Productivity and Exchange Rate Dynamics: Supporting the Harrod-Balassa-Samuelson Hypothesis through an ‘Errors in Variables’ Analysis

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Abstract: Standard tests of the Harrod-Balassa-Samuelson (HBS) hypothesis treat productivity levels in and across countries as fixed and observable, and offer little empirical support for the hypothesis. If productivity follows a jump-diffusion process, these standard tests will generate biased estimates, measuring productivity levels with error. This paper instead proposes an ‘errors in variables’ approach to correct this bias, and finds support for the HBS hypothesis assuming a jump-diffusion process in productivity. Empirical results are obtained for a data set available for the United States, Japan, West Germany and France over the period 1960 to 1996.

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1. Introduction

The Harrod-Balassa-Samuelson (HBS) hypothesis explains the deviation of the real exchange rate from purchasing power parity (PPP) using productivity differentials between tradable and non-tradable goods sectors and between countries (Harrod 1933, Balassa 1964, Samuelson 1964). As usually expressed, for example, an increase in productivity in the home country’s tradable (non-tradeable) goods sector will lead to an appreciation (depreciation) of the domestic currency. An increase in productivity in the foreign tradeable (non-tradeable) goods sector will result in a depreciation (appreciation) of the domestic currency.

Despite the popularity of the HBS hypothesis, there are few empirical tests that support the hypothesis and there are no empirical results that could be termed conclusive. One possible problem is that conventional tests of the HBS hypothesis treat productivity levels, given by some ratio of output to inputs over a period of time (usually a year), as fixed and observable variables. However, if productivity follows a ‘jump-diffusion process’ (with, for example, a jump in productivity when a discovery is made or introduced), standard estimation procedures will measure both average and end-of-period productivity with some error. For example, the end of period measure of productivity with a recent large negative jump (or an increase in productivity in the domestic non-tradable goods sector) may thus be far different than any calculated average of productivity levels over a period of time, given the various and discrete nature at which new technologies are often introduced, and the average itself may only represent actual productivity with some error. If a jump-diffusion process for productivity does hold, so that productivity at a point in time is difficult to observe directly, standard estimates of the HBS hypothesis will necessarily be biased. Estimates of the HBS hypothesis thus call for an ‘errors in variables’ (EIV) approach. Using this approach we find substantial support for the HBS hypothesis. Empirical results are obtained for a data set available for the United States, Japan, West Germany and France over the period 1960 to 1996.

Section 2 of the paper provides some background and reviews the existing empirical tests of the HBS hypothesis and section 3 illustrates how an EIV approach can provide potentially better estimates of the hypothesis. Section 3 (and the Appendix) also shows that the EIV approach can be written in common factor form when the errors in variables are normally distributed, a Johansen cointegration analysis (Johansen 1988, 1991) shows that all variables in the model are integrated of order one, and there is at least one co-integrating vector. The parameters of the EIV model can thus be obtained by transforming the Johansen cointegration analysis into their common factor components, or, more simply, following Jeong and Maddala (1991), using the maximum likelihood estimator (or factor analysis) to obtain consistent estimators of the EIV model. Section 4 describes the data and econometric pre-tests for orders of integration and section 5, showing the equivalence between a Lisrel model and the EIV approach in our context, provides the empirical results. Section 6 concludes. An Appendix collects technical details.

2. Previous Literature

As first presented (Harrod 1933, Balassa 1964, Samuelson 1964), the simplest form of the HBS hypothesis can be written in the form:
\[ \ln(q) = \beta_0 + \beta_1 \ln \left( \frac{\gamma_T}{\gamma_N} \right) + \beta_2 \left( \frac{\gamma_T^*}{\gamma_N^*} \right) \]  

where \( \gamma_T \) and \( \gamma_N \) are the productivity levels in the tradable and non-tradable sectors in the home or domestic country and \( \gamma_T^* \) and \( \gamma_N^* \) are the corresponding productivity variables for the foreign trading partner, and \( q \) is the real exchange rate in the form of the value of 1 unit of domestic currency in terms of a foreign currency. The HBS hypothesis predicts coefficient values \( 0 < \beta_1 < 1 \) and \( -1 < \beta_2 < 0 \), so that an increase (decrease) in productivity in the domestic (foreign) traded goods sector results in a potential depreciation of the domestic currency. There are two main approaches to testing the HBS hypothesis in the prevailing literature: the one-step and two-step approaches.

### 2.1 One-step approaches

The one-step approach carries out estimates of equation (1), or some form of equation (1), either in a cross-sectional or time series analysis. The approach is inspired by Balassa’s (1964) pioneering paper, and a follow-up work (Balassa 1973), which estimates equation (1) indirectly by OLS estimates of

\[ \frac{PPP_i}{ER_i} = \beta_0 + \beta_1 PROD_i + \epsilon \]  

where \( PPP_i \) stands for purchasing power parity calculated in terms of national currencies of country \( i \) per standard currency (USD in this case), \( ER_i \) is the exchange rate of country \( i \) as the amount of domestic currency needed to buy one unit of standard currency, and \( PROD_i \) is a productivity measure for country \( i \), which is proxied by Balassa (1964) by per capita income in the standard currency. To keep equation (2) consistent with the HBS hypothesis, Balassa (1964) assumes that the productivity growth in the tradable sector is higher than that of the non-tradable sector.\(^1\) Therefore, if the productivity measure of a country increases, it is likely that the productivity difference (between tradable and non-tradable sectors) also increases. Balassa (1964:586) thus argues that the currency of a country with a higher productivity level will “appear to be overvalued in terms of purchasing power parity” and coefficient \( \beta_1 \) will have an expected positive sign. Original estimates of equation (2) were encouraging. With the pooled data for 12 industrial countries (with the USA as the standard country) in 1960, \( \beta_1 \) has the expected positive sign and is significant at the 1 percent level (see Balassa 1964: 590, figure 1). In a later article, Balassa (1973) re-estimates equation (2) with the productivity differences proxied by per capita Gross National Product (GNP) and also finds \( \beta_1 \) positive and significant (at the 1 percent level).

Nevertheless, attempts to replicate Balassa’s results with different data sets have not been successful. For example, Clague and Tanzi (1972) do not find a significant value for \( \beta_1 \) when they examine data from 19 Latin American countries. Officer (1976) expands Balassa’s

\(^1\)Balassa (1964: 593, table 5) provides support for this assumption by showing that the productivity growth in the service sector (i.e., non-tradable sector) lags behind productivity growth in agriculture and industry (i.e., tradable sector) using data from seven industrial countries in the 1950s.
experiment with productivity differences proxied by two more indicators: GDP per employed worker, and the ratio of productivity in the tradable sector to productivity in the non-tradable sector. However, even though a comprehensive database is employed, which includes data for 15 countries over 23 years (1950-1973), none of the estimated $\beta_i$ are found to be significant at the 5 percent level.

