THE TRANSITION FROM CONCRETE
TO FORMAL THINKING

A thesis submitted in partial fulfilment of the requirements for the degree of Doctor of Philosophy.

by

Susan Clare Page (née Somerville)

Australian National University
December, 1970.

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CHAPTER 1

PIAGET'S ACCOUNT OF THE STRUCTURES OF CONCRETE AND FORMAL THOUGHT

1.1 Introduction

The introduction has indicated that this thesis is concerned with only one small part of Piaget's theory of development, namely the change in operational structures taking place during the transition from the stage of concrete to the stage of formal thought. The accounts given here of his theoretical and empirical work will thus be limited to those aspects of direct relevance to this transition.

Piaget's account of the transition from concrete to formal thinking is by no means explicit in every detail. Moreover, this particular transition must be seen in the context of his overall theory of the development of intelligence in the child. In this chapter, a familiarity with his basic theory is assumed, and after a brief overview of the early sensori-motor and preoperational stages, discussion proceeds immediately to the logico-mathematical structures which form his account of the stage of concrete operations. These structures are illustrated by detailed reference to only one particular "grouping", and there are thus many aspects of this stage of thought not mentioned explicitly. The transition from concrete to formal thinking is then outlined in terms of the changes taking place in the operational structures. Again, only one extended example is used to illustrate the differences between stages. Thus this chapter consists of statements of the logico-mathematical models of the two final stages of thought, with minimum illust-
rative detail in the sense of performance in test situations. This is done for the sake of brevity and because Piaget himself has only spelled out the transition in terms of one or two formal examples. It would be a major task to relate every feature of the operational structures to a variety of potentially relevant situations. In fact, experimental work is only beginning to do this. The subsequent chapter will be devoted to a discussion of such experimental evidence, relating it as closely as possible to the abstract models described here.

1.2 Overview and the Sensori-Motor and Preoperational Stages

The development of intelligence in the child proceeds, according to Piaget (1928, 1950, 1953a) and Inhelder and Piaget (1958, 1964), by a series of transitions from one stage to another which is derived from it, but which represents a qualitatively different adaptation to the environment. By intelligence is meant the cognitive structures on which interaction with the world is based. These structures derive in a very direct way, according to Piaget, from actions which the child performs on aspects of the environment.

In the first developmental stage, that of Sensori-Motor Intelligence (from 0 to 2 years), physical actions (mostly reflex in nature) are progressively coordinated with one another. Thought is absent, since actions are not internally represented, but coordination of the child's physical movements does bring about an organisation of spatial relationships, with objects having permanence despite their displacement within, or even disappearance from, the perceptual field. The coordinations of movements in space (comprising an ability to return to a starting point, and to take alternative routes to the
same place, amongst others) display a "structure" (that of a "group") which is characteristic of "intelligence". The same and similar structurings but with very different elements describe later stages of thought. At the age of about two years transition to the stage of Preoperational Thought occurs. Between the ages of two and about seven or eight years, two substages are distinguished: Preconceptual Thought (2 to 4 years), and Intuitive Thought (4 to 7 or 8 years). The "symbolic function" (or, more generally, "semiotic function"), beginning to appear at about eighteen months, is developed rapidly in the period from two to four years, through such activities as symbolic play and imitation. It equips the child with signs and symbols in the forms of language and of mental imagery. Deferred imitation, in particular, provides the foundations for the representation of actions in thought. A change from representation of only figural aspects of the environment to the first representations of actions marks the beginning of Intuitive Thought, at about four to five years. The stage of Intuitive Thought is the subsequent period of three to four years, during which represented action is distorted because of the "egocentrism" of the child. Not until the stage of Concrete Operational Thought (beginning at 7 or 8 years) is reached is the representation of actions as operations adequately achieved.

The shortcomings of intuitive thought can be attributed partly to a tendency for figural aspects to dictate conclusions which must be reached by "thought". Bruner (1966) provides evidence that a correct answer in a test of conservation, given when perceptual changes are "screened" from the subject, may be changed to an incorrect one once the screen is removed. A similar point is made by Piaget (1964). The fact that the child is
misled, in this and other reasoning problems, by limited perceptual aspects of the situation, leads Piaget to say that actions which the child can perform (and represent) are not adequately internalised as "possible actions" or coordinated with others related to them. In a sense, once a child at this stage "thinks of" performing an action, it is as if it has been carried out. Thus there is no way in which it can be "reversed" by another "possible action" in thought, nor the possibility of coordinating its result with that of any other (possibly incompatible) action, considered simultaneously.

Thus it is, for example, that a preoperational child presented with twenty wooden beads (Class B), most of which are brown (Class A) and a few white (Class A'), and asked "Are there more wooden beads or more brown beads?" will answer "More brown beads". He cannot do mentally what is not possible physically: that is he cannot combine and separate two classes at the same time, in order to compare the extension of the whole (B) with that of one of its parts (A). Thus his thought, like real action, lacks reversibility. At any given moment the beads are either all grouped together as wooden or else separated into two sub-groups of brown and white. Therefore when the number of brown beads is considered (Class A'), the only available class for comparison is Class A', and so the reason typically given by the child for his answer is "...... because there are only a few white ones".

A coordination of the internalised action of adding classes together to make a higher-order class (written as A + A' = B), with its inverse of subtracting one class from the higher-order class (written B - A' = A), is necessary before the question can be answered correctly. When such coordination of the representation of actions takes place, the child is said
1.3 Concrete Operational Thought and the "Grouping" and "Semi-lattice" Structures.

1.3.1 The nature of an "operation".

The name "operation" is not given to an internalised action until it is reversible and coordinated into structures with other related operations. Inhelder and Piaget (1964) insist that the expression \((A + A^1 = B)\) should only be used to represent operations which are reversible, since writing it in this way automatically implies that the inversion \((B - A^1 = A)\) is possible. Since this is not the case for the preoperational child, his representations of classificatory actions may not be described by expressions such as \((A + A^1 = B)\). To suggest that a child may be able to add classes operationally \((A + A^1 = B)\) before he understands inversion of this operation, and consequently class inclusions, as is suggested by Kofsky (1966), is not compatible with Piaget's use of the term operation. Piaget states:

"Psychologically, operations are actions which are internalizable, reversible, and coordinated into systems characterized by laws which apply to the system as a whole. They are actions, since they are carried out on objects before being performed on symbols. They are internalizable, since they can also be carried out in thought without losing their original character of actions. They are reversible as against simple actions which are irreversible. In this way, the operation of combining can be inverted immediately into the operation of dissociating, whereas the act of writing from left to right cannot be inverted to one of writing from right to left without a new habit being acquired differing from the first. Finally, since operations do not exist in isolation they are connected in the form of structured wholes. Thus, the construction of a class implies a classificatory system and the construction of an asymmetrical transitive relation, a system of serial relations, etc" (Piaget, 1953b, p.8).
The original formulation of the "Grouping" Structure.

The model which Piaget puts forward as a description of the "structured wholes" of concrete operational thought is a "groupement" or "grouping". It is so named because it has many similarities to, but a few differences from, the mathematical or logical "group" structure. Eight different concrete operational groupings are described, each one applicable to operations of a specific type.

These are classified by Beth and Piaget (1966) as follows:

"This grouping structure is found in eight distinct systems, all represented at different degrees of completion in the behaviour of children of 7-8 to 10-12 years of age, and differentiated according to whether it is a question of classes or relations, additive or multiplicative classifications, and symmetrical (or bi-univocal) or asymmetrical (co-univocal) correspondences:

<table>
<thead>
<tr>
<th>Classes</th>
<th>Relations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Additives</td>
<td></td>
</tr>
<tr>
<td>(asymmetrical I)</td>
<td>V</td>
</tr>
<tr>
<td>(symmetrical II)</td>
<td>VI</td>
</tr>
<tr>
<td>Multiplicatives</td>
<td></td>
</tr>
<tr>
<td>(co-univocal III)</td>
<td>VII</td>
</tr>
<tr>
<td>(bi-univocal IV)</td>
<td>VIII</td>
</tr>
</tbody>
</table>

(Beth and Piaget, 1966, p.174).

The interpretation of the axioms obeyed by Groupings I to VIII is different, depending on the type of operations involved in each. The formulation of the axioms given below will be illustrated by examples appropriate to Grouping I, known as the Primary Addition of Classes. Although the elements of a grouping strictly should be thought of as operations (such as \( A + A^1 = B \) for Grouping 1) and indeed Flavell (1963) gives an account of each of the axioms in terms of elements of this kind, Piaget (1950, 1953b) is inclined to give accounts using classes themselves (\( A, B, \) etc.) as the
elements. Insofar as any class, \( A \), may be understood as the internalised action of putting together its constituents, be they single elements or two or more subclasses, the latter procedure seems acceptable. Since it provides a much more concise formulation, it will be used here.

**Axioms of a Grouping** (Illustrated by Grouping I: the Primary Addition of Classes. A set of inclusions is involved - Class \( A \) is included in Class \( B \), Class \( B \) in Class \( C \) and so on).

1. **Combinativity** A combination of two elements of a grouping results in an element of the same grouping.

   \[ A + A^1 = B; \quad B + B^1 = C \text{ etc.} \]

   **Note:** The axiom does not state "A combination of any two elements ........." since combination is restricted to that of contiguous classes (classes of the same rank) such as \( A \) and \( A^1 \), \( B \) and \( B^1 \), \( C \) and \( C^1 \) etc.

2. **Identity Element** There exists an element of the grouping, called the Identity Element, such that any element, when combined with the Identity Element, remains the same.

   \[ \text{eg. } 0, \text{ the null class, is the Identity Element for Grouping I, and we can write } A + 0 = A. \]

Piaget tends to use the equation \( A - A = 0 \) to illustrate the meaning of the Identity Element, expressing it in words as "An operation combined with its converse is annulled" (Piaget 1950, p.41). It is, however, more usual to define the Identity Element by its property than to define it as the result of a combination of inverse elements. Inverse elements may then be defined (as in Axiom 4) as elements whose combination results in the Identity Element.

The difficulty which led Piaget to define the Identity Element by
means of \( A - A = 0 \) is that, for groupings of classes and relations, there are "special identities", expressed under Axiom 3 below. Thus the Identity Element is not the only \( X \) for which \( A + X = A \) is true. ("\( X \)" is used to stand for any of the elements, \( A, B, C \) of the grouping).

3. Tautology (or Special Identity) (i) The combination of an element of a grouping with itself, leaves the element unchanged (Tautology).

\[ \text{eg. } A + A = A; \quad B + B = B \text{ etc.} \]

(ii) The combination of an element of a grouping with an element in which it is contained (some ordering of the elements, such as by class inclusion, is understood here) results in the containing element (Absorption).

\[ \text{eg. } A + B = B; \quad B + C = C \]

Beth and Piaget (1966) present a more general principle of "reabsorption", of which tautology is a special case, in place of (i) and (ii) above. It is part of the reformulation of the grouping devised by Grize (1960). This is set out in section 1.3.3 below.

4. Reversibility To each element of a grouping corresponds an Inverse Element, such that a combination of the two results in the Identity Element.

\[ \text{eg. (i) If single classes are used as elements, then the operation of forming the class ("A") and that of destroying it ("-A") must be accepted as inverses. Thus:-} \]

\[ A - A = 0 \text{ (or, strictly, } A + (-A) = 0) \text{, where } 0 \text{ represents the null class, or Identity Element. (Strictly speaking } 0 \text{ represents the internalised action of forming the null class,)} \]

\[(\text{ii) Since the expression } -A \text{ presents difficulties of interpretation, it is more usual to illustrate the reversibility axiom} \]
by means of \((A + A^1 = B)\) as an element and \((A = B - A^1)\) or \((B - A^1 = A)\) or even \((-A - A^1 = -B)\), all equivalent, as its inverse. In this account, the result of combining an operation and its inverse is not the null class, but a return to the original state of affairs. The same difficulties of conceiving "negative" combinations of classes are still present, however, in this second account, and recently Grize (1960) has offered a way out, by the definition of the grouping as a structure of elements with two binary operations defined on them (+ and - in this case). A very careful distinction must be maintained between the operations which are the elements of the structure and the binary operation or operations (e.g. addition, subtraction) under which they combine.

5. **Associativity** If \(X, Y, Z\) are three elements of a grouping, not necessarily distinct, then \((X + Y) + Z = X + (Y + Z)\)

\[
eg\, A + (A^1 + B^1) = (A + A^1) + B^1
\]

**Note:** As with Axiom 1, Combinativity, this axiom may not be stated "If \(X, Y, Z\) are any three elements .......", since there are certain combinations for which the principle of associativity does not hold e.g. :-

\[
A + (A + (-A)) \neq (A + A) + (-A)
\]

Since it is preferable to have such restrictions to the principles of associativity and combinativity stated in the formal account of the grouping structure, the reformulation proposed by Grize (1960) is more rigorous than the above. This reformulation is not, as yet, well known but can readily be understood in terms of the set of axioms, and the difficulties of definition and restriction, outlined above. The English translation of it presented by Beth and Piaget (1966) will therefore be quoted in full.
It is applicable to each of the eight groupings.

1.3.3 A more rigorous reformulation of the "grouping"

"Assume a system \((M, \rightarrow, +, -)\) where \(M\) is a non-empty set, \(\rightarrow\) a relation, \(+\) and \(-\) two binary operations. Let us designate by \(X, Y, Z\), variables which take their values from \(M\) and state two definitions 1:

\[(D_1) \quad X \rightarrow Y = df. \quad X \rightarrow Y \rightarrow X.\]

\[(D_2) \quad X \rightarrow Y = df. \quad X \rightarrow Y \rightarrow (X \rightarrow Y) \wedge (Z) (X \rightarrow Z \rightarrow Y, \exists . X \rightarrow Z \rightarrow Y \vee Z).\]

The relation \(\rightarrow\) can therefore be read "is contained in" and, as far as \(\leftrightarrow\) indicates an equivalence, it is a relation of partial order. The relation \(\rightarrow\) may be read "is immediately contained in".

The system \((M, \rightarrow, +, -)\) is then a grouping if the following conditions are satisfied:

\[(Refl) \quad X \rightarrow X.\]

\[(Trans) \quad X \rightarrow Y \rightarrow Z, \exists . X \rightarrow Z.\]

\[(G_0) \quad \text{if } Y \in M \text{ and if } X \rightarrow Y \text{ then } (a) \ X \in M\]
\[\quad \text{if } X \in M \text{ and if } X \rightarrow Y \text{ then } (b) \ Y = X \in M,\]
\[\quad (c) \ X + (Y - X) \in M.\]

We see that \(G_0\) (b) and (c) serve to restrict the possible combinations.

\[(G_1) \quad X + (Y + Z) \leftrightarrow (X + Y) + Z.\]

\[(G_2) \quad X + Y \leftrightarrow Y + X.\]

\[(G_3) \quad X \rightarrow Y, \exists . X + Z \rightarrow Y + Z.\]

1. By making use of the following logical signs: \(\neg\) (negation), \(\wedge\) (conjunction)
\(\rightarrow\) (implication), \(\equiv\) (equivalence), \(\vee\) (disjunction), ( ) (universal quantifier), (E) (existential quantifier), \(\in\) (membership).
\( (G_4) \ X \rightarrow Y \iff X + Y \rightarrow Y. \)

\( G_4 \) therefore states a principle of reabsorption, of which tautology is a particular case.

\( (G_5) \ Y \rightarrow X + Z \iff Y - X \rightarrow Z \)

\( G_5 \) allows the operation (\(-\)) to be considered as the inverse of (\(+\)) in spite of tautology and without introducing negative classes.

\( (G_6) \ Y \rightarrow X + (Y - X). \)

\( G_6 \) serves to restrict the associativity of the system without weakening the latter too much.

\( (G_7) \ X \rightarrow Y \iff X \rightarrow Y - (Y - X). \)

\( G_7 \) allows the difference between the "contiguous" elements to be restricted (combination step by step).

