Interaction of Hydra A Jets with the Intracluster Medium

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To my Parents...
Disclaimer

I hereby declare that the work in this thesis is original research work carried out by me. The work was undertaken between February 2010 and January 2015 at the Australian National University, Canberra. It has not been submitted in whole or in part for any other degree at this or any other university.

In this thesis, i) I Initialised all numerical models preparatory to conducting numerical simulations using publicly available code PLUTO. ii) I analysed the output from the simulations and prepared images and movies using the visualisation software VISIT. iii) I analysed the simulation output in order to interpret different features of the extragalactic radio source Hydra A. iv) In preparation for the simulation I estimated the jet kinetic power associated with the 50 kpc radio lobes of the Hydra A, using the 6cm VLA data presented in Taylor et al. (1990). v) I established pressure and density profiles for the intracluster medium of Hydra A, utilising the X-ray data published in David et al. (2001).

Chapter 3 : Fig. 3.1 was generated by using the 6cm VLA data of Hydra A kindly provided by Professor Gregory Taylor.

Chapter 4 : My supervisor, Prof. Geoffrey Bicknell, wrote the argument for the neglect of magnetic field in my simulations (§ 2.4.1). This was first appeared in our joint paper Nawaz et al. (2014).

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Abstract

The physics of jets from Active Galactic Nuclei (AGN) and their interaction with the interstellar medium (ISM) and intracluster medium (ICM) is an important research area of modern astronomy. Over the last few decades theoreticians have studied AGN jets both analytically and numerically. However, to date, a complete understanding on the jet energetics and composition, jet velocity, complex jet morphology and jet-ICM interaction is lacking. This thesis aims to understand the energetics and composition of the jet near its origin, its interaction with the galaxy and cluster, focusing on detailed models of the inner structure of the Hydra A radio source. Analysing radio observations of the inner lobes of Hydra A by Taylor et al. (1990), I confirm jet power estimates $\sim 10^{45}$ ergs s$^{-1}$ derived by Wise et al. (2007) from dynamical analysis of the X-ray cavities. With this result and a model for the galaxy atmosphere, I explore the jet-ICM interactions occurring on a scale of 20 kpc using relativistic hydrodynamic simulations. The key features of my modelling are that i) I identify the four bright knots in the northern jet at approximately 4, 7, 11 and 16 kpc (deprojected) from the radio core as biconical reconfinement shocks, which result when an over pressured jet starts to come into equilibrium with the galactic atmosphere ii) the curved morphology of the source and the turbulent transition of the jet to a plume are produced by the dynamical interaction of a precessing jet with the ICM.

I study the inner 10 kpc of the northern jet utilising two dimensional axisymmetric simulations. Through an extensive parameter space study I deduce that the jet velocity is approximately $0.8c$ at a distance 0.5 kpc from the black hole. The combined constraints of jet power, the observed jet radius profile along the jet, and the estimated jet pressure and jet velocity imply a value of the jet density parameter $\chi \approx 13$ for the northern jet.

To study the complex source morphology within 20 kpc (on the northern side) I generalise my axisymmetric model to a three dimensional jet-ICM interaction model incorporating jet precession. Utilising the jet parameters obtained from the best fit axisymmetric model, a range of precession periods and two values of the
precession angle I produce a set of 3D models. With the precessing jet model I successfully reproduce key features of the inner 20 kpc of the Hydra A northern jet: i) Four bright knots along the jet trajectory at approximately correct locations ii) The curvature of the jet within 10 kpc iii) Turbulent transition of the jet to a plume iv) A misaligned bright knot in the turbulent flaring zone. The best matching model for the Hydra A northern jet gives a precession period $\sim 1$ Myr and a precession angle $\sim 20^\circ$. A low Mach number $\approx 1.85$ of the forward shock indicates a gentle heating of the ICM by the source in its early stages.
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Jets from Active Galactic Nuclei (AGN) are collimated streams of magnetised plasma emanating from the centre of the AGN near the supermassive black hole (SMBH) at speeds close to the speed of light. They form large plumes or lobe structures extending tens to several hundreds of kilo parsecs in the intergalactic medium or intracluster medium (ICM). Many giant elliptical galaxies harbour SMBH (their typical mass being $\sim 10^5 - 10^{10} \, M_\odot$) at their centres and it is now believed that accretion onto the SMBH powers the bipolar jets. The fraction of galaxies that host radio-loud AGN is a sensitive function of the galaxy mass and for the central bright elliptical galaxies of galaxy clusters the fraction exceeds 30% (Best et al., 2005). The jets emanating from the central radio galaxies in clusters are also thought to be responsible for balancing the cooling of the ICM and preventing the occurrence of massive accretion flows of cooled gas (cooling flows). The AGN in a cluster therefore functions like a thermostat, regulating the cluster gas temperature, keeping it nearly isothermal at $\approx 10^7 - 8 \, K$ (Fabian, 2005).

Morphologically AGN radio sources are classified into two groups: i) Edge-darkened Fanaroff-Riley I (FRI) sources and ii) Edge-brightened Fanaroff-Riley II (FRII) sources. The lower-powered FRI sources have bright radio jets near the core which quickly decelerate and flare out to form large plumes. The deceleration and the turbulent transition of the jet can be caused by recollimation shocks and entrainment of the ambient medium by the jets (Bicknell, 1984). Interaction of the jet with
ambient clouds is another potential cause of jet deceleration (Perucho et al., 2014). On the other hand, higher-powered FRII jets remain supersonic and collimated and produce hot spots at the edge of the source. The division in jet power between FRI and FRII sources lies at approximately $\sim 10^{43}$ erg s$^{-1}$ (Bicknell, 1995; Ledlow & Owen, 1996) although there are sources of either class on either side of the divide. The precise reasons for the two morphological types of radio sources is not fully understood, but the rate of deceleration of the jets as they propagate through the ambient medium is an important factor (Bicknell, 1995; Kawakatu et al., 2009).

To understand the morphology of radio jets, lobes, and plumes on tens to hundreds of kpc scales it is vital to understand the energetics, composition of the jet and the dynamical interaction of the jet with the ISM/ICM near its origin. In this thesis, I present detailed models of knot formation and radial oscillations of jets (in particular, the northern jet), in the central 10 kpc of the Hydra A radio source. Combining this with a careful extrapolation of the ICM thermodynamic profile toward the core and a proper estimate of the pressure in the jet-fed lobe, I constrain the power, composition, density, and velocity of the jet near its origin. The constrained jet parameters are then used in three dimensional precessing jet simulations to understand the physics of the inner 20 kpc of the northern jet and constrain the precession period and precession angle. The results from this multifaceted approach provide a new reliable basis from which to perform large-scale simulations and understand mechanisms of energy and mass transport by AGN jets, and the inhibition of cooling flows in the ICM.

1.1. Bright knots

Bright knots are a prominent feature in many classical AGN jets, for example, M87 (Owen et al., 1989), Cygnus A (Steenbrugge & Blundell, 2007), Centaurus A (Goodger et al., 2010), and the spectacular source, Hydra A, which exhibits a high degree of S-symmetry of its structure (Taylor et al., 1990) (see Fig. 1.1). There is no general theory of the formation of bright knots that can be applied in any source. Rather, different interpretations of bright knots are appropriate for
1.1 Bright knots

Figure 1.1 VLA image of the Hydra A (Taylor et al., 1990), the M87 (Owen et al., 1980), the Cygnus A (Perley et al., 1984) and the Centaurus A (Burns et al., 1983). Bright knots, several of which are highlighted with arrows, can be clearly seen along the collimated jet.

different sources. In the following I describe the theories proposed in order to interpret the bright knots near the core of some prominent extragalactic jets.

1.1.1. Shock resulting from velocity variation

The first theoretical explanation for the bright knots in astrophysical jets was proposed by Rees (1978), who interpreted the bright knots of M87 as enhanced synchrotron emission from shocks resulting from a variable flow velocity. The conditions for the formation of shock waves in this theory are 1) A faster region of the jet plasma overtakes an slower region, and 2) The relative velocities are supersonic.

1.1.2. Jet-cloud interaction

Blandford & Koenigl (1979) presented an alternative shock model for the explanation of the knots of M87. According to their model the supersonic M87 jet hits
dense interstellar clouds of sufficiently large size along its path. This collision results in bow shaped shocks behind the cloud. The shock accelerated jet plasma downstream of the shocks are responsible for the bright knots. Using numerical models Coleman & Bicknell (1985) later modelled the interaction of a supersonic jet with a cylindrical cloud and showed the optical spectral index of the knots may be explained with this model.

1.1.3. Shocks by Kelvin-Helmholtz (KH) instabilities

Bicknell & Begelman (1996) proposed another shock model for the formation of the knots of M87 jet. They proposed that the bright knots in M87 are oblique shocks produced by helical modes of the KH instability, produced when a light jet interacts with the ambient medium. The increasing brightness of the knots with distance form the black hole was attributed to the increase in the shock strength due to the growth of the KH instability. They also showed that for a relativistic jet with Lorentz factor $\Gamma = 5 – 7$, the velocities of the shocks are consistent with the pattern speeds of the bright knots.

1.1.4. Plasmon model

The bright knots of AGN jets have also been modelled as blobs of magnetised plasma, plasmons, ejected periodically from the central black hole (Shklovskii, 1977, 1980). The plasmons of mass $\approx 0.1 M_\odot$ move with a relativistic velocity $\beta > 0.65$ through the dense ambient medium. Deceleration of the plasmons by the interaction with the ambient medium constantly accelerates the electrons within the cloud and forms the bright knots.

This model was used to interpret the moving knots of the superluminal quasar 3C 345 (Qian et al., 1992). Because of the motion of the knots, the plasmon model is also popular in the interpretation of observations of some protostellar jets (Goodson et al., 1997) and VLBI AGN jets (Hough, 2013).
1.1.5. Reconfinement shock model

![Diagram of supersonic jet dynamics](image)

Figure 1.2 Structures developed near the base of a supersonic jet. The initial over pressured jet (marked by the high pressure zone) expands freely in the environment. It quickly reaches pressure equilibrium, over-expands and collimated by the external pressure via a reconfinement shock. The change in flow direction caused by the reconfinement shocks are shown in two zones A and B. The reconfinement shock converges towards the jet axis to form a biconical shock structure. The recollimated jet becomes over-pressured again and the cycle repeats. The jet boundary follow the oscillations of the reconfinement shocks. Shock deceleration by a number of biconical shock makes the jet subsonic and a turbulence gradually develops.

The structures of reconfinement shocks in supersonic flows were first observed in laboratory jets more than a century ago (see Krehl (2009) for the history of supersonic laboratory jets). When a supersonic jet interacts with its surroundings, the dynamics of the jet is affected by the external pressure. Fig. 1.2 shows the structures developed near the base of a supersonic jet. The initially over-pressured jet (marked by high pressure zone) expands freely in the ambient medium. It soon reaches pressure equilibrium with the environment and is recollimated by the ambient pressure. Since the jet is supersonic the recollimation occurs through oblique reconfinement shocks. The reconfinement shocks change the flow direction as indicated at points A and B. The recollimated jet becomes over-pressured again and the cycle repeats. The reconfinement shocks periodically converge to either: i) points on the jet axis to form biconical shocks if the jet is only slightly over-pressured with respect to the ambient medium or, ii) planar shocks, known as Mach disks, transverse to the flow if the jet is highly over-pressured. The jet boundary also oscillates following the oscillation of the reconfinement shocks (see Fig. 1.2).

Norman et al. (1982) first drew attention to reconfinement shocks as an explanation for the bright knots of AGN jets. With a 2D hydrodynamical numerical model they explored the structures of a supersonic jet- i) Reconfiment shocks along the jet, ii) A working surface at the end of the jet, iii) A cocoon. They argued that
these structures could be the possible explanation for the following features of astrophysical jets respectively- i) Bright knots ii) Hot spots at the edge of the source iii) Radio lobes. Subsequently, Falle & Wilson (1985) showed (qualitatively) that the spacings of reconfinement shocks of numerical jet model closely matches with the knot spacing of M87. The reconfinement shock model has also been well explored in analytical form (Cántó et al., 1989; Kaiser & Alexander, 1997; Komissarov & Falle, 1997). Stawarz et al. (2006), for example, showed analytically that the HST 1 bright knot of M87 may be a reconfinement shock. From the study presented in this thesis I propose that the bright knots of the Hydra A northern jet are a consequence of reconfinement shocks that appear naturally in hydrodynamic models.

Interpreting bright knots as periodic reconfinement shocks is further motivated by the theoretical relationship between the natural wavelength of a non relativistic, supersonic flow and its Mach number described in Birkhoff & Zarantonello (1957). According to this relationship, the natural wavelength $\Lambda$ of a non-relativistic, supersonic jet with radius $r_{\text{jet}}$ and Mach number $M$, in near pressure equilibrium is given by

$$\frac{\Lambda}{r_{\text{jet}}} \approx 2.6 \sqrt{M^2 - 1}$$

(1.1)

This is indicative of the spacing between shocks of the jet.

Another feature of AGN jets that can be interpreted in terms of reconfinement shocks is the oscillating jet boundary. Oscillations of the jet boundary are a natural consequence of periodic reconfinement shocks (Prandtl, 1907). For example, Sanders (1983) applied a reconfining jet model to show that the periodic structure of the jet width of NGC 315 occurs as a result of the oscillation of the jet boundary resulting from the reconfinement shocks.

The key observed features in the inner 10 kpc structure of the the Hydra A northern jet are the two features that can be simultaneously explained by the reconfinement shock model: i) An oscillating jet boundary, and ii) The appearance of the bright knots at 3.7, 7.0, and 11.0 kpc (deprojected distances from the core)(see Figure 1.3). In this work, I exploit both the observed locations of the bright knots and the jet radius profile with distance from the core to constrain the jet parameters of the Hydra A.
1.2. Complex morphology of extragalactic radio sources

Morphologically, extragalactic radio sources have either straight or complex curved morphologies with C or S shaped symmetry\(^1\) (Zaninetti & van Horn, 1988).

In general, the bent structures of the C-symmetric sources (commonly known as head-tail sources) are attributed to the motion of the host galaxy with respect to the intergalactic medium (IGM) (Begelman et al., 1984; Morsony et al., 2013). The ram pressure resulting from the motion of the galaxy through the IGM causes the jets to bend in a direction opposite to their motion.

Two different theories have been proposed yet in order to explain the peculiar structure of the S (or X, or Z) symmetric sources:

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\(^1\)this is also referred to as X or Z symmetry.
1.2.1. Jet deflection by back flow and buoyancy:

Worrall et al. (1995) proposed a model for the peculiar winged morphology of NGC 326. According to their model, the backflowing jet plasma from the forward bow shock evolves buoyantly along the directions of steep ambient pressure gradient and forms the wings; the jets in the active lobe advance supersonically while the buoyant wings rise subsonically. Based on this model, Hodges-Kluck & Reynolds (2011) performed 3D simulations which produced X-shaped morphologies. They related the simulated source morphologies to some observed radio sources, such as, 3C 192, 3C 315, 4C -06.26 etc.

1.2.2. Jet precession

An alternative explanation for the S-symmetric morphologies is the dynamical interaction between a precessing jet and the ambient intracluster medium. The idea of jet precession was first introduced by Ekers et al. (1978) who interpreted the S-shaped structure of the radio morphology of NGC326 as a result of the precessional motion of the jets. Subsequently, utilising an analytical model, Gower et al. (1982) showed that the curved jet morphologies of a number of radio galaxies may be attributed to jet precession. In a similar fashion, (Klein et al., 1995) proposed a precessing jet model in order to explain the X-morphology of the source 0828+32.

1.2.3. Reasons for jet precession

It is generally accepted that extragalactic jets are emitted along the black hole spin axes. Hence, precession of the black hole is a natural explanation for the jet precession. There are two theories that relate the jet precession to the black hole precession.

Precession associated with binary black hole: Begelman et al. (1980) proposed a theory of jet precession caused by a binary black hole in the galactic core. If the spin axes of the binary black holes are not aligned with their total angular
momentum, both black holes will undergo geodesic precession about the total angular momentum. Using a ballistic jet precession model (Caproni et al., 2013) showed that the periodic variation of the structural position angle of the BL Lacertae 2200+420 could be attributed to a binary black hole at the galactic centre.

**Precession associated with the accretion disk:** If the spin axis of the black hole is misaligned with the angular momentum of the accretion disk, the disk surrounding to the hole is forced to realign with the black hole’s spin due to the combined effect of Lense-Thirring (LT) frame dragging and viscosity, known as the Bardeen-Petterson effect (Bardeen & Petterson, 1975). The short ranged LT frame dragging is effective only within a critical radius, the Bardeen-Petterson radius $r_{BP} \approx \text{few hundreds gravitational radii}$. Outside the Bardeen-Petterson radius the accretion disk retain its angular momentum. Viscous torques in the outer accretion disk force the black hole and the inner disk to precess until they align with the angular momentum of the outer disk (Rees, 1978; Scheuer & Feiler, 1996; Natarajan & Pringle, 1998; Caproni et al., 2007). Caproni et al. (2007) and Morales-Teixeira et al. (2012) used this model to study the precession of the jets in BL Lacertae (2200+420) and the radio galaxy 3C 84 respectively.

### 1.2.4. Numerical modelling of jet precession

Several attempts have been made to model the interaction between a precessing jet and the ambient medium numerically. Using three dimensional numerical simulations, Cox et al. (1991) showed that multiple hotspots of jets in many radio sources produced when the jets change their direction as a result of precessional motion. Hardee et al. (2001) computed 3D models of a precessing cylindrical jet and discussed the jet knots as a result of the wave-wave interactions of the body mode and surface mode of the Kelvin-Helmholtz (KH) instability. They applied their model to the inner knots of M87. Kurosawa & Proga (2008) modelled a precessing jets originating from a precessing accretion disk with a range of precession periods and precession angles. They showed that jet precession is able to produce S- or Z-shaped structures. In this thesis, I show that the internal 20 kpc S-symmetric
structure of the Hydra A jet can also be modelled by a precessing jet and on the basis of a parameter space study I estimate the precession period and precession angle.

1.3. Hydra A: An example of jet-ICM interaction

Comprehensive radio and X-ray observations (see the reviews by McNamara & Nulsen 2007, 2012, and Fabian 2012, and references therein) and numerical models (Gaspari et al., 2011; Dubois et al., 2010) indicate that the interactions between radio jets and the intracluster medium (ICM) counteract the cooling by X-rays in galaxy clusters, in which “cooling flows” would develop without the energy input by the AGN of the central cluster galaxy. This form of feedback, termed “radio-mode” or “maintenance-mode” feedback, is invoked in semi-analytic models and cosmological hydrodynamic simulations of galaxy formation to regulate the growth of the most massive galaxies and explain their deficit in present-day galaxy-luminosity functions (Croton et al., 2006; Okamoto et al., 2008; Dubois et al., 2013). The Hydra A cluster (Abell 780) is a well-studied, relatively nearby cool core cluster at a distance to the central radio galaxy of approximately 230 Mpc (z = 0.054). There exists a wealth of radio and X-ray observations of the jets of Hydra A and of the ambient ICM (Taylor et al., 1990; McNamara et al., 2000; David et al., 2001). Therefore, detailed models of the evolution of the radio jets in the Hydra A environment have the potential to provide valuable insights into the physics of radio-mode feedback.

1.3.1. Hydra A in X-rays

Using high-resolution Chandra data David et al. (2001) showed that Hydra A is a cooling flow galaxy cluster with a mass accretion rate at radii beyond 30 kpc of approximately \( \dot{M} \sim 300 \, M_\odot \, yr^{-1} \) as determined from the integrated X-ray emission. However, inside 30 kpc the mass accretion rate inferred from the X-ray spectroscopy drops sharply indicating that a heating mechanism is active near the cluster centre.
A discontinuity in the X-ray surface brightness and temperature profiles indicates the existence of a large scale weak shock front at $\sim 200 - 300$ kpc (Nulsen et al., 2005). X-ray surface brightness deficiencies in the atmosphere were identified as a chain of X-ray cavities associated with radio bubbles (Wise et al., 2007). In the 0.5-7 keV X-ray image of Hydra A, Figure 1.4 (this figure is the same as Fig.2 in Wise et al. (2007) with an additional length scale at the bottom left), the cavities are labelled by A, B, C, D, E and F.

### 1.3.2. Hydra A in radio

Hydra A has also been observed at a wide range of radio frequencies. Low frequency Very Large Array (VLA) observations reveal the remnant bubbles of the early epochs of radio activity (Lane et al., 2004), while GHz observations reveal active jets and inner radio lobes in the central $\sim 50$ kpc (Taylor et al., 1990). Fig. 1.3
is a reproduction of the 4.635 GHz image from Fig. 1 and Fig. 3 by Taylor et al. (1990). In the inner region of the radio source both jets flare, producing plumes at a deprojected distance of approximately 10 kpc from the core, assuming an inclination angle $\theta = 42^\circ$ derived from rotation measure asymmetries (Taylor et al., 1990; Taylor & Perley, 1993).

In the northern jet, a bright knot at a deprojected distance of $\sim 7$ kpc from the core is apparent just before the jet flares. At approximately 3.7 kpc from the core, another fainter knot is visible. Two more bright knots within the turbulent region at approximately 11.0 kpc and 16.0 kpc from the core are also visible. The turbulent flaring zone is marked by a shaded ellipse in the panel (b) of Fig. 1.3. A mis-aligned bright knot (which is not aligned with the jet path inferred by following the ridge line and connecting the four bright knot) is also shown.

In the southern jet, four bright knots can be seen at approximately 2.5, 3.9, 5.4, and 6.7 kpc from the core. The bright knots and the flaring points are enlarged and clearly seen in the zoomed-in region shown in panels b and c of Fig. 1.3.

The trajectories of the northern and southern jets in the inner 20 kpc from the radio core exhibit a spectacular S-shaped morphology which persists in the morphology of the plumes. The symmetrical S-structure is also visible in the spatially extended low frequency images at 74 MHz and 330 MHz (Lane et al., 2004).

The spatial anti-correlation between radio and X-ray emission in Hydra A strongly indicates that the radio jets impact large volumes of the ICM gas and regulate the cooling flow in the Hydra A. The correlation between jet power and X-ray luminosity in the Bîrzan et al. (2004) sample of sixteen galaxy clusters supports such a scenario in cooling flow clusters in general.

1.4. Models of Hydra A

In recent years, several models of the Hydra A radio source and the ICM have been published. Simionescu et al. (2009a) proposed, using hydrodynamic simulations, that the interaction of very powerful jets ($\sim 6 \times 10^{46}$ erg s$^{-1}$) with a spherically symmetric hydrodynamic environment can reproduce the observed large scale
shock front with Mach number $M \sim 1.3$. In order to explain an offset of 70 kpc between the centre of the shock ellipse and the cluster core, the interaction was deemed to take place in two stages: First, active jets propagate through a hydrostatic environment within 100 kpc from the core; second, the jets turn off and buoyant bubbles rise through a background environment that has a bulk velocity of 670 km s$^{-1}$ relative to the central galaxy. In that study, the base of the jet in the hydrodynamic simulations was located at approximately 10 kpc from the core where the jet radius is approximately 6 kpc. The inner 10 kpc region, where the jet has not yet transitioned to a turbulent flow, was not explored.

Refaelovich & Soker (2012) also modelled the Hydra A using axisymmetric, hydrodynamic simulations and showed that a single outburst can produce a series of X-ray deficient bubbles. In their model, the vortex shedding and the Kelvin-Helmholtz instabilities at the contact discontinuity of the shocked ICM and the shocked jet plasma are responsible for multiple X-ray cavities.

So far, the theoretical modelling of Hydra A discussed above has focused on the large-scale structures, such as the cavities and the shock fronts bounding the expanding bubbles. However, no numerical simulations have related the outer structure of the radio source to the structure within $\sim 20$ kpc of the radio core. In particular, the oscillating nature of the jet boundary inside 10 kpc and the bright knots in the central 20 kpc demand attention in order to construct a reliable physical model of the Hydra A jets. Two other key features that demand attention are the curvature of the jet and the jet-plume transitions in the northern and southern jets which mark a dramatic change in the flow properties of the jets.

1.5. Outline of models

In this thesis, I study the radio source Hydra A and its interaction with the intracluster medium utilising numerical models. As discussed earlier, in my study of the Hydra A jets I adopt and develop the model of reconfinement of the jet by the external medium. I begin my study focussing on the oscillation of the jet boundary and two bright knots in the northern jet at a distance of 3.7 and 7 kpc from the
core. I then concentrate on the complex morphology of the central 20 kpc of the northern jet.

