

# Surface multi-gap vector solitons

Ivan L. Garanovich, Andrey A. Sukhorukov, and Yuri S. Kivshar

*Nonlinear Physics Centre and Centre for Ultra-high bandwidth Devices for Optical Systems (CUDOS),  
Research School of Physical Sciences and Engineering, Australian National University,  
Canberra, ACT 0200, Australia*  
[ilg124@rsphysse.anu.edu.au](mailto:ilg124@rsphysse.anu.edu.au)

Mario Molina

*Departamento de Física, Facultad de Ciencias, Universidad de Chile, Santiago, Chile*  
[mmolina@uchile.cl](mailto:mmolina@uchile.cl)

**Abstract:** We analyze nonlinear collective effects near surfaces of semi-infinite periodic systems with multi-gap transmission spectra and introduce a novel concept of *multi-gap surface solitons* as mutually trapped surface states with the components associated with different spectral gaps. We find numerically discrete surface modes in semi-infinite binary waveguide arrays which can support simultaneously two types of discrete solitons, and analyze different multi-gap states including the soliton-induced waveguides with the guided modes from different gaps and composite vector solitons.

© 2006 Optical Society of America

**OCIS codes:** (190.4420) Nonlinear optics, transverse effects in; (190.5940) Self-action effects

---

## References and links

1. S. G. Davidson and M. Steslicka, *Basic theory of surface states* (Oxford Science Publications, New York, 1996).
2. I. E. Tamm, "A possible kind of electron binding on crystal surfaces," *Z. Phys.* **76**, 849–850 (1932).
3. P. Yeh, A. Yariv, and C. S. Hong, "Electromagnetic propagation in periodic stratified media. I. General theory," *J. Opt. Soc. Am.* **67**, 423–438 (1977).
4. P. Yeh, A. Yariv, and A. Y. Cho, "Optical surface waves in periodic layered media," *Appl. Phys. Lett.* **32**, 104–105 (1978).
5. W. J. Tomlinson, "Surface wave at a nonlinear interface," *Opt. Lett.* **5**, 323–325 (1980).
6. A. D. Boardman, P. Egan, F. Lederer, U. Langbein, and D. Mihalache, "Third-order nonlinear electromagnetic TE and TM guided waves," in *Nonlinear Surface Electromagnetic Phenomena*, Vol. 29 of *Modern Problems in Condensed Matter Sciences*, H. E. Ponath and G.I. Stegeman, eds., (North-Holland, Amsterdam, 1991), pp. 73–287.
7. D. Mihalache, M. Bertolotti, and C. Sibilia, "Nonlinear wave propagation in planar structures," in *Progress in Optics*, E. Wolf, ed., (North-Holland, Amsterdam, 1989), Vol. 27, pp. 229–313.
8. Yu. S. Kivshar, F. Zhang, and S. Takeno, "Multistable nonlinear surface modes," *Physica D* **119**, 125–133 (1998).
9. K. G. Makris, S. Sunstov, D. N. Christodoulides, G. I. Stegeman, and A. Hache, "Discrete surface solitons," *Opt. Lett.* **30**, 2466–2468 (2005).
10. S. Sunstov, K. G. Makris, D. N. Christodoulides, G. I. Stegeman, A. Hache, R. Morandotti, H. Yang, G. Salamo, and M. Sorel, "Observation of discrete surface solitons," *Phys. Rev. Lett.* **96**, 063901–4 (2006).
11. Yu. S. Kivshar and G. P. Agrawal, *Optical Solitons: From Fibers to Photonic Crystals* (Academic Press, San Diego, 2003).
12. M. I. Molina, R. A. Vicencio, and Yu. S. Kivshar, "Discrete solitons and nonlinear surface modes in semi-infinite waveguide arrays," *Opt. Lett.* **31**, 1693–1695 (2006).
13. Y. V. Kartashov, V. A. Vysloukh, and L. Torner, "Surface gap solitons," *Phys. Rev. Lett.* **96**, 073901–4 (2006).
14. C. R. Rosberg, D. N. Neshev, W. Krolikowski, Yu. S. Kivshar, A. Mitchell, R. A. Vicencio, and M. I. Molina, "Observation of surface gap solitons in semi-infinite waveguide arrays," arXiv physics/0603202 (2006).
15. J. Hudock, S. Sunstov, D. N. Christodoulides, and G. I. Stegeman, "Vector discrete nonlinear surface waves," *Opt. Express* **13**, 7720–7725 (2005), <http://www.opticsexpress.org/abstract.cfm?URI=OPEX-13-20-7720>.