An obvious problem with the cross-sectional approach is that it fails to account for long term productivity differences and country-specific factors when estimated in ‘pooled format’. Hsieh (1982) is among the first to use a time series analysis to verify the HBS hypothesis in a country specific framework, by estimating:

$$\Delta r = c_0 + c_1[\Delta a_T - \Delta a_N] - c_2[\Delta a_r^* - \Delta a_s^*] + c_3[\Delta w - \Delta s - \Delta w^* + \Delta a_r^* - \Delta a_r] + u$$  \hspace{1cm} (3)$$

where $\Delta r$ is the growth in the real exchange rate, defined as the price of domestic currency in terms of foreign currency, $\Delta a_j^*$ is the productivity growth rate in sector $j$ (either tradable or non-tradable) in country $i$ (either home or foreign country), $\Delta w^*$ stands for the growth of unit labour cost in country $i$, and $\Delta s$ is the growth in the nominal exchange rate. Estimated results here favour the HBS hypothesis but, clearly, only in terms of a short-run adjustment process. The long-run relationship between the exchange rate and productivity differences is untested.

In these terms, cointegration analysis provides a preferred and more direct long-run test by estimating

$$r = c_0 - c_1[a_T - a_N] + c_2[a_r^* - a_s^*]$$  \hspace{1cm} (4)$$

with lowercase letters representing the log of the level variables, as defined in equation (3). Unfortunately, recent results here are mixed and estimated parameters do not always have the correct sign or significance Strauss (1996), for example, analysed data for six OECD countries: Belgium, Canada, Finland, France, the UK, and the US over the period 1960-1990, showing, with a Johansen cointegration rank test, and in support of the HBS hypothesis, at least one cointegration vector between the real exchange rate and productivity differences. In, all but one of the estimated Johansen coefficients ($c_1$ and $c_2$ in equation (4)) also had the correct sign, with significance levels at least at 5 percent. Unfortunately, in about half of the cases, Strauss’s (1996) estimated coefficients are significantly greater than 1 in absolute value, contradicting the HBS hypothesis. Alexius and Nilsson (2000) confirm the cointegration relationship for 15 OECD countries, and although the estimated coefficients are within the [-1,1] interval, one third of the estimated coefficients have the wrong sign.

Chinn (2000) also finds largely inconclusive evidence for the HBS hypothesis in 9 Asia-Pacific countries: China, Indonesia, Japan, Korea, Malaysia, the Philippines, Singapore, Taiwan and Thailand, based on a one-step error correction model of the form:

$$\Delta q_t = \mu + \sum_{i=1}^{k} \lambda_i \Delta q_{t-i} + \sum_{i=1}^{k} \delta_i \Delta a_{t-i} + \phi [q_{t-1} + \alpha a_{t-1}] + \nu_t$$  \hspace{1cm} (5)$$

$$\tilde{a} = (a_T^* - a_N^*) - (a_r^* - a_s^*)$$  \hspace{1cm} (6)$$
where $\tilde{a}$ is a measure of the natural log of labour productivity and $a$ is restricted to -0.5 for every country except Singapore and Taiwan. Here, the real exchange rate cointegrates with productivity differences in only three cases: Japan, Malaysia and the Philippines (see Chinn 2000:32, table 2). The estimated coefficient on the long-run productivity difference is also a source of concern. When it is allowed to be freely estimated (the case of Singapore and Taiwan), it is either insignificant (the case of Singapore), or significant with a wrong sign (the case of Taiwan). To exploit the cross-sectional information, Chinn (2000) also runs a panel cointegration analysis. Better evidence for a cointegration is found when China is excluded from the model, although the point estimates for productivity coefficients are not significant. They become marginally significant at the 20 percent level when 3 more countries, Singapore, Taiwan and Thailand, are excluded from the model. Chinn (2000) concludes that there is only limited panel evidence to support the HBS hypothesis. In a follow-up paper, using the same model, Alquist and Chinn (2002) find supporting evidence for the hypothesis from the real Dollar-Euro exchange rate data for the period from 1985-2001. However, the estimated productivity coefficient is too high in absolute value (-5) to be bounded by [-1,1] as the HBS hypothesis requires.

2.2 Two-step approaches

The two-step approach attempts to verify the HBS hypothesis by first examining the relationship between productivity differences between tradable and non-tradable sectors and their relative prices, given by

$$P = \beta_0 + \beta_1 P_T + \beta_2 P_N + \epsilon$$

(7)

where $P$ is the relative price of non-tradable goods, and $P_T, P_N$ are the productivity levels of tradable and non-tradable sectors. The HBS hypothesis again predicts $0: \beta_1 : 1$ and $-1: \beta_2 : 0$. The second step is then to check whether PPP is expected to hold in the tradable goods sector. Together, if the two steps show positive results, the real exchange rate is expected to move together with differences in the relative productivity of tradable over non-tradable sectors between countries. In this regard, Gregorio et al. (1994) study a simplified form of equation (7) by combining $P_T$ and $P_N$ into one variable, a productivity difference, and by adding demand shift variables (government expenditure and income) and the rate of inflation. Using a cross-sectional analysis of the first differenced data from 14 OECD countries, in this context, over the period from 1970-85, gives a strong relationship between productivity differences and the relative price of non-tradable goods. Nevertheless, because the study is done with first differenced data only, it suffers from the same deficiency as Hsieh (1982) in that it does not show the long-run relationship between relative prices and productivity differences. Alternatively, Asea and Mendoza (1994), examining data from 14 OECD countries, find evidence that the relative price of non-tradable goods is determined by the differences in the ratio of long-run sectoral marginal products of labour, but they fail to establish the relationship between long-run relative prices and cross country differences in the level of real exchange rates.

