

Bloch oscillations in chirped layered structures with metamaterials

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Abstract: We analyze the Bloch oscillations of electromagnetic waves in chirped layered structures with alternating layers of negative-index metamaterial and conventional dielectric under the condition of the zero average refractive index. We consider the case when the chirp is introduced by varying the thickness of the layers linearly across the structure. We demonstrate that such structures can support three different types of the Bloch oscillations for electromagnetic waves associated with either propagating or evanescent guided modes. In particular, we predict a novel type of the Bloch oscillations associated with coupling between surface waves excited at the interfaces separating the layers of negative-index metamaterial and the layers of the conventional dielectric.

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1. Introduction

Periodic oscillations of electrons induced by a constant electric field were predicted by Bloch in 1928 [1]. Such Bloch oscillations become possible due to beating of the localized eigenmodes of the structure corresponding to the equidistant eigenstates of the spectrum known as the Wannier-Stark ladder [2]. Experimental verification of the theory was impossible at that time, since dephasing time of electrons in crystals is shorter than the period of the electron Bloch oscillation. Later, the electron Bloch oscillations were observed in semiconductor superlattices [3] for which the period was reduced due to mini-bands of fabricated periodic structures.

Dephasing processes for electromagnetic waves are negligible, and this makes the observation of the Bloch oscillations in photonic systems much easier. The experimental observations of optical Wannier-Stark ladder and Bloch oscillations of electromagnetic waves were reported in Refs. [4, 5, 6] for linearly chirped Bragg gratings and linearly growing effective refractive index. Later, several groups reported both theoretical and experimental studies of the Bloch oscillations of electromagnetic waves in various photonic structures [7, 8, 9, 10].

Recent experimental realization of negative-index (or left-handed) metamaterials [11] has opened up many unique opportunities to explore novel effects of the electromagnetic wave propagation in the structures with negative refractive index. In this paper we study, for the first time to our knowledge, the Bloch oscillations of electromagnetic waves in linearly chirped layered structures composed of alternating layers of negative-index and conventional dielectric materials. We choose the material parameters in such a way that the average refractive index \bar{n} across pair of the neighboring layers vanishes, thus fulfilling the condition for the existence of a novel type of the specific zero- \bar{n} bandgap [12, 13]. To model the chirp, we change the layer thickness linearly in the structure and observe an analogue of the Wannier-Stark ladder in the eigenmode spectrum of electromagnetic waves, and the corresponding Bloch oscillations in the resonant transmission bands. We reveal that in such layered structures the Bloch oscillations of electromagnetic waves can be observed in the three different regimes. Compared to the optical Bloch oscillations in conventional dielectric structures, the structures with metamaterial can support a novel type of the Bloch oscillations associated with coupling of surface waves at the interfaces between negative-index and conventional dielectric layers.

2. Structure and bandgap diagram

We study one-dimensional chirped layered structures shown schematically in Fig. 1, where the slabs of a negative-index metamaterial (with the width b_i) are separated by the layers of the conventional dielectric (with the width a_i). Variation of the refractive index in the i -th pair of layers can be described as follows:

$$n(z) = \begin{cases} n_r = \sqrt{\epsilon_r \mu_r} & z \in (z_i, z_i + a_i) \\ n_l = -\sqrt{\epsilon_l \mu_l} & z \in (z_i + a_i, z_i + \Lambda_i) \end{cases} \quad (1)$$

where n_l and n_r are the refractive indices of metamaterial and dielectric, respectively. We consider TE-polarized waves with the electric field having one component $\mathbf{E} = (E_x, 0, 0)$, and the

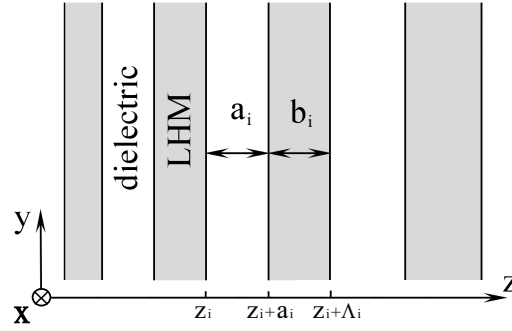


Fig. 1. Schematic of linearly chirped one-dimensional structure with alternating layers of negative-index metamaterial and conventional dielectric.

waves propagating in the plane (y, z) . In this case, the field distribution can be described by the scalar Helmholtz equation:

$$\Delta_2 E_x(y, z) + n^2(z) E_x(y, z) - \frac{1}{\mu} \frac{d\mu(z)}{dz} \frac{\partial E_x(y, z)}{\partial z} = 0, \quad (2)$$

where Δ_2 is the two-dimensional Laplacian, and the coordinates are normalized to c/ω .