16. O. Cohen, T. Schwartz, J. W. Fleischer, M. Segev, and D. N. Christodoulides, "Multiband vector lattice solitons," *Phys. Rev. Lett.* **91**, 113901–4 (2003).
17. A. A. Sukhorukov and Yu. S. Kivshar, "Multigap discrete vector solitons," *Phys. Rev. Lett.* **91**, 113902–4 (2003).
18. O. Cohen, G. Bartal, H. Buljan, T. Carmon, J. W. Fleischer, M. Segev, and D. N. Christodoulides, "Observation of random-phase lattice solitons," *Nature* **433**, 500–503 (2005).
19. C. R. Rosberg, B. Hanna, D. N. Neshev, A. A. Sukhorukov, W. Krolikowski, and Yu. S. Kivshar, "Discrete interband mutual focusing in nonlinear photonic lattices," *Opt. Express* **13**, 5369–5376 (2005), <http://www.opticsexpress.org/abstract.cfm?URI=OPEX-13-14-5369>.
20. A. A. Sukhorukov and Yu. S. Kivshar, "Discrete gap solitons in modulated waveguide arrays," *Opt. Lett.* **27**, 2112–2114 (2002).
21. A. A. Sukhorukov and Yu. S. Kivshar, "Generation and stability of discrete gap solitons," *Opt. Lett.* **28**, 2345–2347 (2003).
22. R. Morandotti, D. Mandelik, Y. Silberberg, J. S. Aitchison, M. Sorel, D. N. Christodoulides, A. A. Sukhorukov, and Yu. S. Kivshar, "Observation of discrete gap solitons in binary waveguide arrays," *Opt. Lett.* **29**, 2890–2892 (2004).
23. D. E. Pelinovsky, A. A. Sukhorukov, and Yu. S. Kivshar, "Bifurcations and stability of gap solitons in periodic potentials," *Phys. Rev. E* **70**, 036618–17 (2004).
24. D. Mandelik, H. S. Eisenberg, Y. Silberberg, R. Morandotti, and J. S. Aitchison, "Observation of mutually trapped multiband optical breathers in waveguide arrays," *Phys. Rev. Lett.* **90**, 253902–4 (2003).

## 1. Introduction

Interfaces separating different physical media can support a special class of transversally localized waves known as *surface waves*. Linear surface waves have been studied extensively in many branches of physics [1], and the structure of surface states in periodic systems is associated with the specific properties of the corresponding Bloch waves. Such linear surface waves with *staggered* profiles are often referred to as *Tamm states* [2], first identified as localized electronic states at the edge of a truncated periodic potential, and then found in other systems, e.g. for an interface separating periodic and homogeneous dielectric optical media [3, 4].

*Nonlinear surface waves* have been studied most extensively in optics where both TE and TM surface modes were predicted and analyzed for the interfaces separating two different homogeneous nonlinear dielectric media [5, 6, 7]. In addition, nonlinear effects are known to stabilize surface modes in discrete systems generating different types of states localized at and near the surface [8]. Self-trapping of light near the boundary of a self-focusing photonic lattice has recently been predicted theoretically [9] and demonstrated in experiment [10] through the formation of *discrete surface solitons* at the edge of a waveguide array. Such *unstaggered* discrete surface modes can be treated as discrete optical solitons [11] trapped at the edge of a waveguide array when the beam power exceeds a certain critical value associated with a strong repulsive surface energy [12]. Staggered surface gap solitons in defocusing semi-infinite periodic media have also been introduced theoretically [13] and recently observed experimentally [14], and they provide a direct nonlinear generalization of the familiar electronic surface Tamm states.

