

# Incoherent multi-gap optical solitons in nonlinear photonic lattices

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**Abstract:** We demonstrate numerically that partially incoherent light can be trapped in the spectral band gaps of a photonic lattice, creating partially incoherent multi-component spatial optical solitons in a self-defocusing nonlinear periodic medium. We find numerically such incoherent multi-gap optical solitons and discuss how to generate them in experiment by interfering incoherent light beams at the input of a nonlinear periodic medium.

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**OCIS codes:** (190.5940) Self-action effects, (030.1640) Coherence, (190.3270) Kerr effect

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## 1. Introduction

Gap solitons are nonlinear localized waves which are associated with band-gaps in the transmission spectra of nonlinear photonic structures with a periodically modulated refractive index [1, 2, 3], such as fiber Bragg gratings [4], waveguide arrays [5, 6], and optically-induced lattices [7, 8, 9]. Such gap solitons are composed of counter-propagating waves which experience Bragg reflection from the periodic structure and remain trapped at the nonlinearly-induced lattice defect. It was recently predicted theoretically [10, 11, 12] and demonstrated experimentally [13] that gap solitons can be composed of many modes which are localized in multiple band-gaps of a single periodic lattice. When several modes are excited simultaneously with a coherent light source, they can trap together in the form of a periodic or quasi-periodic multi-band breather [14]. On the other hand, stationary multi-gap solitons with fixed profile along the propagation direction may form if the modes are made mutually incoherent, and non-instantaneous nonlinear response is defined by the time-averaged light intensity. It was suggested theoretically [15] that multi-gap solitons can be generated by a partially coherent light source which simultaneously excites multiple modes in different gaps of the lattice spectrum.

A multi-gap soliton can be generated in a *self-focusing nonlinear periodic medium* by a single partially coherent beam incident at normal angle on a periodic photonic structure, as was recently demonstrated in experiment for optically-induced photonic lattices [16]. The self-trapping mechanism of such a multi-gap optical soliton is mainly associated with the index guiding due to the total internal reflection experienced by the strongest modes, which support much weaker guided modes in the Bragg-reflection gaps of the spectrum. However, from the general concept of the multi-gap solitons [10, 11, 12] it follows that such multi-component solitons can also exist in *self-defocusing nonlinear periodic media*, but this is only possible when all the modes are localized in the Bragg-reflection gaps of the lattice spectrum.

In this paper, we demonstrate numerically that partially incoherent multi-gap solitons can indeed exist in self-defocusing nonlinear media, and we suggest a simple approach for creating an input light field with special coherence properties, which allow for excitation of the modes in particular gaps, and highly efficient generation of incoherent multi-gap solitons.

## 2. Model and multi-gap optical solitons

Propagation of partially coherent light beams through a nonlinear medium with a slow response can be described by a nonlinear parabolic equation for the envelope  $A$  of the electric field,

$$i \frac{\partial A}{\partial z} + D \frac{\partial^2 A}{\partial x^2} + k_0^2 [v^2(x) + \delta n^2(I)] A = 0, \quad (1)$$