First, we assume that the structure is periodic, i.e. no linear chirp is introduced. In this case, the electric field can be represented as a superposition of Bloch eigenmodes [14], with the electric field envelopes $U(z + \Lambda) = U(z)$, where Λ is the structure period. The dispersion relation for the Bloch waves can be found by the transfer matrix method [14],

$$2 \cos(K_B \Lambda) = 2 \cos(k_{zr} a) \cos(k_{zl} b) - \left(\frac{k_{zl} \mu_r}{k_{zr} \mu_l} + \frac{k_{zr} \mu_l}{k_{zl} \mu_r} \right) \sin(k_{zr} a) \sin(k_{zl} b), \quad (3)$$

where K_B is the Bloch wavenumber, $k_{z,l,r} = \mp \sqrt{n_{l,r}^2 - k_y^2}$ and k_y is the normalized propagation constant along the y axis. According to this relation an infinite layered structure containing metamaterials exhibits a non-resonant gap for $\bar{n} \equiv \Lambda^{-1} \int_0^\Lambda n(z) dz = 0$, where \bar{n} is the average refractive index of the structure, i.e. $n_r a = |n_l b|$. This condition is easy to fulfil for the metamaterials with the negative refractive index.

The bandgap diagram of the layered structure is shown in Fig. 2 for the parameter plane (Λ, k_y) . Here we assume that dielectric is vacuum, $\epsilon_r = \mu_r = 1$, and that it is two times thicker than the second layer, $a/b = 2$. We choose the parameters of the left-handed media as follows: $\epsilon_l = -5$ and $\mu_l = -0.8$. This set of parameters allows surface waves to exist at the interfaces between metamaterial and vacuum [15]. As follows from Fig. 2, for the zero \bar{n} structure the bandgap spectrum differs substantially from the case of conventional periodic structures made of conventional dielectrics [14]. Stack with the average zero refractive index possesses a complete gap with the transmission resonances [12, 13] when the optical path of the wave in either layer of the period coincides with a half of the wavelength in the corresponding medium. Thus for the normal incidence ($k_y = 0$) transmission is observed only when $n_r a = |n_l b| = \pi m$, where m is integer. For the slabs of equal thickness the regions of the transmission resonances in (Λ, k_y) plane degenerate into infinitely thin lines.

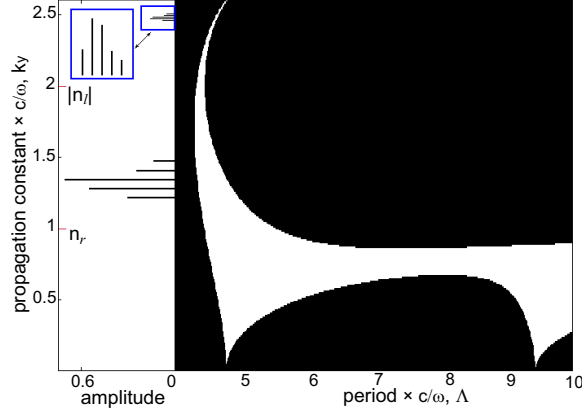


Fig. 2. Bandgap diagram for the TE-polarized waves in structure with alternating dielectric ($\epsilon_r = \mu_r = 1$) and metamaterial ($\epsilon_l = -5$, $\mu_l = -0.8$) in case of zero average refractive index. Black and white areas correspond to gaps and bands, respectively. Two spectra of the excited Bloch oscillations are shown on the left. The inset shows a magnified part of the spectrum.

