

PART I.

OBJECTIVES OF THE STUDY, AND
AN ANALYSIS OF LINEAR MEASUREMENT.

This thesis comprises four parts. Part I presents the problem. Part II reviews relevant literature. Part III describes an empirical study and presents a statistical analysis of the results. Finally, Part IV presents an interpretation of these results.

CHAPTER 1.

OBJECTIVES OF THE STUDY.

Acquisition of the number concept, and attaining skill in using numerical operations, are considered important intellectual achievements of childhood, because they enable the child to use mental operations in lieu of physical actions. For example, when the child is able to perform addition operations, he no longer needs to count physically. Similarly, after the child has acquired an understanding of linear measurement, he no longer has to align two objects physically, in order to find out which is the longer.

Psychological research has contributed substantially to an understanding of how the number concept, and numerical operations, develop. In comparison, however, much less research effort has been expended on finding out how numerical knowledge is linked with knowledge in other concept domains, such as length and distance. One way of examining such linkages between concepts is to study the development of an activity which draws upon knowledge associated with each concept. The development of linear measurement provides an example.

The present study had two main objectives. The first was to identify the 'higher-level' knowledge necessary for a child to understand linear measurement. The second was to chart the growth of linear measurement in terms of the development of its components. In this context, 'higher-level' knowledge refers to skills such as counting an array of objects, as distinct from 'lower-level' skills such as attending to an object in an array.

A major empirical study was carried out to meet these objectives. The main contribution of the thesis is in this empirical work.

CHAPTER 2.

AN ANALYSIS OF LINEAR MEASUREMENT.

2.1 SELECTION OF AN APPROACH.

2.1.1 TWO POSSIBLE APPROACHES.

One objective of the present research was to chart the development of linear measurement in young children. It was necessary to decide whether the investigation would focus on linear measurement at a 'concept level', or at a 'component level'.

2.1.2 CONCEPT-LEVEL APPROACH.

In a concept-level approach, the researcher would focus on the development of different levels of performance in the concept being studied. In the present case, he would define operationally a series of levels of achievement in linear measurement. For example:

- (a) an ability to determine by direct comparison which of two objects is the longer;
- (b) an ability to use correctly a measuring rule;
- (c) an ability to iterate a unit length; and,
- (d) an ability to predict the effect of changing the length of the of the unit.

The researcher would devise tests appropriate to each level. These would be administered to a selected population, and subjects categorised accordingly. The statistical analysis would involve correlating conceptual levels with developmental variables, such as age.

2.1.3 COMPONENT-LEVEL APPROACH.

The component-level approach extends the concept-level approach by also investigating the development of knowledge components of the concept being studied. In the present case, the researcher would devise tests for each component, as well as for different levels of performance, in linear measurement. An example of a component would be knowing how to count. In this approach, concept development would be described in terms of both levels of achievement in the concept, and the progressive acquisition of its components.

2.1.4 ADVANTAGES AND DISADVANTAGES OF EACH APPROACH.

Each approach has advantages and disadvantages. The chief advantage of the concept-level approach is that the levels of the concept may be defined operationally. The main disadvantage is that empirical evidence yielded by a study of that kind would provide little information on how, in a step-by-step sense, the concept develops.

The chief advantage of the component-level approach is that it has the potential to contribute to an understanding of how the concept develops. Moreover, this approach links more readily with existing psychological theory, and its associated empirical data. The main disadvantage is that it involves making a number of a priori assumptions regarding the list of components.

2.1.5 CHOICE OF COMPONENT LEVEL APPROACH.

A motivation of the study was to find information useful in a practical sense in the field of education. In that regard, the component-level approach is the more useful, because it provides information on which components are necessary for linear measurement and, hence assists primary school curriculum development. It was decided to adopt the component-level approach.

Additionally, with this practical purpose in mind, it was decided to restrict the investigation to children in the first two years of primary school, because it is probable that it is during those years that most of the growth in understanding of linear measurement occurs. Against this background, the identification of components of linear measurement will now be discussed.

2.2 IDENTIFICATION OF THE COMPONENTS OF LINEAR MEASUREMENT.

2.2.1 METHODS OF LINEAR MEASUREMENT.

In general, a young schoolchild would know of two ways of measuring the length of an object. The first would be to use a measuring rule and to read off numbers which in some way would represent the object's length. The second would be to select a shorter object for use as a unit length, and to iterate that unit along the longer object, counting the number of iterations. Obviously, the notions underlying the first method correspond to those employed in the second. It would be possible, however, for

a child to use the first method - because it had been trained to - without having an understanding of the underlying principles. Similarly, it would be possible for another child to use the second method - iteration of a unit length - without that child knowing what effect substitution of a longer or shorter unit length would have on the number of iterations required to measure the object. Yet another child might know the effect of substituting a longer or shorter unit length but, when asked to work out the combined length of two previously measured objects, that child might need to join the two objects together physically, and then measure their combined length. It could be argued that such a child did not understand that numerical operations of addition and subtraction can be used instead of measurement operations.

Each of these three children could be said to have a different level of understanding of linear measurement. Intuitively, it would seem reasonable to conjecture that these different levels of understanding of linear measurement result from differences in, or co-ordination between, the number and length concepts of such children.

2.2.2 UNIT ITERATION AND THE LENGTH CONCEPT.

An examination of the operations involved in the act of measuring the length of an object by unit iteration reveals many of the components of linear measurement. In the following discussion the word 'measurer' refers to a person who has a full understanding of iterative linear measurement. By marking off sections of the object being measured (A) into segments (A₁, A₂, A₃, etc.) equal in length to the unit (B), the measurer implies that he holds the following beliefs regarding length:-

- (a) the length of B remains constant throughout the measurement operation, even though the orientation of B to the measurer might change;
- (b) because the length of A1 is equal to the length of B, and the length of A2 is equal to the length of B, the length of A1 must be the same as the length of A2;
- (c) the length of an object is the same as the length of its concatenated parts.

2.2.3 THE CONSERVATION OF LENGTH.

The first of these beliefs (a), involves several components. For example, the measurer must believe that the length of any object does not change when it is displaced in space. Similarly, the measurer must believe that the relation of equivalence of length that holds between B and A1 when B is momentarily aligned with A1, does not change when B is not longer so positioned. These beliefs would seem to rest on the belief that a relation between two lengths can only change when something is added to, or taken away from, one, or other, or both, of the two lengths involved. Beliefs of this kind are usually referred to as the conservation of length.

It seems plausible that, prior to achieving an understanding of linear measurement, the child must know that the length of an object, and length relations between objects, are conserved under various kinds of transformation. However, how can the child acquire the conservation of length without being able to empirically verify it by linear measurement? For example, the child might know that length P is equal to length Q in

position R. How does the child know that P is still equal to Q when P has been moved to R'? Obviously, one answer would be that the child uses visual evidence that nothing has been added or subtracted from P and/or Q. But, how is this rule acquired without an understanding that length is measurable? Furthermore, P in its new position R' may well look longer or shorter than Q. One way around the problem would be for the measurer to move Q to R' so that the kind of direct comparison of P and Q made at R can be repeated at R'. However, how does the child know that Q does not change in length, to exactly the same extent as might have P, whilst being moved from R to R'? The knowledge that P equals Q at both R and R', does not necessarily ensure that P equals Q when Q is at R and P and R'.

The situation becomes more complex when the transformation of the measuring instrument (or unit) involves changing its shape, as distinct from changing its position. An example of such an instrument is a piece of string. How does the child know that the length of the string is the same whether it is placed in the form of a straight line or, say, a circle?

The preceding discussion suggests that there is an interdependence between conservation and measurement. However, for the moment, it will be taken as a working hypothesis that the child acquires the conservation of length before an understanding of linear measurement.

2.2.4 TRANSITIVE REASONING.

The second belief (b), is based upon what is usually known as 'transitive inference'. The length relation between A1 and A2 is inferred, rather than determined by direct comparison. The fact that this inference is made, means that the measurer also holds the conservation beliefs discussed above. The measurer's argument is: (A1=B in position 1) and (A2=B in position 2) implies (A1=A2). For this conclusion on the relative magnitude of A1 and A2 to be drawn, the measurer must believe that the length of B has not changed following its change in position. If the measurer did not hold this belief, then the premises contained in the transitive inference would not apply.

Equally, however, the conservation belief seems itself to imply a transitive inference. The argument is: (A1=B in position 1) and, (A2=B in position 2) implies (A1=A2), which implies (A1=B when B is in position 2).

2.2.5 PART/WHOLE RELATIONS OF LENGTH.

The third belief (c), concerns the measurer's understanding of the distinction between an object and one of its attributes - length. That is, a child may know that an object may be arbitrarily divided into parts, and those parts recombined to form the object. However, the child may not extend that knowledge to encompass the object's length. If that knowledge does extend to length, then it would find expression in such beliefs as: if length A is greater than length B then length A may be considered as length B concatenated with some other length.

2.2.6 UNIT ITERATION AND THE NUMBER CONCEPT.

In addition to beliefs concerning length, the measurement operation also requires the measurer to hold certain beliefs concerning, and to have certain skills with number. Firstly, the measurer must be able to 'numerate': that is, to co-ordinate ordinal position and cardinal value. Secondly, the measurer must believe that the numerosity of a collection of objects remains unchanged when the spatial arrangement of the collection is altered. For example, the measurer must believe that an object found by iteration to contain, say, six unit parts, will always contain six unit parts irrespective of the spatial location of the object. This belief is usually referred to as the conservation of number.

2.2.7 UNIT ITERATION AND INTER-CONNECTION OF LENGTH AND NUMBER CONCEPTS.

In addition to having certain beliefs concerning length and number, the measurer must also have some degree of interconnection of those beliefs. For example, a child may conserve number and length, and may be able to numerate, but unless there is some connection between that child's number and length concepts it is unlikely that the child could understand linear measurement. Similarly, if the measurer has a mature understanding of linear measurement, then he would also have an ability to perform arithmetical operations of addition and subtraction in lieu of measurement operations. For example, the difference in length of two objects can be determined by subtracting one numerical length measurement from the other, and thereby obviating the need to align both objects and measure the difference directly.

2.3 DEVELOPMENTAL PERIOD COVERED.

In any developmental study it is necessary to establish a point at which to start - a lower boundary - and a point at which to finish - an upper boundary. These decisions determined the age range of the subjects used in the study, and the levels of task difficulty employed.

The lower boundary selected for the present study was: determination by direct comparison of ordinal length relations between two objects. This study, therefore, has excluded from consideration a host of perceptual skills and cognitive attainments achieved in the early years of life. Some of those attainments seem to be linked with estimation processes that appear to be perceptually-based. For example, very young children appear to have some idea of what constitutes a distance. They can determine whether an object within view can be grasped without having to move any part of their bodies other than arms or shoulders, or whether an object can be reached without using something to stand on. It is quite possible that processes of that kind could either inform, or confuse, the five or six year old child who is in the course of developing more precise, conceptually-based skills for determining length and distance. However, the contributions made by such perceptual processes to the development of the linear measurement knowledge of young children are outside the scope of the present research.

The upper boundary selected for the present study was measurement of straight line lengths of small magnitude (approximately 30 cms.), and of distances between objects (of similar separation)

located on a plane, not a curved surface. This study, therefore, does not follow the development of linear measurement through to its completion. For example, a child may be able to measure correctly a stick 20 cms in length, but not understand how to measure, or what it would mean to measure, the length of a piece of curved plastic pipe. The former ability is within the scope of this study, the latter is not. Similarly, a child may be able to measure the separation between objects located on his play-room table, but not understand what it would mean to measure the distance between his home and his school. This study is concerned with the former, but not with the latter ability.

2.4 LIST OF COMPONENTS OF LINEAR MEASUREMENT. (1)

The following is a list of the components required for a full understanding of iterative linear measurement, as suggested by the foregoing discussion. It will be evident that the components listed are not independent. This matter will be discussed in later paragraphs.

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1. There is a widely cited formal theory of measurement due to Suppes and Zinnes (1963). The theory formally demonstrates that the empirical relational system of measurement of length or distance is an isomorphic image of a particular numerical relational system, the real number system. It is this isomorphism that is the formal basis of everyday activities of, for example, applying arithmetical operations such as addition and subtraction to length measurement.
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(A) NUMBER. The following assumes that the child has a number concept though it may be in the early stages of development.

- (i) Knowing how to use a 1-to-1 matching rule. This is necessary because such a rule is implicit in counting and in unit iteration.
- (ii) Knowing the natural number order. This is necessary because each 1-to-1 pairing during unit iteration must be identified separately, as the first, second, third, etc.
- (iii) Knowing how to count arrays of small numerosity, where 'count' implies the co-ordination of ordinal position and cardinal value. This is necessary because the number of unit iterations must be determined.
- (iv) Knowing how to make transitive inferences of equivalence and non-equivalence with respect to discrete quantity. This appears to be necessary for the conservation of number.
- (v) Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number). This is necessary to relate a collection of 'n' non-contiguously arranged unit parts to a collection of 'n' contiguously arranged unit parts.
- (vi) Knowing how to perform the arithmetical operations of addition, subtraction, solving for a difference, and balancing numerosities. This is necessary if arithmetical operations are to be used in lieu of measurement operations.

(B)LENGTH. The following assumes that the child has a concept of length, as an attribute of an object. That concept may be incomplete, in that not all of the properties of length may be known to the child.

- (i) Knowing that if length A is greater than length B then A may be considered as B concatenated with some other length. This is necessary for a unit length, (B, in this case) to be employed in measurement.
- (ii) Knowing that any length may be considered as a concatenation of arbitrarily selected sub-lengths. This is necessary because a precise statement of any object's length, expresses that length in terms of a number of object parts of shorter length joined together.
- (iii) Knowing that the length of an object can be altered only by adding something to it or subtracting something from it (setting aside, for present purposes, processes of expansion and contraction). This is necessary because the unit part changes position during measurement. However, as nothing is added to or taken away from it, its length remains constant.
- (iv) Knowing that the length relation between two objects can be changed only by adding to, or taking away from, one, or other, or both, of the objects (setting aside processes of expansion and contraction). This is necessary because the unit part changes position during measurement but, its length remains constant, and so also must the relation of equivalence between the lengths of the unit part and the object parts.

- (v) Knowing that the length relation between objects A and B does not change when the spatial relation between A and B changes. This is necessary because, during measurement, the unit part changes position, but the parts of the objects marked off as equal in length to that of the unit part do not. Hence, the length relations between them are constant.
- (vi) Knowing that objects may be ordered according to their lengths. This is necessary for transitive inference.
- (vii) Knowing that transitive inferences of equivalence and non-equivalence can be applied to length relations. This is necessary because it is implied in relating unit parts of an object to each other, so that an understanding is reached that those parts are equal in length.
- (viii) Knowing that the ordinal length relation between two objects is the same as the cardinal numerical relation between the collection of parts comprising those objects (provided that the lengths of these parts are the same). This forms the basis of the isomorphism between counting and measurement that enables arithmetical operations to be used in lieu of measurement (Suppes and Zinnes 1963).
- (ix) Knowing that length relations between objects can be deduced by applying transitive reasoning to the collections of unit parts. This is necessary because it is implied in comparing the lengths of objects by comparing the number of unit parts contained in them.

- (x) Knowing that length is invariant under certain transformations (the conservation of length). This is necessary because the accuracy of unit iteration depends upon the length relations between the unit part and the object parts remaining constant.

(C) LENGTH MEASUREMENT.

- (i) Knowing how to iterate a unit part along an object. This is necessary for the operation to be accurate - e.g. units must be marked off accurately and in a non-overlapping fashion.
- (ii) Knowing that if the length of the unit part is changed, the number yielded by unit iteration also changes. This is necessary because, although in linear measurement the measurer can arbitrarily choose a unit part, the answer given by unit iteration depends upon the length of that unit part.
- (iii) Knowing that the length relation between two objects can be determined by carrying out a linear measurement operation, using unit iteration. (The difference between this requirement and B(viii) above is that the latter refers to unspecified numerosity: that is, the B(viii) relation is expressed in terms of 'more' or 'less' or 'same', not in terms of precise numbers of parts, arithmetical comparison of which yields the answer.)

- (iv) Knowing that arithmetical addition of linear measurements may be used to determine the length of concatenated objects. This is necessary because a main purpose of linear measurement is the derivation of a number that may be used arithmetically in lieu of carrying out another measurement operation.

(D) DISTANCE. The following assumes that the child has a concept of distance as a spatial relation between two points.

That concept may be incomplete, because all of the properties of distance may not be known to the child. There is an isomorphism between the properties of length and the properties of distance (Suppes and Zinnes, 1963). Hence, the properties previously listed for length will not be listed again for distance, except in the case of conservation. This is mentioned again, because of the importance of conservation in psychological theory concerning concept development.

- (i) Knowing that distance is invariant under certain transformations (the conservation of distance). See comment against B(x).

(E) DISTANCE MEASUREMENT.

- (i) Knowing how to compare indirectly two distances by a measurement operation not involving unit iteration. This is necessary because distances cannot be compared by aligning their end points and making direct comparisons.

- (ii) Knowing how to measure distance between two points, using unit iteration. This is necessary because distance cannot be subdivided directly, as can length.

2.5 NON-INDEPENDENCE OF COMPONENTS OF LINEAR MEASUREMENT.

As previously mentioned, the components listed above are not independent. There is an assumption of an hierarchical arrangement in each concept domain, and of cross linkages between concepts. For example, in the number concept, component (ii) implies component (i), and component (iii) implies components (i) and (ii), together with other 'rules' not mentioned here (Gelman and Gallistel, 1978). Similarly, there are several different levels of arithmetical ability listed, each presumably based upon prior acquisition of less complex arithmetical abilities. All are included in the list because, at this stage, it is not known which are necessary for the demonstration of different levels of understanding of linear measurement. Similarly, for both number and length, distinctions are made between transitive inferences concerning relations of equivalence and transitive inferences concerning relations of non-equivalence (greater than, and less than). This is because it is not known whether only the former are involved in linear measurement - which would be suggested by a theoretical analysis - or whether both must be present. With respect to length concepts, distinctions are made between components concerned with the length of an object, and components concerned with the relations between lengths of objects. This is because it is not known what influence each component might exercise in the development of linear measurement.

Further, transitive inferences regarding length imply the conservation of length, and the conservation of length implies transitive inference. If an attempt were made to draw up a non-redundant list of independent components of linear measurement, it is not at all clear whether conservation and/or transitive inference should be included.

It may be possible to set down a minimal list of independent components of linear measurement. However, for the present study, such a list may not be as useful as a list of the kind given here. This is because the present study is concerned with charting the course of development of linear measurement in terms of the progressive emergence of its components. If only independent components -that is, components representing axioms in a linear measurement system - were studied, the emergence of other components derived logically, but not necessarily psychologically, from those axioms would not be detected.

2.6 NATURE OF THE EMPIRICAL QUESTIONS ASKED BY THE PRESENT STUDY.

2.6.1 WHICH COMPONENTS ARE NECESSARY FOR LINEAR MEASUREMENT?

The components given in the above list provided the framework within which the empirical part of the study was conducted. Various empirical questions relevant to the general issue of the development of linear measurement were framed in terms of those components.

The following operational definition of linear measurement was used as a benchmark: 'a child may be said to have a mature understanding of linear measurement, if he demonstrates a capacity to use correctly arithmetical operations instead of carrying out physical measurement operations'.

According to the preceding analysis, this would only be possible if the child possessed the knowledge listed. Hence, the first question is: is that analysis correct? In other words: are these components necessary for linear measurement?

2.6.2 IS THERE AN ORDER IN WHICH THE COMPONENTS EMERGE?

Developmental research shows that certain of these components emerge at different times in the child's thinking. Hence, other questions are: is there a specific order in which the components emerge? Is the development of the components a continuous or discontinuous function? What is the relationship between levels of achievement in linear measurement and the progressive acquisition of the components? Does development in one concept prompt development in the other?

These empirical questions involve consideration of the difference between a child knowing that arithmetical operations may be substituted for physical measurement operations, and a child understanding why arithmetical operations may be so used. It seems quite possible that a child could possess the former, but not the latter. On the other hand, the reverse would seem unlikely - that is, that a child could possess the latter but not the former. This distinction is conveyed in the terms sometimes used in connection with these two different kinds of knowledge. The former is

often referred to as algorithmic or rule-based (Gagne, 1968), while the latter is sometimes referred to as operational (Piaget, 1953). The present study is concerned with both kinds of knowledge. Some of the developmental precursors of operational knowledge - what might be called its components - could be expressed as algorithms or rules. However, the present study does not assume that operations are nothing more than a particular organisation of rules, or that rules 'grow' into operations.

PART II.

LITERATURE REVIEW.

On the basis of evidence from a number of sources it is possible to make predictions concerning the course of development of components of linear measurement. These sources are Piagetian theory, and a large body of empirical work carried out in that tradition.

Developmental questions concerning the components of linear measurement have been of major and long term interest to Piaget and his followers (Brainerd, 1978; Flavell, 1963). Consequently, all of the predictions outlined in Part II were derived directly from Piagetian theory, or from empirical research conducted within the Piagetian tradition. In this connection, it is noteworthy that, notwithstanding the vast amount of information emanating from Piagetian research and relevant to the questions posed in Part I, it is still entirely reasonable to ask those questions. This is so because of difficulties in interpreting the results of previous studies, and of collating the results of many different studies, each concerned with perhaps only one or two aspects of the general issue of the development of linear measurement.

As the work of Piaget and his followers provides the main theoretical framework for the present study, it is necessary to present briefly those aspects of Piagetian theory which are relevant to the present topic. Chapter 3 reviews this material and derives predictions. Following this, Chapters 4 to 7 review the empirical evidence for these predictions.

CHAPTER 3.

PIAGETIAN VIEW ON COMPONENTS OF LINEAR MEASUREMENT AND THEIR ORDER OF DEVELOPMENT.

3.1 PREDICTED ORDER OF DEVELOPMENT.

Piaget lists a number of abilities which he believes the child must possess before demonstrating an understanding of linear measurement.

They are:-

- . the ability to conserve number, length and distance;
- . the ability to make transitive inference judgements with respect to number and length;
- . the ability to use a unit of length for purposes of iteration;
- . the ability to carry out arithmetical operations of addition and subtraction (Piaget, Inhelder and Szeminska, 1960).

Piaget also believes that each of these abilities only emerges in the child's reasoning after he has mastered more basic skills, such as numeration, seriation of length, and understanding of part/whole relations. (Piaget, 1968).

Additionally, Piaget argues that both the high-order abilities (e.g. the conservation of number), and the more basic skills (e.g. numeration) emerge in a predictable order in the development of intelligence (Piaget, 1968).

The following summarises those predictions. The headings refer to aspects of Piaget's theory which are most directly responsible for the predictions which follow.

3.1.1 PARALLEL DEVELOPMENT.

- . The ability to conserve number emerges at about the same time as the ability to make transitive inferences with respect to discrete quantity;
- . the ability to conserve length emerges at about the same time as the ability to make transitive inferences with respect to length.

3.1.2 THREE SUB-STAGE MODEL.

- . The ability to conserve length emerges earlier than the ability to measure length;
- . the ability to conserve distance emerges earlier than the ability to measure distance;
- . the ability to perform the arithmetical operations of addition and subtraction emerges earlier than the ability to measure length or distance;
- . the ability to conserve number emerges at about the same time as the ability to perform the arithmetical operations of addition and subtraction;
- . the ability to seriate length emerges earlier than the ability to make transitive inferences with respect to length;
- . the ability to order discrete quantity emerges earlier than the ability to make transitive inferences with respect to discrete quantity.

3.1.3 HORIZONTAL DECALAGE.

- . The ability to conserve number emerges earlier than the ability to conserve length;
- . the ability to conserve length emerges at about the same time as the ability to conserve distance;
- . the ability to measure length emerges at about the same time as the ability to measure distance;
- . the ability to seriate length emerges earlier than the ability to numerate.

The abilities referred to in the above predictions cover nearly all the components of linear measurement listed in Chapter 2. It would seem, therefore, that Piagetian theory provides a framework which embraces virtually all of the empirical questions asked in this study. Consequently, it is necessary to present that theory briefly. In doing so, an attempt will be made to link Piagetian theoretical statements with the empirical aspects of the present research, so as to make clear the origin and status of the predictions given above.

As a preface, however, a caveat needs to be made explicit. The present research is not aimed at testing Piaget's theory. It is emphasised that Piagetian theory is consulted because it provides the richest source of relevant theoretical statements and empirical data.

3.2 OVERVIEW OF PIAGET'S THEORY OF COGNITIVE DEVELOPMENT.

3.2.1 NATURE OF THE THEORY.

Piaget's theory of cognitive development is structural, holistic, constructionist, and descriptive.

It is structural because it conceptualises "mental operations" as forming patterns that exhibit properties which change in the course of development. Development is seen primarily as a matter of change in cognitive structure.

It is holistic because it asserts that, as every cognitive act is related in some fashion to all other cognitive acts, an understanding of intelligence can only be gained by an understanding of its organisation as a total system. The total system and its component structural elements are said to change over time, and as a function of experience. Such change is believed to be directed by two broad principles, "organisation" and "adaptation". Because these principles do not change during development they are referred to as "functional invariants".

The theory is constructionist because it declares that, while experience permanently alters intelligence, intelligence modifies its own construction of reality in the process of interpreting it.

Finally, the theory is more descriptive than explanatory. Its structural and functional elements provide a way of classifying and charting ontogenetic development, rather than a system of explanations. Thus, the theory provides a rich and detailed account of the state of intelligence at various stages of development. However, it provides only general and exceedingly abstract principles to account for the processes at work in the formation of, and transition between, such states (Brainerd, 1978).

The following account of the theory is highly condensed and selective. However, it is only intended as a context within which a more detailed discussion can be presented of the period of concrete operations, because that is when the number, length and distance concepts under examination emerge.

3.2.2 COGNITIVE STRUCTURES.

The existence of cognitive structures is inferred from the person's behaviour. Thus, cognitive structure is an hypothetical construct. More specifically, Piaget regards cognitive structures as being systems of operations.

Piaget states:-

"Psychologically, operations are actions which are internalizable, reversible and co-ordinated into systems characterized by laws which apply to the system as a whole. They are actions, since they are carried out on objects before being performed on symbols. They are internalizable, since they can also be carried out in thought without losing their original character of actions. They are reversible as against simple actions

which are irreversible. In this way, the operation of combining can be inverted immediately into the operation of dissociating, whereas the act of writing from left to right cannot be inverted to one of writing from right to left without a new habit being acquired differing from the first. Finally, since operations do not exist in isolation they are connected in the form of structured wholes. Thus, the construction of a class implies a classificatory system and the construction of an asymmetrical transitive relation, a system of serial relations,"

(Piaget, 1953 b, p.8).

3.2.3 CONCEPT OF SCHEME.

Piaget's concept of "scheme" is related to these notions of structure. Schemes consist of sequences of actions. Structures consist of systems of operations.

Schemes are defined in terms of overt behaviour (Piaget & Inhelder, 1969;p4) Thus, Piaget talks of sensory-motor schemes of grasping, reaching, seeing, tasting, and so on. Schemes are said to change as a consequence of cognitive functioning.

3.2.4 COGNITIVE FUNCTIONS.

Development is seen mainly in terms of changes in cognitive structures. Changes occur as a result of experience, and are said to be always under the control of the two functional invariants, organisation and adaptation.

Organisation is the cognitive function Piaget holds responsible for the similarities that exist in intellectual behaviour at all levels of development. Adaptation provides the mechanisms responsible for the changes within cognitive structures. Hence, organisation and adaptation are complementary. The former ensures that the reorganisation of cognitive structures produces an ordered totality. The latter ensures that cognitive structures grow internally as elements of the total system, and that new and different kinds of relationships grow between these elements. Brainerd said:-

"The organization principle presumably is responsible for the organism's cognitive continuity across short or long periods of time. That is, cognitive organization accounts for the fact that there is some degree of sameness in intelligence across time. In contrast, the adaptive side of intelligence presumably is the chief instrument of discontinuity."

(Brainerd, 1978, pp.23).

The mechanisms used by adaptation to guide cognitive growth are assimilation and accommodation. Assimilation is the taking in of experience, and its interpretation by existing cognitive structures. Accommodation is the changing of those cognitive structures in such a manner as to make subsequent interpretations reflect reality more accurately. Hence, assimilation and accommodation are complementary. For Piaget, every cognitive act implies both mechanisms:-

"Accom^modation of mental structures to reality implies the existence of assimilatory (schemes) apart from which any structure would be impossible. Inversely, the formation of (schemes) through assimilation entails the utilization of external realities to which the former must accom^modate..."
 (Piaget, 1954, pp.352-353).

3.2.5 STRUCTURAL CHANGE.

Piaget has described these and other aspects of structural adjustment in terms of an equilibrium model. Expressed most simply, the model refers to a balance between assimilation and accom^modation, that leads to a state of "equilibrium" - one in which the cognitive structures are said to be equilibrated. This process of equilibration has been defined by Piaget (1972) as: "a compensation for an external disturbance" (p.120). When the system is not in equilibrium, it has a tendency to adjust itself continuously to move toward a state of equilibrium. Thus, it is a dynamic process leading to successively higher and higher levels of equilibration. Inhelder (1962) described it as:-

"a constant progression from a less to a more complete equilibrium and manifest therein the organism's steady tendency toward a dynamic integration. This equilibrium is not a static state, but an active system of compensations - not a final conclusion, but a new starting point to higher forms of mental development."

Equilibrium is central to Piaget's stage concepts. It accounts for the structural characteristics of invariant order, of acquisition, hierarchical inclusion, and overall integration that define a stage. It also accounts for the transition between stages as periods when intelligence is in a state of disequilibrium: that is, when its cognitive structures are poorly equilibrated. It also accounts for the changes that occur within a stage. In other words, it represents the organisational and adaptational principles which account for the continuity and discontinuity aspects of Piaget's stage theory (Brainerd, 1978).

3.3 STAGES OF DEVELOPMENT.

An important aspect of intellectual behaviour is that the nature of reasoning changes with age. Piaget uses behavioural data as evidence that intellectual development involves a progression through four distinct and major stages. Each stage is characterized by different reasoning. Piaget has argued that there are:-

"four great stages, or four great periods, in the development of intelligence: first, the sensory-motor period before the appearance of language; second, the period from about two to seven years of age, the pre-operational period which precedes real operations; third, the period from seven to twelve years of age, the period of concrete operations (which refer to concrete objects); and finally after twelve years of age, the period of formal operations, or propositional operations."
(Piaget, 1968 p356).

3.3.1 SENSORY-MOTOR STAGE.

During the sensory-motor stage, the child learns that the world is a permanent place, which may be explored by his senses, and by physical movement. Physical movements are co-ordinated, and are internalized into rudimentary cognitive structures. As these structures develop, the child's behaviour becomes more purposive and goal directed. From about 18 months onwards, language development is apparent, and simple symbolic behaviour appears. These two developments herald the emergence of the next stage.

3.3.2 PRE-OPERATIONAL STAGE.

When compared with its precursor, the pre-operational stage is one of vast growth in the child's capacity to reason. Consequently, it is difficult to summarise the pre-operational stage without conveying the impression that it is 'simply' a period during which the foundations are laid for the development of concrete operational thought.

The pre-operational stage is marked by the development of the "semiotic function":-

"At the end of the sensori-motor period, at about one and a half to two years, there appears a function that is fundamental to the development of later behavior patterns. It consists in the ability to represent something (a signified something: object, event, conceptual scheme, etc.) by means of a "signifier" which is differentiated and which

serves only a representative purpose: language, mental image, symbolic gesture, and so on. Following H. Head and the specialists in aphasia, we generally refer to this function that gives rise to representation as "symbolic." However, since linguists distinguish between "symbols" and "signs," we would do better to adopt their term "semiotic function" to designate those activities having to do with the differentiated signifiers as a whole." (Piaget and Inhelder, 1969, p.51)

During this stage the child's symbolic behaviour encompasses complex activities such as drawing, reading and writing:-

"In spite of the astonishing diversity of its manifestations, the semiotic function presents a remarkable unity. Whether it is a question of deferred imitation, symbolic play, drawing, mental images and image-memories or language, this function allows the representative evocation of objects and events not perceived at that particular moment. The semiotic function makes thought possible by providing it with an unlimited field of application, in contrast to the restricted boundaries of sensori-motor action and perception."
(Piaget and Inhelder, 1969, p.91)

3.3.3 CONCRETE OPERATIONAL STAGE.

The concrete operational stage is characterized by the child's ability to reason logically, provided that the task makes reference to concrete objects - though such objects need not be present. It is also necessary that any hypothesis testing involve only simple extrapolations or interpolations. It is during this stage that the child acquires his ordinal and cardinal concepts of number; develops his ability to argue transitively; exhibits a capacity to classify objects simultaneously on two or more dimensions; is able to handle class inclusion problems in logic; displays an understanding that spatial transformations of objects, or collections of objects, leaves certain properties of the objects, or collections, unaffected; demonstrates an understanding of projective and Euclidean geometry; and learns to apply mathematical concepts to a range of concretely-based problems, such as distance and length measurement.

The child's thinking, however, is still limited by a dependence on concretely based information; by an inability to carry out concurrently the two reversibility operations of negation and reciprocation - although they can be applied independently; and by severe limitations in his ability to control for the effects of variables in multi-variable situations.

