USE OF THESES

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DEVELOPMENT OF

LINEAR MEASUREMENT.

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ABSTRACT.

The study had two objectives. The first was to identify the 'higher-level' knowledge necessary for a child to understand linear measurement. The second was to chart the growth of linear measurement in terms of the development of its components. In this context, 'higher-level' knowledge refers to skills such as counting an array of objects, as distinct from 'lower-level' skills such as attending to an object in an array.

An analysis of measurement operations yielded a list of components which it was argued would underlie linear measurement. Piagetian theory and related empirical literature were consulted as sources of information on the emergence of these components in the child's thinking. This led to the formulation of a number of predictions concerning the components of linear measurement, and their order of emergence.

A battery of 34 number, length, and distance tasks was developed to assess the presence of these components. It was administered to 100 children aged between 63 and 78 months, and drawn from kindergarten and grade one. The results were analyzed using scalogram techniques. The main contribution of the thesis is in this empirical work.

It was found that children who possessed a mature level of understanding of linear measurement also possessed the following:
Knowing how to make transitive inferences of equivalence, with respect to discrete quantity, and length.

Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).

Knowing that length is invariant under certain transformations (the conservation of length).

Knowing how to carry out numerical addition operations.

Knowing how to obtain a linear measurement by counting iterations of a unit of length.

Knowing how to make transitive inferences of non-equivalence, with respect to discrete quantity.

There appeared to be a substantial developmental delay between acquisition of these components and emergence of a mature grasp of linear measurement.

It was also found that the collections of components for the number and length domains formed scaled sets. However, within each domain the pattern of development was marked by discontinuities (abrupt changes in the slopes of the task performance gradients).

It was suggested that the discontinuities might be due to differences in short-term-memory (STM) demands made by tasks which differed significantly in difficulty. An information-processing analysis, using Pascual-Leone’s M-Space model, did not confirm this.
A production-system analysis of certain of the number tasks also failed to reveal differences in demands made on STM by tasks differing in difficulty.

The discontinuities in development were interpreted as being associated with the need to re-organise number and length concepts.

Length of schooling, but not age, was found to be a predictor of performance on the task battery. No sex differences were found.
This thesis comprises four parts. Part I presents the problem. Part II reviews relevant literature. Part III describes an empirical study and presents a statistical analysis of the results. Finally, Part IV presents an interpretation of these results.
Acquisition of the number concept, and attaining skill in using numerical operations, are considered important intellectual achievements of childhood, because they enable the child to use mental operations in lieu of physical actions. For example, when the child is able to perform addition operations, he no longer needs to count physically. Similarly, after the child has acquired an understanding of linear measurement, he no longer has to align two objects physically, in order to find out which is the longer.

Psychological research has contributed substantially to an understanding of how the number concept, and numerical operations, develop. In comparison, however, much less research effort has been expended on finding out how numerical knowledge is linked with knowledge in other concept domains, such as length and distance. One way of examining such linkages between concepts is to study the development of an activity which draws upon knowledge associated with each concept. The development of linear measurement provides an example.
The present study had two main objectives. The first was to identify the "higher-level" knowledge necessary for a child to understand linear measurement. The second was to chart the growth of linear measurement in terms of the development of its components. In this context, "higher-level" knowledge refers to skills such as counting an array of objects, as distinct from "lower-level" skills such as attending to an object in an array.

A major empirical study was carried out to meet these objectives. The main contribution of the thesis is in this empirical work.
CHAPTER 2.

AN ANALYSIS OF LINEAR MEASUREMENT.

2.1 SELECTION OF AN APPROACH.

2.1.1 TWO POSSIBLE APPROACHES.

One objective of the present research was to chart the development of linear measurement in young children. It was necessary to decide whether the investigation would focus on linear measurement at a "concept level", or at a "component level".

2.1.2 CONCEPT-LEVEL APPROACH.

In a concept-level approach, the researcher would focus on the development of different levels of performance in the concept being studied. In the present case, he would define operationally a series of levels of achievement in linear measurement. For example:

(a) an ability to determine by direct comparison which of two objects is the longer;
(b) an ability to use correctly a measuring rule;
(c) an ability to iterate a unit length; and,
(d) an ability to predict the effect of changing the length of the unit.
The researcher would devise tests appropriate to each level. These would be administered to a selected population, and subjects categorised accordingly. The statistical analysis would involve correlating conceptual levels with developmental variables, such as age.

### 2.1.3 COMPONENT-LEVEL APPROACH.

The component-level approach extends the concept-level approach by also investigating the development of knowledge components of the concept being studied. In the present case, the researcher would devise tests for each component, as well as for different levels of performance, in linear measurement. An example of a component would be knowing how to count. In this approach, concept development would be described in terms of both levels of achievement in the concept, and the progressive acquisition of its components.

### 2.1.4 ADVANTAGES AND DISADVANTAGES OF EACH APPROACH.

Each approach has advantages and disadvantages. The chief advantage of the concept-level approach is that the levels of the concept may be defined operationally. The main disadvantage is that empirical evidence yielded by a study of that kind would provide little information on how, in a step-by-step sense, the concept develops.

The chief advantage of the component-level approach is that it has the potential to contribute to an understanding of how the concept develops. Moreover, this approach links more readily with existing psychological theory, and its associated empirical data. The main disadvantage is that it involves making a number of a priori assumptions regarding the list of components.
2.1.5 CHOICE OF COMPONENT LEVEL APPROACH.

A motivation of the study was to find information useful in a practical sense in the field of education. In that regard, the component-level approach is the more useful, because it provides information on which components are necessary for linear measurement and, hence assists primary school curriculum development. It was decided to adopt the component-level approach.

Additionally, with this practical purpose in mind, it was decided to restrict the investigation to children in the first two years of primary school, because it is probable that it is during those years that most of the growth in understanding of linear measurement occurs. Against this background, the identification of components of linear measurement will now be discussed.

2.2 IDENTIFICATION OF THE COMPONENTS OF LINEAR MEASUREMENT.

2.2.1 METHODS OF LINEAR MEASUREMENT.

In general, a young schoolchild would know of two ways of measuring the length of an object. The first would be to use a measuring rule and to read off numbers which in some way would represent the object's length. The second would be to select a shorter object for use as a unit length, and to iterate that unit along the longer object, counting the number of iterations. Obviously, the notions underlying the first method correspond to those employed in the second. It would be possible, however, for
a child to use the first method - because it had been trained to - without having an understanding of the underlying principles. Similarly, it would be possible for another child to use the second method - iteration of a unit length - without that child knowing what effect substitution of a longer or shorter unit length would have on the number of iterations required to measure the object. Yet another child might know the effect of substituting a longer or shorter unit length but, when asked to work out the combined length of two previously measured objects, that child might need to join the two objects together physically, and then measure their combined length. It could be argued that such a child did not understand that numerical operations of addition and subtraction can be used instead of measurement operations.

Each of these three children could be said to have a different level of understanding of linear measurement. Intuitively, it would seem reasonable to conjecture that these different levels of understanding of linear measurement result from differences in, or co-ordination between, the number and length concepts of such children.

2.2.2 UNIT ITERATION AND THE LENGTH CONCEPT.

An examination of the operations involved in the act of measuring the length of an object by unit iteration reveals many of the components of linear measurement. In the following discussion the word 'measurer' refers to a person who has a full understanding of iterative linear measurement. By marking off sections of the object being measured (A) into segments (A1, A2, A3, etc.) equal in length to the unit (B), the measurer implies that he holds the following beliefs regarding length:-
(a) the length of B remains constant throughout the measurement operation, even though the orientation of B to the measurer might change;
(b) because the length of A1 is equal to the length of B, and the length of A2 is equal to the length of B, the length of A1 must be the same as the length of A2;
(c) the length of an object is the same as the length of its concatenated parts.

2.2.3 THE CONSERVATION OF LENGTH.

The first of these beliefs (a), involves several components. For example, the measurer must believe that the length of any object does not change when it is displaced in space. Similarly, the measurer must believe that the relation of equivalence of length that holds between B and A1 when B is momentarily aligned with A1, does not change when B is not longer so positioned. These beliefs would seem to rest on the belief that a relation between two lengths can only change when something is added to, or taken away from, one, or other, or both, of the two lengths involved. Beliefs of this kind are usually referred to as the conservation of length.

It seems plausible that, prior to achieving an understanding of linear measurement, the child must know that the length of an object, and length relations between objects, are conserved under various kinds of transformation. However, how can the child acquire the conservation of length without being able to empirically verify it by linear measurement? For example, the child might know that length P is equal to length Q in
position R. How does the child know that P is still equal to Q when P has been moved to R'? Obviously, one answer would be that the child uses visual evidence that nothing has been added or subtracted from P and/or Q. But, how is this rule acquired without an understanding that length is measurable? Furthermore, P in its new position R' may well look longer or shorter than Q. One way around the problem would be for the measurer to move Q to R' so that the kind of direct comparison of P and Q made at R can be repeated at R'. However, how does the child know that Q does not change in length, to exactly the same extent as might have P, whilst being moved from R to R'? The knowledge that P equals Q at both R and R', does not necessarily ensure that P equals Q when Q is at R and P and R'.

The situation becomes more complex when the transformation of the measuring instrument (or unit) involves changing its shape, as distinct from changing its position. An example of such an instrument is a piece of string. How does the child know that the length of the string is the same whether it is placed in the form of a straight line or, say, a circle?

The preceding discussion suggests that there is an interdependence between conservation and measurement. However, for the moment, it will be taken as a working hypothesis that the child acquires the conservation of length before an understanding of linear measurement.
2.2.4 TRANSITIVE REASONING.

The second belief (b), is based upon what is usually known as "transitive inference". The length relation between $A_1$ and $A_2$ is inferred, rather than determined by direct comparison. The fact that this inference is made, means that the measurer also holds the conservation beliefs discussed above. The measurer's argument is: $(A_1 = B$ in position 1) and $(A_2 = B$ in position 2) implies $(A_1 = A_2)$. For this conclusion on the relative magnitude of $A_1$ and $A_2$ to be drawn, the measurer must believe that the length of $B$ has not changed following its change in position. If the measurer did not hold this belief, then the premises contained in the transitive inference would not apply.

Equally, however, the conservation belief seems itself to imply a transitive inference. The argument is: $(A_1 = B$ in position 1) and, $(A_2 = B$ in position 2) implies $(A_1 = A_2)$, which implies $(A_1 = B$ when $B$ is in position 2).

2.2.5 PART/WHOLE RELATIONS OF LENGTH.

The third belief (c), concerns the measurer's understanding of the distinction between an object and one of its attributes - length. That is, a child may know that an object may be arbitrarily divided into parts, and those parts recombined to form the object. However, the child may not extend that knowledge to encompass the object's length. If that knowledge does extend to length, then it would find expression in such beliefs as: if length $A$ is greater than length $B$ then length $A$ may be considered as length $B$ concatenated with some other length.
2.2.6 UNIT ITERATION AND THE NUMBER CONCEPT.

In addition to beliefs concerning length, the measurement operation also requires the measurer to hold certain beliefs concerning, and to have certain skills with number. Firstly, the measurer must be able to 'numerate': that is, to co-ordinate ordinal position and cardinal value. Secondly, the measurer must believe that the numerosity of a collection of objects remains unchanged when the spatial arrangement of the collection is altered. For example, the measurer must believe that an object found by iteration to contain, say, six unit parts, will always contain six unit parts irrespective of the spatial location of the object. This belief is usually referred to as the conservation of number.

2.2.7 UNIT ITERATION AND INTER-CONNECTION OF LENGTH AND NUMBER CONCEPTS.

In addition to having certain beliefs concerning length and number, the measurer must also have some degree of interconnection of those beliefs. For example, a child may conserve number and length, and may be able to numerate, but unless there is some connection between that child's number and length concepts it is unlikely that the child could understand linear measurement. Similarly, if the measurer has a mature understanding of linear measurement, then he would also have an ability to perform arithmetical operations of addition and subtraction in lieu of measurement operations. For example, the difference in length of two objects can be determined by subtracting one numerical length measurement from the other, and thereby obviating the need to align both objects and measure the difference directly.
2.3 DEVELOPMENTAL PERIOD COVERED.

In any developmental study it is necessary to establish a point at which to start - a lower boundary - and a point at which to finish - an upper boundary. These decisions determined the age range of the subjects used in the study, and the levels of task difficulty employed.

The lower boundary selected for the present study was: determination by direct comparison of ordinal length relations between two objects. This study, therefore, has excluded from consideration a host of perceptual skills and cognitive attainments achieved in the early years of life. Some of those attainments seem to be linked with estimation processes that appear to be perceptually-based. For example, very young children appear to have some idea of what constitutes a distance. They can determine whether an object within view can be grasped without having to move any part of their bodies other than arms or shoulders, or whether an object can be reached without using something to stand on. It is quite possible that processes of that kind could either inform, or confuse, the five or six year old child who is in the course of developing more precise, conceptually-based skills for determining length and distance. However, the contributions made by such perceptual processes to the development of the linear measurement knowledge of young children are outside the scope of the present research.

The upper boundary selected for the present study was measurement of straight line lengths of small magnitude (approximately 30 cms.), and of distances between objects (of similar separation).
located on a plane, not a curved surface. This study, therefore, does not follow the development of linear measurement through to its completion. For example, a child may be able to measure correctly a stick 20 cms in length, but not understand how to measure, or what it would mean to measure, the length of a piece of curved plastic pipe. The former ability is within the scope of this study, the latter is not. Similarly, a child may be able to measure the separation between objects located on his playroom table, but not understand what it would mean to measure the distance between his home and his school. This study is concerned with the former, but not with the latter ability.

2.4 LIST OF COMPONENTS OF LINEAR MEASUREMENT. (1)

The following is a list of the components required for a full understanding of iterative linear measurement, as suggested by the foregoing discussion. It will be evident that the components listed are not independent. This matter will be discussed in later paragraphs.

1. There is a widely cited formal theory of measurement due to Suppes and Zinnes (1963). The theory formally demonstrates that the empirical relational system of measurement of length or distance is an isomorphic image of a particular numerical relational system, the real number system. It is this isomorphism that is the formal basis of everyday activities of, for example, applying arithmetical operations such as addition and subtraction to length measurement.
(A) NUMBER. The following assumes that the child has a number concept though it may be in the early stages of development.

(i) Knowing how to use a 1-to-1 matching rule. This is necessary because such a rule is implicit in counting and in unit iteration.

(ii) Knowing the natural number order. This is necessary because each 1-to-1 pairing during unit iteration must be identified separately, as the first, second, third, etc.

(iii) Knowing how to count arrays of small numerosity, where "count" implies the co-ordination of ordinal position and cardinal value. This is necessary because the number of unit iterations must be determined.

(iv) Knowing how to make transitive inferences of equivalence and non-equivalence with respect to discrete quantity. This appears to be necessary for the conservation of number.

(v) Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number). This is necessary to relate a collection of "n" non-contiguously arranged unit parts to a collection of "n" contiguously arranged unit parts.

(vi) Knowing how to perform the arithmetical operations of addition, subtraction, solving for a difference, and balancing numerosities. This is necessary if arithmetical operations are to be used in lieu of measurement operations.
(B) LENGTH. The following assumes that the child has a concept of length, as an attribute of an object. That concept may be incomplete, in that not all of the properties of length may be known to the child.

(i) Knowing that if length A is greater than length B then A may be considered as B concatenated with some other length. This is necessary for a unit length, (B, in this case) to be employed in measurement.

(ii) Knowing that any length may be considered as a concatenation of arbitrarily selected sub-lengths. This is necessary because a precise statement of any object's length, expresses that length in terms of a number of object parts of shorter length joined together.

(iii) Knowing that the length of an object can be altered only by adding something to it or subtracting something from it (setting aside, for present purposes, processes of expansion and contraction). This is necessary because the unit part changes position during measurement. However, as nothing is added to or taken away from it, its length remains constant.

(iv) Knowing that the length relation between two objects can be changed only by adding to, or taking away from, one, or other, or both, of the objects (setting aside processes of expansion and contraction). This is necessary because the unit part changes position during measurement but, its length remains constant, and so also must the relation of equivalence between the lengths of the unit part and the object parts.
(v) Knowing that the length relation between objects A and B does not change when the spatial relation between A and B changes. This is necessary because, during measurement, the unit part changes position, but the parts of the objects marked off as equal in length to that of the unit part do not. Hence, the length relations between them are constant.

(vi) Knowing that objects may be ordered according to their lengths. This is necessary for transitive inference.

(vii) Knowing that transitive inferences of equivalence and non-equivalence can be applied to length relations. This is necessary because it is implied in relating unit parts of an object to each other, so that an understanding is reached that those parts are equal in length.

(viii) Knowing that the ordinal length relation between two objects is the same as the cardinal numerical relation between the collection of parts comprising those objects (provided that the lengths of these parts are the same). This forms the basis of the isomorphism between counting and measurement that enables arithmetical operations to be used in lieu of measurement (Suppes and Zinnes 1963).

(ix) Knowing that length relations between objects can be deduced by applying transitive reasoning to the collections of unit parts. This is necessary because it is implied in comparing the lengths of objects by comparing the number of unit parts contained in them.
(x) Knowing that length is invariant under certain transformations (the conservation of length). This is necessary because the accuracy of unit iteration depends upon the length relations between the unit part and the object parts remaining constant.

(C) LENGTH MEASUREMENT.

(i) Knowing how to iterate a unit part along an object. This is necessary for the operation to be accurate – e.g. units must be marked off accurately and in a non-overlapping fashion.

(ii) Knowing that if the length of the unit part is changed, the number yielded by unit iteration also changes. This is necessary because, although in linear measurement the measurer can arbitrarily choose a unit part, the answer given by unit iteration depends upon the length of that unit part.

(iii) Knowing that the length relation between two objects can be determined by carrying out a linear measurement operation, using unit iteration. (The difference between this requirement and B(viii) above is that the latter refers to unspecified numerosity: that is, the B(viii) relation is expressed in terms of 'more' or 'less' or 'same', not in terms of precise numbers of parts, arithmetical comparison of which yields the answer.)
(iv) Knowing that arithmetical addition of linear measurements may be used to determine the length of concatenated objects. This is necessary because a main purpose of linear measurement is the derivation of a number that may be used arithmetically in lieu of carrying out another measurement operation.

(D) DISTANCE. The following assumes that the child has a concept of distance as a spatial relation between two points. That concept may be incomplete, because all of the properties of distance may not be known to the child. There is an isomorphism between the properties of length and the properties of distance (Suppes and Zinnes, 1963). Hence, the properties previously listed for length will not be listed again for distance, except in the case of conservation. This is mentioned again, because of the importance of conservation in psychological theory concerning concept development.

(i) Knowing that distance is invariant under certain transformations (the conservation of distance). See comment against B(x).

(E) DISTANCE MEASUREMENT.

(i) Knowing how to compare indirectly two distances by a measurement operation not involving unit iteration. This is necessary because distances cannot be compared by aligning their end points and making direct comparisons.
(ii) Knowing how to measure distance between two points, using unit iteration. This is necessary because distance cannot be subdivided directly, as can length.

2.5 NON-INDEPENDENCE OF COMPONENTS OF LINEAR MEASUREMENT.

As previously mentioned, the components listed above are not independent. There is an assumption of an hierarchical arrangement in each concept domain, and of cross linkages between concepts. For example, in the number concept, component (ii) implies component (i), and component (iii) implies components (i) and (ii), together with other 'rules' not mentioned here (Gelman and Gallistel, 1978). Similarly, there are several different levels of arithmetical ability listed, each presumably based upon prior acquisition of less complex arithmetical abilities. All are included in the list because, at this stage, it is not known which are necessary for the demonstration of different levels of understanding of linear measurement. Similarly, for both number and length, distinctions are made between transitive inferences concerning relations of equivalence and transitive inferences concerning relations of non-equivalence (greater than, and less than). This is because it is not known whether only the former are involved in linear measurement - which would be suggested by a theoretical analysis - or whether both must be present.

With respect to length concepts, distinctions are made between components concerned with the length of an object, and components concerned with the relations between lengths of objects. This is because it is not known what influence each component might exercise in the development of linear measurement.
Further, transitive inferences regarding length imply the conservation of length, and the conservation of length implies transitive inference. If an attempt were made to draw up a non-redundant list of independent components of linear measurement, it is not at all clear whether conservation and/or transitive inference should be included.

It may be possible to set down a minimal list of independent components of linear measurement. However, for the present study, such a list may not be as useful as a list of the kind given here. This is because the present study is concerned with charting the course of development of linear measurement in terms of the progressive emergence of its components. If only independent components—that is, components representing axioms in a linear measurement system—were studied, the emergence of other components derived logically, but not necessarily psychologically, from those axioms would not be detected.

2.6 NATURE OF THE EMPIRICAL QUESTIONS ASKED BY THE PRESENT STUDY.

2.6.1 WHICH COMPONENTS ARE NECESSARY FOR LINEAR MEASUREMENT?

The components given in the above list provided the framework within which the empirical part of the study was conducted. Various empirical questions relevant to the general issue of the development of linear measurement were framed in terms of those components.
The following operational definition of linear measurement was used as a benchmark: "a child may be said to have a mature understanding of linear measurement, if he demonstrates a capacity to use correctly arithmetical operations instead of carrying out physical measurement operations".

According to the preceding analysis, this would only be possible if the child possessed the knowledge listed. Hence, the first question is: is that analysis correct? In other words: are these components necessary for linear measurement?

2.6.2 IS THERE AN ORDER IN WHICH THE COMPONENTS EMERGE?

Developmental research shows that certain of these components emerge at different times in the child's thinking. Hence, other questions are: is there a specific order in which the components emerge? Is the development of the components a continuous or discontinuous function? What is the relationship between levels of achievement in linear measurement and the progressive acquisition of the components? Does development in one concept prompt development in the other?

These empirical questions involve consideration of the difference between a child knowing that arithmetical operations may be substituted for physical measurement operations, and a child understanding why arithmetical operations may be so used. It seems quite possible that a child could possess the former, but not the latter. On the other hand, the reverse would seem unlikely - that is, that a child could possess the latter but not the former. This distinction is conveyed in the terms sometimes used in connection with these two different kinds of knowledge. The former is
often referred to as algorithmic or rule-based (Gagne, 1968), while the latter is sometimes referred to as operational (Piaget, 1953). The present study is concerned with both kinds of knowledge. Some of the developmental precursors of operational knowledge - what might be called its components - could be expressed as algorithms or rules. However, the present study does not assume that operations are nothing more than a particular organisation of rules, or that rules "grow" into operations.
PART II.

LITERATURE REVIEW.

On the basis of evidence from a number of sources it is possible to make predictions concerning the course of development of components of linear measurement. These sources are Piagetian theory, and a large body of empirical work carried out in that tradition.

Developmental questions concerning the components of linear measurement have been of major and long term interest to Piaget and his followers (Brainerd, 1978; Flavell, 1963). Consequently, all of the predictions outlined in Part II were derived directly from Piagetian theory, or from empirical research conducted within the Piagetian tradition.

In this connection, it is noteworthy that, notwithstanding the vast amount of information emanating from Piagetian research and relevant to the questions posed in Part I, it is still entirely reasonable to ask those questions. This is so because of difficulties in interpreting the results of previous studies, and of collating the results of many different studies, each concerned with perhaps only one or two aspects of the general issue of the development of linear measurement.

As the work of Piaget and his followers provides the main theoretical framework for the present study, it is necessary to present briefly those aspects of Piagetian theory which are relevant to the present topic. Chapter 3 reviews this material and derives predictions. Following this, Chapters 4 to 7 review the empirical evidence for these predictions.
Piaget lists a number of abilities which he believes the child must possess before demonstrating an understanding of linear measurement. They are:

- the ability to conserve number, length and distance;
- the ability to make transitive inference judgements with respect to number and length;
- the ability to use a unit of length for purposes of iteration;
- the ability to carry out arithmetical operations of addition and subtraction (Piaget, Inhelder and Szeminska, 1960).

Piaget also believes that each of these abilities only emerges in the child's reasoning after he has mastered more basic skills, such as numeration, seriation of length, and understanding of part/whole relations. (Piaget, 1968).

Additionally, Piaget argues that both the high-order abilities (e.g. the conservation of number), and the more basic skills (e.g. numeration) emerge in a predictable order in the development of intelligence (Piaget, 1968).
The following summarises those predictions. The headings refer to aspects of Piaget's theory which are most directly responsible for the predictions which follow.

3.1.1 PARALLEL DEVELOPMENT.

. The ability to conserve number emerges at about the same time as the ability to make transitive inferences with respect to discrete quantity;
. the ability to conserve length emerges at about the same time as the ability to make transitive inferences with respect to length.

3.1.2 THREE SUB-STAGE MODEL.

. The ability to conserve length emerges earlier than the ability to measure length;
. the ability to conserve distance emerges earlier than the ability to measure distance;
. the ability to perform the arithmetical operations of addition and subtraction emerges earlier than the ability to measure length or distance;
. the ability to conserve number emerges at about the same time as the ability to perform the arithmetical operations of addition and subtraction;
. the ability to seriate length emerges earlier than the ability to make transitive inferences with respect to length;
. the ability to order discrete quantity emerges earlier than the ability to make transitive inferences with respect to discrete quantity.
3.1.3 **HORIZONTAL DECALAGE.**

- The ability to conserve number emerges earlier than the ability to conserve length;
- the ability to conserve length emerges at about the same time as the ability to conserve distance;
- the ability to measure length emerges at about the same time as the ability to measure distance;
- the ability to seriate length emerges earlier than the ability to numerate.

The abilities referred to in the above predictions cover nearly all the components of linear measurement listed in Chapter 2. It would seem, therefore, that Piagetian theory provides a framework which embraces virtually all of the empirical questions asked in this study. Consequently, it is necessary to present that theory briefly. In doing so, an attempt will be made to link Piagetian theoretical statements with the empirical aspects of the present research, so as to make clear the origin and status of the predictions given above.

As a preface, however, a caveat needs to be made explicit. The present research is not aimed at testing Piaget's theory. It is emphasised that Piagetian theory is consulted because it provides the richest source of relevant theoretical statements and empirical data.
3.2 OVERVIEW OF PIAGET'S THEORY OF COGNITIVE DEVELOPMENT.

3.2.1 NATURE OF THE THEORY.

Piaget's theory of cognitive development is structural, holistic, constructionist, and descriptive.

It is structural because it conceptualises "mental operations" as forming patterns that exhibit properties which change in the course of development. Development is seen primarily as a matter of change in cognitive structure.

It is holistic because it asserts that, as every cognitive act is related in some fashion to all other cognitive acts, an understanding of intelligence can only be gained by an understanding of its organisation as a total system. The total system and its component structural elements are said to change over time, and as a function of experience. Such change is believed to be directed by two broad principles, "organisation" and "adaptation". Because these principles do not change during development they are referred to as "functional invariants".

The theory is constructionist because it declares that, while experience permanently alters intelligence, intelligence modifies its own construction of reality in the process of interpreting it.
Finally, the theory is more descriptive than explanatory. Its structural and functional elements provide a way of classifying and charting ontogenetic development, rather than a system of explanations. Thus, the theory provides a rich and detailed account of the state of intelligence at various stages of development. However, it provides only general and exceedingly abstract principles to account for the processes at work in the formation of, and transition between, such states (Brainerd, 1978).

The following account of the theory is highly condensed and selective. However, it is only intended as a context within which a more detailed discussion can be presented of the period of concrete operations, because that is when the number, length and distance concepts under examination emerge.

3.2.2 COGNITIVE STRUCTURES.

The existence of cognitive structures is inferred from the person's behaviour. Thus, cognitive structure is an hypothetical construct. More specifically, Piaget regards cognitive structures as being systems of operations.

Piaget states:—

"Psychologically, operations are actions which are internalizable, reversible and co-ordinated into systems characterized by laws which apply to the system as a whole. They are actions, since they are carried out on objects before being performed on symbols. They are internalizable, since they can also be carried out in thought without losing their original character of actions. They are reversible as against simple actions
which are irreversible. In this way, the operation of combining can be inverted immediately into the operation of dissociating, whereas the act of writing from left to right cannot be inverted to one of writing from right to left without a new habit being acquired differing from the first. Finally, since operations do not exist in isolation they are connected in the form of structured wholes. Thus, the construction of a class implies a classificatory system and the construction of an asymmetrical transitive relation, a system of serial relations, .......

(Piaget, 1953 b, p.8).

3.2.3 CONCEPT OF SCHEME.

Piaget's concept of "scheme" is related to these notions of structure. Schemes consist of sequences of actions. Structures consist of systems of operations.

Schemes are defined in terms of overt behaviour (Piaget & Inhelder, 1969; p4) Thus, Piaget talks of sensory-motor schemes of grasping, reaching, seeing, tasting, and so on. Schemes are said to change as a consequence of cognitive functioning.

3.2.4 COGNITIVE FUNCTIONS.

Development is seen mainly in terms of changes in cognitive structures. Changes occur as a result of experience, and are said to be always under the control of the two functional invariants, organisation and adaptation.
Organisation is the cognitive function Piaget holds responsible for the similarities that exist in intellectual behaviour at all levels of development. Adaptation provides the mechanisms responsible for the changes within cognitive structures. Hence, organisation and adaptation are complementary. The former ensures that the reorganisation of cognitive structures produces an ordered totality. The latter ensures that cognitive structures grow internally as elements of the total system, and that new and different kinds of relationships grow between these elements. Brainerd said:—

"The organization principle presumably is responsible for the organism's cognitive continuity across short or long periods of time. That is, cognitive organization accounts for the fact that there is some degree of sameness in intelligence across time. In contrast, the adaptive side of intelligence presumably is the chief instrument of discontinuity."

(Brainerd, 1978, pp.23).

The mechanisms used by adaptation to guide cognitive growth are assimilation and accommodation. Assimilation is the taking in of experience, and its interpretation by existing cognitive structures. Accommodation is the changing of those cognitive structures in such a manner as to make subsequent interpretations reflect reality more accurately. Hence, assimilation and accommodation are complementary. For Piaget, every cognitive act implies both mechanisms:—
"Accommodation of mental structures to reality implies the existence of assimilatory (schemes) apart from which any structure would be impossible. Inversely, the formation of (schemes) through assimilation entails the utilization of external realities to which the former must accommodate..."

3.2.5 STRUCTURAL CHANGE.

Piaget has described these and other aspects of structural adjustment in terms of an equilibrium model. Expressed most simply, the model refers to a balance between assimilation and accommodation, that leads to a state of "equilibrium" - one in which the cognitive structures are said to be equilibrated. This process of equilibration has been defined by Piaget (1972) as: "a compensation for an external disturbance" (p.120). When the system is not in equilibrium, it has a tendency to adjust itself continuously to move toward a state of equilibrium. Thus, it is a dynamic process leading to successively higher and higher levels of equilibration. Inhelder (1962) described it as:-

"a constant progression from a less to a more complete equilibrium and manifest therein the organism's steady tendency toward a dynamic integration. This equilibrium is not a static state, but an active system of compensations - not a final conclusion, but a new starting point to higher forms of mental development."
Equilibrium is central to Piaget's stage concepts. It accounts for the structural characteristics of invariant order, of acquisition, hierarchical inclusion, and overall integration that define a stage. It also accounts for the transition between stages as periods when intelligence is in a state of disequilibrium: that is, when its cognitive structures are poorly equilibrated. It also accounts for the changes that occur within a stage. In other words, it represents the organisational and adaptational principles which account for the continuity and discontinuity aspects of Piaget's stage theory (Brainerd, 1978).

3.3 STAGES OF DEVELOPMENT.

An important aspect of intellectual behaviour is that the nature of reasoning changes with age. Piaget uses behavioural data as evidence that intellectual development involves a progression through four distinct and major stages. Each stage is characterized by different reasoning. Piaget has argued that there are:

"four great stages, or four great periods, in the development of intelligence: first, the sensory-motor period before the appearance of language; second, the period from about two to seven years of age, the pre-operational period which precedes real operations; third, the period from seven to twelve years of age, the period of concrete operations (which refer to concrete objects); and finally after twelve years of age, the period of formal operations, or propositional operations."

3.3.1 SENSORY-MOTOR STAGE.

During the sensory-motor stage, the child learns that the world is a permanent place, which may be explored by his senses, and by physical movement. Physical movements are co-ordinated, and are internalized into rudimentary cognitive structures. As these structures develop, the child's behaviour becomes more purposive and goal directed. From about 18 months onwards, language development is apparent, and simple symbolic behaviour appears. These two developments herald the emergence of the next stage.

3.3.2 PRE-OPERATIONAL STAGE.

When compared with its precursor, the pre-operational stage is one of vast growth in the child's capacity to reason. Consequently, it is difficult to summarise the pre-operational stage without conveying the impression that it is "simply" a period during which the foundations are laid for the development of concrete operational thought.

The pre-operational stage is marked by the development of the "semiotic function":

"At the end of the sensori-motor period, at about one and a half to two years, there appears a function that is fundamental to the development of later behavior patterns. It consists in the ability to represent something (a signified something: object, event, conceptual scheme, etc.) by means of a "signifier" which is differentiated and which
serves only a representative purpose: language, mental image, symbolic gesture, and so on. Following H. Head and the specialists in aphasia, we generally refer to this function that gives rise to representation as "symbolic." However, since linguists distinguish between "symbols" and "signs," we would do better to adopt their term "semiotic function" to designate those activities having to do with the differentiated signifiers as a whole." (Piaget and Inhelder, 1969, p.51)

During this stage the child’s symbolic behaviour encompasses complex activities such as drawing, reading and writing:

"In spite of the astonishing diversity of its manifestations, the semiotic function presents a remarkable unity. Whether it is a question of deferred imitation, symbolic play, drawing, mental images and image-memories or language, this function allows the representative evocation of objects and events not perceived at that particular moment. The semiotic function makes thought possible by providing it with an unlimited field of application, in contrast to the restricted boundaries of sensori-motor action and perception."

(Piaget and Inhelder, 1969, p.91)
3.3.3 CONCRETE OPERATIONAL STAGE.

The concrete operational stage is characterized by the child's ability to reason logically, provided that the task makes reference to concrete objects - though such objects need not be present. It is also necessary that any hypothesis testing involve only simple extrapolations or interpolations. It is during this stage that the child acquires his ordinal and cardinal concepts of number; develops his ability to argue transitively; exhibits a capacity to classify objects simultaneously on two or more dimensions; is able to handle class inclusion problems in logic; displays an understanding that spatial transformations of objects, or collections of objects, leaves certain properties of the objects, or collections, unaffected; demonstrates an understanding of projective and Euclidean geometry; and learns to apply mathematical concepts to a range of concretely-based problems, such as distance and length measurement.

The child's thinking, however, is still limited by a dependence on concretely based information; by an inability to carry out concurrently the two reversibility operations of negation and reciprocation - although they can be applied independently; and by severe limitations in his ability to control for the effects of variables in multi-variable situations.

3.3.4 FORMAL OPERATIONAL STAGE.

The formal operational stage represents the highest level of intellectual development, and marks the emergence of the ability to think about thinking. Thought is no longer confined to concretely-based information, no
longer restricted by the force of reality, but is free to generate possibilities and hypotheses whose only immediate referents are prior elements of cognition. Piaget said:—

"It is the power of forming operations of operations which enables knowledge to transcend reality."

(Piaget, 1972).

It is during this stage that the power of hypothetico-deductive reasoning can be used to gain full understanding of complex concepts in mathematics and science, and where proof of a proposition involves consideration of all possibilities, in isolation and in combination.

3.4 CONCRETE OPERATIONS.

3.4.1 LOGICAL AND INFRALOGICAL OPERATIONS.

A more detailed presentation of the stage of concrete operations is needed because most of the components of linear measurement listed in Chapter 2 appear in the child's reasoning during that stage. For example, it is then that the child demonstrates an ability to make transitive inferences, and to conserve number, length and distance.

Most of the abilities which emerge during the concrete operational stage fall into two broad categories. They are those concerned with: (a) relations between objects; and, (b) with relations within objects (Piaget and Inhelder, 1969). The former involve logical operations, and the latter, infralogical operations. The distinction is based upon the kinds of in-
formation each provide. Logical operations are concerned with information about collections of objects, and are independent of spatio-temporal location. Infralogical operations bear upon objects, and their parts. The logical operation of dividing a class of objects into a number of sub-classes is, analogous to the infralogical operation of sub-dividing a length into component elements. However, they are different in important respects. The former does not require that the elements of the class or of the sub-classes be in spatial or temporal proximity and, so, is called a logical operation. The latter does require that the elements of the whole (the length) be in spatial proximity, and, hence, is termed an infralogical operation.

3.4.2 GROUPING AND GROUP STRUCTURES.

Logical and infralogical operations form components of the cognitive structures of the concrete operational stage. The principles under which operations in the logical and infralogical domains combine may be stated in the form of axioms. Those axioms bear a close resemblance to the logical laws that define two particular mathematical structures: groups and lattices. Consequently, Piaget has employed logico-mathematical models to describe the organisation of concrete operational thought.

Piaget called his structures "groupings", if they modelled systems which embodied laws of mathematical groups and lattices. The structures were called "groups", if the systems they modelled reflected only the laws of mathematical groups.
3.4.3 TYPES OF GROUPING STRUCTURE.

Piaget posited eight major groupings:-

"This grouping structure is found in eight distinct systems, all represented at different degrees of completion in the behaviour of children of 7-8 to 10-12 years of age, and differentiated according to whether it is a question of classes or relations, additive or multiplicative classifications, and symmetrical (or bi-univocal) or asymmetrical (co-univocal) correspondences:

<table>
<thead>
<tr>
<th>Classes</th>
<th>Relations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asymmetrical</td>
<td>1</td>
</tr>
<tr>
<td>Symmetrical</td>
<td>11</td>
</tr>
<tr>
<td>Co-univocal</td>
<td>111</td>
</tr>
<tr>
<td>Multiplicatives</td>
<td>1IV</td>
</tr>
</tbody>
</table>

(Beth and Piaget, 1966, p.174).

Examples of the behaviours associated with these groupings are given below:-
<table>
<thead>
<tr>
<th>GROUPING</th>
<th>BEHAVIOUR.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Primary addition of classes</td>
<td>Addition and subtraction of classes;</td>
</tr>
<tr>
<td></td>
<td>class inclusion.</td>
</tr>
<tr>
<td>11 Secondary addition of classes</td>
<td>Ability to classify a set of objects in several different ways.</td>
</tr>
<tr>
<td>111 Bi-univocal multiplication of classes</td>
<td>Ability to find the intersect of two or more classes.</td>
</tr>
<tr>
<td>1V Co-univocal multiplication of classes</td>
<td>Ability to set two series of classes in one-to-many correspondence.</td>
</tr>
<tr>
<td>V Addition of asymmetrical relations</td>
<td>Ability to construct a transitive asymmetrical series.</td>
</tr>
<tr>
<td>VI Addition of symmetrical relations</td>
<td>Ability to deduce symmetrical relationships from a genealogical hiérarchy.</td>
</tr>
<tr>
<td>VlBi-univocal multiplication of relations</td>
<td>Ability to set in 1-to-1 correspondence elements of two asymmetrical series.</td>
</tr>
<tr>
<td>Vl1 Co-univocal multiplication of relations</td>
<td>Ability to find the result of multiplying a symmetrical relation and an asymmetrical relation of the kind found in genealogical hiérarchies.</td>
</tr>
</tbody>
</table>

Flavell (1963) succinctly expressed the connection between these groupings and behaviour:
"thus, if Piaget says that the classificatory behaviour of the eight year old indicates that he possesses the grouping of logical class addition, he means the child's thought organisation in the classificatory area has formal properties (reversibility, associativity, composition, tautology, etc.) very like those which define this logico-algebraic structure. The latter has certain specific and definable system properties; we infer from his behaviour that the child's cognitive structure has similar properties." (Flavell, 1963, p.169).

Reflecting the distinction between logical and infralogical operations, Piaget has argued that each of the groupings of logical operations has its infralogical counterpart. Thus, there is a grouping of infralogical operations corresponding to grouping 1 for logical operations. However, in the case of the former, the operations relate not to the act of combining and dissociating classes, but to the act of dividing a whole (say, a length) into its constituent parts, and combining those parts to reconstitute the whole.

3.4.4 TYPES OF GROUP STRUCTURE.

In addition to these grouping structures, Piaget posits two further structures, called groups. The two groups are: (a) the additive group of positive and negative whole numbers; and, (b) the multiplicative group of whole or fractional positive numbers. Just as there are groupings in the domain of logical operations, and in the domain of infralogical operations, there are groups in each domain.
Moreover, these groups are said to come out of a synthesis of grouping structures: the additive group from a synthesis of class addition and addition of asymmetrical relations; and, the multiplicative group from a synthesis of class multiplication and multiplication of symmetrical relations. In other words, understanding of number implies an understanding of the cardinal (how many), and ordinal (ordered series), aspects of the concept. Piaget (1952) asserted, in respect of the additive group of numbers, that:

"..... class, asymmetrical relation and number are three complementary manifestations of the same operational construction applied either to equivalences, differences or to both together. It is, in fact, when the child's intuitive evaluations have become mobile and he has therefore reached the level of the reversible operation, that he becomes capable of inclusions, seriations and counting.

".....class and number are mutually dependent, in that while number involves class, class in its turn relies implicitly on number.

".....number is at the same time a class and an asymmetrical relation." (Piaget, 1952, p. 184).
3.4.5 QUANTIFICATION.

The grouping structures in the domains of logical and infralogical operations permit what Piaget calls "intensive" quantification to be performed. The group structures permit "extensive" quantification to be carried out. Intensive quantification enables judgements such as bigger than, more than, longer than, etc. to be made. Extensive quantification enables such judgements to be more precise, by expressing how much bigger, how many more, how much longer, etc. Piaget also argues that, when the elements of the logical grouping structures (that is, logical operations) become connected to the elements of the numerical group structures (that is, numbers), the result is numerical operations. At that time, the child becomes capable of understanding arithmetic. Similarly, when the elements of the infralogical grouping structures (that is, infralogical operations) become connected to the elements of the numerical group structures, the result is measurement operations. At that time, the child becomes capable of understanding the nature of measurement. Diagramatically, the argument may be summarised as follows:
3.5 PARALLEL DEVELOPMENT HYPOTHESIS.

The structures of the concrete operational stage are linked developmentally, as well as logically. It is argued that all of these structures develop contemporaneously, and emerge in parallel, as distinct from emerging sequentially. This is sometimes termed the "parallel development hypothesis". (Brainerd, 1978: p.86).

The parallel development hypothesis is important in the context of the present research. It predicts that the ability to use transitive reasoning with respect to discrete quantity - which marks the completion of grouping V (addition of asymmetrical relations) - appears in the child's behaviour at about the same time as the ability to conserve number - which marks the synthesis of groupings I (primary addition of classes) and V. Similarly, in the domain of infralogical operations, the parallel development hypothesis predicts that the ability to make transitive inferences with respect to length emerges at about the same time as the ability to
conserve length. It will be recalled from Chapter 2 that knowing how to make transitive inferences with respect to number and length, and knowing of the conservation of number and length, were assumed to be necessary for a child to be able to measure length. Consequently, the above two statements of developmental synchrony form the first two predictions of the order of emergence of components of linear measurement.

3.6 THREE SUB-STAGE MODEL.

Piaget also maintains that the structures do not emerge all of a sudden, when the child enters the concrete operational stage. Instead, he identifies, generally, three sub-stages in the development of each structure. As with his stage concept, he maintains that all children must pass through each sub-stage in a fixed invariant order. Moreover, Piaget also maintains that children move synchronously through these sub-stages. Thus, a child at sub-stage 1 in class concept development should also be at sub-stage 1 with respect to relations and number.

The three sub-stage model of development is relevant to the present research, because Piaget is most explicit regarding the necessary components of linear measurement when discussing sub-stage growth. Also the model yields predictions concerning the order of emergence of some of the higher-order abilities assumed to be necessary for linear measurement. For example, the model posits developmental asynchronies between the conservation and the measurement of length, and also between the emergence of proficiency in numerical operations and the demonstration of an understanding of measurement operations. Additionally, it posits dev-
elopmental dependencies between certain of the more basic skills, such as length seriation, and higher order skills such as transitive inference reasoning. In order that predictions of that kind may be seen in an appropriate theoretical context, a brief account of the three sub-stage model follows.

3.6.1 CLASSES.

The growth of class logic is relevant because, in Piagetian theory, the emergence of an operational grasp of number marks the synthesis of logical groupings 1 and V.

Piaget uses three kinds of task to assess a child's progress through the posited three substages of development. They are, in increasing order of complexity: (a) classification; (b) multiple classification; and, (c) class inclusion.

In the classification task, the child is asked to sort collections of objects on one dimension only - e.g. to sort a collection of beads of differing size and colour on colour only. In the multiple classification task, the child is asked to sort the collection on two dimensions simultaneously - e.g. to sort on size and colour. In the class inclusion task, the child is presented with the information that class A, say green plastic buttons, is contained in class B, say green and blue plastic buttons, and asked if there are more green buttons (assume there are more greens than blues) than plastic buttons. Correct performance on the class inclusion task represents achievement of the class concept, and attainment of the related grouping structures. Progression through these tasks is summarised in the following:-
Piaget uses three kinds of task to assess a child’s progress through the posited three substages of development. They are, in increasing order of difficulty: (a) seriation; (b) multiple seriation; (c) transitive inference.
In the seriation task, the child is asked to arrange a number of objects in order along a particular dimension - e.g. arrange a set of five lengths of wooden dowel in order of increasing length. In the multiple seriation task, the child is asked to arrange the objects in order along two dimensions simultaneously - e.g. to order on length and diameter. In the transitive inference task, the child is presented with the information that A is greater than B, and that B is greater than C, but not directly with information on the quantitative relationship between A and C. The child is then asked: what is the relationship between A and C. Correct performance on the transitive inference task indicates achievement of the relations concept, and attainment of the relations grouping structures. Progression through these tasks is summarised in the following:-

<table>
<thead>
<tr>
<th>TEST</th>
<th>SUB-STAGE 1</th>
<th>SUB-STAGE 2</th>
<th>SUB-STAGE 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiple Seriation</td>
<td>Fails. As for seriation.</td>
<td>Achieves partial success</td>
<td>Passes</td>
</tr>
<tr>
<td>Transitive Inference</td>
<td>Fails. Usually asserts A=C because they look alike.</td>
<td>Fails. As for sub-stage 1.</td>
<td>Passes</td>
</tr>
</tbody>
</table>

The three sub-stage model predicts that an ability to seriate length will emerge earlier, in the child's reasoning, than an ability to make transitive inferences with respect to length. The model also predicts that an ability to order discrete quantity will emerge earlier than an ability to make transitive inferences with respect to discrete quantity.

3.6.3 NUMBER CONSERVATION.
Piaget's (1952) classical test of number is termed conservation of number. Typically, it involves showing a child two rows of objects set in one-to-one correspondence; having the child agree that each row contains the same number of objects; transforming one row so that it is either, shorter and more dense, or longer and less dense, with the child watching; then asking the child if the rows still contain the same number of objects. Again, Piaget sees three sub-stages:-

. Sub-Stage 1 - the child's judgements are dominated by relative lengths, hence, he asserts that the longer row is more numerous.

. Sub-Stage 2 - the child considers both relative length and relative density, but cannot co-ordinate the two; hence, if a correct judgement requires that both cues be taken into account, the child fails.

. Sub-Stage 3 - the conservation of number is achieved.

At a theoretical level, Piaget sees the child's acquisition of the conservation of number as marking the completion, and synthesis, of logical groupings 1 and V - in other words, as reflecting the co-ordination of the
cardinal and ordinal aspects, respectively, of number. Piaget also sees
the growth in those areas of class logic as being linked with growth in the
understanding of conservation. Consequently, the three sub-stage model
predicts that the ability to form one-to-one correspondences, and the
ability to construct ordinal series, appear earlier in the child's reasoning
than the conservation of number. Additionally, the model predicts
that the conservation of number emerges at about the same time as an
understanding of addition and subtraction operations.

3.6.4 CONSERVATION, MEASUREMENT AND ARITHMETICAL OPERATIONS.

The three sub-stage model also posits a developmental sequence between
the conservation of length and linear measurement, and between linear
measurement and arithmetical operations.

Tasks, analogous to that used to assess number conservation, have been
designed for conservation of length, distance, weight, volume, and quan-
tity. In all cases, Piaget sees three sub-stages:

- Sub-Stage 1 - no conservation, child maintains that the quantitative
  relationship has changed.
- Sub-Stage 2 - intermediate reactions, child gives conserving response
  if deformation is small, or will predict conservation
  before deformation. However, if the subsequent deformation
  is large child, the child will reverse its judgement
  and assert that the relationship has changed.
- Sub-Stage 3 - conservation response.
Piaget also sees three sub-stages of development in the measurement of length and distance. It is the last sub-stage which is most relevant to the empirical work of this thesis. Piaget divides the last sub-stage into two parts:

- Sub-Stage IIIA— refers to the emergence of conservation of length and distance; while
- Sub-Stage IIIB— refers to the emergence of measurement of length and distance, via the process of unit iteration.


In addition to positing a developmental dependency between conservation and measurement, Piaget sees a parallel between the development of number and arithmetical operations in the domain of logical groupings and groups, and the development of measurement operations in the domain of infralogical groupings and groups:

"It is perfectly clear that, by Stage IIIB the subjects .......have finally achieved the construction of measurement by fusing or synthesizing the operations of subdivision and change of position. Both operations were required at level IIIA for the notion of conservation, but there they were still complementary to one another instead of being fused into a single operation. There is a parallel to this elaboration of measurement out of two qualitative operations which are at first distinct but which must be synthesized to yield one integral operation. The parallel
is in the elaboration of number. This cannot surprise us since there are isomorphic relations between the iteration of metrical units and the series of whole numbers (as also between the fractioning of metrical units and fractional numbers), likewise between subdivision and composition of parts on the one hand, and nesting class hierarchies on the other, and finally, between change of position and seriation of asymmetrical relations. Thus, measurement, in the field of sublogical operations is the exact equivalent to number in that of logical operations, since number is a synthesis involving the logical grouping of nesting classes and the seriation of asymmetrical relations. The only difference is that the whole number series is constructed at level IIIA and so follows immediately on these two logical groupings, while measurement is delayed for some while after notions of conservation have been mastered - although these are similarly dependent on its constituent operations: subdivision and change of position. We have tried to show how that delay is not unexpected, since numerical unity is something which may be perceived intuitively because any collection of discontinuous items consists of such unique elements, while choosing a unit of length is to make an arbitrary fragmentation of a whole which is continuous." (Piaget Inhelder & Szeminska, 1960, p400).
It will be evident from the above that Piaget's views on the developmental dependencies between conservation, arithmetical operations, and linear measurement are of central importance to the present research. Specifically, the three sub-stage model asserts that the following are necessary components of linear measurement:

- the ability to conserve number, length and distance;
- the ability to make transitive inference judgements with respect to number and length;
- the ability to use a unit of length for purposes of iteration;
- the ability to carry out arithmetical operations of addition and subtraction.

Additionally, the three sub-stage model predicts that:

- the ability to conserve length emerges earlier than the ability to measure length;
- the ability to conserve distance emerges earlier than the ability to measure distance;
- the ability to perform arithmetical operations of addition and subtraction emerges earlier than the ability to measure length or distance.
3.7 HORIZONTAL DECALAGE.

It has been noted in connection with the parallel development hypothesis, and the three sub-stage model, that Piaget claims that the final sub-stages in classes and relations emerge at about the same time. Similarly, attainment of sub-stage 3 in conservation marks the synthesis of the logical (or infralogical, depending upon concept type) groupings and number groups. Since it is these structures which determine the nature of the child's reasoning, it might be supposed that, for example, conservation of number would appear at about the same time as conservation of weight. This is because the logic of the argument is the same in both cases. However, Piaget says not. Instead, he argues that logical and infralogical operations emerge synchronously within any given concept. Thus, while transitivity of number and conservation of number emerge in parallel, and transitivity of weight and conservation of weight are attained at about the same time, there is a developmental lag between number and weight. More specifically, Piaget found that sub-stage 3 of classes, relations, conservation and measurement is reached in the following order: number, length, quantity, weight and volume. Piaget refers to this phenomenon as horizontal decalage.

However, Piaget has also found that length and distance concepts emerge in parallel. His explanation reflects his conception of space. When a child is presented with two lengths of dowel in the course of a conservation experiment, he compares two extents of occupied space. In a distance conservation experiment he compares two extents of unoccupied space. Piaget regards space as a network of sub-spaces linked together
in a one, two, or three dimensional co-ordinate system. He argues that, so far as conservation and measurement are concerned, considerations of whether such sub-spaces are occupied by concrete objects or not, are irrelevant.

In summary, Piaget claims that;

. the ability to conserve number emerges earlier than the ability to conserve length;
. the ability to conserve length emerges at about the same time as the ability to conserve distance;
. the ability to measure length emerges at about the same time as the ability to measure distance.

Finally, Piaget also found that an ability to seriate length emerged earlier than an ability to numerate. Since an ability to seriate length, and an ability to numerate, are assumed to be low-order components of linear measurement, that finding should also constitute a prediction of the present study.

3.8 SUMMARY.

Piaget's theory provides a structural account of cognitive development. Cognitive structures are defined as systems of operations. Intellectual growth is directed by the functional invariants of organisation and adaptation, and the mechanisms of structural change are described in terms of an equilibrium model.
Four stages of development are identified. The third of these stages, that of concrete operations, marks the emergence of behaviour and cognitive structures concerned with logic, number, and certain physical properties of objects and events in the world. Consequently, it is the stage of development most relevant to this thesis.

The theory, and the empirical base of the theory, provide general predictions regarding the composition, and the order of development, of the aspects of linear measurement examined empirically in this thesis. Moreover, the theory provides a descriptive rationale for that predicted order of development. These contributions of Piagetian theory to the derivation of specific hypotheses will be returned to in Part III.
CHAPTER 4.

METODOLOGICAL CONSIDERATIONS.

4.1 PIAGET’S MODIFIED CLINICAL METHOD.

The predictions made in Chapter 3 concerning the order of emergence of components of linear measurement stem from Piagetian theory. Most of the empirical studies discussed in the following review were undertaken with a view to testing various aspects of that theory. Difficulties arise in interpreting much of this research, however, because many of the studies did not use precisely the same tasks as did Piaget. Also, many did not use Piaget’s "modified clinical" style of questioning to draw out from the subject verbal justifications for his answer. In the modified clinical method the experimenter explores the subject's reasoning processes by asking the subject to justify his answers verbally. The experimenter does not adhere to a particular form, or to a fixed sequence, of questioning.

4.2 PERFORMANCE/COMPETENCE ISSUE

A major advantage of the modified clinical method is that it provides evidence of the particular form of reasoning used by the child. Critics of that approach, however, claim that it is too dependent on the child's verbal skills. This issue is part of a long and continuing debate known as the "performance/competence" distinction. More specifically, "competence" refers to the subject having the particular ability in question. "Perfor-
mance refers to the subject's capacity to apply and demonstrate that ability in a particular situation.

This methodological issue needs to be be considered before assessing empirical evidence, and in relation to the design of the tasks and administration procedures employed in this study.

4.3 PERFORMANCE/COMPETENCE CRITICISM OF PIAGETIAN CONCRETE OPERATIONAL TASKS.

Bryant (1974) has criticised some of Piaget's concrete operational tasks on the grounds that they do not sufficiently control performance variables. These variables might either mask the concept being explored, or they might falsely give the impression that the child has acquired that concept. That kind of criticism has most frequently been made of the transitive reasoning and class inclusion tasks. (Ahr and Youniss, 1970; Braine, 1959; Brainerd, 1973, 1974; Brainerd and Kaszor, 1974; Brainerd and Vanden Heuvel 1974; Bryant and Trabasso, 1971; De Boysson-Bardies and O'Regan, 1973; De Soto, London and Handel, 1965; Flavell, 1977; Jennings, 1970; Klahr and Wallace, 1972; Miller, 1976; Riley and Trabasso, 1974; Roodin and Gruen, 1970; Siegel, 1971a; 1971b; Winer, 1974; Winer and Kronberg, 1974; Wohlwill, 1968; Youniss and Dennison, 1971; Youniss and Murray, 1970).

It was argued in the earlier analysis that transitive reasoning is a necessary component of linear measurement. Hence it was decided that studies of transitive inference would be used to convey the important features of the argument.
4.4 CRITICISM OF THE PIAGETIAN TRANSITIVE REASONING TASKS

The Piagetian test for transitive inferences concerning length relations involves presenting objects A and B, then objects B and C, and then objects A and C. The AB and BC pairings are presented in such a manner as to permit the child to determine perceptually which is the longer or shorter. Objects A and C are usually presented in such a manner as to create a misleading perception, the intention being to force the child to use principles of transitive reasoning. Finally, the experimenter questions the child to ensure that the given answer was derived by making a transitive inference. Unless the child is able to justify his answer verbally, he is not credited with having transitive reasoning for length (Beth and Piaget, 1966). The following four kinds of criticism have been made of this procedure:

(a) young children can make transitive inferences, but are compelled by the visual illusion to give non-transitive answers;
(b) young children lack the verbal skills necessary to provide appropriate verbal justification for their answers;
(c) young children can make transitive inferences, but fail the task because they forget the premises;
(d) young children who pass the task may use non-transitive strategies (Brainerd, 1978).
4.4.1 STUDIES CONTROLLING VISUAL ILLUSION, MEMORY CAPACITY AND VERBAL SKILL FACTORS.

Studies which have not used visual illusions and have not required verbal justifications have claimed that kindergarten children of about five years of age can make transitive inferences (Brainerd, 1973; 1974; Brainerd and Vanden Heuvel, 1974). Similar findings have been reported from studies which have not employed visual illusions, have not required verbal justifications, and have nullified the memory factor by using either visual or verbal feedback in a preliminary learning phase (Bryant, 1974; Bryant and Trabasso, 1971; Riley and Trabasso, 1974; Siegel, 1971a; 1971b). However, a recent study that employed the standard three term series procedure found that 64% of the subjects who remembered the premises, still could not make a transitive inference with respect to length (Halford and Galloway, 1977; Halford, 1979; Grieve and Nesdale, 1979).

All of these studies used an adequate sample size, appropriate stimulus materials, and procedures which appeared to eliminate, or control, the extraneous factors. All of the studies, except that of Halford and Galloway (1977), concluded that children can make transitive inferences two to three years earlier than Piaget has claimed. In the case of Bryant's studies, it was found, using five term series problems, that three and four year old children could make transitive inferences.

Methodological rigour, however, does not carry guarantees of conceptual soundness. The resolution of the performance/competence issue rests on discovering the form of internal representation used by young children in solving Piagetian tasks.
Many critics have argued that some children who pass the transitive inference task may not be using transitive reasoning principles (De Boysson-Bardies and O'Regan, 1973; De Soto et. al., 1965; Flavell, 1977; Riley and Trabasso, 1974; Youniss and Dennison, 1971; and Youniss and Murray, 1970). However, it will be seen that these arguments can be applied either in support of the Piagetian position, or in support of the critics of that position. This is because they rest on assumptions concerning the form of internal representation used by the child.

4.4.2 THE ROLE OF LINGUISTIC CODING IN TRANSITIVE INFERENCE.

Youniss and Dennison (1971) and Youniss and Murray (1970) suggested that Piaget's standard three term series problem could be solved by employing a non-transitive, linguistic coding strategy. Their argument is that the standard procedure enables subjects to code linguistically - A is coded as 'big' during the AB pairing, and C as 'small' during the BC pairing, leading to the non-transitive judgement that, since A is 'big' and C is 'small' A must be bigger than C. Their solution to this problem of false positives (correct answers reached by non-transitive processes) was to introduce two additional objects X and Y, and to introduce them in such a manner as to make both A and C both 'big' and 'small'. For example, in the following set of pairings A is both bigger than B and smaller than X, while C is both bigger than Y and smaller than B: A>B; B>C; X>A; C>Y; X?Y. The same argument was applied by De Boysson-Bardies and O'Regan (1973) to Bryant and Trabasso's (1971) procedure.
4.4.3 THE ROLE OF MENTAL IMAGERY IN TRANSITIVE INFERENC.

DeSoto et al. (1965), and Riley and Trabasso (1974) advanced a similar argument, suggesting that false positives might be produced by a strategy involving mental imagery. They argued that a false positive solution would be produced if the child imag(in)ed absolute values for each stimulus item. However, differences between items in the stimulus pairings in transitive reasoning tasks are usually small (e.g., .5cm in the case of length). Furthermore, in general, people are not competent at estimating length (Schiff and Saarni, 1976). It seems unlikely, therefore, that young children would use a strategy requiring absolute coding of stimulus-attribute values as mental images.