3. Three types of Bloch oscillations

We study the propagation of electromagnetic waves in the layered structure shown in Fig. 1 with zero average refractive index in each pair of layers. We assume that the thickness of layers is chirped linearly, i.e. $\Lambda_q = \Lambda_0 + q\delta\Lambda$, where integer q numerates the layers. We are looking for localized electromagnetic waves in the structure, and for numerical simulations we consider a finite stack of the layers with perfect metal boundary conditions, $E(z=0) = E(z=L) = 0$, where L is the total length of the structure. We start from the Gaussian field distribution in the plane $y=0$ across the layers, and in order to find the electromagnetic field distribution in the whole stack we look for its eigenmodes by solving the Helmholtz equation (2). Then we decompose the initial field using the eigenmode basis and find the solution in the structure.

To find the eigenmodes of Eq. (2), we employ the following discretization scheme [16]:

$$\frac{2}{\mu_{m-1}^{-1} + \mu_{m+1}^{-1}} \left[\frac{U_{m+1} - U_m}{\mu_{m+1}} - \frac{U_m - U_{m-1}}{\mu_{m-1}} \right] \frac{1}{h^2} + \frac{\epsilon_{m-1} + \epsilon_{m+1}}{\mu_{m-1}^{-1} + \mu_{m+1}^{-1}} U_m = k_y^2 U_m, \quad (4)$$

where $z_m = mh$ are the mesh points with the discretization step h and $E_x(y, z) = U(z)e^{-jk_y y}$. Such a discretization scheme provides an algorithm convergence [16], and it avoids excitation of spurious modes in the structure.

In negative-index metamaterials the energy flow, defined as $\int [\mathbf{E} \times \mathbf{H}] dz$, can become negative, i.e. the energy can propagate in the opposite direction to the propagation constant \mathbf{k}_y [17]. Consequently, we determine the direction of the energy flow of each eigenmode and choose the sign of the propagation constant such that the energy goes in the positive y -direction. Decomposition of the initial condition in the plane $y=0$ in the eigenmode basis is made using the least squares method.

To find the values of the propagation constant k_y which correspond to the Bloch oscillations, we analyze the spectrum of eigenvalues of electromagnetic waves which can propagate in this layered structure. The Bloch oscillations are expected to appear in the regions of spectrum where the spectrum is equidistant. Practically for all gradients of a linear ramp we observe several sets of equidistant states. The equidistant eigenvalues of k_y correspond to a spatial optical

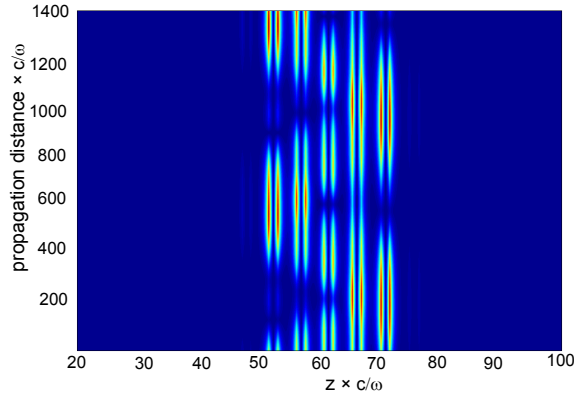


Fig. 3. Field distribution in the case of surface-wave-assisted Bloch oscillations. The Wannier-Stark ladder appears for the propagation constants centered around $k_{y0} = 2.47$, normalized period is $L_y = 820$.

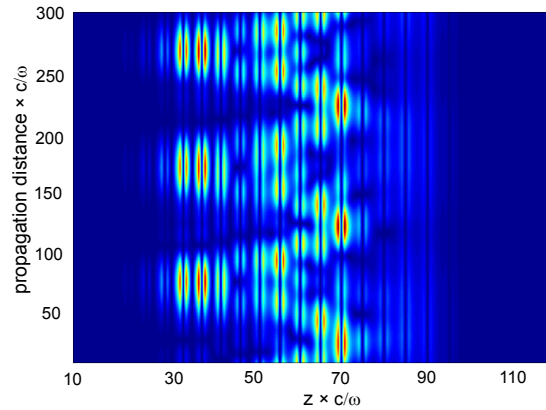


Fig. 4. Field distribution for the case of guided waves. The Wannier-Stark ladder appears for the propagation constants centered around $k_{y0} = 1.34$, period is $L_y \cong 100$.

equivalent of the Wannier-Stark ladder which is associated with the Bloch oscillations.