3.3.4 FORMAL OPERATIONAL STAGE.

The formal operational stage represents the highest level of intellectual development, and marks the emergence of the ability to think about thinking. Thought is no longer confined to concretely-based information, no

longer restricted by the force of reality, but is free to generate possibilities and hypotheses whose only immediate referents are prior elements of cognition. Piaget said:-

"It is the power of forming operations of operations which enables knowledge to transcend reality."

(Piaget, 1972).

It is during this stage that the power of hypothetico-deductive reasoning can be used to gain full understanding of complex concepts in mathematics and science, and where proof of a proposition involves consideration of all possibilities, in isolation and in combination.

3.4 CONCRETE OPERATIONS.

3.4.1 LOGICAL AND INFRALOGICAL OPERATIONS.

A more detailed presentation of the stage of concrete operations is needed because most of the components of linear measurement listed in Chapter 2 appear in the child's reasoning during that stage. For example, it is then that the child demonstrates an ability to make transitive inferences, and to conserve number, length and distance.

Most of the abilities which emerge during the concrete operational stage fall into two broad categories. They are those concerned with: (a) relations between objects; and, (b) with relations within objects (Piaget and Inhelder, 1969). The former involve logical operations, and the latter, infralogical operations. The distinction is based upon the kinds of in-

formation each provide. Logical operations are concerned with information about collections of objects, and are independent of spatio-temporal location. Infralogical operations bear upon objects, and their parts. The logical operation of dividing a class of objects into a number of sub-classes is, analogous to the infralogical operation of sub-dividing a length into component elements. However, they are different in important respects. The former does not require that the elements of the class or of the sub-classes be in spatial or temporal proximity and, so, is called a logical operation. The latter does require that the elements of the whole (the length) be in spatial proximity, and, hence, is termed an infralogical operation.

3.4.2 GROUPING AND GROUP STRUCTURES.

Logical and infralogical operations form components of the cognitive structures of the concrete operational stage. The principles under which operations in the logical and infralogical domains combine may be stated in the form of axioms. Those axioms bear a close resemblance to the logical laws that define two particular mathematical structures: groups and lattices. Consequently, Piaget has employed logico-mathematical models to describe the organisation of concrete operational thought.

Piaget called his structures "groupings", if they modelled systems which embodied laws of mathematical groups and lattices. The structures were called "groups", if the systems they modelled reflected only the laws of mathematical groups.

3.4.3 TYPES OF GROUPING STRUCTURE.

Piaget posited eight major groupings:-

"This grouping structure is found in eight distinct systems, all represented at different degrees of completion in the behaviour of children of 7-8 to 10-12 years of age, and differentiated according to whether it is a question of classes or relations, additive or multiplicative classifications, and symmetrical (or bi-univocal) or asymmetrical (co-univocal) correspondences:

		Classes	Relations
Additives	Asymmetrical	I	V
	Symmetrical	II	VI
Multiplicatives	Co-univocal	III	VII
	Bi-univocal	IV	VIII"

(Beth and Piaget, 1966, p.174).

Examples of the behaviours associated with these groupings are given below:-

<u>GROUPING</u>	<u>BEHAVIOUR.</u>
I Primary addition of classes	Addition and subtraction of classes; class inclusion.
II Secondary addition of classes	Ability to classify a set of objects in several different ways.
III Bi-univocal multiplication of classes	Ability to find the intersect of two or more classes.
IV Co-univocal multiplication classes.	Ability to set two series of classes in one-to-many correspondence.
V Addition of asymmetrical relations	Ability to construct a transitive asymmetrical series.
VI Addition of symmetrical relations.	Ability to deduce symmetrical relationships from a geneological hiérarchy.
VII Bi-univocal multiplication of relations.	Ability to set in 1-to-1 correspondence elements of two asymmetrical series.
VIII Co-univocal multiplication of relations.	Ability to find the result of multiplying a symmetrical relation and an asymmetrical relation of the kind found in geneological hierarchies.

Flavell (1963) succinctly expressed the connection between these groupings and behaviour:

"thus, if Piaget says that the classificatory behaviour of the eight year old indicates that he possesses the grouping of logical class addition, he means the child's thought organisation in the classificatory area has formal properties (reversibility, associativity, composition, tautology, etc.) very like those which define this logico-algebraic structure. The latter has certain specific and definable system properties; we infer from his behaviour that the child's cognitive structure has similar properties." (Flavell, 1963, p.169).

Reflecting the distinction between logical and infralogical operations, Piaget has argued that each of the groupings of logical operations has its infralogical counterpart. Thus, there is a grouping of infralogical operations corresponding to grouping 1 for logical operations. However, in the case of the former, the operations relate not to the act of combining and dissociating classes, but to the act of dividing a whole (say, a length) into its constituent parts, and combining those parts to reconstitute the whole.

3.4.4 TYPES OF GROUP STRUCTURE.

In addition to these grouping structures, Piaget posits two further structures, called groups. The two groups are: (a) the additive group of positive and negative whole numbers; and, (b) the multiplicative group of whole or fractional positive numbers. Just as there are groupings in the domain of logical operations, and in the domain of infralogical operations, there are groups in each domain.

Moreover, these groups are said to come out of a synthesis of grouping structures: the additive group from a synthesis of class addition and addition of asymmetrical relations; and, the multiplicative group from a synthesis of class multiplication and multiplication of symmetrical relations. In other words, understanding of number implies an understanding of the cardinal (how many), and ordinal (ordered series), aspects of the concept. Piaget (1952) asserted, in respect of the additive group of numbers, that:-

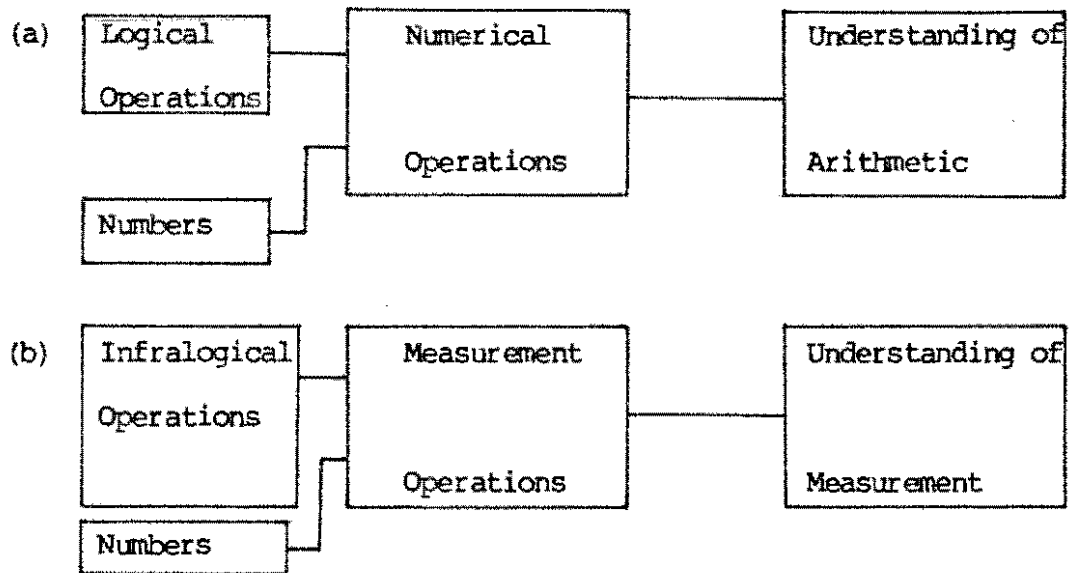
"..... class, asymmetrical relation and number are three complementary manifestations of the same operational construction applied either to equivalences, differences or to both together. It is, in fact, when the child's intuitive evaluations have become mobile and he has therefore reached the level of the reversible operation, that he becomes capable of inclusions, seriations and counting.

".....class and number are mutually dependent, in that while number involves class, class in its turn relies implicitly on number.

".....number is at the same time a class and an asymmetrical relation." (Piaget, 1952, p. 184).

3.4.5 QUANTIFICATION.

The grouping structures in the domains of logical and infralogical operations permit what Piaget calls "intensive" quantification to be performed. The group structures permit "extensive" quantification to be carried out. Intensive quantification enables judgements such as bigger than, more than, longer than, etc. to be made. Extensive quantification enables such judgements to be more precise, by expressing how much bigger, how many more, how much longer, etc. Piaget also argues that, when the elements of the logical grouping structures (that is, logical operations) become connected to the elements of the numerical group structures (that is, numbers), the result is numerical operations. At that time, the child becomes capable of understanding arithmetic. Similarly, when the elements of the infralogical grouping structures (that is, infralogical operations) become connected to the elements of the numerical group structures, the result is measurement operations. At that time, the child becomes capable of understanding the nature of measurement. Diagrammatically, the argument may be summarised as follows:



3.5 PARALLEL DEVELOPMENT HYPOTHESIS.

The structures of the concrete operational stage are linked developmentally, as well as logically. It is argued that all of these structures develop contemporaneously, and emerge in parallel, as distinct from emerging sequentially. This is sometimes termed the "parallel development hypothesis". (Brainerd, 1978: p.86).

The parallel development hypothesis is important in the context of the present research. It predicts that the ability to use transitive reasoning with respect to discrete quantity - which marks the completion of grouping V (addition of asymmetrical relations) - appears in the child's behaviour at about the same time as the ability to conserve number - which marks the synthesis of groupings I (primary addition of classes) and V. Similarly, in the domain of infralogical operations, the parallel development hypothesis predicts that the ability to make transitive inferences with respect to length emerges at about the same time as the ability to

conserve length. It will be recalled from Chapter 2 that knowing how to make transitive inferences with respect to number and length, and knowing of the conservation of number and length, were assumed to be necessary for a child to be able to measure length. Consequently, the above two statements of developmental synchrony form the first two predictions of the order of emergence of components of linear measurement.

3.6 THREE SUB-STAGE MODEL.

Piaget also maintains that the structures do not emerge all of a sudden, when the child enters the concrete operational stage. Instead, he identifies, generally, three sub-stages in the development of each structure. As with his stage concept, he maintains that all children must pass through each sub-stage in a fixed invariant order. Moreover, Piaget also maintains that children move synchronously through these sub-stages. Thus, a child at sub-stage 1 in class concept development should also be at sub-stage 1 with respect to relations and number.

The three sub-stage model of development is relevant to the present research, because Piaget is most explicit regarding the necessary components of linear measurement when discussing sub-stage growth. Also the model yields predictions concerning the order of emergence of some of the higher-order abilities assumed to be necessary for linear measurement. For example, the model posits developmental asynchronies between the conservation and the measurement of length, and also between the emergence of proficiency in numerical operations and the demonstration of an understanding of measurement operations. Additionally, it posits dev-

developmental dependencies between certain of the more basic skills, such as length seriation, and higher order skills such as transitive inference reasoning. In order that predictions of that kind may be seen in an appropriate theoretical context, a brief account of the three sub-stage model follows.

3.6.1 CLASSES.

The growth of class logic is relevant because, in Piagetian theory, the emergence of an operational grasp of number marks the synthesis of logical groupings I and V.

Piaget uses three kinds of task to assess a child's progress through the posited three substages of development. They are, in increasing order of complexity: (a) classification; (b) multiple classification; and, (c) class inclusion.

In the classification task, the child is asked to sort collections of objects on one dimension only - e.g. to sort a collection of beads of differing size and colour on colour only. In the multiple classification task, the child is asked to sort the collection on two dimensions simultaneously - e.g. to sort on size and colour. In the class inclusion task, the child is presented with the information that class A, say green plastic buttons, is contained in class B, say green and blue plastic buttons, and asked if there are more green buttons (assume there are more greens than blues) than plastic buttons. Correct performance on the class inclusion task represents achievement of the class concept, and attainment of the related grouping structures. Progression through these tasks is summarised in the following:-

<u>TEST</u>	<u>SUB-STAGE 1</u>	<u>SUB-STAGE 2</u>	<u>SUB-STAGE 3</u>
Classification.	Fails. Groups objects into spatial configurations. Called graphic collections.	Passes. Will sort objects into two or more mutually exclusive categories.	Passes
Multiple Classification.	Fails. Produces graphic collections.	Partial success. Will sort on two dimensions successively but not simultaneously.	Passes
Class Inclusion	Fails. If A is contained in B, child asserts $A > B$.	Fails. Any success is matter of trial and error discovery, not axiomatic assertion.	Passes

(Inhelder & Piaget, 1964) (Piaget, 1952).

3.6.2 RELATIONS.

Piaget uses three kinds of task to assess a child's progress through the posited three substages of development. They are, in increasing order of difficulty: (a) seriation; (b) multiple seriation; (c) transitive inference.

In the seriation task, the child is asked to arrange a number of objects in order along a particular dimension - e.g. arrange a set of five lengths of wooden dowel in order of increasing length. In the multiple seriation task, the child is asked to arrange the objects in order along two dimensions simultaneously - e.g. to order on length and diameter. In the transitive inference task, the child is presented with the information that A is greater than B, and that B is greater than C, but not directly with information on the quantitative relationship between A and C. The child is then asked: what is the relationship between A and C. Correct performance on the transitive inference task indicates achievement of the relations concept, and attainment of the relations grouping structures. Progression through these tasks is summarised in the following:-

<u>TEST</u>	<u>SUB-STAGE 1</u>	<u>SUB-STAGE 2</u>	<u>SUB-STAGE 3</u>
Seriation	Fails. Can make pairwise comparisons only.	Passes. Can construct series with 5 to 10 elements.	Passes.
Multiple Seriation	Fails. As for seriation.	Achieves partial success. Will order on two dimensions successively not simultaneously.	Passes
Transitive Inference	Fails. Usually asserts A=C because they look alike.	Fails. As for sub-stage 1.	Passes

(Inhelder & Piaget, 1964) (Piaget, Inhelder, & Szeminska, 1960).

The three sub-stage model predicts that an ability to seriate length will emerge earlier, in the child's reasoning, than an ability to make transitive inferences with respect to length. The model also predicts that an ability to order discrete quantity will emerge earlier than an ability to make transitive inferences with respect to discrete quantity.

3.6.3 NUMBER CONSERVATION.

Piaget's (1952) classical test of number is termed conservation of number. Typically, it involves showing a child two rows of objects set in one-to-one correspondence; having the child agree that each row contains the same number of objects; transforming one row so that it is either, shorter and more dense, or longer and less dense, with the child watching; then asking the child if the rows still contain the same number of objects. Again, Piaget sees three sub-stages:-

- . Sub-Stage 1 - the child's judgements are dominated by relative lengths, hence, he asserts that the longer row is more numerous.
- . Sub-Stage 2 - the child considers both relative length and relative density, but cannot co-ordinate the two; hence, if a correct judgement requires that both cues be taken into account, the child fails.
- . Sub-Stage 3 - the conservation of number is achieved.

At a theoretical level, Piaget sees the child's acquisition of the conservation of number as marking the completion, and synthesis, of logical groupings I and V - in other words, as reflecting the co-ordination of the

cardinal and ordinal aspects, respectively, of number. Piaget also sees the growth in those areas of class logic as being linked with growth in the understanding of conservation. Consequently, the three sub-stage model predicts that the ability to form one-to-one correspondences, and the ability to construct ordinal series, appear earlier in the child's reasoning than the conservation of number. Additionally, the model predicts that the conservation of number emerges at about the same time as an understanding of addition and subtraction operations.

3.6.4 CONSERVATION, MEASUREMENT AND ARITHMETICAL OPERATIONS.

The three sub-stage model also posits a developmental sequence between the conservation of length and linear measurement, and between linear measurement and arithmetical operations.

Tasks, analogous to that used to assess number conservation, have been designed for conservation of length, distance, weight, volume, and quantity. In all cases, Piaget sees three sub-stages:-

- . Sub-Stage 1 - no conservation, child maintains that the quantitative relationship has changed.
- . Sub-Stage 2 - intermediate reactions, child gives conserving response if deformation is small, or will predict conservation before deformation. However, if the subsequent deformation is large child, the child will reverse its judgement and assert that the relationship has changed.
- . Sub-Stage 3 - conservation response.

Piaget also sees three sub-stages of development in the measurement of length and distance. It is the last sub-stage which is most relevant to the empirical work of this thesis. Piaget divides the last sub-stage into two parts:-

- . Sub-Stage IIIA— refers to the emergence of conservation of length and distance; while
- . Sub-Stage IIIB-- refers to the emergence of measurement of length and distance, via the process of unit iteration.

(Piaget, Inhelder & Szeminksa, 1960).

In addition to positing a developmental dependency between conservation and measurement, Piaget sees a parallel between the development of number and arithmetical operations in the domain of logical groupings and groups, and the development of measurement operations in the domain of infralogical groupings and groups:-

"It is perfectly clear that, by Stage IIIB the subjectshave finally achieved the construction of measurement by fusing or synthesizing the operations of subdivision and change of position. Both operations were required at level IIIA for the notion of conservation, but there they were still complementary to one another instead of being fused into a single operation. There is a parallel to this elaboration of measurement out of two qualitative operations which are at first distinct but which must be synthesized to yield one integral operation. The parallel

is in the elaboration of number. This cannot surprise us since there are isomorphic relations between the iteration of metrical units and the series of whole numbers (as also between the fractioning of metrical units and fractional numbers), likewise between subdivision and composition of parts on the one hand, and nesting class hierarchies on the other, and finally, between change of position and seriation of asymmetrical relations. Thus, measurement, in the field of sublogical operations is the exact equivalent to number in that of logical operations, since number is a synthesis involving the logical grouping of nesting classes and the seriation of asymmetrical relations. The only difference is that the whole number series is constructed at level IIIA and so follows immediately on these two logical groupings, while measurement is delayed for some while after notions of conservation have been mastered - although these are similarly dependent on its constituent operations: subdivision and change of position. We have tried to show how that delay is not unexpected, since numerical unity is something which may be perceived intuitively because any collection of discontinuous items consists of such unique elements, while choosing a unit of length is to make an arbitrary fragmentation of a whole which is continuous." (Piaget Inhelder & Szeminska, 1960, p400).

It will be evident from the above that Piaget's views on the developmental dependencies between conservation, arithmetical operations, and linear measurement are of central importance to the present research. Specifically, the three sub-stage model asserts that the following are necessary components of linear measurement:-

- . the ability to conserve number, length and distance;
- . the ability to make transitive inference judgements with respect to number and length;
- . the ability to use a unit of length for purposes of iteration;
- . the ability to carry out arithmetical operations of addition and subtraction.

Additionally, the three sub-stage model predicts that:-

- . the ability to conserve length emerges earlier than the ability to measure length;
- . the ability to conserve distance emerges earlier than the ability to measure distance;
- . the ability to perform arithmetical operations of addition and subtraction emerges earlier than the ability to measure length or distance.

3.7 HORIZONTAL DECALAGE.

It has been noted in connection with the parallel development hypothesis, and the three sub-stage model, that Piaget claims that the final sub-stages in classes and relations emerge at about the same time. Similarly, attainment of sub-stage 3 in conservation marks the synthesis of the logical (or infralogical, depending upon concept type) groupings and number groups. Since it is these structures which determine the nature of the child's reasoning, it might be supposed that, for example, conservation of number would appear at about the same time as conservation of weight. This is because the logic of the argument is the same in both cases. However, Piaget says not. Instead, he argues that logical and infralogical operations emerge synchronously within any given concept. Thus, while transitivity of number and conservation of number emerge in parallel, and transitivity of weight and conservation of weight are attained at about the same time, there is a developmental lag between number and weight. More specifically, Piaget found that sub-stage 3 of classes, relations, conservation and measurement is reached in the following order: number, length, quantity, weight and volume. Piaget refers to this phenomenon as horizontal decalage.

However, Piaget has also found that length and distance concepts emerge in parallel. His explanation reflects his conception of space. When a child is presented with two lengths of dowel in the course of a conservation experiment, he compares two extents of occupied space. In a distance conservation experiment he compares two extents of unoccupied space. Piaget regards space as a network of sub-spaces linked together

in a one, two, or three dimensional co-ordinate system. He argues that, so far as conservation and measurement are concerned, considerations of whether such sub-spaces are occupied by concrete objects or not, are irrelevant.

In summary, Piaget claims that;

- . the ability to conserve number emerges earlier than the ability to conserve length;
- . the ability to conserve length emerges at about the same time as the ability to conserve distance;
- . the ability to measure length emerges at about the same time as the ability to measure distance.

Finally, Piaget also found that an ability to seriate length emerged earlier than an ability to numerate. Since an ability to seriate length, and an ability to numerate, are assumed to be low-order components of linear measurement, that finding should also constitute a prediction of the present study.

3.8 SUMMARY.

Piaget's theory provides a structural account of cognitive development. Cognitive structures are defined as systems of operations. Intellectual growth is directed by the functional invariants of organisation and adaptation, and the mechanisms of structural change are described in terms of an equilibrium model.

Four stages of development are identified. The third of these stages, that of concrete operations, marks the emergence of behaviour and cognitive structures concerned with logic, number, and certain physical properties of objects and events in the world. Consequently, it is the stage of development most relevant to this thesis.

The theory, and the empirical base of the theory, provide general predictions regarding the composition, and the order of development, of the aspects of linear measurement examined empirically in this thesis. Moreover, the theory provides a descriptive rationale for that predicted order of development. These contributions of Piagetian theory to the derivation of specific hypotheses will be returned to in Part III.

CHAPTER 4.

METHODOLOGICAL CONSIDERATIONS.

4.1 PIAGET'S MODIFIED CLINICAL METHOD.

The predictions made in Chapter 3 concerning the order of emergence of components of linear measurement stem from Piagetian theory. Most of the empirical studies discussed in the following review were undertaken with a view to testing various aspects of that theory. Difficulties arise in interpreting much of this research, however, because many of the studies did not use precisely the same tasks as did Piaget. Also, many did not use Piaget's "modified clinical" style of questioning to draw out from the subject verbal justifications for his answer. In the modified clinical method the experimenter explores the subject's reasoning processes by asking the subject to justify his answers verbally. The experimenter does not adhere to a particular form, or to a fixed sequence, of questioning.

4.2 PERFORMANCE/COMPETENCE ISSUE

A major advantage of the modified clinical method is that it provides evidence of the particular form of reasoning used by the child. Critics of that approach, however, claim that it is too dependent on the child's verbal skills. This issue is part of a long and continuing debate known as the "performance/competence" distinction. More specifically, "competence" refers to the subject having the particular ability in question. "Perfor-

mance refers to the subject's capacity to apply and demonstrate that ability in a particular situation.

This methodological issue needs to be considered before assessing empirical evidence, and in relation to the design of the tasks and administration procedures employed in this study.

4.3 PERFORMANCE/COMPETENCE CRITICISM OF PIAGETIAN CONCRETE OPERATIONAL TASKS.

Bryant(1974) has criticised some of Piaget's concrete operational tasks on the grounds that they do not sufficiently control performance variables. These variables might either mask the concept being explored, or they might falsely give the impression that the child has acquired that concept. That kind of criticism has most frequently been made of the transitive reasoning and class inclusion tasks. (Ahr and Youniss, 1970; Braine, 1959; Brainerd, 1973,1974; Brainerd and Kaszor, 1974; Brainerd and Vanden Heuvel 1974; Bryant and Trabasso, 1971; De Boysson-Bardies and O'Regan, 1973; De Soto, London and Handel, 1965; Flavell, 1977; Jennings, 1970; Klahr and Wallace, 1972; Miller, 1976; Riley and Trabasso, 1974; Roodin and Gruen, 1970; Siegel, 1971a;1971b; Winer, 1974; Winer and Kronberg, 1974; Wohlwill, 1968; Youniss and Dennison, 1971; Youniss and Murray, 1970).

It was argued in the earlier analysis that transitive reasoning is a necessary component of linear measurement. Hence it was decided that studies of transitive inference would be used to convey the important features of the argument.

4.4 CRITICISM OF THE PIAGETIAN TRANSITIVE REASONING TASKS

The Piagetian test for transitive inferences concerning length relations involves presenting objects A and B, then objects B and C, and then objects A and C. The AB and BC pairings are presented in such a manner as to permit the child to determine perceptually which is the longer or shorter. Objects A and C are usually presented in such a manner as to create a misleading perception, the intention being to force the child to use principles of transitive reasoning. Finally, the experimenter questions the child to ensure that the given answer was derived by making a transitive inference. Unless the child is able to justify his answer verbally, he is not credited with having transitive reasoning for length (Beth and Piaget, 1966). The following four kinds of criticism have been made of this procedure:-

- (a) young children can make transitive inferences, but are compelled by the visual illusion to give non-transitive answers;
- (b) young children lack the verbal skills necessary to provide appropriate verbal justification for their answers;
- (c) young children can make transitive inferences, but fail the task because they forget the premises;
- (d) young children who pass the task may use non-transitive strategies (Brainerd, 1978).

4.4.1 STUDIES CONTROLLING VISUAL ILLUSION,
MEMORY CAPACITY AND VERBAL SKILL FACTORS.

Studies which have not used visual illusions and have not required verbal justifications have claimed that kindergarten children of about five years of age can make transitive inferences (Brainerd, 1973; 1974; Brainerd and Vanden Heuvel, 1974). Similar findings have been reported from studies which have not employed visual illusions, have not required verbal justifications, and have nullified the memory factor by using either visual or verbal feedback in a preliminary learning phase (Bryant, 1974; Bryant and Trabasso, 1971; Riley and Trabasso, 1974; Siegel, 1971a; 1971b). However, a recent study that employed the standard three term series procedure found that 64% of the subjects who remembered the premises, still could not make a transitive inference with respect to length (Halford and Galloway, 1977; Halford, 1979; Grieve and Nesdale, 1979).

All of these studies used an adequate sample size, appropriate stimulus materials, and procedures which appeared to eliminate, or control, the extraneous factors. All of the studies, except that of Halford and Galloway (1977), concluded that children can make transitive inferences two to three years earlier than Piaget has claimed. In the case of Bryant's studies, it was found, using five term series problems, that three and four year old children could make transitive inferences.

Methodological rigour, however, does not carry guarantees of conceptual soundness. The resolution of the performance/competence issue rests on discovering the form of internal representation used by young children in solving Piagetian tasks.

Many critics have argued that some children who pass the transitive inference task may not be using transitive reasoning principles (De Boysson-Bardies and O'Regan, 1973; De Soto et. al., 1965; Flavell, 1977; Riley and Trabasso, 1974; Youniss and Dennison, 1971; and Youniss and Murray, 1970). However, it will be seen that these arguments can be applied either in support of the Piagetian position, or in support of the critics of that position. This is because they rest on assumptions concerning the form of internal representation used by the child.

4.4.2 THE ROLE OF LINGUISTIC CODING IN TRANSITIVE INFERENCE.

Youniss and Dennison (1971) and Youniss and Murray (1970) suggested that Piaget's standard three term series problem could be solved by employing a non-transitive, linguistic coding strategy. Their argument is that the standard procedure enables subjects to code linguistically - A is coded as 'big' during the AB pairing, and C as 'small' during the BC pairing, leading to the non-transitive judgement that, since A is 'big' and C is 'small' A must be bigger than C. Their solution to this problem of false positives (correct answers reached by non-transitive processes) was to introduce two additional objects X and Y, and to introduce them in such a manner as to make both A and C both 'big' and 'small'. For example, in the following set of pairings A is both bigger than B and smaller than X, while C is both bigger than Y and smaller than B: $A > B$; $B > C$; $X > A$; $C > Y$; $X > Y$. The same argument was applied by De Boysson-Bardies and O'Regan (1973) to Bryant and Trabasso's (1971) procedure.

4.4.3 THE ROLE OF MENTAL IMAGERY IN TRANSITIVE INFERENCE.

De Soto et.al. (1965), and Riley and Trabasso (1974) advanced a similar argument, suggesting that false positives might be produced by a strategy involving mental imagery. They argued that a false positive solution would be produced if the child imag(in)ed absolute values for each stimulus item. However, differences between items in the stimulus pairings in transitive reasoning tasks are usually small (e.g. .5cm in the case of length). Furthermore, in general, people are not competent at estimating length (Schiff and Saarni, 1976). It seems unlikely, therefore, that young children would use a strategy requiring absolute coding of stimulus-attribute values as mental images.

4.4.4 THE FORM OF INTERNAL REPRESENTATION.

These arguments concerning the form of internal representation used by children have two curious aspects. Firstly, if the stimulus-attribute values are sufficiently different to enable the child to adopt an absolute linguistic or imaginal coding strategy, a false-positive solution could be produced. However, it is also possible that a comparative linguistic (e.g. bigger, smaller) or ordered imaginal (e.g. big on left, small on right) coding strategy could be adopted. Each would produce a true positive solution. Short of asking the child which strategy he used, it is impossible to resolve the issue.

Secondly, suppose that objections to the standard procedure are raised on the grounds that the child's verbal skills are not sufficient to permit him to justify his answer orally. Surely, an objection of equal force could be raised against asking the child whether he used an absolute linguistic or comparative linguistic, absolute imaginal or ordered imaginal, form of internal representation.

Additionally, even if there were some way of ascertaining which of these two forms of internal representation were used by the child, the value of the argument could still be questioned. Suppose that the absolute linguistic or absolute imaginal form were used. It would be possible to solve problems without using transitive inference if the child constructed a linguistic or imaginal series, and 'read off' the answer. In this case, he would be demonstrating a capacity to seriate, which is substage 1, not substage 3, level of functioning. In order to use transitive reasoning the child would need to transform the problem to the canonical form - if a.R.b. and b.R.c. then a.R.c. It may be that the production of a linguistic or imaginal series is only the first step in the production of an internal form of representation akin to the canonical form. Wallace (1972) argued that the findings of latency studies support this possibility.

4.5 SUMMARY OF CRITICISM REGARDING TRANSITIVE REASONING TASKS.

In summary, a number of critics have insisted that at least some of the subjects used in the Piagetian studies who failed to make transitive inferences were limited, not by the absence of a transitivity rule, but by memory capacity or some other factor. Such failures are instances of

false negatives. In support of those contentions, methodologically rigorous studies were undertaken which, it was claimed, demonstrated that children of four and five years of age could make transitive inferences. However, arguments based on the form of internal representation used suggest that children in these studies could have produced correct answers without employing transitive reasoning. Such passes are instances of false positives.

However, it is also the case that arguments resting on the form of internal representation can be employed to either attack, or defend, the findings of any transitive reasoning study which does not require the subject to provide an appropriate verbal justification. Anderson (1978,1979) has pointed out in another context that it is impossible to determine what kind of internal representation a subject employs in problem solving. Furthermore, Miller (1976) has argued that: "The problem is that, since it is inherently impossible to find a perfect operational definition of the concepts such as transitivity, inclusion (really disjunction), and conservation on which the controversy turns, it cannot be resolved by finding the perfect test (p.430)."

With these considerations in mind, it was decided that, when reviewing evidence concerning the components of linear measurement, greater weight would be given to those studies which adhered to the Piagetian approach, with respect to tasks used and insistence upon verbal justifications.

CHAPTER 5.PARALLEL DEVELOPMENT: EMPIRICAL
EVIDENCE CONCERNING ORDER OF
EMERGENCE OF CONSERVATION AND
TRANSITIVE REASONING.5.1 PREDICTIONS.

It was argued earlier that conservation of number and length, and transitive reasoning involving number and length, are necessary components of linear measurement. Therefore, the order of development of these components might provide insight into the growth of linear measurement.

It was asserted in Chapter 3, that the parallel development hypothesis yields the predictions that:-

- (a) the ability to conserve number emerges at about the same time as the ability to make transitive inferences with respect to discrete quantity;
- (b) the ability to conserve length emerges at about the same time as the ability to make transitive inferences with respect to length.

Empirical evidence concerning these predictions will now be discussed.

5.2 ASSESSMENT CRITERIA.

If a study is to provide clear empirical evidence concerning these predictions, it should employ the same group of subjects for all tasks. This is because of the wide variation in the age at which children attain components of the number and length concepts. For example, a particular study using group A might conclude that the mean age for attainment of length conservation is six years three months, and using group B that the mean age for attainment of transitive reasoning involving length is five years two months. Such findings provide only weak evidence of asynchronous emergence of length conservation and transitive reasoning. This is because such a study does not provide evidence that for each group the component not tested does not emerge in synchrony with the component tested. This is a general limitation of between-group experimental designs.

In addition, Piagetian theory posits a lag in development between number and length with respect to the acquisition of conservation and transitive inference. It is necessary, therefore, that, in a study aimed at testing the prediction of synchronous emergence of conservation and transitive inference, the tasks should all test the same concept. For example, the emergence of conservation of length should be located in relation to the appearance of transitive inference reasoning with respect to length. Unless this strategy is adopted, difficulties will arise in interpreting findings. For example, a finding that transitive reasoning involving length emerged after conservation of number would be expected simply because of the horizontal decalage between number and length.

These considerations, together with the factors already mentioned in Chapter 4, specify the kind of study which could test the predictions presently under review. Specifically, such a study should satisfy the following criteria:-

- (a) the tasks should be administered to all subjects;
- (b) comparisons between emergence of conservation and transitive reasoning should be based on the same concept - e.g. number or length or weight but not number and length;
- (c) The procedures employed should be essentially the same as those used by Piaget, especially in connection with the insistence upon verbal justification, and clinical style questioning of the subject.