1.5.1. Axisymmetric model

The Hydra A northern jet is mildly bent in the inner 10 kpc. Therefore, an axisymmetric model with a straight jet approximation is appropriate and a useful first step to study this region of the source. Hence, in order to study the inner 10 kpc of the Hydra A northern jet I employ a two dimensional axisymmetric model. In outline, my model of the inner 10 kpc Hydra A northern jet is as follows (the details are provided in chapter 5): The jet is initially ballistic with a constant jet velocity $v_{\text{jet}}$ and expands conically until it starts to come into equilibrium with the interstellar medium. The computations of the jet interaction begin at 0.5 kpc from the black hole, at which point, the jet is assumed to have a given over-pressure ratio (a free parameter). The over-pressured jet starts to expand. As a result, its pressure decreases and when the jet pressure reaches the pressure of the ambient environment it starts to collimate via reconfinement shocks. Depending on the pressure ratio between the jet and the environment the reconfinement shocks appear either as transverse Mach disks, or, biconical shocks, or, a combination of the two. The particle acceleration associated with the shock dissipation of the jet kinetic energy causes an enhancement in the brightness in the shocked region, producing the bright knots. The jet boundary oscillates following the periodic structure of the reconfinement shocks (see Fig. 1.2).

1.5.2. Precessing jet model

Guided by the results of the axisymmetric model, I further develop a three dimensional precessing jet model. In outline, the model for the inner 20 kpc of the Hydra A northern jet is as follows (the details are provided in Chapter 8): The curvature of the jet is caused by its precessional motion. The precessing jet interacts with the environment and produces reconfinement shocks which manifest themselves as bright knots, which appear along the jet path. The collimated jet starts to become
turbulent and produces a plume when it is sufficiently decelerated by the recollimation shocks. The jet hits the cocoon wall near the fourth knot and the back flowing jet plasma creates a strong turbulent dissipative zone at approximately 10 to 20 kpc from the core.

1.6. The contribution of this thesis

This thesis makes a significant contribution to the understanding of the internal dynamics and small- and large-scale morphology of AGN radio jets, relating these to the interaction of the jets with the ambient intracluster environment. Here I briefly describe the key contributions of this thesis.

1.6.1. Bright knots of Hydra A

Utilising 2D axisymmetric and three dimensional models of jet interacting with the intracluster medium I interpret the bright knots of the Hydra A as biconical reconfinement shocks produced when an over pressured jet comes into pressure equilibrium with the ambient medium.

1.6.2. Estimation of jet velocity

In appropriate cases the velocity of AGN jets may be determined or constrained by relativistic beaming. The jet velocity $\beta$ (in units of the speed of light) may be estimated from the brightness ratio $R$ of the jet to counter jet and the angle between the jet and the line of sight $\theta$:

$$\beta = \frac{R^{1/2 + \alpha} - 1}{R^{1/2 + \alpha} + 1} \times \frac{1}{\cos \theta} \quad (1.2)$$

The estimate of the jet velocity using equation (1.2) assumes that the jet and counter jet are equally powerful and equally fast and are pointing at exactly opposite directions. Therefore, Doppler beaming estimates involve large uncertainties—i) If the intrinsic brightnesses of the two jets are different. For example, the Hydra A has
four knots on the southern side compared to two on the northern side leading to an intrinsic difference in brightness asymmetry. ii) If the jets have curved structure, such as, in the Hydra A iii) If the counter jet is not visible, for example, M87. Sometimes the Doppler beaming estimate may be misleading if there is some asymmetry in the structure of the jet and the counter jet (Kovalev et al., 2007).

In this thesis, I explored the possibility of an alternative theoretical approach to estimate the jet velocity. Here, I show that in an appropriate case, for instance, if the bright knots of a jet can be modelled by reconfinement shocks, the bright knot locations and the radius profile of the jet near the core can be used to estimate the jet velocity.

1.6.3. Complex morphology of Hydra A northern jet

Utilising a three dimensional model of jet-ICM interaction, I show that the curvature of the inner 20 kpc northern jet of Hydra A can be attributed to the precessional motion of the jet. The precessing jet model is also successful interpreting other key morphological features of the inner 20 kpc of the northern side of Hydra A. For example,

1. The turbulent transition of the jet to a plume.
2. The turbulent flaring zone.
3. A misaligned knot in the turbulent flaring zone.

1.6.4. Constraining jet parameters

Apart from the reproduction of the morphological features of Hydra A northern jet, I also constrain the jet parameters at approximately 0.5 kpc from the core. Fitting the simulated and observed knot spacing and the oscillation of the jet boundary I estimate i) The jet velocity, \( v_{\text{jet}} \approx 0.8c \), ii) The jet inlet radius, \( r_{\text{jet}} \approx 0.1 \text{kpc} \), iii) The jet density parameter, \( \chi = \rho_{\text{jet}}c^2/(\epsilon_{\text{jet}} + p_{\text{jet}}) \approx 12.75 \) (where \( \rho_{\text{jet}} \) and \( p_{\text{jet}} \) are the density and pressure of the jet, \( \epsilon_{\text{jet}} \) is the internal energy density and \( c \) is the speed of light)), and iv) The jet over pressure ratio \( \approx 5 \).
1.7 Structure of this thesis

From the 3D precessing jet models, matching the simulated jet curvature and the jet to plume transition with the observation, I estimate the precession period $P \approx 1$ Myr and precession angle $\psi \approx 20^\circ$ of the Hydra A jets.

1.6.5. Heating of the atmosphere by the jets

In the case of precessing jet models, the momentum of the jet is distributed over a much wider area in comparison to a straight jet. This results in a lower advance speed of the jet head and hence a lower Mach number. For example, from the optimal 3D precessing jet model for Hydra A northern jet I obtain a Mach number $\approx 1.85$ of the forward bow shock (see § 8.2.4). I also obtain a mild pressure jump at the forward shock. The low Mach forward shock and the mild pressure jump across it indicates a gentle heating of the ambient medium by the source in its early stages. This is consistent with the gentle heating of the ambient medium inferred by McNamara & Nulsen (2012).

1.6.6. Viscosity parameter of the accretion disk

Relating the precession of the jet to the precession of the accretion disk (as described in § 1.2.3), I estimate the viscosity parameter of the accretion disk $0.03 \leq \alpha \leq 0.15$ surrounding the black hole in the nucleus of the Hydra A galaxy.

1.7. Structure of this thesis

This thesis consists of ten chapters. In this introductory chapter, I have given an overview of the structures of AGN jets, key features of the Hydra A inner jets, models for the formation of bright knots, and my model of the Hydra A jet-ICM interaction.

In the second chapter, a brief description of the computational code PLUTO, used in this thesis, is provided. Here I also present the numerical setup and the strategies used to solve the two dimensional axisymmetric, and three dimensional models of the jet-ICM interaction.
In the third chapter, I present the study of the energetics and composition of Hydra A jets and inner lobes using the radio data provided by Taylor et al. (1990). I also present here an analytical approach to the construction of a spherically symmetric hydrostatic cluster environment from the X-ray data provided by David et al. (2001).

In the fourth chapter, I present my study of the Hydra A northern jet based on the radio data presented in Taylor et al. (1990). In that contour image, within the first 10 kpc of the northern jet only one bright knot is apparent, which I modelled as a Mach disk. However, later, examining the actual VLA data of the source (G. Taylor, priv. comm.) in greater detail, I realised that an additional fainter knot is apparent near the core. Hence, I revise my study on the northern jet incorporating two bright knots.

In the fifth chapter, I present my jet model and the results of the two dimensional axisymmetric simulations based on two knots in the first 10 kpc of the northern jet.

In the sixth chapter, a verification of the axisymmetric model for the jet-ICM interaction is presented.

In the seventh chapter, an axisymmetric model for the inner 10 kpc of the Hydra A southern jet is presented.

In the eighth chapter, I present the results of precessing jet models.

In the ninth chapter, I summarise the results of my models and discuss them.

In the last chapter, I describe future work that may be carried out by using the study presented in this thesis as a basis.
CHAPTER 2

Hydrodynamical simulations with PLUTO

For my simulations, I use the publicly available PLUTO code (Mignone et al., 2007) to produce two dimensional axisymmetric and three dimensional hydrodynamic models of the jet-ICM interaction in the Hydra A. PLUTO is a highly efficient code for the study of supersonic, astrophysical jets because it uses a high resolution shock capturing Godunov-type scheme. Hence, I use this code to study the structures of the Hydra A northern jet, including two bright knots (which I assume to be reconfinement shocks), curvature of the jet, the jet-plume transition and the interaction of the radio source with the cluster atmosphere. The detail of the models are given in Chapters 5 and 8.

PLUTO is a finite volume hydrodynamic code for computational astrophysics written in C. This software is modular and highly user friendly. It provides an interactive interface (written in python) to select problem dependent physics module and algorithms. Using the message passing interface (MPI) this code can run on multiple processors in parallel. The scalability of the PLUTO code based on one of my three-dimensional jet-ICM interaction models (run A of Chapter 8) is described in this chapter (see § 2.3). I ran my models of Hydra A jets with a maximum of 2048 processors using the National Computing Infrastructure supercomputers VAYU and RAJIN at ANU.
2.1. Relativistic hydrodynamic equations

Since my models involve relativistic velocities, I use the relativistic hydrodynamic (RHD) module available in PLUTO to solve the relativistic fluid equations.

Let \( \rho \) be the proper density, \( p \) the pressure, \( \mathbf{v} = (v_1, v_2, v_3) \) the velocity, \( D \) the laboratory density, \( c \) the speed of light, \( \mathbf{m} = (m_1, m_2, m_3) \) the momentum density, \( E \) the total energy density. The conservative quantities \( \mathbf{U} = (D, \mathbf{m}, E) \) is related to the primitive quantities \( \mathbf{V} = (\rho, p, \mathbf{v}) \) by:

\[
D = \rho \Gamma, \\
\mathbf{m} = \rho h \Gamma^2 \frac{\mathbf{v}}{c^2}, \\
E = \rho h \Gamma^2 - p,
\]

where \( \Gamma = 1/ \sqrt{1 - v^2/c^2} \) is the Lorentz factor, \( h = (e + p)/\rho \) is the specific enthalpy and \( e \) is the proper internal energy density.

The relativistic Euler equations in conservative form are (Mignone, 2012):

\[
\frac{\partial D}{\partial t} + \nabla \cdot (D \mathbf{v}) = 0, \tag{2.2}
\]

\[
\frac{\partial \mathbf{m}}{\partial t} + \nabla \cdot (\mathbf{m} \mathbf{v} + p \mathbf{I}) = D \mathbf{g}, \tag{2.3}
\]

\[
\frac{\partial E}{\partial t} + \nabla \cdot m c^2 = \mathbf{m} \cdot \mathbf{g}. \tag{2.4}
\]

In these equations the first term represents time derivative of the conservative variables, the second term represents the divergence of the fluxes \( \mathbf{F} = (D \mathbf{v}, \mathbf{m} \mathbf{v} + p \mathbf{I}, \mathbf{m}) \) and the right hand side represents the source term \( \mathbf{S} \), arising from gravitation; \( \mathbf{g} \) is the gravitational acceleration vector.

An additional relation between the thermodynamic quantities, the equation of state (EOS), is also available:

\[
h = h(p, \rho). \tag{2.5}
\]

In my models I use the Taub (1948) equation of state, a quadratic approximation to the exact Synge–Jüttner relativistic perfect gas equation of state (Jüttner, 1911;
Synge, 1957), which yields \( \gamma \to 5/3 \) in the low temperature limit, and \( \gamma \to 4/3 \) in the high temperature limit. For the Taub equation of state the enthalpy equation becomes:

\[
h = \frac{5p}{2\rho} + \sqrt{\frac{9}{4} \left(\frac{p}{\rho}\right)^2} + 1.
\] (2.6)

To obtain the temporal evolution of the states at each cell, the equations (2.2)–(2.4) together with the equation of state (equation 2.6) need to be solved numerically. Below, I briefly describe how the numerical code PLUTO solves these special relativistic fluid equations.

2.2.  PLUTO

PLUTO solves the system of conservation equations (2.2-2.4) using a finite volume formalism based on Godunov-type schemes. It performs the numerical integration of the fluid equations in three major steps: (1) Reconstruction, (2) Evolution of fluxes and (3) Update cell averages.

Prior to the reconstruction the conservative quantities are transformed to primitive quantities. This transformation is required because primitive quantities are used to solve the Riemann problem. In PLUTO, a routine called "mappers.c" performs the transformation between the conservative and primitive quantities.

2.2.1.  Reconstruction

This is the first stage of computation where piecewise polynomial approximations to the primitives are estimated from the cell averages. In my models, I use the piecewise parabolic method available in PLUTO (Mignone et al., 2005) to reconstruct the states at the cell interfaces.

2.2.2.  Estimation of fluxes

In this step, a suitable Riemann solver is used to estimate the fluxes at the cell boundaries. The input data for the Riemann solver are the left and right cell-edge
states obtained from the reconstruction.

Among several Riemann solvers that PLUTO provides, I use the least diffusive two shock Riemann solver (Mignone et al., 2005) for the axisymmetric models (presented in chapters 4, 5, 6 and 7). This solver solves Riemann problems at each zone by approximating rarefaction waves as shocks. At locations of shocks the two-shock solver is switched to a more diffusive HLL solver to avoid artificial oscillations.

For the three dimensional precessing jet model (presented in Chapter 8) I used the HLL Riemann solver.

2.2.3. Update cell averages

In the final step, the fluxes at the cell boundaries are used to estimate the cell averages of the states of the next time step utilising a time marching scheme.

Among the time-marching schemes provided by PLUTO, I used the characteristic tracing scheme with i) A directionally split method to solve the two dimensional models ii) A directionally unspilt method to solve the three dimensional models. A directionally split method is computationally least expensive, since it solves $n$ Riemann problems for an $n$-dimensional problem. On the other hand, a directionally unsplit method utilises the corner transport upwind method (Colella, 1990) and is computationally expensive because it solves more Riemann problems. With a directionally unsplit method in two and three dimensional problems, solutions for 4 and 12 Riemann problems are required (instead of 2 and 3). However, directionally unsplit methods are preferable to directionally split methods because they avoid errors resulting from operator splitting.

The time step $\Delta t$ is restricted by the Courant-Friedrichs-Lewy (CFL) condition. Let $\lambda_{\text{min}}^d$ and $\lambda_{\text{max}}^d$ be the smallest cell width and largest signal velocity in direction $d$. Then the courant number $C_a$ is defined as (Mignone et al., 2007)

$$C_a = \Delta t \max \left( \frac{\lambda_{\text{max}}^d}{\Delta l_{\text{min}}^d} \right)$$

(2.7)
The limit for the Courant number, $C_a$, is constrained by the stability analysis on the constant coefficient advection-diffusion equation (see Table 2.1 of Mignone (2012)). I used CFL = 0.4 and 0.2 for the cases of two and three dimensional models respectively.

2.2.4. Shock capturing

![Image](image)

**Figure 2.1** Inner 6 kpc of one of my two dimensional axisymmetric jet-ICM interaction models (model Civ with $r = 0.1$ kpc, $p_{\text{jet}}/p_a = 5$, $\beta = 0.8$, $\chi = 12.75$). This pressure image shows the ability of the PLUTO to capture the Reconfinement shocks. The arrows represents the flow direction.

PLUTO is an excellent hydrodynamic code to simulate supersonic flow, especially when capturing shocks is a requirement. Fig. 2.1 shows the inner 6 kpc of one of my two dimensional jet-ICM interaction model solved by PLUTO (model Civ of Chapter 5 with jet inlet radius $r = 0.1$ kpc, overpressure ratio $p_{\text{jet}}/p_a = 5$, jet velocity in units of the speed of light $\beta = 0.8$, and jet density parameter $\chi = 12.75$). In this pressure image (overlaid with the flow vectors), we see the reconfinement shocks (marked by black arrows) are well captured. This is an illustration of the
performance of PLUTO in shock capturing.

2.3. Scaling of PLUTO code

I performed a scaling test for the PLUTO code using my three dimensional jet-ICM interaction model (resolution = 256$^3$) with different numbers of CPUs. I ran a three dimensional model (run A of Chapter 7) for a fixed wall clock time = 300 sec with 64, 128, 256, 512, 1048 and 2056 cpus and record the average time (wall clock) required for a step, the computed number of cells per second (wall clock), and the computed number of cells per second (wall clock) per cpu (see Table. 2.1).

![Simulation speed](image)

**Figure 2.2** Scaling of PLUTO. The points represent the computed number of cells per second (wall clock) for my three dimensional jet-ICM interaction model. A power law fit to the points with a power law index 0.86 is shown by a blue line. For a comparison an ideal speedup of a code is also shown (black line).

<table>
<thead>
<tr>
<th>No. of CPU</th>
<th>average time/step (s)</th>
<th>Cells/sec</th>
<th>Cells/sec/cpu</th>
</tr>
</thead>
<tbody>
<tr>
<td>64</td>
<td>2.82</td>
<td>3.57×10⁶</td>
<td>5.58×10⁴</td>
</tr>
<tr>
<td>128</td>
<td>1.48</td>
<td>6.82×10⁶</td>
<td>5.32×10⁴</td>
</tr>
<tr>
<td>256</td>
<td>7.85×10⁻¹</td>
<td>1.28×10⁷</td>
<td>5.00×10⁴</td>
</tr>
<tr>
<td>512</td>
<td>4.05×10⁻¹</td>
<td>2.49×10⁷</td>
<td>4.86×10⁴</td>
</tr>
<tr>
<td>1024</td>
<td>2.36×10⁻¹</td>
<td>4.27×10⁷</td>
<td>4.17×10⁴</td>
</tr>
<tr>
<td>2048</td>
<td>1.47×10⁻¹</td>
<td>6.86×10⁷</td>
<td>3.35×10⁴</td>
</tr>
</tbody>
</table>

**Table 2.1** Scaling test for PLUTO.
Fig. 2.2 shows the number of cells computed per second (vertical axis) for different numbers of processors (horizontal axis). Fitting a power law to the points shown in Fig. 2.2 I obtained a power law index 0.86, which indicates a nearly linear speedup of the code. For a comparison the ideal speedup of the code is shown by a black line (power law index = 1).

2.4. Problem initialisation for the simulations

The main aim of this thesis is to study the key features of the inner 20 kpc on the northern side of Hydra A. I perform my study in two stages- i) First, model the inner 10 kpc of the northern jet using an axisymmetric model and obtain best fit jet parameters (presented in Chapter 5); ii) Second, utilising the best fit jet parameters model the inner 20 kpc with a three dimensional precessing jet model (presented in Chapter 8). Here I first describe why I neglect magnetic field in modelling this source. Then I present details of both the axisymmetric and the three-dimensional precessing jet model.

2.4.1. Magnetic field

In the simulations I neglect the magnetic field. Is this a reasonable approximation given the popular notion that jets may be collimated by the toroidal field, which develops as a result of the rotation of the flow ejected from the accretion disk (Blandford & Payne, 1982) or from the ergosphere (Blandford & Znajek, 1977)? In this case one expects the magnetic and particle pressures to be comparable. Moreover, this would argue against the assumption of invoking an over-pressured jet on the parsec scale (see below). Self-collimation by a toroidal magnetic field is an appealing mechanism for the region of jets just outside the Alfvén surface. However, the fact that the jet expands by a factor of over 200 between the parsec scale and the kiloparsec scale indicates that self-collimation does not occur in this region. For example the self-similar models of Li et al. (1992) and Vlahakis & Königl (2003) indicate that asymptotically the flow becomes cylindrical when the jet is magnetically collimated. A different model has been proposed by Spruit
(2011), who has argued that three-dimensional effects lead to reconnection of the magnetic field and that the loss of magnetic energy produces a pressure gradient, which is responsible for the acceleration of jets to high Lorentz factors. Moll (2009, 2010) has carried out numerical simulations based on this concept, in the context of protostellar jets. There is also observational support for sub-equipartition magnetic fields on the sub-parsec scale in a substantial fraction of gamma ray blazars. In a recent paper Zhang et al. (2014) modelled the spectral energy distributions of a number of BL Lac objects and flat spectrum radio quasars (FSRQs) and found that they divide along the line magnetic power = electron power with most of the BL Lac objects being below this dividing line (see their Fig. 13(b)). The respective powers are proportional to the energy densities of the various components (their section 4) so that the ratio of the magnetic power to electron power informs us of the ratio of the respective energy densities. Hence, the magnetic energy densities in many of the BL Lac objects are well below the electron energy density (but with some members of the sample approaching equality). Thus there is good justification, in the first instance, for neglecting the magnetic field with the implication that the beamed counterpart of Hydra A would be a BL Lac object rather than a quasar.

What values of the jet density parameter, $\chi$ are relevant in this context? Two main options for jet composition are generally discussed – electron-positron or electron-proton. Let $m_e$ be the electron mass and $m_+$ the mass of the positively charged component, $m_+$ for a positron and $m_p$ for a proton. The parameter $\chi$ is then given by:

$$\chi = 0.75(a - 2)(a - 1)^{-1} \frac{m_+}{m_e} \gamma_1^{-1},$$

(2.8)

where $a$ and $\gamma_1$ are defined in § 3.1.3. Note that, for an electron-positron jet with $a = 2.4$ and $\gamma_1 \gtrsim 10$, $\chi \ll 1$. The theory of jet production from black holes (Blandford & Znajek, 1977) and X-ray observations of the lobes of both FR1 and FR2 radio galaxies (Croston et al., 2005; Croston & Hardcastle, 2014) make the concept of electron-positron jets appealing. However, the issue of jet composition is by no means settled. In an electron-proton jet, low values of $\chi$ require the low energy cutoff, $\gamma_1 \gg 1$. 

2.4 Problem initialisation for the simulations

![Diagram of jet initialisation](image)

**Figure 2.3** Schematic diagram of the initialisation of a conical jet (marked by dashed lines) with half cone angle \( \alpha \) into the \((r, z)\) computation domain. An initial quarter circular jet inlet with radius \( r_{\text{jet}} \) is used to initialise the jet in the computation domain. This initial jet inlet is useful to avoid reverse shocks running across the ghost zones. The velocity components inside the jet inlet are: \( v_r = r / \sqrt{(L + z)^2 + r^2} \) and \( v_z = (L + z) / \sqrt{(L + z)^2 + r^2} \).

2.4.2. Axisymmetric model

The main focus of this study is the two bright knots within the central 10 kpc of the northern jet. As I discussed in Chapter 1, these bright knots are considered as reconfinement shocks. In the numerical study of AGN jets, the jet can be considered as either initially parallel (Sutherland & Bicknell, 2007a) or initially conical (Komissarov & Falle, 1998; Krause et al., 2012). In both cases, jets are recollimated by reconfinement shocks. However, the VLBI pc scale (Taylor, 1996) and VLA kpc scale (Taylor et al., 1990) jets of Hydra A indicate an initial expansion of the jet by a factor of \( \sim 200 \) between approximately 20 pc and 0.5 kpc. This expansion of the jet suggests that the initially conical jet model is more realistic. Hence, I use an initially conical jet model such as used by Komissarov & Falle (1998).\(^1\)

---

\(^1\)Note that, models presented in Chapter 4 are based on a single bright knot inside 10 kpc of the northern jet. I used an initially parallel jet model for those runs (see details of initialisation in § 4.1).
The $(r, z)$ computational domain for the axisymmetric simulations is a cylinder of radius $r = 25$ kpc and height $z = 50$ kpc. The conical jet marked by dashed lines in Fig. 2.3 with a half cone angle $\alpha$, enters into the computation domain at a distance $L = 0.5$ kpc away from its origin. To initialise the jet in the computation domain, I use a quarter circular jet inlet with radius $r_{\text{jet}}$. The velocity components inside the jet inlet are:

\begin{align*}
    v_r &= \frac{vr}{\sqrt{(L + z)^2 + r^2}} \\
    v_z &= \frac{v(L + z)}{\sqrt{(L + z)^2 + r^2}}
\end{align*}

where $v = \sqrt{v_r^2 + v_z^2}$ is the magnitude of the jet velocity. The other component of the velocity $v_\theta$ is initially set at 0. The initial quarter-circular jet inlet in the computation box prevents reverse shocks running through the ghost zones.

The cluster environment is constructed using analytical fits for the density, pressure and temperature data derived from the X-ray data presented by David et al. (2001) (see Chapter 3 for details).

