Signal Processing for Distributed Nodes in Smart Networks

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A thesis submitted for the degree of Doctor of Philosophy of The Australian National University.

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Except where otherwise indicated, this thesis is my own original work.

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Preface

This thesis has been submitted to the College of Engineering and Computer Science of The Australian National University (ANU) in fulfillment of the requirement for the degree of Doctor of Philosophy (Ph.D.). The studies were carried out over a period of three years and eight months, from April 2009 to December 2012. The research was funded by National ICT Australia (NICTA) through NICTA tuition fee scholarship, NICTA Ph.D. scholarship, NICTA Ph.D. supplementary scholarship, and NICTA Ph.D. assignment scholarship. My supervisors have been Dr. David B. Smith, Dr. Jian A. Zhang, Dr. Tharaka A. Lamahewa, and Dr. Thushara D. Abhayapala.

Dedication

This thesis is dedicated to my mother Mrs. Shahana Ferdous.

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Abstract

With increasing environmental concern for energy conservation and mitigating climate change, next generation smart networks are bound to provide improved performance in terms of security, reliability, and energy efficiency. For instance, future smart networks will work in highly complex and dynamic environments and will have distributed nodes that need to interact with each other and may also interact with an energy provider in order to improve their performance. In this context, advanced signal processing tools such as game theory and distributed transmit beamforming can yield tremendous performance gains in terms of energy efficiency for demand management and signal transmission in smart networks.

The central theme of this dissertation is the modeling of energy usage behavior of self-seeking distributed nodes in smart networks. The thesis mainly looks into two key areas of smart networks: 1) smart grid networks and 2) wireless sensor networks, and contains: an analytical framework of the economics of electric vehicle charging in smart grids in an energy constrained environment; a study of a consumer-centric energy management scheme for encouraging the consumers in a smart grid to voluntarily take part in energy management; an outage management scheme for efficiently curtailting energy from the consumers in smart grids in the event of a power outage; a comprehensive study of power control of sensors in a wireless sensor network using game theory and distributed transmit beamforming; and finally, an energy aware distributed transmit beamforming technique for long distance signal transmission in a wireless sensor network.

This thesis addresses the challenges of modeling the energy usage behavior of distributed nodes through studying the propriety of energy users in smart networks, 1) by capturing the interactions between the energy users and energy provider in smart grids using non-cooperative Stackelberg and generalized Nash games, and showing that the socially optimal energy management for users can be achieved at the solution of the games, and 2) by studying the power control of sensors in wireless sensor networks, using a non-cooperative Nash game and distributed transmit beamforming that demonstrates significant transmit energy savings for the sensors. To foster energy efficient transmission, the thesis also studies a distributed transmit beamforming technique that does not require any channel state information for long distance signal transmission in sensor networks.

The contributions of this dissertation are enhanced by proposing suitable system models and appropriate signal processing techniques. These models and techniques can capture the different cost-benefit tradeoffs that exist in these networks. All the proposed schemes in this dissertation are shown to have significant performance improvement when compared with existing solutions. The work in this thesis demonstrates that modeling power usage behavior of distributed nodes in smart networks is both possible and beneficial for increasing the energy efficiency of these networks.
List of Publications

Much of the work in this thesis has been published or has been submitted for publication in refereed journals and conference proceedings. The following is a list of publications (Tushar2012a–Tushar2010).


[Tushar2010] W. Tushar and D. B. Smith,”Distributed Transmit Beamforming Based on
a 3-Bit Feedback System,” in Proc. of the IEEE International Workshop on Signal Processing Advances in Wireless Communications (SPAWC), Marrakech, Morocco, Jun., 2010, pp. 1-5.
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Glossary

Abbreviations

PEVG  Plug-in electric vehicle group.
PEV   Plug-in electric vehicle.
BEV   Battery only electric vehicle.
PHEV  Plug-in hybrid electric vehicle.
SG    Smart grid.
SEM   Smart energy manager.
MWh   Mega Watt hour.
GNEP  Generalized Nash equilibrium problem.
GNE   Generalized Nash equilibrium.
GSG   Generalized Stackelberg game.
GSE   Generalized Stackelberg equilibrium.
VE    Variational equilibrium.
VI    Variational Inequality.
KKT   Karush-Kuhn-Tucker.
S-S   Solodov and Svaiter.
V2G   Vehicle to grid.
USD   US dollar.
PSO   Particle swarm optimization.
ED    Equal distribution.
CPS   Central power station.
EC    Energy consumer.
G2V   Grid to vehicle.
FIT   Feed-in tariff.
EV    Electric vehicle.
SLMFG Single-leader multiple-follower game.
EMES  Energy management equilibrium solution.
SSHPM S-S hyperplane projection method.
kWh   kilo Watt hour.
ECG   Energy curtailment game.
ES    Energy source.
EU    Energy user.
LAN   Local area network.
CPP   Customer preference parameter.
VIP   Variational inequality problem.
LPM   Leading principal minor.
EEC   Equal energy curtailment.
TDMA  Time division multiple access.
SINR  Signal to interference-noise ratio.
QoS   Quality of service.
BS    Base station.
NPG   Non-cooperative power control game.
IP    Interference power.
DTB   Distributed transmit beamforming.
SEP   Symbol error probability.
CS    Cooperative sensor.
CH    Cluster head.
CM    Cluster member.
DF    Directional flood.
PSK   Phase shift keying.
i.i.d  Independently and identically distributed.
CSI   Channel state information.
ML    Maximum likelihood.
SNR   Signal to noise ratio.
RP    Receiving point.
BPSK  Binary phase shift keying.
QPSK  Quadrature phase shift keying.
RSS   Received signal strength.
BER   Bit error rate.
SON   Self organizing network.

Symbols

Some symbols used in this dissertation have different meanings in different chapters. Hence, we do not provide global definition here. All symbols will be defined in the context of each chapter in which they appear.
This thesis addresses the problem of modeling the power usage behavior of self-interested nodes in distributed smart networks, asking how we can— as well as why we need to— manage and control nodes’ power so as to leverage the socially optimal energy management in smart grids, and further, to increase the battery life-time of sensors in an energy constrained wireless sensor network.

1.1 Definitions

We start by briefly defining the use of the related terms signal processing, distributed node, and smart network in the title of this thesis. We use signal processing to refer to an enabling technology which uses mathematical, statistical and heuristic techniques for representation, modeling, analysis and extraction of signals [Moura, 2009]. We define distributed node as that each node can control its own decision making process, and may unable to access the resources belong to other nodes in the system. Finally, with the term smart network, we exemplify an intelligent network that can be customized based on the devices it has, and can optimize its applications to adapt itself with the changing network environment and constraints by modifying the activities of its connected devices. In general, signal processing for a smart network with distributed nodes includes a wide range of applications. However, the focus of this thesis has been mainly on two key application areas in smart networks: 1) energy management in smart grids; and 2) power control of sensors in wireless sensor networks.

1.2 Motivation and Scope

The awareness for efficient usage and conservation of energy, in both power and communication sectors, has increased overwhelmingly over the past few decades. This is mainly due to 1) the depletion of fossil fuel, 2) climate change, and 3) the urgency for longer network life-time for communication in areas with less accessibility. As most power equipment, e.g., electric vehicle, is owned by self-seeking individual personnel [Fang et al., 2011], and also, due to the self-centred nature of wireless communicating devices, e.g., sensors [Sengupta et al., 2010], it is a challenging task to motivate the energy users to effectively manage their energy at the same time achieving the quality of service they are expecting.
To reduce the environmental impacts through improvement in energy efficiency, and by enabling the integration of a large percentage of renewable energy sources, smart grid emerges with the vision of two way flow of information and electricity to create an automated and distributed advanced energy delivery network. One of the main attributes of smart grids is the introduction of electric vehicles and a number of countries including European Union member states, Japan, South Korea, Canada, China, Israel and USA have established electric vehicle target, policies and plan to succeed in deploying electric vehicles on the road [Foley et al., 2010b]. This large scale penetration of electric vehicles will certainly pose a number of challenges on the charging infrastructure, storage management, energy exchange, and importantly, on the development of charging strategies [Rotering and Ilic, 2011; Samadi et al., 2010; Wu et al., 2012a; Pan et al., 2010; Sojoudi and Low, 2011; Turitsyn et al., 2010; Clement-Nyns et al., 2010; Käbisch et al., 2010; Coullet et al., 2012; Su and Chow, 2011; Foley et al., 2011, 2010a, 2009]. While, the problem of electric vehicle charging and its impact on the distribution grid has been address in the literature [Rotering and Ilic, 2011; Bauer et al., 2010; Foley et al., 2010b; Turitsyn et al., 2010; Sojoudi and Low, 2011], little has been done to develop distributed models and algorithms to capture the interaction between the vehicles and the grid, in a grid-to-vehicle scenario. Hence, there is a need of modeling the energy exchange between the grid and vehicles to successfully capture the energy trading so as to achieve a charging profile for the vehicles that is socially optimal for both the vehicles and the grid.

It is important to note that the energy exchange in smart grid largely depends on the voluntary participation of energy users of the network, which enhance the grid’s reliability and could significantly improve the social benefits for the whole system [Walawalkar et al., 2010]. Therefore, if the energy consumers are not interested in taking part in voluntarily energy management programs, e.g., if they are not happy with the incentives, the benefits of smart grids will be incomplete [Liu et al., 2011]. Hence, it is of great importance to make the consumers an integral part of any energy management scheme [Liu et al., 2011], and thus, to develop an energy management scheme that is consumer-centric whereby the final recipients of the smart grid benefits will be the energy consumers.

Besides energy management, another important aspect of smart grid that requires consumers’ voluntary participation is the outage management. While most of the research in this area has been devoted to fault location identification and service restoration [Calderaro et al., 2011; Overman et al., 2011; Chertkov et al., 2011; Kezunovic, 2011; He and Zhang, 2011; Russell and Benner, 2010], the immediate post outage management, i.e., during the blackout, is seemed to have received less attention. Managing blackout is very important to reduce its disastrous impact on the grid. For instance, in 2002, the annual cost of power outage in USA was estimated to be on the order of $79 billion [Moslehi and Kumar, 2010], rising to $100 billion in 2007 [NETL, 2007]. Hence, the outage management in smart grid in the event of a power disruption so as to reduce the catastrophic impact on the whole grid system is worth investigating.

Successful energy/outage management of smart grids is closely related to smart communication infrastructure [Fang et al., 2011]. Wireless sensor networks in particular, given their low cost, can provide a feasible and cost-effective sensing and communication platform for smart grids [Gungor et al., 2010]. In outage management, for example, a single system contingency in the power grid could be detected and isolated before it causes cascading effects and leads to more catastrophic system-wide breakdowns with the help of
1.3. Contribution

sensor networks [Gungor et al., 2010]. However, sensor nodes are often resource limited devices. This is due to the fact that sensors are powered by batteries for which it is often infeasible to frequently replace due to their remote deployment in areas which are not easily accessible. Hence, it is often important to conserve the energy of the sensor nodes while transmitting its information to the intended receiver [Betz and Poor, 2008]. To this end, novel transmission and routing protocols need to be devised, which would enable control of the transmit power of sensor nodes that would increase the life-time of the sensor network.

Motivated from the above discussion it is clear that there are many possible novel contributions and improvements in smart networks via signal processing. I have been involved in two main research directions: Resource management in smart grid that covers both energy and outage management, and power control for sensor networks including both protocols and techniques with respect to signal transmission. These are the research directions that have helped me to shape my Ph.D. thesis.

1.3 Contribution

To model the energy usage behavior of distributed nodes in smart networks, which are often self-centred in nature [Fang et al., 2011; Sengupta et al., 2010], the questions one must ask are: “How to design the incentives for selfish nodes (i.e., nodes in smart grids) so as to motivate them to agree on the energy trading parameters that are socially optimal for all the energy users of the system?”, and “What are the most appropriate choice of power for sensor nodes to not only increase the life-time of the sensor networks but also to lead the sensors achieving the desired quality of service?”

To answer these questions, the main contributions of this thesis have been made by investigating the research scopes as described in Section 1.2. To that end, the aim of my research has been to identify the socially optimal solution vectors for energy and outage managements in smart grids, and to determine the power vector for sensor nodes for increasing the life-time of sensor network as well as to meet their desired quality of service requirements by making the following contributions:

1. Economics of electric vehicle charging in smart grids has been investigated systematically-

   i) A non-cooperative Stackelberg game is proposed in which the smart grid acts as the leader and the vehicles are assumed to be the followers of the game. The smart grid needs to decide on its price so as to optimize its revenue as well as ensuring vehicles participation in the game, whereby the electric vehicles need to decide on their charging strategies so as to optimize a tradeoff between the benefits from charging batteries and the associated costs.

   ii) It is shown that the solution of the game is socially optimal in which the grid optimizes its price and the vehicles choose their equilibrium strategies.

   iii) Modeling the game as a variational inequality problem, a novel distributed algorithm is proposed that enables both the grid and vehicles to reach the optimal solution.
iv) The game is extended to a time-varying case, which can handle the slowly time-varying environment including such things as arrival and departure of vehicles, and is shown to possess the team-optimal solution.

2. To encourage widespread participation of consumers in smart grid’s energy management, a consumer-centric energy management scheme is studied by

i) Providing a comprehensive analytical framework, using a single-leader multiple-follower game, which is suitable for designing the decentralized decision making process of the energy consumers in smart grids by capturing the interactions between a centralized power station and the energy consumers in the network.

ii) Analyzing the solution of the game, it is shown that the non-cooperative game leads to a solution which is socially optimal for the consumers, and also, in which, the total cost incurred by the power station possesses the minimum value.

iii) Formulating the game between the consumers as a jointly convex Nash equilibrium problem, a distributed algorithm is proposed that guarantees the convergence of the game to the socially optimal solution.

3. Taking the advantages of two-way communication structure of smart grids, an efficient outage management scheme is proposed for cutting off energy from the energy users in the event of a power outage in the system. The aim of the study is to encourage the users to voluntarily decide on an amount of energy to be curtailed from them so as to keep the total cost incurred by the system due to the power outage at the minimum. An energy curtailment game, using a non-cooperative generalized Nash game, is proposed where the energy users are the players who play the game to strategically decide on the amount of energy to be curtailed from them. By studying the properties of the game it is shown that there exists an efficient solution in which the total cost incurred by the system reaches a global minimum. A novel distributed algorithm is proposed for the energy users to reach the optimal solution by playing the game.

4. As a powerful tool for controlling the power usage behavior of distributed sensors in wireless sensor networks, power control via cooperative communications is studied for clusters of sensors. Power control of sensors is shown for two different cases:

i) For controlling the power of multiple source sensors transmitting towards multiple cooperative clusters:

- We formulate a non-cooperative game with multiple sources and multiple receiving clusters, in which each source sensor is a player who decides on its transmit power, and determine the Nash equilibrium of the game.
- The best response dynamics of the proposed game is analyzed and uniqueness is proved under certain constraints.
- The dependency of the equilibrium on the interference power, distance between the source and receiving cluster, and on the path loss exponent is determined analytically.
For controlling the transmit power of multiple cooperative sensors in a cluster to send information, received from the sources, to a distant receiver:

- While both distributed transmit beamforming and data funneling can improve the energy efficiency of signal transmission, we have combined both the techniques and have proposed a novel cross-layer data transmission scheme to save the total system energy of the sensor network.
- We show that significant energy savings are possible for the sensors in the distant cluster. This is while considering the energy spent by each sensor for transmission from its cluster over a long distance compared to direct beamforming and direct single link transmission.
- We show that, for the proposed scheme, to achieve the similar quality of service in terms of symbol error probability at the receiver, the battery energy of the sensor can be saved by up to 36% compared to direct beamforming. Consequently, the network life-time increases significantly.
- We demonstrate improved performance in terms of routing by adopting this cross-layer scheme.

For long distance data transmission, we study a distributed transmit beamforming scheme which does not require any channel state information, but can still achieve phase coherence at the receiver. In developing the beamforming scheme:

i) We propose a three-bit feedback scheme for synchronizing the phases of signals from different transmitters, so as to combine the signals coherently at the receiver. We show that the proposed scheme can considerably increase the signal gain in terms of received signal strength at the receiver, and the received signal strength is shown to increase quadratically with respect to an increasing number of transmitters.

ii) We improve the three-bit feedback scheme by reducing the number of feedback bits, and subsequently propose a two-bit feedback scheme for phase synchronization. The improved scheme is shown to be applicable for both static and time-varying channels, and requires fewer antennas to achieve the same signal strength at the receiver compared to existing schemes.

iii) We investigate the improvement of the speed of phase convergence in the beamforming scheme so as to speed up the convergence of signals to a steady state value. We show that the combination of a single bit with a phase feedback can significantly accelerate the convergence of receive signal strengths to steady state.

This thesis details contributions in each of the above mentioned areas including a detailed literature review to provide an overview of the current state of research in all related fields.

1.4 Thesis Outline

The body of this dissertation is structured around two key elements arising from the contributions listed in the previous section: 1) smart grid’s energy management, and 2)
power control in wireless sensor networks. Smart grid energy management is discussed first followed by the techniques for controlling power of sensors in wireless sensor networks.

Chapter 2 provides an overview of smart networks, game theory and distributed transmit beamforming, and surveys relevant signal processing literature in smart grid energy management and power control in sensor networks.

Chapters 3-5 detail contributions in energy management in smart grid networks. Chapter 3 describes the economics of battery charging of electric vehicles in an energy constrained environment and provides a socially optimal solution for both the grid and vehicles in the network. Chapter 4 studies a consumer-centric energy management scheme for encouraging the energy consumers in smart grid to voluntarily take part in energy management. The optimal solution is obtained which is consumer-centric, and thus, solely benefits the consumers of the network. In Chapter 5, we discuss an efficient energy curtailment scheme, in the event of a power disruption in smart grid, so as to minimize the total cost incurred by the system due to the outage.

In Chapter 6-7, this thesis’s contributions are discussed in the field of power control of sensors in wireless sensor networks. Chapter 6 shows mechanisms of power control for sending information from multiple sources to a distant receiver via multiple clusters with cooperative sensors. The first part of the chapter shows the power control of source sensors for sending their signals to the intended cooperative clusters, while the second part discusses the power control of cooperative sensors in clusters to send the signal towards the final receiver using distributed transmit beamforming via data funneling. In Chapter 7, a distributed transmit beamforming technique is proposed to achieve higher signal gain at the intended receiver, when compared to some recent techniques, without any channel state information. Further, improvement of the technique in terms of number of feedback bits and convergence speed are also demonstrated.

Finally, Chapter 8 concludes the thesis describing how contributions have shown that signal processing for smart networks with distributed nodes is both beneficial and feasible, and identifies future research directions.
This chapter provides an up-to-date literature review with respect to the application of signal processing in distributed smart networks. The main objective is to provide a brief background to the techniques developed and described in this dissertation, and also to discuss relevant recent development in the literature. We begin with a short introduction to: distributed smart networks, particularly smart grid and wireless sensor networks; and signal processing techniques such as the use of game theory and distributed transmit beamforming. Then, we show how such signal processing tools have been described in literature for modeling energy usage of the distributed nodes in a smart network.

2.1 Introduction to distributed networks and distributed signal processing

A distributed network system, i.e., a smart network, consists of a collection of autonomous nodes, i.e., smart nodes, connected through a network that enables the nodes to coordinate their activities and share system resources. Unlike a centralized system where the system’s components are shared by all users and there is only a single point of control, a distributed system provides more flexibility with multiple process controls by autonomous users, and resources available to one user may not be accessible to others. A relevant example of a distributed system is a smart grid system, in which a number of autonomous distributed nodes such as smart homes, electric vehicles, wind farms, industries and power stations coexist in the network, and are connected to each other via appropriate, often simple, communication links. The main characteristics and advantages of a distributed system include [Foster et al., 2002]:

i) A distributed system allows an interaction between the resource provider and the users through appropriate resource sharing models;

ii) New components can easily be integrated with existing network components, and thus provides the opportunity for future system expansion;

iii) A distributed node can decide on its own action (for example, whether or not to collaborate with other) so as to improve its quality of service;

iv) The system is scalable, and hence can accommodate more users in the system;

v) The nodes have fault tolerance and self-healing capabilities that can be achieved using techniques for recovery and redundancy;
vi) The system possesses improved reliability; and

vii) Distributed network models are suitable for many applications that are inherently distributed, for example surveillance using wireless sensor networks or managing a chain of supermarkets.

What follows here is an overview of two emerging smart distributed network systems, one such network system being smart grid and the other being a wireless sensor network.

2.1.1 Introduction to smart grid

A smart grid (SG), also known as an intelligent grid or a smart power grid, is a significant enhancement to the standard 20th century power grid. It can be regarded as an electrical system that uses information, with two-way, cyber-secure communication technologies, and computational intelligence in an integrated fashion across electricity generation, transmission, distribution and consumption so as to achieve a system that is clean, safe, secure, reliable, resilient, efficient and sustainable [Fang et al., 2011]. Although the initial concept of SG started with the idea of advanced metering infrastructure (AMI) for improving demand-side management, energy efficiency and reliable grid protection against malicious sabotage [Rahimi and Ipakchi, 2010], it has been expanded from its initial scope due to growing demands for quality of service and all the requirements related to achieving a specified quality of service. Although a precise and comprehensive definition of SG has not been proposed up till now, the National Institute of Standards and Technology (NIST) [Locke and Gallagher, 2010] has anticipated the following benefits of, and requirements for, future smart grid systems [Locke and Gallagher, 2010]:

i) Improving power reliability and quality, and optimizing facility utilization and hence averting construction of back up power plants;

ii) Enhancing capacity and efficiency of existing electric power networks;

iii) Improving resilience to disruptions, and enabling predictive maintenance and self-healing responses to system disturbance;

iv) Accommodating distributed power sources and facilitating expanded deployment of renewable energy sources;

v) Automating maintenance and operations;

vi) Reducing greenhouse gas emission by enabling electric vehicles and new power sources, and reducing oil consumption by reducing the need for inefficient generation during peak usage periods;

vii) Presenting opportunities to improve grid security and enabling transition to plug-in electric vehicles and new energy storage options;

viii) Increasing consumers choices and interactions, and enabling new products, services and markets.
In order to realize the emerging smart grid paradigm, NIST provided a conceptual model, as shown in Fig. 2.1, that consists of seven domains. They are: i)- Customers; ii)- Markets; iii)- Service providers; iv)- Operations; v)- Bulk generations; vi)- Transmission; and vii)- Distribution [Locke and Gallagher, 2010]. Each of these domains consist of one or more actors\(^1\). The details of each domain and the role of the domain’s actors can be found in [Fang et al., 2011; Locke and Gallagher, 2010].

For the development of SG, governments, academia, industry and research organizations have made a large amount of effort into pilot projects, programs and field trials. These include smart meters, AMI, transmission grids, distribution grids, distributed resources, virtual power plants, home applications, micro-grids, electric vehicles, and integrated systems [Fang et al., 2011]. Meanwhile, the key challenges, as elaborated in [Fang et al., 2011], for the realization of smart grid technologies can be summarized as follows:

i) One principal challenge is to effectively model renewable energy resources due to the intermittent nature of wind and solar generation by introducing reliable and efficient forecasting and scheduling. Both long term and short term renewable resource patterns and likely behaviors must be explored and understood.

ii) Electric vehicle charging will introduce a significant new load on existing distribution grids where many grid circuits do not have any spare capacity. Moreover, for vehicle to grid (V2G) systems, there is an uncertainty with respect to power to be supplied by vehicles as such vehicles can only deliver power when they are parked and connected to the grid.

iii) Another challenging task is to determine the information that should be stored in any smart grid control system so that meaningful system, or user, history can be

\(^1\)Who make the decisions, and exchange information that is necessary for performing their application-specific roles.
constructed from the large amount of information collected from distributed smart meters, sensors and phase measurement units in the network.

iv) Since many different communication technologies and protocols will be used in any smart grid, and each technology, most likely, will use its own protocol [Fang et al., 2011] and algorithm, the interoperability of communication protocols will pose a challenge for effective communication between smart grid nodes.

v) With the emergence of new markets, for example micro-grids\(^2\), guaranteeing a truthful auction for bidding for energy between consumers is a high challenging task as some users may make untruthful bids to cheat the seller in order to obtain benefits they can not get with truthful bidding.

vi) The utilization of renewable resources, such as wind and solar, makes management of energy difficult due to their fluctuating and intermittent nature. A management system needs to maintain reliability and satisfy operational requirements, at the same time as taking into account the uncertainty and variability of energy resources.

vii) Whereas the realization of reliable system operation, resistance to attacks and failures, and preservation of failures are some principal characteristics of the vision for smart grids, realizing these objectives poses many challenges including: 1)- interpretability between cryptographic systems, 2)- conflict between privacy preservation and information accessibility , 3)- determining the impact of increased system complexity and expanded communication paths, 4)- studying the impact of increasing energy consumption and asset utilization, and 5)- analyzing complicated decision making processes.

However, as discussed in [Giordano et al., 2011], most of the technologies that are needed to realize these objectives are known. The main challenge is to integrate them together under the common smart grid framework.

2.1.2 Introduction to sensor networks

Due to advancements in wireless communications and electronics over past few decades, the development of low-cost, low-power and multi-functional sensors has received immense attention. A sensor is a transducer that converts a physical phenomenon such as heat, light, sound or motion into electrical or other signals that may be further manipulated by other apparatus [Zhao and Guibas, 2004]. A sensor network is a connectivity graph of sensors, including routing nodes, and communication links, as shown in Fig. 2.2, which detects events or phenomena, collects and processes data, and transmits information to interested users via an appropriate communication protocol. The main characteristics of a sensor network, which imply that it is a smart network, are:

1. Self-organising capabilities.

2. Short-range broadcast communication and multi-hop routing.

\(^2\)A micro-grid is a networked group of distributed energy sources located at on the distribution side of a power system, and the micro-grid can provide energy to a small geographical area.
3. Dense deployment and cooperative efforts of sensor nodes.
4. Frequently changing topology due to channel fading and node failures.
5. Limitations in energy, transmit power, memory and computing power.

Figure 2.2: Overview of a wireless sensor network [Purelink, 2012].

Networked sensing offers unique advantages over traditional centralized approaches. Dense networks of distributed communicating sensors can improve signal-to-noise ratio (SNR) by reducing average distances from sensor to source of signal, or alternatively to the target. Increased energy efficiency in communications is enabled by a multi-hop topology of the network [Pottie and Kaiser, 2000]. Moreover, additional relevant information from other sensors can be aggregated during this multi-hop transmission through in-network processing [Intanagonwiwat et al., 2000]. However, the greatest advantages of networked sensing are in improved robustness and scalability [Zhao and Guibas, 2004]. A decentralized sensing system is inherently more robust against individual sensor node or link failures, because of redundancy in the network. Decentralized algorithms are also far more scalable in practical deployment and an effective way to achieve the large scale needed for some applications.

Although the potential of sensor networks is almost unlimited, the design of a sensor network and its applications face a number of challenges such as [Stankovic, 2004]:

i) Wireless sensor networks operate in real world environments. Hence, in many cases, sensor data must be delivered within time constraint, i.e., an acceptable delay, so that appropriate observations can be made or actions can be taken. Consequently, there is need to develop real-time protocols for wireless sensor networks in conjunction with sensor network analysis.
ii) Limited processor bandwidth and small memory are two important constraints in sensor networks. Moreover, sensors are also limited by processing and communication capabilities [Zhao and Guibas, 2004]. These issues are need to be resolved via the development of novel and efficient fabrication techniques [Stankovic, 2004].

iii) Energy constraints for sensors is one of the most important research challenges related to the deployment of sensor networks. This is mainly due to the random deployment of sensors over hazardous sensing environments (for example, in forests), which preclude battery replacement as a feasible solution. In fact, most sensor network applications require long life-times for sensors. Hence, it is very important to provide energy-efficient protocols (e.g., for energy efficient transmission) for prolonging the life-time of sensor networks.

iv) Sensor nodes are often deployed in accessible areas, which subsequently increases the risk of physical attacks. Furthermore, sensor networks often interact closely with their physical environment and also with people, which poses additional security problems. Hence, these constraints present new challenges, in sensor network’s design and deployment, including secrecy, authentication, privacy, robustness to denial-of-service attacks, secure routing and node capture.

To address the many issues in realizing smart networks, distributed signal processing has been proven to be a very effective tool, and consequently has been used extensively in literature.

2.1.3 Brief overview of distributed signal processing in smart grids and sensor networks

For efficient and robust operation of different heterogeneous and large-scale distributed systems such as smart grids and sensor networks, it is necessary to study suitable distributed signal processing techniques that enables one to capture the behavior of the distributed smart nodes of the network. In essence, distributed signal processing is an enabling technology that encompasses the fundamental theories, applications, algorithms, and implementations contained in many different physical, symbolic and abstract forms of signals from distributed sources [Moura, 2009]. Moreover, distributed signal processing uses mathematical, statistical, computational representation, formalism and techniques for presentation, modeling, analysis, learning and security [Moura, 2009]. In this context, to study the application of signal processing for smart networks, we have mainly focused on two distributed signal processing techniques: i)- game theory, and ii)- distributed transmit beamforming (DTB). First, we show how game theory can capture the decision making process, of managing energy, by distributed energy users in smart grid systems. Then, we show the application of game theory for choosing the transmit power of distributed sensor nodes in a sensor network so as to increase the network’s life-time. Finally, by using distributed transmit beamforming, we study the autonomous choice of transmit power by distributed sensors during the transmission of signals over long distances. We seek to give a brief introduction to all of these distributed signal processing techniques, and also provide the current state of their application in smart grids and sensor networks. We mainly emphasize energy management techniques in smart grid systems, and power control for wireless sensor network systems respectively.
2.2 Game theory and its application

Game theory provides a formal analytical structure with a set of mathematical tools to study the complex interactions among rational players. Game theory has made a revolutionary impact on a large diverse number of disciplines including engineering, economics, political science, philosophy and psychology [Myerson, 1991]. Recently, there has been significant growth in research activities that use game theory to analyze smart networks. This is mainly due to the nature of smart networks in which each node is autonomous, flexible and distributed, and is enabled to make its own independent and rational strategic decisions [Saad et al., 2009b]. In this section, we provide a brief introduction to game theory and discuss how game theory has been applied to different application areas of smart networks in the literature including smart grids and wireless sensor networks.

2.2.1 Introduction to game theory

Basic game theoretic concept: Game theory is a mathematical framework that can be divided into two main branches: i)- non-cooperative game theory and ii)- cooperative game theory. Non-cooperative game theory can be used to analyze the strategic decision making processes of a number of independent entities that have partially or totally conflicting interests over the outcome of a decision process which is affected by their actions [Başar and Olsder, 1999]. Non-cooperative games can be grouped into two categories: static games and dynamic games. In the first category, static games are games in which the notion of time or information does not affect the action choices of players, and can be formally defined as follows:

Definition 2.2.1. A static non-cooperative static game is defined as a situation that involves three components: the set of players $\mathcal{N}$, the actions set $(\mathcal{E}_n)_{n \in \mathcal{N}}$, and the utility function $(u_n)_{n \in \mathcal{N}}$. In such a non-cooperative game, each player $n$ wants to choose an action $e_n \in \mathcal{E}_n$ so as to optimize its utility function $u_n(e_n, e_{-n})$, which depends not only on player $n$’s action choice $e_n$ but also on the vector of actions taken by the other players in $\mathcal{N} \setminus \{n\}$, denoted by $e_{-n}$.

On the other hand, when the game is dynamic, one needs to define, as part of the game, additional components such as information sets, time and sets of past actions, which are usually reflected in the utility function. The strategy of players in dynamic games are also loosely bound to the information available to each player [Başar and Olsder, 1999].

Solution concept: One of the most important solution concepts, particularly for non-cooperative games, is that of a Nash equilibrium\(^3\). The Nash equilibrium characterizes a state in which no player $n$ can improve its utility by unilaterally changing its strategy, given that the strategies of other players are fixed. For a static game, the Nash equilibrium with a pure strategy can be formally defined as follows:

Definition 2.2.2. A pure-strategy Nash equilibrium of a static non-cooperative game is a vector of actions $e^* \in \mathcal{E}$ ($\mathcal{E}$ is the Cartesian product of the action sets) such that $\forall n \in \mathcal{N}$, the following

\[^{3}\text{Which is also the general solution concept for game theory.}\]
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condition holds:

\[ u_n(e^*_n, e^{*-n}) \geq u_n(e_n, e^{*-n}), \forall e_n \in E_n. \]  \hspace{1cm} (2.1)

A Nash equilibrium does not always exist for pure strategies. However, it is guaranteed to exist for mixed strategies\[4\] [Nash, 1950]. Also a game can possess one or multiple Nash equilibrium solutions [Başar and Olsder, 1999].

Cooperative game: Another branch of game theory is cooperative game theory in which, unlike non-cooperative games, the players are allowed to communicate and to receive side payments (e.g., share utilities). Cooperative game theory allows one to investigate how to provide an incentive for independent decision makers to act together as a single entity so as to improve their position in the game. This branch of game theory encompasses two parts: coalitional games and Nash bargaining. Coalition game theory deals with the formation of cooperative groups or coalitions, while Nash bargaining deals with the situation in which a number of players need to agree on the terms under which they cooperate. A description of cooperative game theory can be found in [Saad et al., 2009b].

Learning in game: Another important aspect of a game is to develop learning algorithms. Such development is essentially an iterative process in which each iteration involves three key steps to be performed by each player: (i)- observing the environment and the current game state; (ii)- estimating prospective utility; and (iii)- updating the strategy based on observations. Numerous learning algorithms have been proposed in the literature [Başar and Olsder, 1999; Nisan et al., 2007; Young, 2005; Fudenberg and Levine, 1998; Shamma and Arslan, 2004; Rose et al., 2011], these include best response dynamics, fictitious play and regret matching amongst many others. A learning algorithm is important as it is a challenging task to reach a certain equilibrium and the players must follow well-defined rules that enables them to observe the current game state and make a decision on their strategy choices. An elaborate description of learning algorithms can be found in [Başar and Olsder, 1999; Nisan et al., 2007; Young, 2005; Fudenberg and Levine, 1998; Shamma and Arslan, 2004; Rose et al., 2011].

2.2.2 Application of game theory in smart grids

Within the context of smart grids, the applications of non-cooperative games are numerous. On the one hand, it can be used to perform distributed demand-side management and real-time monitoring, and to control and deploy micro-grids. On the other hand, economic factors such as markets and dynamic pricing, which are essential parts of smart grids, can also be captured through non-cooperative games. Moreover, with the deployment of advanced communication technologies [Fang et al., 2011], it is often possible to enable a limited form of communication between the nodes that paves the way for introducing cooperative game-theoretic approaches. Consequently, game theory has recently been used to study different management and control services that pertain to smart grids [Fang et al., 2011; Wu et al., 2012a; Couillet et al., 2012; Mohsenian-Rad et al., 2010] to ensure an efficient and robust operation of such heterogeneous systems. Here we discuss the general application of game theory in micro-grid systems and communications

\[4\] In which the players choose their strategies randomizing over a set of available actions according to some probability distribution.
followed by a particular application in the context of demand-side management of smart grids.

### 2.2.3 General application of game theory in smart grid

In this section, we provide an overview of game theoretic applications in deploying micro-grids in future power grids, and also in addressing the challenges of integrating communication technologies in smart grids.

1) **Game theoretic approach in micro-grids:** A micro-grid can be defined as a networked group of distributed energy sources such as solar panel or wind turbines located at the distribution network side of a power grid system, and can provide energy to a small geographic area. The network of a micro-grid is envisioned to operate both in conjunction with the main grid as well as autonomously in isolated mode (also known as island mode). In this respect, game theory has been used for controlling the operation of micro-grids and also to address the technical challenges of integrating them in smart grids [Maity and Rao, 2010; Alibhai et al., 2004; Saad et al., 2011c; Duan and Deconinck, 2008; Karangelos and Bouffard, 2011].

The future smart grid is envisioned to encompass a large number of micro-grids. Hence, sometime it might be beneficial for these micro-grids to exchange power amongst each other instead of requesting energy from the main grid (if possible) which would reduce transmission costs and reliance on the main grid. In [Saad et al., 2011a], the authors have addressed this issue by introducing a cooperative energy exchange mechanism using cooperative game theory. They defined a transferable utility to evaluate the pay-off (i.e., utility) of each micro-grid, and proposed a coalition formation game for energy exchange among the micro-grids in the network. As shown in [Saad et al., 2011a], each micro-grid can decide on whether to form a coalition or not by: i)- agreeing on the procedure for matching sellers to buyers (e.g., using the heuristic of [Saad et al., 2011a]) and ii)- computing their prospective pay-off which is found through a mapping that associates with utility of the coalition. It is shown in [Saad et al., 2011a] that a group of micro-grids would cooperate and form a single, larger coalition if this formation increases the pay-off (reduces the power losses) of at least one of the involved micro-grids without decreasing the pay-off of any of the others. Similarly, a coalition of micro-grids can decide to split and divide itself into smaller coalitions if it is beneficial to do so.

To compensate any mismatch between the demand and supply, a static non-cooperative game model is proposed in [Weaver and Krein, 2009] for controlling both the loads and energy sources in a small scale power system such as a micro-grid. In the proposed game, on the one hand, the strategy of each source is chosen to regulate the voltage, and on the other hand, the strategy of each load is to choose a shunt resistance. Essentially, [Weaver and Krein, 2009] shows two things: i)- the use of non-cooperative games can adequately model the interactions between sources and loads in a small-scale power system, and ii)- advanced analytic techniques and algorithms are still needed to enable operation at Nash equilibria as well as to improve the efficiency of these equilibria.