\( (G_8) \ \text{There is an } O \in M, \text{ such that } O \rightarrow X." \) (Beth and Piaget, 1966, pp.172 - 3)

The reformulation given above expresses the five axioms stated in section 1.3.2 (Combinativity, \( G_0 \) (a), with restrictions expressed in \( G_0 \) (b) and (c) and \( G_7 \); Unit Element, expressed by \( G_8 \); Reversibility, \( G_5 \), not in terms of Inverse Elements, but in terms of the inverse (\(-\)) of the operation (\(+\)) under which they combine; a general principle of reabsorption, \( G_4 \), of which Tautology (or Special Identity) is a special case; and Associativity \( G_1 \), with restrictions imposed by \( G_6 \). It also states that the system obeys the Commulative Law (\( G_2 \) which is an axiom not specifically included by Piaget in earlier works (eg. Piaget, 1950, p.42).

While this reformulation appears to meet the criticisms which logicians made of the original model, it does not alter the nature of the
description of structures of thought which Piaget is attempting to give. Since the elements of the structure are still to be seen as operations (internalised actions) and the axioms serve merely to state the principles under which they combine, McLaughlin's (1963) statement that the terms no longer seem to have any direct psychological referents is not justified.

1.3.4 The grouping structure and the semi-lattice.

A partially ordered set of elements is said to have a lattice structure if, for each pair of elements, a third element may be defined as their "join" and a fourth as their "meet" (for a detailed discussion, see Birkhoff and MacLane 1953, pp.351 - 354). For the purposes of discussing classificatory structures (whose elements can be regarded as a partially ordered set because of the progressive inclusions of classes), the "join" of two classes, A and B, is defined as the smallest class in which both A and B are contained, and their "meet" as the largest class which is contained in both A and B.

While it is clear that the elements of a grouping do not conform completely to a lattice structure as described above, Gonseth and Piaget (1946) claim that they may be considered to form a "semi-lattice". The term "semi-lattice" is applied to partially ordered sets for which one but not both of "join" and "meet" is defined. It should be noted also that, unless the restriction of combinativity to contiguous elements is removed, a definition of the "join" or "meet" will not exist for every pair of the set. Grouping 1 for the Primary Addition of Classes can thus be seen as a semi-lattice in which a "join" but not a "meet" is defined (although in fact the meet of each pair of contiguous classes is the null class). Grouping
III for the Co-univocal Multiplication of Classes, on the other hand, is a semi-lattice in which a "meet" but not a "join" is defined. The restrictions of each of these structures appear to arise because Grouping I consists only of operations for the addition of classes, and Grouping III only of operations for their multiplication. If operations of the other type were introduced into either structure, then both a meet and a join could be found for each pair of elements. Piaget's point seems to be that, at the level of concrete operations, the two operations (multiplication and addition) are not co-ordinated, but found in separate structures. There is, however, one type of operation in which the beginning of such coordination appears. These operations, known as "Vicariances" are the elements of Grouping II, for the Secondary Addition of Classes. If a single class, B, is divided into $A_1$ and $A_1'$, and also into $A_2^2$ and $A_2^1$; and also into $A_3$ and $A_3^1$ (and the divisions are such that $A_1^1 = A_2 + A_3, A_2^1 = A_1 + A_3, A_3^1 = A_1 + A_2$); then these operations ($A_1 + A_1' = B; A_2 + A_2' = B$ etc), called vicariance operations, have a structure in which both a meet and a join are defined. It can be seen that the simultaneous division of the same class, B, in two different ways introduces the possibility of the multiplication of classes.

This last mentioned grouping (Grouping II) lays the basis for the transition from concrete to formal operational thought. If the "vicariance" operations of Grouping II are applied to the multiplicative products of Grouping III, then a complete combinatorial system is derived. Piaget (1953b) describes the result as a "second-order grouping". There is, however, another very important change which takes place simultaneously. The multiplicative products of Grouping III are interpreted now in terms of conjunctions.
of propositions, and so the elements of the second-order grouping are propositions, not classes. Piaget remarks that the construction of a complete combinatorial system is perfectly possible using classes, but that it does not occur in concrete operational thought. When it does occur, a simultaneous change to formal operations (which are propositional in nature) is found. This transition will be discussed further in section 1.4.

1.3.5 Forms of reversibility in concrete operational structures.

Whereas the "grouping" and "semi-lattice" structures described in previous sections have been applied exclusively to classificatory operations in this discussion, Piaget intends them equally well to describe operations of ordering. While details of their applications to relations need not be given here, one aspect should be brought out before the transition to Formal Operational Thought is discussed. This is the type of reversibility which applies to an order relation. An order relation (e.g. \( X \succ Y \)) is "reversed by its reciprocal (\( X \prec Y \)) and the result is a relation of equivalence (\( X = Y \)). This is to be contrasted with the reversibility found in groupings of classes where the combination of an operation with its inverse yields the null class (or possibly, the combination of an operation dividing class \( B \) into classes \( A \) and \( A^1 \), with its inverse combining \( A \) and \( A^1 \) into \( B \), results in the original class, \( B \)). Piaget lays great stress on the fact that:-

"............. various groupements exhibit two very distinct forms of reversibility.
(a) Inversion, which consists in negating a class (\(-A\)) or an inclusion (\(\colon A\)). The product of an operation and its inverse is therefore either the null class (\(A - A = 0\)) or the most general class of the system (\(A : A = Z\), since \(A\) is a subdivision of \(Z\) and if this subdivision is eliminated we arrive back at \(Z\))."
(b) Reciprocity, which consists in eliminating, not a class, or an inclusion (subdivision), but a difference. The product of an operation and its reciprocal gives us not a null class or a universal class but a relation of equivalence: \((A \rightarrow B) + (A \leftarrow B) = (A = B)\). .............

Inversion is the form of reversibility concerned with the operations of classes, and reciprocity the form concerned with the operations of relations. No groupements are present at the level of concrete operations to combine these two kinds of reversibility into a single system. From the standpoint of mental development, inversion (negation or elimination) and reciprocity (symmetry) form two kinds of reversibility, whose beginnings are already to be seen at the lower developmental levels. At the level of concrete operations, they appear in the form of two distinct operational structures (groupements of classes and groupements of relations), and finally form a unique system at the level of propositional operations" (Piaget, 1953b, pp. 28 - 9).

Thus the major importance of the change from concrete operations as elements of the structures of thought (these concrete operations being addition and multiplication of classes, ordering of relations, etc), to formal operations as elements (these being propositional statements), is the consequent coordination of two forms of reversibility. A proposition has both a reciprocal and an inverse proposition. It is this aspect of formal logical structures (such as the Sixteen Binary Operations) which leads Piaget to see them as an appropriate model of adult thought. A coordination of these two forms of reversibility would not be possible, using operations on classes as the elements, even if the operations of addition and multiplication were defined on the same structure: hence the need to regard the elements of formal thought as operations whose form and structure is that of propositional logic. The transition from concrete operational to formal operational thought will now be discussed in more detail.
1.4 The Transition from Concrete to Formal Operational Thought.

The structure of formal operations, to which those of concrete operations tend, is anticipated by them in a number of ways. Formal operations have a structure which is at once that of a group (the grouping without its restrictions) and that of a complete lattice. The two forms of reversibility (by inversion and by reciprocity), each present at the stage of concrete operations, but in separate structures, are present together in the group structure of formal operations. The "join" and "meet" of pairs of elements, only one of which is defined for the semi-lattice structures of some groupings, are both defined for the complete lattice structure of formal operations. As mentioned in section 1.3.4 the one grouping structure, Grouping II, in which both "join" and "meet" are defined, provides a route to the formal structure. Piaget's account of what is involved in this transition will now be discussed in detail. The transition is said to begin at about 11 or 12 years of age, but to be incomplete until about 14 or 15 years.

1.4.1 The class and propositional interpretations of Boolean Algebra.

In a discussion of operations performed on elements of a set or structure, an analogy may be drawn between the arithmetical operations of addition (+), subtraction (-), multiplication (x) and division (÷) as applied to numbers, and the logical operations such as conjunction (\&), disjunction (\lor), implication (\Rightarrow) and negation (\neg) applied to propositions. In particular, the logical operations of conjunction and disjunction are commonly referred to as logical addition and logical multiplication respectively. These two, together with negation (\neg), provide a basis in terms of
which any other logical operation such as implication, equivalence, reciprocal exclusion, can be expressed. Boolean Algebra is the two-valued (0 and 1 for false and true, respectively) algebra of propositions. A number of interpretations may be made of abstract Boolean Algebra, and Piaget’s theory claims that a class interpretation is that available to the child at concrete operations, whereas a propositional interpretation is used in formal operational thought.

In the logic of classes, two distinct systems are described by Piaget, for addition and multiplication separately, and the conventional arithmetic symbols, + and \( \times \), are used to denote these logical operations (and a prime to denote “not”) in his account. Illustration of these two systems, given by Mays in Piaget (1953b) as a summary of Piaget’s (1949) original account, are as follows:

1.4.1.1 The class interpretation, at the level of concrete operations

(a) The Addition of Classes If a class, \( C \), may be subdivided into classes \( B_1 \) and \( B_1^1 \), which in turn may be subdivided into \( A_1 \) and \( A_1^1 \), and \( A_2 \) and \( A_2^1 \), respectively, then the logical addition of these classes may be illustrated as below:

Example

\[
\begin{array}{c}
\text{animal} \\
\text{vertebrate} \\
\text{mammal, non-mammal} \\
\text{insect, non-insect}
\end{array}
\]
(b) The Multiplication of Classes

If a class, B, may be subdivided in two distinct ways simultaneously, then B₁ may be used to represent the class with the first subdivision (into $A₁$ and $A₁^1$ say, thus $B₁ = A₁ + A₁^1$) and $B₂$ to represent that class with the second subdivision (into $A₂$ and $A₂^1$ say, thus $B₂ = A₂ + A₂^1$); then the simultaneous subdivision of the class in two ways is called the logical product of $B₁$ and $B₂$ and written:

$$B₁ \times B₂ = A₁ \cdot A₂ + A₁^1 \cdot A₂ + A₁ \cdot A₂^1 + A₁^1 \cdot A₂^1.$$

For example, if $B₁$ is the class of animals divided into $A₁$ (vertebrates) and $A₁^1$ (invertebrates), and $B₂$ the class of animals divided into $A₂$ (terrestrial) and $A₂^1$ (aquatic), then the multiplication of $B₁ \times B₂$ gives simultaneous classification on both bases. This is illustrated below:

```
  \begin{array}{c|c|c|c}
    & A₁ & A₁^1 & \text{Description} \\
  \hline
  \text{A₂ terrestrial} & A₁ \cdot A₂ & A₁^1 \cdot A₂ & \text{vertebrates, terrestrial} \\
  \text{A₂ aquatic} & A₁ \cdot A₂^1 & A₁^1 \cdot A₂^1 & \text{vertebrates, aquatic} \\
  \end{array}
```

Class     Description

- $A₁ A₂$     vertebrates, terrestrial
- $A₁^1 A₂$     vertebrates, aquatic
- $A₁ A₂^1$     invertebrates, terrestrial
- $A₁^1 A₂^1$     invertebrates, aquatic

(Examples from Mays, in Piaget 1953b, pp. xi, xii)
The way in which such classificatory schemes may be employed by a concrete operational child in a reasoning situation is best illustrated by the experiments reported by Inhelder and Piaget (1958). These experiments provide children with materials to manipulate, the aim being to discover a number of lawful relationships among variables. Piaget's "grouping" structures are then intended to describe the kind of logical system which the child at the stage of concrete operations uses to describe and analyse the events which he observes.

For example, in an experiment where a child is expected to discover (among other things) that the smaller volume a ball has (for the same weight) the further it will travel along a horizontal plane, children may classify events in four possible categories as in the table below:-

<table>
<thead>
<tr>
<th>Size of Ball</th>
<th>Small $A_1$</th>
<th>Large $A_1^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Long $A_2$</td>
<td>$A_1 A_2$</td>
<td>$A_1^1 A_2$</td>
</tr>
<tr>
<td>Short $A_2^1$</td>
<td>$A_1 A_2^1$</td>
<td>$A_1^1 A_2^1$</td>
</tr>
</tbody>
</table>

The child at Concrete Operations will see the events in terms of the multiplication of two classes:

Class $B_1 : A_1 = \text{small balls}, A_1^1 = \text{large balls}; B_1 = A_1 + A_1^1$

Class $B_2 : A_2 = \text{long distances}, A_2^1 = \text{short distances}; B_2 = A_2 + A_2^1$

$B_1 \times B_2 = A_1 A_2 + A_1^1 A_2 + A_1 A_2^1 + A_1^1 A_2^1$

He will thus be able to classify the events reliably and give such
descriptions as "this one is a little ball and it has gone a long way". If he discovers a correspondence between the size of the ball and the distance travelled, it is by ordering the two variables separately, and then noticing that a correspondence can be made. He thus may say as a summary "there are small balls and large balls, and they can go a long way and a short way".

According to Piaget, the concrete operational child does not see the situation as one in which the relationship between two variables (such as the size of ball and distance travelled, discussed above) may be any one of the sixteen which are logically possible. His task is not conceptualised as one of deducing, rigorously, which one of the "possible" relationships obtains, but rather simply as a task where he may "look and see", relying on the "situation" to make apparent any correspondences or regularities of importance. He knows that he has the ability to classify objects and events, to put them in order where applicable, and make correspondences between them, but he does not yet see that he must control other variables and arrange experiments in such a way that relationships are unambiguously revealed. Another way of saying this is that the system of all possible relationships between two variables is not available to him at all. This system, or structure, that of the Sixteen Binary Operations, is the structure of formal thought. Its elements are propositions, not classes, for, as was pointed out earlier, although such a structure could be built for classes, this is not done by the child.

1.4.1.2 The propositional interpretation, at the level of formal operations.

Two changes are apparent in the transition from concrete to
formal thinking. The first is a change in the nature of the operations, or elements of the structure. Whereas operations at the concrete level are those "...... occurring in the manipulation of objects, or in their representation accompanied by language", formal operations are "..... concerned solely with propositions or verbal statements" and independent of any actual manipulation (Beth and Piaget, 1966, p.172). Operations of classifying, ordering etc. do not necessarily have to be confined to real objects which allow actual manipulation (or to the internalisation of such real objects by representation, and language), but the child does not in fact extend them to the world of "possible objects and events", in a purely hypothetico-deductive way.

When he does move to the world of purely formal, hypothetico-deductive thought, the transition is accompanied by a change to propositional statements as the new elements of thought.

Thus in the problem described above, the classification, $B_1$, into small balls ($A_1$) and large balls ($A_1^1$), is replaced by the propositions:

\[ p - \text{"that the ball is small"} \]
\[ \overline{p} - \text{"that the ball is not small" (i.e. that it is large if only two values are used).} \]

and similarly the classification, $B_2$, into long distances ($A_2$) and short distances ($A_2^1$), is replaced by the propositions:

\[ q - \text{"that the distance (travelled) is long"} \]
\[ \overline{q} - \text{"that the distance (travelled) is not long" (i.e. that it is short, if only two values are used).} \]

Thus the child at formal operations sees the situation as involving two propositions and their interrelation:
The correspondence existing between the interpretation in terms of the multiplication of classes $B_1$ and $B_2$, and that in terms of the logical multiplication of two propositions $p$ and $q$, can be seen in the account below from Piaget (1953b).

"Classes: $(A_1 + A_1^1) x (A_2 + A_2^1)$

$$= A_1 A_2^1 + A_1^1 A_2 + A_1^1 A_2^1$$

Propositions: $(p v \overline{p}) \cdot (q v \overline{q})$

$$= (p.q) v (p.q) v (p.q) v (p.q)$$

Product Number

| 1 | 2 | 3 | 4 |

Propositional operations are thus constructed simply by combining these four basic conjunctions. The 16 binary operations of two-valued propositional logic therefore result from the combinations given below (written in numerical form):

$$0; 1; 2; 3; 4; 12; 13; 14; 23; 24; 34; 123; 124; 134; 234; 1234$$

(Piaget, 1953b, p.30).