4.4.4 THE FORM OF INTERNAL REPRESENTATION.

These arguments concerning the form of internal representation used by children have two curious aspects. Firstly, if the stimulus-attribute values are sufficiently different to enable the child to adopt an absolute linguistic or imaginal coding strategy, a false-positive solution could be produced. However, it is also possible that a comparative linguistic (e.g., bigger, smaller) or ordered imaginal (e.g., big on left, small on right) coding strategy could be adopted. Each would produce a true positive solution. Short of asking the child which strategy he used, it is impossible to resolve the issue.
Secondly, suppose that objections to the standard procedure are raised on the grounds that the child's verbal skills are not sufficient to permit him to justify his answer orally. Surely, an objection of equal force could be raised against asking the child whether he used an absolute linguistic or comparative linguistic, absolute imaginal or ordered imaginal, form of internal representation.

Additionally, even if there were some way of ascertaining which of these two forms of internal representation were used by the child, the value of the argument could still be questioned. Suppose that the absolute linguistic or absolute imaginal form were used. It would be possible to solve problems without using transitive inference if the child constructed a linguistic or imaginal series, and "read off" the answer. In this case, he would be demonstrating a capacity to seriate, which is substage 1, not substage 3, level of functioning. In order to use transitive reasoning the child would need to transform the problem to the canonical form - if a.R.b. and b.R.c. then a.R.c. It may be that the production of a linguistic or imaginal series is only the first step in the production of an internal form of representation akin to the canonical form. Wallace (1972) argued that the findings of latency studies support this possibility.

4.5 SUMMARY OF CRITICISM REGARDING TRANSITIVE REASONING TASKS.

In summary, a number of critics have insisted that at least some of the subjects used in the Piagetian studies who failed to make transitive inferences were limited, not by the absence of a transitivity rule, but by memory capacity or some other factor. Such failures are instances of
false negatives. In support of those contentions, methodologically rigorous studies were undertaken which, it was claimed, demonstrated that children of four and five years of age could make transitive inferences. However, arguments based on the form of internal representation used suggest that children in these studies could have produced correct answers without employing transitive reasoning. Such passes are instances of false positives.

However, it is also the case that arguments resting on the form of internal representation can be employed to either attack, or defend, the findings of any transitive reasoning study which does not require the subject to provide an appropriate verbal justification. Anderson (1978, 1979) has pointed out in another context that it is impossible to determine what kind of internal representation a subject employs in problem solving. Furthermore, Miller (1976) has argued that: "The problem is that, since it is inherently impossible to find a perfect operational definition of the concepts such as transitivity, inclusion (really disjunction), and conservation on which the controversy turns, it cannot be resolved by finding the perfect test (p. 430)."

With these considerations in mind, it was decided that, when reviewing evidence concerning the components of linear measurement, greater weight would be given to those studies which adhered to the Piagetian approach, with respect to tasks used and insistence upon verbal justifications.
CHAPTER 5.

PARALLEL DEVELOPMENT: EMPIRICAL EVIDENCE CONCERNING ORDER OF EMERGENCE OF CONSERVATION AND TRANSITIVE REASONING.

5.1 PREDICTIONS.

It was argued earlier that conservation of number and length, and transitive reasoning involving number and length, are necessary components of linear measurement. Therefore, the order of development of these components might provide insight into the growth of linear measurement.

It was asserted in Chapter 3, that the parallel development hypothesis yields the predictions that:--

(a) the ability to conserve number emerges at about the same time as the ability to make transitive inferences with respect to discrete quantity;

(b) the ability to conserve length emerges at about the same time as the ability to make transitive inferences with respect to length.

Empirical evidence concerning these predictions will now be discussed.
5.2 ASSESSMENT CRITERIA.

If a study is to provide clear empirical evidence concerning these predictions, it should employ the same group of subjects for all tasks. This is because of the wide variation in the age at which children attain components of the number and length concepts. For example, a particular study using group A might conclude that the mean age for attainment of length conservation is six years three months, and using group B that the mean age for attainment of transitive reasoning involving length is five years two months. Such findings provide only weak evidence of asynchronous emergence of length conservation and transitive reasoning. This is because such a study does not provide evidence that for each group the component not tested does not emerge in synchrony with the component tested. This is a general limitation of between-group experimental designs.

In addition, Piagetian theory posits a lag in development between number and length with respect to the acquisition of conservation and transitive inference. It is necessary, therefore, that, in a study aimed at testing the prediction of synchronous emergence of conservation and transitive inference, the tasks should all test the same concept. For example, the emergence of conservation of length should be located in relation to the appearance of transitive inference reasoning with respect to length. Unless this strategy is adopted, difficulties will arise in interpreting findings. For example, a finding that transitive reasoning involving length emerged after conservation of number would be expected simply because of the horizontal decalage between number and length.
These considerations, together with the factors already mentioned in Chapter 4, specify the kind of study which could test the predictions presently under review. Specifically, such a study should satisfy the following criteria:

(a) the tasks should be administered to all subjects;
(b) comparisons between emergence of conservation and transitive reasoning should be based on the same concept — e.g. number or length or weight but not number and length;
(c) The procedures employed should be essentially the same as those used by Piaget, especially in connection with the insistence upon verbal justification, and clinical style questioning of the subject.

5.3 EVIDENCE THAT ACQUISITION OF CONSERVATION PRECEDES ACQUISITION OF TRANSITIVE INFERENCE.

5.3.1 LENGTH AND WEIGHT.

In two studies, Smedslund (1961, 1963) found evidence suggesting that conservation appears before transitive reasoning for each concept. In the 1961 study, five to seven year old children were given pre-tests of conservation of quantity, conservation of weight, and transitive reasoning involving weight. These pre-tests were followed by a training phase. Smedslund found that, in the pre-tests, the correlation between conservation and transitive reasoning for weight was very low, after partialling out age variances. He also found that transitive reasoning for weight was more difficult to train than conservation of weight (this finding, may simply reflect relative effectiveness of the training techniques). In the
1963 study, Smedslund found that conservation of length appeared earlier than transitive reasoning for length. His assessment procedures closely followed those of Piaget, except for one aspect of the transitive reasoning task. After presenting the AB and BC pairings, but before presenting the AC test comparison, Smedslund checked that the subject remembered the outcomes of the two earlier comparisons. If the subject couldn't remember them, he re-presented the earlier pairings. This procedure increases the potential for non-transitive solutions.

Mc.Mannis (1969) also studied the order of acquisition of conservation and transitive reasoning, for both length and weight, using 90 normals and 90 retardates, matched for mental age between 5 and 10. The results are given in Table 5.1.

TABLE 5.1: RELATIONSHIP BETWEEN CONSERVATION AND TRANSITIVITY: WEIGHT AND LENGTH:

<table>
<thead>
<tr>
<th>NUMBER OF SUBJECTS.</th>
<th>WEIGHT</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NORMALS</td>
</tr>
<tr>
<td>CONSERVATION</td>
<td>TRANSITIVE REASONING</td>
</tr>
<tr>
<td>Absent</td>
<td>Absent</td>
</tr>
<tr>
<td></td>
<td>30</td>
</tr>
<tr>
<td>Present</td>
<td>13*</td>
</tr>
<tr>
<td>Length</td>
<td></td>
</tr>
<tr>
<td>Absent</td>
<td>33</td>
</tr>
<tr>
<td>Present</td>
<td>22**</td>
</tr>
</tbody>
</table>

*  P < .025
** P < .001
These findings appear to be strong evidence that conservation appears before transitive reasoning in the development of each concept. Mc.Mannis' procedures closely followed Piaget's, except in respect of one aspect of the transitive reasoning tasks. Like Smedslund (1963), he introduced a learning factor by requiring the subject to recall correctly the outcomes of the initial AB and BC pairings, before he moved on to present the AC test comparison. His results are consistent with those of Smedslund (1963).

The variations from Piaget's procedures in these studies do not weaken their findings. This is because the change in the standard transitive reasoning procedure would decrease task difficulty. Therefore, if conservation and transitive reasoning emerge synchronously, the predicted outcome in the Smedslund and Mc.Mannis studies would be for transitive reasoning to precede conservation. If, on the other hand, conservation precedes transitive reasoning, the reduction in difficulty of the transitive reasoning task should have masked that asynchrony. In other words the effect of the change in the transitive reasoning test procedure made by Smedslund and Mc.Mannis would be to reduce, not increase, the probability of finding a conservation followed by transitive reasoning sequence. Their finding may, therefore, be construed as evidence that conservation does emerge before transitive reasoning for each concept.
5.3.2 NUMBER AND LENGTH.

Achenbach and Weisz (1975) carried out a longitudinal study of developmental synchrony between conceptual identity (which was equated with conservation), and transitive reasoning for colour, number and length, using a sample of 102 pre-school age children. The children were tested on two occasions, six months apart. Mean age at the first testing was 50 months. Their results for identity and transitive reasoning, with respect to number and length, are given in Table 5.2.

TABLE 5.2:

PERCENTAGE OF SUBJECTS PASSING IDENTITY AND TRANSITIVE REASONING TESTS FOR NUMBER AND LENGTH.

<table>
<thead>
<tr>
<th>CONCEPT</th>
<th>TEST</th>
<th>IDENTITY</th>
<th>TRANSITIVE REASONING</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number</td>
<td>Testing 1</td>
<td>58</td>
<td>9</td>
</tr>
<tr>
<td></td>
<td>Testing 2</td>
<td>73</td>
<td>12</td>
</tr>
<tr>
<td>Length</td>
<td>Testing 1</td>
<td>35</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Testing 2</td>
<td>70</td>
<td>6</td>
</tr>
</tbody>
</table>

Achenbach and Weisz (1975) interpreted these findings as evidence that, for both number and length, conservation precedes transitive reasoning. Even though this study is cited and accepted by influential scholars (e.g. Brainerd, 1978), the conclusion reached is questionable, for the following reasons:-
(a) The identity tasks were concerned with the conservation of a quantitative attribute of a single stimulus. The standard Piagetian conservation task involves the quantitative equivalence of two stimuli. The identity task should not be equated with a conservation task.

(b) The test for the presence of conservation was whether or not the child was "surprised" on the second presentation of the stimulus.

(c) The transitive reasoning tasks were based on the five term presentations of Youniss and Murray (1970) and Roodin and Gruen (1970), and not on the standard three term procedure.

Moreover, it is not unexpected that identity should have emerged before transitive reasoning for number. This is because Gelman and Gallistel (1972) found in a number of studies, that three and four year old children judged that an array of n items still contained n items after the length, colour, identity and spatial arrangement of the array elements had been surreptitiously altered. The same children, however, did not pass the standard two-array conservation test. Gelman and Gallistel (1972) argue that this is because pre-school children know that certain operations change the numerosity of an array while others do not, but do not know what effect the former group of operations have on the relations between arrays of unspecified numerosity. Consequently, it would be expected that identity would emerge before conservation. Because Piagetian theory predicts that conservation and transitive reasoning for each concept emerge in synchrony, the Achenbach and Weisz (1975) finding that identity appears before transitive reasoning is consistent with, and not in opposition to, Piaget's claims.
5.4 EVIDENCE THAT ACQUISITION OF TRANSITIVE INERENCE PRECEDES ACQUISITION OF CONSERVATION.

5.4.1 WEIGHT.

Lovell and Ogilvie (1961) studied the order of acquisition of transitive reasoning for weight and conservation of weight, and found that 53% of those subjects who could not pass the conservation test did pass the transitive inference test.

This study suffers from two major deficiencies which make interpretation difficult. Firstly, in the transitive inference task the third object in the three object series was not presented physically, but described verbally. Hence, the task required a transitive inference connecting a concrete object (A) with a verbal symbol (C), linked by a second concrete object (B). The most likely effect of this mixed mode of presentation would be to increase the difficulty of the task. Secondly, when scoring the subject's protocol for the transitive inference task, the experimenters did not require the AB and BC pairings to be verbalised. This laxity in scoring would tend to reduce task difficulty.

5.4.2 LENGTH AND WEIGHT.

Brainerd (1973) studied the order of acquisition of transitive reasoning and conservation, for both length and weight, in two experiments. In the first experiment, two samples each of 60 subjects (mean ages of seven years seven months and seven years and five months) were used. All sub-
jects received all tests. In one sample, Brainerd found that transitive reasoning emerged before conservation. In the other, Brainerd found that they emerged synchronously, for both length and weight. Because of this equivocal finding, Brainerd (1973) carried out a second experiment using the same materials and procedure as for the first, but employing three groups each of 60 subjects with mean ages of 5 years 4 months, 6 years 4 months, and 7 years 10 months. In the second experiment he found that, for both length and weight, transitive reasoning emerged before conservation. Brainerd (1973) interpreted these results as being damaging to Piaget's parallel development argument. Brainerd's conclusions may be questioned on methodological grounds because his transitive reasoning tasks employed the A.EQ.B.NE.C. paradigm of Youniss and Murray (1970), in lieu of the standard procedure. Also, subject's responses were not scored on the basis of verbal justifications.

5.4.3 LENGTH.

In a later study, Brainerd (1974) examined the effects of training, and transfer of training, on transitive reasoning and conservation, for length. He found that transitive reasoning was easier to train than conservation. He interpreted these results as infirming the parallel development argument. However, again, his transitive reasoning tasks used one of the non-standard paradigms developed by Youniss and Murray (1970). Also, as with his earlier study, Brainerd did not require his subjects to provide verbal justifications. In addition to these departures from standard assessment procedures, interpretation is difficult because the results may simply reflect the relative effectiveness of the training technique used (verbal feedback) for transitive reasoning and conservation.
As part of a series of experiments concerned with the relationship between geometric imagery and operational thought in children, Brainerd and Vanden Heuvel (1974) tested a group of 60 second grade school children (mean age of eight years two months) for presence of transitive reasoning and conservation, for length. They found that 17 subjects passed the transitive reasoning test and failed conservation, but only two passed conservation and failed transitive reasoning. These findings are consistent with those of Brainerd's (1973) earlier study. However, as the assessment procedures used were the same in both studies, interpretation of the data suffers from the same difficulties as those noted above in connection with the 1973 study.

5.5 SUMMARY.

In summary, none of the studies discussed above meet all of the assessment criteria given at the beginning of this chapter.

The well known studies of Brainerd and his colleagues, which concluded that transitive reasoning precedes conservation in the development of each concept, present two difficulties. Firstly, the task used to assess the presence of transitive reasoning may offer subjects the opportunity to pass the tasks without making transitive inferences. Secondly, the studies did not score the subject's protocols. Both of these factors tend to reduce the difficulty of the transitive reasoning tasks. Hence, the studies may have been biased in favour of the conclusions they reached.
The Smedslund and Mc.Mannis studies, which concluded that conservation is achieved before transitive reasoning for each concept, also departed from standard assessment procedures. In particular, the tasks used to assess the presence of transitive reasoning were easier than the standard form. However, this did not bias those studies towards their eventual conclusions.

In conclusion, the evidence does not favour the view of synchronous emergence of conservation and transitive reasoning. It is considered that the evidence is more consistent with the opinion that, in each concept, conservation appears in the child's thinking before transitive reasoning.
It was argued in Chapter 3 that the three sub-stage model yields the predictions that:—

(a) the ability to conserve length emerges earlier than the ability to measure length;

(b) the ability to conserve distance emerges earlier than the ability to measure distance;

(c) the ability to perform the arithmetical operations of addition and subtraction emerges earlier than the ability to measure length or distance;

(d) the ability to conserve number emerges at about the same time as the ability to perform the arithmetical operations of addition and subtraction;

(e) the ability to seriate length emerges earlier than the ability to make a transitive inference with respect to length;

(f) the ability to order discrete quantity emerges earlier than the ability to make transitive inferences with respect to discrete quantity.
Developmental sequences of the kind referred to in the first four of these predictions should not be confused with causal chains in concept acquisition (Flavell (1971, and 1972) discusses this issue in detail). However, they do provide indirect evidence concerning the composition of concepts. Consequently, these predictions, together with those linking transitive reasoning and conservation, are central to the task of identifying the components of linear measurement.

Considered together, the first four of these predictions link number conservation and arithmetical proficiency with the various conservations and measurement. Diagrammatically, the linkages can be represented as in Figure 6.1.

**FIGURE 6.1:**

SCHEMATIC REPRESENTATION OF PREDICTED ORDER OF EMERGENCE OF ARITHMETICAL PROFICIENCY AND CONSERVATION OF NUMBER, LENGTH AND DISTANCE.
With this diagram in mind, the review of evidence concerning these predictions will commence with a discussion of the developmental relationship between number conservation and arithmetical proficiency. It will then move onto length/distance conservation, length/distance measurement, and arithmetical proficiency. Finally, the evidence regarding the lower order abilities referred to in predictions (e) and (f) above, will be discussed.

6.2 THE CONSERVATION OF NUMBER AND ARITHMETICAL PROFICIENCY.

6.2.1 THE NUMBER CONCEPT AND ARITHMETICAL OPERATIONS

It will be recalled that Piagetian theory states that the conservation of number emerges in the child's reasoning as a consequence of the synthesis of the cognitive structures concerned with the logic of classes and relations. Hence, conservation is seen as marking the integration of the ordinal and cardinal aspects of number. The theory also asserts that an understanding of the arithmetical operations of addition and subtraction emerges as a consequence of that synthesis, and in concert with the conservation of number. Hence, understanding of arithmetic is based upon a prior understanding of some aspects of Boolean logic.

6.2.2 DEFINING AN UNDERSTANDING OF ARITHMETIC.

The principal difficulty in testing the prediction that the conservation of number and an understanding of arithmetic emerge contemporaneously is in deciding upon an acceptable definition of arithmetical understanding. It could be said that possession of an algorithmic-like ability to per-
form certain addition operations by counting sets of objects constitutes a level of understanding. It could also be argued that possession of the knowledge that the natural numbers can be composed in a variety of ways (e.g. \((6) = (3+3) = (4+2) = (5+1)\) etc.) constitutes another, and, perhaps, higher level of understanding. Yet another level of understanding would require demonstration of the subject’s knowledge of the associative, distributive, commutative, etc. laws of arithmetic.

6.2.3 EQUIVOCAL FINDINGS OF STUDIES LINKING CONSERVATION OF NUMBER AND ARITHMETIC.

This problem of settling upon a widely acceptable definition of understanding of arithmetic is reflected in the results of studies that have sought to identify the developmental relationship between the conservation of number and an understanding of arithmetic. Among the most influential studies undertaken in the last decade are those of Brainerd and his associates (Brainerd, 1973(a); 1973(b); 1974; Brainerd and Fraser, 1975; and Siegel, 1971(a); 1971(b); 1974), and those of Gelman and her colleagues (Gelman, 1972; Gelman and Gallistel, 1972; and Gelman and Tucker, 1975). In general, these studies concluded that children first develop an ability to count, then to perform addition and subtraction on sets of small numerosity, and that this provides a basis for the understanding of the cardinal aspects of number (including conservation of number). In contrast, other less recent studies (Beard, 1963; Dodwell, 1960; 1961; and Hood, 1962) concluded that attainment of the conservation of number is required for an operational grasp of number in the child’s thinking, and it is only at this stage that the child can have an understanding of arithmetic.
The more recent studies can be criticised on the grounds that the tasks used do not tap the mental abilities for which they were designed. Brainerd's studies, for instance, used a transitive reasoning task involving length relations to assess the child's understanding of ordinal number, and a one-to-one correspondence task to assess understanding of cardinal number. His measures of arithmetical proficiency ranged from knowledge of the first 16 number facts (i.e. \( n+m=? \) and \( n-m=? \) where \( n \) and \( m \) range from 1 to 4) in his 1973(a) study, to the conservation of number task in his 1975 study. Schaeffer (1980) found Brainerd's methods to be: "so seriously flawed logically, psychologically and experimentally as to be incapable of justifying his claims...." (p.556). Of course, other commentators would take issue with Schaeffer's criticisms.

Gelman's work has drawn less criticism. However, for reasons which need not be examined here in detail, it is possible to argue that her findings do not provide information on the developmental relationship between the conservation of number and an understanding of arithmetic. What her studies do suggest is that very young children have (at least for small numerosities) the capacity to count, and to base number judgements of equivalence and operations of addition and subtraction upon that counting procedure.

The older studies of Beard (1963), Dodwell (1960,1961), and Hood (1962) found, in general, that children who conserved number performed at a high level of proficiency on addition and subtraction tasks. The procedures used in these studies closely resemble Piagetian methods, in respect of both the tasks employed, and insistence upon verbal justifications.
6.2.4 STUDIES LINKING THE
LAWS OF ARITHMETIC AND THE
LAWS OF BOOLEAN ALGEBRA.

A different approach to the order of development of the conservation of
number and an understanding of arithmetic, is to examine the development-
al relationship between the laws of Boolean algebra, and the laws of
arithmetic. This approach has led some critics to enter the lists agai­
nst Piaget, on the grounds that class addition on the one hand, and nat­
ural number addition on the other, are so vastly dissimilar that psychol­
ogically, the latter could not possibly be built upon the former. Brain­
erd (1973a, 1978), Langford (1978, 1981), MacNamara (1975) and Osherson
(1974) are leading critics of this aspect of Piagetian theory. Since,
in respect of this issue, they endorse a common view, only MacNamara's
(1975) criticism, and Langford's (1981) evidence, will be discussed.

MacNamara (1975) made the point that, for purposes of counting, and/or of
applying arithmetic operations to the results of counting, any thing could
be grouped with any other thing to form the set of things counted. Such
things need share no common property, save the fact that the person doing
the counting could discriminate one from the other. For example, the num­
btor of people living in town A, and the number of motor vehicles regis­
tered in town B, could be counted as one set. The only property that the
elements of that set would share would be that they were picked out to be
counted. Such a property is inherent of neither the people in town A, nor
the motor vehicles in town B. In contrast, the members of a class - using
that word in the same sense as Piaget - do share properties; properties
inherent to the members of the class. It is this distinction between sets
and classes that gives rise to the radically different nature of the notion of a unit in a set, and the idea of a unit in a class. That, in turn, leads to the radically different outcomes of class and set operations.

MacNamara (1975) provided the following illustration of some of these differences:

"In some sense, 5 includes 4 and 1, animals includes dogs and cats, and animals includes dogs and animals other than dogs. But in what sense of include? Four and 1 together equal 5. There is nothing in 5 over and above what is in 4 and 1 taken together. But dogs and cats together do not equal animals. There are other animals, such as horses and cows. So 5 does not include 4 and 1 in the same sense that animals includes dogs and cats. However, dogs and animals other than dogs, taken together, do equal animals in something like the sense that 4 plus 1 equals 5. Notice, however, that the relationship between dogs and animals other than dogs is quite different from that between 4 and 1. The number "1" cannot be expressed as "numbers other than 4" or as "numbers less than 5 and other than 4". If the latter were its meaning, it would be 1 plus 2 plus 3, which equals 6, and when added to 4 would make 10, not 5. It is clear that the relationship between 4 and 1 is different from that between dogs and animals other
than dogs. In short, the relationships among numbers are quite unlike those among hierarchically arranged classes." (MacNamara, 1975, p.427).

Langford (1981) supported that kind of theoretical argument with empirical evidence from a longitudinal study of the development of children's understanding of logical laws in arithmetic and Boolean algebra. He tested children's knowledge of eight logical laws and 15 arithmetic laws, measuring gains in knowledge over a two year period. Assessment procedures took account of the child's verbal justifications. The tasks used were appropriate behavioural equivalents of the operations being investigated. Analysis of the results included determination of statistical dependencies between items: for example, pass/fail patterns relating to the logical law $A \cup B = B \cup A$ and its arithmetical counterpart $A+B = B+A$.

In general terms, the results did not support the Piagetian view that laws in arithmetic appear later in development than corresponding laws in Boolean algebra.

6.2.5 SUMMARY OF DISCUSSION: THE CONSERVATION OF NUMBER AND UNDERSTANDING OF ARITHMETIC.

Piagetian theory provides the prediction that the conservation of number and an understanding of arithmetic emerge in the child's thinking at about the same time. Empirical verification of that prediction is difficult, because of problems inherent in deriving a widely acceptable definition of what constitutes an understanding of arithmetic. Consequently, the evidence is equivocal.
Against this background, the most conservative policy would be to predict that, for sets of small numerosity, the ability to count and carry out operations of addition and subtraction based upon counting, emerge in the child's thinking before the conservation of number; and, that the latter emerges before more complex forms of addition and subtraction. The question of whether these abilities imply an "understanding of arithmetic" will be discussed in later Chapters.

6.3 THE CONSERVATION OF LENGTH/DISTANCE, AND MEASUREMENT OF LENGTH/DISTANCE.

6.3.1 EMPIRICAL STUDIES OF LENGTH/DISTANCE, CONSERVATION AND MEASUREMENT.

There have been very few empirical studies concerned with the developmental relationships between conservation of length or distance, and measurement of length or distance. Most of the available empirical evidence is still due to Piaget et.al. (1960). Most of the relevant studies that have been conducted were concerned with assessing the role played by measurement in the formation of conservation. In that context, measurement was meant to include counting, and referred to an algorithmic kind of knowledge.

Beilin (1969) found that children with an appropriate measurement algorithm did not conserve number or area. Wohlwill and Lowe (1962) found that the ability to count did not ensure that the child would conserve number. Some of their subjects placed greater weight on perceptual cues, such as row length, than on the cardinal value given by counting, when the two were in conflict.
On the other hand, Bearison (1969), Gruen (1965), Lifschitz and Langford (1977), and Wallach, Wall and Anderson (1967) all found that training in counting and measuring was effective in producing conserving responses, and that the effect was durable.

6.3.2 IDENTITY, INVERSION AND COMPENSATION

ARGUMENTS.

Wallach (1969) has argued that the three main verbal justifications (identity, inversion and compensation) given by conservers, and accepted by Piagetians, as evidence of attainment of conservation, could not be responsible for producing conservation. Langford's (1978) arguments are essentially the same as Wallach's. In addition, he argued that counting and measurement provide an important means by which children come to conserve.

The identity operation preserves a particular property (e.g. length) of an object, as distinct from the object itself. The argument based on this notion of quantitative identity is that the property concerned must be the same before and after transformation, because nothing has been added or taken away. Wallach (1969) agreed that this is true of transformations that do not change the property in question, but insisted that the identity argument could not possibly be a sufficient basis for attainment of conservation. This is because there is nothing in the child's experience to tell him that the quantity in question does not change on the first transformation, or change back again on the second transformation.
Similarly, with respect to inversion and compensation, Wallach (1969) agreed that the conserving child may carry out these operations but, she also argued, that they do not provide sufficient mechanisms for attainment of conservation. For example, addition during the first transformation can be reversed by subtraction during the second transformation, but neither operation is quantity-conserving. Hence, some reversible operations are quantity-conserving, others are not. Consequently, Wallach (1969) argued that understanding of reversibility cannot be a sufficient mechanism for learning conservation. In connection with compensation, she pointed out:—

".....This not only becomes fantastically complicated with any but the simplest containers, but also exact compensation by differences in width for differences in height cannot in any case be directly perceived." (Wallach, 1969, p 192).

In addition to providing these theoretical arguments against quantitative identity, inversion and compensation, she also summarised evidence from a number of studies dealing with different conservations. All of those studies demonstrated, that mere possession of these operations does not ensure that children will conserve.

6.3.3 ROLE OF MEASUREMENT IN ACQUISITION OF CONSERVATION.

Langford (1978) argues that, given that these operations cannot be considered sufficient mechanisms for acquisition of conservation, counting and measurement must be implicated. He proposed that the accretion of experience with counting beads and stones, and, measuring sticks and blocks, etc., under different conditions, leads to the discovery of the
conservation of number and length. This then leads, via generalization, to other conservations such as quantity, weight and volume.

6.3.4 SUMMARY OF DISCUSSION AND CONCLUSION.

In summary, there is empirical evidence that the possession of quantitative identity, inversion and compensation operations does not ensure conservation; that the possession of counting and measurement skills does not ensure conservation; but that training on counting and measurement is effective in promoting conservation responses. Additionally, there are sound theoretical arguments against the proposal that quantitative identity, inversion and compensation operations provide a sufficient mechanism for explaining the acquisition of conservation. There are intuitively appealing arguments for explaining conservation acquisition in terms of counting and measurement skills.

On the other hand, Piaget and Inhelder (1969) have insisted that conservation is a logical, not an infralogical attainment. That is, conservation is not a matter of measurement, but a logical conviction. In part, this assertion refers to their beliefs that: (a) conservers do not, in reaching their answer, resort to infralogical operations; and, (b) that they produce the identity and reversibility arguments only as after-the-event justifications for their answers.
The operational definition of linear measurement adopted in Chapter 2 was: "a person may be said to have a mature understanding of linear measurement if he demonstrates a capacity to use correctly arithmetical operations instead of carrying out physical measurement operations". With that definition in mind, and having regard to the above discussion, there seem to be insufficient grounds to warrant departing from the prediction that the developmental sequence is the conservation of length (or distance) followed by measurement of length (or distance).

6.4 SERIATION, ORDINATION AND TRANSITIVE INFERENCE.

The last two predictions drawn from the three sub-stage model to be discussed in this chapter are concerned with seriation, ordination and transitive reasoning. It was argued that all are necessary for linear measurement. Specifically, the predictions are that: (a) the ability to seriate length emerges earlier in the child's thinking than the ability to make transitive inferences with respect to length; and, (b) the ability to order discrete quantity emerges earlier than the ability to make transitive inferences with respect to discrete quantity. Hence, both predictions refer to the same developmental sequence of seriation-then-transitive reasoning, for the number and length concepts.

Because any transitive inference is, itself, a kind of ordering, it would be illogical to assert that transitive reasoning emerges before seriation. However, it may be that they emerge synchronously. Consideration of this possibility again raises the performance/competence issue, because tran-
sitive reasoning provided a major focus for that controversy. It will be recalled from the discussion of that issue in Chapter 4, that it was concluded that greater weight should be accorded those studies of transitive reasoning which used standard Piagetian assessment methods. With that in mind, a search of the literature failed to uncover any studies using standard Piagetian procedures that found synchronous attainment of seriation and transitive reasoning for either the number or length concepts. Hence, it seems safe to agree with Klahr and Wallace (1976) and predict that seriation emerges before transitivity.

6.5 SUMMARY.

The predictions listed at the beginning of this Chapter were examined against the empirical evidence, and/or in the light of theoretical analysis. In each case it was found that there were insufficient grounds to justify modifying those predictions.
It was argued in Chapter 3 that the horizontal decalage model is the basis for the predictions that, in the child's thinking:

(a) the ability to conserve number emerges earlier than the ability to conserve length;
(b) the ability to conserve length emerges at about the same time as the ability to conserve distance;
(c) the ability to measure length emerges at about the same time as the ability to measure distance;
(d) the ability to seriate length emerges earlier than an ability to numerate.
7.2 EVIDENCE THAT ACQUISITION
OF THE CONSERVATION OF
NUMBER PRECEDES ACQUISITION
OF THE CONSERVATION OF
LENGTH.

A search of the literature failed to find a study that assessed number and length conservation within the same child, using standard Piagetian procedures, and containing a sufficiently large number of within-subject comparisons to enable statistical treatment of data. There are, however, a number of studies that provide indirect evidence. Bearison’s (1969) study is an example. His experiment was concerned with the effects of training in certain counting-based measurement operations upon the child’s ability to conserve continuous quantity, area, mass, number and length. On a seven-month post-test, the percentage of subjects passing the number and/or length conservation tests are given in Table 7.1.

| TABLE 7.1: PERCENTAGE OF SUBJECTS PASSING NUMBER AND LENGTH CONSERVATION TESTS. |
|---------------------------------|---------------------------------|----------------|
| EXPERIMENTAL GROUP | CONTROL GROUP |
| Number | 75 | 38 |
| Length | 63 | 19 |

Because of difficulties (relative effectiveness of training on different tasks) in interpreting the effects of training on Piagetian tasks, even when, as in this case, standard Piagetian procedures and assessment forms are used, only the figures for the control group should be considered.
Whilst those results indicate that the children found number conservation easier than length, two inter-related points should be noted. Firstly, only percentage pass/fail figures are provided, so that within-subject developmental patterns have to be inferred. Secondly, the control group contained only 16 subjects, too few to enable valid inferences to be drawn. For example, 38% of 16 subjects is six subjects: 19% of 16 subjects is three subjects; but the three subjects who passed length conservation may not have been among the six subjects who passed number conservation.

Strauss and Ilan (1975) studied the effects of training on length conservation and speed concepts, and, in both pre-and post-testing of the control group, assessed number and length conservation using the standard Piagetian approach. On pre-testing, of the 10 subjects in the control group, nine conserved number while only three conserved length. On post-testing, all 10 conserved number, but only four conserved length. Although within-subject pass/fail patterns are not reported, the differences between the proportions passing number and length, on the pretest (.9 to .3), and the post-test (1.0 to .4), suggest a number-then-length development pattern. However, as the sample size was only 10 subjects, the results should be treated with caution.

Goldsmidt (1967), in a correlation study linking 10 different types of conservation with age, sex, IQ, MA and vocabulary, also provides some evidence on number-length conservation development patterns. In addition to using standard Piagetian procedures, this study has the merit of a large sample size of 102 subjects. However, some 20% of the subjects were classified as emotionally disturbed: the effect of that disturbance on
cognitive functioning is unknown. The results provided the following difficulty level-ranking (least to most) for the 10 conservations assessed: mass, number, continuous quantity, two-dimensional space, discontinuous quantity, weight, area, length, three dimensional space and distance. Unfortunately, insufficient information was reported on the data transformations used in the ranking procedures to assess the statistical significance of the separation between number and length conservation ranks. However, the data suggest that length conservation was much more difficult than number conservation.

7.2.1 CONCLUSION.

The little empirical evidence available suggests that the conservation of number is achieved before the conservation of length.

7.3 EVIDENCE THAT LENGTH AND DISTANCE CONSERVATION EMERGE SYNCHRONOUSLY.

A search of the literature did not uncover a comprehensive study of within-subject developmental patterns of the emergence of length and distance conservation. The best available evidence comes from the Goldsmidt (1967) study. In that study, it was found that distance conservation was more difficult than length conservation. However, the cautionary comments made above in connection with the Goldsmidt (1967) study extend with equal force to this particular finding.
The Piagetian finding of no horizontal decalage between length and distance conservation could seem counter-intuitive. It would seem harder to acquire the conservation of distance than the conservation of length. This is because length is an attribute of a single object, but distance is a relation between at least two objects. Moreover, the distance relation changes, if the position of one of the objects changes, while length is transportable.

A study by Schiff and Saarni (1976) replicated in most important respects, an earlier study by Piaget and Taponier. The experiment required adults and five and eight year old children to judge small differences in lengths of objects perceptually. The objects were parallel, but their end points were offset. It was found that, when the differences in length were small, both adults and children were not very good at judging relative length. For example, when the difference was ± or − 1 mm, less than 10% of adults made correct judgements. As the differences in length increased, the five year old children became better at judging relative length than the adults. For example, when the difference was ± or − 5 mm, less than 50% of the adults, but more than 70% of the five year old children, made correct judgments. In contrast, 100% of the adults conserved length, whilst most of the five year old children did not. Schiff and Saarni (1976) argued that these findings demonstrate that conservation reflects the interplay of perceptual and conceptual factors. These findings suggest that the conservation of length is not based on perceptually given information. They are consistent with Piaget's views concerning length/distance synchrony.
7.3.1 CONCLUSION.

Against this background, it seems unnecessary to modify the prediction of approximately synchronous emergence of length and distance conservation.

7.4 EVIDENCE THAT LENGTH AND DISTANCE MEASUREMENT EMERGE SYNCHRONOUSLY.

The Piagetian claim that length and distance measurement would emerge synchronously has not been tested empirically. However, if the claim of a synchronous emergence of length and distance conservation is accepted, then there would be no grounds for expecting asynchrony in the attainment of length and distance measurement.

7.5 EVIDENCE THAT ACQUISITION OF SERIATION PRECEDES ACQUISITION OF NUMERATION.

Piagetian theory claims that seriation of length is mastered before the child can numerate (i.e. co-ordinate ordinal position and cardinal value). In the traditional Piagetian demonstration of this claim, the child is asked to seriate sticks of varying lengths to build a staircase. Then the child is asked to insert additional sticks into the series. A toy, such as a doll, is introduced, and the child asked to work out, starting at different positions, how many stairs the doll would have to climb to reach a particular level. Piaget found that seriation was achieved before numeration in this task. Elkind (1964) obtained a similar result using equivalent materials and procedure.
7.5.1 CONCLUSION.

There are no empirical grounds for departing from the Piagetian view that seriation emerges before numeration.

7.6 SUMMARY OF CONCLUSIONS.

The little empirical evidence available supports the Piagetian claims, however, it is evident that more work is necessary.
A discussion of linear measurement was presented in Part I. That discussion analyzed the components of linear measurement and raised several questions which were examined empirically in the present research. The questions were concerned with identifying the necessary components of linear measurement, and describing its development in terms of the growth of those components.

An examination in Part II of relevant literature yielded several predictions regarding the growth of the components of linear measurement.

In Part III, the empirical study is reported. In Chapter 8, the strategy represented in the design is discussed, and a number of hypotheses stated. In Chapter 9, subjects involved, tasks used, and procedures adopted in the study are described. In Chapter 10, a statistical analysis of the results is given.
CHAPTER 8.

THE STRATEGY OF THE STUDY
AND STATEMENT OF
HYPOTHESES.

8.1 THE STRATEGY OF THE STUDY.

8.1.1 QUESTIONS ASKED IN THE STUDY.

The analysis presented in Chapter 2 provided a list of components assumed to be required for a full understanding of iterative linear measurement. Chapter 2 also provided the following operational definition of linear measurement: 'A child may be said to have a mature understanding of linear measurement, if he demonstrates a capacity to use correctly arithmetical operations instead of carrying out physical measurement operations.' It would be possible for a child who does not possess all of the assumed components to demonstrate linear measurement, according to this definition. This would be the case if the child simply resorted to previously learned rules for substituting arithmetical operations for physical measurement operations. Such a child may be said to know how to 'measure' length but not know why arithmetical operations may be substituted for physical measurement operations. Indeed, it is possible that some adults would not know why arithmetical operations may be used in deriving length measurements. With these issues in mind, several empirical questions were then posed in Chapter 2. They may be summarised under three headings.

(a) Which of the components are necessary for mature linear measurement?
(b) Is there an order in which those components emerge in the child's thinking?
(c) What is the relationship between the growth of linear measurement and the growth of those components?

Piagetian theory and associated empirical evidence were consulted in Part II as a source of information regarding these questions. This yielded several predictions concerning the order of emergence of the components.

8.1.2 TYPE OF DESIGN.

The nature of these questions dictated the kind of study needed. The first question - which components are necessary - could be explored by two types of study: (a) a training study; and, (b) a comparative study. The other questions - which are concerned with the order in which the abilities emerge - can only be answered by a particular kind of comparative study: one that examines the development of component abilities.

8.1.3 TRAINING STUDY.

A training study would attempt to teach subjects who could not measure length, those abilities deemed necessary. Pre-tests would identify missing elements in each subject's repertoire, and instruction would focus on developing those elements. Post-tests would assess whether (presumably as a consequence of training) linear measurement skills had emerged. A study of that kind would present substantial problems of interpretation. In particular, failure to meet the criterion of linear measurement could indicate that the skills taught to the subject were not necessary components of linear measurement. Alternatively, it could indicate only that the method of instruction used, whilst adequate for conveying skill in using a particular algorithm, did not promote understanding of the component abilities.
8.1.4 COMPARATIVE STUDY.

In contrast, a comparative study would set down a list of abilities which might be necessary for linear measurement. The study would then locate two groups of subjects:

(a) those who could measure; and
(b) those who could not.

Subjects in both groups would then be tested to assess the presence or absence of the assumed underlying abilities. Comparisons between the two groups would yield information on which abilities appear to be necessary components of linear measurement.

8.1.5 DEVELOPMENTAL STUDY.

A developmental study differs from the comparative approach mainly in that subjects are tested at various ages.

8.1.6 PREFERRED APPROACH.

Because a developmental study has the potential to answer the two kinds of question asked in the present research, it was decided that it would be the most appropriate. Developmental studies can employ either cross-sectional, longitudinal or scalogram methods.
8.1.7 CROSS-SECTIONAL METHOD.

The cross-sectional method yields average ages at which particular tasks are mastered. Although developmental progressions may be inferred, they are based on age-related differences between groups not on age-related changes within subjects. Moreover, when theoretical considerations suggest that a number of different though related capacities will emerge asynchronously, but within a comparatively brief interval of time, overlapping distributions of scores between groups pose difficulties in interpretation. Consequently, the cross-sectional method can provide only indirect evidence of developmental progressions.

8.1.8 LONGITUDINAL METHOD.

On the other hand, the longitudinal method has the potential to yield direct evidence of developmental progressions because its basic datum is within-subject change over time. Unfortunately, it also carries substantial disadvantages with respect to time, cost, testing effects, selective survival and drop-out rate, and so on.

8.1.9 SCALOGRAM METHOD.

A method that overcomes the disadvantages of the cross-sectional approach, and does not incur the time and cost penalties of the longitudinal procedure, is the scalogram technique. This technique involves administering a battery of tests to a group which includes subjects at varying developmental levels. Analysis of the resultant data focuses on within-subject patterns of passes and fails across the test battery, in order to deter-
mine whether the tasks form a scalable set. In this context, a scalable set is one in which passing a particular task presupposes passing all tasks of lower difficulty ranking. Provided that the tasks have construct validity, demonstrating that they form a scalable set is tantamount to demonstrating that the capabilities being assessed emerge sequentially in the course of development.

8.1.10 CONCLUSION.

Scalogram methods have been applied in a number of Piagetian-type studies, especially noteworthy are those of Wohlwill (1960) and Kofsky (1966). As the logic of those two studies closely resembles that of the present research, it was decided to adopt the scalogram method. All experimental designs, however, have inherent disadvantages. Wohlwill (1960) identified the two main problems arising from use of scalogram techniques for cognitive development research. They result from the fact that the technique scales both the subject and the tasks on the same basis, namely, the pattern of passes and fails across the test battery. Firstly, inferences drawn from such an analysis can only be justified if the researcher is assured that the tasks represent an underlying psychological dimension. This requirement has been met in the present study by selecting tasks drawn from a body of theory that has an extensive empirical base, and by formulating a set of specific and testable hypotheses reflecting the operation of a developmental process. Secondly, it is necessary for the researcher to be able to demonstrate a correlation between age (or some age-related factor, such as length of schooling) and scale-type. In the present study, this desideratum was met by applying multiple regression analysis to subjects’ scores, using age and length of schooling as predictors.
8.2 STATEMENT OF HYPOTHESES.

8.2.1 COMPONENTS OF LINEAR MEASUREMENT.

The research cited in Part II has produced highly equivocal findings. In addition, the empirical research reported in this thesis was carried out essentially as a data gathering exercise. In view of this, the hypotheses stated in the following paragraphs should be regarded as being only tentative in nature rather than expressions of commitment.

An hypothesis concerning the composition of the linear measurement concept could refer to a long list of abilities of varying levels of complexity, or to a smaller list of higher-order abilities. An example of the former would be that presented in Chapter 2 in association with an analysis of linear measurement. It will be recalled that that list contained non-independent entries, because it also referred to the growth of linear measurement. A smaller list of higher-order abilities could be drawn up in order to avoid, or reduce, redundancy of that kind.

When formulating an hypothesis for this study, it was decided to express the composition of linear measurement in terms of a list of higher-order abilities. The extent to which the entries on the list are independent is an open question. The hypothesis which follows is drawn from the analysis given in Chapter 2, and takes account of the views of Piaget et al. (1960) on the development of linear measurement.

HYPOTHESIS 1.

A subject demonstrating a mature understanding of linear measurement will also demonstrate the following:
• knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number);
• knowing that length is invariant under certain transformations (the conservation of length);
• knowing that distance is invariant under certain transformations (the conservation of distance);
• knowing how to make transitive inferences of equivalence and non-equivalence with respect to discrete quantity;
• knowing how to make transitive inferences of equivalence and non-equivalence with respect to length;
• knowing how to carry out the arithmetical operations of addition and subtraction;
• knowing how to obtain a linear measurement by counting iterations of a unit of length.

In this context, a mature understanding of linear measurement was operationally defined as: "a person will be said to have a mature understanding of linear measurement, if he demonstrates a capacity to use correctly arithmetical operations, instead of carrying out physical measurement operations." The ability to use a unit of length was operationally defined as: "a person will be said to be able to use a unit of length, if he can determine, by a process of iteration, how many of the given unit are contained in a given length, and (without resorting to further unit iteration) can determine the effect of changing unit size." The present study has only considered the case where the given length contains a whole number of units.
8.2.2 ORDER OF DEVELOPMENT OF COMPONENTS
OF LINEAR MEASUREMENT.

The literature reviewed in Part II only yields partial order predictions for the total set of number and length components. However full orderings can be predicted for each domain separately. Therefore, the question of whether there is an order in which the components of linear measurement emerge in the child's thinking was examined by separating number from length components. Each domain includes both late-emerging components (e.g. arithmetical addition), and early-emerging abilities (e.g. counting). Piagetian theory and the associated empirical evidence suggest that, in each domain, development is orderly and predictable: that is, that there is a high probability that A will emerge before B; and that possession of B implies, with a high degree of probability, possession of A. This is the aspect of the study at which the scalogram analyses were directed.

These analyses were carried out to test the following specific hypotheses:-

Order in the Growth of the
Number Concept.

HYPOTHESIS 2.
The collection of components of the number concept form a scalable set.

Order in the Growth of the
Length Concept.

Hypothesis 3.
The collection of components of the length concept form a scalable set.
Piagetian theory and the associated empirical evidence also suggest that as well as a particular kind of order, there is a particular pattern of development exhibited by the components of linear measurement.

Predictable patterns of development are especially useful for gaining insight into the growth of a concept, and of the emergence of linkages between associated concepts. Since linear measurement involves knowledge contained in the number, length and distance concepts, and the co-ordination of that knowledge, identification of particular development patterns is relevant to gaining an understanding of its growth. The first two of the hypotheses which follow are concerned with development patterns for the number, length and distance concepts. The remainder are concerned with linkages between these concepts. All hypotheses are drawn directly from the conclusions reached in Part II.

Growth of the Number Concept.

Hypothesis 4.

For the number concept, the order of emergence of component elements (from earliest to latest) will be the following:-
<table>
<thead>
<tr>
<th>Rank</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Knowing how to use a one-to-one matching rule.</td>
</tr>
<tr>
<td></td>
<td>Knowing the natural number order.</td>
</tr>
<tr>
<td>2.</td>
<td>Knowing how to count arrays of small numerosity, where to count implies the co-ordination of ordinal position and cardinal value (numeration).</td>
</tr>
<tr>
<td>3.</td>
<td>Knowing how to add, when the objects are visible and small numbers are involved.</td>
</tr>
<tr>
<td></td>
<td>Knowing how to subtract when the objects are visible and small numbers are involved.</td>
</tr>
<tr>
<td>4.</td>
<td>Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).</td>
</tr>
<tr>
<td>5.</td>
<td>Knowing how to find the numerical difference between two collections, when the objects are visible.</td>
</tr>
<tr>
<td></td>
<td>Knowing how to make two collections equal in number, when the objects are visible.</td>
</tr>
<tr>
<td>6.</td>
<td>Knowing how to make transitive inferences of equivalence with respect to discontinuous quantity.</td>
</tr>
<tr>
<td></td>
<td>Knowing how to make transitive inferences of non-equivalence with respect to discontinuous quantity.</td>
</tr>
</tbody>
</table>
7. Knowing how to add, when the objects are are not visible.
   Knowing how to subtract, when the objects are not visible.

8. Knowing how to co-ordinate addition and subtraction operations, when the objects are not visible.

This ordering was derived mainly from the following partial orderings referred to in Part II:
Brainerd (1973a, 1973b) - [2→3; 3→4];
Gelman and Gallistel (1978) - [1→2; 2→3; 3→4; 3→5];
Siegel (1971a, 1971b) [2→3; 3→4];
Smedslund (1963) - [4→6].

Growth of the Length Concept.

Hypothesis 5.

For the length concept, the order of emergence of component elements (from earliest to latest) will be the following:

<table>
<thead>
<tr>
<th>Rank</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Knowing that if length A is greater than length B, then A may be considered as B concatenated with some other length.</td>
</tr>
<tr>
<td>2.</td>
<td>Knowing that the length of an object can be altered only if something is added to, or taken away from, it.</td>
</tr>
</tbody>
</table>
3. Knowing that the length relation between two objects only changes when something is added to or taken away from one, or other, or both, of the objects.
4. Knowing that the length relation between two objects does not change when the spatial relation is changed.
5. Knowing how to order objects according to their lengths (length seriation).
6. Knowing that any length may be considered as a concatenation of arbitrarily selected sub-lengths.
7. Knowing that length is invariant under certain transformations (the conservation of length).
8. Knowing that the ordinal length relation between two objects is the same as the cardinal numerical relation between the parts comprising those objects.
9. Knowing that length relations between objects can be deduced by applying transitive reasoning to the collections of unit parts.
10. Knowing that transitive reasoning can be applied to relations of equivalence between lengths of objects.
10. Knowing how to make quantitative estimates of length, in terms of the number of "unit" lengths.

11. Knowing how to iterate a unit part along an object.
   - Knowing that if the length of the unit part is changed, the number yielded by unit iteration also changes.

12. Knowing that the length relation between two objects can be determined by carrying out a linear measurement operation, using unit iteration.
   - Knowing that arithmetical addition of linear measurements may be used to determine the length of concatenated objects.
   - Knowing how to add length relations (e.g. given the ordered length series \(a - b - c - d\), what is the relation between lengths \((a+c)\) and \((b+d)\), where the increment in length is constant).

This ordering was derived mainly from the following partial orderings referred to in Part II:-
- McManis (1969) - [6→9];
- Piaget et al. (1960) - [1→2; 2→3; 3→4; 1→4; 1→5; 4→6; 6→10; 6→11; 6→12; 10→11; 11→12];
- Smedslund (1963) - [6→9].
Growth of the Distance Concept.

Hypothesis 6.

For the distance concept, the order of emergence of component elements (from earliest to latest) will be the following:

<table>
<thead>
<tr>
<th>Rank</th>
<th>Component</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Knowing how to compare indirectly two distances by a measurement operation not involving unit iteration.</td>
</tr>
<tr>
<td>2.</td>
<td>Knowing that distance is invariant under certain transformations (the conservation of distance)</td>
</tr>
<tr>
<td>3.</td>
<td>Knowing how to estimate distance between two points in terms of the number of &quot;unit&quot; distances.</td>
</tr>
<tr>
<td>4.</td>
<td>Knowing how to measure distance between two points, using unit iteration.</td>
</tr>
</tbody>
</table>

This ordering was derived by analogy with length.

Linkages between Concepts.

Hypothesis 7.

Knowing how to seriate length emerges earlier than knowing how to numerate. (Elkind, 1964; Piaget, 1952).
Hypothesis 8.
The conservation of number emerges earlier than the conservation of length. (Bearison, 1969; Goldsmidt, 1967; Piaget et al., 1960; Strauss and Ilan, 1975).

Hypothesis 9.
The conservation of length emerges at about the same time as the conservation of distance. (Piaget et al., 1960).

8.3 Age, Sex and Length of Schooling Factors.

Subjects differing in age and in length of schooling were used in the study. Additionally, the male/female distribution in the sample was about 50:50. It was possible, therefore, to analyse subjects' performances according to age, length of schooling, and sex.

However, the age range (63 to 78 months) of the subjects was relatively narrow. Given the magnitude of individual differences, it was unlikely that differences in performances between younger and older subjects, after removal of length of schooling (kindergarten to grade one) effects, would be significant. The situation with respect to the length of schooling factor was a little different, because the older subjects were likely to have had one more year of schooling. Consequently, it seemed reasonable to expect that, after removal of any age effect, subjects with more school experience would perform at a higher level than subjects with less. Regarding the sex factor, as discussed by Goldsmidt (1967), research has not generally revealed sex differences.
9.1.1 AGE.

Because the research was aimed at identifying developmental sequences, and because it is known that there are wide individual differences with respect to age at which different capacities emerge (Goldsmidt, 1967), it was necessary to choose an age range within which floor and ceiling effects would be minimized. The evidence reported in Part II suggested that 63 to 78 months (five years three months to six years six months) would be appropriate.

Choosing that age range had certain consequences. Firstly, it was expected that, if the subjects were to be evenly distributed in the 63-78 months age range, then it would be almost certain that they would be spread over two classes, namely kindergarten and grade one. This was because local schools’ admissions policies precluded the possibility that sufficient numbers of the younger subjects would be found in grade one to permit all subjects to be taken from that class. Additionally, if all subjects had been drawn from kindergarten, then it is possible that a substantial number of the older subjects would have been repeating kindergarten, due to
lack of progress the previous year. Lack of progress in early school years is not necessarily a reflection of I.Q. Moreover, tasks such as the various conservations correlate only moderately with IQ (Goldsmidt, 1967). However, it was decided not to risk importing into the study factors affecting a perhaps small, but unknown, proportion of subjects.

Secondly, if the subjects were to be drawn from two classes, this would present an opportunity to compare mean performance levels between classes so as to evaluate the length of schooling effect. Although such matters were not of concern in relation to testing the main hypotheses given in Chapter 8, it was considered that they might yield information relevant to educational practice.

9.1.2 SEX.

An attempt was made to equate numbers of males and females in each age by length of schooling group. However, for practical reasons, it was not possible to obtain exactly equal numbers of each.

9.1.3 SCHOOL CURRICULUM.

If subjects had to be drawn from different schools, it was considered important that there be no substantial difference between schools in emphasis upon use of materials such as cuisenaire rods, and Montessori counting spindles, and that equal emphasis be given to traditional training in counting and memorising number facts. This was because a number of the assessment tasks resemble classroom problems set by teachers.
9.1.4 SAMPLING FACTORS.

Pilot testing of tasks and procedures suggested that assessment of the capacities under investigation would require about two to three hours testing for each subject. This requirement posed difficulties for the ACT Schools Authority, as the ACT public primary schools are heavily utilized for routine teacher training and research. Similar difficulties obtain in gaining access to public primary schools in areas of New South Wales adjoining the ACT. Practical considerations dictated that subjects be drawn from private primary schools in the ACT. For statistical purposes, a minimum of 100 subjects was required. They were drawn from a number of different schools, because no single available school had sufficient enrolments in kindergarten and grade one.

A consequence of using subjects from private schools is that the sample selected may not be representative of the general ACT population. However, as the research was not intended to be a normative study, this was not considered to be an important factor. As the school population sampled contained a substantial number of migrant and refugee children, it was decided that teachers' rating of language understanding and performance would be sought before including children in the study.

9.1.5 SUMMARY.

100 subjects were drawn from four primary schools in the ACT. The schools were: one non-denominational private institution, the AME school at Weston; and three Catholic convent schools, St.Thomas More's at Campbell, St. Joseph's at O'Connor, and St.Brigid's at Dickson. The three Catholic sch-
Schools are located in affluent inner-city suburbs, and draw their students from local households. The AME school tends to attract students from all parts of the ACT and, in general, from a highly affluent sector of the population. All four schools followed the broad curriculum guidelines of the ACT Schools Authority for the early primary years, and all appeared to give the same emphasis to memorisation of basic number facts. The distribution of subjects within age, sex and school grade categories across the four schools is given in Table 9.1.

**Table 9.1: Subject Sample: Age, Sex and Length of Schooling Distribution.**

<table>
<thead>
<tr>
<th>SCHOOL</th>
<th>AGE CATEGORY</th>
<th>TOTALS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>YOUNGER*</td>
<td>OLDER**</td>
</tr>
<tr>
<td></td>
<td>MALE</td>
<td>FEMALE</td>
</tr>
<tr>
<td>AME</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>CAMPBELL</td>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>O’CONNOR</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>DICKSON</td>
<td>5</td>
<td>2</td>
</tr>
<tr>
<td>TOTALS</td>
<td>23</td>
<td>27</td>
</tr>
<tr>
<td>TOTALS</td>
<td>50</td>
<td>25</td>
</tr>
</tbody>
</table>

Notes:
* Subjects were in the eight month age range, 63 to 70 months.
** Subjects were in the eight month age range, 71 to 78 months.
9.2 TASKS.

Full descriptions of the 34 tasks used in the study are given in Appendix 1. The tasks were developed from the analysis given in Chapter 2, and were designed to produce data for testing the hypotheses specified in Chapter 8. Where standard Piagetian forms were available they were used. Every attempt was made to keep the tasks as simple as possible. Additionally, a number of variants of the tasks were pilot-tested in order to determine the most effective forms of presentation. In all cases, simple forms of questioning were employed. Subjects' evaluative responses and verbal justifications were recorded.

All tasks were scored as '1' for a pass and '0' for a fail. Pass and fail criteria for each task are specified in Appendix 1. Every effort was made to ensure that subjects did not guess their answers. Conservative pass/fail criteria were adopted.

Brief statements of the assessment objectives of the tasks are given in the following paragraphs. Number tasks are described first, then length, and then distance. In the interests of brevity in the ensuing discussion, an acronym is given for each task. The first letter of the acronym identifies the task as relating to the number (N), length (L) or distance (D) type. A second letter (R) before the hyphen indicates that the task was concerned with the relations between at least two lengths or distances, as distinct from an evaluation of the outcome of a transformation upon one length or distance. Subsequent letters refer to the operations involved in the task. For example, task LR-TI-NE was concerned with length; relations between length; transitive inferences; between non-equal lengths.
9.2.1 NUMBER TASKS.

N-(1-TO-1). This task assessed the child's ability to determine the numerical relation of equality between three collections of unspecified cardinal value, via the operation of one-to-one correspondence. [See Chapter 2, (A)(i)]

N-ORD. The subjects had to demonstrate that they could form an ordered series of collections of objects of unequal but unspecified numerosities. That is, the subjects had to form a series of the following kind: a, b, c, d, e, where the relation between any two elements could be determined accurately, but not in terms of specific numerosity. [(A)(ii)]

N-CNT. This task assessed the subject's ability to count small arrays. The child had to demonstrate that he co-ordinated ordinal position and cardinal value while counting nine objects. [(A)(iii)]

N-TI-EQ. The subjects had to make transitive inferences of equality of the following kind: a = b; b = c; a ? c; where a, b and c represent discontinuous quantities. [(A)(iv)]

N-TI-NE. The subjects had to make transitive inferences of inequality of the following kind: a > b; b > c; a ? c; where R represents greater than, and less than; and where a, b, and c represent discontinuous quantities. [(A)(iv)]
N-CONS. This was the standard Piagetian number conservation task involving two rows of objects with various relative length and density patterns. [(A)(v)]

N-ADD-V. This task assessed the subject's ability to predict the results of addition operations when the two collections to be added together were visible. The subject was not permitted to put the collections together and count the number of objects in the combined collection; and nor was the subject permitted to use a pointer, such as a finger, to count one collection and then to move onto the other. The subject was allowed to count out loud, or 'in his head'. Collections of up to 12 objects were used. [(A)(vi)]

N-SUB-V. As for N-ADD-V, but in this task the operation involved was subtraction (e.g. work out how many would be left if 'n' were taken away). The subject was required to predict the outcome. The collections were visible. [(A)(vi)]

N-SOL-V. In this task, the subject had to find the numerical difference between two collections (a and b), and, by addition, or subtraction, make the collections equal (this is usually called solving for a difference). Collections a and b were visible. A third collection was available to draw objects from, or give objects to, in order to solve the problem. The subjects had to solve the problem in one move, and were not allowed to use pointers, such as fingers, during any counting operation. Collections of up to 14 objects were used. [(A)(vi)]
N-BAL-V. The subjects had to solve problems having the following form:
if a > b then \([a-(a-b/2)]=[b+(a-b/2)]\). (That is, balance the two collections by sharing the difference between them.) Collections a and b were visible. Subjects had to balance the two numerosities in one move. Collections of up to 14 objects were used. [(A)(vi)]

N-ADD-NV. This task assessed the subject's ability to determine the outcome of adding \(n\) objects to a collection of similar objects of known numerosity, but where this latter collection is not visible to the subject. Collections of up to 12 objects were used. The important difference between this task and N-ADD-V is that, in the latter, the objects were visible to the subject. [(A)(vi)]

N-SUB-NV. As for N-ADD-NV, but in this task the operation involved was subtraction. [(A)(vi)]

N-CYC-NV. This task assessed the subject's ability to work concurrently on two collections, in a situation where adding to collection (a) meant subtracting from collection (b) - that is, the objects cycled from one collection to the other. The objects were not visible, except when in transit between collections. As the first step in the task, the subjects found, by counting, the total number of objects (12) in collections (a) and (b). [(A)(vi)]
9.2.2 LENGTH TASKS.