3. The Errors in Variables Approach

3.1 The HBS hypothesis and an ‘errors in variables’ model

Conventional tests of equation (1) treat productivity as a fixed an observable variable. This will not be the case, however, if productivity follows a jump-diffusion process and in
particular when productivity levels take discrete jumps following the introduction of a new
technology. To see this, let \( p \equiv \ln(\gamma_T / \gamma_N) \) be the log of the productivity ratio between traded
and non-traded goods for the domestic economy. (The value \( p^* \) would be the comparable
measure for the foreign country.) By taking the log of the productivity ratio we can exclude
the trend in the productivity level, and the errors can reasonably be assumed to follow some
independent distribution with mean zero and some strictly positive variance as the relative
productivity level can rise or fall over time. Since \( \gamma_T \) and \( \gamma_N \) are assumed jump-diffusion
processes, during a unit of time from moment \( t \), the increase in productivity level is

\[
d\gamma_i^t = \kappa_i \gamma_i^t d\mathbf{R}_i^t
\]

where \( i \) indexes both traded and non-traded sectors, \( d\mathbf{R}_i^t \) is the number of jumps in the
productivity level during the given unit of time, \( \kappa_i \) is the magnitude of the jumps and \( \gamma_i^t \) is
the initial productivity level. Following Cyganowski et al. (2002) we can integrate (8) to give

\[
\gamma_i^{t+1} = \gamma_i^t (1 + \kappa_i)^{d\mathbf{R}_i^t}
\]

so that the productivity ratio at \( t+1 \) is

\[
p_{t+1} = \ln\left( \frac{\gamma_T^{t+1}}{\gamma_N^{t+1}} \right) = \ln\left( \frac{\gamma_T^t}{\gamma_N^t} \right) + \ln(1 + \kappa_T) d\mathbf{R}_T^t - \ln(1 + \kappa_N) d\mathbf{R}_N^t
\]

and the log of productivity ratio rises when the tradable goods sector introduces a new
productivity-enhancing discovery, and falls with such discoveries in the non-tradable sector.

In order to know the exact level of \( p \) we have to know the exact nature of technological
progress during the period of time that \( d\mathbf{R}_i^t \) spans. However, as we have already mentioned,
we can only measure (with error) the average productivity level during the unit of time, or the
mean level of \( d\mathbf{R}_i^t \) within that unit of time. More precisely, the measure of productivity
(everything else constant) can be represented by the following sum

\[
\ln \tilde{\gamma}_i^{t+1} = \sum_{j=1}^t \ln\left( \gamma_i^{t+1-j} \right) \Delta t_j = \ln \gamma_i^{t+1} - \ln(1 + \kappa_i) \sum_{j=1}^t j \Delta t_j = \ln \gamma_i^{t+1} - \ln(1 + \kappa_i) \overline{d\mathbf{R}_i^t}
\]

where \( \Delta t_j \) is the period of time, over which the productivity level of sector \( i \) stays constant
(after a jump), and \( \overline{d\mathbf{R}_i^t} \) can be considered as an average number of jumps within a unit
period of time. As such, the measured productivity ratio is

\[
\tilde{p}_{t+1} = \ln(\frac{\tilde{\gamma}_T^{t+1} / \tilde{\gamma}_N^{t+1}}{\gamma_T^{t+1} / \gamma_N^{t+1}}) = p_{t+1} - \ln(1 + \kappa_T) \overline{d\mathbf{R}_T^t} + \ln(1 + \kappa_N) \overline{d\mathbf{R}_N^t} = p_{t+1} + \varepsilon_{t+1}
\]
where \( \varepsilon_{t+1} \) is an iid process. If we thus assume (by the Central Limit Theorem), that \( \varepsilon_{t+1} \) is normally distributed with zero mean, we can thus only measure productivity \( p \) with some normally distributed error.

In practical terms the measurement error in productivity can occur as a result of both the presence of a jump-diffusion process and the choice of ‘end-of-period’ measurement given an underlying stochastic process. Figure 1 illustrates a single realization of a sample productivity ratio over a number of days, for jumps in both the traded (positive jump) and non-traded (negative jump) sectors. At the end of the year, if we measure productivity as a ratio of yearly output over input, the average productivity ratio is considered fixed during the year at roughly 1.27. However, because this level is an average of the productivity ratio during the year, it will measure both the actual and end-of-period productivity ratio (about 1.19 in this realization) with some error.

With errors in productivity measurement in mind, the correct econometric model (dropping time subscripts for convenience) is

\[
\begin{align*}
\ln(q) &= \beta_0 + \beta_1 p + \beta_2 p^* + u \\
\bar{p} &= p + e \\
\bar{p}^* &= p^* + e^*
\end{align*}
\] (13)

where (again) \( p \) is the log of the domestic ratio of productivity in the traded to non-traded goods sector and \( p^* \) is the comparable foreign measure, for \( u, e, \) and \( e^* \) assumed normally distributed residuals with zero mean. As usual with errors in variables approaches, it is clear that if equation (13) is the correct model, then OLS estimates of

\[
\ln(q) = \beta_0 + \beta_1 \bar{p} + \beta_2 \bar{p}^* + \varepsilon
\] (14)

will imply the error term \( \varepsilon = u - \beta_1 e - \beta_2 e^* \) is correlated with \( \bar{p} \) and \( \bar{p}^* \) and the OLS estimator will be biased.

### 3.2 Cointegration, common trend and the dynamic factor model

As we have seen, the HBS hypothesis with measurement errors in productivity can be expressed in a factor model form, or equation (13). The original ‘factor model’ requires the factors to be normally distributed with zero means and a positive definite covariance matrix. However, this assumption is too strict for our context, since the productivity ratio tends to be integrated of order one in practice. If so, our factor model becomes a ‘dynamic factor’ or common trends model given by

\[
\begin{align*}
y_t &= \beta_0 + \beta p_t + u_t \\
p_t &= p_{t-1} + v_t
\end{align*}
\] (15, 16)
where $y = (\ln(q), \tilde{p}, \tilde{p}^*)$ is a vector of dimension $N = 3$ of I(1) observed variables and, following Stock and Watson (1988), $p = (p, p^*)$ is a vector of dimension $K = 2$ of I(1) common factors or common stochastic trends, for $u$, and $v_i$ iid variables.

Escribano and Pena (1994:581) show that the common stochastic trend model, the common factor model and cointegration are in fact equivalent representations of a single model. Following their proof, we can easily see that if equations (15) and (16) hold then we can find the null space of vector $\beta$, vector $\beta_\perp$, such that $0 = \beta_\perp \beta$, and

$$\beta_\perp y = \beta_\perp \beta_0 + \beta_\perp \tilde{pp} + \beta_\perp u = \beta_\perp \beta_0 + \beta_\perp u$$

and thus $\beta_\perp y$ (is I(0)) and $y$ are cointegrated with cointegration vector $\beta_\perp$, and the cointegration analysis can be applied directly. Using Monte Carlo simulation Fisher (1990) indeed finds that if the residuals ($u$, $e$ and $e'$) are I(0), $\ln(q)$, $\tilde{p}$ and $\tilde{p}^*$ can still be cointegrated, but at the expense of Type 1 errors. Note that the above transformation in equation (19) is not unique, since it is also true that if the vector $\beta_\perp$ can be replaced by $H\beta_\perp$ for any comparable nonsingular matrix $H$. Hence, the Johansen cointegration analysis (see Johansen, 1988, 1991) should be more efficient than the conventional OLS cointegration analysis, since the former does not impose any parameter restriction on the cointegration vector $\beta_\perp$ and makes no exogeneity or endogeneity assumptions on the observed variables $y$.