The jet power is fixed for all models $P_{\text{jet}} = 10^{45}$ erg s$^{-1}$. The estimation of jet power is described in Chapter 3. Analysing the pressure of the VLBI jet of Hydra A (Taylor, 1996) and the X-ray atmosphere pressure (David et al., 2001) I choose initial jet overpressure ratio 2 and 5. Based on the data provided for the full width half maximum (FWHM) of the northern Hydra A jet (Taylor et al., 1990), the jet inlet radius is chosen to lie between 80 and 180 pc. The jet velocity $\beta$ (in unit of the speed of light) is a free parameter and is chosen in the range 0.4-0.95. The remaining jet parameter, the density parameter $\chi = \rho_{\text{jet}}c^2/(\epsilon_{\text{jet}} + p_{\text{jet}})$ (where $\rho_{\text{jet}}$ and $p_{\text{jet}}$ are the density and pressure of the jet, $\epsilon_{\text{jet}}$ is the internal energy density and $c$ is the speed of light) is determined by the other parameters according to the relationship among relativistic jet parameters provided by (Sutherland & Bicknell, 2007a) (see § 5.1 for details of the model). For each model with different jet parameters I record data for the jet boundary and the locations of the reconfinement shocks. Fitting both of these simulated jet boundary and the shock location with the observed jet boundary and bright knots locations I estimate a theoretical velocity for the northern Hydra A jet. This procedure is explained in Chapter 5.
A tracer $\lambda$ for the jet is used to track the jet plasma in the various regions of the computation domain. The tracer obeys an advection law of the form:

$$\frac{\partial \rho \lambda}{\partial t} + \nabla \cdot (\rho \lambda \mathbf{v}) = 0$$  \hspace{1cm} (2.11)

Because the radiative cooling time of the ambient gas and the synchrotron cooling time of the jet plasma are both large compared to the simulation time (which is equivalent to the jet crossing time), I do not include radiative cooling in the simulations.

### 2.4.3. Precessing jet model

The main focus of this part of my thesis is the curvature of the jet and the jet to plume transition on the northern side of Hydra A. As I discussed in Chapter 1, the complex morphology of the Hydra A jets is a result of a dynamical interaction between precessing jets and the intracluster medium (see Chapter 8 for details of the model).

The geometrical configuration of the precessing jet model for the Hydra A northern jet is shown in Fig. 2.4. The jet originates near the central black hole (marked as the jet origin in panel (a)) and is initially ballistic and conically expanding (Komissarov & Falle, 1998; Krause et al., 2012; Nawaz et al., 2014). It precesses around the $z$-axis with a precession period $P$ and a precession angle $\psi$. The best fit axisymmetric model (see Chapter 5) gives a jet radius $r_{\text{jet}} = 0.1$ kpc at a distance $L = 0.5$ kpc from the black hole. The half cone angle of the jet cone is then $\alpha = \tan^{-1}(r_{\text{jet}}/L) = 11.3^\circ$.

The jet cone intersects the $xy$ plane at a distance $L$ from the central black hole in an ellipse. As a result of precession the elliptical jet inlet follows a circular path (marked in panel (a)) on the $xy$ plane. The elliptical jet base is determined from the geometry shown in panels (b) and (c) of Fig. 2.4 as described below.

Let $(u, v)$ be a rotating frame fixed on the elliptical jet inlet. The semi-major axis $a$ and semi-minor axis $b$ of the ellipse lie on the $u$ and $v$ axes respectively (see panel
Figure 2.4  Geometry of the precessing jet model. Panel (a) shows the conical jet originating at a distance $L$ below the $x-y$ plane of the computational domain. The precessing jet cone intersects the $x-y$ plane in an elliptical jet inlet which moves on the (dashed) circular path. The coordinates $(u,v)$, defined by the intersection of the cone and the $x-y$ plane at a precession azimuth $\phi = 0^\circ$ and an arbitrary $\phi$ are shown in panel (b). The dotted circular line is the intersection of the cone when the precession angle $\psi = 0^\circ$. The jet semi-minor axis of the jet inlet $b$ is equal to the jet radius $r_{\text{jet}}$. In panel (c) the angles defined by the lines joining the jet origin and the left and right edges of the inlet ellipse are shown. These define the semi-major axis of the ellipse.

(b) of Fig. 2.4). The centre of the ellipse lies at

$$u_0 = L[\tan(\psi + \alpha) - \tan(\psi - \alpha)]/2,$$

$$v_0 = 0.$$  \hspace{1cm} (2.12) \hspace{1cm} (2.13)

From the geometry described in panels (b) and (c) we obtain

$$a = L[\tan(\psi + \alpha) - \tan(\psi - \alpha)]/2,$$

$$b = r_{\text{jet}}.$$  \hspace{1cm} (2.14) \hspace{1cm} (2.15)
2.4 Problem initialisation for the simulations

Therefore, in the rotating \((u, v)\) coordinate system the jet inlet is defined by

\[
(u - u_0^2)/a^2 + v^2/b^2 \leq 1.
\] (2.16)

For a counter-clockwise rotation of the jet inlet the coordinates \(uv\) are related to the computational coordinates \(xy\):

\[
u = x \cos \phi + y \sin \phi,
\] (2.17)
\[
v = -x \sin \phi + y \cos \phi,
\] (2.18)

here \(\phi = 2\pi t/P\) is the azimuth angle of the precession.

In order to avoid reverse shocks running through the ghost zones I initialise the jet in the computational domain with a semi-ellipsoidal cap above the jet inlet with semi-principle axes \(a, b\) and \(c(=a)\). This semi-ellipsoidal jet inlet follows the elliptical entrance of the conical jet into the computational domain (see Figure 2.4).

As the jet precesses, the semi-ellipsoidal jet inlet rotates on a circular path (such as the dashed circle shown in panel (a) of Figure 2.4, marked as 'path of jet inlet') with a period equal to the precession period of the jet.

As in the axisymmetric model, the three-dimensional cluster environment is constructed using analytical fits for the density, pressure and temperature data derived from the X-ray data published by David et al. (2001).

2.4.4. Grid

**Axisymmetric model:** PLUTO provides two different grid structures- uniform grid and stretched grid. If \(x_R\) and \(x_L\) are the leftmost and rightmost points of the grid patch and \(N\) is the number of points then a uniform grid is constructed with cell spacing \(\Delta x\)

\[
\Delta x = \frac{x_R - x_L}{N}
\] (2.19)
Figure 2.5  A small segment of the computation domain \((r = 0.8 - 1.2 \text{ kpc}, z = 9.8 - 10.4 \text{ kpc})\) of an axisymmetric model (run cv of Chapter 5) at the intersection where the stretching ratios change in the \(r\) and \(z\) directions. The bold vertical and the bold horizontal lines are the boundaries of the uniform grid in \(r\) and \(z\) directions respectively. These two bold lines divide the computation domain into four differently resolved zones marked by \(S_z\), \(S\), \(U\), and \(S_r\): i) \(S_z\): uniform in \(r\) and stretched in \(z\) direction, ii) \(S\): stretched in both directions, iii) \(U\): uniform in both directions, iv) \(S_r\): stretched in \(r\) direction and uniform in the \(z\) direction. Since I focus the jet structures along its axis, I use less stretching in jet direction \(z\).

Provided that a uniform grid patch is present, the grid can be stretched in any direction with a stretching ratio \(s\) and maximum number of points \(N\)

\[
\Delta x_i = \Delta x s^i
\]  

(2.20)

where \(i = 1, 2, ..., N\) indicates the cell position in the stretched grid, \(\Delta x_i\) is the respective cell width and \(\Delta x\) is the cell width of the closest uniform grid. From the following relationship the stretching ratio is obtained using a Newton algorithm.

\[
s \frac{1 - s^N}{1 - s} = \frac{x_R - x_L}{\Delta x}
\]  

(2.21)

where \(x_L\) and \(x_R\) are the leftmost and rightmost points.

Using a combination of uniform and stretched grids described above I define a
2.4 Problem initialisation for the simulations

high resolution uniform grid within the central 1 kpc × 10 kpc region and a lower resolution stretched grid in the outer region. The structures of the different grid patches are as follows:

- **Uniform grid, \( U \):** In the inside 1×10 kpc zone a uniform grid with 100×1000 cells is used. The cell width for this grid patch is 0.01 kpc.

- **Grid stretched in \( r \), \( S_r \):** The grid patch in the domain 1 to 25 kpc in \( r \) and 0 to 10 kpc in \( z \) direction is stretched in the \( r \) direction and uniform in the \( z \) direction. 90 points are used to stretch the grid along \( r \). The stretching ratio is 1.009.

- **Grid stretched in \( z \), \( S_z \):** The grid patch in the domain 0 to 1 kpc in \( r \) and 10 to 50 kpc in \( z \) direction is stretched in the \( z \) and uniform in the \( r \) direction. 400 points are used to stretch the grid along \( z \). The stretching ratio is 1.055.

- **Stretched grid, \( S \):** The grid patch in the domain 1 to 25 kpc in \( r \) and 10 to 50 kpc in \( z \) direction is stretched in both \( r \) and \( z \) direction.

Since I focus on the structures of the Hydra A northern jet along its axis \( z \), I choose less stretching along the jet direction.

Fig. 2.5 shows a small segment (\( r = 0.8 - 1.2 \) kpc, \( z = 9.8 - 10.4 \) kpc) of the computation domain. Here we see two bold lines, the bold vertical line (boundary of the uniform grid along \( r \)) and the bold horizontal line (boundary of the uniform grid along \( z \)) divide the computation domain into four zones marked by \( S_z \), \( S \), \( U \), and \( S_r \): i) \( S_z \): uniform in the \( r \) and stretched in the \( z \) direction, ii) \( S \): stretched in both directions, iii) \( U \): uniform in both directions, iv) \( S_r \): stretched in the \( r \) and uniform in the \( z \) direction.

**Precessing jet model:** For the 3D precessing jet model I set up the computational grid as follows. I use a high resolution \( 156^3 \) uniform grid for the inner 5 kpc\(^3 \) (\(-2.5 < x < 2.5, -2.5 < y < 2.5, \) and \( 0.5 < z < 20.5 \), where the units here are kpc), thereby resolving the jet base by six cells. For the remaining computational
domain I use a stretched grid with 100 additional cells along each of the coordinate directions. The stretching ratio along the $-x$, $x$, $-y$, and $y$ axes is 1.05 and the stretching ratio along the $z$-axis is 1.03.

### 2.4.5. Boundary conditions

**Axisymmetric model:** I choose an axisymmetric boundary condition (available in PLUTO) for the boundary $r = 0$ (axis of symmetry). For an axisymmetric boundary condition velocity components, $v'_r$, $v'_z$ and $v'_\theta$, and other states $q'$ (pressures or, density) in the ghost zones are:

\[
\begin{align*}
    v'_r &= -v_r \quad (2.22) \\
    v'_z &= v_z \quad (2.23) \\
    v'_\theta &= -v_\theta \quad (2.24) \\
    q' &= q \quad (2.25)
\end{align*}
\]

where $v_r$, $v_z$, and $v_\theta$ are the velocity components in the computation domain.

I impose a reflective boundary condition for $z = 0.5$ by setting

\[
\begin{align*}
    v'_r &= -v_r \quad (2.26) \\
    v'_z &= v_z \quad (2.27) \\
    v'_\theta &= v_\theta \quad (2.28) \\
    q' &= q \quad (2.29)
\end{align*}
\]

The remaining boundaries are chosen to be outflowing boundaries (available in PLUTO) satisfying the conditions:

\[
\begin{align*}
    \frac{\partial v}{\partial n} &= 0 \quad (2.30) \\
    \frac{\partial q}{\partial n} &= 0 \quad (2.31)
\end{align*}
\]

where $v$ is the velocity vector in the ghost zone and $n$ is the coordinate direction orthogonal to the boundary.
Precessing jet model: I set up the boundary condition for the 3D precessing jet model as follows. I use a reflective boundary condition for the $z = 0.5$ plane by setting the velocity components in the ghost zones $v'_x, v'_y, v'_z$ and other states $q'$ as

\begin{align*}
v'_x &= v_x \quad \text{(2.32)} \\
v'_y &= v_y \quad \text{(2.33)} \\
v'_z &= -v_z \quad \text{(2.34)} \\
q' &= q \quad \text{(2.35)}
\end{align*}

where, $v_x, v_y$ and $v_z$ are the velocity components in the computation domain.

For the remaining boundaries I choose outflowing boundary conditions.
CHAPTER 3

Jet Kinetic Energy and the Hydrostatic Intracluster Medium

In order to construct physically realistic models of the interaction of the radio-jets with the environment of Hydra A, we require good estimates of the jet kinetic power and the spatial profiles of density, temperature and pressure in the cluster atmosphere. Previous estimates of the jet power (Nulsen et al., 2005; Wise et al., 2007) are based on X-ray observations of the outer shock and the cavities produced by the radio source. In this section I both supplement and confirm these estimates by utilising radio data of the inner lobes of Hydra A. Here I also establish a profile for the pressure and density of the ambient medium utilising the high resolution X-ray data provided by David et al. (2001).

3.1. Estimates of Jet kinetic power

3.1.1. Jet power based on a model for the outer shock

Nulsen et al. (2005) focused on the outer shock evident in the X-ray image and used a spherically symmetric hydrodynamic model of a point explosion in an initially isothermal and hydrostatic environment to produce theoretical X-ray surface
Figure 3.1  Radio intensity map of Hydra A at 4.635 GHz. This figure is almost identical to Fig. 1 in Taylor et al. (1990). Contour levels are at 1.5, 2.7, 3.7, 5.1, 10, 21, 37, 51, 103, 154, 311, and 466 mJy arcsec$^{-2}$. The elliptical areas, shaded in blue, outline the approximate volume of the corresponding X-ray A and B cavities and are used to estimate the contribution to the jet kinetic power.

brightness profiles for Hydra A. Their best fit to the observed X-ray brightness profile gives a shock age $\sim 1.4 \times 10^8$ Myr and an explosion energy $\sim 10^{61}$ erg. The estimated power of the outburst is $\sim 2 \times 10^{45}$ erg s$^{-1}$. On the basis of this model, I would associate $10^{45}$ erg s$^{-1}$ with each jet.

3.1.2. Jet power based on X-ray cavities

Wise et al. (2007) used the observations of three pairs of X-ray cavities revealed in Chandra images to estimate the power of the Hydra A jets. The inner cavities A and B correspond to the 4.6 GHz radio lobes (McNamara et al., 2000), the cavities C and D correspond to the middle lobe in the 1.4 GHz radio image (Lane et al., 2004) and the outer cavities E and F correspond to the outer lobes in the 330 MHz
image (Wise et al., 2007). Wise et al. consider that the three cavities on each side are interconnected and use the sum of the enthalpies, $h_{\text{tot}}$

$$h_{\text{tot}} = \frac{\gamma}{\gamma - 1} p_{\text{lobe}} V_{\text{lobe}}$$

where $\gamma$ is the polytropic index of the radio emitting plasma, $p_{\text{lobe}}$ is the pressure of the lobe and $V_{\text{lobe}}$ is the volume of the cavity, in all cavities, to calculate a total outburst energy. From this, the combined jet power is calculated, $P_{\text{jet}} = 4p_{\text{lobe}} V/t_{\text{cav}}$, where $t_{\text{cav}}$ is the age of each cavity. The average of three different cavity age estimates was used: the time required for the X-ray cavity to expand to its present position if expanding at the sound speed, the refilling time of the X-ray cavity, and the time required for the X-ray cavity to rise buoyantly to the present position. Assuming pressure equilibrium of the lobes with the atmosphere they obtained powers for the inner and middle lobes of $\sim 2 \times 10^{44}$ erg s$^{-1}$ and for the outer lobes, $\sim 6 \times 10^{44}$ erg s$^{-1}$, which gives a combined jet power $\sim 2 \times 10^{45}$ erg s$^{-1}$.

The authors find that a power of $1 \times 10^{45}$ erg s$^{-1}$ for the northern jet is consistent with the supposition that the jet is still filling the outermost of the X-ray cavities at $\sim 200$ kpc (the corresponding radio lobe is visible at 330 Mhz) and driving the large-scale shock. An independent estimate of the jet power from the expansion rate of the outermost cavity, assuming a self-similar evolution of the radius of the cavity wall and the large scale shock agrees with their first estimate to within a factor of 2. This value of the jet power $1 \times 10^{45}$ erg s$^{-1}$ is also consistent with the estimate of the jet power obtained by Nulsen et al. (2005) noted above.
3.1.3. Estimates of the jet power from synchrotron minimum energy

I revisit the calculation of the cavity powers of the two innermost cavities (see Wise et al., 2007) by using the synchrotron minimum energy estimate for the pressure and synchrotron ages of the lobes. The main difference between this method and that using the X-ray cavities is that the former introduces a strong dependence of the lobe pressure on the particle content of the lobe, whereas the X-ray cavity pressure only depends weakly on the particle content through the adiabatic index.

The work by Croston et al. (2005) on the lobes of classical double (FRII) radio galaxies shows that using a synchrotron minimum energy estimate is a feasible approach. However, since Hydra A is an FRI source this requires further justification. Croston et al. (2005) used observations of the inverse Compton emission in their sample to show that the lobes are close to equipartition when the inverse ratio of energy in relativistic electrons/positrons to that of “other” particles, \( k = 0 \). They use this fact to rule out an energetic relativistic proton component since the existence of such a component would imply that the magnetic field is in equipartition with the relativistic electrons only. While the authors did not state this directly, their argument can also be used to exclude an energetic thermal component. This is evident for powerful FRII sources, for which we do not expect much entrainment to occur. However, I argue that the turbulent processes leading to equipartition are independent of the plasma composition and that in the case where the plasma has a substantial thermal content these processes also lead to a minimum energy state between all particles and the magnetic field. This conclusion is supported by the work of Birzan et al. (2008) discussed below.

I have approximated the shapes of the lobes with ellipsoidal volumes as shown by the shaded elliptical regions in Fig. 3.1; the plasma depth is taken to be equal to the minor axis, \( L \). The lobe centres are located at \( \sim 30 \) kpc from the core. Let \( I_\nu \) be the central surface brightness of each lobe, where \( \nu = 4.6 \) GHz is the frequency of the Taylor et al. (1990) observations. Let \( m_e \) be the electron mass, \( e \) the elementary charge, \( a \) the electron index, \( \alpha = (a - 1)/2 \) the spectral index (a positive sign
3.1 Estimates of Jet kinetic power

convention is used for both $a$ and $\alpha$, $k$ the ratio of energy in particles other than electron to that of relativistic electrons, and $\gamma_1$ and $\gamma_2$ the lower and upper cutoff Lorentz factors respectively. Then, the minimum energy magnetic field (in Gauss) (e.g. Bicknell, 2013) of the synchrotron radiating plasma is given by:

$$B_{\text{min},E} = \frac{m_e c}{e} \left[ \frac{a + 1}{2} (1 + k) C^{-1}(a) \frac{c}{m_e} f(a, \gamma_1, \gamma_2) \frac{I_\nu \nu^\alpha}{L} \right]^{\frac{2}{a+5}}, \quad (3.2)$$

where

$$f(a, \gamma_1, \gamma_2) = (a - 2)^{-1} \gamma_1^{-(a-2)} \left[ 1 - \left( \frac{\gamma_2}{\gamma_1} \right)^{-(a-2)} \right], \quad (3.3)$$

and

$$C(a) = 3^{a/2} 2^{-(a+7)/2} \pi^{-(a+3)/2} \frac{\Gamma \left( \frac{a}{4} + \frac{19}{12} \right) \Gamma \left( \frac{a}{4} - \frac{1}{12} \right)}{a + 1} \sqrt{\pi} \frac{\Gamma \left( \frac{3a}{4} \right)}{2 \Gamma \left( \frac{7a}{4} \right)}, \quad (3.4)$$

In Eqn. (3.4) $\Gamma$ is the Gamma-function. Values adopted for $a$, $\gamma_1$, $\gamma_2$, $I_\nu$, and $L$ are shown in Table 3.1. I choose a spectral index $\alpha \approx 0.7$ (hence $a = 2.4$), which is representative of the low frequency spectral index of the radio emission (Cotton et al., 2009). I choose a lower Lorentz cutoff $\gamma_1 = 100$, in view of numerous studies of radio galaxies finding $\gamma \sim 100 – 10^3$ (Carilli et al., 1991; Hardcastle, 2001; Godfrey et al., 2009). Also, $\gamma_2 \approx 10^6$, since this corresponds to emission frequencies well above the microwave range. Minimum energy estimates are insensitive to $\gamma_2$ and only weakly dependent on $\gamma_1$.

The minimum total (particles + field) energy density of the lobe is given by

$$\varepsilon_{\text{tot}} = \varepsilon_p + \frac{B_{\text{min},E}^2}{8\pi} = \frac{a + 5}{a + 1} \frac{B_{\text{min},E}^2}{8\pi}, \quad (3.5)$$

and the total pressure of the lobe corresponding to the minimum total energy density is

$$p_{\text{tot}} = \frac{1}{3} \varepsilon_e (1 + k) + \frac{B_{\text{min},E}^2}{8\pi}. \quad (3.6)$$

The major uncertainty in this calculation arises from the lack of observational constraints on the parameter $k$. The value of $k$ determines whether the lobe (in
Table 3.2 Parameters calculated for three values of $k$ using synchrotron minimum energy for both lobes of Hydra A.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$B_{\text{min,E}}$ ($10^{-6}$ Gauss)</th>
<th>$\varepsilon_{\text{tot}}$ ($10^{-10}$ erg cm$^{-3}$)</th>
<th>$p_{\text{tot}}$ ($10^{-10}$ dyne cm$^{-1}$)</th>
<th>$p_{\text{tot}}/p_a$</th>
<th>$P_{\text{cav}}$ ($10^{44}$ erg s$^{-1}$)</th>
<th>$t_{\text{rad}}$ (Myr)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Northern Lobe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>22</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>49</td>
</tr>
<tr>
<td>10</td>
<td>42</td>
<td>1.5</td>
<td>1.2</td>
<td>0.9</td>
<td>1.8</td>
<td>18</td>
</tr>
<tr>
<td>100</td>
<td>76</td>
<td>4.9</td>
<td>3.9</td>
<td>3.0</td>
<td>14.5</td>
<td>7</td>
</tr>
<tr>
<td></td>
<td>Southern Lobe</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>21</td>
<td>0.4</td>
<td>0.3</td>
<td>0.2</td>
<td>0.1</td>
<td>51</td>
</tr>
<tr>
<td>10</td>
<td>40</td>
<td>1.4</td>
<td>1.1</td>
<td>0.9</td>
<td>2.0</td>
<td>19</td>
</tr>
<tr>
<td>100</td>
<td>73</td>
<td>4.6</td>
<td>3.7</td>
<td>3.0</td>
<td>16.3</td>
<td>8</td>
</tr>
</tbody>
</table>

I estimate the age of the source from the curvature in the spectrum derived by Cotton et al. (2009) who showed that the spectra of the inner lobes steepen for frequencies $\geq 300$ MHz. Let $B = B_{\text{min,E}}$ be the minimum energy magnetic field and $\nu_b \approx 300$ MHz be the break frequency, then the synchrotron age of the source is

$$t_{\text{rad}} \approx \frac{3^{5/2}}{8\pi^{1/2}} \left( \frac{m_e^3 c^5}{e^3} \right)^{1/2} B^{-3/2} \nu_b^{-1/2}.$$  \hspace{1cm} (3.7)

Hence, the power associated with each of the inner cavities is

$$P_{\text{cav}} = \frac{\gamma}{(\gamma - 1)} \frac{p_{\text{lobe}} V_{\text{lobe}}}{t_{\text{rad}}}.$$  \hspace{1cm} (3.8)

I use the total pressure $p_{\text{tot}}$ for minimum energy conditions as the lobe pressure. Values of $P_{\text{cav}}$ for different values of $k$ are given in Table 3.2.

Table 3.2 shows, for both the northern and southern lobes, the estimation of the minimum energy magnetic field, $B_{\text{min,E}}$, the total energy density $\varepsilon_{\text{tot}}$, the total pressure of the lobe, $p_{\text{tot}}$, the ratio between the total lobe pressure and the atmospheric pressure $p_{\text{tot}}/p_a$, the cavity power for $\gamma = 4/3$, and the radiative ages of the lobes for values of the parameter $k = 0$, 10 and 100.
For the same value of $k$, the cavity powers of the northern and southern lobes are comparable. Moreover, for $k = 10$ the lobes are in approximate pressure equilibrium with the atmosphere and the cavity powers ($1.8 \times 10^{44}$ and $2.0 \times 10^{44}$ ergs s$^{-1}$ respectively) agree with the Wise et al. (2007) estimates of $2.1 \times 10^{44}$ ergs s$^{-1}$ and $2.0 \times 10^{44}$ ergs s$^{-1}$ respectively. For $k = 0$ the lobes appear to be significantly under-pressured and for $k = 100$ significantly over-pressured.

This high value of $k$, i.e., energy dominated by the heavy or thermal particles, is supported by a recent study performed by (Bîrzan et al., 2008). They estimate $k$ for a group of galaxies including Hydra A assuming pressure equilibrium between the radio lobe and the atmosphere. For the 1.4GHz inner lobe of Hydra A they obtained a value of $k \approx 13$. In their study of the inverse-Compton X-ray emission of the outer lobe of Hydra A Hardcastle & Croston (2010) estimate a moderate value of $k = 17 - 23$.

The major uncertainty associated with these radio-based estimates of the cavity power is that there is no direct estimate of the lobe pressure and I have assumed that the lobe pressure is determined by the total pressure of the lobe when the lobe is in its minimum energy state. This assumption gives a lower limit of the lobe energy, and hence a lower limit on the cavity power.

