Nonlinear phase matching locking via optical readout

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Abstract: For optimal $\chi^{(2)}$ nonlinear interaction the phase matching condition must be satisfied. For type I and type II phase matched materials, this is generally achieved by controlling the temperature of the nonlinear media. We describe a technique to readout the phase-matching condition interferometrically, and experimentally demonstrate feedback control in a degenerate optical parametric amplifier (OPA) which is resonant at both the fundamental and harmonic frequencies. The interferometric readout technique is based on using the cavity resonances at the fundamental and harmonic frequencies to enable the readout of the phase mismatch. We achieve relatively fast temperature feedback using the photothermal effect, by modulating the amplitude of the OPA pump beam.

OCIS codes: (190.4410) Nonlinear Optics, parametric processes; (190.4360) Nonlinear Optics, devices

References and links

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1. Introduction

The $\chi^{(2)}$ interaction is responsible for some of the most important devices in nonlinear optics: the second harmonic generator (SHG), the optical parametric oscillator (OPO) and optical parametric amplifier (OPA). These devices are used in a myriad of applications, for example tunable laser sources [1, 2] and quantum optics [3].

An elementary requirement for $\chi^{(2)}$ nonlinear interaction is the conservation of momentum, also referred to as *phase matching* in this context. The phase matching condition imposes the relation on wavevectors; $k_b = k_a + k_{a'}$, where $k_i = n_i \omega_i / c_0$ and n_i is the refractive index of the i^{th} photon in the nonlinear medium and c_0 is the speed of light in vacuum. For small (optical) phase mismatch nonlinear interaction still occurs, however with reduced efficiency. For optimum nonlinear interaction the phase matching condition must be met precisely. Similarly, it is important that the phase matching condition be precisely controlled for low noise and long term applications, such as the generation of squeezing for gravitational wave detection [7] and to minimize intensity noise in second harmonic generators.

In birefringent materials phase matching can be achieved using *type I*, *type II* or *quasi-phase matching* [10]. Consider a SHG or degenerate OPO that is type I uniaxial crystal phase matched via temperature tuning such as MgO:LiNbO$_3$. Here, in SHG or degenerate OPO: $\omega_a = \omega_{a'}$ and the phase matching condition simplifies to: $n_b = n_a$. Here, and henceforth, we refer to the field with frequency ω_a as the fundamental field and to ω_b as the (second) harmonic field. To achieve phase matching in this system, the polarization of the fundamental field is set to the ordinary axis, the harmonic to the extraordinary axis, and the crystal temperature is tuned. Temperate tuning changes the ordinary and extraordinary refractive indices differentially according to the respective Sellmeier equations [2], until they become equal at the phase matched temperature.

In many experiments the nonlinear medium is temperature controlled to maintain phase matching condition. Typically, the temperature of the nonlinear medium is sensed by a nearby thermister and actuated by a peltier (thermo-electric) element or resistive heater. Stabilizing the phase matching condition using a temperature sensor on the exterior of the nonlinear medium has inherent disadvantages. An external sensor reads out the external crystal temperature, rather than temperature of the optical path through the crystal where the nonlinear interaction occurs. A temperature change in the nonlinear interaction region will not be suppressed by the temperature control loop. Instead a temperature gradient will arise between the interaction region and the boundary of the nonlinear medium. Thus, if a external temperature readout and control system is used, the oven temperature must be reset for each laser power used to achieve phase matching. This is a problem as lasers have power drift and decay over time.