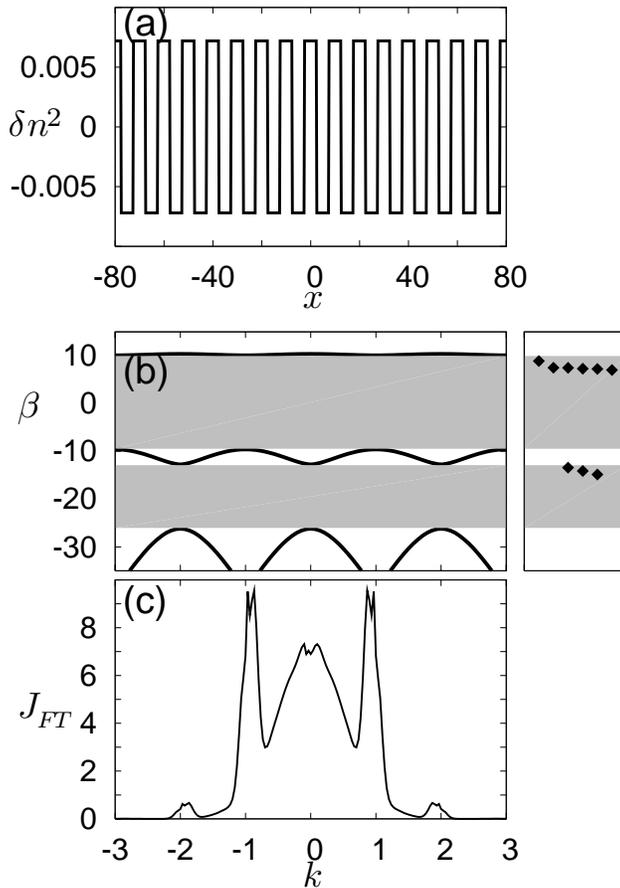


Fig. 1. (a) Refractive index and (b) structure of the bandgap spectrum of the photonic lattice. The first and second gap are shaded. Right panel shows the propagation constants of the different components of the multi-gap soliton shown in Fig. 2. Six of them lie in the first gap, and three in the second band-gap. The power spectrum of the soliton, found as a sum of Fourier power spectra of individual components, is shown in (c).

where  $z$  and  $x$  are the propagation and transverse coordinates, respectively,  $D = 1/(2k_0 n_e)$  is the diffraction coefficient,  $k_0$  is the vacuum wavenumber,  $n_e$  is the average refractive index of the unperturbed crystal,  $v(x)$  is the periodic modulation of the refractive index, and  $\delta n(I)$  is the nonlinearity-induced change of the refractive index that depend on the time-averaged light intensity  $I$ . We consider the case of slow *self-defocusing nonlinearity*,  $\delta^2 n(I) \simeq -\gamma I$ , where  $\gamma > 0$  is the nonlinear coefficient, and we neglect the effect of the nonlinearity saturation.

For a partially coherent light source, the phase of the light field envelope  $A(x, t)$  changes very rapidly (i.e. on a time scale much faster than the response time of the crystal). There exist several equivalent approaches to analyze such an incoherent light theoretically. The coherent density approach [17] and the self-consistent multimode theory [18] are based on the fact that incoherent light can be decomposed into the components,  $A(x, t) = \sum_m \phi_m(x) \exp[i\gamma_m(t)]$ , which are mutually incoherent due to the random phase factors  $\gamma_i(t)$ .

In this paper, we follow Ref. [15] and employ the self-consistent multimode theory to find

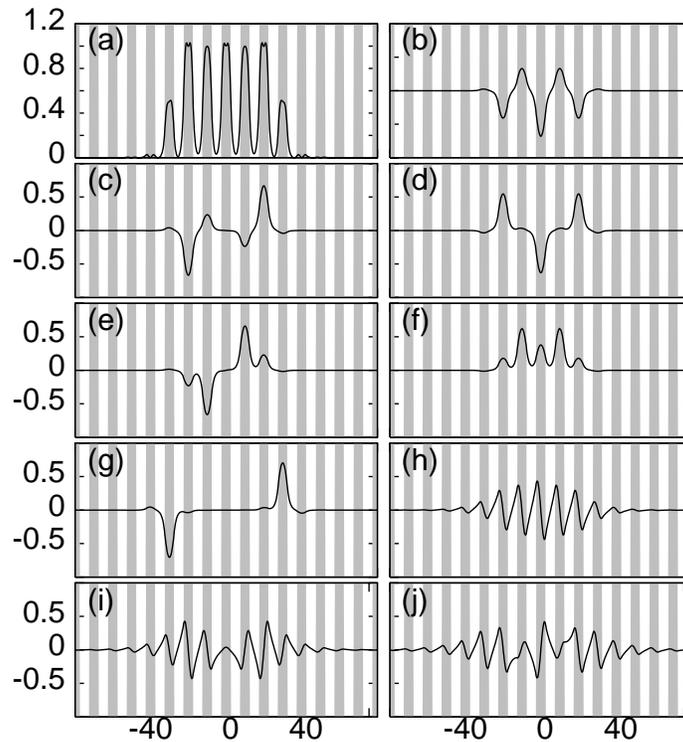


Fig. 2. The numerically obtained incoherent gap soliton. Shown are (a) the total soliton intensity, and its separate components in (b-g) the first gap and (h-j) second spectral gap.

multi-component solutions for incoherent gap solitons. The refractive index distribution associated with such a soliton is created by several localized modes. Because the soliton profile does not change under propagation, all the functions  $\phi_m(x)$  should correspond to the modes of the soliton-induced waveguide. These may include both fundamental and higher-order modes localized in multiple band-gaps. To find incoherent solitons numerically, we use the following procedure. First, we start with a refractive index distribution and calculate, by means of linear algebra, all localized and radiation modes of this refractive index distribution, using Eq. (1). Second, we construct the solutions based on a set number of lowest order localized modes in the first one or two Bragg-reflection band-gaps. We scale the mode amplitudes to match the chosen intensity and power levels, and calculate the time-average refractive index distribution for the multi-mode light field. Then, we repeat the calculations again until a self-consistent multi-component solution is reached with a certain accuracy.