LR-BinA. This task assessed the subject's understanding that if length (a) is greater than length (b) then (a) may be considered as a concatenation of (b) and some other length (that is, a sense in which (b) is included in (a)). [See Chapter 2, (B)(i)]

L-P/W. The subjects had to demonstrate an understanding that any length may be considered as a concatenation of arbitrarily selected sublengths that is, an understanding of part-whole relations of length. [(B)(ii)]

Note: The next three tasks are all concerned with aspects of the conservation of length. They differ, however, from the standard Piagetian conservation of length task insofar as the materials used, and questions asked, are directed at a particular explanation such as: "nothing was added"; or "it only changed its place, that doesn't make it bigger". The rationale for including them, in addition to the standard task, was given in the discussion in Chapter 2 on the components of linear measurement.

L-INVAR-ADD. This task assessed the subject's understanding that the length of an object is invariant unless something is added to or subtracted from it - setting aside expansion and contraction processes. [(B)(iii)]

LR-INVAR-ADD. This task assessed the subject's understanding that the length relation between two objects is invariant unless something is added to, or subtracted from, one of the objects - setting aside expansion and contraction processes. [(B)(iv)]
LR-INVAR-SP. The subjects were required to demonstrate an understanding that the length relation between two objects is invariant under transformations involving only change of spatial position. [(B)(v)]

LR-ORD. This task assessed the subject's ability to order objects according to their lengths. [(B)(vi)]

LR-TI-EQ. This task assessed the subject's ability to make transitive inferences of equivalence with respect to length. [(B)(vii)]

LR-TI-NE. As for LR-TI-EQ, but with respect to objects of unequal lengths, and, hence, relations of greater than and less than. [(B)(vii)]

LR-CARD. The subjects were required to demonstrate an understanding that the ordinal length relation between two objects is the same as the cardinal numerical relation between the collection of parts comprising those objects (provided that the lengths of those parts are the same). [(B)(viii)]

LR-TI-CARD. The subjects had to deduce length relations between objects by applying transitive inference reasoning to the cardinal number relations between the collections of unit parts. [(B)(ix)]

L-CONS. This was the standard Piagetian conservation of length task, using two pieces of string of equal length. [(B)(x)]

L-UNIT. This task required the subject to iterate a unit part along the length of an object. [(C)(i)]
L-EST. This task assessed the subject's ability to estimate length in terms of: "how many of (a) would you need to put together to make a stick as long as this?" [(C)(i)]

L-UNIT-CH. This task assessed the subject's ability to predict the direction in which the number given by unit iteration would change, if the length of the unit part were to change. [(C)(ii)]

LR-M-CARD. This task assessed the subject's ability to determine the length relation between two objects on the basis of a measurement operation involving unit iteration, and comparison of cardinal numbers. (Notice that the difference between this task and LR-CARD is that, in the latter, the subject does not have to measure the length of each object using unit iteration, because he is told which object has the greater number of parts. Additionally, in LR-CARD the length relation is expressed in terms of "more" or "fewer" parts, not in terms of specific numbers of unit parts.) [(C)(iii)]

L-M-ADD. This task assessed the subject's understanding that numbers representing lengths of objects may be added together, and that the resultant number represents the length of the two objects joined together. [(C)(iv)]

L-ADD. This task assessed the subject's ability to add lengths in the following (semi-algebraic) fashion: given an ordered series, a-b-c-d, where the increment in length is constant, what is the relation between the combined lengths (a+c) and (b+d)? [(C)(iv)]
9.2.3 DISTANCE TASKS.

D-CONS. This was the standard Piagetian conservation of distance task. Two variants were used. The first involved the comparison of distances traversed along a path between two fixed points. The comparison was between a journey from A to B, and one from B to A. On the B to A journey a wall with a door was placed across the path. The second variant of the task involved the comparison of distances traversed between fixed points for: (a) a journey along a straight path; and (b) a journey along a non straight path. [(D)(i)]

D-EST. This task assessed the subject's ability to estimate distance between two objects in terms of: "how many of these small ones would you need to build a path across there?" [(E)(i)]

DR-M. This task assessed the subject's ability to compare indirectly two distances by carrying out a measurement operation, but not necessarily using unit iteration. [(E)(i)]

D-M The subject had to demonstrate an ability to measure the distance between two points using unit iteration. [(E)(ii)]
9.3 PROCEDURE.

9.3.1 ORDER OF ADMINISTRATION.

Because there is some similarity between certain tasks - for example between L-EST (length estimation) and D-EST (distance estimation) - the order of administration was arranged so as to minimise carryover effects. The following is a list of task sequences where carryover effects would be most expected, but undesired:

- N-TI-EQ with N-TI-NE
- N-TI-NE with N-ORD
- N-ORD- with N-CONS
- L-INVAR-ADD with LR-INVAR-SP
- LR-INVAR-SP with LR-INVAR-ADD
- LR-ORD with L-ADD
- L-ADD with L-P/W
- L-L/W with LR-CARD
- LR-TI-EQ with LR-TI-NE
- LR-TI-EQ with N-TI-EQ
- LR-TI-EQ with N-TI-NE
- LR-TI-NE with N-TI-EQ
- LR-TI-NE with N-TI-NE
- LR-TI-NE with LR-TI-CARD
- LR-TI-NE with LR-TI-CARD
- LR-TI-CARD with N-TI-EQ
- LR-TI-CARD with N-TI-NE
- L-EST with D-EST
- L-UNIT-CH with LR-M-CARD
- (LR-M-CARD and L-M-ADD) with (N-M and LR-M)
The following is a list of sequences which should go together, because the second task can be presented as an extension of the first:—

- L-UNIT and L-UNIT-CH
- LR-M-CARD and L-M-ADD

The order of administration was arranged so that any two tasks which should not be presented sequentially were separated by at least two other tasks. Because of the large number of tasks in the battery the whole sequence was divided into the following four sections, with the order of administration within sections being as indicated below:—

Section 1

- N-CNT; N-ADD-V; N-SUB-V; N-SOL-V; N-BAL-V; N-ADD-NV; N-SUB-NV;
  N-CYC-NV; N-I-TO-I.

Section 2.

- LR-M-CARD; L-M-ADD; LR-INVAR-ADD; N-TI-NE; L-INVAR-ADD; LR-BinA;
  N-ORD; LR-INVAR-SP; LR-ORD.

Section 3.

- L-ADD; LR-TI-NE; L-CONS; D-CONS; L-P/W; LR-TI-CARD; D-M; DR-M;
  LR-TI-EQ.

Section 4.

- D-EST; L-UNIT; L-UNIT-CH; N-TI-EQ; N-CONS; L-EST; LR-CARD.
9.3.2 TESTING SESSIONS.

The experience gained from pilot testing the tasks suggested that each subject would take from two to three hours to complete the whole battery. With that in mind, it was decided to test each subject over four sessions, each of 30 to 45 minutes in duration: one for each of the sections given in the preceding paragraphs.

All subjects were tested individually in a quiet room, free from distractions, at the subject's school. Typically, the room contained a small low table, two chairs and a cupboard where the experimental materials were stored. All subjects in a class were tested individually, on Section 1 tasks, then on Section 2 tasks, and so on. After all subjects in the class had completed all sessions, the subjects in the next class were then tested on Section 1 tasks, then Section 2 tasks, and so on. This approach meant that no subject was tested twice on any one day, and that there was usually an interval of a few days between sessions for each subject.

Before commencing testing with subjects from each class, the experimenter was introduced to the class by the teacher, and spent some time with the class, so that the subjects became familiar with the experimenter. The same procedure was adopted at all schools. Testing commenced in early April, 1980 and continued through to December, 1980. Subjects' ages were recorded to the nearest month, as at date of testing on Session 1. The longest period of elapsed time between commencement of testing on Session 1 and completion of testing on Session 4, for any one subject, was 14 days. The same experimenter was used throughout the study.
CHAPTER 10.

RESULTS OF THE STUDY.

10.1 SUMMARY DATA.

The responses (scored 1 or 0) of all subjects on all tasks are given in Appendix 2. The distributions of total scores for all subjects, and of the number of subjects passing each task, are given in Figures 10.1 and 10.2, respectively.

It is evident that both floor and ceiling effects have been avoided. The distribution of total scores in Figure 10.1 shows only a minor floor effect. This is confirmed by the pattern of task difficulty shown in Figure 10.2.

10.2 COMPONENTS OF LINEAR MEASUREMENT.

Hypothesis 1 predicted that subjects who demonstrated an operationally defined level of understanding of linear measurement would also demonstrate that they possessed certain other knowledge assumed to underlie linear measurement.

Table 10.1 shows the number of subjects passing the tasks designed to assess level of understanding of linear measurement (LR-M-CARD and
L-M-ADD), and the number passing the tasks assessing possession of the assumed underlying knowledge. The McNemar test was used to compare the proportions passing each of the linear measurement tasks with the proportions passing each of the component tasks. The chi-squared co-efficients for each test are shown in Table 10.1.

All of the component tasks, except N-SUB-NV, were significantly easier than either LR-M-CARD or L-M-ADD. This finding is consistent with Hypothesis 1.

However, since only a small proportion of the subjects passed the two linear measurement tasks, it is possible that a substantial number of those subjects could have failed the easier tasks. This would not be consistent with Hypothesis 1. Table 10.2 shows the number of subjects who passed both the linear measurement tasks and each of the easier tasks.

The data in Table 10.2 give general statistical support for the hypothesis that the components are pre-requisites for linear measurement. All of the 13 subjects who passed L-M-ADD also passed LR-M-CARD. This confirms their validity as indices of linear measurement. There is a high probability that a subject who passed LR-M-CARD and L-M-ADD will also have passed each of the easier tasks, in all but three cases. The exceptions are D-CONS, LR-TI-NE and N-SUB-NV. The reasons for these exceptions are discussed in the next Chapter. If they are excluded from consideration, eight of the 14 subjects who passed LR-M-CARD also passed all of the component tasks.
It is concluded, that these findings are generally in agreement with Hypothesis 1.

### TABLE 10.1: NUMBER OF SUBJECTS PASSING LINEAR MEASUREMENT TASKS AND HIGH ORDER COMPONENT TASKS TOGETHER WITH ASSOCIATED CHI-SQUARED VALUES.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>CHI-SQUARED</td>
<td>P</td>
</tr>
<tr>
<td>N-TI-EQ</td>
<td>100</td>
<td>84.01</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>LR-TI-EQ</td>
<td>100</td>
<td>84.01</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>N-CONS</td>
<td>78</td>
<td>62.02</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>L-CONS</td>
<td>74</td>
<td>56.15</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>N-ADD-NV</td>
<td>58</td>
<td>38.52</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>L-UNIT</td>
<td>53</td>
<td>37.03</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>L-UNIT-CH</td>
<td>49</td>
<td>28.20</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>D-CONS</td>
<td>48</td>
<td>22.69</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>N-TI-NE</td>
<td>41</td>
<td>20.48</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>29</td>
<td>5.94</td>
<td>&lt;.025</td>
</tr>
<tr>
<td>N-SUB-NAV</td>
<td>21</td>
<td>1.89</td>
<td>N.S.</td>
</tr>
<tr>
<td>LR-M-CARD</td>
<td>14</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>L-M-ADD</td>
<td>13</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
### TABLE 10.2: NUMBER OF SUBJECTS WHO PASSED BOTH THE LINEAR MEASUREMENT TASKS AND EACH OF THE HIGH ORDER COMPONENT TASKS.

<table>
<thead>
<tr>
<th>TASK</th>
<th>NUMBER OF SUBJECTS WHO PASSED LR-M-CARD AND TASKS LISTED IN COLUMN 1 *</th>
<th>%</th>
<th>NUMBER OF SUBJECTS WHO PASSED L-M-ADD AND TASKS LISTED IN COLUMN 1 **</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-TI-EQ</td>
<td>14</td>
<td>100</td>
<td>13</td>
<td>100</td>
</tr>
<tr>
<td>LR-TI-EQ</td>
<td>14</td>
<td>100</td>
<td>13</td>
<td>100</td>
</tr>
<tr>
<td>N-CONS</td>
<td>14</td>
<td>100</td>
<td>13</td>
<td>100</td>
</tr>
<tr>
<td>L-CONS</td>
<td>13</td>
<td>93</td>
<td>12</td>
<td>92</td>
</tr>
<tr>
<td>N-ADD-NV</td>
<td>12</td>
<td>86</td>
<td>12</td>
<td>92</td>
</tr>
<tr>
<td>L-UNIT</td>
<td>14</td>
<td>100</td>
<td>12</td>
<td>92</td>
</tr>
<tr>
<td>L-UNIT-CH</td>
<td>11</td>
<td>79</td>
<td>10</td>
<td>77</td>
</tr>
<tr>
<td>D-CONS</td>
<td>7</td>
<td>50</td>
<td>7</td>
<td>54</td>
</tr>
<tr>
<td>N-TI-NE</td>
<td>11</td>
<td>79</td>
<td>10</td>
<td>77</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>5</td>
<td>36</td>
<td>5</td>
<td>38</td>
</tr>
<tr>
<td>N-SUB-NV</td>
<td>8</td>
<td>57</td>
<td>8</td>
<td>62</td>
</tr>
<tr>
<td>LR-M-CARD</td>
<td>-</td>
<td>-</td>
<td>13</td>
<td>100</td>
</tr>
<tr>
<td>L-M-ADD</td>
<td>13</td>
<td>93</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Notes:  
* - Of the 14 subjects who passed LR-M-CARD the numbers who also passed each of the component tasks are shown in Column 2. Hence, the maximum number is 14.  
** - Of the 13 subjects who passed L-M-ADD the numbers who also passed each of the component tasks are shown in Column 4. Hence, the maximum number is 13.
Hypothesis 2 predicted that the collection of number tasks would form a scalable set. This hypothesis was tested by applying scalogram analysis techniques to the response matrix given in Table 1 of Appendix 2. Guttman (Edwards, 1957) and Loevinger (1947) scaling indices were calculated using a computer program written by the experimenter.

The Guttman analysis yielded a co-efficient of reproducibility of .91; a minimum marginal reproducibility of .7754; and a co-efficient of scalability of .569 (this last statistic is also known as Green’s index of consistency).

The co-efficient of reproducibility is a measure of the extent to which a subject’s scale score predicts that subject’s scale pattern. A co-efficient of greater than .9 is usually considered to be necessary to indicate a valid scale. The minimum marginal reproducibility is the minimum co-efficient of reproducibility that could have occurred given the proportion of subjects passing and failing each item (in this case, task). The co-efficient of scalability takes account of the minimum marginal reproducibility and the co-efficient of reproducibility. As a composite measure it provides a more reliable guide to the scaling characteristics of a set of items. A co-efficient of scalability of greater than .5 is required to indicate a unidimensional and cumulative set.

In the present case, the computed co-efficient of reproducibility exceeds .9, and the co-efficient of scalability exceeds .5. Hence, the Guttman analysis suggests that the collection of 13 number tasks is a scalable set.
However, the Guttman technique has been criticised (Green, 1954;1956) for relying too heavily on marginal row and column totals of passes and fails. In contrast, the Loevinger technique takes account of individual patterns of pass/fail across the whole test battery. In situations where the test battery contains a large number of items with a high probability of yielding tied scores - for both items and subjects - the Loevinger technique seems better suited (Kofsky, 1966; Wohlwill, 1960). For those reasons it seemed prudent, in the present case, to place greater emphasis on the Loevinger indices.

The Loevinger analysis yielded an index of homogeneity of .570. The interpretation of this index is the same as for Green's index of consistency. It measures the extent to which the set is unidimensional and cumulative. Again, an index of greater than .5 is required to indicate a scalable set. It would appear, therefore, that, whether measured by Guttman or Loevinger techniques, the collection of tasks is a scalable set.

Loevinger's technique also requires that a matrix of indices be calculated. Each entry is a value of $H(it)$, the "index of homogeneity of an item (i) with a test (t)". $H(it)$ measures the extent to which the item contributes to overall test homogeneity. An item is regarded as perfectly homogeneous with a test if all subjects passing the item have higher scores on the test as a whole, than all of those failing the item. A perfectly homogeneous item would have a $H(it)$ of 1, but a $H(it)$ of .7 is regarded (Kofsky, 1966) as acceptable. Table 10.3 sets out the $H(it)$'s computed for the number tasks.
Table 10.3 shows that all 13 H(it)'s have a discriminant efficiency of greater than .7, and that 10 out of the 13 have a discriminant efficiency of greater than .8. In comparison, Kofsky (1966) found that 2 out of her 11 classification tasks had H(it) values of less than .7. Hence, the present H(it) values support the hypothesis that the collection of number tasks is a scalable set.
Loevinger’s third statistic, H(ii), called "homogeneity of two items", deals with the relationship between two items in a perfectly homogeneous test. In such a test all those who pass the harder also pass the easier item. In contrast, the H(it) statistic only measures the extent to which those passing an item have higher scores on the test overall, than those failing the item. Hence, H(it) does not identify those who pass harder but fail easier items. For example, H(it) does not discriminate the subject who passes the 10th ranked item and at least nine other items, but fails one or more of the items ranked 1 to 9, from the subject who achieves a score of 10 and passes only (but all of) the items ranked 1 to 10.

To complete the analysis of homogeneity, therefore, it is necessary to inspect the matrix of H(ii)’s and find the proportion having values greater than .5, chance level of responding. Table 10.4 contains the matrix of H(ii)’s for the number tasks.

- Inspection of the matrix of H(ii) values for the number tasks reveals that all but 8 of the 78 indices exceed .5 (chance level). That is, only 10% of all item pairs show reversal or chance level responding. In comparison, Kofsky (1966) found that 36 of her 55 inter-item comparisons were less than .5.

Hence, the impression of scalability is supported by the Guttman co-efficient of reproducibility, Green’s index of consistency, Loevinger’s index of homogeneity of the test as a whole, Loevinger’s index of homogeneity of an item with a test, and Loevinger’s index of homogeneity of an item with an item. It is concluded, that technically the collection of number tasks is a scalable set. This provides statistical support for Hypothesis 2.
<table>
<thead>
<tr>
<th></th>
<th>N-CNT</th>
<th>N-TI-EQ</th>
<th>N-(1-TO-1)</th>
<th>N-ADD-V</th>
<th>N-SUB-V</th>
<th>N-ORD</th>
<th>N-CNS</th>
</tr>
</thead>
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<td>1.00</td>
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<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>N-(1-TO-1)</td>
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<td>.55</td>
<td>.91</td>
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<td></td>
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<td>.52</td>
<td>.33</td>
<td>.49</td>
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<tr>
<td>N-SUB-V</td>
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<td>.33</td>
<td>.33</td>
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<tr>
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**TABLE 10.4 cont.**

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</thead>
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<td>1.00</td>
<td>1.00</td>
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<tr>
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TABLE 10.4 cont.

<table>
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<th></th>
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<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
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<tr>
<td>N-SOL-V</td>
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<td>1.00</td>
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<tr>
<td>N-SUB-NV</td>
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<td>.65</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td></td>
</tr>
</tbody>
</table>

Note: Tasks are arranged left to right and top to bottom in order of increasing difficulty.

The fact that it is not perfectly scalable means that some subjects exhibited reverse ordering. This could represent real heterogeneity in order of emergence or it might reflect error of measurement due, for example, to fluctuation in attention.

10.4 ORDER IN THE GROWTH OF THE LENGTH CONCEPT

Hypothesis 3 predicted that the collection of 17 length tasks would form a scalable set. This prediction was also examined using scalogram analysis. The following four statistics provide an indication that the collection is a scalable set.

- Guttman's co-efficient of reproducibility = .89
- Guttman's minimal marginal reproducibility = .78
- Green's index of consistency = .48
- Loevinger's index of homogeneity = .58
It will be remembered from the discussion on the scalability of the number tasks that the above Guttman values, and Green's index, are very close to the levels required for the collection to be considered a scaled set. Also, the Loevinger index of homogeneity of .58 exceeds the .5 criterion level. Bearing in mind the arguments favouring the Loevinger technique, it is reasonable to conclude that there is substantial order in the collection of length tasks.

Table 10.5 sets out the computed H(it) values for the length tasks.

It will be seen from Table 10.5 that all but 3 of the 17 H(it) values exceed .7, and that 2 of those 3 are among the easiest of the tasks (the easier the task the greater the effect on the computed H(it) value of a chance fail by a subject). This result confirms the impression of order given by the indices relating to overall test scalability.

Table 10.6 sets out the matrix of computed H(ii) values for the length tasks.

Examination of the matrix of inter-item comparisons reinforces the impression of order, as all but 35 of the 136 pairings have H(ii)'s exceeding .5, chance level. Additionally, two items, L-P/W and LR-CARD, together account for 15 of the 35 chance level or reversal type indices. It is noteworthy that the H(it) values for these tasks were below .7 and that these are among the easiest of the tasks in the length subset. These two factors suggest that the reversal rates for these two tasks are unduly affected by a small number of chance failures.
<table>
<thead>
<tr>
<th>TASKS *</th>
<th>INDEX OF HOMOGENEITY</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR-TI-EQ</td>
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</tr>
<tr>
<td>LR-CARD</td>
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</tr>
<tr>
<td>LR-BinA</td>
<td>.79</td>
</tr>
<tr>
<td>L-P/W</td>
<td>.65</td>
</tr>
<tr>
<td>LR-INVAR-ADD</td>
<td>.82</td>
</tr>
<tr>
<td>LR-ORD</td>
<td>.76</td>
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<tr>
<td>L-INVAR-ADD</td>
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<tr>
<td>LR-INVAR-SP</td>
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<tr>
<td>L-CONS</td>
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<td>L-EST</td>
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<td>L-UNIT</td>
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<td>L-UNIT-CH</td>
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<tr>
<td>L-M-ADD</td>
<td>.90</td>
</tr>
<tr>
<td>L-ADD</td>
<td>.81</td>
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</table>

Note: * Listed in order of increasing difficulty.
TABLE 10.6: LENGTH TASKS: - INDEX OF HOMOGENEITY OF AN ITEM WITH AN ITEM.

<table>
<thead>
<tr>
<th></th>
<th>LR-CARD</th>
<th>LR-BinA</th>
<th>L-P/W</th>
<th>LR-INVAR -ADD</th>
<th>LR-ORD</th>
<th>L-INVAR -ADD</th>
<th>LR-INVAR -SP</th>
<th>L-CONS</th>
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</thead>
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<td>1.00</td>
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<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
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<td>-.35</td>
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<td>.76</td>
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<td>.25</td>
<td>.46</td>
<td>.73</td>
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<td>L-P/W</td>
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<td>.55</td>
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<tr>
<td>LR-INVAR -SP</td>
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<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>L-EST</td>
<td>L-UNIT</td>
<td>L-UNIT- CH</td>
<td>LR-TI- EQ</td>
<td>LR-TI- CARD</td>
<td>LR-TI- NE</td>
<td>LR-M- CARD</td>
<td>L-M-ADD</td>
</tr>
<tr>
<td>------------------</td>
<td>-------</td>
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<td>-ADD</td>
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<td>.53</td>
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<td></td>
</tr>
</tbody>
</table>

Note: Tasks are listed left to right and top to bottom in order of increasing difficulty.
On the basis of the Guttman, Green and Loevinger indices, it is concluded that technically the collection of length tasks is a scaled set. This provides statistical support for Hypothesis 3. However, some subjects did exhibit reverse ordering. As with the number tasks, this could represent real heterogeneity in order of emergence, or it might reflect error of measurement.

10.5 EXPECTED PATTERN OF DEVELOPMENT OF THE NUMBER CONCEPT.

The predicted (Hypothesis 4) and the observed orders of difficulty of all tasks in the number collection are given in Table 10.7.

Inspection of Table 10.7 indicates that there is substantial agreement between the predicted and observed rankings. The degree of association between the two rankings was assessed by computing the Spearman rank correlation statistic $Rs$. (corrected for ties), which is .72. This is significant at the .01 level (sample $t = 3.4389$, criterion $t = 2.718$ at alpha = .01 and 11 d.f. for a 1 tailed test).

The three main differences between the rankings are the following:—

(a) It was expected that more subjects would pass N-(1-TO-1) (one-to-one correspondence) and N-ORD (number name order) than the N-CNT (numeration) task, but the latter is considerably easier.

(b) N-ORD was expected to be easier than N-ADD-V and N-SUB-V (addition and subtraction when objects are visible), but they are of approximately equal difficulty.
TABLE 10.7: NUMBER TASKS: PREDICTED AND OBSERVED ORDER OF DIFFICULTY
OF TASKS.

<table>
<thead>
<tr>
<th>TASK</th>
<th>No. of Ss. PASSING*</th>
<th>OBSERVED RANK</th>
<th>PREDICTED RANK**</th>
</tr>
</thead>
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<td>N-CNT</td>
<td>100</td>
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<tr>
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</tr>
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</tr>
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</tr>
<tr>
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<td>1</td>
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</tr>
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</tr>
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<td>11</td>
<td>8</td>
</tr>
</tbody>
</table>

Notes:
* Maximum of 100.

** These are the rankings predicted by Hypothesis 4.
(c) N-TI-EQ (transitive inferences concerning equivalence relations) was expected to emerge synchronously with N-TI-NE, and to be substantially more difficult than N-CONS (conservation), but it is one of the two easiest tasks.

It is concluded that the data generally support the pattern of development of the number concept predicted by Hypothesis 4.

10.6 EXPECTED PATTERN OF DEVELOPMENT OF THE LENGTH CONCEPT.

Table 10.8 sets out the predicted (Hypothesis 5) and observed orders of difficulty of all tasks in the length collection.

Table 10.8 shows that there is substantial agreement between the two orders of difficulty. The Spearman rank correlation statistic Rs (corrected for ties) is .74. This is significant at the .01 level (sample t = 4.2686, criterion t = 2.602 at alpha = .01 and 15 d.f. for a 1 tailed test.

The main differences between the observed and predicted orders of difficulty are the following:-

(a) It was expected that attainment of LR-CARD (ordinal length relation between objects is the same as the cardinal numerical relation between the collections of unit parts comprising those objects would be delayed
### TABLE 10.8: LENGTH TASKS: PREDICTED AND OBSERVED ORDER OF DIFFICULTY OF TASKS.

<table>
<thead>
<tr>
<th>TASK</th>
<th>No. of SUBJECTS PASSING*</th>
<th>OBSERVED RANK</th>
<th>PREDICTED RANK**</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR-TI-EQ</td>
<td>100</td>
<td>1</td>
<td>9</td>
</tr>
<tr>
<td>LR-CARD</td>
<td>98</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>LR-BinA</td>
<td>95</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>L-P/W</td>
<td>94</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>LR-INVAR-ADD</td>
<td>85</td>
<td>5</td>
<td>3</td>
</tr>
<tr>
<td>LR-ORD</td>
<td>82</td>
<td>6</td>
<td>4</td>
</tr>
<tr>
<td>L-INVAR-ADD</td>
<td>80</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>LR-INVAR-SP</td>
<td>74</td>
<td>8</td>
<td>3</td>
</tr>
<tr>
<td>L-CONS</td>
<td>74</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>L-EST</td>
<td>56</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>L-UNIT</td>
<td>53</td>
<td>10</td>
<td>11</td>
</tr>
<tr>
<td>L-UNIT-CH</td>
<td>49</td>
<td>12</td>
<td>11</td>
</tr>
<tr>
<td>LR-TI-CARD</td>
<td>48</td>
<td>13</td>
<td>8</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>29</td>
<td>14</td>
<td>9</td>
</tr>
<tr>
<td>LR-M-CARD</td>
<td>14</td>
<td>15</td>
<td>12</td>
</tr>
<tr>
<td>L-M-ADD</td>
<td>13</td>
<td>16</td>
<td>12</td>
</tr>
<tr>
<td>L-ADD</td>
<td>10</td>
<td>17</td>
<td>13</td>
</tr>
</tbody>
</table>

**Notes:**

* Maximum of 100.

** These are the rankings predicted by Hypothesis 5.
until L-CONS (conservation of length) had emerged, yet the former is the second easiest of all length tasks.

(b) L-TI-EQ (transitive inferences concerning equivalence relations) was also expected to emerge after L-CONS, and synchronously with L-TI-NE (transitive inferences involving non-equivalence relations), but it is the easiest of all length tasks.

It is concluded that the data generally support the pattern of development of the length concept predicted by Hypothesis 5.

10.7 EXPECTED PATTERN OF DEVELOPMENT OF THE DISTANCE CONCEPT.

Table 10.9 sets out the order of difficulty for the distance tasks predicted by Hypothesis 6, and that observed in the study.

<table>
<thead>
<tr>
<th>TASK</th>
<th>No. of SUBJECTS PASSING*</th>
<th>OBSERVED RANK</th>
<th>PREDICTED RANK**</th>
</tr>
</thead>
<tbody>
<tr>
<td>DR-M</td>
<td>53</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>D-CONS</td>
<td>48</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>D-EST</td>
<td>34</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>D-M</td>
<td>26</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Notes: * Maximum of 100.

** These are the rankings predicted by Hypothesis 6.
It is apparent from Table 9 that the predicted and observed rankings are the same at all four points in the sequence. Hence, it is concluded that the data support the pattern of development of the distance concept predicted by Hypothesis 6.

10.8 LINKAGES BETWEEN CONCEPTS.

10.8.1 LENGTH SERIATION AND NUMERATION.

Hypothesis 7 predicted that knowing how to seriate lengths would emerge earlier than knowing how to numerate. The former was tested by task LR-ORD, and the latter by N-CNT. The number of subjects passing LR-ORD was 82, while all 100 subjects passed N-CNT. The McNemar chi-squared value for the difference in proportions passing LR-ORD and N-CNT is 16.06, which is significant at the .001 level and 1 d.f. Hence, N-CNT is significantly easier than LR-ORD. The data, therefore, do not support Hypothesis 7, they show that numeration precedes seriation of length.

10.8.2 NUMBER AND LENGTH CONSERVATION.

Hypothesis 8 predicted that the conservation of number would emerge earlier than the conservation of length. N-CONS was passed by 78 subjects, and L-CONS by 74 subjects. The associated McNemar chi-squared co-efficient of 0.50 is not significant (.05 level). Hence, the data do not support Hypothesis 8.
The low chi-squared coefficient means that there is no difference between the proportions passing N-CONS and L-CONS. This could be interpreted as indicating that N-CONS and L-CONS emerge synchronously. Alternatively, it could indicate that both emerge at a much younger age and, hence, that the present data offer no evidence on their order of emergence. The number of subjects passing and failing each task is shown in Table 10.10.

<table>
<thead>
<tr>
<th>LENGTH CONSERVATION</th>
<th>NUMBER CONSERVATION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>No. of SUBJECTS</td>
</tr>
<tr>
<td></td>
<td>PASS</td>
</tr>
<tr>
<td>PASS</td>
<td>67</td>
</tr>
<tr>
<td>FAIL</td>
<td>11</td>
</tr>
<tr>
<td>TOTAL</td>
<td>78</td>
</tr>
</tbody>
</table>

Table 10.10 shows that most of the subjects who passed one of the tasks also passed the other, and that most of those who failed one task also failed the other. Furthermore, of the 22 subjects who failed N-CONS, seven passed L-CONS, and of the 26 who failed L-CONS, 11 passed N-CONS. This is the pattern that would be expected for synchronous emergence. It is concluded, therefore, that the data suggest that number and length conservation emerge at about the same time.
10.8.3 LENGTH AND DISTANCE CONSERVATION.

Hypothesis 9 predicted that the conservation of length would emerge at about the same time as the conservation of distance. L-CONS was passed by 74 subjects, but only 48 passed D-CONS. The associated McNemar chi-squared coefficient of 15.63 is significant at alpha = .001 and with 1 d.f. Therefore, the data do not support Hypothesis 9.

The number of subjects passing and failing each task is shown in Table 10.11.

**Table 10.11: Number of Subjects Passing and Failing Length and Distance Conservation Tasks.**

<table>
<thead>
<tr>
<th></th>
<th>Length Conservation</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PASS</td>
<td>FAIL</td>
</tr>
<tr>
<td>Distance</td>
<td>PASS</td>
<td>42</td>
</tr>
<tr>
<td>CONSERVATION</td>
<td>FAIL</td>
<td>32</td>
</tr>
<tr>
<td>TOTAL</td>
<td>74</td>
<td>26</td>
</tr>
</tbody>
</table>

Table 10.11 shows that of the 74 subjects who passed L-CONS, 32 failed D-CONS, and of the 48 who passed D-CONS, only six failed L-CONS. This is the pattern that would be expected for a length-then-distance sequence. It is concluded that the data show that the conservation of length emerges before the conservation of distance.
10.9 THE EFFECTS OF AGE, LENGTH OF SCHOOLING AND SEX.

The age, length of schooling, and sex classifications of the sample of 100 schoolchildren used in the study are given in Table 10.12.

<table>
<thead>
<tr>
<th>AGE</th>
<th>SCHOOL GRADE</th>
<th>SEX</th>
<th>TOTALS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Younger*</td>
<td>Kindergarten</td>
<td>MALE</td>
<td>23</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FEMALE</td>
<td>27</td>
</tr>
<tr>
<td>Older**</td>
<td>Kindergarten</td>
<td>MALE</td>
<td>13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FEMALE</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>Year 1</td>
<td>MALE</td>
<td>10</td>
</tr>
<tr>
<td></td>
<td></td>
<td>FEMALE</td>
<td>15</td>
</tr>
</tbody>
</table>

Notes:  *  63 to 70 months.
        **  71 to 78 months.

10.9.1 DIFFERENCES BETWEEN GROUP MEANS.

Table 10.13 sets out the means and standard deviations of the total scores (number of passes out of 34) for each of the six groups identified in Table 10.12.
TABLE 10.13: TOTAL SCORES ON ALL TASKS - GROUP MEANS AND STANDARD DEVIATIONS.

<table>
<thead>
<tr>
<th>GROUP</th>
<th>Characteristics</th>
<th>MEANS*</th>
<th>STANDARD DEVIATIONS</th>
<th>No. of SUBJECTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Younger-Kinder-Male</td>
<td>18.00</td>
<td>7.02</td>
<td>23</td>
</tr>
<tr>
<td>2.</td>
<td>Younger-Kinder-Female</td>
<td>16.70</td>
<td>6.36</td>
<td>27</td>
</tr>
<tr>
<td>3.</td>
<td>Older-Kinder-Male</td>
<td>18.77</td>
<td>6.22</td>
<td>13</td>
</tr>
<tr>
<td>4.</td>
<td>Older-Kinder-Female</td>
<td>22.17</td>
<td>3.87</td>
<td>12</td>
</tr>
<tr>
<td>5.</td>
<td>Older-Year 1-Male</td>
<td>28.90</td>
<td>2.95</td>
<td>10</td>
</tr>
<tr>
<td>6.</td>
<td>Older-Year 1-Female</td>
<td>27.67</td>
<td>4.14</td>
<td>15</td>
</tr>
</tbody>
</table>

Note: * Maximum score = 34.

10.9.2 MULTIPLE REGRESSION ANALYSIS.

ALL TASKS.

A multiple regression analysis was carried out using the heirarchical method of decomposition. Age, then length of schooling, and then the interaction term, age by length of schooling, were taken into the regression equation. The results are summarised in Table 10.14.

Table 10.14 shows that the multiple correlation of performance on age and length of schooling is .60289. This is highly significant (P<.001). The table also shows that there is no significant age effect, but there is a significant (P<.01) length of schooling effect. There is no significant interaction between age and length of schooling.
The results of the multiple regression analysis carried out on the number task scores are summarised in Table 10.15.

The multiple correlation of performance on age and length of schooling for the number tasks is .516. This is highly significant (P<.001).

Table 10.15 shows that there is a significant (P<.05) length of schooling effect, but no significant age effect.
LENGTH TASKS.

Table 10.16 contains a summary of the multiple regression analysis carried out on the length task scores.

**Table 10.16: LENGTH TASKS - SUMMARY OF MULTIPLE REGRESSION ANALYSIS.**

<table>
<thead>
<tr>
<th>VARIABLE</th>
<th>MULT.R</th>
<th>R-SQUARED CHANGE</th>
<th>BETA</th>
<th>F-RATIO</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>AGE</td>
<td>.555</td>
<td>.309</td>
<td>.26779</td>
<td>4.329</td>
<td>&lt;.05</td>
</tr>
<tr>
<td>LENGTH OF SCHOOLING</td>
<td>.602</td>
<td>.054</td>
<td>.37030</td>
<td>8.279</td>
<td>&lt;.01</td>
</tr>
</tbody>
</table>

The multiple correlation of performance on age and length of schooling for the length tasks is .602. This is highly significant (P<.001). Table 10.16 shows that both the age and length of schooling effects are significant (P<.05 and <.01, respectively).

10.9.3 SUMMARY.

The preceding analyses show that length of schooling is a significant predictor of performance on all tasks, on the number tasks, and on the length tasks. In contrast, age is a significant predictor of performance only for the length tasks. This should be treated with caution, because age is not significant for all tasks - due to the added variance of total scores. There is no evidence of a sex effect.
The size of the length of schooling effect, as compared with the age effect, is interesting. It implies that length of schooling is more effective than age in promoting intellectual growth in the number and length concepts, within the age range used in the present study. This finding is relevant to primary schools' admissions policies concerning the minimum age at which children may commence school. Under present policies controlling entry to many Australian primary schools, a considerable number of children are delayed for periods of up to six months in commencing school, because their fifth birthday happens to fall after certain cut-off dates. Although level of intellectual development and capacity to learn are only two of the many factors that need to be considered when assessing a child's readiness for school, the findings of the present study suggest that there are many children who are being delayed in commencing school but who are capable of making intellectual progress through school experience.

10.10 THE EFFECT OF SCORING CRITERIA ON THE FINDINGS.

The preceding analysis is based on data derived from a clinical-style of assessment and scored using a strict pass/fail criterion. The criterion is strict because correct answers were required to all questions before a child was credited with possessing the knowledge the task was designed to tap.

Additionally, because a clinical-style of assessment was used, there is considerable variation between tasks in the number and type of questions asked. It is possible, therefore, that the observed differences in task difficulty might stem, to some extent, from the approach adopted to scoring subjects' responses. This possibility is explored in Appendix 5.
Firstly, in the case of the number tasks there is no correlation between the number of questions asked and task difficulty (Spearman Rs = .37). For the length tasks there is a significant correlation (Spearman Rs = -.42), but in the opposite direction to that which might have been predicted. In other words, the length data analysis reveals that the fewer the number of questions asked the greater the difficulty of the task.

Secondly, subjects’ responses were reanalysed using moderate and weak scoring criteria (defined at paragraphs A5.2.1 and A5.2.2 of Appendix 5). The orders of difficulty obtained under each scoring procedure were compared using a Spearman rank correlation analysis. This indicated that for all tasks taken as one collection, for the collection of number tasks, and for the collection of length tasks the correlations obtained under the strict, moderate and weak criteria are high. The orderings obtained under the three scoring procedures are low, however, for the collection of distance tasks. This finding reflects the fact that the distance collection contains only four tasks with similar pass rates. Hence the correlations are very sensitive to small fluctuations in pass rates.

Thirdly, Guttman and Loevinger scalogram analyses were also carried out on the data derived from the moderate and weak scoring criteria. It was found that the effect of adopting less stringent criteria is to reduce marginally overall test homogeneity, and to increase marginally the incidence of chance-level responding. However, the number and length task collections form scaled sets, whether assessment is based upon a strict, moderate or weak criterion.
10.11 SUMMARY OF FINDINGS.

The following is a summary of the main findings reported in this analysis.

10.11.1 COMPONENTS OF LINEAR MEASUREMENT.

Subjects who demonstrated a mature understanding of linear measurement also demonstrated the following:

- Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
- Knowing that length is invariant under certain transformations (the conservation of length).
- Knowing how to make transitive inferences of equivalence and non-equivalence, with respect to discrete quantity.
- Knowing how to make transitive inferences of equivalence, with respect to length.
- Knowing how to carry out numerical addition operations.
- Knowing how to obtain a linear measurement by counting iterations of a unit of length.

10.11.2 ORDER OF DEVELOPMENT OF LINEAR MEASUREMENT.

For number and length the collections of component tasks form scalable sets. That is, development in the number and length concepts is orderly and predictable.
10.11.3 EXPECTED PATTERN OF DEVELOPMENT.

For number, length and distance the order of emergence of the components is, in general, that predicted by Piagetian theory, and the empirical evidence reviewed in Part II. There are exceptions, however. The most notable is the emergence of the conservation of number before transitive inference concerned with non-equivalent relations between discrete quantities. This same lag in development between conservation and transitive inference also occurs with respect to length. Additionally, certain components of the length concept emerge earlier than corresponding components (e.g. conservation) of the distance concept.
Chapter 11 interprets the results of the statistical analysis described in Chapter 10, in the light of Piagetian theory and previous empirical evidence. That interpretation raises questions regarding the role played by short-term-memory-capacity limitations in forming the observed developmental patterns. These questions are examined in Chapter 12, using an information processing analysis, and, in Chapter 13, by computer modelling. Additionally, Chapter 13 argues that a detailed process model of linear measurement needs to be developed. Chapter 13 also presents a number of production systems that constitute a beginning of that project. The conclusions reached in the study are then summarised in Chapter 14.
CHAPTER 11

DISCUSSION OF RESULTS

11.1 THE COMPONENTS OF LINEAR MEASUREMENT.

The findings reported in Part III are generally consistent with both the analysis of linear measurement presented in Chapter 2 and Piagetian theory. However, some aspects of these findings need closer examination. They are the results relating to the connections between linear measurement and the following:

(a) arithmetical proficiency;
(b) transitive inferences regarding length relations of non-equivalence;
(c) the conservation of length and the conservation of distance;
(d) the use of a 'unit' of length;
(e) the estimation of length;
(f) the lag in development between length and distance.
11.1.1 ARITHMETICAL PROFICIENCY AND LINEAR MEASUREMENT.

The operational definition of linear measurement employed in this study required the child to substitute arithmetical operations for measurement operations. That definition immediately raised the problem of defining arithmetical proficiency. In this study, arithmetical proficiency was indicated by the ability to carry out addition and subtraction operations concerning objects that are not visible. These operations were assessed by tasks N-ADD-NV and N-SUB-NV. The child could not pass those tasks simply by rearranging objects and counting. In order to pass them, he had to know an addition and a subtraction algorithm, and to be able to apply them to an internal representation of the problem.

A reason for requiring that a child pass both tasks was that Piagetian theory argues that arithmetical proficiency marks the synthesis of the logical grouping structures and the elements of the numerical group structures. The result of that synthesis is said to be numerical operations. The hallmarks of numerical operations are that they are reversible - implying addition and subtraction - and that they can be carried out on symbols - implying that the objects involved need not be visible to the child.

It will be recalled that, in the present study, only 14 of the 100 subjects passed LR-M-CARD, and only 13 passed L-M-ADD, the tasks assessing mature linear measurement knowledge. Of the 14 who passed LR-M-CARD, 12 passed N-ADD-NV, but only eight passed N-SUB-NV. Of the 13 who passed
L-M-ADD, 12 passed N-ADD-NV, but only eight passed N-SUB-NV. On that basis, it could not be said that arithmetical proficiency was a pre-requisite for linear measurement.

Moreover, 58 subjects passed N-ADD-NV, and 21 passed N-SUB-NV, yet only 14 passed LR-M-CARD. All subjects who passed N-SUB-NV, also passed N-ADD-NV. Hence, there were 21 subjects who passed N-ADD-NV and N-SUB-NV. Of those 21, 13 failed LR-M-CARD. This would suggest that arithmetical proficiency is not sufficient for linear measurement. That, however, would be consistent with the Piagetian view, because that theory argues that linear measurement ability appears after the emergence of arithmetical proficiency.

In short, the findings suggest that there are some subjects who can measure length but are not proficient at arithmetic, and others who are proficient at arithmetic but cannot measure length. The explanation probably lies in the tasks used. Neither LR-M-CARD, nor L-M-ADD, require the subject to carry out a subtraction operation. The former requires a numerical comparison to be performed, and the latter, an addition operation. Hence, not knowing how to subtract would not constitute a barrier to passing either of the tasks used to assess linear measurement. This analysis, however, is not consistent with the Piagetian view, because that theory argues that number mastery - knowing how to add and subtract - should precede attainment of linear measurement. The analysis would, though, be consistent with Gagne’s (1968) componential theory, since that view of human learning argues that what is important in determining whether a child can solve a particular cognitive problem is whether or not it has the components or rules required, as distinct from the concepts implicated in the problem solution. It may be said, therefore, that from Gagne’s
viewpoint a better study would have included a length subtraction task analogous to L-M-ADD in the test battery. The operational definition could then have been made more stringent, by requiring that subjects pass all three tasks to demonstrate possession of linear measurement ability.

11.1.2 TRANSITIVE REASONING AND LINEAR MEASUREMENT.

It was argued in Chapter 2, that transitive reasoning with respect to number and length were involved in linear measurement. Piagetian theory makes the same claim.

In the present study, five tasks were used to assess various kinds of transitive reasoning. Two were concerned with number, N-TI-EQ and N-TI-NE; and three with length, LR-TI-EQ, LR-TI-NE and LR-TI-CARD. Distinctions were made between transitive inferences concerning equivalence (EQ) and non-equivalence relations (NE), because it was not known whether both forms were implicated in linear measurement. Piagetian theory is silent on that matter. LR-TI-CARD is a composite task. It requires subjects to make transitive inferences regarding length relations. However, the premises are expressed in terms of the number of unit parts contained in each object, not in terms of whole lengths.

It was found that all subjects passed N-TI-EQ and LR-TI-EQ; 48 passed LR-TI-CARD; 41 passed N-TI-NE; 29 passed LR-TI-NE; and 14 and 13 passed the linear measurement tasks, LR-M-CARD and L-M-ADD, respectively. Hence, the transitive reasoning tasks were easier than the linear measurement tasks. It might seem, therefore, that these data are consistent with the predictions emanating from theoretical analyses.
However, of the 13 subjects who passed L-M-ADD, 10 passed LR-TI-CARD and 10 passed N-TI-NE, but only 5 passed LR-TI-NE. These figures suggest that transitive inferences concerning number relations of equivalence and non-equivalence, and transitive inferences concerning length relations of equivalence, are implicated in linear measurement. They also suggest that transitive inferences concerning length relations of non-equivalence are not implicated.

A closer examination of the operations involved in linear measurement indicates that this finding could have been anticipated. The transitive reasoning implicated in unit iteration is concerned only with equivalence relations. The transitive reasoning implicated in the comparison phase (length A with length B) may be concerned with non-equivalent relations, but with respect to number, not length. This is because at the stage that the comparison is made the subject is working with numbers not lengths, or, at most, only indirectly with lengths.

11.1.3 CONSERVATION AND LINEAR MEASUREMENT.

The conservation of number task (N-CONS) was passed by 78 subjects, and the conservation of length task (L-CONS) by 74 subjects. Only 14 subjects passed LR-M-CARD, and only 13 passed L-M-ADD. Moreover, of the 14 subjects who passed LR-M-CARD, 14 passed N-CONS, and 13 passed L-CONS. Of the 13 subjects who passed L-M-ADD, 13 passed N-CONS, and 12 passed L-CONS. Clearly, the linear measurement tasks were much harder than the conservation tasks. Similarly, 48 subjects passed distance conservation (D-CONS) but only 26 passed distance measurement (D-M).
These data provide strong support for the Piagetian view that number conservation and length conservation are pre-requisites for linear measurement. Also, they are consistent with the findings of Beilin (1969) and Wolhwill and Lowe (1962), but inconsistent with the conclusions of Bearison (1969) and Gruen (1965) regarding conservation - measurement asynchrony.

11.1.4 USE OF A UNIT IN LINEAR MEASUREMENT.

Understanding of the notion of a unit in linear measurement was assessed using tasks, L-UNIT and L-UNIT-CH. They were passed by 53 and 49 subjects, respectively. Of the 14 subjects who passed LR-M-CARD, 14 passed L-UNIT, and 11 passed L-UNIT-CH. Of the 13 subjects who passed L-M-ADD, 12 passed L-UNIT, and 10 passed L-UNIT-CH. These data clearly indicate that the ability to employ a unit, and understand its use, are pre-requisites for linear measurement. Again, that is consistent with Piagetian theory.

Piagetian theory argues that a major difficulty confronting a child learning linear measurement is acquiring a grasp of a unit of length. This is because, unlike with number, a unit of length is not perceptually given, but decided arbitrarily. This argument is well illustrated by considering the tasks, LR-CARD and L-UNIT. In the former, the child does not have to invent a unit of length when either assembling or disassembling the rods - the unit of length is the length of the plastic block. In that sense, it is analogous to a counting task, insofar as the unit is perceptually given. In the latter, on the other hand, the child has to use
the small 3cm strip to invent, in an abstract sense, a unit to be successively imposed along the length of the longer strip. Intuitively, it seems that the latter ought to be more difficult. As 98 subjects passed LR-CARD, and only 53 passed L-UNIT, the data support that view.

It is also noteworthy that the conservation of length was passed by a significantly larger proportion of subjects than passed L-UNIT. This is also consistent with the analysis presented in Chapter 2. That is, linear measurement implies the selection and use of a unit, and the use of a unit implies the conservation of length.

11.1.5 ESTIMATION AND LINEAR MEASUREMENT.

Length estimation seems to be substantially easier than linear measurement. All 14 of the subjects who passed LR-M-CARD, also passed the length estimation task (L-EST), which was passed by 56 subjects.

This finding should not be surprising, as the requirements of L-EST closely resemble those of L-UNIT, the major exception being that the answer in the latter is precisely determined. The data also reveal this similarity - 56 subjects passed L-EST, and 53 passed L-UNIT. This suggests that the skill of estimating how many of "a" there are in "b" develops hand-in-hand with the understanding of a unit of length.
11.1.6 LENGTH AND DISTANCE.

It was argued in Part II that corresponding components in the length and distance concepts, such as measurement by unit iteration, would emerge synchronously. The findings of the present study do not support that view.

In the length concept, 74 subjects passed L-CONS, 56 passed L-EST, and 53 passed L-UNIT. The numbers of subjects passing the corresponding component tasks in the distance concept were: 48 passed D-CONS; 34 passed D-EST; and 26 passed D-M (this is the distance task which most closely resembles L-UNIT). Hence, the order of emergence is the same in both concepts, but the length components emerge earlier than the corresponding distance components. On that basis, developmental distinctions could be made between the acquisition of measurement of length, and the acquisition of the measurement of distance.

11.2 INTER-CONNECTION OF THE COMPONENTS OF LINEAR MEASUREMENT.

On the basis of the preceding discussion, the kind of proficiency in subtraction assessed by N-SUB-NV, and the ability to make transitive inferences concerning length relations of non-equality and the ability to conserve distance, are not needed for linear measurement (of length). However, the other high order components set out in Hypothesis 1 should be pre-requisites for linear measurement.
The most difficult of those other components was found to be transitive reasoning concerning non-equivalent numerical relations, which was assessed by N-TI-NL. This was passed by 41 subjects. That is, even though only 14 subjects passed LR-M-CARD, 41 subjects passed the most difficult component task. Also, of the 100 subjects in the study, 13 passed all high-order component tasks, but failed LR-M-CARD and L-M-ADD. Moreover, all of those subjects were unable to commence LR-M-CARD and L-M-ADD. Hence, for those subjects, the difficulty was not in executing correctly a solution strategy. The evidence suggests that they didn’t have a strategy to invoke when confronted with the requirements of LR-M-CARD and L-M-ADD.

It would appear, therefore, that a proportion of subjects possessed all the components, but could not measure length. That is, the components may be necessary, but not sufficient, to ensure linear measurement. There appears to be a delay between acquiring the underlying components, and being able to demonstrate a mature understanding of linear measurement. Given the differences in proportions passing the most difficult component task and the linear measurement tasks, the delay appears to be substantial. The question then arises: what is the cause of this delay?

Clearly, it would not be expected that mere possession of the listed components would be sufficient for linear measurement. They would need to be co-ordinated in some fashion, even if only in the same sense that an algorithm orders operations in a computation. Hence, the delay might occur because, even though all the components are present, some subjects might not have been taught how to apply them to the task of linear measurement. This supposition would be consistent with the finding that length of schooling is a predictor of a subject’s overall score.
Alternatively, those subjects possessing the components, but not passing the linear measurement tasks, might have been instructed in linear measurement. However, those subjects might not have been able to co-ordinate the components, because of a structural limitation. An obvious structural limitation would be insufficient short term memory (STM) capacity.

A linear measurement strategy might be permanently represented in long-term-memory (LTM), or it might be generated by other LTM structures to solve a particular problem. In the latter case, the strategy would be simply a transient assembly of knowledge elements. In both cases, STM would be involved in controlling the execution of the strategy. Hence, it could be expected that STM capacity limitations would be manifested in breakdowns, or errors, in execution of the strategy. However, in the present study, all 13 of the subjects who possessed all the components but failed the linear measurement tasks, could not even commence those tasks. This suggests lack of an appropriate strategy, not faulty execution. It also suggests that STM capacity limitations are not responsible for the observed delay between acquisition of the components and mastery of linear measurement. Moreover, since STM capacity increases with age, this conclusion is consistent with the finding in the present study that age is not a predictor of a subject's overall performance.

Piagetian theory would account for the observed delay by asserting that it coincides with a re-organisation of cognitive structures that results in better co-ordination of underlying components. However, such an account would not say why the assumed re-organisation should be a lengthy process. In the present case, one explanation might be that the child
needs to be exposed to a large number of experiences of the appropriate kind before he can deduce the strategy that underlies linear measurement. Moreover, the child might not be able to benefit from these experiences until he has acquired all of the underlying components. This implies that the observed delay corresponds to an active period of learning. This would be consistent with the present finding that length of schooling is a predictor of performance.

11.3 THE IMPLICATIONS OF THE ORDER OF EMERGENCE OF COMPONENTS OF THE NUMBER AND LENGTH CONCEPTS.

The finding that the number and length tasks form scaled sets is significant. It seems that many of the components of the concepts are acquired sequentially.

It would not be prudent, however, to claim that this sequential order is the only pattern that number and length development could exhibit. The earlier discussion of the possible causal links between conservation, transitive reasoning, and measurement hints of the difficulty of maintaining such a position.

Moreover, it should be borne in mind that most of the components assessed in the present study are closely related to, if not synonymous with, skills taught to children in school. Most of the teaching in schools,
especially in arithmetic, is predicated on the assumption that such skills are hierarchically organised. Hence, the observed pattern of development in the number and length concepts may reflect nothing more than the curriculum sequence used in the schools the subjects were drawn from. The finding that length of schooling is a predictor of overall performance is consistent with that suggestion, and would be predicted by Gagne's (1968) theory of learning. Of course, the curriculum sequence may well reflect the logical contingencies between the components tested in this study.

The order of emergence of the components does not necessarily reflect the order in which they began to develop. It may be that the development of component B commences before, or in synchrony with, the development of A. However, if B takes longer to develop then it would emerge after A. In that case, it would be inaccurate to claim a developmental dependency.

Flavell (1971, 1972) has pointed out the distinctions between developmental sequences and developmental dependencies at considerable length, and has proposed schemes for classifying observed developmental patterns. However, in the main, those schemes require identification of the time at which each component started to develop, and the time at which its development was completed. Given the difficulty in assessing cognitive skills, these requirements seem unrealistic. For example, determining the time at which a subject gave his first behavioural evidence of rudimentary counting skill, is probably, impossible.
11.4 THE ORDER OF EMERGENCE OF COMPONENTS OF

THE NUMBER CONCEPT.

The order of emergence of the components of the number concept is indicated by the numbers of subjects passing each task. Table 11.1 shows these tasks in rank order, together with the McNemar chi-squared coefficients for adjacently ranked tasks. The full matrix of chi-squared coefficients is given in Appendix 3.

TABLE 11.1: NUMBER TASKS: CHI-SQUARED VALUES FOR ADJACENTLY RANKED ITEM PAIRS.

<table>
<thead>
<tr>
<th>TASK</th>
<th>NO. OF SUBJECTS PASSING</th>
<th>MC. NEMAR CHI-SQUARED VALUES</th>
<th>P.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-CNT</td>
<td>100</td>
<td>0.00</td>
<td>NS</td>
</tr>
<tr>
<td>N-TI-EQ</td>
<td>100</td>
<td>12.07</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>N-(1-TO-1)</td>
<td>86</td>
<td>-&gt; 0.06</td>
<td>NS</td>
</tr>
<tr>
<td>N-ADD-V</td>
<td>84</td>
<td>-&gt; 0.06</td>
<td>NS</td>
</tr>
<tr>
<td>N-SUB-V</td>
<td>81</td>
<td>-&gt; 0.06</td>
<td>NS</td>
</tr>
<tr>
<td>N-ORD</td>
<td>80</td>
<td>-&gt; 0.06</td>
<td>NS</td>
</tr>
<tr>
<td>N-CONS</td>
<td>78</td>
<td>-&gt; 10.24</td>
<td>&lt;.005</td>
</tr>
<tr>
<td>N-SOL-V</td>
<td>61</td>
<td>-&gt; 0.24</td>
<td>NS</td>
</tr>
<tr>
<td>N-BAL-V</td>
<td>58</td>
<td>-&gt; 0.05</td>
<td>NS</td>
</tr>
<tr>
<td>N-ADD-NV</td>
<td>58</td>
<td>-&gt; 6.56</td>
<td>&lt;.025</td>
</tr>
<tr>
<td>N-TI-NE</td>
<td>41</td>
<td>-&gt; 12.86</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>N-SUB-NV</td>
<td>21</td>
<td>-&gt; 3.20</td>
<td>NS</td>
</tr>
<tr>
<td>N-CYC-NV</td>
<td>16</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The chi-squared values in Table 11.1 and Appendix 3 suggest a stepped performance gradient— that is to say that there are abrupt changes to the slope of this gradient. As indicated in Table 11.2 and Figure 11.1, that gradient can be divided into five levels.

**TABLE 11.2: LEVELS ON THE PERFORMANCE GRADIENT FOR THE NUMBER TASKS.**

<table>
<thead>
<tr>
<th>LEVEL</th>
<th>TASKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>N-CNT</td>
</tr>
<tr>
<td></td>
<td>N-TI-EQ</td>
</tr>
<tr>
<td>2.</td>
<td>N-(1-TO-1)</td>
</tr>
<tr>
<td></td>
<td>N-ADD-V</td>
</tr>
<tr>
<td></td>
<td>N-SUB-V</td>
</tr>
<tr>
<td></td>
<td>N-ORD</td>
</tr>
<tr>
<td></td>
<td>N-CONS</td>
</tr>
<tr>
<td>3.</td>
<td>N-SOL-V</td>
</tr>
<tr>
<td></td>
<td>N-BAL-V</td>
</tr>
<tr>
<td></td>
<td>N-ADD-NV</td>
</tr>
<tr>
<td>4.</td>
<td>N-TI-NE</td>
</tr>
<tr>
<td>5.</td>
<td>N-SUB-NV</td>
</tr>
<tr>
<td></td>
<td>N-CYC-NV</td>
</tr>
</tbody>
</table>

All tasks lying on one level on the gradient are significantly easier than those on the next higher level, and significantly more difficult than those on the next lower level. However, all tasks on the same level do not differ significantly from each other. This does not necessarily mean that those tasks do not differ in difficulty since significance tests for
FIGURE 11.1
NUMBER TASKS - ORDER OF DIFFICULTY

NO. OF SUBJECTS PASSING

N-CNT N-TID EQ N-IND N-ADV N-REV N-SUBY N-ORD N-ONE N-SILY N-DAY N-MIN Y N-TIME N-SUBY N-ORD

TASKS
adjacent tasks in a sequence are of low power. Indeed, the conclusion that the collection of number tasks is a scalable set implies that they are ordered. The extent of that ordering may be gauged by inspecting the relevant H(ii) indices. These indices for the number tasks were given in Table 10.4 in Chapter 10. An inspection of Table 10.4 reveals that the tasks on level 2 and on level 3 are poorly ordered. This confirms the conclusion drawn from the significance tests suggesting no gradient in these regions of the curve. On the other hand, in spite of the results of the significance tests, the tasks at level 5 are perfectly ordered.

It may appear that a conclusion of a stepped performance gradient for the number task would contradict a conclusion that the collection of number tasks is a scaled set (i.e. the components emerge sequentially). However, abrupt changes in the slope of a performance gradient are not necessarily incompatible with a sequential order of development. An abrupt change in the slope of a performance gradient indicates a change in the rate of increase of component difficulty. If the change is large, it may be statistically significant. A sequential order of development is one in which the components appear in a fixed order, with the easier components emerging earlier than the more difficult. However, the increase in difficulty between adjacent components in a developmental sequence need not be statistically significant, though it may be. In the present case, as indicated in Figure 11.1, the levels on the performance gradient are not perfectly flat. Similarly, the collection of number tasks is not a perfectly scaled set. Hence, these two conclusions of a sequential order of development and a stepped performance gradient are not incompatible.
Assessing the developmental implications of a stepped performance gradient can be complex. The form of the performance gradient for any set of tasks is a joint function of two factors, namely, the distribution of intellectual growth levels in the sample of subjects, and the distribution of task difficulty level (i.e. the level of intellectual growth required to pass each task). An apparently stepped performance function could result from inhomogeneities in the distribution of intellectual growth levels in the subject sample, and/or from inhomogeneities in the distribution of the difficulty levels of the tasks.