In this paper we introduce a technique to readout the phase matching condition interferometrically using the interacting optical fields, in a doubly resonant (DR) OPA [4]. This enables fast readout of the phase matching condition, exactly where the nonlinear process is occurring. The readout involves monitoring the cavity resonant frequencies at both the fundamental and harmonic frequencies using the Pound-Drever-Hall (PDH) locking technique [5]. Since the resonant frequencies are proportional to the optical path length (OPL), the cavity resonant frequencies change differentially when there is some phase-mismatch (i.e. a refractive index

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difference $\Delta n = n_a - n_b$). At the phase matching temperature there is no refractive index difference ($\Delta n = 0$) and the two cavities are both resonant [6]. Thus, when operating sufficiently close to the phase matching temperature the cavity error signals can be used to derive the phase mismatch condition. In addition to phase matching readout, we demonstrate fast actuation of the crystal temperature using the photothermal effect [11, 12, 13, 14] for phase matching control. The phase matching error signal is used to modulate the harmonic (pump) field amplitude, which is partially absorbed in the nonlinear medium. This actuation can be extremely fast ($\sim$100kHz) compared to using an temperature actuation external to the crystal, which has bandwidth limitations due to the time delay associated with thermal conductivity of the nonlinear medium.

This paper is structured as follows: in section 2 we derive the nonlinear gain of a DROPA system with the harmonic cavity held on resonance as a function of temperature offset from phase matching. Our model shows that the full width at half maximum (FWHM) temperature of the nonlinear gain is significantly smaller than in the singly resonant (SR) or single pass OPA, motivating the requirement for phase matching control. In section 3 the experimental setup and results demonstrating phase matching readout and control are presented. In section 4 we discuss limitations of this demonstration and discuss the implications of this work.

2. Theory

The classical $\chi^{(2)}$ nonlinear optic equations of motion are [8];

$$\dot{a} = -(\kappa^a + i\Delta^a)a + \varepsilon^* a^* b + \sqrt{2\kappa^a_{in}}A_{in}, \quad (1)$$

$$\dot{b} = -(\kappa^b + i\Delta^b)b - \frac{\varepsilon a^2}{2} + \sqrt{2\kappa^b_{in}}B_{in}, \quad (2)$$

where a and b are proportional to the the intra-cavity fundamental and second harmonic fields, respectively; κ^a and κ^b are the total resonator decay rates for each field; ε is the nonlinear coupling parameter; and A_{in} and B_{in} are the driving fields with the respective input coupling rates are κ^a_{in} and κ^b_{in}. The angular frequency detuning of the fundamental and harmonic cavities with respect to the driving field frequencies are given by Δ^a and Δ^b.

We limit our calculation to OPA in the non-pump depleted regime where $\varepsilon a^2/2 \ll \sqrt{2\kappa^b_{in}}B_{in}$. The steady state intra-cavity field amplitudes are given by;

$$a = \frac{\sqrt{2\kappa^a_{in}}A_{in}((\kappa^a + i\Delta^a) + \varepsilon^* b^*)}{(\kappa^a)^2 + (\Delta^a)^2 - |\varepsilon b|^2}, \quad (3)$$

$$b \approx \frac{\sqrt{2\kappa^b_{in}}B_{in}}{\kappa^b - i\Delta^b}. \quad (4)$$

The detunings of the cavities due to change in the OPL, δp^j, are given by the following equations [9],

$$\Delta^j = \frac{-\delta p^j}{\lambda^j}\omega_{fsr} \quad (5)$$

where the superscript $\{j = a, b\}$. The free spectral range is given by $\omega_{fsr} = 2\pi c_0/p$, with the total OPL; $p = L + nL_c$ with L the round trip length in free space and L_c the length of the crystal with refractive index, n and λ_j is the wavelength in vacuum and c_0 the speed of light in vacuum.

Change in the round trip OPL, δp, can come from change to the free space OPL and from change in the crystal OPL. The crystal OPL is a function of crystal temperature change, δT, arising from two mechanisms, thermal expansion and refractive index change, i.e.;

$$\delta p^j = \delta L + nL_c \left(\frac{1}{n}\frac{dn_j}{dT} + \alpha_j\right)\delta T \tag{6}$$

where dn^j/dT are the crystal's photorefractive constants and α_j are the crystal's thermal expansion constants. We define the total detuning as;

$$\Delta^j = \Delta_{fs} + \Delta^j_{cr}(\delta T) \tag{7}$$

where Δ_{fs} is the due to the change in free space OPL and Δ^j_{cr} is due to change in crystal OPL.