Further, to enable the micro-grids to decide on whether to store or use energy so as to meet the predicted demand of their consumers, a game theoretic framework is proposed by Maity et al. in [Maity and Rao, 2010]. The essence of this framework is based on two types of games. The first type of game is a non-cooperative solution for the Pot-luck prob-
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Problem, which is essentially a formulation of a situation that involves two types of players: players that have certain goods to supply and players that have a certain need for these goods. The second type of game is an auction game for determining pricing in micro-grid networks. The authors have proposed a learning algorithm to capture the non-rational behavior of the players where the algorithm can also reach the desired operating point of the system. Other studies of issues like wind turbine control, pricing, and cooperation in micro-grids can be found in [Alibhai et al., 2004; Saad et al., 2011c; Duan and Deconinck, 2008; Karangelos and Bouffard, 2011; Saad et al., 2011a; Weaver and Krein, 2009; Wu, 2010].

2) Game theoretic approach in communications: One of the main characteristics of a smart grid is its ability to ensure a reliable information flow between a number of heterogeneous nodes. For example, the smart meters of nodes need to communicate with nearby control centers so as to exchange information such as meter readings, pricing or other control data [Pavlidou et al., 2011]. Moreover, the communication of a load signal from the utility operator to electric vehicles is expected to be an inherent component of smart grid communications [Markel et al., 2009]. In fact, the enabling of many smart grid applications, such as micro-grid coordination, and integration of new elements, such as electric vehicles, is contingent upon the deployment of an efficient and reliable communication architecture that can truly allow smart operation of future power systems.

The integration of communication networks into a large-scale system, such as a smart grid, increases network design complexity, and thus motivates the use of advanced tools such as game theory to ensure its proven efficiency in wireless and wireline communications [Han et al., 2011]. For example, narrowband power-line communications (PLC)\textsuperscript{5} is one of the candidate technologies that can be used for data communications between smart grid elements, and has been deployed over advanced metering infrastructure in Europe [Markel et al., 2009]. However, narrowband PLC has limited channel capacity that decreases rapidly with increasing communication distance. To improve this communication delay, Saad \textit{et al.} has proposed a multi-hop network formation game in [Saad et al., 2011b], assuming that all smart elements in the network are physically interconnected such that PLC is possible. The objective of each smart element is to find its preferred partner for forwarding the packets, and each smart element is assumed to act on its own, without coordinating its strategy with neighboring meters. It has been shown in [Saad et al., 2011b] that a multi-hop network formation game has strong potential to reduce delay during PLC communication, and several challenges are proposed, beyond the work done in [Saad et al., 2011b], as follows:

i) Introducing a dynamic approach for jointly performing network formation and channel allocation in a PLC network.

ii) Enabling the smart elements to make strategic decisions based on a long term observation goal.

iii) Analyzing implementation and deployment issues for allowing multi-hop PLC using a network formation game.

With the development of smart communication infrastructures, within the framework of SG, many management tools have become possible and easy-to-implement. In the

\textsuperscript{5}A version of PLC which operates on narrow band frequencies.
following, we discuss the application of game theory for demand management in smart grids.

### 2.2.4 Focused application of game theory on demand management

Demand management is an essential characteristic of current and future smart grid systems that enables a utility company to control energy consumption at consumers' premises [Fang et al., 2011]. In fact, robust demand-side management techniques, which consist of both demand-side management\(^6\) and demand response models\(^7\), are expected to play a very important role in enabling the interconnection of consumers, electric cars, microgrids, and utility companies. The essence of demand management revolves around the interactions between various entities, with specific objectives, in smart grids. Such interactions are reminiscent of players’ interactions in game theory. As a consequence, game theory has been extensively used for both demand-side management and demand response models in smart grids such as in [Chen et al., 2010; Mohsenian-Rad et al., 2010; Saad et al., 2011c; Wu et al., 2012a; Ibars et al., 2010; Fahrioglu and Alvarado, 2000; Li et al., 2011a,c].

For example, in [Mohsenian-Rad et al., 2010], the authors have shown that it is better to develop a demand-side management approach that optimizes the properties of the aggregate load of the users instead of focusing only on an individual user’s energy. This could be enabled by the development of communication technologies that allow the users to coordinate their energy usage, when this is beneficial [Mohsenian-Rad et al., 2010]. To this end, the authors have devised a static non-cooperative game in which each user \(n\) needs to select its energy vector \(e_n\) so as to optimize a utility function \(u_n(e_n, e_{-n})\). The utility depends on how the utility company performs the billing as well as on the type and energy requirement of the users’ appliances. To this context, in [Mohsenian-Rad et al., 2010], the exact expression for this utility was derived under the assumption that each user is billed proportional to its consumption. Based on the game in [Mohsenian-Rad et al., 2010], the following remarks on the properties of the game can be made, as shown in [Mohsenian-Rad et al., 2010]:

i) A Nash equilibrium for the proposed game always exists, and all equilibria coincide with the optimal scheduling policy that minimizes the overall cost to the utility company.

ii) The Nash equilibrium of the game corresponds to a unique set of total loads for each user \(n\).

iii) For the utility function considered in [Mohsenian-Rad et al., 2010], consumers’ appliances are indifferent to when they are being scheduled to be used as every schedule would always correspond to the minimum total cost incurred on the utility company.

\(^6\)A program that attempts to make the users more energy efficient on a longer time scale [Mohsenian-Rad et al., 2010].

\(^7\)Programs that utility companies use to encourage grid users to dynamically change their electricity load [Chen et al., 2010].
Similar to scheduling, storage is another key component that has a strong impact on demand-side management [Perukrishnen et al., 2010]. For example, a user may decide to store energy during off-peak hours and use this stored energy to schedule the use of its appliances, instead of obtaining the energy directly from the substation during peak hours. In [Perukrishnen et al., 2010], the authors proposed a scheme to decide on how to use the storage devices of the appliances and when to buy the energy. To this end, a non-cooperative game, to determine the choices of the consumers on storage, is formulated in which the players are the users, and the strategies are the storage profiles chosen at every time. The utilities are assumed to be the costs incurred on the users over the whole day. The feasible storage profile of each player is optimized subject to three main characteristics of the storage device, which includes its maximum capacity, its storage efficiency\(^8\) and its running cost. The work in [Perukrishnen et al., 2010] shows that, under the considered constraints of storage devices with homogeneous characteristics, the Nash equilibria of the game correspond to the storage profiles that minimize the global generation costs. The result of the game is also tested on empirical data from the UK market. With simulation, it has been shown that the learning scheme converges to a Nash equilibrium, while reducing the peak demand that further leads to reduction in costs and carbon emissions. The results also show the benefit of storage and its impact on the social welfare of the system.

Further applications of game theory in demand management can be found in [Ibars et al., 2010; Chen et al., 2010; Saad et al., 2011c; Fahrioglu and Alvarado, 2000; Li et al., 2011a,c]. For example, in [Ibars et al., 2010], the authors have applied a simple class of non-cooperative game, the so-called congestion game, as a means for performing dynamic pricing so as to control the power demand in an effort to achieve not only the net energy saving but also an efficient utilization of energy. The authors have discussed the key characteristics of the demand-side management congestion game, and have also shown how equilibrium can be reached using a distributed algorithm. Another non-cooperative game based on a double auction is proposed in [Saad et al., 2011c] considering the electric vehicles as the smart nodes of a smart grid network. The proposed game focus on the energy trading between the grid connected plug-in hybrid electric vehicles (PHEVs) and the distribution grid. In the game, each PHEV group is shown to make a decision on the maximum amount of energy surplus that it is willing to sell so as to maximize a utility function that captures the tradeoff between the economic benefits from energy trading and the associated costs. The trading price governing the energy exchange market between the PHEVs and the smart grid network is determined using a strategy-proof double auction. It is shown that a Nash equilibrium exists in the game, which can be reached by each PHEV group in a distributed manner\(^9\), and the utility of each PHEV group is maximized at the equilibrium.

Thus, as shown in the above discussion, game theory can be a very effective tool in modeling the interactive behavior of smart agents in smart grids for different applications including demand management, communications and applications related to micro-grids. Moreover, as shown in [Perukrishnen et al., 2010], game theory not only has impact theoretically but also is very significant in practice for future smart grids. However, it is

\(^8\)Which reflects the fraction of energy that can be extracted after storage.

\(^9\)By adapting a best response dynamics algorithm.
important to note that the application of game theory is not only limited to smart grids, but can also be extended to other smart networks. For example, in wireless sensor networks where performance of sensors is constrained by limited battery power, and also by the other signals’ interference due to the multiple access channel [Sengupta et al., 2010], game theory can effectively be used to model sensors behavior in different applications’ scenarios within these constraints. In this respect, what follows here is an overview of game theory in the context of wireless sensor networks, where game theoretic modeling of transmit power usage of sensors is emphasized.

2.2.5 Application of game theory in sensor networks

Wireless sensor networks (WSNs), comprising of small, power-constrained sensor nodes, can be used in a wide variety of environments like monitoring of environmental attributes, intrusion detection, and various military and civilian applications. The suitability of using game theory to study security, optimize node-level as well as network-wide performance, energy efficiency problems, and pursuit-evasion scenarios of WSNs stems from the nature of strategic interactions between nodes and the distributed decision-making capabilities of WSNs [Machado and Tekinay, 2008]. Consequently, game theory has been very widely applied in wireless sensor networking.

2.2.6 General application of game theory in sensor networks

In this section, we discuss the application of game theory in the context of wireless sensor networks. We briefly address how game theory has been used for energy conservation, routing, load balancing, target tracking and security in sensor networks:

a) Energy conservation: Since sensors are equipped with non-replenishable energy sources, they should be programmed to achieve energy efficiency in their sensing operations, routing and computational capabilities. In [Byers and Nasser, 2000; Felegyhazi et al., 2005], the authors studied, using game theory, how sensors can adapt to different roles such as idle, sensing, routing and routing/sensing to maximize the utility of nodes and networks. Besides, the authors also analyzed the effect of cooperation to achieve an increase in utility of individual network payoffs and thereby optimized energy consumption. In addition, a game theoretic approach is proposed in [Yuan and Yu, 2006] to perform distributed cross-layer optimization for power control at the physical layer and for rate allocation at the application layer.

b) Routing: The choice of efficient routing algorithms in WSNs involves reducing the number of hops, cluster formation, directed diffusion [Dorigo et al., 1996] and randomized algorithms [Lima and Barros, 2007]. In [Miller et al., 2005], the authors have used game theory to design incentives as tokens to encourage cooperation between sensors who belong to different authorities. The authors also have stated and proved the existence of various Nash equilibria for varying conditions of acceptance/rejection of contract. A query based game theoretic approach for routing is also explained in [Kannan and Iyengar, 2004].

c) Load balancing: The distributed nature of sensor networks can lead to a high workload for the sensors in the network. The authors in [Sadagopan et al., 2006] have modeled
this load balancing problem by using mechanism design and game theory, and have proposed an algorithm to design the utility functions of individual nodes such that when the utility functions are optimized by the sensor nodes, the overall network objectives are met. Further investigations on load balancing using game theory are conducted in [Dai and Han, 2003; Raicu et al., 2005].

d) **Target tracking:** A group of sensor nodes (i.e., sensor network) can be used to track a moving target by taking the advantages of large spatial coverage, robustness, and cooperation. However, the implementation of these capabilities in sensor networks is still challenging. In [Gu, 2011], the authors have proposed a zero-sum game for tracking the target in a sensor network. They have devised a minimax filter to estimate the target position in worst-case noise. The use of a pursuit-evasion game was investigated in [Vidal et al., 2002] for target tracking in an unknown environment. Other target tracking approaches using game theory are studied in [Theodor et al., 1994; Simon, 2006; Pack et al., 2009; Wang et al., 2009b].

e) **Security:** The set of security issues for a sensor network is diverse [Machado and Tekinay, 2008], and tremendous efforts have been made, as described in various literature [Chen et al., 2009], to address this issue. For example, the authors in [Mohi et al., 2009] have modeled the interaction of nodes in WSNs and intrusion detection systems as a Bayesian game formulation, and have used this idea to make a secure routing protocol. By this approach, nodes are motivated to act normally and gain reputation, and the intrusion detection can be performed better by using the history of the game. As the game approaches its Nash equilibrium, it leads to a defense strategy for the network. In [Agah et al., 2005, 2004], the authors show that, by using game theory, a service provider can protect its nodes from malicious attacks by intruders. A coalition game theoretic approach has been used in [Saad et al., 2009a] to increase the information secrecy by distributed cooperation among sensors in the network.

### 2.2.7 The use of game theory for power control in sensor networks

Inefficient use of transmit power for signal transmission over multiple access channels, for example to achieve a desired quality of service (QoS), may create undesirable cascading effects over the whole network [Sengupta et al., 2010]. Consequently, the nature of usage of transmit power by sensors has been studied extensively in recent years. To this end, game theory has been proven to be a very effective tool for efficient power control for sensor networks as found in [Sengupta et al., 2010; Niyato et al., 2007a,b; Ren and Meng, 2009; Wang et al., 2009a; Kandeepan et al., 2012; Huang et al., 2008; Shen et al., 2012; Nguyen and Le-Ngoc, 2011; Ileri et al., 2005; Ren and van der Schaar, 2011; Wu et al., 2011b; Tsiropoulou et al., 2012; Rasti et al., 2009; Tan et al., 2010; Feng et al., 2004; Long et al., 2007; Meshkati et al., 2009b, 2006, 2009a; Kucera et al., 2008; Nadkar et al., 2012; Li et al., 2011b; Buzzi and Saturnino, 2011; Le Treust and Lasaulce, 2010; Lee, 2011; Brown and Fazel, 2011; Wu et al., 2012b, 2011a; Park and van der Schaar, 2012; Xiao et al., 2012]. In this section, in order to provide a better overview of how game theory can be applied by sensors to control their transmit power, we categorize and discuss relevant contributions in this area as following:
2.2. Game theory and its application

i) *Power control via pricing:* Starting with [Saraydar et al., 2002], pricing of services in wireless networks emerges as an effective tool for radio resource management because of its ability to change the user’s behavior toward a more efficient operating point. For instance, the authors in [Ren and van der Schaar, 2011] have studied a power control mechanism using non-cooperative game theory for wireless relay networks. In the game, both the relay and the source-destination sensor pairs (i.e., the relay users) are benefited from their choice of strategies. With the proposed game, the authors have shown that the solution is unique and that it can be achieved in a distributed fashion. However, incomplete information about the network leads to minor performance degradation. Pareto-efficient power control solutions for wireless sensor networks are derived in [Rasti et al., 2009; Tsiropoulou et al., 2012]. Other power control schemes, using pricing, can be found in [Tan et al., 2010; Feng et al., 2004; Nguyen and Le-Ngoc, 2011].

ii) *Power control with QoS constraints:* Efficient power control has been proven as an effective approach [Long et al., 2007] to achieve target QoS such as average source rate, transmission delays [Meshkati et al., 2009b], and the latency associated with the sleep-to-waking up state of the sensor nodes [Schurgers et al., 2002]. In [Meshkati et al., 2009b], the authors have proposed a joint power control and rate control game to model the tradeoff between energy efficient transmission and QoS constraints in wireless networks, and have showed that there could be infinite number of solutions of the game in which the user’s utility is maximized. A reinforcement learning algorithm, with the theory of stochastic fictitious play, is proposed in [Long et al., 2007] to model the users’ choice of power dynamically. Scheduling of sleep and wake up of sensor nodes, for power control, and asynchronous power control for ad hoc networks, are proposed in [Schurgers et al., 2002] and [Kucera et al., 2008] respectively.

iii) *Power control for users co-existence:* With the emergence of cognitive radio, there have been many game theoretic approaches to model the co-existence of primary and secondary users in the same frequency band, and thus alleviate the imbalance between spectrum allocation and use [Nadkar et al., 2012; Wang et al., 2007; Lu et al., 2008; Pang et al., 2010; Yang et al., 2010; Nadkar et al., 2012; Li et al., 2011b; Buzzi and Saturnino, 2011; Le Treust and Lasaulce, 2010]. For instance, in [Nadkar et al., 2012], the authors have considered an overlay spectrum sharing approach, using non-cooperative game theory, in which the unlicensed users (i.e., the players of the game) access the licensed spectrum when the licensed users (i.e., the primary users) are not using it, and this bandwidth sharing is done by using Frequency Division Multiple Access (FDMA). They have proposed an algorithm, which is shown to be standard, and which converges to a unique Nash equilibrium solution. Power control schemes for the cognitive radio paradigm are also investigated in [Wang et al., 2007; Lu et al., 2008; Pang et al., 2010; Yang et al., 2010; Nadkar et al., 2012; Li et al., 2011b; Buzzi and Saturnino, 2011; Le Treust and Lasaulce, 2010].

iv) *Power allocation via negotiation:* Negotiations such as auction and bargaining are very effective mechanisms in resource allocation with QoS constraints [Huang et al., 2008; Kandeepan et al., 2012]. In [Kandeepan et al., 2012], Kandeepan et al. have used a sealed bid procurement auction based game theory to model the power trading for
cooperative communications in ad-hoc network with QoS constraints. They show that the solution of the game, i.e., the Nash equilibrium, changes with the environment and with the residual battery charge of nodes’ batteries. In [Shen et al., 2012; Brown and Fazel, 2011], power allocation among sensors is studied using Nash bargaining, and the possibility of achieving a less computationally expensive solution, which is very close to the optimal solution, is shown.

v) **Power control by forming coalition:** Coalitions are the joining of forces of two or more parties during a conflict of interest with each other [Dugatkin, 1998], and a coalition formation game is a branch of game theory where both network structure and also cost of cooperation play an important role [Saad et al., 2009b; Wu et al., 2012b, 2011a]. In [Wu et al., 2011a], with the aim of increasing the life-time of a sensor network, a coalition formation game was proposed considering a non-transferable utility function [Saad et al., 2009b] for each coalition member. The algorithm proposed, based on a merge-and-split rule with the Pareto order, showed that at least one of the sensor nodes could improve its pay-off (i.e., life-time), without hurting the other sensors, by forming a coalition which is homogeneous and globally stable. A coalition formation game for power control in sensor networks is also discussed in [Wu et al., 2012b].

vi) **Optimal power control solutions for sensor networks:** Various research has been conducted to derive an optimal power control solution for sensor networks, e.g., [Sengupta et al., 2010; Ren and Meng, 2009; Niyato et al., 2007b]. In [Sengupta et al., 2010], a game theoretic formulation with incomplete information is studied to optimally control the transmit power used by a sensor in a CDMA-based distributed sensor network. The authors proved that a Nash equilibrium of the proposed game exists for static channel conditions if the nodes agree to abide by both an upper and also a lower power threshold level. On the other hand, for a time-varying channel a Nash equilibrium exists if the sensors in the network act rationally and transmit only when the channel’s condition is better than a threshold value [Sengupta et al., 2010]. Another optimal power control scheme is studied by Wang et al., in [Wang et al., 2009a], for a wireless sensor network using a Stackelberg game. More literatures on optimal solutions for power control in wireless sensor networks can be found in [Niyato et al., 2007a,b; Ren and Meng, 2009].

Thus, game theory has been used in a significant amount of literature as a solid mathematical framework for controlling sensor’s transmit power in order to increase the life-time of the sensor network. However, besides game theory, distributed transmit beamforming is another distributed signal processing technique that has been extensively used for power control and other applications such as increasing transmission range and rate in sensor networks. To this end, what follows in the next section is a brief introduction to this technique and its applications in the context of wireless sensor networks.

### 2.3 Distributed transmit beamforming and its application

Distributed transmit beamforming (DTB) is a form of cooperative communication in which two or more information sources simultaneously transmit a common message and control the phase of their transmission so as to constructively combine the signals at the intended
2.3. Distributed transmit beamforming and its application

DTB provides benefits in terms of increasing transmission range and transmission rate, energy efficiency, physical layer security, interference reduction etc. [Mudumbai et al., 2009]. As a consequence, it has widely been used in the literature for realizing these benefits [Mudumbai et al., 2009]. In this section, we provide an overview of distributed transmit beamforming, in the context of sensor networks, emphasizing the aspect of phase synchronization.

2.3.1 Introduction to distributed transmit beamforming

Transmit beamforming is a technique in which an information source transmits a radio frequency signal over two or more antennas and align the phases of transmission across antennas such that, after propagation, the signals combine constructively at the receiver [Mudumbai et al., 2009]. For this constructive combination of signals, the sources must: i)- agree on a common message; ii)- transmit the message at the same time; iii)- synchronize their carrier frequencies; and iv)- control their carrier phases. For instance, let us assume that there are \( N \) sources in the network and they are cooperating to send a common message \( m(t) \) towards an intended receiver, as considered in [Mudumbai et al., 2010]. Then, as a distributed beamformer, each source \( n \in [1, 2, ..., N] \) would transmit the common message \( m(t) \) together with transmit power \( \sqrt{E_n} \), after multiplying it with a complex beamforming weight \( A_n e^{j\theta_n} \), towards the destination. If the complex channel gain of transmitter \( n \) to the receiver is \( h_n = a_n e^{j\psi_n} \), where \( a_n > 0 \) represents the attenuation and \( \psi_n \) is the phase response of the wireless channel, the received signal from transmitter \( n \) is given by

\[
y_n(t) = A_n a_n e^{j(\theta_n + \psi_n)} \sqrt{E_n} m(t),
\]

and the total received signal, from \( N \) transmitters, at the receiver is

\[
y = \sum_{n=1}^{N} y_n = \sum_{n=1}^{N} A_n \sqrt{E_n} a_n e^{j(\theta_n + \psi_n)} m(t).
\]

Now, by adopting this distributed transmit beamforming technique, the following key advantages can be achieved in terms of signal strength, power efficiency, transmission rate and transmission range [Mudumbai et al., 2009]:

i) Ideal transmit beamforming with \( N \) antennas results in an \( N^2 \)-fold gain in received power.

ii) Transmit beamforming can achieve an \( N \)-fold increase in transmission range with transmission from \( N \) antennas.

iii) The transmission rate can be increase up to \( N^2 \) with \( N \) transmitters.

iv) Power efficiency is increased. For example, for a desired received power that is fixed, an \( N \) fold decrease in the net transmitted power can be achieved with \( N \) transmitters.

\[\text{The complex gain } A_n e^{j\theta_n} \text{ is adjusted to achieve phase coherence at the receiver.}\]
Due to power efficiency and other benefits, as described above, distributed transmit beamforming has been extensively applied in wireless sensor networks, especially due to the reason that the transmitting nodes need to limit their power usage because it is not practical to replace the batteries frequently.

2.3.2 General application of distributed transmit beamforming in sensor networks

The principal potential benefits of distributed beamforming are two-fold: i) full diversity and ii) $N$-fold increase in energy efficiency. Therefore, it has widely been used in wireless sensor networks for different applications as stated in the following.

i) One key application area is the improvement of transmission rate in sensor networks. It has been shown in [Mukkavilli et al., 2003; Love et al., 2003; Zhou et al., 2005; Liu and Jafarkhani, 2006; Xia and Giannakis, 2006] that even with a limited feedback link, transmit beamforming can provide significant array processing gain, and consequently increase the wireless transmission rate.

ii) Multi-user scheduling is an opportunistic transmission technique to exploit multi-user diversity in wireless communications. Multi-user scheduling has been investigated in the literature addressing multiple-input-multiple-output (MIMO) broadcast systems [Sharma and Ozarow, 2005]. For example, [Wang and Yeh, 2010] discussed a zero-forcing transmit beamforming scheduler combined with multi-user scheduling, where the beamforming weights are multiplied at the transmit antennas and the transmissions are opportunistically scheduled to exploit multi-user diversity. Relevant game theoretic contributions with respect to scheduling in beamforming can be found in [Chen and Wang, 2007; Viswanath et al., 2002; Sharma and Ozarow, 2005].

iii) Distributed transmit beamforming with a uniform power constraint on each sensor node is one of the most investigated areas in sensor networks, e.g., [Zhou et al., 2005; Rashid-Farrokhi et al., 1998; Knopp and Caire, 2002; Zheng et al., 2009]. This is due to the power constraints on the battery operated sensor nodes and infeasibility of frequent battery replacements. Hence, energy efficient transmission in wireless networks and efficient usage of power of sensor nodes has been extensively discussed in [Zhou et al., 2005; Rashid-Farrokhi et al., 1998; Knopp and Caire, 2002; Zheng et al., 2009; Zhang et al., 2009; Joung and Sayed, 2010]. For example, in [Joung and Sayed, 2010], the authors proposed an optimized MIMO transceiver processing technique in which the optimization is based on both zero forcing and minimum mean square error criteria under relay power constraints. The proposed scheme is shown to eliminate efficiently both co-channel interference and self-interference.

While distributed transmit beamforming results in significant performance improvements in wireless networks in terms of energy efficiency, transmission gains and increased transmission range and rate, multi-user carrier synchronization (i.e., frequency and phase) is very important for implementing distributed transmit beamforming in practice [Brown

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11 As the sensors are generally deployed randomly and in a distributed fashion over a large geographical area.
and Poor, 2008]. Hence, tremendous research effort has been made recently to enable the synchronization of carrier signals’ phase and frequency so as to ensure constructive combination of their transmitted signals at the receiver [Mudumbai et al., 2009, 2007; Brown and Poor, 2008; Mudumbai et al., 2010]. To give an overview of this research, in the following section we discuss the relevant literature that has focussed on the carrier phase synchronization of distributed beamforming.

2.3.3 Phase synchronization for distributed transmit beamforming in sensor networks

The benefits of distributed transmit beamforming (DTB) in sensor networks largely depend on the constructive combining of signals from multiple sensors at the receiver. The biggest challenge for this combining is the proper realization of phase and frequency synchronization of high frequency carrier signals [Mudumbai et al., 2007]. This is due to the fact that each sensor in a distributed setting has separate carrier signals supplied by separate local oscillator circuits and these carrier signals are not initially synchronized. Hence, in the absence of carrier synchronization, it is not possible to estimate and pre-compensate the channel phase responses so as to assure phase coherence of all signals at the receiver. To that end, in various literature synchronization of signals in sensor networks has been considered, and in this literature algorithms have been devised to capture to enable coherent transmissions.

In [Mudumbai et al., 2007], the authors show that a large fraction of beamforming gains can be realized even with imperfect synchronization corresponding to phase errors with moderately large variance. To this end, a master-slave architecture is proposed in [Mudumbai et al., 2007], in which a designated master transmitter coordinates the synchronization of other distributed transmitters in the cluster (namely, slave transmitters [Mudumbai et al., 2007]) for beamforming. The authors focused on the effect of phase errors on the average receive signal-to-noise-ratio (SNR), as appropriate for power-limited sensor networks, and made the following comments on the change of receive SNR with the number of transmitters:

i) When the total transmit power of the transmitting nodes is kept constant, the received signal power increases linearly with the number of transmitters as this number increases towards infinity.

ii) For a finite number of transmitters, the expected value of the received signal power linearly increases with the number of transmitters in the cluster.

iii) When the number of transmitters is large enough, such that the central limit theorem can be applied, the variance of the received signal power increases linearly with an increasing number of transmitters.

They have proposed a flexible method\(^{12}\) for carrier phase synchronization in [Mudumbai et al., 2007], which can achieve a beamforming gain with minimal coordination with the base station using channel reciprocity. In fact, in the proposed scheme the master sensor

\(^{12}\)The flexibility comes at the price of complexity, and the necessity of synchronizing each of the slaves individually.
measures the round-trip phase offset, and uses it to estimate the unknown phase offset for each slave sensor, assuming reciprocity of the forward and reverse channel to the slave nodes. The authors presented a stochastic analysis to demonstrate the robustness of beamforming gains with imperfect synchronization, and also demonstrated the trade-off between synchronization overhead and beamforming gains [Mudumbai et al., 2007].

A new round-trip synchronization protocol for enabling distributed beamforming in multiuser wireless communication systems is proposed by Brown et al. in [Brown and Poor, 2008]. In their proposed approach, the authors designed the protocol for multiple sources in which a single frequency is used for all the beacons (from destinations) and carriers. This is beneficial for maintaining channel reciprocity in multipath propagation scenarios as discussed in [Brown and Poor, 2008]. For the time-slotted synchronization protocol that is proposed, the following key assumptions have been made in [Brown and Poor, 2008]: i) there is no a priori phase or frequency synchronization among the oscillators; ii) periodic resynchronization is necessary to avoid unacceptable phase drift during beamforming; and iii) the sensor nodes have no knowledge of their precise position, either absolutely or even relative to each other. The authors proposed the synchronization protocol for a two-source scenario and have then extended it to an $N$-sources case. They have demonstrated that the proposed protocol needs $2N$ time slots for execution. They have also presented how to analytically derive frequency and phase estimation error during beamforming. They also show, via numerical analysis, that the parameters of the synchronization protocol can be selected such that i) a desired level of phase accuracy and reliability can be achieved; ii) the synchronization overhead is small with respect to the amount of time that reliable beamforming is achieved; and iii) reliable beamforming can be achieved with mobile sources [Brown and Poor, 2008].

To achieve the phase alignment at the receiver using distributed phase adaptation, Mudumbai et al. proposed a distributed beamforming algorithm in [Mudumbai et al., 2010] whereby the transmitters require minimal coordination with the receiver through a feedback bit. The algorithm is based on an iterative procedure in which each transmitter adjusts its phase randomly at each iteration and the receiver broadcasts one bit of feedback per iteration indicating whether its net SNR is better or worse than before. If it is better, all transmitters update their best phase perturbations according to their latest used phase perturbations. Otherwise, the transmitters discard the latest used perturbation and keep their previous best phase perturbation. The process continues until phase coherence is achieved at the receiver. The iterative phase convergence of the one-bit feedback algorithm is shown in Fig. 2.3. The authors developed an analytical framework for characterizing the dynamics of the algorithm and for optimizing the algorithm’s parameters. In [Mudumbai et al., 2010], the authors analytically i) show that the effect of random phase perturbations is in fact an additive Gaussian perturbation to the received signal amplitude if the number of transmitters becomes large; ii) derived a probability distribution which can be applied to the received phases under feedback algorithm; and iii) derived a simple expression for the expected convergence rate of the algorithm, and showed that the convergence time of algorithm is linear in the number of transmitters. The authors also demonstrated the following key advantages of their proposed scheme [Mudumbai et al., 2010]:

i) The scheme avoids the need for coordination among the transmitters for training
ii) Using the feedback algorithm, the receiver only needs to estimate the strength of the aggregate signal, not the strength of the signal from each transmitter separately. This is particularly useful when the signal from individual transmitters is very weak.

iii) The feedback algorithm does not require a dedicated training phase. As a consequence, the transmitters can send data to the receiver during the beamforming process, and the receiver can estimate the SNR using the data-carrying signal.

An extension of the algorithm in [Mudumbai et al., 2010] is proposed by Song et al. in [Song et al., 2012], namely the Hybrid algorithm, in which the authors exploited the negative feedback information for the one-bit feedback beamforming algorithm. The major focus of [Song et al., 2012] is to improve the convergence speed of the one-bit feedback algorithm [Mudumbai et al., 2010] retaining its advantages, and, also, to make the algorithm robust in time-varying channels with variable rates of phase shifts. To this end, the authors proposed a hybrid algorithm which employs two schemes to speed up the convergence process including: i) exploiting the negative feedback information in a single time slot (scheme-1), and ii) exploiting negative feedback information in successive time slots (scheme-2). On the one hand, in scheme-1, the authors have made use of the negative feedback information in a single time slot to enhance the probability of generating better phase changes. This is due to the fact that, as shown in [Song et al., 2012], if a positive perturbation leads to performance degradation, usually, a negative perturbation of the same phase offset will lead to performance improvement and vice-versa. On the
other hand, scheme-2 of the proposed algorithm use the negative feedback information in successive time slots to adjust perturbation size.

An important aspect of [Song et al., 2012] is that the hybrid algorithm can maintain its fast convergence speed while tracking the very sensitive phase changes in time-varying channels. The authors have provided two solutions in [Song et al., 2012] to track the time-varying channel without reducing the convergence speed. First, when the phase drift due to time variation is very small compared to the phase perturbation, the effect of channel variation on the received signal strength can be assumed to be small, and the hybrid algorithm is shown to become the one-bit feedback algorithm [Mudumbai et al., 2010] to track time-variation. However, this process requires the knowledge of the speed of phase drift. The second solution does not require any knowledge of phase drift speed and operates in two modes: i)- normal mode and ii)- testing mode. A detailed explanation of both modes can be found in [Song et al., 2012]. Based on the hybrid algorithm, the following improvements are demonstrated in [Song et al., 2012]:

i) The hybrid algorithm can easily be applied in practical implementations, and does not require any more information exchange or hardware changes than the one-bit feedback scheme [Mudumbai et al., 2010].

ii) The hybrid algorithm can enhance the convergence speed of phase alignment by over 40% compared to the one-bit feedback algorithm [Mudumbai et al., 2010].

iii) The hybrid algorithm can easily be extended to track a time-varying channel without the knowledge of channel state information by adding one time slot per size period.

iv) The hybrid algorithm (with an additional simple modification) has the ability to adjust perturbation sizes adaptively according to the rate of phase drift in channel variations.

Besides the above discussed literature, more studies on phase synchronization for distributed transmit beamforming can be found in [Thibault et al., 2011; Song et al., 2010a; Preuss and Brown, 2010; Lin et al., 2010; Ochiai et al., 2005b; Bucklew and Sethares, 2007; Song et al., 2010b; Fertl et al., 2008; Mudumbai et al., 2005], amongst others.

2.4 Concluding Remarks

In this chapter, we have provided an overview of two distributed signal processing techniques, i.e., game theory and distributed transmit beamforming, and their application in smart networks that includes both smart grids and wireless sensor networks. By studying the relevant existing literature we have shown that game theory is a very effective tool to study the competition and cooperation of the distributed nodes of a smart network, and can capture the decision making process of rational users in the network. We have also shown the effectiveness of distributed transmit beamforming for signal transmission in wireless sensor networks.

We have mainly focused on two main application areas of smart networks. These application areas are energy management in smart grids and power control in wireless sensor networks. For energy management, we have explained the relevant game theoretic

\footnote{Which includes energy users in smart grids and the sensor nodes in wireless sensor networks.}
approaches shown to be effective for demand-side management in smart grids. Additionally, we have also discussed the game theoretic approaches in the existing literature on sensor networks for controlling the transmit power of sensors in the network. We, further, extend our focus on distributed transmit beamforming, which is also an effective technique for power control of sensor networks and have discussed the relevant literature on phase synchronization, which is very important to achieve efficient use of DTB. However, although a lot has been done for power control and energy management in the context of smart networks, as shown in the above literature review, still there are ample opportunities in areas of energy management and power control where novel contributions are needed. This dissertation makes various important novel contributions in these areas, starting with devising an optimal charging strategy for electric vehicles in a smart grid when the energy at the grid is limited, as explained in the next chapter.
Economics of Electric Vehicle Charging in Smart Grid

In this chapter, a non-cooperative Stackelberg game is used to study the problem of grid-to-vehicle energy exchange between a smart grid and plug-in electric vehicle groups (PEVGs), e.g., a parking lot or groups of vehicles in the same vicinity. In this game, the smart grid acts as a leader, which decides on its selling price per unit of energy so as to optimize its revenue while ensuring the PEVGs’ participation. On the other hand, the PEVGs act as the followers of the game, which need to decide on their charging strategies so as to optimize the tradeoff between the benefit from battery charging and associated cost. Using variational inequality, it is shown that the proposed game possesses a socially optimal Stackelberg solution in which the grid optimizes its price while the PEVGs choose their equilibrium strategies. A novel distributed algorithm is proposed that enables the PEVGs and the smart grid to reach this optimal solution, and this is assessed by extensive simulation. Further, the model is extended to time-varying case that can incorporate and handle slowly varying environments.

3.1 Motivation

With the increasing concerns for energy conservation and the environment, it is expected that plug-in electric vehicles (PEVs), which includes both battery only electric vehicles (BEVs) and plug-in hybrid electric vehicles (PHEV) [Bauer et al., 2010], will play a major role in the future smart grid (SG) [Rotering and Ilic, 2011]. Hence, several countries are working on establishing novel PEV policies and plans because of their significant environmental advantages and cost savings [Foley et al., 2010b]. However, the large scale deployment of PEVs will introduce new challenges in the design of future SGs. These challenges includes developing optimal charging strategies for connected PEVs, ensuring efficient communications between PEVs and the grid, optimal and cost effective deployment of charging points at charging stations, electric storage management, and managing energy exchange between regular loads of the grid and the PEVs [Rotering and Ilic, 2011; Samadi et al., 2010; Wu et al., 2012a; Pan et al., 2010; Sojoudi and Low, 2011; Turitsyn et al., 2010; Clement-Nyns et al., 2010; Käisisch et al., 2010; Couillet et al., 2012; Su and Chow, 2011; Foley et al., 2011, 2010a, 2009].

One of the key challenges of widespread penetration of the PEVs in the power network is the choice of an optimal charging strategy for the PEVs. This is mainly due to
the fact that the integration of PEVs into the network has a major impact on the power grid, and can potentially double the average load [Samadi et al., 2010; Zhenpo and Peng, 2010]. The simultaneous charging of several PEVs in a particular area can overload the network, and thus, lead to an interruption of services for other consumers. The problem of PEV charging and its impact on the power distribution grid and electricity market have been addressed in [Rotering and Ilic, 2011; Bauer et al., 2010; Foley et al., 2010b; Turtysyn et al., 2010; Sojoudi and Low, 2011] and [Stroehle et al., 2011]. However, little has been done to develop distributed models and algorithms that can capture the interactions between PEVs and the grid, in a grid-to-vehicle scenario. Hence, there is a need to develop solutions that capture the often conflicting objectives between the SG, which seeks to maximize its revenue, and the PEVs, which seek to optimize their charging behavior. Because of the limited grid capacity and PEVs’ energy demands, it is of interest to develop a model that can capture the decision making process of the PEVs and the grid when the grid’s limited energy needs to be allocated among the PEVs based on their needs.

