Quantum measurements of spatial conjugate variables: Displacement and tilt of a Gaussian beam

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We consider the problem of measurement of optical transverse profile parameters and their conjugate variable. Using multi-mode analysis, we introduce the concept of detection noise-modes. For Gaussian beams, displacement and tilt are a pair of transverse profile conjugate variables. We experimentally demonstrate their optimal encoding and detection with a spatial homodyning scheme. Using higher order spatial mode squeezing, we show the sub-shot noise measurements for the displacement and tilt of a Gaussian beam.

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Quantum information protocols rely on the use of conjugated variables of a physical system for information encoding. For single mode continuous variable systems, there are only a very limited number of choices for such conjugate variable pairs. For example, phase and amplitude quadrature measurements with a balanced homodyne detector, or polarization Stokes parameter measurements using polarization discriminating detectors,[1] are the common conjugate variables used experimentally.

By not restricting ourselves to single mode analysis, we can use the ability of a laser beam to transmit high multimode information by extending these protocols to the transverse spatial domain. The transverse profile of the beam is then described by a set of orthonormal modes that potentially allows a parallel treatment of information. Recently this parallel processing scheme was used in single photon experiments to extend *q-bits* to *q-dits* using modes with higher angular momentum.[2]

Such an improvement requires the perfect matching of the detection system to the spatial information contained in the light beam. Indeed, we have shown that a single detector extracts information from only one specific transverse mode of the beam.[3] We call this mode the *noise-mode of detection* since it is the only mode contributing to the measurement noise. As a consequence, information encoded in any other mode orthogonal to the detection noise-mode is undetected. Moreover, noise-modes of detection are the transverse spatial modes whose modulation in magnitude is transferred perfectly to the detected output as a photocurrent.

The use of classical resources sets a lower bound to detection performances, which is called the quantum noise limit (QNL) and arises from the random time arrival of photons on the detector. In the case of displacement measurement of a laser beam, the transverse displacement d_{QNL} of a TEM$_{00}$ laser beam corresponding to a signal to noise ratio of 1, is given by $d_{QNL} = \frac{w_0}{2\sqrt{N}}$, where w_0 is the waist of the beam, and N its total number of photons in the interval $\tau = 1/RBW$, where RBW is the resolution bandwidth.[4] Note that the ability to resolve the signal relative to the noise can be further improved by averaging with the spectrum analyzer, by reducing the video bandwidth (VBW) and thus increasing the number of photons detected in the measurement interval, if the system has enough stability. For a 100 μm waist, 1 mW of power at a wavelength of $\lambda = 1\ \mu m$, with $RBW = 100\ kHz$ and $VBW = 100Hz$, the quantum noise limit is for instance given by $d_{QNL} = 0.2\ nm$, and the minimum measurable transverse displacement is $d_{min} = 7\ pm$.

In order to achieve a measurement sensitivity beyond the QNL, it is a necessary and sufficient condition to fill the noise-mode of detection with squeezed light.[3] As required by commutation relations, a measurement of the conjugate variable shows excess noise above the QNL.

In this paper, we first explain how spatial information can be encoded onto a beam, and how optimized measurement of spatial properties of a beam can be achieved classically. As an example, we use the displacement and tilt of a Gaussian laser beam[5] (which are two spatial conjugate variables) to show the quantitative results of signal-to-noise-ratio (SNR) measurements that surpass the quantum noise limit.

Encoding information in the transverse plane of a laser beam can be achieved by modulating any of its scalar parameters p around a mean value p_0. This parameter can correspond to any deformation of the transverse profile, such as displacement and tilt, which are properties easy to visualize and to use in practice. In the simple case of a TEM$_{00}$ mode, the parameterized beam can then be written in the general form by considering the first order Taylor expansion for small modulations $(p - p_0)/p_0 \ll 1$

$$u_{00}(p) \approx u_{00}(p_0) + (p - p_0)\frac{\partial(u_{00})}{\partial p} \qquad (1)$$

where we have used u_{ij} to denote the TEM$_{ij}$ Hermite

Gauss modes and $u_{ij}(p)$ denotes the same mode that experienced the modification induced by p. Specifically, a transversely displaced and tilted beam along the x-direction is given by

$$u_{00}(d) = u_{00} + d\frac{\partial(u_{00})}{\partial x} = u_{00} + \frac{d}{w}u_{10} \quad (2)$$

$$u_{00}(\theta) = u_{00} + \theta\frac{\partial(u_{00})}{\partial \theta} = u_{00} + i\frac{\pi\theta w}{\lambda}u_{10} \quad (3)$$

where d, θ, and w are the displacement, tilt, and waist diameter of the beam in the plane of observation, respectively.[5] These expressions show that small displacement information of a Gaussian beam is encoded in the amplitude quadrature of the co-propagating TEM_{10} mode. Whilst small tilt modulation is directly coupled to the phase quadrature of the TEM_{10} mode.