Equation (17) is a key point in the estimates to follow. The true parameter set for the HBS hypothesis, or model (13), is $\beta$, not its null space $\beta_\perp$. In this sense, the current empirical literature focuses on finding the $\beta_\perp$ cointegration vector of $y$, not the true parameter set for the HBS hypothesis given by $\beta$, with (perhaps not surprisingly) little resulting evidence to support the HBS hypothesis.

It is also important to note that the Johansen cointegration vector error correction model (ECM) is preferable because its estimated coefficients can be transformed into their common trend counterparts by finding their null space vectors. According to Gonzalo and Granger (1995), if $N$ 1 vector $y_i$ is cointegrated with rank $r = N - K = 1$ then the error correction model is of the form

$$\Delta y_i = \gamma \alpha' y_{i-1} + \sum_{j=1}^{\infty} \Gamma_j \Delta y_{i-j} + \eta_i$$

and the element of $y_i$ can be represented in a combination of a smaller number of $K$ common trends

$$y_i = A_1 f_i + A_2 z_i$$

where $A_1 = \alpha_\perp (\gamma_\perp \alpha_\perp)'^{-1}$, $A_2 = \gamma (\alpha' \gamma)^{-1}$, the common trends $f_i = \gamma_\perp y_i$ are all I(1):
\[ f_t = f_{t-1} + \epsilon_t \]
\[
K \times 1 \quad K \times 1 \quad K \times 1
\]

and where the errors \( z_t = \alpha'y_t \) are I(0), since \( \alpha' \) is the long-run cointegration vector of \( y_t \).

Note as well that the above transformation in not unique since we can choose any non-singular \( K \times K \) matrix \( H \) and equation (19) can be represented with the new set of factors without affecting \( A_z z_t \). This ambiguity is not a concern in our estimations, since in our EIV case we can choose vector \( H \) equal to the inverse of the last \( K \) lines of \( A_t \) and equation (20) can be uniquely transformed to model (13). \(^3\)

Johansen’s ECM estimation provides a crucial conclusion, namely that it confirms that \( y_t \) are cointegrated of rank \( r \). As Gonzalo and Granger (1995) show, if \( \text{det}(\alpha') = 0 \), then the ‘true’ common trend model is given by model (13), with errors \( u \) and \( v \) being I(0). In addition, as Jeong and Maddala (1991:434) argue, the factor analysis or ‘Full Information Maximum Likelihood estimator’ of model (13) is consistent and efficient when all of the error terms are independently and normally distributed with zero mean (see the Appendix). Jeong and Maddala (1991:436-7) also conduct a Monte Carlo study and find that factor analysis outperforms both OLS and traditional cointegration analysis in the presence of the errors in variables, especially in small samples. Therefore, when all errors in variables are I(0), we will employ a factor analysis to estimate the coefficients of model (13).

4. Data and Pre-Tests

For our purposes, estimates and bilateral comparisons are obtained for the following countries: the US, West Germany, France and Japan. Data for calculating productivity levels by sector is drawn from the Groningen Growth and Development Centre’s ten-industry database (GGDC 2004), for the period 1947 to 1997, originally published and described in van Ark (1996). Industry data is aggregated into tradeable and non-tradeable sectors, where a sector is defined as tradeable if more than 10 per cent of total production is exported (Gregorio et al. 1994). Following this convention (also invoked by Stockman and Tesar (1995) and Kakkar (2002)), agriculture, manufacturing, mining, retail and transportation are all defined as tradeable sectors. The remainder produce non-tradeable goods. Following Canzoneri et al. (1996) and Chinn (2000), for convenience, we also proxy overall productivity levels by labour productivity (as annual output per employee), thus avoiding the errors in estimating capital stocks that typically distort productivity measures.

Following convention in empirical testing, we define the exchange rate as \( q = e P^* / P \) where \( e \) is the nominal exchange rate defined as the price of foreign in terms of local currency, and where \( P^* \) and \( P \) are the foreign and domestic price levels. For the HBS hypothesis, we thus now expect estimates of productivity ratios of traded to non-traded goods (domestic and foreign) to be given by the following restrictions on the coefficients in equation (1):

\(^3\)The transformation suffers from a deficiency in the sense that there is no explicit formula for the variance of the elements in \( A_z \). This leaves no room for hypothesis testing, and therefore we will rely on another method to estimate our EIV model.
\[-1: \beta_1 \leq 0 \text{ and } 0: \beta_2 \leq 1.\] Data for the nominal exchange rate and price level (given by the GDP deflator index) are taken from the IMF (2004) dataset and in terms of final estimates covers 37 years (from 1960 to 1996), given that the GDP deflator is available for West Germany only from 1960. There are six bilateral real exchange rates in the dataset and (again) productivity is defined as the ratio of labour productivity in tradeable relative to non-tradeable sectors. All series are in log form.

Table 1 provides unit root tests. Lags are chosen on the highest significant lag order. Following Lopez et al. (2005) we also include the Elliott-Rothenberg-Stock DF-GLS test statistic to prevent the short lag from being chosen by the Augmented Dickey-Fuller test. Results show that the unit root hypothesis is accepted for all level variables at least at the 1 percent significance level, except for the case of the cross US-France real exchange rate, where the Augmented Dickey-Fuller and Elliott-Rothenberg-Stock DF-GLS tests reject the unit root hypothesis for the level data at the 5 percent significance level. In the case of differenced data, the unit root hypothesis is rejected for every series at least at the 5 percent significance level, except for the case of the Japanese productivity ratio. In the case of the Japanese productivity ratio, the Augmented Dickey-Fuller test fails to reject the hypothesis. Nevertheless, the hypothesis is strongly rejected with the two other tests. This provides some evidence supporting the hypothesis that the Japanese productivity ratio is an I(1) process. In short, from Table 1 we have evidence to confirm that all variables are I(1) except for the case of cross US-France real exchange rate, which may be I(0). However, since productivity ratios are I(1), a single cointegration vector can be confirmed if the productivity ratios are cointegrated for the US-France case. If so, the maximum likelihood (or factor analysis) estimator of equations (15) and (16) is consistent.