The main contribution of this chapter is to provide a comprehensive analytical framework that is suitable for capturing the interactions between an SG and a number of PEV groups (PEVGs), e.g., a parking lot, which must decide on their charging profiles. We model the problem as a generalized Stackelberg game in which the SG is the leader and the PEVGs are the followers. The objective of the PEVGs is to strategically choose the amount that they need to charge, so as to optimize a utility that captures the tradeoff between the charging benefits and the associated costs, given various practical constraints on the PEVGs and the main grid. Based on the strategy choices of the PEVGs, the leader aims to optimize its price so as to maximize its revenues. We analyze the properties of the resulting game within the studied model, including existence of an equilibrium and optimality, and show that there exists an efficient (i.e., socially optimal) generalized Stackelberg equilibrium. We show that, due to the coupled capacity constraints between the PEVGs, the non-cooperative followers’ game leads to a generalized Nash equilibrium, and the solution enables the capture of not only the charging behavior of the vehicles but also the decisions made by the SG. We propose a novel algorithm that the PEVGs and the grid can use, in a distributed manner, so as to reach the desired equilibrium. We also show that the proposed algorithm enables the system to adapt to time-varying environmental conditions such as arrival/departure of PEVs. Using extensive simulations, we assess the properties of the proposed scheme.

The rest of this chapter is organized as follows: Section 3.2 describes the system model. In Section 3.3, we formulate the non-cooperative generalized Stackelberg game and we discuss its properties. In Section 3.4, we propose a distributed algorithm for finding the equilibrium. Adaptation of the proposed game to time-varying conditions is shown in Section 3.5. Numerical results are analyzed in Section 3.6, and finally some concluding remarks are drawn in Section 3.7.

### 3.2 System Model

Consider a power system, as shown in Fig. 3.1, consisting of a single power grid (i.e., an SG), several primary and secondary load subscribers, or consumers, and a smart energy manager (SEM). Here, the SG refers to the main electric grid, which is connected to the
area of interest via one or more substations. Further, we consider that the SG is servicing a certain area or groups of primary consumers such as industries, houses, schools and offices. After meeting the demands of the primary consumers the grid wishes to sell its excess of energy (if any) to the secondary users, such as PEVs, connected to it in that area. Consider a number of groups of PEVs (from here on, we use the term PEVG to denote a group of PEVs acting as a single PEV entity), which are connected to the grid at peak hours of energy demand, e.g., from 12 pm to 4 pm\footnote{The proposed scheme is not only restricted to the considered period but can also apply for any time duration.} [Galus and Andersson, 2008a]. The total charging period for the PEVGs is considered to be divided into multiple time slots. Each time slot has a duration of anywhere between 5 minutes and half an hour based on the changing traffic conditions of the PEVs in the group [Lin, 2001]. For a particular time slot, the power grid has a maximum energy $C$ that it can sell to the connected $N$ PEVGs, allowing them to meet their demand\footnote{While this chapter is focused on the interactions between PEVGs and the grid, when the grid has energy that can be provided to the PEVGs, it can also be extended to the cases in which there are multiple energy sources beyond the main grid.}. The power grid will set an appropriate price $p$ (per unit of energy) for selling its surplus so as to optimize its revenue.

Each PEVG $n \in N$, where $N$ is the set of all $N$ PEVGs, will request a certain amount of energy $e_n$ from the grid so as to meet its energy requirements (e.g., to go back home after office work). This demand of energy may vary based on different parameters such as the battery capacity $b_n$ of the PEVG, the available energy in the PEVG’s battery at the time of plug-in to the grid, the price $p$ per unit of electricity and the nature of usage (e.g., two identical PEVGs with different travel plans may need different amounts of energy). During peak hours, the available energy for servicing PEVGs is often limited [Galus and Andersson, 2008a], and, thus, the PEVGs will request only the amount needed with respect to their immediate need for charging. Since the net energy $C$ available for the PEVGs at
the grid is fixed, the demands of the PEVGs must satisfy:

$$\sum_{n} e_n \leq C. \quad (3.1)$$

Given the amount requested by the PEVGs, the SG sets a price $p$ per unit of energy so as to optimize its revenue from selling energy by strategically choosing $p$. Although the SG can choose the price $p$ within any range to maximize its total revenue, a very large $p$ may compel a PEVG to withdraw its demand from the grid and to search for alternate markets or even wait until the prices drop. Therefore, an optimal price needs to be chosen by the grid operator, which is not very high (to avoid losing customers), and also not very low (to avoid losing revenue), in an effort to maximize its profit.

To successfully complete the energy trading, the PEVGs and the grid interact with each other and agree on the energy exchange parameters, such as the selling price and the amount of energy demanded, that meet the objectives of the both sides. It is assumed that PEVGs maintain their privacy, and hence do not inform each other of the amount of energy they will request from the SG. The amount of requested energy $e_n$ is determined by the physical characteristics of PEVG $n$ as well as by the tradeoff between the potential benefit that PEVG $n$ is expecting from buying $e_n$ and the selling price $p$ of the grid. Moreover, the selling price $p$ per unit of energy is strongly dependent upon the amount demanded by the PEVGs as well as the number of PEVGs that are connected to the grid. This is due to the fact that the amount of energy at the grid is fixed and, thus, as the number of PEVGs increases, the amount that each PEVG can acquire becomes smaller. As a result, the grid can set a higher price to increase its revenues.

It is clear that the demands of the connected PEVGs are coupled through the energy constraint in (3.1), and are also dependent on the physical characteristics and capabilities of the PEVGs. Also it is important to note that the price set by the grid is dependent on the demand of each PEVG. Thus, the main challenges faced when developing an approach that can successfully capture the decision making process of both the PEVGs and the grid are:

i) modeling the decision making processes and the interactions between the connected PEVGs in the network given the constraint in (3.1);

ii) developing an algorithm that enables the PEVGs in the network to strategically decide on the amount of energy that they will request from the grid so as to optimize their satisfaction levels given the constraint in (3.1);

iii) enabling the grid to optimize its price while capturing the tradeoff between the PEVGs’ participation and revenue maximization.

To address these challenges we propose a framework based on non-cooperative game theory.
3.3 Non-cooperative generalized Stackelberg game

3.3.1 Game formulation

To formally study the interactions between the SG and the PEVGs, we use a Stackelberg game [Başar and Olsder, 1999], which is a type of non-cooperative game that deals with the multi-level decision making process of a number of independent decision makers or players (followers) in response to the decision taken by a leading player (leader) [Başar and Olsder, 1999]. Hence, we formulate a non-cooperative Stackelberg game in which the SG is the leader and the PEVGs are the followers. This game is defined by its strategic form, \( \Gamma = \{ \mathcal{N} \cup \{ \text{SG} \} \}, \{ E_n \}_{n \in \mathcal{N}}, \{ U_n \}_{n \in \mathcal{N}}, L(p), p \}, \) having the following components:

i) The PEVGs in \( \mathcal{N} \) act as the followers in the game and respond to the price set by the SG.

ii) The strategy of each PEVG \( n \in \mathcal{N} \), which corresponds to the amount of energy demanded \( e_n \in E_n \) from the grid satisfying the constraint \( \sum_n e_n \leq C \).

iii) The utility function \( U_n \) of each PEVG \( n \) that captures the benefit of consuming the demanded energy \( e_n \).

iv) The utility function \( L(p) \) for the SG (leader of the game), which captures the total profit that the grid can receive by selling the surplus of energy with price \( p \).

v) The price \( p \) per unit of energy charged by the SG.

Utility Function of a PEVG

For each PEVG \( n \in \mathcal{N} \), we define a utility function \( U_n(e_n, e_{-n}, s_n, b_n, p) \), which represents the level of satisfaction that a PEVG obtains as a function of the energy it consumes. Here, \( b_n \) is the battery capacity of PEVG \( n \), and \( e_n \) is the requested energy from the grid; \( s_n \) is the satisfaction parameter of PEVG \( n \), which is a measure of the satisfaction a PEVG can achieve from consuming one unit of energy. This \( s_n \) may depend on the PEVG’s battery state at the time of plug-in to the grid, the available energy at the grid and/or the travel plan of the corresponding PEVG. For example, a PEVG 1 having less need for energy than another PEVG 2 (e.g., due to having a fuller battery) will need less energy than PEVG 2 to attain the same satisfaction level (i.e., \( s_2 < s_1 \)). The energy demand may vary based on the battery capacity and/or the satisfaction parameter of each PEVG. The price per unit of energy can also affect the demand of the PEVG. Thus, the properties that the utility of a PEVG must satisfy are as follows:

i) The utility functions of the PEVGs are considered to be non-decreasing because each PEVG is interested in consuming more energy if possible, unless it reaches its maximum consumption level. Mathematically,

\[
\frac{\delta U_n(e_n, e_{-n}, s_n, b_n, p)}{\delta e_n} \geq 0. \tag{3.2}
\]
ii) The marginal benefit of a PEVG is considered as a non-decreasing function, as the level of satisfaction of the PEVGs gradually gets saturated as more energy is consumed, i.e.,
\[
\frac{\delta^2 U_n(e_n, e_{-n}, s_n, b_n, p)}{\delta e_n^2} \leq 0.
\] (3.3)

iii) Hereinafter, we consider that, for a fixed consumption level \(e_n\), a larger \(b_n\) implies a larger \(U_n(e_n, e_{-n}, s_n, b_n, p)\) and a larger \(s_n\) leads to a smaller \(U_n(x_n, e_{-n}, s_n, b_n, p)\). So we have
\[
\frac{\delta U_n(e_n, e_{-n}, s_n, b_n, p)}{\delta b_n} > 0,
\] (3.4)
and
\[
\frac{\delta U_n(e_n, e_{-n}, s_n, b_n, p)}{\delta s_n} < 0.
\] (3.5)

iv) The price per unit of energy set by the grid affects the utilities of the PEVGs and the utility of a PEVG decreases for a higher price. That is,
\[
\frac{\delta U_n(e_n, e_{-n}, s_n, b_n, p)}{\delta p} < 0.
\] (3.6)

In this work we assume that, for a fixed price \(p\), the utility achieved by any PEVG \(n\) consuming energy \(e_n\) is
\[
U_n(e_n, e_{-n}, s_n, b_n, p) = b_n e_n - \frac{1}{2} s_n e_n^2 - p e_n,
\] (3.7)
where \(e_n \in \left[0, C - \sum_{k=1,k\neq n}^{N} e_k\right]\) and \(e_{-n} = [e_1, e_2, \ldots, e_{n-1}, e_{n+1}, \ldots, e_N]\). A sample utility function for this case is shown in Fig. 3.2 for three PEVs with different capacity and satisfaction parameter. A PEV with capacity \(b_n\) receives a utility \(U_n\) after consuming \(e_n\) amount of energy. From Fig. 3.2 and (3.7), the utility of PEVG \(n\) is affected by its battery capacity \(b_n\). This is due to the fact that a PEVG with higher \(b_n\) will have higher marginal utility and thus, needs to consume more energy to reach its maximum satisfaction level [Fahrioglu and Alvarado, 1999]. The utility also depends on the satisfaction parameter \(s_n\) of the PEVG. PEVGs with the same capacity but with a different satisfaction parameter will have different marginal utilities and, thus, will be satisfied by different amounts of energy. To this end, we assume that a PEVG does not consume any energy beyond its maximum satisfaction level, i.e., \(U_n = 0\) if \(e_n > (e_n^* - e_{n\text{ini}})\), where \(e_{n\text{ini}}\) is the initial energy in PEVG’s battery at the time of plug-in to the grid, and \(e_n^*\) is the energy that maximizes its utility within the given constraint in (3.1).
Utility Function of the Power Grid

A PEVG \( n \) that consumes \( e_n \) MWh of electricity during a designated period of time at a rate \( p \) per MWh is charged \( p e_n \), which is the cost imposed by the SG on the PEVG. The objective of the SG is to maximize its revenue by selling the available energy surplus to the PEVGs after meeting the demand of its primary consumers, and also to control the nature of energy consumption of the PEVGs. While this energy surplus \( C \) is fixed, the SG wants to set a price \( p \) per unit of energy so as to optimize its revenue, given the demands of the PEVGs. Thus, we assume the utility function for the grid is

\[
L(p, e_n(p)) = p \sum e_n,
\]

which captures the total revenue of the grid when selling the energy required by all PEVGs at a price \( p \) per unit of energy.

In this proposed game, the SG can control the price \( p \) per unit of energy it wishes to sell. Connected PEVGs respond to the price by demanding a certain amount of energy, given the constraint in (3.1), so as to maximize their utilities. Thus, for a fixed price \( p \), the objective of any PEVG \( n \) is

\[
\max_{e_n \in (C - e_n)} U_n(e_n, e_{-n}, s_n, b_n, p), \quad \text{s. t.} \quad \sum e_n \leq C.
\]
Here, we can see that the amount of energy demanded by each PEVG \( n \) depends, not only on its own strategies and price, but also on the demand of other PEVGs in the network through (3.1) and this constraint is the same for all players. This leads connected PEVGs to engage in a non-cooperative resource sharing game, which is a jointly convex generalized Nash equilibrium problem (GNEP) due to the same shared constraint (3.1). Note that, in game theory, a non-cooperative game in which the players’ actions are coupled solely through the constraints, such as in the proposed model, is a special class of games whose solution is the generalized Nash equilibrium [Facchinei and Kanzow, 2007; Arganda et al., 2011], and hence, the proposed followers’ game, for any price \( p \), is a non-cooperative resource sharing game whose solution is the generalized Nash equilibrium (GNE). Then, given all the PEVGs’ demands are at the GNE, the leader, i.e., the grid, optimizes the price to maximize its revenue. Thus, for the given GNE demands of the PEVGs, the objective of the grid is

\[
\max_p L(p) = \max_p \sum_n p e_n(p). \tag{3.10}
\]

Thus, one suitable solution for the formulated game \( \Gamma \) is the Stackelberg equilibrium at which the leader reaches its optimal price, given the followers’ optimal response at their GNE. At this equilibrium, no player (leader or follower) can improve its utility by unilaterally changing its strategy. In classical Stackelberg games, the followers typically choose their Nash equilibrium strategies. In our model, due to the coupled strategies as per (3.1), the PEVGs need to seek a GNE instead of a classical Nash equilibrium. To this end, hereinafter, we refer to our game as a generalized Stackelberg game (GSG) whose solution is the generalized Stackelberg equilibrium (GSE) in which the followers reach a GNE.

**Definition 3.3.1.** Consider the GSG \( \Gamma = \{ (N \cup \{SG\}), \{ E_n \}_{n \in N}, \{ U_n \}_{n \in N}, L(p), p \} \) defined in 3.3.1 where \( U_n \) and \( L(p) \) are given by (3.7) and (3.8) respectively. A set of strategies \( (e^*, p^*) \) constitutes the GSE of this game, if and only if it satisfies the following set of inequalities:

\[
U_n(e^*_n, e_{-n}^*, s_n, b_n, p^*) \geq U_n(e_n, e_{-n}^*, s_n, b_n, p^*), \quad \forall e^*_n \in e^*, \quad n \in N, \quad \sum_n e_n \leq C \tag{3.11}
\]

and

\[
L(p^*, e^*) \geq L(p, e^*). \tag{3.12}
\]

Thus, when all the PEVGs’ demands are at the GSE, no PEVG can improve its utility by deviating from its GSE demand and similarly, no price other than the optimal price\(^\text{16}\) set by the grid at the GSE, can improve the utility for the grid.

### 3.3.2 Existence and efficiency of GSE

In a non-cooperative game, the existence of an equilibrium solution (in pure strategies) is not always guaranteed [Başar and Olsder, 1999]. Therefore, for our follower game, we

\(^{16}\)The optimal price \( p^* \) maximizes the utility of the SG for the given GNE demand vector \( e^* \) of the PEVGs in the smart grid network.
need to investigate the existence of the GNE in response to a price $p$. Specifically, we are interested in investigating the existence and the property of a variational equilibrium (VE), which is also a GNE [Facchinei and Kanzow, 2007], for our case. This is due to the fact that a VE is more socially stable than another GNE (if there exists any), and thus, is a desirable target for any algorithm to achieve [Arganda et al., 2011]. Particularly for the proposed case, where a number of PEVGs in the SG network are demanding energy from a constrained reserve, an efficient VE would be the most appropriate solution to be considered. Hereinafter, we use VE and GNE interchangeably.

**Theorem 3.3.1.** For a fixed price $p$, a socially optimal VE exists in the proposed game $\Gamma$ between the PEVGs connected to the grid.

**Proof.** First, clearly, by adding the quantity $\sum_{m \neq n} (b_m e_m - \frac{1}{2} s_m e_m^2) - \sum_{m \neq n} p e_m$ to $U_n$ in (3.7) and treating the resulting utility function as the new objective function for PEVG $n$ will not affect the solution [Başar and Srikant, 2002]. Hence, the original game is equivalent to one in which all PEVGs have the identical utility function,

$$U(e_1, \ldots, e_N; s_1, \ldots, s_N; b_1, \ldots, b_N; p) = \sum_{m=1}^{N} \left( b_m e_m - \frac{1}{2} s_m e_m^2 \right) - p \sum_{m=1}^{N} e_m.$$  

(3.13)

Hence, to determine the socially stable outcome of the game, the existence of a solution that maximizes (3.13) is our main concern.

Using the method of Lagrange multiplier [Bertsekas, 1995], the Karush-Kuhn-Tucker (KKT) conditions for the $n^{th}$ player GNEP is given by,

$$\langle F(e_n), e - z^* \rangle \geq 0,$$

$$\lambda_n \left( \sum n e_n - C \right) = 0, \lambda_n \geq 0$$

(3.14)

where $\lambda_n$ is the Lagrange multiplier for PEVG $n$.

First, we note that, for a fixed price $p$, the followers’ game admits a jointly convex GNEP, hence, the solution of the GNEP with (3.1) can be found via a variational inequality VI$(E, F)$. This essentially reduces to determining a vector $z^* \in E \subset R^n$, such that $(F(z^*), z - z^*) \geq 0$, for all $z \in E$ where $E$ is the set in the definition of joint convexity and $F(e) = -(\nabla U_n(e))_{n=1}^{N}$ [Arganda et al., 2011]. The solution of VI$(E, F)$ is a variational equilibrium (VE).

Now the KKT conditions can be written as [Facchinei and Kanzow, 2007],

$$F(e) + \lambda \nabla e \left( \sum n e_n - C \right) = 0,$$

$$\lambda \left( \sum n e_n - C \right) = 0, \lambda \geq 0.$$  

(3.15)

Note that the subscript on Lagrange multiplier $\lambda$ is dropped in (3.15). This is due to the
3.4. Proposed solution and algorithm

fact that the solution of a jointly convex GNEP is a VE if and only if the shared constraint has the same multiplier $\lambda$ for all players [Facchinei and Kanzow, 2007].

Now from the definition of $F$ [Arganda et al., 2011]

$$
F = \begin{bmatrix}
    s_1 e_1 + p - b_1 \\
    s_2 e_2 + p - b_2 \\
    \vdots \\
    s_n e_n + p - b_n 
\end{bmatrix}.
$$

(3.16)

Therefore, the Jacobian of $F$ is,

$$
JF = \begin{bmatrix}
    s_1 & 0 & \cdots & 0 \\
    0 & s_2 & \cdots & 0 \\
    \vdots & \vdots & \ddots & \vdots \\
    0 & 0 & \cdots & s_n
\end{bmatrix}.
$$

(3.17)

$JF$ is a diagonal matrix with all positive diagonal elements. Hence, $JF$ is positive definite on $E$ and $F$ is strictly monotone. Thus, the GNEP admits a unique global VE solution [Facchinei and Kanzow, 2007].

Because of the jointly convex nature of the GNEP the VE is the unique global maximizer of (3.13) [Facchinei and Kanzow, 2007], which completes the proof.

As a result, from Theorem 3.3.1, the GSE, in which the SG sets its optimal price in response to the VE demands of the PEVGs, admits the socially optimal solution of the proposed game.

3.4 Proposed solution and algorithm

In this section, we formulate the GNEP among the followers as a VI problem$^{17}$ and propose an algorithm that leads to the socially optimal VE. Note that the VE further leads to the GSE state of the game as defined in Definition 3.3.1. Now, we first state the following corollary and then explain the solution method for the considered GNEP.

**Corollary 3.4.1.** The VI associated with the proposed GNEP of the connected PEVGs for a price $p$ is a strongly monotone VI and thus, the unique VE can be calculated by solving a monotone VI.

**Proof.** By Theorem 3.3.1, we know that the VI associated with the proposed GNEP of the connected PEVGs for any fixed price $p$ is a strongly monotone VI and the VE is unique. It is shown in [Facchinei and Kanzow, 2007] that the solution of a VE can be calculated by solving a monotone VI. Hence, the unique VE solution of the PEVGs’ GNEP of energy demand within constraint (3.1) can be calculated by solving the strongly monotone VI($E, F$).

---

$^{17}$Given $E \subseteq \mathbb{R}^n$ and $F : \mathbb{R}^n \to \mathbb{R}^n$, the VI($E, F$) consists of finding a vector $z^* \in E$ such that $(F(z^*), z - z^*) \geq 0$, for all $z \in E$. 
For solving the monotone VI in our proposed game, we consider the Solodov and Svaiter (S-S) hyperplane projection method [Solodov and Svaiter, 1999; Tinti, 2003]. In the S-S method, two projections per iteration are required using a geometric interpretation (see [Solodov and Svaiter, 1999]). This hyperplane projection algorithm works as follows [Solodov and Svaiter, 1999]: Suppose we have \( e^k \), which is a current approximation to the solution of \( VI(E, F) \). First, the point \( r^e_k = \text{Proj}_E[e^k - F(e^k)] \) is computed\(^\text{18}\). Next, the line segment between \( e^k \) and \( \text{Proj}_E[e^k - F(e^k)] \) is searched for a point \( z^k \) such that the hyperplane \( \partial H_k := \{ e \in \mathbb{R}^n : \langle F(z^k), e - z^k \rangle = 0 \} \) strictly separates \( e^k \) from any solution \( e^* \) of the problem. A computationally inexpensive Armijo-type procedure [Armijo, 1966] is used in this S-S algorithm to find such \( z^k \). Once the hyperplane is constructed, the next iterate \( e^{k+1} \) is computed by projecting \( e^k \) onto the intersection of the feasible set \( E \) with the hyperspace \( H_k := \{ e \in \mathbb{R}^n : \langle F(z^k), e - z^k \rangle \leq 0 \} \), i.e., \( E \cap H_k \), which contains the solution set [Solodov and Svaiter, 1999] [Tinti, 2003].

Next, we show how the PEVGs reach the VE, for a price \( p \), following the optimization of price by the grid when all PEVGs are in VE. Then, we detail the algorithm at the end of this section.

### 3.4.1 GNE for a fixed \( p \)

From (3.13) and (3.15), for any PEVG \( n \), the solution of the KKT system of variational inequalities is

\[
\begin{align*}
  b_n - s_n e_n - p - \lambda &= 0, \\
  \lambda \left( \sum_n e_n - C \right) &= 0; \quad \lambda \geq 0.
\end{align*}
\]

(3.18) \hspace{1cm} (3.19)

For \( \lambda > 0 \), the inequality constraint in (3.19) becomes as equality and hence at the VE,

\[
\sum_n e_n = C.
\]

(3.20)

Thus, for a fixed \( p \) at the grid, the sum of demands of all the PEVGs connected to the grid at the VE is equal to the total energy \( C \) available at the grid.

At the peak hours of demand [Galus and Andersson, 2008a], energy in the grid is a scarce commodity and, hence, all the PEVGs compete with one another for a fair allocation of the available grid’s energy. Thus, for the formulation of the proposed game between the PEVGs the available energy at the grid should be less than the total energy consumption capacity of the connected PEVGs. This is essential for avoiding the trivial case in which all the PEVGs should get an allocation equal to their capacity. From (3.18), we have

\[
\begin{align*}
  b_n - s_n e_n - p &> 0 \\
  \text{i.e., } b_n &> s_n e_n + p.
\end{align*}
\]

\(^\text{18}\)\( \text{Proj}_E(z) = \arg \min \{ ||w - z||, \ w \in E \} \forall z \in \mathbb{R}^n \).
3.4. Proposed solution and algorithm

Taking all \( N \) PEVGs connected to the grid into considerations (3.21) becomes

\[
\sum_n b_n > pN + \sum_n s_n e_n, \tag{3.22}
\]

which leads to the following proposition:

**Proposition 3.4.1.** To achieve the maximum utilities at the VE, within the constraint in (3.1), the total capacities of the \( N \) grid connected PEVGs must be greater than their total VE demand plus a constant equal to \( pN \).

For the special case in which the PEVGs have different capacities (i.e., \( b_n \) is different for each \( n \)) but the same satisfaction parameter (i.e., \( s_n = s \) for all \( n \in N \)), (3.22) becomes

\[
\sum_n b_n > pN + sC, \tag{3.23}
\]

where \( \sum_n e_n = C \), from (3.20).

Now, while Proposition 3.4.1 holds for a price \( p \) at the grid, from (3.18), the demand of the PEVGs at the VE is given by

\[
e_n^*(p) = \frac{b_n - (p + \lambda)}{s_n}, \tag{3.24}
\]

where

\[
\lambda = b_n - s_n e_n^* - p \text{ for any } n \in N. \tag{3.25}
\]

3.4.2 Price Optimization

Having analyzed the followers’ game, we now show how the SG can set its optimal price \( p^* \) given the VE of the PEVGs.

For the KKT system of VI described in (3.18) and (3.19), the selling price for per unit energy is

\[
p \leq b_n - s_n e_n. \tag{3.26}
\]

Again, from (3.24), the demand for energy by PEVG \( n \) at the VE is \( e_n^* \). So the price per unit of energy satisfies

\[
p \leq b_n - s_n e_n^*. \tag{3.27}
\]

Now, with the condition in (3.27) and the utility of the SG from (3.8), which is \( L(p) = p \sum_n e_n \) over \( p \geq 0 \), this dictates that the revenue-maximizing price of the grid should be
Algorithm 3.1: Algorithm to reach GSE

1. Solving VI
   Each PEVG $n \in N$ submits its initial demand $e_{n,ini}$ to the SEM.
   
   repeat
   
   a) The SEM checks $\lambda_n^k$ for the demand $e_n^k$ of all $n \in N$ using (3.15).
   b) Each PEVG $n \in N$ updates its demand $e_n^{k+1}$ using the S-S hyperplane projection method [Tinti, 2003].
      
      i) The PEVG $n$ computes the projection $r(e_n^k)$.
      ii) The PEVG $n$ updates $e_n^{k+1}$ equal to $r(e_n^k)$
      
      if $r(e_n^k) = 0$ and submit to the SEM.
      
      otherwise
      
      iii) The PEVG $n$ determine the hyperplane $z_n^k$ and the half space $H_n^k$ from the projection.
      iv) The PEVG $n$ updates its demand $e_n^{k+1}$ from the projection of its previous demand $e_n^k$ on to $E \cap H_n^k$ and submit to the SEM.
   
   until all $\lambda_n$ converges to $\lambda \geq 0$.

   The SEM determines the VE demand of the PEVGs.

2. Optimizing Price
   a) The SEM submits the VE demand of the PEVGs to the grid.
   b) The grid optimizes its price $p$ to $p^*$ using (3.28).

   The VE demand and price of the GSG are achieved.

the upper limit of (3.27). Thus, the optimal price\(^{19}\) of the proposed GSG is

$$p^* = b_n - s_ne_n^*.$$  \hfill (3.28)

3.4.3 Proposed Algorithm

In order to reach the equilibrium, the PEVGs and the smart grid must make their strategy choices with little communication between one another. To this end, we propose an algorithm that all the PEVGs and the grid can implement in a distributed fashion and reach the efficient GSE of the game. We note that, in a jointly convex GNEP where $F(e) = -\left(\nabla U_n(e)\right)_{n=1}^N$ is strongly monotone, such as our proposed game, the solution of the VI converges to a unique VE [Facchinei and Kanzow, 2007] when the demand of each PEVG $n$ is such that the parameter $\lambda_n$ in (3.15) for all $n \in N$ possesses the same value $\lambda \geq 0$. In other words, if the parameter $\lambda_n$ for all $n \in N$ converges to a single value $\lambda \geq 0$, $e^*$, the demand vector of the PEVGs contains the VE demand of the PEVGs.

For our game, we can use this property and propose an algorithm in which each PEVG updates its demand iteratively, until all $\lambda_n$ converges to a single value $\lambda \geq 0$. In this algorithm we use the hyperplane projection method to solve the proposed VI problem. By using this technique, we guarantee that our algorithm always converges to a non-empty solution if $F$ is strongly monotone [A. Nedic and U. V. Shanbhag, 2008], which is always verified in our game, as previously shown. Thus, the proposed algorithm is guaranteed to converge to a unique solution of the game, given the demand constraints of the PEVGs and the grid’s capacity $C$. As we explained in Section 3.3, this convergence implies that the proposed GSG reaches its GSE as soon as the grid optimizes the price using (3.28) for the given VE demand of the PEVGs.

\(^{19}\)From (3.25), at $p = p^*$, the slack variable $\lambda = 0$. Hence $\lambda_n$, $\forall n$ converges to $\lambda = 0$ as the game reaches its GSE.
3.5. Adaptation to Time-Varying Conditions

The proposed algorithm uses the S-S hyperplane projection method [Tinti, 2003] to calculate the demand at the VE for a price \( p \). Each PEVG and the SG can implement the proposed algorithm to reach the GSE in a distributed fashion with the assumption that the SEM can communicate with both the grid and the PEVGs. The SEM can use any vehicle to grid (V2G) infrastructure technique, [Pan et al., 2010], for this communication. As soon as any PEVG \( n \) is connected to the grid, the SEM receives the utility parameters \( b_n \) and \( s_n \) using V2G. The algorithm starts with the announcement of the available energy \( C \) and the price per unit energy \( p \) by the grid. At any given iteration \( k \), in response to this price \( p \), each PEVG \( n \) updates its demands for a particular amount of energy \( e_{nk} \) from the fixed amount \( C \) of the grid using the S-S method. The SEM gets the price \( p \) from the grid and checks the parameter \( \lambda_{nk} \) using (3.15). To enable the SEM to check the value of \( \lambda_{nk} \), each PEVG \( n \) submits its demand \( e_{nk} \) to the SEM at the end of iteration \( k \). The process continues until all the PEVGs make their demand such that \( \lambda_n = \lambda \geq 0 \) for all \( n \in \mathcal{N} \). The demand of the PEVGs reaches a VE as the SEM determines that \( \lambda_n = \lambda \geq 0 \) for all \( n \in \mathcal{N} \). Then, the SEM submits the VE demand of the PEVGs to the grid and the grid sets the optimal price \( p^* \) using (3.28) and, thus, the proposed GSG reaches the GSE.

After the execution of the algorithm the demand of each PEVG \( n \) reaches its equilibrium value \( e^*_n \), which is given by

\[
e^*_n(p^*) = \frac{b_n - p^*}{s_n}, \tag{3.29}
\]

with the optimal price \( p^* \). This is the equilibrium of the game.

3.5 Adaptation to Time-Varying Conditions

Here, we extend our approach so as to accommodate time-varying conditions using a discrete-time feedback Stackelberg game model with dependent followers [Nie et al., 2006]. We assume that the number of vehicles at a given location, e.g., a parking lot, changes gradually in real time over a moderate time duration (for example, 5 minutes to 30 minutes) [Lin, 2001]. We also assume that the available energy from the grid varies across moderate time intervals, e.g., once in an hour [Derin and Ferrante, 2010]. Hence, using a discrete-time feedback Stackelberg game, we can capture changes in system variables from one time instant to another. Prior to discussing how the proposed scheme can be adapted in a time-varying environment, we define the following parameters:

- \( C_t \) : the state variable of the game, which indicates the state of available charge at time instant \( t \).
- \( b_n^t \) : the battery capacity of PEVG \( n \), which depends on the aggregate quantity of the number of PEVs in the group at instant \( t \).
- \( s_n^t \) : the satisfaction parameter\(^{20} \) of PEVG \( n \) at \( t \).
- \( e_n^t \) : the energy demand by PEVG \( n \) at \( t \).
- \( p_t \) : the price per unit of energy at instant \( t \).
- \( \mathbf{e}^t = (e_1^t, e_2^t, ..., e_N^t) \) : the vector of the demand of all the PEVGs in the network at time \( t \).
- \( \mathbf{e}_{-n}^t = (e_1^t, ..., e_{n-1}^t, e_{n+1}^t, ..., e_N^t) \) : the vector of strategies of all PEVGs except PEVG \( n \) at

\(^{20}\)Here, we consider that the satisfaction parameter changes randomly between consecutive time slots due to the random change of vehicles in the parking lot from one time slot to the next.
instant $t$. 

$L_{T-t} = \sum_t p^t \sum_n e^t_n$: the pay off function of the grid which it wants to maximize over the entire peak hour duration.

$U_{T-t} = \sum_t \sum_n \left( b_n^t e^t_n - \frac{1}{2} s_n^t e^t_n + p_n^t e^t_n \right)$: the joint utility function of the PEVGs in the network.

Consequently, the state transition equation for the time-varying system can be defined as [Staňková and Schutter, 2011],

$$C_{t+1} = f_t(C_t, p^t, e^t), \ t = 0, 1, 2, ..., T - 1,$$

where $t$ is an integer time index, $T$ is the entire peak hour duration. For a feedback Stackelberg game, $C_t$ is the information about the available energy supply at time $t$, which is gained by the grid from (3.30) and fed back to the PEVGs through the SEM [Staňková and Schutter, 2011]. The state of available energy supply at any instant $t$ is a function of the demand of energy by the PEVGs, and the energy available at the previous time slot. The state transition from $t$ to $t + 1$ takes place when a change occurs either in the number of PEVs in the parking lot, or in the available energy from the grid. The objective of both the SG and the PEVGs is to choose their strategies so as to maximize their utilities in each time interval, and thus to maximize their total pay offs in the entire time horizon of peak hours. Hence, the problems of pay off maximization of the players for a discrete time feedback Stackelberg game can be formally expressed as

$$\max L_{T-t} = \sum_t \max_{p^t} \sum_n e^t_n,$$

for the SG, and

$$\max U_{T-t} = \sum_t \max_{e^t_n} \sum_n \left( b_n^t e^t_n - \frac{1}{2} s_n^t e^t_n + p_n^t e^t_n \right)$$

$$\sum_n e^t_n \leq C_t \ at \ t = 0, 1, 2, ..., T,$$

for the PEVG $n$. The objective functions in (3.31) and (3.32) refer to a feedback Stackelberg game with Nash game constraints in the lower level decision making process [Nie et al., 2006] (similar to the game proposed in Section 3.3 for a single time instant) over the whole time horizon. Here, we assume that the leader of the game can perfectly gain the information about $C_t$ from (3.30) [Tolwinski, 1983]. At any instant $t$, the SG gets the information about the volume of the parking lot (i.e., $b_n^{t-1}$ and $s_n^{t-1}$ for all $n = 1, 2, ..., N$), the demand of the PEVGs in the previous time slot $t - 1$ as well as per unit price $p_{t-1}$. Then, it estimates the amount of energy that needs to be provided to the PEVGs at $t$ through (3.30). The SG feeds back $C_t$ to the PEVGs through the SEM and the PEVGs play the jointly convex generalized Nash game of Algorithm 3.1 for the allocation of energy within $\sum_n e^t_n \leq C_t$.

Here, $(e^{t_1}, e^{t_2}, ..., e^{T*})$ and $(p^{t_1}_*, p^{t_2}_*, ..., p^{T*}_*)$ constitute the solution of the discrete time Stackelberg game under a feedback information structure with the corresponding state information $(C_1, C_2, ..., C_T)$, if $(e^{t_1}, e^{t_2}, ..., e^{T*})$ comprises the solution of the GNE among the followers for price $(p^{t_1}_*, p^{t_2}_*, ..., p^{T*}_*)$ at each $t = 1, 2, ..., T$ [Nie et al., 2006]. The solution
will be team optimal if the solution of GNE is optimal, that is if the GNE is a VE [Facchinei and Kanzow, 2007], for the sub-game in each \( t = 1, 2, ..., T \) [Nie et al., 2006]. Now, given that the Stackelberg game described in Section 3.3 constitutes a sub-game in each time interval \( t \) of the feedback Stackelberg game, the sub-game will reach its optimal solution (as shown in Section 3.3.2) within the constraint \( \sum_{n} c_{n} \leq C_{t} \) at each \( t = 1, 2, ..., T \). Therefore, the solution of the discrete-time feedback Stackelberg game possesses a team optimal solution.

3.6 Numerical analysis

![Figure 3.3: Convergence of the demand of each PEVG to the GSE.](image)

For our simulations, we consider a number of PEVGs that are connected to the grid during peak hours. Here, a single PEVG entity represents 1000 vehicles at a specific location [Rotering and Ilic, 2011], where each single vehicle is assumed to require 22 kWh for every 100 miles [Silva et al., 2009; Motors, 2011]. The maximum battery capacity of any single vehicle is chosen between 150 and 300 miles. Hence, the maximum capacity \( b_{n} \) of any PEVG \( n \) ranges between 35 MWh and 65 MWh, which is chosen randomly for each PEVG. The available energy at the grid is chosen as 99 MWh. Initially, the grid sets the selling price to 17 USD per MWh. We note that the chosen parameters correspond to a typical PEVG use case [Rotering and Ilic, 2011; Silva et al., 2009; Motors, 2011]. However, we highlight that these parameters can vary considerably according to PEVG usage and type, and economic conditions within a given city, or country.

The satisfaction parameter \( s_{n} \) is chosen randomly in the range of \([1, 2]\). The range of \( s_{n} \) is chosen based on the assumption that the PEVG with the lowest satisfaction parameter
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Figure 3.4: Convergence of the utility of each PEVG to the GSE.

$s_n = 1$ will get full satisfaction from each unit of energy it will consume, whereas the PEVG with the highest parameter i.e., $s_n = 2$, will reach the same satisfaction level from consuming half the amount due to its smaller limitations in terms of its initial battery state and travel plan. We do not consider the PEVGs with $s_n > 2$ as they do not have an immediate need for energy at peak hour. All statistical results are averaged over all possible random values of the PEVGs’ capacities using 1000 independent simulation runs.