The estimation of the power associated with the inner radio lobes and the power of the corresponding X-ray cavities given in § 3.1.2 for a nearly pressure equilibrium situation are consistent. This indicates the total jet power obtained by summing the powers of all X-ray cavities presented presented by Wise et al. (2007) is reliable and provides a sound basis for numerical models of Hydra A. I therefore adopt a jet power of $10^{45}$ erg s$^{-1}$ in the simulations presented in chapter 4, 5, 6, 7 and 8.

### 3.2. Cluster Environment

In order to construct definitive simulations of the inner jet propagation, we require knowledge of the distribution of the ambient density and pressure on a 20 kpc scale. In this section I present useful analytical fits for the density, temperature, and pressure in the cluster environment, which I use to estimate the pressure and
Figure 3.2 Radial thermodynamic profiles for the Hydra A galaxy cluster. (a) Electron density (data points and dashed line), and corresponding total particle density (solid line) assuming a fully ionized plasma; Error bars are smaller than the points. (b) Electron temperature (data points and dashed line) and the temperature profile obtained from the total particle density and the pressure profiles (solid line); (c) Total pressure. The points in (a) and (b) are data for the electron density and electron temperature, respectively, obtained by David et al. (2001) from X-ray observations of the Hydra A atmosphere. The dashed curves are fits to the data points using Eqn. (3.9) for (a) and a power-law for (b). The points in (c) were calculated from the density data points and the temperature fit. The stars represent the additional data points that I obtained through this method inside of 10 kpc. These two data points are important in constraining the profiles in the innermost region. The line in (c) is a fit to the data with Eqn. (3.11). Finally the temperature profile (solid line in panel b)) is obtained from the total particle density and the pressure profile.
Table 3.3  Atmosphere profile parameters. Fits to data by David et al. (2001) and extrapolation to $r < 10$ kpc.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Best fit value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Density profile</td>
<td></td>
</tr>
<tr>
<td>$r_\rho$0</td>
<td>15.94 kpc</td>
</tr>
<tr>
<td>$\rho_0$</td>
<td>$1.49 \times 10^{-25}$ g cm$^{-3}$</td>
</tr>
<tr>
<td>$\alpha_\rho$</td>
<td>0.67</td>
</tr>
<tr>
<td>Temperature profile</td>
<td></td>
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<td>$a_T$</td>
<td>5.66</td>
</tr>
<tr>
<td>$b_T$</td>
<td>$8.4 \times 10^{-2}$</td>
</tr>
<tr>
<td>Pressure profile</td>
<td></td>
</tr>
<tr>
<td>$r_p0$</td>
<td>18.21 kpc</td>
</tr>
<tr>
<td>$p_0$</td>
<td>$6.58 \times 10^{-10}$ dyne cm$^{-2}$</td>
</tr>
<tr>
<td>$\alpha_p$</td>
<td>0.65</td>
</tr>
</tbody>
</table>

density in the inner 20 kpc.

I assume that Hydra A’s atmosphere prior to the passage of the jet is hydrostatic, following a spherically-symmetric density distribution of the assumed analytic form

$$
\rho(r) = \frac{\rho_0}{(1 + r^2/r_\rho^2)^{\alpha_\rho}}. \quad (3.9)
$$

$\rho_0$, $r_\rho$, and $\alpha_\rho$ are determined through a least-squares fit of the function given in Eqn. (3.9) to the models for the cluster density inferred from the X-ray surface brightness by David et al. (2001). The data and the fitted density profile are shown in Fig. 3.2 (a). The pressure distribution of the atmosphere depends on the temperature distribution through

$$
p = \rho k_B T / \mu m \quad (3.10)
$$

where $k_B$ is Boltzmann’s constant, $\mu$ is the molecular weight and $m$ is the atomic mass unit. However, there is no observational data for the temperature inside of 10 kpc. I therefore use a power law temperature fit, $\log T = a_T + b_T \log r$ (as shown in Fig. 3.2 (b)), to the David et al. (2001) data and the density profile given by Eqn. (3.9) to obtain corresponding pressure values for two additional points within a radius of 10 kpc; these are distinguished from the other data points by the star symbols in Fig. 3.11 (c). The two additional extrapolated data points are important in constraining the shape of the flattening pressure profile toward the core of the galaxy. I adopt the following analytic expression for the pressure profile
of the ICM

\[ p(r) = \frac{p_0}{(1 + r^2/r_0^2)^{\alpha_p}}. \]  

(3.11)

A least squares fit to the pressure data points is used to obtain the parameters \( p_0, r_0, \) and \( \alpha_p \). We then obtain the final temperature fit (solid line in panel (b)) using the total particle density (solid line in panel (a)) and the pressure profile (solid line in panel (c)). The best-fit parameters for the fits to the density, temperature, and pressure data are summarised in Table 3.3.

For a hydrostatic environment I now have the gravitational acceleration as a function of radius

\[ g(r) = -\frac{1}{\rho} \frac{dp}{dr} = -2\alpha_p \frac{p_0}{\rho_0 r_0} \frac{r}{(1 + r^2/r_0^2)^{\alpha_p}}. \]  

(3.12)
CHAPTER 4

Jet model based on a single knot

My initial work on modelling Hydra A was based upon the published images of (Taylor et al., 1990) in which a single bright knot is evident in the first 10 kpc of the northern jet (see Fig. 4.1). Consequently the work described here is based upon the production of a single bright knot in the first 10 kpc. Subsequent to this work, Prof, Gregory Taylor kindly provided a FITS image of Hydra A so that I could determine the brightness ratio of the northern and southern jets. When I constructed the detailed contour map of the source I realised that there are in fact two knots in the northern Hydra A jet (see Fig. 5.1), first of which could with hindsight be faintly discerned in the original published images. Consequently, the following chapter contains a series of models devoted to the modelling of two knots. This chapter describes my work on the single knot interpretation. The comparison between the two models is of interest since it shows the different parameters that are obtained when different assumptions are made.

My initial guess is that the bright knot in the northern jet is a consequence of a Mach disk at approximately 6 kpc produced by the interaction of an over pressured jet and the cluster environment. See § 1.1.5 for the description of reconfinement shock model of jet knots. Depending on the pressure ratio between the jet and the atmosphere, there are two different types of reconfinement shock structures that can occur: strong shocks perpendicular to the jet flow, often referred to as Mach disks, and conical shocks. Fig. 4.2 shows a comparison of the morphology
Figure 4.1  Fig. 3 of Taylor et al. (1990). Central 10 kpc jets of Hydra A. Contour levels are at 1.0, 2.0, 7.8, 15, 32, 51, 103, 208, and 417 mJy arc sec$^{-2}$. The bright knot is marked with black arrow. The location of the Mach disk (at approximately 6 kpc, marked by a ×) which I originally interpret as the reason for the bright knot, is also indicated.
Figure 4.2 Logarithmic pressure maps of the jet and ambient medium in the inner region of the 2D simulations. Left: A jet with an over-pressure ratio of 10 producing a Mach disk (Simulation Cvia). Middle: A jet with an over-pressure ratio of 5 producing a reconfinement shock. (Simulation Cviib) Right: A nearly pressure equilibrium jet ($p_{jet}/p_a = 2$) producing a very weak conical reconfinement shock (Simulation Cviic). In each case, jet is originated at the bottom left corner.
of a jet that is significantly over-pressured (left panel), mildly over-pressured, and in nearly pressure equilibrium (right panel) with respect to the ambient medium. It is well known that a supersonic jet may display a sequence of conical shocks, also known as “diamond shocks”, if the jet is in nearly pressure-equilibrium with respect to the ambient medium. The jet repeatedly expands and contracts towards the jet axis resulting in a sequence of conical shocks. When the pressure ratio of the jet and the atmosphere is sufficiently high (e.g., $p_{\text{jet}}/p_a \gtrsim 10$), a Mach disk occurs normal to the flow. Such shocks occur in the following way: The propagation of the supersonic jet produces a lobe of shocked gas whose pressure is initially higher than the jet pressure. As the jet propagates, the lobe pressure decreases. When the lobe pressure becomes lower than the jet pressure, the jet expands rapidly sideways and the subsequent reconfinement produces a Mach disk. Behind the Mach disk, a transition to turbulence occurs, which is more rapid than that associated with conical shocks. A Mach disk is very disruptive; it drastically reduces the jet velocity in the inner section of the jet and also establishes a strongly sheared flow between the inner jet and its outer layer as shown in Fig. 4.4. Oblique conical shocks are not as disruptive as the Mach disk.

4.1. Simulation Parameters

In the models presented here, I initialise a parallel jet into the computation domain following Sutherland & Bicknell (2007a); Wagner & Bicknell (2011). The $(r,z)$ computational domain for the two dimensional axisymmetric simulations is a cylinder of radius $r = 25$ kpc and height $z = 50$ kpc. Using a stretched grid I define a high resolution grid ($1600 \times 160$ cells) within the central 20 kpc $\times$ 2 kpc region, giving us 30 cells across the jet, and a lower resolution in the outer regions. I impose an axisymmetric boundary condition for the boundary $r = 0$, and a reflective boundary condition for $z = 1$. The remaining boundaries are set to outflowing boundaries.

The jet is initiated within a quarter circle with radius 0.38 kpc and centre at $(r,z) = (0, 1)\, kpc$. The time invariant jet parameters are jet power $P_{\text{jet}}$, jet radius $r_{\text{jet}}$, inlet jet
## 4.1 Simulation Parameters

Table 4.1 Simulation parameters. In all simulations, $r_{\text{jet}} = 0.38 \text{kpc}.$

<table>
<thead>
<tr>
<th>Model</th>
<th>$p_{\text{jet}}/p_a$</th>
<th>$\beta$</th>
<th>$\chi$</th>
<th>$\eta$</th>
<th>$\phi$ (rad cm$^{-2}$)</th>
<th>$\Psi_{6\text{cm}}$ (rad)</th>
<th>$\Psi_{20\text{cm}}$ (rad)</th>
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<td>Ai</td>
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<td>0.09</td>
<td>1029</td>
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</tr>
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<td>$9.0 \times 10^{-4}$</td>
<td>0.03</td>
<td>0.36</td>
</tr>
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<td>$4.3 \times 10^{-4}$</td>
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<td>$5.7 \times 10^{-4}$</td>
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<td>$6.6 \times 10^{-4}$</td>
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<td>$3.7 \times 10^{-4}$</td>
<td>0.01</td>
<td>0.15</td>
</tr>
<tr>
<td>Cvic</td>
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<td>0.50</td>
<td>71</td>
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<td>$4.4 \times 10^{-4}$</td>
<td>0.02</td>
<td>0.18</td>
</tr>
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<td>Cviia</td>
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<td>$1.5 \times 10^{-4}$</td>
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Table 4.2: Inferred jet velocities for Hydra A northern jet as a function of jet power and pressure ratio

<table>
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<tr>
<th>$P_{\text{jet}}$ (erg s$^{-1}$)</th>
<th>$P_{\text{jet}}/p_a$</th>
<th>$\chi$</th>
<th>$v$</th>
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<td>0.25-0.55</td>
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<td>2</td>
<td>$3 	imes 10^6$</td>
</tr>
<tr>
<td>0.4-0.55</td>
<td>$3 \times 10^6$</td>
<td>5</td>
<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>0.7-1</td>
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<td>5</td>
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<td>1.0</td>
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<td>$3 \times 10^6$</td>
</tr>
<tr>
<td>1.3</td>
<td>$1 \times 10^6$</td>
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<td>$1 \times 10^6$</td>
</tr>
<tr>
<td>1.6</td>
<td>$1 \times 10^6$</td>
<td>0.1</td>
<td>$1 \times 10^6$</td>
</tr>
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<td>1.9</td>
<td>$1 \times 10^6$</td>
<td>0.1</td>
<td>$1 \times 10^6$</td>
</tr>
</tbody>
</table>

Jet model based on a single knot
pressure \( p_{\text{jet}} \), inlet jet velocity \( \beta \), and the proper density parameter

\[
\chi = \frac{\rho_{\text{jet}} c^2}{\varepsilon + p_{\text{jet}}} = (\gamma - 1)p_{\text{jet}} c^2 / \gamma p_{\text{jet}} \tag{4.1}
\]

where \( c \) is the speed of light, \( \varepsilon \) is the energy density, \( \rho_{\text{jet}} \) is the jet density, and \( \gamma \) is the polytropic index. The jet velocity vectors at the jet base are parallel to the positive \( z \)-direction.

Let \( A_{\text{jet}} = \pi r_{\text{jet}}^2 \) be the jet cross-sectional area and \( \Gamma = (1 - \beta^2)^{-1/2} \) be the jet Lorentz factor. Then the jet kinetic power is given by Sutherland & Bicknell (2007a):

\[
P_{\text{jet}} = \frac{\gamma}{\gamma - 1} c p_{\text{jet}} \Gamma^2 \beta A_{\text{jet}} \left(1 + \frac{\Gamma - 1}{\Gamma} \chi\right). \tag{4.2}
\]

We determine the density parameter by solving Eqn. (4.2):

\[
\chi = \frac{\Gamma}{\Gamma - 1} \left(\frac{\gamma - 1}{\gamma} \frac{P_{\text{jet}}}{c p_{\text{jet}} \Gamma^2 \beta A_{\text{jet}}} - 1\right). \tag{4.3}
\]

The initial conditions for the ambient medium representing the hot ICM surrounding Hydra A are the hydrostatic thermodynamic profiles found in § 3.2. The ratio, \( \eta = \rho_{\text{jet}} / \rho_a \), of the jet to ambient density is always an important parameter in jet theory and since I am allowing for a significant thermal component in the simulations it is important to estimate this parameter. If \( T \) is the core temperature of the cluster atmosphere, \( \rho_a \) and \( p_a \) are the ambient density and pressure at the jet inlet, \( k_B \) the Boltzmann constant and \( m \) be the atomic mass unit, \( \eta \) is given by

\[
\eta = \frac{\rho_{\text{jet}}}{\rho_a} = \frac{\chi}{\gamma - 1} \frac{p_{\text{jet}}}{p_a} \frac{k_B T}{m c^2}. \tag{4.4}
\]

Equation (4.3) for \( \chi \) and equation (4.4) for \( \eta \) typically lead to values of \( \chi \gg 1 \) (especially for low velocities) and correspondingly large values of \( \eta \approx 10^{-2} \) in comparison to the typical value for AGN jets (\( \eta \approx 10^{-4} \)). The implications of this are discussed further below.

The simulations presented in the next section cover an extensive region in parameter space. The simulation parameters that are varied, and the ranges they span are:
the jet power, $P_{\text{jet}} = 1 \times 10^{45}, 3 \times 10^{44}, 3 \times 10^{45}$ erg s$^{-1}$, the ratio of the jet pressure to that of the atmosphere at the jet base $p_{\text{jet}}/p_a = 2, 5, 10$, and the jet velocity in units of the speed of light $\beta = 0.04 - 0.55 \, c$, are presented in Table 4.1. The range in jet velocity is restricted to fairly low values because of the results presented in § 4.2: the location of the first internal jet shock constrains the velocity to relatively low values. The jet radius is fixed at $r_{\text{jet}} = 0.38$ kpc. This value is measured from the deconvolved FWHM at approximately 1 kpc from the core of the northern jet.

Some derived parameters, namely the density parameter $\chi$, the density ratio $\eta$ of the jet and the atmosphere at the jet base, the rotation measure (RM) $\phi$, and the Faraday rotation angle $\Psi$ at 6 cm ($\Psi_{6\text{cm}}$) and 20 cm ($\Psi_{20\text{cm}}$) are also presented in Table 4.1. The rotation measure and Faraday rotation of the central jet with electron density $n_{e,\text{jet}} = \rho_{\text{jet}}(1 + 2 \, n_{\text{He}}/n_{\text{H}})/u(1 + 4n_{\text{He}}/n_{\text{H}})$, where $u$ is an atomic mass unit), magnetic field along the line of sight $B_z$ (we use 35 $\mu$G, approximately the minimum energy magnetic field near the jet base), differential plasma depth $dl$, jet radius $R_{\text{jet}}$, total plasma depth $L = 2R_{\text{jet}}$, and wavelength $\lambda$ are calculated from

$$\phi = 8.1 \int n_{e,\text{jet}} B_z dl \text{ rad cm}^{-2}$$

$$= 8.1 \times 10^{-5} \left( n_{e,\text{jet}} \right) \left( \frac{B_z}{\mu\text{G}} \right) \left( \frac{2R_{\text{jet}}}{\text{kpc}} \right) \text{rad cm}^{-2} \quad (4.5)$$

where the units of $B_z$ and $l$ are Gauss and cm, respectively. The total Faraday rotation through the jet is given by:

$$\Psi_{\text{rad}} = \phi \lambda^2 \quad (4.6)$$

I calculate these quantities as an additional check to ensure that the jet parameters are consistent with the fact that the jets are polarised along their length. The internal Faraday rotation should be much less than unity for consistency between the models and the observations. Note however, that the values given in Table 4.1 are maximum values and do not take into account the angle between the magnetic field and the line of sight, the possibility that the magnetic field may be sub-equipartition and the occurrence of field reversals.

I group the runs into three sets as set out in Table 4.1; Set A groups simulations
with a jet kinetic power of $1 \times 10^{45}$ erg s$^{-1}$, which is the estimated jet kinetic power for Hydra A (see § 3.1); Sets B and C group simulations with a jet kinetic powers of $3 \times 10^{44}$ erg s$^{-1}$ and $3 \times 10^{45}$ erg s$^{-1}$, respectively.

It is evident from Eqns. (4.3) and (4.4), that the dependence of the density parameter, $\chi$, and the density ratio, $\eta$, on the other jet parameters have implications for the allowable range of the pressure ratio $p_{\text{jet}}/p_a$. For $p_{\text{jet}} = 1 \times 10^{45}$ ergs$^{-1}$ and $3 \times 10^{44}$ ergs$^{-1}$ (simulation set A and B), the density ratio $\eta$ is too large for $p_{\text{jet}}/p_a < 10$. A jet with too large a value of $\eta$ pierces the atmosphere with very little lateral expansion; the back-flowing jet plasma does not form a wide lobe, inconsistent with observations. Therefore there are fewer models in simulation set A and B, while in set C I can explore a more extensive range of pressure ratios $2 \leq p_{\text{jet}}/p_a \leq 10$. Nevertheless, in most runs, the value of $\eta$ is still 10 to 100 times higher than the typical light jet assumption of $\eta \sim 10^{-4} – 10^{-3}$. This could indicate that the jet entrained a substantial amount of ambient material. For example, it is possible that the jet has interacted strongly with HI gas near the radio core, detected by Dwarakanath et al. (1995).

### 4.2. Simulation Results

In this section I present the results of two dimensional axisymmetric hydrodynamic simulations of the interaction of the Hydra A radio jets with the ICM. I have conducted a large number of simulations to cover the parameter space described in table 4.1. I first describe the association of the bright knot in the northern side of Hydra A with a transition to turbulence. As noted above a possible model for the bright knot and the turbulent transition is that it is the result of a normal shock (Mach disk). However, this remains to be confirmed by three dimensional simulations and I also describe here how conical shocks in the case of a less over-pressured jet may also explain these features. I then turn to the results of the parameter space study that enable us to constrain the jet velocity and other jet parameters by the location at which the first jet shock develops.
4.2.1. Morphological Structures: The Bright Knot in the Northern Jet and the Transition to Turbulence

Fig. 4.3 shows two different lobe structures, one fed by a jet displaying shock diamonds (left panel), the other fed by a jet containing a Mach disk (right panel). These are snapshots from runs Cvb and Cva, respectively. Here two completely different types of jet and lobe morphology are seen. For the case of a less over-pressured jet \( (p_{\text{jet}}/p_a = 5) \), the jet reconfinement proceeds through a series of shock diamonds. The jet remains coherent well inside the lobe and a number of conical shocks mediate a gradual transition to turbulence. In the other case of a jet over-pressured significantly by a factor of 10 \( (p_{\text{jet}}/p_a = 50) \), the reconfinement results in a Mach disk. Although the association of a Mach disk with the northern jet bright knot consistently explain the morphology of the northern jet, I do not completely disregard the possibility of the association of a conical shock with the bright knot. Hence, in the following I determine the location of the first shock, either conical or normal, when I examine the dependence of the location of the northern bright knot on jet velocity.

Fig. 4.4 shows the density image of the simulation Av at 24 Myr. I select this particular model (while modelling a single knot in the northern jet) as the optimal...
Figure 4.4  Logarithmic density map snapshot for run Av at 24 Myr, labelling the main features of the jet-ICM interaction seen in simulations that exhibit a Mach disk. The right panel is a magnification of the central region marked with a box in the left panel.
model for the jets of Hydra A with a jet kinetic power of $P_{\text{jet}} = 1 \times 10^{45} \text{ erg s}^{-1}$, because it clearly reproduces the transition of the jet to a turbulent plume at $\sim 10$ kpc as indicated by the observations of the Hydra A northern jet. While this simulation snapshot represents the earliest phase of the development of the Hydra A plumes, I will assume that it is indicative of the dynamics in the inner $\sim 10$ kpc of the jet during the subsequent evolution of the source.

In this simulation, a Mach disk appears at $\sim 6$ kpc from the jet inlet. I associate this Mach disk with the bright knot in the Hydra A northern jet. The shock-accelerated electrons and the compression of the magnetic field increase the synchrotron emission immediately behind the Mach disk and produce a bright radio knot. Downstream of the Mach disk, the jet flow is further decelerated and becomes turbulent. The deceleration distance behind the Mach disk is $\sim 4$ kpc, in agreement with the observed distance between the bright knot at $\sim 6$ kpc and the beginning of the turbulent plume at a distance of $\sim 10$ kpc from the radio core.

Other standard features of simulation Av include a large-scale bow-shock advancing through the ICM, and an entrainment layer which develops between the contact discontinuity separating the shocked ICM and the shocked jet plasma. This develops as a result of the Kelvin-Helmholtz instability.

4.2.2. Results of parameter-space study

The main aim of the parameter space study is to determine the relationship between the position of the first jet shock and jet parameters, in particular, the jet velocity and the jet power. Adopting a shock location at a deprojected distance of $\sim 6$ kpc from the core, as inferred from the radio image of Fig. 3.1 and the assumed inclination angle of $\theta = 42^\circ$ (Taylor et al., 1990), I use these relationships to constrain the jet velocity of the northern Hydra A jet.

An incentive for this approach comes from the (non-relativistic) expression for the natural wavelength of a supersonic jet, $\Lambda$, with diameter, $D$, and Mach number, $M$, $\Lambda/D = 1.3 \sqrt{M^2 - 1}$ (Birkhoff & Zarantonello, 1957). This relationship indicates that the spacing of jet shocks should be a function of the Mach number and hence of the velocity of the jet (given that other parameters are constrained by the jet
4.2 Simulation Results

Figure 4.5 Position of the first reconfinement shock (Mach disk for significantly over-pressured jets $p_{jet}/p_a = 10$ in panel a), b), and c) or the first conical shock for a relatively less over-pressured jet $p_{jet}/p_a = 5$ and 2 in panel d) and e)) as a function of $\beta$, the jet velocity in units of the light speed, measured for three sets of simulations. Each point represents the average of 25 measurements of the position of the Mach disk or first conical shock, which fluctuates with time about a mean value. The error bars represent the standard deviation of the measurements. The dashed green line at 5 kpc (the first shock is assumed at 6 kpc and the jet inlet in the simulation is at a distance $\sim 1$ kpc from the galaxy centre) in each panel represents the location of the observed southern edge of the bright knot, which I assume to be either a Mach disk or a conical shock. Measurements are for: a) parameter set A; b) parameter set B, c) parameter set Ca, d) parameter set Cb and e) parameter set Cc. These five relationships along with the assumed location of the Mach disk or the first conical shock in the northern jet at $\sim 6$ kpc from the radio core lead to different acceptable jet velocities $0.17 \, c$, $0.05 \, c$, $0.45 \, c$, $0.4 - 0.55 \, c$ and $0.25 - 0.55 \, c$ with three different jet kinetic powers $1 \times 10^{45}$ erg s$^{-1}$ (estimated value), as well as lower and higher values of $3 \times 10^{44}$ erg s$^{-1}$, and $3 \times 10^{45}$ erg s$^{-1}$. 

- Panel a) $p_{jet}=1 \times 10^{45}$ erg s$^{-1}$, $p_{jet}/p_{a}=10$
- Panel b) $p_{jet}=3 \times 10^{45}$ erg s$^{-1}$, $p_{jet}/p_{a}=10$
- Panel c) $p_{jet}=3 \times 10^{45}$ erg s$^{-1}$, $p_{jet}/p_{a}=10$
- Panel d) $p_{jet}=3 \times 10^{45}$ erg s$^{-1}$, $p_{jet}/p_{a}=5$
- Panel e) $p_{jet}=3 \times 10^{45}$ erg s$^{-1}$, $p_{jet}/p_{a}=2$
power). I, therefore, vary the jet velocity and at the same time vary the density parameter $\chi$ to maintain a constant jet kinetic power, noting the location of the first jet shock (if one exists) for each run.