5. Empirical Results

Given the unit root tests, we will employ the following estimation strategy. First we perform an OLS cointegration analysis as the benchmark case. Johansen’s (1988, 1991) cointegration analysis follows, and if it confirms at least one cointegration vector for each currency, we then will be able to do a factor analysis to find the consistent estimator of the long-run parameter set.

5.1 OLS cointegration analysis

In the OLS cointegration analysis, we regress the end of period log of the real exchange rate with the ‘observed’ (or measured) productivity ratio for a given period, without considering the errors in productivity measurement and a constant term. As usual, the residuals of the regression are then tested to see if they are integrated of order zero. The results are reported in Table 2. The value \(\hat{p}\) is the productivity ratio of traded to non-traded goods domestic and \(\hat{p}^*\) is the value of the productivity ratio of traded to non-traded goods foreign.

Although the OLS cointegration analysis supports the HBS hypothesis in the sense that the three variables (the real exchange rate, the domestic productivity differential and the foreign productivity differential) are all cointegrated at the 5 percent significance level, the OLS estimated long-run coefficients are not satisfactory. Given the definition of the real exchange rate used here, we expect the coefficient on \(\hat{p}\) to be in the range [-1,0] and \(\hat{p}^*\) to be in the range [0,1]. Among the twelve estimated coefficients of the productivity differentials, there
are only eight with the correct sign, of which five are statistically significant at the 5 percent level. In addition, there are three cases where the coefficients exceed 1 in absolute value.

5.2 Johansen cointegration analysis
As mentioned, given the ‘errors in variables’, OLS cointegration may not be as efficient as a Johansen cointegration analysis. The aim of performing the Johansen trace test in this section is to confirm the hypothesis that at least one cointegration vector exists for each cross country exchange rate.

The lag order is very important in the Johansen cointegration analysis, therefore we follow Alexius and Nilsson (2000) and choose the lag order based on the usual information criteria (e.g., AIC, SIC, and the value of the log-likelihood function), and whether the lag order removes serial correlation in the residuals or not. Since we are interested in the hypothesis that there is one cointegration vector for each cross exchange rate case, we select the lags included in the Johansen cointegration test by looking at the information criteria and the serial correlation of the residuals of the model that is being imposed with one cointegrating vector. Results are summarised in Table 3. Given the chosen lag order, we estimate the long-run cointegration vector, with results reported in Table 4. The results support the hypothesis of at least 1 cointegration vector for every pair of the cross exchange rates examined at the 1 percent significance level. However, the estimated coefficients are clearly inferior to the OLS cointegration analysis: the coefficients are frequently found to be greater than 1 in absolute value and more than half of them have the wrong sign.

5.3 Factor analysis of the HBS hypothesis
With the Johansen cointegration analysis confirming one cointegration vector for each cross exchange rate, model (13) is the appropriate model (see Gonzalo and Granger (1995)), with \( u \) and \( v \) both I(0). In the section, we pool all four cross exchange rate cases into one single factor model.\(^4\) The model is estimated using a Lisrel representation and software (see Wansbeek and Meijer (2000)). In a Lisrel representation, there are two kinds of observed variables, \( x \) and \( y \), and two corresponding latent variables \( \xi \) and \( \eta \), or

\[
\begin{align*}
1 &= B1 \cdot \Gamma \xi + \zeta \\
y &= \Lambda y + \epsilon \\
x &= \Lambda x \xi + \delta
\end{align*}
\]

(21)

In our context the \( y \) variables represent the real exchange rate series, or \( \ln(q) \), and the \( x \) variables represent the log of measured productivity ratios. Note that we analyse here only the case where \( x \) and \( y \) are ‘demeaned’. By demeaning the observed data we can ignore the constant terms in model (13), recovering them by an appropriate formula (see equation (A5))

\(^4\)To have an identified factor model, as Fuller (1987) points out, the number of dependent variables (\( r \)) and the number of variables which are subject to measurement errors (\( k \)), must at least satisfy the following relationship: \( r^2 \geq r + k \). If the number of countries is \( k \), then the maximum number of cross exchange rates is \( r = k(k-1)/2 \), so that the minimum number of countries that satisfies the Fuller-inequality is three. We have four countries, or more than enough for the model to be identified.
in the Appendix). The covariance structure decomposes into four components: \( \Phi = E(\xi'\xi') \), \( \Psi = E(\xi'\xi') \), \( \Theta = E(\varepsilon'\varepsilon') \), and \( \delta = E(\delta'\delta') \).

The Lisrel model is estimated by maximizing:

\[
L = \ln(|\sum|) + \text{tr}(m_{y,x}^{-1}) - \ln(|m_{y,x}|) - N
\]

where:

\[
\sum = \begin{bmatrix}
\Lambda_y [(I-B)^{-1}(\Gamma \Phi' + \Psi)(I-B')^{-1}] \Lambda_y + \Theta_x & \Lambda_y (I-B)^{-1} \Gamma \Phi \Lambda_x^x \\
\Lambda_x \Phi ' (I-B')^{-1} \Lambda_y^x & \Lambda_x \Phi \Lambda_x + \Theta_\delta
\end{bmatrix}
\]

and \( m_{y,x} \) is the actual sample covariance matrix of \((y, x)\) (see Wansbeek and Meijer 2000:201).

Our EIV model fits this context nicely by setting \( y \) as the exchange rate series, \( x \) as the series for the productivity ratios, \( B = 0 \), \( \Lambda_x \) and \( \Lambda_y \) as identity matrices, \( \Theta \) as the matrix in model (13), \( \Theta_x = 0 \), \( \Theta_\delta \) as a symmetric matrix of free parameters to be estimated, \( \Theta_\delta \) as the diagonal matrices of the variance of errors in variables, and \( \Theta_\delta \) as the covariance matrix of the residuals from the last two equations in model (13). The assumptions behind the choices of \( \Theta_\delta \) and \( \Theta_\delta \) allow contemporaneous correlation between the residuals of the exchange rate equations, or the first equations in model (13), but do not allow the error in variables terms to be correlated with each other. With these assumptions, the population covariance matrix can be written in the form:

\[
\sum = \begin{bmatrix}
\Gamma \Phi' + \Psi & \Gamma \Phi \\
\Phi ' & \Phi + \Theta_\delta
\end{bmatrix} = (\Gamma, I) \Phi (\Gamma, I)' + \begin{bmatrix}
\Psi & 0 \\
0 & \Theta_\delta
\end{bmatrix}
\]

Defined in this way, the Lisrel model is equivalent to the ‘errors in variables’ model. We can easily see that if \( \delta = 0 \), model (21) is equivalent to model (13). Furthermore, the maximum likelihood analysis of the Lisrel Model, equation (22), also provides the same result as the factor analysis of the ‘errors in variables’ model (see the Appendix). More precisely, the maximum likelihood problem (22) provides exactly the same estimated parameter values as the solution of the factor analysis problem given by equation (A7) in the Appendix.