In Fig. 3.3 and Fig. 3.4, we show the demand and the utility at the GSE for a network with $N = 5$ PEVGs. Here, we can see that, a similar demand by different PEVGs does not always lead to a similar utility for the PEVGs. For example, although PEVGs 2 and 5 in Fig. 3.3 have almost the same demand at the GSE, their utilities are different from one another as shown in Fig. 3.4. This is due to their different battery capacities and satisfaction parameters. From the utility in (3.7), we can see that the maximum utility level of a PEVG varies significantly for different values of $b_n$ and $s_n$. Therefore, with the same energy consumption, different PEVGs may obtain a different utility (e.g., PEVGs 2 and 5). Fig. 3.3 and 3.4 show that, after the 10th iteration, all the PEVGs reach their maximum utility, and, thus, their demands converge to the GSE.

In Fig. 3.5, we analyze the convergence speed of the proposed algorithm by plotting the values of $\lambda_n$ as a function of the number of iterations for $N = 5$ PEVGs. First, recall that the demand of all PEVGs converges to GSE for the optimal price $p^*$ when all the PEVGs reach their VE and at this VE $\lambda_n = \lambda \geq 0$. Therefore, our algorithm converges to the GSE after 9 iterations (i.e., the PEVGs reach their VE). Hence, as shown

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21 Although a PEVG may not be able to demand the amount of energy equal to its total battery capacity due to the scarcity of energy at peak hour, it can ask for an amount that must be satisfied to reach its destination.
3.6. Numerical analysis

by Fig. 3.5, the convergence speed of our algorithm is reasonable.

In Fig. 3.6, we show how the price set by the grid converges to its optimal value as the strategies of the PEVGs converge to the VE for networks of various sizes ($N = 5, 10, \text{ and } 15 \text{ PEVGs}$) with an initial SG price set to 17 USD per unit of energy. Fig. 3.6 shows that the price converges to an approximate optimal value within 5 iterations. This is due to the fact that the SG sets its price in response to the demand strategies of the PEVGs. In fact, the PEVGs reach an approximate GSE within five iterations, as shown for 5 PEVGs in Fig. 3.3, and, then, they optimize their demand within constraint (3.1). The PEVGs eventually reach the unique GSE that maximizes their utility under this constraint. Hence, the price $p$ converges quickly to its optimal value $p^*$. Furthermore, Fig. 3.6 shows that the variation of the grid’s price is more noticeable when fewer PEVGs are present. This is due to the fact that, for a fixed grid capacity, as the number of PEVGs increases, there are fewer possibilities of variations in the demands due to (3.1).

In Fig. 3.7, we show the effect of the number of PEVGs on the optimal price choice of the grid. To do so, we increase the number of PEVGs in the network for different grid capacities $C = 60, 80 \text{ and } 90 \text{ MWh}$. Fig. 3.7 shows that the average optimal price increases with the number of PEVGs in the network because of the increasing energy demand on the SG’s limited resources. In contrast, increasing the grid’s capacity leads to a decrease in the optimal price $p^*$. This is due to the fact that, as the total available capacity of the grid increases, the grid has more energy to sell and, thus, it can decrease its price while maintaining desirable revenues. Moreover, for a fixed number of PEVGs, as the available

Figure 3.5: Convergence of the values of $\lambda_n$, $\forall n$ to $\lambda \geq 0$ as the solution of the GSG converges to GSE.
Figure 3.6: Convergence of the price per unit of energy to the optimal price for a network with different numbers of PEVGs.

Figure 3.7: Effect of increasing the number of PEVGs $N$ in the network and the grid energy $C$ on the average Stackelberg price.
3.6. Numerical analysis

energy at the grid increases, the SG reduces its optimal price to encourage the PEVGs to demand more energy.

Fig. 3.8 presents the computational complexity of the proposed scheme by showing the average and maximum number of iterations needed to reach the GSE of the proposed game with respect to the number of PEVGs. In Fig. 3.8, we can see that, whenever the number of PEVGs in the network increases, for the same amount of capacity from the grid, the PEVGs require more iterations to find their optimal demands. For example, when the number of PEVGs in the network increases from 15 to 25, the average number of iterations needed to reach the GSE increases from 52 to 79. A similar behavior is also seen for the maximum number of iterations.

In Fig. 3.9, we compare the results of the proposed scheme with a particle swarm optimization (PSO) [Su and Chow, 2011] and an equal distribution (ED) scheme [Chan and Wong, 2004]. In a PSO scheme, a group of random solutions (i.e., particles) are scattered over the search space, and the particles converge to a near optimal solution after a number of iterations. For the PSO algorithm, the particle size is considered to be 40. The parameters are updated in such a way that the constraint in (3.1) is satisfied. For the ED scheme, the available grid energy is distributed equally among the connected PEVGs in the network. That is, if the available energy at the grid is $C$ MWh and there are $N$ PEVGs connected to the grid, each PEVG receives an allocation of $C/N$ MWh of energy from the grid as long as this allocation does not exceed the maximum that a PEVG can be charged.

We assume that the initial price is chosen based on a statistical estimation such as in [Chen et al., 2011], so that the grid can maintain a revenue with optimized price as the grid capacity increases.

In this case the energy demand of a PEVG.
Fig. 3.9 shows that the average utility achieved by the proposed scheme is better for most of the PEVGs in the network, except PEVG 9 and 10, when compared to the PSO scheme. This is due to the fact that the PSO scheme optimizes the energy for each PEVG according to the better particle position in the available energy space. This may lead to a better utility for PEVG 9 and 10 at the expense of much lower utilities for the rest of the PEVGs. However, our proposed scheme allocates the energy among the PEVGs so that the socially optimal solution is achieved. Therefore, most of the PEVGs in the network achieve improved utilities compared with the PSO scheme. From Fig. 3.9, the proposed scheme has a total utility, on the average, 1.3 times the utility achieved by the PSO scheme. Moreover, the proposed scheme has, on the average, twice the utility achieved by the ED scheme, which is a significant improvement.

![Figure 3.9: Comparison of the average utilities of the PEVGs at the GSE for the proposed scheme, the PSO scheme and ED scheme.](image)

Fig. 3.10 shows the average demand per PEVG as the number of PEVGs varies. In Fig. 3.10, we can see that the average demand per PEVG decreases as the number of PEVGs increases. This is a direct result of (3.1) and also because the optimal price set by the grid increases as the number of PEVGs increases. From Fig. 3.10, we can see that average PEVG demand for our scheme, the PSO scheme, and the ED case decreases as the number of PEVGs increases. Fig. 3.10 shows that the energy demand for the proposed scheme is lower than that for both the PSO and ED scheme for all $N$. This is because, in our proposed scheme, each PEVG demands a required amount of energy within constraint (3.1) based on its satisfaction parameter and available grid energy. Therefore, each PEVG requests a socially optimal amount of energy from the grid rather than a predefined amount (as in the ED scheme), or near optimal amount according to a random search space (as in the PSO scheme). Clearly, Fig. 3.10 shows that the proposed scheme leads to
a better energy utilization than both the PSO and ED schemes.

Figure 3.10: Effect of the number of PEVGs in the network on the average demand of a PEVG.

Figure 3.11: Effect of the number of PEVGs in the network on the average utility per PEVG.

In Fig. 3.11, we show the average utility achieved by all three schemes as a function
of the number of PEVGs. In this figure, we can see that the average utility per PEVG decreases as \( N \) increases for all three schemes. This is due to the fact that the benefit extracted by each PEVG decreases as more PEVGs share the fixed available energy from the grid. However, importantly, Fig. 3.11 shows that the proposed scheme has a performance advantage at all network sizes that is, on the average, 1.6 times that of the PSO scheme. When compared to the ED scheme, the proposed scheme shows significant improvements for all network sizes, reaching an improvement of up to 3.5 times over the ED scheme for \( N = 25 \) PEVGs.

![Graph showing average energy demand per PEVG](image)

**Figure 3.12:** Average demands of the PEVGs at the GSE for the proposed scheme in a dynamic environment.

In Fig. 3.12, we assess the average demand of energy per PEVG in a time-varying environment. Here, the state transition of the variable \( C_t \) in (3.30) is modeled as an independent stochastic process [Staňková and Schutter, 2011], due to the random changes in traffic conditions and grid energy from one time instant to another [Pan et al., 2010]. The state of available energy at the grid at any time \( t \) is assumed to be a uniformly distributed random variable in the range \([0.5, 1.5]\) times the average available energy [Pan et al., 2010]. The capacity of the PEVGs \( b_n \forall n \in N \) at \( t = 1, 2, \ldots, T \), is assumed to be a uniformly distributed random variable in the range \([0.5, 1.5]\) times the average battery capacity [Pan et al., 2010]. Assuming that the traffic conditions in any parking lot change every 30 minutes, we have run independent simulations for allocating energy to the PEVGs for eight time slots in peak hours (from 12 pm to 4 pm). The average available energy in the grid is 66 MWh (assuming a range from 33 to 99 MWh), and the variation of energy across time slots is modeled by a uniformly distributed random variable between 0.5 and 1.5 of this amount [Pan et al., 2010]. Similarly, the random variation in PEVGs' capacity at different time slots is captured assuming a maximum of 55 MWh (for 1000 PEVs...
3.7. Concluding Remarks

In a PEVG) and a minimum of 9.9 MWh (for 300 PEVs) [Galus and Andersson, 2008a].

In Fig. 3.12 it is shown that the demand of any PEVG varies across time slots due to variation in both satisfaction parameters of the PEVGs and the available energy. However, the minimum energy demand by any PEVG is always well above its minimum battery requirement. For example, the minimum demand by PEVG 4 is 5.5 MWh in time slot 1, which exceeds the minimum battery requirement of a mid-size car, assuming 1000 PEVs are in that PEVG [Roussean et al., 2007].

![Figure 3.13: Comparison of the average utility per PEVG in the dynamic case.](image)

The change of utility per PEVG in a time-varying environment is shown in Fig. 3.13 for the proposed, PSO and ED schemes. Fig. 3.13 shows that the team optimal solution achieved by the proposed scheme leads to an improved average utility for the PEVG when compared to the PSO and ED schemes. However, the improvement varies across time slots due to variations in available energy and the number of PEVs in the smart grid network. As shown in Fig. 3.13, the average utility achieved per PEVG by our proposed scheme is, on the average, 1.6 times the utility achieved by the PSO scheme and 3.8 times the ED scheme.

3.7 Concluding Remarks

In this chapter, we have formulated a non-cooperative Stackelberg game to study the problem of energy trading between an SG and a number of PEV groups (PEVGs). In this game, the SG chooses its price to maximize its revenue, whereas the PEVGs strategically choose the amount of energy they wish to buy from the grid so as to optimize a tradeoff between the benefit of battery charging and associated costs. We have studied
the properties of this solution and we have shown that this game admits a socially optimal generalized Stackelberg equilibrium. We have also extended the proposed game in a time-varying environment, and we have provided relevant analysis for this case. To reach the equilibrium of the game, we have proposed a novel algorithm that can be adopted by the PEVGs, in a distributed manner. Simulation results have shown that the proposed approach yields improved performance gains, in terms of the average utility per PEVG, compared to a particle swarm optimization and an equal distribution scheme. It is important to note that this successful energy trading solely depends on the spontaneous participation of consumers (here, the electric vehicles) in energy trading with the grid. Hence, they need to be provided with proper motivation so as to encourage them to voluntarily take part in energy trading with the grid. To that end, in the next chapter we investigate a consumer-centric energy trading scheme that gives primary concern to consumers benefits in the SG.
This chapter proposes an energy management technique for a consumer-to-grid system in smart grid. The benefit to consumers is made the primary concern so as to encourage consumer participation in energy trading with the central power station (CPS) of the grid. A single-leader multiple-follower game, similar to the game considered in Chapter 3, is studied to model the interaction between the CPS and a number of energy consumers (ECs). The CPS is considered as a leader seeking to minimize its total cost of buying energy from the ECs, and the ECs are the followers who decide on their amounts of energy to be sold to the CPS for maximizing their utilities. The properties of the resulting game are studied, including the optimality and existence of a solution. It is shown that the game possesses a socially optimal solution, in which the benefits-sum to all consumers is maximized. The total cost incurred by the CPS is also shown to be minimized at the socially optimal solution. Further, after suitably modeling the game as a variational inequality problem, a distributed algorithm is proposed that guarantees the optimal solution when it is implemented. Numerical analysis is used to assess the properties of the game, and the effectiveness of the game is confirmed by comparing it with an existing standard Feed-in Tariff energy trading technique.

4.1 Introduction

Smart grid is a visionary consumer-centric system that will convert conventional power systems into systems that are capable of enabling energy consumers to actively take part in electricity supply chains through the integration of demand-side resources such as distributed generation, storage and renewable energy resources [Zhu and Başar, 2011; Chuang and Gellings, 2009]. One of the features key to smart grid implementation is the enabling of consumers participation by encouraging them to provide ancillary services to the main grid [Kim et al., 2011]. The development of new energy management applications and services, based on consumers’ active participation, will leverage the technology and capability upgrades available from the smart grid through its advanced infrastructure [Fang et al., 2011].

In a constrained energy market, the engagement of consumers in energy management can greatly enhance the grid’s reliability, and significantly improve the social benefit for the overall system [Walawalkar et al., 2010]. For instance, a study by McKinsey & Com-
pany shows that 10 – 15 billion US dollars (USD) in annual benefit can be achieved from large-scale USA-wide active participation of all the customers in energy management programs [Walawalkar et al., 2010]. Consequently, energy management research, in the context of smart grid, has received considerable attention recently. The focus of such research has been mainly in modeling the energy usage behavior of electric vehicles in vehicle-to-grid (V2G) and grid-to-vehicle (G2V) scenarios [Wu et al., 2012a; Sousa et al., 2012; Ma et al., 2012; Han et al., 2010; Saber and Venayagamoorthy, 2009], and in designing effective energy demand management tools, such as in [Mohsenian-Rad et al., 2010; Mohsenian-Rad and Leon-Garcia, 2010; Conejo et al., 2010; Ibars et al., 2010; Asad et al., 2011; Medina et al., 2010; Palensky and Dietrich, 2011]. However, one of the key challenges for successful energy management in smart grids is to motivate the consumers to actively and voluntarily participate in such management programs. If the energy consumers are not interested in actively taking part in energy management, the benefit of smart grid will not fully be realized [Liu et al., 2011]. Therefore, to make the consumers an integral part of any energy management scheme, the design of the scheme needs to be consumer-centric [Liu et al., 2011], whereby the main recipients of smart grid benefits should be energy consumers as both buyers from, and sellers to, the energy grid.

In this chapter, a consumer-centric energy management scheme is proposed for a consumer-to-grid system that gives significant benefit to consumers who actively participate in the smart grid. The idea of consumer-centric smart grid (CCSG) was first introduced by Liu et al. [Liu et al., 2011], in which the concept of a CCSG is explained and its effect on grid modernization is discussed. Further, in [Zafar et al., 2010], customer domain analysis of smart grid is studied along with the tasks designated to this domain. However, in general there have not been detailed studies of consumer-centric energy management schemes. Hence, there is a need for developing energy management methods for CCSG. These methods would successfully capture the benefit to consumers in the network that encourages them to participate in energy trading with the central power unit of the smart grid. To that end, the proposed energy management scheme in this chapter complements the existing work on CCSG in the following manner:

1. A theoretical model, using a single-leader multiple-follower game, is proposed to design the decentralized decision making process of the energy consumers in smart grid by capturing the interaction between the CPS and the ECs in the network.

2. By analyzing the solution of the game, it is shown that playing the game leads to a solution which is socially optimal for ECs, and also that the total cost incurred by the CPS possesses the minimum value.

3. Suitably formulating the game between the consumers as a variational inequality problem, a distributed algorithm is proposed that guarantees convergence to the optimal solution.

4. The proposed management scheme is compared with a standard Feed-in Tariff (FIT) scheme, and considerable performance improvements in terms of both achieved utility and reduction in total cost are demonstrated.

The rest of the chapter is organized as follows: the system model is presented in Section 4.2. The design of the energy management scheme is formulated as two optimization
4.2 System Model

Consider a smart grid network that consists of a central power station (CPS) and multiple energy consumers (ECs) as shown in Fig. 4.1. Here, on the one hand, the CPS refers to a power generating unit that is connected to its area of interest (i.e., to the ECs of the network) by means of power lines. On the other hand, ECs are the energy entities of the network such as electric vehicles (EVs), smart homes and bio-gas plants, owned by individual personnel who may decide on buying energy from or selling to, the CPS, or alternatively using its generated energy for its own energy usage activities. It is assumed that each EC is equipped with a smart meter that has a management capability for trading energy with the CPS [Mohsenian-Rad et al., 2010]. All the consumers can communicate with the CPS using an appropriate communication protocol [Mohsenian-Rad et al., 2010].

Throughout the chapter, the energy management at peak hours of demand in the day, i.e., 12 pm to 4 pm [Galus and Andersson, 2008b], is considered when the CPS is assumed to serve a group of energy consumers such as offices, cities, factories and EVs in city car parks. Energy management in smart grids needs to be carried out frequently, and hence the total peak hours operation can be divided into several time slots, each time slot typically lasting for 30 minutes [Anderson and Fuloria, 2010; Pierce, 2012]. Due to the massive demands of consumers at peak hours, the CPS may be unable to meet the needs of some consumers during some of these time slots. Meanwhile, the CPS needs to buy energy, if possible, from other energy entities such as smart homes, idle electric vehicles,
solar farms, wind farms, and biogas plants in the network. Such entities may voluntarily take part in trading their excess energy with the CPS with appropriate incentives, and thus provide the CPS with energy for its uninterrupted supply. It is assumed that the energy deficiency at the CPS for a particular time slot is fixed. However, this deficiency may vary between different time slots. For the rest of the chapter, we will consider the energy management for a single time slot some time during the peak hours of energy demand.

It is important to note that this energy management scheme completely depends on the voluntary participation of the energy consumers who have surplus energy to sell to the CPS during the considered time period. Hence, the management scheme should be of enough benefit to all consumers to encourage them to take part in a voluntary energy trading program. Besides, as the requested energy by the CPS in any time slot is fixed, the CPS is not interested in buying more than that energy so as to minimize its purchasing cost. Moreover, the CPS may need to provide different incentives to different ECs for buying energy from the ECs. This is due to the fact that each EC may have a different energy surplus to sell to the CPS. Therefore, a lower incentive may not affect the intended revenue of an EC with higher energy surplus (as it can sell more energy), but may severely degrade the intended revenue of an EC with lower energy surplus (as it has less energy to sell).

To this end, let us consider a system model with \( N \) ECs in the set \( \mathcal{N} \) in the smart grid network, which provide ancillary services by supplying their energy to the CPS. Each EC \( n \in \mathcal{N} \) represents a group of similar energy customers of the smart grid acting as a single entity (a smart home group can, for example, represent a geographical area with a number of smart homes connecting via an aggregator). At a particular time slot of energy deficiency \( E_{\text{def}} \), EC \( n \) has an amount of energy \( E_n \) available to sell to the CPS. \( E_n \) for different \( n \) may be different based on parameters such as the type of EC, the current weather (e.g., a solar farm may wish to sell a large amount of energy on a sunny day compared to other cloudy or rainy days) and the capacity of the storage device. Since the amount of energy \( E_{\text{def}} \) needed by the CPS for the considered time slot is assumed to be fixed, the energy supplied by all ECs to the CPS during that time slot needs to satisfy the constraint

\[
\sum_n e_n \leq E_{\text{def}}; \quad e_n \leq E_n, \quad \forall n \in \mathcal{N},
\]  

(4.1)

where \( e_n \) is the energy supplied by the EC \( n \).

On the other hand, the CPS pays a price \( p_n \) per unit of energy to EC \( n \) for its offered energy \( e_n \). The CPS wants to minimize its total cost of purchasing energy by suitably setting \( p_n \) for different \( n \). This is mainly because: i)- reduction in total cost of purchasing energy would encourage the CPS to trade more energy, when needed, with the ECs in the network rather than setting up more expensive generators or bulk capacitors for meeting its excess needs; and ii)- by minimizing total cost of buying energy, the CPS may be able to sell the energy to its consumers at a cheaper rate, which in turn will benefit the consumers. To this end, we introduce a “total price per unit of energy” parameter \( P \) which will be used by the CPS to optimize the purchase price \( p_n \) per unit of energy to be paid to each EC \( n \) so as to minimize its total cost. We note that an analogous hypothesis “total cost per unit production” has widely been used in economics [Farris et al., 2010], in which the total cost per unit varies with the variation of the total production. However, here the
total amount of energy required by the CPS is fixed for any given time slot, and hence the total price per unit $P$ is assumed to be fixed. $P$ can be estimated\(^2\) by the CPS using any real-time price estimation such as that proposed in [Yun et al., 2008]. The CPS pays $p_n$ to each EC $n$ based on its offered energy $e_n$, while maintaining the constraint

$$\sum_n p_n = P, \quad p_n^{\min} \leq p_n \leq p_n^{\max}. \quad (4.2)$$

The equality constraint in (4.2) is to establish the fact that the announced total price per unit of energy $P$ must be paid by the CPS to the whole set of ECs $N$ when buying the required energy, and thus provide an incentive\(^2\) to the ECs to take part in energy trading with the CPS. Although $P$ is fixed, the individual price per unit energy $p_n$ could be different for different ECs within the bounds $p_n^{\min} \leq p_n \leq p_n^{\max}$ based on the amount of energy they offer to the CPS. Here, $p_n^{\min}$ is the minimum price per unit energy that the CPS needs to pay any EC to keep it trading its energy. This is due to the fact that a much lower price may not be beneficial to an EC, and consequently may deter an EC from energy trading with the CPS. $p_n^{\max}$ is the maximum price per unit energy that the CPS can pay to any EC. For example, the CPS may choose to buy its total required energy from a single EC, and thus could pay $p_n^{\max} = P$ to it.

### 4.3 Problem Formulation

For successful consumer-centric energy trading in smart grids, the CPS and the ECs interact with each other, and agree on the energy trading parameters such as $p_n$ and $e_n$. Both the CPS and the ECs choose their respective energy trading parameters so as to optimize their own objective functions, as discussed in the following subsections.

#### 4.3.1 Objective of the ECs

The ECs in the network voluntarily sell their energy to the CPS so as to get revenue according to the price offered by the CPS. Thus, for the price $p_n$, each EC $n$ chooses its proposed amount of energy $e_n$ to maximize its benefit. To model this benefit, we choose a utility function for EC $n$ that is a function of its energy $E_n$ available for sale, the amount of energy $e_n$ to be sold to the CPS, and the price per unit of energy $p_n$ paid by the CPS. To that end, we propose to set the utility of any EC $n$ as

$$U(e_n, E_n, p_n) = E_n e_n - \frac{1}{2} e_n^2 + p_n e_n, \quad (4.3)$$

which captures the benefit of the EC for selling the energy $e_n$ at price $p_n$. The choice of the utility function is based on a linearly decreasing marginal benefit, which has recently been shown to be appropriate for users of power [Samadi et al., 2010]. Besides, the utility function $U(e_n, E_n, p_n)$ also possesses the following properties:

\(^2\)Based on the assumption that the statistical estimation would lead to a total cost that is minimal for the CPS with respect to its chosen $P$.

\(^2\)An alternate way of encouraging each EC is to set a Quality-of-Service (QoS) constraint for it.
1. The utility of EC \( n \) increases as the amount of energy \( E_n \) available to the EC for sale increases. That is,

\[
\frac{\delta U(e_n, E_n, p_n)}{\delta E_n} > 0. \tag{4.4}
\]

2. The utility achieved by an EC \( n \) increases as \( p_n \) increases. Therefore,

\[
\frac{\delta U(e_n, E_n, p_n)}{\delta p_n} > 0. \tag{4.5}
\]

It is important to note that the addition of the quantity \( \sum_{m \neq n} E_m e_m - \frac{1}{2} e_m^2 + p_m e_m \) to \( U \) in (4.3) does not affect the solution [Basar and Srikant, 2002]. Thus, the ECs jointly maximize

\[
\tilde{U}(e, E, p) = \sum_n U(e_n, E_n, p_n) \tag{4.6}
\]

subject to the constraint \( \sum_n e_n \leq E_{\text{def}} \). Here, \( e = [e_1, e_2, \ldots, e_N]^T \), \( E = [E_1, E_2, \ldots, E_N]^T \) and \( p = [p_1, p_2, \ldots, p_N]^T \). That is, to decide on a consumer-centric energy trading parameter, EC \( n \) chooses an amount of energy, \( e_n \leq E_n \), to supply to the CPS so as to maximize the sum of utilities in (4.6); i.e., social benefit, of all the ECs in the network. Thus, \( e_n \), for all \( n \), is the amount of energy that benefits all the consumers in the smart grid network.

### 4.3.2 Objective of the CPS

While the objective of an EC is to maximize the social benefit from its chosen amount of energy, the CPS wants to minimize a cost function \( L(p) = \sum_n L(p_n) \), representing its total cost of purchasing energy from the ECs for the reasons described in Section 4.2. The individual cost function \( L(p_n) \) needs to capture the effect of paying \( p_n \) per unit energy of \( e_n \) from the EC \( n \), and also other associated costs such as the transmission cost and the artificial tariff cost for buying energy at a price \( p_n \). To this end, we consider a convex cost function, analogous to the practical convex cost functions of some utility companies [Mohsenian-Rad et al., 2010] that has the following properties:

1. The cost to the CPS increases as the price per unit of energy increases for any EC \( n \). That is, for each \( n \in \mathcal{N} \),

\[
L(\hat{p}_n) < L(\hat{p}_n), \forall \hat{p}_n < \hat{p}_n. \tag{4.7}
\]

2. The cost function is strictly convex. Thus, for each \( n \in \mathcal{N} \), \( \hat{p}_n, \hat{p}_n \geq 0 \) and any real number \( 0 < \theta < 1 \), the cost function satisfies

\[
L(\theta \hat{p}_n + (1 - \theta)\hat{p}_n) < \theta L(\hat{p}_n) + (1 - \theta) L(\hat{p}_n). \tag{4.8}
\]

Such a cost function for buying an amount of energy \( e_n \) from EC \( n \) is then specified as

\[
L(p_n) = e_n p_n^2 + a_n p_n + b_n, \tag{4.9}
\]
where $a_n$, $b_n \geq 0$, $\forall n \in \mathcal{N}$ [Mohsenian-Rad et al., 2010]. In (4.9), the first term $e_n p_n^2$ is the cost of purchasing energy with price $p_n$ per unit of energy. It is important to note that as the price per unit of energy would affect the amount of energy to be sold by the EC $n$ (as the incentive changes), it would also affect the associated costs, such as transmission cost, of purchasing this energy. These associated costs are reflected in the last two terms, $a_n p_n + b_n$ in (4.9). The objective of the CPS is to determine a suitable price $p_n$, within the constraint in (4.2), so as to minimize this total cost. Thus, the goal of the CPS is

$$\min_{p} \hat{L}(p) = \min_{p} \sum_{n} L(p_n),$$

such that $\sum_{n} p_n = P$, $p_{\min} \leq p_n \leq p_{\max} \forall n$.

The optimization problems in (4.6) and (4.10) are joined by the price factor $p_n$. The CPS can find solutions by jointly optimizing (4.6) and (4.10) in the case where the CPS has full control over the decision making processes of the ECs. However, in practice, the CPS does not have any direct control over the ECs’ decisions as these are made directly by each customer [Wu et al., 2012a]. Therefore, a decentralized control mechanism is required for the ECs to decide on the energy they sell to the CPS, within the constraint in (4.1), so as to maximize (4.6). The mechanism also needs to successfully capture the interaction between the ECs and the decision making of the CPS for the prescribed energy trading. We propose such an energy management mechanism, using game theory, in the next section.

### 4.4 Non-Cooperative Game Formulation

To decide on energy trading parameters, the ECs in the smart grid interact with the CPS, and a single-leader multiple-followers game (SLMFG) is proposed to study this interaction. It is important to note that although other decentralized optimization techniques might also be applicable to model the decision making process of the ECs and the CPS, we use a SLMFG for its ability to capture both the behaviour of self-interested nodes and the interaction between the higher level and lower level players without directly controlling their decision making process [Başar and Olsder, 1999]. In the proposed SLMFG, the CPS is the leader of the game, which decides on the price per unit of energy $p_n$, within constraint (4.2), to be paid to the EC $n$ for the amount of energy offered by it. And each EC $n \in \mathcal{N}$ is a follower, playing a generalized Nash game [Facchinei and Kanzow, 2007] with other ECs in the network to decide on the amount of energy it will sell to the CPS, within constraint (4.1) in response to the price $p_n$. Thus, the SLMFG can be formally defined by its strategic form as

$$\Gamma = \{\mathcal{N} \cup \{\text{CPS}\}), \{E_n\}_{n \in \mathcal{N}}, \tilde{U}, \tilde{L}, p\},$$

where $\mathcal{N} \cup \{\text{CPS}\}$ is the total set of players in the game, $E_n$ is the strategy vector of each EC $n \in \mathcal{N}$ satisfying the constraint in (4.1), i.e., $\sum_{n} e_n \leq E_{\text{def}}, e_n \in E_n, \forall n \in \mathcal{N}$; $\tilde{U}$ is the objective function that each EC $n$ wants to maximize; $\tilde{L}$ is the objective function of the CPS, and $p$ is the strategy vector of the CPS.
In the proposed game, it is assumed that ECs maintain their privacy, and hence do not inform each other of the amount of energy they will offer to the CPS. Thus, the proposed game leads to a non-cooperative SLMFG in which the followers do not communicate with each other, but they may interact with the leader by controlled signaling through their smart meters [Mohsenian-Rad et al., 2010]. Importantly, in this game, the decision making process of an EC $n$ depends not only on its own strategies but also on the strategies of other ECs in the network via (4.1). Thus, the generalized Nash game amongst the ECs, to decide on the amount of energy to be supplied to the CPS by each EC $n$, is a jointly convex generalized Nash equilibrium problem (GNEP), in which the ECs’ actions are coupled solely through the constraint (4.1) [Facchinei and Kanzow, 2007]. The solution of a GNEP is the generalized Nash equilibrium (GNE) [Facchinei and Kanzow, 2007].

The game is initiated as soon as the ECs in the network start playing a GNEP for a price $p_n = p$, $\forall n \in \mathcal{N}$ announced by the CPS. The ECs play the GNEP and offer, according to their GNE, the amount of energy they wish to sell to the CPS for the offered price $p$. Then, having an insight into the capacity of each EC’s energy supply, the CPS decides on its optimal price vector $p^* = [p_1^*, p_2^*, ..., p_N^*]^T$ to pay the ECs by solving the constrained optimization problem in (4.10) using a convex optimization tool [Mohsenian-Rad et al., 2010]. Thereafter, as soon as the ECs decide on their GNE energy vector $e^* = [e_1^*, e_2^*, ..., e_N^*]^T$, after playing the GNEP for the optimal price vector $p^*$, the proposed SLMFG reaches the equilibrium solution. From here on, the solution of the proposed SLMFG $(e^*, p^*)$ will be referred to as an energy management equilibrium solution (EMES) in which the CPS will decide on an optimized price vector $p^*$ to pay to the ECs in the network, and the ECs will agree on a GNE energy vector $e^*$ to be supplied to the CPS for the given $p^*$.

**Definition 4.4.1.** Consider the SLMFG $\Gamma = (\mathcal{N} \cup \{CPS\}), \{E_n\}_{n \in \mathcal{N}}, \tilde{U}, \tilde{L}, p$ where $\tilde{U}$ and $\tilde{L}$ are defined by (4.6) and (4.10) respectively. A set of strategies $(e^*, p^*)$ constitute the EMES of this game if and only if it satisfies the following set of inequalities:

$$\tilde{U}(e_n^*, e_{-n}^*, E, p) \geq \tilde{U}(e_n, e_{-n}^*, E, p),$$

$$\forall e_n \in e, \ n \in \mathcal{N}, \sum_n e_n \leq E_{ef} \tag{4.12}$$

and

$$L(p_n^*, p_{-n}^*) \leq L(p_n, p_{-n}^*),$$

$$\forall n \in \mathcal{N}, \forall p_n \in p, p_{min} \leq p_n \leq p_{max}, \tag{4.13}$$

where $e_{-n}$ is the GNE energy vector of all the ECs in set $\mathcal{N} \setminus \{n\}$, and $p_{-n}$ is the price vector set by the CPS for all the ECs in the set $\mathcal{N} \setminus \{n\}$.

Thus, at EMES, no EC can improve its utility by deviating from its EMES strategy provided all other ECs are playing their EMES strategies. Similarly, deviation from EMES

---

26For example, the CPS can send a single bit to the EC $n$ if its offered energy is beyond the constraint in (4.1) given the energy offered by other ECs in the network.

27For a similar price $p$, each EC receives a similar incentive, and thus their offered energies reflect their capacity of supply.
price $p_n^*, \forall n \in \mathcal{N}$ can not lower the total cost for the CPS once the SLMFG reaches the EMES.

4.5 Properties of the Game

In a non-cooperative game, the existence of an equilibrium (in pure strategies) is not always guaranteed [Başar and Olsder, 1999]. Moreover, for consumer-centric smart grids, it is important that the solution is beneficial for all the consumers in the network [Liu et al., 2011]. Therefore, the existence and optimality of a solution of the proposed SLMFG needs to be determined.

4.5.1 Existence and optimality of the solution

**Lemma 4.5.1.** A solution exists for the proposed SLMFG if and only if a solution exists for the GNEP amongst the ECs in the smart grid network. The solution will be optimal if the GNE of the GNEP is optimal.

**Proof.** As the game is formulated, the proposed SLMFG reaches the EMES as soon as the ECs in the network agree on a GNE energy vector to be supplied to the CPS in response to the optimized price vector $p^*$ set by the CPS. The cost function for the CPS in (4.10) is a strictly convex function, and thus, a unique solution always exists for the CPS’s optimization problem in choosing a price per unit of energy for the EC $n$, $\forall n \in \mathcal{N}$ [Dattorro, 2005]. Subsequently, the existence of a solution for the GNEP among the ECs, for this unique price vector, would guarantee the existence of an EMES in the proposed SLMFG. Similarly, the solution will be an optimal solution if the GNE of the GNEP amongst the ECs lead to an optimal GNE.

To investigate the existence and the optimality of the solution of proposed GNEP, first, we formulate the GNEP as a variational inequality (VI) problem $\text{VI}(\mathbf{E}, \mathbf{F})$ [Arganda et al., 2011], which is essentially to determine a vector $\mathbf{e}^* \in \mathbf{E} \subset \mathbb{R}^n$, such that $(\mathbf{F}(\mathbf{e}^*), \mathbf{e} - \mathbf{e}^*) \geq 0$, for all $\mathbf{e} \in \mathbf{E}$. Here, $\mathbf{E}$ is the set of strategies of all ECs satisfying (4.1), and $\mathbf{F} = -\left(\nabla_{\mathbf{e}} U(e_n, E_n, p_n)\right)_{n=1}^N$ [Arganda et al., 2011], where

$$
\mathbf{e} = \begin{bmatrix} e_1 & e_2 & \ldots & e_N \end{bmatrix}^T \in \mathbf{E}.
$$

(4.14)

The solution of the VI($\mathbf{E}, \mathbf{F}$) is a variational equilibrium (VE) [Facchinei and Kanzow, 2007]. In the proposed scheme, we are particularly interested in investigating the existence and the efficiency of the VE due to the fact that the proposed GNEP is a jointly convex GNEP due to the coupled constraint (4.1), and hence the VE is the socially optimal solution among all the GNEs [Facchinei and Kanzow, 2007]. Therefore, in designing a socially optimal consumer-centric energy management scheme, it is our primary interest to investigate the existence and efficiency of a VE solution. In the rest of this chapter, we will use the terms “GNEP” and “variational inequality” interchangeably.

**Theorem 4.5.1.** The GNEP amongst the ECs in response to the CPS’s decision vector, i.e., price vector, possesses a socially optimal VE.
Proof. Recall that the joint utility function of the ECs in the network can be defined as (4.6), and therefore, the pseudo-gradient of \( \tilde{U}(e, E, p) \) is [Arganda et al., 2011]

\[
\mathbf{F} = \begin{bmatrix}
e_1 - E_1 - p_1 \\
e_2 - E_2 - p_2 \\
\vdots \\
e_N - E_N - p_N
\end{bmatrix},
\]

(4.15)

whose Jacobean matrix is [Arganda et al., 2011]

\[
\mathbf{JF} = \begin{bmatrix}
1 & 0 & \ldots & 0 \\
0 & 1 & \ldots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \ldots & 1
\end{bmatrix},
\]

(4.16)

From (4.16), \( \mathbf{JF} \) is an identity matrix, and hence, \( \mathbf{JF} \) is positive definite\(^{28}\) on \( E \). Therefore, \( \mathbf{F} \) is strictly monotone. Consequently, \( \text{VI}(E, \mathbf{F}) \) admits a unique VE solution [Facchinei and Kanzow, 2007]. Further, as the proposed GNEP is jointly convex and the VE is the unique solution, the VE is also the unique global maximizer of (4.6) [Facchinei and Kanzow, 2007]. As a result, the VE is the socially optimal solution of the GNEP for energy supply management amongst the ECs.

Remark 4.5.1. From Theorem 4.5.1, the GNEP among the ECs in response to the unique optimized price paid to them by the CPS admits a socially optimal solution. As a consequence, as proved in Lemma 4.5.1, the proposed single-leader multiple-follower game of consumer-centric energy management possesses a socially optimal solution.