5.3 EVIDENCE THAT ACQUISITION OF CONSERVATION PRECEDES ACQUISITION OF TRANSITIVE INFERENCE.

5.3.1 LENGTH AND WEIGHT.

In two studies, Smedslund (1961,1963) found evidence suggesting that conservation appears before transitive reasoning for each concept. In the 1961 study, five to seven year old children were given pre-tests of conservation of quantity, conservation of weight, and transitive reasoning involving weight. These pre-tests were followed by a training phase. Smedslund found that, in the pre-tests, the correlation between conservation and transitive reasoning for weight was very low, after partialling out age variances. He also found that transitive reasoning for weight was more difficult to train than conservation of weight (this finding, may simply reflect relative effectiveness of the training techniques). In the

1963 study, Smedslund found that conservation of length appeared earlier than transitive reasoning for length. His assessment procedures closely followed those of Piaget, except for one aspect of the transitive reasoning task. After presenting the AB and BC pairings, but before presenting the AC test comparison, Smedslund checked that the subject remembered the outcomes of the two earlier comparisons. If the subject couldn't remember them, he re-presented the earlier pairings. This procedure increases the potential for non-transitive solutions.

McMannis (1969) also studied the order of acquisition of conservation and transitive reasoning, for both length and weight, using 90 normals and 90 retardates, matched for mental age between 5 and 10. The results are given in Table 5.1.

TABLE 5.1:

RELATIONSHIP BETWEEN CONSERVATION AND TRANSITIVITY: WEIGHT AND LENGTH:
NUMBER OF SUBJECTS.

CONSERVATION	WEIGHT			
	NORMALS		RETARDATES	
	TRANSITIVE REASONING	TRANSITIVE REASONING	TRANSITIVE REASONING	TRANSITIVE REASONING
	Absent	Present	Absent	Present
Absent	30	4*	36	3**
Present	13*	43	34**	19
Length				
Absent	33	1**	37	0**
Present	22**	34	45**	8

* $P < .025$

** $P < .001$

These findings appear to be strong evidence that conservation appears before transitive reasoning in the development of each concept. Mc.Mannis' procedures closely followed Piaget's, except in respect of one aspect of the transitive reasoning tasks. Like Smedslund (1963), he introduced a learning factor by requiring the subject to recall correctly the outcomes of the initial AB and BC pairings, before he moved on to present the AC test comparison. His results are consistent with those of Smedslund (1963).

The variations from Piaget's procedures in these studies do not weaken their findings. This is because the change in the standard transitive reasoning procedure would decrease task difficulty. Therefore, if conservation and transitive reasoning emerge synchronously, the predicted outcome in the Smedslund and Mc.Mannis studies would be for transitive reasoning to precede conservation. If, on the other hand, conservation precedes transitive reasoning, the reduction in difficulty of the transitive reasoning task should have masked that asynchrony. In other words the effect of the change in the transitive reasoning test procedure made by Smedslund and Mc.Mannis would be to reduce, not increase, the probability of finding a conservation followed by transitive reasoning sequence. Their finding may, therefore, be construed as evidence that conservation does emerge before transitive reasoning for each concept.

5.3.2 NUMBER AND LENGTH.

Achenbach and Weisz (1975) carried out a longitudinal study of developmental synchrony between conceptual identity (which was equated with conservation), and transitive reasoning for colour, number and length, using a sample of 102 pre-school age children. The children were tested on two occasions, six months apart. Mean age at the first testing was 50 months. Their results for identity and transitive reasoning, with respect to number and length, are given in Table 5.2.

TABLE 5.2:

PERCENTAGE OF SUBJECTS PASSING IDENTITY AND TRANSITIVE REASONING TESTS FOR NUMBER AND LENGTH.

CONCEPT	TEST	
	IDENTITY	TRANSITIVE REASONING
Number		
Testing 1	58	9
Testing 2	73	12
Length		
Testing 1	35	6
Testing 2	70	6

Achenbach and Weisz (1975) interpreted these findings as evidence that, for both number and length, conservation precedes transitive reasoning. Even though this study is cited and accepted by influential scholars (e.g. Brainerd, 1978), the conclusion reached is questionable, for the following reasons:-

- (a) The identity tasks were concerned with the conservation of a quantitative attribute of a single stimulus. The standard Piagetian conservation task involves the quantitative equivalence of two stimuli. The identity task should not be equated with a conservation task.
- (b) The test for the presence of conservation was whether or not the child was "surprised" on the second presentation of the stimulus.
- (c) The transitive reasoning tasks were based on the five term presentations of Youniss and Murray (1970) and Roodin and Gruen (1970), and not on the standard three term procedure.

Moreover, it is not unexpected that identity should have emerged before transitive reasoning for number. This is because Gelman and Gallistel (1972) found in a number of studies, that three and four year old children judged that an array of n items still contained n items after the length, colour, identity and spatial arrangement of the array elements had been surreptitiously altered. The same children, however, did not pass the standard two-array conservation test. Gelman and Gallistel (1972) argue that this is because pre-school children know that certain operations change the numerosity of an array while others do not, but do not know what effect the former group of operations have on the relations between arrays of unspecified numerosity. Consequently, it would be expected that identity would emerge before conservation. Because Piagetian theory predicts that conservation and transitive reasoning for each concept emerge in synchrony, the Achenbach and Weisz (1975) finding that identity appears before transitive reasoning is consistent with, and not in opposition to, Piaget's claims.

5.4 EVIDENCE THAT ACQUISITION OF TRANSITIVE
INFERENCE PRECEDES ACQUISITION OF CONSERVATION.

5.4.1 WEIGHT.

Lovell and Ogilvie (1961) studied the order of acquisition of transitive reasoning for weight and conservation of weight, and found that 53% of those subjects who could not pass the conservation test did pass the transitive inference test.

This study suffers from two major deficiencies which make interpretation difficult. Firstly, in the transitive inference task the third object in the three object series was not presented physically, but described verbally. Hence, the task required a transitive inference connecting a concrete object (A) with a verbal symbol (C), linked by a second concrete object (B). The most likely effect of this mixed mode of presentation would be to increase the difficulty of the task. Secondly, when scoring the subject's protocol for the transitive inference task, the experimenters did not require the AB and BC pairings to be verbalised. This laxity in scoring would tend to reduce task difficulty.

5.4.2 LENGTH AND WEIGHT.

Brainerd (1973) studied the order of acquisition of transitive reasoning and conservation, for both length and weight, in two experiments. In the first experiment, two samples each of 60 subjects (mean ages of seven years seven months and seven years and five months) were used. All sub-

jects received all tests. In one sample, Brainerd found that transitive reasoning emerged before conservation. In the other, Brainerd found that they emerged synchronously, for both length and weight. Because of this equivocal finding, Brainerd (1973) carried out a second experiment using the same materials and procedure as for the first, but employing three groups each of 60 subjects with mean ages of 5 years 4 months, 6 years 4 months, and 7 years 10 months. In the second experiment he found that, for both length and weight, transitive reasoning emerged before conservation. Brainerd (1973) interpreted these results as being damaging to Piaget's parallel development argument. Brainerd's conclusions may be questioned on methodological grounds because his transitive reasoning tasks employed the A.EQ.B.NE.C. paradigm of Youniss and Murray (1970), in lieu of the standard procedure. Also, subject's responses were not scored on the basis of verbal justifications.

5.4.3 LENGTH.

In a later study, Brainerd (1974) examined the effects of training, and transfer of training, on transitive reasoning and conservation, for length. He found that transitive reasoning was easier to train than conservation. He interpreted these results as infirming the parallel development argument. However, again, his transitive reasoning tasks used one of the non-standard paradigms developed by Youniss and Murray (1970). Also, as with his earlier study, Brainerd did not require his subjects to provide verbal justifications. In addition to these departures from standard assessment procedures, interpretation is difficult because the results may simply reflect the relative effectiveness of the training technique used (verbal feedback) for transitive reasoning and conservation.

As part of a series of experiments concerned with the relationship between geometric imagery and operational thought in children, Brainerd and Vandenberg (1974) tested a group of 60 second grade school children (mean age of eight years two months) for presence of transitive reasoning and conservation, for length. They found that 17 subjects passed the transitive reasoning test and failed conservation, but only two passed conservation and failed transitive reasoning. These findings are consistent with those of Brainerd's (1973) earlier study. However, as the assessment procedures used were the same in both studies, interpretation of the data suffers from the same difficulties as those noted above in connection with the 1973 study.

5.5 SUMMARY.

In summary, none of the studies discussed above meet all of the assessment criteria given at the beginning of this chapter.

The well known studies of Brainerd and his colleagues, which concluded that transitive reasoning precedes conservation in the development of each concept, present two difficulties. Firstly, the task used to assess the presence of transitive reasoning may offer subjects the opportunity to pass the tasks without making transitive inferences. Secondly, the studies did not score the subject's protocols. Both of these factors tend to reduce the difficulty of the transitive reasoning tasks. Hence, the studies may have been biased in favour of the conclusions they reached.

The Smedslund and Mc.Mannis studies, which concluded that conservation is achieved before transitive reasoning for each concept, also departed from standard assessment procedures. In particular, the tasks used to assess the presence of transitive reasoning were easier than the standard form. However, this did not bias those studies towards their eventual conclusions.

In conclusion, the evidence does not favour the view of synchronous emergence of conservation and transitive reasoning. It is considered that the evidence is more consistent with the opinion that, in each concept, conservation appears in the child's thinking before transitive reasoning.

CHAPTER 6.SEQUENTIAL DEVELOPMENT: EMPIRICAL
EVIDENCE CONCERNING ORDER OF
EMERGENCE OF CONSERVATION, ARITHMETICAL
PROFICIENCY AND MEASUREMENT.6.1 PREDICTIONS.

It was argued in Chapter 3 that the three sub-stage model yields the predictions that:-

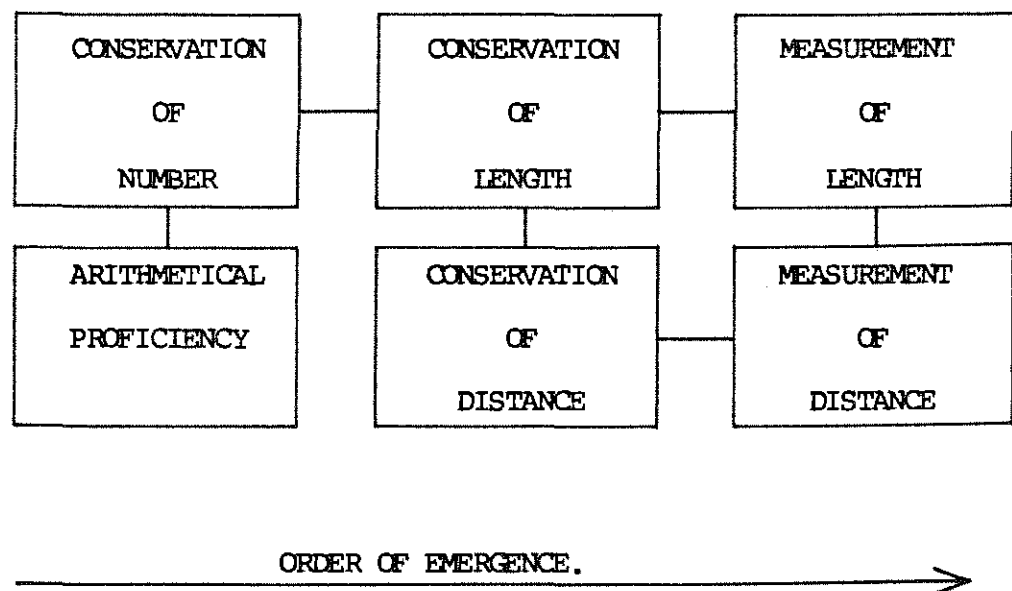
- (a) the ability to conserve length emerges earlier than the ability to measure length;
- (b) the ability to conserve distance emerges earlier than the ability to measure distance;
- (c) the ability to perform the arithmetical operations of addition and subtraction emerges earlier than the ability to measure length or distance;
- (d) the ability to conserve number emerges at about the same time as the ability to perform the arithmetical operations of addition and subtraction;
- (e) the ability to seriate length emerges earlier than the ability to make a transitive inference with respect to length;
- (f) the ability to order discrete quantity emerges earlier than the ability to make transitive inferences with respect to discrete quantity.

Developmental sequences of the kind referred to in the first four of these predictions should not be confused with causal chains in concept acquisition (Flavell (1971, and 1972) discusses this issue in detail). However, they do provide indirect evidence concerning the composition of concepts. Consequently, these predictions, together with those linking transitive reasoning and conservation, are central to the task of identifying the components of linear measurement.

Considered together, the first four of these predictions link number conservation and arithmetical proficiency with the various conservations and measurement. Diagrammatically, the linkages can be represented as in Figure 6.1.

FIGURE 6.1:

SCHEMATIC REPRESENTATION OF PREDICTED ORDER OF EMERGENCE OF ARITHMETICAL PROFICIENCY AND CONSERVATION OF NUMBER, LENGTH AND DISTANCE.



With this diagram in mind, the review of evidence concerning these predictions will commence with a discussion of the developmental relationship between number conservation and arithmetical proficiency. It will then move onto length/distance conservation, length/distance measurement, and arithmetical proficiency. Finally, the evidence regarding the lower order abilities referred to in predictions (e) and (f) above, will be discussed.

6.2 THE CONSERVATION OF NUMBER AND ARITHMETICAL PROFICIENCY.

6.2.1 THE NUMBER CONCEPT AND ARITHMETICAL OPERATIONS

It will be recalled that Piagetian theory states that the conservation of number emerges in the child's reasoning as a consequence of the synthesis of the cognitive structures concerned with the logic of classes and relations. Hence, conservation is seen as marking the integration of the ordinal and cardinal aspects of number. The theory also asserts that an understanding of the arithmetical operations of addition and subtraction emerges as a consequence of that synthesis, and in concert with the conservation of number. Hence, understanding of arithmetic is based upon a prior understanding of some aspects of Boolean logic.

6.2.2 DEFINING AN UNDERSTANDING OF ARITHMETIC.

The principal difficulty in testing the prediction that the conservation of number and an understanding of arithmetic emerge contemporaneously is in deciding upon an acceptable definition of arithmetical understanding. It could be said that possession of an algorithmic-like ability to per-

form certain addition operations by counting sets of objects constitutes a level of understanding. It could also be argued that possession of the knowledge that the natural numbers can be composed in a variety of ways (e.g. $(6)=(3+3)=(4+2)=(5+1)$ etc.) constitutes another, and, perhaps, higher level of understanding. Yet another level of understanding would require demonstration of the subject's knowledge of the associative, distributive, commutative, etc. laws of arithmetic.

6.2.3 EQUIVOCAL FINDINGS OF STUDIES

LINKING CONSERVATION OF NUMBER AND ARITHMETIC.

This problem of settling upon a widely acceptable definition of understanding of arithmetic is reflected in the results of studies that have sought to identify the developmental relationship between the conservation of number and an understanding of arithmetic. Among the most influential studies undertaken in the last decade are those of Brainerd and his associates (Brainerd, 1973(a); 1973(b); 1974; Brainerd and Fraser, 1975; and Siegel, 1971(a); 1971(b); 1974), and those of Gelman and her colleagues (Gelman, 1972; Gelman and Gallistel, 1972; and Gelman and Tucker, 1975). In general, these studies concluded that children first develop an ability to count, then to perform addition and subtraction on sets of small numerosity, and that this provides a basis for the understanding of the cardinal aspects of number (including conservation of number). In contrast, other less recent studies (Beard, 1963; Dodwell, 1960; 1961; and Hood, 1962) concluded that attainment of the conservation of number is required for an operational grasp of number in the child's thinking, and it is only at this stage that the child can have an understanding of arithmetic.

The more recent studies can be criticised on the grounds that the tasks used do not tap the mental abilities for which they were designed. Brainerd's studies, for instance, used a transitive reasoning task involving length relations to assess the child's understanding of ordinal number, and a one-to-one correspondence task to assess understanding of cardinal number. His measures of arithmetical proficiency ranged from knowledge of the first 16 number facts (i.e. $n+m=?$ and $n-m=?$ where n and m range from 1 to 4) in his 1973(a) study, to the conservation of number task in his 1975 study. Schaeffer (1980) found Brainerd's methods to be: "so seriously flawed logically, psychologically and experimentally as to be incapable of justifying his claims...." (p.556). Of course, other commentators would take issue with Schaeffer's criticisms.

Gelman's work has drawn less criticism. However, for reasons which need not be examined here in detail, it is possible to argue that her findings do not provide information on the developmental relationship between the conservation of number and an understanding of arithmetic. What her studies do suggest is that very young children have (at least for small numerosities) the capacity to count, and to base number judgements of equivalence and operations of addition and subtraction upon that counting procedure.

The older studies of Beard (1963), Dodwell (1960,1961), and Hood (1962) found, in general, that children who conserved number performed at a high level of proficiency on addition and subtraction tasks. The procedures used in these studies closely resemble Piagetian methods, in respect of both the tasks employed, and insistence upon verbal justifications.

6.2.4 STUDIES LINKING THE
LAWS OF ARITHMETIC AND THE
LAWS OF BOOLEAN ALGEBRA.

A different approach to the order of development of the conservation of number and an understanding of arithmetic, is to examine the developmental relationship between the laws of Boolean algebra, and the laws of arithmetic. This approach has led some critics to enter the lists against Piaget, on the grounds that class addition on the one hand, and natural number addition on the other, are so vastly dissimilar that psychologically, the latter could not possibly be built upon the former. Brainerd (1973a, 1978), Langford (1978, 1981), MacNamara (1975) and Osherson (1974) are leading critics of this aspect of Piagetian theory. Since, in respect of this issue, they endorse a common view, only MacNamara's (1975) criticism, and Langford's (1981) evidence, will be discussed.

MacNamara (1975) made the point that, for purposes of counting, and/or of applying arithmetic operations to the results of counting, any thing could be grouped with any other thing to form the set of things counted. Such things need share no common property, save the fact that the person doing the counting could discriminate one from the other. For example, the number of people living in town A, and the number of motor vehicles registered in town B, could be counted as one set. The only property that the elements of that set would share would be that they were picked out to be counted. Such a property is inherent of neither the people in town A, nor the motor vehicles in town B. In contrast, the members of a class - using that word in the same sense as Piaget - do share properties; properties inherent to the members of the class. It is this distinction between sets

and classes that gives rise to the radically different nature of the notion of a unit in a set, and the idea of a unit in a class. That, in turn, leads to the radically different outcomes of class and set operations.

MacNamara (1975) provided the following illustration of some of these differences:-

"In some sense, 5 includes 4 and 1, animals includes dogs and cats, and animals includes dogs and animals other than dogs. But in what sense of include? Four and 1 together equal 5. There is nothing in 5 over and above what is in 4 and 1 taken together. But dogs and cats together do not equal animals. There are other animals, such as horses and cows. So 5 does not include 4 and 1 in the same sense that animals includes dogs and cats. However, dogs and animals other than dogs, taken together, do equal animals in something like the sense that 4 plus 1 equals 5. Notice, however, that the relationship between dogs and animals other than dogs is quite different from that between 4 and 1. The number "1" cannot be expressed as "numbers other than 4" or as "numbers less than 5 and other than 4". If the latter were its meaning, it would be 1 plus 2 plus 3, which equals 6, and when added to 4 would make 10, not 5. It is clear that the relationship between 4 and 1 is different from that between dogs and animals other

than dogs. In short, the relationships among numbers are quite unlike those among hierarchically arranged classes." (MacNamara, 1975, p.427).

Langford (1981) supported that kind of theoretical argument with empirical evidence from a longitudinal study of the development of children's understanding of logical laws in arithmetic and Boolean algebra. He tested children's knowledge of eight logical laws and 15 arithmetic laws, measuring gains in knowledge over a two year period. Assessment procedures took account of the child's verbal justifications. The tasks used were appropriate behavioural equivalents of the operations being investigated. Analysis of the results included determination of statistical dependencies between items: for example, pass/fail patterns relating to the logical law $A \cup B = B \cup A$ and its arithmetical counterpart $A+B = B+A$.

In general terms, the results did not support the Piagetian view that laws in arithmetic appear later in development than corresponding laws in Boolean algebra.

6.2.5 SUMMARY OF DISCUSSION: THE CONSERVATION OF NUMBER AND UNDERSTANDING OF ARITHMETIC.

Piagetian theory provides the prediction that the conservation of number and an understanding of arithmetic emerge in the child's thinking at about the same time. Empirical verification of that prediction is difficult, because of problems inherent in deriving a widely acceptable definition of what constitutes an understanding of arithmetic. Consequently, the evidence is equivocal.

Against this background, the most conservative policy would be to predict that, for sets of small numerosity, the ability to count and carry out operations of addition and subtraction based upon counting, emerge in the child's thinking before the conservation of number; and, that the latter emerges before more complex forms of addition and subtraction. The question of whether these abilities imply an "understanding of arithmetic" will be discussed in later Chapters.

6.3 THE CONSERVATION OF LENGTH/DISTANCE, AND MEASUREMENT OF LENGTH/DISTANCE.

6.3.1 EMPIRICAL STUDIES OF LENGTH/DISTANCE, CONSERVATION AND MEASUREMENT.

There have been very few empirical studies concerned with the developmental relationships between conservation of length or distance, and measurement of length or distance. Most of the available empirical evidence is still due to Piaget et.al. (1960). Most of the relevant studies that have been conducted were concerned with assessing the role played by measurement in the formation of conservation. In that context, measurement was meant to include counting, and referred to an algorithmic kind of knowledge.

Beilin (1969) found that children with an appropriate measurement algorithm did not conserve number or area. Wohlwill and Lowe (1962) found that the ability to count did not ensure that the child would conserve number. Some of their subjects placed greater weight on perceptual cues, such as row length, than on the cardinal value given by counting, when the two were in conflict.

On the other hand, Bearison (1969), Gruen(1965), Lifschitz and Langford (1977), and Wallach, Wall and Anderson (1967) all found that training in counting and measuring was effective in producing conserving responses, and that the effect was durable.

6.3.2 IDENTITY, INVERSION AND COMPENSATION

ARGUMENTS.

Wallach (1969) has argued that the three main verbal justifications (identity, inversion and compensation) given by conservers, and accepted by Piagetians, as evidence of attainment of conservation, could not be responsible for producing conservation. Langford's (1978) arguments are essentially the same as Wallach's. In addition, he argued that counting and measurement provide an important means by which children come to conserve.

The identity operation preserves a particular property (e.g. length) of an object, as distinct from the object itself. The argument based on this notion of quantitative identity is that the property concerned must be the same before and after transformation, because nothing has been added or taken away. Wallach (1969) agreed that this is true of transformations that do not change the property in question, but insisted that the identity argument could not possibly be a sufficient basis for attainment of conservation. This is because there is nothing in the child's experience to tell him that the quantity in question does not change on the first transformation, or change back again on the second transformation.

Similarly, with respect to inversion and compensation, Wallach (1969) agreed that the conserving child may carry out these operations but, she also argued, that they do not provide sufficient mechanisms for attainment of conservation. For example, addition during the first transformation can be reversed by subtraction during the second transformation, but neither operation is quantity-conserving. Hence, some reversible operations are quantity-conserving, others are not. Consequently, Wallach (1969) argued that understanding of reversibility cannot be a sufficient mechanism for learning conservation. In connection with compensation, she pointed out:-

".....This not only becomes fantastically complicated with any but the simplest containers, but also exact compensation by differences in width for differences in height cannot in any case be directly perceived." (Wallach, 1969, p 192).

In addition to providing these theoretical arguments against quantitative identity, inversion and compensation, she also summarised evidence from a number of studies dealing with different conservations. All of those studies demonstrated, that mere possession of these operations does not ensure that children will conserve.

6.3.3 ROLE OF MEASUREMENT IN ACQUISITION OF CONSERVATION.

Langford (1978) argues that, given that these operations cannot be considered sufficient mechanisms for acquisition of conservation, counting and measurement must be implicated. He proposed that the accretion of experience with counting beads and stones, and, measuring sticks and blocks, etc., under different conditions, leads to the discovery of the

conservation of number and length. This then leads, via generalization, to other conservations such as quantity, weight and volume.

6.3.4 SUMMARY OF DISCUSSION AND CONCLUSION.

In summary, there is empirical evidence that the possession of quantitative identity, inversion and compensation operations does not ensure conservation; that the possession of counting and measurement skills does not ensure conservation; but that training on counting and measurement is effective in promoting conservation responses. Additionally, there are sound theoretical arguments against the proposal that quantitative identity, inversion and compensation operations provide a sufficient mechanism for explaining the acquisition of conservation. There are intuitively appealing arguments for explaining conservation acquisition in terms of counting and measurement skills.

On the other hand, Piaget and Inhelder (1969) have insisted that conservation is a logical, not an infralogical attainment. That is, conservation is not a matter of measurement, but a logical conviction. In part, this assertion refers to their beliefs that: (a) conservers do not, in reaching their answer, resort to infralogical operations; and, (b) that they produce the identity and reversibility arguments only as after-the-event justifications for their answers.

The operational definition of linear measurement adopted in Chapter 2 was: 'a person may be said to have a mature understanding of linear measurement if he demonstrates a capacity to use correctly arithmetical operations instead of carrying out physical measurement operations'. With that definition in mind, and having regard to the above discussion, there seem to be insufficient grounds to warrant departing from the prediction that the developmental sequence is the conservation of length (or distance) followed by measurement of length (or distance).

6.4 SERIATION, ORDINATION AND TRANSITIVE INFERENCE.

The last two predictions drawn from the three sub-stage model to be discussed in this chapter are concerned with seriation, ordination and transitive reasoning. It was argued that all are necessary for linear measurement. Specifically, the predictions are that: (a) the ability to seriate length emerges earlier in the child's thinking than the ability to make transitive inferences with respect to length; and, (b) the ability to order discrete quantity emerges earlier than the ability to make transitive inferences with respect to discrete quantity. Hence, both predictions refer to the same developmental sequence of seriation-then-transitive reasoning, for the number and length concepts.

Because any transitive inference is, itself, a kind of ordering, it would be illogical to assert that transitive reasoning emerges before seriation. However, it may be that they emerge synchronously. Consideration of this possibility again raises the performance/competence issue, because tran-

sitive reasoning provided a major focus for that controversy. It will be recalled from the discussion of that issue in Chapter 4, that it was concluded that greater weight should be accorded those studies of transitive reasoning which used standard Piagetian assessment methods. With that in mind, a search of the literature failed to uncover any studies using standard Piagetian procedures that found synchronous attainment of seriation and transitive reasoning for either the number or length concepts. Hence, it seems safe to agree with Klahr and Wallace (1976) and predict that seriation emerges before transitivity.

6.5 SUMMARY.

The predictions listed at the beginning of this Chapter were examined against the empirical evidence, and/or in the light of theoretical analysis. In each case it was found that there were insufficient grounds to justify modifying those predictions.

CHAPTER 7

HORIZONTAL DECALAGE: EMPIRICAL EVIDENCE
CONCERNING ORDER OF EMERGENCE OF
CORRESPONDING COMPONENTS OF THE
NUMBER, LENGTH AND DISTANCE CONCEPTS.

7.1 PREDICTIONS.

It was argued in Chapter 3 that the horizontal decalage model is the basis for the predictions that, in the child's thinking:-

- (a) the ability to conserve number emerges earlier than the ability to conserve length;
- (b) the ability to conserve length emerges at about the same time as the ability to conserve distance;
- (c) the ability to measure length emerges at about the same time as the ability to measure distance;
- (d) the ability to seriate length emerges earlier than an ability to numerate.

7.2 EVIDENCE THAT ACQUISITION
OF THE CONSERVATION OF
NUMBER PRECEDES ACQUISITION
OF THE CONSERVATION OF
LENGTH.

A search of the literature failed to find a study that assessed number and length conservation within the same child, using standard Piagetian procedures, and containing a sufficiently large number of within-subject comparisons to enable statistical treatment of data. There are, however, a number of studies that provide indirect evidence. Bearison's (1969) study is an example. His experiment was concerned with the effects of training in certain counting-based measurement operations upon the child's ability to conserve continuous quantity, area, mass, number and length. On a seven-month post-test, the percentage of subjects passing the number and/or length conservation tests are given in Table 7.1.

TABLE 7.1:

PERCENTAGE OF SUBJECTS PASSING NUMBER AND LENGTH CONSERVATION TESTS.

	EXPERIMENTAL GROUP	CONTROL GROUP
Number	75	38
Length	63	19

Because of difficulties (relative effectiveness of training on different tasks) in interpreting the effects of training on Piagetian tasks, even when, as in this case, standard Piagetian procedures and assessment forms are used, only the figures for the control group should be considered.

Whilst those results indicate that the children found number conservation easier than length, two inter-related points should be noted. Firstly, only percentage pass/fail figures are provided, so that within-subject developmental patterns have to be inferred. Secondly, the control group contained only 16 subjects, too few to enable valid inferences to be drawn. For example, 38% of 16 subjects is six subjects: 19% of 16 subjects is three subjects; but the three subjects who passed length conservation may not have been among the six subjects who passed number conservation.

Strauss and Ilan (1975) studied the effects of training on length conservation and speed concepts, and, in both pre-and post-testing of the control group, assessed number and length conservation using the standard Piagetian approach. On pre-testing, of the 10 subjects in the control group, nine conserved number while only three conserved length. On post-testing, all 10 conserved number, but only four conserved length. Although within-subject pass/fail patterns are not reported, the differences between the proportions passing number and length, on the pretest (.9 to .3), and the post-test (1.0 to .4), suggest a number-then-length development pattern. However, as the sample size was only 10 subjects, the results should be treated with caution.

Goldsmidt (1967), in a correlation study linking 10 different types of conservation with age, sex, IQ, MA and vocabulary, also provides some evidence on number-length conservation development patterns. In addition to using standard Piagetian procedures, this study has the merit of a large sample size of 102 subjects. However, some 20% of the subjects were classified as emotionally disturbed: the effect of that disturbance on

cognitive functioning is unknown. The results provided the following difficulty level-ranking (least to most) for the 10 conservations assessed: mass, number, continuous quantity, two-dimensional space, discontinuous quantity, weight, area, length, three dimensional space and distance. Unfortunately, insufficient information was reported on the data transformations used in the ranking procedures to assess the statistical significance of the separation between number and length conservation ranks. However, the data suggest that length conservation was much more difficult than number conservation.

7.2.1 CONCLUSION.

The little empirical evidence available suggests that the conservation of number is achieved before the conservation of length.

7.3 EVIDENCE THAT LENGTH AND DISTANCE CONSERVATION EMERGE SYNCHRONOUSLY.

A search of the literature did not uncover a comprehensive study of within-subject developmental patterns of the emergence of length and distance conservation. The best available evidence comes from the Goldsmidt (1967) study. In that study, it was found that distance conservation was more difficult than length conservation. However, the cautionary comments made above in connection with the Goldsmidt (1967) study extend with equal force to this particular finding.

The Piagetian finding of no horizontal decalage between length and distance conservation could seem counter-intuitive. It would seem harder to acquire the conservation of distance than the conservation of length. This is because length is an attribute of a single object, but distance is a relation between at least two objects. Moreover, the distance relation changes, if the position of one of the objects changes, while length is transportable.

A study by Schiff and Saarni (1976) replicated in most important respects, an earlier study by Piaget and Taponier. The experiment required adults and five and eight year old children to judge small differences in lengths of objects perceptually. The objects were parallel, but their end points were offset. It was found that, when the differences in length were small, both adults and children were not very good at judging relative length. For example, when the difference was + or - 1 mm, less than 10% of adults made correct judgements. As the differences in length increased, the five year old children became better at judging relative length than the adults. For example, when the difference was + or - 5 mm, less than 50% of the adults, but more than 70% of the five year old children, made correct judgements. In contrast, 100% of the adults conserved length, whilst most of the five year old children did not. Schiff and Saarni (1976) argued that these findings demonstrate that conservation reflects the interplay of perceptual and conceptual factors. These findings suggest that the conservation of length is not based on perceptually given information. They are consistent with Piaget's views concerning length/distance synchrony.

7.3.1 CONCLUSION.

Against this background, it seems unnecessary to modify the prediction of approximately synchronous emergence of length and distance conservation.

7.4 EVIDENCE THAT LENGTH AND DISTANCE MEASUREMENT EMERGE SYNCHRONOUSLY.

The Piagetian claim that length and distance measurement would emerge synchronously has not been tested empirically. However, if the claim of a synchronous emergence of length and distance conservation is accepted, then there would be no grounds for expecting asynchrony in the attainment of length and distance measurement.

7.5 EVIDENCE THAT ACQUISITION OF SERIATION PRECEDES ACQUISITION OF NUMERATION.

Piagetian theory claims that seriation of length is mastered before the child can numerate (ie. co-ordinate ordinal position and cardinal value). In the traditional Piagetian demonstration of this claim, the child is asked to seriate sticks of varying lengths to build a staircase. Then the child is asked to insert additional sticks into the series. A toy, such as a doll, is introduced, and the child asked to work out, starting at different positions, how many stairs the doll would have to climb to reach a particular level. Piaget found that seriation was achieved before numeration in this task. Elkind (1964) obtained a similar result using equivalent materials and procedure.