The means by which the sixteen binary operations are derived from the four products $(p.q); (p.q); (p.q)$ and $(p.q)$ is described by Piaget as the application of Grouping II (vicariances) to the multiplicative products of Grouping III. It is suggested that this may be understood as follows. A vicariance operation consists in the division of some set of elements into two parts. There are sixteen different ways of dividing the four conjunctions $(p.q); (p.q); (p.q)$ and $(p.q)$ into two groups. The numbers set out by
Piaget, as above, identify the conjunctions in one of two such groups, for each of the sixteen possible divisions. If those conjunctions referred to by the numbers are then asserted to be true, and the others false, for each case, the set of sixteen binary operations of formal logic is obtained.

Thus the vicariance operations, performed on the four conjunctions divide them, in all possible ways, into some that are true and some that are false. It should be emphasised that this elaboration of Piaget's statements is given by the present author and not by Piaget himself.

The sixteen possible relationships thus may be more readily understood from the truth table set out below, in which the names of the relationships are also specified, for future reference.

The Sixteen Binary Propositions

<table>
<thead>
<tr>
<th>Number</th>
<th>Title</th>
<th>Truth Values of Products</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Complete Negation</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>2.</td>
<td>Conjunction</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>3.</td>
<td>Non-implication</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>4.</td>
<td>Negation of Reciprocal Implication</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>5.</td>
<td>Conjunctive Negation</td>
<td>F</td>
<td>F</td>
</tr>
<tr>
<td>6.</td>
<td>Affirmation of p (independently of q)</td>
<td>T</td>
<td>T</td>
</tr>
<tr>
<td>7.</td>
<td>Affirmation of q (independently of p)</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>8.</td>
<td>Equivalence</td>
<td>T</td>
<td>F</td>
</tr>
<tr>
<td>9.</td>
<td>Reciprocal Exclusion</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>10.</td>
<td>Negation of q (independently of p)</td>
<td>F</td>
<td>T</td>
</tr>
<tr>
<td>Number</td>
<td>Name</td>
<td>Truth Values of Products</td>
<td>Expression</td>
</tr>
<tr>
<td>--------</td>
<td>------------------------------------</td>
<td>--------------------------</td>
<td>------------</td>
</tr>
<tr>
<td>11.</td>
<td>Negation of p (independently of q)</td>
<td>p, q</td>
<td>F F T T</td>
</tr>
<tr>
<td></td>
<td></td>
<td>p, q</td>
<td>p(q)</td>
</tr>
<tr>
<td>12.</td>
<td>Disjunction</td>
<td>p, q</td>
<td>T T T F</td>
</tr>
<tr>
<td>13.</td>
<td>Reciprocal Implication</td>
<td>p, q</td>
<td>T T F T</td>
</tr>
<tr>
<td>14.</td>
<td>Implication</td>
<td>p, q</td>
<td>T F T T</td>
</tr>
<tr>
<td>15.</td>
<td>Incompatibility</td>
<td>p, q</td>
<td>F T T T</td>
</tr>
<tr>
<td>16.</td>
<td>Complete Affirmation</td>
<td>p, q</td>
<td>T T T T</td>
</tr>
</tbody>
</table>

In summary, then, Piaget maintains that the construction of the sixteen binary relationships is achieved by application of the vicariance operations of Grouping II to the four products resulting from the multiplication of two propositions (i.e. \((p \lor \neg p) \times (q \lor \neg q)\)). He says:

"In classifying the products \(p, q; p, \neg q; \neg p, q; \) and \(\neg p, \neg q\) in all possible ways, using the operation of vicariance we obtain a combinatorial system \(n \times n\) and a set of all subsets.

We can therefore say that the characteristic combinatorial structure of propositional operations forms a groupement of the second order, and consists in applying classification generalised by vicariance to the product sets of the multiplicative groupement. In other words, elementary groupements are groupements of the first order; consisting of (a) simple classifications, (b) vicariences or reciprocal substitutions within the classifications and (c) the multiplication of two or \(n\) classifications. On the other hand, the combinatorial structure of propositional operations which applies operations (a) and (b) to the products of operation (c), is a groupement of the second order; and hence of a more general form and corresponds to later mental structures."

(Piaget, 1953b, pp. 31–32).
The latter part of this discussion has expressed the second fundamental difference between concrete and formal thinking. The first was in the nature of the elements of the structures, the second is in the nature of the structure itself. From the preceding outline of the sixteen binary operations it can be seen that they conform to a complete lattice structure, with any two elements having a "join" expressed by \((pq)\) and a "meet" expressed by \((p,q)\). The symbols \(p\) and \(q\) must be extended here to refer to binary propositions amongst the set of sixteen.

The second aspect of the structure of formal operations, which according to Piaget (1953b) was not previously well-known, is that certain sets of four (sometimes fewer) amongst the sixteen binary operations can be seen as elements of a logical group. Each of the sixteen belongs to one such "group" structure because each has an inverse, a reciprocal and a correlate proposition amongst the other members of the set of sixteen. The "group" structure is the structure of the transformations involved:

1. **Identity** (I), **Negation** (N), **Taking the Reciprocal** (R) and **Taking the Correlate** (C)

These are defined as follows (the account is adapted from Mays, in Piaget 1953b, p. xiv)

1. **The Inverse** (N) of a proposition is obtained by negating the proposition.
   
   e.g. If the proposition is \((pq)\) it has as its inverse (or complementary) \((\overline{p},\overline{q})\). (If we negate \((p,q)\) we arrive at \((pq)\).)

2. **The Reciprocal** (R) of \((pq)\) is the same proposition with negation signs \((\overline{p},\overline{q})\).

3. **The Correlate** (C) of a proposition is the proposition such that \(v\) has been substituted for \(v\) throughout, and vice versa. Thus for \((pq)\)
The correlate is \((p,q)\).

4. **The Identity Operation** (I) is an operation which when performed on a proposition leaves it unchanged.

Operations 1, 2, 3 above are related to each other as in the table below (one particular set of four elements from the sixteen is used as an illustration):

<table>
<thead>
<tr>
<th>Disjunction ((p \lor q))</th>
<th>Incompatibility ((p \land q))</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>Conjunction ((p \land q))</td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>N</td>
</tr>
<tr>
<td></td>
<td>R</td>
</tr>
<tr>
<td></td>
<td>C</td>
</tr>
</tbody>
</table>

The above set of operations, together with the identity operation, I, constitute an abstract logical group, known as "The INRC Group." A definition of a group structure is not offered here, but left until Chapter 4, in the context of a mathematical task.

Whereas a grouping composed of *Classes* as elements has **Inverses** (e.g., mammals and non-mammals); and a grouping composed of *Relations* between elements (A is twice as long as B) has **Reciprocals** (B is twice as long as A); the group composed of *Propositions* (e.g., \(p\) implies \(q\)) has both an **Inverse** (\(p\) does not imply \(q\)) and a **Reciprocal** (\(q\) implies \(p\)).

For the formal operational child, then, the problem of discovering the role of the size of the ball in the conservation of motion experiment discussed earlier, "reduces" to a problem of discovering that there is an equivalence relationship (one of the sixteen possible). To do this he must see the problem as follows:-
If \( p \land q \) and/or \( \neg p \land \neg q \) are true \((p \land q) \lor (\neg p \land \neg q)\),
and if \( p \land q \) and/or \( \neg p \land \neg q \) are false \((\neg p \land \neg q) \lor (p \land q)\);
then the two propositions \( p \) and \( q \) are equivalent \((p = q)\).

To understand this relationship of equivalence, then, Piaget says the child must understand:

1. That one proposition is the reciprocal of another.

\( A. \) \((p \land q)\) is the reciprocal of \((\neg p \land \neg q)\). They thus both support the same relationship between \( p \) and \( q \) (namely equivalence \((p = q)\), in words: If, and only if, \( p \); then \( q \): the two propositions \( p \) and \( q \) are always both true or both false).

\( B. \) \((p \land q)\) is the reciprocal of \((\neg p \land \neg q)\). They thus both support the same relationship between \( p \) and \( q \) (namely reciprocal exclusion \((p \lor q)\), in words: If, and only if, \( p \); then \( q \): of the two propositions \( p \) and \( q \), if one is true, then the other must be false.

2. That one proposition is the inverse of another.

The proposition \((p = q)\), Equivalence (which is \((p \land q) \lor (\neg p \land \neg q)\) as above) is the inverse of the proposition \((p \lor q)\), Reciprocal Exclusion (which is \((p \land q) \lor (\neg p \land \neg q)\) as above). An inverse proposition is the negation of the proposition of which it is the inverse, thus \((p \lor q) = (\neg p \land \neg q)\), and \((p = q) = (p \lor q)\):

\[
\text{in detail } (p \land q) \lor (\neg p \land \neg q) = (\neg p \land \neg q) \lor (p \land q)
\]
and
\[
(p \lor q) \lor (\neg p \lor \neg q) = (\neg p \lor \neg q) \lor (p \lor q).
\]

It is clear that evidence for any proposition is evidence against its inverse, (and vice versa).

Thus at the stage of formal operations the child is expected to use the numbers in the four categories as evidence for and against the two
inverse propositions \((p=q)\) and \((p \lor q)\). At the simplest level he may take the number in \((p.q)\) plus the number in \((p.q)\), and subtract, from this total, the number in \((p.q)\) plus the number in \((p.q)\); the result of this subtraction should be taken as a proportion of the total number of cases. Hence if the numbers of cases in the four cells are given by \(a, b, c, d\) as in the table (see page 22) an index of the strength of the equivalence relation (loosely speaking) would be:

\[
\frac{(a + d) - (b + c)}{a + b + c + d}
\]

It should be noted that Piaget's statistical approach to the assessment of the "truth" or "falsity" of propositions is not, strictly speaking, compatible with a two-valued propositional logic. However, his logical model is intended to describe the processes of thought, rather than vice versa.

In summary, the Sixteen Binary Operations, whose elements are propositions, and whose structural relationships conform to those of a group as well as those of a complete lattice, constitute Piaget's model of formal operational thought. A subject at this stage of development, faced with a problem involving relationships between variables, will conceive of all possible relationships which could exist, and conduct systematic and controlled experiments to decide between them. If the instructions direct him to look for the existence of a particular relationship, he will be aware of the other possible ones which he has to eliminate. Thus the experiment discussed above, where he investigates the role of different variables in the conservation of motion in a horizontal plane, requires that he establish that the relationship of the size of the ball to distance travelled is one of equivalence. The same type of relationship is involved, in another experiment,
between the length of string and the frequency of oscillation of a pendulum. This equivalence relationship may also be examined, by direct questioning about the number of cases in cells a, b, c, d of a 2 x 2 table, to see whether the child understands the reciprocal and inverse relationships involved.

These last two experiments (the frequency of oscillation of a pendulum; and an experiment devised specifically to test the understanding of equivalence, using faces with two colours of hair and two colours of eyes) were both performed initially by Inhelder and Piaget (1958).

It is worth noting that, although Inhelder and Piaget's (1958) accounts of the formal operations involved in solution of the problems consider variables only two at a time, formal thought is not seen as limited to necessity to binary operations. The child at the stage of formal thought is considered capable of considering ternary relationships (or higher) if necessary, although the number involved seems prohibitively high (the number of ternary propositions is \(2^8 = 256\)). In practice, the experimental technique of holding constant all factors except one always reduces the problem to one where only the sixteen binary operations need be considered. Such a procedure, however, neglects the possibility of interaction effects among variables, which could only be discovered by simultaneous variation of at least two.

No theory or experimental evidence is available to say whether discovery of such interaction effects is possible as soon as competence with binary operations is achieved, or whether further development must occur. Piaget's discussions seem to imply that no further mental development would be required (Piaget 1950, Inhelder and Piaget 1958). However McLaughlin (1963), who gives an alternative account to Piaget's psycho-logic in terms of the number
of concepts which may be considered simultaneously in each developmental stage, suggests that a formal thinker may be able to process no more than eight concepts simultaneously. It is not clear, however, whether McLaughlin's requirement of the simultaneous retention, or consideration, of a number of concepts is equivalent to Piaget's statement that a child knows he must determine, experimentally, which of a number of relationships is the one that holds. The apparent conflict between McLaughlin's upper limit of 8 concepts and Piaget's minimum structure of 16 binary operations might thus be resolved by further analysis of the theories advanced. It is surprising that McLaughlin's formulation has received no subsequent attention.

The experimental work advanced by Piaget, Inhelder and others to support the models of concrete and formal operational structures, outlined in this chapter, will be discussed in Chapter 2. Piaget's (1928, 1932) early verbal descriptions of the stages, of which the foregoing logical models are a formalisation, will be presented in the context of experimental work which has continued on this verbal plane. The more recent work, based on Inhelder and Piaget's (1958) experiments and logical analyses, will form a second section. As a conclusion to the theoretical background of the thesis, methodological considerations relating to studies of the development of thought will be discussed.

In Chapter 3, a study with a different theoretical background (Dienes and Jeeves, 1965) is isolated for interpretation in Piagetian terms, and a report of an experimental investigation using one of their tasks, and a task taken from Inhelder and Piaget (1958), forms the rest of the thesis.
CHAPTER 2

EXPERIMENTAL EVIDENCE RELATED TO PIAGET'S THEORY

2.1 Introduction

This chapter aims to provide an account of experimental evidence relevant to Piaget's theory of the transition from concrete to formal thought. Apart from studies carried out in Geneva (Inhelder and Piaget 1958), and direct replications of them (Lovell 1961; Smedslund 1963), there is very little work which tests, or is directly relevant to, this section of his theory. Some experimental work (Peel 1959, 1966) has been based on Piaget's earlier analyses of judgments made about verbal "story" material (Piaget 1928, 1932). The chapter will therefore be structured as follows.

Firstly, an overview will indicate the main aspects of Piaget's theory which have received experimental attention. Apart from a few methodological considerations, it will be clear that there is not much to be gleaned of direct relevance to the transition from concrete to formal thought. Experimental work specifically designed to examine performances of subjects at these stages of development will be presented in a second section. This section will be subdivided into a first part, concerned with work based on Piaget's earlier techniques, employing story material and verbal responses; and a second part, concerned with work stemming from Inhelder and Piaget's (1958) experiments, using a variety of physical problems. The latter type of study has more direct relevance to the logico-mathematical models discussed in Chapter 1. A number of studies have been published recently
which derive more from the logical structures of the model itself, than from Inhelder and Piaget's (1958) empirical work (Peel 1967; Wason 1968; Johnson-Laird and Tagart 1969). These will be discussed in the same section.

Finally, a summary of findings, placed in the context of a recent theoretical analysis of transition periods (Flavell and Wohlwill 1969), will provide a methodological basis for the present study.

2.2 General Trends in Approach and Methodology

Experimental work based on Piaget's ideas has arisen, and recently become prolific, in at least three main areas: that of cross-cultural comparisons (and comparisons of socially, physically or mentally handicapped children with those of the normal population); that of curriculum development and the education of the child (and of the teacher); that of the nature of child development as a theoretical problem to be investigated by traditional experimental methods. Piaget's clinical techniques of interrogating the child (being almost always in the context of concrete materials and their manipulation) have been translated, put into non-verbal forms, structured into "more objective questionnaires" and commonly coaxed into sets of "items" each of which may be scored 0 or 1 yielding a total (more or less quantitative) index of "developmental level" within some specified range. While feeling compelled to adopt such "refinements", and the present study is no exception in this regard, most experimenters have sought to preserve a "real Piagetian approach", in the sense of attempting to investigate basic structures of thought rather than test for individual differences among subjects who are
all "thinking" in the same sort of way. This radical difference in emphasis between work inspired by Piaget and that of the mainstream of psychological testing is pointed out by Hunt (1961), Heron (1969) and Wohlwill (1970).