In each simulation the positions of shock structures in the jet vary with time. They oscillate about a mean position. For each run, I have therefore measured the position of the jet shock at 25 snapshots with 100 kyr time difference between each two adjacent snapshot. The points in Fig. 4.5 show the mean position of the jet shock, and the extent of the oscillation is indicated by the by error bars which represent the standard deviation of the measurements.

Fig. 4.5 shows the dependence of the distance of the first jet shock from the jet base upon the jet velocity for three different values of the jet kinetic powers:
4.3 Summary and discussion

\[ P_{\text{jet}} = 1 \times 10^{45} \text{ erg s}^{-1} \] (the estimated value, see § 3.1), \[ P_{\text{jet}} = 3 \times 10^{44} \text{ erg s}^{-1} \] and \[ P_{\text{jet}} = 3 \times 10^{45} \text{ erg s}^{-1} \]. These are results from the runs in Table 4.1 set A, set B, set Ca, set Cb, and Cc respectively. In panels (a), (b) and (c) the first shock is a Mach disk; in panel (d) and (e) the first shock is a conical shock. Since the jet inlet in the simulation is at a distance 1 kpc from the centre of the galaxy I locate the first shock at 5 kpc in each panel to determine the jet velocities. The inferred jet velocities for each result of Fig. 4.5 is summarised in Table 4.2.

The variations in the shock positions occur because the pressure field in the backflow adjacent to the jet changes continuously as a result of the turbulence in the cocoon. An example of this is shown in Fig. 4.6 which shows the position of a Mach disk at five different time steps.

4.3. Summary and discussion

Here I summarise the results of the axisymmetric models based on the assumption that the only bright knot, apparent in Fig. 3 of Taylor et al. (1990), of the inner 10 kpc Hydra A northern jet, is a consequence of a Mach disk at approximately 6 kpc.

1. Among the models with the estimated jet kinetic power \( 1 \times 10^{45} \text{ erg s}^{-1} \) (set A), the optimal model for Hydra A northern jet (which provide a Mach disk at approximately 6 kpc from the core) is A\text{v}. The jet velocity of the optimal model is \( 0.17 \, c \). Using a jet-to-counterjet brightness ratio \( R = 1.9 \) (Taylor, 1996), and an inclination of the source to the line of sight of \( \theta = 42^\circ \) (Taylor & Perley, 1993), in the formula of Doppler beaming, \( R = ((1 + \beta \cos \theta)/(1 - \beta \cos \theta))^{2-a} \), I estimate a jet velocity \( 0.17 \, c \).

Therefore two completely different approaches, one based on the interaction of the jet and the atmosphere and the other based on relativistic Doppler beaming, give the same estimate for the jet velocity of Hydra A jets. However, later, using the 6 cm VLA data of Hydra A I estimate a higher jet-to-counterjet brightness ratio \( R = 7 \). Attributing this ratio to Doppler beaming I obtain a moderately relativistic jet velocity \( \approx 0.5 \, c \).
A mild relativistic jet velocity for Hydra A $\approx 0.17\,c$ is also inconsistent with the theoretical estimate of jet velocity for FR I radio jets $\approx 0.8\,c$ provided by (Laing & Bridle, 2014).

2. In the optimal model for a single knot (run Av), the density ratio between the jet and the ambient medium is $\eta \approx 10^{-2}$. This density ratio is relatively higher than the typical assumption light AGN jets $\eta \approx 10^{-4}$. This relatively heavy jet can be attributed to the entrainment of the pc scale jets with a heavy gas (Dwarakanath et al. (1995) detected HI gas near the core of Hydra A galaxy) near the core.

3. In model Av the internal Faraday rotation $\Psi_{20\text{cm}} = 1.42$ is also high, which is inconsistent with the observed polarisation of the jet (Taylor & Perley, 1993). Three possible explanation for this uncomfortably high value of $\Psi_{20\text{cm}}$ are: i) Equipartition between the magnetic field energy in the radio plasma and the particle energy does not obtain. The magnetic field may be less than the equipartition value, which would imply lower internal Faraday rotation; ii) A preferentially perpendicular orientation of the magnetic field with respect to the line of sight, e.g. due to a toroidal magnetic field, will give smaller values of the rotation measure. c) For random magnetic field distributions in the radio plasma, the rotation measure is decreased by a factor of $N^{1/2}$, where $N$ is the number of magnetic field reversals across the jet.

4. If lower jet kinetic powers are assumed, the best values for the jet velocity require values of the density parameter $\chi$ that are uncomfortably large, e.g., in the case in which $P_{\text{jet}} = 3 \times 10^{44}\,\text{erg}\,\text{s}^{-1}$, $\beta = 0.04$ and $\chi = 3143$ (run Bi) give the best fit for the location of the jet shock. In this case the internal Faraday rotation at 20 cm is 17.76 radians, clearly inconsistent with the polarisation of the jets.

5. The scenario of higher jet kinetic power and a comparatively less over-pressured jet suggest (runs Civb, Cvb, Cvib and Cviib) a range of jet velocity from $\sim 0.45\,c$ to $\sim 0.55\,c$. In the case of higher jet kinetic power and a nearly pressure equilibrium jet (runs Cic to Cviic) a mildly decreasing trend of the first conical
shock position with increasing jet velocity indicates an wide range of possible jet velocities from $\sim 0.25 \, c$ to $\sim 0.55 \, c$. These two cases involve conical shocks so that the jet remains supersonic after the first shock.

As I stated in the beginning of the chapter, the results presented here are based on the study of the contour image of the northern jet presented in (Taylor et al., 1990). A careful study of the original data (which was used to produce Fig. 3 of Taylor et al. (1990)) shows that a fainter knot is also apparent near the core (see Fig. 5.1). Therefore, in the following chapter I revised my study of the Hydra A northern jet with a further improvement of the axisymmetric model.

Although the models presented here are inappropriate to the Hydra A northern jet and provide unusual values for the jet velocity, the density ratio of the jet and the ambient medium and the internal Faraday rotation, they shed some light on the physics of the jet-ICM interaction. For example,

1. Formation of different reconfinement shocks, diamond shocks or normal shocks, depending on the pressure ratio between the jet and the ambient medium (see Fig. 4.2).

2. Formation of two different jet-lobe morphologies: one is a lobe fed by a jet with biconical shocks and the other is a lobe fed by a jet with a disruptive Mach disk (see § 4.2.1 and Fig. 4.3)

3. A correlation between the jet velocity and the location of the inner jet knot. This correlation can be easily demonstrated by the relationship between the jet kinetic power $P_{\text{jet}}$ and Mach number $M$ for a non-relativistic flow

$$P_{\text{jet}} = \frac{\gamma}{\gamma - 1} p_{\text{jet}} v_{\text{jet}} A_{\text{jet}} \left(1 + \frac{\gamma - 1}{2} M^2\right),$$

(4.7)

Sutherland & Bicknell (2007a), and the relationship for the natural wavelength of a non-relativistic supersonic jet (equation 1.1):

$$\Lambda/r_{\text{jet}} = 2.6 \sqrt{M^2 - 1}.$$
According to the latter equation, for a lower wavelength and hence shock spacing we require a lower Mach number. Then equation (4.7) implies that, for a fixed jet kinetic power, jet pressure and jet inlet radius, lowering the Mach number results an increase in the jet velocity. Hence there is an inverse relationship between the shock spacing and the jet velocity for a given jet power. This relationship suggests that the appearance of a bright knot near the core may be a remedy for the unusually low velocity for the Hydra A jets estimated in this chapter.

In the following chapter, based on two knots in the inner 10 kpc of Hydra A northern jet, I estimate a jet velocity \( \approx 0.8 \, c \), which is reasonable for an FRI source (Laing & Bridle, 2014).

4. The Mach number of the jet is related to the jet velocity \( v_{\text{jet}} \), the jet pressure \( p_{\text{jet}} \) and the jet density \( \rho_{\text{jet}} \):

\[
M = \frac{v_{\text{jet}} \rho_{\text{jet}}}{\gamma p_{\text{jet}}}. \tag{4.8}
\]

According to equation (4.7) a fixed jet kinetic power, jet pressure and jet inlet radius, increase in the jet velocity results in a decrease in the jet density and hence a decrease in the Faraday rotation of the source. Therefore, as above, the bright knot near the core will provide lower values of jet density and Faraday rotation. In the next chapter, modelling the Hydra A jet knot with two bright knots, I obtain a density ratio between the jet and the ambient medium \( \eta \approx 10^{-4} \), which is a typical value of an AGN jet, and a Faraday rotation \( \Phi \approx 10^{-2} \), which is comfortably less than unity and consistent with the observed polarisation of the jet (Taylor et al., 1990).
CHAPTER 5

Jet velocity from knot locations and radial oscillations

When I finished my modelling of the Hydra A northern jet based on the data presented in Taylor et al. (1990) (where only one knot is evident within the central 10 kpc, see Fig. 4.1), I requested Professor Gregory Taylor to provide the original 6 cm VLA data, which I could use to study the jets near the core region more carefully and to estimate the brightness ratio of the jets. After constructing more finely contoured maps from the original data I realised that a fainter knot is apparent at approximately 3.7 kpc from the core in the northern jet. Therefore, a revision of the parameter space study was required in order to model both of these internal jet knots inside 10 kpc of the northern jet. In this section, I present my study of 2D axisymmetric jet-ICM interactions focusing on the two inner jet knots in the Hydra A northern jet. Fig. 5.1 shows two bright knots marked by arrows, location of the shocks marked by ×, and the location of the turbulent transition of the jet to a plume marked by an arrow. Apart from incorporating two bright knots inside 10 kpc of the northern jet, I improve the axisymmetric model presented in the previous chapter with the following two additional features:

**Conical opening jet:** In the numerical study of AGN jets, jets are considered to be either initially parallel (Sutherland & Bicknell, 2007a; Wagner & Bicknell, 2011) or
Figure 5.1  Radio intensity map of the central 20 kpc region of the Hydra A northern jet at 4.635 GHz. Contour levels are at 1.5, 2.7, 3.7, 5.1, 6.3, 7.5, 8.8, 10, 21, 37, 51, 72, 90, 103, 154, 311, and 466 mJy arcsec$^{-2}$. Two bright knots and the turbulent transition of the jet to a plume are marked by arrows. The location of the reconfinement shocks which I interpret as the cause of bright knots are marked by $\times$.

initially conical (Komissarov & Falle, 1998; Krause et al., 2012). In the study of global effects of the jet-ambient medium interaction, for example, the mass or energy transport by the jet, it is not important whether the jet is initially parallel or conical. However, structures such as, reconfinement shocks along the jet axis, are sensitive to the initial jet radius and the opening angle of the jet. Moreover, the fact that AGN jets are emitted from the black hole implies that they are initially expanding. For instance, the VLBI pc scale data (Taylor, 1996) and VLA kpc scale data (Taylor et al., 1990) of the Hydra A indicate that the jets expand from approximately 1 pc to approximately 200 pc. Therefore, for the modelling of jet structures near the core, a conical opening jet is more realistic. In the models presented in this chapter I use initially conical jet model following Komissarov & Falle (1998).

Oscillatory nature of the jet boundary: Oscillation of the jet boundary is a natural consequence of periodic reconfinement shocks (Prandtl, 1907; Sanders, 1983). From the deconvolved FWHM of the jets of Hydra A (Taylor et al., 1990), a radial
oscillation is apparent (see Fig. 6 of Taylor et al. (1990) and Fig. 5.2 in this chapter). I did not consider the radial oscillation in the study presented in Chapter 4. In this chapter, I consider both the knot locations and the oscillation of the jet boundary in modelling the northern jet.

Here I present a model of a conically expanding jet entering the computational domain and interacting with the environment. I record the location of the shocks and the oscillation of the jet radius for a large number of models with different jet parameters. Both the shock locations and the oscillation of the jet boundary are used as constraints on the jet parameters: the jet radius \( r_{\text{jet}} \), the over-pressure ratio \( p_{\text{jet}}/p_a \), the jet density parameter \( \chi \) and the jet velocity \( v_{\text{jet}} \). The results are then analysed to obtain a best fit model for the Hydra A northern jet. The results of this study have been published in the journal MNRAS (Nawaz, M. A., Wagner, A. Y., Bicknell, G. V., Sutherland, R. S., and McNamara, B. R., 2014, MNRAS, 444, 1600).

5.1 Jet parameters

In this section I describe the selection of the initial jet parameters, the jet cross-sectional area \( A_{\text{jet}} = \pi r_{\text{jet}}^2 \), where \( r_{\text{jet}} \) is the jet inlet radius), the jet pressure \( p_{\text{jet}} \), the jet density parameter \( \chi = \rho_{\text{jet}} c^2 / (\epsilon_{\text{jet}} + p_{\text{jet}}) \), where \( \rho_{\text{jet}} \) and \( \epsilon_{\text{jet}} \) are the rest mass density and the energy density of the jet respectively) and the jet Lorentz factor \( \Gamma = (1 - \beta^2)^{-1/2} \). These parameters are assigned so as to be consistent with the expression for the jet power (equation (4.2)).

5.1.1 Over-pressured jets

A key feature of the jet model is that the bright knots beginning at \( \sim 3.7, 7.0 \) and \( 11.0 \) kpc from the core in the northern jet and at \( \sim 2.5, 3.9, 5.4 \) and \( 6.7 \) kpc in the southern jet are the result of consecutive biconical shocks following recollimation of over-pressured jets. I have identified the points where the surface brightness gradient markedly increases, as the location of the upstream side of each knot (see Fig. 5.1 and Fig. 7.1). The third knot in the northern jet occurs just as the jet
merges into the lobe so that I might expect the location of this knot to be affected somewhat by the jet’s transition to turbulence.

Norman et al. (1982) first drew attention to the production of biconical and normal shocks (Mach discs) in over-pressured astrophysical jets. An initially over-pressured jet expands laterally and its thermal pressure and ram pressure decrease with distance along the direction of propagation. When the jet pressure reaches the ambient pressure, the jet begins to recollimate. The jet periodically expands and recollimates, producing a series of biconical or normal shocks along the jet axis. This phenomenon had been known to laboratory hydrodynamicists for some time and Birkhoff & Zarantonello (1957) associated it with the natural wavelength of a supersonic jet $\Lambda$ (see equation (1.1)).

It is feasible that the Hydra A jets are initially over-pressured since the minimum energy pressure in the parsec scale northern jet, 27 pc from the central black hole (Taylor, 1996) is $1.33 \times 10^{-7}$ and $1.26 \times 10^{-7}$ dynes cm$^{-2}$ for $\beta = 0.2$ and $0.9$ respectively (a jet diameter of 26 pc was used in these estimates). These minimum energy pressure estimates are about a factor of 200 times higher than the central pressure $\approx 6.6 \times 10^{-10}$ dynes cm$^{-2}$ of the modelled interstellar medium (see § 3.2). Moreover, using these pressures underestimates the jet kinetic power at $\approx 2.4 \times 10^{44}$ erg s$^{-1}$ for $\beta = 0.8$ and $\chi \sim 10^{-2}$ compared to the value $10^{45}$ erg s$^{-1}$ used in the models by a factor $\approx 4$. Therefore the value of the jet kinetic power of the models implies a jet pressure $6 \times p_{\text{min}} \approx 7.0 \times 10^{-7}$ dynes cm$^{-2}$ at 27 pc. This is approximately 100 times the central atmosphere pressure. If I assume that the jet expands adiabatically, i.e., the jet pressure decreases with the jet radius according to $p_{\text{jet}} \propto r_{\text{jet}}^{-8/3}$, I obtain an over-pressured jet $5 p_a$ at 0.5 kpc from the core (where I initialise the jet in the computational domain) with a jet radius 100 pc.

Interpreting the jet as over-pressured on the parsec scale implies that from the parsec to the kiloparsec scale it is freely expanding. I also note here that, in a detailed analysis of protostellar jets, Cabrit (2007) has concluded that those jets are initially magnetically collimated but are freely expanding at some distance ($\sim 50$ AU) from the star. Of course, these scales are not directly commensurable with Hydra A, but a long held view is that the physics of protostellar and AGN
5.1 Jet parameters

Figure 5.2  Plot of the jet radius of the northern jet and the location of shocks as a function of the deprojected distance from the core. The radius is estimated from the deconvolved FWHM (see text). The vertical dashed lines represent the location of the southern edge of the first two bright knots in the northern jet, which correspond to the assumed locations of the shocks. In the bottom two panels the simulated logarithmic density and pressure slices show the periodically expanding and reconfining morphology and the shocks produced in the best-fit model for the northern jet. Both the radius plot and the images are stretched in the radial direction, emphasising the wave-like nature of the jet boundary. The vertical solid lines represent the shock positions in the simulations. The colourbar represents \( \log \rho \) on the left and \( \log p \) on the right.

outflows are similar in many respects.

The proposition of the jet bright knots as biconical shocks is further reinforced by the observed wave-like nature of the northern jet boundary. Fig 5.2 shows the radius profile (dots) of the northern jet, which I obtain by assuming the jet to be a homogeneous cylinder and utilising the deconvolved FWHM of the jet (Taylor et al., 1990) \( \Phi_{\text{jet}} \) together with \( r_{\text{jet}} = \Phi_{\text{jet}}/\sqrt{3} \). In order to illustrate the association of biconical shocks with the sinusoidal radius profile I attach the logarithmic density and pressure images (panels marked with \( \log \rho \) and \( \log p \) respectively) of one of the best fit models Ciii for the northern jet. In the simulated radius profile I see the jet boundary oscillates and at \( \sim 0.7 \) kpc before each radius minimum biconical shocks appear. These are clearly indicated by the large increase in pressure. The
observed and simulated shock locations are marked with dashed and solid vertical lines respectively.

I construct models of the northern jet for which data on the jet FWHM are more complete. The modelling strategy for this jet is as follows. I conduct a parameter space study searching for numerical models which can successfully reproduce the correct shock locations and the radius profile of this jet.

In the axisymmetric numerical models of the jet-ICM interaction I deal with straight jets whereas the Hydra A jets are curved. However since the curvature of the jets are modest within the central 10 kpc, I expect an approximation by a straight jet to be reasonable.

As stated above, in order to model the jets of Hydra A I require five jet parameters, the jet kinetic power $P_{\text{jet}}$, the initial jet radius $r_{\text{jet}}$, the initial jet pressure $p_{\text{jet}}$, the initial jet velocity $\beta$ (in units of the speed of light), and the jet density parameter $\chi$, of which four are independent. In the previous section I established a value for the jet kinetic power $10^{45}$erg s$^{-1}$. In the following I describe how I choose the other three independent jet parameters and set their values.

The first parameter is the jet kinetic power, which is reasonably well-determined by the radio and X-ray observations. The jet radius is the second parameter; this affects the downstream scale of the oscillating jet boundary and is not known ab initio. The third parameter is the jet pressure ratio; this affects both the amplitude of the radial oscillations and the knot spacing. The fourth parameter is the jet velocity, $\beta$. Then the parameter $\chi$ is determined using equation (4.3).

Referring to the expression for the natural wavelength $\Lambda$ of a supersonic non-relativistic jet in near pressure equilibrium $\Lambda/r_{\text{jet}} \approx 2.6 \sqrt{M^2 - 1}$ (Birkhoff & Zarantonello, 1957), I note that the selection of the velocity and density parameters is equivalent to defining the Mach number,

$$M = (2 + 3\chi)^{1/2} \Gamma \beta$$

(Bicknell, 1994).

Following Komissarov & Falle (1998) and Krause et al. (2012), I model the jet as
5.2 Grid of models

ballistic and conically expanding in the first 0.5 kpc, which represents the base
of the computational domain. Komissarov & Falle (1998) used an identical setup
in their simulations to show that an initially conical jet may be collimated by the
ambient pressure. Krause et al. (2012) performed simulations, also with identical
initial conditions to provide a theoretical basis for the FRI/FRII classification of
radio sources based on the half cone angle of the initial jet cone.

To summarize, I set up my simulations with an initially over-pressured (in one case
equilibrium pressure) conically expanding jet with cross-sectional radius \( r_{\text{jet}} \) and
centre at \((r, z) = (0, 0.5) \) kpc, where \( r \), and \( z \) are the radial and height coordinate
of the axisymmetric cylindrical domain. The independent jet parameters are jet
power \( P_{\text{jet}} = 10^{45} \) erg s\(^{-1} \), jet radius \( r_{\text{jet}} \), inlet jet pressure \( p_{\text{jet}} \), and inlet jet velocity \( \beta \).

The remaining jet parameter \( \chi \) is determined from equation (4.3). The components
of the jet velocity at a points \((r, z)\) within the initial conically expanding jet cross-
section are \( v_r = \beta z / \sqrt{r^2 + z^2} \) and \( v_z = \beta r / \sqrt{r^2 + z^2} \).

5.2. Grid of models

For my simulations I use the the publicly available PLUTO code (Mignone et al.,
2007) and produce two dimensional axisymmetric hydrodynamic models of the
jet-ICM interaction in Hydra A. Since my models involve relativistic velocities
I use the relativistic hydrodynamic (RHD) module available in PLUTO. Detail
description of the code and the problem initialisation are given in Chapter 2.

To determine the optimal values for the three initial jet parameters \( r_{\text{jet}}, p_{\text{jet}}, \) and
\( \beta \) for the Hydra A northern jet, I compare the radius profile of the jet and the
locations and spacing of the reconfinement shocks in my simulations with the
observed radius profile and shock positions as indicated by the locations of the two
bright knots. The thirty three sets of parameters that I have used are summarised
in Table 5.1. I have not utilised every possible combination of parameters since
I have restrictions on the jet radius minimum of 160 pc. I have not used models
with five times over-pressured jet with jet inlet radius 180 pc and two times over-
pressured jet with inlet radius 100 pc since they will produce much larger or smaller
minimum in the radius profile than 160 pc. Experimenting with models with lower jet velocities I obtain significantly large shock spacing compared to the observed shock spacing. Therefore, I have not presented models with $\beta < 0.4$. A grid of models with jet $\beta = 0.5$ which exhibit larger shock spacings is presented in § 5.3.4.

For an additional check for the consistency of the jet parameters, some derived parameters, namely the density parameter $\chi$, the density ratio $\eta$ of the jet and the atmosphere at the jet base, the rotation measure (RM) $\phi$, and the Faraday rotation angle $\Psi$ at 6 cm ($\Psi_{6\text{cm}}$) and 20 cm ($\Psi_{20\text{cm}}$) are also summarised in Table 5.1. Unlike models with a single knot (presented in Chapter 4), all of the Faraday rotation values are comfortably less than unity and in the best models, Ciii, Civ and Cv, much less than unity.

I group the runs into four sets as set out in Table 5.1; Sets A, B and C correspond to simulations with initial jet radii of 0.18kpc, 0.15kpc and 0.10kpc, respectively. Set D corresponds to model with jet $\beta = 0.5$ and initial jet radii 0.12, 0.10, 0.08 and 0.06kpc.
Table 5.1  Simulation parameters. In all simulations, \( P_{\text{jet}} = 10^{45} \text{ erg s}^{-1} \).

<table>
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<th>( \chi )</th>
<th>( \eta )</th>
<th>( \phi ) (\text{rad cm}^{-2})</th>
<th>( \Psi_{60m} ) (rad)</th>
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5.3. Simulation Results

In this section I present the results of my two-dimensional axisymmetric hydrodynamic simulations, including the parameter study described above. I have conducted a series of simulations to cover the parameter space described in Table 5.1. I first describe the results of my parameter space study, which enable us to constrain the jet velocity and other jet parameters at 0.5 kpc from the black hole. These provide best fit models for the northern Hydra A jet. Using one of the best fit models, Civ, I then discuss the association of biconical shocks with the bright knots, the turbulent transition of the jet, and the flux density ratio between the northern and southern jet of Hydra A. Finally, based on the discrepancy between the simulated and the observed flux density ratio, I explore the possibility of varying the angle of inclination within the range defined by Taylor & Perley (1993).

5.3.1. Parameter space study for the northern jet

The aim of the parameter space study is to obtain optimal values for the jet parameters, in particular, the jet radius, the jet pressure and the jet velocity at 0.5 kpc from the core.

As discussed in § 5.1, the natural wavelength for the occurrence of reconfinement shocks in a supersonic jet is directly related to the jet velocity. I vary the jet velocity, at the same time consistently varying the density parameter $\chi$ to maintain a constant jet kinetic power, noting the location of the first two reconfinement shocks in the jet for each run. As the cocoon pressure decreases with increasing size the locations of the reconfinement shocks of each run evolve with time. The shocks gradually shift downstream and reach asymptotic values at approximately 20 Myr. I take these asymptotes as the location of the shocks. Figure 5.3 shows the evolution of the location of the first (blue dots) and second (blue crosses) shocks with time and the observed location of the shocks (green lines) for run Civ.