Final EIV estimates of model (13) are reported in Table 5. Again, the value \( \hat{p} \) is the domestic productivity ratio of traded to non-traded goods. The value \( \hat{p}^* \) is the foreign productivity ratio of traded to non-traded goods. Given the definition of the real exchange rate, the HBS hypothesis predicts the coefficient on \( \hat{p} \) to be in the range [-1,0] and \( \hat{p}^* \) to be in the range [0,1]. The top part of the table estimates the first equation of model (13), and the bottom part represents the last two equations of model (13). The latter estimates matrix \( \Psi \), and we also report the fitness coefficient \( R^2 \). (Estimates of the symmetric matrix \( \Theta_\delta \) are not reported, but are available from the authors on request.)
The EIV estimation shows considerable support for the HBS hypothesis. Nine out of twelve estimated productivity differentials have the correct sign, and six of them are significant at the 1 percent level. None of the estimated productivity coefficients exceeds 1 in absolute value.

6. Closing Remarks

In the existing literature, both OLS and Johansen long-run estimated parameters are frequently found to be insignificant, or have the wrong sign and magnitudes that exceed one in absolute value, thus contradicting the HBS hypothesis. Existing ‘short-run adjustment’ tests of the HBS hypothesis are more successful in the current literature, but these fail to uncover the desired long-run relationships. Assuming a jump-diffusion process for productivity, the EIV approach used in this paper to test the HBS hypothesis provides a clear improvement over conventional ‘short-run adjustment’ and OLS and Johansen cointegration analysis. We show that the common stochastic trend model is preferable and the factor analysis method obtains estimates that offer far better support of the HBS hypothesis.

However, a number of further improvements are needed. Although this paper uses the best available data set on cross-country productivity levels, the dataset itself is still limited in annual observations. Put simply, a more extensive dataset is needed. It may also be preferable to test using measures of productivity that are broader than the relative labour productivity measures used here, and across many more comparison countries. In other words, the current comparison across six pair-wise cases is also limited, and should be expanded, and broader measures of total factor productivity in traded and non-tradeable goods sectors would be desirable. That said, it must be noted that including a larger number of countries places even more severe demands on the dataset. The factor analysis used in this paper is a nonlinear optimization process. Including a large number of countries in the data set is equivalent to an increase in the number of parameters to be estimated. Without adequate data, the nonlinear procedure will be less reliable. In this regard, if and when the data becomes available, it would be interesting to see if the estimation technique used in this paper to support the HBS hypothesis is robust to more than four countries, and six pair-wise comparisons.
APPENDIX

Recall that a factor analysis is used to estimate parameters of model (13), and in factor analysis form is given by

\[ y = \beta_0 + \beta p + u \]  

(A1)

where \( y = (\ln(q), \tilde{p}, \tilde{p}^*) \) is a vector of dimension \( N = 3 \) of I(1) observed variables and \( p = (p, p^*) \) is a vector of unobserved variables of dimension \( K = 2 \) of I(1) common factors. Following Fuller (1987:354), the density of \( y \) is

\[
(2\pi)^{(p+k)2} \left| \Sigma_y \right|^{-0.5} \exp\{-0.5(y - \mu_y)\Sigma_y^{-1}(y - \mu_y)'\}
\]

(A2)

where \( \Sigma_y = (\beta, I_K)\Sigma_p (\beta, I_K)' + \Sigma_u \), \( \Sigma_p \) is the covariance matrix of \( p \), \( \Sigma_u \) is the covariance matrix of \( u \), and \( \mu_y \) is the mean vector of observed variables \( y \). The matrix \( \Sigma_y \) can be estimated from observed data, while \( \Sigma_u \) and \( \mu_y \) are the matrices with unknown parameters.

If we have \( T \) observations then our maximum likelihood function becomes:

\[
\ln(L) = -0.5(p + k)T \ln(2\pi) - 0.5T \ln\left| \Sigma_y \right| - 0.5 \sum \Sigma_y^{-1}(y - \mu_y)'(y - \mu_y)
\]

\[
= -0.5(p + k)T \ln(2\pi) - 0.5T \ln\left| \Sigma_y \right| - 0.5 \sum \Sigma_y^{-1}(y - \bar{y})'(y - \bar{y})
\]

\[
-0.5 \sum \Sigma_y^{-1}(y - \mu_y)'
\]

(A3)

which, following Bollen (1989) we can transform into

\[
\ln(L) = -0.5(p + k)T \ln(2\pi) - 0.5T \ln\left| \Sigma_y \right| - 0.5 \sum \text{tr}\{[y - \bar{y}]\Sigma_y^{-1}[y - \bar{y}]\}
\]

\[
-0.5 \sum \Sigma_y^{-1}(y - \mu_y)'
\]

\[
= -0.5(p + k)T \ln(2\pi) - 0.5T \ln\left| \Sigma_y \right| - 0.5 \sum \text{tr}\{[y - \bar{y}]'[y - \bar{y}]\Sigma_y^{-1}\}
\]

\[
-0.5 \sum \Sigma_y^{-1}(y - \mu_y)'
\]

(A4)
where \( m_y = [y - \bar{y}'][y - \bar{y}]/(T - 1) \) are the sample covariances of the observed variables adjusted for degrees of freedom, and where \( [y - \bar{y}]\Sigma_y^{-1}[y - \bar{y}]' \) is a scalar and \( tr(CAC') = tr(C'CA) \) when \( C' \) and \( CA \) are comparable.

As Fuller (1987:354) indicates, the maximum likelihood estimator of \( \mu_y \) is \( \hat{\mu}_y = \bar{y} \), or

\[
\hat{\mu}_y = \bar{y} = (\hat{\beta}_0 + \hat{\beta}\hat{\mu}_p, \hat{\mu}_p)
\] (A5)

and substituting equation (A5) into (A4) gives a reduced form of the likelihood function, or

\[
ln(L) = -0.5(p + k)Tln(2\pi) - 0.5Tln(|\Sigma_y|) - 0.5(T - 1)tr\left(m_y \Sigma_y^{-1}\right)
\] (A6)

This reduced form of the likelihood function is very convenient since we can solve for \( \beta \) with demeaned data \([y - \bar{y}]\). Furthermore, since \( 0.5ln(|m_y|) \) is a constant we can add it to function \( (A6) \), so that

\[
ln(L) = -0.5(p + k)Tln(2\pi) - 0.5Tln\left(|\Sigma_y|\right) + 0.5ln(|m_y|) - 0.5(T - 1)tr\left(m_y \Sigma_y^{-1}\right)
\] (A7)

The maximization process can now be viewed as a process of matching elements of the ‘theoretical variance’ \( \Sigma_y \) with elements of the sample variance \( m_y \). In this way, the maximization of the reduced likelihood function is sometimes called ‘covariance analysis’.

