How does an inclined holding beam affect discrete modulational instability and solitons in nonlinear cavities?

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Abstract: We study light propagation in arrays of weakly coupled nonlinear cavities driven by an inclined holding beam. We show analytically that both discreteness and inclination of the driving field can dramatically change the conditions for modulational instability in discrete nonlinear systems. We find numerically the families of resting and moving dissipative solitons for an arbitrary inclination angle of the driving field, both in the discrete and a quasi-continuous limits. We analyze a crossover between resting and moving cavity solitons, and also observe novel features in the soliton collision.

OCIS codes: (190.4420) Nonlinear optics, transverse effects in; (190.5530) Nonlinear optics: Pulse propagation and solitons; (190.5940) Self-action effects; (190.1450) Bistability.

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1. Introduction

The study of light propagation in nonlinear periodic structures has developed as an active research direction which allows for addressing the important issues of light control and switching in novel types of discretized systems supporting optical self-trapped beams (or optical solitons) [1]. Many linear and nonlinear effects, including diffraction management, modulational instability (MI), and discrete soliton formation, have been mainly studied for discrete Hamiltonian systems where the energy is conserved upon light propagation [1, 2].

On the other hand, self-localized states in *dissipative nonlinear systems* have attracted a special attention because they exhibit many novel and unique features introduced by a balance between linear/nonlinear gain and loss, which is additionally required to the effects of diffraction and nonlinearity [3]. These spatial dissipative optical solitons may exist in different transversely homogeneous nonlinear systems such as Fabry-Pérot cavities (see [4, 5] and for an overview [6]) and for unidirectional propagation [7, 8].

A paradigm for describing discrete nonlinear dissipative systems is the discrete cubic complex Ginzburg-Landau equation. For this model, MI was studied analytically [9]. Discrete dark and bright soliton solutions were found for the dissipative version of the discrete Ablowitz-Ladik model [10]. All soliton solutions of these models with a cubic nonlinearity turned out to be unstable. More realistic physical models, such as an array of coupled semiconductor lasers, exhibit saturation mechanisms resulting in additional stabilizing quintic terms. As a result, it was shown that discrete dissipative solitons can be stable in the discrete cubic-quintic Ginzburg-Landau model with both local [11] and nonlocal [12, 13] nonlinearities. The MI and pattern formation were investigated as well [14].

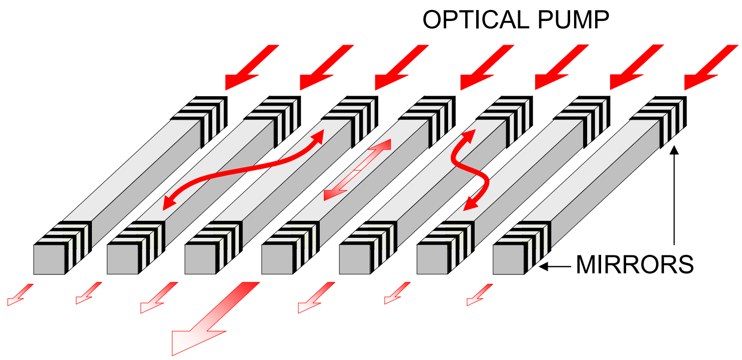

Fig. 1. Array of coupled waveguide cavities endowed with a Kerr nonlinearity.

Recently introduced and analyzed examples of *discrete dissipative solitons* are associated with the study of arrays of weakly coupled nonlinear optical cavities driven by an external coherent optical field, where *discrete cavity solitons* (DCSs) have been predicted for Kerr [15] and quadratic [16] nonlinear waveguide arrays in the simplest case of normal incidence of a wide holding beam. Beyond controllable diffraction the attractive feature of DCSs is the possibility to route and drag them externally into a desired position along the cavity array. For these aims, the mobility of DCSs, driven by an inclined holding beam, have been analyzed in the limit of strong coupling between adjacent waveguides [17].