So far, unlike the only case of vector discrete nonlinear surface waves [15], discrete surface solitons have been described by scalar fields, and they were associated either with the semi-infinite total-internal reflection optical gap, such as discrete surface solitons [9], or with the Bragg-reflection photonic gap, such as surface gap solitons [13], being treated completely independently. However, as was already shown for infinite nonlinear periodic and discrete systems [16, 17, 18, 19], nonlinear collective effects in photonic systems with multi-gap transmission spectra may lead to mutual cross-band focusing effects and can support *multi-gap solitons* as composite modes with the components associated with different spectral gaps.

In this paper, we introduce the concept of *multi-gap surface waves* as composite states consisting of mutually trapped components from different gaps localized at the surface. Such composite surface states can be created either by a surface soliton that traps linear guided modes from other spectral gaps, or as vector surface solitons with the major components from differ-

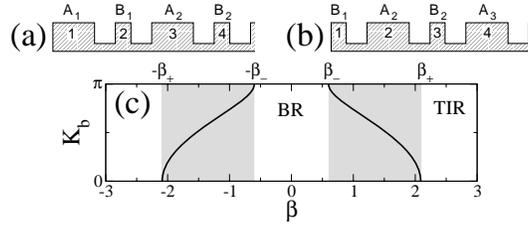


Fig. 1. (a,b) Two types of a semi-infinite binary array truncated at wide or narrow waveguides. (c) Dispersion diagram of the binary array with the bands shaded and gaps marked. Detuning between the A and B-type waveguides is  $\rho = 0.6$ .

ent gaps. To the best of our knowledge, such states have never been mentioned in any field of physics, and they become possible due to interaction of multi-gap discrete solitons [17] with the surface. As a specific example allowing us to demonstrate the basic concept, we consider here the discrete surface modes in semi-infinite binary waveguide arrays, earlier introduced theoretically [20, 21] and then studied experimentally [22], and find numerically different classes of multi-gap surface states also discussing their existence and stability.

## 2. Surface discrete solitons in binary arrays

We consider a binary array composed of two types (A and B, wide and narrow) of separated optical waveguides [20, 21], that can be fabricated by etching waveguides on top of a Al-GaAs substrate [22]. To study the surface effects, we assume that the array is truncated at either waveguide, as shown in Figs. 1(a,b). We analyze interactions between several mutually incoherent components with the electric field envelopes  $E^{(j)}(x, z)$ . Then, we employ the tight-binding approximation and present the total field  $\mathbf{E}$  as a superposition of the guided modes supported by individual guides,  $\mathbf{E}(x, z) = \sum_n [\mathbf{A}_n(z) \psi_{A_n}(x) + \mathbf{B}_n(z) \psi_{B_n}(x)] \exp(ikz)$ , where we introduce the vector notations  $\mathbf{F} = (F^{(1)}, F^{(2)}, \dots)$ . Here  $\psi_{A_n}(x)$  and  $\psi_{B_n}(x)$  are the mode profiles of the individual waveguides,  $k$  is the average propagation constant of A and B modes, and  $A_n^{(j)}$  and  $B_n^{(j)}$  are the mode amplitudes for the field component number  $j$ . Finally, we derive a system of coupled discrete equations [17] for the normalized amplitudes  $\mathbf{a}_n$  and  $\mathbf{b}_n$ ,

$$i \frac{d\mathbf{a}_n}{dz} + \rho \mathbf{a}_n + \mathbf{b}_{n-1} + \mathbf{b}_n + \|\mathbf{a}_n\|^2 \mathbf{a}_n = 0, \quad i \frac{d\mathbf{b}_n}{dz} - \rho \mathbf{b}_n + \mathbf{a}_n + \mathbf{a}_{n+1} + \|\mathbf{b}_n\|^2 \mathbf{b}_n = 0, \quad (1)$$