In order to extract this spatial information out of the modulated beam, let us consider the example of homodyne detection. This device selects the particular mode of the incoming beam which is matched to the local oscillator transverse profile. Thus, the detection noise mode is the one imposed by the local oscillator. By changing the transverse distribution and phase of the local oscillator Φ_{LO}, one can, at will, tune the noise mode of detection, to any spatial information of the incoming beam. In addition, by squeezing the noise-mode of the incoming beam, one can improve the measurement sensitivity. This apparatus, which we call a spatial homodyne detector, is therefore a perfect tool for multi-mode quantum information processing.

In the case of small displacement and tilt measurement, a homodyne detector with a TEM_{10} local oscillator can measure the TEM_{10} component of an incoming beam with up to 100% efficiency. Hence, the detector precisely matches the displacement and tilt conjugate observables of a TEM_{00} incident beam. A TEM_{10} spatial homodyne detector, as shown on (Fig. 1), is in this sense an optimal small displacement and tilt detector. Note that this scheme is not only more efficient by 25% than the conventional split detector to measure a displacement,[6] it is also sensitive to tilt, which is not accessible in the plane of a split detector.

As the TEM_{10} mode is the noise-mode of the spatial homodyne detector, we can experimentally improve the detection sensitivity by filling the TEM_{10} mode of the input beam with squeezed light. This non-classical beam is produced with an optical parametric amplifier (OPA), that emits 3.6 dB of vacuum squeezing in the TEM_{00} mode at 1064 nm (note that a ring cavity - not represented on the diagram - spatially filters the laser beam to a pure TEM_{00} mode used as the main TEM_{00} mode, as well as produces a shot noise limited beam for frequencies greater than 1 MHz used to seed the OPA). A phase mask converts the vacuum squeezed beam into a TEM_{10} mode, with an efficiency of 80%,[7] which brings the squeezing level in the TEM_{10} down to 2 dB. This vacuum squeezed TEM_{10} beam is combined, with less than 5% losses, with the main bright TEM_{00} beam, by means of a modified Mach-Zehnder interferometer.[4] This beam interacts with a PZT actuator that induces simultaneously displacement and tilt at RF frequencies (4 MHz). Note that the relative amount of tilt and displacement is fixed here by the characteristics of the actuator. This beam is analyzed with a homodyne detector, whose TEM_{10} local oscillator beam is produced via a misaligned ring cavity resonant for the TEM_{10} mode. Note that the mode matching between these two beams is achieved in a preliminary step by measuring a fringe visibility of 97% between the bright TEM_{00} mode, and the TEM_{00} mode generated when the cavity is locked on resonance for the TEM_{00} mode instead of the TEM_{10} mode.

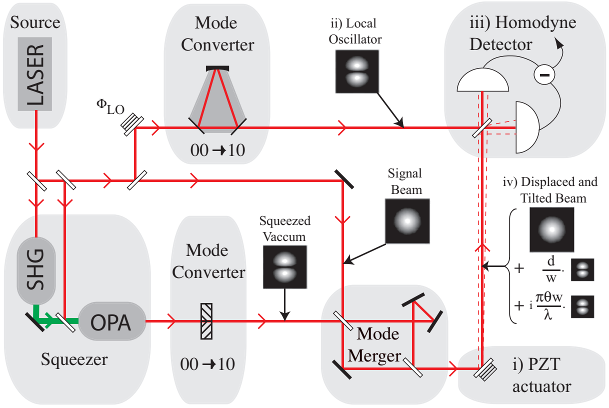

Fig. 1. Schematic diagram of the experiment for optimal displacement and tilt measurements with a spatial homodyne detector. A TEM_{00} mode, which is displaced and tilted using a PZT actuator (i), is mode-matched to the TEM_{10} local oscillator (ii) of a balanced homodyne detector (iii). The TEM_{10} local oscillator selects the quadratures amplitude of the TEM_{10} component of the input beam that contains the small displacement and tilt information of the incident TEM_{00} beam (iv).

The experimental results are presented in Fig. 2(a) and Fig. 2(b), when the TEM_{10} local oscillator phase is scanned and locked for displacement ($\phi_{LO} = 0$) and tilt ($\phi_{LO} = \pi/2$) measurement. Note that without the use of squeezed light, the displacement modulation is masked by quantum noise. Improvement of the signal-to-noise ratio for displacement measurement beyond the quantum noise limit is achieved when the squeezed quadrature of the TEM_{10} mode is in phase with the displacement measurement quadrature (i.e. in phase with the incoming TEM_{00} mode). Since we are dealing with conjugated variables, improving displacement measurement degrades the tilt measurement of the same beam, as required by the anti-squeezing of the other quadrature. Displacement measurement is improved by the 2 dB of squeezing, whereas the tilt measurement is degraded by the 8 dB of anti-squeezing. Theoretical curves calculated with 2 dB of noise reduction, and 90 % of tilt modulation and 10 % of displacement modulation - continuous