Consider the total OPL of the harmonic cavity is locked to resonance by controlling the free space length to suppress any cavity detuning ($\Delta^b \to 0$). If the temperature of the crystal is changed, the control system will have to change the free space OPL to compensate for the change in crystal OPL ($\Delta_{fs} = -\Delta^b_{cr}(\delta T)$) to maintain cavity resonance. Substituing this into the total detuning of the fundamental cavity we find;

$$\Delta^a = \Delta_{fs} + \Delta^a_{cr}(\delta T) \tag{8}$$

$$= \Delta^a_{cr}(\delta T) - \Delta^b_{cr}(\delta T) \tag{9}$$

Thus the detuning for the fundamental cavity is given by the difference detunings of the change in *crystal OPL* at the fundamental and harmonic cavities due to temperature tuning of the crystal. We can extend Eq. 9 to considered more than one longitudinal mode of the fundamental cavity. Also, for completeness, we add an arbitrary differential phase shift between the fundamental and harmonic fields, θ, which adds an detuning of $\Delta^\theta = \theta\omega_{fsr}$. Thus the total detuning then becomes;

$$\Delta^a_l = \Delta^a_{cr}(\delta T) - \Delta^b_{cr}(\delta T) - \Delta^\theta - l\omega_{fsr}, \tag{10}$$

where l is the cavity mode number. The intra-cavity amplitude of an lth mode is:

$$a_l = \frac{\sqrt{2\kappa^a_{in}}A_{in}[(\kappa^a + i\Delta^a_l) + |\varepsilon^* b^*|e^{i\phi_l}]}{(\kappa^a)^2 + (\Delta^a_l)^2 - |\varepsilon b|^2} \tag{11}$$

where the phase ϕ_l is the combined angle $\angle(\varepsilon^* b^*)$, which can be varied to control the sign of the parametric gain. Here $\phi_l = \pm\angle(\kappa^a + i\Delta^a_l)$ is set to maximize (+) or minimize (−) the parametric gain.

The nonlinear coupling parameter dependence on temperature is due to the phase mismatch, Δk, in the following form [10];

$$\varepsilon = \varepsilon_0 z e^{i\frac{\Delta k L_c}{2}} \operatorname{sinc}\frac{\Delta k L_c}{2}, \tag{12}$$

where ε_0 is a constant. The Sellmeier equation [2] at the fundamental frequency is $\Delta k = \xi(\delta T)$ where ξ is a constant whose value depends on the crystal's properties, and δT is the crystal's temperature offset from the phase matched temperature. The nonlinear gain is found when the transmitted power with/without the pump field is compared;

$$\frac{P_{out}}{P_{out}|_{b=0}} = \frac{|a_l|^2}{|a_{l|b=0}|^2} \tag{13}$$

The nonlinear gain as a function of temperature is shown in Fig. 1 with parameters given in the figure caption. The compromise between nonlinear gain and temperature stability requirements