In the present study, there are two reasons for believing that it is unlikely that the stepped performance gradient is due to the subject sample. They are the narrow age range of the subjects, and the finding that age is not a predictor of performance. Each of these factors suggest homogeneity, not inhomogeneity, in the subject sample.

Regarding the distribution of task difficulty levels, it is possible that the stepped performance gradient results simply from taking a small random sample of tasks from a larger population. The distribution of difficulty levels in this population could be continuous. The apparent discontinuity could be a consequence of sampling error. Alternatively, the distribution might be discontinuous. Piaget's stage theory of development asserts that this is the case.

In any case it might be possible to explain the observed performance gradient by analyzing the information-processing demands of the tasks. This analysis is given in Chapters 12 and 13.
The observed pattern of development for the number tasks contains certain other features which need comment.

Transitive Reasoning. N-TI-EQ was significantly easier than N-TI-NE. It is usually assumed that the components assessed by these tasks emerge together. However, Langford (1981) also found that the transitive law for the greater-than relation was more difficult than the transitive law for the equal-to relation, with respect to number.

Conservation and Transitive Reasoning. N-TI-EQ was significantly easier than N-CONS, but N-CONS was significantly easier than N-TI-NE. Additionally, the Loevinger indices of homogeneity of an item with an item for N-CONS and N-TI-EQ, and for N-CONS and N-TI-NE, are 1.00 and .89, respectively. That is, almost all subjects who passed N-TI-NE also passed N-CONS, and very few of the subjects who failed N-CONS passed N-TI-NE. Thus, there is a developmental asynchrony between the components assessed by N-CONS and N-TI-NE.

This finding is not consistent with Piagetian theory, because the latter claims that, in each concept domain, conservation and transitive reasoning emerge in parallel. This finding is consistent, however, with Gagne's (1968) componential theory, since the form of transitive reasoning implied in the N-CONS task is N-TI-EQ, not N-TI-NE. Hence, lack of the ability assessed by N-TI-NE would not be a barrier to a child passing N-CONS. For the ability assessed by N-TI-NE to be implicated, the conservation of number task would need to have included tests for the conservation of the numerical relations of greater-than and less-than. That comment aside, however, the present finding of a development asynchrony between conservation and transitive reasoning is consistent with the Smedslund (1963) and Mc.Mannis (1969) data.
Arithmetical Proficiency. The components assessed by the more difficult arithmetical tasks (e.g. N-SOL-V, N-BAL and N-ADD-NV) emerge after conservation, rather than at the same time. The most difficult of these components (assessed by N-SUB-NV and N-CYC-NV) emerge much later than the three which follow the appearance of conservation. Moreover, there are no reversal-type responses in the data concerning these five arithmetical tasks - that is, they are a perfectly ordered set.

The reason for the delay in achieving the kind of arithmetical proficiency assessed by N-SUB-NV is not apparent. It may be a reflection of the additional time needed for reversibility of the numerical operations implied in N-ADD-NV to be achieved. That kind of Piagetian argument, however, is not consistent, because the emergence of the conservation of number is also supposed to indicate that the property of reversibility has been achieved. The inconsistency stems from the finding that the conservation of number emerges much earlier than arithmetical proficiency, as assessed by N-SUB-NV. Hence, Piagetian theory does not offer a ready explanation.

11.5 THE ORDER OF EMERGENCE
OF COMPONENTS OF THE
LENGTH CONCEPT.

The order of emergence of the components of the length concept is indicated by the number of subjects passing each task. Table 11.3 shows these tasks in rank order, together with the McNemar chi-squared coefficients for adjacently ranked tasks. The full matrix of chi-squared coefficients is given in Appendix 3.
### Table 11.3: Length Tasks: Chi-Squared Values for Adjacently Ranked Item Pairs

<table>
<thead>
<tr>
<th>TASK</th>
<th>No. of Subjects</th>
<th>McNemar Chi-Squared Values</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR-TI-EQ</td>
<td>100</td>
<td>→ 0.50</td>
<td>NS</td>
</tr>
<tr>
<td>LR-CARD</td>
<td>98</td>
<td>→ 0.57</td>
<td>NS</td>
</tr>
<tr>
<td>L-BinA</td>
<td>95</td>
<td>→ 0.00</td>
<td>NS</td>
</tr>
<tr>
<td>L-P/W</td>
<td>94</td>
<td>→ 3.76</td>
<td>NS</td>
</tr>
<tr>
<td>LR-INVAR-ADD</td>
<td>85</td>
<td>→ 0.27</td>
<td>NS</td>
</tr>
<tr>
<td>LR-ORD</td>
<td>82</td>
<td>→ 0.03</td>
<td>NS</td>
</tr>
<tr>
<td>L-INVAR-ADD</td>
<td>80</td>
<td>→ 2.50</td>
<td>NS</td>
</tr>
<tr>
<td>LR-INVAR-SP</td>
<td>74</td>
<td>→ 0.04</td>
<td>NS</td>
</tr>
<tr>
<td>L-CONS</td>
<td>74</td>
<td>→ 9.03</td>
<td>&lt;.005</td>
</tr>
<tr>
<td>L-EST</td>
<td>56</td>
<td>→ 0.19</td>
<td>NS</td>
</tr>
<tr>
<td>L-UNIT</td>
<td>53</td>
<td>→ 0.41</td>
<td>NS</td>
</tr>
<tr>
<td>L-UNIT-CH</td>
<td>49</td>
<td>→ 0.00</td>
<td>NS</td>
</tr>
<tr>
<td>LR-TI-CARD</td>
<td>48</td>
<td>→ 11.17</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>29</td>
<td>→ 5.94</td>
<td>&lt;.025</td>
</tr>
<tr>
<td>LR-M-CARD</td>
<td>14</td>
<td>→ 0.00</td>
<td>NS</td>
</tr>
<tr>
<td>L-M-ADD</td>
<td>13</td>
<td>→ 0.36</td>
<td>NS</td>
</tr>
<tr>
<td>L-ADD</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
The chi-squared values in Table 11.3 and Appendix 3 suggest a stepped performance gradient. As indicated in Table 11.4 and Figure 11.2, that gradient can be divided into four levels.

TABLE 11.4: LEVELS ON THE PERFORMANCE GRADIENT FOR THE LENGTH TASKS.

<table>
<thead>
<tr>
<th>LEVELS</th>
<th>TASKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>LR-TI-EQ</td>
</tr>
<tr>
<td></td>
<td>LR-CARD</td>
</tr>
<tr>
<td></td>
<td>L-BINa</td>
</tr>
<tr>
<td></td>
<td>L-P/W</td>
</tr>
<tr>
<td>1B</td>
<td>LR-INVAR-ADD</td>
</tr>
<tr>
<td></td>
<td>LR-ORD</td>
</tr>
<tr>
<td></td>
<td>L-INVAR-ADD</td>
</tr>
<tr>
<td></td>
<td>LR-INVAR-SP</td>
</tr>
<tr>
<td></td>
<td>L-CONS</td>
</tr>
<tr>
<td>2.</td>
<td>L-EST</td>
</tr>
<tr>
<td></td>
<td>L-UNIT</td>
</tr>
<tr>
<td></td>
<td>L-UNIT-CH</td>
</tr>
<tr>
<td></td>
<td>LR-TI-CARD</td>
</tr>
<tr>
<td>3.</td>
<td>LR-TI-NE</td>
</tr>
<tr>
<td>4.</td>
<td>LR-M-CARD</td>
</tr>
<tr>
<td></td>
<td>L-M-ADD</td>
</tr>
<tr>
<td></td>
<td>L-ADD</td>
</tr>
</tbody>
</table>
All tasks lying on one level on the gradient are significantly easier than those on the next higher level, and significantly more difficult than those on the next lower level. With the exception of the level 1 tasks, all tasks on the same level do not differ significantly from each other. In the case of the level 1 tasks, there is no significant difference in difficulty between adjacent tasks (ranked in order of difficulty as in Table 11.3), but there are significant differences between tasks more widely separated in difficulty ranking. The tasks on level 1 could be divided into two sub-levels, namely, level 1A containing LR-TI-EQ, LR-CARD, L-BinA, and L-P/W, and level 1B containing the remainder of the level 1 tasks. With that division, most of the tasks on level 1B are significantly (P<.05) more difficult than those on 1A, but all tasks on the same sub-level do not differ significantly from each other.

It will be recalled from the earlier discussion of the stepped performance gradient observed for the number tasks that a chi-squared analysis of the proportions of subjects passing and failing tasks on the same difficulty level does not provide information on whether those tasks form a developmental sequence. The extent of any ordering of tasks on the same level can be assessed by inspecting the relevant H(ii) indices in Table 10.6 (Chapter 10). Table 10.6 shows that tasks on the same level are, in general, poorly ordered. This confirms the impression of a stepped performance gradient for the length tasks.

There are other features of the stepped performance gradient for the length tasks which need comment.
Transitive Reasoning. As for the corresponding number tasks, transitive inferences concerned with equivalent relations appear much earlier than those concerned with non-equivalent relations.

Conservation and Transitive Reasoning. Again there is a similarity between number and length with respect to the observed patterns concerning conservation and transitive reasoning. Specifically, the component assessed by LR-TI-EQ appears before that assessed by L-CONS, which appears before that assessed by LR-TI-NE.

This finding is not consistent with Piagetian theory. However, as has been noted previously in connection with the corresponding number tasks, this kind of asynchrony is consistent with Gagne's (1968) theory. Clearly, that theory, with its emphasis upon componential structure, is consistent with this finding, because the form of transitive reasoning implied in L-CONS is that assessed by LR-TI-EQ, not that assessed by LR-TI-NE.

11.6 ORDERING ACROSS NUMBER AND LENGTH TASKS.

There is a similarity between the observed patterns in the number and length concepts. In general, the tasks located at abrupt changes of slope on the performance gradient for the number tasks were of major theoretical interest (e.g. N-CONS). That is also the case for the length tasks (e.g. L-CONS). It was seen in Chapter 10 (Hypothesis 8) that the conservation of number appears at about the same time as the conservation of length. It may be that these abrupt changes in slope of the performance gradients reflect a re-organisation of the child's number and length concepts.
This suggestion stems from two facts. Firstly, following the emergence of N-CONS and L-CONS, there is a considerable delay before the emergence of the next components in the length domain. Secondly, those components, L-EST and L-UNIT, implicate the numerical representation of length. It may be that until that development occurs the child's reasoning about length is restricted, because the length concept is unconnected (or only loosely connected) to the number concept. However, the appearance of conservation for each domain may enable a re-organisation which results in the child's reasoning about length being augmented by a number-based, or number-connected form, of internal representation.

The next major discontinuity (i.e. abrupt change in slope of the length performance gradient) in the length domain occurs after the development of the capacity assessed by LR-TI-CARD. That task requires the child to reason transitively about non-equivalent relations between lengths on the basis of the number of unit parts contained in each object's length. It is noteworthy that this capacity implicates numerical forms of reasoning about length relations, and that it emerges at about the same time as the capacity to reason transitively about relations of non-equivalence concerning number (assessed by N-TI-NE). Indeed, the capacity assessed by N-TI-NE is implied in the capacity assessed by LR-TI-CARD. This suggests that further enhancement of the length concept by numerical forms of reasoning has occurred.

Following the emergence of the capacity assessed by LR-TI-CARD, there is another delay before the capacity assessed by LR-TI-NE emerges. The latter does not implicate numerical forms of reasoning about length. However, it
may be that there is a developmental dependency between LR-TI-CARD and LR-TI-NE, in that it is the possession of a capacity to reason numerically about length (LR-TI-CARD) which provides the 'proof' of the inference required in LR-TI-NE. Once that has been established the child no longer need depend upon numerical representations of length in order to make transitive inferences concerning non-equivalent length relations.

In more general terms, these speculations about the developmental discontinuities in each concept domain, and the interconnection of those domains, imply that an advance in one concept domain prompts development in another.

11.7 SUMMARY.

The findings of the study suggest that the necessary components of linear measurement are the following:-

- Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
- Knowing that length is invariant under certain transformations (the conservation of length).
- Knowing how to make transitive inferences of equivalence and non-equivalence, with respect to discrete quantity.
- Knowing how to make transitive inferences of equivalence, with respect to length.
- Know how to carry out numerical addition operations.
- Knowing how to obtain a linear measurement by counting iterations of a unit of length.
The data also imply a delay between acquisition of these components and the emergence of an understanding of linear measurement. Piagetian theory would suggest that the delay is associated with a re-organisation of cognitive structures that results in better co-ordination between the components.

Inspection of the development sequence in each concept domain reveals that each is characterised by discontinuities. These discontinuities coincide with the emergence of components of major theoretical interest, such as the conservation of length. Additionally, there are concordances between the discontinuities in the number and length domains. Examination of these concordances suggests that the discontinuities occur during periods of development when new forms of co-ordination are being established between the number and length concepts.

Some elements of the observed sequences of development in the number and length concept domains are not predicted by Piagetian theory. In particular, the asynchronies between conservation and transitive inferences of non-equivalence are not consistent with the Piagetian view, though they are consistent with Gagne's (1968) model of development.
AN INFORMATION-PROCESSING ANALYSIS OF CERTAIN NUMBER AND LENGTH TASKS, USING PASCUAL-LEONE'S M-SPACE MODEL.

12.1 INTRODUCTION.

It was seen in Chapter 11 that the number tasks could be organised into five levels of difficulty, and the length tasks into four levels of difficulty. It was argued that tasks at a similar level of difficulty should make similar information-processing demands. Hence, it was thought that an information-processing analysis of the tasks which fall on the boundaries of the levels might reveal the reasons for the sharp changes in task difficulty that occur between levels. That analysis is given in this Chapter.

The information-processing model used in the analysis was developed by Pascual-Leone (1970). Firstly, his model is described. Secondly, the application of his model to the present study is discussed.
12.2 PASCUAL-LEONE'S M-SPACE MODEL.

12.2.1 NATURE OF THE MODEL.

Pascual-Leone (1970) constructed a functional or process model of development, complementary to Piaget's structural model. The model predicts performance on a range of Piagetian and other cognitive tasks, given prior estimates of the values of two structural variables - namely, "M-space" and "field independence/dependence."

An information-processing approach has been adopted by Pascual-Leone, and his main collaborator, Case (1972). Unlike some other information-processing theorists, they don't write computer programs. Their level of analysis is that of "scheme". Their use of scheme is the same as that of Piaget. Pascual-Leone identified three categories of scheme: "figurative", "operative", and "executive".

12.2.2 FIGURATIVE SCHEMES.

Figurative schemes are the internal representations of "declarative-type knowledge" (e.g. properties of objects or relations between objects). They are proposed as active, functional units, akin to Neisser's (1967) pattern recognition devices. Case (1974) gave the following example of a figurative scheme:-
"If, for example, a subject looked at a photograph and asserted that it was a picture of his house, one would say that he did so by transforming the raw sensory input into a network of perceptual features which were readily associated in his mind with a conceptual response of the order, 'that is my house.' More simply, one would say that he assimilated the sensory input to his (figurative) 'house scheme'."

12.2.3 OPERATIVE SCHEMES.

Operative schemes are the internal representations of "procedural-type knowledge" (e.g. rules applied to properties of objects, or relations between objects). Both figurative schemes, and operative schemes are assumed to be active processes. Hence, the internal distinction between these two categories of scheme is blurred, they are distinguished by what they are used for. Operative schemes act on figurative schemes to generate new figurative schemes, but figurative schemes do not act on other figurative schemes.

12.2.4 EXECUTIVE SCHEME.

Executive schemes are proposed as the internal representations of the lists of rules and procedures to be assembled, sequenced and actioned in order to reach some desired goal. They represent strategies to be employed in solving a particular class of problem. They are also proposed as active processes, but they differ from operative schemes insofar as they don't directly act on figurative schemes to generate new figurative schemes. Their function is to direct and control solution processes by deciding upon and activating operative scheme sequences.
ne argues that, in the course of problem solving, a person's thought is constituted by the assembly of schemes that are currently activated. It follows, therefore, that for Pascual-Leone a principal limitation on thought processes is the number of discrete schemes that may be activated and co-ordinated at any given time. He refers to this limitation as "M-space", which he defines as the set measure of Piaget's field of centration. Using the scheme construct as the fundamental unit of analysis, Pascual-Leone produced descriptions of various Piagetian tasks in terms of schemes invoked and co-ordinated in the subject's M-space. The following two examples may help to illustrate the approach:—

"Conservation of identity. The age at which this task is first passed is 5-6 years. In solving it, children appear to activate the following schemes:
E(IS): An executive scheme representing the instructions ("Does the ball still have the same amount of clay in it?") and directing an appropriate perceptual scan of the ball as it is transformed;
F(1): A figurative scheme representing the fact that "nothing has been added or taken away";
F(2): A figurative scheme representing the rule a "if nothing is added or taken away, then the amount remains the same";
If children do not co-ordinate the above schemes, they fail the task, apparently because they activate another scheme already present in their repertoires, which is misleading in the conservation situation.

Call it

$F(M)$: A figurative scheme representing the rule that "things which look bigger, contain more".

Conservation of equivalence. The age at which this task is first passed is 7-8 years. In solving it, children appear to activate the following schemes:

$E(IS)$: An executive scheme representing the instructions ('Do the balls still have the same amount of clay in them?') and directing an appropriate perceptual scan of the ball as it is transformed;

$F(1)$: A figurative scheme representing the fact that 'nothing has been added to or taken away from the ball which was transformed;

$F(2)$: A figurative scheme representing the rule that if nothing is added or taken away, then the amount remains the same;

$F(3)$: A figurative scheme representing the information that "the balls originally were equal in amount;"
If children do not co-ordinate the above schemes, they usually fail the task, apparently because they activate:

F(M): A figurative scheme representing the rule that "things which look bigger contain more."

(Case, 1972; pp340-341).

(Note: To be consistent with Pascual-Leone's own classification of schemes as figurative, operative and executive, schemes F(2) and F(M), in each of the above examples, should have been classified as operative.)

12.3 DEVELOPMENTAL PROGRESSIONS.

These two examples illustrate an important feature of Pascual-Leone's model which is that the number of schemes children co-ordinate in approaching a task is related to the age at which they first succeed at the task. Pascual-Leone argued that a different value of M (for M-space) is associated with each substage of intellectual development. The values he proposed are as follows:-
<table>
<thead>
<tr>
<th>DEVELOPMENTAL STAGE</th>
<th>AGE (yrs)</th>
<th>MAXIMUM VALUE OF M (a + k)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Early Pre-operational</td>
<td>3-4</td>
<td>a + 1</td>
</tr>
<tr>
<td>Late Pre-operational</td>
<td>5-6</td>
<td>a + 2</td>
</tr>
<tr>
<td>Early Concrete</td>
<td>7-8</td>
<td>a + 3</td>
</tr>
<tr>
<td>Late Concrete</td>
<td>9-10</td>
<td>a + 4</td>
</tr>
<tr>
<td>Early Formal</td>
<td>11-12</td>
<td>a + 5</td>
</tr>
<tr>
<td>Middle Formal</td>
<td>13-14</td>
<td>a + 6</td>
</tr>
<tr>
<td>Late Formal</td>
<td>15-16</td>
<td>a + 7</td>
</tr>
</tbody>
</table>

(Note: The constant (a) refers to the space required by the executive scheme. The numeral represents the maximum number (k) of additional schemes which can be co-ordinated.)

According to Pascual-Leone, the M-space model provides a functional explanation of developmental progressions. For example, children usually fail conservation of amount until the age of 7-8 years, because their M-space, until that age, can only co-ordinate the concurrent activation of a + 2 schemes. However, the conservation task requires, under normal conditions, the concurrent activation of (a + 3) schemes.

12.4 INDIVIDUAL DIFFERENCES.

Whilst Pascual-Leone sees M-space as a structural limitation on performance, his model also makes provision for other variables. Specifically, he argued that the following conditions must be met before a task can be successfully handled:-
(a) the child must possess the necessary schemes;
(b) if necessary, the full capacity of the available M-space must be used;
(c) the child must attend to other than only the perceptually dominant
and potentially misleading cues;
(d) if two incompatible schemes are activated by the perceptual features
of the problem, the child must resolve the conflict in favour of that
scheme which is compatible with the greatest number of other associated schemes.

The first of these conditions can be satisfied only by learning. The
third is related to the perceptual/cognitive style variable known as field
independence/dependence (Witkin, 1959). The fourth is identified with a
component of Piaget's equilibration model. It reflects the operation of
an individual differences variable, and is assumed to be highly correlated
with the third factor. Hence, Pascual-Leone's model posits that perform­
ance is determined by M-space and learning, and is moderated by field
independence/dependence.

12.4.1 LEARNING.

Learning is defined as the acquisition of new schemes. This is accompli­
shed in two ways:-

(a) by incorporation of new information into old schemes, in a manner
analogous to Piaget's differentiation; and
(b) by combining formerly discrete schemes into a new compound or
superordinate scheme, in a manner analogous to Piaget's reciprocal
assimilation.

Both processes have the effect of increasing performance, because they
lead to more efficient utilisation of M-space.
12.4.2 FIELD-INDEPENDENCE/DEPENDENCE.

The field-independence/dependence factor is said to explain much of the variance attributable to individual differences. The field-independent subject is less likely to focus on perceptual cues inherent in the task situation, more likely to attend to the task instructions, and will tend to utilize available M-space fully. The field-dependent subject who is faced with two incompatible schemes, one activated by perceptual cues and the other by task instructions, is more likely to resolve the conflict by acting on the former than the latter. He is also less likely to make full use of available M-space.

12.5 EMPIRICAL EVIDENCE FOR THE M-SPACE MODEL.

12.5.1 EARLY STUDIES.

Pascual-Leone (1970) tested the model by teaching children a series of novel responses to particular visual stimuli. He then measured their capacity to activate these "S-R" connections concurrently when confronted with a compound visual stimulus. The children's capacity to produce the appropriate compound response was taken as a measure of their M-space. It was found that there was a high correlation between that measure, and the M-space factor inferred from an analysis of Piagetian tasks previously passed by the children. The findings were interpreted as a demonstration that the M-space model had construct and predictive validity. Pascual-Leone and Smith (1969) reported another investigation of children's classification concepts, and presented findings consistent with the M-space model.
12.5.2 METHODOLOGICAL CRITICISMS.

Pascual-Leone's (1970) experimental methods, and techniques of model evaluation, have been severely criticised by Trabasso and Pollinger (1978) and Trabasso (1978) on two main grounds. Firstly, in Pascual-Leone's (1970) study only some of the children were trained and tested on all the S-R associations; the number of associations used increased with increasing group mean age; the S-R associations used varied in terms of inherent difficulty; and the subject's stage of development (eg. pre-operational) was inferred from the subject's age and not directly assessed. Secondly, Pascual-Leone did not use statistical goodness-of-fit tests to estimate the predictive accuracy of his model, relying instead on visual inspection of averaged probability distributions. Also, Pascual-Leone did not compare his model's predictive accuracy against that achieved by a variety of other stochastic (eg. Monte Carlo) models. Pascual-Leone (1978) defended his approach on the grounds that he was concerned with testing a "general" not a "local" model, and that his critics' objections were appropriate only in the context of verifying empirically local models of limited scope.

12.5.3 LATER STUDIES.

The studies by Case (1972, 1972a) appear to have overcome most of the methodological objections raised against Pascual-Leone's experimental work, and have yielded findings consistent with the M-space model. In the 1972 study, a more carefully designed version of the Pascual-Leone and Smith (1969) compound stimulus task was employed. Dale's (1975) large-scale investigation of performance on Piaget's bending rods task also produced findings consistent with the M-space model.
Pascual-Leone's approach has attracted the attention of educational psychologists, because it offers a bridging construct between developmental theory (i.e. Piaget's) and human learning theory (i.e. Gagne's).

"There are clear parallels between Gagne's model and Pascual-Leone's. Gagne's model interprets cognitive problems as requiring the application of certain rules. Pascual-Leone's model interprets these same problems as requiring the co-ordination of certain schemes (some of which are merely the internal representation of such rules)........Both theorists agree that children will not be able to solve cognitive problems if they do not have the appropriate internalized items of information in their repertoires.

Both theorists agree that children can often be enabled to solve such problems if they are helped to acquire the appropriate repertoires, i.e. if they are instructed. The difference lies in the role assigned to development. For Gagne, the process of development is largely one of cumulative learning, within the confines of whatever (unspecified) limitations may be imposed by 'growth'.......For Pascual-Leone, the process of development is equally one of cumulative learning. However, one of the major limitations imposed by 'growth' is explicitly defined. It is a limitation in mental space." (Case, 1972, p356).
12.6 M-SPACE ANALYSIS OF CERTAIN NUMBER TASKS.

12.6.1 SELECTION OF NUMBER TASKS.

Tasks from each level of the performance gradient for the number collection were selected for analysis. The tasks chosen were those of least and most difficulty, within a level. Hence, the most difficult task from level \( n \), was compared with the least difficult task from level \( n+1 \). Tasks could have been randomly sampled from each level. However, the sampling method used was more conservative because it selected pairs of tasks adjacent in rank order of difficulty, and adjacent in ungrouped order of difficulty. The tasks selected from the number collection (see Table 11.2 in Chapter 11) are shown in Table 12.1.

**TABLE 12.1: NUMBER TASKS SELECTED FOR M-SPACE ANALYSIS.**

<table>
<thead>
<tr>
<th>LEVEL ( N )</th>
<th>MOST DIFFICULT TASK AT LEVEL ( N )</th>
<th>LEVEL ( N+1 )</th>
<th>LEAST DIFFICULT TASK AT LEVEL ( N+1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>N-TI-EQ</td>
<td>2</td>
<td>N-(1 TO 1)</td>
</tr>
<tr>
<td>2</td>
<td>N-CONS</td>
<td>3</td>
<td>N-SOL-V</td>
</tr>
<tr>
<td>3</td>
<td>N-ADD-NV</td>
<td>4</td>
<td>N-TI-NE</td>
</tr>
<tr>
<td>4</td>
<td>N-TI-NE*</td>
<td>5</td>
<td>N-SUB-NV</td>
</tr>
</tbody>
</table>

Note: * - N-TI-NE is the only task at level 4.
The selected tasks were analyzed into co-activated executive, figurative, and operative schemes, in accordance with Pascual-Leone's model. That analysis is set out below. Entries for executive schemes are not given, because all tasks must activate an executive scheme representing the task instructions, and the solution strategy used. To show an executive scheme entry against all tasks would be redundant. Figurative schemes are prefixed by an 'F', and operative schemes by an 'O'.

12.6.2 SPECIFICATION OF THE
CO-ACTIVATED SCHEMES FOR THE
SELECTED NUMBER TASKS.

N-TI-EQ.

F1: A figurative scheme representing the fact that there are as many blue blocks as there are red blocks.

F2: A figurative scheme representing the fact that there are as many red blocks as there are green blocks.

O1: An operative scheme representing the canonical form of the transitive law: if (a.R.b) and (b.R.c), then (a.R.c).

N-(1-TO-1)

F3: Every bolt had a nut, or every bolt had a washer, or every nut had a washer, depending on the question being considered.
F4: Every nut had a bolt, or every washer had a bolt, or every washer had a nut, depending on the question being considered.

02: If every "a" had a "b", and every "b" had an "a", then the number of a's equals the numbers of b's; where a and b refer to bolt and nut, or bolt and washer, or nut and washer, depending on the question being considered.

N-CONS.
ON Phase 1:

F5: Nothing has been added or taken away from the line of blocks which was transformed.

03: If nothing has been added or taken away from the line of blocks, then it contains the same number of blocks that it did before transformation.

On Phase 2.

F6: The line of blocks which was transformed contains the same number of blocks as it did before the transformation occurred.

F7: The two lines of blocks originally contained the same number of blocks.

04: The numerical relation between two collections of objects is invariant unless the numerosity of one collection is changed.
On Phase 1.

F8: The experimenter's collection contains $n$ blocks.

F9: The subject's collection contains $m$ blocks.

05: If two collections have a different number of blocks, then the collection with more blocks is the one whose number name appears later in the number name order.

The application of 05 to F8 and F9 yields F10:

On Phase 2.

F10: The experimenter's collection has more blocks.

06: If one collection has more blocks than another, then how many more can be found by subtraction.

The application of 06 to F10 initiates a subtraction process carried out on F8 and F9 and resulting in F11.

On Phase 3.

F11: The experimenter's collection has $(n-m)$ more blocks than the subject's.

07: If one collection has more blocks than the other, then they can be made equal in number by taking the difference $(n-m)$ away from the collection with more.

Notice that the same phases would be involved in solving the problem by adding to the collection with fewer blocks.
The subtraction process would be common to both, and would involve the co-activation of F8 and F9, together with a subtraction rule, such as:

08: To subtract m from n, start counting at one more than m, stop at n, and the number of number names mentioned is the answer.

\[ N-ADD-NV. \]

F12: The bottom jar contained \( \text{`m'} \) balls before more balls were sent down the tube.

F13: \( \text{`m'} \) balls were sent down the tube.

09: If more objects are added to a collection of similar objects, the number of objects then in the collection is given by an addition operation.

\[ \text{The addition operation (call it, OLO) would apply to F12 and F13. It may be based on a counting method - using fingers, for example, to represent the balls - or on a `table-look-up' method.} \]

\[ N-TI-NE. \]

As for N-TI-EQ, except that the relations `greater-than' and `less-than', are substituted for `equal-to'.
N-SUB-NV.

FL4: The top jar contained "n" balls before more balls were sent down the tube.

FL5: "m" balls were sent down the tube.

011: If some objects are taken away from a collection of similar objects, then the number of objects left in the collection is given by a subtraction operation.

-- The subtraction operation (call it, 012) would apply to FL4 and FL5. It may be based on a counting method - using fingers, for example, to represent the balls - or on a "table-look-up" method. Notice that 012 may be different from 08, as in N-SOL-V the objects were visible to the subject throughout the task.

12.6.3 NUMBER OF CO-ACTIVATED SCHEMES REQUIRED FOR THE SELECTED NUMBER TASKS.

For all of the tasks listed above, the maximum number of co-activated schemes needed at any stage of the solution process is three; two of which are figurative, and one, operative. Hence, in terms of Pascual-Leone's model, all tasks are of the type that can be solved by children at the early concrete sub-stage of development. Therefore, an M-space analysis of the number tasks does not reveal any structural limitation corresponding to the steps in the observed performance gradient.
12.7 M-SPACE ANALYSIS OF CERTAIN LENGTH TASKS.

12.7.1 SELECTION OF LENGTH TASKS.

Tasks were selected for analysis from each of the levels in the performance gradient for the length collection. The basis for selection was the same as that described earlier for the number collection. The tasks selected from the length collection (see Table 11.4 in Chapter 11) are shown in Table 12.2.

**Table 12.2: Length Tasks Selected for M-Space Analysis.**

<table>
<thead>
<tr>
<th>Level N</th>
<th>Most Difficult Task at Level N</th>
<th>Level N+1</th>
<th>Least Difficult Task at Level N+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>L-CONS</td>
<td>2</td>
<td>L-EST*</td>
</tr>
<tr>
<td>2</td>
<td>LR-TI-CARD</td>
<td>3</td>
<td>LR-TI-NE</td>
</tr>
<tr>
<td>3</td>
<td>LR-TI-NE*</td>
<td>4</td>
<td>LR-M-CARD</td>
</tr>
</tbody>
</table>

Note: * - Instead of giving an analysis of L-EST as the least difficult task at level 2, an analysis is given of L-UNIT. This is because L-EST and L-UNIT are of, essentially, the same difficulty, but the latter is of greater theoretical importance and, hence, of more interest.

** - LR-TI-NE is the only task at level 3.

The tasks listed in Table 12.2 were analysed in accordance with the M-space model. That analysis is set out below. Executive schemes are not shown, because it is assumed that there is only one executive scheme activated for each task.
12.7.2 SPECIFICATION OF THE
CO-ACTIVATED SCHEMES FOR THE
SELECTED LENGTH TASKS.

L-CONS

On Phase 1.

F16: Nothing has been added or taken away from the piece of string which was transformed.

O13: If nothing has been added or taken away from the piece of string, then it is the length it was before the transformation.

--- The application of O13 to F16 yields F17:

On Phase 2.

F17: The piece of string which was transformed is the length it was before the transformation.

F18: The two pieces of string were originally the same length.

O14: The length relation between objects is invariant unless the length of one object is changed.

L-UNIT

This task may be divided into three processes. Process 1 is responsible for marking off equal units. Process 2 is responsible for counting the
units. Each cycle of process 1 is followed by a cycle of process 2. Process 3 is responsible for producing the answer. It is initiated after the last cycle of Process 2 has been executed.

Process 1.

F19: The location of the right hand border (assume left-to-right movement) of the previous segment marked off is known.

F20: The left hand border of the unit coincides with the right hand border of the previous segment marked off.

C15: If the unit length equals the previous segment length, and the unit length equals the current segment length, then the previous segment length equals the current segment length.

Process 2.

F21: Number name mentioned when the previous segment length was marked off.

F22: List of number names.

C16: When the current segment length has been marked off, mention the next number name on the number name list.

Process 3.

F23: The whole object has been divided into segments of equal length.

F24: Last number name mentioned.

C17: The number of equal segments represents the length of the whole object.
LR-TI-CARD

On Phase 1.

F25: There are more red blocks than green.
F26: There are more blue blocks than red.
O1: The canonical form of the transitive law with respect to number.

The application of O1 to F25 and F26 yields F27:

On Phase 2

F27: There are more blue blocks than red.
F28: All blocks are the same size.
O18: A is longer than B, if A contains more parts than B, and if the parts are the same length.

LR-TI-NE.

As for Phase 1 of LR-TI-CARD, except that the figurative and operative schemes refer to length, not number.

LR-M-CARD.

There are two parts to this task. The first part is responsible for unit iteration. The second part is responsible for comparison of object lengths, and production of the answer.
The first part is exactly the same as L-UNIT, described above. The second part uses the same operative scheme (018) as that employed on phase 2 of LR-TI-CARD. Hence, the schemes co-activated on the second part of "LR-M-CARD are:—

F29: Length A contains \( n \) unit parts.
F30: Length B contains \( m \) unit parts.
018: A is longer than B, if A contains more parts than B, and if the parts are the same length.

12.7.3 NUMBER OF CO-ACTIVATED SCHEMES REQUIRED FOR THE SELECTED LENGTH TASKS.

Inspection of the above analysis reveals that the maximum number of co-activated schemes needed at any stage of the solution process is three: two figurative and one operative. Hence, the analysis does not reveal any structural limitation corresponding to the steps in the observed performance gradient. This conclusion is the same as that reached with respect to the number tasks.

12.8 SUMMARY.

An M-space analysis of the steps in the number and length task performance gradients does not support the suggestion that those steps represent developmental discontinuities which stem from structural limitations, such as mental processing capacity. However, the M-space analysis does not explicitly provide for differences in complexity of control processes,
because all directive information is assumed to be stored in a single executive scheme. Because it would seem likely that there are differences in complexity between control processes associated with these tasks and, hence, differences between the demands these processes place on STM, this assumption of a unitary executive structure limits the value of the analysis.

Moreover, in Pascual-Leone's model the level of analysis of component processes and knowledge elements into operative and figurative schemes is somewhat arbitrary. In the present case, an attempt was made to conform with the examples of M-space analysis given by Pascual-Leone. However, there is no certainty that a different analyst would derive the same list of co-activated schemes. Hence, this apparently arbitrary aspect of Pascual-Leone's model also limits the value of the analysis.

If these two criticisms of Pascual-Leone's model are set aside, a conclusion which could be drawn from this analysis is that the stepped performance gradients for the number and length tasks reflect, simply, delays inherent in the accretion of a large number of rules. This conclusion would be consistent with the finding of the present study that length of schooling is a predictor of performance, whilst age is not.
The analysis of linear measurement given in Chapter 2 drew upon the actions involved in linear measurement operations, and the logical structure of linear measurement. The empirical work reported here has broadly supported this analysis. However, as a psychological theory it has two weaknesses.

The first is that the analysis was intuitive. It was not formally demonstrated that the listed components are sufficient to enable a person to substitute measurement operations for actual operations on objects.

The second is that the analysis did not concern itself with the psychological processes at work in linear measurement.

The only psychologically-orientated theory that considers linear measurement is Piaget's. That theory offered general guidance to the analysis in Chapter 2, but its usefulness was limited, because it is only concerned with describing the gross psychological structures needed for linear measurement. The Piagetian analysis is not made at a sufficient level of detail for the present purposes.
A more satisfactory account of linear measurement would give a detailed process-analysis of what is involved in each component, and of precisely how the components are co-ordinated.

The objectives of that analysis would be, firstly, to express linear measurement in terms of a minimum number of psychologically primitive operations and, secondly, to show that the account does, indeed, generate linear measurement. An account of that kind could be provided by constructing a “production-system” model of linear measurement.

Production systems are collections of condition-action rules, called productions. The rules are expressed in the form: “if conditions A(1), and A(2), and A(3), …… and A(n) hold, then take actions B(1), and B(2), and B(3), …… and B(n)” If the rules, or productions, are written in a computer language, they may be “executed” (Newell and Simon, 1972).

Klahr and Wallace’s (1976) work provides an example of how the production-system approach can be used to construct a detailed account of aspects of cognition related to the present study. Their objective was to develop a theory of cognitive development at a level of precision that would enable the theory to be expressed as an executable computer model.

Klahr and Wallace’s (1976) strategy was to construct, firstly, state models. Each model depicted the performance of a child at a particular stage of development on a range of Piagetian tasks. Secondly, they constructed transitional models accounting for changes between stages (i.e. between state models). Their state models were expressed as executable production systems.
Klahr and Wallace (1976) present models of various quantification processes (subitization, counting, estimation, relative magnitude determination), class inclusion, conservation of quantity, and transitive reasoning. Their models are constructed so that groups of productions used in less complex tasks, such as counting, may be used in more complex tasks, such as addition. This reflects the view that development stems from the accretion of experience, coupled with periodic re-organisation of the internal representation of that experience.

An approach similar to Klahr and Wallace’s could be taken to the problem of providing a satisfactory account of the composition and growth of linear measurement. That approach would make explicit the theory implicit in the list of components of linear measurement given in Chapter 2. A full production-system analysis of linear measurement will not be attempted here (2). Nevertheless, in order to illustrate the possibilities offered by this approach, this Chapter presents an analysis of three of the tasks used in the present study. The core productions concerning counting, subitization, addition, control processes, and so on, given in these models, are all directly relevant to the longer-term objective of providing a production-system model of mature linear measurement.

2. The original intention of the present research project was to develop such an analysis. However, the lack of an appropriate and detailed theoretical framework, and the paucity of directly relevant empirical data, necessitated that, first, the empirical work reported here be undertaken. The position has now been reached where a production-system model of mature linear measurement could be developed. Development of such a
model of linear measurement would, itself, be a substantial undertaking, and is beyond the scope of the present research. The production-system model would provide a sufficient account of the data reported in the present study, and would constitute a formal theory of what is involved in linear measurement. A speculative outline of a possible approach to the development of such a model is given in Appendix 6.

Additionally, the three tasks selected for analysis were chosen because they have the potential to yield further information on the question examined in Chapter 12. It was suggested there that the discontinuities in the observed performance gradients could be explained by changes in STM capacity. An analysis of selected tasks, using Pascual-Leone's model, failed to support this suggestion. However, it was argued in the concluding paragraphs of Chapter 12 that the assumption by the Pascual-Leone model of a unitary control structure had a significant drawback. It implied that variations in the complexity of control information have no effect on STM load. Production-system modelling does make explicit provision for representing control information in STM. Hence, a production-system analysis of the selected tasks should complement the analysis given in Chapter 12 of the information-processing demands of tasks drawn from different regions of the performance gradient, and could yield different conclusions.

Before describing the task models it will be necessary to amplify the descriptions given above of production-system modelling. Section 13.2 provides a brief overview of a production-system language.
In addition to Klahr and Wallace (1976), Anzai and Simon (1979), Baylor and Gascon (1974), Newell and Simon (1972), and O’Shea and Young (1978) provide examples of production-system models of aspects of cognition relevant to the present topic. Hunt and Poltrock (1974), Klahr and Wallace (1976), Newell and Simon (1972) and Winston (1979) provide well documented introductions to the technique. The following passage from Klahr and Wallace (1976) introduces a production-system language, and its operating rules.

"The models are posed in the form of a collection of ordered condition-action links, called productions, that together form a production system. The condition side of a production refers to the symbols in short-term memory (STM) that represent goals and knowledge elements existing in the system's momentary knowledge state; the action consists of transformations on STM including the generation, interruption, and satisfaction of goals, modification of existing elements, and addition of new ones. A production system obeys simple operating rules:-

1. The condition of each production is matched against the symbols in STM. If all of the elements in a condition can be matched with elements (in any order) in STM, then the condition is satisfied.
2. If no conditions are satisfied, then the system halts. If more than one condition is satisfied, then some conflict resolution principle must select which production to "fire." Typically, conflict is resolved by choosing the earliest production in the production system. Other resolutions are possible, but that is the one we will use at first.

3. When a production "fires," the actions associated with it are taken. Actions can change the state of goals, replace elements, apply operators, or add elements to STM.

4. After a production has fired, the production system is re-entered from the top; that is, the first production's condition is tested, then the second, and so on.

5. The STM is a stack in which new elements appear at the top (or front), pushing all else in the stack down one position. Since STM is limited in size, elements may be lost.

6. When a condition is satisfied, all those STM elements that were matched are moved to the front of STM. This provides a form of automatic rehearsal."

(Klahr and Wallace, 1976, pp 6-7)
The task models constructed in the present study contain productions that make use of a small number of actions. In every instance, these actions cause the contents of STM to change. The actions used are the following:

- **INS(X)** - Inserts the expression X into STM.
- **DEL(X)** - Deletes the expression X from STM.
- **REPL(X;Y)** - Replaces the expression X in STM with the expression Y.
- **SAY(X)** - Prints the expression X. This provides an interface with the user.
- **USER()** - Asks the user if he has any information. The user's response is stored in STM.
- **DO(X)** - Transfers control to the list of productions labelled X, but only for a single cycle.

(Ohlsson, 1980)

Typically, the expression (X) would be a representation of information provided by the task, or information retrieved from LTM, or information needed to control the solution process.

13.3 **TASKS SELECTED FOR MODELLING.**

The three tasks selected for modelling were: N-ADD-NV; N-SUB-NV; and N-CYC-NV. The three tasks are closely related. Although 58 subjects passed N-ADD-NV, only 21 passed N-SUB-NV, and only 16 passed N-CYC-NV. The proportion of subjects who passed N-ADD-NV was significantly higher than the proportions who passed N-SUB-NV and N-CYC-NV. However, N-SUB-NV,
and N-CYC-NV were of about the same difficulty. It will be apparent that
N-CYC-NV requires a co-ordination between two major components: namely,
those assessed by N-ADD-NV and N-SUB-NV. The difficulty data suggest,
therefore, that the delay between acquisition of the N-ADD-NV and
N-CYC-NV components is due to a delay in acquisition of the N-SUB-NV
component, and not to a delay in co-ordinating the N-ADD-NV and N-SUB-NV
components. It was thought that a comparison of models for these three
tasks would provide an appropriate example of the application of the
technique, and it would enable a re-examination of the role of STM
capacity limitations.

A factor that influenced the selection of these tasks for modelling was
the existence of a substantial empirical literature on the strategies
used by young children when subitizing, counting, adding and subtracting.
Most of that literature has been reviewed recently by Klahr and Wallace
(1976). Their conclusions are reflected in the subitization and count­
ing productions found in several of their models.

A broad outline of each of the models developed will be given. This will
be followed by annotated listings for the addition and subtraction models
which use a counting strategy. Finally, performance statistics for the
six models will be summarised, and conclusions drawn regarding the inform­
ation-processing demands of the three tasks.
Alternative models were developed for two of the tasks. These were designed to simulate various strategies children had been observed to use. Six models were developed. Three were of N-ADD-NV, two of N-SUB-NV, and one of N-CYC-NV. Listings of all models, and traces of their execution showing the contents of STM on every cycle, are given in Appendix 4. Descriptions of all models are also given in Appendix 4. Additionally, to illustrate the technique, listings and detailed descriptions of two of the models will be given in this Chapter. They are the counting models for addition (simulating performance on N-ADD-NV) and subtraction (simulating performance on N-SUB-NV).

13.4.1 ADDITION MODELS.

Three models constructed to simulate performance on the N-ADD-NV task are provided in Appendix 4. Each carries out the addition operation in a different manner.

The first model (ADD6.PSS) is described in Appendix 4, and listed in full in Addendum 1. It is based on a simple table-look-up procedure. The number of balls in the bottom jar is used as a key for accessing the appropriate entry. For example, if the bottom jar has two balls in it, and the subject sends four more down the tube, the model uses 'two' to access the (2+4=6) entry.
The second model (ADD2.PSS) will be described in detail in Section 13.5 (a full listing and execution trace are provided at Appendix 4, Addendum 2). It is based on a counting method. It simulates performance by a subject who carries out addition by co-ordinating two counting operations. Assume that memory contains an ordered list of number names ("1", "2", "3", ...). Subjects count by placing names from this list into STM.

Two counts are maintained. Count A is a measure of the number of balls counted to date in the bottom jar. Count B is a measure of the increment in count A that has been made at a given point in the counting process. The subject's procedure is:

Step 1 - Set count A to the number of balls initially in the bottom jar.

Step 2 - Set count B to zero.

Step 3 - Compare number in count B to number of balls sent down the tube. If equal, then the answer is given by the number in count A—FINISH. Otherwise, continue to Step 4.

Step 4 - Move count A forward one.

Step 5 - Move count B forward one.

Step 6 - Go back to Step 3.

The third model (ADD3.PSS) is described in Appendix 4, and listed in full in Addendum 3. It is also based on a counting method. The first step in this method, however, is finding out which of the two addends is the larger. This becomes the initial value of count A. This method reduces the number of iterations of Steps 3 to 6, above. However, it also incurs the overhead involved in first finding the larger addend.
13.4.2 SUBTRACTION MODELS.

For the N-SUB-NV task, the first model (ADD5.PSS) is based on a table-look-up procedure similar to that used for addition. The access key in this case is the number of balls in the top jar. The model is described in Appendix 4 and listed in full in Addendum 4.

The second method (ADD4.PSS) will be described in detail in Section 13.5 (a full listing and execution trace are provided at Appendix 4, Addendum 5). It is based on the counting method illustrated schematically below:

Step 1 -- Set count A to the number of balls initially in the top jar.

Step 2 -- Set count B to zero.

Step 3 -- Compare number in count B to number of balls sent down the tube. If equal, then the answer is given by the number in count A→FINISH.

Otherwise, continue to Step 4.

Step 4 -- Move count A backward one.

Step 5 -- Move count B forward one.

Step 6 -- Go back to Step 3.

13.4.3 ADDITION AND SUBTRACTION MODEL.

For the N-CYC-NV task, the model (ADD7.PSS) is based on the table-look-up procedures used in the addition and subtraction models. It is described in Appendix 4, and listed in full in Addendum 6.
13.5 ANNOTATED LISTINGS OF THE
COUNTING-BASED ADDITION
AND SUBTRACTION MODELS.

Listings of the counting-based addition (ADD8.PSS) and subtraction
(ADD4.PSS) models and traces of their execution are given in Appendix 4,
Addenda 2 and 5, respectively. Sections 13.5.1 and 13.5.2 present annotated
listings of the productions constituting these models.

In many production-system models goal manipulation procedures are group-
ed together in one or two common-servicing productions that are always
tested at the beginning of every cycle. In the present models, they have
been located separately in productions which trigger particular goal act-
ivation, re-activation, suspension and deletion operations. This approach
makes the production systems easier to follow, and has been adopted to
assist the reader who is not familiar with production-system languages.
It reduces the programming elegance of the models, but it does not result
in any greater demands being placed on STM, and does not increase the
total number of productions fired.

The models are written in PSS (Ohlsson, 1980), a variant of PSG.
13.5.1 THE COUNTING-BASED MODEL
OF N-ADD-NV.

In the first phase, the subject is asked to send \( n \) balls to the bottom jar by pressing the button. After the balls have gone down the tube, the subject is asked how many were in the bottom jar before, how many more had he just sent down, and how many were now in the bottom jar.

The productions PO00, P00 and P0 model the entry of task information into STM, and initiate the run.

\[
(P000 \ (GOAL \ * \ ATTEND) \ (OLD \ SEND \ C)) \\
\rightarrow \\
REPL((GOAL \ * \ ATTEND) ; (GOAL + ATTEND)); \\
DEL((OLD \ SEND \ C)); \\
USER() \\
\]

\[
(P00 \ (GOAL \ * \ ATTEND) \ (GO)) \\
\rightarrow \\
REPL((GOAL \ * \ ATTEND) ; (GOAL + ATTEND)); \\
DEL((GO)) \\
\]

\[
(P0 \ (GOAL \ * \ ATTEND)) \\
\rightarrow \\
USER() \\
\]
When that information is entered, the model responds by simulating the subjects button-pressing and counting behaviour. The productions responsible are labelled P1 to P8.

(P1 (GOAL + ATTEND) (TOP A) (BOTTOM B) (SEND C) 

==>
REPL((GOAL + ATTEND) ; (GOAL *S* ATTEND));
INS((GOAL * SEND C));
INS((Y 1 Z)) )

(P2 (GOAL * SEND C)

==>
REPL((GOAL * SEND C) ; (GOAL *S* SEND C));
INS((GOAL * PUSH BUTTON)) )

(P3 (GOAL * PUSH BUTTON)

==>
DEL((GOAL * PUSH BUTTON));
INS((ELM A));
INS((GOAL * SUBIT)) )

(P4 (GOAL * SUBIT) (ELM A)

==>
DEL((GOAL * SUBIT));
INS((QS 1));
DEL((ELM A));
DEL((ELM A));
INS((GOAL * COUNT)) )
(P5 (GOAL * COUNT) (QS 1) (Y <X> Z))

==> REPL(((GOAL * COUNT) ; (GOAL + COUNT)));
DEL((QS 1));
SAY(<X>);
REPL((Y <X> Z) ; (SAID <X>));
INS((GOAL * MARK))}

(P6 (GOAL * MARK) (SAID 1))

==> REPL((GOAL * MARK) ; (GOAL + MARK));
INS((Y 2 Z))}

(P6A (GOAL * MARK) (SAID 2))

==> REPL((GOAL * MARK) ; (GOAL + MARK));
INS((Y 3 Z))}

(P6B (GOAL * MARK) (SAID 3))

==> REPL((GOAL * MARK) ; (GOAL + MARK));
INS((Y 4 Z))}

(P7 (GOAL + MARK) (GOAL + COUNT) (GOAL *S* SEND C) (SAID C))

==> REPL(((GOAL *S* SEND C)));
DEL((GOAL + MARK));
DEL((GOAL + COUNT));
DEL((SAID C));
REPL(((GOAL *S* ATTEND) ; (GOAL * ATTEND))}
P1 inserts an active goal of sending "n" balls down the tube. In the service of that goal, P2 inserts the subordinate goal of pushing the button. P3 notices the ball going down the tube and inserts the subitization goal. P4 simulates subitization of the ball(s) noticed (by P3) going down the tube. P5 carries out a counting operation by accessing and 'saying' the next name on a number-name-list. P6 marks the name 'said' by P5, and inserts the next number name on the list into STM. P6A to P6C simulate similar marking and moving operations. P7 simulates a checking operation. If the last name 'said' by P5 is the same as the number-name given in the instruction to send "n" balls down the tube (represented in STM by (SEND C)), then the model "knows" that it has finished that part of the task. In that event, it re-activates the goal 'to attend' and the next production to fire is P0. If not, the goal manipulation in P8 ensures that P2 will be the next production to fire, and that a new cycle of button pressing, subitizing, and counting will be entered.

This procedure can be followed by reading the listing for the model concurrently with the trace of the model's execution, given in Appendix 4, Addendum 2.
When control is passed back to P0, the user then asks: how many balls were in the bottom jar before? This is represented by the STM elements (MANY BOTTOM BEFORE). P9 and P10 simulate the answering of this question, after which control is passed back to P0. The user then asks: how many were sent down the tube? This is represented in STM by the element (MANY TUBE). This question is answered by P11 and P12, after which control is again passed back to P0.

(P9 (GOAL * ATTEND) (MANY BOTTOM BEFORE))

==>
REPL((GOAL * ATTEND) ; (GOAL *S* ATTEND));
DEL((MANY BOTTOM BEFORE));
INS((GOAL * RECALL BOTTOM) )

(P10 (GOAL * RECALL BOTTOM) (BOTTOM B))

==>
SAY((BOTTOM B));
DEL((GOAL * RECALL BOTTOM));
REPL((GOAL *S* ATTEND) ; (GOAL * ATTEND)) )

(P11 (GOAL * ATTEND) (MANY TUBE))

==>
REPL((GOAL * ATTEND) ; (GOAL *S* ATTEND));
DEL((MANY TUBE));
INS((GOAL * RECALL TUBE) )

(P12 (GOAL * RECALL TUBE) (SEND C))

==>
DEL((GOAL * RECALL TUBE));
REPL((GOAL *S* ATTEND) ; (GOAL * ATTEND));
SAY((SEND C)) )
By entering (MANY BOTTOM NOW) into STM, the user causes control to be given to P13 on the next cycle.

The direct counting procedure involves the co-ordination of step-by-step movement through two sequences, each of which is the number-name-list. Items from the first sequence are represented in STM by elements having the form \((Y <P> Z)\), and those from the second sequence by \((W <P> V)\). The letters \(Y, Z, W,\) and \(V\) imbedded in these elements are of technical significance only. They constitute a method of marking locations in a list. The symbol \(<P>\) assumes numerical values.

P13 activates the addition goal (GOAL * ADD). It sets the location in the first sequence at the point corresponding to the number of balls in the bottom jar before the last series of button presses. P13 also sets the location in the second sequence, just before the first element in the number-name-list. P14 checks to see if the location in the second sequence is the same as the value in the STM element representing the instruction to send \(n\) balls down the tube. That is, P14 asks: is the value of \(<P>\) in the element \((W <P> V)\), the same as the value of \(<P>\) in the element \((SEND <P>)\)? If so, P14 fires, and the answer is extracted from the \((Y <P> Z)\) element currently in STM. If not, either P15 or P16 fires. They control the movement through the two sequences. P17 to P17J carry out the moves from place to place in the first sequence. P18 to P18C perform the same function for the second sequence.
(P13 (GOAL * ATTEND) (MANY BOTTOM NOW) (BOTTOM B))

==>

REPL((GOAL * ATTEND); (GOAL *S* ATTEND));
DEL((MANY BOTTOM NOW));
INS((GOAL * ADD));
INS((W 0 V));
INS((Y B Z))

(P14 (GOAL * ADD) (W <P> V) (SEND <P>) (Y <X> Z))

==>

SAY(<X>);
DEL((GOAL * ADD));
INS((GOAL * PURGE))

(P15 (GOAL * ADD) (W <P> V) (Y <X> Z))

==>

REPL((GOAL * ADD); (GOAL *S* ADD));
INS((GOAL * NEXT ALONG));
DO(GET-NEXT)

(P16 (GOAL * NEXT UP))

==>

DO(STEPUP)

(P17 (GOAL * NEXT ALONG) (Y ZERO Z))

==>

REPL((Y ZERO Z); (Y ONE Z));
DEL((GOAL * NEXT ALONG));
INS((GOAL * NEXT UP))

NOTE: Productions P17A to P17J have the same form as P17. Each inserts the next number-name symbol (e.g. (Y FOUR Z)) in SIM.
(P18 (GOAL * NEXT UP) (W 0 V))

==>

REPL((W 0 V) ; (W 1 V));
DEL((GOAL * NEXT UP));
REPL((GOAL *S* ADD) ; (GOAL * ADD)) )

NOTE: Productions P18A to P18C have the same form as P18. Each replaces the current number-name symbol (e.g. (W 2 V)) with the next number-name symbol (e.g. (W 3 V)).

An example may clarify the operation of P13 to P18C. Suppose that the bottom jar had two balls in it, and the subject sent down four more. STM would contain the elements (BOTTOM 2) and (SEND 4). P13 would insert the elements (Y 2 Z) and (W 0 V). On the first cycle after the firing of P13, P14 would not fire because <P> would be set to 0 in (W 0 V). P15 would initiate an entry to the P17 to P17J group of productions. Specifically, P17B would fire and insert (Y 3 Z) into STM, and set a goal causing P16 to fire on the next cycle. P16 would then initiate an entry to the P18 to P18C group of productions. Specifically, P18 would fire and insert (W 1 V) into STM, and set a goal causing P14's conditions to be examined on the next cycle. Again, P14 would not fire, because <P> would be set to 1 in (W 1 V). Hence, P15 would fire again, and the P15 to P18C procedure would be re-entered. This pattern would continue until on one cycle P14 found <P> set to 4 in (W 4 V). At that time, the (Y <P> Z) element would contain (Y 6 Z). P14 would then extract the answer (6) from that element.

The remaining productions P24A to P27 perform housekeeping functions needed to prepare the model to receive further input.
13.5.2 **THE COUNTING-BASED MODEL**

OF N-SUB-NV.

A complete listing of this model (ADD4.PSS) is provided in Appendix 4, Addendum 5. Productions P000 to P12 are identical in form and function to the same-numbered productions for the addition model. They simulate the subject's behaviour up to the point where he is asked how many balls are left in the top jar.

The productions P13 to P18C also carry out functions analogous to the same-numbered productions in the addition model. The differences are that:

- P13 sets \(<P> in (Y <P> Z) to equal the number of balls in the top jar, initially.
- P17 to P17J moves down the number-name-list, not up it, as in the case of the addition model (ADD8.PSS).

13.6 **PERFORMANCE STATISTICS.**

Table 13.1 sets out the number of productions fired, and the maximum number of elements held in SIM during the execution of each of the six models. The entries in Table 13.1 relating to SIM represent the maximum amount of SIM used by the respective models - that is, if SIM allocations of lesser capacity were to be made, the models would not execute correctly.
<table>
<thead>
<tr>
<th>TASK MODELLING</th>
<th>N-ADD-NV</th>
<th>N-SUB-NV</th>
<th>N-CYC-NV</th>
</tr>
</thead>
<tbody>
<tr>
<td>PROCESS MODELLING</td>
<td>ADDITION</td>
<td>SUBTRACTION</td>
<td>ADDITION AND SUBTRACTION</td>
</tr>
<tr>
<td>METHODS USED</td>
<td>TABLE-LOOK-UP</td>
<td>COUNTING</td>
<td>COUNTING</td>
</tr>
<tr>
<td></td>
<td>(ADD6.PSS)</td>
<td>(ADD8.PSS)</td>
<td>FROM LARGER ADDEND (ADD3.PSS)</td>
</tr>
<tr>
<td>No. of PRODUCTIONS FIRED</td>
<td>130</td>
<td>162</td>
<td>190</td>
</tr>
<tr>
<td>MAX. No. of ELEMENTS HELD IN SIM</td>
<td>9</td>
<td>9</td>
<td>12</td>
</tr>
</tbody>
</table>

The 'number of productions fired' is the number of steps required - at this level of analysis - to simulate a successful subject's performance on the modelled task. The table-look-up procedure is the more efficient for addition and subtraction. The straight-forward counting procedure for addition (ADD8.PSS) is more efficient than the alternative (ADD3.PSS). Subtraction, by table-look-up, or by counting, involves exactly the same number of steps as the corresponding addition procedure. The small increase in the number of productions fired for N-CYC-NV, over N-ADD-NV and N-SUB-NV (174 versus 130), suggests that the bulk of the effort is expended on simulating aspects of the tasks not directly concerned with addition or subtraction.
The more efficient, table-look-up models require no more STM space than the counting-based models. All but one of the models required up to 9 elements of STM to store the control information and data used during execution. The exception is the counting-based addition model (ADD3.PSS), which first finds the larger of the two addends. That model requires a maximum of 12 elements of STM.

According to this analysis, the delay in development of STM capacity could not be the factor causing the observed delay in acquisition of the components assessed by N-SUB-NV and N-CYC-NV. Additionally, this analysis reinforces the suggestion given in Section 13.3 that the delay in acquisition of the component assessed by N-CYC-NV is due to delay in acquisition of the component assessed by N-SUB-NV, rather than a need to co-ordinate the components assessed by N-ADD-NV and N-SUB-NV.