4.5.2 Analysis of the solution

For a clear understanding of the decision making process of the players at EMES, we formulate Karush-Kuhn-Tucker (KKT) conditions, using the method of Lagrange multipliers [Bertsekas, 1995], for the ECs’ optimization problem. We also study the decision making process of the CPS at EMES through the Lagrange multiplier approach.

ECs’ decisions

The KKT condition for ECs’ GNEP, i.e., variational inequality problem, is given by [Facchinei and Kanzow, 2007]

\[
\mathbf{F}(e) + \xi \nabla_e (\sum_n e_n - E_{\text{def}}) = 0,
\]

\[
\xi (\sum_n e_n - E_{\text{def}}) = 0, \quad \xi \geq 0,
\]

(4.17)

where \( \mathbf{F} \) is defined in (4.15) and \( \xi \) is the Lagrange multiplier. It is important to note that the same multiplier \( \xi \) is used for all the ECs in the network in (4.17) (i.e., \( \xi_n = \xi, \ \forall n \in \mathcal{N} \)).

\(^{28}\)All eigenvalues are positive.
4.5. Properties of the Game

This is due to the fact that the GNEP amongst the ECs is a jointly convex GNEP, and the solution of this jointly convex GNEP is a VE [Facchinei and Kanzow, 2007]. Now, the solution of the KKT condition for any EC $n$ is

$$E_n - e_n + p_n - \xi = 0,$$
$$\xi(\sum_n e_n - E_{\text{def}}) = 0; \, \xi \geq 0.$$  \hfill (4.18)

Therefore, for $\xi \geq 0$

$$E_n - e_n + p_n \geq 0,$$
$$\text{i.e., } e_n \leq E_n + p_n.$$  \hfill (4.19)

**Remark 4.5.2.** From (4.19), the decision making process of an EC can be interpreted as follows: if $N$ ECs are playing a GNEP to decide on the amount of energy to be sold to the CPS, the maximum amount of energy that an EC may voluntarily offer to the CPS changes with the change of the energy available to it for sale, or with the price per unit of energy it is receiving from the CPS, or with the change of both.

**CPS’s decision**

Using the Lagrange multiplier approach

$$\Omega = \sum_n (e_n p_n^2 + a_n p_n + b_n) + \lambda (P - \sum_n p_n),$$ \hfill (4.20)

where $\lambda$ is the Lagrange multiplier. From (4.20),

$$\left. \frac{\delta \Omega}{\delta p_n} \right|_{p_n} = 2e_n p_n + a_n - \lambda = 0,$$ \hfill (4.21)

and

$$\left. \frac{\delta \Omega}{\delta \lambda} \right|_{\lambda} = P - \sum_n p_n = 0.$$ \hfill (4.22)

Now assuming that associated costs are same for all the ECs, i.e., $a_1 = a_2 = ... = a_n = a$ and $b_1 = b_2 = ... = b_n = b$, from (4.21)

$$2e_1 p_1 = 2e_2 p_2 = ... = 2e_n p_n,$$ \hfill (4.23)

and hence, in general

$$\frac{p_{n_1}}{p_{n_2}} = \frac{e_{n_2}}{e_{n_1}}; \, n_1, n_2 \in [1, N]; \, n_1 \neq n_2.$$ \hfill (4.24)

Thus, if $N$ ECs are connected to the CPS and play a GNEP to decide on their amount of energy to be sold to the CPS, at equilibrium the price per unit of energy paid to an EC by
the CPS is inversely proportional to the amount being offered by the EC\textsuperscript{29}. This is because if the CPS pays more to the customer with more energy, it would enable customers with more energy surplus to supply all the required energy by the CPS, and therefore, \textit{customers with less energy would refrain} from participating in the game. Hence, for a socially optimal solution, in which all the ECs participate in energy trading with the CPS and benefit from that solution, the consumers with less available energy are given greater incentive to take part in the game.

### 4.6 Algorithm

To reach the EMES of the proposed game, an algorithm is proposed in this section that can be implemented by the CPS and the ECs in a distributed fashion with limited communication between one another. It has already been shown that the GNEP of the decision making process of the ECs is a variational inequality problem, where $F$ is strongly monotone. Therefore, the solution of the energy supply game among the ECs, in response to the price set by the CPS, can be obtained by solving a strongly monotone variational inequality problem. On the other hand, the CPS decides on its price per unit of energy to pay to each EC by solving a constraint convex optimization problem using any standard convex optimization technique.

For a strongly monotone variational inequality problem, the slack variable, $\xi_n = E_n - e_n + p_n$, possesses the same value for all the ECs, i.e., $\xi_n = \xi, \ n = 1, 2, ..., N$, when their choice of supply amount of energy reaches the variational equilibrium (VE) [Facchinei and Kanzow, 2007]. This property is being used in the algorithm to check the convergence of the solution of the proposed GNEP to the VE. A hyperplane projection method, which is among the simplest ones concerning the solution methods for monotone variational inequalities [Arganda et al., 2011], is used to solve the monotone variational inequality. The hyperplane projection method is always guaranteed to converge to a non-empty solution if $F$ is strongly monotone [Facchinei and Kanzow, 2007], which is the case for the proposed algorithm. Furthermore, for the offered energy amount by the ECs, the optimization problem of the CPS also always converges to a unique solution due to its strict convexity. Thus, the proposed algorithm is guaranteed to converge to an optimal solution for the given constraints in (4.1) and (4.2).

The algorithm is executed in two steps assuming that all the information exchanges between the CPS and the ECs are done through their smart meters via the two-way communication structure of the smart grid [Mohsenian-Rad et al., 2010]. It starts with the announcement of the required energy by the CPS for the considered time slot, and the total amount of price per unit of energy the CPS is going to pay to the ECs. In the first step, each EC $n$ in the network assumes its own equally distributed price per unit of energy $p_n = \frac{P}{N}$, and plays a GNEP to decide on the amount of energy it would offer to the CPS for this price, within constraint (4.1). Having the offered energy from the ECs, the CPS gets insight into the capacity of each EC\textsuperscript{30} $n$, and optimizes the price per unit of energy $p^*_n$ for each $n$, within constraint (4.2), by using any standard convex optimization technique.

\textsuperscript{29}An analogous approach is used in consumer-to-grid Feed-in Tariff schemes [Couture et al., 2010].

\textsuperscript{30}As the utility of each EC is a linear function of price per unit energy, the energy offered by each EC for similar $p_n = p$ would reflect its capacity for supplying energy for the given incentives.
4.6. Algorithm to reach EMES

Algorithm 4.1 Algorithm to reach EMES

Step-1
(i) The CPS announces \( E_{\text{init}} \) and \( P \).
(ii) Each EC \( n \) calculates \( p_n = \frac{E_{\text{init}}}{z_{k+1}} \), and the ECs play a GNEP to determine VE energy \( e_n \), for \( p_n \), using the SSHPM.
(iii) Each EC \( n \) submits the offered energy for \( p_n \), to the CPS.
(iv) The CPS optimizes (4.10) using a standard convex optimization technique [Boyd and Vandenberghe, 2004], and determines \( p_n = p^*_n \), \( \forall n \in N \).

The optimized price vector \( p^* \) is obtained.

Step-2
(v) Each EC \( n \) receives the optimized price \( p^*_n \) offered by the CPS.
(vi) Each EC \( n \) calculates \( e_n^* \) using the SSHPM to determine the VE energies \( e^*_n \) to supply to the CPS.

The VE energy vector \( e^* \) for \( p^* \) is obtained.

The game reaches the EMES.

S-S Hyperplane Projection Method (SSHPM)
1) At iteration \( k \), each EC \( n \) computes the hyperplane projection \( r(e_n^k) \) and updates \( e_n^{k+1} = r(e_n^k) \).
   - if \( r(e_n^k) = 0 \)
     a) The EC \( n \) determines \( e_n = e_n^{k+1} \) to offer to the CPS.
     b) The EC \( n \) sends \( \xi_n = E_n - e_n + p_n \) to the CPS.
   - else
     The EC \( n \)
     a) Determines the hyperplane \( z_n^k \) and the half space \( H^k_n \) from the projection.
     b) Updates the amount \( e_n^{k+1} \) from the projection of its previous energy amount \( e_n^k \) on \( E \cap H^k_n \).
     c) Determines \( e_n = e_n^{k+1} \) to offer to the CPS.
     d) Sends \( \xi_n = E_n - e_n + p_n \) to the CPS.
   end if
2) The CPS checks \( \xi_n, \forall n \in N \).
   - if \( \xi_n = \xi \forall n \in N \)
     The CPS determines the offered energy at VE for the estimated price \( p_n \).
   - else
     The CPS acknowledges the ECs to update their offered energy at next iteration \( k = k + 1 \).
   end if
End of SSHPM

In the second step, each EC \( n \) receives the optimized price \( p^*_n \) from the CPS, and amends the offered energy \( e_n = e^*_n \) to be supplied to the CPS by playing a GNEP for the price \( p^*_n \). The GNEP, in both steps, reaches the VE as soon as the slack variable \( \xi_n, \forall n \in N \) reaches the same value \( \xi_n = \xi \). However, the SLMFG reaches the EMES when the GNEP amongst the ECs reaches the VE for the optimized price vector \( p^* \).

As a hyperplane projection method for solving the GNEP, we use the S-S hyperplane projection method (SSHPM) [Solodov and Svaiter, 1999] in the proposed algorithm. In SSHPM [Solodov and Svaiter, 1999], a geometrical interpretation is used and two projections per iteration are required. Suppose \( e^k \) is the current approximation of the solution of VI(\( E, F \)). First, the projection \( r(e^k) = \text{Proj}_E[e^k - F(e^k)] \) is computed\(^{31}\). Then, a point \( z^k \) is sought in the line segment between \( e^k \) and \( r(e^k) \) such that the hyperplane \( \partial H^k := \{ e \in \mathbb{R}^n \mid \langle F(z^k), e - z^k \rangle = 0 \} \) strictly separates \( e^k \) from any solution \( e^* \) of the problem. In SSHPM, an Armijo-type process [Armijo, 1966] is used to find such a \( z^k \). Once the hyperplane is constructed, \( e^{k+1} \) is computed in the next iteration onto the intersection of the feasible set \( E \) with the hyperspace \( H^k = \{ e \in \mathbb{R}^n \mid \langle F(z^k), e - z^k \rangle \leq 0 \} \), which contains

\(^{31}\text{Proj}_E(z) = \arg \min \{ \| \omega - z \|, \omega \in E \}, \omega \in \mathbb{R}^n \).
the solution set [Solodov and Svaiter, 1999]. The details are Algorithm 4.1.

### 4.7 Numerical Results

To simulate the proposed consumer-centric energy management scheme, we consider a number of ECs in the network that are assumed to take part in trading energy with the CPS. It is assumed that each EC represents a group of 20 similar energy entities. Here, on the one hand, a group of smart homes, in which each home is equipped with a solar panel of battery capacity of 3.6 kilowatt-hour (kWh) [UNISUN, 2012], is assumed as the EC with lowest energy capacity (i.e., the EC with the lowest available energy to supply). On the other hand, a wind farm with a group of RPI080 wind turbines, each with rated capacity around 12.25 kWh [Inc., 2010], is assumed as the EC with the highest energy capacity. The available energy of any EC is assumed to be a uniformly distributed random variable in the range of [64, 240] kWh. We assume that, in the time slot of interest, the energy deficiency of the CPS is 700 kWh, and that there are five ECs in the smart grid network, which are taking part in energy trading to meet this demand. The total price per unit of energy (i.e., per kWh), $P$, is assumed to be 185 US cents\(^{32}\). $p_{\text{min}}$ and $p_{\text{max}}$ are set as 8.45 [Choice, 2012] and 185 cents respectively unless stated otherwise. All results averaged in the simulation are averaged over all possible random values of the ECs’ capacities, using 1000 independent simulation runs.

\(^{32}\text{For five ECs connected to the grid, the average individual price per unit of energy for each EC is 37 US cents per kWh, which is a standard electricity tariff in the U.S.A. [Choice, 2012].}\)}
4.7. Numerical Results

4.7.1 Convergence to the equilibrium

In Fig. 4.2 and Fig. 4.3, we show the convergence of the utility achieved by each EC from selling its energy to the CPS, and the amount of energy sold by each EC to the EMES after a number of iterations. As shown in Fig. 4.2 and Fig. 4.3, as the amount of energy sold by each EC increases towards the equilibrium, the utility achieved for selling this energy also increases to the EMES in a similar fashion, and selling of more energy gives more utility to the EC. Moreover, an EC with more available energy to sell achieves more utility than the other ECs in the network. This is due to the fact that each EC is being paid for what it sells, and thus as the amount of energy sold increases the utility achieved from selling increases. Both the energy supplied by the ECs, and the utility achieved from supplying this energy, converge to the EMES after the 6th iteration.

An interesting property, which is also an objective of the proposed scheme, is presented in Fig. 4.4. By comparing Fig. 4.4 with Fig. 4.3, it can be seen that the cost incurred by the CPS at EMES is more for buying energy from an EC that has less energy available to supply. In other words, the CPS pays more price per unit of energy to an EC that voluntarily takes part in energy trading with the CPS, with a lower available energy to supply. In fact, in a consumer-centric energy management scheme, the recipients of the smart grids’ energy management benefits need to be the energy consumers [Liu et al., 2011], and due to the social optimality of the benefits, all the ECs in the network need to benefit from their energy trading with the CPS. Therefore, as the scheme is formulated, the CPS pays less, up to a certain limit, to the ECs that have more energy for trading, whereas it pays more to the ECs with less energy. This would encourage the ECs with lower available energy to remain engaged in energy trading so as to help the CPS to reduce its total
cost at EMES, as shown in Fig. 4.4.

4.7.2 Effect of the amount of energy required by the CPS on EC’s utility

We show in Fig. 4.5 how the total amount of energy required by the CPS affects the average utility achieved by each EC in the smart grid network. Considering the cases of 5, 10 and 15 ECs connected to the grid, it is shown that the average utility achieved by each EC increases as the amount of energy required by the CPS increases, in all the considered cases. This is because as the energy deficiency of the CPS increases it needs to buy more energy from each of the ECs in the smart grid. This would enable each EC to sell more energy, within its capacity, to the CPS, and thus increase the utility.

4.7.3 Effect of the number of ECs on average utility and cost

In Fig. 4.6, we show the effect of the number of ECs in the smart grid network on the average utility achieved per EC. To show this, the number of ECs in the network is increased from 5 to 25 and the achieved average utility per EC is plotted against the number of ECs. As the number of ECs in the network increases, the CPS can buy its required energy from more ECs in the network, and consequently the amount of energy sold by each EC to the CPS decreases. Hence, the utility achieved by each EC decreases.

An interesting property is shown in Fig. 4.7, where the effect of the number of ECs is considered on the total cost incurred by the CPS. As can be seen from Fig. 4.7, the total cost incurred by the CPS gradually decreases as the number of ECs increases from 5 to 15. Then, the cost again starts increasing as the number of ECs changes from 20 to 25. This can
4.7. Numerical Results

Figure 4.5: Effect of the amount of energy required by the CPS on the average utility achieved by each EC.

Figure 4.6: Effect of the number of ECs on the average utility achieved by each EC.

be explained as follows. For a fixed energy requirement, as the number of ECs increases
from 5 to 15, this would allow the CPS to buy the energy from more ECs, and hence it can pay at a cheaper rate (i.e., more options for buying energy enable the CPS to pay less), and so the total cost gradually reduces. However, the CPS purchases energy from all the available customers to keep them engaged in the energy trading, and thus needs to pay at least the minimum amount (e.g., a minimum amount of $p_{\text{min}} = 8.45$ cents/kWh) to each customer. Therefore, when the number of consumers increases from 20 to 25, although the price per unit energy reduces, the total cost increases due to the mandatory minimum payment to a higher number of ECs in the network.

4.7.4 Effect of the upper bound on the price per unit of energy on the total cost incurred by the CPS

We show the effect of upper bound $p_{\text{max}}$, on the price per unit of energy, on the average total cost incurred by the CPS in Fig. 4.8. We assume that the CPS can pay a maximum of between $\frac{P}{N}$ and $P$ cents per kWh to any EC for buying its energy. The total number of ECs $N = 5$, and the total price per unit of energy $P$ is 185, 210 and 260 US cents respectively. As shown in Fig. 4.8, the average total cost incurred by the CPS eventually decreases as the threshold on the price per unit of energy increases, and reaches a stable state that is not affected further by any change in price. In fact, as proposed in the algorithm, the CPS gains insight into the capacity of the ECs’ energy supply by offering them an equal price $\frac{P}{N}$ in the first step of the algorithm. Then, in the final step, the CPS optimizes its price, given the energy offered by each EC, so as to minimize its total cost. In the course of this optimization, as we have seen in Section 4.5, the EC with less capacity is paid more and vice-versa. However, keeping the threshold at $\frac{P}{N}$ restricts the degree of freedom of
the CPS in choosing its price per unit of energy to pay to any EC, and consequently a higher total cost is incurred by it. On the other hand, as the threshold increases the CPS can choose a higher price, bounded by the threshold, to pay to the EC with less energy, which in turn enables the CPS to pay a lower price to other ECs in the network. Hence, as shown in Fig. 4.8, the total cost incurred by the CPS decreases. Nevertheless, at a particular threshold of price per unit energy, the CPS is enabled to optimize the price to incur the lowest cost to itself, and hence further change in the threshold does not lead to any change in the average total cost, as shown in Fig. 4.8. Furthermore, as the total price per unit energy increases, the total cost incurred by the CPS increases, i.e., the plot moves upwards, because the CPS pays more to the ECs for their energy.

4.7.5 Computational cost measured by speed of convergence

The number of ECs in the network affects the computational cost of the proposed scheme measured by speed of convergence of the game to the EMES as shown in Fig. 4.9. We show the average and maximum number of iterations required to reach the EMES of the game versus the number of ECs in the network. As Fig. 4.9 shows, when the number of ECs in the network increases from 5 to 25, the ECs need more iterations to find their optimal strategies (the amount of energy to be supplied within constraint (4.1)). For example, if the number of ECs in the network increases from 15 to 25, the average number of iterations required to reach the EMES increases from 20 to 29. A similar behavior can be found for the maximum number of iterations. Hence, as the number of ECs in the smart grid increases the convergence of the algorithm to the solution slows.
Figure 4.9: Effect of the number of ECs on the average and maximum number of iterations to reach the equilibrium.

4.7.6 Comparison with a Feed-in tariff (FIT) scheme

In Fig. 4.10, Fig. 4.11, and Fig. 4.12, the performance of the proposed scheme is compared with a standard Feed-in tariff (FIT) scheme [Couture et al., 2010]. An FIT is a long-term incentive based energy trading scheme designed to encourage the uptake of renewable energy systems that provide the main grid with power (e.g., when the main grid fails to meet the demands of its consumers from its own supply). A higher tariff is paid to the electricity producers as an incentive to take part in the FIT scheme. For comparison, it is assumed that the contract between the energy sources and the CPS is such that the sources are capable of providing the energy the CPS requires. For the FIT scheme, the per unit tariff is considered to be 60 US cents/kWh [Choice, 2012].

The average utility per EC for the proposed scheme and the FIT scheme are shown in Fig. 4.10 as the number of ECs increases in the network. For both cases, the average utility reduces as the number of ECs in the network increase. However, the utility for the proposed scheme is always shown to be better than the utility achieved by the ECs for the FIT scheme. This is due to the fact that the proposed scheme allocates the amount of energy for each EC, using an SLMFG, in such a way that the consumers benefit is maximized. On the other hand, the FIT is a contract based scheme that enables the customers to supply the amount stipulated in their contract irrespective of what they would choose with respect to the current situation (e.g., generation cost, price, transmission cost). As shown in Fig. 4.10, each EC in the network achieves an improved utility for the proposed scheme which is, on average, 1.5 times better than the utility achieved by adopting the FIT scheme.
Assuming the same total price per unit energy for both the proposed and the FIT schemes, the change in the average total cost of the CPS for buying energy from the ECs is shown in Fig. 4.11 as the total price per unit of energy increases. As shown in Fig. 4.11, the average total cost increases for both the proposed and the FIT schemes as the total per unit price increases, as explained for Fig. 4.8. However, due to the optimal allocation of per-unit price for each EC, the average total cost for the proposed scheme is always lower than that of the FIT scheme in all price ranges. The performance benefit of the proposed scheme is also shown to increase as the total price per unit of energy increases. It is due to the real-time price optimization by the CPS of the proposed scheme in response to the VE energy demand of the ECs, in contrast with the contract-based payment of the FIT scheme.

In Fig. 4.12, the average total cost incurred to the CPS for the proposed scheme is compared with the FIT scheme as the number of ECs in the network increases, assuming the similar total price per unit of energy for both schemes. For both cases the cost decreases as the number of ECs in the network increases from 5 to 15, and then gradually increases again as the number increases from 20 to 25. The reason for this change of cost with ECs is discussed in Fig. 4.7. However, the proposed scheme always shows a lower average total cost compared to the FIT scheme. As shown in Fig. 4.12, for 5 ECs in the smart grid network the average total cost for the proposed scheme is 0.85 times that of the FIT scheme, and for 25 ECs it becomes 0.93 times that of the FIT scheme. This is due to the fact that as the number of ECs in the network increases, the CPS needs to optimize the per unit price for a greater number of ECs, and also needs to pay a minimum amount to every EC in the network. Hence, because of the constraint in (4.2), having more ECs
Figure 4.11: Comparison of the effect of the total per unit price on the cost to the CPS for the proposed scheme and the FIT scheme.

Figure 4.12: Comparison of the average total cost incurred to the CPS at the equilibrium for the proposed scheme and the FIT scheme.
induces the CPS to choose a price closer to this minimum individual payment, and thus
the total cost incurred to the CPS falls closer to the cost incurred with the FIT scheme.
Nonetheless, for relatively small numbers of ECs in the network, the proposed scheme
shows considerable performance improvement over the FIT scheme. The cost to the CPS
for the proposed scheme is, on average, 0.7 times the cost for the FIT scheme.

4.8 Concluding Remarks

In this chapter, a consumer-centric energy management scheme for smart grids has been
studied, in which a number of energy consumers and a centralized power station take
part in an energy management game whereby they set energy trading parameters accord-
ing to their choice of optimal strategies. The proposed scheme is based on maximizing the
end user’s benefit, as well as keeping the total cost to the power station at a minimum.
It has been shown that the game admits a socially optimal solution, and the properties
of the solution have been studied for the considered system. Moreover, an algorithm has
been proposed that can be implemented in a distributed fashion by the energy consumers
and the centralized power station with limited communication between each other. Sim-
ulation results have shown the effectiveness of the proposed scheme, with noticeable
performance improvements over a standard Feed-in Tariff energy management scheme.
Certainly, this spontaneous participation of consumers in the smart grid not only has en-
ergy management benefits, but also can be used for outage management in the event of a
power outage in the grid. Hence, in the following chapter, we show how voluntary par-
ticipation of consumers can efficiently manage power usage during an outage in a smart
grid power system.
Efficient Energy Curtailment Scheme for Smart Grid

In this chapter, an efficient energy curtailment scheme for future smart grids is studied. The proposed scheme enables the power users in a smart grid network to decide on the reduction in energy supplied to them in the event of a power outage in the system so as to minimize the total cost incurred to them. A non-cooperative generalized Nash game is proposed\(^{33}\), using the advantages of the two-way communication infrastructure in future smart grids, where the power consumers of the network are considered as the players of the game. The strategy of each player is to choose the reduction in energy supplied to it based on its energy requirements during the period of power outage. The game is modeled as a variational inequality problem, and it is shown that the socially optimal solution is obtained at the variational equilibrium of the game. A novel algorithm is proposed that enables the players to reach the optimal solution. In addition, the performance improvement of the proposed scheme is demonstrated by comparing it with a standard equal power curtailment scheme, and it is shown that the proposed scheme yields an improvement of about 15\%, on average, in terms of total cost reduction of the system during a power outage.

5.1 Introduction

Increased expectations of customers, limited energy resources and the expensive process of exploiting new resources have put the reliability of the power grid in danger [Samadi et al., 2010]. Especially in the event of a power outage in the grid system, lack of “situational awareness” is often the main reason that leads to a large scale fault event which may cause an extensive cost to the whole system [He and Zhang, 2011]. For instance, the annual cost of power outages in the USA in 2002 was estimated to be around $79 billion [Moslehi and Kumar, 2010], rising to $100 billion in 2007 [NETL, 2007]. Therefore, the study of an efficient outage management scheme in the event of a power disruption in the grid (e.g., a scheme for optimal curtailment of electricity from the customers) to reduce its catastrophic impact on the whole grid system is of paramount importance to improve the system’s reliability.

\(^{33}\)In contrast to Chapter 4, where we adopted a single-leader multiple-follower game for energy management, here we simply use a generalized Nash game.
It is envisioned that a smart grid will transform the current power grid into one that functions more intelligently, giving better situational awareness and providing resilience against component failures and disastrous impacts on the grid [Moslehi and Kumar, 2010]. In a smart grid network, energy consumers are able to actively take part in the decision making process regarding various grid management issues, and then agree on parameters that better protect the grid from any undesirable fault event [Fang et al., 2011]. Hence, in recent years, extensive research has been devoted to system reliability and pre and post-failure protection of the smart grid, e.g., [Calderaro et al., 2011; Overman et al., 2011; Chertkov et al., 2011; Kezunovic, 2011; He and Zhang, 2011; Russell and Benner, 2010].

One of the key challenges for reliable smart grid operation is the post-outage management of power among the users immediately after a power disruption in the network. A power outage may occur due to a fault in the power line or due to the intermittent nature of renewable energy sources [Fang et al., 2011]. Hence, there might be a need of curtailment of power from the users in the network so as to manage the activities of the grid effectively during the period of outage until the whole system can operate normally again. Of paramount importance is efficient curtailment of energy so that there is minimum total cost to the whole system. It is important to note that even a reduction in the total cost by 1% can significantly benefit the users in the system. For example, the annual cost of outage in the U.S.A. in 2002 would be reduced by up to $790 million with only 1% reduction in total cost of outage [Moslehi and Kumar, 2010]. Thus, there is a need to develop solutions that will be able to optimally reduce the total cost incurred to a system during periods of power outage, and thereby to benefit the power system users.

The main contribution of this chapter is to model an efficient outage management scheme for smart grid by taking advantage of its two way communications infrastructure. An efficient energy curtailment scheme for reducing energy supplied to users in the event of a power outage is proposed to minimize the total cost incurred to the system for this outage. We study an energy curtailment game (ECG) where the users play a generalized Nash game amongst themselves to decide on the amount of energy to be curtailed from them in the event of a power disruption. We investigate the properties of the game and show that there exists a socially optimal solution. The socially optimal solution is where the total cost incurred reaches a global minimum. An algorithm is proposed to obtain this optimal solution. Through simulation, the performance of the game is assessed and the effectiveness of the scheme is demonstrated by comparing it with a standard equal energy curtailment scheme.

The rest of the chapter is organized as follows: the system model is presented in Section 5.2. A non-cooperative generalized Nash game is formulated and its properties are studied in Section 5.3. In Section 5.4, the solution method of the game is discussed and an algorithm is proposed to reach the equilibrium. Numerical results for the proposed scheme are given in Section 5.5. Finally, conclusions are drawn in Section 5.6.

5.2 System model

Consider a smart grid system, as shown in Fig. 5.1, consisting of a single energy source (ES) and multiple energy users (EUs). The ES can be a single energy generating unit or
Chapter 5. Efficient Energy Curtailment Scheme for Smart Grid

Figure 5.1: System model for outage management in smart grid.

an aggregation of multiple distributed renewable energy generating units in the network such as wind farms, smart homes, solar farms, bio-gas plants and plug-in hybrid electric vehicles (PHEVs) acting as a single virtual power plant in the system. It is assumed that each EU is equipped with a smart meter that has a decision making capability on the amount of energy to be curtailed and each EU is also connected to the ES by means of a power line [Mohsenian-Rad et al., 2010]. The smart meters are also connected to the ES through a local area network (LAN). All communications between the ES and EUs take place using an appropriate communication protocol (e.g., Zigbee [Iqbal et al., 2011]).

Throughout the chapter, it is assumed that $\mathcal{N}$ denotes the set of EUs in the network where the number of EUs is $\mathcal{N} = |\mathcal{N}|$. Each EU $n \in \mathcal{N}$ is a single user in the network, for instance a smart home. Renewable energy generation is subjected to wide fluctuations and the available energy for the consumers may vary significantly with time [Molderink et al., 2010]. In the event of an unfavorable circumstance, e.g., a cloudy day making solar energy generation unproductive, or the failure of a few energy generating units, a power disruption may occur in the smart grid system. Therefore, the ES would be unable to meet the total energy demand of its customers in the network for a particular period of time, e.g., before restoration of full service. Let us assume that, for a particular duration of time of the day, $E_d$ is the total energy demand of the consumers and $E_a$ is the available energy to the ES in the event of a power outage. If $E_d > E_a$, the ES will be unable to meet the excess demand,

$$E_x = E_d - E_a,$$

of the power users in the network for that period of time. So this amount of energy must be curtailed from the EUs and the users will experience a black-out. If $e_n$ is the amount of energy, which is to be curtailed from user $n$, the curtailment of energy $e_n$ from each user
According to constraint (5.2), the total deficiency of energy $E_x$ is overcome by suitable curtailting of energy from all users, which is necessary for reliable power distribution in the smart grid network.

Energy requirements of EUs may vary based on different factors, such as the time of the day or the type of EU. For example, a school requires less energy, and maintaining full energy supply is less important, during vacation than during term time. Hence, such factors must be taken into account when designing an energy curtailment scheme for the EUs. Thus, the main challenges faced during the decision making process for energy curtailment in a smart grid in the event of a power outage are:

1. modeling the decision making process of energy curtailment from the energy users given the constraint in (5.2);
2. capturing the EUs’ requirements for energy during the decision making process for curtailment; and
3. developing an algorithm that enables the EUs to optimize the amount of energy to be curtailed from them so as to minimize the total cost incurred to the system.

To address the above challenges, first we define a cost function for each EU in Section 5.2.1 and then formulate the decision making process as a constrained optimization problem in Section 5.2.2.

### 5.2.1 EU’s cost function

To capture the effects of energy outage on the overall smart grid system, we define a cost function $c_n$ for each EU $n \in \mathcal{N}$ in the network, which represents the cost incurred by the EU $n$ due to the curtailment of $e_n$ from it. The choice of cost function is based on a linearly decreasing cost with decrease in energy supply, which has recently been shown to be appropriate for power users [Samadi et al., 2010]. The cost function $c_n$ for EU $n$ is defined as, [Fahrioglu and Alvarado, 1999],

$$c_n(e_n, \theta_n) = k_1 e_n^2 + k_2 (e_n - \theta_n e_n),$$

(5.3)

where $k_1, k_2 > 0$ are the scaling factors and $\theta_n$ is the customer preference parameter (CPP) of EU $n$ [Fahrioglu and Alvarado, 1999]. The CPP, $\theta_n$, is a measure of each EU’s preference for the amount of energy to be curtailed from it. For example, curtailment of 1 kWh of energy may have far worse impact on industry than on a residential home. Thus, the CPP could be very different for a residential home than for an industry for the same energy curtailment. As from (5.3), a higher CPP leads to a lower cost for an EU and hence, an EU with higher CPP would be able to endure the cost of more curtailment of energy than the EU with lower CPP. To this end, the cost function in (5.3) is assumed to possess the following properties:
Chapter 5. Efficient Energy Curtailment Scheme for Smart Grid

i) the cost for EU \( n \) increases as the amount of energy to be curtailed from it increases. That is
\[
\frac{\delta c_n}{\delta e_n} > 0.
\] (5.4)

ii) an EU with higher CPP will experience less cost compared to an EU with lower CPP for the same energy curtailment. Therefore,
\[
c_n(e_n, \tilde{\theta}_n) < c_n(e_n, \hat{\theta}_n), \forall \tilde{\theta}_n > \hat{\theta}_n.
\] (5.5)

5.2.2 Problem formulation

It is clear from (5.3) that, for a fixed amount of energy curtailed from an EU, the cost incurred by the EU changes as the CPP of the EU changes. Hence, each EU \( n \) in the smart grid network needs to optimally choose an amount of energy \( e_n \) to be curtailed from it so as to minimize the overall cost \( c = \sum_n c_n(e_n, \theta_n) \) incurred to the whole system. Thus, the objective of each EU is
\[
\min_{e_n} \sum_n c_n(e_n, \theta_n) = \min_{e_n} \sum_n \left(k_1 e_n^2 + k_2 (e_n - \theta_n e_n)\right),
\] subject to \( \sum_n e_n = E_x, \forall n \in \mathcal{N}. \) (5.6)

The optimization problem in (5.6) can be solved by the ES in a centralized fashion if the ES knows all the parameters. However, in smart grid, the ES may not have full control over the decision making process of the EUs [Wu et al., 2012a], and in particular the ES may not know the preference parameter \( \theta_n \) when each EU \( n \) keeps \( \theta_n, k_1 \) and \( k_2 \) confidential. Therefore, a decentralized decision making scheme is required for the EUs to voluntarily choose an amount of energy to be curtailed from them for the whole system’s social benefit. Next, we will show that the optimization can be achieved with limited coordination by the ES and without letting the ES know the \( e_n \) that each EU contributes.

5.3 Generalized Nash game

To study the energy curtailment scheme for outage management in a smart grid, we use the framework of a generalized Nash game [Facchinei and Kanzow, 2007]. A generalized Nash game (GNG) is a type of game that allows joint constraints for all players involved in the game [Facchinei and Kanzow, 2007]. In the proposed game, the players are the energy users in the network, which choose the amount of energy to be curtailed from them subject to the joint constraint in (5.2). The game is formally defined by its strategic form
\[
\Gamma = \{\mathcal{N}, \mathbf{E_n}, c\},
\] (5.7)

which has the following components:

i) the set of energy users in the smart grid network \( \mathcal{N} \).
ii) the strategy vector $E_n$ of each player $n \in \mathcal{N}$, which refers to the amount of energy to be curtailed $e_n \in E_n$ from $n$ satisfying the constraint $\sum_n e_n = E_x$.

iii) the total cost $c$ incurred by all the EUs due to the curtailment of energy $e_n$ from the EU $n$ for all $n \in \mathcal{N}$.

It is important to note that the action of each EU $n$, to optimize (5.6), affects the choice of actions of other EUs in the network due to the presence of (5.2). Thus, the proposed GNG is a jointly convex generalized Nash equilibrium problem (GNEP) with coupled constraint (5.2) [Facchinei and Kanzow, 2007]. To solve this jointly convex GNEP, we formulate the game as a variational inequality problem (VIP) and investigate the existence of the variational equilibrium (VE) of the game [Arganda et al., 2011]. In fact, the VE is the socially optimal outcome of a GNEP, and hence a valuable target solution for the proposed energy curtailment scheme. This is due to the fact that a socially optimal solution leads to the minimum total cost for the whole system. To this end, we investigate the existence of a socially optimal solution of the proposed GNG in the following section.

5.3.1 Existence of a socially optimal solution

The existence of a pure strategy equilibrium in a non-cooperative game is not always guaranteed [Başar and Olsder, 1999]. Therefore, in this section, we investigate the existence of a solution of the proposed GNG and study its optimality, if it exists. Now, the joint cost function in (5.6) of the proposed game can be expressed as, [Krawczyk and Tidball, 2006],

$$c(e) = \sum_{n=1}^{N} c_n(e_n, \theta_n),$$

where

$$e = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{bmatrix} \in E,$$  

and $E$ is included in the definition of joint convexity [Facchinei and Kanzow, 2007]. Due to the jointly convex nature of the GNEP, the proposed GNG can be formulated as a variational inequality problem VIP($E, F$) [Facchinei and Kanzow, 2007] where $F = (\nabla c_n(e))_{n=1}^{N}$ is the pseudo-gradient of (5.8). Now, to show the existence of an optimal solution, we will prove the following theorem:

**Theorem 5.3.1.** A variational equilibrium (VE) exists for the proposed VIP($E, F$) and the VE is unique.

**Proof.** To prove this, we need only to prove that the pseudo-gradient of $c(e)$ monotonically increases with $e_n$ for fixed $k_1, k_2$ and $\theta_n$ [Krawczyk and Tidball, 2006]. Clearly from
(5.8), the pseudo-gradient of $c(e)$ is

$$F = \begin{bmatrix} 2k_1 e_1 + k_2 (1 - \theta_1) \\ 2k_1 e_2 + k_2 (1 - \theta_2) \\ \vdots \\ 2k_1 e_N + k_2 (1 - \theta_N) \end{bmatrix}, \quad (5.10)$$

and the Jacobian of $F$ is [Arganda et al., 2011],

$$JF = \begin{bmatrix} 2k_1 & 0 & \cdots & 0 \\ 0 & 2k_1 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 2k_1 \end{bmatrix}. \quad (5.11)$$

In (5.11), the scaling factor $k_1$ is always positive. Now, considering the $i^{th}$ leading principal minor (LPM) $JF_i$ of the leading principal sub-matrix, it can be shown that $JF_i$ is always positive (i.e., $|JF_1| > 0$, $|JF_2| > 0$, and so on). Hence, $JF$ is positive definite on $E$, and thus, $F$ is strictly monotone. So, VIP$(E,F)$ admits a unique VE solution [Facchinei and Kanzow, 2007].

**Remark 5.3.1.** An important result of Theorem 5.3.1 is that the VE is the socially optimal solution of the proposed GNG. This can be explained as follows: from Theorem 5.3.1, the VE is unique for the proposed VIP$(E,F)$. On the other hand, the proposed GNEP has already been shown to be a jointly convex GNEP. Hence, the VE is the unique global minimizer of (5.8) [Facchinei and Kanzow, 2007]. Thus, in other words, it is also the socially optimal solution of the proposed GNG.

For the rest of the chapter, we will use the VE to indicate the solution of the proposed GNG.