Hunt says:-

"Intelligence tests consist essentially of samplings of behaviour. In traditional tests, what is sampled is typically named in terms of such skill categories as verbal or arithmetic skill. The attempts by factor analysts, including Spearman's (1927) g, Thurstone's (1938) primary abilities, and Guilford's (1956, 1957) factor structures of intellect, to specify what is sampled yield what is probably best conceived as systems of coordinates which simplify the comparing of people in their test performance and perhaps facilitate making predictions about the efficiency of people. These systems of coordinates, regardless of the names given to them, may - yes, probably - have little or nothing to do with the natural structures, schemata, operations, and concepts organized within individuals that determine their problem-solving. It is the merit of Piaget to give attention to the natural structures of the central processes that mediate problem-solving" (Hunt, 1961, p.311).

Wohlwill (1970) has drawn attention to a number of important issues in the study of behaviour which changes with age. He recognises that the work of theoretical developmental psychologists, such as Piaget, is purely descriptive in nature and sees it as a painstaking but necessary task. The goal of this task is not to arrive at experimental data which provides tests of the significance of the difference between performance at different age groups, but a complete description (in mathematical terms if possible) of the form of the relationship to age. Few studies have either seen this as their goal, or made very much progress towards it, with the exception of a small number in the areas of perceptual and motor development. Wohlwill points out the lack of integration of this type of work with that of the differential psychologists as follows:-
"Developmental theorists such as Piaget and Werner and researchers studying age differences generally have not been noted for paying systematic attention to individual differences in behaviour. Conversely, differential psychologists have rarely attempted to integrate developmental changes into their work or their thinking."
(Wohlwill, 1970, p.60.)

Wohlwill notes that there are some exceptions to this, mostly in the field of personality development, but speaks of the need for the developmental psychologist to "..... integrate the roles of the differentialist and experimentalist which are generally separated by a wide (and widening) gulf ...." (Ibid, p.62). He points out that many of the research needs can only be met through longitudinal rather than cross-sectional studies.

Although the third mentioned area of interest, child development as a theoretical problem, is the one most relevant for the present studies, the first two deserve brief mention. It must be recognised that information from work with such groups as the subnormal (Mannix 1960; Hood 1962; Woodward 1962; Jacksen 1965) and deaf (Furth 1966) and peoples of other cultures (deLemos 1966; Heron 1969) is a valuable way to investigate the roles played by language, environmental experience, and social and cultural variables in the development of the child. A detailed discussion of the findings of such studies is, however, beyond the scope of this thesis. The most important general conclusion arising from such work is that, even in circumstances where progress through Piaget's developmental stages is severely retarded, the same sequence of stages is detectable. There do appear to be instances where progress beyond, say, concrete operations is not made, even by adult members of a culture, and this raises the issue of
critical maturational periods in which certain experiences and developments must occur for further progress to be possible. No such considerations arise in studies of the present kind, where the aim is simply to describe the course of development typical in one section of the Australian culture.

Work done primarily with educational objectives in mind, or to answer the question of whether progress through the stages may be accelerated by certain kinds of learning experiences, is a little more relevant to the present investigation. The results of the extensive amount of work done in this area are not easy to summarise. Most of the studies have been concerned with the development of concrete operations, and with conservation problems in particular. Some techniques of training appear to be successful, and some do not, but perhaps the kind of conclusions that Smedslund advanced after a series of discussions and experiments (Smedslund 1961a, 1961b, 1961c) are still generally accepted. These are that conservations achieved as a result of training are relatively unstable, impermanent and susceptible to extinction. Further studies by Smedslund (1961d, 1961e) and others (Beilin and Franklin 1962; Feigenbaum and Sulkin 1964) on the acquisition of conservations and by a variety of authors on number conservation in particular (Wohlwill 1960; Wohlwill and Lowe 1962; Gruen 1965; Smedslund 1964) have produced predominantly negative, although by no means unequivocal, evidence on the effects of training procedures. Piaget (Duckworth, 1964) holds the view that it may be detrimental to provide the child with a ready-made or verbal solution to problems, the derivation of which he does not understand. He says:-
"Experience is always necessary for intellectual development.... But I fear that we may fall into the illusion that being submitted to an experience (a demonstration) is sufficient for a subject to disengage the structure involved. But more than this is required. The subject must be active, must transform things and find the structure of his own actions on the objects."

"Words are probably not a short-cut to a better understanding .... The level of understanding seems to modify the language that is used, rather than vice versa .... Mainly language serves to translate what is already understood; or else language may even present a danger if it is used to introduce an idea which is not yet accessible". (Piaget, quoted in Duckworth 1964, pp. 4,5.)

More recent studies (Smedslund 1962, 1963, 1966; Wallach, Wall and Anderson 1967; Roberts 1969), where training on a completely different problem is shown to transfer to a conservation test, are more difficult to dismiss. In these cases, training is on the understanding of such concepts as transitivity or reversibility of operations, these skills being involved in conservation problems according to Elkind (1967) and Wallach (1969). The child appears to be learning a logical argument which he applies, himself, without any specific instruction to do so, in the conservation situation. This changes his performance from a preoperational to a concrete operational one. It seems clear then, that at least on the point of transition, children may progress to the next stage as a result of training on appropriate aspects of the logical structures.

Some writers investigating the effects of training (e.g. Kohnstamm 1967, 1969) have done so not to examine the possible acceleration through stages, but because of a disagreement with Piaget about what is revealed by performance on the tests. In particular, concern has often been expressed (Braine and Shanks 1965a, 1965b; Ausubel 1965, 1968) that a child may be
failing because of inadequacies of language, either in his own verbal expression, or perhaps in understanding of the terms used by the experimenter. Sinclair-de-Zwart (1969) has provided experimental evidence that language is determined by operational level, rather than vice-versa. She demonstrates that training in verbal expression does not produce a change in performance on conservation tests of operational level. She says:-

"Verbal training leads subjects without conservation to direct their attention to pertinent aspects of the problem (co-variance of the dimensions), but it does not ipso facto bring about the acquisition of operations" (Sinclair-de-Zwart, 1969, p.35).

This evidence provides important justification for the clinical questioning techniques used in the great majority of Piagetian studies. As long as an experimenter is careful not to provide both the questions and their answers in a verbal form to the child, his detailed questioning of him will not be sufficient to change the child's level of operational understanding. It may well change his verbal expression, and direct his attention to aspects of the task which he has previously overlooked, but the ideas expressed should be of the same level, typical of the same developmental stage.

Turning next to the studies with a primarily theoretical orientation, it is clear that many have aimed principally at a replication of Piaget's findings in a particular area (Dodwell 1960, 1962, 1963; Lovell and Ogilvie 1960, 1961; Lovell 1959, 1961, 1965; Lovell, Mitchell and Everett 1962; Uzgiris 1964; Almy, Chittenden and Miller 1966). Some authors attempt, among other things, an extension of the notion of conservation to other concepts (Braine and Shanks 1965a, 1965b); a number have paid particular attention to whether there is a fixed order of development of
certain abilities within such areas as the different conservations (Goldschmid 1968); number conservation in particular (Wohlwill 1960), classification (Kofsky 1966), the concept of probability (Davies 1965), perceptual tasks and judgments about historical and moral situations (Peel 1959), and causal thinking (Laurendeau and Pinard 1962). Somewhat rarely has an investigation attempted to test whether different tasks said by Piaget to detect the presence of the same structure (in these cases, concrete operations) are in fact related, using the same sample of subjects (Smedslund 1964; Shantz 1967). Beilin (1965) has made an approach to this problem by an examination of the effects of training in bringing about greater "convergence" of performance, in subjects who previously had performed at operational level in only one or some of the tasks. On the whole the relationship of one Piagetian test to another (with controls for their separate relationships to age) has not been demonstrated convincingly.

To the extent that different conservation items form a scale of the Guttman type, they are clearly measuring something in common, but attempts to show the same type of consistency amongst classification problems (Smedslund 1964; Kofsky 1966; Shantz 1967) have been only moderately successful. A demonstration of the relationship of performance on tasks concerned with very different areas (say conservations and classification tasks) has apparently not been attempted, except by Smedslund (1964), but must be considered important for a theory which accounts for performance on each in terms of the same thought structures.

To conclude this overview, before proceeding to studies of direct relevance to the transition from concrete to formal thought, a few methodol-
ogical points deserve attention. Much of the argument in the literature, and apparent disagreement between findings, can be tied to differences in questioning and scoring procedures. Laurendeau and Pinard (1962), make an excellent analysis of studies of precausal thinking in this regard. They point out that whether or not a study reproduces Piaget's findings depends on whether or not there was departure from his method of collecting and analysing data. The changes made by those whose results disagree with his are all in the spirit of standardising and objectifying procedures. For example, there is firstly the decision to score only on the basis of an initial "yes/no" answer to questions, rather than taking into account the reasons and justifications given subsequently, by the child, in response to questioning; and secondly the tendency to regard each question as an independent "item" so that some sort of "total score" on all items will be the most valid index of the stage of thought. Laurendeau and Pinard argue that such "objective" procedures lead an investigator to ignore the subtleties and unreliabilities present in the answers (particularly of young children) to questions about the world. They maintain that, whether or not it is methodologically "impure", one must make allowances for the fact that some answers to questions will not be indicative of the level of thought at all (being merely stereotyped answers, or related to some chance event or observance which "sticks" in the child's mind), and also for the fact that a "yes" and "no" answer can usually each be given for either a precausal or a causal reason. Thus they argue that responses to further questioning must be taken into account. It would seem from these
kinds of considerations, that one must accept a clinical questioning approach and the attendant unavoidable subjectivity of the judgments made by the investigator. Those who favour more rigorous methods should probably be arguing not that Piaget's theory (based on his observations) is wrong and theirs right, but that such investigations are impossible to conduct, because no meaningful method of collecting data is available. The arguments of Laurendeau and Pinard (1962) are supported by Gruen (1966).

2.3 Experimental Evidence Relating to the Transition from Concrete to Formal Operations.

Although most of the experimental work on the transition from concrete to formal thinking has occurred in the nineteen sixties and in nineteen seventy, following the publication, in English translation, of "The Growth of Logical Thinking: from Childhood to Adolescence" (Inhelder and Piaget, 1958), Peel (1959) reports some work on the judgment of adolescents about historical and moral situations. This work is based on early investigations by Piaget (1928, 1932) into the developmental changes in judgments made by the child about situations depicted verbally by the experimenter. Studies along these lines have been continued by Case and Collinson (1962), Goldman (1965) and a number of other investigators whose work is reported by Peel (1966).

2.3.1 Experimental work using verbally presented material and purely verbal responses.

Lodwick (reported in Peel, 1959) devised a method of categorising
logical judgments about historical situations (e.g., King Alfred and the Cakes) into levels of intuitive, concrete and formal thought. The presentation method draws mainly on Piaget (1932), but the categorisation of answers is determined by the logical operations evident in the child's response. Thus an "intuitive" answer focuses on one aspect of the situation; a "concrete" answer combines single relations between concrete objects and is very much tied to the data; whereas a "formal" answer relies on the logical relationships between statements and is not distorted by their content. The analysis he makes is thus drawing on Piaget's (1928) conclusions that ..... there are, in our opinion, three genetically distinct types of mental experiment; that which we find in the child before 7-8, that which we find between 7-8 and 11-12 and finally, that of the adult. We also believe it necessary to point out that this third type of mental experiment is accompanied by an experiment which might be called "logical experiment" (Piaget, 1928, p.235).

The three stages are described by Piaget as follows. Before 7-8 years the child is said to reason "transductively", from one particular to another particular:--

"..... the judgments of children before the age of 7-8 do not imply each other, but simply follow one another, often the manner of successive actions or perceptions which are psychologically determined without being logically necessitated by each other. For transduction is nothing but a mental experiment unaccompanied by logical experiment. It is either a single account of events in succession, or a sequence of thoughts grouped together by one and the same aim or by one and the same action; it is not yet a reversible system of judgments, such that each will be found to have remained identical with itself after no matter what kind of transformation" (Ibid, p.237).
"After the age of 7-8, however, comes a stage lasting till about the age of 11-12, and during which the following fundamental changes take place. Little by little the child becomes conscious of the definition of the concepts he is using, and acquires a partial aptitude for introspecting his own mental experiments. Henceforward a certain awareness of implications is created in his mind, and this gradually renders these experiments reversible, removing at least such contradictions as are the result of condensation. Does this mean that the child is now fit to reason formally, i.e., from given or merely hypothetical premisses? We have shown that this is not the case, and that formal thought does not appear till about the age of 11-12. From 7-8 till 11-12 syncretism, contradiction by condensation, etc., all reappear independently of observation, upon the plane of verbal reasonings ... It is not until about the age of 11-12 that we can really talk of logical experiment. The age of 7-8, nevertheless, marks a considerable advance for logical forms have entered upon the sense of the mind in perception. Within the sphere of direct observation the child becomes capable of amplifying induction and of necessary deduction" (Ibid, pp.243 - 4).

The shortcomings still present in the concrete operational stage follow from the fact that mental experimentation is still tied to actual or possible events.

"Mental experiment is a reproduction in thought of events as they actually succeed one another in the course of nature; or again, it is an imagined account of events in the order which they would follow in the course of an experiment which one could carry out, if it were technically possible to do so. As such, mental experiment knows nothing of the problem of contradiction; it simply declares that a given result is possible or actual, if we start from a given point, but it never reaches the conclusion that two judgments are contradictory of each other ...... And these defects in mental experiment are the same as those which characterise childish reasoning, for the latter is content to imagine or to reproduce mentally actual physical experiments or external sequences of fact.

The logical experiment which intervenes from the age of 11-12 is certainly derivative from this process and has no other material than that of the mental experiment itself ... But this logical experiment which comes as the completion of mental experiment and which alone confers upon it the quality of true "experiment", introduces nevertheless, a
new element which is of fundamental importance, it is an experiment upon the subject himself as a thinking-subject .... It is, therefore an attempt to become conscious of one's own operations (and not only of their results), and to see whether they imply, or whether they contradict one another" (Ibid, pp.235 - 6).

Lodwick succeeded in locating a child's response in one of five stages (the three described above, plus two, transitional ones) with reliability coefficients between raters of +.67, +.78, +.62 for the three stories used. In an attempt to confirm Piaget's developmental stage sequence, subjects were categorised into three groups on three different criteria: (a) chronological age (b) mental age, derived from Raven's Progressive Matrices test and (c) total score on the three stories (each scored 0-4). Each of these three sets of criteria was then used as a basis for predicting the level of answer to each of the stories, and the percentage of errors taken as an index of departure from scalability in Guttman's (1944, 1950) sense. The coefficients of reproducibility were all low, being highest for total score (69.8%), next for mental age (64.2%) and lowest for chronological age (59.7%) as the criterion. The use of mental and chronological ages as criteria instead of total score may have introduced some anomalies in a Guttman scaling, but, despite this, it is clear overall that only a moderate amount of support is given by the data to Piaget's "stages": there is a good deal of inconsistency in the level of answers given to three different stories by an individual child. This however may be a result of the uncertainties involved in the categorisation of purely verbal responses to verbal descriptions of a situation. Comparable values of reproducibility coefficients, derived
in an identical way, were obtained by Goldman (1965) in analysis of responses to religious story data.

In a study which uses similar materials to those above, but attempts to make categorisation of the verbal responses more reliable, Peel (1966) specifies the form of logical argument which is used by the child in each of the stages, following Suppes and Hill (1964). One of the stories used in this study is quoted below, and the types of argument he distinguishes illustrated in reference to it.