Spectrum of k_y can be divided into three different regions. First, when $k_y < n_r < |n_l|$, electromagnetic waves propagate in both left- and right-handed materials. In the second region, $n_r < k_y < |n_l|$, waves propagate in metamaterial only being evanescent in the vacuum layers. In this regime, our structure can be considered as an array of coupled left-handed waveguides. When $k_y > |n_l| > n_r$, only surface waves may propagate along the different interfaces.

We find that the Bloch oscillations can be observed in all three regimes of the wave propagation when the corresponding set of the equidistant values of the propagation constant is excited. For definiteness, we consider a stack containing 36 pairs of negative-index metamaterial and conventional dielectric layers and the normalized period Λ varying from 3.7 to 6. First, we excite the eigenstates corresponding to the regime of surface waves with the center of the spectrum at $k_{y0} = 2.47$. Figure 3 presents the intensity distribution for the electric field which shows clearly spatially periodic oscillations of the beam position in the chirped layered structure. The corresponding spectrum of eigenmodes is shown on the left side of Fig. 2. We note that the beam reconstructs its shape after each period of oscillations. The field is highly confined to

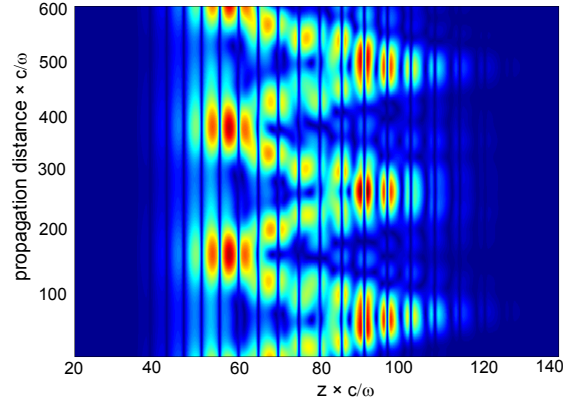


Fig. 5. Field distribution for the case when waves propagate in both type of materials. The Wannier-Stark ladder appears with average propagation constant $k_{y0} = 0.8$, period of oscillations is $L_y = 210$.

the interfaces between metamaterial and vacuum, demonstrating that such Bloch oscillations appear solely due to the coupling between surface waves exited at different interfaces of the structure. The distance between the Wannier-Stark eigenstates Δk_y defines the period of oscillations, $L_y = 2\pi/\Delta k_y$. For this case, we find $L_y = 820$, and this value agrees well with Fig. 3.

Example of the Bloch oscillations of the beam with the spectrum corresponding to the coupled waveguide regime, $n_r < k_y < |n_l|$, are shown in Fig. 4. The equidistant spectrum of eigenstates corresponding to the Wannier-Stark ladder is also shown in Fig. 2 (top, left). We notice that oscillations are strongly anharmonic, but they are still periodic with the period defined well by the relation $L_y = 2\pi/\Delta k_y$; this period is less than the period of the Bloch oscillations associated with surface waves.

The regime of the Bloch oscillations corresponding to the waves propagating in both media can be found in a different structure with a wider transmission resonance. We analyze the structure consisting of 36 periods with the normalized period varying linearly from the value 2.5 to 7.5, which correspond to the gradient $\delta\Lambda = 0.14$. The ratio of the layer thicknesses in each period remains the same as in the previous case. We choose the values $\varepsilon = -3.6$ and $\mu = -1.11$, preserving the zero average refractive index of the structure. The calculated field distribution in this case is shown in Fig. 5, and the center of the equidistant spectrum appears at $k_{y0} \sim 0.8$.

4. Conclusions

We have studied the propagation of electromagnetic waves in chirped layered structures with alternating layers of negative-index (left-handed) metamaterials and conventional dielectric with the zero average refractive index. We have revealed that linearly chirped structures the excitation spectra are equidistant manifesting a similarity with the so-called optical Wannier-Stark ladder. Using direct numerical numerical simulations, we have demonstrated that such chirped structures can support three types of Bloch oscillations for electromagnetic waves, including a novel type of unusual Bloch oscillations associated with the coupling of surface modes.

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