### 5.4 Game solution and algorithm

Each EU plays a GNG, to choose an amount of energy to be curtailed, by solving the variational inequality problem. Each EU $n$ wants to minimize the total cost incurred to the system by suitably choosing its strategy $e_n$ subject to (5.2). The total cost is minimized as soon as the solution of the game reaches the VE.

**Definition 5.4.1.** Consider the GNG $\Gamma$ given in Section 5.3 where the joint cost function $c$ is defined as in (5.8). A vector of strategies $e^*$ constitutes the VE of the game if and only if it satisfies the following set of inequalities:

$$c(e^*) \leq c(e), \forall e_n \in E_n, n \in N, \quad (5.12)$$

where $e^* = [e_1^*, e_2^*, \ldots, e_N^*]^T$ and $e = [e_1^*, e_2^*, \ldots, e_{n-1}^*, e_n, e_{n+1}^*, \ldots, e_N^*]$ for one or more $n \in N$.

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34 The $i^{th}$ order principal submatrix $A_i$ can be created by deleting the last $g - i$ rows and last $g - i$ columns from $g \times g$ matrix $A$. 
5.4. Game solution and algorithm

Algorithm 5.1 Algorithm to reach VE
1. The ES announces $E_s$ to the EUs in the network.
2. Each EU $n$ estimates an amount $e_n$ to be curtailed using the S-S method [Solodov and Svaiter, 1999]:
   
   **S-S method**
   
   a) At iteration $k$, EU $n \in N$ computes the hyperplane projection $r(e^k_n)$ and updates $e^{k+1}_n = r(e^k_n)$.
   
   b) The EU checks: if $r(e^k_n) = 0$
      
      a) The EU chooses the energy $e_n$ to submit to the ES.
      
      else
      
      a) the EU $n$ determines the hyperplane $z^k_n$ and the half space $H^k_n = \{e_n \in \mathbb{R}^n | \langle F(z^k_n), e_n - z^k_n \rangle \leq 0 \}$ from the projection [Solodov and Svaiter, 1999].
      
      b) the EU updates the amount $e^{k+1}_n$ from the projection of its previous energy $e^k_n$ on to $X \cap H^k_n$ and choose to submit to the ES.
   
   end if
3. The EU $n$ calculates $\lambda_n$ using Proposition 5.4.1 and submits it to the ES.
4. The ES checks $\lambda_n, \forall n \in N$.
   
   if $\lambda_1 = \lambda_2 = ... = \lambda_N = \lambda$
   
      The ES determines the VE energy vector $e^*$ of all the EUs in the network.
   
   else
   
      The ES directs the EUs to Repeat step - 2 and Step - 3.
   
   end if

The VE energy choices of all the energy users in the network $e^*$ are obtained.

Thus, the VE defines a state of the game in which the total cost incurred by the system cannot be reduced if any EU deviates from its VE strategy and chooses a different amount of energy to be curtailed from it, given the other EUs are playing their VE strategies. To find the VE of the game, the EUs in the network have to solve a VIP. As shown in Section 5.3, the VIP of the proposed scheme is strictly monotonic and thus the EUs can reach the VE of the game by solving a monotone VIP. To find the solution of the proposed monotone VIP, it will be useful to state the following proposition which characterizes the solution of a strictly monotone VIP [Facchinei and Kanzow, 2007].

**Proposition 5.4.1.** For a strongly monotone VIP, the slack variable $\lambda_n = \delta_{c_n}/\delta e_n$ possesses the same value $\lambda$ at the VE solution for all the EUs in the smart grid network. That is, $\lambda_1 = \lambda_2 = ... = \lambda_N = \lambda, \forall n \in N$.

This property is used to determine the VE of the proposed GNG. We use a hyperplane projection method, particularly the iterative S-S method [Solodov and Svaiter, 1999], to solve the proposed monotone variational inequality problem to determine the EUs' decisions on the energy vector. The algorithm, as detailed in Algorithm 5.1, requires limited communication between the EUs and the ES in the network. The algorithm starts with an announcement by the ES of the total energy deficiency in the network and the EUs play a GNG amongst themselves to reach an optimal solution. Each EU $n$ uses the S-S hyperplane projection method to choose its energy curtailment and calculates $\lambda_n$ for its choice using Proposition 5.4.1. The EUs submit these $\lambda_n, \forall n \in N$, to the ES. The ES checks $\lambda_n$ for each EU $n$ and informs the EUs as to whether all $\lambda_n$'s are equal. The algorithm converges to the optimal VE and the curtailment of energy takes place as soon as $\lambda_n$ converges to

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35Projection $r(z) = \arg \min \{||w - z||, w \in E\}, \forall z \in \mathbb{R}^n$.
36For instance, a single bit can be sent by the ES to the EU $n$ if the EU's strategy does not satisfy (5.2) given the strategies of other EUs.
the single value $\lambda_n = \lambda$, $\forall n \in \mathcal{N}$. For a strongly monotone VIP, the hyperplane projection method is guaranteed to converge to a non-empty solution [A. Nedic and U. V. Shanbhag, 2008], and consequently, the proposed algorithm is guaranteed to always converge to a non-empty optimal solution.

### 5.5 Numerical Simulation

To simulate the proposed energy curtailment scheme, we consider a number of EUs in the network and simulate the scheme for different scenarios. The consumer preference parameter is chosen from a uniform random distribution between 0 and 1 [Fahrioglu and Alvarado, 1999]. The value of $k_1$ and $k_2$ is chosen in such a way that $\frac{k_1}{k_2} = 0.5$ is maintained [Fahrioglu and Alvarado, 1999]. The minimum deficiency in energy is chosen as 2 kWh and the maximum deficiency is chosen as 10 kWh for the duration of the power outage.

To show the convergence of the proposed algorithm to the VE, we assume a network with five EUs in which they are playing a generalized Nash game amongst themselves to reduce the effect of a total energy outage of an amount of 10 kWh on the system. In Fig. 5.2 and Fig. 5.3, we show the convergence of the amount of energy that has to be curtailed from each EU to the VE and also the convergence of the cost due to this curtailment. From these figures, it can be seen that as the amount of energy to be curtailed reduces, the cost incurred to the corresponding EU decreases and vice versa. This is due to the

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Clearly this energy deficiency range is highly variable and could be different for many scenarios.
5.5. Numerical Simulation

variation in the customer preference parameter, $\theta_n$, of each EU $n$, as the choice of energy curtailment is highly dependent on $\theta_n$. Hence, by choosing more curtailment by the EU with higher CPP and lower curtailment by the EU with lower CPP, the total cost incurred to the system can be reduced to the minimum at the VE. As shown in Fig. 5.2 and Fig. 5.3, both the choice of energy and the cost incurred by each EU converges to the VE after the 10th iteration of the algorithm.

The total cost incurred by the system at VE is shown in Fig. 5.4, where we see that the total cost is reduced to its minimum value after 10 iterations. Although the cost, incurred at variational equilibrium, to each EU does not achieve a minimum value for all EUs, as seen in Fig. 5.3, the total cost to the system is minimized at equilibrium. This is due to a preference based choice of energy by the EUs in the network. An EU with higher CPP can endure the impact of more energy curtailment (i.e., more cost) compared to the other EUs with lower CPP. Thus, higher cost to an EU with higher CPP does not affect the system, due to its higher tolerance to reductions in energy supply.

The performance benefit of the proposed scheme is assessed in Fig. 5.5 and Fig. 5.6. The effect of total energy deficiency on the average cost incurred by any EU in the network is shown in Fig. 5.5. The effect of the number of EUs on the total average cost of the system is shown in Fig. 5.6. In both Fig. 5.5 and Fig. 5.6, we compare our proposed scheme with a standard equal energy curtailment (EEC) scheme [Costa et al., 2004] in which an equal amount of energy is curtailed from each of the users in the network to mitigate the effect of total system energy deficiency.

In Fig. 5.5, for a fixed number of EUs in the network, the effect of variation in total system energy deficiency on the average cost per EU is observed. It is shown for both the EEC and the proposed case that the average cost per EU increases as the system’s total
energy deficiency increases. Because of more energy curtailment allowed by the EUs the average cost incurred per EU increases. However, the average cost per EU is less for our
scheme than for the EEC scheme. As shown in the figure, the average costs per EU for the proposed case are 0.76, 0.86, 0.78, 0.85 and 0.88 times the cost of the EEC scheme for energy deficiencies of 2, 4, 6, 8 and 10 kWh respectively. Therefore, there is a 17% average reduction in cost for our scheme. The CPP of each EU enables it to decide on the amount to be curtailed that reduces the total cost to the system and thus obtains a performance benefit in terms of average cost per EU relative to the EEC scheme.

The effect of the number of EUs on the average total cost for the system is shown in Fig. 5.6. Assuming a total 10 kWh energy deficiency, we increase the number of EUs in the network from 5 to 25 and observe the impact on total cost for both the proposed scheme and the EEC scheme. In Fig. 5.6, the total cost for the system decreases as the number of EUs in the network increases. The reason behind this phenomenon is that, as the number of EUs in the network increases, less energy can be curtailed from each EU to address the total system energy deficiency, and consequently the average total cost decreases. It is also observed in Fig. 5.6 that the average total cost for our proposed scheme is between 5.6% and 26% less than the EEC scheme, and on average 15% less than the EEC scheme, with increasing performance benefit for our scheme as the number of EUs increases. The performance benefit is due to the fact that in the proposed scheme the EUs decide on their energy curtailment amount based on their preferences, and optimally choose the amount to be curtailed from them. On the other hand, in the EEC scheme the energy is curtailed uniformly from all the users in the network. Hence, this optimal selection of energy leads to an improvement for the proposed scheme, in terms of total cost incurred to the smart grid system, compared to the EEC scheme. Furthermore, such improvement in total cost becomes more pronounced as the number of EUs increases.
5.6 Concluding Remarks

In this chapter, we have introduced an approach for outage management in smart grid to address the effect of power disruption in the system. We have formulated a non-cooperative generalized Nash game among the energy users in the network. In the game, each user strategically chooses an amount of energy to be curtailed from them based on their preference parameter. We have studied the properties of the game and we have shown that the game leads to an optimal solution for curtailment. By formulating the game as a variational inequality problem, we have proposed a novel algorithm that enables the power users in the network to choose their strategies of energy curtailment, and thus reach the optimal solution for the game. With simulation, it has been shown that the total cost to the smart grid system due to this energy outage converges to a minimum at the variational equilibrium. We have also compared our scheme with an equal energy curtailment scheme and we have demonstrated improvement in terms of reduction in average total cost as well as reduction in average cost per user in the system. Interestingly, this energy curtailment scheme, or any other energy management scheme, for smart grids are solely dependent on efficient information infrastructure based on a smart sensor network [Fang et al., 2011]. However, sensors are typically battery powered and thus, energy constrained entities. Hence, power control of sensors, for example during their radio signal transmission, is very important for increasing a wireless sensor networks life-time, and consequently, to ensure the robust and efficient operation of smart grids. To that end, we study power control schemes for distributed wireless sensor networks in the next chapter.
In this chapter, we study two separate power control mechanisms, for signal transmission in wireless sensor networks. Firstly, in Part 1, we propose a non-cooperative power control game in which each sensor in a cluster is considered as a player that chooses its power independently to minimize its cost of power usage, and also to achieve a target signal to interference noise ratio at the master sensor of the target cluster. It is shown that the game has a Nash equilibrium that is unique under certain constraints, and is subjected to change according to distance dependent attenuation and variations in path-loss exponent. Then, for transmitting the signal from a sensor cluster to the intended receiver, we propose a novel cooperative cross-layer transmission scheme in Part 2 of this chapter, which is shown to reduce the required energy for transmission of sensor nodes while communicating over long distances in a wireless sensor network. It is shown that the proposed transmission scheme requires less transmit power when compared to direct beamforming and direct single link transmission. Considering battery energy consumption, we show that the scheme is capable of increasing the lifetime of sensor nodes that need to communicate over long distances. Improved performance in terms of a routing metric is also demonstrated.


#### 6.1.1 Motivation

In a multiple-source wireless sensor network, due to the use of a time division multiple access (TDMA) scheme, the information sharing time that is required for node collaboration increases proportionally with the number of sources [Dong et al., 2007]. As a result, there is an increase in overall transmission delay. However, a transmission scheme where all the sources transmit their information simultaneously, and non-cooperatively, towards their intended clusters can alleviate this delay in transmission whereupon, any given cluster can beamform (or otherwise) the information from a particular source to the receiver (which is further discussed in Part 2 of the chapter). Nevertheless, simultaneous transmission causes interference at each receiving cluster, and subsequently reduces the signal-to-interference-noise ratio (SINR) [Larsson et al., 2009]. Due to energy constraints,
which are typical in a sensor network, and in order to have an acceptable SINR at the receiving cluster, power control is used for resource allocation and interference management [Betz and Poor, 2008].

In a multi-source multi-cluster wireless sensor network, as presented in this part of the chapter, we use non-cooperative game theory to study the power control of sensors in a sensor network. Essentially, non-cooperative game is a very powerful tool to study power control in data networks [Betz and Poor, 2008; Meshkati et al., 2005; Betz and Poor, 2006; Koskie and Gajic, 2005; Bacci et al., 2007; Meshkati et al., 2007]. In this study, we assume that the source sensors are non-cooperative and rational. We choose the SINR as a quality-of-service (QoS) measure. Each source aims to maximize its SINR at the receiving cluster as well as to minimize the transmit power to achieve that. The sources are aware of the choices of power of other sources and they can choose their own transmit power. Therefore, in this power control game, each source aims to choose a power to be in equilibrium so as to meet the target SINR at the receiving cluster by maintaining the minimum cost of energy (or transmit power).

To this end, we choose a cost function which was first introduced in [Koskie and Gajic, 2005]. We note that a star topology was considered in [Koskie and Gajic, 2005], with one base station (BS) and multiple mobile station for power control. In comparison, in the proposed case, we investigate a different topology for sensor networks, with power control from multiple source sensors to multiple clusters (as opposed to one base station). Thus, with the proposed power control game, the main contributions of this work are:

i) We formulate a non-cooperative game for a distributed multi-source wireless sensor network with multiple receiving clusters, and determine the Nash equilibrium of the game based on the extreme value theorem.

ii) We explain the best response of the proposed game, and comment on the uniqueness of the Nash equilibrium. We show that the analysis in [Koskie and Gajic, 2005] is also valid in our system. In addition, we derive the dependence of the Nash equilibrium on the distance between the source sensor and target cluster.

iii) With simulation, we show the effectiveness of the proposed game in an environment where distance (and exponent) dependent path loss affects the received SINR.

The rest of Part 1 of the chapter are organized as follows. We describe the system model in Section 6.1.2. The non-cooperative power control game is formulated in Section 6.1.3 including the Nash equilibrium of the game and bounds on source power at equilibrium. Properties of equilibrium, which include uniqueness of the solution and dependence of the equilibrium on distance are discussed in Section 6.1.4. Finally, simulation results and performance analysis are given in Section 6.1.5 considering a network with two clusters and two source sensors.

### 6.1.2 System Model

We consider a system with $N$ source-receiving cluster pairs, as shown in Fig. 6.1, in which the sources are transmitting their signals simultaneously towards the respective clusters. Each receiving cluster consists of multiple cooperative sensors with a master sensor node. We index each source sensor with $n$ and each cluster with $j$. We assume a time-slotted

Figure 6.1: System Model for power allocation game in multi-source multi-cluster wireless sensor network.

data transmission scheme where, at the first time slot, each source \( n \) sends its information to the master sensor of the non-cooperating cluster \( j = n \), and in the next time slot, each cluster beamforms, or otherwise, the received signal to a distant receiver utilizing the multiple cooperative sensors in each cluster. Here, we focus on the power allocation of the sources for transmission in the first time slot. Due to the broadcast nature of the channel, the master sensor of each cluster experiences a collision of signals from all the sources [Dong et al., 2007]. Subsequently, at cluster \( j = n \), we treat the signals from the sources other than the target source (i.e., \( n \neq j \)) as additive interference [Larsson et al., 2009]. Now, the signal at the master sensor of cluster \( j \) can be written as

\[
x^j = \sum_{n=1}^{N} \sqrt{P_n s_n^{\text{inf}}} h_{n}^j + w_j,
\]

where \( P_n (\geq 0) \) is the transmit power of the \( n \)-th source, \( s_n^{\text{inf}} \) is the unit energy information symbol from source \( n \), \( h_{n}^j \) is the channel gain from source \( n \) to the master sensor of cluster \( j \), and \( w_j \) is the zero mean additive white Gaussian noise at master sensor of cluster \( j \) with variance \( \sigma_w^2 \).

At the master sensor of cluster \( j \), the desired signal is the signal from source \( j \). Therefore, signal from other sources, i.e., \( n \neq j \), will be treated as noise at the receiving cluster. So, SINR at the master sensor of cluster \( j \) is,

\[
\gamma_j = \frac{P_j |h_j^j|^2}{\sum_{n=1, n \neq j} P_n |h_n^j|^2 + \sigma_w^2}.
\]
It is obvious from (6.2) that the SINR at the cluster \( j \) can be maximized if the source \( j \) transmits at its full power. However, it will cause more interference at the neighboring clusters, and will also reduce its battery life-time. Hence, the net cost of transmission will be increased. Therefore, to meet this trade off, optimal power has to be allocated to all the sources \( n = 1, 2, \ldots, N \), so that a target SINR (\( \gamma_{\text{tar}} \)) can be obtained at the master sensor of cluster \( j \) with the minimum expense of power \( P_j \).

In the rest of Part 1, we focus on the \( i \)-th source and master sensors in the cluster, and we will use \( j, j \in [1, N] \) as a variable to denote any source and master sensors other than \( i \).

### 6.1.3 The Non-cooperative Power Control Game

Here, we propose a non-cooperative power control game (NPG), in which source sensor \( i \) is a player that seeks to minimize its own cost by choosing its transmit power \( P_i \). Let, \( \Gamma = [\mathcal{N}, P_i, J_i] \) denotes the proposed NPG where \( \mathcal{N} \) is the set of the players, and \( P_i = [0, P_{\text{max}}] \) is the strategy set for the \( i \)-th player. \( P_{\text{max}} \) is the maximum allowed power for transmission, and \( J_i \) is the cost for player \( i \). The cost function \( J_i \) is defined as [Koskie and Gajic, 2005]

\[
J_i(P_i, \gamma_i) = b_i P_i + c_i (\gamma_{\text{tar}} - \gamma_i)^2,
\]

where \( b_i \) and \( c_i \) are constant non-negative weighting factors, and \( b_i \) has units of inverse power. \( \gamma_{\text{tar}} \) is the target SINR at cluster \( i \), and is considered to be the same for all clusters in the network. To that end, from (6.3), the source has two conflicting objectives: higher power results in higher SINR but with higher power there is increased cost due to battery energy consumption and increased interference to non-target clusters\(^{38}\). So, the resulting NPG can be expressed as the following minimization problem:

\[
\min_{P_i} J_i(P_i, \gamma_i) = \min_{P_i} \left( b_i P_i + c_i (\gamma_{\text{tar}} - \gamma_i)^2 \right).
\]

For the rest of the discussion, we will use player and sensor interchangeably to indicate a sensor in the network.

### Nash Equilibrium

A Nash equilibrium is a set of strategies of all players in the game, in which no player can unilaterally minimize its own cost by deviating from its equilibrium strategy provided all the other players are playing their Nash equilibrium strategies [Meshkati et al., 2007]. Therefore, at Nash equilibrium

\[
J_i(P_i^*, (P_{i-1}^*), \gamma_i) \leq J_i(P_i^*, P_2^*, \ldots, P_{(i-1)}^*, P_{(i+1)}^*, \ldots, P_N^*), \gamma_i),
\forall P_i, \forall i = 1, 2, \ldots, N
\]

\(^{38}\)The increased interference in non-target clusters causes their respective sources to increase their transmit power, which subsequently increases interference at the target cluster.
where $\mathbf{P}^*_{-i}$ is the power vector of sources in $\mathcal{N}\setminus \{i\}$ at Nash equilibrium. $J_i(\mathbf{P}_i(\mathbf{P}^*_{-i}), \gamma_i)$ is the cost of source $i$ for transmitting at power $P_i$ while other sensors are transmitting with $P^*_{-i} \in \mathbf{P}^*_{-i}$.

**Theorem 6.1.1.** A Nash equilibrium exists in the proposed NPG $\Gamma$.

*Proof.* For the proposed game $\Gamma$, the existence of an equilibrium can be shown from the extreme value theorem [Jerome, 1986]39. According to this theorem, a Nash equilibrium exists in a game $\Gamma = [\mathcal{N}, \mathbf{P}_i, J_i]$ for all $i = 1, ..., N$ if:

1. $\mathbf{P}_i$ is a nonempty, convex and compact subset of some Euclidean space $\mathcal{R}^N$.
2. The function $J_i : \mathbf{P}_i \rightarrow \mathcal{R}$ is continuous in $\mathbf{P}_i = [0, P_{\text{max}}]$ ∀$i$, where $P_i$ is the strategy of player $i$.

Note that, in this game, each sensor $i$ has a strategy space that is defined by a minimum power, a maximum power, and all the power values in between. We also assume that the maximum power is larger than or equal to the minimum power. Thus, the first condition is satisfied.

Now, to show that the function $J_i(\mathbf{P}_i, \gamma_i)$ is continuous in $[0, P_{\text{max}}]$, it is sufficient to show that the first order derivative $\frac{\partial J_i}{\partial P_i}$ is defined in the interval $[0, P_{\text{max}}]$. From (6.3),

$$\frac{\partial J_i}{\partial P_i} = b_i - 2c_i \left( \gamma_{\text{tar}} - \gamma_i \right) \left( \frac{\partial \gamma_i}{\partial P_i} \right),$$

(6.6)

and from (6.2),

$$\frac{\partial \gamma_i}{\partial P_i} = \frac{|h_i|^2}{\sum_{j=1, j\neq i}^N P_j |h_j|^2 + \sigma^2_w}.$$  

(6.7)

So, (6.6) can be expressed as,

$$\frac{\partial J_i}{\partial P_i} = b_i - 2c_i \gamma_{\text{tar}} \frac{|h_i|^2}{\sum_{j=1, j\neq i}^N P_j |h_j|^2 + \sigma^2_w} - P_i |h_i|^4 \frac{(\sum_{j=1, j\neq i}^N P_j |h_j|^2 + \sigma^2_w)^2}{(\sum_{j=1, j\neq i}^N P_j |h_j|^2 + \sigma^2_w)^2},$$  

(6.8)

which is real for $[0, P_{\text{max}}]$. Hence, an equilibrium exists in $\Gamma$, and this completes the proof of Theorem 6.1.1. □

---

39 which says that if a real valued function $f$ is continuous in the closed and bounded interval $[a,b]$, then it must attain its minimum value at least once.
Bounds on source power at Nash Equilibrium

At Nash equilibrium, the cost function (6.3) should be at its minimum. Therefore, to have a minimum $J_i$:

$$\frac{\partial J_i}{\partial P_i} = 0. \quad (6.9)$$

Now, the resulting quadratic equation from (6.8), in terms of

$$\left(\sum_{j=1, j\neq i}^{N} P_j |h_{ij}|^2 + \sigma_w^2\right),$$

has a real solution if

$$4c_i^2 |h_{ii}|^4 (\gamma_{\text{tar}})^2 - 8b_i c_i |h_{ii}|^4 P_i \geq 0. \quad (6.10)$$

**Condition on $P_i$**

From (6.10), when $i = j$, the condition for the transmit power of source sensor $i$, i.e., the power from the sensor $i$ when transmitting to it’s target cluster $j = i$, that must be maintained to be in Nash equilibrium should satisfy

$$P_i \leq \frac{(\gamma_{\text{tar}})^2 c_i}{2b_i}. \quad (6.11)$$

It is thus evident that the maximum value of $P_i$ must satisfy the condition,

$$P_i^{\text{max}} \leq \frac{(\gamma_{\text{tar}})^2 c_i}{2b_i}, \quad (6.12)$$

where $P_i^{\text{max}}$ is the maximum value of $P_i$. Note that the condition on $P_i$ in (6.12) is the upper bound for the power of source sensor $i$ to be in the Nash equilibrium.

**Condition on $P_j$ when $j \neq i$**

If the noise power is assumed negligible, with respect to total interference powers at the receiving cluster, from (6.2) and (6.11), the Nash equilibrium condition on source powers $P_i$ with respect to non-target clusters $j \neq i$ is

$$\sum_{j=1, j\neq i}^{N} P_j |h_{ij}|^2 \leq \frac{(c_i \gamma_{\text{tar}})^2 |h_{ii}|^2}{2b_i \gamma_i}. \quad (6.13)$$

**Condition on $b_i$ and $c_i$**

From (6.3), the cost function depends on the weighting coefficients $c_i$ and $b_i$. Now, from (6.12), for a given maximum value of $P_i^{\text{max}}$, the ratio of $b_i$ and $c_i$ that needs to be maintained at the Nash equilibrium is

$$\frac{b_i}{c_i} \leq \frac{(\gamma_{\text{tar}})^2}{2P_i^{\text{max}}}. \quad (6.14)$$
To this end, Theorem 6.1.1 can be redefined as:

**Theorem 6.1.2.** A Nash equilibrium exists for the proposed game $\Gamma$, and is given by $P^* = [p^*_1, p^*_2, \ldots, p^*_N]$ provided (6.12), (6.13) and (6.14) are satisfied for all $i = 1, 2, \ldots, N$.

6.1.4 Properties of equilibrium

Here, we discuss the properties of the Nash equilibrium of the proposed game. First, we discuss the best response correspondence of a sensor in the game.

**Proposition 6.1.1.** In the NPG $\Gamma$, the sensor $i$'s best response to a given interference vector $P_{-i}$, where $P_{-i}$ is the vector of all source sensors in $\mathcal{N}\setminus\{i\}$, is given as

$$r_i(P_{-i}) = \min(P_{\text{max}}, P^*_i),$$  \hspace{1cm} (6.15)

where $P^*_i = \arg \min_{P_i \in \mathbb{R}^+} J_i(P_i, \gamma_i)$ is the unconstrained minimizer of the cost function in (6.3), and $P^*_i$ is unique.

**Proof.** From (6.2), for a given interference, the transmit power $P^*_i$ corresponds to $\gamma^*_i$ is given by,

$$P^*_i = \frac{\gamma^*_i \left( \sum_{j=1, j \neq i}^N P_j |h_{ij}|^2 + \sigma^2_w \right)}{|h_{ii}|^2}.$$  \hspace{1cm} (6.16)

Since, $\gamma^*_i$ is the unique minimizer of the cost, and as there is a one-to-one correspondence between the transmit power and SINR, the transmit power $P^*_i$ that minimizes cost for fixed interference is also unique. If $P^*_i \notin P_i$ for some user $i$, then its not a feasible point, $P^*_i$ can not be the best response to given $P_{-i}$. Therefore, in this case, $\gamma_i \approx \gamma^*_i$ for $P_i > P_{\text{max}}$. As $P_{\text{max}}$ is the maximum power in the strategy space, to achieve the minimum cost for all $P_i \in [0, P_{\text{max}}]$, it is the best response to the given $P_{-i}$.

### Uniqueness of Nash Equilibrium

The key aspect of the uniqueness proof is to realize that the best response correspondence $r_i(P_{-i})$ is a standard function [Saraydar et al., 2002]. If we assume the same power update function as in [Koskie and Gajic, 2005], the function can be shown to be standard with the constraints on interference power (IP) and noise power [Koskie and Gajic, 2005] as given in Table 6.1. Hence, the proposed NPG has a constrained unique Nash equilibrium.

**Table 6.1:** Constraints on the interference and noise power to maintain a unique Nash equilibrium of the game.

<table>
<thead>
<tr>
<th>Properties</th>
<th>constraints</th>
</tr>
</thead>
<tbody>
<tr>
<td>Positivity &amp; monotonicity</td>
<td>$IP \leq \frac{c_i</td>
</tr>
<tr>
<td>Scalability</td>
<td>$\sigma^2_w &lt; \frac{2 \nu_i</td>
</tr>
</tbody>
</table>


Dependence of equilibrium on distance

When either the sources or the receiving clusters (or both) are mobile, their positions are not fixed any more. To that end, we show that the Nash equilibrium also changes with the change of position of the sensors in the system. The relation of the equilibrium with distance can be derived as follows.

Consider two sources and two receiving clusters. From Table 6.1, the positivity and monotonicity condition for this case can be expressed as,

\[
A^2((r_0/r_1)^{\alpha/2})^2 P_2 \leq \frac{c_1 \gamma_{\text{tar}}(A^2((r_0/r_1)^{\alpha/2})^2)}{b_1}
\]

i.e. \( r_1^1 \leq \left( \frac{c_1 \gamma_{\text{tar}}}{b_1 P_2} \right)^{1/\alpha} r_2^1. \) \( (6.17) \)

Here, \( r_1^1 \) and \( r_2^1 \) is the distance from source 1 and 2 to cluster 1 respectively. Note that (6.17) gives the upper bound of distance from source 1 to cluster 1 for being in Nash equilibrium, and it is dependent on other sources’ (in this case source 2) transmit power and distance from cluster 1\(^{40}\).

6.1.5 Performance Analysis

For simulation, we consider a network with 2 clusters and 2 source sensors \( (N = 2) \). The maximum transmit power of each sensor is considered as \( P_{\text{max}}^{\text{1,2}} = 800 \text{mW} \), and the target SINR at the cluster’s master sensor is assumed to be \( \gamma_{\text{tar}} = 5 \) (6.9 dB). The weighting coefficients \( b_{1,2} \) and \( c_{1,2} \) are chosen as 2 and 160 respectively so that \( \frac{b_{1,2}}{c_{1,2}} \) satisfies the condition in (6.14) for the chosen \( P_{\text{max}}^{\text{1,2}} \) and \( \gamma_{\text{tar}} \). The noise variance is assumed to be \( \sigma_w^2 = 8 \times 10^{-12} \text{mW} \) [Betz and Poor, 2008]. While simulating the system for distant dependent path loss conditions, we consider the fixed channel gain as \( h_j^i = A(r_0/r_j)^{\alpha/2} \) (path loss is \( 1/|h_j^i|^2 \)), where \( A \) is a constant, \( \alpha \) is path loss exponent, \( r_j^i \) is the distance between the source sensor \( j \) and the master sensor of cluster \( i \), and \( r_0 \) is the reference distance. For our simulation, we consider \( A = 10^{-3} \) and \( r_0 = 100 \text{m} \). With simulation, we show that the Nash equilibrium is conditioned on the distance between the source sensors and the target clusters as well as on the path loss exponent.

In Fig. 6.2, we show the cost for both source sensors with respect to their transmit power. Considering \( r_1^1 = r_1^2 = 200 \text{m} \), first we assume that sensor 2 is transmitting at equilibrium power \( P_2^* = 100 \text{ mW} \), and observe that only at power \( P_1 = 500.58 \text{ mW} \) sensor 1 can have the minimum value for its cost function \( J_1 \) while achieving the target SINR of 5 at cluster 1. Therefore, \( P_1^* = 500.58 \text{ mW} \) is its Nash equilibrium power. Then, we consider the cost for sensor 2, i.e. \( J_2 = b_2 P_2 + c_2 (\gamma_{\text{tar}} - \gamma_2)^2 \), to achieve \( \gamma_{\text{tar}} = 5 \) at the master sensor of cluster 2. Here, we observe that while sensor 1 is transmitting at its Nash equilibrium power \( P_1^* \), sensor 2 can not minimize its cost by violating its choice of equilibrium power, and consequently has to choose the power \( P_2 = 100 \text{ mW} \). So, the Nash equilibrium power of the system, in current set up, is \( (P_1^*, P_2^*) = (500.58 \text{ mW}, 100 \text{ mW}) \).

---

\(^{40}\)This dependence on distance can also be expressed in terms of sensor’s velocity, if we assume that all the sensors are moving with same acceleration.
Figure 6.2: Cost for the both source sensors as a function of transmit power to achieve the target SINR at the respective receiving clusters.

Figure 6.3: The effect of interference power on the Nash equilibrium to achieve a target SINR at the receiving cluster.
However, the Nash equilibrium is constrained by the interference power of the system as given in (6.13). We show this effect of interference power on the Nash equilibrium in Fig. 6.3. As shown in Fig. 6.3, the Nash equilibrium exists, i.e., a minimum of cost $J_1$ exists as long as $P_{j\neq i} \leq 200$ mW (here $P_2$). Since, $P_{j\neq i}|h_{ij}|^2$ is the interference at cluster $i$, the Nash equilibrium is no longer valid in the system, within the limit of transmit power of sensor $i$ (here sensor 1), when $P_{j\neq i}|h_{ij}|^2$ is beyond the limit in (6.13).

Now, we show that the Nash equilibrium is highly dependent on the distances of the sources from the receiving clusters. First, we follow the power update algorithm in [Koskie and Gajic, 2005] for our system, and show in Fig. 6.4 that the Nash equilibrium is achieved after the 1st power update in time slot 2 for source 1. It is further shown in Fig. 6.4 that the sources do not alter their power for the rest of the time so that minimum cost is maintained. However, this Nash equilibrium is constrained by the distance from the source sensor $i$ to the target cluster $i$. To show this, we change the distance $r_{11}$ from source sensor 1 to cluster 1 while maintaining a fixed $r_{12} = 200$ m for $P_2 = 100$mW and $\alpha = 2$ (free-space transmission). The effect of variation of distance is given in Fig. 6.5.

Fig. 6.5 shows that when the constraints on the distance between the source sensor and target cluster do not satisfy the bound (6.17) anymore the cost does not reach to its minimum within the maximum limit of the sensor power $P_1$. As a result, no Nash equilibrium exists in the game within the power range of sensor 1. Further, from (6.17), the Nash equilibrium is conditioned on path loss exponent $\alpha$. We observe the effect of $\alpha$ on the Nash equilibrium and show the cost for different values of $\alpha$ in Fig. 6.6.

In Fig. 6.6, we show that the Nash equilibrium exists in the system as long as $\alpha \leq 3$. From (6.17), changing the value of $\alpha$ is similar to the change of the value of $r_{11}$. Therefore,

Figure 6.5: Dependence of Nash equilibrium on the distance between the source sensor and the cluster.

Figure 6.6: The effect of path loss exponent $\alpha$ on the cost.

...the effect is also similar on the Nash equilibrium (i.e., the minimum cost shifts horizon-
tally with the change of $\alpha$).

6.2 Part 2: Distributed Transmit Beamforming Via Data Funneling

6.2.1 Motivation

Communication via on-board radio is the most power expensive operation of sensors [Heinzelman et al., 1999]. For example, in free space radio communications, the signal strength decreases proportionally to the square of propagation distance [Lee et al., 2006]. Due to the fact that the individual nodes in sensor networks are typically powered by batteries that may be difficult to replace or recharge [Betz and Poor, 2008], protocol design for sensor networks that allows the reduction in energy consumption for communications so as to increase the network lifetime has been widely researched recently [Fasolo et al., 2007].

Two popular transmission schemes for energy savings in sensor networks are i)- distributed transmit beamforming (DTB), and ii)- data aggregation (or data funneling) [Petrović et al., 2003]. DTB is a transmission scheme where two or more radios cooperatively form a virtual antenna array, and obtain diversity gains against channel fading and array gains from increased diversity. DTB allows $M$-fold energy efficiency of wireless sensor networks for $M$ transmitters [Mudumbai et al., 2006] whereas, data funneling saves the sensors’ energy for transmission by routing data packets through low cost communication paths for long distance communication [Petrović et al., 2003]. Numerous data aggregation techniques [Petrović et al., 2003; Lindsey et al., 2002; Manjhi et al., 2005; Sharaf et al., 2004; He et al., 2004] and DTB [Wang et al., 2001; Heinzelman et al., 2002; Mudumbai et al., 2010, 2007, 2006; Dong et al., 2007] have been proposed in literature, which are directly or partially related to the increase in lifetime of wireless sensor networks. Motivating by the fact that both techniques can be used separately for power saving of sensor nodes, in this work, we incorporate DTB with data funneling, and propose a novel cross-layer scheme for transmission of signals over long distances. The proposed scheme essentially uses layered cooperative beamforming. We show that layered beamforming gives significantly better performance in terms of energy savings to achieve a particular symbol error probability (SEP) at the receiver with respect to direct beamforming and direct single link transmission for the same source-destination pair.

To adapt DTB, a clustered network is assumed where each cluster consists of multiple cooperative sensors (CS). We consider that any signal is funneled through clusters from a source to the receiver. DTB is used for transmission of signals from one layer to another, and one or more clusters are assigned to a given layer for DTB collaboration. Thus, the main contribution of this work is the combination of DTB and data funneling for long-range communication in sensor networks to save total system energy. We show that significant energy savings are possible for the sensors in the distant cluster when considering the energy spent by each sensor for transmitting from its cluster over long distances compared with direct beamforming and single link transmission. With simulation, we show that the battery energy consumed by the sensor can be saved up to 36% to achieve a SEP of $10^{-3}$ compared to direct beamforming. This energy saving will significantly increase network lifetime. Further, we show improved performance of the
6.2. Part 2: Distributed Transmit Beamforming Via Data Funneling

The rest of this part of the chapter is organized as follows. We describe the system model in Section 6.2.2. The funneling of data and the performance measure are explained in Section 6.2.3. Simulations results and performance comparison with other transmission techniques are given in Section 6.2.4, both at the physical OSI layer\footnote{Here we use the term ‘layer’ interchangeably to mean layers of sensor clusters for DTB with data funneling and Open Systems Interconnection (OSI) layers (e.g., PHY, MAC, routing, application), hence for the latter we always use the term OSI layer.} and routing (or network) OSI layer. Finally, some concluding remarks of this chapter are given in Section 6.3.

