Attraction of nonlocal dark optical solitons

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We study the formation and interaction of spatial dark optical solitons in materials with a nonlocal nonlinear response. We show that unlike in local materials, where dark solitons typically repel, the nonlocal nonlinearity leads to a *long-range attraction* that enables the formation of stable bound states of dark solitons.

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Dark solitons, which have an intensity profile in the form of a dip in an otherwise uniform background, are topological objects because of their nontrivial phase structure. It appears that dark solitons *always repel*[1], unless special external perturbations are imposed. Already early theoretical studies of the self-defocusing nonlinear Schrödinger (NLS) equation indicated the repulsive nature of dark soliton interaction[2]. The first systematic investigations were conducted by Zhao and Bourkoff[3], who numerically studied the propagation of closely placed dark temporal solitons in optical fibers and found that their interaction was repulsive and weak compared to bright solitons. Subsequent experimental studies of temporal[4] and spatial dark solitons[5–7] confirmed that their repulsive interaction is generic.

To suppress the repulsion of dark solitons, Afanasjev *et al.* perturbed the Kerr response of a nonlinear medium by incorporating higher order gain terms[8]. Ostrovskaya *et al.* proposed the use of "solitonic gluons"[9], i.e., two out-of-phase weak bright beams guided by closely spaced dark solitons.

Here we discuss propagation of dark solitons in a non-dissipative self-defocusing *nonlocal* Kerr-like medium. We show that the nonlocality drastically modifies the interaction of dark solitons by inducing a *long-range attraction* between them, thereby enabling the formation of stable dark soliton bound states for the first time in a homogeneous medium without external perturbations.

We consider the evolution of a one-dimensional optical beam $u = u(x, z)$ described by the nonlocal NLS equation

$$i\partial_z u + \partial_x^2 u + \Delta n(I)u = 0, \qquad (1)$$
$$\Delta n(I) = -\int_{-\infty}^{\infty} R(x-\tau)I(\tau)d\tau,$$

where $I{=}I(x, z){=}|u|^2$ is the intensity, z is the evolution coordinate, x represents a transverse coordinate, and Δn is the nonlinear refractive index change of the medium. The function $R(x)$ characterizes the nonlocal response of the medium and is assumed to be real and symmetric.

The nonlocal term $\Delta n(I)$ in Eq. (1) represents a general phenomenological model for self-defocusing Kerr-like media, in which the change in the refractive index induced by an optical beam involves a transport process. This may include diffusion of molecules or atoms in atomic vapours[10] or thermal effects in plasma[11]. A

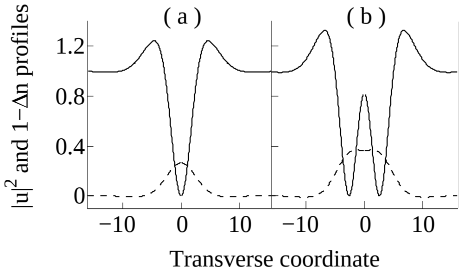

Fig. 1. Intensity profile $|u(x)|^2$ (solid line) and induced waveguide structure: $1+\Delta n(x)$ (dashed line) of a single nonlocal dark soliton (a) and a bound state (b) for λ=1. The degree of nonlocality is σ=4.

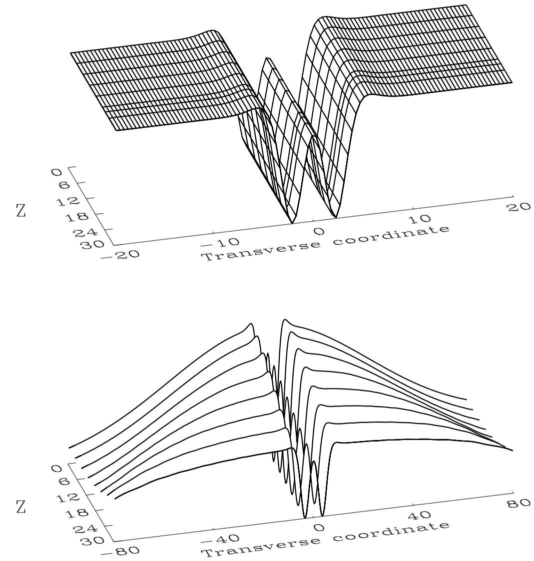

Fig. 2. Evolution of the intensity profile of a bound state of two nonlocal dark solitons for σ=1 and λ=1 on a cw background (top) and a Gaussian background (bottom).

nonlocal response in the form (1) appears naturally due to many body interaction processes in the description of Bose-Einstein condensates[12]. It was shown recently that such a nonlocal nonlinearity prevents collapse of multidimensional beams and supports formation of bright solitons[13–15].

Without loss of generality[16,17] we consider the normalized response function $R(x) = (2\sigma)^{-1}\exp(-|x|/\sigma)$. This form allows us to write Eq. (1) as two coupled equations

$$i\partial_z u + \partial_x^2 u + \Delta n u = 0, \tag{2}$$

$$\Delta n - \sigma^2 \partial_x^2 \Delta n = -|u|^2, \tag{3}$$

where Eq. (3) is the diffusion equation for a defocusing material. In this context the degree of nonlocality σ is then the diffusion parameter. In particular, for $\sigma \to 0$ the model (1) approaches the local Kerr nonlinearity.