7.5.1 CONCLUSION.

There are no empirical grounds for departing from the Piagetian view that seriation emerges before numeration.

7.6 SUMMARY OF CONCLUSIONS.

The little empirical evidence available supports the Piagetian claims, however, it is evident that more work is necessary.

PART III

THE EMPIRICAL STUDY:

DISCUSSION OF METHODOLOGY

AND PRESENTATION OF RESULTS.

A discussion of linear measurement was presented in Part I. That discussion analyzed the components of linear measurement and raised several questions which were examined empirically in the present research. The questions were concerned with identifying the necessary components of linear measurement, and describing its development in terms of the growth of those components.

An examination in Part II of relevant literature yielded several predictions regarding the growth of the components of linear measurement.

In Part III, the empirical study is reported. In Chapter 8, the strategy represented in the design is discussed, and a number of hypotheses stated. In Chapter 9, subjects involved, tasks used, and procedures adopted in the study are described. In Chapter 10, a statistical analysis of the results is given.

CHAPTER 8.THE STRATEGY OF THE STUDYAND STATEMENT OFHYPOTHESES.8.1 THE STRATEGY OF THE STUDY.8.1.1 QUESTIONS ASKED IN THE STUDY.

The analysis presented in Chapter 2 provided a list of components assumed to be required for a full understanding of iterative linear measurement. Chapter 2 also provided the following operational definition of linear measurement: 'A child may be said to have a mature understanding of linear measurement, if he demonstrates a capacity to use correctly arithmetical operations instead of carrying out physical measurement operations.' It would be possible for a child who does not possess all of the assumed components to demonstrate linear measurement, according to this definition. This would be the case if the child simply resorted to previously learned rules for substituting arithmetical operations for physical measurement operations. Such a child may be said to know how to 'measure' length but not know why arithmetical operations may be substituted for physical measurement operations. Indeed, it is possible that some adults would not know why arithmetical operations may be used in deriving length measurements. With these issues in mind, several empirical questions were then posed in Chapter 2. They may be summarised under three headings.

- (a) Which of the components are necessary for mature linear measurement?
- (b) Is there an order in which those components emerge in the child's thinking?

- (c) What is the relationship between the growth of linear measurement and the growth of those components?

Piagetian theory and associated empirical evidence were consulted in Part 11 as a source of information regarding these questions. This yielded several predictions concerning the order of emergence of the components.

8.1.2 TYPE OF DESIGN.

The nature of these questions dictated the kind of study needed.

The first question - which components are necessary - could be explored by two types of study: (a) a training study; and, (b) a comparative study.

The other questions - which are concerned with the order in which the abilities emerge - can only be answered by a particular kind of comparative study: one that examines the development of component abilities.

8.1.3 TRAINING STUDY.

A training study would attempt to teach subjects who could not measure length, those abilities deemed necessary. Pre-tests would identify missing elements in each subject's repertoire, and instruction would focus on developing those elements. Post-tests would assess whether (presumably as a consequence of training) linear measurement skills had emerged. A study of that kind would present substantial problems of interpretation. In particular, failure to meet the criterion of linear measurement could indicate that the skills taught to the subject were not necessary components of linear measurement. Alternatively, it could indicate only that the method of instruction used, whilst adequate for conveying skill in using a particular algorithm, did not promote understanding of the component abilities.

8.1.4 COMPARATIVE STUDY.

In contrast, a comparative study would set down a list of abilities which might be necessary for linear measurement. The study would then locate two groups of subjects:

- (a) those who could measure; and
- (b) those who could not.

Subjects in both groups would then be tested to assess the presence or absence of the assumed underlying abilities. Comparisons between the two groups would yield information on which abilities appear to be necessary components of linear measurement.

8.1.5 DEVELOPMENTAL STUDY.

A developmental study differs from the comparative approach mainly in that subjects are tested at various ages.

8.1.6 PREFERRED APPROACH.

Because a developmental study has the potential to answer the two kinds of question asked in the present research, it was decided that it would be the most appropriate. Developmental studies can employ either cross-sectional, longitudinal or scalogram methods.

8.1.7 CROSS-SECTIONAL METHOD.

The cross-sectional method yields average ages at which particular tasks are mastered. Although developmental progressions may be inferred, they are based on age-related differences between groups not on age-related changes within subjects. Moreover, when theoretical considerations suggest that a number of different though related capacities will emerge asynchronously, but within a comparatively brief interval of time, overlapping distributions of scores between groups pose difficulties in interpretation. Consequently, the cross-sectional method can provide only indirect evidence of developmental progressions.

8.1.8 LONGITUDINAL METHOD.

On the other hand, the longitudinal method has the potential to yield direct evidence of developmental progressions because its basic datum is within-subject change over time. Unfortunately, it also carries substantial disadvantages with respect to time, cost, testing effects, selective survival and drop-out rate, and so on.

8.1.9 SCALOGRAM METHOD.

A method that overcomes the disadvantages of the cross-sectional approach, and does not incur the time and cost penalties of the longitudinal procedure, is the scalogram technique. This technique involves administering a battery of tests to a group which includes subjects at varying developmental levels. Analysis of the resultant data focuses on within-subject patterns of passes and fails across the test battery, in order to deter-

mine whether the tasks form a scalable set. In this context, a scalable set is one in which passing a particular task presupposes passing all tasks of lower difficulty ranking. Provided that the tasks have construct validity, demonstrating that they form a scalable set is tantamount to demonstrating that the capabilities being assessed emerge sequentially in the course of development.

8.1.10 CONCLUSION.

Scalogram methods have been applied in a number of Piagetian-type studies, especially noteworthy are those of Wohlwill (1960) and Kofsky (1966). As the logic of those two studies closely resembles that of the present research, it was decided to adopt the scalogram method. All experimental designs, however, have inherent disadvantages. Wohlwill (1960) identified the two main problems arising from use of scalogram techniques for cognitive development research. They result from the fact that the technique scales both the subject and the tasks on the same basis, namely, the pattern of passes and fails across the test battery. Firstly, inferences drawn from such an analysis can only be justified if the researcher is assured that the tasks represent an underlying psychological dimension. This requirement has been met in the present study by selecting tasks drawn from a body of theory that has an extensive empirical base, and by formulating a set of specific and testable hypotheses reflecting the operation of a developmental process. Secondly, it is necessary for the researcher to be able to demonstrate a correlation between age (or some age-related factor, such as length of schooling) and scale-type. In the present study, this desideratum was met by applying multiple regression analysis to subjects' scores, using age and length of schooling as predictors.

8.2 STATEMENT OF HYPOTHESES.

8.2.1 COMPONENTS OF LINEAR MEASUREMENT.

The research cited in Part 11 has produced highly equivocal findings. In addition, the empirical research reported in this thesis was carried out essentially as a data gathering exercise. In view of this, the hypotheses stated in the following paragraphs should be regarded as being only tentative in nature rather than expressions of commitment.

An hypothesis concerning the composition of the linear measurement concept could refer to a long list of abilities of varying levels of complexity, or to a smaller list of higher-order abilities. An example of the former would be that presented in Chapter 2 in association with an analysis of linear measurement. It will be recalled that that list contained non-independent entries, because it also referred to the growth of linear measurement. A smaller list of higher-order abilities could be drawn up in order to avoid, or reduce, redundancy of that kind.

When formulating an hypothesis for this study, it was decided to express the composition of linear measurement in terms of a list of higher-order abilities. The extent to which the entries on the list are independent is an open question. The hypothesis which follows is drawn from the analysis given in Chapter 2, and takes account of the views of Piaget et al. (1960) on the development of linear measurement.

HYPOTHESIS 1.

A subject demonstrating a mature understanding of linear measurement will also demonstrate the following:-

- . knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number);
- . knowing that length is invariant under certain transformations (the conservation of length);
- . knowing that distance is invariant under certain transformations (the conservation of distance);
- . knowing how to make transitive inferences of equivalence and non-equivalence with respect to discrete quantity;
- . knowing how to make transitive inferences of equivalence and non-equivalence with respect to length;
- . knowing how to carry out the arithmetical operations of addition and subtraction;
- . knowing how to obtain a linear measurement by counting iterations of a unit of length.

In this context, a mature understanding of linear measurement was operationally defined as: 'a person will be said to have a mature understanding of linear measurement, if he demonstrates a capacity to use correctly arithmetical operations, instead of carrying out physical measurement operations.' The ability to use a unit of length was operationally defined as: 'a person will be said to be able to use a unit of length, if he can determine, by a process of iteration, how many of the given unit are contained in a given length, and (without resorting to further unit iteration) can determine the effect of changing unit size'. The present study has only considered the case where the given length contains a whole number of units.

8.2.2 ORDER OF DEVELOPMENT OF COMPONENTS
OF LINEAR MEASUREMENT.

The literature reviewed in Part II only yields partial order predictions for the total set of number and length components. However full orderings can be predicted for each domain separately. Therefore, the question of whether there is an order in which the components of linear measurement emerge in the child's thinking was examined by separating number from length components. Each domain includes both late-emerging components (e.g. arithmetical addition), and early-emerging abilities (e.g. counting). Piagetian theory and the associated empirical evidence suggest that, in each domain, development is orderly and predictable: that is, that there is a high probability that A will emerge before B; and that possession of B implies, with a high degree of probability, possession of A. This is the aspect of the study at which the scalogram analyses were directed.

These analyses were carried out to test the following specific hypotheses:-

Order in the Growth of the
 Number Concept.

HYPOTHESIS 2.

The collection of components of the number concept form a scalable set.

Order in the Growth of the
 Length Concept.

Hypothesis 3.

The collection of components of the length concept form a scalable set.

8.2.3 EXPECTED PATTERN OF DEVELOPMENT.

Piagetian theory and the associated empirical evidence also suggest that as well as a particular kind of order, there is a particular pattern of development exhibited by the components of linear measurement.

Predictable patterns of development are especially useful for gaining insight into the growth of a concept, and of the emergence of linkages between associated concepts. Since linear measurement involves knowledge contained in the number, length and distance concepts, and the co-ordination of that knowledge, identification of particular development patterns is relevant to gaining an understanding of its growth. The first two of the hypotheses which follow are concerned with development patterns for the number, length and distance concepts. The remainder are concerned with linkages between these concepts. All hypotheses are drawn directly from the conclusions reached in Part II.

Growth of the
Number Concept.

Hypothesis 4.

For the number concept, the order of emergence of component elements (from earliest to latest) will be the following:-

<u>Rank</u>	<u>Component</u>
1.	<ul style="list-style-type: none">. Knowing how to use a one-to-one matching rule.. Knowing the natural number order.
2.	<ul style="list-style-type: none">. Knowing how to count arrays of small numerosity, where to count implies the co-ordination of ordinal position and cardinal value (numeration).
3.	<ul style="list-style-type: none">. Knowing how to add, when the objects are visible and small numbers are involved.. Knowing how to subtract when the objects are visible and small numbers are involved.
4.	<ul style="list-style-type: none">. Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
5.	<ul style="list-style-type: none">. Knowing how to find the numerical difference between two collections, when the objects are visible.. Knowing how to make two collections equal in number, when the objects are visible.
6.	<ul style="list-style-type: none">. Knowing how to make transitive inferences of equivalence with respect to discontinuous quantity.. Knowing how to make transitive inferences of non-equivalence with respect to discontinuous quantity.

7. . Knowing how to add, when the objects are are not visible.
- . Knowing how to subtract, when the objects are not visible.
8. . Knowing how to co-ordinate addition and subtraction operations, when the objects are not visible.

This ordering was derived mainly from the following partial orderings referred to in Part II:-

Brainerd (1973a, 1973b) - [2→3; 3→4];
 Gelman and Gallistel (1978) - [1→2; 2→3; 3→4; 3→5];
 Siegel (1971a, 1971b) [2→3; 3→4];
 Smedslund (1963) - [4→6].

Growth of the
 Length Concept.

Hypothesis 5.

For the length concept, the order of emergence of component elements (from earliest to latest) will be the following:-

<u>Rank</u>	<u>Component</u>
1.	. Knowing that if length A is greater than length B, then A may be considered as B concatenated with some other length.
2.	. Knowing that the length of an object can be altered only if something is added to, or taken away from, it.

3. . Knowing that the length relation between two objects only changes when something is added to or taken away from one, or other, or both, of the objects.
- . Knowing that the length relation between two objects does not change when the spatial relation is changed.
4. . Knowing how to order objects according to their lengths (length seriation).
5. . Knowing that any length may be considered as a concatenation of arbitrarily selected sub-lengths.
6. . Knowing that length is invariant under certain transformations (the conservation of length).
7. . Knowing that the ordinal length relation between two objects is the same as the cardinal numerical relation between the parts comprising those objects.
8. . Knowing that length relations between objects can be deduced by applying transitive reasoning to the collections of unit parts.
9. . Knowing that transitive reasoning can be applied to relations of equivalence between lengths of objects.
- . Knowing that transitive reasoning can be applied to relations of non-equivalence between lengths of objects.

10. . Knowing how to make quantitative estimates of length, in terms of the number of "unit" lengths.
11. . Knowing how to iterate a unit part along an object.
- . Knowing that if the length of the unit part is changed, the number yielded by unit iteration also changes.
12. . Knowing that the length relation between two objects can be determined by carrying out a linear measurement operation, using unit iteration.
- . Knowing that arithmetical addition of linear measurements may be used to determine the length of concatenated objects.
- . Knowing how to add length relations (e.g. given the ordered length series $a - b - c - d$, what is the relation between lengths $(a+c)$ and $(b+d)$, where the increment in length is constant).

This ordering was derived mainly from the following partial orderings referred to in Part II:-

Mc.Manis (1969) - [6→9];

Piaget et.al. (1960) - [1→2; 2→3; 3→4;

1→4; 1→5; 4→6; 6→10; 6→11; 6→12;

10→11; 11→12];

Smedslund (1963) - [6→9].

Growth of the
Distance Concept.

Hypothesis 6.

For the distance concept, the order of emergence of component elements (from earliest to latest) will be the following:-

<u>Rank</u>	<u>Component</u>
1.	. Knowing how to compare indirectly two distances by a measurement operation not involving unit iteration.
2.	. Knowing that distance is invariant under certain transformations (the conservation of distance)
3.	. Knowing how to estimate distance between two points in terms of the number of "unit" distances.
4.	. Knowing how to measure distance between two points, using unit iteration.

This ordering was derived by analogy with length.

Linkages between
Concepts.

Hypothesis 7.

Knowing how to seriate length emerges earlier than knowing how to numerate. (Elkind, 1964; Piaget, 1952).

Hypothesis 8.

The conservation of number emerges earlier than the conservation of length. (Bearison, 1969; Goldsmidt, 1967; Piaget et.al, 1960; Strauss and Ilan, (1975).

Hypothesis 9.

The conservation of length emerges at about the same time as the conservation of distance. (Piaget et.al., 1960).

8.3 AGE, SEX AND LENGTH OF
SCHOOLING FACTORS.

Subjects differing in age and in length of schooling were used in the study. Additionally, the male/female distribution in the sample was about 50:50. It was possible, therefore, to analyse subjects' performances according to age, length of schooling, and sex.

However, the age range (63 to 78 months) of the subjects was relatively narrow. Given the magnitude of individual differences, it was unlikely that differences in performances between younger and older subjects, after removal of length of schooling (kindergarten to grade one) effects, would be significant. The situation with respect to the length of schooling factor was a little different, because the older subjects were likely to have had one more year of schooling. Consequently, it seemed reasonable to expect that, after removal of any age effect, subjects with more school experience would perform at a higher level than subjects with less. Regarding the sex factor, as discussed by Goldsmidt (1967), research has not generally revealed sex differences.

CHAPTER 9.SUBJECTS, TASKS AND
PROCEDURE.9.1 SUBJECTS.9.1.1 AGE.

Because the research was aimed at identifying developmental sequences, and because it is known that there are wide individual differences with respect to age at which different capacities emerge (Goldsmidt, 1967), it was necessary to choose an age range within which floor and ceiling effects would be minimized. The evidence reported in Part II suggested that 63 to 78 months (five years three months to six years six months) would be appropriate.

Choosing that age range had certain consequences. Firstly, it was expected that, if the subjects were to be evenly distributed in the 63-78 months age range, then it would be almost certain that they would be spread over two classes, namely kindergarten and grade one. This was because local schools' admissions policies precluded the possibility that sufficient numbers of the younger subjects would be found in grade one to permit all subjects to be taken from that class. Additionally, if all subjects had been drawn from kindergarten, then it is possible that a substantial number of the older subjects would have been repeating kindergarten, due to

lack of progress the previous year. Lack of progress in early school years is not necessarily a reflection of I.Q. Moreover, tasks such as the various conservations correlate only moderately with IQ (Goldsmidt, 1967). However, it was decided not to risk importing into the study factors affecting a perhaps small, but unknown, proportion of subjects.

Secondly, if the subjects were to be drawn from two classes, this would present an opportunity to compare mean performance levels between classes so as to evaluate the length of schooling effect. Although such matters were not of concern in relation to testing the main hypotheses given in Chapter 8, it was considered that they might yield information relevant to educational practice.

9.1.2 SEX.

An attempt was made to equate numbers of males and females in each age by length of schooling group. However, for practical reasons, it was not possible to obtain exactly equal numbers of each.

9.1.3 SCHOOL

CURRICULUM.

If subjects had to be drawn from different schools, it was considered important that there be no substantial difference between schools in emphasis upon use of materials such as cuisenaire rods, and Montessori counting spindles, and that equal emphasis be given to traditional training in counting and memorising number facts. This was because a number of the assessment tasks resemble classroom problems set by teachers.

9.1.4 SAMPLING FACTORS.

Pilot testing of tasks and procedures suggested that assessment of the capacities under investigation would require about two to three hours testing for each subject. This requirement posed difficulties for the ACT Schools Authority, as the ACT public primary schools are heavily utilized for routine teacher training and research. Similar difficulties obtain in gaining access to public primary schools in areas of New South Wales adjoining the ACT. Practical considerations dictated that subjects be drawn from private primary schools in the ACT. For statistical purposes, a minimum of 100 subjects was required. They were drawn from a number of different schools, because no single available school had sufficient enrolments in kindergarten and grade one.

A consequence of using subjects from private schools is that the sample selected may not be representative of the general ACT population. However, as the research was not intended to be a normative study, this was not considered to be an important factor. As the school population sampled contained a substantial number of migrant and refugee children, it was decided that teachers' rating of language understanding and performance would be sought before including children in the study.

9.1.5 SUMMARY.

100 subjects were drawn from four primary schools in the ACT. The schools were: one non-denominational private institution, the AME school at Weston; and three Catholic convent schools, St.Thomas More's at Campbell, St. Joseph's at O'Connor, and St.Brigid's at Dickson. The three Catholic sch-

ools are located in affluent inner-city suburbs, and draw their students from local households. The AME school tends to attract students from all all parts of the ACT and, in general, from a highly affluent sector of the population. All four schools followed the broad curriculum guidelines of the ACT Schools Authority for the early primary years, and all appeared to give the same emphasis to memorisation of basic number facts. The distribution of subjects within age, sex and school grade categories across the four schools is given in Table 9.1.

TABLE 9.1: SUBJECT SAMPLE: AGE, SEX AND LENGTH OF SCHOOLING DISTRIBUTION.

SCHOOL	AGE CATEGORY						TOTALS
	YOUNGER*		OLDER**				
	KINDERGARTEN		KINDERGARTEN		GRADE ONE		
	MALE	FEMALE	MALE	FEMALE	MALE	FEMALE	
AME	2	7	0	1	2	5	17
CAMPBELL	2	9	4	2	3	1	21
O'CONNOR	14	9	5	9	3	5	45
DICKSON	5	2	4	0	2	4	17
TOTALS	23	27	13	12	10	15	100
TOTALS	50		25		25		100

Notes: * Subjects were in the eight month age range, 63 to 70 months.

** Subjects were in the eight month age range, 71 to 78 months.

9.2 TASKS.

Full descriptions of the 34 tasks used in the study are given in Appendix 1. The tasks were developed from the analysis given in Chapter 2, and were designed to produce data for testing the hypotheses specified in Chapter 8. Where standard Piagetian forms were available they were used. Every attempt was made to keep the tasks as simple as possible. Additionally, a number of variants of the tasks were pilot-tested in order to determine the most effective forms of presentation. In all cases, simple forms of questioning were employed. Subjects' evaluative responses and verbal justifications were recorded.

All tasks were scored as '1' for a pass and '0' for a fail. Pass and fail criteria for each task are specified in Appendix 1. Every effort was made to ensure that subjects did not guess their answers. Conservative pass/fail criteria were adopted.

Brief statements of the assessment objectives of the tasks are given in the following paragraphs. Number tasks are described first, then length, and then distance. In the interests of brevity in the ensuing discussion, an acronym is given for each task. The first letter of the acronym identifies the task as relating to the number (N), length (L) or distance (D) type. A second letter (R) before the hyphen indicates that the task was concerned with the relations between at least two lengths or distances, as distinct from an evaluation of the outcome of a transformation upon one length or distance. Subsequent letters refer to the operations involved in the task. For example, task LR-TI-NE was concerned with length; relations between length; transitive inferences; between non-equal lengths.

9.2.1 NUMBER TASKS.

N-(1-TO-1). This task assessed the child's ability to determine the numerical relation of equality between three collections of unspecified cardinal value, via the operation of one-to-one correspondence.

[See Chapter 2, (A) (i)]

N-ORD. The subjects had to demonstrate that they could form an ordered series of collections of objects of unequal but unspecified numerosities. That is, the subjects had to form a series of the following kind: a, b, c, d, e , where the relation between any two elements could be determined accurately, but not in terms of specific numerosity. [(A) (ii)]

N-ONT. This task assessed the subject's ability to count small arrays. The child had to demonstrate that he co-ordinated ordinal position and cardinal value while counting nine objects. [(A) (iii)]

N-TI-EQ. The subjects had to make transitive inferences of equality of the following kind: $a=b; b=c; a?c$; where a, b and c represent discontinuous quantities. [(A) (iv)]

N-TI-NE. The subjects had to make transitive inferences of inequality of the following kind: $a.R.b; b.R.c; a?c$; where R represents greater than, and less than; and where a, b , and c represent discontinuous quantities. [(A) (iv)]

N-CONS. This was the standard Piagetian number conservation task involving two rows of objects with various relative length and density patterns. [(A) (v)]

N-ADD-V. This task assessed the subject's ability to predict the results of addition operations when the two collections to be added together were visible. The subject was not permitted to put the collections together and count the number of objects in the combined collection; and nor was the subject permitted to use a pointer, such as a finger, to count one collection and then to move onto the other. The subject was allowed to count out loud, or 'in his head'. Collections of up to 12 objects were used. [(A) (vi)]

N-SUB-V. As for N-ADD-V, but in this task the operation involved was subtraction (e.g. work out how many would be left if 'n' were taken away). The subject was required to predict the outcome. The collections were visible. [(A) (vi)]

N-SOL-V. In this task, the subject had to find the numerical difference between two collections (a and b), and, by addition, or subtraction, make the collections equal (this is usually called solving for a difference). Collections a and b were visible. A third collection was available to draw objects from, or give objects to, in order to solve the problem. The subjects had to solve the problem in one move, and were not allowed to use pointers, such as fingers, during any counting operation. Collections of up to 14 objects were used. [(A) (vi)]

N-BAL-V. The subjects had to solve problems having the following form: if $a > b$ then $[a - (a - b/2)] = [b + (a - b/2)]$. (That is, balance the two collections by sharing the difference between them.) Collections a and b were visible. Subjects had to balance the two numerosities in one move. Collections of up to 14 objects were used. [(A) (vi)]

N-ADD-NV. This task assessed the subject's ability to determine the outcome of adding n objects to a collection of similar objects of known numerosity, but where this latter collection is not visible to the subject. Collections of up to 12 objects were used. The important difference between this task and N-ADD-V is that, in the latter, the objects were visible to the subject. [(A) (vi)]

N-SUB-NV. As for N-ADD-NV, but in this task the operation involved was subtraction. [(A) (vi)]

N-CYC-NV. This task assessed the subject's ability to work concurrently on two collections, in a situation where adding to collection (a) meant subtracting from collection (b) - that is, the objects cycled from one collection to the other. The objects were not visible, except when in transit between collections. As the first step in the task, the subjects found, by counting, the total number of objects (12) in collections (a) and (b). [(A) (vi)]

9.2.2 LENGTH TASKS.

LR-BinA. This task assessed the subject's understanding that if length (a) is greater than length (b) then (a) may be considered as a concatenation of (b) and some other length (that is, a sense in which (b) is included in (a)). [See Chapter 2, (B) (i)]

L-P/W. The subjects had to demonstrate an understanding that any length may be considered as a concatenation of arbitrarily selected sublengths - that is, an understanding of part-whole relations of length. [(B) (ii)]

Note: The next three tasks are all concerned with aspects of the conservation of length. They differ, however, from the standard Piagetian conservation of length task insofar as the materials used, and questions asked, are directed at a particular explanation such as: 'nothing was added'; or 'it only changed its place, that doesn't make it bigger'. The rationale for including them, in addition to the standard task, was given in the discussion in Chapter 2 on the components of linear measurement.

L-INV-ADD. This task assessed the subject's understanding that the length of an object is invariant unless something is added to or subtracted from it - setting aside expansion and contraction processes. [(B) (iii)]

LR-INV-ADD. This task assessed the subject's understanding that the length relation between two objects is invariant unless something is added to, or subtracted from, one of the objects - setting aside expansion and contraction processes. [(B) (iv)]

LR-INV-SP. The subjects were required to demonstrate an understanding that the length relation between two objects is invariant under transformations involving only change of spatial position. [(B) (v)]

LR-ORD. This task assessed the subject's ability to order objects according to their lengths. [(B) (vi)]

LR-TI-EQ. This task assessed the subject's ability to make transitive inferences of equivalence with respect to length. [(B) (vii)]

LR-TI-NE. As for LR-TI-EQ, but with respect to objects of unequal lengths, and, hence, relations of greater than and less than. [(B) (vii)]

LR-CARD. The subjects were required to demonstrate an understanding that the ordinal length relation between two objects is the same as the cardinal numerical relation between the collection of parts comprising those objects (provided that the lengths of those parts are the same). [(B) (viii)]

LR-TI-CARD. The subjects had to deduce length relations between objects by applying transitive inference reasoning to the cardinal number relations between the collections of unit parts. [(B) (ix)]

L-CONS. This was the standard Piagetian conservation of length task, using two pieces of string of equal length. [(B) (x)]

L-UNIT. This task required the subject to iterate a unit part along the length of an object. [(C) (i)]

L-EST. This task assessed the subject's ability to estimate length in terms of: "how many of (a) would you need to put together to make a stick as long as this?" [(C)(i)]

L-UNIT-CH. This task assessed the subject's ability to predict the direction in which the number given by unit iteration would change, if the length of the unit part were to change. [(C)(ii)]

LR-M-CARD. This task assessed the subject's ability to determine the length relation between two objects on the basis of a measurement operation involving unit iteration, and comparison of cardinal numbers. (Notice that the difference between this task and LR-CARD is that, in the latter, the subject does not have to measure the length of each object using unit iteration, because he is told which object has the greater number of parts. Additionally, in LR-CARD the length relation is expressed in terms of "more" or "fewer" parts, not in terms of specific numbers of unit parts.) [(C)(iii)]

L-M-ADD. This task assessed the subject's understanding that numbers representing lengths of objects may be added together, and that the resultant number represents the length of the two objects joined together. [(C)(iv)]

L-ADD. This task assessed the subject's ability to add lengths in the following (semi-algebraic) fashion: given an ordered series, $a-b-c-d$, where the increment in length is constant, what is the relation between the combined lengths $(a+c)$ and $(b+d)$? [(C)(iv)]

9.2.3 DISTANCE TASKS.

D-CONS. This was the standard Piagetian conservation of distance task. Two variants were used. The first involved the comparison of distances traversed along a path between two fixed points. The comparison was between a journey from A to B, and one from B to A. On the B to A journey a wall with a door was placed across the path. The second variant of the task involved the comparison of distances traversed between fixed points for: (a) a journey along a straight path; and (b) a journey along a non straight path. [(D) (i)]

D-EST. This task assessed the subject's ability to estimate distance between two objects in terms of : "how many of these small ones would you need to build a path across there?" [(E) (i)]

DR-M. This task assessed the subject's ability to compare indirectly two distances by carrying out a measurement operation, but not necessarily using unit iteration. [(E) (i)]

D-M The subject had to demonstrate an ability to measure the distance between two points using unit iteration. [(E) (ii)]

9.3 PROCEDURE.9.3.1 ORDER OF ADMINISTRATION.

Because there is some similarity between certain tasks - for example between L-EST (length estimation) and D-EST (distance estimation) - the order of administration was arranged so as to minimise carryover effects. The following is a list of task sequences where carryover effects would be most expected, but undesired:-

- . N-TI-EQ with N-TI-NE
- . N-TI-NE with N-ORD
- . N-ORD with N-CONS
- . L-INVAR-ADD with LR-INVAR-SP
- . LR-INVAR-SP with LR-INVAR-ADD
- . LR-ORD with L-ADD
- . L-ADD with L-P/W
- . L-L/W with LR-CARD
- . LR-TI-EQ with LR-TI-NE
- . LR-TI-EQ with N-TI-EQ
- . LR-TI-EQ with N-TI-NE
- . LR-TI-NE with N-TI-EQ
- . LR-TI-NE with N-TI-NE
- . LR-TI-EQ with LR-TI-CARD
- . LR-TI-NE with LR-TI-CARD
- . LR-TI-CARD with N-TI-EQ
- . LR-TI CARD with N-TI-NE
- . L-EST with D-EST
- . L-UNIT-CH with LR-M-CARD
- . (LR-M-CARD and L-M-ADD) with (D-M and LR-M)

The following is a list of sequences which should go together, because the second task can be presented as an extension of the first:-

- . L-UNIT and L-UNIT-CH
- . LR-M-CARD and L-M-ADD

The order of administration was arranged so that any two tasks which should not be presented sequentially were separated by at least two other tasks. Because of the large number of tasks in the battery the whole sequence was divided into the following four sections, with the order of administration within sections being as indicated below;-

Section 1

- . N-CNT; N-ADD-V; N-SUB-V; N-SOL-V; N-BAL-V; N-ADD-NV; N-SUB-NV;
N-CYC-NV; N-1-TO-1.

Section 2.

- . LR-M-CARD; L-M-ADD; LR-INVAR-ADD; N-TI-NE; L-INVAR-ADD; LR-BinA;
N-ORD; LR-INVAR-SP; LR-ORD.

Section 3.

- . L-ADD; LR-TI-NE; L-CONS; D-CONS; L-P/W; LR-TI-CARD; D-M; DR-M;
LR-TI-EQ.

Section 4.

- . D-EST; L-UNIT; L-UNIT-CH; N-TI-EQ; N-CONS; L-EST; LR-CARD.

9.3.2 TESTING SESSIONS.

The experience gained from pilot testing the tasks suggested that each subject would take from two to three hours to complete the whole battery. With that in mind, it was decided to test each subject over four sessions, each of 30 to 45 minutes in duration: one for each of the sections given in the preceding paragraphs.

All subjects were tested individually in a quiet room, free from distractions, at the subject's school. Typically, the room contained a small low table, two chairs and a cupboard where the experimental materials were stored. All subjects in a class were tested individually, on Section 1 tasks, then on Section 2 tasks, and so on. After all subjects in the class had completed all sessions, the subjects in the next class were then tested on Section 1 tasks, then Section 2 tasks, and so on. This approach meant that no subject was tested twice on any one day, and that there was usually an interval of a few days between sessions for each subject. Before commencing testing with subjects from each class, the experimenter was introduced to the class by the teacher, and spent some time with the class, so that the subjects became familiar with the experimenter. The same procedure was adopted at all schools. Testing commenced in early April, 1980 and continued through to December, 1980. Subjects' ages were recorded to the nearest month, as at date of testing on Session 1. The longest period of elapsed time between commencement of testing on Session 1 and completion of testing on Session 4, for any one subject, was 14 days. The same experimenter was used throughout the study.

CHAPTER 10.

RESULTS OF THE STUDY.

10.1 SUMMARY DATA.

The responses (scored 1 or 0) of all subjects on all tasks are given in Appendix 2. The distributions of total scores for all subjects, and of the number of subjects passing each task, are given in Figures 10.1 and 10.2, respectively.

It is evident that both floor and ceiling effects have been avoided. The distribution of total scores in Figure 10.1 shows only a minor floor effect. This is confirmed by the pattern of task difficulty shown in Figure 10.2.