6.2.2 System Model

As shown in Fig. 6.7, the architecture of the considered sensor network is divided into different layers, and all the sensors in each layer are organized into different clusters. Each cluster consists of $M$ number of cooperative sensors (CSs) with a single cluster head (CH), and $(M-1)$ cluster members (CM). The CH in each cluster is assumed to have more computational ability, and is responsible for reception and detection of the beamformed signal. All the CSs in a cluster are uniformly distributed, and can hear each other. The clusters can be formed using any general cluster formation scheme that ends up with a single CH with multiple cooperative CMs in each cluster [Guo et al., 2009; Yu et al., 2007]. The layers can be defined as the regions that the receiver ($R_0$ in Fig. 6.7) wishes to monitor. We consider the location of $R_0$ as the origin and each layer is numbered from the origin as $z_1, z_2, ..., z_l$. Similar to [Petrović et al., 2003], we assume that CHs will be informed by a directional flood (DF), initiating from $R_0$, about the layers they belong to, and the CH will share this information with the fellow CMs.

Here, we consider a time-slotted uni-directional transmission scheme, which is ap-
propriate for wireless sensor networks. At time slot $t$ a sensor node $i$ of cluster $j$ in layer $z_i$ wishes to transmit a message $s^\text{inf}(t)$ to the receiver, where $s^\text{inf}(t)$ is a $m$-ary phase shift keying (PSK) signal which consists of $L$ symbols. All the sensors access $s^\text{inf}(t)$ and collaborate with each other to beamform the signal towards the CH of cluster $k$ in layer $z_{l-1}$ (towards $R_0$ if $l = 1$). For beamforming the signal, each CS independently synchronizes itself and adjusts its initial phase to a beacon sent from the CH in $z_{l-1}$ (from $R_0$ if $l = 1$) as in a closed loop case [Dong et al., 2008]. The channel between the CS $i$ in cluster $j$ of $z_l$ and the receiving CH in cluster $k$ of $z_{l-1}$ is considered as

$$g_{j,z_l}^{k,z_{l-1}}(i,t) = \sqrt{b_m h_{j,z_l}^{k,z_{l-1}}(i,t)},$$  

(6.18)

where $b_m$ gives the large scale path loss effect (i.e., $b_m \approx G/(d^k_j)^{\alpha/2}$ [Dong et al., 2008], where $\alpha$ is the path loss exponent and $G$ is a constant and $d^k_j$ is the distance from the center of cluster $j$ to CH in $k$ of $z_{l-1}$ ($R_0$ if $l = 1$) and is independent of $i$ and $h_{j,z_l}^{k,z_{l-1}}(i,t)$ is independently and identically distributed (i.i.d.) complex Gaussian random variable with mean zero and variance $\sigma^2$. The CH of cluster $k$ in $z_{l-1}$ receives the signal, detects it and shares with the CMs in its own cluster. Then in the next time slot, upon receiving the beacon, the CSs of cluster $k$ beamform their own information along with the information received in the previous time slot from cluster $j$ towards the closest CH in the next layer $z_{l-2}$. Eventually the message from the source reaches $R_0$.

**Transmit beamforming**

For transmit beamforming, we assume that perfect channel state information (CSI) is available at the transmitting sensors, and we also consider an interference free system. Now at time slot $t$, the transmitted signal from each CS $i$ in cluster $j$ of $z_l$ towards the CH of cluster $k$ in $z_{l-1}$ is

$$x_{j,z_l}^{k,z_{l-1}}(i,t) = s^\text{inf}(i,t) \sqrt{E_s \mu_i} a_{j,z_l}^{k,z_{l-1}}(i,t),$$  

(6.19)

where $E_s$ is the transmit energy, $\mu_i$ is a scalar of the order of $1/\sqrt{M}$ used to adjust the transmit energy [Dong et al., 2008] and is the same for all CSs, $a_{j,z_l}^{k,z_{l-1}}(i,t)$ is the beamforming weight which is the conjugate of the channel gain from CS $i$ of cluster $j$ to the CH in cluster $k$, i.e., $a_{j,z_l}^{k,z_{l-1}}(i,t) = (g_{j,z_l}^{k,z_{l-1}}(i,t))^\ast$.

The received signal at the CH of cluster $k$ in layer $z_{l-1}$ is,

$$y_{j,z_l}^{k}(CH,t) = \sum_{i=1}^{M} x_{j,z_l}^{k,z_{l-1}}(i,t) g_{j,z_l}^{k,z_{l-1}}(i,t) + w(t),$$  

(6.20)

where $w(t)$ is the zero mean additive white Gaussian noise with variance $\sigma^2$. The noise variance is considered the same for all communication links.

From (6.18), (6.19) and (6.20),

$$y_{j,z_l}^{k}(CH,t) = \sum_{i=1}^{M} \sqrt{E_s \mu_i} s^\text{inf}(i,t) b_m |h_{j,z_l}^{k,z_{l-1}}(i,t)|^2 + w(t).$$  

(6.21)
Therefore, the received signal will be the sum of the scaled version of the signal of node \( i \). It is assumed that the signal is demodulated and decoded at the CH of \( z_{l-1} \) (at \( R_0 \) for \( l = 1 \)) using the maximum likelihood (ML) method [Proakis, 2000].

### 6.2.3 Data Funneling and Performance Measures

The proposed data funneling with DTB protocol consists of two phases: (1) Setup phase and (2) communication phase. The setup phase starts with the initiation of DF from \( R_0 \) towards the network area. The DF packet consists of the location of the receiver and the specific time period for the sensors in each layer to beamform their signal towards the next CH. Upon receiving the DF, each CH records the distance of the closest CH, updates the packet with its location and sends it towards the next layer. The setup phase ends as soon as the DF reaches the last layer in the sensor network.

Communication phase initiates when a sensor in a cluster has information to send to the receiver \( R_0 \). The message is funneled through the clusters using DTB as described in Section 6.2.2. The communication phase ends as soon as the message reaches \( R_0 \). The proposed protocol for the two phases of the data funneling using DTB is summarized as a flow chart in Fig. 6.8 and Fig. 6.9 respectively.

![Flow Chart](Figure 6.8: Setup phase of the data funneling using distributed transmit beamforming protocol.)
Chapter 6. Power Control in Wireless Sensor Networks

**Receiving CH**
1. Detect the received signal
2. Share the detected signal with CM

---

**Beamforming towards the next CH**
1. Receive beacon from adjacent CH
2. Calculate beamforming weights

---

**Aggregated own data with detected signal**

---

**Is the receiving CH = R0 ?**

---

**Start**

---

**End**

---

**Figure 6.9: Communication phase of the data funneling using distributed transmit beamforming protocol.**

**Performance measures**

From (6.21) the instantaneous signal to noise ratio (SNR) at the receiving point (RP) (which is a CH in \( z_{l-1} \) or \( R_0 \) if \( l = 1 \)) is,

\[
\gamma = \frac{\mu_i^2 b_m^2 E_s \left( \sum_{i=1}^{M} |h_{j,z_{l-1}}^{k,z_{l-1}}|^2 \right)^2}{\sigma_t^2} = \frac{b_m^2 \mu_i^2 E_s \xi^2}{\sigma_t^2}, \tag{6.22}
\]

where \( \xi = \sum_{i=1}^{M} |h_{j,z_{l-1}}^{k,z_{l-1}}|^2 \). Since \( |h_{j,z_{l-1}}^{k,z_{l-1}}| \) is Rayleigh distributed, \( \xi \) follows an Erlang distribution with shape parameter \( M \) and rate parameter \( \sigma_t^2 \) [Dong et al., 2008].

Now following [Dong et al., 2008], an upper bound for the average SEP for m-PSK can be derived for the proposed scheme as,

\[
P_s \leq \frac{m - 1}{m} \left( 1 + \frac{\sin^2 \left( \frac{\pi}{m} \right)}{\sin^2 \left( \frac{(m-1)\pi}{m} \right)} \right)^{-M} \tag{6.23}
\]

where, \( \xi_0 > 0 \) and \( \frac{\mu_i^2 b_m^2 E_s}{\sigma_t^2/\xi_0} \) can be expressed as \( \frac{\mu_i^2 G^2 E_s}{\sigma_t^2/\xi_0} \).

From (6.23), it can be seen that SEP is a function of \( b_m \) and the number of collaborating nodes \( M \) in the cluster. Since \( b_m \) is distant dependent path loss, average SEP \( P_s \) is a function of the distance between the source cluster and RP, and \( M \), i.e., \( P_s \leq f(d_s, M) \), where with \( d_s \) we indicate the distance between the source cluster center and RP. Note that for fixed \( d_s \), the SEP decreases as the number of cooperative sensors \( M \) at the transmitting
cluster increases. As a result, direct beamforming from a cluster with multiple sensors \((M > 1)\) always gives a lower SEP at \(R_0\) compared to single link transmission.

However, the incorporation of data funneling with distributed transmit beamforming provides even better results in terms of SEP at \(R_0\) than direct beamforming alone. In the data funneling algorithm, the distance from the source cluster to \(R_0\) is divided into multiple small distances by placing intermediate clusters between them. The distance between the intermediate clusters are smaller than the source to receiver distance \(d_{R_0}^s\). Assuming all the clusters have identical number of CSs, from (6.23), SEP at RP is dominated by the distance between the clusters. Therefore, beamforming and detection along these small distances leads to performance improvement, which is explained as follows.

Equation (6.23) can be expressed as,

\[
P_s \leq \frac{m-1}{m} \left( 1 + \frac{G'}{(d_s^M)^\alpha} \right)^{-M},
\]

where \(G' = E_s \left( \frac{\sin(\frac{\pi}{m})}{\sin(\frac{\pi}{m} - \frac{\pi}{2m})} \right)^2 \frac{\sigma_n^2}{\sigma_n^2} G^2 \mu_i^2 \xi_0 \). From (6.24), for a particular \(m\)-ary modulation scheme and constant number of sensors \(M\) in each cluster, SEP at a RP depends on the distance \(d_s\) between the source and the RP and the SEP increases with the distance \(d_s\). Therefore, transmission over multiple small distances performs better than beamforming over a long distance.

### 6.2.4 Results and Performance Analysis

#### Performance at the Physical OSI Layer

We run a Monte Carlo simulation for the proposed scheme to transmit \(10^4\) bits \((N_b = 10^4)\) in each transmission, and compare the results with direct beamforming and single link transmission for the same source-destination pair. The variance of channel gain between the clusters is scaled by the square of distance \((i.e., \alpha = 2)\) between them to incorporate the path loss effect. The variance of noise power of the shortest link is chosen, and is assumed to be the same across all transmission links.

We divide the network area into three different layers \(z_1, z_2\) and \(z_3\). As shown in Fig. 6.7, \(z_1\) contains cluster 1, \(z_2\) contains cluster 2 and 3, and finally \(z_3\) contains cluster 4 and 5. We assume that each cluster contains 5 sensors including the CH. The distance from clusters 1, 2, 3, 4 and 5 to the receiver \(R_0\) are \(d_{1R_0}^R = 400\lambda, d_{2R_0}^R = 700\lambda, d_{3R_0}^R = 1000\lambda, d_{4R_0}^R = 1400\lambda\) and \(d_{5R_0}^R = 1300\lambda\), respectively, where \(\lambda\) is the wavelength of the signal. Here, we consider the transmission from cluster 4 to the receiver \(R_0\). The distance between the clusters along the funneling path of cluster 4 are \(d_3^R = 487.2\lambda, d_2^R = 613.24\lambda, d_1^R = 400\lambda\), respectively, where the distances are calculated from the polar coordinates of the clusters with respect to the receiver \(R_0\).

We consider two examples. In the first example, we run the simulation for our proposed data funneling scheme, direct single link transmission, and direct beamforming from cluster 4 to the receiver for the same noise and fading environment. We show that the proposed scheme provides considerable improvements over the other two transmission schemes in terms of overall system’s transmit energy savings while achieving the same SEP at the receiver. In the second example, we run the simulation considering the
energy $\sqrt{E_s}$ spent by the sensors at the furthest distant cluster to send its signal to the receiver $R_0$ over a long distance, and show that significant improvement in transmit energy savings is possible with the proposed data funneling scheme.

Fig. 6.10 shows the SEP at the receiver with respect to system SNR for binary phase shift keying (BPSK) and quadrature phase shift keying (QPSK) modulation schemes. The system SNR is defined as the ratio of the system’s total transmit power to the noise power. To make the system SNR same for all transmission schemes, the transmit energy of each sensor is scaled as follows for the three different type of transmissions:

Single link energy: $E_{\text{single}} = M \sqrt{E_s \mu_m L_{\text{path}}}$

Direct Beamforming: $E_{\text{BF}} = \sqrt{E_s \mu_m L_{\text{path}}}$

Data funneling: $E_{\text{Dfun}} = \sqrt{E_s \mu_m}$, \hspace{1cm} (6.25)

where $L_{\text{path}}$ is the number of funneling hops from cluster 4 to the receiver for the proposed scheme. As shown in the figure, for both modulation schemes, data funneling provides transmit energy\textsuperscript{42} savings of 1 dB and 10 dB for the whole system to achieve a SEP of around $10^{-3}$ with respect to direct distributed beamforming and direct single link transmission, respectively.

We now consider the energy spent by the sensors in a distant cluster (i.e., cluster 4) for transmitting its signal to the receiver $R_0$. It is seen that significant energy savings are possible for the proposed scheme compared to direct beamforming and single link

\textsuperscript{42}For the same noise variance at all links, SNR can be translated to transmit energy.
transmission. The observed output for both BPSK and QPSK modulation schemes are plotted in Fig. 6.11. From this figure, the transmit energy savings for data funneling over direct beamforming is around 11.5 dB for BPSK to achieve a SEP of $10^{-3}$ at the receiver. For QPSK modulation scheme, it is of the order of 10 dB to achieve the same SEP. This improvement is due to the fact that for direct beamforming the transmitters in cluster 4 have to overcome a distance of $d_{4R}^{R_0}$ whereas for data funneling the transmitters only need to transmit over a distance of $d_{4}^{a}$ where $d_{4R}^{R_0} >> d_{4}^{a}$. This saving is particularly useful for the application scenarios where the location of distant cluster 4 is not easily accessible for battery replacement but has easy radio access to other clusters. This improvement of SEP can be explained from (6.24). From (6.24), for direct beamforming from cluster 4 to $R_0$, the upper bound for SEP is,

$$p_{s,dir}^{R_0} \leq \frac{m - 1}{m} \left( \frac{1}{1 + \frac{G^2}{(d_{4R}^{R_0})^2}} \right)^M .$$

(6.26)

Now, if we assume that the SEP is very small in the intermediate paths between the clusters, then, for data funneling, the upper bound of SEP, from cluster 4 to the receiver $R_0$,
can be expressed as,

\[ P_{s,4}^{R_0, df} \leq P_{s,4}^3 + P_{s,3}^1 + P_{s,1}^{R_0} \]

\[ \leq \left( \frac{m-1}{m} \right) \left[ \left( 1 + \frac{G'}{(d_3^3)^2} \right)^{-M} + \left( 1 + \frac{G'}{(d_1^3)^2} \right)^{-M} \right. \]

\[ \left. + \left( 1 + \frac{G'}{(d_{R_0}^1)^2} \right)^{-M} \right] . \tag{6.27} \]

Considering all other parameters are unchanged (i.e., \( E_s, \sigma^2, M \) and \( m \)), from (6.27), \( P_{s,4}^{R_0, df} < P_{s,4}^{R_0, dir} \) for \( d_3^2, d_1^1, d_{R_0}^1 \ll d_{R_0}^4 \). As a result, data funneling using distributed transmit beamforming, demonstrates significant performance improvements compared to direct beamforming for long distance communication in terms of SEP at the receiver.

Figure 6.12: Upper bounds of SEP at the receiver for BPSK modulation scheme for single link transmission, distributed transmit beamforming and data funneling with beamforming from the distant cluster 4.

In Fig. 6.12, we plot the upper bound of SEP based on (6.26) and (6.27) (considering \( G' = 7 \times 10^5 \)) as well as the SEP at the receiver for BPSK, for all three transmission schemes, with respect to the transmit SNR of the sensors in cluster 4. It can be seen from the figure that the upper bounds are consistent with our analysis in (6.26) and (6.27). The received SEP at the receiver, for all the transmission schemes, are also well below the upper bound of SEP.

We now compare the battery energy consumption of the sensors in the distant cluster. To that end, we consider the power consumption characteristics of Rockwell’s WINS node
which represents a high-end sensor node, and is equipped with a powerful StrongARM SA-1100 Intel processor [Raghunathan et al., 2002]. Though relation of the transmit SNR to the battery power is application specific, we assume that the noise power is such that the 0 dB transmit SNR in the proposed system is equivalent to a transmit power of 0 dBm (1 mW) [Raghunathan et al., 2002]. With this assumption, in order to achieve a SEP of $10^{-3}$ at the receiver, the battery energy savings for data funneling with distributed transmit beamforming with BPSK modulation scheme is 36% over direct beamforming, which is a significant energy saving for the sensor’s battery. Therefore, the reduction in transmit power as well as battery energy savings, when compared to single link transmission and direct beamforming, to achieve the same SEP at the receiver with typical modulation schemes, clearly emphasizes the effectiveness of the proposed scheme for low energy data transmission over large distances.

**Routing OSI Layer Performance**

We also compare the performance of the proposed scheme with direct beamforming (from Fig. 6.11, the improved performance over direct beamforming is also an indication of the improvement over direct transmission as well) based on a routing metric given in [Yu et al., 2007]. An energy efficient and power aware routing protocol should have a lower cost value for the link cost function [Yu et al., 2007]:

$$D_i = \varrho_i E_{tr}^i,$$  \hspace{1cm} (6.28)

where $D_i$ is the cost for the transmitting sensor node $i$, $E_{tr}^i$ is the transmit energy of node $i$, and $\varrho_i$ is a dimensionless coefficient defined by,

$$\varrho_i = \frac{B_{0i}}{B_{0i} - B_{tx}}.$$ \hspace{1cm} (6.29)

In (6.29), $B_{0i}$ is the new battery power of sensor $i$ and $B_{tx}$ is the required battery power for a transmission of signal. It is easily observed from (6.29) that less remaining battery energy (i.e., more energy consumption for transmission) results in a much bigger value for the coefficient $\varrho_i$.

In Table 6.2, we list the values of the coefficient $\varrho_i$ for both direct beamforming and the proposed scheme for a list of particular values of SEP at the receiver. We consider that each sensor is using a 4W-E27 motion sensor battery and the consumption characteristics

<table>
<thead>
<tr>
<th>SEP</th>
<th>$\varrho_i$</th>
<th>$\varrho_i$</th>
<th>$\varrho_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.13</td>
<td>0.2245</td>
<td>0.3029</td>
<td></td>
</tr>
<tr>
<td>0.08</td>
<td>0.25</td>
<td>0.3289</td>
<td></td>
</tr>
<tr>
<td>0.043</td>
<td>0.2579</td>
<td>0.3559</td>
<td></td>
</tr>
<tr>
<td>0.0180</td>
<td>0.2658</td>
<td>0.4063</td>
<td></td>
</tr>
<tr>
<td>0.0063</td>
<td>0.2739</td>
<td>0.6273</td>
<td></td>
</tr>
<tr>
<td>0.0014</td>
<td>0.2903</td>
<td>0.6273</td>
<td></td>
</tr>
</tbody>
</table>
Chapter 6. Power Control in Wireless Sensor Networks

of the sensors are equivalent to Rockwell’s WINS node [Raghunathan et al., 2002]. From Table 6.2, the value of the coefficient of the proposed scheme is always lower than the value of the direct beamforming case, which refers to the energy saving of the battery of the proposed scheme.

![Figure 6.13: Comparison of the cost to achieve the same SEP at the receiver.](image)

In Fig. 6.13, we show the cost of achieving the same SEP at the receiver for both the proposed scheme and the direct beamforming scheme. It shows that the cost of transmission for a sensor node can be significantly reduced for long distance transmission using our proposed scheme. For instance, to achieve a SEP of $10^{-3}$, the cost incurred by the proposed scheme is 0.076 times the cost incurred by the direct beamforming scheme. This is clearly a significant cost reduction in terms of energy savings. Hence, collaborative beamforming across multiple clusters, and data funneling through clusters, gives significant improvement in terms of routing performance.

6.3 Concluding Remarks

In this chapter, we provided two power control mechanisms for wireless sensor networks. In the first part of the chapter, we formulated a power control game between the sources in the network in which each source chooses its transmit power independently so as to achieve a desired SINR at the intended receiving cluster. We demonstrated that the game possesses a Nash equilibrium that is constrained by the interference and noise power. The equilibrium is also shown to vary according to distance between the source and destination cluster, and also according to path loss exponent. Then in the second part of the chapter, for transmitting the signals from the clusters to the intended receiver, we pro-
posed a cross-layer transmission scheme, which incorporated both data funneling and distributed transmit beamforming. We have shown that the cross-layer scheme demonstrates significant energy savings for a sensor in a distant cluster achieve a desired SEP at the receiver. The proposed transmission scheme also gives overall system energy savings. The transmit energy savings should significantly lengthen the battery life of sensor networks, when our proposed scheme is adopted, as well as improve the efficiency of routing. However distributed transmit beamforming, as described in part 2 of this chapter, requires channel state information for its phase synchronization, which makes such distributed beamforming computationally expensive. To address this, in the next chapter, we propose a distributed transmit beamforming technique for wireless sensor networks that does not require any channel state information and uses a very simple limited feedback model for the phase synchronization of signals at the receiver.
CHAPTER 7

Distributed Transmit Beamforming with Feedback

For energy efficient transmission of signals over long distance, in this chapter we propose distributed transmit beamforming techniques for both static and time-varying channels. We show beamforming techniques that enable signals from multiple transmitters to add coherently at the receiver without any channel state information. To that end, first, we propose a distributed transmit beamforming technique that can achieve the coherent phase alignment of signals at the receiver after a large number of iterations. At each iteration, the transmitters receive three bits of feedback of information from the receiver so as to modify their phase angles accordingly for transmission in the following time slot. The feedback is set based on the improvement or degradation of the signal strength at the receiver from one time slot to the next, and on the movement of the receiver. With simulation, we show that the received power of the proposed three-bit feedback scheme increases quadratically with respect to an increasing number of transmitters, and there is a substantially improved performance over the existing one-bit feedback scheme. Further, we improve the three-bit feedback scheme by reducing the number of feedback bits to two bits, and demonstrate further improved performance over the existing feedback schemes including one-bit feedback and improved one-bit feedback. Finally, we investigate the speed of phase convergence of signals at the receiver, and show that the use of an additional phase feedback in conjunction with a single bit feedback from the receiver can significantly improve the speed of convergence for transmitting a known training sequence from the transmitters to the receiver.

7.1 Motivation

Energy is a scarce commodity in wireless sensor networks as sensors are typically operated by using batteries, which often need a long life-time. Distributed transmit beamforming (DTB) has the potential to increase the life-time of these distributed sensors by energy efficient transmission of signals over long distance. Essentially, DTB is a technique in which two or more radios cooperatively form a virtual antenna array so as to obtain the diversity gain from channel diversity, and an array gain from increased directivity [Mudumbai et al., 2006]. Consequently, the signal gain from DTB is large, and the communication distance can be considerably increased by adding more transmitting sensors [Barriac et al., 2004]. In fact, cooperative transmission from \( N \) transmitters can in-
7.1. Motivation

crease the signal-to-interference-noise ratio (SINR) gain by up to a factor of $N$ compared
to transmission from a single transmitter [Ochiai et al., 2005a]. However, destructive in-
terference of signals can significantly degrade the received signal strength (RSS). Hence,
there is a need for proper phase and frequency synchronization of high frequency car-
rier signals sent from different transmitters so as to guarantee coherent signal addition at
the receiver. Hereinafter, we will use sensor and transmitter interchangeably to refer to a
sensor in the network.

Effective and proper design of phase synchronization of signals from distributed trans-
mitters has been given immense attention over the past few years due to the need for
obtaining the highest possible RSS at the receiver. On the one hand, various research
has been conducted to devise protocols for phase synchronization which would enable
the signals from different transmitters to coherently combine at the receiver [Ochiai et al.,
2005a; Brown and Poor, 2008; Pun et al., 2009; Ahmed and Vorobyov, 2009; Koyuncu et al.,
2008; Johnson et al., 2008; Barriac et al., 2004; Mudumbai et al., 2005, 2006, 2010]. On the
other hand, means to improve the speed of convergence of signals from different trans-
mitters at the receiver has also been investigated recently in [Song et al., 2010a,b; Thibault
et al., 2010]. In this chapter, our work complements these existing works on phase syn-
chronization of distributed transmit beamforming by making the following contributions:

i) Firstly, we propose a three-bit feedback scheme for phase synchronization of sig-
nals in time-varying channels, and show that the received signal strength increases
quadratically in proportion to the number of transmitters. The proposed scheme is
shown to improve the RSS considerably when compared to the existing one-bit feed-
back algorithm [Mudumbai et al., 2006].

ii) Secondly, we improve the three-bit feedback scheme, in terms of reducing the num-
ber of feedback bits from the receiver, and subsequently propose a two-bit feedback
scheme for phase synchronization. The use of a second bit as feedback, in addi-
tion to the one bit in [Mudumbai et al., 2006], captures the mobility of the receiver,
and thus, is equally applicable for both static and time-varying channels. The im-
proved scheme is shown to require fewer antennas, e.g., 0.47 times less antennas
than [Mudumbai et al., 2006], to achieve the same RSS, as well as demonstrating an
improved RSS at the receiver for any number of antennas.

iii) Finally, we study how to improve the speed of convergence of the transmitted sig-
nals’ phases at the receiver so as to accelerate the convergence of RSS to a steady
state value. We show that the use of an additional phase feedback, to the one-bit
feedback [Mudumbai et al., 2006] from the receiver, can significantly accelerate the
convergence speed of RSS, with 33% faster convergence than the one-bit feedback
scheme for the coherent alignment of signals at the receiver.

The rest of this chapter is organised as follows: The system model is presented in
Section 7.2. We propose the three-bit feedback protocol for phase synchronization in Sec-
tion 7.3, and its improvement, in terms of the number of feedback bits, in Section 7.4.
The improvement of convergence speed of signals at the receiver, using training data sent
from the transmitters to the receiver, is shown in Section 7.5. Finally, some concluding
remarks are made in Section 7.6.
Chapter 7. Distributed Transmit Beamforming with Feedback

7.2 System Model

Consider the wireless system shown in Fig. 7.1, which consists of \( N \) static sensors in a cluster and a moving receiver. Each sensor consists of a single antenna, and all sensors cooperate with each other to send a common message \( m(t) \) to the receiver. It is assumed that all the transmitters are frequency locked to a reference signal from the master sensor using a master-slave architecture [Mudumbai et al., 2006], and there is no phase offset between the transmitters. However, there is an arbitrary phase difference between the transmitted signals owing to the unknown propagation delay in the master-slave architecture. Hence, the carrier signal of transmitter \( n \) is

\[
c_n(t) = \Re(e^{j(2\pi f_c t + \gamma_{ph}^n)}),
\]

(7.1)

where \( \Re(\cdot) \) is the real part of the argument, and \( \gamma_{ph}^n \) is the arbitrary phase offset of the slave sensor \( n \). After modulating the message signal \( m(t) \) and multiplying with beam-forming weight\(^4\) \( \omega_n = e^{j\theta_n} \), each transmitter \( n \) transmits the modulated signal \( s_{n}^{\text{mod}}(t) = \Re \left( m(t)e^{j\theta_n}e^{j(2\pi f_c t + \gamma_{ph}^n)} \right) \) on the wireless channel. If the channel response of the \( n \)th sensor is \( h_n = a_n e^{j\psi_n} \), the total received signal at the receiver is

\[
r_{\text{rec}}(t) = \Re \left( m(t)e^{j2\pi f_c t} \sum_{i=1}^{N} a_i e^{j(\gamma_{ph}^i + \theta_i + \psi_i)} \right).
\]

(7.2)

\(^4\)Complex weights’ amplitudes are normalized to 1.
Hence, the RSS at the receiver is

\[ P_{\text{rss}}^r = \left| \sum_n a_n e^{j\eta_n} \right|. \quad (7.3) \]

Here, \( \eta_n = \gamma_{n}^{\text{ph}} + \theta_n + \psi_n = \phi_n + \psi_n \), and \( \phi_n \) is the transmission angle of the signal from transmitter \( n \).

From (7.3), we can see that the coherent addition of signals from different transmitters leads to the maximum RSS at the receiver. However, due to the random time varying channel response the RSS can change in a random manner. Therefore, the phase rotations prior to transmissions from different transmitters need to be adjusted so as to achieve phase coherence at the receiver. Here, it is assumed that the received signal phase \( \eta_n \) \( \forall n \) is completely unknown before the feedback control algorithm is executed. The initial values of \( \eta_n \) are assumed to be uniformly distributed random variables in the range \([-\pi, \pi]\), and the angle of arrival of the plane waves are considered to be varied between time slots if the receiver is moving. From here on, we use “received signal power” and RSS interchangeably to indicate RSS.

### 7.3 Three-bit Feedback Protocol for Phase Synchronization

In the proposed three-bit feedback control protocol, it is assumed that the variation of channel response is due to the relative motion between the transmitters and the receiver\(^{44}\). Whenever the receiver is moving, there is a change in carrier frequency due to the Doppler effect [Proakis, 2000] which leads to a random change in the phase of the signal from sensor \( n \) at its receiver, i.e., \( 2\pi f_c t + \theta_n + \gamma_{n}^{\text{ph}} \pm \psi_n \). To mitigate this random effect of the time-varying channel on the received signal phase, the receiver sends three bits \( b_{1}^{\text{bit}}, b_{2}^{\text{bit}} \) and \( b_{0}^{\text{bit}} \), as feedback to the transmitters at the end of each time slot. Firstly, the receiver always keeps a record of the best RSS, and \( b_{0}^{\text{bit}} \) conveys the information on the improvement or the degradation of this RSS at the current time slot compared to the previous time slot as in [Mudumbai et al., 2006]. The receiver sets the bits \( b_{1}^{\text{bit}}, b_{2}^{\text{bit}} \) based on the mobility of the receiver. If \( b_{1}^{\text{bit}} \oplus b_{2}^{\text{bit}} = 1 \), where \( \oplus \) is the exclusive-OR operation, the receiver is assumed to be mobile, and hence the receiver is either moving towards the cluster or moving away from the cluster. To this end, in the proposed algorithm, \( b_{1}^{\text{bit}} = 0, b_{2}^{\text{bit}} = 1 \) indicates that the receiver’s motion is towards the transmitting cluster, and \( b_{1}^{\text{bit}} = 1, b_{2}^{\text{bit}} = 0 \) indicates vice-versa. The transmitters get the feedback bits from the receiver at the end of each time slot, and perturbed their transmitted signal’s phase accordingly so as to achieve coherent phase alignment at the receiver. The working principle of the proposed algorithm is explained in a step-wise fashion as follows:

1. At time slot \( t \), the receiver measures the RSS \( P_{\text{rss}}(t) \) and keeps a record of the best RSS \( P_{\text{rss},\text{best}}(t) \) of its previously observed RSS:

\[ P_{\text{rss},\text{best}}(t) = \max \left( P_{\text{rss}}(t), P_{\text{rss},\text{best}}(t - 1) \right), \quad \text{where} \quad (7.4) \]

\(^{44}\)Due to the mobility of the receiver.
\[ P_{\text{rss}}(t) = \left| \sum_n a_n e^{j\eta_n(t)} \right| + \left( \hat{b}_1 \oplus \hat{b}_2 \right) P_{t,\text{rand}}. \] (7.5)

Here, the time variation is captured by adding the parameter \( P_{t,\text{rand}} \), which is an appropriately scaled parameter from a uniform random distribution in the range \([-1, 0] \) and \([0, 1]\) for \((\hat{b}_1, \hat{b}_2) = (1, 0)\) and \((\hat{b}_1, \hat{b}_2) = (0, 1)\) respectively, to the RSS. This variation in RSS due to the time-variation of the channel is only applicable if there is a movement in the receiver, i.e., \((\hat{b}_1 \oplus \hat{b}_2) = 1\).

ii) At time slot \( t + 1 \), each transmitter \( n \) generates a random phase perturbation \( \delta_{n,r} \) and adds this to the phase of the original signal \( \phi_n(t + 1) \). Therefore,

\[ \phi_n(t + 1) = \phi_{n,\text{best}}^t + \delta_{n,r}(t + 1), \] (7.6)

\[ \delta_{n,r}(t + 1) = \delta_{n,\text{random}}(t + 1) + \left( \hat{b}_1 \oplus \hat{b}_2 \right) \left( \hat{b}_1 - \hat{b}_2 \right) \delta_{n,\text{per}}. \] (7.7)

Here, \( \phi_{n,\text{best}}^t \) is the phase for which (7.4) is satisfied and \( \delta_{n,\text{per}} \) is a randomly generated perturbation chosen randomly in the range \([0, \pi]\), which is added to the transmission phase based on \( \hat{b}_1 \) and \( \hat{b}_2 \). \( \delta_{n,\text{random}} \) is the random phase perturbation generated uniformly randomly in the range of \([-\frac{\pi}{20}, \frac{\pi}{20}]\), and is added to the phase of the transmitted signal \( \phi_n(t + 1) \).

iii) The receiver measures the RSS and generates three bits of feedback depending on the RSS and the direction of motion of the receiver. The combination of different types of feedback and their interpretations are summarized in Table 7.1.

iv) The transmitter \( n \) receives the feedback \((b_{0,\text{bit}}, b_{1,\text{bit}}, b_{2,\text{bit}})\), and adds a random perturbation \( \delta_{n,\text{random}} \) to \( \delta_{n,r} \). The transmitter generates and adds another perturbation \( \delta_{n,\text{per}} \) to the phase \( \delta_{n,r} \), if \((b_1 \oplus b_2) = 1\).

v) The receiver updates its best RSS value to \( P_{\text{rss}}^{r,\text{best}}(t + 1) \), and the transmitter \( n \) updates its best phase value to \( \phi_{n,\text{best}}^t(t + 1) \), if the feedback bit \( b_{0,\text{bit}} = 1 \), and discards it otherwise.

vi) The process is repeated in the next time slot.

### 7.3.1 Performance Measures for the Three-Bit Feedback Scheme

In order to verify the performance benefit of the distributed transmit beamforming strategy with three-bit feedback, the algorithm described in Section 7.3 is simulated for both static and time-varying conditions, and the performance comparison between the one-bit feedback scheme [Mudumbai et al., 2006] and the proposed scheme is made. It is shown that the RSS increases quadratically in proportion to the number of sensors in the cluster. The time span between each time slot is considered as 577 micro-seconds [Olgaard and Yeung, 2004]. Depending on the direction of motion of the receiver, the frequency shift...
7.3. Three-bit Feedback Protocol for Phase Synchronization

Table 7.1: Feedback from the receiver to transmitters.

<table>
<thead>
<tr>
<th>(b_0)</th>
<th>RSS</th>
<th>Type of channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>000</td>
<td>(P_{\text{RSS}}(t + 1) &lt; P_{\text{RSS}}(t))</td>
<td>The channel is static ((b_1^{\text{bit}} + b_2^{\text{bit}} = 0))</td>
</tr>
<tr>
<td>001</td>
<td>(P_{\text{RSS}}(t + 1) &gt; P_{\text{RSS}}(t))</td>
<td>The channel is static ((b_1^{\text{bit}} + b_2^{\text{bit}} = 0))</td>
</tr>
<tr>
<td>010</td>
<td>(P_{\text{RSS}}(t + 1) &lt; P_{\text{RSS}}(t))</td>
<td>The channel is time-varying ((b_1^{\text{bit}} + b_2^{\text{bit}} = 1))</td>
</tr>
<tr>
<td>011</td>
<td>(P_{\text{RSS}}(t + 1) &gt; P_{\text{RSS}}(t))</td>
<td>The channel is time-varying ((b_1^{\text{bit}} + b_2^{\text{bit}} = 1))</td>
</tr>
<tr>
<td>100</td>
<td>(P_{\text{RSS}}(t + 1) &gt; P_{\text{RSS}}(t))</td>
<td>The channel is time-varying ((b_1^{\text{bit}} + b_2^{\text{bit}} = 1))</td>
</tr>
<tr>
<td>101</td>
<td>(P_{\text{RSS}}(t + 1) &lt; P_{\text{RSS}}(t))</td>
<td>The channel is static ((b_1^{\text{bit}} + b_2^{\text{bit}} = 0))</td>
</tr>
<tr>
<td>110</td>
<td>(P_{\text{RSS}}(t + 1) &lt; P_{\text{RSS}}(t))</td>
<td>The channel is static ((b_1^{\text{bit}} + b_2^{\text{bit}} = 0))</td>
</tr>
<tr>
<td>111</td>
<td>(P_{\text{RSS}}(t + 1) &gt; P_{\text{RSS}}(t))</td>
<td>The channel is static ((b_1^{\text{bit}} + b_2^{\text{bit}} = 0))</td>
</tr>
</tbody>
</table>

Figure 7.2: Mathematical relationship of the RSS as the number of transmitters \(N\) varies.

is positive (causing an increase in frequency) if the receiver is moving towards the transmitters (i.e., \((b_1^{\text{bit}}, b_2^{\text{bit}}) = (1, 0)\)), and the frequency shift is negative for \((b_1^{\text{bit}}, b_2^{\text{bit}}) = (0, 1)\). To this end, compensation of the phase shift is made according to (7.6) and (7.7) based on \(b_1^{\text{bit}}\) and \(b_2^{\text{bit}}\).

In Fig. 7.2, we show the change of RSS at the receiver with the number of sensors (i.e., the transmitters) at the transmitting cluster. As Fig. 7.2 demonstrates, due to the increased transmit diversity corresponds to the increased number of the transmitters, the RSS increases quadratically at the receiver with the number of transmitters. Due to this increment in signal strength with increasing number of transmitters, the proposed scheme shows substantial performance improvement with respect to the one-bit feedback scheme [Mudumbai et al., 2006] and a system without feedback as shown in Fig. 7.3.
Chapter 7. Distributed Transmit Beamforming with Feedback

Fig. 7.3: Comparison of the RSS of the three-bit feedback system with a one-bit feedback system [Mudumbai et al., 2006] and a system without any feedback at time-varying channel.