13.7 CONCLUSIONS.

The performance similarities between the corresponding addition and subtraction models suggest that the observed development sequence is not a function of STM capacity. It seems to reflect, simply, the order of acquisition of certain rules. An inspection of the productions used in the models of the table-look-up procedures (Appendix 4, Addenda 1 and 4) supports this conclusion. For the table-look-up procedure, the only difference between the addition and subtraction models is in the data tables. The addition model makes use of entries of the form: \(a+b=c\). The subtraction model makes use of entries of the form: \(a-b=c\). For the child using this procedure, the observed developmental sequence may simply reflect the fact that the former are usually learned before the latter.
In the case of the counting procedure, the addition model controls the co-ordinated movement in the same direction of two pointers through a single number name list. The subtraction model controls the co-ordinated movement in opposite directions of two pointers through the same list. The former procedure contains less potential for confusion because the two pointers are rarely in close proximity in the list. The latter procedure does contain potential for confusion when the two pointers co-incide in the list. For example, in the N-SUB-NV task, at one time the bottom jar holds six balls, and the top jar six balls. Hence, one pointer would be moving: "6 \rightarrow 7 \rightarrow 8 \ldots \text{ etc}". The other would be moving: "6 \rightarrow 5 \rightarrow 4 \text{ etc}". Consequently, in the case of subtraction, even if the child has the appropriate counting-based rules, the probability of a breakdown during execution of those rules would be greater than for the corresponding addition operation.

It is noteworthy that these conclusions are consistent with the finding that length of schooling is a better predictor, than age, of a subject's performance. That is, performance is a function of experience with the appropriate rules, and not a function of STM capacity.

Finally, it is emphasized that the six models discussed in this Chapter were developed for two purposes. Firstly, to examine in more detail the questions discussed in Chapter 12, and, secondly, to illustrate an approach to the longer-term objective of deriving a detailed process account of the composition and growth of linear measurement. The core productions relating to subitization, counting, addition and subtraction contained in the present models are all pertinent to that enterprise. However, they constitute only a small fragment of the work that needs to be done to realise the longer-term objective.
Chapter 14.

SUMMARY OF CONCLUSIONS.

The present study had two main objectives. The first was to identify the 'higher-level' knowledge necessary for a child to understand linear measurement. The second was to chart the growth of linear measurement in terms of the development of its components.

An analysis of measurement operations yielded a list of components which it was argued would underlie linear measurement. Piagetian theory and related empirical literature were consulted as sources of information on the emergence of these components in the child's thinking. This led to the formulation of a number of predictions concerning the components of linear measurement, and their order of development.

A battery of 34 number, length and distance tasks was developed to assess the presence of these components. It was administered to 100 children aged between 63 and 78 months, and drawn from kindergarten and grade one. The results were analyzed using scalogram techniques. The main contribution of the thesis is in this empirical work.

The main conclusions are summarised in the following paragraphs.
14.1 COMPONENTS OF LINEAR MEASUREMENT.

It was found that children who possessed a mature level of understanding of linear measurement also possessed the following:

. Knowing how to make transitive inferences of equivalence, with respect to discrete quantity, and length.
. Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
. Knowing that length is invariant under certain transformations (the conservation of length).
. Knowing how to carry out numerical addition operations.
. Knowing how to obtain a linear measurement by counting iterations of a unit of length.
. Knowing how to make transitive inferences of non-equivalence, with respect to discrete quantity.

14.2 ORDER OF DEVELOPMENT OF LINEAR MEASUREMENT.

For linear measurement, the components emerge in the order in which they are listed above.

The data imply a delay between acquisition of the components and emergence of an understanding of linear measurement. It was also noted that those children who possessed all the necessary components, but could not
demonstrate an understanding of linear measurement, could not commence the linear measurement task. It was argued that this was evidence that short-term-memory capacity limitations were not implicated in the delay, because such limitations are expressed usually in breakdowns in performance of a strategy. The delay was interpreted as being associated with the need for a re-organisation of the relevant cognitive structures, resulting in better co-ordination between components. That is, the delay was associated with the formation of new long-term-memory links between components.

14.3 ORDER OF DEVELOPMENT

OF COMPONENTS IN THE
NUMBER, LENGTH AND
DISTANCE DOMAINS.

For the number and length domains, the collections of components form scalable sets. That is, the components emerge sequentially in each domain.

In general, the order of emergence of the components is that predicted by Piagetian theory, and the empirical evidence reviewed in Part II. The most important exceptions to that pattern are noted below.

Conservation of number emerges significantly before transitive inference concerned with non-equivalent relations between discrete quantities. The same lag in development occurs in respect of length. It was argued that this finding could have been expected, because the form of transitive reasoning involved in the conservation task is concerned with equivalent, and not with non-equivalent, relations.
The observed lag in development between the emergence of corresponding components in the length and distance domains, such as the conservation of length and the conservation of distance, was not predicted.

14.4 DISCONTINUITIES IN NUMBER AND LENGTH CONCEPT DEVELOPMENT.

It was found that the patterns of development in the number and length concepts were marked by discontinuities.

It was suggested that these discontinuities might have been due to capacity limitations of short term memory. However, an information-processing analysis, using the M-space model, failed to find evidence supporting that suggestion. Furthermore, a production-system analysis that explicitly accounted for strategy control information also failed to find evidence of a short-term-memory barrier.

In general, the discontinuities co-incide with the emergence of components of major theoretical interest, such as the conservation of length. Additionally, there are concordances between discontinuities in the number and length growth patterns. It was suggested that these concordances co-incide with periods of development during which new forms of co-ordination are being established between the two concept domains. New inter-connections of that kind would be represented by long-term memory linkages.

In summary, the general impression is that growth in one concept domain prompts growth in the other.
It was stated that the analysis of linear measurement given in Chapter 2 was largely intuitive and informal due to the lack of a detailed psychological theory of linear measurement, and the paucity of directly relevant empirical data. It was argued that a more satisfactory account of linear measurement could be given in the form of an executable production-system model. A model of that kind would be a formal theory, and would demonstrate that the components listed in Chapter 2 are necessary for linear measurement. It would also provide a sufficient account of the empirical data reported in this study. Development of that model would be a substantial undertaking, and beyond the scope of the present research. However, the six production systems constructed in this study provide a small start on that larger, longer-term project.

It was found that length of schooling is a predictor of a subject's score, but age is not. This latter finding, though, must be due to the narrow age range of the subjects used in the study.

These findings are consistent with the general pattern of development observed in the study. This is because delays and discontinuities in development during the period studied were thought to be associated not with short-term-memory barriers - an age related factor - but with the forging of new long-term-memory links - an experience related factor.
For many children, experiences of the appropriate kind are provided most frequently at school. Hence, length of schooling could be expected to be a better predictor of performance, than age, on the tasks used, and for the period of development covered, in this study.

14.7 SUGGESTIONS FOR FURTHER RESEARCH.

It is considered that further research is needed in order to elaborate upon the kinds of inter-connection between components that represent solution strategies for linear measurement tasks. It is suggested that such further research incorporate the construction of production-system models depicting mature levels of performance in linear measurement. Additionally, further research is needed to identify the reasons for the lag in development between corresponding components in the length and distance domains.

The findings of the present study, together with those from such further research, could considerably inform early primary school curriculum planning.
APPENDIX 1.

TASK DESCRIPTIONS.

NUMBER TASKS,

Al.l.l N-(1-TO-1).

Purpose: Assess the subject's ability to determine the numerical equivalence relation between three collections each of unspecified cardinal value, using a one-to-one correspondence operation.

Materials: Ten plastic bolts; 10 plastic nuts; 10 plastic washers; cardboard box; 3 opaque plastic jars with screw-on lids. Two of the bolts were white and were 4 cms in diameter; two were blue and 3.5 cms in diameter; two were green and 3 cms in diameter; two were red and 2.5 cms in diameter; and two were yellow and 2 cms in diameter. The nuts and washers fitted the bolts.

Procedure: The experimenter emptied the bolts, nuts, and washers out of the cardboard box onto the table. He unscrewed the nut and washer off one of the white (largest diameter) bolts, and asked the subject if he knew the names of the three objects. If the subject did not know the names, he was told them. The experimenter then drew the subject's attention to the fact that every bolt also had a nut and a washer, every nut had a bolt and washer, and every washer had a nut and bolt. The experimenter then asked him to unscrew all the nuts; to take all the nuts and all the washers
off the bolts; to place all the nuts in one jar, all the bolts in another jar, and all the washers in a third jar. He was told not to count the nuts, bolts or washers. He was then asked to place the tops on the jars, and to tell the experimenter the names of the parts in each jar. The experimenter then reminded the subject that, for every bolt, there was also a nut, and a washer. The experimenter then went through the following questions:-

(i) "Pick up the jar with the bolts in one hand, and the jar with the nuts in the other. Are there more nuts in there than bolts in there, the same number of nuts there as bolts there, or less nuts than bolts? You can shake the jars, but you can't look inside. How did you work that out? Did you count them?"

(ii) "Pick up the jar with the bolts in one hand and the jar with the washers in the other. Are there the same number of bolts there as washers there, more bolts than washers, or more washers than bolts? You can shake them, but don't look inside. How did you know that? Did you count them."

(iii) "Pick up the jar with the nuts in one hand and the jar with the washers in the other. Are there less nuts than washers, more nuts than washers, or the same number of nuts as washers? You can shake them, but don't look inside. How did you know that? Did you count them?"

(iv) "O.K., make a guess - how many nuts are there in here? Did you count them?"

If the subject gave correct answers to questions (i), (ii), and (iii), he was told he was correct and congratulated, provided that he indicated that he hadn't counted. If the subject gave the incorrect answer, or indicated that he counted, the experimenter moved onto the next question, without comment.
Scoring: To pass the subject had to answer questions (i) to (iii) correctly; to give appropriate verbal justifications; and to indicate by answers to question (iv) that he had not counted. Appropriate verbal justifications mentioned some aspect of one-to-one matching, for example: "there was one each when I unscrewed them"; and "every bolt had one."

Al.1.2 N-ORD.

Purpose: Assess the subject's capacity to form an ordered series of collections of objects.

Materials: Four semi-transparent plastic jars, each of which was 10 cms in height and 7 cms in diameter; four lids, one each of blue, red, green, yellow; three blue, five red, seven green and nine yellow plastic blocks, each 2x2x2 cms. The blocks were placed in the jars with the correspondingly coloured lids. The jars were known as the blues, reds, greens and yellows.

Procedure: The experimenter handed the blues, reds and yellows to the subject, and said: "See if you can place these in order. Go from here to here. Put the one with the most here, the one with least here, and the other one in the middle. You cannot count them, but if you pick them up and shake them, you will be able to guess which one has most." (A number of children didn't understand the word "least" but "smallest number" sufficed as a synonym.) When the subject completed that task, he was handed the greens, and asked to find out where they went in the line so that "they all stay(ed) in order." After the subject made the insertion, the
experimenter picked up the greens and moved them to the right of the yellows. He placed his hands on the greens and yellows, and said: "How can you move the blues and reds down this end, so that they will still be in order?" If the subject did not transpose the blues and reds when moving them to the other end of the line, he was asked to think about it again, and was given another opportunity to order the jars. The following illustrates the moves involved:

(i) Subject sets up series:

3 5 9
blues reds yellows

(ii) Subject inserts greens:

3 5 7 9
blues reds greens yellows

(iii) Experimenter shifts greens:

3 5 9 7
blues reds yellows greens

(iv) Subject moves blues and reds:

9 7 5 3
yellows greens reds blues

Scoring: To pass the task, the subject had to construct the first three term series, insert the greens in the correct place, and invert the series.

Al.1.3 N-CNT.

Purpose: Assess the subject's ability to count an array of n objects and to demonstrate co-ordination of ordinal position and cardinal value.
Materials: Collection of nine coloured plastic building blocks, each measuring 2x2x2 cms: two each of white, red, green, and blue, and one yellow.

Procedure: The collection of nine blocks was handed to the subject, and the subject asked to find out how many blocks there were. After the subject had counted the blocks, he was asked to count them again but, this time, as he counted each block he was asked to push it across the table to the experimenter. As the subject was pushing across the fourth block the experimenter stopped him, and asked if he knew how many the experimenter already had in his pile (three blocks). This was repeated on the eighth block.

Scoring: If a subject gave an incorrect answer he was given another opportunity, but was not told that his answer was wrong. The usual comment was: "let's try that again." A subject had to give correct answers to all questions to pass the task.

Al.1.4 N-TI-EQ.

Purpose: Assess the subject's ability to make a transitive inference of equality with respect to discrete quantity.

Materials: Three semi-transparent plastic jars, each of which was 10 cms in height and 7 cms in diameter; three lids, one blue, one red, and one green, for the jars; and seven blue blocks, seven red blocks, and seven green blocks, each 2x2x2 cms. The blue blocks were in the jar with the blue lid, the red
blocks in the jar with the red lid, and the green blocks in the jar with
the green lid. The jars were known as the "blues," the "reds", and the
"greens." It was possible to see the blocks inside the jars, but not suffi-
ciently clearly to enable the subject to count them.

Procedure: The experimenter handed the subject the blues and the reds,
and said: "Hold the blues in your left hand and the reds in your right
hand. Shake the jars and see whether you can guess whether they each have the
same number of blocks, or has one got more than the other. You will not be
able to count them, so don't try that." If the subject said "the same,"
the experimenter agreed, and moved on to the red and green
pairing. Otherwise, the experimenter prompted him until he agreed that
the jars contained the same number of blocks. If the subject mentioned
a specific number of blocks he was told that that was wrong. He was then
told not to guess how many were in the jars, just whether the jars had the
same number of blocks or not. The same procedure was followed for the red
and green jars. After both comparison had been presented, the experimenter
placed the reds behind his back, held the blues in his left hand, and the
greens in his right hand, so that only the lids were visible to the sub-
ject. He then said: "O.K., now you said the blues had the same as the reds,
and the reds had the same as the greens. Is that right? Well, what about
the blues and the greens, can you work out whether they have the same
number of blocks, or has one got more? How did you know?"

Scoring: To pass the task, the subject had to give a correct verbal
statement of transitive reasoning.
Al.1.5 N-TI-NE.

Purpose: Assess the subject's ability to make transitive inferences of inequality with respect to discrete quantity.

Materials: Six semi-transparent plastic jars, each of which was 10 cms in height and 7 cms in diameter; six lids, one each of blue, red, green, yellow, white, and black; five blue, seven red, nine green, nine yellow, seven white and five black blocks. The blocks were placed in the jars with the correspondingly coloured lids.

Procedure: As for N-TI-EQ, except that the first presentation tested the relation of less-than, and the second presentation the relation of greater-than. The first presentation was:-

(i) five blues (less than) seven reds, and
(ii) seven reds (less than) nine greens, so
(iii) five blues...? nine greens.

The second presentation was:-

(i) nine yellows (greater than) seven whites, and
(ii) seven whites (greater than) five blacks, so
(iii) nine yellows ....? five blacks.

Scoring: To pass the task, the subject had to give a correct verbal statement of transitive reasoning.
Purpose: Assess the subject's ability to conserve number.

Materials: 16 green plastic building blocks, each 2x2x2 cms.

Procedure: The experimenter handed a collection of eight blocks to the subject, and said: "I'm going to put my blocks down in a line across here, so that they are about that far apart (5 cms). I would like you to put yours down in a line opposite mine, so that one of yours is opposite one of mine. Don't count them as you do it, or we will have to start all over again." When the subject and experimenter completed building the lines of blocks, the experimenter said: "We have the same number of blocks in each line, haven't we? Don't count them, just see if you can tell by looking." When the subject confirmed that each line contained the same number of blocks, the experimenter said: "Now I'm going to spread mine out like that, but don't move yours. O.K., now do you think that my line has more blocks than your line, or do you think that they both still have the same number of blocks? Why? How did you figure that out?" The experimenter then reset his line, so that it again stood, block for block, opposite the subject's. He then shortened his line, and asked the subject: "Now, do you still think that my line has the same number of blocks as your line, or do you think your line has more? Why? How did you know that?"

Scoring: The subject was required to pass both versions and to give appropriate verbal justifications, to pass the task. Appropriate verbal justifications were those in the category of either inversion, compensation, or quantitative identity.
Purpose: Assess the subject's ability to predict the results of addition operations, when the two collections to be added together are visible.

Materials: Twelve red plastic building blocks, each measuring 2x2x2 cms.

Procedure: The experimenter placed the collection of 12 blocks on his side of the table, and told the subject to take two blocks from that collection to his side of the table. The subject was then asked how many blocks he had. When the subject answered "two", he was asked if he knew how many he would have, if he took one more from the experimenter's pile. If the subject attempted to take the block before answering, he was told not to. If he attempted to count using his fingers, or some other object, as a counter, he was also told not to. When the subject answered "three", he was told "that was good, take the block over to your pile." The experimenter then said: "O.K., you've got three blocks now, how many would you have if you took another three blocks from my pile and put them with your's?" This pattern was followed for the following operations:- 2+1; 3+3; 6+2; 8+4.

Scoring: If the subject gave an incorrect answer the experimenter said: "how about doing that again". To pass the task the subject had to give correct answers to all questions.
Al.1.8 N-SUB-V,

Purpose: Assess the subject's ability to predict the outcome of subtraction operations when the two collections are visible.

Materials: Twelve blue plastic building blocks, each measuring 2x2x2 cms.

Procedure: The procedure was the same as for N-ADD-V, except that the subject was asked if he knew how many the experimenter would have left in his pile, if the subject were to take n blocks away. The subtraction operations tested were: 12-3; 9-4; 5-2; 3-1.

Scoring: As for N-ADD-V.

Al.1.9 N-SOL-V.

Purpose: Assess the subject's ability to find the numerical difference between two collections (a and b), and, by addition or subtraction, make the collections equal - this is usually called solving for a difference.

Materials: 16 green plastic building blocks, each measuring 2x2x2 cms.

Procedure: The experimenter handed the collection of 16 blocks to the subject, asked him to give five to the experimenter, four to himself, and to place those left over in the middle of the table. The experimenter then asked who had more blocks. After the subject said that the experimenter did, the experimenter told him that the objective was to make the experimenter's and the subject's collections have the same number of blocks. The subject was told the rules of the game. They were:
(i) Blocks could only be moved from or to the middle, and from or to the experimenter's or subject's piles. They could not be moved from the experimenter's pile to the subject's pile, or vice-versa.

(ii) Only one move was allowed, but, if the subject wanted to move more than one block he had to move them at the same time.

(These rules were designed to force the subject to work out the answer "in his head", and, thereby avoid trial and error solutions). If the subject moved one block from the middle to his pile so that both piles contained five blocks, the experimenter congratulated him and then said: "That's one way of doing it. Let's set it back the way it was, four for you and five for me. Now, can you see another way of doing it?" If the subject took one away from the experimenter's pile so that both piles contained four blocks, he was congratulated. If the subject could not work out the answer, he was prompted by the experimenter saying: "It's all right to take some of mine, or give more to me, if that would help." If he still couldn't solve the problem, the experimenter demonstrated the solution. After this preamble, which was not scored, the test phase was entered. Using the same format, the subject was required to solve the following problems: 7-4=? and 4+?=7; 6-4=? and 4+?=6; and, 6-2=? and 2+?=6.

Scoring: All test questions had to be answered correctly for the subject to pass the task. If the subject made an incorrect move, the experimenter said: "let's put them back and you think about it again." If the subject's second move was also incorrect, he failed that question.
Purpose: Assess the subject's ability to solve problems of the form:
if \( a > b \) then \([a-(a-b)/2)] = [b+(a-b)/2]\), when all objects are visible
(that is, balance the two collections by sharing the difference between
them.)

Materials: 16 yellow plastic building blocks, each measuring 2x2x2 cms.

Procedure: The experimenter handed the collection of 16 blocks to the
subject, asked him to give six blocks to the experimenter, four to himself,
and to put the remainder back in the box and out of sight. The experimenter
then asked who had more blocks. After the subject answered correctly,
he was told that the objective was to make the piles the same, in that each
had the same number of blocks. The subject was told that the rules of
the game were:

(i) blocks could only be moved one pile to the other, they could not be
moved to one side; and
(ii) only one move was allowed, but as many blocks as the subject wished
could be moved at the same time.

(These rules were designed to avoid trial and error solutions.) If the
subject answered the practice item correctly the experimenter moved onto
the test phase. If not, he was prompted by the experimenter saying:
"it's all right to take some of mine across to your side. Do you think
that would help?" If he still could not solve the problem, the solution
was demonstrated by the experimenter, and the test phase then entered.
The preamble was not scored. In the test phase, he was required to solve
the following problems: \( a=7 \) and \( b=3 \), hence \( a'=5 \) and \( b'=5 \); \( a=9 \) and
\( b=3 \), hence \( a'=6 \) and \( b'=6 \). The questioning and prompting format used in
the preamble was employed in the test phase.

Scoring: Both test questions had to be answered correctly for the subject
to pass the task. If the subject made an incorrect move, the experimenter
said: "let's put them back and you think about it again." If the subject's
second move was also incorrect, he failed that question.

Al.1.11 N-ADD-NV.

Purpose: Assess the subject's ability to determine the outcome of adding
\( n \) objects to a collection of similar objects but where that collection
of objects is not visible.

Materials: A counting tube apparatus consisting of one vertical wooden
post attached to a rectangular wooden base; an opaque plastic jar attached
to the top of that post; another opaque plastic jar attached to the wooden
base; a length of transparent plastic tubing connecting the bottom of the
top jar with the side of the bottom jar; a machined metal mechanism
containing a spring loaded button attached to the bottom of the top jar
and the connecting tube; and 12 silver coloured metal ball bearings of
1 cm diameter (see Figure Al.1 for dimensions of apparatus.) The apparatus
was designed so that the ball bearings could be placed in the top jar and
then sent down the tube to the bottom jar, one by one, by pressing the
button. Each button press released one ball bearing. The ball bearings
were only visible whilst they were rolling down the tube.
Procedure: The experimenter asked the subject to take the lid off the top jar and check that there were no metal balls inside, and to do the same with the bottom jar. The experimenter then handed the 12 metal balls to the subject, and asked him count them into the top jar. Then, he was asked how many metal balls were in the jar. The subject was then told how the apparatus worked. The experimenter demonstrated it by pressing the button twelve times, and counting the balls as they rolled down the tube into the bottom jar. The subject was then asked how many balls were in the the bottom jar, and how many were left in the top jar. This preamble was repeated, if necessary, until the subject clearly understood what was happening. The subject's responses during the preamble were not scored as part of the task. Following the preamble, the experimenter again asked the subject to verify that there were no balls in either jar or in the tube. The experimenter then handed the 12 balls to the subject, and asked him to count them into the top jar. The subject was then asked to send two balls down to the bottom jar. The experimenter then said: "right, there were none in the bottom jar and you sent two down the tube, so how many balls are there in the bottom jar now?" If the subject gave the wrong answer, the question was repeated. If the answer was again wrong the the experimenter asked the subject to take the top off the bottom jar, look inside and count how many were there. Next, the experimenter said: "O.K., there are two balls in the bottom jar, send down four more." The experimenter then asked the subject how many were in the bottom jar, reminding him that there were two there before, and that he had sent four more down the tube. This format was followed until all of the following addition operations had been performed: 0+2; 2+4; 6+3; 9+1.

Scoring: The subject had to give correct answers to all questions to pass (a correct answer on the second attempt was considered a pass).
Al.1.12 N-SUB-NV.

Purpose: Assess the subject's ability to determine the outcome of subtracting n objects from a collection of similar objects where that collection but where that collection is not visible to the subject.

Materials: As for N-ADD-NV.

Procedure: The preamble and questioning format used in N-ADD-NV was followed in this task, except that the question related to how many balls were left in the top jar. The subject was reminded of how many were in the top jar before sending the last batch down the tube. The operations tested were: 12-2; 10-4; 6-3; 3-1.

Scoring: As for N-ADD-NV.

Al.1.13 N-CYC-NV.

Purpose: Assess the subject's ability to concurrently work on two collections in a situation where adding to collection (a) meant subtracting from collection (b), and where the objects in those collections were not visible to the subject: that is, the objects cycled from one collection to the other.

Materials: As for N-ADD-NV.

Procedure: The preamble and questioning format used in N-ADD-NV was followed in this task, except that the question related to how many balls were left in the top jar, and how many were in the bottom jar. Also, in posing the question, the subject was reminded that there were 12 balls in total, and that before sending n down there were m left in the top jar. The operations tested were those used in N-ADD-NV and N-SUB-NV.

Scoring: As for N-ADD-NV.
Al.2 LENGTH TASKS.

Al.2.1 LR-B-IN-A.

Purpose: Assess the subject's understanding that if length (a) is greater than length (b), then (a) may be considered as (b) concatenated with some other length (that is, a sense in which (b) is included in (a)).

Materials: A perspex box having the following dimensions was used: 21.5 cms in length and width and 15 cms in height, and having walls and a top surface made of perspex 6 cms in thickness. Set vertically into the box was a hollow plastic tube of the following dimensions: 10 cms in length, outer diameter of 1.4 cms, inner diameter of .9 cms. The top of the tube was flush with the top surface of the box. Three lengths of wooden dowel, each of .8 cms in diameter, and of lengths 7 cms, 10 cms, and 13 cms, were also used. See Figure Al.2 for details.

Procedure: The experimenter placed the box on the table, and drew the subject's attention to the hollow tube. The experimenter then placed the three pieces of dowel on the table so that they were in order of length, and drew his attention to that order. The experimenter then asked him to pick up the middle stick, and to push it into the tube so that it went all the way to the bottom. The experimenter then drew his attention to the fact that the top of the stick was flush with the top of the box, and reminded him that that stick was the middle one of the three on the table. The subject was then asked whether he thought that the longer stick of the two left on the table would project above the top surface of the box if it were inserted in the tube, or would it come just to the top, or would it not reach the top. The same question was asked in relation to the shorter stick. The subjects were asked to justify their answers.
FIGURE A1.2

LR - 211A
PERSPEX BOX WITH TUBE
Scoring: Both questions had to be answered correctly, and appropriate verbal justifications given, to pass.

Al.2.2 L-P/W.

Purpose: Assess the subject's knowledge that any object may be considered as a concatenation of arbitrarily selected sub-lengths (that is, an understanding of part/whole relations of length).

Materials: Eight red and eight blue plastic building blocks, each measuring 2x2x2 cm. The red blocks were pushed together to make a red rod, and the blue blocks to make a blue rod.

Procedure: The experimenter placed the red and blue rods side by side, end points aligned, on the table, and sought the subject's agreement that they were the same length. The experimenter then dismantled the red rod, forming a pile of red blocks, and said: "If I put all these together again, would my red rod be as long as that blue one? How do you know?"

Scoring: As well as the correct evaluative response, the subject was required to give an appropriate verbal justification, to pass the task.

Al.2.3 L-INVAR-ADD.

Purpose: Assess the subject's understanding that the length of an object is invariant unless something is added to or subtracted from it (setting aside expansion and contraction processes).

Materials: Ten red plastic building blocks pushed together to make a red rod 20 cm in length; two red blocks; and a perspex box 15 cm high.
Procedure: The experimenter placed the red rod in front of the subject and asked him to indicate with his hands the length of the rod. The experimenter then rotated the rod 90 degrees, and asked the subject whether it was still the same length, and how he knew the answer. Then the experimenter stood the rod on the table and repeated the question. Then the experimenter placed the rod on top of the box and again repeated the question. Then the experimenter placed the two extra red blocks on the table, and asked the subject how he could make the rod shorter; and then, how he could make it longer. In each case, the subject was asked if that was the only way the rod could be made shorter or longer.

Scoring: All questions had to be answered correctly, and the appropriate verbal justifications given. If the subject gave an incorrect answer, he was asked to show the experimenter where the length of the rod started and where it finished, and then to think again about the answer. If the subject again gave the wrong answer, he was asked to explain how he worked it out. Then the experimenter moved onto the next question. If the experimenter gave the correct answer on the second attempt, and an adequate verbal justification, he passed that question.

Al.2.4 LR-INVAR-ADD

Purpose: Assess the subject's understanding that the length relation between two objects is invariant unless something is added to or subtracted from one of the objects (setting aside expansion and contraction processes).
Materials: Ten blue plastic building blocks pushed together to make one blue rod, and ten green blocks making one green rod; two blue and two green blocks: one perspex box 15 cms in height.

Procedure: The two rods were placed side by side, end points aligned, on the table. The experimenter sought the subject’s agreement that they were the same length. The blue rod was then moved through the positions described in the similar one-length task L-INVAR-ADD. The questioning format used in that task was also used in this task, except that, in this case, the questions concerned the length relations between the blue and green rods.

Scoring: As for L-INVAR-ADD.

Al.2.5 LR-INVAR-SP.

Purpose: Assess the subject’s understanding that the length relation between two objects is invariant under transformations involving only change of position.

Materials: Two pieces of wooden dowel, each 20 cms in length.

Procedure: The experimenter placed the two pieces of dowel flat on table so that they formed a "V", with the apex pointing toward the subject. The experimenter then asked the subject to find out which stick was the longer, or if they were the same length, telling the subject that he could move the sticks if he needed to. When the subject agreed that they were the same length, the experimenter placed one stick flat on the table with
its long axis parallel to the subject's side of the table. The other stick
was placed half way along the length of the first, but vertically on the
surface of the table. The experimenter then asked the subject if he still
thought that the sticks were the same length, and how he knew that. The
positions of the sticks were then reversed, and the questions repeated.
The sticks were then placed at opposite ends of the table. The one on the
subject's left was parallel with the long side of the table. The one on
the subject's right was parallel with the short side of the table. The
question was then repeated.

Scoring: All questions had to be answered correctly, and appropriate
verbal justifications given, for the subject to pass the task. If an
incorrect response was given, the subject was asked to think again. If
the subject gave the correct answer on the second attempt, he
passed the task.

Al.2.6 LR-ORD.

Purpose: Assess the subject's ability to order objects according to
their lengths.

Materials: Five pieces of wooden dowel - one each of the following lengths:
16.5, 17.0, 17.5, 18.0, 18.5 cms.

Procedure: The experimenter handed the three shorter pieces and the long-
est piece of dowel to the subject, and asked him to arrange them in order
of increasing length. After the subject had completed the ordering, the
experimenter handed the subject the fifth piece (18 cms) and asked him
to insert it in the correct place in the series. If the subject indicated
that he had completed the series, but the sticks were not ordered, the experimenter asked if he was sure that he had finished. If the subject was adamant that he had finished, and that the sticks were in order, the experimenter went through the series asking: "Is this longer than than that? No, well maybe you haven't finished yet. I'll pick them up, shuffle them again, and you have another go at it." This was repeated up to three times. If, on the third attempt, the subject still hadn't built a length-ordered series, he failed the task. If he succeeded on or before the third attempt, he passed that phase. Subjects were given only one opportunity on the insertion phase, and were asked to explain why they had placed the stick in the position chosen.

Scoring: To pass, subjects had to construct a length-ordered, four-term series, insert the fifth stick in the correct position, and give an appropriate verbal justification.

Al.2.7 LR-TI-EQ.

Purpose: Assess the subject's ability to make transitive inferences of equivalence with respect to length.

Materials: Three pieces of dowel, each 25 cms in length, one red, one green, and one blue.

Procedure: The experimenter handed the red and green sticks to the subject, and asked him to find out which was longer, or whether they were the same length. The experimenter then took back the red stick and handed the
subject the blue stick, and repeated the question. The experimenter then took back both sticks, and placed the red and blue sticks in front of the subject in the form of a "T", e.g.

```
  Subject
  red
  blue.
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The experimenter then asked: "remember the red was the same as the green, and the green the same as the blue, but what about the red and the blue? Without moving them, can you tell whether they are the same length, or is one longer? How did you know that?"

Scoring: The correct evaluative response and an appropriate verbal justification was required to pass.

Al.2.8 LR-TI-NE.

Purpose: Assess the subject's ability to make transitive inferences of greater than, and less than, with respect to length.

Materials: Six pieces of dowel of the following lengths and colours:
- red - 15 cms; green - 15.5 cms; blue - 16 cms; yellow - 16.5 cms;
- white - 17 cms; and grey - 17.5 cms.

Procedure: The procedure used in LR-TI-EQ was also followed in this task. The presentations were:

Longer Than:
(i) blue. longer. green;
(ii) green. longer. red;
(iii) blue. ?. red, where the blue and red were arranged in the form of a "T", with the shorter one (red) forming the vertical arm pointing toward the subject.
Shorter Than:

(i) yellow. shorter. white;
(ii) white. shorter. grey;
(iii) yellow. ? grey, where the yellow and grey were arranged in the form of a "T", with the shorter one (yellow) forming the vertical arm pointing toward the subject.

Scoring: Correct evaluative responses and appropriate verbal justifications were required on both phases of the task to pass.

Al.2.9 LR-CARD.

Purpose: Assess the subject's understanding that the ordinal length relation between two objects is the same as the cardinal numerical relation between the collections of parts comprising those objects (provided that the lengths of the respective parts are the same).

Materials: Eleven red and 12 green building blocks, each 2x2x2 cms; two semi-transparent plastic jars, 10 cms x 7 cms; two screw-top lids, one red and one green.

Procedure: The experimenter placed the blocks on the table, and asked the subject to find out whether:

(a) all the red blocks were the same size;
(b) all the green blocks were the same size;
(c) the red blocks and the green blocks were the same size.

The experimenter then showed the subject that the blocks could be pushed together to form rods. The experimenter
then placed seven red blocks in the jar with the red top, and five green blocks in the jar with the green top, without allowing the subject to count how many went in each jar. The experimenter then handed the jars to the subject, and asked him to guess which jar had more blocks, but not to take the tops off and count them. When the subject gave the correct answer, he was reminded that all blocks were the same length, and then asked:
"if you took all the red ones out and pushed them together, you would have a red rod. If you did that with the green ones, you would have a green rod. Which rod would be longer, the red one or the green one? How do you know?"
This procedure was repeated using eight blocks in each jar, after which all the blocks were placed back on the table. The experimenter then pushed the 12 green blocks across to the subject, and asked him to put them together to make a green rod while he (the experimenter) made a red rod out of the 11 red blocks. When that was done, the experimenter asked the subject to find out which one was longer. After the subject responded, he was asked to disassemble his rod and to place all the green blocks in the jar with the green lid, while the experimenter did the same with the red rod. When that was done, the experimenter then took both jars to his side of the table, covered the jars with his hands, and asked the subject if he knew which jar, red or green, had the greater number of blocks, and how he knew the answer. This procedure was repeated using 10 blocks in each jar. At all times the subject was observed for signs of counting, and was requested not to do so.

Scoring: All four phases had to be answered correctly, with appropriate verbal justifications, to pass.
Al2.10 LR-TI-CARD.

Purpose: Assess the subject’s ability to deduce length relations between objects by applying transitive reasoning to the cardinal number relations between the collections of unit parts.

Materials: Seven red, nine green, and eleven blue plastic building blocks, each 2x2x2 cms; three semi-transparent plastic jars; three screw-top lids, being one each of red, green and blue.

Procedure: The experimenter placed seven red, seven green and seven blue blocks on the table, and asked the subject to check that all blocks were the same length. He then demonstrated that they could be pushed together to make a rod. The experimenter then took back the blocks and placed seven reds in the jar with the red lid, nine greens in the jar with the green lid, and eleven blues in the jar with the blue lid, making sure that the subject could not count the number of blocks in each. The experimenter then handed the blue and green jars to the subject, and said: "Hold the blues in your left hand, the greens in your right, shake the jars and see whether you can guess whether they each have the same number of blocks? Or has one got more than the other? You will not be able to count them, so don’t try that."

After the subject made the correct response, the experimenter took the blue jar back, handed the subject the red jar, and repeated the previous instructions. The experimenter then placed the green jar behind his back, held the red jar in his left hand and the blue jar in his right, so that only the lids were visible to the subject, and said: "O.K., you said that there were more greens than reds, and more blues than greens, what about the reds and the blues? If you put all the reds together you would have a red
rod. If you put all the blues together you would have a blue rod. Which rod would be longer? How did you work it out?"

Scoring: To pass the subject had to give the correct evaluative response, and an appropriate verbal justification.

Al.2.11 L-CONS.

Purpose: Assess the subject's capacity to conserve length.

Materials: Two pieces of string, each 20 cms in length.

Procedure: The experimenter showed the subject the two pieces of string. He asked the subject to open one piece out so that it formed a straight line, and to put a finger on each end. The experimenter did the same with the other piece of string, placing it side by side with the subject's, so that the end points were aligned. The experimenter sought the subject's agreement that the two pieces of string were the same length. He then asked the subject to leave his string as it was, while the experimenter bent his into the form of a circle. The experimenter then asked the subject if he thought that the pieces of string were still the same length or not, and to explain his answer. The experimenter then restored his piece of string to its former position side by side with the subject's, and again sought the subject's agreement that they were the same length. He then said that he wanted to form a "T" with the two pieces of string, by leaving his in its present position, and by the subject turning his 90 degrees so that it formed the vertical arm. The same questions were then put to the subject. The two variants of the task are illustrated below:-
(i) (a) first presentation -
subject
standard form
variable form
experimenter

(i) (b) test presentation -
subject
standard form
variable form
experimenter

(ii) (a) first presentation - as for (i) (a) above

(ii) (b) test presentation -
subject
variable form
standard form
experimenter

Scoring: Both sets of questions had to be answered correctly, and appropriate verbal justifications given, to pass.

Al.2.12 L-UNIT

Purpose: Assess the subject's ability to iterate a unit part along the length of an object.

Materials: Strip of white cardboard 24 by 3 cms, and a second strip 3 by 3 cms; pencil; rubber.
Procedure: The experimenter placed the materials on the table and said: "can you work out how many of these small pieces you would need to make a piece as long as this one? You can move the pieces and use the pencil to put marks on them, if you want to?"

Scoring: Provided the subject attempted to iterate by marking off equal units, and correctly counted them, he passed. (A subject who lacked sufficient motor skill to mark off unit lengths exactly ended up with a portion of the unit measure projecting beyond the end of the strip. Nevertheless, he passed if he attempted to mark off adjacent and adjoining unit lengths.)

Al.2.13 L-EST.

Purpose: Assess the subject's ability to estimate length in terms of a number of unit lengths.

Materials: Four pieces of dowel, of the following lengths - 1 x 10 cms, 1 x 15 cms, 1 x 22 cms, 1 x 30 cms, and 1 red plastic building block 2x2x2 cms.

Procedure: The experimenter placed the 15 cm and the 30 cm long stick in front of the subject in the following pattern:-

(i) 

----------- 15 cm

----------------- 30 cm

subject
He then asked the subject: "how many of these (15cm) would you need to put together to make one as long as that (30cm). How did you work it out without moving them?" The same approach was used for the following two presentations:

(ii) \[ \begin{array}{c}
\text{subject} \\
10 \text{ cm} \\
30 \text{ cm}
\end{array} \]

(iii) \[ \begin{array}{c}
\text{subject} \\
2 \text{ cm} \\
22 \text{ cm}
\end{array} \]

The subject was not allowed to move the materials, but could point with his finger to the longer object, as he mentally iterated the shorter object.

Scoring: Acceptable answers were:

(i) two and three;
(ii) two and four;
(iii) eight to fourteen.

Acceptable answers were required to all three questions for the subject to pass. Verbal justifications were not scored.
Al.2.14 L-UNIT-CH.

Purpose: Assess the subject's ability to predict the direction in which the number given by unit iteration would change, if the length of the unit part were to change.

Materials: As for L-UNIT, plus two extra strips of white cardboard, one 4x4 cms, and the other 2x2 cms.

Procedure: This task was presented as an extension of L-UNIT. After the subject had worked out that it took eight of the 3 cm pieces to make one 24 cm strip - or, alternatively, had that demonstrated by the experimenter - the experimenter showed the subject the 4 cm long strip and demonstrated that it was longer than the 3 cm strip. He then asked: "Would it take more, less, or the same number of these (4 cm) as these (3 cm) to make one long one? How did you work it out?" The same procedure was followed with the 2 cm piece.

Scoring: Correct evaluative responses to both questions, and appropriate verbal justifications, were required to pass.

Al.2.15 LR-M-CARD.

Purpose: Assess the subject's ability to determine the length relation between two objects on the basis of a measurement operation involving unit iteration and comparison of cardinal numbers.

Materials: Three strips of white cardboard, one 3 x 24 cms, the other 3 x 27 cms, and the third 3 x 3 cms; pencil, rubber.
Procedure: The experimenter placed the two longer strips of cardboard on the table in the form of a "T", with the vertical arm (24 cms) pointing towards the subject, e.g:-

