}_{t+1} \right) \] \[ = (4.9) \]

\[ \hat{\Gamma} (\hat{\omega}_t) = G_y \hat{y}_t + G_w \hat{w}_t + G_r \dot{r}_t^e \]  
\[ = (4.10) \]

\[ \dot{\theta}_t = T_y \dot{y}_{t+1} + T_w \dot{w}_{t+1} + T_r \dot{r}_t^e \]  
\[ = (4.11) \]

\[ \dot{\hat{\omega}}_t = \dot{\hat{\lambda}}_t + \alpha \dot{\hat{k}}_t + (1 - \alpha) \dot{l}_t \]  
\[ = (4.12) \]

\[ \dot{\hat{\omega}}_t = \dot{\hat{\lambda}}_t + \alpha \dot{\hat{k}}_t + (1 - \alpha) \dot{l}_t \]  
\[ = (4.13) \]

\[ \dot{\hat{y}}_t = C \dot{\hat{c}}_t + \frac{C^e}{Y} \dot{\hat{c}}_t + \frac{I}{Y} \dot{\hat{i}}_t \]  
\[ = (4.14) \]
4. LEVERAGE CYCLES AND HOUSING PRICES

\[ B \hat{b}_{t+1} = \theta \left( K + P^h H^e \right) \frac{1}{R^e} \left( \hat{\theta}_t - \hat{r}^e_{t+1} \right) \]  
\[ + \theta \frac{1}{R^e} \left[ K \left( \hat{q}_{t+1} + \hat{k}_{t+1} \right) + P^h H^e \left( \hat{p}^h_{t+1} + \hat{h}^e_{t+1} \right) \right] \]  
\[ (4.15) \]

\[ \hat{b}_{t+1} = \hat{d}_{t+1} \]  
\[ (4.17) \]

\[ WL \left( \hat{w}_t + \hat{l}_t \right) + RD \left( \hat{r}_t + \hat{d}_t \right) = C\hat{c}_t + D\hat{d}_{t+1} \]  
\[ (4.18) \]

\[ k_{t+1} = (1 - \delta) k_t + \delta \hat{k}_t \]  
\[ (4.19) \]

\[ \hat{z}_{t+1} = \rho \hat{z}_t + \varepsilon^z_{t+1} \]  
\[ (4.20) \]

4.8 Appendix D: log-linearization of \( \Gamma \left( \bar{\omega}_{t+1} \right) \) and \( \theta_t \)

From section 3.3, we know that, \( \Gamma \left( \bar{\omega}_{t+1} \right) \) and \( \theta_t \) are implicit functions of \( Y_{t+1}, B_{t+1}, \) and \( R^e_{t+1}. \) (Equations (4.31) and (4.30) in the text.):

\[ \Gamma \left( \bar{\omega}_t \right) = \psi \left( \bar{Y}_t, \bar{B}_t, \bar{R}^e_t \right) \]  
\[ (4.21) \]

\[ \theta_t = \phi \left( Y^+_t, B^+_t, R^e_{t+1} \right) \]  
\[ (4.22) \]

We take Taylor’s first-order extensions on the implicit functions of \( \Gamma \left( \bar{\omega}_t \right) = \psi \left( Y_t, B_t, R^e_t \right) \) and \( \theta_t = \phi \left( Y_{t+1}, B_{t+1}, R^e_{t+1} \right) \) with respect to \( Y_t, B_t \) and \( R^e_t \) at the steady state, then use the log-linearization approximation formula, we have:

\[ \hat{\Gamma} \left( \bar{\omega}_t \right) = G_y \hat{y}_t + G_b \hat{b}_t + G_r \hat{r}^e_t \]  
\[ (4.23) \]

\[ \hat{\theta}_t = T_y \hat{y}_{t+1} + T_b \hat{b}_{t+1} + T_r \hat{r}^e_{t+1} \]  
\[ (4.24) \]
4.8 Appendix D: log-linearization of $\Gamma(\bar{\omega}_{t+1})$ and $\theta_t$

where

$$G_y = \frac{1}{\psi(Y_t, B_t, R_t^e)} \left. \frac{\partial \psi(Y_t, B_t, R_t^e)}{\partial Y_t} \right|_{Y_t = Y, B_t = B, R_t^e = R}$$

$$= \left. \frac{1}{\psi(Y_t, B_t, R_t^e)} \left( - \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \frac{R_t^e B_t Y_t^2}{Y_t} \right) \right|_{Y_t = Y, B_t = B, R_t^e = R}$$

$$= -\frac{1}{\Gamma(\bar{\omega})} \frac{R^e B}{Y^2} \left. \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \right|_{\bar{\omega}_t = \bar{\omega}}.$$