Fig. 7.3 shows the comparison of RSS of the proposed three-bit feedback system in a time-varying channel for $N = 100$, and the one-bit feedback system [Mudumbai et al., 2006] for the same environment. The graph of mean value over 200 iterations, and the graph of values for a typical iteration are both given. In case of a time-varying channel, our proposed three-bit feedback system gives substantial improvement in performance. Fig. 7.3 demonstrates the large performance gains of a three-bit feedback system over the one-bit feedback system and the system without feedback. As shown in Fig. 7.3, the RSS of the one-bit and three-bit feedback schemes increases towards its steady state as the number of time slots increases from 1 to 800, and there is a substantial difference between the mean RSSs of the two schemes. Comparing the mean RSS of the one-bit feedback system with the three-bit feedback, it is observed that the RSS of the three-bit feedback system is twice that of the one-bit feedback at time slot 600. This performance improvement is more impressive at time slot 800 (at steady state). Meanwhile, in the case of the system without feedback, there is no improvement in RSS, and large fluctuations are observable over all RSS.

In Fig. 7.4, we show a comparison of feedback systems for a static channel. The response of the system without feedback is also shown in the figure. Fig. 7.4 shows that the three-bit feedback system converges in almost the same manner as the 1-bit feedback system with a small improvement at the steady state. From Fig. 7.4, at time slot 150, the RSS of the three-bit feedback system shows an improvement of 11.2% over the 1-bit feedback system, and on average, the improvement is around 14%. Although there is no significant random perturbation in received power from one time slot to another in a static channel, there is some minor phase perturbation of the received signal that the three-bit
7.4 Improved Feedback Protocol for Phase Synchronization

In this section, we discuss an improved feedback scheme for distributed transmit beamforming, in terms of the number of feedback bits from the receiver at the end of each time slot, and propose a two-bit feedback algorithm for DTB in a time varying channel so as to improve the signal gains in terms of RSS at the receiver. We note that a random algorithm for distributed beamforming is proposed in [Thibault et al., 2010], which uses two-bit feedback (2BF). However, our proposed scheme is different from the one in [Thibault et al., 2010] in many respects. Firstly, the authors in [Thibault et al., 2010] focussed on the improvement of the convergence speed of the transmitted signal at the receiver whereas
our scheme improves the signal gain in terms of RSS at the receiver. Secondly, the second feedback bit in the 2BF scheme was used to carry the information of the quality of the RSS [Thibault et al., 2010]. In contrast, in our case the second feedback bit manifests the mobility of the receiver. However, the first bit serves the same purpose as [Mudumbai et al., 2006] in both the schemes. Thirdly, a number of threshold RSS levels is considered in 2BF [Thibault et al., 2010]. However, our scheme is simpler, and only requires an additional perturbation to the one-bit feedback scheme when the receiver is in motion.

### 7.4.1 Two-bit Feedback Algorithm

#### Algorithm 7.1 Two-bit feedback algorithm

**At the receiver**

1. The receiver measures the RSS and updates the best RSS.
   
   - if \( P_{\text{rss}}(t) > P_{\text{rss, best}}(t-1) \) then
     
     \[ P_{\text{rss, best}}(t) = P_{\text{rss}}(t) \]
   
   - else
     
     \[ P_{\text{rss, best}}(t) = P_{\text{rss, best}}(t-1) \]

2. The receiver updates the bits \( b_{\text{bit}0} \) and \( b_{\text{bit}1} \) as follows:

   - if \( P_{\text{rss}}(t) > P_{\text{rss, best}}(t-1) \) then
     
     \[ b_{\text{bit}0} = 1 \]
   
   - else
     
     \[ b_{\text{bit}0} = 0 \]

   - and
   
   - if receiver in motion then
     
     \[ b_{\text{bit}1} = 1 \]
   
   - else
     
     \[ b_{\text{bit}1} = 0 \]

3. The receiver sends the feedback bits to the transmitters.

**At the transmitter**

4. The transmitter gets the feedback \( b_{\text{bit}0} \) and \( b_{\text{bit}1} \) from the receiver and updates its best transmission angle.

   - if \( b_{\text{bit}0} = 1 \) then
     
     \[ \phi_{\text{best}}(t) = \phi(t) \]
   
   - else
     
     \[ \phi_{\text{best}}(t) = \phi_{\text{best}}(t-1) \]

5. The transmitter \( n \) randomly generates \( \delta_{n, \text{random}} \in [-\pi/20, \pi/20] \) and \( \delta_{n, \text{pp}} \in [-\pi, \pi] \), which is based on \( b_{\text{bit}1} \), and add them to its best known transmission phase, \( \phi_{\text{best}} \) for transmission in time slot \( t + 1 \). So,

   \[ \phi_n(t + 1) = \phi_{\text{best}}(t) + \delta_{n, \text{random}} + b_{\text{bit}1} \delta_{n, \text{pp}}. \]

6. Each transmitter \( n \) sends their signal towards the receiver.

**The process is repeated in subsequent time slots**

In the two-bit feedback algorithm, the receiver sends two bits, \( b_{\text{bit}0} \) and \( b_{\text{bit}1} \), as feedback to the transmitters at the end of each time slot. The bit \( b_{\text{bit}0} \) conveys the information of the RSS of the signal at the current time slot as in [Mudumbai et al., 2006, 2010], and \( b_{\text{bit}1} \) is set

\[45\]To acquire information on the increase or decrease of the RSS.
7.4. Improved Feedback Protocol for Phase Synchronization

At time slot $t$, upon receiving the signal, the receiver compares the RSS $P_r(t)$ with its previously measured best RSS $P_{r,\text{best}}(t-1)$. If $P_{r,\text{best}}(t) > P_{r,\text{best}}(t-1)$, the receiver makes the feedback bit $b_{0}^{\text{bit}} = 1$ and updates the value $P_{r,\text{best}}(t)$ with the RSS $P_{r,\text{best}}(t)$. Otherwise, the receiver sets $b_{0}^{\text{bit}} = 0$ and maintains the previously measured best RSS. The bit $b_{1}^{\text{bit}}$ is set to 1 if the receiver is in motion and to 0 otherwise. Upon receiving the feedback from the receiver, each transmitter $n$ checks the feedback bit $b_{0}^{\text{bit}}$ and updates its best transmission angle $\phi_{n,\text{best}}$ if $b_{0}^{\text{bit}} = 1$. The transmitter keeps its previous best angle, $\phi_{n,\text{best}}(t) = \phi_{n}(t-1)$ if $b_{0}^{\text{bit}} = 0$. The transmitter also checks the value of $b_{1}^{\text{bit}}$ and adds a further perturbation $\delta_{n,\text{pp}}$ in its next transmission if it finds $b_{1}^{\text{bit}} = 1$, but adds no further perturbation if $b_{1}^{\text{bit}} = 0$. Thus, while always looking for the best transmission angle, the angle alignment at the receiver always results in better RSS with each progressive time-slot. In Table 7.2, we show all combinations of feedback bits sent from the receiver, and how these feedback bits are interpreted in the proposed scheme.

In Fig. 7.5, we show the working method of the proposed two-bit feedback algorithm for a two-antenna transmission system in a time-varying environment. As shown in Fig. 7.5 (a), two transmit signal vectors, $x_1$ and $x_2$, are added at the receiver to give $(x_1 + x_2)$, and the RSS is given by the magnitude of $|x_1 + x_2|$. Due to the variation of channel response over time, the signals alignment is disturbed at the receiver, and hence the RSS is reduced due to the misalignment of the signals as shown in Fig. 7.5 (b). However, in our proposed algorithm, a random perturbation $\delta_{n,\text{pp}}, \forall n$ is added in addition to the perturbation $\delta_{n,\text{random}}$ [Mudumbai et al., 2006], which further leads to a performance improvement in terms of RSS as shown in Fig. 7.5 (c). The proposed algorithm is formally stated in Algorithm 7.1.

Table 7.2: Feedback from receiver to the transmitters

<table>
<thead>
<tr>
<th>$b_{0}^{\text{bit}}$</th>
<th>RSS</th>
<th>Receivers mobility</th>
</tr>
</thead>
<tbody>
<tr>
<td>00</td>
<td>$P_{r,\text{best}}(t) &lt; P_{r,\text{best}}(t-1)$</td>
<td>Static</td>
</tr>
<tr>
<td>01</td>
<td>$P_{r,\text{best}}(t) &gt; P_{r,\text{best}}(t-1)$</td>
<td>Static</td>
</tr>
<tr>
<td>10</td>
<td>$P_{r,\text{best}}(t) &lt; P_{r,\text{best}}(t-1)$</td>
<td>Mobile</td>
</tr>
<tr>
<td>11</td>
<td>$P_{r,\text{best}}(t) &gt; P_{r,\text{best}}(t-1)$</td>
<td>Mobile</td>
</tr>
</tbody>
</table>
7.4.2 Performance Improvement Using Two-Bit Feedback Scheme

In this section, we verify the validity of the proposed two-bit feedback scheme using Monte-Carlo simulations under various conditions. The total received signal strength (RSS), as specified in (7.3), may be reinforced or degraded in any particular time slot due to the relative motion of the receiver with respect to the transmitters (and any scatterers). Thus, the time-varying channel response is captured with respect to all transmitters by adding a quantity, which is an appropriately scaled parameter from a uniform random distribution in the range −1 to 1, to the RSS. This is done only when the feedback bit $b^t_i = 1$. We first show the behavior of the proposed scheme as the number of transmitters is changed. Then, we compare our scheme with the one-bit feedback [Mudumbai et al., 2006] and the improved one-bit feedback algorithm [Song et al., 2010b] schemes to demonstrate the improved performance of the proposed scheme.

In Fig. 7.6, we show the RSS at the receiver for different number of transmitters at the transmit side. Similar to any standard cooperative communication schemes, the RSS increases as the number of transmitters increases in the transmitting cluster. For example, at time slot 700, the RSS for $N = 400$ is 2.5 times better than the RSS for $N = 100$ transmitters. This is because of the increased transmit diversity corresponding to an increased number of the transmitters.

To show the effectiveness of the proposed scheme, we compare our scheme with the one-bit feedback, [Mudumbai et al., 2006], and the improved one-bit feedback algorithm, [Song et al., 2010b], for similar time-varying conditions. In Fig. 7.7, we compare the RSS at the receiver for the proposed scheme with one-bit feedback and the improved one-bit feedback scheme in a time-varying channel, when $N = 100$ distributed transmitters...
are beamforming to the receiver. Fig. 7.7 demonstrates a substantial gain of the two-bit feedback scheme over both the one-bit feedback and improved one-bit feedback. For example, at time slot 500, the RSS of the proposed scheme is 1.74 times that of one-bit feedback, and at time slot 800 this gain is almost 2 times. In case of improved one-bit feedback scheme, these improvements are 1.9 and 2.23 times respectively. The reason for this improved performance is that the proposed scheme tracks the time variation by using an additional bit from the receiver. This additional bit assists the transmitters to decide on the nature of channel, i.e., whether time-varying or static, and incorporates an additional perturbation to the transmitter phases accordingly.

In Fig. 7.8, we show the improvement of the RSS at the receiver as the number of transmitters increases for the proposed scheme and compare this improvement with the one-bit and improved one-bit feedback schemes. Because of the transmit-side cooperation, each one of the analyzed schemes shows an improvement in terms of the RSS at the receiver as the number of transmitters increases. The use of the additional bit in feedback of the proposed scheme significantly enhances the RSS when compared to the other schemes, as the number of transmitters increases. From Fig. 7.8, with increasing number of transmitters, the proposed scheme achieves an RSS that is, on average, 1.7 and 2.1 times that of the RSS achieved by the one-bit and the improved one-bit feedback schemes respectively.

In Fig. 7.9, we plot the number of transmitters required to achieve the same RSS at the receiver, for the proposed scheme, the one-bit feedback scheme and the improved one-bit feedback scheme. Fig. 7.9 shows that the use of the proposed two feedback bits significantly reduces the number of transmitters required to achieve the same RSS at the
receiver, compared to the one-bit and improved one-bit schemes. From Fig. 7.9, on average, 0.47 times and 0.4 times less transmitters are required for the proposed scheme, relative to the one-bit feedback and improved one-bit feedback schemes respectively, to achieve the same RSS at the receiver. This is a large improvement in terms of scaling of transmitters that is due to the considerable improvement in terms of RSS, for any given number of transmitters, for the proposed scheme.

7.5 Improving The Speed of Phase Convergence

In this section, we study the improvement of the speed of convergence of the received signals’ phases at steady state so as to accelerate the coherent addition of signals from multiple transmitters at the receiver. To that end, we assume that the phase offset $\gamma_{\text{ph}}$ due to different transmitter location is known, and hence, the misalignment of the multiple transmitted signals’ phases at the receiver is due to the time-varying channel response. Here, we study a scheme in which the transmitters can track the average channel effect on the transmitted signals so as to compute the perturbation to add to their beamforming weights’ phase at each time slot. We derive a simple formula, using a phase feedback from the receiver, that each transmitter can use to compute this phase perturbation. We propose an Enhanced one-bit Feedback algorithm by introducing a phase feedback in conjunction with a one-bit feedback to capture the effect of the average time-varying channel response on the transmitted signals. The feedback phase conveys the information of the average
7.5. Improving The Speed of Phase Convergence

channel effect on the signals from transmitters in the previous time slot, and the one-bit feedback is adopted as in [Mudumbai et al., 2006].

To this end, the phase perturbation at each time slot added by each transmitter $n$ is considered as $\theta_n(t) = \rho(t) + \zeta_n(t)$, where $\rho(t)$ is the phase perturbation based on the phase feedback from the receiver at the previous time slot $(t-1)$, and $\zeta_n(t)$ is another perturbation considered from a specific pseudo-random sequence $\zeta(t)$ of length $N$ (or larger) in the interval $[-\zeta_0, \zeta_0]$ from a specific sequence table $\Xi$. With numerical experiments, we show that the use of the perturbation, using the phase feedback, can significantly improve the convergence speed of the alignment of received signal phases. In the specific sequence table $\Xi$, each row of the table has the same values contained in the fixed sequence, $\zeta(t)$, but the sequence is indexed differently with respect to different transmitters for each $t$; i.e. within any given column (or row) of $\Xi$ there are no identical perturbation values within that column (or row). The transmitters and the receiver use the same deterministic function [Petrović et al., 2003] to choose $\zeta_n(t)$, for transmitter $n$ in each time slot $t$, from $\Xi$.

\footnote{Where, typically $\zeta_0 << \pi$.}
Chapter 7. Distributed Transmit Beamforming with Feedback

7.5.1 Phase Formulation as Part of Perturbation

Here, we present the basis of the proposed enhanced one-bit feedback. From (7.2) and (7.3), the received signal at time slot \( t \) is,

\[
r_{\text{rec}}(t) = m(t) \left[ \Phi^T(t) H(t) \right],
\]

where \( \Phi^T(t) = [e^{\phi_1(t)}, e^{\phi_2(t)}, \ldots, e^{\phi_N(t)}]^T \) is the transmit phase vector from the transmitters, \( H(t) = [h_1(t), h_2(t), \ldots, h_N(t)]^T \) is the channel effect on the signal, and \( | \cdot |^T \) is the transpose of the vector. Now, if the average distance between the transmitters \( d_{\text{ms}} \) is much lower than the average distance from the transmitting cluster to the receiver \( d_{\text{TR}} \), i.e., \( d_{\text{ms}} \ll d_{\text{TR}} \), it is reasonable to assume that the channel attenuation experienced by the signals from different transmitters to the receiver are of same order of magnitude [Mudumbai et al., 2006]. As a consequence, an average channel attenuation effect from all the transmitters can be considered upon the transmitted signal [Dong et al., 2007]. So, the average effect of a time-varying channel on the signal from each transmitter \( n \) at time \( t \) is

\[
h_c(t) = \frac{1}{N} \sum_{n=1}^{N} h_n(t).
\]

(7.10)

Now, due to \( d_{\text{ms}} \ll d_{\text{TR}} \), it is reasonable to approximate \( h_c(t) \) as the channel gain \( h_n(t) \) from each transmitter \( n \) [Dong et al., 2007]. To this end, the channel vector \( H \) in (7.9) appears as \( H_c(t) = h_c(t)1_N \), where \( 1_N \) is a column vector of 1 of length \( N \). From section 7.2, the transmitted phase \( \phi_n(t) \) consists of a known phase offset \( \gamma_n^{\text{ph}} \), and the phase perturbation \( \theta_n \), which is based on the previous phase feedback from the receiver and the known pseudu-random perturbation \( \zeta_n(t) \), from \( \Xi \), for each \( n \). Therefore, for a known \( m(t) \), the channel effect \( h_n(t) \approx h_c(t) \) is the only unknown to the receiver. Hence, from (7.9) and (7.10),

\[
h_c(t) = \frac{r_{\text{rec}}(t)}{m(t) \left[ \Phi(t)^T 1_N \right]},
\]

(7.11)

and the estimated average change of the signal phase at the receiver at time \( t \) is

\[
\psi(t) = \arg h_c(t), \ \forall n.
\]

(7.12)

For the first transmission \( \rho(t) \) is assumed to be zero, and therefore, the received signal phase at the first time slot is

\[
\eta_n(t) = \gamma_n^{\text{ph}} + \zeta_n(t) + \psi(t), \ \text{for} \ t = 1.
\]

(7.13)

A time-varying channel causes the received signal phase to change on average by \( \psi \), which leads to a misalignment of the received signal phase. The receiver sends the estimation of this phase information \( \psi \) to the master transmitter as phase feedback along with the bit\(^{\text{bit}}\) through a controlled channel [Dong et al., 2007]. The feedback bit\(^{\text{bit}}\) is

\(^{47}\text{bit}^{\text{bit}}\) is the one bit feedback as in [Mudumbai et al., 2006].
7.5. Improving The Speed of Phase Convergence

Figure 7.10: (a) Two transmit signal vectors $a$ and $b$ are added as vectors at the receiver and the received signal is $r_{\text{rec}}$. (b) i.i.d random channel variation $\psi_a$ and $\psi_b$ leads to misalignment of the phase at the receiver that results in poor RSS shown by the solid line. The estimate of the channel variation from the previous time slot $\rho$ leads to performance improvement which is shown separately in (c).

sent as 1 if the beamforming gain improves in the current time slot, and 0 otherwise. The phase feedback, $\psi$, can be transmitted by a quantized phase-feedback scheme [Liu and Jafarkhani, 2005]. The channel phase conveys the information of the average channel effect on the received signal, whereas $\psi_{\text{bit}}$ represents the improvement of RSS at the receiver [Mudumbai et al., 2006].

Upon receiving the feedback, the master transmitter derives this phase information, $\psi$, using a phase extraction algorithm (for example, the APES algorithm [Guo et al., 2006]) and broadcasts it to the other transmitters. All the transmitters use the conjugate of this phase as the perturbation of the beamforming vector in time slot $(n + 1)$. Therefore, the perturbation at time slot $(t + 1)$ is

$$e^{j\rho_{(t+1)}} = e^{-j\psi(t)}.$$ (7.14)

Thus, from (7.13) and (7.14)

$$\rho(t + 1) = -\arg h_c(t).$$ (7.15)

7.5.2 Enhanced One-Bit Feedback Algorithm

Here, we summarize the enhanced one-bit feedback algorithm, which can be implemented by the sensors in a distributed fashion with the assumption that the master sensor calculates $\rho$ using (7.15), and broadcasts it to all the sensors in the cluster. The algorithm captures the average changes in the transmitted signals’ phases due to the time-varying channel response, and use it to estimate the effect of channel on the signals in the next time slot. In Fig. 7.10, we show the working principle of our algorithm for a two antenna system in a time-varying environment. Fig. 7.10(a) shows the RSS in the absence of a time-varying channel. While the signal from both antennas goes through the time-varying channel the misalignment of the phases caused by the channel is $\psi_a$ and $\psi_b$ respectively (see Fig. 7.10(b)). However addition of the conjugate of the estimated average channel phase response $\rho$ in the previous time slot balances the misalignment up to $-\rho$ and leads to a performance improvement as shown in Fig. 7.10(c). Thus, the phases eventually converge to a value that aligns the signal phases at the receiver, and consequently,
the maximum RSS at the receiver is achieved. The details of the algorithm is shows in Algorithm 7.2.

**Algorithm 7.2 Enhanced One-Bit Feedback**

1. At time slot $t$, the transmitter $n$ receives the bit $b_0$, and adopts its best known carrier phase as follows.
   - if $b_0 = 1$
     - $\phi_{best}^n(t) = \phi_n(t)$
   - else
     - $\phi_{best}^n(t) = \phi_{best}^n(t-1)$
   - end if
2. The master sensor detects $\psi(t)$ from channel feedback and broadcasts to other sensors in the cluster.
3. The sensors determine $\rho(t)$ from $\psi(t-1)$ using (7.15), and adds it to the beamforming weight’s phase.
4. Each sensor $n$ computes a perturbation $\theta_n(t)$ based on $\psi(t-1)$.
   - if $\psi(t-1) \neq 0$
     - $\theta_n(t) = \zeta_n(t) + \rho(t)$
   - else
     - $\theta_n(t) = \zeta_n(t)$
   - end if
   - $\zeta_n(t)$ is from a fixed pseudo-random sequence $\zeta(t)$ in $[-\zeta_0, \zeta_0]$, and is different from each other at time slot $t$.
5. Sensor $n$ computes the transmission angle $\phi_n(t)$ from its best known carrier phase i.e., $\phi_n(t) = \phi_{best}^n(t) + \theta_n(t)$
6. The receiver measures the RSS, $P_{r, rss}(t)$, and updates the best RSS in its memory: $P_{r, best}(t) = \max(P_{r, best}(t-1), P_{r, rss}(t))$.
7. The receiver sets the bit $b_0$ based on $P_{r, rss}(t)$.
   - if $P_{r, rss}(t) > P_{r, best}(t-1)$ then
     - $b_0 = 1$
   - else
     - $b_0 = 0$
   - end if
8. The receiver extracts the average channel effect using (7.11), and updates its phase feedback $\psi(t)$.
9. The receiver sends $b_0$ and $\psi(t)$ to the master sensor through a noise free control channel [Dong et al., 2007].

The process is repeated in subsequent time slots until convergence.

### 7.5.3 Performance Measures of Enhanced One-Bit Feedback

Here, we show the performance enhancement of enhanced one-bit feedback algorithm for a number of channel conditions. We compare our results with [Mudumbai et al., 2006] for the parameter settings stated in [Mudumbai et al., 2006]. For the time-varying channel response, we adopt Jakes model [Jakes, 1974] to generate temporarily-correlated Rayleigh fading coefficient, which are $CN(0, 1)$ distributed at a specified normalized fading rate $f_{\text{rate}}$. We consider $f_{\text{rate}} = F_d T_s = 0.0048$ and 0.0962, where $F_d$ is Doppler spread and $T_s$ is 577 $\mu$s. For these specified fading rates the channel can be approximated as constant within one time slot. We show that, at higher fading rate, the use of the derived perturbation, as given in Section 7.5.1, shows better performance in terms of the speed of

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For simulation of method in [Mudumbai et al., 2006], initial phase is considered uniformly distributed in $[-\pi, \pi]$ and the random perturbation from a uniform distribution in $[-\pi/20, \pi/20]$.

$f_{\text{rate}} = 0.0048$ and $f_{\text{rate}} = 0.0962$ corresponds to Doppler velocity $5 \text{ kmh}^{-1}$ and $100 \text{ kmh}^{-1}$ respectively, with carrier frequency $1800 \text{ MHz}$. 

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7.5. Improving The Speed of Phase Convergence

We conduct a Monte Carlo simulation for $N = 100$ transmitters, and plot the RSS with respect to time slots in Fig. 7.11 for both the proposed enhanced one-bit and the one-bit feedback algorithm. From Fig. 7.11, the enhanced one-bit feedback performs better than the one-bit feedback in both the convergence speed and the achievable RSS at the receiver. In Fig. 7.11, for $\text{fad}_{\text{rate}} = 0.0048$, the enhanced one-bit feedback system achieves the steady state\(^{51}\) value around 550 time slots, whereupon the one-bit feedback system achieves the steady state value after 750 time slots, which is a 25% improvement over the one-bit feedback system. In case of a faster fading rate, e.g. $\text{fad}_{\text{rate}} = 0.0962$, the additional phase estimate $\psi$ shows more impressive convergence speed, and the enhanced one-bit feedback scheme shows a performance improvement of 33% over the one-bit feedback system at this fading rate.

From Fig. 7.12, at lower fading rate $\text{fad}_{\text{rate}} = 0.0048$, the enhanced one-bit algorithm achieves a scalability of 11.8% on the average, in number of transmit antennas, to achieve the same RSS at the receiver as the one-bit feedback scheme. When the fading rate is higher, i.e. $\text{fad}_{\text{rate}} = 0.0962$, the achieved average scalability in number of transmit an-

\(^{50}\)We consider the BER for binary phase shift keying (BPSK) signal.

\(^{51}\)Steady state is considered to be achieved in the region where the slope of the curve is approximately constant, e.g., one may consider the instant when the variation in RSS over 50 time slots is not more than 1.5%.
Figure 7.12: Comparison of the enhanced one-bit and the one-bit feedback algorithm [Mudumbai et al., 2006] for the number of distributed transmitters required for achieving the same RSS at the receiver after $t = 600$ time slots.

The number of antennas for the enhanced one-bit system is 18.0%. Since, for both original and enhanced one-bit feedback algorithms, the required number of transmit antennas is a function of target RSS at the receiver, from Fig. 7.12, the scalability of the number of transmit antennas ($S$) can be expressed as

$$S = \frac{N_1 - N_E}{N_1} \times 100\%,$$

(7.16)

where $N_E$ and $N_1$ are the number of distributed transmit transmitters for enhanced one-bit feedback and the one-bit feedback scheme respectively.

We now compare the achievable theoretical BER at the receiver for both the enhanced one-bit feedback algorithm and the one-bit feedback algorithm. In doing so, we record the RSS at the receiver for each time slot, and then measure the signal to noise ratio (SNR) for a constant noise power ($\sigma_n^2 = -10$ dB). Assuming binary phase shift keying (BPSK) modulation, we calculate the theoretical BER using [Proakis, 2000]:

$$\text{BER} = \frac{1}{2} \left( 1 - \sqrt{\frac{\text{SNR}}{\text{SNR} + 1}} \right),$$

(7.17)

and plot them in Fig. 7.13. It is observed that the enhanced one-bit feedback system achieves the target theoretical BER faster than the one-bit feedback system for both fading rates. This is due to the fact that the enhanced one-bit feedback system achieves the steady state RSS faster than the one-bit feedback for both fading rates and, consequently
the speed to achieve the theoretical BER is also faster for enhanced one-bit feedback. From Fig. 7.13, for \( fad_{rate} = 0.0048 \), the speed to achieve theoretical BER is 50\% faster in enhanced one-bit feedback, while it is 54\% faster for \( fad_{rate} = 0.0962 \).

### 7.6 Concluding Remarks

In this chapter, distributed transmit beamforming techniques for signal transmission over long distances in both static and time-varying channels have been studied. We have proposed techniques that enable the signals from multiple transmitters to add coherently at the receiver without any channel state information. One technique uses only three bits feedback of information on the mobility of the receiver and received signal strength at the end of each time slot. Whereas, the received signal strength has been shown to increase quadratically with the number of transmitters at the receiver, we have also shown that the improvement of performance also holds for a technique using a reduced number of feedback bits, i.e., two bits as feedback, as well. Further, we have also sought improve the speed of convergence over one-bit feedback and we have shown that a technique that uses an additional phase feedback with the original one-bit feedback scheme can significantly accelerate the speed of convergence of received signal strength. The effectiveness of the proposed schemes has been confirmed by comparing them with existing feedback-controlled distributed transmit beamforming techniques.
Conclusion

The benefits of smart networks, as an integrated entity of smart grids and smart sensor networks, are demonstrated by the tremendous research efforts that are being made in this area throughout the world. Although critically required, their widespread application is still far from universal, however, improvements are critical in some key application areas, for example in energy efficiency, before their extensive deployment.

This thesis seeks to bring the possible advantages of smart networks in respect of their energy efficiency by modeling their energy usage behavior in both energy management (in smart grids), and signal transmission (in sensor networks). Although both technologies can act in an integrated fashion in smart networks, this thesis has discussed their properties emphasizing their particular nature with respect to energy usage. This is motivated by the fact that both technologies can act as an individual entity so as to provide energy efficient services individually. This thesis shows how the self-seeking distributed nodes in smart grids can be motivated in energy management, which is socially optimal so as to benefit all the energy users in the network. The thesis also demonstrates efficient energy allocation for smart nodes, and also an efficient signal transmission technique for their transmission of signal over long distances so as to improve the network lifetime.

The first problem we investigated and provided a solution for is how to manage the optimal charging of a large number of electric vehicles in smart grids particularly at the peak hour of energy demand when the supply of electricity from the central power station is a scarce commodity. Our answer to this problem was based on a non-cooperative Stackelberg game. In the game, both the energy provider and electric vehicles strategically chose their decision on their energy trading parameters and reached an equilibrium solution that is shown to be optimal for both the grid and the vehicles. Our findings highlight the fact that as each electric vehicle is owned by a self-seeking individual each utility from trading energy has to be maximized at the optimal solution of the game.

As our second problem, to answer the question as to how the energy consumers can be motivated to voluntarily take part in energy management, as they are an integral part of energy management in smart grids, we have studied a consumer-centric energy management scheme. Using single-leader-multiple-followers game we have shown that the proposed consumer-centric management scheme leads to a socially optimal solution for the consumers in the network, and thus, enhance their utility through energy trading either as a buyer or seller of energy in the network. As a consequence, the consumers are motivated to take part in the energy trading with a central power station.

As a third problem, using the benefits of voluntary participation of energy consumers in smart grids, we proposed an outage management scheme for efficiently curtailed en-
energy from the consumers in the event of a power outage in the system. This work emphasizes the fact that socially optimal curtailment of energy can lead to the minimal total cost to the whole system due to any outage, and can be obtained by the users in the network playing a generalized Nash game. However, a smart communications infrastructure, for example a wireless sensor network, is a precondition for these energy managements.

Motivated by the importance of sensors in smart grids, and due to the fact that energy efficient sensor networks are an essential element of any smart communication infrastructure, we have investigated the power control of sensor networks as our fourth contribution in this thesis. First, we have played a power control game for multiple source sensors, which are transmitting their signals towards multiple clusters with cooperative sensors, and we have showed that at equilibrium of the game the cost to sensors in terms of usage power is at the minimum required for a target quality of service. Then, we investigate the power usage pattern of the cooperative sensors in the cluster to improve network life-time. It is demonstrated that a cross-layer scheme using distributed transmit beamforming via data funnelling, as proposed here, can significantly improve the energy usage pattern of cooperative sensors in clusters so as to improve the life-time of the sensor network.

After showing the effectiveness of transmit beamforming in saving energy, we have studied distributed transmit beamforming techniques with feedback in the context of sensor networks. We have mainly studied the problem of phase synchronization of signals in transmit beamforming, which is essential to coherent addition of signals at the receiver that significantly increases the signal gain (i.e., energy efficient transmission). It has been highlighted here that transmit beamforming of a signal by distributed sensors, using feedback information from a receiver, can significantly improve the signal gain at a receiver and hence achieve energy efficient signal transmission. The subsequent improvements of the beamforming techniques in terms of feedback bits, and, in the case of the enhanced one-bit feedback technique, convergence speed are also demonstrated.

The main contributions of this thesis not only lie in bringing out several energy efficient solutions for smart networks. But the main contributions also lie in having successfully analyzed and demonstrated the behavior of self-seeking distributed nodes, including their competition and cooperation, in the network and having provided some methodologies for modeling these distributed nodes. This is of help to shape future research directions, as discussed in the following section.

8.1 Future Work

Most of the work in this thesis has the potential for further extension, from extending effective demand management schemes in smart grids to more dynamic scenarios with more complex environments; to incorporating analysis of choice of sensor nodes in distributed transmit beamforming to enable transmission of signals with even greater energy efficiency. To this end, the following two sections focus on potential future research directions that should have significant impact on performance improvements for smart networks.
8.1.1 Energy management in smart grids

Dynamics of electric vehicle charging

Electric vehicle charging in smart grids in an energy constrained environment has been discussed in this thesis. Extension of this work to more practical scenarios, including the dynamics of charging and the incorporation of battery states into charging profiles, is important. One potential way of incorporating the dynamics of charging is to incorporate a second game into the model that has been proposed in this thesis to enable the modeling of interactions between aggregators that control the groups of electric vehicles and individual electric vehicle so as to account for characteristics of individual vehicles. Another interesting extension of this work would be to investigate the effect of price discrimination among electric vehicles, within certain bounds, on their respective utilities and costs.

Incorporating cost operation with consumers preference

The need of energy for a PEVG and its motivation to pay the specified price per unit of energy to the grid may be affected by various preferences. For example, consumers driving an electric vehicle for business may be willing to pay more per unit of energy compared to consumers driving an electric vehicle for pleasure. Hence, it would be an interesting extension of the work in Chapter 3 to consider the cost of operation of each PEVG based on consumer preferences, preferences that also depend on the total number of active vehicles that are connected to the smart grid to charge their batteries.

Deployment of charging stations in urban areas

Deployment of charging stations for electric vehicles in urban areas is important as typically car parking spaces in urban areas are occupied for around 4–7 hours per day [Fang et al., 2011]. However, the installation and deployment of charging stations depends on a large number of factors including the churn/idle time of cars, costs, strategic locations, number of cars etc [Fang et al., 2011]. These factors constitute a very interesting research problem with respect to optimally selecting the location and the number of charging stations in urban areas, and also the cost of vehicle charging. One potential tool that can be used to address this problem is the framework of a facility location game that allows one to strategically deploy the charging/installation stations at the same time as minimizing costs.

Scheduling of supply of energy to consumers

In the context of consumer-centric smart grids, as explained in Chapter 3 of this dissertation, very important further work is the scheduling of devices in the network to provide their surplus energy to the grid. As different energy consumers use their energy at different times, their energy supply time needs to be scheduled based on their physical characteristics and how they use energy [Fang et al., 2011]. One potential way to do this scheduling is to introduce an auction based max-min energy game in which the consumers play the game amongst themselves to maximize the least amount of profit they can make from trading their energy.
8.1. Future Work

Designing incentives for energy volunteers

As shown for the outage management schemes described in Chapter 5 of this thesis, the voluntary participation of users in energy curtailment from themselves is crucial for the efficiency of schemes. Hence, the design of appropriate incentives (e.g., prices) for taking part in such voluntary curtailment schemes is important. For example, a smart home may pay less price per unit of energy for the duration for which it has been agreed to curtailed its energy due to power outages in the grid.

Game theoretic approaches for energy management

Energy management schemes will always face technical challenges such as pricing, regulations, adaptive decision making, users' interactions, and operation in dynamic environments. All of these issues are very well-suited to the application of game theory. In this regard, several potential energy management schemes using game theory can be studied:

i) Developing online algorithms for learning the Nash equilibria of demand-side management non-cooperative games. These involve short-term optimization of demand to match supply, i.e., demand-response models. These algorithms can be based on stochastic games [Başar and Olsder, 1999].

ii) Studying the use of cooperative games for enabling a coordinated load management among the users that can, subsequently, lead to a more efficient load distribution and low costs on the utility operator. Such a model can be a coalition formation for the micro-grids integrated into the demand-side management of the power grid (i.e., applied at the user-level instead of the micro-grid or the smart grid level).

iii) Studying the application of Bayesian games [Başar and Olsder, 1999] (i.e., games with imperfect or incomplete information) to develop non-cooperative techniques that the consumers can use when little is known about other consumers’ behavior.

iv) Investigating the impact of privacy on demand-side management games.

8.1.2 Power management in sensor networks

Opportunistic selection of sensors

Distributed transmit beamforming can significantly improve the energy efficiency of sensor networks. However, this efficiency can be further improved by opportunistically choosing sensors to perform beamforming in different time slots [Choi, 2011]. This is due to the fact that as the channel conditions between the sources and the receiver are subject to change from one time slot to the next, the participation of all cooperative sensors in a cluster are not always required. An example of this is when the channel condition is good, for signal transmission to achieve a target quality of service. Furthermore, an interesting research extension would be devising novel signal processing techniques in order to opportunistically select the particular cooperative sensors so as to reduce the usage of energy in signal transmission by sensors.
Application of game theory in data funneling

Due to dynamic channel conditions, transmitted signals in data funneling may be required to choose the transmission path dynamically. In this regard, incorporation of dynamic game theory can play a significant role in finding the optimal funneling path. Moreover, when the clusters of sensors are mobile the location of the clusters are another important aspect which must be addressed in modeling data funneling techniques. Stochastic [Başar and Olsder, 1999] and facility location games [Pan et al., 2010] could be two very interesting signal processing tools for addressing these issues.

Self-organization of networks

Beyond transmission techniques and routing protocols, means for organization of sensor networks can play a significant role in increasing energy efficiency of complete networks. This is due to the fact that sensors, like any other smart nodes, have the capability of forming self-organizing-networks (SONs), and hence they can use advance signal processing tools to organize themselves so as to maximize energy efficiency. In this context, it would be interesting to apply advanced signal processing techniques such as applying Memetic algorithms [Fasolo et al., 2007] and coalition game theory [Saad et al., 2009b] for self-organization of sensor network so as to reduce the overall system’s cost in terms of energy usage.

Dynamics of beamforming gain

Another important extension of applying distributed transmit beamforming would be to investigate the combined effect of moving speed and direction, at the same time as distance, on beamforming gain. In addition, how to track and adjust the beamforming vector when the receiver is mobile would be a valuable investigation. The effect of delay of feedback in this scenario also needs to be considered.
Bibliography


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