```
27 cm
```

```
  24 cm
   subject
```

He asked the subject to guess which one was longer, but not to move the strips. The experimenter then handed the 3 cm strip and the pencil to the subject, and said: "O.K., now let's see how good you are at guessing. Without moving these two strips, can you use this little one, and the pencil, to work out whether that one really is longer than this one? You can put marks on them with the pencil if you like."

Scoring: To pass, the subject had to iterate the unit along both longer strips, compare the cardinal numerons, and give the correct answer.

Al.2.16 L-M-ADD.

Purpose: Assess the subject's understanding that numbers representing lengths of objects may be added together, and that the resultant number represents the length of the two objects joined together.

Materials: As for LR-M-CARD.
Procedure: This task was presented as an extension of LR-M-CARD. After the subject had determined that the longer strip required nine units, and the shorter eight units - or, alternatively, had that demonstrated by the experimenter - the experimenter asked: "O.K., so it takes nine to make that one and eight to make that one. How many would it take to make a strip as long as these two put together? How did you work it out?" When working out the answer the subject was not permitted to reposition the strips, but was allowed to count.

Scoring: The subject had to demonstrate that he knew that the answer was given by adding nine and eight. Subjects demonstrating that knowledge, but having difficulty in carrying out correctly the addition operation, were encouraged to count to get the answer.

Al.2.17 L-ADD.

Purpose: Assess the subject's ability to add lengths in the following (semi-algebraic) fashion: given an ordered series, - (a), (b), (c), (d) - where the increment in length is constant, what is the relation between the combined lengths (a+c) and (b+d)?

Materials: Four pieces of dowel - green, 16.5 cms; blue, 17.0 cms; yellow, 17.5 cms; red, 18.0 cms. Four pieces of dowel of the same colours and lengths and arranged in order of increasing lengths, 2 cms apart, and mounted vertically on a piece of pine-board. One small piece of dowel, .5 cms in length.
Procedure: The experimenter showed the subject the ordered series (the standard), and asked him to construct the same series with the four sticks lying flat on the table. The experimenter called the series a stair-case, and demonstrated with the .5 cm piece that the length of each "step" was the same (i.e. \( b-a = c-b = d-c = .5 \text{ cms} \)). The experimenter asked the subject to give the first one (16.5 cm.) to the experimenter, the second (17.0 cm) to the subject, the third (17.5 cm) to the experimenter, and the fourth (18.0 cm) to the subject. The experimenter then said: "if I put my two together like this, and you put your two together, who would have the longer stick? How did you work it out?" Notice that the standard was present throughout the presentation, and that the experimenter held his two sticks in such a manner as to prevent the subject from making a visual comparison.

Scoring: The subject passed the task if he gave the correct evaluative response, and appropriate verbal justifications.

**Al.3 DISTANCE TASKS.**

**Al.3.1 D-CONS.**

Purpose: Assess the subject's ability to conserve distance.

Materials: One red and one blue plastic building block, each measuring 2x2x2 cm; a path pattern drawn on cardboard (see Figure A1.5); a strip of white cardboard 8 x 13 cm to use as a wall, with a "door" drawn on both sides.
Procedure: There were three parts to the task.

(i) The experimenter placed the two blocks on the table about 50 cms. apart, and said: "I want you to pretend that that's a red house, and that's a blue house, and that there is a path between the two houses. I want you to pretend that on Monday you walk from the red house to the blue house along the path (experimenter demonstrated with his fingers). On Tuesday you walk back from the blue house to the red house along the path, but when you get to here (20 cms from the red house), you find that somebody has put a wall across the path. But the wall has got a door in it, so you open the door, go through, and walk onto the red house. O.K., now let's see if you remember that. You show me where you walk on Monday. Good. Now, where do you walk on Tuesday? When is the wall on the path? Tuesday. Right. Would you walk further on Monday, or on Tuesday, or the same both days? Why?"

(ii) The experimenter placed the path pattern on the table and asked the subject if it would be further to walk from A to B, or further to walk from A' to B', or would it be the same distance along the respective paths. The experimenter then asked the same question but in connection with the reverse direction - i.e., comparing (B → A) with (B' → A').

(iii) The experimenter then placed the red block at A, and asked the subject to place the blue block at A'. The experimenter then asked the subject to pretend that the blocks were cars, and asked the subject to move his car along his path the same distance as the experimenter moved his car along his path. The experimenter then moved the red block along the path in increments of about 4 - 5 cms, and noted the subject's movement of his block.
Scoring: All three questions had to be answered correctly and appropriate verbal justifications given to pass.

Al.3.2 D-EST.

Purpose: Assess the subject's ability to estimate distance between two points in terms of a number of unit distances.

Materials: Three sheets of white cardboard on two of which were drawn four squares and on one of which were drawn three squares, 2x2 cms in size, as shown on Figure Al.6; four plastic building blocks, red, blue, green and yellow, each 2x2x2 cms.

Procedure: There were three questions:

(i) The first pattern was placed on the table, and blocks placed in the squares. The subject was then asked to pretend that the blocks were houses, and that there were paths between the houses. The experimenter then asked the subject to guess how many times the shorter path would go into the longer path. If "times" was not understood, he was asked how many of the shorter paths would he need to put together to make one longer path.

(ii) The second pattern was placed on the table and the same questions asked.

(iii) The third pattern was placed on the table, a red and blue block placed over the left and right squares, and a green block placed on the middle square. The subject was asked to pretend that the red and blue blocks were houses, that there was a path between the houses, and that the path was made of green blocks. The subject's task was to estimate the number of blocks needed to build the path, without moving the blocks.

Scoring: As for L-EST.
Al.3.3 D-M

Purpose: Assess the subject's ability to compare two distances indirectly on the basis of measurement operations, but not necessarily using unit iteration.

Materials: The horizontal distance plate described in D-M above, together with a vertical distance plate of the same construction, except that the pieces of dowel were separated vertically, one being directly above the other; one strip of white cardboard 3 x 35 cms; pencil; rubber. Figure Al.4 illustrates the vertical distance plate.

Procedure: The experimenter placed the two plates on the table, and said: "I want you to look at the distance between here and here (24 cms between arms on the horizontal plate) and the distance between here and here (27 cms between arms on the vertical plate). Without moving them, can you work out whether it is bigger from here to here, than from here to here? You can use this long strip if you want to. You can write on it with this pencil, if you need to."

Scoring: The subject had to use the long cardboard strip in an appropriate manner, and give the correct answer, to pass.

Al.3.4 D-M

Purpose: Assess the subject's ability to measure the distance between two objects using unit iteration.

Materials: A horizontal distance plate consisting of a wooden base to which were attached two pieces of dowel, each cm in length, projecting
out from the edge of the base 3 cm; two strips of white cardboard, one 3 x 35 cms, and the other 3 x 3 cms; pencil; rubber. The pieces of dowel were 1 cm apart and could be moved in the horizontal plane so as to adjust the distance between their end points. Figure A1.3 illustrates the apparatus.

Procedure: The experimenter set the arms of the horizontal plate so that the separation between their end points was 24 cms. He placed it on the subject's side of the table, so that the edge of the plate was coincident with the edge of the table. The experimenter then said: "I want you to pretend that you are going to build a bridge from here (a) to here (b). You want it to go exactly from here to here, so that it doesn't stick out at all. Pretend that you are going to build the bridge out of these small strips of cardboard (3 cms). Your job is to find out how many of these small strips you'll need to build the bridge. If you want to, you can use this long piece (35 cms) to help you work it out. You can use this pencil to put marks on them, if you need to."

Scoring: The subject had to use the longer strip and the shorter strip to measure the distance between the arms. The quality of the subject's attempt to iterate the unit was taken into account. Hence, a subject didn't have to be exact in marking off units to pass the task. Provided that the subject attempted to be accurate and precise in marking off units, and counting units, he passed.
HORIZONTAL MEASURING PLATE.
APPENDIX 2.

RAW DATA SCORES.
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| TOTANS |
CHI-SQUARED MATRIX - ALL TASKS.

APPENDIX 3.
### APPENDIX 3. ALL TASKS

(The names of the tasks identified by number in the matrix are given below)

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APPENDIX 3 cont.

A list of all tasks in order of increasing difficulty, as shown in the chi-squared matrix, is given below.

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APPENDIX 4.

DESCRIPTIONS OF PRODUCTION-SYSTEM MODELS.

A4.1 INTRODUCTION.

Listings and execution traces for all models are attached at Addenda 1 to 6. All models are written in PSS (Ohlsson, 1980), a variant of PSG. PSS is a Stanford LISP preprocessor.

The models were processed on a DEC-10 system at the Coombs Computer Centre, Australian National University.

In these models, goal manipulation procedures are not grouped together in one or two common-servicing productions, as is usual with production systems. Instead, they are located separately in productions which trigger particular goal activation, re-activation, suspension and deletion operations. This has been done to assist the reader, who is not familiar with LISP-type languages, to gain an appreciation of the operation of the models. This reduced the programming elegance of the models. However, it did not result in any greater demands being placed on STM, and did not increase the total number of productions fired during execution.

Firstly, the segments common to each model are described briefly. Secondly, the segments specific to each model are commented upon.
A4.2 COMMON SEGMENTS.

The productions which simulate the first phase of N-ADD-NV, N-SUB-NV, and N-CYC-NV are common to all models. In the first phase, the subject is asked to send "n" balls to the bottom jar by pressing the button. When "n" balls have been sent to the bottom jar, the subject is asked how many were there before (and/or in the top jar, depending upon the particular task), and how many more had just been sent down.

An understanding of the simulation of the first phase may be gained by reading the listing for ADD6.PSS (Addendum 1). This is a performance model of N-ADD-NV that uses a table-look-up procedure.

The productions P000, P00 and P0 control the entry of task information into STM, and initiate the run. When that information is entered, the model responds by simulating the subject's button pressing and counting behaviour. The productions responsible are labelled P1 to P8.

P1 inserts an active goal of sending "n" balls down the tube. In the service of that goal, P2 inserts the subordinate goal of pushing the button. P3 notices the ball going down the tube and inserts the subitization goal. P4 simulates subitization of the ball(s) noticed (by P3) going down the tube. P5 carries out a counting operation by accessing and "saying" the next name on a number-name-list. P6 marks the name "said" by P5, and inserts the next number name on the list into STM. P6A to P6C simulate similar marking and moving operations. P7 simulates a checking operation. If the last name "said" by P5 is
the same as the number-name given in the instruction to send "n" balls down the tube (represented in STM by \((\text{SEND C})\)), then the model "knows" that it has finished that part of the task. In that event, it re-activates the goal "to attend" and the next production to fire is be P0. If not, the goal manipulation in P8 ensures that P2 will be the next production to fire, and that a new cycle of button pressing, , subitizing, and counting will be entered. This procedure can be followed by reading the listing for each model concurrently with the trace of the model's execution.

When control is passed back to P0, the user then asks: how many balls were in the bottom jar before? This is represented by the STM element \((\text{MANY BOTTOM BEFORE})\). P9 and P10 simulate the answering of this question, after which control is passed back to P0. The user then asks: how many were sent down the tube? This is represented in STM by the element \((\text{MANY TUBE})\). This question is answered by P11 and P12, after which control is again passed back to P0. The user then asks: how many balls are now in the bottom jar? This is represented in STM by \((\text{MANY BOTTOM NOW})\). That marks the end of the first phase.

In the subtraction models (ADD5.PSS and ADD4.PSS listed in Addenda 4 and 5, respectively), P9 and P10 answer the question: how many were in the top jar? This is represented in STM by the element \((\text{MANY TOP BEFORE})\). In the addition and subtraction model (ADD7.PSS listed in Addendum 6), P9 and P10 answer the \((\text{MANY BOTTOM BEFORE})\) question, and P49 and P410 answer the \((\text{MANY TOP BEFORE})\) question.

The segments specific to each model are described briefly in the following sections.
A4.3 TABLE-LOOK-UP PROCEDURES.

The addition model (ADD6.PSS) and the subtraction model (ADD5.PSS) use table-look-up procedures. These procedures are also used in the addition and subtraction model (ADD7.PSS).

A4.3.1 ADDITION.

An understanding of the addition procedure may be gained by reading the listing for ADD6.PSS, in conjunction with the trace of its execution (Addendum 1).

By entering the question (MANY BOTTOM NOW) into STM the user causes control, on the next cycle, to be given to P13, which activates the addition goal (GOAL * ADD). P14 activates the goal of retrieving from LTM the list containing the results of adding 'n' balls to the 'm' already in the bottom jar. P15 and P15C simulate the entry of those lists to STM.

P16 'reads off' the answer. For example, suppose the bottom jar contained two balls and the subject sent four more down the tube. Then P14 would activate the goal of retrieving from LTM the relevant list (LIST 2), and P15A would insert the entries (1->3),(2->4),(3->5),(4->6),etc. P16 would select the entry (4->6), and extract from it the answer, 6.

The remaining productions, P24A to P27 are responsible for housekeeping functions needed to prepare the model to receive more input.
A4.3.2 SUBTRACTION.

An understanding of the subtraction procedure may be gained by reading the listing for ADD5.PSS in conjunction with the trace of its execution (Addendum 4).

Productions P14 to P16 are responsible for the subtraction table-look-up procedure. They are similar in form to their counterparts in the addition model. Hence, an example should be sufficient to explain their function.

Suppose the top jar had six balls in it, and then the subject sent three balls to the bottom jar. P14 would activate the goal of retrieving (LIST 6). P15A would insert the entries from that list, such as (2->4),(3->3), etc. P16 would select the entry (3->3), and "read-off" the answer, 3.

A4.4 DIRECT COUNTING PROCEDURE.

The addition model, ADD8.PSS, and the subtraction model, ADD4.PSS, use direct counting procedures. They are used instead of table-look-up methods for simulating the answers to the (MANY BOTTOM NOW) and (MANY TOP NOW) questions.
The direct counting procedure involves the co-ordination of step-by-step movement through two sequences, each of which is the number-name-list. Items from the first sequence are represented in STM by elements having the form \((Y <P> Z)\), and those from the second sequence by \((W <P> V)\). The letters \(Y, Z, W\) and \(V\) imbedded in these elements are of technical significance only. They constitute a method of marking locations in a list. The symbol \(<P>\) assumes numerical values.

Pl3 activates the addition goal (GOAL * ADD). It sets the location in the first sequence at the point corresponding to the number of balls in the bottom jar before the last series of button presses. Pl3 also sets the location in the second sequence, just before the first element in the number-name-list. Pl4 checks to see if the location in the second sequence is the same as the value in the STM element representing the instruction to send "n" balls down the tub. (Essentially, is the value of \(<P>\) in the element \((W <P> V)\) the same as the value of \(<P>\) in the element \((SEND <P>)\)?) If so, Pl4 fires, and the answer is extracted from the \((Y <P> Z)\) element currently in STM. If not, either Pl5 or Pl6 fires. They control the movement through the two sequences. Pl7 to Pl7J carry out the moves from place to place in the first sequence. Pl8 to Pl8C perform the same function for the second sequence.

An example may clarify the operation of Pl3 to Pl8C. Suppose that the bottom jar had two balls in it, and the subject sent down four more. STM would contain the elements (BOTTOM 2) and (SEND 4). Pl3 would insert the ele-
ments \((Y 2 Z)\) and \((W 0 V)\). On the first cycle after the firing of P13, P14 would not fire because \(\llcorner P \lrcorner\) would be set to 0 in \((W 0 V)\). P15 would initiate an entry to the P17 to P17J group of productions. Specifically, P17B would fire and insert \((Y 3 Z)\) into STM, and set a goal causing P16 to fire on the next cycle. P16 would then initiate an entry to the P18 to P18C group of productions. Specifically, P18 would fire and insert \((W 1 V)\) into STM, and set a goal causing P14's conditions to be examined on the next cycle. Again, P14 would not fire, because \(\llcorner P \lrcorner\) would be set to 1 in \((W 1 V)\). Hence, P15 would fire again, and the P15 to P18C procedure would be re-entered. This pattern would continue until on one cycle P14 found \(\llcorner P \lrcorner\) set to 4 in \((W 4 V)\). At that time, the \((Y <P> Z)\) element would contain \((Y 6 Z)\). P14 would then extract the answer (6) from that element.

The remaining productions P24A to P27 perform housekeeping functions needed to prepare the model to receive further input.

A4.4.2 SUBTRACTION.

The productions P13 to P18C in the subtraction model (ADD4.PSS) carry out analogous functions. The differences are that:

- P13 sets \(\llcorner P \lrcorner\) in \((Y <P> Z)\) to equal the number of balls in the top jar, initially.
- P17 to P17J moves down the number-name-list, not up it, as in the case of the addition model (ADD8.PSS).
The addition model ADD3.PSS uses a modified-counting procedure. In this model, the larger of the two numbers to be added (represented in STM by the elements (BOTTOM B) and (SEND C) is found, and used to set the initial location in the first sequence. Groen and Parkman (1972) argued that this procedure is used by young children, and some adults.

In this model, productions P13 to P16 control the finding of the smaller and larger of the two addends. Productions P17 to P17K simulate movement through the first sequence. Productions P230 to P230C simulate movement through the second sequence. Productions P180 to P210 extract the smaller and larger addends and set the initial locations in the two sequences. P220 fires when movement through the two sequences has resulted in the answer being held in the \((Y \lessdot P > Z)\) STM element. The remaining productions simulate either special cases or housekeeping functions. The two special cases are:

- equal addends - P130A
- one addend of zero - P130B
APPENDIX 5.

COMPARATIVE ANALYSIS OF RESULTS USING ASSESSMENT CRITERIA VARYING IN DEGREE OF STRICTNESS.

A5.1 CORRELATION BETWEEN NUMBER OF QUESTIONS ASKED AND ORDER OF TASK DIFFICULTY.

The analysis in Chapter 10 was based on a strict assessment criterion. In order to pass a task the child had to answer every question correctly and, in many instances, give a correct verbal statement of the reasoning employed. However, every child was also given a number of opportunities to reconsider an incorrect answer. The child was prodded with comments like: "Are you sure? Think about it again. Slowly this time, and see if you get the same answer." Thus, a clinical style of questioning was used to assess the child's knowledge.

An important aspect of this approach is that there is substantial variation between tasks in the number of questions asked. This is because it was necessary to relate the number and type of question asked to the nature of the task. Some tasks (eg. DR-M) required the child to demonstrate by physical action certain kinds of knowledge. Other tasks required only an evaluative answer (eg. "longer" or "shorter"), supported by verbal statements of the reasoning employed. Still other tasks required knowledge of number facts within a certain range.
In these circumstances, it may be thought that the observed orders of difficulty of the collections of number and length tasks reported in Chapter 10 could reflect simply the variation between tasks in the number of questions asked. A correlation analysis was carried out to investigate this possibility.

A5.1.1 NUMBER TASKS

In the case of the number tasks, the number of questions asked ranged from 2 to 8. The degree of association between the number of questions asked and the rank order of difficulty was assessed by computing the Spearman rank correlation statistic Rs (corrected for ties), which is .37. This is not significant at the .05 level (sample t=1.32, criterion t=1.8 at alpha = .05 and 11 d.f. for a one tailed test).

A5.1.2 LENGTH TASKS

In the collection of length tasks, the number of questions asked ranged from 1 to 10. The relevant Spearman rank correlation coefficient is -.42. This is significant at the .05 level (sample t=1.8, criterion t=1.75, at alpha=.05 and 15 d.f. for a one tailed test) but the direction of association between number of questions and task difficulty is the opposite to that which might have been expected. In other words, this analysis reveals that the fewer the number of questions the greater the difficulty of the task.

It may be concluded, therefore, that the orders of difficulty for the number and length task collections do not reflect simply variations in the number of questions asked.
A5.2 EFFECT OF VARYING STRINGENCY OF SCORING CRITERION ON ORDER OF TASK DIFFICULTY.

This conclusion notwithstanding, it may still be objected that the use of a strict assessment criterion in developmental research is inappropriate because, as Brainerd (1978) argues, it increases the risk of false negatives. By the same token, the use of a lax criterion increases the risk of false positives. As Langford (1981) has noted, the former is a fairer test of hypotheses concerning synchronous development, while the latter is more effective in detecting sequential development. In order to gauge the effect of the use of a strict criterion in the present research, the data were also scored using "moderate" and "weak" criteria.

A5.2.1 MODERATE CRITERION.

Under the moderate criterion a child was assessed as having passed the task if he answered the first evaluative question correctly, and gave the correct verbal explanation. For those tasks where verbal explanations were not requested the child was required to answer the first two evaluative questions correctly.

A5.2.2 WEAK CRITERION.

Under the weak criterion a child was assessed as having passed the task if he gave the correct evaluative response to the first question asked.

A5.2.3 DISTRIBUTION OF TOTAL SCORES.

The distribution of total scores for all subjects, and of the number of subjects passing each task, for both moderate and weak scoring procedures,
are depicted in Figures A5.1 to A5.4. When these distributions are compared with the corresponding distributions obtained under the strict scoring regime (given at Figures 10.1 and 10.2, pages 126a and b of the main text), it is apparent that the minor floor effect evident under the strict criterion assessment is exaggerated. For example, under the weak criterion 64% of the subjects scored 24 or more (out of 34); under the moderate criterion, 48% scored 24 or more; and under the strict criterion, 36% scored 24 or more. This outcome is, of course, predictable, since increasing the probability of false positives increases the probability of a floor effect.

A5.2.4 ORDER OF TASK DIFFICULTY

The orders of difficulty obtained under each scoring procedure are given in Table A5.1 for all tasks, in A5.2 for the number tasks, in A5.3 for the length tasks, and in A5.4 for the distance tasks. The general impression obtained from inspection of these tables is that a similar ranking is obtained under all three procedures. This impression is confirmed by the Spearman rank correlation statistics given in Table A5.5. These values indicate that for all tasks taken as one collection, for the collection of number tasks, and for the collection of length tasks, the rank orderings obtained under the strict, moderate and weak criteria are not significantly different. The orderings obtained under the three scoring procedures are significantly (alpha=.05) different, however, for the collection of distance tasks.

The differences for the distance tasks are due to the small number of tasks involved. Since the collection contains only four tasks, a reversal of order between two adjacently ranked tasks, which may differ only slightly in the number of subjects passing each, has the effect of changing 50% of the ranks. This effect is obvious in the case of DR-M and D-QNS.
FIGURE A5.1

DISTRIBUTION OF TOTAL SCORES

MODERATE CRITERION

Each bar indicates no. of subjects achieving a total score of at least $N$. 

No. of Subjects

Total Scores

N

39 38 37 36 35 34 33 32 31 30 29 28 27 26 25 24 23 22 21 20 19 18 17 16 15 14 13 12 11 10 9 8 7 6

297
FIGURE A5.2

DISTRIBUTION OF TASK DIFFICULTY
MODERATE CRITERION

No. of Subjects Passing

100

80

60

40

20

0

TASGS
FIGURE A5.6

DISTRIBUTION OF TOTAL SCORES.

WEAK CRITERION

Each bar indicates no. of subjects achieving a total score of at least N.
Under the strict scoring criterion 53 subjects passed the former while 48 passed the latter, yielding ranks of 1 and 2, respectively. Under the moderate scoring criterion, 53 subjects passed DR-M and 65 passed D-CONS, yielding a reversal of order in ranking.

<table>
<thead>
<tr>
<th>TABLE A5.1: ALL TASKS: ORDER OF TASK DIFFICULTY OBTAINED UNDER THE STRICT, MODERATE AND WEAK SCORING CRITERIA.</th>
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* Observed number of subjects passing the task.

NOTES: *Maximum of 100
### TABLE A5.2: NUMBER TASKS: ORDER OF TASK DIFFICULTY OBTAINED UNDER THE STRICT, MODERATE AND WEAK SCORING CRITERIA.

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<th>MODERATE CRITERION</th>
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<td>58</td>
<td>8</td>
<td>58</td>
<td>8</td>
<td>68</td>
</tr>
<tr>
<td>N-ADD-NV</td>
<td>58</td>
<td>8</td>
<td>63</td>
<td>7</td>
<td>77</td>
</tr>
<tr>
<td>N-TI-NE</td>
<td>41</td>
<td>9</td>
<td>43</td>
<td>9</td>
<td>77</td>
</tr>
<tr>
<td>N-SUB-NV</td>
<td>21</td>
<td>10</td>
<td>22</td>
<td>10</td>
<td>27</td>
</tr>
<tr>
<td>N-CYC-NV</td>
<td>16</td>
<td>11</td>
<td>18</td>
<td>11</td>
<td>18</td>
</tr>
</tbody>
</table>

**NOTES:** * Maximum of 100.
### TABLE A5.3: LENGTH TASKS: ORDER OF TASK DIFFICULTY OBTAINED UNDER THE STRICT, MODERATE AND WEAK SCORING CRITERIA.

<table>
<thead>
<tr>
<th>TASK</th>
<th>STRICT CRITERION</th>
<th>MODERATE CRITERION</th>
<th>WEAK CRITERION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NO. OF Ss.</td>
<td>RANK</td>
<td>NO. OF Ss. RANK</td>
</tr>
<tr>
<td></td>
<td>PASSING *</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LR-TI-DQ</td>
<td>100</td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>LR-CARD</td>
<td>98</td>
<td>2</td>
<td>98</td>
</tr>
<tr>
<td>LR-BinA</td>
<td>95</td>
<td>3</td>
<td>96</td>
</tr>
<tr>
<td>L-P/W</td>
<td>94</td>
<td>4</td>
<td>94</td>
</tr>
<tr>
<td>LR-INVAR-ADD</td>
<td>85</td>
<td>5</td>
<td>92</td>
</tr>
<tr>
<td>LR-ORD</td>
<td>82</td>
<td>6</td>
<td>86</td>
</tr>
<tr>
<td>L-INVAR-ADD</td>
<td>80</td>
<td>7</td>
<td>88</td>
</tr>
<tr>
<td>LR-INVAR-SP</td>
<td>74</td>
<td>8</td>
<td>77</td>
</tr>
<tr>
<td>L-CONS</td>
<td>74</td>
<td>8</td>
<td>74</td>
</tr>
<tr>
<td>L-EST</td>
<td>56</td>
<td>9</td>
<td>66</td>
</tr>
<tr>
<td>L-UNIT</td>
<td>53</td>
<td>10</td>
<td>53</td>
</tr>
<tr>
<td>L-UNIT-CH</td>
<td>49</td>
<td>11</td>
<td>54</td>
</tr>
<tr>
<td>LR-TI-CARD</td>
<td>48</td>
<td>12</td>
<td>48</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>29</td>
<td>13</td>
<td>34</td>
</tr>
<tr>
<td>LR-M-CARD</td>
<td>14</td>
<td>14</td>
<td>14</td>
</tr>
<tr>
<td>L-M-ADD</td>
<td>13</td>
<td>15</td>
<td>13</td>
</tr>
<tr>
<td>L-ADD</td>
<td>10</td>
<td>16</td>
<td>10</td>
</tr>
</tbody>
</table>

**NOTES:** *Maximum of 100.*
### TABLE A5.4: DISTANCE TASKS: ORDER OF TASK DIFFICULTY OBTAINED UNDER THE STRICT, MODERATE AND WEAK SCORING CRITERIA.

<table>
<thead>
<tr>
<th>TASK</th>
<th>STRICT CRITERION</th>
<th>MODERATE CRITERION</th>
<th>WEAK CRITERION</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>NO. OF Ss. PASSING</td>
<td>RANK</td>
<td>NO. OF Ss. PASSING</td>
</tr>
<tr>
<td>DR-M</td>
<td>53</td>
<td>1</td>
<td>53</td>
</tr>
<tr>
<td>D-CONS</td>
<td>48</td>
<td>2</td>
<td>60</td>
</tr>
<tr>
<td>D-EST</td>
<td>34</td>
<td>3</td>
<td>45</td>
</tr>
<tr>
<td>D-M</td>
<td>26</td>
<td>4</td>
<td>26</td>
</tr>
</tbody>
</table>

NOTES: * Maximum of 100

### TABLE A5.5: COMPARISON OF RANK ORDERINGS OBTAINED UNDER THE STRICT, MODERATE AND WEAK SCORING CRITERIA.

<table>
<thead>
<tr>
<th>TASK COLLECTION</th>
<th>RANK ORDERINGS COMPARED</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STRict WITH MODERATE</td>
</tr>
<tr>
<td></td>
<td>Rs</td>
</tr>
<tr>
<td>ALL TASKS</td>
<td>.989</td>
</tr>
<tr>
<td>NUMBER TASKS</td>
<td>.997</td>
</tr>
<tr>
<td>LENGTH TASKS</td>
<td>.99</td>
</tr>
<tr>
<td>DISTANCE TASKS</td>
<td>.80</td>
</tr>
</tbody>
</table>
A5.3 EFFECT OF VARYING STRINGENCY OF SCORING CRITERION ON
SCALABILITY OF NUMBER AND LENGTH TASK COLLECTIONS.

Since it is evident that adopting a less stringent criterion has the
effect of distorting the distribution of total scores, though without
altering significantly overall task difficulty ranking, the possibility was
investigated that the number and length task collections may not form scaled
sets under the moderate and weak scoring criteria. The results are summar­
ised in Table A5.6 which sets out the relevant Guttman and Loewinger indices.
If more weight is accorded the Loewinger index of homogeneity than the
Guttman co-efficient of scalability, the effect of adopting a less string­
ent criterion is to reduce marginally overall test homogeneity, and to
increase the incidence of chance level responding. However, as is shown
in Table A5.6, the number and length task collections form scaled sets,
whether assessment is based upon a strict, moderate or weak criterion.
### TABLE A5.6: SCALING INDICES OBTAINED FROM DATA DERIVED FROM STRICT, MODERATE AND WEAK SCORING CRITERIA.

<table>
<thead>
<tr>
<th>TASK</th>
<th>CRITERION</th>
<th>GUTTMAN CO-EFFICIENT OF SCALABILITY</th>
<th>LOEVINGER INDEX OF HOMOGENEITY</th>
<th>LOEVINGER INDEX OF HOMOGENEITY OF AN ITEM WITH A TEST</th>
<th>LOEVINGER INDEX OF HOMOGENEITY OF AN ITEM WITH AN ITEM</th>
</tr>
</thead>
<tbody>
<tr>
<td>NUMBER</td>
<td>STRICT</td>
<td>.589</td>
<td>.570</td>
<td>ALL 13 EXCEED .7 (a)</td>
<td>70 OUT OF 78 EXCEED .5 (b)</td>
</tr>
<tr>
<td></td>
<td>MODERATE</td>
<td>.568</td>
<td>.685</td>
<td>ALL 13 EXCEED .7</td>
<td>69 OUT OF 78 EXCEED .5</td>
</tr>
<tr>
<td></td>
<td>WEAK</td>
<td>.401</td>
<td>.540</td>
<td>12 OUT OF 13 EXCEED .7 (c)</td>
<td>56 OUT OF 78 EXCEED .5</td>
</tr>
<tr>
<td>LENGTH</td>
<td>STRICT</td>
<td>.48</td>
<td>.58</td>
<td>14 OUT OF 17 EXCEED .7</td>
<td>101 OUT OF 136 EXCEED .5</td>
</tr>
<tr>
<td></td>
<td>MODERATE</td>
<td>.455</td>
<td>.563</td>
<td>13 OUT OF 17 EXCEED .7</td>
<td>93 OUT OF 136 EXCEED .5</td>
</tr>
<tr>
<td></td>
<td>WEAK</td>
<td>.431</td>
<td>.539</td>
<td>16 OUT OF 17 EXCEED .7 (d)</td>
<td>98 OUT OF 136 EXCEED .5</td>
</tr>
</tbody>
</table>

**NOTES:**
(a) A perfectly homogeneous item would have a \( H(it) \) value of 1 but values of .7 and higher are regarded as acceptable.
(b) Chance level responding is indicated by a \( h(ii) \) value of .5.
(c) The \( H(it) \) for N-TI-NE = .66.
(d) The \( H(it) \) for LR-TI-NE = .33.
Much of the controversy surrounding the use of a strict or weak scoring criterion has centred on the classical Piagetian tests for transitivity and conservation. Therefore, the effect of varying scoring criterion on those tasks was examined separately from the issue of overall task difficulty ranking. Table A5.7 sets out the number of subjects passing the various conservation and transitivity tests under each scoring procedure. When the results obtained under the strict criterion are compared with those for the moderate criterion, the same orderings of task difficulty and the same differences in task difficulty - with the exception of D-CONS - emerge. In the case of D-CONS, an analysis of subject protocols revealed that the majority of those failing that task had most difficulty with the second and third variant of the task, rather than in giving verbal accounts of the reasoning involved.

If the orderings of task difficulty obtained under the weak criterion are compared with those gained under the strict and moderate procedures, large differences emerge for the transitivity but not the conservation tasks. This is most evident in the case of LR-TI-NE. Assessed under the strict criterion only 29 subjects passed the task, under the moderate criterion 34 passed, but under the weak criterion 90 passed. This indicates the difficulty subjects experienced in giving a verbal account of the reasoning they employed in reaching their answers.
TABLE A5.7: NUMBER OF SUBJECTS PASSING CONSERVATION AND TRANSITIVITY TASKS ACCORDING TO SCORING CRITERION USED.

<table>
<thead>
<tr>
<th>TASKS</th>
<th>NO. OF SUBJECTS PASSING</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>STRICT CRITERION</td>
</tr>
<tr>
<td>N-CONS</td>
<td>78</td>
</tr>
<tr>
<td>N-TI-NE</td>
<td>41</td>
</tr>
<tr>
<td>L-CONS</td>
<td>74</td>
</tr>
<tr>
<td>LR-TI-CARD</td>
<td>48</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>29</td>
</tr>
<tr>
<td>D-CONS</td>
<td>48</td>
</tr>
</tbody>
</table>

A5.5 SUMMARY OF EFFECTS OF VARYING STRINGENCY OF SCORING CRITERION ON TASK DIFFICULTY RANKINGS.

The main findings of the analysis based upon the strict scoring criterion are summarised at Section 10.11 (pages 152 to 154) of the main report, under the headings "Components of Linear Measurement", "Order of Development of Linear Measurement", and "Expected Pattern of Development." It will be apparent from the immediately preceding discussion of the effects of varying scoring criterion, that the conclusions reported in Chapter 10 do not rest solely, or even in large measure, upon the particular scoring approach adopted.
The conclusions reached in Chapter 10 were further examined in Chapter 11. This examination focussed, in the main, on the delay between acquisition of the assumed components of linear measurement and the emergence of an understanding of linear measurement. That examination also highlighted the discontinuities in the growth of the number and length concept domains. The extent to which these findings depend upon the scoring criterion used should also be examined.

A5.6.1 DELAY IN ATTAINMENT OF LINEAR MEASUREMENT.

Regarding the components of linear measurement, an inspection of Table A5.1 indicates that the components listed at Section 11.7 (page 180) of the main text (derived from the strict scoring criterion) are the same as those which would be yielded by an analysis based on moderate criterion. If a weak criterion were to be used then "knowing how to make transitive inferences of non-equivalence, with respect to length" would have to be added to that list.

Additionally, as noted at page 163 of the main text, under the strict scoring criterion it was found that 13 subjects passed all high-order component tasks but failed the two linear measurement benchmark tasks, LR-M-CARD and L-M-ADD. This was interpreted as indicating a delay between acquiring the underlying components and being able to demonstrate a mature understanding of linear measurement. The same interpretation could be made under the moderate and weak scoring procedures. Using the former, 22 subjects passed all high-order components but failed LR-M-CARD and L-M-ADD, whilst, using the latter, 29 passed the component tasks but failed the two benchmark tasks.
A5.6.2 DISCONTINUITIES IN GROWTH PATTERNS.

Regarding the discontinuities in the growth of the number and length concept domains, Tables A5.8 to A5.11 set out the orders of emergence of the assumed components of the number and length concepts, assessed under moderate and weak criteria. These tables should be compared with Tables 11.1 (page 167) and 11.3 (page 174) in the main text.

Considering the number concept first, under the strict scoring criterion a stepped performance gradient having five levels was detected. Under the moderate criterion, as is indicated by the chi-squared values in Table A5.8, a stepped performance gradient is also apparent but the five levels are reduced to four. Under the weak criterion (Table A5.9), the stepped performance gradient is reduced to three levels.

In the case of the length concept, under the strict criterion four levels of a stepped performance gradient were detected. Under both moderate (Table A5.10) and weak (Table A5.11) scoring criterion the stepped performance gradient is reduced to three levels.
<table>
<thead>
<tr>
<th>TASK</th>
<th>NO. OF SUBJECTS</th>
<th>McNEMAR CHI-SQUARED VALUES</th>
<th>P.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>PASSING</td>
<td></td>
<td></td>
</tr>
<tr>
<td>N-CNT</td>
<td>100</td>
<td>0.00</td>
<td>NS</td>
</tr>
<tr>
<td>N-TI-EQ</td>
<td>100</td>
<td>13.90</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>N-1tol</td>
<td>87</td>
<td>0.17</td>
<td>NS</td>
</tr>
<tr>
<td>N-ADD-V</td>
<td>85</td>
<td>0.33</td>
<td>NS</td>
</tr>
<tr>
<td>N-SUB-V</td>
<td>82</td>
<td>0.29</td>
<td>NS</td>
</tr>
<tr>
<td>N-ORD</td>
<td>79</td>
<td>0.00</td>
<td>NS</td>
</tr>
<tr>
<td>N-CONS</td>
<td>79</td>
<td>2.13</td>
<td>NS</td>
</tr>
<tr>
<td>N-SOL-V</td>
<td>70</td>
<td>1.10</td>
<td>NS</td>
</tr>
<tr>
<td>N-ADD-NV</td>
<td>63</td>
<td>0.52</td>
<td>NS</td>
</tr>
<tr>
<td>N-BAL-V</td>
<td>58</td>
<td>4.50</td>
<td>&lt;.05</td>
</tr>
<tr>
<td>N-TI-NE</td>
<td>43</td>
<td>10.05</td>
<td>&lt;.005</td>
</tr>
<tr>
<td>N-SUB-NV</td>
<td>22</td>
<td>0.50</td>
<td>NS</td>
</tr>
<tr>
<td>N-CYC-NV</td>
<td>18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
TABLE A5.9  NUMBER TASKS -WEAK CRITERION: CHI-SQUARED VALUES FOR ADJACENTLY RANKED ITEM PAIRS.

<table>
<thead>
<tr>
<th>TASKS</th>
<th>NO. OF SUBJECTS</th>
<th>McNEMAR CHI-SQUARED VALUES</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>N-CNT</td>
<td>100</td>
<td>0.0</td>
<td>NS</td>
</tr>
<tr>
<td>N-TI-EQ</td>
<td>100</td>
<td>7.25</td>
<td>&lt;.01</td>
</tr>
<tr>
<td>N-ADD-V</td>
<td>93</td>
<td>2.00</td>
<td>NS</td>
</tr>
<tr>
<td>N-1to1</td>
<td>87</td>
<td>0.36</td>
<td>NS</td>
</tr>
<tr>
<td>N-SUB-V</td>
<td>84</td>
<td>0.31</td>
<td>NS</td>
</tr>
<tr>
<td>N-CONS</td>
<td>81</td>
<td>0.03</td>
<td>NS</td>
</tr>
<tr>
<td>N-ORD</td>
<td>80</td>
<td>0.27</td>
<td>NS</td>
</tr>
<tr>
<td>N-ADD-NV</td>
<td>77</td>
<td>0.00</td>
<td>NS</td>
</tr>
<tr>
<td>N-TI-NE</td>
<td>77</td>
<td>0.03</td>
<td>NS</td>
</tr>
<tr>
<td>N-SOL-V</td>
<td>76</td>
<td>1.59</td>
<td>NS</td>
</tr>
<tr>
<td>N-BAL-V</td>
<td>68</td>
<td>33.70</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>N-SUB-NV</td>
<td>27</td>
<td>2.32</td>
<td>NS</td>
</tr>
<tr>
<td>N-CYC-NV</td>
<td>18</td>
<td></td>
<td></td>
</tr>
<tr>
<td>TASK</td>
<td>NO. OF SUBJECTS PASSING</td>
<td>McNEMAR CHI-SQUARED VALUES</td>
<td>P</td>
</tr>
<tr>
<td>-----------------</td>
<td>--------------------------</td>
<td>-----------------------------</td>
<td>-----</td>
</tr>
<tr>
<td>LR-TI-EQ</td>
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<td>2.02</td>
<td>NS</td>
</tr>
<tr>
<td>LR-CARD</td>
<td>98</td>
<td>0.69</td>
<td>NS</td>
</tr>
<tr>
<td>LR-BinA</td>
<td>95</td>
<td>0.42</td>
<td>NS</td>
</tr>
<tr>
<td>L-P/W</td>
<td>94</td>
<td>0.31</td>
<td>NS</td>
</tr>
<tr>
<td>LR-INVAR-ADD</td>
<td>92</td>
<td>0.89</td>
<td>NS</td>
</tr>
<tr>
<td>L-INVAR-ADD</td>
<td>88</td>
<td>0.18</td>
<td>NS</td>
</tr>
<tr>
<td>LR-ORD</td>
<td>86</td>
<td>2.69</td>
<td>NS</td>
</tr>
<tr>
<td>LR-INVAR-SP</td>
<td>77</td>
<td>0.24</td>
<td>NS</td>
</tr>
<tr>
<td>L-CONS</td>
<td>74</td>
<td>1.52</td>
<td>NS</td>
</tr>
<tr>
<td>L-EST</td>
<td>66</td>
<td>3.00</td>
<td>NS</td>
</tr>
<tr>
<td>L-UNIT-CH</td>
<td>54</td>
<td>0.02</td>
<td>NS</td>
</tr>
<tr>
<td>L-UNIT</td>
<td>53</td>
<td>0.50</td>
<td>NS</td>
</tr>
<tr>
<td>LR-TI-CARD</td>
<td>48</td>
<td>4.05</td>
<td>&lt;.05</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>34</td>
<td>10.96</td>
<td>&lt;.005</td>
</tr>
<tr>
<td>LR-M-CARD</td>
<td>14</td>
<td>0.04</td>
<td>NS</td>
</tr>
<tr>
<td>L-M-ADD</td>
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<td>0.44</td>
<td>NS</td>
</tr>
<tr>
<td>L-ADD</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
### TABLE A5.11

LENGTH TASKS - WEAK CRITERION: CHI-SQUARED VALUES FOR ADJACENTLY RANKED ITEM PAIRS.

<table>
<thead>
<tr>
<th>TASK</th>
<th>NO. OF SUBJECTS PASSING</th>
<th>MONEMAR CHI-SQUARED VALUES</th>
<th>P</th>
</tr>
</thead>
<tbody>
<tr>
<td>LR-TI-EQ</td>
<td>100</td>
<td>1.01</td>
<td>NS</td>
</tr>
<tr>
<td>LR-CARD</td>
<td>99</td>
<td>1.02</td>
<td>NS</td>
</tr>
<tr>
<td>LR-Bina</td>
<td>97</td>
<td>1.05</td>
<td>NS</td>
</tr>
<tr>
<td>L-P/W</td>
<td>94</td>
<td>0.00</td>
<td>NS</td>
</tr>
<tr>
<td>LR-INVAR-ADD</td>
<td>94</td>
<td>0.08</td>
<td>NS</td>
</tr>
<tr>
<td>L-INVAR-ADD</td>
<td>93</td>
<td>0.07</td>
<td>NS</td>
</tr>
<tr>
<td>LR-ORD</td>
<td>92</td>
<td>0.24</td>
<td>NS</td>
</tr>
<tr>
<td>LR-TI-NE</td>
<td>90</td>
<td>0.44</td>
<td>NS</td>
</tr>
<tr>
<td>LR-TI-CARD</td>
<td>87</td>
<td>0.95</td>
<td>NS</td>
</tr>
<tr>
<td>LR-INVAR-SP</td>
<td>82</td>
<td>0.13</td>
<td>NS</td>
</tr>
<tr>
<td>L-EST</td>
<td>80</td>
<td>0.47</td>
<td>NS</td>
</tr>
<tr>
<td>L-CONS</td>
<td>76</td>
<td>0.10</td>
<td>NS</td>
</tr>
<tr>
<td>L-UNIT-CH</td>
<td>54</td>
<td>10.64</td>
<td>&lt;.005</td>
</tr>
<tr>
<td>L-UNIT</td>
<td>53</td>
<td>0.02</td>
<td>NS</td>
</tr>
<tr>
<td>LR-M-CARD</td>
<td>14</td>
<td>34.14</td>
<td>&lt;.001</td>
</tr>
<tr>
<td>L-M-ADD</td>
<td>13</td>
<td>0.04</td>
<td>NS</td>
</tr>
<tr>
<td>L-ADD</td>
<td>10</td>
<td>0.44</td>
<td>NS</td>
</tr>
</tbody>
</table>

### A5.6.3 SUMMARY

In summary, the two main conclusions reached in Chapter 11 - delay between acquisition of assumed components and attainment of concept, and discontinuities in growth of the number and length concept domains - are essentially unaffected by adopting less stringent criteria for identifying item pairs.
Finally, on the basis of the evidence discussed in this Appendix, it is reasonable to conclude that the results and findings of the present research as presented in Chapters 10 and 11 of the main text, do not stem in any significant fashion from the use of a strict scoring criterion incorporating wide variation between tasks in the number and type of question asked.
APPENDIX 6.

SPECULATIVE OUTLINE OF FURTHER WORK ON PRODUCTION SYSTEM MODELLING OF THE DEVELOPMENT OF LINEAR MEASUREMENT.

A6.1 INTRODUCTION.

The production systems described in Chapter 13 were constructed largely as a means of testing a STM hypothesis concerning developmental discontinuities. It was also stated that those systems constituted a beginning of a much larger project aimed at constructing a production system model of the development of iterative linear measurement. This Appendix outlines briefly the direction in which such further work could proceed.

A production system model of the development of linear measurement should exhibit the developmental patterns observed in this study. For example, the model should display a capacity to conserve length before it demonstrates a capacity to 'construct' a unit of length for purposes of iteration. A separate sub-system would be constructed for each assumed component of linear measurement. These sub-systems would be state models constituting a developmental hierarchy. The major task would then be to construct a learning, or transition, model which accounted for that hierarchy, and, in particular, for the delay between acquisition of the assumed components and eventual attainment of iterative linear measurement.
A6.2 TYPES OF STATE MODEL REQUIRED.

The nature of these state models of the assumed components of linear measurement requires some elaboration. As indicated in Chapter 2, there is a sense in which, for both number and length, conservation implies transitive reasoning, and transitive reasoning implies conservation. Reflecting that observation, a state model of conservation could provide explicitly for the activation of productions encoding transitive reasoning rules, and a state model of transitive reasoning could provide explicitly for the processing of conservation rules.

Although such models might provide an admirable formal specification of conservation and transitivity, it is unlikely that they would provide an accurate description of the cognitive processes actually invoked by a subject in making a conservation response, or responding correctly to a transitive reasoning task. This is because it is not usually the case that the conserving subject, for example, resorts to "first principles" in reaching his decision. Instead, it seems that he relies upon a general rule covering a certain class of transformation. First principle statements of the kind that are necessary in the formal derivation of, say, conservation are only given if the subject is required to justify, post hoc, his answer. Piaget and Inhelder (1969) capture this distinction by asserting that conservation is a matter of logical necessity not empirical determination.
Hence, for both number and length, it would seem that, in respect of both conservation and transitive reasoning, at least two types of state model are required: one that contains first principle rules; and one that contains only a general rule. The nature of the developmental relationship between these models should be consistent with the empirical evidence. For example, for both number and length, the first conservation model would incorporate explicit rules concerning transitive processing of equivalence relations (because, in this study, the order of appearance was the capacity assessed by N-TI-EQ then that assessed by N-CONS; LR-TI-EQ then L-CONS). The second conservation model would not contain these rules. In two senses, therefore, the second model would be more mature: firstly, it would appear later in development; and, secondly, it would have had redundant processing - the first principle rules - removed from it, and, to that extent, it would be more efficient. That greater efficiency would be expressed behaviourally in much faster response times. Consequently, the impression would be conveyed that the subject was making a logical assertion, rather than expressing a ponderously determined decision concerning an empirical regularity. It should also be noted that the first model - the one concerning the first principle rules - may not be expressed behaviourally before the subject’s responses provide evidence that the second model has been constructed. This is so even though the former is cast as the developmental precursor of the latter. Indeed, the first model may only ever be expressed behaviourally when the subject is required to justify his answers. This point will be returned to in later paragraphs.
This speculation concerning the two-phase development of conservation is also relevant to the development of linear measurement, as assessed by LR-M-CARD. When a subject solves the linear measurement problem represented by that task, it is extremely unlikely that, in the absence of prior, detailed and complete instruction, he would invoke the conservation and/or transitive reasoning models as part of his solution strategy. That is to say, the subject would not ordinarily work out LR-M-CARD from first principles. The conservation and transitive reasoning models would only be invoked if the subject was required to justify his answer. Thus, as with conservation, development of linear measurement would proceed through two (at least) phases. The developmental relationship between the two production system models representing each phase should be consistent with the empirical evidence. That is, the first model would incorporate productions corresponding to conservation and transitivity rules, whilst the second model would not. The latter would be more mature than the former in-so-far as: (a), it would appear later in development; and (b), it would have had redundant processing - the conservation and transitive reasoning rules - removed from it. To that extent it would also be more efficient, and yield faster solution times.

Again, as was the case with conservation, the first model of linear measurement - the one containing the conservation and transitive reasoning rules - may only be expressed behaviourally when the subject is required to justify his answers. Additionally, the conservation and transitive reasoning capacities incorporated in that model would be the more mature, later-developing versions of those sub-systems, not the versions which incorporate first principle rules.
The character of the proposed developmental process will be apparent from the above discussion. Development is seen as proceeding through the assembly of lower-level sub-systems into higher-level sub-systems which, in turn, are assembled into still higher-level sub-systems. This process of vertical integration is also characterised by the progressive refinement of newly-assembled sub-systems until appropriate levels of efficiency have been achieved. This gain in efficiency would be achieved essentially by removal of redundant processing, with a concomitant reduction in solution path step length and process supervisor complexity.

In addition to this vertical integration, development would also be characterised by a horizontal spread of the range of objects and events to which specific sub-systems can be applied, and a progressive differentiation of those objects and events to which specific sub-systems cannot be applied.

Temporal discontinuities could be expected to be manifested in both the vertical and horizontal patterns of development. The discontinuities in the vertical pattern - such as those reported in the present study - would be attributed to the processing involved in, firstly, the assembly, and, secondly, the refinement of the constituent sub-systems.

The proposed developmental processes are not new in character. They reflect the same kind of approach that characterises Piaget's ideas regarding differentiation and integration of schemes. They also underlie Gagne's notions of cumulative learning, and are at the heart of Pascual-Leone's
view of learning. The other notable factor all these theoretical formulations have in common is the absence of a detailed description of the mechanisms required to give effect to them. However, the longer term enterprise of constructing a production system model of the development of linear measurement must proceed within a theoretical context that provides for such mechanisms. Since, at the present time, the most detailed account of these mechanisms is that provided by Klahr and Wallace (1976), their theoretical proposals would appear to offer the most appropriate framework within which to proceed.

A6.4 KLahr AND wallace'S model.

A6.4.1 organisation of memory.

The organisation of Klahr and Wallace's (1976) memory model, and the flow of information within the model, are illustrated in Figure A6.1.

The following list of features serves to elaborate the schematic information provided by Figure A6.1:

(i) All productions are stored in long term memory (LTM).

(ii) The condition elements of productions are tested against the contents of up to two of the short term memory buffers, and/or "echoic" and/or "iconic memory stores".

(iii) The action elements of productions are applied to the contents of the short term memory buffers or specified parts of LTM, but not to echoic or iconic memory stores.

(iv) Different timing and capacity parameters apply to each memory component.
Figure A6.1: Schematic Outline of Klahr and Wallace Memory Model. (From: Klahr & Wallace (1976), pp. 175).

Note: Dashed lines represent functional information flow.
Klahr and Wallace (1976) represent LTM as a "disconnected graph structure". Each graph consists of a network of nodes. Each node consists of a production or production system; a description list connecting that node with other nodes; and an experience list that provides a trace of the involvement of that node in previous processing. The model also makes provision for an episodic component of LTM, called the "Time Line" (TL). Critical aspects of discrete processing sequences are stored in the Time Line. Essentially, they are the initiating conditions, the terminating condition, and production and production systems invoked during processing.

Klahr and Wallace (1976) propose that LTM is divided into "three tiers", with each tier having multiple levels. The tiers are searched in sequence and, within each tier, the levels are searched sequentially. However, within each level, parallel searching is carried out for the next appropriate production.

Productions and production systems relating to specific objects, events and situations previously encountered are stored at tier 1 of LTM. Productions controlling strategic procedures, such as "means-end analysis", and "predicate-led functioning", are stored at tier 2. Productions responsible for learning are held in tier 3.

A6.4.3 PROCESSING ECONOMY PRINCIPLE.

Klahr and Wallace (1976) see development largely in terms of rule generation and periodic LTM re-organisation. These processes operate in accordance with a "processing economy principle". That principle sets out the
conditions under which "specific situation sequences", encapsulated in time line episodic recording, become cast as "consistent sequence" productions and production systems. Concomitant with consistent sequence detection, the developing system also searches out, and eliminates, instances of redundancy.

These processes are not modelled by Klahr and Wallace in the form of executable computer programs. Rather, they posit the existence of certain innate systemic productions at each tier of LTM, but particularly at tier 3, which operate in accordance with the general systemic principles of consistency detection and redundancy elimination.

A6.5 APPLICATION OF KLAHR AND WALLACE’S MODEL.

The component of Klahr and Wallace’s model that offers particular promise to the longer term project referred to earlier is their proposal concerning time line recording. Together with their proposed innate systemic productions, time line recording provides a mechanism that could give effect to developmental processes resulting in the kinds of delay and discontinuity observed in the present study.

The following example should be sufficient to convey a general understanding of the kind of processing envisaged. Consider the developing system that has already constructed general-rule models of conservation and transitive reasoning, for number and length. When faced with a problem of the kind represented by LR-M-CARD, the system would respond firstly by searching LTM for an appropriate common consistent sequence production system. This search would proceed under the control of a tier 2 innate
systemic production. After failing in the search, the initial goal might be replaced with a lesser goal, one concerning the length of one of the objects. This would initiate another search of LTM for a common consistent sequence appropriate to that new, lesser goal. If that search is also unsuccessful, the current goal would be replaced with one concerned with the properties of the length of an object. The next search of LTM might locate the production systems concerning the conservation of length and the transitive processing of length relations. (In this context, the properties of length could be construed as declarative or procedural knowledge. For present purposes, however, this distinction, and the consequences it implies for differing memorial representations, can be set aside). Though this processing would not initially result in a solution to the original problem, it does have the effect of constructing partial solutions. Moreover, the processing involved is recorded as a series of time line episodes. Further exposure to problems of the LR-M-CARD kind would result in additional time line recording. Subsequently examination of this time line information by the tier 3 innate systemic productions responsible for learning would identify these partial solutions, and recast them as consistent specific production systems that move progressively closer to eventual solutions.

Additionally, review of time line information in this manner would identify redundant processing of the kind represented in the first principle models discussed earlier in connection with conservation and linear measurement, and would lead to the eventual emergence of the more efficient, second phase models.
As Klahr and Wallace (1976) illustrate, using quantification processes as an example, this processing of experience, stored as time-sequenced information, by innate systemic productions can result in a pattern of development marked by temporal discontinuities. The main difficulty with the present proposal is that it could reasonably be expected that there would be behavioural evidence of the developing system assembling progressively closer approximations to the eventual solution strategy. That is, there should be behavioural evidence of faulty and incomplete execution of solution strategies. That was the case with the quantification evidence cited by Klahr and Wallace (1976). However, as far as the present study is concerned that was not the case — those subjects who could not solve LR-M-CARD did not even begin the task, while those who could solve the problem did so perfectly. On the surface this suggests a sudden insightful assembly of components into a solution strategy which is executed perfectly the first time it is applied. Intuitively, this seems less likely than the gradual — though marked by discontinuities — process sketched in the above paragraphs.

How then can the present study's lack of behavioural evidence of partial solutions be accounted for? Currently the most appealing suggestion is that, in the initial stages of the development of linear measurement, the components are still represented in LTM by first principle systems. Because these first principle component systems would have longer step paths than the later-developing, general rule component systems, this would mean extended step paths at two (or more) levels. If that were the case, the strategy could impose supervisory processing demands beyond the developed capacity of the system. In that event, breakdown could occur before a point was reached where cognitive processes would be expressed behaviourally.
Clearly, these notions are somewhat imprecise. Though they carry some (usually dimly glimpsed) prospect of accounting for the developmental patterns observed in the present study, they will remain conjectural until they can be demonstrated in action. The most compelling demonstration would be observation of the behaviour of a production system model of the proposed system. A minimum outcome would be that examination of the trace of execution of a model of that kind would prompt the construction of other information-processing orientated hypotheses which would be both consistent with the behavioural data and amenable to being modelled as executable systems.
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USE OF THESES

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LISTINGS AND EXECUTION TRACES OF SIX
PRODUCTION-SYSTEM MODELS OF THREE
NUMBER TASKS.

ADDITION TO THE MAIN REPORT.

Kenneth Vine
January, 1983.
ADDENDUM 1.

LISTING AND EXECUTION TRACE FOR
N-ADD-NV: TABLE-LOOK-UP METHOD.
* AOO
(VARIABLES:
(A GENC)
(B GENC)
(C GENC)
(D GENC)
(E GENC)
(<X> SENDC)
(<P> SENDC)
)

(PRODUCTIONS:
(P000 (GOAL + ATTEND) (OLD SEND C)
USER() )
(P001 (GOAL + ATTEND) (TOP A) (BOTTOM B) (SEND C)
USER() )
(REPL (GOAL + ATTEND) (GOAL + ATTEND))
(DEL (OLD SEND C) )
(USER() )
(P005 (GOAL + ATTEND) (GOAL) (SEND C)
USER() )
(REPL (GOAL + ATTEND) (GOAL + ATTEND))
(DEL (GOAL) )
(USER() )
(PF1 (TOP A) (BOTTOM B) (SEND C)
USER() )
(REPL (GOAL + ATTEND) (GOAL + SEND C))
(INS (GOAL + SEND C) )
(INS ((Y 1 ?)) )
(PF2 (GOAL + SEND C)
USER() )
(REPL (GOAL + SEND C) (GOAL + SEND C))
(INS ((GOAL + PUSH BUTTON)) )
(PF3 (GOAL + PUSH BUTTON)
USER() )
(DEL (GOAL + PUSH BUTTON) )
(INS (ELM A) )
(INS (GOAL + SUBIT) )
(PF4 (GOAL + SUBIT) (ELM A)
USER() )
(DEL (GOAL + SUBIT) )
(INS (OS 1) )
(DEL (ELM A) )
(INS (GOAL + COUNT) )
(PF5 (GOAL + COUNT) (OS 1) (Y <X> X)
USER() )
(REPL (GOAL + COUNT) (GOAL + COUNT))
(DEL (OS 1) )
(SAY (OS 1) )
(REPL ((Y <X> 2) ) (SATD <X>))
(IN ((GOAL + MARK)) )
(PF6 (GOAL + MARK) (SATD 1)
USER() )
(REPL (GOAL + MARK) (GOAL + MARK))
(IN ((Y 2 ?)) )
(PF7 (GOAL + MARK) (SATD 2)
USER() )
(REPL (GOAL + MARK) (GOAL + MARK))
(IN ((Y 3 ?)) )
(PF8 (GOAL + MARK) (SATD 3)
USER() )
(REPL (GOAL + MARK) (GOAL + MARK))
(IN ((Y 4 ?)) )
(PF9 (GOAL + MARK) (SATD 4)
USER() )
(REPL (GOAL + MARK) (GOAL + MARK))

INS(C(5 Z Z))

(P7 (GOAL + MARK) (GOAL + COUNT) (GOAL + SEND C) (SAID C))

DEL((GOAL + SEND C))
DEL((GOAL + MARK))
DEL((GOAL + COUNT))
DEL((SAID C))

RPL((GOAL + SEND C) (GOAL + ATTEND))
DEL((GOAL + MARK))
DEL((GOAL + COUNT))
DEL((SAID C))

(P9 (GOAL + ATTEND) (MANY BOTTOM BEFORE))

RPL((GOAL + ATTEND) (GOAL + SEND C))
DEL((GOAL + MARK))
DEL((GOAL + COUNT))
DEL((SAID C))

(P10 (GOAL + RECALL BOTTOM) (BOTTOM B))

INS((GOAL + RECALL BOTTOM))

(SAY((BOTTOM B)));
INS((GOAL + RECALL BOTTOM))

RPL((GOAL + SEND C) (GOAL + ATTEND))
DEL((GOAL + MARK))
DEL((GOAL + COUNT) (SEND C))

(P12 (GOAL + RECALL TUBE) (SEND C))

INS((GOAL + TUBE))

RPL((GOAL + ATTEND) (GOAL + SEND C))

(SAY((SEND C)));

(P13 (GOAL + ATTEND) (MANY BOTTOM NOW))

RPL((GOAL + ATTEND) (GOAL + ADD))
DEL((MANY BOTTOM NOW))
INS((GOAL + ADD))

(P14 (GOAL + ADD) (BOTTOM B) (ABD (GOAL + ADD))

INS((GOAL + ADD))

(P15 (GOAL + ADD))

INS((GOAL + ADD))

RPL((GOAL + ADD) (GOAL + ADD))

INS((2 TUBE Z))
INS((3 TUBE Z))

(P15A (GOAL + LIST TWO))

RPL((GOAL + ADD) (GOAL + ADD))

INS((3 TUBE Z))
INS((4 TUBE Z))

(P15B (GOAL + LIST SIX))

RPL((GOAL + ADD) (GOAL + ADD))

INS((2 TUBE Z))
INS((3 TUBE Z))

(P15C (GOAL + LIST NINE))

RPL((GOAL + ADD) (GOAL + ADD))

INS((2 TUBE Z))
INS((1 TUBE Z))

(P16 (GOAL + ADD) (SEND C) (ADD 2) (BOTTOM B))
SAY(<X>);
DEL((GOAL * CID));
DEL((GOAL * LIST B));
INS((GOAL * PURGE));
(P24A (GOAL * PURGE) (SEND C) (C <X> Z) (BOTTOM B))
 REPL((BOTTOM B) ; (BOTTOM <X>));
 REPL((SEND C) ; (OLD SEND C));
 DEL((C <X> Z));
(P24 (GOAL * PURGE) (Y <X> Z))
 DEL((Y <X> Z));
(P25 (GOAL * PURGE) (C <P> Z))
 DEL((C <P> Z));
(P26 (GOAL * PURGE) (BOTTOM 12))
 STOP;
(P27 (GOAL * PURGE))
 DECL((GOAL * PURGE));
 REPL((GOAL * ATTEND) ; (GOAL * ATTEND))
(POLISHE!
N-ADD-INV
P9 P11 P13
P13 P11 P9
P1 P2 F3 P4 P3 P6
P6A P4B P5C
P7 P8 P10 P12
P13 P14 P16 P15
P15a P15b P15c
P1246
P24 P25 P24 P27
)

(REFL((GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2)))

***END
RUN BEGINS:

(((GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
INPUT TO STM:
$2(GO)$

STM: ((GO) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P0

STM: ((GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STM: ((Y 1 Z) (GOAL & SEND 2) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STM: ((GOAL & BUTTON) (GOAL & SEND 2) (Y 1 Z) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STM: ((GOAL & SHOW) (ELM A) (GOAL & SEND 2) (Y 1 Z) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P4

STM: ((GOAL & COUNT) (DD 1) (GOAL & SEND 2) (Y 1 Z) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P5

**END**

STM: ((GOAL & MARK) (GOAL COUNT) (SAID 1) (GOAL & SEND 2) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK)
P6

STM: ((Y 2 Z) (GOAL, MARK) (SAID 1) (GOAL COUNT) (GOAL & SEND 2) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK)
P7
STH1: ((GOAL & PURGE) (OLD SEND 2) (BOTTOM TWO) (Y THREE Z) (GOAL #& ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P25

STH1: ((GOAL & PURGE) (OLD SEND 2) (BOTTOM TWO) (GOAL #& ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P27

STH1: (OLD SEND 2) (BOTTOM TWO) (GOAL & ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P30

OUTPUT TO STH1:
SEND FOUR JUNK JUNK
(SEND 4)

STH1: (SEND 4) (GOAL ATTEND) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P1

STH1: ((Y 1 Z) (GOAL & SEND 4) (GOAL #& ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P2

STH1: ((GOAL & PUSH BOTTOM) (GOAL #& SEND 4) (Y 1 Z) (GOAL #& ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P3

STH1: (GOAL & RUSH) (GOAL & SEND 4) (Y 1 Z) (GOAL #& ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P4

STH1: (GOAL & COURT) (GOAL & SEND 4) (Y 1 Z) (GOAL #& ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P5

STH1: ((GOAL & SEND 3) (GOAL SEND 3) (GOAL COURT) (GOAL & SEND 3) (GOAL #& ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P6
STM: (GOAL * SEND 4) (Y 2 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK

STM: (GOAL * PUSH BUTTON) (GOAL *% SEND 4) (Y 2 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK

STM: (GOAL * COUNT) (GOAL *% SEND 4) (Y 2 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK

PREPARE 2

STM: (GOAL * MARK) (GOAL * COUNT) (GOLD? 2) (GOAL *% SEND 4) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK

STM: (Y 2 Z) (GOAL MARK) (GOLD? 2) (GOAL * COUNT) (GOAL *% SEND 4) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK

STM: (GOAL * SEND 4) (Y 3 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK

STM: (GOAL * PUSH BUTTON) (GOAL *% SEND 4) (Y 3 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK

STM: (GOAL * QUIT) (GOLD AT) (GOAL *% SEND 4) (Y 3 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK

STM: (GOAL * SEND 4) (Y 3 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK
**F16**

```
**SIX**

STM: ((GOAL * PURGE) (SEND 1) (SIX 2) (BOTTOM TWO) (SIX FIVE 2) (GOAL *% ATTEND) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P24

STM: ((GOAL * PURGE) (OLD SEND 4) (BOTTOM SIX) (SIX FIVE 2) (GOAL *% ATTEND) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P24

STM: ((GOAL * PURGE) (OLD SEND 4) (BOTTOM SIX) (SIX FIVE 2) (GOAL *% ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P27

STM: ((OLD SEND 4) (BOTTOM SIX) (GOAL * ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P30

INPUT TO STM:
*/(SEND 3)

STM: */(SEND 3) (GOAL ATTEND) (BOTTOM SIX) (TOP TWELVE) JUNK JUNK JUNK
JUNK JUNK JUNK JUNK JUNK
P1

STM: */(Y 1 Z) (GOAL * SEND 3) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P2

STM: */(GOAL * PUSH BUTTON) (GOAL *% SEND 3) (Y 1 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P3

STM: */(GOAL * SUBIT) (ELH A) (GOAL *% SEND 3) (Y 1 Z) (GOAL *% ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK
P4

