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GENTZEN SYSTEMS AND DECISION PROCEDURES
FOR RELEVANT LOGICS

by

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Doctor of Philosophy, in the Australian
National University, February, 1983.
The research contained herein was carried out either by me independently or in conjunction with Dr. R.K. Meyer. In particular, the semantic results of §1.7 and §1.8 are based on collaborative work performed by Dr. Meyer and myself.

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This dissertation is primarily a proof theoretic investigation of the positive fragments and boolean extensions of two of the principal relevant logics T and R, with and without contraction, and of the corresponding positive semilattice relevant logics. In addition to motivational and syntactic preliminaries, Chapter 1 contains some new semantic results which are useful in the later chapters. In particular, $T^\circ$, $T^\circ_7$, and $R^\circ_7$ are proved to be complete with respect to their boolean semantics, and are then shown to be conservative extensions of $T^\circ$, TW and RW, respectively. In Chapter 2 we develop subscripted Gentzen systems for four positive semilattice logics. Appropriate Cut Theorems are proved, and one system is shown to be equivalent to $^uR_+$. Decision procedures are then given for the two contractionless systems. In Chapter 3 Gentzen systems are given for $TW_+$, $T_+$, $RW_+$ and $R_+$, Cut Theorems and equivalences are proved, and $TW_+$ and $RW_+$ are shown to be decidable. The sequent calculi that are used are multiply structured as required for relevant logics. Chapter 4 begins by collecting decision procedures for fragments of $TW_+$ and $RW_+$. We then discuss and make some progress toward solving some open problems, viz., the decision questions for $EW_+$, TW and RW, and the question of equivalence between $RW_+$ and its semilattice counterpart $^uRW_+$. 
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