10.2 COMPONENTS OF LINEAR

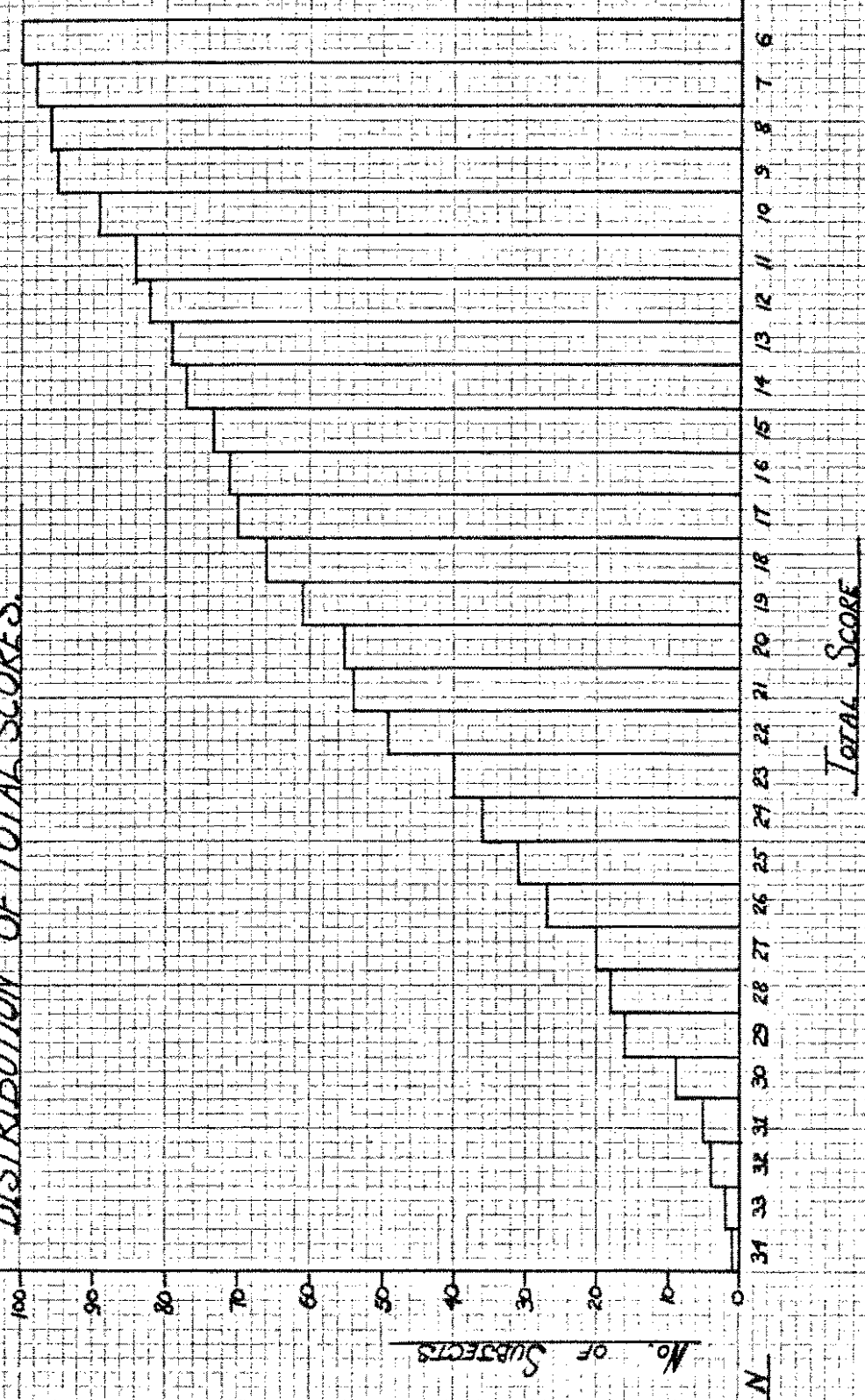
MEASUREMENT.

Hypothesis 1 predicted that subjects who demonstrated an operationally defined level of understanding of linear measurement would also demonstrate that they possessed certain other knowledge assumed to underlie linear measurement.

Table 10.1 shows the number of subjects passing the tasks designed to assess level of understanding of linear measurement (LR-M-CARD and

FIGURE 10.1

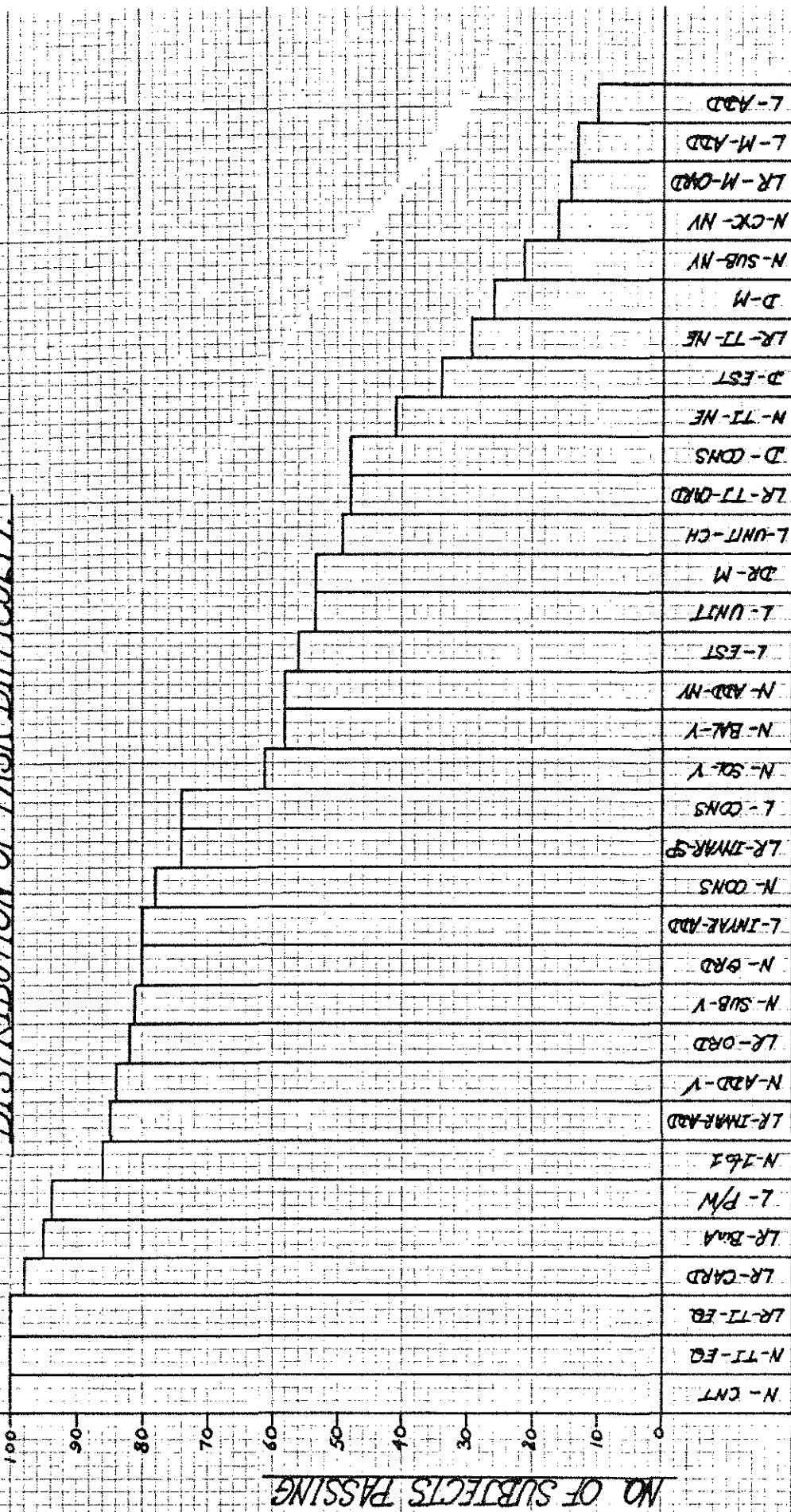
DISTRIBUTION OF TOTAL SCORES.



Each bar indicates no. of subjects achieving a total score of at least N.

FIGURE 10.2

DISTRIBUTION OF TASK DIFFICULTY



TASKS

NO OF SUBJECTS PASSING

L-M-ADD), and the number passing the tasks assessing possession of the assumed underlying knowledge. The McNemar test was used to compare the proportions passing each of the linear measurement tasks with the proportions passing each of the component tasks. The chi-squared coefficients for each test are shown in Table 10.1.

All of the component tasks, except N-SUB-NV, were significantly easier than either LR-M-CARD or L-M-ADD. This finding is consistent with Hypothesis 1.

However, since only a small proportion of the subjects passed the two linear measurement tasks, it is possible that a substantial number of those subjects could have failed the easier tasks. This would not be consistent with Hypothesis 1. Table 10.2 shows the number of subjects who passed both the linear measurement tasks and each of the easier tasks.

The data in Table 10.2 give general statistical support for the hypothesis that the components are pre-requisites for linear measurement. All of the 13 subjects who passed L-M-ADD also passed LR-M-CARD. This confirms their validity as indices of linear measurement. There is a high probability that a subject who passed LR-M-CARD and L-M-ADD will also have passed each of the easier tasks, in all but three cases. The exceptions are D-CONS, LR-TI-NE and N-SUB-NV. The reasons for these exceptions are discussed in the next Chapter. If they are excluded from consideration, eight of the 14 subjects who passed LR-M-CARD also passed all of the component tasks.

It is concluded, that these findings are generally in agreement with Hypothesis 1.

TABLE 10.1: NUMBER OF SUBJECTS PASSING LINEAR MEASUREMENT
TASKS AND HIGH ORDER COMPONENT TASKS
TOGETHER WITH ASSOCIATED CHI-SQUARED VALUES.

TASK	NO. OF SUBJECTS WHO PASSED	MC.NEMAR CHI-SQUARED VALUES:		MC.NEMAR CHI-SQUARED VALUES:	
		LR-M-CARD AND TASKS LISTED IN COLUMN 1.	CHI-SQUARED	P	L-M-ADD AND TASKS LISTED IN COLUMN 1.
N-TI-EQ	100	84.01	<.001	85.01	<.001
LR-TI-EQ	100	84.01	<.001	85.01	<.001
N-CONS	78	62.02	<.001	63.02	<.001
L-CONS	74	56.15	<.001	57.14	<.001
N-ADD-NV	58	38.52	<.001	39.51	<.001
L-UNIT	53	37.03	<.001	38.02	<.001
L-UNIT-CH	49	28.20	<.001	29.17	<.001
D-CONS	48	22.69	<.001	24.60	<.001
N-TI-NE	41	20.48	<.001	21.44	<.001
LR-TI-NE	29	5.94	<.025	7.03	<.01
N-SUB-NV	21	1.89	N.S.	2.72	N.S.
LR-M-CARD	14	-	-	-	-
L-M-ADD	13	-	-	-	-

TABLE 10.2: NUMBER OF SUBJECTS WHO PASSED BOTH THE LINEAR MEASUREMENT
TASKS AND EACH OF THE HIGH ORDER COMPONENT
TASKS.

TASK	NUMBER OF SUBJECTS WHO PASSED LR-M-CARD AND TASKS LISTED IN COLUMN 1 *	%	NUMBER OF SUBJECTS WHO PASSED L-M-ADD AND TASKS LISTED IN COLUMN 1 **	%
N-TI-EQ	14	100	13	100
LR-TI-EQ	14	100	13	100
N-CONS	14	100	13	100
L-CONS	13	93	12	92
N-ADD-NV	12	86	12	92
L-UNIT	14	100	12	92
L-UNIT-CH	11	79	10	77
D-CONS	7	50	7	54
N-TI-NE	11	79	10	77
LR-TI-NE	5	36	5	38
N-SUB-NV	8	57	8	62
LR-M-CARD	-	-	13	100
L-M-ADD	13	93	-	-

Notes: * - Of the 14 subjects who passed LR-M-CARD the numbers who also passed each of the component tasks are shown in Column 2. Hence, the maximum number is 14.

** - Of the 13 subjects who passed L-M-ADD the numbers who also passed each of the component tasks are shown in Column 4. Hence, the maximum number is 13.

10.3 ORDER IN THE GROWTH OF THE
NUMBER CONCEPT.

Hypothesis 2 predicted that the collection of number tasks would form a scalable set. This hypothesis was tested by applying scalogram analysis techniques to the response matrix given in Table 1 of Appendix 2. Guttman (Edwards, 1957) and Loevinger (1947) scaling indices were calculated using a computer program written by the experimenter.

The Guttman analysis yielded a co-efficient of reproducibility of .91; a minimum marginal reproducibility of .7754; and a co-efficient of scalability of .589 (this last statistic is also known as Green's index of consistency).

The co-efficient of reproducibility is a measure of the extent to which a subject's scale score predicts that subject's scale pattern. A co-efficient of greater than .9 is usually considered to be necessary to indicate a valid scale. The minimum marginal reproducibility is the minimum co-efficient of reproducibility that could have occurred given the proportion of subjects passing and failing each item (in this case, task). The co-efficient of scalability takes account of the minimum marginal reproducibility and the co-efficient of reproducibility. As a composite measure it provides a more reliable guide to the scaling characteristics of a set of items. A co-efficient of scalability of greater than .5 is required to indicate a unidimensional and cumulative set.

In the present case, the computed co-efficient of reproducibility exceeds .9, and the co-efficient of scalability exceeds .5. Hence, the Guttman analysis suggests that the collection of 13 number tasks is a scalable set.

However, the Guttman technique has been criticised (Green, 1954;1956) for relying too heavily on marginal row and column totals of passes and fails. In contrast, the Loevinger technique takes account of individual patterns of pass/fail across the whole test battery. In situations where the test battery contains a large number of items with a high probability of yielding tied scores - for both items and subjects - the Loevinger technique seems better suited (Kofsky, 1966; Wohlwill, 1960). For those reasons it seemed prudent, in the present case, to place greater emphasis on the Loevinger indices.

The Loevinger analysis yielded an index of homogeneity of .570. The interpretation of this index is the same as for Green's index of consistency. It measures the extent to which the set is unidimensional and cumulative. Again, an index of greater than .5 is required to indicate a scalable set. It would appear, therefore, that, whether measured by Guttman or Loevinger techniques, the collection of tasks is a scalable set.

Loevinger's technique also requires that a matrix of indices be calculated. Each entry is a value of $H(it)$, the "index of homogeneity of an item (i) with a test (t)". $H(it)$ measures the extent to which the item contributes to overall test homogeneity. An item is regarded as perfectly homogeneous with a test if all subjects passing the item have higher scores on the test as a whole, than all of those failing the item. A perfectly homogeneous item would have a $H(it)$ of 1, but a $H(it)$ of .7 is regarded (Kofsky, 1966) as acceptable. Table 10.3 sets out the $H(it)$'s computed for the number tasks.

TABLE 10.3: NUMBER TASKS: INDEX OF HOMOGENEITY OF AN ITEM WITH A TEST

TASK *	INDEX OF HOMOGENEITY.
N-CNT	1.0
N-TI-EQ	1.0
N(1-TO-1)	.88
N-ADD-V	.95
N-SUB-V	.90
N-ORD	.78
N-CONS	.85
N-SOL-V	.90
N-BAL-V	.93
N-ADD-NV	.77
N-TI-NV	.76
N-SUB-NV	.99
N-CYC-NV	.98

Notes: * - Listed in order of increasing difficulty.

Table 10.3 shows that all 13 $H(it)$'s have a discriminant efficiency of greater than .7, and that 10 out of the 13 have a discriminant efficiency of greater than .8. In comparison, Kofsky (1966) found that 2 out of her 11 classification tasks had $H(it)$ values of less than .7. Hence, the present $H(it)$ values support the hypothesis that the collection of number tasks is a scalable set.

Loevinger's third statistic, $H(ii)$, called "homogeneity of two items", deals with the relationship between two items in a perfectly homogeneous test. In such a test all those who pass the harder also pass the easier item. In contrast, the $H(it)$ statistic only measures the extent to which those passing an item have higher scores on the test overall, than those failing the item. Hence, $H(it)$ does not identify those who pass harder but fail easier items. For example, $H(it)$ does not discriminate the subject who passes the 10th ranked item and at least nine other items, but fails one or more of the items ranked 1 to 9, from the subject who achieves a score of 10 and passes only (but all of) the items ranked 1 to 10. To complete the analysis of homogeneity, therefore, it is necessary to inspect the matrix of $H(ii)$'s and find the proportion having values greater than .5, chance level of responding. Table 10.4 contains the matrix of $H(ii)$'s for the number tasks.

- Inspection of the matrix of $H(ii)$ values for the number tasks reveals that all but 8 of the 78 indices exceed .5 (chance level). That is, only 10% of all item pairs show reversal or chance level responding. In comparison, Kofsky (1966) found that 36 of her 55 inter-item comparisons were less than .5.

Hence, the impression of scalability is supported by the Guttman co-efficient of reproducibility, Green's index of consistency, Loevinger's index of homogeneity of the test as a whole, Loevinger's index of homogeneity of an item with a test, and Loevinger's index of homogeneity of an item with an item. It is concluded, that technically the collection of number tasks is a scalable set. This provides statistical support for Hypothesis 2.

TABLE 10.4: NUMBER TASKS: INDEX OF HOMOGENEITY OF AN ITEM WITH AN ITEM.

	N-TI-EQ	N-(1-TO-1)	N-ADD-V	N-SUB-V	N-ORD	N-CONS
N-CNT	1.00	1.00	1.00	1.00	1.00	1.00
N-TI-EQ		1.00	1.00	1.00	1.00	1.00
N-(1-TO-1)			.40	.38	.55	.91
N-ADD-V				.77	.38	.52
N-SUB-V					.34	.33
N-ORD						.49

TABLE 10.4 cont.

	N-SOL-V	N-BAL-V	N-ADD-NV	N-TI-NE	N-SUB-NV	N-CYC-NV
N-CNT	1.00	1.00	1.00	1.00	1.00	1.00
N-TI-EQ	1.00	1.00	1.00	1.00	1.00	1.00
N-(1-TO-1)	.77	.75	.63	.83	1.00	1.00
N-ADD-V	1.00	1.00	.89	.85	1.00	1.00
N-SUB-V	.91	1.00	.82	.74	1.00	1.00
N-ORD	.67	.66	.66	.76	.76	.69

TABLE 10.4 cont.

	N-SOL-V	N-BAL-V	N-ADD-NV	N-TI-NE	N-SUB-NV	N-CYC-NV
N-CONS	.70	.69	.53	.89	1.00	1.00
N-SOL-V		.69	.43	.62	1.00	1.00
N-BAL-V			.59	.65	1.00	1.00
N-ADD-NV				.36	1.00	1.00
N-TI-NE					.68	.68
N-SUB-NV						1.00

Note: Tasks are arranged left to right and top to bottom in order of increasing difficulty.

The fact that it is not perfectly scalable means that some subjects exhibited reverse ordering. This could represent real heterogeneity in order of emergence or it might reflect error of measurement due, for example, to fluctuation in attention.

10.4 ORDER IN THE GROWTH OF THE LENGTH CONCEPT

Hypothesis 3 predicted that the collection of 17 length tasks would form a scalable set. This prediction was also examined using scalogram analysis. The following four statistics provide an indication that the collection is a scalable set.

- . Guttman's α -efficient of reproducibility = .89
- . Guttman's minimal marginal reproducibility = .78
- . Green's index of consistency = .48
- . Loevinger's index of homogeneity = .58

It will be remembered from the discussion on the scalability of the number tasks that the above Guttman values, and Green's index, are very close to the levels required for the collection to be considered a scaled set. Also, the Loevinger index of homogeneity of .58 exceeds the .5 criterion level. Bearing in mind the arguments favouring the Loevinger technique, it is reasonable to conclude that there is substantial order in the collection of length tasks.

Table 10.5 sets out the computed $H(it)$ values for the length tasks.

It will be seen from Table 10.5 that all but 3 of the 17 $H(it)$ values exceed .7, and that 2 of those 3 are among the easiest of the tasks (the easier the task the greater the effect on the computed $H(it)$ value of a chance fail by a subject). This result confirms the impression of order given by the indices relating to overall test scalability.

Table 10.6 sets out the matrix of computed $H(ii)$ values for the length tasks.

Examination of the matrix of inter-item comparisons reinforces the impression of order, as all but 35 of the 136 pairings have $H(ii)$'s exceeding .5, chance level. Additionally, two items, L-P/W and LR-CARD, together account for 15 of the 35 chance level or reversal type indices. It is noteworthy that the $H(it)$ values for these tasks were below .7 and that these are among the easiest of the tasks in the length subset. These two factors suggest that the reversal rates for these two tasks are unduly affected by a small number of chance failures.

TABLE 10.5: LENGTH TASKS: INDEX OF HOMOGENEITY OF AN ITEM WITH A TEST.

TASKS *	INDEX OF HOMOGENEITY
LR-TI-EQ	1.00
LR-CARD	.49
LR-BinA	.79
L-P/W	.65
LR-INVAR-ADD	.82
LR-ORD	.76
L-INVAR-ADD	.75
LR-INVAR-SP	.82
L-CONS	.73
L-EST	.83
L-UNIT	.90
L-UNIT-CH	.81
LR-TI-CARD	.77
LR-TI-NE	.64
LR-M-CARD	.89
L-M-ADD	.90
L-ADD	.81

Note: * Listed in order of increasing difficulty.

TABLE 10.6 cont.

	L-EST	L-UNIT	L-UNIT-CH	LR-TI- CARD	LR-TI-NE	LR-M-CARD	L-M-ADD	L-ADD
LR-TI-EQ	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
LR-CARD	.11	1.00	-.02	-.04	1.00	1.00	1.00	1.00
LR-BinA	.64	.62	1.00	1.00	.31	1.00	1.00	1.00
L-P/W	.70	.69	.66	.31	1.00	1.00	1.00	1.00
L-INVAR	.52	.75	.73	.86	.54	1.00	1.00	1.00
-ADD								
LR-ORD	.50	.79	.77	.77	.81	1.00	1.00	1.00
L-INVAR	.64	.81	.80	.58	.83	.64	.62	.00
-ADD								
LR-INVAR	.66	.85	.76	.60	.60	.73	.70	.23
-SP								
L-CONS	.52	.78	.53	.60	.60	.73	.70	.23
L-EST		.61	.63	.48	.61	1.00	1.00	1.00
L-UNIT			.61	.47	.49	1.00	1.00	.79
L-UNIT- CH				.51	.46	.58	.55	.61
LR-TI -CARD					.67	.45	.56	1.00
LR-TI-NE						.09	.13	.58
LR-M-CARD							1.00	.53
L-M-ADD								.54

Note: Tasks are listed left to right and top to bottom in order of increasing difficulty.

On the basis of the Guttman, Green and Loevinger indices, it is concluded that technically the collection of length tasks is a scaled set. This provides statistical support for Hypothesis 3. However, some subjects did exhibit reverse ordering. As with the number tasks, this could represent real heterogeneity in order of emergence, or it might reflect error of measurement.

10.5 EXPECTED PATTERN OF DEVELOPMENT OF THE NUMBER CONCEPT.

The predicted (Hypothesis 4) and the observed orders of difficulty of all tasks in the number collection are given in Table 10.7.

Inspection of Table 10.7 indicates that there is substantial agreement between the predicted and observed rankings. The degree of association between the two rankings was assessed by computing the Spearman rank correlation statistic R_s . (corrected for ties), which is .72. This is significant at the .01 level (sample $t = 3.4389$, criterion $t = 2.718$ at $\alpha = .01$ and 11 d.f. for a 1 tailed test).

The three main differences between the rankings are the following:-

- (a) It was expected that more subjects would pass N-(1-TO-1) (one-to-one correspondence) and N-ORD (number name order) than the N-CNT (numeration) task, but the latter is considerably easier.
- (b) N-ORD was expected to be easier than N-ADD-V and N-SUB-V (addition and subtraction when objects are visible), but they are of approximately equal difficulty.

TABLE 10.7: NUMBER TASKS: PREDICTED AND OBSERVED ORDER OF DIFFICULTY
OF TASKS.

TASK	No.of Ss.PASSING*	OBSERVED RANK	PREDICTED RANK**
N-CNT	100	1.	2.
N-TI-EQ	100	1	6
N-(1-TO-1)	86	2	1
N-ADD-V	84	3	3
N-SUB-V	81	4	3
N-ORD	80	5	1
N-CONS	78	6	4
N-SOL-V	61	7	5
N-BAL-V	58	8	5
N-ADD-NV	58	8	7
N-TI-NE	41	9	6
N-SUB-NV	21	10	7
N-CYC-NV	16	11	8

Notes: * Maximum of 100.

** These are the rankings predicted by Hypothesis 4.

(c) N-TI-EQ (transitive inferences concerning equivalence relations) was expected to emerge synchronously with N-TI-NE, and to be substantially more difficult than N-CONS (conservation), but it is one of the two easiest tasks.

It is concluded that the data generally support the pattern of development of the number concept predicted by Hypothesis 4.

10.6 EXPECTED PATTERN OF
DEVELOPMENT OF THE
LENGTH CONCEPT.

Table 10.8 sets out the predicted (Hypothesis 5) and observed orders of difficulty of all tasks in the length collection.

Table 10.8 shows that there is substantial agreement between the two orders of difficulty. The Spearman rank correlation statistic R_s (corrected for ties) is .74. This is significant at the .01 level (sample $t = 4.2686$, criterion $t = 2.602$ at $\alpha = .01$ and 15 d.f. for a 1 tailed test).

The main differences between the observed and predicted orders of difficulty are the following:-

(a) It was expected that attainment of LR-CARD (ordinal length relation between objects is the same as the cardinal numerical relation between the collections of unit parts comprising those objects) would be delayed

TABLE 10.8: LENGTH TASKS: PREDICTED AND OBSERVED ORDER OF DIFFICULTY OF
TASKS.

TASK	No. of SUBJECTS PASSING*	OBSERVED RANK	PREDICTED RANK**
LR-TI-EQ	100	1	9
LR-CARD	98	2	7
LR-BirA	95	3	1
L-P/W	94	4	5
LR-INVAR-ADD	85	5	3
LR-ORD	82	6	4
L-INVAR-ADD	80	7	2
LR-INVAR-SP	74	8	3
L-CONS	74	8	6
L-EST	56	9	10
L-UNIT	53	10	11
L-UNIT-CH	49	12	11
LR-TI-CARD	48	13	8
LR-TI-NE	29	14	9
LR-M-CARD	14	15	12
L-M-ADD	13	16	12
L-ADD	10	17	13

Notes: * Maximum of 100.

** These are the rankings predicted by Hypothesis 5.

until L-CONS (conservation of length) had emerged, yet the former is the second easiest of all length tasks.

(b) L-TI-EQ (transitive inferences concerning equivalence relations) was also expected to emerge after L-CONS, and synchronously with L-TI-NE (transitive inferences involving non-equivalence relations), but it is the easiest of all length tasks.

It is concluded that the data generally support the pattern of development of the length concept predicted by Hypothesis 5.

10.7 EXPECTED PATTERN OF
DEVELOPMENT OF THE
DISTANCE CONCEPT.

Table 10.9 sets out the order of difficulty for the distance tasks predicted by Hypothesis 6, and that observed in the study.

TABLE 10.9: DISTANCE TASKS - PREDICTED AND OBSERVED ORDER OF DIFFICULTY
OF TASKS.

TASK	No. of SUBJECTS PASSING*	OBSERVED RANK	PREDICTED RANK**
DR-M	53	1	1
D-CONS	48	2	2
D-EST	34	3	3
D-M	26	4	4

Notes: * Maximum of 100.

** These are the rankings predicted by Hypothesis 6.

It is apparent from Table 9 that the predicted and observed rankings are the same at all four points in the sequence. Hence, it is concluded that the data support the pattern of development of the distance concept predicted by Hypothesis 6.

10.8 LINKAGES BETWEEN CONCEPTS.

10.8.1 LENGTH SERIATION AND NUMERATION.

Hypothesis 7 predicted that knowing how to seriate lengths would emerge earlier than knowing how to numerate. The former was tested by task LR-ORD, and the latter by N-CNT. The number of subjects passing LR-ORD was 82, while all 100 subjects passed N-CNT. The McNemar chi-squared value for the difference in proportions passing LR-ORD and N-CNT is 16.06, which is significant at the .001 level and 1 d.f. Hence, N-CNT is significantly easier than LR-ORD. The data, therefore, do not support Hypothesis 7, they show that numeration precedes seriation of length.

10.8.2 NUMBER AND LENGTH CONSERVATION.

Hypothesis 8 predicted that the conservation of number would emerge earlier than the conservation of length. N-CONS was passed by 78 subjects, and L-CONS by 74 subjects. The associated McNemar chi-squared coefficient of 0.50 is not significant (.05 level). Hence, the data do not support Hypothesis 8.

The low chi-squared coefficient means that there is no difference between the proportions passing N-CONS and L-CONS. This could be interpreted as indicating that N-CONS and L-CONS emerge synchronously. Alternatively, it could indicate that both emerge at a much younger age and, hence, that the present data offer no evidence on their order of emergence. The number of subjects passing and failing each task is shown in Table 10.10.

TABLE 10.10: NUMBER OF SUBJECTS PASSING AND FAILING NUMBER
AND LENGTH CONSERVATION TASKS.

	NUMBER CONSERVATION			TOTAL
	No. of SUBJECTS			
		PASS	FAIL	
LENGTH CONSERVATION	PASS	67	7	74
	FAIL	11	15	26
TOTAL		78	22	100

Table 10.10 shows that most of the subjects who passed one of the tasks also passed the other, and that most of those who failed one task also failed the other. Furthermore, of the 22 subjects who failed N-CONS, seven passed L-CONS, and of the 26 who failed L-CONS, 11 passed N-CONS. This is the pattern that would be expected for synchronous emergence. It is concluded, therefore, that the data suggest that number and length conservation emerge at about the same time.

10.8.3 LENGTH AND
DISTANCE CONSERVATION.

Hypothesis 9 predicted that the conservation of length would emerge at about the same time as the conservation of distance. L-CONS was passed by 74 subjects, but only 48 passed D-CONS. The associated McNemar chi-squared coefficient of 15.63 is significant at $\alpha = .001$ and with 1 d.f. Therefore, the data do not support Hypothesis 9.

The number of subjects passing and failing each task is shown in Table 10.11.

TABLE 10.11: NUMBER OF SUBJECTS PASSING AND FAILING LENGTH AND
DISTANCE CONSERVATION TASKS.

	LENGTH CONSERVATION			TOTAL
	No. of SUBJECTS			
		PASS	FAIL	
DISTANCE CONSERVATION	PASS	42	6	48
	FAIL	32	20	52
TOTAL		74	26	100

Table 10.11 shows that of the 74 subjects who passed L-CONS, 32 failed D-CONS, and of the 48 who passed D-CONS, only six failed L-CONS. This is the pattern that would be expected for a length-then-distance sequence. It is concluded that the data show that the conservation of length emerges before the conservation of distance.

10.9 THE EFFECTS OF AGE, LENGTH OF SCHOOLING
AND SEX.

The age, length of schooling, and sex classifications of the sample of 100 schoolchildren used in the study are given in Table 10.12.

TABLE 10.12: GROUP CHARACTERISTICS - No. OF SUBJECTS BY GROUP.

AGE	SCHOOL GRADE	SEX		TOTALS
		MALE	FEMALE	
Younger*	Kindergarten	23	27	50
Older**	Kindergarten	13	12	25
	Year 1	10	15	25
TOTALS		46	54	100

Notes: * 63 to 70 months.

** 71 to 78 months.

10.9.1 DIFFERENCES BETWEEN

GROUP MEANS.

Table 10.13 sets out the means and standard deviations of the total scores (number of passes out of 34) for each of the six groups identified in Table 10.12.

TABLE 10.13: TOTAL SCORES ON ALL TASKS - GROUP MEANS AND STANDARD DEVIATIONS.

GROUP		MEANS*	STANDARD DEVIATIONS	No. of SUBJECTS
No.	Characteristics			
1.	Younger-Kinder-Male	18.00	7.02	23
2.	Younger-Kinder-Female	16.70	6.36	27
3.	Older-Kinder-Male	18.77	6.22	13
4.	Older-Kinder-Female	22.17	3.87	12
5.	Older-Year 1-Male	28.90	2.95	10
6.	Older-Year 1-Female	27.67	4.14	15

Note: * Maximum score = 34.

10.9.2 MULTIPLE REGRESSION ANALYSIS.

ALL TASKS.

A multiple regression analysis was carried out using the heirarchical method of decomposition. Age, then length of schooling, and then the interaction term, age by length of schooling, were taken into the regression equation. The results are summarised in Table 10.14.

Table 10.14 shows that the multiple correlation of performance on age and length of schooling is .60289. This is highly significant ($P < .001$). The table also shows that there is no significant age effect, but there is a significant ($P < .01$) length of schooling effect. There is no significant interaction between age and length of schooling.

TABLE 10.14: ALL TASKS - SUMMARY OF MULTIPLE REGRESSION ANALYSIS.

VARIABLE	MULT.R.	R-SQUARED CHANGE	BETA	F-RATIO	P
AGE	.54939	.30183	.24311	3.571	>.05
LENGTH OF SCHOOLING	.60289	.06164	.39427	9.393	<.01

NUMBER TASKS.

The results of the multiple regression analysis carried out on the number task scores are summarised in Table 10.15.

TABLE 10.15: NUMBER TASKS - SUMMARY OF MULTIPLE REGRESSION ANALYSIS.

VARIABLE	MULT. R	R-SQUARED CHANGE	BETA	F-RATIO	P
AGE	.485	.235	.27024	3.826	>.05
LENGTH OF SCHOOLING	.516	.030	.27670	4.011	<.05

The multiple correlation of performance on age and length of schooling for the number tasks is .516. This is highly significant ($P < .001$). Table 10.15 shows that there is a significant ($P < .05$) length of schooling effect, but no significant age effect.