In this paper, we provide a brief but comprehensive analysis of light localization and dynamics in arrays of weakly coupled *Kerr nonlinear cavities* driven by an inclined holding beam. The aim of our study is twofold. First, we analyze MI for an arbitrary inclination angle of the holding beam and provide a comparison of the respective domains and related properties of MI for dissipative and conservative discrete models. Second, we find numerically the families of resting and moving (bright and dark) DCSs, and employ a quasi-continuous model, which accounts for discrete diffraction in terms of a Taylor expansion, to provide a deeper physical insight into the properties of *moving discrete dissipative solitons*. We then study briefly collisions between resting and moving cavity solitons which may exist simultaneously in this system.

The paper is organized as follows. In Sec. 2, following our earlier studies [15], we introduce the model describing light dynamics in an array of coupled waveguide cavities. The analysis of MI in this model is presented in Sec. 3 for an arbitrary inclination of the holding beam. Sections 4 and 5 are devoted to the study of resting and moving DCSs, respectively. In Sec. 6 we discuss briefly novel features observed in collisions between coexisting resting and moving DCSs. Finally, Sec. 7 concludes the paper.

2. Discrete model

We consider an array of weakly coupled nonlinear waveguides where mirrors at the input and output facets backfold the light path, thus forming an array of coupled-waveguide resonators, which is excited by an external driving field (see Fig. 1).

We assume that the operating frequency is close to a resonance of identical high finesse cavities and that the nonlinearly-induced field variation is small per round trip. Then a mean-field approach can be applied (for details, see [15]). The appropriately scaled evolution equations for the slowly varying envelopes read as

$$i\frac{\partial u_n}{\partial T}+C(u_{n+1}+u_{n-1}-2u_n)+(i+\Delta)u_n+\gamma|u_n|^2u_n=E_0e^{iqn}. \tag{1}$$

In Eq.(1) all quantities are dimensionless where the evolution time are scaled by the photon lifetime and the field amplitudes by the effective nonlinear coefficient. Now Δ accounts for the detuning from the cavity resonance, C for the next-neighbor evanescent coupling between cavities, T for time and γ for the sign of nonlinearity ($+1$ for the focusing and -1 for the self-defocusing case, respectively). We also allow for an inclination of the holding beam with the amplitude E_0 and introduce the phase shift q between the field at adjacent cavity inputs.

Equation (1) exhibits a stationary plane wave (PW) solution, $u_n = b\exp(iqn)$. The external phase gradient q can be absorbed in effective detuning as $\Delta' = \Delta + 2C(\cos q - 1)$. Then the PW amplitude does not depend on q provided that the effective detuning Δ' remains constant. The PW amplitude b can be easily obtained from the corresponding cubic algebraic equation [18]. For $\gamma\Delta' \geq -\sqrt{3}$, the steady-state solution is single-valued, whereas for $\gamma\Delta' < -\sqrt{3}$, we observe the well-known hysteresis-like dependencies and multistability [see Fig. 2(a)].

3. Modulational instability in discrete dissipative systems

Modulational instability in nonlinear optical systems manifests itself in a growth of periodically modulated perturbations, and in continuous media it is responsible for a breakup of broad beams into multiple filaments under the action of self-focusing nonlinearity [1]. In periodic systems as e.g. waveguide arrays, the extended states are Floquet-Bloch modes, which can experience both normal and anomalous effective diffraction depending on the band structure [2]. The specific properties of MI in discrete systems were analyzed for an arbitrary phase difference between adjcent guides for both the discrete Klein-Gordon model and nonlinear Schrödinger equation [19, 20, 21]. It was shown that discreteness can drastically change the conditions for MI, e.g., at small wave numbers a nonlinear carrier wave becomes unstable to all possible modulations provided the wave amplitude exceeds a certain threshold value. In contrast to Kerr nonlinear waveguide arrays, a saturable nonlinearity bounds the existence domain for discrete MI and decreases both the MI gain and the critical spatial frequency of perturbation [22]. Theoretical studies of MI in discrete lattices have been confirmed experimentally for waveguide arrays with Kerr [23] and quadratic [24] nonlinearity as well as for a photovoltaic defocusing nonlinearity [25].

MI is often related to the formation of a train of equally spaced solitons, with the soliton spacing being inversely proportional to the spatial frequency of the highest MI gain [1]. In particular, in a discrete system the development of MI can lead to the formation of periodic arrays of weakly interacting discrete solitons [26, 27].