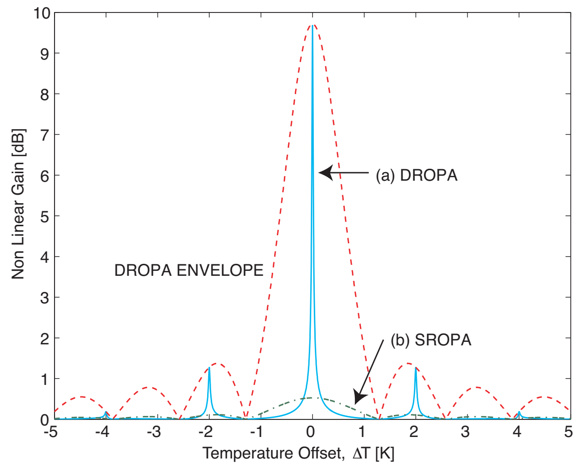

Fig. 1. Examples of nonlinear gain (Eq. 13) vs temperature offset from phase matching. Trace (a) is a DROPA with harmonic cavity held on resonance ($\Delta^b = 0$) and the fundamental detuning (Δ_l^a) given by Eq. 10. Trace (b) is a SROPA with the fundamental cavity held on resonance ($\Delta_l^a = 0$). The dashed curve gives the nonlinear gain envelope of the DROPA found with both fundamental and harmonic cavities are held on resonance ($\Delta^b = \Delta_l^a = 0$). Parameters: $P_{in}^b = 0.35$ W, $\lambda_a = 2\lambda_b = 1064$ nm, n = 2.23, $L = .65$ m, $L_c = .0065$ m, $\frac{dn_a}{dT} = 3.3\times10^{-6}$ 1/K, $\frac{dn_b}{dT} = 37.0\times10^{-6}$ 1/K, $\alpha_a = \alpha_b = 5\times10^{-6}$ 1/K, $\kappa_{in}^a = 1\times10^4$ 1/s, $\kappa^a = 2.26\times10^7$ 1/s, $\kappa_{in}^b = 6.77\times10^6$ 1/s, $\kappa^b = 6.82\times10^6$ 1/s, $\xi = 749$ 1/m/K, $\varepsilon_0 = 30$ 1/s. Parameter mostly taken from [14]

can be seen in the comparison between the DROPA, trace (a), with a high reflectivity input coupler at the harmonic frequency, compared to the SROPA, trace (b), which has 0% reflectivity. The resonant enhancement of the nonlinearity gives the DROPA additional nonlinear gain over the SROPA however the FWHM of the nonlinear gain is significantly smaller than for the SROPA.

In addition to the resonant enhancement of the nonlinear gain, cavities at both the fundamental and harmonic frequencies provide a phase reference between the two frequencies. This phase reference can be used to readout the phase matching condition interferometrically. Near the phase matching temperature, the cavity resonance conditions can be monitored using standard cavity readout techniques, and this readout can be used to produce a phase matching error signal.

3. Experiment

A schematic of the experimental setup is shown in Fig. 2. The SHG [15] produced the OPA pump beam at 532nm. The OPA crystal was 6.5mm long, type-I phase-matched MgO:$LiNbO_3$ with 7% doping. The optical surfaces were flat and coated for anti-reflection at both wavelengths. The crystal was placed in a peltier driven oven held at $\sim63^o$C to approximately 5mK accuracy. The DROPA bow-tie cavity configuration consisted of three high reflectivity (HR) mirrors at both wavelengths and the input/output coupler with transmission of 10% and 3% at 1064nm and 532nm, respectively. The incident pump power was 100mW, giving a circulating pump power of ∼2.7W, corresponding to a parametric gain of just under 3dB. The OPA was seeded through a HR mirror with ∼ 10mW. The input harmonic field was phase modulated

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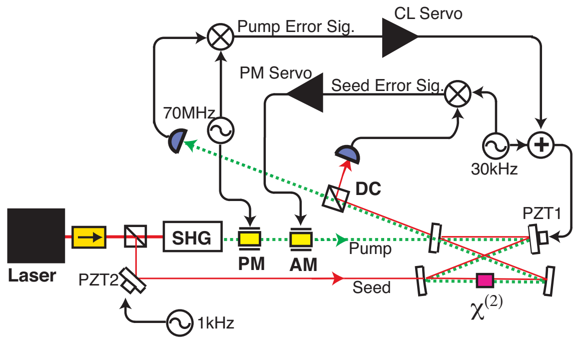

Fig. 2. The experimental layout. Most of the laser power is used to produce the harmonic beam for the OPA. The harmonic beam is passed though a broad band amplitude modulator (AM) and a resonant (70MHz) phase modulator (PM) before being incident on the OPA. The cavity length error signal is derived from the reflected harmonic field and fed to the PZT1. The phase matching error signal is derived from the transmitted fundamental field and sent to the AM on the harmonic field. The temperature of the crystal oven is actively controlled to 63^{o}C.