LENGTH TASKS.

Table 10.16 contains a summary of the multiple regression analysis carried out on the length task scores.

TABLE 10.16: LENGTH TASKS - SUMMARY OF MULTIPLE REGRESSION ANALYSIS.

VARIABLE	MULT.R	R-SQUARED CHANGE	BETA	F-RATIO	P
AGE	.555	.309	.26779	4.329	<.05
LENGTH OF SCHOOLING	.602	.054	.37030	8.279	<.01

The multiple correlation of performance on age and length of schooling for the length tasks is .602. This is highly significant ($P < .001$). Table 10.16 shows that both the age and length of schooling effects are significant ($P < .05$ and $< .01$, respectively).

10.9.3 SUMMARY.

The preceding analyses show that length of schooling is a significant predictor of performance on all tasks, on the number tasks, and on the length tasks. In contrast, age is a significant predictor of performance only for the length tasks. This should be treated with caution, because age is not significant for all tasks - due to the added variance of total scores. There is no evidence of a sex effect.

The size of the length of schooling effect, as compared with the age effect, is interesting. It implies that length of schooling is more effective than age in promoting intellectual growth in the number and length concepts, within the age range used in the present study. This finding is relevant to primary schools' admissions policies concerning the minimum age at which children may commence school. Under present policies controlling entry to many Australian primary schools, a considerable number of children are delayed for periods of up to six months in commencing school, because their fifth birthday happens to fall after certain cut-off dates. Although level of intellectual development and capacity to learn are only two of the many factors that need to be considered when assessing a child's readiness for school, the findings of the present study suggest that there are many children who are being delayed in commencing school but who are capable of making intellectual progress through school experience.

10.10 THE EFFECT OF SCORING CRITERIA ON THE FINDINGS.

The preceding analysis is based on data derived from a clinical-style of assessment and scored using a strict pass/fail criterion. The criterion is strict because correct answers were required to all questions before a child was credited with possessing the knowledge the task was designed to tap.

Additionally, because a clinical-style of assessment was used, there is considerable variation between tasks in the number and type of questions asked. It is possible, therefore, that the observed differences in task difficulty might stem, to some extent, from the approach adopted to scoring subjects' responses. This possibility is explored in Appendix 5.

Firstly, in the case of the number tasks there is no correlation between the number of questions asked and task difficulty (Spearman $R_s = .37$). For the length tasks there is a significant correlation (Spearman $R_s = -.42$), but in the opposite direction to that which might have been predicted. In other words, the length data analysis reveals that the fewer the number of questions asked the greater the difficulty of the task.

Secondly, subjects' responses were reanalysed using moderate and weak scoring criteria (defined at paragraphs A5.2.1 and A5.2.2 of Appendix 5). The orders of difficulty obtained under each scoring procedure were compared using a Spearman rank correlation analysis. This indicated that for all tasks taken as one collection, for the collection of number tasks, and for the collection of length tasks the correlations obtained under the strict, moderate and weak criteria are high. The orderings obtained under the three scoring procedures are low, however, for the collection of distance tasks. This finding reflects the fact that the distance collection contains only four tasks with similar pass rates. Hence the correlations are very sensitive to small fluctuations in pass rates.

Thirdly, Guttman and Loevinger scalogram analyses were also carried out on the data derived from the moderate and weak scoring criteria. It was found that the effect of adopting less stringent criteria is to reduce marginally overall test homogeneity, and to increase marginally the incidence of chance-level responding. However, the number and length task collections form scaled sets, whether assessment is based upon a strict, moderate or weak criterion.

10.11 SUMMARY OF FINDINGS.

The following is a summary of the main findings reported in this analysis.

10.11.1 COMPONENTS OF LINEAR MEASUREMENT.

Subjects who demonstrated a mature understanding of linear measurement also demonstrated the following:-

- . Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
- . Knowing that length is invariant under certain transformations (the conservation of length).
- . Knowing how to make transitive inferences of equivalence and non-equivalence, with respect to discrete quantity.
- . Knowing how to make transitive inferences of equivalence, with respect to length.
- . Knowing how to carry out numerical addition operations.
- . Knowing how to obtain a linear measurement by counting iterations of a unit of length.

10.11.2 ORDER OF DEVELOPMENT OF LINEAR MEASUREMENT.

For number and length the collections of component tasks form scalable sets. That is, development in the number and length concepts is orderly and predictable.

10.11.3 EXPECTED PATTERN
OF DEVELOPMENT.

For number, length and distance the order of emergence of the components is, in general, that predicted by Piagetian theory, and the empirical evidence reviewed in Part 11. There are exceptions, however. The most notable is the emergence of the conservation of number before transitive inference concerned with non-equivalent relations between discrete quantities. This same lag in development between conservation and transitive inference also occurs with respect to length. Additionally, certain components of the length concept emerge earlier than corresponding components (e.g. conservation) of the distance concept.

PART IV.INTERPRETATION OF RESULTS, DISCUSSIONAND CONCLUSIONS.

Chapter 11 interprets the results of the statistical analysis described in Chapter 10, in the light of Piagetian theory and previous empirical evidence. That interpretation raises questions regarding the role played by short-term-memory-capacity limitations in forming the observed developmental patterns. These questions are examined in Chapter 12, using an information processing analysis, and, in Chapter 13, by computer modelling. Additionally, Chapter 13 argues that a detailed process model of linear measurement needs to be developed. Chapter 13 also presents a number of production systems that constitute a beginning of that project. The conclusions reached in the study are then summarised in Chapter 14.

CHAPTER 11DISCUSSION OF RESULTS11.1 THE COMPONENTS OF LINEAR MEASUREMENT.

The findings reported in Part III are generally consistent with both the analysis of linear measurement presented in Chapter 2 and Piagetian theory. However, some aspects of these findings need closer examination. They are the results relating to the connections between linear measurement and the following:-

- (a) arithmetical proficiency;
- (b) transitive inferences regarding length relations of non-equivalence;
- (c) the conservation of length and the conservation of distance;
- (d) the use of a 'unit' of length;
- (e) the estimation of length;
- (f) the lag in development between length and distance.

11.1.1 ARITHMETICAL PROFICIENCY
AND LINEAR MEASUREMENT.

The operational definition of linear measurement employed in this study required the child to substitute arithmetical operations for measurement operations. That definition immediately raised the problem of defining arithmetical proficiency. In this study, arithmetical proficiency was indicated by the ability to carry out addition and subtraction operations concerning objects that are not visible. These operations were assessed by tasks N-ADD-NV and N-SUB-NV. The child could not pass those tasks simply by rearranging objects and counting. In order to pass them, he had to know an addition and a subtraction algorithm, and to be able to apply them to an internal representation of the problem.

A reason for requiring that a child pass both tasks was that Piagetian theory argues that arithmetical proficiency marks the synthesis of the logical grouping structures and the elements of the numerical group structures. The result of that synthesis is said to be numerical operations. The hallmarks of numerical operations are that they are reversible - implying addition and subtraction - and that they can be carried out on symbols - implying that the objects involved need not be visible to the child.

It will be recalled that, in the present study, only 14 of the 100 subjects passed LR-M-CARD, and only 13 passed L-M-ADD, the tasks assessing mature linear measurement knowledge. Of the 14 who passed LR-M-CARD, 12 passed N-ADD-NV, but only eight passed N-SUB-NV. Of the 13 who passed

L-M-ADD, 12 passed N-ADD-NV, but only eight passed N-SUB-NV. On that basis, it could not be said that arithmetical proficiency was a pre-requisite for linear measurement.

Moreover, 58 subjects passed N-ADD-NV, and 21 passed N-SUB-NV, yet only 14 passed LR-M-CARD. All subjects who passed N-SUB-NV, also passed N-ADD-NV. Hence, there were 21 subjects who passed N-ADD-NV and N-SUB-NV. Of those 21, 13 failed LR-M-CARD. This would suggest that arithmetical proficiency is not sufficient for linear measurement. That, however, would be consistent with the Piagetian view, because that theory argues that linear measurement ability appears after the emergence of arithmetical proficiency.

In short, the findings suggest that there are some subjects who can measure length but are not proficient at arithmetic, and others who are proficient at arithmetic but cannot measure length. The explanation probably lies in the tasks used. Neither LR-M-CARD, nor L-M-ADD, require the subject to carry out a subtraction operation. The former requires a numerical comparison to be performed, and the latter, an addition operation. Hence, not knowing how to subtract would not constitute a barrier to passing either of the tasks used to assess linear measurement. This analysis, however, is not consistent with the Piagetian view, because that theory argues that number mastery - knowing how to add and subtract - should precede attainment of linear measurement. The analysis would, though, be consistent with Gagne's (1968) componential theory, since that view of human learning argues that what is important in determining whether a child can solve a particular cognitive problem is whether or not it has the components or rules required, as distinct from the concepts implicated in the problem solution. It may be said, therefore, that from Gagne's

viewpoint a better study would have included a length subtraction task analogous to L-M-ADD in the test battery. The operational definition could then have been made more stringent, by requiring that subjects pass all three tasks to demonstrate possession of linear measurement ability.

11.1.2 TRANSITIVE REASONING AND LINEAR MEASUREMENT.

It was argued in Chapter 2, that transitive reasoning with respect to number and length were involved in linear measurement. Piagetian theory makes the same claim.

In the present study, five tasks were used to assess various kinds of transitive reasoning. Two were concerned with number, N-TI-EQ and N-TI-NE; and three with length, LR-TI-EQ, LR-TI-NE and LR-TI-CARD. Distinctions were made between transitive inferences concerning equivalence (EQ) and non-equivalence relations (NE), because it was not known whether both forms were implicated in linear measurement. Piagetian theory is silent on that matter. LR-TI-CARD is a composite task. It requires subjects to make transitive inferences regarding length relations. However, the premises are expressed in terms of the number of unit parts contained in each object, not in terms of whole lengths.

It was found that all subjects passed N-TI-EQ and LR-TI-EQ; 48 passed LR-TI-CARD; 41 passed N-TI-NE; 29 passed LR-TI-NE; and 14 and 13 passed the linear measurement tasks, LR-M-CARD and L-M-ADD, respectively. Hence, the transitive reasoning tasks were easier than the linear measurement tasks. It might seem, therefore, that these data are consistent with the predictions emanating from theoretical analyses.

However, of the 13 subjects who passed L-M-ADD, 10 passed LR-TI-CARD and 10 passed N-TI-NE, but only 5 passed LR-TI-NE. These figures suggest that transitive inferences concerning number relations of equivalence and non-equivalence, and transitive inferences concerning length relations of equivalence, are implicated in linear measurement. They also suggest that transitive inferences concerning length relations of non-equivalence are not implicated.

A closer examination of the operations involved in linear measurement indicates that this finding could have been anticipated. The transitive reasoning implicated in unit iteration is concerned only with equivalence relations. The transitive reasoning implicated in the comparison phase (length A with length B) may be concerned with non-equivalent relations, but with respect to number, not length. This is because at the stage that the comparison is made the subject is working with numbers not lengths, or, at most, only indirectly with lengths.

11.1.3 CONSERVATION AND LINEAR MEASUREMENT.

The conservation of number task (N-CONS) was passed by 78 subjects, and the conservation of length task (L-CONS) by 74 subjects. Only 14 subjects passed LR-M-CARD, and only 13 passed L-M-ADD. Moreover, of the 14 subjects who passed LR-M-CARD, 14 passed N-CONS, and 13 passed L-CONS. Of the 13 subjects who passed L-M-ADD, 13 passed N-CONS, and 12 passed L-CONS. Clearly, the linear measurement tasks were much harder than the conservation tasks. Similarly, 48 subjects passed distance conservation (D-CONS) but only 26 passed distance measurement (D-M).

These data provide strong support for the Piagetian view that number conservation and length conservation are pre-requisites for linear measurement. Also, they are consistent with the findings of Beilin (1969) and Wolhwill and Lowe (1962), but inconsistent with the conclusions of Bearison (1969) and Gruen (1965) regarding conservation - measurement asynchrony.

11.1.4 USE OF A UNIT IN LINEAR MEASUREMENT.

Understanding of the notion of a unit in linear measurement was assessed using tasks, L-UNIT and L-UNIT-CH. They were passed by 53 and 49 subjects, respectively. Of the 14 subjects who passed LR-M-CARD, 14 passed L-UNIT, and 11 passed L-UNIT-CH. Of the 13 subjects who passed L-M-ADD, 12 passed L-UNIT, and 10 passed L-UNIT-CH. These data clearly indicate that the ability to employ a unit, and understand its use, are pre-requisites for linear measurement. Again, that is consistent with Piagetian theory.

Piagetian theory argues that a major difficulty confronting a child learning linear measurement is acquiring a grasp of a unit of length. This is because, unlike with number, a unit of length is not perceptually given, but decided arbitrarily. This argument is well illustrated by considering the tasks, LR-CARD and L-UNIT. In the former, the child does not have to invent a unit of length when either assembling or disassembling the rods - the unit of length is the length of the plastic block. In that sense, it is analogous to a counting task, insofar as the unit is perceptually given. In the latter, on the other hand, the child has to use

the small 3cm strip to invent, in an abstract sense, a unit to be successively imposed along the length of the longer strip. Intuitively, it seems that the latter ought to be more difficult. As 98 subjects passed LR-CARD, and only 53 passed L-UNIT, the data support that view.

It is also noteworthy that the conservation of length was passed by a significantly larger proportion of subjects than passed L-UNIT. This is also consistent with the analysis presented in Chapter 2. That is, linear measurement implies the selection and use of a unit, and the use of a unit implies the conservation of length.

11.1.5 ESTIMATION AND

LINEAR MEASUREMENT.

Length estimation seems to be substantially easier than linear measurement. All 14 of the subjects who passed LR-M-CARD, also passed the length estimation task (L-EST), which was passed by 56 subjects.

This finding should not be surprising, as the requirements of L-EST closely resemble those of L-UNIT, the major exception being that the answer in the latter is precisely determined. The data also reveal this similarity - 56 subjects passed L-EST, and 53 passed L-UNIT. This suggests that the skill of estimating how many of 'a' there are in 'b' develops hand-in-hand with the understanding of a unit of length.

11.1.6 LENGTH AND

DISTANCE.

It was argued in Part II that corresponding components in the length and distance concepts, such as measurement by unit iteration, would emerge synchronously. The findings of the present study do not support that view.

In the length concept, 74 subjects passed L-CONS, 56 passed L-EST, and 53 passed L-UNIT. The numbers of subjects passing the corresponding component tasks in the distance concept were: 48 passed D-CONS; 34 passed D-EST; and 26 passed D-M (this is the distance task which most closely resembles L-UNIT). Hence, the order of emergence is the same in both concepts, but the length components emerge earlier than the corresponding distance components. On that basis, developmental distinctions could be made between the acquisition of measurement of length, and the acquisition of the measurement^{s/} of distance.

11.2 INTER-CONNECTION OF THE COMPONENTS OF LINEAR MEASUREMENT.

On the basis of the preceding discussion, the kind of proficiency in subtraction assessed by N-SUB-NV, and the ability to make transitive inferences concerning length relations of non-equivalence and the ability to conserve distance, are not needed for linear measurement (of length). However, the other high order components set out in Hypothesis 1 should be pre-requisites for linear measurement.

The most difficult of those other components was found to be transitive reasoning concerning non-equivalent numerical relations, which was assessed by N-TI-NE. This was passed by 41 subjects. That is, even though only 14 subjects passed LR-M-CARD, 41 subjects passed the most difficult component task. Also, of the 100 subjects in the study, 13 passed all high-order component tasks, but failed LR-M-CARD and L-M-ADD. Moreover, all of those subjects were unable to commence LR-M-CARD and L-M-ADD. Hence, for those subjects, the difficulty was not in executing correctly a solution strategy. The evidence suggests that they didn't have a strategy to invoke when confronted with the requirements of LR-M-CARD and L-M-ADD.

It would appear, therefore, that a proportion of subjects possessed all the components, but could not measure length. That is, the components may be necessary, but not sufficient, to ensure linear measurement. There appears to be a delay between acquiring the underlying components, and being able to demonstrate a mature understanding of linear measurement. Given the differences in proportions passing the most difficult component task and the linear measurement tasks, the delay appears to be substantial. The question then arises: what is the cause of this delay?

Clearly, it would not be expected that mere possession of the listed components would be sufficient for linear measurement. They would need to be co-ordinated in some fashion, even if only in the same sense that an algorithm orders operations in a computation. Hence, the delay might occur because, even though all the components are present, some subjects might not have been taught how to apply them to the task of linear measurement. This supposition would be consistent with the finding that length of schooling is a predictor of a subject's overall score.

Alternatively, those subjects possessing the components, but not passing the linear measurement tasks, might have been instructed in linear measurement. However, those subjects might not have been able to co-ordinate the components, because of a structural limitation. An obvious structural limitation would be insufficient short term memory (STM) capacity.

A linear measurement strategy might be permanently represented in long-term-memory (LTM), or it might be generated by other LTM structures to solve a particular problem. In the latter case, the strategy would be simply a transient assembly of knowledge elements. In both cases, STM would be involved in controlling the execution of the strategy. Hence, it could be expected that STM capacity limitations would be manifested in breakdowns, or errors, in execution of the strategy. However, in the present study, all 13 of the subjects who possessed all the components but failed the linear measurement tasks, could not even commence those tasks. This suggests lack of an appropriate strategy, not faulty execution. It also suggests that STM capacity limitations are not responsible for the observed delay between acquisition of the components and mastery of linear measurement. Moreover, since STM capacity increases with age, this conclusion is consistent with the finding in the present study that age is not a predictor of a subject's overall performance.

Piagetian theory would account for the observed delay by asserting that it coincides with a re-organisation of cognitive structures that results in better co-ordination of underlying components. However, such an account would not say why the assumed re-organisation should be a lengthy process. In the present case, one explanation might be that the child

needs to be exposed to a large number of experiences of the appropriate kind before he can deduce the strategy that underlies linear measurement. Moreover, the child might not be able to benefit from these experiences until he has acquired all of the underlying components. This implies that the observed delay corresponds to an active period of learning. This would be consistent with the present finding that length of schooling is a predictor of performance.

11.3 THE IMPLICATIONS OF THE
ORDER OF EMERGENCE OF
COMPONENTS OF THE
NUMBER AND LENGTH CONCEPTS.

The finding that the number and length tasks form scaled sets is significant. It seems that many of the components of the concepts are acquired sequentially.

It would not be prudent, however, to claim that this sequential order is the only pattern that number and length development could exhibit. The earlier discussion of the possible causal links between conservation, transitive reasoning, and measurement hints of the difficulty of maintaining such a position.

Moreover, it should be borne in mind that most of the components assessed in the present study are closely related to, if not synonymous with, skills taught to children in school. Most of the teaching in schools,

especially in arithmetic, is predicated on the assumption that such skills are hierarchically organised. Hence, the observed pattern of development in the number and length concepts may reflect nothing more than the curriculum sequence used in the schools the subjects were drawn from. The finding that length of schooling is a predictor of overall performance is consistent with that suggestion, and would be predicted by Gagne's (1968) theory of learning. Of course, the curriculum sequence may well reflect the logical contingencies between the components tested in this study.

The order of emergence of the components does not necessarily reflect the order in which they began to develop. It may be that the development of component B commences before, or in synchrony with, the development of A. However, if B takes longer to develop then it would emerge after A. In that case, it would be inaccurate to claim a developmental dependency.

Flavell (1971, 1972) has pointed out the distinctions between developmental sequences and developmental dependencies at considerable length, and has proposed schemes for classifying observed developmental patterns. However, in the main, those schemes require identification of the time at which each component started to develop, and the time at which its development was completed. Given the difficulty in assessing cognitive skills, these requirements seem unrealistic. For example, determining the time at which a subject gave his first behavioural evidence of rudimentary counting skill, is probably, impossible.

11.4 THE ORDER OF EMERGENCE OF COMPONENTS OF
THE NUMBER CONCEPT.

The order of emergence of the components of the number concept is indicated by the numbers of subjects passing each task. Table 11.1 shows these tasks in rank order, together with the McNemar chi-squared coefficients for adjacently ranked tasks. The full matrix of chi-squared coefficients is given in Appendix 3.

TABLE 11.1: NUMBER TASKS: CHI-SQUARED VALUES FOR ADJACENTLY RANKED ITEM
PAIRS.

TASK	NO. OF SUBJECTS PASSING	MC.NEMAR CHI-SQUARED VALUES	P.
N-CNT	100	0.00	NS
N-TI-EQ	100	12.07	<.001
N-(1-TO-1)	86	→ 0.06	NS
N-ADD-V	84	→ 0.44	NS
N-SUB-V	81	→ 0.00	NS
N-ORD	80	→ 0.06	NS
N-CONS	78	→ 10.24	<.005
N-SOL-V	61	→ 0.24	NS
N-BAL-V	58	→ 0.05	NS
N-ADD-NV	58	→ 6.56	<.025
N-TI-NE	41	→ 12.86	<.001
N-SUB-NV	21	→ 3.20	NS
N-CYC-NV	16		

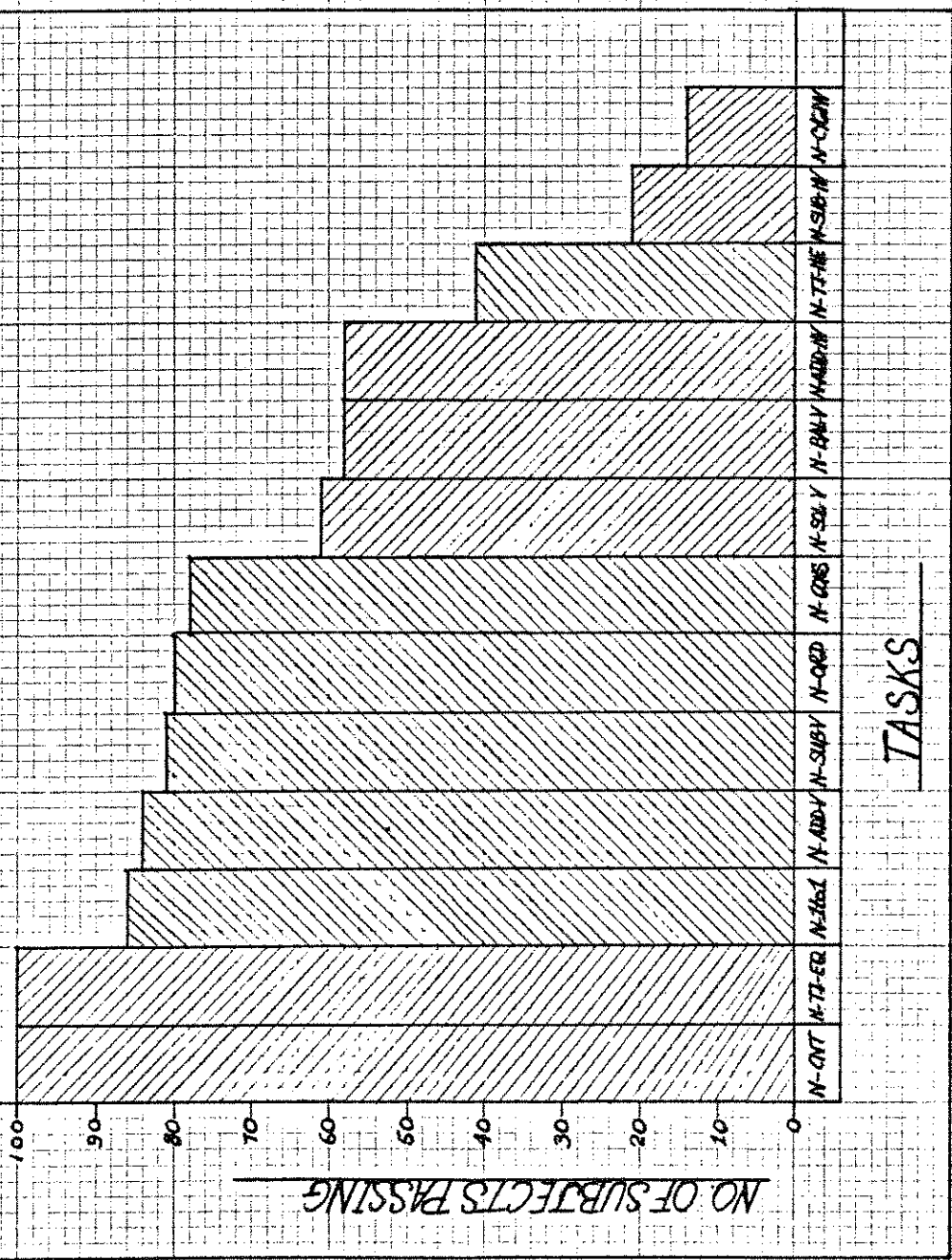
The chi-squared values in Table 11.1 and Appendix 3 suggest a stepped performance gradient - that is to say that there are abrupt changes to the slope of this gradient. As indicated in Table 11.2 and Figure 11.1, that gradient can be divided into five levels.

TABLE 11.2: LEVELS ON THE PERFORMANCE GRADIENT FOR THE NUMBER TASKS.

LEVEL	TASKS
1.	N-CNT N-TI-EQ
2.	N-(1-TO-1) N-ADD-V N-SUB-V N-ORD N-CONS
3.	N-SOL-V N-BAL-V N-ADD-NV
4.	N-TI-NE
5.	N-SUB-NV N-CYC-NV

All tasks lying on one level on the gradient are significantly easier than those on the next higher level, and significantly more difficult than those on the next lower level. However, all tasks on the same level do not differ significantly from each other. This does not necessarily mean that those tasks do not differ in difficulty since significance tests for

FIGURE 11.1
NUMBER TASKS - ORDER OF DIFFICULTY



adjacent tasks in a sequence are of low power. Indeed, the conclusion that the collection of number tasks is a scalable set implies that they are ordered. The extent of that ordering may be gauged by inspecting the relevant $H(ii)$ indices. These indices for the number tasks were given in Table 10.4 in Chapter 10. An inspection of Table 10.4 reveals that the tasks on level 2 and on level 3 are poorly ordered. This confirms the conclusion drawn from the significance tests suggesting no gradient in these regions of the curve. On the other hand, in spite of the results of the significance tests, the tasks at level 5 are perfectly ordered.

It may appear that a conclusion of a stepped performance gradient for the number task would contradict a conclusion that the collection of number tasks is a scaled set (i.e. the components emerge sequentially). However, abrupt changes in the slope of a performance gradient are not necessarily incompatible with a sequential order of development. An abrupt change in the slope of a performance gradient indicates a change in the rate of increase of component difficulty. If the change is large, it may be statistically significant. A sequential order of development is one in which the components appear in a fixed order, with the easier components emerging earlier than the more difficult. However, the increase in difficulty between adjacent components in a developmental sequence need not be statistically significant, though it may be. In the present case, as indicated in Figure 11.1, the levels on the performance gradient are not perfectly flat. Similarly, the collection of number tasks is not a perfectly scaled set. Hence, these two conclusions of a sequential order of development and a stepped performance gradient are not incompatible.

Assessing the developmental implications of a stepped performance gradient can be complex. The form of the performance gradient for any set of tasks is a joint function of two factors, namely, the distribution of intellectual growth levels in the sample of subjects, and the distribution of task difficulty level (i.e. the level of intellectual growth required to pass each task). An apparently stepped performance function could result from inhomogeneities in the distribution of intellectual growth levels in the subject sample, and/or from inhomogeneities in the distribution of the difficulty levels of the tasks.

In the present study, there are two reasons for believing that it is unlikely that the stepped performance gradient is due to the subject sample. They are the narrow age range of the subjects, and the finding that age is not a predictor of performance. Each of these factors suggest homogeneity, not inhomogeneity, in the subject sample.

Regarding the distribution of task difficulty levels, it is possible that the stepped performance gradient results simply from taking a small random sample of tasks from a larger population. The distribution of difficulty levels in this population could be continuous. The apparent discontinuity could be a consequence of sampling error. Alternatively, the distribution might be discontinuous. Piaget's stage theory of development asserts that this is the case.

In any case it might be possible to explain the observed performance gradient by analyzing the information-processing demands of the tasks. This analysis is given in Chapters 12 and 13.

The observed pattern of development for the number tasks contains certain other features which need comment.

Transitive Reasoning. N-TI-EQ was significantly easier than N-TI-NE. It is usually assumed that the components assessed by these tasks emerge together. However, Langford (1981) also found that the transitive law for the greater-than relation was more difficult than the transitive law for the equal-to relation, with respect to number.

Conservation and Transitive Reasoning. N-TI-EQ was significantly easier than N-CONS, but N-CONS was significantly easier than N-TI-NE. Additionally, the Loevinger indices of homogeneity of an item with an item for N-CONS and N-TI-EQ, and for N-CONS and N-TI-NE, are 1.00 and .89, respectively. That is, almost all subjects who passed N-TI-NE also passed N-CONS, and very few of the subjects who failed N-CONS passed N-TI-NE. Thus, there is a developmental asynchrony between the components assessed by N-CONS and N-TI-NE.

This finding is not consistent with Piagetian theory, because the latter claims that, in each concept domain, conservation and transitive reasoning emerge in parallel. This finding is consistent, however, with Gagne's (1968) componential theory, since the form of transitive reasoning implied in the N-CONS task is N-TI-EQ, not N-TI-NE. Hence, lack of the ability assessed by N-TI-NE would not be a barrier to a child passing N-CONS. For the ability assessed by N-TI-NE to be implicated, the conservation of number task would need to have included tests for the conservation of the numerical relations of greater-than and less-than. That comment aside, however, the present finding of a development asynchrony between conservation and transitive reasoning is consistent with the Smedslund (1963) and Mc.Mannis (1969) data.

Arithmetical Proficiency. The components assessed by the more difficult arithmetical tasks (e.g. N-SOL-V, N-BAL and N-ADD-NV) emerge after conservation, rather than at the same time. The most difficult of these components (assessed by N-SUB-NV and N-CYC-NV) emerge much later than the three which follow the appearance of conservation. Moreover, there are no reversal-type responses in the data concerning these five arithmetical tasks - that is, they are a perfectly ordered set.

The reason for the delay in achieving the kind of arithmetical proficiency assessed by N-SUB-NV is not apparent. It may be a reflection of the additional time needed for reversibility of the numerical operations implied in N-ADD-NV to be achieved. That kind of Piagetian argument, however, is not consistent, because the emergence of the conservation of number is also supposed to indicate that the property of reversibility has been achieved. The inconsistency stems from the finding that the conservation of number emerges much earlier than arithmetical proficiency, as assessed by N-SUB-NV. Hence, Piagetian theory does not offer a ready explanation.

11.5 THE ORDER OF EMERGENCE OF COMPONENTS OF THE LENGTH CONCEPT.

The order of emergence of the components of the length concept is indicated by the number of subjects passing each task. Table 11.3 shows these tasks in rank order, together with the McNemar chi-squared coefficients for adjacently ranked tasks. The full matrix of chi-squared coefficients is given in Appendix 3.

TABLE 11.3: LENGTH TASKS: CHI-SQUARED VALUES FOR ADJACENTLY RANKED ITEM
PAIRS.