In order to probe the stability of plane waves $u_n = b\exp(iqn)$, we study the evolution of small perturbations with amplitude a and wavenumber Q and look for solutions in the form,

$$u_n(T) = \left(b + a\mathrm{e}^{\lambda T + iQn}\right)\mathrm{e}^{iqn}. \tag{2}$$

Substituting the ansatz (2) and the corresponding expression for $u_n^*(T)$ into Eq. (1) and neglecting all nonlinear terms in the perturbation amplitude a, we obtain a system of coupled algebraic equations for a and a^*. The solvability conditions define the eigenvalue $\lambda(Q,q)$:

$$\lambda(Q,q) = -1 \pm i\sqrt{(\Lambda+\gamma|b|^2)(\Lambda+3\gamma|b|^2)} - 2iC\sin Q\sin q, \tag{3}$$

where the abbreviation: $\Lambda \equiv 2C(\cos Q\cos q - 1) + \Delta$ was used. The PW solution becomes unstable if the real part of any eigenvalue $\lambda(Q,q)$ is positive. Therefore, the stability boundary is

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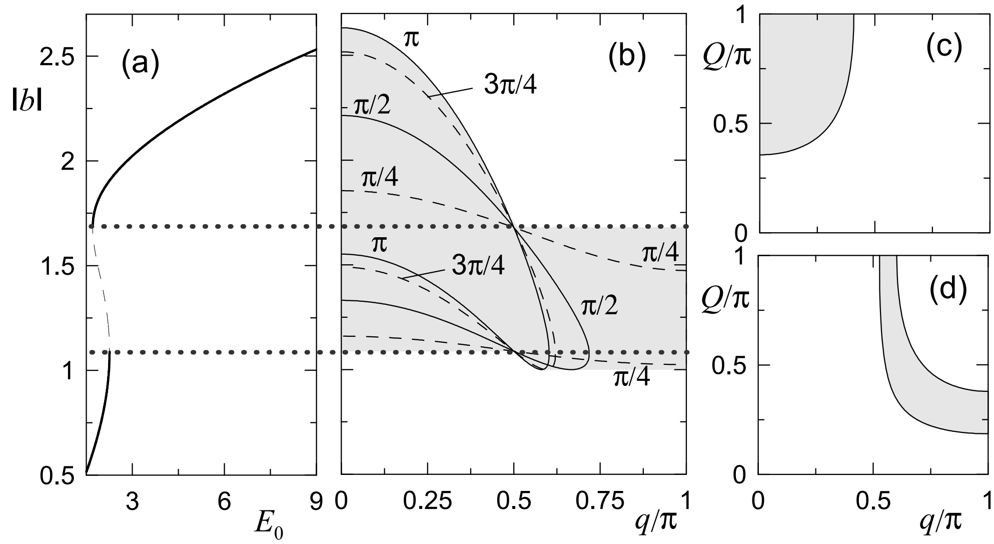

Fig. 2. (a) Modulus $|b|$ of the plane wave amplitude vs. the holding beam amplitude E_0 (the dotted section indicates homogeneous instability - $Q = 0$); (b) modulationally unstable plane wave domains in $|b| - q$ parameters space; shaded regions between the full and dashed lines depict MI domains for various Q values, whereas the dotted lines mark the boundaries of homogenous instability. Regions of MI in $q - Q$ parameter space for (c) the upper ($|b| = 2$) and (d) the lower ($|b| = 1.05$) branch; parameters: $\Delta' = -3$, $C = 1$.

defined as the point where the real part of the eigenvalue vanishes. Using this marginal stability condition and solving the quadratic equation for $|b|^2$, we find the interval of the PW amplitudes, $|b_2|^2 < |b|^2 < |b_1|^2$, for which the PW solution is unstable, where

$$|b_{1,2}|^2 = \frac{1}{3\gamma}\left(-2\Lambda \pm \sqrt{\Lambda^2 - 3}\right). \tag{4}$$

First, to analyze MI we exclude the PW solutions that are unstable against homogeneous perturbations ($Q = 0$); these solutions correspond to the portion of the branch with a negative slope in Fig. 2(a). According to Eq. (4), these instability boundaries coincide with the turning points of multistable PW solutions for $Q = 0$ [see Fig. 2(b)].