For our study, we choose a step-like modulation of the refractive index with a period of  $10\mu\text{m}$  and varying in the value between  $v_+ = 7.20 \times 10^{-3}$  and  $v_- = -v_+$ . As for other parameters, we assume light of a vacuum wavelength of  $532\text{nm}$  and an unperturbed refractive index  $n_e = 2.3$ . The bandgap spectrum of such a periodic structure is shown in Fig. 1. Note that we choose a rather strong modulation of the refractive index ('a deep lattice'). This results in a considerable width of the first two band gaps. The third band gap, however, is still rather narrow and can only support weakly localized modes. In the following, we consider solitons based on strongly localized modes in the first and second gaps.

Using the self-consistent multimode theory, we find the incoherent gap solitons numerically. We were able to find solitary solutions containing only localized modes from the first gap, but

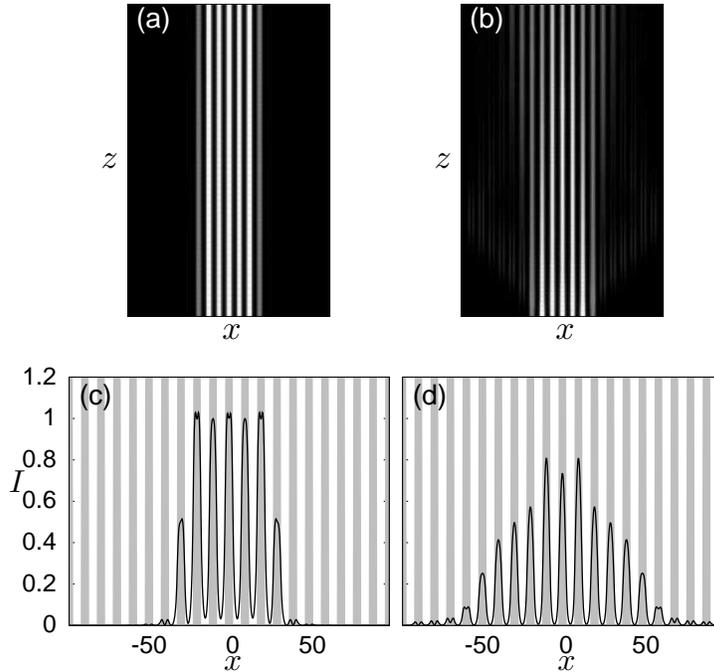


Fig. 3. (a,b) Propagation of an incoherent gap soliton for 20mm in the nonlinear and linear regimes, respectively. The corresponding output beam profiles are shown in (c,d).

also solutions where a considerable amount of the light intensity is trapped in the second gap. In Fig. 1, we show an example of that second type presented by the propagation constants of the localized modes that form the incoherent gap soliton. The soliton consists of six modes localized in the first gap and three modes localized in the second gap.

Figure 2 shows the profile of the incoherent gap soliton and the corresponding set of single modes that create the soliton. The components from the first gap [Figs. 2(b)-(g)] were chosen to have equal power, the components from the second spectral gap [Figs. 2(h)-(j)] were chosen to have 75% of that power. The power was chosen such that we obtained a soliton with a peak intensity of around 1. The resulting propagation constants are shown in Fig. 1(b). For components from the first gap [Figs. 2(b)-(g)], the corresponding propagation constants are 8.89284, 7.48452, 7.47812, 7.29089, 7.22568, 7.02095, and in the second gap [Figs. 2(h)-(j)] the values are -13.4242, -14.1175, -14.8541.

We notice two remarkable differences comparing to the case of *coherent gap solitons* studied earlier. The first feature of the incoherence is that the gap soliton is very broad with a rather flat top. The soliton has to be broad, otherwise it could not support so many localized modes, especially in the second bandgap where localized modes tend to have a large spatial extent. The second feature introduced by the incoherence is that there is some light intensity even in the regions of lower refractive index. Near the center of the soliton the lowest intensity in the lower index region is still about 3-4% of the maximum intensity of the soliton. Although this might be a small effect, it is still a qualitative difference to the coherent case, where the intensity inevitably drops to zero somewhere in the lower index region. We note that coherent gap solitons can form stable bound states for a particular phase difference [19], however the interacting properties of multi-mode gap solitons are expected to be more complex due to nontrivial effect of coherence [20].