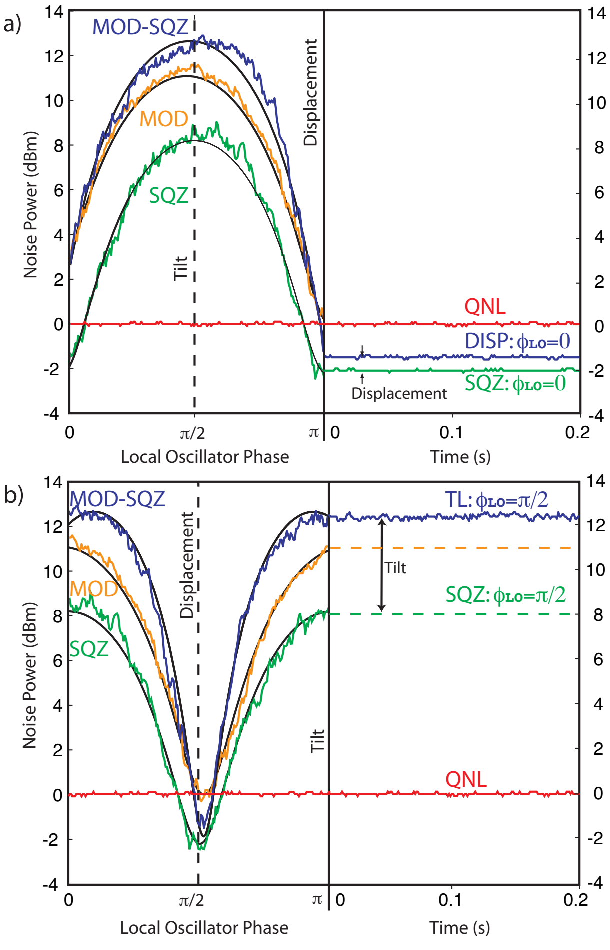

Fig. 2. Demonstration of sub-shot noise measurements of (a) displacement and (b) tilt modulations using the spatial homodyne detector. The figures show an example where there was 90% of tilt, and only 10% of displacement modulations. Left hand side of figures shows the scanning of local oscillator phase $\phi_{\rm LO}$ that continuously access the pure displacement (at $\phi_{\rm LO} = 0$ and π) to pure tilt (at $\phi_{\rm LO} = \pi/2$ and $3\pi/2$) information of the beam. QNL: quantum noise limit. SQZ: quadrature noise of squeezed light with 2dB of squeezing and 8dB of anti-squeezing on the TEM$_{10}$ mode. MOD: measured modulation with coherent light. MOD-SQZ: measured modulation with squeezed light. Right hand side of figures shows the corresponding locked local oscillator phase to the (a) displacement or (b) tilt measurement. SQZ: at $\phi_{LO} = 0$ is and the squeezed noise level 2 dB below the shot noise and at $\phi_{LO} = \pi/2$ is 8 dB of anti-squeezing noise. DISP: $\phi_{LO} = 0$ displacement measurement. TL: $\phi_{LO} = \pi/2$ tilt measurement. Displacement measurement is improved by the 2 dB of squeezing, while the tilt measurement is degraded by the 8 dB of anti-squeezing.

curves on Fig. 2(a) - are in very good agreement with experimental data. In our experiment, we have a TEM$_{00}$ waist size of $w_0 = 106\ \mu m$ in the PZT plane, a power of 170 μW, $RBW = 100\ kHz$ and $VBW = 100\ Hz$, corresponding to a Quantum Noise Limit of $d_{QNL} = 0.6\ nm$. The measured displacement lies 1.5 dB below the shot noise, yielding a value of 0.4 nm. The ratio between displacement and tilt modulations can be inferred from the theoretical fit in figure 2, giving a measured tilt of $10^{-7}\ rad$.

We have found and demonstrated a technique for encoding and extracting CW quantum information on multiple co-propagating optical modes. We use spatial modulation as a practical technique to couple two transverse modes and have devised a detection system whose noise mode perfectly matches beam position and momentum variables. We have performed quantum measurements of a pair of conjugate transverse variables, and have verified our predictions with experiments that show the detection of a displacement modulation below the quantum noise limit.

This work shows that in principle a large set of orthogonal multi-mode information is accessible. We can already simultaneously encode and detect information in x and y directions,[8] which corresponds to a simultaneous use of TEM$_{10}$ and TEM$_{01}$ modes. The possible extension to array detectors and higher order spatial modes will be investigated. This technique, demonstrated here in the context of quantum imaging, leads to the feasibility of parallel quantum information processing.

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