The spatial modulation of the PW solution ($Q \neq 0$) results in an additional resonance detuning which can lead to the resonant amplification of periodic waves on the modulationally unstable background. The boundary of MI depends strongly on the inclination of the holding beam q. Indeed, while at normal incidence ($q = 0$) and for $\gamma = +1$ effective self-focusing appears out-of-phase pumping ($q = \pi$) evokes an effective self-defocusing because discrete diffractions changes the sign at $q = \pi/2$ [28]. It is clear that in both regimes MI, pattern formation, and the formation of localized solutions will differ. For normal diffraction ($q < \pi/2$) the MI boundaries shift towards larger values of the PW amplitude $|b|$, where the maximal shift occurs for an "out-of-phase" (or staggered) perturbation ($Q = \pi$) [see Fig. 2(b)]. Therefore, the upper branch of the PW solutions is modulationally unstable for the effective self-focusing regime ($q < \pi/2$). In the strong-coupling limit ($C \gg 1$) and for a normally incident holding beam ($q = 0$) this finding coincides with the well-known results for the continuous one-dimensional nonlinear cavity [18]. Unlike the continuous case the upper branch becomes stable for large pump amplitudes in the discrete model [Fig. 2(b)].

Discrete diffraction changes its sign at $q = \pi/2$. As a result, the lower branch of the PW

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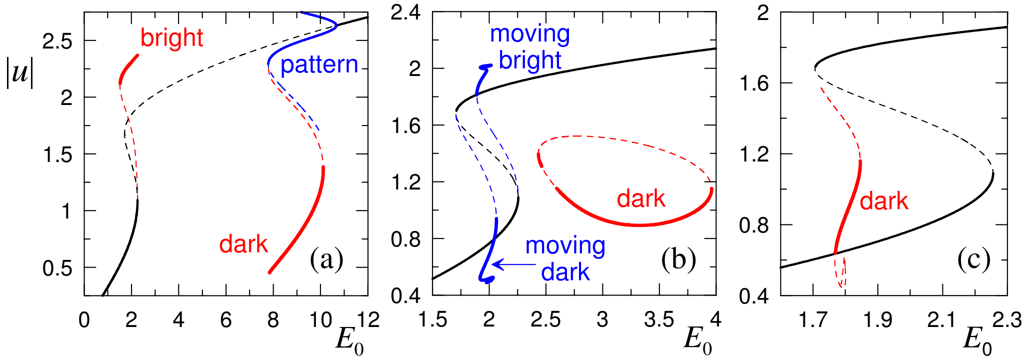

Fig. 3. Families of discrete cavity solitons shown as the modulus of the field amplitude at the center $|u|$ vs. the amplitude of the holding beam E_0 for different phase shifts (holding beam inclination) q: (a) $q = 0$, (b) $q = \pi/2$, and (c) $q = \pi$. Plane wave solutions are shown by black lines where the soliton branches emanate subcritically. The solid parts of these branches correspond to stable and the dashed ones to unstable solitons; parameters: $\Delta' = -3$ and $C = 1$.

solution becomes modulationally unstable. At the marginal point $q = \pi/2$ the MI domain disappears completely because for $u_n = b\exp(iqn)$ the coupling term $(u_{n+1} + u_{n-1})$ vanishes [see Eq. (1)]. The MI regions are presented in Figs. 2(c) and 2(d) for an arbitrary amplitude at the upper and lower branch of Fig. 2(a), respectively. Unlike the conservative case [19, 20] the PW solutions are stable for long-wavelength perturbations ($Q << 1$) in the dissipative case, except the branch with the negative slope which remains always unstable.