where  $\mathbf{b}_0 \equiv 0$  or  $\mathbf{a}_1 \equiv 0$  for arrays terminated at a wide or narrow waveguide, respectively. We consider the case of a positive Kerr-type medium response proportional to the total field intensity  $\|\{\mathbf{a}, \mathbf{b}\}_n\|^2 = \sum_j |\{a, b\}_n^{(j)}|^2$ , neglecting the small differences in the effective nonlinear coefficients at the A and B sites;  $\rho$  is proportional to the detuning between the propagation constants of the A and B-type guided modes.

According to Eqs. (1), the linear Bloch-wave dispersion is defined as  $K_b = \cos^{-1}(-\eta/2)$ , where  $\eta = 2 + \rho^2 - \beta^2$ . The transmission bands correspond to real  $K_b$ , and they appear when  $\beta_- \leq |\beta| \leq \beta_+$ , where  $\beta_- = |\rho|$  and  $\beta_+ = (\rho^2 + 4)^{1/2}$ . A characteristic dispersion relation and the corresponding band-gap structure are presented in Fig. 1(c). The upper gap at  $\beta > \beta_+$  is due to the effect of total internal reflection (TIR gap), whereas additional BR gap appears for  $|\beta| < \beta_-$  due to the resonant Bragg reflection.

First, we study scalar discrete surface solitons in the truncated binary arrays and look for spatially localized solutions of Eqs. (1) in the form  $(a_n, b_n) = (u_{2n-1-n_0}, u_{2n-n_0}) \exp(i\beta z)$ ,  $n = 1, 2, \dots$ , where  $\beta$  is the propagation constant,  $n_0 = 0, 1$  for the structure termination at the A or B site, respectively, and the function  $u_n$  describes the soliton profiles. The soliton power

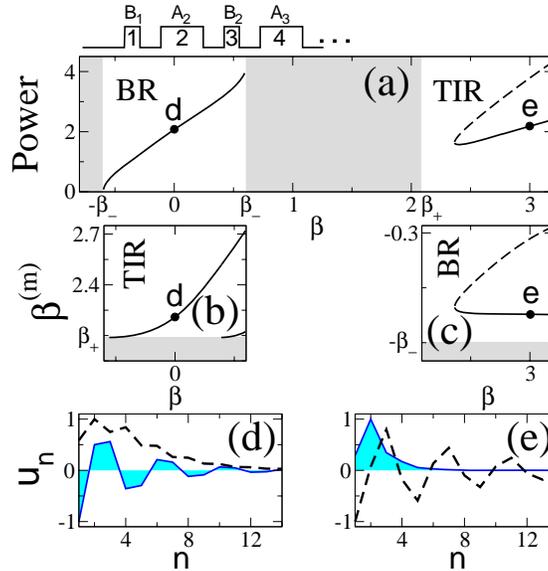


Fig. 2. (a) Families of the discrete surface solitons in the BR and TIR gaps, and (b,c) propagation constants of the localized linear modes from the other gaps guided by these solitons, in the binary array truncated at the narrow waveguide. Dashed curves in (a-c) mark unstable soliton branches. (d,e) Examples of two types of the discrete surface solitons (solid) and their guided linear modes localized in the other gaps (dashed), corresponding to the marked points "d" and "e" in (a-c). Detuning parameter of the binary array is  $\rho = 0.6$ .

is  $P = \sum_n |u_n|^2$ . Discrete solitons can appear in the TIR gap, and gap solitons can form inside the BR gap.