"Jane is a very clever 15-year-old girl who is preparing for her final examinations. One evening, as Jane was doing her homework, her mother asked her to look after her younger brother, Teddy, while she went out. Teddy wandered from the living room into the kitchen, got hold of a jar of jam, ate a lot of it, covered his clothes with it, and spilled it on to some clean washing in a basket so that it was ruined.

Now answer the following questions:

(1) Was Jane a careless person? .......
(2) Why do you think so?..........." (Peel, 1966, p.78).

Five categories of answer were identified and they are described below, with examples, and Peel's analysis of the logical reasoning they exemplify.

Category a: Prelogical answers, characterised by "irrelevance (she's clever) and denial of terms (it's Teddy who was so careless)" (Ibid, p.79)

Category bi: Answers of the form "yes, because she let Teddy wander".

"The one piece of content, or circumstantial evidence, namely that Teddy wanders ...., is seized upon to support the single unqualified inference that Jane is careless. It is judgment in terms of content which is correlated with the statement implied in the question ..... Even though
it is limited, this judgment by content consists of a proposition. It is understandable that verbal material should give rise to propositional statements and, as we shall see, the change in quality of thinking as we pass up to higher categories is marked by increased inference, transformation, complexity and coordination of these propositions" (Ibid, p.82).

This form of answer is thus seen by Peel to be logically equivalent to the implication:

"Teddy wanders implies that Jane is careless" and the form of argument is modus ponens (Suppes and Hill, 1964) where "assertion of the antecedent leads to affirmation of the consequent" (Ibid, p.82).

Category bii: This category differs from bi only in the form of the propositions used. Their content remains the same. Answers are of the types "Yes, she should have watched Teddy"; "If she wasn't careless, she would have watched Teddy; or If she had watched Teddy he would not have wandered" (Ibid, p.79).

Logically, the argument has the form modus tollens (Suppes and Hill, 1964) where the subject "considers a denial of the consequences and infers a negation of the antecedent." (Ibid, p.83). A conditional form of argument is involved.

Category ci: Reference is made to extenuating circumstances and answers are of the form "Yes, although she was doing her homework, she should have watched Teddy". The answer "implies that doing homework and watching her brother are not incompatible and that watching would prevent him from wandering" (Ibid, p.83).
Category cii: An answer is given which takes account of possible extenuating circumstances (Importance of examinations, mobility of younger brothers, impossibility of the commission). Such an approach involves the combination of propositions and an attempt to reconcile them, resulting finally in the "recognition of incompatibility and a .... concentration upon extenuating possibilities" (Ibid., p.83).

In summarising the findings from a sample including American, English and Irish children, ranging in age from 9 to 15 years, Peel says:-

"..... we may note that at mean chronological and mental ages of 13-13½ years, the pupils from all three groups are still judging circumstantially, even though their judgment is expressed in the more sophisticated conditional form (Category bii). Generally, only by 14 years do the pupils show a firm tendency to make more circumspect and comprehensive judgments, by first realising the limited nature of the evidence available and the subsequent invocation of imagined alternative possibilities. This transition from partially descriptive to imagined explanatory responses marks the major change in the quality of the pupil's judgment during mid-adolescence" (Ibid., p.83).

Roughly speaking, the mean ages associated with responses in each of the categories appear to be as set out below:-

<table>
<thead>
<tr>
<th>Category</th>
<th>Mean age (Years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>10+</td>
</tr>
<tr>
<td>bi</td>
<td>12+</td>
</tr>
<tr>
<td>bii</td>
<td>13 to 13+</td>
</tr>
<tr>
<td>ci</td>
<td>13+ to 14</td>
</tr>
<tr>
<td>cii</td>
<td>14 to 14+</td>
</tr>
</tbody>
</table>

It is clear, however, that there is a good deal of overlap between the chronological ages associated with each category (standard
deviations were from 10 to 20 months) and Peel points out that the separation is much better in terms of mental ages, where these were available.

It is interesting that investigations of this kind have continued to validate Piaget's early descriptions of stages in the development of judgment. Undeniably the verbal presentation of story material, combined with the difficulties of categorising purely verbal answers, contributes to a reduction in the clarity of findings. However recent writers such as Lodwick, Rhys and Stones (unpublished works referred to by Peel, 1966) and Hooper (1968) have pointed to the need to "extend" the Piagetian stages to concepts in the humanities and social sciences, and themselves made attempts to do so. At present, work relies mostly on the categorisation of verbal answers, but Peel's (1966) attempt to specify the logical structure of verbal statements suggests further refinements along these lines. It is also possible that recent developments in psycholinguistics could assist in more meaningful categorisation of verbal material (e.g. on the basis of "deep structure", Chomsky 1957, 1965).

Rather than an "extension" of Piaget's theory to new areas, these investigations can be seen as a return to his earlier kind of analysis (Piaget 1928, 1932). The difference probably lies in the attempt to adopt a more "experimental" approach, following the example of Inhelder and Piaget (1958) who used a variety of scientific and mathematical problems. Probably a major reason for turning to these experiments was a desire to observe the child's method of approach to a problem, rather than relying only on a verbal statement of his conclusions. It is much easier to externalise and observe a problem-solving process in the
context of physical objects and their manipulation, than in one requiring purely verbal answers. Certainly, in Piaget's view, the method of investigation employed by the child is no less valuable an index of the stage to which he belongs than is the content of his conclusions. It has even been suggested by some authors that it may be a superior index:

"A persistent question ... (has) to do with the ways the child's thinking in the interview resembles or differs from his thinking elsewhere. In "The Child's Conception of the World", Piaget (1929) devoted considerable discussion to the safeguards that the adult must use in order to avoid suggesting certain answers to the child and encouraging him to perseverate. The more recent "Growth of Logical Thinking" (Inhelder and Piaget, 1958) implies that the child's own demonstration or experimentation provides a better index to the nature of his thinking than do his verbal responses to questions" (Almy, Chittenden and Miller, 1966, p.7)

The experiments reported by Inhelder and Piaget (1958) will now be described, since they constitute the most important empirical evidence supporting Piaget's theory of the transition from concrete to formal thought. Their importance lies, in particular, in the detailed interpretations of performance made in terms of the logical models of concrete and formal operational structures. Subsequent relevant experimental work by other authors will also be discussed in the next section.

2.3.2 Experimental investigations involving scientific, mathematical and logical concepts.

Inhelder and Piaget (1958) describe, as the aim of the first section of their work, to "analyse how children or adolescents at Stage III go about solving problems which appear purely concrete but which experiments indicate can be solved only at Stage III and which actually
By Stage III is meant the stage of Formal Operational Thought, comprising substages IIIA (from 11-12 years to 14-15 years) and IIIB (from 14-15 years onwards). The authors then give a detailed account of six experiments involving physical laws, accompanied firstly by a description, and secondly by an analysis in terms of Piaget's psychology, of performance in each stage and substage of development. To indicate the variety of experiments, and the specific logical relationships involved, a description of each problem, together with the operations necessary for solution, is given below.

**EXPERIMENT 1. The Equality of Angles of Incidence and Reflection.**

**Materials:** A kind of billiard game in which balls are launched with a tubular spring device that can be pivoted and aimed in various directions around a fixed point. They rebound from a wall to the interior of the apparatus.

**Stage of Solution:** IIIA, with adequate formulation only at IIIB.
EXPERIMENT

2. The Law of Floating Bodies.
Materials: Buckets of water, suitable objects, some having the same volume but different densities.
Stage of Solution: IIA; with more general formulation at IIB.

3. Flexibility of a Rod
Materials: Rods whose lengths can be varied, made of different materials, and with "weights" (dolls) which can be put on the end. The rods also vary in thickness and in the nature of cross-section. The rods are held in clamps at the edge of a tray of water and flexibility is measured by whether or not the rod will bend to meet the water, with a given weight on it.
Stage of Solution: Partial solution at IIIA, complete at IIB.

OPERATIONS NECESSARY FOR SOLUTION

Equivalence (p = r) where "p" is the assertion that a given object floats and "r" is the assertion that it is heavier than its equal volume of water.

Negation of implication (p\Rightarrow q), where "p" is as above and "q" is an assertion that the object is light (in terms of absolute weight).

There are "compensations" involved, e.g., other things being equal a thin steel bar has the same flexibility as a thicker brass bar. This can be written:

p, \neg q \Rightarrow x  \quad \text{and}  \quad \neg p, q \Rightarrow x

where "p" is an assertion of thinness, "q" is an assertion of steel composition and "x" is an assertion of a degree of flexibility. Together these amount to Reciprocal Exclusion (pvq)\Rightarrow x.

The method of holding all other variables constant, in order to study the effects of one, is essential.
EXPERIMENT

4. The Oscillation of a Pendulum.

**Materials:** A string whose length may be varied, a set of weights which may be suspended on it, from a rod which is mounted on a backing board.

**Stage of Solution:** The role of the change in the length of string (Equivalence) is discovered Formally at IIIA, but the exclusion of other factors (Complete Affirmation) not until IIIB.

5. Falling Bodies on an Inclined Plane.

**Materials:** A pegboard allowing an inclined track to be raised or lowered to different heights, marbles of varying size which can be rolled down the track, and a device at the bottom for measuring the distance travelled.

**Stage of Solution:** IIIB.

OPERATIONS NECESSARY FOR SOLUTION

**Equivalence** \((p=x)\) where "p" is an assertion of change in the length of the string, "x" is an assertion of change in the frequency of oscillation, **Complete Affirmation** \((q*x)\), where "q" is an assertion of change in the weight, or in the dropping height, or in the push given; "x" is as above.

**Disjunction** "(avb)" in a more complex form, where, in order to isolate the variable of height of dropping point, the subject must relate a conservation of height \((r_o)\) to the disjunction of two other variables, slope \((p)\) and distance along the track \((q)\). Thus

\[(p,q) v (p_o,q_o)\]

**Equivalence** \((r=x)\) where "r" is an assertion of change in the height, "x" an assertion of change in distance travelled.
EXPERIMENT

6. The Role of Invisible Magnetisation.

Materials: A metal bar is attached to a nonmetallic, rotating disc.

When spun the disc stops with the bar pointing to one pair of boxes instead of any others around the disc. The crucial pair contain magnets concealed in wax. The disc is placed on a board divided into equal sectors of different colours. The boxes, which differ in weight, and all contain wax, may be moved from one sector to another, but matching pairs must always be put opposite.

Stage of Solution: IIIB

The second section of Inhelder and Piaget's (1958) work is devoted to experiments which bring out structural characteristics ("group" and "lattice" properties) of the sixteen binary operations, rather than emphasising the type of binary operation required for solution, as in section one described above. The experiments of the second section will now be described, combining some experiments because they illustrate the same structural characteristics of the operations.
EXPERIMENT

7. Combination of Coloured and Colourless Chemical Bodies.

Materials: Four similar flasks containing different liquids (perceptually identical). A smaller flask, labelled "g". Two glasses are shown to the subject. One contains liquid No. 2. One contains liquids 1 and 3. The experimenter adds "g" to each, and the one containing 1 + 3 turns yellow, the one containing 2 remains colourless. The subject is asked to reproduce the colouring effect.

Stage of Solution: IIIA

S. The Conservation of Motion in a Horizontal Plane.

Materials: A spring device launches balls, of differing volumes and weights, along a horizontal track. The subject has to predict and explain the stopping points.

STRUCTURAL PROPERTIES INVOLVED

The ability to generate all possible combinations of the liquids is required. This derives from the lattice structure of the 16 binary operations. (The operations which need to be understood are:-

Complete Affirmation (p*x)
Reciprocal Exclusion (pvvx)
Conjunction x-(s,t,g), where "p", "q", "s", "r", "g", are the liquids, "x" the colour.

Reversibility by inversion (a property of the group structure) must be understood for solution. Once the subject has established, by observation, the implication:-

p-(qvrvsvt), he must infer by negation:-

\overline{q}.r.\overline{s}.\overline{t}.\overline{p}. ("p" is the stopping of the ball, and "q", "r", "s", "t", "g")
EXPERIMENT

Stage of Solution: IIIB (IIIA subjects see that they have to explain why it stops, rather than why it moves (Stage II), but fail to achieve conservation of motion).

10. Equilibrium in the Hydraulic Press

Materials: Communicating open vessels containing water for 9; and a closed hydraulic press system (also containing water) for 10. The levels of water in each arm (equal for 9, unequal for 10 depending on the weight of the piston) must be explained by the subject.

Stage of Solution: IIIA, with the principle generalised further at IIIB.


STRUCTURAL PROPERTIES INVOLVED

"t" are factors such as friction, air resistance etc. He must infer that it would NOT STOP ($\overline{P}$) if it were not for the operation of these factors.

In both of these tasks, the essential ability for solution is the co-ordination of the two forms of reversibility (by inversion, N, and reciprocity R) of the INRC group. The equilibrium reached by the liquids, "$r"", must be explained by the equality of two pressures, "$p" and "$q" and their counterbalancing pressures "$\overline{p}$" and "$\overline{q}$".

$$r = x (p_v q) = y (\overline{p_v q})$$, where "x" and "y" represent the magnitudes of the pressures represented by (pvq) and (pvq) respectively.

In each of these experiments, the schema of proportionality, derived from the INRC group, is essential.
EXPERIMENT

13. The Projection of Shadows.

14. Centrifugal Force and Compensations

Materials: 11. A simple balance on which a number of different weights may be hung, at different distances from the fulcrum. 12. An inclined track of fixed length, with a car (to which weight may be added) to be hauled up by a counter weight. The vertical height to which the track is raised may be varied. 13. A number of rings of different sizes, a light source and a screen on which shadows may be thrown. 14. A variable-speed, rotating disc on which balls may be placed at different distances from the centre.

Stage of Solution: 11. IIIA.

12. IIIA, but the law is only stated at IIIB.

13. IIIA, but the law is only generalised to all possible cases at IIIB.

STRUCTURAL PROPERTIES INVOLVED

for solution. Since \( NR = IC \) and \( RC = IN \) and \( NC = IR \) it follows that

\[
I_x = R_x \quad \text{or} \quad \frac{R_x}{N_x} = \frac{C_x}{N_x}
\]

(where \( x \) is the operation transformed by I, N, R, C).

These logical relationships between proportions are applied to the specific variables involved in each of the experiments.

11. \( W_1 = \frac{L_1}{L} = \frac{H_1}{H} \) where \( W \), \( L \), and \( H \), are the weight, distance and height of arm 1; \( W_1 \), \( L_1 \), \( H_1 \) similarly for arm 2.

12. \( W = h \) where \( W \) is the counter weight, \( M \) the hauled weight, \( h \) the vertical height and \( H \) the length of the track.

13. \( r_0 \circ (p,q) \vee (p,q) \), where "\( r_0 \)" is conservation of size of shadow, "\( p \)" is an increase in diameter of ring, and "\( q \)" is an increase in the distance of the light source from the ring.

14. \( r_0 \circ (p,q) \vee (p,q) \) where "\( r_0 \)" is the same time of leaving the disc, "\( p \)" is an increase in weight, "\( q \)"
EXPERIMENT

14. IIIA, but real quantification only at IIIB.


Materials: Faces, with two colours of hair, two colours of eyes represented. The subject has to decide whether there is a correlation between the two attributes.

Stage of Solution: IIIB.

In order to infer the level of thought from protocols of subjects performing these experiments, Inhelder and Piaget rely on two aspects of the performance. The first of these is the method of experimentation and manipulation of variables used by the child. It is said repeatedly, in connection with a number of the experiments, that the method of "all other things being equal" is not put into practice until Stage IIIB. In some situations a subject at Stage IIIA is able to make correct inferences, if the appropriate tests and experiments are set up for him to watch, but he is unable to devise these tests for himself (see, for example, the Pendulum Problem). With this limitation of Stage IIIA goes a lack of generalisability of findings from one specific set of conditions to others (see, for example, several of the Equilibrium and Proportionality problems, Nos. 9-14). Thus, although Stage IIIA is characterised by a realisation that careful experimentation is needed

STRUCTURAL PROPERTIES INVOLVED

is an increase in distance from the centre.