STM: */(GOAL * COUNT) (US 1) (GOAL *% SEND 3) (Y 1 Z) (GOAL *% ATTEND)
(TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK
P5
```
1

TH: (GOAL ≠ MARK) (GOAL COUNT) (SAID 1) (GOAL ≠ SEND 3) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK
P6

TH: (Y 2 Z) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL ≠ SEND 3) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK
P3

TH: (GOAL ≠ SEND 3) (Y 2 Z) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P2

TH: (GOAL ≠ SEND 3) (Y 2 Z) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK
P3

TH: (GOAL ≠ SEND 3) (Y 2 Z) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK
P4

TH: (GOAL ≠ SEND 3) (Y 2 Z) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK
P5

2

TH: (GOAL ≠ MARK) (GOAL COUNT) (SAID 2) (GOAL ≠ SEND 3) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK
P6

TH: (Y 3 Z) (GOAL MARK) (SAID 2) (GOAL COUNT) (GOAL ≠ SEND 3) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK
P3

TH: (GOAL ≠ SEND 3) (Y 3 Z) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK
P7

TH: (GOAL ≠ PUSH BUTTON) (GOAL ≠ SEND 3) (Y 3 Z) (GOAL ≠ ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK
P3
STM: ((GOAL * RECALL TABLE) (GOAL ** ATTEND) (BOTTOM SIX) (Y 4 Z) (TOP TWELVE) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P12

******** (SEND 3)

STM: ((SEND 3) (GOAL & ATTEND) (BOTTOM SIX) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0

INPUT TO STM:
*(MANY BOTTOM NOW)

STM: ((MANY BOTTOM NOW) (GOAL & ATTEND) (SEND 3) (BOTTOM SIX) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P13

STM: ((GOAL & ADD) (GOAL ** ATTEND) (SEND 3) (BOTTOM SIX) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P14

STM: ((GOAL & LIST SIX) (GOAL ** ADD) (BOTTOM SIX) (GOAL ** ATTEND) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P15B

STM: ((3 NINE 7) (2 EIGHT Z) (GOAL & LIST SIX) (GOAL & ADD) (BOTTOM SIX) (GOAL ** ATTEND) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P16

******** NINE

STM: ((GOAL & PURGE) (SEND 3) (3 NINE 7) (BOTTOM SIX) (2 EIGHT Z) (GOAL ** ATTEND) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P24A

STM: ((GOAL & PURGE) (OLD SEND 3) (BOTTOM NINE) (2 EIGHT Z) (GOAL ** ATTEND) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P24

STM: ((GOAL & PURGE) (OLD SEND 3) (BOTTOM NINE) (2 EIGHT Z) (GOAL ** ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P25

STM: ((GOAL & PURGE) (OLD SEND 3) (BOTTOM NINE) (GOAL ** ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P26
P27

STH: ((SEND 3) (BOTTOM NINE) (GOAL % ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK)

FOCA

INPUT TO STH:

2(SEND 1)

STH: ((SEND 1) (GOAL ATTEND) (BOTTOM NINE) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P1

STH: ((Y 1 Z) (GOAL % SEND 1) (GOAL % ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STH: ((GOAL & PUSH BUTTON) (GOAL % SEND 1) (Y 1 Z) (GOAL % ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

P3

STH: ((GOAL % SUBIT) (ELM A) (GOAL % SEND 1) (Y 1 Z) (GOAL % ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

P4

STH: ((GOAL % COUNT) (DS 1) (GOAL % SEND 1) (Y 1 Z) (GOAL % ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

P5

********** 1

STH: ((GOAL % MARK) (GOAL COUNT) (SAID 1) (GOAL % SEND 1) (GOAL % ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

P6

STH: ((Y 2 Z) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL % SEND 1) (GOAL % ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P7

STH: ((Y 2 Z) (GOAL % ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P8

INPUT TO STH:

2(SEND BOTTOM BEFORE)
STM: ((MANY BOTTOM BEFORE) (GOAL * ATTEND) (Y 2 Z) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL * RECALL BOTTOM) (GOAL *# ATTEND) (Y 2 Z) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

****** (BOTTOM NINE)

STM: ((BOTTOM NINE) (GOAL & ATTEND) (Y 2 Z) (TOP TWELVE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

*INPUT TO STM: *(MANY TUBE)

STM: ((MANY TUBE) (GOAL * ATTEND) (BOTTOM NINE) (Y 2 Z) (TOP TWELVE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL * RECALL TUBE) (GOAL *# ATTEND) (BOTTOM NINE) (Y 2 Z) (TOP TWELVE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

****** (SEND 1)

STM: ((SEND 1) (GOAL * ATTEND) (BOTTOM NINE) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

*INPUT TO STM: *(MANY BOTTOM NOW)

STM: ((MANY BOTTOM NOW) (GOAL * ATTEND) (SEND 1) (BOTTOM NINE) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL & ADD) (GOAL *# ATTEND) (SEND 1) (BOTTOM NINE) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL * LIST NINE) (GOAL *# ADD) (BOTTOM NINE) (GOAL *# ATTEND) (SEND 1) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)

P15C
STM: ((GOAL * PURGE) (SEND 1) (1 TEN Z) (BOTTOM HlUE) (GOAL *5# ATTEND) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
F24

STM: ((GOAL * PURGE) (OLD SEND 1) (BOTTOM TEN) (0 NINE Z) (GOAL *$# ATTEND) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
F25

STM: ((GOAL * PURGE) (OLD SEND 1) (BOTTOM TEN) (GOAL *$# ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK)
F27

STM: ((OLD SEND 1) (BOTTOM TEN) (GOAL * ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
POOL
INPUT TO STM:
*
*
*

STM: (HIL (GOAL ATTEND) (BOTTOM TEN) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

END OF RUN

NUMBER OF PRODUCTIONS FIRED : 292
NUMBER OF OPERATIONS EXECUTED : 557
NUMBER OF PRODUCTIONS TRIED : 3310
NUMBER OF SINGLE-ELEMENT COMPARISONS : 52057
SIMULATED TIME : 6
COMPUTER TIME : 2753.3 MIPS
NUMBER OF TRANS-OPERATIONS : 514441
LISTING AND EXECUTION TRACE FOR

N-ADD-NV: COUNTING-BASED METHOD.
P 500
(VARIABLES:
(A SEND)
(B SEND)
(C SEND)
(D SEND)
(<X> SEND)
(<P> SEND)
)
(PRODUCTIONS:
(P0) (GOAL * ATTEND)(OLD SEND C)

REPL((GOAL * ATTEND) ; (GOAL + ATTEND)))
DEL((OLD SEND C));
USER();

<P0 (GOAL & ATTEND)(GO)

REPL((GOAL * ATTEND) ; (GOAL + ATTEND));
DEL((GO));
(P0 (GOAL * ATTEND))

USER();

<P1 (GOAL + ATTEND)(TOP A)(BOTTOM B)(SEND C)

REPL((GOAL * ATTEND) ; (GOAL + ATTEND));
INS((GOAL * SEND C));
INS((Y Z 2));
(P2 (GOAL * SEND C))

REPL((GOAL * SEND C) ; (GOAL + SEND C));
INS((GOAL * PUSH BUTTON))

<P3 (GOAL * PUSH BUTTON))

DEL((GOAL * PUSH BUTTON));
INS((ELM A));
INS((GOAL * SUBIT))
(P4 (GOAL * SUBIT)(ELM A))

DEL((GOAL * SUBIT));
INS((GS 1));
DEL((ELM A));
INS((GOAL * COUNT));
(P5 (GOAL * COUNT)(GS 1)(Y <X> Z))

REPL((GOAL * COUNT) ; (GOAL + COUNT));
DEL((GS 1));
SY((ZX));

REPL((Y <X> Z) ; (SAID <X>));
INS((GOAL * MARK))

<P6 (GOAL * MARK)(SAID 1))

REPL((GOAL & MARK) ; (GOAL + MARK));
INS((Y Z 2));
(P6A (GOAL & MARK)(SAID 2))

REPL((GOAL & MARK) ; (GOAL + MARK));
INS((Y 3 Z));
(P5B (GOAL & MARK)(SAID 3))

REPL((GOAL & MARK) ; (GOAL + MARK));
INS((Y 4 Z));
(P5C (GOAL & MARK)(SAID 4))

REPL((GOAL & MARK) ; (GOAL + MARK));
REPL((Y TWO Z) ; (Y THREE Z));
DEL((GOAL * NEXT UP));
INS((GOAL ; NEXT ALONG));
G137C (GOAL * NEXT ALONG) (Y THREE Z)

REPL((Y THREE Z) ; (Y FOUR Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G137B (GOAL * NEXT ALONG) (Y FOUR Z)

REPL((Y FOUR Z) ; (Y FIVE Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G138G (GOAL ; NEXT ALONG) (Y FIVE Z)

REPL((Y FIVE Z) ; (Y SIX Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G137F (GOAL ; NEXT ALONG) (Y SIX Z)

REPL((Y SIX Z) ; (Y SEVEN Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G138G (GOAL ; NEXT ALONG) (Y SEVEN Z)

REPL((Y SEVEN Z) ; (Y EIGHT Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G137H (GOAL ; NEXT ALONG) (Y EIGHT Z)

REPL((Y EIGHT Z) ; (Y NINE Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G137J (GOAL ; NEXT ALONG) (Y NINE Z)

REPL((Y NINE Z) ; (Y TEN Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G136O (GOAL ; NEXT ALONG) (Y TEN Z)

REPL((Y TEN Z) ; (Y ELEVEN Z));
DEL((GOAL ; NEXT ALONG));
INS((GOAL ; NEXT UP));
G138A (GOAL ; NEXT UP) (W 0 V)

REPL((W 0 V) ; (W 1 V));
DEL((GOAL ; NEXT UP));
REPL((GOAL #$ ADD) ; (GOAL # ADD));
(P1A (GOAL ; NEXT UP) (W 1 V))

REPL((W 1 V) ; (W 2 V));
DEL((GOAL ; NEXT UP));
REPL((GOAL #$ ADD) ; (GOAL # ADD));
(P18B (GOAL ; NEXT UP) (W 2 V))

REPL((W 2 V) ; (W 3 V));
DEL((GOAL ; NEXT UP));
REPL((GOAL #$ ADD) ; (GOAL # ADD));
(P16C (GOAL ; NEXT UP) (W 3 V))

REPL((W 3 V) ; (W 4 V));
DEL((GOAL ; NEXT UP));
REPL((GOAL #$ ADD) ; (GOAL # ADD));
(P14A (GOAL ; FURGELY (Y X) Z) (BOTTOM B) (SEND C))
REPL (BOTTOM Z) ; (BOTTOM <X>)
REPL (SEND C) ; (OLD SEND C)
DEL ((Y <X> Z)) ;
(P24 (GOAL & PURGE) (Y <Z> Z))
DEL ((Y <X> Z)) ;
(P25 (GOAL & PURGE) (W <P> V))
DEL ((W <P> V)) ;
(P26 (GOAL & PURGE) (BOTTOM Z))
STOP)
(P27 (GOAL & PURGE))
(REPL ((GOAL & PURGE)) ; (GOAL & PURGE))
(FDLISTS)
(N-ADD-HV2
P9 P11 P13
F00 P00 P0
F1 P2 P3 P4 P5 P6
F6A F6B F6C
F7 P8 P10 P12
P13 P14 P15 P16
P24
P25 P26 P27)
(DET-NEXT
P17 P17A P17B P17C P17D P17E P17F
P17G P17H P17I P17J)
(EPUP
P18 P18A P18B P18C)

(DEF: STR ((GOAL & ATTEND) (TOP 1) (NEW 5) (BOTTOM ZERO) (SEND 2)))
***END
***FILE
[EGIPTL Filed : DSGK1.A4FOG.P13]
RUN BEGINS:

((GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P0

INPUT TO STM:

*GO

STM: ((GO) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P0

STM: ((GOAL ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P1

STM: ((Y 1 Z) (GOAL * SEND 2) (GOAL #X ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STM: ((GOAL * PUSH BUTTON) (GOAL #X SEND 2) (Y 1 Z) (GOAL #X ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P3

STM: ((GOAL * SUBIT) (EM 1) (GOAL #X SEND 2) (Y 1 Z) (GOAL #X ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P4

STM: ((GOAL * COUNT) (PS 1) (GOAL #X SEND 2) (Y 1 Z) (GOAL #X ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P5

RESTARTS 1

STM: ((GOAL * HARK) (GOAL COUNT) (SAID 1) (GOAL #X SEND 2) (GOAL #X ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P6

STM: ((Y 2 Z) (GOAL HARK) (SAID 1) (GOAL COUNT) (GOAL #X SEND 2) (GOAL #X ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P7
STH: ((GOAL * SEND) (Y 2 Z) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P2

STH: ((GOAL * PUSH BUTTON) (GOAL * SEND) (Y 2 Z) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK) P3

STH: ((GOAL * SUBIT) (ELH A) (GOAL * SEND) (Y 2 Z) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK) P4

STH: ((GOAL * COUNT) (Z 3 1) (GOAL * SEND) (Y 2 Z) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK) P5

STH: ((Y 3 Z) (GOAL MARK) (GOAL COUNT) (SAID 2) (GOAL * SEND) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK) P6A

STH: ((Y 3 Z) (GOAL MARK) (GOAL COUNT) (SAID 2) (GOAL * SEND) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK) P7

STH: ((Y 3 Z) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P8

INPUT TO STH:

STH: ((GOAL * MAKES BOTTOM BEFORE) (GOAL * ATTEND) (Y 3 Z) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P9

STH: ((GOAL * RECALL BOTTOM) (GOAL * SEND) (Y 3 Z) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK) P10

STH: ((GOAL * SEND) (BOTTOM 0))
STH: ((BOTTOM ZERO) (GOAL & ATTEND) (Y 3 Z) (TOP TWELVE) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
F0
INPUT TO STH:
*(MANY MORE*)

STH: ((MANY MORE) (GOAL & ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P11

STH: ((GOAL & RECALL MORE) (GOAL & ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P12

******* (SEND 2)

STH: ((SEND 2) (GOAL & ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STH:
*(MANY BOTTOM NOW*)

STH: ((MANY BOTTOM NOW) (GOAL & ATTEND) (SEND 2) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK)
P13

STH: ((Y ZERO Z) (W 0 V) (GOAL & ADD) (GOAL & ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P15

STH: ((GOAL & NEXT ALONG) (GOAL & ATTEND) (W 0 V) (Y ZERO Z) (GOAL & ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P17

STH: ((GOAL & NEXT UP) (Y ONE Z) (GOAL & ATTEND) (W 0 V) (GOAL & ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P16

STH: ((GOAL & NEXT UP) (Y ONE Z) (GOAL & ATTEND) (W 0 V) (GOAL & ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P16

STH: ((W 1 V) (Y ONE Z) (GOAL & ADD) (GOAL & ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P15
SYN: ((GOAL 2 NEXT ALONG) (GOAL 3*- ADD) (U 1 V) (Y ONE 2) (GOAL 3*- ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
F1A

STH: ((GOAL 2 NEXT UP) (Y TUG Z) (GOAL 3*- ADD) (U 1 V) (GOAL 3*- ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P16

STH: ((GOAL 2 NEXT UP) (Y TUG Z) (GOAL 3*- ADD) (U 1 V) (GOAL 3*- ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P16A

STH: ((U 2 V) (Y TUG Z) (GOAL 2 ADD) (GOAL 3*- ATTEND) (BOTTOM ZERO) (SEND 2) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P14

***** **** TWO

STH: ((GOAL 2 PURGE) (U 2 V) (SEND 2) (Y TUG Z) (GOAL 3*- ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P24A

STH: ((GOAL 2 PURGE) (BOTTOM TUG) (OLD SEND 2) (W 2 V) (GOAL 3*- ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P24

STH: ((GOAL 2 PURGE) (BOTTOM TUG) (OLD SEND 2) (W 2 V) (GOAL 3*- ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P25

STH: ((GOAL 2 PURGE) (BOTTOM TUG) (OLD SEND 2) (GOAL 3*- ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
P27

STH: ((BOTTOM TUG) (OLD SEND 2) (GOAL 3*- ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P200

INPUT TO STH:
((SEND 4)

STH: ((SEND 4) (GOAL ATTEND) (BOTTOM TUG) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P1
STM: ((GOAL 4 MARK) (GOAL COUNT) (SAID 2) (GOAL 23% SEND 4) (GOAL 23% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
F6A

STM: ((Y 3 Z) (GOAL MARK) (SAID 2) (GOAL COUNT) (GOAL 23% SEND 4) (GOAL 23% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
F8

STM: ((GOAL 4 SEND 4) (Y 3 Z) (GOAL 82% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
F3

STM: ((GOAL 4 PUSH BUTTON) (GOAL 82% SEND 4) (Y 3 Z) (GOAL 82% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
F3

STM: ((GOAL 4 SUBIT) (ELM A) (GOAL 85% SEND 4) (Y 3 Z) (GOAL 85% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
F3

STM: ((GOAL 4 COUNT) (DS 1) (GOAL 85% SEND 4) (Y 3 Z) (GOAL 85% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
F5

STM: (E3)

STM: ((GOAL 4 MARK) (GOAL COUNT) (SAID 3) (GOAL 23% SEND 4) (GOAL 23% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
F6B

STM: ((Y 4 Z) (GOAL MARK) (SAID 3) (GOAL COUNT) (GOAL 23% SEND 4) (GOAL 23% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
F8

STM: ((GOAL 4 SEND 4) (Y 4 Z) (GOAL 23% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
F2

STM: ((GOAL 4 PUSH BUTTON) (GOAL 23% SEND A1) (Y 4 Z) (GOAL 23% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
F3
Status: (Goal: Get Count) (Goal: A) (Goal: Get Send 1) (Y: 5 Z: 2) (Goal: Get Attend) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk

Status: (Goal: Get Mark) (Goal: Count) (Goal: Get Send 1) (Goal: Get Attend) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk

Status: (Y: 5 Z: 2) (Goal: Mark) (Goal: Count) (Goal: Get Send 1) (Goal: Get Attend) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk

Status: (Y: 5 Z: 2) (Goal: Attend) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk Junk

Input to Status: (Many Bottom Before) (Goal: Attend) (Y: 5 Z: 2) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk Junk

Status: (Many Bottom Before) (Goal: Attend) (Y: 5 Z: 2) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk Junk

Status: (Many Bottom Before) (Goal: Attend) (Y: 5 Z: 2) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk Junk

Status: (Bottom: Two) (Goal: Attend) (Y: 5 Z: 2) (Top: Twelve) (Send: 4) Junk Junk Junk Junk Junk Junk

Input to Status: (Many Tube)

Status: (Many Tube) (Goal: Attend) (Bottom: Two) (Y: 5 Z: 2) (Top: Twelve) (Send: 4) Junk Junk Junk Junk Junk Junk

Status: (Goal: Get Mark) (Goal: Count) (Goal: Get Send 1) (Goal: Get Attend) (Top: Twelve) (Bottom: Two) (Send: 4) Junk Junk Junk Junk Junk
P TWELVE: (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F12

** (SEND 4)

STH: ((SEND 4) (GOAL * ATTEND) (BOTTOM TWO) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PC INPUT TO STH
*(MANY BOTTOM NOW)

STH: ((MANY BOTTOM NOW) (GOAL * ATTEND) (SEND 4) (BOTTOM TWO) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
F13

STH: ((Y TWO Z) (U O U) (GOAL * ADD) (GOAL *S% ATTEND) (BOTTOM TWO) (SEND 4) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
F16

STH: ((GOAL * NEXT ALONG) (GOAL *S% ADD) (U O U) (Y YUG Z) (GOAL *S% ATTEND) (BOTTOM TWO) (SEND 4) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK)
F17B

STH: ((GOAL * NEXT UP) (Y THREE Z) (GOAL *S% ADD) (U O U) (GOAL *S% ATTEND) (BOTTOM TWO) (SEND 4) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)
F16

STH: ((GOAL * NEXT UP) (Y THREE Z) (GOAL *S% ADD) (U O U) (GOAL *S% ATTEND) (BOTTOM TWO) (SEND 4) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
F18

STH: ((U 1 U) (Y THREE Z) (GOAL * ADD) (GOAL *S% ATTEND) (BOTTOM TWO) (SEND 4) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F15

STH: ((GOAL * NEXT ALONG) (GOAL *S% ADD) (U 1 U) (Y THREE Z) (GOAL *S% ATTEND) (BOTTOM TWO) (SEND 4) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F17C

STH: ((GOAL * NEXT UP) (Y THREE Z) (GOAL *S% ADD) (U 1 U) (GOAL *S% ATTEND) (BOTTOM TWO) (SEND 4) (Y S Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F16
STH: ((GOAL * NEXT UP) (Y FOUR Z) (GOAL *&% ADD) (U 1 V) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F15A

STH: ((W 2 V) (Y FOUR Z) (GOAL *&% ADD) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F15

STH: ((GOAL * NEXT ALONG) (GOAL *$% ADD) (W 2 V) (Y FOUR Z) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F17D

STH: ((GOAL * NEXT UP) (Y FIVE Z) (GOAL *$% ADD) (U 2 V) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F16

STH: ((GOAL * NEXT UP) (Y FIVE Z) (GOAL *&% ADD) (U 3 V) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F12B

STH: ((W 3 V) (Y FIVE Z) (GOAL * ADD) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F15

STH: ((GOAL * NEXT ALONG) (GOAL *$% ADD) (W 3 V) (Y FIVE Z) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F17E

STH: ((GOAL * NEXT UP) (Y SIX Z) (GOAL *$% ADD) (W 3 V) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F16

STH: ((GOAL * NEXT UP) (Y SIX Z) (GOAL *$% ADD) (U 3 V) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F18C

STH: ((W 4 V) (Y SIX Z) (GOAL * ADD) (GOAL *$% ATTEND) (BOTTOM TWO) (SEND 4) (Y 5 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
F14

#### SIX
#### (SEND 3)

STH: ((SEND 3) (GOAL & ATTEND) (BOTTOM SIX) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

**INPUT TO STH:**

*(HARY BOTTOM HOW)*

STH: ((HARY BOTTOM HOW) (GOAL & ATTEND) (SEND 3) (BOTTOM SIX) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

**P13**

STH: ((Y SIX Z) (W 0 V) (GOAL & ADD) (GOAL & FT ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)

**P15**

STH: ((GOAL & NEXT ALONG) (GOAL & ADD) (W 0 V) (Y SIX Z) (GOAL & ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

**P17F**

STH: ((GOAL & NEXT UP) (Y SEVEN Z) (GOAL & ADD) (W 0 V) (GOAL & ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

**P16**

STH: ((GOAL & NEXT UP) (Y SEVEN Z) (GOAL & ADD) (W 0 V) (GOAL & ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

**P18**

STH: ((W 1 V) (Y SEVEN Z) (GOAL & ADD) (GOAL & ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

**P15**

STH: ((GOAL & NEXT ALONG) (GOAL & ADD) (W 1 V) (Y SEVEN Z) (GOAL & ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK)

**P176**

STH: ((GOAL & NEXT UP) (Y EIGHT Z) (GOAL & ADD) (W 1 V) (GOAL & ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

**P16**
STH: ((GOAL * NEXT UP) (Y EIGHT Z) (GOAL *# ADD) (W 2 V (GOAL *# ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P18A

STH: ((W 2 V) (Y EIGHT Z) (GOAL * ADD) (GOAL *# ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P15

STH: ((GOAL * NEXT ALONG) (GOAL *# ADD) (W 2 V) (Y EIGHT Z) (GOAL *# ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P17B

STH: ((GOAL * NEXT UP) (Y NINE Z) (GOAL *# ADD) (W 2 V) (GOAL *# ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P16

STH: ((GOAL * NEXT UP) (Y NINE Z) (GOAL *# ADD) (GOAL *# ATTEND) (BOTTOM SIX) (SEND 3) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P14

********** NINE

STH: ((GOAL & PURGE) (W 3 V) (SEND 3) (Y NINE Z) (GOAL *# ATTEND) (BOTTOM SIX) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P24A

STH: ((GOAL & PURGE) (BOTTOM NINE) (OLD SEND 3) (W 3 V) (GOAL *# ATTEND) (Y 4 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P24

STH: ((GOAL & PURGE) (BOTTOM NINE) (OLD SEND 3) (W 3 V) (GOAL *# ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P25

STH: ((GOAL & PURGE) (BOTTOM NINE) (OLD SEND 3) (GOAL *# ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P27

STH: ((BOTTOM NINE) (OLD SEND 3) (GOAL & ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
INPUT TO STM:
* (SEND 1)

STM: ((SEND 1) (GOAL_NMTEND) (BOTTOM NINE) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STM: ((Y 1 Z) (GOAL_ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STM: ((GOAL_PUSH_BUTTON) (GOAL_ATTEND) (Y 1 Z) (GOAL_ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK)
P3

STM: ((GOAL_SUBIT) (ELM_A) (GOAL_ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK)
P4

STM: ((GOAL_COUNT) (GOAL_ATTEND) (GOAL_ATTEND) (Y 1 Z) (GOAL_ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK)
P5

STM: ((GOAL_COUNT) (GOAL_ATTEND) (SAID 1) (GOAL_ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK)
P6

STM: ((Y 2 Z) (GOAL_COUNT) (SAID 1) (GOAL_ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)
P7

STM: ((Y 2 Z) (GOAL_ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)
P8

INPUT TO STM:
* (HANDY BOTTLE BEFORE)

STM: ((HANDY BOTTLE BEFORE) (GOAL_ATTEND) (Y 2 Z) (TOP TWELVE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)
STH: ((GOAL $ NEXT UP) (Y TEN Z) (GOAL $& ADD) (Y 0 V) (GOAL $& ATTEND) (BOTTOM NINE) (SEND 1) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK) P10

STH: ((W 1 V) (Y TEN Z) (GOAL $ ADD) (GOAL $& ATTEND) (BOTTOM NINE) (SEND 1) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK) P14

$$$$$$ TEN

STH: ((GOAL $ PURGE) (W 1 V) (SEND 1) (Y TEN Z) (GOAL $& ATTEND) (BOTTOM NINE) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK) P24A

STH: ((GOAL $ PURGE) (BOTTOM TEN) (OLD SEND 1) (W 1 V) (GOAL $& ATTEND) (Y 2 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK) P24

STH: ((GOAL $ PURGE) (BOTTOM TEN) (OLD SEND 1) (W 1 V) (GOAL $& ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK) P25

STH: 3 (BOTTOM TEN) (OLD SEND 1) (GOAL $ ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

PO00 INPUT TO STH:

()

STH: NIL (GOAL ATTEND) (BOTTOM TEN) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

END OF RUN

*

°C
ADDENDUM 3.

LISTING AND EXECUTION TRACE FOR

N-ADD-NV: MODIFIED COUNTING METHOD.
P 500

(VARIABLES:
(A SEND)
(B SEND)
(C SEND)
(D SEND)
(E SEND)
(CX> SEND)
(<P> SEND)

(PRODUCTIONS:
(P000 (GOAL * ATTEND)(OLD SEND C)

REPL((GOAL * ATTEND) ; (GOAL + ATTEND))
DEL((GOAL SEND C))
USER() )
(P000 (GOAL * ATTEND)(GO)

REPL((GOAL * ATTEND) ; (GOAL + ATTEND))
DEL((GO))
(P0 (GOAL * ATTEND)

USER() )
(P1 (GOAL + ATTEND)(TOP A)(BOTTOM B)(SEND C)

REPL((GOAL + ATTEND) ; (GOAL + ATTEND))
INS((GOAL SEND C))
INS((Y ONE Z)) )
(P2 (GOAL * SEND C)

REPL((GOAL * SEND C) ; (GOAL * SEND C))
INS((GOAL * PUSH BUTTON))
(F3 (GOAL * PUSH BUTTON)

DEL((GOAL * PUSH BUTTON))
INS((ELM A))
INS((GOAL * SUBIT)) )
(P4 (GOAL * SUBIT)(ELM A)

DEL((GOAL * SUBIT))
INS((Y SUBIT))
DEL((ELM A))
INS((GOAL * COUNT))
(F5 (GOAL * COUNT)(Y SEND Z)

REPL((GOAL * COUNT) ; (GOAL + COUNT))
DEL((Y SEND Z))
SAY((Y SEND Z))

REPL((Y <X> Z) ; (SAID <X>))
INS((GOAL * MARK))
(F6 (GOAL * MARK)(SAID ONE)

REPL((GOAL * MARK) ; (GOAL + MARK))
INS((Y TWO Z))
(F7A (GOAL * MARK)(SAID TWO)

REPL((GOAL * MARK) ; (GOAL + MARK))
INS((Y THREE Z))
(F7B (GOAL * MARK)(SAID THREE)

REPL((GOAL * MARK) ; (GOAL + MARK))
INS((Y FOUR Z))
(F7c (GOAL * MARK)(SAID FOUR)

REPL((GOAL * MARK) ; (GOAL + MARK))
INS((Y FIVE Z))

39a
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y EIGHT 2))
(F17E (GOAL # MARK)(NUMBER FOUR)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y FIVE 2))
(F17D (GOAL # MARK)(NUMBER FIVE)
==>
REPL((GOAL # MARK)) ; (GOAL #MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y SIX 2))
(F17F (GOAL # MARK)(NUMBER SIX)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y SEVEN 2))
(F17G (GOAL # MARK)(NUMBER SEVEN)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y EIGHT 2))
(F17H (GOAL # MARK)(NUMBER EIGHT)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y NINE 2))
(F17I (GOAL # MARK)(NUMBER NINE)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y TEN 2))
(F17J (GOAL # MARK)(NUMBER TEN)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y ELEVEN 2))
(F17K (GOAL # MARK)(NUMBER ELEVEN)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y TWELVE 2))
(F17L (GOAL # MARK)(NUMBER TWELVE)
==>
REPL((GOAL # MARK)) ; (GOAL # MARK))
REPL((GOAL ## COMPARE) ; (GOAL # COMPARE))
INS((Y THIRTEEN 2))
(P24A (GOAL # PURGE)(Y <X> Z)(BOTTOM D)(SEND C)
==>
REPL((BOTTOM E)) ; (BOTTOM <X>))
REPL((SEND C)) ; (OLD SEND C))
DEL((Y <X> Z))
(P24A (GOAL # PURGE)(Y <X> Z)
==>
DEL((Y <X> Z))
(P26 (GOAL # PURGE)(D <X> Z)
==>
DEL((D <X> Z))
(P26 (GOAL # PURGE)(BOTTOM TWELVE)
==>
STOP)
(P27 (GOAL # PURGE)
==>
DEL((GOAL # PURGE))
REPL((GOAL ## ATTEND) ; (GOAL # ATTEND))
}
(P130A (GOAL * SMALL) (BOTTOM B) (SEND B)

DEL ((GOAL * SMALL));
REPL ((GOAL * SMALL)) ; (GOAL * ADD));
INS ((SMALLER B));
INS ((LARGER B));
(P130B (GOAL * SMALL) (BOTTOM ZERO) (SEND C)

DEL ((GOAL * SMALL));
DEL ((GOAL * MOVE));
DEL ((GOAL * MARK));
DEL ((GOAL * SMALL));
DEL ((GOAL * ADD));
INS ((GOAL * PURGE));
REPL ((BOTTOM ZERO); (BOTTOM C));
REPL ((SEND C)); (OLD SEND C));
SAY (C));
(P130 (GOAL * COMPARE) (GOAL * SMALL) (BOTTOM B) (NUMBER B)

REPL ((GOAL * COMPARE)); (GOAL * COMPARE));
REPL ((GOAL * SMALL)); (GOAL * LARGE));
REPL ((GOAL * MOVE)); (GOAL * MOVE));
REPL ((NUMBER B)); (LARGER B));
(P130 (GOAL * COMPARE) (GOAL * SMALL) (SEND C) (NUMBER C)

REPL ((GOAL * COMPARE)); (GOAL * SMALL));
DEL ((GOAL * LARGE));
REPL ((GOAL * ADD)); (GOAL * ADD));
REPL ((NUMBER B)); (LARGER B));
INS ((W ONE V));
(P230 (GOAL * COMPARE) (GOAL * LARGE) (SEND C) (NUMBER C)

REPL ((GOAL * COMPARE)); (GOAL * SMALL));
DEL ((GOAL * LARGE));
REPL ((GOAL * ADD)); (GOAL * ADD));
REPL ((NUMBER C)); (LARGER C));
INS ((W ONE V));
(P230 (GOAL * COMPARE) (GOAL * ADD) (NUMBER E) (ABS (W <P> V)) (ABS (GOAL * FIND PLACE))

REPL ((GOAL * COMPARE)); (GOAL * SMALL));
DEL ((NUMBER E));
(P230 (GOAL * ADD) (W <P> V) (SMALLER <P>) (Y <X> Z) (LARGER B)

DEL ((GOAL * ADD));
DEL ((GOAL * MOVE));
DEL ((GOAL * MARK));
DEL ((GOAL * SMALL));
DEL ((GOAL * SMALL));
DEL ((LARGER E));
INS ((GOAL * PURGE));
DEL ((W <P> V));
SAY (X));
(P230 (GOAL * ADD) (W ONE V)

REPL ((GOAL * COMPARE)); (GOAL * SMALL));
REPL ((GOAL * MOVE)); (GOAL * MOVE));
RUN BEGINS:

((GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

PO
INPUT TO STI:
$1 (GO)

STI: ((GO) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

POO

STI: ((GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P1

STI: ((Y ONE Z) (GOAL & SEND TWO) (GOAL &% ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STI: ((GOAL & PUSH BUTTON) (GOAL &% SEND TWO) (Y ONE Z) (GOAL &% ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P3

STI: ((GOAL & SUBIT) (ELH A) (GOAL &% SEND TWO) (Y ONE Z) (GOAL &% ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P4

STI: ((GOAL & COUNT) (AS 1) (GOAL &% SEND TWO) (Y ONE Z) (GOAL &% A TEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P5

***** ONE

STI: ((GOAL & MARK) (GOAL & COUNT) (SAY ONE) (GOAL &% SEND TWO) (GOAL &% ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P6

STI: ((Y TWO Z) (GOAL & MARK) (SAY ONE) (GOAL & COUNT) (GOAL &% ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P7
(GOAL #2: ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P3

STM1: ((GOAL #5: SEND TWO) (Y TWO Z) (GOAL #5: ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P2

STM1: ((GOAL # PUSH BUTTON)) (GOAL #5: SEND TWO) (Y TWO Z) (GOAL #5: AT

11 TwO) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P4

STM1: ((GOAL # COUNT) (IN A)) (GOAL #5: SEND TWO) (Y TWO Z) (GOAL #5: A

10 VEL) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P5

**###** TWO

STM1: ((GOAL # MARK)) (GOAL COUNT) (SEND TWO) (GOAL #5: SEND TWO) (GOAL #5: ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P6

STM1: ((Y THREE Z) (GOAL MARK) (SEND TWO) (GOAL COUNT) (GOAL #5: SEND TWO) (GOAL #5: ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P7

STM1: ((Y THREE Z) (GOAL # ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P8

INPUT TO STM1

STM1: (GOAL # MARK BEFORE) (GOAL # ATTEND) (Y THREE Z) (TOP TWELVE) (BO

TTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P9
STM: ((GOAL * RECALL TUBE) (GOAL *# ATTEND) (Y THREE Z) (TOP TWELVE) (BOTTOM ZERO) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P10

***** (BOTTOM ZERO)

STM: ((BOTTOM ZERO) (GOAL * ATTEND) (Y THREE Z) (TOP TWELVE) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P0

INPUT TO STM:
*MANY TUBE*

STM: ((MANY TUBE) (GOAL * ATTEND) (BOTTOM ZERO) (Y THREE Z) (TOP TWELVE) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P11

STM: ((GOAL * RECALL TUBE) (GOAL *# ATTEND) (BOTTOM ZERO) (Y THREE Z) (TOP TWELVE) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P12

***** (SEND TWO)

STM: ((SEND TWO) (GOAL * ATTEND) (BOTTOM ZERO) (Y THREE Z) (TOP TWELVE) (SEND TWO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P0

INPUT TO STM:
*MANY BOTTOM NOW*

STM: ((MANY BOTTOM NOW) (GOAL * ATTEND) (SEND TWO) (BOTTOM ZERO) (Y THREE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P13

STM: ((Y ONE Z) (GOAL *# ADD) (GOAL *# COMPARE) (GOAL *# MARK) (GOAL *# MOVE) (GOAL * SMALL) (GOAL *# ATTEND) (SEND TWO) (BOTTOM ZERO) (Y THREE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P13D

***** TWO

STM: ((GOAL * PURGE) (BOTTOM TWO) (OLD SEND TWO) (Y ONE Z) (GOAL *# ATTEND) (Y THREE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P24

STM: ((GOAL * PURGE) (BOTTOM TWO) (OLD SEND TWO) (GOAL *# ATTEND) (Y
THREE (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P24

STM: ((GOAL $ PURGE) (BOTTOM TWO) (OLD SEND FOUR) (GOAL $5* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P27

STM: ((BOTTOM TWO) (OLD SEND TWO) (GOAL $ ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) 000
INPUT TO STM:
#(SEND FOUR)

STM: ((SEND FOUR) (GOAL ATTEND) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P1

STM: ((Y ONE Z) (GOAL $ SEND FOUR) (GOAL $5* ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P2

STM: ((GOAL $ PUSH BUTTON) (GOAL $5* SEND FOUR) (Y ONE Z) (GOAL $5* ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P3

STM: ((GOAL $ SUBIT) (SLH A) (GOAL $5* SEND FOUR) (Y ONE Z) (GOAL $5* ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P4

STM: ((GOAL $ COUNT) (OS 1) (GOAL $5* SEND FOUR) (Y ONE Z) (GOAL $5* ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P5

***** ONE

STM: ((GOAL $ HARK) (GOAL COUNT) (SAID ONE) (GOAL $5* SEND FOUR) (GOAL $5* ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P6

STM: ((Y TWO Z) (GOAL HARK) (SAID ONE) (GOAL COUNT) (GOAL $5* SEND FO)
0) (GOAL $5* ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK
JUNK JUNK JUNK JUNK JUNK

PS

STIM: ((GOAL & SEND FOUR) (Y TWO Z) (GOAL &ST ATTEND) (TOP TWELVE) (BOT TON TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STIM: ((GOAL & PUSH BUTTON) (GOAL &ST SEND FOUR) (Y TWO Z) (GOAL &ST ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P3

STIM: ((GOAL & SEND FOUR) (GOAL &ST SEND FOUR) (Y TWO Z) (GOAL &ST ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P4

STIM: ((GOAL & COUNT) (PS 1) (GOAL &ST SEND FOUR) (Y TWO Z) (GOAL &ST ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P5

**********

TWO

STIM: ((GOAL & MARK) (GOAL COUNT) (SAID TWO) (GOAL &ST SEND FOUR) (GOAL &ST ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

PS

STIM: ((Y THREE Z) (GOAL MARK) (SAID TWO) (GOAL COUNT) (GOAL &ST SEND FOUR) (GOAL &ST ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P8

STIM: ((GOAL & SEND FOUR) (Y THREE Z) (GOAL &ST ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STIM: ((GOAL & PUSH BUTTON) (GOAL &ST SEND FOUR) (Y THREE Z) (GOAL &ST ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P3
STH: ((GOAL + COUNT) (G6 1) (GOAL 18% SEND FOUR) (Y THREE Z) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK)
P5

******* THREE

STH: ((GOAL + MARK) (GOAL COUNT) (SAID THREE) (GOAL 25% SEND FOUR) (GOAL 18% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK)
P6

STH: ((Y FOUR Z) (GOAL MARK) (SAID THREE) (GOAL COUNT) (GOAL 18% SEND FOUR) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P8

STH: ((GOAL + SEND FOUR) (Y FOUR Z) (GOAL 18% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STH: ((GOAL + PUSH BUTTON) (GOAL 18% SEND FOUR) (Y FOUR Z) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STH: ((GOAL + SUBIT) (FLA 1) (GOAL 18% SEND FOUR) (Y FOUR Z) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P4

STH: ((GOAL + COUNT) (G6 1) (GOAL 18% SEND FOUR) (Y FOUR Z) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK)
P5

******* FOUR

STH: ((GOAL + MARK) (GOAL COUNT) (SAID FOUR) (GOAL 18% SEND FOUR) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK)
P6

STH: ((Y FOUR Z) (GOAL MARK) (SAID FOUR) (GOAL COUNT) (GOAL 18% SEND FOUR) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK)
P7

STH: ((GOAL + MARK) (GOAL COUNT) (SAID FOUR) (GOAL 18% SEND FOUR) (GOAL 25% ATTEND) (TOP TWELVE) (BOTTOM TWO) (SEND FOUR) JUNK JUNK JUNK JUNK JUNK JUNK)
P8
F13

STH: ( (Y ONE Z) (GOAL *S* ADD) (GOAL *S* COMPARE) (GOAL *S* MARK) (GOAL *S* MOVE) (GOAL *S* SMALL) (GOAL *S* ATTEND) (SEND FOUR) (BOTTOM TWO) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F14

STH: ( (GOAL *S* SMALL) (Y ONE Z) (GOAL *S* ADD) (GOAL *S* COMPARE) (GOAL *S* MARK) (GOAL *S* MOVE) (GOAL *S* SMALL) (GOAL *S* ATTEND) (SEND FOUR) (BOTTOM TWO) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F16

STH: ( (GOAL *S* MOVE) (NUMBER ONE) (GOAL *S* SMALL) (GOAL *S* ADD) (GOAL *S* COMPARE) (GOAL *S* ATTEND) (SEND FOUR) (BOTTOM TWO) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F17

STH: ( (GOAL *S* COMPARE) (GOAL *S* ADD) (Y TWO Z) (GOAL *S* MOVE) (GOAL *S* SMALL) (GOAL *S* SMALL) (GOAL *S* ATTEND) (SEND FOUR) (BOTTOM TWO) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F16A

STH: ( (GOAL *S* MOVE) (NUMBER TWO) (GOAL *S* COMPARE) (GOAL *S* ADD) (GOAL *S* MARK) (GOAL *S* SMALL) (GOAL *S* ATTEND) (SEND FOUR) (BOTTOM TWO) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F17A

STH: ( (Y THREE Z) (GOAL *S* MARK) (NUMBER TWO) (GOAL *S* MOVE) (GOAL *S* COMPARE) (GOAL *S* ADD) (GOAL *S* SMALL) (GOAL *S* ATTEND) (SEND FOUR) (BOTTOM TWO) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F19

STH: ( (GOAL *S* COMPARE) (GOAL *S* LARGE) (BOTTOM TWO) (SMALLER TWO) (Y THREE Z) (GOAL *S* MARK) (GOAL *S* MOVE) (GOAL *S* ADD) (GOAL *S* ATTEND) (SEND FOUR) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F16

STH: ( (GOAL *S* MOVE) (NUMBER THREE) (GOAL *S* COMPARE) (GOAL *S* LARGE) (BOTTOM TWO) (SMALLER TWO) (GOAL *S* MARK) (GOAL *S* ADD) (GOAL *S* ATTEND) (SEND FOUR) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F17B
STH: ((Y FOUR Z) (GOAL "$# MARK) (NUMBER THREE) (GOAL "$# MOVE) (GOAL "$# COMPARE) (GOAL "$# LARGE) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ADD) (GOAL "$# ATTEND) (SEND FOUR) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK)
P210

STH: ((GOAL "$# COMPARE) (GOAL "$# ADD) (Y FOUR Z) (GOAL "$# MARK) (GOAL "$# MOVE) (GOAL "$# LARGE) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (SEND FOUR) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P116

STH: ((GOAL "$# MOVE) (NUMBER FOUR) (GOAL "$# COMPARE) (GOAL "$# ADD) (GOAL "$# MARK) (GOAL "$# LARGE) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (SEND FOUR) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P170

STH: ((Y FIVE Z) (GOAL "$# MARK) (NUMBER FOUR) (GOAL "$# MOVE) (GOAL "$# COMPARE) (GOAL "$# ADD) (GOAL "$# LARGE) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (SEND FOUR) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK)
P210

STH: ((U ONE V) (GOAL "$# COMPARE) (SEND FOUR) (LARGER FOUR) (Y FIVE Z) (GOAL "$# MARK) (GOAL "$# MOVE) (GOAL "$# ADD) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK)
P230

STH: ((GOAL "$# ADD) (U TWO V) (GOAL "$# COMPARE) (SEND FOUR) (LARGER FOUR) (Y FIVE Z) (GOAL "$# MARK) (GOAL "$# MOVE) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK)
P16

STH: ((GOAL "$# MOVE) (NUMBER FIVE) (GOAL "$# ADD) (U TWO V) (GOAL "$# COMPARE) (SEND FOUR) (LARGER FOUR) (GOAL "$# MARK) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK)
P170

STH: ((Y SIX Z) (GOAL "$# MARK) (NUMBER FIVE) (GOAL "$# MOVE) (GOAL "$# ADD) (U TWO V) (GOAL "$# COMPARE) (SEND FOUR) (LARGER FOUR) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK)
P210

STH: ((GOAL "$# COMPARE) (U TWO V) (Y SIX Z) (GOAL "$# MARK) (GOAL "$# MOVE) (GOAL "$# ADD) (SEND FOUR) (LARGER FOUR) (BOTTOM TWO) (SMALLER TWO) (GOAL "$# ATTEND) (Y FIVE Z) (TOP TWELVE) JUNK JUNK JUNK)
P230

* * * * * * * SIX
STH: ((GOAL * PURGE) (Y SIX 2) (SEND FOUR) (BOTTOM TWO) (GOAL #3* ATTEND) (Y FIVE 2) (TOP THIRTY) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
F24A

STH: ((GOAL * PURGE) (BOTTOM SIX) (OLD SEND FOUR) (GOAL #3* ATTEND) (Y FIVE 2) (TOP THIRTY) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
F24

STH: ((GOAL * PURGE) (BOTTOM SIX) (OLD SEND FOUR) (GOAL #3* ATTEND) (TOP THIRTY) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
F006
INPUT TO STH:
#(SEND THREE)

STH: ((SEND THREE) (GOAL ATTEND) (BOTTOM SIX) (TOP THIRTY) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STH: (Y ONE 2) (GOAL # SEND THREE) (GOAL #3* ATTEND) (TOP THIRTY) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F2

STH: ((GOAL # PUSH BUTTON) (GOAL #3% SEND THREE) (Y ONE 2) (GOAL #3* ATTEND) (TOP THIRTY) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F3

STH: ((GOAL # SUBMIT) (ELM A) (GOAL #3% SEND THREE) (Y ONE 2) (GOAL #3* ATTEND) (TOP THIRTY) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F4

STH: ((GOAL # COUNT) (OS 1) (GOAL #3% SEND THREE) (Y ONE 2) (GOAL #3% ATTEND) (TOP THIRTY) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F5

# # # # # # ONE
S8th: (GOAL & PUSH BUTTON) (GOAL 888 SEND THREE) (Y THREE Z) (GOAL 88 ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
S9th: (GOAL & COUNT) (US 1) (GOAL 888 SEND THREE) (Y THREE Z) (GOAL 88 ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
# 9
S10th: (GOAL 886 MARK) (GOAL COUNT) (SAID THREE) (GOAL 888 SEND THREE) (GOAL 88 ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
S11th: (Y FOUR Z) (GOAL MARK) (SAID THREE) (GOAL COUNT) (GOAL 888 SEND THREE) (GOAL 88 ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F7
S12th: (Y FOUR Z) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F8
INPUT TO S13th:
# (MANY BOTTOM BEFORE)
S13th: (MANY BUTTON BEFORE) (GOAL & ATTEND) (Y FOUR Z) (TOP TWELVE) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F9
S14th: (GOAL & RECALL BOTTOM) (GOAL 888 ATTEND) (Y FOUR Z) (TOP TWELVE) (BOTTOM SIX) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
F10
# 10
S15th: (BOTTOM SIX) (GOAL & ATTEND) (Y FOUR Z) (TOP TWELVE) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
INPUT TO 5TH:
* (MANY TIMES)

5TH: (GOAL 3 X ATTEND) (GOAL 2 X ATTEND) (GOAL 1 X ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P11

5TH: (GOAL 2 X RECALL TUBE) (GOAL 3 X ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) (SEND THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P12

* * * * * * (SEND THREE)

5TH: (SEND THREE) (GOAL 3 X ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
INPUT TO 5TH:
* (MANY BOTTOM NOW)

5TH: (MANY BOTTOM NOW) (GOAL 3 X ATTEND) (SEND THREE) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P13

5TH: (Y ONE Z) (GOAL 2 X ADD) (GOAL 3 X COMPARISON) (GOAL 1 X MARK) (GOAL 3 X MOVE) (GOAL 3 X SMALL) (GOAL 3 X ATTEND) (SEND THREE) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK
P14

5TH: (GOAL 3 X SMALL) (Y ONE Z) (GOAL 2 X ADD) (GOAL 3 X COMPARISON) (GOAL 3 X MARK) (GOAL 3 X MOVE) (GOAL 3 X SMALL) (GOAL 3 X ATTEND) (SEND THREE) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK
P15

5TH: (GOAL 3 X MOVE) (NUMBER ONE) (GOAL 2 X SMALL) (GOAL 2 X ADD) (GOAL 2 X COMPARISON) (GOAL 2 X MARK) (GOAL 3 X ATTEND) (SEND THREE) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK
P16

5TH: (Y TWO Z) (GOAL 2 X MARK) (NUMBER ONE) (GOAL 3 X MOVE) (GOAL 3 X SMALL) (GOAL 2 X ADD) (GOAL 3 X COMPARISON) (GOAL 3 X ATTEND) (SEND THREE) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK
P17

5TH: (GOAL 3 X COMPARISON) (GOAL 2 X ADD) (Y TWO Z) (GOAL 2 X MARK) (GOO
STH: ((GOAL #5% MOVE) (NUMBER FIVE) (GOAL #5% COMPARE) (GOAL #5% ADD) (GOAL #5% MARK) (GOAL #5% LARGE) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((Y SIX Z) (GOAL #5% MARK) (NUMBER FIVE) (GOAL #5% MOVE) (GOAL #5% COMPARE) (GOAL #5% ADD) (GOAL #5% LARGE) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((GOAL #5% COMPARE) (GOAL #5% ADD) (Y SIX Z) (GOAL #5% MARK) (GOAL #5% LARGE) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((GOAL #5% MARK) (NUMBER FIVE) (GOAL #5% COMPARE) (GOAL #5% ADD) (GOAL #5% LARGE) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((Y SEVEN Z) (GOAL #5% MARK) (NUMBER SIX) (GOAL #5% MOVE) (GOAL #5% COMPARE) (GOAL #5% ADD) (GOAL #5% LARGE) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (BOTTOM SIX) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((W ONE Y) (GOAL #5% COMPARE) (BOTTOM SIX) (LARGER SIX) (Y SEVEN Z) (GOAL #5% MARK) (GOAL #5% MOVE) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((GOAL #5% ADD) (W TWO Y) (GOAL #5% COMPARE) (BOTTOM SIX) (LARGER SIX) (GOAL #5% MARK) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((GOAL #5% MOVE) (NUMBER SEVEN) (GOAL #5% ADD) (W TWO Y) (GOAL #5% COMPARE) (BOTTOM SIX) (LARGER SIX) (GOAL #5% MARK) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)

STH: ((Y EIGHT Z) (GOAL #5% MARK) (NUMBER SEVEN) (GOAL #5% MOVE) (GOAL #5% ADD) (W TWO Y) (GOAL #5% COMPARE) (BOTTOM SIX) (LARGER SIX) (SEND THREE) (SMALLER THREE) (GOAL #5% ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
STH: ((GOAL #9 COMPARE) (U TWO V) (Y EIGHT Z) (GOAL #9 MARK) (GOAL #9 MOVE) (GOAL # ADD) (BOTTOM SIX) (LARGER SIX) (SEND THREE) (SMALLER THREE) (GOAL #9 ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK) P229

STH: ((GOAL #9 ADD) (U THREE V) (GOAL #9 COMPARE) (Y EIGHT Z) (GOAL #8 MARK) (GOAL # MOVE) (BOTTOM SIX) (LARGER SIX) (SEND THREE) (SMALLER THREE) (GOAL #9 ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK) P16

STH: ((GOAL #9 MOVE) (NUMBER EIGHT) (GOAL #9 ADD) (U THREE V) (GOAL #9 COMPARE) (BOTTOM SIX) (LARGER SIX) (SEND THREE) (SMALLER THREE) (GOAL #9 ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK) P176

STH: ((Y NINE Z) (GOAL #9 MARK) (NUMBER EIGHT) (GOAL #9 MOVE) (GOAL # ADD) (U THREE V) (GOAL # COMPARE) (BOTTOM SIX) (LARGER SIX) (SEND THREE) (SMALLER THREE) (GOAL #9 ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK) P240

STH: ((GOAL #9 COMPARE) (U THREE V) (Y NINE Z) (GOAL #9 MARK) (GOAL #9 MOVE) (GOAL # ADD) (BOTTOM SIX) (LARGER SIX) (SEND THREE) (SMALLER THREE) (GOAL #9 ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK) P220

***NINE***

STH: ((GOAL # PURGE) (Y NINE Z) (BOTTOM SIX) (SEND THREE) (GOAL #9 ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P244

STH: ((GOAL # PURGE) (BOTTOM NINE) (OLD SEND THREE) (GOAL #9 ATTEND) (Y FOUR Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P24

STH: ((GOAL # PURGE) (BOTTOM NINE) (OLD SEND THREE) (GOAL #9 ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P27

STH: ((BOTTOM NINE) (OLD SEND THREE) (GOAL #9 ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P220
INPUT TO STM:
* (SEND ONE)

STM: ((SEND ONE) (GOAL ATTEND) (BOTTOM NINE) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STM: ((Y ONE Z) (GOAL * SEND ONE) (GOAL ** ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STM: ((GOAL * PUSH BUTTON) (GOAL ** SEND ONE) (Y ONE Z) (GOAL *** ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STM: ((GOAL * SUBIT) (SLH A) (GOAL ** SEND ONE) (Y ONE Z) (GOAL *** ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P4

STM: ((GOAL * COUNTER) (AS 1) (GOAL ** SEND ONE) (Y ONE Z) (GOAL *** ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P5

* * * * * * * ONE

STM: ((GOAL * MARK) (GOAL COUNT) (SAID ONE) (GOAL ** SEND ONE) (GOAL *** ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK)
P6

STM: ((Y TWO Z) (GOAL HARK) (SAID ONE) (GOAL COUNT) (GOAL ** SEND ON C) (GOAL *** ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK)
P7

STM: ((Y TUN Z) (GOAL *** ATTEND) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P8

INPUT TO STM:
* (GOAL BOTTOM BEFORE)
TTDD: (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P9

STH: ((GOAL # RECALL BOTTOM) (GOAL % ATTEND) (Y TWD Z) (TOP TWELVE) (BOTTOM NINE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P10

#### (BOTTOM NINE)

STH: ((BOTTOM NINE) (GOAL % ATTEND) (Y TWD Z) (TOP TWELVE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P11

STH: ((GOAL # RECALL TUBE) (GOAL % ATTEND) (BOTTOM NINE) (Y TWD Z) (TOP TWELVE) (SEND ONE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P12

#### (SEND ONE)

STH: ((SEND ONE) (GOAL % ATTEND) (BOTTOM NINE) (Y TWD Z) (TOP TWELVE) (SEND ONE) (BOTTOM NINE) (Y TWD Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P13

STH: ((Y ONE Z) (GOAL % ADD) (GOAL #4 COMPARE) (GOAL %2 MARK) (GOAL % ADD) (GOAL % SMALL) (GOAL % ATTEND) (SEND ONE) (BOTTOM NINE) (Y TWD Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P14

STH: ((GOAL #4 SMALL) (Y ONE Z) (GOAL % ADD) (GOAL #4 COMPARE) (GOAL % ADD) (GOAL % SMALL) (GOAL % ATTEND) (SEND ONE) (BOTTOM NINE) (Y TWD Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P15
STH: ((GOAL #S% MOVE) (NUMBER ONE) (GOAL #S% SMALL) (GOAL #S% ADD) (GOAL #S% COMPARE) (GOAL #S% MARK) (GOAL #S% ATTEND) (SEND ONE) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F17

STH: ((GOAL #S% MOVE) (NUMBER ONE) (GOAL #S% MARK) (GOAL #S% ADD) (GOAL #S% COMPARE) (GOAL #S% ATTEND) (SEND ONE) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F190

STH: ((GOAL #S% COMPARE) (GOAL #S% LARGE) (SEND ONE) (SMALLER ONE) (Y TWO Z) (GOAL #S% MARK) (GOAL #S% MOVE) (GOAL #S% ADD) (GOAL #S% ATTEND) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F16

STH: ((GOAL #S% MOVE) (NUMBER TWO) (GOAL #S% COMPARE) (GOAL #S% LARGE) (SEND ONE) (SMALLER ONE) (GOAL #S% ADD) (GOAL #S% ATTEND) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F17A

STH: ((GOAL #S% COMPARE) (GOAL #S% LARGE) (SEND ONE) (GOAL #S% MARK) (GOAL #S% MOVE) (GOAL #S% ADD) (GOAL #S% ATTEND) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F16

STH: ((GOAL #S% COMPARE) (GOAL #S% LARGE) (SEND ONE) (SMALLER ONE) (GOAL #S% ADD) (GOAL #S% ATTEND) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F17B

STH: ((GOAL #S% MOVE) (NUMBER THREE) (GOAL #S% COMPARE) (GOAL #S% ADD) (GOAL #S% MARK) (GOAL #S% LARGE) (SEND ONE) (SMALLER ONE) (GOAL #S% ATTEND) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F210A

STH: ((GOAL #S% COMPARE) (GOAL #S% ADD) (Y FOUR Z) (GOAL #S% MARK) (GOAL #S% MOVE) (GOAL #S% LARGE) (SEND ONE) (SMALLER ONE) (GOAL #S% ATTEND) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F16

STH: ((GOAL #S% COMPARE) (GOAL #S% ADD) (Y FOUR Z) (GOAL #S% MARK) (GOAL #S% MOVE) (GOAL #S% LARGE) (SEND ONE) (SMALLER ONE) (GOAL #S% ATTEND) (BOTTOM HINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)  
F210A
STH: ((GOAL *% MOVE) (NUMBER FOUR) (GOAL *% CHANGE) (GOAL *% ADD)
(GOAL * MARK) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (GOAL *% AT
TEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P176

STH: ((Y FIVE Z) (GOAL *% MARK) (NUMBER FOUR) (GOAL *% MOVE) (GOAL
& COMPARE) (GOAL *% ADD) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE)
(GOAL *% ATTEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P210A

STH: ((GOAL *% COMPARE) (GOAL *% ADD) (Y FIVE Z) (GOAL *% MARK) (G
OAL * MOVE) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (GOAL *% AT
TEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P16

STH: ((GOAL *% MOVE) (NUMBER FIVE) (GOAL *% COMPARE) (GOAL *% ADD)
(GOAL * MARK) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (GOAL *% AT
TEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P17B

STH: ((Y SIX Z) (GOAL *% MARK) (NUMBER FIVE) (GOAL *% MOVE) (GOAL *
COMPARE) (GOAL *% ADD) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (G
OAL *% ATTEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P210A

STH: ((GOAL *% COMPARE) (GOAL *% ADD) (Y SIX Z) (GOAL *% MARK) (GO
AL * MOVE) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (GOAL *% ATTEN
D) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P16

STH: ((GOAL *% MOVE) (NUMBER SIX) (GOAL *% COMPARE) (GOAL *% ADD)
(GOAL * MARK) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (GOAL *% AT
TEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P17E

STH: ((Y SEVEN Z) (GOAL *% MARK) (NUMBER SIX) (GOAL *% MOVE) (GOAL
& COMPARE) (GOAL *% ADD) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (G
OAL *% ATTEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P210A

STH: ((GOAL *% COMPARE) (GOAL *% ADD) (Y SEVEN Z) (GOAL *% MARK) (G
OAL * MOVE) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (GOAL *% ATTEN
D) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
P16

STH: ((GOAL *% MOVE) (NUMBER SEVEN) (GOAL *% COMPARE) (GOAL *% ADD)
(GOAL * MARK) (GOAL *% LARGE) (SEND ONE) (SMALLER ONE) (GOAL *% AT
TEND) (BOTTOM NINE) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK)
<table>
<thead>
<tr>
<th>Page 17F</th>
</tr>
</thead>
<tbody>
<tr>
<td>P210A</td>
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<tr>
<td>P16</td>
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<td>P200</td>
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<td>P220</td>
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<td>TEN</td>
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</tbody>
</table>
STH1: ((GOAL & PUNGE) (BOTTOM TEN) (OLD SEND ONE) (GOAL & SRT ATTEND) (Y TWO Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P24

STH1: ((GOAL & PUNGE) (BOTTOM TEN) (OLD SEND ONE) (GOAL & SRT ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P27

STH1: ((BOTTOM TEN) (OLD SEND ONE) (GOAL & ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P300

INPUT TO STH1:

*()

STH1: (NIL (GOAL ATTEND) (BOTTOM TEN) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

END OF RUN

NUMBER OF PRODUCTIONS FIRED : 276
NUMBER OF OPERATIONS EXECUTED : 1161
NUMBER OF PRODUCTIONS TRIED : 5661
NUMBER OF SINGLE-ELEMENT COMPARISONS : 135141
SIMULATED TIME : 0
COMPUTER TIME: 652045 MSEC
NUMBER OF CONS-OPERATIONS : 1521701

* * *
ADDENDUM 4.

LISTING AND EXECUTION TRACE FOR

N-SUB-NV: TABLE-LOOK-UP METHOD.
V.500
(VARIABLES:
(A GENC)
(B GENC)
(C GENC)
(D GENC)
(E GENC)
(<X> SENC)
(<P> SENC)
)
(PRODUCTIONS:
(F000 (GOAL & ATTEND))(OLD SEND C)
REPL((GOAL & ATTEND) ; (GOAL & ATTEND))
DEL((OLD SEND C))
USER()
(F000 (GOAL & ATTEND))(GO)
REPL((GOAL & ATTEND) ; (GOAL & ATTEND))
DEL((GO))
(F00 (GOAL & ATTEND))
USER()
(P1 (GOAL & ATTEND))(TOP A)(BOTTOM B)(SEND C)
REPL((GOAL & ATTEND) ; (GOAL & ATTEND))
INS((GOAL & SEND C));
INS((Y 1 Z))
(P2 (GOAL & SEND C))
REPL((GOAL & SEND C) ; (GOAL & SEND C))
INS((GOAL & PUSH BUTTON))
(D3 (GOAL & PUSH BUTTON))
DEL((GOAL & PUSH BUTTON))
INS((ELM A))
INS((GOAL & SUBIT))
(P4 (GOAL & SUBIT))(ELM A)
DEL((GOAL & SUBIT))
INS((GS 1))
DEL((ELM A))
INS((GOAL & COUNT))
(P5 (GOAL & COUNT))(GS 1)(Y <X> Z)
REPL((GOAL & COUNT) ; (GOAL & COUNT))
DEL((GS 1))
SAY(:GOAL)
REPL((Y <X> Z) ; (SAID <X>))
INS((GOAL & MARK))
(P6 (GOAL & MARK))(SAID 1)
REPL((GOAL & MARK) ; (GOAL & MARK))
INS((Y 2 Z))
(P7A (GOAL & MARK))(SAID 2)
REPL((GOAL & MARK) ; (GOAL & MARK))
INS((Y 3 Z))
(P8A (GOAL & MARK))(SAID 3)
REPL((GOAL & MARK) ; (GOAL & MARK))
INS((Y 4 Z))
(P9A (GOAL & MARK))(SAID 4)
REPL((GOAL & MARK) ; (GOAL & MARK))
INS((Y 5 Z))
)
(P2 (GOAL + MARK) (GOAL + COUNT) (GOAL * SEND C) (SAID C))

DEL ((GOAL * SEND C));
DEL ((GOAL + MARK));
DEL (GOAL + COUNT));
DEL ((SAID C));
REPL ((GOAL * SEND C) ; (GOAL * ATTEND));
(REPL (GOAL + MARK) (GOAL + COUNT) (GOAL * SEND C) (SAID D))

REPL ((GOAL * SEND C) ; (GOAL * SEND C));
DEL (GOAL + MARK));
DEL (GOAL + COUNT));
DEL ((SAID D));
(REPL (GOAL + ATTEND) (GOAL + COUNT) (GOAL * SEND C) (GOAL + MANY TOP BEFORE)

(REPL ((GOAL * ATTEND) ; (GOAL * ATTEND));
DEL (MANY TOP BEFORE));
INS ((GOAL * RECALL TOP));
(P10 (GOAL + RECALL TOP) (TOP A))

SAY ((TOP A));
DEL ((GOAL + RECALL TOP));
REPL ((GOAL * ATTEND) ; (GOAL * ATTEND));
(P11 (GOAL + ATTEND) (MANY TUBE))

REPL ((GOAL * ATTEND) ; (GOAL * ATTEND));
DEL (MANY TUBE));
INS ((GOAL + RECALL TUBE));
(P12 (GOAL + RECALL TUBE) (SEND C))

DEL ((GOAL + RECALL TUBE));
REPL ((GOAL * ATTEND) ; (GOAL * ATTEND));
SAY (SEND C));
(P13 (GOAL + ATTEND) (MANY TOP NOW))

REPL ((GOAL * ATTEND) ; (GOAL * ATTEND));
DEL (MANY TOP NOW));
INS ((GOAL * SUBTRACT));
(C14 (GOAL + SUBTRACT) (TOP A) (ABS (GOAL * LIST A))

REPL ((GOAL * SUBTRACT) ; (GOAL * SUBTRACT));
INS ((GOAL + LIST A))
(P15 (GOAL + LIST THREE))

REPL ((GOAL * SUBTRACT) ; (GOAL * SUBTRACT));
INS ((1 TUNE Z));
INS ((2 ONE Z))
(F15A (GOAL + LIST SIX))

REPL ((GOAL * SUBTRACT) ; (GOAL * SUBTRACT));
INS ((2 FOUR Z));
INS ((3 THREE Z))
(F15B (GOAL + LIST TEN))

REPL ((GOAL * SUBTRACT) ; (GOAL * SUBTRACT));
INS ((7 SEVEN Z));
INS ((6 SIX Z))
(F15C (GOAL + LIST TWELVE))

REPL ((GOAL * SUBTRACT) ; (GOAL * SUBTRACT));
INS ((1 ELEVEN Z));
INS ((2 TEN Z))
(P16 (GOAL + SUBTRACT) (SEND C) (C <X> Z) (TOP A))

SAY (<X>));
•

68

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''_. L.

f

?JTTD-!D)(TOP TWELVE)(I<OTTOM ZEf::Q)(SEND 2)))


RUN BEGINS:

((GOAL $ ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK)
JUNK JUNK JUNK JUNK JUNK JUNK

INPUT TO STM:
$<GO>)

STM: ((GO) (GOAL $ ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK
JUNK JUNK JUNK JUNK JUNK)
POO

STM: ((GOAL ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK
JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STM: ((Y 1 2) (GOAL $ SEND 2) (GOAL $ ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STM: ((GOAL $ PUSH BUTTON) (GOAL $ SEND 2) (Y 1 2) (GOAL $ ATTEND)
JUNK JUNK JUNK JUNK JUNK)
P3

STM: ((GOAL $ SUBIT) (ELM A) (GOAL $ SEND 2) (Y 1 2) (GOAL $ ATTEND)
JUNK JUNK JUNK JUNK JUNK)
P4

STM: ((GOAL $ COUNT) (GG 1) (GOAL $ SEND 2) (Y 1 2) (GOAL $ ATTEND)
JUNK JUNK JUNK JUNK JUNK JUNK)
P5

STM: ((GOAL $ MARK) (GOAL COUNT) (SAID 1) (GOAL $ SEND 2) (GOAL $ ATTEND)
JUNK JUNK JUNK JUNK JUNK)
P6

STM: ((Y 2 2) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL $ SEND 2) (GOAL $ ATTEND)
JUNK JUNK JUNK JUNK JUNK)
P7
SYM: ((GOAL & SEND 2) (Y 2 Z) (GOAL &#3 ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P2

SYM: ((GOAL & PUSH BUTTON) (GOAL &#3 SEND 2) (Y 2 Z) (GOAL &#3 ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P3

SYM: ((GOAL & SUBIT) (FLA A) (GOAL &#3 SEND 2) (Y 2 Z) (GOAL &#3 ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P3

###

SYM: ((GOAL & COUNT) (OR 1) (GOAL &#3 SEND 2) (Y 2 Z) (GOAL &#3 ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK) P5

SYM: ((GOAL & MARK) (GOAL COUNT) (SAID 2) (GOAL &#3 SEND 2) (GOAL &#3 ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P6

SYM: ((Y 3 Z) (GOAL MARK) (SAID 2) (GOAL COUNT) (GOAL &#3 SEND 2) (GOAL &#3 ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P6A

SYM: ((GOAL & SEND 2) (Y 2 Z) (GOAL &#3 ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P7

SYM: ((Y 2 Z) (GOAL & ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P8

INPUT TO SYM:

* (MANY TOP BEFORE)

SYM: ((MANY TOP BEFORE) (GOAL & ATTEND) (Y 3 Z) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK) P9

SYM: ((GOAL & RECALL TOP) (GOAL &#4 ATTEND) (Y 3 Z) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P10

###

SYM: ((TOP TWELVE) (GOAL & ATTEND) (Y 2 Z) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
K JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P0
INPUT TO STM
*
(MANY TUBE)

STM: ((MANY TUBE) (GOAL ≠ ATTEND) (TOP TWELVE) (Y 3 Z) (BOTTOM ZERO)
(SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P11

STM: ((GOAL ≠ RECALL TUBE) (GOAL ≠# ATTEND) (TOP TWELVE) (Y 3 Z) (BOTTOM ZERO)
(SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P12

****** *(SEND 2)*

STM: ((SEND 2) (GOAL ≠ ATTEND) (TOP TWELVE) (Y 3 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*
(MANY TOP NOW)

STM: ((MANY TOP NOW) (GOAL ≠ ATTEND) (SEND 2) (TOP TWELVE) (Y 3 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P13

STM: ((GOAL ≠ SUBTRACT) (GOAL ≠# ATTEND) (SEND 2) (TOP TWELVE) (Y 3 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P14

STM: ((GOAL ≠ LIST TWELVE) (GOAL ≠# SUBTRACT) (TOP TWELVE) (GOAL ≠# ATTEND) (SEND 2) (Y 3 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK)
P15C

STM: ((2 TEN Z) (1 ELEVEN Z) (GOAL ≠ LIST TWELVE) (GOAL ≠ SUBTRACT) (TOP TWELVE) (GOAL ≠# ATTEND) (SEND 2) (Y 3 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK)
P16