4. Resting discrete cavity solitons

In a previous paper we have shown that for normal incidence of the holding beam ($q = 0$), the discrete model (1) gives rise to a family of stable *bright DCS* solutions emanating from the stable lower branch of the PW solutions [15]. These DCSs bifurcate from the turning point of the bistable PW branch, as shown in Fig. 3. *The important question is whether stable DCSs exist also for an inclined holding beam.* In continuous dissipative systems, solitons start moving as soon as a small inclined driving field is applied [29]. In discrete systems, solitons are usually trapped by discreteness (lack of translational symmetry and the respective neutral Goldstone mode) and remain at rest unless the driving force exceeds some critical value. This critical value was found in the limit of strong coupling and it was shown to be inversely proportional to the coupling parameter [17]. In this paper, we find a family of resting bright DCSs for moderate coupling depending on the inclination parameter [see Fig. 4(a) bottom]. For the selected parameters the DCS is localized at a few array sites only [see Fig. 4(b)] and, therefore, effects of discreteness play a major role. This type of stationary solutions exists below a critical inclination ($q \approx 0.2\pi$).

The PW solution becomes again stable for larger amplitudes of the holding beam. For this case, we expect bifurcations of periodical patterns or localized solutions [30]. We find a staggered pattern emanating supercritically from this boundary of MI [see Fig. 3(a) top]. The unstable mode at the bifurcation point ($Q = \pi$) determines the period and staggered structure of this pattern. We note that staggered dark DCSs can bifurcate from this boundary of MI in quadratically nonlinear cavity arrays [16]. In contrast to that case, stable dark DCSs in Kerr media originate from the turning point of the pattern [see the branch of dark solitons in Fig. 3(a)]. Unlike other localized modes which exist on a stable PW background, this dark DCS is ”frozen”

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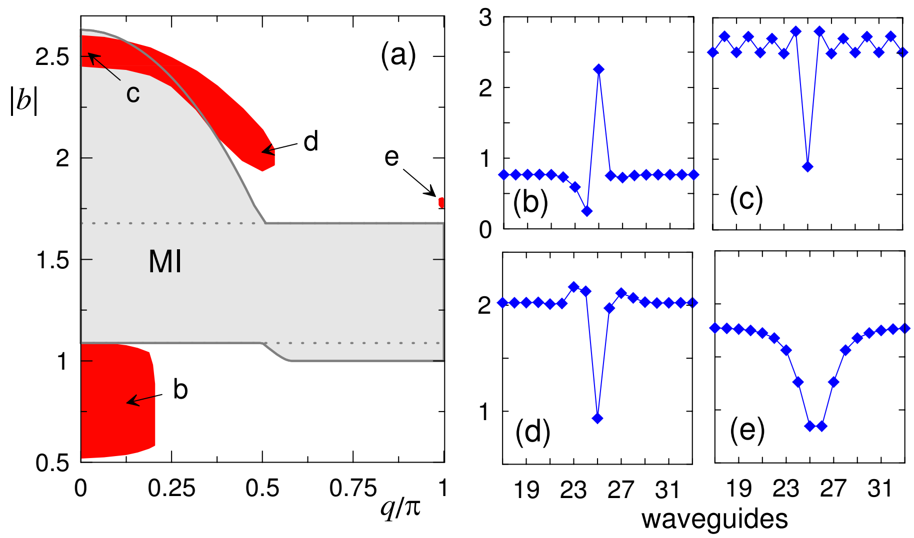

Fig. 4. (a) Existence domains of stable resting DCSs (red areas) in $q - |b|$ (modulus of background amplitude) parameter space. Arrows point at the site where the respective DCS shown in the right panel exist. Profiles of stable resting DCSs: (b) Bright DCS for $E_0 = 2$, $q = 0.12\pi$, (c) dark DCS for $E_0 = 9.4$, $q = 0$; (d) dark DCS for $E_0 = 3$, $q = \pi/2$; and (e) dark DCS for $E_0 = 1.8$, $q = \pi$. Other parameters are: $\Delta' = -3$ and $C = 1$. The shaded area displays the MI domain and the dotted lines mark the limits of homogeneous instability.

into a stable periodic pattern [Fig. 4(c)]. This type of resting DCS exists up to a large inclination of the holding beam ($q \approx 0.53\pi$), i.e. even entering the regime of effective self-defocusing [Fig. 4(a)]. The qualitative difference is that in this case the PW solution is stable; therefore, dark DCSs exist on the stable homogeneous background [Fig. 4(d)]. The soliton branch is closed [see ”dark” branch in Fig. 3(b)], and it does not have any connection with PW solutions, as it would happen for usual cavity solitons [6].