Equivalence \( p = q \), where "p" is one hair colour, "q" is one eye colour; involving an understanding of both:-

Reciprocity \( (p, q) = R (\overline{p}, \overline{q}) \);
\( (\overline{p}, q) = R (p, \overline{q}) \); and:-

Inversion \( (\overline{p} = \overline{q}) = (p \lor \neg q) \);
\( (p = q) = (\overline{p} \lor \neg q) \).
to decide between the logical possibilities, it is not until the later substage IIIB that this understanding can be put into practice. The distinction between Stage III as a whole and the preceding stage, Stage II, of Concrete Operational Thought, is that the situation is formalised into "variables" at Stage III, but not at Stage II. The Stage II child is content to observe events, classify and order them and "look" for any correspondences which may be there. He does not conceive of possible logical relationships and see the need to test for them experimentally.

The second aspect of performance which locates a subject in a stage is the content and nature of the conclusions he reaches, and the logical "routes" used to arrive at them. In some of the experiments, valid conclusions can be drawn by a concrete operational child, from his observations of correspondences. Thus, for example, in the Pendulum Problem, he can arrive at the correct role for the length of the string. He is also liable, however, to attribute the same role to any other variable (e.g. the weight) which he varies simultaneously with the length of the string - thus it is not a relationship deduced logically in the case of either variable, but merely an attempt to describe what he has seen. (Peel 1964) has called this "descriptor-thinking" and contrasted it with Stage III where "the induction of laws and formation of further concepts constitutes a bridge in thought between description and explanation" Peel, (1964, p. 102). At the stage of formal thought, then, a child is said to be using the evidence he obtains from experiments as a method of deciding between logical possibilities of which he is already
aware. There is some evidence, in the work of Inhelder and Piaget (1958), that certain logical relationships are easier for him to assess, or to arrive at, than others. For example, in the Pendulum Problem, the equivalence relationship is discovered at Stage IIIA, and the exclusions of other factors not until Stage IIIB. It is possible, then, that certain parts of the "complete lattice structure" of formal operations become usable before other parts — and this is an important qualification to be made to Piaget's insistence that what is acquired is the "structured whole".

The fact that there is improvement in the ability to make general statements, and to turn them into quantitative laws, from Stage IIIA to Stage IIIB of formal operations, shows that further development takes place after the transition from Stage II. Whether this development is simply increased sophistication and assuredness in the use of newly acquired structures, or a change in the nature of thought itself, is not well established. Piaget seems to imply the former, by saying that a child at Stage IIIA will reason in the same way, given pre-organised material, as the child at Stage IIIB. The only difference is in his ability to organise events appropriately to provide himself with a basis for making deductions. Such a difference could well be due to differences in the degree of experience in the practical application of thought to problems, but that it is really so needs adequate demonstration.

Investigations carried out subsequently by other authors have thrown some light on a few of the issues raised above. Lovell (1961)
repeated ten of Inhelder and Piaget's (1958) experiments (Nos. 3, 4, 5, 6, 7, 8, 10, 11, 13, 15) keeping their form as close as possible to the original ones, in an attempt to replicate the original findings.

Apparently some difficulties were experienced in constructing apparatus which behaved in the Genevan way, but the performances of the subjects (200, ranging in age from 8 to 18 years) were comparable with the performances typical of the stages described by the original authors. Since each of Lovell's subjects was tested on four of the ten experiments (a different four for different groups of subjects), his study was able to provide direct evidence that performances on different problems are related. Kendall's coefficient of concordance was used to assess the degree of relationship, and Lovell concludes:-

"......there is considerable agreement between the levels of thinking that the subjects display in the four experiments. Moreover, the value of the coefficient of concordance, $W$, declines as the population becomes more homogeneous with respect to mental age. Naturally there is no exact correspondence since the experiments and "intelligence" tests do not measure exactly the same thinking skills"(Lovell, 1961, p.149.)

In the same investigation, Lovell also examined the relative difficulty of each of the experiments used and concluded that :-

"......eight of the ten experiments may be regarded as samples drawn from the same population of experiments. The Correlations experiment is too easy for secondary, but not for primary pupils, compared with the other eight experiments; while the Projection of Shadows test placed too many subjects at Stage IIB" (Ibid, p.149).

Although, taken as a whole, Lovell interpreted his results as providing confirmation of Inhelder and Piaget's stages, there are a
number of experiments where he reported disagreement. For example, there were Stage II (performance) subjects who, on being questioned, could state the Law of Conservation of Motion in a horizontal plane in some form such as that the balls would "go on for ever", whereas this law is not understood until Stage IIIB according to the original authors. Lovell also commented that, on the Pendulum Problem, he "...found subjects who could serially order particular variables but drew the wrong conclusions. Perception seemed to be at fault in these instances" (Ibid., p.150). This last comment will be important in a discussion of some of the findings in the present study.

At some age levels, Lovell (1961) was able to compare groups of different educational level and ability, and he suggested that performance varied directly with these factors. He also surmised that the children tested by Inhelder and Piaget may have been rather more able than "the average population" since, for example, many of the less able 15 year-olds in his study were still at Stages IIA and IIB. These findings were borne out by Jackson (1965), who compared a group of forty eight normal children (I.Q between 90 and 110; four boys and four girls in each of the age groups 5, 7, 9, 11, 13, and 15 years), with a group of forty educationally sub-normal children (E.S.N. means with an I.Q between 60 and 80; four boys and four girls in each of the age groups 7, 9, 11, 13 and 15 years), most of the children in each group being from families of low socio-economic status. Each of Jackson's subjects was tested on six of Inhelder and Piaget's (1958) tasks (Nos. 2, 4, 5, 8, 9 and 11) and assigned to one of the stages IA, IB, IIA, IIB, IIIA, IIIB on each (except for three of the six experiments where a substage IA is not distinguished from IB). He reports that none of the
educationally subnormal subjects reached Stage III on any problem and that, in fact, 95% of the responses of these subjects fell in category IIA or below. In summarising his results he says:

"E.S.N. children showed very little increase in scores beyond the age of 9 and there was a suspicion of a deterioration between 13 and 16. Fifteen-year-olds achieved no higher level of logical thinking than did normal 8 - year olds" (Jackson, 1965, p.238).

Reporting on the performance of the sample of normal subjects, Jackson states:

"Apart from the failure of many older subjects to reach stage 3B, the responses of the normal sample generally confirmed the ages at which Piaget suggests certain levels of logical thinking are reached. But this study also indicates that wide variation in levels of logical thinking may exist among children of similar mental age" (Ibid., p.258).

In fact, in three of the experiments he used, no subject reached Stage IIIB; the experiment with the greatest number of Stage IIIB performances was the Pendulum Problem. Using a "logical thinking score", derived from performance on all six tests, the rank correlation obtained with chronological age was +.608; and that with Raven's Progressive Matrices score was +.859. There was no significant difference in performance between the sexes.

There have been a number of further studies, which depart somewhat from the experiments of Inhelder and Piaget (1958), but which add evidence about the development of formal operations. Rogers (1967) makes a distinction between the concept of historical time and a more general concept of integral time (which includes the former) and suggests that the former should appear at an earlier age than the latter. A study conducted
with 330 children in four age groups (mean ages 12.6, 13.6, 14.6 and 15.6 years) revealed no difference in performance between the four groups on a test of historical time, but a significant difference between the groups of average ages 13.6 and 14.6 years on the test of integral time. Other significant differences found on the test of integral time were between ages 12.6 and 14.6 years; between 12.6 and 15.6 years and between 13.6 and 15.6 years; the overall conclusion being that a major advance in understanding of this concept occurs at about the age of 14 years. Boys were found to perform significantly better than girls on the test of integral time, and this difference was attributed to their superior ability in numerical and spatial tasks. Rogers interprets the significance of his findings as follows:

"Hunt observes elsewhere that the factor analysts have not revealed the existence of Piagetian Structures and suggests that this may be so because their studies have not ranged across age groups and, therefore, that they were dealing with subjects who all shared the same basic structures (Hunt, 1961, p.257). Placed in such a context, the present findings could well be interpreted to show that the historical concept is a schema absorbed about a year later into a higher schema, the integral concept of time, and that, whatever the I.Q or age of the population tested, the sequence would remain the same. Finally, if the nature of intelligence is as Piaget's work implies, then the exploration of the more difficult concepts such as time could well play a part in the delimitation of intelligence of age levels beyond the 11 plus, which has hitherto been regarded as a watershed. Such speculations underline the need to extend this present type of enquiry to populations of lower I.Q and indeed, of greater age" (Rogers, 1967, p. 107.)

The schema of proportionality has been investigated by Pumfrey (1968), using one test taken from Inhelder and Piaget (1958), (Equilibrium in the Balance), and two additional tests. One of the additional tests made use of structured arithmetic material (the Cuisenaire rods), and the other required prediction of the amplitude and direction of the movement of one
pointer with respect to the other on a pantograph. Four boys and four girls at each of the ages 5 to 15 years were tested. Pumfrey reports very high split-half reliability coefficients for the three tests (+.960, +.928, +.953) and high relationships of performance on the tests to chronological age (+.562, +.684, +.898) and to mental age (+.615, +.739, +.918): figures in each case refer to tests 1, 2, and 3 in the order in which they were described above. Intercorrelations of performance on the three tests of proportionality were: Tests 1 and 2, +.401; Tests 1 and 3, +.599; Tests 2 and 3, +.627. Whereas all the correlations reported above reach the .01 level of significance, only that of Test 1 and Test 3 remains significant at the .05 level, when chronological age is partialled out of the inter-test correlations.

The order of difficulty of the tests was, from easiest to hardest, Test 2 (structured arithmetic materials), Test 3 (Pantograph), Test 1 (Equilibrium in the Balance, from Inhelder and Piaget, 1958). Thus despite the lack of correlation between the tests when age is partialled out, Pumfrey considers that they do all measure the schema of proportionality, and concludes that this concept may be taught to children younger than Secondary School age, providing that the most suitable materials are used.

Fischbein, Pampu and Monzat (1970) also suggest that proportionality may be understood before the stage of formal thought. Children in three age groups (5-0 to 6-4; 9-0 to 10-0 and 12-4 to 13-7) were tested on problems in which they had to choose, after inspection of both, the box from which they would have the best chance of drawing a marble of a specified colour (black or white). A number of different combinations of proportions
of white and black marbles in the two boxes were presented to each child. Uninstructed performance differed markedly between the age groups; instruction, however, was found to produce improvement in both the (12-4 to 13-7) and the (9-0 to 10-0) age groups, but not in the youngest, (5-0 to 6-4), group. It is on the basis of the improvement after instruction in the (9-0 to 10-0) age group that these authors recommend the teaching of proportionality to children of primary school age. They point out that Piaget's assertion that the schema of proportionality does not appear until the stage of formal thought is based on evidence gleaned only from the child's spontaneous responses. It should be noted, however, that their testing situation differed greatly from that of Inhelder and Piaget (1958) and that there is no guarantee that a child who succeeds in their choice task would be able to succeed on tasks such as the balance problem. In fact, their probable failure on the latter can be predicted from Pumfrey's (1968) finding, that the balance problem was more difficult than the two other tasks which he used. The point emerging from these studies is that a comparison of performance on tasks which appear to "measure" the same concept cannot be made, meaningfully, without an analysis of the precise cognitive abilities involved. To what extent the instruction of a child in the solution of a particular problem contributes to a real cognitive understanding can probably only be tested in terms of the transfer of this understanding to a new situation. In the same, or similar situation, the possibility that only limited rules for solution have been learned is very real. This is not to deny that instruction can be effective, but merely to point out that the implications of this for a theory of the development of thought structures
are by no means clear.

Whereas the group of experimental studies discussed to this point has concentrated on the replication of, or addition of information to, the stage by stage development of thought postulated by Piaget, there have been a number of studies concentrating on the final stage as such, that of Formal Thought. Questions have been raised by a number of authors as to the applicability of Piaget's logico-mathematical model to adult human thinking. Both Smedslund (1963) and Wason (1969) have suggested that formal operational thought "may be specific to problems with which the intelligent adolescent is customarily involved, but not a general property of cognitive functioning which can be applied to any problem whatsoever" (Wason, 1969, p.480).

Smedslund (1963) found that nurses did not evaluate correctly evidence for an equivalence relationship between the presence of a symptom and that of a disease, when presented (in a number of different ways) with frequencies of cases in the four possible categories. Rather than exhibiting formal operational thought, the performance of the majority of his subjects was much closer to that of subjects in Piaget's concrete operational stage. Most of the judgments, in one particular experiment, could be accounted for by saying that the degree of the perceived relationship depended solely on the absolute number of cases having both the symptom and the disease. He says:-

"It is concluded that adult subjects with no statistical training apparently have no adequate concept of correlation (based on the ratio of the two pairs of diagonal frequencies), and that, in so far as they reason statistically at all, they tend to depend exclusively on the frequency of ++ cases in judging relationship" (Smedslund, 1963; in Duncan, 1967, p.432)

Wason's (1969) conclusions need to be set against the background
of quite a large body of experiments on the solution of "conceptual
tasks." The work of Bruner, Goodnow and Austin (1956) revived interest
in a number of specific aspects of concept formation; the effects of
redundancy and the number of relevant and irrelevant dimensions (Bourne
and Haygood 1959; 1961; Archer 1962; Haygood and Bourne 1964), the
relative difficulty of conjunctive and disjunctive concepts (Hunt and
Hovland 1960; Neisser and Weene 1962; Wells, 1963; Haygood and Bourne
1965; King 1966; Wason and Johnson-Laird 1969; Seggie 1969), the
differential utilization of information couched in positive and negative
forms (Donaldson 1959; Wason 1959, 1961, 1965; Freibergs and Tulving
1961; Huttenlocker 1962; Wason and Jones 1963; Wales and Grieve 1969;
Chlebek and Dominowski 1970), and the processes of hypothesis testing
and elimination which can be seen to occur in the solution of problems
1963, 1965; Duncan 1964; Suppes 1965; Johnson-Laird and Tagart 1969;
Wason and Johnson-Laird 1969). The notion that hypothesis testing, or
the rules of logical deduction, can serve as an appropriate model for
the processes of human thought has been supported by such writers as
Henle (1961), Wason (1964) and E. B. Hunt (1962). The general trend of
findings has been, however, that human thought does not adhere closely
to correct logical principles. Henle (1961) suggested that the processes
of deduction were sound, but the initial adoption of premises at fault,
due to the influence of factors such as emotional involvement in the
issues concerned.

Inadequacies in the understanding of binary relationships of
implication, incompatibility and disjunction have been found in children (ages from 5+ to 11+ years) by Peel (1967). The situation he presented to the child was as follows:-

"The three kinds of binary proposition, implication, incompatibility and disjunction, were modelled in a game played between the experimenter and each individual child. The experimenter has before him a collection of many beads in a tin, there being at least 10 each of the four colours, blue, yellow, red and green. In another shallow tin is a similar collection of counters, also made up of some 10 each of the same colours. The experimenter and the subject face each other with their beads and counters before each respectively, and an empty box is placed on the table between them into which they make their play. The game consists essentially of the experimenter drawing a certain bead, and then the subject drawing a counter depending upon the rule of the particular game. The rules for each of the three games are briefly as follows:-

1. Implication: "We may each pick any colour we like, but in this game, if and whenever I draw a red bead, you also draw a red counter".

2. Incompatibility: "We may each draw any colour we like, but in this game, if and whenever I draw a red bead, you are not to draw a red counter.

3. Disjunction: "We both draw so that there is at least one red in the box between us. I shall draw first, and put my bead in, and then you draw a counter and put that in".

The box is emptied after each draw in each game and the counters and beads put back in their respective boxes. Ten pairs of draws are made with each child in a set random order consisting of six reds and four non-reds. The 10 bead draws are initial draws made by the experimenter, to which the child has to make the response by drawing an appropriate counter. The child's draws are recorded in relation to the initial draws made by the experimenter and so a complete record of all possible conjunctions of bead and counter draws is kept.