The shock positions also vary on a short time scale, oscillating about a mean position.
5.3 Simulation Results

Figure 5.3  Evolution of the locations of the first (blue dots) and second (blue crosses) shocks with time for run Civ. The horizontal lines represent the observed shock locations. This figure shows that the first two reconfinement shocks move downstream with time and asymptote towards 3.6 and 7.4 kpc at approximately 20 Myr.

These variations occur because the pressure field in the backflow adjacent to the jet changes intermittently as a result of the turbulence in the cocoon. Hence, for each run, I have measured the position of the jet shock at five time steps separated by 100 kyr in time. Figure 5.4 shows the jet radius profiles and reconfinement shock positions from selected simulations. The results from simulations in set A, B, and C are shown in the top, middle two, and bottom rows, respectively. I compare the simulated shock positions (solid vertical lines) with the observed shocks in Hydra A (dashed vertical lines) and also compare the simulated jet radius profiles (solid green lines and squares) with the observed jet radius profile (solid blue line and circles).

In assessing these models, one first notes a strong dependence of shock location on jet speed, as expected, and I use this as the first discriminant in selecting candidate best fit models. This narrows the choice to Aiii, Biii, Ciii, Civ, and Cv. Then,
Figure 5.4  Jet radius profiles and shock positions along the jet extracted from selected hydrodynamic simulations. The green line with squares, and the blue lines with circles represent simulated and observation data of radius, respectively. The blue dashed and green solid vertical lines represent the observed and simulated shock locations, respectively. The top row of panels are simulations from set A, the second and third rows of panels are simulations from set B, and the bottom row of panels are simulations from set C. In the simulations shown in the upper two rows of panels, $p_{\text{jet}}/p_{\text{ICM}} = 2$, whereas in those shown in the lower two rows of panels, $p_{\text{jet}}/p_{\text{ICM}} = 5$. The left, middle, and right column of panels, show simulations for which $\beta = 0.75$, 0.80, and 0.85, respectively. A visual comparison of the jet radius profiles and shock positions between the simulations and observations shows that, of the models, Ciii, Civ and Cv give the good fit models.
focusing on the radius profile, in models Aii, Bii, the jet radius does not contract sufficiently at large distances, which make these two models less appealing. At the same time, I note that the remaining models Cii, Ciii and Civ provide poor radius fits within 3 kpc. However, the first three data points are derived from a region, which is affected by the emission from the core (see Taylor et al., 1990, Fig. 3). It is also possible that the models do not capture the details of the initial jet-ISM interaction in this region.

Hence, I concentrate on the data points further out from the core. Consequently the choice for the best fit models are Ciii, Civ and Cv. My preference for these three models is based on the fact that the simulated radius shows larger excursions between minima and maxima as exhibited by the data. The parameters for the best fit models Ciii, Civ and Cv are \( r_{\text{jet}} = 100 \text{pc}, \frac{p_{\text{jet}}}{P_{\text{ISM}}} = 5 \) and \( \beta = 0.75, 0.80, \) and \( 0.85 \), respectively. I also note that the last point in the observed radius profile jumps significantly. I attribute this to the onset of turbulence in the jet where it makes a transition to a plume. The third knot/shock may be affected by this transition so that in deciding between models I have mainly concentrated on the first two knots.

### 5.3.2. The surface brightness of the knots in the Northern Jet

To strengthen the association of biconical shocks with the bright knots in the Hydra A northern jet I present a synthetic radio image of one of the best fit models, Civ, based on an assumed synchrotron emissivity \( j_\nu \approx \lambda \delta^{2+\alpha} p^{(3+\alpha)/2} \), where \( \lambda \) is the relativistic gas tracer, the Doppler factor \( \delta = 1/\Gamma(1 - \beta \cos 42^\circ) \) and the pressure dependence assumes that the magnetic pressure is proportional to the non-thermal particle pressure (see Sutherland & Bicknell, 2007b, §5.4). Integrated along rays \( I_\nu = \int j_\nu ds \), this emissivity provides a semi-quantitative estimate of the surface brightness corresponding to this model.

Fig. 5.5 (left panel) shows the synthetic surface brightness of the simulated jet. The contour image of the synthetic surface brightness is shown in the right panel. Here we see that, in the shocked zone beyond each biconical shock, the pressure increases, producing bright knots in each region. This image reproduces some qualitative features of the data: The second and third knots are significantly brighter and
Figure 5.5 Synthetic surface brightness for model Civ based on an emissivity $j_\nu = \delta^{(2+\alpha)} P^{(3+\alpha)/2} \nu^{-\alpha}$, where $\delta = 1/\Gamma(1 - \beta \cos 42^\circ)$ is the Doppler factor. The right panel shows the surface brightness contours of the left panel. The contour levels are 4, 8, 11, 14, 25, 35, 40 in arbitrary units.
5.3 Simulation Results

Figure 5.6 Logarithmic density snapshot for run Civ at \( t = 20 \) Myr in the left panel. The right panel shows the zoomed in central zone marked with black rectangle in the left panel. This is one of the best fit models, which yields the correct location of the first two biconical reconfinement shocks in the northern jet of Hydra A. A transition to turbulence occurs due to significant shock deceleration of the jet in the reconfinement shocks and the developing Kelvin-Helmholtz instability.

more extended than the first knot. However, the brightness ratios of the knots are not reproduced. Observationally (corrected for resolution) the second knot is 8.7 times brighter than the first and the third knot is 3 times brighter than the second. The model values are 2.5 and 1.14 respectively. In addition, in the observed jet, the FWHM extent of the second knot in the jet direction is 3.3 kpc compared to 0.6 kpc for the model. These differences may possibly be attributed to the approximate magnetic field model, which I have used, or the lack of turbulent three dimensional structure in the simulations. These are aspects to which I can return with three-dimensional simulations with magnetic field.

5.3.3. Transition to turbulence

These two-dimensional models cannot adequately reproduce the structure of the entire source, in particular the plume like regions beyond approximately 7.4". 
These are probably the result of three-dimensional turbulence and/or precession, and these effects will be addressed in future study of three dimensional precessing jet model. However, I note that the numerical models qualitatively reproduce the turbulent transition of the jets to plumes, albeit at a distance of 14 kpc compared to approximately 11 kpc deprojected in Hydra A. In the density image snapshot at approximately 20 Myr of run Civ, Fig. 5.6 (the left panel shows the full computational domain and the right panel is the zoom in section indicates by the rectangle in the left panel), a series of biconical shocks appears in the jet. Deceleration of the jet occurs at these shocks and the jet becomes subsonic after the fourth shock at ~ 14 kpc (see the variation of Mach number of the flow with distance along the jet axis in Fig. 5.7). Beyond 14 kpc the jet transitions to turbulence as a result of the axisymmetric Kelvin-Helmholtz instability, which becomes stronger as the Mach number decreases.) Although the axisymmetric jet simulations shed some light on the turbulent transition of the jet, it is well known that turbulence and the formation of plumes are three dimensional phenomena, especially in supersonic
flows. I study the details of these features of the inner 20 kpc of the Hydra A jets in the ensuing three dimensional study.

5.3.4. Brightness ratio of the jets

I have used the 6cm VLA data of Taylor et al. (1990) to determine the flux density ratio of the northern and southern jets within the first 10 kpc, obtaining a value $\approx 7.0$. Attributing this ratio to Doppler beaming, and using the inclination estimated by Taylor et al. (1990), implies a moderately relativistic jet $\beta \approx 0.5$. However, my parameter space study produces a higher jet velocity $\sim 0.8$ which, on the basis of a simple estimate, would give a brightness ratio $\approx 40$. However, in my model, the emissivity is dominated by the decelerated post-shock regions of the jet, so that I estimate the brightness ratio from the synthetic brightness images of approaching and receding jets. With this approach, I obtain a simulated flux density ratio of 33 which still differs significantly from the observed value by a factor $\approx 5$.

I ran several additional models with jet $\beta = 0.5$, different jet inlet radii 120 pc,

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1Taylor et al. (1990) quote a value of 1.9, which is close to the observed ratio within 1 kpc.
Jet velocity from knot locations and radial oscillations

100 pc, 80 pc and 60 pc and different pressure ratio 5, 10, and 15, keeping the jet kinetic power constant at $10^{45}$ ergs s$^{-1}$. I have not decreased the jet radius below 60 pc because that would require an even more highly over-pressured jet to obtain the correct radius profile. These $\beta = 0.5$ models are summarised in Table 5.1 (set D) and the comparison of the simulated and observed shock positions and radius profiles are shown in Fig. 5.8. It is evident that no model with the given jet kinetic power and jet $\beta = 0.5$ is able to produce good fits for both the shock position and the jet radius. The shock spacings are all significantly larger than the observed shock spacing and the radius profiles are mismatched with these models.

From the above I can say that if I fix the inclination angle at the Taylor et al. (1990) value of $42^\circ$ and fix the jet kinetic power at $10^{45}$ ergs s$^{-1}$, then the jet pressure, jet velocity and the inlet jet radius at 0.5 kpc away from the core of the Hydra A northern jet are well-constrained by both the jet radius profile and the first two knot/shock spacings. The best-fit values are $\beta = 0.75 - 0.85$ and $r_j = 100$ pc. Thus, there is a discrepancy in the flux density ratio between the simulated and observed jets. Two potential explanations of the low flux density ratio are: i) Since my models do not include the magnetic field, I employ the assumption $p \propto B^2/8\pi$ which gives a brightness ratio 33. If I further assume that the magnetic field is 2.5 times stronger in the southern jet of Hydra A I would obtain a lower brightness ratio $\sim 7$. ii) The southern jet is more dissipative since it is more twisted and produces more shocks producing a larger intrinsic emissivity than the northern jet.

5.3.5. Variation of the inclination angle

In the above models I have used the angle between the jet and the line of sight, $\theta \approx 42^\circ$, estimated by Taylor & Perley (1993) from the rotation measure asymmetry of Hydra A. However, there is a fairly large uncertainty in their estimate of $\theta$ with $30^\circ \leq \theta \leq 60^\circ$. Increasing $\theta$ from $42^\circ$, would reduce the brightness ratio? However, for larger inclinations, the deprojected knot separation would decrease, and as I have seen with the above models, this would require a higher velocity than 0.8c, tending to increase the brightness ratio. Similar considerations apply if I decrease the inclination. Nevertheless, is it possible that notwithstanding opposing effects,
5.3 Simulation Results

Figure 5.9 Comparison of the observed spacing of the first two shocks (blue curves) with the corresponding simulated shock spacing (red points for model set A, black points for model set B, and green points for model set C) as a function of the 4-velocity $\Gamma \beta$. The dashed vertical lines represent the upper limits of $\Gamma \beta$ for each model (set A – red, set B – black, and set C – green). These limits are estimated for $\chi = 0$ using equation (4.3).

In order to assess this possibility I adopted the following procedure: For $\beta$ within the range, $0.35 < \beta < 0.98$ (the lower limit being defined by the brightness ratio, $R = 7$) I first estimate the value of $\theta$ corresponding to $R = 7$, using the standard Doppler beaming formula, $\theta(\beta) = \beta^{-1}(R^{1/2.7} - 1)(R^{1/2.7} + 1)^{-1}$. For these values of $\theta(\beta)$ I determine the deprojected spacing between the first two shocks $D_{1,2}(\beta) = 2.25/\sin \theta(\beta)$ kpc given the observed spacing of 2.25 kpc. This is shown, as a function of the 4-velocity, $\Gamma \beta$, as the blue curve in Fig. 5.9. I then compare the observed deprojected knot spacings with the values inferred from the simulations so that in Fig. 5.9 the simulated shock spacing, for model sets A, B and C, are also plotted as functions of $\Gamma \beta$. The upper limits on the 4-velocity for each model...
Jet velocity from knot locations and radial oscillations

(estimated from equation (4.3)), associated with a zero density parameter, $\chi = 0$, are also shown as dashed vertical lines.

The first point to note with this comparison is that for most of allowable range of $\beta$

Fig. 5.9 shows that the calculated shock spacing exceeds the observed, deprojected value. At the upper end of the $\beta$ range the simulated shock spacings for each model asymptote to $\approx 2.85 \text{kpc}$ for values of $\Gamma\beta \gtrsim 3$, i.e., $\beta \gtrsim 0.95$. However the asymptote of the observed shock spacing $\approx 2.4 \text{kpc}$. Hence, there is an offset of approximately 0.5 kpc between the asymptotes of the simulated and observed shock spacing for $\Gamma\beta \gtrsim 3$.

At the other end of the allowable range of velocity, $\beta \approx 0.345 (\Gamma\beta = 0.368)$, it could be inferred that the simulations and observations intersect at approximately this limiting value. However, this is the result of the steepness at $\beta \approx 0.345$ of the blue curve representing the observed deprojected shock spacing as a function of 4-velocity, rather than a real physical correspondence between observed and simulated values. It would be fortuitous if the jet initial velocity were to be almost exactly the same as the lower limit on the jet velocity implied by beaming. Hence I reject a solution at this end of the $\beta$ range on the basis of the “fine-tuning” that would be involved in accepting it. Another unappealing feature of a low-$\beta$ solution is that the jet would be initially heavy with $\chi \gtrsim 300$. As I noted above, observations and modelling of X-ray observations of the lobes of radio galaxies indicate that jets are initially electron-positron in composition (Croston et al., 2005; Croston & Hardcastle, 2014) and $\chi \gtrsim 300$ is inconsistent with this.

Another way of looking at the issue of reconciling shock spacing and flux ratios is the following: Consider the simulation points near the upper end of the $\beta$ range in Fig. 5.9, where the discrepancy between the observed jet and simulated jets with $\chi \sim 1$ is the least. By way of example, consider the (green) point in simulation series C with $\beta = 0.95 (\Gamma\beta = 3.04)$. The simulated flux ratio (see § 5.3.4) for this model is 26.5, a factor of 3.8 higher than the observed value. Thus, even for these models there is an implication of intrinsic differences in the northern and southern jet rest-frame emissivities. Moreover, this ratio is not very different from the value of 33 for the $\beta = 0.8, \theta = 42^\circ$ model considered earlier.
In view of the above, I conclude that, taking into account the modelling of shock spacing, radius evolution and surface brightness ratios, the most likely situation is that of fast, $\beta \gtrsim 0.8$, jets with an intrinsic difference between the rest-frame emissivities of northern and southern jets.
CHAPTER 6

A verification of the axisymmetric model: model of a naked jet

The simulations of jets propagating into the cluster atmosphere conducted in the previous two chapters typically evolve to an extent of around 50 kpc, which is the extent of the simulation domain. The jet evolution occurs over a timescale of approximately 20 Myr. While the shock structures in the jet, the turbulent transition of the jet, and global structures, e.g. the turbulent backflows in the cocoon and forward shock of the bubble, are well captured in the simulations, the physical simulation time is less than the age of the source, which is approximately 50 Myr. It is possible, therefore, that the properties of the flow adjacent to the simulated jet are not representative of the medium the real jet is engulfed in. For example, the simulated jet may be strongly affected by the backflow generated as the jet propagates within this restricted domain.

The purpose of this chapter is, thus, to ascertain whether the shock positions seen in the simulations are affected by the backflow originating from the head of the jet down along the jet. To this end, I look at the evolution of the shock positions with time, and also perform a simulation of an unbounded jet – a jet spanning the entire domain and not bounded by a termination shock. I refer to such a jet as a “naked jet”. This is more representative of conditions in the inner regions of an evolved radio source, such as that of Hydra A. The fact that the shock positions are
not affected by the backflowing ambient plasma, will reassure us that the results presented of in the previous chapter are applicable to the inner Hydra A jet during later stages of its evolution.

### 6.1. Simulation results

The simulation of an unbounded “naked” jet is an axisymmetric relativistic hydrodynamic simulation. I set up the jet inlet with the same jet parameters and geometry as in run Cvi, but I also initiate the full extent of a jet column spanning the length of the domain along the jet axis at the start of the simulation. The radial profiles for the hot atmosphere, the boundary conditions, and the numerical scheme are the same as those in the simulations presented in Chapter 5.

Since the jet is initially overpressured, it expands, then contracts, sending a nearly axisymmetric shock wave into the ambient medium, and forming reconfinement shocks along the jet axis. Kelvin-Helmholtz instabilities develop in the shear layer between the jet and the ambient medium, but other than the initial adjustment, the flow in the entire simulation box is broadly time-independent. Without a jet backflow, a turbulent cocoon does not develop.

Figure 6.1 shows the features developed by a naked jet model at time \( t = 19 \) Myr. The right panel shows a closeup of the central \( 5 \times 20 \) kpc region. The jet is embedded in a relatively steady ambient medium exhibits the first two reconfinement shocks at approximately 3 and 6 kpc.

Simulations of naked jets allow us to measure the quasi-steady reconfinement shock positions because, in the absence of a turbulent cocoon, the global structure of the flow in the simulation domain and the pressure field of the medium surrounding the jet remain fairly steady. Thus, the naked jets, as presented in the simulations here, can be thought of the state of the inner jet for a source that has evolved to much larger spatial extents than the region contained in the simulation box, largely unperturbed by the distant turbulent cocoon. One restriction is, of course, that the nearly steady state structure of the ambient hot atmosphere would not obtain beyond its cooling time.
Figure 6.1 Logarithmic density image of a snapshot in during the simulation of the interaction of an unbounded “naked” jet with a non-turbulent ICM. The shock positions are almost time-independent as the jet expands and reconfines multiple times before escaping through the top boundary. No transition to turbulent flow is observed. The right panel is a close-up of the central region marked with a box in the left panel. In both panels the physical scales are in units of kpc.
Figure 6.2 shows the evolution of the first (circles) and second (crosses) shock positions for model Cvi, respectively, as a function of time. The shock positions increase with time, but appear to asymptote to values very close to 3 and 6 kpc, respectively. The locations of the two reconfinement shocks in the naked jet model are also shown in Fig. 6.2 with two horizontal lines. These shock locations agree with the asymptotic values of the shock position in run Cvi.

Simulations of unbounded naked jets are useful for accurately measuring nearly time-independent positions of shocks in a jet, provided the top outflowing boundary is not affecting the structure of the jet. However, these models are not useful in explaining other important features of the Hydra A northern jet, in particular, the turbulent transition of the jet and the formation of the plume structure. As we see in the naked jet simulation the deceleration through the biconical shocks and the entrainment in the shear layer is insufficient to produce a turbulent transition of
the jet to a plume. From this, I surmise that the ram pressure of the turbulent back flow in the cocoon plays an important role in further decelerating and disrupting of the jet. We see this phenomenon, at least qualitatively, in the simulations of an evolving jet, in which the jet is surrounded by a cocoon of entrained, turbulent, backflowing plasma, and is clearly affected by the surrounding flow.
In Chapter 5 I presented the detail study of the Hydra A northern jet focussing two bright knots and the radial oscillation of the central 10 kpc jet. Here I consider the implications of the knot structure in the southern jet. In this case there are four knots compared to two in the northern jet and the jet is more curved. Moreover there has been no determination of the jet radius versus distance from the core. Hence, the southern jet data do not provide as good a basis for parameter estimation. Nevertheless, it is of interest to apply the previous approach, which reinforces the velocity estimates for the northern jet. There is one interesting difference in that the optimal model has an initial overpressure ratio of unity. Four bright knots and the locations of the associated reconfinement shocks are shown in Fig. 7.1.

Guided by the models for the northern jet, I perform a parameter study for the southern jet on the basis of the same proposition that the four bright knots in the southern jet are the consequence of four consecutive biconical shocks at 2.5, 3.9, 5.4, and 6.7 kpc. As I discussed earlier, because of the lack of observational data for the radius profile of the jet within 5 kpc from the core, I only consider the shock positions of this jet.
Figure 7.1  Radio intensity map of the central 20 kpc region of the Hydra A southern jet at 4.635 GHz. Contour levels are at 1.5, 2.0, 2.2, 2.7, 3.7, 5.1, 5.5, 6.0, 6.3, 6.8, 7.5, 8.8, 10, 21, 37, 51, 72, 90, 103, 154, 311, and 466 mJy arcsec$^{-2}$. Four bright knots are marked by arrows. The location of the reconfinement shocks which we interpret as the cause of bright knots are marked by $\times$.

7.1. Parametric study for the southern jet

We fix the jet power and jet radius to the values of the best-fit model for the northern jet at $P_{\text{jet}} = 10^{45}$ erg s$^{-1}$ and $r_{\text{jet}} = 100$ pc, respectively, and vary only the two other independent jet parameters, the jet velocity and the pressure ratio between the jet and the ambient medium. Because the spacing of the shocks in the southern jet is smaller than that in the northern jet, I explored slightly higher ranges of the

Table 7.1  Simulation parameters. In all simulations, $P_{\text{jet}} = 10^{45}$ erg s$^{-1}$.

<table>
<thead>
<tr>
<th>Simulation</th>
<th>$p_{\text{jet}}/p_a$</th>
<th>$\beta$</th>
<th>$\chi$</th>
<th>$\eta$</th>
<th>$\phi$ (rad cm$^{-2}$)</th>
<th>$\Psi_{6\text{cm}}$ (rad)</th>
<th>$\Psi_{20\text{cm}}$ (rad)</th>
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<tr>
<td>Ei</td>
<td>5</td>
<td>0.95</td>
<td>0.57</td>
<td>$3.18\times10^{-3}$</td>
<td>$1.61\times10^{-3}$</td>
<td>$5.80\times10^{-4}$</td>
<td>$6.45\times10^{-5}$</td>
</tr>
<tr>
<td>Eii</td>
<td>5</td>
<td>0.96</td>
<td>0.15</td>
<td>$8.33\times10^{-6}$</td>
<td>$4.22\times10^{-6}$</td>
<td>$1.52\times10^{-4}$</td>
<td>$1.69\times10^{-3}$</td>
</tr>
<tr>
<td>Eiii</td>
<td>2</td>
<td>0.80</td>
<td>35.63</td>
<td>$7.97\times10^{-4}$</td>
<td>$4.04\times10^{-4}$</td>
<td>$1.45\times10^{-2}$</td>
<td>$1.61\times10^{-1}$</td>
</tr>
<tr>
<td>Eiv</td>
<td>1</td>
<td>0.75</td>
<td>113.85</td>
<td>$1.27\times10^{-3}$</td>
<td>$6.45\times10^{-4}$</td>
<td>$2.32\times10^{-2}$</td>
<td>$2.58\times10^{-1}$</td>
</tr>
<tr>
<td>Ev</td>
<td>1</td>
<td>0.80</td>
<td>73.77</td>
<td>$8.25\times10^{-4}$</td>
<td>$4.18\times10^{-4}$</td>
<td>$1.50\times10^{-2}$</td>
<td>$1.67\times10^{-1}$</td>
</tr>
<tr>
<td>Evi</td>
<td>1</td>
<td>0.85</td>
<td>44.66</td>
<td>$4.99\times10^{-4}$</td>
<td>$2.53\times10^{-4}$</td>
<td>$9.10\times10^{-3}$</td>
<td>$1.01\times10^{-1}$</td>
</tr>
</tbody>
</table>
7.1 Parametric study for the southern jet

Figure 7.2 Shock positions, marked by points, for different jet velocity (bottom x-axis) and different pressure ratio (top x-axis). The horizontal solid lines represent the observed shock locations of the southern jet. The models presented here are deviated from the best fit model for Hydra A northern jet (Civ, in Chapter 5), in the jet velocity or, in the over pressure ratio. A visual comparison of the shock locations between the simulations and the observations shows that the model of the southern jet is with $p_{\text{jet}}/p_a = 1$, and $\beta = 0.8$.

jet velocity for a given value of $p_{\text{jet}}/p_a$. I also performed runs with pressure ratios $p_{\text{jet}}/p_a = 2$ and $1$. Simulation parameters for the different models are presented in Table 7.1.

The parameter $\chi$, which is determined by the other jet parameters through equation (4.3) becomes negative for jet velocities greater than 0.96, so that this represents the maximum possible jet velocity. I therefore explored values of the pressure ratio $p_{\text{jet}}/p_a = 2$ and $1$, keeping the jet velocity fixed at $\beta = 0.8$ (run Eiii, Eiv, Ev, and Evi).

The results of different runs are shown in Fig. 7.2. In this figure the vertical dashed lines represent individual models (labeled by model names) with increasing order from left to right. The over pressure ratio and velocity of each model are presented on the top and bottom axes. The points on each model line represent the shock
Figure 7.3 Logarithmic pressure image of the best fit model for the southern jet (run Dvi) shows the four biconical shocks along the jet marked by 1, 2, 3 and 4.