The second step uses the result from section 3.3: equation [3.27]. Similarly, we can get the values for the other five parameters using the properties of $\Gamma(\bar{\omega}_{t+1})$ and $\theta_t$ from the section 3.3:

$$G_b = \frac{1}{\Gamma(\bar{\omega})} \left. \frac{\partial \psi(Y_t, B_t, R_t^e)}{\partial B_t} \right|_{Y_t = Y, B_t = B, R_t^e = R}$$

$$= \left. \frac{1}{\Gamma(\bar{\omega})} \frac{R^e}{Y} \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \right|_{\bar{\omega}_t = \bar{\omega}} \quad (4.25)$$

$$G_r = \frac{1}{\Gamma(\bar{\omega})} \left. \frac{\partial \psi(Y_t, B_t, R_t^e)}{\partial R_t^e} \right|_{Y_t = Y, B_t = B, R_t^e = R}$$

$$= \left. \frac{1}{\Gamma(\bar{\omega})} \frac{B}{Y} \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \right|_{\bar{\omega}_t = \bar{\omega}} \quad (4.26)$$

$$T_y = \frac{1}{\theta} \left. \frac{\partial \phi(Y_{t+1}, B_{t+1}, R_{t+1}^e)}{\partial Y_t} \right|_{Y_{t+1} = Y, B_{t+1} = B, R_{t+1}^e = R}$$

$$= \frac{1}{\theta} \frac{R^e B}{Y (K + P^H H^e)} \quad (4.27)$$

$$T_b = \frac{1}{\theta} \left. \frac{\partial \phi(Y_{t+1}, B_{t+1}, R_{t+1}^e)}{\partial Y_t} \right|_{Y_{t+1} = Y, B_{t+1} = B, R_{t+1}^e = R}$$

$$= -\frac{1}{\theta} \frac{R^e}{K + P^H H^e} \quad (4.28)$$

$$T_r = \frac{1}{\theta} \left. \frac{\partial \phi(Y_{t+1}, B_{t+1}, R_{t+1}^e)}{\partial Y_t} \right|_{Y_{t+1} = Y, B_{t+1} = B, R_{t+1}^e = R}$$

$$= -\frac{1}{\theta} \frac{B}{K + P^H H^e} \quad (4.29)$$

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And it is clearly from the formulas above that the values of these six parameters depend on the steady state values of \( \Gamma(\bar{\omega}_t) \), \( \theta_t \), \( Y_t \), \( B_t \), \( K_t \), \( P^h H^e \) and \( R^e_t \) (whose values are listed in appendix B) as well as \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \).

4.9 Appendix E: Sensitivity Analysis

This appendix reports the sensitivity analysis for the six parameters in these two equations:

\[
\hat{\Gamma}(\bar{\omega}_t) = G_y \hat{y}_t + G_b \hat{b}_t + G_r \hat{r}_t^e \tag{4.30}
\]

\[
\hat{\theta}_t = T_y \hat{y}_{t+1} + T_b \hat{b}_{t+1} + T_r \hat{r}_{t+1}^e \tag{4.31}
\]

Table 1 reports the values of \( G_y, G_b, G_r \) and \( T_y, T_b, T_r \) for different values of \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \). It is clear from this table that the higher value of \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \), the larger the gap between \( G_y \) and \( G_b \), \( G_r \) as well as \( T_y \) and \( T_b, T_r \).