The game is also played in the reverse form in which the child is asked to make an initial bead draw and the experimenter then makes a consequent counter draw. The child is then asked to say whether the experimenter's draw is right or wrong. By this technique the investigator is able to
concentrate on the rarer occurring conjunctions made by the child in the binary games of implication, incompatibility and disjunction" (Peel, 1967, p.83.)

Peel's results indicated, in particular, that a relation of implication was misunderstood as one of equivalence. His conclusion was "It seems that the children reacted to the game instruction "If ....then" as if it meant "If and only when ......then" (Ibid., p.87). This replicates an earlier finding of Matalon (1962) and parallels some of the work by Wason (1968a, 1969) and Johnson-Laird and Tagart (1969) with adult subjects. Wason's (1968a, 1969) experiments have shown that adult subjects do not make correct choices of stimuli in order to test the validity of a relationship of implication, stated between two attributes (such as colour and shape) of the stimuli presented to them. Specifically, if the implication to be "investigated" is "If p, then q", a typical response is to check only the truth of p.q (and possibly of q.p since the order of appearance of the attributes is varied in the experimental procedure), neglecting to check the falsity of q.p. The need to check on the truth of p.q and q.p is not mentioned by the author.

Thus, apparently, the correct, "truth table" for implication is not available to adult subjects in Wason's experiment, nor was it (much less surprisingly) to the children tested on Peel's (1967) task. Wason concludes that "either Piaget's theory requires modifications because not all the highly intelligent subjects in this experiment gave evidence of thinking in terms of "formal operational thought"; or there is something about the task which predisposes some individuals to regress.
temporarily to earlier modes of cognitive functioning" (Wason, 1969, p.478). One aspect of the task possibly provides a clue to the anomaly, especially when it is linked with other evidence about the effect of the form in which verbal instructions are given (Johnson-Laird and Tagart, 1969). In Wason’s experiment, the "value" of one of the propositions (say "red" or "not red", i.e., $p$ or $\overline{p}$) appeared on one side of a card and the value of the other proposition (say "blue" or "not blue", i.e., $q$ or $\overline{q}$) appeared on the other side, not immediately visible to the subject. In order to discover which "cells" of the table were considered important for verification of the implication, the subject was asked to say which cards he thought it essential to turn over.

On some cards the $q$ value was initially uppermost, on others the $p$ value, and Wason reports that some subjects had difficulty in seeing "on the other side" as a "reversible operation". It was for this reason that some subjects, on being presented with an implication "If $p$, then $q$" to evaluate, did not feel that a card with $\overline{q}$ visible should be turned over, to ensure that the other side was $\overline{p}$. According to Wason, this difficulty reflects a basic lack of understanding of the "reversibility" of an operation such as $p.q$. By this he appears to mean that the operation $q.p$ is its "inverse", although this is certainly not the sense in which the term would be used by Piaget.

It could be suggested that the difficulty encountered is not a logical one, but one concerning the subject's interpretation of the instructions given. When presented with stimuli of which only one side
is visible and asked to consider the proposition that a red triangle implies a blue circle "on the other side", a subject may well take "the other side" to refer only to the one which he cannot see at present. The difficulty could then be seen as arising from a failure to understand the instructions. Indeed Wason and Johnson-Laird (1969) have found that wording the same problem as a disjunction "either not-p or q" makes it easier to solve than as an implication "if p then q". They say:

"It is as if the "either .... or" expression itself creates uncertainty. It breaks up the "direction" which seems to be strongly imposed by the conditional, "if .... then", sentence. With a conditional the individual is likely to be confident but wrong; with a disjunction he is more likely to be unconfident but right. The meaning of a conditional gives no hint of the negation or falsity which underlies its logic. The disjunction expression makes this element explicit, but this seems to weaken the grounds upon which any inference may be made. .... There would seem to be inherent difficulties in the concept of implication, but the words in which it is expressed also affect both its ease of understanding and the manner in which it is construed". (Wason and Johnson-Laird, 1969, p. 20).

A similar study by Johnson-Laird and Tagart (1969) lends additional weight to this conclusion. Four wordings of a statement of implication were presented to subjects and they were asked (indirectly) to fill in a truth table for the four binary propositions. The four wordings and predicted truth tables are shown below:-

<table>
<thead>
<tr>
<th>Sentence Type</th>
<th>Predicted Truth Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. If p then q.</td>
<td>p,q p,q p,q p,q</td>
</tr>
<tr>
<td>2. There isn't p if there isn't q.</td>
<td>T,F ?</td>
</tr>
<tr>
<td>3. Either there isn't p, or there is q (or both).</td>
<td>T,F T,T</td>
</tr>
<tr>
<td>4. There is never p without there being q.</td>
<td>T,F T,T</td>
</tr>
</tbody>
</table>

(Adapted from Johnson-Laird and Tagart, 1969, p.368).
The question marks are intended to convey that the subject would consider the truth value of these propositions irrelevant to the logical relationship in question. An example used for type-one sentences is the statement "If it is raining, I will go to the pictures". The authors claim that a typical response to this includes an interpretation that no indication of the intentions, should it not be raining, has been given. However, it could well be argued that a statement of "no knowledge" is really equivalent to an understanding that both are possible, or equally likely. It is not much of an extension to call this an assignment of "True" to each of the conjunctions. Johnson-Laird and Tagart (1969) do not interpret it in this way, however, and the fact that type-four sentences, as well as those of type one, are commonly assigned the truth table predicted for type one leads them to conclude that "undergraduates in the present experiment failed to treat them (conditionals) as any sort of truth-functional connective" (Johnson-Laird and Tagart, 1969, p.372). This study also confirms Wason and Johnson-Laird's (1969) finding that a disjunctive wording of the same implication results in more accurate assignment of truth values to the four conjunctions than a conditional wording. Most of the subjects gave the predicted, correct, set of values for type-three sentences. The authors conclude, in a similar way to Wason (1968) (a) and Wason and Johnson-Laird (1969):-

"The results showed that disjunction ('not-p or q') yielded the greatest number of classifications in accordance with the truth-values of implication. The remaining sentences ('if p then q', 'not-p if not-q', 'never p without q') were not classified in a truth-functional way; stimuli
were judged irrelevant when they falsified the antecedents of these sentences. The results would seem to raise some difficulties for Piaget's notion of the developmental level of formal operations" (Johnson-Laird and Tagart, 1969, p.373)

In view of the role attributed, by these authors themselves, to the form in which the problem is worded, it is difficult to regard their challenge of Piaget's theory as having very extensive ramifications. That such studies may be reflecting difficulties in verbal communication, rather than the weaknesses of formal thought, is confirmed by a number of recent studies in related areas mentioned earlier. The comparative difficulty of processing negative information has been shown to decrease with practice (Chlebek and Dominowski, 1970); and can perhaps be dismissed as an effect altogether, when the hidden differences in ambiguity and confusibility between positive and negative information are removed (Wales and Grieve 1969). It can be suggested that any task situation which attempts to instruct subjects by means of logical or propositional statements, and any involving negations in particular, will engender difficulties of interpretation; and that these difficulties alone, rather than inferred faults in logical thinking, may account for the results. Seggie (1969) has provided startling evidence of this kind on the relative difficulty of learning conjunctive and disjunctive concepts. He has argued that, when a subject learns a conjunctive concept (exemplified by the "positive instances") in a typical Bruner, Goodnow and Austin experiment, he also automatically learns a disjunctive concept (exemplified by the "negative instances"). If the claim that disjunctive concepts are more difficult to learn than conjunctive ones
is re-examined in this light, it appears that the only difference between them is in the labelling of "positive" and "negative" instances. Seggie (1969) replaced the words "negative" and "positive" "instances" with neutral category labels and showed that the difference between so-called conjunctive and disjunctive concept learning disappeared. This view of the task is much more consistent with Piaget's model of thought, since, in his theory, a binary proposition (such as a conjunction, p\&q) is always understood in relation to its negation (in this case, the disjunction, p\lor q); and in relation to the others of the possible sixteen binary propositions.

A more general, and possibly more serious, reservation about the relevance of studies reported in the foregoing discussion to Piaget's theory relates to a distinction made by Bruner, Goodnow and Austin (1956), E. B. Hunt (1962), Kendler (1964) and Crager and Spriggs (1969) amongst others. This is a distinction between concept identification or utilization and concept development or formation, although the terms are by no means used unambiguously. The kind of distinction envisaged is between a task which requires that a subject discover which one of a number of concepts, already familiar to him, is the "solution" to a particular problem; and a task which requires the development or discovery of a new concept, by the subject, for its solution. Thus many authors concerned with experimental work on conceptual tasks acknowledge that their theory (Hunt 1962) or tests (Crager and Spriggs, 1969) will have little to say about Piagetian work. Hunt observes:—
"Piaget, in his monumental research on the development of thought in children, has attempted to develop a logic system suited to the description of human mental processes (Piaget, 1957). He uses the term "concept" within this system. But he appears to mean something quite different from the meaning implied by the studies of concept learning, in American experimental psychology. For Piaget, a "concept" is an explanatory rule, or law, by which a relation between two or more events may be described (e.g. the concept of causation)." (B. B. Hunt, 1962, pp. 7-8)

Clearly, a developmental study, where the main comparisons made are between the capabilities of different age groups, will reveal different aspects of thinking from those revealed by a study of performance within any age group. J. McV. Hunt's (1961) view of the difference in aim has already been mentioned. It is important, however, not to neglect the relevance of studies such as those of Smedslund (1964), Peel (1967), Wason (1968) and Johnson-Laird and Tagart (1969) as clarifications of the range of performance which may be found, on a specified problem, at any given age (or, perhaps, at a given developmental stage). To see them as raising problems for Piaget's theory is questionable, however, when the experimental materials, and questioning procedures, have usually been quite different from his and in particular when the subject has been instructed to look for evidence of a particular relationship, in standardised, verbal (sometimes propositional), terms.

It can be argued that these features destroy the comparability of studies such as those discussed earlier with Piaget's work, in some important respects. Firstly, in those studies, the subject was tested on his ability to understand and "identify", a logical structuring of the situation which is communicated verbally, by the experimenter. In a
Piagetian experiment, stress would be laid on the way in which the subject isolated variables and structured the situation for himself. At least equal weight would be given to the kind of relationships which he attempted to test, as was given to whether or not his final analysis of the problem was correct. Thus although, in theory, specific instruction to look for a relationship of equivalence, or implication, should not alter the formal operational subject's tendency to consider these as part of a "structured whole", it may in fact have this effect. The leading and misleading effects of verbal statements of the problem, by the experimenter, appear to be very strong. Secondly, even if the form of verbal instruction given did not act to restrict the subject's "logical experimentation" with the problem, the nature of the materials presented may have tended to do so.

Wason's (1969) experiment, in particular, restricted itself to four presented instances only, which did little more than provide the subject with concrete tags for the verbal propositions about which he was asked to reason. It would seem preferable to provide the subject with a number of instances to categorise and examine (as in Smedslund's (1964) procedure), in order that his structuring of the situation into variables (represented by propositions) should derive from his own experimentation. While it is true that Inhelder and Piaget (1958) claim that formal operational thought dictates a scientific method of investigation of variables, they do not go so far as to say that instruction to look for an implication will evoke the appropriate truth table, immediately, in the subject's mind. They concentrate more on a demonstration that he knows how to control variables and to experiment adequately in order to discover what
the form of the relationship is.

Perhaps the most alarming feature of some studies which question Piaget's theory of formal thinking is the lack of a validation task taken from the work of Piaget and Inhelder (1958) themselves. Without evidence that subjects perform at a formal level on one or more of these latter tasks, information about their performances on tasks which appear to relate to formal thinking is of limited use. Apart from the study of Neimark (1970), to be discussed now, authors seem to have been wary about making predictions from Piagetian to markedly different problem-solving tasks. Studies have commonly employed a task whose logical structures seem intuitively to locate it as a "formal operational problem" and have made minimum attempts to state, in detail, the theoretical connections or to verify them empirically. Neimark's (1970) approach is in contrast with this, in that she takes a problem-solving task (see Neimark and Lewis 1967) which cannot be described as involving "implications", or "disjunctions" or any other immediately obvious logical relationship, and tries to discover what difference a formal operational approach will make. She includes two of Inhelder and Piaget's (1958) tasks as validation of her findings.

Neimark (1970) describes the need for correlational studies as follows:

"....although, for example, Inhelder and Piaget (1958) report the results of a number of studies of tasks assumed to be diagnostic of formal operations thinking - which may or may not have been on the same S's - there is no evidence of the intercorrelations among these tasks which would support the assumption that they do, in fact, reflect a
coherent structure. Recently Shantz (1967) and Goldschmid (1967) have reported data which support the assumption of intercorrelation among tasks reflecting logical multiplication and conservation, respectively. There are, as yet no available data concerning the more abstract, formal operations level of thinking. The present experiment represents an attempt to fill that gap by providing evidence on the intercorrelation among two Piaget formal operations tasks (correlation and the chemistry experiment) and a problem solving task which seems to get at formal operations thinking" (Neimark, 1970, p.223).

While her review of the situation seems to neglect the correlational information provided by Peel (1959), Lovell (1961), and Pumfrey (1968), it is true that these studies constitute only limited empirical support for the interrelationships implied by Piaget's theory. Thus her investigation does represent a much needed addition to previous work in this vein, but it is felt by the present author that a more significant contribution is her attempt to relate performance on two of Inhelder and Piaget's (1958) tasks to performance on a rather different problem-solving task. Neimark's "problem" requires that the subject decide which of eight alternative patterns of binary elements (circles which may be black or white) is the one covered up by movable shutters on a board. In this task (which she refers to as PS and which is described in detail by Neimark and Lewis (1967)) four scores are available which reflect the subject's information-gathering strategy, in addition to two time scores and a rating of the subject's own verbal description of his strategy. The subject has an answer sheet, showing the eight alternatives amongst which he must decide, and he indicates which elements of the covered pattern he wants to see. The problems are arranged so that elements differ in the amount of information they convey. Half of the elements
(referred to as "safe moves") will eliminate half of the patterns which are possible solutions. The other half will eliminate, at best, only one pattern from further consideration, and will possibly give no information at all (referred to in the latter case as "O moves"). In Neimark's (1970) experiment, each subject solved eight such problems, and the strategy scores obtained were as follows:-

**PS, Strategy:** The sum of the expected informational outcomes for each move, divided by the number of moves - maximum possible score, 8.

**PS, O Moves:** The number of shutter openings which yielded no information.

**PS, Safe first:** The number of initial shutter choices (out of eight) on which the subject chose an element which eliminated half of the patterns.

**PS, Ideal:** The number of problems solved by a series of three shutter openings, each of which eliminated half of the remaining possible patterns.

The two time scores were:- Time I (to the first shutter opening and Time II (to completion of the problem).