In Figs. 2(a) and 3(a) we summarize our findings and show the results for the families of the nonlinear surface states for the two different cases. In the first case, when the edge waveguide is narrow, the nonlinear surface modes start to appear in the BR gap (as the nonlinear Tamm states) at low powers, whereas the existence of the surface modes in the TIR gap requires the beam power to exceed some threshold value. In the second case, when the array is truncated at the wide waveguide, the existence of nonlinear surface modes in both the gaps requires similarly to exceed a threshold power. We employ the beam propagation method to investigate the soliton stability. In Figs. 2(a) and 3(a), dashed branches of the curves indicate unstable surface modes. Whereas oscillatory instabilities [23] may arise for the solid branches, we have verified that the solitons corresponding to marked points on the solid curves demonstrate stable propagation for more than 100 coupling lengths even with input perturbations of 1%.

### 3. Waveguides induced by surface solitons

The mutual trapping of the modes localized in different gaps and the physics of surface multi-gap vector solitons can be understood in terms of the soliton-induced waveguides. We search for the linear guided modes supported by a scalar soliton in other gaps, and consider two types of surface solitons and two different waveguide truncations. In Figs. 2(b,c) and 3(b,c), we plot the eigenvalues of the linear guided modes supported by the BR (b) and TIR (c) surface solitons, respectively. In Figs. 2(d,e) and 3(d,e) we show several characteristic examples of the BR and TIR surface solitons together with linear surface modes they guide in the other gap. We find that surface solitons may be able to guide linear surface modes, and the number of guided modes

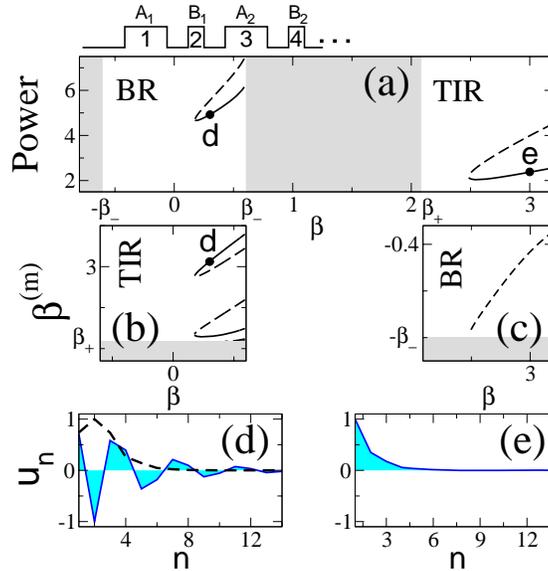


Fig. 3. Same as in Fig. 2, but for the binary array truncated at the wide waveguide.

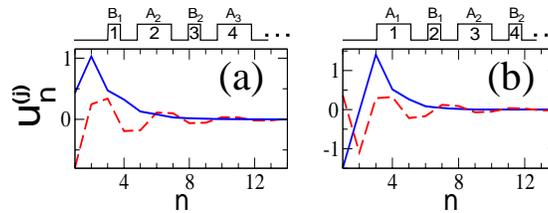


Fig. 4. Examples of surface multi-gap vector solitons for different structure terminations; solid and dashed curves show the profiles of the soliton components from the TIR and BR gaps, respectively. For the termination on the narrow site (a), BR component propagation constant is  $\beta = -0.1$  and its power is  $P = 0.9$ . TIR component has  $\beta = 2.6$  and  $P = 1.6$ . For the termination on the wide site (b), BR component propagation constant and power are  $\beta = 0$  and  $P = 1.6$ , respectively, while TIR component has  $\beta = 3.0$  and  $P = 4.6$ .

supported by a soliton-induced surface waveguide depends on the lattice termination and the soliton distance from the edge, indicating the possibility of the effective engineering of the interband interactions. For example, in contrast to the case of infinite waveguide arrays where linear modes are always present [16, 17, 23], in the case of the structure termination on the A-type (narrow) waveguide stable fundamental TIR solitons do not support BR modes [note the absence of a solid curve in Fig. 3(c)].