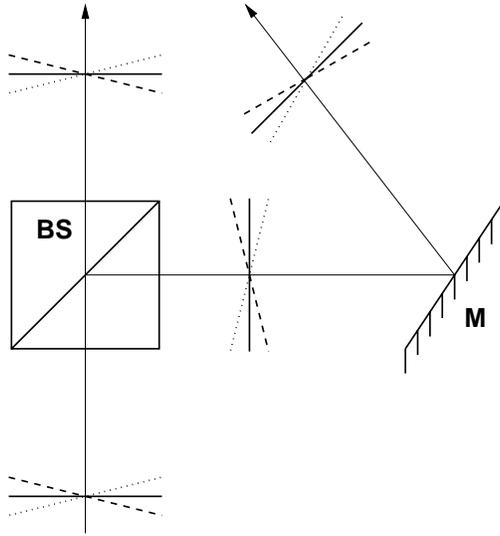


Fig. 4. Schematic for the generation of multi-gap solitons. First, an input partially incoherent beam splits into two beams by a beam splitter and then two beams are recombined at the input of the nonlinear periodic medium. Solid, dashed, and dotted lines stand for the mutually incoherent plane waves that partially incoherent light can be decomposed into. The plane waves interfere at the input face of the nonlinear medium.

In order to verify that the incoherent gap soliton is stable and that the nonlinearity is essential to keep it localized, we have numerically propagated the soliton, once with the nonlinearity switched off and once with the nonlinearity switched on, but in the presence of an initial perturbation. The results are summarized in Figs. 3(a-d). We observe that the initial perturbation does lead to internal oscillations of the soliton as it propagates, and the soliton retains its structure, demonstrating robustness under the action of perturbations. However, when the nonlinearity is switched off, we observe that diffraction sets in. Due to the deep lattice used for this example, the light only diffracts slowly. However, the length scales are still small enough to make possible experiments to check these numerical results seem feasible.

### 3. Generation of incoherent gap solitons

We now discuss the approaches which can be used to generate partially coherent gap solitons in self-defocusing nonlinear media. According to the coherent density method, a uniform incoherent light beam can be regarded as a superposition of many mutually incoherent plane waves that are all tilted at small angles with respect to each other. In order to realize the efficient generation of stationary multi-gap solitons, one has to selectively excite only the Floquet-Bloch modes corresponding to the upper gap edges, which experience *anomalous diffraction* in the periodic medium. Since the Bloch waves are composed of counter-propagating waves with a particular phase difference, their controlled excitation can be performed by a partially coherent input, where all pairs of waves propagating at the angles  $\alpha + \alpha_B$  and  $\alpha - \alpha_B$  are fully correlated, and form interference patterns matching the Bloch-wave profiles (here  $\alpha_B$  is the Bragg angle). The angle  $\alpha$  is introduced, because the interference patterns do not necessarily have to propagate parallel to the  $z$ -direction. Hence,  $\alpha$  is the angle between the direction of propagation of the interference pattern and the  $z$ -axis. Such angles need to be considered, since incoherent light can be most conveniently described mathematically as an incoherent superposition of fully

coherent components, all propagating in slightly different directions. These requirements can be satisfied by using a simple set-up illustrated in Fig. 4. First, a light beam is made partially incoherent, e.g. by passing a laser light through a rotating diffuser [21]. Then, the input beam is split into two beams, which are now fully correlated with respect to each other. If optical paths of these beams are made equal until they are incident on the crystal under the angles  $\theta$  and  $-\theta$ , and additionally the beams exhibit different number of reflections, then the components with the opposite propagation angles will be fully correlated, so that the light amplitude at the input face can be presented in the form

$$A(x, t) = \sum_j G(\alpha_j) e^{ik_0(z + \alpha_j x)} \cos[k_0 \theta (x - x_j)] e^{iY_j(t)}. \quad (2)$$

We thus have a superposition of interference patterns with the same lattice constant  $d_i = (k_0 \theta)^{-1}$  but propagating at slightly tilted angles  $\alpha_j$ . These patterns will excite Bloch waves at a particular band edge which symmetry is selected by a choice of the parameters  $d_i$  and  $x_i$ . The suggested generation scheme provides a generalization of the approach which was recently used for generating coherent spatial gap solitons [6, 9]. It is also reminiscent of experimental setup which was recently used to investigate the interaction between partially incoherent solitons [20]. Note, however, that in contrast to those experiments, the beams have to fully overlap on the input face of the medium in our case and that they have to do so under a certain angle.