According to soliton theory in continuous systems [1], the self-defocusing regime is usually associated with the existence of dark solitons. Indeed, the branch of dark DCSs emanates from the upper limiting point of bistable PW solutions for the out-of-phase pumping of the array [Fig. 3(c)]. As it was shown above, the upper branch is stable and such solitons do exist, whereas the lower branch of the PW solutions is modulationally unstable [Fig. 2(b)]. However, this stationary solution disappears for small deviations of the inclination parameter q from the stationary value ($q = \pi$), at least for the set of parameters studied here. This particular dark DCS [Fig. 4(e)] is broader than other solitons considered above, therefore, discreteness effects such as trapping in a Peierls-Nabarro potential [27] are weaker.

5. Moving discrete cavity solitons

Resting DCSs can start moving for increasing inclination q of the holding beam. In continuous systems, the transverse motion of localized solutions is associated with translational symmetry or the existence of a zero frequency Goldstone mode [29].

In order to facilitate the physical understanding first we derive a quasi-continuous model which retains the peculiarities of discrete diffraction (this is the decisive difference to a simple continuous model) and can be employed for describing the motion of wide DCSs. Thus, this quasi-continuous model decribes with a reasonable accuracy the field evolution in a dis-

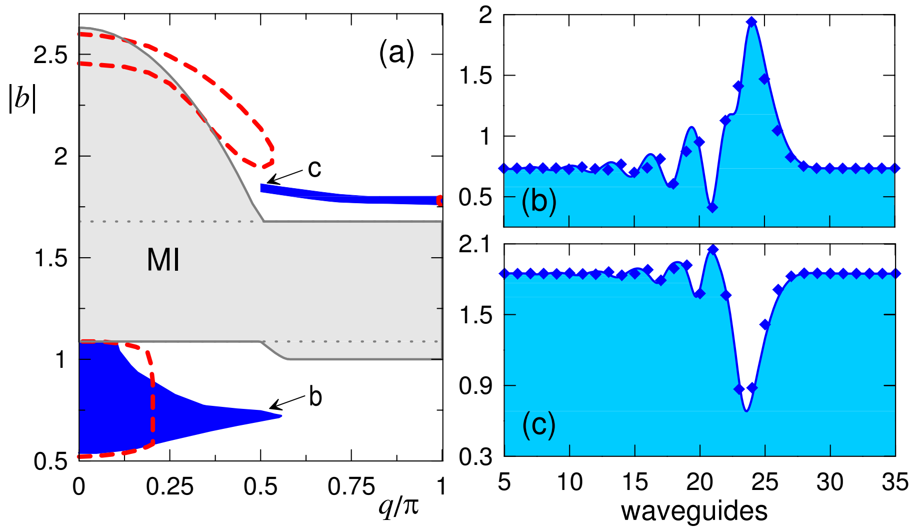

Fig. 5. (a) Existence domains of moving DCSs (blue areas) in $q-|b|$ parameter space (dashed lines encircle domains of resting DCSs). Arrows point to the sites where DCSs shown on the right exist. (b,c) Profiles of bright (b) and dark moving DCSs (c) for the inclination parameter $q=\pi/2$, calculated both in the quasi-continuous (solid) and discrete models (diamonds). A movie (2.1MB) shows propagation of bright DCS (b). Other parameters are: $E_0 = 1.95$, $\Delta' = -3$, and $C = 1$

crete system for any inclination but with a sufficient width of the beam. We introduce a new transverse coordinate x and the continuous function $u(x)$, which describes the envelope of the discrete solution of Eq. (1) with the central spatial frequency q: $u(x=nh)\exp(iqn) \equiv u_n$, and expand the functions $u_{n\pm1}$ into Taylor series up to the third order. As a consequence we may replace the set of ordinary equations (1) by an effective partial differential equation,

$$i\frac{\partial u}{\partial T} + iD^{(1)}\frac{\partial u}{\partial x} + D^{(2)}\frac{\partial^2 u}{\partial x^2} + iD^{(3)}\frac{\partial^3 u}{\partial x^3} + (i+\Delta')u + \gamma|u|^2 u = E_0, \tag{5}$$

where $D^{(1)} = 2Ch\sin q$, $D^{(2)} = Ch^2\cos q$, and $D^{(3)} = \frac{1}{3}Ch^3\sin q$.