#### 4. Multi-gap surface solitons

When the amplitude of the guided mode grows, the mode interacts with the surface soliton waveguide creating a coupled multi-gap state in the form of a *surface multi-gap vector soliton*. The eigenvalues of the linear guided modes define the point where such vector solitons bifurcate from their scalar counterparts [16, 17]. In the vicinity of the bifurcation point, the soliton symmetry and stability are defined mostly by the large-amplitude soliton component. However, as the power in the second component grows, the soliton properties change dramatically, and

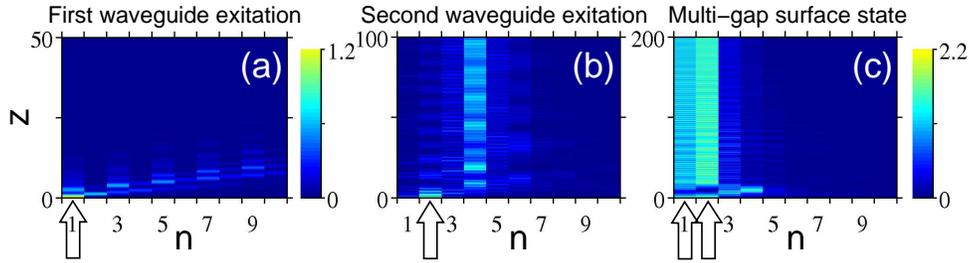


Fig. 5. Density plots for the total intensity demonstrating (a,b) nonlinear dynamics of individual beams incident on the first or second waveguide with input powers  $P = 1.3$  and  $P = 2.3$ , respectively; and (c) generation of a multi-gap state through the combined excitation of the first and second waveguides with the same two mutually incoherent beams. The array is truncated at a narrow waveguide.

the surface multi-gap vector solitons may demonstrate quite complicated structure and behavior. Figures 4(a,b) show two examples of such two-gap surface solitons found numerically for the two different truncations of the binary waveguide array. In both cases, the composite state is created by coupling the components residing in the TIR and BR spectral gaps. Properties of the surface multi-gap solitons can be engineered by controlling geometry and parameters of the semi-infinite array. For example, in the case of the termination of the binary array on the wide site, TIR component of the stable multi-gap soliton has a twisted structure [see Fig. 4(b)] which reflects the fact that when the array is truncated on the wide site, a fundamental TIR soliton can not support any guided mode in the other gap [Fig. 3(c)]. We find that these multi-gap solitons are stable for propagation over more than 500 coupling lengths under initial perturbations on the order of 1%, whereas in the absence of mutual trapping, the individual TIR and BR components experience breathing and decay.

One of the advantages of binary waveguide arrays is that the preferential excitation of the first or second spectral bands can be achieved by focusing beams at the wide or narrow waveguides, respectively. This allows us to suggest a simple scheme for the generation of multi-gap surface states from two mutually incoherent beams incident at the first and second waveguides, see an example in Fig. 5(c). The weak oscillations of the total intensity are due to the excitation of the soliton internal mode, and we have confirmed that the spectrum of propagation constants  $\beta$  contains dominant components in both the TIR ( $\beta \simeq 2.9$ ) and BR ( $\beta \simeq 0.1$ ) gaps, realizing an intermediate state between coherent multi-band breathers [24] and stationary multi-gap solitons. We emphasize that it is the collective beam trapping that enables the formation of a nonlinear surface state, whereas individual beams exhibit repulsion from the interface [see Fig. 5(a,b)].

## 5. Conclusion

We have introduced a novel type of nonlinear surface waves created by mutual trapping of several components from different spectral gaps in semi-infinite systems with multi-gap transmission spectra. We have studied surface modes in truncated binary waveguide arrays and found various multi-gap surface states including the surface soliton-induced waveguides with the guided modes from different spectral gaps and multi-gap composite vector solitons.

## Acknowledgements

The authors acknowledge support from the Australian Research Council and Fondecyt (grants 1050193 and 7050173).