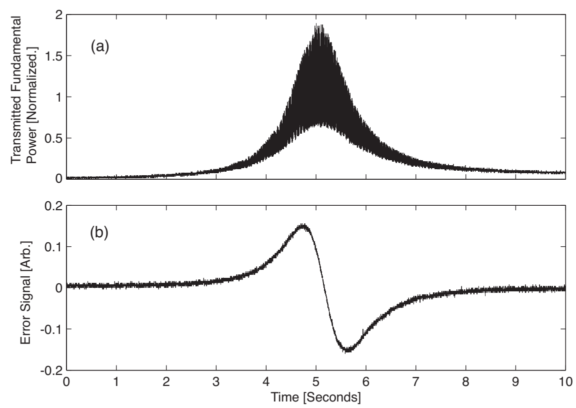

Fig. 3. (a) The transmitted fundamental power from the OPA as the temperature of the crystal is varied by sweeping the oven temperature. This is normalized to the resonant transmitted power through the OPA without parametric gain. The nonlinear gain is being dithered at approx. 1kHz, which shows the nonlinear gain envelope (amplification and de-amplification). (b) the corresponding error signal of the phase matching condition. The dither signal of the nonlinear gain is filtered out of the error signal.

at 70MHz. The harmonic field reflected from the OPA was demodulated to produce an error signal for the cavity length. This error signal was fed back to a piezo-electric tranducer (PZT1) bonded to a OPA cavity mirror. The PZT1 also had a modulation signal added at 30kHz to produce phase modulation on the intra-cavity fields. The transmission at the seed was detected and demodulated (at 30kHz) to produce an error signal also using the PDH technique. Figure 3 shows the transmission of the cavity at the fundamental frequency, plot (a), and the associated error signal, plot (b), as the temperature of the crystal was swept across the phase matching temperature. These data was taken with the harmonic cavity locked on resonance. The vertical axis of the Figure 3(a) is normalized to the resonant power transmitted through the OPA without parametric gain. Thus on resonance, below 1 indicates parametric de-amplification and above 1

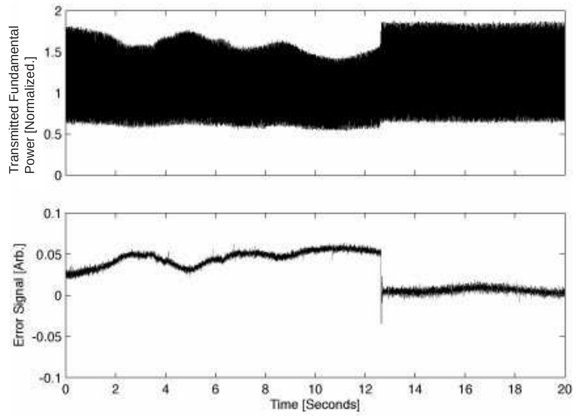

Fig. 4. (a) The transmitted fundamental power of the OPA as a function of time. This is normalized to the resonant transmitted power through the OPA without parametric gain. (b) the phase matching error signal. At 12.5 seconds the control loop is closed and the nonlinear gain is optimized.

parametric amplification. The relative phase of the harmonic and fundamental frequencies was swept rapidly by dithering PZT2 at 1kHz in order to sample amplification and de-amplification, to show the nonlinear gain envelope. The error signal here has been low pass filtered to remove any component associated with the nonlinear gain at 1kHz. [1] This (phase matching) error signal was sent to an amplitude modulator (AM) in the input harmonic fields' path to actuate on the crystal temperature via the photothermal effect. Photothermal actuation was proven to be very effective since most of the harmonic field was absorbed in the crystal, as the cavity is nearly impedance matched and the crystal represents the dominant loss mechanism.

The behavior of the nonlinear gain as a function of time is shown in Fig. 4(a). Up until 12.6 seconds the crystal has temperature control only from a oven with temperature sensor on crystal exterior. Even though a high precision temperature controller was used, the nonlinear gain and the error signal (Fig. 4(b)) are seen to drift significantly. 12.6 seconds into the data run the phase matching temperature control loop was closed, and the nonlinear gain moves to its maximum value and the error signal to zero. A comparison of the temperature stability with/without phase matching control showed the mean temperature offset was 0.5mK/3mK and standard deviations of 0.3mK/0.72mK, measured over a 5 second interval.

