# RELEVANT LOGICS, MODAL LOGICS AND THEORY CHANGE

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Except where otherwise acknowledged,
this thesis is my own work.

# *Abstract*

This thesis is a contribution to applied relevant logics. In Part One relevant logics are presented proof-theoretically and semantically. These logics are then extended to modal logics. Completeness proofs for all of the logics presented in Part One are provided. In Part Two, the logics of Part One are applied to certain problems in philosophical logic and Artificial Intelligence. Deontic and epistemic logics based on relevant logics are presented in chapter three and chapter four contains an extensive investigation of the logic of theory change (or database updating).

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My wife Susanne ... *SINE QUA NON.* I would have dedicated this thesis to her, had my parents not a prior claim to this small token of gratitude. So I dedicate this thesis to my parents, Wolfgang and Irmgard Fuhrmann.

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*Contents*

# *Introduction*

This dissertation is a contribution to the study of relevant logics. Its emphasis is on applications. Such an emphasis, I believe, is timely. For the purely philosophical debate about the notion of entailment has reached a deadlock. It has issued on the one side in an elaborate classical epicycle[1] and on the other side in a rich fundus of well-investigated alternatives to classical logic.[2] The divide between these two sides is unlikely to become permeable by further reflections on the elusive notion of entailment or introspection of one's linguistic intuitions about *if... then....* Progress, however, can perhaps be made by observing the contenders "in use" rather than *in vacuo*.

Almost coinciding with the decline of the entailment debate within the philosophical community is the increasing interest in non-classical logics among researchers in Artificial Intelligence (AI). It has become plain in recent years that for the solution of many problems in AI, classical logic is either not suited at all or an extremely cumbersome tool to use. Thus, in AI, alternatives to classical logic are now considered and evaluated free from the philosophical prejudices hardened in a seven decades spanning debate about "deviant" logics – non-classical logics suddenly get a "fair go".

The present dissertation attempts to take advantage of the open-minded attitude with which various logics are now considered in AI. Thus, the applications of relevant logics in Part Two of this dissertation are presented with a view to problems in AI. These problems fall under the heading of database theory. Chapter three offers some tools for reasoning about databases in a fixed state; chapter four treats the problem of database updating. In more traditional terms, however, these chapters contain also contributions to philosophical logic: chapter three presents some epistemic and deontic logics based on relevant logics, and chapter four is an exercise in the logic of theory change. The discussion in Part Two will frequently switch between philosophy and AI. Such a transfer of ideas, I believe, is beneficial to both disciplines.

Chapter one provides a grounding in the proof theory and semantics of relevant logics. We give axiomatic formulations of a group of logics, starting from a very weak system **BM** and proceeding to classical logic **K** via the comparatively strong relevant logics of Anderson and Belnap

[1] See e.g. Jackson (1987).

[2] See e.g. Routley, Meyer et al. (1982).

(1975) and the semi-relevant systems **RM** ("Mingle") and **RM3**. All of these logics will be proved complete with respect to appropriate classes of model structures (frames) of the kind used in Routley, Meyer, et al. (1982). The aim of this chapter is to provide a self-contained completeness argument for all of the major relevant logics (and a few more) as a background to the following chapters. In presenting this argument I have benefited from Dunn's survey article on relevant logics (1986).

In chapter two we shall consider extensions of the systems presented in chapter one in a language including a unary modal operator. The resulting modal systems will be proved sound and complete with respect to two extensions of the semantics introduced in chapter one. The two extensions are, first, a Kripke-style semantics, modelling the modal operator by means of a binary accessibility relation, and, secondly, a Montague-Scott-style semantics in which the modal operator is modelled by means of a so-called neighbourhood function.

In Part Two, we shall put the systems of Part One to use. The modal logics of chapter two will be used in chapter three as a means to represent and reason about the *static* properties of theories of various kinds. We shall consider in some detail two kinds of theories: sets of sentences an agent is committed to accept as true at a particular point of time ("acceptance sets"), and sets of sentences an agent is committed to make true at a particular point of time ("norm sets"). As a result of these considerations, logics of acceptance (or commitment-to-believe) and of obligation will emerge. We shall refrain from enshrining in these logics idealising assumptions about acceptance sets and norm sets; in particular, we shall not assume that such sets are always consistent. The possibility, and indeed actuality, of inconsistent but non-trivial acceptance and norm sets will motivate the move towards epistemic and deontic logics based on a paraconsistent logic. The concern with representing correctly the deductive dependencies within acceptance sets and norm sets will motivate a move towards epistemic and deontic logics based on a relevant logic.

Chapter four focuses on certain *dynamic* aspects of theories. The study of the formal aspects of theory change – though a natural complement to the investigations of Tarski (1930) – has been curiously neglected for a long time. A beginning has only recently been made in the work of Alchourron, Gärdenfors and Makinson. Though squarely based within the framework provided by these three authors, the present contribution to the theory of theory change differs in a number of aspects from their work. First, Alchourron, Gärdenfors and Makinson (AGM) consider changes of theories by one sentence at a time. I consider multiple changes: changes by sets of sentences at a time. Changes by single sentences will emerge as a special case of multiple changes, namely as changes by singleton sets of sentences. Secondly, AGM think of theories as sets of sentences closed under logical consequence; theories are thus rather amorphous objects. I think of theories as sets of sentences generated from a distinguished set of sentences (the base of the theory in question) by means of a logical consequence operation. As I shall argue in chapter four, the base of a theory does play an important role in changing a theory. Thirdly, a central concern for AGM is that changes to theories ought to be minimal: a changed theory should be as big a subset of the original theory as possible under the circumstances. I shall argue that minimality of change is a rule of thumb that may easily be overridden by other constraints on theory change. One such constraint – not recognised in the work of AGM – is that if a sentence $B$ is in a theory just because $A$ is in that theory, then $B$ should not remain in the theory after $A$ has been removed. I call this constraint on theory change 'the filtering condition'. Fourthly, for AGM, theories are closed under a consequence operation provided by classical logic. In view of classical theses like $A \rightarrow . {\sim} A \rightarrow B$ and $A \rightarrow B \rightarrow A$, the change of inconsistent theories and the removal of logical truths from a theory receive a rather special treatment in AGM's theory. The theory advanced in this dissertation will be more general: any one of the logics of chapter one may provide the consequence operation theories are closed under. However, as I shall argue in chapter four, only if theories are closed under a non-classical, relevant, consequence operation, does a satisfactory account of how inconsistent theories ought to change and how to remove logical truths from a theory emerge.

The chapters of Part Two complement each other in a straightforward sense: while chapter three provides a formal framework for reasoning about theories at a particular point of time, theories as they "move" along a time axis are the subject of formal investigations in chapter four. The formal tools employed in these chapters are, however, quite distinct. Whereas modal logics provide the background for chapter three, Tarski's theory of consequence operations is the unifying theory behind the considerations in chapter four. In the final section of this dissertation, an outlook on one way of bringing to bear modal logic on the theory of theory change will be given by employing the resources of dynamic logic in order to formulate a logic of theory change.