We numerically solve Eq. (5) for different inclinations q. For normal incidence ($q=0$ and $D^{(1)}, D^{(3)} = 0$) it is obvious that this model must collapse to the ordinary continuous one (mean field equation for a Kerr cavity) [18]. The upper S-shaped branch of PW solutions is modulationally unstable allowing for pattern formation whereas the stable lower branch is appropriate for the bright CSs background. DCSs of the quasi-continuous model (5) bifurcate from the limiting point of bistable PW solutions. Such CSs start moving for any small field inclination due to the translational symmetry of the quasi- continuous model. Therefore, we look for stationary solutions of Eq. (5) in a coordinate system moving with a constant but unknown velocity, which itself is an eigenvalue of our stationary problem. Moving bright DCSs exist up to $q \approx 0.56\pi$ in the quasi-continuous limit [Fig. 5(a), blue area below MI domain] whereas moving dark DCSs exist in the effective self-defocusing regime ($\pi/2 < q < \pi$) where the upper PW branch becomes modulationally stable [Fig. 5(a), blue area above MI domain].

As mentioned above, although the model (5) is continuous, its soliton solutions substantially differ from those in a 1D nonlinear Kerr cavity [cf. Eq. (5) with the basic equation of Ref. [18]]. For example, the second-order diffraction completely disappears in the center of the Brillioun zone ($q=\pi/2$) where the first- and third-order diffraction terms define the DCS profiles. As a

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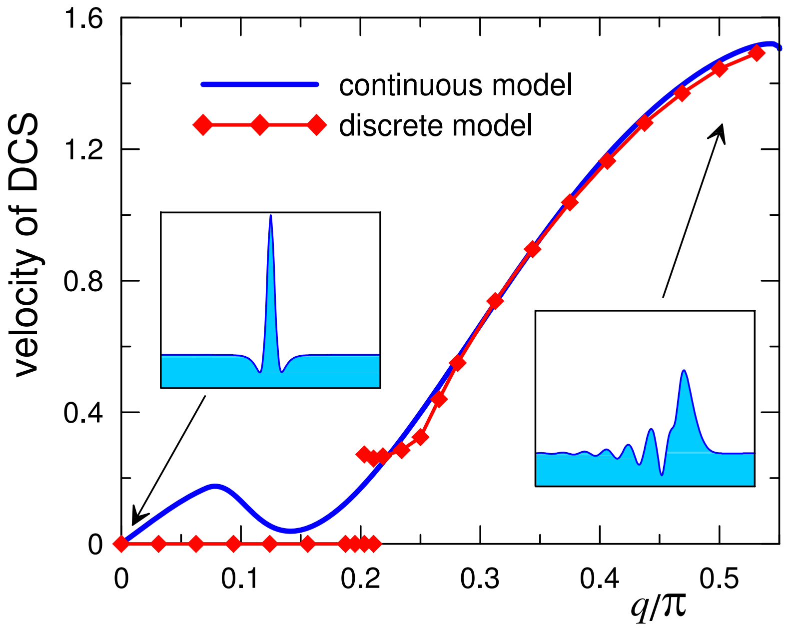

Fig. 6. Dependence of the velocity of bright DCS vs. the inclination q of the holding beam, calculated for the effective continuous model (5) (solid) and discrete model (1) (rhombuses). Parameters are: $E_0 = 1.92$, $\Delta' = -3$, and $C = 1$. Inserts show the soliton profiles.

result, the PW solution becomes modulationally stable. Moreover, both bright and dark DCSs can be found for the same set of system parameters. Moving bright DCSs belong to the branch which emanates subcritically from the lower limit point of bistable PW solutions, whereas moving dark DCSs bifurcate from the upper PW solution [see Fig. 3(b)]. In Fig. 5(b,c) we compare the profiles of DCSs obtained from the discrete (1) and quasi-continuous (5) models. We find a very good agreement of moving DCSs of both models. Both bright and dark soliton branches are multistable. We note that a special case of such solitons, so-called ”midband” solitons, with a spatial spectrum centered at the zero-diffraction point was recently reported for lasers with saturable absorber in which the refractive index is laterally modulated [31].