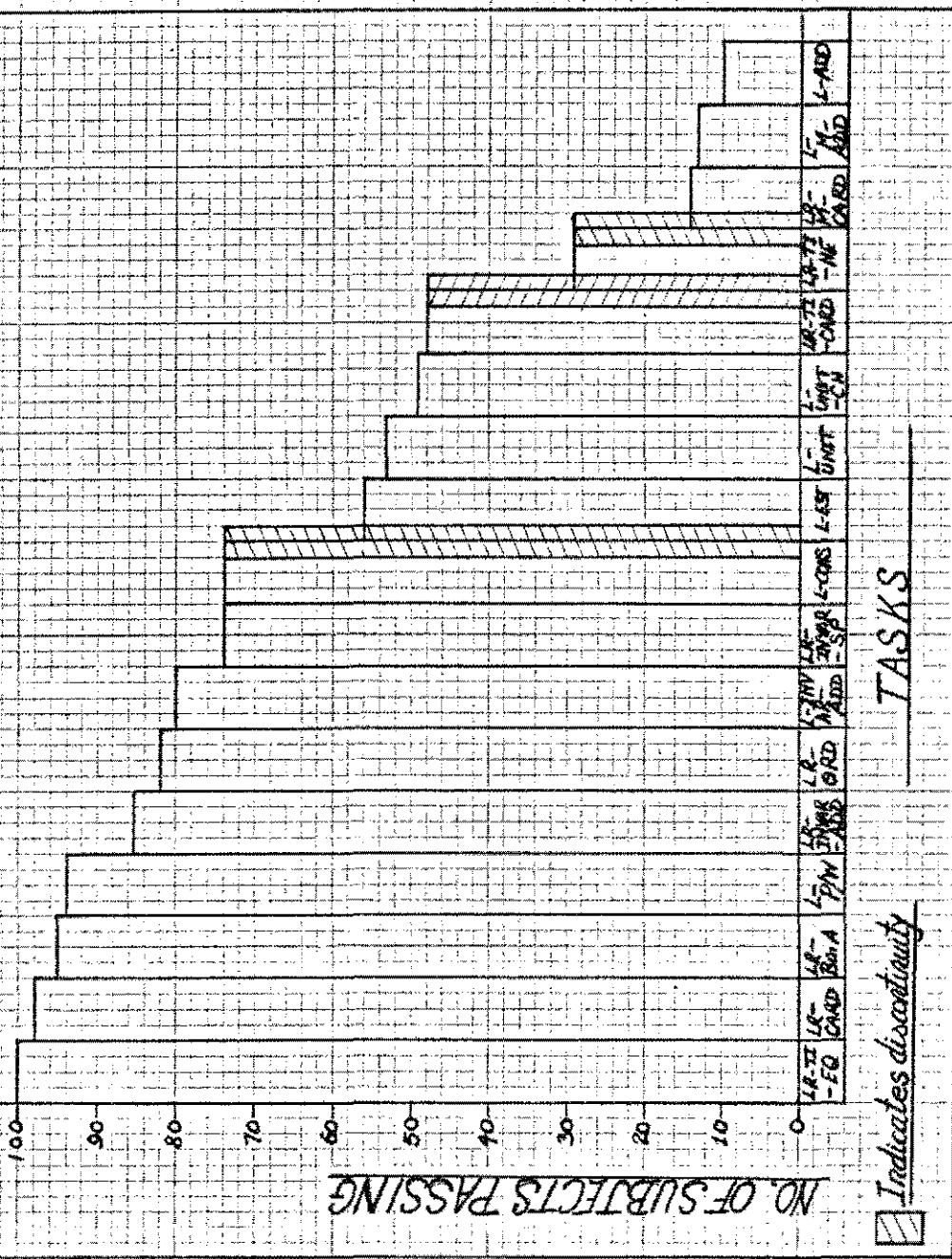
TASK	No. of SUBJECTS PASSING	MC.NEMAR CHI-SQUARED VALUES	P
LR-TI-EQ	100	→ 0.50	NS
LR-CARD	98	→ 0.57	NS
L-BinA	95	→ 0.00	NS
L-P/W	94	→ 3.76	NS
LR-INVAR- ADD	85	→ 0.27	NS
LR-ORD	82	→ 0.03	NS
L-INVAR- ADD	80	→ 2.50	NS
LR-INVAR- SP	74	→ 0.04	NS
L-CONS	74	→ 9.03	<.005
L-EST	56	→ 0.19	NS
L-UNIT	53	→ 0.41	NS
L-UNIT-CH	49	→ 0.00	NS
LR-TI- CARD	48	→ 11.17	<.001
LR-TI-NE	29	→ 5.94	<.025
LR-M-CARD	14	→ 0.00	NS
L-M-ADD	13	→ 0.36	NS
L-ADD	10		

The chi-squared values in Table 11.3 and Appendix 3 suggest a stepped performance gradient. As indicated in Table 11.4 and Figure 11.2, that gradient can be divided into four levels.

TABLE 11.4: LEVELS ON THE PERFORMANCE GRADIENT FOR THE LENGTH TASKS.

LEVELS	TASKS
1A	LR-TI-EQ LR-CARD L-BinA L-P/W
1B	LR-INVAR-ADD LR-ORD L-INVAR-ADD LR-INVAR-SP L-CONS
2.	L-EST L-UNIT L-UNIT-CH LR-TI-CARD
3.	LR-TI-NE
4.	LR-M-CARD L-M-ADD L-ADD

FIGURE 112
LENGTH TASKS - ORDER OF DIFFICULTY



Indicates discontinuity

TASKS

All tasks lying on one level on the gradient are significantly easier than those on the next higher level, and significantly more difficult than those on the next lower level. With the exception of the level 1 tasks, all tasks on the same level do not differ significantly from each other. In the case of the level 1 tasks, there is no significant difference in difficulty between adjacent tasks (ranked in order of difficulty as in Table 11.3), but there are significant differences between tasks more widely separated in difficulty ranking. The tasks on level 1 could be divided into two sub-levels, namely, level 1A containing LR-TI-EQ, LR-CARD, L-BinA, and L-P/W, and level 1B containing the remainder of the level 1 tasks. With that division, most of the tasks on level 1B are significantly ($P < .05$) more difficult than those on 1A, but all tasks on the same sub-level do not differ significantly from each other.

It will be recalled from the earlier discussion of the stepped performance gradient observed for the number tasks that a chi-squared analysis of the proportions of subjects passing and failing tasks on the same difficulty level does not provide information on whether those tasks form a developmental sequence. The extent of any ordering of tasks on the same level can be assessed by inspecting the relevant $H(ii)$ indices in Table 10.6 (Chapter 10). Table 10.6 shows that tasks on the same level are, in general, poorly ordered. This confirms the impression of a stepped performance gradient for the length tasks.

There are other features of the stepped performance gradient for the length tasks which need comment.

Transitive Reasoning. As for the corresponding number tasks, transitive inferences concerned with equivalent relations appear much earlier than those concerned with non-equivalent relations.

Conservation and Transitive Reasoning. Again there is a similarity between number and length with respect to the observed patterns concerning conservation and transitive reasoning. Specifically, the component assessed by LR-TI-EQ appears before that assessed by L-CONS, which appears before that assessed by LR-TI-NE.

This finding is not consistent with Piagetian theory. However, as has been noted previously in connection with the corresponding number tasks, this kind of asynchrony is consistent with Gagne's (1968) theory. Clearly, that theory, with its emphasis upon componential structure, is consistent with this finding, because the form of transitive reasoning implied in L-CONS is that assessed by LR-TI-EQ, not that assessed by LR-TI-NE.

11.6 ORDERING ACROSS NUMBER AND LENGTH TASKS.

There is a similarity between the observed patterns in the number and length concepts. In general, the tasks located at abrupt changes of slope on the performance gradient for the number tasks were of major theoretical interest (e.g. N-CONS). That is also the case for the length tasks (e.g. L-CONS). It was seen in Chapter 10 (Hypothesis 8) that the conservation of number appears at about the same time as the conservation of length. It may be that these abrupt changes in slope of the performance gradients reflect a re-organisation of the child's number and length concepts.

This suggestion stems from two facts. Firstly, following the emergence of N-CONS and L-CONS, there is a considerable delay before the emergence of the next components in the length domain. Secondly, those components, L-EST and L-UNIT, implicate the numerical representation of length. It may be that until that development occurs the child's reasoning about length is restricted, because the length concept is unconnected (or only loosely connected) to the number concept. However, the appearance of conservation for each domain may enable a re-organisation which results in the child's reasoning about length being augmented by a number-based, or number-connected form, of internal representation.

The next major discontinuity (i.e. abrupt change in slope of the length performance gradient) in the length domain occurs after the development of the capacity assessed by LR-TI-CARD. That task requires the child to reason transitively about non-equivalent relations between lengths on the basis of the number of unit parts contained in each object's length. It is noteworthy that this capacity implicates numerical forms of reasoning about length relations, and that it emerges at about the same time as the capacity to reason transitively about relations of non-equivalence concerning number (assessed by N-TI-NE). Indeed, the capacity assessed by N-TI-NE is implied in the capacity assessed by LR-TI-CARD. This suggests that further enhancement of the length concept by numerical forms of reasoning has occurred.

Following the emergence of the capacity assessed by LR-TI-CARD, there is another delay before the capacity assessed by LR-TI-NE emerges. The latter does not implicate numerical forms of reasoning about length. However, it

may be that there is a developmental dependency between LR-TI-CARD and LR-TI-NE, in that it is the possession of a capacity to reason numerically about length (LR-TI-CARD) which provides the 'proof' of the inference required in LR-TI-NE. Once that has been established the child no longer need depend upon numerical representations of length in order to make transitive inferences concerning non-equivalent length relations.

In more general terms, these speculations about the developmental discontinuities in each concept domain, and the interconnection of those domains, imply that an advance in one concept domain prompts development in another.

11.7 SUMMARY.

The findings of the study suggest that the necessary components of linear measurement are the following:-

- . Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
- . Knowing that length is invariant under certain transformations (the conservation of length).
- . Knowing how to make transitive inferences of equivalence and non-equivalence, with respect to discrete quantity.
- . Knowing how to make transitive inferences of equivalence, with respect to length.
- . Know how to carry out numerical addition operations.
- . Knowing how to obtain a linear measurement by counting iterations of a unit of length.

The data also imply a delay between acquisition of these components and the emergence of an understanding of linear measurement. Piagetian theory would suggest that the delay is associated with a re-organisation of cognitive structures that results in better co-ordination between the components.

Inspection of the development sequence in each concept domain reveals that each is characterised by discontinuities. These discontinuities coincide with the emergence of components of major theoretical interest, such as the conservation of length. Additionally, there are concordances between the discontinuities in the number and length domains. Examination of these concordances suggests that the discontinuities occur during periods of development when new forms of co-ordination are being established between the number and length concepts.

Some elements of the observed sequences of development in the number and length concept domains are not predicted by Piagetian theory. In particular, the asynchronies between conservation and transitive inferences of non-equivalence are not consistent with the Piagetian view, though they are consistent with Gagne's (1968) model of development.

CHAPTER 12.AN INFORMATION-PROCESSING ANALYSIS OF CERTAINNUMBER AND LENGTH TASKS, USING PASCUAL-LEONE'S M-SPACE MODEL.12.1 INTRODUCTION.

It was seen in Chapter 11 that the number tasks could be organised into five levels of difficulty, and the length tasks into four levels of difficulty. It was argued that tasks at a similar level of difficulty should make similar information-processing demands. Hence, it was thought that an information-processing analysis of the tasks which fall on the boundaries of the levels might reveal the reasons for the sharp changes in task difficulty that occur between levels. That analysis is given in this Chapter.

The information-processing model used in the analysis was developed by Pascual-Leone (1970). Firstly, his model is described. Secondly, the application of his model to the present study is discussed.

12.2 PASCUAL-LEONE'S M-SPACE MODEL.

12.2.1 NATURE OF THE MODEL.

Pascual-Leone (1970) constructed a functional or process model of development, complementary to Piaget's structural model. The model predicts performance on a range of Piagetian and other cognitive tasks, given prior estimates of the values of two structural variables - namely, "M-space" and "field independence/dependence."

An information-processing approach has been adopted by Pascual-Leone, and his main collaborator, Case (1972). Unlike some other information-processing theorists, they don't write computer programs. Their level of analysis is that of "scheme". Their use of scheme is the same as that of Piaget. Pascual-Leone identified three categories of scheme: "figurative", "operative", and "executive".

12.2.2 FIGURATIVE SCHEMES.

Figurative schemes are the internal representations of "declarative-type knowledge" (e.g. properties of objects or relations between objects). They are proposed as active, functional units, akin to Neisser's (1967) pattern recognition devices. Case (1974) gave the following example of a figurative scheme:-

"If, for example, a subject looked at a photograph and asserted that it was a picture of his house, one would say that he did so by transforming the raw sensory input into a network of perceptual features which were readily associated in his mind with a conceptual response of the order, 'that is my house.' More simply, one would say that he assimilated the sensory input to his (figurative) 'house scheme'."

12.2.3 OPERATIVE SCHEMES.

Operative schemes are the internal representations of "procedural-type knowledge" (e.g. rules applied to properties of objects, or relations between objects). Both figurative schemes, and operative schemes are assumed to be active processes. Hence, the internal distinction between these two categories of scheme is blurred, they are distinguished by what they are used for. Operative schemes act on figurative schemes to generate new figurative schemes, but figurative schemes do not act on other figurative schemes.

12.2.4 EXECUTIVE SCHEME.

Executive schemes are proposed as the internal representations of the lists of rules and procedures to be assembled, sequenced and actioned in order to reach some desired goal. They represent strategies to be employed in solving a particular class of problem. They are also proposed as active processes, but they differ from operative schemes insofar as they don't directly act on figurative schemes to generate new figurative schemes. Their function is to direct and control solution processes by deciding upon and activating operative scheme sequences.

SPACE CONSTRUCT.

He argues that, in the course of problem solving, a person's thought is constituted by the assembly of schemes that are currently activated. It follows, therefore, that for Pascual-Leone a principal limitation on thought processes is the number of discrete schemes that may be activated and co-ordinated at any given time. He refers to this limitation as "M-space", which he defines as the set measure of Piaget's field of centration. Using the scheme construct as the fundamental unit of analysis, Pascual-Leone produced descriptions of various Piagetian tasks in terms of schemes invoked and co-ordinated in the subject's M-space. The following two examples may help to illustrate the approach:-

"Conservation of identity. The age at which this task is first passed is 5-6 years. In solving it, children appear to activate the following schemes:

E(IS): An executive scheme representing the instructions ('Does the ball still have the same amount of clay in it?') and directing an appropriate perceptual scan of the ball as it is transformed;

F(1): A figurative scheme representing the fact that 'nothing has been added or taken away';

F(2): A figurative scheme representing the rule a 'if nothing is added or taken away, then the amount remains the same';

If children do not co-ordinate the above schemes, they fail the task, apparently because they activate another scheme already present in their repertoires, which is misleading in the conservation situation.

Call it

F(M): A figurative scheme representing the rule that 'things which look bigger, contain more'.

Conservation of equivalence. The age at which this task is first passed is 7-8 years. In solving it, children appear to activate the following schemes:

E(IS): An executive scheme representing the instructions ('Do the balls still have the same amount of clay in them?') and directing an appropriate perceptual scan of the ball as it is transformed;

F(1): A figurative scheme representing the fact that 'nothing has been added to or taken away from the ball which was transformed;

F(2): A figurative scheme representing the rule that if nothing is added or taken away, then the amount remains the same;

F(3): A figurative scheme representing the information that 'the balls originally were equal in amount;'

If children do not co-ordinate the above schemes, they usually fail the task, apparently because they activate:

F(M): A figurative scheme representing the rule that "things which look bigger contain more."

(Case, 1972; pp340-341).

(Note: To be consistent with Pascual-Leone's own classification of schemes as figurative, operative and executive, schemes F(2) and F(M), in each of the above examples, should have been classified as operative.)

12.3 DEVELOPMENTAL

PROGRESSIONS.

These two examples illustrate an important feature of Pascual-Leone's model which is that the number of schemes children co-ordinate in approaching a task is related to the age at which they first succeed at the task. Pascual-Leone argued that a different value of M (for M-space) is associated with each substage of intellectual development. The values he proposed are as follows:-

<u>DEVELOPMENTAL</u> <u>SUBSTAGE</u>	<u>AGE</u> <u>(yrs)</u>	<u>MAXIMUM VALUE OF M</u> <u>(a + k)</u>
Early Pre-operational	3-4	a + 1
Late Pre-operational	5-6	a + 2
Early concrete	7-8	a + 3
Late concrete	9-10	a + 4
Early formal	11-12	a + 5
Middle formal	13-14	a + 6
Late formal	15 -16	a + 7

(Note: The constant (a) refers to the space required by the executive scheme. The numeral represents the maximum number (k) of additional schemes which can be co-ordinated.)

According to Pascual-Leone, the M-space model provides a functional explanation of developmental progressions. For example, children usually fail conservation of amount until the age of 7-8 years, because their M-space, until that age, can only co-ordinate the concurrent activation of a + 2 schemes. However, the conservation task requires, under normal conditions, the concurrent activation of (a + 3) schemes.

12.4 INDIVIDUAL DIFFERENCES.

Whilst Pascual-Leone sees M-space as a structural limitation on performance, his model also makes provision for other variables. Specifically, he argued that the following conditions must be met before a task can be successfully handled:-

- (a) the child must possess the necessary schemes;
- (b) if necessary, the full capacity of the available M-space must be used;
- (c) the child must attend to other than only the perceptually dominant and potentially misleading cues;
- (d) if two incompatible schemes are activated by the perceptual features of the problem, the child must resolve the conflict in favour of that scheme which is compatible with the greatest number of other associated schemes.

The first of these conditions can be satisfied only by learning. The third is related to the perceptual/cognitive style variable known as field independence/dependence (Witkin, 1959). The fourth is identified with a component of Piaget's equilibration model. It reflects the operation of an individual differences variable, and is assumed to be highly correlated with the third factor. Hence, Pascual-Leone's model posits that performance is determined by M-space and learning, and is moderated by field independence/dependence.

12.4.1 LEARNING.

Learning is defined as the acquisition of new schemes. This is accomplished in two ways:-

- (a) by incorporation of new information into old schemes, in a manner analogous to Piaget's differentiation; and
- (b) by combining formerly discrete schemes into a new compound or superordinate scheme, in a manner analogous to Piaget's reciprocal assimilation.

Both processes have the effect of increasing performance, because they lead to more efficient utilisation of M-space.

12.4.2 FIELD-INDEPENDENCE/DEPENDENCE.

The field-independence/dependence factor is said to explain much of the variance attributable to individual differences. The field-independent subject is less likely to focus on perceptual cues inherent in the task situation, more likely to attend to the task instructions, and will tend to utilize available M-space fully. The field-dependent subject who is faced with two incompatible schemes, one activated by perceptual cues and the other by task instructions, is more likely to resolve the conflict by acting on the former than the latter. He is also less likely to make full use of available M-space.

12.5 EMPIRICAL EVIDENCE FOR THE M-SPACE MODEL.

12.5.1 EARLY STUDIES.

Pascual-Leone (1970) tested the model by teaching children a series of novel responses to particular visual stimuli. He then measured their capacity to activate these "S-R" connections concurrently when confronted with a compound visual stimulus. The children's capacity to produce the appropriate compound response was taken as a measure of their M-space. It was found that there was a high correlation between that measure, and the M-space factor inferred from an analysis of Piagetian tasks previously passed by the children. The findings were interpreted as a demonstration that the M-space model had construct and predictive validity. Pascual-Leone and Smith (1969) reported another investigation of children's classification concepts, and presented findings consistent with the M-space model.

12.5.2 METHODOLOGICAL CRITICISMS.

Pascual-Leone's (1970) experimental methods, and techniques of model evaluation, have been severely criticised by Trabasso and Follinger (1978) and Trabasso (1978) on two main grounds. Firstly, in Pascual-Leone's (1970) study only some of the children were trained and tested on all the S-R associations; the number of associations used increased with increasing group mean age; the S-R associations used varied in terms of inherent difficulty; and the subject's stage of development (eg. pre-operational) was inferred from the subject's age and not directly assessed. Secondly, Pascual-Leone did not use statistical goodness-of-fit tests to estimate the predictive accuracy of his model, relying instead on visual inspection of averaged probability distributions. Also, Pascual-Leone did not compare his model's predictive accuracy against that achieved by a variety of other stochastic (eg. Monte Carlo) models. Pascual-Leone (1978) defended his approach on the grounds that he was concerned with testing a 'general' not a 'local' model, and that his critics' objections were appropriate only in the context of verifying empirically local models of limited scope.

12.5.3 LATER STUDIES.

The studies by Case (1972, 1972a) appear to have overcome most of the methodological objections raised against Pascual-Leone's experimental work, and have yielded findings consistent with the M-space model. In the 1972 study, a more carefully designed version of the Pascual-Leone and Smith (1969) compound stimulus task was employed. Dale's (1975) large-scale investigation of performance on Piaget's bending rods task also produced findings consistent with the M-space model.

Pascual-Leone's approach has attracted the attention of educational psychologists, because it offers a bridging construct between developmental theory (i.e. Piaget's) and human learning theory (i.e. Gagne's).

"There are clear parallels between Gagne's model and Pascual-Leone's. Gagne's model interprets cognitive problems as requiring the application of certain rules. Pascual-Leone's model interprets these same problems as requiring the co-ordination of certain schemes (some of which are merely the internal representation of such rules).....Both theorists agree that children will not be able to solve cognitive problems if they do not have the appropriate internalized items of information in their repertoires. Both theorists agree that children can often be enabled to solve such problems if they are helped to acquire the appropriate repertoires, ie. if they are instructed. The difference lies in the role assigned to development. For Gagne, the process of development is largely one of cumulative learning, within the confines of whatever (unspecified) limitations may be imposed by 'growth'.....For Pascual-Leone, the process of development is equally one of cumulative learning. However, one of the major limitations imposed by 'growth' is explicitly defined. It is a limitation in mental space." (Case, 1972, p356).

12.6 M-SPACE ANALYSIS OF CERTAIN

NUMBER TASKS.

12.6.1 SELECTION OF NUMBER TASKS..

Tasks from each level of the performance gradient for the number collection were selected for analysis. The tasks chosen were those of least and most difficulty, within a level. Hence, the most difficult task from level n , was compared with the least difficult task from level $n+1$. Tasks could have been randomly sampled from each level. However, the sampling method used was more conservative because it selected pairs of tasks adjacent in rank order of difficulty, and adjacent in ungrouped order of difficulty. The tasks selected from the number collection (see Table 11.2 in Chapter 11) are shown in Table 12.1.

TABLE 12.1: NUMBER TASKS SELECTED FOR M-SPACE ANALYSIS.

LEVEL N	MOST DIFFICULT TASK AT LEVEL N	LEVEL N+1	LEAST DIFFICULT TASK AT LEVEL N+1
1	N-TI-EQ	2	N-(1 TO 1)
2	N-CONS	3	N-SOL-V
3	N-ADD-NV	4	N-TI-NE
4	N-TI-NE*	5	N-SUB-NV

Note: * - N-TI-NE is the only task at level 4.

The selected tasks were analyzed into co-activated executive, figurative, and operative schemes, in accordance with Pascual-Leone's model. That analysis is set out below. Entries for executive schemes are not given, because all tasks must activate an executive scheme representing the task instructions, and the solution strategy used. To show an executive scheme entry against all tasks would be redundant. Figurative schemes are prefixed by an 'F', and operative schemes by an 'O'.

12.6.2 SPECIFICATION OF THE
CO-ACTIVATED SCHEMES FOR THE
SELECTED NUMBER TASKS.

N-TI-EQ.

- F1: A figurative scheme representing the fact that there are as many blue blocks as there are red blocks.
- F2: A figurative scheme representing the fact that there are as many red blocks as there are green blocks.
- O1: An operative scheme representing the canonical form of the transitive law: if (a.R.b) and (b.R.c), then (a.R.c).

N-(1-TO-1)

- F3: Every bolt had a nut, or every bolt had a washer, or every nut had a washer, depending on the question being considered.

- F4: Every nut had a bolt, or every washer had a bolt, or every washer had a nut, depending on the question being considered.
- O2: If every 'a' had a 'b', and every 'b' had an 'a', then the number of a's equals the numbers of b's; where a and b refer to bolt and nut, or bolt and washer, or nut and washer, depending on the question being considered.

N-CONS.

ON Phase 1:

- F5: Nothing has been added or taken away from the line of blocks which was transformed.
- O3: If nothing has been added or taken away from the line of blocks, then it contains the same number of blocks that it did before transformation.
- The application of O3 to F5 leads to the creation of F6:-

On Phase 2.

- F6: The line of blocks which was transformed contains the same number of blocks as it did before the transformation occurred.
- F7: The two lines of blocks originally contained the same number of blocks.
- O4: The numerical relation between two collections of objects is invariant unless the numerosity of one collection is changed.

N-SOL-V

On Phase 1.

- F8: The experimenter's collection contains n blocks.
- F9: The subject's collection contains m blocks.
- O5: If two collections have a different number of blocks, then the collection with more blocks is the one whose number name appears later in the number name order.
- The application of O5 to F8 and F9 yields F10:-

On Phase 2.

- F10: The experimenter's collection has more blocks.
- O6: If one collection has more blocks than another, then how many more can be found by subtraction.
- The application of O6 to F10 initiates a subtraction process carried out on F8 and F9 and resulting in F11.

On Phase 3.

- F11: The experimenter's collection has $(n-m)$ more blocks than the subject's
- O7: If one collection has more blocks than the other, then they can be made equal in number by taking the difference $(n-m)$ away from the collection with more.
- Notice that the same phases would be involved in solving the problem by adding to the collection with fewer blocks.

The subtraction process would be common to both, and would involve the co-activation of F8 and F9, together with a subtraction rule, such as:

- O8: To subtract m from n , start counting at one more than m , stop at n , and the number of number names mentioned is the answer.

N-ADD-NV.

- F12: The bottom jar contained ' n ' balls before more balls were sent down the tube.
- F13: ' m ' balls were sent down the tube.
- O9: If more objects are added to a collection of similar objects, the number of objects then in the collection is given by an addition operation.
- The addition operation (call it, O10) would apply to F12 and F13. It may be based on a counting method - using fingers, for example, to represent the balls - or on a 'table-look-up' method.

N-TI-NE.

As for N-TI-EQ, except that the relations 'greater-than' and 'less-than', are substituted for 'equal-to'.

N-SUB-NV.

F14: The top jar contained 'n' balls before more balls were sent down the tube.

F15: 'm' balls were sent down the tube.

O11: If some objects are taken away from a collection of similar objects, then the number of objects left in the collection is given by a subtraction operation.

-- The subtraction operation (call it, O12) would apply to F14 and F15. It may be based on a counting method - using fingers, for example, to represent the balls - or on a 'table-look-up' method. Notice that O12 may be different from O8, as in N-SOL-V the objects were visible to the subject throughout the task.

12.6.3 NUMBER OF CO-ACTIVATED SCHEMES REQUIRED FOR
THE SELECTED NUMBER TASKS.

For all of the tasks listed above, the maximum number of co-activated schemes needed at any stage of the solution process is three; two of which are figurative, and one, operative. Hence, in terms of Pascual-Leone's model, all tasks are of the type that can be solved by children at the early concrete sub-stage of development. Therefore, an M-space analysis of the number tasks does not reveal any structural limitation corresponding to the steps in the observed performance gradient.

12.7 M-SPACE ANALYSIS OF CERTAIN
LENGTH TASKS.

12.7.1 SELECTION OF LENGTH TASKS.

Tasks were selected for analysis from each of the levels in the performance gradient for the length collection. The basis for selection was the same as that described earlier for the number collection. The tasks selected from the length collection (see Table 11.4 in Chapter 11) are shown in Table 12.2.

TABLE 12.2: LENGTH TASKS SELECTED FOR M-SPACE ANALYSIS.

LEVEL N	MOST DIFFICULT TASK AT LEVEL N	LEVEL N+1	LEAST DIFFICULT TASK AT LEVEL N+1
1	L-CONS	2	L-EST*
2	LR-TI-CARD	3	LR-TI-NE
3	LR-TI-NE*	4	LR-M-CARD

Note: * - Instead of giving an analysis of L-EST as the least difficult task at level 2, an analysis is given of L-UNIT. This is because L-EST and L-UNIT are of, essentially, the same difficulty, but the latter is of greater theoretical importance and, hence, of more interest.

** - LR-TI-NE is the only task at level 3.

The tasks listed in Table 12.2 were analysed in accordance with the M-space model. That analysis is set out below. Executive schemes are not shown, because it is assumed that there is only one executive scheme activated for each task.

12.7.2 SPECIFICATION OF THE
CO-ACTIVATED SCHEMES FOR THE
SELECTED LENGTH TASKS.

L-CONS

On Phase 1.

F16: Nothing has been added or taken away from the piece of string which was transformed.

O13: If nothing has been added or taken away from the piece of string, then it is the length it was before the transformation.

— The application of O13 to F16 yields F17:

On Phase 2.

F17: The piece of string which was transformed is the length it was before the transformation.

F18: The two pieces of string were originally the same length.

O14: The length relation between objects is invariant unless the length of one object is changed.

L-UNIT

This task may be divided into three processes. Process 1 is responsible for marking off equal units. Process 2 is responsible for counting the

units. Each cycle of process 1 is followed by a cycle of process 2. Process 3 is responsible for producing the answer. It is initiated after the last cycle of Process 2 has been executed.

Process 1.

- F19: The location of the right hand border (assume left-to-right movement) of the previous segment marked off is known.
- F20: The left hand border of the unit coincides with the right hand border of the previous segment marked off.
- O15: If the unit length equals the previous segment length, and the unit length equals the current segment length, then the previous segment length equals the current segment length.

Process 2.

- F21: Number name mentioned when the previous segment length was marked off.
- F22: List of number names.
- O16: When the current segment length has been marked off, mention the next number name on the number name list.

Process 3.

- F23: The whole object has been divided into segments of equal length.
- F24: Last number name mentioned.
- O17: The number of equal segments represents the length of the whole object.

LR-TI-CARD

On Phase 1.

F25: There are more red blocks than green.

F26: There are more blue blocks than red.

O1: The canonical form of the transitive law with respect
to number.

-- The application of O1 to F25 and F26 yields F27:

On Phase 2

F27: There are more blue blocks than red.

F28: All blocks are the same size.

O18: A is longer than B, if A contains more parts than B,
and if the parts are the same length.

LR-TI-NE.

As for Phase 1 of LR-TI-CARD, except that the figurative and operative schemes refer to length, not number.

LR-M-CARD.

There are two parts to this task. The first part is responsible for unit iteration. The second part is responsible for comparison of object lengths, and production of the answer.

The first part is exactly the same as L-UNIT, described above. The second part uses the same operative scheme (O18) as that employed on phase 2 of LR-TI-CARD. Hence, the schemes co-activated on the second part of LR-M-CARD are:-

- F29: Length A contains n unit parts.
- F30: Length B contains m unit parts.
- O18: A is longer than B, if A contains more parts than B,
and if the parts are the same length.

12.7.3 NUMBER OF CO-ACTIVATED
SCHEMES REQUIRED FOR THE
SELECTED LENGTH TASKS.

Inspection of the above analysis reveals that the maximum number of co-activated schemes needed at any stage of the solution process is three: two figurative and one operative. Hence, the analysis does not reveal any structural limitation corresponding to the steps in the observed performance gradient. This conclusion is the same as that reached with respect to the number tasks.

12.8 SUMMARY.

An M-space analysis of the steps in the number and length task performance gradients does not support the suggestion that those steps represent developmental discontinuities which stem from structural limitations, such as mental processing capacity. However, the M-space analysis does not explicitly provide for differences in complexity of control processes,

because all directive information is assumed to be stored in a single executive scheme. Because it would seem likely that there are differences in complexity between control processes associated with these tasks and, hence, differences between the demands these processes place on STM, this assumption of a unitary executive structure limits the value of the analysis.

Moreover, in Pascual-Leone's model the level of analysis of component processes and knowledge elements into operative and figurative schemes is somewhat arbitrary. In the present case, an attempt was made to conform with the examples of M-space analysis given by Pascual-Leone. However, there is no certainty that a different analyst would derive the same list of co-activated schemes. Hence, this apparently arbitrary aspect of Pascual-Leone's model also limits the value of the analysis.

If these two criticisms of Pascual-Leone's model are set aside, a conclusion which could be drawn from this analysis is that the stepped performance gradients for the number and length tasks reflect, simply, delays inherent in the accretion of a large number of rules. This conclusion would be consistent with the finding of the present study that length of schooling is a predictor of performance, whilst age is not.

CHAPTER 13.AN EXAMPLE OF A PRODUCTION-SYSTEM ANALYSIS
OF CERTAIN COMPONENTS OF LINEAR MEASUREMENT.13.1 THE NEED FOR A DETAILED PROCESS-
ANALYSIS OF LINEAR MEASUREMENT.

The analysis of linear measurement given in Chapter 2 drew upon the actions involved in linear measurement operations, and the logical structure of linear measurement. The empirical work reported here has broadly supported this analysis. However, as a psychological theory it has two weaknesses.

The first is that the analysis was intuitive. It was not formally demonstrated that the listed components are sufficient to enable a person to substitute measurement operations for actual operations on objects.

The second is that the analysis did not concern itself with the psychological processes at work in linear measurement.

The only psychologically-orientated theory that considers linear measurement is Piaget's. That theory offered general guidance to the analysis in Chapter 2, but its usefulness was limited, because it is only concerned with describing the gross psychological structures needed for linear measurement. The Piagetian analysis is not made at a sufficient level of detail for the present purposes.

A more satisfactory account of linear measurement would give a detailed process-analysis of what is involved in each component, and of precisely how the components are co-ordinated.

The objectives of that analysis would be, firstly, to express linear measurement in terms of a minimum number of psychologically primitive operations and, secondly, to show that the account does, indeed, generate linear measurement. An account of that kind could be provided by constructing a "production-system" model of linear measurement.

Production systems are collections of condition-action rules, called productions. The rules are expressed in the form: 'if conditions A(1), and A(2), and A(3), and A(n) hold, then take actions B(1), and B(2), and B(3), and B(n)'. If the rules, or productions, are written in a computer language, they may be 'executed' (Newell and Simon, 1972).

Klahr and Wallace's (1976) work provides an example of how the production-system approach can be used to construct a detailed account of aspects of cognition related to the present study. Their objective was to develop a theory of cognitive development at a level of precision that would enable the theory to be expressed as an executable computer model.

Klahr and Wallace's (1976) strategy was to construct, firstly, state models. Each model depicted the performance of a child at a particular stage of development on a range of Piagetian tasks. Secondly, they constructed transitional models accounting for changes between stages (i.e. between state models). Their state models were expressed as executable production systems.

Klahr and Wallace (1976) present models of various quantification processes (subitization, counting, estimation, relative magnitude determination), class inclusion, conservation of quantity, and transitive reasoning. Their models are constructed so that groups of productions used in less complex tasks, such as counting, may be used in more complex tasks, such as addition. This reflects the view that development stems from the accretion of experience, coupled with periodic re-organisation of the internal representation of that experience.

