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DEVELOPMENT OF  
LINEAR MEASUREMENT.

Kenneth Vine

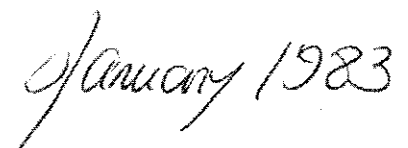
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Kenneth Vine.



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ABSTRACT.

The study had two objectives. The first was to identify the 'higher-level' knowledge necessary for a child to understand linear measurement. The second was to chart the growth of linear measurement in terms of the development of its components. In this context, 'higher-level' knowledge refers to skills such as counting an array of objects, as distinct from 'lower-level' skills such as attending to an object in an array.

An analysis of measurement operations yielded a list of components which it was argued would underlie linear measurement. Piagetian theory and related empirical literature were consulted as sources of information on the emergence of these components in the child's thinking. This led to the formulation of a number of predictions concerning the components of linear measurement, and their order of emergence.

A battery of 34 number, length, and distance tasks was developed to assess the presence of these components. It was administered to 100 children aged between 63 and 78 months, and drawn from kindergarten and grade one. The results were analyzed using scalogram techniques. The main contribution of the thesis is in this empirical work.

It was found that children who possessed a mature level of understanding of linear measurement also possessed the following:-

- . Knowing how to make transitive inferences of equivalence, with respect to discrete quantity, and length.
- . Knowing that the numerosity of an array of objects is invariant under certain transformations (the conservation of number).
- . Knowing that length is invariant under certain transformations (the conservation of length).
- . Knowing how to carry out numerical addition operations.
- . Knowing how to obtain a linear measurement by counting iterations of a unit of length.
- . Knowing how to make transitive inferences of non-equivalence, with respect to discrete quantity.

There appeared to be a substantial developmental delay between acquisition of these components and emergence of a mature grasp of linear measurement.

It was also found that the collections of components for the number and length domains formed scaled sets. However, within each domain the pattern of development was marked by discontinuities (abrupt changes in the slopes of the task performance gradients).

It was suggested that the discontinuities might be due to differences in short-term-memory (STM) demands made by tasks which differed significantly in difficulty. An information-processing analysis, using Pascual-Leone's M-Space model, did not confirm this.



A production-system analysis of certain of the number tasks also failed to reveal differences in demands made on STM by tasks differing in difficulty.

The discontinuities in development were interpreted as being associated with the need to re-organise number and length concepts.

Length of schooling, but not age, was found to be a predictor of performance on the task battery. No sex differences were found.