There are two more assumptions of relevance: (1) the covariances are assumed to be strictly positive definite (see Bollen 1989:107), so there is no multicollinearity problem among observed variables. More precisely, there should be no linear combination of productivity ratios \( p \) which is normally distributed (see Kapteyn and Wansbeek 1983) and, (2) the model should be identified, meaning that the number of parameters \( (\beta, \Sigma_u) \) should be less than or equal to the number of unique elements of \( m_y \). When all parameters are estimated, following Fuller (1987:294) we can thus estimate the residual vector \( u \) using the formula

\[
\hat{u} = y(I_1 - \beta')[(I_1 - \beta')\Sigma_u^{-1}(I_1 - \beta')]'^{-1}(I_1 - \beta')\Sigma_u
\] (A8)
References


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Figure 1: Simulated measures of a productivity ratio following a jump-diffusion process
Table 1: Unit root tests

<table>
<thead>
<tr>
<th></th>
<th>Japan-US</th>
<th>US-Germany</th>
<th>Germany-France</th>
<th>France-Japan</th>
<th>Japan-Germany</th>
<th>France-US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Unit root test for the level real exchange rate data (constant and trend included):</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Elliott-Rothenberg-Stock DF-GLS test statistic</td>
<td>-0.3337</td>
<td>-1.2324</td>
<td>-1.1941</td>
<td>-0.8073</td>
<td>-1.2209</td>
<td>-2.0424**</td>
</tr>
</tbody>
</table>

| **Unit root test for the differenced data:** |          |            |               |              |               |          |
| Augmented Dickey-Fuller T statistic | -4.8044*** | -4.1134*** | -6.4837***    | -5.9529***   | -7.0719***    | -3.9401*** |
| Elliott-Rothenberg-Stock DF-GLS test statistic | -5.2157*** | -4.1593*** | -6.0521***    | -5.8279***   | -6.9754***    | -3.9895*** |

| Productivity ratio data: |          |            |               |              |               |          |
| Augmented Dickey-Fuller T statistic | -2.3401  | -1.2190    | -1.9128       | -1.9926      |               |          |
| Elliott-Rothenberg-Stock DF-GLS test statistic | 0.6234   | -0.2015    | 1.3549        | -0.1228      |               |          |
| Phillips-Perron T statistic | -1.5467  | -0.8189    | -1.9128       | -1.8305      |               |          |

| Productivity ratio data: |          |            |               |              |               |          |
| Augmented Dickey-Fuller T statistic | -2.4855** | -1.1529    | -2.3873**     | -5.4107***   |               |          |
| Elliott-Rothenberg-Stock DF-GLS test statistic | -4.4061*** | -4.2211*** | -4.2693***    | -6.7805***   |               |          |
| Phillips-Perron T statistic | -2.3090** | -3.3361*** | -2.1480**     | -5.4495***   |               |          |

* Significant at 10 percent level, ** 5 percent level and *** 1 percent level.

Note: Productivity measures are the ratios of labour productivities in traded to non-traded goods sectors.
Table 2: OLS Cointegration Estimation

<table>
<thead>
<tr>
<th></th>
<th>Japan-US</th>
<th>US-Germany</th>
<th>Germany-France</th>
<th>France-Japan</th>
<th>Japan-Germany</th>
<th>France-US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cointegration vector estimation:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tilde{p}$</td>
<td>-0.7391***</td>
<td>0.4031</td>
<td>-0.5182*</td>
<td>0.0343</td>
<td>-1.1079***</td>
<td>-0.7153</td>
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<tr>
<td></td>
<td>(0.2241)</td>
<td>(0.3400)</td>
<td>(0.2637)</td>
<td>(0.3992)</td>
<td>(0.1776)</td>
<td>(0.4388)</td>
</tr>
<tr>
<td>$\tilde{p}^*$</td>
<td>-0.9077**</td>
<td>1.0951***</td>
<td>-0.0362</td>
<td>0.7448***</td>
<td>1.3305***</td>
<td>0.0494</td>
</tr>
<tr>
<td></td>
<td>(0.4223)</td>
<td>(0.4010)</td>
<td>(0.1791)</td>
<td>(0.2645)</td>
<td>(0.3947)</td>
<td>(0.5479)</td>
</tr>
<tr>
<td>constant</td>
<td>4.6981***</td>
<td>-0.3869**</td>
<td>-1.3213***</td>
<td>-2.7436***</td>
<td>4.1096***</td>
<td>1.7376***</td>
</tr>
<tr>
<td></td>
<td>(0.1825)</td>
<td>(0.1684)</td>
<td>(0.06243)</td>
<td>(0.1053)</td>
<td>(0.04199)</td>
<td>(0.1255)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.9169</td>
<td>0.6969</td>
<td>0.7060</td>
<td>0.9019</td>
<td>0.8232</td>
<td>0.5529</td>
</tr>
</tbody>
</table>

Unit root test of the residuals (no trend included):

<table>
<thead>
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<th>Germany-France</th>
<th>France-Japan</th>
<th>Japan-Germany</th>
<th>France-US</th>
</tr>
</thead>
</table>

*Significant at 10 percent level, **5 percent level and ***1 percent level.