****** TEN

STM: ((GOAL ≠ PURGE) (SEND 2) (2 TEN Z) (TOP TWELVE) (1 ELEVEN Z) (GOAL ≠# ATTEND) (Y 3 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK)
P24A

STM: ((GOAL ≠ PURGE) (SEND 2) (TOP TEN) (1 ELEVEN Z) (GOAL ≠# ATTEND) (TOP TWELVE) (1 ELEVEN Z) (GOAL ≠# ATTEND) (SEND 2) (Y 3 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK)
P24A
THI: ((GOAL * PURGE) (OLD SEND 2) (TOP TEN) (1 ELEVEN 2) (GOAL *$* ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 24

THI: ((GOAL * PURGE) (OLD SEND 2) (TOP TEN) (GOAL *$* ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 25

THI: ((GOAL * PURGE) (OLD SEND 2) (TOP TEN) (GOAL *$* ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 26

THI: ((Send 4) (GOAL * ATTEND) (TOP TEN) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 1

THI: ((Y 1 Z) (GOAL * SEND 4) (GOAL *$* ATTEND) (TOP TEN) (BOTTOM ZERO) (Send 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 2

THI: ((GOAL * PUSH BUTTON) (GOAL *$* SEND 4) (Y 1 Z) (GOAL *$* ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 3

THI: ((GOAL * SUBIT) (ELM A) (GOAL *$* SEND 4) (Y 1 Z) (GOAL *$* ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 4

THI: ((GOAL * COUNT) (CS 1) (GOAL *$* SEND 4) (Y 1 Z) (GOAL *$* ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 5

****

THI: ((GOAL * JUNK) (GOAL COUNT) (SAID 1) (GOAL *$* SEND 4) (GOAL *$* ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P 6

THI: ((Y 2 Z) (GOAL HIGH) (SAID 1) (GOAL COUNT) (GOAL *$* SEND 4) (GOAL *$* ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
SYN: ((GOAL & SEND 4) (Y 2 Z) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P2

SYN: ((GOAL & PUSH BUTTON) (GOAL #& SEND 4) (Y 2 Z) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P3

SYN: ((GOAL & SUBIT) (TEL A) (GOAL #& SEND 4) (Y 2 Z) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P4

SYN: ((GOAL & MARK) (GOAL COUNT) (SAY 2) (GOAL #& SEND 4) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P6A

SYN: ((Y 3 Z) (GOAL MARK) (SAY 2) (GOAL COUNT) (GOAL #& SEND 4) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P8

SYN: ((GOAL #& SEND 4) (Y 3 Z) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P2

SYN: ((GOAL & PUSH BUTTON) (GOAL #& SEND 4) (Y 3 Z) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P3

SYN: ((GOAL & SUBIT) (TEL A) (GOAL #& SEND 4) (Y 3 Z) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P4

SYN: ((GOAL & COUNT) (TEL 1) (GOAL #& SEND 4) (Y 3 Z) (GOAL #& ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P5

***** 2
STM: ((GOAL ≠ MARK) (GOAL COUNT) (SEND 3) (GOAL ≠2 SEND 1) (GOAL ≠3
ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
PA5

STM: ((Y 4 Z) (GOAL MARK) (SEND 3) (GOAL COUNT) (GOAL ≠3 SEND 4) (GO
AL ≠3 ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK)
PA5

STM: ((GOAL ≠ SEND 4) (Y 4 Z) (GOAL ≠3 ATTEND) (TOP TEN) (BOTTOM ZER
O) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA5

STM: ((GOAL ≠ PUSH BUTTON) (GOAL ≠3 SEND 4) (Y 4 Z) (GOAL ≠3 ATTEND
) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
PA5

STM: ((GOAL ≠ COUNT) (GO 1) (GOAL ≠3 SEND 4) (Y 4 Z) (GOAL ≠3 ATTEN
D) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
PA5

********** 4

STM: ((GOAL ≠ MARK) (GOAL COUNT) (SEND 4) (GOAL ≠3 SEND 4) (GOAL ≠3
ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
PA6C

STM: ((Y 5 Z) (GOAL MARK) (SEND 4) (GOAL COUNT) (GOAL ≠3 SEND 4) (GO
AL ≠3 ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK)
PA7

STM: ((Y 5 Z) (GOAL ≠ ATTEND) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK J
UNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA7

INPUT TO STM: 6(MONY TOP BEFORE)

STM: ((MONY TOP BEFORE) (GOAL ≠ ATTEND) (Y 5 Z) (TOP TEN) (BOTTOM ZER
O) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA7
STH: ((GOAL * RECALL TOP) (GOAL *# ATTEND) (Y S Z) (TOP TEN) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P10

******** (TOP TEN)

STH: ((TOP TEN) (GOAL * ATTEND) (Y S Z) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STH:
$(MANY TUBE)

STH: ((MANY TUBE) (GOAL * ATTEND) (TOP TEN) (Y S Z) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P11

STH: ((GOAL * RECALL TUBE) (GOAL *# ATTEND) (TOP TEN) (Y S Z) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P12

******** (SEND 4)

STH: ((SEND 4) (GOAL * ATTEND) (TOP TEN) (Y S Z) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STH:
$(MANY TOP NOW)

STH: ((MANY TOP NOW) (GOAL * ATTEND) (SEND 4) (TOP TEN) (Y S Z) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P13

STH: ((GOAL * SUBTRACT) (GOAL *# ATTEND) (SEND 4) (TOP TEN) (Y S Z) (BOTTOM ZERO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P14

STH: ((GOAL * LIST TEN) (GOAL *# SUBTRACT) (TOP TEN) (GOAL *# ATTEND) (SEND 4) (Y S Z) (BOTTOM ZERO) (SEND 4) (Y S Z) (BOTTOM ZERO) (JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P15

STH: ((4 SIX Z) (3 EVEN Z) (GOAL * LIST TEN) (GOAL * SUBTRACT) (TOP TEN) (GOAL *# ATTEND) (SEND 4) (Y S Z) (BOTTOM ZERO) (JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P16

******** SIX
STM: (GOAL # PURGE) (SEND 4) (A SIX Z) (TOP TEN) (3 SEVEN Z) (GOAL #3 # ATTEND) (Y 5 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK

STM: (GOAL # PURGE) (OLD SEND 4) (TOP SIX) (3 SEVEN Z) (GOAL #3 # ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK

STM: (GOAL # PURGE) (OLD SEND 4) (TOP SIX) (3 SEVEN Z) (GOAL #3 # ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK

STM: (OLD SEND 4) (TOP SIX) (GOAL # ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK

INPUT TO STM:
#(SEND 3)

STM: (SEND 3) (GOAL ATTEND) (TOP SIX) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK

STM: (Y 1 Z) (GOAL # SEND 3) (GOAL #3 # ATTEND) (TOP SIX) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK

STM: (GOAL # PUSH BUTTON) (GOAL #3 # SEND 3) (Y 1 Z) (GOAL #3 # ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK

STM: (GOAL # SPLIT) (CLH A) (GOAL #3 # SEND 3) (Y 1 Z) (GOAL #3 # ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK

STM: (GOAL # COUNT) (CB 1) (GOAL #3 # SEND 3) (Y 1 Z) (GOAL #3 # ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK

#222222 1
STH: ((GOAL % MARK) (GOAL COUNT) (SAY 1) (GOAL 2% SEND 3) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P6

STH: ((Y 2 Z) (GOAL MARK) (SAY 1) (GOAL COUNT) (GOAL 3% SEND 3) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P8

STH: ((GOAL % SEND 3) (Y 2 Z) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P2

STH: ((GOAL % PUSH BUTTON) (GOAL %% SEND 3) (Y 2 Z) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P3

STH: ((GOAL % SEND 3) (Y 2 Z) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P4

STH: ((GOAL % COUNT) (GO 1) (GOAL %% SEND 3) (Y 2 Z) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P5

********** 2

STH: ((GOAL % MARK) (GOAL COUNT) (SAY 2) (GOAL %% SEND 3) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P6

STH: ((Y 3 Z) (GOAL MARK) (SAY 2) (GOAL COUNT) (GOAL %% SEND 3) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P8

STH: ((GOAL % SEND 3) (Y 3 Z) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P2

STH: ((GOAL % PUSH BUTTON) (GOAL %% SEND 3) (Y 3 Z) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P3

STH: ((GOAL % SEND 3) (Y 3 Z) (GOAL %% ATTEND) (TOP SIX) (BOTTOM ZERO) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P4
STH: ((SEND 3) (GOAL #2 ATTEND) (TOP SIX) (Y 4 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) PO
INPUT TO STH:
* (MANY TOP NOW)

STH: ((MANY TOP NOW) (GOAL # ATTEND) (SEND 3) (TOP SIX) (Y 4 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) F13

STH: ((GOAL # SUBTRACT) (GOAL #2# ATTEND) (SEND 3) (TOP SIX) (Y 4 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) P14

STH: ((GOAL # LIST SIX) (GOAL #2# SUBTRACT) (TOP SIX) (GOAL #2# ATTEND) (SEND 3) (Y 4 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) P15A

STH: ((3 THREE Z) (2 FOUR Z) (GOAL # LIST SIX) (GOAL #2# SUBTRACT) (TOP SIX) (GOAL #2# ATTEND) (SEND 3) (Y 4 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) P16

THREE

STH: ((GOAL # PURGE) (SEND 3) (3 THREE Z) (TOP SIX) (2 FOUR Z) (GOAL #2# ATTEND) (Y 4 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) P24A

STH: ((GOAL # PURGE) (OLD SEND 3) (TOP THREE) (2 FOUR Z) (GOAL #2# ATTEND) (Y 4 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) P24

STH: ((GOAL # PURGE) (OLD SEND 3) (TOP THREE) (2 FOUR Z) (GOAL #2# ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) P25

STH: ((GOAL # PURGE) (OLD SEND 3) (TOP THREE) (2 FOUR Z) (GOAL #2# ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK) P27

STH: (OLD SEND 3) (TOP THREE) (GOAL # ATTEND) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
INPUT TO 5TH:
(*SEND 1)

5TH: ((SEND 1) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK)

F1

5TH: ((Y 1 Z) (GOAL SEND 1) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

F2

5TH: ((GOAL PUSH BUTTON) (GOAL ATTEND) (Y 1 Z) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

F3

5TH: ((GOAL SUBIT) (SEND 1) (GOAL SEND 1) (Y 1 Z) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

F4

5TH: ((GOAL COUNT) (SEND 1) (GOAL SEND 1) (Y 1 Z) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

F5

********** 1

5TH: ((GOAL MARK) (GOAL COUNT) (SEND 1) (GOAL SEND 1) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

F6

5TH: ((Y 2 Z) (GOAL MARK) (SEND 1) (GOAL COUNT) (GOAL SEND 1) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

F7

5TH: ((Y 2 Z) (GOAL ATTEND) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F8

INPUT TO 5TH:
(*SEND TOP BEFORE)

5TH: ((SEND TOP BEFORE) (GOAL ATTEND) (Y 2 Z) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

F9
STH: ((GOAL % RECALL TOP) (GOAL #* ATTEND) (Y 2 Z) (TOP THREE) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK)

******** (TOP THREE)

STH: ((TOP THREE) (GOAL * ATTEND) (Y 2 Z) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

INPUT TO STH:
*MANY TUBE*

STH: ((GOAL * ATTEND) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((SEND 1) (GOAL * ATTEND) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

INPUT TO STH:
*MANY TOP NOW*

STH: ((GOAL % ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL % ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

******** (SEND 1)

STH: ((GOAL & ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
SYN: ((GOAL, PUSHE) (SEND 1) (TOP TWO) (2 ONE 2) (GOAL # ATTEMPT) (Y 2 2) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK)

SYN: ((GOAL, PUSHE) (OLD SEND 1) (TOP TWO) (2 ONE 2) (GOAL # ATTEMPT) (Y 2 2) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK)

SYN: ((GOAL, PUSHE) (OLD SEND 1) (TOP TWO) (2 ONE 2) (GOAL # ATTEMPT) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK)

SYN: ((OLD SEND 1) (TOP TWO) (GOAL # ATTEMPT) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK)

INPUT TO SYN:

SYN: ((HIL, GOAL, ATTEMPT) (TOP TWO) (BOTTOM ZERO) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

END OF RUN

NUMBER OF PRODUCTIONS FIRED : 202
NUMBER OF OPERATIONS EXECUTED : 557
NUMBER OF PRODUCTIONS TRIED : 3310
NUMBER OF SINGLE-ELEMENT COMPARISONS : 52047
SIMULATED TIME : 0
COMPUTER TIME: 274100 MSECS
NUMBER OF CONS-OPERATIONS : 516576

#
ADDENDUM 5.

LISTING AND EXECUTION TRACE FOR
N-SUB-NV: COUNTING-BASED METHOD.
P 500
(VARIABLES:
(A SEND)
(B SEND)
(D SEND)
(E SEND)
(<X> SEND)
(<P> SEND)
)
(PRODUCTIONS:
(P000 (GOAL # ATTEND) (OLD SEND C))
(REPL ((GOAL # ATTEND) ; (GOAL # ATTEND)) ;
DEL ((OLD SEND C)) ;
USER())
(P000 (GOAL # ATTEND) (GO))
(REPL ((GOAL # ATTEND) ; (GOAL # ATTEND)) ;
DEL ((GO)) ;
(P000 (GOAL # ATTEND))
)
(USER())
(F1 (GOAL # ATTEND) (TOP A) (BOTTOM B) (SEND C))
(REPL ((GOAL # ATTEND) ; (GOAL # SEND C)) ;
INS ((GOAL # PUSH BUTTON)) )
(F3 (GOAL # SEND C))
(REPL ((GOAL # SEND C) ; (GOAL # SEND C)) ;
INS ((GOAL # SUBR)) )
(F4 (GOAL # SUBR) (ELM A))
(REPL ((GOAL # SUBR) ; (GOAL # SEND C)) ;
INS ((GOAL # PUSH BUTTON)) ;
DEL ((ELM A)) ;
INS ((GOAL # COUNT)) )
(F5 (GOAL # COUNT) (GS 1) (Y <X> 2))
(REPL ((GOAL # COUNT) ; (GOAL # COUNT)) ;
DEL ((GS 1)) )
(SAY <X>)
(REPL ((Y <X> Z) ; (SAID <X>)) ;
INS ((GOAL # MARK)) )
(F6 (GOAL # MARK) (SAID 1))
(REPL ((GOAL # MARK) ; (GOAL # MARK)) ;
INS ((Y 2 7)) )
(F7 (GOAL # MARK) (SAID 2))
(REPL ((GOAL # MARK) ; (GOAL # MARK)) ;
INS ((Y 3 2)) )
(F8 (GOAL # MARK) (SAID 3))
(REPL ((GOAL # MARK) ; (GOAL # MARK)) ;
INS ((Y 4 2)) )
(F9 (GOAL # MARK) (SAID 4))
(REPL ((GOAL # MARK) ; (GOAL # MARK)) ;
INS ((Y 5 2)) )
INS ((GOAL # MARK) ; (GOAL # MARK)) ;
INS ((Y 6 2)) )
(P25 (GOAL * PURGE) (W <P> V))

DEL((W <P> V))

(P26 (GOAL * PURGE) (BOTTOM 12))

STOP)

(P27 (GOAL * PURGE))

REPL((GOAL *RET * ATTEND) ; (GOAL * ATTEND))

(FINISH)

(H-SUB-RU3)

P9 P11 P13

P000 P00 P0

P1 P2 P3 P4 P5 P6

P6A P6I P6C

P7 P8 P10 P12

P13 P14 P15 P16

P24A

P24 P25 P26 P27

(STEPDOWN)

P17 P17A P17B P17C P17D P17E P17F

P17G P17H P17I

(STEPUP)

P18 P18A P18B P18C

(DEF: STM:(GOAL * ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2))

**END

**C
RUN BEGINS:

((GOAL * ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P0

INPUT TO STM:

((GO)

STM: ((GO) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P00

STM: ((GOAL ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P1

STM: ((Y 1 2) (GOAL * SEND 2) (GOAL *# ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STM: ((GOAL * PUSH BUTTON) (GOAL #$ SEND 2) (Y 1 2) (GOAL *# ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK)

P3

STM: ((GOAL * SUBIT) (ELM A) (GOAL #$ SEND 2) (Y 1 2) (GOAL *# ATTEND B) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK)

P4

STM: ((GOAL * COUNT) (BS 1) (GOAL #$ SEND 2) (Y 1 2) (GOAL *# ATTEND B) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK)

P5

STM: ((GOAL * MARK) (GOAL COUNT) (SATB 1) (GOAL #$ SEND 2) (GOAL *# ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK)

P6

STM: ((Y 2 7) (GOAL Mark) (SATB 1) (GOAL COUNT) (GOAL #$ SEND 2) (GOAL *# ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK)

P7
STM: ((GOAL 2 SEND 2) (Y 2 Z) (GOAL #3% ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STM: ((GOAL 3 PULL BUTTON) (GOAL #3% SEND 2) (Y 2 Z) (GOAL #3% ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STM: ((GOAL # SUSIT) (ELM A) (GOAL #3% SEND 2) (Y 2 Z) (GOAL #3% ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P4

STM: ((GOAL # COUNT) (OS 1) (GOAL #3% SEND 2) (Y 2 Z) (GOAL #3% ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P5

******** 2

STM: ((GOAL # MARK) (GOAL COUNT) (Said 2) (GOAL #3% SEND 2) (GOAL #3% ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P6

STM: ((Y 3 Z) (GOAL MARK) (Said 2) (GOAL COUNT) (GOAL #3% SEND 2) (GOAL #3% ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P7

STM: ((Y 3 Z) (GOAL # ATTEND) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P9

INPUT TO STM:
S(MANY TOP BEFORE)

STM: ((MANY TOP BEFORE) (GOAL # ATTEND) (Y 3 Z) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P9

STM: ((GOAL # RECALL TOP) (GOAL #3% ATTEND) (Y 3 Z) (TOP TWELVE) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P10

******** (TOP TWELVE)

STM: ((TOP TWELVE) (GOAL # ATTEND) (Y 3 Z) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P9

INPUT TO STM:
S(MANY TOP BEFORE)
STM: (MANY TOP) (GOAL & ATTEND) (TOP TWELVE) (Y 3 Z) (BOTTOM 0) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P12

STM: (GOAL & RECALL TOP) (GOAL & ATTEND) (TOP TWELVE) (Y 3 Z) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P12

******** (SEND 2)

STM: (SEND 2) (GOAL & ATTEND) (TOP TWELVE) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK
PO INPUT TO STM:
$ (MANY TOP NOW)

STM: (MANY TOP NOW) (GOAL & ATTEND) (SEND 2) (TOP TWELVE) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK
P13

STM: (Y TWELVE Z) (W 0 V) (GOAL & SUBTRACT) (GOAL & ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK
P15

STM: (GOAL & NEXT DOWN) (GOAL & SUBTRACT) (W 0 V) (GOAL & ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK
P17

STM: (GOAL & NEXT UP) (Y ELEVEN Z) (GOAL & SUBTRACT) (W 0 V) (GOAL & ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK
P16

STM: (GOAL & NEXT UP) (Y ELEVEN Z) (GOAL & SUBTRACT) (W 0 V) (GOAL & ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK
P18

STM: (W 1 V) (Y ELEVEN Z) (GOAL & SUBTRACT) (GOAL & ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK
P15
STH: ((GOAL % NEXT DOWN) (GOAL % SUBTRACT) (U 1 V) (Y ELEVEN Z) (GOAL % ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P17A

STH: ((GOAL % NEXT UP) (Y TEN Z) (GOAL % SUBTRACT) (U 1 V) (GOAL % ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)

P16

STH: ((GOAL % NEXT UP) (Y TEN Z) (GOAL % SUBTRACT) (U 1 V) (GOAL % ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P18A

STH: ((U 2 V) (Y TEN Z) (GOAL % SUBTRACT) (GOAL % ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)

P14

******** TIE

STH: ((GOAL % PURGE) (U 2 V) (SEND 2) (Y TEN Z) (GOAL % ATTEND) (TOP TWELVE) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P21A

STH: ((GOAL % PURGE) (TOP TEN) (OLD SEND 2) (U 2 V) (GOAL % ATTEND) (Y 3 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P24

STH: ((GOAL % PURGE) (TOP TEN) (OLD SEND 2) (U 2 V) (GOAL % ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P25

STH: ((GOAL % PURGE) (TOP TEN) (OLD SEND 2) (U 2 V) (GOAL % ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P27

STH: ((TOP TEN) (OLD SEND 2) (GOAL % ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

POPO

INPUT TO STH;

2 (SEND 4)

STH: ((SEND 4) (GOAL ATTEND) (TOP TEN) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P1
STH: ((Y 1 Z) (GOAL # SEND 4) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STH: ((GOAL # PUSH BUTTON) (GOAL # SEND 4) (Y 1 Z) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STH: ((GOAL # SUBIT) (EML A) (GOAL # SEND 4) (Y 1 Z) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P4

STH: ((GOAL # COUNT) (OS 1) (GOAL # SEND 4) (Y 1 Z) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P5

**WARNING 1

STH: ((GOAL # MARK) (GOAL COUNT) (SAID 1) (GOAL # SEND 4) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P6

STH: ((Y 2 Z) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL # SEND 4) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P8

STH: ((GOAL # SEND 4) (Y 2 Z) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STH: ((GOAL # PUSH BUTTON) (GOAL # SEND 4) (Y 2 Z) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STH: ((GOAL # SUBIT) (EML A) (GOAL # SEND 4) (Y 2 Z) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P4

STH: ((GOAL # COUNT) (OS 1) (GOAL # SEND 4) (Y 2 Z) (GOAL # ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P5

**WARNING 2
STH: ((GOAL & COUNT) (GS 1) (GOAL &% SEND 4) (Y 4 Z) (GOAL &% ATTEND)
TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
P9
$$$$

STH: ((GOAL & MARK) (GOAL COUNT) (SAID 4) (GOAL &% SEND 4) (GOAL &% ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
P7

STH: ((Y S Z) (GOAL MARK) (SAID 4) (GOAL COUNT) (GOAL &% SEND 4) (GOAL &% ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
P6
INPUT TO STH:
1(MANY TOP BEFORE)

STH: (MANY TOP BEFORE) (GOAL & ATTEND) (Y S Z) (TOP TEN) (BOTTOM 0)
(SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P5

STH: ((GOAL & RECALL TOP) (GOAL &% ATTEND) (Y S Z) (TOP TEN) (BOTTOM
0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P4
$$$$ (TOP TEN)

STH: ((TOP TEN) (GOAL & ATTEND) (Y S Z) (BOTTOM 0) (SEND 4) JUNK JUNK
JUNK JUNK JUNK JUNK JUNK JUNK)
P3
INPUT TO STH:
1(MANY TUBE)

STH: (MANY TUBE) (GOAL & ATTEND) (TOP TEN) (Y S Z) (BOTTOM 0) (SEND
4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STH: ((GOAL & RECALL TUBE) (GOAL &% ATTEND) (TOP TEN) (Y S Z) (BOTTOM
0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
$$$$ (SEND 4)

STH: ((GOAL & MARK) (GOAL COUNT) (SAID 4) (GOAL &% SEND 4) (GOAL &% ATTEND) (TOP TEN) (BOTTOM 0) (SEND 4) JUNK JUNK JUNK JUNK JUNK)
P11

STH: ((GOAL & RECALL TUBE) (GOAL &% ATTEND) (TOP TEN) (Y S Z) (BOTTOM
0) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P12
$$$$ (SEND 4)
STH1: ((SEND 4) (GOAL #& ATTEND) (TOP TEN) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)

INPUT TO STH1
$ (MANY TOP NOW)

STH1: ((MANY TOP NOW) (GOAL & ATTEND) (SEND 4) (TOP TEN) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P13

STH1: ((Y TEN 7) (U 0 V) (GOAL & SUBTRACT) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P15

STH1: ((GOAL #& NEXT DOWN) (GOAL #& SUBTRACT) (U 0 V) (Y TEN 7) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P17B

STH1: ((GOAL #& NEXT UP) (Y NINE 2) (GOAL #& SUBTRACT) (U 0 V) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P16

STH1: ((GOAL #& NEXT UP) (Y NINE 2) (GOAL #& SUBTRACT) (U 0 V) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P16

STH1: ((U 1 V) (Y NINE 2) (GOAL #& SUBTRACT) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P13

STH1: ((GOAL #& NEXT DOWN) (GOAL #& SUBTRACT) (U 1 V) (Y NINE 2) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P17C

STH1: ((GOAL #& NEXT UP) (Y EIGHT 7) (GOAL #& SUBTRACT) (U 1 V) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P16

STH1: ((GOAL #& NEXT UP) (Y EIGHT 7) (GOAL #& SUBTRACT) (U 1 V) (GOAL #& ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)
P16
STH: ((W 2 V) (Y EIGHT Z) (GOAL $8% SUBTRACT) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK)

P12

STH: ((GOAL $ NEXT DOWN) (GOAL $5% SUBTRACT) (W 2 V) (Y EIGHT Z) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)

P179

STH: ((GOAL $ NEXT UP) (Y SEVEN Z) (GOAL $5% SUBTRACT) (W 2 V) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK)

P188

STH: ((W 3 V) (Y SEVEN Z) (GOAL $5% SUBTRACT) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P19

STH: ((GOAL $ NEXT DOWN) (GOAL $5% SUBTRACT) (W 3 V) (Y SEVEN Z) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P17E

STH: ((GOAL $ NEXT UP) (Y SIX Z) (GOAL $5% SUBTRACT) (W 3 V) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P16

STH: ((GOAL $ NEXT UP) (Y SIX Z) (GOAL $5% SUBTRACT) (W 3 V) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P18C

STH: ((U 4 V) (Y SIX Z) (GOAL $ SUBTRACT) (GOAL $5% ATTEND) (TOP TEN) (SEND 4) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P18

STH: ((GOAL $ FUDGE) (W 4 V) (SEND 4) (Y SIX Z) (GOAL $5% ATTEND) (TOP TEN) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P3
STH: ((GOAL * PURGE) (TOP SIX) (OLD SEND 3) (Y 1 Y) (GOAL ** ATTEND) (Y 5 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK)
P24

STH: ((GOAL * PURGE) (TOP SIX) (OLD SEND 3) (Y 4 Y) (GOAL ** ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK)
P25

STH: ((GOAL * PURGE) (TOP SIX) (OLD SEND 3) (GOAL ** ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK)
P27

STH: ((TOP SIX) (OLD SEND 3) (GOAL * ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P30
INPUT TO STH:
*(SEND 3)

STH: ((SEND 3) (GOAL ATTEND) (TOP SIX) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STH: ((Y 1 Z) (GOAL * SEND 3) (GOAL ** ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STH: ((GOAL * PUSH BUTTON) (GOAL ** SEND 3) (Y 1 Z) (GOAL ** ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STH: ((GOAL * SUBST) (ELH A) (GOAL ** SEND 3) (Y 1 Z) (GOAL ** ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P4

STH: ((GOAL * COUNT) (ps 1) (GOAL ** SEND 3) (Y 1 Z) (GOAL ** ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P5

********************

STH: ((GOAL * MARK) (GOAL COUNT) (SALT A) (GOAL ** SEND 3) (GOAL ** ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK)
P6
STH: ((G 2) (GOAL MARK) (SAYD 1) (GOAL COUNT) (GOAL % SEND 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK]

P8

STH: ((GOAL % SEND 3) (G 2) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P2

STH: ((GOAL % PUSH BUTTON) (GOAL % SEND 3) (G 2) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P3

STH: ((GOAL % BURNT) (EML A) (GOAL % SEND 3) (G 2) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P4

STH: ((GOAL % COUNT) (OS 1) (GOAL % SEND 3) (G 2) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P5

********* 2

STH: ((GOAL % MARK) (GOAL COUNT) (SAYD 2) (GOAL % SEND 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P6

STH: ((G 3) (GOAL MARK) (SAYD 2) (GOAL COUNT) (GOAL % SEND 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P8

STH: ((GOAL % SEND 3) (G 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P2

STH: ((GOAL % PUSH BUTTON) (GOAL % SEND 3) (G 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P3

STH: ((GOAL % BURNT) (EML A) (GOAL % SEND 3) (G 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P4

STH: ((GOAL % COUNT) (OS 1) (GOAL % SEND 3) (G 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P8

STH: ((GOAL % MARK) (GOAL COUNT) (SAYD 2) (GOAL % SEND 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P6

STH: ((G 3) (GOAL MARK) (SAYD 2) (GOAL COUNT) (GOAL % SEND 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]

P8

STH: ((GOAL % SEND 3) (G 3) (GOAL % ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK]
STH: ((GOAL & MARK) (GOAL COUNT) (SAID 3) (GOAL #5 SEND 3) (GOAL #5 ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK)

STH: ((Y 4 Z) (GOAL MARK) (SAID 3) (GOAL COUNT) (GOAL #5 SEND 3) (GOAL #5 ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK)

STH: ((Y 4 Z) (GOAL # ATTEND) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

INPUT TO STH:
#(MANY TOP BEFORE)

STH: ((MANY TOP BEFORE) (GOAL & ATTEND) (Y 4 Z) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)

STH: ((GOAL & RECALL TOP) (GOAL #5 ATTEND) (Y 4 Z) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)

STH: ((GOAL & RECALL TOP) (GOAL #5 ATTEND) (Y 4 Z) (TOP SIX) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)

INPUT TO STH:
#(MANY TOP BEFORE)

STH: ((MANY TOP BEFORE) (GOAL & ATTEND) (TOP SIX) (Y 4 Z) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)

STH: ((GOAL & RECALL TOP) (GOAL #5 ATTEND) (TOP SIX) (Y 4 Z) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)

STH: ((GOAL & RECALL TOP) (GOAL #5 ATTEND) (TOP SIX) (Y 4 Z) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)

STH: ((GOAL & RECALL TOP) (GOAL #5 ATTEND) (TOP SIX) (Y 4 Z) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)

STH: ((GOAL & RECALL TOP) (GOAL #5 ATTEND) (TOP SIX) (Y 4 Z) (BOTTOM 0) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK)
INPUT TO STN:
\texttt{SIXY TOP NOW}

STN: \texttt{(SIXY TOP NOW) (GOAL \& ATTEND) (SEND 3) (TOP SIX) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK}

STN: \texttt{(Y SIX Z) (W 0 V) (GOAL \& SUBTRACT) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}

STN: \texttt{(GOAL \& NEXT DOWN) (GOAL \& SUBTRACT) (W 0 V) (Y SIX Z) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}

STN: \texttt{(GOAL \& NEXT UP) (Y FIVE Z) (GOAL \& SUBTRACT) (W 0 V) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}

STN: \texttt{(GOAL \& NEXT UP) (Y FIVE Z) (GOAL \& SUBTRACT) (W 0 V) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}

STN: \texttt{(GOAL \& NEXT DOWN) (GOAL \& SUBTRACT) (W 1 V) (Y FIVE Z) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}

STN: \texttt{(GOAL \& NEXT UP) (Y FOUR Z) (GOAL \& SUBTRACT) (W 1 V) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}

STN: \texttt{(GOAL \& NEXT UP) (Y FOUR Z) (GOAL \& SUBTRACT) (W 1 V) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}

STN: \texttt{(Y 2 V) (Y FOUR Z) (GOAL \& SUBTRACT) (GOAL \& ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK}
STM: ((GOAL # NEXT DOWN) (GOAL #54 SUBTRACT) (W 2 V) (Y FOUR Z) (GOAL #54 ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P17H

STM: ((GOAL # NEXT UP) (Y THREE Z) (GOAL #54 SUBTRACT) (W 2 V) (GOAL #54 ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P16

STM: ((GOAL # NEXT UP) (Y THREE Z) (GOAL # SUBTRACT) (GOAL #54 ATTEND) (TOP SIX) (SEND 3) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK)

P14

THREE

STM: ((GOAL # PURGE) (W 3 V) (SEND 3) (Y THREE Z) (GOAL #54 ATTEND) (TOP SIX) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK)

P24A

STM: ((GOAL # PURGE) (TOP THREE) (OLD SEND 3) (W 3 V) (GOAL #54 ATTEND) (Y 4 Z) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK)

P24

STM: ((GOAL # PURGE) (TOP THREE) (OLD SEND 3) (GOAL #54 ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P25

STM: ((GOAL # PURGE) (TOP THREE) (OLD SEND 3) (GOAL #54 ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P27

STM: ((TOP THREE) (OLD SEND 3) (GOAL #54 ATTEND) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P25O

INPUT TO STM:
#(SEND 1)

STM: ((SEND 1) (GOAL ATTEND) (TOP THREE) (BOTTOM 0) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
(GOAL * SEND 1) (GOAL * SEND 1) (Y 1 Z) (GOAL * ATTEND) (TOP THREE) (BOTTOM 0) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P2

(PEEK) (GOAL * PUSH BUTTON) (GOAL * SEND 1) (Y 1 Z) (GOAL * ATTEND) (TOP THREE) (BOTTOM 0) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P3

(ENABLE) (GOAL * COUNT) (GOAL * SEND 1) (Y 1 Z) (GOAL * ATTEND) (TOP THREE) (BOTTOM 0) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P5

(ENABLE) (GOAL * MARK) (GOAL COUNT) (SAID 1) (GOAL * SEND 1) (GOAL * ATTEND) (TOP THREE) (BOTTOM 0) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P6

(Y 2 Z) (GOAL * MARK) (SAID 1) (GOAL COUNT) (GOAL * SEND 1) (GOAL * ATTEND) (TOP THREE) (BOTTOM 0) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P7

(Y 2 Z) (GOAL * ATTEND) (TOP THREE) (BOTTOM 0) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P8

INPUT TO STM:
* (SAME TOP BEFORE)

P9

(y 2 Z) (GOAL * ATTEND) (TOP THREE) (BOTTOM 0) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

P10

#=#=#=#=# (TOP THREE)
STH: (TOP THREE) (GOAL & ATTEND) (Y Z Z) (BOTTOM O) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK
PO
*INPUT TO STH
*(MANY TUBE)

STH: (MANY TUBE) (GOAL & ATTEND) (TOP THREE) (Y Z Z) (BOTTOM O) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK
F11

STH: (GOAL & RECALL TUBE) (GOAL & ATTEND) (TOP THREE) (Y Z Z) (BOTTOM O) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK
F12

STH: (SEND 1) (GOAL & ATTEND) (TOP THREE) (Y Z Z) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK
PO
*INPUT TO STH
*(MANY TOP NOW)

STH: (MANY TOP NOW) (GOAL & ATTEND) (SEND 1) (TOP THREE) (Y Z Z) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK
F13

STH: (Y THREE Z) (O V) (GOAL & SUBTRACT) (GOAL & ATTEND) (TOP THREE) (SEND 1) (Y Z Z) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK
F15

STH: (GOAL & NEXT DOWN) (GOAL & SUBTRACT) (O V) (Y THREE Z) (GOAL & ATTEND) (TOP THREE) (SEND 1) (Y Z Z) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK
F171

STH: (GOAL & NEXT UP) (Y TWO Z) (GOAL & SUBTRACT) (O V) (GOAL & ATTEND) (TOP THREE) (SEND 1) (Y Z Z) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK
F16

STH: (GOAL & NEXT UP) (Y TWO Z) (GOAL & SUBTRACT) (O V) (GOAL & ATTEND) (TOP THREE) (SEND 1) (Y Z Z) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK
F18

STH: (Y1 Y) (Y TWO Z) (GOAL & SUBTRACT) (GOAL & ATTEND) (TOP THREE)
Ef!}

22

Z (TOP TWO) (OLD SEND 1) (W 1 V) (GOAL %* ATTEND) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK
P25

STM: ((GOAL % PURGE) (TOP TWO) (OLD SEND 1) (W 1 V) (GOAL %* ATTEND) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK)
P27

STM: ((GOAL % PURGE) (TOP TWO) (OLD SEND 1) (W 1 V) (GOAL %* ATTEND) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL % PURGE) (TOP TWO) (OLD SEND 1) (W 1 V) (GOAL %* ATTEND) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL % PURGE) (TOP TWO) (OLD SEND 1) (W 1 V) (GOAL %* ATTEND) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL % PURGE) (TOP TWO) (OLD SEND 1) (W 1 V) (GOAL %* ATTEND) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK)

STM: ((GOAL % PURGE) (TOP TWO) (OLD SEND 1) (W 1 V) (GOAL %* ATTEND) (BOTTOM O) JUNK JUNK JUNK JUNK JUNK JUNK)

End of Run

Number of Productions Fired: 242
Number of Operations Executed: 703
Number of Productions Tried: 3750
Number of Single-Element Comparisons: 66666
Simulated Time: 0
Computer Time: 340332 MSEC
Number of Cons-Operations: 614253

S

C
ADDENDUM 6.

LISTING AND EXECUTION TRACE FOR
N-CYC-NV: TABLE-LOOK-UP METHOD.
EDIT KVADD7.PSS

(VARIABLES:
* P 500
(VARIABLES:
(A GENC)
(B GENC)
(C GENC)
(D GENC)
(E GENC)
(<X> SEQC)
(<P> SEQC)
)

(PREDICATES:
(P00 (GOAL * ATTEND) (OLD SEND C)

REPL' (GOAL * ATTEND) ; (GOAL + ATTEND));
DEL' (OLD SEND C));
USER();
(P00 (GOAL * ATTEND) (GO)

REPL' (GOAL * ATTEND) ; (GOAL + ATTEND));
DEL' (GO));
(P0 (GOAL * ATTEND)

USER();
(P1 (GOAL + ATTEND) (TOP A) (BOTTOM B) (SEND C)

REPL' (GOAL + ATTEND) ; (GOAL ** ATTEND));
INS'((GOAL * SEND C));
INS'((Y 1 Z));
(P2 (GOAL * SEND C)

REPL' (GOAL * SEND C) ; (GOAL ** SEND C));
INS'((GOAL * PUSH BUTTON));
(P3 (GOAL * PUSH BUTTON)

DEL' (GOAL * PUSH BUTTON));
INS'((ELM A));
INS'((GOAL * SUBIT))
(P4 (GOAL * SUBIT) (ELM A)

DEL' (GOAL * SUBIT));
INS'((GS 1));
DEL' (ELM A));
INS'((GOAL * COUNT));
(P5 (GOAL * COUNT) (GS 1) (Y <X> Z)

REPL' (GOAL * COUNT) ; (GOAL + COUNT));
DEL' (GS 1));
SAY'(<X>);
REPL'((Y <X> Z) ; (SAID <X>));
INS'((GOAL * MARK));
(P6 (GOAL * MARK) (SAID 1)

REPL'((GOAL * MARK) ; (GOAL + MARK));
INS'((Y 2 Z));
(P6A (GOAL * MARK) (SAID 2)

REPL'((GOAL * MARK) ; (GOAL + MARK));
INS'((Y 3 Z));
(P6B (GOAL * MARK) (SAID 3)
\texttt{REPLC((GOAL \*S\* ADD) \&\& (GOAL \* ADD));}
\texttt{INS((GOAL \* ADD));}
\texttt{INS((1 \* NINE \* Z));}
\texttt{(P16 (GOAL \* ADD) (SEND C) (C <X> \* Z) (BOTTOM B))}

\texttt{REPLC((GOAL \* ADD));}
\texttt{INS((GOAL \* LIST B));}
\texttt{INS((GOAL \* UPDATE));}
\texttt{(P24 (GOAL \* UPDATE) (SEND C) (C <X> \* Z) (BOTTOM B))}

\texttt{REPLC((GOAL \* UPDATE));}
\texttt{INS((GOAL \* PURGE));}
\texttt{DEL((C <X> \* Z));}
\texttt{(P24 (GOAL \* PURGE) (Y <X> \* Z))}

\texttt{DEL((Y <X> \* Z));}
\texttt{(P25 (GOAL \* PURGE) (C <P> \* Z))}

\texttt{DEL((C <P> \* Z));}
\texttt{(P26 (GOAL \* PURGE) (BOTTOM 12))}

\texttt{STOP}
\texttt{(P27 (GOAL \* PURGE))}

\texttt{DEL((GOAL \* PURGE));}
\texttt{REPLC((GOAL \*S\* ATTEND) \&\& (GOAL \* ATTEND));}
\texttt{(P29 (GOAL \* ATTEND) (MANY TOP BEFORE))}

\texttt{REPLC((GOAL \* ATTEND) \&\& (GOAL \*S\* ATTEND));}
\texttt{DEL((MANY TOP BEFORE));}
\texttt{INS((GOAL \* RECALL TOP));}
\texttt{(P310 (GOAL \* RECALL TOP) (TOP A))}

\texttt{STOP}
\texttt{(P311 (GOAL \* RECALL TOP));}
\texttt{REPLC((GOAL \*S\* ATTEND) \&\& (GOAL \* ATTEND));}
\texttt{(P313 (GOAL \* ATTEND) (MANY TOP NOW))}

\texttt{REPLC((GOAL \* ATTEND) \&\& (GOAL \*S\* ATTEND));}
\texttt{DEL((MANY TOP NOW));}
\texttt{INS((GOAL \* SUBTRACT));}
\texttt{(P314 (GOAL \* SUBTRACT) (TOP A) (ABS (GOAL \* LIST A))}

\texttt{REPLC((GOAL \* SUBTRACT) \&\& (GOAL \*S\* SUBTRACT));}
\texttt{INS((GOAL \* LIST A));}
\texttt{(P315 (GOAL \* LIST THREE) (GOAL \*S\* SUBTRACT))}

\texttt{REPLC((GOAL \*S\* SUBTRACT) \&\& (GOAL \* SUBTRACT));}
\texttt{INS((1 \* TWO \* Z));}
\texttt{INS((2 \* ONE \* Z));}
\texttt{(P315A (GOAL \* LIST SIX) (GOAL \*S\* SUBTRACT))}

\texttt{REPLC((GOAL \*S\* SUBTRACT) \&\& (GOAL \* SUBTRACT));}
\texttt{INS((2 \* FOUR \* Z));}
\texttt{INS((3 \* THREE \* Z));}
\texttt{(P315B (GOAL \* LIST TEN) (GOAL \*S\* SUBTRACT))}

\texttt{REPLC((GOAL \*S\* SUBTRACT) \&\& (GOAL \* SUBTRACT));}
\texttt{INS((3 \* SEVEN \* Z));}
\texttt{INS((4 \* SIX \* Z));}
\texttt{(P315C (GOAL \* LIST TWELVE) (GOAL \*S\* SUBTRACT))}

\texttt{REPLC((GOAL \*S\* SUBTRACT) \&\& (GOAL \* SUBTRACT));}
INS((1 ELEVEN Z));
INS((2 TEN Z));

(PA16 (GOAL * SUBTRACT)(SEND C)(C <X> Z)(TOP A))

SAY(<X>);
DEL((GOAL * SUBTRACT));
DEL((GOAL * LIST 4));
INS((GOAL * PURGE));

(PA24A (GOAL * PURGE)(SEND C)(C <X> Z)(TOP A))

REPL((TOP A) ; (TOP <X>));
REPL((SEND C) ; (OLD SEND C));
DEL((C <X> Z));
)

PDLISTS:

(N-CYC-MV1

P9 PA9 P11 P13 PA13
P000 P00 P0
P1 P2 P3 P4 P5 P6
PA6A PA6B PA6C
P7 PA8 P10 PA10 P12
P13 P14 PA16 P15
PA15A PA15B PA15C
P24A
PA24A
-24 P25 P26 P27
)

(DEF: STM((GOAL * ATTEND)(TOP TWELVE)(BOTTOM ZERO)(SEND 2)))

***END
RUN
UNDEFINED FUNCTION
BACKTRACE
?-*EVAL

*(RUN: N-CYC-NV1 STM 15 500)

RUN BEGINS:

(((GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P0
INPUT TO STM:
*(GO).

STM: ((GO) (GOAL * ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P00

STM: ((GOAL ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P1

STM: ((Y 1 Z) (GOAL * SEND 2) (GOAL *S* ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P2

STM: ((GOAL * PUSH BUTTON) (GOAL *S* SEND 2) (Y 1 Z) (GOAL *S* ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STM: ((GOAL * SUBIT) (ELM A) (GOAL *S* SEND 2) (Y 1 Z) (GOAL *S* ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P4

STM: ((GOAL * COUNT) (QS 1) (GOAL *S* SEND 2) (Y 1 Z) (GOAL *S* ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P5

********** 1

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 1) (GOAL *S* SEND 2) (GOAL *S* ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P6
STM: ((Y 2 Z) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL ** SEND 2) (GOAL ** ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P8

STM: ((GOAL * SEND 2) (Y 2 Z) (GOAL ** SEND 2) (GOAL && ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK)
P2

STM: ((GOAL * PUSH BUTTON) (GOAL ** SEND 2) (Y 2 Z) (GOAL ** ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P3

STM: ((GOAL * SUBIT) (ELM A) (GOAL ** SEND 2) (Y 2 Z) (GOAL ** ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P4

********** 2

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 2) (GOAL ** SEND 2) (GOAL ** ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK)
P6A

STM: ((Y 3 Z) (GOAL MARK) (SAID 2) (GOAL COUNT) (GOAL ** SEND 2) (GOAL ** ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK)
P7

STM: ((Y 3 Z) (GOAL ** ATTEND) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P9

INPUT TO STM:
*(MANY BOTTOM BEFORE)

STM: ((MANY BOTTOM BEFORE) (GOAL ** ATTEND) (Y 3 Z) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P9

STM: ((GOAL * RECALL BOTTOM) (GOAL ** ATTEND) (Y 3 Z) (TOP TWELVE) (BOTTOM ZERO) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK)
P10
******** (BOTTOM ZERO)

STM: ((BOTTOM ZERO) (GOAL * ATTEND) (Y 3 Z) (TOP TWELVE) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PO
INPUT TO STM:
*(MANY TUBE)

STM: ((MANY TUBE) (GOAL * ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P11

STM: ((GOAL * RECALL TUBE) (GOAL *S* ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) (SEND 2) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P12

******** (SEND 2)

STM: ((SEND 2) (GOAL * ATTEND) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TOP BEFORE)

STM: ((MANY TOP BEFORE) (GOAL * ATTEND) (SEND 2) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P19

STM: ((GOAL * RECALL TOP) (GOAL *S* ATTEND) (SEND 2) (BOTTOM ZERO) (Y 3 Z) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P10

******** (TOP TWELVE)

STM: ((TOP TWELVE) (GOAL * ATTEND) (SEND 2) (BOTTOM ZERO) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY BOTTOM NOW)

STM: ((MANY BOTTOM NOW) (GOAL * ATTEND) (TOP TWELVE) (SEND 2) (BOTTOM ZERO) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P13

STM: ((GOAL * ADD) (GOAL *S* ATTEND) (TOP TWELVE) (SEND 2) (BOTTOM ZERO) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
STM: ((GOAL * LIST ZERO) (GOAL *S* ADD) (BOTTOM ZERO) (GOAL *S* ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P15

STM: ((3 THREE Z) (2 TWO Z) (GOAL * LIST ZERO) (GOAL * ADD) (BOTTOM ZERO) (GOAL *S* ATTEND) (TOP TWELVE) (SEND 2) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK)
P16

********** TWO

STM: ((GOAL * UPDATE) (SEND 2) (2 TWO Z) (BOTTOM ZERO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P24A

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P24

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P25

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P27

STM: ((SEND 2) (BOTTOM TWO) (GOAL * ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TOP NOW)

STM: ((MANY TOP NOW) (GOAL * ATTEND) (SEND 2) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P413

STM: ((GOAL * SUBTRACT) (GOAL *S* ATTEND) (SEND 2) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P414

P415

STM: ((GOAL * UPDATE) (SEND 2) (2 TWO Z) (BOTTOM ZERO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P42A

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P42

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P425

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P427

STM: ((SEND 2) (BOTTOM TWO) (GOAL * ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TOP NOW)

STM: ((MANY TOP NOW) (GOAL * ATTEND) (SEND 2) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P413

STM: ((GOAL * SUBTRACT) (GOAL *S* ATTEND) (SEND 2) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P414

P415

STM: ((GOAL * UPDATE) (SEND 2) (2 TWO Z) (BOTTOM ZERO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P42A

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P42

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P425

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P427

STM: ((SEND 2) (BOTTOM TWO) (GOAL * ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TOP NOW)

STM: ((MANY TOP NOW) (GOAL * ATTEND) (SEND 2) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P413

STM: ((GOAL * SUBTRACT) (GOAL *S* ATTEND) (SEND 2) (BOTTOM TWO) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P414

P415

STM: ((GOAL * UPDATE) (SEND 2) (2 TWO Z) (BOTTOM ZERO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P42A

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) (Y 3 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P42

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (3 THREE Z) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P425

STM: ((GOAL * PURGE) (SEND 2) (BOTTOM TWO) (GOAL *S* ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P427

STM: ((SEND 2) (BOTTOM TWO) (GOAL * ATTEND) (TOP TWELVE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TOP NOW)
D: (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK

********** 1

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 1) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)

P6

STM: ((Y 2 Z) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK)

P8

STM: ((GOAL * SEND 4) (Y 2 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STM: ((GOAL * PUSH BUTTON) (GOAL *S* SEND 4) (Y 2 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P3

STM: ((GOAL * SUBIT) (ELM A) (GOAL *S* SEND 4) (Y 2 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P4

STM: ((GOAL * COUNT) (GS 1) (GOAL *S* SEND 4) (Y 2 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P5

********** 2

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 2) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P6A

STM: ((Y 3 Z) (GOAL MARK) (SAID 2) (GOAL COUNT) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P8

STM: ((GOAL * SEND 4) (Y 3 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P2

STM: ((GOAL * PUSH BUTTON) (GOAL *S* SEND 4) (Y 3 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)

P3
STM: ((GOAL * SUBIT) (ELM A) (GOAL *S* SEND 4) (Y 3 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P4

STM: ((GOAL * COUNT) (QS 1) (GOAL *S* SEND 4) (Y 3 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P5

******** 3

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 3) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P&6

STM: ((Y 4 Z) (GOAL MARK) (SAID 3) (GOAL COUNT) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P8

STM: ((GOAL * SEND 4) (Y 4 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P2

STM: ((GOAL * PUSH BUTTON) (GOAL *S* SEND 4) (Y 4 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P3

STM: ((GOAL * SUBIT) (ELM A) (GOAL *S* SEND 4) (Y 4 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P4

STM: ((GOAL * COUNT) (QS 1) (GOAL *S* SEND 4) (Y 4 Z) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P5

******** 4

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 4) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P&6

STM: ((Y 5 Z) (GOAL MARK) (SAID 4) (GOAL COUNT) (GOAL *S* SEND 4) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK) P7
STM: ((Y 5 Z) (GOAL * ATTEND) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY BOTTOM BEFORE)

STM: ((MANY BOTTOM BEFORE) (GOAL * ATTEND) (Y 5 Z) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P9

STM: ((GOAL * RECALL BOTTOM) (GOAL *S* ATTEND) (Y 5 Z) (TOP TEN) (BOTTOM TWO) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P10

********* (BOTTOM TWO)

STM: ((BOTTOM TWO) (GOAL * ATTEND) (Y 5 Z) (TOP TEN) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TOP BEFORE)

STM: ((MANY TOP BEFORE) (GOAL * ATTEND) (BOTTOM TWO) (Y 5 Z) (TOP TEN) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P9

STM: ((GOAL * RECALL TOP) (GOAL *S* ATTEND) (BOTTOM TWO) (Y 5 Z) (TOP TEN) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P10

********* (TOP TEN)

STM: ((TOP TEN) (GOAL * ATTEND) (BOTTOM TWO) (Y 5 Z) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TUBE)

STM: ((MANY TUBE) (GOAL * ATTEND) (TOP TEN) (BOTTOM TWO) (Y 5 Z) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P11

STM: ((GOAL * RECALL TUBE) (GOAL *S* ATTEND) (TOP TEN) (BOTTOM TWO) (Y 5 Z) (SEND 4) JUNK JUNK JUNK JUNK JUNK JUNK)
P12
******** (SEND 4)

STM: ((SEND 4) (GOAL * ATTEND) (TOP TEN) (BOTTOM TWO) (Y 5 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P0

INPUT TO STM:
*(MANY BOTTOM BEFORE)*

STM: ((MANY BOTTOM NOW) (GOAL * ATTEND) (SEND 4) (TOP TEN) (BOTTOM TWO) (Y 5 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P13

STM: ((GOAL * ADD) (GOAL *S* ATTEND) (SEND 4) (TOP TEN) (BOTTOM TWO) (Y 5 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P14

STM: ((GOAL * LIST TWO) (GOAL *S* ADD) (BOTTOM TWO) (GOAL *S* ATTEND) (SEND 4) (TOP TEN) (Y 5 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P15A

STM: ((4 SIX Z) (3 FIVE Z) (GOAL * LIST TWO) (GOAL * ADD) (BOTTOM TWO) (GOAL *S* ATTEND) (SEND 4) (TOP TEN) (Y 5 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P16

******** SIX

STM: ((GOAL * UPDATE) (SEND 4) (4 SIX Z) (BOTTOM TWO) (3 FIVE Z) (GOAL *S* ATTEND) (TOP TEN) (Y 5 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P24A

STM: ((GOAL * PURGE) (SEND 4) (BOTTOM SIX) (3 FIVE Z) (GOAL *S* ATTEND) (TOP TEN) (Y 5 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P24

STM: ((GOAL * PURGE) (SEND 4) (BOTTOM SIX) (3 FIVE Z) (GOAL *S* ATTEND) (TOP TEN) JUNK JUNK JUNK JUNK JUNK JUNK)
P25

STM: ((GOAL * PURGE) (SEND 4) (BOTTOM SIX) (GOAL *S* ATTEND) (TOP TEN) JUNK JUNK JUNK JUNK JUNK JUNK)
P27

STM: ((SEND 4) (BOTTOM SIX) (GOAL * ATTEND) (TOP TEN) JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY TOP NOW)

STM: ((MANY TOP NOW) (GOAL * ATTEND) (SEND 4) (BOTTOM SIX) (TOP TEN) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA13

STM: ((GOAL * SUBTRACT) (GOAL * 4* ATTEND) (SEND 4) (BOTTOM SIX) (TOP TEN) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA14

STM: ((GOAL * LIST TEN) (GOAL *S* SUBTRACT) (TOP TEN) (GOAL *S* ATTEND) (SEND 4) (BOTTOM SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA15B

STM: ((4 SIX Z) (3 SEVEN Z) (GOAL * LIST TEN) (GOAL * SUBTRACT) (TOP TEN) (GOAL *S* ATTEND) (SEND 4) (BOTTOM SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA16

******** SIX

STM: ((GOAL * PURGE) (SEND 4) (4 SIX Z) (TOP TEN) (3 SEVEN Z) (GOAL *S* ATTEND) (BOTTOM SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA24A

STM: ((GOAL * PURGE) (OLD SEND 4) (TOP SIX) (3 SEVEN Z) (GOAL *S* ATTEND) (BOTTOM SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P25

STM: ((GOAL * PURGE) (OLD SEND 4) (TOP SIX) (GOAL *S* ATTEND) (BOTTOM SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P27

STM: ((OLD SEND 4) (TOP SIX) (GOAL * ATTEND) (BOTTOM SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P000

INPUT TO STM:
*(SEND 3)

STM: ((SEND 3) (GOAL ATTEND) (TOP SIX) (BOTTOM SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
F1
STM: (Y 1 Z) (GOAL * SEND 3) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P2

STM: (GOAL * PUSI BUTTON) (GOAL * SEND 3) (Y 1 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P3

STM: (GOAL * SUBIT) (ELM A) (GOAL * SEND 3) (Y 1 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P4

STM: (GOAL * COUNT) (QS 1) (GOAL * SEND 3) (Y 1 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P5

********* 1

STM: (GOAL * MARK) (GOAL COUNT) (SAID 1) (GOAL * SEND 3) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P6

STM: (Y 2 Z) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL * SEND 3) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P7

STM: (GOAL * SEND 3) (Y 2 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P8

STM: (GOAL * PUSI BUTTON) (GOAL * SEND 3) (Y 2 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P9

STM: (GOAL * SUBIT) (ELM A) (GOAL * SEND 3) (Y 2 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P10

STM: (GOAL * COUNT) (QS 1) (GOAL * SEND 3) (Y 2 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P11

********* 2

STM: (GOAL * MARK) (GOAL COUNT) (SAID 2) (GOAL * SEND 3) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P12
STM: ((Y 3 Z) (GOAL MARK) (SAID 2) (GOAL COUNT) (GOAL *S* SEND 3) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P8

STM: ((GOAL * SEND 3) (Y 3 Z) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P2

STM: ((GOAL * PUSH BUTTON) (GOAL *S* SEND 3) (Y 3 Z) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P3

STM: ((GOAL * SUB1) (ELM A) (GOAL *S* SEND 3) (Y 3 Z) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P4

STM: ((GOAL * COUNT) (OS 1) (GOAL *S* SEND 3) (Y 3 Z) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P5

********* 3

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 3) (GOAL *S* SEND 3) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P6

STM: ((Y 4 Z) (GOAL MARK) (SAID 3) (GOAL COUNT) (GOAL *S* SEND 3) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P7

STM: ((Y 4 Z) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P0

INPUT TO STM:
*(MANY BOTTOM BEFORE)*

STM: ((MANY BOTTOM BEFORE) (GOAL * ATTEND) (Y 4 Z) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P9

STM: ((GOAL * RECALL BOTTOM) (GOAL *S* ATTEND) (Y 4 Z) (TOP SIX) (BOTTOM SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P10
******** (BOTTOM SIX)

STM: ((BOTTOM SIX) (GOAL * ATTEND) (Y 4 Z) (TOP SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PO
INPUT TO STM:
*(MANY TOP BEFORE)

STM: ((MANY TOP BEFORE) (GOAL * ATTEND) (BOTTOM SIX) (Y 4 Z) (TOP SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0

STM: ((GOAL * RECALL TOP) (GOAL *S* ATTEND) (BOTTOM SIX) (Y 4 Z) (TOP SIX) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P10

******** (TOP SIX)

STM: ((TOP SIX) (GOAL * ATTEND) (BOTTOM SIX) (Y 4 Z) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PO
INPUT TO STM:
*(MANY TUBE)

STM: ((MANY TUBE) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (Y 4 Z) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P11

STM: ((GOAL * RECALL TUBE) (GOAL *S* ATTEND) (TOP SIX) (BOTTOM SIX) (Y 4 Z) (SEND 3) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P12

******** (SEND 3)

STM: ((SEND 3) (GOAL * ATTEND) (TOP SIX) (BOTTOM SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0
INPUT TO STM:
*(MANY BOTTOM NOW)

STM: ((MANY BOTTOM NOW) (GOAL * ATTEND) (SEND 3) (TOP SIX) (BOTTOM SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P13
STM: ((GOAL * ADD) (GOAL *S* ATTEND) (SEND 3) (TOP SIX) (BOTTOM SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P14

STM: ((GOAL * LIST SIX) (GOAL *S* ADD) (BOTTOM SIX) (GOAL *S* ATTEND) (SEND 3) (TOP SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK)
P15B

STM: ((3 NINE Z) (2 EIGHT Z) (GOAL * LIST SIX) (GOAL * ADD) (BOTTOM SIX) (GOAL *S* ATTEND) (SEND 3) (TOP SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P16

******** NINE

STM: ((GOAL * UPDATE) (SEND 3) (3 NINE Z) (BOTTOM SIX) (2 EIGHT Z) (GOAL *S* ATTEND) (TOP SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P24A

STM: ((GOAL * PURGE) (SEND 3) (BOTTOM NINE) (2 EIGHT Z) (GOAL *S* ATTEND) (TOP SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P24

STM: ((GOAL * PURGE) (SEND 3) (BOTTOM NINE) (2 EIGHT Z) (GOAL *S* ATTEND) (TOP SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P25

STM: ((GOAL * PURGE) (SEND 3) (BOTTOM NINE) (GOAL *S* ATTEND) (TOP SIX) (Y 4 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P27

STM: ((SEND 3) (BOTTOM NINE) (GOAL * ATTEND) (TOP SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
P0

INPUT TO STM!
*(MANY TOP NOW)

STM: ((MANY TOP NOW) (GOAL * ATTEND) (SEND 3) (BOTTOM NINE) (TOP SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA13

STM: ((GOAL * SUBTRACT) (GOAL *S* ATTEND) (SEND 3) (BOTTOM NINE) (TOP SIX) JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
PA14

STM: ((GOAL * LIST SIX) (GOAL *S* SUBTRACT) (TOP SIX) (GOAL *S* ATTEN
D) (SEND 3) (BOTTOM NINE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK

STM: (((3 THREE Z) (2 FOUR Z) (GOAL * LIST SIX) (GOAL * SUBSTITUTE) (TOP SIX) (GOAL *S* ATTEND) (SEND 3) (BOTTOM NINE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) PA16

******** THREE

STM: (((GOAL * PURGE) (SEND 3) (3 THREE Z) (TOP SIX) (2 FOUR Z) (GOAL *S* ATTEND) (BOTTOM NINE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) PA24A

STM: (((GOAL * PURGE) (OLD SEND 3) (TOP THREE) (2 FOUR Z) (GOAL *S* ATTEND) (BOTTOM NINE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P25

STM: (((GOAL * PURGE) (OLD SEND 3) (TOP THREE) (GOAL *S* ATTEND) (BOTTOM NINE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P27

STM: (((OLD SEND 3) (TOP THREE) (GOAL * ATTEND) (BOTTOM NINE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P600

INPUT TO STM:
*(SEND 1)

STM: (((SEND 1) (GOAL ATTEND) (TOP THREE) (BOTTOM NINE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P1

STM: (((Y 1 Z) (GOAL * SEND 1) (GOAL *S* ATTEND) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P2

STM: (((GOAL * PUSH BUTTON) (GOAL *S* SEND 1) (Y 1 Z) (GOAL *S* ATTEND) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P3

STM: (((GOAL * SUBSTITUTE) (ELK A) (GOAL *S* SEND 1) (Y 1 Z) (GOAL *S* ATTEND) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P4

STM: (((GOAL * COUNT) (QS 1) (GOAL *S* SEND 1) (Y 1 Z) (GOAL *S* ATTEND) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK) P
D) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK
P5

******** 1

STM: ((GOAL * MARK) (GOAL COUNT) (SAID 1) (GOAL *S* SEND 1) (GOAL *S* ATTEND) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK
P6

STM: ((Y 2 Z) (GOAL MARK) (SAID 1) (GOAL COUNT) (GOAL *S* SEND 1) (GOAL *S* ATTEND) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK
P7

STM: ((Y 2 Z) (GOAL * ATTEND) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P8

INPUT TO STM:\n*(MANY BOTTOM BEFORE)

STM: ((MANY BOTTOM BEFORE) (GOAL * ATTEND) (Y 2 Z) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P9

STM: ((GOAL * RECALL BOTTOM) (GOAL *S* ATTEND) (Y 2 Z) (TOP THREE) (BOTTOM NINE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P10

******** (BOTTOM NINE)

STM: ((BOTTOM NINE) (GOAL * ATTEND) (Y 2 Z) (TOP THREE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P0

INPUT TO STM:\n*(MANY TOP BEFORE)

STM: ((MANY TOP BEFORE) (GOAL * ATTEND) (BOTTOM NINE) (Y 2 Z) (TOP THREE) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK
P11

******** (TOP THREE)
STH: ((TOP THREE) (GOAL * ATTEND) (BOTTOM NINE) (Y 2 Z) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK)

P0

INPUT TO STH:
*(MANY TUBE)

STH: ((MANY TUBE) (GOAL * ATTEND) (TOP THREE) (BOTTOM NINE) (Y 2 Z) (SEND 1) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P11

*********  (SEND 1)

STH: ((SEND 1) (GOAL * ATTEND) (TOP THREE) (BOTTOM NINE) (Y 2 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P0

INPUT TO STH:
*(MANY BOTTOM NOW)

STH: ((MANY BOTTOM NOW) (GOAL * ATTEND) (SEND 1) (TOP THREE) (BOTTOM NINE) (Y 2 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P13

STH: ((GOAL * ADD) (GOAL *S* ATTEND) (SEND 1) (TOP THREE) (BOTTOM NINE) (Y 2 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P14

STH: ((GOAL * LIST NINE) (GOAL *S* ADD) (BOTTOM NINE) (GOAL *S* ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P15C

STH: ((1 TEN Z) (0 NINE Z) (GOAL * LIST NINE) (GOAL * ADD) (BOTTOM NINE) (GOAL *S* ATTEND) (SEND 1) (TOP THREE) (Y 2 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P16

*********  TEN

STH: ((GOAL * UPDATE) (SEND 1) (1 TEN Z) (BOTTOM NINE) (0 NINE Z) (GOAL *S* ATTEND) (TOP THREE) (Y 2 Z) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)

P24A

STH: ((GOAL * PURGE) (SEND 1) (BOTTOM TEN) (0 NINE Z) (GOAL *S* ATTEND)
STM: ((GOAL * PURGE) (SEND 1) (BOTTOM TEN) (0 NINE Z) (GOAL *S* ATTEND) (TOP THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
STM: ((GOAL * PURGE) (SEND 1) (BOTTOM TEN) (GOAL *S* ATTEND) (TOP THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK)
STM: ((SEND 1) (BOTTOM TEN) (GOAL * ATTEND) (TOP THREE) JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUNK JUN
END OF RUN

NUMBER OF PRODUCTIONS FIRED : 256
NUMBER OF OPERATIONS EXECUTED : 733
NUMBER OF PRODUCTIONS TRIED : 6120
NUMBER OF SINGLE-ELEMENT COMPARISONS : 115536
SIMULATED TIME : 0
COMPUTER TIME : 503505 MSEC
NUMBER OF CONS-OPERATIONS : 1121342