The model with a pressure equilibrium jet and jet (run Ev) produces the required locations.
four shocks within the central 7 kpc. To see the dependence of the shock positions on the jet velocity I ran models with two other jet velocities 0.75 and 0.85. From the comparison of the simulated and observed shock positions I conclude that the best fit model for the southern jet is with jet velocity $\beta = 0.8$.

Figure 7.3 shows the logarithmic pressure image of the central $2 \times 8$ kpc domain of the best fit model for the southern jet (run Eiv). This image shows the four consecutive conical shocks marked with 1, 2, 3 and 4.

7.2. summary

![Image](image_url)

**Figure 7.4** Logarithmic pressure image for an over-pressed jet (run Ei, left panel) and a pressure equilibrium jet (run Ev, right panel) imposed with the flow velocity. The flow velocity appears to be converging at the reflected shock zone in the case of over-pressed jet and to be diverging in the case of pressure equilibrium jet. In the pressure equilibrium case the difference in the expansion rate of the jet in the outer and inner part of the reflected shock zone (marked by the pentangle) cause an extra biconical shock in the reflected shock domain.

The axisymmetric models with pressure equilibrium jets produce four bright knots within 8 kpc. This result is consistent with the number of shocks and their locations
of the southern jet of Hydra A. A visual comparison of the shock locations between the simulations and the observations shows the best fit model of the southern jet is with $p_{jet}/p_a = 1$ and $\beta = 0.8$. Therefore, axisymmetric models of the northern and southern jets based on the knot location (for both cases) and radial oscillation (only for the northern jet) provide a consistent jet velocity for the Hydra A jets.

I now compare in more detail the flow structure of an over-pressured jet with that of a pressure-equilibrium jet, and describe why in the former case we see two knots and in the latter case four knots. In Fig. 7.4, the flow pattern of the over-pressured jet (left panel, run Ei) and the pressure equilibrium jet (right panel, run Ev) overlaid on the pressure image is shown in Fig. 7.4. Here we see that an over-pressured jet expands more and the inclination of the incident reconfinement shock is larger than in the case of a pressure equilibrium jet. Hence, in the pressure equilibrium case, the flow is diverging (see right panel of Fig. 7.4) at the shocked zone (marked by pentangle). The diverging flow causes rapid expansion, which is counteracted by the formation of another reconfinement shock. Therefore, in the pressure equilibrium case, we have knots in the intermediate spaces of the knot locations of the over-pressured jet and have twice the number of shocks in the pressure equilibrium jet.

One concern with the modelling of the southern jet is that the observational data is noisy and not well-resolved. It is also possible that the first knot is an observational artefact. Therefore, in the next stage of my study of Hydra A jets with three dimensional models I focus on the northern jet.
CHAPTER 8

Complex morphology of the northern jet: An effect of jet precession

Hydra A, located at the centre of the galaxy cluster Abell 780, shows a spectacular S-shaped morphology within the central 20 kpc. The symmetrical S-structure is also visible in the extended low frequency images at 74 and 330 MHz (Lane et al., 2004). The radio source is extended by approximately 340 kpc in the north and by 190 kpc in the south. Modelling the entire source is computationally impractical and I have adopted the approach of modelling the innermost structures (inner 10 kpc of the northern jet) first in order to constrain jet parameters (see Chapter 5), then utilising these parameters model the intermediate scale structure (inner 20 kpc of the northern jet).

I have studied the kinetic power of the Hydra A jets and two key features of the inner 10 kpc of the northern jet: i) the oscillatory jet boundary and ii) two bright knots at approximately 3.7 kpc and 7.0 kpc (see chapters 3 and 5). Since the jet is mildly bent within 10 kpc, I have used two dimensional axisymmetric simulations and have modelled the inner two bright knots as biconical reconfinement shocks. By fitting the knot location and the radius profile of the modelled and observed jet I have estimated the jet velocity at 0.5 kpc to be approximately 0.8c, the jet over
pressure ratio with respect to the ICM approximately 5, the jet density parameter approximately 13.

In this chapter I address the following additional key features of the inner 20 kpc of the northern Hydra A jet: i) the curved jet morphology, ii) two additional bright knots beyond 10 kpc and iii) the turbulent transition of the jet to a dissipative plume. In Fig. 8.1 I show the radio structure of the northern jet and indicate these features. This figure is produced using the 4.635 GHz VLA data (G. Taylor, priv. comm.). The detailed description of these data is available in Taylor et al. (1990). In order to model the curved jet morphology I use a three dimensional hydrodynamical model of a precessing jet based on the jet and interstellar medium (ISM) parameters obtained in Chapter 5. The results presented in this chapter will be submitted in a new paper by Nawaz, M. A., Bicknell, G. V., Wagner, A. Y., Sutherland, R. S., and McNamara, B. R..

8.1. Details of the model

The motivation for this study is to understand the dynamical interaction of the inner Hydra A northern jet with the interstellar medium and cluster environment and to understand the reason for the source morphology. Therefore, I mainly focus on the features of the inner 20 kpc including i) the curved jet ii) the four bright knots at approximately 3.7 kpc, 7.0 kpc, 11.0 kpc and 16.0 kpc from the core (deprojected) iii) the turbulent transition of the jet to a plume at approximately 10 kpc from the core, and iv) the bright radio emission region at approximately 10 to 20 kpc from the core.

In Chapter 5 using axisymmetric straight jet simulations I modelled the first two bright knots of the northern jet as biconical reconfinement shocks. Here I develop this model by introducing precession of the jet and this necessitates three dimensional simulations. According to my model, the jet is initially ballistic and conically expands in the first 0.5 kpc. It then starts to interact with the ISM and is collimated by the ambient pressure. A series of bright knots are produced along the jet path at the locations of the biconical reconfinement shocks.
8.1 Details of the model

Figure 8.1  Radio intensity map of the central 20 kpc of the Hydra A northern jet at 4.635 GHz. Contour levels are at 1.5, 2.7, 3.7, 5.1, 6.3, 7.5, 8.8, 10, 21, 37, 51, 72, 90, 103, 150, 180, 200, 205, 220, 240 and 249 mJy arcsec$^{-2}$. Four bright knots are marked with black arrows. The locations of the biconical reconfinement shocks which I interpret as the cause of the bright knots (Nawaz et al., 2014) are marked with $\times$. An imaginary jet path is traced by a dotted line following the ridge line and joining the four knots. Near the third knot there is a bright knot, marked as ‘misaligned knot’, which is not aligned with the jet trajectory. The turbulent transition of the jet starts at the location marked by an arrow. The elliptical shaded area outlines a dissipative flaring zone.

The initially supersonic jet is decelerated significantly by the first two reconfinement shocks and the jet starts to form a turbulent plume at approximately 11 kpc from the core. The jet strongly interacts with the ISM and produces further reconfinement shocks at approximately 11 kpc and 16 kpc. Some jet plasma is deflected by the dense cocoon wall near the fourth knot and a highly turbulent zone is established in the region approximately 11-20 kpc from the core.

Note that in the northern Hydra A jet there is a bright knot (marked by ‘misaligned knot’) that is not aligned with the jet path (the black dotted line in Fig. 8.1 inferred by following the ridge line and connecting the four bright knots). Prior to conducting the simulations there was no indication as to how this misaligned knot actually formed.

My modelling strategy is as follows: I conduct a small parameter space study...
with jet parameters derived from the best fit axisymmetric model presented in Chapter 5, a range of precession periods and two values of the precession angle. I then construct synthetic surface brightness images of the models and compare the source morphology obtained from my models with the observations. Matching the key features, namely, the curvature of the jet, the locations of the bright knots and the turbulent transition of the jet to a plume, I select a best model.

The input jet parameters, the jet kinetic power $P_{\text{jet}} = 1 \times 10^{45}$ erg s$^{-1}$, the jet over-pressure ratio $p_{\text{jet}}/p_a = 5$, the jet velocity $\beta = 0.8$, and the jet density parameter $\chi = 12.75$ are chosen from the best fit axisymmetric model presented in Chapter 5. I explore a range of values for the precession period $P = 1, 5, 10, 15, 20, 25$ Myr and the precession angle $\theta = 15^\circ$ and $20^\circ$. The grid of models is presented in Table 8.1. Since the radiative cooling timescale of the jet plasma and ambient medium are large compared to the simulation time, I do not include cooling in my models.

The numerical setup, grid construction and the boundary condition of the 3D precessing jet model have been presented in § 2.4.3, § 2.4.4 and § 2.4.5 respectively.

### 8.1.1. Synthetic surface brightness

In order to compare the morphologies derived from the models with the radio observations I produce synthetic surface brightness images for each model. Following Sutherland & Bicknell (2007a), I use a synchrotron rest-frame emissivity $j_\nu \propto p^{(3+\alpha)/2}$ where $\alpha$ is the spectral index. In this expression, the magnetic pressure is assumed to be proportional to the non-thermal particle pressure. The northern
Figure 8.2 Dependencies of the jet morphology on the line of sight and the viewing direction. (a) A cartoon of a spiral jet, an arbitrary line of sight and a viewing cone with cone axis aligned with the jet axis and cone angle equal to the line of sight angle are shown. Any line of sight lying on the viewing cone has the same inclination $\theta$ with the jet axis. The observed source morphology depends on both the line of sight inclination $\theta$ and the viewing direction. (b) The image cube, the data cube and the line of sight (marked by rays) are shown. The data cube is rotated with respect to the image cube to obtain any line of sight and a viewing direction.

Hydra A jet is approaching towards the observer and hence the emissivity $j_\nu$ is modified by the Doppler factor $\delta = 1/\Gamma(1 - \beta \cos \theta)$, where $\Gamma$ is the bulk Lorentz factor and $\theta$ is the angle between the jet axis and the line of sight. In addition, to isolate the jet plasma from the ambient medium I use a tracer $\lambda$, which is the mass concentration of plasma at each cell. I initialise the jet plasma with a value $\lambda = 1$. Hence the emissivity $j_\nu$ becomes (in arbitrary units)

$$j_\nu = \lambda \delta^{2+\alpha} \rho^{(3+\alpha)/2} \quad (8.1)$$

Integrating the synchrotron emissivity along rays, parallel to the line of sight, $I_\nu = \int j_\nu ds$, I obtain images of the synthetic surface brightness (in arbitrary units) of the modelled jets.

It is noted that the source morphology depends on both the angle between the jet axis and the line of sight and the viewing direction in azimuth. For instance, Fig. 8.2 shows an arbitrary spiral jet structure about the jet axis and an arbitrary line of sight (making an angle $\theta$ with the jet axis). In this figure a viewing cone is also shown. The axis of the viewing cone lies along the jet axis and its cone angle is equal to the inclination of the line of sight $\theta$. Any line of sight lying on the viewing cone has the same inclination $\theta$ but different azimuthal direction. It is clear from this figure that the jet morphology is different if either $\theta$ or the azimuth direction...
or both change. Therefore, I scan the synthetic images for different lines of sight and azimuth until I obtain the best match of the synthetic surface brightness to the observations.

In using the VisIt visualisation software\(^1\), it proved to be expedient to work with a fixed image cube and to rotate the computed emissivity cube with respect to this image cube in order to investigate the dependence of the synthetic image on viewing direction. The data cube is rotated so that the line of sight along which the surface brightness is calculated is the Y-axis of the image cube. I perform four successive rotations of the data cube \((xyz)\) with respect to the image cube \((XYZ)\) to obtain a desired line of sight and viewing direction. Details of the transformations are presented in the Appendix A.

Let \(\mathbf{v}'\) and \(\mathbf{v}\) be the velocity vector of the fluid in the image cube and data cube respectively. Then the velocity \(\mathbf{v}'\) is given by

\[
\mathbf{v}' = R\mathbf{v}
\]

where \(R\) is the transformation matrix (see Appendix A for the description of \(R\)).

The angle between the line-of-sight (Y-axis) and the fluid velocity at a cell is given by

\[
\theta' = \cos^{-1}\frac{v'_Y}{\mathbf{v}'}
\]

where \(v'_Y\) and \(\mathbf{v}'\) are the Y component and magnitude of the velocity in the image cube, respectively. In order to obtain the correct Doppler factor \(\text{for each cell}\) I use \(\theta'\) in the expression for the Doppler factor.

Since I am considering the Doppler beaming for individual cell in the simulation data cube, changing the line of sight or viewing direction not only changes the radio morphology of the synthetic image, but the relative brightness of different regions in the source changes as well. In Appendix B I present a collage of surface brightness images of the optimal model (Fig. B.1) of the Hydra A northern jet for different lines of sight.

\(^1\)https://wci.llnl.gov/simulation/computer-codes/visit/
8.2 Simulation Results

In this section I present the results of the three-dimensional precessing jet models. As expected all jets exhibit curvature with the degree of curvature depending upon the precession period and the precession angle. Hence the degree of curvature provides an important diagnostic of the precession parameters, which I discuss below (§8.2.1). I also present other morphological features produced by the various models and compare them with the observations.

8.2.1 Curvature of the jet

My aim is to match the simulated jet curvature within 10 kpc from the core to the observed curvature of the Hydra A northern jet. Fig. 8.3 shows the synthetic surface brightness images for models A, B, C, D, E, and G. The snapshots are taken when the jet is fully developed in the computational domain. Since I am comparing the curvatures of jets with different parameters all images in Fig. 8.3 are produced for $\theta = 90^\circ$. 

Figure 8.3 Synthesis surface brightness of models A, B, C, D, E and G. The snapshots are chosen for a simulation time at which the jet is fully developed in the computation domain.
In Fig. 8.3 it is evident that the curvature of the jet increases as the precession period decreases. Models with longer precession periods produce straight jets within the first 10 kpc. For example, jets produced by the models C, D, E, and G with precession periods 5, 10, 15 and 25 Myr are straight in the inner 10 kpc. The jet with a precession period 1 Myr and a precession angle 15° is also nearly straight within this region. We see a mild curvature inside 10 kpc for model A with a precession period 1 Myr and a precession angle 20°. This curvature is comparable to the curvature of the Hydra A northern jet. Therefore, on the basis of this curvature comparison alone, model A is the best match for Hydra A. This choice is confirmed by other observational features reproduced by the model. In particular, in model A, no additional knots are produced downstream of the fourth knot. However models with longer precession periods produce more than four bright knots along the jet trajectory.

### 8.2.2. Bright knots and the turbulent transition of the jet

Fig. 8.4 compares the optimal view ($\theta = 70^\circ$, and $\chi = 45^\circ$) of the simulated jet of model A (left panel, at a simulation time 22 Myr) and the Hydra A northern jet (middle and right panel). It is evident that the simulated jet successfully reproduces
the following key features and processes occurring in the source within the central 20 kpc;

**Bright knots:** The moderately over-pressured precessing jet interacts with the ambient medium and produces four reconfinement shocks at approximately 4.0 kpc, 7 kpc, 10.0 kpc and 14.0 kpc from the core. Since the synchrotron emissivity is \( j_\nu \propto p^{(3+\alpha)/2} \), downstream of the reconfinement shocks the pressure and therefore the surface brightness increase producing four bright knots. The locations of the bright knots produced with this model agree well with the locations of bright knots in the Hydra A northern jet located at approximately 3.7 kpc, 7.0 kpc, 11.0 kpc and 16.0 kpc (deprojected) from the core and shown in the middle and right panel. This is consistent with the result of the axisymmetric models presented in Chapter 5.

**Turbulent transition of the jet:** In the simulated jet a turbulent transition of the jet to a plume occurs approximately after the second bright knot, which is consistent with the observations. This figure also shows that in the optimal model, the turbulent jet starts to forms a dissipative flaring zone (marked by an ellipse in the left panel). This is the beginning of a large plume structure as observed in the Hydra A northern jet (marked by an ellipse in the mid panel).

In the Hydra A northern jet, the flaring region within approximately 11 to 20 kpc from the core where the plume starts, is bright compared to the inner collimated jet. The corresponding region in the optimal model does not reach the same level of brightness. However, the flaring region is strongly turbulent (see § 8.2.3). The amplification of the magnetic field resulting from this turbulence may be responsible for the increase in the source brightness. Since, my model is purely hydrodynamic, and the amplification of the magnetic field is not reflected in the synthetic surface brightness images. In order to produce more accurate synthetic brightness images magnetohydrodynamic (MHD) models are required. Therefore, further development of this model with the inclusion of magnetic field is of interest.
8.2.3. Turbulent flaring zone

Fig. 8.5 shows the logarithmic density of run A (at a simulation time 26 Myr) sliced by a cone with a cone angle of 17° (left panel) and cone axis aligned with the precession axis (z axis). To obtain a clear view of the jet and the flow direction the cone is projected onto the x − y plane (right panel of Fig. 8.5) and overlaid with the flow vectors. A zoom in of the region marked by a rectangle in the middle panel is shown in the right panel. It is noted here that, although the precession angle in model A is 20°, the jet is mostly visible along the conic slice with a cone angle 17°. This is the result of the reflective boundary condition at the lower z boundary. The reflection of the back flow on the side of the jet closest to the boundary pushes the jet towards the precession axis. Therefore, the jet is maximally visible along a conic slice with cone angle less than 20°.

In Fig. 8.5 we see that after the turbulent transition of the jet some jet plasma hits the cocoon plasma and produces a strong back flow (shown in the right panel). This turbulent back flow establishes a flaring region. Such a flaring region is apparent at approximately 10 to 20 kpc from the core in the northern jet of Hydra A. Moreover, in the polarisation image of Hydra A (Taylor et al., 1990) the polarisation significantly falls from 40% (in the collimated jet) to 10% in the flaring region. This reduction in polarisation suggests that the flaring region of the northern jet is turbulent and this is consistent with the simulations.
8.2 Simulation Results

8.2.4. Forward shock

In the optimal model the radio jet-ICM interactions are bounded by an advancing forward shock. Here I estimate the Mach number of that forward shock.

The forward bow shock is shown in the logarithmic density snapshot of model A (left panel of Fig. 8.6). In the right panel of Fig. 8.6 the location of the forward shock along the $z$-axis at five different time steps is indicated. A least square fit to the shock positions gives a shock advance speed $\approx 1630$ km s$^{-1}$ of the forward shock. The sound speed at approximately 15 kpc from the core is $\approx 880$ km s$^{-1}$. Hence, the Mach number of the forward shock is $\approx 1.85$. There is a mild pressure jump $\approx 3.4$ at the forward shock. The low Mach number and mild pressure jump indicate that the heating of the atmosphere by the radio AGN in its earlier stage is gentle. This general feature of the heating of cooling flows was inferred by (McNamara & Nulsen, 2012). A straight jet would give a much higher advance speed and a larger pressure jump. The low Mach number and pressure jump derived here can be attributed to the jet precession depositing its momentum over a much wider area.

8.2.5. Misaligned bright knot

In the turbulent flaring region of the Hydra A northern jet there is a knot, which is not aligned with the main trajectory of the jet. This mis-aligned knot is approxim-
ately two kpc north of the third knot and is marked as 'misaligned knot’ in Fig. 8.1. This knot is formed as a consequence of the transition to turbulence of the jet. The jet temporarily splits in two forming the misaligned knot and then returns to its final trajectory through the plume region (see Figure 8.7). This happens only with the optimal model (A).

8.2.6. Implication of precession: Estimate of viscosity parameter of the AGN disk

Knowledge of the precession period of the jet provides us with information that can be used to estimate the well known accretion disk viscosity parameter $\alpha$. A black hole (BH) whose spin is misaligned with the angular momentum of the accretion disk aligns the surrounding inner part of the disc to the BH spin axis via Lense-Thirring precession and internal viscosity up to a critical radius known as the Bardeen-Petterson radius, $r_{BP}$ (Bardeen & Petterson, 1975). Beyond $r_{BP}$, the disk retains its original structure because of its dominant angular momentum.
8.2 Simulation Results

Viscous torques in the outer accretion disk force the spin axis of the black hole to precess until it is aligned with the angular momentum of the outer disk (Rees, 1978; Scheuer & Feiler, 1996; Natarajan & Pringle, 1998; Caproni et al., 2007). The alignment time-scale can be considered equivalent to the precession period of the jet. Using the alignment timescale, a jet precession period $\approx 10^8$-10$^{10}$ yr has been estimated for the source, NGC 4258 (Caproni et al., 2007).

Natarajan & Pringle (1998) estimated the alignment time scale in terms of accretion parameters, including the disk viscosity parameter, $\alpha$. Using their theory, I use the precession period of the optimal model of Hydra A to estimate $\alpha$. Let $t_{\text{align}}$ be the alignment time-scale, $a$ the spin parameter of the black hole, $\alpha$ the viscosity parameter of the accretion disk, $L$ the total power provided by the black hole, $L_E$ the Eddington luminosity, $M_{\text{BH}}$ the mass of the black hole, $M_\odot$ the solar mass, and $\epsilon$ the accretion efficiency of the black hole. Then, from the equation for the alignment time of the black hole (Natarajan & Pringle 1998, equation 2.16) I obtain:

$$\alpha = 0.04 \left( \frac{a}{0.8} \right)^{-11/26} \left( \frac{L}{0.02L_E} \right)^{7/13} \times \left( \frac{M}{10^9 M_\odot} \right)^{1/26} \left( \frac{\epsilon}{0.1} \right)^{-7/13} \left( \frac{P}{\text{Myr}} \right)^{8/13}. \tag{8.4}$$

where $P(= t_{\text{align}})$ is the precession period of the jet.

The mass of the supermassive black hole in Hydra A is approximately $10^9 M_\odot$ (Fujita et al., 2013). The total jet power provided by the black hole is $L = 2L_{\text{jet}} \approx 2 \times 10^{45}$ erg s$^{-1} \approx 0.02L_E$ and I equate this to the total disk luminosity resulting from accretion. Using $\epsilon = 0.1$, $P = 1$ Myr, and a range of $a$ (= 0.1 to 1) I obtain $0.03 \leq \alpha \leq 0.15$. The upper end of the range of $\alpha \approx 0.15$ (for $a \approx 0.1$) is consistent with the range of values typically inferred from observations of dwarf novae $0.1 \leq \alpha \leq 0.4$ (King et al., 2007). However, in general, there is a discrepancy between values of $\alpha$ derived from observations and those derived from numerical magneto-hydrodynamic (MHD) simulations; the latter are generally an order of magnitude lower than the former. For instance, the quasi-steady disk MHD models by Parkin & Bicknell (2013) imply $\alpha \approx 0.04$. Such a low value is consistent with the lower bound of the estimate of $\alpha$ (for $a \approx 0.9$). The lowest value $\alpha = 0.03$ in the range is consistent with the estimates
Complex morphology of the northern jet: An effect of jet precession

by Starling et al. (2004) from observations of AGN disks.
APPENDIX A

Rotation of the data cube for a desired line of sight

Let $XYZ$ be the coordinates (shown in panel (a) of Fig. A.1) associated with the synthetic image cube, which is used for ray-traced integrations of the surface brightness. Let $Y$ be the direction of the line of sight. To begin with, the synthetic image cube and the simulation data cube have the same orientation. The initial direction the jet is defined by the precession angle $\psi$ and an azimuthal angle $\phi$ defined with respect to the $X$-axis (see panel (a) of Fig. A.1). Let $\chi$ be the angle defining viewing direction, an angle about the line of sight. Let $\theta$ be the angle between the jet axis and the line of sight $Y$-axis (see panel (e) of Fig. A.1).

In order to prescribe a line of sight that is inclined at an angle $\theta$ to the jet direction and an azimuthal angle, $\chi$ about this line of sight, I perform the rotations of the data cube with respect to the image cube. Since the visualisation software VISIT restricts the choice of the line of sight along any one axis of the image cube (in my case I choose the $Y$-axis), we require four rotations; two rotations associated with $\psi$ and $\phi$ to align the jet axis with the line of sight and two rotations associated with $\chi$ and $\theta$ to obtain the desired azimuthal angle and angle of inclination. These rotations are depicted in Fig. A.1. In this figure angles are depicted by arcs and rotations are depicted by arcs with arrowheads.
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Figure A.1 Transformations associated with the rotations of points of the simulation data cube with respect to the synthetic image cube. Panel (a): At a given instant, both the data cube and the image cube have the same orientation and they can be represented by the same coordinates XYZ. Panel (b): Transformation associated with the angle $\phi$, to bring the jet axis on the YZ plane. Panel (c): Transformation associated with the angle $\psi$, to align the jet axis with the line of sight $Y$ axis. Panel (d): Transformation associated with the angle $\chi$, to obtain a desired viewing direction. Panel (e): Transformation associated with the angle $\theta$, to obtain a desired line of sight. In panels (b), (c) and (e) the jet axes before transformations are shown in dashed blue arrows and after transformations with solid blue arrows. In panel (d), the jet does not change its location. It only rotates about its axis.