The subject's verbal description of his strategy was scored as follows:- "0, no plan or an irrelevant one; 1, at the point at which two possible answers remained, comparing the two patterns and selecting that shutter with respect to which they differed; 2, a plan for the last move plus one subsequent (? antecedent) one; 3, selecting on each move in such a way as to halve the alternative possible solutions" (Neimark, 1970, p.227. Suggested substitution of antecedent for subsequent by the present author).
The two tasks taken from Inhelder and Piaget (1958) were scored in a similar manner to the PS task (relying mostly on the subject's procedure, rather than on the content of his conclusions). Neimark's (1970) description of the scoring methods cannot be abbreviated and so will be quoted in full. This is done to enable comparison, subsequently, with some of those used in the present study. The letters CH and CO are used to refer to the chemistry experiment (No. 7 of section 2.3.2) and the correlations experiment (No. 15 of section 2.3.2) respectively. The numbers 1, 2, 3, 4, and the letter "g" are used similarly in this account and the account of CH in section 2.3.2, but in the case of the CO problem, different materials were used by Neimark (1970) from those of Inhelder and Piaget (1958). Neimark's (1970) correlation task concerns the relationship between the presence or absence of germs and the presence or absence of a disease, a problem very similar to that used by Smedslund (1964). The problem was presented in the form of a number of cards, bearing faces, with indications of disease or no disease, and germs or no germs. The subject was asked to look through the cards and decide "whether or not green germs cause the spot disease" (ibid., p. 225). The scoring systems for the two tasks were as follows:-

"We initially planned to use the scoring system of Wynn (7) for the CH task, but had to modify it slightly. Scoring was as follows: O-, does not use G each time until reminded and then produces purely random combinations; O, purely random combinations of one or more chemicals with G; O+, random combinations (as above) but with occasional evidence of primitive order (eg. G + 1 + 2 + 3 + 4); 1, primarily random combinations, but with evidence of keeping track of past trials (by taking notes or absence of duplications); 2, systematic pairing of each single ingredient with G but no plan beyond this; 3, systematic
pairing of each single ingredient with G followed by systematic testing of pairs of ingredients; 4, procedure as in 3 but going on to test for uniqueness of the combinations. For the CO task three aspects of performance were rated: sorting through the data, and response to each of the two subsequent questions. Scoring for sorting through the deck initially was as follows: 0, no systematic procedure so that S cannot even tell where he began (eg, putting the top card on the bottom and proceeding in this fashion); 0, no sorting but looking at each only once; 1, sorting into two piles (either healthy vs. sick or germs vs. no germs, or some combination of these vs. all others); 2, sorting into four piles in a row; 3, sorting into four piles arranged in a 2 x 2 contingency array. The deck produced in answer to the request to remove the cards which "don't fit" and to produce a deck which shows that germs and disease are related was scored as follows: 0, for the four cards illustrating sickness and germs combined; 1, for the four showing sickness and germs plus the four showing health and no germs; 2, for all else. Similarly, the deck produced in response to the request to assemble instances showing that germs and disease are unrelated was scored: 0, for either of the two discrepant instances (germs and health or sickness and no germs); 1, for both of the discrepant pairs; 2, for a deck containing two instances of each of the four possible combinations. Total score was the sum of the three ranks" (Ibid., p.226).

Neimark's sample consisted of 61 subjects, with a mean chronological age of 140.20 months, and standard deviation 10.45 months. The mean IQ was 112.13, with standard deviation 10.19; and the mean mental age 157.23 months, with standard deviation 18.79 months. The correlations she obtained between scores on the three "formal operations" tasks were very disappointing. In particular, CO and CH were not correlated at all, r = 0.08. High correlations (most reaching the .01 level of significance) were found between the various measures on the PS task, but since they all show similar patterns of correlation with measures on the two Piagetian tasks, and with CA, MA and IQ, only one
will be reported here in a summary table. This one is "PS, verbal", based on the subject's own statement of his strategy.

Table 1 below presents some of the correlations obtained by Neimark (1970). It is a section of her Table 3 (Ibid., p.229).

<table>
<thead>
<tr>
<th>Measure</th>
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<th>4</th>
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<th>6</th>
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<tbody>
<tr>
<td>1. PS, verbal</td>
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<td>2. CO</td>
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<td>3. CH</td>
<td>.07</td>
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<td>4. CA</td>
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<td>.15</td>
<td>.12</td>
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<tr>
<td>5. MA</td>
<td>.30*</td>
<td>.08</td>
<td>.31*</td>
<td>.59*</td>
<td></td>
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</tr>
<tr>
<td>6. IQ</td>
<td>.24</td>
<td>.04</td>
<td>.29*</td>
<td>.07</td>
<td>.81**</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Correlations obtained by Neimark (1970) between performance on formal operational tasks (PS verbal, CO and CH scores) and CA, MA and IQ. * indicates significance at the .05 level, ** at the .01 level.

The lack of any correlation between CH performance and performance on the other tasks (CO and PS) is taken as showing that the chemistry experiment (CH) is a poor measure of formal operations, rather than as evidence against the notion of a structure of formal thought. Neimark says:-

"We have the impression that the properties of the CH task provide an irresistible stimulus to manipulation and that this incitement to a more primitive level of operation overcomes any problem-solving set which a child might normally bring to the experimental situation" (Ibid, p.231).

No such difficulty with the task was encountered by Dale (1970), who has investigated the possibilities of presenting it in two new forms,
one involving keys in a lock and one a paper and pencil test.

The correlation of CO with PS measures, other than the one reported in Table 1 above, is consistently about .25, reaching the .05 level of significance. CH does not correlate with any of the PS measures. Neimark also reports that the PS task appears to be easier than either CH or CO and that, in fact, no children in the study attained the concept of correlation (i.e., none would be at Stage IIIB on Inhelder and Piaget's (1958) criteria). Such differences in difficulty of the tasks, combined with the restricted ranges of possible scores on each, would partly explain the low correlations obtained. Neimark suggests also, in explanation, that the whole set of sixteen binary operations has not been adequately sampled by the two Piagetian tests used. A further comment which might be made is that her PS task, as it stands, is capable of revealing only methodological aspects of a subject's performance (there is no content for him to structure and "explain"). Because of this fact, and the bias of the scoring systems on the two Piagetian tasks towards methodology also, the study may be neglecting important aspects of the nature of formal thought. To overcome this objection, a way of introducing some content into the PS task would have to be devised, and something closer to Inhelder and Piaget's (1958) qualitative stages of performance used to describe the results on all three tasks.

2.3.3 Summary of the experimental findings and some methodological considerations.

Taking the experimental findings reported in the previous section
as a whole, there is considerable uncertainty about the nature of the transition from concrete to formal thinking, and about the performance which can be expected from the "formal thinker" in various experimental situations. While no author goes further than saying that his results "raise problems for", or show that a particular test "may not be a good measure of" what Piaget means by formal operational thought, it is clear that a good deal of confusion exists with regard to the relationship of theory to findings.

This is due, at least in part, to lack of clarity about the exact nature of a "stage", and about what happens during the transition from one stage to another.

Piaget's logical models describe the structures to which the thought of the child "tends" in each stage, and his substages within stages represent degrees of approximation to the final equilibrium. To some extent he has also spelled out implications for the successive acquisition of concepts during the same operational stage. This is particularly so for the so-called "horizontal décalages" of conservation problems, and to some extent can be found in the correct handling of equivalences before exclusions and disjunctions at the level of formal operational thought. The form in which a problem is presented to the child is also acknowledged to affect its difficulty (see for example a number of the equilibrium problems in Inhelder and Piaget (1958)).

Recently, Flavell and Wohlwill (1969) have attempted to give a precise account of the types of changes occurring during transition
from one stage to the next, and to say what implications their account has for correlations between different tasks in particular. They point out firstly that, if a study limits itself to children either well below or well beyond a transition period, the consistency of performance on different tasks (which test for attainment of the second stage involved) will be a foregone conclusion: failure on all tasks for the first group, and success on all tasks for the second group.

Flavell and Wohlwill (1969) thus consider that, to a certain extent, consistency of performance across tasks can be taken for granted, but they do point out that they restrict this to tasks measuring exactly the same kind of concept (eg. different conservation tasks or different classification tasks). They do not consider that Piaget needs to show that the different structures of concrete operations, for example, all develop in synchrony, but argue that, separate, although parallel, developments of conservation and classification structures could be envisaged (or even separate development of each of the different concrete "Groupings" of operations on classes).

They then argue that, within a single structure,

"... we can expect to find departure from intertask consistency during the transition period. For it is precisely during this period in which the newly emerging structures are in process of formation that the child's responses may be expected to oscillate from one occasion to the next, to be maximally susceptible to the effects of task-related variables, and accordingly to evince a relative absence of consistency" (Flavell and Wohlwill, 1969, p.95).

They point out that Piaget has, at least implicitly, recognised that there is a period of stabilisation of the new structure at the beginning of each stage; and they see his notion of the "horizontal décalage" of
different problems as designed to account for the effects of "task-related variables."

The emphasis which these authors place on the transition phase from one stage to the next, as the appropriate place to investigate relationships between different tasks, reflects the spirit in which the present study was undertaken. Their detailed model of the transition period (which they see as a necessary extension and elaboration of Piaget's model) provides a rationale for the present investigation. Although Flavell and Wohlwill's (1969) analysis was not available when this study was designed and undertaken, it will be used now as a formal account of the aims of, and needs for, studies of this kind.

Research conducted by Nassefat (1963) and reported by Flavell and Wohlwill (1969) provides the foundation for the latter authors' model of transitional periods. Nassefat tested 150 subjects ranging in age from 9 to 13 years, on a total of 48 items, measuring a wide variety of concrete and formal operational tasks. Each item was classified as concrete (C) or formal (F) according to the nature of the operations required for its solution, although roughly one-third of the items had to be regarded as in an intermediate (I) category, because of ambiguities or inconsistencies in the responses of subjects to them. The important part of Nassefat's analysis, from Flavell and Wohlwill's (1969) point of view, is his analysis of the scalability of items separately for each age level and for each of the three item categories C, I and F. They report, referring to his results -
we find consistency generally highest at the age level at which the discriminative power of each item category is maximal, i.e. at age nine for the C items, at age eleven for the I, at age twelve for the F. (actually, the consistency of items in the F category never exceeds .25, apparently reflecting the fact that even at the oldest age level only a minority of S's passed them)" (Flavell and Wohlwill, 1969, p. 97).

Thus the scalability of the items is investigated, not as a means of demonstrating a developmental sequence of tasks, but as a means of studying their interrelationships at given points of development. The way in which the relationships between items change from age to age is the main interest of the findings. Nassefat's point of view is that, only when a certain amount of stabilization of the stage reached by the child has occurred, will consistent relationships between tasks be found. Flavell and Wohlwill (1969) point out that an analogy can be made with the scaling of attitudes, where a Guttman scale is normally obtained only for subjects with stable, well-articulated attitudes on the issue concerned.

Taking Nassefat's results as their major justification (while expressing reservations about the validity of his statistical treatment of data), Flavell and Wohlwill (1969) propose that three parameters may be used, which jointly determine the performance of a child in transitional periods. These parameters are as follows:

\[ P_a \] -- "the probability that the operation will be functional in a given child" (Ibid., p.98). This refers to one of two determinants of the child's performance in a cognitive task; the rules, structures or "mental operations" necessary for its solution.
\(P_b\) -- "a coefficient applying to a given task or problem, and determining whether, given a functional operation, the information will be correctly coded and processed" (Ibid., p.98).

\(k\) -- "a parameter expressing the weight to be attached to the \(P_b\) factor in a given child". (Ibid., p. 98). \(P_b\) and \(k\) are thus both parameters relating to the second determinant of the child's performance; the skills and mechanisms necessary for processing the input and producing an appropriate output, for the specific task concerned.

\(P_a\) is taken as being a probability, in literal terms, of the functioning of an operation. Its value changes, with development, from 0 to 1 and, at intermediate stages, a value of 0.4 (for example) implies that the operation will sometimes be used, more often not.

\(P_b\) is a value (again between 0 and 1) attached to the task itself, and this value is determined by a large number of factors known to be related to "task difficulty". \(P_b\) is thus roughly the probability that the problem in question will be solved "by the child".

The parameter \(k\), is introduced, by raising \(P_b\) to the power \((1-k)\), in an attempt to weight the difficulty of the task appropriately for a given child. By "a given child" Flavell and Wohlwill (1969) appear to mean a child of a given age, not a specific individual. The value of \(k\) is also said to change from 0 to 1 with the progressive establishment of the new operational stage. Thus the influence of \(P_b\) (raised to the power \((1-k)\)) decreases and eventually becomes zero.

The description given by Flavell and Wohlwill (1969) of the transitional process is thus in terms of an equation, whose parameters
change in the way outlined above. They say (referring their discussion to the transition from preoperational to concrete operational thought):-

"We may now write the equation

\[ P(+) = P_a \times P_b \]

(1-k)

to show the probability of a given child, characterized by particular values \( P \) and \( k \), solving a task with some particular value of \( aP_b \). The course of the formation of a new cognitive structure may accordingly be described in terms of a four-phase process:

Phase 1: In the initial phase, \( P = 0 \), i.e. the child, lacking a given operation, must fail all the problems demanding that operation. This corresponds clearly to Piaget's stage of preoperational thought (designated as Stage 1 in his writings).

Phase 2: In this transitional stage, \( P \) changes from 0 to 1.0, while \( k \) is assumed to remain equal to 0, or close to it. (This would seem to be a reasonable assumption, not only because the role of situational and task-related variables would be expected to be maximal during the period in which the operation is still in process of becoming established, but also because the abstraction of relevant information is necessarily dependent on the establishment of the operation.) Thus \( P(+) = P_a \times P_b \). This means that for the most part, during this phase, the child will still fail most tasks based on the operation, for instance, if \( P = .5 \) (e.g. in the middle of this phase), and for a task of medium difficulty for which \( P_b = .5 \), then \( P(+) \approx .25 \). Thus in terms of any criterion of operation-level responses, the child at this phase must still be considered preoperational. He should, moreover, manifest the kinds of oscillations and intermediary forms of reasoning characteristic of this transitional period, which Piaget designates by II-A and II-B.

Phase 3: This is the period of stabilization and consolidation of the operation, which has now become functional, i.e. \( P \leq 1.0 \). At the start of this period, however, success will still vary with the demands placed on the subject by the particular task, i.e. the value of \( P \) for that task. During the course of this phase the contribution of this factor progressively decreases, i.e. \( 1-k \) decreases from 1.0 towards an ideal state of 0. This would seem to correspond most closely to the stage designated by Piaget as III-A.
Phase 4: This is the terminal phase of unequivocal success, in which $P^*_b = 1.0$ and $k = 1.0$ i.e. $S$ is able to bring the operation to bear on the problem successfully, regardless of the situational and task variables involved. In practice one may presume that $k$ always falls somewhat short of 1.0, even at the most advanced levels; but unless $P^*_b$ is very low (i.e. task difficulty very great, as in a task of ordering a series of tones, for instance), the net result will still be an expectation of success, $P(+)$, close to 1.0" (Ibid., pp. 100-101)

After a general discussion of the plausibility and implications of the model, the authors turn to an examination of the expected relationships between tasks during a transitional period. Their account is:

"Specifically, the picture with respect to the interrelationship among operationally equivalent tasks now shapes up as follows: In Phase 1 there is failure across the board: correlations may thus be expected to be low, for lack of variance. During Phase 2, correlations should be low, due to the oscillations and inconsistencies in response implied by intermediate values of $P$. Towards the end of this phase and the beginning of Phase 3, consistency should become more apparent, with items with equivalent $P^*_b$'s being passed about equally often, and with items with discrepant $P^*_b$'s exhibiting a Guttman-type pattern. Finally in Phase 4 (the terminal phase) there is success across the board, so that inter-item correlation should again drop near to zero" (Ibid., pp. 105-106).

This model, presented by Flavell and Wohlwill (1969), has been quoted extensively here because it provides a remarkably apt foundation for the present study. Although the authors refer their discussion exclusively to the transition from preoperational to concrete operational thought, it requires no adaptation to become an account of the later transition from concrete to formal operations. In talking of the kind of experimental work for which their model calls, they say:

"This formulation, incidentally, points up the need for studies such as Nassefat's and Uzgiris's, in which, rather than being lumped together over an extended age range, these interrelationships are studied separately within fairly narrowly defined age groups" (Ibid., p.106).
The work to which they refer, by Uzgiris (1962, 1964), was in fact reanalysed by the authors to provide direct evidence for their model, and the reanalysis presented as an appendix to their paper (Flavell and Wohlwill, 1969, p. 116).

The investigation to be described here conforms to the recommendations made by Flavell and Wohlwill above, although, as mentioned previously, their analysis had not been published at the time when this study was undertaken.