An approach similar to Klahr and Wallace's could be taken to the problem of providing a satisfactory account of the composition and growth of linear measurement. That approach would make explicit the theory implicit in the list of components of linear measurement given in Chapter 2. A full production-system analysis of linear measurement will not be attempted here (2). Nevertheless, in order to illustrate the possibilities offered by this approach, this Chapter presents an analysis of three of the tasks used in the present study. The core productions concerning counting, subitization, addition, control processes, and so on, given in these models, are all directly relevant to the longer-term objective of providing a production-system model of mature linear measurement.

2. The original intention of the present research project was to develop such an analysis. However, the lack of an appropriate and detailed theoretical framework, and the paucity of directly relevant empirical data, necessitated that, first, the empirical work reported here be undertaken. The position has now been reached where a production-system model of mature linear measurement could be developed. Development of such a

model of linear measurement would, itself, be a substantial undertaking, and is beyond the scope of the present research. The production-system model would provide a sufficient account of the data reported in the present study, and would constitute a formal theory of what is involved in linear measurement. A speculative outline of a possible approach to the development of such a model is given in Appendix 6.

Additionally, the three tasks selected for analysis were chosen because they have the potential to yield further information on the question examined in Chapter 12. It was suggested there that the discontinuities in the observed performance gradients could be explained by changes in STM capacity. An analysis of selected tasks, using Pascual-Leone's model, failed to support this suggestion. However, it was argued in the concluding paragraphs of Chapter 12 that the assumption by the Pascual-Leone model of a unitary control structure had a significant drawback. It implied that variations in the complexity of control information have no effect on STM load. Production-system modelling does make explicit provision for representing control information in STM. Hence, a production-system analysis of the selected tasks should complement the analysis given in Chapter 12 of the information-processing demands of tasks drawn from different regions of the performance gradient, and could yield different conclusions.

Before describing the task models it will be necessary to amplify the descriptions given above of production-system modelling. Section 13.2 provides a brief overview of a production-system language.

13.2 OVERVIEW OF A

PRODUCTION-SYSTEM LANGUAGE.

In addition to Klahr and Wallace (1976), Anzai and Simon (1979), Baylor and Gascon (1974), Newell and Simon (1972), and O'Shea and Young (1978) provide examples of production-system models of aspects of cognition relevant to the present topic. Hunt and Poltrock (1974), Klahr and Wallace (1976), Newell and Simon (1972) and Winston (1979) provide well documented introductions to the technique. The following passage from Klahr and Wallace (1976) introduces a production-system language, and its operating rules.

"The models are posed in the form of a collection of ordered condition-action links, called productions, that together form a production system. The condition side of a production refers to the symbols in short-term memory (STM) that represent goals and knowledge elements existing in the system's momentary knowledge state; the action consists of transformations on STM including the generation, interruption, and satisfaction of goals, modification of existing elements, and addition of new ones. A production system obeys simple operating rules:-

1. The condition of each production is matched against the symbols in STM. If all of the elements in a condition can be matched with elements (in any order) in STM, then the condition is satisfied.

2. If no conditions are satisfied, then the system halts. If more than one condition is satisfied, then some conflict-resolution principle must select which production to "fire." Typically, conflict is resolved by choosing the earliest production in the production system. Other resolutions are possible, but that is the one we will use at first.
3. When a production "fires," the actions associated with it are taken. Actions can change the state of goals, replace elements, apply operators, or add elements to STM.
4. After a production has fired, the production system is re-entered from the top; that is, the first production's condition is tested, then the second, and so on.
5. The STM is a stack in which new elements appear at the top (or front), pushing all else in the stack down one position. Since STM is limited in size, elements may be lost.
6. When a condition is satisfied, all those STM elements that were matched are moved to the front of STM. This provides a form of automatic rehearsal."

(Klahr and Wallace, 1976, pp 6-7)

The task models constructed in the present study contain productions that make use of a small number of actions. In every instance, these actions cause the contents of STM to change. The actions used are the following:-

- INS(X) - Inserts the expression X into STM.
- DEL(X) - Deletes the expression X from STM.
- REPL(X;Y) - Replaces the expression X in STM with the expression Y.
- SAY(X) - Prints the expression X. This provides an interface with the user.
- USER() - Asks the user if he has any information. The user's response is stored in STM.
- DO(X) - Transfers control to the list of productions labelled X, but only for a single cycle.

(Ohlsson, 1980)

Typically, the expression (X) would be a representation of information provided by the task, or information retrieved from LTM, or information needed to control the solution process.

13.3 TASKS SELECTED FOR

MODELLING.

The three tasks selected for modelling were:- N-ADD-NV; N-SUB-NV; and N-CYC-NV. The three tasks are closely related. Although 58 subjects passed N-ADD-NV, only 21 passed N-SUB-NV, and only 16 passed N-CYC-NV. The proportion of subjects who passed N-ADD-NV was significantly higher than the proportions who passed N-SUB-NV and N-CYC-NV. However, N-SUB-NV,

and N-CYC-NV were of about the same difficulty. It will be apparent that N-CYC-NV requires a co-ordination between two major components: namely, those assessed by N-ADD-NV and N-SUB-NV. The difficulty data suggest, therefore, that the delay between acquisition of the N-ADD-NV and N-CYC-NV components is due to a delay in acquisition of the N-SUB-NV component, and not to a delay in co-ordinating the N-ADD-NV and N-SUB-NV components. It was thought that a comparison of models for these three tasks would provide an appropriate example of the application of the technique, and it would enable a re-examination of the role of STM capacity limitations.

A factor that influenced the selection of these tasks for modelling was the existence of a substantial empirical literature on the strategies used by young children when subitizing, counting, adding and subtracting. Most of that literature has been reviewed recently by Klahr and Wallace (1976). Their conclusions are reflected in the subitization and counting productions found in several of their models.

A broad outline of each of the models developed will be given. This will be followed by annotated listings for the addition and subtraction models which use a counting strategy. Finally, performance statistics for the six models will be summarised, and conclusions drawn regarding the information-processing demands of the three tasks.

13.4 OUTLINE OF THE MODELS.

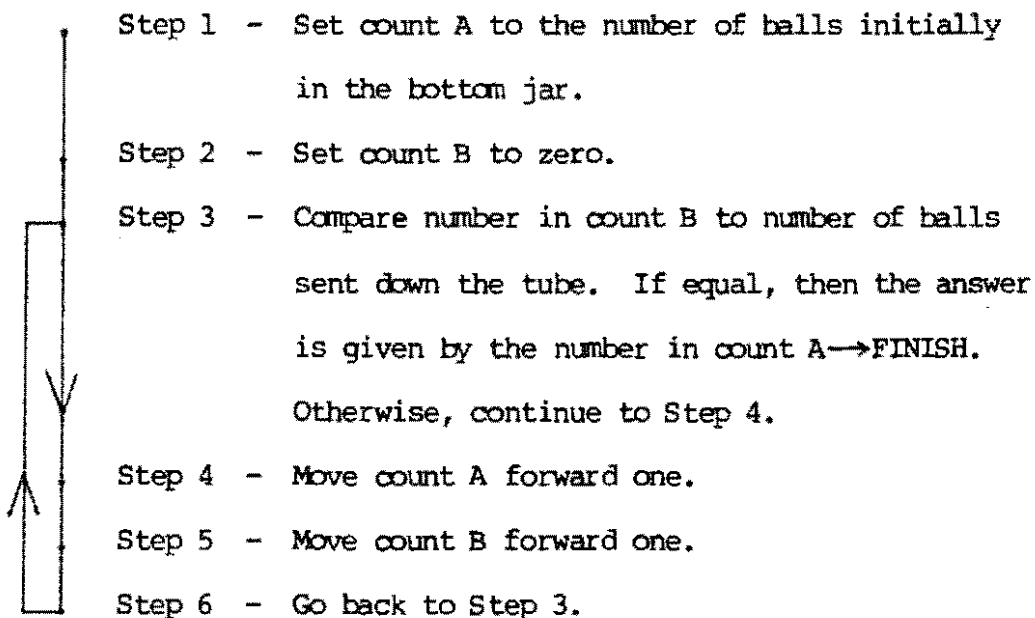
Alternative models were developed for two of the tasks. These were designed to simulate various strategies children had been observed to use. Six models were developed. Three were of N-ADD-NV, two of N-SUB-NV, and one of N-CYC-NV. Listings of all models, and traces of their execution showing the contents of STM on every cycle, are given in Appendix 4. Descriptions of all models are also given in Appendix 4. Additionally, to illustrate the technique, listings and detailed descriptions of two of the models will be given in this Chapter. They are the counting models for addition (simulating performance on N-ADD-NV) and subtraction (simulating performance on N-SUB-NV).

13.4.1 ADDITION MODELS.

Three models constructed to simulate performance on the N-ADD-NV task are provided in Appendix 4. Each carries out the addition operation in a different manner.

The first model (ADD6.PSS) is described in Appendix 4, and listed in full in Addendum 1. It is based on a simple table-look-up procedure. The number of balls in the bottom jar is used as a key for accessing the appropriate entry. For example, if the bottom jar has two balls in it, and the subject sends four more down the tube, the model uses 'two' to access the (2+4=6) entry.

The second model (ADD8.PSS) will be described in detail in Section 13.5 (a full listing and execution trace are provided at Appendix 4, Addendum 2). It is based on a counting method. It simulates performance by a subject who carries out addition by co-ordinating two counting operations. Assume that memory contains an ordered list of number names ("1", "2", "3",....). Subjects count by placing names from this list into STM. Two counts are maintained. Count A is a measure of the number of balls counted to date in the bottom jar. Count B is a measure of the increment in count A that has been made at a given point in the counting process. The subject's procedure is:-

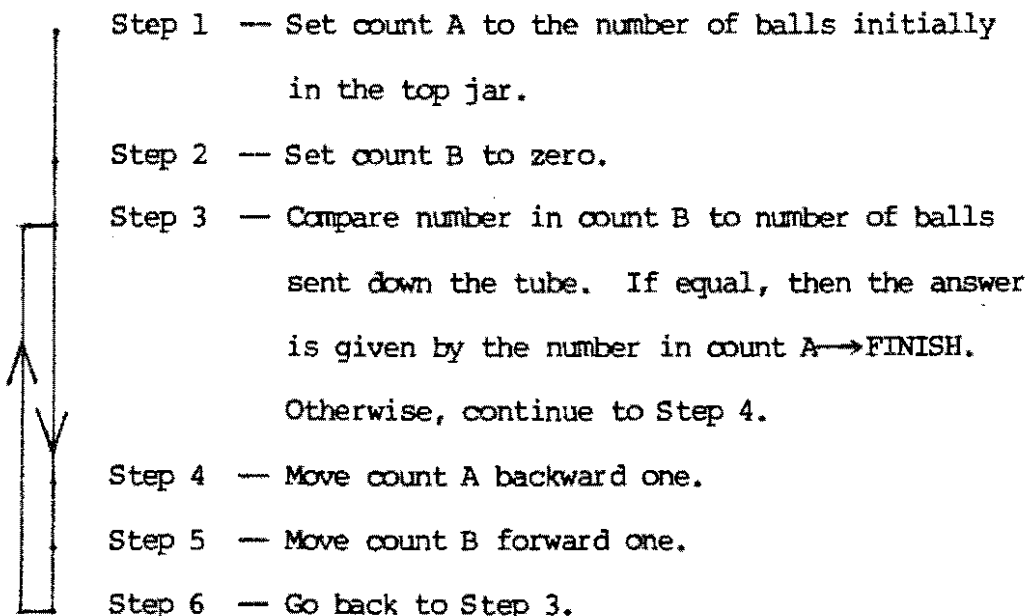


The third model (ADD3.PSS) is described in Appendix 4, and listed in full in Addendum 3. It is also based on a counting method. The first step in this method, however, is finding out which of the two addends is the larger. This becomes the initial value of count A. This method reduces the number of iterations of Steps 3 to 6, above. However, it also incurs the overhead involved in first finding the larger addend.

13.4.2 SUBTRACTION MODELS.

For the N-SUB-NV task, the first model (ADD5.PSS) is based on a table-look-up procedure similar to that used for addition. The access key in this case is the number of balls in the top jar. The model is described in Appendix 4 and listed in full in Addendum 4.

The second method (ADD4.PSS) will be described in detail in Section 13.5 (a full listing and execution trace are provided at Appendix 4, Addendum 5). It is based on the counting method illustrated schematically below:-



13.4.3 ADDITION AND SUBTRACTION MODEL.

For the N-CYC-NV task, the model (ADD7.PSS) is based on the table-look-up procedures used in the addition and subtraction models. It is described in Appendix 4, and listed in full in Addendum 6.

13.5 ANNOTATED LISTINGS OF THE
COUNTING-BASED ADDITION
AND SUBTRACTION MODELS.

Listings of the counting-based addition (ADD8.PSS) and subtraction (ADD4.PSS) models and traces of their execution are given in Appendix 4, Addenda 2 and 5, respectively. Sections 13.5.1 and 13.5.2 present annotated listings of the productions constituting these models.

In many production-system models goal manipulation procedures are grouped together in one or two common-servicing productions that are always tested at the beginning of every cycle. In the present models, they have been located separately in productions which trigger particular goal activation, re-activation, suspension and deletion operations. This approach makes the production systems easier to follow, and has been adopted to assist the reader who is not familiar with production-system languages. It reduces the programming elegance of the models, but it does not result in any greater demands being placed on STM, and does not increase the total number of productions fired.

The models are written in PSS (Ohlsson, 1980), a variant of PSG.

13.5.1 THE COUNTING-BASED MODELOF N-ADD-NV.

In the first phase, the subject is asked to send n balls to the bottom jar by pressing the button. After the balls have gone down the tube, the subject is asked how many were in the bottom jar before, how many more had he just sent down, and how many were now in the bottom jar.

The productions P000, P00 and P0 model the entry of task information into STM, and initiate the run.

```
(P000 (GOAL * ATTEND) (OLD SEND C)
```

```
==>
```

```
REPL((GOAL * ATTEND) ; (GOAL + ATTEND));
```

```
DEL((OLD SEND C));
```

```
USER() )
```

```
(P00 (GOAL * ATTEND) (GO)
```

```
==>
```

```
REPL((GOAL * ATTEND) ; (GOAL + ATTEND));
```

```
DEL((GO) )
```

```
(P0 (GOAL * ATTEND)
```

```
==>
```

```
USER() )
```

When that information is entered, the model responds by simulating the subjects button-pressing and counting behaviour. The productions responsible are labelled P1 to P8.

```
(P1 (GOAL + ATTEND) (TOP A) (BOTTOM B) (SEND C)
```

```
==>
```

```
REPL((GOAL + ATTEND) ; (GOAL *S* ATTEND));
```

```
INS((GOAL * SEND C));
```

```
INS((Y 1 Z) )
```

```
(P2 (GOAL * SEND C)
```

```
==>
```

```
REPL((GOAL * SEND C) ; (GOAL *S* SEND C));
```

```
INS((GOAL * PUSH BUTTON) )
```

```
(P3 (GOAL * PUSH BUTTON)
```

```
==>
```

```
DEL((GOAL * PUSH BUTTON));
```

```
INS((ELM A));
```

```
INS((GOAL * SUBIT) )
```

```
(P4 (GOAL * SUBIT) (ELM A)
```

```
==>
```

```
DEL((GOAL * SUBIT));
```

```
INS((QS 1));
```

```
DEL((ELM A));
```

```
INS((GOAL * COUNT) )
```

(P5 (GOAL * COUNT) (QS 1) (Y <X> Z)

==>

REPL((GOAL * COUNT) ; (GOAL + COUNT));

DEL((QS 1));

SAY(<X>);

REPL((Y <X> Z) ; (SAID <X>));

INS((GOAL * MARK))

(P6 (GOAL * MARK) (SAID 1)

==>

REPL((GOAL * MARK) ; (GOAL + MARK));

INS((Y 2 Z))

(P6A (GOAL * MARK) (SAID 2)

==>

REPL((GOAL * MARK) ; (GOAL + MARK));

INS((Y 3 Z))

(P6B (GOAL * MARK) (SAID 3)

==>

REPL((GOAL * MARK) ; (GOAL + MARK));

INS((Y 4 Z))

(P7 (GOAL + MARK) (GOAL + COUNT) (GOAL *S* SEND C)

(SAID C)

==>

DEL((GOAL *S* SEND C));

DEL((GOAL + MARK));

DEL((GOAL + COUNT));

DEL((SAID C));

REPL((GOAL *S* ATTEND) ; (GOAL * ATTEND))

```
(P8 (GOAL + MARK) (GOAL + COUNT) (GOAL *S* SEND C) (SAID D)
```

```
==>
```

```
REPL((GOAL *S* SEND C) ; (GOAL * SEND C));
```

```
DEL((GOAL + MARK));
```

```
DEL((GOAL + COUNT));
```

```
DEL((SAID D) )
```

P1 inserts an active goal of sending `n` balls down the tube. In the service of that goal, P2 inserts the subordinate goal of pushing the button. P3 notices the ball going down the tube and inserts the subitization goal. P4 simulates subitization of the ball(s) noticed (by P3) going down the tube. P5 carries out a counting operation by accessing and `saying` the next name on a number-name-list. P6 marks the name `said` by P5, and inserts the next number name on the list into STM. P6A to P6C simulate similar marking and moving operations. P7 simulates a checking operation. If the last name `said` by P5 is the same as the number-name given in the instruction to send `n` balls down the tube (represented in STM by (SEND C)), then the model `knows` that it has finished that part of the task. In that event, it re-activates the goal `to attend` and the next production to fire is P0. If not, the goal manipulation in P8 ensures that P2 will be the next production to fire, and that a new cycle of button pressing, subitizing, and counting will be entered. This procedure can be followed by reading the listing for the model concurrently with the trace of the model's execution, given in Appendix 4, Addendum 2.

When control is passed back to P0, the user then asks: how many balls were in the bottom jar before? This is represented by the STM elements (MANY BOTTOM BEFORE). P9 and P10 simulate the answering of this question, after which control is passed back to P0. The user then asks: how many were sent down the tube? This is represented in STM by the element (MANY TUBE). This question is answered by P11 and P12, after which control is again passed back to P0.

```
(P9 (GOAL * ATTEND) (MANY BOTTOM BEFORE)
```

```
==>
```

```
REPL((GOAL * ATTEND) ; (GOAL *S* ATTEND));
```

```
DEL((MANY BOTTOM BEFORE));
```

```
INS((GOAL * RECALL BOTTOM) )
```

```
(P10 (GOAL * RECALL BOTTOM) (BOTTOM B)
```

```
==>
```

```
SAY((BOTTOM B));
```

```
DEL((GOAL * RECALL BOTTOM));
```

```
REPL((GOAL *S* ATTEND) ; (GOAL * ATTEND) )
```

```
(P11 (GOAL * ATTEND) (MANY TUBE)
```

```
==>
```

```
REPL((GOAL * ATTEND) ; (GOAL *S* ATTEND));
```

```
DEL((MANY TUBE));
```

```
INS((GOAL * RECALL TUBE) )
```

```
(P12 (GOAL * RECALL TUBE) (SEND C)
```

```
==>
```

```
DEL((GOAL * RECALL TUBE));
```

```
REPL((GOAL *S* ATTEND) ; (GOAL * ATTEND));
```

```
SAY((SEND C) )
```

By entering (MANY BOTTOM NOW) into STM, the user causes control to be given to P13 on the next cycle.

The direct counting procedure involves the co-ordination of step-by-step movement through two sequences, each of which is the number-name-list. Items from the first sequence are represented in STM by elements having the form (Y <P> Z), and those from the second sequence by (W <P> V). The letters Y,Z,W, and V imbedded in these elements are of technical significance only. They constitute a method of marking locations in a list. The symbol <P> assumes numerical values.

P13 activates the addition goal (GOAL * ADD). It sets the location in the first sequence at the point corresponding to the number of balls in the bottom jar before the last series of button presses. P13 also sets the location in the second sequence, just before the first element in the number-name-list. P14 checks to see if the location in the second sequence is the same as the value in the STM element representing the instruction to send 'n' balls down the tube. That is, P14 asks: is the value of <P> in the element (W <P> V), the same as the value of <P> in the element (SEND <P>)? If so, P14 fires, and the answer is extracted from the (Y <P> Z) element currently in STM. If not, either P15 or P16 fires. They control the movement through the two sequences. P17 to P17J carry out the moves from place to place in the first sequence. P18 to P18C perform the same function for the second sequence.

(P13 (GOAL * ATTEND) (MANY BOTTOM NOW) (BOTTOM B)

==>

REPL((GOAL * ATTEND) ; (GOAL *S* ATTEND));

DEL((MANY BOTTOM NOW));

INS((GOAL * ADD));

INS((W 0 V));

INS((Y B Z)))

(P14 (GOAL * ADD) (W <P> V) (SEND <P>) (Y <X> Z)

==>

SAY(<X>);

DEL((GOAL * ADD));

INS((GOAL * PURGE)))

(P15 (GOAL * ADD) (W <P> V) (Y <X> Z)

==>

REPL((GOAL * ADD) ; (GOAL *S* ADD));

INS((GOAL * NEXT ALONG));

DO(GET-NEXT))

(P16 (GOAL * NEXT UP)

==>

DO(STEPUP))

(P17 (GOAL * NEXT ALONG) (Y ZERO Z)

==>

REPL((Y ZERO Z) ; (Y ONE Z));

DEL((GOAL * NEXT ALONG));

INS((GOAL * NEXT UP)))

NOTE: Productions P17A to P17J have the same form as P17. Each inserts the next number-name symbol (e.g. (Y FOUR Z)) in STM.

```
(P18 (GOAL * NEXT UP) (W 0 V)
```

```
==>
```

```
REPL((W 0 V) ; (W 1 V));
```

```
DEL((GOAL * NEXT UP));
```

```
REPL((GOAL *S* ADD) ; (GOAL * ADD)) )
```

NOTE: Productions P18A to P18C have the same form as P18. Each replaces the current number-name symbol (e.g. (W 2 V)) with the next number-name symbol (e.g. (W 3 V)).

An example may clarify the operation of P13 to P18C. Suppose that the bottom jar had two balls in it, and the subject sent down four more. STM would contain the elements (BOTTOM 2) and (SEND 4). P13 would insert the elements (Y 2 Z) and (W 0 V). On the first cycle after the firing of P13, P14 would not fire because <P> would be set to 0 in (W 0 V). P15 would initiate an entry to the P17 to P17J group of productions. Specifically, P17B would fire and insert (Y 3 Z) into STM, and set a goal causing P16 to fire on the next cycle. P16 would then initiate an entry to the P18 to P18C group of productions. Specifically, P18 would fire and insert (W 1 V) into STM, and set a goal causing P14's conditions to be examined on the next cycle. Again, P14 would not fire, because <P> would be set to 1 in (W 1 V). Hence, P15 would fire again, and the P15 to P18C procedure would be re-entered. This pattern would continue until on one cycle P14 found <P> set to 4 in (W 4 V). At that time, the (Y <P> Z) element would contain (Y 6 Z). P14 would then extract the answer (6) from that element.

The remaining productions P24A to P27 perform housekeeping functions needed to prepare the model to receive further input.

13.5.2 THE COUNTING-BASED MODEL

OF N-SUB-NV.

A complete listing of this model (ADD4.PSS) is provided in Appendix 4, Addendum 5. Productions P000 to P12 are identical in form and function to the same-numbered productions for the addition model. They simulate the subject's behaviour up to the point where he is asked how many balls are left in the top jar.

The productions P13 to P18C also carry out functions analogous to the same-numbered productions in the addition model. The differences are that:-

- . P13 sets <P> in (Y <P> Z) to equal the number of balls in the top jar, initially.
- . P17 to P17J moves down the number-name-list, not up it, as in the case of the addition model (ADD8.PSS).

13.6 PERFORMANCE STATISTICS.

Table 13.1 sets out the number of productions fired, and the maximum number of elements held in SIM during the execution of each of the six models. The entries in Table 13.1 relating to SIM represent the maximum amount of SIM used by the respective models - that is, if SIM allocations of lesser capacity were to be made, the models would not execute correctly.

TABLE 13.1: PERFORMANCE STATISTICS FOR EACH MODEL.

TASK MODELLED	N-ADD-NV			N-SUB-NV		N-CYC-NV
PROCESS MODELLED	ADDITION			SUBTRACTION		ADDITION AND SUBTRACTION
METHODS USED	TABLE-LOOK -UP (ADD6.PSS)	COUNTING (ADD8.PSS)	COUNTING FROM LARGER ADDEND (ADD3.- PSS)	TABLE-LOOK -UP (ADD5.PSS)	COUNTING (ADD4.PSS)	TABLE-LOOK -UP (ADD7.PSS)
No. of PRODUCT- IONS. FIRED	130	162	190	130	162	174
MAX.No of ELE- MENTS HELD IN SIM	9	9	12	9	9	9

The 'number of productions fired' is the number of steps required - at this level of analysis - to simulate a successful subject's performance on the modelled task. The table-look-up procedure is the more efficient for addition and subtraction. The straight-forward counting procedure for addition (ADD8.PSS) is more efficient than the alternative (ADD3.PSS). Subtraction, by table-look-up, or by counting, involves exactly the same number of steps as the corresponding addition procedure. The small increase in the number of productions fired for N-CYC-NV, over N-ADD-NV and N-SUB-NV (174 versus 130), suggests that the bulk of the effort is expended on simulating aspects of the tasks not directly concerned with addition or subtraction.

The more efficient, table-look-up models require no more STM space than the counting-based models. All but one of the models required up to 9 elements of STM to store the control information and data used during execution. The exception is the counting-based addition model (ADD3.PSS), which first finds the larger of the two addends. That model requires a maximum of 12 elements of STM.

According to this analysis, the delay in development of STM capacity could not be the factor causing the observed delay in acquisition of the components assessed by N-SUB-NV and N-CYC-NV. Additionally, this analysis reinforces the suggestion given in Section 13.3 that the delay in acquisition of the component assessed by N-CYC-NV is due to delay in acquisition of the component assessed by N-SUB-NV, rather than a need to co-ordinate the components assessed by N-ADD-NV and N-SUB-NV.

13.7 CONCLUSIONS.

The performance similarities between the corresponding addition and subtraction models suggest that the observed development sequence is not a function of STM capacity. It seems to reflect, simply, the order of acquisition of certain rules. An inspection of the productions used in the models of the table-look-up procedures (Appendix 4, Addenda 1 and 4) supports this conclusion. For the table-look-up procedure, the only difference between the addition and subtraction models is in the data tables. The addition model makes use of entries of the form: $a+b=c$. The subtraction model makes use of entries of the form: $a-b=c$. For the child using this procedure, the observed developmental sequence may simply reflect the fact that the former are usually learned before the latter.

In the case of the counting procedure, the addition model controls the co-ordinated movement in the same direction of two pointers through a single number name list. The subtraction model controls the co-ordinated movement in opposite directions of two pointers through the same list. The former procedure contains less potential for confusion because the two pointers are rarely in close proximity in the list. The latter procedure does contain potential for confusion when the two pointers co-incide in the list. For example, in the N-SUB-NV task, at one time the bottom jar holds six balls, and the top jar six balls. Hence, one pointer would be moving: "6 -> 7 -> 8 ... etc". The other would be moving: "6 ->5 -> 4 etc". Consequently, in the case of subtraction, even if the child has the appropriate counting-based rules, the probability of a breakdown during execution of those rules would be greater than for the corresponding addition operation.

It is noteworthy that these conclusions are consistent with the finding that length of schooling is a better predictor, than age, of a subject's performance. That is, performance is a function of experience with the appropriate rules, and not a function of STM capacity.

Finally, it is emphasized that the six models discussed in this Chapter were developed for two purposes. Firstly, to examine in more detail the questions discussed in Chapter 12, and, secondly, to illustrate an approach to the longer-term objective of deriving a detailed process account of the composition and growth of linear measurement. The core productions relating to subitization, counting, addition and subtraction contained in the present models are all pertinent to that enterprise. However, they constitute only a small fragment of the work that needs to be done to realise the longer-term objective.

CHAPTER 14.SUMMARY OF CONCLUSIONS.

The present study had two main objectives. The first was to identify the "higher-level" knowledge necessary for a child to understand linear measurement. The second was to chart the growth of linear measurement in terms of the development of its components.

An analysis of measurement operations yielded a list of components which it was argued would underlie linear measurement. Piagetian theory and related empirical literature were consulted as sources of information on the emergence of these components in the child's thinking. This led to the formulation of a number of predictions concerning the components of linear measurement, and their order of development.

A battery of 34 number, length and distance tasks was developed to assess the presence of these components. It was administered to 100 children aged between 63 and 78 months, and drawn from kindergarten and grade one. The results were analyzed using scalogram techniques. The main contribution of the thesis is in this empirical work.

The main conclusions are summarised in the following paragraphs.

14.1 COMPONENTS OF LINEAR
MEASUREMENTS.

It was found that children who possessed a mature level of understanding of linear measurement also possessed the following:-

- . Knowing how to make transitive inferences of equivalence, with respect to discrete quantity, and length.
- . Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
- . Knowing that length is invariant under certain transformations (the conservation of length).
- . Knowing how to carry out numerical addition operations.
- . Knowing how to obtain a linear measurement by counting iterations of a unit of length.
- . Knowing how to make transitive inferences of non-equivalence, with respect to discrete quantity.

14.2 ORDER OF DEVELOPMENT
OF LINEAR MEASUREMENT.

For linear measurement, the components emerge in the order in which they are listed above.

The data imply a delay between acquisition of the components and emergence of an understanding of linear measurement. It was also noted that those children who possessed all the necessary components, but could not

demonstrate an understanding of linear measurement, could not commence the linear measurement task. It was argued that this was evidence that short-term-memory capacity limitations were not implicated in the delay, because such limitations are expressed usually in breakdowns in performance of a strategy. The delay was interpreted as being associated with the need for a re-organisation of the relevant cognitive structures, resulting in better co-ordination between components. That is, the delay was associated with the formation of new long-term-memory links between components.

14.3 ORDER OF DEVELOPMENT
OF COMPONENTS IN THE
NUMBER, LENGTH AND
DISTANCE DOMAINS.

For the number and length domains, the collections of components form scalable sets. That is, the components emerge sequentially in each domain.

In general, the order of emergence of the components is that predicted by Piagetian theory, and the empirical evidence reviewed in Part II. The most important exceptions to that pattern are noted below.

Conservation of number emerges significantly before transitive inference concerned with non-equivalent relations between discrete quantities. The same lag in development occurs in respect of length. It was argued that this finding could have been expected, because the form of transitive reasoning involved in the conservation task is concerned with equivalent, and not with non-equivalent, relations.

The observed lag in development between the emergence of corresponding components in the length and distance domains, such as the conservation of length and the conservation of distance, was not predicted.

14.4 DISCONTINUITIES IN NUMBER AND LENGTH CONCEPT DEVELOPMENT.

It was found that the patterns of development in the number and length concepts were marked by discontinuities.

It was suggested that these discontinuities might have been due to capacity limitations of short term memory. However, an information-processing analysis, using the M-space model, failed to find evidence supporting that suggestion. Furthermore, a production-system analysis that explicitly accounted for strategy control information also failed to find evidence of a short-term-memory barrier.

In general, the discontinuities co-incide with the emergence of components of major theoretical interest, such as the conservation of length. Additionally, there are concordances between discontinuities in the number and length growth patterns. It was suggested that these concordances co-incide with periods of development during which new forms of co-ordination are being established between the two concept domains. New inter-connections of that kind would be represented by long-term memory linkages.

In summary, the general impression is that growth in one concept domain prompts growth in the other.

14.5 PRODUCTION-SYSTEM

MODELS OF LINEAR

MEASUREMENT.

It was stated that the analysis of linear measurement given in Chapter 2 was largely intuitive and informal due to the lack of a detailed psychological theory of linear measurement, and the paucity of directly relevant empirical data. It was argued that a more satisfactory account of linear measurement could be given in the form of an executable production-system model. A model of that kind would be a formal theory, and would demonstrate that the components listed in Chapter 2 are necessary for linear measurement. It would also provide a sufficient account of the empirical data reported in this study. Development of that model would be a substantial undertaking, and beyond the scope of the present research. However, the six production systems constructed in this study provide a small start on that larger, longer-term project.

14.6 THE EFFECTS OF AGE AND LENGTH OF SCHOOLING.

It was found that length of schooling is a predictor of a subject's score, but age is not. This latter finding, though, must be due to the narrow age range of the subjects used in the study.

These findings are consistent with the general pattern of development observed in the study. This is because delays and discontinuities in development during the period studied were thought to be associated not with short-term-memory barriers - an age related factor - but with the forging of new long-term-memory links - an experience related factor.

For many children, experiences of the appropriate kind are provided most frequently at school. Hence, length of schooling could be expected to be a better predictor of performance, than age, on the tasks used, and for the period of development covered, in this study.

14.7 SUGGESTIONS FOR FURTHER RESEARCH.

It is considered that further research is needed in order to elaborate upon the kinds of inter-connection between components that represent solution strategies for linear measurement tasks. It is suggested that such further research incorporate the construction of production-system models depicting mature levels of performance in linear measurement. Additionally, further research is needed to identify the reasons for the lag in development between corresponding components in the length and distance domains.

The findings of the present study, together with those from such further research, could considerably inform early primary school curriculum planning.