4. Discussion

Our control loops performance was limited by the phase delay inherent in photothermal actuation. This can be seen in the measured photothermal response of the crystal shown in Fig. 5. The measured trace (solid line) was fitted with a theoretical curve [13] (dashed line) which has a corner frequency of 140Hz. These data was taken by measuring the transfer function from the AM on the harmonic field to the fundamental error signal. This was done whilst the harmonic field was locked to the cavity resonance and the crystal temperature set so that the fundamental field was also on resonance, however detuned 8 Kelvin from the phase matching temperature to eliminate any $\chi^{(2)}$ effects.

The photothermal phase delay limited the control bandwidth to approximately 10 Hz, since the controller did not take this into accout. We expect that a high bandwidth ($\sim$100kHz), high

[1] When using this technique for applications one may lock the phase of the fundamental / harmonic fields to amplification or deamplification.

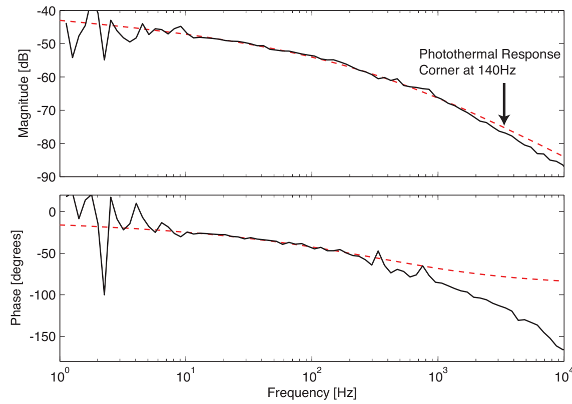

Fig. 5. Measured (solid line) and theoretical (dashed line) photothermal response of the nonlinear crystal. (Top) the amplitude response and (bottom) the phase response.

gain control loop could be implemented if the photothermal response was considered and an appropriate controller electronics designed. Also, an integrator would be useful to increase low frequency gain and drive residual phase mismatch (and temperature deviation) lower. These improvements to the electronics could potentially suppress the deviation from the phase matching temperature even further.

Using the photothermal feedback via the harmonic (pump) field is convenient and potentially very fast. An issue that may arise using this type of actuator is that the phase matching error signal changes the pump power, which is coupled to the amplitude of the nonlinear gain. However, this is a second order effect compared to operating with a phase mis-match. Ideally, photothermal actuation would be used in parallel with a slow feedback to the crystal oven to null any low frequency temperature variation, thus keeping the power of the harmonic field constant.

We note that the phase matching error signal is obtained by using the fundamental cavity resonance condition, and therefore any rms locking error of the length of the cavity will couple directly in to phase matching error signal. In this case, where the control bandwidth of the cavity length is a factor of a thousand larger than the phase matching control bandwidth, the coupling of the rms locking error of the cavity length is negligible in comparison the the phase matching error.

5. Conclusions

We have introduced a technique to interferometrically readout the phase matching condition in a DROPA. High precision readout of the phase matching condition is obtained by differencing cavity error signals of the fundamental and harmonic frequencies. An experimental demonstration of phase matching locking was performed, showing temperature control to a mean value of 0.5mK from the phase matching temperature, with a standard deviation of 0.3mK. With the phase matching locked we obtain a substantial improvement of nonlinear gain, and of nonlinear gain stability. As a temperature actuator for the nonlinear interaction region we used the photothermal effect, via electro-optic modulation of the harmonic field amplitude. This enabled a unity gain bandwidth of the phase matching control loop of approximately 10Hz. With servo design taking the photothermal phase response into account, we expect that a unity gain bandwidth of $\sim$100kHz could be achieved using photothermal actuation.

Optical readout of the phase matching condition is also applicable to other nonlinear systems

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such as SHGs and non-degenerate OPOs. This technique could also be extended to atom optics, similar to the work in [16].

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