To illustrate the generation of the gap solitons with multiple modes trapped in the first gap, we perform numerical simulations and select the following parameters:  $d_i = d/2$  and  $x_i = 0$ . The soliton dynamics is summarized in Fig. 5, where we use a shallow lattice ( $v_+ = -v_- = 1.2 \times 10^{-3}$ ) to make all effects more visible. The input and output (after 16mm) intensity profiles are presented in in Fig. 5(a). We observe highly efficient generation of immobile multi-gap soliton with minimal amount of radiation, in a sharp contrast to the case of a homogeneous self-defocusing medium.

What happens if only the power spectrum of the soliton is matched at the input, for example, by performing Fourier filtering of a single partially-coherent beam, whereas the phases of waves with opposite propagation angles remain uncorrelated? In this case, the Bloch waves corresponding to the top and bottom gap edges would be excited simultaneously, however they experience self-trapping or enhanced broadening in self-defocusing media, respectively.

We perform numerical simulations using the same overlapping beams at the input as in Fig. 5(b), but making counter-propagating waves *uncorrelated*, see Fig. 5(d). Although the power spectrum at the input remains almost the same, the interference fringes disappear, and the whole beam diffracts strongly due to self-defocusing of the modes at the bottom gap edge which carry about 50% of the beam power. We note that soliton generation may be observed under similar conditions in a very deep lattice, where the band-1 modes diffract much slower than the higher order bands, however substantial radiation losses may still occur.

On the other hand, if the input is made fully coherent, strong focusing is observed leading to the formation of a breathing state, as shown in Fig. 5(c). These results illustrate the importance to engineer the coherence properties of the input light for the efficient generation of gap solitons especially in shallow lattices, in contrast to the random-phase lattice solitons [16] where such engineering is not required.

#### 4. Conclusions

We have demonstrated that, in self-defocusing media with a periodically modulated refractive index, partially incoherent light can be trapped in the gaps of the structure bandgap spectrum, and it can even propagate in the form of bright incoherent gap solitons. The interplay between the nonlinearity and the lattice discreteness is essential for this to be possible. Furthermore, we

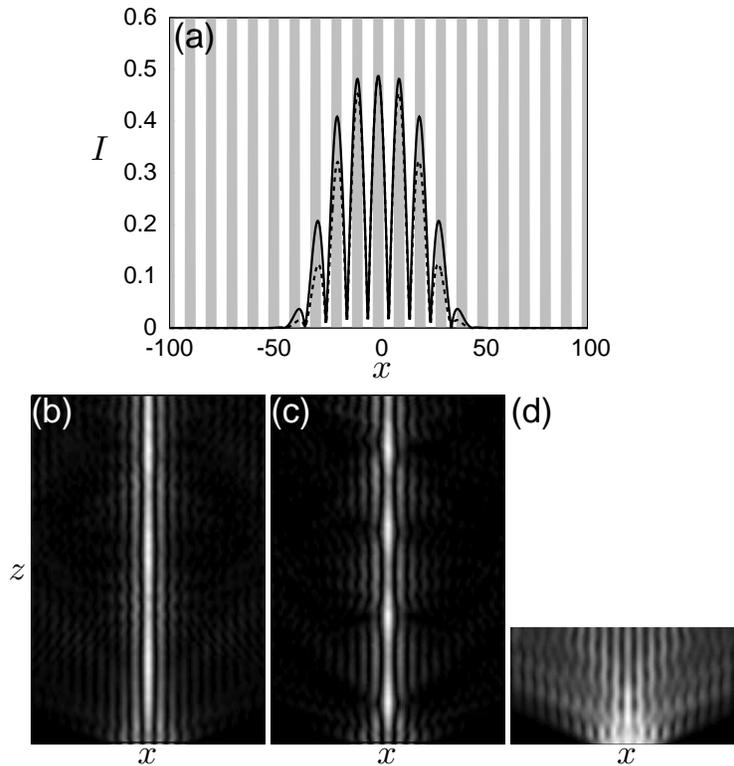


Fig. 5. Example of the generation of an incoherent multi-gap soliton. (a) Output (dashed) and input (solid) intensity profiles corresponding to case (b); (b-d) intensity patterns along the propagation direction for input excitation by inclined overlapping beams with the same profiles but different coherence properties: (b) partially incoherent, correlated counter-propagating Fourier components; (c) fully coherent; (d) partially incoherent, uncorrelated Fourier components.

have shown that it should be possible to generate such incoherent gap solitons in experiments by splitting an incoherent plane wave into two beams and then recombining these beams under an appropriate angle at the input of a nonlinear periodic medium.