Let us focus on fundamental dark solitons $u(x,z) = u(x)\exp(i\lambda z)$, where the real profile $u(x)$ has zero center amplitude, $u(0)$=0, and a π phase jump at the center, and where λ is a propagation constant. It was shown recently[18–20] that Eqs. (2-3) written for these stationary dark solitons are identical to the equations describing spatial solitons in quadratic nonlinear (or $\chi^{(2)}$) materials. The $\chi^{(2)}$ system predicts[21] the existence of stationary dark solitons with monotonic tails for $\sigma < 1/\sqrt{8}$ and nonmonotonic tails for $\sigma > 1/\sqrt{8}$. This implies that bound states of two or more dark solitons can be formed in sufficiently nonlocal materials with $\sigma > 1/\sqrt{8}$.

In Fig. 1 we show a numerically found dark soliton solution and the corresponding two-soliton bound state for σ=4. The dashed line shows the soliton-induced waveguide $(1+\Delta n)$, in which the soliton is guided.

It is important to note that although the profiles of stationary nonlocal spatial dark solitons and their bound states are equivalent to those of the $\chi^{(2)}$ system, the $\chi^{(2)}$ dark soliton bound states are *unstable* and break-up when propagating, while the corresponding nonlocal bound states are *stable, robust entities.* We demonstrate the stability by numerical integration of Eq. (1) with the exact numerically found solution as initial condition. Our simulations confirm stable propagation of nonlocal dark solitons and their bound states over tens of diffraction lengths, as the example shown in Fig. 2.

For a better verification of stability we used the same initial condition, but now embedded in a broad Gaussian beam, as in typical experiments. The propagation of this structure is shown in Fig. 2(b). The solitons remain bounded even though the background beam experiences strong deformation due to the self-defocusing. As the intensity of the background beam decreases the solitons *adiabatically* follow, adjusting their widths and mutual separation.

The ability of nonlocal dark solitons to form stable bound states is a direct consequence of the *long-range attraction* induced by the nonlocality. This effect may be explained using the self-guiding concept. In a local defocusing Kerr medium the refractive index distribution corresponding to two dark solitons always has the form of two waveguides separated by a region of lower refractive index which acts as a potential barrier. This potential barrier leads to their repulsion. In the presence of a sufficiently strong nonlocality, for which the width of the response function (σ) is comparable to or larger than the separation between the solitons, the convolution integral in Eq. (1) is able to even out this central dip completely and create a single broad waveguide (see Fig. 1), which traps the solitons and enable them to form a bound state.

Dark solitons are generated experimentally using odd or even boundary conditions[5,6]. In the first case a broad laser beam passes through a phase-mask with a single phase jump, leading to formation of an odd number of dark solitons. In the second case the beam passes through an amplitude mask (a thin wire), which imposes a dark

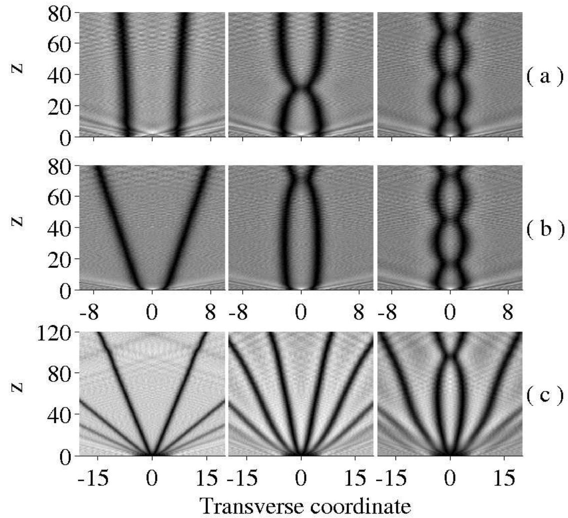

Fig. 3. Dark nonlocal solitons formed by phase (a,b) or intensity (c) modulation of a cw background. In (a) the phase jump is π and the degree of nonlocality is σ=2, while the initial soliton separation is x_0=5.5, 4, 2.5 from left to right. In (b) the phase jump is 0.95π and x_0=2.5, while σ=0.1, 1, 2. In (c) the width of the intensity gap is 7.5, while σ=0.1, 3, 6.

notch in the beam and excites an even number of dark solitons with opposite transverse velocities[22].

In the simulations shown in Figs.3(a-b) the solitons are created by imposing two opposite phase jumps in a cw background. In Fig.3(a) the degree of nonlocality σ=2 is fixed, while the initial separation x_0 between phase jumps (and thus the solitons) decreases from left to right. For distant solitons the interaction is weak and repulsive and their separation gradually increases. However, when the solitons are brought closer the nonlocality-induced attraction becomes strong enough to overcome the repulsion and force the solitons to collide. For even smaller initial separation the solitons trap each other in an oscillatory transversal motion.

In Fig. 3(b) σ increases from left to right, while x_0=2.5 is fixed. For σ=0.1 the nonlocality is too weak to affect the repulsive interaction of dark solitons, so they separate. As the degree of nonlocality increases the effective range of attraction expands until it overcomes the repulsion. For a strong nonlocality the solitons are again mutually trapped and propagate together as a bound state exhibiting transversal oscillations.

Figure 3(c) illustrates the interaction of solitons created by a narrow gap in the incident cw wavefront. Here the width of the gap is constant while the degree of nonlocality increases from left to right. For small σ the inherent repulsive force dominates and the solitons separate. As σ increases the nonlocal attraction becomes stronger and the outward motion is slowed down. A strong nonlocality again leads to their mutual trapping.

In conclusion, we have investigated the interaction of dark solitons in Kerr-like nonlocal nonlinear media. We have shown that unlike local dark solitons, which repel, nonlocal dark solitons exhibit strong long-range attraction, which enables formation of stable bound states. We also demonstrated numerically that bound states of dark solitons can be created dynamically in sufficiently nonlocal materials.

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