6. Collisions between discrete cavity solitons

The quasi-continuous model (5) correctly describes the profiles and velocities of moving DCSs for sufficiently large inclination of the holding beam (see Fig. 6). Our studies reveal that discreteness essentially affects those DCSs the transverse kinetic energy of which compares to or is less than the Peierls-Nabarro barrier. Therefore, slowly moving DCSs can be trapped at a relatively small inclination of the holding beam (Fig. 6, $q < 0.2\pi$, note the difference between the quasi-continuous and discrete models). Thus, we expect that in the discrete system DCSs start moving only for an inclination beyond some critical value. More importantly, both moving and resting bright DCSs may exist in the same region of system parameters because their existence domains overlap (see Fig. 6). Both DCS types can be excited by appropriately inclined holding beams. Therefore, for some inclination one DCS can be at rest whereas the other one moves. Figure 7 shows an example of collisions between these two DCSs. We numerically simulate that case where the resting soliton, trapped by the array, behaves like a barrier that stops the moving DCSs creating a multiple-hump DCS with additionally peaks.

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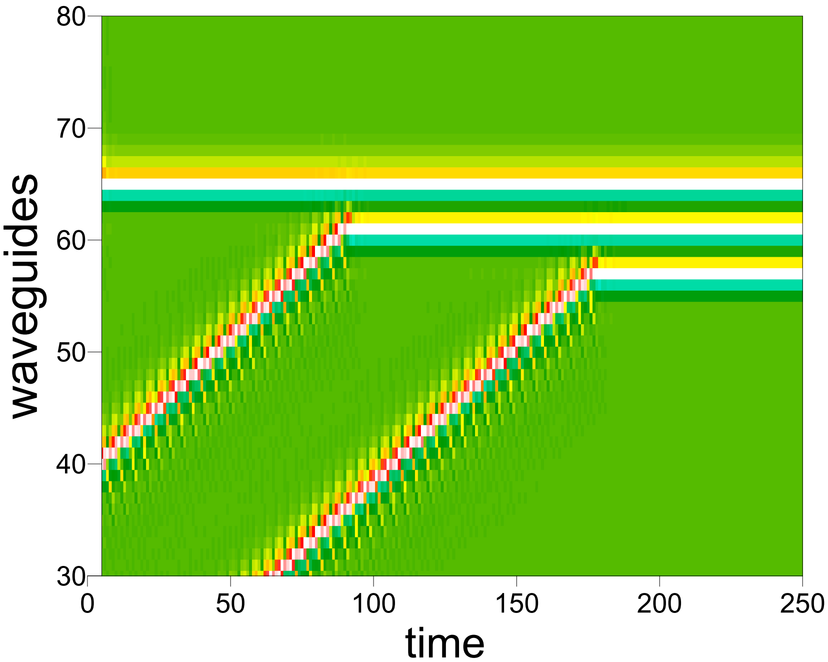

Fig. 7. A movie (2.5MB) shows collision between a resting and two moving DCSs and the formation of multi-soliton bound state. Parameters are: $q = 0.204\pi$, $E_0 = 1.92$, $\Delta' = -3$, and $C = 1$.

7. Conclusions

We have studied the dissipative dynamics of light in arrays of weakly coupled optical cavities endowed with a Kerr nonlinearity and driven by an inclined holding beam. We have analyzed modulational instability of discrete plane waves in such a discrete system for an arbitrary inclination angle of the holding beam, and identified substantial differences between the results for dissipative and conservative discrete systems. In particular, we have found that modulational instability disappears for some intermediate values of the inclination angle of the holding beam. We have also numerically found different families of resting and moving (bright and dark) discrete cavity solitons, including those which do not have their counterparts in continuous dissipative models. We have described a crossover between the resting and moving solitons and employed a quasi-continuous model with higher-order diffraction to analyze their properties. We have demonstrated that both resting and moving solitons may coexist in the same parameter domain, and we discussed novel features of their inelastic collisions.

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