Note: The value $\tilde{p}$ is the domestic productivity ratio of traded to non-traded goods. The value $\tilde{p}^*$ is the foreign productivity ratio of traded to non-traded goods. Given the definition of the real exchange rate used here, the HBS hypothesis predicts the coefficient on $\tilde{p}$ to be in the range [-1,0] and $\tilde{p}^*$ to be in the range [0,1].
Table 3: Choosing the lag order in Johansen cointegration estimation

<table>
<thead>
<tr>
<th>Country Pair</th>
<th>AIC</th>
<th>BIC</th>
<th>Log Likelihood</th>
<th>LM-Stat Lag 1</th>
<th>LM-Stat Lag 4</th>
<th>Preferred lag order</th>
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<tbody>
<tr>
<td>Japan-US</td>
<td>6</td>
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<td>6</td>
<td>13.23854</td>
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<td>US-Germany</td>
<td>6</td>
<td>1</td>
<td>6</td>
<td>6.211617</td>
<td>9.375372</td>
<td>6</td>
</tr>
<tr>
<td>Germany-France</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>8.911519</td>
<td>15.53088*</td>
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</tr>
<tr>
<td>France-Japan</td>
<td>2</td>
<td>1</td>
<td>6</td>
<td>10.25185</td>
<td>10.54803</td>
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<td>Japan-Germany</td>
<td>1</td>
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<td>6</td>
<td>10.54803</td>
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<td>France-US</td>
<td>3</td>
<td>1</td>
<td>6</td>
<td>14.83465*</td>
<td>4.297055</td>
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</table>

*Significant at 10 percent level, **5 percent level and ***1 percent level.

Note: Serial correlation LM test statistics equal the number of observation times R-squared, and have \( \chi^2 \) distribution under a null hypothesis of no serial correlation at the lag order under the test. The tests are calculated for the chosen lag.
### Table 4: Johansen Cointegration Estimation

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td><strong>Unrestricted Cointegration Rank Test (Trace statistics):</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Number of cointegration equations: None</td>
<td>43.9211***</td>
<td>67.3152***</td>
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<td>46.2074***</td>
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<tr>
<td>Number of cointegration equations: At most 1</td>
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<td>33.3650***</td>
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<td>21.0976**</td>
<td>20.9222**</td>
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<td>3.6917</td>
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</tbody>
</table>

**Normalized co-integrating equations**

(derivative of the real exchange rate is -1):

<table>
<thead>
<tr>
<th></th>
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<th>US-Germany</th>
<th>Germany-France</th>
<th>France-Japan</th>
<th>Japan-Germany</th>
<th>France-US</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\tilde{p}$</td>
<td>-1.2829***</td>
<td>14.0132***</td>
<td>-0.8072***</td>
<td>36.4752</td>
<td>-2.3392***</td>
<td>-5.5139***</td>
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<tr>
<td></td>
<td>(1.0357)</td>
<td>(3.3611)</td>
<td>(0.1623)</td>
<td>(110.836)</td>
<td>(0.47238)</td>
<td>(1.56436)</td>
</tr>
<tr>
<td>$\tilde{p}^*$</td>
<td>-1.4527***</td>
<td>-19.8087***</td>
<td>-0.0254</td>
<td>-59.9630</td>
<td>3.5256***</td>
<td>5.0265***</td>
</tr>
<tr>
<td></td>
<td>(2.0020)</td>
<td>(6.01627)</td>
<td>(0.0938)</td>
<td>(73.1803)</td>
<td>(1.0402)</td>
<td>(1.95830)</td>
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<tr>
<td>constant</td>
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<td>-1.3996***</td>
<td>6.877</td>
<td>4.3568***</td>
<td>1.1984</td>
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<tr>
<td></td>
<td>(0.8178)</td>
<td>(1.2203)</td>
<td>(0.0268)</td>
<td>(27.5221)</td>
<td>(0.11183)</td>
<td>(0.40369)</td>
</tr>
</tbody>
</table>

*Significant at 10 percent level, **5 percent level and ***1 percent level.

Note: The value $\tilde{p}$ is the domestic productivity ratio of traded to non-traded goods. The value $\tilde{p}^*$ is the foreign productivity ratio of traded to non-traded goods. Given the definition of the real exchange rate used here, the HBS hypothesis predicts the coefficient on $\tilde{p}$ to be in the range [-1,0] and $\tilde{p}^*$ to be in the range [0,1].
### Table 5: EIV Cointegration Estimation

<table>
<thead>
<tr>
<th></th>
<th>Japan-US</th>
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<th>Germany-France</th>
<th>France-Japan</th>
<th>Japan-Germany</th>
<th>France-US</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cointegration vector estimation:</strong></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\hat{p}$</td>
<td>-0.9245***</td>
<td>0.5668**</td>
<td>-0.9262***</td>
<td>-0.2368</td>
<td>-0.9317***</td>
<td>-0.2376</td>
</tr>
<tr>
<td></td>
<td>(0.1402)</td>
<td>(0.2798)</td>
<td>(0.2974)</td>
<td>(0.2002)</td>
<td>(0.1377)</td>
<td>(0.1996)</td>
</tr>
<tr>
<td>$\tilde{p}^*$</td>
<td>-0.5725**</td>
<td>0.9245***</td>
<td>0.2396</td>
<td>0.9264***</td>
<td>0.9342***</td>
<td>-0.5678**</td>
</tr>
<tr>
<td></td>
<td>(0.2869)</td>
<td>(0.2974)</td>
<td>(0.1993)</td>
<td>(0.1368)</td>
<td>(0.3008)</td>
<td>(0.2801)</td>
</tr>
<tr>
<td>constant</td>
<td>4.5483***</td>
<td>-0.4596***</td>
<td>-1.4143***</td>
<td>-2.6728***</td>
<td>4.0869***</td>
<td>1.8738***</td>
</tr>
<tr>
<td></td>
<td>(0.1161)</td>
<td>(0.1271)</td>
<td>(0.0703)</td>
<td>(0.0600)</td>
<td>(0.0392)</td>
<td>(0.0620)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.9194</td>
<td>0.7045</td>
<td>0.7219</td>
<td>0.8990</td>
<td>0.8168</td>
<td>0.5452</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Productivity US</th>
<th>Productivity Japan</th>
<th>Productivity France</th>
<th>Productivity Germany</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Measurement errors in productivity ratios:</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Variance of the measurement errors</td>
<td>0.0005</td>
<td>0.0006</td>
<td>0.0014</td>
<td>0.0002</td>
</tr>
<tr>
<td></td>
<td>(0.0012)</td>
<td>(0.0008)</td>
<td>(0.0016)</td>
<td>(0.0003)</td>
</tr>
<tr>
<td>R-squared</td>
<td>0.9875</td>
<td>0.9953</td>
<td>0.9747</td>
<td>0.9930</td>
</tr>
</tbody>
</table>

*Significant at 10 percent level, **5 percent level and ***1 percent level.

Note: The value $\hat{p}$ is the domestic productivity ratio of traded to non-traded goods. The value $\tilde{p}^*$ is the foreign productivity ratio of traded to non-traded goods. Given the definition of the real exchange rate used here, the HBS hypothesis predicts the coefficient on $\hat{p}$ to be in the range $[-1,0]$ and $\tilde{p}^*$ to be in the range $[0,1]$.  