<table>
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<th>( \frac{\partial \Gamma(\bar{\omega})}{\partial \bar{\omega}} )</th>
<th>( G_y )</th>
<th>( G_b )</th>
<th>( G_r )</th>
<th>( T_y )</th>
<th>( T_b )</th>
<th>( T_r )</th>
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<td>3.5671</td>
<td>1.3424</td>
<td>-0.0053</td>
<td>-0.9204</td>
</tr>
</tbody>
</table>

Table E1: \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \) and the six parameters

Figure E1 reports the impulse response of \( \theta_t \) and \( Y_t \) for different values of \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} \). The first row is for \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} = 0.0005 \), the second row is for \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} = 0.007 \). The results of \( \frac{\partial \Gamma(\bar{\omega}_t)}{\partial \bar{\omega}_t} \bigg|_{\bar{\omega}_t=\bar{\omega}} = 0.005 \) are reported in the main text.
4.9 Appendix E: Sensitivity Analysis

Figure E1. Impulse response of $Y_t$ and $\theta_t$ for different values of $\frac{\partial \Gamma (\omega_t)}{\partial \omega_t} \bigg|_{\omega_t=\bar{\omega}}$.

Figure E2 presents the impulse response of $\Gamma_t$ for different values of $\frac{\partial \Gamma (\omega_t)}{\partial \omega_t} \bigg|_{\omega_t=\bar{\omega}}$. The first row is for $\frac{\partial \Gamma (\omega_t)}{\partial \omega_t} \bigg|_{\omega_t=\bar{\omega}} = 0.005$, the second row is for $\frac{\partial \Gamma (\omega_t)}{\partial \omega_t} \bigg|_{\omega_t=\bar{\omega}} = 0.007$. The results of $\frac{\partial \Gamma (\omega_t)}{\partial \omega_t} \bigg|_{\omega_t=\bar{\omega}} = 0.005$ are reported in the main text.
4. LEVERAGE CYCLES AND HOUSING PRICES

The sensitivity analysis shows that, first, the qualitative relationship is robust to the values of \( \frac{\partial \Gamma (\hat{\omega}_t)}{\partial \hat{\omega}_t} \bigg|_{\hat{\omega}_t = \bar{\omega}} \) as long as it is quite low.
5

Conclusion

This thesis attempts to improve the ability of flexible price models to produce higher asset price volatilities by introducing frictions in the money and credit markets.

The second chapter integrates a segmented money market with a credit constraint into a stochastic dynamic general equilibrium model to explain the volatile fluctuations of asset prices in a stable macroeconomic environment. The integrated model, though sticks to the flexible price assumption, is capable to produce similar impulse responses to monetary shocks with sticky price models in the sense that in response to positive monetary shocks, production and asset prices increase dramatically. In addition, the simulations show that the integrated model matches the data better in general than a standard cash-in-advance model or a model with only credit constraint or segmented money market. In particular, the integrated model produces much larger asset price volatilities, a fact explained poorly by standard real business models.

The third chapter develops a real dynamic stochastic general equilibrium (DSGE) model with a credit constraint of which the loan-to-value (LTV) ratio is endogenized. Banks are allowed to optimally determine the loan-to-value (LTV) ratio. A positive productivity shock motivates banks to increase the optimal LTV ratio. The endogenous LTV ratio functions as the accelerator of the financial accelerator incurred by credit constraints. Compared with a constant LTV ratio model, the endogenous LTV ratio model produces much more significant and persistent impulse responses to productivity shocks. Asset price movements are much more volatile in the endogenous LTV ratio model as well, which is an improvement to match the data.
The fourth chapter expands the models developed in chapter 3 to include houses as collateral and studies the impacts of endogenous LTV ratio on housing price behaviors in a real dynamic stochastic general equilibrium model. The simulations show that the endogenous LTV ratio can produce much larger volatilities in housing prices as well, therefore improve the ability of flexible price models in explaining volatile housing price movements.

A future research direction would be to combine segmented market, credit constraint with an endogenous loan-to-value ratio into one model to see whether this combination could produce even larger volatilities in asset prices.

After the global financial crisis, the interests of economists in developing a DSGE model with a serious (that is, micro-founded) financial sector have been surging. The financial sector in chapter 2 is just a tool to introduce financial frictions. The financial sectors in chapter 3 & 4 are more rigorous and can be seen as belonging to the new strand of literature on micro-founded financial sectors. However, the financial sectors in chapter 3 & 4 are still too simple and more micro-foundations are still needed. This would be on my future research agenda.
References


REFERENCES


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