1. First I rotate the simulation data cube (anticlockwise) about the Z-axis by an angle $\pi/2 - \phi$ (shown in panel (b)). This rotation brings the jet axis on the YZ plane. The rotation matrix for this rotation $R_{Z,\phi}^{(1)}$ is given by

$$R_{Z,\phi}^{(1)} = \begin{pmatrix} \cos(\pi/2 - \phi) & -\sin(\pi/2 - \phi) & 0 \\ \sin(\pi/2 - \phi) & \cos(\pi/2 - \phi) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} \sin \phi & -\cos \phi & 0 \\ \cos \phi & \sin \phi & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

(A.1)
2. I rotate the simulation data cube second time (clockwise) about the X-axis by an angle $\pi/2 - \theta$ (shown in panel (c)). This rotation makes the jet axis aligned with the line of sight Y-axis. The rotation matrix for this rotation $R_{X,\psi}^{(2)}$ is given by

$$R_{X,\psi}^{(2)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos(\pi/2 - \psi) & \sin(\pi/2 - \psi) \\ 0 & -\sin(\pi/2 - \psi) & \cos(\pi/2 - \psi) \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & \sin \psi & \cos \psi \\ 0 & -\cos \psi & \sin \psi \end{bmatrix} \quad (A.2)$$

3. Now, in order to prescribe the azimuth of the viewing direction I rotate the simulation data cube with respect to the Y-axis by an angle $\chi$ (shown in panel (d)). The rotation matrix for this rotation $R_{Y,\chi}^{(3)}$ is given by

$$R_{Y,\chi}^{(3)} = \begin{bmatrix} \cos \chi & 0 & \sin \chi \\ 0 & 1 & 0 \\ -\sin \chi & 0 & \cos \chi \end{bmatrix} \quad (A.3)$$

4. Finally, I rotate the simulation data cube about the X-axis by an angle $\theta$ (shown in panel (e)). This rotation relocates the jet axis at an angle $\theta$ with respect to the line of sight axis (Y). The rotation matrix associated with this rotation $R_{X,\theta}^{(4)}$ is given by

$$R_{X,\theta}^{(4)} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta \\ 0 & \sin \theta & \cos \theta \end{bmatrix} \quad (A.4)$$

The velocity of the fluid in the image cube $\mathbf{v}'$ after the transformations described above is calculated from the velocity in the simulation data cube using the rotation matrix $R = R_{X,\theta}^{(4)}R_{Y,\chi}^{(3)}R_{X,\psi}^{(2)}R_{Z,\phi}^{(1)}$

$$\mathbf{v}' = R\mathbf{v} \quad (A.5)$$
where $R$ is the combined transformation matrix.

Let $s_1 = \sin \psi$, $s_2 = \sin \phi$, $s_3 = \sin \chi$, $s_4 = \sin \theta$, $c_1 = \cos \psi$, $c_2 = \cos \phi$, $c_3 = \cos \chi$, and $c_4 = \cos \theta$. Then the transformation matrix $R$ is given by:

$$R = R_{Y, \theta}^{(4)} R_{X, \psi}^{(3)} R_{X, \chi}^{(2)} R_{Z, \phi}^{(1)}$$

$$= \begin{pmatrix}
    c_3 s_2 - s_3 c_1 c_2 & -c_3 c_2 - s_3 c_1 s_2 & s_3 s_1 \\
    c_4 s_1 c_2 + s_4 s_3 s_2 & c_4 s_1 s_2 + s_4 s_3 c_2 & c_4 c_1 - s_4 c_3 s_1 \\
    s_4 s_1 c_2 - c_4 s_3 s_2 & s_4 s_1 s_2 + c_4 s_3 c_2 & s_4 c_1 + c_4 c_3 s_1 \\
    -c_4 c_3 c_1 c_2 & -c_4 c_3 c_1 s_2 &
  \end{pmatrix}$$
APPENDIX B

Synthetic surface brightness of the source at different $\theta$ and $\chi$

Figure B.1  Synthetic surface brightness images of the best match model for different line of sights $\theta$ and viewing directions $\chi$. For comparison the observed radio image of the inner 20 kpc of the Hydra A northern jet is shown at the third column of second row.
CHAPTER 9

Summary and Discussion

The main aim of the study presented in this thesis has been to understand the physics of the inner jets in the Hydra A radio galaxy with a view to understanding the spectacular morphology of the source and to infer parameters such as the jet kinetic power, the jet pressure, the jet density and the jet velocity. From the larger scale radio structure of Hydra A, we aimed to constrain the jet precession period and the precession angle, and explain in detail the jets interaction with the cluster atmosphere. This study was performed in two stages. First, I studied the inner 10 kpc of the the northern jet, where the jet is nearly straight, using two dimensional axisymmetric relativistic hydrodynamical models. Second, I generalised the axisymmetric model to a three dimensional precessing jet model in order to study i) The complex morphology of the northern part of the source within 20 kpc from the core and ii) The heating of the cluster environment by the radio source during its early stages of evolution.

9.1. Jet kinetic power and the ambient medium

To ensure that I used reasonable values for the jet parameters in my simulations, I have estimated the powers associated with the inner radio lobes of Hydra A corresponding to the inner 50 kpc X-ray cavities. Utilising the synchrotron minimum energy estimate for the pressure and synchrotron ages of the 4.6 GHz radio
lobes (Taylor et al., 1990), I have estimated the power of the inner X-ray cavities. I have compared my estimates with those by Wise et al. (2007) for the same cavities based on the X-ray data. I obtain powers for the northern and southern cavities $\approx 1.8 \times 10^{44}$ ergs s$^{-1}$ and $2.0 \times 10^{44}$ ergs s$^{-1}$, respectively. These estimates are consistent with the Wise et al. (2007) estimates of $\sim 2 \times 10^{44}$ ergs s$^{-1}$ for both cavities. Hence, I adopt the total jet power obtained by Wise et al. (2007) $P_{\text{jet}} = 10^{45}$ erg s$^{-1}$ from the summation of powers of all X-ray cavities as the value for the jet power in the numerical models. The other jet parameters, namely, the jet pressure $p_{\text{jet}} = 2p_{a}$ and $p_{\text{jet}} = 5p_{a}$, where $p_{a}$ is the ambient pressure, and the jet inlet radius $r_{\text{jet}} = 180, 150$, and $100$ pc are chosen based on the 23 cm VLBA and 6 cm VLA data of Hydra A (Taylor et al., 1990).

Using a minimum pressure estimate, I conclude that, in the lobes, $k$, the ratio of energy in other particles to that in relativistic electrons $\sim 10$. Moderate values of this parameter are supported by other recent studies: Birzan et al. (2008) estimated $k$ for a group of radio galaxies assuming that the radio lobes are in pressure equilibrium with the ambient medium. Their estimates include the Hydra A radio lobes at 1.4 GHz for which they obtained a value of $k \approx 13$. Hardcastle & Croston (2010) studied the inverse-Compton X-ray emission from the outer Hydra A radio lobes and obtained values of $k \sim 17$ and 23 for minimum Lorentz factor cut-offs of $\gamma_{1} = 1$ and 10 respectively. These estimates are all comparable given the different techniques used to derive them.

For the X-ray atmosphere used in my simulations, I have constructed hydrostatic profiles for the Hydra A atmosphere by fitting and extrapolating the density and temperature data from the X-ray observations of David et al. (2001).

### 9.2. Axisymmetric model

For the two-dimensional axisymmetric relativistic hydrodynamic models, I have mainly focused on the following key features of the northern radio jet inside 10 kpc from the core: i) the bright knots in the northern jet at $\sim 3.7$, and 7.0 kpc from the black hole, and ii) the wave-like boundary, i.e., oscillating radius, of the northern jet.
To this end, I have performed a series of two dimensional axisymmetric relativistic hydrodynamic simulations of the interaction of the northern Hydra A jet with the interstellar medium, concentrating on the central 10 kpc. For the simulations, I have utilised the relativistic hydrodynamic code PLUTO (Mignone et al., 2007) to produce the 2D axisymmetric models.

The results of my numerical models of the interaction of an initially conical and ballistic jet with the ambient medium support the idea that consecutive biconical shocks are responsible for the bright knots in the northern jets of Hydra A. With appropriate values of the initial jet pressure ratio and velocity the observed knot spacings and variation in jet radius are reproduced along a considerable section of the jet up to the flaring region.

From the comprehensive parameter study in Chapter 5 covering a range of values in jet pressure, jet velocity, jet radius, and jet density parameter, I have selected models Ciii, CIV and CV as the best fit models for the inner ~ 10 kpc radio structure of the northern jet. These jet models with an initially conical and ballistic jet that is over-pressured with respect to the environment by a factor of 5 produce four successive biconical reconfinement shocks before the jets becomes fully turbulent. The location of the first three shocks and the radius profile of the jet along the direction of its propagation closely match the location of the southern edges of the first two bright knots in the Hydra A northern jet and the radius profile of the jet. Constructing synthetic surface brightness images, I have shown that the biconical shocks produced in the simulated jet are associated with bright knots. For the best fit models of the northern jet, the jet parameters are a jet kinetic power $P_{\text{jet}} = 10^{45}$ ergs$^{-1}$, a jet inlet radius $r_{\text{jet}} = 100$ pc, a jet over pressure ratio = 5, a jet density parameter $\chi = 20.41, 12.75, 7.24$ and a jet velocity (in units of the speed of light) $\beta = 0.75, 0.8$ and 0.85. The estimated jet velocity for the northern jet of Hydra A $\approx 0.8$ c is consistent with recent observational and theoretical estimates of jet velocities in FRI jets determined by Laing & Bridle (2014). Their models of the surface brightness of 10 FRI radio sources yielded a typical kpc scale jet velocity $\approx 0.8$ c.

I have also performed some axisymmetric “naked” jet models - models of unboun-
ded jets that span the computational domain from the onset of the simulation. These models confirmed that the shock spacings are largely independent of the flow structure in the cocoon surrounding the jet stream, and that the shock spacings and radius variations derived from models of a propagating jet bounded by the jet cocoon are applicable to sources at later times in their evolution, when the backflow in the cocoon at small distances from the core has become weak. A turbulent transition of the jet stream does not occur in the naked jet simulations, indicating that deceleration in addition to that in the reconfineiment shocks may be provided by the ram pressure of backflows in the cocoon that impinge on the jet.

The brightnesses of the knots in the best fit model gradually increase with distance from the core, in a way that is qualitatively consistent with the observed jet. However the brightness ratio between the second and first knot and between the third and second knot for the simulated jet (run Civ) ≈ 2.5 and 1.14 respectively, differ from the observed brightness ratios of ~ 8.7 and ~ 3. This discrepancy may arise as a result of the magnetic field increasing faster than the pressure along the jet and hence the assumption that $B^2 / 8\pi \propto p_{jet}$ in the emissivity model would underestimate the emissivity increase along the jet.

The inferred relativistic jet velocity of $\approx 0.8 \, c$ differs from the estimate based on Doppler beaming, which is $\approx 0.5 \, c$. The associated flux density ratio between the approaching and receding jet as obtained from the simulations is 33, and is much larger than the observed value of 7. The additional parameter study in § 5.3.4 shows that the combination of parameters $\beta = 0.5$, jet kinetic power $10^{45} \, \text{erg s}^{-1}$ and an inclination angle $\theta = 42^\circ$ is unable to produce the correct shock locations and the profile of the jet boundary for any feasible combination of the jet inlet radius and jet pressure. Hence, one possibility is to adopt $\beta = 0.8$ and to attribute the different flux density ratios to a difference in intrinsic rest-frame emissivities. For example, the flux density ratio may be overestimated in the best fit model since I assume that the magnetic field is the same in both jets. If I assume that the magnetic field is 2.5 times stronger in the southern jet, the flux density ratio would be 7. Another possibility is that the observed value of the flux density ratio is low since the southern jet is more dissipative as a result of its greater bending and the
greater number of shocks.

A further possible explanation for the discrepancy between estimated and measured flux density ratios is that the angle, $\theta$, between the jet and the line of sight, inferred from the rotation measure asymmetry (see Taylor & Perley, 1993) differs from $42^\circ$. This is certainly possible given the range $30^\circ \lesssim \theta \lesssim 60^\circ$ estimated by Taylor & Perley (1993). Hence, I have used the jet velocity as a parameter, calculated the inclination required to give a northern to southern flux ratio of 7, calculated the deprojected spacing between the first and second knots and compared this with the simulated spacing. The result of this comparison was that the simulated and observed spacings do not agree except at the lowest possible jet velocities, consistent with a beaming interpretation, $\beta \approx 0.35$. I have argued that a solution for the jet velocity at around $\beta = 0.35$ is unappealing since it is unlikely that the optimal velocity for knot spacing would be fortuitously close to the lower limit from beaming.

Taking into consideration the modelling of the shock spacing, the radius evolution of the jet, and the surface brightness ratio, I conclude that the jet velocities $\approx 0.8c$ and that there is an intrinsic asymmetry between the rest-frame emissivities of the northern and southern jets. This may be a result of different magnetic fields (by about a factor of 2.5) or higher dissipation in the southern jet.

The initial value (at 0.5 kpc from the core) of the density parameter $\chi = \frac{\rho c^2}{4p}$ derived from the simulations is also of interest for the value this parameter would have on the parsec scale. Assuming that the jet has constant velocity from the parsec scale outwards, $\rho \propto r_{\text{jet}}^{-2}$ and $p \propto r_{\text{jet}}^{-8/3}$ so that $\chi \propto r_{\text{jet}}^{2/3}$. From the VLBI images of Taylor (1996) $r_{\text{jet}} \approx 1$ pc in the 15.4 GHz image. Hence, the best fit value of $\chi = 12.75$ extrapolates to 0.59 – consistent with an electron-positron jet with Lorentz factor lower cutoff, $\gamma_1 \sim 1$ or an electron proton jet with $\gamma_1 \sim 700$.

The conclusions of the axisymmetric models are subject to the assumption of a low magnetic pressure in the jet, and I have provided some justification for this assumption on the sub-parsec scale in § 3 as well as some justification for the lack of magnetic collimation from the parsec to kiloparsec scale. Nevertheless, the magnetic field evolves along a jet, and its downstream strength and influence on
the dynamics is an interesting issue. Moreover, the magnetic field is important for an accurate calculation of the synchrotron emission and, thus, its evolution is important for the construction of surface brightness images. Hence, the inclusion of a magnetic field in future simulations is an important next step. However, as Spruit (2011) has shown there is a lot more physics to consider in this case, in particular reconnection of a three-dimensional magnetic field. Thus, while magnetic effects are important for future work, their consideration is beyond the scope of this thesis.

9.3. Precessing jet model

With the axisymmetric models I have successfully reproduced the correct oscillations of the jet boundary and the first two bright knots inside 10 kpc of the Hydra A northern jet. In order to study features beyond 10 kpc, where the jet curves significantly and begins a turbulent transition and enters a flaring region, I have generalised the axisymmetric model to a three dimensional precessing jet model. The three dimensional precessing jet model successfully reproduces the prominent features of the complex inner 20 kpc jet-lobe morphology in the northern side of Hydra A.

I have run a series of three-dimensional relativistic hydrodynamic simulations of a precessing jet, which constitute a parameter space study using parameters obtained from the best fit axisymmetric model, a range of precession periods (1, 5, 10, 15, 20, and 25 Myr) and two precession angles (15° and 20°). From the parameter study presented in Chapter 8 I find that model A with a precession period of 1 Myr and a precession angle of 20° produces the correct jet curvature, the correct number of knots, and the jet to plume transition at approximately the correct locations. Therefore I choose this model as the optimal model.

The optimal model reproduces:

1. Four bright knots along the direction of the jet. The bright knots appear at the locations of the biconical shocks resulting from reconfinement shocks associated with recollimation of the jet by the ambient medium. This is consistent with the axisymmetric models. The locations of the knots at
9.3 Precessing jet model

approximately 4, 7, 10 and 14 kpc coincide reasonably well with the observed bright knots at approximately (3.7, 7.0, 11.0 and 16.0 kpc).

2. The turbulent transition of the jet to a plume at approximately 9 kpc compared to the observed transition location at 10 kpc. The initially supersonic jet is significantly decelerated by the first two reconfinement shocks and the transition to turbulence begins after the second knot.

3. A turbulent flaring zone at approximately 10-20 kpc from the core. The back flowing jet plasma from the cocoon wall near the fourth knot produces strong turbulence in this region. The turbulence is responsible for the widening of the flow at approximately 10 kpc from the core. This simulated feature is consistent with the following observed feature of Hydra A. From the polarisation image (Taylor et al., 1990) we see that the polarisation drops from 40% in the collimated jet (until 10 kpc from the core) to 10% in the flared region (10-20 kpc from the core) on the northern side of Hydra A. This drop in polarisation in this region is consistent with an increase in turbulence there.

4. A misaligned knot in the turbulent flaring zone. This feature is only produced in model A, supporting the choice of that model as the best match to Hydra A.

I have estimated the Mach number of the forward shock to be \( \approx 1.85 \) from our optimal model. This low Mach number and the pressure jump (\( \approx 3.4 \)) of the ambient medium associated with the forward shock suggest a gentle heating of the of the ICM by the radio AGN in its initial phases of evolution as noted by McNamara & Nulsen (2012). A low Mach forward shock can be attributed to the jet precession depositing its momentum over a much wider area.

Two features associated with the optimal precessing jet model for Hydra A (precession period = 1 Myr) are i) The gentle heating of the atmosphere (described above) and ii) The continuous dissipation of jet kinetic energy in the turbulent flaring region due to shock deceleration and turbulence (see § 8.2.3). These two features suggest that a precessing jet with a lower precession period can be capable
of maintaining a long term balance between heating and cooling in a wider region of the ICM of a cooling flow cluster, such as Hydra A (see further details in § 10.5). Therefore, inclusion of cooling in the precessing jet model is of interest to study the process of maintenance-mode feedback, in which heating of the cluster gas by the radio source counteracts the thermal cooling. However, this is beyond the scope of this thesis.

Finally, interpreting the realignment time-scale estimate by Natarajan & Pringle (1998) as the precession period of the jet, I have also estimated the viscosity parameter of the accretion disk of Hydra A to be \(0.03 \leq \alpha \leq 0.15\). The lowest end of the range of viscosity parameter is consistent with the estimates by Starling et al. (2004) from observations of AGN disks. Recent quasi-steady disk MHD models by also predicts a lower value of \(\alpha \approx 0.04\) (e.g., Parkin & Bicknell, 2013).

As for the 2D axisymmetric models, inclusion of magnetic fields in the study using 3-dimensional precessing jets would be interesting, mainly for the production of more realistic synthetic surface brightness images. For instance, magnetic field amplification in the turbulent flaring region (10-20 kpc) of the northern jet may be a possible explanation for the increase in brightness there; a purely hydrodynamic model does not capture this effect. This is a likely reason for the fact that, in the optimal model, the ratio of the brightness between the initial jet (up to 10 kpc from the core) and the turbulent plume (10-20 kpc from the core) is not reproduced correctly.

Modelling the Hydra A southern jet is complicated, because the initial 5 kpc trajectory of the jet is not well determined observationally. I presented a series of axisymmetric simulations for the southern jet using the same strategy for fitting the shock spacing and locations as for the northern jet, which yielded the same jet velocity for the southern jet as obtained for the northern jet \((\beta = 0.8)\), and an overpressure ratio of 1. The lower overpressure ratio is directly related to the smaller shock spacings in the southern jet compared to the northern jet. Ultimately, however, adequate modelling of the southern side of the source as I have done for the northern side requires better signal to noise high resolution observations of the source.
In the models presented here I have considered Hydra A’s northern jet morphology up to 20 kpc from the core. Since the initial jet radius (0.1 kpc) is very small compared to the extent of the inner lobe (50 kpc), modelling the entire inner lobe for a large range of parameters is unrealistic. However the parameters I have obtained through modelling the inner 20 kpc northern jet morphology can be used as input into future large scale studies of this source.
The study of the Hydra A inner 10 kpc jet presented in this thesis uses an innovative method for the estimation of jet velocity by combining the information of the inner knot locations and the oscillation of the jet boundary.

The detailed parameter study using axisymmetric relativistic hydrodynamic simulations presented in this thesis using this technique provides best fit jet parameters, which were used for a even more realistic three dimensional modelling of the dynamical interaction of the Hydra A jets and the cluster atmosphere.

In the following I describe the studies that could be done in future guided by the methodology I have used and the results we obtained in this thesis.

10.1. The large scale morphology of the Hydra A

In this thesis I have modelled the inner 20 kpc structures of the Hydra A northern jet, including the width profile of the jet, the bright knots along its axis, the curvature of the jet and the jet to plume transition. This is a good starting point for a bottom up approach to study different scales of a very extended source like Hydra A.

Following the evolution of the jet plasma toward larger radii, one could first investigate the full development of the inner 50 kpc radio plume, then study the series of X-ray cavities, and finally attempt to explain the large scale structures in
the X-ray images including the outer shock at approximately 200 kpc, as well as
the oldest and largest radio lobes seen in the low-frequency extent of the source.
Energy and mass transport measurements from three-dimensional hydrodynamic
simulations, in particular the transport of metal rich gas from the galaxy center,
across these scales will provide answers to how cooling flows are suppressed (see
e.g. Gaspari et al., 2012), and allow direct comparisons with ICM temperature and
metal distribution maps (e.g. Simionescu et al., 2009b), and observations of cold
filaments (e.g. Pope et al., 2008). Simulations on large scales ($\gtrsim 50$ kpc) can be
aided by mapping the boundary conditions known from smaller scale simulations
and by the use of the CHOMBO adaptive mesh refinement package supported by
the PLUTO code.

10.2. Magnetohydrodynamic models

In this thesis I calculate the radio emissivity as a function of pressure derived by
(Sutherland & Bicknell, 2007a) to obtain the synthetic surface brightness images
of the modelled jet. However, a proper calculation of synchrotron emission re-
quires magnetic fields. Therefore, an improved comparison between the simulated
and observed radio jets and the lobes requires magneto-hydrodynamic (MHD)
modelling of the source. One can incorporate magnetic field into the purely hy-
drodynamic model I developed in this work and study the radio features in an
even more realistic way. The PLUTO code contains robust MHD modules, and the
generalization to MHD simulations would therefore also be a natural next step
from a computational point of view.

10.3. The modelling of AGN jets displaying inner knots

In this thesis, I showed that simultaneously modelling the data of the locations of
the inner knots and the oscillation of the boundary of the Hydra A northern jet
can be used to estimate the velocity of the jet. This method of estimation of the jet
velocity can in principle be applied with only small modifications to other radio
10.4. Modelling complex morphologies of other radio sources

In my precessing jet models I notice a generic morphological feature of the precessing jet. In the first half cycle of the jet precession, we see a narrow jet structure.

Figure 10.1  A comparison between the simulated source morphology (left panel) for model G (precession period $P = 25$ Myr) and the radio morphology of Centaurus A (inset of the right panel). Credit for the Centaurus A image: CSIRO.

Figure 10.2  A comparison between the simulated source morphology (left panel) for model G (precession period $P = 25$ Myr) and the radio morphology of m87 (inset of the right panel). Credit for the M87 image: NRAO.
with a trailing balloon shaped lobe. This type of jet-lobe morphology is common in radio sources, for example, Centaurus A, M87 (inner lobe), J0116-473 etc.

Fig. 10.1 and 10.2 shows the morphological similarities between the simulated radio image of the source (in the left panels) for model G (precession period $P = 25$ Myr) and the observed radio morphology of Centaurus A and M87 (in the right panels). The snapshots are captured when the jet reaches approximately a half (Fig. 10.1) and approximately a quarter (Fig. 10.2) of its precession cycle. Therefore, using (magneto-)hydrodynamic models precessing jets can potentially be a useful strategy to study sources with this type of jet-lobe morphology.

### 10.5. Maintenance mode feedback

In the context of cosmological galaxy formation, studying a galaxy like Hydra A is important to understand the details of “maintenance-mode” feedback, whereby the effect of the feedback from jets, in this case suppressing cooling flows, maintains the galaxy in a state of low star-formation rate. The precession of jets may be an important effect for efficiently maintaining a hot cluster atmosphere, suppressing cooling flows, and ensuring a low star formation rate in the cluster central galaxy.

In case of a straight jet, most of the jet energy is deposited at large distances from the core as the jet quickly makes its way through the atmosphere, its momentum is always aligned in the same direction. Hence, for a straight jet it is difficult to prevent a catastrophic cooling flow from developing over all solid angles (Vernaleo & Reynolds, 2006). However, a precessing jet, especially one with a small precession period, always strongly interacts with the environment near the core. For example, in the optimal model for the Hydra A northern jet (precession period = 1 Myr, run A of Chapter 8), the jet maintains a turbulent dissipative zone at approximately 10 kpc to 20 kpc from the core (the extent of the source is approximately 340 kpc in the north). Hence, a long-term balance between the heating and cooling appears more feasible by a precessing jet model. Incorporating cooling in the precessing jet model presented here, running simulations over a timescale of order Gyr, and systematically comparing mass and energy transport in precessing and non-
precessing jet models, one can study this form of maintenance-mode feedback by AGN sources.
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